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Optimal Monetary and Fiscal Policies in a Heterogeneous Country-size Monetary Union

por

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Tese de Doutoramento em Economia

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Biographical Note

Paulo Vieira was born in May 1974, in Amarante, Portugal. After completing the undergraduate course in Economics at FEP in the University of Porto, he entered the Master's program in Economics at the same university and obtained the master's degree in 2004, with the thesis "Análise Bi-etápica da Eficiência dos Municípios Portugueses". Then, he entered the PhD program in Economics, again at the same university.

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Resumo

Na sequência da recente crise económica e financeira, a dívida pública cresceu consideravelmente em muitos países da União Económica e Monetária (UEM). Um nível elevado de endividamento público pode ampliar os custos de estabilização, bem como agravar a sua distribuição desigual entre pequenos e grandes países da união monetária.

Adaptando um modelo *standard* de economia aberta Novo-Keynesiano a uma união monetária com países de dimensão heterogénea, onde muitas pequenas economias coexistem com uma grande, começou-se por investigar de que forma o nível de endividamento público determina as políticas discricionárias ótimas e afeta a performance da estabilização macroeconómica sob diferentes regimes de política.

Os resultados numéricos mostram que, em geral, maiores níveis de endividamento público agravam a estabilização macroeconómica da união como um todo e tendem a penalizar, principalmente, os países pequenos. Concluímos também que as interações estratégicas de política têm diferentes consequências para o bem-estar de pequenos e grandes países. Para os pequenos países e para a união existem claros ganhos de estabilização em promover a cooperação das políticas económicas, num cenário de dívida alta. No entanto, num cenário de dívida alta para a união, o país grande prefere um regime de liderança fiscal, podendo ser difícil obter a sua aprovação para um regime de cooperação. Ao invés, para a união e para os países pequenos, o regime de liderança monetária é preferível ao de liderança fiscal.

Derivaram-se também regras simples ótimas (OSR) para países grandes e para países muito pequenos. Ainda que subótimas, as OSR facilitam a comunicação dos objetivos de política e a verificação dos seus resultados. A nossa análise incide sobre os custos de estabilização associados a regras orçamentais alternativas e sobre os coeficientes ótimos, de reação ao ciclo e à dívida, que a regra orçamental deve observar em cenários de dívida alta e baixa, num regime cooperativo. Os resultados confirmam que os custos de estabilização para a união são superiores num cenário de dívida alta. Concluímos também que, num cenário de elevada (baixa) dívida média, a política orçamental de um país pequeno deverá reagir mais (menos) fortemente à dívida do que a de um país grande.

Abstract

After the recent financial and economic crisis, public debt has increased considerably in many European and Monetary Union (EMU) countries. A high level of government indebtedness may enlarge stabilization costs and aggravate their uneven distribution across small and large union country-members.

Extending a standard New Keynesian open-economy model to a heterogeneous country-size monetary union, where very small economies coexist with a large country, we first explore how the level of government indebtedness shapes full-optimal discretionary policies and affects macroeconomic stabilization performance, under different policy regimes.

Numerical results show that, in general, higher public debt levels hamper business cycle stabilization, for the union as a whole and are more likely to penalize the stabilization performance of small country-members. They also indicate that strategic policy interactions disclose different welfare consequences for large and small countries. For small countries and the union as a whole, there are clear stabilization gains from promoting policy cooperation in a high-debt scenario. However, in such scenario, the big country prefers fiscal leadership, suggesting that it may be hard to get its approval for a cooperative policy arrangement. In contrast, monetary leadership is preferable to fiscal leadership, for the union as whole and for the small countries.

Optimal simple rules (OSR) for large and very small economies are also derived. Tough suboptimal, OSR facilitate the communication of policy objectives and the verification of their outcomes. Our analysis focus on the welfare stabilization costs of alternative fiscal rules and on the optimal countercyclicality and debt feedback degrees of a fiscal rule, in low- and high-debt scenarios, under a cooperative regime. Results confirm that the stabilization costs for the union as a whole are higher under a high-debt scenario. Moreover, in a high-debt (low-debt) scenario, fiscal instruments in a small country should react more (less) to debt than in a big country.

Index

Table Index

Figure Index

1 Introduction

In the sequence of the recent financial and economic crisis, public debt has increased considerably in many European and Monetary Union (EMU) countries, triggering for some of them even fears of unsustainability of public finances. A higher level of government indebtedness may enlarge the budgetary consequences of the shocks and restrain more considerably business cycle stabilization policies. The consequences for macroeconomic stabilization are also highly dependent on the conduct of fiscal and monetary policy authorities. In fact, as Leith and Wren-Lewis (2013) have shown, they are expected to be more serious under discretionary policymaking, as policy is set to make debt return to its pre-shock level, than under commitment, with public debt following a random walk. Furthermore, as the level of government debt also affects the absolute and relative effectiveness of the fiscal and monetary policy instruments to promote debt-stabilization, it is possible that stabilization costs evolve non-monotonically with debt under discretion, as shown by Blake and Kirsanova (2011). Depending on whether these potential efficiency stabilization gains surpass or not the costs of stabilizing larger budgetary consequences of shocks, a higher level of debt may either improve or worsen business cycle stabilization, under discretionary policymaking.

Moreover, in a monetary union such as the EMU, a common monetary policy coexists with decentralized fiscal policies. We expect business cycle stabilization to be seriously hampered by strategic interactions between non-coordinated policies, particularly when policymakers are also tied to debt stabilization in high-debt environments.

Additionally, in a heterogeneous country-size monetary union, small and large countries are expected to face different incentives, and incur in different macroeconomic stabilization performances. A small, rather open, country is more likely to suffer to a larger extent the effects of country-specific shocks and, thus, to experience a worse stabilization performance than a large, rather closed, one. Furthermore, as its policy spillovers are smaller, the government of a small economy is less prone to internalize the external consequences of its policy than a government of a large country. These divergent incentives add to those already referred to determine macroeconomic stabilization consequences under different public debt scenarios.

Yet, the research on how the level of debt affects the macroeconomic stabilization performance of small and large countries tied to a common monetary policy is still limited. Literature on optimal debt-constrained stabilization policies in a monetary union environments includes, among others, Kirsanova et al. (2005, 2007), Ferrero (2009), Leith and Wren-Lewis (2007b, 2011) and Blueschke and Neck (2011). However, most of the existing literature relies either in a two-country model or in a multi-country model of a continuum of small open economies. Moreover, only a small branch of this literature has considered the case of non-cooperation.

We intend to contribute to this literature through building a micro-founded DSGE model for a multi-country monetary union formed by a large country and a continuum of small open economies. This setup is useful for analyzing interactions between country-members that produce either negligible or meaningful impacts on the union's outcomes as well as on the counterparts'. The model, presented in Chapter 2, also encompasses, without loss of generality, the two cases already studied in the literature: the two-country case and that of multiple small countries. The model considers three policy instruments that are generally considered for stabilization purposes: (1) the conventional monetary policy through the control of nominal interest rate; (2) fiscal policy, in the form of expenditures in home-biased public goods; and (3) the income tax rate, that determines to what extent government should rely on debt to finance current expenditure. Thus, in Chapter 2 we derive the structural equations of the model, the efficient equilibrium solution and the social union-wide loss function. We also define the policy games considered in the analysis of non-cooperative full optimal policies, deriving the corresponding numerical algorithms. Additionally, we also present in this chapter the baseline calibration of the model.

After a detailed review of the literature, we proceed, in Chapter 3, with the derivation and the analysis of the performance and the implications of fully optimal policies across different public debt levels and policy regimes. The analysis is focused on discretionary polices – first, we consider the full-cooperation setup, as a benchmark, and then we move to non-cooperative scenarios, where the conflict of policy objectives allows for strategic interactions between policymakers and for different equilibria, depending on the timing structure of the policy games. Under non-cooperation, national fiscal authorities are assumed to be exclusively concerned with the national counterparts of the union-wide welfare.

The main goals of this chapter are (i) to assess how the average level of public debt shapes full optimal discretionary policies in a heterogeneous country-size monetary union and affects macroeconomic stabilization performance under different policy regimes; (ii) to appraise the implications for a very small and a large country; and, (iii) to check whether the implementation of, e.g., a full policy cooperative arrangement could be welfare-improving for the union, while having the political support of small and large countries.

To address these issues, we compute numerically optimal solutions under different debt levels, using appropriate algorithms to mimic cooperative outcomes and also to reflect the different timing structures of the (non-cooperative) policy games: Nash, monetary leadership and fiscal leadership. We follow the methodology developed in the seminal work of Söderlind (1999) and in the recent work of Kirsanova and co-authors (e.g., Blake and Kirsanova, 2011, for a closed-economy setup, and Kirsanova et al., 2005, for an open-economy setup) and further developed by Machado and Ribeiro (2010). The rankings of the policy regimes are provided by using union-wide and country-specific welfare criterions.

Though this first part of our work focuses only on full optimal discretionary policies, we take optimal cooperative policy under commitment as a benchmark to understand the nature of time-consistency problems arising from the need to fulfil the government's budget constraint. Besides helping to understand the welfare implications from the strategic interactions between policy authorities, full-optimal policies are a useful benchmark analysing rather simpler and more implementable set of rules.

There is now a vast literature focusing on optimal simple rules (OSR), as they reflect more accurately institutional rigidities than full optimization. In addition, full-optimal policies imply that monetary/fiscal instruments would optimally and immediately react to contemporaneous shocks (Kirsanova and Wren-Lewis, 2012). Moreover, the communication, and thus the ability to influence expectations, as well as the monitoring of rather complex full-optimal policy reaction functions are very difficult. It is thus advisable policymakers to commit themselves to simple decision rules, where policy instruments react to a small set of meaningful and observable variables. On the one hand, being easier to implement, simple rules reinforce the policy makers' ability to react timely to business cycle fluctuations. On the other hand, through an increased monitoring of policy makers' commitment, simple rules lessen the time-consistency problem, reinforcing credibility, i.e., enhancing policy makers' "[...] ability to anchor the expectations of households and firms" (Vogel et al., 2013, p. 173). Although not full optimal, simple rules are expected to not substantially deplete welfare; for that, optimal feedback parameters should differ across countries, indexed to the particular structure of the economies, including debt levels. Additionally, a supranational device relying on simple fiscal feedback rules on broadly observed variables, such as output gap or debt, would be a way to operationally enforce cooperative outcomes in a monetary union where fiscal policies are still country-fragmented, as in the EMU.

Unlike monetary policy, there is much less literature on simple fiscal policy rules. Some authors advocate that the role of fiscal policy should be limited to letting automatic stabilizers work, as is the case, among others, of Taylor (2000), Schmitt-Grohé and Uribe (2007) and Kollman (2008). Exceptions are usually made for the cases of nominal interest rates approaching the zero lower bound or in a fixed exchange rate regime (Taylor, 2000).

In practice, and in order to prevent procyclicality, several countries have been adopting numerical rules in terms of cyclically-adjusted indicators and for multiyear or medium-term horizons (see Kopits, 2007, for details on the adoption of numerical rules by several countries). But as Bi and Kumhof (2011) argue, what is the optimal countercyclicality of a fiscal rule? This refinement is even more acute for a small country member of a monetary union that relies solely on own fiscal policy to react to specific/asymmetric shocks. Even though this vulnerability has been recognized since the work of Mundell (1961), and the prescription for the use of countercyclical fiscal policy being also advocated both in the literature and among the policy bodies of the EMU, micro-founded theoretical analysis on the specific design of fiscal policies is still limited. In the context of a monetary union setup, several recent papers have already studied the use of optimal monetary and fiscal simple rules-based policies with welfare maximization purposes (e.g., Beetsma and Jensen, 2003, van Aarle et al., 2004, Ferrero, 2009, Pappa, 2012, and Vogel et al., 2013). However, the case of a more general structure for a currency union, where few large countries may coexist with many small countries, as is the case, e.g., of the EMU, has not yet been addressed. After a brief review of the literature, in Chapter 4 we extend the analysis of OSR for fiscal policy to a small country within a heterogeneous country-size monetary union. Since we are interested in the particular role of fiscal policy in the stabilization of asymmetric shocks, we consider that the common monetary policy follows a Taylor-type feedback rule, and focus on the design of optimal simple fiscal rules. Hence, and assuming full coordination, our goals are to: (i) derive optimal countercyclicality and debt feedback degrees of a fiscal rule; (ii) compare across alternative fiscal rule instruments; (iii) provide insights on how rules should differ with different structural features, particularly in low- and high-debt scenarios. Since we expect large and very small economies to face different macroeconomic stabilization performances in face of asymmetric shocks, we compare the optimal countercyclicality and debt feedbacks obtained in our model with those obtained for a standard symmetric-size two-country monetary union.

Finally, Chapter 5 draws the main conclusions of the thesis, identifies some limitations and explores future research paths.

2 A Currency Union Model

We model the currency union as a closed system, represented by the unit interval, and made up of two blocks of countries. One of these blocks is a big country, indexed by **B**, with a relative size of $(1-n)$, $n \in [0,1]$. The other block, indexed by **S**, has dimension n and is made up of a continuum of small countries. Each small country, indexed by $s \in [0,n]$, is of measure zero and, as a result, its domestic policy does not have any impact on the rest of the union. Each country has a separate fiscal authority, but there is a common monetary policy, defined by a common central bank. Although subject to idiosyncratic shocks, countries are assumed to have identical preferences, technology and market structure.

We assume that the big country (**B**) is made up of a continuum of small geographic units, indexed by b, on the interval $[n,1]$ [1]. In terms of population (households and firms), each one of these geographic units is equivalent to a small country, and, accordingly, has a relative size of zero with respect to the union. Notice, however, that countries belonging to the **S** block have independent fiscal authorities and are subject to imperfectly correlated shocks, while the geographic units constituting country **B** have the same fiscal authority and are subject to the same shocks.

Each small country (geographic unit) in **S** (**B**) is populated by a continuum of agents (households/firms - indexed by h) on the interval $[0,1]$. Moreover, we assume that each household specializes in the production of a differentiated good (indexed by h).

Firms, which are owned by domestic households, constitute a monopolistic competitive sector that produces a continuum of differentiated goods. In addition to imperfect competition, we introduce nominal price stickiness a la Calvo (1983) in the goods market.

With regard to factor markets, labor is the only input of production, and it is assumed that households are monopolistic competitive labor suppliers. Labor is immobile across countries. Each firm act as a wage-taker in segmented labor markets - each household supplies a differentiated labor input, specializing in the production of a specific final good. Wages are settled by workers of each type and are assumed to be perfectly flexible.

We consider a cashless economy as in Woodford (2003, Chapter 2) abstracting

[1] A small country is made up of one geographic unit only.

from any monetary frictions.

2.1 Households

Each country is inhabited by an infinitely-lived representative household h seeking to maximize lifetime utility $U_0(h)$. We assume full asset markets, such that, through risk sharing all the households inhabiting a given country face the same budget constraint and make the same consumption plans.

Given the asymmetry between blocks **S** and **B**, we describe separately the problem of the representative household living in one of the small economies and that of the representative household living in the big country.

The representative household h inhabiting a small country in the **S** block (say, country $i \in [0, n]$), seeks to maximize

$$U_0^i(h) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^i\right) + V\left(G_t^i\right) - v\left(L_t^i(h)\right)\right] \right\}, \tag{1}$$

$$\begin{aligned}
&where: \\
u\left(C_t^i\right) &= \frac{\sigma}{\sigma - 1}\left(C_t^i\right)^{\frac{\sigma-1}{\sigma}}, \\
V\left(G_t^i\right) &= \psi_0 \frac{\psi}{\psi - 1}\left(G_t^i\right)^{\frac{\psi-1}{\psi}}, \psi_0 \geq 0, \\
v\left(L_t^i(h)\right) &= \chi_0 \frac{1}{1+\chi}\left(L_t^i(h)\right)^{1+\chi}, \chi_0 > 0.
\end{aligned}$$

The utility function is additively separable and C_t^i, G_t^i and $L_t^i(h)$ denote, respectively, real private consumption, real *per capita* public consumption and hours of work. We allow consumer preferences to depend on government spending.[2] The

[2] It is common in the literature to include government expenditures on the utility function while conducting stabilization analysis, e.g., Beetsma and Jensen (2005), Leith and Wren-Lewis (2007b, 2011), Galí and Monacelli (2008), Forlati (2009), Evi Pappa (2009), Natvick (2009), Blake and Kirsanova (2011), Kirsanova and Wren-Lewis (2012) and Vogel et al. (2013).

More generally, we think there is both theoretical and empirical motivation for including utility-augmenting public spending. Both these lines of reasoning have led to seminal works of Barro (1981, 1990), Kormendi (1983) and Barro and King (1984), Aschauer (1985), among others. Christiano and Eichenbaum (1988) argue that utility-augmenting public consumption has significantly improved the empirical performance of the models.

Criticisms to this assumption refer that it prevents isolating the quantitative effects exclusively derived from the stabilization role of public consumption. However, Lambertini (2006, p. 94) representative household derives utility both from private and public consumption (though they are not perfect substitutes) but experiences disutility from his work effort.

$\beta < 1$ stands for the intertemporal preferences discount factor, $\frac{1}{\chi}$ represents the elasticity of labor supply with respect to real wage, σ is the intertemporal elasticity of substitution of private consumption ($\frac{1}{\sigma}$ is the constant-relative-risk-aversion coefficient), and ψ is the intertemporal elasticity of substitution of public consumption. The assumed functional forms for the utility components are common in the literature (e.g., Benigno and Benigno, 2006, Galí, 2008, Forlati, 2009, Kirsanova and Wren-lewis, 2012, and Vogel *et al.*, 2013), representing isoelastic preferences.[3]

Because labor is differentiated, there exists a continuum of labor types, indexed by $h \in [0,1]$. Thus, we identify the quantity of labor supplied by household h (the quantity of h-type labor supplied in country i) by $L_t^i(h)$, and further on we identify the nominal wage rate of h-type labor as $W_t^i(h)$.

Similarly, for the representative household h living in geographic unit b of the big country B, we have:

$$U_0^{B,b}(h) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^B\right) + V\left(G_t^B\right) - v\left(L_t^b(h)\right) \right] \right\}, \forall b \in \mathbf{B}. \tag{2}$$

Household-specific hours of work is denoted by $L_t^b(h)$ because there exists, in each one of country B's geographic units b a continuum of differentiated h-type labor inputs, $\forall h \in [0,1]$,which in turn is differentiated from the labor varieties existing in others geographic units.

C_t^i, C_t^B, G_t^i and G_t^B are composite consumption indexes, described below.

2.1.1 Consumption

From the viewpoint of a representative household inhabiting small country $i \in \mathbf{S}$, C_t^i is an index representing a consumption "bundle" (or composite good). This composite

notices that, for sufficient low weights on public consumption in the utility function [ψ_0 in $V(G_t)$], all the results remain true regarding how monetary and fiscal policies can better stabilize output and inflation.

[3]Functions $u(C_t)$ and $V(G_t)$ are strictly increasing and strictly concave, and $v(L_t(h))$ is increasing and strictly convex in L_t.

consumption index is an Dixit-Stiglitz (1977) aggregator of home and foreign goods as

$$C_t^i \equiv \left[\lambda_S^{\frac{1}{\gamma}} \left(C_{i,t}^i \right)^{\frac{\gamma-1}{\gamma}} + (1-\lambda_S)^{\frac{1}{\gamma}} \left(C_{-i,t}^i \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{3}$$

where $\gamma > 0$ is the intratemporal elasticity of substitution, measuring the substitutability between domestic and foreign goods from the perspective of domestic consumer. $C_{i,t}^i$ is a composite consumption index of domestic goods (i.e., home-produced goods), and $C_{-i,t}^i$ is a composite consumption index of imported goods - we use "$-i$" to denote "not i". Following Benigno and De Paoli (2009) and Galí and Monacelli (2005, 2008), λ_S and $(1-\lambda_S)$ are the weights of domestic and imported goods baskets, respectively, on total consumption[4]; in this sense, $(1-\lambda_S)$ represents a natural index of openness. λ_S and $(1-\lambda_S)$ are function of the relative size of the small economy with respect to the union, (in this case, zero), and of parameter $\alpha \in [0,1]$. $(1-\alpha)$ is an index of home bias: $(1-\lambda_S) = (1-0)\alpha = \alpha$ and $\lambda_S = 1-(1-\lambda_S) = (1-\alpha)$.[5]

We assume that parameter λ_S is the same across the continuum of small economies. It is imperative to notice that in the absence of home bias, and given the infinitesimal weight of a small economy in the union, the household inhabiting a small economy would attach only an infinitesimally small (and, hence, negligible) fraction of total consumption expenditures to home-produced goods, Galí (2008, p. 152).Moreover, the presence of home bias in private consumption implies that households in different countries will have different consumption bundles, Galí and Monacelli (2008, p. 118).

The composite consumption index of domestic goods, $C_{i,t}^i$, is defined by the CES function

$$C_{i,t}^i \equiv \left(\int_0^1 \left[C_{i,t}^i(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

$C_{i,t}^i(h)$ is the quantity of domestic good h consumed by country i's representative household, and $\epsilon > 1$ is the elasticity of substitution between goods produced within a given country (assumed to be the same for all countries).

[4]In a symmetric steady state, where the price indexes for domestic and foreign goods are equal, $(1-\lambda_S)$ corresponds to the share of domestic consumption allocated to imported goods.

[5]The specification of λ_S and $(1-\lambda_S)$ gives rise to home bias in consumption: a strictly positive value for $(1-\alpha)$, or $\alpha < 1$, reflects the presence of home bias in private consumption ($\alpha = 0$ characterizes a closed economy, i.e., complete home bias).

The composite consumption index of foreign-produced goods, $C^i_{-i,t}$, is given by

$$C^i_{-i,t} \equiv \left[(1-n)^{\frac{1}{\gamma}}\left(C^i_{B,t}\right)^{\frac{\gamma-1}{\gamma}} + (n)^{\frac{1}{\gamma}}\left(C^i_{S,t}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \tag{5}$$

$$where :$$

$$C^i_{B,t} \equiv \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}}\int_n^1 \left[C^i_{b,t}\right]^{\frac{\epsilon-1}{\epsilon}} db\right)^{\frac{\epsilon}{\epsilon-1}}; C^i_{b,t} \equiv \left(\int_0^1 \left[C^i_{b,t}(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}},$$

$$C^i_{S,t} \equiv \left(\left(\frac{1}{n}\right)^{\frac{1}{\gamma}}\int_0^n \left[C^i_{s,t}\right]^{\frac{\gamma-1}{\gamma}} ds\right)^{\frac{\gamma}{\gamma-1}}; C^i_{s,t} \equiv \left(\int_0^1 \left[C^i_{s,t}(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}}.$$

$C^i_{B,t}$ is a composite index of imported goods from country **B** and $C^i_{S,t}$ is a composite index of imported goods from the continuum of small countries (block **S**); $(1-n)$ and n are the weights of imported goods from blocks **B** and **S**, respectively, on total consumption of imported goods. $\gamma > 0$ is the elasticity of substitution between goods produced in different foreign countries, assumed to be equal to the elasticity of substitution between domestic and foreign goods. In turn, $C^i_{b,t}$ is a composite index of imported goods from geographic unit $b \in \mathbf{B}$, where $C^i_{b,t}(h)$ is the quantity of geographic unit b's good h consumed by country i's representative household, and $C^i_{s,t}$ is a composite index[6] of imported goods from country $s \in \mathbf{S}$, where $C^i_{s,t}(h)$ is the quantity of country s's good h consumed by country i's representative household.

Notice that since all geographic units in country **B** have identical preferences, technology and market structure, and are subject to the same shocks and fiscal policies, their behavior is identical, so the distribution of prices and, hence, the distribution of quantities is the same across geographic units[7]. The only distinctive feature across country **B**'s geographic units is the fact that each one produces a differentiated continuum of goods. We can take a representative geographic unit (say, $b \in [n, 1]$), and define $C^i_{B,t} \equiv (1-n)\left[C^i_{b,t}\right], \forall b \in \mathbf{B}$.[8]

[6] Notice that because each small country has a zero measure, the presence of country i (the country being modeled) has a negligible impact on integrals over the continuum of small countries, Galí (2008, p. 152).

[7] Further on we will assume that firms set prices in a staggered fashion, following a partial adjustment rule a la Calvo (1983), in which each firm may reset its price with probability $(1-\theta_B)$ in any given period, independently of the time elapsed since the last adjustment.

If the law of large number holds, in each geographic unit a fraction $(1-\theta_B)$ of randomly selected firms will choose prices optimally at each point in time, while a fraction θ_B will maintain its prices unchanged. Hence, the distribution of prices in each geographic unit corresponds to the distribution of prices in country **B** as a whole, though with total mass reduced to zero.

[8] Notice that $C^i_{b,t}(h)$ and $C^i_{B,t}(h)$ are not equivalent: good h, produced in geographic unit b, is

From the viewpoint of a representative household inhabiting country **B**, independently of the geographic unit where she or he lives, C_t^B is a composite consumption index, which aggregates home and foreign goods as

$$\begin{aligned} C_t^B &\equiv \left[\lambda_B^{\frac{1}{\gamma}} \left(C_{B,t}^B\right)^{\frac{\gamma-1}{\gamma}} + (1-\lambda_B)^{\frac{1}{\gamma}} \left(C_{S,t}^B\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \qquad (6) \\ where &: \\ C_{B,t}^B &\equiv \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 \left[C_{b,t}^B\right]^{\frac{\epsilon-1}{\epsilon}} db\right)^{\frac{\epsilon}{\epsilon-1}}; C_{S,t}^B \equiv \left(\left(\frac{1}{n}\right)^{\frac{1}{\gamma}} \int_0^n \left[C_{s,t}^B\right]^{\frac{\gamma-1}{\gamma}} ds\right)^{\frac{\gamma}{\gamma-1}}. \end{aligned}$$

λ_B and $(1-\lambda_B)$ are the weights of domestic and imported goods (produced in block **S**) baskets, respectively, on total consumption[9], and are function of the relative size of economy **B** with respect to the whole union, $(1-n)$, and of parameter α: $(1-\lambda_B) = [1-(1-n)]\,\alpha = n\alpha$ and $\lambda_B = 1-(1-\lambda_B) = 1-n\alpha$. The specification of λ_B and $(1-\lambda_B)$ gives rise to home bias in consumption. Though we assume the same α for all countries, the large economy presents a higher degree of home bias $((1-n\alpha)$ instead of $(1-\alpha))$.

$C_{B,t}^B$ is a composite consumption index of domestic goods while $C_{b,t}^B$ is a composite index of domestic goods produced in geographic unit $b \in \mathbf{B}$, defined in the same way as $C_{b,t}^i$. $C_{S,t}^B$ is a composite consumption index of imported goods whereas $C_{s,t}^B$ is the specific composite index of imported goods from country $s \in \mathbf{S}$, defined in the same way as $C_{s,t}^i$.

Taking a representative geographic unit $b \in \mathbf{B}$, $C_{B,t}^B$ can be defined as $C_{B,t}^B \equiv (1-n)\left[C_{b,t}^B\right], \forall b \in \mathbf{B}$.

2.1.2 Prices

Next, we define the producer and consumer price indexes that correspond to the above specifications of consumption preferences.

The aggregate consumer price index (CPI) for a small economy $i \in \mathbf{S}$, that corresponds the above consumption preferences (C_t^i), is given by

$$P_{c,t}^i \equiv \left[\lambda_S \left(P_t^i\right)^{1-\gamma} + (1-\lambda_S)\left(P_t^{-i}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \qquad (7)$$

only produced in that particular geographic unit.

[9] $(1-\lambda_B)$ represents a natural index of country **B**'s degree of openness.

where $P_{c,t}^i$ is defined as the minimum expenditure required to purchase one unit of the composite consumption index, C_t^i. P_t^i is the aggregate price index for goods produced in country i, and P_t^{-i} is the aggregate price index for foreign products[10]. Because there is a common currency (all prices are expressed in units of the single currency) and there are no trade barriers, the price of each good produced in the union is the same in all countries – the law of one price holds. As a consequence of the law of one price, combined with the fact that preferences are identical in the entire union, the aggregate price index for the bundle of goods imported from a given country is equal to the aggregate price index of the latter's domestic price. However, given the home biased preferences, the purchasing power parity does not hold for aggregate consumer price indexes.

P_t^i is given by

$$P_t^i \equiv \left(\int_0^1 \left[P_t^i(h) \right]^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}}, \tag{8}$$

where $P_t^i(h)$ is the price of country i's home-produced good h.

The aggregate price index for foreign products is defined as

$$\begin{aligned} P_t^{-i} &\equiv \left[(1-n) \left(P_t^B \right)^{1-\gamma} + n \left(P_t^S \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \qquad (9) \\ where &: \\ P_t^B &\equiv \left(\frac{1}{1-n} \int_n^1 \left[P_t^b \right]^{1-\epsilon} db \right)^{\frac{1}{1-\epsilon}}; P_t^b \equiv \left(\int_0^1 \left[P_t^b(h) \right]^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}}, \\ P_t^S &\equiv \left(\frac{1}{n} \int_0^n \left[P_t^s \right]^{1-\gamma} ds \right)^{\frac{1}{1-\gamma}}; P_t^s \equiv \left(\int_0^1 \left[P_t^s(h) \right]^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}}. \end{aligned}$$

P_t^B is the aggregate price index for the bundle of goods imported from country **B**, as well as the latter's domestic price index, and P_t^S is the aggregate price index for the bundle of goods imported from the continuum of small countries. In turn P_t^b is an aggregate price index for the bundle of goods imported from country **B**'s geographic unit b, where $P_t^b(h)$ is the price of geographic unit b's home-produced good h, and P_t^s is an aggregate price index for the bundle of goods imported from country $s \in \mathbf{S}$, where $P_t^s(h)$ is the price of country s's home-produced good h. Taking a representative geographic unit, we get that $P_t^B \equiv P_t^b, \forall b \in \mathbf{B}$.

[10] Since small country i has a zero measure, P_t^{-i} also corresponds to the aggregate price index of the union as a whole.

Analogously, the aggregate consumer price index for country **B**, that corresponds the above consumption preferences (C_t^B), is given by

$$P_{c,t}^B \equiv \left[\lambda_B \left(P_t^B\right)^{1-\gamma} + (1-\lambda_B)\left(P_t^S\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$
$$= \left[\lambda_B \left(P_t^B\right)^{1-\gamma} + (1-\lambda_B)\frac{1}{n}\int_0^n \left[P_t^s\right]^{1-\gamma} ds\right]^{\frac{1}{1-\gamma}}, \forall b \in \mathbf{B}, \qquad (10)$$

where $P_{c,t}^B$ is defined as the minimum expenditure required to purchase one unit of the composite consumption index, C_t^B. Recall that the aggregate price index for goods produced within a geographic unit of country **B** is the same across all geographic units. Combined with identical preferences and the law of one price, we also have that the aggregate consumer price index is the same across all geographic units, i.e., $P_{c,t}^b \equiv P_{c,t}^B$, $\forall b \in \mathbf{B}$.

2.1.3 Implied Demands for Individual Goods

Private consumption demand for good h is the sum of two components: the demand of domestic and the demand of foreign households.

First, we deduce individual household (domestic and foreign) demand for good h produced in a small country $i \in \mathbf{S}$, and then the demand for good h produced in some geographic unit of country **B**.

The household problem can be solved in three steps. To exemplify, consider the case of the representative household h, inhabiting country $i \in \mathbf{S}$. The first step involves the household optimal choice of the hours worked and the intertemporal allocation of consumption. Households maximize (1) with respect to C_t^i, D_{t+1}^i[11] and $L_t^i(h)$, subject to a sequence of constraints, which will be presented in the following subsections. This intertemporal trade-off determines C_t^i, $t = 0, 1, 2, ...$ Given C_t^i, the second step implies solving the optimal allocation of any given expenditure across the bundle of goods produced in different countries, that is, the household chooses $C_{i,t}^i$, $C_{B,t}^i$ and $C_{s,t}^i$, $\forall s \in \mathbf{S}$, in order to minimize $P_{c,t}^i C_t^i$, under restriction (3). Finally, the household optimally allocates spending over the varieties of each bundle, by minimizing $P_t^i C_{i,t}^i$, $P_t^B C_{B,t}^i$, and $P_t^s C_{s,t}^i$, $\forall s \in \mathbf{S}$, under restrictions (4) and (5).

The implied private consumption demand of the representative household h, inhabiting country $i \in \mathbf{S}$, for domestic good h is

[11] D_{t+1}^i is the nominal payoff in period $t+1$ of a portfolio of state-contingent securities held at the end of period t by the representative household of country i.

$$
\begin{aligned}
C_{i,t}^{i}(h) &= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} C_{i,t}^{i} \\
&= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} \lambda_S C_t^i, \forall h \in [0,1], \qquad (11) \\
with &: \\
C_{i,t}^{i} &= \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} \lambda_S C_t^i.
\end{aligned}
$$

Good h, produced in country i, is also demanded by households inhabiting other small countries and households inhabiting country $\mathbf{B}$.

The demand of the representative household of country $s \in \mathbf{S}$, $s \neq i$, for good h is given by[12]:

$$
\begin{aligned}
C_{i,t}^{s}(h) &= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} C_{i,t}^{s} \\
&= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_{c,t}^s}\right)^{-\gamma} (1-\lambda_S)\, C_t^s, \forall h \in [0,1], \qquad (12) \\
with &: \\
C_{i,t}^{s} &= \left(\frac{P_t^i}{P_t^S}\right)^{-\gamma} \frac{1}{n} C_{S,t}^{s}; C_{S,t}^{s} = \left(\frac{P_t^S}{P_{c,t}^s}\right)^{-\gamma} (1-\lambda_S)\, n C_t^s.
\end{aligned}
$$

In turn,

$$
\begin{aligned}
C_{i,t}^{B}(h) &= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} C_{i,t}^{B} \\
&= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_{c,t}^B}\right)^{-\gamma} \frac{1}{n} (1-\lambda_B)\, C_t^B, \forall h \in [0,1], \qquad (13) \\
with &: \\
C_{i,t}^{B} &= \left(\frac{P_t^i}{P_t^S}\right)^{-\gamma} \frac{1}{n} C_{S,t}^{B}; C_{S,t}^{B} = \left(\frac{P_t^S}{P_{c,t}^B}\right)^{-\gamma} (1-\lambda_B)\, C_t^B.
\end{aligned}
$$

Next, we deduce individual household (domestic and foreign) demand for good $h \in [0,1]$ produced in some geographic unit b of country $\mathbf{B}$. The implied private

[12] Notice that we are making use of equations for foreign country s's consumption preferences similar to those specified for country i; thus, variables related to country s have an analogous interpretation

consumption demand of the representative household h, inhabiting country $\mathbf{B}$, for domestic good $h \in b$ is

$$\begin{aligned} C^B_{b,t}(h) &= \left(\frac{P^b_t(h)}{P^b_t}\right)^{-\epsilon} C^B_{b,t} \\ &= \left(\frac{P^b_t(h)}{P^b_t}\right)^{-\epsilon} \left(\frac{P^B_t}{P^B_{c,t}}\right)^{-\gamma} \frac{1}{1-n}\lambda_B C^B_t, \forall h \in [0,1], \forall b \in \mathbf{B}, \qquad (14) \\ with &: \\ C^B_{b,t} &= \frac{1}{1-n} C^B_{B,t}, ; C^B_{B,t} = \left(\frac{P^B_t}{P^B_{c,t}}\right)^{-\gamma} \lambda_B C^B_t. \end{aligned}$$

The demand of the representative household of country $s \in \mathbf{S}$, for good $h \in b$ is:

$$\begin{aligned} C^s_{b,t}(h) &= \left(\frac{P^b_t(h)}{P^b_t}\right)^{-\epsilon} C^s_{b,t} \\ &= \left(\frac{P^b_t(h)}{P^b_t}\right)^{-\epsilon} \left(\frac{P^B_t}{P^s_{c,t}}\right)^{-\gamma} (1-\lambda_S) C^s_t,\ \forall h \in [0,1], \forall b \in \mathbf{B}, \qquad (15) \\ with &: \\ C^s_{b,t} &= \frac{1}{1-n} C^s_{B,t}; C^s_{B,t} = \left(\frac{P^B_t}{P^s_{c,t}}\right)^{-\gamma} (1-\lambda_S)(1-n) C^s_t. \end{aligned}$$

2.1.4 Budget Constraint

The representative household inhabiting a small country $i \in \mathbf{S}$ maximizes equation (1) subject to a sequence of budget constraints of the form:

$$\begin{aligned} &\int_0^1 P^i_t(h) C^i_{i,t}(h) dh + \int_0^n \int_0^1 P^s_t(h) C^i_{s,t}(h) dh ds + \\ &\int_n^1 \int_0^1 P^b_t(h) C^i_{b,t}(h) dh db + E_t \left\{ Q_{t,t+1} D^i_{t+1} \right\} \\ \leq\ & D^i_t + W^i_t(h) L^i_t(h) + \Gamma^i_t - T^i_t. \qquad (16) \end{aligned}$$

In each period t, $t = 0, 1, 2, ...$, combined expenditure on goods and on the net accumulation of financial assets must equal its disposable income. $W^i_t(h)$ is the nominal wage settled by household h living in country i in period t, D^i_{t+1} is the nominal payoff in period $t+1$ of a portfolio of state-contingent securities held at the end of period t by the representative household of country i (whether private issued or claims on the government), which may include local and foreign shares in firms, Γ^i_t

stands for after-tax nominal profits from ownership of the firms, and $Q_{t,t+1}$denotes the stochastic discount factor for one-period ahead nominal payoffs, common across countries. It is assumed that households have access to a complete set of state-contingent securities that span all possible states of nature and are traded across the union (financial markets are complete both at the domestic and at the international level). T_t^i denotes per capita lump sum taxes in country i.

Analogously, the representative household h living in geographic unit $b \in [n, 1]$ of the big country **B** maximizes (2), subject to a sequence of budget constraints of the form:

$$\begin{aligned} & \int_n^1 \int_0^1 P_t^b(h) C_{b,t}^B(h) dhdb + \int_0^n \int_0^1 P_t^s(h) C_{s,t}^B(h) dhds + E_t \left\{ Q_{t,t+1} D_{t+1}^B \right\} \\ \leq \ & D_t^B + W_t^b(h) L_t^b(h) + \Gamma_t^B - T_t^B, \forall b \in \mathbf{B}, \end{aligned} \tag{17}$$

for $t = 0, 1, 2, ...$

Conditional on an optimal allocation of expenditures, and after taking in consideration aggregate consumption and price indexes, the period budget constraints (16) and (17)[13] can be rewritten, respectively, as:

$$P_{c,t}^i C_t^i + E_t \left\{ Q_{t,t+1} D_{t+1}^i \right\} \leq D_t^i + W_t^i(h) L_t^i(h) + \Gamma_t^i - T_t^i, \tag{18}$$

$$and$$

$$P_{c,t}^B C_t^B + E_t \left\{ Q_{t,t+1} D_{t+1}^B \right\} \leq D_t^B + W_t^b(h) L_t^b(h) + \Gamma_t^B - T_t^B, \forall b \in \mathbf{B}. \tag{19}$$

2.1.5 Labor Supply, Wage Setting and Optimal Consumption

Now, we focus on the household optimal choice of the hours worked and the intertemporal allocation of consumption.

The representative household h inhabiting a small country $i \in \mathbf{S}$, maximizes lifetime utility (1) with respect to C_t^i, D_{t+1}^i and $L_t^i(h)$ (or, equivalently, $W_t^i(h)$), subject to (18). In turn, the representative household h, inhabiting geographic unit

[13]Notice that price and consumption indexes are such that at the optimum:

$$\begin{aligned} P_{c,t}^i C_t^i &= P_t^i C_{i,t}^i + P_t^S C_{S,t}^i + P_t^B C_{B,t}^i \\ P_{c,t}^B C_t^B &= P_t^B C_{B,t}^B + P_t^S C_{S,t}^B \end{aligned}$$

$b \in \mathbf{B}$, maximizes lifetime utility (2) with respect to C_t^B, D_{t+1}^B and $L_t^b(h)$, subject to (19). The first-order conditions are, respectively,

$$\begin{aligned}
\frac{W_t^i(h)}{P_{c,t}^i} &= -\left(1+\mu_{w,t}^i\right)\frac{v_{L_t}\left(L_t^i(h)\right)}{u_{C_t}\left(C_t^i\right)} = \left(1+\mu_{w,t}^i\right)\left(C_t^i\right)^{\frac{1}{\sigma}}\chi_0\left(L_t^i(h)\right)^{\chi}, \\
\frac{W_t^b(h)}{P_{c,t}^B} &= -\left(1+\mu_{w,t}^B\right)\frac{v_{L_t}\left(L_t^b(h)\right)}{u_{C_t}\left(C_t^B\right)}, \forall b \in \mathbf{B}, && \text{(20a)} \\
Q_{t,t+1} &= \beta\left\{\frac{u_{C_{t+1}}\left(C_{t+1}^j\right)}{u_{C_t}\left(C_t^j\right)}\frac{P_{c,t}^j}{P_{c,t+1}^j}\right\}, j = \mathbf{B}, s(including\ i) \in \mathbf{S}, && \text{(20b)}
\end{aligned}$$

which are assumed to hold for all periods and states of nature (at t and $t+1$, in the case of equation (20b)).Under the assumption of complete markets for state-contingent securities across the union, equation (20b) will hold for the representative household in any country j.

Equation (20a) is the optimal condition for labor supply, reflecting household's market power. Labor markets are characterized by an exogenous country-specific wage markup. $\mu_{w,t}^j > 0$ is the optimal net wage-markup, capturing monopolistic distortions in input supply. Following Clarida, Galí and Gertler (2002), we allow for exogenous variation in the wage markup. This allows having different wage markups across countries and is used as a device to introduce the possibility of "pure cost-push shocks" that affects the equilibrium price behavior but does not change the efficient output, as in Benigno and Woodford (2003). Notice that in the symmetric steady state the net wage-markup is the same across all countries. These parameters only differ across countries as a consequence of idiosyncratic shocks. In particular, the deviation of $\mu_{w,t}^j$ from its steady state value captures "pure cost-push shocks" affecting country j. In the absence of cost push shocks, $\mu_{w,t}^i = \mu_{w,t}^s = \mu_{w,t}^B = \mu_w$ (i.e., the steady state net markup).

Equation (20b) reflects the optimal intertemporal allocation of consumption, where the intertemporal marginal rate of substitution of consumption between periods t and $t+1$ times the price ratio $(\frac{P_{c,t}^j}{P_{c,t+1}^j})$ should equalize the stochastic discount factor $Q_{t,t+1}$.

Let R_t^* denote the gross nominal yield on a riskless one-period discount bond. Then by taking the expectations of each side of equation (20b), we obtain the following conventional stochastic Euler equation:

$$1 = \beta R_t^* E_t \left\{ \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\frac{1}{\sigma}} \frac{P_{c,t}^j}{P_{c,t+1}^j} \right\}, \tag{21}$$

$\forall j = \mathbf{B}$, $s \in \mathbf{S}$, where $(R_t^*)^{-1} = E_t \{Q_{t,t+1}\}$ is the price of the riskless one-period discount bond. For future reference, a "star" will be used to denote the currency union (as a whole) variables.

Rewriting in log-linearized form equations (20a) and (21), respectively, we have that

$$\begin{aligned} w_t^i(h) - p_{c,t}^i &= \log(1 + \mu_{w,t}^i) + \frac{1}{\sigma} c_t^i + \log \chi_0 + \chi l_t^i(h), \\ w_t^b(h) - p_{c,t}^B &= \log(1 + \mu_{w,t}^B) + \frac{1}{\sigma} c_t^B + \log \chi_0 + \chi l_t^b(h), \forall b \in \mathbf{B}, \qquad (22) \\ & \quad \mathit{and} \\ c_t^j &= E_t \left\{ c_{t+1}^j \right\} - \sigma \left(r_t^* - E_t \left\{ \pi_{c,t+1}^j \right\} - \rho \right), j = \mathbf{B}, s \in \mathbf{S}, \qquad (23) \end{aligned}$$

where lowercase letters denote (natural) logs of the corresponding variables (i.e., $r_t^* \equiv \log R_t^*$, $c_t^j \equiv \log C_t^j$, $p_t^j \equiv \log P_t^j$), $\rho = -\log(\beta)$ is the time discount rate, and $\pi_{c,t+1}^j = \left(p_{c,t+1}^j - p_{c,t}^j\right)$ is the consumer price index (CPI) inflation rate in period $t+1$.

To complete the set of optimality conditions we must add to the sequence of period budget constraints the no Ponzi schemes and transversality condition on borrowing, i.e., a solvency condition of the form:

$$\lim_{k \to \infty} E_t \left\{ Q_{t,t+k} D_{t+k}^j \right\} = 0, \forall t, j = \mathbf{B}, s \in \mathbf{S}. \tag{24}$$

2.1.6 Terms of Trade, Domestic Inflation and the CPI Inflation

In this section, we introduce some assumptions and definitions that will be useful in future analysis. We start by defining bilateral terms of trade between a small country $i \in \mathbf{S}$ and country j of the currency union, $j = \mathbf{B}$, $s \in \mathbf{S}$, as the price of country j's goods in terms of country i's goods, i.e., $TT_{j,t}^i \equiv \frac{P_t^j}{P_t^i}$. Similarly, the bilateral terms of trade between country $\mathbf{B}$ and country j is defined as $TT_{j,t}^B \equiv \frac{P_t^j}{P_t^B}$.

The effective terms of trade for country $i \in \mathbf{S}$ is given by

$$TT_t^i \equiv \frac{P_t^{-i}}{P_t^i} = \frac{\left[(1-n)\left(P_t^B\right)^{1-\gamma} + \int_0^n \left[P_t^s\right]^{1-\gamma} ds\right]^{\frac{1}{1-\gamma}}}{P_t^i}$$
$$= \left[(1-n)\left(TT_{B,t}^i\right)^{1-\gamma} + \int_0^n \left(TT_{s,t}^i\right)^{1-\gamma} ds\right]^{\frac{1}{1-\gamma}}, \qquad (25)$$

where we use the definitions of P_t^{-i} and P_t^S from (9). Notice that since country i has a zero measure, $P_t^{-i} = P_t^*$, i.e., the aggregate price index of the union as a whole (average price).

Following Galí (2008, p. 155), TT_t^i can be approximated up to a first-order log-linear approximation around a symmetric (zero-inflation) steady state[14] in which domestic and foreign goods are equal, i.e., $TT_j^i = 1$ and $TT^j = 1$, $\forall\, j = \mathbf{B}$, $s \in \mathbf{S}$, by

$$tt_t^i = (1-n)\, tt_{B,t}^i + \int_0^n tt_{s,t}^i ds, \qquad (26)$$

where $tt_t^i \equiv \log TT_t^i = p_t^{-i} - p_t^i = p_t^* - p_t^i$. Similarly, $tt_t^s \equiv \log TT_t^s = p_t^{-s} - p_t^s = p_t^* - p_t^s$.

Regarding country $\mathbf{B}$, the effective terms of trade are given by

$$TT_t^B \equiv \frac{P_t^S}{P_t^B} = \frac{\left(\frac{1}{n}\int_0^n \left[P_t^s\right]^{1-\gamma} ds\right)^{\frac{1}{1-\gamma}}}{P_t^B}$$
$$= \left(\frac{1}{n}\int_0^n \left[TT_{s,t}^B\right]^{1-\gamma} ds\right)^{\frac{1}{1-\gamma}}, \qquad (27)$$

which can also be approximated up to a first-order around the symmetric steady state by

$$tt_t^B = \frac{1}{n}\int_0^n tt_{s,t}^B ds, \qquad (28)$$

where $tt_t^B \equiv \log TT_t^B = p_t^S - p_t^B$.

The effective terms of trade permit to establish a relation between domestic and consumer price indexes. Following Galí (2008, p. 155), we log-linearize the consumer

[14] We assume that the steady-state inflation is zero.

price indexes from (7) and (10) around the same symmetric (zero-inflation) steady state, yielding, respectively:

$$
\begin{aligned}
p^i_{c,t} &= \lambda_S(p^i_t) + (1-\lambda_S)\left(p^{-i}_t\right) \\
&= (1-\lambda_S)\left[p^{-i}_t - p^i_t\right] + p^i_t \\
&= (1-\lambda_S)\, tt^i_t + p^i_t, \qquad (29) \\
& \quad \textit{and} \\
p^B_{c,t} &= \lambda_B(p^B_t) + (1-\lambda_B)\left(p^S_t\right) \\
&= (1-\lambda_B)\left[p^S_t - p^B_t\right] + p^B_t \\
&= (1-\lambda_B)\, tt^B_t + p^B_t. \qquad (30)
\end{aligned}
$$

From (29)-(30) it is possible to establish a relation between domestic inflation and the CPI inflation. Country i's domestic inflation is $\pi^i_t \equiv \left(p^i_t - p^i_{t-1}\right)$,while country i's CPI inflation is given by $\pi^i_{c,t} \equiv \left(p^i_{c,t} - p^i_{c,t-1}\right)$, therefore[15]

$$
\begin{aligned}
\pi^i_{c,t} &\equiv \left(p^i_{c,t} - p^i_{c,t-1}\right) \\
&= (1-\lambda_S)\, tt^i_t + p^i_t - \left[(1-\lambda_S)\, tt^i_{t-1} + p^i_{t-1}\right] \\
&= \pi^i_t + (1-\lambda_S)\, \Delta tt^i_t, \qquad (31)
\end{aligned}
$$

while for country B

$$
\begin{aligned}
\pi^B_{c,t} &\equiv \left(p^B_{c,t} - p^B_{c,t-1}\right) \\
&= (1-\lambda_B)\, tt^B_t + p^B_t - \left[(1-\lambda_B)\, tt^B_{t-1} + p^B_{t-1}\right] \\
&= \pi^B_t + (1-\lambda_B)\, \Delta tt^B_t, \qquad (32)
\end{aligned}
$$

For future reference, notice that when we use the term inflation without discriminating the type of inflation we are talking about domestic inflation.

With regard to the currency union as a whole, there is no distinction between CPI and producer (domestic) price indexes, neither between their corresponding inflation rates. The producer price index for the union as a whole is

[15] The gap between the CPI and the domestic inflation is proportional to the percent change in the effective terms of trade, with the coefficient of proportionality given by the openness index.

$$P_t^* \equiv \left[\int_0^n (P_t^s)^{1-\gamma} ds + (1-n)\left(P_t^B\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{33}$$

Let the CPI price index for the union as a whole be defined as

$$P_{c,t}^* \equiv \left[\int_0^n \left(P_{c,t}^s\right)^{1-\gamma} ds + (1-n)\left(P_{c,t}^B\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{34}$$

Substituting $P_{c,t}^B$ and $P_{c,t}^s$ (analogous to $P_{c,t}^i$) from (10) and (7) and taking in consideration that $P_t^{-s} = P_t^*, \forall s \in \mathbf{S}$ (including country i), we get that $P_{c,t}^* = P_t^*$.

A first-order log-linear approximation to (33) and to P_t^S in (9), around the symmetric steady state yields

$$\begin{aligned} p_t^* &= \int_0^n p_t^s ds + (1-n)\, p_t^B = n p_t^S + (1-n)\, p_t^B, && (35a) \\ where &: \\ p_t^S &= \frac{1}{n}\int_0^n p_t^s ds, && (35b) \end{aligned}$$

Taking into account equations (35a), (30) and (29) - analogous for country $s, \forall s \in$ **S,** we get, up to a first-order log-linear approximation around the symmetric steady state, that

$$\begin{aligned} p_t^* &= \int_0^n p_{c,t}^s ds + (1-n)\, p_{c,t}^B - \left[\int_0^n (1-\lambda_S)\, tt_t^s ds + (1-n)(1-\lambda_B)\, tt_t^B\right] \\ &= p_{c,t}^*, && (36a) \\ where &: \\ 0 &= \int_0^n (1-\lambda_S)\, tt_t^s ds + (1-n)(1-\lambda_B)\, tt_t^B. && (36b) \end{aligned}$$

2.1.7 International Risk Sharing

Combining the Euler equations for each country under the assumption of complete markets for state-contingent securities across the union, we obtain:

$$\begin{aligned} \frac{u_{C_{t+1}}\left(C_{t+1}^i\right)}{u_{C_t}\left(C_t^i\right)} \frac{P_{c,t}^i}{P_{c,t+1}^i} &= \frac{u_{C_{t+1}}\left(C_{t+1}^s\right)}{u_{C_t}\left(C_t^s\right)} \frac{P_{c,t}^s}{P_{c,t+1}^s} \\ &= \frac{u_{C_{t+1}}\left(C_{t+1}^B\right)}{u_{C_t}\left(C_t^B\right)} \frac{P_{c,t}^B}{P_{c,t+1}^B}, \end{aligned}$$

$\forall i, s \in \mathbf{S}$, resulting in the following international risk sharing condition:

$$\begin{aligned} \frac{1}{u_{C_t}\left(C_t^i\right)} P_{c,t}^i &= \vartheta_s \frac{1}{u_{C_t}\left(C_t^s\right)} P_{c,t}^s = \vartheta_B \frac{1}{u_{C_t}\left(C_t^B\right)} P_{c,t}^B \Leftrightarrow \\ \left(C_t^i\right)^{\frac{1}{\sigma}} P_{c,t}^i &= \vartheta_s \left(C_t^s\right)^{\frac{1}{\sigma}} P_{c,t}^s = \vartheta_B \left(C_t^B\right)^{\frac{1}{\sigma}} P_{c,t}^B, \end{aligned} \tag{37}$$

$\forall i, s \in S$, and all t, and where ϑ_s and ϑ_B are constants which will generally depend on initial conditions. Without loss of generality, we assume symmetric initial conditions (i.e., equal initial debt holdings across countries, combined with an ex-ante identical environment), in which case we have $\vartheta_s = \vartheta_B = \vartheta = 1$:

$$\begin{aligned} \left(C_t^i\right)^{\frac{1}{\sigma}} P_{c,t}^i &= \left(C_t^s\right)^{\frac{1}{\sigma}} P_{c,t}^s = \left(C_t^B\right)^{\frac{1}{\sigma}} P_{c,t}^B \Leftrightarrow \\ C_t^i &= C_t^s \left(\frac{P_{c,t}^s}{P_{c,t}^i}\right)^{\sigma} = C_t^B \left(\frac{P_{c,t}^B}{P_{c,t}^i}\right)^{\sigma}. \end{aligned} \tag{38}$$

These assumptions (complete markets and symmetric initial conditions) imply perfect consumption risk sharing within each country but not between countries, due to home bias in consumption.[16]

The log-linearization of equation (38) yields the following relations

$$c_t^i = c_t^s + \sigma\left(p_{c,t}^s - p_{c,t}^i\right), \tag{39a}$$

$$c_t^i = c_t^B + \sigma\left(p_{c,t}^B - p_{c,t}^i\right), \tag{39b}$$

$$c_t^B = c_t^s + \sigma\left(p_{c,t}^s - p_{c,t}^B\right). \tag{39c}$$

As regards the union as a whole, notice that in nominal terms

$$\int_0^n C_t^s P_{c,t}^s ds + (1-n) C_t^B P_{c,t}^B = C_t^* P_{c,t}^*,$$

where C_t^* is the union-wide consumption index, which can also be interpreted as the union's *per capita* or average consumption. A first-order log-linear approximation to the above equation around the symmetric steady state yields

[16] Even if $\sigma = 1$ and countries are subject to the same exogenous disturbances affecting the demand for private consumption goods, in which case the consumption expenditure is the same across countries, risk sharing is not perfect, because the existence of home bias in consumption implies that the purchasing power parity (PPP) does not hold.

$$c_t^* = \int_0^n c_t^s ds + (1-n)\, c_t^B. \tag{40}$$

By integrating over all households, using equations (39a) and (39b), we get that

$$\begin{aligned} c_t^i &= \int_0^n c_t^s + \sigma\left(p_{c,t}^s - p_{c,t}^i\right) ds + (1-n)\left[c_t^B + \sigma\left(p_{c,t}^B - p_{c,t}^i\right)\right] \\ &= \int_0^n c_t^s ds + (1-n) c_t^B + \sigma\left[\int_0^n \left(p_{c,t}^s - p_{c,t}^i\right) ds + (1-n)\left(p_{c,t}^B - p_{c,t}^i\right)\right] \\ &= c_t^* + \sigma\left(p_{c,t}^* - p_{c,t}^i\right), \end{aligned} \tag{41}$$

where $p_{c,t}^* = p_t^*$. Similarly, $c_t^s = c_t^* + \sigma\left(p_{c,t}^* - p_{c,t}^s\right), \forall s \in S$.

By integrating over all households, using equation (39c),

$$c_t^B = c_t^* + \sigma\left(p_{c,t}^* - p_{c,t}^B\right). \tag{42}$$

Equations (41)-(42) state that the marginal rate of substitution between a country's consumption and the average union's consumption has to be equal to the CPI of that country relative to the union's CPI.

Taking into account equations (30) and (29), and that $p_t^* = p_{c,t}^*$, we have that

$$c_t^i = c_t^* + \sigma(1-\alpha)\, tt_t^i, \tag{43a}$$

$$c_t^s = c_t^* + \sigma(1-\alpha)\, tt_t^s, \tag{43b}$$

$$c_t^B = c_t^* + \sigma(1-\alpha)\, ntt_t^B, \tag{43c}$$

using the fact that

$$\begin{cases} tt_t^i = p_t^* - p_t^i, i \in S, \\ ntt_t^B = \int_0^n p_t^s ds - np_t^B = \int_0^n p_t^s ds + (1-n)p_t^B - p_t^B = p_t^* - p_t^B \end{cases}, \tag{44}$$

and that $(1-\lambda_S) = \alpha$ and $(1-\lambda_B) = n\alpha$.

As a request of the complete markets assumption, domestic consumption is positively linked to union-wide consumption and to the terms of trade.

2.2 Optimal Allocation of Government Purchases

For simplicity, it is assumed that government expenditures are fully allocated to domestically produced goods[17], as in Beetsma and Jensen (2005), Kirsanova *et al.* (2007), Galí and Monacelli (2008), Ferrero (2009), Leith and Wren-Lewis (2007b, 2011), among others.

Thus, for a small country i in the $\mathbf{S}$ block, G_t^i is a composite index representing real *per capita* public consumption of domestic goods, as

$$G_t^i \equiv \left(\int_0^1 \left[G_{i,t}^i(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{45}$$

where $G_{i,t}^i(h)$ is the quantity of domestic good h purchased by the government.

For the big country $\mathbf{B}$, the composite index representing real *per capita* public consumption, G_t^B, is given by

$$G_t^B \equiv \left(\frac{1}{1-n} \int_n^1 \left[G_{b,t}^B \right]^{\frac{\epsilon-1}{\epsilon}} db \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{46}$$

where $G_{b,t}^B$ is in turn a composite index of country $\mathbf{B}$'s government expenditures on goods produced in geographic unit b, defined in the same way as $C_{b,t}^B$ Taking a representative geographic unit b, G_t^B can be defined as $G_t^B = G_{b,t}^B, \forall b \in \mathbf{B}$, that is, *per capita* government spending is the same across country $\mathbf{B}$'s geographic units.

For any given level of public spending, governments are assumed to allocate expenditures across domestic goods in order to minimize total cost. This yields the following government demand for the individual domestic good h at country $i \in \mathbf{S}$ and at country $\mathbf{B}$, respectively [18]:

$$G_{i,t}^i(h) = \left(\frac{P_t^i(h)}{P_t^i} \right)^{-\epsilon} G_t^i, \forall h \in [0,1], \tag{47}$$

$$G_{b,t}^B(h) = \left(\frac{P_t^b(h)}{P_t^b} \right)^{-\epsilon} G_{b,t}^B, \ with\ G_{b,t}^B = G_t^B, \forall h \in [0,1], \forall b \in \mathbf{B}. \tag{48}$$

[17] Galí and Monacelli (2008) refer that there is evidence of strong home bias in government procurement, over and above that observed in private consumption.

