

**FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO
PORTO**

**Continuum damage analysis of
composite materials in presence of fiber
rotation and viscoplasticity**

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October, 2016

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Abstract

There is a growing interest in applications of composite materials. Precise modeling of structures is becoming more critical due to competitiveness of commercial objectives and the need for lighter products, but still with robust design. There is some difficulty in modeling of composite materials due to its complicated nature and the defects during the manufacturing process. In order to propose a comprehensive model by a phenomenological approach, it is necessary to consider different mechanisms and phenomena which are taking place during the loading process. In this study, an analytic solution is presented to model finite fiber rotation which is an important mechanism happening during the course of the loading. Then, this model is incorporated in a meso-scale continuum damage model for the prediction of the onset and evolution of intra-laminar failure mechanisms and the collapse of the structures manufactured by fiber-reinforced plastic laminates. Afterwards, models are implemented using finite element analysis and its results for different cases of loading and materials are compared with experiments. The model reveals a good accuracy in prediction of material behavior and capturing induced nonlinearities. It also improves the capability of the model to capture the fracture plane and failure mechanisms along with fiber scissoring. The studied materials included thermoset and thermoplastic based composites and it is shown that the fiber rotation is a must in modeling of the thermoplastics. Then, a parametric study is conducted and some recommendations for design of composites in order to optimize fiber rotation is proposed. In addition, the proposed constitutive model is used for the application of the modeling for out of plane undulated fiber composites. The geometrical nonlinearity of undulated fibers also included in the model and the models with material nonlinearity and geometrical nonlinearity are compared with each other and the experiments. These comparisons verify the model accuracy and its ability to capture ongoing mechanisms. Finally the viscoplastic behavior of the composite is taken into account and behavior of the composite structures under dynamic conditions is studied. Effects of dynamic conditions on material properties including strength and fracture toughness is considered in the modeling. The model can capture the effects of strain rate on material behavior and has a good capability in prediction of final strength of composite laminates in comparison to the experimental data.

Resumo

As aplicações de materiais compósitos têm suscitado um grande interesse. A correta modelação das estruturas reveste-se, cada vez mais, de uma importância fundamental devido à competitividade dos objetivos comerciais e à necessidade de produtos mais leves, mas que assegurem as solicitações com um design robusto. Há alguma dificuldade na modelação de materiais compósitos, devido à sua natureza complexa e aos defeitos que ocorrem durante o processo de fabricação. A fim de propôr um modelo abrangente através de uma abordagem fenomenológica, é necessário considerar diversos mecanismos e fenómenos que têm lugar durante o processo de carregamento. Neste estudo, apresenta-se uma solução analítica para modelar a rotação finita da fibra que é um mecanismo importante que decorre no curso do carregamento. Desta forma, este fenómeno é incorporado num modelo à meso-escala de dano contínuo para a previsão do aparecimento e evolução dos mecanismos de falha intra-laminares e do colapso das estruturas fabricadas por laminados poliméricos reforçados com fibras. Em seguida, os modelos são implementados, usando análise de elementos finitos, e compara-se experimentalmente os resultados para diferentes casos de carga e materiais. O modelo revela uma boa precisão na previsão do comportamento dos materiais capturando as não-linearidades induzidas. Além disso, melhora a capacidade do modelo para detetar os mecanismos de falha e fratura juntamente com as deslocções da fibra. Os materiais estudados incluíram compósitos termo endurecíveis e termoplásticos e é mostrado que a rotação da fibra é um aspeto importante na modelação dos termoplásticos. Por isso, um estudo paramétrico é efetuado e são propostas algumas recomendações para a projecto de compósitos, a fim de otimizar a rotação da fibra. Além disso, o modelo constitutivo proposto é usado na modelação fora do plano de compósitos de fibra ondulada. A não linearidade geométrica das fibras onduladas também incluídas no modelo e os modelos com não linearidade material e não-linearidade geométrica são comparados entre si e com os resultados experimentais. Estas comparações verificam a precisão do modelo e sua capacidade de analisar os mecanismos em curso. Finalmente, o comportamento viscoplástico do compósito é tomado em consideração e estudado o comportamento das estruturas compósitas sob condições dinâmicas. Os efeitos das condições dinâmicas sobre as propriedades dos materiais, incluindo a força e tenacidade à fratura, são considerado na modelações. O modelo pode captar os efeitos da taxa de deformação sobre o comportamento do material e tem uma boa capacidade de previsão da resistência final de compósitos laminados, em comparação com os dados experimentais.

Acknowledgment

I acknowledge the financial support of the Portuguese Foundation for Science and Technology (FCT) under individual research fellowship SFRH/BD/103436/2014. In addition, the financial support of FCT under the project PTDC/EME-TME/115472/2009 and by the program FEDER/COMPETE is appreciated.

I also would like to give my special thanks to Prof. Francisco Pires who supported me during this long journey and I admire his ability to come up with solution when I faced any kind of problem. He was not only the person who supervised me scientifically, but I could also rely on him for a wide range of obstacles I had during my studies and life abroad. In addition, I appreciate deeply thoughtful ideas and the great research path designed by Prof. Pedro Camanho. His comments on my work and his guidance through the whole process of my PhD added enormous value to my work and doubled my motivation to go forward. His state of the art research field made me proud in research community and among my colleagues. Furthermore, I acknowledge the support of Prof. Torres Marques who helped me to start my career in Portugal. I always felt awesome for the way he treats people in a very respectful and constructive manner.

I express my deepest thanks to my parents and my sister who encouraged me to pursue my PhD and their emotional support touched my heart. Specially my mother who was my very first teacher and she taught me alphabet and the lessons of life. She has been always my best adviser and my best friend.

Additionally, I am thankful of Mrs. Carla Monteiro. She has been very kind and supportive for administrative works and she did much more than she was supposed to do. Her presence and efficiency is a privilege for our research group. Finally, I appreciate all of my friends and colleagues who helped me during my study and life in Portugal.

Sina Eskandari

It is dedicated to my mom
who was my first teacher

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Abbreviations

FRP	Fiber Reinforced Polymer
FRPC	Fiber Reinforced Polymer Composite
CDM	Continuum Damage Mechanics
PFA	Progressive Failure Analysis
VSC	Variable Stiffness Composite
VSCL	Variable Stiffness Composite Laminate
CSCL	Constant Stiffness Composite Laminate
WRot	With fiber Rotation
WORot	WithOut Fiber Rotation
str	straight fiber model
mat	material non-linearity model
geo	geometrical non-linearity model

Chapter 1

Introduction

Composite materials are growing across different industries from aerospace to automotive and wind turbines. It is very important to predict the behavior of this material in order to have a cost effective and reliable design. Because of complicated nature of this material, it is often difficult to develop a model that predicts every mechanism of this material that is ongoing during the loading process, therefore the efforts of researchers in this field is on progress.

By advancing technology in case of computational devices, the computer aided engineering is a tool which can be utilized for analyzing the composite structure behavior. Analysis of composites can be generated in structural level, material level or lamina level. The ideal choice for design would be the structural level. But this kind of analysis has some problems. Mainly, analysis in structural level is costly in case of computation. They are not accurate enough since they cannot capture all the material mechanisms. In addition, they usually need a lot of material properties which do not have generality and it is sensitive to ply sequences and other aspect of structure. Micro-mechanical analysis is also not suitable to be applied on structural level but they can be used for designing the material itself. An optimized solution would be pursuing analysis in lamina level that compensates problems of 2 earlier mentioned methods.

In this research, the analysis has been done in lamina level which is an intermediate length scale level. Composite structure behavior consists of elastic deformation until failure, damage, fiber rotation, plasticity and visco-plasticity under dynamic conditions leading to final collapse of the structure. These different stages of material behavior are addressed in this thesis. The failure criteria has an important role in the modeling of this kind of materials. So it is critical step in the analysis to choose the suitable criterion. There are several failure criteria which are developed and some of the most important of them are introduced and discussed in chapter 2. The criterion that is chosen in this

research is Larc failure criteria that is a phenomenological model. Based on the chosen failure criterion, it is possible to model post-failure behavior. The literature review of it is presented in chapter 3. The damage model that is used in this thesis is Maimi damage model. The used failure criteria and the damage model is introduced in chapter 4.

Fiber rotation is an important mechanism during the loading of the composite structures that stiffens the material and resists damage depending on the loading condition and fiber orientation. In this research, an analytic solution is provided to calculate fiber rotation during the course of the loading. Afterwards, the model is incorporated in the damage and failure constitutive equations to analyze the structure in presence of fiber rotation. It is discussed extensively in chapter 4. Then, the constitutive model is implemented by finite element analysis and different numerical examples are executed. The numerical examples include open hole tension and compression, shear test and unidirectional compression of specimens. These models are compared with the experimental data in case of failure load, fiber rotation, and stress-strain curves. These comparisons are provided in chapters 4 and 5. The importance of inclusion of fiber rotation is also investigated for thermoplastic-based composites. Finally, a parametric study conducted for to see the effect of different variable on the model.

Variable stiffness composites (VSC) are the kind of composites that are developed in order to widen the design parameters and tailor the material specification according to the application needs. This structures are developed by the technology of fiber placement. They can also be as a result of manufacturing defects. The constitutive model that is generated in presence of fiber rotation can also be used for modeling undulated fiber composites which is a common type of VSC. Because the undulation can be assumed as rotated fibers. This modeling and its results along with the comparisons with experiments are discussed in chapter 6. The specifications of VSC including undulated fiber composites and the technology of producing them is shortly presented in chapter 2.

Finally the viscoplasticity of composites are studied. This mechanism is triggered when the structure is under dynamic condition and the matrix is loaded. This analysis are discussed in chapter 7 in details. The same as previous chapters, the experimental data has used to verify the finite element model.

Chapter 2

Polymer composite materials

Composite structure is widely used in aerospace industry and its applications are growing to other industries. Figure 2.1 shows the use of composite in A-320 of European aircraft manufacturer, Airbus. Figure 2.1 also depicts the ratio of composite structure to aircraft structure in Airbus during last decades that shows a significant growth from 5% in A310-300, to 53% in A350XWB model. In this chapter, a short introduction to composite materials is presented. Then, some of the most common failure criteria are discussed and compared. Finally, an introduction to variable stiffness composites and its technology is provided.

The materials scientist's definition of a composite says that a composite material contains a chemically and/or physically distinct phase distributed within another continuous phase and exhibits properties that are different from both of these. A typical composite material is a system of materials composed of two or more materials (mixed and bonded) on a macroscopic scale. Generally, a composite material is composed of reinforcement (fibers, particles, flakes, and/or fillers) embedded in a matrix (polymers, metals, or ceramics). The matrix holds the reinforcement to form the desired shape while the reinforcement improves the overall mechanical properties of the matrix. When designed properly, the new combined material exhibits better strength than would each individual material.

2.1 Introduction

Different materials are suitable for different applications. When composites are selected over traditional materials such as metal alloys or woods, it is usually because of one or more of the following advantages:

- Cost:

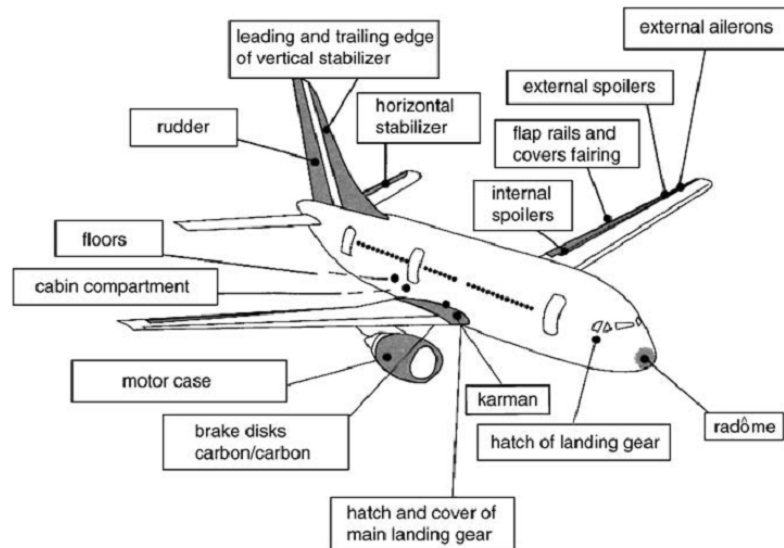


Figure 2.1: Composite Components in an Airbus A-320 (Carl, 2012) .

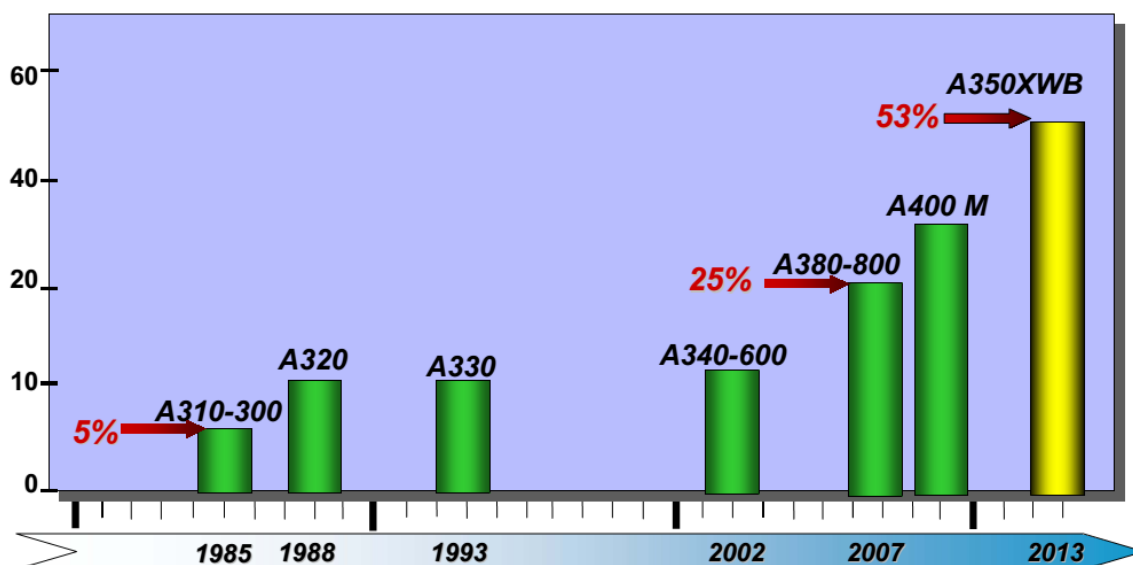


Figure 2.2: A step by step gain of composite experience in Airbus (Hellard, 2013) .

1. Prototypes
 2. Mass production
 3. Part consolidation
 4. Maintenance
 5. Long term durability
 6. Production time
 7. Maturity of technology
- Weight:
 1. Light weight
 2. Weight distribution
 - Strength and Stiffness:
 1. High strength-to-weight ratio
 2. Directional strength and/or stiffness
 - Dimension:
 1. Large parts
 2. Special geometry
 - Surface Properties:
 1. Corrosion resistance
 2. Weather resistance
 3. Tailored surface finish
 - Thermal Properties:
 1. Low thermal conductivity
 2. Low coefficient of thermal expansion
 - Electric Property:
 1. High dielectric strength
 2. Non-magnetic
 3. Radar transparency

Other remarkable properties of these materials include the following:

- Composite materials do not yield Composite materials are very fatigue resistant.
- Composite materials age subject to humidity (epoxy resin can absorb water by diffusion up to 6% of its mass; the composite of reinforcement/resin can absorb up to 2%) and heat.
- Composite materials do not corrode, except in the case of contact “aluminum with carbon fibers” in which case galvanic phenomenon creates rapid corrosion.
- Composite materials are not sensitive to the common chemicals used in engines: grease, oils, hydraulic liquids, paints and solvents, petroleum. However, paint thinners attack the epoxy resins.
- Composite materials have medium to low level impact resistance (inferior to that of metallic materials).
- Composite materials have excellent fire resistance as compared with the light alloys with identical thicknesses. However, the smokes emitted from the combustion of certain matrices can be toxic.

The characteristics of composite materials resulting from the combination of reinforcement and matrix depend on:

- The form of the reinforcement
- The proportions of reinforcements and matrix
- The fabrication process

Depending on the form of reinforcement, composite materials can be categorized in 5 types as below:

- Fibrous composite material in which fibers are long and continuous (Figure 2.3(a)) or short and randomly-distributed (Figure 2.3(b))
- Flake composites (Figure 2.3(c))
- Particulate composite material (Figure 2.3(d))
- Filler composites (Figure 2.3(e))
- Combination of more than one of above types

Materials in the fibrous form exhibit much greater strengths than in any other form, and furthermore the smaller the diameter of the fiber the greater the strength. (Figure 2.4)

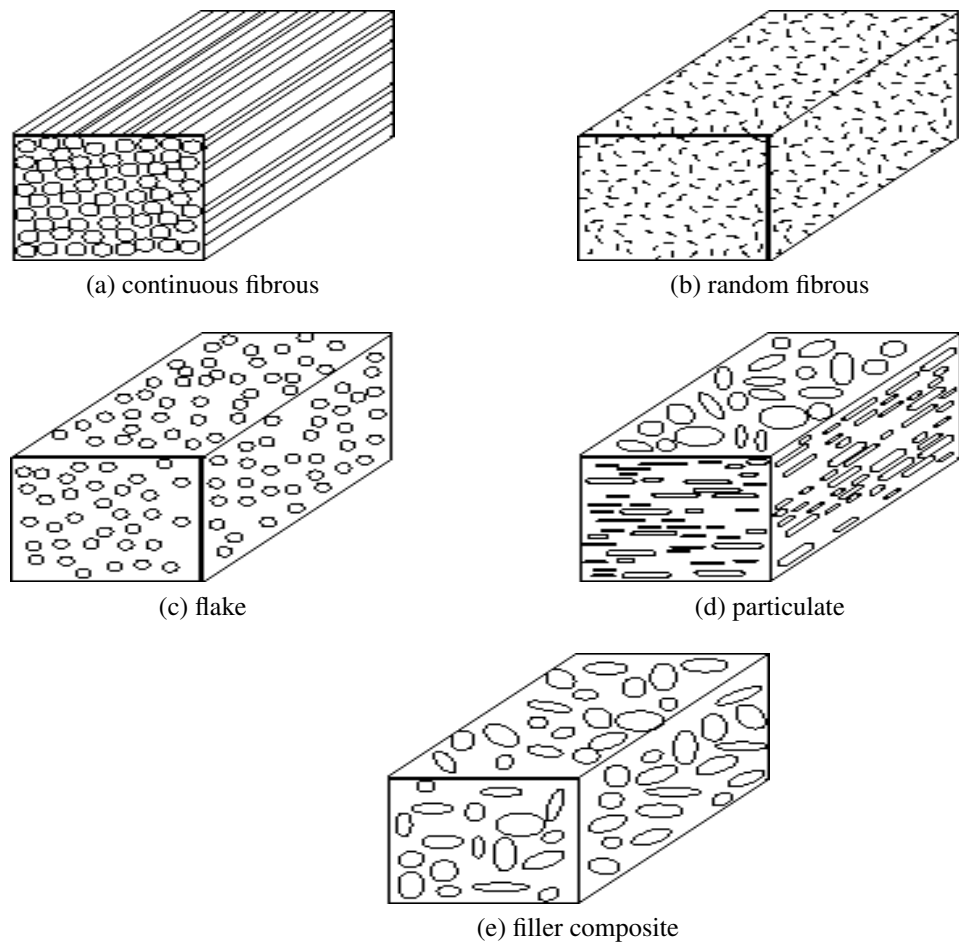


Figure 2.3: Different forms of reinforcement

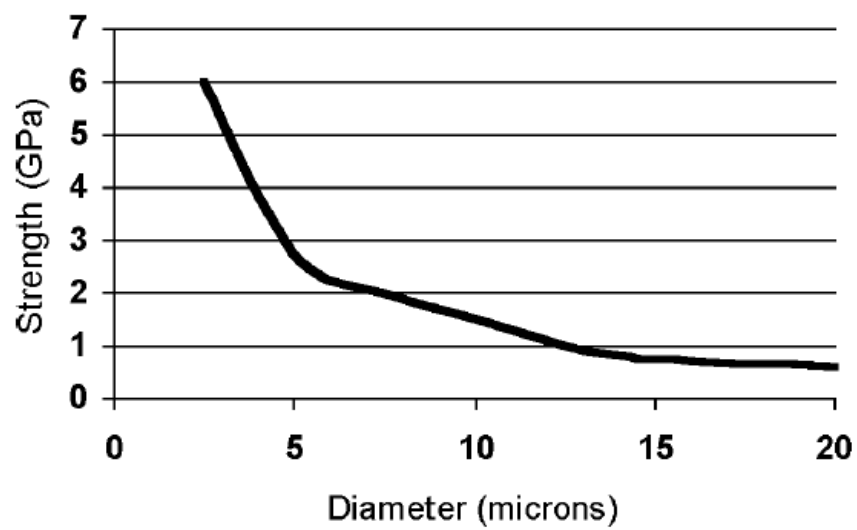


Figure 2.4: Reducing fiber diameter increases tensile strength.

2.1.1 Fibers

Fibers consist of thousands of filaments, each filament having a diameter of between 5 and 15 micrometers, allowing them to be producible using textile machines. Principal fiber materials are:

- Glass
- Aramid or Kevlar (very light)
- Carbon (high modulus or high strength)
- Boron (high modulus or high strength)
- Silicon carbide (high temperature resistant)

In forming fiber reinforcement, the assembly of fibers to make fiber forms for the fabrication of composite material can take the following forms:

- Uni-dimensional: unidirectional tows, yarns, or tapes
- Bi-dimensional: woven or non-woven fabrics (felts or mats)
- Tridimensional: fabrics (sometimes called multidimensional fabrics) with fibers oriented along many directions (> 2)

2.1.2 Matrix

The matrix materials include the following:

- Polymeric matrix: thermoplastic resins (PEEK, polypropylene, polyphenylene sulfone, polyamide, polyetheretherketone, etc.) and thermoset resins (polyesters, phenolics, melamines, silicones, polyurethanes, epoxies).
- Mineral matrix: silicon carbide, carbon. They can be used at high temperatures
- Metallic matrix: aluminum alloys, titanium alloys, oriented eutectics.

Polymer matrix composites amount to 75% of the world composite market by value or by tonnage. The technology for manufacturing both raw materials and finished articles is the most mature of all the composites. Thermoplastic composites (compared to thermoset composites):

- can be remelted without damage to the material

- have higher thermal expansion coefficients
- generally higher material costs
- generally have lower working temperatures
- generally have poorer solvent resistance
- can be economically recycled (although the reinforcements may present problems)

Thermoset composites (compared to thermoplastic composites):

- can't be remelted without damage to the material
- have lower thermal expansion coefficients
- generally lower material costs

2.1.3 Fabrication processes

There are two general divisions of composites manufacturing processes: open molding and closed molding. With open molding, the gel coat and laminate are exposed to the atmosphere during the fabrication process. In closed molding, the composite is processed in a two-sided mold set, or within a vacuum bag. There are a variety of processing methods within the open and closed molding categories:

- Open Molding
 - Hand Lay-Up
 - Spray-up
 - Filament Winding
 - Tow Placement
- Closed Molding
 - Compression molding
 - Pultrusion
 - Reinforced Reaction Injection Molding (RRIM)
 - Resin Transfer Molding (RTM)
 - Vacuum Bag Molding
 - Vacuum Infusion Processing

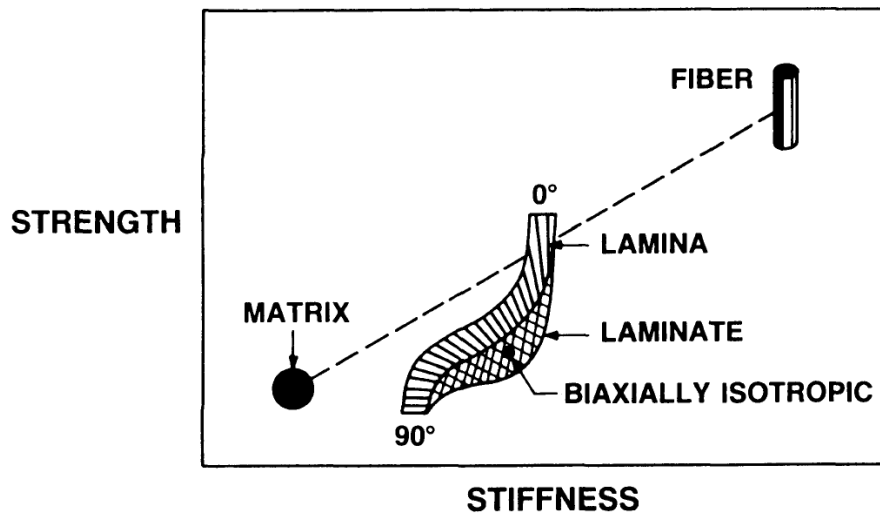


Figure 2.5: Translation from constituent properties to lamina and to laminate (Jones, 1999) .

- Centrifugal Casting
- Continuous Lamination

Selecting which manufacturing process to select depends on a number of factors including cost, materials, size, and most important volume.

2.1.4 Mechanical properties of composites

Composites do not have the well-known, established properties associated with metals. There is therefore no single reference or standard for the mechanical properties of composites. This in part is one of the difficulties of designing with composites. This is also one of composites greatest attractions: the ability to change or tailor the mechanical properties of the composites to what is required for the job in hand.

In a laminated composite material, fibers and matrix contribute to the properties of lamina and consequently lamina affects the properties of laminate. Illustrates the strength of constituents of composite material to the level of lamina and then finally to the level of laminate. Lamina is substantially sensitive to direction of fibers in matrix whose mechanical properties are fiber dominant in 0 direction and they are affected by matrix properties while increasing the direction angle. A laminate is a bonded of variously angled laminas, and we expect laminate properties is not as high as 0 direction lamina nor as low as of those in 90 direction, but something in between them. In the following paragraphs, characteristics and behavior of lamina in response to stress will be discussed. The focus

of the section is on the macro mechanical behavior of lamina which is averaged apparent mechanical properties rather than details of interactions between fibers and matrix. It should be mentioned that the scope of discussion is linear elastic behavior. The generalized Hook's law can be written in contracted notation as below in which σ , τ , and C are stress, strain, and compliance tensors in 3 dimensions.

$$\sigma_{ij} = C_{ijmn} \epsilon_{mn} \quad (2.1)$$

Both of σ and ϵ tensors are 3×3 matrix so the compliance tensor has 81 elements. However, regarding the symmetry of the stress and the strain tensors, number of elements decreases to 21 independent elements. The most general stress-strain relationship is that of the materials without any plane of symmetry, i.e., general anisotropic materials or triclinic materials. If there is a plane of symmetry, the material is termed monoclinic. On that condition, the stress-strain relationship reduces to:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{21} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{31} & C_{32} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{61} & C_{62} & C_{63} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}, \quad (2.2)$$

where the plane of the symmetry is $Z=0$ (or the 1-2 plane). If the number of symmetric planes increases to two, the third orthogonal plane of material symmetry will automatically yield and form a set of principal axes. In this case, the material is known as orthotropic. The stress-strain relation in coordinates aligned with principal material directions are according to:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}, \quad (2.3)$$

This equation implies that there is no interaction between normal stresses and shearing strains such occurs in anisotropic materials. In addition, there is no interaction between shearing stresses and normal strains along with shearing strains. If exists a plane in which the mechanical properties are equal in all directions, the material is called transversely isotropic. The stress-strain relation in this kind of material has just 5 independent constants according to:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}. \quad (2.4)$$

Characteristics of compliance matrix are summarized in Table 2.1 .

Table 2.1: Summary of compliance matrix characteristics for different kinds of materials (Jones, 1999) .

	Independent Constants	Nonzero On-axis	Nonzero Off-axis	Nonzero General
Triclinic	21	36	36	36
Monoclinic	13	20	36	36
Orthotropic	9	12	20	36
Transversely Isotropic	5	12	20	36

For an orthotropic material compliance matrix in terms of engineering constants is presented in:

$$\begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \quad (2.5)$$

Engineering constants are generalized Young's moduli, Poisson ratio, and shear moduli which are measured in simple tests like uniaxial tension or pure shear tests. Different kinds of materials have certain relations for engineering constants that must be satisfied.

2.2 Composite Failure criteria

Failure of a unidirectional laminate begins on the microscopic level. Initial microscopic failures can be represented by local failure modes, such as:

- Fiber failure- breakage, micro-buckling
- Bulk matrix failure- voids, crazing
- Interface/flaw dominated failures- crack propagation and edge delamination

Microscopic failures can become macroscopic and result in catastrophic failure. The general nature of failure for orthotropic materials is more complicated than for an isotropic material. The failure of composite laminates is a complex problem and the modes of failure depend on the loading, the geometry size, the physical and mechanical properties of materials as well as the specimen defects. The composite laminates may fail in the complex failure modes which may potentially interact in a unique pattern (Liu and J.Y. Zheng , 2010) . The failure of the first layer represents the initiation of the failure and damage evolution of the whole composite laminates, but does not represent the ultimate catastrophic damage of structures since the residual load-bearing ability of composite laminates still remains, which is reflected by continuous layer failure and macroscopic stiffness degradation (Reddy and A.K. Pandey, 2008) . Tay et al. (Tay et al., 1987), Davila et al. (Davila et al., 2005), Sleight (Sleight, 1999), Wu and Stachurski (Wu and Z. Stachurski , 1984), Cuntze and Freund (Cuntze and A. Freund, 2004), Orifici et al. (Orifici and I. Herszberg , 2008) performed extensive literature surveys on the existing failure criteria and they classified the existing failure criteria for composite laminates into two types: the non-interactive failure criteria and interactive failure criteria. The non-interactive failure criteria, sometimes also called independent failure criteria, assume no interactions between the stress or strain components and compare the individual stress or strain components with the corresponding material allowable strength values. The maximum stress criteria belong to this category. There is also an evolving trend to develop the fracture energy-based failure criteria to judge the initial failure and damage evolution of composite laminates. Direct mode determining criteria can also be considered another category. Hashin (Hashin, 1980), (Hashin and A. Rotem, 1973) , Puck (Puck, 1969),

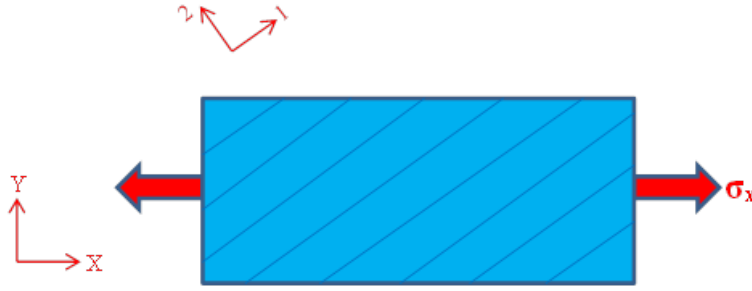


Figure 2.6: A lamina subjected to uniaxial tension.

(Puck and W. Schneider, 1969) Chang (Chang and L.B. Lessard, 1991) , and Larc (Davila et al., 2005) failure criteria are among those failure models. If composite materials are to be used in structural applications, then the understanding of how each failure mode takes place —i.e. having a physical model for each failure mode becomes an important point of concern. These physical models should establish when failure takes place (Davila et al., 2005). Some famous failure criteria are selected to be discussed.

2.2.1 Maximum Stress Criterion

This theory is commonly attributed to C. F. Jenkins (Jenkins, 1920) , and is an extension of the maximum normal stress theory for the failure of orthotropic materials. Consider a lamina subjected to uniaxial tension as shown in Figure 2.6. For a failure to occur according to the maximum stress theory, one of three possible conditions must be met, which reads:

$$\sigma_1 \geq X \text{ or } \sigma_2 \geq Y \text{ or } \tau_{12} \geq S. \quad (2.6)$$

Where, X, Y, S represent maximum strength in fiber, matrix direction and shear strength, respectively. If any one of the inequalities is satisfied, it is assumed that failure occurs. In cases of multi-axial stress, the simple relationships just given are no longer valid. The relationship between applied stress components and principal material direction stresses must be determined through stress transformation, while the inequalities in 2.6 remain valid. The criterion is illustrated in Figure 2.7 where the tension and compression behaviors have been plotted simultaneously for an E-glass-epoxy composite material.

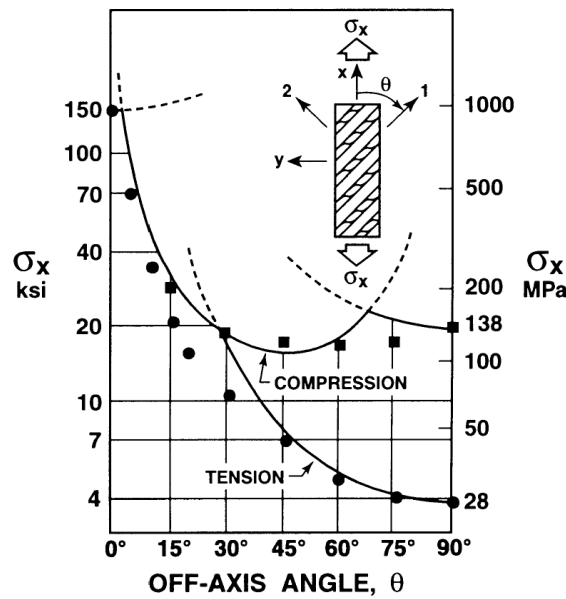


Figure 2.7: Maximum Stress Criterion (Jones, 1999) .

2.2.2 Maximum Strain Criterion

The maximum strain failure criterion is an extension of St. Venant's maximum strain theory to accommodate orthotropic material behavior. The maximum strain failure theory is expressed as:

$$\epsilon_1 \geq \frac{X}{E_1} \text{ or } \epsilon_2 \geq \frac{Y}{E_2} \text{ or } \gamma_{12} \geq \frac{S}{G_{12}}, \quad (2.7)$$

which is illustrated in Figure 2.8.

The maximum stress and strain failure theories generally yield different results and are not extremely accurate. They are often used because of their simplicity. As with the maximum stress theory, a more complex state of stress results in different expressions.

2.2.3 Interactive Failure Criteria

Initial efforts to formulate an interactive failure criterion are credited to Hill (Hill, 1950) in 1950. Since then others have proposed modifications to the initial theory. Some interactive failure theories are complicated, and in certain cases the amount of work needed to define critical parameters for a specific theory exceeds the benefit of using it. The general form of a majority of these theories can be put into one of two categories, each of which has a different form. These criteria and their associated failure theories are expressed in Table 2.3 and Table 2.4 where prime superscript represents compressive strength. The fundamental difference in each is the extent to which the terms F_i and F_{ij} are defined. The

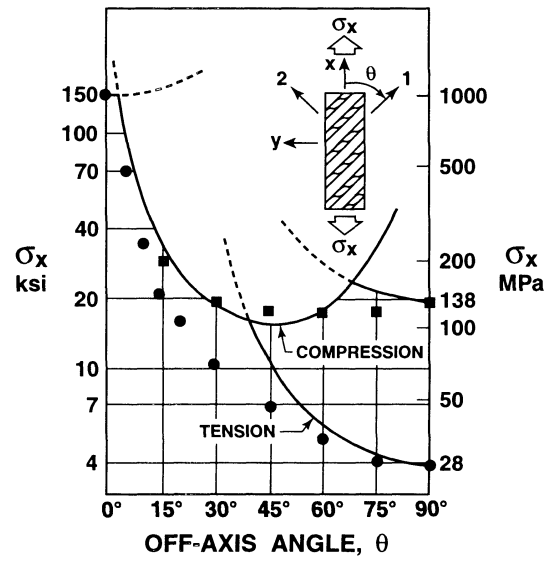


Figure 2.8: Maximum Strain Criterion (Jones, 1999) .

interactive term F_{12} can be influential in predicting failure and is often difficult to experimentally define. The Tsai–Wu failure criterion (Tsai and E. M. Wu, 1971) has a general nature, because it contains almost all other interactive theories as special cases.

Table 2.2: Categories of interactive failure criterion (Reddy, 1997) .

Criterion	Theory
$F_{ij}\sigma_i\sigma_j = 1$	Ashkenazi, Chamis, Fischer, Tsai-Hill, Noms Cowin, Hoffman,
$F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$	Malmeister, Marin, Tsai-Wu, Gol'denblat-Kopnov

Table 2.3: Interactive Theories governed by first category (Jones, 1999) .

Criterion	F_{11}	F_{22}	F_{12}	F_{66}
Ashkenazi	$\frac{1}{X^2}$	$\frac{1}{Y^2}$	$\frac{1}{2} \left[\frac{4}{U^2} - \frac{1}{X^2} - \frac{1}{Y^2} - \frac{1}{S^2} \right]$	$\frac{1}{S^2}$
Chamis	$\frac{1}{X^2}$	$\frac{1}{Y^2}$	$-\frac{K_1 K_1}{2XY}$	$\frac{1}{S^2}$
Fisher	$\frac{1}{X^2}$	$\frac{1}{Y^2}$	$-\frac{K}{2XY}$	$\frac{1}{S^2}$
Tsai-Hill	$\frac{1}{X^2}$	$\frac{1}{Y^2}$	$-\frac{1}{2X^2}$	$\frac{1}{S^2}$
Norris	$\frac{1}{X^2}$	$\frac{1}{Y^2}$	$-\frac{K}{2XY}$	$\frac{1}{S^2}$

Interactive criteria such as Tsai–Wu are often criticized due to their lack of phenomenological basis and origins in theories originally proposed for metals. However,

Table 2.4: Interactive Theories governed by second category (Jones, 1999) .

Criterion	F_1	F_2	F_{11}	F_{22}	F_{12}	F_{66}
Cowin	$\frac{1}{X} - \frac{1}{X'}$	$\frac{1}{Y} - \frac{1}{Y'}$	$\frac{1}{XX'}$	$\frac{1}{YY'}$	$\frac{2S^2\sqrt{F_{11}F_{22}}-1}{2S^2}$	$\frac{1}{S^2}$
Hoffman	$\frac{1}{X} - \frac{1}{X'}$	$\frac{1}{Y} - \frac{1}{Y'}$	$\frac{1}{XX'}$	$\frac{1}{YY'}$	$-\frac{1}{2XX'}$	$\frac{1}{S^2}$
Malmeister	$\frac{1}{X} - \frac{1}{X'}$	$\frac{1}{Y} - \frac{1}{Y'}$	$\frac{1}{XX'}$	$\frac{1}{YY'}$	$\frac{(S_{45})[(F_1-F_2)+(S_{45})(F_{11}+F_{22})-1]}{2S_{25}^2}$	$\frac{1}{S^2}$
Marin	$\frac{1}{X} - \frac{1}{X'}$	$\frac{1}{Y} - \frac{1}{XX'}$	$\frac{1}{XX'}$	$\frac{1}{YY'}$	$\frac{1}{XX'} - \frac{XX' - S[X' - X - X'(\frac{X}{Y}) + Y]}{2S^2XX}$	-
Tsai-Wu	$\frac{1}{X} - \frac{1}{X'}$	$\frac{1}{Y} - \frac{1}{Y'}$	$\frac{1}{XX'}$	$\frac{1}{YY'}$	$\leq \pm\sqrt{F_{11}F_{22}}$	$\frac{1}{S^2}$

they are common in industrial applications and widely available in FE codes (Orifici and I. Herszberg , 2008).

2.2.4 Tsai-Hill (Maximum Work) Criterion

The Tsai-Hill theory is considered an extension of the Von Mises failure criterion. The failure strengths in the principal material directions are assumed to be known. The tensor form of this criterion is $F_{ij}\sigma_i\sigma_j = 1$. If this expression is expanded and the F_i , terms replaced by letters, the failure criterion is

$$F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L(\tau_{23}^2 + M\tau_{13}^2 + N\tau_{12}^2) = 1 \quad (2.8)$$

where F, G, H, L, M, and N are anisotropic material strength parameters. This criterion is also illustrated in Figure 2.9.

2.2.5 Hashin failure criteria

Hashin (Hashin and A. Rotem, 1973) used his experimental observations of failure of tensile specimens to propose two different failure criteria, one related to fiber failure and the other related to matrix failure. The criteria assume a quadratic interaction between the tractions acting on the plane of failure. Later on he developed his model (Hashin, 1980). he introduced fiber and matrix failure criteria that distinguish between tension and compression failure. Given the difficulty in obtaining the plane of fracture for the matrix compression mode, Hashin used a quadratic interaction between stress invariants. Such derivation was based on logical reasoning rather than micromechanics. Although the Hashin criteria were developed for unidirectional laminates, they have also been applied successfully to progressive failure analyses of laminates by using in-situ unidirectional

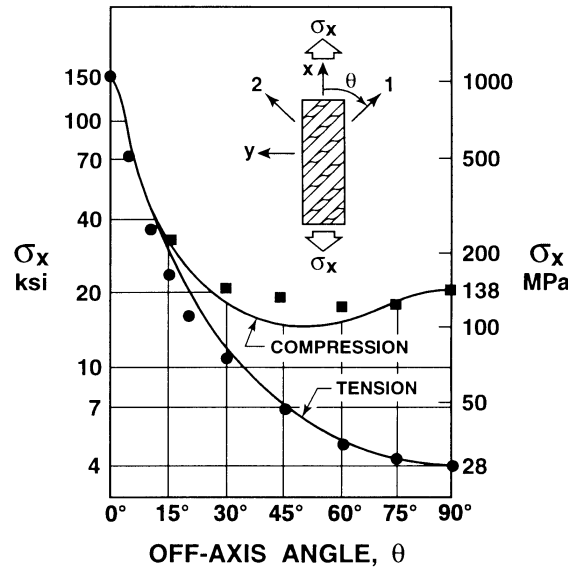


Figure 2.9: Tsai-Hill (Maximum Work) Criterion (Jones, 1999) .

strengths to account for the constraining interactions between the plies. Two dimensional model of Hashin is summarized as below:

1. Fiber failure mode in tension:

$$\left(\frac{\sigma_1}{X^T}\right)^2 + \left(\frac{\tau_{12}}{S^L}\right)^2 = 1 \quad (2.9)$$

2. Fiber failure mode in compression:

$$\frac{\sigma_1}{X^C} = 1 \quad (2.10)$$

3. Matrix failure mode in tension:

$$\left(\frac{\sigma_2}{Y^T}\right)^2 + \left(\frac{\tau_{12}}{S^L}\right)^2 = 1 \quad (2.11)$$

4. Matrix failure mode in compression:

$$\left(\frac{\sigma_2}{2S^T}\right)^2 + \left(\frac{Y^C}{2S^T} - 1\right) \left(\frac{\sigma_2}{Y^C}\right) + \left(\frac{\tau_{12}}{S^L}\right)^2 = 1 \quad (2.12)$$

Numerous studies conducted over the past decade indicate that the stress interactions proposed by Hashin do not always fit the experimental results, especially in the case of matrix or fiber compression. It is well known, for instance, that moderate transverse compression ($\sigma_{22} \leq 0$) increases the apparent shear strength of a ply, which is not predicted by Hashin's criterion. In addition, Hashin's fiber compression criterion does not account for the effects of in-plane shear, which significantly reduces the effective compressive

strength of a ply. Several researchers have proposed modifications to Hashin's criteria to improve their predictive capabilities (Pinho et al., 2005) .

2.2.6 Chang failure criteria

As mentioned earlier, Chang (Chang and L.B. Lessard, 1991) is a direct mode criterion. This category of criteria, provide separate failure equations for each mode of failure. Chang uses five distinct polynomials to describe five modes of failure, which are discussed below:

1. Fiber breakage mode

$$\frac{\sigma_1}{X_{ut}} = 1 \quad (2.13)$$

2. Fiber buckling failure mode

$$\frac{\sigma_1}{X_{uc}} = 1 \quad (2.14)$$

3. Matrix tensile cracking mode

$$\frac{\sigma_1^2}{X_{ut}} + \frac{\frac{\tau_{12}}{2G_{12}} + \frac{3}{4}S_{6666}\tau_{12}^4}{\frac{S^2}{2G_{12}} + \frac{3}{4}S_{6666}S^4} = 1 \quad (2.15)$$

4. Matrix compression failure mode

$$\frac{\sigma_1^2}{Y_{uc}} + \frac{\frac{\tau_{12}}{2G_{12}} + \frac{3}{4}S_{6666}\tau_{12}^4}{\frac{S^2}{2G_{12}} + \frac{3}{4}S_{6666}S^4} = 1 \quad (2.16)$$

5. Fiber–matrix shearing failure mode

$$\frac{\sigma_1^2}{X_{ut}} + \frac{\frac{\tau_{12}}{2G_{12}} + \frac{3}{4}S_{6666}\tau_{12}^4}{\frac{S^2}{2G_{12}} + \frac{3}{4}S_{6666}S^4} = 1 \quad (2.17)$$

or

$$\frac{\sigma_1^2}{X_{uc}} + \frac{\frac{\tau_{12}}{2G_{12}} + \frac{3}{4}S_{6666}\tau_{12}^4}{\frac{S^2}{2G_{12}} + \frac{3}{4}S_{6666}S^4} = 1 \quad (2.18)$$

It should be noted that S_{6666} is a constant in Chang failure criterion and it should be measured by experiment that its details are available in (Chang and L.B. Lessard, 1991).

2.2.7 Comparison of different failure criteria

Comparing the different failure criteria with each other is a difficult task, which requires extensive experimental data across the full range of possible loading scenarios. This process is further complicated by the vast number of theories and criteria available in the literature, the necessary use of curve-fitting or “tuning” in any analysis to some degree, and the subjective nature of comparison itself (Orifici and I. Herszberg, 2008). Word Wide Failure Exercise (WWFE) (Hinton and P. G. Soden, 2002) was conducted to assess the real predicting capability of the current available failure criteria. Leading researchers in failure of composites were invited to participate in a round-robin in which they presented their approaches and predictions. This exercise spanned 12 years and compared 19 leading failure theories for the analysis of 14 biaxial tests cases covering different materials, laminates and load cases. Numerical predictions were made both without and with access to the experimental data, and the organizers ranked each criterion in a range of categories, including the accuracy of the prediction, necessity for ‘tuning’ and applicability across the different scenarios. Pinho et al. (Pinho et al., 2005) assessed this effort. Several lessons can be learned from the WWFE. Firstly, most criteria were unable to capture some of the trends in the failure envelopes of the experimental results. Secondly, on what concerns phenomenological failure criteria, most expressions proposed to predict each failure mode are still to some extent empirical. It is somewhat difficult to choose between the criteria due to the lack of experimental data needed to validate them against each other. Despite several efforts to develop sound phenomenological criteria, non-phenomenological criteria like Tsai-Wu (Tsai and E. M. Wu, 1971) are often better prediction tools than some phenomenological criteria (Liu and S. W. Tsai, 1998). Although test results are not provided in the WWFE for several stress combinations that remain open for discussion, significant progress was made. From the limited predictive capabilities of the most accurate analyses available, it is clear that further developments in failure model theories and criteria are required before any analysis approach can be used with confidence to predict the strength of a typical aerospace composite component (Pinho et al., 2005). After this evaluation Pinho et al. proposed Larc failure criteria which has attracted attention of researchers that is the basis of the current thesis and will be discussed extensively on the following chapters.

2.3 Micro-Mechanical properties of composites

The study of composite materials at the fiber and matrix level is referred to as micromechanics. It is desired to predict the overall effective (or average) elastic properties and

inelastic response of the composite based upon the known properties, arrangement and volume fraction of the constituent phases. Table 2.5 shows Common Fiber Volume Fractions in Different Processes.

The development of micromechanics models for predicting the effective properties of composites experienced a flurry of activity beginning in the late 1950s and extending through the 1960s and into the 1970s. The earlier works were concerned with the prediction of effective properties for materials (both solids and fluids) consisting of inclusions in a carrier material. Table 2.6 summarizes the development in the field of micro mechanical approach.

Table 2.5: Common Fiber Volume Fractions in Different Processes (Gray et al., 2003) .

Molding Process	Fiber Volume Fraction
Contact Molding	30%
Compression Molding	40%
Filament Winding	60% – 85%
Vacuum Molding	50% – 80%

Modulus of elasticity along the direction of the fiber E_1 is given by:

$$E_1 = E_f V_f + E_m (1 - V_f) \quad (2.19)$$

In practice, this modulus depends essentially on the longitudinal modulus of the fiber, E_f , because $E_m \ll E_f$ (as $\frac{E_{mresin}}{E_{fglass}} \approx 6\%$). Modulus of elasticity in the transverse direction to the fiber axis, E_t , is presented in:

$$E_t = E_m \left[\frac{1}{((1 - v_f) + \frac{E_m}{E_{ft}} V_f)} \right], \quad (2.20)$$

in that, E_{ft} represents the modulus of elasticity of the fiber in the direction that is transverse to the fiber axis.

$$G_{lt} = G_m \left[\frac{1}{((1 - v_f) + \frac{G_m}{G_{lft}} V_f)} \right], \quad (2.21)$$

also gives shear module in which G_{flt} represents the shear modulus of the fiber. The Poisson coefficient equation, represents the contraction in the transverse direction t when

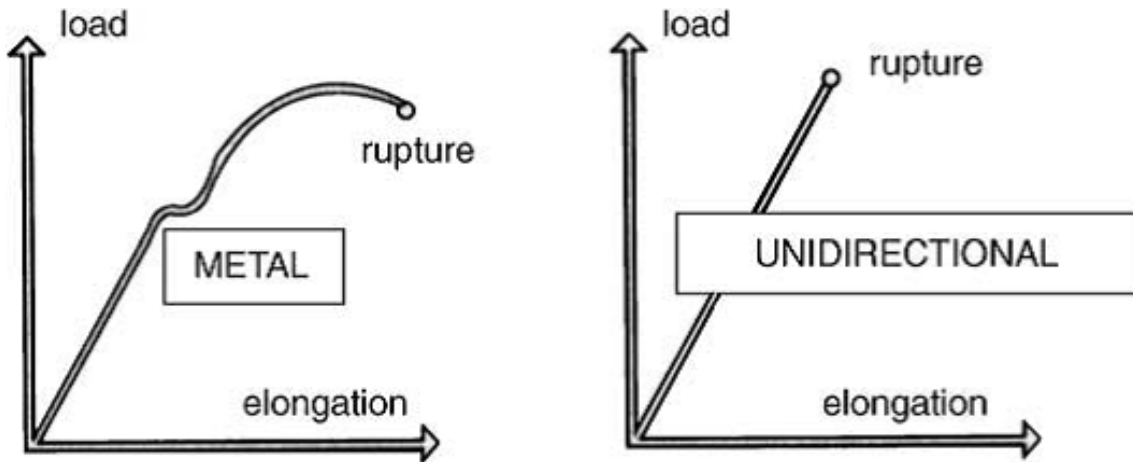


Figure 2.10: Loading Curves of Metal and Unidirectional Composite(Carl, 2012).

a ply is subjected to tensile loading in the longitudinal direction, as:

$$v_t = v_f V_f + v_m V_m. \quad (2.22)$$

The curves in Figure 2.10 show the important difference in the behavior between classical metallic materials and the unidirectional plies. These differences can be summarized in a few points as:

- There is lack of plastic deformation in the unidirectional ply. (This is a disadvantage.)
- Ultimate strength of the unidirectional ply is higher. (This is an advantage.)
- There is important elastic deformation for the unidirectional ply. (This can be an advantage or a disadvantage, depending on the application; for example, this is an advantage for springs, arcs, or poles.)

When the fibers break before the matrix during loading along the fiber direction, one can obtain equation for the composites as:

$$\sigma_{1rupture} = \sigma_{frupture} \left[V_f (1 - V_f) \frac{E_m}{E_f} \right]. \quad (2.23)$$

Off-axis Rupture Strength is also shown in Figure 2.11 .

Table 2.6: Developments in micromechanics (Carl, 2012) .

Year	Development	Author(s)
1837	Strain energy density defined - 21 elastic constants	Green
1887-1889	Uniform strain modulus prediction	Voigt
1929	Uniform stress modulus prediction	Reuss
1957	Determination of the elastic field of an ellipsoidal inclusion and related problems	Eshelby
1960	Prediction of Elastic Constants of Multiphase Materials	Paul
1962	The Elastic Moduli of Heterogeneous Materials	Hashin
1963	Elastic Properties of Reinforced solids: some theoretical principles	Hill
1963	Variation Approach to the Theory of The Elastic Behavior of Multiphase Materials	Hashin & Shtrikman
1964	Theory of Mechanical Behavior of Heterogeneous Media	Hashin
1964	Theory of Mechanical Properties of Fiber-Strengthened Materials - I. Elastic Behavior	Hill
1964	The Elastic Moduli of Fiber-Reinforced Materials	Hashin & Rosen
1965	The Principle of the Fiber Reinforcement of Metals	Kelly
1967	Modern Composite Materials	& Davies Broutman & Krock Tsai, Halpin
1968	Composite Materials Workshop	& Pagano
1972	Theory of Fiber Reinforced Materials	Hashin
1972	On the Effective Moduli of Composite Materials: Slender Rigid Inclusions at Dilute Concentrations	Russel & Acrivos
1975	A Theory of Elasticity with Micro-structure for Directionally Reinforced Composites	Achenbach
1979	Analysis of Properties of Fiber Composites with Anisotropic Constituents	Hashin
1991	Mechanics of Composite Materials: A Unified Micromechanical Approach	Aboudi
1993	Micromechanics: overall properties of heterogeneous materials	Nasser & Hori

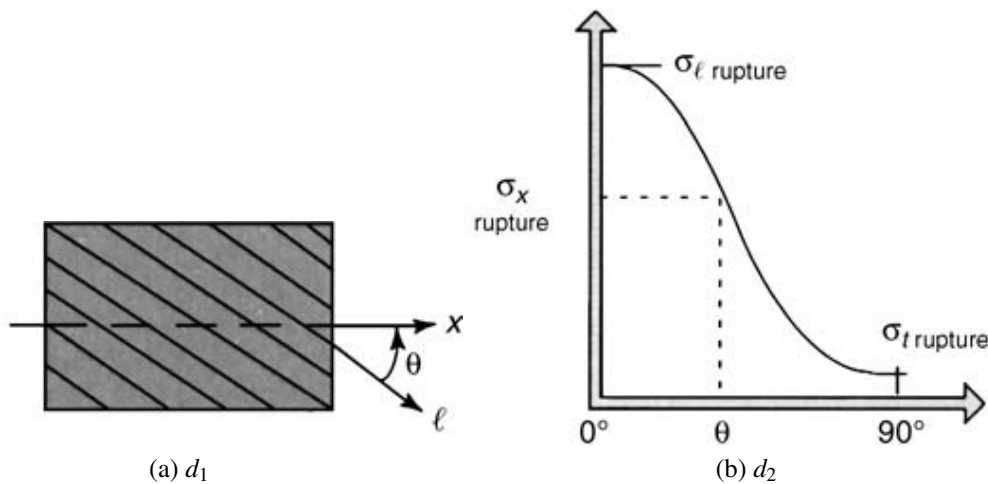


Figure 2.11: Off-axis Rupture Strength.

2.4 Variable stiffness composites

Lamina consisting of high-stiffness high-strength fibers can be combined in different orientations and thicknesses to create a composite material tailored to specific loading conditions. The ability to customize the mechanical properties and structural response of a composite material allows for the most efficient structure to be determined and economically produced. The challenge to the engineer is to determine an effective means of achieving the most efficient structure. While many techniques for optimizing traditional composite materials have been developed, modern processing methods allow for more complex composite structures to be conceived and created. Traditional composite laminates consist of multiple plies of composite lamina. Each composite ply has specific material properties, thickness and orientation angles that remain constant throughout the ply. Traditional composite laminates have a specific loading response that remains constant throughout each ply. Composite plies with different material properties, thicknesses and/or orientation angles can be stacked together to tailor the response to a specific loading condition resulting in a very efficient structure. In some cases, however, it is advantageous to have a higher level of tailor-ability in order to reduce overall manufacturing costs or weight. Variable-stiffness composite laminates (VSCL) differ from traditional straight-fiber composite laminates called constant stiffness composite lamina (CSCL) by varying material properties, thickness and/or fiber orientation spatially throughout the laminate. Variations in layer material properties, thickness and/or fiber orientation results in a spatially varying response. Variable-stiffness designs allow for the composite to be tailored to a wider range of potential applications than straight-fiber laminates. Figure 2.12 shows a typical example of VSCL in comparison with CSCL.

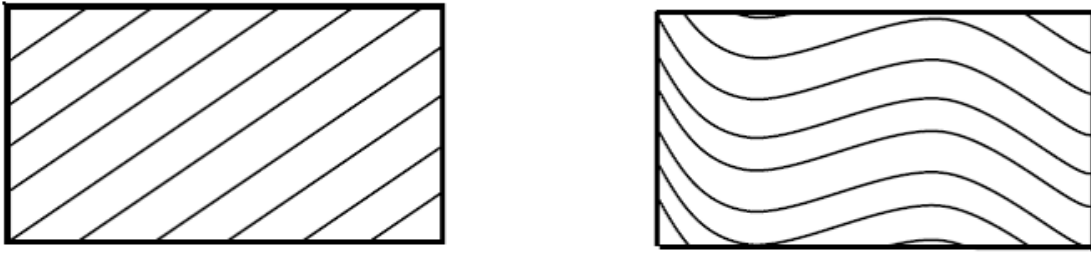


Figure 2.12: Constant stiffness composite lamina (CSCL) versus variable-stiffness composite laminates (VSCL) .

