# Core-Periphery model with a CES utility function

by

Alexandre Manuel da Rocha Freitas

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Faculdade de Economia do Porto

Supervisor: João Oliveira Correia da Silva

Co-Supervisor: Sofia Balbina Santos Dias de Castro Gothen

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## Biography

Alexandre Manuel da Rocha Freitas was born in Guimarães, on 27 October 1992. In 2010, aware of his passion for the area, initiated a Bachelor (BSc) degree in Economics at the School of Economics and Management of the University of Porto (FEP), that was completed in June 2013, with a final average of fifteen (15) points out of twenty (20). During this period, he was co-founder and finance director of an association created within the University of Porto, called *Academia de Politica Apartidária*, which aims to connect college students to aspects of politics and policies aside from any party institution. Also, in the third and final year, he had the opportunity to study abroad, in Bucharest, Romania. In September 2013, he continued his studies in the Master of Science (MSc) in Economics at the same institution. In the first year, he finished the curricular part of the course, with an average of sixteen (16) points in twenty (20). Alongside, he gained experience in volunteering, tutoring children in the school domain, but also foreign students in the adaptation process to a new cultural setting.

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## Abstract

Recently, thanks to new techniques of economic modeling, a renewed literature emerged in the treatment of spatial aspects of economic activity in an area known as new economic geography. We contribute to the field by developing a core-periphery model that assumes CES preferences, rather than a Cobb-Douglas utility function as in the original model of Krugman (1991). The main purpose of this work is to articulate the widely-studied agglomeration and dispersion equilibria with the study of changes in the elasticity of substitution between agricultural and industrial goods, which is made possible by the generalization of the utility function. Through numerical simulations, mainly conducted in a core-periphery state, complemented by a section dedicated to the economic interpretation, we develop an analysis that was not previously possible. In general terms, an increase of the elasticity of substitution promotes agglomeration (and a decrease, dispersion) because qualifying the two types of goods as substitutes (or complements) changes the share of expenditure on industrial goods (*i.e.,* the demand) differently in the two regions, which, ultimately, also modifies the magnitude of the effects discussed in the literature: market-size, cost-of-living and market-crowding effects. However, these considerations are based on a price index of industrial varieties significantly lower than the price of agricultural goods, which, if tested inversely, leads to different conclusions.

**Keywords:** new economic geography; core-periphery model; agglomeration; dispersion; constant elasticity of substitution

**JEL Classification Numbers:** R10 R12 R23

## Resumo

Recentemente, graças a novas técnicas de modelação económica, uma literatura renovada emergiu no tratamento de aspetos espaciais da atividade económica numa área conhecida como nova economia geográfica. Com o intuito de enriquecer o campo, foi desenvolvido um modelo centro-periferia que assume preferências CES, em vez de uma função de utilidade Cobb-Douglas como no modelo original de Krugman (1991). O principal objetivo deste trabalho é articular os equilíbrios de aglomeração e dispersão com o estudo de alterações na elasticidade de substituição entre bens agrícolas e industriais, que surge pela generalização da função de utilidade. Através de simulações numéricas, realizadas sobretudo numa economia de centro-periferia, complementada com uma seção dedicada a interpretação económica, desenvolvemos uma análise que não era possível ser providenciada anteriormente. Em termos gerais, um aumento da elasticidade de substituição promove a aglomeração (e uma diminuição, a dispersão), porque ao qualificar os dois tipos de bens como substitutos (ou complementares) alteramos a percentagem de despesa em bens industriais (isto é, a procura) diferentemente nas duas regiões, o que, em última análise, modifica também a magnitude dos efeitos discutidos na literatura: *market-size*, *cost-of-living* e *market-crowding effect*s. No entanto, estas considerações têm por base um índice de preços do setor industrial significativamente mais baixo que o preço de bens agrícolas, que, se testados de forma inversa, originam conclusões diferentes.



**Palavras-chave:** nova economia geográfica; modelo centro-periferia; aglomeração; dispersão; *constant elasticity of substitution*

**Códigos JEL:** R10 R12 R23

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# 1 Introduction

This dissertation falls within the recent strand of neoclassical models developed in the field of new economic geography. Nevertheless, concepts such as distance or space have remained, surprisingly, on the outskirts of Economics until very recently, and it was only following Krugman (1991) that mainstream economists placed geography at the center of economic activity analysis. These theoretical advances, in regard to the well-established ideas developed in traditional location theory, were made possible due to a number of modeling tricks and numerical and computational methods, which enabled the new literature to embrace models of *general equilibrium*, "and in which spatial structures emerge from *invisible-hand* processes" (Krugman, 1998, p. 9).

Economic Geography is the field that is responsible for the study of the spatial organization of economic activities and the intrinsic reasons behind it. Despite the emergence of so-called core-periphery models in the last two decades, it is important to remember the roots of location theory, the authors and the grounds that justify their position as an economic model.

## 1.1 The roots of economic geography

Johann Heinrich von Thünen is, according to Walter Isard (1956, p. 27), the father of location theorists, which is the same as saying that economic geography is rooted in *The Isolated State* (1826). In a scenario of a single isolated town (market), with homogeneously fertile land and able to generate several cultures, von Thünen showed that competition among farmers leads to an increase in land costs from the outside limit of cultivation, where rent is nil, to the center. As shipping costs differ between cultures, agricultural goods subject to a higher cost of transportation will be closer to the city. The result is a pattern of concentric rings of production and a study that marked the first ever location theory.

However, the work of von Thünen, conducted at the beginning of the Industrial Revolution in Germany, was focused on agricultural land use, getting barely within concepts such as industry or agglomeration. Thus, it was only in the end of the nineteenth century and the first part of the twentieth century, that a number of economists studied more thoroughly aspects related to the location of industry.

In the late nineteenth century in England, Alfred Marshall in chapter 10 of the fourth book of *Principles of Economics* (1890) analyzed the reasons for the concentration of specialized industries in distinct locations. According to Marshall (1890, p 225.) "(...) if one man starts a new idea, it is taken up by others and combined with suggestions of their own; and thus it becomes the source of further new ideas". Marshallian externalities, as they became known, attempt to explain agglomeration as the result of a snowball effect in which an increasing number of agents wants to organize themselves to benefit from a wider range of activities and a higher degree of specialization. Regardless of a remarkable job, Marshall lacks economic modeling and does not provide the microeconomic mechanisms behind these externalities, but no one had managed to do that thence far.

At the beginning of the twentieth century, the German Max Weber was the first in a series of scholars in the field that became known as German geometry. Weber (1909), according to Fujita (2010), examined the localization of production according to a triangle formed by the market and two points of supply inputs. The location of the firm, whose production requires a given amount of both inputs, will be the one that minimizes the transportation cost per unit of output. Despite having contributed to the development of a new general location theory, his review was completely devoid of price analysis and market structure.

Meanwhile, some models were constructed relating location to firms' pricing policy. The emphasis goes to the different conclusions between the works of Hotelling (1929) and Palander (1935), according to Fujita (2010). Based on the same location model of Hotelling, where two companies sell the same product in a linear market, both competitors choose their locations and then their prices, considering that consumers bear the transportation cost, moving to the store which carries lower costs. While Hotelling concluded that both companies tend to agglomerate in the center of the market, Palander, years later, in a game with two stages, argued that competitors diverge from the center to prevent a severe price competition. Somehow, Palander's criticism has shown that a new location theory was needed to explain more adequately the phenomenon of agglomeration.

And so it happened in 1940, when August Lösch, heir of the German geography school, published his celebrated work *The Economics of Location*, where he developed the broadest general location theory to date. Its main contributions consisted in the definition of market areas as hexagonal, the consequent analysis of market networks and the maximization of profit, instead of shipping cost minimization, as the determinant of location choice. The patterns of distribution of market areas in space, with the significant help of Christaller's (1933) central place theory, managed to explain the existence of industrial clusters or the formation of cities. However, as mentioned in Fujita *et al.* (2001), the work of these authors (particularly central place theory) does not constitute an economic model since it fails to explain the phenomenon of agglomeration by the interaction of individuals or firms' decisions. Ultimately, "It is at best a description, rather than an explanation, of the economy's spatial structure" (Fujita et al., 2001, p.27).

## 1.2 The new economic geography

In 1991, Paul Krugman wrote *Increasing returns and economic geography.* Developing an abstract model seeking to explain the agglomeration of economic activity, the author managed to draw the attention of mainstream economists to the treatment of spatial aspects of economics. His work in the early 90s demonstrated that agglomeration can be achieved without exogenous regional asymmetries or external economies, or even without being related to climate or to resource endowments (the so-called first-nature geography, where the traditional neoclassical literature focused on). According to Krugman (1998, p.7), "This [new] approach inevitably has much in common with older approaches. Nevertheless, it also has a number of distinctive features that do qualify as a new departure". The structure of the reminder of this section is taken from Krugman (1998).

#### (i) Modelling Principles

There is no doubt that some of the ideas of Von Thünen, Christaller and Lösch are incorporated in new economic geography. Incidentally, in this sense, Krugman adds very little with respect to concepts and theories: the traditional literature is simply updated, and "the so-called new theories do not add any new variables, nor do they establish different relations or reach original interpretations" (Ruiz, 2001). Krugman's real asset is the flaw of the others. As mentioned above, some of the most influential works neither take into account the decisions of individuals and firms, the "microagents", nor explained the process of emergence of spatial structures. In new economic geography, however, individuals choose location maximizing their welfare given what other individuals are doing (Krugman, 1998). The result is the emergence of spatial structures arising from invisible-hand processes, a self-organizing system in which the "micromotives" of the agents are the key (Ruiz, 2001).

#### (ii) Modelling Tricks

Paul Krugman in a conversation with Masahita Fujita, in The new economic geography: Past, present and the future (2003), describes the model with a peculiar slogan: "Dixit-Stiglitz, icebergs, evolution and the computer" (p. 142). *Dixit-Stiglitz* is relative to the monopolistic competition model by Avinash Dixit and Joseph Stiglitz (1977), adopted nowadays by various fields of study. According to Krugman, "it has the virtue of producing in the end a picture of an economy, in which there are increasing returns [and existence of monopoly power], (...) not get into the fascinating but messy issues posed by realistic oligopoly." (Fujita & Krugman, 2003, p.143). The expression *icebergs* concerns a transportation model by Paul Samuelson (1952), in which the costs are introduced by imagining that a fraction of the product melts in the road. This avoids the analysis of the transportation service as an isolated industry, while it simplifies the perception of companies' costs when setting their monopoly price. On the behalf of agents' behavior, the *evolution* slogan refers to the decision not to assess location by future expectations, but solely on current conditions. Finally, *computer* is no more than the use of new numerical and computational methods, as models of new economic geography "turn out to be a bit beyond the reach of paper-and pencil analysis" (Krugman, 1998, p.11).

### (iii) Modelling Strategy

Above all, Krugman's framework represents a tension between forces that promote and forces that oppose agglomeration, *i.e,* between centripetal and centrifugal forces, respectively.

In the introduction of *Increasing returns and economic geography*, Krugman first stresses that, due to economies of scale, the production of each industrial good will be limited to a small number of locations. *Ceteris paribus*, the location of these production plants will be next to markets with high demand, in order to minimize transportation costs. Then he asks where that demand will be higher. Since part of consumption of manufactured goods is directed for industrial workers, the demand will be bigger the larger the sector. This is a clear example of the theory of circular cumulative causation of Myrdal (1957): industrial production tends to concentrate where there is greater demand, but the market will be bigger as more concentrated the industry is (market-size effect).

The circularity created by backward linkages is reinforced, in turn, by forward linkages: other things equal, it is more pleasant to live in the center, where the goods are not subject to shipping costs (Krugman, 1991). It is known as the cost-of-living effect, and is more beneficial (for the locals) the more crowded the region. This brings us to the last property, the only one that acts against agglomeration, the market-crowding effect. The market-crowding effect is the benefit of the region with a smaller industry, which, by having a smaller number of firms and workers, faces for the local market less competition than if it was located in the other region. Obviously, the effect is felt more when workers are concentrated in one region, because by moving they are able to get a clear comparative advantage over goods imported from the other region. The outcome of these three forces, which determines the location of industry, depends on a number of parameters that arise from the modeling of the main actors in the economic geography.

## 1.3 The core-periphery model

The core-periphery model is based on two regions separated in space, each with two sectors, agriculture and industry. The agriculture sector produces a homogeneous good in perfect competition (*numéraire*), whereas the industry sector is characterized by monopolistic competition where each firm produces a different variety of the same good. Consumers, although products are symmetric in the sense that they do not prefer one to another (horizontally differentiated goods), have preference for variety (inversely related to the elasticity of substitution between different industrial varieties, $\sigma > 1$).

There are two types of labor: workers and farmers. Workers are the factor of production of industrial goods and travel to the region that offers them a higher real wage. Farmers sell the agricultural good and in turn are immobile, their distribution being fixed evenly between regions. Agricultural goods are perfectly tradable across regions, while the industrial varieties are subject to transportation costs. Krugman (1991), as already mentioned, conveniently introduces iceberg costs, assuming that only an exogenous fraction of the goods reaches its destination ($0 < \tau \leq 1$).

In terms of behavioral assumptions, consumers maximize their utility function given a budget constraint. In Krugman (1991), the consumers' utility is represented by a Cobb-Douglas function of an agglomerate of manufactures, $C_M$, and agricultural goods, $C_A$: $U = C_M^{\mu} C_A^{1-\mu}$. This choice of utility function establishes that the percentage of expenditure in industrial goods is constant and given by $\mu$. Finally, market entry is free, *i.e.*, in equilibrium, firms do not enjoy profits.

In equilibrium, due to industrial workers' mobility, both regions exhibit the same real wage or the entire industry will be concentrated in the region with the highest one. In the latter case, a firm may have the incentive to migrate to the other region, referred as periphery, by the absence of competition. Yet, to move from the core of the economy entails some downsides, the net result being a counterbalance of forces emphasized in the role of transportation costs and economies of scale.