According to Beetsma and Jensen (2005), p. 325, "While this is an extreme situation, fiscal policy remains effective at stabilizing the individual economies in the face of asymmetric disturbances as long as the public spending indices remain biased towards nationally produced goods".

[18] Notice that:

2.3 Total Demand for Individual Goods

We assume that the production of each good is completely absorbed by private and public consumption. Thus, in each country, the demand for home-produced good h is the sum of three components: the demands of domestic and foreign households (private consumption) and government (public consumption). Given individual household's demand functions for good h, we have to aggregate over domestic and foreign households to obtain total private consumption demand.

Hence, total demand for the generic good h, produced in a small country $i \in \mathbf{S},$ is given by

$$Y_{i,t}^{d}(h) = C_{i,t}^{i}(h) + \int_{0}^{n} \left[C_{i,t}^{s}(h) \right] ds + (1-n) \left[C_{i,t}^{B}(h) \right] + G_{i,t}^{i}(h), \tag{49}$$

$\forall h \in [0,1]$, where we take into account the identical behavior of households within each country/geographic units - symmetric equilibrium, given identical preferences and initial conditions. Using equations (11)-(13) and (47), we get that

$$Y_{i,t}^{d}(h) = \left(\frac{P_t^i(h)}{P_t^i} \right)^{-\epsilon} \left\{ \begin{array}{l} \left(\frac{P_t^i}{P_{c,t}^i} \right)^{-\gamma} \left[\begin{array}{l} \lambda_S C_t^i \\ + (1-\lambda_S) \int_0^n \left(\frac{P_{c,t}^i}{P_{c,t}^s} \right)^{-\gamma} C_t^s ds \\ + \frac{(1-n)}{n} (1-\lambda_B) \left(\frac{P_{c,t}^i}{P_{c,t}^B} \right)^{-\gamma} C_t^B \end{array} \right] \\ + G_t^i \end{array} \right\}, \tag{50}$$

$\forall h \in [0,1]$.

$$\begin{aligned} \int_0^1 P_t^i(h) G_{i,t}^i(h) dh &= P_t^i G_t^i, \\ \int_0^1 P_t^b(h) G_{b,t}^B(h) dh &= P_t^b G_{b,t}^B, \\ & \textit{and} \\ \int_n^1 P_t^b G_{b,t}^B db &= (1-n) P_t^B G_t^B, \ \textit{since} \ \ P_t^b = P_t^B \ \ \textit{and} \ \ G_{b,t}^B = G_t^B, \forall b \in \mathbf{B}. \end{aligned}$$

Similarly, total demand for good h produced in geographic unit $b \in \mathbf{B}$ is given by:

$$Y_{b,t}^d(h) = (1-n)\left[C_{b,t}^B(h)\right] + \int_0^n \left[C_{b,t}^s(h)\right] ds + G_{b,t}^B(h), \tag{51}$$

$\forall h \in [0,1]$ and $\forall b \in \mathbf{B}$. Using equations (14)-(15) and (48),

$$Y_{b,t}^d(h) = \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} \left\{ \begin{array}{l} \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma} \left[\begin{array}{l} \lambda_B C_t^B \\ +(1-\lambda_S) \int_0^n \left(\frac{P_{c,t}^B}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \end{array} \right] \\ +G_t^B \end{array} \right\}, \tag{52}$$

$\forall h \in [0,1]$ and $\forall b \in \mathbf{B}$.

2.4 Aggregate Demand

Aggregate demand is normalized by population size, i.e., it is expressed in *per capita* terms. In the case of small countries or geographic units in $\mathbf{B}$, there is no difference between total and *per capita* values, since they are populated by a continuum of agents on the interval $[0,1]$; however, in the case of country $\mathbf{B}$ as a whole, we must take into consideration the population size $(1-n)$. To obtain aggregate demand for countries $i \in \mathbf{S}$ and $\mathbf{B}$, we need to aggregate over all varieties h in equations (50) and (52), using Dixit-Stiglitz aggregators.

Hence, aggregate demand in country $i \in \mathbf{S}$ is given by

$$
\begin{aligned}
Y_{i,t}^{d} &\equiv \left(\int_0^1 \left[Y_{i,t}^{d}(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left\{ \begin{array}{l} \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} \left[\begin{array}{l} \lambda_S C_t^i \\ +\left(1-\lambda_S\right)\int_0^n \left(\frac{P_{c,t}^i}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \\ +\frac{(1-n)}{n}\left(1-\lambda_B\right)\left(\frac{P_{c,t}^i}{P_{c,t}^B}\right)^{-\gamma} C_t^B \end{array} \right] + \\ +G_t^i \end{array} \right\} \times \\
&\quad \times \left(\int_0^1 \left[\left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon}\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} \left[\begin{array}{l} \lambda_S C_t^i \\ +\left(1-\lambda_S\right)\int_0^n \left(\frac{P_{c,t}^i}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \\ +\frac{(1-n)}{n}\left(1-\lambda_B\right)\left(\frac{P_{c,t}^i}{P_{c,t}^B}\right)^{-\gamma} C_t^B \end{array} \right] + G_t^i, \qquad (53)
\end{aligned}
$$

where we make use of the definition of P_t^i, (8), in the second equality.

Total demand for the generic good h, produced in country $i \in \mathbf{S}$, is a function of relative prices and aggregate demand:

$$
Y_{i,t}^{d}(h) = \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} Y_{i,t}^{d}, \forall h \in [0,1]. \qquad (54)
$$

With regard to country $\mathbf{B}$, we can define the aggregate demand for a geographic unit b, and the aggregate demand for country $\mathbf{B}$ as a whole. In geographic unit b, the aggregate demand is defined as

$$
\begin{aligned}
Y_{b,t}^d &\equiv \left(\int_0^1 \left[Y_{b,t}^d(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left\{ \begin{array}{l} \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma} \left[\begin{array}{l} \lambda_B C_t^B \\ +(1-\lambda_S)\int_0^n \left(\frac{P_{c,t}^B}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \end{array} \right] \\ +G_t^B \end{array} \right\} \times \\
&\quad \times \left(\int_0^1 \left[\left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon}\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma} \left[\begin{array}{l} \lambda_B C_t^B + \\ +(1-\lambda_S)\int_0^n \left(\frac{P_{c,t}^B}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \end{array} \right] + G_t^B, \forall b \in \mathbf{B}.
\end{aligned}
\tag{55}
$$

For country $\mathbf{B}$ as a whole, aggregate demand (normalized by population size) is $Y_{B,t}^d = Y_{b,t}^d, \forall b \in \mathbf{B}$, making use of the fact that aggregate demand is the same across all geographic units. In turn, total aggregate demand is $(1-n)Y_{B,t}^d$.

Total demand for the generic good h, produced in geographic unit $b \in \mathbf{B}$, is a function of relative prices and aggregate demand:

$$
Y_{b,t}^d(h) = \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} Y_{B,t}^d, \forall h \in [0,1], \forall b \in \mathbf{B}. \tag{56}
$$

2.5 Firms

2.5.1 Technology

In each country of the currency union there is a continuum of monopolistic competitive firms sharing the same technology and using labor as the only input.

In country $i \in \mathbf{S}$, there is a continuum of firms indexed by $h \in [0,1]$. Each one of these firms produces a differentiated good h with a linear technology represented by the production function

$$
Y_t^i(h) = A_t^i L_t^i(h), \forall h \in [0,1], \tag{57}
$$

where $Y_t^i(h)$ denotes country i's production of good h (or, equivalently, firm h's production), A_t^i represents the level of technology of country i, assumed to be common to all i-firms and to evolve exogenously over time, and $L_t^i(h)$ is the firm-specific labor input.

In country $\mathbf{B}$, each geographic unit b is populated by a continuum of firms indexed by $h \in [0,1]$ producing differentiated goods. Independently of the geographic unit where they are located, all firms in country $\mathbf{B}$ have access to the same linear technology represented by

$$Y_t^b(h) = A_t^B L_t^b(h), \forall h \in [0,1], \forall b \in \mathbf{B}, \tag{58}$$

where $Y_t^b(h)$ denotes geographic unit b's production of good h (which corresponds to country $\mathbf{B}$'s total production of this good), $L_t^b(h)$ is the firm-specific labor input and A_t^B represents the level of country $\mathbf{B}$'s technology.

Let A_t^j, $j = \mathbf{B}$, $s \in \mathbf{S}$, including small country i, represent country j's technology. It is assumed that $a_t^j \equiv \log A_t^j$ follows the stationary AR(1) process

$$a_t^j = \rho_a a_{t-1}^j + \varepsilon_t^j,\ j = \mathbf{B}, s \in \mathbf{S}, \tag{59}$$

where $\rho_a \in [0,1]$ and ε_t^j is a zero mean white noise process.

2.5.2 Aggregate Output

Like aggregate demand, aggregate output (GDP) is normalized by population size, i.e., it is expressed in *per capita* terms.

Let aggregate domestic output of a small country $i \in \mathbf{S}$ be defined as

$$Y_t^i \equiv \left(\int_0^1 \left[Y_t^i(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{60}$$

and aggregate domestic output of country $\mathbf{B}$ as

$$\begin{aligned} Y_t^B &\equiv \left(\frac{1}{1-n} \int_n^1 \left[Y_t^b \right]^{\frac{\epsilon-1}{\epsilon}} db \right)^{\frac{\epsilon}{\epsilon-1}} = Y_t^b, \forall b \in \mathbf{B}, \\ where &: \\ Y_t^b &\equiv \left(\int_0^1 \left[Y_t^b(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \forall b \in \mathbf{B}. \end{aligned} \tag{61}$$

Given the same technology and market structure, the same price setting mechanism (explained in next section), and the fact that all the firms in country **B** are subject to the same shocks and fiscal policies, the aggregate output produced within a geographic unit is the same independently of the geographic unit we take. Hence, country **B**'s total aggregate output is given by $(1-n)Y_t^B$.

2.5.3 Price Setting

It is assumed that firms set prices in a staggered fashion, following a partial adjustment rule a la Calvo (1983), in which $(1-\theta)$ is the probability a firm may reset its price in any given period, independently of the time elapsed since the last adjustment, and θ is the probability a firm keeps its price unchanged, where probability draws are i.i.d. over time. Hence, within a country, and at each point in time, a fraction $(1-\theta)$ of randomly selected firms will choose prices optimally while a fraction θ will maintain its prices unchanged. The average duration of a price is given by $\frac{1}{(1-\theta)}$, with θ being a natural index of price rigidity.

Notice that since the average duration of a price might not be the same across countries, we introduce the possibility of the average duration of a price in a small country $s \in \mathbf{S}$ (including country i) being different from the average duration of a price in country **B**. Let country j's index of price rigidity, θ^j, be defined as

$$\theta^j = \begin{cases} \theta_S, for\ j = s, \forall s \in \mathbf{S} \\ \theta_B,\ \ for\ j = \mathbf{B} \end{cases} .$$

In each country j, $j = \mathbf{B}, s \in \mathbf{S}$, a firm re-optimizing in period t will choose the price $\overset{o\,j}{P_t}$ that maximizes the current market value of the future profits that would be collected if the optimal price could not be changed. The fraction $(1-\theta^j)$ of country j's firms that can optimal set a new price in period t will choose the same price because they face an identical problem. As to the fraction θ^j of firms that maintain prices unchanged, they just adjust output to meet demand, assuming that these firms operate with a non-negative net markup.

Formally, a wage-taker firm able to set a new price in period t solves the following optimization problem

$$\max_{\overset{o}{P}{}^{j}_{t}} \sum_{k=0}^{\infty} \left(\theta^{j}\right)^{k} E_t \left\{ Q_{t,t+k} \left[(1-\tau^{j}_{t+k}) \overset{o}{P}{}^{j}_{t} \overset{o}{Y}{}^{j}_{t+k|_t} - (1-\varsigma^{j}_{w}) \overset{o}{W}{}^{j}_{t+k} \frac{\overset{o}{Y}{}^{j}_{t+k|_t}}{A^{j}_{t+k}} \right] \right\}, \tag{62}$$

for $j = \mathbf{B}, s \in \mathbf{S},$ subject to the sequence of demand constraints, given by (54) and(56),

$$\overset{o}{Y}{}^{j}_{t+k|_t} = \left(\frac{\overset{o}{P}{}^{j}_{t}}{P^{j}_{t+k}} \right)^{-\epsilon} Y^{d}_{j,t+k},$$

for $k = 0, 1, 2, ...,$ where $Q_{t,t+k} = \beta^{k} \left\{ \left(\frac{C^{j}_{t+k}}{C^{j}_{t}} \right)^{-\frac{1}{\sigma}} \frac{P^{j}_{c,t}}{P^{j}_{c,t+k}} \right\}$ is the stochastic discount factor for nominal payoffs, τ^{j}_{t+k} is a proportional tax rate on sales (revenue tax) with the non-zero steady state level τ^{j}, and ς^{j}_{w} is an employment subsidy. $\overset{o}{Y}{}^{j}_{t+k|_t}$ represents the output in period $t+k$ for a firm that last reset its price in period t, $\frac{Y^{j}_{t+k|_t}}{A^{j}_{t+k}}$ is the amount of labor employed, and $Y^{d}_{j,t+k}$ represents country j's aggregate demand in period $t+k$ (in equilibrium $Y^{d}_{j,t+k} = Y^{j}_{t+k}$). Since firms reoptimizing their price in period t will choose the same price (given the model symmetry), they will also produce the same quantity $\overset{o}{Y}{}^{j}_{t+k|_t}$ and employ the same amount of labor in period $t+k$, until next price adjustment takes place; hence, there is no need to distinguish across different firms/(varieties of goods) from the same country in the above maximization problem. As firms reoptimizing their price in period t will employ the same amount of labor in period $t+k$, they also will pay the same nominal wage $\overset{o}{W}{}^{j}_{t+k}$, which results from the optimal condition for labor supply (20a). However, since labor market is segmented, firms that not reoptimize its price employ a different amount of labor and, hence, pay a different nominal wage. Consequently, they face different nominal marginal costs. It is assumed that each firm receives a subsidy of ς^{j}_{w} percent of its wage bill, which is not supposed to vary over the business cycle, and that there is a lump-sum tax available to finance it. This employment subsidy ς^{j}_{w} is designed to allow the flexible price equilibrium to be efficient (or, equivalently, to support the assumption that the steady state level of output is efficient). Following Leith and Wren-Lewis (2007a, 2007b), the employment subsidy is used to eliminate the steady state distortion associated to monopolistic competition and distortionary revenue taxes. ς_w^j eliminates linear terms in the social welfare function without losing the possibility of using the revenue tax τ_t^j as fiscal instrument.

After dividing the optimality condition of the firm problem by P_{t-1}^j, as in Galí (2008, p. 45), it takes the form

$$\sum_{k=0}^{\infty}\left(\theta^j\right)^k E_t\left\{Q_{t,t+k}\overset{o}{Y}{}^{j}_{t+k|t}(1-\tau_{t+k}^j)\left[\frac{\overset{o}{P}{}^{j}_t}{P_{t-1}^j}-(1+\mu_p)\overset{o}{MC}{}^{j}_{t+k}\frac{P_{t+k}^j}{P_{t-1}^j}\right]\right\} = 0, \tag{63}$$

where $\mu_p \equiv \frac{1}{\epsilon-1} > 0$ is the optimal net price-markup and $\overset{o}{MC}{}^{j}_{t+k} \equiv \frac{(1-\varsigma_w^j)\overset{o}{W}{}^{j}_{t+k}}{A_{t+k}^j(1-\tau_{t+k}^j)P_{t+k}^j}$ is the real marginal cost of firms reoptimizing price in period t. Taking into account the optimal condition for labor supply (20a), the production function and the sequence of demand constraints, given by (54)-(56),

$$\begin{aligned}
\overset{o}{MC}{}^{j}_t &\equiv \frac{(1-\varsigma_w^j)P_{c,t}^j(1+\mu_{w,t}^j)(C_t^j)^{\frac{1}{\sigma}}\chi_0(\overset{o}{L}{}^{j}_t)^{\chi}}{A_t^j(1-\tau_t^j)P_t^j} \\
&= \frac{(1-\varsigma_w^j)P_{c,t}^j(1+\mu_{w,t}^j)(C_t^j)^{\frac{1}{\sigma}}\chi_0}{(A_t^j)^{(1+\chi)}(1-\tau_t^j)P_t^j}(\overset{o}{Y}{}^{j}_t)^{\chi} \\
&= MC_t^j\left(\frac{\overset{o}{P}{}^{j}_t}{P_t^j}\right)^{-\epsilon\chi}, \\
\textit{with} &: \\
MC_t^j &= \frac{(1-\varsigma_w^j)P_{c,t}^j(1+\mu_{w,t}^j)(C_t^j)^{\frac{1}{\sigma}}\chi_0}{(A_t^j)^{(1+\chi)}(1-\tau_t^j)P_t^j}\left(Y_t^j\right)^{\chi}
\end{aligned} \tag{64}$$

where MC_t^j is the average/aggregate real marginal cost in country j, considering an index analogous to the one used in the definition of aggregate domestic prices, and that $Y_{j,t}^d = Y_t^j$ in equilibrium.

It is possible to derive the standard log-linear optimal price-setting rule using a first-order approximation of (63) around the symmetric zero-inflation steady state. Taking into account (64) and that in the (symmetric) zero-inflation steady state $Q_{t,t+k} = \beta^k$, we get that

$$\overset{oj}{p}_t = \frac{\log(1+\mu_p)}{(1+\epsilon\chi)} + (1-\theta^j\beta)\sum_{k=0}^{\infty}(\theta^j\beta)^k E_t\left\{\frac{mc_{t+k}^j}{(1+\epsilon\chi)} + p_{t+k}^j\right\}, j = \mathbf{B}, s \in \mathbf{S}, \tag{65}$$

where, as before, lowercase letters denote the logs of the corresponding variables. In the zero-inflation steady state, the real marginal cost faced by country j's firms is constant[19], and accordingly to the optimality condition (63), $MC^j = \frac{1}{(1+\mu_p)}$. Since $MC^j = \frac{1}{(1+\mu_p)}$, $mc^j = -\log(1+\mu_p) \approx -\mu_p$. Notice that in the absence of price rigidities (or flexible prices, i.e., $\theta^j = 0$), profit maximization behavior (63) implies that $MC_t^j = \frac{1}{(1+\mu_p)}$, that is, firms would choose a constant markup over the (nominal) marginal cost, MCn_t^j. Hence, μ_p can be interpreted as the desired net markup in the absence of price-adjustment constraints, or equivalently, the net markup prevailing in a steady state with zero-inflation [Galí and Monacelli (2008, p. 121)].

2.6 Equilibrium

2.6.1 Goods Market Clearing Conditions

In equilibrium:

$$\begin{cases} Y_t^s(h) = Y_{s,t}^d(h), \forall h \text{ produced in country } s \in \mathbf{S} \text{ (including country } i), \\ Y_t^s = Y_{s,t}^d, \forall s \in \mathbf{S}, \end{cases}$$
$$\begin{cases} Y_t^b(h) = Y_{b,t}^d(h), \forall h \text{ produced in geographic unit } b, \forall b \in \mathbf{B}, \\ \left[Y_t^b = Y_t^B\right] = \left[Y_{B,t}^d = Y_{b,t}^d\right], \forall b \in \mathbf{B}. \end{cases} \tag{66}$$

Using the above equilibrium conditions and equations (54) and (56), both total demand for a specific individual good and the output of a specific firm can be written simply as a function of relative prices and national GDP.

Since in equilibrium $Y_t^j = Y_{j,t}^d$, $j = \mathbf{B}, s \in \mathbf{S}$ (including country i), it follows from equations (53) and (55), respectively, that

[19] Given the constancy of prices in that steady state, we have that $\overset{o}{P}{}_t^j = P_{t-1}^j = P_{t+k}^j = P^j$, $k = 0, 1, 2, ...$, implying that $Y_{t+k|t}^j = Y^j$, $k = 0, 1, 2, ...$ (P^j and Y^j denote, respectively, the aggregate domestic price and the aggregate output of country j in the zero-inflation steady state). In addition, nominal wages are constant in the zero-inflation steady state, from which it follows that nominal marginal cost faced by firms is also constant.

$$
Y_t^i = \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} C_t^i \left[\begin{array}{l} \lambda_S + \\ +(1-\lambda_S)\int_0^n \left(\frac{P_{c,t}^i}{P_{c,t}^s}\right)^{-\gamma+\sigma} ds + \\ +\frac{(1-n)}{n}(1-\lambda_B)\left(\frac{P_{c,t}^i}{P_{c,t}^B}\right)^{-\gamma+\sigma} \end{array}\right] + G_t^i, \tag{67a}
$$

$$
Y_t^B = \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma} C_t^B \left[\begin{array}{l} \lambda_B + \\ +(1-\lambda_S)\int_0^n \left(\frac{P_{c,t}^B}{P_{c,t}^s}\right)^{-\gamma+\sigma} ds \end{array}\right] + G_t^B, \tag{67b}
$$

where we take in consideration the risk sharing condition in (38). A first-order log-linear approximation to (67a) and (67b) around the symmetric steady state yields, respectively,

$$
\begin{aligned}
\widehat{y}_t^i &= (1-\varphi)\,\widehat{c}_t^i + (1-\varphi)\,\Phi tt_t^i + \varphi\widehat{g}_t^i, &(68a)\\
\widehat{y}_t^B &= (1-\varphi)\,\widehat{c}_t^B + (1-\varphi)\,n\Phi tt_t^B + \varphi\widehat{g}_t^B, &(68b)\\
\textit{where} &: \\
\Phi &\equiv \alpha\left[\gamma-(1-\alpha)(-\gamma+\sigma)\right].
\end{aligned}
$$

The symbol "^" is used to denote the log deviation of a variable from its steady state value (the symmetric zero-inflation steady state value), e.g., $\widehat{y}_t^j = y_t^j - y^j$, with y^j representing the steady state value, and $\varphi \equiv \frac{G^j}{Y^j}$ denotes the steady state government spending share. Notice that in the symmetric steady state $P_c^j = P^j$ and $Y^j = C^j + G^j$ (from (67a) and (67b)), implying a balanced trade balance across all countries. Thus, $\frac{G^j}{Y^j} \equiv \varphi$ and $\frac{C^j}{Y^j} \equiv 1-\varphi$, $j = \mathbf{B}, s \in \mathbf{S}.$

The above approximations make use of equations (29) - and analogous equations for each country $s \in \mathbf{S}$ - (30) and (36a), taking into account that $(1-\lambda_S) = \alpha$ and $(1-\lambda_B) = n\alpha$. In addition, (68b) takes in consideration condition (36b), (35a) and the definition of tt_t^B in (44). A condition analogous to (68a) holds for each country $s \in \mathbf{S}.$

Substituting tt_t^i and ntt_t^B from (44) in (68a) and (68b), respectively, we get a similar condition for each country j in the union, $j = \mathbf{B}, s \in \mathbf{S}$:

$$
\widehat{y}_t^j = (1-\varphi)\,\widehat{c}_t^j + (1-\varphi)\,\Phi\left(p_t^* - p_t^j\right) + \varphi\widehat{g}_t^j. \tag{69}
$$

Country j's output is positively related to private and public consumption and inversely related to domestic prices relative to union's average prices. Alternatively,

$$\widehat{y}_t^j = (1-\varphi)\,\widehat{c}_t^* + (1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]\left(p_t^* - p_t^j\right) + \varphi\widehat{g}_t^j. \tag{70}$$

where we take into consideration equations (43a) and (43c), (44), and that in the symmetric steady state $C^j = C^*$, $j = \mathbf{B}, s \in \mathbf{S}$, so that

$$\begin{cases} \widehat{c}_t^i = \widehat{c}_t^* + \sigma(1-\alpha)\,tt_t^i \\ \widehat{c}_t^B = \widehat{c}_t^* + \sigma(1-\alpha)\,ntt_t^B \end{cases}, \tag{71}$$

$$or$$

$$\widehat{c}_t^j = \widehat{c}_t^* + \sigma(1-\alpha)\left(p_t^* - p_t^j\right), j = \mathbf{B}, s \in \mathbf{S}.$$

Aggregating (70) over all countries, we obtain the union-wide goods market clearing condition[20]

$$\begin{aligned} \widehat{y}_t^* &= (1-\varphi)\,\widehat{c}_t^* + \varphi\widehat{g}_t^*, \\ where &: \\ \widehat{y}_t^* &= \int_0^n \widehat{y}_t^s ds + (1-n)\,\widehat{y}_t^B, \\ \widehat{c}_t^* &= \int_0^n \widehat{c}_t^s ds + (1-n)\,\widehat{c}_t^B, \\ \widehat{g}_t^* &= \int_0^n \widehat{g}_t^s ds + (1-n)\,\widehat{g}_t^B. \end{aligned} \tag{72}$$

2.6.2 Aggregate Price Dynamics

As a result of firm's price-setting decisions, combined with the law of large number, the aggregate domestic price index of a given country j, $j = \mathbf{B}, s \in \mathbf{S}$, evolves accordingly to:

[20] Notice that in nominal terms $Y_t^* P_t^* = \int_0^n Y_t^s P_t^s ds + (1-n)\,Y_t^B P_t^B$, and up to a first-order log-linear approximation around the symmetric steady state (where $Y^j = Y^*$, $j = \mathbf{B}, s \in \mathbf{S}$), $y_t^* = \int_0^n y_t^s ds + (1-n)\,y_t^B$.

Since in the symmetric steady state the output is the same across countries, i.e., $y^j = y^*$, $j = \mathbf{B}, s \in \mathbf{S}$, and the government spending share is also the same, then $g^j = g^*$, $j = \mathbf{B}, s \in \mathbf{S}$. Thus, $g_t^* = \int_0^n g_t^s ds + (1-n)\,g_t^B$.

$$P_t^j = \left[\theta^j \left(P_{t-1}^j\right)^{1-\epsilon} + \left(1-\theta^j\right)\left(\overset{o}{P}{}_t^j\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \; j = \mathbf{B}, s \in \mathbf{S}, \tag{73}$$

where $\overset{o}{P}{}_t^j$ is the optimal price settled in period t by a fraction $(1-\theta^j)$ of country j's firms. The fraction θ^j of country j's firms unable to reset their prices in period t, maintain prices unchanged. Notice that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, although with total mass reduced to θ^j [Galí (2008, p. 62)].[21]

Given the assumption about firm's price-setting decisions, traduced in equation (65), and the evolution of the aggregate domestic price index (73), a New Keynesian relationship between inflation and real marginal cost aggregate over all goods can be obtained[22]:

$$\begin{aligned} \pi_t^j &= \beta E_t\left\{\pi_{t+1}^j\right\} \\ &\quad + \left(1-\theta^j\beta\right)\frac{\left(1-\theta^j\right)}{\theta^j}\left[\frac{mc_t^j}{(1+\epsilon\chi)} + \frac{\log(1+\mu_p)}{(1+\epsilon\chi)}\right], j = \mathbf{B}, s \in \mathbf{S}, \end{aligned} \tag{74}$$

Recall that the log of the (average) real marginal cost zero-inflation steady state value is $mc^j = -\log(1+\mu_p)$, hence $\left[\log(1+\mu_p) + mc_t^j\right] = \left[mc_t^j - mc^j\right]$. Letting $\widehat{mc}_t^j \equiv \left(mc_t^j - mc^j\right)$ denote the log deviation of real marginal cost from its steady state value mc^j, equation (74) can be rewritten as

[21] In the case of country $\mathbf{B}$ the law of large number ensures that this price-adjustment mechanism also takes place at the geographic unit level, i.e.,

$$P_t^b = \left[\theta_B \left(P_{t-1}^b\right)^{1-\epsilon} + (1-\theta_B)\left(\overset{o}{P}{}_t^b\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \quad with \; \overset{o}{P}{}_t^b = \overset{o}{P}{}_t^B, \quad \forall b \in \mathbf{B}.$$

Given the symmetry across all country $\mathbf{B}$'s geographic units, P_t^b is the same for all geographic units, so $P_t^B = P_t^b$, for all $b \in \mathbf{B}$.

[22] The derivation of the Phillips curve is standard in the literature (see, e.g., Woodford, 2003, Galí, 2008), reason why it is not repeated here.

$$\begin{aligned}
\pi_t^j &= \beta E_t\left\{\pi_{t+1}^j\right\} + \phi^j \widehat{mc}_t^j, j = \mathbf{B}, s \in \mathbf{S}, & (75)\\
where &: \\
\phi^j &\equiv \frac{\left(1-\theta^j\beta\right)\left(1-\theta^j\right)}{\theta^j\left(1+\epsilon\chi\right)} \\
&= \begin{cases} \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)}, for\ j = s, \forall s \in \mathbf{S} \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)}, for\ j = \mathbf{B} \end{cases},
\end{aligned}$$

where $\phi^j \widehat{\mu}_{w,t}^j$ represents the "cost-push shock". Following Clarida, Galí and Gertler (2002), we assume that the "cost push shock" obeys to a stationary AR(1) process:

$$\phi^j \widehat{\mu}_{w,t}^j = \rho_\mu\left(\phi^j \widehat{\mu}_{w,t-1}^j\right) + \varepsilon_t \tag{76}$$

with $0 < \rho_\mu < 1$ and where ε_t is the zero mean white noise process.

From the definition of the real marginal cost in (64), in logs

$$\begin{aligned}
mc_t^j &= \log\left(1-\varsigma_w^j\right) - \log(1-\tau_t^j) + \mu_{w,t}^j + \frac{1}{\sigma}c_t^j + \log\chi_0 + \chi y_t^j \\
&\quad + \left(p_{c,t}^j - p_t^j\right) - (1+\chi)\, a_t^j. & (77)
\end{aligned}$$

Thus,

$$\widehat{mc}_t^j = -\widehat{\log(1-\tau_t^j)} + \widehat{\mu}_{w,t}^j + \frac{1}{\sigma}\widehat{c}_t^j + \chi\widehat{y}_t^j + \left(p_{c,t}^j - p_t^j\right) - (1+\chi)\, a_t^j, \tag{78a}$$

where $\widehat{\log(1-\tau_t^j)} = \log(1-\tau_t^j) - \log(1-\tau^j)$, $\widehat{\left(p_{c,t}^j - p_t^j\right)} = \left(p_{c,t}^j - p_t^j\right)$ and $\widehat{a}_t^j = a_t^j$, $j = \mathbf{B}, s \in \mathbf{S}$.[23]

As expected, marginal cost increases both with the revenue tax and the net wage-markup, while there is a negative relationship between technology and marginal cost through its direct impact on labor productivity. An increase in domestic output raises marginal cost through its impact on employment and, consequently, the real wage (22). Private consumption increases marginal cost through the wealth effect

[23] In the symmetric steady state, $tt^j = 0$ and A^j is the same across all countries. Without loss of generality it is assumed that $A^j = 1$, $j = \mathbf{B}, s \in \mathbf{S}$.

on labor supply (22). As to the terms of trade, tt_t^j (which increase with $\left(p_{c,t}^j - p_t^j\right)$- see equations (29) and (30)), marginal cost increases in the terms of trade, given the positive effect of the terms of trade on private consumption (71). Notice also that from (29) and (30) changes in the terms of trade have a positive direct impact on the product wage $(w_t^j - p_t^j)$ for any given consumption wage $(w_t^j - p_{c,t}^j)$, affecting positively marginal cost [Galí (2008), p. 163].

Alternatively, we can remove private consumption and the terms of trade from (78a). By combining expressions (71) and (72) and rewriting expressions (68a) and (68b), we obtain

$$\begin{aligned}
\widehat{mc}_t^j &= \widehat{\mu}_{w,t}^j + \left(\frac{1}{(1-\varphi)\left[\sigma\left(1-\alpha\right)+\Phi\right]} + \chi\right)\widehat{y}_t^j \\
&\quad -\widehat{\log(1-\tau_t^j)} - \frac{\varphi}{(1-\varphi)\left[\sigma\left(1-\alpha\right)+\Phi\right]}\widehat{g}_t^j \\
&\quad + \left(\frac{1}{\sigma\left(1-\varphi\right)} - \frac{1}{(1-\varphi)\left[\sigma\left(1-\alpha\right)+\Phi\right]}\right)\widehat{y}_t^* \\
&\quad -\varphi\left(\frac{1}{\sigma\left(1-\varphi\right)} - \frac{1}{(1-\varphi)\left[\sigma\left(1-\alpha\right)+\Phi\right]}\right)\widehat{g}_t^* - (1+\chi)\,a_t^j.
\end{aligned} \tag{79}$$

2.7 Monetary and Fiscal Authorities

Following Woodford (2003, Chapter 2), we abstract from any monetary frictions and consider the limit of a "cashless economy". Hence, there are no seigniorage revenue transfers for national governments resulting from the monetary policy. However, monetary policy has important implications for fiscal policy, since the interest rate determines the debt burden and inflation affects the real value of debt, and for real activity, given nominal price rigidity. The monetary policy instrument is the nominal interest rate r_t^* ($\equiv \log R_t^*$), which is set for the whole union by the common central bank.

As for fiscal policy, we assume that national governments choose the mix between government spending, revenue taxation and one-period nominal risk-free debt. The lump-sum taxation is assumed to fully finance the employment subsidy, while government spending and the revenue tax are instruments of fiscal policy stabilization, responding to shocks. It is assumed that government spending is financed either by debt issuance or revenue taxation[24] so that Ricardian equivalence holds.

[24] In comparison to be financed by lump-sum taxes, this affects the supply-side of the economy

In period t, the national government of a small country $i \in \mathbf{S}$ faces the following primary budget (total value in nominal terms):

$$\int_0^1 \tau_t^i P_t^i(h) Y_t^i(h) dh + \int_0^1 T_t^i dh - \int_0^1 \varsigma_w^i W_t^i(h) L_t^i(h) dh - \int_0^1 P_t^i(h) G_{i,t}^i(h) dh$$

It is assumed that lump-sum taxation is used exclusively[25] to finance the steady state employment subsidy in all countries at all points in time, i.e., $\int_0^1 T_t^i dh = \int_0^1 \varsigma_w^i W_t^i(h) L_t^i(h) dh$, allowing to simplify the budget constraint. Moreover, using the expression for the demand of good h, (54), and the goods market clearing conditions, the definition of the domestic (producer) price index (8) and the optimal allocation of government purchases expressed by (47), previous equation can be rewritten as

$$\tau_t^i P_t^i Y_t^i - P_t^i G_t^i \tag{80}$$

Hence, the flow government budget constraints are

$$\begin{aligned} D_{g,t}^i &= R_{t-1}^* D_{g,t-1}^i - \left(\tau_t^i P_t^i Y_t^i - P_t^i G_t^i\right) \\ &= R_{t-1}^* D_{g,t-1}^i - P_t^i \left(\tau_t^i Y_t^i - G_t^i\right) \end{aligned} \tag{81}$$

$\forall t$, where $D_{g,t}^i$ represents the end of period *per capita*[26] issues in nominal terms of country i's risk-free bonds.

In the case of country $\mathbf{B}$, the primary budget (total value in nominal terms) is

$$\begin{aligned} &\int_n^1 \int_0^1 \tau_t^B P_t^b(h) Y_t^b(h) dh db + \int_n^1 \int_0^1 T_t^B dh db \\ &- \int_n^1 \int_0^1 \varsigma_w^B W_t^b(h) L_t^b(h) dh db - \int_n^1 \int_0^1 P_t^b(h) G_{b,t}^B(h) dh db \end{aligned}$$

Assuming that lump-sum taxation is used exclusively to finance the steady state employment subsidy at all points in time, and taking into account the expression for

and potentially diminishing the stimulating effect of an increase in government spending (Beestma and Jensen (2005), p. 325).

[25]Lump-sum taxation cannot be used to alter the employment subsidy or to finance any other government activities.

[26]In the case of small countries, there is no difference between total and *per capita* values.

the demand of good h, (56), and the goods market clearing conditions, the definition of the domestic (producer) price index in (9) and the optimal allocation of government purchases expressed by (48), previous equation can be rewritten as

$$(1-n)\left(\tau_t^B P_t^B Y_t^B - P_t^B G_t^B\right) \tag{82}$$

Thus, the flow government budget constraints are

$$\begin{aligned}(1-n)D_{g,t}^B &= R_{t-1}^*(1-n)D_{g,t-1}^B - (1-n)\left(\tau_t^B P_t^B Y_t^B - P_t^B G_t^B\right) \Leftrightarrow \\ D_{g,t}^B &= R_{t-1}^* D_{g,t-1}^B - P_t^B\left(\tau_t^B Y_t^B - G_t^B\right)\end{aligned} \tag{83}$$

$\forall t$, where $D_{g,t}^B$ represents the end of period *per capita* issues in nominal terms of country $\mathbf{B}$'s risk-free bonds. $(1-n)D_{g,t}^B$ represents total issues.

In real terms, expressed in the consumer price indexes, the flow budget constraints for country j's national government are given by

$$\frac{D_{g,t}^j}{P_{c,t}^j} = R_{t-1}^* \frac{D_{g,t-1}^j}{P_{c,t}^j} - \frac{P_t^j}{P_{c,t}^j}\left(\tau_t^j Y_t^j - G_t^j\right), j = \mathbf{B}, s \ (including\ country\ i) \in \mathbf{S}, \forall t.$$

Defining $d_{g,t}^j \equiv \frac{R_t^* D_{g,t}^j}{P_{c,t}^j}$, which denotes the real value of country j's debt (expressed in consumer prices) at maturity in *per capita* terms, we can rewrite the flow budget constraints in real terms as

$$\begin{aligned}\frac{d_{g,t}^j}{R_t^*} &= d_{g,t-1}^j\left(\frac{P_{c,t-1}^j}{P_{c,t}^j}\right) + \frac{P_t^j}{P_{c,t}^j}\left(G_t^j - \tau_t^j Y_t^j\right) \Leftrightarrow \\ d_{g,t}^j &= R_t^*\left[d_{g,t-1}^j\left(\frac{P_{c,t-1}^j}{P_{c,t}^j}\right) + \frac{P_t^j}{P_{c,t}^j}\left(G_t^j - \tau_t^j Y_t^j\right)\right], j = \mathbf{B}, s \in \mathbf{S}, \forall t\end{aligned} \tag{84}$$

Taking a first-order log-linear approximation to the symmetric zero-inflation steady state, the national flow budget constraints (84) can be rewritten as

$$\begin{aligned}\widehat{\log(d_{g,t}^j)} &= \widehat{r}_t^* + \frac{1}{\beta}\left\{\widehat{\log(d_{g,t-1}^j)} - \pi_t^j + \frac{Y^j}{d_g^j}\left[\varphi\widehat{g}_t^j - \tau^j\widehat{y}_t^j - \tau^j\widehat{\log(\tau_t^j)}\right]\right. \\ &\qquad \left. +\alpha\left(p_{t-1}^* - p_{t-1}^j\right) - \left(\frac{1}{1+r^*}\right)\alpha\left(p_t^* - p_t^j\right)\right\}, \\ j &= \mathbf{B}, s \in \mathbf{S}, \forall t.\end{aligned} \tag{85}$$

where $r^* \equiv \log R^*$ and $R^* = \frac{1}{\beta}$, d_g^j, G^j, τ^j and Y^j are the steady state values for the corresponding variables, and we use the fact that $G^j = \varphi Y^j$.

From (84), the steady state value d_g^j is

$$\begin{aligned} d_g^j &= R^* \left[d_g^j + G^j - \tau^j Y^j \right] \Leftrightarrow \\ d_g^j &= \frac{1+r^*}{r^*} (\tau^j - \varphi) Y^j \end{aligned} \tag{86}$$

Given that $d_{g,t}^j \equiv \frac{R_t^* D_{g,t}^j}{P_{c,t}^j}$, the steady sate value D_g^j is

$$D_g^j = \frac{P^j}{R^*} d_g^j = \frac{1}{r^*} (\tau^j - \varphi) P^j Y^j \tag{87}$$

Thus, in order to maintain its debt, country j's nominal surpluses must pay the debt service, i.e, $r^* D_g^j = (\tau^j - \varphi) P^j Y^j$.

2.8 Model Equations

In order to solve for the optimal policies, monetary and fiscal authorities have to take into account both the private sector behavior, obtained from optimization of (1), (2) and (63), as well as the budget constraints described above. These conditions can be log-linearized and written in gap form, as presented below[27]. Notice that for a generic variable X_t, its gap is defined as $\widetilde{x}_t = \widehat{x}_t - \overline{\overline{x}}_t$, where $\widehat{x}_t$ and $\overline{\overline{x}}_t$ denote, respectively, their effective and efficient values in log-deviations from the zero-inflation efficient steady-state (see section 2.9 below). A union-wide variable, x_t^*, is defined as $x_t^* = nx_t^S + (1-n)x_t^B, where\ x_t^B$ represents the big country (block) variable and x_t^S the block S variable, defined as an average of small countries variables: $x_t^S = \frac{1}{n} \displaystyle\int_0^n x_t^s ds$.

2.8.1 Equations of the Forward Looking Variables

(Euler Equation)

Aggregating (23) over all countries, and rewriting in gaps, yields

$$\widetilde{c}_t^* = E_t \left\{ \widetilde{c}_{t+1}^* \right\} - \sigma \left(\widetilde{r}_t^* - E_t \left\{ \pi_{t+1}^* \right\} \right), \tag{88}$$

where $\pi_{c,t+1}^* = p_{c,t+1}^* - p_{c,t}^* = \pi_{t+1}^*$ (since $p_{c,t}^* = p_t^*$). We assume that the efficient union-wide inflation level is zero.

[27] The derivations of these equations are available upon request.

(Phillip's Curves)

$$\pi_t^j = \beta E_t \left\{\pi_{t+1}^j\right\} + \phi^j \left(\frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]} + \chi\right) \widetilde{y}_t^j$$
$$- \phi^j \frac{\varphi}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]} \widetilde{g}_t^j$$
$$+ \phi^j \left(\frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\right) \widetilde{y}_t^*$$
$$- \phi^j \varphi \left(\frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\right) \widetilde{g}_t^* + \frac{\phi^j}{(1-\tau^j)} \widetilde{\tau}_t^j,$$
$$for\ j = s, \forall s \in \mathbf{S}, \tag{89}$$

$$\begin{aligned} where &: \\ \phi^j &\equiv \frac{\left(1-\theta^j\beta\right)\left(1-\theta^j\right)}{\theta^j\left(1+\epsilon\chi\right)} \\ &= \left\{ \begin{array}{l} \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)}, for\ j = s, \forall s \in \mathbf{S} \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)}, for\ j = \mathbf{B} \end{array} \right. . \end{aligned}$$

2.8.2 Equations of the Predetermined Variables

(Market Clearing)

$$\widetilde{y}_t^s = (1-\varphi)\widetilde{c}_t^* + (1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]\widetilde{tt}_t^s + \varphi\widetilde{g}_t^s, \forall s \in \mathbf{S}, \tag{90a}$$
$$\widetilde{y}_t^B = (1-\varphi)\widetilde{c}_t^* + (1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]n\widetilde{tt}_t^B + \varphi\widetilde{g}_t^B, \tag{90b}$$
$$\begin{aligned} where &: \\ \Phi &\equiv \alpha\left[\gamma-(1-\alpha)(-\gamma+\sigma)\right]. \end{aligned}$$

(International risk sharing conditions)

$$\widetilde{c}_t^s = \widetilde{c}_t^B + \sigma(1-\alpha)\left[\widetilde{tt}_t^s - n\widetilde{tt}_t^B\right], \forall s \in \mathbf{S}, \tag{91a}$$
$$\widetilde{c}_t^s = \widetilde{c}_t^i + \sigma(1-\alpha)\left[\widetilde{tt}_t^s - \widetilde{tt}_t^i\right], \forall s, i \in \mathbf{S}. \tag{91b}$$

(Law of motion for the terms-of-trade gaps)

Taking into consideration expressions (29), (30), (31), (32) and (44), we obtain

$$\left(\widetilde{tt}_t^s + \overline{\overline{tt}}_t^s\right) - \left(\widetilde{tt}_{t-1}^s + \overline{\overline{tt}}_{t-1}^s\right) = \pi_t^* - \pi_t^s, \forall s \in \mathbf{S}, \tag{92a}$$

$$\left(\widetilde{tt}_t^B + \overline{\overline{tt}}_t^B\right) - \left(\widetilde{tt}_{t-1}^B + \overline{\overline{tt}}_{t-1}^B\right) = \frac{\pi_t^* - \pi_t^B}{n}. \tag{92b}$$

(Flow Budget Constraints)

Taking into account (44), expression (85) can be rewritten in gaps as

$$\begin{gathered} \widehat{\log(d_{g,t}^s)} = \\ \widetilde{r}_t^* + \frac{1}{\beta}\left\{\widehat{\log(d_{g,t-1}^s)} - \pi_t^s + \frac{Y^s}{d_g^s}\left[\varphi\widetilde{g}_t^s - \tau^s\widetilde{y}_t^s - \widetilde{\tau}_t^s\right] + \alpha\widetilde{tt}_{t-1}^s - \left(\frac{1}{1+r^*}\right)\alpha\widetilde{tt}_t^s\right\} \\ +\overline{\overline{r}}_t^* + \frac{1}{\beta}\left\{\frac{Y^s}{d_g^s}\left[\varphi\overline{\overline{g}}_t^s - \tau^s\overline{\overline{y}}_t^s - \overline{\overline{\tau}}_t^j\right] + \alpha\overline{\overline{tt}}_{t-1}^s - \left(\frac{1}{1+r^*}\right)\alpha\overline{\overline{tt}}_t^s\right\}, \forall s \in \mathbf{S}. \end{gathered} \tag{93a}$$

$$\begin{gathered} \widehat{\log(d_{g,t}^B)} = \\ \widetilde{r}_t^* + \frac{1}{\beta}\left\{\widehat{\log(d_{g,t-1}^B)} - \pi_t^B + \frac{Y^B}{d_g^B}\left[\varphi\widetilde{g}_t^B - \tau^B\widetilde{y}_t^B - \widetilde{\tau}_t^B\right]\right. \\ \left. +\alpha n\widetilde{tt}_{t-1}^B - \left(\frac{1}{1+r^*}\right)\alpha n\widetilde{tt}_t^B\right\} \\ +\overline{\overline{r}}_t^* + \frac{1}{\beta}\left\{\frac{Y^B}{d_g^B}\left[\varphi\overline{\overline{g}}_t^B - \tau^s\overline{\overline{y}}_t^B - \overline{\overline{\tau}}_t^B\right] + \alpha n\overline{\overline{tt}}_{t-1}^B - \left(\frac{1}{1+r^*}\right)\alpha n\overline{\overline{tt}}_t^B\right\}. \end{gathered} \tag{93b}$$

In what follows, we consider variable $\widehat{b}^j = \widehat{\log(d_{g,t}^j)} \times \left(\frac{d_g^j}{Y^j}\right)$ when referring to country j's debt, procedure adopted by Kirsanova and Wren-Lewis (2011). $\widehat{b}^j$ denotes the log deviation of $d_{g,t}^j$ from its steady state value multiplied by the steady-state debt ratio $\left(\frac{d_g^j}{Y^j}\right)$, i.e., the absolute change in debt.

Finally, we must take into consideration the AR(1) processes governing technology and cost-push shocks, respectively:

(Technology shocks)

$$a_t^j = \rho_a a_{t-1}^j + \varepsilon_t^j \;\; j = \mathbf{B}, s \in \mathbf{S} \tag{94}$$

(Cost-push shocks)

$$\phi^j\widehat{\mu}_{w,t}^j = \rho_\mu\left(\phi^j\widehat{\mu}_{w,t-1}^j\right) + \varepsilon_t, \;\; j = \mathbf{B}, s \in \mathbf{S} \tag{95}$$

2.9 The Social Planner's Problem

In order to develop an objective function with which to evaluate optimal policy, we start by analyzing the social planner's problem. The social planner is concerned with real allocations, ignoring nominal inertia and distortionary taxation. Thus, she or he simply decides how to allocate private and public consumption and production of goods in each economy within the union, subject to the existent technology, the resources constraints and all the constraints that arise from operating in a monetary union, e.g., the international risk sharing condition.

In any given period t, the optimal allocation for the currency union as a whole can be described as the solution to the following social planner's problem, where the single policy authority is willing to maximize the discounted sum of the utility flows of the households belonging to the union (W^*):

$$\underset{\substack{C^s_{s,t},\, C^s_{i,t},\, C^s_{B,t},\, G^s_t,\, \forall s, i \in S, i \neq s \\ C^B_{B,t},\, C^B_{s,t} (\forall s \in S),\, G^B_t}}{Max} \quad W^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\int_0^n W^s_t ds + (1-n) W^B_t \right] \right\}, \tag{96}$$

$$with:$$

$$W^s_t = u(C^s_t) + V(G^s_t) - v(L^s_t), \forall s \in S,$$

$$W^B_t = u\left(C^B_t\right) + V\left(G^B_t\right) - v\left(L^B_t\right),$$

$$s.t.:$$

$$Y^j_t = A^j_t L^j_t, j = \mathbf{B}, s \in \mathbf{S},$$

$$consumption\ indexes: (3)\ and\ (6),\ \forall s, i \in S$$

$$international\ risk\ sharing\ condition:\ (38)$$

$$resource\ constraints:$$

$$\begin{cases} Y^s_t = C^s_{s,t} + \int_0^n C^i_{s,t} di + (1-n) C^B_{s,t} + G^s_t, \forall s \in S \\ (1-n) Y^B_t = (1-n) C^B_{B,t} + \int_0^n C^s_{B,t} ds + (1-n) G^B_t \end{cases},$$

Notice that in the absence of nominal rigidities there is no price dispersion and, hence, the social planner will chose to produce equal quantities of the different goods in each country. Consequently, all households work the same number of hours in each country. Moreover, the aggregation over all agents (households, governments and monetary authority) cancels out the budget constraints and, thus, the social planner's solution is not constrained by them.

Optimization of problem (96) yields the following optimality conditions:

$$\begin{aligned}
(1-\alpha)^{\frac{1}{\gamma}} \left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{s,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s \in S, \\
(\alpha)^{\frac{1}{\gamma}} \left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{i,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^i\right)^{\chi}}{A_t^i}, \forall s, i \in S, i \neq s, \\
\left[\alpha\left(1-n\right)\right]^{\frac{1}{\gamma}} \left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{B,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^B\right)^{\chi}}{A_t^B}, \forall s \in S, \\
\psi_0 \left(G_t^s\right)^{-\frac{1}{\psi}} &= \chi_0 \frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s \in S, \\
(1-n\alpha)^{\frac{1}{\gamma}} \left(C_t^B\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{B,t}^B\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^B\right)^{\chi}}{A_t^B}, \\
(\alpha)^{\frac{1}{\gamma}} \left(C_t^B\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{s,t}^B\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s \in S, \\
\psi_0 \left(G_t^B\right)^{-\frac{1}{\psi}} &= \chi_0 \frac{\left(L_t^B\right)^{\chi}}{A_t^B},
\end{aligned}$$

Efficient Equilibrium

In a symmetric efficient steady state equilibrium, it follows that

$$\begin{aligned}
Y^s &= Y^B = Y, \forall s \in S, \\
L^s &= L^B = L = Y, \forall s \in S, \\
C^s &= C^B = C, \forall s \in S, \\
C_s^s &= (1-\alpha)\, C^s, \forall s \in S, \\
C_i^s &= \alpha C^s, \forall s, i \in S, i \neq s, \\
C_B^s &= \alpha\,(1-n)\, C^s, \forall s \in S, \\
C_B^B &= (1-n\alpha)\, C^B, \\
C_s^B &= \alpha C^B, \forall s \in S, \\
G^B &= G^s = G, \forall s \in S, \\
Y &= C + G.
\end{aligned}$$

The complete solution for the efficient equilibrium is given by the following expressions, in deviations from the steady state (see Appendix A.1, for the main steps in their derivation):

(Private consumption)

$$\begin{aligned}\bar{\bar{c}}_t^j &= \qquad (97)\\ &\frac{(1-\alpha)(1+\chi)\sigma}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\ &+\frac{\alpha(1+\chi)\sigma\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma-(\sigma-\gamma)(1-\alpha)\right]\right)\right\}}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\ &j=\mathbf{B},s\in\mathbf{S},\end{aligned}$$

and

$$\bar{\bar{c}}_t^* = \frac{(1+\chi)\sigma}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^*.$$

(Public consumption)

$$\begin{aligned}\bar{\bar{g}}_t^j &= \qquad (98)\\ &\frac{(1+\chi)\psi}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\ &-\frac{(1+\chi)\psi(1-\varphi)\chi(\sigma-\gamma)(2\alpha-\alpha^2)}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\ &j=\mathbf{B},s\in\mathbf{S},\end{aligned}$$

and

$$\bar{\bar{g}}_t^* = \frac{(1+\chi)\psi}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^*.$$

(Output)

$$\begin{aligned}\bar{\bar{y}}_t^j &= \qquad (99)\\ &\frac{(1+\chi)\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\ &+\frac{(1+\chi)(1-\varphi)(\sigma-\gamma)(2\alpha-\alpha^2)}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\ &j=\mathbf{B},s\in\mathbf{S},\end{aligned}$$

and

$$\overline{\overline{y}}_t^* = \frac{(1+\chi)\left[\varphi\psi + (1-\varphi)\sigma\right]}{1+\chi\left[\varphi\psi + (1-\varphi)\sigma\right]} a_t^* = (1-\varphi)\overline{\overline{c}}_t^* + \varphi\overline{\overline{g}}_t^*.$$

In order to fully define the gap variables described in previous section, we need to determine the efficient terms-of-trade and interest rate levels. The efficient terms-of-trade levels follow from a combination of the international risk sharing condition with the efficient levels of private consumption ($\overline{\overline{c}}_t^*$ and $\overline{\overline{c}}_t^j, j = \mathbf{B}, s \in \mathbf{S}$), while the efficient interest rate follows from the Euler equation, assuming that the efficient union-wide inflation is zero. Thus,

(Terms-of-trade)

$$\begin{gathered}(\overline{\overline{p}}_t^* - \overline{\overline{p}}_t^j) = \\ \frac{(1+\chi)}{1+\chi\left\{\varphi\psi + (1-\varphi)\left[\gamma + (\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}(a_t^j - a_t^*), \\ j = \mathbf{B}, s \in \mathbf{S},\end{gathered} \tag{100}$$

with

$$\overline{\overline{tt}}_t^s = (\overline{\overline{p}}_t^* - \overline{\overline{p}}_t^s), s \in \mathbf{S}, \tag{101a}$$

$$\overline{\overline{tt}}_t^B = \frac{1}{n}(\overline{\overline{p}}_t^* - \overline{\overline{p}}_t^B). \tag{101b}$$

(Interest rate)

$$\overline{\overline{r}}_t^* = \frac{(1+\chi)}{1+\chi\left\{\varphi\psi + (1-\varphi)\sigma\right\}} E\left\{a_{t+1}^* - a_t^*\right\}, \tag{102}$$

where it is implicit that the steady state nominal (and real) interest rate is $r^* = \rho = -\log(\beta)$.

Recall that it is assumed that each firm receives an employment subsidy of ς_w^j designed to allow the flexible price equilibrium to be efficient (or, equivalently, to support the assumption that the steady state level of output is efficient). Additionally, we assume that the efficient revenue tax rate eliminates marginal cost deviations from its steady state level arising from pure cost-push shocks. Thus,

(Revenue tax rate)

$$\overline{\overline{\tau}}_t^j = -(1-\tau^j)\widehat{\mu}_{w,t}^j, j = \mathbf{B}, s \in \mathbf{S}, \tag{103}$$

where τ^jrepresents the revenue tax steady-state value.

In a debt-unconstrained scenario, this efficient equilibrium would correspond to the flexible-price equilibrium if an employment subsidy was used to eliminate the steady state distortion associated to monopolistic competition and distortionary revenue taxes, and governments spending respected the optimal rules derived under the optimization of problem (96). However, under our debt-constrained scenario fiscal sustainability may require fiscal policy instruments to deviate from the efficient rules, though the two steady state equilibriums coincide.

Steady State Equilibrium and the Employment Subsidy

As discussed above, we assume the existence of an employment subsidy ς_w^j that removes steady state distortions and is fully financed by lump sum taxes. Besides avoiding the traditional inflationary bias problem arising from an inefficiently low steady-state output level, this employment subsidy eliminates linear terms in the social welfare function without losing the possibility of using the revenue tax τ_t^j as fiscal instrument.

To compute the employment subsidy, observe that in the absence of price rigidities profit maximization behavior (63) implies that real marginal cost $MC_t^j = \frac{1}{(1+\mu_p)}$, $j = \mathbf{B},s \in \mathbf{S}$, that is, firms would choose a constant markup over the nominal marginal cost, with $\mu_p \equiv \frac{1}{\epsilon-1}$.

From (64), we know that in the steady state

$$MC^j = \frac{(1-\varsigma_w^j)(1+\mu_w^j)}{(1-\tau^j)} \left(C^j\right)^{\frac{1}{\sigma}} \chi_0 \left(Y^j\right)^{\chi}, j = \mathbf{B},s \in \mathbf{S},$$

and from the optimality conditions of the social planner problem we get that in the steady state

$$\left(C^j\right)^{-\frac{1}{\sigma}} = \chi_0 \left(Y^j\right)^{\chi}, j = \mathbf{B},s \in \mathbf{S}.$$

Thus,

$$MC^j = \frac{(1-\varsigma_w^j)(1+\mu_w^j)}{(1-\tau^j)}, j = \mathbf{B},s \in \mathbf{S}.$$

To ensure that in the steady state $MC^j = \frac{1}{(1+\mu_p)}$, the employment subsidy in country $j = \mathbf{B},s \in \mathbf{S}$, is assumed to take the value

$$\varsigma_w^j = 1 - \frac{(1-\tau^j)}{(1+\mu_p)\left(1+\mu_w^j\right)} = 1 - \frac{(\epsilon-1)(1-\tau^j)}{\epsilon\left(1+\mu_w^j\right)}, j = \mathbf{B}, s \in \mathbf{S}. \tag{104}$$

2.10 Policy Objectives: The Social Loss Function

Since the social planner ignores nominal inertia and distortionary taxation in describing optimal allocations, the solution to the social planner problem can be used as a benchmark for optimal policy. Following a shock, it is often optimal to deviate from the efficient steady-state, but stabilization policy must ensure that economies follow a path as close to the efficient solution as possible, given distortionary taxation and nominal rigidities and the need to ensure fiscal solvency.

Under full cooperation, benevolent authorities seek to maximize welfare for the currency union as a whole (W^*) - the discounted sum of the utility flows of the households belonging to the union, given the set of equations describing the dynamic structure of the economies (88)-(95). Following Woodford (2003), we compute the second-order approximation of the welfare objective (W^*) around a deterministic steady state. Ignoring an irrelevant proportionality factor as well as terms independent of policy or of third or higher order, we obtain the following union-level loss function :

$$L^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t^* \right\},$$

where the per-period social loss function (L_t^*) is defined as

$$L_t^* = \int_0^n L_t^s ds + (1-n) L_t^B, \tag{105}$$

and

$$
L_t^s = (\frac{1}{2}) \left\{ \begin{array}{c} \frac{\epsilon}{\phi_S}(\pi_t^s)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^s)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^s)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](\widetilde{tt}_t^s)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^s\widetilde{g}_t^s \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^s\widetilde{tt}_t^s \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^s\widetilde{tt}_t^s \end{array} \right\}, \forall s \in \mathbf{S}, \tag{106a}
$$

$$
L_t^B = (\frac{1}{2}) \left\{ \begin{array}{c} \frac{\epsilon}{\phi_B}(\pi_t^B)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^B)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^B)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](n\widetilde{tt}_t^B)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^B\widetilde{g}_t^B \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^B(n\widetilde{tt}_t^B) \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^B(n\widetilde{tt}_t^B) \end{array} \right\}, \tag{106b}
$$

$$
where : \left\{ \begin{array}{l} \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)} \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)} \\ \Phi \equiv \alpha\left[\gamma-(1-\alpha)(-\gamma+\sigma)\right] \end{array} \right. .
$$

The main steps in the derivation of the loss function are presented in Appendix A.2.