This stiffness variation may be discrete by defining several different patches within a laminate or continuous by varying the fiber angle orientation continuously, within a ply's domain. Schematic examples of discrete variable stiffness laminates is shown in Figure 2.13 .

There are three primary methods for varying the material properties, thickness and orientation angle of a laminate spatially. The primary methods include varying volume fraction of fibers, dropping or adding of plies to the laminate and varying the orientation angle through a ply. An example of such a lamina is shown in Figure 2.14 . A second method of creating a variable-stiffness laminate is to change the thickness within the laminate structure. Figure 2.15 is a simple example of how dropping composite plies leads to a thickness change of the laminate. The final method of creating a variable-stiffness laminate is to vary the fiber orientations throughout each lamina, also known as a curvilinear-fiber lamina which has been considered in this research.

2.4.1 Fiber placement technology

The processing of complex composite structures is currently possible with automated tow-placement machines. This emerging technology is characterized by a using a computer-

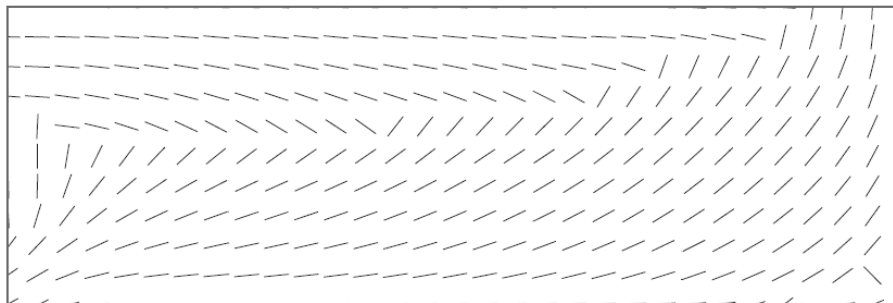


Figure 2.13: Example of discrete fiber angle distribution .

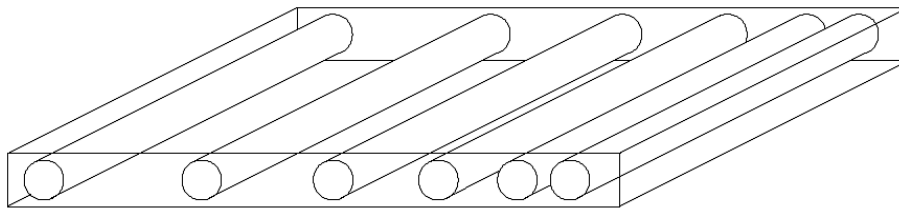


Figure 2.14: Lamina with varying volume fraction ([Enders and P. C. Hopkins , 1991](#)) .

controlled multi-axis delivery head to place numerous flat narrow strips of composite material known as tows. This process allows design and production of parts that would be extremely difficult or impossible with other methods such as tape lay-up or filament winding. Figure 2.16 shows a basic schematic of a Hercules tow placement machine and the seven axes of motion in it ([Langley, 1999](#)) .

Fiber placement technology has made it possible to produce large yet intricate structures while helping to reduce labor costs, the amount of scrap material produced, and it has increased product quality. However, Automated Fiber Placement (AFP) remains an expensive technology and therefore is only applied when it can be economically justified. Aircraft often have service lives in excess of 60000 flight hours and millions of nautical miles. Large operating cost savings can be achieved by reducing structural weight and increasing maintenance intervals. The aerospace sector, where relatively large, expensive parts are produced in small series, has been an ideal industry to introduce and apply fiber placement technology, however, as machine costs start to decrease and production rates increase fiber placement technology becomes attractive for other end users, such as the automotive, maritime and wind-energy industries, which will or are starting to integrate fiber placement into their production environment ([Ijsselmuiden, 2011](#)) . The development of AFP machines started in the late 1980s; AFP is a unique composite manufacturing technology which combines individual tow control found in filament winding machines with compaction and cut-restart capabilities of automated tape laying. Modern fiber placement

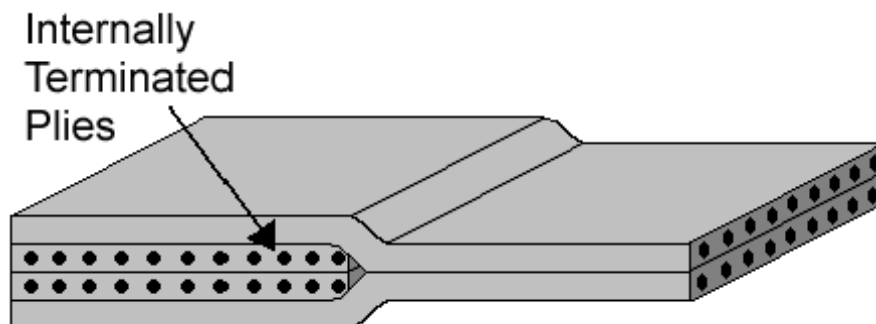


Figure 2.15: Laminate with internally dropped plies ([DiNardo and P. A. Lagace , 1989](#)) .

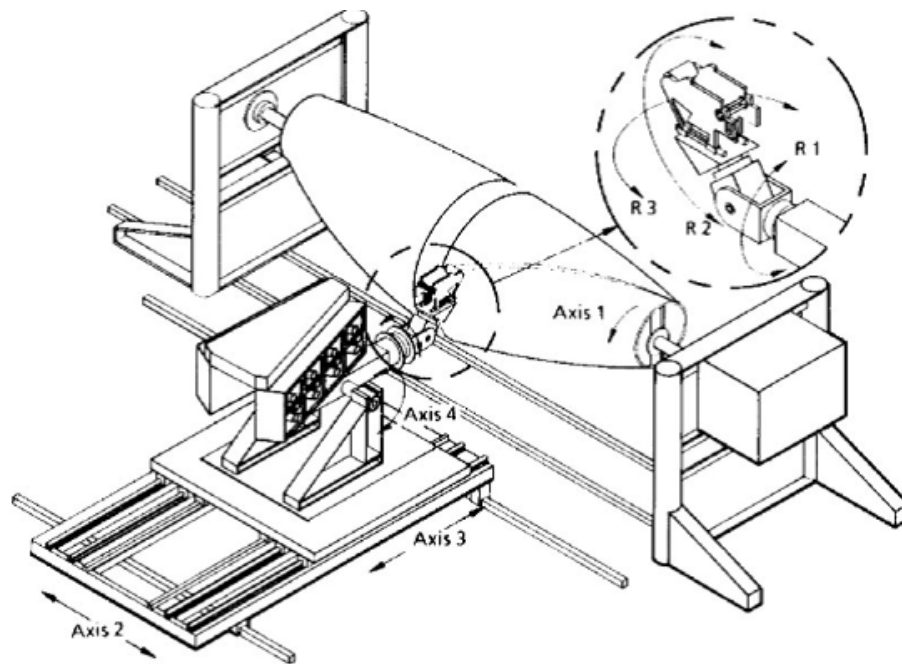


Figure 2.16: Hercules tow placement machine ([Langley, 1999](#)) .

systems consist of seven axes, a 6-axis robotic arm or gantry assembly and a rotational axis of the mandrel. Current AFP machines allow up to 32 individually controlled tows, collectively known as a course or band, to be placed on an arbitrary surface. Tows can be added or dropped at any point along the path of the machine head, to increase or decrease the total course width as required. Tows typically vary in width from an eighth of an inch (3.2 mm) to an inch (25.4 mm). AFP machines can be adapted to accommodate the wide range of fibrous materials in use in the aerospace industry, such as carbon, aramid and glass. The majority of AFP machines currently in service process thermoset pre-impregnated materials. Thermo-plastic fiber placement and dry-fiber placement systems are showing great promise for future applications. Tows are placed on a mandrel surface via a tow placement head, shown schematically in Figure 2.17 . The tows are typically guided from a climatically controlled creel chamber, where pre-impregnated material is stored on bobbins, to the mold surface via a network of individual tow guidance rollers and tensioning mechanisms. Current fiber placement machines use passive tow-feeding, where friction between tow, compaction roller and tool surface provides the required force. Tows can be cut and restarted individually via a cutting mechanism and restart rollers. A heating unit is placed prior to material deposition to activate material tackiness which improves adhesion to the mandrel surface and finally a compaction roller places the fibers securely on the mold's surface ([IJsselmuiden, 2011](#)) .

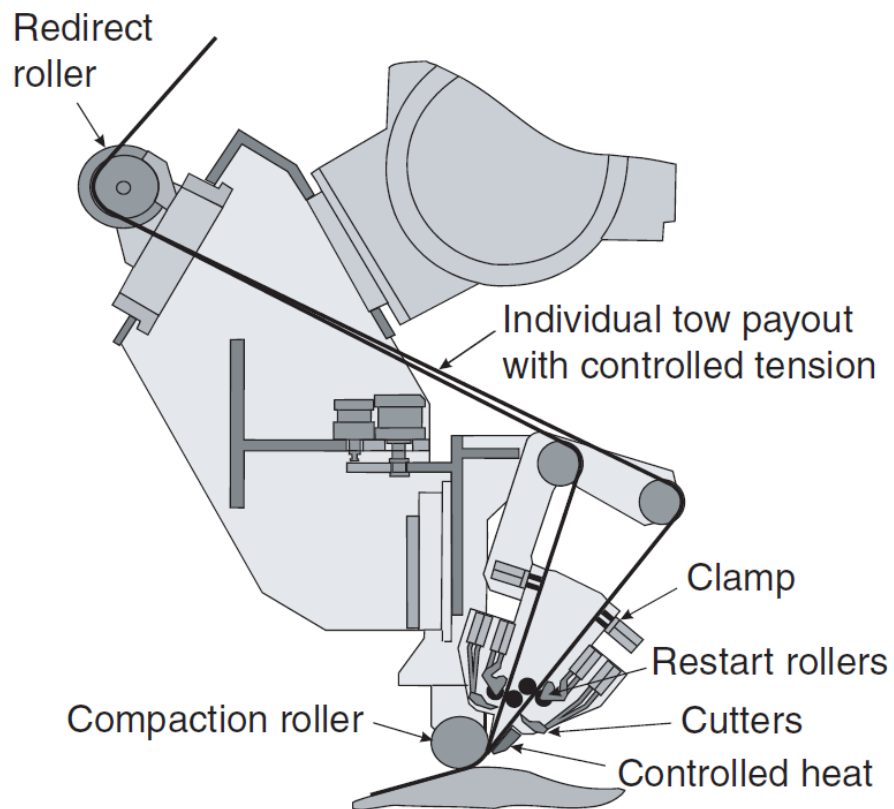


Figure 2.17: Schematization of a fiber deposition head used for automated fiber placement (Langley, 1999) .

2.4.2 Fields of Application

Fiber placement technology has improved drastically over the past three decades and is being used by an increasing number of aerospace manufacturers ([Evans, 2001](#)) . Boeing Helicopters was the first company to apply fiber placement in a production environment in the early 1990's. Boeing and Hercules were funded to develop a process to fiber place the aft fuselage section for the Bell/Boeing V22 Osprey. Initially the aft fuselage section was built up of nine individual panels built using hand layup, with redesigning for fiber placement a single monolithic structure could be manufactured allowing the required amount of fasteners to be reduced by 34%. The trim and assembly labor was reduced by 53% and the amount of material scrap produced was reduced by 90%. Fiber placement was later applied for the production of fuselage skins of the F/A-18 Super Hornet, allowing Northrop Grumman to reduce labor costs by 38% with respect to hand layup ([IJsselmuiden, 2011](#)) . Now, well into the 21st century, fiber placement had become mature enough to be applied on a large scale in the commercial aviation environment. The Boeing 787 Dreamliner was the first commercial airliner to be manufactured primarily from composite materials. Several manufactures are working together with Boeing to fabricate wing and fuselage sections at multiple locations around the world. The cured parts are subsequently transported to the Boeing Everett plant for final assembly. For example, Alenia Aeronautica manufactures section 44 and 46 in their new composite facility in Grottagie. Each section is in the region of 10m long, has a diameter of approximately 6m and contains roughly 2000kg of carbon fiber. The sections are manufactured using the latest generation fiber placement machines made by Ingersoll Machine Tools. Similarly for the next generation wide-body aircraft from Airbus Industries, the A350- XWB, large portions of the fuselage barrel and wings are and will be manufactured using fiber placement technology ([IJsselmuiden, 2011](#)) .

Chapter 3

Modeling of damage in composite materials

3.1 Introduction

It is important to model the complex material behavior in order to predict its response under different loading conditions. With the rapid development of the calculation performance of computer, the finite element analysis has become a powerful tool to deal with the complex boundary value problems. Composite materials reveal different modes of failure under different loading and based on that the damage initiates and evolves that leads to stiffness degradation. In the analysis of composite structures, various approaches are used to characterize the onset and progression of damage. This typically involves monitoring a particular type of parameter to predict and monitor damage development and growth. Though there are a variety of damage characterization approaches, these can be generally categorized as being based on theories of strength or fracture mechanics. The strength, as defined by the allowable stresses for a material, can be used to characterize the initiation and growth of all types of damage. The application of the strength approach is usually fairly simple, with one or more strength criteria defined, and the material deemed to have been irreversibly damaged once these criteria are satisfied. The criteria themselves can range from single stress parameter limits, combinations of various stress terms, or normalization of stress terms using structural or material values. Also, strength criteria can be applied so that each damage mechanism has a distinct criterion, or a more general damage criterion can be applied. There are also a number of parameters similar to stress that have been used to characterize damage including strain, force, displacement or rotation among others. It is important to note that strength-based characterization of damage is most commonly applied to define the damage initiation, and not the progression of an

existing damage region, and this is especially relevant for delamination (Orifici and I. Herszberg , 2008).

3.2 Progressive failure analysis

Progressive failure analysis using continuum damage mechanisms (CDM) is a common method to simulate damage in materials. By continuum damage mechanisms (CDM), the mechanical properties of the material in undamaged form will gradually decreases. In the CDM theory, the loss of stiffness can be physically considered the macroscopic representation of a series of distributed micro-cracks and micro-voids. Firstly, (Kachanov, 1958) used this method to analyze the creep rupture in metals. (Liu and J.Y. Zheng , 2010) has provided a review on recent studies on composite damage using CDM. For the damaged laminates, stress can be written according to:

$$\boldsymbol{\sigma} = (1 - \mathbf{D})\bar{\boldsymbol{\sigma}}. \quad (3.1)$$

Where $\mathbf{D}$ is damage tensor and $\bar{\boldsymbol{\sigma}}$ is nominal stress. When the material is undamaged $\mathbf{D}$ is equal to zero and by damage evolution it increases while it is below 1. The thermo-mechanical behavior of the materials can be described by the Helmholtz free energy and it can define the link between stress and strain. The Helmholtz free energy per unit mass ψ for elastic–plastic materials under isothermal conditions reads:

$$\rho \psi = \rho \psi^e(\varepsilon_{ij} - \varepsilon_{ij}^p, D_{ij}) + \rho \psi^p(\kappa) + \rho \psi^d(\kappa). \quad (3.2)$$

where ψ^e , ψ^p and ψ^d are the free energy which represents the elastic, plastic deformations and damage hardening, respectively. κ is an internal variable and ρ is the density. The thermodynamic conjugate forces (Y_{ij}, B, R) corresponding to the internal variables (D_{ij}, κ, α) are denoted as:

$$\sigma_{ij} = \rho \frac{\partial \psi}{\partial \varepsilon_{ij}}, Y_{ij} = -\rho \frac{\partial \psi}{\partial D_{ij}}, B_{ij} = \rho \frac{\partial \psi}{\partial \kappa}, R_{ij} = \rho \frac{\partial \psi}{\partial \alpha} \quad (3.3)$$

The thermodynamic formulation gives the thermodynamic forces and the dissipation inequality equation but no information about the evolution laws for internal variables. The only restriction imposed by the continuum thermodynamics on the evolution laws is that the Clausius–Duhem dissipation inequality must be fulfilled, which takes the following form under isothermal conditions.

$$\gamma = \dot{\psi} + \sigma_{ij} \dot{\varepsilon}_{ij} \geq 0. \quad (3.4)$$

where γ is the power of dissipation due to damage and a dot denotes the rate with respect to time. From Eqs.3.2-3.4 the dissipation inequality is expressed as:

$$\gamma = \sigma_{ij}\dot{\epsilon}_{ij}^p + Y_{ij}\dot{D}_{ij} - B\dot{\kappa} - R\dot{\alpha} \geq 0. \quad (3.5)$$

If the dissipation γ reaches the maximum, the damage evolution law is given by:

$$\dot{D}_{ij} = \dot{\lambda}^d \frac{\partial F^d}{\partial Y_{ij}}, \dot{\kappa} = -\dot{\lambda}^d \frac{\partial F^d}{\partial B}, \dot{\epsilon}_{ij}^p = -\dot{\lambda}^p \frac{\partial F^p}{\partial \sigma_{ij}}, \dot{\alpha}^p = -\dot{\lambda}^p \frac{\partial F^p}{\partial R}. \quad (3.6)$$

where $F_d(Y_{ij}, D, B)$ and $F_p(\sigma_{ij}, D, R)$ are the damage potential functions. $\dot{\lambda}^d \geq 0$ and $\dot{\lambda}^p \geq 0$ are called the consistency parameters and they are assumed to obey the Kuhn–Tucker consistency requirements as:

$$\dot{\lambda}^d \geq 0, \quad F^d \leq 0, \quad F^d = 0, \quad \dot{\lambda}^p \geq 0, \quad F^p \leq 0, \quad F^p = 0. \quad (3.7)$$

In the associative plastic flow criterion, the functions F^d and F^p in Eq.3.7 are equivalent to the corresponding potential function G^d and G^p . Only after the activation functions $F_d(Y_{ij}, D, B)$ and $F_p(\sigma_{ij}, D, R)$ are determined can the damage evolution information be acquired. Then, as the consistency parameters $\dot{\lambda}^d$ and $\dot{\lambda}^p$ are derived, the damage tensor and internal variable as well as the consistent constitutive equations are obtained. Based on the CDM theory, Schapery (Schapery, 1990), Murakami and Kamiya (Murakami and K. Kamiya, 1997), Hayakawa et al. (Hayakawa et al., 1998), Tang et al. (Tang et al., 1998), Brünig (Brünig, 2003), Olsson and Ristinmaa (Olsson and M. Ristinmaa, 1998), Basu et al. (Basu et al., 2007) and Maimí et al (Maimí et al., 2007a), (Maimí et al., 2007b) proposed the isotropic/anisotropic stiffness degradation models and damage evolution models respectively by introducing a two-order or four-order damage tensor. In these models, the relationships between the damage dissipation potential F , the conjugate force Y and the damage tensor D were addressed, which were explained by different damage evolution laws. The last model will be discussed more in following chapter because it is the basis of the current work.

Besides, the micromechanics theory which assumed a representative volume element (RVE) was associated with the CDM to describe the evolvement of damage properties. Specially, Matzenmiller et al. (Matzenmiller et al., 2007), Kwon and Liu (Matzenmiller et al., 1997), Schipperen (Schipperen, 1997), Maa and Cheng (Maa and J.H. Cheng, 2002), Camanho et al. (Camanho et al., 2007), Barbero and Vivo (Barbero and L.D. Vivo, 2001) established the thermodynamic models to describe the progressive failure properties and to interpret the stiffness degradation of composite laminates. In their models, the various failure modes were assumed and the relationships between the damage tensor, con-

jugate forces and internal stresses/strains were further formulated. However, these thermodynamic models were all limited to the plane structures. For the aluminium–carbon fiber/epoxy composite cylindrical laminates, Perreux et al. (Perreux and F. Thiebaud, 1995) and Ferry et al. (Ferry et al., 1998) derived the constitutive relationships of the damaged materials using the CDM and fracture mechanics. However, these models cannot explain different failure modes of FRCs because they defined only the individual thermodynamic conjugate force and damage tensor. Further, Perreux et al. (Perreux et al., 2006), Liu and Zheng (Liu and J.Y. Zheng, 2008) described the damage evolution process by defining three independent damage tensors to explain three kinds of failure modes: fiber breakage, matrix cracking and shear failure.

3.3 Plasticity incorporation

The elastic/damage coupling damage constitutive model may be insufficient in order to accurately acquire the damage initiation and evolution information of composite laminates. Therefore, the damage/plasticity coupling nonlinear models develop to describe the interactive effect of the plastic deformations on the damage properties of composite materials in terms of the dissipative energy concept according to the CDM. This is generally realized by introducing the thermodynamic conjugate forces into the damage surfaces or plasticity potential functions. Chow and Yang (Chow and Yang, 1997) outlined the nonlinear constitutive models for the mechanical response due to damage for inelastic composite materials. The damage/plasticity model formulation was based on the concept of damage surface and on the assumption that the change of material behavior is independent of loading path over which the damage state of material develops, but is dependent on the current stress state and the energy dissipated.

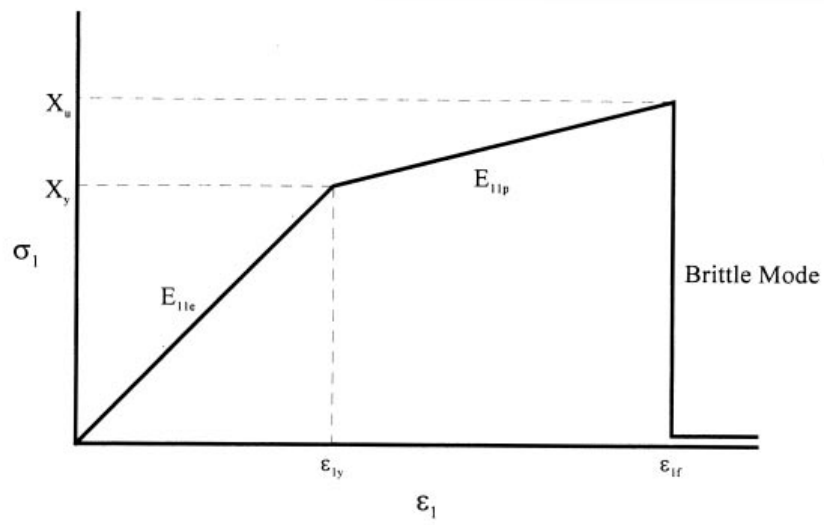
Lin and Hu (Lin et al., 2002) proposed a nonlinear elasticity–plasticity/damage coupling constitutive model together with a mixed failure criterion to simulate the behavior of composite laminates under uniaxial tension. In the model, fiber and matrix are assumed to behave elastic–plastic and the in-plane shear to behave nonlinear with a variable shear parameter. The damage onset for individual lamina is detected by a mixed failure criterion, which is composed of Tsai–Wu criterion and maximum stress criterion. After damage is taken place within the lamina, fiber and in-plane shear are assumed to exhibit brittle behavior and matrix to exhibit degrading behavior. It uses 2 approaches for modeling comprising Idealized Stress–Strain Curve and Post-Damage Model along with Nonlinear Constitutive Model of the Lamina. In the first approach, it is assumed that the material response can be adequately represented by bilinear stress–strain curves in

the principal material directions, 1-direction (fiber direction) and 2-direction (transverse direction) of the lamina, and by a nonlinear stress–strain curve for in-plane shear in 1-2 direction. During the nonlinear stage in shear direction, the nonlinear shear modulus, G_{12n} , is depending on the shear strain, γ_{12} . In the pre-damage regions, the elastic modulus for elastic stage is denoted by $E_{iie}(i = 1, 2)$, and the elastic modulus for plastic stage is denoted by $E_{iip}(i = 1, 2)$, in the principal material directions, respectively. In the post-damage region, the elastic stiffnesses are dropped to zero (brittle modes) in 1-direction and 1-2 direction. However, the elastic stiffness is assumed to have a negative modulus, E_{22f} (degrading mode) in 2-direction. This means that the damaged lamina unloads in the transverse direction through a negative tangent modulus until no load remains in the lamina. In Nonlinear Constitutive model, the stress–strain relations for an orthotropic lamina in the material coordinates (1, 2) is written as:

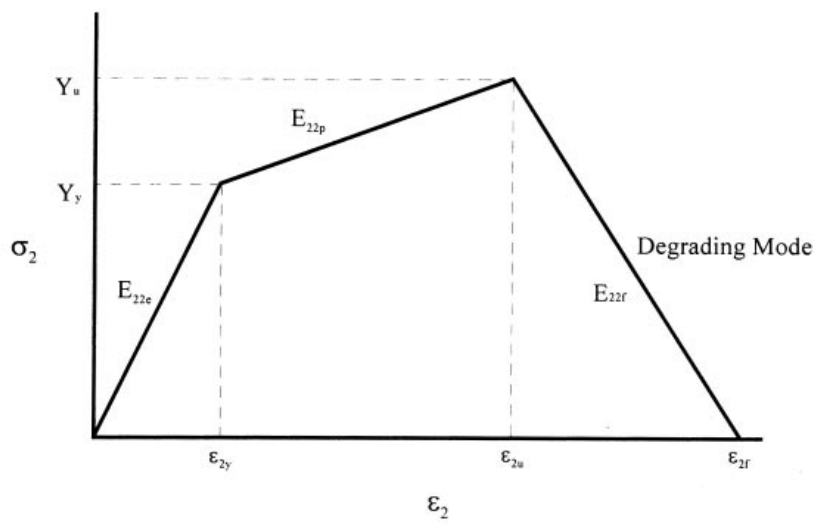
$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & \frac{-\nu_{21}}{E_{22}} & 0 \\ \frac{-\nu_{12}}{E_{22}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} + S_{6666} \tau_{12}^2 \begin{bmatrix} 0 \\ 0 \\ \tau_{12} \end{bmatrix}. \quad (3.8)$$

In the elastic stage, then $E_{11} = E_{11e}$ or $E_{22} = E_{22e}$. And if the material is in the plastic stage in 1-direction or 2-direction, then $E_{11} = E_{11p}$ or $E_{22} = E_{22p}$. The G_{12} is the shear modulus and S_{6666} is a shear parameter to account for the in-plane shear nonlinearity. The S_{6666} is a function of shear strain and is determined by fitting the stress–strain curve of pure shear test data. Upon damage within the lamina occurring, the material begins to degrade its property. Material degradation within the damaged area was evaluated based on the mode of failure predicted by the failure criterion. Therefore, the residual stiffnesses of composites strongly depend on the mode of failure in each layer. For the brittle mode, the material is assumed to lose its entire stiffness and strength in the dominant stress direction, whereas for the ductile mode the material retains its strength but loses all of its stiffness in the failure direction. For the degrading mode the material is assumed to lose its stiffness and strength in the failure direction gradually until the stress in that direction is reduced to zero. The illustration of the nonlinear model is shown in Fig 3.1 < (a)-(c).

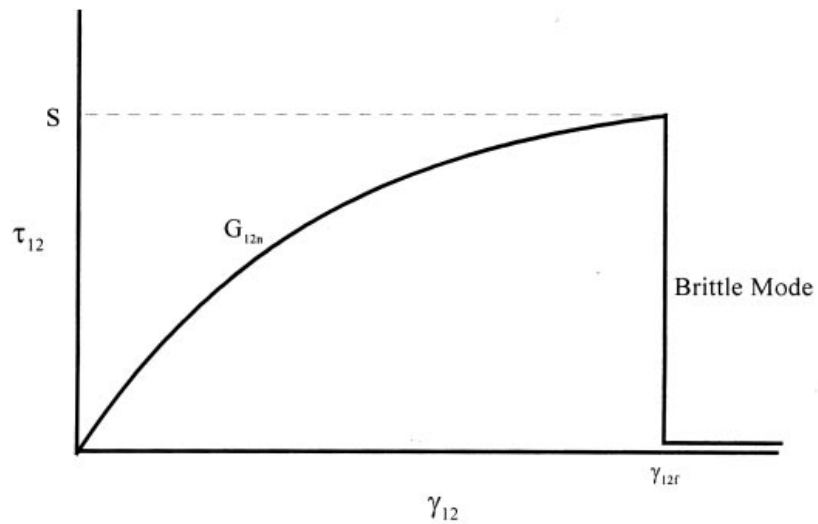
(Lin et al., 2006) established an elastoplastic damage model by applying a homogenization method at the microscopic scale. It is modeled by an anisotropic yield criterion which takes into account isotropic and nonlinear kinematic hardening. However, plasticity develops only in the matrix and its flow is blocked in the direction of the fibers. Barbero (Barbero, 2002) proposed a damage/plasticity coupling model by combining the



(a) For 1 direction



(b) For 2 direction



(c) For 1-2 direction

Figure 3.1: Stress-strain curves of the proposed nonlinear model (Lin et al., 2002).

CDM and classical thermodynamic theory to predict the inelastic effects as the reduction of stiffness and the increments of damage and unrecoverable deformation. Unrecoverable deformations and Damage are coupled by the concept of effective stress. New expressions of damage and unrecoverable deformation domains are presented so that the number of model parameters is small. The damage variables are (a) a second order symmetric damage tensor D , defined to account for the anisotropic evolution of defects along matrix and fiber directions and (b) a scalar damage hardening parameter p that controls the size of the damage surface. The evolution law is assumed to be isotropic for simplicity and due to the lack of experimental observations indicating anisotropic evolution of the damage surface. The unrecoverable deformation variables are (c) a second order symmetric tensor p to account for the accumulated, unrecoverable deformations and (d) a scalar unrecoverable deformation hardening parameter p that controls the size of the unrecoverable deformation surface. The evolution law is assumed to be isotropic for simplicity and due to the lack of experimental observations indicating otherwise.

The dissipation energy $\pi(p, \delta)$ is separated into two terms, which express the evolution of the damage and unrecoverable deformation surfaces as:

$$\pi(p, \delta) = R(p) + \gamma(\delta). \quad (3.9)$$

By considering:

$$R(p) = -\frac{1}{2}c_1^p p^2. \quad (3.10)$$

and,

$$\gamma(\delta) = c_1 \exp\left(\frac{\delta}{c_2}\right) \quad (3.11)$$

where c_1^p , c_1 and, c_2 are material parameters that should be calculated by curve fitting. The unrecoverable strain evolution is realized by the classical plasticity formulation. It is also important to incorporate finite fiber rotation in the continuum damage model. It will be discussed in this thesis in the following sections. In addition, in cases that there is a sensible strain rate and consequently dynamic effects, the viscoplastic effect should be regarded, that is also addressed in this thesis.

3.4 Damage and fracture mechanics

In fracture mechanics, it is assumed that the crack is existing in the material and it will analyze the propagation of that. In finite element analysis, the crack propagation needs

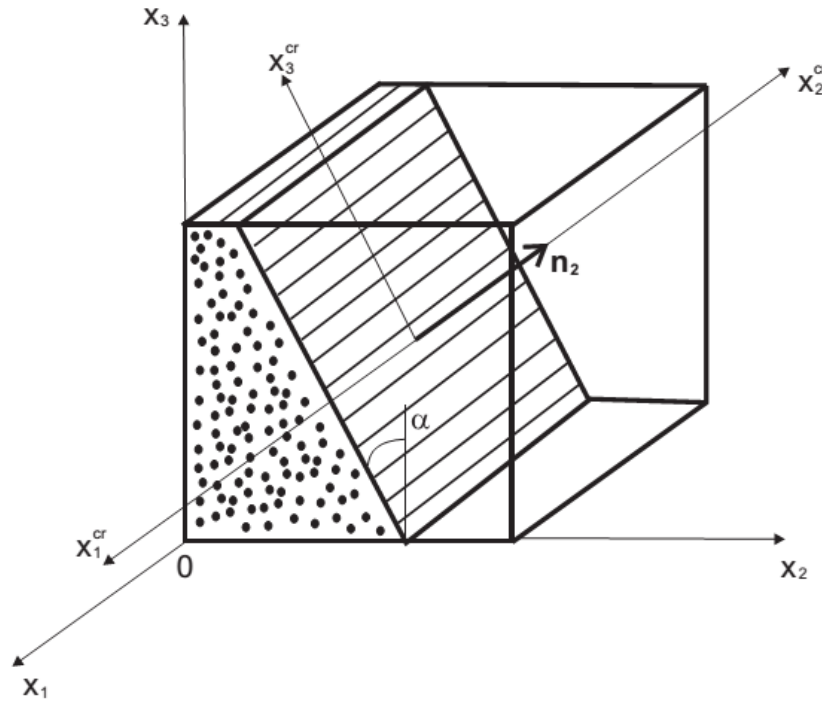


Figure 3.2: Defined crack coordinate system (Camanho et al., 2013) .

reconfiguration of meshes or very fine that increases computational costs. Different techniques can be utilized to analyze damage using fracture mechanics. J-integral method (Rice, 1968), virtual crack closure technique (VCCT) (Rybicki and M.F. Kanninen, 1977), (Rybicki, 1977), and cohesive zone models (Dugdale, 1960), (Barenblatt, 1962) are among those techniques.

Camanho et al. (Camanho et al., 2013) proposed a 3 dimensional model that is able to predict the onset of crack, orientation of fracture plane and crack opening displacement. The assumed that fracture orientation is fixed during analysis. It considers smeared crack model in transverse direction comprise and a bilinear softening in fiber direction similar to (Maimi et al., 2007a). The mechanism of transverse failure is comprised of matrix cracking and fiber-matrix decohesion. Total strain is as:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_c = \boldsymbol{\varepsilon}_e + \mathbf{R}\boldsymbol{\varepsilon}_c^{cr}\mathbf{R}^T. \quad (3.12)$$

In which, $\boldsymbol{\varepsilon}_c^{cr}$ is cracking strain tensor in crack coordinate system and $\mathbf{R}$ is a rotation matrix that relates this coordinate system to global system (Fig. 3.2).

The key variable in Equation 3.12 is crack strain and it is calculated using a system of nonlinear equation including cohesive law and interlaminar damage relations in the context of smeared crack model. The cohesive law that relates displacement jump ω^{cr}

and traction on fracture plane $\mathbf{t}^{cr}$ can be written generally as:

$$\mathbf{t}^{cr} = g(\boldsymbol{\omega}^{cr}). \quad (3.13)$$

The suitable cohesive law is used from the model developed by Turon (Turon et al., 2006) that proposed a linear softening law. Displacement jump is also driven from finite element implementation method for smeared crack model as:

$$\boldsymbol{\omega}^{cr} = [(2\boldsymbol{\epsilon}_c^{cr} : \mathbf{n}_1^{cr} \otimes \mathbf{n}_2^{cr})\mathbf{n}_1^{cr} + (\boldsymbol{\epsilon}_c^{cr} : \mathbf{n}_2^{cr} \otimes \mathbf{n}_2^{cr})\mathbf{n}_2^{cr} + (2\boldsymbol{\epsilon}_c^{cr} : \mathbf{n}_2^{cr} \otimes \mathbf{n}_3^{cr})\mathbf{n}_3^{cr}]l^*. \quad (3.14)$$

Where, l^* is characteristic length of finite element, and n_i represents normal vectors of fracture plane.

Leone (Leone, 2015) has also proposed a new method for CDM, recently. In his model instead of additive decomposition of strain which was the case in previous discussed model, the deformation gradient tensor, $\mathbf{F}$, is decomposed in 2 components which represent crack deformation and bulk material deformation, as:

$$\mathbf{F} = \mathbf{F}_B \cdot \mathbf{F}_{cr}. \quad (3.15)$$

In order to derive the formulation that accounts for mentioned components, the geometry of crack is defined in the coordinate system that is illustrated in Fig.3.3 . Based on the figure components, $\mathbf{F}_B$ reads:

$$\mathbf{F}_B = \mathbf{F} - \mathbf{R}_{cr} \mathbf{D} \mathbf{X}_R^{-1}. \quad (3.16)$$

Where $\mathbf{D}$ is a matrix that stores cohesive displacement-jump, $\boldsymbol{\delta}$, which reads:

$$\mathbf{D} = \begin{bmatrix} \delta_F \\ \delta_N \\ \delta_T \end{bmatrix} \begin{bmatrix} 0 \\ 3-q \\ q-2 \end{bmatrix}^T, \quad (3.17)$$

where q is a scalar that defines which axis of coordinate system intersects crack. Additionally, $\mathbf{R}_{cr}$ is the matrix of unit vector basis of crack coordinate system, $[\hat{e}_F, \hat{e}_N, \hat{e}_T]$ (Fig.3.3).

In order to calculate stress in the material, it is required to find distribution of stress between cohesive crack and bulk material. The cohesive stress, $\boldsymbol{\tau}$, is calculated by means

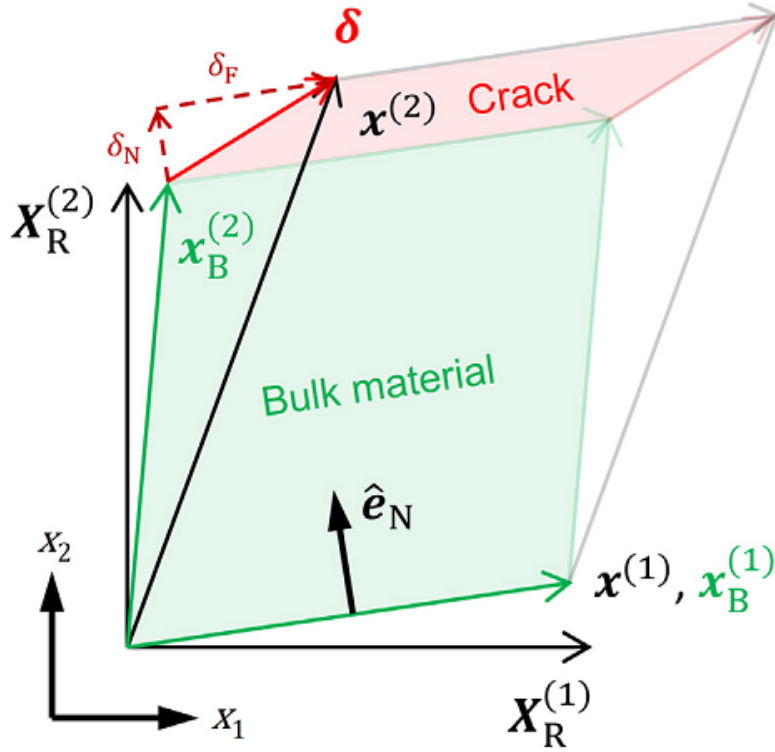


Figure 3.3: Schematic of crack and bulk material (Leone, 2015) .

of cohesive damage law as:

$$\boldsymbol{\tau} = \begin{bmatrix} k_F (1 - d_M) \delta_F \\ k_N (1 - d_M) \delta_N - k_N d_M \langle -\delta_N \rangle \\ k_T (1 - d_M) \delta_T \end{bmatrix}. \quad (3.18)$$

Where, k_F , k_N , and k_T are cohesive penalty stiffness components, d_M is the scalar cohesive damage variable, and $\langle x \rangle$ is McCauley operator. The stress formulation, in the bulk is driven by large strain formulation in continuum mechanics as:

$$\boldsymbol{\sigma}_B = \mathbf{F}_B \mathbf{S} \mathbf{F}_B^T |\mathbf{F}_B|^{-1}. \quad (3.19)$$

In sets of proposed equation, the components of δ are unknown. In order to solve the system of equations, the equilibrium that should be applied is the relation between cohesive stress and the stress vector $\mathbf{t} = \boldsymbol{\sigma}_B \cdot \hat{\mathbf{e}}_N$ acting on cohesive interface. By solving this equilibrium a nonlinear equation will be achieved that by minimizing it δ components will be revealed.

Chapter 4

Constitutive model in presence of damage and fiber rotation

The nonlinear behavior of fiber-reinforced composite materials is investigated in the presence of both damage evolution and fiber rotation. While damage mechanisms usually decrease the stiffness of the material and cause premature failure, fiber rotation can lead to a stiffening of the material, due to the reorientation of the fiber towards the loading direction, and delay failure. In this chapter, fiber rotation is modeled by an analytical solution and incorporated in Maimi damage model. The predictions of the model with and without fiber rotation are analyzed and compared with the experimental results of IM7/8552 carbon-epoxy composite. The results show the importance of including fiber rotation in the constitutive equations and the accuracy of the overall model in predicting final failure of the specimens.

4.1 Introduction

The presence of fiber rotation in fiber-reinforced polymer composites (FRP) can increase the apparent axial modulus and intra-ply stresses of the material due to the reorientation of the fibers towards the loading direction. This effect can increase the stiffness of the material and suppress the evolution of damage mechanisms that normally cause premature failure. The opposite of this damage delay also can take place when the loading is in a way that the fibers are being more diverged from loading direction. In this case the fiber rotation phenomenon speeds up the damage. Therefore, the consideration of both sources of non-linearity in the constitutive description of the material is crucial for an accurate prediction of the material response.

The effects of deformation-induced fiber orientation change and processing-induced thermal residual stresses on the non-linear behavior of IM7/5260 and AS4/PEEK composite laminates have been investigated by Sun et al. [Sun and C. Zhu \(2000\)](#). An over-stress viscoplasticity model was used to describe the rate-dependent non-linear behavior of the composite lamina, and the laminate was modeled with the classical laminated-plate theory. Mauguet et al. [Mauguet et al. \(1998\)](#) used a laminate analogy model with very low stiffness matrix to compute the finite element properties. The assumptions made are that fibers do not elongate, the laminate is made of symmetrically woven fibers and that during a finite element step the properties can be assumed as constant. Based on these assumptions, a simple way to compute the fiber orientation changes is proposed since it is observed that a box with fibers in the diagonal will deform into a box of different aspect ratio. Both of these authors analyzed fiber rotation under large deformation of composite materials but the damage evolution is not taken into account. They also utilized a very simple fiber rotation relation of $\Delta\theta = a \tan \frac{1+\varepsilon_{xx}}{1+\varepsilon_{yy}}$ that is based on several simplifications.

Herakovich et al. [Herakovich and D. Robert \(2000\)](#) incorporated fiber rotation effects in Ladeveze mesoscale damage model ([Ladeveze, 1993](#)) to study the large strain response of [45]_s laminates of IM7-K3B, graphite-fiber/polyimide-matrix composite. This study also used the same solution for fiber rotation [Mauguet et al. \(1998\)](#) in the analysis, which does not consider complications of fiber of rotation. The utilized damage model is also Ladeveze mesoscale damage model. Thijs et al. [Thijs et al. \(2007\)](#) presented a scheme that was implemented in an Lagrangian formulation, which decomposes the deformation gradient in such a way that the constitutive equations are defined as invariant terms, but there was no consideration of damage evolution like in [Sun and C. Zhu \(2000\)](#) and [Mauguet et al. \(1998\)](#). They just considered the rigid rotation in their calculations which is defined using the kinematics of the body.

In this chapter, an analytic solution is used to model fiber rotation during the loading process for orthotropic materials before and after damage initiation. Continuum mechanics relations and kinematics of the object are used in order to analyze stress and strain in presence of fiber rotation. Failure of composites is predicted by means of Larc criteria, and their damage evolution is modeled by Maimi continuous damage model. Therefore, the overall model includes several building blocks, which are conveniently coupled. Several numerical examples are analyzed and their results are compared with experimental data.

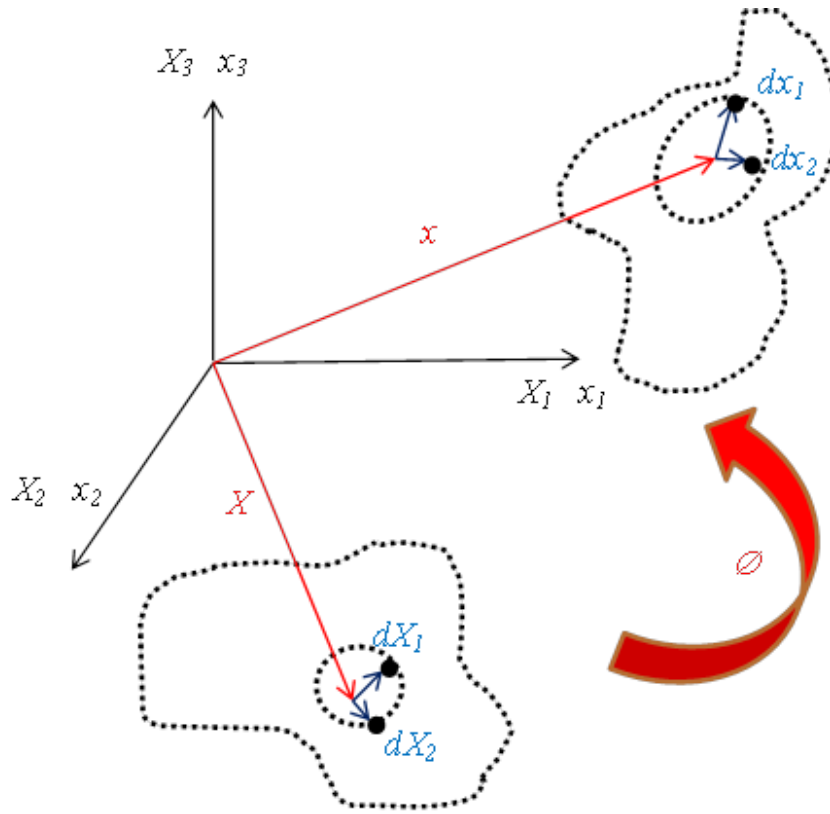


Figure 4.1: General motion of a deformable body.

4.2 Fiber rotation

A key quantity in finite deformation analysis is the deformation gradient $\mathbf{F}$, which is involved in the relation of quantities before deformation to corresponding quantities after or during deformation. The deformation gradient $\mathbf{F}$ maps the initial configuration into the current configuration. Defining the deformation gradient tensor as:

$$\mathbf{F} = \frac{\partial \phi}{\partial \mathbf{X}} = \nabla_0 \phi. \quad (4.1)$$

Where ϕ represents the mapping of the general motion of a deformable body (Figure 4.1). The deformation gradient consists of rotation and stretch parts that can be decomposed by polar decomposition to a symmetric stretch, $\mathbf{U}$, and an orthogonal rotation tensor, $\mathbf{R}_t$:

$$\mathbf{F} = \mathbf{R}_t \mathbf{U}. \quad (4.2)$$

The second order tensor $\mathbf{R}_t$ is a combination of 2 kinds of rotation. The first is rigid rotation, which does not change the shape or size of the body, and has no effect on the

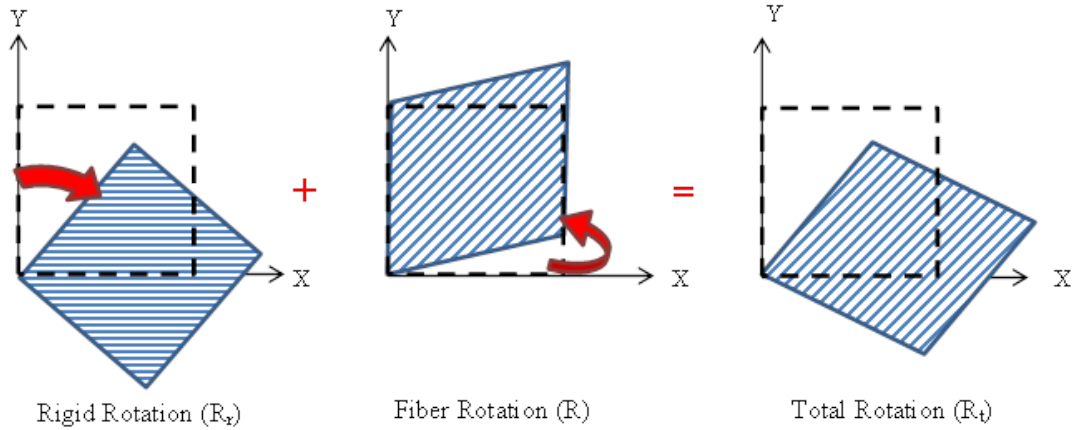


Figure 4.2: total rotation decomposition.

stresses of the object. Nevertheless, if the constitutive equation is established in the reference coordinate system, rigid rotation may spuriously cause stress or strain. Therefore, the development of constitutive equations for anisotropic materials is usually made using invariant quantities (Thije et al., 2007), (Huetink, 2006). If the constitutive equation is defined in a corotational coordinate system, where the basis system rotates with the material, rigid rotation effects will be omitted. This is the strategy that is employed to handle rigid rotation effects. Nevertheless, the change of orientation caused by material deformation modifies the axis of anisotropy within the material system and consequently the strain and stress state of the material. This rotation which can be substantial, should be calculated and included in the development of anisotropic materials constitutive laws. In order to incorporate this rotation in the model, it is necessary to decompose the total rotation $\mathbf{R}_t$ into rigid rotation, $\mathbf{R}_r$, and fiber rotation, $\mathbf{R}$. (see Figure 4.2). The relation between total rotation, rigid rotation ($\mathbf{R}_r$), and finite fiber rotation, ($\mathbf{R}$), reads:

$$\mathbf{R}_t = \mathbf{R}_r \mathbf{R}. \quad (4.3)$$

Since all of 3 rotation tensors are orthogonal, they map a vector onto another vector with the same length. As a linear transformation, an orthogonal matrix preserves the dot product of vectors, and therefore acts as an isometry of Euclidean space, such as a rotation or reflection. In other words, it is a unitary transformation. Therefore orthogonal tensors can be visualized physically as rotations. It also necessarily invertible and satisfies the property of $\mathbf{R}^{-1} = \mathbf{R}^T$.

In order to compute fiber rotation, $\mathbf{R}$, from Equation (4.3), we will firstly compute the total rotation, $\mathbf{R}_t$, using the polar decomposition and then compute the rigid rotation, $\mathbf{R}_r$, using the body kinematics. By means of the polar decomposition, the right Cauchy-Green

tensor $\mathbf{C}$, reads:

$$\mathbf{C} = \mathbf{F}^T \mathbf{F} = \mathbf{U}^T \mathbf{R}_t^T \mathbf{R}_t \mathbf{U}. \quad (4.4)$$

The right Cauchy-Green tensor is a symmetric tensor and $\mathbf{R}_t$ is an orthogonal tensor, i.e. $\mathbf{R}_t^T \mathbf{R}_t = \mathbf{R}_t \mathbf{R}_t^T = \mathbf{I}$. Taking into account that $\mathbf{U}$ is a symmetric tensor yields a unique definition of the material stretch tensor $\mathbf{U}$ in terms of $\mathbf{C}$:

$$\mathbf{C} = \mathbf{U} \mathbf{U} = \mathbf{U}^2. \quad (4.5)$$

To actually obtain $\mathbf{U}$ from (4.5), it is first necessary to evaluate the principal directions of $\mathbf{C}$, denoted here by the eigenvector triad $\{\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3\}$ and their corresponding eigenvalues λ_1^1, λ_2^2 and λ_3^3 , which results in:

$$\mathbf{C} = \sum_{\alpha=1}^3 \lambda_{\alpha}^2 \mathbf{N}_a \otimes \mathbf{N}_a. \quad (4.6)$$

Due to the symmetry of $\mathbf{C}$, the triad $\{\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3\}$ is composed of orthogonal unit vectors. In addition, due to the symmetry of $\mathbf{C}$, λ_{α}^2 is real and positive. By combining (4.5) and (4.6), the material stretch tensor $\mathbf{U}$ can be obtained as:

$$\mathbf{U} = \sum_{\alpha=1}^3 \lambda_{\alpha} \mathbf{N}_a \otimes \mathbf{N}_a. \quad (4.7)$$

In this relation, λ_{α} represents eigenvalues of the stretch tensor, $\mathbf{U}$, and because of the fact that $\mathbf{U}$ is symmetric, λ_{α} has a real and positive value. Once the stretch tensor $\mathbf{U}$ is known, the total rotation tensor can also be found from Equation (4.2) as:

$$\mathbf{R}_t = \mathbf{F} \mathbf{U}^{-1}. \quad (4.8)$$

After the calculation of the total rotation, $\mathbf{R}_t$, we will have to calculate the rigid rotation tensor, $\mathbf{R}_r$. In order to do that, the deformation gradient matrix should be decomposed in a different way. In this case, it will be splitted into an orthogonal matrix of $\mathbf{R}_r$ and a non-symmetrical matrix $\mathbf{G}$ which is constant under arbitrary rigid rotation, expressed by:

$$\mathbf{F} = \mathbf{R}_r \mathbf{G}. \quad (4.9)$$

Therefore the velocity gradient reads:

$$\mathbf{L} = \mathbf{V} \vec{\nabla} = \dot{\mathbf{F}} \mathbf{F}^{-1} = \dot{\mathbf{R}}_r \mathbf{R}_r^T + \mathbf{R}_r \dot{\mathbf{G}} \mathbf{G}^{-1} \mathbf{R}_r^T. \quad (4.10)$$

The first term of the velocity gradient, $\mathbf{L}$, is the skew symmetric spin tensor ($\mathbf{\Omega}$) and the second term represents invariant non-symmetric rate of deformation tensor ($\mathbf{D}$):

$$\mathbf{\Omega} = \dot{\mathbf{R}}_r \mathbf{R}_r^T, \quad (4.11)$$

$$\mathbf{D} = \mathbf{R}_r \dot{\mathbf{G}} \mathbf{G}^{-1} \mathbf{R}_r^T, \quad (4.12)$$

$$\mathbf{L} = \mathbf{\Omega} + \mathbf{D}. \quad (4.13)$$

By having deformation gradient, $\mathbf{F}$, the velocity gradient, $\mathbf{L}$, can be calculated. Then, the skew symmetric part of it should be driven that provides spin tensor, $\mathbf{\Omega}$.

This term will also allow to compute the rigid rotation matrix, $\mathbf{R}_r$, using finite differences that is described in Section 5. It is possible to rewrite Equation (4.11), as:

$$\dot{\mathbf{R}}_r = \mathbf{\Omega} \mathbf{R}_r. \quad (4.14)$$

Equation (4.14) will be used in order to update rigid rotation which is a path dependent parameter, during numerical modeling. With $\mathbf{R}_r$ and $\mathbf{R}_t$ at hand, the fiber rotation matrix, $\mathbf{R}$, can be obtained as:

$$\mathbf{R} = \mathbf{R}_t \mathbf{R}_r^T, \quad (4.15)$$

which will be used to model fiber rotation effects on composite materials behavior. $\mathbf{R}$ is a second order tensor that represents general rotation of the fiber in 3D.

4.3 Failure criteria

The failure criteria which is used in this work is Larc. Larc failure criteria is a set of criteria for laminated fiber-reinforced composites presented by Davila et al. (Davila et al., 2005) The criteria are based on physical models for each failure mode and take into consideration non-linear matrix shear behavior. The model for matrix compressive failure is based on the Mohr-Coulomb criterion and it predicts the fracture angle. Fiber kinking is triggered by an initial fiber misalignment angle and by the rotation of the fibers during compressive loading. The four damage activation functions, F_N , associated with failure

mechanisms in the longitudinal ($N = 1+, 1-$) and transverse ($N = 2+, 2-$) directions, are defined as:

$$\begin{aligned} F_{1+} &= \phi_{1+} - r_{1+} \leq 0; & F_{1-} &= \phi_{1-} - r_{1-} \leq 0, \\ F_{2+} &= \phi_{2+} - r_{2+} \leq 0; & F_{2-} &= \phi_{2-} - r_{2-} \leq 0. \end{aligned} \quad (4.16)$$

where the loading functions $\phi_N (N = 1+, 1-, 2+, 2-)$ depend on the strain tensor and material constants. The elastic domain thresholds $r_N (N = 1+, 1-, 2+, 2-)$ take an initial value of 1 when the material is undamaged, and they increase with damage. The loading function for matrix tension can be expressed in terms of the ply stresses and in situ strengths Y_T^{is} and S_L^{is} by:

$$\phi_{2+} = (1 - g) \frac{\sigma_{22}}{Y_T^{is}} + g \left(\frac{\sigma_{22}}{Y_T^{is}} \right)^2 + g \left(\frac{\tau_{12}^m}{S_L^{is}} \right)^2, \quad (4.17)$$

where g is toughness ratio which is the ratio of fracture toughness in the first and the second mode, defined by $g = \frac{G_I}{G_{II}}$ and superscript $(^m)$ is a coordinate system of the fiber misalignment. The subscript “*is*” indicates that for general laminates the in situ longitudinal shear strength rather than the strength of a unidirectional laminate should be used. The constraining effect of adjacent plies substantially increases the effective strength of a ply. The in-situ strengths are calculated from the closed-form equations proposed in the research of Ref. [Camanho et al. \(2006\)](#).

