



The valuation of the buyback option in football contracts

by

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Dissertation for the Master in Finance

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Year of Graduation: 2015

BIOGRAPHICAL INTRODUCTION OF THE AUTHOR

I, Hoa Quynh Nga Nhu, was born in Hanoi, Vietnam on June 08, 1989. In the year 2012, I became a holder of a Bachelor's degree in Business Administration (five-year program) from Hanoi University of Science and Technology, Vietnam. During my last semester of my bachelor's degree, I interned at the Tax Function of KPMG Vietnam. After graduation from Hanoi University of Science and Technology, I spent a semester on exchange at Warsaw School of Economics in Poland, between the periods of September 2012 to February 2013, under a full Erasmus Mundus scholarship which was funded by the European Commission. I was then awarded a full scholarship to conduct a full master program again under the Erasmus Mundus program by the European Commission. I decided to choose the Master in Finance at FEP, University of Porto, Portugal, as my next destination with a strong belief that the program would provide me with a sound background in finance to pursue my dream of becoming a successful financial specialist in the near future.

As our Vietnamese people are great fans of football, I have developed an interest in “The King of Sports” since I was a child. My first football matches were during World Cup 1998: “France 98”. The aim of this dissertation therefore is to explore a completely new area of research, under a new research theme, that is, “real options”. Additionally, I hope to inspire other researchers that have an interest in real options, to explore this interesting topic, football, in the context of real options.

ACKNOWLEDGEMENT

I would like to express my deepest gratitude and appreciation to my supervisor, Professor Paulo Pereira, who has inspired and supported me all the way. His passion for real options has been one of the main factors that motivated me to explore this new and interesting topic from the very beginning without questions.

I also greatly appreciate the advice and timely feedback I received from Professor Natércia Fortuna and Dr. Gonçalo Faria which significantly contributed to my work.

I would like to deliver special thanks to my professors at the Master in Finance program for the inspiration, knowledge and skills that they provided me within the last two years. Being equipped with all of that, I feel more confident in pursuing the challenging career ahead.

I wish to express my deepest acknowledgement to my parents, my brother for their unconditional support and encouragement throughout this lengthy period, as well as for enabling me to develop a love for football since I was a child. I would also like to specially thank my friends, namely; Daaliya Headlie, Tran Thi Hien, Nguyen Minh Phuong, José Alves, Hoang Duc Anh, Mai Ngoc Luan, José Osório, Vitor Minhoto and Inês Trigo for providing me with their valuable advice, ideas, knowledge and comments along the way.

I am grateful to be surrounded with very friendly classmates at FEP in particular and at University of Porto in general. The warm welcome and friendliness allowed me to quickly adapt and feel comfortable in a new environment. Thank you Ana Carla and your family, Irina Balasyan and your family, Daniela Rodrigues and your family, ChengChingYu, Nguyen Thi Hai, and Hang Dao for always being by my side in my toughest times, and for having shared the same passion in finance.

I also would like to thank Ms. Paula Isabel Carvalho, Ms Susana Pereira and all academic staffs at FEP library and FEP Academic Service for their enthusiastic assistance during the time I conducted this report.

My deep appreciations are also for Ms. Thuy Tien de Oliveira and her husband Manuel de Casimiro who have been supporting me from the beginning of my stay in Porto until now.

Especially, I would like to extend my warmest thanks to the European Commission and Erasmus Mundus Lotus Project for giving me this chance to advance a step further in my career. This two-year program has been so far the most remarkable milestone of my life, opening doors for the future development and further tightening the relationship between Europe and Asia in general, and Europe and Vietnam in particular.

ABSTRACT

The valuation of the buyback option in football contracts

The study aimed to find proper option pricing models among many in financial theories, to be able to value the buyback option attached in football transfer deals. To the best of our knowledge, this area of research had not been studied before in finance. In order to get a sense of the price of the buyback option that could be exercised at maturity, the traditional Black-Scholes-Merton (1973) model for valuing European Options was employed. For pricing options that had two deterministic strike prices and maybe exercised either at the end or at one predefined moment during its maturity, the Carr (1995) model for valuing American Exchange Options with Application to Real Options was modified in this study.

Keywords: *real option, option pricing, football, buyback option, Black-Scholes-Merton Model, modified Carr (1995) model*

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CHAPTER 1:

INTRODUCTION

Financial option pricing models that have initially been developed for the sole purpose of pricing “financial assets” have now been applied to the case of “real assets” in the world of business in the year 1977 by Myers as a new capital budgeting technique. This application, which is commonly referred to as “real options”, was first used to value investment opportunity in the oil and gas industry, and then later expanded into mining, R&D, technology and other areas. A few literatures in the past had employed option pricing models in the area of sports in order to price a player’s value or betting. However, to the best of our knowledge, until before this study was conducted, there had not been any research papers dedicated to valuing the buyback option on football players or buyback options alike in other sports.

The main motivation for this topic was to find proper option pricing models among many in financial option theories to apply in a new area of research that had not been explored before: valuing buyback option in football transfer contracts.

Football managers, investors, football players and football fans worldwide are the main target audience for this dissertation. By the information captured from sport news available to them, they can now have a better sense of how much selling clubs should discount their young talents and how much buying clubs should ask for discount when they agree to embed the buyback option in the deal.

In this dissertation, tools for both selling and buying clubs to determine a fair value for the buyback option when they enter such a deal were suggested. The results achieved were then expected to assist both parties in the process of making informed decisions.

It is to be hoped that this study would be able to contribute to the world of academia by filling a gap in the academic world, as this area of study has not been previously explored. Though it is a small step to solving real world problems, it is expected that this study would inspire other researchers to explore the field.

The topic is of great economic relevance. What is happening in football nowadays is that the buyback option is dynamically embedded in transfer deals, especially in deals targeting

young players. Biggest clubs in the world such as Real Madrid, FC Barcelona and Arsenal tend to be the most dynamic users of buyback option. The buyback option involves significant amount of money, making the selling clubs to give up millions of Euros when selling their players. The buyback option provides the selling clubs with the flexibility to repurchase their players at predefined moments for predefined prices while enables the buying clubs to save significant amounts of money when buying the football players.

The research questions are: How much buyback option is worth? In other words, how much should the selling club give up and how much should the buying club agree when a buyback option is included? However, the most important question is: Which models are proper to value such an option? There is a need for using proper valuation tools in finance theory to estimate the fair value of the buyback option.

In order to answer these questions above, the dissertation was developed as follows:

The first chapter, Chapter 1, is the introduction to the topic. Chapter 2 explains briefly what buyback option is and the history of buyback option. In Chapter 3, a summary of research in finance applied in sports is presented. In Chapter 4, the proper models for the valuation of the buyback options in football contracts were suggested. The traditional Black-Scholes-Merton (1973) model for valuing European Option was reviewed for the purpose of pricing buyback options that had only one possible exercising moment. For buyback options that gave the holders the possibility to exercise at two predefined moments for two predefined strike prices, Carr (1995) model for the valuation of American Exchange Option with Application to Real Option was modified. Chapter 5 is the data collection where the sources of deals and player's historical values and the method of linear interpolation were presented. In Chapter 6, the models were applied and the results were discussed. Finally, Chapter 7 entails the conclusion and suggestions of this study.

CHAPTER 2:

BUYBACK OPTION

2.1. About buyback option

A buyback option in a football transfer deal is the right, but not the obligation for the selling club¹ to repurchase the player for a certain amount of money in a particular period, usually within one (1) to four (4) years. In such transactions, the buying club is obligated to resell the players to the selling club. This buyback option is commonly referred to as a “call option” in financial option literature. In such instances, the selling club takes a long position in the deal and is required to pay a “premium” for having this option. This premium is usually in the form of a discount when the player is sold to the buying club.



Figure 2.1: An example of buyback option of Carlos Vela²

Leading European Football clubs, which are often the sellers, usually participate in numerous championships. These football clubs have several qualified players for each position whereby each player has a different style of playing to fit the different strategies and tactics of the team in order to be able to competitively compete with different clubs in different competitions. Therefore, each

position in a big club is usually competitive which makes it even harder for young players to play and gain experience in such reputable clubs. The solution to this problem is for the reputable clubs to sell or lend these players to less reputable clubs in order for the player to train and play in a high professional environment. For buying clubs, young players are attractive as they are usually full of potential, enthusiastic, have a great desire to learn, and are quickly able to adapt to the new team. More interestingly, these young players are

¹ In this study, the “selling club” is defined as the club who sells a player with a buyback option to repurchase him in the future. The other party is referred to as the “buying club”.

² When selling 50% of Carlos Vela’s value to Real Sociedad in 2012 (Arsenal kept 50% of Vela), Arsenal embedded a buyback option into the deal so that within two years, they can repurchase Vela back with the strike price (comparable to 50% of Vela) of 3.5ME. As such option was embedded into the deal, this then gave Arsenal the flexibility of repurchasing the players after two years. Arsenal discounted the transfer price of Vela and only received 2.5ME (instead of 3.5ME which was comparable to the 50% value of Vela at that moment)

usually less risky when compared to the more reputable players who not only have higher transfer prices, but also have higher salaries and are not able quickly adapt to new clubs, new coaches, new fans and new weather.

In a financial aspect, including a buyback option in deals involving young players are quite advantageous to both the selling club and buying club. Buyback options provide the selling club with three main advantages. Firstly, in the case of repurchasing the player, the amount that the selling club would be required to pay is much lower than the prevailing market value of the player. It is believed that overtime, with increased experience the value of a player would increase. If the selling club was to sell the young player without including a buyback option, and now wants to repurchase the player, the selling club would be forced to pay a significant amount of money. Secondly, in case the player becomes injured or does not perform well, the buyback option gives the selling club the option of not repurchasing the player. Lastly, embedding a buyback option means giving a discount to the buying club at the purchasing date. This therefore makes it easier for the selling club to sell the players, while still achieving the benefits discussed above. Buying clubs on the hand benefit by accepting the buyback option as it gives them a chance to have young football talent at a lower cost.

2.2. Buyback option history³

The practice of buyback options took its roots in the UK nearly 40 years ago. Buyback options were first embedded in the deals of British clubs even though the selling clubs were not certain as to whether they would repurchase the players in the future. In 1977, the buyback option was included in the deal of Kevin Keegan between Liverpool and Hamburg. This was due to the wish of the player, Kevin Keegan. In the deal, Keegan wanted to maintain his position in England and promised that he would not do anything without the permission of Liverpool. At that time, Liverpool did not exercise the option despite the player wanting to return to England in 1980. After that, when he wanted to join Southampton, he had to ask for permission from Liverpool's chairman in order to move.

³ Information taken from <http://www.theguardian.com/football/2013/dec/04/the-knowledge-football-clubs-activating-Buyback-clauses> last accessed on November 25, 2014

The period of 1983 was the first time a buyback option was exercised. The buyback option took place when Watford sold Luther Blissett to Milan for a deal value worth 1M£. In the twenty two (22) appearances in Milan, Blissett was not as successful as only three (3) goals were scored. Blissett attributed the poor performance to the lack of cooperation from his colleagues. As Blissett was aware that there was a buyback option in the contract when he was sold from Watford to Milan, Watford, where he had his best memories, was the first place Blissett wanted to return to. However, Blissett was unsure as to whether or not Watford could afford to repurchase him. Fortunately, Watford was able to carry out the transaction and Blissett was able to return to Watford and shine again. At that time, Watford paid Milan £550,000, thus earning a smart profit of £450,000.

Following Blissett's buyback case, there were many other small cases that took place. It was not until the summer of 1991, when the next high profile buyback occurred. The deal involved Rangers selling Trevor Steven to Marseille with a massive fee of 5.0M£, this being one of the highest fee transfers at the time. If Ranger was interested in repurchasing Trevor Steven they would have needed to pay 5.0M£. Due to the financial turmoil the French club was facing, Marseille was only able to pay an amount of 2.8M£. The British club therefore agreed to write off the difference of 2.2M£ for the return of the player.

CHAPTER 3:

LITERATURE REVIEW

To date, there have been not been any research papers dedicated to valuing buyback options in football deals or buyback options alike in any other sports. Most research papers on football have explored the area of betting and team performance. Case in point, the studies of Coles et al(1996); Rue et al(2000); Crowder et al(2002); and Carmichael et al (2000).

Subsequent to the study undertaken by Black and Scholes (1973), there have been numerous research dedicated towards developing models to value financial options. In later years, the models used to value financial options were then utilized to value “real investment opportunities” as they have similar features to that of financial options. However, limited research has been dedicated to sports. The two papers that have applied option pricing models in the context of football are those of Tunaru et al (2005) and Kanyinda et al (2012). These two studies propose models that can be used to determine the value of a football player. In sports such as cricket and baseball, past studies have focused on the valuing of players and game betting. In order to value the buyback option in the football deals in this study, there is a need to review and employ models from financial option pricing studies.

Table 3.1 provides a summary of previous financial studies applied in sports

			Name of the paper	Authors	Contribution
	Football	Team Performance	Team Performance: The Case of English Premiership Football	Carmichael, Thomas et al. (2000)	A production function model for association football utilizing Opta index
		Betting	Modelling association football scores and inefficiencies in the football betting market	Dixon and Coles (1997)	A simple bivariate Poisson distribution model for the numbers of goals scored by each team, with parameters based on past performance
			Prediction and Retrospective Analysis of Soccer Matches in a League	Rue and Salvesen (2000)	A Bayesian dynamic generalized linear model to estimate the time dependent skills of all teams in a league, and to predict next weekend's soccer matches
			Dynamic modelling and prediction of English Football League matches for betting	Crowder, Dixon et al. (2002)	A fast computational approach for predicting the results of football matches with a view to betting in the primary market on the chances of winning, drawing or losing outcomes, using refinements of the independent Poisson model of Dixon and Coles
Real option		Valuing football players	An option pricing framework for valuation of football players	Tunaru, Clark et al. (2005)	Two RO models for valuing football players, based on Opta index, Jump effects taken into consideration

			Assessing the financial value of football players on the basis of real options theory	Aloïs, Bouteiller et al. (2012)	Two models: - One for a typical player - Other for a atypical player
	Cricket	Valuing Cricket players	Option on Human Performance-A Case Study of Indian Premier League	Suleman and Saeed (2009)	Using binomial option pricing model to valuing cricket players based on SS index of India
	Baseball	Betting	A real options approach to the baseball game betting	Wang (2013)	Model of Switching Options of the Baseball Game
		Empirical application of RO on baseball contracts	Major League Baseball Player Contracts: An Investigation of the Empirical Properties of Real Options	Clayton and Yermack (2001)	Research shows that players receive higher compensation when allowing the team to take options on their future services, and lower compensation when receiving options to extend their own contracts

Table 3.1: Studies on sports in finance⁴⁴ Authors' own analysis

CHAPTER 4:

METHODOLOGY

After considering a number of models in financial option literature, two models were found to be suitable for valuing the buyback option included in the football transfer deals.

4.1. Black-Scholes-Merton (1973)

The Black-Scholes-Merton (B-S-M) model is considered to be the foundation of the option pricing theory. The purpose of B-S-M model was to price European Options which give the option holder the right but not the obligatory to exercise the option at maturity. For buyback options which have one possible future date to exercise, the B-S-M model can be used.

The B-S-M formula for the value of the call option is:

$$C(S, K_T, \sigma, \delta, r_f, T) = Se^{-\delta_s T} N_1(d_1) - K_T e^{-r_f T} N_1(d_2) \quad (4.1)$$

$$d_1 = \frac{\ln\left(\frac{S}{K_T}\right) + (r_f - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \quad (4.1a)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (4.1b)$$

where S , K_T , T , δ_s , r_f and σ are the spot price of the underlying asset, strike price of the option, time to maturity, dividend yield of the underlying asset, risk-free interest rate and the volatility of the value of the underlying asset respectively. $N_1(x)$ is the cumulative probability distribution function of a standard normal distribution.

There are two components on the right side of the formula. The first component, $Se^{-\delta_s T} N_1(d_1)$, is the expected value of the underlying asset that the holder may receive, and the second component $K_T e^{-r_f T} N_1(d_2)$ represents the expected present value of the payment of the exercise price.