As expected welfare losses associated to inflation are larger for a higher degree of nominal rigidity (θ^j), vanishing only when prices are fully flexible. Moreover, the cost of inflation increases with the elasticity of substitution between goods produced in the same country and with the inverse of the labor supply elasticity (χ). A lower elasticity of labor supply (higher χ) results in higher stabilization costs since fluctuations in work effort arising from misallocations caused by inflation are more costly. Fluctuations in private and public consumption gaps imply welfare losses in line with the household's risk aversions ($\frac{1}{\sigma}$ and $\frac{1}{\psi}$, respectively) and with the inverse of the labor supply elasticity (χ). Since deviations of the terms-of-trade from their respective efficient level imply misallocation of goods at the monetary union level, there is a cost associated with this distortion, which increase with the intertemporal elasticity of substitution between domestic and foreign goods (γ), with the steady-state share of private consumption on output $(1-\varphi)$ and with the inverse of the labor supply elasticity, and decreases with the intertemporal elasticity of substitution of private consumption (σ), with the degree of home bias (increase with α), and, in the particular case of the large economy, decreases with her dimension. Besides the work effort fluctuations caused by private and public consumption *per se*, positive co-movements between them cause additional undesirable fluctuations, which are captured by the cross-term between their respective gaps. Finally, notice the cross-terms between the terms-of-trade gap and private and public consumption gaps. Both these terms increase with the intertemporal elasticity of substitution between domestic and foreign goods (γ) and with the inverse of the labor supply elasticity, and decrease with the intertemporal elasticity of substitution of private consumption, with the degree of home bias, and, in the particular case of the large economy, decreases with her dimension. While the cross-term between the terms-of-trade gap and private consumption increases with the steady-state share of private consumption on output, the cross-term between the terms-of-trade gap and public consumption decreases with it (for steady-state shares of private consumption on output higher than 50%). A positive terms-of-trade gap for economy j rises her competitiveness which, combined with a positive private/public consumption gap, shifts demand towards j-produced goods. As a consequence, work effort shifts from the other households in the union towards j-households (see, e.g., Beetsma and Jensen, 2004, 2005).

2.11 Policy Games

Monetary and fiscal authorities are assumed to set their policies in order to minimize their respective loss functions, given the dynamic structure of the economies, and are able to engage in various policy games. As a benchmark, we assume that policymakers are benevolent and cooperate under discretion, sharing the same per-period social loss function L_t^*. Alternatively, we consider that governments may have divergent policy objectives. While the benevolent monetary authority seeks to maximize the union-wide welfare, it is reasonable to assume that national fiscal authorities are exclusively concerned with their own citizens and, hence, their objective functions should only include their national counterparts. We approximate the national welfare counterparts through welfare losses obtained from splitting the union-wide loss function. Following Leith and Wren-Lewis (2011), we set the linear terms contained in the country-specific loss functions to zero. These linear terms capture the desire of national governments to manipulate their terms-of-trade to obtain additional national gains, but this manipulation is ineffective if all union members proceed in the same manner, reason why they are not present in the union-wide loss function.[28]

Accordingly, for benevolent cooperative policymakers we have that

$$L_t^{central\ bank} = L_t^{big\ country} = L_t^{small\ country\ s} = L_t^*, \forall s \in S,$$

defined in (105). As for benevolent non-cooperative policymakers

$$\begin{aligned} L_t^{central\ bank} &= L_t^*, \\ L_t^{big\ country} &= L_t^B, \\ L_t^{small\ country\ s} &= L_t^s, \forall s \in S, \end{aligned}$$

as defined in (106a-106b).

The conflict of policy objectives allows for strategic interactions between policymakers and different equilibriums, depending on the timing structure of the policy games. Our analysis considers both simultaneous move (Nash) and leadership (monetary and fiscal leadership) equilibria. In these different setups, the timing of the events is as follows: 1) private sector forms expectations; 2) the shocks are realized; 3) depends on the policy scenario: under Nash equilibrium, the central bank sets the interest rate and fiscal authorities define fiscal policy instruments simultaneously; under monetary leadership, the central bank sets the interest rate before fiscal authorities choose the right amount of fiscal instruments (fiscal authorities play a Nash between them), while the reverse occurs under fiscal leadership.

In order to acess the importance of time-consistency, we compute both the commitment and the discretonary solutions under cooperation. To solve these two optimizing problems, we use the methodology developed in the seminal work of Söderlind (1999). As to the dynamic policy games, we follow the recent work of Kirsanova and co-authors (e.g., Blake and Kirsanova (2011), for a closed-economy setup, and Kirsanova et al. (2005), for an open-economy setup) and further developed by Machado

[28] Forlati (2009) provides fully micro-founded welfare criteria for the case of non-cooperative fiscal and monetary policies in a monetary union.

and Ribeiro (2010). To work within our multi-country monetary union setup, we implemented considerable changes to the algorithms developed by these authors. The derivation of a numerical algorithm for the solution of the non-cooperative monetary leadership discretionary game is presented in Appendix A.3.[29]

2.12 Baseline Calibration

Relative to the structure of the monetary union, we assume that the large economy and the block made up of small countries have identical dimension, each representing 50% of the union (thus $n = 0.5$). To operationalize the algorithms, we assume that the dimension of a small country is $ii_$dim=0.00001 (*c.f.* Appendix A.3, Modeling the behavior of block S). In a robust section below, we test different dimensions for the large economy.

The model is calibrated at a quarterly frequency. We set the discount factor of the private sector (and policymakers) to β=0.99 (as, e.g., Kirsanova *et al.*, 2005, Benigno and De Paoli, 2009, and Forlati, 2012), which implies a 4% annual basis steady-state interest rate. Since we assume the same α parameter both for the large and for the small economies, we choose α=0.4, which implies a 40% share of domestic consumption allocated to imported goods for the small countries (a degree of home bias of 60%), but for the large economy the share of domestic consumption allocated to imported goods is only of 20%. This is in line with the fact that small economies are much more open than large economies. Leith and Wren-Lewis (2007b, 2011) and Forlati (2009) also consider a degree of home bias of 60% for small countries. Forlati (2012) considers a degree of home bias of 60% for small countries, while assuming a degree of home bias of 85% between economic areas structural identical to our blocks **S** and **B**. Benigno and De Paoli (2009) consider a degree of home bias of 80% for a small country. In a two-country currency union model, Kirsanova *et al.* (2007) consider a 70% share of consumption of domestic goods, while Corsetti *et al.* (2010) consider a home bias of private absorption of 86.5%. We assume $\sigma = \psi = 0.4$, which implies a coefficient of risk aversion for private and public consumption equal to 2.5, as in Beetsma and Jensen (2005). The inverse of the labor supply elasticity, χ, is set to 3, following Blake and Kirsanova (2011), Kirsanova and Wren-Lewis (2012) and Forlatti (2012). The elasticity of substitution between goods produced in the same country, ε, is equal to 11, implying a price mark-up of 10%, as in Ferrero (2009)

[29] The derivation of the other algorithms is available upon request.

and Corsetti *et al.* (2010). In turn, the elasticity of substitution between domestic and foreign goods, γ, is set at 4.5, following Benigno and Benigno (2006), Forlatti (2009) and Ferrero (2009). The steady-state share of public consumption in output, φ, is set to 0.25, a value commonly used in the literature (e.g., Beetsma and Jensen, 2005, Galí and Monacelli, 2008, Leith and Wren-Lewis, 2007b, 2011, and Blake and Kirsanova, 2011). We consider that $\theta_S = \theta_B = 0.75$, in order to get an average length of price contracts equal to one year.

As to the steady-state debt-to-output level ($\frac{D_g^j}{4P^jY^j}$), we consider several debt levels within the range [0%;80%]. In particular, we adress a low-debt scenario, illustrated for a debt-to-output ratio of 15%, and a high-debt scenario, illustrated for a debt-to-output ratio of 60%.

Finally, we assume that technology and cost-push shocks are independent. Technology shocks follow an AR(1) process with common persistence of 0.85 (for instance, Ferrero, 2009, considers $\rho_a = 0.815$), while cost-push shocks are assumed to be non-persistent ($\rho_\mu = 0$). It is common in the literature to consider a 0.7% or 0.8% standard deviation for the technology shocks[30] and a 0.5% standard deviation for the cost-push shocks, tough it is also common to assume the same standard deviation. For example, Kirsanova *et al.* (2007) assume that the standard deviations of cost-push and taste/technology shocks are 0.5%, while Chadha and Nolan (2007) consider that the standard deviations of the productivity, fiscal and monetary innovations are equal to 1%. We follow the latter and set the standard deviation for technology and cost-push shocks to 1%.

The key baseline parameter values that we adopt to simulate the model are summarized in Table A, in Appendix A.4.

[30] A standard parameterization in the one-country RBC literature considers the standard deviation of a quarterly productivity innovation of about 0.007–0.008 (see, e.g., Chadha and Nolan, 2007).

3 Optimal Stabilization Policies

There exists a vast literature on the joint macroeconomic stabilization by monetary and fiscal authorities in a monetary union environment. However, most of the existing literature does not consider a realistic monetary union environment, where large economies coexist with several small economies. In this scenario, we expect large and very small countries to face different incentives as well as different stabilization costs. Furthermore, it is possible that these divergent incentives may even have different macroeconomic stabilization consequences under different public debt scenarios, or under different strategic interactions between non-coordinated policies.

We intend to contribute to the literature by studying full-optimal policies in a monetary union environment described by the baseline model presented in chapter 2. Fiscal authorities preserve their policy autonomy and engage in discretionary policy games with a common benevolent central bank that maximizes the union-wide welfare. Our specific goals are to analyze demand and supply-side policies interactions in a monetary union environment, (i) across different country-sizes, (ii) constrained by different steady-state debt-to-output levels and (iii) where strategic interactions arise not from different relative weights attached for common policy targets but, instead, from fiscal policy being biased towards country-specific stabilization objectives. Moreover, we intend to assess how the level of public debt shapes optimal discretionary policies and affects macroeconomic stabilization performance under different policy regimes, considering the cooperative solution, as benchmark, and three distinctive non-cooperative scenarios: simultaneous move (Nash), monetary leadership and fiscal leadership.

Optimal discretionary solutions are computed numerically using appropriate algorithms to mimic cooperative outcomes and also to reflect the different time structures of the (non-cooperative) policy games. We follow the methodology developed in the seminal work of Söderlind (1999) and in the recent work of Kirsanova and co-authors (*e.g.*, Blake and Kirsanova, 2011, for a closed-economy setup, and Kirsanova *et al.*, 2005, for an open-economy setup) and further developed by Machado and Ribeiro (2010). To work within our heterogeneous country-size monetary union setup, we implemented considerable changes to the algorithms developed by these authors (*c.f.* Appendix A.3).

The chapter is organized as follows. Section 3.1 reviews the relevant theoretical literature and highlights the main contributions of our work. Then, we study the design, performance and implications of full optimal discretionary policies across different public debt levels and policy regimes. Our analysis is focused on the effects of asymmetric technology shocks which, by their persistence and the trade-offs they generate, are the welfare-dominating shocks in our setup. We appraise the consequences for a very small and a large country member of the monetary union. In section 3.2, we consider the cooperative setup, comparing discretionary and commitment solutions. In section 3.3 we move to the analysis of discretionary policies under non-cooperative scenarios (Nash, monetary leadership and fiscal leadership). An analysis of the welfare stabilizations costs across different debt levels and policy regimes is presented in section 3.4. In section 3.5, we analyze the sensitivity of our results to cost-push shocks, and then we conduct a robustness analysis to some structural model parameters. Section 3.6 reviews the main results of this chapter.

3.1 Overview of the Literature

There are many examples on the relevant scientific literature addressing the joint economic stabilization by monetary and fiscal authorities. As regards to monetary and fiscal policy interactions in a currency union, most of the existing literature relies either in a two-country model (e.g., Beetsma and Jensen, 2004, 2005, van Aarle *et al.*, 2004, Kirsanova *et al.,* 2005, 2007, Ferrero, 2009, and Machado and Ribeiro, 2010, 2011) or in a multi-country model where the union is made up of a continuum of small open economies (*e.g.*, Gali and Monacelli, 2008, Argentiero, 2009, Forlati, 2009, 2012, and Leith and Wren-Lewis, 2011). This allows for the analysis of policy interactions between economies where domestic shocks and policy decisions have either significant impact on the counterpart and on the union as a whole (two-country model)[1] or

[1] Other studies consider somewhat different structures. For instance, Orjasniemi (2009) uses a DSGE model where the world consists of three countries of equal size to study the effects of implementation of an open monetary union on international fluctuations.

negligible impact on other member states and on the union (continuum of small open economies model).

Given that countries within a monetary union lost the ability to conduct independent monetary and exchange rate policies, autonomous and active fiscal policies play a central role in the stabilization of asymmetric shocks, particularly for very small country members since the stabilization costs of country-specific shocks are entirely on them. Thus, in the context of multi-country models where the union is made up of a continuum of small open economies, it is not surprising that the country-specific stabilization role is attributed to fiscal policy, while the optimal policy mix requires inflation to be stabilized at the union level by the common central bank (see, e.g., Galí and Monacelli, 2008). In a two-country monetary union model, Pappa (2012) also argues that regional fiscal policy flexibility is crucial for regional stabilization, in accordance with Ferrero (2009). Another important conclusion of this study is that regional fiscal policy should focus on regional output stabilization. This is in line with the works of Beetsma and Jensen (2004, 2005) and Canzoneri *et al.* (2005). Beetsma and Jensen (2004, 2005) attest the key role of fiscal policy in the stabilization of inflation differences and the terms-of-trade, while with identical union members monetary authority aims to stabilize the union-wide economy (Beetsma and Jensen, 2005). Also, Canzoneri *et al.* (2005) argue that the primary concern of the monetary policy in a monetary union is with stability of union-wide inflation, while fiscal policy should focus on the stabilization of asymmetric shocks.

A small branch of this literature also considers country-size asymmetry (*e.g.*, Canzoneri *et al.*, 2005, Machado and Ribeiro, 2010, 2011, and Mykhaylova, 2011) and analyzes how this shapes policy interactions and stabilization outcomes. Vogel *et al.* (2013) considers a different two-country[2] monetary union model, constituted by a very small open economy and a very large economy – the rest of the union. In this case, a shock at the large economy is similar to a union-wide shock, and monetary policy is country-specific, which contrasts to our model. More recently, some works address richer monetary union environments, as is the case of Orjasniemi (2014), Andersen and Seneca (2010) and Forlati (2012), which we discuss below. However, none, as far as we

[2] Focusing on the fiscal and monetary policy in a small open economy, Benigno and De Paoli (2010) also consider a different two-country framework, a very small open economy and the rest of the world.

know, has yet addressed the case of a more general structure for a monetary union, where few large countries may coexist with many small countries, as is the case, *e.g.*, of the EMU. We contribute to fill this gap. Our heterogeneous country-size model is useful for discussing issues concerning interactions between country-members either with negligible and meaningful impact on the union's outcomes as well as on the counterparts'. Moreover, our model's strategy allows matching the two-country asymmetric monetary-union, where the relative size of the two countries is inversely related, as well as the case of a multi-country monetary union where small countries could have negligible size even when the large country has only a middle size.

Orjasniemi (2014) constructs a DSGE model with nominal rigidities and monopolistic competition partly based on Galí and Monacelli (2005, 2008), but in contrast to those works the monetary union does not consist of a continuum of countries. Instead, the monetary union is formed by n ($n \geq 2$) "small" open economies. It is shown that the spillovers caused by asymmetric technology shocks depend on the relative size of the country hit by the shocks. However, given the symmetry assumption among the monetary union countries, all domestic shocks and policy decisions have non negligible impact on the counterpart and on the union as a whole (unless n is too big, but in that case the model approaches the monetary union made up of a continuum of small open economies). This obviously contrasts to our model[3].

In a different context, where there is no room for monetary and fiscal policy interactions, Andersen and Seneca (2010) analyze how a monetary union performs in the presence of labor market asymmetries, considering a monetary union made up of I separate and independent countries/regions that may have different sizes. The relative size of country i is given by v_i so that $\sum_{i=1}^{I} v_i = 1$. To illustrate the model's properties the authors consider the case of a two-country monetary union, with country-size asymmetry, since the complexity of the model makes it difficult to extract analytical results.

The structure of our monetary union shares some common aspects with the open economy model of Forlati (2012), though we have derived our model autonomously. In

[3] Additionally, Orjasniemi (2014) assumes a balanced budget, where public spending and an employment subsidy are financed by lump sum taxes. Neither the effects associated with distortionary taxation nor the dynamic implications of public debt in terms of macroeconomic stabilization performance are contemplated in the analysis.

Forlati (2012), the world is split in two areas of equal size, each one made up of a continuum of small open regions/countries. The main objective is to compare welfare costs and benefits of a monetary union. Indeed, Forlati (2012) solves the optimal monetary policy problems assuming no monetary and fiscal policy interactions. One of the possible structures for the world consists on a monetary union with a common central bank. Though close to our currency union model, there are still substantial differences: all regions are very small countries, with autonomous fiscal authorities; moreover, our analysis focuses on monetary and fiscal policy interactions, which are enriched by elements like distortionary taxation and government debt dynamics.

Regarding the implications of different debt levels, there is substantial literature on how public debt shapes optimal stabilization policies in the presence of New Keynesian frictions, under commitment and under discretion. Notice that the assumption of no debt implies that we abstract from monetary and fiscal interactions operating through government budgetary constraints (see, *e.g.*, Díaz-Giménez *et al.*, 2008). Furthermore, recall that the relationship between monetary and fiscal policy is often characterized as an assignment regime. Under the "conventional assignment" (Kirsanova et al., 2009) monetary policy is assigned short-run stabilization of output and inflation, while fiscal policy is assigned control of public debt.

In a closed economy setting, Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2004) and Leith and Wren-Lewis (2013) show that the efficient steady-state debt follows a random walk under commitment, while Leith and Wren-Lewis (2013) show that, under discretion, debt is returned to its pre-shock level. Indeed, Leith and Wren-Lewis (2013) demonstrate that the welfare consequences of public debt are negligible for precommitment policies, though they can be significant for discretionary policy. The steady-state debt-to-output level has shown to be critical for monetary and fiscal policy interactions and for the optimal policy-mix that allows for debt-stabilization under discretion (*e.g.*, Stehn and Vines, 2008, Blake and Kirsanova, 2011, and Leith and Wren-Lewis, 2013).

It is now emerging an expressive literature on optimal debt-constrained stabilization policies in open economy settings (e.g., Benigno and De Paoli, 2010, and Corsetti *et al.,* 2010) and in a monetary union environment (*e.g.*, van Aarle *et al.*, 2004, Kirsanova *et* *al.,* 2005, 2007, Argentiero, 2009, Ferrero, 2009, Leith and Wren-Lewis, 2007b, 2011, Machado and Ribeiro, 2010, 2011, Blueschke and Neck, 2011, Pappa, 2012, and Vogel *et al.*, 2013). For instance, Blueschke and Neck (2011) consider the case of a two-block monetary union with different steady-state levels of public debt, and examine how these affect macroeconomic stabilization policies under cooperative and non-cooperative policy regimes. Using a monetary union model calibrated to represent typical countries in the Euro area, namely, an "average" (small) country, a high debt country, and a large country, Canzoneri *et al.* (2005) show that a common monetary policy, responding to union-wide inflation, produces asymmetric effects on countries within the union, depending on whether they are large or small, or whether they have high or low levels of government indebtedness. They study the implications of these asymmetries both for countries' fiscal policies and welfare. Results show that the monetary policy favors larger countries in the Euro area, as their inflation rates are more highly correlated with the union-wide inflation. They also conclude that high debt levels lead to welfare costs.

Notice that though the majority of the works, we referred to, studies full-optimal policies, some consider the use of simple rules, as is the case of van Aarle *et al.* (2004), Kirsanova *et al.* (2007), Argentiero (2009), Pappa (2012) and Vogel *et al.* (2013). Anyhow, as regards strategic interactions between different policy authorities, a significant number of these papers have analyzed the nature of optimal policy only under cooperation, as is the case of Gali and Monacelli (2008), in a no debt-constrained policy scenario, and of Ferrero (2009) in a debt-constrained policy scenario. Another important branch of this literature has considered the case of non-cooperation but still a scant part uses dynamic models, which are more appropriate to analyze the role of public debt in policy interactions. In this spirit, van Aarle *et al.* (2002), Beetsma and Jensen (2005) and Forlati (2009), for instance, analyze non-cooperative monetary and fiscal policies under Nash, Kirsanova *et al.* (2005) and Orjasniemi (2014) examine the case of monetary leadership, while Machado and Ribeiro (2010, 2011), Blueschke and Neck (2011) and Adam and Billi (2008, 2014) consider both Nash and leadership solutions.[4]

[4] Clarida *et al.* (2002) and Benigno and Benigno (2006) analyze cooperative and non-cooperative (Nash) games between monetary authorities in a two-country model of the world.

Adam and Billi (2008) study monetary and fiscal policy games, considering Nash and leadership (monetary and fiscal) equilibria, without commitment. For a wide range of model parametrizations, they argue that under fiscal leadership the monetary authority should focus exclusively on stabilizing inflation. However, besides referring to a closed economy, some relevant elements that introduce additional interactions between monetary and fiscal policy are missing in the analysis, as distortionary taxation and government debt dynamics. Based on this model, Adam and Billi (2014) consider distortionary income taxes to finance public goods, confirming that full conservatism entirely eliminates the welfare losses from discretionary monetary and fiscal policymaking under a fiscal leadership scenario, while it is severely suboptimal under the Nash or monetary leadership regimes. Nonetheless, they still impose a balance-budget constraint and set the initial level of public debt equal to zero.

Using a two-country monetary union model, van Aarle *et al.* (2002) argues that EMU increases the need for macroeconomic policy cooperation. They study the effects of policy cooperation, comparing three alternative policy regimes: (i) non-cooperative monetary and fiscal policies (Nash), (ii) full cooperation, and (iii) partial cooperation. Though cooperation is found to be often efficient for governments, in many simulations full cooperation does not induce a Pareto improvement for the ECB. van Aarle *et al.* (2002, p. 23) conclude that "[…] the stronger the asymmetry of the bargaining powers is, the less probability of coalitions among players becomes".

Also using a two-country monetary union model, which incorporates inflation persistence, Kirsanova *et al.* (2005) investigate the importance of fiscal policy in providing macroeconomic stabilization. It is assumed that the monetary authority and national governments have different objectives (ad hoc loss functions) and that they interact strategically in a monetary leadership game. The authors found active fiscal stabilization to be welfare improving, which results mainly from the ability of fiscal policy to mitigate the effects of asymmetric cost-push shocks.

Considering an asymmetric monetary union model consisting of two blocks with different initial levels of public debt, Blueschke and Neck (2011) analyze how these affect macroeconomic stabilization policies under cooperative and non-cooperative scenarios, contemplating both Nash and leadership solutions. For a symmetric demand shock, they conclude that slightly active cooperative countercyclical policies dominate non-cooperative solutions.

Even this short overview reveals a voluminous literature on monetary and fiscal policy interactions in a currency union. However, there is still a scant research on how the level of government indebtedness affects the macroeconomic stabilization performance of small and large countries under a common monetary policy. Thus, we propose to extend the work of Machado and Ribeiro (2010, 2011) to allow for the analysis of monetary and fiscal policy interactions in a richer and more realistic monetary union environment, where fiscal authorities of a large country and small countries coexist and engage in discretionary policy games with a common monetary authority that maximizes the union-wide welfare.

3.2 Cooperation

In what follows, we study how monetary and fiscal authorities optimally interact when they share the same policy objective function – the union-wide loss function – under different levels of government indebtedness. We find optimal discretionary policies and compute the incurred welfare stabilization costs, from the union's and each country's perspective. Optimal policy under commitment is used as a benchmark to understand the nature of time-consistency problems arising from the need to fulfil the government's budget constraint. To assess this issue, we first examine the optimal policy response to a symmetric technology shock, which is only costly to the extent that it produces budgetary consequences. We then analyse policy responses to asymmetric technology shocks which, given the trade-offs they cause, are the welfare-dominating shocks. We defer to a robustness section the analysis of cost-push shocks which, given the availability of a supply-side policy instrument such as the tax rate, only cause stabilization costs because of their budgetary consequences.

3.2.1 Symmetric Technology Shocks

Before analysing optimal stabilization policies, it is helpful to assess the nature of the trade-offs generated by shocks. To do so, we begin by examining their impact on the efficient equilibrium.

A negative symmetric technology shock, by increasing work effort to produce a given output, leads to a decrease in efficient output and requires a reduction of both private and public consumption on impact. To guarantee the lower efficient level of private consumption, the efficient interest rate has to rise and so do debt service costs, except in the case of a zero steady-state debt level. Therefore, achieving the efficient equilibrium produces negative budgetary consequences and debt rises. To support the resulting higher debt service costs, the policy instruments have to be permanently adjusted, leading to long-lasting effects on real welfare-relevant variables (negative private and public consumption gaps). As these permanent effects can only be lessened at expenses of higher short-run volatility, debt-constrained policies face a policy trade-off between short-run and long-run stabilization.

Given the discounting structure embedded in welfare, the optimal policy solution (commitment) requires to incur in costs from permanent effects on debt, in order to accomplish a better short-run stabilization. Nevertheless, in the first period, given that private sector expectations have already been formed, it is optimal to implement a policy-mix that generates higher inflation volatility but smaller debt consequences, thus allowing for smaller consumption and government spending gaps thereafter. This policy is time-inconsistent because, at any later stage, policymakers would face the same incentive as that of the first period. As shown analytically and numerically by Leith and Wren-Lewis (2013), time inconsistency vanishes only when permanent effects are fully eliminated and all variables, including debt, return to their efficient pre-shock levels (discretion). Thus, the discretionary solution exhibits a so-called debt-stabilization bias, as policy reveals to be overzealous in stabilizing debt. Naturally, this reflects higher welfare stabilization costs compared to commitment; under the latter, policy eliminates only partially the budgetary consequences of the shock.

Moreover, the effectiveness of monetary policy in promoting debt-stabilization increases with the initial level of public debt, because of the higher first-order effects of interest and inflation rates on debt. Conversely, fiscal policy instruments – particularly, the tax rate – become relatively less effective to achieve debt-stabilization, while becoming progressively more apt to offset inflationary consequences, the larger is the steady-state level of public debt[5] (*cf.* Leith and Wren-Lewis, 2013). As a consequence, and differently from commitment, the optimal stabilization policy-mix that emerges under discretion crucially depends on the steady-state debt level.

Under discretion, for sufficiently small debt levels (debt-to-output ratio < 28%, under our calibration), in face of a shock simultaneously boosting debt and inflation, the conventional monetary and fiscal policy assignments apply: in the first period, the interest rate gap increases to control for inflation ("active" monetary policy) and government spending gaps diminish while the tax rate gaps increase to stabilize debt. However, if the level of government indebtedness is large enough (debt ≥ 28%), monetary policy moves towards debt-stabilization and the interest rate gap decreases in the first period ("passive" monetary policy).[6] In turn, as the steady-state debt-to-output ratios increase, fiscal policy becomes relatively less effective in promoting debt-adjustment and eventually, for high enough debt levels, moves towards an inflation-stabilization assignment. Hereafter, "low-debt" refers to an environment where conventional policy assignments apply (debt < 28%), while in the "high-debt" scenario, monetary policy promotes debt-stabilization (debt ≥ 28%).

Figure 3.1 depicts the responses to a one-percent negative symmetric technology shock for a low and a high steady-state debt-to-output ratios (debt = 15% and debt = 60%, respectively) under optimal cooperative discretionary and commitment policies.[7] Notice that in all figures "Kdebt j" represents variable $\hat{b}_t^j$.

While under commitment the pattern of the policy instruments is the same under both debt levels, under discretion the direction of response of the interest rate gap crucially depends on the steady-state debt level. As expected, debt follows a random walk under commitment, in line with the findings of Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004) or Leith and Wren-Lewis (2013), while it returns to the pre-shock levels under discretion. It is clear from Figure 3.1 that the discretionary solution exhibits

[5] Higher steady-state debt levels lead to higher steady-state tax rates and, consequently, a negative tax rate gap has a larger negative impact on inflation (*c.f.* Phillip's curves in Chapter 2).

[6] Following Leeper's (1991) categorization, a policy is said to be "passive" when it promotes debt-stabilization. Otherwise, when policy instruments promote short-run stabilization, policy is said to be "active".

[7] Figure B1, in Appendix B, shows the impulse responses of the main variables under optimal cooperative discretionary policy, together with the respective efficient values and implied gaps (Gap = Effective – Efficient), all expressed in deviations from the steady-state.

higher short-run volatility than commitment and that the stabilization costs of the shock increase with the debt levels, in both policy regimes. Table 3.1 complements these results, by presenting the welfare losses under both regimes, for key debt levels.

Figure 3.1: Responses to a 1% negative symmetric technology shock under cooperation: discretionary (Coop_disc) *versus* commitment (Coop_com) (debt-to-output ratios: 15% and 60%)

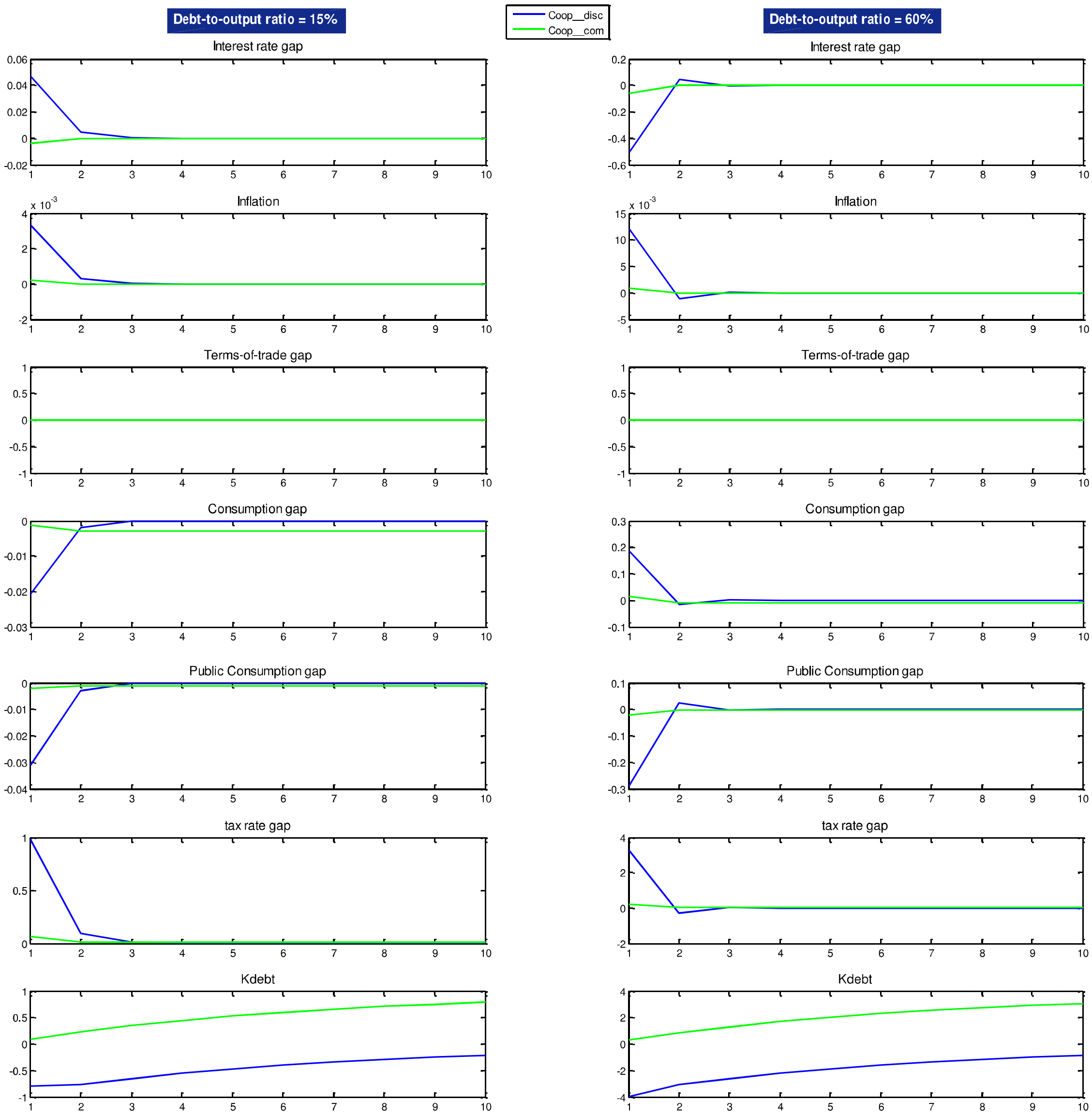


Table 3.1: *Per capita* welfare losses under cooperation: symmetric technology shock

	Debt-to-Output Ratio		
	0%	15%	60%
Commitment	0	0.0016	0.0268
Discretion	0	0.0259	0.3792

3.2.2 Asymmetric Technology Shocks

In our setup, an asymmetric technology shock generates policy trade-offs because of both its budgetary consequences and the existence of nominal rigidities. We first examine its impact on the efficient equilibrium, to assess the different budgetary consequences it produces domestically and abroad. Because an asymmetric shock hitting a big country has different consequences than one hitting a small country, we provide a separate analysis for each case.

Negative Technology Shock at the Big Country

A negative technology shock at the big country B, by increasing the work effort to produce a given output, leads to a decrease, on impact, of the domestic efficient levels of the output and, to a lesser extent, of the utility-enhanced government spending. The terms-of-trade also fall, since the domestically produced goods (B-goods) become relatively more expensive than external goods (S-goods). As domestic and foreign goods are taken as substitutes in the utility function, the reduction in the terms-of-trade increases output in the small countries.[8] Therefore, achieving efficient outcomes produces opposite budgetary consequences domestically and abroad: a government primary budget deficit at the big country B, where output and tax revenues decrease, and a surplus at the small countries, where the reverse occurs.

Moreover, as the efficient interest rate increases, on impact, to ensure a lower efficient level of private consumption debt service costs raise. This further enlarges the domestic budgetary deficit (in B) while mitigates external budgetary surplus (in S). As a result, except for the case of a zero steady-state debt level, the total budgetary

[8] Domestic and foreign goods are taken as substitutes (complements) in the utility function when the trade elasticity is larger (smaller) than the intertemporal elasticity of substitution.

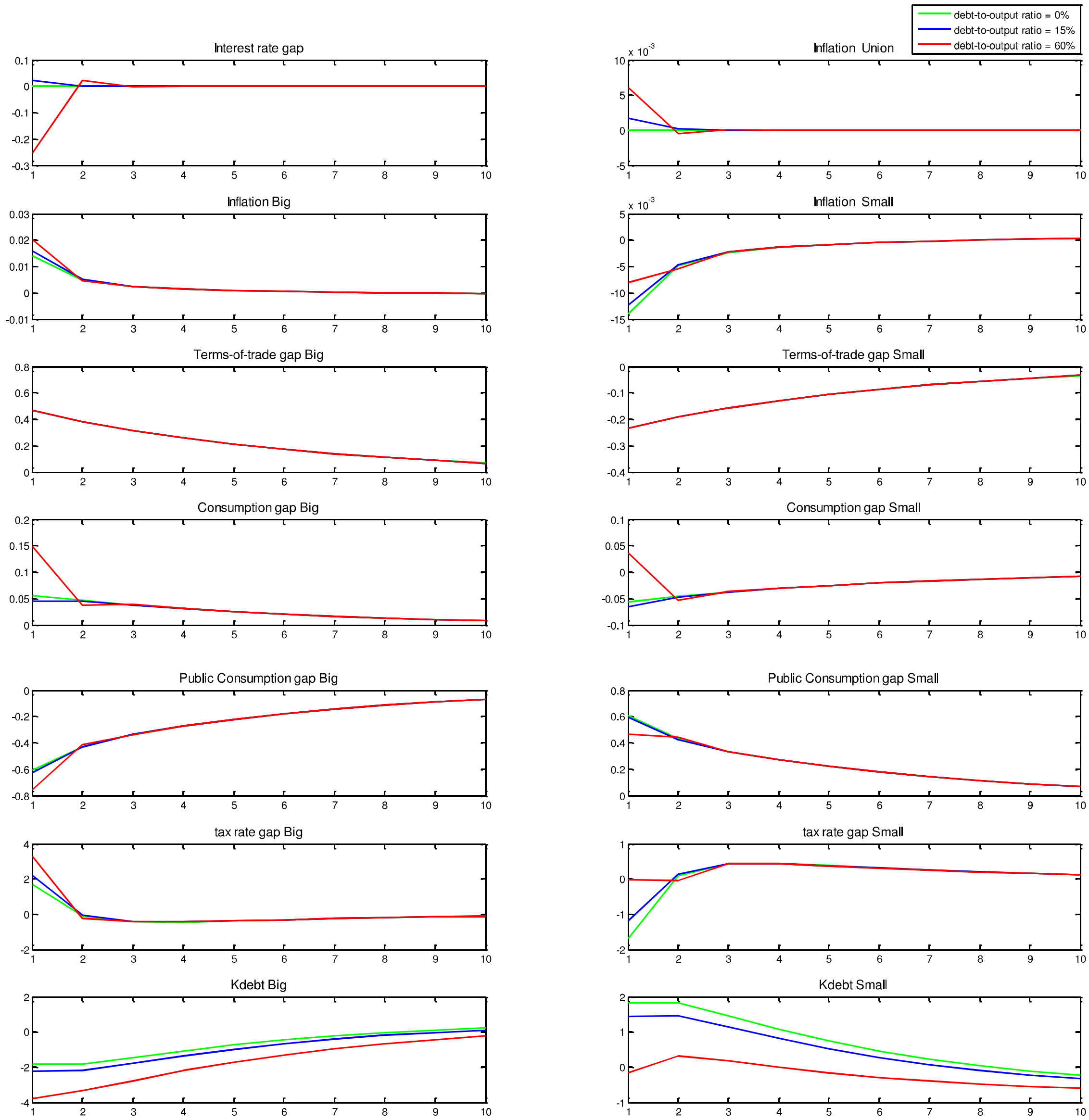
consequences of the shock are higher for country B than for the small countries and, thus, the union-wide debt increases on impact. In turn, higher steady-state debt levels increase debt-service costs and require higher steady-state tax rates which amplify the magnitude of changes in the tax base. Hence, due to the larger interest payments and to the larger decrease in tax revenues, the budgetary consequences of the shock increase with the steady-state debt level at country B. Instead, this only occurs for sufficiently high debt levels for the small countries, *i.e.*, when debt-service costs surpass primary budget surplus.

Furthermore, due to the existence of nominal rigidities, the terms-of-trade fall by less, on impact, than the efficient level. This leads to a positive terms-of-trade gap that inefficiently shifts demand from S-goods to B-goods, causing a positive output gap and reinforcing inflation in country B while the opposite occurs in the small countries. These effects could be mitigated through negative (positive) public spending and tax rate gaps at the big (small) country (ies), as it occurs under commitment (see Figure B2, in Appendix B). However, the budgetary consequences of the shock and the elimination of its long-lasting effects under discretion induce a somewhat different response from the fiscal policy.

Figure 3.2 details the adjustments under discretion, for three debt parametrizations: debt = 0% and debt = 15%, representing a low-debt scenario, and debt = 60%, illustrative of a high-debt scenario.

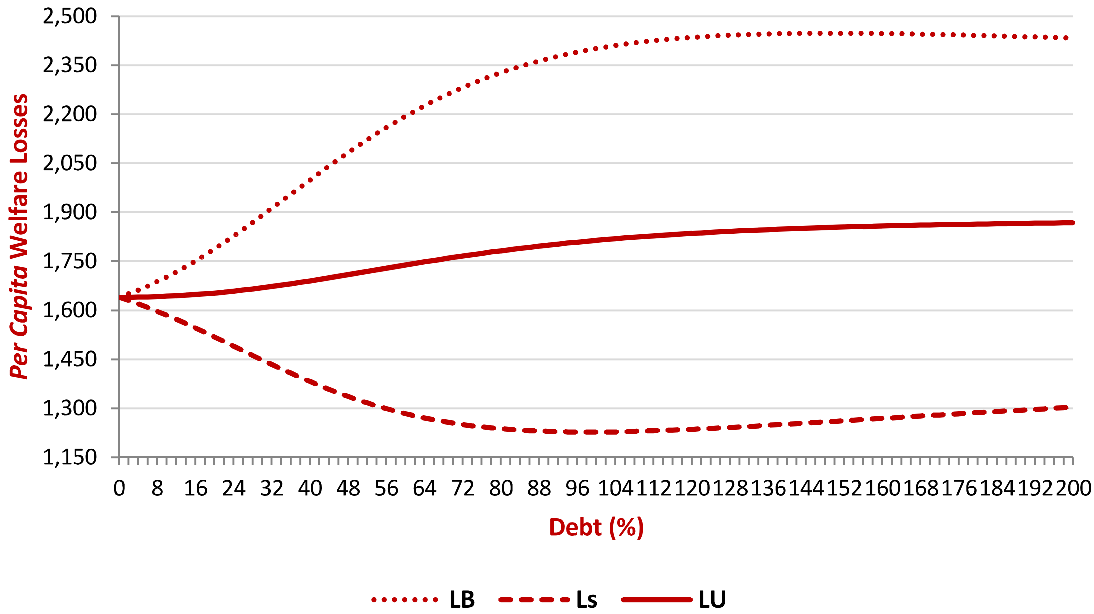
As the shock causes a primary budget deficit domestically (country B) and a surplus externally (small countries), the reaction of the fiscal policy in the first period requires a positive tax rate gap and a negative government spending gap at country B, while the reverse occurs at the small countries, for all debt parametrizations. The higher (lower) distortionary tax rate gap at country B (small countries) fuels (cuts) firms' marginal costs and further increases (decreases) inflation. The initial debt level is critical for monetary policy response and for the magnitude of fiscal policy responses under discretion:

Figure 3.2: Responses to a 1% negative technology shock at the **Big** country under optimal cooperative discretionary policy (debt-to-output ratios: 0%, 15% and 60%)



- In a **low-debt** monetary union (and except for the zero-debt case where symmetric primary and total budget effects require symmetric fiscal policies), the reaction of domestic (country B) fiscal policy dominates. To lessen the subsequent union-wide inflation, the interest rate gap increases in the first period and, by raising debt-service costs, further enlarges domestic (country B) budgetary deficit while mitigates external (small countries) surplus. Moreover, as the pre-shock debt level increases, the extent to which this shock affects national debts further amplifies this asymmetry and, thus, country B's fiscal policy becomes progressively more debt-adjusting in response to such a shock while the reverse occurs for small countries' fiscal policies.[9] Therefore, the welfare stabilization costs increase in country B while they decrease in the small countries, as the steady-state debt level increases in a low-debt scenario. The first effect dominates and welfare deteriorates for the union as a whole. This is shown in Figure 3.3 that depicts, for different debt levels, the welfare stabilization losses for the big and a representative small country s (L_B, L_s) and for the union as a whole (L_U) in face of a technology shock at B, under cooperative discretionary policies.

Figure 3.3: Union-wide and country-specific *per capita* welfare losses across debt levels, under cooperative discretionary policy responses to a technology shock at the **Big** country



- In a **high-debt** monetary union, monetary policy moves towards debt-stabilization: in order to reduce the union-wide debt level, the interest rate gap decreases in the first period.[10] The union-wide fiscal consequences of the shock grow with the level of government debt and, given its increasing effectiveness,

[9] *Cf.* Figure B3, in Appendix B, which illustrates the impact of a negative technology shock hitting country B considering three different debt-to-output ratios: 5%, 15% and 25%.

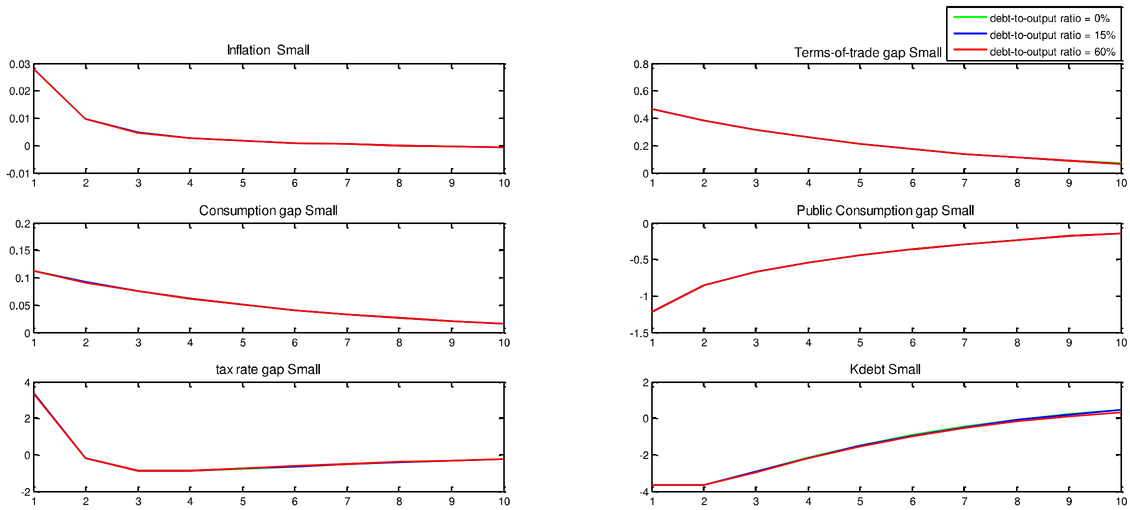
[10] This reduction on debt allows for monetary policy to raise the interest rate gap in the second period which, by lowering inflation expectations, contributes to a lower current union-wide inflation.

monetary policy becomes progressively more active towards debt stabilization. Such monetary policy response mitigates the budgetary consequences of the shock at country B but aggravates those at the small countries. These effects only dominate the budgetary consequences of the shock for sufficiently high debt levels. Therefore, fiscal policy of country B (small countries) still remains progressively more (less) debt-adjusting with the increase of the steady-state debt level, until a threshold debt value. Above this threshold, fiscal policy becomes progressively less (more) debt-adjusting at country B (at the small countries) – *cf.* Figure B4, in Appendix B. As a consequence, welfare stabilization costs begin to decrease in country B and to increase in small countries with government indebtedness, above a sufficiently high steady-state debt level. The net effect is, for the union as a whole, a monotonically increase of welfare stabilization costs as debt rises (*cf.* Figure 3.3).

Negative Technology Shock at a Small Country

A shock at a small country (with zero-dimension) has no external effects, bringing only domestic implications; thus there is no monetary policy adjustment, nor fiscal policy reactions from any other country.

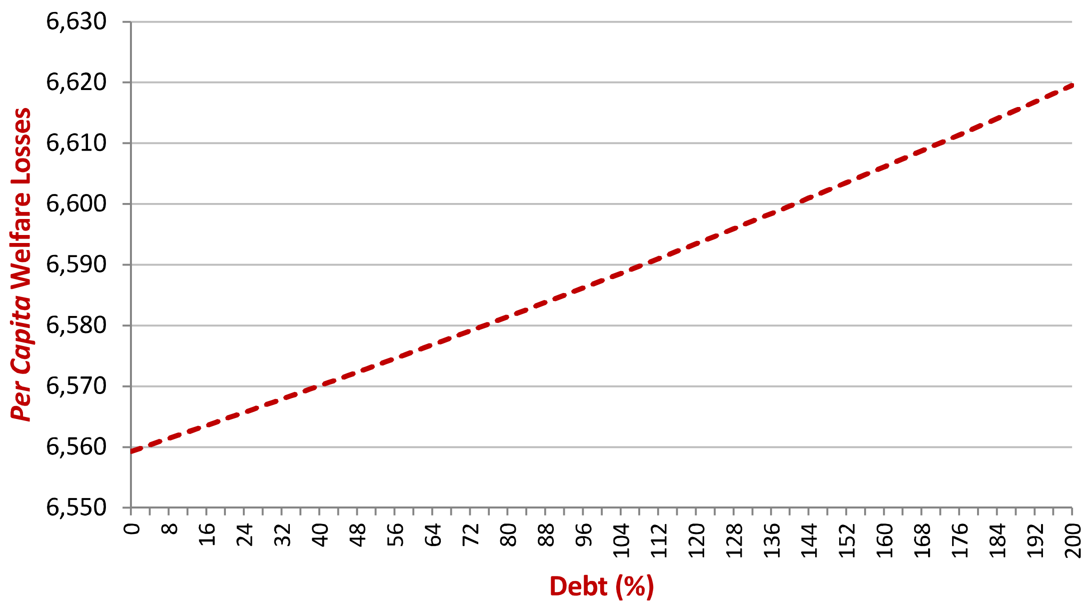
Figure 3.4: Responses to a 1% negative technology shock at a **small** country under optimal cooperative discretionary policy (debt-to-output ratios: 0%, 15% and 60%)



In the sequence of a negative technology shock hitting a small country s, its efficient output falls, on impact, as well as its public consumption, but to a lesser extent. The resulting primary and total budgetary deficit determines fiscal policy reaction under discretion. To make debt returning to its initial (and efficient) level, the government spending gap decreases and the tax rate gap increases in country s, in the first period (*cf.* Figure 3.4).

As a higher steady-state debt level also requires higher steady-state tax rates, the budgetary consequences of the shock enlarge with the initial debt level and so do the welfare stabilization costs under discretion (*cf.* Figure 3.5). However, since the country is too small to change efficient and effective interest rate, the budgetary consequences of the shock are relatively small and progress slightly with the steady-state debt level; welfare costs also progress slightly. Unsurprisingly, a small country s experiences much higher stabilization costs in face of a domestic shock than of one at country B. Instead, a shock at a small country does not produce welfare costs in any other country.

Figure 3.5: Small country welfare loss across debt, under cooperative discretionary policy response to a domestic technology shock



In sum, there are non-negligible welfare consequences from having debt-constrained policies, especially when they lack commitment. At the union level, welfare losses increase with the level of government indebtedness, since the aggregate budgetary consequences of the technology shocks also enlarge. Therefore, the putative efficiency stabilization gains, resulting from the higher effectiveness of monetary policy on debt-stabilization and of fiscal policy on short-run stabilization, are not sufficiently large to overcome the costs of stabilizing the larger budgetary consequences of shocks in high-debt environments.

As for the perspective of the big and the small countries in a monetary union, there are meaningful differences on the level and evolution with debt of their welfare stabilization costs. In line with the findings of Canzoneri *et al.* (2005) or Machado and Ribeiro (2010), a small country always faces higher welfare stabilization costs than the big one, in face of asymmetric non-correlated shocks.[11] Furthermore, for reasonable debt levels, welfare improves at small countries while it worsens at the big country, when the level of government indebtedness increases, uniformly, in a monetary union.[12] The reverse occurs, but only for sufficiently high debt levels. These results are in accordance with those of Machado and Ribeiro (2011) for the case of an asymmetric two-country monetary union.

3.3 Non-Cooperation

In our non-cooperative setup, the benevolent monetary authority still seeks to maximize the union-wide welfare but, differently from cooperation, national fiscal authorities are exclusively concerned with their national counterparts. This conflict of policy objectives allows for strategic interactions between policymakers and different equilibriums, depending on the timing structure of the policy games.

We consider three non-cooperative discretionary policy regimes which are, according to the order of moving: 1) Nash, when all policymakers move simultaneously; 2) monetary leadership, when the monetary authority moves first and fiscal authorities move, simultaneously, afterwards; 3) fiscal leadership, when fiscal authorities move, simultaneously, first, leading the monetary authority.

[11] In a setting close to Canzoneri *et al.* (2005), Mykhaylova (2011) found, in a monetary union calibrated to the EMU, that welfare costs are virtually the same for small and large union-member countries, due to highly correlated technological processes and trade openness.

[12] Notice that in our model, all government debt has a one-period maturity, lending monetary policy high leverage over debt service. A given one-period debt level of our setup should correspond to a higher debt level in a more realistic structure of debt.

Since asymmetric shocks occurring at small countries produce no external effects and no reaction from the central bank, there are no significant differences between alternative policy regimes. Hence, in the following subsections, we focus on the implications of asymmetric technology shocks hitting the big country.

3.3.1 Simultaneous Policy Decisions (Nash Equilibrium)

In the non-cooperative setup, policy outcomes can diverge from those under cooperation only because fiscal authorities do not internalize the cross-border effects of their policies. Relative to cooperation, nationally-oriented fiscal policies react more (less) intensively to a shock when they cause negative (positive) externalities. Naturally, the magnitude of these deviations is expected to be different for small and large countries, since the externalities they cause are also of different size. As each of the small countries has virtually zero-dimension, its externalities are negligible and it is not expectable that its fiscal authority has any incentive to meaningfully deviate from the cooperative policy. It is practically the same having the union-wide welfare or its national counterpart as a policy objective, when externalities are trivial. This arises because a country-specific social loss is given by the national counterparts of the union-wide social loss and, thus, there is no difference between policy objectives beyond that of not including the remainder counterparts. In this respect, we follow the arguments of Leith and Wren-Lewis (2011) who, differently from us, have explicitly derived the social loss for the individual union country-members, but got rid of the linear terms that introduce a permanent conflict between national and union-wide objectives.[13]

Effectively, since the cross-border effects and the externalities on aggregate variables produced by a small country are not significant, a small country's fiscal policy only deviates from the cooperation setup if the big country's fiscal authority or the monetary authority deviates. On the other hand, since the objective of the monetary authority is the same as in the cooperation setup, deviations of monetary policy relative to cooperation can only be justified as a reaction to divergent fiscal policies, in order to

[13] The divergence between national and union-wide objectives follows from the incentive of national governments of manipulating their terms-of-trade at their benefit, in a non-cooperative environment. As this reveals to be futile collectively, Leith and Wren-Lewis (2011) assume that individual union members resist to this temptation. Hence, non-cooperative policies only diverge from cooperative because national fiscal authorities set their policies ignoring their impact on the union-wide economy.

correct their deviation from the cooperative solution. Hence, the incentives faced by the big country's fiscal authority are crucial to understand the outcome of non-cooperation regimes. If the big country's fiscal policy diverges relative to cooperation, the monetary policy reacts and will also diverge. Small countries' fiscal authorities react to those policies. Naturally, as a block, small countries influence the response of both the big country and the monetary authority.

Consider the case of a negative technology shock at country B. As seen before, this causes a primary budget deficit domestically, requiring a positive tax rate gap and a negative government spending gap to promote debt stabilization, under discretion.

As for the first-period tax rate response, it further increases inflation at country B and contributes to mitigate the nominal rigidity's distortion, as it allows for a lower terms-of-trade gap. This allows effective demand for small countries' goods to increase by more and helps to close the negative gap between effective and efficient output. This positive externality coexists with a negative one, since it also enlarges the positive budgetary consequences of the shock at the small countries.

The reverse occurs for the government spending response. Effectively, the first-period negative government spending gap contributes to lower inflation at country B, aggravating the nominal rigidity's distortion, as it leads to a higher terms-of-trade gap and, consequently, to a higher negative demand gap for small countries' goods. This negative externality cohabits with a positive one, resulting from attenuating the budgetary consequences of the shock at the small countries. In what follows, we denote by "standard" externality, the former externality and by "debt-related" externality, the latter.

The initial debt level is critical for the externality that dominates, and determines how country's B fiscal policy response under non-cooperation diverges from cooperation.

- In a **low-debt** monetary union, where the budgetary consequences are less relevant, the "standard" externality prevails. Therefore, relative to cooperation, a negative technology shock at country B requires a smaller tax rate gap domestically and a larger government spending gap, in the first period, as the two policy instruments cause opposite externalities. Compared with cooperation, this allows for a lower inflation at B and a higher terms-of-trade gap that reduces demand for small countries' goods. Consequently, small countries' fiscal policies need to be relatively less debt-adjusting, as their primary budget surpluses are smaller. In turn, since the lower inflation of country B allows for lower inflation at the union level, the monetary policy turns to be less "active" under Nash relative to cooperation, in the first period. These differences between Nash and cooperative policy reactions can be assessed in Figure 3.6 that plots the impulse response to a one-percent negative technology shock at country B, in a low-debt case (debt = 15%). From its inspection, it is apparent that, except for the private and public consumption of country B, all the other welfare-relevant variables display shorter volatility under Nash compared to cooperation. The *per capita* welfare losses of this shock, reported in Table 3.2, show the welfare-superiority of Nash for all countries, in a low-debt parametrization. This finding is in accordance with a more general result that policy cooperation can be counterproductive in the presence of pre-existing distortions.[14]

Table 3.2: *Per capita* welfare losses under cooperation and non-cooperation (Nash) – asymmetric technology shock at the **Big** country

		Debt-to-Output Ratio	
		15%	60%
Big Country Loss (L_B)	Cooperation	1.7421	2.1939
	Nash	1.7032	1.8826
Small Country Loss (L_S)	Cooperation	1.5525	1.2836
	Nash	1.5290	2.2403
Union-wide Loss (L_U)	Cooperation	1.6473	1.7388
	Nash	1.6161	2.0615

Note: grey cells show the best result for each "country-debt" combination.

[14] The argument follows from the key contribution of Rogoff (1985), according to which cooperation among a subset of players (all policymakers) could lead to such an adverse reaction of the outsiders (the private sectors of the countries) that all players would be better off by not cooperating. Non-cooperation may alleviate time-consistency problems. See Beetsma *et al.* (2001) for a review on the literature on the desirability of policy coordination.