Matrix failure under compression loading is assumed to result from a quadratic interaction between the effective shear stresses acting on the fracture plane. The failure index for a failure mode is written as an equality stating that stress states that violate the inequality are not physically admissible. The loading function is:

$$\phi_{2-} = \left(\frac{\tau_{eff}^T}{S^T} \right)^2 + \left(\frac{\tau_{eff}^L}{S_{is}^L} \right)^2. \quad (4.18)$$

The subscript “*eff*” represents stress components acting on the fracture plane. In the presence of high transverse compression combined with moderate fiber compression, matrix damage can occur without the formation of kink bands or damage to the fibers. This matrix damage mode is calculated using the stresses in the misaligned frame which gives:

$$\phi_{2-} = \left(\frac{\tau_{eff}^{mT}}{S^T} \right)^2 + \left(\frac{\tau_{eff}^{mL}}{S_{is}^L} \right)^2. \quad (4.19)$$

The LaRC04 criterion for fiber tensile failure is a non-interacting maximum allowable stress criterion. Consequently, the LaRC04 failure index for fiber tensile failure is simply:

$$\phi_{1+} = \frac{\sigma_{11}}{X^T}, \quad (4.20)$$

where σ_{11} is stress and X^T is strength of ply in fiber direction. Fiber compression failure by the formation of a kink band is predicted using the stresses in a coordinate system (m) and the failure criterion for matrix tension or matrix compression as:

$$\phi_{1-} = \left\langle \frac{\tau_{12}^m + \eta^L \sigma_{22}^m}{S_{is}^L} \right\rangle, \quad (4.21)$$

where $\langle x \rangle$ is the McCauley operator defined as $\langle x \rangle = (x + |x|)/2$. For fiber compression with matrix tension, the transformed stresses are substituted into the matrix tensile failure criterion to yield the following criterion for fiber kinking:

$$\phi_{1-} = (1 - g) \frac{\sigma_{22}^m}{Y_{is}^T} + g \left(\frac{\sigma_{22}^m}{Y_{is}^T} \right)^2 + g \cdot \left(\frac{\tau_{12}^m}{S_{is}^L} \right)^2. \quad (4.22)$$

More details about the equations in Larc failure criteria can be found in Refs. (Davila et al., 2005) and Pinho et al. (2005). The Larc criteria is developed according to the stress and strain in fiber direction. Obviously, as a result of fiber rotation, these directions change during loading. So it is important to consider the fiber scissoring effects on them. This will be discussed on the next section.

4.4 Continuum damage model

A continuum damage model for the prediction of the onset and evolution of intra-laminar failure mechanisms and the collapse of structures manufactured in fiber-reinforced plastic laminates was proposed by Maimi et al. Maimi et al. (2007a), Maimi et al. (2007b), Camanho et al. (2007). The elastic domain is assumed to be bounded by four distinct damage activation functions based on the Larc failure criteria Davila et al. (2005) longitudinal and transverse fracture under tension and compression. After satisfying the failure, the damage will be regarded in the material behavior model. The complementary free energy density of a ply was defined as:

$$G = \frac{\sigma_{11}^2}{2(1 - d_1)E_1} + \frac{\sigma_{22}^2}{2(1 - d_2)E_2} - \frac{\nu_{12}}{E_1} \sigma_{11} \sigma_{22} + \frac{\sigma_{12}^2}{2(1 - d_6)G_2} + (\alpha_{11} \sigma_{11} + \alpha_{22} \sigma_{22}) \Delta T + (\beta_{11} \sigma_{11} + \beta_{22} \sigma_{22}) \Delta M, \quad (4.23)$$

where d_1 and d_2 are the damage variables associated with longitudinal (fiber) failure and transverse matrix cracking, respectively. The damage variable d_6 is influenced by longitudinal and transverse cracks; α_{ii} and β_{ii} , ($i = 1, 2$) are respectively the coefficients of thermal and hygroscopic expansion in the longitudinal ($i = 1$) and transverse directions ($i = 2$); ΔT and ΔM are the temperature and moisture content variations with respect to the corresponding reference values. The strain tensor is equal to the derivative of the complementary free energy density with respect to the stress tensor:

$$\boldsymbol{\varepsilon} = \frac{\partial G}{\partial \boldsymbol{\sigma}} = \mathbf{H} : \boldsymbol{\sigma} + \alpha \Delta T + \beta \Delta T. \quad (4.24)$$

The lamina compliance tensor can be represented in Voigt notation as:

$$\mathbf{H} = \frac{\partial G^2}{\partial \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & \frac{-\nu_{21}}{E_2} & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_6)G_{12}} \end{bmatrix}. \quad (4.25)$$

The longitudinal and transverse damage variables are identified using:

$$\begin{aligned} d_1 &= d_{1+} \frac{\langle \sigma_{11} \rangle}{|\sigma_{11}|} + d_{1-} \frac{\langle -\sigma_{11} \rangle}{|\sigma_{11}|}, \\ d_2 &= d_{2+} \frac{\langle \sigma_{22} \rangle}{|\sigma_{22}|} + d_{2-} \frac{\langle -\sigma_{22} \rangle}{|\sigma_{22}|}. \end{aligned} \quad (4.26)$$

Damage caused by tension loads (d_+) is tracked separately from damage caused by compression loads (d_-). In this way, the eventual closure of transverse cracks under load reversal is taken into account. Depending on the sign of the corresponding normal stress, a damage mode can be either active or passive. The damage evolution laws promote strain softening after meeting any damage activation criterion. A combination of exponential and linear damage evolution laws are used for the longitudinal damage variable in tension and for the rest only the exponential is used to simulate the cohesive response. The damage laws are defined as:

$$\begin{aligned} d_{1+} &= 1 - (1 - d_{1+}^L)(1 - d_{1+}^E), \\ d_{1-}^* &= 1 - \frac{1}{r_{1-}} \exp[A_{1-}(1 - r_{1-})], \\ d_{2+} &= 1 - \frac{1}{f_{2+}(r_{2+})} \exp[A_{2+}(1 - f_{2+}(r_{2+}))], \\ d_{2-} &= 1 - \frac{1}{r_{2-}} \exp[A_{2-}(1 - r_{2-})], \\ d_6^*(r_{2+}) &= 1 - \frac{1}{r_{2-}} \exp[A_6(1 - r_{2+})], \end{aligned} \quad (4.27)$$

where $f_{2+}(r_{2+})$ is the function which forces softening and is the same order as activation function and is defined as:

$$f_{2+}(r_{2+}) = \frac{1}{2g} [g - 1 + \sqrt{(1-g)^2 + 4gr_{2+}^2}]. \quad (4.28)$$

In the definition of d_{1+} , two variables of d_{1+}^L and d_{1+}^E represent linear damage that is before stress pull-out and exponential damage that is after it, and are defined as:

$$\begin{aligned} d_{1+}^L &= 1 + \frac{K_1}{E_1} - \left(\frac{K_1}{E_1} + 1\right) \frac{1}{r_{1+}^L}, \\ d_{1+}^E &= 1 - \frac{1}{r_{1+}^E} \exp[A_{1+}(1 - r_{1+}^E)], \end{aligned} \quad (4.29)$$

While the auxiliary damage thresholds are calculated as:

$$\begin{aligned} r_{1+}^L &= \max[1, \min(r_{1+}, r_{1+}^F)], \\ r_{1+}^E &= \max[1, (1 - d_{1+}^F) \frac{X_T}{X_{PO}} r_{1+}]. \end{aligned} \quad (4.30)$$

K_1 is the slope of the linear softening law and A_{1+} is a parameter that defines the exponential softening law. Damage evolution in the direction of fiber is illustrated in Figure 4.3. As can be seen in the picture, The softening is linear until it reaches the pull-out stress X_{PO} . The scalar g_M is the dissipation energy per volume area that is calculated by integration the rate of dissipation according to:

$$g_M = \int_0^\infty Y_M d_M dt = \int_1^\infty \frac{\partial G}{\partial d_M} \frac{\partial d_M}{\partial r_M} dr_M, M = 1+, 1-, 2+, 2-, 6. \quad (4.31)$$

4.5 Constitutive equation in presence of finite fiber rotation

In a fiber reinforced material, where the elastic response and damage evolution are affected by the orientation of the fibers, it is crucial to consider this change under large deformations. While the fiber orientation will change continuously during loading, this is seldom taken into account and the undeformed fiber configuration is normally used. Fiber scissoring changes the principal material direction which affects strain and stress, leading to influences on the elastic behavior, damage onset and damage evolution of the

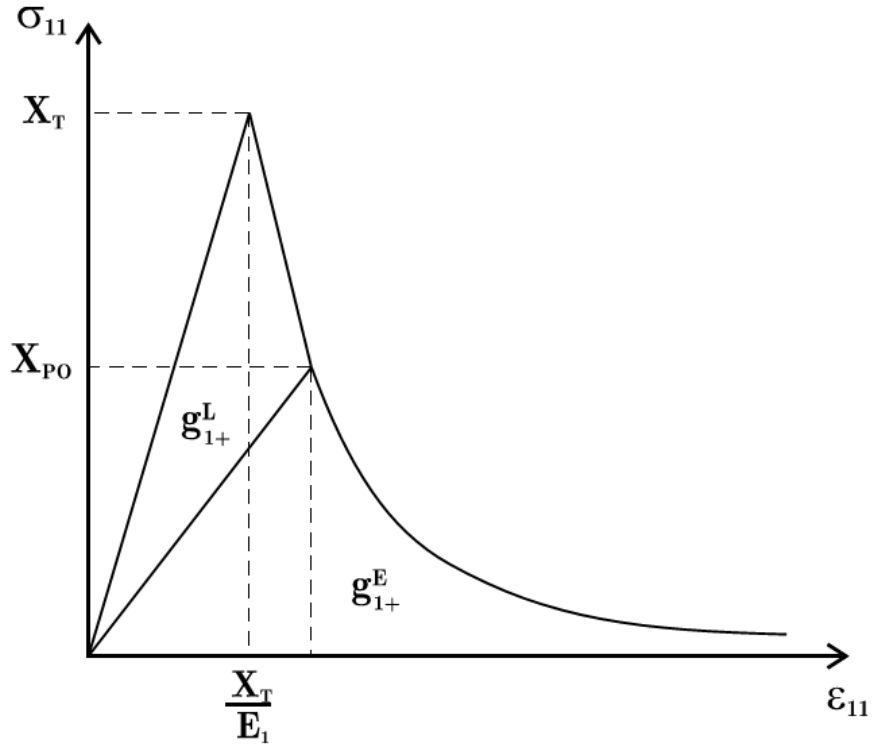


Figure 4.3: Damage evolution in fiber direction. (Maimi et al., 2007b)

composite material. By deriving fiber rotation, $\mathbf{R}$, in Section 2, it is now possible to incorporate fiber rotation in the elastic and damage model for composites. Therefore, in order to do so, the strain tensor is rotated from coordinate system which coincides initial fiber direction, $\mathbf{X}_i$, to a new coordinate system, $\mathbf{x}_i$, that is defined with respect to real direction of fibers. The failure criteria and damage evolution mechanisms are analyzed in this new coordinate system. This coordinate system which is illustrated in Figure 4.4, is obtained by applying the rotation tensor, $\mathbf{R}$, on initial coordinate system in material direction. If we consider $\mathbf{e}_i$ as unique vectors of the new coordinate system $\mathbf{x}_i$ and $\mathbf{E}_i$ as unique vectors of initial coordinate system, $\mathbf{X}_i$, the relation between them reads:

$$\mathbf{e}_i = \mathbf{R}\mathbf{E}_i, \quad (4.32)$$

$$\mathbf{e}_j = \mathbf{E}_j\mathbf{R}^T. \quad (4.33)$$

Now the strain should be rotated from the initial coordinate system to the new coordinate system. The strain can be defined using $\mathbf{E}_i$ base vectors as:

$$\boldsymbol{\epsilon}^X = \mathbf{E}_i\epsilon_{ij}\mathbf{E}_j, \quad (4.34)$$

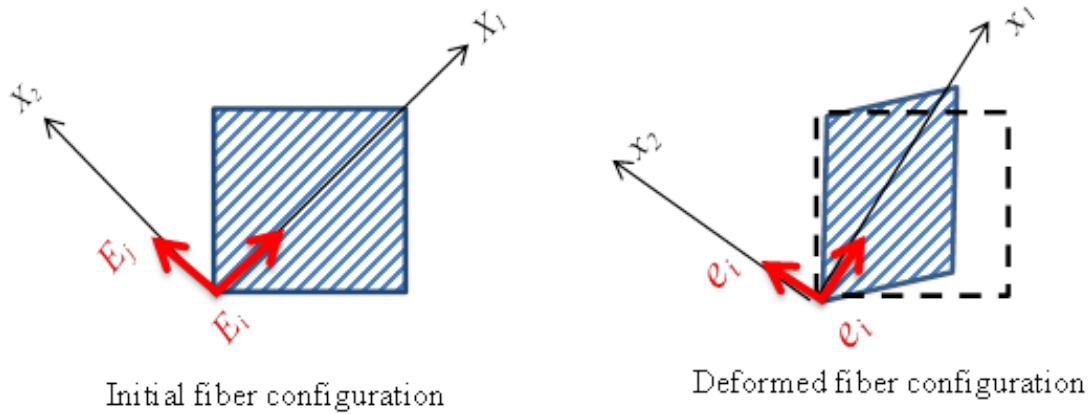


Figure 4.4: Fibers in deformed configuration.

and in the new base vectors as:

$$\boldsymbol{\epsilon}^x = \mathbf{e}_i \epsilon_{ij} \mathbf{e}_j. \quad (4.35)$$

By replacing Equations (4.32) and (4.33) in (4.35), we will have:

$$\boldsymbol{\epsilon}^x = \mathbf{R} \mathbf{E}_i \epsilon_{ij} \mathbf{E}_j \mathbf{R}^T = \mathbf{R} \boldsymbol{\epsilon}^X \mathbf{R}^T. \quad (4.36)$$

The shear strain also reads:

$$\frac{\gamma^x}{2} = \mathbf{R}^T \frac{\gamma^X}{2} \mathbf{R}. \quad (4.37)$$

It is clear that $\mathbf{R}$ is a source of nonlinearity in the previous equations. After mapping the strain tensor to the real material orientation with Equation (4.36), it is possible to compute the elastic strains for an orthotropic material using Hook's law in the real material orientation:

$$\boldsymbol{\sigma}^x = \mathbf{Q} \boldsymbol{\epsilon}^x. \quad (4.38)$$

The components of $\mathbf{Q}$ can be arranged based on engineering constants according to orthotropic materials based on Jones notation Jones (1999). After calculating the stress tensor in the new coordinate system, it is necessary to rotate back the stress to the initial fiber direction. In this way, the stress tensor will be consistent with the applied strain tensor. The stress should be defined in undeformed configuration, $\boldsymbol{\sigma}^X$, in order to obtain stress which is the result of the applied strain, $\boldsymbol{\epsilon}^X$. Stress components can also be expressed in the deformed configuration in the same way of strain components which were

defined earlier by considering $\mathbf{e}_i = \mathbf{R}\mathbf{E}_i$.

$$\boldsymbol{\sigma}^X = \mathbf{E}_i \sigma_{ij} \mathbf{E}_j, \quad (4.39)$$

$$\boldsymbol{\sigma}^x = \mathbf{e}_i \sigma_{ij}^x \mathbf{e}_j, \quad (4.40)$$

$$\boldsymbol{\sigma}^x = \mathbf{R} \mathbf{E}_i \sigma_{ij} \mathbf{E}_j \mathbf{R}^T = \mathbf{R} \boldsymbol{\sigma}^X \mathbf{R}^T. \quad (4.41)$$

Now the stress in the initial fiber direction is required. From Equation (4.41) write:

$$\boldsymbol{\sigma}^X = \mathbf{R}^T \boldsymbol{\sigma}^x \mathbf{R}. \quad (4.42)$$

Therefore, with Equations (4.36) , (4.38) and (4.42) we can have:

$$\boldsymbol{\sigma}^X = \mathbf{R}^T \boldsymbol{\sigma}^x \mathbf{R} = \mathbf{R}^T \mathbf{Q} \boldsymbol{\epsilon}^x \mathbf{R} = \mathbf{R}^T \mathbf{Q} \mathbf{R} \boldsymbol{\epsilon}^X \mathbf{R}^T \mathbf{R} = \mathbf{R}^T \mathbf{Q} \mathbf{R} \boldsymbol{\epsilon}^X. \quad (4.43)$$

Hence, the constitutive equation taking into account the real deformed material orientation for shells in elastic domain will be:

$$\begin{bmatrix} \sigma_1^X \\ \sigma_2^X \\ \sigma_3^X \\ \sigma_{12}^X \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 & R_{21} \\ 0 & R_{22} & 0 & R_{12} \\ 0 & 0 & R_{33} & 0 \\ 0 & R_{21} & 0 & R_{11} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 \\ 0 & 0 & Q_{33} & 0 \\ 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \end{bmatrix}, \quad (4.44)$$

and for solids:

$$\begin{bmatrix} \sigma_1^X \\ \sigma_2^X \\ \sigma_3^X \\ \sigma_{12}^X \\ \sigma_{13}^X \\ \sigma_{23}^X \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 & R_{21} & R_{31} & 0 \\ 0 & R_{22} & 0 & R_{12} & 0 & R_{32} \\ 0 & 0 & R_{33} & 0 & R_{31} & R_{23} \\ 0 & R_{21} & 0 & R_{11} & 0 & R_{31} \\ 0 & 0 & R_{31} & 0 & R_{11} & R_{21} \\ 0 & 0 & R_{32} & 0 & R_{12} & R_{22} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} R_{11} & 0 & 0 & R_{12} & R_{13} & 0 \\ 0 & R_{22} & 0 & R_{21} & 0 & R_{23} \\ 0 & 0 & R_{33} & 0 & R_{31} & R_{32} \\ 0 & R_{12} & 0 & R_{11} & 0 & R_{13} \\ 0 & 0 & R_{13} & 0 & R_{11} & R_{12} \\ 0 & 0 & R_{23} & 0 & R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \\ \epsilon_{13}^X \\ \epsilon_{23}^X \end{bmatrix}.$$

(4.45)

In both equations the components of the rotation and transverse rotation matrix are written in a way to be consistent with voigt notation and the related component to be multiplied. The relation between stress and strain after damage onset is also affected by fiber orientation during loading process. The Maimi constitutive damage model was developed in the undeformed fiber orientation. Nevertheless, since fiber rotation is being taken into account in this work, the complementary free energy, Equation (4.23), should be rewritten in real material orientation. Therefore, the strain ϵ^X should be rotated to deformed material orientation, ϵ^x . The stresses are also should be in real material orientation, σ^x . If the strain rotates to the deformed material orientation, the stress after applying the damaged stiffness will be in that orientation as well. Damage parameters are also affected by fiber rotation during damage evolution. They are being activated through loading functions and threshold values based on Larc criteria using stress and strain in real material orientation. The effective stress in loading functions should be calculated at time t by:

$$\tilde{\sigma}_t^x = \mathbf{Q} \mathbf{R}_t \epsilon_t^X \mathbf{R}_t^T, \quad (4.46)$$

based on stress and strain in real material orientation, ϵ_t^x and σ_t^x , at time t respectively. Where $\mathbf{Q}$ is the stiffness tensor for orthotropic material. The damage parameters also

evolve based on the stress in real material direction, as:

$$\begin{aligned} d_1^x &= d_{1+} \frac{\langle \sigma_{11}^x \rangle}{|\sigma_{11}^x|} + d_{1-} \frac{\langle -\sigma_{11}^x \rangle}{|\sigma_{11}^x|}, \\ d_2^x &= d_{2+} \frac{\langle \sigma_{22}^x \rangle}{|\sigma_{22}^x|} + d_{2-} \frac{\langle -\sigma_{22}^x \rangle}{|\sigma_{22}^x|}. \end{aligned} \quad (4.47)$$

Since the damage laws are developed based on fracture energy which is not affected by fiber rotation, then $d_i^\pm$ parameters can be calculated based on Eqs. (4.27)-(4.31). After applying the damage model, and strength reduction, the calculated stress σ^x is in deformed material orientation. Then, it will also rotate to be defined in undeformed material orientation σ^X . Stress-Strain relation for shells is redefined as:

$$\begin{aligned} \begin{bmatrix} \sigma_1^X \\ \sigma_2^X \\ \sigma_3^X \\ \sigma_{12}^X \end{bmatrix} &= \begin{bmatrix} R_{11} & 0 & 0 & R_{21} \\ 0 & R_{22} & 0 & R_{12} \\ 0 & 0 & R_{33} & 0 \\ 0 & R_{21} & 0 & R_{11} \end{bmatrix} \begin{bmatrix} \frac{Q_{11}}{1-d_1^x} & Q_{12} & 0 & 0 \\ Q_{12} & \frac{Q_{22}}{1-d_2^x} & 0 & 0 \\ 0 & 0 & \frac{Q_{33}}{1-d_3^x} & 0 \\ 0 & 0 & 0 & \frac{Q_{66}}{1-d_6^x} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \end{bmatrix}, \quad (4.48) \\ &\quad \begin{bmatrix} R_{11} & 0 & 0 & R_{12} \\ 0 & R_{22} & 0 & R_{21} \\ 0 & 0 & R_{33} & 0 \\ 0 & R_{12} & 0 & R_{11} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \end{bmatrix} \end{aligned}$$

where d_i^x represents damage parameters in real material direction. The rotation matrix is organized in a way to be compatible with Voigt notation. The constitutive equation for

solids after damage onset reads:

$$\begin{aligned}
 \begin{bmatrix} \sigma_1^X \\ \sigma_2^X \\ \sigma_3^X \\ \sigma_{12}^X \\ \sigma_{13}^X \\ \sigma_{23}^X \end{bmatrix} &= \begin{bmatrix} R_{11} & 0 & 0 & R_{21} & R_{31} & 0 \\ 0 & R_{22} & 0 & R_{12} & 0 & R_{32} \\ 0 & 0 & R_{33} & 0 & R_{31} & R_{23} \\ 0 & R_{21} & 0 & R_{11} & 0 & R_{31} \\ 0 & 0 & R_{31} & 0 & R_{11} & R_{21} \\ 0 & 0 & R_{32} & 0 & R_{12} & R_{22} \end{bmatrix} \begin{bmatrix} \frac{Q_{11}}{1-d_1^x} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & \frac{Q_{22}}{1-d_2^x} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & \frac{Q_{33}}{1-d_3^x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Q_{44}}{1-d_4^x} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{Q_{55}}{1-d_5^x} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Q_{66}}{1-d_6^x} \end{bmatrix} \\
 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} R_{11} & 0 & 0 & R_{12} & R_{13} & 0 \\ 0 & R_{22} & 0 & R_{21} & 0 & R_{23} \\ 0 & 0 & R_{33} & 0 & R_{31} & R_{32} \\ 0 & R_{12} & 0 & R_{11} & 0 & R_{13} \\ 0 & 0 & R_{13} & 0 & R_{11} & R_{12} \\ 0 & 0 & R_{23} & 0 & R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_1^X \\ \varepsilon_2^X \\ \varepsilon_3^X \\ \varepsilon_{12}^X \\ \varepsilon_{13}^X \\ \varepsilon_{23}^X \end{bmatrix} \\
 & \quad (4.49)
 \end{aligned}$$

Fiber rotation also has effect on internal and dissipation energy due to damage. The effect is through strain and stress which both should be in real material orientation, $\boldsymbol{\varepsilon}^x$ and $\boldsymbol{\sigma}^x$, respectively.

4.6 Finite element implementation

The numerical implementation has been done in ABAQUS non-linear finite element code (ABAQUS, 2007). In most models, final structural failure is identified at the load level at which static equilibrium can no longer be reached. At failure or close to, static equilibrium ceases to be achieved because fracture is in itself a dynamic process, with sudden dissipation of energy and conversion into kinetic energy of a great part of the accumulated strain energy. Therefore, structural collapse due to fracture is often possible to be numerically simulated through dynamic analyses. So the analyses have been done in explicit conditions and also the computations are executed by double precision. Explicit analysis has the advantage that it is not required to calculate inverse of stiffness matrix which is a big asset regarding nonlinearities of this problem. Based on it, a VUMAT subroutine is developed in order to analyze finite rotation under elastic and damaged conditions. It is important to choose the right subroutine in order to have access to required information for calculation and analysis of fiber rotation and possibility to update affected variable by it. The deformation gradient is available at the beginning and at the end of time step, Δt ,

in the subroutine so the total rotation, $\mathbf{R}_t$, can be calculated using the procedure described in section 2. The numerical model is developed for both of shells and solid elements. In shells the non-symmetrical tensor of deformation gradient has 5 elements which are $F_{11}, F_{22}, F_{33}, F_{12}, F_{21}$, while in 3D it has 9 elements of $F_{11}, F_{22}, F_{33}, F_{12}, F_{23}, F_{31}, F_{21}, F_{32}, F_{13}$ which is in different ordering than shells. In order to calculate velocity gradient, time derivative of deformation gradient is required. It can be calculated by numerical formulation that is driven using taylor series, as:

$$\dot{\mathbf{F}} = \frac{\mathbf{F}^{n+1} - \mathbf{F}^n}{\Delta t}. \quad (4.50)$$

Following the procedure provided in section 2, symmetric spin tensor, $\dot{\mathbf{R}}_r \mathbf{R}_r^T$, can be obtained. Now rigid rotation matrix ($\mathbf{R}_r$) should be found by having, $\dot{\mathbf{R}}_r \mathbf{R}_r^T$, using numerical techniques as:

$$\dot{\mathbf{R}}_r = \frac{\mathbf{R}_r^{n+1} - \mathbf{R}_r^n}{\Delta t}. \quad (4.51)$$

By replacing $\dot{\mathbf{R}}_r$ using Eq. (4.51), in Eq. (4.14) we will have the relation to update the rigid rotation tensor:

$$\mathbf{R}_r^{n+1} = \boldsymbol{\Omega} \mathbf{R}_r^n \Delta t + \mathbf{R}_r^n. \quad (4.52)$$

Material orientation of each ply is defined in the input file based on undeformed conditions and the fiber rotation is being controlled by subroutine VUMAT. The subroutine is called in every integration point and collects deformation gradients tensor which is passed by ABAQUS and calculates rotation tensor which will be affected on total strain. By having the total strain in deformed fiber orientation, the calculation of stress in real material orientation will be carried out. Then, it will analyze whether the damage has started and if so the damage parameters, internal energy and dissipation energy will be calculated. The calculated stress also rotates to be defined in the same coordination that strain had defined in order to have the constitutive equation in global coordination system.

Material points that satisfy Larc failure criterion will be deleted from the model and pass zero stress and strain after deletion. The 20th state variable controls material deletion process in solids mod and 18th one controls it in 2D. The algorithm is implemented as:

- (i) Collect deformation gradient matrix, $\mathbf{F}$.
- (ii) Calculate, $\dot{\mathbf{F}}$.
- (iii) Calculate the stretch tensor, $\mathbf{U}$.
- (iv) Calculate, $\mathbf{U}^{-1}$.
- (v) Calculate total rotation, $\mathbf{R}_t$.
- (vi) Calculate, $\mathbf{R}_t^T$.
- (vii) Calculate, $\mathbf{L}$.
- (viii) Calculate skew symmetric part of velocity gradient, $\dot{\mathbf{R}}, \mathbf{R}_t^T$.
- (ix) Calculate rigid rotation tensor, $\mathbf{R}_r$.
- (x) Calculate fiber rotation tensor, $\mathbf{R}$.
- (xi) Rotate strain to deformed fiber configuration, $\boldsymbol{\varepsilon}^x = \mathbf{R}\boldsymbol{\varepsilon}^X\mathbf{R}^T$.
- (xii) Check if the element is failed or not based on Larc failure criteria.
- (xiii) Calculate damage parameters and effective stress, $\tilde{\boldsymbol{\sigma}}_i^x$, if element has failed d_i based on effective stress.
 - (a) Calculate stress by elastic model (Shell/Solid) if damage has not started.
 - (b) Calculate stress by damage model (Shell/Solid) if damage has started.
- (xiv) Calculate internal energy at the end of the increment.
- (xv) Calculate dissipated energy at the end of the increment.
- (xvi) Rotate stress to undeformed material orientation, $\boldsymbol{\sigma}^X = \mathbf{R}^T\boldsymbol{\sigma}^x\mathbf{R}$.
- (xvii) Delete the element if $d_i = 0.999$; $i = 1-6$.

Some numerical models are developed in order to analyze fiber rotation effects on continuum damage evolution. The examples are conducted under condition of incorporating the rotation and without it. The material which is chosen for the models is IM7-8552 carbon/epoxy. Elastic properties of IM7-8552 are presented in Table 4.1 and its unidirectional strengths are also shown in Table 4.2. In numerical modeling the material is

considered transversely isotropic. The elastic properties and strengths were measured by [Camanho et al. \(2007\)](#) using ASTM test standards [ASTM D 3039 \(a\)](#), [ASTM D 3410-87 \(b\)](#), and [ASTM D 3518 \(c\)](#). Five specimens were used for each test performed.

Table 4.1: IM7-8552 elastic properties.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
291.714	9.08	5.29	0.32

Table 4.2: IM7-8552 unidirectional strengths (MPa).

X_T	X_C	Y_T	Y_C	S_L
2323.5	1200.1	62.3	199.8	92.3

The fracture toughness for the different damage models are required for both the determination of the in-situ strengths and of the adjusting parameters A_M , $M = \pm 1, \pm 2, \pm 6$, that needs to be calculated using the corresponding component of the fracture toughness G_M . The fracture energy per unit area for longitudinal tensile fracture, G_{1+} , is the sum of the fracture energy per unit area associated with linear softening, G_E^{1+} , with the one associated with exponential softening, G_{1+}^L . These energies can be measured using the Compact Tension (CT) and Compact Compression (CC) tests in cross-ply laminates proposed by Pinho et al. [S.T. Pinho \(2006\)](#) The values of fracture toughness used in the model are measured by [Camanho et al. \(2007\)](#) which is shown in Table 4.3.

Table 4.3: Fracture toughness($\frac{N}{mm}$).

G_1^+	G_1^-	G_2^+	G_2^-	G_6
81.5	106.3	0.28	1.31	0.79

The damage activation functions are established in terms of the in-situ strengths. The resulting in-situ strengths by proposed method in [Camanho et al. \(2006\)](#) are shown in Table 4.4 which are based on the mode I fracture toughness, G_{2+} , and on the mode II fracture toughness, G_6 .

The coefficients of thermal expansion of the material used are $\alpha_{11} = -5.5 \times 10^{-6}/^\circ C$ and $\alpha_{22} = 2.58 \times 10^{-5}/^\circ C$, and the temperature change is $\Delta T = -152^\circ C$ [Camanho et al. \(2007\)](#). The effects of moisture absorption are neglected in the model. The interaction

Table 4.4: In situ strengths (MPa).

	Embedded ply	Outer ply	Thick ply
Y_T	160.2	101.4	113.3
S_L	130.2	106.9	106.9

between laminae has cohesive and friction behavior. A single strength-based damage parameter is applied in a softening law to track the damage state of the interface. Delamination damage is initiated by an interactive criterion based on the out-of-plane components of stress tensor and its evolution is governed by energy law and linear softening. The softening law is applied in the three-parameter Benzeggagh-Kenane [Kenane and Benzeggagh \(1997\)](#) mode interaction criterion to predict mixed-mode delamination propagation. Stiffness coefficients for cohesive behavior are uncoupled and has presented in Table 4.5. Damage initiation stresses are also 62.3, 92.3, 92.3 MPa that the first one is for normal stress and the others for the shear. In addition, parameters for damage evolution include fracture energy that is shown in Table 4.6 and Benzeggagh-Kenane exponent for mixed mode that is 1.45. Friction coefficient is also defined 0.5.

Table 4.5: Stiffness coefficients for cohesive behavior($\frac{N}{mm^3}$).

K_{nn}	K_{ss}	K_{tt}
10^6	10^6	10^6

Table 4.6: Fracture energy for delamination($\frac{KJ}{m^2}$).

Normal	First shear	Second Shear
0.28	0.28	0.79

Regarding the fact that analysis are in explicit, the quasi static condition should be meet in order to the results become comparable with experimental data. Therefore, the load rate increase technique has been utilized in order to keep inertia forces insignificant. The loading period is very short and mass scaling technique is applied. 20 state variables are allocated in order to store analysis solution dependent data. 8-node linear brick, reduced integration, hexagonal elements is utilized in order to mesh the model (ABAQUS C3D8R). Their family is 3D stress and their geometrical order is linear.

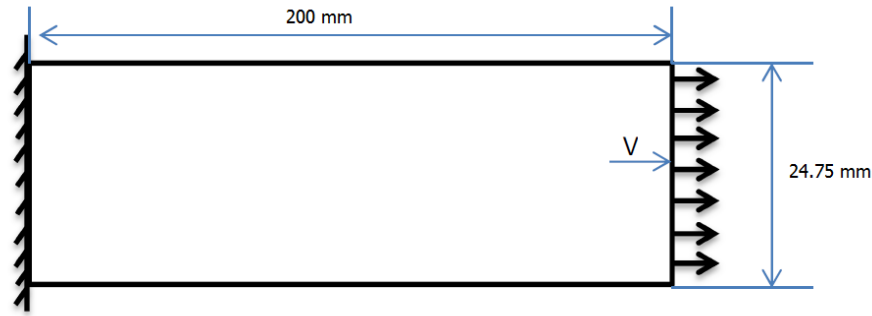


Figure 4.5: Geometry and boundary conditions of the shear test specimen.

The first example which is developed is the shear test according to ASTM standard D-3518 [ASTM D 3518 \(c\)](#). The stacking sequence of the specimen is $[\{\pm 45\}]_{8s}$ and it is under tension which is applied by velocity. Its geometry and boundary condition is depicted in [4.5](#).

A 3D solid plate with a central hole is also studied under tensile and compressive velocities. The load is applied through boundary condition of velocity which is $1000(\frac{mm}{s})$ in a very short step time of 0.0019 s. The geometry and applied boundary conditions are illustrated in [4.7](#).

The number of nodes that are used to mesh the specimen is 602400 while the number of elements are 296000. Total number of variables in the model is 1807200. The mesh structure is shown in [Figure 4.6](#).

The plate is divided to 3 partitions of 12 mm including the left and right supports with the laminate in the center which is consisted of 12 plies of IM7-8552 carbon fiber reinforced epoxy with different orientations by nominal ply thickness of 0.125 mm. The stacking sequence of laminate is $[\{90/0/+45/-45\}]_{3s}$. The left and right parts are supports for the central specimen part. The test specimen is user defined material by 29 constants while the supports are orthotropic elastic. Therefore the damage and fiber orientation is assumed to be only on the test specimen. The size of elements in support parts is relatively coarse and in the test specimen is finer. [Figure \(4.8\)](#) depicts the mesh structure in the model. Number of elements in the model is 42216 and number of nodes is 78246. Total number of variables in the model is 234738.

4.7 Results and discussions

In this section the result of finite element analysis which is based on proposed analytical model will be discussed and the data will be compared with the numerical model

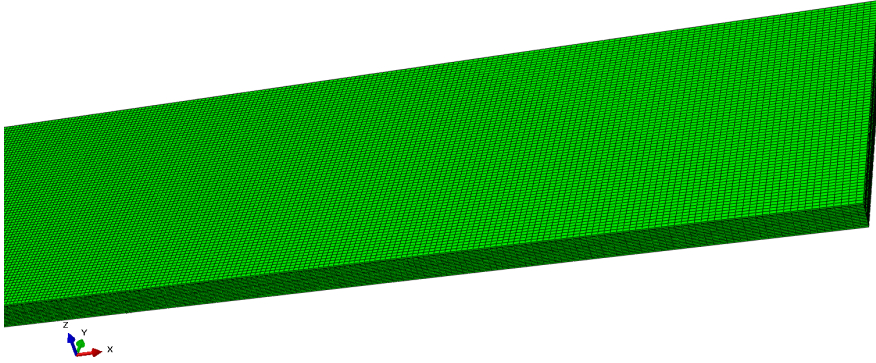


Figure 4.6: The mesh structure of the shear test specimen.

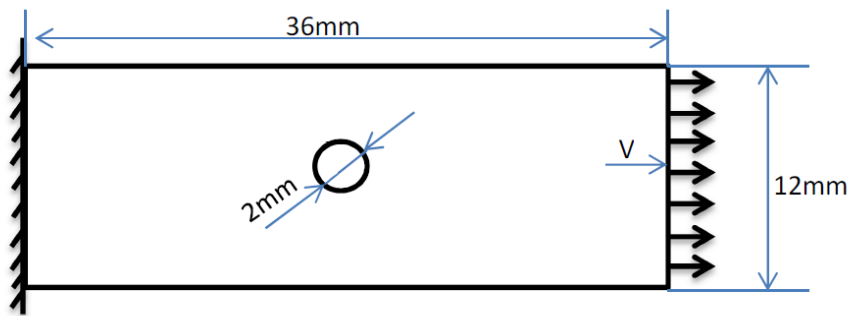


Figure 4.7: Geometry and boundary conditions of the laminate.

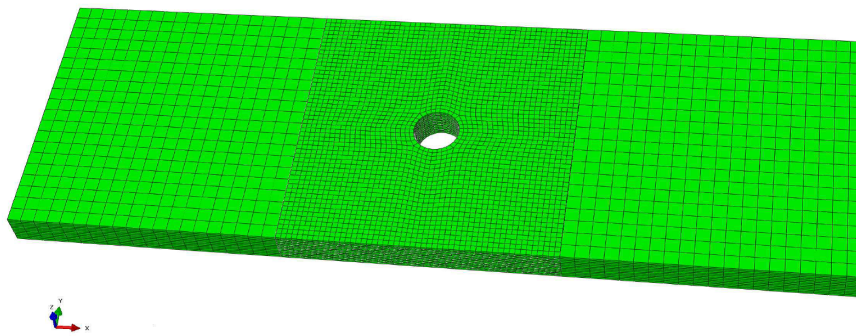


Figure 4.8: The mesh structure of the notched plate.

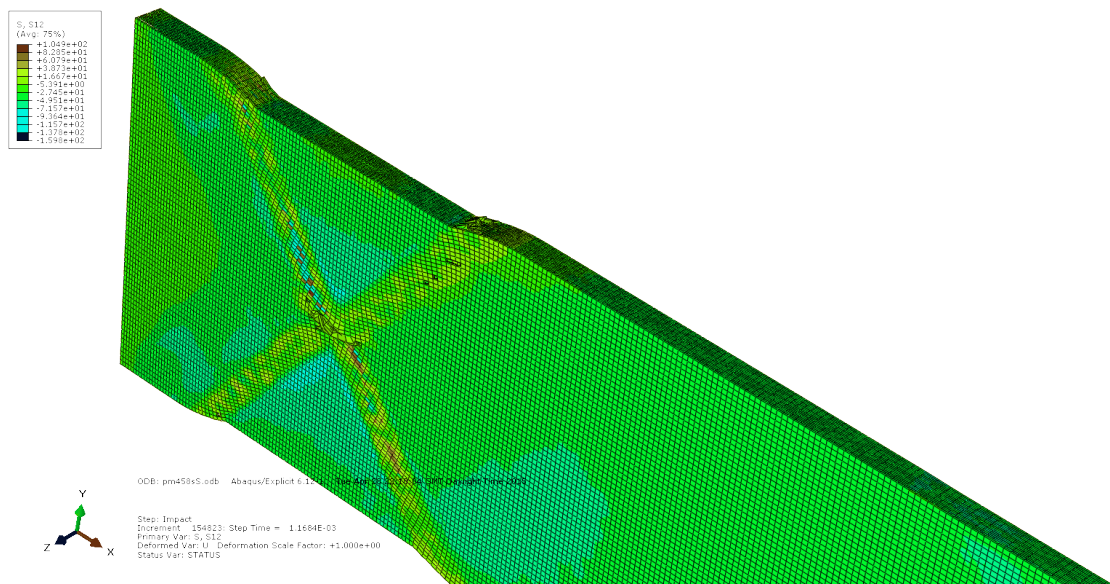


Figure 4.9: The fracture of the shear test specimen/fiber scissoring.

without considering fiber rotation and experimental data. For the first example which is based on ASTM standard D-3518 [ASTM D 3518 \(c\)](#), the results will be compared to the shear test results. In the numerical models, the damage initiates next to the fixed side of the plate by revealing the necking. The failed model is shown in Figure 4.9. It shows that simulation can grab fiber scissoring phenomenon. The force-displacement diagram are obtained for the both models including (W/Rot) and excluding (Wo/Rot) fiber rotation that is shown in Figure 4.10. Where RF1 is the reaction sum of reaction forces (N) and U is the end displacement (mm). According to the standard, the in-plane shear strength is calculated based on force-displacement diagram by expression of $S_L = \frac{P_{Max}}{2A}$ where $P_{max}(N)$ is the maximum load before failure, and $A(m^2)$ is the average area of the transverse section of the specimen. Considering this relation, by having the maximum load from the force-displacement diagram which is equal for both numerical models, the in-plane shear strength will be 85.96 MPa. In the experiment the shear strength is calculated 92.34 MPa from the average of 5 specimens with correlation of 0.67%. So the numerical models predict the strength by error of 6%. In can be understood from the diagrams that the damage is suppressed in presence of fiber rotation and the model has the capability of showing this damage delay.

On the other hand, ultimate shear strain, $\bar{\gamma}_{12}$, obtained by the equation $\epsilon_{xx} - \epsilon_{yy}$ where ϵ_{xx} corresponds to the longitudinal normal strain at failure and ϵ_{yy} is the transverse normal strain at failure. In the numerical models without considering fiber rotation, the longitudinal normal strain at failure is 2 % and the transverse normal strain is -5.4 %, so the ultimate shear strain is 7.43 %. In the other numerical model which considers fiber rotation, the

longitudinal normal strain at failure is 1.71 % and the transverse normal strain is -6.32 %, Consequently, the ultimate shear strain is 8.03 %. In the 5 shear tests, the ultimate shear strain varies between 6.79 to 8.31 which have the average of 7.41 and correlation of 9.81 %. The results are summarized in Table 4.7.

Table 4.7: Comparison between experimental and numerical results for shear test specimen.

	S_L (MPa)	Error (%)
Experiment	92.34	
Without rotation	85.96	-6
With rotation	85.96	-6

According to discussed results, there is a good agreement between experiments and the numerical models. In addition, the model which includes fiber rotation, show more stiffness after maximum load which models the increase in stiffness well, as a result of fiber rotation in diverse effect to damage. In the second example that is notched plate, by increasing the displacement, damage initiates at the vicinity of the hole, the failure criteria is met and then it propagates until final structural failure. The final failure load is calculated in the numerical models by summing reaction forces at the fixed edge nodes on the collapse moment for models with rotation and without it. Having failure load, P^u , the nominal failure stress can be calculated by $\sigma_N = P^u / wt$. Finally, nominal failure stresses are compared with the experimental data, presented by [Camanho et al. \(2007\)](#). The comparison of the data is shown in Table 4.8. The table consists of the results for open-hole tension tests (OHT) and open-hole compression tests (OHC). The experimental data is average of 5 specimens test. This table shows that the proposed analytical and numerical solution provides a good prediction of stress failure and improves the continuum damage model. The relation between applied end displacement and load for incorporated fiber rotation model (W/Rot) and for the model without considering rotation (Wo/Rot) is also illustrated in Figure 4.11 in tension (OHT) and compression loading (OHC) of notched plates.

Although Failure load in the models seems to be close, there are different active damage mechanisms in the same elements of 2 different model. Besides, The model including fiber rotation, can simulate the total failure while in the other model the force does not reach zero point. The damage parameters also show different trends in the damaged elements though most of them have higher amount in the model with fiber rotation, but there are also some elements with higher values of damage variable in the model without rotation. Therefore, the effect of the rotation on damage is complicated and can not be

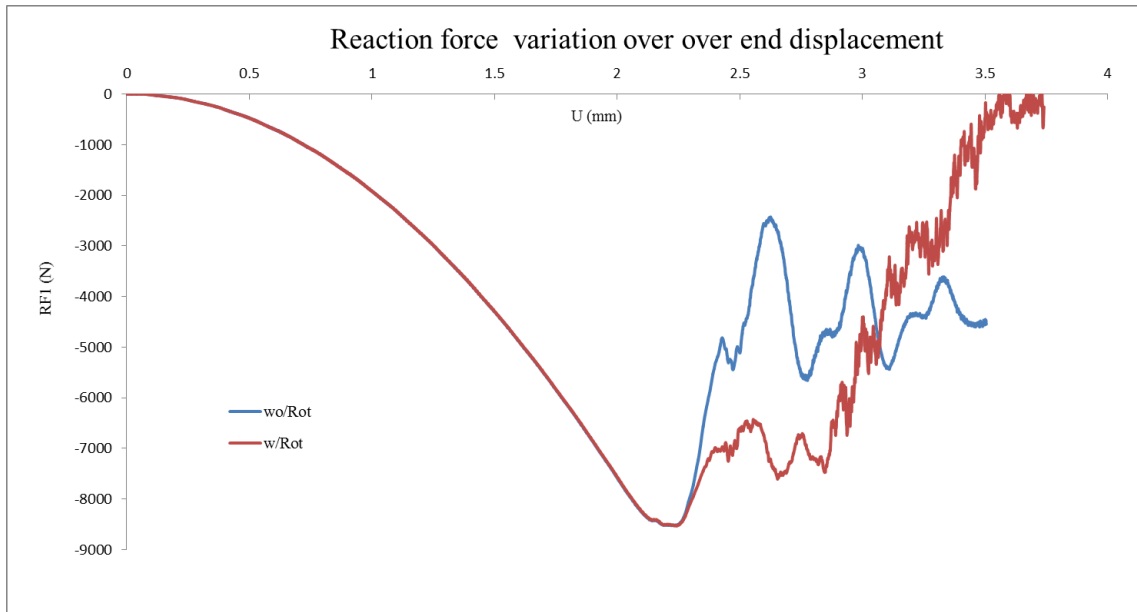


Figure 4.10: Load- End displacement chart.

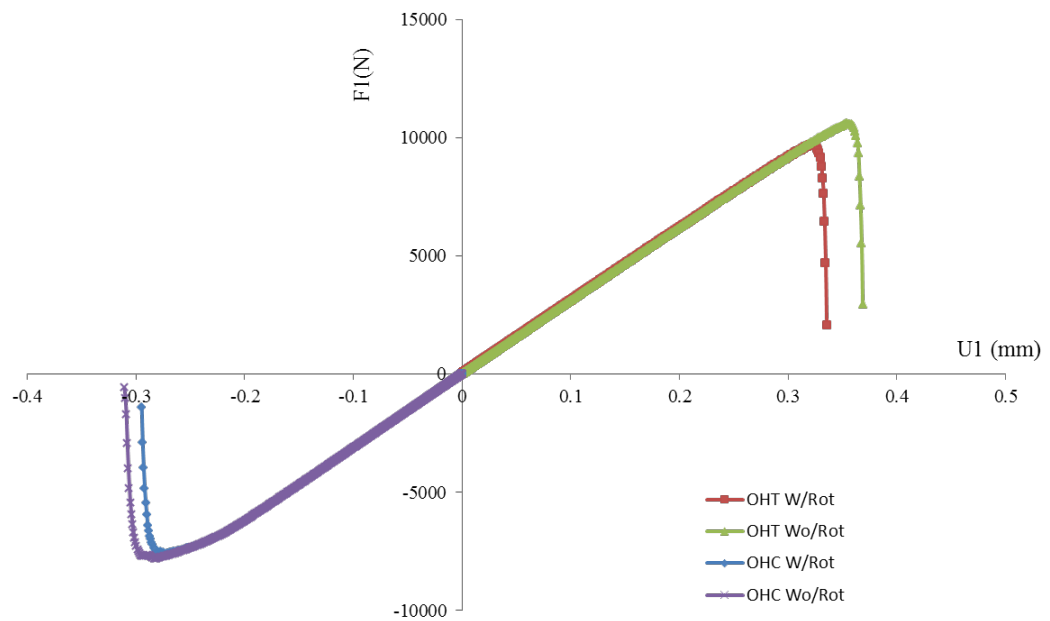


Figure 4.11: Load- End displacement chart for notched plates.

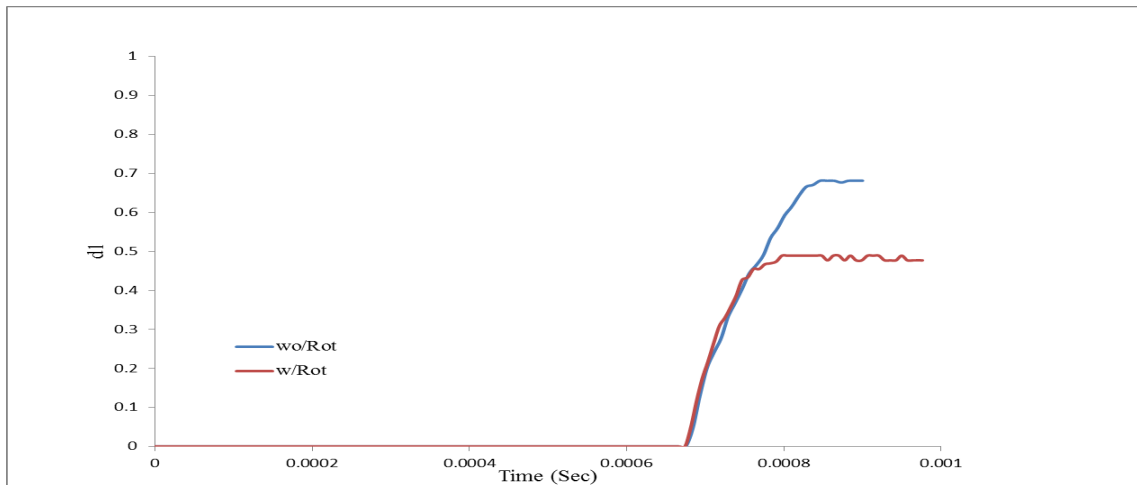
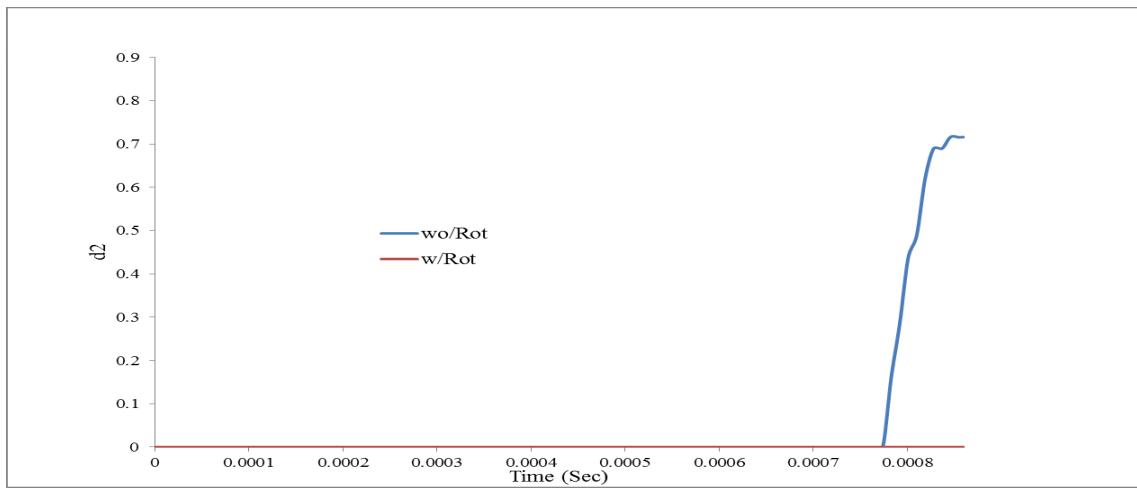
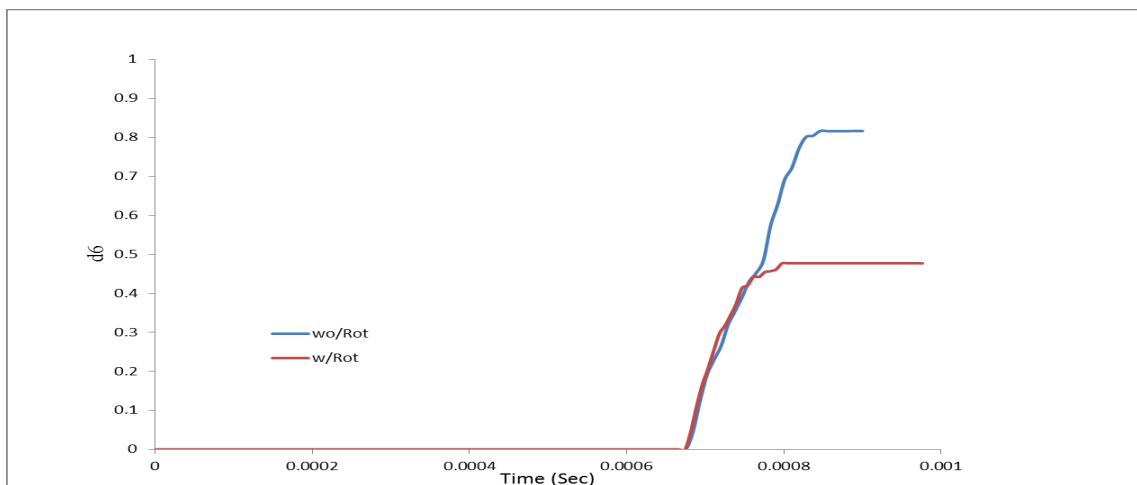
Table 4.8: Comparison between experimental and numerical failure stresses in notched plates.

