Introducing a new High-Tech product considering Market Expansion: A Real Options approach

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Biographical note

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Abstract

The technology sector offers a wide range of products and services for both customers and other businesses. It is a high growth market, but with high uncertainty and an aggressive competition with hidden competitors that have probability of entrance.

The main aim of this dissertation is, by using a real options approach, to know the optimal time to launch a new generation product using an adapted model that includes market expansion. Although the existing works about the optimal timing to enter a market, there are some specific markets with specific characteristics that cannot follow these main works. For example, the high-tech market has always new products emerging and it is very important to take in consideration the expansion of dimension that happens in the market every time a new generation is launched.

This dissertation will be based on similar studies, but it has in consideration aspects that they do not take into consideration as the expandable markets under uncertainty and as the loss in incremental terms. This means that when a new generation is launched, there are some specific markets, such as the technological market, where a market expansion occurs that leads to changes in the market structure, for example changes in the market share of products and companies. It is also important to have in consideration the loss the company has if is the first to introduce the new generation in the market in comparison with the loss if a competitor is the first to introduce the new generation in the market which lead to a loss that is considered the loss in incremental terms. The model developed in the dissertation can be applied to real case scenarios and to demonstrate it, we will present a model implementation in two different situations: the iPhone and the iPad, being useful to determine the optimal timing to launch the new version of a technology.

Key-words: Real Options; Technological Industry; Expandable Markets.
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1. Introduction

In the past years, we have seen a big increase in the value of the technological industry market. According to Forbes\(^1\) three of the top five listed world companies in the public exchanges with more market value are related with technology (Apple, Google and Microsoft). This tells us that investor society are getting more and more into the usage of this technological industry, that is, in the usage of high-tech goods and services. For example, at the end of 2008 Facebook had a hundred million users, a number that increased every quarter until the end of 2015 where it reaches one thousand and six hundred million users\(^2\). Another example is the total number of smartphones sold that in 2008 was near the 140 million units\(^3\) and in 2015 it increased to almost 1,424 million units\(^4\) which demonstrates the growing importance of technology nowadays. Also in the global video game industry, there was an increase with a worth of 56 billion dollars in 2010 and a worth of 82 billion dollars in 2015\(^5\).

The technology industry is always in flux with new products and innovations appearing continually and redefining the sector’s constantly shifting landscape. Increasingly, technology firms are re-examining the structure of their businesses and taking bold steps to squeeze out better financial performance due to profit margins and market shares under siege from disruptive, often well-funded startups and other aggressive competitors. This competition has made customers more demanding by seeking greater performance, better features, more platform independence and flexibility at the lowest price possible. The volatility is shown in a flurry of attempted and consummated mergers, acquisitions, and divestitures. For example, in 2014 HP announced that it was splitting in two, separating its computer and printer hardware business (HP Inc.) from its enterprise hardware, software, and services units (Hewlett-Packard Enterprise). Another example is that IBM has invested heavily in the cloud computing and big data sectors to drive revenue growth in a new market segment. Also,

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3 See http://www.gartner.com/newsroom/id/910112
4 See http://www.gartner.com/newsroom/id/3215217
5 See http://www.economist.com/node/21541164
Symantec, a much smaller firm best known for its antivirus software, is splitting into two companies: cybersecurity and information management with a focus on data storage.

By being a highly-competitive market based upon research and development, it is common to see faster discoveries being made and large improvements over predecessors. This happens when the product already in the market start to become obsolete because there has been some technological evolution that justifies the improvement. This implies that the sales of the old product starts to decrease and the new product sales starts to increase at a high growth rate, which sometimes lead the companies to lower the price of the old product in order to soften the decrease in sales. Some companies consider the option to sell this old product in emerging countries where consumers are not capable of buying the new one. For example, this happens with the DVD player that when a second generation was released the older one saw a decrease in their prices, which led them to reach a wider range of consumers that means an increase of demand. But another phenomenon can happen that is by the entry of the new product, the market can expand by increasing its cash flows as will be presented.

Every time a new product is released, which means a new generation is sent to the market, the launcher gains the monopoly of that generation until a competitor enters that market because most of the consumers will stop buying the last generation products and will buy the only current-generation available product or will wait for its competitors and choose from a wider range of products. Also, some disadvantages can appear like the rivals know what technology was used and have the choice to replicate it at a lower price or even have a more advanced product.

This business has particularities like high uncertainty (depends on the results of research and development), high growth (keep up with the pace of the changes taking place), aggressive competition (probability of entrance of hidden competitors) and patent wars (common court proceedings). Also, when two different companies/products are competing for the same markets usually one will be accepted and gain the majority of the market, which means that decisions taken by the companies become of a great importance. A decision that can have great effects on the market share is the optimal timing to introduce the new product because if they launch too soon that means that the sales of the old product will be affected and competition can replicate or gain advantage
and if they launch to late the reputation of the company will be affected and won’t have the advantages of being the first mover in the market.

An example of how a new product affects the sales of an old one is the Apple iPhone. The second iPhone, the iPhone 3G, was released in 2008 and its sales increase until the launch of the third iPhone, the iPhone 3GS. This happens in every generation until the last two that were released in 2014 that are the iPhone 6 and the iPhone 6 Plus where the sales start by being near 10 million in the third quarter of 2014. It is possible to see this and other effects in the Figure 1 where is presented the sales of every iPhone generation in sales between the third quarter of 2008 and the first quarter of 2015:

**Figure 1- Historical iPhone Sales** – Thousands of iPhones shipped between the third quarter of 2008 and the first quarter of 2015.

![Graph showing iPhone sales over time](image)

*Source: Asymco.*

By the analysis of the Figure 1 we can see that when a new generation is launch the previous generation has decreased in its sales, but continues in the market because Apple choose to sell it at a lower price. With this, Apple gains the more demanding segment of the market and sacrifices a little more to have new features as well as gains
the segment that want a lower price product. Another effect that can be observed by analysing Figure 1 is the expansion when the new generation is released that can result from a market expansion or from an increase in the market share. When the new generation appears in the market, the iPhone in general suffers a big incremental. For example, when the iPhone 4S was launched the global sales of all iPhones in that quarter reach higher values that ever seen reaching almost 40 million sales that is iPhone 3GS, 4 and 4S contrary to the previous high 15 million reached when the iPhone 4 was launched and was in the market with the iPhone 3GS.

The main objective of the dissertation is to determine the optimal timing to launch a new generation of a product giving that when the new generation enters the market this one can expand which means that the new product will affect the general cash flows. Although there is extensive literature about the optimal timing to enter a market in Real Options, some markets that have special characteristics need special studies in this case the lack of the studies is that they do not have in consideration the idea that when a new generation is launch the market expands becoming larger. It is very important to a company to predict the optimal timing to launch this new generation to gain some advantages in the market. To do so, we must have in consideration not only the expandable market already referred but also have in mind that when the new generation product is launched, the old generation product already in the market will see its sales starting to decrease which means that they only increase until a certain point after which they will start to decrease.

The new concept brought in this dissertation is by the introduction of a scenario where the whole market cash-flows increases with the entry of the new generation in the market and the percentage of each product (old and new) being different from each other in the market. We will valuate the option to enter the market as a call option where the company has the option to launch the new generation.

The results show that the model can be applied to real case scenarios and can be useful to predict the optimal timing to launch the new version of a technology. It is important to perform a sensitivity analysis having in consideration the changes in input values that can lead to great changes in the results as we demonstrate especially with the
market expansion and the loss in incremental terms as well as with volatility and market share.

