Bertrand Model Under Incomplete Information

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Abstract. We consider a Bertrand duopoly model with unknown costs. The firms' aim is to choose the price of its product according to the well-known concept of Bayesian Nash equilibrium. The chooses are made simultaneously by both firms.

In this paper, we suppose that each firm has two different technologies, and uses one of them according to a certain probability distribution. The use of either one or the other technology affects the unitary production cost. We show that this game has exactly one Bayesian Nash equilibrium. We analyse the advantages, for firms and for consumers, of using the technology with highest production cost *versus* the one with cheapest production cost. We prove that the expected profit of each firm increases with the variance of its production costs. We also show that the expected price of each good increases with both expected production costs, being the effect of the expected production costs of the rival dominated by the effect of the own expected production costs.

Keywords: Industrial Organization; Bertrand duopoly; uncertainty; Bayesian-Nash equilibrium.
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INTRODUCTION

Let F_1 and F_2 be two firms, each producing a differentiated product. Both firms simultaneously choose the price for the corresponding good with the purpose to maximize their expected profit. We consider an economic model in which we suppose that each firm has two different technologies, and uses one of them according to a certain probability distribution. The use of either one or the other technology affects the unitary production cost. We suppose that firm F_1's unitary production cost is c_A with probability ϕ and c_B with probability $1-\phi$ (where $c_A > c_B$), and firm F_2's unitary production cost is c_H with probability θ and c_L with probability $1-\theta$ (where $c_H > c_L$). Both probability distributions of unitary production costs are common knowledge. In this work, we determine the prices in the Bayesian Nash equilibrium for the above model, and we analyze the advantages, for firms and for consumers, of using the technology with highest production cost *versus* the one with cheapest production cost. We prove that the expected profit of each firm increases with the variance of its production costs. We also show that the expected price of each good increases with both expected production costs, being the effect of the expected production costs of the rival dominated by the effect of the own expected production costs.

THE MODEL AND THE EQUILIBRIUM

We consider an economy with a monopolistic sector with two firms, F_1 and F_2. Firm F_i produces a substitutable product i at a constant marginal cost, for $i \in \{1,2\}$. The firms simultaneously choose prices, respectively, $p_1 \geq 0$ and $p_2 \geq 0$. The direct demands are given by

$$q_i = a - p_i + bp_j,$$

where q_i stands for quantity, $a > 0$ is the intercept demand parameter and $b \geq 0$ is a constant representing how much the product of one firm is a substitute for the product of the other. For simplicity, we assume $b \leq 1$. These demand functions are unrealistic in that one firm could conceivably charge an arbitrary high price and still have a positive demand provided the other firm also charges a high enough price. However, this function is chosen to represent a linear approximation to the "true" demand function, appropriate near the usual price settings where the equilibrium is

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reached. Usually, in the case of complete information, the literature considers firm F_i's profit, π_i, given by

$$\begin{aligned}\pi_i(p_i,p_j) &= q_i(p_i-c)\\ &= (a-p_i+bp_j)(p_i-c),\end{aligned}$$

where $0<c<a$ is the unitary production cost for both firms. Here, we suppose that each firm has two different technologies, and uses one of them following a certain probability distribution. The use of either one or the other technology affects the unitary production cost. The following probability distributions of the firms' production costs are common knowledge among both firms:

$$C_1 = \begin{cases} c_A & \text{with probability } \phi \\ c_B & \text{with probability } 1-\phi \end{cases},$$

$$C_2 = \begin{cases} c_H & \text{with probability } \theta \\ c_L & \text{with probability } 1-\theta \end{cases}.$$

We suppose that $c_A > c_B$, $c_H > c_L$ and $c_A, c_B, c_H, c_L < a$. Moreover, we suppose that the highest unitary production cost of any firm is greater than the lowest unitary production cost of the other one, that is, $c_A > c_L$ and $c_H > c_B$.

Firms' profits, π_1 and π_2, are given by

$$\pi_1(p_1(c_1),p_2(c_2)) = (a-p_1(c_1)+bp_2(c_2))(p_1(c_1)-c_1),$$

$$\pi_2(p_1(c_1),p_2(c_2)) = (a-p_2(c_2)+bp_1(c_1))(p_2(c_2)-c_2),$$

where the price $p_i(c_i)$ depends on the unitary production cost c_i of firm F_i, for $i\in\{1,2\}$.