Figure 3.6: Responses to a 1% negative technology shock at the **Big** country – cooperation (Coop) *versus* non-cooperation (Nash): low debt-to-output ratio = 15%

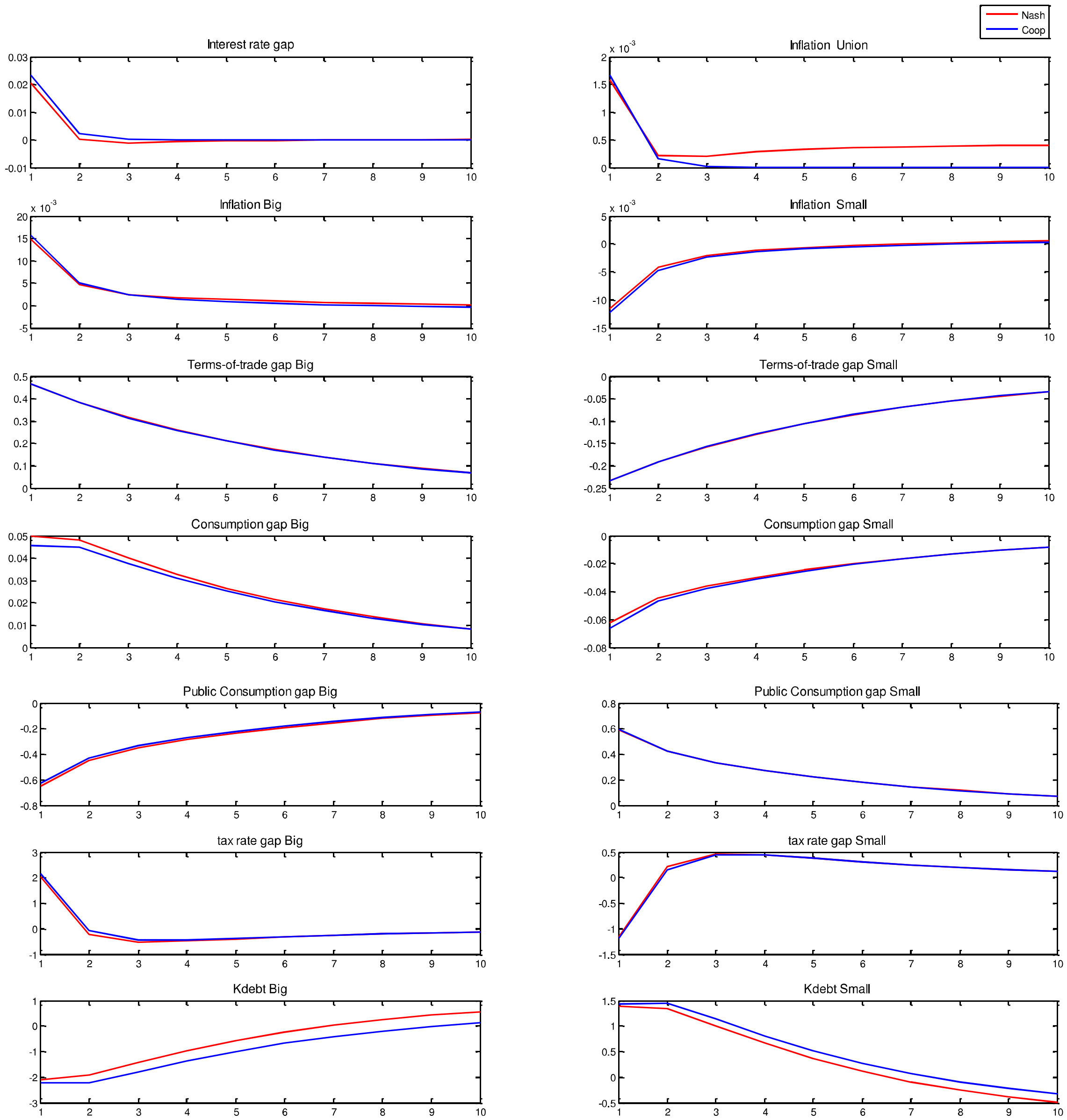
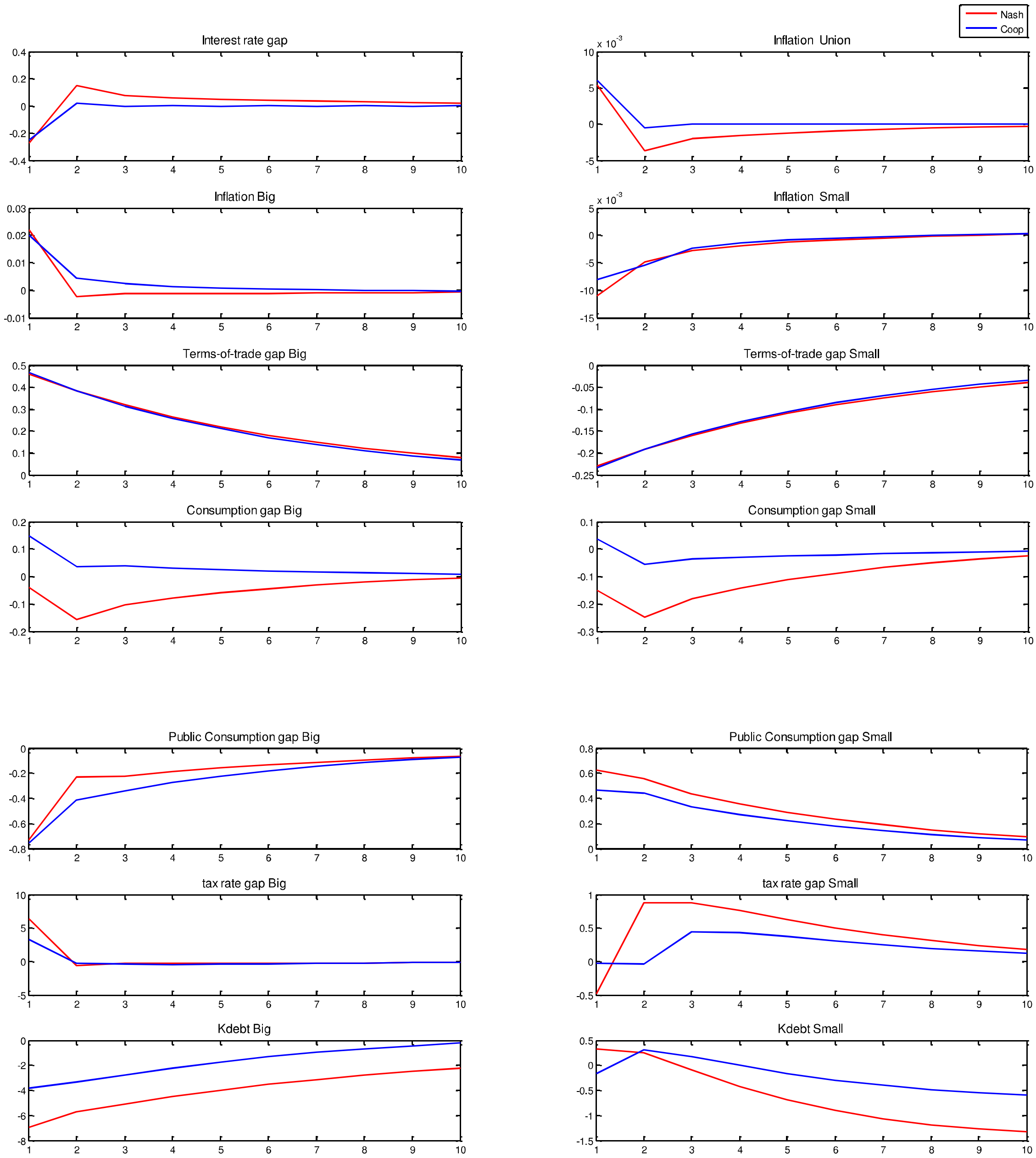


Figure 3.7: Responses to a 1% negative technology shock at the **Big** country – cooperation (Coop) *versus* non-cooperation (Nash): high debt-to-output ratio=60%



- In a **high-debt** monetary union, where the budgetary consequences are meaningful, the "debt-related" externality dominates. Therefore, relative to cooperation, a negative technology shock at country B requires, domestically, a larger (smaller) tax rate gap (government spending gap) in the first period, as the dominant "debt-related" externality is negative (positive). Compared with cooperation, this allows for a higher inflation at B and a lower terms-of-trade gap that increases demand for small countries' goods and enlarges their primary budget surpluses. As a result, small countries' fiscal policies are required to be more debt-adjusting, under Nash. At the aggregate level, fiscal policy turns to be less debt-adjusting than in cooperation and, thus, the central bank chooses to reduce the interest rate gap by more, in the first period, to ensure the stabilization of aggregate debt. This allows the central bank to raise the interest rate by more, in the second period, without an adverse effect on debt and helping to lower country B's inflation. Figure 3.7 illustrates these differences in a high-debt scenario calibrated to a steady state debt-to-output ratio of 60%. It shows that the small countries' welfare-relevant variables display higher volatility under the simultaneous-move regime than under cooperation. Therefore, for the small countries cooperation is welfare-superior to Nash, in a high-debt monetary union, as it is shown in Table 3.2. This result also holds for the union as a whole, despite the preferences of big country B for the non-cooperative regime, under which he fails to internalize the cross-border consequences of its fiscal policy (*cf.* Table 3.2).

3.3.2 Different-time decisions (Leadership)

Consider now the possibility of the fiscal or monetary authority to react as a first mover. In what follows, the analysis under (fiscal or monetary) leadership to a negative technology shock at country B will take as reference the policy reaction under the simultaneous-move regime (Nash).[15] Since the main differences between non-cooperation and cooperation were explored in the previous section, we will now focus on the differences arising from exploiting a first-moving advantage.

Fiscal Leadership (FL)

Relative to the simultaneous-move regime, consider the case where fiscal authorities take advantage of moving first relative to the monetary authority. However, this is only relevant for the fiscal authority of the big country. Given the zero-dimension of a small

[15] In this set up, and concerning the behavior of fiscal authorities, we consider that the authorities of the big country (B) and of the small countries (S) play Nash. Results (not reported) do not change substantially for the benchmark debt levels if we consider B as the first-mover.

country, fiscal policy reaction deviates from the cooperative setup only if the big country's fiscal authority or the monetary authority deviates. Hence, the big country's fiscal policy is determinant to the outcome of the fiscal leadership regime, to which both monetary policy and the small countries' fiscal policies react.

- Compared to Nash, in a **low-debt** monetary union, the leading big country's fiscal authority chooses to be relatively more debt-adjusting (especially through the costless fiscal instrument – the tax rate) [16] , relying on the monetary policy to control for excessive inflation at the union-wide level. Thus, relative to the simultaneous-move regime, the big country's fiscal policy requires a larger increase in the tax rate gap, which generates higher domestic (and union-wide) inflation, further enlarged by a smaller decrease in the public consumption gap. This results in a lower terms-of-trade gap that increases demand for the small countries' goods which produces larger primary budget surpluses. In turn, a higher union-wide inflation is expected to induce a higher interest rate gap, which would mitigate the small countries' budgetary surpluses. For sufficiently low debt levels, the latter effect is less effective than the former and, thus, the small countries' fiscal policies are required to be more debt-adjusting, under FL relative to Nash. The fiscal policy-mix that emerges at the aggregate level – higher positive tax rate gap and lower negative government spending gap – aggravates union-wide inflation and compels the monetary authority to set a higher interest rate gap, in the first-period. Figure 3.8 illustrates (green lines), for a low-debt case, the differences of responses to a negative technology shock at country B under fiscal leadership relative to the ones under the simultaneous-move regime (*cf.* Figure 3.6, above). Table 3.3, reporting the *per capita* welfare losses incurred with this shock across all policy regimes, reveals that fiscal leadership delivers, for all countries, higher stabilization costs than Nash, for the low-debt case. For the small country and the union as a whole, FL reveals to be the welfare-costliest equilibrium.

[16] Note that, as Leith and Wren-Lewis (2013) have shown, the effectiveness of the tax rate in promoting debt-stabilization is higher for lower levels of public debt.

Figure 3.8: Responses to a 1% negative technology shock at the **Big** country: fiscal leadership (FL) and monetary leadership (ML) relative to simultaneous-move regime (low debt-to-output ratio = 15%)

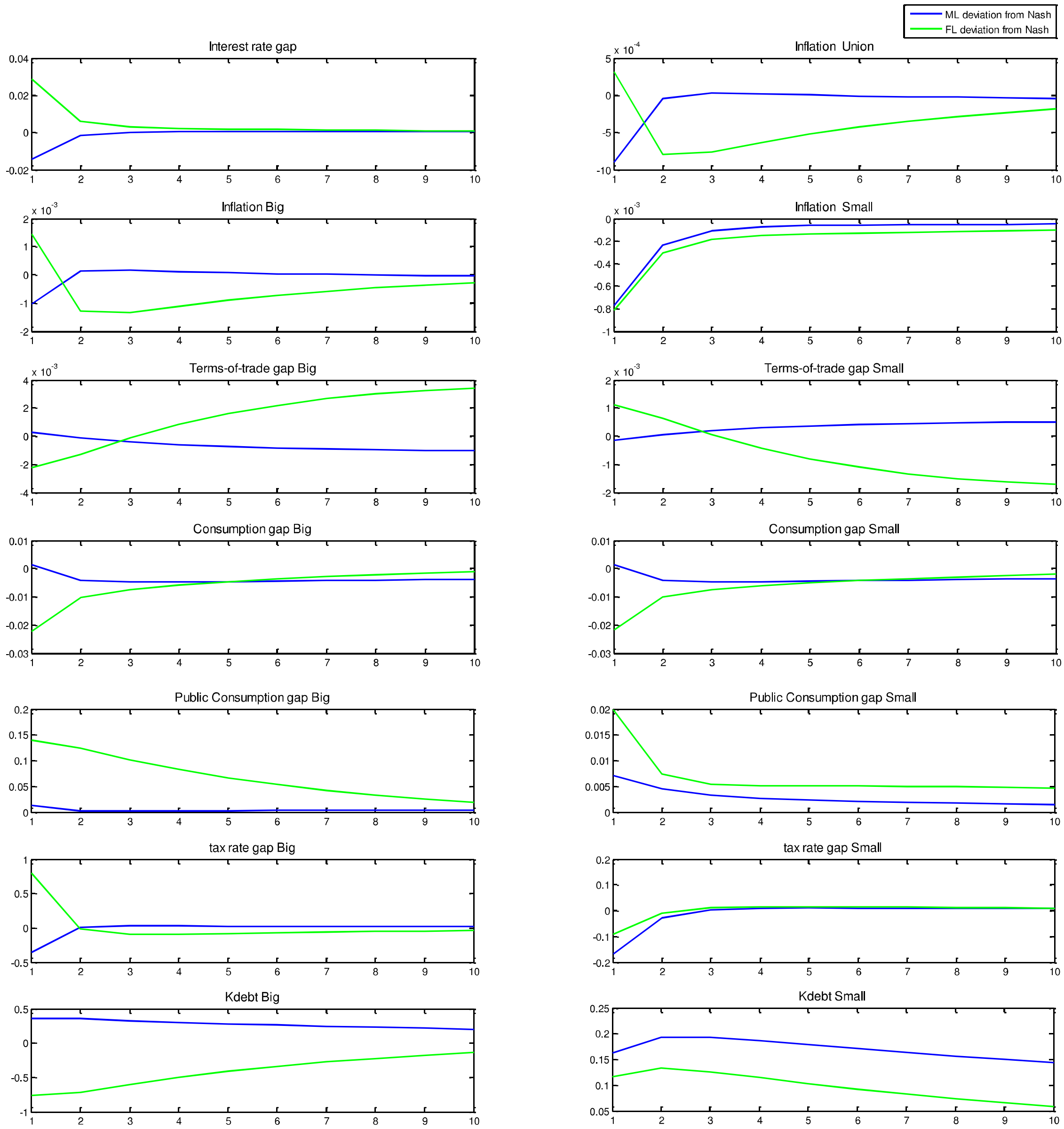


Figure 3.9: Responses to a 1% negative technology shock at the **Big** country: fiscal leadership (FL) and monetary leadership (ML) relative to simultaneous-move regime (high debt-to-output ratio = 60%)

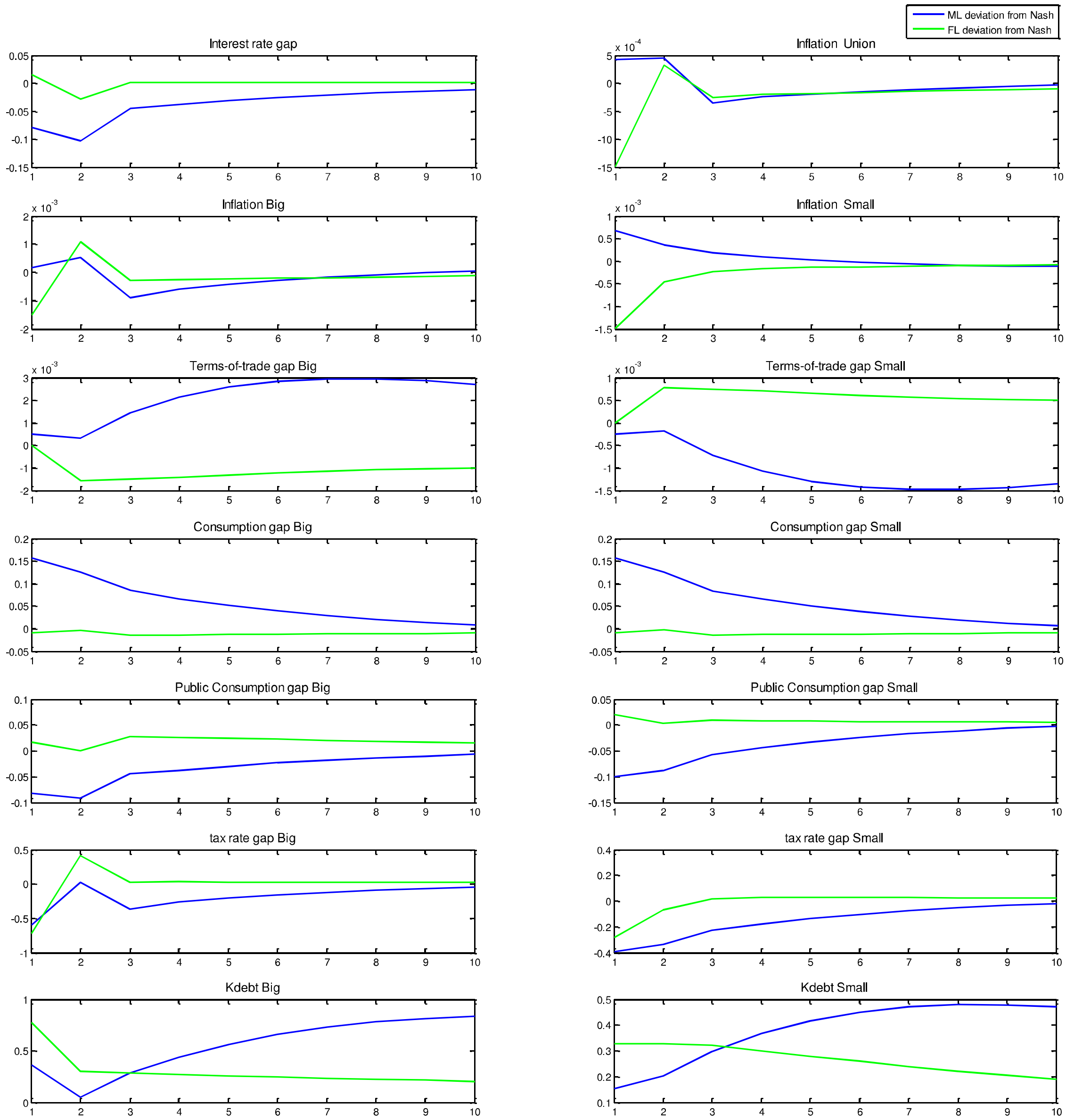


Table 3.3: *Per capita* welfare losses under cooperation and non-cooperation – asymmetric technology shock at the **Big** country

		Debt-to-Output Ratio	
		15%	60%
Big Country Loss (L_B)	Cooperation	1.7421	2.1939
	Nash	1.7032	1.8826
	Fiscal Leadership	1.7068	1.7086
	Monetary Leadership	1.6287	2.2202
Small Country Loss (L_S)	Cooperation	1.5525	1.2836
	Nash	1.5290	2.2403
	Fiscal Leadership	1.6276	2.3987
	Monetary Leadership	1.5850	1.6460
Union-wide Loss (L_U)	Cooperation	1.6473	1.7388
	Nash	1.6161	2.0615
	Fiscal Leadership	1.6672	2.0537
	Monetary Leadership	1.6069	1.9331

Note: grey (blue) cells show the best (worst) result for each "country-debt" combination.

- In a **high-debt** monetary union, and differently from the low-debt case, the leading big country's fiscal authority chooses to be relatively less debt-adjusting compared with Nash, and forces a more debt-adjusting monetary policy. Therefore, relative to Nash, the big country's fiscal policy requires a lower positive tax rate gap and a smaller negative public consumption gap, which globally contribute to reduce domestic (and union-wide) inflation and to enlarge the positive terms-of-trade gap. This allows for a smaller increase of small countries' outputs and, consequently, of primary budget surpluses relative to the outcomes under Nash. However, it is also expected a larger decrease of the interest rate gap which would enlarge budgetary consequences at the small countries. For sufficiently high debt levels, this effect dominates and the small countries' fiscal policies are required to be more debt-adjusting under FL relative to Nash. The fiscal policy-mix that emerges at the aggregate level either allows for a relatively less debt-adjusting monetary policy, as is the case in Figure 3.9 (green lines) for the debt level of 60%, or for a relatively more debt-adjusting monetary policy, if debt is high enough. It is clear, from inspection of Figure 3.9

and confrontation with Figure 3.7, that country B's inflation displays lower volatility than under Nash, as it raises by less in the first period and falls by less subsequently, while the reverse occurs for the small countries' inflation. As shown in Table 3.3, FL delivers the best stabilization outcome for the big country, B, and the worst for small countries, for the high-debt benchmark level. For the union as a whole FL is slightly better than the simultaneous-move regime.

Monetary Leadership (ML)

Under monetary leadership, the first-mover is the benevolent central bank. When making its decision, he is aware of fiscal authorities' incentives to deviate from cooperation, in particular those for the big country's fiscal authority. Since the externalities produced by a small country are negligible[17], the monetary authority recognizes that the big economy has larger incentives than a small economy to deviate from cooperation. This deviation from cooperation may either hamper or improve the welfare of the union. Thus the monetary authority sets monetary policy accordingly with the different expected consequences for the union's welfare.

- In a **low-debt** monetary union, the leading monetary authority anticipates that the incentives faced by the fiscal authority of country B under non-cooperation, in face of a negative technology shock at country B, will result in a union-wide welfare-superior equilibrium. Hence, the interest rate gap rises even less than in Nash and, thus, by lowering debt-service costs, further enlarges external (small countries) budgetary surplus while it mitigates domestic (country B) deficit. As a consequence, relative to the simultaneous-move regime, the fiscal policy of small countries (country B) has to be more (less) debt-adjusting under the monetary leadership regime (*cf.* Figure 3.8, blue lines). From confrontation of Figures 3.6 and 3.8, it is clear that, in general, country-specific variables of country B display lower volatility under ML than under Nash, while the reverse occurs for the small countries. So, relative to Nash, ML reveals to be welfare-improving for the big country B and welfare-deteriorating for small countries (*cf.* Table 3.3). For the

[17] Fiscal policy in the small country faces incentives to deviate from cooperation only if the big country's fiscal authority or the monetary authority deviates.

union as a whole, ML is the welfare-superior regime, for the benchmark low-debt level.

- In a **high-debt** monetary union, the monetary authority, recognizing that the country B's fiscal policy reaction is welfare-decreasing for the union, acts as to influence its incentives. Therefore, the monetary authority chooses to reduce by more the interest rate gap in the first period, in order to induce the big country's fiscal authority to be relatively less debt-adjusting. Effectively, as this monetary policy response mitigates country B's budgetary deficit while it enlarges small countries' surplus, the fiscal authority of country B (small country) implements a relatively less (more) debt-adjusting policy-mix, under the monetary leadership regime (*cf.* Figure 3.9, blue lines). The resulting equilibrium ends up by delivering, relative to the Nash outcomes, higher volatility for country B's specific variables while the reverse occurs for the small countries.[18] In the high-debt case, small countries and the union as a whole experience welfare stabilization gains from having a leading monetary authority under non-cooperation (*cf.* Table 3.3).

3.4 Welfare Stabilization Costs across Different Debt Levels and Policy Regimes

In the previous sections, we have examined optimal discretionary monetary and fiscal policy responses to a negative technology shock at country B and consequences for welfare stabilization costs under two parametrizations of the steady-state debt-to-output ratio (15% and 60%). Here, we extend the debt range and we consider all asymmetric technology shocks, to appraise how the welfare stabilization costs of the whole union and of the different-size country members evolve across debt levels. This analysis is meaningful within the context of the Euro Area, where formal limits to debt are set on a supranational basis. We conjecture, from previous results, that the institutional environment of both monetary and fiscal policy in the EMU is not welfare neutral for defining limits on public accounts. We first look at the *per capita* welfare losses of the union as a whole and then at the country-specific *per capita* welfare losses.

[18] For this result it is crucial the relatively higher (lower) government spending gap at country B (s).

Union-wide Welfare Losses

Figure 3.10: Union-wide welfare loss (L_U) across different policy regimes and debt levels (all asymmetric technology shocks included)[19]

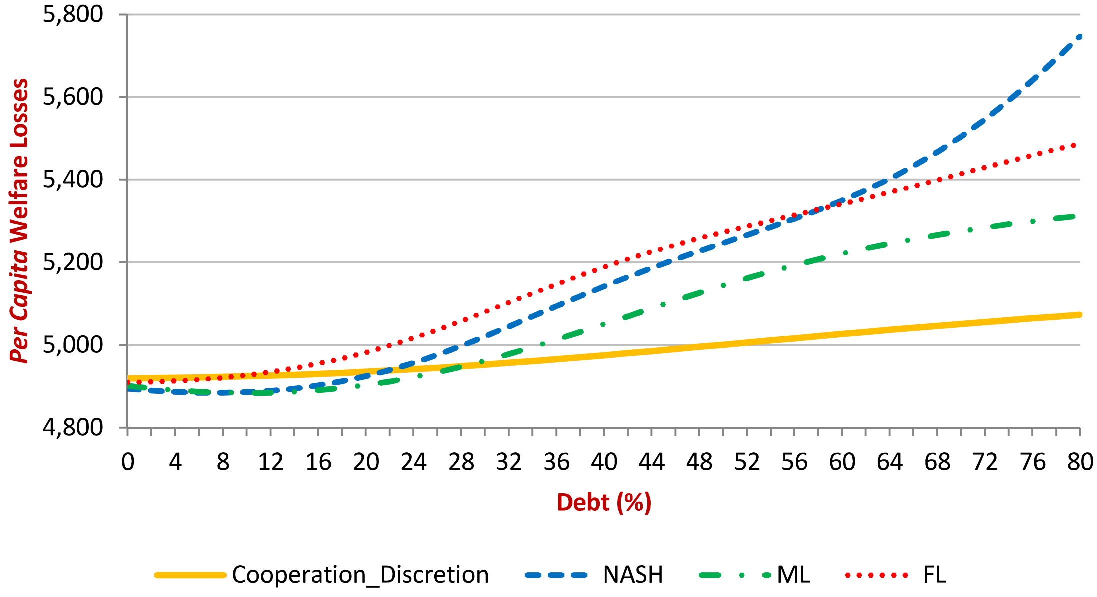


Figure 3.10 plots union-wide welfare losses for a range of debt levels for which the system converges to a unique solution under all policy regimes. From its inspection we can conclude that, in general, higher government indebtedness hampers business cycle stabilization. Moreover, there seems to be clear stabilization gains from promoting policy cooperation in a high-debt monetary union but this could be counterproductive in a low-debt monetary union.

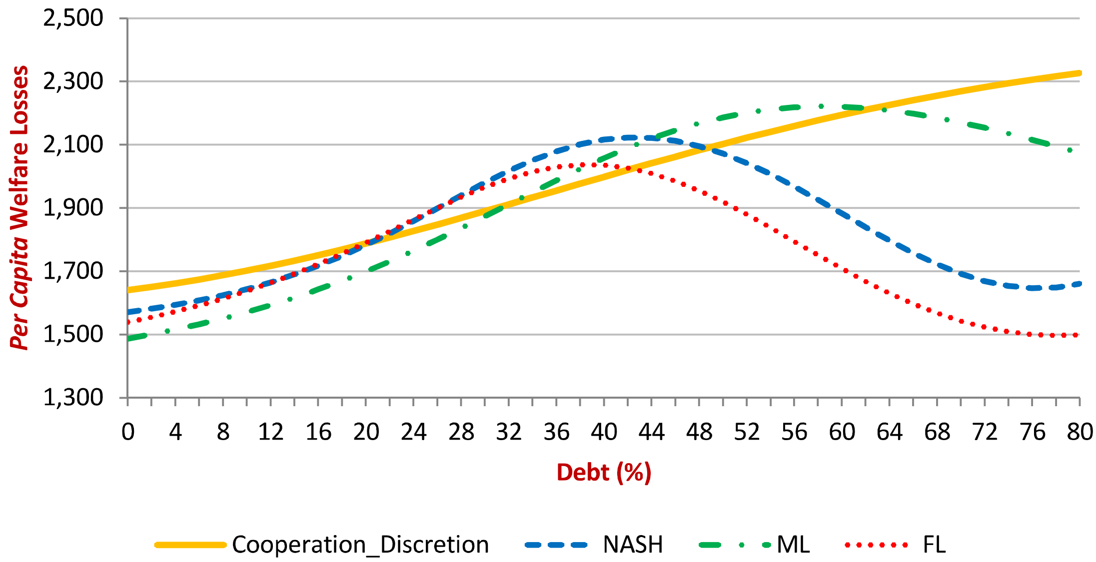
Non-cooperation may alleviate time-consistency problems of optimal discretionary policies and allow for a better stabilization performance than cooperation, when monetary policy is attached to an inflation-stabilization assignment in low-debt environments. Conversely, for high debt levels, cooperation is welfare superior relative to non-cooperation at the union level, as the gains of having a less debt-adjusting fiscal policy at the aggregate level, under non-cooperation, are surpassed by the costs of having a more debt-adjusting monetary policy.

[19] Although an asymmetric shock at a small economy (with zero dimension) produces negligible welfare costs at the union level, we consider that all small countries will face this kind of shock at any point in time, so union's welfare (L_U) takes into account the big country's welfare (L_B) and one representative small country's welfare (L_S). Hence, assuming that the big country's dimension is 0.5, we have that: $L_U = 0.5L_B + 0.5L_S$.

In turn, monetary leadership delivers a better stabilization performance than fiscal leadership, independently of the level of government indebtedness.

Big Country Welfare Losses

Figure 3.11: **Big** country welfare loss (L_B) across different policy regimes and debt levels (all asymmetric technology shocks included)



Domestic shocks are the only that cause stabilization costs for the big country B, as a shock at a country with zero-dimension produces no external effects.[20] The dominance of domestic shocks is crucial for the observed non-monotonic relationship between welfare stabilization costs and debt across policy regimes, despite the increasing budgetary consequences of the shocks with debt (*cf.* Figures 3.11 and 3.3). This follows from the fact that, when debt becomes high enough, the increased effectiveness of monetary policy in promoting debt-stabilization may allow for domestic fiscal policy to be less progressively debt-adjusting. When the gains of alleviating domestic fiscal policy from time-consistency problems are sufficiently large, welfare stabilization costs decrease with the level of government indebtedness at country B.

As for the union as a whole, for sufficiently low-debt levels, non-cooperation dominates cooperation and monetary leadership is the welfare-superior regime for the

[20] Symmetric technology shocks over a significant branch of the small countries would also produce effects at the big country. We have not considered such shocks.

big country. Fiscal leadership allows for a better stabilization performance than Nash, only for very low debt levels (*cf.* Figure 3.11).

Non-cooperation still dominates cooperation for sufficiently high-debt levels but not for an intermediate range of debt levels. Only above a sufficiently high threshold for the steady-state debt-to-output, the positive "debt-related" externality identified before dominates for government spending and allows for a lower country B's government spending gap under non-cooperation. For intermediate-debt levels, still dominates the negative "standard" externality and the negative government spending gap is larger under non-cooperation than cooperation, at country B.[21]

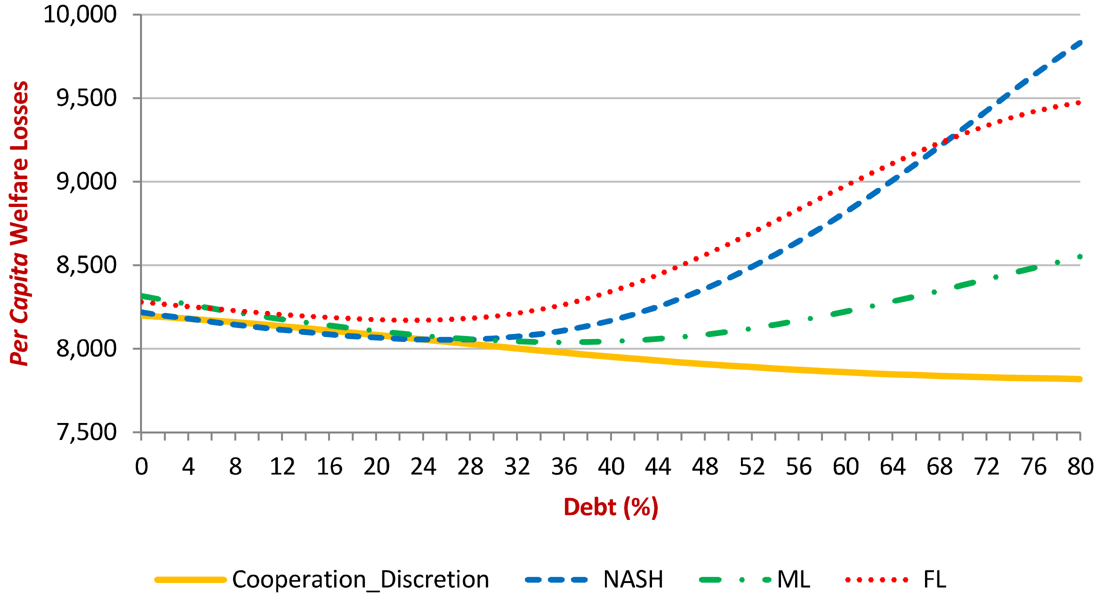
Clearly, the big country B takes advantage of having its government leading a union-wide benevolent monetary authority, in a high-debt monetary union (*cf.* Figure 3.11).

Small Country Welfare Losses

For a small country, only external shocks at a big country matter for the differences between cooperation and non-cooperation; domestic shocks, despite their higher stabilization costs, do not produce relevant externalities. Furthermore, external shocks at country B reveal to be determinant for the non-monotonic relationship observed between welfare stabilization costs and debt at a small country (*cf.* Figure 3.12 and Figure 3.3). For low-debt levels, the extent to which such shock affects the small country's debt diminishes with the pre-shock debt level, as seen previously. Moreover, the reaction of the monetary policy under low-debt levels further mitigates the budgetary consequences for the small country. Therefore, the welfare stabilization costs decrease in small countries, as the steady-state debt level increases in a low-debt scenario. However, in a high-debt scenario, the "passive" monetary policy reaction to the shock enlarges the small country's primary budget surplus and its fiscal policy becomes progressively more debt-adjusting with the increase of the steady-state debt level, above a sufficiently high threshold for the debt-to-output ratio. As a consequence, above a sufficiently high steady-state debt level, welfare stabilization costs increase with government indebtedness for a small country (*cf.* Figure 3.12).

[21] This is probably a consequence of the different evolution, with debt, of the effectiveness of the two fiscal policy instruments – government spending and tax rate – in promoting debt-stabilization, shown by Leith and Wren-Lewis (2013).

Figure 3.12: **Small** country welfare loss (L_s) across different policy regimes and debt levels (all asymmetric technology shocks included)



In general, a small country experiences lower stabilization costs under cooperation than under non-cooperation. Only for a small range of low debt levels, Nash is welfare-superior. In a non-cooperative setting, small countries benefit from having a leading union-wide benevolent monetary authority, in a high-debt monetary union. Fiscal leadership is the welfare-inferior regime for a considerable range of debt levels (*cf.* Figure 3.12).

3.5 Sensitivity Analysis

Previous analysis has considered only the welfare-dominant technology shocks. In this section, we extend the analysis to cost-push shocks. Then, we test the robustness of our results across a wider range of structural model parameters, beyond that for the debt-GDP emphasized so far.

3.5.1 Cost-push Shocks

In what follows, we focus the analysis on a cost-push shock at the big country B, because a shock at a small country only brings domestic implications.

In face of a cost-push shock at country B, the domestic tax rate should fall efficiently to fully offset the impact of the shock. The subsequent primary budget deficit forces fiscal policy of country B to deviate from efficient outcomes and lets inflation increase at home. This has a positive effect on demand for *s*-goods, leading to primary budget surpluses at the small countries.

At the union-wide level, inflation and debt increase, leading to opposite monetary policy reactions under low- and high-debt scenarios. As for the case of technology shocks, these different monetary policy reactions either enlarge or mitigate the domestic and external budgetary consequences of a cost-push shock at country B. The first-period increase in the interest rate, in a low-debt scenario, further enlarges the budgetary consequences of the shock for country B while it mitigates those for small countries. Conversely, in a high-debt scenario, the fall in the interest rate in the first period mitigates the budgetary consequences of the shock domestically (country B), while enlarging them for the small countries.

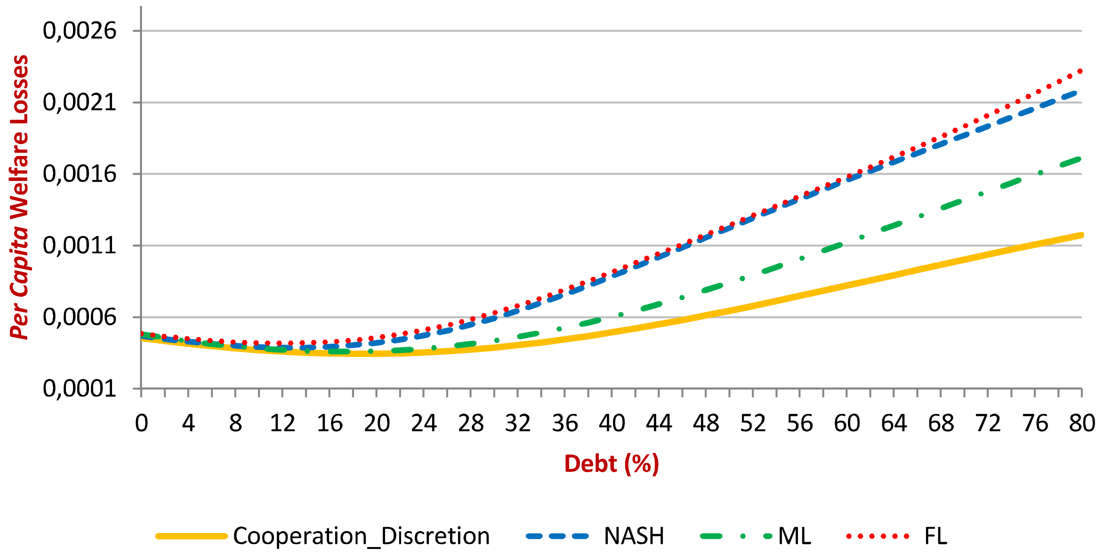
Such asymmetric budgetary consequences are amplified with the raise of the pre-shock debt level and the subsequent enlargement of the interest rate's leverage over debt service. Independently of the policy regime (cooperation or non-cooperation), the welfare stabilization costs increase for country B and decrease for the small countries, as the level of public debt increases in a low-debt scenario, while the reverse occurs for a sufficiently high level of debt (*cf.* Figures 3.13a and 3.13b).

Joint inspection of Figure 3.13a and Figure 3.12 suggests that the welfare rankings for a small country are robust across different sources of shocks.[22] In general, and independently of the shocks: (i) a small country experiences lower stabilization costs under cooperation than under non-cooperation; and (ii) monetary leadership is welfare-superior to fiscal leadership for the small country.

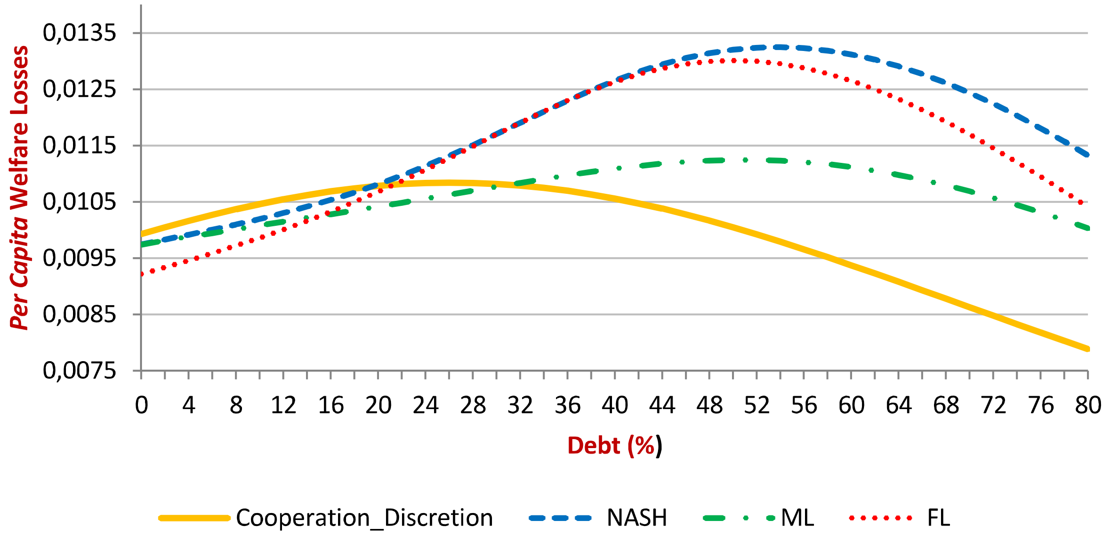
[22] The consideration of all asymmetric cost-push shocks when computing country-specific welfare losses would have changed the scale, but not the pattern, of the small country's welfare losses presented in Figure 3.13a. Thus, the patterns of the welfare losses plotted in Figure 3.13a can be directly compared with those in Figure 3.12.

Figure 3.13: Country-specific welfare losses across different policy regimes and debt levels (cost-push shock at the **Big** country)

a. Small country welfare loss (L_S)

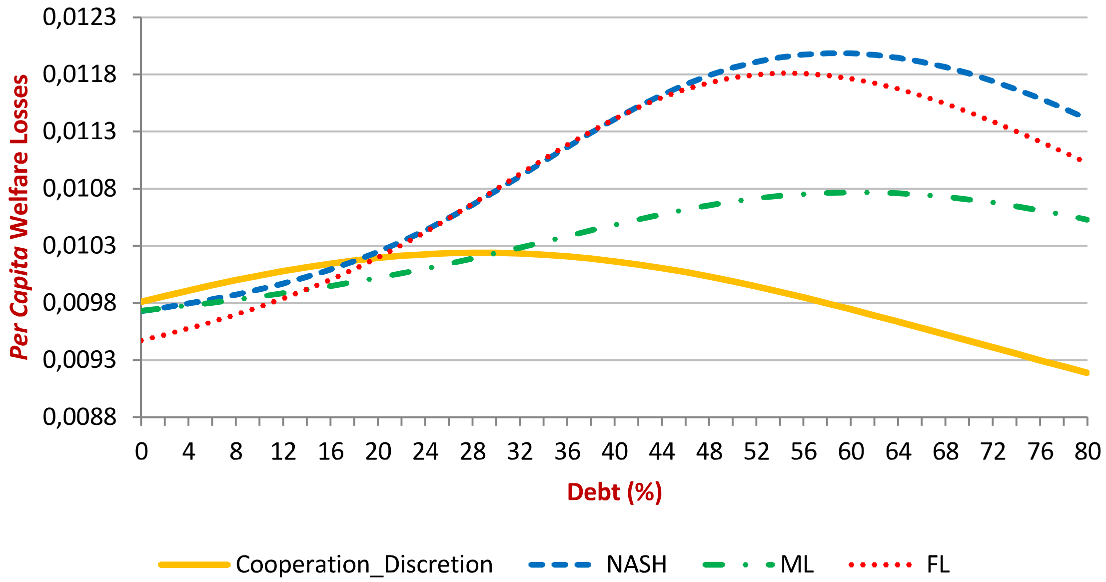


b. Big country welfare loss (L_B)



In a low-debt scenario, the big country B also achieves lower welfare costs under non-cooperation than under cooperation, independently of the shocks. Still, there are important differences between the stabilization performance of cost-push and technology shocks under different policy regimes. First, for sufficiently high debt levels, cooperation is preferable for the big country for the stabilization of cost-push shocks, while it delivers the worst stabilization performance of technology shocks. Furthermore, in face of cost-push shocks, and at odds with technology shocks, fiscal leadership is welfare-inferior to monetary leadership for the big country in a high-debt monetary union, while the reverse occurs, in a low-debt monetary union (*cf.* Figure 3.13b and Figure 3.11).

Figure 3.14: Union-wide welfare loss (L_U) across different policy regimes and debt levels (all asymmetric cost-push shocks included)



From comparing results depicted in Figure 3.14 with those in Figure 3.10, it is clear that the stabilization of cost-push and technology shocks under different policy regimes delivers identical welfare rankings for the union as a whole: i) in a low-debt monetary union, non-cooperation is welfare-superior to cooperation; ii) in a high-debt monetary union, non-cooperation is welfare-inferior to cooperation and monetary leadership is preferable to fiscal leadership. However, at odds with the results fort the technology shocks, the welfare losses of cost-push shocks decrease with government indebtedness when the steady-state debt rises above a sufficiently high level.[23]

[23] The efficient response to a cost-push shock produces decreasing budgetary consequences and also decreasing welfare losses, as the steady-state levels of debt and taxation increase. When both the steady-state levels of debt and taxation increase, the tax rate becomes progressively more apt to control for inflationary consequences and, so, the budgetary consequences of an idiosyncratic cost-push shock diminish, because smaller reductions in the efficient tax rate are required to offset the impact of the shock.

In sum, cost-push shocks deliver lower stabilization costs than technology shocks. This follows both from the use of the tax rate as a policy instrument and from the lower persistence embedded. The welfare rankings obtained for a small country and the union as a whole are, in general, robust to both supply-side shocks. This is not the case for a big country: in a high-debt scenario, a better stabilization performance may be achieved under cooperation or under non-cooperation, depending on the nature of the shock.

3.5.2 Structural Features of the Model

In this subsection we conduct a robustness analysis to some structural features of the model. Specifically, we focus our analysis on the degree of nominal rigidity, θ, the inverse of the elasticity of labor supply, χ, and the elasticity of substitution between home- and foreign-produced goods, γ. Additionally, we will test welfare effects from considering alternative dimensions for the big country. Given its welfare-dominance, we keep the focus on the stabilization costs of asymmetric technology shocks.

Nominal Rigidity

A reduction on the degree of nominal rigidity impinges directly on welfare, because it reduces the penalization of inflation variability, but also affects welfare indirectly by changing the relative volatility of welfare-related variables and the effectiveness of policy instruments to promote stabilization. Furthermore, the degree of nominal rigidity has non-negligible budgetary consequences and, as shown by Leith and Wren-Lewis (2011, 2013), is proved to have consequences on the absolute and relative contribution of the different policy instruments to debt stabilization. Therefore, there are arguments for not expecting clear-cut conclusions on the welfare consequences of increased price flexibility.

In fact, focusing on optimal cooperative discretionary outcomes across different debt levels and a wide range of degrees of nominal rigidity, we find a non-monotonic relationship between welfare stabilization costs and nominal rigidity (see Figure B5, in Appendix B).

Simulations of cooperative and non-cooperative outcomes for an alternative calibration of the degree of nominal rigidity ($\theta = 2/3$), does not change meaningfully the welfare rankings between policy regimes relative to the baseline calibration (*cf.* Figure B6, in Appendix B).

Elasticity of Labor Supply

Under our baseline calibration the inverse of the labor elasticity is set to $\chi = 3$. We now assess the welfare implications if we assume two alternative values for χ: $\chi = 1.5$ and $\chi = 5$ (as in Erceg *et al.*, 2010).

It is expected that a lower elasticity of labor supply (higher χ) results in higher stabilization costs since fluctuations in work effort, arising from misallocations caused by inflation, become more costly. Moreover, this calls for a stronger response of monetary policy to dampen the fluctuations in the work effort observed at the aggregate level. A lower value of the elasticity of labor supply reduces the cost of private consumption fluctuation relative to inflation fluctuation and, thus, it reduces the incentives for monetary policy to stabilize debt. Consequently, the threshold debt levels for which monetary policy becomes "passive" increase with smaller values for the elasticity of labor supply (higher χ). Thus cooperation becomes welfare superior to non-cooperation for higher debt levels as the elasticity of labor supply decreases, but the key qualitative results are preserved (*cf.* Figure B7, in Appendix B).

Elasticity of Substitution between National and Foreign Goods

If the elasticity of substitution between domestic and foreign goods (γ) is bigger (smaller) than the intertemporal elasticity of substitution (σ), goods are substitutes (complements) in the utility function. We now examine the implications of the goods being complements, by setting $\gamma = 0.2$ ($< \sigma = 0.4$).

As a consequence of this parametrization, one should expect that a negative technology shock at country B would cause opposite budgetary consequences, both domestically and abroad, of those observed if goods were assumed substitutes as in baseline. According to our experiment, the subsequent differences on policy responses do not change the qualitative results obtained under the benchmark calibration of this parameter (*cf.* Figure B8, in Appendix B).

Alternative Dimension for the Big Country

Finally, we test for different dimensions of the big country. Under the baseline calibration, the large economy (country B) represents 50% of the monetary union. We will now consider two alternative dimensions for country B: $n_B = 0.35$ and $n_B = 0.65$.[24]

The bigger is the large economy, the larger are the externalities produced to small countries and also the larger is the impact on aggregate variables to which monetary policy reacts. Our experiments show that this renders the distribution of welfare costs across countries even more asymmetric: the welfare stabilization costs of a small (big) country are higher and increasing (lower and decreasing) with the size of country B, independently of the debt level (*cf.* Figure B9 and B10, in Appendix B). Our results also suggest that it may be harder to get the approval of a bigger country for a cooperative arrangement.[25] Conversely, and for a wide range of debt levels, cooperation is beneficial for the small countries and the union as a whole, the larger the big country is. In general, the welfare rankings of the different policy regimes reveal to be robust to the relative size of the big country.

3.6 Concluding Remarks

We have explored how the level of government indebtedness shapes the interactions between monetary and fiscal optimal stabilization policies using a micro-founded New-Keynesian macroeconomic model for a heterogeneous monetary union composed by a continuum of small countries and a big country.

In accordance with the findings of Stehn and Vines (2008), Blake and Kirsanova (2011) and Leith and Wren-Lewis (2013), in a closed economy context, or Machado and Ribeiro (2010, 2011), in an open economy setup, the union average debt level proved to be non-neutral to policy interactions and, thus, to business cycle stabilization performance. The size of the steady-state level of debt qualifies both (i) the budgetary consequences of the shocks and (ii) the effectiveness of the different policy instruments to debt-adjustment. The policy-mix that emerges under discretion, under which long-

[24] In the Euro area, for instance, Germany's GDP represents approximately 28.5% of the union's GDP; in turn, the three major economies (France, Germany and Italy) represent approximately 66.2% of the Euro area's GDP (Eurostat, 2013).

[25] The threshold debt level for which cooperation delivers the worst solution for the big country is lower the larger the big country is (see Figure B9, in Appendix B).

lasting budgetary effects are eliminated, presents meaningful differences in low- and high-debt environments with monetary policy shifting from inflation- to debt-stabilization assignment. At the union level, the potential efficiency stabilization gains, resulting from the higher effectiveness of monetary policy on debt-stabilization and of fiscal policy on short-run stabilization in high-debt environments, are not sufficiently large to overcome the costs of stabilizing the larger budgetary consequences of technology shocks. According to our results, in general, higher government indebtedness hampers union-wide business cycle stabilization.

In turn, shocks at the large economy, due to the externalities they cause and to the reaction from monetary policy, are crucial to how country-specific welfare stabilization costs depend on the average government indebtedness of the union. They determine that the stabilization costs are distributed unevenly between small and big countries, in low and high debt environments: welfare costs decrease with debt for a small country and increase for the big country, in a low-debt monetary union; the reverse occurs in a high-debt monetary union.[26] Thus, a higher level of government indebtedness, as the one experienced in the EMU is more likely to penalize the stabilization performance of the small country-members than that of the large.

Strategic policy interactions, arising from different policy objectives of nationally-oriented fiscal authorities, disclose different welfare consequences for the large and the small countries. In general, the non-internalization, under non-cooperative policy regimes, of the externalities produced by the big country's fiscal policy imposes higher stabilization costs to the small countries. The large country can benefit from non-cooperation, for extreme debt ranges – sufficiently low or high debt levels – but not for intermediate ranges. As a whole, the union achieves a worse stabilization performance under non-cooperation in a high-debt scenario, but the reverse occurs in a low-debt scenario.

The leadership structure has also critical stabilization consequences. Fiscal leadership imposes higher stabilization costs than monetary leadership, for the union as whole and for the small countries. In turn, a highly indebted big country clearly prefers

[26] Machado and Ribeiro (2011) obtained similar result.

fiscal leadership, where it can explore a larger strategic power *vis-a-vis* a debt-accommodative monetary authority.

4 Optimal Fiscal Simple Rules for Small and Large Countries of a Monetary Union

There is now a vast literature on optimal simple rules (OSR) given their practical advantages over full optimal policies. Relative to monetary policy rules, which commonly assume that monetary policy follows a class of Taylor (1993)-type inflation targeting rule, there is much less literature and consensus on the optimal reaction function for simple fiscal policy rules, although the need for fiscal rules has gained importance in recent years. According to Schaechter *et al.* (2012), numerical fiscal rules (as defined in Kopits and Symanski, 2001) have evolved from being adopted by only 5 countries in 1990 up to 76 countries in March 2012.

Several recent papers have already studied the use of optimal simple rules in a monetary union environment. However, the case of a more general structure for a currency union, where few large countries may coexist with many small countries, has not yet been addressed. In such an environment, fiscal policy assumes a prominent role, since a very small country relies exclusively on its own fiscal policy to promote stabilization when hit by country-specific shocks.

We intend to contribute to the literature through (i) deriving and compare across alternative optimal simple monetary and fiscal policy rules in the above-mentioned monetary union scenario. Moreover, we attempt to (ii) address how optimal feedback parameters should differ with structural features of the economies, such as average government debt. We rely on the baseline model presented in chapter 2, where multiple fiscal authorities, representing a large country as well as each of several small economies, coexist with a common benevolent monetary union. In particular, we extend the analysis of OSR for fiscal policy to a very small country within a heterogeneous country-size monetary union. As we are interested in the particular role of fiscal policy in the stabilization of asymmetric shocks, our analysis focuses on the design of optimal simple fiscal rules, assuming a Taylor-type feedback rule for the monetary policy. Moreover, we consider a cooperative scenario where the union-wide welfare function is optimized and, thus, no strategic interactions apply.

This chapter is organized as follows. Section 4.1 reviews the literature on OSR for monetary and, in particular, for fiscal policy. In section 4.2, we describe the simulation procedure for OSR used in our model setup and discuss alternative fiscal policy rules. Section 4.3 provides an analysis of the results, first considering a standard symmetric-size two-country monetary union and then considering a heterogeneous country-size union model. We focus on the welfare stabilization costs of alternative fiscal rules with countercyclicality and debt feedback degrees optimally derived for both low- and high-debt scenarios. OSR feedback parameters are expected to differ across countries, indexed to the particular structure of the economies, including debt levels. In Section 4.4, we analyze the sensitivity of our results to changes in the calibration of the shocks, particularly that of cost-push shocks, and to alternative model parameters, in order to provide insights on how rules should differ with different structural features of the economies. Section 4.5 presents the summary of main results.

4.1 Overview of the Literature

Several papers have recently examined joint stabilization by monetary and fiscal authorities through the use of optimal simple rules. In some cases, OSR are derived both for monetary and fiscal policies (*e.g.,* Beetsma and Jensen, 2003, van Aarle *et al.*, 2004, Argentiero, 2009, Ferrero, 2009, Pappa, 2012, Vogel *et al.*, 2013, in a monetary union context; Schmitt-Grohé and Uribe, 2007, Chadha and Nolan, 2007, Marattin and Marzo, 2008, Rossi, 2009, Bi and Kumhof, 2011, Canzoneri *et al.*, 2011, Motta and Tirelli, 2012, Cantore *et al.*, 2013, Kliem and Kriwoluzky, 2014, in a closed economy; Kumhof and Laxton, 2009, in a two-open economies setup). In other studies, one authority follows a full optimal policy while the other relies on OSR (*e.g.*, Kirsanova *et al.* , 2007, in a monetary union context, and Kirsanova and Wren-Lewis, 2012, in a closed economy, consider the monetary policy to be full optimal and focus on the derivation of fiscal OSR; in turn, Lambertin, 2007, studies full-optimal fiscal policy in a two-country monetary union assuming that the central bank follows a simple Taylor rule, while Eser, 2009, considers an optimal Taylor rule and assumes that fiscal policy is set through Ramsey optimal outcome in a closed economy model).

Despite the increasing OSR literature, particularly in the context of a monetary union, the case of a more general structure for a currency union, where few large countries may coexist with many small countries, as is the case, *e.g.*, of the EMU, has not yet been addressed.

Moreover, while, in amongst the vast majority of the literature on rules-based policy, monetary policy follows a class of Taylor (1993)-type conventional inflation targeting rule, there is fewer consensus in setting theoretical OSR for fiscal policy.

Monetary Taylor-type Rules: the Background

In the literature, monetary policy is usually defined either through a strict inflation targeting rule or through a broader class of flexible-inflation targeting rules. Strict inflation targeting refers to the situation where inflation is the only variable entering the loss function, while flexible-inflation targeting allows for other target variables. A targeting rule will be mimicked by an implicit instrument rule (Rudebusch and Svensson, 1999), such as feedback rules belonging to the class of Taylor-type rules (see Taylor, 1999a, for a historical analysis on monetary policy rules). Taylor's (1993) original monetary policy rule sets the short-term nominal interest rate as a feedback rule on current output gap and deviations of current inflation from an inflation target. The original rule has been enriched, for instance, by introducing interest rate smoothing, forward-looking expectations or a target for money growth (*e.g.*, Schmitt-Grohé and Uribe, 2007, Vogel *et al.*, 2013 and Kliem and Kriwoluzky, 2014). Besides offering simplicity and transparency relative to a full-optimal monetary policy, empirical research suggests that a Taylor rule is able to explain rather well actual interest rate adjustments (van Aarle *et al.*, 2004). Moreover, according to Rudebusch and Svensson (1999), simple Taylor-type rules may deliver, in many cases, a degree of macroeconomic volatility and welfare losses that approach those of the full-optimal policy. "An interest rate rule is generally considered "simple" if it makes the policy instrument a function of observable variables only, and does not require any precise knowledge of the exact model or the values taken by its parameters" (Galí, 2008, p. 81).

Although there is now a widespread consensus on the reaction function of a monetary policy rule, there are still divergences about the value of the coefficients, the adequate measure of the output gap and that of the natural interest rate (van Aarle *et al.*, 2004). Taylor (1999b) shows that the interest rate response coefficient to deviations of inflation from its target should be larger than one to ensure model stability and that interest-rate smoothing is destabilizing if expectations are not rational.

Fiscal Rules: Theoretical and Practical Definitions

Even though optimal fiscal policy can also be approximated by simple rules, these have received less attention when compared with monetary rules[1]; thus, a consensus about their design is yet to come. Nevertheless, a wide range of numerical targets and procedures have been adopted by governments. Schaechter *et al.* (2012) compile a dataset of fiscal rules in use around the world, covering 81 countries from 1985 to end-March 2012. Their study reveals that many fiscal rules have been adopted/strengthened in response to the recent financial crisis, that the number of fiscal rules, as well as the comprehensiveness of their design, in emerging countries has caught up those in advanced countries[2], and that the "next-generation" fiscal rules tend to be more complex as they explicitly combine the objectives of sustainability with more flexibility to accommodate economic shocks, thereby creating new challenges for communication, monitoring and effective implementation. Some of these "new-generation" fiscal rules include the use of cyclically-adjusted targets and well-defined escape clauses, together with stronger legal and enforcement arrangements.[3]

Obviously, the choice of some type of fiscal rule cannot be dissociated from the question on what should be the role of fiscal policy. Besides general desirable properties, such as flexibility, clear operational guidance, being easy to communicate, implement and monitor, a fiscal rule should combine the merits of a short-run stabilization device with the fulfillment of long-run sustainability.[4] Relative to the latter, the different types of numerical rules perform distinctively (for an exhaustive analysis on the pros and cons of the different rules see, *e.g.*, Schaechter *et al.*, 2012 and Portes and Wren-Lewis, 2014).

[1] There are many factors that explain the less interest in the study of theoretical fiscal policy rules. For a discussion on the issues related to the design of fiscal policy rules, see Portes and Wren-Lewis (2014).

[2] Since the early 2000s, developing countries outnumbered advanced economies as users of fiscal rules (Bova *et al.*, 2014).

[3] Bova *et al.* (2014) document the spread of fiscal rules in the developing world and analyze the relation between fiscal rules and procyclical fiscal policy. The authors find partial evidence that some aspects of these "new-generation" fiscal rules may be associated with less procyclicality.

[4] Kopits and Symansky (1998) identify eight properties to define a good fiscal rule: well-defined, transparent, simple, flexible, adequate relative to the goal, enforceable, consistent and efficient. Creel and Saraceno (2010) discuss these properties in the context of the reform of the Stability Growth Pact (SGP).

