The risk variable in a sequential investment option: a Real Options' approach

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Abstract

Foreign Direct Investment (FDI) represents, today, about more than half of the total international investments, and is one of the main drivers of the process of globalization. Companies now take advantage of the opening of the cross-borders to expand to other countries and reach new costumers and have access to more efficient resources, creating the definition of Multinational Companies (MNC). Throughout the our work we present a model that allows us to value the option of a company to invest in a foreign market through a staged investment taking into consideration a risk parameter, namely the market's reaction to the exercise of each of the investment stages of the company. We aim to demonstrate that this risk parameter might have a significant impact on the exercise or delay of the following stages of the investment.

Key words: Real Options, FDI, Market Reaction, Uncertainty, Capital Budgeting

JEL-Codes: G11, G15, G31, O16

Bibliographical note

Jiguita Manish Kumar was born (1992) and raised in Cedofeita, Porto. Since 2006, she works as a sales assistant for as a part-time, where she found out about her interest for business. In 2010 she joined Faculdade de Economia da Universidade do Porto to start her Bachelor Degree in Business Administration.

During her bachelor course, she developed a great interest for the subject of finance, which lead her to pursue the Master Degree in Finance, in 2013, in the same educational institution.

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1. Introduction

Foreign Direct Investment (FDI) is the action of an investor or an entity to take control or obtains a significant influence on a company residing in a different country. The FDI can be made through the creation of a new company in the foreign country or through an existing company in the market (M&A, joint-ventures, among others).

This phenomena has been gaining importance as from the 1980s, since which it began growing exponentially. The United Nations Conference on Trade and Development (also known as UNCTAD) recently published a report "World Investment Report 2016"[1], which illustrates the evolution of FDI throughout the years, and we can observe that in 1990 the FDI inflows amounted to $207 Billion, while as of 2015 the same indicator amounted to $1,762 Billion (which represents an increase of 751%) worldwide. As of 2015, FDI flows accounted for over 55% of the global investment flows.

This international investment method revolutionized the financial markets, leading it into a more integrated framework. Now-a-days the companies expand their businesses cross-borders in order to reach new customers and to be acquire its resources from the most efficient locations, allowing to decrease the acquisition prices and avoid transportation costs. We can summarize by saying that a FDI not only allows the company to become more competitive and reach higher profits (creating more value for its shareholders) as well as it does boost the growth of the foreign economies.

The choice between a staged investment or an investment in only one stage, in a FDI context, can depend upon many factors, such as competition to be the first to enter the market, take advantage of a limited advantage, such as a license, the level of uncertainty of the cash-flows, among others. Taking into consideration the last example, the uncertainty, when it is high, a sequential strategy would be preferable once it would allow the investor to divide its investment (which in the majority of the cases is a sunk cost) into milestones, and according to the outcome of each stage, the company can decide whether to proceed or not.

[1]http://unctad.org/en/PublicationsLibrary/wir2016_en.pdf

This investment method is very frequent in industries such as the pharmaceutical and mining, and most of the capital budgeting models are designed in order to respond to the need to value the projects in such industries. But when we take into consideration an investment project in foreign market whose success depends on exogenous events, such as an economic depression in the foreign economy, these variables must also be taken into consideration, once its impact in the outcome of the first stage of the investment might influence the decision-making regarding the exercise of the second stage, for instance.

Through a Real Option's approach we intend to incorporate this exogenous parameter in to the existing models of capital budgeting, in this dissertation our work will be mainly based upon Dixit & Pindyck (1994)'s work, in order to create a more realistic model that allows us to reach a more accurate value of an option to invest.

Our work is organized as follows: Chapter 2 relies upon the literature review used to better understand the concept of real options, compound option, and analyzes the past literature regarding FDI and the incorporation of the risk parameters, from a Real Options' approach; Chapter 3 we introduce the generic framework created to develop the model, the mathematical background of the model, the different stages of the investment and the respective valuation models; Chapter 4 includes a numerical example of the model developed in the previous chapter and analyzes the impact of the variance of some parameters in the results; in Chapter 5 we discuss the different applications of the method; and Chapter 6 summarizes the conclusions of the dissertation.

2. Literature Review

2.1 The Origins of Real Options

The NPV method, the neoclassical theory of investment, is the most used tool for project valuation until the present, despite of the several limitations it presents. The standard model relies upon a set of unrealistic assumptions, such as the passive attitude of the management throughout the lifetime of the investment, not taking into consideration the option to wait and invest later (if the NPV is negative), assuming that the cash in-flows and the cash out-flows recorded throughout the project will be in accordance with the management predictions, among others.

Ross (1995) explains, through numerical examples, the several limitations of this traditional method and explains that despite of its limitations, the NPV is still useful for investment opportunities with significant little time to be undertaken and that, once undertaken, do not create additional options, characteristics that do not match with most investment opportunities.

These rough assumptions have been subject of analysis by many authors, such as Jorgenson (1963), Tobin (1969), Abel (1990), among others, who attempted to adapt the original model in order to overcome the existing gaps.

Real Options arises from the need of overcoming those limitations in order to reach a more realistic value of a project, and its origins date back to Myers (1977), who introduced the idea of a company's value comprises two distinct components: (1) the present value of the assets in place, and (2) the present value of the investments that the firm will make in the future (options to invest, growth opportunities, among others).