On the one hand, firms (workers) that deviate will have to consume a large percentage of industrial products that must be imported from the center, affecting their cost of living. To offset the transportation costs, firms will have to pay higher nominal wages. Furthermore, most of the market served remains in the other region, which implies more transport, but, in the opposite direction, serving the population in the peripheral region becomes cheaper. Thus, agglomeration arises only if $\tau$ is below unity (with no transportation costs location is irrelevant), but moderate enough so that there are no temptations to serve the peripheral market locally.

The size of the industrial sector, measured by the percentage of expenditure on industrial goods, is relevant. The higher the value of $\mu$, the worse the living conditions for workers who are considering to move will be, since the share of industrial goods that need to be imported is also higher, but nevertheless, from the firm's point of view, the demand in the peripheral region increases.

Finally, the elasticity of substitution between industrial varieties also plays an important role on centrifugal and centripetal forces. At equilibrium, $\frac{\sigma}{\sigma-1}$ is equal to the ratio between average and marginal cost, a common measure of economies of scale (Krugman, 1991). A low elasticity of substitution can give rise to significant economies of scale, making it less attractive to serve the peripheral market locally.

In short, with reduced transport costs (low $\tau$), a dominant industry sector (high $\mu$) and important economies of scale (lower $\sigma$), agglomerations become more robust (Schmutzler, 1999).

## 1.4 Modifications to the original model

While Krugman's model captures important aspects of the development of spatial patterns, it is based on various assumptions which, if relaxed, lead to additional insights, as Schmutzler (1999, p.364) indicates.

As Krugman points out in *What's new about new economic geography* (1998), beyond the desire of firms to serve the periphery, there are other centrifugal forces of agglomeration that are not incorporated in the base model. Barkman *et al.* (1994) modifies Krugman's model by introducing effects of congestion. The study concludes that, in the presence of a negative externality, a complete concentration rarely occurs because production in the core becomes too costly.

Assuming n-regions, the results are complex and show, most of the time, multiple agglomerations as equilibrium (Schmutzler, 1999). With only one more region, such as Castro *et al.* (2012) shows, a model with three regions favors concentration in relation to the initial setup. A work by Krugman and Livas-Elizondo, in 1996, extended the analysis of a multiple regions' model in order to study the spatial distribution as a function of the degree of openness to trade. In addition, they introduce urban rents and commuting costs, which represents an additional centrifugal force. This occurs because the agglomeration of industry causes an increase in the size of the city, resulting in the growth of rents and commuting costs, which ultimately reduces the likelihood of agglomeration.

### An analytically solvable model

Forslid & Ottaviano in *An analytically core-periphery model* (2003) address a major limitation of Krugman's core-periphery model. By considering that mobile skilled workers are a fixed cost in the industrial sector, while immobile unskilled workers are the variable input in the industrial sector (as well as the single input in agriculture), the model allows an analytical treatment that is not possible in Krugman's setting, while maintaining the properties of the original work. This model is, as well, the starting point for obtaining the model explored in this dissertation.

## 1.5 The CES core-periphery model

The theoretical field known as new economic geography studies issues related to the location of industry, in a core-periphery model in which it is frequently assumed that agents have preferences of Cobb-Douglas type. Given that Cobb-Douglas can be seen as a particular case of a CES (constant elasticity of substitution) function, we investigate if the aspects described in the literature persist in the more general case of a model using a CES utility function. We also address the impact of the elasticity of substitution between agricultural and industrial goods ($\rho$, a new parameter) on the long-term stability of the traditional agglomeration and dispersion equilibria.

The methodology of this investigation combines analytical and computational treatment. We generalize the Forslid & Ottaviano (2003) model to obtain the relevant equations when consumers have CES type preferences, later implemented in a numerical computing software, MatLab. Here, the work partitioned in ensuring that the behavior observed for a Cobb-Douglas value appears in the new model when $\rho$ tends to zero, beyond the illustration of the model's behavior for positive and negative values of $\rho$, which unfolds new insights.

In a core-periphery economy, regarding the elasticity of substitution between agricultural and industrial goods, we found that, when the elasticity is positive, its increase promotes agglomeration, when compared to the Cobb-Douglas setup. This is because the increase of substitution between agricultural and industrial goods raises the demand for industrial goods more proportionately in the core than at the periphery, while the cost of living is lower. The ratio of utilities increases in favor of the central region, and therefore, of agglomeration. On the contrary, when the elasticity is negative, the consumption of industrial goods decreases more strongly in the central region. An agent in the peripheral region increases its relative utility by consuming a greater share of industrial goods (even with transportation costs), acting against concentration. However, the latter case only reveals itself a force of dispersion given the marked difference in prices between agricultural and industrial cheaper goods.

Finally, the results of the model also differ when the price of the agricultural good is lower than the price of manufactured goods. Negative values of $\rho$ increase the share of expenditure on industrial goods more in the peripheral region, which given the higher price of industrial goods in relation to agricultural goods, affect the consumption in this region compared to the core. The second case, when the goods in the two sectors are substitutes, again, depends on the degree of the differences between prices. As it was tested, if prices are distinct, then the substitution harms agglomeration because the demand for agricultural goods is higher in the peripheral region.

The present dissertation is organized as follows. In the next section, the model is described, containing all the relevant functions based on the model of Forslid & Ottaviano (2003), which, obviously, is not restricted merely to the amendment of the utility function. In Section 3, we begin by introducing the concepts of agglomeration and dispersion equilibria, stating the relevant parameters to our study and characterizing the migration mechanism of skilled workers stemming from the indirect utility between regions. Still in the same chapter, we complement our analysis with a numerical example, using distinct values of transportation costs in order to corroborate the same conclusions of the new economic geography literature when we set $\rho = 0$, which corresponds to the Cobb-Douglas case. We finish Section 3 with a reflection on our efforts to develop analytical conditions for both known equilibria. Section 4 is focused on simulation of the model in a number of different scenarios, which, with the assistance of a series of figures, are intended to show the effects of changes in preferences, on a first instance, in a general form, and later on, detailed for each relevant parameter. Section 5, in turn, presents the economic interpretation, duly substantiated, of the results obtained in Section 4. Here the main inferences of our work are described, which are reiterated in Section 6, of conclusions.

# 2 The model

This core-periphery model is based on two regions, each with two sectors, agriculture and industry. Based on the framework of Forslid & Ottaviano (2003), there are two factors of production, skilled ($H$) and unskilled labour ($L$), both employed in manufactured production. Unskilled workers are perfectly mobile between sectors but spatial immobile and assumed to be evenly distributed across regions ($L_i = L/2$, $i = 1{,}2$), unlike skilled ones that are geographically mobile and therefore choose to reside in the region that offers a higher well-being ($H = H_1 + H_2$).

**(i) Demand Side**

Preferences are defined over two final goods, agricultural (a homogenous good, $A$) and manufactured (a differentiated good, $X$). Unlike Krugman (1991), the preference ordering of the representative consumer in region $i = 1{,}2$ is captured by a constant elasticity of substitution (CES) utility function:

$$U_i = \frac{\mu X_i^{\rho} + (1-\mu) A_i^{\rho}}{\rho}, \tag{1}$$

where $X_i$ is the consumption of the manufactures aggregate, $A_i$ is the consumption of agricultural products, $\mu \in (0,1)$ is the strength of preference for manufactured goods relatively to the agricultural good and $\rho \in (-\infty, 1)$ is a measure of the elasticity of substitution between agricultural and manufactured goods[1]. The first order condition for the problem of maximizing $U_i$ with respect to $x_i$ (the demand of a representative consumer), subject to $P_i x_i + P_A a_i = y$, gives us:

$$P_i x_i = \frac{y}{1 + (\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}}, \tag{2}$$

where $P_i$ is the local price index of manufactures and $y$ each consumer's income. Define $\bar{\mu}_i \in (0,1)$, different between regions, as the share of expenditure in manufactured goods:

$$\bar{\mu}_i = \frac{1}{1 + (\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}} \tag{3}$$

The calculations leading to (3) are given in Appendix A. It is also important to stress that, relatively to Krugman (1991) or Forslid & Ottaviano (2003), the percentage of expenditure on industrial goods is no longer a parameter. The consumption of the manufactures

[1] Note that the standard Cobb-Douglas utility function is recovered for $\rho \to 0$. The elasticity of substitution is represented by $\frac{1}{1-\rho}$.

aggregate, $X_i$, is defined by:

$$X_i = \left(\int_{s\in N} d_i(s)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $d_i(s)$ is the consumption of variety $s$ of good $X$, $N$ is the mass of varieties ($N = n_i + n_j$) and $\sigma > 1$ is the elasticity of substitution between any two varieties of good $X$. Demand by residents in location $i$ for a variety produced in location $j$ is:

$$d_{ji}(s) = \frac{p_{ji}(s)^{-\sigma}}{P_i^{1-\sigma}} \bar{\mu}_i Y_i,\ i,j = 1,2, \tag{5}$$

where $p_{ji}$ is the consumer price of a variety produced in $j$ and sold in $i$ and $Y_i$ the local income consisting of skilled ($w_i$) and unskilled wages ($w_i^L$), defined by:

$$Y_i = w_i H_i + w_i^L \frac{L}{2} \tag{6}$$

The local price index of manufactures $P_i$ associated with (4) is:

$$P_i = \left(\int_{s\in n_i} p_{ii}(s)^{1-\sigma} ds + \int_{s\in n_j} p_{ji}(s)^{1-\sigma} ds^{\frac{1}{1-\sigma}}\right) \tag{7}$$

The representative consumer in region $i$ has the following budget constraint when maximizing utility (1):

$$Y_i = \int_{s\in n_i} p_{ii}(s) d_{ii}(s) ds + \int_{s\in nj} p_{ji}(s) d_{ji}(s) ds + p_i^A A_i \tag{8}$$

**(ii) Supply Side**

Firms in the industrial sector are monopolistically competitive and employ both skilled and unskilled workers under increasing returns to scale. The total cost of production of a firm, in location $i$, is:

$$TC_i(s) = w_i\alpha + w_i^L \beta x_i(s), \tag{9}$$

meaning that in order to produce $x(s)$ units of variety $s$, a firm must employ $\alpha$ units of skilled labour (fixed cost) and a marginal input of $\beta x$ units of unskilled labour. Trade of manufactures is subject to a transportation cost, modeled as iceberg costs ($1 \leq \tau < +\infty$)[2]. Given the fixed input requirement $\alpha$, skilled labour market clearing implies that in equilibrium the number of firms is determined by:

[2]Differently to Krugman (1991), where $0 < \tau \leq 1$

$$n_i = \frac{H_i}{\alpha} \tag{10}$$

Firms in agriculture produce a homogenous good under perfect competition and employ only unskilled labour, in a way that one unit of output requires one unit of unskilled labour. Thus, perfect competition implies $p_i^A = w_i^L$, $i = 1,2$. Good A is freely traded so that its price is the same everywhere ($p_1^A = p_2^A$). Due to marginal cost pricing, this also implies inter-regional wage equalization ($w_1^L = w_2^L$). This suggests choosing good A as *numéraire*, so that $p_1^A = w_1^L = p_2^A = w_2^L = 1$.

A typical manufacturing firm located in region $i$ maximizes its profit function:

$$\pi_i(s) = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta\left[d_{ii}(s) + \tau d_{ij}(s)\right] - \alpha w_i, \tag{11}$$

where $\tau d_{ij}(s)$ represents total supply to the distant location $j$ inclusive of the fraction of product that melts away in transit due to the iceberg costs. The first order condition of maximization of (11), for every $i$ and $j$, is:

$$p_{ii}(s) = \frac{\beta\sigma}{\sigma-1}, \quad p_{ij}(s) = \frac{\tau\beta\sigma}{\sigma-1} \tag{12}$$

After using (12), the price index (7) simplifies to:

$$P_i = \frac{\beta\sigma}{\sigma-1}\left[n_i + \phi n_j\right]^{\frac{1}{1-\sigma}}, \tag{13}$$

where $\phi = \tau^{1-\sigma} \in [0,1]$ is the ratio of total demand by domestic residents for each foreign variety to their demand for each domestic variety, therefore a measure of freeness of trade. Replacing (10) in (13), the price indices of regions 1 and 2 are:

$$P_1 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\left[h + \phi(1-h)\right]^{\frac{1}{1-\sigma}} \tag{14}$$

$$P_2 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\left[1 - h + \phi h\right]^{\frac{1}{1-\sigma}} \tag{15}$$

Due to free entry and exit, there are no profits in equilibrium. A firm's operating profits are entirely absorbed by the wage bill of its skilled workers: $\alpha w_i = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta\left[d_{ii}(s) + \tau d_{ij}(s)\right]$. Given (12), we obtain:

$$w_i = \frac{\beta x_i}{\alpha(\sigma-1)} \tag{16}$$

where $x_i = d_{ii}(s) + \tau d_{ij}(s)$ is total production by a typical firm in location $i$. Using (5), (12), (13) and (16), we can determine the output of firms in both regions:

$$x_i = \frac{\sigma - 1}{\beta\sigma}\left(\frac{\bar{\mu}_i Y_i}{n_i + \phi n_j} + \frac{\phi\bar{\mu}_j Y_j}{\phi n_i + n_j}\right) \tag{17}$$

Using (9) and (13), (17) can be equivalently written as:

$$w_i = \frac{1}{\sigma}\left(\frac{\bar{\mu}_i Y_i}{H_i + \phi H_j} + \frac{\phi\bar{\mu}_j Y_j}{\phi H_i + H_j}\right) \tag{18}$$