The reminder of the paper is as follow: the second chapter is a literature review about Real Options, about technological industry, about expandable markets under uncertainty and about other subjects that contribute to this theme; the third chapter includes the development of the model, where we evaluate the optimal timing of a new product release, assuming the company has the control of the technological leap as base but also having in consideration that a competitor can also be the first to introduce the new generation in the market; the fourth chapter includes two practical and numerical examples of the model applied to the real world in this case to the example of the iPhone and the iPad; the fifth chapter includes the conclusion of the paper and some discussions.
2. Literature Review

Since a first and very important appearance made by Black and Scholes (1973), the Options theory has followed a very high growth as an area of research and as a tool of analysis to a point where today is one of the most important fields of finance and its usage are expanding between many decision making processes in the area. They were the firsts to close a formula solution for option pricing that is still widely used today and regarded as one of the best methods of determining fair prices of options.

The term Real Options was coined in a paper by Myers (1977) and after that the study about the topic start to rapidly grow. It started being used in many topics for example, as an auxiliary in determining investment strategies and related often with Games Theory. Also the increasing flow of research start to present some results such as: flexibility and information have a real and sometimes substantial value and option theory can help identify and measure options embedded in real assets. It is important to highlight the work of Dixit and Pindyck (1994) that brought academic research together into one reference and in their book develop a model based on the stochastic value of the project evolving randomly, and present a model where there is a leader-follower competition.

The next step was studying how the investment under uncertainty combines with the investment under competition and it was presented by Huisman and Kort (1999). Their conclusions were that when there is uncertainty the option value of waiting must be considered because when the company is investing it is also giving up the possibility of doing it later and if the company does not invest quickly its competition may do it which means loss of value.

The importance of investment strategies in Real Option is explained by Tsekrekos (2003). This is a very interesting topic because Real Options may be held by a few investors at the same time and exercising of one can affect the value of other so, the author builds a model where there is a big threat of preemption and being a first mover may have advantages. This concept may be applicable in different markets, including the high-tech industry where is common seeing the first mover gaining advantage. Also Paxson and Pinto (2003) approach this, but introduces an innovation that is the change in market share of a company due to changes in consumers which means they create a model where the
leader and follower real value function assumes that the leader’s market share evolves according to a birth and death process for a duopoly environment and the conclusion was that the follower is less sensitive to changes and to the ratio arrivals/departures until the cash-flows exceed their trigger.

Following the work of this previous authors, Armada, Kryzanowski et al. (2011) consider the hypothesis of temporary or permanent advantages adding the possibility of hidden competition which affect the decision to invest. It is important to refer that all of these models have in common the usage of a value of the project/company evolving accordingly to a Geometric Brownian Motion.

Some studies instead of using an expected growth of the value of the project, study an option valuation framework where demand is governed by a product life cycle which is a very useful approach when valuing the introduction of goods. An example is the study of Bollen (1999). Also Rodrigues (2009) in his work, develop a model where demand can only grow up to a limit determined by segment size and it can behave differently before and after the investment in each segment.

In terms of the technology industry is important to refer a very important definition in this area that is innovation. One of the first and most cited definitions is the one made by Schumpeter (1934) because it is believed that he was the first one to propose an explicit definition. The timing of innovation and examination of the combination of expected benefits, the cost of R&D and competing firms was studied by Reinganum (1989) by analysing the incentives for individual companies to invest in R&D extended based on the rivalry interactions between companies in the market, as well as the effect of licensing the technology discovered and the diffusion of innovation and Chaney, Devinney et al. (1991) study how the introduction of a new product affects the market value of the companies concluding that there is a small increase over short time, but the impact of introduction varies negatively with the magnitude of the systematic risk and with the number of announcements made over the decade. Also Chou and Yang (2011) examine companies’ innovation orientation, market orientation and new products performance concluding that new product performance when derived from the interaction between innovation orientation and responsive market orientation show that the
interaction effect is contributory to firm performance until an optimal level is reached and then becomes adverse thereafter.

Other important topics deserve to be present like for example the importance of strategies when releasing a new product which was studied by *Takeuchi and Nonaka (1986)* concluding that more and more consumers tend to choose new products instead of the old ones. Also *Azevedo and Paxson (2009)* studied the case of complementary technology in a world of uncertainty concluding that given the uncertainty about revenues and the price of the two technologies, it might be optimal for the leader and follower to adopt the two technologies asynchronously that is the first one whose price is decreasing at lower rate and then the other one. *Rubera, Griffith et al. (2012)* identify the key dynamics that contribute to a successful new product rollouts within a region, that means a significant product release often accompanied by a marketing campaign to generate a lot of consumer hype and conclude that as economic openness will make it easier for market entrance, it will also increase competition as easier market entry applies to all new entries so the company will benefit from gaining the pioneer advantages.

*Munnukka and Järvi (2011)* studied the perceived economic value of high-tech consumer products this means that they studied the influence of costumer-value hierarchy factors on consumers value perceptions of high-tech products. This is important to understand the degree of adoption of a new product over the old which determine the market split between the two products. They identify five categories that can influence the perceived value of a high-tech product and test every one of them concluding that the most important are the product’s direct attributes like layout, size, design, quality and price; the benefits perceived by the customer like reliability, ease use, availability and quality/cost ratio; and the extent to which the purchased high-tech product is in use in the buyers social network.

Also a research on the factors that influence the consumer about the product to be an object of interest which will influence the diffusion of new high-tech products in the market was conducted by *Lee, Khan et al. (2013)*. The conclusion of the study is that the unexpected and non-essential attributes have a stronger impact than expected, but the non-essential attributes and the expected and essential ones.
To understand the importance of expandable markets under uncertainty is important to know that investment decisions in the real world rarely occur under monopolistic settings and also perfect competitive markets are not typical structure for the majority of the businesses. As exposed by Bouis, Huisman et al. (2009) the recent wave of mergers and acquisitions contributed for a new oligopoly structures in several sectors with a few number of operating firms. In their study, they study the optimal decision to invest in markets where more than two competitors are allowed to enter making a basic assumption that market demand remains relatively stable and so as firms enters the market a reallocation of the market shares occurs which implies that none of the entries is considered to have some sort of impact on the market dimension itself. From that they derive the value and the triggers for an oligopoly of three firms and then extend the model to n firms.

This oligopolistic market structure that appears in many markets also appears in some regulated markets where the regulator determines the number of firms allowed to operate. All of this justifies the recent growing interest of real option approaches in oligopolistic structures. It is critical to refer some of them like the ones that deal with duopoly markets, which are the most of them. For example, Smets (1991) for start and Grenadier (1995) follow a real-options approach to endogenously derive the entire term structure of lease rates, by developing a unified framework for pricing a wide variety of leasing contracts and show how the model is flexible enough to determine equilibrium lease rates for leases of any term and practically any structure. Another example is Pawlina and Kort (2002) that analyses the impact of investment cost asymmetry on the optimal real option exercise strategies and the value of firms in a duopoly with both firms have an opportunity to invest in a project enhancing the profit flow and show that three types of equilibrium strategies exist and express the critical levels of cost asymmetry delineating the equilibrium regions as functions of basic economic variables. Also, they discuss the welfare implications of the optimal exercise strategies and show that the presence of identical firms can result in a socially less desirable outcome than if one of the competitors has a significant cost advantage.

