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Essays on Antitrust Issues for Horizontal and Vertical Mergers

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Tese de Doutoramento em Economia

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For my grandfather (in memoriam).

Biographical Note

Mariana Alves da Cunha was born on October 18, 1987, in Porto, Portugal.

She concluded her undergraduate studies in Economics in 2008 and Master studies in Economics in 2010, both from Faculdade de Economia e Gestão, Universidade Católica - Centro Regional do Porto. Her Master's dissertation entitled "Eficiência do Sector de Energia Eléctrica na União Europeia" has been supervised by Professor Carlos Santos.

During her Master studies, Mariana has taught at Universidade Católica - Centro Regional do Porto (2008-2010): Microeconomics, Econometrics and Mathematics.

In September of 2010, she started her PhD in Economics at Faculdade de Economia da Universidade do Porto, under the supervision of Professors Hélder Vasconcelos and Paula Sarmento. Her most recent research on mergers has been presented in conferences, as the 4th and 5th edition UECE Lisbon Meetings: Game Theory and Applications (both in Lisbon, Portugal) and the 8th Annual Meeting of the Portuguese Economic Journal (Braga, Portugal). Her research has also resulted in two publications in the Journal of Industry Competition in Trade.

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Mariana Cunha, Porto, 2014.

Resumo

A presente tese inclui quatro ensaios sobre questões de política de concorrência na análise de fusões horizontais e verticais. No primeiro ensaio estudamos fusões horizontais quando as empresas têm comportamentos estratégicos assimétricos. Além disso, analisamos uma fusão sequencial num cenário onde: (i) as empresas concorrem à Stackelberg; (ii) inicialmente as empresas são simétricas, mas as fusões podem dar origem a ganhos de eficiência e, por consequência, a assimetrias de custos entre as empresas; e (iii) todas as fusões têm de ser submetidas a aprovação pela Autoridade da Concorrência. Em particular, consideramos dois tipos de Autoridades da Concorrência: uma Autoridade da Concorrência míope, que avalia a proposta de fusão sem antecipar que poderão existir outras fusões subsequentes, e uma Autoridade da Concorrência com visão futura, que antecipa a estrutura final de mercado, no caso da primeira fusão ser aprovada.

O segundo ensaio utiliza um modelo semelhante ao caracterizado no primeiro ensaio. Neste ensaio estudamos a lucratividade das fusões, o problema do *free-riding* e os efeitos induzidos das fusões no bem estar social e no bem estar dos consumidores, quando as empresas concorrem à Stackelberg e as fusões geram assimetrias de custos entre as empresas existentes na indústria. Em particular, mostramos que as conclusões sobre a lucratividade das fusões, os efeitos no bem estar e a existência do problema do *free-ridding* depende de forma crucial dos ganhos de eficiência gerados pelas fusões.

No terceiro ensaio exploramos o papel da incerteza na decisão de fusão e no seu controlo pelas autoridades de concorrência. Num cenário de concorrência à Cournot, consideramos que as fusões podem gerar ganhos de eficiência incertos e que todas a fusões têm de ser

submetidas para aprovação da Autoridade da Concorrência. O objectivo deste ensaio é não só estudar o impacto da incerteza na probabilidade da fusão ser proposta pelas empresas e aceite pela Autoridade da Concorrência mas também analisar de que forma a incerteza influencia os incentivos das empresas que ficam de fora da fusão a prosseguirem estratégias de *free-riding*.

Por fim, no quarto ensaio analisamos de que forma a integração vertical aumenta os incentivos ao conluio das empresas retalhistas. O principal contributo deste ensaio é realçar a relação entre dois tipos de estratégias das empresas: a integração vertical e o conluio, num cenário em que nem todas as empresas aceitam o acordo colusivo (conluio incompleto).

Abstract

This thesis comprises four essays on antitrust issues for horizontal and vertical mergers. In the first essay we study horizontal mergers when firms have asymmetric strategic roles. In this essay we assume a sequential merger formation game in a setting where: (i) firms compete à la Stackelberg; (ii) all firms are ex-ante symmetric, but as mergers may give rise to endogenous efficiency gains, cost asymmetries between the remaining firms in the industry may be created; and (iii) every merger has to be submitted for approval to the Antitrust Authority. In particular, we consider two possible types of Antitrust Authorities: a myopic Antitrust Authority, that evaluates a merger proposal without anticipating possible subsequent mergers, and a forward looking Antitrust Authority, which is able to anticipate the ultimate market structure if the merger is approved.

The second essay uses a framework similar to the previous one. In this essay we investigate mergers' profitability, free-riding problem and induced effects on social and consumer welfare when firms are in a Stackelberg market and mergers can create cost heterogeneity between the remaining firms in the industry. In particular, we show that conclusions about the merger profitability, the social welfare effects and the existence of a free-riding problem crucially depend on whether the merger creates synergies.

In the third essay we explore the role of uncertainty in merger control and in merger decisions. In a Cournot setting, we consider that mergers may give rise to uncertain efficiency gains and that every merger has to be submitted for approval to the Antitrust Authority. We assume that both the Antitrust Authority and the firms in the industry face the same uncertainty about the future efficiency gains induced by the merger. The purpose of this essay is

not only to study the impact of the existence of uncertainty in the likelihood of the merger being proposed by the firms and accepted by the Antitrust Authority but also to analyze how the uncertainty affects outsider firms' incentives to free-ride on it.

Finally, the fourth essay analyzes how vertical integration increases downstream firm's incentives to collude. The main contribution of this essay is to highlight the importance of considering, simultaneously, two types of firms' strategies: vertical integration and collusion, in a context where some firms have no incentives to collude (incomplete collusion).

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Chapter 1

Introduction

The present thesis is organized in five chapters. This first chapter gives an overview of the thesis's organization and familiarizes the reader with the main research topics. Each research question is answered in its own chapter as an independent essay, consisting of introduction that includes the literature review and some practical applications, methodology, discussion of the results, conclusion and policy implications. Some of these individual essays are, however, related to each other and some overlap in content is inevitable, especially with regard to the assumptions of the theoretical models.

Horizontal mergers in concentrated industries are an important issue for antitrust authorities. Mergers can reduce industry competition and hence result in higher prices, which could harm consumers. However, a merger can also be welfare-enhancing if it generates efficiency gains, increases incentives for innovation or creates synergies and scale economies among firms. Then, the efficiency gains might offset the increased market power and result in higher social welfare. Therefore, the horizontal merger could actually lead to more efficient firms and to lower prices, which benefits consumers. Usually, the welfare analysis of mergers assumes that firms have incentives to merge because this could increase their profits by increasing the prices or reducing the costs. Stigler (1950) argued that firms that do not merge (outsiders) may also benefit more than the participants (insiders). With the merger, the new firm will reduce its production and then the industry price will increase. However, the nonparticipants firms will expand their output and therefore will have higher profits due to the high industry price.

The first essay (Chapter 2) is concerned with horizontal mergers, in a setting where the firms in the market have asymmetric strategic roles. In this essay we assume a sequential merger formation game in a setting where: (i) firms compete à *la* Stackelberg; (ii) all firms are ex-ante symmetric, but mergers may give rise to endogenous efficiency gains and, therefore, cost asymmetries between the remaining firms in the industry; and (iii) every merger has to be submitted for approval to the Antitrust Authority. In particular, we consider two possible types of Antitrust Authority: a myopic Antitrust Authority, that evaluates a merger proposal without anticipating that this merger might trigger other subsequent mergers, and a forward looking Antitrust Authority, which is able to anticipate the ultimate market structure a merger will lead to, if approved. We find that these two types of Antitrust Authority adopt similar decisions whenever a merger would not trigger the exit of outsider firms. The Antitrust Authorities' decisions are, however, shown to be very different when evaluating exit-inducing merger proposals.

The second essay (Chapter 3) is related to the previous one and is concerned with mergers' profitability, free-riding problem and induced effects on social and consumer welfare of mergers, when firms are in a Stackelberg market and mergers can create cost heterogeneity between the remaining firms in the industry. We find that under certain conditions regarding the cost benefits resulting from mergers, the so called "free-riding" problem is eliminated and mergers are not only profitable but also welfare enhancing, even with linear costs.

The assessment of the efficiency gains resulting from a merger usually raises an information issue for antitrust authorities. Although some mergers can actually generate significant efficiency gains, these are usually difficult to measure and verify. In practice, it is often the case where both the firms and the antitrust authority cannot predict exactly the post-merger efficiency gains, implying that they are not aware of all the conditions they are going to face after the merger. Sometimes, only after the merger firms and antitrust authorities will understand the true level of the merger's induced efficiency gains. For instance, some pharmaceutical firms may adopt merger decisions without knowing whether their R&D efforts will be successful or not. Also, any type of firms' investment could generate uncertainty about future costs and, sometimes, a merger could actually occur before the uncertainty is resolved.

In the third essay (Chapter 4), we explore the role of uncertainty in merger control and in merger decisions. In a Cournot setting, we consider that mergers may give rise to uncertain efficiency gains and that every merger has to be submitted for approval to the Antitrust Authority. We assume that both the Antitrust Authority and the firms in the industry face the same uncertainty about the future efficiency gains induced by the merger. The purpose of this essay is to contribute to the literature that studies the efficiency gains' role in merger decisions, departing from a deterministic environment by considering a setting in which there is (symmetric) uncertainty. The present essay then contributes to fill the gap in the extant literature by assuming that, when firms propose the merger to the Antitrust Authority, all the players (insider, outsiders and Antitrust Authority) are uncertain about the post-merger efficiency gains and therefore they decide by considering the expectations on those gains. Once the merger is consummated, both insider and outsider firms can observe the efficiency gains and compete *à la* Cournot. This analysis is useful to the Antitrust Authority in order to more properly evaluate merger proposals, when there is uncertainty about the cost savings that mergers may induce. It is shown that an increase in the degree of uncertainty benefits both insider and outsider firms but also the consumers. Further, when uncertainty is high, there is a greater likelihood that firms propose a merger to the Antitrust Authority and the Antitrust Authority accepts it. Interestingly, however, although uncertainty enhances merger approval chances, it also decreases merger's stability, by increasing outsiders' incentives to free-ride on it.

Theoretical literature on vertical integration has also gained a great contribution from researchers on industrial economics for many years. The analysis of the anticompetitive effects of vertical integration has been a debated topic of research in Industrial Organization. Vertical integration can have negative impacts on welfare due to coordinated effects, that is, vertical integration may improve the sustainability of collusion among competing firms. Antitrust authorities have remained concerned that vertical integration might facilitate collusion at upstream and downstream levels. Vertical integration facilitates collusion if, after the merger, upstream or downstream firms are able to coordinate in a more effectively way than if they were operating separately.

The fourth essay (Chapter 5) is about how vertical integration increases firm's incentives to collude. The literature suggests that there is a relationship between vertical integration and collusion. Particularly, vertical integration might ease the collusive agreements, by supporting the punishment and monitoring mechanisms and by allowing the agreement between firms. Moreover, the models presented in the literature study if vertical integration facilitates upstream or downstream collusion and differ mainly on the assumptions chosen. It is possible to verify that most of these studies concluded that vertical integration facilitates collusion in downstream or upstream level. The main contribution of the fourth essay is to highlight the importance of considering, simultaneously, two types of firms' strategies: vertical integration and collusion, in a context where some firms have no incentives to collude (incomplete collusion). Then, we try to answer the following question: does vertical integration promote incomplete collusion? This research topic could be very important for antitrust authorities because they are concerned with the impacts of vertical integration on collusion and how these two types of strategies might affect social welfare. We find that a vertical merger between an upstream firm and a downstream cartel or fringe firm promotes downstream collusion, under certain conditions on the market size. Moreover, a welfare analysis shows that consumer surplus increases with the vertical merger since it partially eliminates the double marginalization problem. The results obtained could modify the formulation of policies towards vertical mergers.

All of these questions are interesting and also bring some important insights that could be helpful for the antitrust authorities' intervention. In this context, all these issues seem to be important for antitrust authorities when deciding whether or not to accept a horizontal or vertical merger.

Chapter 2

Efficiency gains of mergers in Stackelberg markets with an active Antitrust Authority

2.1 Introduction

The large body of previous literature on the effects of horizontal mergers has argued that Antitrust Authorities should pay particular attention to the balance between anticompetitive price (market power) effects and pro-competitive merger related efficiency improvements (see e.g. Williamson (1968) and Farrell and Shapiro (1990)).[1]

The European Commission (EC), in practice, has so far never used efficiency gains arguments to clear a merger. When the merging parties use the "efficiency defence argument" reporting that the merger would generate cost reductions, this claim usually has been dismissed by the EC. The EC uses the efficiency offence argument arguing that although the merger would benefit the consumers in the short-run, in the long-run rival firms have incentives to exit the industry and then the merging firm would become a monopolist, thereby

[1]The analysis of merger-induced efficiencies was introduced into the US Merger in 1997 (Section 4) and into the European Merger Guidelines in 2004 (European Commission Horizontal Merger Guidelines, 2004/03, Article 7).

harming consumers' welfare. This efficiency offence argument has received some attention by economists, providing an informal discussion of efficiency offence arguments in EC practices, such as Padilla (2002). The efficiency offence argument has also been formally analyzed on a static and a dynamic framework (Motta, 2004; Motta and Vasconcelos, 2005). However, to the best of our knowledge, scarce attention has been devoted to the study of efficiency defence arguments, in contexts where merging parties have asymmetric strategic power.

In this paper, we contribute to close this gap in the literature by studying an endogenous merger formation game wherein: (i) merging parties compete à la Stackelberg; (ii) all firms are ex-ante symmetric, but mergers may give rise to endogenous efficiency gains and, therefore, cost asymmetries between the remaining firms in the industry; and (iii) every merger must be submitted for approval to an Antitrust Authority (AA). In particular, we consider two possible types of AA: a myopic AA, that evaluates a merger proposal without anticipating that the corresponding merger might trigger other subsequent mergers, and a forward looking AA, which is able to anticipate the ultimate market structure a merger will lead to if approved.

Although in reality there are merger waves (or sequential mergers), usually the AAs rarely adopt forward-looking decisions when analyzing a merger case, since it is hard to predict the future behavior of the firms. Also AAs might make mistakes in predicting that following a merger, outsiders might leave. However, the economic literature (eg. Motta and Vasconcelos (2005) and Nocke and Whinston (2010)) has emphasized the need for those AAs to predict which is the ultimate market structure, after the first merger takes place. Recently, the Centre on Regulation in Europe (CERRE) published a dossier to the European Commission (EC) where it includes some recommendations on the future regulatory policy in network industries (CERRE, 2014). This report highlights that the EC and national competition authorities should pay particular attention to the dynamic issues when investigating mergers such as, in electronic communications and media markets and concludes that the merger policy is currently not very good at assessing the dynamic effects of mergers, given its static focus on prices. In this sense our paper is a theoretical contribution that suggests that when predicting the possible impact of the merger, possible reactions by the outsiders (defensive mergers) should be properly taken into account. More particularly, before concluding that a merger will create such a more efficient merged entity that rivals would not be able to compete with it, AAs should consider whether in the industry at hand there exists room for further mergers allowing outsiders to attain similar efficiency levels, and/or whether the outsiders might be able to enhance efficiency through internal growth.

Within this theoretical structure, we find that, when evaluating a two-firm merger proposal, both myopic and forward-looking AAs adopt similar decisions if the proposed merger would not trigger the exit of outsider firms in case it was approved (and no further mergers take place). Nevertheless, their decisions are shown to be in sharp contrast when evaluating exit-inducing merger proposals, i.e, in the region where efficiency gains and the fixed costs are sufficiently high such that the merger would trigger the exit of outsider firms. In particular, under some circumstances, while the myopic AA blocks the merger under an efficiency offence argument, the forward looking AA approves the very same merger proposal, since it correctly anticipates that this merger will be followed by a subsequent defensive merger involving the outsider firms of the first merger. This will lead to a duopoly situation wherein even though firms have asymmetric strategic power, they end up being equally sized and equally efficient, to the benefit of consumers.

This paper is mainly related to three strands in the extant literature. Firstly, it is related to the literature on endogenous horizontal mergers, since we explicitly model the merger formation process. Important references in this literature are Kamien and Zang (1990), Gowrisankaran (1999), Faulí-Oller (2000), Horn and Persson (2001a,b), Motta and Vasconcelos (2005), Fumagalli and Vasconcelos (2008), Fumagalli and Nilssen (2008), Banal-Estañol et al. (2008), Vasconcelos (2010) and Nocke and Whinston (2010), to name a few.[2]

Secondly, our paper is also related to the previous literature that incorporates an active AA into the merger formation game. Few studies have introduced the AA as an active player, such as, Motta and Vasconcelos (2005), Fumagalli and Nilssen (2008), Fumagalli and Vasconcelos

[2]Other papers also assumed sequential game but with exogenous mergers, such as Nilssen and Søgard (1998) and Salvo (2006).

(2008), Vasconcelos (2010) and Nocke and Whinston (2010).

Thirdly, this paper is also related to the branch of the literature that deals with the economic analysis of merger control by taking into account the efficiency defence argument. The efficiency defence argument refers to the case where the AA approves the merger if the efficiencies caused by the merger more than compensate for the induced increase of market power. Some papers include the AA's efficiency defence argument when evaluating a merger and some also analyze the role of structural remedies introduced by the AA after the merger takes place, such as, Röller et al. (2000), Motta and Vasconcelos (2005), Fumagalli and Nilssen (2008), Fumagalli and Vasconcelos (2008), Banal-Estañol et al. (2008), Vasconcelos (2010) and Nocke and Whinston (2010).[3]

Our paper is closely related to Motta and Vasconcelos (2005)'s paper, who also studied endogenous mergers, where each merger has to be approved by the AA. Motta and Vasconcelos (2005) show that if the AA does not anticipate subsequent merging, such a myopic AA could actually make wrong decisions. However, if the AA correctly anticipates that the merger is followed by other mergers, no efficiency offence argument is justified. Like Motta and Vasconcelos (2005), we also consider that the AA maximizes consumer welfare and, therefore, the AA approves both mergers if consumer surplus increases (i.e. if prices decrease).[4]

There are two major differences between Motta and Vasconcelos (2005) framework and the setting used in this paper. First, Motta and Vasconcelos (2005) assume that the mergers

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[3]The inclusion of an efficiency defence argument could bring an asymmetric information problem with respect to the merger's efficiency gains between the AA and the merging firms. Some papers consider the issue of asymmetric information about merger-specific efficiencies, such as, Gonzalez (2004), Medvedev (2004), Lagerlöf and Heidhues (2005), Cosnita and Tropeano (2009), however, this analysis is far beyond the scope of this paper.

[4]By assuming that the AA evaluates mergers according to a consumer surplus standard this does not mean that this is always better than the total welfare standard. However, as Lyons (2002) argued, the consumer surplus standard is applied in most antitrust jurisdictions. Other papers also study how the AA should apply the consumer surplus standard when challenging a merger, such as Besanko and Spulber (1993), Neven and Röller (2005), Vasconcelos (2010), Nocke and Whinston (2010), Jovanovic and Wey (2012), among others.

occur under symmetric Cournot competition. In contrast, in this paper is assumed that the mergers occur when firms have asymmetric strategic power and, therefore, it is assumed that different types of mergers may occur in an industry characterized by Stackelberg competition.[5] This assumption implies that, in the setting proposed in the current paper, when the efficiency gains are very small (or even when there are no efficiency gains) a merger between two firms will always be proposed because it still is profitable, which is not the case under Cournot. When there is no strategic power advantage for any firm we obtain the well known Salant et al. (1983) merger's paradox: a merger between two firms with the same strategic power that creates a firm of the same type is always unprofitable for the merging firms, unless more than 80% of the firms in the industry merge or there are sufficiently high fixed costs.

Also, we found that in an asymmetric strategic power industry, the AA will be more severe when blocking the mergers since the interval of efficiency gains where mergers are usually accepted is greater under Stackelberg than under Cournot. Second, Motta and Vasconcelos (2005) consider, under the forward looking AA, that after the defensive merger occurs, the firms of the first and the second merger are allowed to seek a merger to a monopoly. However, in this paper, we rule out merger to monopoly since we want to analyze the competitive effects of merger proposals in which both at the status quo industry structure and in the merger induced industry structure firms do have asymmetric strategic power.

Although this paper encompasses a theoretical exercise, it is also motivated by the profitability of real-world mergers involving firms with asymmetric strategic power. A case in point is the DRAM (Dynamic Random Access Memory) industry, where the leading manufacturers announce their production plans in advance and manufacturers, which enter the market later, respond by adjusting their quantity of DRAM produced. In addition, in this specific industry, the last decades have witnessed a wave of mergers.[6] Another example is Microsoft's dominance in software markets. In this case, although Microsoft usually makes

[5] See Daughety (1990), Feltovich (2001), Huck et al. (2001), Escrihuela-Villar and Faulí-Oller (2007), Heywood and McGinty (2007, 2008) and Brito and Catalão–Lopes (2011) on the profitability of mergers involving Stackelberg firms operating in a context without merger induced synergies.

[6] For details, see Escrihuela-Villar and Faulí-Oller (2007) and the references cited therein.

decisions first, other smaller companies typically react to Microsoft's actions when making their own decisions. Obviously, these subsequent followers' actions, in turn, affect Microsoft (see e.g. Graham (2013)). In both examples, it seems reasonable and plausible to analyze both firms and AA decisions when firms have asymmetric strategic power.

The remainder of the paper is organized as follows. Section 2.2 introduces the baseline assumptions of the model. Section 2.3 presents the results before the merger. Section 2.4 analyzes the post-merger industry structure under two different scenarios regarding the behaviour of the AA (myopic and forward looking), for the case in which the merger involves two leaders. In Section 2.5, we discuss two extensions of the benchmark model. In particular, we extend the analysis by assuming that now both AAs consider the effects of the two-leader merger on social welfare, instead of considering only the effects on consumer surplus. Further, we also present the results obtained for alternative types of mergers. Finally, Section 2.6 concludes the paper by discussing the results obtained. All proofs and details on the calculations are relegated to the Appendix.

2.2 Basic Model

We consider a market with $N = 4$ firms. We assume a two stage-game. First, all leaders simultaneously choose their output levels. Second, all followers simultaneously choose their output levels, after observing the leaders' choices. We assume that there are two leaders and two followers that compete in quantities over a linear demand $P = 1 - Q$, where $Q = Q_L + Q_F$.

What distinguishes firms is the amount of capital they own. Hence, the cost function of a firm which owns k_i units of the industry capital and produces q_i units of output is given by:

$$C(q_i, k_i, \alpha) = \tfrac{\alpha K}{k_i} q_i + k_i f$$

where $\alpha \geq 0$, $\sum\limits_{i=1}^{N=4} k_i = K = 4$ and $f > 0$.