Plugging expression (6) into (18) generates a system of two linear equations in $w_i$ and $w_j$, that can be solved to obtain the equilibrium skilled wages as explicit functions of the spatial distribution of skilled workers $H_i$, whose expression is obtained in Appendix B and displayed as:

$$w_i = \frac{L}{2\sigma}\frac{\phi H_i(\bar{\mu}_i + \bar{\mu}_j) + \left[\bar{\mu}_i - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma} + (1 + \frac{\bar{\mu}_i}{\sigma})\bar{\mu}_j\phi^2\right] H_j}{\phi(H_i^2 + H_j^2) - (H_i^2\bar{\mu}_i + H_j^2\bar{\mu}_j)\frac{\phi}{\sigma} + \left[1 + \frac{1}{\sigma^2} - \frac{\bar{\mu}_i + \bar{\mu}_j}{\sigma} + (1 - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma^2})\phi^2\right] H_i H_j}, \tag{19}$$

for $i, j = 1,2$. The ratio between the equilibrium skilled wages, defining $h = H_1/H$ as the share of skilled workers that reside in region 1, is:

$$\frac{w_1}{w_2} = \frac{\phi h(\bar{\mu}_1 + \bar{\mu}_2) + \left[\bar{\mu}_1 - \frac{\bar{\mu}_1\bar{\mu}_2}{\sigma} + (1 + \frac{\bar{\mu}_1}{\sigma})\bar{\mu}_2\phi^2\right](1 - h)}{\phi(1 - h)(\bar{\mu}_1 + \bar{\mu}_2) + \left[\bar{\mu}_2 - \frac{\bar{\mu}_1\bar{\mu}_2}{\sigma} + (1 + \frac{\bar{\mu}_2}{\sigma})\bar{\mu}_1\phi^2\right] h} \tag{20}$$

It is important to emphasize that this expression, despite being formally very similar to the equivalent (17) in Forslid & Ottaviano (2003), underlies a major distinction. The share of expenditure in manufactured goods $\bar{\mu}_i$, $i = 1,2$, before just a parameter, depends on the elasticity of substitution between agricultural and manufactured goods $\rho$ and on the region's price index.[3] The indirect utility, i.e., the maximal utility attainable in region $i$ for given prices and wages, is:

$$V_i = \frac{\mu^{\frac{1}{\rho}}}{P_i}\left[1 + \left(\frac{1 - \mu}{\mu}\right)^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}} w_i, \tag{21}$$

or, simply, $V_i = \frac{\mu^{\frac{1}{\rho}}}{\bar{\mu}_i^{\frac{1-\rho}{\rho}}}\left(\frac{w_i}{P_i}\right)$ (see Appendix C). Finally, the ratio between the indirect utilities gives us a starting point to study agglomeration and dispersion patterns:

$$\frac{V_1}{V_2} = \frac{P_2}{P_1}\left(\frac{\bar{\mu}_2}{\bar{\mu}_1}\right)^{\frac{1-\rho}{\rho}}\frac{w_1}{w_2} \tag{22}$$

[3]This will have implications in the analytical treatment of the long-run equilibrium patterns.

# 3 Long-run equilibria

Let us start the study of agglomeration and dispersion equilibria, focusing our analysis on the ratio between indirect utilities (22), an indicator that triggers the mobility of skilled workers between regions (see also Section 3.1). The utility of any skilled worker in region $i = 1{,}2$ will depend on his wage ($w_i$), on the price index associated ($P_i$) and on the share of expenditure in manufactured goods, $\bar{\mu}_i$. Contrary to the original model, the latter depends not only on $\mu$ (that now only represents a measure of preference for manufactured goods relatively to the agricultural good), but also on the elasticity of substitution between agricultural and manufactured goods, $\rho$, and on the region's price index. Indeed, $P_i$ incorporated in $\bar{\mu}_i$ makes fixed cost, $\alpha$, marginal cost, $\beta$, and number of skilled workers, $H$, that on Krugman's model were eliminated by choice of units, parameters relevant to our study. Finally, and to complete the description of all the parameters that affect the model's behavior, the ratio of indirect utilities also depends on the elasticity of substitution between industrial varieties, $\sigma$, and on transportation costs, $\tau$.

Assuming symmetry regarding the production factors, the immobility of unskilled workers makes them evenly split between regions. The issue has to do then with industry and skilled workers: in the long term, the industry can be fragmented between the two regions, or concentrated in one, creating an economy divided between an industrial center (or core) and an agricultural periphery. Herein, we need to introduce the concept of long-term equilibrium (from now on, just equilibrium), as the distributions of skilled workers that remain unchanged over time. An equilibrium is stable, as Gaspar (2012, p.11) explains, "if after occurrence of some small exogenous migration of skilled workers to any of the regions, the spatial distribution of skilled workers is pulled back to the initial one."

Thereby, a dispersion equilibrium is observed when the skilled workers are equally divided and is stable if, after a small migration, the economy returns to a symmetry of industries. By moving, for example, from region $i$ to region $j$, a skilled worker causes the real wage of region $j$ to become comparatively lower than the one in region $i$, and consequently, a different skilled worker will do the opposite movement. On the other hand, an agglomeration equilibrium is stable when the real wage of the region in which all entrepreneurs are located is larger than the real wage in the region without manufacturing. To determine when we have a stable or unstable equilibrium, let us analyze both situations from the analytical and numerical point of view.

## 3.1 Equilibrium patterns

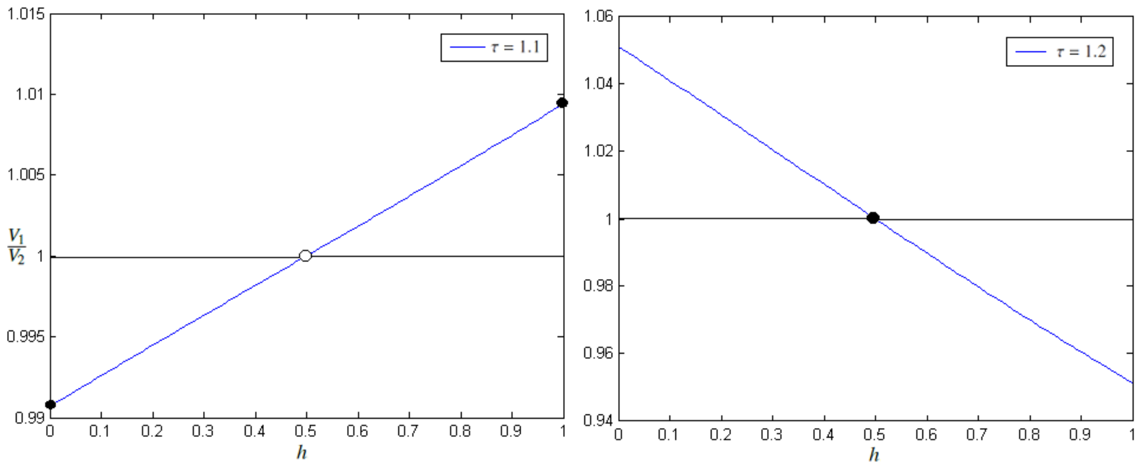
As explained before, workers are short-sighted and choose their location by maximizing their indirect utility. According to Forslid & Ottaviano (2003, p.234), skilled worker migration follows a simple Marshallian adjustment, where the rate of change of skilled workers in one region only depends on the indirect utility differential (or ratio, in our model) between the two regions:

$$\dot{h} \equiv \frac{dh}{dt} = \begin{cases} \frac{V_1}{V_2} - 1 & if \quad 0 < h < 1 \\ min\left\{0, \frac{V_1}{V_2} - 1\right\} & if \quad h = 1 \\ max\left\{0, \frac{V_1}{V_2} - 1\right\} & if \quad h = 0 \end{cases}$$

where $t$ is time. The share of skilled workers in region 1, expressed as $h$, becomes an essential indicator in the study of the long-run dynamics, as it allows us to work with only one region, while providing the necessary information to understand the inter-regional dynamics ($1 - h$ is the implied share of workers in the region 2). Furthermore, as skilled workers are a fixed productive factor in industry, the share of skilled workers in region 1 is equivalent to the share of industry (as a whole) in region 1.

The level of transportation costs is a determinant parameter for the stability of each available equilibrium and hence, for observed equilibrium patterns. Figure 1 shows the ratio of indirect utilities, against $h$, for low transport costs ($\tau = 1.1$) and for high transport costs ($\tau = 1.2$). The results, using $\rho \rightarrow 0$, remain loyal to Krugman's (1991) conclusions. In respect to other parameters, we assume $\sigma = 4$, $\mu = 0.3$ (similarly with Krugman), besides $\alpha = 1$, $\beta = 0.1$ and $H = 100$.

Figure 1: Ratio of utilities over $h$, for $\rho \rightarrow 0$, for $\tau = 1.1$ and $\tau = 1.2$
$\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$.
Full circles represent stable equilibria and empty circles denote instability.

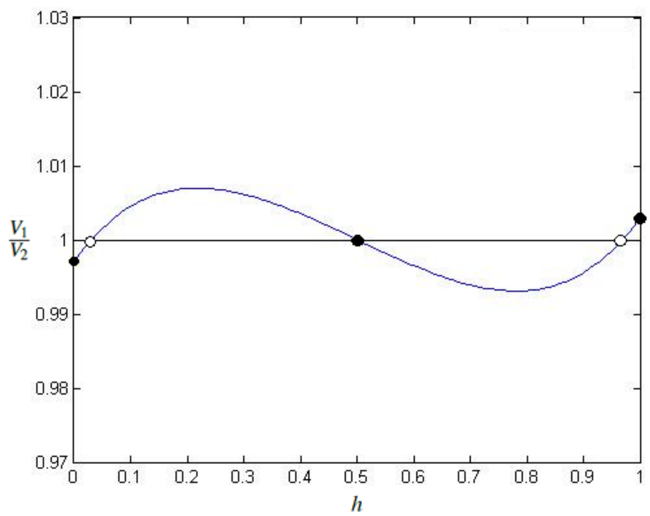



For $\tau = 1.2$, the ratio of utilities slopes downwards in $h$. In the long-term, the economy converges to a stable dispersion equilibrium where the industry is divided equally between the two regions. At $h = \frac{1}{2}$, a deviation of any worker is not useful, given that the ratio of utilities raises in favor of the region from where they left. For example, if a entrepreneur moves from region 1 to 2, $h$ reduces and, therefore, $\frac{V_1}{V_2}$ increases, improving the utility of the region 1 compared to the other region.

On the contrary, for $\tau = 1.1$, the ratio is positive for $h > \frac{1}{2}$, negative for $h < \frac{1}{2}$ (at $h = \frac{1}{2}$ wages are always equal due to the symmetry). When a region contains the majority of the manufacturing labor force, workers from the other region have the motivation to shift towards the most industrialized zone. Thus, in the long-term, workers will all move to the same region: an agglomeration equilibrium in region 1 if $h > \frac{1}{2}$, and in region 2 if $h < \frac{1}{2}$ (unlike the latter, agglomeration equilibria are always stable). This upward slope results both from market-size effects (see modelling strategy in Section 1.2) and, of course, low transport costs. Let us recall that without transportation costs location is irrelevant, and an agglomeration equilibrium only happens in a situation in which transport costs are moderate enough that serving the peripheral market from the center is not prohibitive, a situation observed through the first example for $\tau = 1.2$.

Surprisingly, a unique agglomeration or dispersion equilibria are not the only possibilities. For certain intermediate values of transport costs, we may observe both equilibria for one given set of parameters. Figure 2, for $\tau = 1.125$, reveals this special case that displays multiple stable equilibria.

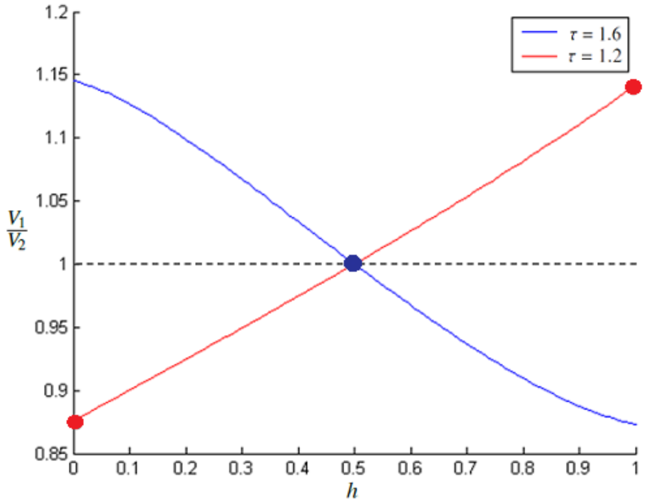
Figure 2: Ratio of utilities over $h$, for $\rho \to 0$, for $\tau = 1.125$
$\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$.
Full circles represent stable equilibria and empty circles denote instability.




The graph shows a total of five equilibria, three of which are stable, as shown also in Fujita, et al. (2001, p.67). The dispersion equilibrium is locally stable at $h = \frac{1}{2}$, as in the case of high transport costs, but lies now between two unstable equilibria. That is, for an initial value of $h$ sufficiently low or high, the economy can also converge to a agglomeration state in region 2, at $h = 0$, or region 1, at $h = 1$.

These statements are in line with the literature, which gives us the legitimacy to begin the study of the model when $\rho \neq 0$, the main objective of this work and what differentiates it from all the others. As an introduction, Figure 3 shows $\frac{V_1}{V_2}$ over $h$, for $\rho = 0.5$ and also for two different values of the transportation cost.

Figure 3: Ratio of utilities over $h$, for $\rho = 0.5$, for $\tau = 1.2$ and $\tau = 1.6$
$\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$
Full circles represent stable equilibria.




Notice that the figure presents both equilibria, a dispersion equilibrium where workers are distributed equitably (function that decreases with $h$) and an agglomeration state in any of the two regions, depending on the initial $h$. Transportation costs, however, are not the same, especially as in the Cobb-Douglas limit ($\rho \rightarrow 0$), the value $\tau = 1.2$ produces a dispersion equilibrium, unlike what is observed here. This reflects the effect of the elasticity of substitution between agricultural and industrial goods on the spatial distribution of industry. This issue is continued in Section 4.

## 3.2 The search for analytical solutions

From the beginning, our goal was to prepare both analytical and numerical solutions. Unfortunately, from the analytical point of view, we faced unpromising results, for not being able to develop a simple condition neither for the agglomeration equilibrium, nor for the dispersion one. In the remaining of this section, we illustrate our difficulties.