This insight given by Myers (1977) is supported by the study made by Kester (1984), who concludes that most of the companies are overvalued on the market when compared to the value that we reach when using the capitalized value of earnings only, meaning that investors take into consideration the growth opportunities when valuing a company (that is, they take into consideration real options). As referred by Dixit and Pindyck (1994, p. 9), "Indeed, for most firms, a substantial part of the market value is attributable to their options to invest and grow in the future, as opposed to the capital they already have in place", it would not be realistic to only consider the present economic and financial situation of a certain company in order to reach its value.

Since Myers (1977), Real Options have been object of study by several other authors and is giving a valuable contribution to the evolution of the area of Corporate Finance and Capital Budgeting. One of the reasons for this method to be an added value to these areas of expertise is due to the fact that it takes into account management's flexibility. This is a factor that has been gaining importance throughout the years, mainly due to the rapid pace of development of the industries and of the technologies, constantly new information arrives to the market and the managers' active role gain a particular relevance (a fact ignored by methods such as the NPV).

Authors such as Brennan and Schwartz (1985), McDonald and Siegel (1986) and Majd and Pindyck (1987) explain the importance of such flexibility, namely through the exercise of the option to defer an investment and wait for new information which might impact the decision to invest or not and the expenditure pattern. The authors study the optimal moment to invest in a project and show that it is not optimal to invest when the gross project value matches the investment costs, but, instead, when it exceeds it by the value of the option being exercised, the option which is being "killed" by not deferring the investment, in other words, the value of the option to defer is null.

By ignoring the management's active role and possible unexpected market developments, the traditional methods also ignore options generated once the investment is made, such as the option to abandon, growth options, the option to switch, and so on.

Dixit and Pindyck (1994), one of the main important literature regarding Real Options, in which the authors introduce and explain a set of valuation methods, the rationale underlying the models presented and some numerical examples. Their work guides us through the models computation, which is mainly based upon the work done by McDonald and Siegel (1986), its mathematical background and the different applications of the models. Dixit and Pindyck (1994) is the basis for the calculation method of the value of a project and the decision taking of undertaking a certain investment, through a Real Options' perspective.

Luerhman (1998), present an alternative approach of a Real Options' model, by explaining and illustrating a numerical example of the adaptation of the famous Black-Scholes (1973) model to the investment decision (since the financial options and the real options have so many characteristics in common). The author suggests that a manager should see a company as a call option, and attempts to value it through the adaption of the model created by Black and Scholes (1973) (for example, the stock price would in this case be equivalent to the present value of the project and the exercise price would be represented this time by the expenditure incurred into in order to acquire the assets for the project).

According to Luerhman (1998), by making these adjustments this model will help us to reach a more realistic value. And, despite of the fact that the output that we get from this method is not precise, Luehrman (1998, p. 3) assertively states that it is "good enough", which is still better than a valuation solely based on a Discounted Cash-Flow's approach.

In a world of globalization and increasing uncertainty, flexibility is a very important factor, mainly in the investment strategy of a company. Therefore, Real Options is a very suitable method as it takes into consideration possible future investments which impact the value of a firm today and assumes a management's active attitude throughout the life of the investment, as new information arrives, the manager can decide whether to proceed with the investment, expand or abandon it. This allows the manager to explore the future possible in-flows (if the new information is beneficial) or limit the losses by abandoning the investment (if the new information is harmful to the company).

2.2 Literature review on Compounding options

There are many different types of options: option to invest, defer, abandon, among others, and there are also options which are prerequisites to create other options, the compounding options. The staged option, or time to build option, is a compound option in which investments are interrelated, the first investment gives the right, but not the obligation, to make a second investment (to scale up the investment or abandon), by exercising the second investment the investor acquires the right to make a third, and so on.

The staged options are more common when investing in the pharmaceutical industry, the mining industry or R&D, which are risky investments with higher uncertainty. In these industries the flexibility embedded in real options is an important factor, once the manager has the option to wait until uncertainty is reduced (more information arrives) or they can stage the investment into critical milestones of the project, in which the manager can decide whether it is worth to keep investing or abandon. The staged investments allow the manager to limit the losses and have an upside value potential.

The first studies regarding the notion and valuation of the concept of compounding options was made by Geske (1979) and (1977). In both papers the author had as the main objective the calculation of the value of a compounding option, by adapting the Black-Scholes (1973) formula.

Pindyck and Majd (1987) introduce the valuation method of a sequential investment with irreversible investment, this time based on the paper written by McDonald and Siegel (1986), the authors incorporate the concept of flexibility in the investment decision. Pindyck and Majd (1987, p. 25) demonstrate the advantage of such investments, which "(…) is that the pattern of expenditures can be adjusted as new information arrives", and explain why the traditional methods would not be useful for this sort of investments.

Different models have been suggested to value these sort of options, Pindyck and Majd (1982) adapted the econometric model presented by Kyndland and Prescott (1982), Carr (1988) developed a model for finite exchange options with an exercise price that can be hedged by traded assets, and the most recent contribution, Dixit and Pindyck (1994) who introduce the valuation approach to be adopted in the present model (which is described in chapter 3).

2.3 Real Options & FDI and the risk parameter

The Foreign Direct Investment (or FDI) is an investment made by a company or a person in a foreign country, with the objective of either establishing a business or acquiring one. Many studies refer this phenomena as the reason for some countries to be more developed than others, the reason for the specialization of the companies regarding their activities (products / services sold) and creation of the concept of the multinationals enterprises.