The total supply of capital is assumed to be fixed to the industry (and equal to $K = 4$ units) and k_i is firm i's capital holdings. By assuming that the total quantity of capital available in the industry is fixed, we are also assuming that entry is very difficult in this industry. This cost structure was proposed by Motta and Vasconcelos (2005). Each firm operates with a constant marginal cost of production, but the level of its marginal cost is a decreasing function of its capital holdings, k_i. In addition, it is assumed that there exists a plant specific fixed cost f, which has to be paid for each unit of industry's capital owned by the firm. This fixed cost could be, for instance, a plant specific fixed fee associated with an energy suppliance contract. Also, at the status quo industry structure, each firm is endowed with a single unit of capital $(k_i = 1)$, implying that all firms are symmetric in terms of costs. However, cost asymmetries may arise as a result of a merger (or a series of mergers), that is, assuming that two firms merge, the merged firm has now two units of capital and the non-merged firms still have one unit of capital each. With this type of cost structure, we capture two distinct cost effects induced by a merger. First, a merger brings the capital of merging parties into a single larger entity and, therefore, gives rise to endogenous efficiency gains. The higher the value of α is, the stronger the efficiency gains induced by a merger are. Second, by creating a larger firm, the merger has also the effect of increasing fixed costs proportionally. This effect is captured by the parameter f in the cost function.

We assume that there is Stackelberg competition and that firms are allowed to merge before competition in the product market. However, when firms merge, they will have to ask the Antitrust Authority (AA) for authorisation. Following Motta and Vasconcelos (2005) we also assume that there are two different scenarios. First, we assume that the AA is myopic and, hence, when deciding to accept or not the merger it does not take into account that the merger can be followed by other mergers (Section 2.4.1). Second, we assume that the AA is forward looking and anticipates the ultimate market structure a merger will lead to (Section 2.4.2). Further, in order to keep the analysis simple, we assume a consumer-surplus-maximizer AA.

2.3 Before the merger

As mentioned, at the status quo industry structure, firms are symmetric and each firm has one unit of the industry capital. In addition, since there is a sequential output choice: first we have leader firms decision and then the followers decide, we solve the game following the usual backward induction procedure. So, each follower firm chooses q_i^F that maximizes their profit, taking the output of the leader as given. The profit function of follower firm 1 is given by:

$$\max \pi_1^F = \left(1 - q_1^F - \sum_{j=1}^{2} q_j^L - q_2^F\right) q_1^F - 4\alpha q_1^F - f$$

where q_1^F, $q_2^F$and q_j^L, with $j = 1, 2$, represent, respectively, the outputs produced by each follower and each leader firm. By symmetry, all follower firms choose the same output and, therefore, the best reply function of each follower firm is given by $q^F = \frac{1-q_1^L-q_2^L-4\alpha}{3}$. Acting as a Stackelberg leader against the follower firms, each leader firm is going to produce $q_j^L = \frac{1-4\alpha}{3}$. Hence, we find that, at the symmetric initial market structure (L,L,F,F), the equilibrium quantities, profits, market price, Consumer Surplus (CS), Producer Surplus (PS) and Social Welfare (SW) are given by:

$$q_j^L\,(\text{L,L,F,F}) = \frac{1-4\alpha}{3}, \text{with } j = 1, 2 \tag{2.1}$$

$$q_i^F\,(\text{L,L,F,F}) = \frac{1-4\alpha}{9}, \text{with } i = 1, 2 \tag{2.2}$$

$$\pi_j^L\,(\text{L,L,F,F}) = \frac{1}{3}\left(\frac{1-4\alpha}{3}\right)^2 - f \tag{2.3}$$

$$\pi_i^F\,(\text{L,L,F,F}) = \left(\frac{1-4\alpha}{9}\right)^2 - f \tag{2.4}$$

$$P\,(\text{L,L,F,F}) = \frac{1+32\alpha}{9} \tag{2.5}$$

$$CS\,(\text{L,L,F,F}) = 32\left(\frac{1-4\alpha}{9}\right)^2 \tag{2.6}$$

$$PS\,(\text{L,L,F,F}) = 8\left(\frac{1-4\alpha}{9}\right)^2 - 4f \tag{2.7}$$

$$SW\,(\text{L,L,F,F}) = 40\left(\frac{1-4\alpha}{9}\right)^2 - 4f \tag{2.8}$$

From the analysis of equations (2.1) and (2.2) it can be demonstrated that when $\alpha \geq \frac{1}{4}$, then both leader and follower firms would not produce, i.e. $q_j^L = q_i^F \leq 0$. Also, for leader and follower firms to produce, their fixed costs should not be very high otherwise their profits would be zero, that is, $\pi^{\text{L}} > 0 \Leftrightarrow f < \frac{1}{3}\left(\frac{1-4\alpha}{3}\right)^2 \equiv \overline{f_1}$ and $\pi^{\text{F}} > 0 \Leftrightarrow f < \left(\frac{1-4\alpha}{9}\right)^2 \equiv \overline{f_2}$. Since $\overline{f_1} > \overline{f_2}$, we only assume that $f < \overline{f} = \overline{f_2}$. If $f > \overline{f}$, follower firms will not produce and could exit the market.

Assumption 2.1: Let:[7]

$$\alpha < \frac{1}{4} \equiv \overline{\alpha};\;\; f < \left(\frac{1-4\alpha}{9}\right)^2 \equiv \overline{f} \tag{2.9}$$

This assumption is imposed to exclude the case in which firms do not produce at the status quo industry structure.

2.4 Merger involving the two leader firms

Suppose that there is a merger proposal between the two leader firms in the industry. If the merger occurs, then a larger and more efficient firm is created, owning $k_i = 2$ units of the industry capital.[8] In this section, we analyze the results obtained for the two leader merger case under two different scenarios regarding the behaviour of the AA: a myopic AA and a forward-looking AA.

[7] This is the same as in Motta and Vasconcelos (2005).

[8] In Section 2.5, we discuss the results obtained for two other merger cases.

2.4.1 Merger of two leaders under a Myopic AA

For a **myopic AA** the game is the following:

- **First Stage**: the two leader firms decide whether to propose a merger (they will do so, if the merger gives higher profits).
- **Second Stage**: the AA decides whether to authorise or not the merger of two leaders and it will not take into account that other mergers might occur.

In the post-merger equilibrium, the merged leader firm and the outsider follower firms will choose their levels of output that maximize their profits. Hence, the equilibrium levels of output for the merged leader firm and for outsider follower firms are given by:

$$Q^L\,(\text{2L,F,F}) = \frac{1+2\alpha}{2} \tag{2.10}$$

$$q_i^F\,(\text{2L,F,F}) = \frac{1-10\alpha}{6}, \text{ with } i = 1,2 \tag{2.11}$$

Remark 2.1: $q_i^F = 0$, if $\alpha \geq \frac{1}{10}$ and/or $f > \left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L}$.

From equations (2.10) and (2.11) we can observe that level of efficiency gains has a positive effect on insider's quantity and a negative effect on outsiders' quantity. When the merger generates high cost savings, the insider firm produces more and therefore, the outsider follower firms react by producing less quantity. Further, if the merger gives rise to high synergies, the two follower outsider firms are constrained to exit the market. Also, if $f > \widetilde{f}_{2L}$, follower firms are not able to cover the fixed costs and make positive profits.

Suppose for the moment that $\alpha < \frac{1}{10}$. From the equilibrium outputs above, one can obtain, by substitution, the equilibrium levels of profits for the merged leader firm and for each outsider follower firms:

$$\pi^{2L}\,(\text{2L,F,F}) = \frac{1}{3}\left(\frac{1+2\alpha}{2}\right)^2 - 2f \tag{2.12}$$

$$\pi_i^F\,(\text{2L,F,F}) = \left(\frac{1-10\alpha}{6}\right)^2 - f \tag{2.13}$$

Additionally, the equilibrium price, the CS, the PS and the SW are given by:

$$P\,(2\text{L,F,F}) = \frac{1+14\alpha}{6} \tag{2.14}$$

$$CS\,(2\text{L,F,F}) = \frac{1}{2}\left(\frac{5-14\alpha}{6}\right)^2 \tag{2.15}$$

$$PS\,(2\text{L,F,F}) = \frac{212\alpha^2-28\alpha+5}{36} - 4f \tag{2.16}$$

$$SW\,(2\text{L,F,F}) = \frac{620\alpha^2-196\alpha+35}{72} - 4f \tag{2.17}$$

After the merger, the market structure is (2L,F,F) or simply (2L), the monopoly industry structure where only the merged firm is active. The resultant post-merger market structure depends on follower firms' ability to make positive profits or not. These two different scenarios are analyzed in the equilibrium analysis of stage 2 that we discuss in turn.

Analysis of Stage 2 At the second stage of the game, the AA has to decide whether or not to allow the two-leader merger, if the merger has been submitted for approval. The behaviour of the AA, that depends on the two possible scenarios presented above, is as follows:

- If $\alpha < \frac{1}{10}$ and $f < \left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L}$, then outsider follower firms are able to make positive profits after the merger has taken place. If this is the case, then the AA decides to authorise the submitted merger only if the prices after the merger are lower or equal than the prices before the merger, that is, if Consumer Surplus (CS) increases with the merger:

  $P\,(2L,F,F) = \frac{1+14\alpha}{6} \leq P(L,L,F,F) = \frac{1+32\alpha}{9}$

  which is equivalent to

  $$\frac{1}{22} \approx 0.04545 \leq \alpha < \frac{1}{10} \tag{2.18}$$

  Hence, in order to authorise the merger, the AA will require that the efficiency gains are sufficiently high.

- If, instead, (i) $\alpha \geq \frac{1}{10}$; or (ii) $\alpha < \frac{1}{10}$ and $\left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L} < f < \overline{f} \equiv \left(\frac{1-4\alpha}{9}\right)^2$ then, from Remark 2.1, one has that the merger induces the two outsider follower firms to exit the market. Therefore, the industry is characterized by a single monopolist endowed with two units of capital. The equilibrium profit, price, CS, PS and SW for the monopolist firm are given by:

$$\pi\left(2L\right) = \left(\frac{1-2\alpha}{2}\right)^2 - 2f \tag{2.19}$$

$$P(2L) = \frac{1+2\alpha}{2} \tag{2.20}$$

$$CS(2L) = \frac{1}{2}\left(\frac{1-2\alpha}{2}\right)^2 \tag{2.21}$$

$$PS(2L) = \left(\frac{1-2\alpha}{2}\right)^2 - 2f \tag{2.22}$$

$$SW(2L) = \frac{3}{2}\left(\frac{1-2\alpha}{2}\right)^2 - 2f \tag{2.23}$$

- Now, the AA faced with such a merger proposal inducing the exit by outsiders, will decide to reject it if the following inequality holds:

  $P(2L) = \frac{1+2\alpha}{2} > P(L, L, F, F) = \frac{1+32\alpha}{9}$

$$\alpha < \frac{7}{46} \approx 0.15217 \tag{2.24}$$

  This implies that a merger would not be authorised by the (myopic) AA if efficiency gains induced by the merger are sufficiently low.

Let us now turn to the analysis of the firms' decisions at the first stage of the game.

Analysis of Stage 1 Following Motta and Vasconcelos (2005), we also assume that firms have no administrative costs when submitting the merger to the AA.[9] Also, when firms anticipate that the merger will be blocked, they are indifferent between asking or not the AA for

[9] Although the assumption that firms have zero administrative costs from submitting a merger is not very realistic, assuming that these costs are zero does not matter much, since the equilibrium outcome would not change if we assumed positive filing costs.

authorisation. Firms are assumed to propose a merger to the AA even in the case of indifference.

In Stage 1, firms decide whether or not to submit a merger. Again we have to distinguish two scenarios, depending on whether the merger triggers exit by outsiders or not.

- If $\alpha < \frac{1}{10}$ and $f < \left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L}$, then outsider follower firms are able to make positive profits after the merger has taken place. Anticipating this, insider leader firms will then merge if the merger is profitable, that is if:

$$\pi^L(2L, F, F) = \frac{1}{3}\left(\frac{1+2\alpha}{2}\right)^2 - 2f \geq 2\pi^L(L, L, F, F) = 2\left[\frac{1}{3}\left(\frac{1-4\alpha}{3}\right)^2 - f\right], \tag{2.25}$$

which in turn implies that the merger is submitted to the AA for all parameter values in Assumption 2.1.

Note, however, that from eq. (2.18) we have that the AA will only approve the merger if $\alpha \geq \frac{1}{22}$. This means that for low values of the efficiency parameter, i.e. for $\alpha < \frac{1}{22}$, the two-leader merger will always be submitted but blocked by the myopic AA.

- If, instead, (i) $\alpha \geq \frac{1}{10}$; or (ii) $\alpha < \frac{1}{10}$ and $\left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L} < f < \overline{f} = \left(\frac{1-4\alpha}{9}\right)^2$, then the two-leader merger would trigger exit by outsiders if approved. Therefore, the post-merger industry structure is characterized by a single monopolist endowed with half of the monopoly capital. Consequently, the two leader firms will decide to merge if:

$$\pi(2L) = \left(\frac{1-2\alpha}{2}\right)^2 - 2f > 2\pi^L(L, L, F, F) = 2\left[\frac{1}{3}\left(\frac{1-4\alpha}{3}\right)^2 - f\right], \tag{2.26}$$

which holds for all $\alpha \in [0, \overline{\alpha}]$. Thus, the two leader firms will always decide to submit the merger to the AA.

The behaviour of a myopic AA, when deciding whether or not to authorise a merger which would trigger the exit by the outsiders, can then be summarised as follows:

- If $\alpha \geq \frac{7}{46}$, then the two-leader merger will always be authorised. Outsider follower firms would be pushed out of the market after the merger has taken place but efficiency gains are so high that consumers would gain.

- If, instead, $\alpha < \frac{7}{46}$, then the two-leader merger would not be authorised. Outsiders are not able to survive and consumers would be worse off.

Figure 2.1 illustrates, for each possible region of parameters, the equilibrium outcome obtained when there is a merger between two leaders, and thereby summarizes the results obtained when the AA is characterized by a myopic behaviour. Inside the $\{\}$ we identify the type of market structure for each region, for instance, $\{$2L,F,F$\}$ means that the market is composed by a merged firm (merger of two leaders) and two independent outsider follower firms.

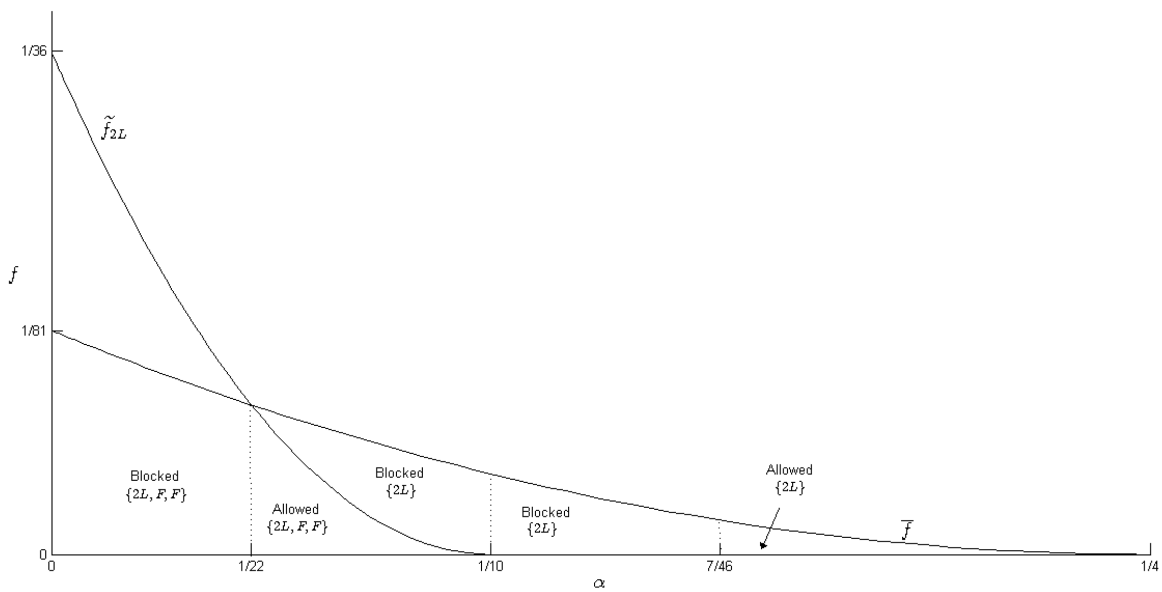

Figure 2.1: Merger of two leaders: equilibrium outcomes with a myopic AA.

2.4.2 Merger of two leader firms under a forward looking AA

For a forward looking AA the game is the following:

- **First Stage**: the two leader firms decide whether or not to propose a merger (they will do so, if the merger increases profits). If they decide to merge, they will have to ask to the AA for authorisation.

- **Second Stage**: the AA decides whether to authorise the merger or not. If AA does not authorise it, the game ends and firms stay in Stackelberg competition.

- **Third Stage**: if the AA decides to authorise the merger of two leaders at stage 2, it is the turn of the next two outsider follower firms to decide if they want to merge or not. If they do not propose a (defensive) merger, then the merger game stops and market realisation occurs. If they do want to merge, they have to ask the AA for authorisation.

- **Fourth Stage**: the AA decides whether it wants to authorise the defensive merger between outsiders of the first merger. If the AA rejects the merger, the merger game stops and the product market stage occurs. If the AA accepts the new merger then we have only two firms in the market.

In the present scenario, when making a decision on whether to allow or not the merger, the AA takes into account that the merger may be followed by other mergers. Hence, after the merger between two leaders, the market structure could be (2L,F,F), (2L) or (2L,2F).

As in the previous section, we proceed by solving the game by backward induction, and thus we start by analysing Stage 4.

Analysis of Stage 4 At the fourth stage, the AA decides whether it wants to authorise or not the defensive merger between outsiders (followers) of the first merger. Here, we have to distinguish two situations:

- If: (i) $\alpha \geq \frac{1}{10}$; or (ii) $\alpha < \frac{1}{10}$ and $\left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L} < f < \overline{f} \equiv \left(\frac{1-4\alpha}{9}\right)^2$, the followers would exit the market if the defensive merger was rejected. The resultant market structure would then be {2L}. Hence, the defensive merger is always approved if $P(2L,2F) < P(2L)$, that is, if:

$$P(2L,2F) = \tfrac{1+6\alpha}{4} \leq P(2L) = \tfrac{1+2\alpha}{2},$$

which is always true in the region of parameter values defined by Assumption 2.1.

- If, instead, $\alpha < \frac{1}{10}$ and $f < \left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L}$, the followers would not exit the market if the defensive merger was rejected. So:

  - If the AA blocks the defensive merger, the resultant market structure is {2L, F, F}, where the equilibrium price and profits are given by:

$$P(2L,F,F) = \tfrac{1+14\alpha}{6}$$
$$\pi^L(2L,F,F) = \tfrac{1}{3}\left(\tfrac{1+2\alpha}{2}\right)^2 - 2f$$
$$\pi_i^F(2L,F,F) = \left(\tfrac{1-10\alpha}{6}\right)^2 - f$$

  - If, instead the AA decides to approve the defensive merger, then the resulting structure will be {2L, 2F}, that is, a duopoly with one leader and one follower, owning half of the industry available capital each. Thus, the equilibrium price, profits, CS, PS and SW in this market structure are given by:

$$P(2L,2F) = \frac{1+6\alpha}{4} \tag{2.27}$$

$$\pi^L(2L,2F) = \frac{1}{2}\left(\frac{1-2\alpha}{2}\right)^2 - 2f \tag{2.28}$$

$$\pi^F(2L,2F) = \left(\frac{1-2\alpha}{4}\right)^2 - 2f \tag{2.29}$$

$$CS(2L,2F) = \frac{9}{2}\left(\frac{1-2\alpha}{4}\right)^2 \tag{2.30}$$

$$PS(2L.2F) = 3\left(\frac{1-2\alpha}{4}\right)^2 - 4f \tag{2.31}$$

$$SW(2L,2F) = \frac{15}{2}\left(\frac{1-2\alpha}{4}\right)^2 - 4f \tag{2.32}$$

Therefore, the AA will **decide to block** the merger between two followers if:

$$P\left(2L,2F\right)=\tfrac{1+6\alpha}{4}\geq P(2L,F,F)=\tfrac{1+14\alpha}{6},$$

or equivalent if:

$$\alpha\leq\frac{1}{10}. \tag{2.33}$$

Therefore, the defensive merger will always be blocked if $\alpha<\frac{1}{10}$ and $\widetilde{f}_{2L}<f<\overline{f}$, which is the case in the region under analysis.

Analysis of Stage 3 In this stage, we have to check whether the outsider follower firms will decide to propose the merger or not.

- If: (i) $\alpha\geq\frac{1}{10}$; or (ii) $\alpha<\frac{1}{10}$ and $\left(\frac{1-10\alpha}{6}\right)^2\equiv\widetilde{f}_{2L}<f<\overline{f}\equiv\left(\frac{1-4\alpha}{9}\right)^2$, the followers would leave the market if the defensive merger was rejected. The resultant market structure is then $\{2\text{L}\}$. Hence, the defensive merger is always proposed if:
$$\pi_F(2L,2F)\geq 0\Leftrightarrow\left(\frac{1-2\alpha}{4}\right)^2-2f\geq 0. \tag{2.34}$$
which is always true in the region of parameter values obtained in Assumption 2.1.
- If, instead, $\alpha<\frac{1}{10}$ and $f<\left(\frac{1-10\alpha}{6}\right)^2\equiv\widetilde{f}_{2L}$, the defensive merger is not going to be proposed because outsider follower firms anticipate that the defensive merger will be rejected by the AA in the following stage.

Analysis of Stage 2 In the second stage, the AA has to decide whether or not to allow the merger between the two leaders, if the merger has been submitted for approval.

- If: (i) $\alpha\geq\frac{1}{10}$; or (ii) $\alpha<\frac{1}{10}$ and $\left(\frac{1-10\alpha}{6}\right)^2\equiv\widetilde{f}_{2L}<f<\overline{f}\equiv\left(\frac{1-4\alpha}{9}\right)^2$, the AA anticipates that the merger between two followers is approved. Hence, the AA will authorise the merger between two leaders if

  $P(2L,2F)=\frac{1+6\alpha}{4}\leq P(L,L,F,F)=\frac{1+32\alpha}{9}$

$$\alpha\geq\frac{5}{74}\approx 0.06757 \tag{2.35}$$

- If, instead, $\alpha < \frac{1}{10}$ and $f < \left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L}$, the followers will not leave the market, the AA anticipates that the merger between two followers is blocked. Thus, the AA will authorise the merger between two leaders if:

  $P(2L, F, F) = \frac{1+14\alpha}{6} \leq P(L, L, F, F) = \frac{1+32\alpha}{9}$

$$\alpha \geq \frac{1}{22} \tag{2.36}$$

  Therefore, the merger will be authorised if $\alpha \geq \frac{1}{22}$.

Analysis of Stage 1 In Stage 1, leader firms decide whether or not to submit a merger. Again we have to distinguish two scenarios, depending on whether or not the merger triggers outsiders to exit the market.

- If: (i) $\alpha \geq \frac{1}{10}$; or (ii) $\alpha < \frac{1}{10}$ and $\left(\frac{1-10\alpha}{6}\right)^2 \equiv \widetilde{f}_{2L} < f < \overline{f} \equiv \left(\frac{1-4\alpha}{9}\right)^2$, leader firms anticipate that the follower firms will merge. Therefore, the leaders will propose a merger if:

$$\pi^L(2L, 2F) = \frac{1}{2}\left(\frac{1-2\alpha}{2}\right)^2 - 2f \geq 2\pi^L(L, L, F, F) = 2\left[\frac{1}{3}\left(\frac{1-4\alpha}{3}\right)^2 - f\right], \tag{2.37}$$

  which is always true in the region where $\alpha < \overline{\alpha}$ (Assumption 2.1).