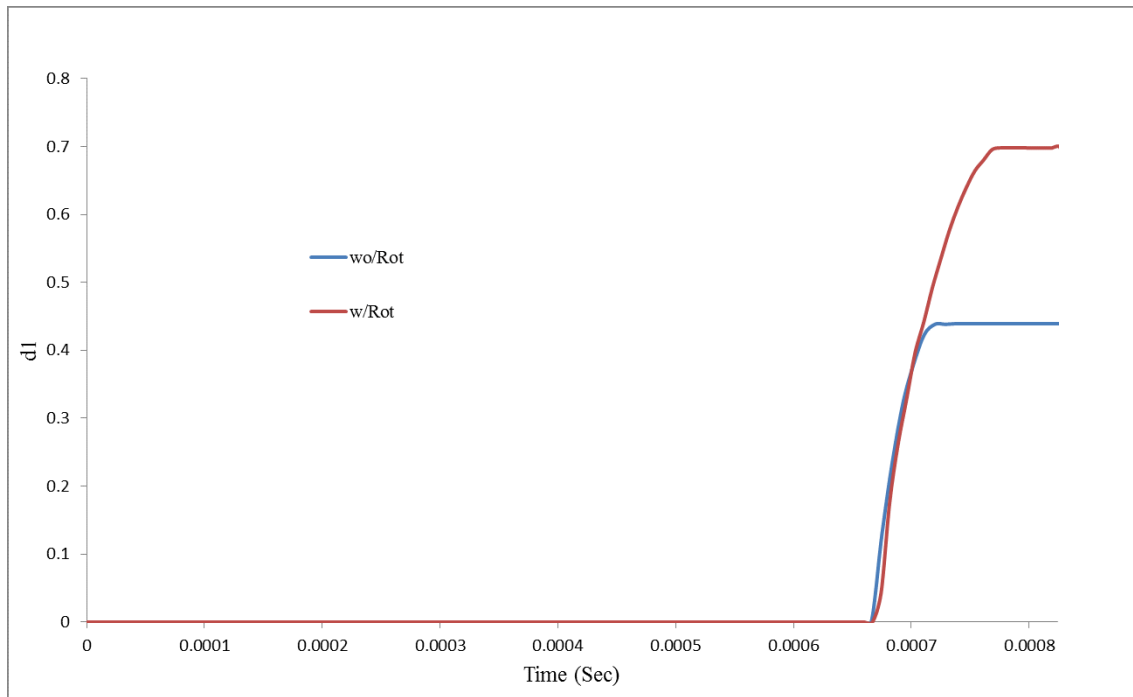
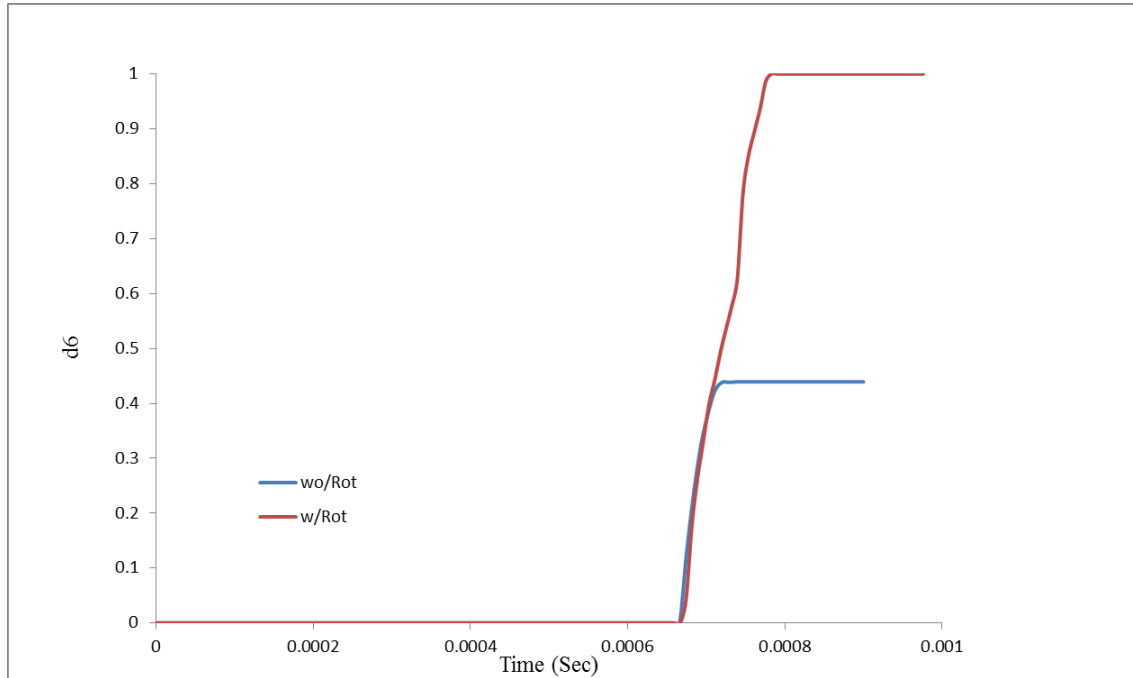
	OHT		OHC	
	Failure stress (MPa)	Error (%)	Failure stress (MPa)	Error (%)
Experiment	555.7		383	
Without rotation	588.9	5.9	433.5	13.2
With rotation	539.1	-2.9	422.8	10.4

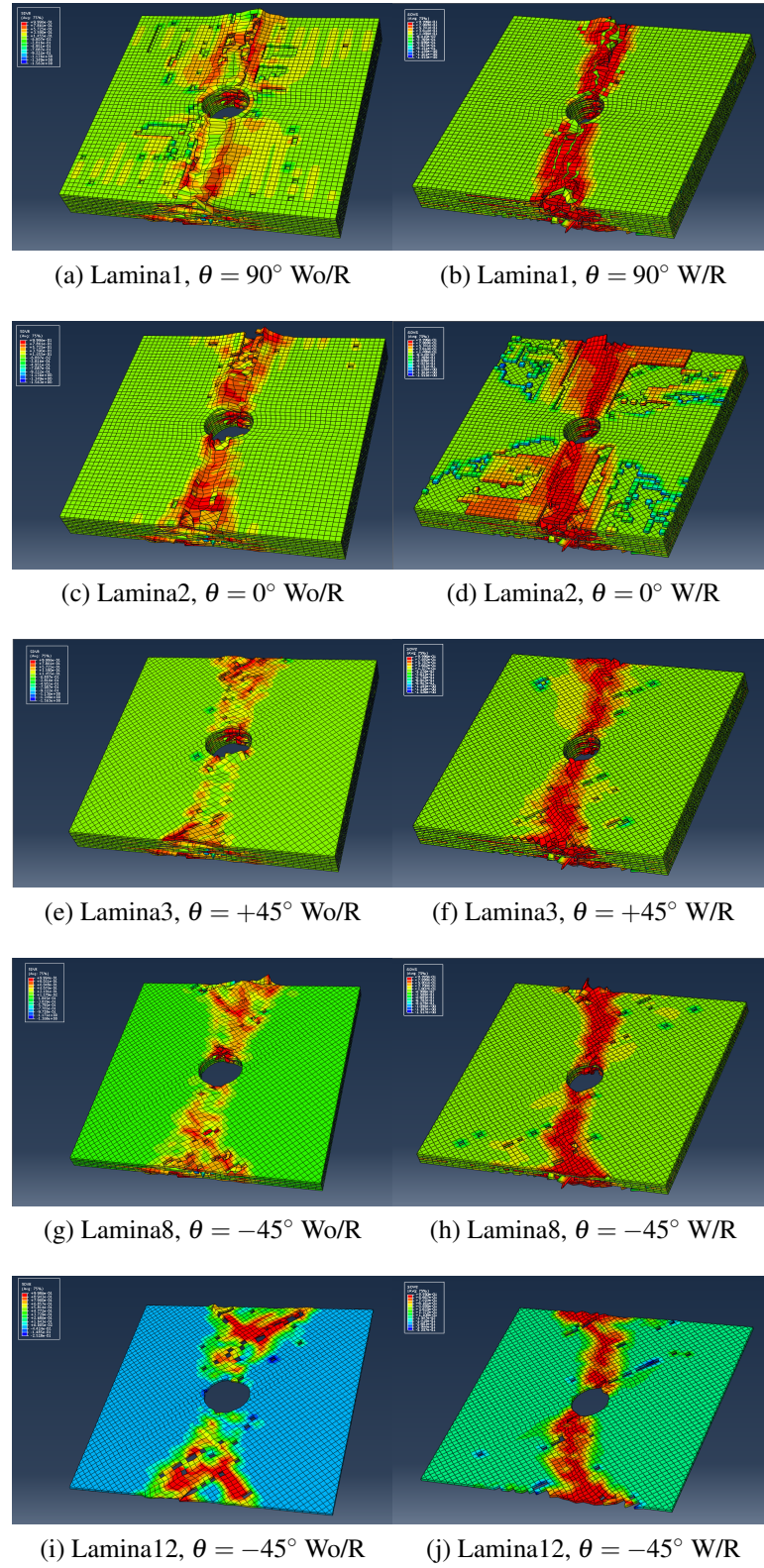
generalized. Figures 4.12 and 4.13 show different damage evolution trends in different layers comparing the model with rotation (W/Rot) and the one without it (Wo/Rot). The fiber rotation incorporation also provides a better illustration of failure propagation. Figure 4.14 depicts different plies after structural failure. Inclusion of fiber rotation results in significant changes in the nonlinear response and the intra-ply stresses. These curves also approves the results of Herakovich and D. Robert (2000) qualitatively. In the following chapter, more justification will be provided that highlights importance of the fiber rotation presence in the model. The calculated fiber rotations in different specimens will be compared by experimental observations. It will include the study on the damage mechanisms in the numerical models with considering the fiber rotation and without it, and also comparing the results with the experiments. Furthermore, the significance of considering fiber scissoring in composites with thermoplastic matrix will be discussed.

4.8 Conclusion

The Maimi damage model is elaborated by inclusion of fiber rotation effects by an analytical method using continuum mechanics approach. Based on the developed analytical model, the numerical model is developed. The numerical model considers the nonlinearities of the fiber rotation in the specimen. The damage evolution in IM7/8552 carbon/epoxy is analyzed for a shear test using the developed damaged model and the results are compared to experimental data and the Maimi model. It has been shown that the developed model has a good agreement with experiments and also it is necessary to consider fiber rotation in order to increase the accuracy of the results. The developed model shows that the stiffness increases as a result of fiber rotation. It also captures the fracture line better and changes the damage onset and evolution in different areas of the specimen.

(a) d_1 (b) d_2 (c) d_6 Figure 4.12: Evolution of damage parameters on element on Lamina1, $\theta = 90^\circ$ near hole

(a) d_I (b) d_6 Figure 4.13: Evolution of damage parameters on element on Lamina6, $\theta = 0^\circ$ near hole

Figure 4.14: Comparison of damage parameter of d_1 after fracture

Chapter 5

Numerical models and validation

In this chapter the effects of fiber rotation inclusion in the continuum damage model are studied. Firstly, transverse compression case for unidirectional laminate composites is considered and the ability of the model to predict the final failure strength and fiber rotation angle for different initial fiber angles is analyzed using experimental data, and compared to the damage model excluding fiber rotation. Some damage analysis also carried out by means of X-ray images of damaged specimens. In addition, the ability of the models to be used for failure analysis of the thermoplastic matrix composites is evaluated. This type of composites reveals high level of fiber rotation, that makes them a good candidate to be investigated under this context. The study suggests the importance of fiber rotation inclusion in the damage analysis and highlights its advantages over not regarding it. Besides, it reveals the capability of the rotation considered model to be used for thermoplastic-based composites while the original damage model is not useful in this case.

5.1 Introduction

There is a contrary relation between damage of composite materials and the fiber rotation as a result of large deformation. In order to predict failure strength, it is necessary to consider the effect of fiber rotation known also as fiber scissoring. In the previous chapter, an analytic solution was provided to include change of fiber direction in continuum damage mode. The numerical implementation of the developed constitutive model was also described. In this chapter, the necessity of this inclusion will be elaborated. Two different subroutine is used to analyze the same sort of examples and the results are compared to the experimental observations to see which subroutine shows more robustness

and accuracy. The subroutine that models damage in presence of fiber rotation is abbreviated in this chapter as WROT (With ROTation) and the other routine that does not regard fiber scissoring is called WOROT (WithOut ROTation). The models are evaluated in the cases of different materials, different configuration and loading conditions. One section discusses the transverse compression in carbon/epoxy IM7-8552 for unidirectional laminates. Different initial fiber directions are regarded and analyzed. Their results for final strength, stress-strain curves and fiber rotation angle are compared to the available experimental data. Another section focuses on thermoplastic matrix composite of AS4-PEEK while it is under tension loading. It is known that thermoplastic based composites show more ductile behavior and reveal high strains. As a result, it is expected that they show high fiber rotation. Therefore, in this case the effect of the developed constitutive model would be even more important.

5.2 Constitutive model

The details of constitutive model for continuous damage model of plastic matrix composites in presence of fiber rotation are described in previous chapter. There are different failure criteria for composite materials including interactive methods, non-interactive methods and failure-based methods. Larc criteria is among those failure criteria that are based on failure modes which introduced by [Davila et al. \(2005\)](#). The summary of the Larc criteria is presented in Table 5.1 for fiber failure and in Table 5.2 for matrix failure mode. In the Table, *is* index refers to in-situ strength of composite, *g* is toughness ratio and η is friction parameter. More details about the parameters in the tables and calculation procedures for them can be found in ([Davila et al., 2005](#)), ([Pinho et al., 2005](#)).

Table 5.1: Larc04 failure criteria in fiber failure mode.

Loading condition	FI_F
$\sigma_{11} \geq 0$	$\frac{\sigma_{11}}{X^T}$
$\sigma_{11} < 0$ and $\sigma_{2m2m} < 0$	$\left\langle \frac{\tau_{12}^m + \eta^L \sigma_{22}^m}{S_{is}^L} \right\rangle$
$\sigma_{11} < 0$ and $\sigma_{2m2m} \geq 0$	$(1 - g) \frac{\sigma_{22}^m}{Y_{is}^T} + g \left(\frac{\sigma_{22}^m}{Y_{is}^T} \right)^2 + g \left(\frac{\tau_{12}^m}{S_{is}^L} \right)^2$

The Maimi model ([Maimi et al., 2007a](#)), ([Maimi et al., 2007b](#)), ([Camanho et al., 2007](#)) is a continuous damage model for intralaminar failure based on Larc failure criteria and it defines damage variables of d_1 , d_2 , and d_6 which are associated with fiber, matrix and in-plane shear failure respectively. Every damage variable is triggered by meeting the

Table 5.2: Larc04 failure criteria in matrix failure mode.

Loading condition	FI_M
$\sigma_{22} \geq 0$	$(1 - g) \frac{\sigma_{22}}{Y_{is}^T} + g \left(\frac{\sigma_{22}}{Y_{is}^T} \right)^2 + g \left(\frac{\tau_{12}^m}{S_{is}^L} \right)^2$
$\sigma_{22} < 0$ and $\sigma_{11} < -Y^C$	$\left(\frac{\tau_{eff}^{mT}}{S^T} \right)^2 + \left(\frac{\tau_{eff}^{mL}}{S_{is}^L} \right)^2$
$\sigma_{22} < 0$ and $\sigma_{11} > -Y^C$	$\left(\frac{\tau_{eff}^T}{S^T} \right)^2 + \left(\frac{\tau_{eff}^L}{S_{is}^L} \right)^2$

related failure criteria and in the course of loading, the damage is accumulated until the final structural collapse. Crack closure effects under load reversal are taken into account by using damage variables that are established as a function of the sign of the components of the stress tensor.

The damage model is extended to take fiber rotation into account which is caused by the nonlinear geometrical change. The details of derivation of the rotation matrix R and its effect on damage evolution before and after damage onset is described in previous chapter . The relation between stress and strain by incorporating the reorientation of the fibers is defined as:

$$\begin{bmatrix} \sigma_1^X \\ \sigma_2^X \\ \sigma_3^X \\ \sigma_{12}^X \\ \sigma_{13}^X \\ \sigma_{23}^X \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 & R_{21} & R_{31} & 0 \\ 0 & R_{22} & 0 & R_{12} & 0 & R_{32} \\ 0 & 0 & R_{33} & 0 & R_{31} & R_{23} \\ 0 & R_{21} & 0 & R_{11} & 0 & R_{31} \\ 0 & 0 & R_{31} & 0 & R_{11} & R_{21} \\ 0 & 0 & R_{32} & 0 & R_{12} & R_{22} \end{bmatrix} \begin{bmatrix} \frac{Q_{11}}{1-d_1^X} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & \frac{Q_{22}}{1-d_2^X} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & \frac{Q_{33}}{1-d_3^X} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Q_{44}}{1-d_4^X} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{Q_{55}}{1-d_5^X} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Q_{66}}{1-d_6^X} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} R_{11} & 0 & 0 & R_{12} & R_{13} & 0 \\ 0 & R_{22} & 0 & R_{21} & 0 & R_{23} \\ 0 & 0 & R_{33} & 0 & R_{31} & R_{32} \\ 0 & R_{12} & 0 & R_{11} & 0 & R_{13} \\ 0 & 0 & R_{13} & 0 & R_{11} & R_{12} \\ 0 & 0 & R_{23} & 0 & R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \\ \epsilon_{13}^X \\ \epsilon_{23}^X \end{bmatrix}, \quad (5.1)$$

where R_{ij} represents components of fiber rotation tensor, and Q_{ij} denotes components

of orthotropic material stiffness. d_i^x also shows damage variables in real material orientation which are being activated after meeting related failure criteria and then evolve based on damage laws, discussed in previous chapter. Some numerical issues arise in modeling damage variables which are handled according to procedures introduced in ref. [Maimi et al. \(2007b\)](#). Rotation Matrix, models an arbitrary 3D rotation that can be thought of a sequence of three rotations, one about each principle axis. These three rotations can be described in terms of rotation angles about each axis, α , β , γ , which are called Euler angles and denote rotation about, the first, the second, and third principal axis, respectively. By multiplying the standard rotation matrix about each axis, the general rotation matrix reads:

$$\mathbf{R} = \mathbf{R}_{X3}\mathbf{R}_{X2}\mathbf{R}_{X1} = \begin{bmatrix} \cos \beta \cos \gamma & \cos \gamma \sin \alpha \sin \beta - \cos \alpha \sin \gamma & \cos \alpha \cos \gamma \sin \beta + \sin \alpha \sin \gamma \\ \cos \beta \sin \gamma & \cos \alpha \cos \gamma + \sin \alpha \sin \gamma \sin \beta & -\cos \gamma \sin \alpha + \cos \alpha \sin \gamma \sin \beta \\ -\sin \beta & \cos \beta \sin \alpha & \cos \alpha \cos \beta \end{bmatrix}. \quad (5.2)$$

Equation(5.2) can be used in order to approximate the fiber rotation angle. If we assume the rotation happens in plane, angle β will be approximately zero and the component R_{11} expresses $\cos \gamma$. Then the fiber rotation angles can be obtained by:

$$\gamma = \arccos(R_{11}). \quad (5.3)$$

According to developed analytic model, a VUMAT subroutine is implemented in ABAQUS explicit that is explained in previous chapter. Another routine just implements progressive damage without considering fiber scissoring that will be used as control data to analyze the effects of fiber rotation.

5.3 Transverse compression and in-plane shear in unidirectional composites

In this section, transverse compression in thermoset matrix composites for unidirectional laminates is analyzed. In compression, fiber rotation will be different in comparison to tension that was shown in chapter 4. This difference will be investigated in this chapter. In order to do so, different initial fiber orientation is modeled. The results are going to be compared with experiments carried out by [Körber et al. \(2010\)](#), therefore the material which is chosen is carbon-epoxy IM7-8552. Each model is consisted of 32 plies with the

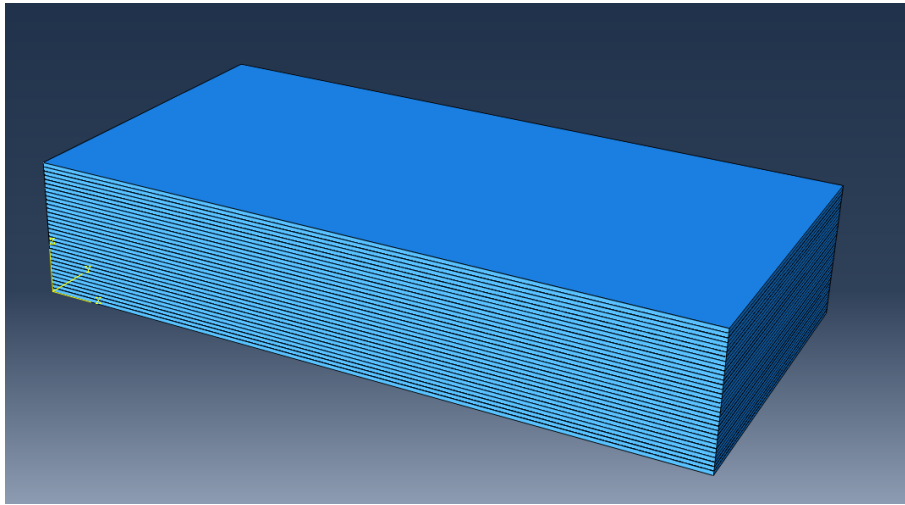


Figure 5.1: Geometry of a 32-ply plate with unidirectional fiber angle.

thickness of 0.125 mm. The geometry of the model is $20 \times 10 \times 4 \text{ mm}^3$ that is shown in 5.1. An in-plane aspect ratio of 2 was chosen to obtain a larger homogeneous strain field to not be affected by edge effects at the specimen end-surfaces.

Table 5.3: IM7-8552 elastic properties.

E_1 (Gpa)	E_2 (Gpa)	G_{12} (Gpa)	ν_{12}
291.714	9.08	5.29	0.32

Table 5.4: IM7-8552 unidirectional strengths (MPa).

X_T	X_C	Y_T	Y_C	S_L
2323.5	1200.1	62.3	199.8	92.3

6 specimens are analyzed by unidirectional fiber orientations of 15° , 30° , 45° , 60° , 75° , and 90° . The boundary condition is chosen in a way to satisfy the experimental setup in Körber et al. (2010) which is according to Fig. 5.2 . The plate is under compressive loading in X direction while it can slip freely in Y direction. Therefore in-plane shear can also occur in this model under described loading. The applied boundary condition is compressive velocity in X direction. The schematic of boundary condition is shown in Fig. 5.3 . Regarding the short length of the specimens, compression will not cause buckling.

The number of nodes in finite element model is 553344 and the number of elements is 270400. Total number of the variables in the model is also 1660032. The mesh shape

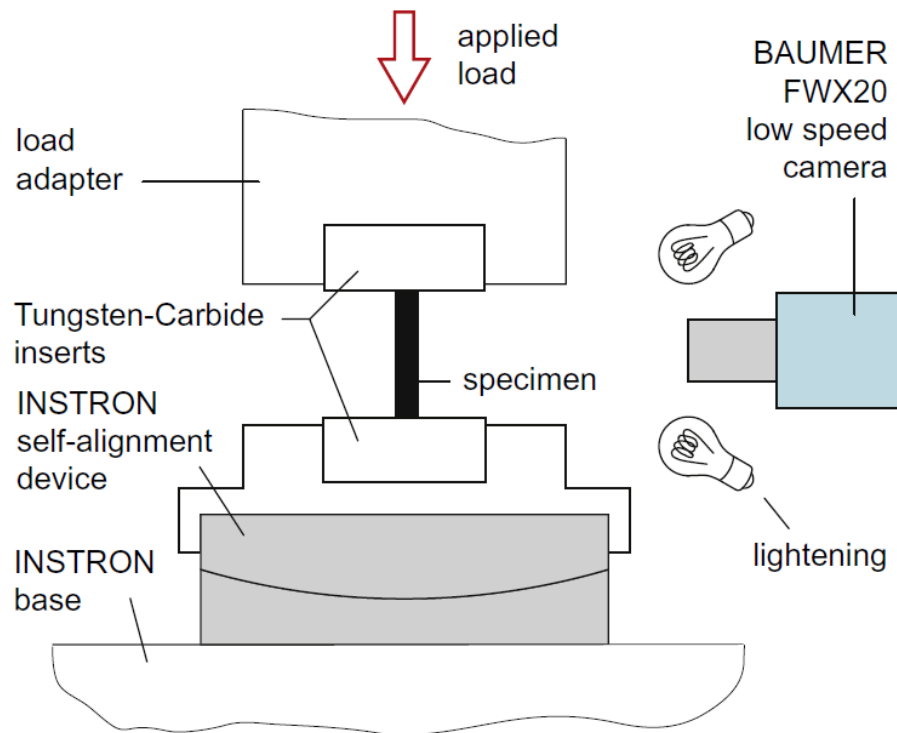


Figure 5.2: Experimental setup of in-plane shear and compressive loading. [Körber et al. \(2010\)](#)

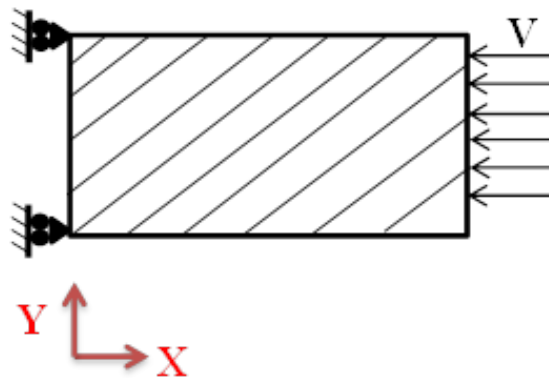


Figure 5.3: Schematic of boundary condition.

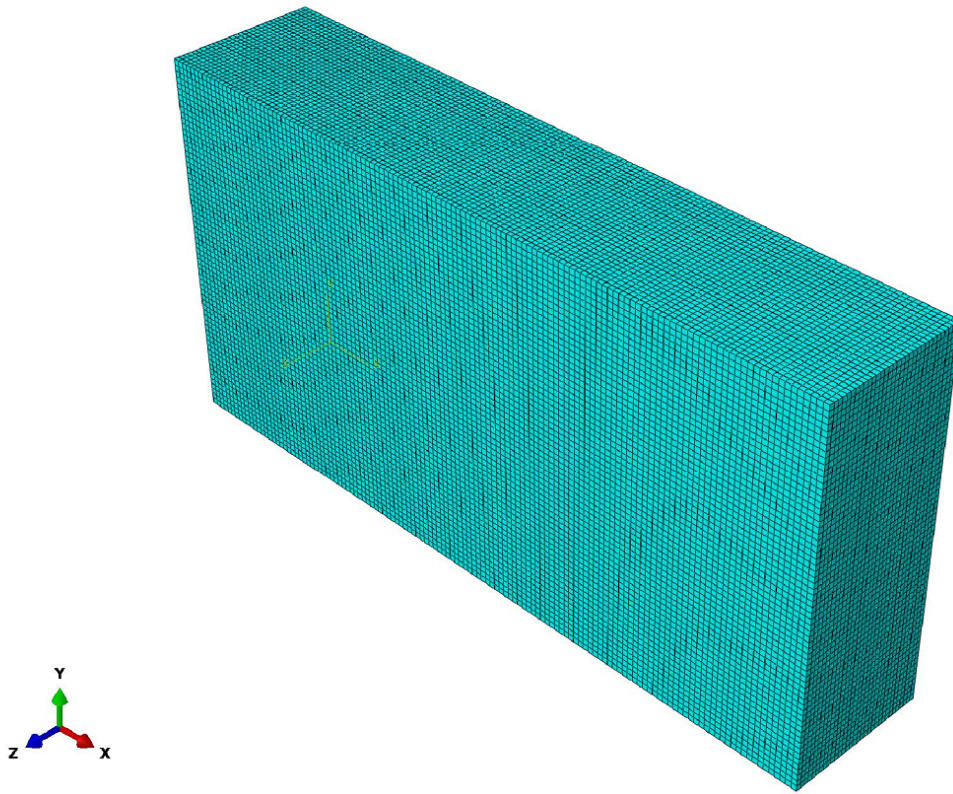


Figure 5.4: Mesh structure of the model.

is chosen as hex and structured, while the size of its diagonal is approximately 0.153 mm. This size is chosen according to:

$$l^* \leq \frac{2E_M G_M}{X_M^2} \quad M = 1\pm, 2\pm, 6. \quad (5.4)$$

proposed by (Maimi et al., 2007b). Where E_M is elasticity modulus, G_M is fracture energy and X_M is strength. M also indicates the relevant damage law. The Equation (5.4) must be met in order to avoid local snap-back in the stress-strain relation. It is driven based on the fact that the elastic energy of an element must be lower or equal to fracture energy at the onset of localization. This equation guaranties the dissipation of the energy in softening. The mesh structure of the model is illustrated in Figure 5.4.

The analysis has been done in explicit solution and in order to maintain numerical stability mass scaling technique is utilized in every $10^{-6}sec$. Mass scaling reduces the CPU cost and increases the performance of the computation by increasing the step time in each cycle through scaling up the specific elements which would be useful in explicit analysis due to very small time step. Mass scaling is a tried and proven method for

reducing run times in quasi-static analyses where the velocity is low and the kinetic energy is very small relative to the internal energy. Totally 12 different numerical examples executed that their results are going to be discussed. The models are executed on the cluster by 32 CPUs and it took about 3 h for each model to be done.

5.3.1 Results and discussion

The results of the laminate damage analysis show that the importance of the fiber rotation is different for different fiber orientation in unidirectional laminates and the ability of the models to predict failure varies in different cases. When the fiber rotation is negligible, according to the experimental observations, the both modeling approaches reveal the same results. The model can predict the amount of fiber rotation angle quite well.

In the laminate which is consisted of 15° plies fails at 13600 N in the model with fiber rotation while the pure damage model reveals failure at 15500 N. Figure 5.5 depicts the reaction forces in X direction versus displacement for both modeling approaches. Because of the compression, the fibers rotate in the way that decrease the stiffness, So the WROT model shows softer behavior in comparison to WOROT model. Figure 5.6 also shows the stress-strain behavior for the both models, WOROT and WROT, and the comparison of them to experimental results. The end of the curve is where the specimen fails. Since this model is for compression and in compression fibers diverge from loading direction, the consideration of fiber rotation should increase damage as can be seen in the diagrams that it is the case and the model which regards fiber rotation, has faster softening. In addition, in the area of prior to structural collapse where fiber rotation starts, WROT model has a better prediction of stress.

According to the experiments, this specimen has 1.9° rotation at failure which has good agreement by the prediction of the model. Figure 5.7 compares the rotation angle that predicted by WROT model, $\Delta\theta$, with experimental results. The rotation angle is calculated by Equation (5.3).

Conducted experiments show that 2 modes of the failure can occur in 15° direction laminate including fiber kinking and in plane shear. This is observed in both works of (Körber et al., 2010) and (Kawai and Saito, 2009) (Figure 5.8). While (Körber et al., 2010) reported no difference in the strength of the specimens with different failure modes, however (Kawai and Saito, 2009) found that the strength of the T800-2500 specimen failing in the kink-band mode was higher than the strength of the specimen failing in in-plane shear dominated failure mode.

In numerical analysis, both damage modes of fiber kinking and in-plane shear are triggered and the in-plane shear is progressed more (see Figure 5.9 and 5.10). The fracture

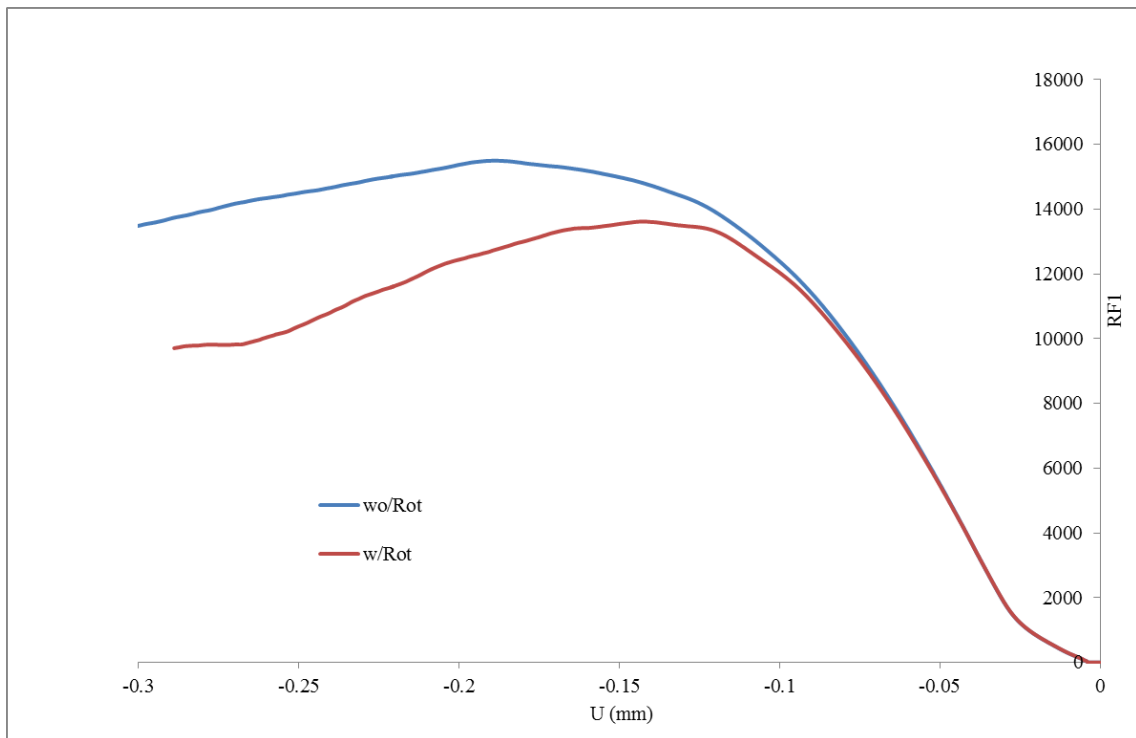


Figure 5.5: Reaction force in X direction for 15 Deg-ply laminate.

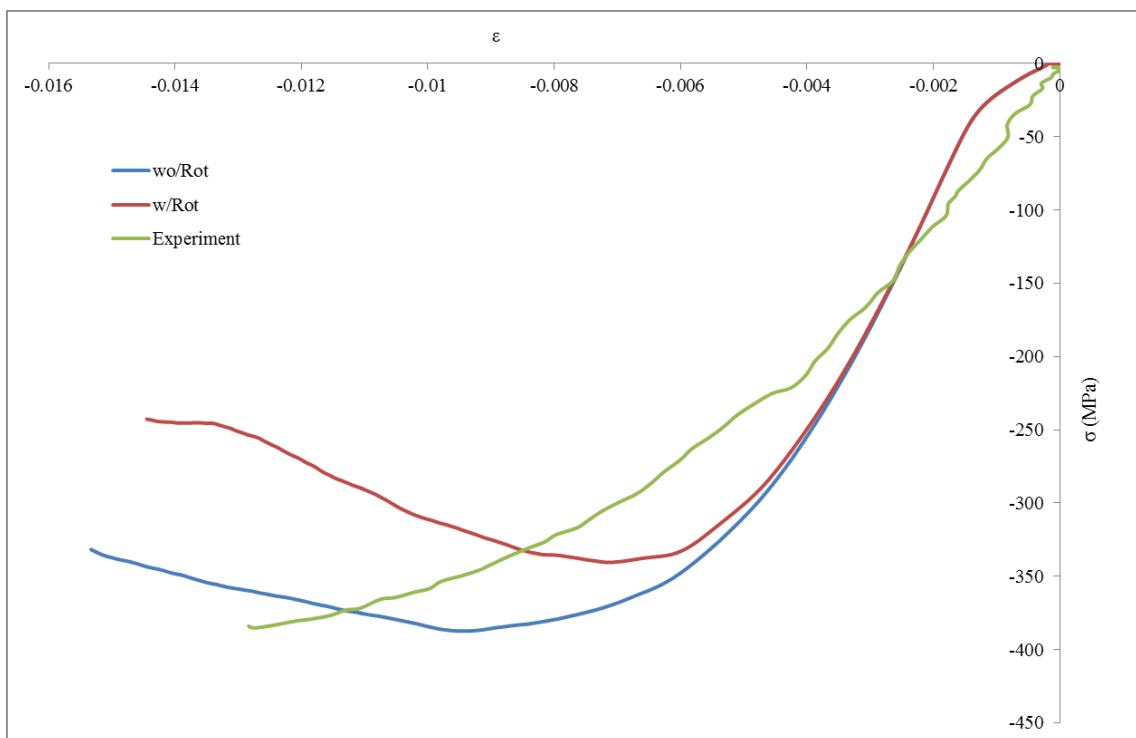


Figure 5.6: Stress-Strain diagram for 15 Deg-ply laminate.

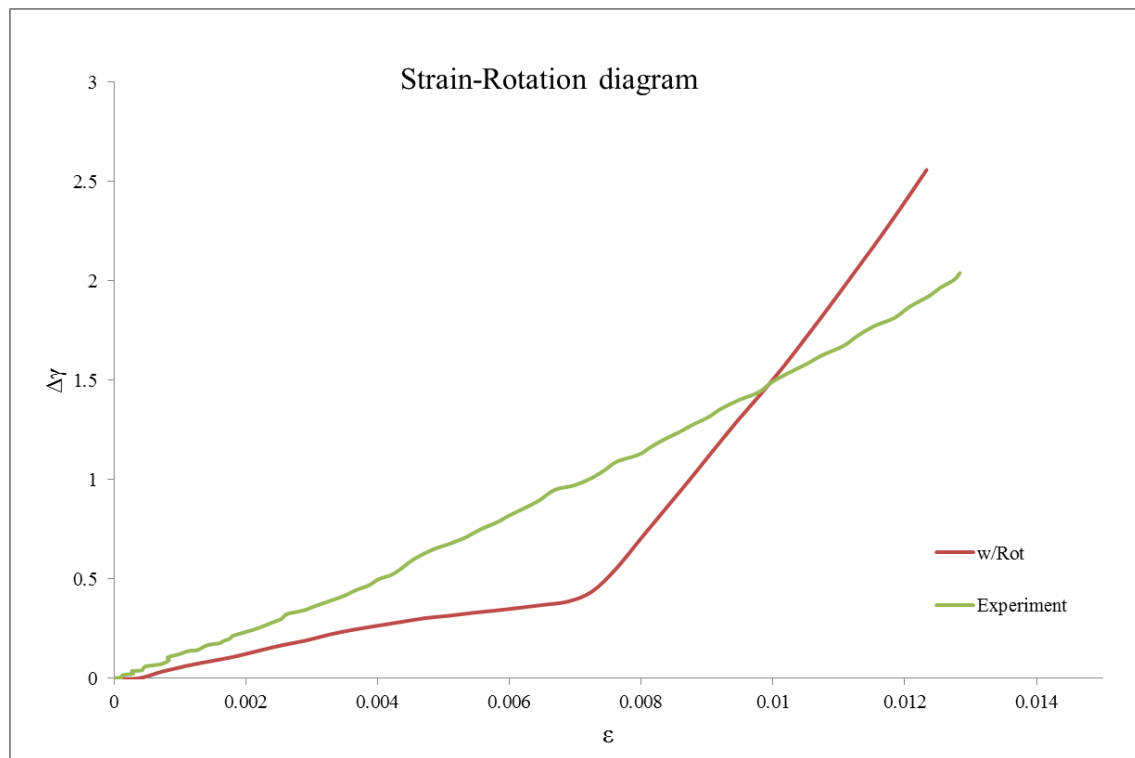
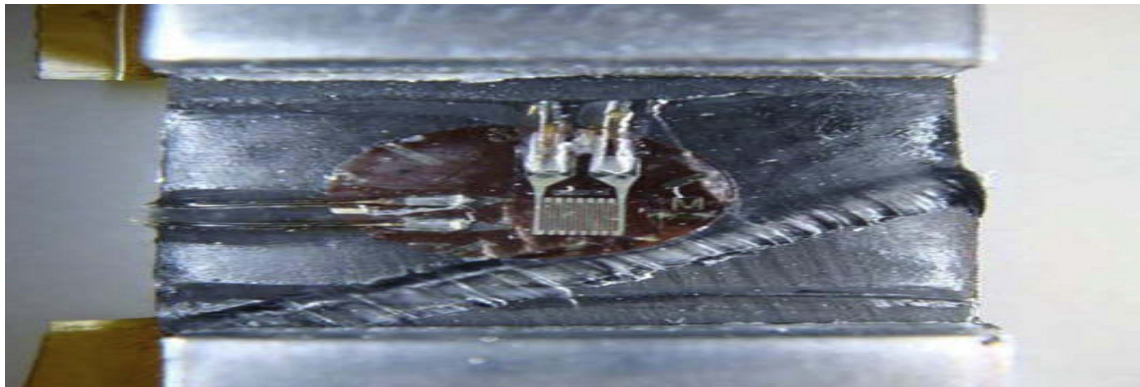


Figure 5.7: Rotation angle for 15 Deg-ply laminate..

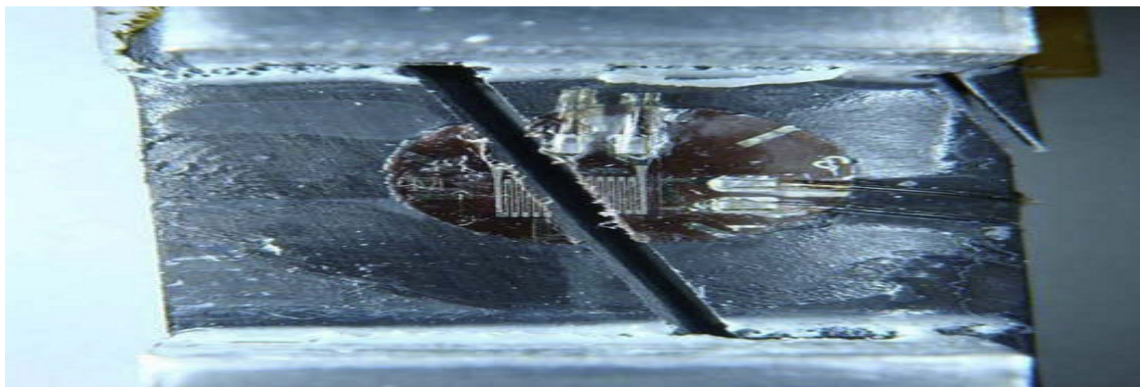
plane in fiber rotation inclusion is formed despite the model excluding it which exposes dispersed damaged elements. The fracture plane is also through thickness, the same as experimental results (see Figure 5.10 and 5.11), while the same thing does not happen in the other model. In the models, the failure ensued at the bottom the surface of the laminate which is confirmed by experiments. Besides, fiber rotation is uniform in the WROT numerical model until failure on which superimposed fiber rotation happens along with the direction of the fracture plane. This phenomenon is also observed in the experiments that shows necessity and importance of the fiber scissoring inclusion in the model once more (see Figure 5.12).

For the specimen by 30° plies, the same trend as 15° plies was observed. The compression reduces its stiffness and the model with fiber rotation can capture this. Also both models predict the failure stress with good agreement to the experiments. Figure 5.13 presents the evolution of the reaction forces in the case of fiber scissoring inclusion and exclusion. The comparison of the experimental results with the models are shown in Figure 5.14. The elements are chosen from the uniform area.

The rotation of the fiber in WROT model is compared to the experimental observations in Figure 5.15. It shows the capability of the WROT model to predict the rotation angle. Among the analyzed examples, 30° ply laminate has the highest fiber scissoring

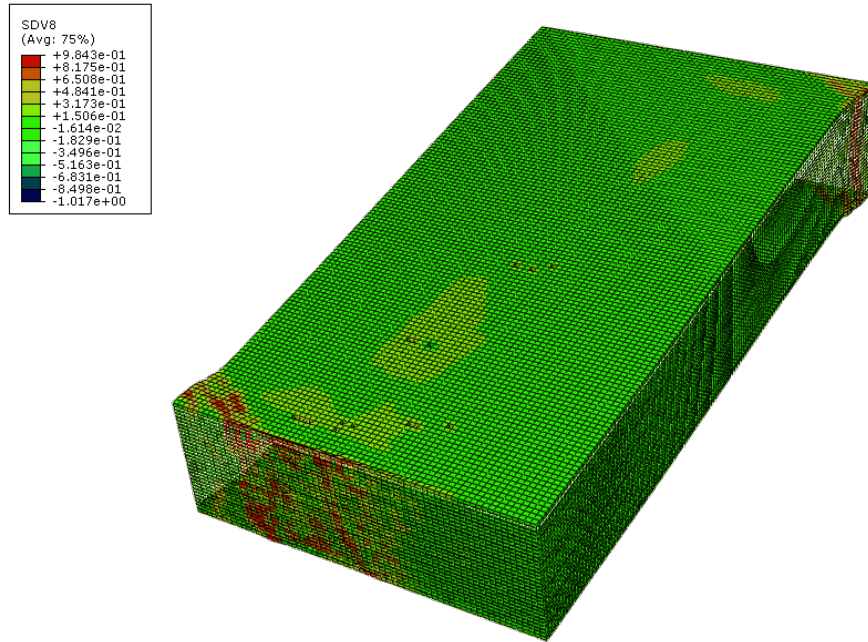


(a) Fiber kinking like failure mode

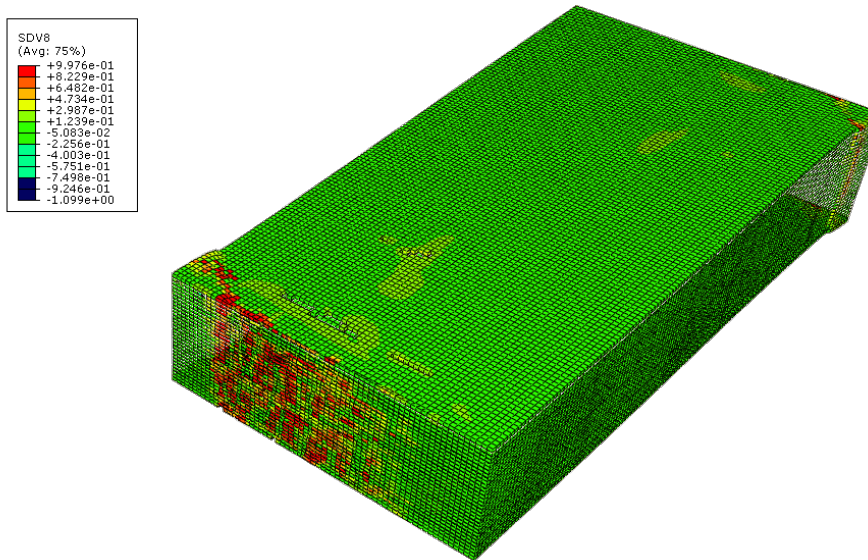


(b) In-plane shear like failure mode

Figure 5.8: Dominant failure modes in carbon-epoxy in 15° unidirectional laminate (Kawai and Saito, 2009)

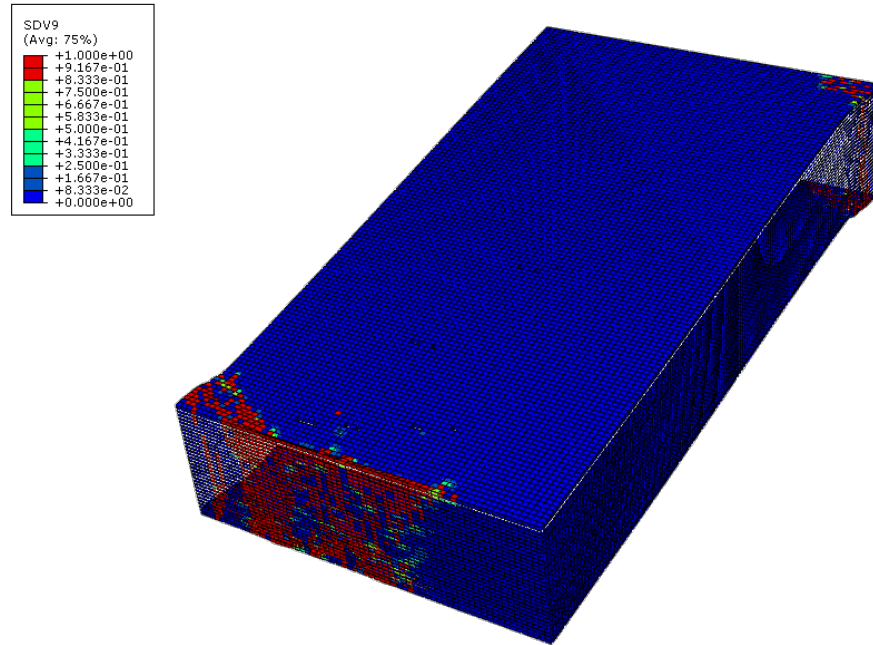


(a) Without rotation model

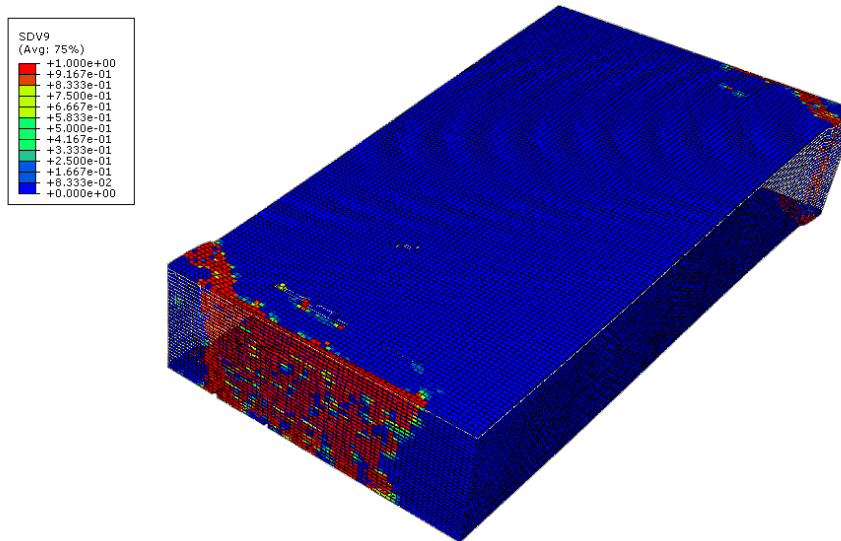


(b) With rotation model

Figure 5.9: Damage variable of d_1 in 15° unidirectional laminate

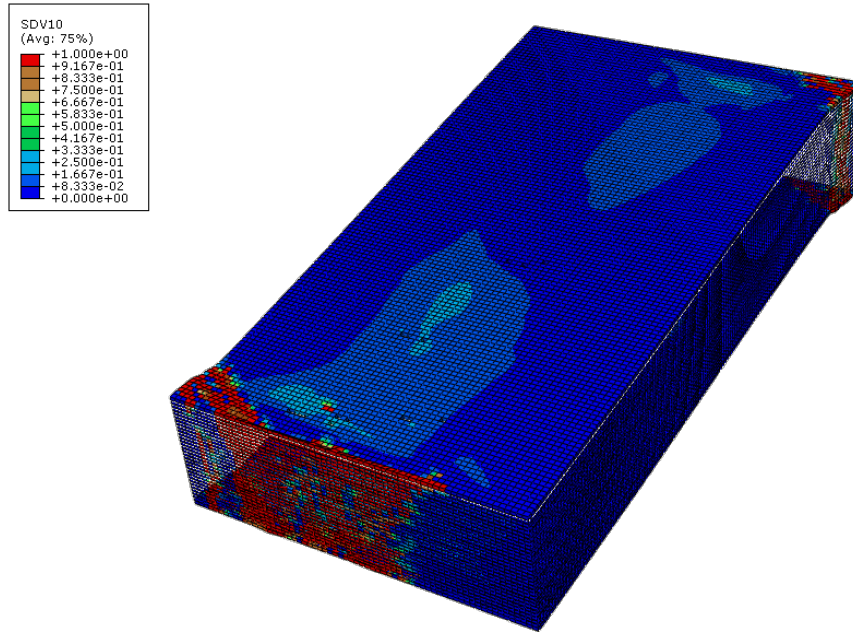


(a) Without rotation model

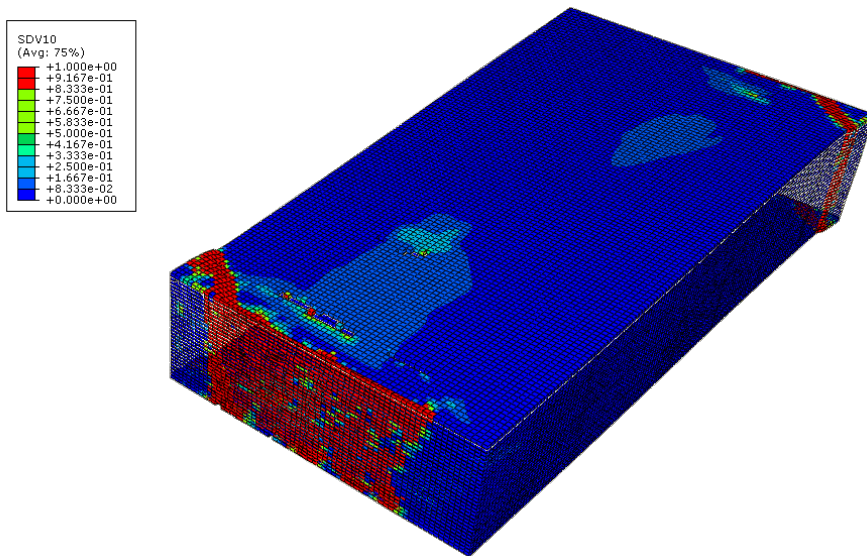


(b) With rotation model

Figure 5.10: Damage variable of d_2 in 15° unidirectional laminate

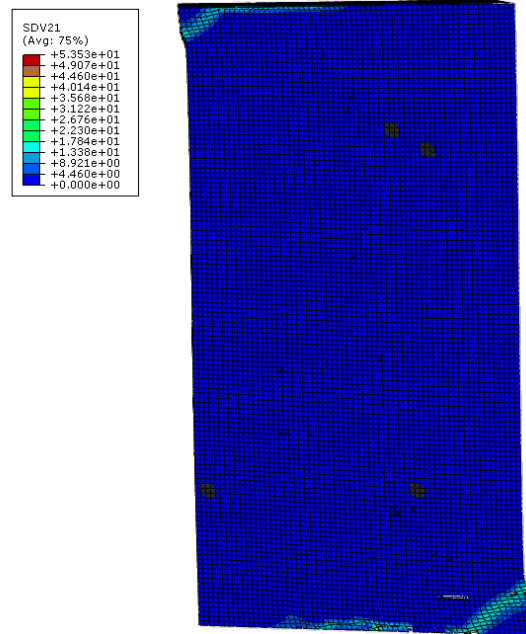


(a) Without rotation model

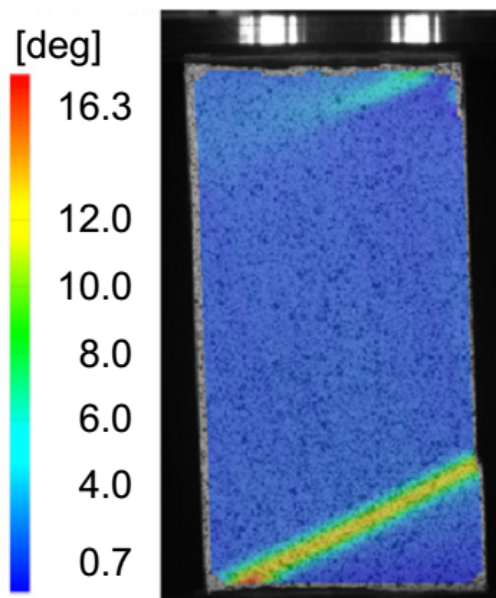


(b) With rotation model

Figure 5.11: Damage variable of d_6 in 15° unidirectional laminate



(a) Numerical model (WRot)



(b) Experiment (Körber et al., 2010)

Figure 5.12: Super imposed fiber rotation in direction of fracture plane

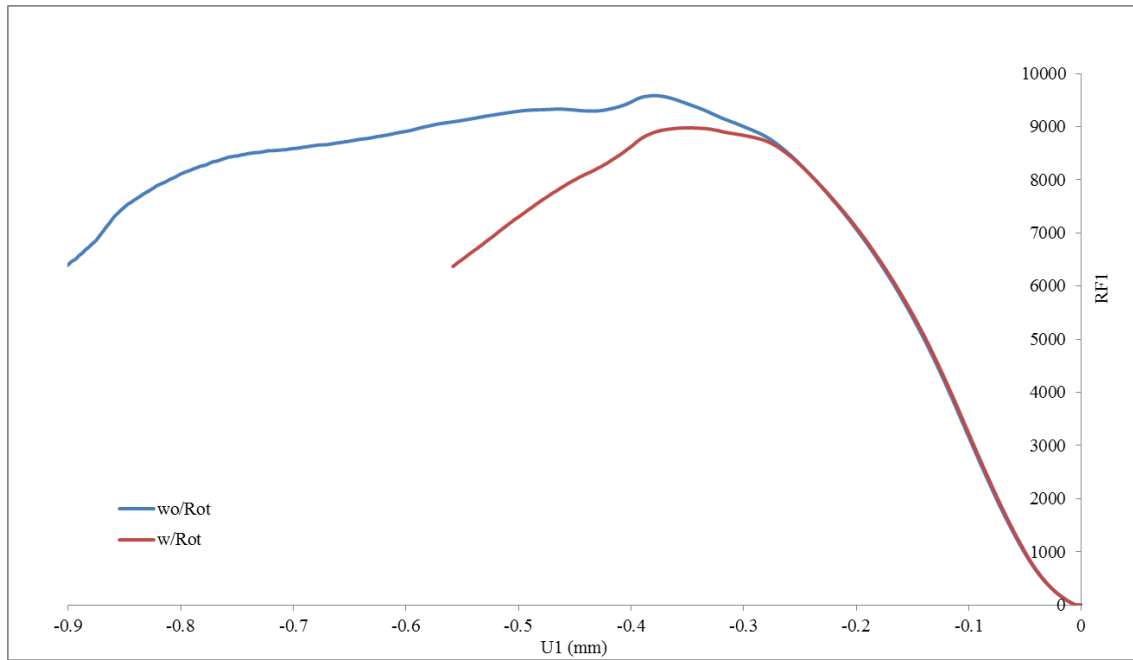


Figure 5.13: Reaction force in X direction for 30 Deg-ply laminate.

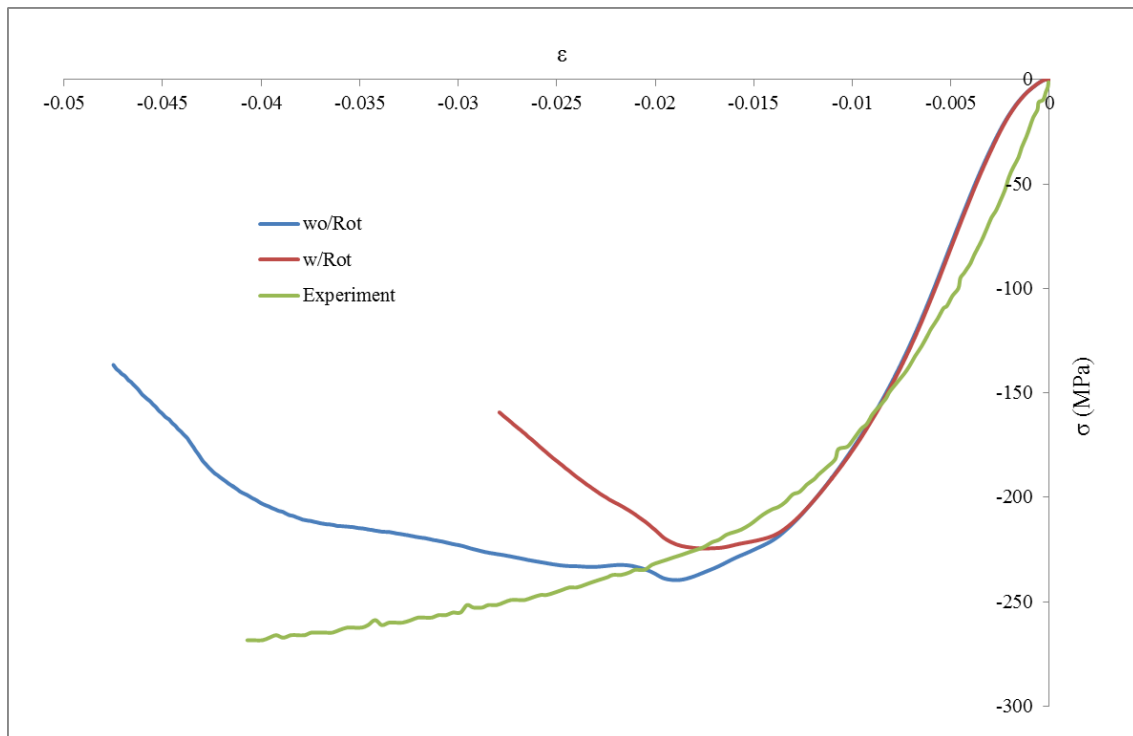


Figure 5.14: Stress-Strain diagram for 30 Deg-ply laminate.