There are also other studies that concern about market structures with more than two active players. The major example is Grenadier (2002) that provides a tractable approach for deriving equilibrium investment strategies in a continuous-time Cournot–
Nash framework being the impact of competition on exercise strategies dramatic like while standard real options models emphasize that a valuable “option to wait” leads firms to invest only at large positive net present values, the impact of competition drastically erodes the value of the option to wait and leads to investment at very near the zero net present value threshold.

_Fernandes and Pereira (2013)_ study the optimal time to launch a new generation product, but having in consideration a specific condition of the technology market, which means that the sales of the old generation start to decrease after a new product enters the market which means that the cash flows will increase only until a certain point after which they will start to decrease. This was the innovation brought in this study where the cash-flow of the old generation growth rate differ along the time and the cash-flow of the new generation growth rate will be different from the old because the market is more mature and has a higher demand. A lack in this study is that it does not consider that when a new product or generation enters the market, the market could suffer an expansion and that will be considered in our model as well as the product’s cash-flows evolving stochastically according to a Geometric Brownian Motion.

After taking into account the theoretical background that influenced our work, we present our own model.
3. Model Development

In this chapter, we introduce a model where the company, as an assumption, has the control of the technology leap, and with the capacity to determine when the next generation starts. This is the case when the product is a new-born and the company launching it has the know-how and has the lead in research and development. This often occurs when a new product is released, which creates a new market, and it takes some time before competitors reach the market. But we will also have in consideration the case where the control of the technology leap is in possession of the competitor by introducing a variable that measures the loss in incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor.

In the model presented, we have in basis two different models and with them we build a hybrid model that means a model that is the joint of the two. The first model is the one developed by Fernandes and Pereira (2013) that consists in determining the optimal timing to launch a new generation model under a monopolistic setting having in consideration that sales of the old generation start to decrease after a new product enters the market which means that the cash flows will increase only until a certain point after which they will start to decrease. The second model is the one developed by Pereira and Rodrigues (2010) that consists in considering the hypothesis that a new entry of a competitor in the market produces a market expansion having in consideration some effects like the likelihood of entry, the impact on the firms’ market shares and how the dimension of the expansion influences the behaviour of the firms already in the market. The authors, in this paper, proposed a real options model for a duopoly that faced the entry of a third competitor that can expand the market.

Assuming that company has one product in a market that meets the assumptions mentioned above, and the whole market cash-flows will be denominated as $X$. This $X$ evolves following a Geometric Brownian Motion:

$$dX = \alpha X \, dt + \sigma X \, dz$$  \hspace{1cm} (3.1)

where $X > 0$, $\alpha$ represents the risk neutral expected growth rate of the cash-flows, $\sigma$ is the volatility and it was assumed that the volatility of the cash-flows is the same no matter what generation is available in the market. $dz$ is the increment of a standard Wiener
process. This means, by the analysis of the equation, that one may know the value of the current cash-flows, but its future values are lognormally distributed with linear growing over the time variance.

We develop a two period model:

- The first period where there is only one product to sale by the company, that is, the continuation region because it is not yet optimal for the company to launch the product at that moment. We will refer to this as the Old Generation moment;

- The second moment where the new product is released as it is optimal to invest that is the stopping region. We will refer to this as the New Generation moment.

So, in the Old Generation period, the Net Present Value (NPV) will be:

\[ G1: \theta_i^1 X \]  

where \( \theta_i^1 \) represents the market share of the generation \( i \) in the market where only that \( i \) generation is in the market. It is important to refer that there are other products in the market so the market share will not be 100%.

At some moment in time the company will introduce a new and upgraded version of its product. This upgraded version suits more costumers and may expand the market. With this new upgraded version, the company may choose to sell the old product at a lower price, reaching to more consumers as referred before. Sometimes companies choose the launch of a new product to introduce the old product into new markets or segments with lower purchase power, trying to increase or at least maintain its sales. With two products now being sold, it is likely that the market will expand. With the old generation of being sold at a lower price, the number of products sold will, most likely, increase because new consumer segments will be reached.

The cash-flows of the company will now have two sources: its old product and its new product. Since the new is an improvement is probable to see a cut-down on the old product’s sales. It is also expected that the new product will have some new features when
compared to the old one, and will be released in a more mature market since the old one had already made its way into the market, thus it is expected that sales will have a boost.

So, after the release of the new product, the Net Present Value (NPV) of each product will be:

\[
G2: \begin{cases} 
\theta_i^1 X (1 + \emptyset) \\
\theta_i^{i+1} X (1 + \emptyset) 
\end{cases}
\]  

where \( \theta_i^1 \) represents the market share of the generation \( i \) in the market where both generations are present (generation \( i \) and generation \( i + 1 \)); \( \theta_i^{i+1} \) represents the market share of the generation \( i + 1 \) in the market where both generations are present (generation \( i \) and generation \( i + 1 \)); \( \emptyset \) represents the expansion, corresponding to the percentage of the net cash flow growth by the entry of the new product in the market. By the analysis, we can see that the two products share the market, but this market or the cash-flow represented by \( X \) suffer an increase of \( \emptyset \) due to the fact of a market expansion created by the launching of the new product. It is important to have in consideration that other products are also in the market, which means that the market share of these two generations will probably not be 100% since this is a very competitive market.

The model contemplates some possibilities, for example: when the old generation disappears, we set \( \theta_i^1 = 0 \); when the new generation is not accepted in the market and in extreme cases, no one buys it, we set \( \theta_i^{i+1} = 0 \); when the entry of the new generation does not expand the market, we set \( \emptyset = 0 \); when the entry of the new generation expands the market to twice bigger than before we set \( \emptyset = 1 \).

When the company has only one product, its cash-flows will come from that product and the company could stay there forever. But the knowledge that the release of a second product will boost the market, probably increasing its cash-flows, makes the company wonder about the option to launch a new product but to exercise the option the company will have to pay \( K \). This launch of the new product will make it steal part or all the sales of the older one and the company will see its cash-flows coming from two different products.

In the continuation region, following the study by Dixit and Pindyck (1994) and having in consideration that the company already is in the market we get:
\[ rFdt = E(dF) \quad (3.4) \]

where \( Xdt \) is the instantaneous cash-flows of the market. \( rFdt \) is the total expected return on the investment opportunity under risk uncertainty, which is equal to the expected rate of capital appreciation. Using Ito’s Lemma to expand the equation becomes:

\[ dF = F'(X)dX + \frac{1}{2} F''(X)(dX)^2 \quad (3.5) \]

Substituting for \( dX \) and given that \( E(dz) = 0 \):

\[ E(dF) = \alpha XF'(X)dt + \frac{1}{2} \sigma X^2 F''(X)dt \quad (3.6) \]

The ordinary differential equation will by this way become:

\[ \frac{1}{2} \sigma^2 X^2 F''(X) + \alpha F'(X) - rF(X) = 0 \quad (3.7) \]

And the general solution to this equation will be:

\[ F(X) = A_1X^{\beta_1} + A_2X^{\beta_2} \quad (3.8) \]

Knowing this, it’s now important know that \( F(X) \) will also have to satisfy three boundary conditions that are the following:

\[ \lim_{X \to 0} F(X) = 0 \quad (3.9) \]

\[ \lim_{X \to X^*} F(X) = \theta_{i+1}^i X^*(1 + \emptyset) + \theta_{i+1}^{i+1} X^*(1 + \emptyset) - w \theta_{i}^i X^* - K \quad (3.10) \]

\[ \lim_{X \to X^*} F'(X) = \theta_{i+1}^i (1 + \emptyset) + \theta_{i+1}^{i+1} (1 + \emptyset) - w \theta_{i}^i \quad (3.11) \]

where \( \emptyset \in \{0,1\} \) is a parameter that measures the loss in incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor. This means that this parameter measures the loss by having in consideration if the company is the first to introduce the new generation in the market or if a competitor is the first to introduce it. Both situations lead to a loss that is considered in incremental terms.