- If, instead, $\alpha < \frac{1}{10}$, the leader firms anticipate that followers will not merge. Hence, the merger of two leaders is always proposed and accepted by the AA in the following stage if:

$$\pi^L(2L, F, F) = \frac{1}{3}\left(\frac{1+2\alpha}{2}\right)^2 - 2f \geq 2\pi^L(L, L, F, F) = 2\left[\frac{1}{3}\left(\frac{1-4\alpha}{3}\right)^2 - f\right], \tag{2.38}$$

  which, again, always holds under Assumption 2.1.

Figure 2.2 illustrates these results by presenting the full equilibrium outcome of the proposed merger game wherein that the AA not only is an active player, but it also anticipates the final equilibrium outcome a merger will lead to if approved.

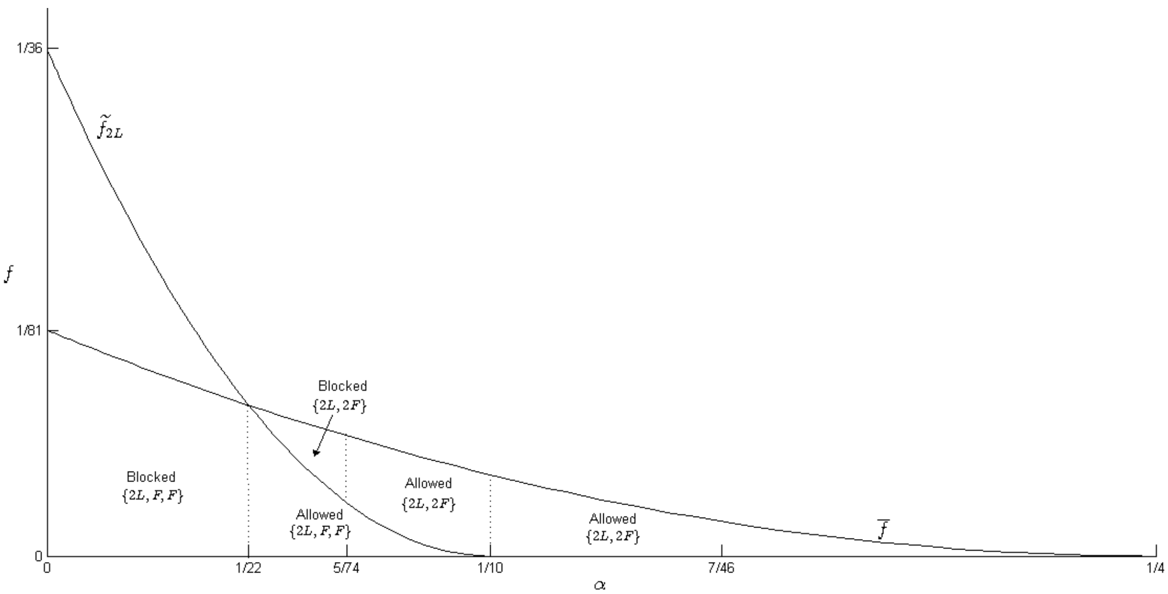

Figure 2.2: Merger of two leaders: equilibrium outcomes with a forward looking AA.

Comparing the results obtained in Figures 2.1 and 2.2, we conclude that the AA decision regarding the merger between two leaders only differs in the region of the efficiency parameter (α) values where, after the merger, the (follower) outsiders would be constrained to exit in the absence of a subsequent merger. In particular, while the myopic AA would only approve the merger between leaders for very high efficiency gains levels (expecting that the merged entity would be the monopolist of the market), the forward looking AA correctly anticipates that monopoly will not be the final induced market structure resulting from the merger, if it is approved. The first merger involving the two leaders will instead be followed by a defensive merger formed by the two outsider follower firms, and the resulting market structure will be composed of two symmetric firms (endowed with two units of capital each) with asymmetric strategic power (one leader and one follower). This being the case, the forward looking AA will only reject the merger between the two leaders if the induced efficiency gains are low (i.e. if $\alpha < \frac{5}{74} \approx 0.06757$). Hence, there exist circumstances wherein while the myopic AA would want to block a merger between two leaders, under an efficiency offence argument, the forward looking AA authorises the very same merger proposal, since it correctly anticipates that this first merger is going to be followed by a defensive merger, to the benefit of consumers.

2.5 Extensions

In this section we study two possible extensions of the benchmark model. First, we investigate the impact on our main results if we consider a total-welfare-maximizer AA. Further, we also extend the analysis to other merger cases: merger between two followers and merger between a leader and a follower.

2.5.1 Social Welfare standard

By considering an extended version of our endogenous merger formation game where the AA adopts a total welfare standard, we show, in Figure 2.3, the decisions adopted by a myopic AA.

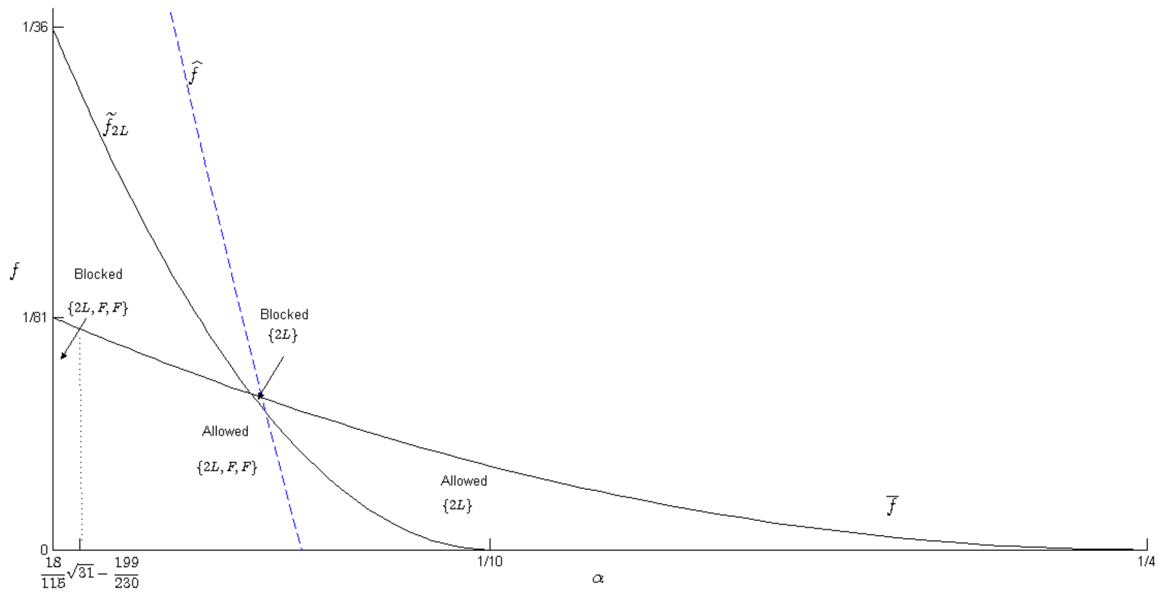

Figure 2.3: Merger of two leaders: equilibrium results with a myopic SW-maximizer AA.

By contrasting the results in Figures 2.1 and 2.3, one concludes that the merger decisions obtained for the myopic AA under the SW standard are similar to those obtained under the CS standard. In particular, in both the "exit" and the "no exit" regions, the myopic AA blocks the merger between two leaders, for low levels of the efficiency gains. However, by adopting the SW standard, the myopic AA allows the merger of two leaders for a larger range of the efficiency parameter. In the region where outsider follower firms are not constrained to exit the market, the AA only rejects the merger if $\alpha < \frac{18}{115}\sqrt{31} - \frac{199}{230} \approx 0.00626$, and before it rejected it if $\alpha < \frac{1}{22} \approx 0.04545$. When outsider follower firms are constrained to exit the market, the AA rejects the merger of two leaders to monopoly if $\widetilde{f}_{2L} < f < \overline{f}$ and $f < \widehat{f}$. Under the CS standard, the AA rejects the merger in a larger region, that is, $\frac{1}{22} \leq \alpha < \frac{7}{46} \approx 0.15217$, due to the fact that this type of AA does not take into account that the merger also increases firms' profits, which contributes to increase the social welfare.

Further, under the SW approach, the decisions of a forward-looking AA are illustrated in Figure 2.4.[10]

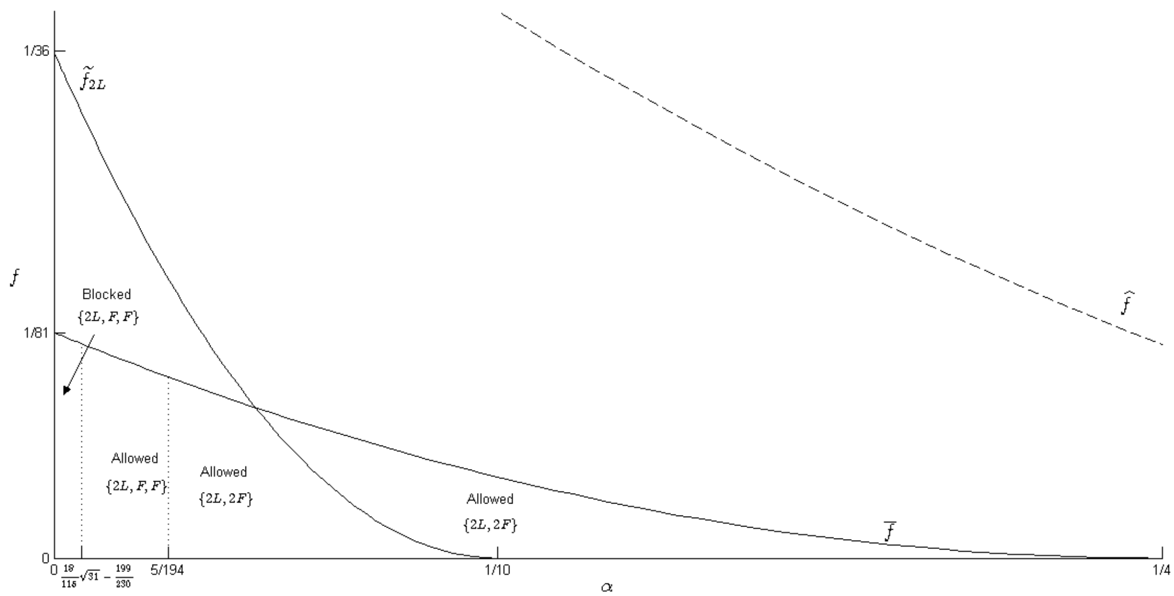

Figure 2.4: Merger of two leaders: equilibrium results with a forward-looking SW-maximizer AA.

So, contrary to what happened in the benchmark model (see Figure 2.2), the forward looking AA, adopting a SW standard, always approves both mergers in the region where outsider follower firms are constrained to exit the market. Under a CS standard and in the region of $\frac{1}{22} < \alpha < \frac{5}{74}$ and $\widetilde{f}_{2L} < f < \overline{f}$, the AA blocked the merger between the two leaders because it decreased the CS. However, in the same region and under the SW standard, the AA

[10]For more details on the calculations see Appendix A.

allows the merger because the induced increase in producers' surplus more than compensates for the decrease of CS and, therefore, the net effect is an increase in the SW.

2.5.2 Other merger possibilities

In this section, we analyze a modified version of the proposed merger formation game, wherein the merger proposal involves two followers, where the resultant firm behaves as follower or one leader and one follower, where the new firm behaves as leader. When a leader merges with a follower in a market where firms compete in quantities, it is reasonable to expect that the merged entity will behave as a leader for two reasons: (1) the merged firm can still use the old commitment technology of the former leader firm and (2) it can be checked that the merged firm would always rather be a leader than a follower. However, when two followers merge it is not so straightforward to explain why two followers should gain this commitment power by merging. Daughety (1990) studies mergers between two followers that give rise to a leader using linear costs, however the author does not address why the merger changes the strategic power of the insider firm neither considers the possibility that the merger could generate efficiency gains. With convex costs, this case is analyzed by Brito and Catalão–Lopes (2011). Hence, in this paper we assume that when two follower firms merge, the resultant firm behaves as follower. In this section we also want to investigate if the decisions of the two types of AAs change, when evaluating different mergers cases.[11]

First, we analyze the results obtained for the **merger involving two follower firms**, where the resulting firm behaves as follower. Assuming that $\alpha < \frac{1}{6}$, in order to avoid that the outsider leader firms exit the market and that $\widetilde{f}_{2F} \equiv \frac{1}{2}\left(\frac{1-2\alpha}{3}\right)^2$. The decisions of both types of AAs are summarised in Figures 2.5 and 2.6, respectively.

[11] More details on the calculations can be provided upon request to the authors.

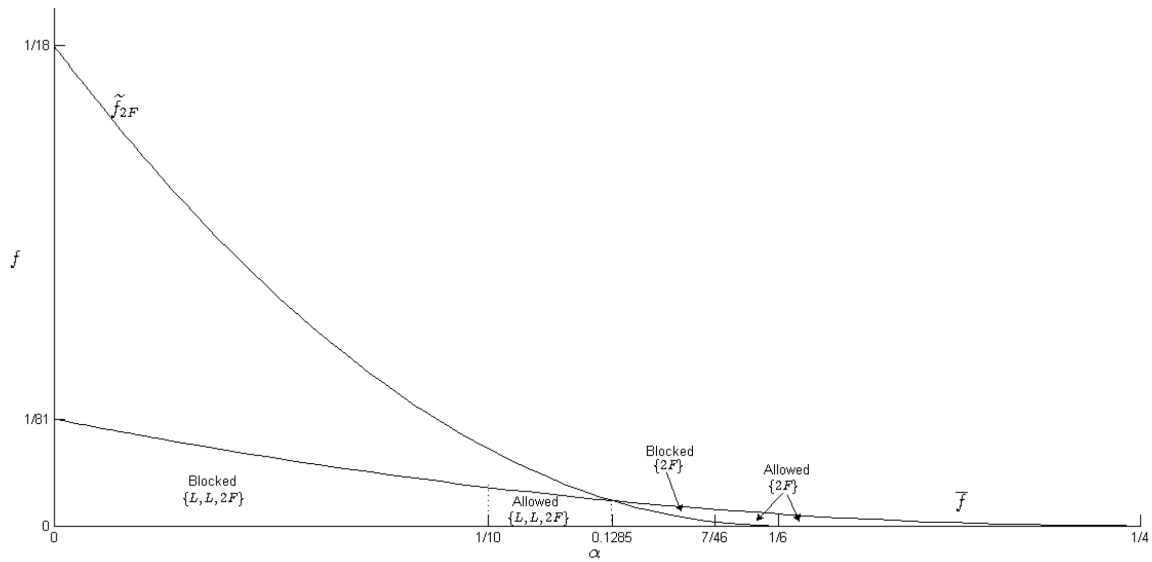

Figure 2.5: Merger of two followers: equilibrium outcomes with a myopic AA.

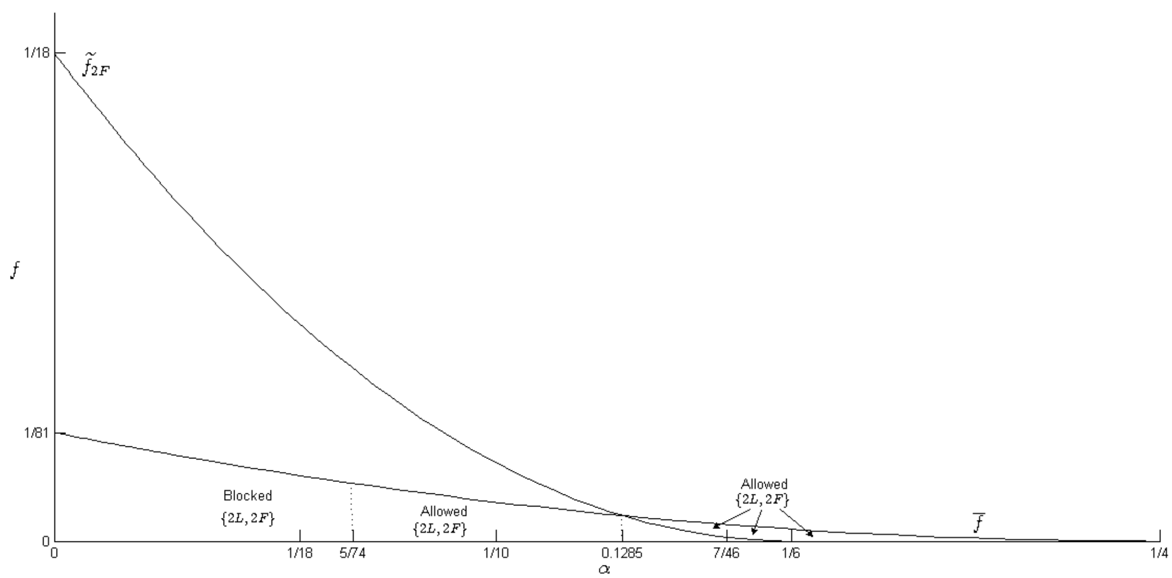

Figure 2.6: Merger of two followers: equilibrium outcomes with a forward looking AA.

By comparing Figures 2.5 and 2.6, we conclude that, in the region where outsider leader firms are not constrained to exit the market, both AAs approve the two-follower merger when the efficiency gains are high (i.e. $\alpha > \frac{1}{10}$). It turns out, however, that decisions are very different in the "exit" region. In this region, while the myopic AA would only approve the merger for high efficiency gains (expecting that the resulting firm with half of the industry capital would become a monopolist), the forward looking AA correctly anticipates that a wave of two mergers will take place instead (leading to a market structure where a unique leader and a unique follower exist with half of the industry capital each) and, as a result, always approves the first merger between the two followers, even if the efficiency gains are low.

Suppose now, that there is a **merger between one leader and one follower**, where the new firm behaves as a leader after the merger takes place. In this case, we assume that $\alpha < \frac{1}{8}$, in order for the two outsider firms not be constrained to exit the market and that $\widetilde{f}_{LF} \equiv \left(\frac{1-8\alpha}{6}\right)^2$. For this merger case, the decisions from the two types of AAs are summarised in Figures 2.7 and 2.8, respectively.

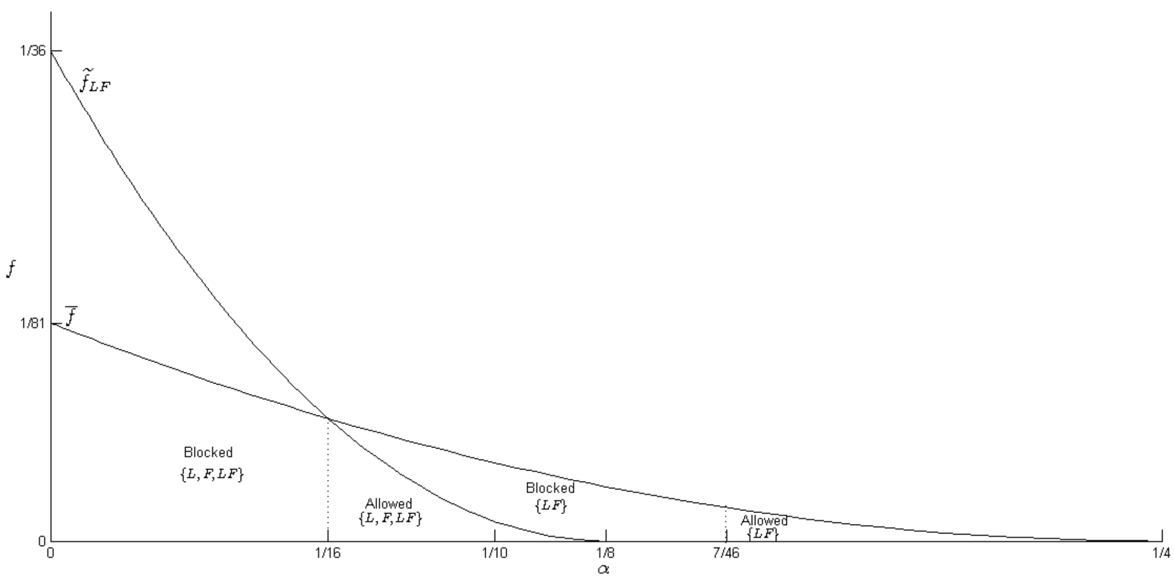

Figure 2.7: Merger of one leader and one follower: equilibrium results with a myopic AA.

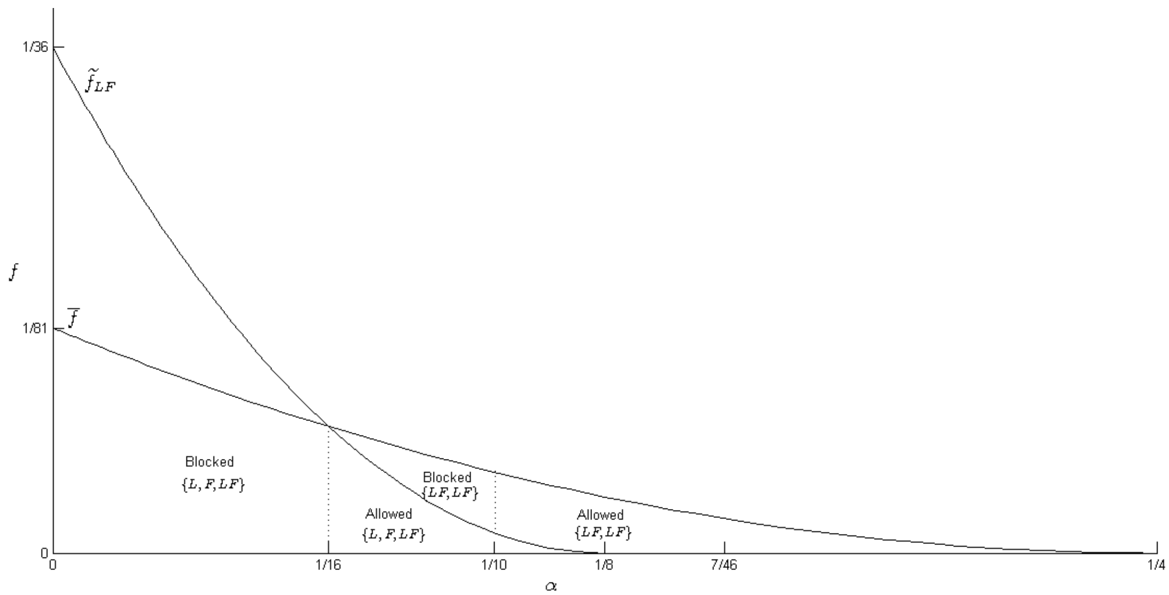

Figure 2.8: Merger of one leader and one follower: equilibrium results with a forward looking AA.

Analysing Figures 2.7 and 2.8, again, we conclude that, in the region of the efficiency parameter values where the outsider firms would not be constrained to exit the market, both AAs block the merger between one leader and one follower if the efficiency gains are low (i.e. for $\alpha < \frac{1}{16}$). The results are slightly different in the"exit" region: when the efficiency gains are small, both AAs block the leader-follower merger, however, the myopic AA will block the merger to monopoly if $\alpha < \frac{7}{46}$ and the forward looking anticipates that this merger will trigger a defensive merger from the outsider firms and will only block it for $\alpha < \frac{1}{10}$.

The following table 2.1 summarizes the results obtained for each merger case.