Theorem 1. Let $E(C_1)=\phi c_A+(1-\phi)c_B$ be the expected unitary production cost of firm F_1, and let $E(C_2)=\theta c_H+(1-\theta)c_L$ be the expected unitary production cost of firm F_2. For the Bertrand model with uncertainty costs considered, the Bayesian Nash equilibrium is

$$((p_1^*(c_A),p_1^*(c_B)),(p_2^*(c_H),p_2^*(c_L))),$$

where

$$p_1^*(c_A) = \frac{2a(2+b)+(4-b^2)c_A+b^2E(C_1)+2bE(C_2)}{2(4-b^2)}, \tag{1}$$

$$p_1^*(c_B) = \frac{2a(2+b)+(4-b^2)c_B+b^2E(C_1)+2bE(C_2)}{2(4-b^2)}, \tag{2}$$

$$p_2^*(c_H) = \frac{2a(2+b)+(4-b^2)c_H+b^2E(C_2)+2bE(C_1)}{2(4-b^2)}, \tag{3}$$

$$p_2^*(c_L) = \frac{2a(2+b)+(4-b^2)c_L+b^2E(C_2)+2bE(C_1)}{2(4-b^2)}. \tag{4}$$

Proof. If firm F_1's unitary production cost is high, $p_1^*(c_A)$ is the solution of

$$\max_{p_1\geq 0}\left(\theta(a-p_1+bp_2^*(c_H))(p_1-c_A)+(1-\theta)(a-p_1+bp_2^*(c_L))(p_1-c_A)\right);$$

and if it is low, $p_1^*(c_B)$ is the solution of

$$\max_{p_1\geq 0}\left(\theta(a-p_1+bp_2^*(c_H))(p_1-c_B)+(1-\theta)(a-p_1+bp_2^*(c_L))(p_1-c_B)\right).$$

If firm F_2's unitary production cost is high, $p_2^*(c_H)$ is the solution of

$$\max_{p_2\geq 0}\left(\phi(a-p_2+bp_1^*(c_A))(p_2-c_H)+(1-\phi)(a-p_2+bp_1^*(c_B))(p_2-c_H)\right);$$

and if it is low, $p_2^*(c_L)$ is the solution of

$$\max_{p_2\geq 0}\left(\phi(a-p_2+bp_1^*(c_A))(p_2-c_L)+(1-\phi)(a-p_2+bp_1^*(c_B))(p_2-c_L)\right).$$

Solving these maximization problems, we obtain equalities (1)-(4). ■

From this theorem we get Corollaries 1 and 2.

Corollary 1. The expected price, $E(p_1^*)$, of the good produced by firm F_1 is given by

$$E(p_1^*) = \frac{a(2+b)+2E(C_1)+bE(C_2)}{2(4-b^2)}$$

and the expected price, $E(p_2^*)$, of the good produced by firm F_2 is given by

$$E(p_2^*) = \frac{a(2+b)+bE(C_1)+2E(C_2)}{2(4-b^2)}.$$

Proof. The expected market prices, $E(p_1^*)$ and $E(p_2^*)$, of the goods produced by firms F_1 and F_2 are determined, respectively, by $E(p_1^*) = p_1^*(c_A)\phi + p_1^*(c_B)(1-\phi)$ and $E(p_2^*) = p_2^*(c_H)\theta + p_2^*(c_L)(1-\theta)$. Using equalities (1)-(4), we get the results. ■

The effect of the probabilities ϕ and θ over the expected prices is shown in Figure 1, for some parameter region of the model. The expected prices are lower in the case of independent goods than in the case of differentiated goods.

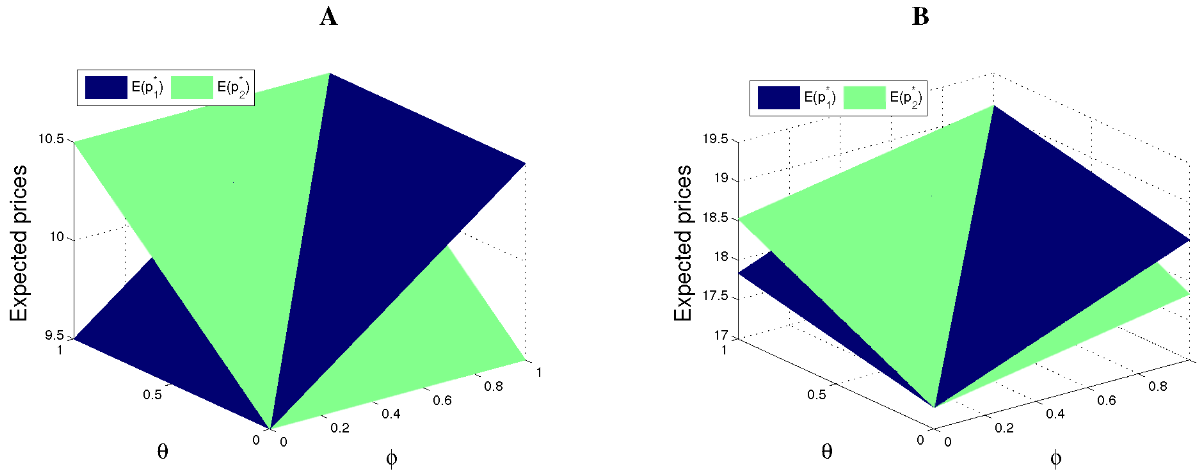

FIGURE 1. Expected prices, $E(p_1^*)$ and $E(p_2^*)$, in the case of: (**A**) firms producing independent goods ($b=0$); and (**B**) firms producing differentiated goods ($b=0.9$). Other parameters values: $a=15$, $c_A=c_H=6$, $c_B=c_L=4$.