Among all of B-S-M's variables, the level of volatility (σ) is the only variable that cannot be directly observed in the market, as it is the future volatility of the underlying asset in the period. Traditionally, there are two methods used to estimate the level of volatility. The first method involves using historical data to derive the historical volatility, while the second method entails using a forward-looking approach to derive the so-called implied volatility.

Despite its mathematical complexity in deriving the closed-form solution, the B-S-M formula is quite straightforward and easy to apply. The use of B-S-M therefore, is not only limited to financial options, but can also be applied to real options, and in this case the buyback option of football contracts.

Table 4.1 compares the inputs for the financial option to the inputs used in the buyback option in football contracts

	Financial option	Buyback option
S	Price of the underlying asset	Current value of the football player
K_T	Exercise price	Buyback price
σ	Volatility of the underlying asset	Volatility of the football player's value
δ_S	Dividend yield	Opportunity cost of deferring exercising buyback option
r_f	Risk-free interest rate	Risk-free interest rate
T	Time to maturity	Time until Buyback option expires

Table 4.1: Financial option inputs compared to the inputs for the buyback option⁵

4.2. The modified Carr (1995) model

In reality, buyback options are not limited to only one possible exercising date. In fact, there are cases where the option can have two moments where it can be possible to exercise the option. The B-S-M model therefore cannot be applied to such cases. There must be a model dedicated to valuing such an option. The Carr (1995)⁶ model for valuing an American Exchange Option is a prominent example.

In brief, the American Exchange Option, according to Carr (1995), is an option that gives its holder the right but not the obligation to exchange one risky asset (optioned asset), for another risky asset (delivery asset), at any time prior or on the expiration date.

The difference between the Carr (1995) model and the B-S-M model is that unlike the B-S-M that assumes only one stochastic process of the spot price S of the optioned asset, the

⁵ Authors' own analysis

⁶ Carr (The valuation of American Exchange Options with Applications to Real Options)

Carr (1995) model in addition, also assumes the stochastic process of the delivery asset. The following equations describe the Carr model:

$$\frac{dS_t}{S_t} = (\alpha_S - \delta_S)dt + \sigma_S dz_t^S \quad (4.2a)$$

$$\frac{dK_t}{K_t} = (\alpha_K - \delta_K)dt + \sigma_K dz_t^K \quad (4.2b)$$

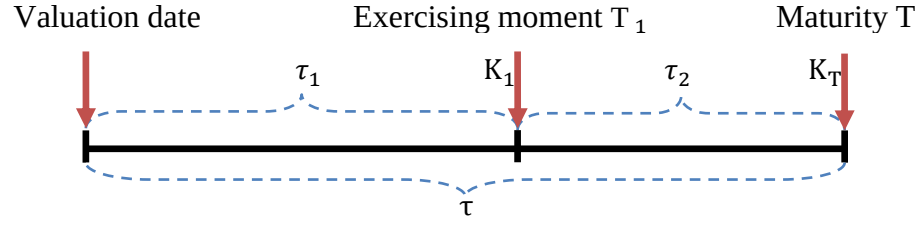
α_S and α_K are the drift rates of assets S and K respectively; δ_S and δ_K are the corresponding dividend yields; σ_S and σ_K are the volatilities of S and K respectively. The two stochastic process are correlated with the rate σ_{SK} .

Since the values of the strike price of the football players can be varied and are deterministic, the Carr model would be modified, with the volatility of the delivery asset (σ_K) being equal to zero, the drift rate (α_K) and the dividend yield (δ_K) being equal to the risk free rate and thus $\frac{dK_t}{K_t} = 0$. This then becomes the American Option.

Since the selling club cannot exercise the buyback option anytime during maturity, and can only exercise the option on two pre-deterministic moments, the pricing of Pseudo American Option was considered.

In this study, the methodology proposed by Carr to value a Pseudo American Exchange Option with two possible exercising moments was used. The methodology includes two steps. The first step is to value the European Option which has already been done in the previous section. The second step is to value the Pseudo American Option that has two exercising dates.

Carr suggested a model for options that can be exercised at either T or $\frac{T}{2}$. However, in this study, the Carr model was modified in a more general context so that the pricing of the option that can be exercised at T or one moment during its life but not limited to only moment $\frac{T}{2}$. In the modified model the strike price can have two different values for each exercising moment. The following diagram illustrates this idea.

Figure 4.1: The modified Carr model illustration⁷

$E_2(\tau)$ is denoted as the value of the Pseudo American Option that has two possible exercising moments. If on the first exercising date, the option value is greater than the cash proceeds when exercising, the option should be kept alive. This is expressed in the following formula:

$$S e^{-\delta_s \tau_2} N_1 \left(d_1 \left(\frac{S}{K_T}, \delta, \tau_2, \sigma^2 \right) \right) - K_T e^{-r_f \tau_2} N_1 \left(d_2 \left(\frac{S}{K_T}, \delta, \tau_2, \sigma^2 \right) \right) > S - K_1 \quad (4.3)^8$$

$$d_{1\tau_2} \left(\frac{S}{K_T}, \delta, \tau_2, \sigma^2 \right) = \frac{\ln \left(\frac{S}{K_T} e^{-\delta \tau_2} \right) + \frac{1}{2} \sigma^2 \tau_2}{\sqrt{\sigma^2 \tau_2}} \quad (4.3a)$$

$$d_{2\tau_2} \left(\frac{S}{K_T}, \delta, \tau_2, \sigma^2 \right) = d_{1\tau_2} \left(\frac{S}{K_T}, \delta, \tau_2, \sigma^2 \right) - \sigma \sqrt{\tau_2} \quad (4.3b)$$

$$\delta = \delta_S - r_f \text{ and } \sigma = \sigma_S \quad (4.3c)$$

where $N_1(x)$ is the cumulative probability distribution function of a standard normal distribution.

Denoting $P_T = \frac{S}{K_T}$ and dividing both sides of Equation (4.3) by K_T , the expression for P_T is derived as follow:

$$P_T e^{-\delta_s \tau_2} N_1 \left(d_1(P_T, \delta, \tau_2, \sigma^2) \right) - e^{-r_f \tau_2} N_1 \left(d_2(P_T, \delta, \tau_2, \sigma^2) \right) > P_T - \frac{K_1}{K_T} \quad (4.4)^9$$

⁷ For the sake of convenience, in this study, τ_1 and T_1 , τ and T will be used interchangeably

⁸ Authors' own analysis based on Formula (6.6), Trigeorgis, L. (1995). Real Options in Capital Investment: Models, Strategies, and Applications, Praeger; Third printing edition

⁹ Authors' own analysis based on Formula (6.7), Trigeorgis, L. (1995). Real Options in Capital Investment: Models, Strategies, and Applications, Praeger; Third printing edition

In order to find the trigger ratio to be able to determine when the option is worth exercising on the first exercising date, let P^* be the unique value of the ratio P_T which makes (4.4) an equality, P^* satisfies the following equation:

$$P^* e^{-\delta_s \tau_2} N_1(d_1(P^*, \delta, \tau_2, \sigma^2)) - e^{-r_f \tau_2} N_1(d_2(P^*, \delta, \tau_2, \sigma^2)) = P^* - \frac{K_1}{K_T} \quad (4.5)^{10}$$

Whenever P_T is greater than P^* , the left hand side of Equation (4.5) becomes smaller than the right hand side. In other words, the expected value of the option is smaller than the payoff if it was to be exercised on the first exercising date, as such, making it rational for the option holder to exercise the option to be able to gain an amount of $S - K_1$, if not, the option should be kept alive until maturity.

To be able to understand the payoff of this Pseudo American Option better, a portfolio comprising of three European Options was constructed. The first composition of the portfolio entailed using a long position in a European Option with a strike price of K_T and time to maturity of T . The second composition was a long position in a European Option with a strike price of $P^* K_T$ which matures on the first exercising date. While the last composition comprised of one short position in a compound European Exchange Option which also matures on the first exercising date exchanging $(P^* - \frac{K_1}{K_T})$ unit of K_T for the first European Option which has a strike price of K_T and maturity of T .

<i>Evaluation date</i>	<i>Exercising date 1</i>	
Long $c(S, K_T, T)$	If $P_T \leq P^*$ $c(S, K_T, T)$	If $P_T > P^*$ $c(S, K_T, T)$
Long $c(S, P^* K_T, \tau_1)$	0	$S - P^* K_T$
Short cpd $(c(S, K_T, T); (P^* - \frac{K_1}{K_T}) K_T; \tau_1)$	0	$-c(S, K_T, T) + (P^* - \frac{K_1}{K_T}) K_T$
	$c(S, K_T, T)$	$S - K_1$

Table 4.2: Portfolio that replicates the payoff of the Pseudo American Option with two exercising dates¹¹

If $c(S, K_T, T) = S - K_1$ equation (4.5) is derived in order to solve P^* .

¹⁰ Authors' own analysis based on Formula (6.8), Trigeorgis, L. (1995). *Real Options in Capital Investment: Models, Strategies, and Applications*, Praeger; Third printing edition

¹¹ Authors' own analysis based on Armada, M. Kryzanowski, L. Pereira, P (2006). "A modified finite-lived American exchange option methodology applied to real options valuation" *Global Finance Journal* 17 (2007), 419-438

The value of the Pseudo American Option in the modified Carr model is equivalent to the value of this replicated portfolio.

The value of the first option which is a European Option with a strike price of K_T and time to maturity of T is:

$$c(S, K_T, T) = Se^{-\delta_s T} N_1(d_1(P_T, \delta, T, \sigma^2)) - K_T e^{-r_f T} N_1(d_2(P_T, \delta, T, \sigma^2)) \quad (4.6)$$

$$d_{1T}(P_T, \delta, T, \sigma^2) = \frac{\ln(P_T e^{-\delta T}) + \frac{1}{2} \sigma^2 T}{\sqrt{\sigma^2 T}} \quad (4.6a)$$

$$d_{2T}(P_T, \delta, T, \sigma^2) = d_1(P_T, \delta, T, \sigma^2) - \sigma \sqrt{T} \quad (4.6b)$$

The value of the second option which is a European Option with a strike price of $P^* K_T$ maturing on the first exercising date is:

$$c(S, P^* K_T, \tau_1) = Se^{-\delta_s \tau_1} N_1\left(d_1\left(\frac{P_T}{P^*}, \delta, \tau_1, \sigma^2\right)\right) - P^* K_T e^{-r_f \tau_1} N_1\left(d_2\left(\frac{P_T}{P^*}, \delta, \tau_1, \sigma^2\right)\right) \quad (4.7)$$

$$\begin{aligned} d_{1\tau_1}\left(\frac{P_T}{P^*}, \delta, \tau_1, \sigma^2\right) &= \frac{\ln\left(\frac{S}{P^* K_T} e^{-\delta \tau_1}\right) + \frac{1}{2} \sigma^2 \tau_1}{\sqrt{\sigma^2 \tau_1}} \\ &= \frac{\ln\left(\frac{P_T}{P^*} e^{-\delta \tau_1}\right) + \frac{1}{2} \sigma^2 \tau_1}{\sqrt{\sigma^2 \tau_1}} \end{aligned} \quad (4.7a)$$

$$d_{2\tau_1}\left(\frac{P_T}{P^*}, \delta, \tau_1, \sigma^2\right) = d_1\left(\frac{P_T}{P^*}, \delta, \tau_1, \sigma^2\right) - \sigma \sqrt{\tau_1} \quad (4.7b)$$

The value of the third option which is a compound European Exchange Option which matures on the first exercising date and exchanges $(P^* - \frac{K_1}{K_T})$ unit of K_T for the first European Option which has a strike price of K_T and maturity of T is:

$$cpd\left(c(S, K_T, T); \left(P^* - \frac{K_1}{K_T}\right) K_T; \tau_1\right) = Se^{-\delta_s T} N_2(d_{1\tau_1}, d_{1T}, \rho) - K_T e^{-r_f T} N_2(d_{2\tau_1}, d_{2T}, \rho) \quad (4.8)^{12}$$

$$- \left(P^* - \frac{K_1}{K_T}\right) K_T e^{-r_f \tau_1} N_1(d_{2\tau_1})$$

$$\rho = \sqrt{\frac{\tau_1}{T}} \quad (4.8a)$$

$$N_2(x_1, x_2, \rho) \equiv \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \frac{\exp\left\{-\frac{1}{2(1-\rho^2)}[z_1^2 - 2\rho z_1 z_2 + z_2^2]\right\}}{2\pi\sqrt{1-\rho^2}} dz_2 dz_1 \quad (4.8b)$$

$N_2(d_{2\tau_1}, d_{2T}, \rho)$ is the cumulative distribution function of the standard bivariate normal distribution, this being the probability that $z_1 < d_{2\tau_1}$ and $z_2 < d_{2T}$ occur simultaneously with the correlation of $\rho = \sqrt{\frac{\tau_1}{T}}$

Combining the value of three options in this portfolio, the value of Pseudo American Option with two possible exercising dates is:

$$\begin{aligned} E_2(\tau) &= c(S, K_T, T) + c(S, P^* K_1, \tau_1) - cpd\left(c(S, K_T, T); \left(P^* - \frac{K_1}{K_T}\right) K_T; \tau_1\right) \quad (4.9) \\ &= Se^{-\delta_s T} N_1(d_{1T}) - K_T e^{-r_f T} N_1(d_{2T}) \\ &\quad + Se^{-\delta_s \tau_1} N_1(d_{1\tau_1}) - P^* K_T e^{-r_f \tau_1} N_1(d_{2\tau_1}) \\ &\quad - Se^{-\delta_s T} N_2(d_{1\tau_1}, d_{1T}, \rho) + K_T e^{-r_f T} N_2(d_{2\tau_1}, d_{2T}, \rho) \\ &\quad + P^* K_T e^{-r_f \tau_1} N_1(d_{2\tau_1}) - K_1 e^{-r_f \tau_1} N_1(d_{2\tau_1}) \end{aligned}$$

As $N_1(b) - N_2(a, b, \rho) = N_2(-a, b, -\rho)$, the equation for the price of Pseudo American Option with two possible exercising dates is derived as follows:

¹² Authors' own analysis based on Carr, P. (1988). "The valuation of Sequential Exchange Opportunities" The Journal of Finance, Volume 43, Issue 5 (Dec 1988), 1235-1256

$$E_2(\tau) = Se^{-\delta_s T} N_2(-d_{1\tau_1}, d_{1T}, -\rho) - K_T e^{-r_f T} N_2(-d_{2\tau_1}, d_{2T}, -\rho) \quad (4.10)$$

$$\begin{aligned} &+ Se^{-\delta_s \tau_1} N_1(d_{1\tau_1}) - K_1 e^{-r_f \tau_1} N_1(d_{2\tau_1}) \\ &= S \left[e^{-\delta_s \tau_1} N_1(d_{1\tau_1}) + e^{-\delta_s T} N_2(-d_{1\tau_1}, d_{1T}, -\rho) \right] \\ &- K_T \left[\frac{K_1}{K_T} e^{-r_f \tau_1} N_1(d_{2\tau_1}) + e^{-r_f T} N_2(-d_{2\tau_1}, d_{2T}, -\rho) \right] \end{aligned} \quad (4.11)$$

$N_1(d_{2\tau_1})$ is the probability that the option is exercised on the first exercising date. $K_1 e^{-r_f \tau_1} N_1(d_{2\tau_1})$ is therefore the present value of the expected payment and $Se^{-\delta_s \tau_1} N_1(d_{1\tau_1})$ is the expected value of the underlying asset on the first exercising date.

$N_2(-d_{2\tau_1}, d_{2T}, -\rho)$ is the probability that the option is exercised at maturity. $K_T e^{-r_f T} N_2(-d_{2\tau_1}, d_{2T}, -\rho)$ is thus the present value of the expected payment and $Se^{-\delta_s T} N_2(-d_{1\tau_1}, d_{1T}, -\rho)$ is the expected income of the option at maturity.

In conclusion, in this section, the Carr (1995) model has been modified to price options with two possible exercising dates with the first date being at any moment during the maturity life and no longer limited to only the moment $\frac{T}{2}$ like that of the original Carr (1995) model. In addition, in the modified model the strike prices can have two different predetermined values, that is K_1 and K_T .