Table 2.1: Efficiency gains results, by region and by AA for each merger case

	Myopic AA		Forward Looking AA	
	No Exit	**Exit**	**No Exit**	**Exit**
Cournot	$0 < \alpha < \frac{1}{14}$: No Proposal $\frac{1}{14} < \alpha < \frac{1}{6}$: Allow $\{2,1,1\}$	$\frac{1}{14} < \alpha < \frac{3}{22}$: Block $\{2\}$ $\alpha > \frac{3}{22}$: Allow $\{2\}$	$0 < \alpha < \frac{1}{14}$: $\{1,1,1,1\}$ $\frac{1}{14} < \alpha < \frac{1}{6}$: Allow $\{2,2\}$	$\frac{1}{14} < \alpha < \frac{1}{5}$: Allow $\{2,2\}$ $\alpha > \frac{1}{5}$: Allow $\{1\}$
2L (CS)	$0 < \alpha < \frac{1}{22}$: Block {2L,F,F} $\frac{1}{22} < \alpha < \frac{1}{10}$: Allow {2L,F,F}	$\frac{1}{22} < \alpha < \frac{7}{46}$: Block {2L} $\alpha > \frac{7}{46}$: Allow {2L}	$0 < \alpha < \frac{1}{22}$: Block {2L,F,F} $\frac{1}{22} < \alpha < \frac{1}{10}$: Allow {2L, F,F}	$\frac{1}{22} < \alpha < \frac{5}{74}$: Block {2L,2F} $\alpha > \frac{5}{74}$: Allow {2L,2F}
2L (SW)	$0 < \alpha < 0.00626$: Block {2L,F,F} $0.02334 < \alpha < \frac{1}{10}$: Allow {2L,F,F}	$\alpha > \frac{1}{22} \wedge f > \hat{f}$: Block {2L} $f > \hat{f}$: Allow {2L}	$0 < \alpha < 0.00626$: Block {2L,F,F} $0.00626 < \alpha < \frac{5}{194}$: Allow {2L, F,F} $\frac{5}{194} < \alpha < \frac{1}{10}$: Allow {2L, 2F}	$\alpha > \frac{1}{22}$: Allow {2L,2F}
2F	$0 < \alpha < \frac{1}{10}$: Block {L,L,2F} $\frac{1}{10} < \alpha < \frac{1}{6}$: Allow {L,L,2F}	$0.12848 < \alpha < \frac{7}{46}$: Block {2F} $\alpha > \frac{7}{46}$: Allow {2F}	$0 < \alpha < \frac{5}{74}$: Block {L,L,2F} $\frac{5}{74} < \alpha < \frac{1}{6}$: Allow {L,L,2F}	$\alpha > 0.12848$: Allow {2L,2F}
LF	$0 < \alpha < \frac{1}{16}$: Block {L,F,LF} $\frac{1}{16} < \alpha < \frac{1}{8}$: Allow {L,F,LF}	$\frac{1}{16} < \alpha < \frac{7}{46}$: Block {LF} $\alpha > \frac{7}{46}$: Allow {LF}	$0 < \alpha < \frac{1}{16}$: Block {L,F,LF} $\frac{1}{16} < \alpha < \frac{1}{8}$: Allow {L,F,LF}	$\frac{1}{16} < \alpha < \frac{1}{10}$: Block {LF,LF} $\alpha > \frac{1}{10}$: Allow {LF,LF}

2.6 Conclusion

In this paper we investigate the role of efficiency defence argument in a setting where mergers involve firms with asymmetric strategic power and the merger formation game encompasses the AA as an active player.

We study and compare the decisions of two different types of AA: first, we assume a myopic AA, which accepts or rejects a given merger without considering that this merger may be followed by other mergers; and, second, a forward looking AA, which anticipates the final industry structure a merger will give rise to if approved. By so doing, we conclude that the decisions of these two types of AAs turn out to be very different, for all studied two-firm merger cases, when the proposed merger would induce outsiders to exit the market, in the absence of a subsequent merger. When this is the case, the myopic AA would not authorise any merger proposal when the associated efficiency gains are sufficiently low, since it assumes that the resulting merged entity would monopolize the industry. The forward-looking AA, however, correctly anticipates that the first (proposed) merger will be followed by a merger involving the outsider remaining firms and, therefore, makes a decision anticipating the final industry structure the first merger will lead to if approved. By so doing, the forward looking AA decides to approve a merger involving two leaders or a leader and a follower if the induced efficiency gains are high enough, and decides to approve a merger between two followers even if the resulting efficiency gains turn out to be low. Notice that when two leader firms or one leader and one follower firm merge, the resulting merged entity will be a leader in the post-merger industry structure, implying that it can explore more its enhanced efficiency by making use of its first-mover advantage. Even if for the three merger cases the final market structure is the same (2L, 2F) the decisions of a forward looking AA when deciding whether or not to accept the merger of two leaders knowing that this is going to be followed by a merger of two followers are different than when the first merger is of two followers and followed by a defensive merger of two leaders. Interestingly, the two types of AAs are shown to instead adopt similar behaviour when a merger is not supposed to trigger exit by outsider firms if approved (and no further merger takes place). When this is the case, and for all possible two-firm merger proposals, the AA will, regardless of its type, require that induced efficiency gains are sufficiently high, so as to approve the proposed merger. Obviously, the forward looking AA anticipates whether a subsequent merger will occur or not, but the decisions of the two types of AAs regarding the first merger proposal end up being qualitatively very similar.

Comparing these results with those obtained with symmetric strategic power (see Motta and Vasconcelos (2005)), we conclude that, when firms compete à la Cournot, the two-firm merger is not going to be proposed to the myopic AA, for low levels of the efficiency gains. A different outcome is obtained when firms compete instead à la Stackelberg. In this case, firms propose the merger for all levels of the efficiency gains. Further, differently from what happens under Cournot competition, we find that, even without efficiency gains, a two-firm merger is always profitable under Stackelberg. Additionally, the forward-looking AA has similar behaviour under Cournot and in the two-follower Stackelberg merger case. However, we note that in the two leaders or one leader and one follower merger cases the AA is more severe and restrictive in a Stackelberg industry than in a Cournot industry due to the fact that, in some specific circumstances, it does not allow the defensive merger or it blocks the merger for a larger range of the efficiency parameter.

Further, we find that the myopic AA decisions, when evaluating a two-leader merger, are very similar when it considers the SW or the CS standards. Regardless of the adopted standard, the myopic AA always blocks the merger involving the two leaders for low levels of the efficiency gains. Moreover, the decisions of the forward looking AAs are quite different for each adopted standard, when outsider follower firms are confined to exit the market. In this case, although the SW-maximizer AA always accepts the merger, the CS-maximizer AA blocks it for low levels of the efficiency gains. Also, we conclude that the CS-maximizer AA (myopic or forward-looking) is more restrictive when evaluating merger proposals than the SW-maximizer AA.

Finally, not only the merger's cost savings but also the fact that firms have different strategic power may affect AA decision on merger cases. The framework and the assumptions we have assumed are of a particular kind. Further research on the analysis of AA intervention should consider what happens if there is an asymmetric information problem between the AA and the merging firms concerning efficiencies due to mergers. Also, it would be interesting to assume more than two leaders or two followers in the market. This would change the results of merger profitability and the intervention of an AA that assesses mergers according to a consumer surplus standard. We think that these are very interesting and useful subjects for further research.

Appendix

A.1. Social Welfare Approach

A.1.1. Merger of two leaders under a Myopic AA

Analysis of Stage 2 First the AA has to decide whether or not to allow the merger, if the merger has been submitted for approval.

- If $\alpha < \frac{1}{10}$ and $f < \widetilde{f}_{2L}$, then outsider follower firms are able to make positive profits after the merger has taken place. If this is the case, then the AA decides to authorise the submitted merger if the total social welfare increases:

  $$SW(2L, F, F) \geq SW(L, L, F, F),$$

  which is equivalent to

  $$\alpha > \tfrac{18}{115}\sqrt{31} - \tfrac{199}{230} \approx 0.00626.$$

  Since we are in region where $\alpha < \frac{1}{10}$, hence the AA will require that the efficiency gains are sufficiently high in order to authorise the merger between to leaders, that is $\frac{18}{115}\sqrt{31} - \frac{199}{230} \approx 0.00626 \leq \alpha < \frac{1}{10}$.

- If, instead, $\alpha \geq \frac{1}{10}$ or $\alpha < \frac{1}{10}$ and $\widetilde{f}_{2L} < f < \overline{f}$, hence the merger induces outsiders to exit the industry and therefore the industry is characterized by a single monopolist. Now the AA will decide to reject the merger of two leaders if the social welfare decreases, that is:

  $$SW(2L) < SW(L, L, F, F) \Leftrightarrow f < \tfrac{4148\alpha^2 - 1588\alpha + 77}{1296} \equiv \widehat{f}.$$

  Hence, the AA rejects the merger if $\widetilde{f}_{2L} < f < \overline{f}$ and $f < \widehat{f}$.

Analysis of Stage 1 The analysis of Stage 1 is the same as before.

- If $\alpha < \frac{1}{10}$ and $f < \widetilde{f}_{2L}$, then outsider follower firms are able to make positive profits after the merger has taken place. Hence, insider leader firms will always submit the merger however this will be blocked by the myopic AA for low levels of the efficiency gains.

- If, instead, $\alpha \geq \frac{1}{10}$ or $\alpha < \frac{1}{10}$ and $\widetilde{f}_{2L} \leq f < \overline{f}$, hence the merger gives rise to very high synergies that leads to the two outsider leader firms to want to exit the market. Also after the merger, the outsider fringe firms will have negative profits. Hence, the merger induces outsiders to exit the industry and therefore the industry is characterized by a single monopolist. Therefore, the two leader firms will always submit the merger to AA.

A.1.2. Merger of two leader firms under a forward looking AA

Analysis of Stage 4 The AA decides whether it wants to authorise the defensive merger between outsiders (followers) of the first merger. Here we have two situations:

- If $\alpha > \frac{1}{10}$ or $\alpha < \frac{1}{10}$ and $\widetilde{f}_{2L} < f < \overline{f}$, the followers will exit the market if the defensive merger is rejected. The resultant market structure is $\{2\text{L}\}$. Hence, the defensive merger is always approved if:

$$SW(2L, 2F) > SW(2L) \Leftrightarrow f < \tfrac{3(1-2\alpha)^2}{64} \equiv \widehat{f}.$$

  However, $\widehat{f}$ is always above $\overline{f}$ and $\widetilde{f}_{2L}$. Hence, the defensive merger is always approved in this region.

- If, instead, $\alpha < \frac{1}{10}$ and $f < \widetilde{f}_{2L}$, the followers will not exit the market if the defensive merger is rejected. The AA will **decide to block** the merger between two followers if:

$$SW(2L, 2F) \leq SW(2L, F, F) \Leftrightarrow \alpha < \tfrac{5}{194} \approx 0.02577$$

Hence, this implies that the defensive merger will always be blocked if $\alpha < \frac{5}{194}$ and $f < \widetilde{f}_{2L}$.

Analysis of Stage 3 The analysis of Stage 3 is the same as before.

- If $\alpha \geq \frac{1}{10}$ or $\alpha < \frac{1}{10}$ and $\widetilde{f}_{2L} < f < \overline{f}$, the followers will leave the market if the defensive merger is rejected. Hence, the defensive merger is always proposed if

$$\pi^F(2L, 2F) \geq 0 \Leftrightarrow \left(\tfrac{1-2\alpha}{4}\right)^2 - 2f \geq 0,$$

which is true for all α.

- If $\alpha < \frac{1}{10}$ the followers are not constrained to exit the market if the defensive merger is rejected. Therefore, followers will merge if:

$$\pi^F(2L, 2F) = \left(\tfrac{1-2\alpha}{4}\right)^2 - 2f \geq 2\pi^F(2L, F, F) = 2\left[\left(\tfrac{1-10\alpha}{6}\right)^2 - f\right] \Leftrightarrow$$
$$\alpha \leq \tfrac{12}{191}\sqrt{2} + \tfrac{31}{382} = 0.17.$$

Thus, the defensive merger is always going to be proposed however the follower firms anticipate that the defensive merger is blocked by the AA in the following stage when $\alpha < \frac{5}{194}$.

Analysis of Stage 2 In this stage, the AA has to decide whether or not to allow the merger of two leaders, if the merger has been submitted for approval.

- If $\alpha \geq \frac{1}{10}$ or $\alpha < \frac{1}{10}$ and $\widetilde{f}_{2L} < f < \overline{f}$, the AA anticipates that the merger between two followers is approved. Hence, the AA will authorise the merger between two leaders if:

$$SW(2L, 2F) \geq SW(L, L, F, F) \Leftrightarrow$$
$$0.01254 \simeq -\tfrac{72}{781}\sqrt{3} + \tfrac{269}{1562} \leq \alpha \leq \tfrac{72}{781}\sqrt{3} + \tfrac{269}{1562} \simeq 0.33189.$$

- If $\alpha < \frac{1}{10}$ and $f < \widetilde{f}_{2L}$, the followers will not leave the market, the AA anticipates that the merger between two followers is blocked if $\alpha < \frac{5}{194}$. Hence, the AA will authorise the merger between two leaders if:

$$\alpha \geq \frac{18}{115}\sqrt{31} - \frac{199}{230} \approx 0.00626$$

Analysis of Stage 1 Also, Stage 1 is the same as before. Leader firms will always submit the merger in both regions.

Chapter 3

Mergers in Stackelberg Markets with Efficiency Gains

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3.1 Introduction

Horizontal mergers are often seen as anticompetitive and harmful to social welfare because they increase market concentration by eliminating at least one competitor in the market and by creating entry barriers. However, a merger can also be welfare-enhancing if it generates efficiency gains, increases incentives for innovation or creates synergies and scale economies among firms.

In this paper, we analyze mergers' profitability, the so called free-riding problem and the merger induced effects on social and consumer welfare in a setting where: (i) firms are in a Stackelberg market; and (ii) mergers can create cost heterogeneity between the remaining firms in the industry. The literature on profitability and welfare effects of horizontal mergers involving firms competing *à la* Stackelberg includes important contributions by: Daughety (1990), Feltovich (2001), Huck et al. (2001), Heywood and McGinty (2007, 2008),

[1]This chapter benefited a lot from comments of anonymous reviewers from JICT and of the seminar participants at the UECE Lisbon Meetings: Game Theory and Application in Lisbon, 2013.

Escrihuela-Villar and Faulí-Oller (2007) and Brito and Catalão–Lopes (2011).[2] The main message of these papers is that different types of mergers may occur in an industry characterized by Stackelberg competition, that is, mergers may occur when firms have asymmetric strategic power. In particular, one can observe mergers between leaders, mergers between followers or even mergers between leaders and followers that give rise to a leader. It should be highlighted, however, that these papers differ on the type of cost function assumed by the authors. Daughety (1990), Feltovich (2001), Huck et al. (2001), Escrihuela-Villar and Faulí-Oller (2007) considered a linear cost function while Heywood and McGinty (2007, 2008) and Brito and Catalão–Lopes (2011) assumed a convex cost function.[3] To the best of our knowledge, however, none of these previous studies has addressed the role played by efficiency gains when mergers involve firms in Stackelberg markets with linear costs.

In this paper, we contribute to cover the gap in this literature by showing that conclusions about merger profitability, social welfare effects and the existence of a free-riding problem crucially depend on whether the merger creates synergies, allowing firms to produce using a more efficient technology after the merger takes place. In particular, we show that allowing for merger induced synergies in Stackelberg markets (i) increases the incentive for mergers by reducing the free rider problem associated with mergers (that outsiders benefit more than insiders when a merger take place), and (ii) implies that profitable mergers can be welfare improving also in a setting with linear costs. The intuition is that variable cost synergies

[2]Other strands in the literature also tried to solve the Salant et al. (1983)'s merger's paradox and studied the induced welfare effects and the free-riding problem of horizontal mergers by adopting Bertrand competition with product differentiation (Deneckere and Davidson, 1985), Cournot competition with convex costs (Perry and Porter, 1985), spatial competition (Rothschild et al., 2000) or by changing the properties of the demand function (Faulí-Oller, 2002).

[3]Heywood and McGinty (2008) showed that the combination of convex costs and leadership can largely eliminate the merger paradox. Daughety (1990) showed that a merger between two followers is potentially profitable and that this merger may be welfare-improving. Huck et al. (2001) and Feltovich (2001) concluded that if a leader merges with a follower, the merger is unambiguously profitable. Further, Escrihuela-Villar and Faulí-Oller (2007) assumed that followers are less efficient than leaders and showed that leaders rarely have incentives to merge and that mergers among followers become profitable when the followers are inefficient enough.

associated with mergers imply that the merged entity increases its output and that rivals then expand less, which makes the merger more profitable. Moreover, the more aggressive behaviour by the merged entity implies that consumers will be less hurt by the merger.

Although this paper encompasses a theoretical exercise, it is also motivated by the profitability of real-world mergers involving firms with asymmetric strategic power. A case in point is the DRAM (Dynamic Random Access Memory) industry, where the leading manufacturers announce their production plans in advance and manufacturers, which enter the market later, respond by adjusting their quantity of DRAM produced. In addition, in this specific industry, the last decades have witnessed a wave of mergers.[4] Another example is Microsoft's dominance in software markets. In this case, although Microsoft usually makes decisions first, other smaller companies typically react to Microsoft's actions when making their own decisions. Obviously, these subsequent followers' actions, in turn, affect Microsoft (see e.g. Graham (2013)). In both examples, it seems reasonable and plausible to analyze the induced impacts of Stackelberg mergers.

The remainder of the paper is organized as follows. Section 3.2 introduces the baseline assumptions of the model and presents the pre-merger results. Section 3.3 shows the effects induced by the mergers. In Section 3.4 we present and discuss the results of the numerical simulation. Finally, Section 3.5 concludes. All proofs are relegated to the Appendix.

3.2 Baseline Model

We consider an industry with n firms producing a homogeneous product, where m are leader firms and $n-m$ are follower firms. We assume a two-stage game. The sequence of firms' decisions is as follows. First, all leaders simultaneously choose their quantity, then all followers simultaneously choose their quantity, after observing the leaders' decision. Firms compete in quantities over a linear demand $P = 1 - Q$, where $Q = Q_m + Q_{n-m}$, Q_m and Q_{n-m} denote, respectively, the total quantity produced by leader and follower firms and P is the industry price.

[4]For details, see Escrihuela-Villar and Faulí-Oller (2007) and the references cited therein.

What distinguishes firms is not only their strategic power (being leader or follower) but also the amount of capital they own. Hence, the cost function of firm i, which owns k_i units of the industry capital and produces q_i units of output, is given by:[5]

$$C(q_i, k_i, \alpha) = \frac{\alpha K}{k_i} q_i \tag{3.1}$$

where $\alpha \geq 0$ and $\sum_{i=1}^{n} k_i = K$.

The total supply of capital is assumed to be fixed to the industry (and equal to K units, where $K = n$). By assuming that the total quantity of capital available in the industry is fixed, we are also assuming that, in this industry, entry is very difficult. Further, we are also going to assume that there is no exit (see Assumption 3.2). We assume that each firm operates with a constant marginal cost of production, but the level of its marginal cost is a decreasing function of its capital holdings, k_i.

Before the merger, although firms have asymmetric strategic roles they are cost symmetric, holding one unit of capital each. After a two firms merger, however, firms still have asymmetric strategic roles but are cost asymmetric: outsiders still own one unit of capital each, whereas insiders are endowed with two units of capital.[6] Hence, in this setting, the merger brings the capital of merging parties into a single larger firm and, therefore, gives rise to efficiency gains by decreasing the cost of the merging firms.[7] The insider firm benefits from some merger-specific cost savings, captured by the parameter α. The higher the value of α is, the stronger the efficiency gains induced by a merger are and, therefore, the higher is the cost reduction after the merger. Although merged firm's level of capital is exogenous and equal to the number of insider firms ($k_i = 2$), the level of efficiency gains obtained will

[5] This is a simplified version of the cost structure proposed by Motta and Vasconcelos (2005) and captures the specific case studied in Horn and Persson (2001a,b).

[6] Differently from this paper, Catalão–Lopes (2007, 2008) studied mergers' effects when firms have symmetric strategic roles but are pre-merger cost asymmetric. Similar to this paper, however, Catalão–Lopes (2007, 2008) assumes that, after the merger, the merged firm becomes more efficient than outsiders.

[7] This feature of a merger was proposed by Perry and Porter (1985). In their framework firms' marginal cost is linear in output and mergers reduce the variable costs of production. For a discussion on the literature that models mergers as the pooling of capacities see, for instance, Brito and Catalão–Lopes (2006).

depend on the type of the merger. Also, note that the marginal cost (C') variation is given by $\Delta C'(q_i, k_i, \alpha) = -\frac{\alpha K}{k_i} = -\frac{\alpha K}{2}$. Since K is fixed and $k_i = 2$, then $|\frac{\partial \Delta C'(q_i,k_i,\alpha)}{\partial \alpha}| > 0$, i.e., the higher is the level of cost savings, the lower is the cost after the merger, and therefore the greater is the absolute change in the marginal cost.

Additionally, we assume that the fixed costs are zero, such that the elimination of a firm does not create a further incentive to merge.[8] Therefore, by setting the fixed costs to zero, we are isolating the impact of cost savings on firms' incentives to merge, on the free-riding incentives and on both consumer and social welfare.

3.2.1 Pre-merger equilibrium

We consider that, before the merger, there are $m < n$ Stackelberg leaders who independently and simultaneously decide their individual quantity. The remaining $n - m$ firms are Stackelberg followers who decide upon their quantity after learning about the total quantity supplied by the leaders. Before the merger (BM), the equilibrium profits for leader and follower firms are, respectively, given by:[9]

$$\pi_{L_i}^{BM}(n, m, \alpha) = \frac{(1 - n\alpha)^2}{(m + 1)^2 (n - m + 1)}, \text{ for } i = 1, .., m. \tag{3.2}$$

$$\pi_{F_j}^{BM}(n, m, \alpha) = \frac{(1 - n\alpha)^2}{(m + 1)^2 (n - m + 1)^2}, \text{ for } j = 1, .., n - m. \tag{3.3}$$

Additionally, the consumer surplus, the producer surplus and the social welfare are given by:

$$CS^{BM} = \frac{1}{2}\frac{[n + m(n - m)]^2 (1 - n\alpha)^2}{(m + 1)^2 (n - m + 1)^2} \tag{3.4}$$

[8]Usually, in constant marginal cost models it is assumed that all firms in the market have identical costs and that there are no fixed costs. Differently, in a Cournot setting, Catalão–Lopes (2008) studies the formation and stability of mergers where firms are asymmetric and face fixed production costs. After a two firms merger, only one fixed cost subsists. Catalão–Lopes (2008) shows that the existence of fixed costs increases the desirability of merging and may also increase its stability. Perry and Porter (1985), on the other hand, showed that assuming a positive fixed cost does not change the incentives to merge, since the insider firm would keep the fixed costs of each of its pre-merger constituent firms.

[9]For $\alpha = 0$, these results are the same as those obtained by Huck et al. (2001) and Feltovich (2001).

$$PS^{BM} = \frac{[n + m(n-m)]\,(1 - n\alpha)^2}{(m+1)^2\,(n-m+1)^2} \tag{3.5}$$

$$SW^{BM} = \frac{1}{2}\frac{[n + m(n-m)]\,[n + m(n-m) + 2]\,(1 - n\alpha)^2}{(m+1)^2(n-m+1)^2} \tag{3.6}$$

Assumption 3.1. Let us assume that

$$\alpha < \frac{1}{n} \equiv \overline{\alpha} \tag{3.7}$$

This condition is imposed in order to exclude the case in which firms will not produce at the status quo industry structure.