Debt rules (DR), which set upper limits for debt over GDP, are directly linked to debt sustainability and, hence, are the most appropriate to accomplish long-run budgetary sustainability; though, they do not explicit ensure stabilization (actually, they can even be pro-cyclical) and are embedded with no clear operational guidance. Besides the time lag involved in the reaction to fiscal instruments, debt developments can be affected by variables beyond the control of policymakers (such as interest or exchange rate changes). Balance-budget rules (BBR), targeting overall budget, are closely linked to debt sustainability but, as debt rules, can also be pro-cyclical. Structural, or cyclically-adjusted balance, and balance "over the cycle" rules have the advantage of allowing for short-run stabilization, while being closely linked to debt sustainability; however, the correction for cycle and the identification of the structural elements is complex, making it more difficult to communicate and monitor. Additionally, an "over the cycle" balance budget rule has the disadvantage that remedial measures could be postponed to the end of the cycle (Schaechter *et al.*, 2012, p. 7). In contrast, expenditure (ER) and revenue ceiling rules (RR) are not directly linked to debt sustainability since they focus only on one side of the budget. While expenditure rules allow for economic stabilization, revenue rules combine the inability to stabilize debt with the no stabilization feature (*i.e.*, can also be pro-cyclical), characteristics that certainly contribute to explain the limited use of the latter (Schaechter *et al.*, 2012, pp. 7-9).

All types of rules have some kind of limitation, what explains why most governments have adopted a combination of them. According to Schaechter *et al.* (2012), DR and BBR are the most frequently used, often in combination, particularly in the case of monetary unions with supranational rules (exception is made for the Eastern Caribbean Currency Union, which only has a BBR). In the case of national rules, expenditure rules are also commonly used in advanced economies, often "[…] combined with balance budget or debt rules to provide a greater anchor for debt sustainability" (Schaechter *et al.*, 2012, p. 15). Debt rules are the predominant national rules for low-income countries, possibly due to institutional difficulties in implementing more demanding rules like expenditure rules.

Table 4.1: Moving to multiple national fiscal rules (selected EU countries) – types of national fiscal rules in place in 2013

	BBR	DR	ER	RR
France*	✓	✓	✓	✓
Germany	✓			
Greece	✓			
Ireland	✓	✓	✓	
Italy**	✓		✓	
Portugal	✓	✓		
Spain	✓	✓	✓	
United	✓	✓		

Source: European Commission: Fiscal Governance in the EU Member States (database on national numerical fiscal rules).

* The debt rule is applied for social security accounts.

** At the regional/local government level.

For the European Union (EU), Nerlich and Reuter (2013) conclude that the number of countries with any kind of fiscal rule in place increased steadily from 1990 to 2012. According to the authors, many countries have more than one fiscal rule in place today – see Table 4.1 for some examples. The most common rules are expenditure rules and balanced budget rules, followed by debt rules; revenue rules are fairly rare.

This evidence follows theoretical policy prescriptions derived from optimal policy behavior. Most of the theoretical literature use simple debt feedback rules with or without accounting for smoothing instrument (*e.g.*, Marattin and Marzo, 2008, Pappa, 2012, Schmitt-Grohé and Uribe, 2007, and Canzoneri *et al.*, 2011, Kirsanova and Wren-Lewis, 2012, respectively). Simple debt feedback rules relate the deviation of actual government spending (or tax rate) from its optimal level (the natural value of the variable or the value associated with the long-run target for debt) to the deviation of the debt-to-output ratio from its long-run target. If debt is above its long-run target, government spending (and/or the tax rate) is expected to fall below (rise above) its long-run level. Accordingly, prescription to enhance welfare is, in general, for slow adjustments towards the target level of debt.[5] Kirsanova and Wren-Lewis (2012) focus

[5] In simple rules that account for inertia in the adjustment of fiscal instruments, the degree of instrument smoothing determines the persistence of the adjustment of the debt (or deficit) to the target. In cases when the target is close to be reached, fiscal authorities are likely to become more careful with the adjustment. van Aarle *et al.* (2004) refer to the situation, in the EMU, when fiscal deficits approach the ceiling of 3 percent of GDP as a case of deficit smoothing, which can also be explained by institutional reasons.

on simple debt feedback rules in a closed economy, considering both the use of government spending and/or income taxes as instruments. They show that, if the feedback parameter on debt is small, simple rules can perform close to full-optimal policy adjustment, which lends theoretical argument for slow adjustments towards the target level of debt. Debt should act as a shock absorber and, hence, governments should smooth recurrent spending and taxes (Portes and Wren-Lewis, 2014).

Moreover, besides debt, and to reinforce the countercyclical element of simple fiscal rules, several recent studies explicitly consider that fiscal instruments should react to other macroeconomic variables like output gap, inflation, and the terms-of-trade in the case of open economies. This is the case, among others, of van Aarle *et al.* (2004), Kirsanova *et al.* (2007), Argentiero (2009), Ferrero (2009), Rossi (2009), Corsetti *et al.* (2010), Motta and Tirelli (2012), Vogel *et al.* (2013) and Kliem and Kriwoluzky (2014).[6] However, only the first four together with Vogel *et al.* (2013) consider a monetary union environment.

For example, in a two-country monetary union model, Kirsanova *et al.* (2007) assume commitment in the common monetary policy and country-specific fiscal rules. They conclude that welfare is reduced if government spending reacts only to output, ignoring inflation. Fiscal policy can play an important role by reacting to national differences in inflation, on the one hand, and to either national differences in output or changes in the terms-of-trade, one the other hand. Argentiero (2009) considers a currency union model built up of a continuum of small countries in the spirit of Galí and Monacelli (2008), such that country-specific fiscal policy has no effect on the rest of the union. It is assumed a Taylor rule for monetary policy while government consumption feedbacks on its lagged out-of-steady-state level, on the lagged out-of-steady-state real stock of public debt and on the present out-of-steady state output. The author finds that, in the presence of the monetary rule alone, the domestic inflation variance falls more than in the presence of fiscal rules alone, while the output gap smoothing is stronger in the presence of the fiscal rules alone. The welfare stabilization costs are lower when the

[6] In a closed economy environment, Kliem and Kriwoluzky (2014) show that the cyclical movements of labor and capital income tax rates are better described by a contemporaneous response to hours worked and investment, respectively, instead of output. This paper also contributes to the literature by proposing a new way of approximating optimal policy using simple linear rules, by simulating the model and estimating policy feedback rules using Bayesian methods which incorporate all potential explicative variables and then selecting the variables which influences most the tax rate's variance at the optimal location.

two rules are combined. Vogel *et al.* (2013) study a monetary union model constituted by one small open economy and a big economy (the rest of the union, approximated by a closed economy), and consider fiscal rules that imply a response from fiscal instruments (government spending, transfers and taxation) to fluctuations in the terms-of-trade and to output growth (domestic and the rest of the union's output). The advantage of output growth over theoretical output gap is the observability of the former.[7]

Alternative fiscal rules found in the literature target deficit instead of debt, *e.g.*, van Aarle *et al.* (2004) and Pappa (2012), using a two-country currency union model, Marattin and Marzo (2008), considering a highly distorted closed economy, calibrated with the Euro-area data, and Vogel *et al.* (2013), modeling a monetary union formed by one small open economy and a big economy. While Marattin and Marzo (2008) and Pappa (2012) explicitly consider that fiscal instruments react to deficit (or to deficit and some other macroeconomic variables), Vogel *et al.* (2013) assume that government adjusts taxes or transfers payments to stabilize government debt and the budget deficit at their target levels – a budgetary closure rule is introduced. van Aarle *et al.* (2004) assume that fiscal policy is conducted by a rule expressing the net government spending as the sum of the cyclical fiscal stance and the structural fiscal balance. All these works also include instrument smoothing.

Portes and Wren-Lewis (2014) refer two major problems of debt feedback rules: (i) the lack of an incentive for a government to behave in a benevolent manner and fulfill its plans and (ii) the fact that the debt target is unlikely to be achieved within the lifetime of a government. If the policy rule had a fixed operational target (*i.e.*, if government is required to match targets for debt or deficits in particular years) the implementation incentive would be larger, but this type of strict targets would be largely sub-optimal once the economy is hit by shocks. Rules targeting deficits instead of debt are, according to the authors, more robust to shocks and make it easier to achieve the debt target over the lifetime of the government. However, reducing the deficit below target to correct past deficit overruns works very rapidly on debt stabilization but entails

[7] In a closed economy context, Motta and Tirelli (2012) consider alternative specifications for the fiscal rules. In one of those alternatives fiscal policy controls nominal income growth, which, according to the authors, is consistent with empirical evidence that suggests that revenues are more sensitive to output growth than to the output gap and that the real progression of tax rates may be affected by inflation.

a large deviation from optimal policy, so it should only be considered if governments are highly non-benevolent. Nonetheless, since there is not a "natural" target for the budget deficit, the appropriate deficit target will depend on the level of debt and on any initial excess of debt, being a matter of arithmetic to move from one target to the other.

Bi and Kumhof (2011) assume a fiscal policy rule where the government of a closed economy targets the interest-inclusive fiscal surplus to GDP ratio. The interest-inclusive fiscal surplus (in percentage of GDP) deviation from its target is function of a tax revenue (in percentage of GDP) gap and a debt (in percentage of GDP) gap. Structural surplus rules[8] are a special case of this class of rules, when the coefficient associated to the tax revenue gap is one and the coefficient associated to the debt gap is zero. The authors explore alternative structural surplus rules, mainly by varying the coefficient associated with the tax revenue gap between pure balanced budget rules (when the two coefficients are set to zero) and highly countercyclical rules. According to the authors, although superior to a balanced budget rule, the structural surplus rule does not have a business cycle stabilization or welfare objective as its prime concern. With a coefficient associated with the tax revenue gap superior to one, taxes are raised during a boom and therefore act in a more countercyclical way.

Marattin and Marzo (2008) and Pappa (2012) compare the two alternatives: targeting debt or deficit. Using a New Keynesian model of a highly distorted closed economy, calibrated with the Euro-area data, Marattin and Marzo (2008) assume that distortionary tax rates (on consumption, labor income and capital income) react, alternatively, to total liabilities, total deficit, and a linear combination of both targets. According to the authors, targeting deficit follows the prescription of the SGP in the EMU, while the alternative is the most recommended policy standpoint, in line with the pact reform and is the standard policy rule defining "active" or "passive" fiscal stance *cf.* Leeper's (1991) categorization (Marattin and Marzo, 2008, p. 14). They find that targeting public debt is welfare-superior to other specifications provided monetary policy's response to output is not mute (*i.e.*, if monetary authority takes care of short-run economic stabilization).[9] Otherwise, fiscal policy should react to the budgetary deficit. The

[8] Kumhof and Laxton (2009) also consider that fiscal policy follows a structural surplus rule (Chile's structural surplus rule), in a model representing a world constituted by two open economies.

[9] Hughes Hallett and Jensen (2011) consider an intertemporal assignment where fiscal targets are long-term objectives and monetary authority is involved with short-term stabilization. They argue for

combination of both targets ("mixed" feedback rule) is largely suboptimal, though it provides the best smoothing response after a shock. Pappa (2012) considers strict deficit rules (alternative to strict debt rules) that ensure constant deficits in her analysis. In a two-region monetary union, her results reveal that strict debt rules perform better in terms of overall welfare than strict deficit rules when variable taxation is used as fiscal policy instrument, but produce higher output volatility. Regional fiscal policy flexibility is found to be crucial for regional stabilization, in accordance with Ferrero (2009). Another important conclusion of this study is that regional fiscal policy should focus on regional output stabilization.

Marattin and Marzo (2008) conclude that a SGP-like rule seems highly suboptimal and Pappa (2012) results justify the SGP reform since more flexibility in fiscal rules "[…] should result in welfare gains and macroeconomic stability as long as fiscal authorities engage in domestic stabilization policies" (Pappa, 2012, p. 17). Still, the SGP was mainly constraining global budgetary deficits. Moreover, many authors considered that SGP enforcement mechanisms were too weak and that stricter rules were necessary. The revision of the SGP proposed by the European Commission on 29 September 2010 introduced a new set of rules for economic and fiscal surveillance, strongly reinforcing the corrective part of the SGP (see, *e.g.*, European Commission, 2011). The new legislative package, called the "Fiscal Compact", which entered into force on 1 January 2013, reinforces the constraint on structural deficits. Besides insisting on the reduction of public debt, the "Fiscal compact" introduces a new "golden rule" – a structural deficit below 0.5% of GDP (Menguy, 2014). With the new "Fiscal Compact", fiscal rules in terms of global budgetary deficit, of structural budgetary deficit, or of public debt, seem to be mixed in the framework of the EMU (Menguy, 2014). This revived the debate about the most appropriate fiscal rules in a monetary union. Using a dynamic New-Keynesian model, in the spirit of Galí and Monacelli (2008), Menguy (2014) tries to define which of these types of rules are the most efficient for the short-term economic stabilization, assuming that all have the same long-term objective: the inter-temporal sustainability of fiscal policies. The author shows that a goal in terms of public debt is the most adequate in order to decrease the level of government indebtedness, but it could increase the recessionary risks for the

public sector debt targets and provide a mechanism for identifying a stable region within which the debt targeting regime should operate.

most indebted countries. Fiscal rules that stablish their goals in terms of global budgetary deficit or public debt are the most appropriate to stabilize fiscal variables and to limit budgetary activism, but in order to stabilize economic activity in case of asymmetric shocks the most adequate fiscal rules are those that define a goal in terms of structural budgetary deficit. Creel *et al.* (2012) simulate a small-scale macroeconomic model and compare the working of the "Fiscal Compact" – a balanced budget rule (a structural deficit at 0.5% of GDP) and the debt reduction rule – with the Maastricht 3% deficit limit (*status quo*), and an public investment rule (assuming that, over the cycle, governments contract debt only to finance public investment). They find that all rules guarantee long run sustainability of public finances, but the investment rule is the one that robustly displays the lowest output loss, followed by the status quo, while the "Fiscal Compact" rule appear to be the most recessionary and deflationary.

Finally, reflecting institutional delays in fiscal decision making, some of the fiscal rules from the literature assume a lag before fiscal instruments can react to economic variables. For instance, Kirsanova *et al.* (2007), Corsetti *et al.* (2010) and Vogel *et al.* (2013) consider a one-period lag.

4.2 Definition and Simulation Procedure for Optimal Simple Rules (OSR)

4.2.1 Definition of Simple Rules in the Model

We use the same model as in the analysis presented in previous chapter but, instead of fully optimal policies, we take linear feedback rules for the fiscal instruments of each country, as well as for the common nominal interest rate. Feedback parameters on selected variables are optimized such as to maximize the union-wide welfare function (cooperative scenario), yielding OSRs.

As the common central bank cannot influence the cross-sectional variance of outcomes across countries, monetary policy stabilizes only union-wide aggregates. We assume that the nominal interest rate is set according to a contemporaneous Taylor-based rule of the form:

$$\tilde{r}_t^* = \rho_r \tilde{r}_{t-1}^* + (1-\rho_r)\left[\alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^* + \alpha_b \hat{b}_{t-1}^*\right],$$

where once more variables denoted by the symbol "~", the union-wide nominal interest rate (r^*),) and the union-wide output (y^*), are defined in gaps and the target for the union-wide inflation is assumed to be zero (hence, π^* represents the union-wide effective inflation).

We extend the "traditional" Taylor rule by considering that the monetary authority may also be sensitive to the union's average debt level. Besides the effects of monetary policy on the level of government indebtedness, we assume that in maximizing social welfare the monetary authority may take into account the effect that high public debt levels will have on the effectiveness of fiscal instruments (in particular, on distortionary taxes) and, consequently, on inflation and output. Hence, we also consider the possibility of the interest rate gap in period *t* reacts to the union's average debt level deviation from its steady-state level at the end of period *t*-1 ($\hat{b}_{t-1}^*$).[10]

Parameters α_π and α_y are the reaction coefficients on inflation and output deviations from their targets, respectively. ρ_r represents the preference of the central bank for smoothing the interest rate path. In some studies this parameter is calibrated (*e.g.*, Vogel *et al.*, 2013, assume ρ_r=0.75), while in others it is optimally determined. Reasons for the interest rate smoothing include the fear of disrupting capital markets, the loss of credibility from sudden large policy changes and the need for consensus to support policy changes.

As to fiscal policy, we consider that each country's *j* government conducts fiscal policy through government spending, *g*, and revenue tax rate manipulation, τ, according to the following forms:

$$\tilde{g}_t^j = \rho_g \tilde{g}_{t-1}^j + (1-\rho_g)\left[\theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j\right],$$

and

$$\tilde{\tau}_t^j = \rho_\tau \tilde{\tau}_{t-1}^j + (1-\rho_\tau)\left[\delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j\right].$$

[10] Recall that $d_{g,t}^j$ represents the real value of country *j*'s debt (expressed in consumer prices) at maturity in *per capita* terms, and $\hat{b}_t^j = \log\widehat{\left(d_{g,t}^j\right)} \times \left(\frac{d_g^j}{Y^j}\right)$ denotes the log deviation of $d_{g,t}^j$ from its steady state value multiplied by the steady-state debt ratio $\left(\frac{d_g^j}{Y^j}\right)$, i.e., the absolute change in debt.

We assume that government spending gap and the revenue tax rate gap in country j can potentially react to both home and union-wide inflation and output gaps and to its own government debt. In particular, and since the monetary authority tries to stabilize union-wide inflation and output, we restrict fiscal policy to react only to differences relative to union's average values. These restrictions are important because they prevent fiscal authorities from "fighting" the common central bank in dealing with union-wide shocks and avoid some political economy concerns about fiscal stabilization (Kirsanova *et al.*, 2007, p. 1782). We also investigate restricted versions of the above-mentioned fiscal rules.

Because of institutional delays in fiscal decision making, we assume a one-period lag (one quarter) before spending and taxes react, as, *e.g.*, Kirsanova *et al.* (2007), Corsetti *et al.* (2010) and Vogel *et al.* (2013). In the context of a small open economy, Leith and Wren-Lewis (2006) show that longer lags reduce the effectiveness of fiscal policy, but there are still welfare gains from fiscal stabilization, in particular if shocks are persistent as is the case of technology shocks under our baseline calibration.

Similarly to monetary policy rule, we consider the possibility of fiscal instrument smoothing: ρ_g and ρ_τ represent persistence of government spending and tax rate, respectively. Explanations for fiscal instrument smoothing include political difficulty of changing past spending programs or implementing drastic tax reforms. Nonetheless, we start our analysis by considering no inertia in the policy instruments adjustment: $\rho_r = \rho_g = \rho_\tau = 0$. The main reason is that variables are defined in gaps. Smoothing makes more sense when variables are defined in levels. For instance, suppose that the efficient level of public consumption rises after a shock. Since in period t=0 public consumption gap is zero, in period t=1 the smoothing of public consumption gap (for $\rho_g > 0$) would imply a greater adjustment in the effective public consumption (relative to the situation where $\rho_g = 0$) in order to approach its efficient level.[11]

Relative to the design of fiscal rules, perhaps the closest paper to ours is Kirsanova *et al.* (2007), in a two-country monetary union context. They postulate that government spending in each country can potentially react, with a one-period lag, to home and overseas inflations and outputs, the terms-of-trade, and its own debt. Differently from

[11] While public consumption gap is part of the welfare function and therefore smoothness ensures lower welfare costs, there are no microfoundations in the model to assume interest rate smoothing or tax rate smoothing.

them, we consider two fiscal instruments and derive OSR for all policy instruments, including the nominal interest rate.

4.2.2 Methodology for Simulation Procedure of OSR

Policy rules are derived through assuming a cooperative scenario where all agents seek to maximize the union-wide social welfare. Rule parameters are optimized by minimizing the union-wide welfare costs resulting from asymmetric shocks[12], both technology and cost-push shocks which are assumed to be independent. It is common in the literature to assume the same standard deviation for both shocks. We follow Chadha and Nolan (2007) and set standard deviation for technology and cost-push shocks at 1%. The relative impact of these two types of shocks is crucially determined by the assumption concerning persistency. Cost-push shocks are less persistent and cause smaller policy trade-offs than technology shocks, especially when a supply-side instrument, such as the revenue tax rate, can be used for stabilization purposes.[13] Under our calibration, cost-push shocks are modelled as non-persistent, while technology shocks are persistent with autocorrelation set at $\rho_a = 0.85$.[14]

Table 4.2: Grid for initialization values of the coefficients of reaction of monetary and fiscal rules

	Feedback on y	*Feedback on π*	*Feedback on b*
$\tilde{r}_t^*$	{0.25; 0.5; 0.75}	{1; 1.5; 2}	{0}
$\tilde{g}_t$	{-0.5; 0; 0.5}	{-0.5; 0; 0.5}	{-0.2; -0.1; -0.01}
$\tilde{\tau}_t$	{-0.5; 0; 0.5}	{-0.5; 0; 0.5}	{0.01; 0.1; 0.2}

[12] By their nature, symmetric shocks produce far less stabilization costs than asymmetric ones. Thus the latter are the meaningful for welfare analysis.

[13] When the tax rate is available for stabilization purposes, cost-push shocks cause stabilization costs only because of the budgetary constraints. Apart from different budgetary consequences, the process by which a cost-push shock affects the economy is the same as that of a change in the tax rate.

[14] If we assume the same persistency for cost-push shocks as that of technology shocks (*i.e.*, $\rho_\mu = 0.85$), there is a substantial increase in the welfare effects of cost-push shocks but yet still small when compared with the effects of technology shocks. For instance, considering a debt-to-output ratio of 60%, if we assume no persistency in cost-push shocks, the stabilization costs (union's *per capita* welfare losses) of asymmetric cost-push shocks represent only 0.02% and 0.11% of the stabilization costs of asymmetric technology shocks, under commitment and discretionary full-optimal policies, respectively; if we assume persistency, these values rise to 1.28% and 7.68%, respectively.

Since numerical methods do not ensure the convergence to a global extreme, being sensible to the initializing parameter values, we have to set a grid (see Table 4.2) for different initial values for the computation. For the interest rate feedback on debt we choose zero as initialization value, since we are deliberately assessing whether monetary policy should respond or not to the union's average level of public debt.

We have adapted Söderlind (1999) and Giordani and Söderlind (2004) matlab codes to perform simple rules optimization. However, given that, in the first step, we simply need to obtain the initialization values for the rules optimization process and that the loops involved are very time-consuming, we decided to use the optimal simple rule numerical methods of the Dynare software (version 4.4.3) to perform it. Although Dynare software is more user-friendly and simpler to test alternative rules design, it assumes, by default, a planner-discount factor of 1, which makes it impossible to assess the full implications of higher public debt levels. Additionally, we wish to compare optimal simple rules performance with full-optimal policies from chapter 3, for which we assume a planner-discount factor equal to the private discount factor β=0.99. Thus Giordani and Söderlind's (2004) approach is far more consistent.

Moreover, given the large number of parameters to be optimized, the convergence to a global extreme is hardly achievable. Analysis of results shows that there are different rules specifications which practically attain the same welfare results, although some parameter values are consistent across these alternative specifications. Hence, we check for robust solutions for some of the parameters. In particular, we found that the responses of the two fiscal instruments to national output gap deviations from union's and especially to the public debt level are very consistent. On the other hand, their responses to inflation depend on the initialization values and have a minor impact on the objective function value. Thus, for the interest rate rule, we parsimoniously decided to choose a parametrization closer to the "traditional" Taylor rule in relation to the output and inflation feedbacks as alternative specifications provide just about the same results, both in terms of stabilization costs as well as of the impact on main macroeconomic variables.

In short, we proceed as follows:

Step 1: set a grid (Table 4.2) for different initial values and perform OSR computations using Dynare;

Step 2: found robust results across different optimizations and set such parameter values as initialization values. In particular, we set $\theta_y^j = -1$, $\theta_b^j = -0.005$, $\delta_y^j = -1.5$ and $\delta_b^j = 0.015$. As fiscal instruments feedbacks on national inflation deviations from union's depend on the initialization values and have a minor impact on the objective function value, we consider as initialization values $\theta_\pi^j = \delta_\pi^j = 0$. Finally, as discussed above, we decided to choose a parametrization closer to the "traditional" Taylor rule for initialization of the parameters in the interest rate. Hence, we consider the following initialization values for all our optimizations (Table 4.3):

Table 4.3: Initialization values of the coefficients of reaction of monetary and fiscal rules

	Feedback on y	Feedback on π	Feedback on b
$\tilde{r}_t^*$	0.5	1.5	0
$\tilde{g}_t$	-1	0	-0.005
$\tilde{\tau}_t$	-1.5	0	0.015

Step 3: optimization process using matlab codes adapted from Giordani and Söderlind (2004).

Given the distinctive structure of our heterogeneous country-size currency union, formed by a big country and a continuum of small countries (each one with zero dimension), it is not possible to represent each one of the small economies in our computational model. Hence, we consider three model economies in our optimization setup, just as we did for deriving full-optimal policies[15]: a big economy (block B) and two small economies (fictitious country, *i*, and small country *ii*). Country *ii* is very small within block S (*ii*_dim=0.00001) and, hence, when he is hit by an asymmetric (domestic) shock there are only domestic consequences, as expected. As asymmetric shocks at small countries produce no external consequences, they do not affect the behavior of block S, and so the behavior of country *ii* cannot serve as a proxy for the block S's behavior. For that reason, we consider another small country *i*, whose behavior is similar to the other small country with the exception that he is not subject to

[15] See Appendix A.3 (*Modeling the behavior of block S*).

asymmetric shocks. Moreover, in order to mimic the behavior of block S we consider that within block S the size of country *i* is almost 1 ((1-*ii*_dim)=0.99999). This approach makes it possible to obtain optimal simple policy rules for a small country and allows us to define all union-wide variables as an average of block B and block S, which in turn represents the average of small economies and is defined as a simple average of only two small economies (*i* and *ii*).

We proceed as follows:

- first, we implement the OSR optimization procedure described above (steps 1 to 3) considering only asymmetric shocks at the big country, *i.e.*, shocks that produce effects in all countries. At this stage we obtain the monetary policy rule and the fiscal policy rules for the big country and for the block S;
- next, we set the monetary policy rule, the big country's fiscal policy rules and the block S' fiscal policy rules and perform a new OSR optimization through which only the parameters for the fiscal policy rules of the small country *ii* are optimized (again following steps 1 to 3, above). In this second optimization we consider not only asymmetric shocks hitting the big country but also asymmetric shocks hitting the small economy *ii*. Notice that asymmetric shocks at a small country have no external effects and, hence, only influence specific fiscal policy rules for the small country, *ii*. However, since each individual small country is hit by this kind of shock at some point in time, we assume that all small countries share the same fiscal policy rules.

4.3 Analysis of Results

In the following subsections, we consider two different currency union structures. First, we address the case of a currency union made up of two identical countries, which will serve as a benchmark, and then we focus on our heterogeneous country-size currency union model.

4.3.1 Optimal Simple Fiscal and Monetary Rules in a Two-Country Monetary Union – Benchmark Analysis

We consider two identical country blocks (H and F) that are integrated into a currency union. We assume that both countries are defined in a similar way as the big country in our specific heterogeneous country-size currency union model. As the two countries are identical, the optimized fiscal rules are common to both countries.

Expenditure- and Revenue-Based Fiscal Rules

As referred above, we start our analysis by considering no inertia in the policy instruments adjustment: $\rho_r = \rho_g = \rho_\tau = 0$.

Table 4.4 presents the baseline policy rules' optimal feedback coefficients. Our baseline set of rules assumes that the interest rate gap reacts to the union's average public debt level and that both fiscal instruments (public consumption and the tax rate) react to national public debt, and to inflation and to output gap differences from union's values. By default, and for the moment, we assume a high-debt scenario (debt-to-output ratio of 60%).[16]

Table 4.4: Policy rules' optimal feedback coefficients: baseline rules. Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$\hat{b}_{t-1}^*$	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$
Baseline	$\tilde{r}_t^*$	0.6301	0.9109	0.0016	—	—	—
	$\tilde{g}_t^j$	—	—	—	-0.9272	-0.0032	-0.0062
	$\tilde{\tau}_t^j$	—	—	—	-1.6525	0.0015	0.0082

Union's *per capita* welfare loss:
Optimized simple rules: 3.1648
Full-optimal rules under commitment: 2.8705
Full-optimal rules under discretion: 3.4876

Table 4.4 also reports the union's *per capita* welfare losses for the baseline set of simple rules and for the full-optimal rules, both under commitment and discretionary

[16] Notice that we cannot distinguish between debts of different maturities. Hence, although many industrialized countries present effective public debt-to-output levels above 60%, this seems a reasonable value for the model (see, *e.g.*, Kirsanova and Wren-Lewis, 2012).

technologies (*cf.* chapter 3). Results reveal that though the performance of optimal simple rules is worse than full-optimal policies under commitment they perform better than full-optimal policies under discretion.

We additionally tested for the alternative hypothesis where fiscal instruments react to the terms-of-trade instead of inflation differences, but the results were basically the same in terms of the union's *per capita* welfare loss. In both cases, the reaction of fiscal instruments to inflation gaps (or the terms-of-trade) does not seem to be significant.

Table 4.5: Policy rules' optimal feedback coefficients - simpler monetary and fiscal rules. Two-country union model (debt-to-output ratio = 60%; n=0.5)

			$\tilde{y}_t^*$	π_t^*	$\hat{b}_{t-1}^*$	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
Baseline	$\tilde{r}_t = f(\tilde{y}_t^*, \pi_t^*, \hat{b}_{t-1}^*)$	$\tilde{r}_t^*$	0.6301	0.9109	0.0016	—	—	—	
		$\tilde{g}_t^j$	—	—	—	-0.9272	-0.0032	-0.0062	**3.1648**
		$\tilde{\tau}_t^j$	—	—	—	-1.6525	0.0015	0.0082	
Simpler Monetary Rule	$\tilde{r}_t = f(\tilde{y}_t^*, \pi_t^*)$	$\tilde{r}_t^*$	0.0079	4.9207	—	—	—	—	
		$\tilde{g}_t^j$	—	—	—	-0.9279	-0.0060	-0.0064	**3.1650**
		$\tilde{\tau}_t^j$	—	—	—	-1.6532	0.0133	0.0081	
Simpler Fiscal and Monetary Rules		$\tilde{r}_t^*$	0.0050	2.9367	—	—	—	—	
		$\tilde{g}_t^j$	—	—	—	-0.9316	—	-0.0058	**3.1659**
		$\tilde{\tau}_t^j$	—	—	—	-1.6513	—	0.0083	

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^* + \alpha_b \hat{b}_{t-1}^*, \;\; \rho_r = 0$

$\tilde{g}_t^j = \theta_y^j (\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_\pi^j (\pi_{t-1}^j - \pi_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j, \;\; \rho_g = 0$

$\tilde{\tau}_t^j = \delta_y^j (\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_\pi^j (\pi_{t-1}^j - \pi_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j, \;\; \rho_\tau = 0$

Table 4.5 presents the optimal feedback coefficients for two alternative sets of simple rules: the baseline and an alternative hypothesis where the interest rate does not react to public debt. Though the welfare consequences are practically the same, by removing public debt level the interest rate gap reaction to the union-wide inflation significantly increases while the opposite occurs in relation to the union-wide output gap. We conjecture that a larger reaction to inflation relative to output conveys an objective for debt stabilization. Moreover, while the optimal countercyclical (reaction to output gap differences) and debt feedback behavior of fiscal instruments is practically the same, the feedback on inflation differences increases. However, the higher feedback on inflation differences still does not translates into significant welfare losses.

Since the performance of the baseline rules is essentially the same as the alternative hypothesis where the interest rate rule does not react to debt, there are still doubts about the significance of the feedback on inflation differences. Hence, also in Table 4.5, we display a simpler version where the interest rate responds both to the union's output and inflation while fiscal instruments react only to output gap differences and to public debt level.[17] As expected, this alternative performs worse.[18]

Next, we improve the baseline rules by introducing inertia in all policy instruments.[19] The idea is to further approximate the performance of simple rules to those under commitment and where policy instruments present a smoothed behavior. Table 4.6 presents the optimal feedback coefficients for this alternative compared with baseline rules.

[17] We tested for additional simpler rules where i) debt or ii) output gap differences were removed from fiscal rules. Under i), the model becomes unstable, although we tried for different initialization values; under ii), the model performs substantially worse in terms of welfare (union's *per capita* welfare loss goes up to 4.75).

[18] Kirsanova *et al.* (2007), in a two-country union model, conclude that welfare costs are reduced if national fiscal policy reacts only to output, ignoring inflation. However, their analysis considers non-persistent cost-push and preference/technology shocks, a single fiscal instrument – government spending, and full optimal monetary policy.

[19] We considered the same starting values as in the baseline rules and, again proceeding with Step 1, tested robustness for several initializing values for the "inertia" parameters. We then considered most robust values as starting points: $\rho_r = 0; \rho_g = 0.1; \rho_\tau = 0.2$.

Table 4.6: Policy rules' optimal feedback coefficients - comparing baseline rules with the ones including instrument-inertia.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		Last period value	$\tilde{y}_t^*$	π_t^*	$\hat{b}_{t-1}^*$	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$
Baseline	$\tilde{r}_t^*$	—	0.6301	0.9109	0.0016	—	—	—
	$\tilde{g}_t^j$	—	—	—	—	-0.9272	-0.0032	-0.0062
	$\tilde{\tau}_t^j$	—	—	—	—	-1.6525	0.0015	0.0082
Rules with Inertia	$\tilde{r}_t^*$	0.0036*	0.8352	2.1861	0.0021	—	—	—
	$\tilde{g}_t^j$	0.1109	—	—	—	-0.8132	-0.0101	-0.0049
	$\tilde{\tau}_t^j$	0.1808	—	—	—	-1.3361	0.0016	0.0068

Union's *per capita* welfare loss: *Commitment / discretion*: 2.8705 / 3.4876
Baseline: 3.1648
Rules with Inertia: 3.1558

* The results show that the parameter capturing inertia in the interest rate gap is practically zero. Depending on the starting values we use (for the "inertia parameters"), we might even obtain a negative α_r, although close to zero.

As expected, more complex rules (with inertia) present a better performance. Nonetheless, the welfare differences between alternative rules (*cf.* Tables 4.5 and 4.6) are not significant[20] and, hence, from hereafter we will focus on the most parsimonious ones (*Simpler Fiscal and Monetary Rules* as in Table 4.5).[21]

In the simpler set of rules, the optimized monetary rule fulfils the Taylor principle ($\alpha_\pi > 1$), while the reaction of the interest rate gap to the union's output gap is very small.[22] As regards fiscal rules, both fiscal instruments present a consistent feedback on

[20] In a closed economy model, Schmitt-Grohé and Uribe (2007) also find that the welfare gains from interest-rate smoothing are negligible.

[21] Notice, however, that our model fails to capture the welfare costs from having a higher degree of volatility in the policy instruments. For instance, a more comprehensive model would capture that higher tax rate volatility has negative implications on employment. The small values of ρ_r may also be a consequence of the feedback variables being defined in terms of gaps or from changes in the union's output gap and inflation being very small.

[22] From the literature on Taylor rules (see, for example, Woodford, 2003), a feedback coefficient on contemporaneous inflation greater than unity and a feedback close to zero on contemporaneous output have desirable stabilizing properties. Schmitt-Grohé and Uribe (2007) show that interest-rate rules not responding to output (or with a very small feedback on output) are critical from a welfare point of view.

both output gap differences and the level of government indebtedness. As expected, public consumption gap (tax rate gap) reacts negatively (positively) to the level of public debt. Also, both fiscal instruments present small adjustment parameters on debt (in line with commitment solutions) and the adjustment towards the target level of debt is slow, acting as shock absorbers.[23] In a stationary equilibrium near the steady state, deviations of real public debt from its non-stochastic steady-state grow at a rate less than the real interest rate: we simulate our model under OSR for a very large number of periods (T=1000) and confirm that the present discounted value of government liabilities (in deviations from the steady-state) converges to zero (as in Schmitt-Grohé and Uribe, 2007, p. 1707). As to the feedback sign on output differences, government spending responds negatively to positive output gap differences, as expected from the countercyclical behavior of fiscal rules, but the response of the tax rate instrument is not as intuitive, although it mimics the reaction obtained in a full commitment setup (*c.f.* Figure 4.1).

First, notice that asymmetric technology shocks (dominant in determining welfare effects) produce opposite effects for the domestic and for the external economy and, hence, the impact on union-wide variables is mitigated. Thus there is a substantial stabilization effort on fiscal authorities. Second, an asymmetric technology shock generates output gap and inflation co-movement. Thus, the negative sign of the tax revenue rate feedback on output gap differences may be related to an over-negative response of the tax instrument to inflation. To test this hypothesis, we consider an alternative set of simpler rules where both fiscal policy instruments react only to inflation differences. The results (available upon request) confirm our hypothesis, as public consumption and the tax rate instrument present negative feedbacks on national inflation differences, both significantly different from zero (although these alternative rules perform worse than simpler rules from Table 4.5). Finally, notice that since the revenue tax rate is available as a fiscal instrument (which is less costly in comparison to government spending), there is an initial attempt to offset inflationary consequences of domestic (external) asymmetric technology shocks by cutting (rising) taxes.[24]

[23] The reason why very slow adjustment of debt may be optimal is extensively explored in Kirsanova and Wren-Lewis (2012).

[24] Under our calibration the coefficient attached to inflation in the social welfare is the largest. Since a non-optimal provision of public goods is costly given its direct effect on welfare, tax adjustments are

Figure 4.1: Responses to a 1% negative technology shock at country H: parsimonious optimal simple rules *versus* full-optimal rules (commitment/discretion) Two-country union model (debt-to-output ratio = 60%; n=0.5)

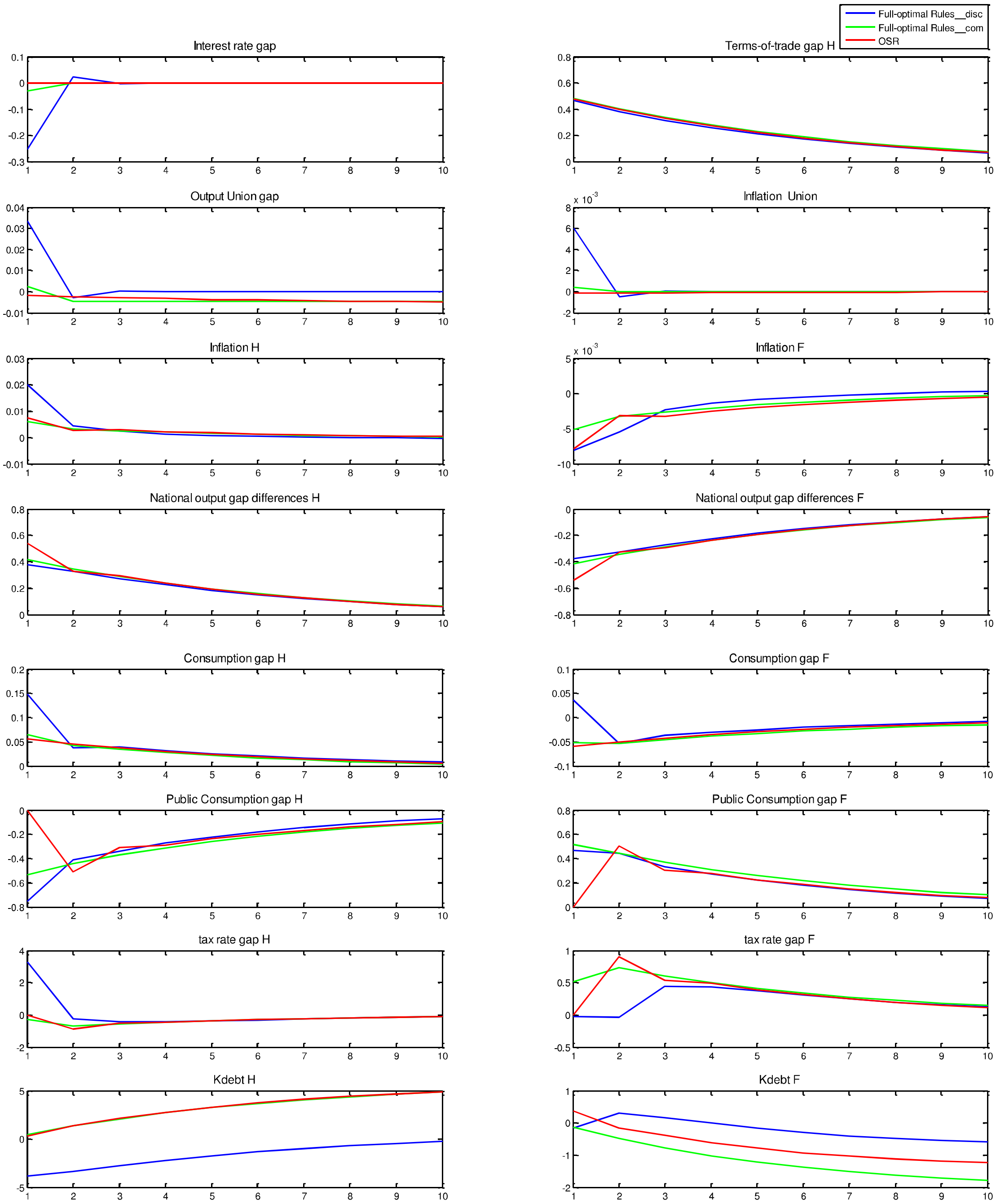


Figure 4.1 illustrates the impact of a one-percent domestic negative technology shock at country H, comparing the most parsimonious set of rules ("*Simpler Fiscal and*

preferable. Other arguments in favor of tax adjustments, although not captured in the model, include the smaller flexibility, and possibly some exogenous components, of government spending.

Monetary Rules") from Table 4.5 with full-optimal rules. Moreover, Figure C1, in Appendix C, shows the impulse responses of the main variables under OSR, together with the respective efficient values and implied gaps (Gap_OSR = Effective_OSR – Efficient), all expressed in deviations from the steady-state. Recall that in all figures "Kdebt j" represents variable $\hat{b}_t^j$.

In face of a domestic shock, Figure C1 shows that the efficient levels of output, private and public consumption fall on impact at country H, as a consequence of the increase of the work effort at a given output. The fall in productivity inherent to the negative technology shock determines an increase in real marginal costs and, consequently domestic prices increase. The efficient terms-of-trade also fall since the domestically-produced goods become relatively more expensive comparative to external goods (positive terms-of-trade-gap). This makes efficient output to increase at F, if goods are substitutes.[25] Thus, the work effort increases in country F leading to a reduction in the efficient levels of private and public consumption. At home, aggregate demand and private consumption decrease due to the increase in domestic prices. Nonetheless, prices and aggregate demand change less than under fully flexible prices, due to nominal rigidities. Hence, the aggregate output decreases by less than in the absence of price rigidities, which explains the positive output gap. The efficient interest rate increases, as required to ensure a lower efficient level of private consumption in both countries.

Following the domestic shock, country H's efficient public consumption falls less than the efficient output (when goods are substitutes) causing a primary budget deficit, which increases with the debt level through output's increasing impact on tax revenue. Conversely, both the increase in the efficient output and the fall in public consumption have a positive impact on F's primary budget, also increasing with debt. The increase in the efficient interest rate raises debt service costs and, thus further enlarges the primary deficit effects at country H, while it mitigates the surplus effects on country F's debt. Since budgetary consequences of the shock are higher for H, the union-wide debt increases on impact.

[25] We will study the implications of goods being complements in a robustness section, below.

Inspection of Figure 4.1 shows that the impulse responses from OSR are closer to commitment. Notice also that the one-period lag in the response of fiscal instruments is determinant for the path of some variables.

Under full optimal policies, country H's reacts to the shock by reducing public consumption gap; tax rate gap increases under discretion (to promote debt stabilization at the expenses of higher inflation) whereas under commitment there is a cut in the tax rate to control for inflation through increased incentives to work. The response of the tax rate gap under the OSR scenario is similar to that under commitment, though tax adjustment occurs only in the second period and with a higher cut relative to commitment. The adjustment of government spending is as expected and also closer to the commitment solution. As in the cases of taxes, adjustment takes place only in the second period through a larger fall relative to commitment.

Clearly, there is a smother adjustment of fiscal instruments under OSR in comparison to the discretionary technology. The short-run trade-off is improved if a persistent deviation of the goal variables from their efficient levels. For this reason, the OSR solution (as the commitment solution) requires incurring in costs from permanent effects on debt in order to achieve a better short-run stabilization. Notice also that the delay in the response of fiscal instruments (mainly the cut in government spending) results in a larger output gap in the first period. Since the cuts in government spending and the tax rate are delayed, in the first period the control of inflation is less effective than under commitment (although the reverse occurs in the second period).

Given the positive impact on F's primary budget, full optimal policies require a positive public consumption gap on F; a negative tax rate gap is implied under discretion in the first period (promoting debt stabilization while accentuating deflation), whereas an increased in the tax gap is prescribed under commitment. Once more, the OSR solution is in line with the commitment solution, controlling for deflation. The delay in the reaction of fiscal instruments results in a larger negative output gap in the first period. Since the increase in government spending and the tax rate is delayed, in the first period the control of deflation is less effective than under commitment (although the reverse occurs in the second period). Moreover, due to debt service costs the stock of public debt increases in the first-period.

Similarly to the commitment solution, we have an overall short-run stabilization of inflation, since policymakers are able to steer the expectations of the private sector. Hence, the nominal interest rate is set below its efficient level, resulting into a negative interest rate gap, with positive effects on the union-wide public debt.[26] There is a gradual response to the shock with the interest rate converging slowly to its steady-state value. This is clearer under the commitment solution; under OSR the interest rate gap is negative but close to zero. Consistently with the Taylor rule, changes in the union's inflation and the nominal interest rate are very small; given the asymmetric nature of the shock, the burden of stabilization largely relies on fiscal policy. From the Euler equation, the union's aggregate private consumption is determined by the real interest rate gap but also by expectations about future private consumption. When the initial debt is zero, the union aggregate private consumption gap is zero and the difference between the two countries' private consumption gaps is explained by the terms-of-trade gap. Thus, monetary policy can set the interest rate at the efficient level without budgetary consequences. However, as public debt level increases the impact of a higher efficient interest rate on debt service costs and, consequently, on governments' budgets is stronger. Given that total budgetary consequences of the shock are higher for the domestic economy, the union-wide debt increases on impact. Globally, households expect that a higher future debt will require policy measures that affect negatively future private consumption, which has a negative impact on current private consumption via consumption smoothing. In turn, in face of a negative interest rate gap current private consumption gap increases. These opposing effects result in a small volatility of the union's aggregate current private consumption.

Private consumption gaps of countries H and F can be explained by the effects in the terms-of-trade gap. In comparison to commitment, under OSR country H presents a smaller positive private consumption gap while country F presents a larger negative private consumption gap, which is explained by higher real interest rate gaps. Under discretion both countries present a positive private consumption gap, higher for H, in the first period. In this scenario, the union's aggregate private consumption gap is solely determined by the real interest rate gap, since permanent effects on private consumption tend to be eliminated. In a high-debt scenario like the one in Figure 4.1, the first-period

[26] This effect partially offsets the negative effects on debt stabilization from the cut in the tax rate in country H.

interest rate gap is negative (in contrast to a low-debt scenario, monetary policy is now "passive" focusing on debt stabilization) and, hence, the union's aggregate private consumption gap is positive. In the second period, the interest rate overshoots and the union's aggregate private consumption gap becomes negative. Thereafter, the union's aggregate private consumption stabilizes at is efficient level, but the international risk sharing condition requires a negative private consumption gap for country F.

Restricted Fiscal Policy Rules

Table 4.7: Union's *per capita* welfare losses - restricted fiscal policy rules *versus* baseline rules under alternative scenarios.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

	Union's *per capita* welfare losses		
	Scenario A (parsimonious rules)	Scenario B	Commitment/Discretion
Simpler Fiscal and Monetary Rules with and without inertia	3.1659	3.1558	**2.8705/3.4876**
Hypothesis I	3.9494	3.8893	**2.8705/3.4876**
Hypothesis II	4.8709	4.3752	**2.8705/3.4876**

Scenario A:
$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^*, \;\; \rho_r = 0; \alpha_b = 0$

Hypothesis I*:
$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*), \;\; \rho_g = 0; \theta_\pi^j = 0$
$\tilde{\tau}_t^j = \delta_b^j \hat{b}_{t-1}^j, \;\; \rho_\tau = 0$

Hypothesis II**:
$\tilde{g}_t^j = \theta_b^j \hat{b}_{t-1}^j, \;\; \rho_g = 0$
$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*), \;\; \rho_\tau = 0; \delta_\pi^j = 0$

Scenario B:
$\tilde{r}_t^* = \rho_r \tilde{r}_{t-1}^* + \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^* + \alpha_b \hat{b}_{t-1}^*$

Hypothesis I:
$\tilde{g}_t^j = \rho_g \tilde{g}_{t-1}^j + \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*)$
$\tilde{\tau}_t^j = \rho_\tau \tilde{\tau}_{t-1}^j + \delta_b^j \hat{b}_{t-1}^j$

Hypothesis II**:
$\tilde{g}_t^j = \rho_g \tilde{g}_{t-1}^j + \theta_b^j \hat{b}_{t-1}^j$
$\tilde{\tau}_t^j = \rho_\tau \tilde{\tau}_{t-1}^j + \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*)$

* Considering as starting value $\delta_b^j = -0.015$ results in an unstable solution. Instead, we consider as starting value $\delta_b^j = -0.0175$.

** Considering as starting value $\theta_b^j = -0.005$ results in an unstable solution. Instead, we consider as starting value $\theta_b^j = -0.05$.

In the previous analysis we assumed that both fiscal instruments are jointly used for economic and debt stabilization purposes. Suppose, instead, that one of the fiscal instruments is devoted to economic stabilization purposes only while the other is used exclusively to stabilize debt. We consider two hypotheses: Hypothesis I assumes that the revenue tax rate responds only to debt (debt stabilization purposes only) while public consumption reacts to output gap differences from average (economic stabilization function); Hypothesis II assumes the reverse.

Table 4.7 – **Scenario A** (parsimonious rules) – shows the union's *per capita* welfare losses inherent to these two restricted hypothesis and compares with the alternative where both fiscal instruments are used for economic and debt stabilization purposes ("*Simpler Fiscal and Monetary Rules*") as in Table 4.5, above. Following previous section, we also present results for a **Scenario B** – a more complex set of rules, where we assume policy-instrument "inertia" and in which policy instruments react to all economic and debt arguments (see *"Rules with Inertia"* in Table 4.6, above).

Restricted rules perform substantially worse than rules where fiscal instruments react to both economic and debt stabilization. Moreover, the stabilization costs become higher than under discretion, even in scenario B. Another relevant aspect is that the restricted version where the tax rate is used only for debt stabilization purposes (Hypothesis I) is superior to the alternative restricted hypothesis. This is in line with Kirsanova and Wren-Lewis (2012) findings that show that fiscal feedback on debt using taxes rather than government spending is preferable.

Focusing on the more parsimonious rules, Table 4.8 depicts the optimal feedback coefficients. As expected, when restricted to react only to debt, the fiscal instruments' feedback on debt is larger.

Table 4.8: Policy rules' optimal feedback coefficients – simpler rules: restricted *versus* unrestricted rules.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
"Simpler Rules" $\alpha_b = \theta_\pi^j = \delta_\pi^j = 0$	$\tilde{r}_t^*$	0.0050	2.9367	—	—	
	$\tilde{g}_t^j$	—	—	-0.9316	-0.0058	**3.1659**
	$\tilde{\tau}_t^j$	—	—	-1.6513	0.0083	
Hypothesis I	$\tilde{r}_t^*$	0.0037	6.1534	—	—	
	$\tilde{g}_t^j$	—	—	-1.3339	—	**3.9494**
	$\tilde{\tau}_t^j$	—	—	—	0.0324	
Hypothesis II	$\tilde{r}_t^*$	0.0006	0.9994	—	—	
	$\tilde{g}_t^j$	—	—	—	-0.0476	**4.8709**
	$\tilde{\tau}_t^j$	—	—	-1.0929	—	

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^*,\ \rho_r = 0; \alpha_b = 0$

Hypothesis I:
$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*),\ \rho_g = 0; \theta_\pi^j = 0$
$\tilde{\tau}_t^j = \delta_b^j \hat{b}_{t-1}^j,\ \rho_\tau = 0$

Hypothesis II:
$\tilde{g}_t^j = \theta_b^j \hat{b}_{t-1}^j,\ \rho_g = 0$
$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*),\ \rho_\tau = 0; \delta_\pi^j = 0$

Relative to simpler rules, Hypothesis I shows a stronger reaction of the tax rate instrument to debt, since now the debt feedback of government spending is zero. In turn, the tax rate is not used to offset inflationary consequences of the shocks. This can explain the larger government spending feedback on output differences (which is highly correlated with inflation differences).

Under Hypothesis II, government spending becomes more reactive to public debt while the tax rate feedback on output differences decreases as it is now used only to offset inflationary consequences of the shocks.

Inspection of impulse responses to a negative asymmetric technology shock at H country (not reported) shows that, under Hypothesis I, the union's output gap and inflation are even more negative than under simpler rules (depicted in Figure 4.1, above). However, under Hypothesis II they are positive. Thus, under Hypothesis I the interest rate gap is negative, whereas the reverse occurs under Hypothesis II to control for inflation.

In conclusion, though the tax rate instrument is used mainly to offset inflationary consequences of the shocks, it is highly welfare penalizing to restrict debt control on government spending (Hypothesis II). Due to the nature of technology shocks, and given the structure of our model, the output gap differences are positively correlated with changes in public debt and, hence, when government spending responds only to output gap differences (Hypothesis I) it ends up, simultaneously, stabilizing debt and national output/inflation differences.

The Impact of Low and High Levels of Government Debt on Policy Rules

Table 4.9: Policy rules' optimal feedback coefficients for different public debt scenarios. Two-country currency union model (debt-to-output ratios: 15%, 60%; n=0,5)

		Debt ratios	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's per capita welfare loss	Commitment/ Discretion
Simpler rules	$\tilde{r}_t^*$	15%	0.0018	2.2813	—	—	**3.1251**	2.8564/3.3056
		60%	0.0050	2.9367	—	—	**3.1659**	2.8705/3.4876
	$\tilde{g}_t^j$	15%	—	—	-0.9246	-0.0113		
		60%	—	—	-0.9316	-0.0058		
	$\tilde{\tau}_t^j$	15%	—	—	-1.7170	0.0069		
		60%	—	—	-1.6513	0.0083		

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^*$,
$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j$
$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j$

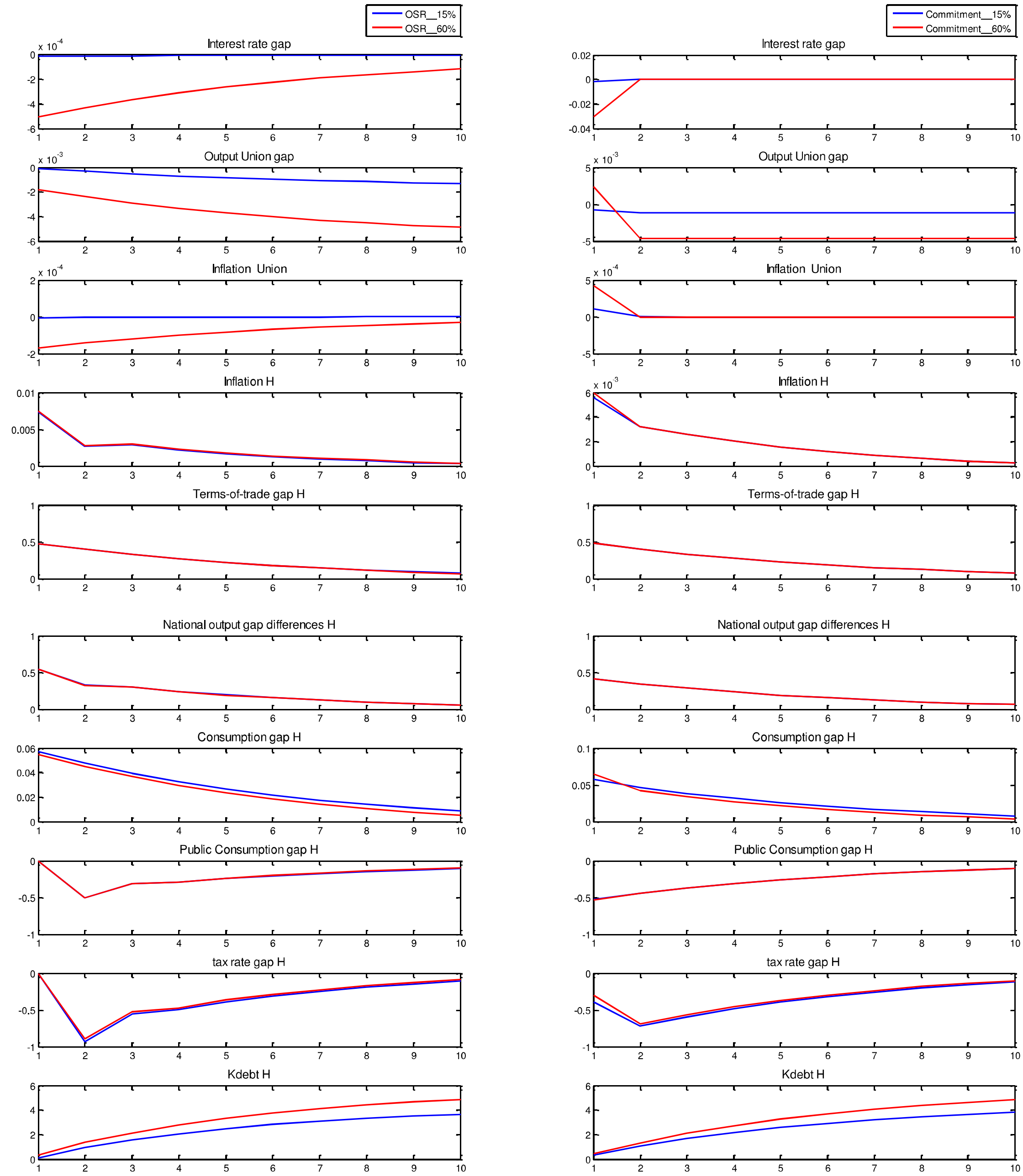
Table 4.9 presents the rules' optimal feedback coefficients for two alternative public debt scenarios (low and high debt-to-output ratios of 15% and 60%, respectively), under simpler rules. We focus on only two debt scenarios as we expect, in contrast with the debt-convergent discretionary scenario, debt to have little impacts on policy reaction coefficients under these pre-commitment, OSR, rules (Leith and Wren-Lewis, 2013).

As expected, the welfare stabilization costs are larger for higher steady-state public debt levels. Relative to the lower-debt scenario, under the high-debt scenario the government spending feedback on debt is smaller in absolute value while the reverse occurs for the tax rate. On the other hand, the tax rate revenue becomes significantly less responsive to output gap differences.

Figures 4.2a and 4.2b show the impact of a one-percent technology shock at country H and the reaction of country F to the external shock, respectively, under OSR and considering the two debt scenarios. It takes full optimal commitment as a reference.