In the following chapter, the sources for the inputs of the models are presented.

CHAPTER 5:

DATA COLLECTION

5.1. Buyback option deals

Information pertaining to buyback options deals was collected through a number of reliable sources, namely; official press releases, official announcements of the buying and selling clubs and qualified websites such as The Telegraph, The Guardian, and the Dailymail.

Sports news are updated daily. As such, finding trusted information of deals in the past was difficult as sports' news websites rarely store past information. In this study, only deals arisen in the last three or four years were collected.

In addition, though buyback options are usually used by football clubs, the targets for such deals are mainly young players. Hence, the news of such deals tend to be less popular in sport news. As such, only deals of prominent young players that attract press attention are assessed.

5.2. Player values

5.2.1. Data sources

Data concerning the market value of players was found on the traditional and popular football website known as "TransferMarkt". However, in some cases the targeted football players in the buyback option are still young, so data for the player was limited, as such in some cases, assumptions were made.

The official value of a player changes only once or twice every year in the transfer season making it at first glance a very discrete variable. However, it is believed that the market value of the player is a continuous variable that follow a stochastic process. This is due to the fact that the value of the player tends to change after each match he plays, after every training session, plus his "extra football" qualities such as the ability to attract the crowd, his image and fashion, as discussed in Aloïs, K., et al. (2012).

5.2.2. Method of Linear Interpolation

As discussed above, the market value of a player usually changes twice or three times per year. Thus, in this study the simple linear interpolation methodology is employed to

estimate the value of the player in other months. These interpolated values are used to calculate the monthly volatility followed by the yearly volatility of the player.

According to Richard G. Brown¹³, “linear interpolation is a method of curve fitting using linear polynomials”. The following formula illustrates the method of linear interpolation.

$$y = y_1 + (x - x_1) \frac{y_2 - y_1}{x_2 - x_1} \quad (5.1)$$

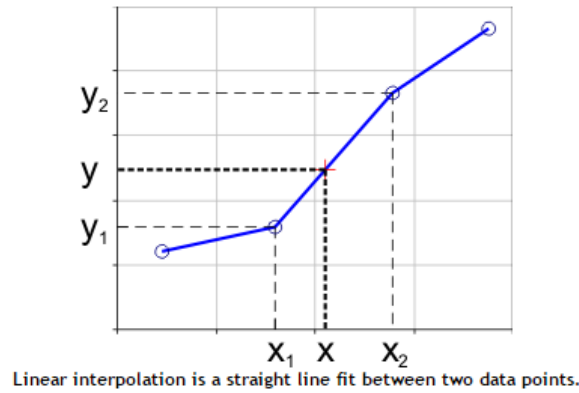


Figure 5.1: Linear interpolation method illustration¹⁴

The historical volatility of the player is calculated as the following:

Define:

$n + 1$: number of montly observations

S_i : price of the player at the end of i^{th} interval, with $i = 0, 1, 2, \dots, n$

Return of each month:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \text{ for } i = 1, 2, 3, \dots, n \quad (5.2)^{15}$$

¹³ Brown, R. (2014). E-Study Guide for Advanced mathematics, Cram101; 1st edition

¹⁴ <http://www.blueleafsoftware.com/Products/Dagra/LinearInterpolationExcel.php>, last accessed on 30 June 2015

¹⁵ Hull, J. (2012). Options, Futures and Other Derivatives

Monthly volatility is the standard deviation of u_i and is given by

$$\sigma_{monthly} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2} \quad (5.3)^{16}$$

where $\bar{u}$ is the mean of the u_i

Yearly volatility

$$\sigma_{annual} = \sigma_{monthly} \times \sqrt{12} \quad (5.4)^{17}$$

¹⁶ Hull, J. (2012). Options, Futures and Other Derivatives

¹⁷ Hull, J. (2012). Options, Futures and Other Derivatives

5.3. Deals in detail

5.3.1. Carlos Vela

5.3.1.1. About Carlos Vela

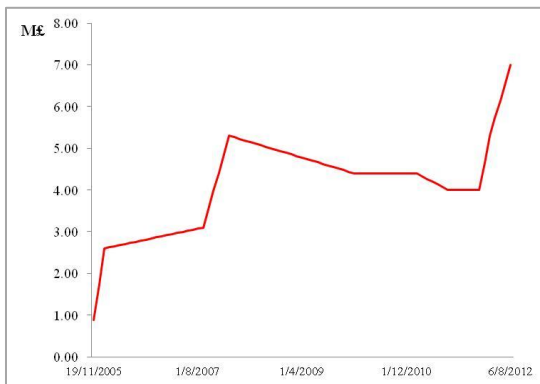


Figure 5.2: The evolution of Vela's value before the deal using the linear interpolation method

Former Number 11 of Arsenal was born on March 01, 1989 in Cancún, Quintana Roo, Mexico. He first joined Arsenal in November 2005 on a five-year deal for a fee of £125,000. His main position in the team is a forward. In Season 2011/2012, before moving to Real Sociedad, Vela scored 12 goals in his 35 appearances.

The Figure 5.2 illustrates the evolution of Vela's value using the linear interpolation method.

5.3.1.2. About the deal¹⁸

Selling club: Arsenal

Buying club: Real Sociedad

When selling 50% of Carlos Vela's value to Real Sociedad in 2012 (Arsenal kept 50% of Vela's value), Arsenal embedded a buyback option into the deal so that within two years, they can repurchase Vela with the strike price of 3.5M£ (comparable to 50% of Vela). By attaching such an option to the deal provided Arsenal with the flexibility of repurchasing

¹⁸ Appendix 5.1

Vela. Arsenal therefore was required to discount the price of Vela by 1M£ and only received 2.5M£

5.3.2. Cecs Fabregas

5.3.2.1. About Cecs Fabregas

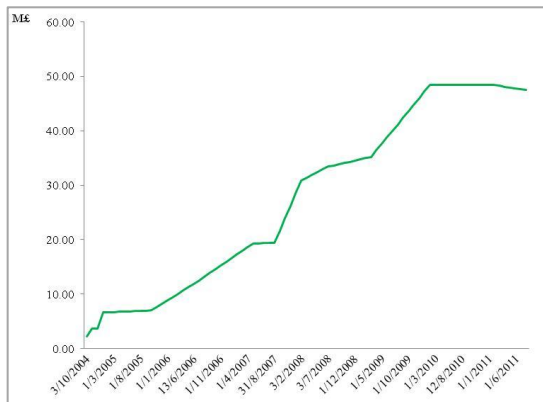


Figure 5.3: The evolution of Fabregas's value before the deal using the linear interpolation method

Former Number 04 of Arsenal was born on May 04, 1987, in Arenys de Mar, Spain. Fabregas was the captain of the Gunners before he left for Barcelona in 2011. He first joined Arsenal on September 11, 2003; this made him the youngest first team player and the youngest first team goal scorer of the club. His main position in the team is a central midfielder or attacking midfielder. During the eight (8) years with Arsenal, Fabregas played 303 games and scored 57 goals in total. Figure 5.3 describes the evolution of F4's market value using linear interpolation method

5.3.2.2. About the deal

Selling club: Arsenal

Buying club: Barcelona

According to the Telegraph website¹⁹, Barcelona had to pay 14.5M€ (12.8m£) up front for Fabregas, with a further 14.5M€ to be paid in October 2011. 1M€ (around 0.88m£) will be released every year until the end of Fabregas's contract in 2016 (5 years). Providing

¹⁹ Appendix 5.2

Fabregas wins one Champions League and two La Liga titles over the next five years, Arsenal will receive a further 6M€. The sum of all the potential fees Arsenal could receive totals to 40M€ (35.9M€).

With Fabregas having a market value of 47.5M€ in 2011, the Gunners offered a discount of 20.5M€ as they were certain to receive the amount of 27M€.

5.3.3. Tiago Ferreira

5.3.3.1. About Tiago Ferreira



The Portuguese player was born on July 10, 1993, in Porto, Portugal. His role is a central defender, playing for Porto B. He joined FC Porto at the age of 14. In the last season with FC Porto, season 2013/2014, he played 35 games and scored one goal.

5.3.3.2. About the deal²⁰

Selling club: FC Porto

Buying club: Zulte-Waregem

On July 15 2014, Tiago Ferreira signed a contract for four years with Zulte-Waregem, in R Belgian League. FC Porto received 250,000 Euros for the transfer of the defender but kept 35% of the ownership and attached a buyback option of 2.5M€. In the summer of 2014, the prevailing value of Ferreira was 1.2M€. However, historical values for Ferreira were limited. However, his case was still of interest as assumptions were made based on similar players historical values to determine his buyback option price.

²⁰ Appendix 5.3

5.3.4. Alvaro Morata

5.3.4.1. About Alvaro Morata

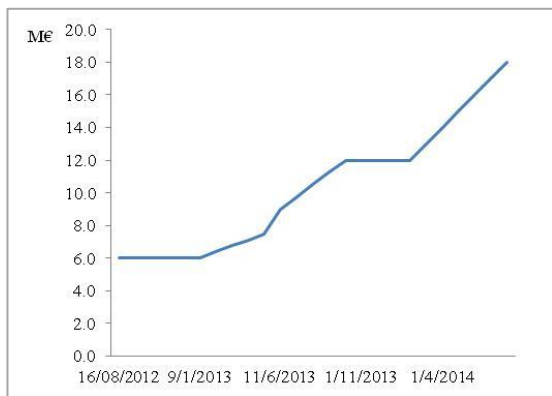


Figure 5.4: The evolution of Morata's value before the deal using the linear interpolation method

Former Number 21 of Real Madrid was born on October 23, 1992 in Madrid, Spain. The Spanish football player had always been in the young teams of Real Madrid. His main position in the team was centre forward. However, Morata can also play at the left wing and the right wing. In season 2013/2014, though Morata came to the field from the reserved bench, he played a total of 974 minutes in that season and scored nine goals. The combination of Morata's talent and young age shows Morata's potential ability to learn and quickly adapt in a new team.

Figure 5.4 illustrates Morata's market value evolution using the linear interpolation method.

5.3.4.2. About the deal

Selling club: Real Madrid

Buying club: Juventus

In July 2014, Morata joined Juventus with a five-year contract which included a buyback option,²¹ so that during the next two and three seasons (Season 2015/2016 and 2016/2017), Real Madrid can repurchase their young talent for 30M€ and 35M€ respectively. According to the news related to the deal, Real Madrid valued Morata for 30M€ without the buyback option. With the buyback option embedded, Juventus purchased Morata for only 20M€.

²¹ Appendix 5.4

5.3.5. Oriol Romeu

5.3.5.1. About Oriol Romeu



The Spanish footballer was born on September 24, 1991 in Uldecona, Spain. Romeu's main position is that of a defensive midfielder, but he can also play as a central defender. Romeu joined Barça's Youth Academy in the year 2004. He made his first appearance in La Liga on May 15, 2011.

5.3.5.2. About the deal²²

Selling club: Barcelona

Buying club: Chelsea

On August 01, 2011, Romeu was sold to Chelsea for 5M€ with a buyback option which could be exercised by Barcelona either in the year 2012 for 10M€ or in the year 2013 for 15M€. Similar to the case of Tiago Ferreira, data for Romeu was limited. Therefore, in this study assumptions were made in order to have an idea of Romeu's buyback option price.

²² Appendix 5.5

CHAPTER 6:
MODEL APPLICATIONS
AND RESULT DISCUSSIONS

6.1. The Black-Scholes-Merton (1973) Model

In this section, the B-S-M model would be applied for the three cases, namely; Carlos Vela, Cesc Fabregas and case of Tiago Ferreira, as these three scenarios are similar to that of a European call option, that is, the three situations have only one exercising date.

The inputs for the model are the following:

The current market value of the player (S), the strike price (K_T) and time to maturity (T) were attained from and reliable sport websites that have been introduced in Chapter 5.

The dividend yield (δ_S) was assumed to be zero²³.

For the value of the risk-free rate (r_f), the German Bonds interest rates were used for deals denominated in Euros, and UK Gilts were utilized for deals denominated in Pounds.²⁴

In this study, it was assumed that the volatility level (σ) is constant during the period of valuation. To derive the volatility value, the historical values of the players were used. For the players that lacked available information, the historical values of players with similar

²³ Given the difficulties to calculate δ_S , in this dissertation, for the sake of convenience, the dividend yield (δ_S) was assumed to be equal to zero. However, a non-zero dividend yield can still be considered in further research. By the nature of the option itself, as selling clubs are usually considered consisting of super stars, there is little chance that young talents have chances to play much time in the season. As such, it is difficult for them to increase their performance and value. By being sold to a less reputable club, the young players have more chance to improve their performance, thus increasing their values. The dividend yield, therefore, can be negative.

$$\delta_S = \alpha_{\text{selling club}} - \alpha_{\text{buying club}} < 0$$

A recent example is Alvaro Morata, In the semi-final matches between Juventus and Real Madrid in Champion League 2014/2015, Morata scored two goals against his old club, Real. If Morata remained with Real Madrid however, it would have been less likely that Morata would have had a chance to score in the Champion League's semi-final match as of recent May. Instead, he would have probably still sit in the bench watching his superstar colleagues CR7, Gareth Bale and Karim Benzema playing.

²⁴ Appendix 6.10

characteristics were utilized. The yearly volatility calculations of the players are calculated based on the formulas (5.1), (5.2), (5.3) and (5.4).

6.1.1. Carlos Vela

In this section, the option would be calculated based on half of Vela's value as Arsenal only sold 50% of Vela and kept the remaining 50%. According to the information available on the UK transfermarkt website, in the year 2012, the value of Vela was 7M£. Therefore, the 50% value of Vela that Arsenal sold to Real Sociedad was worth a value of 3.5M£. This value represented the spot market value of the player (S). The strike price (K_T) for Vela was 3.5M£. Furthermore, the dividend yield (δ_s) was set to be equal to zero. The maturity (T) was set equal to two (2), as Arsenal can only exercise the option after two seasons. Moreover, the risk-free rate (r_f) was equal to 0.08%, as the two year UK gilt as of August 06, 2012 was 0.08%. Lastly, the volatility (σ) was set at 32.49%. The calculations for the volatility can be found in Appendix 6.1.

Since Arsenal embedded buyback options when selling 50% of Vela to Real Sociedad, Arsenal only received 2.5M£ million, instead of the 3.5M£ which was equivalent to the 50% prevailing value of Vela. In other words, Arsenal gave up 1M£ when the buyback option was attached. The following table provides information pertaining to the value of the option in the context of the B-S-M model.

Inputs			Outputs	
S	Current market value	3.5 M£		
K_T	Buyback strike price	3.5 M£	$N1(d1)$	0.59
T	Maturity	2 years	$N1(d2)$	0.41
σ	Volatility	32.49%	Buyback option price	0.64 M£
δ_s	Dividend yield	0%		
r_f	Risk-free interest rate	0.08%		

Table 6.1: Buyback option price of Vela

With a volatility level of 32.49% per year, the value of the buyback option calculated using the B-S-M model was found to be approximately 0.64M£, this value being smaller than that of the actual amount of 1M£ valued by Arsenal.

Sensitivity to K

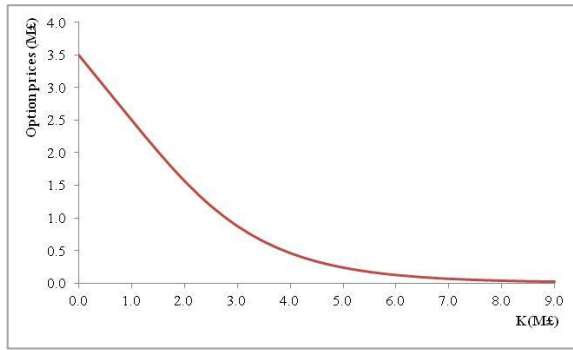


Figure 6.1: The sensitivity of the buyback option price of Vela to strike prices with $S = 3.5\text{M£}$, $T=2$, $\sigma = 32.49\%$, $\delta_S=0$, $r_f = 0.08\%$.