Starting with agglomeration, the simpler of the two, we obtain a stable equilibrium for $\frac{V_1}{V_2} > 1$, for $h = 1$. Recovering expression (22), and taking $h = 1$ as given, the condition is:

$$\phi^{\frac{1}{1-\sigma}} \left(\frac{\bar{\mu}_2}{\bar{\mu}_1}\right)^{\frac{1-\rho}{\rho}} \frac{\phi\,(\bar{\mu}_1+\bar{\mu}_2)}{\bar{\mu}_2 - \frac{\bar{\mu}_1\bar{\mu}_2}{\sigma} + (1+\frac{\bar{\mu}_2}{\sigma})\bar{\mu}_1\phi^2} > 1, \tag{23}$$

with $\bar{\mu}_i = \frac{1}{1+(\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}}$ $(i = 1,2)$, $P_1 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}$ and $P_2 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\phi^{\frac{1}{1-\sigma}}$. Unfortunately, our hopes of getting a simpler condition bump on the difficulty in working with $\bar{\mu}_i$, due to the differences between price indices. Instead of replacing the entire expression, we try to rearrange $\bar{\mu}_1$ and $\bar{\mu}_2$ into the fraction they share in common and the one that differentiates them, whose outcome is exposed in the following expression[4]:

$$\phi^{\frac{2-\sigma}{1-\sigma}} \left(\frac{1+\theta}{1+\phi^*\theta}\right)^{\frac{1-\rho}{\rho}} \frac{1+\frac{1+\theta}{1+\phi^*\theta}}{\frac{1+\theta}{1+\phi^*\theta} + \phi^2 - (1-\phi^2)\frac{1}{(1+\phi^*\theta)\sigma}} > 1, \tag{24}$$

defining the common part as $\theta = \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}}\left(\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\right)^{\frac{\rho}{1-\rho}}$ and the distinctive fraction as $\phi^* = \phi^{\frac{\rho}{(1-\sigma)(1-\rho)}}$. Even so, the problem remains, by not been able to reduce the expression to a condition that explains the agglomeration equilibrium in a straightforward way.

Aggravating, is the case of a dispersion of skilled workers, whose analytical condition for a stable equilibrium is defined by $\frac{\partial V_1}{\partial h} < 0$, for $h = \frac{1}{2}$. The negative first derivative ensures that no worker has the incentive to work in the other region, which is easy to examine in a figure, but difficult to prove analytically. The following expression for $V_1$ makes the point:

$$\begin{aligned} V_1 \quad = \quad & \frac{\mu^{\frac{1}{\rho}}}{\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\left[h+\phi(1-h)\right]^{\frac{1}{1-\sigma}}} \left[1+\left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}}\left(\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\left[h+\phi(1-h)\right]^{\frac{1}{1-\sigma}}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}} \\ & \frac{L}{2}\frac{1}{\sigma}\frac{\phi(\bar{\mu}_1+\bar{\mu}_2)H_1+\left[\bar{\mu}_1-\frac{\bar{\mu}_1\bar{\mu}_2}{\sigma}+(1+\frac{\bar{\mu}_1}{\sigma})\bar{\mu}_2\phi^2\right]H_2}{\phi(H_1^2+H_2^2)-(H_1^2\bar{\mu}_1+H_2^2\bar{\mu}_2)\frac{\phi}{\sigma}+\left[1+\frac{1}{\sigma^2}-\frac{\bar{\mu}_1+\bar{\mu}_2}{\sigma}+(1-\frac{\bar{\mu}_1\bar{\mu}_2}{\sigma^2})\phi^2\right]H_1H_2} \end{aligned}$$

Besides the issue of the size of the expression, many of the $\bar{\mu}_i$ are not replaced, and we were unable to put the whole function depending on the ratio $h = \frac{H_1}{H_1+H_2}$. As the reader will agree, this expression is beyond reasonable analytical treatment.

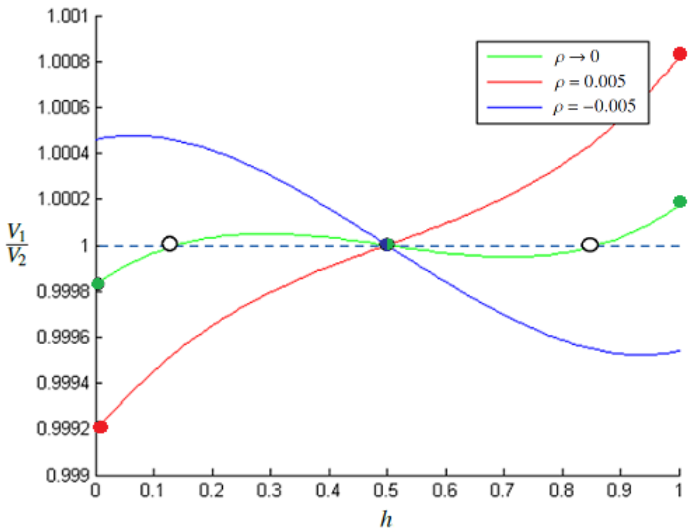
[4]The calculations leading to (24) are given in Appendix D.

# 4 Simulation

In this section, we present, through numerical computations, the effect of the elasticity of substitution between agricultural and manufactured goods (from now on, just elasticity of substitution), on the patterns of equilibria already stated, besides its influence within the parameters already widely-studied by the new economic geography literature. Recall that this parameter is introduced by a CES utility function, that unlike a Cobb-Douglas type, mainstream in the core-periphery models, covers a range between $\rho \in (-\infty, 1)$.

Let us start by picking up the Cobb-Douglas case ($\rho = 0$) and introduce small perturbations in the parameter, in the most usual representation, which opposes the ratio of indirect utilities $\frac{V_1}{V_2}$ to the share of industry (or skilled workers) in region 1. Figure 4 uses a value of $\tau$ corresponding to an intermediate case of transportation costs for $\rho \to 0$ (based on Figure 2), and just by a very small variation, both positive and negative, will give us a first impression on all known equilibria.

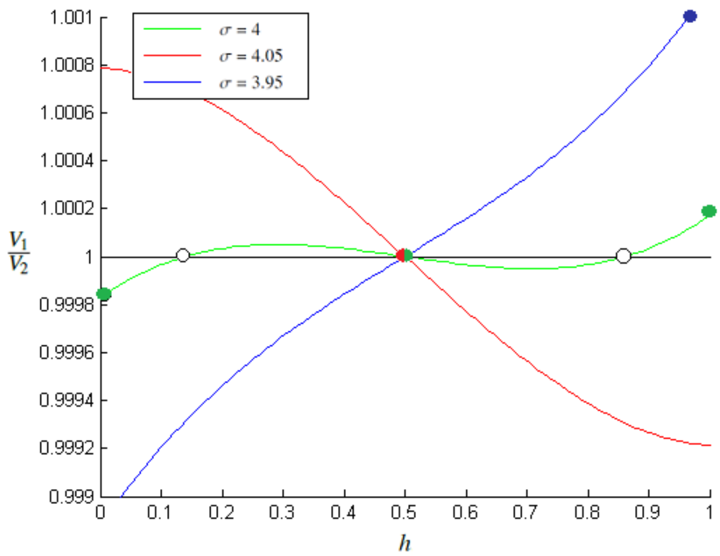
Figure 4: Ratio of utilities over $h$, for $\tau = 1.125$, for small perturbations on $\rho$
$\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$
Full circles represent stable equilibria and empty circles denote instability.




The results, displaying only the effects of small variation in both directions from the Cobb-Douglas case, highlight the shift from an agglomeration equilibrium for $\rho = 0.005$, up to a unique dispersion equilibrium for $\rho = -0.005$, experiencing a situation of multiple equilibria in $\rho \to 0$, a special situation already discussed in Section 3. Although with very small differences on the ratio of indirect utilities, the purpose of the figure is to expose that we do not need large variations of $\rho$ to see major changes concerning equilibrium patterns.

Obviously, we chose a value of transportation costs that would enable us to make this inference, and, no less true, is the fact that we can repeat the exercise for any parameter in the model. Similar results are obtained for variations in $\sigma$ as illustrated next. Keeping with $\tau = 1.125$ and $\rho \to 0$, but shifting the focus to a disturbance in the elasticity of substitution between industrial varieties $\sigma$, we encounter a resembling figure.

Figure 5: Ratio of utilities over $h$, for $\tau = 1.125$, for small perturbations on $\sigma$
$\rho \to 0$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$
Full circles represent stable equilibria and empty circles denote instability.



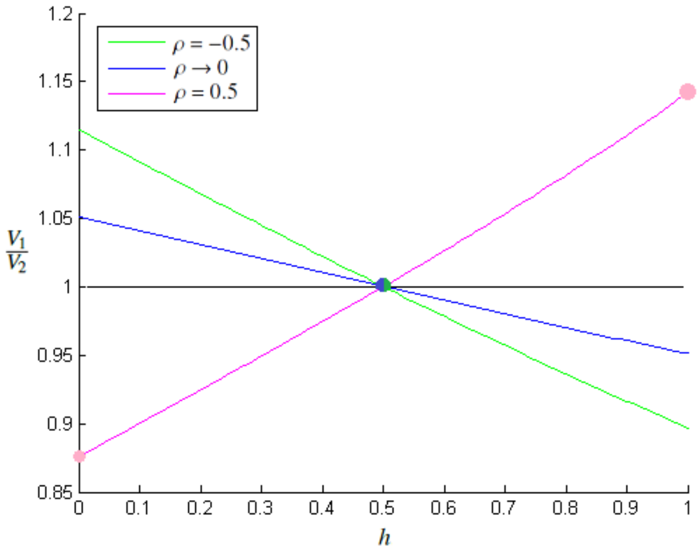

For the initial $\sigma = 4$, we continue to face multiple equilibria, but now the variation of this parameter has the opposite effect with respect to the last figure: an agglomeration equilibrium for a decrease in $\sigma$, and a dispersion state for $\sigma = 4.05$.

Nevertheless, it is obvious that this tells us very little in regard to the model's dynamics, wherein each individual parameter has a different behavior, and where some of them are not strictly monotonic in their interval. Estimating the impact of the elasticity of substitution in the evolution of spatial patterns of the economy is also not an easy task, since it modifies part of the solid framework of the original model, specially the settings of the share of expenditure in industrial goods. Accordingly, we need to be especially cautious on separating effects in order to make more forceful conclusions, but bearing in mind that we have a question that is answered only by cross interaction of multiple parameters.

## 4.1 Elasticity of substitution between agricultural and industrial goods

At this point, it becomes essential to study the elasticity of substitution when it assumes a value other than zero. In this context, we will examine three distinct values of $\rho$, based on the already used setup, $a = 1$, $b = 0.1$, $H = 100$, $\sigma = 4$, $\mu = 0.3$, and finally, $\tau = 1.2$. [5]

Figure 6: Ratio of utilities over $h$, for $\rho \to 0$, $\rho = -0.5$ and $\rho = 0.5$
$\tau = 1.2$, $\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$
Full circles represent stable equilibria.




In Figure 6, which again opposes the ratio of indirect utilities to the share of industry in region 1, we give two opposite values of $\rho$ in addition to the Cobb-Douglas solution, that presents an unique dispersion equilibrium given the downward slope. In $\rho = -0.5$, we continue to observe a decreasing trajectory, with an even steeper slope, moving further away from the possibility of agglomeration. At the other end we have $\rho = 0.5$, which leads to an agglomeration equilibrium, inducing $h = 0$ whenever $h < \frac{1}{2}$ and $h = 1$ when $h > \frac{1}{2}$. From plain observation, we could say that the decrease in the parameter promotes dispersion, while its increase encourages agglomeration, but is the effect the same (in terms of direction and intensity) for any variation? The best way to exhibit the effect of $\rho$ throughout its range, on the stability of agglomeration, is by setting $h = 1$ and evaluating $\frac{V_1}{V_2}$.

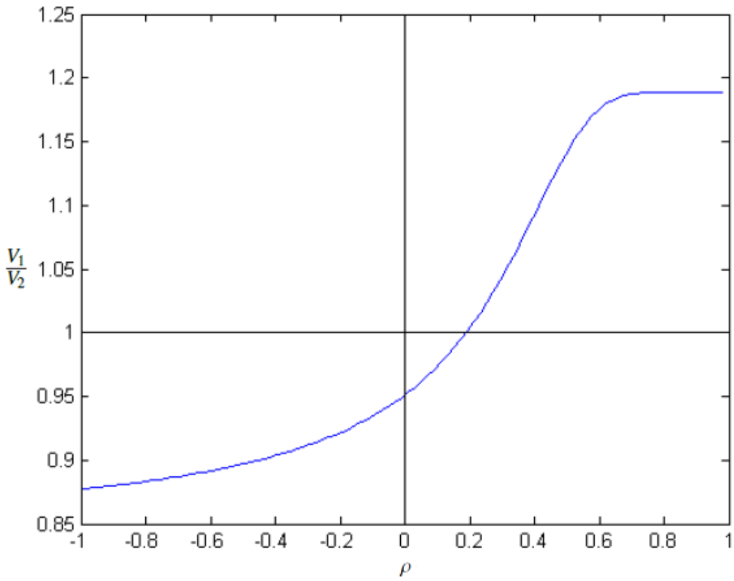
If we assume that industry is all concentrated in region 1, whenever the ratio of utilities is above one ($\frac{V_1}{V_2} > 1$), we encounter an agglomeration equilibrium. The major advantage of setting $h = 1$, thus, is the ability to visualize, for a wide range of a parameter, which values support a concentration of industry in a core-periphery economy. Moreover,

[5]The discussion on other parameters' implications are organized in the next sections.

placing multiple functions in the same figure allows us to study the joint effect of two indicators (one being on the axis, the other resulting in different functions).

