In this work a numerical study has been performed to verify the adequacy of the End Notched Flexure (ENF) test to determine the fracture toughness under mode II loading of cortical bovine bone tissue. In this work a detailed numerical analysis using the finite element method and a cohesive damage model was performed in order to optimize the specimen geometry when applied to bone fracture characterization under mode II loading. A data reduction scheme based on specimen compliance and crack equivalent concept was used to overcome the difficulties inherent to crack monitoring during its growth. It was verified that a judicious selection of the geometry allows a rigorous estimation of toughness in mode II.

**KEY WORDS:** bone, fracture, mode II loading, ENF test

1. **INTRODUCTION**

Fracture of healthy bone tissue needs to be understood in order to predict and reduce fractures due to aging, exercise, over-use and disease. Fracture tests using bovine bone are frequently performed since it provides longer specimens relative to the human case. Although the fracture properties are not equal \[1, 2\], the fracture behavior of bovine bone is similar to the human, which justifies its use in experimental tests. This particularity can be used to identify new test methods using bovine bone.

The majority of the works about fracture characterization of bone tissue are dedicated to mode I loading. Effectively, the mode II fracture characterization is much less studied owing to the difficulties inherent to tests execution. Only a few works were performed dedicated to mode II fracture characterization of bone. Norman et al. \[3\] proposed the compact shear test for mode II fracture characterization of human bone. However, this test presents some disadvantages related to small variation of compliance \(C\) as a function of pre-crack length \(a_0\) which difficult the establishment of the compliance calibration \(C=f(a)\), mixed-mode crack growth instead of pure-mode II, and unstable propagation which means that only crack initiation fracture toughness is available.

In order to overcome these drawbacks, the applicability of the End Notched Flexure test (ENF) to mode II fracture characterization of bone is numerically assessed in this work. This test is especially adequate for mode II fracture characterization due to its simplicity and the possibility to use the beam theory to measure the fracture energy. However, it also presents some limitations related to spurious influence on the fracture process zone (FPZ) of the central loading. In this study, the specimen geometric conditions to be fulfilled in order to provide accurate measurements of bone toughness under mode II loading are identified.

2. **DATA REDUCTION SCHEME**

The classical data reduction schemes used to determine the fracture energy in mode II are usually based on the specimen compliance calibration or on the beam theory. In both cases, a rigorous measurement of crack length during its growth is a fundamental task. However, in the ENF test (Figure 1) the crack tends to close during its growth which difficult the identification of its tip. Moreover, in bone a non-negligible fracture process zone (FPZ) in the vicinity of the crack tip, characterized by the development of toughening mechanisms, exists which also contribute to the above referred difficulty.
To overcome these drawbacks an alternative data reduction scheme based on specimen compliance and crack equivalent concept is proposed in the following.

Using the Timoshenko beam theory, the specimen compliance is given by [4],

\[
C = \frac{3a^3 + 2L^3}{8Bh^2E_L} + \frac{3L}{10BhG_{LT}}
\]  

(1)

In the early stages of loading, the initial values of compliance \(C_0\) and crack length \(a_0\) can be used to estimate a corrected flexural modulus \(E_f\),

\[
E_f = \frac{3a_0^3 + 2L^3}{8Bh^2} \left( C_0 - \frac{3L}{10BhG_{LT}} \right)^{-1}
\]  

(2)

This procedure is quite effective since material variability among different specimens leads to non-negligible scatter on the elastic modulus. The beam theory (equation (1)) does not include root rotation effects and stress concentrations at the crack-tip. Following this approach, the longitudinal modulus is not a measured property but an estimated parameter thus including the above referred effects. During crack growth the current compliance \(C\) is used to estimate an equivalent crack length \(a_c\) through equations (1-2),

\[
a_c = \left[ \frac{C_c}{C_{0c}} a_0^3 + \frac{2}{3} \left( \frac{C_c}{C_{0c}} - 1 \right) L^3 \right]^{1/3}
\]  

(3)

where

\[
C_c = \frac{3L}{10BhG_{LT}} ; \quad C_{0c} = C_0 - \frac{3L}{10BhG_{LT}}
\]  

(4)

Using equation (1), \(G_{II}(a_c)\) can be obtained as

\[
G_{II} = \frac{9P^2a_c^2}{16B^2h^2E_f}
\]  

(5)

In summary, this method is based on the equivalent crack concept and does not require crack length monitoring during its growth. A resistance curve (R-curve) can be easily obtained only from the load-displacement curve.