Strike prices and buyback option prices are inversely related. The higher the strike price, the lower the buyback option price. Figure 6.1 describes the sensitivity of the option price to the strike prices of Carlos Vela. It was found that when the strike price (K_T) changes, whilst keeping the other inputs constant, the buyback option price becomes insignificant when the strike prices are higher than 8M£.

If Arsenal and Real Sociedad had agree upon a strike price of 3M£, they would have had a 1M£ buyback option, as calculated by B-S-M model.

Sensitivity to σ

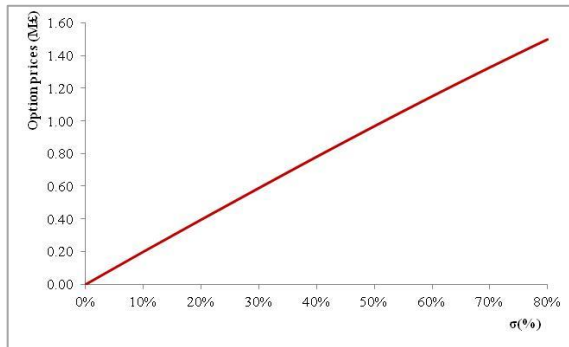


Figure 6.2: The sensitivity of the buyback option price of Vela to volatilities given that $S=K_T = 3.5\text{M£}$, $T=2$, $\delta_S=0$, $r_f = 0.08\%$.

As the volatility of Vela increases, the option price increases. This is in line with financial theory which states that the volatility and the price of an option have a positive relationship. Figure 6.2 describes the sensitivity of the buyback option price to volatilities.

When Arsenal sold Vela with a discount of 1M£ in exchange for an embedded buyback option, Arsenal expected Vela's volatility to be around 51%.

Sensitivity to S

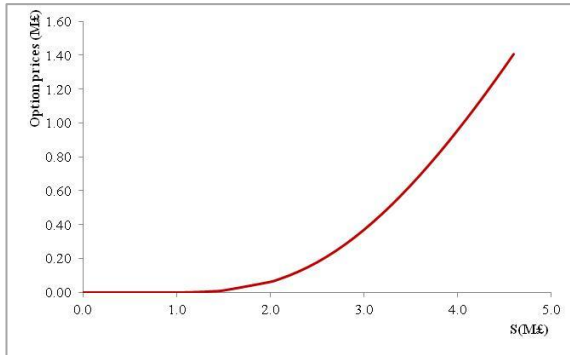


Figure 6.3: The sensitivity of the buyback option price of Vela to current market values

with $K_T = 3.5\text{M£}$, $T=2$, $\sigma = 32.49\%$, $\delta_S = 0$, $r_f = 0.08\%$.

The buyback option price increases when the current market value of Vela increases. Figure 6.3 illustrates the sensitivity of the buyback option price in relation to market values. It was found that when the current market value changes whilst keeping the other inputs constant, the price of the buyback option of Vela becomes significant when his prevailing market value was above 1.3M£.

In the summer of 2014, Arsenal had the right to repurchase 50% of the value of Carlos Vela from Real Sociedad. In order for Real Sociedad to remove the possibility of Arsenal exercising the option, that is, to be able to own 100% of Vela, Real Sociedad was required to pay Arsenal an amount of 12M£²⁵. This 12M£ comprised of 8.8M£ which was equivalent to 50% (17.6M£) of the prevailing market value of Vela at that time²⁶ plus an additional amount of 4M£ which was based on Vela's future performance.

When Arsenal sold Vela in 2012, Arsenal expected that within two years' time Vela would have a yearly volatility of 51% as previously discussed. In reality however, Vela's performance surpassed that which was projected by Arsenal. Between the years of 2012 - 2014, Vela had a yearly volatility of 73%²⁷. As such, it can be seen that Arsenal achieved a significant profit with the deal made between Real Sociedad by including the buyback option when Vela was sold in the year 2012. Arsenal only paid 1M£ for the buyback option but the buyback option gave them the flexibility of repurchasing 50% of Vela's value in the year 2014, for an amount of 3.5M£ which at the time was really worth 8.8M£.

²⁵ Appendix 6.2

²⁶ <http://www.transfermarkt.co.uk/carlos-vela/marktwertverlauf/spieler/35773>, last accessed on 01 June 2015

²⁷ Appendix 6.3

6.1.2. Cesc Fabregas

According to the information provided on the UK transfermarkt website, for the period of August 05, 2011, Fabregas was worth 47.5M£. This 47.5M£ was represented as S . The strike price (K_T) for Fabregas was set at a value of 25M£, while the dividend yield was set equal to zero (0). The level of maturity was set to three (3) years as Arsenal would only be able to exercise the option after three (3) seasons. The level of risk-free rate (r_f) was set at 0.73% as the three (3) year UK gilts as of August 05, 2011 was recorded as being 0.73%. Lastly, in this study, the calculated level of Fabrega's volatility (σ) was found to be 29.91%. This calculation was based on his historical prices and can be found in Appendix 6.4.

Due to the fact that in the year 2011 Fabregas had a prevailing market value of 47.5M£, The Gunners offered a discount of an amount of 20.5M£ as they were certain to receive 27M£ (Refer to Appendix 5.2).

Inputs			Outputs	
S	Current market value	47.5 M£		
K_T	Buyback strike price	25.0 M£	$N1(d1)$	0.94
T	Maturity	3 years	$N1(d2)$	0.85
σ	Volatility	29.91%	Buyback option price	23.86 M£
δ_s	Dividend yield	0%		
r_f	Risk-free interest rate	0.73%		

Table 6.2: Buyback option price of Fabregas

According to the calculations derived from the B-S-M, the value of the buyback option was found to be 23.86M£. This can be seen in Table 6.2 above. As such, in our opinion Arsenal seemed to have benefitted from the deal as the buyback option was worth more than the discounted value of 20.5M£.

Sensitivity to K

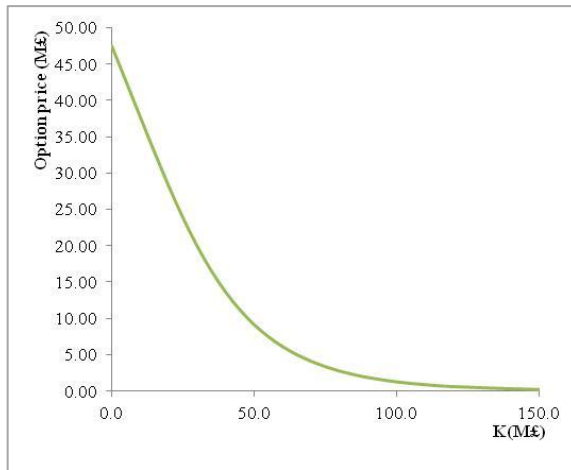


Figure 6.4: The sensitivity of the buyback option price of Fabregas to strike prices with $S=47.5\text{M£}$, $T=3$, $\sigma=29.91\%$, $\delta_S=0$, $r_f=0.73\%$.

Figure 6.4 describes the sensitivity of the prices of Fabregas's buyback option strike prices. It can be seen that there is an inverse relationship between the two variables, which is in line with the financial literature. It was found that when the strike price (K_T) changes while keeping the other inputs constant, the buyback option is worth 20.5M£ when the strike price is set around 29M£.

Sensitivity to σ

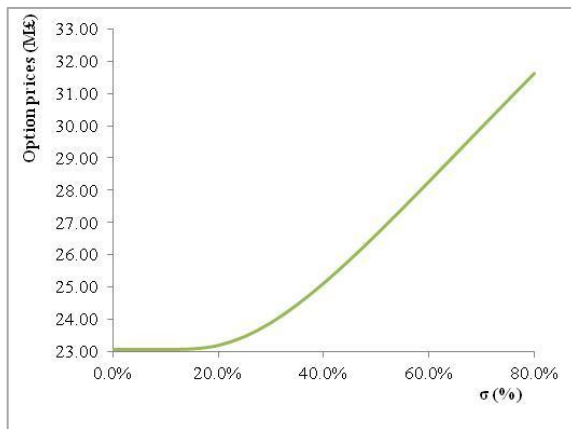


Figure 6.5: The sensitivity of the buyback option price of Fabregas to volatilities with $S=47.5\text{M£}$, $K_T=25\text{M£}$, $T=3$, $\delta_S=0$, $r_f=0.73\%$.

Figure 6.5 illustrates the sensitivity of the prices of the buyback option in relation to the levels of volatility. In the case of Fabregas, regardless of the changes in the volatility, the option price cannot fall below 23M£ as the strike price is well below the prevailing market value of Fabregas. In such a scenario, this is commonly referred to as a “deep-in-the-money option”.

Figure 6.5 illustrates, that with a yearly volatility ranging between 0-20%, the option prices are approximately 23M£. In other words, even if the value of Fabregas increased or decreased in three years following 2011 when the deal was initially made, with a volatility between 0 and 20%, the option prices were still roughly around 23M£.

Sensitivity to S

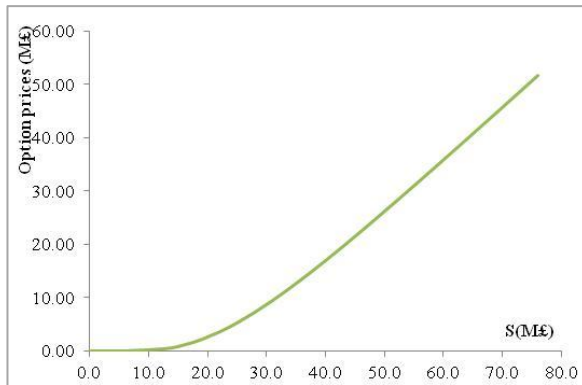


Figure 6.6: The sensitivity of the buyback option price of Fabregas to current market values with $K_T = 25\text{M£}$, $T=3$, $\sigma = 29.91\%$, $\delta_S = 0$, $r_f = 0.73\%$.

It can be seen from Figure 6.6 that the buyback option price tends to increase when the current market value of Fabregas increases. Providing that the current market values vary while holding all other inputs constant, it can be seen that the value of the buyback option becomes significant when the market value is above 6M£, given that other inputs remain constant.

Upon reviewing varying sports news concerning Fabregas deals, we are of the belief that Arsenal valued F4 of around 40M£²⁸. If F4's prevailing market value was worth 40M£ similar to that of Arsenal's valuation, and not a value of 47.5M£ that was provided by the UK transfermarkt website, the results derived from the B-S-M for the price of the buyback option would have been 17M£ as opposed to the 23.8M£ previously calculated. This would then mean, that the option price would not be able to fall below a value of 15M£ regardless of Fabregas's volatilities. The option price of 17M£ is higher than the amount that Arsenal actually paid for the "premium" of their buyback option which was around 13M£ (40-27M£). The reason for the high valuation of the buyback option was due to the very low strike price set by Arsenal, this value being 25M£. It is important to note that when the transaction took place Fabregas was considered to be a mature and qualified athlete that switched to a football club with a similar reputation unlike the traditional deals in which the football player would switch from a higher reputable club to a club with a lower reputation where by buyback deals are concerned. Therefore, Fabregas in this case could be faced with particular risks that could diminish his value. Such risks include; high competition amongst other team players, adapting to the new environment, coaches and the other members of the football team.

²⁸ Appendix 6.5

From the buyers' perspective, though Barcelona agreed to a lower a discount, Barcelona likewise Arsenal was also able to benefit from the deal. As such, by accepting to include a buyback option in the deal, Barcelona was able to purchase a high-profile player at a cheaper price at the time of the transfer.

6.1.3. Tiago Ferreira

The case of Tiago Ferreira is similar to that of Carlos Vela. In the sense that, the selling club, FC Porto, did not sell the 100% rights of Ferreira, instead, the club only sold 65% of the rights. In the summer of 2014, the prevailing value of Ferreira was 1.2M€. Therefore 65% of Ferreira's value was approximately 780,000€. However, FC Porto only received 250,000€ for the deal when the buyback option was included. In the following four (4) years, FC Porto expected that Ferreira's value would be greater than 3.8M€, so that they could repurchase 65% of Ferreira's value with a strike price (K_T) of 2.5M€. The level of maturity was set to four (4) years as FC Porto would only be able to exercise the option after four (4) seasons. The dividend yield was set equal to zero (0). The level of risk-free rate (r_f) was set at 0.14% as the four (4) year German Bunds as of July 14, 2014 was recorded as being 0.14%.

The annual volatility (σ) was set equal to 12% as the historical data for Tiago Ferreira was very limited and could not be observed through the transfermarkt website. As such, it was difficult to calculate the historical volatility of Ferreira. To replace the historical volatility, in this study the comparison method was employed. The comparison method entails using the volatility of players with similar characteristics, in this case, players with similar characteristics with that of Ferreira. Due to the fact that Tiago Ferreira was sold from FC Porto to Zulte-Waregem, a player with similar characteristics from Zulte-Waregem was taken into consideration. The three criteria that needed to be met were firstly, the player had to join Zulte-Waregem approximately at the same age as that of Tiago Ferreira when he joined, secondly, the player was required to have approximately the same market value as Ferreira when he joined Zulte-Waregem, and thirdly, the player was required to be a defender, as Ferreira. Considering the above mentioned criteria, Steve Colpaert fulfilled the necessary requirements. In the year 2008-2012 the annual volatility of Colpaert's was

found to be 12%, hence the reason a volatility of 12% was used for calculating the buyback option for Ferreira (Refer to Appendix 6.6 for detailed calculations of the volatility).

Table 6.4 below illustrates the inputs and outputs used in calculating the buyback option price of Tiago Ferreira using the B-S-M model.

Inputs				Outputs	
S	Current market value	0.78	M€		
K_T	Buyback strike price	2.5	M€	N1(d1)	0.00
T	Maturity	4	years	N1(d2)	0.00
σ	Volatility	12%		Buyback Option Value	0.00 M€
δ_s	Dividend yield	0%			
r_f	Risk-free interest rate	0.14%			

Table 6.3: Buyback option price of Tiago Ferreira

After undertaking the relevant calculations when using the B-S-M, the price of the buyback option was found to be approximately 0M€. In this case, the buyback option does not have any significant value for the selling club as the strike price was set at a considerably high price when the volatility is relatively low.

Sensitivity to K

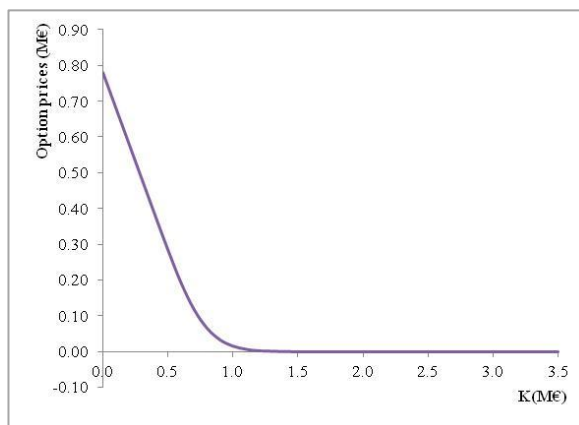


Figure 6.7 describes the sensitivity of the option price in relation to the strike prices. The price of the buyback option falls within an insignificant value range when strike prices are set above 1.2M€.

Figure 6.7: Sensitivity of the buyback option price of Ferreira to strike prices

with $S=0.78\text{M€}$, $T=4$, $\delta_s=0$, $\sigma=12\%$ and $r_f=0.14\%$

Sensitivity to σ

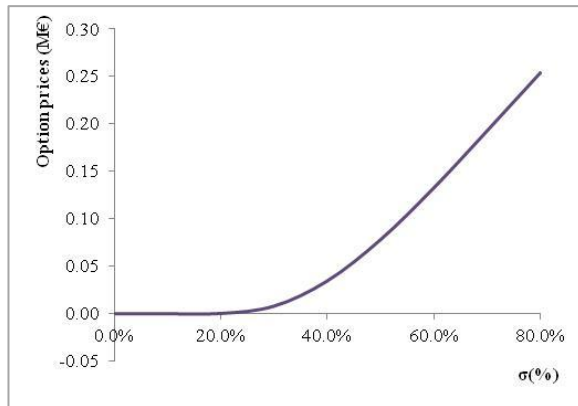


Figure 6.8: The sensitivity of the buyback option price of Ferreira to volatilities

with $S=0.78\text{M€}$, $K_T=2.5\text{M€}$, $T=4$, $\delta_S=0$ and $r_f=0.14\%$

within the next four years following the year 2014 when the deal was made, with a yearly volatility between 0-20%, it would not be worth discounting 530,000€ in return for a buyback option with a strike price of 2.5M€. The option price only becomes significant with a volatility above 20%.