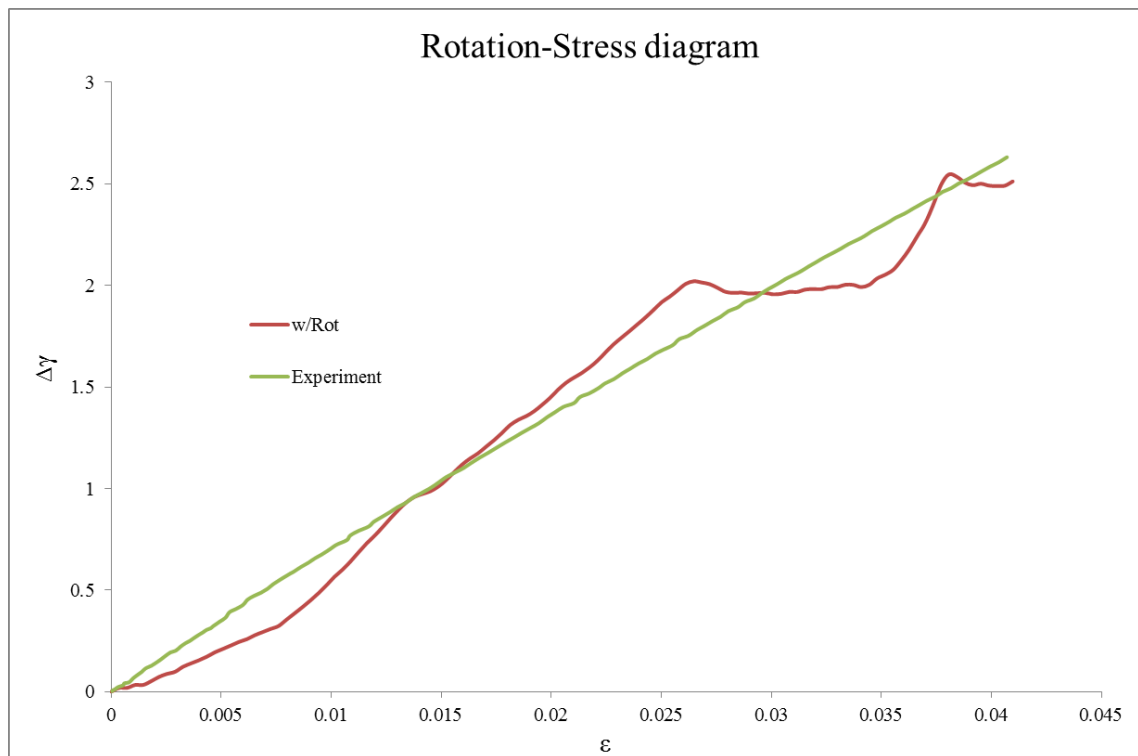


Figure 5.15: Rotation angle for 30 Deg-ply laminate..

and the best prediction of the model for its deviation angle besides to the failure stress. The periodic nature of the diagram for numerical model is due the approximation of Eq. 5.3. The in-plane shear mode in direction of the fiber is identified in this initial fiber direction which determines the fracture plane which is through thickness. This mode and the fracture plane is confirmed by the experimental observations. This fracture plane is not formed in the model that excludes fiber rotation (see Figure 5.16).

In the case that initial fiber angle is 45° , the matrix starts to have more effect on the behavior of the material and fiber plays less role, hence the fiber rotation will be less in this case and both models reveal more similar results. Because of visco-plastic behavior of the polymer matrix, that is not considered in the constitutive model, the error of the final failure prediction increases to 20% in these specimens. Figures 5.18 and 5.19 depict the reaction forces versus displacement and the stress-strain diagrams for 45° unidirectional laminate, respectively. In Maimi damage model when the failure mode is matrix-dominated, laminate collapse occurs as soon as a matrix crack is created (Maimi et al., 2007a). Therefore its prediction for final failure is more conservative and it predicts the failure strength lower than actual strength observed in the experiments. Experimental observations also approves that the transverse compression is dominated mode and the failure occurs either in the middle or initiated at the loading surfaces. Failure strength did

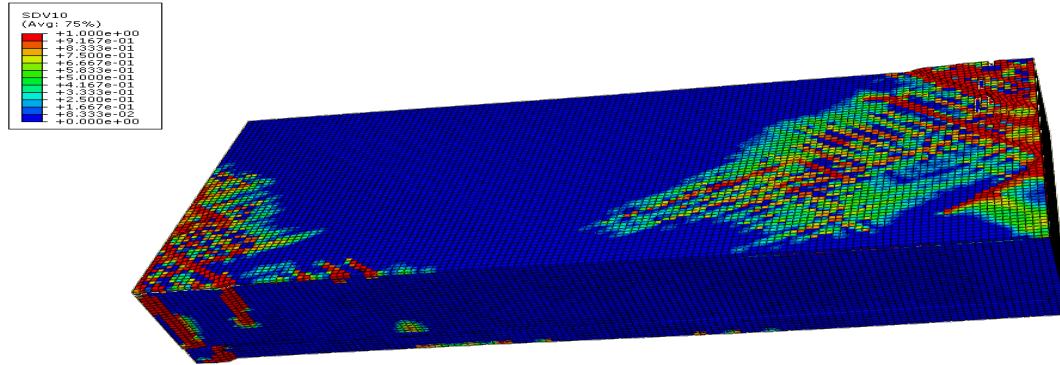
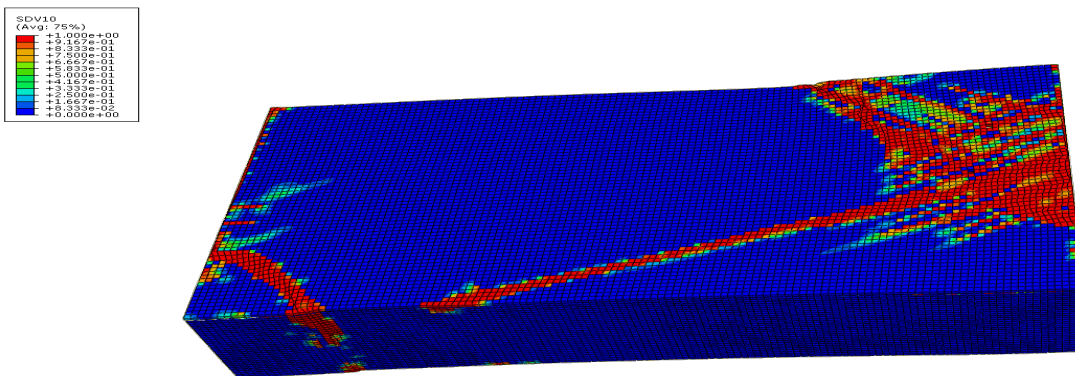
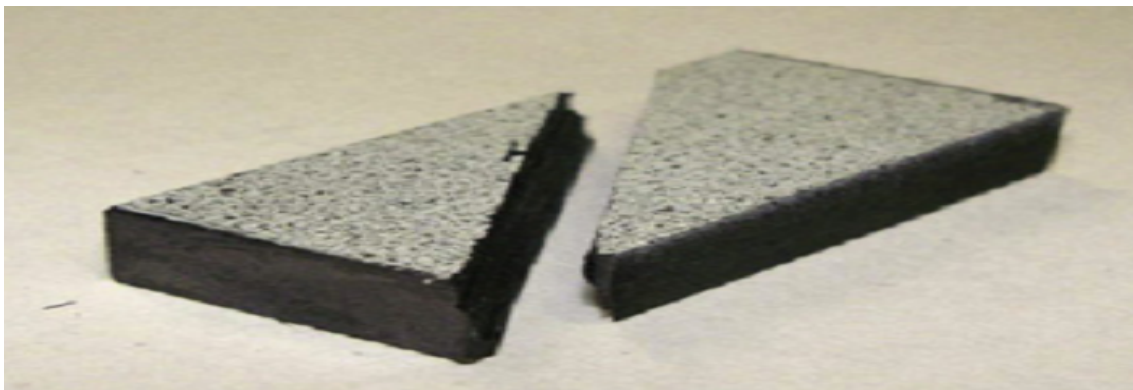
(a) d_6 in WOROT model(b) d_6 in WRot model(c) Experiment ([Körber et al., 2010](#))

Figure 5.16: Fracture plane in 30° fiber direction laminate

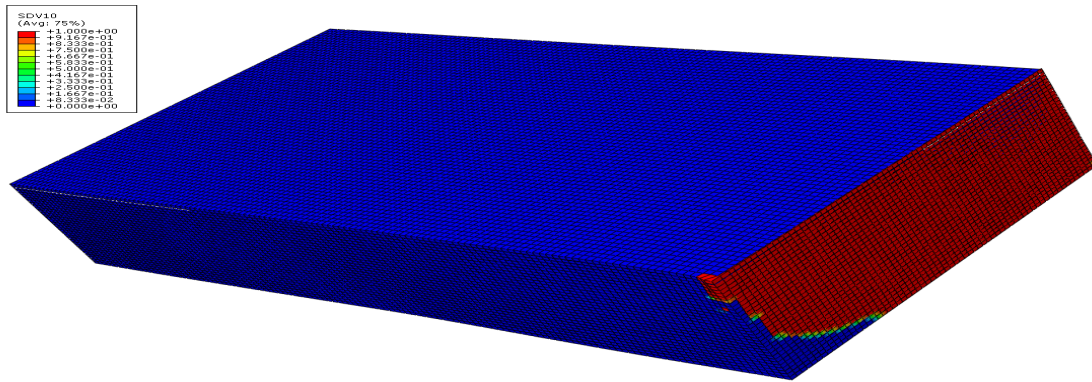
not change in different failure positions. The numerical models can capture the failure mode correctly and the failure position in the loading surface, while in the presence of fiber scissoring, failure at both loading surfaces are captured and without regarding it just one loading surface shows the failure. Figure 5.17 illustrated the damage variable d_2 for both numerical models.

The WROT model still predicts the rotation angle well when its results are compared to the experimental data of fiber rotation angle for 45° initial fiber direction. Figure 5.20 also depicts the rotation angle evolution for numerical and experimental results. Furthermore, both models predict the displacement in Y direction well, which according to the experiments is 0.23 mm while the pure damage model predicts it 0.24 mm and the other model discloses 0.23 mm.

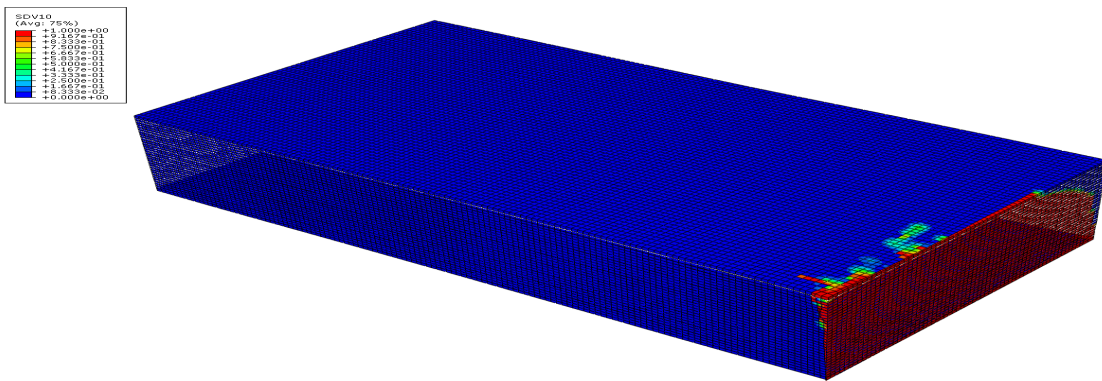
The matrix domination will be more obvious when it comes to higher initial fiber angles. According to the experimental data 45° , 60° , 75° and 90° have almost the same final failure around 255 MPa that shows different fiber angle has neutral effect on their failure. Final failure strength of different initial fiber orientations for unidirectional IM7-8552 composite is presented in Table 5.5. Therefore, there is no surprise that in these cases rotation angle would not be considerable, as it is reported by experiment that they show zero fiber rotation. The WROT model also shows that the fiber rotation is less than 0.3° . This prediction is approved by experimental observations that expresses maximum of fiber rotation is 0.3° . Therefore, in these cases Maimi damage model reveals almost the same results as damage in presence of fiber rotation. Regarding Maimi damage model is conservative in matrix dominated mode, predicted final failure in both models are underestimated, though they are still in the safe area for design purposes despite of causing more costs. It is recommended that the damage model to be revised for matrix dominated mode in compression in order to improve inaccurate results. This can be achieved by inclusion of plasticity in the model. Inclusion of a visco-plastic behavior in the model would improve the prediction accuracy in addition to extension the capability of the model for high strain rates. This will be discussed in details in chapter 7.

Table 5.5: Failure strength for different initial fiber angles.

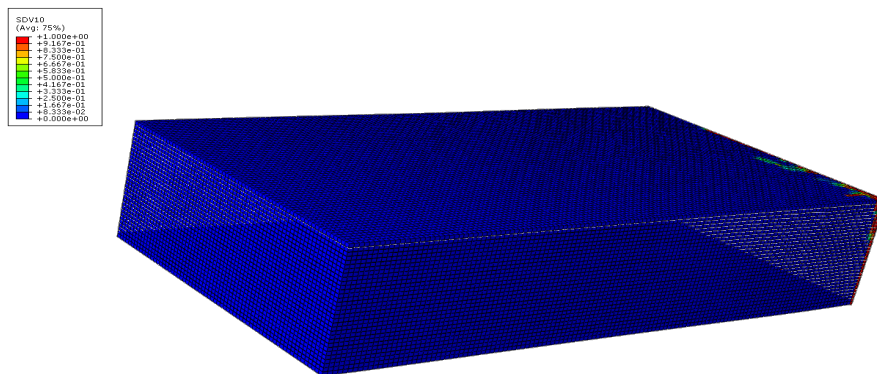
Initial fiber angle (Deg)	Failure strength (MPa)
45	254
60	263
75	252
90	255



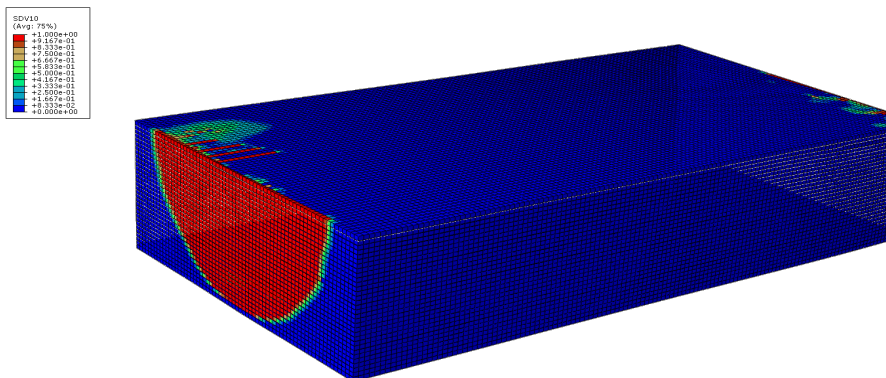
(a) Without rotation/ Right side



(b) With rotation/ Right side



(c) Without rotation/ Left side



(d) With rotation/ Left side

Figure 5.17: Damage variable of d_2 in 45° unidirectional laminate

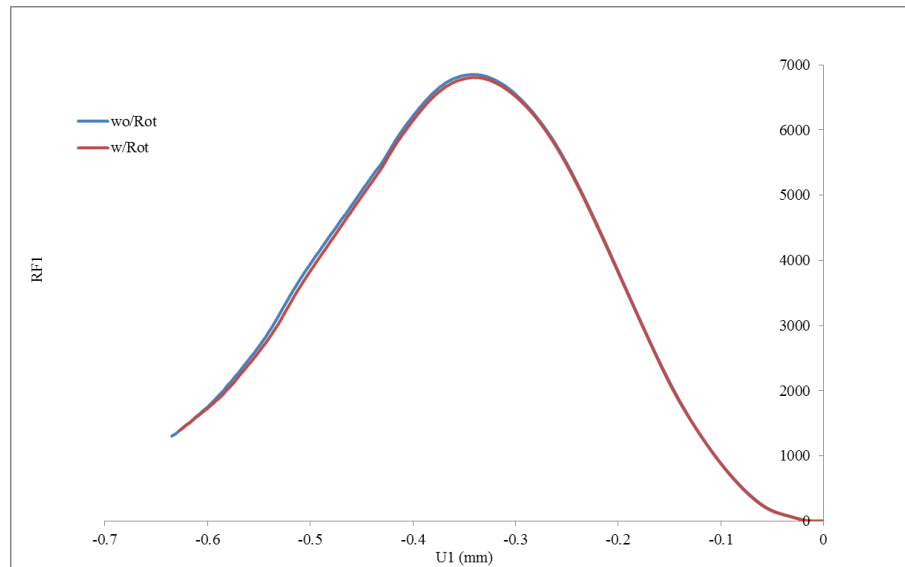


Figure 5.18: Reaction force in X direction for 45 Deg-ply laminate.

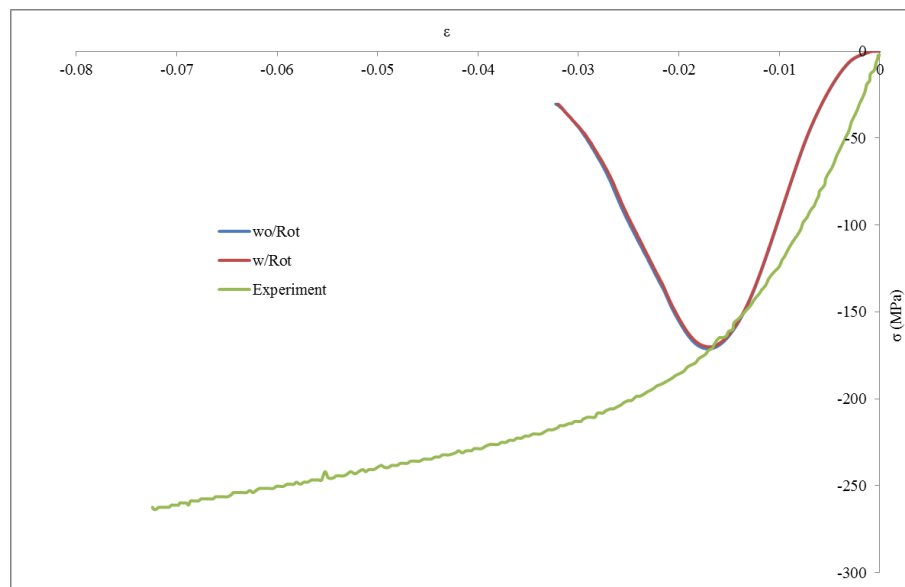


Figure 5.19: Stress-Strain diagram for 45 Deg-ply laminate.

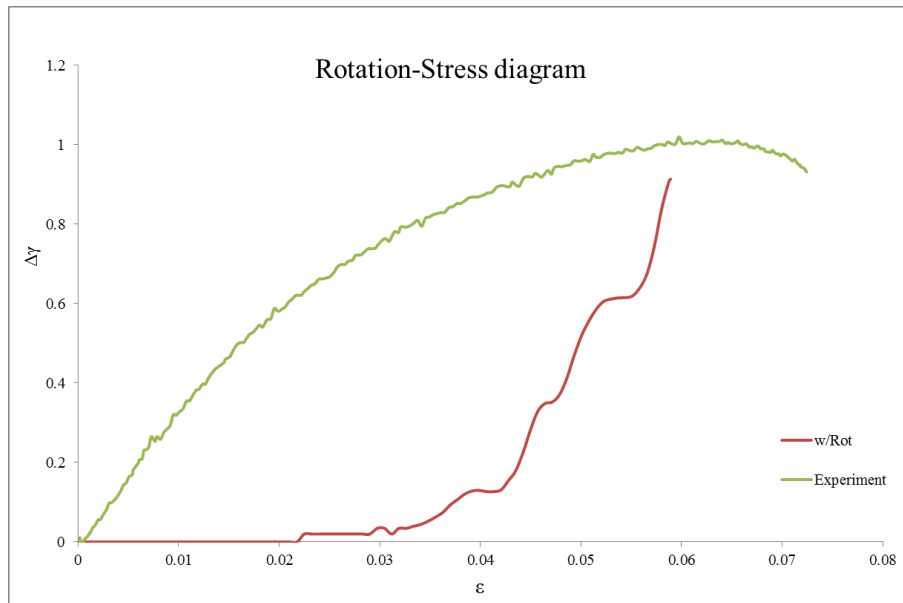


Figure 5.20: Rotation angle for 45 Deg-ply laminate..

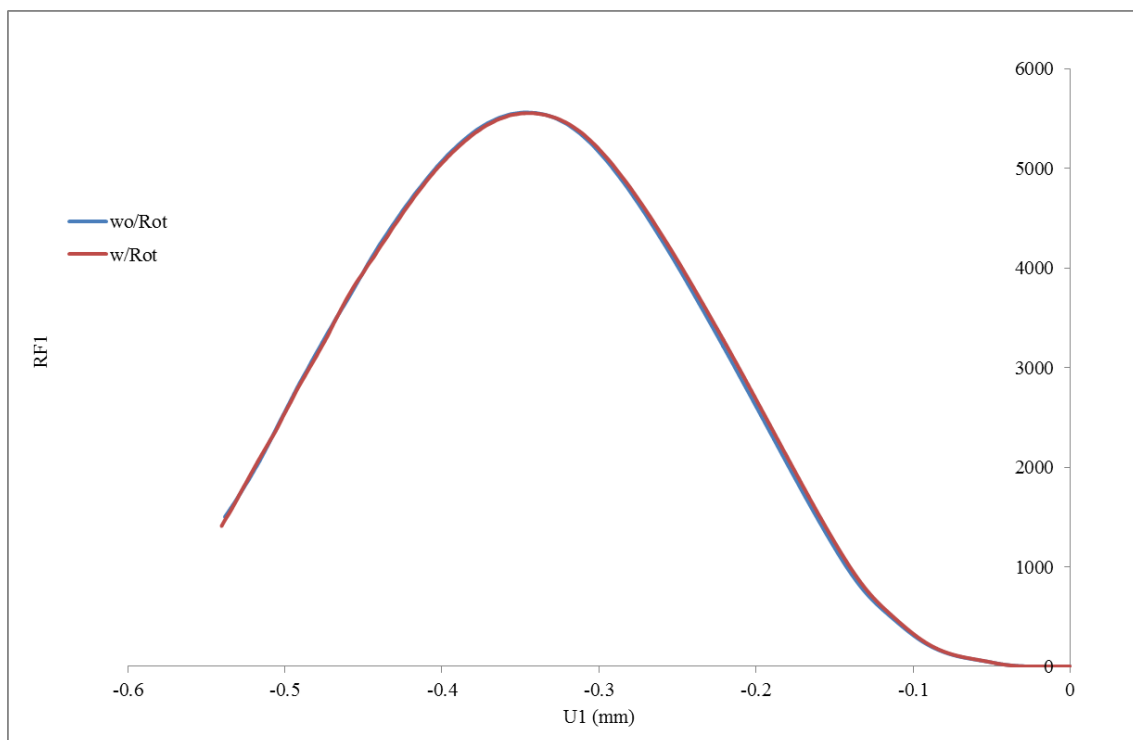


Figure 5.21: Reaction force in X direction for 60 Deg-ply laminate.

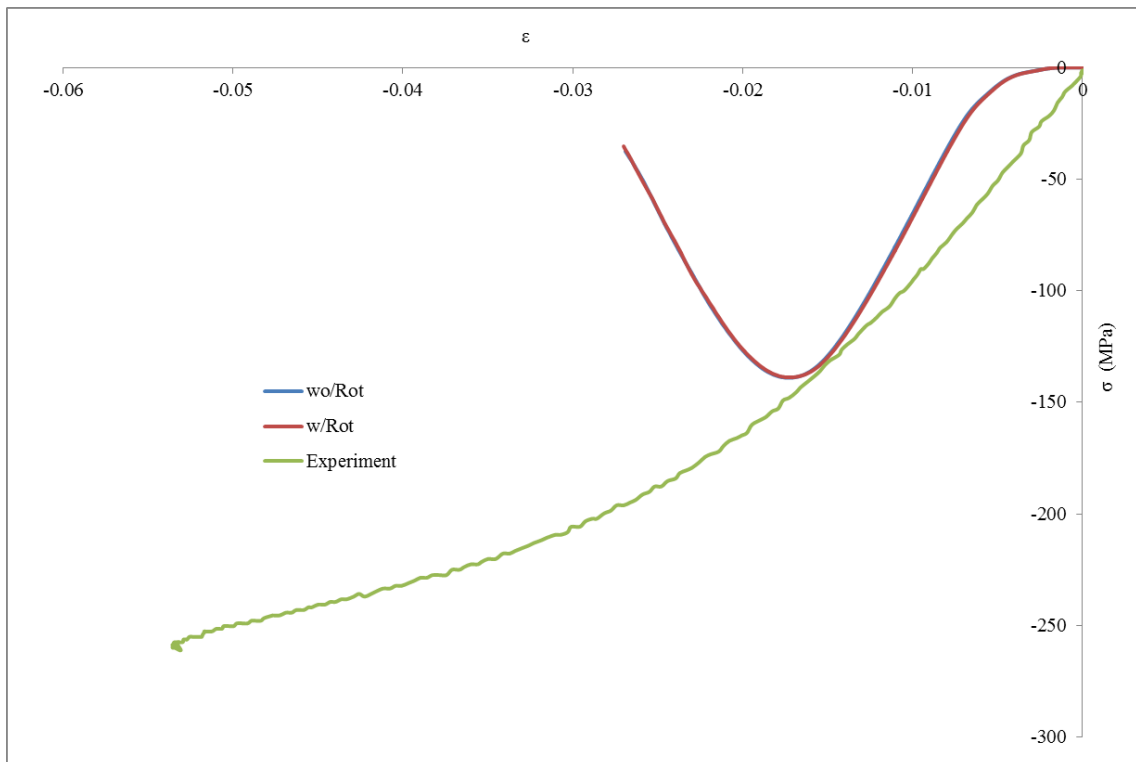


Figure 5.22: Stress-Strain diagram for 60 Deg-ply laminate.

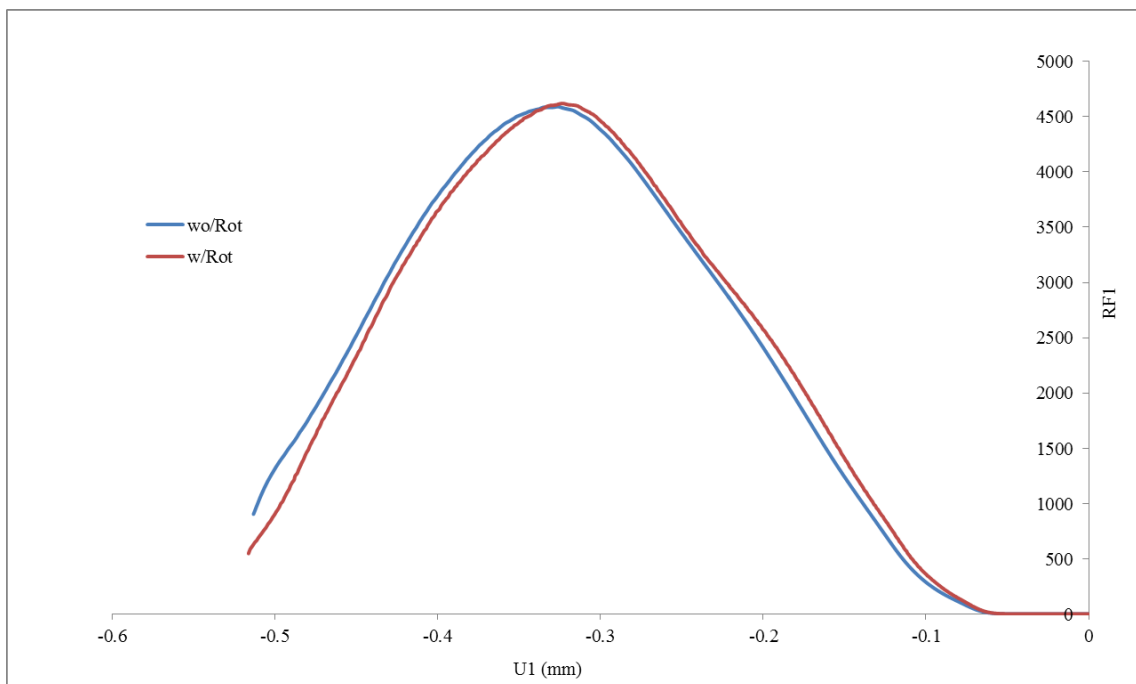


Figure 5.23: Reaction force in X direction for 75 Deg-ply laminate.

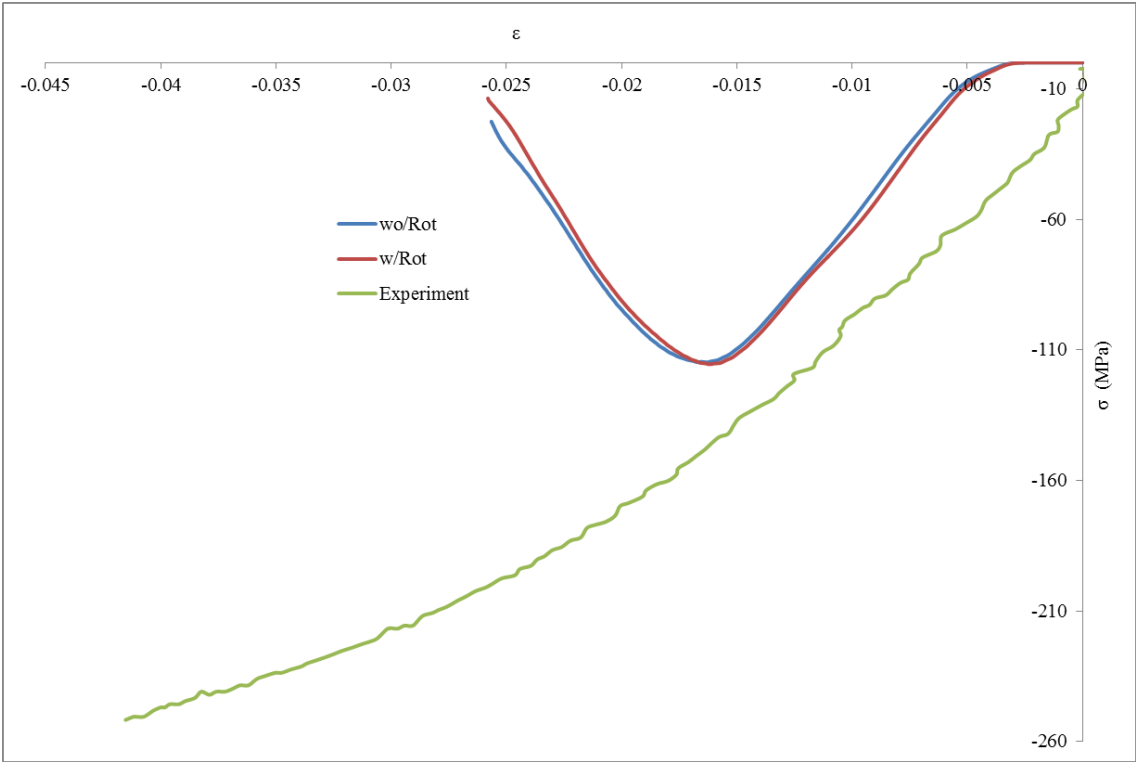


Figure 5.24: Stress-Strain diagram for 75 Deg-ply laminate.

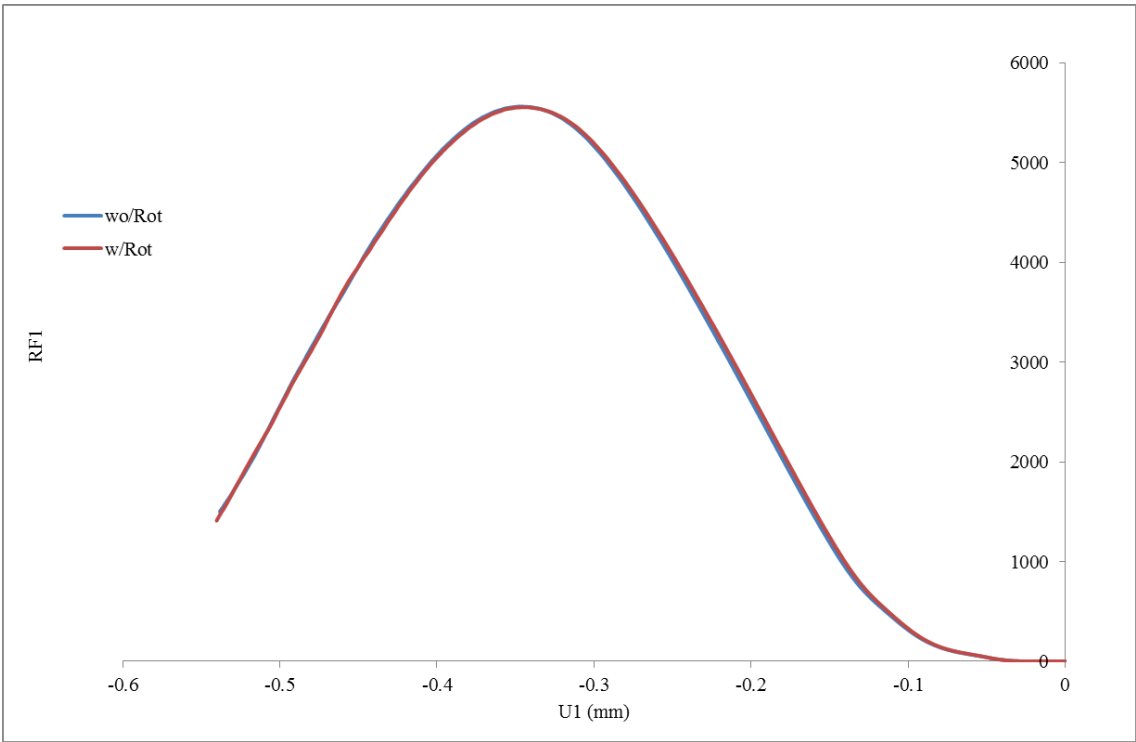


Figure 5.25: Reaction force in X direction for 90 Deg-ply laminate.

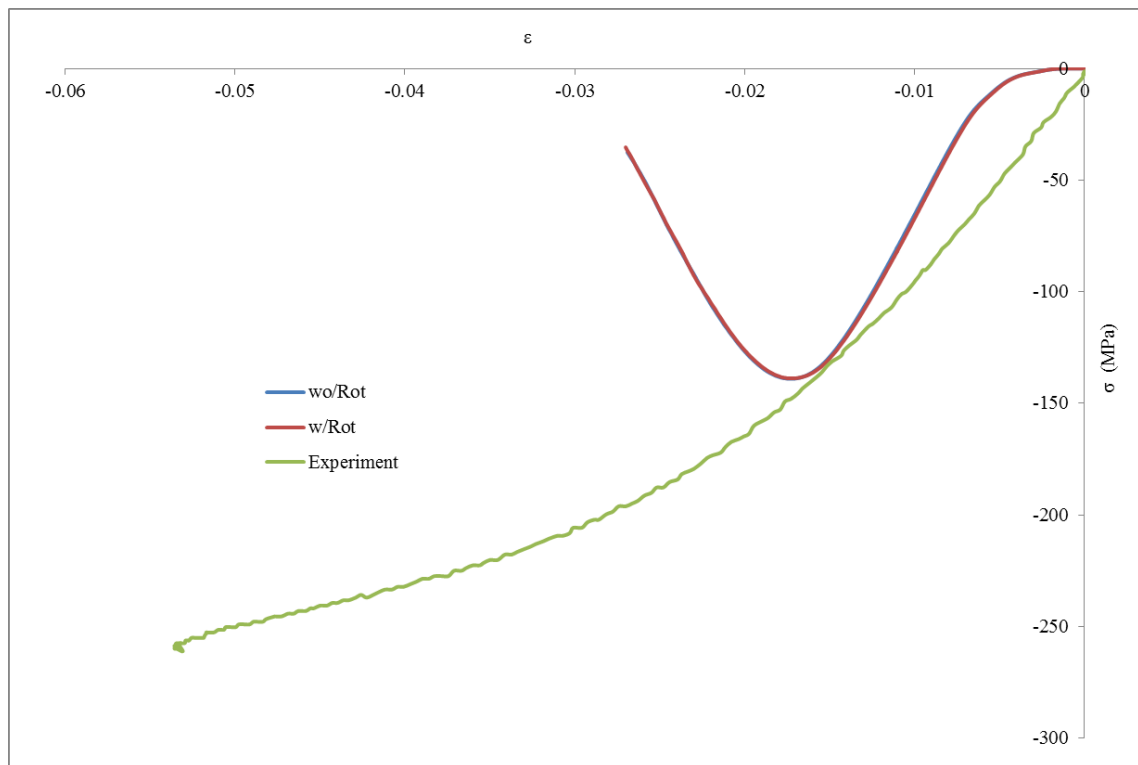


Figure 5.26: Stress-Strain diagram for 90 Deg-ply laminate.

5.4 Tension in thermoplastic-based composite laminates

In the last section, the damage in the presence of the fiber rotation phenomenon studied for a thermoset-based composite laminate. Usually, thermosets appear less ductile behavior and fail at lower elongations, in contrary to the thermoplastics. Thus, the fiber rotation would have more important role in thermoplastic-based composites because of the higher material deformation. In addition, regarding the fact that no chemical bond takes place during curing process of the thermoplastics, they can be recycled completely which leads to eco-friendly manufacturing. This is totally favorable concerning sustainability issues and mandates more study on composites with thermoplastic matrix in order to widen their applications. Automotive sector, increasingly stringent requirements for reduced emissions of CO₂, the maximum return on capital investment and the increase in plastic recycling and reuse, are some of the most important problems that directly influence the development of new materials. The thermoplastic composites appear to be the right alternative to the materials currently used for vehicles ([Boria et al., 2016](#)).

In this study, the damage of AS4-PEEK is studied using the developed numerical models and the results are compared to the experimental data provided by Ref. [Kellas and J.Morton \(1992\)](#). This material has very good fatigue and creep strength and its resistance to fire and environmental problems is outstanding. The matrix material, PEEK,

exhibit high ductility that is expected to accompany with high fiber scissoring in the lamina. The laminate that is chosen for modeling consists of 8 plies with $\pm 45^\circ$ distributed configuration. each ply has a geometry of $12.7\text{mm} \times 127\text{mm}$ with 0.125 mm standard thickness. The velocity boundary condition is applied on the specimen in longitudinal direction and the loading is tensional.

As mentioned before, the model needs elastic, strength and toughness properties of the material. The elastic properties of AS4-PEEK is presented in Table 5.6. The strength of the material in different directions is also provided in Table 5.7. The data is obtained from [Chinesta and Cueto \(2014\)](#).

Table 5.6: AS4-PEEK 3552 elastic properties.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
138	10.3	5.7	0.3

Table 5.7: AS4-PEEK strengths properties(MPa).

X_T	X_C	Y_T	Y_C	S_L
2070	1360	62.3	86	196

The coefficients of thermal expansion of the material used are $\alpha_{11} = -2.4 \times 10^{-7}/^\circ\text{C}$ and $\alpha_{22} = 6.00 \times 10^{-5}/^\circ\text{C}$, and the temperature change is $\Delta T = -152^\circ\text{C}$ [Chinesta and Cueto \(2014\)](#). The effects of moisture absorption are neglected in the model. The transverse fracture toughness in I and II fracture modes are also required for the damage softening prediction which also collected from ([Chinesta and Cueto, 2014](#)) that is presented in Table 5.8. The longitudinal toughness data which is also entailed in the damage model is not available for this material in the literature, so the longitudinal toughness data from carbon-epoxy is used for it. The numerical analysis showed that this data does not have any effect on the final results of this model since the loading is not activate this fracture mode.

It is very important to pay attention to the point that some material properties which are obtained according to the standard procedure may need some modifications in the case that fiber rotation is relatively high, that includes composites based on thermoplastic matrix. Because in the standard the effect of fiber rotation is neglected while it may have effects on the final material properties. Shear strength is one of the material properties which is required in the current damage model and has significant effect on the

final results, especially on cases that in-plane shear damage mechanism is activated. The maximum in-plane shear strength in fiber direction of the polymer matrix composites is usually tested by ASTM D3518/D3518 M method ([ASTM D 3518, c](#)). In this standard method, a tension test is carried on $\pm 45_n$ laminate and the shear strength is calculated by $\tau_{ult} = \sigma_{ult}/2$. This equation could only be true in the case that fiber rotation is negligible. Because the maximum shear stress in fiber direction is only half of the maximum tension stress that fiber direction of lamina is 45° . When the fiber scissoring occurs, the direction is not the same as initial one so $\tau_{12} \neq \sigma_{ult}/2$. Hence, the shear strength should be calculated according to the final fiber direction.

In this study a modification is proposed for the calculation of the real shear strength and it is used in the numerical model of WROT. In order to modify the final shear strength, it is necessary to have the final fiber rotation angle. In order to reduce the cost of the testing, it is possible to approximate final fiber angle. [Kellas and J.Morton \(1992\)](#) and [ASTM D 3518 \(c\)](#) suggest that there is 1° of fiber rotation for every 2% of axial strain. Having the ASTM test results, the final axial strain is defined. So the approximated fiber rotation angle ($\delta\gamma^\circ$) would be $\epsilon_{ult}/2$. Then, having the final fiber direction, either by experiment or approximation, real shear strength (τ_{ult}^{real}) can be achieved as:

$$\tau_{12}^{real} = \frac{1}{2} \sigma_{ult}^{ASTM} \sin(90 - \epsilon_{ult} \%). \quad (5.5)$$

The shear strength is calculated according to the test carried out by ([Kellas and J.Morton, 1992](#)) that is shown in Figure 5.27. The extensometer range of 15% was exceeded its data is extrapolated. Therefore, beyond this point all curves were extrapolated to the maximum measured stress. Since the specimen thickness is important and have effect on the test results, the 16-ply thickness is chosen for picking final strength. Because in this case stress inhomogeneities would be reduced and average applied stress will be closer to the ply stress. Therefore, the measured average strength also will be closer to the ply strength. It should be mentioned that [Kellas and J.Morton \(1992\)](#)'s recommendation for minimum laminate thickness is 24 plies while, in the current study 16 plies results used due to lack of data. According to Figure 5.27, the maximum strength for 16-ply laminate is 365 MPa and the final strain is 17.74%, which approximates the fiber rotation as 8.5° . Finally, putting all together in Eq. 5.5 the modified shear strength will be 173.8 MPa.

In-situ data are calculated based on the modified shear strength and the formulation provided by ([Camanho et al., 2006](#)) along with the available toughness data for AS4-PEEK. The resulting in-situ strength are presented in Table 5.9.

The mesh type used for numerical modeling is hex-structured and its maximum size

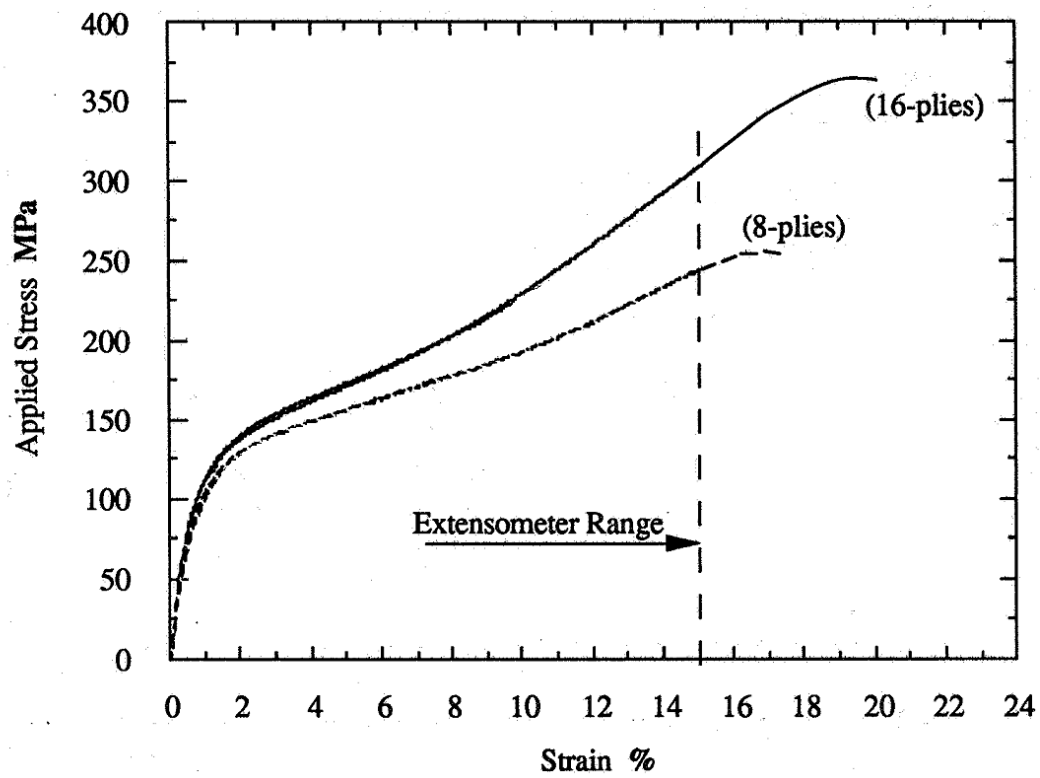


Figure 5.27: ASTM in-plane shear test for 8 and 16 plies AS4-PEEK model (Kellas and J.Morton, 1992) . (Extra data is removed from original diagram)

Table 5.8: Toughness properties of AS4-PEEK($\frac{KJ}{m^2}$).

G_2^+	G_2^-	G_6
1.7	1.8	2

Table 5.9: In situ strengths of AS4-PEEK(MPa).

	Embedded ply	Outer ply	Thick ply
Y_T	422.3	265.8	136.22
S_L	171.29	142.0	207.9

calculated by Equation (5.4). According to the formulation, the maximum allowed mesh size is to dissipate energy is 0.659 mm. The number of the nodes in the model is 1275840 and number of the elements is 630120 with length of 0.144 mm, which generates total number of variables 3827520. The geometry and the meshing of the model is illustrated in 5.28. It is executed on the cluster by 32 CPUs and it took 8 h for each model to be done.

5.4.1 Results and discussion

The reaction force versus displacement diagram for the damage model in presence of fiber rotation (WROT) is shown in Figure 5.29. But the damage model without considering the fiber rotation (WOROT), as can be seen in Figure 5.30, does not have ability to model this case and reveals instability. Because in composites with ductile matrix, elongation is relatively high and fiber will rotates much higher in comparison to more brittle matrix. Therefore, neglecting rotation will cause a significant difference in the results. The WOROT model reveals Super distorted elements that cause premature failure in the modeling process which makes this model inapplicable for cases including high fiber rotation. It should also be added that the velocity boundary condition is applied by a linear amplitude from 0 to its maximum and it is the reason why the load is quite low in the beginning of the time period in force versus time diagram.

In the model that regards fiber scissoring, the failure strength is 280 MPa. The experimental data of Kellas and J.Morton (1992) having the same geometry and configuration reveals that the model predicts the failure stress quite accurate. The failure strength according to experiment is 269 MPa that shows WROT model error is less than 1% . The final true strain of the WOROT model is also 12% that is with good agreement with the

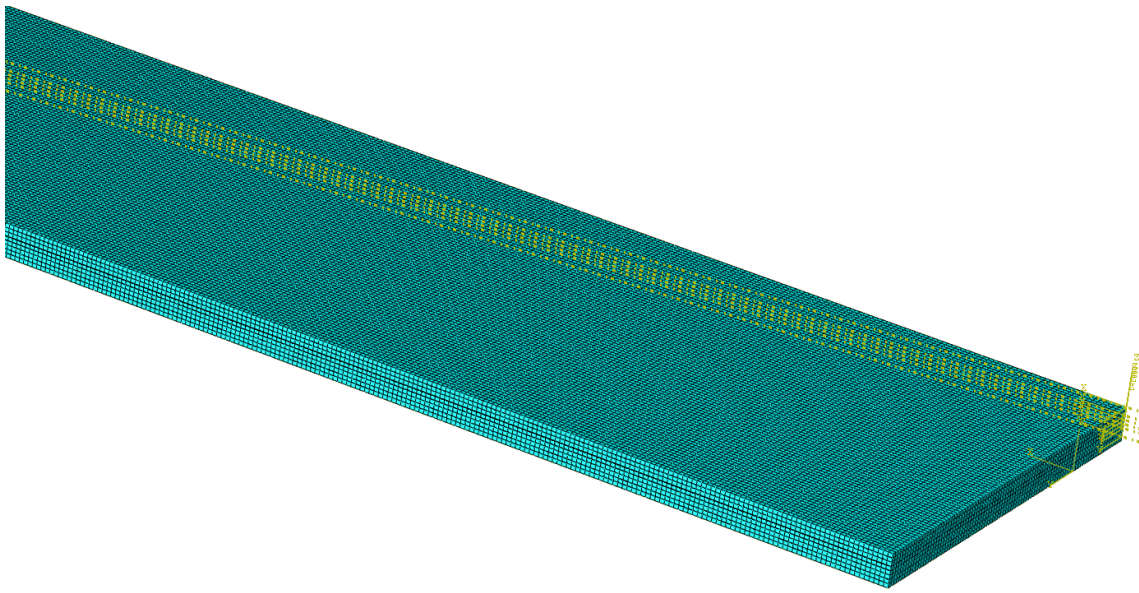


Figure 5.28: Mesh and geometry of AS4-PEEK model.

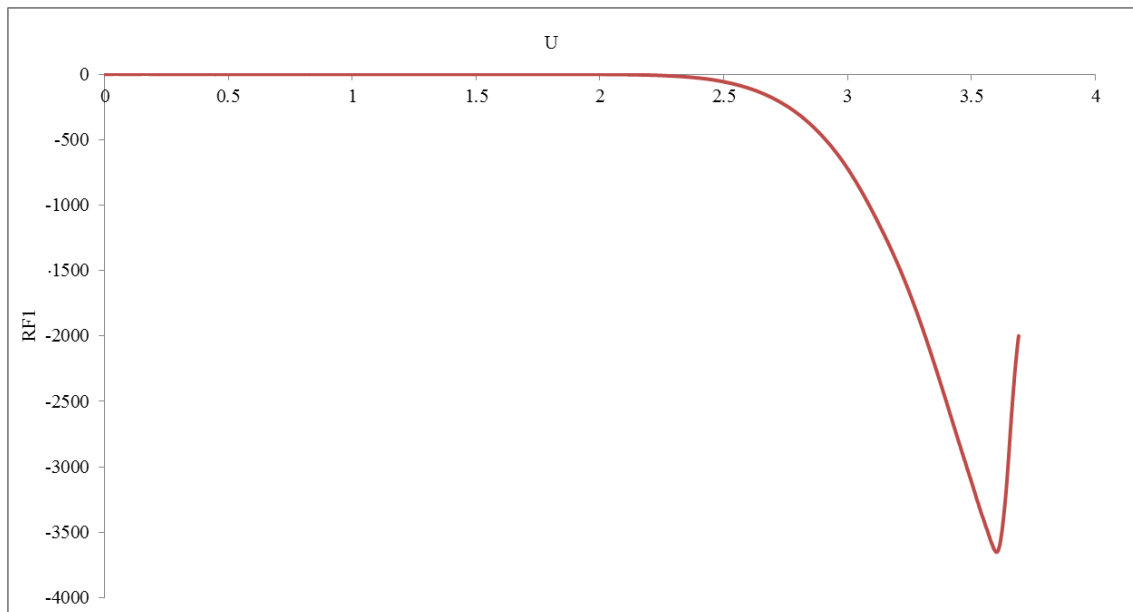


Figure 5.29: Force-displacement in WROT model for AS4-PEEK.

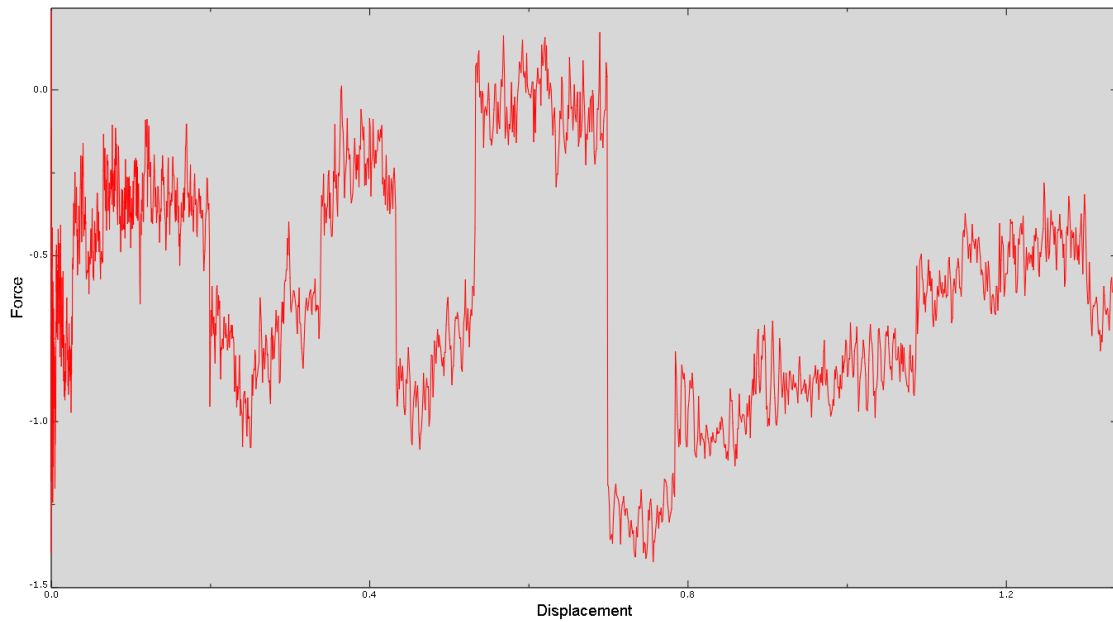


Figure 5.30: Force-displacement in WOROT model for AS4-PEEK.

experimental observations. However, maximum strain in the model happens in the course of the softening and this strain at the highest stress point is less than the measured strain. It could be because of not considering the visco-plastic behavior of PEEK in the model. The final results of these models for AS4-PEEK are concluded in the Table 5.10. The experimental data is the average of three specimens closest to the middle of the panel. The strain data in the experiments represents extrapolated values, since the extensometer range was only 15.0% strain. In the extrapolation the measured values of the stress were used in conjunction with an assumed constant rate of strain.

Table 5.10: Comparison between experimental and numerical results for AS4-PEEK specimen.

	Failure Strength (MPa)	Error (%)	Final strain (%)	Error (%)
Experiment	279		15.99	
With rotation	280	0.3	14	-12

Analysis the damage progress of the WROT model also exhibits interesting results. Damage examination of Kellas and J.Morton (1992) showed that the damage initiated at surface plies. X-ray radiograph images did contain dark lines at 45° which are normally associated with transply cracks. Damage, as detected in the X-ray radiographs, became clear in PEEK matrix specimens which were loaded to a stress of approximately 90 MPa

and beyond. The same thing happens in the numerical model which can be seen in Figure 5.31. However, in mentioned experimental observation, microscopic examination of polished edges of specimens did not produce evidence of transply cracks. Clear evidence of transply cracking in the 8-ply PEEK matrix specimens was observed at stresses of 117 MPa and beyond, as shown in Figure 5.32(a). Such cracks were first observed to occur in the surface plies ($+45^\circ$ direction), and as the X-ray radiograph of Figure 5.32(a) indicates, they co-exist with cracks in -45° direction (Kellas and J.Morton, 1992). The numerical model also can capture this cracks in $+45^\circ$ and -45° that is illustrated in Figure 5.32 (b). At this point the local stress in numerical model is 115 MPa.

5.5 Parametric study

According to standard (ASTM D 3039, a) extreme fiber scissoring can occur for cases of ductile matrices, weak fiber/matrix interfaces, thick specimens with a large number of repeated plies, or a combination of above. Some of these parameters are studied in order to analyze capability of the model. From practical point of view, some of these results can be used for design purposes aiming to reduce or produce fiber rotation. A laminate of 4 plies square lamina with the dimension of 20 mm is considered. The configuration of plies is $\{\pm 45\}_2$ and the regarded material is IM7/8552.