The FDI notion was introduced recently in the Real Options literature, and the field of studies is developing for the most varied reasons, such as understanding the most appropriated entrance method through the application of Real Options (such as Li and Rugman (2007) and Brouthers, et al. (2008) attempt to do in their work), the determinants of the most appropriated model to value such investments (Altomonte (1998), Buckley and Casson (1998), Reuer and Tong (2005), among others), to study the impact of the exogenous events once the investment is undertaken (such as Dixit (1989) and the impact of exchange rate risk and Clark (1997) and Schnitzer (2000) with the implications of political risk / expropriation in the valuation model), among others).

Kemna (1993) and Trigeorgis and Panayi (1998) made a practical approach of the concept. While the first analyzed 3 real cases of Shell (an option to invest in an offshore project, a growth option through a pioneer venture and an abandonment decision of a refinery) in which the option pricing theory is used (using a model base in the Black and Scholes (1973) work), the latter studied the growth option of the Bank of Cyprus, a financial institution that intends to expand overseas, namely to U.S.A. and Canada.

Throughout the literature review, in a FDI compounding option framework, that the authors (such Clark (1997), Schnitzer (2000), Restrepo et al. (2014), among others) as only adapted the model in order to study the impact of exogenous events such as the political risk and the exchange rate risk, and it is our understanding that the model should also take into consideration the impact of the exercise of such options in the gross project value as the first, second or third stage of the sequential investment is exercised.

We call this risk parameter as the "market reaction", which has a great impact on the future of a company once the market is the one who dictate the demand of the services provided / products sold by the company and, influencing the cash in-flows, and, consequently, has an impact in the value of the following investment stages.

The market reaction might be positive or negative towards a certain product, and there are many possible events which might influence the market reaction, such as the increase of the purchase power of the consumers or the exit of a competitor from the foreign market. These events are not controlled by the company, they are exogenous events which dictate the success or failure of an investment.

The main objective of the present dissertation is to fill the gap found regarding the impact of the arrival rate of this exogenous event in the value of the option to expand to another country.

In order to understand the most appropriated method to incorporate this risk parameter in the valuation method we analyzed some recent studies regarding FDI investments with political risk, the risk that the government of the hosting country will nationalize the company once it becomes profitable.

A valuation method for this risk parameter is presented by Clark (1997) who suggests that the political risk is an exogenous variable which occurs at random times according to a Poisson arrival process and by incorporating this risk in the value of the exposure to loss, the investor can calculate the most appropriate value of the insurance policy to be contracted.

Schnitzer (2000) suggests the use of the probability of the company being nationalized by the host country's government. The author values this investment option by summing 2 parts: (i) in which the author multiplies the probability of the company not being nationalized by the profits generated in the host country and (ii) the value of the profits if the company withdraws partially (in order to discourage the nationalization) multiplied by the probability of the company being expropriated.

A more recent contribution to this field was made by Restrepo et al. (2014). The authors adapt the investment option model of McDonald and Siegel (1986), by introducing 2 new components: (i) the value of the private firm at expropriation, which will be the indemnity paid by the government as a counterpart for the nationalization and (ii) the value of the firm at abandonment, which they assume to be zero.

In our dissertation we adapt the Dixit and Pindyck (1994) model for compounding options by including this risk parameter. The method we use to do so is the one introduced by Clark (1997). Assuming that the market reaction follows a Poisson distribution, we attempt to analyze the impact of the exogenous event that impacts the market reaction regarding the exercise of the first option in the value of the second stage of the investment.

In the following section we introduce the assumptions taken into consideration in the present thesis and explain the basis and development of the model.

3. The Model

3.1 General framework

The present dissertation's model relies upon a hypothetical scenario in which a company, who produces and sells a range of products, has the option to expand its sales to a foreign market (an option to invest). Due to the uncertainty associated to the market reaction regarding the products to be sold, the company chooses to make a sequential investment in order to gather more information from the market, this way the management is able to minimize its losses (create a floor) and still be able to maximize its gains.

Once the option to invest is exercised, the company will place a small scale of the products of its portfolio on sale in the new market (what we define as the 1st stage of the investment) and waits for the arrival of more information regarding the market's reaction to its entrance. A positive market reaction results from an exogenous event which can not be controlled by the company, implying that the exercise of the 2nd stage of the investment (which is, operating in a full scale in the foreign market) is not a decision to be made by the management, therefore no flexibility is embedded.

We assume that the company incurs in two types of costs: i) a fixed sunk cost which is paid once the option to invest is exercised and when the exogenous event occurs (and the company expands the range of products sold), and ii) variable costs, which represent the costs incurred in the production of the goods. Also, please note that the present scenario assumes that there is no exchange rate risk.