3.3 Merger Effects

In this section we analyze the effects of a merger between two firms in the industry on the profits of both outsider and insider firms, and on both social welfare and consumer surplus. If the merger occurs, then a larger and more efficient firm is created, owning $k_i = 2$ units of the industry capital.

The effects of a merger in the Stackelberg industry will depend on the types of the merger partners involved. We consider three cases: a) merger of two leaders that form a more efficient leader, b) merger of two followers, where the resulting firm behaves as follower and c) merger of one leader and one follower, where the new firm behaves as leader after the merger takes place. We do not analyze the case where two followers merge and the resultant firm is a leader. When a leader merges with a follower in a market where firms compete in quantities, it is reasonable to expect that the merged entity will behave as a leader, mainly for two reasons: (1) the merged firm can still use the old commitment technology of the former leader firm; and (2) it can be checked that the merged firm would always rather be a leader than a follower. However, when two followers merge it is not so straightforward to explain why two followers should gain this commitment power by merging. Although the resultant firm becomes more efficient than its rivals, this does not mean that the merger allows firms to change their strategic power. Daughety (1990) studies mergers between two followers that give rise to a leader using linear costs, but he does not address why the merger changes the behaviour of the firms neither considers the possibility that the merger could generate efficiency gains. With convex costs, this case is analyzed by Brito and Catalão–Lopes (2011).

This means that in case a) the post-merger market will have $m-1$ leaders but still $n-m$ followers, in case b) the post-merger market will have m leaders but only $n-m-1$ followers, and in case c) the post-merger market will have m leaders and $n-m-1$ followers.[10]

Assumption 3.2 is introduced to ensure that after any merger, the remaining firms in the industry are all active. The motivation behind this assumption is that, in practice, exit inducing mergers are very rare. Relaxing this assumption would significantly complicate the analysis, since we would have to consider not only growth by mergers but also growth by capital accumulation. In this case, we would obtain two regions of analysis: one region where the merger would trigger the exit of outsider firms and the other region where the reverse would occur. In addition, although mergers decrease competition, we exclude the case where the merger leads to industry monopolization.

Assumption 3.2. Assume that after any two firms merger, all firms are active. That is, for all $\alpha > 0$ and knowing that $\alpha_1(n,m) < \alpha_3(n,m) < \alpha_2(n,m)$:

- Let $\alpha < \alpha_1(n,m) \equiv \frac{2}{n(n-m+3)}$, such that all firms produce a positive amount after a merger involving two leaders;[11]
- Let $\alpha < \alpha_2(n,m) \equiv \frac{2}{3n}$, such that all firms produce a positive amount after a merger involving two followers and
- Let $\alpha < \alpha_3(n,m) \equiv \frac{2}{n(n-m+2)}$, such that all firms produce a positive amount after a merger involving one leader and one follower.

[10] See Appendix B.1 for further information on post-merger equilibrium results.

[11] Note that in order for the insider's quantity to be positive, the level of efficiency gains should be higher than a negative threshold. Since we have assumed that $\alpha > 0$, we exclude this threshold. Also, it is straightforward to show that the quantity of insider firms is always positive for any level of efficiency gains. However, in order for both outsider leader or follower firms to produce, α must be lower than α_1. See Remark 3.1, Appendix B.1.

For instance, if the level of efficiency gains exceeds $\alpha_1\left(n,m\right)$, then, in a symmetric equilibrium, both outsider follower and leader firms would exit the market after the merger between two leaders. Consequently, the monopoly would be the merger induced industry structure. In addition, if exit inducing mergers were possible, the total industry capital could change after a merger. This being the case, the model would probably need to be extended so as to allow for firms to buy and sell capital. This is, however, outside the scope of the present paper since we want to focus on growth by acquisition only.

The following subsections discuss the impact of each merger on the profitability of insiders and outsiders and on consumer surplus and social welfare. Further, we compare some of the results obtained with those obtained if firms were competing *à la* Cournot or if there were no efficiency gains generated by the mergers.

3.3.1 Insiders' Profitability

In this subsection we show that mergers in industries where firms have asymmetric strategic power may actually be profitable.

In order to assess mergers' profitability it is necessary to compare, for each merger case, the profits obtained when firms operate independently with the equilibrium profits of the merged entity.

Let the change in insider profits due to the merger be expressed as $g(n,m,\alpha)$, where g is different for the different merger cases. For a specified number of leader firms (m) and follower firms ($n-m$), this expression can be used to determined whether the merger of two firms is profitable. Now, for each merger case, gains from the merger occur if and only if $g(n,m,\alpha)>0$.

The merger between two leaders is profitable if the profit of the insider leader firm ($\pi_{L_I}^{2L}$) is higher than the aggregate profit of the leader firms before the merger, that is, if and only if:

$$g^{2L}(n,m,\alpha)=\pi_{L_I}^{2L}(n,m,\alpha)-2\pi_{L_i}^{BM}(n,m,\alpha)>0$$

$$g^{2L}\left(n,m,\alpha\right)=\frac{\left[2-n\alpha(n-m(n-m+2)+3)\right]^2\left(m+1\right)^2-8m^2\left(1-n\alpha\right)^2}{4m^2\left(n-m+1\right)\left(m+1\right)^2}>0 \qquad (3.8)$$

The merger between two followers is profitable if the profit of the insider follower firm ($\pi_{F_I}^{2F}$) is higher than the aggregate profit of the follower firms before the merger, i.e., iff:

$$g^{2F}(n,m,\alpha) = \pi_{F_I}^{2F}(n,m,\alpha) - 2\pi_{F_j}^{BM}(n,m,\alpha) > 0$$

$$g^{2F}(n,m,\alpha) = \frac{[2-n\alpha(3-(m+1)(n-m))]^2(n-m+1)^2 - 8(n-m)^2(1-n\alpha)^2}{4(n-m+1)^2(m+1)^2(n-m)^2} > 0 \tag{3.9}$$

The merger between a leader and a follower is profitable if the profit of the insider firm (π_I^{LF}) is higher than the aggregate profit of the outsider firms before the merger, that is, iff:

$$g^{LF}(n,m,\alpha) = \pi_I^{LF}(n,m,\alpha) - \left[\pi_{F_j}^{BM}(n,m,\alpha) + \pi_{L_i}^{BM}(n,m,\alpha)\right] > 0$$

$$g^{LF}(n,m,\alpha) = \frac{[2-n\alpha(m(n-m)-2)]^2(n-m+1)^2 - 4(n-m+2)(n-m)(1-n\alpha)^2}{4(n-m+1)^2(m+1)^2(n-m)} > 0 \tag{3.10}$$

Hence, the incentives for two firms to merge depend not only on the number of each type of firms in the market but also on the efficiency parameter α.[12] Propositions 3.1, 3.2 and 3.3 summarize, for each merger case, the profitability results.

Proposition 3.1. For all $m > 1$ and $0 < \alpha < \alpha_1(n,m)$, knowing that $\alpha_4(n,m) < \alpha_1(n,m)$ the merger between two leaders is profitable iff:

i) $\alpha > 0$, for $m = 2$ and

ii) $\alpha > \alpha_4(n,m)$, for all $m > 2$.

Proof. See Appendix B.2. □

[12]By solving $g^{2L}(n,m,\alpha) = 0$, $g^{2F}(n,m,\alpha) = 0$ and $g^{LF}(n,m,\alpha) = 0$, we obtain the roots of α as a function of n and m, that ensure merger profitability. However, some of these roots are negative or higher than $\overline{\alpha}$ and, therefore, we exclude them.

Proposition 3.2. For all $n - m > 1$ and $0 < \alpha < \alpha_2(n, m)$, knowing that $\alpha_5(n, m) < \alpha_2(n, m)$ the merger between two followers is profitable iff:

i) $\alpha > 0$, for $n - m = 2$ and

ii) $\alpha > \alpha_5(n, m)$, for all $n - m > 2$.

Proof. See Appendix B.2. □

Proposition 3.3. For all $n - m > 1$ and $m > 1$ and $0 < \alpha < \alpha_3(n, m)$, the merger between a follower and a leader is always profitable.

Proof. See Appendix B.2. □

Not surprisingly, Propositions 3.1 and 3.2 confirm that a merger is profitable when it involves the only two leaders or the only two followers in the industry. These mergers were already profitable in the no synergies case and, therefore, their profitability is reinforced when efficiency gains are induced by a merger and even when these efficiency gains are low. However, these propositions also show that if there are more than two leader firms or more than two follower firms in the status quo industry structure, mergers involving two leaders or two followers can be profitable if the induced efficiency gains are high enough.[13] Finally, Proposition 3.3 states that there is always profit incentive for a leader and a follower to merge if there are efficiency gains.

The results obtained in Propositions 3.1, 3.2 and 3.3 differ from those obtained without merger-created efficiencies.[14] When mergers do not create any synergies, the results obtained in our framework are the same as those previously shown by Huck et al. (2001) and Feltovich (2001). In particular, a merger of two leaders to form a leader or two followers to form a follower is profitable if and only if the merging firms were the only firms of their type at the status quo industry structure (Huck et al. (2001)'s Proposition 1 and Feltovich (2001)'s

[13]This result is similar to the one obtained by Perry and Porter (1985), where firms compete *à la* Cournot and costs are convex. This is also similar to Heywood and McGinty (2007)'s paper, where they conclude, under convex costs, that mergers involving two leaders or two followers are profitable if the costs savings are sufficiently high.

[14]See Appendix B.3 for further information.

Result 1). However, if one leader and one follower merge to form a leader, the merger is always profitable since now the follower firm benefits from an increase in its strategic power (Huck et al. (2001)'s Proposition 2 and Feltovich (2001)'s Result 3). Moreover, if there is no strategic power advantage for any firm (Cournot competition), we obtain the well known Salant et al. (1983)'s result: a merger between two firms with the same strategic power that creates a firm of the same type is always unprofitable for the merging firms, unless more than 80% of the firms in the industry merge or there are sufficiently high fixed costs. The difference between our results and its analogy to the Cournot model is that, in our setting, in order for a merger to be profitable, the merging firms do not need to be the only two firms in the industry nor the only firms of their type in the industry. Further, it can be shown that when there are no efficiency gains, the incentives for two firms to merge are higher when firms compete *à la* Stackelberg than in the case of Cournot competition. Hence, the asymmetric strategic power combined with merger efficiency gains creates a larger incentive for firms to merge.

Figure 3.1 illustrates the results regarding merger profitability, for each merger case, when $n = 6$ and $m = 3$.

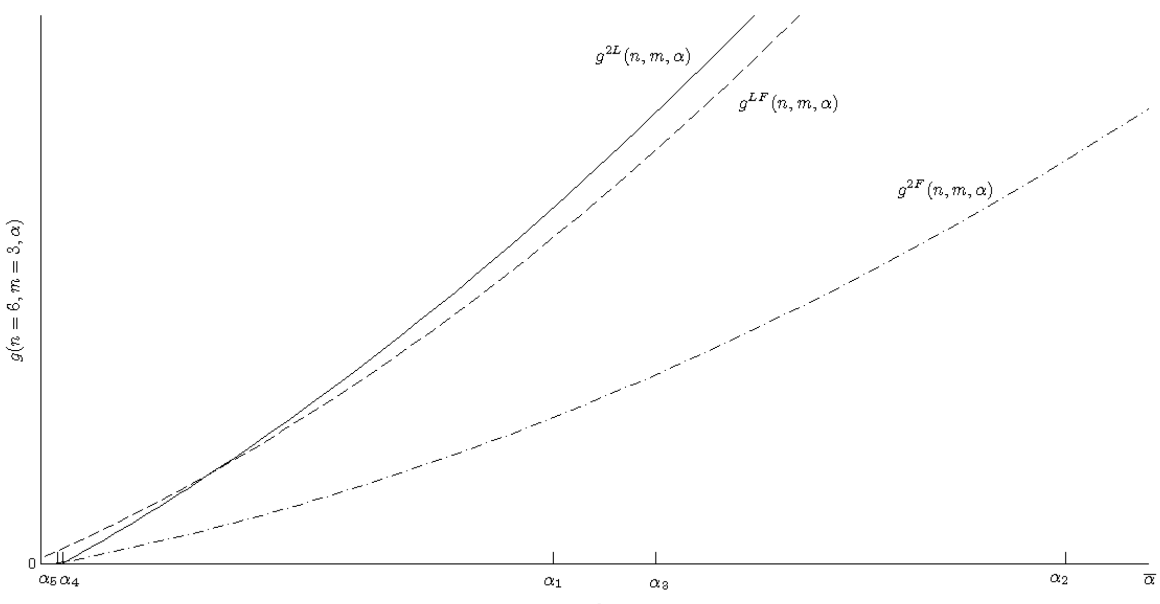


Figure 3.1: Merger profitability for $n = 6$ and $m = 3$

Two remarks should be made regarding this figure. First, merger profitability is positively correlated with the level of efficiency gains induced by the merger, as expected.[15] Second, for a given level of efficiency gains, the merger's impact on insiders' profitability is more significant when (at least one of these) insiders have a leadership position in the market. The same results are obtained if we considered any other feasible values for $n > 2$ and m. Further, when there are efficiency gains, the effect of these efficiency gains on merger profitability is higher when the merger is between two leader firms (or a leader and a follower firms) than when the merger is between two follower firms, that is, $\frac{\partial g^{2L}(n,m,\alpha)}{\partial \alpha} > \frac{\partial g^{2F}(n,m,\alpha)}{\partial \alpha}$ and $\frac{\partial g^{LF}(n,m,\alpha)}{\partial \alpha} > \frac{\partial g^{2F}(n,m,\alpha)}{\partial \alpha}$. The reason why the profitability of a merger involving at least one leader is more positively affected by merger induced synergies than the profitability of a merger involving two followers is simple. When the merged entity will be a leader in the post-merger industry structure, its higher efficiency will be crucial to explore further the Stackelberg (first-mover) advantage. If instead we investigate the impact of merger efficiencies on the profitability of two alternative mergers where a leader firm is always involved, then the conclusion is no-longer clear-cut. If α is low, the effect of the efficiency gains on merger profitability is higher when the merger is between a leader and a follower firm than when the merger is between two leader firms, i.e., $\frac{\partial g^{2L}(n,m,\alpha)}{\partial \alpha} < \frac{\partial g^{LF}(n,m,\alpha)}{\partial \alpha}$. However, the reverse occurs as α increases.

Additionally, the sequentiality of the choices plays an important role in explaining merger profitability. In a merger involving two followers, the insider followers anticipate that outsider (leader and follower) firms will reduce their quantities in response to the merger. This, in turn, implies that the increase in the merged follower firm quantity will not be so great than in the case where two leaders (or a leader and a follower) merge, even if cost savings result from the merger, since now the insider firm has a follower position. If the merger involves, instead, two leader firms, both insider leader and outsider leader firms make their output decisions anticipating the subsequent followers' best response. Thus, as mentioned above, insider firms will be able to enhance merger profitability by combining synergies with their

[15]Note that for all merger cases the $\frac{\partial g(n,m,\alpha)}{\partial \alpha} > 0$ and that $\frac{\partial^2 g(n,m,\alpha)}{\partial \alpha^2} > 0$. See Appendix B.2.

leadership position.

3.3.1.1 Numerical simulation results on merger profitability

By making use of numerical simulations, it can be illustrated that for $m > 2$ and $n - m > 2$, keeping the level of efficiency gains and the number of followers constant, as the number of leaders increases, the merger profitability increases in the three cases. Further, keeping the number of leaders constant, as the number of followers increases, the merger profitability decreases in the cases of a merger between two leaders and in a merger involving a leader and a follower, i.e., $\frac{\partial g^{2L}(n,m,\alpha)}{\partial m} > 0$, $\frac{\partial g^{2L}(n,m,\alpha)}{\partial n-m} < 0$; $\frac{\partial g^{2F}(n,m,\alpha)}{\partial m} > 0$, $\frac{\partial g^{2F}(n,m,\alpha)}{\partial n-m} > 0$; $\frac{\partial g^{LF}(n,m,\alpha)}{\partial m} > 0$, $\frac{\partial g^{LF}(n,m,\alpha)}{\partial n-m} < 0$.

Plugging in specific values of n and m into the $g^{2L}(n,m,\alpha)$, $g^{2F}(n,m,\alpha)$ and $g^{LF}(n,m,\alpha)$ functions, allows solving for the critical value of α. The results of our simulations are presented in Table 3.1. Table entries are values of α that satisfy Propositions 3.1, 3.2 and 3.3 (lower bound) and Assumption 3.2 (upper bound), i.e. they identify the interval for the parameter α, such that the merger is profitable and exit is not induced by the merger. The top entry is for a merger between two leaders, the middle entry is for a merger between two followers, and the bottom entry is for a merger between a leader and a follower.

Table 3.1: Efficiency gains conditions for profitable mergers and no exit

	$m=2$	$m=4$	$m=8$	$m=10$
$n-m=2$	$0<\alpha<0.1$ $0<\alpha<0.1667$ $0<\alpha<0.125$	$0.0047<\alpha<0.0667$ $0<\alpha<0.1111$ $0<\alpha<0.0833$	$0.0024<\alpha<0.04$ $0<\alpha<0.0667$ $0<\alpha<0.05$	$0.0017<\alpha<0.0333$ $0<\alpha<0.0556$ $0<\alpha<0.0417$
$n-m=4$	$0<\alpha<0.0476$ $0.0039<\alpha<0.1111$ $0<\alpha<0.0556$	$0.0021<\alpha<0.0357$ $0.0017<\alpha<0.0833$ $0<\alpha<0.0417$	$0.0012<\alpha<0.0238$ $0.0006<\alpha<0.0556$ $0<\alpha<0.0278$	$0.0009<\alpha<0.0204$ $0.0004<\alpha<0.0476$ $0<\alpha<0.0238$
$n-m=8$	$0<\alpha<0.0182$ $0.0022<\alpha<0.0667$ $0<\alpha<0.02$	$0.0008<\alpha<0.0152$ $0.0011<\alpha<0.0556$ $0<\alpha<0.0167$	$0.0005<\alpha<0.0114$ $0.0005<\alpha<0.0417$ $0<\alpha<0.0125$	$0.0004<\alpha<0.0101$ $0.0003<\alpha<0.037$ $0<\alpha<0.0111$
$n-m=10$	$0<\alpha<0.0128$ $0.0016<\alpha<0.0556$ $0<\alpha<0.0139$	$0.0006<\alpha<0.0110$ $0.0008<\alpha<0.0476$ $0<\alpha<0.0119$	$0.0004<\alpha<0.0086$ $0.0004<\alpha<0.0370$ $0<\alpha<0.0104$	$0.0003<\alpha<0.0077$ $0.0003<\alpha<0.0333$ $0<\alpha<0.0083$

Two remarks are in order at this point. First, keeping the number of leaders (followers) constant, as the number of followers (leaders) increases, the range of the critical region of the level of efficiency gains for two leaders (two followers) to profitably merge decreases. Second, keeping the number of leaders (followers) constant, the range of the critical region of the parameter α for a leader-follower to be profitable decreases as the number of follower (leader) firms increases.

Hence, any two firms merger is more likely to be profitable when there are relatively few firms of the same type of the merging firms but also few firms of the other type. In the two-leader merger case, the merged leader firm wants to have few leader competitors and to enjoy leadership over few follower firms. In the two-follower merger case, the merged follower firm wishes to have few follower competitors (increasing the output in response to their reduction) and to follow few leader firms. Also, as the number of firms of same type (for instance, leaders) falls due to the merger (two-leader firms merger), there is a decrease in the region of parameters α for which the merger is profitable.

Another result is that the range of the critical region of the parameter α required for two leaders to profitably merge without exit does not exceed that necessary for the two followers to profitably merge and also does not exceed that necessary for the merger between one leader and one follower to be profitable. This means that, when there are efficiency gains the merger between two leader firms is less likely to be profitable and satisfy Assumption 3.2 than the mergers between two follower firms or between a leader and a follower firms.

Surprisingly, however, the range of the critical region of the parameter α required for two followers to profitably merge exceeds that necessary for one leader and one follower to profitably merge. Hence, when a merging firm behaves as a follower at the status quo industry structure, it is more likely that it will benefit from a merger (generating efficiency gains) when it merges with a firm of the same type than when it embarks on a merger with a firm of a different type and enjoys leadership in the merger induced industry structure. This result may be due to the fact that in the merger between a leader and a follower, the leader has already a high profit at the status quo industry structure. Hence, the merger induced efficiency gains will have to be substantial so as to convince this leader firm to accept to share its leadership position with its merger partner (a follower) in the market structure induced by the merger.

Further, having two stages of quantity choice does not mitigate the reaction to the quantity reduction undertaken by the merged firm but instead increases it, reducing the opportunities for a two-leader merger, that generates efficiency gains, to be profitable under Stackelberg than a two firms merger under Cournot.[16] However, for the two-follower merger the reverse occurs. Hence, we conclude that, when each merger is profitable, the profitability under Stackelberg could be higher or lower than under Cournot, for different merger cases and levels of efficiency gains.

Hence, the asymmetric strategic power combined with merger induced efficiency gains can indeed increase firms' incentives to merge.

3.3.2 Free-riding problem

In this subsection we analyze whether outsider firms profit more from the merger than the insider firms. If this is the case, although insiders' profits increase with the merger, firms would prefer to wait for their rivals to merge and, thus, benefit from higher prices without having to reduce the quantity. To verify if there is a free-riding problem in our setting, we compare, for each merger case, the profits of the insider firm with the profits of the outsider firms that could also participate in the merger, conditioning on each merger being profitable and excluding the case where outsider firms do not produce.

To identify the free-riding problem, define $f(n, m, \alpha)$ as the difference between the profits of the outsider firms and the profit of the insider firm, that depends on the number of leader (m) and follower ($n - m$) firms and on the level of cost savings (α). For each merger case, outsider firms profit more from the merger than the insider firms if and only if $f(n, m, \alpha) > 0$.

When the merger between two leaders (followers) creates no synergies, outsider leader (follower) firms have always incentives to free-ride and, thus, always would prefer to wait for their rivals to merge, benefiting from higher prices without having to reduce the quantity. Also, when the merger is between a leader and a follower, outsider firms have always incentives to free-ride on it. These results are similar to those obtained under Cournot. In

[16]More details on calculations for a Cournot market can be provided upon request to the authors.

both settings, and without cost savings, the outsider firms benefit more from the merger than the insider firms, and these benefits may be higher under Cournot than under Stackelberg, depending on the number of firms in the market.

For each merger case, let $\alpha_{A3.2}$ be the level of efficiency gains obtained in Assumption 3.2. Proposition 3.4 summarizes under which conditions it is better to free ride than to participate in the merger.