The first condition guarantees that the option component of the value function tends to zero as \( X \) tends to zero also. This means that the option to launch the new generation product will have no value when the market cash-flows tend to zero. In other words the lower the present values of the future cash-flows the lower will be the value of the option because the company will have no incentive to release a new product.
The second condition is the value-matching condition. When the company decides to invest, it will receive the present value of the cash-flows originated by the old product and the present value of the cash-flows that come from selling the new one. This means that the value of the company is given by the present value of the Cash Flows of the current generation ($\theta_i^i X^*$) plus the option to launch the new generation ($A_2 X^\beta_2$) and at the optimal timing that’s is when the trigger value is reached ($X = X^*$) the value condition applies. The first two terms of the equation $[\theta_{i+1}^i X^*(1 + \emptyset) + \theta_{i+1}^{i+1} X^*(1 + \emptyset)]$ represents the present value of the cash flows with the market expansion; the third term ($w\theta_i^i X^*$) represents the incremental losses of cash flows due to launch the new generation in the market. Also, the company will have to pay $K$ in order to exercise the option, which are, in this case, all of the costs required to put the new generation in the market.

The third condition is the smooth-pasting condition, which assures that the function is continuously differentiable along $X$.

So after having in consideration the first boundary condition the solution for $F(X)$ becomes:

$$F(X) = A_1 X^\beta_1 \quad (3.12)$$

Solving the equation according to the second boundary condition, we obtain the value of $A_1$ that is:

$$A_1 = [\theta_{i+1}^i X^*(1 + \emptyset) + \theta_{i+1}^{i+1} X^*(1 + \emptyset) - w\theta_i^i X^* - K] \left(\frac{X}{X^*}\right)^{\beta_1} \quad (3.13)$$

Now we can obtain the complete solution by replacing the $A_1$ in the solution and reach the following:

$$F(X) = \begin{cases} \left[\theta_{i+1}^i X^*(1 + \emptyset) + \theta_{i+1}^{i+1} X^*(1 + \emptyset) - w\theta_i^i X^* - K\right] \left(\frac{X}{X^*}\right)^{\beta_1} & \text{for } X \geq X^* \\ \theta_{i+1}^i X^*(1 + \emptyset) + \theta_{i+1}^{i+1} X^*(1 + \emptyset) - w\theta_i^i X^* - K & \text{for } X < X^* \end{cases} \quad (3.14)$$

The upper part of the equation that is the first branch represents the value that is received by the company for being in the market with the old generation plus the value of the option to release the new generation. The market will likely expand and the market shares of each product will be different. After the release both products will be in the market and the cash-flows will come from both products.
The lower part of the equation that is the second branch represent the value that will be received after the release of the new generation with the cost of the exercise price.

The value from which the company will exercise the option to release the new product that is the trigger value is:

\[
X^* = \frac{\beta_1}{\beta_{1-1}} \frac{K}{(\theta_{i+1}^i + \theta_{i+1}^{i+1})(1 + \phi) - w \theta_i^i}
\]  

(3.15)

where \((\theta_{i+1}^i + \theta_{i+1}^{i+1})(1 + \phi) > w \theta_i^i\) which means that \(\phi > \frac{w \theta_i^i}{\theta_{i+1}^i + \theta_{i+1}^{i+1}} - 1\). Also in the equation:

\[
\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}
\]  

(3.16)
4. Model Implementation

4.1 The case of the Apple iPhone and Smartphone industry

In this section, the model will be applied to a real world situation and for that we needed to collect data about the Apple iPhone and the Smartphone industry in general. Smartphones are a modern iteration of handheld telecommunication devices. They took the market by storm after Apple introduced the first iPhone back in 2007. As basic and feature-phones gradually became obsolete, the industry dynamics changed as newer players took over the market for “smart” mobile devices. Over time, smartphones have become the anchor for sweeping changes across many industries, as they have allowed on-the-go consumption of large volumes of data and information like no other device before.

The situation presented is one where the company introduces the new generation in the market and, in this specific case, the launch of the iPhone 4S that happen at 14th October of 2011. However, this exercise is just for demonstration purposes because we do not intend, to make a final assessment about the firm’s decision. Some assumptions will be made in order to keep the application simple. In the final section of this chapter we will present a comparative statics about the results obtain and the real life results, and a sensitive analysis where we will analyse the change in the more important variables.

It is easy to distinguish between generations because they seem to be launch every year and there is something in common between the products available for that generation. It is important to know the optimal timing to release a product, as the right entry in the market bring some advantages against the competition. In this specific market, the old product continues to be sold (for example the iPhone 4 continues to be sell some years after the release of the iPhone 4S) which is represented in the model and allows to see how it represents reality. Being a technological market, there is high uncertainty as new technologies are constantly being discovered and new competition, including hidden competitors, is always appearing. It is important to refer that the market share of the new generation affects the market share of the older generation which means that in general, when the new generation is launched, the old generation market share goes to the new
generation product that appropriates of the majority of the market share but the older generation product still maintain a market share.

We start by analysing the units of smartphone sales in the last few years summarized in the table below:

**Table 1 - Smartphone Sales – Millions of smartphones sold between 2006 and 2015 and division of sales by brand.**

<table>
<thead>
<tr>
<th>(Millions of units)</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia</td>
<td>345</td>
<td>435</td>
<td>472</td>
<td>441</td>
<td>461</td>
<td>422</td>
<td>334</td>
<td>251</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Motorola</td>
<td>209</td>
<td>164</td>
<td>106</td>
<td>58</td>
<td>39</td>
<td>40</td>
<td>34</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Samsung</td>
<td>116</td>
<td>154</td>
<td>199</td>
<td>236</td>
<td>281</td>
<td>314</td>
<td>385</td>
<td>444</td>
<td>393</td>
<td>320</td>
</tr>
<tr>
<td>Sony Ericsson</td>
<td>74</td>
<td>101</td>
<td>93</td>
<td>55</td>
<td>42</td>
<td>33</td>
<td>-</td>
<td>38</td>
<td>38</td>
<td>-</td>
</tr>
<tr>
<td>LG</td>
<td>62</td>
<td>79</td>
<td>102</td>
<td>122</td>
<td>114</td>
<td>86</td>
<td>58</td>
<td>69</td>
<td>76</td>
<td>-</td>
</tr>
<tr>
<td>Apple</td>
<td>-</td>
<td>2</td>
<td>12</td>
<td>25</td>
<td>47</td>
<td>89</td>
<td>130</td>
<td>150</td>
<td>191</td>
<td>226</td>
</tr>
<tr>
<td>Others</td>
<td>185</td>
<td>218</td>
<td>248</td>
<td>299</td>
<td>614</td>
<td>790</td>
<td>805</td>
<td>855</td>
<td>1.180</td>
<td>878</td>
</tr>
<tr>
<td>TOTAL</td>
<td>991</td>
<td>1.153</td>
<td>1.232</td>
<td>1.236</td>
<td>1.598</td>
<td>1.774</td>
<td>1.746</td>
<td>1.807</td>
<td>1.878</td>
<td>1.424</td>
</tr>
</tbody>
</table>

*Source: Statista – The portal for statistics.*

From the table analysis, we can see that the smartphone sales has increased in a large scale and in 2016 it reach almost the double of the smartphones sold in 2006. This increase was sustained during the past years with a small increase during every year, which indicates a market that is growing. In terms of brands we can see that the brands that have the biggest sales in the first years are now brands with few sales or even brands that disappear of the market. We can see that Nokia and Motorola, that were the leaders in 2006, had a great decrease over the years. The global Smartphone market has slowed down in 2015 and by facing greater challenges, Smartphone vendors are focusing more on emerging markets like India and Africa.