As suggested in the standard (ASTM D 3518, c) lower thickness produces lower fiber rotation. But in the standard this concept is not well explained. According to the experimental work of (Kellas and J.Morton, 1992) the influence of thickness on fiber rotation depends on stacking sequence of the laminate. In distributed plies it is the opposite and thicker plies increase fiber rotation while blocked plies decrease it (See Fig. 5.33 where dark layers represent positive fiber orientation while light layers are showing negative fiber direction). In this model which is distributed case, thin plies produce more fiber rotation and the model can show this trend. This thickness can be result of number of plies or the thickness of ply itself. Both of this type of increase are addressed in this parametric study. Figure 5.34 compares fiber rotation of a 4-ply distributed laminate with a 8-ply distributed laminate. As can be seen in the diagram, reduction in number of plies, increases fiber rotation. The effect of configuration of laminate on fiber rotation is also analyzed. Two different stacking sequence for 8-ply laminate subjected to tension. Figure 5.35 compares fiber rotation in distributed laminate versus blocked ones. Another interesting observation took place here which is the reaction force versus time for these 2 sequences of plies. As reported by (Kellas and J.Morton, 1992) blocked plies show brittle-like behavior and distributed plies reveal behavior like ductile material. This can

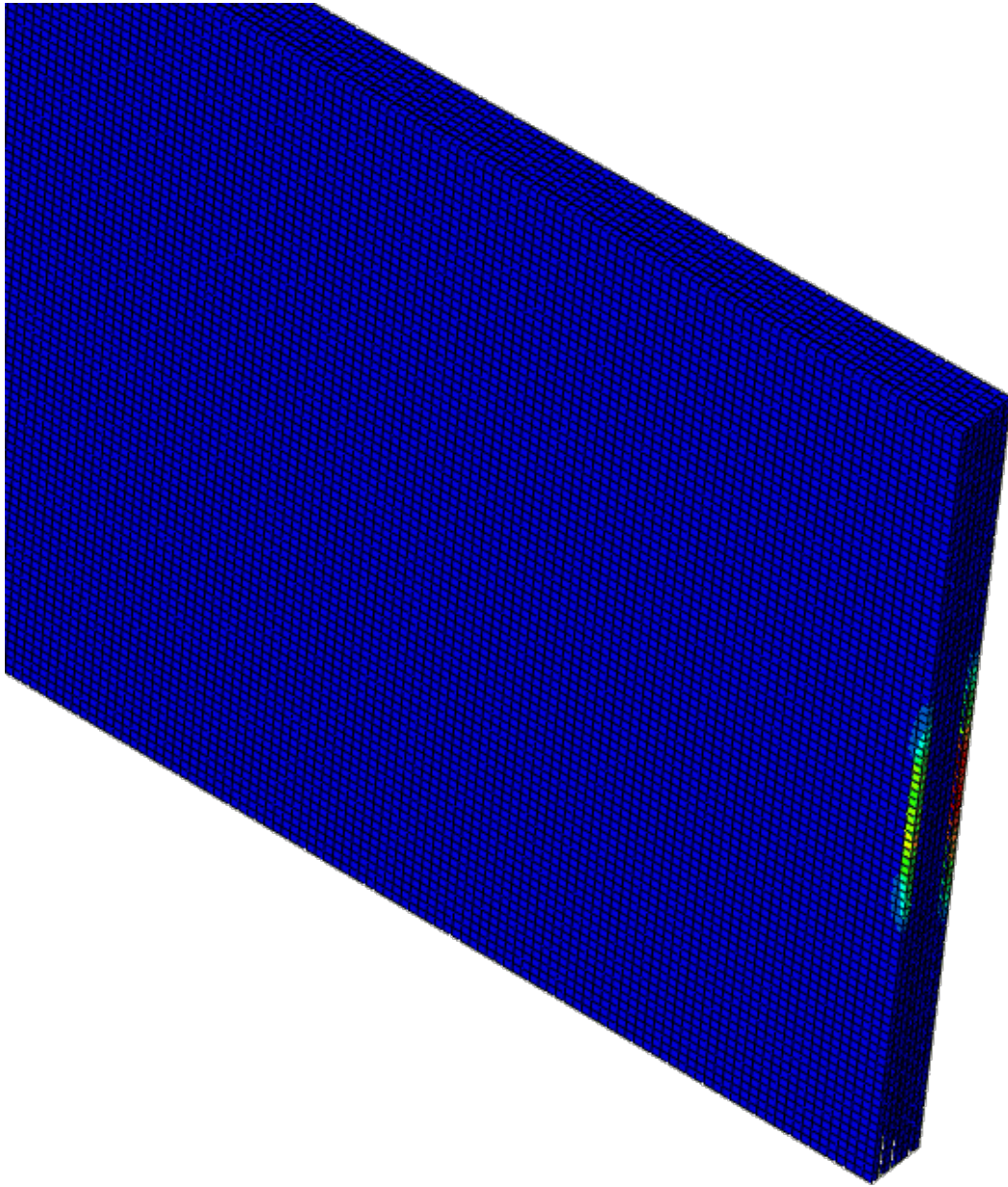


Figure 5.31: Damage initiation in surface plies for AS4-PEEK.

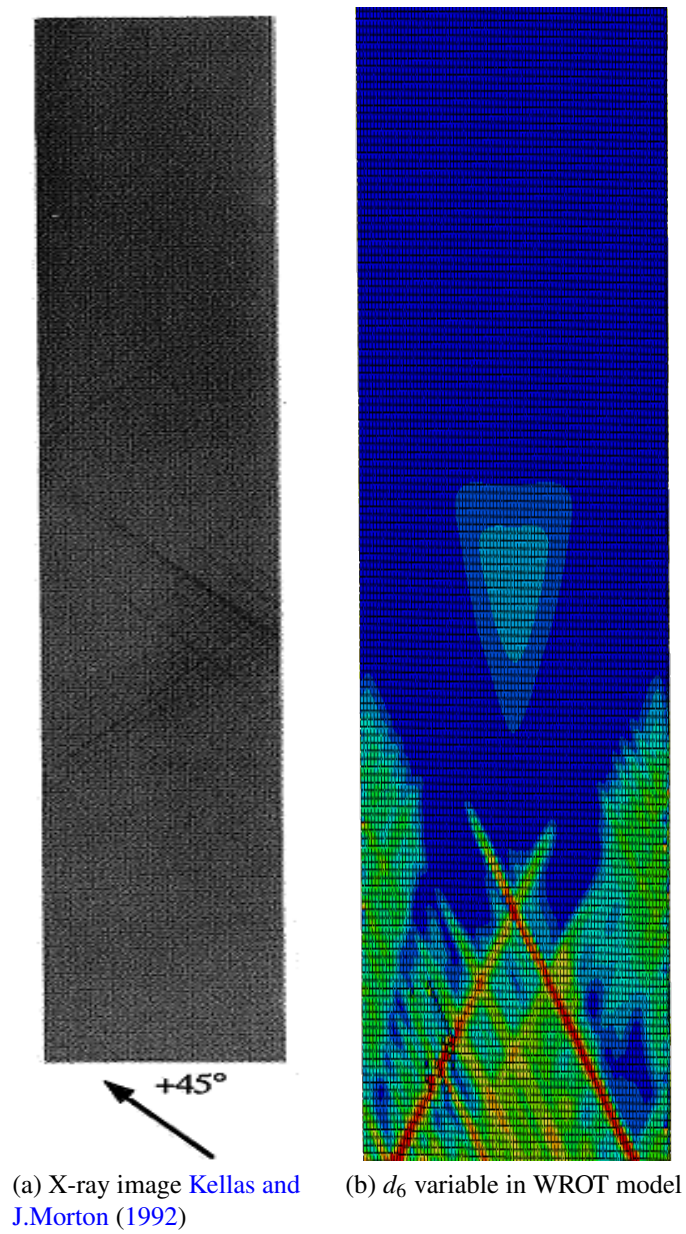
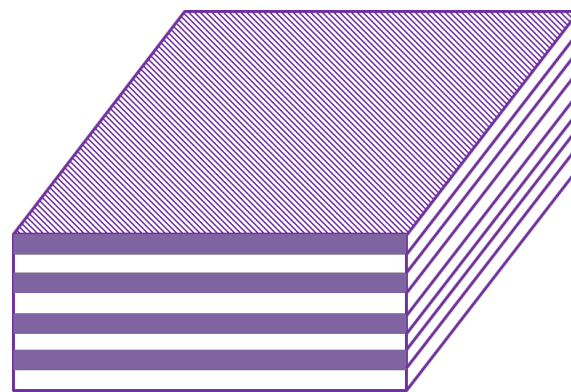
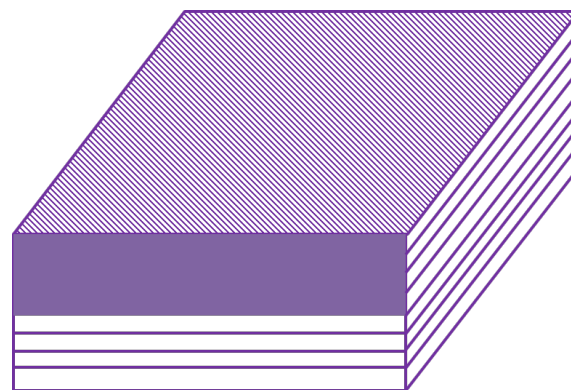


Figure 5.32: cracks in 8-ply AS4-PEEK.



(a) Distributed plies



(b) Blocked plies

Figure 5.33: Distributed ply sequence versus blocked ply sequence.

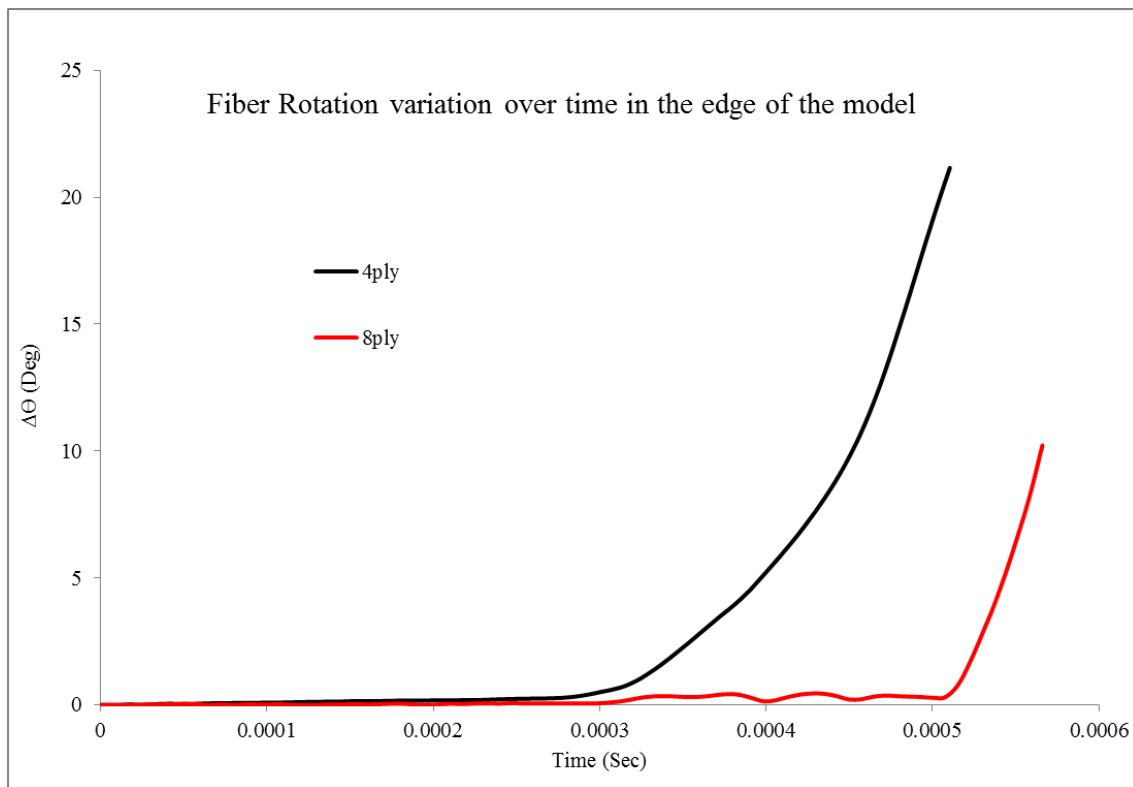


Figure 5.34: Number of plies effects on fiber rotation.

be seen in the diagrams of Fig. 5.36 . It should be mentioned that the very high fiber rotation values are for the time period after final collapse which are not the value which should be measured as fiber rotation, but the value at failure should be considered the fiber rotation.

Then, the thickness of ply effect is investigated. Figure 5.37 shows the reaction forces during the course of loading for different thicknesses subjected to tension and Fig.5.38 is the same values for the specimens under compression. It is obvious that increase of thickness will lead to higher failure capacity.

Figures 5.39 and 5.40 depict the effect of thickness on fiber rotation for tension and compression loading. Similarly, thicker ply laminates reveal higher level of fiber rotation.

Another parameter that standard recommends for fiber rotation is material ductility. This parameter is analyzed in previous section for thermoplastic-based composites where ductility is high. This ductility is applied by fracture toughness in modes I and II. But as mentioned earlier because of lack of data in longitudinal fracture energy, an alternative value from carbon-epoxy is used for it. Here it shown that 5.41 it does not influence on reaction forces in the model. But it does affect on fiber rotation and with higher longitudinal fracture energy experiences less fiber rotation which is also illustrated in Fig. 5.42.

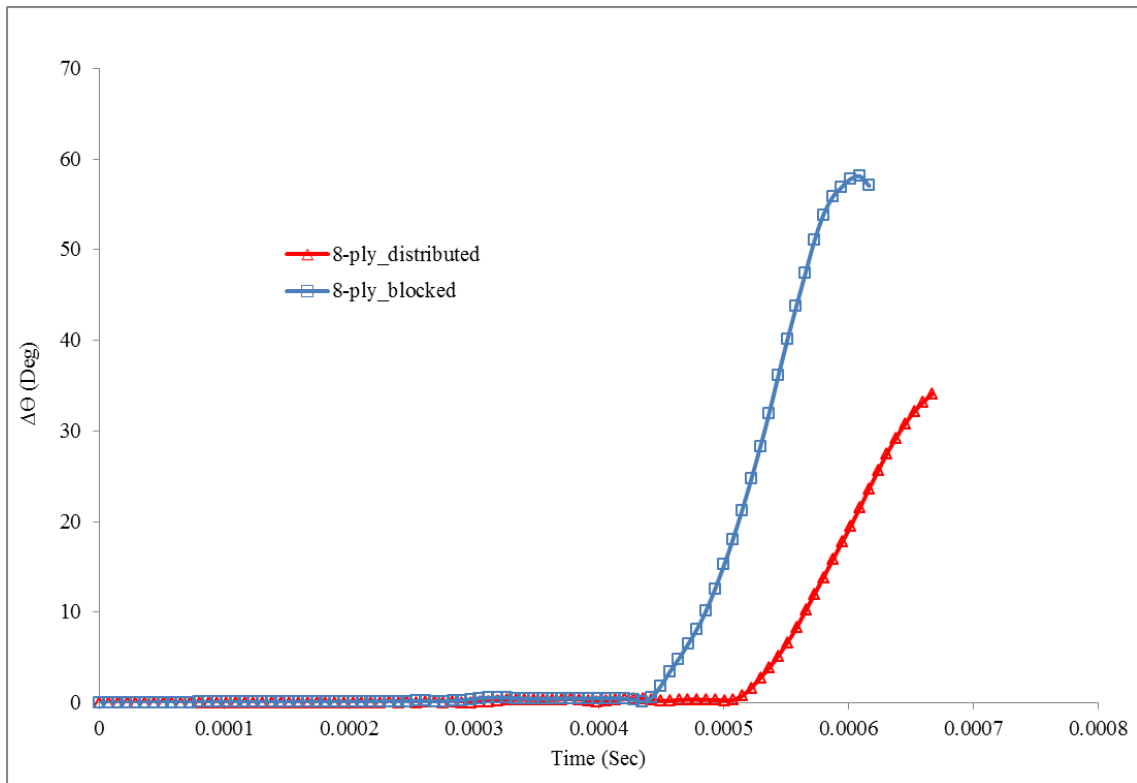


Figure 5.35: Effect of stacking sequence on fiber rotation for 8-ply laminate.

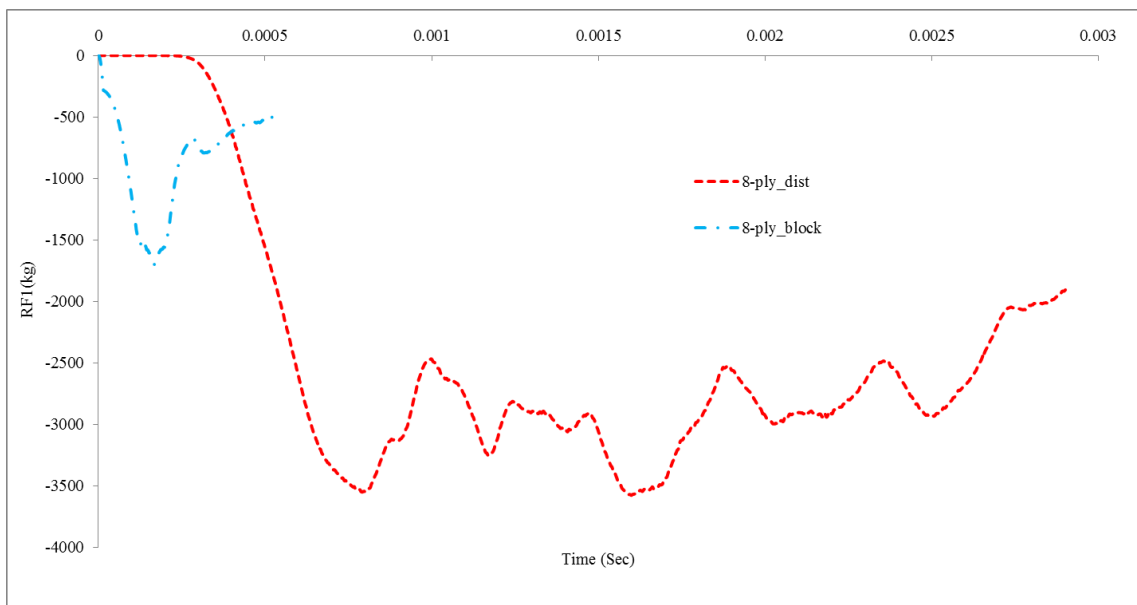


Figure 5.36: Effect of stacking sequence on reaction forces for 8-ply laminate.

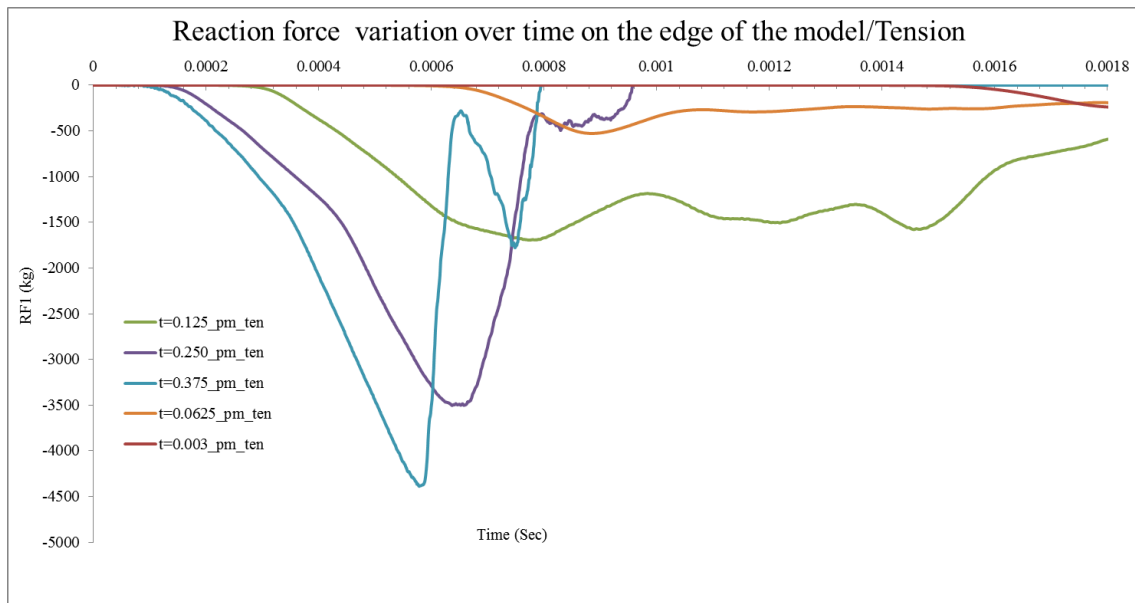


Figure 5.37: Ply thickness effects on reaction forces in tension.

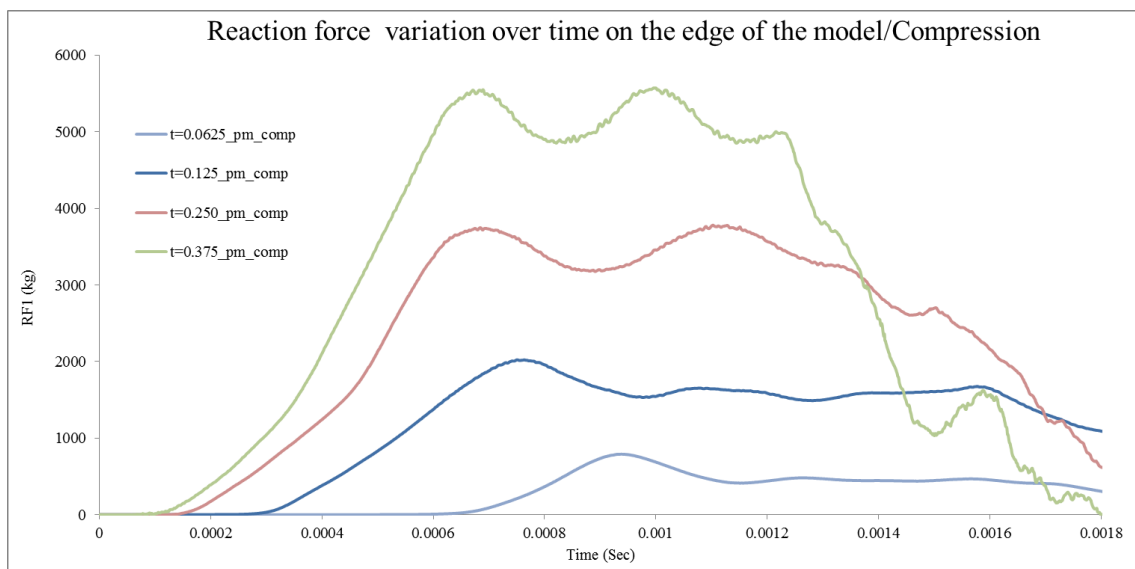


Figure 5.38: Ply thickness effects on reaction forces in compression.

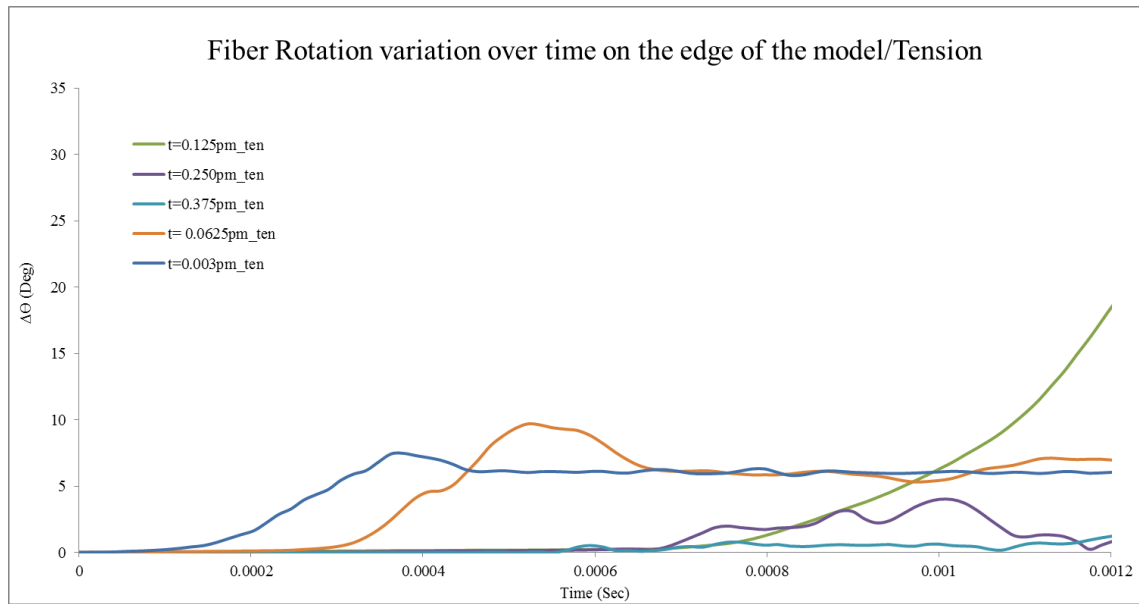


Figure 5.39: Ply thickness effects on fiber rotation in tension.

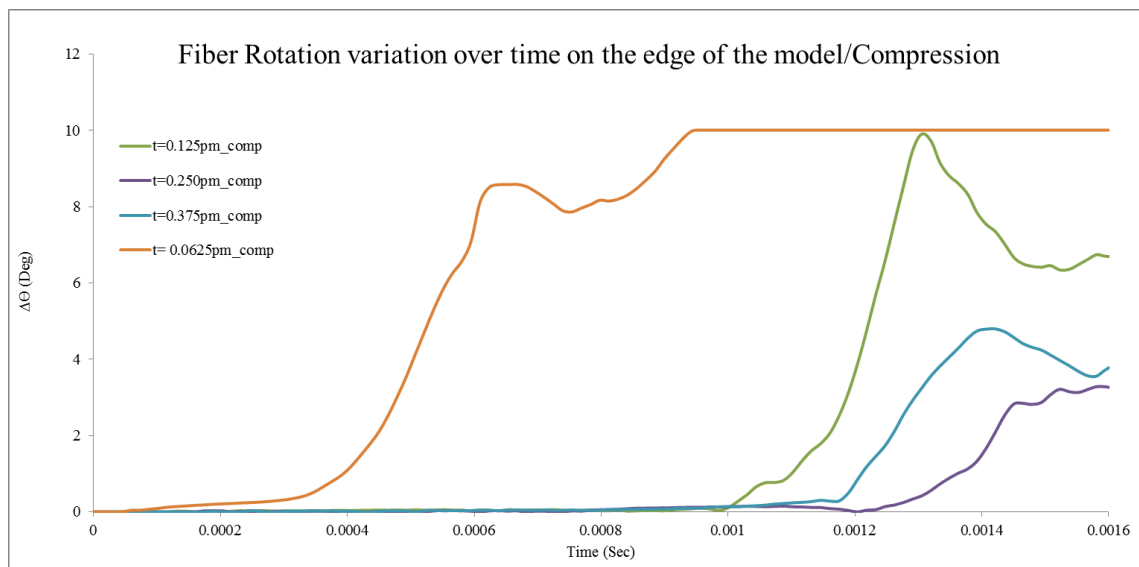


Figure 5.40: Ply thickness effects on fiber rotation in compression.

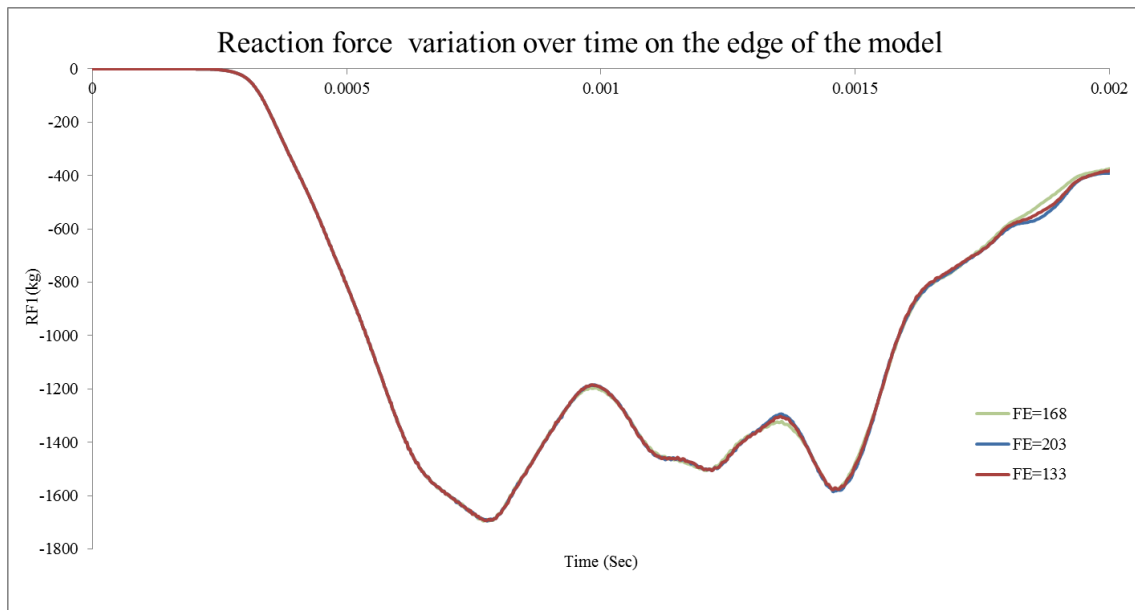


Figure 5.41: Longitudinal fracture energy effects on reaction forces in tension.

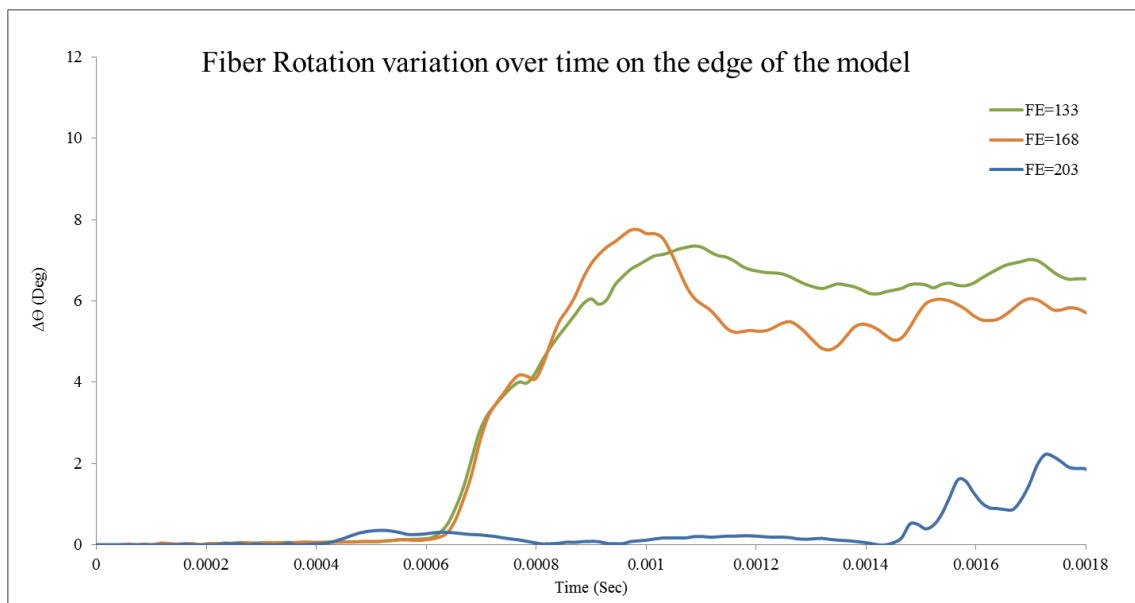


Figure 5.42: Longitudinal fracture energy effects on reaction forces in tension.

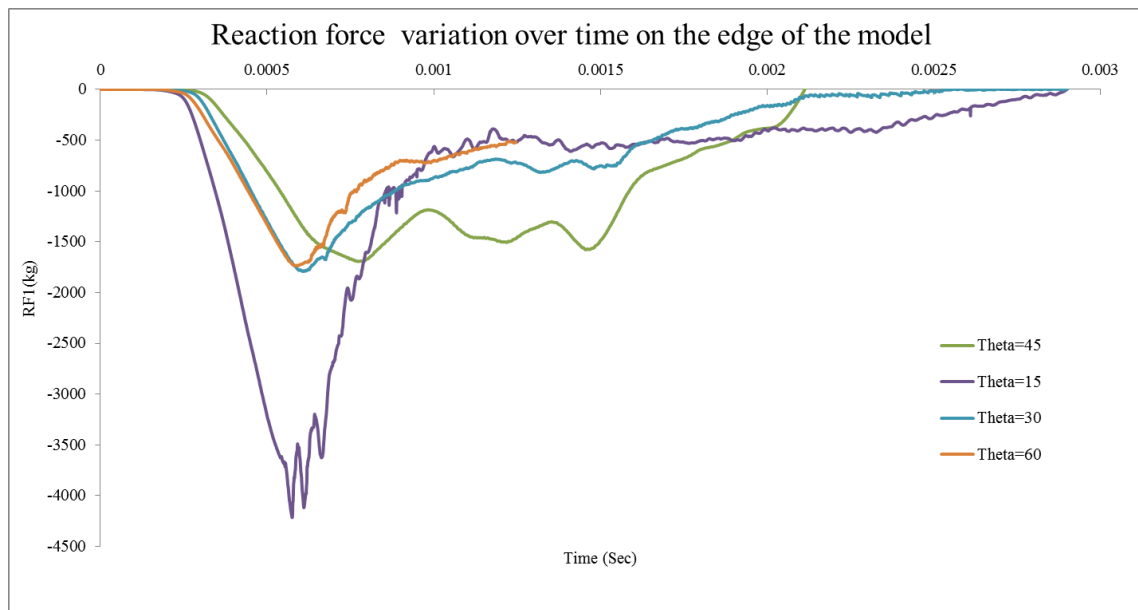


Figure 5.43: Initial fiber orientation effects on fiber rotation in tension.

By increasing the initial fiber orientation in unidirectional composites, the reaction forces will decrease obviously (Fig. 5.43). It is also shown that higher initial value for fiber orientation reduces fiber rotation, except the case in which fiber is aligned with loading direction that does not show fiber rotation. This is reflected in Fig. 5.44 .

5.6 Conclusion

The importance of the fiber rotation inclusion in the constitutive equation of fiber reinforced polymer composites is shown in this chapter. The presented model has the ability to predict the rotation angle and final strength. The model also captures the different fiber rotation behavior in tension and compression with regard to stiffness change. It also improves the ability of the model to predict the damage initiation point and fracture plane. In addition, the model with fiber rotation could analyze the AS4-PEEK laminate as a thermoplastic-based composite and predict its failure strength by less than 1% error, while the model without regarding rotation was not useful in this case.

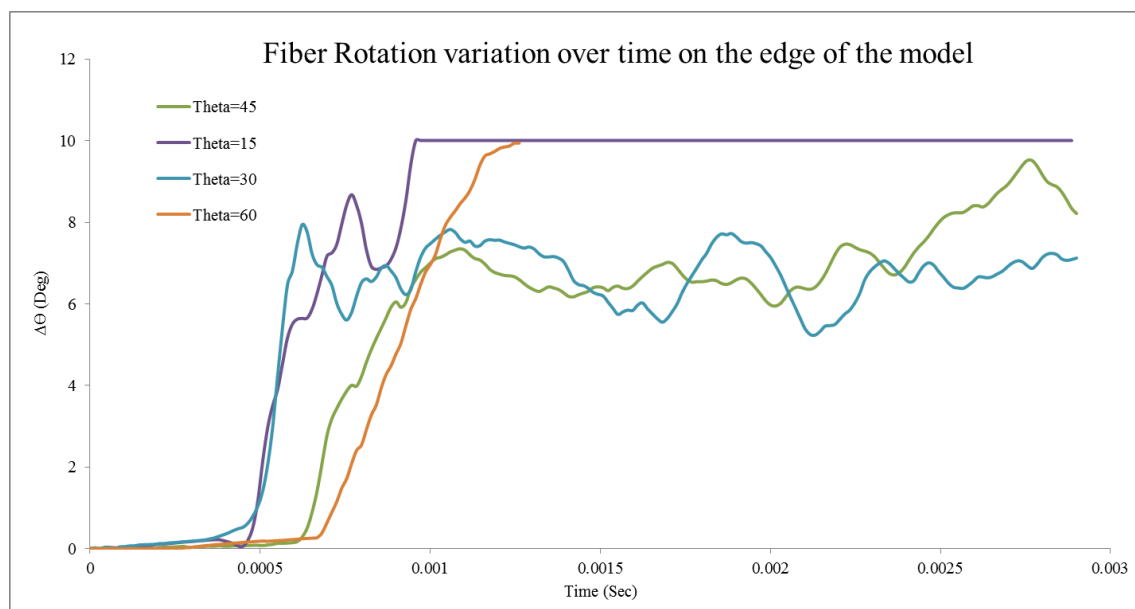


Figure 5.44: Initial fiber orientation effects on fiber rotation.

Chapter 6

Damage analysis of out of plane undulated fiber composites

It is very important to widen the design capacity by analyzing different configurations of composite materials. Out of plane undulated fiber composite is one category of the variable stiffness composites which has attracted research community attention. Waved fibers in this type can be as an unintentional result of production process or a deliberate design purpose. In this research, a 3 dimensional constitutive model is proposed for continuum damage analysis of through thickness curvilinear fiber composites. Larc failure criteria is used in order to determine the damage onset which is a well-established criteria for composite structures. The model is developed by 2 different approaches. In the first approach, it is assumed that the wave shape is fixed during the analysis, while in the second one the geometrical non-linearity is taken into account. Damage evolution is based on the Maimi continuum damage model. Several numerical examples are implemented and results are compared together and the experimental data. There is a good agreement between the second approach and the experiments. Finally a parametric study is conducted and the results are reported.

6.1 Introduction

Exploring the performance of different categories of composite materials provide the opportunity to discover new applications for this kind of materials and enhancing the current usages. Variable stiffness composite structures are a kind of composites which are being used in aerospace and wind turbine industries. They are involved higher design and manufacturing costs. The higher design cost is due to the inordinately large number of design variables required to define variable orientations and thicknesses and additional con-

straints required for maintaining the continuity in the structure, which implies a need for higher computational resources compared to the constant stiffness design (Ghiasi et al., 2009), (Ghiasi et al., 2010). Variable stiffness can be as a result of design desires which are constructed by tow placement machine or unintended consequence of manufacturing process. The variation in the stiffness can be caused by different sources as thickness change across lamina, material change, or fiber undulation. The focus of this research is on the later one. Fiber undulation or waviness can be in plane of lamina or out of plane through thickness.

Part of the research available in the literature has devoted to model the stiffness distribution of the undulated fiber composites. Hyer and Lee (Hyer and H. H. Lee, 1991) approached in-plane undulated fiber composites by modeling the laminate as a group of finite elements with piece-wise continuous fiber directions to implement the waviness. They tried to achieve an optimized design for buckling resistance of the laminate. While a substantial increase of buckling load was produced, the optimal designs had a configurations that would be difficult or impossible to manufacture. Nagendra et al. (Nagendra et al., 1995) looked specifically at the Cincinnati Milacron tow-placement machine and proposed a method for modeling the fiber orientations. The goal of their research was to develop a method of optimizing tow paths. The paths were defined by passing a single curve through a set of control points based on a single cubic NURBS (Non-Uniform Rational B-Splines) curve. Another method of defining the tow paths was presented by Gurdal (Abdalla et al., 2009) and Abdalla ,(Gurdal, 1991). In this method, the fiber-orientation angles varies linearly along the length of the composite laminate. A reference path was defined that passed through the center of a flat rectangular plate. The path was then shifted a uniform distance to define fiber paths for the entire plate. In an extension of that research, Waldhart (Waldhart, 1996) examined two different possible methods of using the formulation proposed by Gurdal. The first method was the same shifted method used by Abdalla et al. (Abdalla et al., 2009) and the second method was to create new paths that are parallel to the reference fiber. An interval-search method was used to determine the distance of the adjacent paths from the reference path. In order to increase the number of design variables and hence expand the available design space, fiber paths can be expressed in terms of more complex functions such as Lobatto polynomials, as has been done by Alhajahmad et al.(Alhajahmad et al., 2008). In the formulation presented by Alhajahmad et al. the definition of non-linear fiber angle variation along both spatial surface coordinates is possible. Klees et al. (Klees et al., 2009) use hierarchical shape functions, based on Lobatto polynomials, to express fiber angle distributions over the plate. Similarly to the formulation presented by Alhajahmad et al. (Alhajahmad et al., 2008), the number of design variables can be increased by increasing the polynomial or-

der. An additional advantage of the formulation presented by Klees et al. (Klees et al., 2009) is that curvature estimates, which are required to apply manufacturing constraints, can be efficiently computed.

Once the structural stiffness distribution has been defined, the desired response information must be calculated which is typically done using analytical, semi-analytical, and numerical methods. The Ritz method, which is a semi-analytical direct method for finding approximate solution to boundary value problems, has been applied extensively in the past, to compute vibration and buckling modes of variable stiffness composites. However, to compute buckling modes of variable stiffness laminates, the complex pre-buckling plane stress state must first be modeled. Martin and Leissa (Leissa and A.F. Martin, 1990) are among the first to develop a procedure based on the Ritz method to determine both stresses and displacements for sheets with arbitrary fiber spacing. Kassapoglou (Kassapoglou, 1990) presents a Rayleigh-Ritz solution to compute buckling loads of panels with an arbitrary number of patches. The energy of a panel is formulated in terms of the out-of-plane displacements. Minimizing the energy with respect to the unknown coefficients in the expression for displacement results in an eigenvalue problem the solution of which yields the buckling load.

Hyer and Lee (Hyer and H. H. Lee, 1991) were the first to use the finite element method to design variable stiffness structures. Due to its general applicability, this approach is used extensively for variable stiffness design. Finite difference discretization has also been implemented by Khani et al. (Khani et al., 2011) to solve several structural problems. Superior structural performance of variable stiffness design versus constant stiffness design have been demonstrated for different properties such as buckling capacity (Hyer and H. H. Lee, 1991), (Abdalla et al., 2009), (Setoodeh et al., 2008), elastic behavior (Gurdal and R. Olmedo, 1993), stiffness (Setoodeh and M.M. Abdalla, 2006), compressive buckling and first-ply failure (Lopes et al., 2008), maximum fundamental frequency (Blom et al., 2008) and post-buckling progressive damage (Lopes et al., 2007). Variable stiffness design also provides flexibility for trade-off between different structural properties (Gurdal et al., 2008). Akhavan et al. Akhavan et al. (2014) also analyzed the damage onset and deflection of in-plane curvilinear fiber under vibration and impact by means of Tsai-Wu criteria. The tow model in this work is also from Abdalla et al. (2009).

Most of the applications of composite materials have been limited to thin sections. However, there is growing interest in thick composite materials especially for primary structures. Fiber waviness is one of the manufacturing defects frequently encountered in thick composite structures. It results from local buckling of prepreg or from wet hoop-wound filament strands under the pressure exerted by the overwrapped layers during the filament winding process or from the lamination residual stress built up during curing

(Hsiao and I.M. Daniel, 1996c) . Its characteristics can be represented by the through thickness undulation of fibers within a thick composite laminate. A number of studies have been conducted on the behavior of composites with out of plane fiber waviness. Shuart (Shuart, 1989) considered both in-plane and out of plane waviness in an analytical and experimental investigation for multi-directional laminates and analyzed failure of selected compression loaded specimens. A general nonlinear theory is presented for predicting a laminate compressive strength and failure mode. The theory also includes fiber scissoring. Bogetti et al. (Bogetti et al., 1993) developed an analytic model, based on a three-dimensional laminated media analysis, to predict the effective nonlinear laminate behavior associated with ply waviness. They studied the influence of ply waviness with nonlinear shear material response on the mechanical performance of composite laminates. The nonlinear analysis revealed significant nonlinearity in the stress/strain response of both the wavy ply and crossover region configurations investigated. Telegadas and Hyer (Telegadas and M.W. Hyer , 1992) conducted finite element analysis to study influence of layer waviness on the stress state in hydrostatically loaded cylinders. Chou et al. (Chou and K. Takahashi , 1987) predicted the tensile stress-strain response of wavy fiber composites. An elastic constitutive model was developed for flexible fiber composites composed of continuous curved fibers and ductile matrices. The prediction of non-linear stress/strain responses of the composites was performed by a stepwise incremental analysis. Hsiao and Daniel (Hsiao and I.M. Daniel, 1996c), (Hsiao and I.M. Daniel, 1996a) , (Hsiao and I.M. Daniel, 1996b) investigated the compressive behavior of thick composites with fiber waviness. An investigation has been done on the effect of fiber waviness on the stiffness and strength reduction of unidirectional composites under compressive loading using constitutive relations developed by Hsiao and Daniel (Hsiao and I.M. Daniel, 1996c). These studies analyzed the elastic behavior of the material by analytic method using integration of the compliance, S , over one period of wave as:

$$\bar{\epsilon}_x = \frac{1}{L} \left\{ \int_0^L S_{xx} dx \right\} \bar{\sigma}_x. \quad (6.1)$$

This integration will lead to 5 relations for integration constants in linear case and 12 constants for non-linear case that makes computer programming tedious and time consuming due to high number of parameters and equations. Hsiao and Daniel (Hsiao and I.M. Daniel, 1996b) also calculated the damage onset in the material based on the elastic formulation developed by (Hsiao and I.M. Daniel, 1996c) . The failure criteria used in that research is Tsai-wu which is not a well-established criteria for composites.

Mandell et al. Mandell et al. (2003) analyzed the effects of fiber waviness on wind turbine blades. Since blades must be produced at a low finished cost, only relatively

inexpensive resins, reinforcement sand processes are competitive. The low cost materials and processes combined with strength-driven designs produce a significant concern with the effects of flaws on the various strength-based properties. They reported the effects of both in-plane and through-thickness fiber waviness under compressive static and fatigue loading.

In this research a continuum damage model is proposed to analyze the behavior of out of plane fiber undulation in fiber reinforced polymer composites. Two constitutive models are developed based on material and geometrical non-linearity of the model. The constitutive models are implemented by finite element method and the results are analyzed and compared with the experimental data.

6.2 Constitutive model

The constitutive equation is developed to analyze elastic and damage behavior of out of plane undulated fiber composites. The method which is used is possible to apply to any initial fiber undulation including in-plane tow-placed fiber composites. The initial configuration of the fibers is assumed to be uniform sinusoidal, while the fibers are shifted parallel to each other. The initial wavelength of the fiber curvature is λ_0 and its amplitude is a_0 . Two approaches for the modeling are established. The first approach assumes that the wave configuration is fixed during the course of the loading, while the second one regards the geometrical non-linearities which arise due to the fiber deformation. In the second approach, the current wavelength is λ_l that is evolved from λ_0 , and the current amplitude is also a_l which similarly evolved from a_0 (Fig. 6.1).

Given the above assumptions, it is possible to calculate the out of axis angle for the fibers, the same way as (Chuna et al., 2001), in order to achieve the material orientation. The out of axis angle, θ_l , for the any point in terms of the first coordinate element of it reads:

$$\theta_l(x) = \tan^{-1} \left(\frac{2\pi a_l}{\lambda_l} \cos\left(\frac{2\pi x}{\lambda_l}\right) \right), \quad (6.2)$$

where, θ_l is fixed during loading for the first approach and it is varying in the second approach that regards geometrical non-linearities. Having θ_l at hand, the undulation matrix, $\mathbf{U}$, which represent the fiber orientation around y-axis will be:

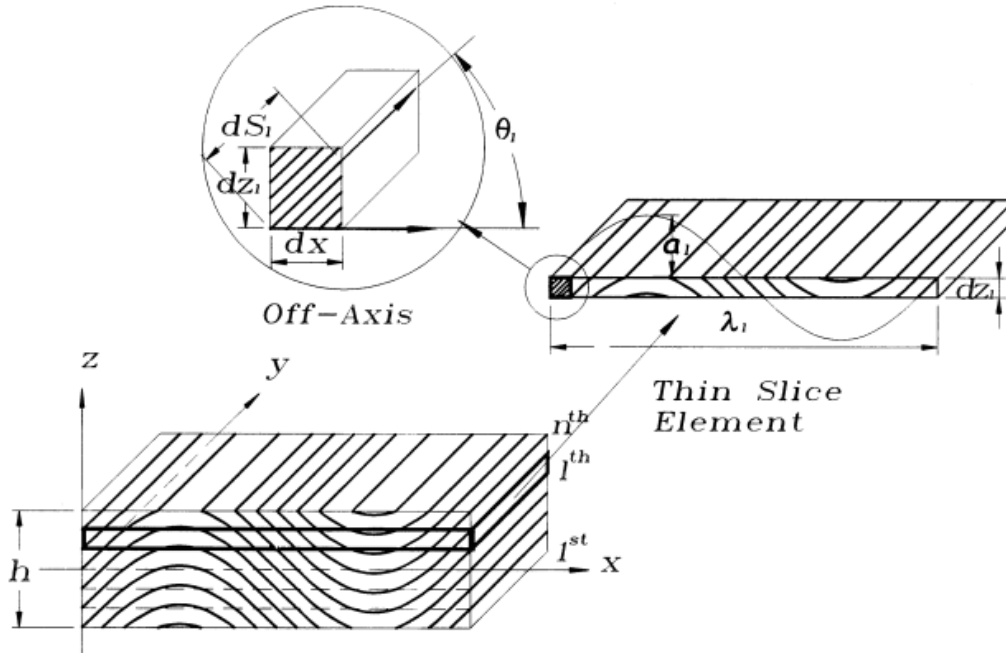


Figure 6.1: Schematic of out of plane undulated fiber composite.(Chuna et al., 2001)

$$\mathbf{U}(x) = \begin{bmatrix} \cos \theta_l & 0 & -\sin \theta_l \\ 0 & 1 & 0 \\ \sin \theta_l & 0 & \cos \theta_l \end{bmatrix}. \quad (6.3)$$

The elastic behavior and the damage of out of plane undulated fiber composites are studied using the model proposed by the authors for the damage in the presence of fiber rotation [Eskandari et al. \(2016a\)](#), [Eskandari et al. \(2016b\)](#). In that work, the fiber rotation and the change in principal material orientation due to deformation is calculated for every point in the specimen and then it is incorporated in Maimi continuum damage model ([Maimi et al., 2007a](#)), ([Maimi et al., 2007b](#)), ([Camanho et al., 2007](#)). In this research, the material direction is already variable at the initial of the analysis because of the manufacturing process. Therefore, the undulation matrix, $\mathbf{U}$, which is proposed in Equation 6.3, is used instead of the fiber rotation matrix at that work. The undulation matrix will be used to rotate strain to the material direction at each point and after calculating of the stiffness whether it is degraded due to damage or not, the stress will be achieved in the material direction. Then, the stress will rotate back to the direction of the strain to have strain and stress in consistence with each other. Hence, the constitutive equation reads:

$$\begin{aligned}
& \begin{bmatrix} \sigma_1^X \\ \sigma_2^X \\ \sigma_3^X \\ \sigma_{12}^X \\ \sigma_{13}^X \\ \sigma_{23}^X \end{bmatrix} = \begin{bmatrix} U_{11} & 0 & 0 & U_{21} & U_{31} & 0 \\ 0 & U_{22} & 0 & U_{12} & 0 & U_{32} \\ 0 & 0 & U_{33} & 0 & U_{31} & U_{23} \\ 0 & U_{21} & 0 & U_{11} & 0 & U_{31} \\ 0 & 0 & U_{31} & 0 & U_{11} & U_{21} \\ 0 & 0 & U_{32} & 0 & U_{12} & U_{22} \end{bmatrix} \begin{bmatrix} \frac{Q_{11}}{1-d_1^X} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & \frac{Q_{22}}{1-d_2^X} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & \frac{Q_{33}}{1-d_3^X} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Q_{44}}{1-d_4^X} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{Q_{55}}{1-d_5^X} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Q_{66}}{1-d_6^X} \end{bmatrix} \\
& \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} U_{11} & 0 & 0 & U_{12} & U_{13} & 0 \\ 0 & U_{22} & 0 & U_{21} & 0 & U_{23} \\ 0 & 0 & U_{33} & 0 & U_{31} & U_{32} \\ 0 & U_{12} & 0 & U_{11} & 0 & U_{13} \\ 0 & 0 & U_{13} & 0 & U_{11} & U_{12} \\ 0 & 0 & U_{23} & 0 & U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \\ \epsilon_{13}^X \\ \epsilon_{23}^X \end{bmatrix} \\
& = \begin{bmatrix} \cos \theta_l & 0 & 0 & 0 & \sin \theta_l & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \theta_l & 0 & \sin \theta_l & 0 \\ 0 & 0 & 0 & \cos \theta_l & 0 & \sin \theta_l \\ 0 & 0 & \sin \theta_l & 0 & \cos \theta_l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{Q_{11}}{1-d_1^X} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & \frac{Q_{22}}{1-d_2^X} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & \frac{Q_{33}}{1-d_3^X} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Q_{44}}{1-d_4^X} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{Q_{55}}{1-d_5^X} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Q_{66}}{1-d_6^X} \end{bmatrix} \\
& \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \cos \theta_l & 0 & 0 & 0 & -\sin \theta_l & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \theta_l & 0 & \cos \theta_l & 0 \\ 0 & 1 & 0 & \cos \theta_l & 0 & -\cos \theta_l \\ 0 & 0 & -\cos \theta_l & 0 & \cos \theta_l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
& \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1^X \\ \epsilon_2^X \\ \epsilon_3^X \\ \epsilon_{12}^X \\ \epsilon_{13}^X \\ \epsilon_{23}^X \end{bmatrix}, \tag{6.4}
\end{aligned}$$

where, Q_{ij} represents the stiffness elements for orthotropic material and d_i^X is variable

that shows the damage parameters. The superscript of X implies that the variable is defined in the reference coordinate system, while the superscript, x , presents the coordinate system corresponding to the fiber orientation. When the material is still in the elastic domain, damage parameters are zero and when the related failure criterion according to Larc criteria (Davila et al., 2005), (Pinho et al., 2005) is met, they will be triggered. The Larc failure criteria is summarized in Tables 6.1 and 6.2. In the Tables, is index refers to in-situ strength of composites proposed by Camanho et al. (Camanho et al., 2006) and g is toughness ratio defined by the fraction of fracture energy in the first and the second mode, $\frac{G_I}{G_{II}}$, while η is friction parameter. The damage parameters are evolved based on the damage laws that are concluded in Table 6.3. Each damage variable should be calculated by means of strain and stress which are rotated to current fiber direction using undulation matrix. More details about the model can be found in (Eskandari et al., 2016a), (Eskandari et al., 2016b). For the first approach, θ_l is fixed and is obtained using Equation 6.2, for a given point. The way to calculate it for the case including geometrical non-linearity, will be discussed later on.

Table 6.1: Larc04 failure criteria in fiber failure mode.

Loading condition	FI_F
$\sigma_{11} \geq 0$	$\frac{\sigma_{11}}{X^T}$
$\sigma_{11} < 0$ and $\sigma_{2m2m} < 0$	$\langle \frac{\tau_{12}^m + \eta^L \sigma_{22}^m}{S_{is}^L} \rangle$
$\sigma_{11} < 0$ and $\sigma_{2m2m} \geq 0$	$(1 - g) \frac{\sigma_{22}^m}{Y_{is}^T} + g \left(\frac{\sigma_{22}^m}{Y_{is}^T} \right)^2 + g \left(\frac{\tau_{12}^m}{S_{is}^L} \right)^2$

Table 6.2: Larc04 failure criteria in matrix failure mode.

Loading condition	FI_M
$\sigma_{22} \geq 0$	$(1 - g) \frac{\sigma_{22}^m}{Y_{is}^T} + g \left(\frac{\sigma_{22}^m}{Y_{is}^T} \right)^2 + g \left(\frac{\tau_{12}^m}{S_{is}^L} \right)^2$
$\sigma_{22} < 0$ and $\sigma_{11} < -Y^C$	$\left(\frac{\tau_{eff}^m}{S^T} \right)^2 + \left(\frac{\tau_{eff}^m}{S_{is}^L} \right)^2$
$\sigma_{22} < 0$ and $\sigma_{11} > -Y^C$	$\left(\frac{\tau_{eff}^T}{S^T} \right)^2 + \left(\frac{\tau_{eff}^L}{S_{is}^L} \right)^2$

During applying of the tension or compression, the geometrical parameters of the undulated fibers will be changed which leads to geometrical non-linearity (Figure 6.2). Chuna et al. (Chuna et al., 2001) proposed a constitutive relation consisting of non-linearities with regard the geometry of the fiber. The method which is used in research for modeling the fiber waviness is similar to (Hsiao and I.M. Daniel, 1996a), (Hsiao and

Table 6.3: Damage evolution laws.

Failure mechanism	d_i	
	Tension	Compression
Fiber	$1 - (1 - d_{1+}^L)(1 - d_{1+}^E)$	$1 - \frac{1}{r_{1-}} \exp[A_{1-}(1 - r_{1-})]$
Matrix	$1 - \frac{1}{f_{2+}(r_{2+})} \exp[A_{2+}(1 - f_{2+}(r_{2+}))]$	$1 - \frac{1}{r_{2-}} \exp[A_{2-}(1 - r_{2-})]$
Shear	$1 - \frac{1}{r_{2-}} \exp[A_6(1 - r_{2+})]$	

I.M. Daniel, 1996b) , (Hsiao and I.M. Daniel, 1996c). The method to model geometrical non-linearity by these authors is also used in the current research.