When the ratio of utilities is below unity no conclusion can be made on the stability of dispersion. The condition for a dispersion equilibrium is $\frac{\partial V_1}{\partial h} < 0$ for $h = \frac{1}{2}$, so, as we set $h = 1$, we do not have the certainty of what happens if the share of skilled workers in region 1 is lower. Therefore, the next subsections, by using this type of tests, are more focused in the study of agglomeration patterns. Nevertheless, most importantly than only observing if the ratio of utilities is above or below the unity, the effects of changes in the elasticity of substitution can be structured into the forces directed towards agglomeration and the forces that originate the dispersion of industry. Figure 7 illustrates the ratio between indirect utilities over $\rho \in (-1,1)$:

Figure 7: Ratio of utilities over $\rho$, for $h = 1$
$\tau = 1.2$, $\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$




Based on our setup, starting on $\rho = -1$, all values up to $\rho \approx 0.2$ lie beneath the condition $\frac{V_1}{V_2} > 1$: agglomeration is not an equilibrium. Past this point, throughout the range of positive values of $\rho$, a pattern of agglomeration in region 1 emerges. An important note is that we are facing a monotonic non-decreasing function, which translates into an increase (or unaltered for high values of $\rho$) of $\frac{V_1}{V_2}$ for every increase of the elasticity of substitution. Therefore, a raise in the elasticity of substitution always works in favor of agglomeration, whereas a decrease works in favor of dispersion. In terms of slope, this starts to slowly increase, reaching a maximum between $0 < \rho < 0.6$, stabilizing from $\rho = 0.7$ onwards. This makes us infer, from the point of view of a Cobb-Douglas benchmark, that the increase of the elasticity of substitution for positive values has a higher proportional effect in the ratio of utilities than its decrease to negative values.

Still, it would be important to examine, within the ratio of utilities' expression, which variables or parameters are more sensitive to the elasticity of substitution. Recovering expressions (20) and (22), the ratio between skilled wages and the ratio of utilities, respectively:

$$\frac{w_1}{w_2} = \frac{\phi h(\bar{\mu}_1 + \bar{\mu}_2) + \left[\bar{\mu}_1 - \frac{\bar{\mu}_1\bar{\mu}_2}{\sigma} + (1 + \frac{\bar{\mu}_1}{\sigma})\bar{\mu}_2\phi^2\right](1-h)}{\phi(1-h)(\bar{\mu}_1 + \bar{\mu}_2) + \left[\bar{\mu}_2 - \frac{\bar{\mu}_1\bar{\mu}_2}{\sigma} + (1 + \frac{\bar{\mu}_2}{\sigma})\bar{\mu}_1\phi^2\right]h}$$

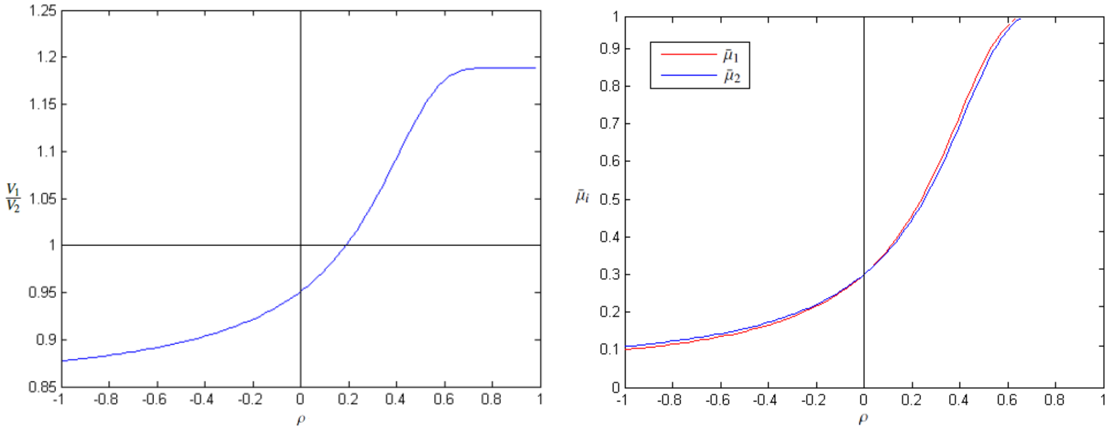
$$\frac{V_1}{V_2} = \frac{P_2}{P_1}\left(\frac{\bar{\mu}_2}{\bar{\mu}_1}\right)^{\frac{1-\rho}{\rho}}\frac{w_1}{w_2}$$

The functions, in relation to Forslid & Ottaviano (2003), appear very similar (the prices indices being exactly the same as before). The only difference is the variable $\bar{\mu}_i$, $i = \{1,2\}$, the share of expenditure in industrial goods, that ceases to be a constant and becomes different between regions. The function is displayed on expression (3):

$$\bar{\mu}_i = \frac{1}{1 + (\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}}$$

Moreover, the results of this model for a Cobb-Douglas utility function are recovered when $\rho$ tends to zero, which makes $\bar{\mu}_i = \mu$. Then, if everything else seems equal to the original framework, understanding the effect of the elasticity of substitution on the ratio of utilities, and therefore, on equilibrium patterns of spatial distribution of industry, must lie on $\bar{\mu}_i$ settings. To this end, Figure 8 shows, in addition to the ratio of utilities, also $\bar{\mu}_1$ and $\bar{\mu}_2$ over $\rho$.

Figure 8: Ratio of utilities over $\rho$ + $\bar{\mu}_1$ and $\bar{\mu}_2$ over $\rho$, for $h = 1$
$\tau = 1.2$, $\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$




The two figures display, unequivocally, the same trend. It can also expose, given the restriction on the share of expenditure on industrial goods $\bar{\mu}_i \in (0,1)$, why the ratio of utilities stagnates from a certain value of the elasticity of substitution. Given the evidence, we can assume that all effects generated by the change of the utility function summed up to the analysis of $\bar{\mu}_i$ according to $\rho$. However, $\bar{\mu}_1$ is not exactly equal to $\bar{\mu}_2$. Setting $h = 1$, the price index of the core region is lower than the one in the periphery, which makes $\bar{\mu}_1$ greater than $\bar{\mu}_2$ for positive values of the elasticity of substitution, with the opposite occurring when $\rho < 0$.

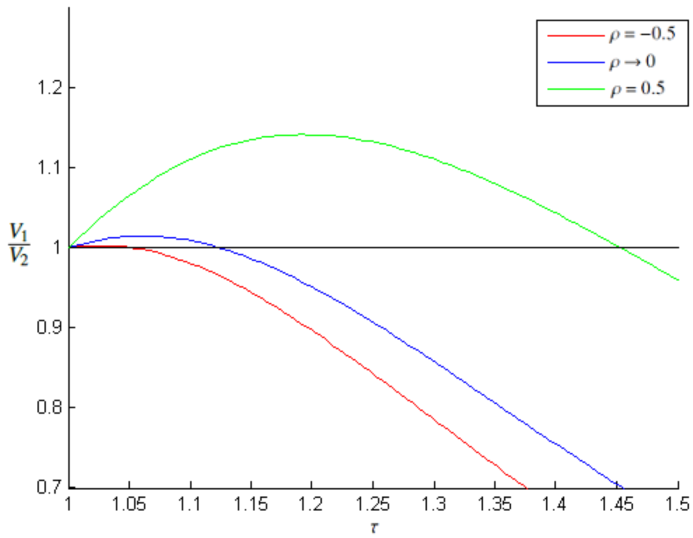
More significantly, it explains the origin of the improvement on the ratio of utilities in favor of region 1, for positive values of $\rho$, which works as an agglomeration force. As $\bar{\mu}_1 > \bar{\mu}_2$, workers from the core region benefit from a higher consumption of industrial goods, compared to the periphery, where, in addition to a lower consumption of industrial goods, the same are subject to shipping costs. When $\rho < 0$ and the share of expenditure drops strongly in region 1, the peripheral population increases their utility in relation to the center by consuming a greater percentage of industrial goods, even in the presence of the same transportation costs. Indeed, transportation costs clarify why the ratio of utilities increases more proportionately for $\rho > 0$, than its decrease for $\rho > 0$. In $h = 1$, industrial goods will always be cheaper in region 1, which intensifies the differences when the percentage of expenditure is higher in this region, and mitigates them when $\bar{\mu}_1 < \bar{\mu}_2$.

On a final note, $\bar{\mu}_i$ is in line with the role of $\mu$ in basic core-periphery models, which associate agglomeration states to dominant industry sectors. However, the elasticity of substitution, which makes agricultural and industrial goods complements or substitutes, causes the analysis to be different from the rest of the literature. Accordingly, the figure will be resumed, in Section 5, to an economic interpretation analysis.

## 4.2 The transportation costs

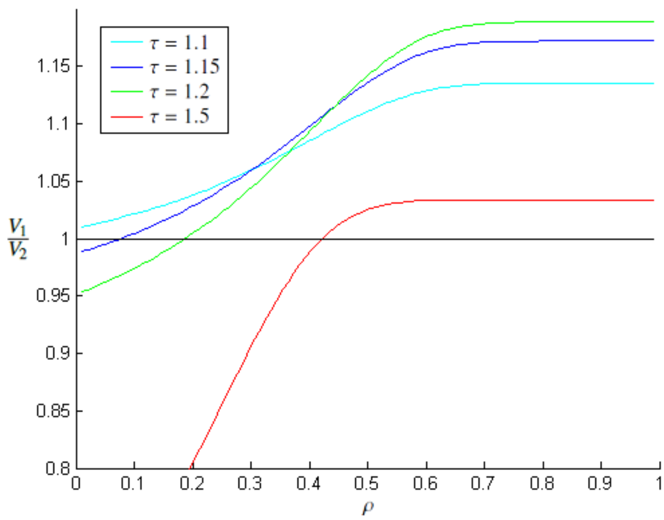
Much of the discussion on transport costs has been clarified in Section 3. In order to promote agglomeration, the transportation costs have to be moderate, while high transport costs lead to a dispersion equilibrium. A CES utility function will not change this conclusion, but it contains some interesting points, that we will again show by setting a core-periphery economy ($h = 1$), in order to vary another parameter in one of the figure axes, in this case, $\tau$.

Figure 9: Ratio of utilities over $\tau$, for $h = 1$, for different $\rho$
$\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$




The graph shows the evolution of the ratio of indirect utilities to a range of transportation costs, $\tau \in (1, 1.5)$, and also for different values of $\rho$. Starting from the Cobb-Douglas case, an increase in the elasticity of substitution, besides rising the $\frac{V_1}{V_2}$ for any level of cost, extends the range of values of $\tau$ supporting an agglomeration equilibrium, given the condition $\frac{V_1}{V_2} > 1$ ($h = 1$). To some extent, the way we define transportation costs might change, if we think of low costs as a promoting factor of an agglomeration economy and high transport costs as a feature of a dispersion equilibrium. For example, when $\rho = 0.5$, $\tau = 1.2$ is a relatively low transportation cost, as it allows the industry to concentrate on only one region, but, when $\rho \to 0$, the ratio of utilities is below one, which means that, using the same reasoning, $\tau = 1.2$ should be regarded as a high transportation cost. The next figure, in addition to providing the same conclusion, offers another characteristic of the interaction of transport costs with the elasticity of substitution that is being studied.

Figure 10: Ratio of utilities over $\rho$, for $h = 1$, for different $\tau$
$\sigma = 4$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$



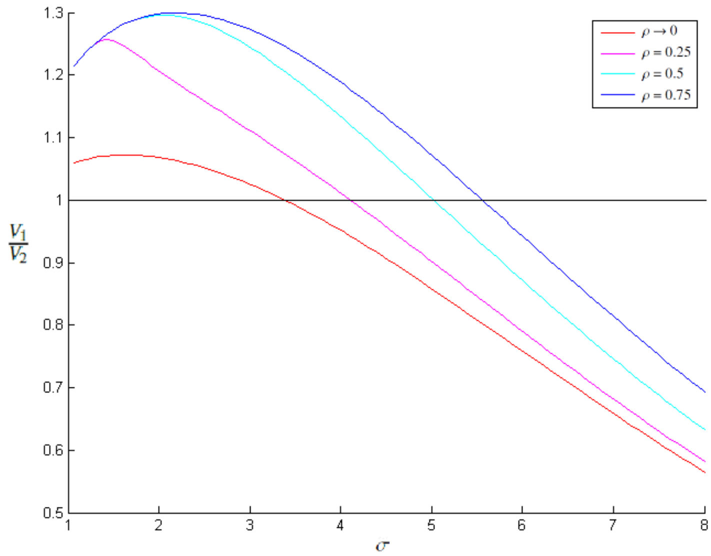

Here, the horizontal axis represents $\rho$ and $\frac{V_1}{V_2}$ is flotted for different values of $\tau$, the reverse of the latter figure. While at $\rho = 0.5$ (and throughout most of the positive range of $\rho$), all drawn functions of $\tau$ are above $\frac{V_1}{V_2} = 1$ ($h = 1$), for $\rho \to 0$ (Cobb-Douglas), only the lowest, $\tau = 1.1$, is a transportation cost compatible with agglomeration. Also, the effect of $\rho$ on the ratio of utilities for relatively low values of transportation costs is rather curious. For instance, $\frac{V_1}{V_2}$ for $\tau = 1.2$, at Cobb-Douglas, is below one and is the lowest in relation to all low transport costs cases represented. Yet, throughout $\rho$, the ratio of utilities eventually overcomes, first, $\tau = 1.1$ (which had already been surpassed by $\tau = 1.15$), and then the latter. This displays that for a high elasticity of substitution, the transportation cost that maximizes the ratio of utilities is a value of $\tau$ that might be considered a high transportation cost in the original model, which reinforces what we stated previously.