Another effect that can be verified is that small brands (represented by “Others” in the table) have become the most important selling force in the market with an increase in the units sold all over the previous years and reaching an amount of more than 60% of the market in 2014. But the two most important brands in this sector that have been growing since their appearance in the market are Samsung and Apple and we can verify their growing in the following figure:
As we can see by analysing the figure, both Samsung and Apple had a large increase in their sales from 2007 to 2013 when Samsung started to decrease but Apple continued its rise. These brands are the two most important brands in the market and they are also the most famous brands. Samsung has the strategy of launching more than one new product per year, while Apple has the strategy of launching only one new version per year, which means that besides both companies have different market strategies, they are the market leaders. Apple also is defined as the leading digital asset Management Company and global smartphone provider and it is the only one who generates and runs both software’s and hardware’s which allows consumers to buy and share contents via Apple devices by an exclusive selling media platform. Due to these specifications and the importance of Apple in this market we choose it as an example to implement our model.

So, as the risk-free rate, we used the 10 years United States Treasury bonds of 14th October 2011 as the Apple iPhone 4S was released in that data (2.26%)7.

To estimate the value of the cash-flows growth or the drift ($\alpha$) is important to have in consideration that we are working in a risk-free environment and so we have to find

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the risk-neutral drift. For that, we first calculated the growth rate of the smartphone between quarters before the launch and calculate the median because when the new version of a smartphone is launched we consider it a market expansion and not a growth rate. So analysing the period between the first quarter of 2009 and the first quarter of 2016 we reach the value of the growth rate \((g)\) of 4.87\%. To obtain the risk premium we observe the risk premium of Apple in 2011 (5.10\%)\(^9\) and subtract the risk-free rate (2.26\%) which give us a risk premium of 2.84\%. So, with the growth rate and the risk premium already find, we can now verify that cash-flow growth rate \((a)\) will be 2.03\%.

The value of the investment \((K)\) was not explicit in the financial reports, so we used the Selling, General and Administrative Expenses, that include expansions in retail, headcount and related expenses, professional services, marketing and advertising and variable costs associated with the growth of net sales, and we made a direct proportion with the iPhone and related products and services in the total of net sales. This means that we multiply the percentage of iPhone and related products and services in the total of net sales \((47.057M$/108.249M$=43.47\%)\) with the value of Selling, General and Administrative Expenses \((43.47\%*7.599M$=3.303,37M$)\(^10\). So, according to our assumptions, the value of investment \((K)\) is 3,303,37 million dollars.

For the project’s volatility \((\sigma)\), as a simplification, we used Apple’s share’s daily return volatility from 2007 (date of the launch of the 1\(^st\) iPhone) until May 2012 (date before the launch of the next iPhone after the iPhone 4S that is the iPhone 5)\(^11\). So, the value of the volatility \((\sigma)\) was considered to be 0.3794.

Now it is important to know the market share of iPhone 4 in the market where iPhone 4S was not yet launch \((\theta_4)\) and the market share when Apple had both iPhone 4 and iPhone 4S in the market \((\theta_4^4+\theta_4^4S)\)\(^12\). So in 2010, when Apple only had the iPhone 4 in the market, its market share was 15.6\% and in 2011 when Apple had both


\(^9\) See [https://www.stock-analysis-on.net/NASDAQ/Company/Apple-Inc/DCF/CAPM](https://www.stock-analysis-on.net/NASDAQ/Company/Apple-Inc/DCF/CAPM)


\(^12\) See [https://gigaom.com/2012/02/07/npd-apple-sold-most-smartphones-in-q4-but-samsung-wins-2011/](https://gigaom.com/2012/02/07/npd-apple-sold-most-smartphones-in-q4-but-samsung-wins-2011/)
products in the market its market shares was 19% being divided in 15,5% of the iPhone 4S and 3,5% of the iPhone 4.

To estimate the market expansion (∅) we analysed the expansion in the market when the new product or generation is launch that is the quarter when the market suffers the highest growth that is the market expansion. This means that in our example, since the iPhone 4S was launched in October of 2011 the market expansion (∅) will be 30% since the market expansion in the fourth quarter of 2011 was 30%. Besides this value presented in the market it is important to make a sensibility analysis because this is a parameter that can be variable every quarter and also we must have in consideration that, due the specifications of these market, the market expansions can be huge and the market can suffer a high growth.

In terms of the loss in incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor (w) it is the most difficult parameter to estimate because we do not have the data to know the real value. So, we decided to make an analysis where we consider alternative values to this incremental loss (w) having in consideration that this value only vary between 0 and 1 which means between 0% and 100% of losses.

Following the rationale mentioned before, the values that were taken into consideration are:

**Table 2 – iPhone Study Case – Parameters of the iPhone study case**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>The Value of the Investment</td>
<td>3,303.37 M$</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Instantaneous Volatility</td>
<td>0.3794</td>
</tr>
<tr>
<td>( r )</td>
<td>Risk-free Interest Rate</td>
<td>2.26%</td>
</tr>
<tr>
<td>( \emptyset )</td>
<td>Market Expansion</td>
<td>30.00%</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Cash-flow Growth Rate</td>
<td>2.03%</td>
</tr>
<tr>
<td>( \theta_4^4 )</td>
<td>Market Share of iPhone 4 in the market where iPhone 4S was not yet launch</td>
<td>15.60%</td>
</tr>
<tr>
<td>( \theta_4^4+4s )</td>
<td>Market Share of iPhone 4 in the market where iPhone 4S was already launched</td>
<td>3.50%</td>
</tr>
<tr>
<td>( \theta_4^s+4s )</td>
<td>Market Share of iPhone 4S</td>
<td>15.50%</td>
</tr>
</tbody>
</table>
The loss in incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor (w), as already was referred, will be analysed in a sensibility analysis table that will be presented next also with a more detailed focus in the market expansion:

**Table 3 - Trigger Values** – different trigger values that vary with the market expansion and the loss in incremental terms (iPhone case)

<table>
<thead>
<tr>
<th>Trigger Values (X*) (Millions of Dollars)</th>
<th>0.00%</th>
<th>20.00%</th>
<th>30.00%</th>
<th>50.00%</th>
<th>100.00%</th>
<th>200.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss in Incremental Terms (w)</td>
<td>0.00%</td>
<td>717,123,54</td>
<td>597,602,95</td>
<td>551,633,49</td>
<td>478,082,36</td>
<td>358,561,77</td>
</tr>
<tr>
<td>10.00%</td>
<td>781,269,92</td>
<td>641,494,69</td>
<td>588,822,27</td>
<td>505,766,42</td>
<td>373,911,84</td>
<td>245,767,45</td>
</tr>
<tr>
<td>17.10%</td>
<td>834,240,23</td>
<td>676,778,89</td>
<td>618,416,40</td>
<td>527,446,91</td>
<td>385,630,58</td>
<td>250,776,45</td>
</tr>
<tr>
<td>30.00%</td>
<td>951,490,73</td>
<td>751,950,74</td>
<td>680,586,78</td>
<td>572,012,90</td>
<td>408,923,99</td>
<td>260,423,31</td>
</tr>
<tr>
<td>60.00%</td>
<td>1,413,417,77</td>
<td>1,013,790,72</td>
<td>888,223,42</td>
<td>711,878,13</td>
<td>475,745,37</td>
<td>286,006,45</td>
</tr>
<tr>
<td>100.00%</td>
<td>4,007,455,09</td>
<td>1,892,409,35</td>
<td>1,497,290,91</td>
<td>1,056,228,47</td>
<td>608,274,43</td>
<td>329,114,67</td>
</tr>
</tbody>
</table>

As we can see from the analysis of the table, when the market expansion (∅) is higher the trigger value is lower which means that when the market expands in higher values, is ideal to the company to launch the new generation early. In terms of the loss in incremental terms (w) the results are the contrary, as expected, that is the trigger value is higher when the loss in incremental terms (∅) is also higher.