Proposition 3.4. For all $n > m$ and $0 < \alpha < \alpha_{A3.2}$, knowing that $\alpha_8 < \alpha_6 < \alpha_7$, when mergers create synergies:

i) The merger between two leaders gives rise to a free-riding problem if outsider leader firms earn more than the insider ($f^{2L}(n,m,\alpha) > 0$), that is, iff:

$$\alpha < \alpha_6(n,m), \text{ for all } m \geq 2;$$

ii) The merger between two followers gives rise to a free-riding problem if outsider follower firms earn more than the insider ($f^{2F}(n,m,\alpha) > 0$), that is, iff:

$$\alpha < \alpha_7(n,m), \text{ for all } n - m \geq 2;$$

iii) The merger of one follower and one leader gives rise to a free-riding problem if outsider firms earn more than the insider ($f^{LF}(n,m,\alpha) > 0$), that is, iff:

$$\alpha < \alpha_8(n,m), \text{ for all } n - m > 1 \text{ and } m > 1;$$

where $\alpha_6(n,m)$, $\alpha_7(n,m)$ and $\alpha_8(n,m)$ are lower than $\alpha_{A3.2}$.

Proof. See Appendix B.4. □

Hence, it turns out that when there are cost savings created by the merger, for some merger cases the free-riding problem is solved whereas for others it is not. While the profit of the merged firm always increases, that of an outsider firm may either increase or decrease as a result of the merger.

In traditional models, without merger cost's savings the profit of the outsider firms increases after the merger. The merged firm increases its profits by decreasing the quantity, which, in turn, also increases outsiders' profits. However, with linear costs and cost synergies the insider firm may actually increase quantity beyond that of its pre-merger constituent firms and, therefore, the quantity and profits of outsider firms may decrease after the merger. For instance, if the merger is between two followers, the insider firm will produce more and both outsider follower and leader firms will react by reducing their quantity. This reduction is higher as the level of efficiency gains increases. Hence, for high level of efficiency gains, outsider firms have no incentives to free ride on the merger.

Hence, we find that when the merger creates cost synergies and firms in the industry have asymmetric strategic power, the free-riding problem might disappear. This result is similar to the one obtained by Heywood and McGinty (2008), with convex costs and Stackelberg leadership. However, this contrasts with Perry and Porter (1985), Huck et al. (2001), Heywood and McGinty (2007) and Brito and Catalão–Lopes (2011). In these papers, outsider firms would always prefer to remain as outsiders, even when the merger is profitable for the insiders.[17]

In sum, we find that when there is a merger between two leaders, two followers or a leader and a follower, outsider firms have no incentives to free-ride if the merger generates sufficiently cost savings.

3.3.3 Consumer Surplus and Social Welfare

In the next subsections we examine the effects of each merger case on both social welfare and consumer surplus.

[17]For instance, the model of Brito and Catalão–Lopes (2011) is significantly different from ours. In particular, they study the effects of the merger between two followers that become a leader. In addition, they consider a convex cost function and that after the merger firms have asymmetric costs. They show that under convex costs there is a free riding problem. The merger between two followers that become leader decreases the number followers. Now, this decrease of the number of followers has a negative effect on the profit of the merging followers because one of them is eliminated, however it has a positive effect for outsider followers. Also, by turning a follower into a leader induces a negative effect for outsider followers since increasing the number of leaders by one always reduces each follower's profit. Hence, Brito and Catalão–Lopes (2011) show that if the cost effect is such that the negative effect on outsiders' profits due to the appearance of more leaders is higher than the positive effect due to the elimination of one follower, the free-riding problem emerges.

Williamson (1968) argued that the goal of competition policy should be to maximize total welfare. Later, Farrell and Shapiro (1990) considered that mergers should be assessed on the basis of their effects on consumers and firms jointly, assuming that the merger will be privately profitable to the merging firms. However, as Lyons (2002) argued, "most antitrust authorities operate under legislation and guidelines that reject the social welfare standard, and no major antitrust authority seems to apply it consistently. Instead, antitrust authorities tend to use the consumer surplus standard when evaluating merger cases".

3.3.3.1 Consumer Surplus Standard

In this subsection we analyze the effects of each merger case on consumer surplus. We also show how the consumer surplus variation depends on with the efficiency parameter α.

The consumer surplus variation is found by solving $\Delta CS = CS^{AM}(n, m, \alpha) - CS^{BM}(n, m, \alpha)$, where the formal expressions regarding consumer surplus after and before the merger (CS^{AM} and CS^{BM}, respectively) are presented in the Appendix.

When mergers create efficiency gains, the effects on consumer surplus are manifold, as summarized in the following proposition.

Proposition 3.5. For all $n > m$ and $0 < \alpha < \alpha_{A3.2}$, when mergers create efficiency gains:

i) The merger between two leaders improves consumer surplus iff:

$$\alpha > \alpha_9(n, m), \text{ for all } m \geq 2 \text{ and } n - m > 1.$$

ii) The merger between two followers improves consumer surplus iff:

$$\alpha > \alpha_{10}(n, n), \text{ for all } n - m \geq 2 \text{ and all } m > 1.$$

iii) The merger between a leader and a follower improves consumer surplus iff:

$$\alpha > \alpha_{11}(n, m), \text{ for } m > 1 \text{ and } n - m > 1.$$

where $\alpha_9(n, m)$, $\alpha_{10}(n, m)$ and $\alpha_{11}(n, m)$ are lower than $\alpha_{A3.2}$.

Proof. See Appendix B.5. □

Analyzing Proposition 3.5 we conclude that the merger between two leaders, two followers or one leader and one follower can actually improve consumer surplus if the level of efficiency gains is high enough.

The merged entity resulting from the merger of two leaders has a lower cost than outsiders and this may actually increase the quantity beyond the quantity of its pre-merger constituent firms. Also, outsider firms will react by reducing their quantity. However, the increase in insider's quantity may more than compensate for the decrease in outsiders' quantity. Hence, the net effect may then be a price decrease if the cost savings are sufficiently high. If the level of efficiency gains are low, the reverse occurs. Hence, if the merger induced cost savings are sufficiently high and satisfy the thresholds above, the merger could generate gains for consumers by decreasing price and increasing total quantity in the market. This result is similar to the one obtained by Farrell and Shapiro (1990) who have shown that, in a Cournot framework, mergers are likely to harm consumers unless cost savings are sufficiently strong.

Interestingly, however, without synergies, all discussed types of mergers have the same effect: the consumer surplus decreases. This is a similar result to the one obtained by Salant et al. (1983) under Cournot competition. Without cost savings, any two firms merger always leads to a decrease in the total quantity and, thus, increases the market price.

In Figure 3.2 we illustrate, for the case in which $n = 6$ and $m = 3$, the interval of cost savings where each type of merger increases the consumer surplus.

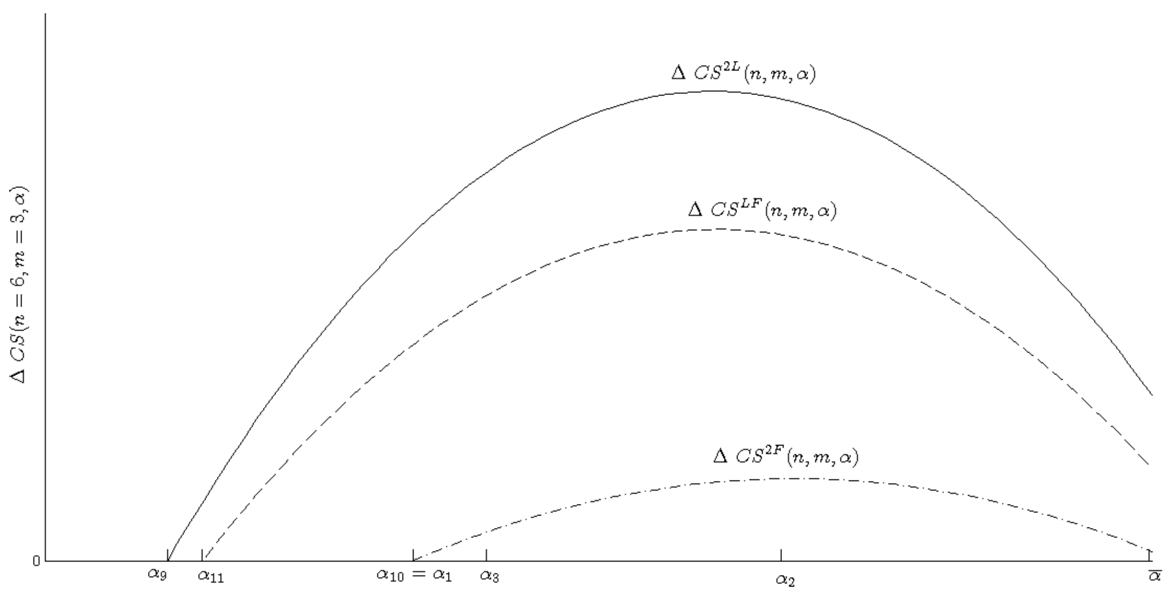


Figure 3.2: ΔConsumer Surplus

As we can observe from Figure 3.2, the variation in the consumer surplus is positive for all merger cases if the level of efficiency gains is high enough. Moreover, this relation is a non-monotonic one.

In any case, in the relevant region of parameter values defined by Assumption 3.2, after any two firms merger and for sufficiently high efficiency gains, the merged entity, due to its increased efficiency, produces more and outsider firms react by decreasing their output. The net effect then is an increase in the industry output and a decrease in the market price, leading to an increase in consumer surplus.[18]

This figure also shows that the merger's cost savings generate higher gains for consumers when the resultant insider firm is a leader than when it is a follower. As already mentioned, when the firm resulting from the merger is not only more efficient, but also enjoys a leadership position, the total industry output will increase more than in a merger wherein the resulting firm becomes more efficient but behaves as a follower in the merger induced industry structure. Additionally, when there are merger induced efficiency gains, there is a greater likelihood that a merger between two leader firms (or a merger between a leader and a follower firms) improves consumer surplus when comparing to the merger between two followers, since the α_9 (or α_{11}) in Proposition 3.5 is always lower than α_{10}.

Let us start by comparing the two mergers which do not affect the strategic power of the firms involved. A merger between two leader (follower) firms, reduces the number of leaders (followers) in the market. However, the insider leader (follower) firm becomes more efficient than its rivals (leaders and followers). Since the strategic power of the insider does not change, here the cost efficiency effect turns out to be stronger than the concentration effect and the merger increases consumer surplus. Now, comparing the two merger cases, the key difference is that the resulting firm has different strategic power in both cases. In the two follower merger case, the insider firm is a follower and, therefore, this explains why the cost savings affect less the increase in consumer surplus.

Consider now a merger involving a leader and a follower firm. When this is the case, there are two effects at play induced by the merger: an increase in concentration (there is

[18]Note that, initially, $\frac{\partial \Delta CS}{\partial \alpha} > 0$ when $\alpha < \alpha_{A3.2}$. Also, $\frac{\partial^2 \Delta CS}{\partial \alpha^2}$ is always negative. See Appendix B.5.

a decrease in the number of follower firms) that tends to hurt consumer surplus, on the one hand, and an increase in the efficiency of a merged entity that enjoys a leadership position which benefits consumer surplus, on the other. As a result, there will be a reduction in the quantity produced by all outsider firms (both followers and leader ones) whereas the merged entity will expand output. The net effect is then an increase in the industry quantity (and a corresponding decrease in market price). It should be noted, however, that the induced cost synergies in this merger do no exceed those regarding a merger between two leaders.

3.3.3.2 Social Welfare Standard

In this subsection we examine the induced welfare effects of each merger and also show how the welfare variation depends on the level of efficiency gains (as measured by α).

The welfare variation is found by solving $\Delta SW = SW_{AM}(n, m, \alpha) - SW_{BM}(n, m, \alpha)$. The solution to this equation, for each merger, is a second order polynomial expression and untractable. However, as we can see from the simulations (in Section 3.4), the results show a consistent pattern.

When mergers create efficiency gains, the effects on social welfare are manifold. Proposition 3.6 summarizes the results obtained for social welfare when the mergers create efficiency gains.

Proposition 3.6. For all $n > m$ and $0 < \alpha < \alpha_{A3.2}$, when mergers create efficiency gains:

i) The merger between two leaders is welfare-enhancing iff:

$$\alpha > \alpha_{12}\,(n, m), \text{ for all } m > 1 \text{ and } n - m > 1;$$

ii) The merger between two followers is welfare-enhancing iff:

$$\alpha > \alpha_{13}\,(n, m), \text{ for } n - m \geq 2;$$

iii) The merger of one leader and one follower is welfare-enhancing iff:

$$\alpha > \alpha_{14}\,(n, m), \text{ for } m > 1 \text{ and } n - m > 1;$$

where $\alpha_{12}(n, m)$, $\alpha_{13}(n, m)$ and $\alpha_{14}(n, m)$ are lower than $\alpha_{A3.2}$.

Proof. See Appendix B.6. □

Proposition 3.6 states that a merger between two leaders, two followers or one leader and one follower can actually improve the social welfare, for certain conditions on the efficiency parameter.[19] When the merger generates sufficiently high cost synergies, the insider firm will produce more and hence the price decreases which, in turn, increases the consumer surplus. On the other hand, the cost savings also give rise to an increase in merger profitability, which, in turn, lead to an increase in the producer surplus of the insiders and a decrease in the producer surplus of the outsiders. Hence, overall, and for certain levels of efficiency gains there is an increase in the social welfare.

In concluding, it should be highlighted that, in a no-synergies scenario, as far as welfare is concerned, all discussed types of mergers have the same effect: the social welfare decreases. This is the same result as in Feltovich (2001)'s and Huck et al. (2001)'s papers. In a model without efficiency gains and linear costs, both Feltovich (2001) and Huck et al. (2001) concluded that a merger between two similar firms that produces a firm of the same type is welfare lowering since, despite the price and total profits increase, the quantity and consumer surplus decrease. Additionally, Huck et al. (2001) show that when one big fish eats one small fish (i.e., when a leader firm merges with a follower one), the price increases and the total output decreases and so the net effect is also a decrease in social welfare. In addition, if firms have the same strategic power, we obtain the Salant et al. (1983)'s Cournot results, which, in this case, is the same as obtained when firms have asymmetric strategic power.[20]

3.4 Simulation Results

In this section we present and discuss the simulation results obtained for the different merger cases, illustrating the results obtained in the propositions above. For simplicity, we assume that there are $n = 12$ firms in the market, such that the critical parameters of α obtained in

[19]This is a similar result to the one obtained by Salant et al. (1983), under Cournot: when firms have the same strategic power, a merger that generates cost savings may also increase social welfare.

[20]In a Cournot setting with asymmetric firms and fixed costs, Catalão–Lopes (2007) shows that mergers may be socially desirable as long as the level of asymmetry is high enough. Also, if firms are not sufficiently asymmetric, there is a decrease in social welfare, unless the fixed cost savings are high enough.

Propositions 3.1 to 3.6 only depend on the number of leader firms (m).

The important point of the simulation results in Tables 3.2, 3.3 and 3.4 is to isolate the fact that it is possible to have mergers in which not only the merging firm may increase its profit but also the merger can improve both the social welfare and the consumer surplus. In particular, these tables also show that it is possible to have profitable mergers in which merger participants do not suffer from a free-riding problem. Note that, in these tables our results are obtained for the case where outsider firms do not exit the market and therefore Assumption 3.2 is satisfied.

Table 3.2: Effects of a merger between two leader firms

Regions	$g^{2L}(n,m,\alpha)$	$f^{2L}(n,m,\alpha)$	$\Delta CS^{2L}(n,m,\alpha)$	$\Delta SW^{2L}(n,m,\alpha)$
I	+	-	+	+
II	+	-	-	+
III	+	+	-	+
IV	-	+	-	+
V	-	+	-	-

The regions correspond to those derived from the critical parameters of α that are presented in Propositions 3.1, 3.4, 3.5 and 3.6 for the two leader merger case and are identified in figure 3.3. Legend: $g^{2L}(n,m,\alpha)$ - merger profitability; $f^{2L}(n,m,\alpha)$ - free riding problem (negative value indicates that there is no free-riding); $\Delta CS^{2L}(n,m,\alpha)$ - change in consumer surplus; $\Delta SW^{2L}(n,m,\alpha)$ - change in social welfare.

Table 3.2 summarizes some results which are also illustrated in figure 3.3 (See Appendix B.7). In particular, Region I gives us the combinations of m and α for which the level of cost savings is high and hence the profit of excluded rivals is lower than the profits of the insider leader firms. In addition, in this region, mergers of two leaders are profitable, welfare enhancing and improve the consumer surplus. In Region II, the merger of two leader firms is profitable but leads to a decrease in consumer surplus. In Regions III, IV, V the free-ridding problem emerges, but in Regions IV and V the merger of two leaders is not profitable. In Region V the results are the same as those obtained in the literature for Stackelberg mergers with symmetric costs: since efficiency gains are very low, mergers between two leader firms are not profitable, face a free-riding problem, and decrease both consumer and social welfare.

Table 3.3: Effects of a merger between two follower firms

Regions	$g^{2F}(n,m,\alpha)$	$f^{2F}(n,m,\alpha)$	$\Delta CS^{2F}(n,m,\alpha)$	$\Delta SW^{2F}(n,m,\alpha)$
I	+	-	+	+
II	+	-	-	+
III	+	+	-	+
IV	-	+	-	+
V	-	+	-	-
VI	+	+	-	-

The regions correspond to those derived from the critical parameters of α that are presented in Propositions 3.2, 3.4, 3.5 and 3.6 for the two follower merger cases and are identified in figure 3.4. Legend: $g^{2F}(n,m,\alpha)$ - merger profitability; $f^{2F}(n,m,\alpha)$ - free riding problem (negative value indicates that there is no free-riding); $\Delta CS^{2F}(n,m,\alpha)$ - change in consumer surplus ; $\Delta SW^{2F}(n,m,\alpha)$ - change in social welfare.

Table 3.3 sums up the main results illustrated in figure 3.4 (See Appendix B.7). Again, Region I gives us the combinations of m and α for which the merger of two followers is profitable, improves both the consumer and social welfare, and the profits of outsider firms are lower than the profits of the merged firm. Regions II, III, IV, V, VI gives us the combinations of m and α for which the merger always decrease the consumer surplus. In Regions III, IV, V, VI the free-ridding problem emerges, but in Regions IV and V the merger of two followers is not profitable. Moreover, in Regions V and VI, since the level of efficiency gains are low, the merger decreases both consumer and social welfare.

Table 3.4: Effects of a merger between a leader and a follower firms

Regions	$g^{LF}(n,m,\alpha)$	$f^{LF}(n,m,\alpha)$	$\Delta CS^{LF}(n,m,\alpha)$	$\Delta SW^{LF}(n,m,\alpha)$
I	+	-	+	+
II	+	-	-	+
III	+	+	-	+
IV	+	+	-	-

The regions correspond to those derived from the critical parameters of α that are presented in Propositions 3.3 to 3.6 for the leader follower merger case and are identified in figure 3.5.

Legend: $g^{LF}(n,m,\alpha)$ - merger profitability; $f^{LF}(n,m,\alpha)$ - free riding problem (negative value indicates that there is no free-riding); $\Delta CS^{LF}(n,m,\alpha)$ - change in consumer surplus ; $\Delta SW^{LF}(n,m,\alpha)$ - change in social welfare.

Table 3.4 summarizes the main results illustrated in figure 3.5 (see Appendix B.7). In all regions, the merger between a leader and a follower is always profitable. Region I gives us the combinations of m and α for which the merger of a leader and a follower is always profitable, welfare enhancing and improves the consumer surplus. Region II, III, IV gives us the combinations of m and α for which the merger always decreases consumer surplus. In Regions III and IV, although the merger between a leader and a follower firm is profitable, outsider firms have incentives to free ride on it.

In concluding this section, we contrast the main qualitative results obtained with this simulation exercise with those regarding the no-synergies benchmark case. In a scenario in which there are no cost savings, although a merger between a leader and a follower is always profitable, mergers between two leaders or two followers are profitable only if the merger involves the only two leaders or the only two followers at the status quo industry. Moreover, the three types of mergers should be objected by antitrust authorities because they reduce both consumer and social welfare.

Some important implications are obtained when there are efficiency gains resulting from the merger. These gains obviously enhance the profitability of any of the studied mergers, by allowing the merged entity to benefit from a cost advantage over its rivals. Some of the results presented on Tables 3.2, 3.3 and 3.4 are similar for the three merger cases. In particular, when the level of efficiency gains is low (Regions IV or V), the merger between two firms is not profitable (with exception in the leader-follower merger case), harms social and consumer welfare and faces a free-riding problem. Therefore, antitrust authorities should discourage such mergers. Further, we can also conclude that, in our framework, whenever a merger is not profitable, it also does not increase consumer surplus. In addition, whenever the studied mergers face free-riding problems, they also hurt consumer surplus.

If, however, efficiency gains induced by the merger are high, then the social welfare actually improves for the three merger cases. In particular, there are some cases in which the high cost saving allows firms to produce more at lower price, which, in turn, increases social welfare by increasing both consumer surplus and producer surplus.

Additionally, we conclude that when efficiency gains are sufficiently high, almost all the mergers that overcome the free-riding problem are also profitable and enhance social welfare, a result that differs from the ones obtained in the previous literature. This contrasts with the models considering Stackelberg leadership with linear costs such as Huck et al. (2001), in which they conclude that all mergers reduce social welfare.

Our results reinforce important policy implications already highlighted in the literature. When evaluating horizontal mergers, antitrust authorities should carefully evaluate whether (i) the proposed merger is expected to give rise to important synergies; and (ii) what is the strategic power of the firms involved in the merger proposal. This is clearly a challenging and difficult task, but, as the previous analysis illustrates, the final induced impacts of a given merger crucially depend on the interplay between these two very important factors.

3.5 Conclusion

This paper extends the literature on the incentives for and welfare effects of mergers in oligopolistic markets where some firms are (Stackelberg) leaders and some are (Stackelberg) followers by allowing for merger synergies.

Within this theoretical framework, we find that mergers involving two leaders, two followers or a leader and a follower may be profitable, even if costs are linear, in a setting where the merger may give rise to efficiency gains.

In case there are no efficiency gains from a given merger, then we show that the profitability of the merger crucially depends on the type of merger the firms in the industry decide to embark on, a result which is consistent with what has been demonstrated by previous literature in the field. In particular, in line with Huck et al. (2001)'s and Daughety (1990)'s papers, we find that while a merger between one leader and one follower firm is always profitable, mergers between two leaders or mergers between two followers are profit-enhancing if there exist exactly two leaders or exactly two followers in the status quo industry structure, respectively. Moreover, any two firms merger is shown to be welfare detrimental.

If instead efficiency gains do result from a merger, then the profitability results associated with the mergers are in sharp contrast with those derived in the extant literature. More specifically, mergers involving any two firms in the industry can actually be profitable, regardless of how many firms of each type (leaders or followers) exist in the status quo industry structure. In addition, under some circumstances regarding the efficiency gains induced by the merger, the merger can actually enhance social welfare. Further, we also conclude that the existence of cost benefits resulting from a merger can actually solve the so called free-riding problem which is usually associated with mergers involving no synergies.

Finally, our analysis also discloses that the combination of cost synergies and Stackelberg markets implies that mergers become more profitable and more welfare improving than under Cournot markets, since cost advantages give rise to a more aggressive behaviour (by the merged entity) in Stackelberg markets.