In order to determine the change in the fiber waviness during the loading, it is assumed that the fibers in each thin slice element maintain a similar shape of fiber waviness while varying only their amplitude (a_l) and wavelength (λ_l). The total fiber length in one period of waviness of the l_{th} thin slice element is obtained from the fiber length (ds_l) in an infinitesimally short sub-element of length dx in the l th thin slice element as:

$$\begin{aligned}
S_l &= \int_0^{\lambda_1} ds_l = \int_0^{\lambda_1} \sqrt{dx^2 + dx^2 \tan^2 \theta_l} = \int_0^{\lambda_1} \sqrt{1 + c_1 \cos^2\left(\frac{2\pi x}{\lambda_1}\right)} \\
&= \frac{\lambda_1}{2\pi} \frac{1}{\sqrt{1 - k_1^2}} \int_0^{2\pi} \sqrt{1 - k_1^2 \sin^2 \beta} d\beta \\
&= \lambda_1 \left(1 + 2\left(\frac{k_1^2}{8}\right) + 13\left(\frac{k_1^2}{8}\right)^2 + 90\left(\frac{k_1^2}{8}\right)^3 \right. \\
&\quad \left. + 644.25\left(\frac{k_1^2}{8}\right)^3 + 4708.5\left(\frac{k_1^2}{8}\right)^5 + \dots\right),
\end{aligned} \tag{6.5}$$

where,

$$c_1 = \left(\frac{2\pi a_1}{\lambda_1}\right)^2, \tag{6.6}$$

$$k_1^2 = \frac{c_l}{c_l + 1}. \tag{6.7}$$

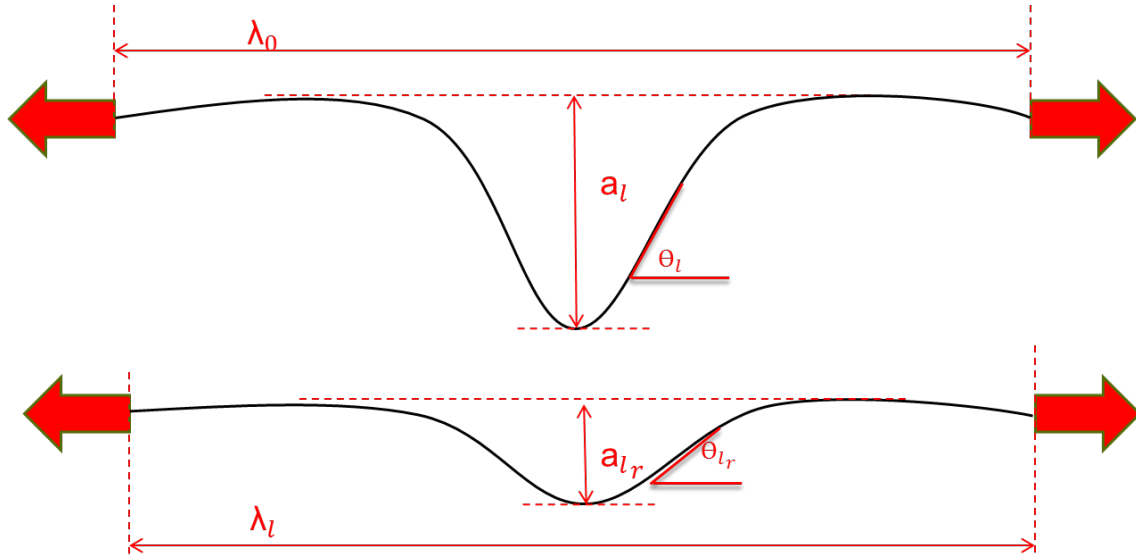


Figure 6.2: The geometrical change in waves caused by loading.

The changed wavelength and length of the fiber in the l_{th} thin slice element are calculated from these strains as:

$$(\lambda_l)_r = (\lambda_l)_0(1 + ((\epsilon_{xx})_l)_r), \quad (6.8)$$

$$(S_l)_r = (S_l)_0(1 + ((\overline{\epsilon_f})_l)_r), \quad (6.9)$$

in which,

$$((\epsilon_{xx})_l)_r = \exp \left(\sum_{i=1}^r ((\Delta \epsilon_{xx})_l)_i \right) - 1, \quad (6.10)$$

$$(\epsilon_f)_l = \cos^2 \theta_l (\epsilon_{xx})_l + \sin^2 \theta_l (\epsilon_{zz})_l + \sin \theta_l \cos \theta_l (\gamma_{xz})_l, \quad (6.11)$$

$$((\overline{\epsilon_f})_l)_r = \exp \left(\sum_{i=1}^r ((\Delta \overline{\epsilon_f})_l)_i \right) - 1, \quad (6.12)$$

The amplitude of waviness in the thin slice element for the r_{th} step loading is determined by Eqs. 6.5-6.7 using the Newton-Raphson method. From these newly determined values of amplitude $((a_l)_r)$ and changed wavelength $((\lambda_l)_r)$ from Equation 6.8, the changed angle $((\theta_l)_r)$ between the tangent to the fiber and the x axis is obtained from Equation 6.2. With this changed angle, calculations from Eqs. 6.5-6.12 are repeated for the $(r+1)_{th}$ step of the loading. These repeated procedures continue till the final value of the loading is reached.

6.3 Material properties

The chosen material in order to pursue the analysis is IM6G/3501-6 Carbon/Epoxy aiming to be able to compare the results with the available data in the literature. The constitutive model needs 4 sets of material properties including elastic properties, unidirectional strengths, in-situ strengths and fracture toughness. The elastic properties and unidirectional strength of the material are presented in Table 6.4 and 6.5 which is collected from Ref. (Hsiao and I.M. Daniel, 1996a).

Table 6.4: IM6G/3501-6 elastic properties.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
169.0	9.0	6.5	0.31

Table 6.5: IM6G/3501-6 unidirectional strengths (MPa).

X_T	X_C	Y_T	Y_C	S_L
2236	1682	62.3	46.2	72.8

Due to the lack of data, the fracture toughness properties of the IM7/8552 Carbon/Epoxy is used in the model. Regarding the similarity of the properties of 2 materials, this would be an acceptable alternative. By means of the formulations provided by (Camanho et al., 2006) and the properties in the Tables 6.4 to 6.6, the in-situ properties have been calculated that are shown in Table 6.7.

Table 6.6: IM7-8552 fracture toughness($\frac{N}{mm}$).

G_1^+	G_1^-	G_2^+	G_2^-	G_6
81.5	106.3	0.28	1.31	0.79

Table 6.7: IM6G/3501-6 in-situ strengths (MPa).

	Embedded ply	Outer ply	Thick ply
Y_T	159.5	100.4	98.68
S_L	114.5	95.5	88.52

6.4 Numerical model

In order to implement the finite element model, commercial software of ABAQUS (?) is used. Three different VUMAT subroutines are developed in order to model different constitutive models discussed earlier. The first subroutine models the polymer composite elastic and damage behavior for straight fibers which is abbreviated by “str”. This routine has been validated in several works by the authors (Maimi et al., 2007a), (Camanho et al., 2007), (Eskandari et al., 2016a) and its results can be used as control data. The second subroutine models the variable stiffness composites when the fibers have out of plane undulation. But this subroutine does not take the wave reconfiguration due to the deformation into account, while it regards material non-linearities. This subroutine will be referred by the acronym of “mat”. The final subroutine uses the second approach in the constitutive model which considers geometrical non-linearities as well, that is called “geo” in this research.

Figure 6.3 illustrates the algorithm for “mat” subroutine. It calculates the stress and strain for any given integration point in the direction of the fiber by calculating the fiber orientation at that particular point in the wavy configuration. It evaluates if the point is failed or not and based on that flows the computation towards elastic or damage section. In the damage section, the active damage parameters in the direction of the fiber, d_i^x , will be calculated which will lead to degrading the stiffness. Then, the final stress will be rotated back to the original direction which the imported strain was in. For the “geo” subroutine the same procedure as “mat” will be followed, but in every integration point instead of θ_l , the updated fiber angle, $(\theta_l)_r$, is calculated for r_{th} step of the loading which varies during the loading unlike the way in “mat” subroutine. Figure 6.4 presents the algorithm to calculate $(\theta_l)_r$ for a given integration point in r_{th} step of the loading. Since

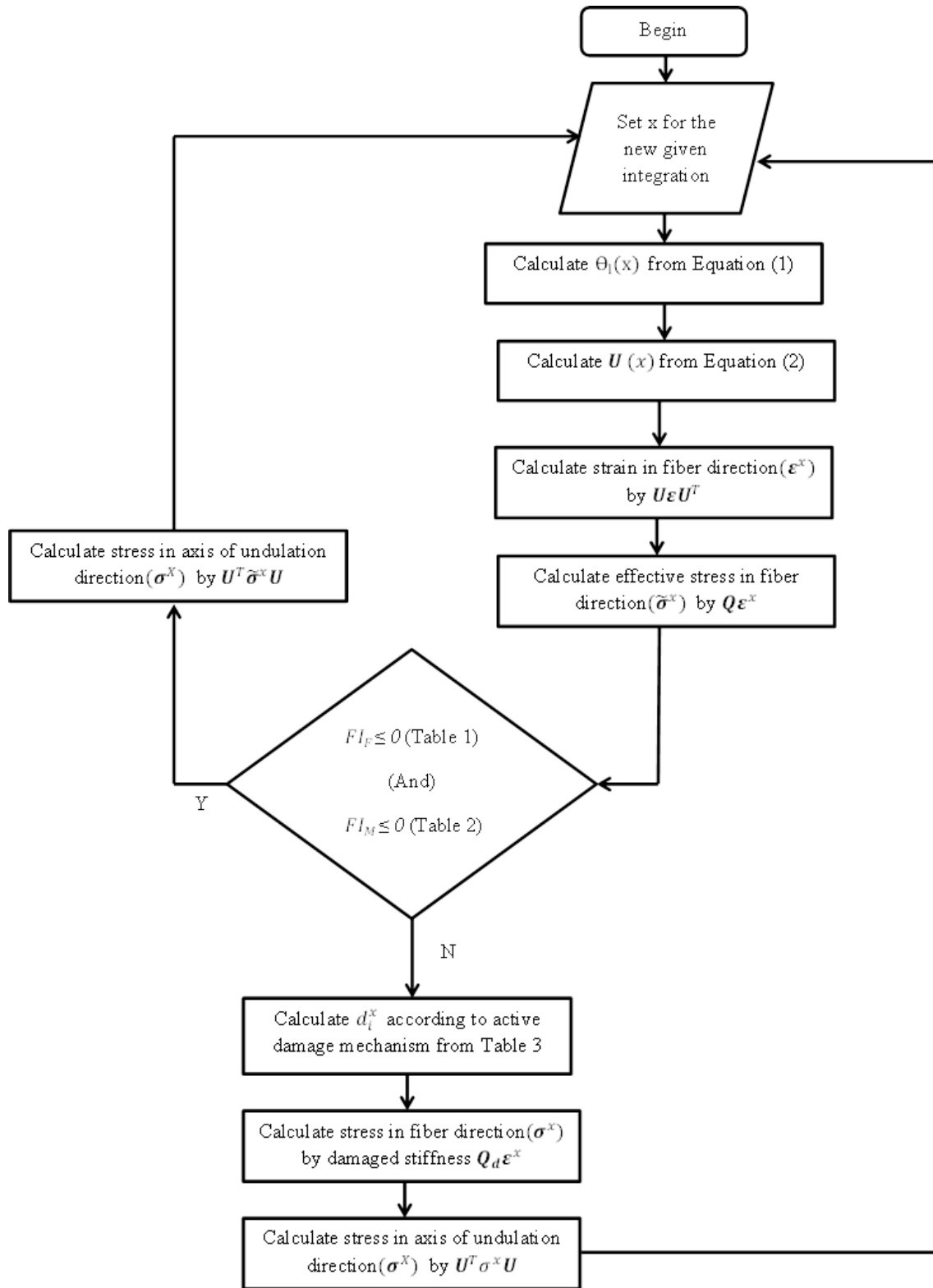


Figure 6.3: Subroutine of damage for out of plane undulated fiber Composite.

the equation is nonlinear, it uses the Newton-Raphson method and it has to be repeated in a loop until reaching in the margin of the error. After calculation of $(\theta_l)_r$, this subroutine will continue the same as the previous subroutine to calculate $(U(x))_r$ and failure along with damage evolution. The process to calculate $(\theta_l)_r$ is being repeated for every loading step.

The models are implemented in explicit analysis regarding the dynamic nature of the fracture. It also enjoys the advantage that there is no need to calculate the inverse of the stiffness matrix which is a big asset in the nonlinear analysis. The geometry of the plies is a squared shape with the dimension of 20 mm. The thickness of plies is the standard thickness of 0.125 mm. The laminates consist of 4 plies with the fiber in the direction of x axis. 40 nodes are devoted for each edge of the specimen that generates meshing with the dimension of 0.5 mm. This size satisfies the general rule that provided in Ref. (Maimi et al., 2007a) in order to ensure energy dissipation. The meshing is 8-node linear brick, reduced integration, hexagonal elements (ABAQUS C3D8R). Their family is 3D stress and their geometrical order is linear. Figure 6.5 depicts the mesh structure of the laminate. The number of total variables in the model is 63,591 and the number of the nodes and the meshes are 18,400 and 21,197, respectively.

6.5 Results and discussions

Different loading conditions for 3 constitutive models are applied with the aim of the analyzing those materials behavior and comparing the results together as well as experimental data. The implemented loading cases include tension and compression in fiber direction and in transverse direction. The amplitude of the wave is 1.19 mm and the wavelength is 27.19 mm. Figure 6.6 shows the force versus displacement during the course of the loading for the polymer composite with different fibers configurations, under tension. As mentioned earlier, the one with straight fibers, referred as “str”, has been validated in several studies and its results can be dealt as a reliable data. It shows that the final failure for “str” is 55,647 N. Waved fibers that considers only material non-linearities is denoted by “mat” and the one which includes reconfiguration of the fibers due to deformation is mentioned by “geo”. Both of these undulated models, as can be seen in the figure, have captured the non-linearities in comparison to “str”. In addition, both of the models reflect that out of plane waviness reduces the failure capacity of the composite materials. This observation has been approved in several research works Chou and K. Takahashi (1987), Hsiao and I.M. Daniel (1996b), (Hsiao and I.M. Daniel, 1996c), (Chuna et al., 2001). This reduction in tension is mainly because the waviness shifts the fiber orientation off

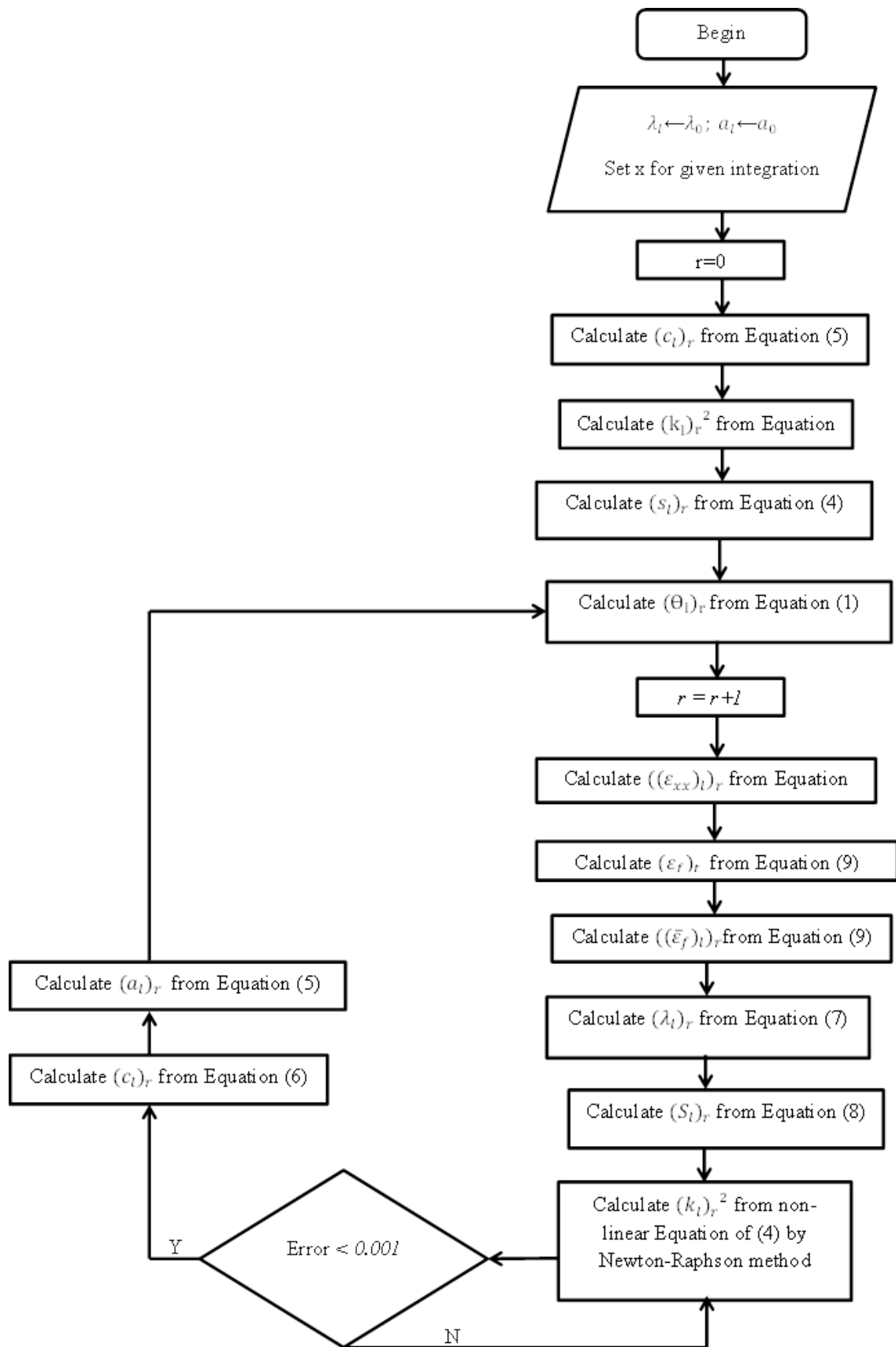


Figure 6.4: Subroutine of geometrical non-linearity.

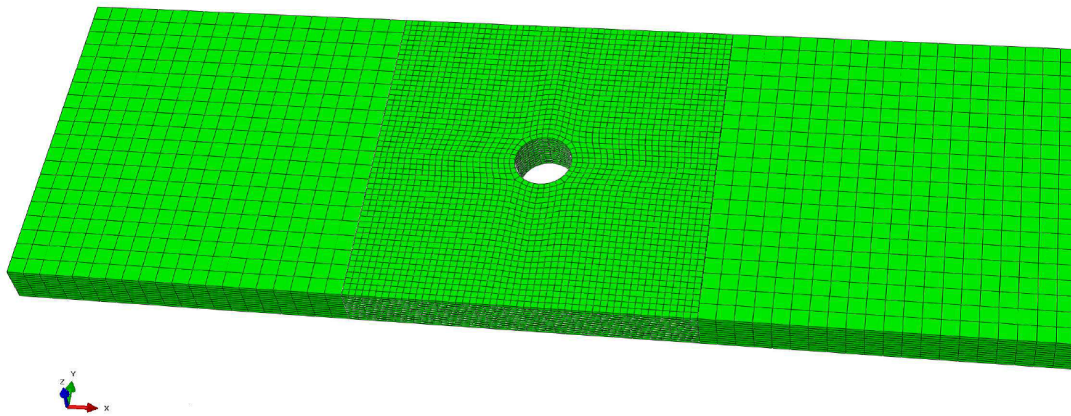


Figure 6.5: Mesh structure of the specimen.

the axis of the ply longitudinal direction, producing matrix dominated failures for plies nominally orientated in the primary load direction, 0° [Mandell et al. \(2003\)](#) .

The first waved constitutive model, “mat”, fails at 34,638.4 N and “geo” fails at 52,447 N. It is reasonable that final failure for “geo” is something between “str” and “mat”, because after applying the tension, the fiber tends to reconfigure in a way that the amplitude of the wave decreases and the wavelength increases, so it tends to be more similar to the straight fiber. Therefore the model which considers this fiber reconfiguration tendency is likely to show a final failure strength that something between the other 2 cases. The “geo” model also should be stiffer than “mat” in the elastic area, which is accomplished in the model, but its stiffness is expected to be lower than “str” while it is not the case in the result. It might be because of the fiber rotation that makes the material stiffer and it is not considered in “str” model while it is going to have effect on it and makes the real material stiffer than what “str” model predicts. In the softening process, the stress also declines faster for the waved cases. Once more, the rate is higher for the case excluding geometrical reconfiguration. Regarding the fact that the “geo” model considers the physical reality of the phenomenon, it is expected to have more reliable results. The out of plane waviness is reflected in the stress pattern on the specimen. Figure 6.7 shows the stress distribution in the “geo” model after failure. The diagram shows the stress across the thickness of the specimen at its edge.

In compression similar trends are also observed, though the difference between the materials is not as high as the tension. Figure 6.8 illustrates the reaction force in relation to the end-displacement for the 3 discussed models in compression. Out of plane undulation decreases the final failure strength for compression as well, which is observed in several experiments [Chou and K. Takahashi \(1987\)](#), [Hsiao and I.M. Daniel \(1996b\)](#) . The reason of this strength reduction in compression is because the fibers maybe put into a geometry

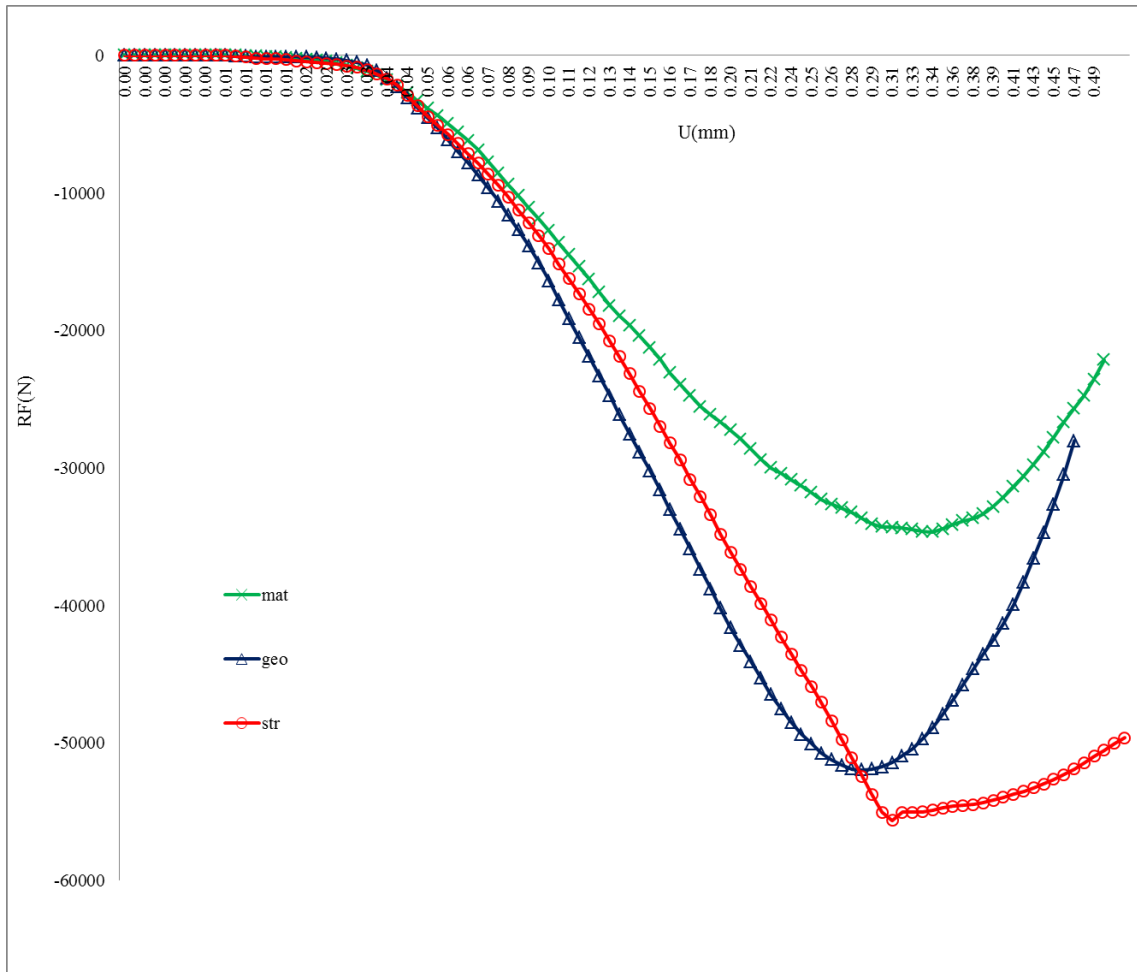


Figure 6.6: Reaction force versus displacement of IMG6/3501-6 in tension for 3 different cases ($\frac{A_0}{L_0} = 0.02$).

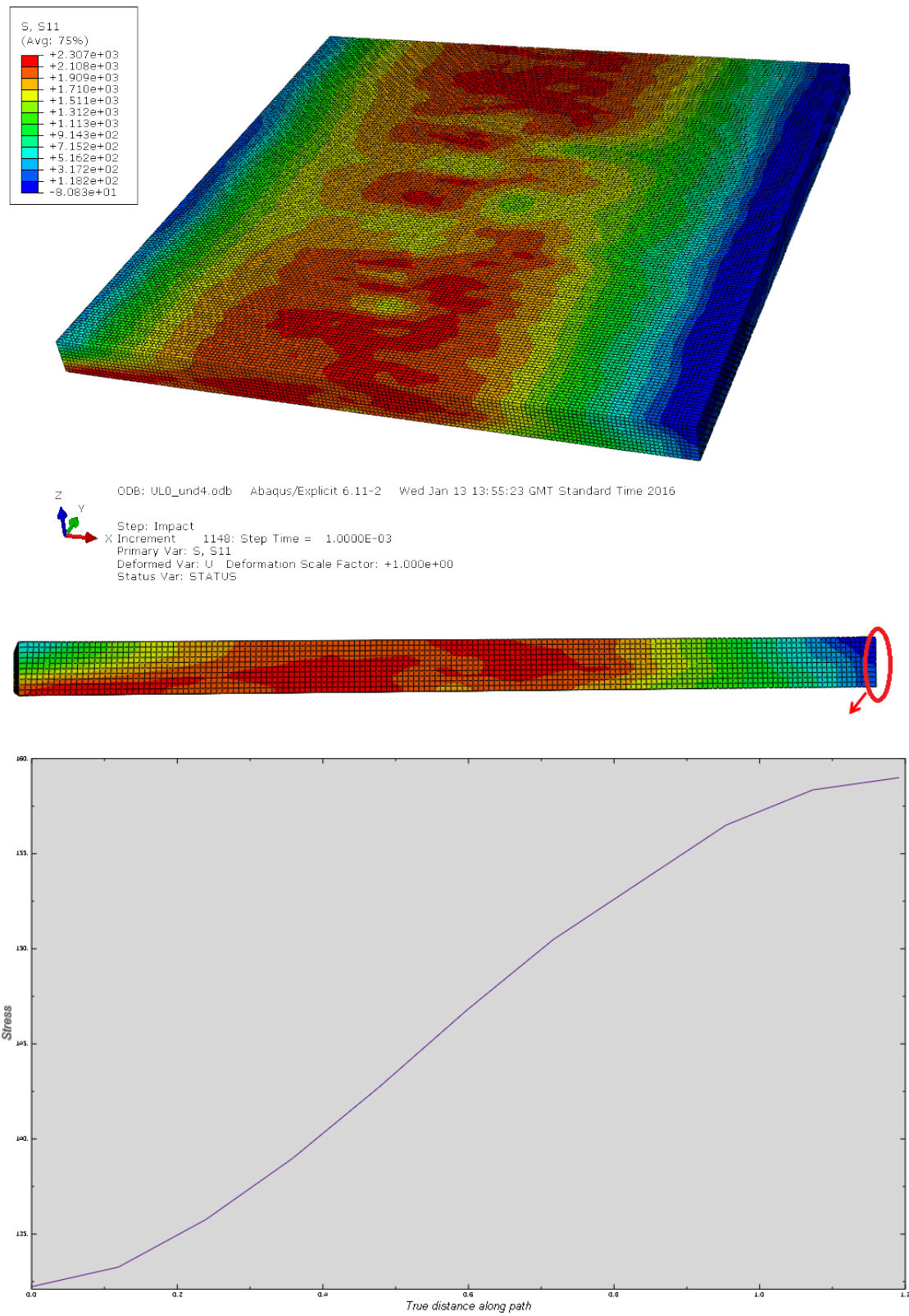


Figure 6.7: Stress in x direction for tension in “geo” model

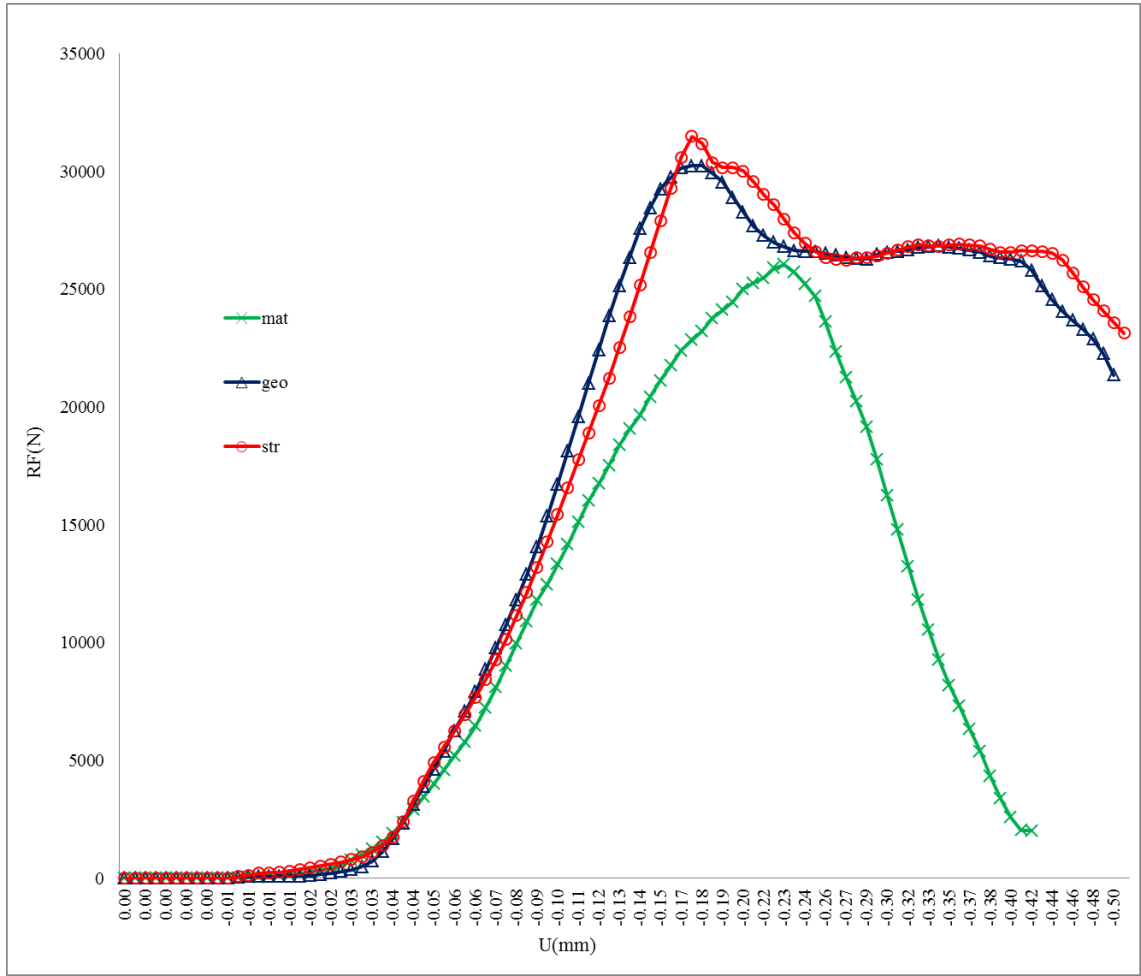


Figure 6.8: Reaction force versus displacement of IMG6/3501-6 in compression for 3 different cases ($\frac{A_0}{L_0} = 0.02$).

which exacerbates the basic fiber, or layer buckling mode of failure [Mandell et al. \(2003\)](#), in addition to what mentioned for the tension. Figure 6.9 compares the stress-strain relation for “geo” model with experimental data in compression. The experimental data is attained from Ref. [Hsiao and I.M. Daniel \(1996b\)](#). There is a good correlation between 2 sets of data. Some oscillations are visible in the softening section of the numerical model curve which is because of data-reduction and in the full data set it is smooth.

Another loading case that is investigated is transverse loading. It is expected that there is no difference between constitutive models in transverse loading since the fiber undulation does not have sensible effect on the transverse direction which is matrix dominant. The results for reaction force-displacement are shown in the Figure 6.10 and as expected, the diagrams are coincidence for straight and undulated fibers. Figure 6.11 presents the comparison between experimental data of stress-strain with the “geo” model for uniform waved undulation under transverse loading, that shows a good agreement between them.

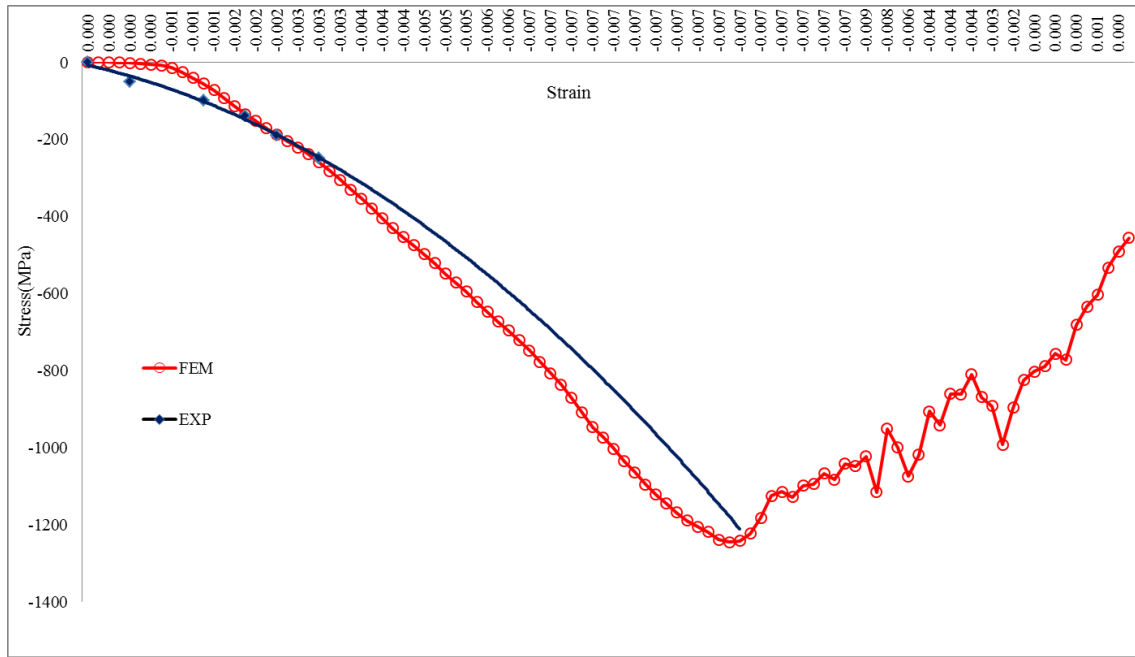


Figure 6.9: Comparison of stress-strain diagram for IMG6/3501-6 in Compression between experiment and “geo” model ($\frac{A_0}{L_0} = 0.04$).

The experimental results are collected from Ref. [Hsiao and I.M. Daniel \(1996b\)](#) . For the same wavelength and amplitude “mat” model does not converge and it reveals premature failure.

In order to analyze capability of “geo” model more in deep, some parametric studies have been conducted. The ratio of the wave amplitude to the wavelength, $\frac{A_0}{L_0}$, is an important parameter in out of plane fiber undulation which is called wave parameter. Several research have found using experiments and analytic studies [Chou and K. Takahashi \(1987\)](#), [Hsiao and I.M. Daniel \(1996b\)](#), ([Hsiao and I.M. Daniel, 1996c](#)), ([Chuna et al., 2001](#)), ([Karami and M. Garnich, 2005](#)) that the increase in this parameter will lead to decrease in stiffness and final failure strength. The constitutive equation is able to capture this trend as can be seen in Fig. 6.12. The Figure shows the results of force-displacement in tension for “geo” model at 4 different $\frac{A_0}{L_0}$. Higher wave parameter leads to higher stiffness in elastic domain and higher final failure, while the softening has an indirect relation with it.

The same trend is observed in the compression while studying parameter variations (Fig. 6.13). It reveals higher stiffness and failure strength as well as lower softening rate when the waviness parameter is higher.

Damage parameter evolution is also studied during the loading process. Figure 6.14 shows d_1 parameter in compression loading for “geo”. It shows that the damage initiates in the loading edge of the specimen and evolves to whole of it. This parameter is between

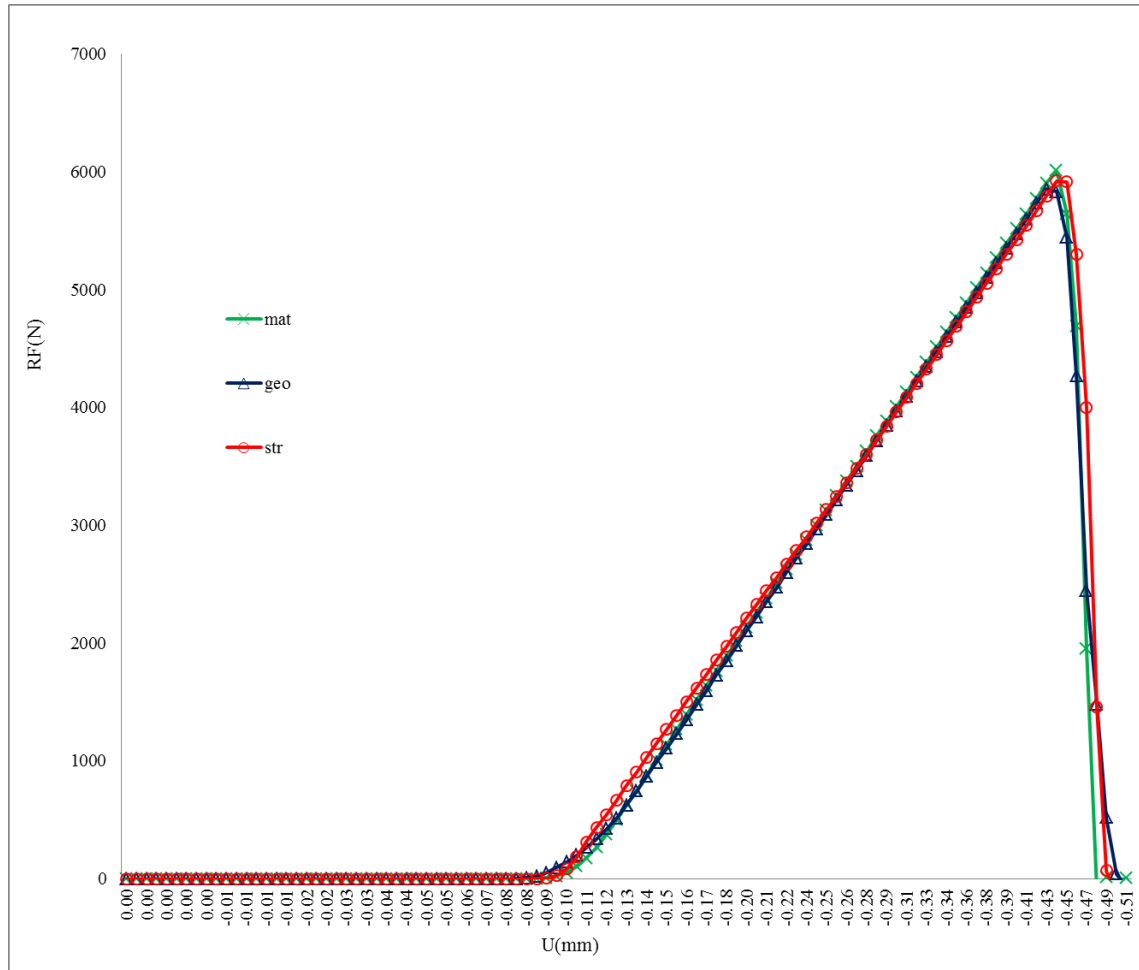


Figure 6.10: Force versus displacement of IMG6/3501-6 in transverse direction for 3 different cases ($\frac{A_0}{L_0} = 0.04$).

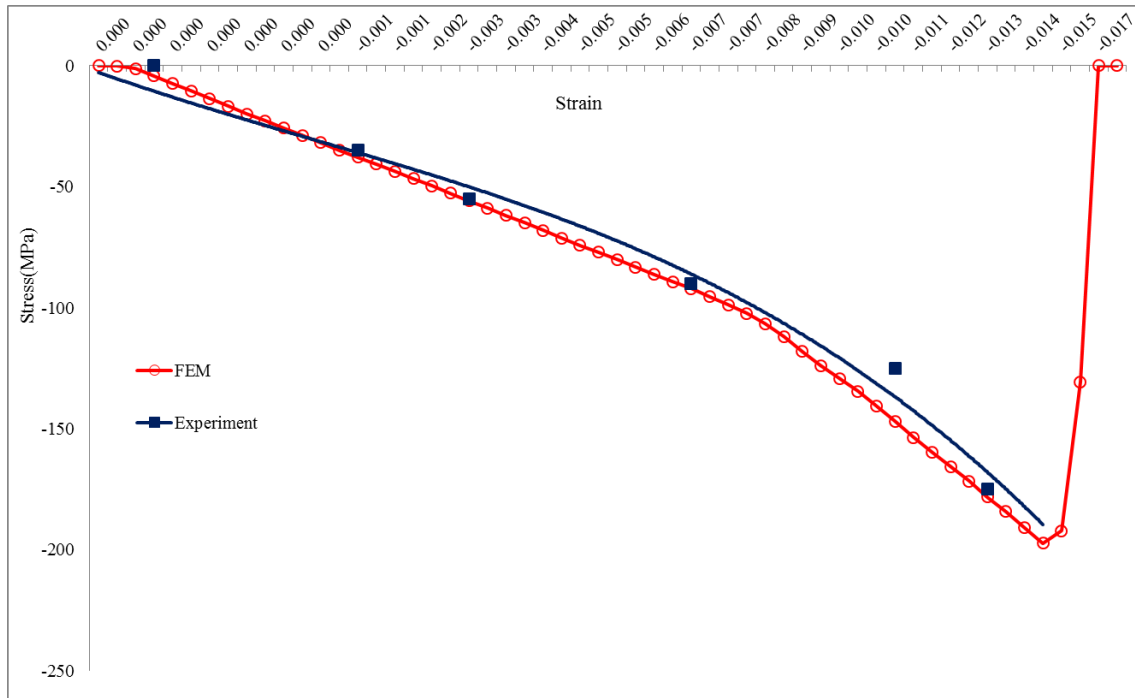


Figure 6.11: Comparison of stress-strain diagram for IMG6/3501-6 in transverse direction between experiment and “geo” model ($\frac{A_0}{L_0} = 0.04$).

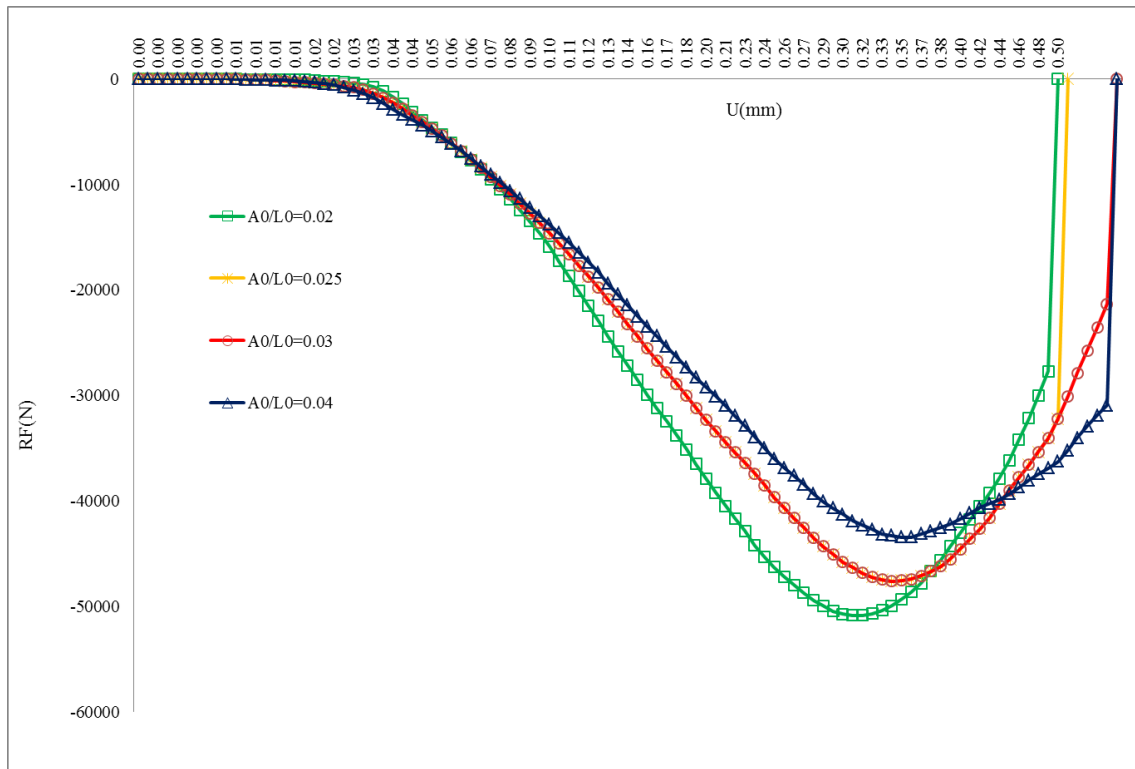


Figure 6.12: Parametrical study of IMG6/3501-6 for “geo” model in tension.

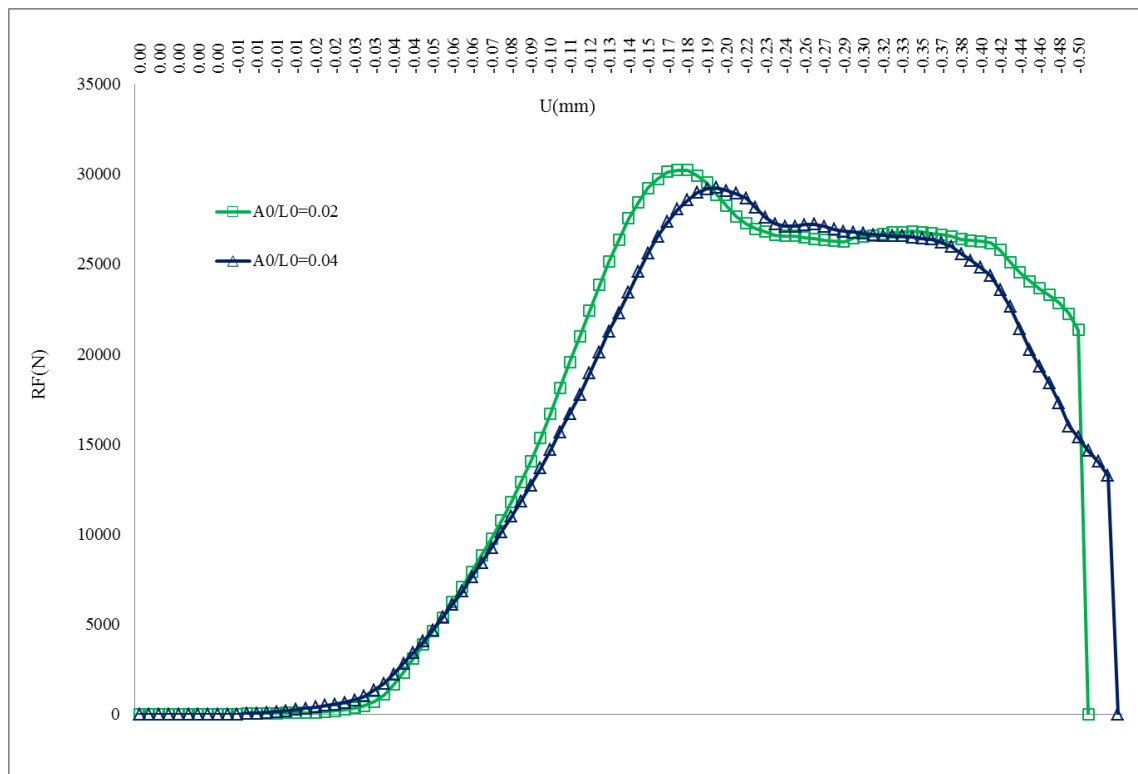


Figure 6.13: Parametrical study of IMG6/3501-6 for “geo” model in compression.

0 and 1 and when the damage parameter for an element becomes 0.999, the subroutine eliminates the given element. In the Figure, the first element deletion starts from t_7 until the final collapse happens that contains several deleted elements.

6.6 Conclusion

A continuum damage model is proposed for out of plane undulated fiber polymer composites. The model in comparison to previous models includes the damage behavior of the material. It is also easier to program because of matrix form and fewer variables in elastic domain. In the modeling 2 different approach are compared. The model in which the geometrical non-linearity is regarded shows better capability in analyzing a wider range of wave parameters. It is also has a good agreement with the experiments.

It is also concluded that the out of plane undulation cause reduction in the final failure strength of the laminates for both tension and compression in comparison to straight fibers. In addition, increasing in wave parameter leads to lower stiffness in elastic domain, lower final failure strength, and faster softening. This chapter is published in ([Eskandari et al., 2016c](#)) .

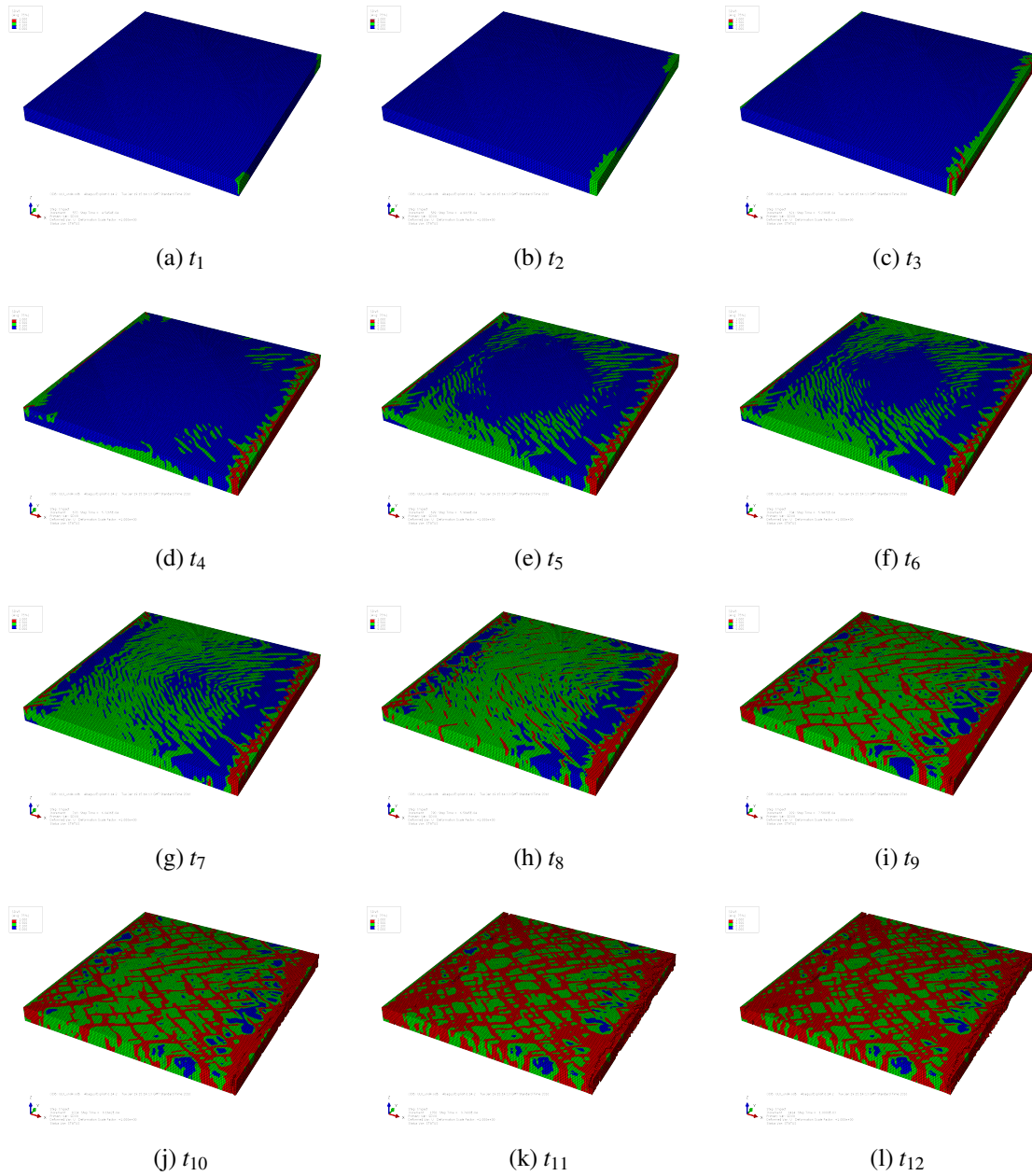


Figure 6.14: Evolution of damage parameter of d_1 during loading in compression specimen for “geo”

Chapter 7

Damage under dynamic loads and in presence of viscoplasticity

In order to design structures which are under dynamic loads using composite materials, it is necessary to develop material models that include different mechanisms which are playing role at higher strain rates. Under sensible strain rates the structures usually reveal visco-plastic behavior. It has reported in several works that strain rate influences composite behavior and arise nonlinearity (Yoon and Sun, 1991) , (Lifshitz, 1976) ,(Daniel, 1993) , (Powers et al., 1995) , (Körber et al., 2010) , (Körber, 2010) . In order to capture the nonlinearities regarding that and achieve a better model, it is important to incorporate viscoplasticity in the progressive damage model. Aboudi (Aboudi, 1982) proposed a 3-dimensional continuum theory for fiber-reinforced elastic-viscoplastic composites. In his model both fiber and matrix are assumed elastic-viscoplastic. He also included work hardening in the model. Weeks et al. (Yoon and Sun, 1991) modeled nonlinear behavior of composites under strain rate. They have used one parameter plastic potential function to analyze rate dependency of the material. It is assumed that the material is linear elastic in fiber direction and is transversely isotropic. They compared the prediction of the model to experimental results for AS4-PEEK which reveal in some cases good results though there is still room for improvement. Saleeb (Saleeb and Wilt, 1993) analyzed the anisotropic viscoplastic-damage response of composite laminates. Lissenden (Lissenden and Herakovich, 1995) implemented numerical modeling of damage development viscoplasticity in metal matrix composites. Source of nonlinearity in this work is metal-matrix that is viscoplastic material and the damage in the interface of fiber and matrix. Viscoplasticity is modeled by inclusion of rate dependency in classical plastic theory. Non-linear lamination theory provides the link between the micro- and macro-level in the study.

Many research have been conducted considering viscoplasticity in the scope of micromechanical analysis (Allen et al., 1994) , (Ohno et al., 2000) ,(Megnis and Varna, 2003), (Pierard et al., 2007) . (Megnis and Varna, 2003) analyzed inelastic behavior of composite materials by Schapery's nonlinear viscoelastic, viscoplastic material model. The viscoelastic behavior is regarded linear and viscoplasticity is characterized nonlinear. Micromechanical Hashin's Concentric Cylinder Assembly model is used in viscoelastic formulation to calculate the coefficients of linear viscoelastic compliance functions of UD composite.

Al-Haik et al. (Al-Haik et al., 2004) compared explicit and implicit viscoplastic models for carbon-fiber/polymer matrix composite. In implicit model, the experimental data is used to train artificial neural networks with truncated Newton algorithm, and in explicit approach, the model which described earlier (Yoon and Sun, 1991) was utilized. Temperature-dependent material constants are also applied.

In previous chapters a constitutive model were used that considers elastic behavior, damage, and fiber rotation phenomena in meso-scale for composite laminates. In this chapter the dynamic loads effects will be incorporated in the model. In order to do so 2 mechanisms should be taken into account. First, the viscoplastic effects that is produced after matrix yield and under strain rate. A 3-dimensional orthotropic viscoplastic model is utilized for modeling of this mechanism. It postpones the damage and final failure. In addition, the elastic properties, in-situ properties, and strengths of material will be varied under dynamic conditions. This is also has effects on the damage behavior of the material. In addition fracture energy also will be used by an updated value for dynamic case.

7.1 Constitutive model

The constitutive model in real material coordinate system ($\mathbf{x}$) is considered as:

$$\boldsymbol{\sigma}^x = \mathbf{K}_d^{x dy} (\boldsymbol{\epsilon}^x - \boldsymbol{\epsilon}_{vp}^x). \quad (7.1)$$

In which, $\mathbf{K}_d^{x dy}$ is damaged stiffness under dynamic loads, $\boldsymbol{\epsilon}_{vp}^x$ is viscoplastic strain tensor, and $\boldsymbol{\epsilon}^x$ is total strain tensor in real material coordinate system. The procedure to relate stress and strain in real material direction and initial material direction were discussed in chapter 4. The damaged stiffness under quasi-static loads, $\mathbf{K}_d^x$, was also introduced. In order to find stiffness under dynamic loads, the elastic properties and final strength of material should be updated based on strain rate. Indeed, the dynamic stiffness

is a function of quasi-static stiffness as:

$$\mathbf{K}_d^{x dy} = \mathbf{K}_d^x(E_{ij}^{dy}, X_{ij}^{dy}), \quad (7.2)$$

where, E_{ij}^{dy} represents elastic properties and X_{ij}^{dy} shows strengths under dynamic loads. Koerber (Körber, 2010) conducted a research to identify the relation of material properties between dynamic and quasi-static modes. The chosen material in that research is carbon/epoxy IM7-8552. According to those analysis E_{22} , E_{33} , G_{12} , G_{13} , G_{23} , X_C , Y_T , Y_C , and S_L are sensitive to strain rate in carbon/epoxy IM7-8552. The relation for a property in quasi-static and dynamic modes which is presented by Koerber (Körber, 2010) based on experimental analysis reads:

$$E_{ij}^{dy} = E_{ij} h(\dot{\epsilon}), \quad (7.3)$$

where, $h(\dot{\epsilon})$ is:

$$h(\dot{\epsilon}) = 1 + m(\dot{\epsilon})^{\frac{1}{n}}, \quad (7.4)$$

in which, m and n are experimental factors that are defined based on experiments and the fact that if the property is for elastic or strength behavior, as summarized for IM7-8552 in Table 7.1 .