Finally, one can also say that the increase of $\rho$ has the effect to smooth out the gap between ratios of utilities, when exhibiting significantly different values of transportation costs. For instance, between $\tau = 1.1$ and $\tau = 1.5$, at Cobb Douglas, the differences between ratios (that even escapes the figure's limits on the case of $\tau = 1.5$, but which is set on $\frac{V_1}{V_2} \approx 0.66$), are wider. As $\rho$ grows, the function with higher transportation costs grows at a faster rate than $\tau = 1.1$ (and all the others function represented), at least up to the stagnation that all functions show for high values of the elasticity of substitution. In summary, what positive values of $\rho$ adds in terms of transportation costs, besides raising the range of $\tau$ that tend to an agglomeration equilibrium, it also removes some sensitivity to large variations in costs: for an increasing $\rho$, the gap between two ratios of utilities, which have distinct transportation costs, reduces, compared to the situation at Cobb-Douglas, where the difference is larger.

## 4.3 Preference for variety

In the introduction of the current section, we have already presented graphically, but briefly, the effect of the elasticity of substitution between industrial varieties ($\sigma$). Namely, it was shown, in a multiple equilibria state, that an increase of $\sigma$, from 4 to 4.05, diminishes the ratio of utilities for $h > \frac{1}{2}$ (leading to a dispersing equilibrium), and a decrease to $\sigma = 3.95$ leads, again when region 1 holds more than half of skilled workers, to a growth of the ratio $\frac{V_1}{V_2}$, favoring an agglomeration output. Nevertheless, a decrease in $\sigma$ does not always lead to an increased ratio in favor of region 1. Figure 11 shows, again for $h = 1$ and $\tau = 1.2$, the ratio of utilities against the elasticity of substitution between industrial varieties, $\sigma > 1$. In order to also study the joint effect with the elasticity of substitution between agricultural and industrial goods, we considered several values of $\rho$.

Figure 11: Ratio of utilities over $\sigma$, for $h = 1$, for different $\rho$
$\tau = 1.2$, $\mu = 0.3$, $\alpha = 1$, $\beta = 0.1$ and $H = 100$




Observing the figure, we can assess that, for any of the represented $\rho$ values, the value of the ratio increases slightly for low values of $\sigma$, followed by a long decline phase for medium and high $\sigma$. Recovering Krugman (1991, p.490), $\frac{\sigma}{\sigma-1}$ is the ratio of the marginal product of labor to its average product, that is, the degree of economies of scale. Then it makes sense, given that economies of scale are associated with low values of $\sigma$, that in a situation where all the entrepreneurs are concentrated, the ratio of utilities in favor of this region is bigger for low values of the elasticity of substitution between industrial varieties.

A rise of the elasticity of substitution between agricultural and industrial goods, as already discussed, increases the ratio of utilities, and here we come to the same conclusion.

Only be noted that, excluding Cobb-Douglas, the functions depart from the same point of origin, overlap for sufficiently low values of $\sigma$, and then diverge according to the value of $\rho$.

In terms of equilibria, all the functions represented present the agglomeration equilibrium for low values of $\sigma$. Yet, for example, on Krugman's benchmark, $\sigma = 4$, a Cobb-Douglas utility function has a ratio of utilities lower than unity. The other represented values, all positive values of the elasticity of substitution between agricultural and industrial goods, by opposite, are above $\frac{V_1}{V_2} = 1$ at $h = 1$.

## 4.4 The price index

The impact of $\sigma$ is not limited to the description of the latter subsection. Here are the price indices of both regions, expressions (14) and (15), identical to a model that assumes Cobb-Douglas preferences:

$$P_1 = \frac{\beta\sigma}{\sigma - 1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}} \left[h + \phi(1-h)\right]^{\frac{1}{1-\sigma}}$$

$$P_2 = \frac{\beta\sigma}{\sigma - 1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}} \left[1 - h + \phi h\right]^{\frac{1}{1-\sigma}}$$

At Krugman (1991) or Forslid & Ottaviano (2003), the price index is represented in the ratio of indirect utilities by $\frac{P_2}{P_1}$, which ultimately nullifies the first part of the function. In our model, we continue to behold $\frac{P_2}{P_1}$, but now each price index generates effects on the share of expenditure on industrial goods in the respective region $\bar{\mu}_i$, $i = 1,2$. So, what's the relevance of $\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}$?

First of all, recall that $\alpha$ is the fixed cost of producing a variety of manufactures and $\beta$ is the marginal cost of producing each unit, while $H$ is the total number of skilled workers in the economy. Skilled labour market clearing implies that, in equilibrium, $\frac{H}{\alpha}$ is the number of firms (expression (10)), that, in turn, is affected by a measure of economies of scale, $\frac{1}{1-\sigma}$. Also, expression $\frac{\beta\sigma}{\sigma-1}$ is equivalent to the price of industrial goods consumed locally (expression (12)). Therefore, it represents the industry's market structure, the production costs, and given that there are no profits in equilibrium, the prices. The rest of the price index's expression is relative to the percentage of industrial goods that needs to be imported from the other region, which differentiates the price index between regions.

What makes the price index relevant in this model is the relationship between the agricultural and industrial sectors, making its goods substitutes or complements. Given our setup, *i.e.,* $\tau = 1.2$, $\mu = 0.3$, $\sigma = 4$, $\alpha = 1$, $\beta = 0.1$, $h = 1$ and $H = 100$, the contrast of prices are illustrated below:

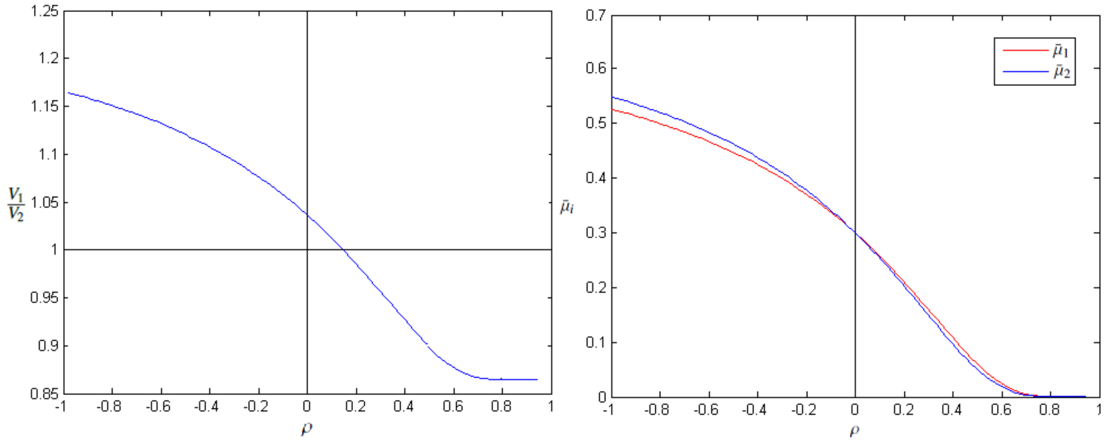
$$P_1 = 0.0287 < P_2 = 0.0345 < \; p_1^A = p_2^A = 1$$

The main implications of these differences are straightforward: When goods become substitutes, agents will exchange a portion of income that was previously intended to agricultural goods, to the consumption of a greater number of industrial ones. In the presence of complementarity (as they are preferably consumed together) the higher price of agricultural goods causes the share of expenditure for this type of goods to increase, in relation to a Cobb-Douglas situation, where the percentage was independent of prices.

This largely explains the results of the simulations presented before. Recall, from Section 4.1, that an increase in the elasticity of substitution (when $h = 1$), raises the share of expenditure on industrial goods, which consequently increases the ratio of utilities in favor of region 1, and finally, in favor of agglomeration. The percentage of industrial goods in consumption grows as a result of a greater demand, whose price is significantly lower.

Mindful of this aspect, it is important to analyze what would happen in the opposite case, that is, when the price of the agricultural good is lower than the price of manufactures. It is expected that an increase in the elasticity of substitution decreases the percentage of expenditure on industrial products, since agents will choose to consume, from the same amount of income, a higher volume of agricultural goods. Figure 12, to this purpose, displays the share of expenditure on industrial goods in both regions over $\rho$, when $P_i > p^A$. In order to achieve a substantial difference, in the same way as in the original setup, we increased $\beta$, the marginal cost of an industrial variety, to a value of 10, but we could do the same for parameters $\alpha$ and $H$.

Figure 12: Ratio of utilities over $\rho$ + $\bar{\mu}_1$ and $\bar{\mu}_2$ over $\rho$, for $h = 1$ and $\beta = 10$
$\tau = 1.2$, $\mu = 0.3$, $\alpha = 1$ and $H = 100$




The results are as expected. Positive values of the elasticity of substitution decrease the share of expenditure on industrial goods, which is beneficial to workers of the peripheral region, as $\bar{\mu}_1 > \bar{\mu}_2$. Even in presence of transportation costs, that makes the consumption of manufactured goods more expensive in relation to region 1, the ratio of utilities suffers the same trend. Hence, in this scenario, an increase of $\rho$ constitutes a dispersion force.

The opposite case, when $\rho$ is negative, the share of expenditure on industrial goods increases less in the center. The utilities of both regions decreases, but the ratio increases in favor of region 1 as they consume a larger share of agricultural goods.

# 5 Economic interpretation

This dissertation would be incomplete if we did not extrapolate information allowing us to understand the economic mechanisms behind the various changes in the ratio of utilities discussed throughout the simulations. In this sense, we will first recall the known centrifugal and centripetal forces, along with a reflection on the same under the new conditions of this model.

**The effects**

Already introduced in the literature review of this work, it is important to reintroduce the effects, from the point of view of the agents' motivation, which drive the mechanism of this model. On the basis of the new economic geography literature, the effects are divided among forces that foster agglomeration and those that tend to disperse industry across regions. First, the market size effect, in which firms intend to agglomerate in order to constitute a larger market, each benefiting from a greater individual demand. This is a example of the theory of circular causation of Myrdal (1957), where the increase of the market (and demand) and the concentration of industry are cause and consequence of each other. Second, in the same direction, there is the cost of living effect, a forward-linkage, which results in a lower price index as more firms join in the region, being favorable from the consumer point of view. Given a basket of goods, a greater variety of locally available goods reduces the percentage that has to be imported, subject to transportation costs. Finally, on the side of geographic dispersion, the market crowding effect. The intense competition in an area of high concentration, which, as mentioned, lowers the price index, causes a negative impact on industrial firms. Thus, firms see the shift to the peripheral region as an opportunity to increase their demand, where they possess a competitive advantage relatively to firms located at the center.

In Krugman's model (1991), the agricultural sector was not that important in the spatial distribution of the industry, since the share of expenditure on each sector was constant and given *ex-ante*. Now, the elasticity of substitution, turning agricultural goods substitutes or complements to industrial ones, causes the share of expenditure on each sector to be endogenous, and therefore, the agriculture to be equally significant in the study of equilibrium patterns. The abovementioned effects need, therefore, to be adapted to the conditions in this model. So, what does the elasticity of substitution changes?

## 5.1 Elasticity of substitution

First, to clarify, when $\rho \to 0$, a Cobb-Douglas production function, the share of expenditure for the two goods is independent: when there are changes on industrial prices, for example, individuals only adjust their demand of this type of goods, not changing their consumption of agricultural ones. Yet, as $\rho$ increases, substitutability between the agricultural and industrial goods enhances, being perfect substitutes at $\rho = 1$. Here, the percentage of expenditure on industrial goods follows an inverse relationship to the price: a rise in manufactured goods' prices reduces the consumption of the same, as they can be replaced with agricultural goods. On the other hand, as $\rho$ reduces, the complementarity increases, reaching perfect complements at $\rho \to -\infty$, a Leontief production function. In this case, the share of expenditure of industrial goods grows in relation to its price: in order to consume the goods together, the agent chooses rather to reduce the percentage of expenditure on agricultural goods.

So, recovering Figures 6 and 7, what is the economic interpretation of the increase of the ratio of utilities with the elasticity of substitution, at $h = 1$, especially in the positive range of $\rho$? We already know, from Section 4.1, that the increase in the elasticity of substitution intensifies the share of expenditure on industrial goods in region 1. This is caused by price differences, as seen in Section 4.4, which lead to the exchange of agricultural goods for industrial ones.

This generates a greater demand for industrial goods, causing, obviously, an increase on firm's demand. Despite not increasing in terms of population or firms, the market-size effect grows as consumers replace part of their consumption of agricultural goods for industrial goods. Nevertheless, in terms of market crowding effect, for the same reasons, the incentive for dispersion also enhances. As the market-size in the core region increases, in the sense that the elasticity of substitution reveals a greater demand for industrial goods, the same happens in the peripheral region. The following we know from the literature: in the absence of local competition, a company has the temptation to deviate in order to gain a competitive advantage and serve the peripheral population more accessible. Yet, we also observed that this increase in demand is more felt in region 1, since $\rho > 0$, makes $\bar{\mu}_1 > \bar{\mu}_2$. Therefore, as the increase of the demand, which is correlated with the market, is larger in the core region, we can infer, for positive elasticities of substitution, that the market-size exceeds the market-crowding effect.

A similar impact occurs, *ceteris paribus*, in the cost-of-living effect. On the one hand, as we are examining the effects from a static point of view, given $h = 1$, changes on $\rho$ do not affect the market structure, the spatial distribution of industry and, from there, the price indices. But, on the other hand, the substitution between agricultural and industrial goods initiates a price competition between sectors, where the consumption of agricultural goods is considerably more expensive. Hence, the substitution of a further part of expenditure for industrial products (*i.e.* an increase in $\bar{\mu}_i$) raises the amount of goods that each consumer may acquire given its income, reducing their cost-of-living. Note that the differences in prices are so wide, that even the workers of the peripheral region, which only produce the agricultural good, benefit from the substitution between sectors, which increases their percentage of expenditure on industrial goods, $\bar{\mu}_2$. However, due to transportation costs, the region's price index is higher, which makes, in comparison, the improvement of living conditions more felt in the core region. Again, we can assert that positive values of $\rho$ favor agglomeration, since the cost-of-living effect improves proportionally more for workers located in the core region. Therefore, for this setup, the net result of the increase in the elasticity of substitution on these three forces works in support of agglomeration, as the ratio of utilities improves in favor of the concentrated region.