Apple launched the iPhone 4S in the year 2011 as already refer. In that year the total sales of smartphones were 1.774 million units with an average selling price\(^1\) of 348.6 dollars, which means that the total sales in 2011 of the smartphone market was 618.416,40 million dollars. So, with this data, we can say that if the loss in incremental terms (w) was 17.10% and the market expansion (∅) was 30%, Apple entered the market at the right moment since the value calculated with our model is the same as the market sales of smartphones. For example, if the loss in incremental terms was 10% and the market expansion was 100%, the trigger value would be 373,911.84 million dollars, which means that Apple did not make the launch at the right moment and should have launched the new version (iPhone 4S) earlier when the market was smaller in order to

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have more benefits. Another example is that if the loss in incremental terms was 60% and the market expansion was 20%, the trigger value would be 1.013.790,72 million dollars, which means that Apple did not make the launch at the right moment and should have launched the new version (iPhone 4S) later when the market was bigger in order to have more benefits.

**Comparative statics and sensitive analysis**

Since the parameters chosen were based on assumptions, we decided to do a sensitivity analysis to find out how the model would behave if the parameters were different from those presented before.

In the following figures we will present and summarize the behaviour of the trigger, according to changes first in volatility ($\sigma$):

*Figure 3 - Sensitivity Analysis on Volatility (iPhone study case)*

As it would be expected the increase in volatility ($\sigma$) also increases the value of the trigger, as the company will exercise the option later.

The second figure is about the Market Share of iPhone 4 in the market where iPhone 4S was not yet launch ($\theta_4^4$):
As it would be expected the increase in Market Share of iPhone 4 in the market where iPhone 4S was not yet launch ($\theta^{4}_{4}$) also increases the value of the trigger, as the company will exercise the option later.

The third figure is about the Market Share of iPhone 4 in the market where iPhone 4S was already launched ($\theta^{4}_{4+4S}$):

*Figure 5 - Sensitivity Analysis on the Market Share of iPhone 4 in the market where iPhone 4S was already launched*
As it would be expected the increase in Market Share of iPhone 4 in the market where iPhone 4S was already launched \((\theta^4_{4S})\) leads to a decrease in the trigger value, as the company will exercise the option sooner.

The fourth and last figure is about the Market Share of iPhone 4S \((\theta^4_{4S})\):

Figure 6 - Sensitivity Analysis on the Market Share of iPhone 4S

As it would be expected the increase in Market Share of iPhone 4S \((\theta^4_{4S})\) leads to a decrease in the trigger value, as the company will exercise the option sooner.

4.2 The case of the Apple iPad and Tablets industry

Now, the model will be applied to other real world situation and for that we needed to collect data about the Apple iPad and the Tablets industry in general. Tablets are mobile computers with a touchscreen display and battery in a single device and come equipped with sensors, including cameras, a microphone, an accelerometer, and the touchscreen display uses the recognition of finger or stylus gestures replacing the usage of the mouse and keyboard. They usually feature on-screen, pop-up virtual keyboards for typing. They may have physical buttons for basic features such as speaker volume and power, and ports for network communications and battery charging. In terms of size, they are typically larger than smartphones or personal digital assistants with screens with 18 centimetres or larger, measured diagonally.
Some special styles of tablets are\textsuperscript{14}:

- **Convertible tablets** - typically have a display that rotates 180 degrees and can be folded to close, screen up, over an integrated hardware keyboard. Convertible models may allow user input through a variety of methods in addition to the hardware keyboard, including natural handwriting with a stylus or digital pen and typing through a screen-based software keyboard.

- **Hybrid tablets** - is like a regular notebook but has a removable display that functions independently as a slate.

- **Rugged tablets** - a slate-like model that is designed to withstand rough handling and extreme conditions and are usually encased in a thick protective shell with shock-protected hard drives.

The situation presented is one where the company introduces the new generation in the market and, in this specific case, it will be analysed the launch of the iPad 4 that happen at 2\textsuperscript{nd} of November of 2012. However, this analysis is just for demonstration purposes of the application of the model, and we do not intend, to make a final assessment about firm’s decision. This means that some assumptions will be made in order to keep the application simple. In the final section of this chapter we will present a comparative statics about the results obtain and the real life results, and a sensitive analysis where we will analyse the change in the more important variables.

We study the launch of the iPad 4 coming from the iPad 2 because iPad 3 lived for only a few months on the market and was discontinued on the day of the announcement of the launch of the iPad 4 while iPad 2 stay in the market before iPad 4 was launched\textsuperscript{15}.

It is easy to distinguish between generations because they seem to be launched on a regular basis and there is something in common between the products available for that generation. It is important to know the optimal timing to release a product, as the right entry in the market bring some advantages against the competition. In this specific

\textsuperscript{14} See https://maxtouuchblog.wordpress.com/2013/03/29/different-types-of-tablets-pc-available-in-the-market/

\textsuperscript{15} See http://www.pcadvisor.co.uk/feature/tablets/what-happened-ipad-3-3443908/
market, the old product continues to be sold (for example the iPad 2 continues to be sold some time after the release of the iPad 4) which is represented in the model and allows to see how it represents reality. Being a technological market, there is high uncertainty as new technologies are constantly being discovered and new competition, including hidden competitors, is always appearing. It is important to refer that the market share of the new generation affects the market share of the older generation which means that in general, when the new generation is launched, the old generation market share goes to the new generation product that appropriates of the majority of the market share but the older generation product still maintain some market share.