Table 7.1: Experimental factor for defining IM7-8552 properties in dynamic mode.

Property type	m	n
Elastic	1.60×10^{-4}	2
Strength	1.13×10^{-4}	4

By having Eq. 7.4, material properties which are sensitive to strain rate can be updated, and based on them the stiffness and damage variables will be defined. The same formulation should be used for in-situ properties. Another approach might be defining the in-situ properties based on the material properties in dynamic condition. This approach is also tested which revealed the same results. Now it is necessary to calculate viscoplastic strain, ϵ_{vp}^x .

In order to define viscoplastic strain a non associative flow rule is used. Therefore, it is necessary to define yield, f , and plastic potential, g , functions. These function and the procedure to calculate viscoplastic strain is followed according to the method proposed

by (Vogler et al., 2005) that is an extension to (Vogler et al., 2013) . By having f and g , the viscoplastic strain will be defined by:

$$\dot{\boldsymbol{\epsilon}}_{vp} = \gamma_{n+1}^{vp} \mathbf{n}_g = \frac{\langle f(\boldsymbol{\sigma}, \bar{\boldsymbol{\epsilon}}_{vp}, \mathbf{A}) \rangle^m}{\eta} \mathbf{n}_g, \quad (7.5)$$

where, η and m are viscoplastic parameters and $\mathbf{A}$ is structural tensor. In developing the yield surface, it is assumed that the stiffness is not symmetric and the anisotropy is applied by so called structural tensor that is dyadic product of preferred direction, $\mathbf{a}$, that is the fiber direction in unidirectional fiber composites, as:

$$\mathbf{A} = \mathbf{a} \otimes \mathbf{a}. \quad (7.6)$$

Since the rest of equations are defined based on symmetric stiffness and in material coordinate system, the viscoplastic strain that is computed using this procedure should be rotated to be compatible with the rest of the equations. The input of this computations also should be in global coordinate system. $\mathbf{n}_g$ is also flow direction and defined by:

$$\mathbf{n}_g = \frac{\partial g(\boldsymbol{\sigma}, \mathbf{A})}{\partial \boldsymbol{\sigma}}, \quad (7.7)$$

Yield function and plastic function are defined by Volger (?) based on defined invariants as:

$$f(\boldsymbol{\sigma}, \bar{\boldsymbol{\epsilon}}_{vp}, \mathbf{A}) = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_{32} I_3^2 - 1 \leq 0, \quad (7.8)$$

and,

$$g(\boldsymbol{\sigma}, \mathbf{A}) = \beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3^2 - 1. \quad (7.9)$$

α_i and β_i will be defined according to experimental tests for the material which is being studied. The utilized invariants read:

$$I_1 = \frac{1}{2} \text{tr}(\boldsymbol{\sigma}^{pind})^2 - \mathbf{a}(\boldsymbol{\sigma}^{pind})^2 \mathbf{a}, \quad (7.10)$$

$$I_2 = \mathbf{a}(\boldsymbol{\sigma}^{pind})^2 \mathbf{a}, \quad (7.11)$$

$$I_3 = \text{tr}(\boldsymbol{\sigma}) - \mathbf{a}(\boldsymbol{\sigma}) \mathbf{a}. \quad (7.12)$$

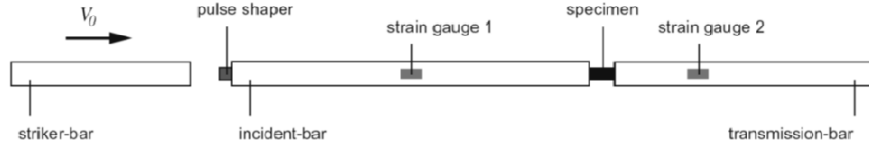


Figure 7.1: Split-Hopkinson pressure bar setup for dynamic tests (Kuhn et al., 2016).

Where, σ^{pind} is:

$$\sigma^{pind} = \sigma - \frac{1}{2}(tr(\sigma) - a(\sigma)a)1 + \frac{1}{2}(tr(\sigma) - 3a(\sigma)a)A. \quad (7.13)$$

Kuhn et al (Kuhn et al., 2016) have found that the fracture toughness increases under dynamic loads. They have conducted an experimental study to measure compressive crack closure resistance of unidirectional laminates IM7-8552 under different strain rates. The dynamic tests are executed by split-Hopkinson pressure bar that is shown schematically in Fig.7.1. Their results show that the fracture toughness is $165.6 \frac{KJ}{m^2}$ when it is subjected to dynamic loads. This updated fracture energy is used in this work.

7.2 Finite element implementation

In order to implement the model incorporating strain rate effects and viscoplasticity, the VUMAT subroutine which was developed in previous chapters is extended to include new mechanisms. Viscoplasticity implementation is pursued based on (Vogler et al., 2005) by solving the nonlinear equations which introduced in previous section. It is necessary to update consistency parameter, $\Delta\gamma^p$, that is done using consistency condition linearization. The root of the linearized residual is:

$$\Delta^2\gamma_{n+1}^{(k+1)} = - \frac{f_{n+1}^{(k)m} - \Delta\gamma_n^{(k+1)} \frac{\eta}{\Delta t}}{m \cdot f_{n+1}^{(k)m-1} - \Delta\gamma_{n+1}^{(k)} \frac{\eta}{\Delta t}}. \quad (7.14)$$

A set of experimental tests conducted by (Körber et al., 2010), (Körber, 2010) for unidirectional composite laminates under dynamic loads. The setup of the experiments are similar to the ones which discussed in chapter 5. The numerical examples also developed to compare the results of the model with them in order to verify the results. A 3D 2-layer laminate of IM7-8552 is generated. The meshing of the model is shown in Fig.7.2.

The numerical model is executed under dynamic condition for different fiber orientation and the results are compared with experimental results and quasi-static model. The chosen material for analysis is IM7-8552 that its characteristics are discussed in previous

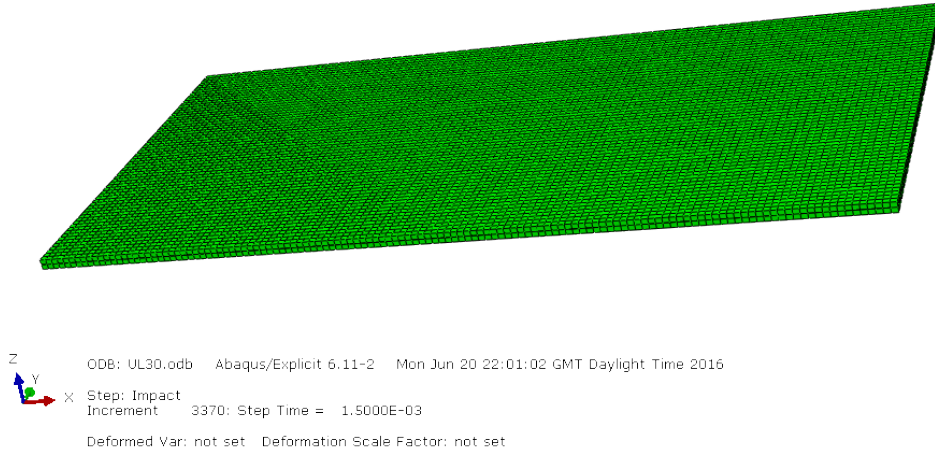


Figure 7.2: Mesh for 2 layers laminates.

chapters. In addition, the method for finding viscoplastic properties and calibration of them is discussed in (Vogler et al., 2005) .

7.3 Results and discussion

Analysis have been made for 0, 30, 45, 60, and, 75 ° of initial fiber orientation. Reaction forces variation, damage, fiber rotation, and viscoplasticity are analyzed for these cases. Figure 7.3 shows the reaction force versus time for the model with fiber orientation in direction of x axis. The Figure includes the data for quasi-static and dynamic conditions. The model for quasi-static condition considers elastic-damage behavior of the material along with bilinear plastic in shear. The dynamic model regards elastic-damage-viscoplastic behavior and it is assumed that strain rate is $246 S^{-1}$. As can be understood from the picture, the quasi-static model fails at 3130 kg while the other one reveals failure at 4103 kg. Regarding the area of $2.5 m^2$ which is under the applied load, the final stress for the first model will be 1256 MPa and strength capacity of high strain rate case will be 1642 MPa. Therefore, the dynamic model is capable to capture experimental observation showing increase in failure strength due to dynamic loading. It is not only qualitatively good but also in comparison to experimental data it is accurate. According to tests conducted by (Körber et al., 2010) for both quasi-static and dynamic loading, the final strength of IM7-8552 with fiber direction of 0° increases from 1050 MPa to 1459 MPa under high strain rate. Therefore, the developed model can predict the failure with a good

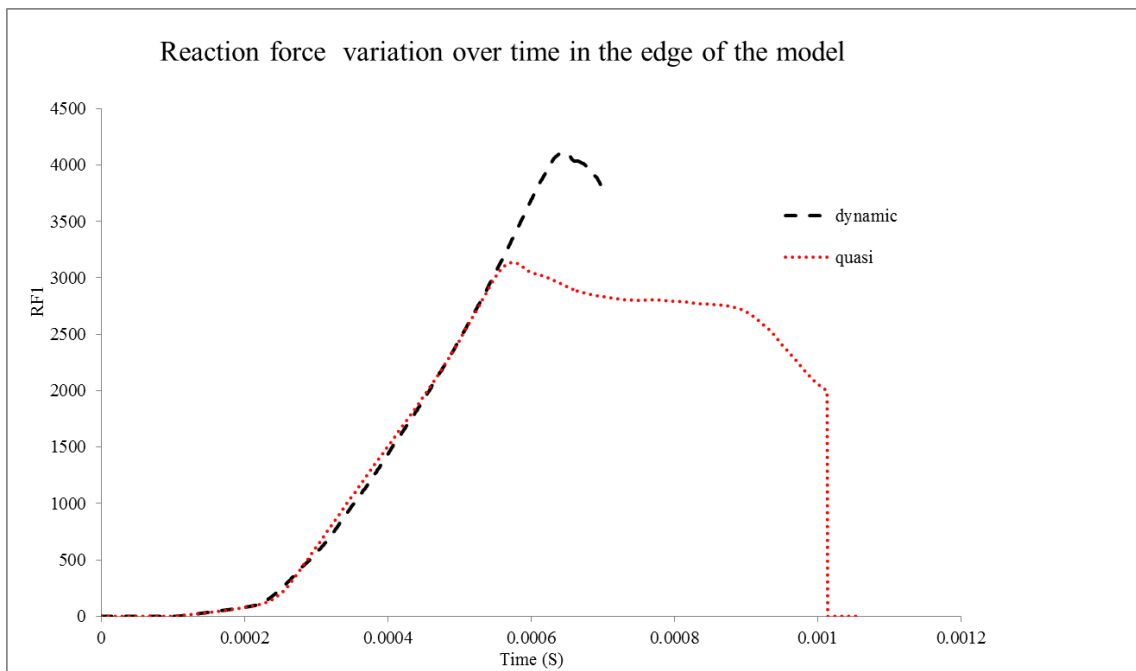


Figure 7.3: Comparisons of reaction forces over time for quasi-static and dynamic conditions for 0° fiber laminate.

accuracy of 13% error. In addition, no viscoplastic behavior is observed in x direction for this model since it is aligned with fibers.

Figure 7.4 depicts reaction forces of the laminate under dynamic compression loads for 30° model. As expected, there is an increase in the forces because of high strain rate which leads to higher failure stresses. The dynamic model fails at 1140 N which means 448 MPa strength. This model is performed under the strain rate of 246 S^{-1} . In comparison to experimental strength that is 370 MPa, the error of the model in 30° for direction of fiber is 21%.

Stress-strain curve for an element in uniform area is chosen for this specimen to compare to experiment. Figure 7.5 illustrates the mentioned comparison which shows a good agreement between them and it can capture the nonlinearity of fiber rotation and viscoplasticity. Figure 7.6 presents the distribution of viscoplastic strain across model in x direction which shows a uniform viscoplastic strain.

In this model, damage initiates at one end of the specimen and evolves through the model. Figure 7.7 illustrates the first damage parameter, d_1 , propagation during the time from t_1 to t_4 . This parameter evolution is illustrated in Fig 7.8. The damage parameter which represents in-plane shear is presented in 7.9.

The same trend between static and dynamic conditions is observed for 45° fibers that is shown in Fig.7.12. The dynamic model yields failure at 902 kg that corresponds to 388

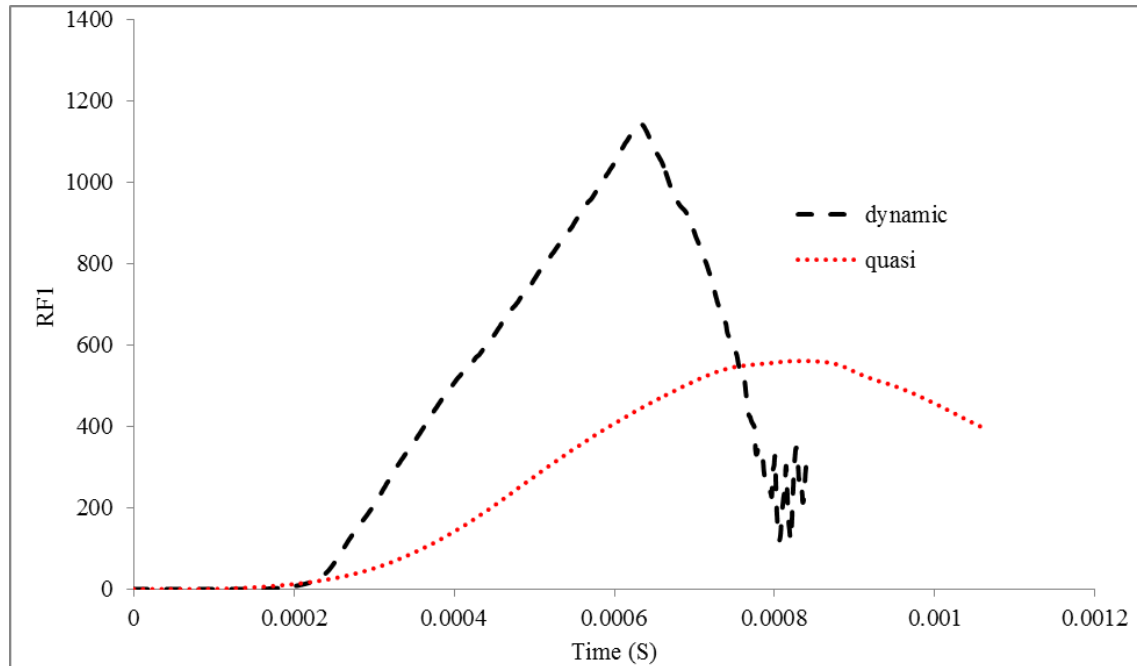


Figure 7.4: Comparisons of reaction forces over time for quasi-static and dynamic conditions for 30° fiber laminate.

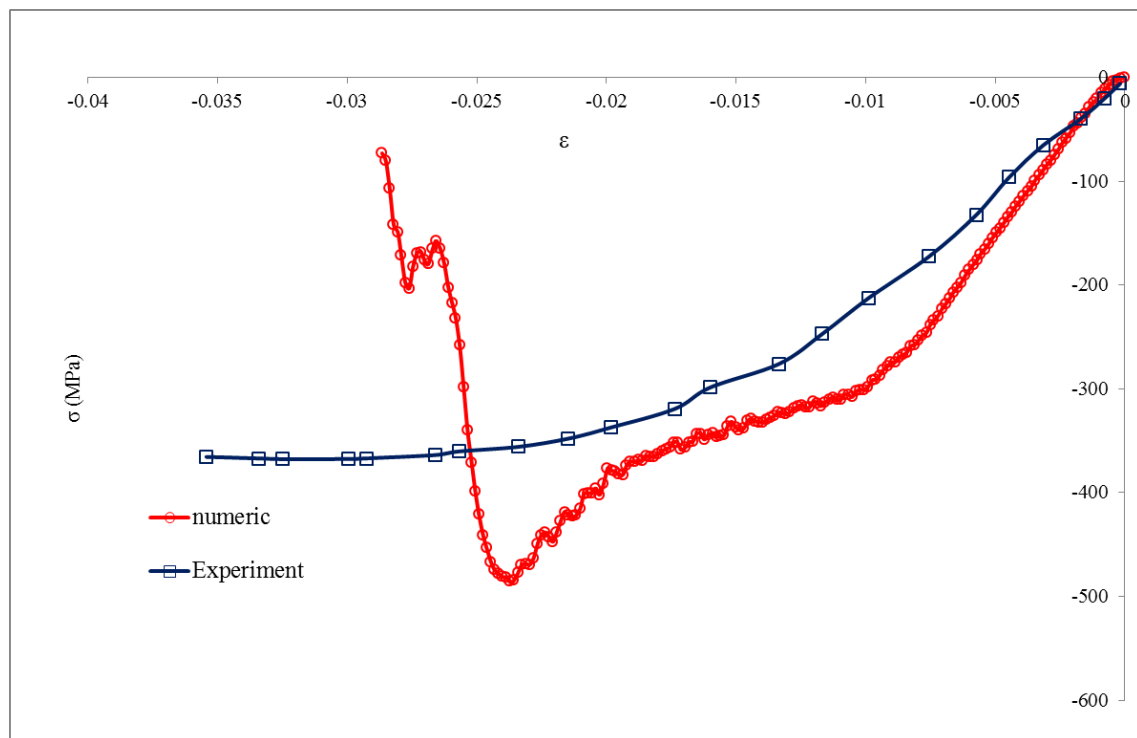
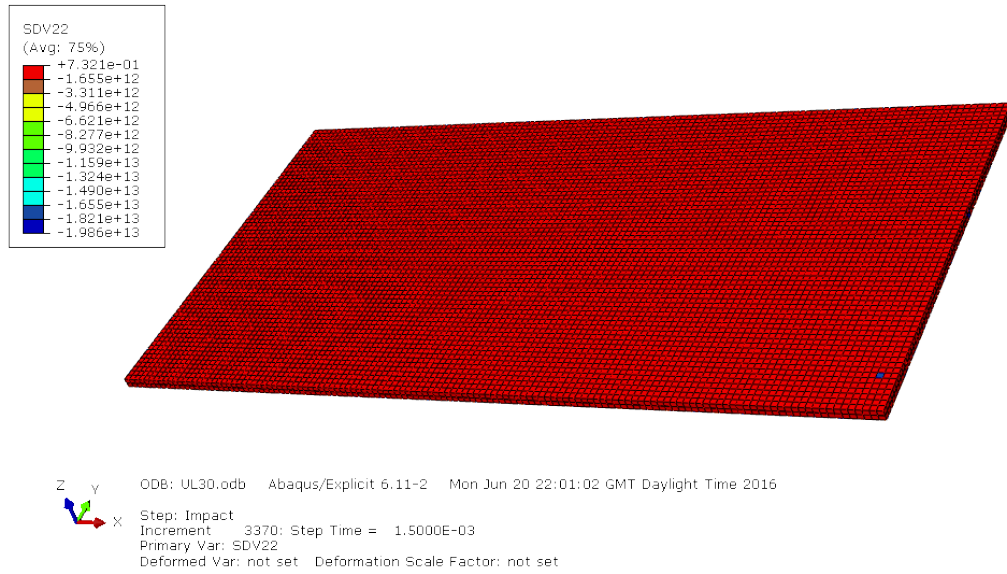
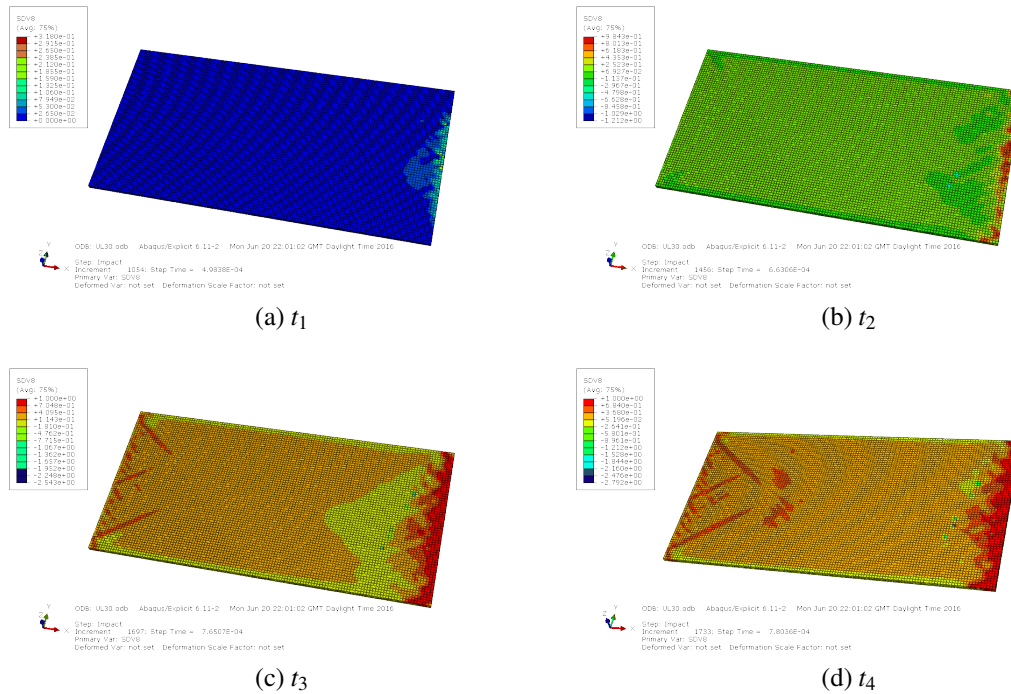


Figure 7.5: Stress-strain curve comparison of dynamic conditions for 30° fiber laminate.

Figure 7.6: plastic strain in x direction for 30° fiber laminate.Figure 7.7: Evolution of d_1 damage parameter for 30° model.

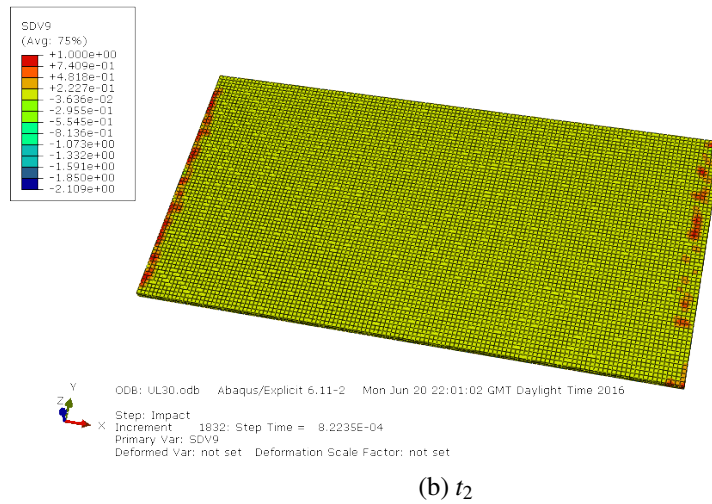
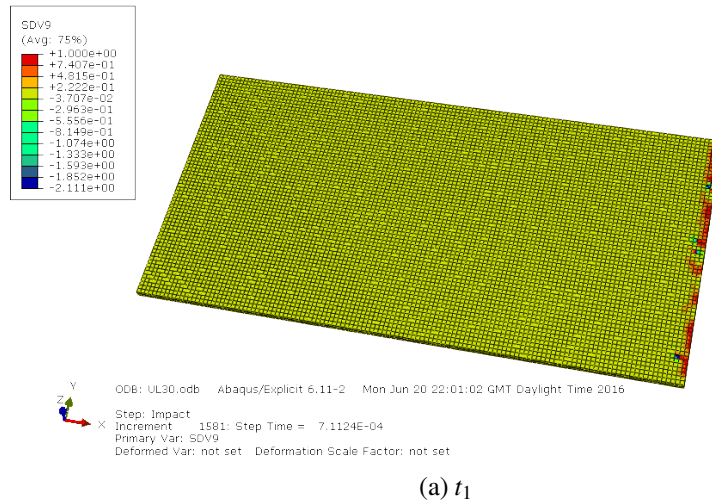


Figure 7.8: Evolution of d_2 damage parameter for 30° model.

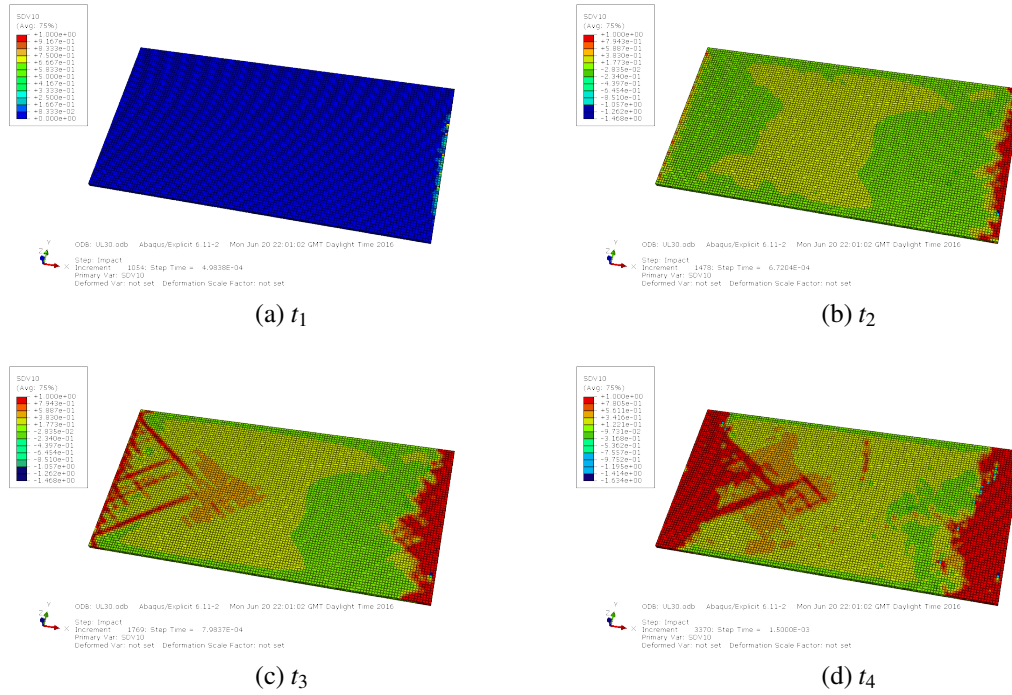


Figure 7.9: Evolution of d_6 damage parameter for 30° model.

MPa failure stress. In comparison to the strength observed in experiment that is 354 MPa, the model demonstrates error of 9% that is very good prediction regarding the complexity of the model and different ongoing mechanisms.

The stress-strain curve for 45° fibers is also expresses good accuracy that is presented in Fig. 7.11 . The damage evolution of this model is also analyzed. The first damage parameter is depicted in Fig 7.12 . It also shows damage initiation in the end-load of the specimen and developing the cracks across the model. The second damage parameter and the in-plane shear are also presented in Figs. 7.13 and 7.14, respectively. In the experimental work of (Körber et al., 2010) also reported that the damage grows from end-load of the specimen. In addition the X-ray image of the failed specimen from that, has a correlation with damage parameters that implies ability of the model in capturing the failure plane. This comparison is shown in Fig. 7.15 . In addition, the fiber rotation at failure for this model is 1.4° that is close to experimental data that is 1.2° . The viscoplastic strain in x direction for this model is also presented n Fig. 7.16 that is unlike 30° model is not uniform and has higher progress in the loading end of the specimen. This strains are higher in comparison to viscoplastic in 30° model.

The model has a good ability to predict the final strength of the laminates for higher fiber orientation as well. Figure 7.17 illustrates the forces in 60° and 75° models. The

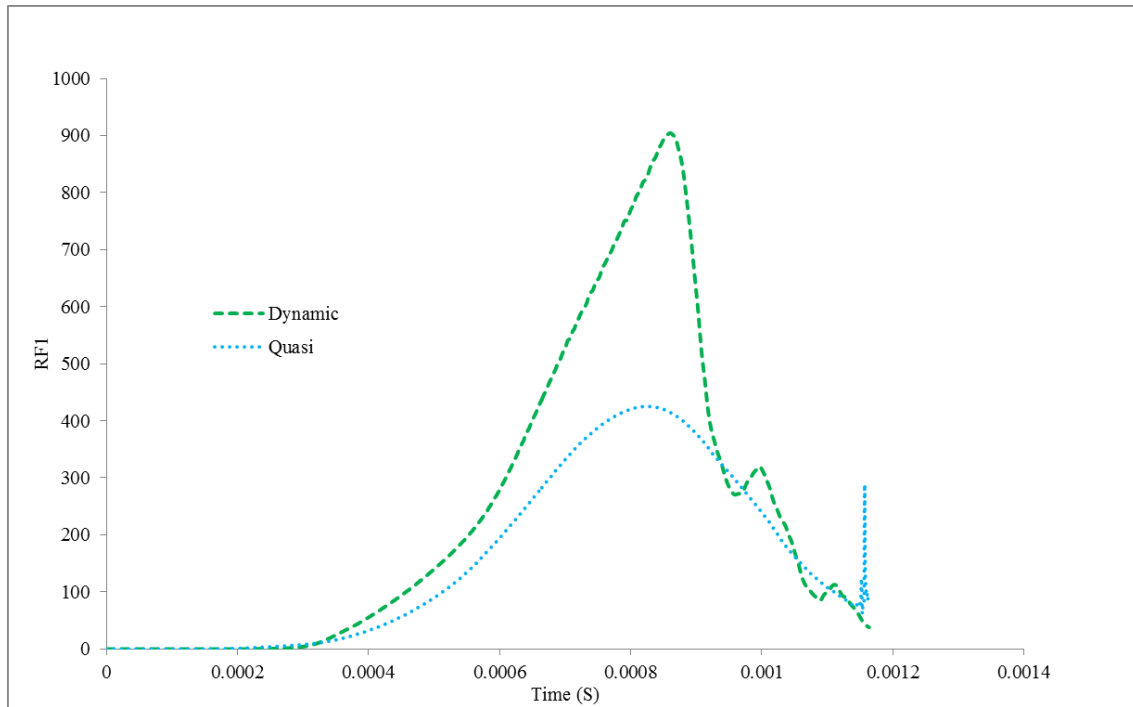


Figure 7.10: Comparisons of reaction forces over time for quasi-static and dynamic conditions for 45° fiber laminate.

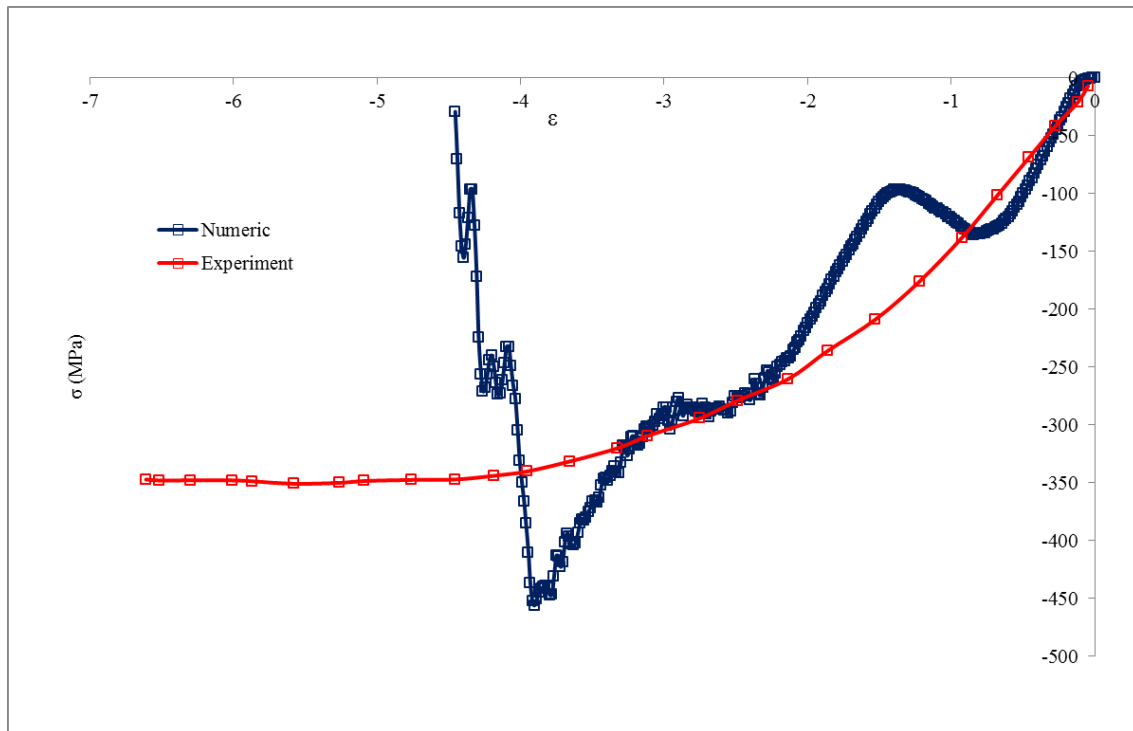
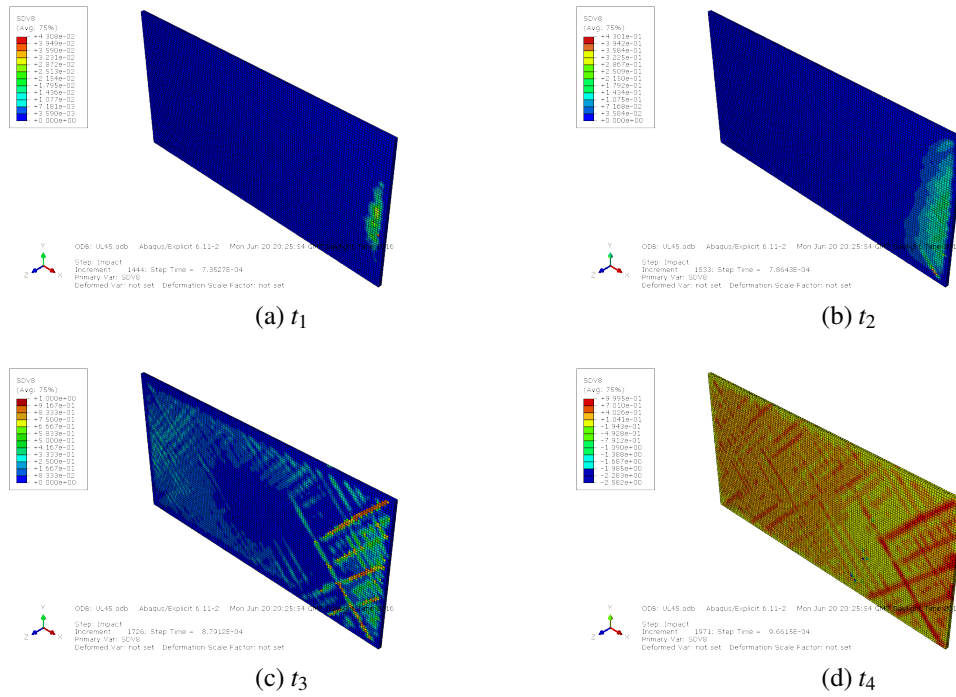


Figure 7.11: Stress-strain curve comparison of dynamic conditions for 45° fiber laminate.

Figure 7.12: Evolution of d_1 damage parameter for 45° model.

first one fails at 891 kg and the second one at 712 kg. Table 7.2 shows the final strength predicted by the model and the ones achieved by experiments that expresses capability of the numerical model to predict final strength under dynamic loads. The model with direction of fiber in 60° has the best accuracy among all the directions which is only 6% error. This chapter is concluded in (Eskandari et al., 2016d).

Table 7.2: Final strength for different fiber orientation under dynamic loads.

Orientation ($^\circ$)	Strain rate (S^{-1})	Experiment (MPa)	Numeric (MPa)	Error
0	246	1450	1642	13%
30	246	370	448	21%
45	280	354	388	9%
60	331	365	340	-6%
75	305	371	286	-21%

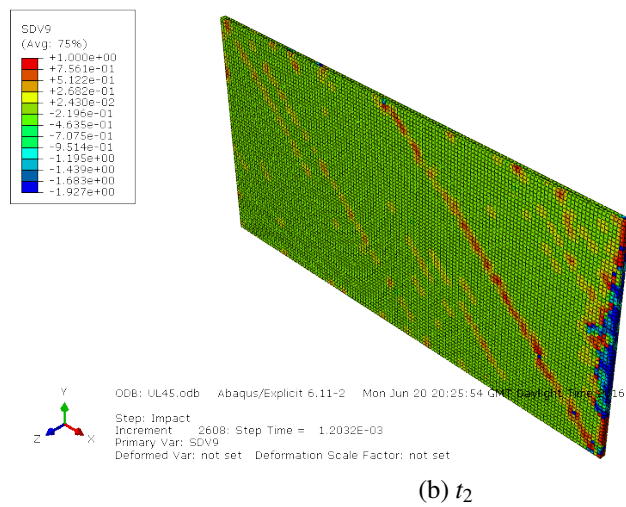
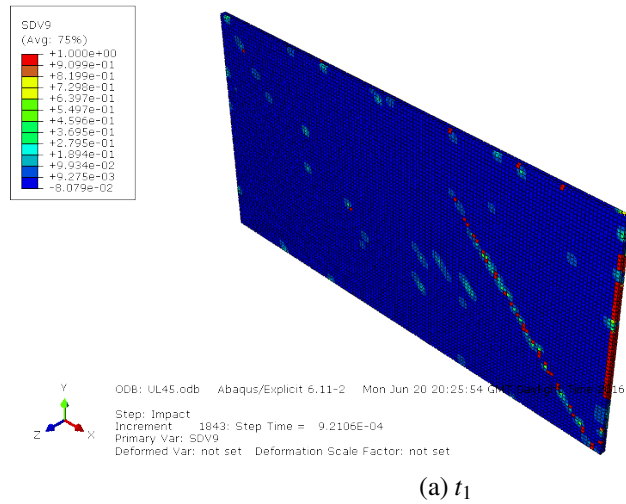
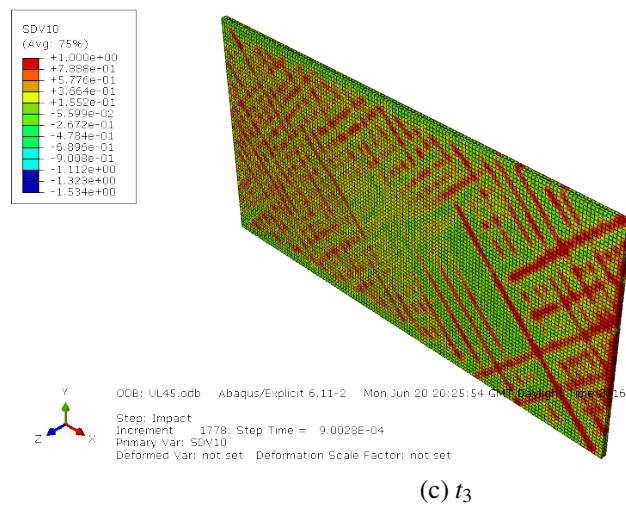
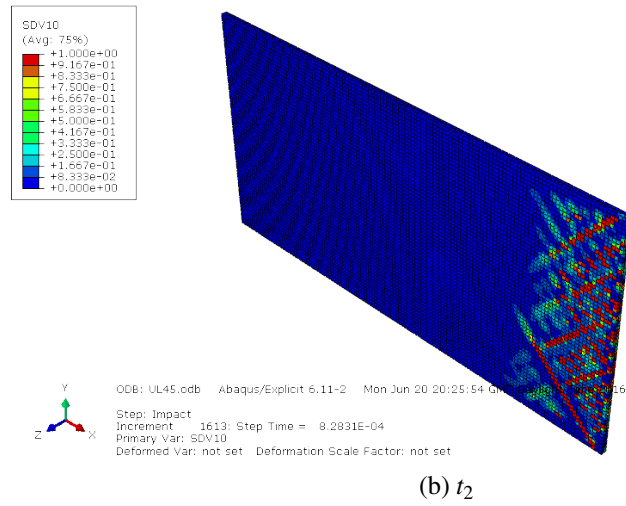
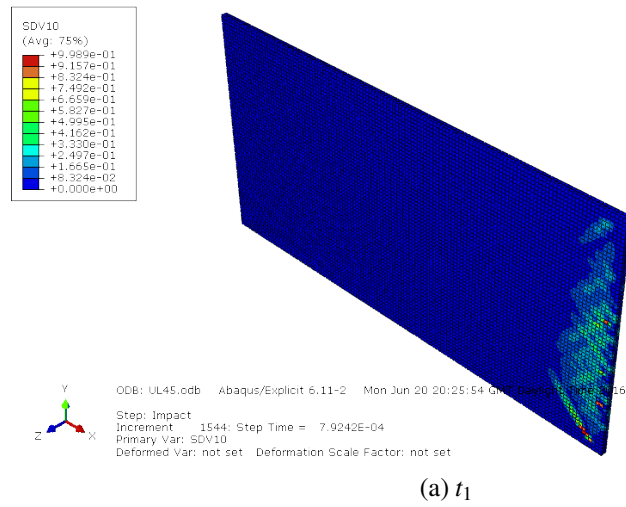


Figure 7.13: Evolution of d_2 damage parameter for 45° model.

Figure 7.14: Evolution of d_6 damage parameter for 45° model.

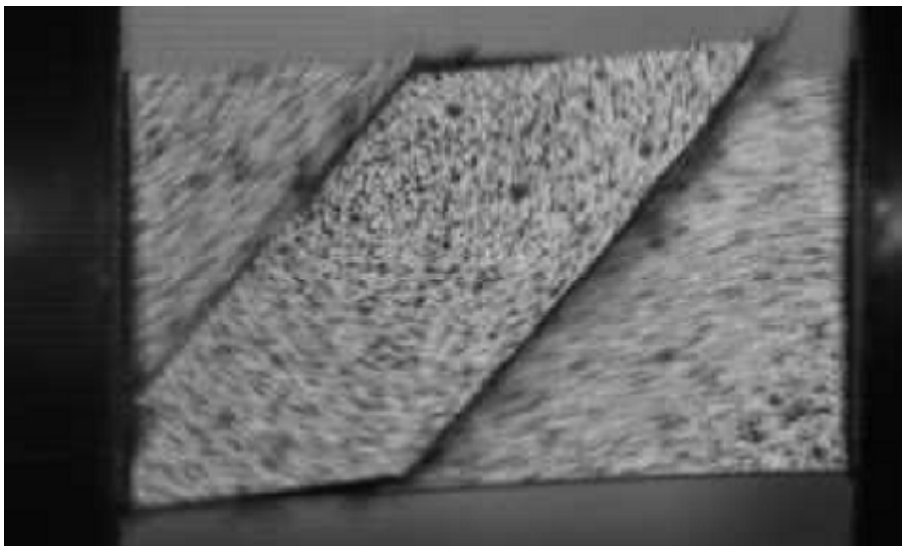
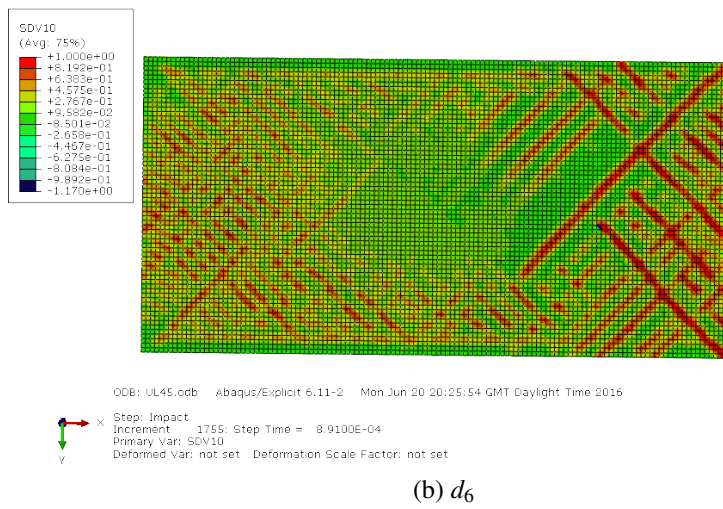
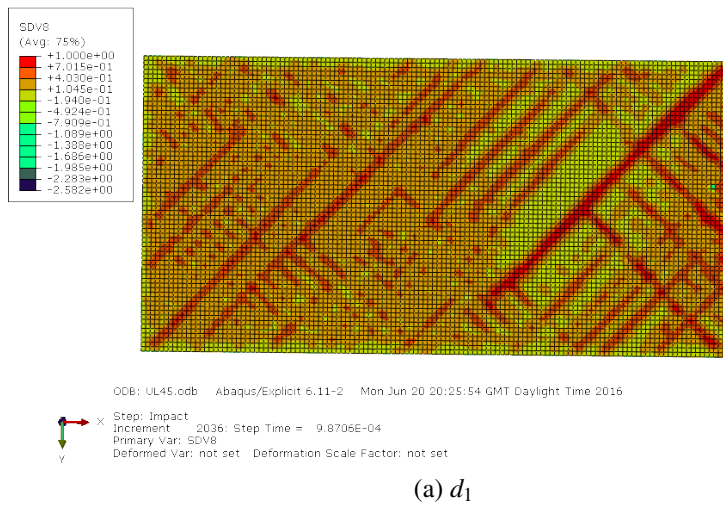


Figure 7.15: Comparison of failed specimen in experiment and numeric model for 45° model.

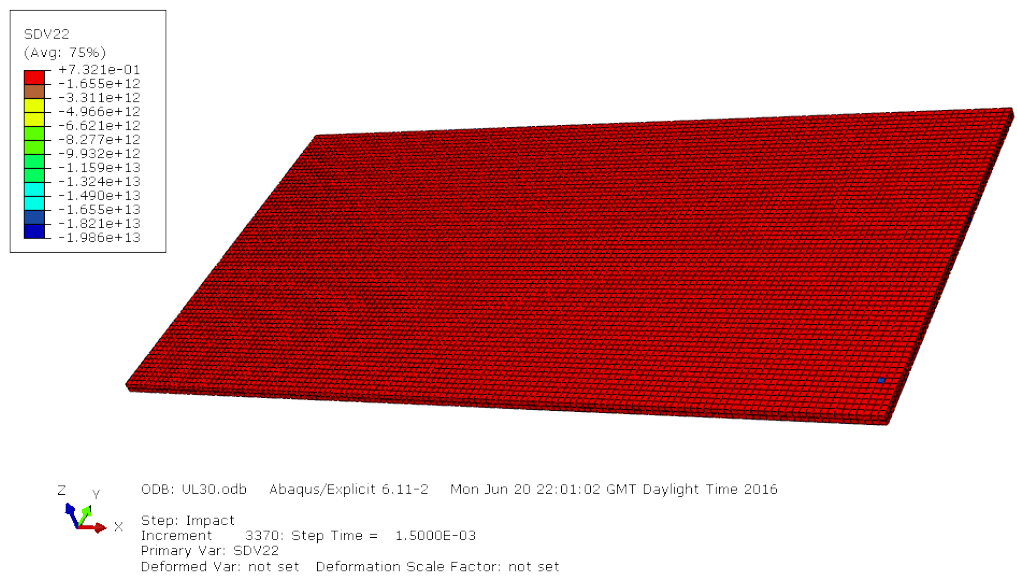


Figure 7.16: plastic strain in x direction for 45° fiber laminate.

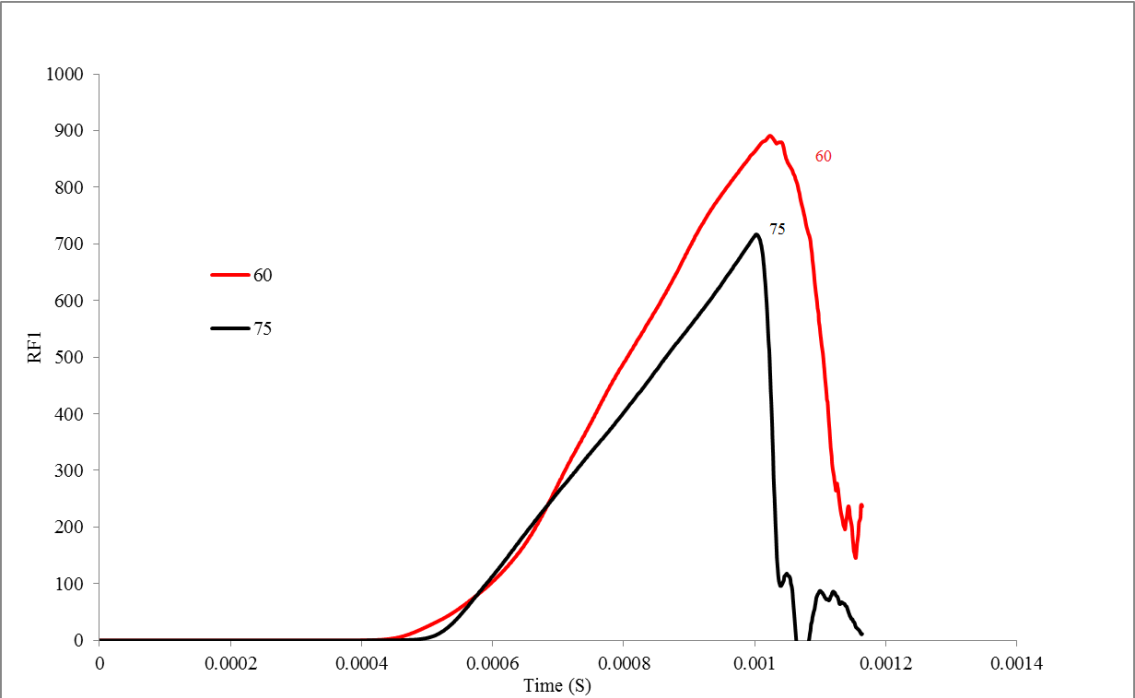


Figure 7.17: Reaction forces versus time under dynamic conditions for 60° and 75° fiber laminate.

Chapter 8

Conclusion and future work

In this thesis an analytic model is proposed for computation of fiber rotation using continuum mechanics during the loading of the composite material laminate. The model is incorporated in material constitutive model in elastic and damage sections. The achieved model is implemented by VUMAT subroutine in commercial Finite element modeling software of ABAQUS. Then, the results of the model are compared with experiments and the model without considering fiber rotation and it is proved that this improves the accuracy of the results. The following conclusions can be summarized from this part of the thesis. The model with fiber rotation is referred as the model.

1. Rotation of continuum includes 2 parts of rigid rotation and fiber rotation. Only fiber rotation has effect on constitutive model and should be calculated and incorporated in it.
2. Inclusion of fiber rotation has effects on force-displacement of the specimen, its final strength and also damage evolution of it.
3. Incorporating fiber rotation generally improves the results in comparison to the model which does not regard it.
4. In laminate level incorporating fiber rotation delays damage in tension while speeds it up in compression.
5. In element level, there is no general rule with regard to damage increase or decrease while incorporating the fiber rotation and it depends on fiber orientation, element vicinity and loading type.
6. In shear test, the model can capture the fiber scissoring phenomenon.

7. In open-hole tension models, the model predicts final strength by 2.9% error and open-hole compression by 13.4%.
8. The model with fiber rotation can capture damage mechanisms of the specimen better when it is compared to X-ray images of cracked experimental samples.
9. The WRot model can capture the fracture plane better.
10. The WRot model can capture stiffening and damage delay due to fiber rotation
11. The results of fiber rotation amounts are in good agreement with experimental data.
12. In compression test of unidirectional composites there is negligible fiber rotation for the cases with initial fiber angle more than 45° . In these cases, the results of both models are the same.
13. It is mandatory to consider fiber rotation in modeling of thermoplastic based composites. The model without fiber rotation diverges for this kind of materials due to high level of fiber rotation and high fracture energy.
14. The model considering fiber rotation can model thermoplastic composites failure strength with accurate results in comparison with experiments. It predicts the strength in tension of AS4-PEEK by error of 0.3% and its final strain by error of 12%.
15. The model can capture cracks in thermoplastic-based composites by comparing with X-ray images.

Using the model a parametric study is conducted to analyzed effects of different variables on final strength and fiber rotation. The results of this study is concluded as below:

1. Increasing of the number of plies decreases fiber rotation in distributed stacking laminates while it increases it in blocked laminates.
2. Blocked ply laminates reveal more fiber rotation in comparison to distributed ones.
3. Increasing the ply thickness in tension and compression will increase the final strength and decrease the fiber rotation.
4. Initial fiber orientation has a direct relation with final failure strength and fiber rotation

The developed constitutive model is also used for analyzing damage in undulated fiber composites. Geometrical nonlinearity which is being induced by loading in these materials also incorporated in the model. Therefore, 3 models were in hand comprising straight fibers, undulated fibers with material nonlinearity, and undulated fiber with material and geometrical nonlinearity. The results of loading for these models are compared under different loading with experimental data and each other. The result can be summarized as:

1. The best constitutive model that reveals the most converged and accurate results is the one which considers material and geometrical nonlinearity.
2. Out of plane fiber undulation decreases the material strength in tension and compression.
3. In transverse loading all the 3 models reveal the same results.
4. Wave parameter has an indirect relation with final strength in tension and compression.
5. The model without considering geometrical nonlinearity diverges in some wave parameters.

The final phase of this research is incorporating viscoplasticity in the constitutive model. The conclusions of this phase are:

1. The strain rate increases the failure capacity of the laminate
2. The model can predict final failure of the laminates
3. The model is capable of capturing nonlinearities due to viscoplasticity.
4. The model is capable to find the damage initiation point and the fracture plane

8.1 Publications

There are several published papers as outcome of this research which are collected in the following subsection.

8.1.1 Journal papers

1. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*Intralaminar damage in polymer composites in the presence of finite fiber rotation: Part I – Constitutive model*” Composite Structures, Volume 151(1) 2016, pp 114–126.
2. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*Intralaminar damage in polymer composites in the presence of finite fiber rotation: Part II – Numerical analysis and validation*” Composite Structures, Volume 151(1), 2016, pp 127–141.
3. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*Damage analysis of out of plane undulated fiber composites*” Composite Structures, Volume 152(15) 2016, pp 464–476.
4. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*Damage analysis of FRP composites in dynamic conditions*” Submitted in Composites Part B, 2016.

8.1.2 Conference papers

1. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*Analysis of Transverse Compression and in-Plane Shear in Unidirectional Composites by a progressive Damage Model in Presence of Fiber Rotation*” ECCM17, Munich, Germany, 2016.
2. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*An analytical solution to incorporate finite fiber rotation in continuous damage model*” MechComp, Porto, Portugal, 2016.
3. S. Eskandari, F.M. Andrade Pires, P.P. Camanho, A.T. Marques “*Finite Element Analysis of Out of Plane Undulated Fiber Composites with Material Nonlinearity*” ICAME, Madrid, Spain, 2015.

8.2 Future work

More numerical examples and experimental tests should be conducted in order to fully analyze different aspects of the developed constitutive models. It is also possible to implement the constitutive model in different finite element platforms and compare the results.

The viscoplastic model in this research has used the fracture energy under dynamic conditions for compression. It can be extended by a comprehensive study on fracture energy under dynamic loads and usage of them in the model.

Regarding the different aspects that are included in the model, it can be used to model a wide variety range of applications. Concerning the desire in industry for composite manufacturing, the machining process of composite material is an interesting application that is possible to be analyzed by this model. Since in the cutting process, an elastic-plastic deformation is taking place until fracture of the material and because of cutting velocity there is always a high strain rate and dynamic conditions. So this phenomenon should be analyzed by elastic-damage-viscoplastic model which is the concept of this research.

Thermoplastic based composites have a promising future and will contribute in sustainability. Regarding the global trends it is highly anticipated that thermoplastic composites to be under spotlight in coming years. Therefore, it is interesting to pursue a research in the field of this kind of materials using the constitutive model including fiber rotation which is relatively high in this type along with plasticity/viscoplasticity under quasi static/dynamic conditions. This research started in this work and it is recommended to be continued by researchers.

In the modeling of materials, research concentrates on different scales from micromechanics to structural level in order to provide an understanding of the processes in the material deformation until structural analysis and prediction. This research focus was on meso-scale modeling. Needless to say that this should be extended to other scales to have more comprehensive study. Hence, it is interesting to analyze fiber rotation in micro scale and analyze its effect on damage in that level. In addition, regarding the fact that fiber rotation and plasticity are dealt with independently in this research, it would provide a better capability for generalization to structural level. Because many researchers just combine these 2 phenomena and derive material parameters by mechanical test. But when this simplified model will be used in structure behavior analysis, it is not necessarily reveal the same behavior and will be a source of error.

Considering the tendencies in modeling damage using fracture mechanics which is discussed in chapter 3 it is recommended to incorporate fiber rotation in smeared crack model and develop a model to understand the effects of fiber rotation on crack evolution and size.

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