By discounting 530,000€ FC Porto assumed that the future volatility of Ferreira would be around 131%.

Sensitivity to S

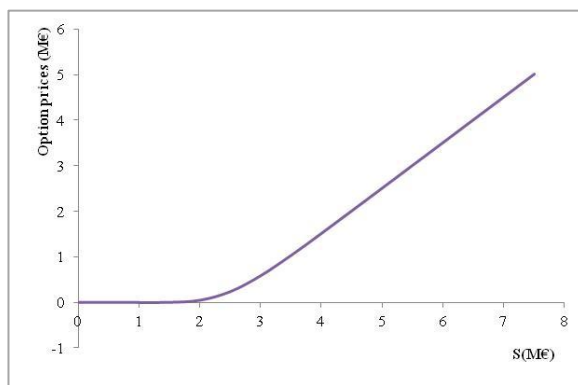


Figure 6.9: The sensitivity of the buyback option price of Ferreira to current market values

with $K_T=2.5\text{M€}$, $T=4$, $\delta_S=0$, $\sigma=12\%$ and $r_f=0.14\%$

Figure (6.8) illustrates the sensitivity of the buyback option values to different yearly volatilities. Unlike the cases of Vela and Fabregas, in the case of Ferreira there is a “deep-out-of-the money option”. Upon analysing the above figure, it can be seen that the option value would be approximately zero (0) if the level of volatility ranges between 0-20%. In other words, if Ferreira’s value was to increase

The sensitivity of the option prices to the current market values described in Figure 6.9 is similar to the cases previously discussed. The price of the buyback option becomes significant when 65% of the current market value of Tiago Ferreira is above 2M€. If the value is below this threshold, the option value is considered to be insignificant.

In short, in this section the B-S-M has been applied to determine the prices of the buyback options embedded in the deals involving Carlos Vela, Cesc Fabregas and Tiago Ferreira. These three cases are examples of B-S-M applied for at-the-money, in-the-money and out-of-the-money buyback options in the football world.

6.2. The modified Carr model

The B-S-M cannot be applied to buyback options that have more than one exercising moment, as the model is only applicable in valuing European Options. The Carr (1995) model that was modified in Chapter 4 of this study is applied to such cases.

This section applies the modified model for two scenarios that is, in the cases of Alvaro Morata and Oriol Romeu. Morata is a striker and has a relatively high expected volatility and Romeu, in contrast, is a defender and has a low expected volatility during the life of buyback options.

The sources for the inputs of the models were similar to those of the B-S-M model. The current market value (S); strike prices (K_1 and K_T); the first exercising moment (T_1) and time to maturity (T) were attained from and reliable sport websites. The dividend yield (δ_S) was assumed to be zero. For the value of the risk-free rate (r_f), the German Bonds interest rates were used for deals denominated in Euros. In this section, it was again assumed that the volatility level (σ) applied for the modified Carr model is constant during the period of valuation.

6.2.1. Alvaro Morata

The modified Carr model was applied to the case of Alvaro Morata as there are two moments when the option can be exercised. These two moments can occur either after the second or third season with two different pre-deterministic prices.

The inputs for the model are as follow:

The current market value (S) of Morata was found to be 30M€. Though the European transfermarkt website did not provide Morata's market value of 30M€, various football news relating to this deal stated that Real Madrid would demand 30M€ for Morata without

the buyback option. The first strike price (K_1) was set at 30M€ after two (2) seasons and the second strike price (K_T) was set at a 35M€ after three (3) seasons. The first exercising moment is two years from the transfer date as such, T_1 was set equal to two (2), while the second exercising moment (T) which was also the duration of the deal was set equal to three (3). The dividend yield (δ_S) as explained in the B-S-M Model is assumed to be zero (0), while the risk free rate (r_f) was equal to 0.05% which is the rate of the three year German Bunds on the date of the deal. The volatility level (σ) was set at 15.2% (Refer to Appendix 6.7 for detailed calculations). This volatility value of 15.2% was based on Morata's historical price when he played for Real Madrid.

Table 6.6 illustrates the price of Morata's buyback option

Modified Carr model		
Inputs		
S	Value of the player	30 M€
K_1	Strike price on the first exercising date	30 M€
K_T	Strike price at maturity	35 M€
T	Time to maturity	3 years
τ_1	Time until the first exercising date	2 years
τ_2	Time between the first exercising date and maturity	1 years
δ_S	Dividend yield of S	0%
r_f	Risk-free rate	0.05%
δ	Dividend yield	-0.05%
σ	Volatility	15.2%
Outputs		
P^*	The first trigger ratio	0.871
S^*	The first trigger value	30.49 M€
$N_1(d_{1\tau_1})$		0.5144
$N_1(d_{2\tau_1})$		0.4290
$N_2(-d_{1\tau_1}, d_{1T}, -\rho)$		0.0292
$N_2(-d_{2\tau_1}, d_{2T}, -\rho)$		0.0234
$E_2(\tau)$	Buyback option price	2.63 M€

Table 6.4: Buyback option price of Alvaro Morata

$P^* = 0.871$ can be interpreted as if Morata's value exceeds 30.49M€ after two seasons, Real Madrid should exercise the option. If on the first exercising date Morata's value does not exceed 30.49M€, Real Madrid should not exercise the option and leave it alive until the date of its maturity to decide.

According to the modified Carr model, the buyback option is expected to be worth 2.63M€. This means that Real Madrid could have sold Morata for around 27M€. However, Real Madrid only sold Morata for a price of 20M€, and discounted an amount of 10M€ for including the buyback option, which then gave the buyer, Juventus, a good deal. Although Real Madrid discounted more than they should, it is of the belief that Real Madrid would still greatly benefit by attaching the buyback option in the deal. If Morata remained with Real Madrid however, it would have been less likely that Morata would have had a chance to score in the Champion League's semi-finals and final. Due to the recent success in the last season, along with the higher chances of now being able to play in higher profile matches in the future, Morata's value is expected to increase significantly. Despite the value of Morata potentially increasing in the future, Real Madrid is only required to pay 30M€ in the year 2016 or 35M€ in the year 2017 to bring him back to Santiago Bernabeu.

Sensitivity to σ

Figure 6.10 illustrates the sensitivity of buyback option price of Morata to different levels of volatility.

By discounting 10M€ for Juventus, Real expected that Morata's volatility in the following two years would be 65%.

In the modified Carr model, a new diagram is used to describe the sensitivity of the first trigger ratio (P^*), which is derived by dividing the first trigger value S^* by the strike price at maturity (K_T), to the changes in the model's inputs. P^* is an important result in this study as it demonstrates the trigger ratio to exercise the option on the first exercising date. Figure 6.11 shows the changes in P^* when volatility varies.

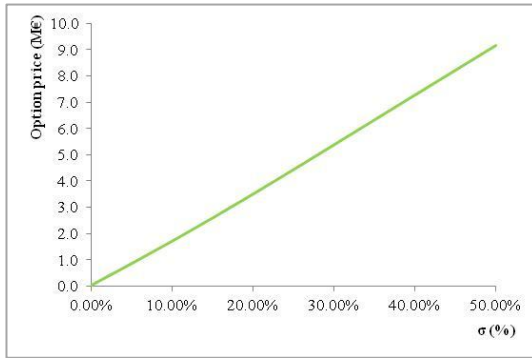


Figure 6.10: The sensitivity of the buyback option price of Morata to volatilities

with $S=30M€$, $K_1 = 30M€$, $K_T = 35M€$, $T_1 = 2$, $T=3$, $\delta_S = 0$ and $r_f = 0.05\%$

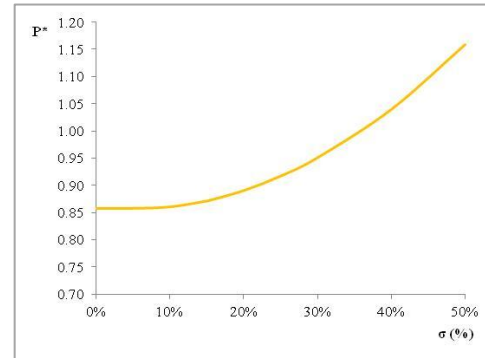


Figure 6.11: The sensitivity of P^* of Morata to volatilities

As volatility increases, the first trigger ratio (P^*) and the first trigger value (S^*) also increase. However, P^* never reaches below 0.86 and S^* never falls below 30M€ which is equal to the strike price on the first exercising date.

Table 6.7 illustrates the results for different P^* and S^* for varying level of volatilities σ

σ	0%	5%	10%	15.2%	20%	30%	40%	50%
P^*	0.86	0.86	0.86	0.87	0.89	0.95	1.04	1.16
S^* (M€)	30.00	30.00	30.09	30.49	31.15	33.27	36.38	40.57

Table 6.5: The sensitivity of P^* and S^* of Morata to volatilities

Sensitivity to K_T

In this section the relationship between the option price, P^* and the strike price at maturity (K_T) is interesting to analyse when using the modified Carr model.

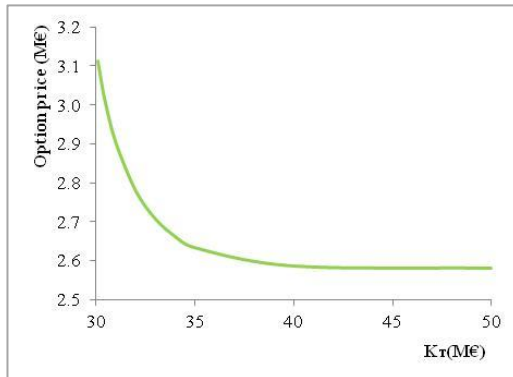


Figure 6.12: The sensitivity of the buyback option price of Morata to K_T

with $S=30M€$, $K_1 = 30M€$, $T_1 = 2$, $T=3$, $\delta_S = 0$, $\sigma = 15.2\%$ and $r_f = 0.05\%$

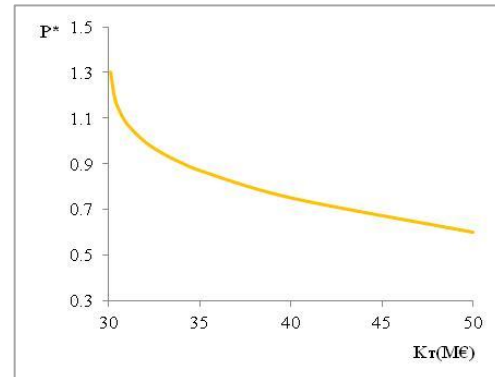


Figure 6.13: The sensitivity of P^* of Morata to K_T

When K_T increases, it is less likely that the option will be kept until maturity, as it is rational for option holders to exercise the option on the first exercising date, providing that the current market value of Morata does not fall below S^* . Figure 6.12 shows that when K_T is growing, the value of the option given by the modified Carr model tends to approach the value of an equivalent European Option with a strike price of K_1 and a maturity of T_1 . Figure 6.13 and Table 6.8 highlight the inverse relationship between the changes of P^* , S^* and K_T . When K_T increases, the first trigger ratio (P^*) and the first trigger value (S^*) decrease, thus encouraging the option holder to exercise the option on the first exercising date providing that $S_1 > S^*$.

K_T (M€)	30	31	32	33	34	35	40	50
P^*	1.30	1.08	1.00	0.94	0.90	0.87	0.75	0.60
S^* (M€)	39.23	33.37	31.89	31.17	30.75	30.49	30.06	30.00

Table 6.6: The sensitivity of P^* and S^* of Morata to K_T

Sensitivity to K_1

In comparison to the B-S-M, one new feature of the modified Carr model involves analysing the sensitivity of the buyback option price to the first strike prices (K_1). As described in Figure 6.14 the values of K_1 have an impact on the prices of the buyback options as well as the first trigger ratio (P^*) and the first trigger value (S^*) on the first exercising date as illustrated in Figure 6.15 and Table 6.9.

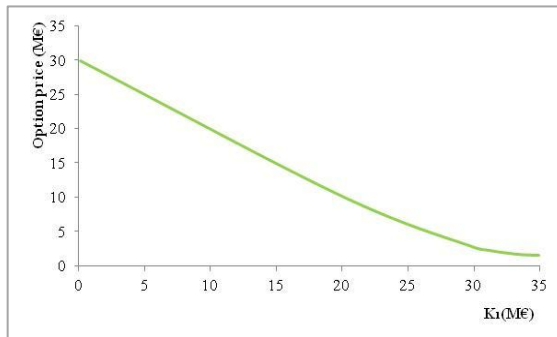


Figure 6.14: The sensitivity of the buyback option price of Morata to K_1

with $S=30\text{M€}$, $K_T = 35\text{M€}$, $T_1 = 2$, $T=3$, $\delta_S = 0$, $\sigma = 15.2\%$ and $r_f = 0.05\%$

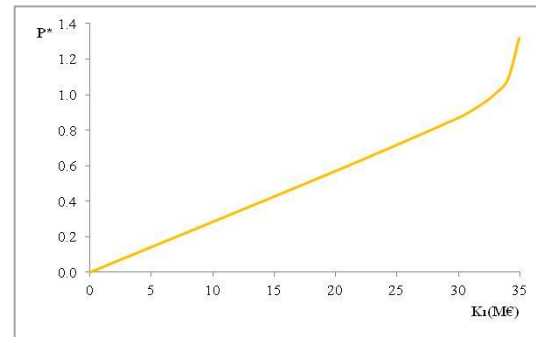


Figure 6.15: The sensitivity of P^* of Morata to K_1

The lower the first strike price (K_1), the higher the price of the buyback option and the greater the chances of the option being exercised on the first exercising date. When K_1 reaches K_T , the value of the buyback option converges to its European counterpart with a maturity of T and a strike price of K_T . In such situations, it is less likely that the option will be exercised on the first exercising date as it is more rational for the option holder to wait until its maturity to have a chance to pay the same amount at a later date.

K_1 (M€)	0.10	20	30	31	32	33	34	35
P^*	0.00	0.57	0.87	0.91	0.95	1.01	1.09	1.32
S^* (M€)	0.10	20.00	30.49	31.82	33.36	35.28	38.15	46.14

Table 6.7: The sensitivity of P^* and S^* of Morata to K_1

Sensitivity to S

The current market value of the football player (S) does not affect the first trigger ratio (P^*). The P^* stays the same regardless of the changes of S . P^* is only dependent on the volatility, time to maturity, the time from the valuation date to the first exercising date, and the risk-free rate.

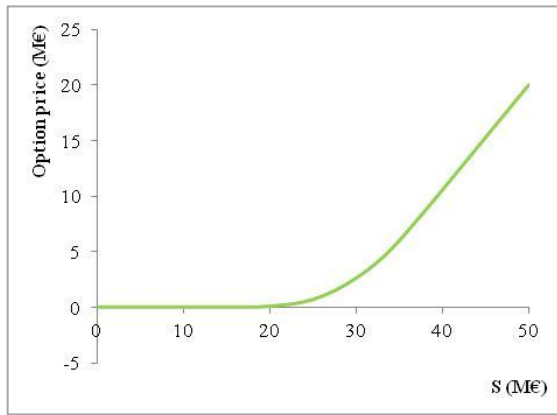


Figure 6.16: The sensitivity of the buyback option price to current market value of Morata with $K_1 = 35\text{M€} = 30\text{M€}$, $K_T = 35\text{M€}$, $T_1 = 2$, $T=3$, $\delta_S = 0$, $\sigma = 15.2\%$ and $r_f = 0.05\%$

Figure 6.16 illustrates the relationship between the current market values (S) and the buyback option prices.

Analysing this figure shows that when Morata's current market value is above 15M€ the buyback option value becomes significant.

6.2.1. Oriol Romeu

The modified Carr model was applied to the case of Romeu as there are two moments that the option can be exercised. These two moments can occur either after the first or second season when Romeu was sold from Barcelona to Chelsea with two different pre-deterministic strike prices.