We start by analysing the units of smartphone sales in the last few years summarized in the table below:

**Table 4 - Tablets Sales** – Millions of tablets sold between 2012 and 2015 and division of sales by brand.

<table>
<thead>
<tr>
<th>(Millions of units)</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>55,35</td>
<td>74,22</td>
<td>63,4</td>
<td>49,5</td>
</tr>
<tr>
<td>Samsung</td>
<td>8,43</td>
<td>39,7</td>
<td>40</td>
<td>33,6</td>
</tr>
<tr>
<td>Asus</td>
<td>2,65</td>
<td>12,2</td>
<td>11,3</td>
<td>3,7</td>
</tr>
<tr>
<td>Amazon</td>
<td>6,11</td>
<td>1,8</td>
<td>2,9</td>
<td>5,2</td>
</tr>
<tr>
<td>Barnes &amp; Noble</td>
<td>3,74</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Others</td>
<td>50,78</td>
<td>91,8</td>
<td>112,5</td>
<td>114,1</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>127,06</td>
<td>219,72</td>
<td>230,1</td>
<td>206,1</td>
</tr>
</tbody>
</table>

*Source: Statista – The portal for statistics.*

From analysing the table we can see that the tablets sales has increased on a large scale and in 2014 it almost the double of the tablets sold in 2012. This increase was sustained during the past years, except the last one, which indicates a market growing. In terms of brands we can see that the brands that had the biggest sales in the first years are now brands that maintain the market domination. We can see that Apple and Samsung, that were the leaders in 2012 continued to dominate the market in 2015. One of the biggest reasons why tablets sales are growing so fast is because they offer an attractive design and portability along with flexible screen, which is expected to fuel the tablet market. However, high prices, poor input speed and weak video capabilities are expected to be barriers to tablets market growth over the next period.
Another effect that can be verified is that small brands (represented by “Others” in the table) became the most important selling force in the market with an increase in the units sold all over the previous years and reaching an amount of more than 55% of the market in 2015. But the two most important brands in this sector that have been growing since their appearance in the market are Samsung and Apple and we can see they are growing in the following figure:

*Figure 7 - Samsung and Apple Tablets Sales* - Millions of tablets sold between 2012 and 2015 by Samsung and Apple.

![Figure 7 - Samsung and Apple Tablets Sales](image)

Source: Statista – The portal for statistics.

As we can see by analysing the figure, both Samsung and Apple had an increase in their sales from 2012 to 2013 when both started to decrease. These brands are the two most important brands in the market and they are also the most famous brands. Apple is defined as the leading digital asset Management Company and global tablet provider and it is the only one who generates and runs both software’s and hardware’s which allows consumers to buy and share contents via Apple devices by an exclusive selling media platform. Due to these specifications and the importance of Apple in this market we choose it as an example to test our model.
As the risk-free rate, we used the 10 years United States Treasury bonds of 2\textsuperscript{nd} November of 2012 as the Apple iPad 4 was released in that data (1.78\%)\textsuperscript{16}.

To know the value of the cash-flows growth or the drift ($\alpha$) is important to have in consideration that we are working in a risk-free environment and so we have to find the risk-neutral drift. For that, we first calculate the growth rate of the tablet industry between quarters before the launch and calculate the median because when the new version of a tablet is launched we consider it a market expansion and not a growth rate. So analysing the period between the second quarter of 2010 and the fourth quarter of 2015 we reach the value of the growth rate ($g$)\textsuperscript{17} of 2.56\%. To obtain the risk premium we observe the risk premium of Apple in 2012 (3\%)\textsuperscript{18} and subtract the risk-free rate (1.78\%) which give us a risk premium of 1.22\%. So, with the growth rate and the risk premium already find, we can now verify that cash-flow growth rate ($\alpha$) will be 1.34\%.

The value of the investment ($K$) was not explicit in the financial reports, so we used the Selling, General and Administrative Expenses, that include expansions in retail, headcount and related expenses, professional services, marketing and advertising and variable costs associated with the growth of net sales, and we made a direct proportion with the iPad and related products and services in the total of net sales. This means that we multiply the percentage of iPad and related products and services in the total of net sales (32.424M$/156.508M$=20.72\%) with the value of Selling, General and Administrative Expenses (20.72\%*7.599M$=1.574,30M$)\textsuperscript{19}. So, according to our assumptions, the value of investment ($K$) is 1.574.3 million dollars.

For the project’s volatility ($\sigma$), as a simplification, we used Apple’s share’s daily return volatility from 2007 (date of appearing of Apple in high markets with the launch of the 1\textsuperscript{st} iPad) until May 2014 (date when the iPad 4 is established clearly in the market with the next generation)\textsuperscript{20}. So, the value of the volatility ($\sigma$) was considered to be 0.3536.

Now it is important to know the market share of iPad 2 when Apple had not yet launched iPad 4 in the market ($\theta_2^2$) and the market share when Apple had already launched

\textsuperscript{16} See http://data.cnbc.com/quotes/US10Y/tab/2
\textsuperscript{17} See http://www.statista.com/statistics/272070/global-tablet-shipments-by-quarter/
\textsuperscript{18} See https://www.stock-analysis-on.net/NASDAQ/Company/Apple-Inc/DCF/CAPM
\textsuperscript{19} See http://investor.apple.com/secfiling.cfm?filingid=1193125-12-444068
\textsuperscript{20} See http://finance.yahoo.com/q/hp?s=AAPL
iPad 4 in the market ($\theta_{2+4}^2$ and $\theta_{2+4}^4$)\textsuperscript{21}. So in 2011, when Apple only had the iPad 2 in the market, its market share was 51.7% and in 2012 when Apple had both products in the market its market shares was 38.2% being divided in 15.28% of the iPad 2 and 22.92% of the iPad 4.

To estimate the market expansion ($\emptyset$) we analysed the expansion in the market when the new product or generation is launch that is the quarter when the market suffers the highest growth that is the market expansion. This means that in our example, since the iPad 4 was launched in November of 2012 the market expansion ($\emptyset$) will be 72.41% since the market expansion in the fourth quarter of 2012 was 72.41%. Besides this value presented in the market it is important to make a sensibility analysis because this is a parameter that can be variable every quarter and also we must have in consideration that, due to the specifications of these market, the market expansions can be huge and the market can suffer a high growth.

In terms of the loss in incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor ($w$) it is the most difficult parameter to estimate because we do not have the data to know the real value. So, we decide to make an analysis where we consider alternative values to this incremental loss ($w$) having in consideration that this value only vary between 0 and 1 which means between 0% and 100% of losses.

Following the rationale mentioned before, the values that were taken into consideration are:

\textsuperscript{21}See \url{http://www.statista.com/statistics/276635/market-share-held-by-tablet-vendors/}
Table 5– iPad Study Case – Parameters of the iPad study case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>The Value of the Investment</td>
<td>1574,30 M$</td>
</tr>
<tr>
<td>σ</td>
<td>Instantaneous Volatility</td>
<td>0,3536</td>
</tr>
<tr>
<td>r</td>
<td>Risk-free Interest Rate</td>
<td>1,78%</td>
</tr>
<tr>
<td>φ</td>
<td>Market Expansion</td>
<td>72,41%</td>
</tr>
<tr>
<td>α</td>
<td>Cash-flow Growth Rate</td>
<td>1,34%</td>
</tr>
<tr>
<td>$\theta_2^2$</td>
<td>Market Share of iPad 2 in the market where iPad 4 was not yet launch</td>
<td>51,70%</td>
</tr>
<tr>
<td>$\theta_2^2+4$</td>
<td>Market Share of iPad 2 in the market where iPad 4 was already launched</td>
<td>15,28%</td>
</tr>
<tr>
<td>$\theta_4^2+4$</td>
<td>Market Share of iPad 4</td>
<td>22,92%</td>
</tr>
</tbody>
</table>

The loss in incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor ($w$), as already was referred, will be analysed in a sensibility analysis table that will be presented next along with a sensitivity analysis made to the market expansion:

Table 6- Trigger Values – different trigger values that vary with the market expansion and the loss in incremental terms (iPad case)