The timeline presented in Figure 1 represents the structure and time horizon of the events occurred in this proposed scenario:

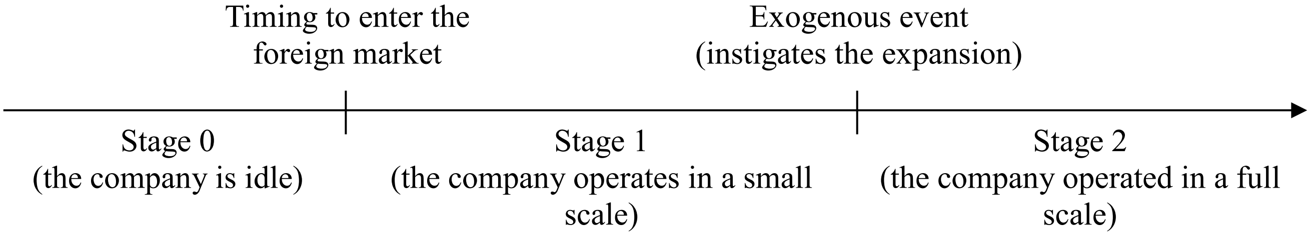

Figure 1: Timeline of the investment. In stage 0 the company is idle in the foreign market, waiting for the optimal moment to exercise the option to invest. In stage 1 the company is operating in a small scale and waiting for the appropriate signal for the expansion of its business in the foreign market. In stage 2 the company is operating in a full scale.

We assume that profit flow pre-variable costs and pre-taxes of the project (π) follows a stochastic process, the Geometric Brownian Motion (GBM):

$$d\pi = \alpha\pi dt + \sigma\pi dz \tag{1}$$

in which π represents the value of the profit flow (before variable costs and taxes) generated by the project, α the drift rate, σ is the standard deviation of the cash-flows, and dz is the increment of the Wiener process, a continuous time stochastic process that implies that the variance increases linearly with the interval of time. Please note that $\alpha = r - \delta$, where r is the risk-free rate and δ represents the opportunity cost, and that $\pi_0 > 0$.

Once we are modeling a sequential option, we must use a backwards induction to solve the problem: section 3.2 we start by deriving the value of the project when the company is operating in a full scale (the 2nd investment stage), and then we move on to the computation of the model when the company is operating in a small scale (the 1st investment stage), and finally we present the model which allows us to compute the value of the investment opportunity when the company is idle and has the option to invest in the 1st stage.

3.2 Stage 2 – operating full scale

Assume that the option to invest and the exogenous favorable event already took place, and the company already incurred into the fixed sunk investment to expand the scale of products sold (K_2). So we are in the 2nd investment stage, the stage in which the company is operating in a full scale.

Ignoring the existence of additional flexibility, the value of the project in this stage can be denoted as $T(\pi)$:

$$T(\pi) = \frac{x\pi(1-c_2)}{r-\alpha}(1-t) \tag{2}$$

In the formula, x represents the multiple by which the sales increase once the company starts operating in a full scale. We assume that $x > 1$, since, with a positive market reaction and the entrance of the new products to the market, the sales of the company are expected to increase, when compared to the volume of sales during the 1st stage of the investment. c_2 is the variable costs incurred with the production of the goods sold (such as energy, raw materials, personnel expenses, among others), which, in the present model, represents a percentage of the sales of the company. K_2, represents the fixed sunk cost already incurred by the company, represents all the initial expenses borne in order to expand the scale of products sold in the foreign market (e.g. acquiring or expanding the facilities of the company in the foreign market, acquiring more transportation and production equipment, among others). Finally, t represents the taxes to be paid to the local authorities.

3.3 Stage 1 – operating in a reduced scale

Taking a step back, now we are assuming that the company already entered in the foreign market and incurred into the necessary expenses (K_1) and that the exogenous event leading to a favorable market reaction did not occur yet. The company is operating in a small scale, only selling part of its products in the new market and is gathering new information from the market.

The value of the project at this stage must satisfy the following Ordinary Differential Equation (ODE):

$$\frac{1}{2}\sigma^2\pi^2F''(\pi) + \alpha\pi F'(\pi) - rF(\pi) + \pi(1-c_1)(1-t) + \lambda\left(\frac{x\pi(1-c_2)}{r-\alpha}(1-t) - K_2 - F(\pi)\right) = 0 \qquad (3)$$

The equation can be divided into 3 different parts: a second-order homogenous differential equation; and two non-homogenous components, which represent:

- $\pi(1-c_1)(1-t)$, the profit flow after variable costs and taxes of the project, this is, the value of the project while operating in a small scale and paying a variable costs of c_1; and

- $\lambda\left(\frac{x\pi(1-c_2)}{r-\alpha}(1-t)-K_2-F(\pi)\right)$, the additional value that comes from the exchange from the current situation (*F(π)*) to the scenario in which the company expands its portfolio of products in the new market (*T(π)*), by investing K_2. The change at which this change occurs is given by λ.

The solution to the ODE is the following:

$$F(V)=A_1\pi^{\beta_1}+A_2\pi^{\beta_2}+\frac{\pi(1-c_1)}{r-\alpha+\lambda}(1-t)+\frac{\lambda}{r-\alpha+\lambda}\frac{x\pi(1-c_2)}{r-\alpha}(1-t)-\frac{\lambda}{r+\lambda}K_2 \quad (4)$$

A_1 and A_2 are constants which can be found through the definition of appropriated boundary conditions. Once the expansion of the scale of products sold depends solely on the occurrence of the exogenous event, there is no additional flexibility[2] in the model, consequently $A_1 = A_2 = 0$.