The inputs for the model are as follow:

As there was limited amount of data available, for the sake of convenience, in this study, it was assumed that the current market value of Romeu (S) was 10M€, this value being equal to the first strike price of 10M€. The second strike price (K_T) was set at 15M€. The first exercising moment is one year from the transfer date as such, T_1 was set equal to one (1), while the second exercising moment (T) which also the duration of the deal was set equal to two (2).

The dividend yield (δ_S) as explained in the B-S-M Model is assumed to be zero (0), while the risk free rate (r_f) was equal to 0.74% which is the rate of the two year German Bunds on the date of the deal. The volatility level (σ) was set at 6.1%.

Similar to the case of Tiago Ferreira, data for Romeu was limited. As such, to replace historical volatility the comparison method was once again employed. Since Romeu was sold from Barça to Chelsea, a player with similar characteristics to that of Romeu in Chelsea was considered. Again, the three criteria that needed to be met were firstly, the player had to join Chelsea at approximately the same age as Romeu, secondly, the player was required to have approximately the same market value as Romeu when he joined Chelsea and thirdly, the player was required to have a main position as a defensive midfielder, and a central midfielder as his side position. Having considered all defensive midfielders and defenders of Chelsea, John Obi Mikel was found to be the most suitable

candidate. After two years of joining Chelsea, Mikel's annual volatility was 6.1%.²⁹ This 6.1% that was applied to Romeu was an interesting result, as this level of volatility was much lower than the previous case of Morata.

Using the above inputs, when calculating the buyback option under the modified Carr (1995) model the value of the option was found to be around 0.3M€, as illustrated in Table 6.8 in the following page. One of the reasons for the low valuation of the buyback option was due to the low expected volatility of Romeu. In reality however, Barça sold Romeu to Chelsea for 5M€, meaning a discount of 5M€ was applied in exchange for attaching the buyback option. Chelsea therefore seemed to have benefited significantly from this deal.

The first trigger ratio (P^*) result was found to be 0.67 therefore making the first trigger value (S^*) 10M€. As such, this suggests that if in the year 2012 Romeu's value reached above 10M€, Romeu could have been repurchased, and Barça did not have to wait until the year 2013 to exercise the option.

²⁹ Mikel joined Chelsea in 2007 when he was 20 years old, the same age with Romeu when he joined Chelsea in 2011. Mikel starting value at Chelsea was 11M€, this is approximately equal to our assumption of 10M€ for Romeu. Both Mikel and Romeu are defensive midfielders. Other players that can be compared with Romeu can be Ramires (joined Chelsea at the age of 23 with 10.5M€) and César Azpilicueta (joined Chelsea at the age of 23 with 9.5M€). Ramires and César Azpilicueta's yearly volatilities are 7% and 12% respectively. Refer to Appendix 6.8 and Appendix 6.9 for volatility calculations.

Modified Carr model

Inputs		
S	Value of the player	10 M€
K_1	Strike price on the first exercising date	10 M€
K_T	Strike price at maturity	15 M€
T	Time to maturity	2 years
τ_1	Time until the first exercising date	1 years
τ_2	Time between the first exercising date and maturity	1 years
δ_S	Dividend yield of S	0%
r_f	Risk-free rate	0.87%
δ	Dividend yield	-0.87%
σ	Volatility	6.10%
Outputs		
P^*	The first trigger ratio	0.667
S^*	The first trigger value	10.000 M€
$N_1(d_{1\tau_1})$		0.5687
$N_1(d_{2\tau_1})$		0.5446
$N_2(-d_{1\tau_1}, d_{1T}, -\rho)$		0.0000
$N_2(-d_{2\tau_1}, d_{2T}, -\rho)$		0.0000
$E_2(\tau)$	Buyback option price	0.29 M€

Table 6.8: Buyback option price of Oriol Romeu

Sensitivity to σ

The figures 6.17 and 6.18 describe the sensitivity of the option price and the first trigger ratio (P^*) to volatility respectively. The curvature of the two figures are similar to the previous case of Morata.

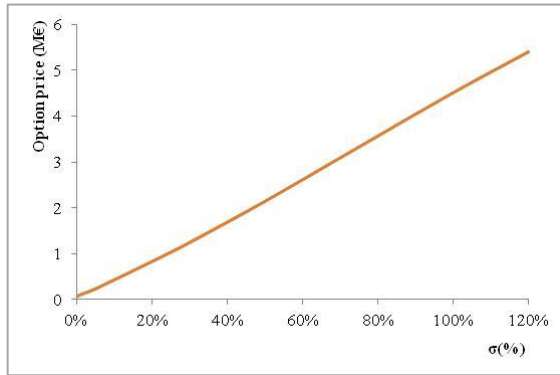


Figure 6.17: The sensitivity of the buyback option price of Romeu to volatilities
with $S = 10M\text{€}$, $K_1 = 10M\text{€}$, $K_T = 15M\text{€}$, $T_1 = 1$, $T = 2$, $\delta_S = 0$ and $r_f = 0.87\%$

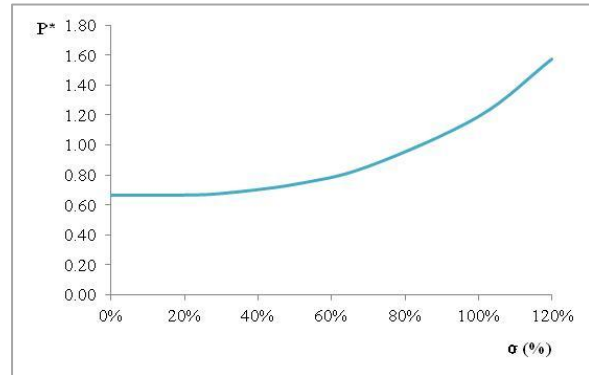


Figure 6.18: The sensitivity of P^* of Romeu to volatilities
with $S = 10M\text{€}$, $K_1 = 10M\text{€}$, $K_T = 15M\text{€}$, $T_1 = 1$, $T = 2$, $\delta_S = 0$ and $r_f = 0.87\%$

When the level of volatility increases, both the option price and the first trigger ratio (P^*) increase. Moreover, when the volatilities are equal to 7% and 12% like that of Ramires and César Azpilicueta in their first two years at Chelsea, the prices of the buyback option are 0.3M€ and 0.5M€ respectively. The price of the option would only fall around 4.5M€ and 5M€, this being equal to the amount that Barça gave up when Romeu's volatility ranges between 100% and 120%.

Table 6.11 below highlights the sensitivities of P^* and S^* to Romeu's volatilities

σ	0%	5%	6%	7%	12%	20%	30%	50%	70%	100%	120%
P^*	0.67	0.67	0.67	0.67	0.67	0.67	0.68	0.74	0.86	1.19	1.57
$S^*(M\text{€})$	10.00	10.00	10.00	10.00	10.00	10.02	10.18	11.08	12.86	17.82	23.55

Table 6.9: The sensitivity of P^* and S^* of Romeu to volatilities

Sensitivity to K_T

Figures 6.19, Figure 6.20 and Table 6.12 below describe the sensitivity of the option prices, the first trigger ratio (P^*) and the first trigger value (S^*) to the strike prices at maturity K_T .

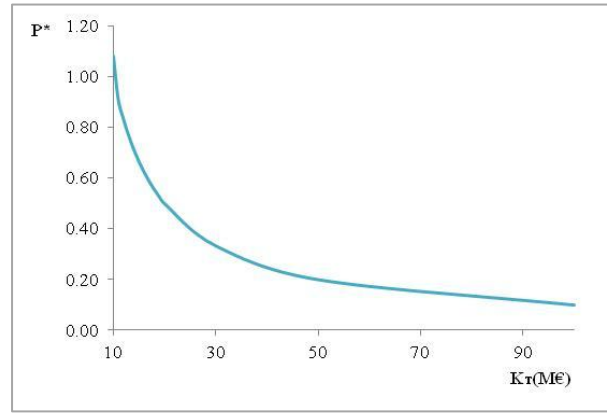
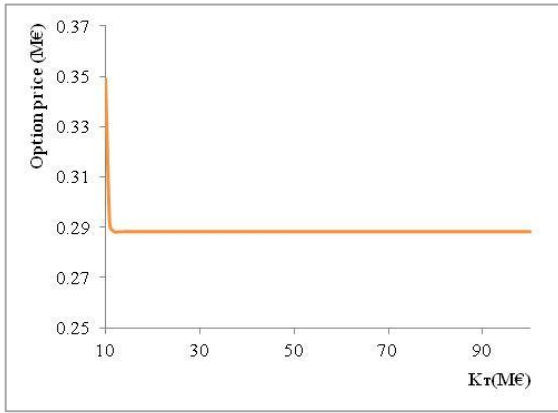


Figure 6.19: The sensitivity of the buyback option price of Romeu to K_T
 with $S = 10\text{M€}$, $K_1 = 10\text{M€}$, $T_1 = 1$, $T = 2$, $\sigma = 6.1\%$, $\delta_S = 0$ and $r_f = 0.87\%$

K_T (M€)	10	11	12	13	14	15	20	30	50	100
P^*	1.08	0.91	0.83	0.77	0.71	0.67	0.50	0.33	0.20	0.10
S^* (M€)	10.89	10.02	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00

Table 6.10: The sensitivity of P^* and S^* of Romeu to K_T

It is worth mentioning that the option prices significantly decrease when K_T goes beyond K_1 and the option prices quickly approach the value of equivalent European Option with a strike price of K_1 . This is due to the fact that the volatility in the case of Romeu is quite small. In the case of Romeu, it is more likely for the buyback option to be exercised on the first exercising date provided that $S_1 > S^*$, instead of waiting to the maturity date.

Sensitivity to K_1

According to Figure 6.21, the lower the value of the first strike price (K_1), the higher the price of the buyback option. In the case of Romeu, with such inputs, when K_1 exceeds 10M€ the price of the buyback option becomes insignificant.

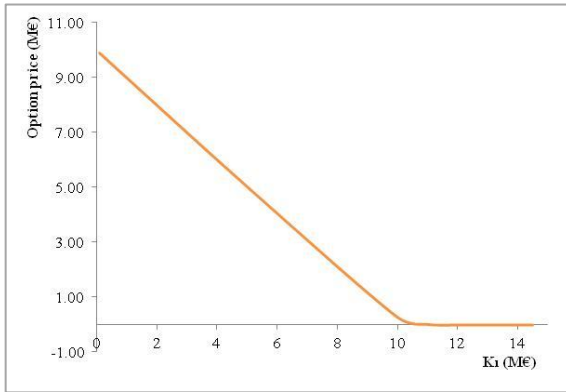


Figure 6.21: The sensitivity of the buyback option price of Romeu to K_1

with $S = 10\text{M€}$, $K_T = 15\text{M€}$, $T_1 = 1$, $T = 2$, $\sigma = 6.1\%$, $\delta_S = 0$ and $r_f = 0.87\%$

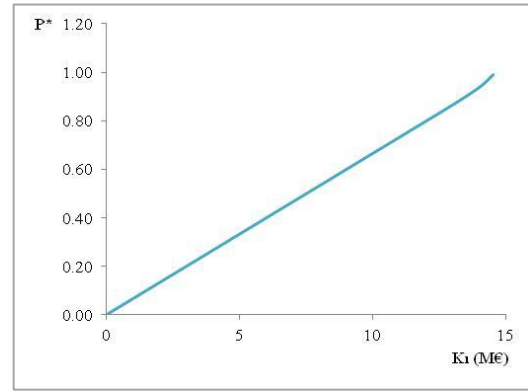


Figure 6.22: The sensitivity of P^* of Romeu to K_1

Figure 6.22 and Table 6.13 show that in Romeu's case, the first trigger value (S^*) almost has the same values as the first strike price (K_1). In other words, if the market value of Romeu on the first exercising date exceeds the first strike price (K_1), buyback option should be exercised.

K_1 (M€)	0	5	10	11	12	13	14	14.5
P^*	0.00	0.33	0.67	0.73	0.80	0.87	0.94	0.99
S^* (M€)	0	5.00	10.00	11.00	12.00	13.00	14.09	14.85

Table 6.11: The sensitivity of P^* and S^* of Romeu to K_1

Sensitivity to S

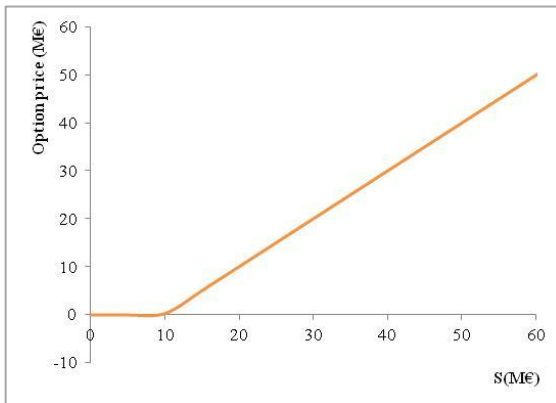


Figure 6.23: The sensitivity of the buyback option price of Romeu to current market values

with $K_1 = 10\text{M€}$, $K_T = 15\text{M€}$, $T_1 = 1$, $T = 2$, $\sigma = 6.1\%$, $\delta_S = 0$ and $r_f = 0.05$

Analysing the figure 6.23, shows that when Romeu's current market value is above 10M€ the buyback option value becomes significant.

CHAPTER 7:

CONCLUSIONS AND SUGGESTIONS

7.1. Summary of findings

Among many financial models in finance theories, two models were found to be able to apply to valuing buyback options in football contracts, these models providing relatively meaningful results. Black-Scholes-Merton Model was implemented to value buyback options that have only one possible exercising date in the future. For buyback options that have two exercising dates in the future, the Carr (1995) model was employed for the purpose of valuing buyback options that are either exercised at the end of the life of the option or at a predefined date during its life. As such, in this study, the Carr model was modified in such a way that the first exercising date of the option was no longer limited to the mid-life period of the option. The advantage of using the Black-Scholes and Merton model was that it was very quick, easy and straightforward to apply, with the inputs being easily attainable through the sports news and Transfermarkt websites. The modified Carr model was also advantageous as it was a straightforward model to apply. For most cases, both models produced results of buyback options that were lower than real world cases but still meaningful.

With the modified Carr model in this study, through the sensitivity analysis, it was found that when the value of K_T increases, Pseudo American Option values converge to its European counterpart with strike price K_1 and maturity T_1 . When K_1 approaches K_T , Pseudo American Option value converges to the equivalent European Option with strike price K_T and maturity T since it is more rational for option holder to wait until its maturity to have a chance to pay the same amount at a later date as it was already assumed for all cases of buyback options that dividend yield of the market value the player equals to zero.

Within the scope of this dissertation, five cases of buyback options were analysed. The first three scenarios included those of Carlos Vela, Cesc Fabregas and Tiago Ferreira, where there was only one possible date at the end of the option life to exercise. There were a few differences amongst each scenario. Case in point, Carlos Vela's buyback option could be considered an at-the-money option, whereas Cesc Fabregas option was a deep in-the-money option, while Tiago Ferreria was a deep out-of-the-money option. The results from

the B-S-M were more in line with that of Vela and Fabregas in reality, in which case the seller is Arsenal. In the case of Tiago Ferreira, which was a deep out of the money option, B-S-M calculation suggested that such the buyback option price was not significant. With relation to the sensitivity of buyback options value to volatility, each case provided a different type of curve shape. The last two cases of Alvaro Morata and Oriol Romeu had two possible dates to exercise the buyback option. The former case, Morata, is a striker and has a relatively high expected volatility and the latter case, Romeu, is a defender and has a lower expected volatility. In both cases, buyback option prices were found to be lower than in reality.