Appendix

Appendix B.1. Post-Merger Equilibrium Results

a) Merger of two leaders

After the merger of two leaders, the equilibrium quantities, profits, the consumer surplus, the producer surplus and the social welfare are given by:

$$q_{L_I}^{2L} = \frac{1}{2}\frac{n\alpha\,(n(m-1)-m(m-2)-3)+2}{m}, \text{ for the insider leader firm;}$$

$$q_{L_i}^{2L} = \frac{1}{2}\frac{n\alpha\,(m-n-3)+2}{m}, \text{ for the } i=1,...,m-1 \text{ outsider leader firms;}$$

$$q_{F_j}^{2L} = \frac{1}{2}\frac{n\alpha\,(m-n-3)+2}{(n-m+1)\,m}, \text{ for the } j=1,...,n-m \text{ outsider follower firms;}$$

$$\pi_{L_I}^{2L}(n,m,\alpha) = \frac{1}{4}\frac{\left[2-n\alpha(n-m(n-m+2)+3)\right]^2}{m^2\,(n-m+1)}, \text{ for the insider leader firm;}$$

$$\pi_{L_i}^{2L}(n,m,\alpha) = \frac{1}{4}\frac{\left[2-n\alpha\,(n-m+3)\right]^2}{m^2\,(n-m+1)}, \text{ for the } i=1,...,m-2 \text{ outsider leader firms;}$$

$$\pi_{F_j}^{2L}(n,m,\alpha) = \frac{1}{4}\frac{\left[2-n\alpha\,(n-m+3)\right]^2}{m^2\,(n-m+1)^2}, \text{ for the } j=1,...,n-m \text{ outsider follower firms;}$$

$$CS^{2L} = \frac{1}{8}\frac{\left[2(m(n-m+1)-1)+n\alpha(n-m(2(n-m)+3)+3)\right]^2}{m^2\,(n-m+1)^2}$$

$$PS^{2L} = \frac{1}{4}\frac{\left[2-n\alpha(n-m(n-m+2)+3)\right]^2}{m^2(n-m+1)} + \frac{1}{4}\frac{\left[-n+m(n-m+2)-2\right]\left[2-n\alpha(n-m+3)\right]^2}{m^2(n-m+1)^2}$$

$$SW^{2L} = CS^{2L} + PS^{2L}$$

Remark 3.1: Again, we assume that:[21]

$$\frac{2}{n\left[m\,(m-2)-n\,(m-1)+3\right]} < \alpha < \frac{2}{n\,(n-m+3)} \equiv \alpha_1.$$

[21] The lower threshold is obtained from setting insider firm's quantity greater than zero ($q_{L_I}^{2L} > 0$). This threshold is always negative, then we can exclude it since $\alpha > 0$. Hence, for any level of efficiency gains, insider leader firms always produce a positive quantity. The upper threshold is obtained by setting the quantity of both outsider leader and follower firms greater than zero. This α upper threshold is the same for both types of outsider firms.

These conditions are imposed to exclude the case where both outsider follower and leader firms do not produce.

b) Merger of two followers

After the merger of two followers, where the resultant firm is still follower, the equilibrium quantities, profits, the consumer surplus, the producer surplus and the social welfare are given by:

$$q_{F_I}^{2F} = \frac{1}{2}\frac{n\alpha\left(\left(m+1\right)\left(n-m\right)-3\right)+2}{\left(n-m\right)\left(m+1\right)}, \text{ for the insider follower firm;}$$

$$q_{F_j}^{2F} = \frac{1}{2}\frac{2-3n\alpha}{\left(n-m\right)\left(m+1\right)}, \text{ for } j=1,...,n-m-2 \text{ outsider follower firms;}$$

$$q_{L_i}^{2F} = \frac{1}{2}\frac{2-3n\alpha}{m+1}, \text{ for } i=1,...,m \text{ outsider leader firms;}$$

$$\pi_{F_I}^{2F}\left(n,m,\alpha\right) = \frac{1}{4}\frac{\left[2-n\alpha\left(3-\left(m+1\right)\left(n-m\right)\right)\right]^2}{\left(m+1\right)^2\left(n-m\right)^2}, \text{ for the insider follower firm;}$$

$$\pi_{L_i}^{2F}\left(n,m,\alpha\right) = \frac{1}{4}\frac{\left(2-3n\alpha\right)^2}{\left(m+1\right)^2\left(n-m\right)}, \text{ for } i=1,...,m \text{ outsider leader firms;}$$

$$\pi_{F_j}^{2F}\left(n,m,\alpha\right) = \frac{1}{4}\frac{\left(2-3n\alpha\right)^2}{\left(m+1\right)^2\left(n-m\right)^2}, \text{ for } j=1,...,n-m-2 \text{ outsider follower firms;}$$

$$CS^{2F} = \frac{1}{8}\frac{\left[2(n+m(n-m-1)-1)+n\alpha(3-2(m+1)(n-m))\right]^2}{\left(m+1\right)^2\left(n-m\right)^2}$$

$$PS^{2F} = \frac{1}{4}\frac{\left(2-3n\alpha\right)^2\left[n+m(n-m-1)-2\right]+\left[n\alpha\left(3-(m+1)(n-m)\right)-2\right]^2}{\left(m+1\right)^2\left(n-m\right)^2}$$

$$SW^{2F} = CS^{2L} + PS^{2L}$$

Remark 3.2: In order to exclude the case where firms do not produce, we impose the following conditions: [22]

$$\frac{2}{n\left[3-\left(m+1\right)\left(n-m\right)\right]} < \alpha < \frac{2}{3n} \equiv \alpha_2.$$

[22] Again, the lower threshold is always negative, and therefore we can exclude it because $\alpha > 0$.

c) Merger of one leader and one follower

After the merger of one leader and one follower, where the resultant firm is now a leader, the equilibrium quantities, profits, the consumer surplus, the producer surplus and the social welfare are given by:

$$q_I^{LF} = \frac{1}{2}\frac{2 + n\alpha\left(m\left(n-m\right)-2\right)}{m+1}, \text{ for the insider firm;}$$

$$q_{L_i}^{LF} = \frac{1}{2}\frac{2 - n\alpha\left(n-m+2\right)}{m+1}, \text{ for } i = 1,..,m-1 \text{ outsider leader firms;}$$

$$q^{LFF_j} = \frac{1}{2}\frac{2 - n\alpha\left(n-m+2\right)}{\left(n-m\right)\left(m+1\right)}, \text{ for } j = 1,..,n-m-1 \text{ outsider follower firms;}$$

$$\pi_I^{LF}\left(n,m,\alpha\right) = \frac{1}{4}\frac{\left[2 - n\alpha\left(2-m\left(n-m\right)\right)\right]^2}{\left(m+1\right)^2\left(n-m\right)}, \text{ for the insider firm;}$$

$$\pi_{L_i}^{LF}\left(n,m,\alpha\right) = \frac{1}{4}\frac{\left[2 - n\alpha\left(n-m+2\right)\right]^2}{\left(m+1\right)^2\left(n-m\right)}, \text{ for } i = 1,..,m-1 \text{ outsider leader firms;}$$

$$\pi_{F_j}^{LF}\left(n,m,\alpha\right) = \frac{1}{4}\frac{\left[2 - n\alpha\left(n-m+2\right)\right]^2}{\left(m+1\right)^2\left(n-m\right)^2}, \text{ for } j = 1,..,n-m-1 \text{ outsider follower firms;}$$

$$CS^{LF} = \frac{1}{8}\frac{\left[(2(n+m(n-m-1)-1) - n\alpha(n+m(2(n-m)-1)-2))\right]^2}{\left(m+1\right)^2\left(m-n\right)^2}$$

$$PS^{LF} = \frac{1}{4}\frac{\left[2 - n\alpha\left(2-m\left(n-m\right)\right)\right]^2}{\left(m+1\right)^2\left(n-m\right)} + \frac{1}{4}\frac{(m-1)(2-n\alpha\left(n-m+2\right))^2}{\left(m+1\right)^2\left(n-m\right)} + \\ + \frac{1}{4}\frac{(n-m-1)(2-n\alpha\left(n-m+2\right))^2}{\left(m+1\right)^2\left(n-m\right)^2}$$

$$SW^{LF} = CS^{LF} + PS^{LF}$$

Remark 3.3: In order to exclude the situation where firms do not produce, we impose the following conditions: [23]

$$\frac{2}{n\left[m\left(m-n\right)+2\right]} < \alpha < \frac{2}{n\left(n-m+2\right)} \equiv \alpha_3.$$

[23] Since the lower threshold is always negative, we can exclude it because $\alpha > 0$.

Appendix B.2. Proofs

Proof Proposition 3.1.

The merger between two leaders is profitable if $g^{2L}(n,m,\alpha) \equiv \pi_{L_I}^{2L}(n,m,\alpha) -$ $-\pi_{L_i}^{BM}(n,m,\alpha) > 0$, that is, iff:

$$g^{2L}(n,m,\alpha) \equiv \frac{(m+1)^2[2-n\alpha(n-m(n-m+2)+3)-2]^2-8m^2(1-n\alpha)^2}{4m^2(n-m+1)(m+1)^2} > 0$$

Since the denominator of g^{2L} is always positive, $4m^2(n-m+1)(m+1)^2 > 0$, we need to solve the numerator with respect to α in order to get the roots that satisfy the merger profitability condition.

Let $\alpha_4(n,m) \equiv \frac{2(m+1)^2n(n-m(n-m+2)+3)-8m^2n+2\sqrt{2}mn(m-1)(m+1)(n-m+1)}{(m+1)^2n^2(n-m(n-m+2)+3)^2-8m^2n^2}$ be the only positive root obtained from the solving the inequality above ($g^{2L} > 0$) that is lower than α_1 and than $\overline{\alpha}$. Hence, two leaders have incentives to merge for all $m \geq 2$, iff:

i) $\alpha > 0$, for $m = 2$; and

ii) $\alpha > \alpha_4$, for all $m > 2$.

■

The first and the second derivatives of g^{2L} with respect to α are, respectively, given by:

$\frac{\partial g^{2L}}{\partial \alpha} = \frac{n\alpha\left(n(m+1)^2(n-m(n-m+2)+3)^2-8m^2n\right)-2n(m+1)^2(n-m(n-m+2)+3)+8m^2n}{2m^2(n-m+1)(m+1)^2} > 0$ and

$\frac{\partial^2 g^{2L}}{\partial \alpha^2} = \frac{n\left(n(m+1)^2(n-m(n-m+2)+3)^2-8m^2n\right)}{2m^2(n-m+1)(m+1)^2} > 0.$

Proof Proposition 3.2.

The merger between two followers is profitable if $g^{2F}(n,m,\alpha) \equiv \pi_{F_I}^{2F}(n,m,\alpha) -$ $-2\pi_{F_j}^{BM}(n,m,\alpha) > 0$, that is, iff:

$$g^{2F}(n,m,\alpha) \equiv \frac{(2-n\alpha((m+1)(m-n)+3))^2(n-m+1)^2-8(1-n\alpha)^2(n-m)^2}{4(m+1)^2(n-m)^2(n-m+1)^2} > 0$$

Let $\alpha_5(n,m) \equiv \frac{2n(3-(m+1)(n-m))(n-m+1)^2-8n(n-m)^2+2\sqrt{2}n(n-m)(n+m(n-m-1)-1)(n-m+1)}{n^2(3-(m+1)(n-m))^2(n-m+1)^2-8(n-m)^2n^2}$ be the only positive roots obtained from the solving the inequality above ($g^{2F} > 0$) that is also lower than α_2 and than $\overline{\alpha}$. Hence, two followers have incentives to merge for all $n-m \geq 2$, iff:

i) $\alpha > 0$, for $n - m = 2$; and

ii) $\alpha_5 < \alpha <$, for $n - m > 2$.

■

The first and the second derivatives of g^{2F} with respect to α are, respectively, given by:

$\frac{\partial g^{2F}}{\partial \alpha} = \frac{8n(1-n\alpha)(n-m)^2 - n(2-n\alpha(3-(m+1)(n-m)))(n-m+1)^2(3-(m+1)(n-m))}{2(m+1)^2(n-m)^2(n-m+1)^2} > 0$ and

$\frac{\partial^2 g^{2F}}{\partial \alpha^2} = \frac{n^2(n-m+1)^2(3-(m+1)(n-m))(3-(m+1)(n-m)) - 8n^2(n-m)^2}{2(m+1)^2(n-m)^2(n-m+1)^2} > 0.$

Proof Proposition 3.3

A merger between a leader and a follower is profitable if $g^{LF}(n,m,\alpha) \equiv \pi_I^{LF}(n,m,\alpha) - \left[\pi_{F_j}^{BM}(n,m,\alpha) + \pi_{L_i}^{BM}(n,m,\alpha)\right] > 0$, that is, iff:

$$g^{LF}(n,m,\alpha) \equiv \frac{(2-n\alpha(m(n-m))+2)^2(n-m+1)^2 - 4(1-n\alpha)^2(n-m+2)(n-m)}{4(m+1)^2(n-m)(n-m+1)^2} > 0$$

In this case, since both α obtained for solving $g^{LF} > 0$ are always negative we exclude them. Hence, the merger between one follower and one leader is always profitable for all $\alpha > 0$, all $n - m > 1$ and $m > 1$.

■

The first and the second derivatives of g^{LF} with respect to α are, respectively, given by:

$\frac{\partial g^{LF}}{\partial \alpha} = \frac{n(n\alpha(m(n-m)-2)+2)(n-m+1)^2(m(n-m)-2) + 4n(1-n\alpha)(n-m+2)(n-m)}{2(m+1)^2(n-m)(n-m+1)^2} > 0$ and

$\frac{\partial^2 g^{LF}}{\partial \alpha^2} = \frac{n^2(n-m+1)^2(m(n-m)-2)(m(n-m)-2) - 4n^2(n-m+2)(n-m)}{2(m+1)^2(n-m)(n-m+1)^2} > 0.$

Appendix B.3. No-synergies benchmark results

Insiders' Profitability

By assuming that $\alpha = 0$, the merger profitability conditions that we obtain and summarize in Proposition 3.1 are the same as those presented in Huck et al. (2001)'s and Feltovich (2001)'s papers. That is, two leaders have only incentives to merge if there are $m = 2$ leaders and, similarly, two followers have only incentives to merge if there are $n - m = 2$ followers (Huck et al. (2001)'s Proposition 1 and Feltovich (2001)'s Result 1). Also, we obtain Huck et al. (2001)'s Proposition 2, that is, a merger between a leader and a follower is always profitable.

Lemma 3.1. For all $n > m$, when mergers do not create any synergies ($\alpha = 0$):

i) The merger between two leaders is profitable, for all $m \geq 2$, iff:

$g^L(n, m, \alpha = 0) > 0 \Leftrightarrow \frac{-\left(m^2-2m-1\right)}{m^2(n-m+1)(m+1)^2} > 0$. This is true only for $m = 2$.

ii) The merger between two followers is profitable, for all $n - m \geq 2$, iff:

$g^F(n, m, \alpha = 0) > 0 \Leftrightarrow \frac{2n-n^2+2mn-m^2-2m+1}{(n-m)^2(m+1)^2(n-m+1)^2} > 0$. This is true only for $n - m = 2$.

iii) The merger of a follower and a leader is profitable, for all $n - m \geq 1$ and $m \geq 1$, iff:

$g^I(n, m, \alpha = 0) > 0 \Leftrightarrow \frac{1}{(n-m)(m+1)^2(n-m+1)^2} > 0$. This is always true.

Consumer Surplus

Lemma 3.2 sums up, for each merger, the results obtained for the consumer surplus impact without efficiency gains.

Lemma 3.2. For all $n > m$, when mergers do not create any synergies, $\alpha = 0$:

i) The merger between two leaders always decreases consumer surplus iff:

$\frac{1}{2}\frac{2m\left(-n-mn+m^2\right)+1}{m^2(m+1)^2(m-n-1)^2} < 0$. This condition is true for all $m \geq 2$ and $n - m$.

ii) The merger between two followers decreases consumer surplus iff:

$\frac{1}{2}\frac{2(m-n)\left(n+mn-m^2\right)-1}{(m-n)^2(m+1)^2(m-n-1)^2} < 0$. This condition is true for all m and $n - m \geq 2$.

iii) The merger of one follower and one leader decreases consumer surplus iff:

$\frac{2(n-m)\left(-n-mn+m^2\right)+1}{2(m-n)^2(m+1)^2(m-n-1)^2} < 0$. This condition is true for all $m > 1$ and $n - m > 1$.

Social Welfare

Lemma 3.3 sums up, for each merger, the results obtained for the social welfare without efficiency gains.

Lemma 3.3. For all $n > m$, when mergers do not create any synergies, $\alpha = 0$:

i) The merger between two leaders decreases social welfare iff:

$-\frac{1}{2}\frac{2m+1}{m^2(m+1)^2(m-n-1)^2} < 0$. This condition is true for all $m \geq 2$ and $n - m$.

iii) The merger between two followers decreases social welfare iff:

$\frac{2m-2n-1}{2(m-n)^2(m+1)^2(m-n-1)^2} < 0$. This condition is true for all m and $n - m \geq 2$.

iii) The merger of one follower and one leader decreases social welfare iff:

$\frac{2m-2n-1}{2(m-n)^2(m+1)^2(m-n-1)^2} < 0$. This condition is always true for all $m > 1$ and $n-m > 1$.

Free-Riding Problem

For all $n > m$, when mergers do not create any synergies, for $\alpha = 0$:

i) The merger between two leaders has a free-riding problem if outsider leader firms earn more than the insiders, that is, iff:

$\pi_I^L(n,m) < 2\pi_O^L(n,m) \Leftrightarrow f_L^L(n,m) = \frac{1}{m^2(n-m+1)} > 0$. This is always true for all $m \geq 2$.

ii) The merger between two followers has a free-riding problem if outsider follower firms earn more than the insiders, that is, iff:

$\pi_I^F(n,m) < 2\pi_O^F(n,m) \Leftrightarrow f_F^F(n,m) = \frac{1}{(n-m)^2(m+1)^2} > 0$. This is always true for all $m > 1$ and $n - m > 1$.

iii) The merger of one follower and one leader has a free-riding problem if outsider firms earn more than the insiders, that is, iff:

$\pi_I(n,m) < \pi_O^L(n,m) + \pi_O^F(n,m) \Leftrightarrow f_{FL}^I(n,m) = \frac{1}{(n-m)^2(m+1)^2} > 0.$ This is always true.

Appendix B.4. Free-Riding Effects

a) Merger between two leaders

There is free-riding problem if $\pi_{L_I}^{2L}(n,m,\alpha) < 2\pi_{L_i}^{2L}(n,m,\alpha)$, that is, iff:

$$f^{2L}(n,m,\alpha) \equiv \frac{2\left(2 - n\alpha\left(n-m+3\right)\right)^2 - \left(2 - n\alpha\left(n - m(n-m+2)+3\right)\right)^2}{4m^2\left(n-m+1\right)} > 0 \tag{3.11}$$

Let $\alpha_6 \equiv \frac{4n(n-m+3)-2n(n-m(n-m+2)+3)-2\sqrt{2}mn(n-m+1)}{2n^2(n-m+3)^2-n^2(n-m(n-m+2)+3)^2}$ and $\alpha \equiv \frac{4n(n-m+3)-2n(n-m(n-m+2)+3)+2\sqrt{2}mn(n-m+1)}{2n^2(n-m+3)^2-n^2(n-m(n-m+2)+3)^2}$[24] be the two roots obtained from solving the inequality (3.11). For all $\alpha > 0$, leader firms have incentives to free-ride on the merger of two leaders iff:

$$\alpha < \alpha_6(n,m), \text{ for } m \geq 2.$$

■

The first and the second derivatives of f^{2L} with respect to α are, respectively, given by:
$\frac{\partial f^{2L}(n,m,\alpha)}{\partial \alpha} = \frac{-4n(-n(m-3)+m(m-4)+9)+2\alpha n^2\left((n-m(n-m+2)+3)^2+2(n-m+3)^2\right)}{4m^2(n-m+1)} > 0$
if $\alpha > \frac{2(-n(m-3)+m(m-4)+9)}{n\left((n-m(n-m+2)+3)^2+2(n-m+3)^2\right)}$ and

$\frac{\partial^2 f^{2L}(n,m,\alpha)}{\partial \alpha^2} = \frac{2n^2\left((n-m(n-m+2)+3)^2+2(n-m+3)^2\right)}{4m^2(n-m+1)} > 0.$

[24] Note that this root is always negative or greater than $\overline{\alpha}$ for $m > 2$. Also, for $m = 2$ this root is higher than α_1 imposed in A2. Thus, we exclude it.

b) Merger between two followers

There is free-riding problem if $\pi_{F_I}^{2F}(n,m,\alpha) < 2\pi_{F_j}^{2F}(n,m,\alpha)$, that is, iff:

$$f^{2F}(n,m,\alpha) \equiv \frac{2(2-3n\alpha)^2 - [2-n\alpha(3-(m+1)(n-m))]^2}{4(m+1)^2(n-m)^2} > 0 \tag{3.12}$$

Let $\alpha_7 \equiv \frac{2n((m+1)(n-m)-3)+12n-2\sqrt{2}n(m+1)(n-m)}{18n^2-n^2((m+1)(n-m)-3)^2}$ be the only root that is positive and that satisfies Assumption 3.1 obtained from solving the inequality (3.12). For all $\alpha > 0$, follower firms have incentives to free-ride on the merger of two followers iff:

$$\alpha < \alpha_7(n,m), \text{ for all } n-m > 2 \text{ and } m > 1.$$

■

The first and the second derivatives of f^{2F} with respect to α are, respectively, given by: $\frac{\partial f^{2F}(n,m,\alpha)}{\partial \alpha} = \frac{2n(-(m+1)(n-m)-3)-n^2\alpha\left((3-(m+1)(n-m))^2-18\right)}{2(m+1)^2(n-m)^2} > 0$ if $\alpha > \frac{2(-(m+1)(n-m)-3)}{n\left((3-(m+1)(n-m))^2-18\right)}$ and

$\frac{\partial^2 f^{2F}(n,m,\alpha)}{\partial \alpha^2} = \frac{-n^2\left((3-(m+1)(n-m))^2-18\right)}{2(m+1)^2(n-m)^2} > 0$, except when $m=1$ and $n-m=2,3$ and $m=n-m=2$.

c) Merger between a leader and a follower

There is free-riding problem if $\pi_I^{LF}(n,m,\alpha) < \pi_{F_j}^{LF}(n,m,\alpha) + \pi_{L_i}^{LF}(n,m,\alpha)$, that is, iff:

$$f^{LF}(n,m,\alpha) \equiv \frac{(2-n\alpha(n-m+2))^2(n-m+1) - [2-n\alpha(2-m(n-m))]^2(n-m)}{4(m+1)^2(n-m)^2} > 0 \tag{3.13}$$

Let $\alpha_8 \equiv \frac{2(n-m+2)(n-m+1)+2(m(n-m)-2)(n-m)+2(m+1)\sqrt[2]{(n-m+1)(n-m)^3}}{n^2(n-m+2)^2(n-m+1)-n^2(m(n-m)-2)^2(n-m)}$ be the only root that is positive and that satisfies Assumption 3.1 obtained from solving the inequality (3.13). For all $\alpha > 0$, outsider firms have incentives to free-ride on the merger of a leader and a follower iff:

$$\alpha < \alpha_8(n,m), \text{ for all } n-m > 1 \text{ and } m > 1.$$

■

The first and the second derivatives of f^{LF} with respect to α are, respectively, given by:
$\frac{\partial f^{LF}(n,m,\alpha)}{\partial\alpha} = \frac{-2n((n-m)((m+1)(n-m)+1)+2)+\alpha n^2\left((n-m+2)^2(n-m+1)-(2-m(n-m))^2(n-m)\right)}{2(m+1)^2(n-m)^2} > 0$ if $\alpha > \frac{2((n-m)((m+1)(n-m)+1)+2)}{n\left((n-m+2)^2(n-m+1)-(2-m(n-m))^2(n-m)\right)}$. Since this $\alpha > \overline{\alpha}$ in some cases and is negative for the others, then $\frac{\partial f^{LF}(n,m,\alpha)}{\partial\alpha} < 0$;
$\frac{\partial^2 f^{LF}(n,m,\alpha)}{\partial\alpha^2} = \frac{n^2\left((n-m+2)^2(n-m+1)-(2-m(n-m))^2(n-m)\right)}{2(m+1)^2(n-m)^2} > 0$, which is true for $m = 2$ and $n-m = 2, 3$, and $m = 3$ and $n-m = 2$.