A decrease in the elasticity of substitution for negative values makes agricultural and industrial goods, in a certain level, become complements. As previously explained, since the goods are preferably consumed together, the percentage of expenditure on industrial (and agricultural) goods has a positive relationship with its price. Since agricultural goods are more costly, the percentage of expenditure on industrial goods decreases with the drop of $\rho$. As a consequence, the demand for industrial goods decreases, damaging the market-size effect. Also, the decrease in $\bar{\mu}_1$ is followed by a similar one in region 2, that, by reducing the demand for industrial goods, relieves the incentive to deviate to the other region, lowering the market-crowding effect. Meanwhile, the peripheral region has a higher price index, which makes $\bar{\mu}_2$ also bigger. In the end, the market crowding prevails over the market size effect, because the decrease in demand for industrial goods is deeper in the central region, which does not act in favor of an agglomeration pattern. Yet, the effect of the elasticity of substitution on the cost-of-living is reserve of the one mentioned above. Industrial goods' prices, due to transportation costs, are always lower in the core region, which translates the effect of $\rho$ on cost-of-living always as an agglomeration force.

In this model, the prices are so significantly distinct, that even with transportation costs, the largest consumption of industrial goods in the peripheral region allows an improvement of its consumers' utility in relation to the core, acting on behalf of a dispersion of skilled workers. Otherwise, if the cost-of-living effect exceeded the difference in demand between the periphery and the core (the combined effect of market-crowding and market-size effects), such as for $\rho > 0$, it is expected that negative values of the elasticity of substitution would also be revealed as a force of agglomeration.

At this point, some conclusions can be made regarding the impact of the elasticity of substitution in the effects behind the behavior of agents. From this first example, we realized that the market-size and the market-crowding effects, for changes on preferences, should be studied together, given that the share of expenditure on manufactured (and agricultural) goods moves in the same direction for both regions. The differences between $\bar{\mu}_1$ and $\bar{\mu}_2$, that change the industrial demands in different magnitudes, are focused on the respective price indices, which dates back to transportation costs that the peripheral region has to bear. The same parameter also determines the differences in the cost-of-living effect, however, contrary to the other effects, is has only one direction. Industrial goods will always be comparatively more affordable in the core region, thus, any change in the elasticity of substitution works as a centripetal force.

## 5.2 The price index

Section 4.4 highlighted the importance of price indices in the distribution of the industry, explaining that the differences between the prices of agricultural and industrial goods was the factor that triggered the above findings. On the contrary, when agricultural goods are a cheaper alternative to industrial ones, the evidence shows us something quite different.

Starting from $\rho > 0$, the elasticity of substitution has the effect of decreasing the ratio of utilities, discouraging agglomeration. This is because agents switch now part of their consumption of industrial goods for agricultural ones, reducing the share of expenditure on manufactured goods in both regions. Given the differences between price indices, workers from the core region, would suffer more by consuming a higher percentage of industrial goods. Indeed, in a somewhat paradoxical way, the fact that the market size exceeds the market-crowding effect, works opposed to the maintenance of an agglomeration state, for the preference of skilled workers, not as firms, but as consumers, to move to the other region in order to increase the consumption of agricultural goods. This is another case where the high price differences overrule the final result.

Finally, when the goods are complements, the percentage of expenditure on industrial goods increases more in the peripheral region. Thus, even if in the traditional definition of market crowding effect the incentive for a deviation has increased (proportionally more than the local market), workers in the center realize that this effect does not testify towards a dispersion equilibrium, as they will spend, in addition to a higher price caused by transportation costs, a greater percentage of income on industrial goods, which makes their utility diminishing.

This analysis turns out to change even more the perception of the effects discussed by the new economic geography literature. Skilled workers, with changes in preference between agricultural and industrial goods, mix the professional with the consumer perspective. With agricultural goods comparatively more affordable, they go so far as to sacrifice the best decision from the firms' point of view in order to reduce their cost of living. That is, while better living conditions in the core region always translate as a force of agglomeration, a higher proportional increase in market size in relation to market-crowding effect, or in this case, a lower proportional decrease, that should have the same direction, is reflected sometimes as a dispersion force, such is the preference for agricultural goods.

# 6 Conclusions

The conclusions of this work should start for the reasons that make it unique. In this model, the agricultural and industrial goods share a connection that did not previously exist. At Cobb-Douglas, the share of expenditure on each type of goods was constant and exogenous. Hence, the cost structure of manufacturing firms was not relevant to the analysis of skilled workers' spatial distribution. That being said, the effects of introducing $\rho$, causing substitution or complementarity among agricultural and industrial goods, must include a reflection of the price index, and more specifically, in the price competitiveness between the two sectors of activity.

As an initial setup, much of the simulation conducted in this thesis was based on the assumption that the price of industrial goods, essentially by assuming low marginal costs, would be considerably lower than agricultural ones, selected as *numéraire.* Thus, when goods become substitutes, the agents' rationality would cause an exchange of consumption of agricultural goods for industrial varieties. Otherwise, when the elasticity of substitution is negative, the percentage of expenditure on industrial goods would decrease. This happens because, in order to be consumed together, the higher relative price of agricultural goods turns its percentage of expenditure also bigger, consequently, reducing the demand for industrial goods.

Nevertheless, the impact of $\rho$ on the share of expenditure on industrial goods is not the same in the two regions, by the difference between price indices. On a core-periphery state, the price index in the center is lower than the one recorded in the periphery, where industrial goods are subject to transportation costs. When $\rho$ is positive, the lower price index causes the share of expenditure on industrial products to be greater in the core. On the other hand, when the goods are complements, the percentage of expenditure on industrial goods features a positive relation to the price index, which makes the consumption of these goods by each individual larger in the periphery.

So, what conclusions can we draw from the elasticity of substitution in relation to agglomeration and dispersion equilibria? Evidently, it depends on the set of selected parameters, but at least we can classify, according to $\rho$ (and prices), the effect as promoter of concentration or dispersion of industry, *i.e.,* in an additional centripetal or centrifugal force.

When the elasticity of substitution is positive, as mentioned, the region in which all the entrepreneurs are concentrated increases the percentage of expenditure on industrial goods more than the periphery, which makes the center, in addition to being already the region with the largest number of workers, the one which has a bigger demand for industrial goods *per* consumer. In this case, the increase of the elasticity of substitution raises the ratio of utilities in favor of region 1, given that the consumption of industrial goods grows both in absolute and relative terms more proportionately in the core than in the periphery, which ultimately ends up acting in favor of agglomeration.

On the other hand, with negative values of the elasticity of substitution, a higher percentage of expenditure on industrial goods in the peripheral region increases a worker's utility in this region compared to the center. It is important to reiterate that this conclusion is valid because the differences between the prices of agricultural and industrial goods are so large that the higher price paid by industrial goods in the peripheral region does not reach the effect of the differences between the share of industrial goods in consumption. At the end, the peripheral region consumes a greater number of goods, encouraging therefore, the dispersion of skilled workers.

Afterwards, we studied the impact of $\rho$ if the prices of agricultural goods were considerably lower than the manufactured goods. When the elasticity of substitution is positive, consumers switch part of their consumption of industrial goods for agricultural ones, reducing the percentage of expenditure on industrial goods in both regions. Given the differences in price indices, workers from the core region suffer more by consuming a higher share of expenditure on industrial goods, although having a better cost-of-living.

Finally, when the goods are complements, the share of expenditure on industrial goods increases in the two regions, which decreases both the utilities, yet more in the peripheral region. Thus, with the price of agricultural goods more affordable, skilled workers concentrated in the core, perceive that a greater demand in the peripheral region does not testify in the direction of a possible deviation, as they will spend, in addition to transportation costs, a greater percentage of their income on industrial goods.

It is interesting to analyze that, when agricultural goods are more competitive, the market-size effect (the same being for the market-crowding effect) operates inversely to the way that is studied in the literature. The higher the percentage of expenditure on industrial products, the greater the size of the market. In the presence of a cheaper alternative good, in this case, the agricultural one, skilled workers, from a consumer position, wish to move to the region where the demand of these type of goods is stronger, which is the region with a lower demand for industrial goods.

Table 1 summarizes the impact of the elasticity of substitution, according to the pricing structure of the two sectors, on the ratio of utilities in relation to the Cobb-Douglas case, and hence, regarding the effect as a centripetal or centrifugal force.

Table 1: Effects of $\rho$, according to prices

$P_i$ is equal to the price index in region $i$ = 1,2, whereas $p^A$ is the price of the agricultural good

| $h = 1$ | $P_i \ll p^A$ | $P_i \gg p^A$ |
|---|---|---|
| $\rho > 0$ | Agglomeration Force | Dispersion Force |
| $\rho < 0$ | Dispersion Force | Agglomeration Force |

A final thought for future research consists on the investigation of the model's results when the two sectors are equally competitive. By having studied the impact of $\rho$ on market-size, cost-of-living and market-crowding effects, we think that, when revealing similar prices, any change in the elasticity of substitution, in relation to the Cobb-Douglas benchmark, should raise the ratio of utilities in favor of the core, and thus, should act on behalf of agglomeration. If this assumption is right, this could be explained by the cost-of-living effect that, regardless of whether the goods are substitutes or complements, tells us that it is more pleasant to live in the center because the industrial goods are cheaper. So, in a core-periphery economy, it may even exist a larger percentage of income intended at the cheaper good in the peripheral region, which creates incentives to the deviation of skilled workers. However, we suppose that this latter effect would not be offset by the differences of living conditions between the periphery and the center, revealing the crucial role of iceberg costs, one of the pillars of the new economic geography literature.

# Appendix A

## Derivation of expression (3):

Given the CES utility function (1), let us maximize with respect to $x_i$, subject to $P_i x_i + P_A a_i = y$, $(P_A = 1)$ (8). So, replacing $a_i = y - x_i P_i$ in the function, we obtain an unconstrained maximization of $U_i = \mu x_i^{\rho} + (1-\mu)(y - x_i P_i)^{\rho}$. The first order condition results in the following steps:

$$
\begin{aligned}
&\frac{dU_i}{dx_i} = 0 \Leftrightarrow \\
\Leftrightarrow &\mu x_i^{\rho-1} + (1-\mu)(y - P_i x_i)^{\rho-1}(-P_i) = 0 \Leftrightarrow \\
\Leftrightarrow &\mu^{\frac{1}{\rho-1}} x_i = (1-\mu)^{\frac{1}{\rho-1}} P_i^{\frac{1}{\rho-1}} (y - P_i x_i) \Leftrightarrow \\
\Leftrightarrow &x_i(\mu^{\frac{1}{\rho-1}} + P_i^{\frac{\rho}{\rho-1}}(1-\mu)^{\frac{1}{\rho-1}}) = (1-\mu)^{\frac{1}{\rho-1}} P_i^{\frac{1}{\rho-1}} y \Leftrightarrow \\
\Leftrightarrow &x_i = \frac{(1-\mu)^{\frac{1}{\rho-1}} P_i^{\frac{1}{\rho-1}}}{\mu^{\frac{1}{\rho-1}} + P_i^{\frac{\rho}{\rho-1}}(1-\mu)^{\frac{1}{\rho-1}}} y \Leftrightarrow \\
\Leftrightarrow &P_i x_i = \frac{(1-\mu)^{\frac{1}{\rho-1}} P_i^{\frac{\rho}{\rho-1}}}{\mu^{\frac{1}{\rho-1}} + P_i^{\frac{\rho}{\rho-1}}(1-\mu)^{\frac{1}{\rho-1}}} y \Leftrightarrow \\
\Leftrightarrow &P_i x_i = \frac{(1-\mu)^{\frac{1}{\rho-1}}}{\mu^{\frac{1}{\rho-1}} P_i^{-\frac{\rho}{\rho-1}} + (1-\mu)^{\frac{1}{\rho-1}}} y \Leftrightarrow \\
\Leftrightarrow &P_i x_i = \frac{1}{1 + (\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}} y,
\end{aligned}
$$

Given this equation, $\bar{\mu}_i = \frac{1}{1+(\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}}$, is the share of expenditure in manufactured goods.