7.2. Main contributions

Main contributions of this study to the world of academic knowledge lay on these aspects:

Firstly, to the best of our knowledge, the study explored a problem that had not been researched to date. In today's society, buyback options are quite popular in the world of football as it has significant economic value, mainly as it involves the most famous football clubs worldwide. Selling clubs had experienced a numerous set of advantages as a result of buyback options. In this study, besides the case of Fabregas, it was evident that the selling clubs were discounting by far more than what were derived from the Black Scholes and Merton model and the modified Carr model. However, though the selling clubs were discounting more than what the models suggested, the selling clubs had been seen to be experiencing numerous benefits over the years. Reasons being, these selling clubs were usually the more renowned clubs worldwide, whose main teams were full of popular stars. Therefore, by selling their talents along with including an embedded buyback option, they allow the young footballers' time to enhance their performance, while at the same time providing themselves with the flexibility of buying back the players at a cheaper price in the future. For buying clubs, buyback options enabled them to save significant amount of money when purchasing young talents. This study was done to determine whether or not the buyback option possesses significant value and therefore it needs to be priced fairly so that both parties will have a sense of its worth. This contributed to much of the informed decision process of both clubs.

Secondly, a very simple tool from financial theory was applied to solve a real world problem. B-S-M option model can now extend its application to the world of football.

Thirdly, in this study, the Carr (1995) model was successfully modified to fit the needs of valuing the buyback option. The model was modified in such a way that the first exercising date is no longer limited to the midpoint of the time to maturity, but can now be exercised at any one point during its maturity life, providing that it has been predefined. The model was also modified so that two different deterministic strike prices could be set. This modified model will be useful in assisting researchers who would like to pursue the topic of valuing buyback options in the future or other similar types of real options in a real-world context.

Fourthly, this dissertation is hoped to provide more information in terms of simple valuing tools used in football for football managers, interested investors, football players and football fans worldwide.

Fifthly, the method of linear interpolation was employed to deal with very discrete variable observations like that of football player's values. By the nature of football transfer market, the value of football players is only updated once or twice per year. However, it was of belief that value of football players in fact is a continuous variable that changes day to day, depending on their level of training each day, their performance in each match and their ability to attract the crowds based on their image and style. This method of linear interpolation can also be applied to other kind of data that have the same features.

Lastly, the authors of this study would like to inspire other academic researchers to explore a very new but interesting area of research which are real options and real options applied in football industry in general; and in buyback option in football contracts in particular.

7.3. Limitations and suggestions for further research

Due to time constraints, in this study, only buyback option valuations where there are either one or two predefined exercising dates in the future were focused. In reality however, there can be instances whereby there might be three possible predefined exercising dates during

the life of buyback option. Case in point, the study of Dani Carvajal from Real Madrid to Bayern Leverkusen in 2012. It should be noted that Carr (1995) model can be used to value such an option; however, applying the model in such a case would require an intensive knowledge of mathematics. As previously mentioned, due to time constraints, such application in this study was not presented in this study. The authors of this study are however looking forward to cooperating with interested researchers and publishing it in our futures research.

An assumption of the B-S-M model and the Carr model that is far from reality is that there is constant volatility of the underlying asset movement, especially when valuing buyback options of football players. As already be known, the value of football players' changes overtime with very different volatility that is nondeterministic, dependent on each stage of his career. Normally, the volatility of the player will largely depend on his age. When the players are young, his volatility tends to be higher than when he grows older and matures in his career. Within the scope of this dissertation, it was assumed that the volatility of the player was constant and would be the same as his historical volatility or the same as his counterparts in the new team. However, this is in fact not true in reality. Therefore, it is suggested further research on this theme should focus on solving the problem of football players' volatility using a stochastic volatility model. Such models have been existed in financial options such as Heston (1993) model or recently Faria and Correia da Silva (2014) model. There was an attempt from the authors of this study to use these two models to approach this topic. However, more time would be needed. Therefore, within the scope of this dissertation, the applications of these two models were not mentioned. The authors of this study are looking forward to presenting them in our further work and we welcome any researchers who are interested in the topic to cooperate.

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APPENDICES

Appendix 5.1: Official Carlos Vela's Deal released by Real Sociedad (in Spanish)

Source: <http://www.realsociedad.com/document/view/spa/156467/carlos-vela---me-siento-como-en-casa--como-uno-mas->, last accessed on May 29th 2015

El futbolista ha estado arropado en la presentación por el presidente Jokin Aperribay y el director deportivo Loren. El mandatario txuri urdin ha apuntado que el fichaje de Carlos Vela asciende a tres millones de euros (around 2.5M£³⁰) y que el Arsenal se guarda una opción de recompra por cuatro millones (around 3.5M£³¹) ejecutable al final de cada temporada y sólo aplicable por razones exclusivamente deportivas. En caso de un traspaso a un tercer club, los dos clubes se repartirían los ingresos al 50% siempre y cuando la cantidad del traspaso superase los tres millones de euros.

Appendix 5.2:

Arsenal agree £35m Cesc Fàbregas deal with Barcelona

<http://www.telegraph.co.uk/sport/football/teams/arsenal/8701337/Arsenal-agree-35m-Cesc-Fabregas-deal-with-Barcelona.html> Last accessed on 28 April 2015

Published on 14 August 2011

“Barcelona were last night claiming they will only have to pay €14.5 million up front for Fabregas, with a further €14.5 million to be paid in October. It is then understood that

€1 million will be released every year until the end of his contract in 2016. Providing Fabregas wins one Champions League and two La Liga titles over the next five years, Arsenal will receive a further €6 million. This takes the overall potential fee to €40 million (£35 million)”.

³⁰ Author's note

³¹ Author's note

Cesc Fabregas on Arsenal's radar in buy-back as Barcelona star set to leave... with Manchester City and Chelsea also interested

<http://www.dailymail.co.uk/sport/football/article-2644744/Cesc-Fabregas-puts-Arsenal-Chelsea-Manchester-City-red-alert-told-leave-Barcelona-summer.html>, last accessed on 07 July 2015

Published on 31 May 2014

In selling Fabregas to Barcelona, the north London club secured a first-option clause on their former captain.

It is also understood that the clause is set at a fixed price of **£25million**.

Appendix 5.3:

Tiago Ferreira: «Saí do FC Porto para continuar a evoluir»

Original news in Portuguese at <http://www.maisfutebol.iol.pt/zulte-waregem/entrevista/tiago-ferreira-sai-do-fc-porto-para-continuar-a-evoluir>, last accessed on 16 April 2015

Published on 20 July 2014

“Tiago Ferreira chegou ao FC Porto em 2007 e parte sete anos mais tarde. Apontado como um dos grandes valores da formação portista, nunca jogou pela equipa principal (apenas duas presenças como suplente não utilizado). O central sentiu que era o ciclo tinha terminado.

A um ano do final do contrato, o FC Porto vendeu o seu passe por um valor relativamente baixo mas ficou com parte dos direitos económicos – 35 por cento – e negociou com o Zulte-Wagerem uma cláusula de recompra por 2,5 milhões de euros.”

Tiago Ferreira no Zulte Waregem por 300 mil euros

Original news in Portuguese at:

http://www.dn.pt/desporto/porto/interior.aspx?content_id=4024843, last accessed on 16 April 2015

Published on 13 July 2014

“Central da equipa B do FC Porto vendido ao emblema belga, sendo que os dragões ficam com opção de recompra e 50% do passe.

O FC Porto selou este domingo a transferência do central Tiago Ferreira, de 21 anos, para os belgas do Zulte Waregem, por 300 mil euros. O jogador que foi vice-campeão mundial de sub-20 em 2011, na Colômbia, vai assinar um contrato válido até 2018.

Os dragões ficam, contudo, com o direito de fazer regressar o jogador em qualquer altura, em troca de uma compensação financeira, e ficarão ainda com 50% do passe de Tiago Ferreira”

Appendix 5.4:

<http://www.juventus.com/wps/wcm/connect/06baf795-883a-4ca9-80e9-beba5def3cfd/comunicato+19072014+morata+eng.pdf?MOD=AJPERES>, last accessed on 30 June 2015



www.juventus.com

press release

AGREEMENT WITH REAL MADRID FOR THE DEFINITIVE ACQUISITION OF THE PLAYER ALVARO MORATA

Turin, 19 July 2014 – Juventus Football Club S.p.A. announces that the agreement with Real Madrid Club de Fútbol for the definitive acquisition of the registration rights of the player Alvaro Borja Morata Martin has been reached for a consideration of € 20 million payable in three financial years.

Furthermore, the agreement provides to Real Madrid the option right to buy back the registration rights of the player, at the end of the 2015/16 season or at the end of the 2016/17 season, at predetermined values, up to a maximum of € 30 million, based on the number of matches played by Alvaro Morata with Juventus in the relevant season.

Juventus and the player have signed a 5-year contract of employment.

Real Madrid to Arsenal: €30 million for Alvaro Morata

<http://www.insidespanishfootball.com/100129/real-madrid-to-arsenal-e30-million-for-alvaro-morata/> Last accessed on 02 July 2015

Published on March 8th, 2014

“Real Madrid have reportedly informed Arsenal that if they wish to pry the Spanish U21 striker, Alvaro Morata away from the Santiago Bernabeu, it will cost them a total of €30 million (£25m).

According to English daily, Metro, Morata has already given his consent to make the move to the Emirates, with the two sides even agreeing to personal terms, but the two teams must now negotiate what will be a very costly transfer fee, with the Spanish giants demanding some €30 million for one of their top youth products in recent memory.

The high price tag, **according to the report, is due to the fact that the transfer will not include a buy-back option**, which is often placed in the contract in the sale of Real Madrid youth products

Juventus risk losing Morata to Madrid

<http://www.goal.com/en/news/11/transfer-zone/2015/06/28/13120832/juventus-risk-losing-morata-to-madrid-lawyer>, Last accessed on 14 July 2015

Published on Jun 28, 2015

The Spaniard signed a five-year contract with the Bianconeri last summer but Los Blancos retain the right to buy the forward back in 2016 or 2017

The Spain international joined the Italian champions from Los Blancos for **€20 million** last summer, penning a five-year contract with the Old Lady.

However, Madrid retained the right to buy Morata back, for **€30m in 2016 or €35m in 2017**

Appendix 5.5: Oriol Romeu signs for Chelsea

Official news on Barça official webpage:
<http://arxiu.fcbarcelona.cat/web/english/noticies/futbol/temporada11-12/08/04/n110804118651.html>, last accessed on 01 June 2015

“FC Barcelona and Chelsea have reached agreement for the transfer of Oriol Romeu for **five million Euros**.

The deal includes a buy-back clause of **10 million Euros** after **a year**, rising to **15** after **the second year**”.

Appendix 6.1: Carlos Vela historical volatility

1 month volatility				9.38%	1 year volatility			32.49%
Date	Value (M£)	Return		Date	Value (M£)	Return		
19/11/2005	0.88		transfe rmarkt	1/5/2009	4.7375	-0.008	interpolated value	
1/1/2006	1.74	0.682	interpolated value	1/6/2009	4.70	-0.008	interpolated value	
2/2/2006	2.60	0.402	transfe rmarkt	1/7/2009	4.66	-0.008	interpolated value	
1/3/2006	2.63	0.010	interpolated value	1/8/2009	4.63	-0.008	interpolated value	
1/4/2006	2.65	0.010	interpolated value	1/9/2009	4.59	-0.008	interpolated value	
1/5/2006	2.68	0.010	interpolated value	1/10/2009	4.55	-0.008	interpolated value	
1/6/2006	2.71	0.010	interpolated value	1/11/2009	4.51	-0.008	interpolated value	
1/7/2006	2.73	0.010	interpolated value	1/12/2009	4.48	-0.008	interpolated value	
1/8/2006	2.76	0.010	interpolated value	1/1/2010	4.44	-0.008	interpolated value	
1/9/2006	2.78	0.009	interpolated value	24/1/2010	4.40	-0.008	transfe rmarkt	
1/10/2006	2.81	0.009	interpolated value	1/3/2010	4.40	0.000	interpolated value	
1/11/2006	2.84	0.009	interpolated value	1/4/2010	4.40	0.000	interpolated value	
1/12/2006	2.86	0.009	interpolated value	1/5/2010	4.40	0.000	interpolated value	
1/1/2007	2.89	0.009	interpolated value	1/6/2010	4.40	0.000	interpolated value	
1/2/2007	2.92	0.009	interpolated value	1/7/2010	4.40	0.000	interpolated value	
1/3/2007	2.94	0.009	interpolated value	12/8/2010	4.40	0.000	transfe rmarkt	
1/4/2007	2.97	0.009	interpolated value	1/9/2010	4.40	0.000	interpolated value	
1/5/2007	2.99	0.009	interpolated value	1/10/2010	4.40	0.000	interpolated value	
1/6/2007	3.02	0.009	interpolated value	1/11/2010	4.40	0.000	interpolated value	
1/7/2007	3.05	0.009	interpolated value	1/12/2010	4.40	0.000	interpolated value	
1/8/2007	3.07	0.009	interpolated value	1/1/2011	4.40	0.000	interpolated value	
27/08/2007	3.10	0.009	transfe rmarkt	31/1/2011	4.40	0.000	interpolated value	
1/10/2007	3.54	0.133	interpolated value	1/3/2011	4.33	-0.015	interpolated value	
1/11/2007	3.98	0.117	interpolated value	1/4/2011	4.27	-0.016	interpolated value	
1/12/2007	4.42	0.105	interpolated value	1/5/2011	4.20	-0.016	interpolated value	
1/1/2008	4.86	0.095	interpolated value	1/6/2011	4.13	-0.016	interpolated value	
3/2/2008	5.30	0.087	transfe rmarkt	1/7/2011	4.07	-0.016	interpolated value	
1/3/2008	5.26	-0.007	interpolated value	7/8/2011	4.00	-0.017	transfe rmarkt	
1/4/2008	5.23	-0.007	interpolated value	1/9/2011	4.00	0.000	interpolated value	
1/5/2008	5.19	-0.007	interpolated value	1/10/2011	4.00	0.000	interpolated value	
1/6/2008	5.15	-0.007	interpolated value	1/11/2011	4.00	0.000	interpolated value	
1/7/2008	5.11	-0.007	interpolated value	1/12/2011	4.00	0.000	interpolated value	
1/8/2008	5.08	-0.007	interpolated value	1/1/2012	4.00	0.000	interpolated value	
1/9/2008	5.04	-0.007	interpolated value	4/2/2012	4.00	0.000	transfe rmarkt	
1/10/2008	5.00	-0.007	interpolated value	1/3/2012	4.65	0.151	interpolated value	
1/11/2008	4.96	-0.008	interpolated value	13/4/2012	5.30	0.131	transfe rmarkt	
1/12/2008	4.93	-0.008	interpolated value	1/5/2012	5.73	0.077	interpolated value	
1/1/2009	4.89	-0.008	interpolated value	1/6/2012	6.15	0.072	interpolated value	
1/2/2009	4.85	-0.008	interpolated value	1/7/2012	6.58	0.067	interpolated value	
1/3/2009	4.81	-0.008	interpolated value	6/8/2012	7.00	0.063	transfe rmarkt	
1/4/2009	4.78	-0.008	interpolated value					

Appendix 6.2:

Carlos Vela signs new four-year contract with Real Sociedad

<http://www.theguardian.com/football/2014/jun/24/carlos-vela-real-sociedad-new-contract-arsenal>
accessed on 02 July 2015

Last

News published on Tuesday 24 June 2014 11.39 BST

“Having sold Vela to the Spanish club in 2012, Arsenal were widely expected to activate a £3.5m buy-back clause this summer, but Sociedad have reportedly paid £8.8m, plus a further £4m in possible performance-related bonuses, to remove the clause and ensure that the 25-year-old remains in Spain.

In a statement released on their website, the La Liga club confirmed: “Real Sociedad have reached an agreement with Arsenal FC which will allow Carlos Vela to continue at the club on a permanent basis.”

Carlos Vela: Arsenal to net £12m after passing on buy-back option

<http://www.bbc.com/sport/0/football/27996066>, last accessed on 08 April 2015

News published on Tuesday 24 June 2014. Last updated at 12:54 GMT

“Arsenal will receive £12m from Real Sociedad after passing on the option of re-signing Carlos Vela, who has signed a four-year deal with the La Liga side.

The 25-year-old Mexico international joined Sociedad on loan in 2011 but Arsenal retained a buy-back option when he signed permanently in 2012.

The fee will give Sociedad exclusivity on Vela who has scored 47 goals in 126 matches for the club.

The striker said on the club website: "I've always said this is my home."