<table>
<thead>
<tr>
<th>$X^*$ (Millions of Dollars)</th>
<th>Market Expansion ($\emptyset$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25.00%</td>
</tr>
<tr>
<td>10,00%</td>
<td>70,392,12</td>
</tr>
<tr>
<td>25,00%</td>
<td>86,067,38</td>
</tr>
<tr>
<td>35,03%</td>
<td>101,122,72</td>
</tr>
<tr>
<td>50,00%</td>
<td>136,862,85</td>
</tr>
<tr>
<td>70,00%</td>
<td>259,281,70</td>
</tr>
<tr>
<td>80,00%</td>
<td>469,060,49</td>
</tr>
</tbody>
</table>

As we can see from the analysis of the table, when the market expansion ($\emptyset$) is higher the trigger value is lower which means that when the market expands in higher values, is ideal to the company to launch the new generation early. In terms of the loss in incremental terms ($w$) the results are the contrary, as expected, that is the trigger value is higher when the loss in incremental terms ($\emptyset$) is also higher.
Apple launched the iPad 4 in the year 2012 as already refer. In that year the total sales of tablets was 127.06 million units with an average selling price\(^{22}\) of 494 dollars, which means that the total sales in 2012 of the tablet market was 62.767.64 million dollars. So, with this data, we can say that if the loss in incremental terms (\(w\)) was 35.03\% and the market expansion (\(\varnothing\)) was 72.41\%, Apple entered the market at the right moment since the value calculated with our model is the same as the market sales of tablets. For example, if the loss in incremental terms was 10\% and the market expansion was 100\%, the trigger value would be 42.079.13 million dollars, which means that Apple did not make the change in the right moment and should have launched the new version (iPad 4) earlier when the market was smaller in order to have more benefits. Another example is that if the loss in incremental terms was 70\% and the market expansion was 25\%, the trigger value would be 259.281.70 million dollars, which means that Apple did not make the change in the right moment and should have launched the new version (iPad 4) later when the market was bigger in order to have more benefits.

**Comparative statics and sensitive analysis**

Since the parameters chosen were based on assumptions, we decided to do a sensitivity analysis to find out how the model would behave if the parameters were different from those presented before.

In the following figures we will present and summarize the behaviour of the trigger, according to changes first in volatility (\(\sigma\)):

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As it would be expected the increase in volatility ($\sigma$) also increases the value of the trigger, as the company will exercise the option later.

The second figure is about the Market Share of iPad 2 in the market where iPad 4 was not yet launch ($\theta_2^2$):

**Figure 9 - Sensitivity Analysis on the Market Share of iPad 4 in the market where iPad 4S was not yet launch**

As it would be expected the increase in Market Share of iPad 2 in the market where iPad 4 was not yet launch ($\theta_2^2$) also increases the value of the trigger, as the company will exercise the option later.
The third study is about the Market Share of iPad 2 in the market where iPad 4 was already launched ($\theta_{2+4}^2$):

**Figure 10 - Sensitivity Analysis on the Market Share of iPad 2 in the market where iPad 4 was already launched**

As it would be expected the increase in Market Share of iPad 2 in the market where iPad 4 was already launched ($\theta_{2+4}^2$) leads to a decrease in the trigger value, as the company will exercise the option sooner.

The fourth and last study is about the Market Share of iPad 4 ($\theta_{2+4}^4$):
As it would be expected the increase in Market Share of iPad 4 ($\theta_{2+4}^{4}$) leads to a decrease in the trigger value, as the company will exercise the option sooner.
5. Conclusions

The main objective of this dissertation was to define the optimal time to launch a new generation product using an adapted model that includes market expansion and loss in incremental terms. This model contributes to the Real Options field of investigation because it includes the market expansion, which others studies in the area don’t include and is a very important input in some markets because the market expansion that occurs with the launch of a new product/company, for example, leads to changes in the market structure and complexity.

So we start by analysing the technological industry market, verifying that a big increase in the value of this market has been seen in the past years with three of the first five listed companies in public exchanges with more market value are related with technology. Also, the technology industry is always in flux which lead the companies to take bold steps in order to stay competitive since costumers are more demanding by seeking greater performance and features. This leads to larger volatility with the attempts of mergers, acquisitions or divestitures. The high competition makes companies faster researchers and developers which lead to faster discoveries and improvements which means that being the first mover can make gain a lot of advantages.

We reviewed some research related with Real Options theory and some models that evaluate options when cash-flows are stochastic. We also review studies about the technology markets and essentially about innovation more specifically about what it is, its impact and the reaction of consumers. To conclude our research, we review studies about expandable markets related to Real Options and a specific study that serve as the basis for our work about the optimal time to launch a new generation product, but having in consideration a specific condition of the technology market that is that the sales of the old generation start to decrease after a new product enters the market which means that the cash flows will increase only until a certain point after which they will start to decrease.

We introduced a model where the company had the control of the technology leap which means that the company has the capacity to determine when the next generation starts. But we also had in consideration the case when the control of the technology leap is in possession of the competitor by introducing a variable that measures the loss in
incremental terms taking in consideration the potential loss due to the introduction of the new generation made by a competitor. We assume that the company is acting in the market with only one product in sale and receives their product market share of the total cash-flows of the sector but has the option to launch a new generation and, after launching this new generation, will receive their old product new market share of the total cash-flows (that probably suffer an expansion due to the entry of the new generation) plus their new product market share of this total cash-flows in a market with two products. It is important to have in consideration that the company to exercise the option of launching the new generation will have to pay a price and also we need to have in consideration that being the first mover or follower will lead to a loss in incremental terms due to a market change. Having in consideration these aspects we had been able to formulate a trigger value from which is advantageous for the company to exercise the option to release the new product.

We applied the model to two real world cases: the Apple iPhone case and the Apple iPad case. The objective was to review two different realities in different markets and evaluate if the model suits each market.

In the first case we evaluated the transition from the iPhone 4 to the iPhone 4S that means that we start by analysing the smartphone industry. We made some assumptions in order to complete the example, since some data is not available and also, the loss in incremental terms cannot be calculated because we do not know the concrete value and so, we made a sensitivity analysis and discover the value of the loss at which Apple launch the iPhone 4S that was 17,10%. This means that if the loss in incremental terms was higher, Apple should have launched the new version later when the market become bigger and if the loss in incremental terms was lower, Apple should have launched the new version earlier when the market was smaller. To conclude this example, we conducted a sensitivity analysis and verify that the increase in volatility and in the market share of iPhone 4 in a market where only that iPhone is present also increases the trigger value making the company invest later. The increase in the market share of iPhone 4 in a market where iPhone 4 and iPhone 4S are present and in the market share of iPhone 4S in a market where iPhone 4 and iPhone 4S are present decreases the trigger value making the company invest earlier.
In the second case we evaluated the transition from the iPad 2 to the iPad 4, having also analysed the tablets industry. We made some assumptions in order to complete the example, since some data is not available and also, the loss in incremental terms cannot be calculated because we do not know the concrete value and so, we made a sensitivity analysis and discover the value of the loss at which Apple launch the iPad 4 that was 35.03%. This means that if the loss in incremental terms was higher, Apple should have launched the new version later when the market became bigger and if the loss in incremental terms was lower, Apple should have launched the new version earlier when the market was smaller. To conclude this example, we conducted a sensitivity analysis and verified that the increase in volatility and in the market share of iPad 2 in a market where only that iPad is present also increases the trigger value making the company invest later. The increase in the market share of iPad 2 in a market where iPad 2 and iPad 4 are present and in the market share of iPad 4 in a market where iPad 2 and iPad 4 are present decreases the trigger value making the company invest earlier.

For further studies, we would suggest using the model in different areas and markets different from the technological area or even in the technological area but with another example like the video-game industry.
6. References