Therefore, the non-homogenous part of the solution represents the value of the project in the current stage:

$$F(\pi)=\frac{\pi(1-c_1)}{r-\alpha+\lambda}(1-t)+\frac{\lambda}{r-\alpha+\lambda}\frac{x\pi(1-c_2)}{r-\alpha}(1-t)-\frac{\lambda}{r+\lambda}K_2 \quad (5)$$

The model relies upon the assumption that the variable costs in the first stage of the investment (c_1) are higher than the variable costs once the company is in the second stage of the investment (c_2) due to economies of scale. K_1 represents the initial investment incurred when exercising the option to invest, sunk costs such as the acquisition of the facilities, legal fees to enter the market, among others. And the parameter λ represents the arrival rate of the exogenous event that triggers the expansion and follows a Poisson distribution. Since we are working on an annual basis, λ we can say that the exogenous event is expected to occur within $\frac{1}{\lambda}$ years.

In our model λ is a central parameter, since it is related to the event which has a positive impact in the market reaction towards the entrance of the company's products in the market. An exogenous event with direct impact in the market reaction (an consequently in the profit flow of the company) can be, for instance, a change of the

[2]In a further research, the model can be expanded by considering the option to abandon

government or a change in the local legislation, the exit of a competitor from the foreign market, an effective marketing campaign, an increase of the purchase power of the private consumers, or any other event that makes the product more attractive to the final consumers.

A simple and recent example of what an exogenous event with implication in the sales of the company is, is the case of the #thedress. The #thedress refers to a Tumblr post that become viral, in 2015, as no one agreed on the color of a dress from the company Roman Originals, the public was divided into white and gold or black and blue. According to the website Fortune[3], the sales of the company increased by 347% in the very next day of the post and the dress became the “big seller” of Roman Originals.

Throughout our literature analysis we were able to find a real life case, from Panayi and Trigeorgis (1998), which resembles to our hypothetical scenario. The authors describe the expansion plan of the Bank of Cyprus (BOC), who intended to expand to U.S. or Canada through staged-investment.

The bank, in the first stage of the investment would open a reduced amount of branches in the foreign country chosen, and, expectably in the 10^{th} year of the investment, if the demand to its products / services would prove to be high, the financial entity would open additional branches in other parts of the country or / and would extend the range of products and services sold.

In this real life event, the exogenous event would be any event that would lead to the increase of the demand of the products and services of BOC, such as an increase of the Cypriot people in the foreign market.

[3]For more detail, please refer to http://fortune.com/2015/02/27/the-dress/

3.4 Stage 0 - Prior to the exercise of the option to invest

Let us assume now that the company is idle deciding whether to invest in the foreign market or not.

While waiting for the option to invest to be exercised, the value of the project in the present stage ($G(\pi)$) needs to fulfil the following ODE:

$$\frac{1}{2}\sigma^2\pi^2 G''(\pi) + \alpha\pi G'(\pi) - rG(\pi) = 0 \tag{6}$$

which has the following general solution:

$$G(V) = A_3\pi^{\beta_3} + A_4\pi^{\beta_4} \tag{7}$$

A_3 and A_4 are arbitrary constants which can be computed through the definition of the boundary conditions, and β_3 and β_4 are the root of the fundamental quadratic equation, as denoted infra:

$$\beta_3 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{8}$$

$$\beta_4 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{9}$$

In order to reach the value of the constants A_3 and A_4 and compute the formula which allows us to find π^*, the optimal value of profit flow pre-variable costs and pre-taxes, at which it is optimal to exercise the option to invest (the trigger value), we need to define three boundary conditions:

$$\begin{gathered} G(0) = 0 \\ G(V^*) = \frac{\pi^*(1-c_1)}{r-\alpha+\lambda}(1-t) + \frac{\lambda}{r-\alpha+\lambda}\frac{x\pi^*(1-c_2)}{r-\alpha}(1-t) - \frac{\lambda}{r+\lambda}K_2 - K_1 \\ G'(\pi^*) = \frac{(1-c_1)}{r-\alpha+\lambda}(1-t) + \frac{\lambda}{r-\alpha+\lambda}\frac{x(1-c_2)}{r-\alpha}(1-t) \end{gathered} \tag{10}$$

The first boundary condition implies that when the profit flow is null, the value of the project is null as well (an implication of the stochastic process). Consequently, as $\beta_4 < 0$ and therefore the value of the project would tend to infinity when $\pi = 0$, in order to satisfy the first boundary condition, A_4 needs necessarily to be set equal to 0. So, the solution of the ODE (6) can now be denoted as:

$$G(V) = A_3 \pi^{\beta_3} \tag{11}$$

The second boundary condition is the value-matching condition, which denotes the value of the project once the option to invest is exercised. And the third boundary condition, is the “smooth-pasting” condition, that implies that the value of the project if invested is equal to the value of the project if not undertaken, when $\pi = \pi^*$, and that as π approaches π^*, the transition between the value of both scenarios is smooth.