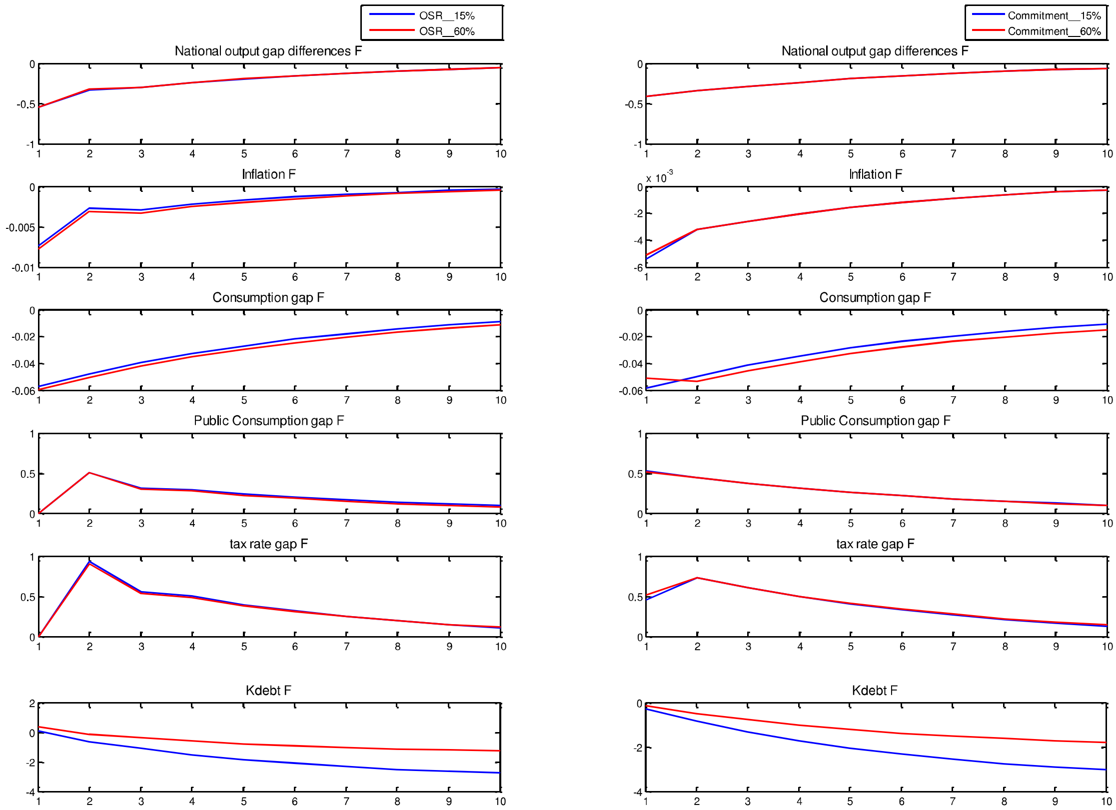
Recall that as the initial debt level increases, the impact of a higher efficient interest rate on debt service costs and, consequently, on governments' budgets, is stronger, enlarging the primary deficit effects at country H while mitigating the surplus effects at F. However, from the impulse responses we see that, similarly to the commitment solution, both monetary and fiscal policies are not substantially different under low- and high-debt scenarios. Relative to the low-debt scenario, the reduction in the interest rate is "larger" to help reducing a larger union-wide debt, but only moderate such that debt stabilization remains gradual. Though the interest rate is more responsive to the output gap and inflation under the high-debt scenario, the decrease in the interest rate gap is very small in comparison to the commitment solution because changes in the union's output gap and inflation also remain small.

Figure 4.2a: Country H's responses to a 1% negative technology shock at country H under low-debt and high-debt scenarios: OSR *versus* commitment solution. Two-country currency union model (n=0.5)



Focusing on fiscal instruments, notice that the adjustment of government spending on both countries is essentially the same under low- and high-debt scenarios, despite the smaller feedback on debt in absolute value under the high-debt scenario. As to the tax rate instrument, it is less (more) responsive to output gap differences (debt) under the high-debt scenario, indicating a higher but moderate concern with debt stabilization following a domestic shock.

Figure 4.2b: Country F's responses to a 1% negative technology shock at country H under low-debt and high-debt scenarios: OSR *versus* commitment solution. Two-country currency union model (n=0.5)



Following the asymmetric technology shock at country H, Figures 4.2a and 4.2b show that, under the high-debt scenario, the response of the tax rate instrument is slightly less active in both countries and, hence, there is a worse stabilization of inflation (more evident in country F). In fact, the tax rate adjustment in country F diverges from the commitment solution where the response of the tax rate is stronger under the high-debt scenario to face higher debt service costs.[27] This is a relevant point and cannot be disassociated from the limitation of simple rules (even when optimized) relative to full optimal policies. By definition, full-optimal policy is reactive to contemporaneous shocks and, hence, the response of fiscal instruments is not the same

[27] In the case of optimal simple rules, the tax rate gap is slightly lower under the high-debt scenario. This changes, however, from period 8 onwards.

if the economy is hit by a domestic or an external shock. Differently, optimal simple rules represent a common reaction to a variety of shocks and the welfare effects they generate. Such reaction minimizes the global welfare effects of all shocks, but not shock-specific welfare consequences.

Finally, note that private consumption gap decreases in both countries in the high-debt scenario. For higher debt levels the debt service costs increase and, thus, to service a higher debt level households expect policy measures with stronger negative effects on future private consumption, depressing current private consumption.

4.3.2 Optimal Simple Fiscal and Monetary Rules in a Heterogeneous Country-size Monetary Union

We now address the case of a currency union formed by a big country (n=0.5) and by a continuum of small countries (each with dimension=0). We focus on simpler fiscal and monetary rules (Table 4.5, above).

Expenditure- and Revenue-Based Fiscal Rules

Table 4.10 presents the policy rules' optimal feedback coefficients for the big country and for small countries, considering a low- and a high-debt scenario (debt-to-output ratios of 15% and 60%, respectively).

A first glance at Table 4.10 reveals that small countries fiscal rules are not substantially different from the big country's fiscal rules. Before a deeper analysis, we should take in consideration two aspects. First, notice that the big country is only affected by domestic shocks and, hence, simple rules are optimized to perform in that specific context. In turn, a small economy is affected both by domestic and external (big country) shocks. This is a distinctive feature that results in different fiscal rules for the small countries relative to the big country, especially because the effects of a domestic shock at a small country are, crucially, welfare dominant. Second, under a cooperative scenario, the impulse responses of a small country to a shock at the big country (B), under our specific model, are the same as the impulse responses of country F (H) to a shock at country H (F) in a two-country currency union with equivalent parametrization. This follows from the fact that variables are expressed in *per capita* terms (so the welfare consequences are the same in *per capita* terms only). Since small countries are identical (the only distinctive feature is the country-specific fiscal policy), when the big country (B) is hit by a shock, under a cooperative scenario, small countries internalize the same behavior and, hence, implement exactly the same fiscal policy (in *per capita* terms) as a big country would[28]. This may help explain why fiscal policy optimal reaction functions are not so different between small and large countries.

Table 4.10: Policy rules' optimal feedback coefficients for different public debt scenarios. Big country *versus* Small countries (debt-to-output ratios: 15%, 60%; n=0.5).

		Debt ratios	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$
Big Country	$\tilde{r}_t^*$	15%	-0.0042	2.1828	—	—
		60%	0.0108	4.2428	—	—
	$\tilde{g}_t^j$	15%	—	—	-0.9198	-0.0083
		60%	—	—	-0.9361	-0.0050
	$\tilde{\tau}_t^j$	15%	—	—	-1.6956	0.0070
		60%	—	—	-1.6737	0.0066
Small Country	$\tilde{r}_t^*$	15%	-0.0042	2.1828	—	—
		60%	0.0108	4.2428	—	—
	$\tilde{g}_t^j$	15%	—	—	-0.9113	-0.0062
		60%	—	—	-0.9324	-0.0184
	$\tilde{\tau}_t^j$	15%	—	—	-1.7330	0.0035
		60%	—	—	-1.6266	0.0105

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^*$

$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j$

$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j$

Though, a more detailed analysis of Table 4.10 reveals some noteworthy aspects. The response of small countries' fiscal policy to debt should be larger than that of the big country, under the high-debt scenario, and the reverse occurs for the low-debt

[28] Recall that under our model specification we assume that the big country is made up of a continuum of small geographic units, which essentially differ from small countries because they share the same fiscal policy.

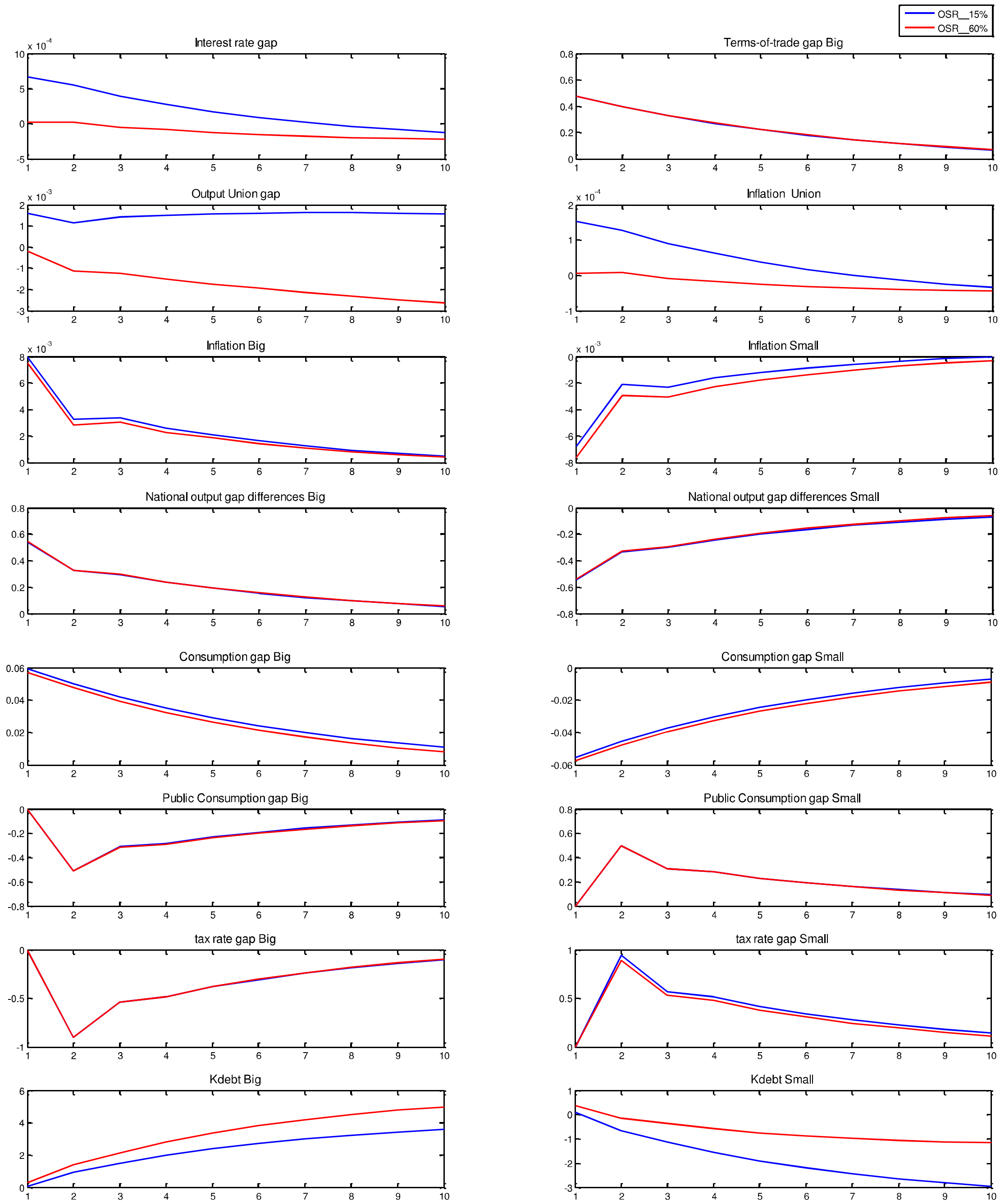
scenario. Interestingly, the big (small) country's fiscal instruments feedback on debt should become weaker (stronger) in a high relative to a low-debt scenario. As to the response of fiscal instruments to output gap differences, there is a slightly stronger response from government spending under the high-debt scenario, while in the case of the tax rate gap the response is weaker, particularly for small countries.

Focusing on the big country's fiscal policy, we can compare the results from Table 4.10 with those from Table 4.9. However, in the current model, the big country is affected by domestic shocks only, since asymmetric shocks at small countries have negligible external repercussions; moreover, we follow a different optimization using algorithms that may not converge to the absolute minimum. Still, if in the case of government spending the differences between the low- and the high-debt scenario are qualitatively similar, the same is not true for the tax revenue instrument. In a two-country union (see Table 4.9) the tax rate gap is more reactive to debt while it is significantly less reactive to output gap differences under the high-debt scenario. In the context of a heterogeneous country-size union (Table 4.10), the tax rate gap is also less reactive to output gap differences under the high-debt scenario, although the reduction is smaller relative to the case of a two-country union. Additionally, the tax rate gap feedback on debt is practically the same under the two debt scenarios.

Figure 4.3 illustrates the impact of a one-percent technology shock at the big country, comparing across the two debt scenarios. Since we are assuming a cooperative setup, the commitment solution is the same as in the two-country model. Additionally, the impact of an asymmetric technology shock at the big country on the efficient equilibrium is identical to that of an asymmetric technology shock at country H in a two-country union. Hence, we explain only significant changes in the impulse responses relative to Figures 4.2a and 4.2b, above.

As Figure 4.3 shows, following a domestic technology shock at the big country, there are no significant changes in the response of the big country's fiscal instruments in the low and the high-debt scenarios. This result quantitatively diverges from the commitment solution (see right panel of Figure 4.2a), where the tax rate adjustment is more debt-adjusting (smaller reduction in the tax rate) under the high-debt scenario – this also happens in Figure 4.3, but is minimal. Consequently, the effects on the big country's inflation are also different: under the high-debt scenario inflation is lower due mainly to the smaller private consumption gap.[29]

Figure 4.3: Responses to a 1% negative technology shock at the **Big** country under OSR: low-debt and high-debt scenarios (n=0.5).



[29] Recall that to service a higher debt level households expect policy measures with stronger negative effects on future private consumption, which, in turn, depresses current private consumption.

In the case of small countries the tax rate adjustment also diverges from the commitment solution (see right panel of Figure 4.2b) where the adjustment of the tax rate is stronger under the high-debt scenario and, hence, the stabilization of inflation remains worse under high-debts.

For the union as a whole, inflation is higher than under commitment, particularly under the low-debt scenario. Hence, the interest rate gap is positive in both debt scenarios, although monetary policy is less active under the high-debt scenario.[30]

Figure 4.4: Responses to a 1% negative technology shock at a **Small** country under low-debt and high-debt scenarios - OSR *versus* commitment solution (n=0.5)

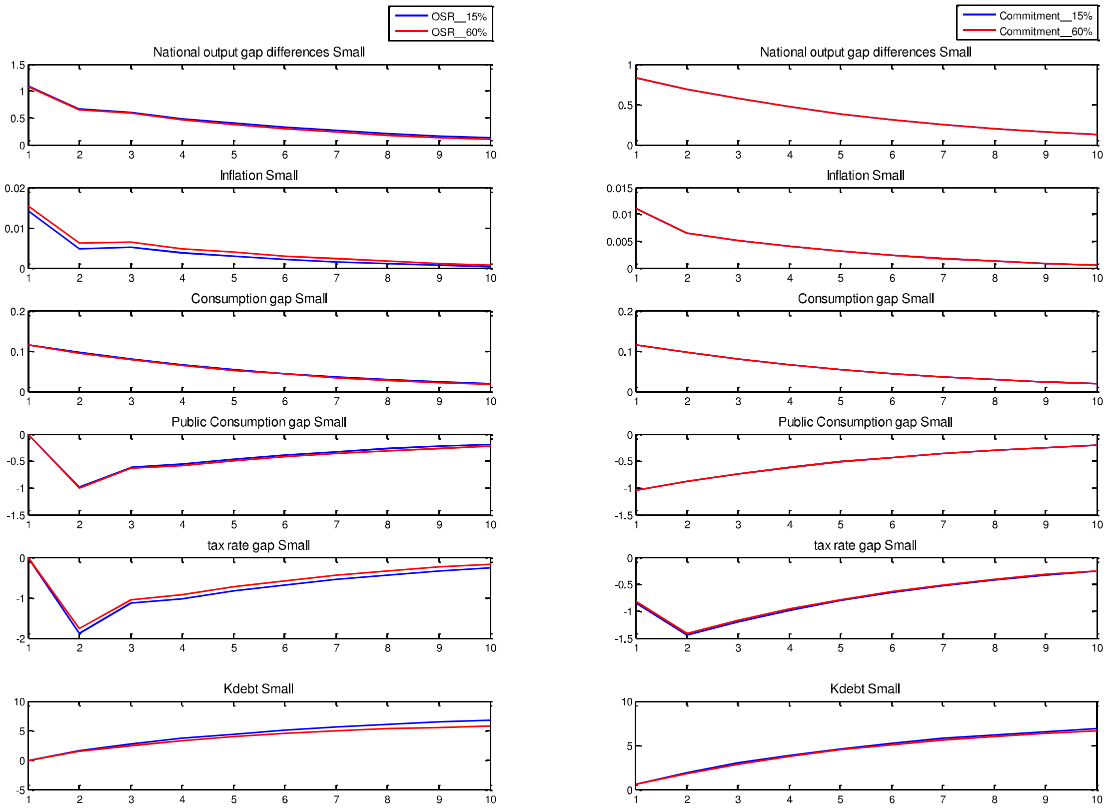


Figure 4.4 illustrates the impact of a one-percent domestic negative technology shock at a small country, comparing the impulse responses under optimal simple rules with those under commitment. Tough a small economy is affected both by domestic and external shocks, the effects of domestic shocks are welfare dominant. Consequently,

[30] Although the Taylor rule parameters are higher under the high-debt scenario, changes in the union's output gap and inflation are smaller.

simple rules perform more closely to commitment in the case of a small-country reaction to domestic shocks.

Given the very small dimension of the economy, there are no external effects and, hence, no reaction from the central bank. There are also no changes on the efficient equilibria of the other countries and on the efficient interest rate. Consequently, the welfare losses are substantial for the small economy because stabilization relies fully on its own policy instruments. The small country's efficient levels of output, private and public consumption fall on impact. The terms-of-trade also fall for the small country since the domestically produced goods become relatively more expensive.

Since the efficient public consumption falls less than the efficient output, a primary budget deficit arises, which increases with the initial steady-state debt level through output's increasing negative impact on tax receipts. Under discretion, the small country's government would raise the tax rate gap and reduce the public consumption gap to accommodate debt at the cost of higher inflation. Differently, under commitment, the objective is to control for inflation. The optimal solution under commitment is thus to cut the tax rate to control for inflation. Furthermore, and also in contrast with the discretionary solution, it is preferable to move fiscal instruments by a small amount permanently than to service a higher public debt level.

Differences between the low- and the high-debt scenarios are small because there are no external effects and, hence, no reaction from the central bank.[31] While under commitment the cut in the tax rate gap is slightly smaller in the high-debt scenario in order to limit the increase in debt service costs, under OSR the tax reduction is more pronounced and, hence, the inflationary consequences of the shock are larger. As to government spending, there is a slightly larger reduction under the high-debt scenario and this reduction is also more significant under OSR.

For small countries the feedbacks on debt are higher under the high-debt scenario both for government spending and the tax rate gaps (*cf.* Table 4.10), in line with the dynamics in Figure 4.4. Still, changes in fiscal instruments' feedback on output gap differences largely explain a more debt-stabilizing fiscal policy, particularly in terms of

[31] Notice however that since variables are defined in gaps the adjustment of the effective value of a variable may be more significant. That is the case of the tax revenue rate, as higher steady-state levels of debt result in higher steady-state tax rates. Hence the same negative tax rate gap represents a higher effective tax rate for higher debt levels.

the tax rate adjustment, and a lesser activeness on inflation stabilization. Actually, changes in fiscal instruments' feedbacks when we move from the low-debt to the high-debt scenario are in line with reactions to (welfare-dominant) domestic shocks.

Table 4.11 presents the welfare implications of the shocks illustrated in Figures 4.3 and 4.4, for the two debt scenarios, comparing the performance of OSR with full optimal policies (commitment/discretion).

Table 4.11: *Per capita* welfare losses: optimal simple rules *versus* full-optimal rules (debt-to-output ratios: 15%, 60%; n=0.5)

	Debt ratios	Simple Rules			Commitment/Discretion		
		Small country loss (L_S)	Big country loss (L_B)	Union-wide loss (L_U)	Small country loss (L_S)	Big country loss (L_B)	Union-wide loss (L_U)
All shocks*	15%	7.72	1.61	4. 66	7.14/8.13	1.43/1.75	4.28/4.94
	60%	7.87	1.60	4.74	7.13/7.87	1.45/2.20	4.29/5.04
Shocks at the big country	15%	1.53	1.61	1.57	1.43/1.55	1.43/1.75	1.43/1.65
	60%	1.57	1.60	1.58	1.42/1.28	1.45/2.20	1.44/1.74
Shocks at a small country	15%	6.19	0	0	5.71/6.57	0/0	0/0
	60%	6.31	0	0	5.71/6.58	0/0	0/0

* Although an asymmetric shock at a small economy (with zero dimension) produces negligible welfare costs at the union level, we consider that all small countries will face this kind of shock at any point in time, so union's welfare (L_U) takes into account the big country's welfare (L_B) and one representative small country's welfare (L_S). Hence, assuming that the big country's dimension is 0.5, we have that: $L_U = 0.5L_B + 0.5L_S$.

While in a two-country monetary union the welfare costs are the same for the two countries (L_H=L_F=L_U), in an heterogeneous country-size model the stabilization costs are significantly larger for the small countries because the stabilization costs generated by domestic shocks run entirely on them. The information in Table 4.11 raises some additional noteworthy aspects. First, most of the results reported in Table 4.11 show that stabilization costs under OSR are higher than those from the commitment solution but lower than those under discretion. Second, in the perspective of the union as a whole, the stabilization costs (L_U) are higher under the high-debt scenario. However, while the increase in costs is marginal at the big country (rather similar to that under commitment), the increase in the stabilization costs for the small countries is substantially larger than under fully optimal policies as debt increases. Finally, for the small countries, the stabilization costs of external shocks are higher under the high-debt scenario under OSR, in contrast with full optimal policies. Since small countries are affected by both domestic and external shocks, higher public debt levels may reduce the effectiveness of fiscal rules to react in face of these two distinctive types of shocks.

4.4 Sensitivity Analysis

Next, we analyze the sensitivity of our results to changes in the calibration of the shocks, particularly cost-push shocks, and to alternative structural features of the model. In order to keep this analysis simple, we take as reference the results obtained for the two-country union model[32] under the simpler set of optimal simple rules, considering that both fiscal instruments are used for debt and economic stabilization purposes.[33]

4.4.1 Cost-push Shocks

Clearly, the design of simple rules and the policy rules' optimal feedback coefficients depend on the nature of the stochastic forces driving the economy. Under our baseline calibration, the impact of cost-push shocks on welfare and, consequently, on the design of optimal simple rules is negligible. Welfare stabilization costs inherent to cost-push shocks represent only 0.07% of the welfare effects generated by technology shocks. Since we assume the same standard deviation for the two types of shocks ($\sigma_a = \sigma_\mu = 0.01$), this huge difference in terms of stabilization costs is explained not only by the assumption of no persistency in cost-push shocks, but mainly because cost-push shocks cause smaller policy trade-offs than technology shocks when the revenue tax rate is available for stabilization purposes.

Given the distinctive nature of cost-push shocks, our results would probably change if they had a larger impact in terms of welfare. To test this hypothesis we now assume persistency in cost-push shocks. Table 4.12 compares our baseline results, where $\rho_\mu = 0$ and $\rho_a = 0.85$, with an alternative specification where $\rho_\mu = \rho_a = 0.85$.

[32] Notice that, under our heterogeneous-country size model, we assume the same parameterization for the small economies and for the big country. Furthermore, as we discussed above, simple fiscal rules for the small countries are similar to those of the big country.

[33] Moreover, the required optimizations are done considering the same initialization values used under the baseline calibration.

Table 4.12: Policy rules' optimal feedback coefficients – simpler rules: persistent ***versus*** non-persistent cost-push shocks.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
Both Shocks $\rho_a = 0.85; \rho_\mu = 0$	$\tilde{r}_t^*$	0.0050	2.9367	—	—	
	$\tilde{g}_t^j$	—	—	-0.9316	-0.0058	**3.1659**
	$\tilde{\tau}_t^j$	—	—	-1.6513	0.0083	
Both Shocks $\rho_\mu = \rho_a = 0.85$	$\tilde{r}_t^*$	0.0079	5.0772	—	—	
	$\tilde{g}_t^j$	—	—	-0.9177	-0.0052	**3.2471**
	$\tilde{\tau}_t^j$	—	—	-1.6131	0.0084	

From Table 4.12, we can see that the welfare depletion relative to baseline is not much. In relation to changes in policy rules' optimal feedback coefficients, fiscal instruments are slightly less responsive to national output gap differences and in the case of government spending the feedback on debt is also slightly smaller. As to the monetary optimal Taylor rule, the interest rate gap becomes more responsive to the union's output gap and inflation. However, notice that these differences are only slightly.

Alternatively, we can exclude technology shocks and consider only cost-push shocks in the optimization procedure. By changing the correlation between output gap and inflation differences to union-average, we should expect changes in fiscal instruments' feedbacks. Table 4.13 shows the results from this alternative specification of shocks in line (2).

A higher feedback on debt for the tax rate instrument relative to that of government spending is expected since the tax rate instrument is relatively more effective in stabilizing cost-push shocks. Note also that under the alternative specification of shocks, where we consider cost-push shocks only, fiscal instruments' feedback on output gap differences becomes positive. In contrast to technology shocks, cost-push shocks generate a negative correlation between output gap and inflation deviations from union average.[34]

Table 4.13: OSR's feedback coefficients (simpler rules) - considering both technology and cost-push shocks ***versus*** cost-push shocks only.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$	**Union's *per capita* welfare loss**
***Both Shocks* (1)**	$\tilde{r}_t^*$	0.0050	2.9367	—	—	—	
	$\tilde{g}_t^j$	—	—	-0.9316	—	-0.0058	**3.1659** (of which cost-push shocks represent 0.0021)
	$\tilde{\tau}_t^j$	—	—	-1.6513	—	0.0083	
***Cost-push only* (2)**	$\tilde{r}_t^*$	0.0084	8.9867	—	—	—	
	$\tilde{g}_t^j$	—	—	0.4103	—	-0.0011	**0.0013196**
	$\tilde{\tau}_t^j$	—	—	0.0228	—	0.0103	

4.4.2 Structural Features of the Model

In this subsection we conduct a robustness analysis to some structural features of the model. Specifically, we focus our analysis on the degree of nominal rigidity, θ, the

[34] Following a domestic positive cost-push shock at country H there are no changes in the efficient equilibrium, apart from that in the domestic tax rate. The efficient level of domestic tax rate falls to fully offset the impact of the domestic cost-push shocks. However, the cut in fiscal revenue results in a primary budget deficit which forces domestic policy to deviate from efficiency and lets inflation increase at home. This has a positive effect on demand at F, penalizing domestic output. Moreover, to control for debt, public consumption falls at home, further reducing domestic output. At country F, the demand increase has a positive effect on output, resulting in a primary budget surplus which, in turn, leads to a cut in the tax rate. Hence, country H faces inflation and a negative output gap while the reverse occurs in country F.

We also investigate if the performance of OSR changes significantly by considering an additional feedback on national inflation deviations from union-wide inflation. The signs of the fiscal instruments' coefficients on national inflation deviations are as expected – cutting government spending and tax rates reduce inflationary pressures –, but they are small in value. Though the response of fiscal instruments to national output gap deviations becomes slightly weaker, it is still dominant and there is no improvement in terms of performance.

elasticity of labor supply, χ, and the elasticity of substitution between home- and foreign-produced goods, γ.

Nominal Rigidity

Under our baseline calibration we set θ=0.75, implying an average length of price contracts of one year. Table 4.14 presents OSRs' feedback coefficients considering two alternative, smaller, values for θ: θ=0.5 and θ=2/3, implying, respectively, an average length of price duration of two and three quarters.

Table 4.14: OSR's feedback coefficients across different average length of price contracts.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

	Price rigidity	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
$\tilde{r}_t^*$	θ=0.5	0.0121	7.0838	—	—	θ=0.5: **2.7202**
	θ=2/3	0.0087	6.4881	—	—	
	θ=0.75	0.0050	2.9367	—	—	
$\tilde{g}_t^j$	θ=0.5	—	—	-0.8930	-0.0042	θ=2/3: **3.0262**
	θ=2/3	—	—	-0.9140	-0.0057	
	θ=0.75	—	—	-0.9316	-0.0058	
$\tilde{\tau}_t^j$	θ=0.5	—	—	-1.6927	0.0086	θ=0.75: **3.1659**
	θ=2/3	—	—	-1.6902	0.0083	
	θ=0.75	—	—	-1.6513	0.0083	

As expected, a higher degree of nominal price rigidity increases stabilization costs. The terms-of-trade gaps are larger for higher degrees of price rigidity. However, this effect lessens for relatively higher levels of θ.

As θ increases, monetary policy should react less to the union's output gap and inflation. As for fiscal instruments, a higher price rigidity requires a weaker reaction of the tax rate to output gap deviations from union's and to debt (only slightly), while government spending should become more responsive to both variables. Nonetheless, globally there are no substantial changes in the OSRs' coefficients, exception made for the Taylor rule coefficients.

Elasticity of Labor Supply

Under our baseline calibration the inverse of the labor elasticity is χ=3. Table 4.15 presents OSR's feedback coefficients considering two alternative values: χ=1 (widely used in the literature) and χ=5 (as in Erceg *et al.*, 2010).[35]

A lower elasticity of labor supply (higher χ) results in higher stabilization costs since fluctuations in work effort, arising from misallocations caused by inflation, are more costly. Consequently, there is a higher control of inflation. Results in Table 4.15 show that, as χ increases, fiscal instruments' feedback on output gap differences are larger, while the feedback on debt should increase in the case of government expenditures but decrease in the case of taxes. The interest rate gap is required to become more responsive to the union's inflation. However, if we further broaden the range of values of χ, only the increased feedback of fiscal instruments on output gap remains rather robust.

Table 4.15: OSR's feedback coefficients across different elasticities of labor supply. Two-country union model (debt-to-output ratio = 60%; n=0.5)

	Different χ	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
$\tilde{r}_t^*$	χ=1	0.0026	1.0021	—	—	χ=1: **2.1470**
	χ=3	0.0050	2.9367	—	—	
	χ=5	0.0043	6.7119	—	—	
$\tilde{g}_t^j$	χ=1	—	—	-0.3744	-0.0030	χ=3: **3.1659**
	χ=3	—	—	-0.9316	-0.0058	
	χ=5	—	—	-1.3474	-0.0066	
$\tilde{\tau}_t^j$	χ=1	—	—	-0.6673	0.0092	χ=5: **4.1360**
	χ=3	—	—	-1.6513	0.0083	
	χ=5	—	—	-2.6339	0.0079	

[35] In fact, we consider χ=0.999. With χ=1 the optimization procedure converges to a different solution for the Taylor rule, particularly for the feedback on the union's output.

Elasticity of Substitution between National and Foreign Goods

We assume that goods produced in different countries are substitutes since under our baseline calibration the trade price elasticity or elasticity of substitution between domestic and foreign goods (γ=4.5) is larger than the intertemporal elasticity of substitution (σ=0.4). We now study the implications of goods still being substitutes (γ=1.5 as in Vogel *et al.*, 2013), or complements (γ=0.2).

Results presented in Table 4.16 show that, when foreign- and domestically-produced goods are complements, fiscal instruments are more responsive to output gap deviations from union's average and the tax rate gap feedback on debt is higher. Furthermore, the feedback of government spending on debt is positive in contrast with what happens when goods are substitutes.

Table 4.16: OSR's feedback coefficients – substitute ***versus*** complementary foreign-domestic goods.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

Case		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
γ=4.5; σ=0.4 *substitutes*	$\tilde{r}_t^*$	0.0050	2.9367	—	—	
	$\tilde{g}_t^j$	—	—	-0.9316	-0.0058	**3.1659**
	$\tilde{\tau}_t^j$	—	—	-1.6513	0.0083	
γ=1.5; σ=0.4 *substitutes*	$\tilde{r}_t^*$	0.0017	1.0027	—	—	
	$\tilde{g}_t^j$	—	—	-1.0669	-0.0046	**2.9022**
	$\tilde{\tau}_t^j$	—	—	-2.3209	0.0086	
γ=0.2; σ=0.4 *complements*	$\tilde{r}_t^*$	0.0168	2.1778	—	—	
	$\tilde{g}_t^j$	—	—	-1.6607	0.0048	**2.1588**
	$\tilde{\tau}_t^j$	—	—	-5.2617	0.0107	

Notice that when domestic and foreign goods are complements, an asymmetric technology shock still produces opposite budgetary consequences for countries H and F, but now in the reverse order relative to when goods are substitutes. For instance, following an asymmetric technology shock at country H, the efficient public consumption falls more than the efficient output generating a government primary budget surplus at home; however, the increase in debt service costs reduces this surplus effect and may even result in a negative global impact on debt for sufficient high steady-state debt levels, as is the case for a debt-to-output ratio=60%. At country F, a government primary budget deficit arises, resulting from the fall in the efficient output and the increase in the efficient public consumption levels, and it is further enlarged by the rise in the service costs of debt.

4.6 Concluding Remarks

In the context of a currency union setup, several recent papers have already studied the use of optimal simple rules with welfare maximization purposes. We derived the optimal countercyclicality and debt feedback degrees of a fiscal rule for the particular case of a very small country-member of a monetary union, and compared the efficiency across alternative rules.

Results reveal that though the performance of OSR is worse than full-optimal policies under commitment, they perform better relative to discretion. In particular, our results point to an active monetary policy and a passive fiscal policy. The interest rate gap responds to both structural variables (output and inflation), and mostly to the union's average inflation, and fiscal instruments (government spending and the revenue tax rate) react to output gap differences and national public debt. The optimizations show that the interest rate gap does not respond significantly to the union's aggregate debt, whereas the responses of fiscal instruments to inflation differences (or the terms-of-trade) are negligible. In fact, the impulse responses under OSR are closer to the commitment solution.

In the particular case of fiscal rules, both fiscal instruments present a consistent feedback on national output gap differences and on the level of government indebtedness. As expected, public consumption gap (tax rate gap) reacts negatively (positively) to the level of public debt, and the size of the adjustment on debt is small, acting as a shock absorber, in line with the commitment solution. Government spending responds negatively to output gap differences, while the negative response of the revenue tax rate is related to a negative response to inflation, since technology shocks (dominant in terms of welfare) generate a positive correlation between inflation and output gap differences.

Restricted optimal fiscal rules (reacting to a single argument) perform substantially worse than unrestricted simple rules and the implied stabilization costs are higher than under discretion. Anyway, the tax rate is better at promoting debt stabilization whereas government spending should target output gap stabilization.

Our results reveal that small countries fiscal rules are not substantially different from those of a big country. Nonetheless, the response of small countries' fiscal policy to debt should be stronger than that for a big country under a high-debt scenario (debt-to-output ratio=60%), but the reverse should occur for a low-debt scenario (debt-to-output ratio=15%). Moreover, the reaction of a big (small) country's fiscal instruments to debt should weaken (strengthen) in high-debt scenarios.

High-debt scenarios also optimally require higher (lower) government spending (taxes) feedback on output gap, particularly for small countries.

For a small country, the stabilization costs are substantially higher under the high-debt scenario when OSR are in place relative to full optimal policies. This difference is almost negligible for a big country: since small countries are affected both by domestic and external shocks, it is possible that higher public debt levels lessen the effectiveness of fiscal policy rules to deal with these two distinctive types of shocks.

The analysis of the impulse responses to technology shocks at the big country and at a small country (which has no external consequences) is very useful to understand the implications of the differences in the fiscal rules coefficients between the big country and small countries and for different public debt scenarios. Following a domestic technology shock at the big country, there are no significant changes in the response of the big country's fiscal instruments between the low- and the high-debt scenarios. In face of external technology shocks, the response of a small country is not substantially different from that of a big country. Following a technology shock at a small country, differences between the low- and high-debt scenarios are small because there are no external effects and, hence, the stabilization costs are entirely on the small country. Nonetheless, the domestic fiscal policy is more debt-stabilizing (less active towards inflation stabilization) under the high-debt scenario, which is explained by the larger feedback on debt of both fiscal instruments, but especially by the changes in fiscal instruments' feedback on output gap differences.

We have also conducted sensitivity analysis regarding alternative shocks and economic structure of the economy. As cost-push shocks gain relative importance, our results prescribe larger feedback of the common interest rate to both union's output gap and inflation. Country-specific fiscal instruments should react slightly and positively to output gap deviations from average, whereas the tax rate (government spending) should be more (less) reactive towards debt.

Under higher nominal rigidity in a monetary union, interest rate should be less reactive to both arguments; in contrast, government expenditures (tax rate) should be slightly more (less) reactive towards output gap and debt. In turn, lower labor supply elasticity requires more union-wide inflation stabilization and a larger output stabilization burden on both country-specific fiscal instruments. Foreign-domestic goods complementarity relative to substitutability also requires larger reactions of fiscal instruments to output gap and larger debt stabilization from taxes.

5 Conclusion

This work is constituted by two separate projects though they share some common aspects. We contribute to the literature on optimal policy interactions by extending a standard New Keynesian open-economy model to a heterogeneous country-size monetary union, where very small economies coexist with a large country sharing a common monetary authority while keeping fiscal policy autonomy. Most of the existing literature relies either in a two-country framework, where domestic shocks and policy decisions have significant impact on the counterpart and on the union as a whole, or in a multi-country model where the union is made up of a continuum of small open economies and where domestic shocks and policy decisions have no external consequences. Our model fills this gap while providing a more realistic and general structure for a monetary union, and allows for the discussion of the optimal policy design for country-members either with negligible or meaningful impact on the union's outcomes as well as on the counterparts'. Furthermore, our model's strategy allows, as a particular case, to model the two-country monetary-union, as well as the case of a multi-country monetary union composed of only very small members.

In this context, in chapter 3 we study full-optimal monetary and fiscal policy interactions and focus on how the level of government indebtedness affects the macroeconomic stabilization performance of small and large countries, assuming that fiscal authorities engage in discretionary policy games with a benevolent union-wide welfare maximizing monetary authority and where strategic interactions may arise if fiscal policy is biased towards country-specific stabilization objectives.

For the union as a whole, our results show that, in general, higher public debt levels hamper business cycle stabilization. Furthermore, a higher level of average government indebtedness, as the one experienced in the Economic and Monetary Union (EMU), is expected to penalize by more the stabilization performance of the small country-members than that of the large. Given its negligible dimension, when hit by an asymmetric shock, the stabilizations costs run entirely on the small country.

We also found that strategic policy interactions disclose different welfare consequences for the large and small countries. The non-internalization of the externalities produced by the fiscal policy of the big country imposes higher stabilization costs on the small countries. For the union as a whole, there are clear stabilization gains from promoting policy cooperation under a high-debt scenario, though this could be counterproductive in a low-debt monetary union. Moreover, under a non-cooperative environment, fiscal leadership imposes higher stabilization costs than monetary leadership, for the union as a whole as well as for the small countries. In a high-debt monetary union, the big country clearly prefers the fiscal leadership regime, suggesting that it may be hard to get the political support from a big country for a union-wide cooperative arrangement.

Yet, full-optimal policy reactions are of no practical tractability. The vast amount of literature on optimal simple rules (OSR) is justified given their practical advantage over full optimal policy solutions. Moreover, simple rules lessen the time-consistency problem, reinforcing credibility: tough suboptimal, OSR facilitate the communication of policy objectives and the verification of their outcomes. In Chapter 4 we derive and discuss alternative optimal simple rules. We extend the analysis of OSR for fiscal policy to both large and very small countries, and, considering cooperation, we focus on how the optimal countercyclicality and debt feedback degrees of the rule should be set according to different structures of the economy, namely in low- and high-debt scenarios.

Though the OSR provide a worse stabilization performance than full-optimal policies under commitment, they perform better relative to discretion. In line with the results obtained for full-optimal policies, the stabilization costs for the union as a whole are higher under a high-debt scenario. Though our results reveal that optimal simple fiscal rules do not substantially differ for the small and the big country, there are still some noteworthy aspects. For instance, the reaction of small countries' fiscal policy to debt should be larger than that of the big country, under the high-debt scenario (debt=60%), but the reverse should occur for the low-debt scenario (debt=15%).

In future research, we intend to check the implications of assuming different structural features for the large and the small economies (*e.g.*, nominal price rigidity, market power, wage flexibility, elasticity of labor supply, elasticity of substitution between domestic goods, level of government indebtedness). For instance, Andersen and Seneca (2010) analyze how a monetary union performs in the presence of labor market functioning asymmetries, considering different wage flexibility, market power and country sizes, concluding that asymmetries can create substantial changes in spillover effects. Moreover, they show that a disproportionate share of the consequences of wage flexibility may fall on small countries.

In the case of optimal simple rules, we have centered our analysis on parsimonious simple fiscal rules, leaving aside an examination of more elaborated/realistic rules for future research. One possibility is to consider both deficit and debt as arguments in the design of fiscal policies. Furthermore, given that under OSR it is preferable to move fiscal instruments by a small amount permanently to service a higher public debt level, it would be interesting to study the implications for the design of simple rules from imposing a ceiling on public debt due to borrowing limits, especially for the case of small countries when hit by asymmetric domestic shocks. Additionally, we intend to conduct our analysis under a richer setup taking into consideration a wider range of stochastic forces, distinctive in their nature, to improve the robustness of our results while also considering a non-cooperative scenario. Finally, both projects would benefit from the derivation of the benevolent non-cooperative country-specific loss functions which would enable capturing different incentives faced by large and very small economies.

Appendix A.1:

Derivation of the Efficient Equilibrium

From the resource constraints, we have that

$$\begin{cases} Y_t^s - G_t^s = C_{s,t}^s + \int_0^n C_{s,t}^i di + (1-n)\, C_{s,t}^B, \forall s \in S \\ Y_t^B - G_t^B = C_{B,t}^B + \frac{1}{(1-n)} \int_0^n C_{B,t}^s ds \end{cases} \tag{1}$$

and by the optimality conditions

$$\begin{aligned}
C_{s,t}^s &= \left[\chi_0 \frac{(L_t^s)^\chi}{A_t^s} (1-\alpha)^{-\frac{1}{\gamma}} (C_t^s)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\right]^{-\gamma}, \forall s \in S, \qquad (2)\\
C_{s,t}^i &= \left[\chi_0 \frac{(L_t^s)^\chi}{A_t^s} (\alpha)^{-\frac{1}{\gamma}} \left(C_t^i\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\right]^{-\gamma}, \forall s, i \in S, i \neq s,\\
C_{s,t}^B &= \left[\chi_0 \frac{(L_t^s)^\chi}{A_t^s} (\alpha)^{-\frac{1}{\gamma}} \left(C_t^B\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\right]^{-\gamma}, \forall s \in S,\\
C_{B,t}^B &= \left[\chi_0 \frac{\left(L_t^B\right)^\chi}{A_t^B} (1-n\alpha)^{-\frac{1}{\gamma}} \left(C_t^B\right)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\right]^{-\gamma},\\
C_{B,t}^s &= \left[\chi_0 \frac{\left(L_t^B\right)^\chi}{A_t^B} [\alpha (1-n)]^{-\frac{1}{\gamma}} (C_t^s)^{\frac{1}{\sigma}-\frac{1}{\gamma}}\right]^{-\gamma}, \forall s \in S,\\
G_t^s &= \left[\frac{\chi_0}{\psi_0} \frac{(L_t^s)^\chi}{A_t^s}\right]^{-\psi} = \left[\frac{\chi_0}{\psi_0} \frac{(Y_t^s)^\chi}{(A_t^s)^{1+\chi}}\right]^{-\psi}, \forall s \in S,\\
G_t^B &= \left[\frac{\chi_0}{\psi_0} \frac{\left(L_t^B\right)^\chi}{A_t^B}\right]^{-\psi} = \left[\frac{\chi_0}{\psi_0} \frac{\left(Y_t^B\right)^\chi}{\left(A_t^B\right)^{1+\chi}}\right]^{-\psi}.
\end{aligned}$$

Substituting these expressions in the resource constraints (1) and taking into account that in the efficient equilibrium $Y_t^j = A_t^j L_t^j, j = \mathbf{B}, s \in S$, we can rewrite the resource constraints as:

$$\begin{aligned}
\overline{Y}_t^s - \overline{G}_t^s &= \alpha (\chi_0)^{-\gamma} \left(\overline{Y}_t^s\right)^{-\gamma\chi} (A_t^s)^{\gamma(\chi+1)} \times \\
&\times \left\{ \left(\frac{1-\alpha}{\alpha}\right) \left(\overline{C}_t^s\right)^{\frac{\sigma-\gamma}{\sigma}} + \int_0^n \left(\overline{C}_t^i\right)^{\frac{\sigma-\gamma}{\sigma}} di + (1-n) \left(\overline{C}_t^B\right)^{\frac{\sigma-\gamma}{\sigma}} \right\},\\
\forall s &\in S,
\end{aligned}$$

$$\overline{Y}_t^B - \overline{G}_t^B = \alpha\left(\chi_0\right)^{-\gamma}\left(\overline{Y}_t^B\right)^{-\gamma\chi}\left(A_t^B\right)^{\gamma(\chi+1)}\times$$
$$\times\left\{\left(\frac{1-\alpha}{\alpha}\right)\left(\overline{C}_t^B\right)^{\frac{\sigma-\gamma}{\sigma}}+\int_0^n\left(\overline{C}_t^i\right)^{\frac{\sigma-\gamma}{\sigma}}di+(1-n)\left(\overline{C}_t^B\right)^{\frac{\sigma-\gamma}{\sigma}}\right\},$$

where $\overline{X}$ represents the efficient value of variable X and $\overline{\overline{x}}_t$ its efficient value in log-deviations from the zero-inflation efficient steady-state. Taking into account that $\overline{g}_t^j = -\psi\ln(\chi_0) + \psi\ln(\psi_0) - \psi\chi\overline{y}_t^j + \psi(1+\chi)a_t^j$, $j = \mathbf{B}$,$s\in S$, we obtain a similar expression for all countries:

$$[1+\varphi(\psi\chi)+(1-\varphi)\gamma\chi]\,\overline{\overline{y}}_t^j = \tag{4}$$
$$(1+\chi)\left[\varphi\psi+(1-\varphi)\gamma\right]a_t^j+(1-\varphi)\left(\frac{\sigma-\gamma}{\sigma}\right)\left\{(1-\alpha)\,\overline{\overline{c}}_t^j+\alpha\overline{\overline{c}}_t^*\right\},j=\mathbf{B},s\in\mathbf{S},$$

where $\overline{c}_t^* = \displaystyle\int_0^n \overline{c}_t^s ds + (1-n)\,\overline{c}_t^B$, up to a first-order log-linear approximation around the efficient symmetric (zero-inflation) steady state.

Integrating expression (4) over the union, and taking into account that $\overline{g}_t^* = \displaystyle\int_0^n \overline{g}_t^s ds + (1-n)\,\overline{g}_t^B$ and that $\overline{y}_t^* = (1-\varphi)\overline{c}_t^* + \varphi\overline{g}_t^*$, we get:

$$\overline{\overline{y}}_t^* = \frac{(1+\chi)\left[\varphi\psi+(1-\varphi)\sigma\right]}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^* \tag{5a}$$

and

$$\overline{\overline{c}}_t^* = \frac{(1+\chi)\,\sigma}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^*, \tag{5b}$$

$$\overline{\overline{g}}_t^* = \frac{(1+\chi)\,\psi}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^*. \tag{5c}$$

Returning to the country specific expressions (4), we take first-order log-linear approximations around the efficient symmetric (zero-inflation) steady state for the consumption indexes

$$C_t^s \equiv \left[(1-\alpha)^{\frac{1}{\gamma}}\left(C_{s,t}^s\right)^{\frac{\gamma-1}{\gamma}}+\right.$$
$$\left[\alpha(1-n)\right]^{\frac{1}{\gamma}}\left(C_{B,t}^s\right)^{\frac{\gamma-1}{\gamma}}+$$
$$\left.(\alpha)^{\frac{1}{\gamma}}\int_0^n\left[C_{i,t}^s\right]^{\frac{\gamma-1}{\gamma}}di\right]^{\frac{\gamma}{\gamma-1}},\forall s,i\in S,i\neq s,$$

and

$$C_t^B \equiv \left[(1-n\alpha)^{\frac{1}{\gamma}} \left(C_{B,t}^B \right)^{\frac{\gamma-1}{\gamma}} + (\alpha)^{\frac{1}{\gamma}} \int_0^n \left[C_{s,t}^B \right]^{\frac{\gamma-1}{\gamma}} ds \right]^{\frac{\gamma}{\gamma-1}},$$

and obtain

$$\begin{aligned} \overline{\overline{c}}_t^s &= (1-\alpha)\,\overline{\overline{c}}_{s,t}^s + \alpha \int_0^n \overline{\overline{c}}_{i,t}^s di + \alpha\,(1-n)\,\overline{\overline{c}}_{B,t}^s, \forall s \in S, &(6) \\ \overline{\overline{c}}_t^B &= (1-n\alpha)\,\overline{\overline{c}}_{B,t}^B + \alpha \int_0^n \overline{\overline{c}}_{s,t}^B ds. &(7) \end{aligned}$$

For the components of the consumption indexes, we log-linearize the expressions obtained from the optimality conditions, so that

$$\begin{aligned} \overline{\overline{c}}_{s,t}^s &= -\chi\gamma\overline{\overline{y}}_t^s + \left(\frac{\sigma-\gamma}{\sigma}\right)\overline{\overline{c}}_t^s + \gamma\,(1+\chi)\,a_t^s, \forall s \in S, \\ \overline{\overline{c}}_{i,t}^s &= -\chi\gamma\overline{\overline{y}}_t^i + \left(\frac{\sigma-\gamma}{\sigma}\right)\overline{\overline{c}}_t^s + \gamma\,(1+\chi)\,a_t^i, \forall s,i \in S, i \neq s, \\ \overline{\overline{c}}_{B,t}^s &= -\chi\gamma\overline{\overline{y}}_t^B + \left(\frac{\sigma-\gamma}{\sigma}\right)\overline{\overline{c}}_t^s + \gamma\,(1+\chi)\,a_t^B, \forall s \in S, \\ \overline{\overline{c}}_{B,t}^B &= -\chi\gamma\overline{\overline{y}}_t^B + \left(\frac{\sigma-\gamma}{\sigma}\right)\overline{\overline{c}}_t^B + \gamma\,(1+\chi)\,a_t^B, \\ \overline{\overline{c}}_{s,t}^B &= -\chi\gamma\overline{\overline{y}}_t^s + \left(\frac{\sigma-\gamma}{\sigma}\right)\overline{\overline{c}}_t^B + \gamma\,(1+\chi)\,a_t^s, , \forall s \in S. \end{aligned} \tag{8}$$

Substituting the expressions for the components of these indexes, we obtain

$$\begin{aligned} \overline{\overline{c}}_t^s &= -(1-\alpha)\,\sigma\chi\overline{\overline{y}}_t^s + (1-\alpha)\,\sigma\,(1+\chi)\,a_t^s + \left[\frac{\alpha\,(1+\chi)\,\sigma}{1+\chi\,[\varphi\psi+(1-\varphi)\sigma]}\right] a_t^*, \\ \forall s &\in S, \\ \overline{\overline{c}}_t^B &= -(1-\alpha)\,\sigma\chi\overline{\overline{y}}_t^B + (1-\alpha)\,\sigma\,(1+\chi)\,a_t^B + \left[\frac{\alpha\,(1+\chi)\,\sigma}{1+\chi\,[\varphi\psi+(1-\varphi)\sigma]}\right] a_t^*. \end{aligned}$$

We get identical expressions for all countries in the union. Finally, substituting the above expressions in (4), as well as the expressions of $\overline{\overline{c}}_t^*$,

$$
\begin{aligned}
&\overline{\overline{y}}_t^j = \quad (9)\\
&\frac{(1+\chi)\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\
&+\frac{(1+\chi)(1-\varphi)(\sigma-\gamma)(2\alpha-\alpha^2)}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\
&j=\mathbf{B},s\in\mathbf{S}.
\end{aligned}
$$

Expressions for $\overline{\overline{c}}_t^j$ and $\overline{\overline{g}}_t^j$ are now easly to obtain. Since

$$
\begin{aligned}
\overline{\overline{c}}_t^j &= -(1-\alpha)\,\sigma\chi\overline{\overline{y}}_t^j+(1-\alpha)\,\sigma\,(1+\chi)\,a_t^j+\left[\frac{\alpha\,(1+\chi)\,\sigma}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}\right]a_t^*,\\
\overline{\overline{g}}_t^j &= -\psi\chi\overline{\overline{y}}_t^j+\psi\,(1+\chi)\,a_t^j,\, j=\mathbf{B},s\in\mathbf{S},
\end{aligned}
$$

we have that

$$
\begin{aligned}
&\overline{\overline{c}}_t^j =\\
&\frac{(1-\alpha)(1+\chi)\sigma}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\
&+\frac{\alpha(1+\chi)\sigma\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma-(\sigma-\gamma)(1-\alpha)\right]\right)\right\}}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\
&j=\mathbf{B},s\in\mathbf{S}. \quad (10)
\end{aligned}
$$

and

$$
\begin{aligned}
&\overline{\overline{g}}_t^j =\\
&\frac{(1+\chi)\psi}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\
&-\frac{(1+\chi)\psi(1-\varphi)\chi(\sigma-\gamma)(2\alpha-\alpha^2)}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\
&j=\mathbf{B},s\in\mathbf{S}. \quad (11)
\end{aligned}
$$

Appendix A.2:

Derivation of the Social Loss Function

The average utility flow of the households belonging to a small country $s \in \mathbf{S}$ and to the large economy B, respectively, is

$$
\begin{aligned}
W_t^s &= \int_0^1 \left[u\left(C_t^s\right) + V\left(G_t^s\right) - v\left(L_t^s(h)\right) \right] dh \\
&= u\left(C_t^s\right) + V\left(G_t^s\right) - \int_0^1 v\left(L_t^s(h)\right) dh, \forall s \in \mathbf{S}, \qquad (12a) \\
&\quad \textit{and} \\
W_t^B &= \int_0^1 u\left(C_t^B\right) + V\left(G_t^B\right) - v\left(L_t^b(h)\right) dh \\
&= u\left(C_t^B\right) + V\left(G_t^B\right) - \int_0^1 v\left(L_t^b(h)\right) dh, \forall b \in \mathbf{B}. \qquad (12b)
\end{aligned}
$$

The welfare criterion of the common central bank (and the coordinating fiscal authorities) is:

$$
W^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\int_0^n W_t^s ds + (1-n) W_t^B \right] \right\}.
$$

We start by making computations for a small economy $s \in \mathbf{S}$ and then we present analogous derivations for the large economy. After that, we combine the expressions to obtain W^*.

Derivation of the terms $u\left(C_t\right)$, $V\left(G_t\right)$ and $v\left(L_t(h)\right)$

Taking into account a small country $s \in \mathbf{S}$,

$$
\begin{aligned}
W_t^s &= u\left(C_t^s\right) + V\left(G_t^s\right) - \int_0^1 v\left(L_t^s(h)\right) dh \\
&= \frac{\sigma}{\sigma - 1} \left(C_t^s\right)^{\frac{\sigma-1}{\sigma}} + \psi_0 \frac{\psi}{\psi - 1} \left(G_t^s\right)^{\frac{\psi-1}{\psi}} - \int_0^1 \chi_0 \frac{1}{1+\chi} \left(L_t^s(h)\right)^{1+\chi} dh, \forall s \in \mathbf{S}, \qquad (13)
\end{aligned}
$$

we perform second-order approximations around the efficient symmetric zero-inflation steady state for the three terms: $u\left(C_t^s\right)$, $V\left(G_t^s\right)$ and $\int_0^1 v\left(L_t^s(h)\right)dh$.

Starting with $u\left(C_t^s\right)$, we get that

$$\begin{aligned} u\left(C_t^s\right) &= u\left(C^s\right)+u_c C^s\left[\widehat{c}_t^s+\frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^s\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right), \forall s\in\mathbf{S}, \\ with &: \\ u_c &= \left(C^s\right)^{-\frac{1}{\sigma}}=\frac{\partial u\left(C_t^s\right)}{\partial C_t^s} \text{ evaluated at the steady state,} \end{aligned} \tag{14}$$

and where $\mathcal{O}\left(\|\zeta\|^3\right)$ represents terms of third or higher order.

The term $V\left(G_t^s\right)$ is approximated by

$$\begin{aligned} V\left(G_t^s\right) &= V\left(G^s\right)+V_g G^s\left[\widehat{g}_t^s+\frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^s\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right), \forall s\in\mathbf{S}, \\ with &: \\ V_g &= \psi_0\left(G^s\right)^{-\frac{1}{\psi}}=\frac{\partial V\left(G_t^s\right)}{\partial G_t^s} \text{ evaluated at the steady state.} \end{aligned} \tag{15}$$

As for the term $\int_0^1 v\left(L_t^s(h)\right)dh$, we get that

$$\begin{aligned} v\left(L_t^s(h)\right) &= v\left(L^s\right)+v_l L^s\left[\widehat{l}_t^s(h)+\frac{1}{2}(1+\chi)\left(\widehat{l}_t^s(h)\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right), \forall h, \forall s\in\mathbf{S}, \\ with &: \\ v_l &= \chi_0\left(L^s\right)^{\chi}=\frac{\partial v\left(L_t^s(h)\right)}{\partial L_t^s(h)} \text{ evaluated at the steady state.} \end{aligned} \tag{16}$$

Taking into account that

$$Y_t^s(h)=A_t^s L_t^s(h)\Longrightarrow \widehat{l}_t^s(h)=\widehat{y}_t^s(h)-a_t^s,\ since\ L^s(h)=L^s=Y^s,$$

we get that

$$
\begin{aligned}
v\left(L_t^s(h)\right) &= v\left(L^s\right)+v_l L^s\left[\widehat{y}_t^s(h)-a_t^s+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^s(h)-a_t^s\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right) \\
&= v\left(L^s\right)+v_l L^s\left\{\widehat{y}_t^s(h)\left[1-(1+\chi)a_t^s\right]+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^s(h)\right)^2\right\} \\
&\quad +v_l L^s\left\{-a_t^s+\frac{1}{2}(1+\chi)(a_t^s)^2\right\}+\mathcal{O}\left(\|\zeta\|^3\right) \\
&= v\left(L^s\right)+v_l L^s\left\{\widehat{y}_t^s(h)\left[1-(1+\chi)a_t^s\right]+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^s(h)\right)^2\right\} \\
&\quad +t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right), \forall h, \forall s \qquad (17)
\end{aligned}
$$

where "t.i.p." stands for "terms independent of policy".