Corollary 2. Let $V(C_i)$ be the variance of the firm F_i's unitary production cost, for $i \in \{1,2\}$. Firm F_1's expected profit $E(\pi_1^*)$ is given by

$$E(\pi_1^*) \quad = \quad \frac{\left(a(2+b)-(2-b^2)E(C_1)+bE(C_2)\right)^2}{(4-b^2)^2} + \frac{V(C_1)}{4}$$

and firm F_2's expected profit $E(\pi_2^*)$ is given by

$$E(\pi_2^*) \quad = \quad \frac{\left(a(2+b)+bE(C_1)-(2-b^2)E(C_2)\right)^2}{(4-b^2)^2} + \frac{V(C_2)}{4}.$$

Proof. The results follow by putting the quantities (1)-(4) into the formulas

$$\begin{aligned} E(\pi_1^*) \quad = \quad & \pi_1(q_1^*(c_A),q_2^*(c_H))\phi\theta + \pi_1(q_1^*(c_A),q_2^*(c_L))\phi(1-\theta)+ \\ & +\pi_1(q_1^*(c_B),q_2^*(c_H))(1-\phi)\theta + \pi_1(q_1^*(c_B),q_2^*(c_L))(1-\phi)(1-\theta) \end{aligned}$$

and

$$E(\pi_2^*) = \pi_2(q_1^*(c_A), q_2^*(c_H))\phi\theta + \pi_2(q_1^*(c_A), q_2^*(c_L))\phi(1-\theta) + \\ +\pi_2(q_1^*(c_B), q_2^*(c_H))(1-\phi)\theta + \pi_2(q_1^*(c_B), q_2^*(c_L))(1-\phi)(1-\theta).$$

■

The effect of the probabilities ϕ and θ over the firms' expected profits is shown in Figure 2A for the case of independent goods ($b = 0$), and in Figure 2B for the case of differentiated goods ($b = 0.9$), for some parameter region of the model. We see that both firms profit more in the case of differentiated goods than in the case of independent goods.

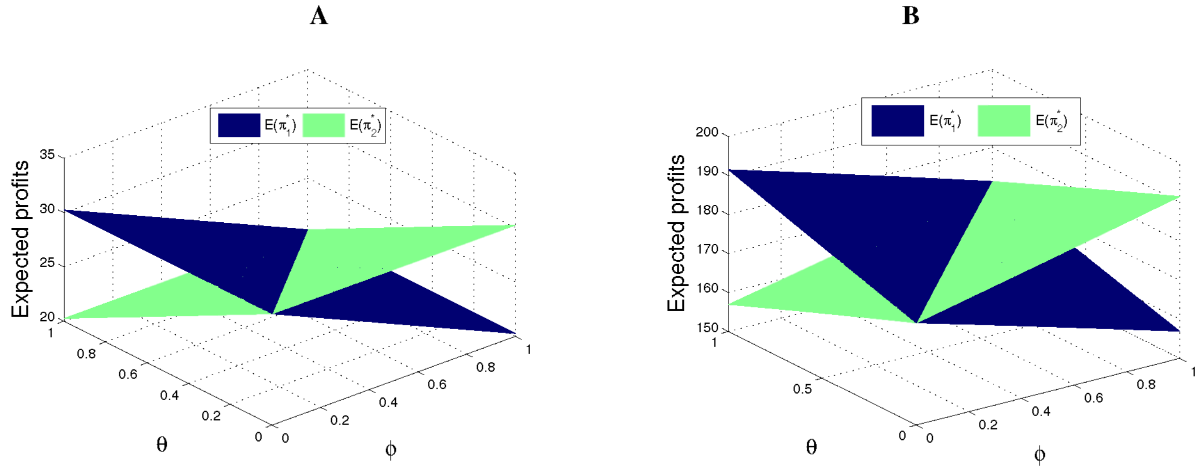

FIGURE 2. Firms' expected profits, $E(\pi_1^*)$ and $E(\pi_2^*)$, in the case of: (**A**) firms producing independent goods ($b = 0$); and (**B**) firms producing differentiated goods ($b = 0.9$). Other parameters values: $a = 15$, $c_A = c_H = 6$, $c_B = c_L = 4$.

CONCLUSIONS

We showed that the expected profit of each firm increases with the variance of its production costs. We also showed that the expected price of each good increases with both expected production costs, being the effect of the expected production costs of the rival dominated by the effect of the own expected production costs.

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