These last two conditions allow us to reach the value of A_3 and π^*, respectively:

$$A_3 = \left(\frac{\pi^*(1-c_1)}{r-\alpha+\lambda}(1-t) + \frac{\lambda}{r-\alpha+\lambda}\frac{x\pi^*(1-c_2)}{r-\alpha}(1-t) - \frac{\lambda}{r+\lambda}K_2 - K_1\right)\left(\frac{1}{\pi^*}\right)^{\beta_3} \tag{12}$$

and

$$\pi^* = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\alpha)(r-\alpha+\lambda)}{(1-t)(r+\lambda)}\frac{\lambda K_2 + K_1(r+\lambda)}{(1-c_1)(r-\alpha)+x\lambda(1-c_2)} \tag{13}$$

With all the necessary inputs, now we are able to compute the formula of the option to invest:

$$G(V) = \begin{cases} \left(\frac{\pi^*(1-c_1)}{r-\alpha+\lambda}(1-t) + \frac{\lambda}{r-\alpha+\lambda}\frac{x\pi^*(1-c_2)}{r-\alpha}(1-t) - \frac{\lambda}{r+\lambda}K_2 - K_1\right)\left(\frac{\pi}{\pi^*}\right)^{\beta_3} & \pi < \pi^* \\ \frac{\pi(1-c_1)}{r-\alpha+\lambda}(1-t) + \frac{\lambda}{r-\alpha+\lambda}\frac{x\pi(1-c_2)}{r-\alpha}(1-t) - \frac{\lambda}{r+\lambda}K_2 - K_1 & \pi \geq \pi^* \end{cases} \tag{14}$$

4. Numerical Example and Sensitivity Analysis

4.1 ***Numerical example***

The present section illustrates a practical example to the theoretical scenario previously introduced. The inputs chosen for this base case can be seen in the table infra:

Variable	Denotation	Value
Risk-free rate	r	0.04
Expected growth rate	α	0.02
Opportunity cost	$\delta = r - \alpha$	0.02
Volatility	σ	0.25
Taxes	t	0.3
Multiple of sales	x	2
Exogenous event's arrival	λ	0.3
Variable costs 1	c_1	0.5
Variable costs 2	c_2	0.4
Profit flow pre-variable costs and pre-taxes	π_0	€8.5M
Investment costs 1	K_1	€100M
Investment costs 2	K_2	€50M

Table 1a: The base case parameters

The table illustrates a hypothetical scenario in which the taxes charged in the foreign market are 30% of the net gains of the company, and the growth rate of the revenues is 2%. The variable costs, in a first stage, amount to 50% of the profit flow, and, if the second option is exercised, decrease to 40%. The initial sunk costs amount to €100M and, if the market sentiment is positive, the fixed costs to be incurred, in order to expand of the range of products sold, amount to €50M, as the revenues duplicate.

Using the models developed in chapter 3, were able to reach the following:

Variable	Denotation	Value
Trigger	π^*	€9.73M
Option value at stage 0	$G(\pi_0)$	€199.9M

Table 2: Base case results

4.2 ***Sensitivity analysis***

In the present section we intend to understand the impact of the variation of main parameters (λ, σ, t and x), which we assume to be given by the market, in the value of the trigger.

For the **change in λ**, we interpret the impact of the change of the arrival rate in the trigger value. Figure 2 illustrates the changes in the value of the trigger as the value of λ increases:

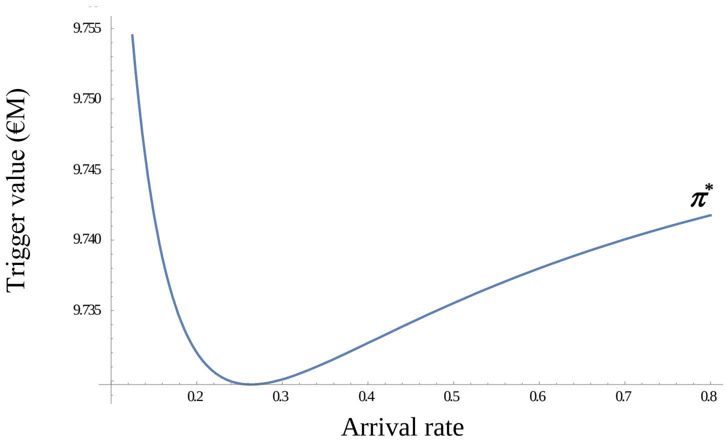

Figure 2: Impact of the change in the arrival rate in the Trigger value

According to the present analysis, if λ tends to zero, the trigger increases up to €15.613M, meaning that the lowest the λ gets (the lower the arrival rate of the event of a good market reaction, and consequently the lower the value of the second investment stage) the higher needs to be the expected revenues of the project in order to the option to invest to be exercised.

As we can observe in the graph above, the trigger hits its lowest value, €9.73M, when the arrival rate is app. 0.26 (the event is expected to occur every 3 years and 10 months), and from this point on it tends to increase (in the limit, reaching the value of €9.758M). This trend can be explained by the fact that as more certain as the occurrence of the exogenous event gets, higher is the probability of the company to expand the range of the products offered in foreign market, which implies the payment of the investment sunk cost of €50M (K_2) and the variable costs associated with the expansion (c_2), therefore a higher trigger value is requested.