Integrating (17) over the population of country $s \in \mathbf{S}$,

$$
\begin{aligned}
\int_0^1 v\left(L_t^s(h)\right)dh &= v\left(L^s\right)+\int_0^1 v_l L^s\left\{\widehat{y}_t^s(h)\left[1-(1+\chi)a_t^s\right]+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^s(h)\right)^2\right\}dh \\
&\quad +t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right) \\
&= v\left(L^s\right)+v_l L^s\left[1-(1+\chi)a_t^s\right]\int_0^1 \widehat{y}_t^s(h)dh \\
&\quad +v_l L^s\frac{1}{2}(1+\chi)\int_0^1\left(\widehat{y}_t^s(h)\right)^2 dh+t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right). \qquad (18)
\end{aligned}
$$

Taking into account that

$$
\begin{aligned}
\int_0^1 \widehat{y}_t^s(h)dh &= E_h\left(\widehat{y}_t^s(h)\right), \\
& \quad and \\
\int_0^1 \left(\widehat{y}_t^s(h)\right)^2 dh &= Var_h\left(\widehat{y}_t^s(h)\right)+\left[E_h\left(\widehat{y}_t^s(h)\right)\right]^2,
\end{aligned}
$$

we can rewrite (18) as

$$\int_0^1 v\left(L_t^s(h)\right) dh = v\left(L^s\right) + v_l L^s\left[1-(1+\chi)a_t^s\right] E_h\left(\widehat{y}_t^s(h)\right) \tag{19}$$
$$+v_l L^s \frac{1}{2}(1+\chi)\left\{Var_h\left(\widehat{y}_t^s(h)\right) + \left[E_h\left(\widehat{y}_t^s(h)\right)\right]^2\right\}$$
$$+t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right).$$

Next, we take a second-order approximation around the efficient symmetric zero-inflation steady state to the aggregate output Y_t^s

$$Y_t^s \equiv \left(\int_0^1 \left[Y_t^s(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}}, \forall s \in \mathbf{S},$$

and get that

$$\begin{aligned}
\widehat{y}_t^s &= \frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right)\left\{\int_0^1 \left(\widehat{y}_t^s(h)\right)^2 dh - \left[\int_0^1 \widehat{y}_t^s(h) dh\right]^2\right\} + \int_0^1 \widehat{y}_t^s(h) dh + \mathcal{O}\left(\|\zeta\|^3\right) \\
&= \frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right) Var_h\left(\widehat{y}_t^s(h)\right) + E_h\left(\widehat{y}_t^s(h)\right) + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \in \mathbf{S}. \qquad (20)
\end{aligned}$$

Given that up to a second-order

$$E_h\left(\widehat{y}_t^s(h)\right) = \widehat{y}_t^s - \frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right) Var_h\left(\widehat{y}_t^s(h)\right),$$

we have that

$$\int_0^1 v\left(L_t^s(h)\right) dh = v\left(L^s\right) + v_l L^s\left[1-(1+\chi)a_t^s\right]\left\{\widehat{y}_t^s - \frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right) Var_h\left(\widehat{y}_t^s(h)\right)\right\}$$
$$+v_l L^s \frac{1}{2}(1+\chi)\left\{Var_h\left(\widehat{y}_t^s(h)\right) + \left[\widehat{y}_t^s - \frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right) Var_h\left(\widehat{y}_t^s(h)\right)\right]^2\right\}$$
$$+t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right).$$

Up to a second-order,

$$\int_0^1 v\left(L_t^s(h)\right)dh = v\left(L^s\right) + v_l L^s\left[1-(1+\chi)a_t^s\right]\widehat{y}_t^s + v_l L^s\left[-\frac{1}{2}(\frac{\epsilon-1}{\epsilon})Var_h\left(\widehat{y}_t^s(h)\right)\right]$$

$$+v_l L^s\frac{1}{2}(1+\chi)\left\{\left(\widehat{y}_t^s\right)^2 + Var_h\left(\widehat{y}_t^s(h)\right)\right\} + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right)$$

$$= v\left(L^s\right) + v_l L^s\left\{\begin{array}{c} \widehat{y}_t^s + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^s\right)^2 + \frac{1}{2}(\frac{1}{\epsilon}+\chi)Var_h\left(\widehat{y}_t^s(h)\right) \\ -(1+\chi)\widehat{y}_t^s a_t^s \end{array}\right\}$$

$$+t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \in \mathbf{S}. \tag{21}$$

Following Beetsma and Jensen (2005),

$$\begin{aligned} Var_h\left(y_t^s(h)\right) &= \epsilon^2 Var_h\left(p_t^s(h)\right) + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \in \mathbf{S} \qquad (22) \\ with &: \\ Var_h\left(\widehat{y}_t^s(h)\right) &= Var_h\left(y_t^s(h)\right), \\ Var_h\left(\widehat{p}_t^s(h)\right) &= Var_h\left(p_t^s(h)\right), \end{aligned}$$

and (see also, e.g., Woodford (2003), chapter 6)

$$\sum_{t=0}^{\infty}\beta^t Var_h\left(p_t^s(h)\right) = \frac{\theta_S}{\left(1-\theta_S\beta\right)\left(1-\theta_S\right)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^s\right)^2 + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \in \mathbf{S}. \tag{23}$$

Thus,

$$\sum_{t=0}^{\infty}\beta^t Var_h(y_t^s(h)) = \frac{\epsilon^2\theta_S}{\left(1-\theta_S\beta\right)\left(1-\theta_S\right)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^s\right)^2 + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \in \mathbf{S}. \tag{24}$$

Substituting (22) in (21), we get

$$
\begin{aligned}
\int_0^1 v\left(L_t^s(h)\right)dh &= v\left(L^s\right)+v_l L^s\left\{\begin{array}{c}\widehat{y}_t^s+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^s\right)^2+\frac{1}{2}(\frac{1}{\epsilon}+\chi)\epsilon^2 Var_h\left(p_t^s(h)\right)\\ -(1+\chi)\widehat{y}_t^s a_t^s\end{array}\right\}\\
&\quad +t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right)\\
&= v\left(L^s\right)+v_l L^s\left\{\widehat{y}_t^s+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^s\right)^2+(1+\epsilon\chi)z_t^s-(1+\chi)\widehat{y}_t^s a_t^s\right\}\\
&\quad +t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right)\\
where &: \\
z_t^s &= \frac{\epsilon}{2}Var_h\left(p_t^s(h)\right),\\
\forall s &\in \mathbf{S}. \qquad (25)
\end{aligned}
$$

Proceeding in a similar fashion for country $\mathbf{B}$,

$$
\begin{aligned}
W_t^B &= \int_0^1 u\left(C_t^B\right)+V\left(G_t^B\right)-v\left(L_t^b(h)\right)dh\\
&= u\left(C_t^B\right)+V\left(G_t^B\right)-\int_0^1 v\left(L_t^b(h)\right)dh, \forall b\in\mathbf{B}. \qquad (26)
\end{aligned}
$$

Second-order approximations around the efficient symmetric zero-inflation steady state for the terms: $u\left(C_t^s\right)$, $V\left(G_t^s\right)$, yield,

$$
\begin{aligned}
u\left(C_t^B\right) &= u\left(C^B\right)+u_c C^B\left[\widehat{c}_t^B+\frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^B\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right), \qquad (27)\\
V\left(G_t^B\right) &= V\left(G^B\right)+V_g G^B\left[\widehat{g}_t^B+\frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^B\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right), \qquad (28)
\end{aligned}
$$

Taking a representative geographic unit b,

$$
\begin{aligned}
v\left(L_t^b(h)\right) &= v\left(L^b\right)+v_l L^b\left[\widehat{l}_t^b(h)+\frac{1}{2}(1+\chi)\left(\widehat{l}_t^b(h)\right)^2\right]+\mathcal{O}\left(\|\zeta\|^3\right)\\
&= v\left(L^b\right)+v_l L^b\left\{\widehat{y}_t^b(h)\left[1-(1+\chi)a_t^B\right]+\frac{1}{2}(1+\chi)\left(\widehat{y}_t^b(h)\right)^2\right\}\\
&\quad +t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right), \forall b. \qquad (29)
\end{aligned}
$$

using the fact that $Y_t^b(h)=A_t^B L_t^b(h)\Longrightarrow \widehat{l}_t^b(h)=\widehat{y}_t^b(h)-a_t^B$, *since* $L^b(h)=L^b=Y^b$, Integrating (29) over the population of geographic unit $b \in \mathbf{B}$,

$$
\begin{aligned}
\int_0^1 v\left(L_t^b(h)\right) dh &= v\left(L^b\right) + v_l L^b \left[1-(1+\chi)a_t^B\right] E_h\left(\widehat{y}_t^b(h)\right) \qquad (30) \\
&\quad + v_l L^b \frac{1}{2}(1+\chi)\left\{Var_h\left(\widehat{y}_t^b(h)\right) + \left[E_h\left(\widehat{y}_t^b(h)\right)\right]^2\right\} \\
&\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right). \\
where &: \\
\int_0^1 \widehat{y}_t^b(h) dh &= E_h\left(\widehat{y}_t^b(h)\right), \\
\int_0^1 \left(\widehat{y}_t^b(h)\right)^2 dh &= Var_h\left(\widehat{y}_t^b(h)\right) + \left[E_h\left(\widehat{y}_t^b(h)\right)\right]^2.
\end{aligned}
$$

After a second-order approximation around the efficient symmetric zero-inflation steady state to the aggregate output $Y_t^b \equiv \left(\int_0^1 \left[Y_t^b(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}}$, we get that

$$
E_h\left(\widehat{y}_t^b(h)\right) = \widehat{y}_t^b - \frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right) Var_h\left(\widehat{y}_t^b(h)\right),
$$

and can rewrite (30) as

$$
\begin{aligned}
\int_0^1 v\left(L_t^b(h)\right) dh &= v\left(L^b\right) + v_l L^b \left\{\begin{array}{c} \widehat{y}_t^b + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^b\right)^2 + \frac{1}{2}\left(\frac{1}{\epsilon}+\chi\right) Var_h\left(\widehat{y}_t^b(h)\right) \\ -(1+\chi)\widehat{y}_t^b a_t^B \end{array}\right\} \\
&\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall b \in \mathbf{B}. \qquad (31)
\end{aligned}
$$

up to a second-order.

Following Beetsma and Jensen (2005),

$$
\begin{aligned}
\int_0^1 v\left(L_t^b(h)\right) dh &= v\left(L^b\right) + v_l L^b \left\{\widehat{y}_t^b + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^b\right)^2 + (1+\epsilon\chi) z_t^b - (1+\chi)\widehat{y}_t^b a_t^B\right\} \\
&\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right) \\
where &: \\
z_t^b &= \frac{\epsilon}{2} Var_h\left(p_t^b(h)\right), \\
\forall b &\in \mathbf{B}, \qquad (32)
\end{aligned}
$$

using the fact that

$$Var_h\left(y_t^b(h)\right) = \epsilon^2 Var_h\left(p_t^b(h)\right) + \mathcal{O}\left(\|\zeta\|^3\right), \forall b \in \mathbf{B} \quad (33)$$

$$and$$

$$\sum_{t=0}^{\infty} \beta^t Var_h\left(p_t^b(h)\right) = \frac{\theta_B}{(1-\theta_B\beta)(1-\theta_B)} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^b\right)^2 + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall b \in \mathbf{B}. \quad (34)$$

Collecting the terms

Using the above derivations, and taking into account that $u(C)$, $V(G)$ and $v(L)$ are $t.i.p.$, we have that

$$\begin{aligned} W_t^s &= u(C_t^s) + V(G_t^s) - \int_0^1 v(L_t^s(h))\, dh \quad (35) \\ &= u_c C^s \left[\widehat{c}_t^s + \frac{1}{2}(\frac{\sigma-1}{\sigma})(\widehat{c}_t^s)^2\right] \\ &\quad + V_g G^s \left[\widehat{g}_t^s + \frac{1}{2}(\frac{\psi-1}{\psi})(\widehat{g}_t^s)^2\right] \\ &\quad - v_l L^s \left\{\widehat{y}_t^s + \frac{1}{2}(1+\chi)(\widehat{y}_t^s)^2 + (1+\epsilon\chi)z_t^s - (1+\chi)\widehat{y}_t^s a_t^s\right\} \\ &\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \end{aligned}$$

$$\begin{aligned} W_t^B &= u\left(C_t^B\right) + V\left(G_t^B\right) - \int_0^1 v\left(L_t^b(h)\right) dh \quad (36) \\ &= u_c C^B \left[\widehat{c}_t^B + \frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^B\right)^2\right] \\ &\quad + V_g G^B \left[\widehat{g}_t^B + \frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^B\right)^2\right] \\ &\quad - v_l L^b \left\{\widehat{y}_t^b + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^b\right)^2 + (1+\epsilon\chi)z_t^b - (1+\chi)\widehat{y}_t^b a_t^B\right\} \\ &\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall b \end{aligned}$$

From the optimality conditions of the social planner problem,

$$\begin{aligned}
(1-\alpha)^{\frac{1}{\gamma}}\left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}}\left(C_{s,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0\frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s\in S,\\
(1-n\alpha)^{\frac{1}{\gamma}}\left(C_t^B\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}}\left(C_{B,t}^B\right)^{-\frac{1}{\gamma}} &= \chi_0\frac{\left(L_t^B\right)^{\chi}}{A_t^B},\\
\psi_0\left(G_t^s\right)^{-\frac{1}{\psi}} &= \chi_0\frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s\in S,\\
\psi_0\left(G_t^B\right)^{-\frac{1}{\psi}} &= \chi_0\frac{\left(L_t^B\right)^{\chi}}{A_t^B},
\end{aligned}$$

Taking into account that in the efficient steady state:

$$\begin{aligned}
Y^s &= Y^b=Y^B=Y,\forall s\in S,\forall b\in B,\\
C^s &= C^b=C^B=C,\forall s\in S,\forall b\in B,\\
G^s &= G^b=G^B=G,\forall s\in S,\forall b\in B,\\
& \quad \textit{and}\\
C_s^s &= (1-\alpha)\,C^s,\forall s\in S,\\
C_B^B &= (1-n\alpha)\,C^B,
\end{aligned}$$

we have that

$$\begin{aligned}
\left(C^s\right)^{-\frac{1}{\sigma}} &= \chi_0\left(L^s\right)^{\chi},\forall s\in S,\\
\left(C^B\right)^{-\frac{1}{\sigma}} &= \chi_0\left(L^B\right)^{\chi},\\
\psi_0\left(G^s\right)^{-\frac{1}{\psi}} &= \chi_0\left(L^s\right)^{\chi},\forall s\in S,\\
\psi_0\left(G^B\right)^{-\frac{1}{\psi}} &= \chi_0\left(L^B\right)^{\chi}.
\end{aligned}$$

Hence, in the efficient steady state $u_c=V_g=v_l$. So, using the fact that $\frac{C}{Y}=(1-\varphi)$, $\frac{G}{Y}=\varphi$ and $L=Y$,

$$\begin{aligned}
u_cC &= \left(C^s\right)^{-\frac{1}{\sigma}}C,\\
V_gG &= \psi_0\left(G^s\right)^{-\frac{1}{\psi}}G=u_c\varphi Y=u_c\frac{\varphi}{(1-\varphi)}C,\\
v_lL &= \chi_0\left(L\right)^{\chi}L=u_cY=u_c\frac{1}{(1-\varphi)}C.
\end{aligned}$$

Using the above relations, we can rewrite (35) and (36) as

$$W_t^s = u_c C \left\{ \begin{array}{c} \left[\widehat{c}_t^s + \frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^s\right)^2\right] \\ +\frac{\varphi}{(1-\varphi)}\left[\widehat{g}_t^s + \frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^s\right)^2\right] \\ -\frac{1}{(1-\varphi)}\left[\widehat{y}_t^s + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^s\right)^2 + (1+\epsilon\chi)z_t^s - (1+\chi)\widehat{y}_t^s a_t^s\right] \end{array} \right\} +t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall s \tag{37}$$

$$W_t^B = u_c C \left\{ \begin{array}{c} \left[\widehat{c}_t^B + \frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^B\right)^2\right] \\ +\frac{\varphi}{(1-\varphi)}\left[\widehat{g}_t^B + \frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^B\right)^2\right] \\ -\frac{1}{(1-\varphi)}\left[\widehat{y}_t^b + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^b\right)^2 + (1+\epsilon\chi)z_t^b - (1+\chi)\widehat{y}_t^b a_t^B\right] \end{array} \right\} +t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall b \tag{38}$$

Working linear terms in W^*

Given that

$$W^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\int_0^n W_t^s ds + (1-n) W_t^B \right] \right\}$$

we can use (23) and (34) to obtain

$$W_t^s = u_c C \left\{ \begin{array}{c} \left[\widehat{c}_t^s + \frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^s\right)^2\right] \\ +\frac{\varphi}{(1-\varphi)}\left[\widehat{g}_t^s + \frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^s\right)^2\right] \\ -\frac{1}{(1-\varphi)}\left[\widehat{y}_t^s + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^s\right)^2 + \frac{1}{2}\frac{\epsilon}{\phi_S}(\pi_t^s)^2 - (1+\chi)\widehat{y}_t^s a_t^s\right] \end{array} \right\} +t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \forall s$$

$$with : \tag{39}$$

$$\phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)},$$

and

$$
\begin{aligned}
W_t^B &= u_c C \left\{ \begin{array}{c} \left[\widehat{c}_t^B + \frac{1}{2}(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^B\right)^2\right] \\ +\frac{\varphi}{(1-\varphi)}\left[\widehat{g}_t^B + \frac{1}{2}(\frac{\psi-1}{\psi})\left(\widehat{g}_t^B\right)^2\right] \\ -\frac{1}{(1-\varphi)}\left[\widehat{y}_t^B + \frac{1}{2}(1+\chi)\left(\widehat{y}_t^B\right)^2 + \frac{1}{2}\frac{\epsilon}{\phi_B}(\pi_t^B)^2 - (1+\chi)\widehat{y}_t^B a_t^B\right] \end{array} \right\} \\
&\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right), \qquad (40) \\
with &: \\
\phi_B &\equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)},
\end{aligned}
$$

where we also use the fact that $Y_t^b = Y_t^B$ and $P_t^b = P_t^B, \forall b \in \mathbf{B}$, and that the dispersion of prices is the same across all geographic units of country $\mathbf{B}$ (as geographic units are identical).

Collecting linear terms and second-order terms from (39) and (40), we have that

$$
\begin{aligned}
&\frac{W^*}{u_c C}(1-\varphi) = \qquad (41) \\
&E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} (1-\varphi)\left[\int_0^n \widehat{c}_t^s ds + (1-n)\widehat{c}_t^B\right] \\ +\varphi\left[\int_0^n \widehat{g}_t^s ds + (1-n)\widehat{g}_t^B\right] \\ -\left[\int_0^n \widehat{y}_t^s ds + (1-n)\widehat{y}_t^B\right] \end{array} \right\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\int_0^n \left\{-\frac{\epsilon}{\phi_S}(\pi_t^s)^2 + (1-\varphi)(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^s\right)^2 \right. \\ \left. +\varphi(\frac{\psi-1}{\psi})\left(\widehat{g}_t^s\right)^2 - (1+\chi)\left[\left(\widehat{y}_t^s\right)^2 - 2\widehat{y}_t^s a_t^s\right]\right\} ds \end{array} \right\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}(1-n)\left\{-\frac{\epsilon}{\phi_B}(\pi_t^B)^2 + (1-\varphi)(\frac{\sigma-1}{\sigma})\left(\widehat{c}_t^B\right)^2 \right. \\ \left. +\varphi(\frac{\psi-1}{\psi})\left(\widehat{g}_t^B\right)^2 - (1+\chi)\left[\left(\widehat{y}_t^B\right)^2 - 2\widehat{y}_t^B a_t^B\right]\right\} \end{array} \right\} \\
&+t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right)
\end{aligned}
$$

Next, we work the linear terms in (41).

The term $\left[\int_0^n \widehat{c}_t^s ds + (1-n)\widehat{c}_t^B\right]$

Note that from the definition of union-wide nominal private consumption:

$$\begin{aligned} C_t^* P_{c,t}^* &= \int_0^n C_t^s P_{c,t}^s ds + (1-n) C_t^B P_{c,t}^B \\ &\Leftrightarrow \\ C_t^* &= \int_0^n C_t^s \left(\frac{P_{c,t}^s}{P_{c,t}^*} \right) ds + (1-n) C_t^B \left(\frac{P_{c,t}^B}{P_{c,t}^*} \right). \end{aligned} \tag{42}$$

Moreover, from the definition of aggregate price indexes,

$$\begin{aligned} P_{c,t}^s &\equiv \left[(1-\alpha) (P_t^s)^{1-\gamma} + \alpha (P_t^*)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \\ &\Leftrightarrow \\ \frac{P_{c,t}^s}{P_t^*} &= \left[(1-\alpha) \left(\frac{P_t^s}{P_t^*} \right)^{1-\gamma} + \alpha \right]^{\frac{1}{1-\gamma}}, \forall s \in \mathbf{S}, \\ \textit{with } P_t^* &= P_{c,t}^*. \end{aligned}$$

$$\begin{aligned} P_t^* &\equiv \left[(1-n) \left(P_t^B \right)^{1-\gamma} + n \left(P_t^S \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \\ &\Leftrightarrow \\ n \left(P_t^S \right)^{1-\gamma} &= \left(P_t^* \right)^{1-\gamma} - (1-n) \left(P_t^B \right)^{1-\gamma}, \end{aligned}$$

$$P_{c,t}^B \equiv \left[(1-n\alpha) \left(P_t^B \right)^{1-\gamma} + n\alpha \left(P_t^S \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \forall b \in \mathbf{B}.$$

Substituting P_t^S in the definition of $P_{c,t}^B$, yields

$$\begin{aligned} P_{c,t}^B &\equiv \left[(1-\alpha) \left(P_t^B \right)^{1-\gamma} + \alpha (P_t^*)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \\ &\Leftrightarrow \\ \frac{P_{c,t}^B}{P_t^*} &\equiv \left[(1-\alpha) \left(\frac{P_t^B}{P_t^*} \right)^{1-\gamma} + \alpha \right]^{\frac{1}{1-\gamma}}, \forall b \in \mathbf{B}. \end{aligned}$$

Using the above equations, we can rewrite (42) as

$$\begin{aligned} C_t^* = & \int_0^n C_t^s \left[(1-\alpha) \left(\frac{P_t^s}{P_t^*} \right)^{1-\gamma} + \alpha \right]^{\frac{1}{1-\gamma}} ds \\ & + (1-n) C_t^B \left[(1-\alpha) \left(\frac{P_t^B}{P_t^*} \right)^{1-\gamma} + \alpha \right]^{\frac{1}{1-\gamma}} \end{aligned} \tag{43}$$

Equation (43) can be approximated around the symmetric zero-inflation steady state up to a second-order, yielding

$$
\begin{aligned}
& \int_0^n \widehat{c}_t^s ds + (1-n)\widehat{c}_t^B \\
= \; & \widehat{c}_t^* + \frac{1}{2}\left(\widehat{c}_t^*\right)^2 - \frac{1}{2}\left[\int_0^n \left(\widehat{c}_t^s\right)^2 ds + (1-n)\left(\widehat{c}_t^B\right)^2\right] \\
& -(1-\alpha)\left[\int_0^n \left(p_t^s - p_t^*\right) ds + (1-n)\left(p_t^B - p_t^*\right)\right] \\
& -\frac{1}{2}(1-\alpha)(1-\alpha\gamma)\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& -(1-\alpha)\left[\int_0^n \widehat{c}_t^s\left(p_t^s - p_t^*\right) ds + (1-n)\widehat{c}_t^B\left(p_t^B - p_t^*\right)\right]. \\
& +\mathcal{O}\left(\|\zeta\|^3\right) \qquad (44)
\end{aligned}
$$

From the definition of P_t^*,

$$
\begin{aligned}
P_t^* & \equiv \left[\int_0^n \left(P_t^s\right)^{1-\gamma} ds + (1-n)\left(P_t^B\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \\
& \Leftrightarrow \\
1 & = \int_0^n \left(\frac{P_t^s}{P_t^*}\right)^{1-\gamma} ds + (1-n)\left(\frac{P_t^B}{P_t^*}\right)^{1-\gamma}.
\end{aligned}
$$

and up to a second-order around the symmetric zero-inflation steady state,

$$
\begin{aligned}
& \int_0^n \left(p_t^s - p_t^*\right) ds + (1-n)\left(p_t^B - p_t^*\right) \\
= \; & -\frac{1}{2}(1-\gamma)\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] + \mathcal{O}\left(\|\zeta\|^3\right). \qquad (45)
\end{aligned}
$$

Substituting (45) in equation (44), yields

$$
\begin{aligned}
& \int_0^n \widehat{c}_t^s ds + (1-n)\widehat{c}_t^B \\
= \; & \widehat{c}_t^* + \frac{1}{2}\left(\widehat{c}_t^*\right)^2 - \frac{1}{2}\left[\int_0^n \left(\widehat{c}_t^s\right)^2 ds + (1-n)\left(\widehat{c}_t^B\right)^2\right] \\
& -\frac{1}{2}\gamma(1-\alpha)^2\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& -(1-\alpha)\left[\int_0^n \widehat{c}_t^s\left(p_t^s - p_t^*\right) ds + (1-n)\widehat{c}_t^B\left(p_t^B - p_t^*\right)\right] \\
& +\mathcal{O}\left(\|\zeta\|^3\right). \qquad (46)
\end{aligned}
$$

Moreover, taking into account

$$
Y_t^* = C_t^* + G_t^*,
$$

a second-order approximation around the symmetric zero-inflation steady state, yields

$$
\begin{aligned}
& \widehat{c}_t^* + \frac{1}{2}\left(\widehat{c}_t^*\right)^2 \\
= \; & \frac{\widehat{y}_t^* - \varphi\widehat{g}_t^*}{1-\varphi} + \frac{1}{2}\left[\frac{\left(\widehat{y}_t^*\right)^2 - \varphi\left(\widehat{g}_t^*\right)^2}{1-\varphi}\right] + \mathcal{O}\left(\|\zeta\|^3\right). \qquad (47)
\end{aligned}
$$

Substituting (47) in (46), we get that

$$
\begin{aligned}
& \int_0^n \widehat{c}_t^s ds + (1-n)\widehat{c}_t^B \\
= \; & \frac{\widehat{y}_t^* - \varphi \widehat{g}_t^*}{1-\varphi} + \frac{1}{2}\left[\frac{(\widehat{y}_t^*)^2 - \varphi\left(\widehat{g}_t^*\right)^2}{1-\varphi}\right] \\
& -\frac{1}{2}\left[\int_0^n \left(\widehat{c}_t^s\right)^2 ds + (1-n)\left(\widehat{c}_t^B\right)^2\right] \\
& -\frac{1}{2}\gamma(1-\alpha)^2\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& -(1-\alpha)\left[\int_0^n \widehat{c}_t^s\left(p_t^s - p_t^*\right) ds + (1-n)\widehat{c}_t^B\left(p_t^B - p_t^*\right)\right] \\
& +\mathcal{O}\left(\|\zeta\|^3\right).
\end{aligned}
\tag{48}
$$

The term $\left[\int_0^n \widehat{g}_t^s ds + (1-n)\widehat{g}_t^B\right]$

Using the fact that in nominal terms

$$
\begin{aligned}
P_t^* G_t^* &= \int_0^n P_t^s G_t^s ds + (1-n)\, P_t^B G_t^B \\
&\Leftrightarrow \\
G_t^* &= \int_0^n \left(\frac{P_t^s}{P_t^*}\right) G_t^s ds + (1-n)\left(\frac{P_t^B}{P_t^*}\right) G_t^B.
\end{aligned}
\tag{49}
$$

Up to a second-order, equation (49) can be approximated around the symmetric zero-inflation steady state, yielding

$$
\begin{aligned}
&\int_0^n \widehat{g}_t^s ds + (1-n)\widehat{g}_t^B \\
= \;& \widehat{g}_t^* + \frac{1}{2}\left(\widehat{g}_t^*\right)^2 - \frac{1}{2}\left[\int_0^n \left(\widehat{g}_t^s\right)^2 ds + (1-n)\left(\widehat{g}_t^B\right)^2\right] \\
& - \left[\int_0^n \left(p_t^s - p_t^*\right) ds + (1-n)\left(p_t^B - p_t^*\right)\right] \\
& - \frac{1}{2}\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& - \left[\int_0^n \widehat{g}_t^s \left(p_t^s - p_t^*\right) ds + (1-n)\widehat{g}_t^B \left(p_t^B - p_t^*\right)\right]. \\
& + \mathcal{O}\left(\|\zeta\|^3\right)
\end{aligned} \tag{50}
$$

Substituting (45) in equation (50), yields

$$
\begin{aligned}
&\int_0^n \widehat{g}_t^s ds + (1-n)\widehat{g}_t^B \\
= \;& \widehat{g}_t^* + \frac{1}{2}\left(\widehat{g}_t^*\right)^2 - \frac{1}{2}\left[\int_0^n \left(\widehat{g}_t^s\right)^2 ds + (1-n)\left(\widehat{g}_t^B\right)^2\right] \\
& - \frac{1}{2}\gamma\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& - \left[\int_0^n \widehat{g}_t^s \left(p_t^s - p_t^*\right) ds + (1-n)\widehat{g}_t^B \left(p_t^B - p_t^*\right)\right] + \mathcal{O}\left(\|\zeta\|^3\right).
\end{aligned} \tag{51}
$$

The term $\left[\int_0^n \widehat{y}_t^s ds + (1-n)\widehat{y}_t^B\right]$

Using the fact that in nominal terms

$$
\begin{aligned}
P_t^* Y_t^* &= \int_0^n P_t^s Y_t^s ds + (1-n) P_t^B Y_t^B \\
&\Leftrightarrow \\
Y_t^* &= \int_0^n \left(\frac{P_t^s}{P_t^*}\right) Y_t^s ds + (1-n) \left(\frac{P_t^B}{P_t^*}\right) Y_t^B. \qquad (52)
\end{aligned}
$$

Up to a second-order, equation (52) can be approximated around the symmetric zero-inflation steady state, yielding

$$
\begin{aligned}
& \int_0^n \widehat{y}_t^s ds + (1-n)\widehat{y}_t^B \\
= \ & \widehat{y}_t^* + \frac{1}{2}\left(\widehat{y}_t^*\right)^2 - \frac{1}{2}\left[\int_0^n \left(\widehat{y}_t^s\right)^2 ds + (1-n)\left(\widehat{y}_t^B\right)^2\right] \\
& - \left[\int_0^n \left(p_t^s - p_t^*\right) ds + (1-n)\left(p_t^B - p_t^*\right)\right] \\
& - \frac{1}{2}\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& - \left[\int_0^n \widehat{y}_t^s \left(p_t^s - p_t^*\right) ds + (1-n)\widehat{y}_t^B \left(p_t^B - p_t^*\right)\right]. \\
& + \mathcal{O}\left(\|\zeta\|^3\right) \qquad (53)
\end{aligned}
$$

Substituting (45) in equation (50), yields

$$
\begin{aligned}
& \int_0^n \widehat{y}_t^s ds + (1-n)\widehat{y}_t^B \\
= \ & \widehat{y}_t^* + \frac{1}{2}\left(\widehat{y}_t^*\right)^2 - \frac{1}{2}\left[\int_0^n \left(\widehat{y}_t^s\right)^2 ds + (1-n)\left(\widehat{y}_t^B\right)^2\right] \\
& - \frac{1}{2}\gamma\left[\int_0^n \left(p_t^s - p_t^*\right)^2 ds + (1-n)\left(p_t^B - p_t^*\right)^2\right] \\
& - \left[\int_0^n \widehat{y}_t^s \left(p_t^s - p_t^*\right) ds + (1-n)\widehat{y}_t^B \left(p_t^B - p_t^*\right)\right] + \mathcal{O}\left(\|\zeta\|^3\right). \qquad (54)
\end{aligned}
$$

Collecting the linear terms in W^{*}

Combining expressions (48), (51) and (54), we get that

$$(1-\varphi)\left[\int_0^n \widehat{c}_t^s ds+(1-n)\widehat{c}_t^B\right]+\varphi\left[\int_0^n \widehat{g}_t^s ds+(1-n)\widehat{g}_t^B\right]-\left[\int_0^n \widehat{y}_t^s ds+(1-n)\widehat{y}_t^B\right]$$

$$=$$

$$\int_0^n \left[\begin{array}{c} -\frac{(1-\varphi)}{2}\left(\widehat{c}_t^s\right)^2-\frac{\varphi}{2}\left(\widehat{g}_t^s\right)^2+\frac{1}{2}\left(\widehat{y}_t^s\right)^2-\frac{(1-\varphi)\gamma\alpha(\alpha-2)}{2}\left(p_t^s-p_t^*\right)^2 \\ -(1-\varphi)(1-\alpha)\widehat{c}_t^s\left(p_t^s-p_t^*\right)-\varphi\widehat{g}_t^s\left(p_t^s-p_t^*\right)+\widehat{y}_t^s\left(p_t^s-p_t^*\right)\end{array}\right]ds$$
$$+(1-n)\left[\begin{array}{c} -\frac{(1-\varphi)}{2}\left(\widehat{c}_t^B\right)^2-\frac{\varphi}{2}\left(\widehat{g}_t^B\right)^2+\frac{1}{2}\left(\widehat{y}_t^B\right)^2-\frac{(1-\varphi)\gamma\alpha(\alpha-2)}{2}\left(p_t^B-p_t^*\right)^2 \\ -(1-\varphi)(1-\alpha)\widehat{c}_t^B\left(p_t^B-p_t^*\right)-\varphi\widehat{g}_t^B\left(p_t^B-p_t^*\right)+\widehat{y}_t^B\left(p_t^B-p_t^*\right)\end{array}\right],$$

and after substituting it in (41),

$$\begin{aligned} & \frac{W_t^*}{u_c C}(1-\varphi) \qquad (55)\\ = \quad & +E_0\sum_{t=0}^{\infty}\beta^t \times \\ & \left\{\int_0^n \left[\begin{array}{c} -\frac{\epsilon}{2\phi_S}(\pi_t^s)^2-\frac{1}{\sigma}\frac{(1-\varphi)}{2}\left(\widehat{c}_t^s\right)^2-\frac{1}{\psi}\frac{\varphi}{2}\left(\widehat{g}_t^s\right)^2 \\ -\frac{\chi}{2}\left(\widehat{y}_t^s\right)^2+(1+\chi)\widehat{y}_t^s a_t^s-\frac{(1-\varphi)\gamma\alpha(\alpha-2)}{2}\left(p_t^s-p_t^*\right)^2 \\ -(1-\varphi)(1-\alpha)\widehat{c}_t^s\left(p_t^s-p_t^*\right) \\ -\varphi\widehat{g}_t^s\left(p_t^s-p_t^*\right)+\widehat{y}_t^s\left(p_t^s-p_t^*\right)\end{array}\right]ds\right. \\ & +(1-n)\left[\begin{array}{c} -\frac{\epsilon}{2\phi_B}(\pi_t^B)^2-\frac{1}{\sigma}\frac{(1-\varphi)}{2}\left(\widehat{c}_t^B\right)^2-\frac{1}{\psi}\frac{\varphi}{2}\left(\widehat{g}_t^B\right)^2 \\ -\frac{\chi}{2}\left(\widehat{y}_t^B\right)^2+(1+\chi)\widehat{y}_t^B a_t^B-\frac{(1-\varphi)\gamma\alpha(\alpha-2)}{2}\left(p_t^B-p_t^*\right)^2 \\ -(1-\varphi)(1-\alpha)\widehat{c}_t^B\left(p_t^B-p_t^*\right) \\ -\varphi\widehat{g}_t^B\left(p_t^B-p_t^*\right)+\widehat{y}_t^B\left(p_t^B-p_t^*\right)\end{array}\right] \\ & +t.i.p.+\mathcal{O}\left(\|\zeta\|^3\right)\end{aligned}$$

Substituting output terms in W^{*}

From the goods market clearing condition, we have that

$$\begin{aligned} \widehat{y}_t^j &= (1-\varphi)\,\widehat{c}_t^j+(1-\varphi)\,\Phi\left(p_t^*-p_t^j\right)+\varphi\widehat{g}_t^j, j=\mathbf{B},s\in\mathbf{S}, \\ \textit{where} &: \\ \Phi &\equiv \alpha\left[\gamma-(1-\alpha)\left(-\gamma+\sigma\right)\right]. \end{aligned}$$

up to a first-order approximation around the symmetric zero-inflation steady state. Substituting this expression into the second-order terms of W^* containing the output variable, we get

$$
\begin{aligned}
& \frac{W^*}{u_c C}(1-\varphi) \qquad (56) \\
= & +E_0 \sum_{t=0}^{\infty} \beta^t \times \\
& \left\{ \int_0^n \left[\begin{array}{c}
-\frac{\epsilon}{2\phi_S}(\pi_t^s)^2 + \left[-\frac{(1-\varphi)}{2\sigma} - \chi\frac{(1-\varphi)^2}{2}\right](\widehat{c}_t^s)^2 \\
+\left[-\frac{\varphi}{2\psi} - \chi\frac{\varphi^2}{2}\right](\widehat{g}_t^s)^2 \\
+\left[-\frac{(1-\varphi)\gamma\alpha(\alpha-2)}{2} - (1-\varphi)\Phi - \frac{\chi(1-\varphi)^2\Phi^2}{2}\right](p_t^s - p_t^*)^2 \\
+\left[-\chi(1-\varphi)\varphi\right]\widehat{c}_t^s\widehat{g}_t^s \\
+\left[(1-\varphi)\alpha + \chi(1-\varphi)^2\Phi\right]\widehat{c}_t^s(p_t^s - p_t^*) \\
+\left[\chi(1-\varphi)\varphi\Phi\right]\widehat{g}_t^s(p_t^s - p_t^*) \\
+(1+\chi)\left[(1-\varphi)\widehat{c}_t^s + \varphi\widehat{g}_t^s - (1-\varphi)\Phi(p_t^s - p_t^*)\right]a_t^s
\end{array} \right] ds \right. \\
& \left. +(1-n) \left[\begin{array}{c}
-\frac{\epsilon}{2\phi_B}(\pi_t^B)^2 + \left[-\frac{(1-\varphi)}{2\sigma} - \chi\frac{(1-\varphi)^2}{2}\right]\left(\widehat{c}_t^B\right)^2 \\
+\left[-\frac{\varphi}{2\psi} - \chi\frac{\varphi^2}{2}\right]\left(\widehat{g}_t^B\right)^2 \\
+\left[-\frac{(1-\varphi)\gamma\alpha(\alpha-2)}{2} - (1-\varphi)\Phi - \frac{\chi(1-\varphi)^2\Phi^2}{2}\right]\left(p_t^B - p_t^*\right)^2 \\
+\left[-\chi(1-\varphi)\varphi\right]\widehat{c}_t^B\widehat{g}_t^B \\
+\left[(1-\varphi)\alpha + \chi(1-\varphi)^2\Phi\right]\widehat{c}_t^B\left(p_t^B - p_t^*\right) \\
+\left[\chi(1-\varphi)\varphi\Phi\right]\widehat{g}_t^B\left(p_t^B - p_t^*\right) \\
+(1+\chi)\left[(1-\varphi)\widehat{c}_t^B + \varphi\widehat{g}_t^B - (1-\varphi)\Phi\left(p_t^B - p_t^*\right)\right]a_t^B
\end{array} \right] \right\} \\
& +t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right) \\
& where \\
\Phi \equiv & \ \alpha\left[\gamma - (1-\alpha)(-\gamma+\sigma)\right].
\end{aligned}
$$

Express everything in terms of gaps

Next, we use the fact that

$$
\begin{aligned}
tt_t^s &= (p_t^* - p_t^s), s \in \mathbf{S}, \\
tt_t^B &= \frac{1}{n}(p_t^* - p_t^B),
\end{aligned}
$$

and express variables in terms of gaps, obtaining

$$
\begin{aligned}
& \frac{W^*}{u_c C}(1-\varphi) && (58)\\
= \quad & -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t \times \\
& \left\{ + \int_0^n \left[\begin{array}{c} \frac{\epsilon}{\phi_S}(\pi_t^s)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^s)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^s)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](\widetilde{tt}_t^s)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^s\widetilde{g}_t^s \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^s\widetilde{tt}_t^s \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^s\widetilde{tt}_t^s \end{array} \right] ds \right. \\
& +(1-n)\left[\begin{array}{c} \frac{\epsilon}{\phi_B}(\pi_t^B)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^B)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^B)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](n\widetilde{tt}_t^B)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^B\widetilde{g}_t^B \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^B(n\widetilde{tt}_t^B) \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^B(n\widetilde{tt}_t^B) \end{array} \right] \\
& \left. +\Psi \right\} \\
& +t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right),
\end{aligned}
$$

where Ψ represents a collection of terms that yield zero (the proof is available upon request).

The loss function

Ignoring an irrelevant proportionality factor, the associated loss function is given by

$$
L^* = E_0\left\{\sum_{t=0}^{\infty}\beta^t L_t^*\right\},
$$

where the per-period social loss function (L_t^*) is defined as

$$
L_t^* = \int_0^n L_t^s ds + (1-n)L_t^B,
$$

and

$$
L_t^s = (\frac{1}{2}) \left\{ \begin{array}{c} \frac{\epsilon}{\phi_S}(\pi_t^s)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^s)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^s)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](\widetilde{tt}_t^s)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^s\widetilde{g}_t^s \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^s\widetilde{tt}_t^s \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^s\widetilde{tt}_t^s \end{array} \right\}, \forall s \in \mathbf{S},
$$

$$
L_t^B = (\frac{1}{2}) \left\{ \begin{array}{c} \frac{\epsilon}{\phi_B}(\pi_t^B)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^B)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^B)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](n\widetilde{tt}_t^B)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^B\widetilde{g}_t^B \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^B(n\widetilde{tt}_t^B) \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^B(n\widetilde{tt}_t^B) \end{array} \right\},
$$

$$
where: \left\{ \begin{array}{l} \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)} \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)} \\ \Phi \equiv \alpha\left[\gamma-(1-\alpha)(-\gamma+\sigma)\right] \end{array} \right. .
$$

Alternative procedure

Instead of aggregating and working the linear terms

$$
\left\{ \begin{array}{c} (1-\varphi)\left[\int_0^n \widehat{c}_t^s ds + (1-n)\widehat{c}_t^B\right] \\ +\varphi\left[\int_0^n \widehat{g}_t^s ds + (1-n)\widehat{g}_t^B\right] \\ -\left[\int_0^n \widehat{y}_t^s ds + (1-n)\widehat{y}_t^B\right] \end{array} \right\}
$$

in expression (41), we consider country specific loss functions and work the linear terms

$$
-\widehat{y}_t^j + (1-\varphi)\widehat{c}_t^j + \varphi\widehat{g}_t^j, j = \mathbf{B}, s \in \mathbf{S}.
$$

After some derivations, we get country specific loss functions identical to the ones in expression (58) plus some additional linear terms. Following Leith and Wren-Lewis (2011), we do not further develop these terms and exclude them from the country specific loss functions, since their aggregate value is zero. These linear terms capture the desire of national governments to manipulate their terms-of-trade to obtain additional national gains, but this manipulation is ineffective if all union members proceed in the same manner.

Appendix A.3:

Monetary Leadership and Nash between the Fiscal Authorities (derivation of a numerical algorithm)

This annex summarizes the iterative dynamic programming algorithm for the discretionary monetary leadership setup when fiscal authorities play a Nash game between them.

The game is played by the monetary authority and each one of the fiscal authorities in the union (explicit players), as well as by the private sector in each country (implicit players). In this type of game, the monetary authority (the leader) moves first and sets the interest rate. Then, the fiscal authorities choose simultaneously the right amount of fiscal policy instruments (government expenditures and the revenue tax rate). Finally, the private sector reacts being the ultimate follower.

Before continuing, we clarify the methodology used to represent the block made up of a continuum of small open economies. Since we consider a multi-country monetary union formed by a big country (with dimension 0.5) and a continuum of small countries (each one with zero dimension), it is not possible to represent each one of the small economies in our computational model. Additionally, we assume that all economies are subject to symmetric (union-wide) and asymmetric shocks. While symmetric and asymmetric shocks at the big country are easy to operate computationally (since shocks produce identical consequences for the small economies, we can consider a representative small country to mimic the behavior of all the small countries), the same is not true when dealing with an asymmetric shock at a small economy as it only produces domestic effects. To overcome this issue, we consider three economies in our optimization setup: a big economy (block **B**) and two small economies (i and ii), although one of the small economies (i) is considered only to serve as a proxy for the behavior of the block constituted by all the small countries (block **S**). This approach makes it possible to obtain fiscal policy rules for a small country.

We define all union-wide variables as an average of block **B** and block **S**, which represents the average of small economies and is defined as a simple average of only two small economies (i and ii). As mentioned, country i is used as a proxy of the behavior of block **S** and, hence, we consider that within block **S** the size of country i is almost 1 (0.99999). In turn, country ii is assumed to be very small within block

S (ii_dim=0.00001) so that when hit by an asymmetric shock there are no external consequences.

Although our approach makes it possible to represent the behavior of very small countries, the definition of block **S** raises some problems under the non-cooperative scenario. Since the definition of block **S** is embedded in the model structure equations, policy games take as given that all small countries behave in the same way because the size of country i is almost 1 within block **S**. This does not constitute a problem under the cooperative setup, but it is problematic for the non-cooperative regime. Thus, for non-cooperative games we introduce changes in the solving algorithms so that, at each iteration, the optimization is performed only for the monetary policy and the fiscal policies of the big and the small country ii, while the reaction of country i's fiscal instruments is set at the end of each iteration according to the optimal feedback rules obtained for country ii. Since economy i mimics the behavior of block **S**, by allowing its response to change at the end of each iteration, we constrain the reaction of the other agents in the following iteration, a process that will converge to the optimal solution. We will address this issue below.

To solve this type of game, the order of playing is inverted and we begin by solving the optimization of the last player, concluding with the optimization of the leader. Notice that the private sector's optimization problem is already solved out - see equations in section 2.8 - and can be represented by the following system, written in a state space form:

$$\widetilde{A}_0 \begin{bmatrix} Y_{t+1} \\ E_t\left(X_{t+1}\right) \end{bmatrix} = \widetilde{A}_1 \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \widetilde{B} \begin{bmatrix} U_t^B \\ U_t^i \\ U_t^{ii} \end{bmatrix} + \widetilde{D} \begin{bmatrix} U_t^M \end{bmatrix} + \widetilde{C}\varepsilon_{t+1}, \qquad (60)$$

where Y_t is an n_1-vector of predetermined state variables, Y_0 is given, and X_t are the effective instruments of the private sector, an n_2-vector of non-predetermined or forward-looking variables ($n = n_1 + n_2$).The policy instruments are represented by U_t^B, U_t^i, U_t^{ii} and U_t^M. U_t^B, U_t^i and U_t^{ii} represent the instruments of the followers - the fiscal authorities, while U_t^M stands for the monetary authority instrument, the nominal interest rate. ε_{t+1} is an n_ε-vector of exogenous zero-mean iid shocks with an identity covariance matrix. Premultiplying (60) by $\left(\widetilde{A}_0\right)^{-1}$, we get

$$\begin{bmatrix} Y_{t+1} \\ E_t\left(X_{t+1}\right) \end{bmatrix} = A \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + B \begin{bmatrix} U_t^B \\ U_t^i \\ U_t^{ii} \end{bmatrix} + D \begin{bmatrix} U_t^M \end{bmatrix} + C\varepsilon_{t+1}, \tag{61}$$

where $A = \left(\widetilde{A}_0\right)^{-1} \widetilde{A}_1$; $B = \left(\widetilde{A}_0\right)^{-1} \widetilde{B}$; $D = \left(\widetilde{A}_0\right)^{-1} \widetilde{D}$; $C = \left(\widetilde{A}_0\right)^{-1} \widetilde{C}$. The covariance matrix of the shocks to Y_{t+1} is CC' and matrices are partioned conformably with Y_t and X_t as:

$$\begin{aligned} A &\equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; B \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix}; \\ D &\equiv \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}; C \equiv \begin{bmatrix} C_1 \\ \mathbf{0} \end{bmatrix}. \end{aligned} \tag{62}$$

The followers' optimization problem

In the discretionary setup the policymakers reoptimize every period by taking the process by which private agents form their expectations as given, and where the expectations are consistent with actual policies (Söderlind (1999)). In turn, fiscal authorities, who play Nash between them, minimize their individual loss functions considering the monetary policy instrument as parametric but incorporating the reaction function of private agents.

We assume that the fiscal authority of the big country **B** has the following objective function:

$$\begin{aligned} &\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(G_t^{B\prime} \mathcal{Q}^B G_t^B\right) = \\ &= \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(Z_t' Q^B Z_t + Z_t' P^B U_t + U_t' P^{B\prime} Z_t + U_t' R^B U_t\right) \end{aligned} \tag{63}$$

where G_t^B is the target variables for country B's fiscal authority and $\mathcal{Q}^B$ is the corresponding $((n_1 + n_2 + k) \times (n_1 + n_2 + k))$ matrix of weights (where k stands for the number of policy instruments), partioned accordingly:

$$\mathcal{Q}^B \equiv \begin{bmatrix} \mathcal{Q}_{11}^B & \mathcal{Q}_{12}^B & \mathcal{Q}_{13}^B \\ \mathcal{Q}_{21}^B & \mathcal{Q}_{22}^B & \mathcal{Q}_{23}^B \\ \mathcal{Q}_{31}^B & \mathcal{Q}_{32}^B & \mathcal{Q}_{33}^B \end{bmatrix} = \\ = \begin{bmatrix} Q^B & P^B \\ P^{B\prime} & R^B \end{bmatrix},$$

where Q^B is $((n_1 + n_2) \times (n_1 + n_2))$, P^B is $((n_1 + n_2) \times k)$ and Q^B is $(k \times k)$.

The target variables can be rewritten in terms of the predetermined and non-predetermined state variables collected on vector $Z_t \equiv (Y_t', X_t')'$, in terms of the policy instruments ($U_t \equiv (U_t^{M\prime}, U_t^{B\prime}, U_t^{i\prime}, U_t^{ii\prime})'$) and in terms of the combinations of these two variables.

The fiscal authority optimizes every period , taking into account that she will be able to reoptimize next period. Since the model is linear-quadratic in the state variables,

$$W_{t+1}^B \equiv \frac{1}{2}\left(Y_{t+1}' S_{t+1}^B Y_{t+1} + w_{t+1}^B\right),$$

where S_{t+1}^B is a positive semidefinite matrix and w_{t+1}^B is a scalar independent of Y_{t+1}. Moreover, the forward looking variables must be linear functions of the state variables, $X_{t+1} = -N_{t+1}Y_{t+1}$. Thus, the value function of the fiscal authority of country **B** in t will then satisfy the Bellman equation:

$$\begin{aligned} W_t^B &= \min_{U_t^B} \frac{1}{2}\left[Z_t' Q^B Z_t + Z_t' P^B U_t + U_t' P^{B\prime} Z_t + U_t' R^B U_t\right] + \beta E_t(W_{t+1}^B) \\ & \quad s.t. \qquad (64) \\ E_t(X_{t+1}) &= -N_{t+1}E_t(Y_{t+1}), \\ W_{t+1}^B &\equiv \frac{1}{2}\left(Y_{t+1}' S_{t+1}^B Y_{t+1} + w_{t+1}^B\right), eq.(61) \text{ and } Y_t \text{ given.} \end{aligned}$$

Rewriting the system by using $E_t(X_{t+1}) = -N_{t+1}E_t(Y_{t+1})$

Using expression $E_t(X_{t+1}) = -N_{t+1}E_t(Y_{t+1})$ to substitute into the upper block of (61), we get

$$E_t(X_{t+1}) = -N_{t+1}(A_{11}Y_t + A_{12}X_t + B_{11}U_t^B + B_{12}U_t^i + B_{13}U_t^{ii} + D_1U_t^M),$$

while the lower block of (61) is

$$E_t(X_{t+1}) = A_{21}Y_t + A_{22}X_t + B_{21}U_t^B + B_{22}U_t^i + B_{23}U_t^{ii} + D_2U_t^M,$$

Equating previous expressions, we obtain (assuming the invertibility of matrix $(A_{22} + N_{t+1}A_{12})$)

$$\begin{aligned}
X_t &= -J_tY_t - K_t^BU_t^B - K_t^iU_t^i - K_t^{ii}U_t^{ii} - K_t^MU_t^M, \qquad (65)\\
with &: \\
J_t &= (A_{22} + N_{t+1}A_{12})^{-1}(A_{21} + N_{t+1}A_{11}),\\
K_t^B &= (A_{22} + N_{t+1}A_{12})^{-1}(B_{21} + N_{t+1}B_{11}),\\
K_t^i &= (A_{22} + N_{t+1}A_{12})^{-1}(B_{22} + N_{t+1}B_{12}),\\
K_t^{ii} &= (A_{22} + N_{t+1}A_{12})^{-1}(B_{23} + N_{t+1}B_{13}),\\
K_t^M &= (A_{22} + N_{t+1}A_{12})^{-1}(D_2 + N_{t+1}D_1),
\end{aligned}$$

where J_t is $(n_2 \times n_1)$, K_t^B is $(n_2 \times k_B)$, K_t^i is $(n_2 \times k_i)$, K_t^{ii} is $(n_2 \times k_{ii})$ and K_t^M is $(n_2 \times k_M)$. k_B, k_i, k_{ii} stand, respectively, for the number of fiscal policy instruments of countries **B**, i and ii (with $k_B = k_i = k_{ii} = 2$), while k_M stands for the number of monetary policy instruments ($k_M = 1$).

The evolution of Y_t

Given expression (65), we can use it in the first n_1 equations in the system (61) to obtain the reduced form evolution of the predetermined variables

$$\begin{aligned}
Y_{t+1} &= O_t^YY_t + O_t^BU_t^B + O_t^iU_t^i + O_t^{ii}U_t^{ii} + O_t^MU_t^M + C_1\varepsilon_{t+1}, \qquad (66)\\
with &: \\
O_t^Y &= A_{11} - A_{12}J_t,\\
O_t^B &= B_{11} - A_{12}K_t^B,\\
O_t^i &= B_{12} - A_{12}K_t^i,\\
O_t^{ii} &= B_{13} - A_{12}K_t^{ii},\\
O_t^M &= D_1 - A_{12}K_t^M,
\end{aligned}$$

where O_t^Y is $(n_1 \times n_1)$, O_t^B is $(n_1 \times k_B)$, O_t^i is $(n_1 \times k_i)$, O_t^{ii} is $(n_1 \times k_{ii})$ and O_t^M is $(n_1 \times k_M)$.

Being a follower, country **B**'s fiscal authority observes monetary authority's actions and reacts to them. In a linear quadratic setup, the optimal solution belongs to the class of linear feedback rules of the form:

$$U_t^B = -F_t^B Y_t - L_t^B U_t^M, \tag{67}$$

where F_t^B denotes feedback coefficients on the predetermined state variables and L_t^B is the leadership parameter.

Small countries' fiscal authorities solve a similar problem and get:

$$\begin{aligned} U_t^i &= -F_t^i Y_t - L_t^i U_t^M, \\ U_t^{ii} &= -F_t^{ii} Y_t - L_t^{ii} U_t^M, \\ &\quad ... \\ &\quad for\ other\ small\ countries\ in\ block\ \mathbf{S} \end{aligned} \tag{68}$$

Notice that being in a Nash game fiscal authorities do not respond to each other's actions. In turn, being the leader, the monetary authority takes into account these fiscal policy reaction functions as well as the private agents' optimal conditions, when solves its optimization problem. Hence, the monetary authority can manipulate the followers by changing its policy instruments.