# Appendix B

## Derivation of expression (19):

Plugging $Y_i = w_i H_i + \frac{L}{2}$ into $w_i = \frac{1}{\sigma}\left[\frac{\bar{\mu}_i Y_i}{H_i+\phi H_j} + \frac{\phi\bar{\mu}_j Y_j}{\phi H_i + H_j}\right]$, generates a system of two linear equations in $w_i$ and $w_j$:

$$\begin{cases} w_i = & \frac{1}{\sigma}\left[\frac{\bar{\mu}_i(\frac{L}{2}+w_i H_i)}{H_i+\phi H_j} + \frac{\phi\bar{\mu}_j(\frac{L}{2}+w_j H_j)}{\phi H_i+H_j}\right] \\ w_j = & \frac{1}{\sigma}\left[\frac{\bar{\mu}_j(\frac{L}{2}+w_j H_j)}{\phi H_i+H_j} + \frac{\phi\bar{\mu}_i(\frac{L}{2}+w_i H_i)}{H_i+\phi H_j}\right] \end{cases},$$

that can be solved to obtain the equilibrium skilled wages as explicit functions of the spatial distribution of skilled workers, $H_i$. We solve the system through Cramer's rule.

$$\begin{cases} a_{ii} w_i + a_{ij} w_j = & b_i \\ a_{ji} w_i + a_{jj} w_j = & b_j \end{cases},$$

where $a_{ii} = 1 - \frac{1}{\sigma}\frac{\bar{\mu}_i H_i}{H_i+\phi H_j}$, $a_{ij} = -\frac{1}{\sigma}\frac{\phi\bar{\mu}_j H_j}{\phi H_i+H_j}$, $a_{ji} = -\frac{1}{\sigma}\frac{\phi\bar{\mu}_i H_i}{H_i+H_j}$, $a_{jj} = 1 - \frac{1}{\sigma}\frac{\bar{\mu}_j H_j}{\phi H_i+H_j}$, $b_i = \frac{1}{\sigma}\frac{L}{2}(\frac{\bar{\mu}_i}{H_i+\phi H_j} + \frac{\phi\bar{\mu}_j}{\phi H_i+H_j})$ and $b_j = \frac{1}{\sigma}\frac{L}{2}(\frac{\phi\bar{\mu}_i}{H_i+\phi H_j} + \frac{\bar{\mu}_j}{\phi H_i+H_j})$.

$$\begin{aligned}
\triangle_i = \; & b_i a_{jj} - b_j a_{ij} \\
= \; & \frac{1}{\sigma}\frac{L}{2}\left(\frac{\bar{\mu}_i}{H_i+\phi H_j} + \frac{\phi\bar{\mu}_j}{\phi H_i + H_j}\right)\left(1 - \frac{1}{\sigma}\frac{\bar{\mu}_j H_j}{\phi H_i + H}\right) - \\
& -\frac{1}{\sigma}\frac{L}{2}\left(\frac{\phi\bar{\mu}_i}{H_i+\phi H_j} + \frac{\bar{\mu}_j}{\phi H_i + H_j})\right)\left(-\frac{1}{\sigma}\frac{\phi\bar{\mu}_j H_j}{\phi H_i + H_j}\right) \\
= \; & \frac{1}{\sigma}\frac{L}{2}\left(\frac{\bar{\mu}_i}{H_i+\phi H_j} + \frac{\phi\bar{\mu}_j}{\phi H_i + H_j}\right) - \left(\frac{1}{\sigma}\right)^2 \frac{L}{2}\left(\frac{\bar{\mu}_i}{H_i+\phi H_j}\right)\left(\frac{\bar{\mu}_j H_j}{\phi H_i + H}\right) + \\
& + \left(\frac{1}{\sigma}\right)^2 \frac{L}{2}\left(\frac{\phi^2\bar{\mu}_i}{H_i+\phi H_j}\right)\left(\frac{\bar{\mu}_j H_j}{\phi H_i + H_j}\right) \\
= \; & \frac{1}{\sigma}\frac{L}{2}\left(\frac{\bar{\mu}_i}{H_i+\phi H_j} + \frac{\phi\bar{\mu}_j}{\phi H_i + H_j}\right) + \left(\frac{1}{\sigma}\right)^2 \frac{L}{2}\left(\frac{\bar{\mu}_j H_j}{\phi H_i + H_j}\right)\left(\frac{\bar{\mu}_i(\phi^2-1)}{H_i+\phi H_j}\right) \\
= \; & \frac{1}{\sigma}\frac{L}{2}\frac{1}{(H_i+\phi H_j)(\phi H_i + H_j)}\left[\bar{\mu}_i(\phi H_i + H_j) + \phi\bar{\mu}_j(H_i+\phi H_j) + \frac{1}{\sigma}\bar{\mu}_i(\phi^2-1)\bar{\mu}_j H_j\right] \\
= \; & \frac{1}{\sigma}\frac{L}{2}\frac{1}{(H_i+\phi H_j)(\phi H_i + H_j)}\left[\phi H_i(\bar{\mu}_i + \bar{\mu}_j) + H_j\left(\bar{\mu}_i + \phi^2\bar{\mu}_j + \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma}(\phi^2-1)\right)\right] \\
= \; & \frac{1}{\sigma}\frac{L}{2}\frac{1}{(H_i+\phi H_j)(\phi H_i + H_j)}\left[\phi H_i(\bar{\mu}_i + \bar{\mu}_j) + H_j\left(\bar{\mu}_i - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma} + \phi^2\bar{\mu}_j(1+\frac{\bar{\mu}_i}{\sigma})\right)\right]
\end{aligned}$$

$$
\begin{aligned}
\triangle = \;& a_{ii}a_{jj} - a_{ij}a_{ji} \\
= \;& \left(1 - \frac{1}{\sigma}\frac{\bar{\mu}_i H_i}{H_i + \phi H_j}\right)\left(1 - \frac{1}{\sigma}\frac{\bar{\mu}_j H_j}{\phi H_i + H_j}\right) - \\
& - \left(-\frac{1}{\sigma}\frac{\phi\bar{\mu}_j H_j}{\phi H_i + H_j}\right)\left(-\frac{1}{\sigma}\frac{\phi\bar{\mu}_i H_i}{H_i + H_j}\right) \\
= \;& 1 - \frac{1}{\sigma}\left(\frac{\bar{\mu}_i H_i}{H_i + \phi H_j} + \frac{\bar{\mu}_j H_j}{\phi H_i + H_j}\right) + \left(\frac{1}{\sigma}\right)^2 \frac{\bar{\mu}_i H_i \bar{\mu}_j H_j}{(H_i + \phi H_j)(\phi H_i + H_j)} - \\
& - \left(\frac{1}{\sigma}\right)^2 \phi^2 \frac{\bar{\mu}_i H_i \bar{\mu}_j H_j}{(H_i + \phi H_j)(\phi H_i + H_j)} \\
= \;& \frac{1}{(H_i + \phi H_j)(\phi H_i + H_j)}\Big[(H_i + \phi H_j)(\phi H_i + H_j) - \\
& - \frac{1}{\sigma}\Big(\bar{\mu}_i H_i(\phi H_i + H_j) + \bar{\mu}_j H_j(H_i + \phi H_j)\Big) + \left(\frac{1}{\sigma}\right)^2 \Big(\bar{\mu}_i H_i \bar{\mu}_j H_j - \phi^2 \bar{\mu}_i H_i \bar{\mu}_j H_j\Big)\Big] \\
= \;& \frac{1}{(H_i + \phi H_j)(\phi H_i + H_j)}\Big[\phi H_i^2 + \phi H_J^2 + (\phi^2 + 1)H_i H_j - \\
& - \frac{1}{\sigma}\Big(\phi\bar{\mu}_i H_i^2 + \bar{\mu}_i H_i H_j + \bar{\mu}_j H_j H_i + \phi\bar{\mu}_j H_j^2\Big) + \left(\frac{1}{\sigma}\right)^2 \Big((1 - \phi^2)\bar{\mu}_i H_i \bar{\mu}_j H_j\Big)\Big] \\
= \;& \frac{1}{(H_i + \phi H_j)(\phi H_i + H_j)}\Big[(H_i^2 + H_j^2)\phi - (\bar{\mu}_i H_i^2 + \bar{\mu}_j H_j^2)\frac{\phi}{\sigma} + \\
& + H_i H_j\left(\phi^2 + 1 - \frac{1}{\sigma}(\bar{\mu}_i + \bar{\mu}_j) + \left(\frac{1}{\sigma}\right)^2 - \left(\frac{1}{\sigma}\right)^2 \phi^2 \bar{\mu}_i \bar{\mu}_j\right)\Big] \\
= \;& \frac{1}{(H_i + \phi H_j)(\phi H_i + H_j)}\Big[(H_i^2 + H_j^2)\phi - (\bar{\mu}_i H_i^2 + \bar{\mu}_j H_j^2)\frac{\phi}{\sigma} + \\
& + H_i H_j\left(1 + \left(\frac{1}{\sigma}\right)^2 - \frac{\bar{\mu}_i + \bar{\mu}_j}{\sigma} + \phi^2\left(1 - \frac{\bar{\mu}_i \bar{\mu}_j}{\sigma^2}\right)\right)\Big]
\end{aligned}
$$

Therefore:

$$
w_i = \frac{\triangle_i}{\triangle} = \frac{1}{\sigma}\frac{L}{2}\frac{\phi H_i(\bar{\mu}_i + \bar{\mu}_j) + \left[\bar{\mu}_i - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma} + (1 + \frac{\bar{\mu}_i}{\sigma})\bar{\mu}_j\phi^2\right] H_j}{\phi(H_i^2 + H_j^2) - (H_i^2\bar{\mu}_i + H_j^2\bar{\mu}_j)\frac{\phi}{\sigma} + \left[1 + \frac{1}{\sigma^2} - \frac{\bar{\mu}_i + \bar{\mu}_j}{\sigma} + (1 - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma^2})\phi^2\right] H_i H_j}
$$

By symmetry, we can simply obtain $\triangle_j$ to get $w_j$:

$$
w_j = \frac{\triangle_j}{\triangle} = \frac{1}{\sigma}\frac{L}{2}\frac{\phi H_j(\bar{\mu}_i + \bar{\mu}_j) + \left[\bar{\mu}_j - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma} + (1 + \frac{\bar{\mu}_j}{\sigma})\bar{\mu}_i\phi^2\right] H_i}{\phi(H_i^2 + H_j^2) - (H_i^2\bar{\mu}_i + H_j^2\bar{\mu}_j)\frac{\phi}{\sigma} + \left[1 + \frac{1}{\sigma^2} - \frac{\bar{\mu}_i + \bar{\mu}_j}{\sigma} + (1 - \frac{\bar{\mu}_i\bar{\mu}_j}{\sigma^2})\phi^2\right] H_i H_j}
$$

# Appendix C

## Derivation of expression (21):

Returning expression (2), replacing $y$ by $w_i$:

$$P_i x_i = \frac{1}{1 + (\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}} w_i,$$

that is equivalent to:

$$x_i = \frac{1}{P_i + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}} P_i^{\frac{1}{1-\rho}}} w_i$$

Back to the utility function (1), subject to $P_i x_i + A_i = w_i$:

$$\begin{aligned}
V_i^* &= \mu x_i^{\rho} + (1-\mu)(w_i - P_i x_i)^{\rho} \\
&= \left[\mu^{\frac{1}{\rho}} + (1-\mu)^{\frac{1}{\rho}} (\frac{w_i}{x_i} - P_i)\right] x_i \\
&= \frac{\mu^{\frac{1}{\rho}} + (1-\mu)^{\frac{1}{\rho}} \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}} P_i^{\frac{1}{1-\rho}}}{P_i + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}} P_i^{\frac{1}{1-\rho}}} w_i \\
&= \frac{\mu^{\frac{1}{\rho}} \left[1 + \frac{1-\mu}{\mu} \left(\frac{1-\mu}{\mu}\right)^{\frac{\rho}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\rho}}}{P_i \left[1 + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}\right]} w_i \\
&= \frac{\mu^{\frac{1}{\rho}}}{P_i} \left[1 + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}} w_i \\
&= \frac{\mu^{\frac{1}{\rho}}}{P_i} \left[\frac{1}{\bar{\mu}_i}\right]^{\frac{1-\rho}{\rho}} w_i \\
V_i^* &= \frac{\mu^{\frac{1}{\rho}}}{\bar{\mu}_i^{\frac{1-\rho}{\rho}}} \left(\frac{w_i}{P_i}\right)
\end{aligned}$$

# Appendix D

## Derivation of expression (24):

Expression (24) was the last step possible of an attempt to simplify $\frac{V_1}{V_2} > 1$ ($h = 1$), a necessary condition for the existence of a agglomeration equilibrium in region 1. Our aim was to prepare a simple condition that would explain the model's dynamics analytically for agglomeration patterns. Thus, returning expression (23), and applying some mathematical manipulation:

$$\begin{aligned}
\phi^{\frac{1}{1-\sigma}} \left(\frac{\bar{\mu}_2}{\bar{\mu}_1}\right)^{\frac{1-\rho}{\rho}} \frac{\phi(\bar{\mu}_1+\bar{\mu}_2)}{\bar{\mu}_2-\frac{\bar{\mu}_1\bar{\mu}_2}{\sigma}+(1+\frac{\bar{\mu}_2}{\sigma})\bar{\mu}_1\phi^2} &> 1 \\
\phi^{\frac{2-\sigma}{1-\sigma}} \left(\frac{\bar{\mu}_2}{\bar{\mu}_1}\right)^{\frac{1-\rho}{\rho}} \frac{1+\frac{\bar{\mu}_2}{\bar{\mu}_1}}{\frac{\bar{\mu}_2}{\bar{\mu}_1}-\frac{\bar{\mu}_2}{\sigma}+(1+\frac{\bar{\mu}_2}{\sigma})\phi^2} &> 1 \\
\phi^{\frac{2-\sigma}{1-\sigma}} \left(\frac{\bar{\mu}_2}{\bar{\mu}_1}\right)^{\frac{1-\rho}{\rho}} \frac{1+\frac{\bar{\mu}_2}{\bar{\mu}_1}}{\frac{\bar{\mu}_2}{\bar{\mu}_1}+\phi^2-(1-\phi^2)\frac{\bar{\mu}_2}{\sigma}} &> 1
\end{aligned}$$

Let us now displace $\bar{\mu}_i$, not using the entire formula $\bar{\mu}_i = \frac{1}{1+(\frac{1-\mu}{\mu})^{\frac{1}{1-\rho}} P_i^{\frac{\rho}{1-\rho}}}$, but rather simplifying things up, writing the expression in a way that reflects the difference between the two regions, expressed in their price indices. Hereupon, at $h = 1$, as the price indices are $P_1 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}$ and $P_2 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H\phi}{\alpha}\right)^{\frac{1}{1-\sigma}}$, we obtain $\frac{P_2}{P_1} = \phi^{\frac{1}{1-\sigma}}$. We cannot simply eliminate the remainder of the price index expression on $\frac{\bar{\mu}_2}{\bar{\mu}_1}$, due to the sum in the denominator of $\bar{\mu}_i$. With that in mind, the best way to simplify $\frac{\bar{\mu}_2}{\bar{\mu}_1}$ is to rearrange as $\frac{1+\theta}{1+\phi^*\theta}$, with $\theta = \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\rho}} \left(\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\right)^{\frac{\rho}{1-\rho}}$ and $\phi^* = \phi^{\frac{\rho}{(1-\sigma)(1-\rho)}}$, the distinctive fraction of the share of expenditure in manufactured goods between regions:

$$\phi^{\frac{2-\sigma}{1-\sigma}} \left(\frac{1+\theta}{1+\phi^*\theta}\right)^{\frac{1-\rho}{\rho}} \frac{1+\frac{1+\theta}{1+\phi^*\theta}}{\frac{1+\theta}{1+\phi^*\theta}+\phi^2-(1-\phi^2)\frac{1}{(1+\phi^*\theta)\sigma}} > 1$$

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