The graph infra allows us to understand the impact of the **change in volatility (σ)** in the optimal profit flow to exercise the option to invest:

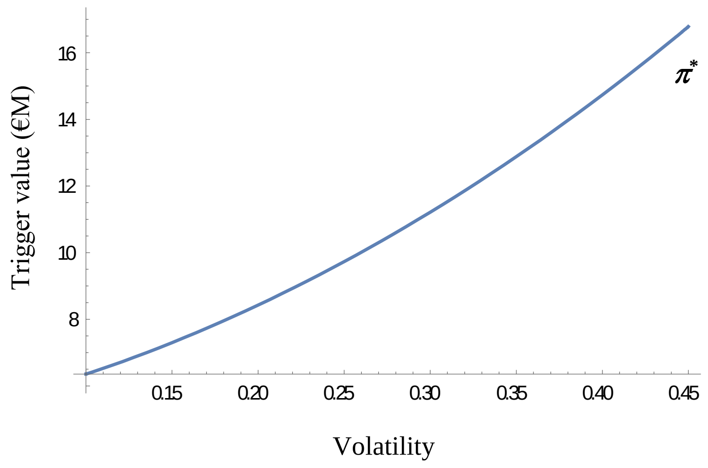

Figure 3: Impact of the change in volatility in the Trigger value

The increase of the volatility leads to an exponential increase of the trigger value. It is an expected behavior once higher volatility means higher uncertainty regarding the future profit flows of the project, therefore the management would rather defer the exercise of the option and gather more information regarding the market in order to reduce the uncertainty.

The **change in taxes (*t*)** in the foreign market are expected to have a similar impact in π^* as the one created by the volatility, once the increase of the taxes leads to a lower net income for the company:

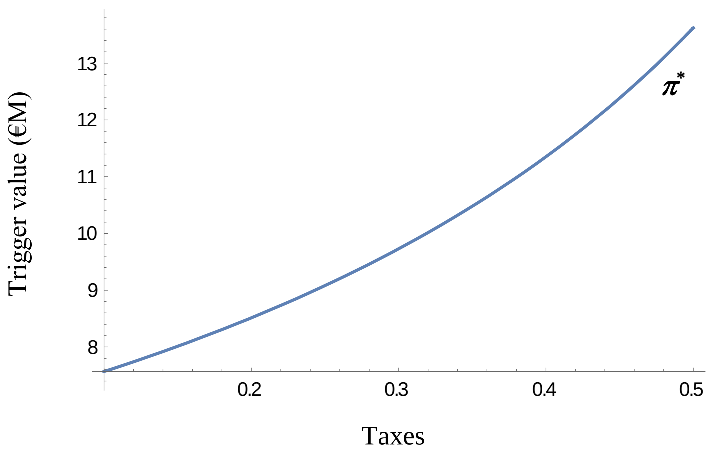

Figure 4: Impact of the change in taxes in the Trigger value

As higher as the taxes charged get, higher needs to be the treshold in order for the option to be exercised. The taxes are very often used to attract Foreign Direct Investment, as we are going to see in the following chapter of the dissertation.

And finally, the impact of the **change in the multiple of the sales (x)**, once the option to expand is exercised:

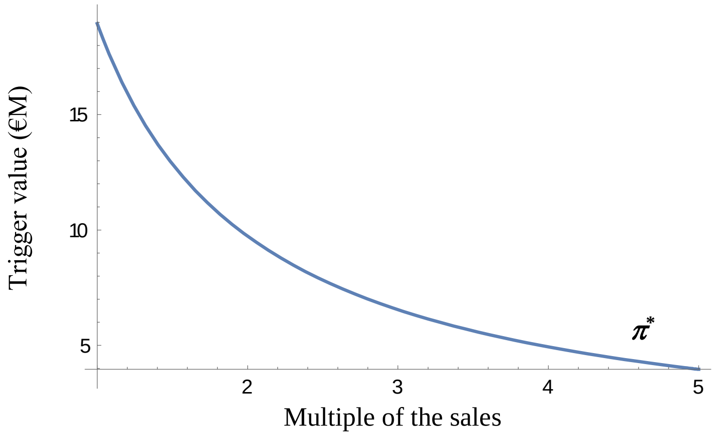

Figure 5: Impact of the change in the multiple of the sales in the Trigger value

As expected, as the multiple of sales increases, the trigger value decreases, as the net pay-off increases, therefore the value of the second option also goes up, leading to an appreciation of the value of the option to invest.

This analysis can be extended to other variables which were not approached in the present chapter, such as the risk-free rate (r), the expected growth rate (α), the investment expenses for both stages (K_1 and / or K_2) and even the variable costs linked to the production of goods of the company (c_1 and / or c_2).

5. Discussion

Throughout this dissertation we presented and illustrated the model as a decision-making resource for the management of the company. The present chapter intends to offer a different, yet also useful, perspective through which the model can also be used.

When we are considering a FDI, we can have two different perspectives of the transaction, we can either see from the point of view of the company and the decision-making process of the optimal moment to invest, either from the point of view of the legal entities of the foreign market. If the government of the market intends to attract Foreign Direct Investments, they can promote the necessary market conditions for the market to become more attractive to foreign investors.

Here, we suggest two different approaches that could be used in order to anticipate the entrance of a foreign entrance in the market: 1) the decrease of the local taxes (t) to the foreign players; and 2) the attribution of subsidies for the initial investment in the country (K_1), (for similar work please refer to Barbosa et al. (2016)).

Assume that the present value of the profit flow pre-variable costs and pre-taxes of a target company is π_0, which is lower than the threshold π^*. The company chooses to defer the investment and wait for a more favorable situation. The local government, in order to anticipate the entrance, adopts a preferential tax treatment.