In a club statement, Sociedad confirmed: "Real Sociedad have reached an agreement with Arsenal FC which will allow Carlos Vela to continue at the club on a permanent basis.”

Appendix 6.3: Carlos Vela' volatility from 2012 to 2014

	1 month volatility	21.1%
	1 year volatility	73.0%
Date	Value (M£)	Return
6/8/2012	7.00	transfermarkt
1/9/2012	7.36	1.051 interpolated value
1/10/2012	7.72	0.048 interpolated value
1/11/2012	8.08	0.046 interpolated value
1/12/2012	8.44	0.044 interpolated value
10/1/2013	8.80	0.042 transfermarkt
1/2/2013	9.70	0.097 interpolated value
6/3/2013	10.60	0.089 transfermarkt
1/4/2013	11.77	0.104 interpolated value
1/5/2013	12.93	0.095 interpolated value
11/6/2013	14.10	0.086 transfermarkt
1/7/2013	15.27	0.079 interpolated value
1/8/2013	16.43	0.074 interpolated value
16/9/2013	17.60	0.069 transfermarkt
1/10/2013	17.60	0.000 interpolated value
1/11/2013	17.60	0.000 interpolated value
1/12/2013	17.60	0.000 interpolated value
1/1/2014	17.60	0.000 interpolated value
22/1/2014	17.60	0.000 interpolated value
1/3/2014	17.60	0.000 interpolated value
1/4/2014	17.60	0.000 interpolated value
1/5/2014	17.60	0.000 interpolated value
1/6/2014	17.60	0.000 interpolated value
1/7/2014	17.60	0.000 interpolated value
19/7/2014	17.60	0.000 transfermarkt

Appendix 6.4: Cesc Fabregas's historical volatility

Montly volatility			8.63%	1-year volatility			29.91%
Date	Value (M£)	Return		Date	Value (M£)	Return	
3/10/2004	2.20	transfe rmarkt		1/4/2008	31.84	0.016	interpolated value
1/11/2004	3.67	0.511	interpolated value	1/5/2008	32.36	0.016	interpolated value
1/12/2004	3.67	0.000	interpolated value	1/6/2008	32.88	0.016	interpolated value
6/1/2005	6.60	0.588	transfe rmarkt	3/7/2008	33.4	0.016	transfe rmarkt
1/2/2005	6.64	0.007	interpolated value	1/8/2008	33.63	0.007	interpolated value
1/3/2005	6.69	0.007	interpolated value	1/9/2008	33.85	0.007	interpolated value
1/4/2005	6.73	0.007	interpolated value	1/10/2008	34.08	0.007	interpolated value
1/5/2005	6.78	0.007	interpolated value	1/11/2008	34.30	0.007	interpolated value
1/6/2005	6.82	0.007	interpolated value	1/12/2008	34.53	0.007	interpolated value
1/7/2005	6.87	0.006	interpolated value	1/1/2009	34.75	0.006	interpolated value
1/8/2005	6.91	0.006	interpolated value	1/2/2009	34.98	0.006	interpolated value
1/9/2005	6.96	0.006	interpolated value	22/2/2009	35.2	0.006	transfe rmarkt
16/10/2005	7.0	0.006	transfe rmarkt	1/4/2009	36.4	0.034	interpolated value
1/11/2005	7.61	0.084	interpolated value	1/5/2009	37.6	0.032	interpolated value
1/12/2005	8.23	0.077	interpolated value	1/6/2009	38.8	0.031	interpolated value
1/1/2006	8.84	0.072	interpolated value	1/7/2009	40	0.030	interpolated value
1/2/2006	9.45	0.067	interpolated value	1/8/2009	41.2	0.030	interpolated value
1/3/2006	10.06	0.063	interpolated value	1/9/2009	42.4	0.029	interpolated value
1/4/2006	10.68	0.059	interpolated value	1/10/2009	43.6	0.028	interpolated value
1/5/2006	11.29	0.056	interpolated value	1/11/2009	44.8	0.027	interpolated value
13/6/2006	11.90	0.053	transfe rmarkt	1/12/2009	46	0.026	interpolated value
1/7/2006	12.57	0.055	interpolated value	1/1/2010	47.2	0.026	interpolated value
1/8/2006	13.25	0.052	interpolated value	24/1/2010	48.4	0.025	transfe rmarkt
1/9/2006	13.92	0.050	interpolated value	1/3/2010	48.4	0.000	interpolated value
1/10/2006	14.59	0.047	interpolated value	1/4/2010	48.4	0.000	interpolated value
1/11/2006	15.26	0.045	interpolated value	1/5/2010	48.4	0.000	interpolated value
1/12/2006	15.94	0.043	interpolated value	1/6/2010	48.4	0.000	interpolated value
1/1/2007	16.61	0.041	interpolated value	1/7/2010	48.4	0.000	interpolated value
1/2/2007	17.28	0.040	interpolated value	12/8/2010	48.4	0.000	transfe rmarkt
1/3/2007	17.95	0.038	interpolated value	1/9/2010	48.4	0.000	interpolated value
1/4/2007	18.63	0.037	interpolated value	1/10/2010	48.4	0.000	interpolated value
23/4/2007	19.30	0.035	transfe rmarkt	1/11/2010	48.4	0.000	interpolated value
1/6/2007	19.33	0.001	interpolated value	1/12/2010	48.4	0.000	interpolated value
1/7/2007	19.35	0.001	interpolated value	1/1/2011	48.4	0.000	interpolated value
1/8/2007	19.38	0.001	interpolated value	31/1/2011	48.4	0.000	transfe rmarkt
31/8/2007	19.40	0.001	transfe rmarkt	1/3/2011	48.25	-0.003	interpolated value
1/10/2007	21.68	0.111	interpolated value	1/4/2011	48.1	-0.003	interpolated value
1/11/2007	23.96	0.100	interpolated value	1/5/2011	47.95	-0.003	interpolated value
1/12/2007	26.24	0.091	interpolated value	1/6/2011	47.8	-0.003	interpolated value
1/1/2008	28.52	0.083	interpolated value	1/7/2011	47.65	-0.003	interpolated value
3/2/2008	30.80	0.077	transfe rmarkt	7/8/2011	47.5	-0.003	transfe rmarkt
1/3/2008	31.32	0.017	interpolated value				

Appendix 6.5

Arsenal's sense of loss from Cesc Fabregas' departure softened by clauses in his Barcelona contract

<http://www.telegraph.co.uk/sport/football/teams/arsenal/8703312/Arsenals-sense-of-loss-from-Cesc-Fabregas-departure-softened-by-clauses-in-his-Barcelona-contract.html>, last accessed on 07 July 2015

Published on 15 Aug 2011

While Barcelona believe the deal is worth no more than €40 million (£35 million), Arsenal value the transfer at almost **£40 million**.

Appendix 6.6: Steve Colpaert's historical volatility

STEVE COLPAERT

Sep 13, 1986 in Etterbeek, Belgium

Main: DF, Left back

Join Zulte-Waregem: 03/08/2008 with 0.75M€

1-month vol 3.4%

1-year vol 12%

	M€	Return			M€	Return	
3/8/2008	0.750		transfe rmarkt	1/8/2010	1.075	-0.02	interpolated value
1/9/2008	0.755	0.01	interpolated value	1/9/2010	1.050	-0.02	interpolated value
1/10/2008	0.760	0.01	interpolated value	1/10/2010	1.025	-0.02	interpolated value
1/11/2008	0.765	0.01	interpolated value	17/10/2010	1.000	-0.02	transfe rmarkt
1/12/2008	0.770	0.01	interpolated value	1/12/2010	1.150	0.14	interpolated value
1/1/2009	0.775	0.01	interpolated value	9/1/2011	1.300	0.12	transfe rmarkt
1/2/1009	0.780	0.01	interpolated value	1/2/2011	1.283	-0.01	interpolated value
1/3/2009	0.785	0.01	interpolated value	1/3/2011	1.267	-0.01	interpolated value
1/4/1010	0.790	0.01	interpolated value	1/4/2011	1.250	-0.01	interpolated value
1/5/2009	0.795	0.01	interpolated value	1/5/2011	1.233	-0.01	interpolated value
28/5/2009	0.800	0.01	transfe rmarkt	1/6/2011	1.217	-0.01	interpolated value
1/7/2009	0.844	0.05	interpolated value	29/6/2011	1.200	-0.01	transfe rmarkt
1/8/2009	0.889	0.05	interpolated value	1/8/2011	1.225	0.02	interpolated value
1/9/2009	0.933	0.05	interpolated value	1/9/2011	1.250	0.02	interpolated value
1/10/2009	0.978	0.05	interpolated value	1/10/2011	1.275	0.02	interpolated value
1/11/2009	1.022	0.04	interpolated value	1/11/2011	1.300	0.02	interpolated value
1/12/2009	1.067	0.04	interpolated value	1/12/2011	1.325	0.02	interpolated value
1/1/2010	1.111	0.04	interpolated value	1/1/2012	1.350	0.02	interpolated value
1/2/2010	1.156	0.04	interpolated value	1/2/2012	1.375	0.02	interpolated value
8/3/2010	1.200	0.04	transfe rmarkt	1/3/2012	1.400	0.02	interpolated value
1/4/2010	1.175	-0.02	interpolated value	1/4/2012	1.425	0.02	interpolated value
1/5/2010	1.150	-0.02	interpolated value	1/5/2012	1.450	0.02	interpolated value
1/6/2010	1.125	-0.02	interpolated value	1/6/2012	1.475	0.02	interpolated value
1/7/2010	1.100	-0.02	interpolated value	24/6/2012	1.500	0.02	transfe rmarkt

Appendix 6.7: Alvaro Morata's historical volatility

1 month volatility		4.4%	
1 year volatility		15.2%	
Date	Value (M€)	Return	
16/08/2012	6.0		transfermarkt
1/9/2012	6.0	0.000	interpolated value
1/10/2012	6.0	0.000	interpolated value
1/11/2012	6.0	0.000	interpolated value
1/12/2012	6.0	0.000	interpolated value
9/1/2013	6.0	0.000	transfermarkt
1/2/2013	6.4	0.061	interpolated value
1/3/2013	6.8	0.057	interpolated value
1/4/2013	7.1	0.054	interpolated value
8/5/2013	7.5	0.051	transfermarkt
11/6/2013	9.0	0.182	transfermarkt
1/7/2013	9.8	0.080	interpolated value
1/8/2013	10.5	0.074	interpolated value
1/9/2013	11.3	0.069	interpolated value
20/10/2013	12.0	0.065	transfermarkt
1/11/2013	12.0	0.000	interpolated value
1/12/2013	12.0	0.000	interpolated value
1/1/2014	12.0	0.000	interpolated value
22/01/2014	12.0	0.000	transfermarkt
1/3/2014	13.0	0.080	interpolated value
1/4/2014	14.0	0.074	interpolated value
1/5/2014	15.0	0.069	interpolated value
1/6/2/14	16.0	0.065	interpolated value
1/7/2014	17.0	0.061	interpolated value
19/7/2014	18.0	0.057	transfermarkt

Appendix 6.8: Mikel's annual volatility after 2 years joining Chelsea

JOHN OBI MIKEL

Apr 22, 1987 in Jos , Nigeria

Main: DMF

Join Chelsea: 24/4/2007, 20yr, 11.6M€

1-month vol 1.75%

1-year vol 6.07%

	M€ Return		
24/4/2007	11.6		transfermarkt
1/6/2007	11.58	-0.002	interpolated value
1/7/2007	11.55	-0.002	interpolated value
1/8/2007	11.53	-0.002	interpolated value
31/8/2007	11.5	-0.002	transfermarkt
1/10/2007	11.7	0.017	interpolated value
1/11/2007	11.9	0.017	interpolated value
1/12/2007	12.1	0.017	interpolated value
1/1/2008	12.3	0.016	interpolated value
3/2/2008	12.5	0.016	transfermarkt
1/3/2008	12.4	-0.008	interpolated value
1/4/2008	12.3	-0.008	interpolated value
1/5/2008	12.2	-0.008	interpolated value
1/6/2008	12.1	-0.008	interpolated value
3/7/2008	12	-0.008	transfermarkt
1/8/2008	12.5	0.041	interpolated value
1/9/2008	13	0.039	interpolated value
1/10/2008	13.5	0.038	interpolated value
1/11/2008	14	0.036	interpolated value
1/12/2008	14.5	0.035	interpolated value
1/1/2009	15	0.034	interpolated value
1/2/2009	15.5	0.033	interpolated value
22/2/1009	16	0.032	transfermarkt
1/4/2009	16.375	0.023	interpolated value
1/5/2009	16.75	0.023	interpolated value
1/6/2009	17.125	0.022	interpolated value
28/6/2009	17.5	0.022	transfermarkt

Appendix 6.9: Ramires and César Azpilicueta's yearly volatilities after 2 years at Chelsea

CÉSAR AZPILICUETA

Aug 28, 1989 in Pamplona, Spain

Main: DF, Left back

Join Chelsea: 08/09/2012, 9M€

1-month vol 3.4%

1-year vol 12%

	M€ Return		
8/9/2012	9		transfermarkt
1/10/2012	10	0.11	interpolated value
1/11/2012	11	0.10	interpolated value
1/12/2012	12	0.09	interpolated value
1/1/2013	13	0.08	Interpolated value
27/1/2013	14	0.07	transfermarkt
1/3/2013	14.8	0.06	interpolated value
1/4/2013	15.6	0.05	interpolated value
1/5/2013	16.4	0.05	interpolated value
1/6/2013	17.2	0.05	interpolated value
1/7/2013	18	0.05	transfermarkt
1/8/2013	18	0.00	interpolated value
1/9/2013	18	0.00	interpolated value
1/10/2013	18	0.00	interpolated value
1/11/2013	18	0.00	interpolated value
1/12/2013	18	0.00	interpolated value
1/1/2014	18	0.00	interpolated value
14/1/2014	18	0.00	transfermarkt
1/3/2014	18.3	0.02	interpolated value
1/4/2014	18.6	0.02	interpolated value
1/5/2014	18.9	0.02	interpolated value
1/6/2014	19.1	0.02	interpolated value
1/7/2014	19.4	0.01	interpolated value
1/8/2014	19.7	0.01	interpolated value
11/8/2014	20	0.01	transfermarkt

RAMIRES

Mar 24, 1987 in Barra do Piraí, Brazil

Main Position: CMF, Side: RMF, DMF

Join Chelsea: 12/7/2010, 23yr, 10.5M€

1-month vol 2.2%

1-year vol 7%

	M€ Return		
12/7/2010	14		transfermarkt
1/8/2010	14	0.00	interpolated value
1/9/2010	14	0.00	interpolated value
1/10/2010	14	0.00	interpolated value
1/11/2010	14	0.00	interpolated value
1/12/2010	14	0.00	interpolated value
1/13/2010	14	0.00	interpolated value
31/1/2010	14	0.00	transfermarkt
1/3/2011	14.83	0.06	interpolated value
1/4/2011	15.67	0.05	interpolated value
1/5/2011	16.50	0.05	interpolated value
1/6/2011	17.33	0.05	interpolated value
1/7/2011	18.17	0.05	interpolated value
7/8/2011	19	0.04	transfermarkt
1/9/2011	20.0	0.05	interpolated value
1/10/2011	21.0	0.05	interpolated value
1/11/2011	22.0	0.05	interpolated value
1/12/2011	23.0	0.04	interpolated value
1/1/2012	24.00	0.04	interpolated value
5/2/2012	25	0.04	transfermarkt
1/3/2012	26.0	0.04	interpolated value
1/4/2012	27.0	0.04	interpolated value
1/5/2012	28.0	0.04	interpolated value
1/6/2012	29.0	0.04	interpolated value
23/6/2012	30	0.03	transfermarkt

Appendix 6.10: Risk-free rates

Source: Investing.com, last accessed on 14 July 2015

	Case	Date	Last
UK Gilt 2y	Carlos Vela	6-Aug-12	0.082
UK Gilt 3y	Cesc Fabregas	5-Aug-11	0.732
German Bunds 4y	Tiago Ferreira	14-Jul-14	0.137
German Bunds 3y	Alvaro Morata	18-Jul-14	0.047
German Bunds 2y	Oriol Romeu	4-Aug-11	0.87