The monetary authority reaction function takes the form:

$$U_t^M = -F_t^M Y_t. \tag{69}$$

Reformulated optimization problem

We can substitute equations (65) and (66) into the Bellman equation (64) to obtain an equivalent minimization problem. Making use of the fact that w_{t+1}^B is independent of Y_{t+1} and $E_t(\varepsilon_{t+1}) = 0$, we get

$$
\begin{aligned}
2\widetilde{W}_t^B = \min_{U_t^B} \{ & Y_t' \left[Q_B + \beta O_t^{Y\prime} E_t(S_{t+1}^B) O_t^Y \right] Y_t + \\
& + U_t^{B\prime} \left[\mathcal{U}_B^{B\prime} + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^Y \right] Y_t + Y_t' \left[\mathcal{U}_B^B + \beta O_t^{Y\prime} E_t(S_{t+1}^B) O_t^B \right] U_t^B \\
& + U_t^{i\prime} \left[\mathcal{U}_i^{B\prime} + \beta O_t^{i\prime} E_t(S_{t+1}^B) O_t^Y \right] Y_t + Y_t' \left[\mathcal{U}_i^B + \beta O_t^{Y\prime} E_t(S_{t+1}^B) O_t^i \right] U_t^i \\
& + U_t^{ii\prime} \left[\mathcal{U}_{ii}^{B\prime} + \beta O_t^{ii\prime} E_t(S_{t+1}^B) O_t^Y \right] Y_t + Y_t' \left[\mathcal{U}_{ii}^B + \beta O_t^{Y\prime} E_t(S_{t+1}^B) O_t^{ii} \right] U_t^{ii} \\
& + U_t^{M\prime} \left[\mathcal{U}_M^{B\prime} + \beta O_t^{M\prime} E_t(S_{t+1}^B) O_t^Y \right] Y_t + Y_t' \left[\mathcal{U}_M^B + \beta O_t^{Y\prime} E_t(S_{t+1}^B) O_t^M \right] U_t^M \\
& + U_t^{B\prime} \left[\mathcal{R}_B^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^B \right] U_t^B + U_t^{i\prime} \left[\mathcal{R}_i^B + \beta O_t^{i\prime} E_t(S_{t+1}^B) O_t^i \right] U_t^i \\
& + U_t^{ii\prime} \left[\mathcal{R}_{ii}^B + \beta O_t^{ii\prime} E_t(S_{t+1}^B) O_t^{ii} \right] U_t^{ii} + U_t^{M\prime} \left[\mathcal{R}_M^B + \beta O_t^{M\prime} E_t(S_{t+1}^B) O_t^M \right] U_t^M \\
& + U_t^{B\prime} \left[\mathcal{P}_{B,i}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^i \right] U_t^i + U_t^{i\prime} \left[\mathcal{P}_{B,i}^{B\prime} + \beta O_t^{i\prime} E_t(S_{t+1}^B) O_t^B \right] U_t^B \\
& + U_t^{B\prime} \left[\mathcal{P}_{B,ii}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^{ii} \right] U_t^{ii} + U_t^{ii\prime} \left[\mathcal{P}_{B,ii}^{B\prime} + \beta O_t^{ii\prime} E_t(S_{t+1}^B) O_t^B \right] U_t^B \\
& + U_t^{B\prime} \left[\mathcal{P}_{B,M}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^M \right] U_t^M + U_t^{M\prime} \left[\mathcal{P}_{B,M}^{B\prime} + \beta O_t^{M\prime} E_t(S_{t+1}^B) O_t^B \right] U_t^B \\
& + U_t^{i\prime} \left[\mathcal{P}_{i,ii}^B + \beta O_t^{i\prime} E_t(S_{t+1}^B) O_t^{ii} \right] U_t^{ii} + U_t^{ii\prime} \left[\mathcal{P}_{i,ii}^{B\prime} + \beta O_t^{ii\prime} E_t(S_{t+1}^B) O_t^i \right] U_t^i \\
& + U_t^{i\prime} \left[\mathcal{P}_{i,M}^B + \beta O_t^{i\prime} E_t(S_{t+1}^B) O_t^M \right] U_t^M + U_t^{M\prime} \left[\mathcal{P}_{i,M}^{B\prime} + \beta O_t^{M\prime} E_t(S_{t+1}^B) O_t^i \right] U_t^i \\
& U_t^{ii\prime} \left[\mathcal{P}_{ii,M}^B + \beta O_t^{ii\prime} E_t(S_{t+1}^B) O_t^M \right] U_t^M + U_t^{M\prime} \left[\mathcal{P}_{ii,M}^{B\prime} + \beta O_t^{M\prime} E_t(S_{t+1}^B) O_t^{ii} \right] U_t^{ii} \}
\end{aligned} \tag{70}
$$

where

$$
\begin{aligned}
Q_B &= Q_{11}^B - J_t' Q_{21}^B - Q_{12}^B J_t + J_t' Q_{22}^B J_t, \\
\mathcal{U}_B^B &= -Q_{12}^B K_t^B + J_t' Q_{22}^B K_t^B + P_{12}^B - J_t' P_{22}^B, \\
\mathcal{U}_i^B &= -Q_{12}^B K_t^i + J_t' Q_{22}^B K_t^i + P_{13}^B - J_t' P_{23}^B, \\
\mathcal{U}_{ii}^B &= -Q_{12}^B K_t^{ii} + J_t' Q_{22}^B K_t^{ii} + P_{14}^B - J_t' P_{24}^B, \\
\mathcal{U}_M^B &= -Q_{12}^B K_t^M + J_t' Q_{22}^B K_t^M + P_{11}^B - J_t' P_{21}^B, \\
\mathcal{R}_B^B &= K_t^{B\prime} Q_{22}^B K_t^B - K_t^{B\prime} P_{22}^B - P_{22}^{B\prime} K_t^B + R_{22}^B, \\
\mathcal{R}_i^B &= K_t^{i\prime} Q_{22}^B K_t^i - K_t^{i\prime} P_{23}^B - P_{23}^{B\prime} K_t^i + R_{33}^B, \\
\mathcal{R}_{ii}^B &= K_t^{ii\prime} Q_{22}^B K_t^{ii} - K_t^{ii\prime} P_{24}^B - P_{24}^{B\prime} K_t^{ii} + R_{44}^B, \\
\mathcal{R}_M^B &= K_t^{M\prime} Q_{22}^B K_t^M - K_t^{M\prime} P_{21}^B - P_{21}^{B\prime} K_t^M + R_{11}^B, \\
\mathcal{P}_{B,i}^B &= K_t^{B\prime} Q_{22}^B K_t^i - K_t^{B\prime} P_{23}^B - P_{22}^{B\prime} K_t^i + R_{23}^B, \\
\mathcal{P}_{B,ii}^B &= K_t^{B\prime} Q_{22}^B K_t^{ii} - K_t^{B\prime} P_{24}^B - P_{22}^{B\prime} K_t^{ii} + R_{24}^B, \\
\mathcal{P}_{B,M}^B &= K_t^{B\prime} Q_{22}^B K_t^M - K_t^{B\prime} P_{21}^B - P_{22}^{B\prime} K_t^M + R_{21}^B, \\
\mathcal{P}_{i,ii}^B &= K_t^{i\prime} Q_{22}^B K_t^{ii} - K_t^{i\prime} P_{24}^B - P_{23}^{B\prime} K_t^{ii} + R_{34}^B, \\
\mathcal{P}_{i,M}^B &= K_t^{i\prime} Q_{22}^B K_t^M - K_t^{i\prime} P_{21}^B - P_{23}^{B\prime} K_t^M + R_{31}^B, \\
\mathcal{P}_{ii,M}^B &= K_t^{ii\prime} Q_{22}^B K_t^M - K_t^{ii\prime} P_{21}^B - P_{24}^{B\prime} K_t^M + R_{41}^B.
\end{aligned} \tag{71}
$$

Hence, the problem faced by the big country (**B**)'s fiscal authority has been transformed to a standard linear-quadratic regulator problem without forward looking variables but with time varying parameters. The first-order condition is

$$
\begin{gathered}
\frac{\partial(2W_t^B)}{\partial U_t^B} = 0 \Leftrightarrow \\
\left[\mathcal{U}_B^{B\prime} + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^Y\right] Y_t + \left[\mathcal{R}_B^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^B\right] U_t^B \\
+ \left[\mathcal{P}_{B,i}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^i\right] U_t^i + \left[\mathcal{P}_{B,ii}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^{ii}\right] U_t^{ii} \\
+ \left[\mathcal{P}_{B,M}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^M\right] U_t^M \\
= 0
\end{gathered}
$$

Using the fact that:

$$
\begin{aligned}
U_t^B &= -F_t^B Y_t - L_t^B U_t^M, \\
U_t^i &= -F_t^i Y_t - L_t^i U_t^M, \\
U_t^{ii} &= -F_t^{ii} Y_t - L_t^{ii} U_t^M,
\end{aligned}
$$

the first-order condition can be solved for the feedback coefficients of the reaction function of country **B**'s fiscal authority:

$$
F_t^B = \left[\mathcal{R}_B^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^B\right]^{-1} \times \left\{ \begin{array}{c} \left[\mathcal{U}_B^{B\prime} + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^Y\right] \\ -\left[\mathcal{P}_{B,i}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^i\right] F_t^i \\ -\left[\mathcal{P}_{B,ii}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^{ii}\right] F_t^{ii} \end{array} \right\}, \tag{72}
$$

$$
L_t^B = \left[\mathcal{R}_B^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^B\right]^{-1} \times \left\{ \begin{array}{c} \left[\mathcal{P}_{B,M}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^M\right] \\ -\left[\mathcal{P}_{B,i}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^i\right] L_t^i \\ -\left[\mathcal{P}_{B,ii}^B + \beta O_t^{B\prime} E_t(S_{t+1}^B) O_t^{ii}\right] L_t^{ii} \end{array} \right\}. \tag{73}
$$

Finding the recursive equation for S_{t+1}^B

Substituting the decision rules

$$
\begin{aligned}
U_t^M &= -F_t^M Y_t, \\
U_t^B &= -F_t^B Y_t - L_t^B U_t^M = -(F_t^B - L_t^B F_t^M) Y_t, \\
U_t^i &= -F_t^i Y_t - L_t^i U_t^M = -(F_t^i - L_t^i F_t^M) Y_t, \\
U_t^{ii} &= -F_t^{ii} Y_t - L_t^{ii} U_t^M = -(F_t^{ii} - L_t^{ii} F_t^M) Y_t,
\end{aligned}
$$

in $2\widetilde{W}_t^B$, we obtain the recursive equation for S_t^B:

$$
S_t^B = T_{0,t}^B + \beta T_t^{B\prime} E_t(S_{t+1}^B) T_t^B \tag{74}
$$

where

$$
\begin{aligned}
T_{0,t}^B = \; & \{Q_B - (F_t^B - L_t^B F_t^M)'\mathcal{U}_B^{B\prime} - \mathcal{U}_B^B(F_t^B - L_t^B F_t^M) \\
& -(F_t^i - L_t^i F_t^M)'\mathcal{U}_i^{B\prime} - \mathcal{U}_i^B(F_t^i - L_t^i F_t^M) \\
& -(F_t^{ii} - L_t^{ii} F_t^M)'\mathcal{U}_{ii}^{B\prime} - \mathcal{U}_{ii}^B(F_t^{ii} - L_t^{ii} F_t^M) - F_t^{M\prime}\mathcal{U}_M^{B\prime} - \mathcal{U}_M^B F_t^M \\
& +(F_t^B - L_t^B F_t^M)'\mathcal{R}_B^B(F_t^B - L_t^B F_t^M) + (F_t^i - L_t^i F_t^M)'\mathcal{R}_i^B(F_t^i - L_t^i F_t^M) \\
& +(F_t^{ii} - L_t^{ii} F_t^M)'\mathcal{R}_{ii}^B(F_t^{ii} - L_t^{ii} F_t^M) + F_t^{M\prime}\mathcal{R}_M^B F_t^M \\
& +(F_t^B - L_t^B F_t^M)'\mathcal{P}_{B,i}^B(F_t^i - L_t^i F_t^M) + (F_t^i - L_t^i F_t^M)'\mathcal{P}_{B,i}^{B\prime}(F_t^B - L_t^B F_t^M) \\
& +(F_t^B - L_t^B F_t^M)'\mathcal{P}_{B,ii}^B(F_t^{ii} - L_t^{ii} F_t^M) + (F_t^{ii} - L_t^{ii} F_t^M)'\mathcal{P}_{B,ii}^{B\prime}(F_t^B - L_t^B F_t^M) \\
& +(F_t^B - L_t^B F_t^M)'\mathcal{P}_{B,M}^B F_t^M + F_t^{M\prime}\mathcal{P}_{B,M}^{B\prime}(F_t^B - L_t^B F_t^M) \\
& +(F_t^i - L_t^i F_t^M)'\mathcal{P}_{i,ii}^B(F_t^{ii} - L_t^{ii} F_t^M) + (F_t^{ii} - L_t^{ii} F_t^M)'\mathcal{P}_{i,ii}^{B\prime}(F_t^i - L_t^i F_t^M) \\
& +(F_t^i - L_t^i F_t^M)'\mathcal{P}_{i,M}^B F_t^M + F_t^{M\prime}\mathcal{P}_{i,M}^{B\prime}(F_t^i - L_t^i F_t^M) \\
& +(F_t^{ii} - L_t^{ii} F_t^M)'\mathcal{P}_{ii,M}^B F_t^M + F_t^{M\prime}\mathcal{P}_{ii,M}^{B\prime}(F_t^{ii} - L_t^{ii} F_t^M\} .
\end{aligned} \tag{75}
$$

and

$$
T_t^B = O_t^Y - O_t^B(F_t^B - L_t^B F_t^M) - O_t^i(F_t^i - L_t^i F_t^M) - O_t^{ii}(F_t^{ii} - L_t^{ii} F_t^M) - O_t^M F_t^M . \tag{76}
$$

Similar formulae can be derived for the other countries.

The leader's (monetary authority) optimization problem

This part of the problem is the standard optimization problem when the system under control evolves as

$$
\begin{bmatrix} Y_{t+1} \\ E_t\left(X_{t+1}\right) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix} \begin{bmatrix} U_t^B \\ U_t^i \\ U_t^{ii} \end{bmatrix} + \\
+ \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \begin{bmatrix} U_t^M \end{bmatrix} + \begin{bmatrix} C_1 \\ \mathbf{0} \end{bmatrix} \varepsilon_{t+1}.
$$

Substituting the followers' reaction functions

$$
\begin{aligned}
U_t^B &= -F_t^B Y_t - L_t^B U_t^M, \\
U_t^i &= -F_t^i Y_t - L_t^i U_t^M, \\
U_t^{ii} &= -F_t^{ii} Y_t - L_t^{ii} U_t^M,
\end{aligned}
$$

we obtain

$$
\begin{aligned}
\begin{bmatrix} Y_{t+1} \\ E_t\left(X_{t+1}\right) \end{bmatrix} &= \begin{bmatrix} A_{11} - B_{11}F_t^B - B_{12}F_t^i - B_{13}F_t^{ii} & A_{12} \\ A_{21} - B_{21}F_t^B - B_{22}F_t^i - B_{23}F_t^{ii} & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \\
&+ \begin{bmatrix} D_1 - B_{11}L_t^B - B_{12}L_t^i - B_{13}L_t^{ii} \\ D_2 - B_{21}L_t^B - B_{22}L_t^i - B_{23}L_t^{ii} \end{bmatrix} \begin{bmatrix} U_t^M \end{bmatrix} + \begin{bmatrix} C_1 \\ \mathbf{0} \end{bmatrix} \varepsilon_{t+1}.
\end{aligned} \tag{77}
$$

The monetary loss function is

$$
\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(G_t^{M\prime} \mathcal{Q}^M G_t^M \right). \tag{78}
$$

However, since the leader integrates the followers' reaction functions into its optimization problem we can rewrite this loss function in terms of the relevant variables for the leadership authority.

Since

$$
\begin{bmatrix} Y_t \\ X_t \\ U_t^M \\ U_t^B \\ U_t^i \\ U_t^{ii} \end{bmatrix} = \underbrace{\begin{bmatrix} I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I \\ -F_t^B & \mathbf{0} & -L_t^B \\ -F_t^i & \mathbf{0} & -L_t^i \\ -F_t^{ii} & \mathbf{0} & -L_t^{ii} \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} Y_t \\ X_t \\ U_t^M \end{bmatrix},
$$

we can set

$$
\begin{aligned}
G_t^{M\prime}\mathcal{Q}^M G_t^M &= \left[\begin{array}{ccc} Y_t' & X_t' & U_t^{M\prime} \end{array}\right] \widetilde{\mathcal{K}}^M \left[\begin{array}{c} Y_t \\ X_t \\ U_t^M \end{array}\right], \\
where &: \\
\widetilde{\mathcal{K}}^M &= \mathcal{C}'\underbrace{C^{M\prime}\mathcal{Q}^M C^M}_{\mathcal{K}^M}\mathcal{C}
\end{aligned}
$$

and $\widetilde{\mathcal{K}}^M$ have to be partioned conformably with $\left[\begin{array}{ccc} Y_t' & X_t' & U_t^{M\prime} \end{array}\right]'$.

The iterative procedure

We start with initial approximation for the monetary policy rule, $F_{(0)}^M$, symmetric positive definite matrices (usually, identity matrices), $S_{(0)}^B$, $S_{(0)}^i$ and $S_{(0)}^{ii}$, some (e.g. a matrix of zeros) $N_{(0)}$ and solve the follower's optimization problem using equations (72)-(76) for the big country (**B**) and equivalent equations for the other countries. At this stage we introduce changes in the solving algorithms so that, at each iteration, the optimization is performed only for the big country and the small country ii's fiscal authorities, while the reaction of country i's fiscal instruments is set at the end of each iteration according to the optimal feedback rules obtained for country ii, as explained in next section. On one hand, this allows representing not only the behavior of block S but also the behavior of the union as a whole, and, on the other hand, avoids assuming *a priori* the same behavior for all small countries, what would replicate the case of a two-country monetary union.

After the optimization is performed only for the big country and the small country ii's fiscal authorities, we get $F_{(0)}^B$, $L_{(0)}^B$, as well as $F_{(0)}^i$, $L_{(0)}^i$, define $F_{(0)}^{ii}$, $L_{(0)}^{ii}$ (according to $F_{(0)}^i$ and $L_{(0)}^i$) and obtain updated matrices $S_{(1)}^B$ and $S_{(1)}^{ii}$. Next, we take into account the policy reaction functions of fiscal authorities and compute new matrices in equation (77), updated target variable ($G_t^M = C^M\mathcal{C}\left[\begin{array}{ccc} Y_t' & X_t' & U_t^{M\prime} \end{array}\right]'$) and solve the problem for the monetary authority. This will give us the monetary policy reaction function, $F_{(1)}^M$, and updated matrices $N_{(1)}$ and $S_{(1)}^M$. Then, we again solve the problem for the fiscal authorities to update $S_{(2)}^B$, $S_{(2)}^{ii}$, $F_{(1)}^B$, $L_{(1)}^B$, $F_{(1)}^i$, $L_{(1)}^i$, $F_{(1)}^{ii}$ and $L_{(1)}^{ii}$, and so on. At each iteration, the same procedure is adopted in what concerns the definition of F^{ii} and L^{ii}. The fixed point is found when the policy rules and the matrices converge towards constants for a given level of tolerance.

Modeling the behavior of block S (and the setting of F^{ii} and L^{ii})

We consider three economies in our optimization setup: a big economy (**B**) and two small economies (i and ii). Country ii is very small within block **S** ($ii\_dim = 0.00001$) and, hence, when he is hit by an asymmetric (domestic) shock there are only domestic consequences, as expected. As asymmetric shocks at small countries produce no external consequences, they do not affect the behavior of block S, and so the behavior of country ii cannot serve as a proxy for the block **S**'s behavior. For that reason, we consider another small country i, whose behavior is similar to the other small country with the exception that he is not subject to asymmetric shocks. Moreover, in order to mimic the behavior of block **S** we consider that within block **S** the size of country i is almost 1 ($1 - ii\_dim = 0.99999$). This approach allows us to define all union-wide variables as an average of block **B** and block **S**, which in turn represents the average of small economies and is defined as a simple average of only two small economies (i and ii).

Taking into account the system of equations written in the state space form (61), we define

$$
Y_t \equiv \begin{bmatrix}
\mu_{w,t}^{B} \\
\mu_{w,t}^{i} \\
\mu_{w,t}^{ii} \\
a_t^{B} \\
a_t^{i} \\
a_t^{ii} \\
a_{t-1}^{B} \\
a_{t-1}^{i} \\
a_{t-1}^{ii} \\
a_{t-1}^{S} \\
a_{t-1}^{*} \\
\pi_{t-1}^{S} \\
\pi_{t-1}^{*} \\
tt_{t-1}^{B} \\
tt_{t-1}^{i} \\
tt_{t-1}^{ii} \\
y_{t-1}^{B} \\
y_{t-1}^{i} \\
y_{t-1}^{ii} \\
y_{t-1}^{S} \\
y_{t-1}^{*} \\
c_{t-1}^{B} \\
c_{t-1}^{i} \\
c_{t-1}^{ii} \\
c_{t-1}^{S} \\
g_{t-1}^{S} \\
g_{t-1}^{*} \\
Kdebt\_B \\
Kdebt\_i \\
Kdebt\_ii
\end{bmatrix}
$$

and

$$X_t \equiv \begin{bmatrix} \widetilde{c}_t^* \\ \pi_t^B \\ \pi_t^i \\ \pi_t^{ii} \end{bmatrix}.$$

All variables representing block **S** are defined as a simple average of the two small economies. As as example, $a_t^S = (1 - ii\_\dim)a_t^i + ii\_\dim a_t^{ii}$. In turn, union-wide variables are defined as an average of block S and block **B**, for instance, $a_t^* = na_t^S + (1-n)a_t^B$.

In relation to the methodology followed in the iterative procedure, the reaction of country i's fiscal instruments is set at the end of each iteration according to the optimal feedback rules obtained for country ii, avoiding to assume *a priori* the same behavior for all small countries.

Taking into account that

$$\begin{aligned} U_t^i &= -F_t^i Y_t - L_t^i U_t^M, \\ U_t^{ii} &= -F_t^{ii} Y_t - L_t^{ii} U_t^M, \end{aligned}$$

at the end of each iteration we set

$$L^i = L^{ii},$$

since the reaction to changes in the monetary policy instrument is the same for all small countries, and

$$
\begin{aligned}
F^{i} &= F^{ii}, in\ general, \\
with &: \\
F^{i}(&: ,2) = F^{ii}(:,3) + F^{ii}(:,2), \\
F^{i}(&: ,5) = F^{ii}(:,6) + F^{ii}(:,5), \\
F^{i}(&: ,8) = F^{ii}(:,9) + F^{ii}(:,8), \\
F^{i}(&: ,15) = F^{ii}(:,16) + F^{ii}(:,15), \\
F^{i}(&: ,18) = F^{ii}(:,19) + F^{ii}(:,18), \\
F^{i}(&: ,23) = F^{ii}(:,24) + F^{ii}(:,23), \\
F^{i}(&: ,29) = F^{ii}(:,30) + F^{ii}(:,29), \\
& and \\
F^{i}(&: ,3) = F^{B}(:,3) \approx (0,0)', \\
F^{i}(&: ,6) = F^{B}(:,6) \approx (0,0)', \\
F^{i}(&: ,9) = F^{B}(:,9) \approx (0,0)', \\
F^{i}(&: ,16) = F^{B}(:,16) \approx (0,0)', \\
F^{i}(&: ,19) = F^{B}(:,19) \approx (0,0)', \\
F^{i}(&: ,24) = F^{B}(:,24) \approx (0,0)', \\
F^{i}(&: ,30) = F^{B}(:,30) \approx (0,0)'.
\end{aligned}
$$

The second set of relations results from the dimension of small countries so that changes in small country ii's variables produce no external effects. As to the first set of relations, it is an outcome of the optimization procedure under cooperation, though it is not a consequence of the policy regime but of the way we modelled the union. Indeed, by considering that within block **S** the size of country i is practically 1, so that his behavior could mimic the behavior of block **S**, the response of country i's fiscal instruments to block **S** variables contemplates two different aspects: on one hand, fiscal instruments react to domestic changes, and, on the other hand, they respond to changes in block **S**, in the same way small country ii responds. Since these relations have an identical interpretation, we take as example $F^{i}(:,5) = F^{ii}(:,6) + F^{ii}(:,5)$, that is $F^{i}(:,a_t^i) = F^{ii}(:,a_t^{ii}) + F^{ii}(:,a_t^i)$. Notice that $a_t^S = (1 - ii\_\dim) a_t^i + ii\_\dim a_t^{ii}$, so the aggregate response of country i to a_t^i reflects the response to changes in his own technology, but at the same time the technology of block **S** changes affecting all small countries in the same way (note that a change in a_t^i traduces a symmetric technology shock at block **S**).[1]

[1] More precisely, the relation is $F^i(:, a_t^i) = F^{ii}(:, a_t^{ii}) + (1 - ii\_\dim)F^{ii}(:, a_t^i)$.

Appendix A.4:

Baseline Calibration

Table A: Baseline Calibration

Symbol	**Value**	**Description**
n	0.5	Block S dimension
ii_ dim	0.00001	Small country (ii)'s dimension within Block S
$(1-\alpha)$	0.6	home bias parameter for small countries
β	0.99	Intertemporal preferences discount factor
ρ	-log(β)	Steady-state real interest rate (4% annual basis)
σ	0.4	Intertemporal elasticity of substitution of private consumption
ψ	0.4	Intertemporal elasticity of substitution of public consumption
χ	3	The inverse of the labor supply elasticity
ϵ	11	Elasticity of substitution between goods produced in the same country
γ	4.5	Intertemporal elasticity of substitution between domestic and foreign goods
φ	0.25	Steady-state share of public consumption in output
θ_S	0.75	Calvo's price adjustment rule for small countries
θ_B	0.75	Calvo's price adjustment rule for the big country
ρ_a	0.85	Autocorrelation parameter of the technology AR(1) process
ρ_μ	0	Autocorrelation parameter of the wage markup AR(1) process
σ_a	0.01	Standard error deviation of the white noise in the technology AR(1) process
σ_μ	0.01	Standard error deviation of the white noise in the wage markup AR(1) process

Appendix B

Figure B1: Responses to a 1% negative symmetric technology shock under cooperative discretionary policy – Gaps, Efficient and Effective variable values: high debt-to-output ratio = 60%

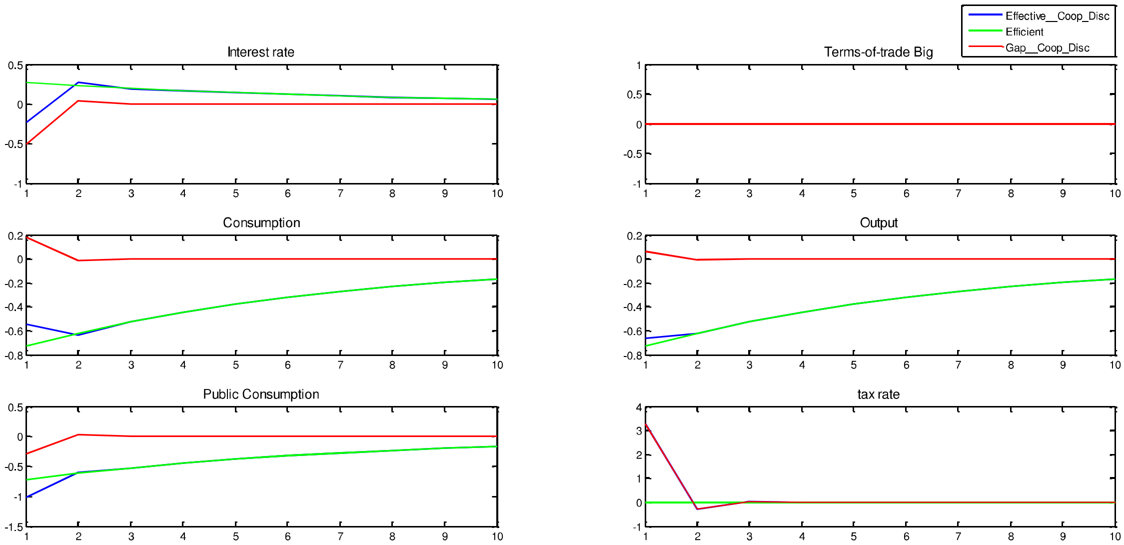


Figure B2: Responses to a 1% negative technology shock at the **Big** country under optimal cooperative commitment policy (debt-to-output ratios: 0%, 15% and 60%)

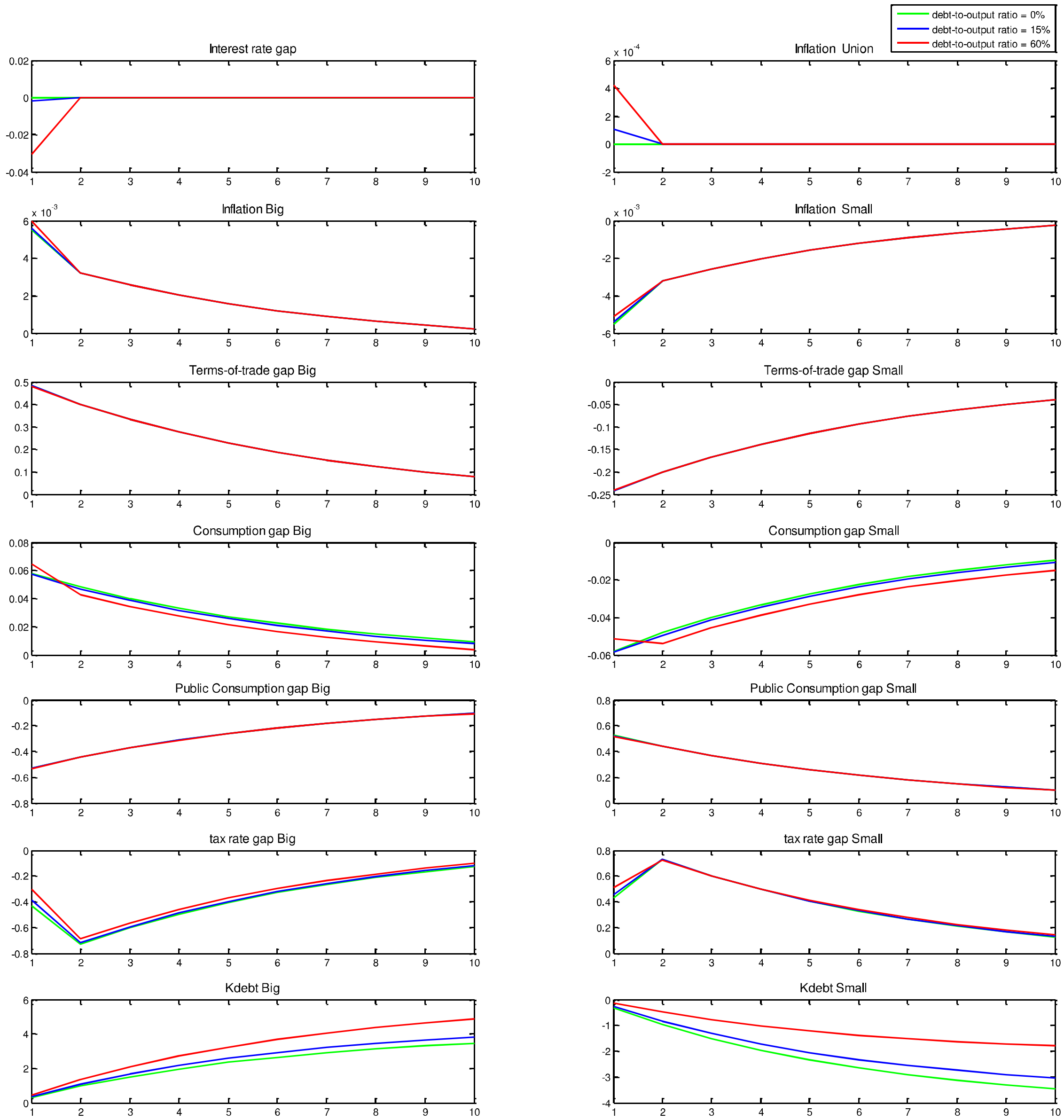


Figure B3: Responses to a 1% negative technology shock at the **Big** country under optimal cooperative discretionary policy: low-debt scenario

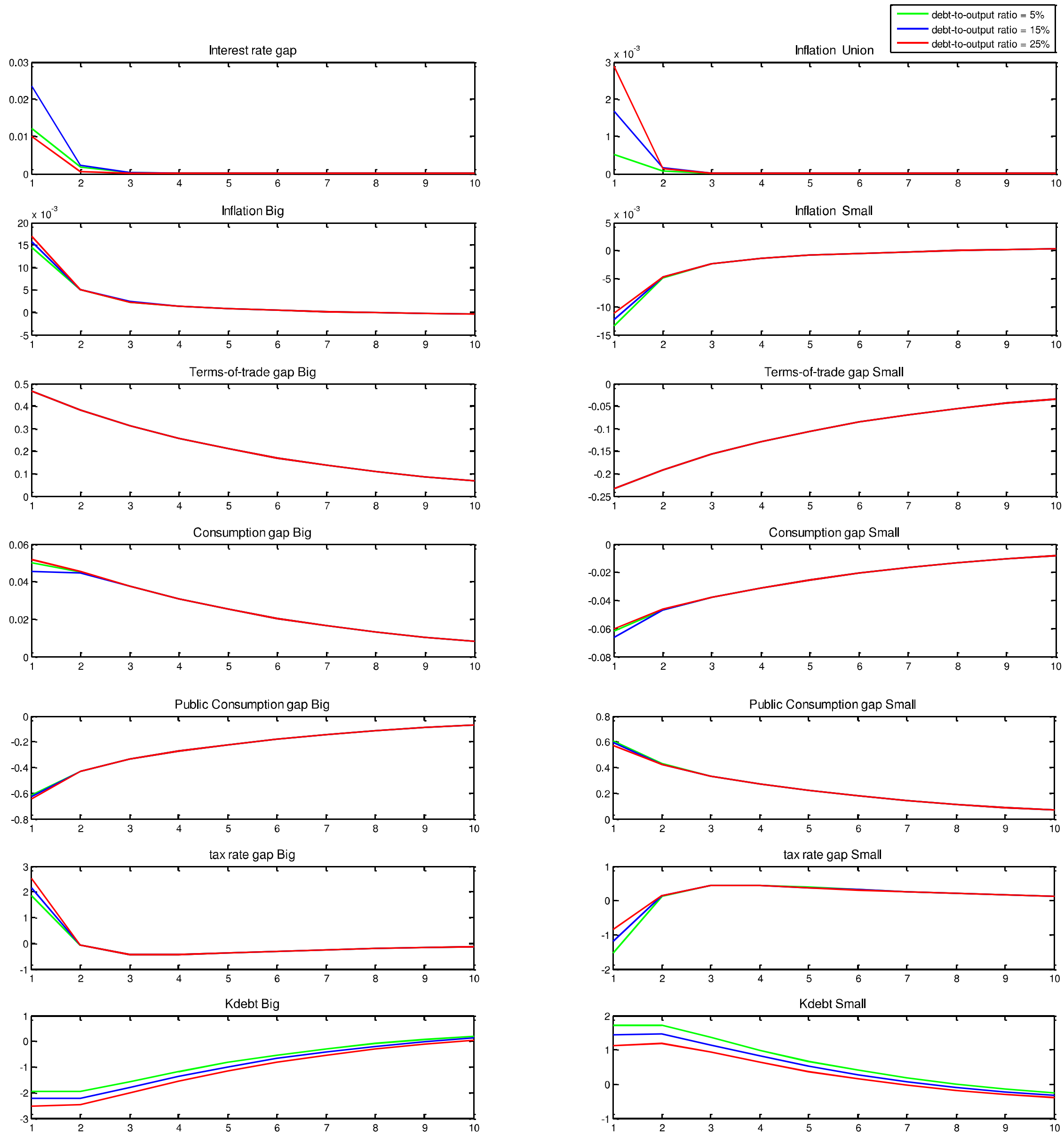


Figure B4: Responses to a 1% negative technology shock at the **Big** country under optimal cooperative discretionary policy: high-debt scenario

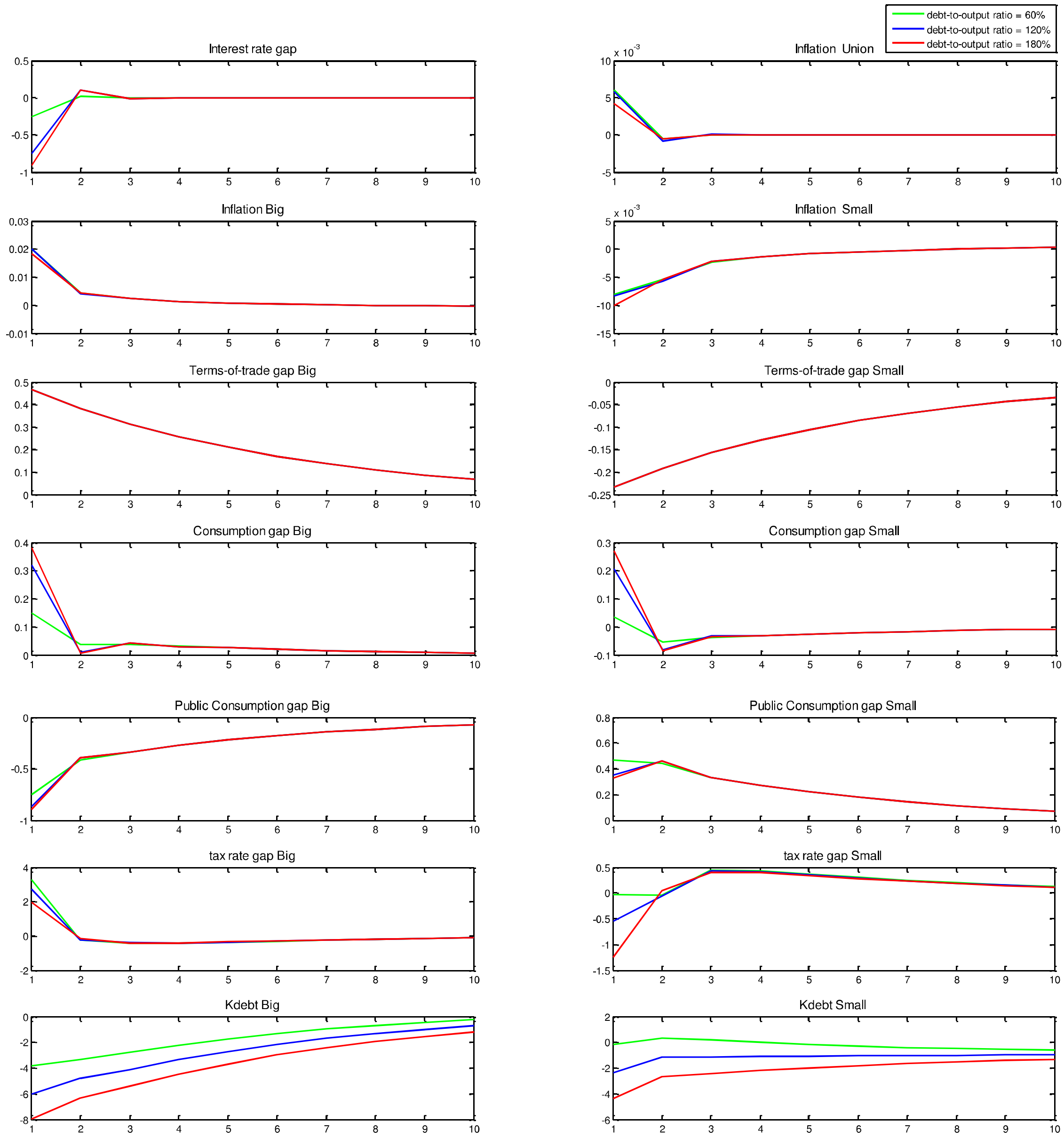


Figure B5: Union-wide welfare loss (L_U) under cooperative discretionary policy, across different debt levels and different degrees of nominal rigidity (all asymmetric technology shocks included)

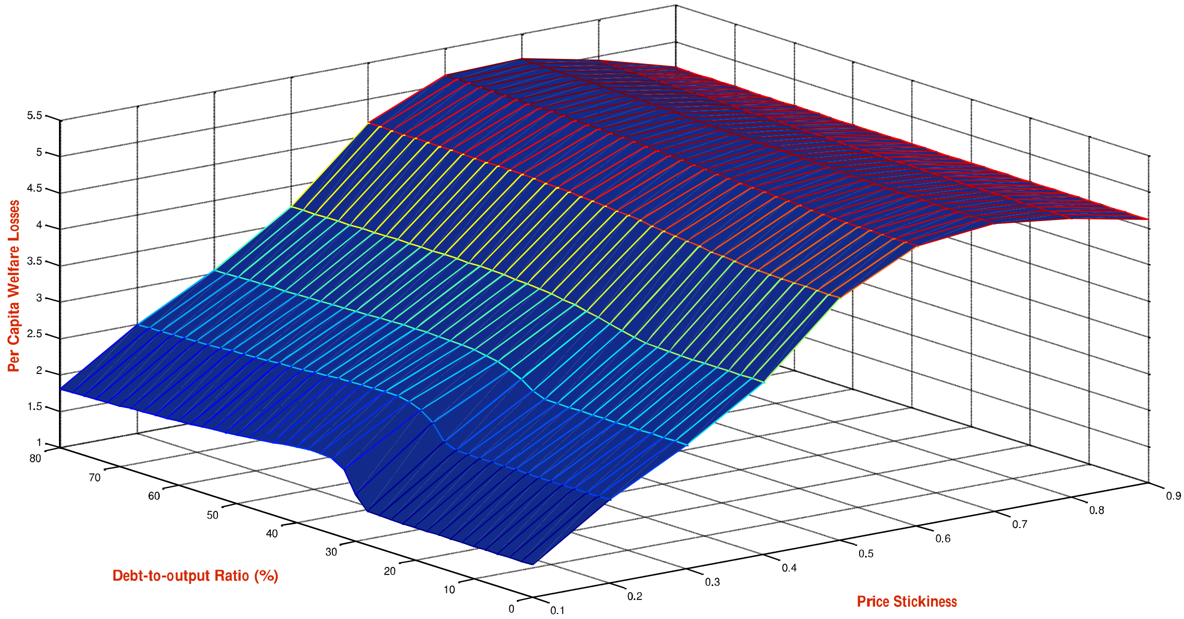


Figure B6: Union-wide welfare loss (L_U) across different policy regimes and debt levels, considering two degrees of nominal rigidity (all asymmetric technology shocks included)

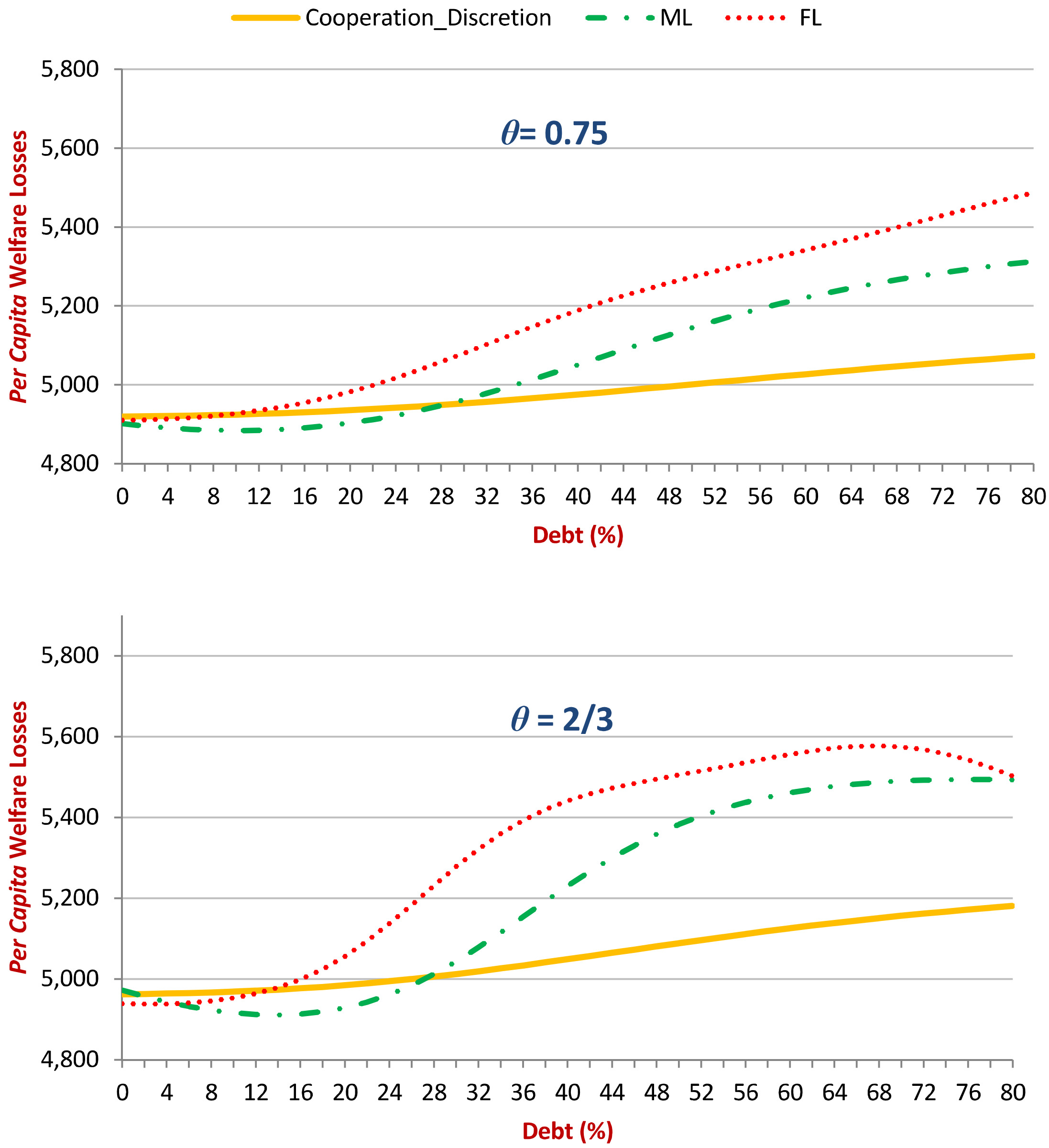


Figure B7: Union-wide welfare loss (L_U) across different policy regimes and debt levels, for alternative elasticities of labor supply (all asymmetric technology shocks included)

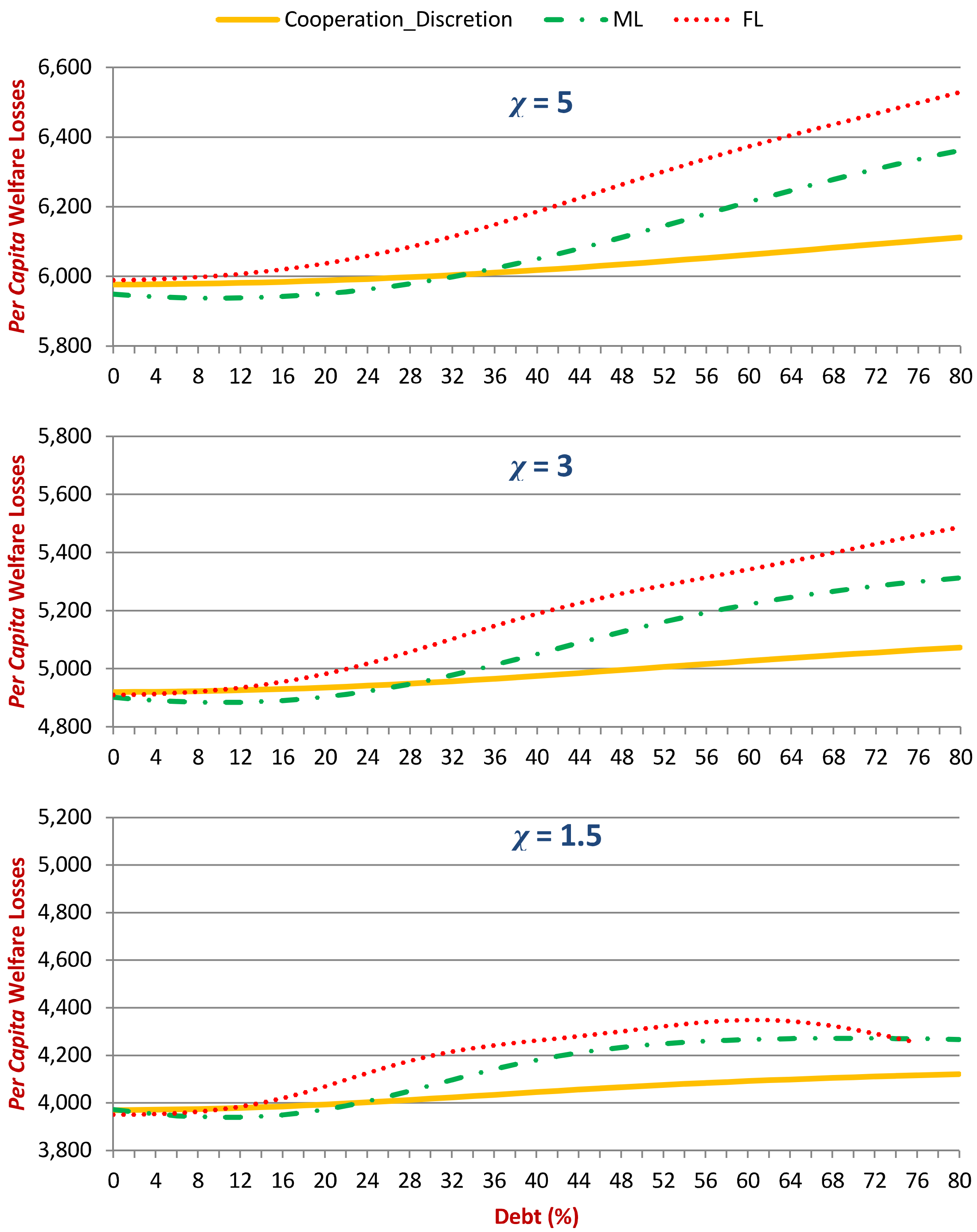


Figure B8: Union-wide welfare loss (L_U) across different policy regimes and debt levels, for alternative elasticities of substitution between domestic and foreign goods (all asymmetric technology shocks included)

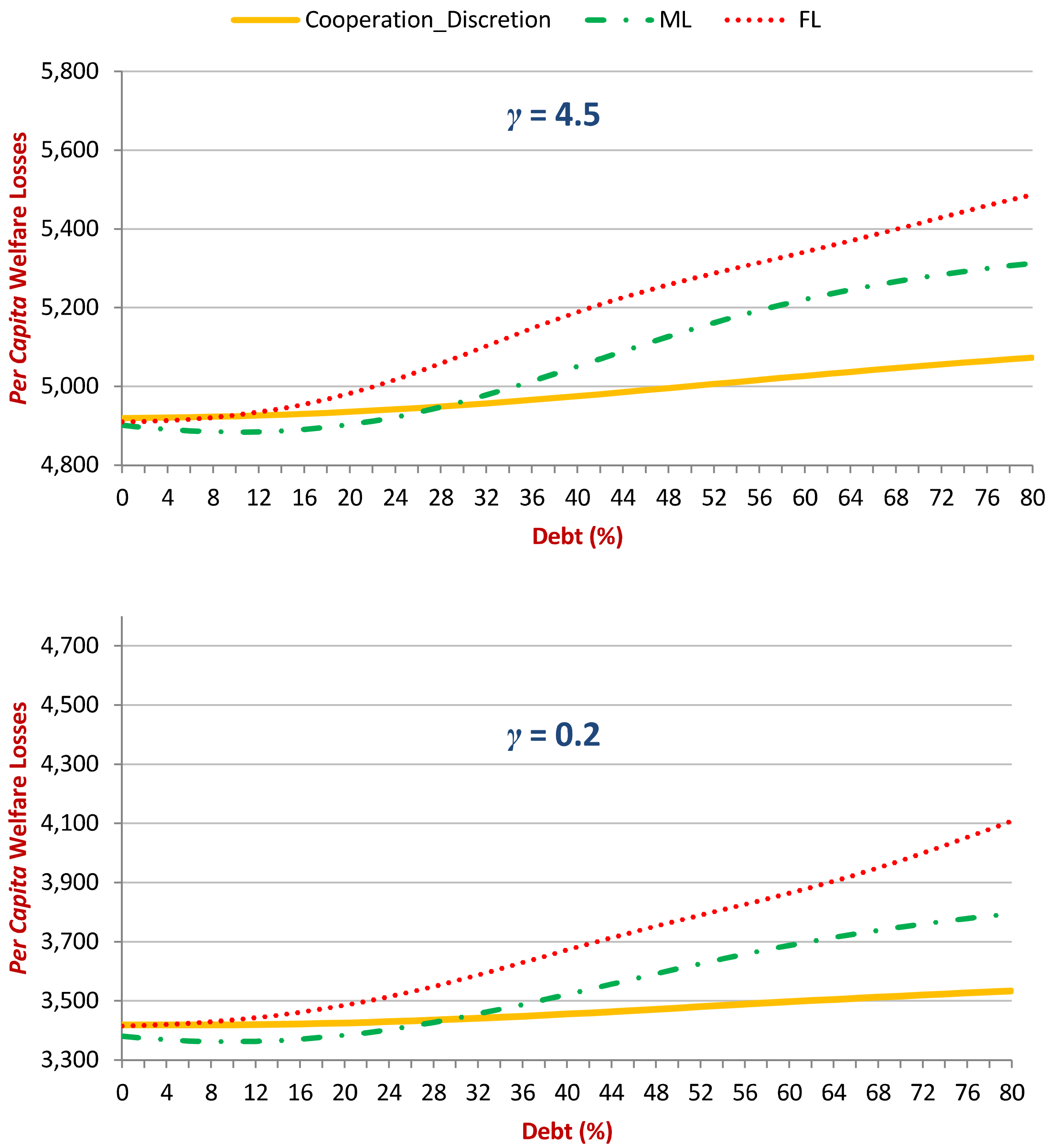


Figure B9: Welfare losses for the **Big** country (L_B) across different policy regimes and debt levels, for alternative dimensions of country **B** (all asymmetric technology shocks included)

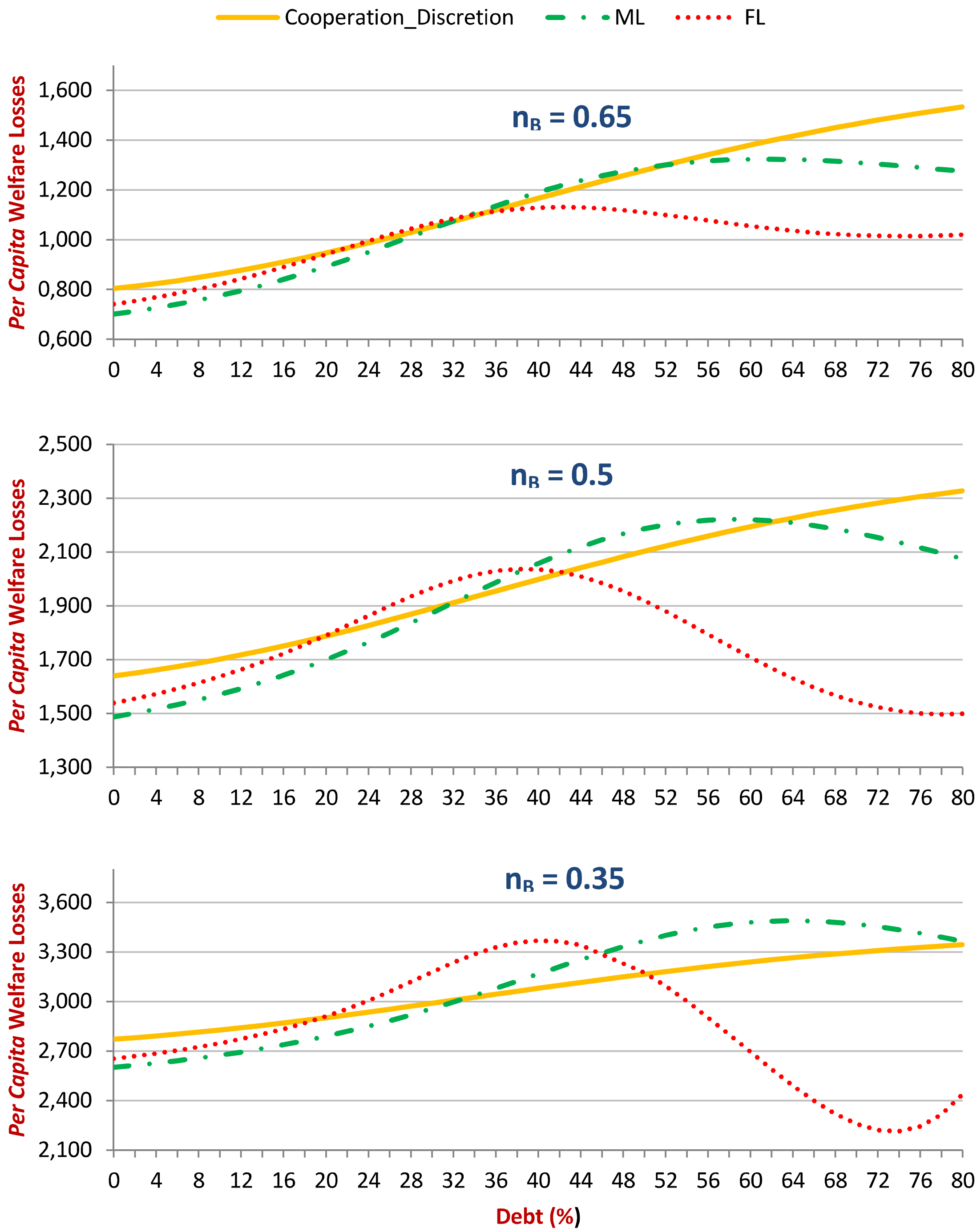
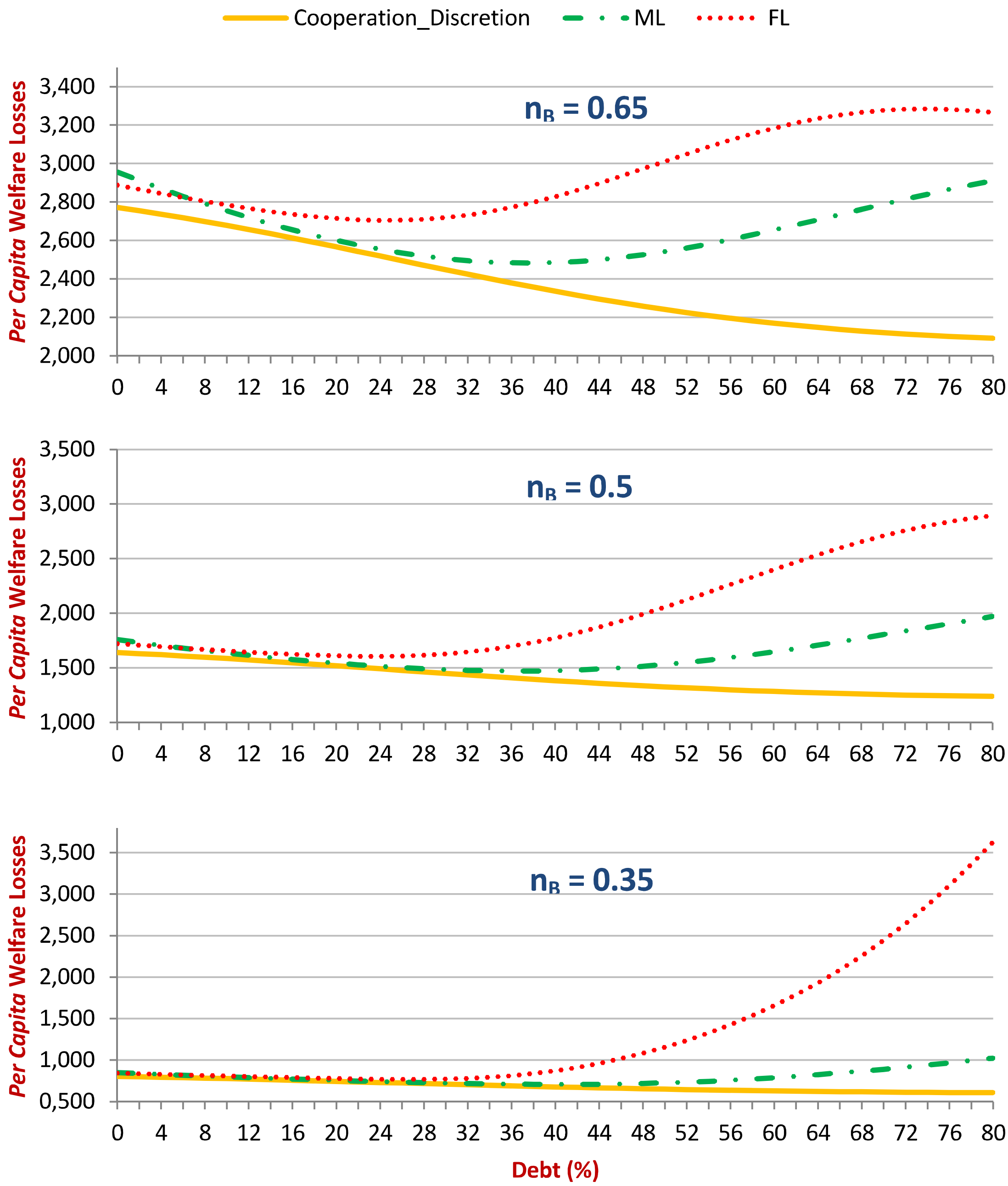
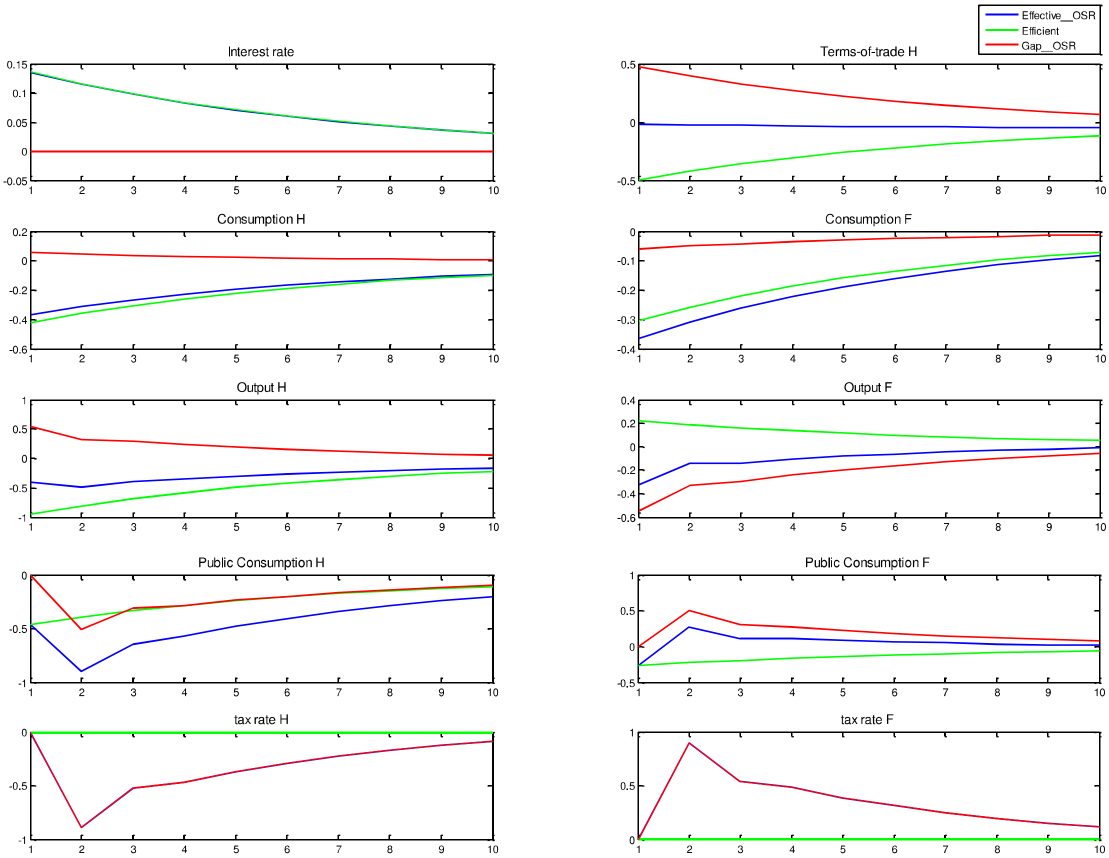


Figure B10: Welfare losses for a **small** country (L_s) across different policy regimes and debt levels, for alternative dimensions of country **B** (all asymmetric technology shocks included)



Appendix C

Figure C1: Responses to a 1% negative technology shock at country H: parsimonious optimal simple rules – Gaps, Efficient and Effective variable values. Two-country currency union model (debt-to-output ratio = 60%; n=0.5)



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