3. NUMERICAL ANALYSIS

The numerical analyses were performed using a cohesive damage mode (Figure 2), allowing both crack initiation and growth [4]. A bilinear relationship is established between stresses \(\sigma\) and relative displacements \(\delta\) at the integration points of the interface finite elements. These elements are designated as cohesive elements in the following discussion. A quadratic stress criterion and a linear energetic criterion were used to simulate damage initiation and growth under mixed-mode loading.

A two-dimensional analysis was implemented in ABAQUS® software using isoparametric plane stress eight-node solid elements (Figure 3). Cohesive elements were disposed at the specimens’ half-height along direction \(x\), allowing the simulation of damage initiation and growth. At the pre-crack length the cohesive elements were considered “opened” which means that they only transmit normal compressive stresses, avoiding spurious interpenetrations. Loading and supporting devices were simulated as rigid bodies and contact conditions assumed to prevent interpenetration. Small increments (0.1%×\(\delta\) – see Figure 1) were considered to provide stable crack growth. A non-linear geometrical analysis was carried out.
4. RESULTS

The maximum specimen length that can be obtained from the bovine femur is about 70 mm. On the other hand, the crack tip during its propagation must be maintained far from the loading point, since the compressive effects tend to affect decisively the measured toughness. In fact, during self-similar crack propagation the fracture process zone (FPZ) at the crack tip must be maintained distant of the central loading point since the compressive effects lead to a spurious toughness enhance. In this context, an important role can be played by the toughness ($G_{IIc}$) and local strength ($\sigma_{u,II}$) under mode II loading. Effectively, high values of $G_{IIc}$ and/or low values of $\sigma_{u,II}$ contribute to increase the size of the FPZ (the descending branch of the triangle in Figure 2 increases) thus aggravating the limitations of the application of the ENF test to fracture characterization of bone under mode II loading. In addition the pre-crack length also influences this behavior. Consequently, three crack lengths ($a_0 = 15, 18$ and $20$ mm), three local typical shear strengths ($\sigma_{u,II} = 35, 40$ and $50$ MPa) and four characteristic mode II toughness values ($G_{IIc} = 4, 5, 6$ and $7$ N/mm) were considered in a parametric study. It must be noted that the used cohesive parameters can be considered pessimistic concerning the performance of the ENF test. In fact, Turner et al. [5] present values ranging between $50.4$ MPa and $51.6$ MPa for human bone strength, which has inferior properties than bovine. On the other hand, Feng [6] evaluated the $G_{IIc}$ of bovine bone as being equal to $2.43\pm 0.836$ N/mm. This means that more favorable conditions (higher strengths and lower fracture energies) are expected to exist in the experiments.

Numerous simulations were carried out to verify which combination of studied parameters could provide accurate estimates of $G_{IIc}$. This verification was executed comparing the measured $G_{IIc}$ with the introduced value in the cohesive model. Considering normalized strain energy (Figure 4) the efficacy of the proposed geometry is assessed when the ratio ($G_{II}/G_{IIc}$) points to the unity at the plateau of the $R$-curve. Moreover, the self-similar crack growth condition was verified by means of the cohesive zone length ($l_{FPZ}$). This parameter must maintain constant for some extent in order to satisfy the required conditions of self-similar crack growth (Figure 4).

It was verified that a judicious selection of the specimen dimensions allows a rigorous estimate of toughness in mode II loading. In fact, considering $a_0 = 20$ mm reasonable plateaus are obtained for all the examined values of $G_{IIc}$ when $\sigma_{u,II} = 50$ MPa. It should be noted that this is an underestimated local shear strength value [5].

This means that higher real values would provide shorter FPZs that do interact with the central loading region for longer crack lengths. Even for the highest toughness value considered ($G_{IIc} = 7$ N/mm) a short plateau can be identified (Figure 5). This means that the ENF test can be applied to bone fracture characterization under mode II loading since the real conditions would be more favorable than the ones used in Figure 5.

5. CONCLUSIONS

The ENF test was analyzed numerically in order to verify its applicability to characterize the bovine bone fracture under mode II loading. A data reduction scheme based on specimen compliance and crack equivalent concept was used to overcome the difficulties inherent to crack monitoring during its growth. A cohesive damage model was used to simulate damage initiation and propagation, thus assessing the influence of several parameters on the measured $G_{IIc}$ and the efficacy of the proposed data reduction scheme. The effects of initial crack length, local strength and toughness were analyzed in order to verify their
influence on the measured $G_{IIc}$ relatively to the inputted value.

The present study allows to identify the limiting aspects of the ENF test and to optimize the geometry leading to a correct estimation of the properties of bovine bone under mode II loading. The critical aspect of the ENF test is related to the eventual spurious effect of the central loading point on the measured $G_{IIc}$. However, the presented work showed that with careful geometry selection, good estimations of $G_{IIc}$ can be provided by the ENF test.

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