In order to know which would be the “optimal” tax treatment (t^*) for the company to exercise its option now, the government assumes the following assumption:

$$\pi^* = \frac{\beta_1}{\beta_1 - 1} \frac{(r-\alpha)(r-\alpha+\lambda)}{(1-t)(r+\lambda)} \frac{\lambda K_2 + K_1(r+\lambda)}{(1-c_1)(r-\alpha)+x\lambda(1-c_2)} = \pi_0 \tag{15}$$

The equation (15) understands that the trigger value is equal to the current present value of the profit flows of the company. Knowing the value of π, we would solve the equation in order to find the value of t^*:

$$t^* = \frac{\beta 1}{(1-\beta 1)} * \frac{\mathrm{K}1 + \frac{\mathrm{K}2\lambda}{r+\lambda}}{\frac{R\left((1-\mathrm{c}1)(r-\alpha)+(1-\mathrm{c}2)x\lambda\right)}{(r-\alpha)(r-\alpha+\lambda)}} + 1 \tag{16}$$

Variable	Denotation	Value
Risk-free rate	r	0.04
Expected growth rate	α	0.02
Opportunity cost	$\delta = r - \alpha$	0.02
Volatility	σ	0.25
Taxes	t	0.3
Multiple of sales	x	2
Exogenous event's arrival	λ	0.3
Variable costs 1	c_1	0.5
Variable costs 2	c_2	0.4
Profit flow pre-variable costs and pre-taxes	π_0	€8.5M
Investment costs 1	K_1	€100M
Investment costs 2	K_2	€50M

Table 1b: The base case parameters

Using the base case values (in Table 1), and assuming that π_0 = €8.5M, our t optimal would amount to 19.87%, this means that for prompting the investment, the local authorities should reduce the taxes from 30% to 19.87%.

Assuming the same scenario, but this time the local government chooses to offer a subsidy, such as lower legal fees to enter the market, discounts in public properties to be leased or acquired by the foreign company to use as facilities, or any other financial support to the initial investment stage (K_1). In order for the public entity to know how much should the subsidy (K^s) amount to in order for the company to invest now, they would have to solve the following equation in order of K^s:

$$\pi^* = \frac{\beta_1}{\beta_1 - 1} \frac{(r-\alpha)(r-\alpha+\lambda)}{(1-t)(r+\lambda)} \frac{\lambda K_2 + (K_1 - K^S)(r+\lambda)}{(1-c_1)(r-\alpha) + x\lambda(1-c_2)} = \pi_0 \tag{17}$$

Reaching the following formula:

$$K^s = \frac{(r+\lambda)}{\beta_1 \lambda}\left(K_1\beta_1 + \frac{K_2\beta_1\lambda}{r+\lambda} + \pi_0(1-t)(1-\beta_1)\frac{(1-c_1)(r-\alpha)+(1-c_2)x\lambda}{(r-\alpha)(r-\alpha+\lambda)}\right) \tag{18}$$

Using the base case values and, again, assuming that π_0 = €8.5M, our subsidy (K^s) should amount to €20.649M, in order for the company to exercise the option to invest now, which corresponds to support about 20% of the initial investment cost.

Now we will analyze the impact of the changes in λ, in the values of t^* and K^s, for a profit flow pre-variable costs and pre-tax in stage 0 of π_0 = €8.5M.

Approach	**0.1**	**0.3**	**0.5**	**0.7**	**1**
t^*	19.46%	19.87%	19.83%	19.79%	19.75%
K^s	€24.9M	€20.6M	€20.1M	€19.8M	€19.7M

Table 3: Impact of the change in λ in the value of t^* and K^s

Based upon the values in Table 3, we can conclude that in a:

- Tax treatment approach, as the arrival rate of the exogenous event increases, higher is the incentive that the local government needs to offer in order to prompt the investment. The tax treatment increase from 19.46% to 19.87% (between 0.1 and 0.3), but then decreases to 19.75%. This results are in accordance with the conclusions we reached when analyzing Figure 2 (the trigger decreases until reaching the arrival rate 0.26, and then increases from that point onwards); and

- Subsidy approach, the percentage of the expenses to be covered by the public entities decreases from 24.9% (when the arrival rate is 0.1) to 19.7% (when the arrival rate is 1). This results from the fact that the higher the λ, the higher is the trigger value, amounting to €9.7M when λ = 1, and as higher as the initial investment costs is (for a fixed λ = 0.3), the higher the trigger is, tending also to €9,7M when K_1 = €100M, decreasing the gap between π_0 and π^*. Therefore, lower needs to be the subsidy.

That said, we can conclude that a tax treatment approach is more effective for higher values of arrival rate, while the subsidy approach is useful for lower values of arrival rate.

6. Conclusion

Throughout our work we have developed a model which allows a company to value sequential investment and helps the management in their decision-making process, in a FDI context.

The past literature in this field take into consideration other matters such as the most appropriated determinants of a model to value such investments, the most appropriated entry mode in a foreign market, the incorporation of the exchange rate risk or the expropriation risk in the Real Options' model or the analysis of real life cases, not taking into consideration the uncertainty surrounding the market reaction towards the entrance of the Company in a new market. In our work we developed a model which allows the company to take into account this risk parameter when computing the value of the option to invest.

Our work is based upon the hypothetical situation in which a company intends to penetrate a new market through a sequential investment, in a first stage the company offers a small scale of its business lines, and, as the management receives the appropriate signal, a good market reaction, they expand the business operating in a full scale.

To conclude, we believe that this risk parameter adds value to the model, as the uncertainty of the market reaction towards the entrance of a new company in a foreign market impacts the value of the option to invest.

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