Appendix B.5. Consumer Surplus Results

a) Merger between two leaders

If the merger between two leaders creates synergies, the consumer surplus increases iff $\Delta CS^{2L} \equiv CS^{2L} - CS^{BM} > 0$, that is, iff:

$$\frac{[n\alpha(n+m(n-m)+3)-2]\,[n\alpha((1-4m)(n+m(n-m))+3)+4m(n+m(n-m))-2]}{8m^2(m+1)^2(n-m+1)^2} > 0 \tag{3.14}$$

Solving the inequality (3.14) we obtain one threshold for α that is positive and satisfies Assumption 3.1, which is given by: $\alpha_9 \equiv \frac{2}{n(3+n+m(n-m))}$.

Hence, the merger between two leaders increases consumer surplus iff:

$$\alpha > \alpha_9\,(n,m), \text{ for all } m \geq 2 \text{ and all } n-m > 1.$$

■

The first and the second derivatives of ΔCS^{2L} with respect to α are, respectively, given by:
$\frac{\partial\Delta CS^{2L}}{\partial\alpha} = \frac{(n(n+m(n-m)+3))(n\alpha((1-4m)(n+m(n-m))+3)+4m(n+m(n-m))-2)+(n\alpha(n+m(n-m)+3)-2)n((1-4m)(n+m(n-m))+3)}{8m^2(m+1)^2(n-m+1)^2} > 0$
if $\alpha < \frac{2n((n-m(m-n))(4m-1)-3)+n(4m(n-m(m-n))-2)(n-m(m-n)+3)}{2n^2((n-m(m-n))(4m-1)-3)(n-m(m-n)+3)}$ and
$\frac{\partial^2 CS^{2L}}{\partial\alpha^2} = \frac{n^2(n+m(n-m)+3)((1-4m)(n+m(n-m))+3)+n^2(n+m(n-m)+3)((1-4m)(n+m(n-m))+3)}{8m^2(m+1)^2(n-m+1)^2} < 0$.

b) Merger between two followers

If the merger between two follower creates synergies the consumer surplus increases if $\Delta CS^{2F} \equiv CS^{2F} - CS^{BM} > 0$, that is, iff:

$$\frac{(2 - n\alpha((n-m)(1-4(n+m(n-m))) + 3) - 4(n-m)(n+m(n-m)))(2 - n\alpha(n-m+3))}{8(m+1)^2(n-m+1)^2(n-m)^2} > 0 \tag{3.15}$$

From solving the inequality (3.15), we obtain one threshold for α that is positive and satisfies Assumption 3.1, given by $\alpha_{10} \equiv \frac{2}{n(n-m+3)}$.

Hence, the merger between two followers increases consumer surplus iff:

$$\alpha > \alpha_{10}\,(n,n), \text{ for all } n-m \geq 2 \text{ and all } m.$$

■

The first and the second derivatives of ΔCS^{2F} with respect to α are, respectively, given by:

$$\frac{\partial \Delta CS^{2F}}{\partial \alpha} = \frac{-n((n-m)(1-4(n+m(n-m)))+3)(2-n\alpha(n-m+3))-n(n-m+3)(2-n\alpha((n-m)(1-4(n+m(n-m)))+3)-4(n-m)(n+m(n-m)))}{8(m+1)^2(n-m+1)^2(n-m)^2} > 0$$

if $\alpha < -\frac{-2n((m-n)(-4n+4m(m-n)+1)-3)+n(4(n-m(m-n))(m-n)+2)(-m+n+3)}{2n^2((m-n)(-4n+4m(m-n)+1)-3)(-m+n+3)}$ and

$$\frac{\partial^2 \Delta CS^{2F}}{\partial \alpha^2} = \frac{n^2((n-m)(1-4(n+m(n-m)))+3)((n-m+3))+n^2(n-m+3)((n-m)(1-4(n+m(n-m)))+3)}{8(m+1)^2(n-m+1)^2(n-m)^2} < 0.$$

c) Merger between a leader and a follower

If the merger between a leader and a follower creates synergies the consumer surplus is improved if $\Delta CS^{LF} \equiv CS^{LF} - CS^{BM} > 0$, that is, iff:

$$\frac{[2(1-(m+1)(n-m)) - n\alpha(2-(2m+1)(n-m))]^2\,(n-m+1)^2 - 4(n+m(n-m))^2(1-n\alpha)^2(n-m)^2}{8(n-m)^2(m+1)^2(n-m+1)^2} > 0 \tag{3.16}$$

Solving the inequality (3.16) we obtain one threshold for α that is positive and satisfies Assumption 3.1, which is given by: $\alpha_{11}\,(n,m) \equiv \frac{2}{n((n-m+1)(n-m)+2)}$.

Hence, the merger between a leader and a follower increases consumer surplus if:

$$\alpha > \alpha_{11}\,(n,m), \text{ for } m > 1 \text{ and } n-m > 1.$$

■

The first and the second derivatives of ΔCS^{LF} with respect to α , respectively, given by:

$\frac{\partial \Delta CS^{LF}}{\partial \alpha} = \frac{-n(2-(2m+1)(n-m))(2(1-(m+1)(n-m))-n\alpha(2-(2m+1)(n-m)))(n-m+1)^2+4n(n+m(n-m))^2(1-n\alpha)(n-m)^2}{4(n-m)^2(m+1)^2(n-m+1)^2} > 0$

if $\alpha < \frac{-4n(n-m(m-n))^2(m-n)^2+n((2m+1)(m-n)+2)(2(m-n)(m+1)+2)(-m+n+1)^2}{n^2((2m+1)(m-n)+2)^2(-m+n+1)^2-4n^2(n-m(m-n))^2(m-n)^2}$ and

$\frac{\partial^2 \Delta CS^{LF}}{\partial \alpha^2} = \frac{n^2(2-(2m+1)(n-m))(2-(2m+1)(n-m))(n-m+1)^2-4n^2(n+m(n-m))^2(n-m)^2}{4(n-m)^2(m+1)^2(n-m+1)^2} < 0.$

Appendix B.6. Social Welfare Results

a) Merger between two leaders

If the merger between two leaders creates synergies, the social welfare increases iff $\Delta SW \equiv SW^{2L} - SW^{BM} > 0$. Solving this inequality, the only threshold for α that is positive and lower than $\overline{\alpha}$ is the following:

$$\alpha_{12} \equiv \frac{-A+\sqrt{A^2+8B(2m+1)}}{2B},$$

where $A = 2n\left(6m + n + mn^2 + 3m^2n - 2m^3n - 2m^4n + 2m^2n^2 + m^3n^2 + 4mn - m^2 - -3m^3 + m^5 + 3\right.$ and $B = -\frac{1}{2}n^2(18m + 6n + 12mn^2 + 4m^2n + 2mn^3 - 22m^3n + 4m^4n + 6m^5n - 6m^6n + 9m^2n^2 + 2m^2n^3 - 8m^3n^2 - 2m^3n^3 - 2m^4n^3 + 6m^5n^2 + 24mn - 10m^2 - 14m^3 + 9m^4 + n^2 + 2m^5 - 4m^6 + 2m^7 + 9)$

Hence, the merger between two leaders increases social welfare iff:

$$\alpha_{12}\left(n, m\right), \text{for the remaining all } m > 1 \text{ and } n - m > 1.$$

■

The signs of the first and the second derivatives of ΔSW^{2L} are, respectively, given by: $\frac{\partial \Delta SW^{2\mathrm{L}}}{\partial \alpha} > 0$ and $\frac{\partial \Delta^2 SW^{2L}}{\partial \alpha^2} > 0$, except when $m = 2$ and $n - m = 1$.[25]

[25]Since formal expressions are very long, we indicate only the signs of the derivatives. A mathematical appendix with all the details is, however, available upon request to the authors.

b) Merger between two followers

If the merger between two followers creates synergies the social welfare increase iff $\Delta SW \equiv SW^{2F} - SW^{BM} > 0$. Solving this inequality, the only threshold for α that is positive and lower than $\overline{\alpha}$ is the following:

$$\alpha_{13}(n,m) \equiv \frac{C+\sqrt{C^2-16D(2m-2n-1)}}{2D},$$

where $C = 4n\left(7m - 7n + mn^2 + m^2n - mn^3 - 3m^3n + 3m^2n^2 + 5mn - 2m^2 - m^3 + \right.$ $\left. + m^4 - 3n^2 - n^3 - 3\right.$ and $D = 18n^2\left(-m + n + mn - m^2 - 2\right)(n - m + 1)^2 -$ $- 8\left(n + mn - m^2\right)(n-m)^2 n^2 + \left(m - n + 4mn^2 - 8m^2n - 4mn + 4m^3 + 4n^2 - 3\right)$ $(m - n - 3)\,n^2 + 2\left((m+1)(m-n) + 3\right)^2 (n - m + 1)^2 n^2$

Hence, the merger between two followers increases social welfare iff:

$$\alpha > \alpha_{13}(n,m), \text{ for all } n - m \geq 2 \text{ and } m > 1.$$

■

The signs of the first and the second derivatives of ΔSW^{2F} are, respectively, given by: $\frac{\partial \Delta SW^{2\text{F}}}{\partial \alpha} > 0$ and $\frac{\partial \Delta^2 SW^{2\text{F}}}{\partial \alpha} > 0$, for all α.

c) Merger between a leader and a follower

If the merger between a leader and a follower creates synergies the social welfare is improved iff $\Delta SW \equiv SW^{LF} - SW^{BM} > 0$. Solving the inequality, the only threshold for α that is positive and lower than $\overline{\alpha}$ is the following:

$$\alpha_{14}(n,m) \equiv \frac{-E+\sqrt{E^2-16G(2m-2n-1)}}{2G},$$

where $E = 4n(-5m + 5n - 8mn^2 + 7m^2n - 2mn^3 + 2m^3n + mn^4 - 4m^4n - 4m^2n^3 + 6m^3n^2 -$ $6mn + 3m^2 - 2m^3 - m^4 + 3n^2 + m^5 + 3n^3 + n^4 + 2)$ and $G = -4n^2\left(-n - mn + m^2\right)^2 (m-n)^2 -$ $8\left(n + mn - m^2\right)(n-m)^2 n^2 + 2(n-m)(n-m+1)^2 (m(n-m) - 2)^2 n^2 +$ $\left(m - n - 2mn + 2m^2 + 2\right)^2 (m-n-1)^2 n^2 + 2(n-m)(m-1)(n-m+1)^2 (n-m+2)^2 n^2 +$ $2(n-m+2)^2 (n-m-1)(n-m+1)^2 n^2$

Hence, the merger between a leader and a follower increases social welfare iff:

$$\alpha > \alpha_{14}(n,m) \text{ , for all } m > 1 \text{ and } n - m > 1.$$

■

The signs of the first and the second derivatives of ΔSW^{LF} are, respectively, given by: $\frac{\partial \Delta SW^{L\mathrm{F}}}{\partial \alpha} < 0$ and $\frac{\partial \Delta^2 SW^{L\mathrm{F}}}{\partial \alpha^2} < 0$, for all α.

Appendix B.7. Figures

Figure 3.3 graphs the critical parameters of α that are presented in Propositions 3.1, 3.4, 3.5 for the two leader merger case with $n = 12$ and identifies five different regions. Also, in these regions, the outsider firms do not exit the market since α_1^* is always greater than α_9^* and therefore, Assumption 3.2 is always satisfied in the regions obtained.

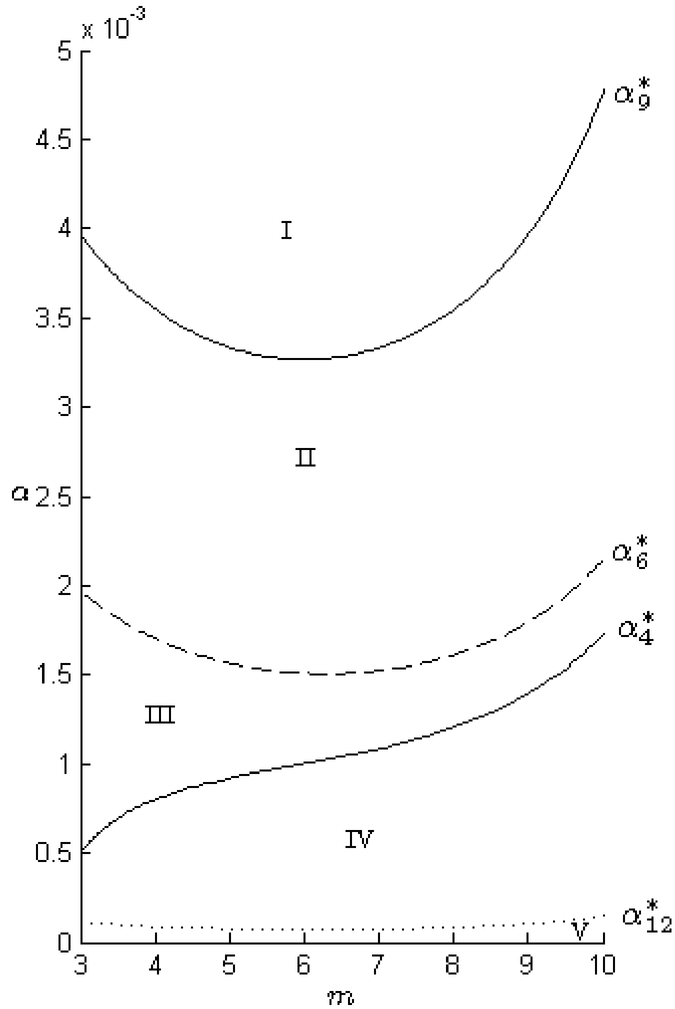


Figure 3.3: Critical parameters of α for a merger between two leader firms

Figure 3.4 graphs the critical parameters of α that are presented in Propositions 3.2, 3.4, 3.5 and 3.6 for the two follower merger case with $n = 12$ and identifies six different regions. Also, in these regions, the outsider firms do not exit the market since α_2^* is always greater than α_{10}^* and therefore, Assumption 3.2 is always satisfied in the regions obtained.

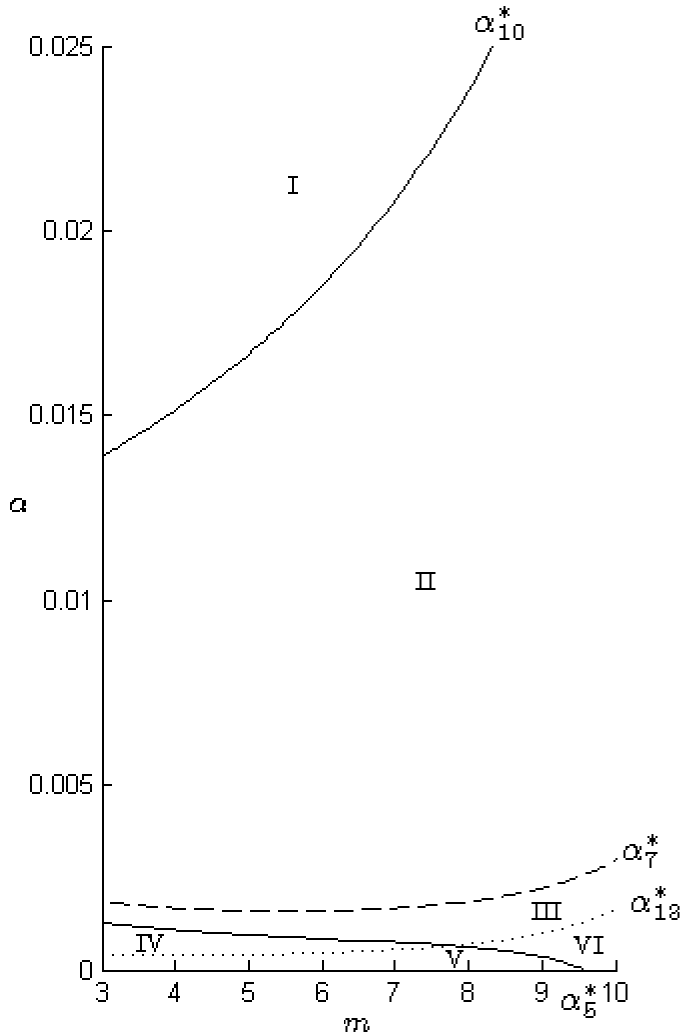


Figure 3.4: Critical parameters of α for a merger between two follower firms

Figure 3.5 graphs the critical parameters of α that are presented in Propositions 3.3 to 3.6 for the leader-follower merger case with $n = 12$ and identifies four different regions. Also, in these regions, the outsider firms do not exit the market since α_3^* is always greater than α_{11}^* and therefore, Assumption 3.2 is always satisfied in the regions obtained.

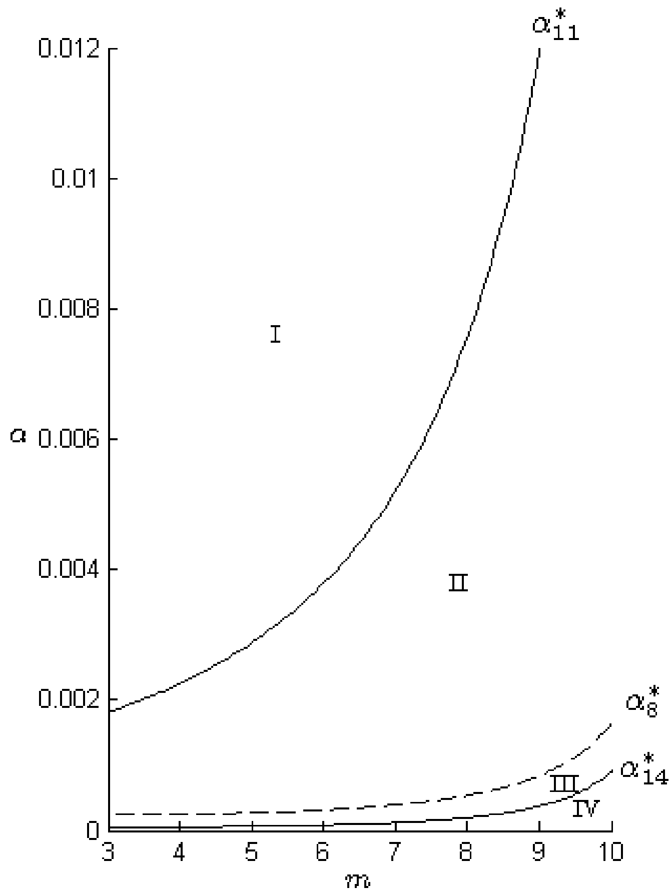


Figure 3.5: Critical parameters of α for a merger between a leader and a follower firms

Chapter 4

Uncertain Efficiency Gains and Merger Policy

[1]

4.1 Introduction

The assessment of the efficiency gains resulting from a merger usually raises an information issue for antitrust authorities. Although some mergers can actually generate significant efficiency gains, these are usually difficult to measure and verify. In practice, it is often the case that both the firms and the antitrust authority (henceforth, AA) cannot predict exactly the post-merger efficiency gains, implying that they are not aware of all the conditions they are going to face after the merger. Sometimes, only after the merger firms and antitrust authorities will understand the true level of induced efficiency gains. For instance, some pharmaceutical firms may adopt merger decisions without knowing whether their R&D efforts will be suc-

[1]This chapter benefited a lot from the seminar participants' comments at the 8th Annual Meeting of the Portuguese Economic Journal, University of Minho, School of Economics and Management and at the 10th Seminar Day of the Doctoral Programme in Economics, Faculty of Economics of University of Porto, 2014. Also, we are very grateful to Professors Joana Pinho, João Correia da Silva, Ricardo Ribeiro and Hélder Valente for their helpful comments and suggestions that substantially improved this paper.

cessful or not. Also, any type of firms' investment could generate uncertainty about future costs and, sometimes, a merger could actually occur before the uncertainty is resolved.

In 1996, Ciba-Geigy AG (hereinafter "Ciba") and Sandoz AG (hereinafter "Sandoz") proposed a merger to the European Commission (EC). Ciba and Sandoz, two Swiss manufacturers, were involved in the research, development and production of biological and chemical products, as well as in the production and marketing of pharmaceutical products. The merger would create a new enterprise, Novartis AG (hereinafter "Novartis"), that would be the world's second largest supplier of pharmaceutical products. This merger raised important issues regarding the effects on future innovation for the EC and the Federal Trade Commission (FTC). The EC was first concerned on whether this merger would lead to a decrease of competition in the market but also on the effects of these two firms combining their R&D programmes into one single firm. The EC concluded that Novartis would succeed in keeping its research expenditure lower, in relative terms, than that of its competitors, maintaining its position as market leader. Nonetheless, the EC concluded that all the Novartis's competitors have the "critical size" necessary for effective R&D activity and that increases in market share would be insignificant in most areas. Further, the real possibility of market entry and the restraining effect of generic pharmaceuticals on pricing were also seen as helping to prevent anticompetitive effects. Both EC and FTC were mainly concerned on the possibility that this merger would generate adverse effects on market power however, little reference was made to the merger's efficiency effects (EC, 1996).

The analysis of merger-induced efficiencies was introduced into the US Horizontal Merger Guidelines in 1997 (DOJ and FTC, Section 10) and into the European Merger Guidelines in 2004 (EC Horizontal Merger Guidelines, 2004/03, Article 7). The US merger guidelines state that "Other efficiencies, such as those relating to research and development, are potentially substantial but are generally less susceptible to verification and may be the result of anticompetitive output reductions". Also, in the European merger guideline the "efficiency defence" argument has been specifically ruled out and the effects on technical and economic progress can only be taken into account "provided it is to the consumers' advantage and does not form an obstacle to competition". The significance of potential welfare benefits from innovation or mergers' induced efficiency gains means it is important to give appropriate weight to this aspect of competition in merger analysis. Both EU and US merger guidelines allow future R&D efficiency gains to be taken into account, however these dynamic effects of mergers are generally disregarded since they are not only difficult to verify and quantify but also very hard to predict when the AAs are deciding on a merger case.

The purpose of this paper is to contribute to the literature that studies the efficiency gains' role in merger decisions, departing from a deterministic environment by considering a setting in which there is (symmetric) uncertainty. In particular, we assume that the decisions of the antitrust authority, when evaluating a merger case, crucially depend on the uncertainty regarding the efficiency gains realization. In the proposed model, firms and the AA are uncertain about the level of efficiency gains and, therefore, this uncertainty is going to influence the decision of the AA but also firms' incentives to merge. This analysis is useful to the AA in order to more properly evaluate merger proposals when there is uncertainty about the cost savings that mergers may induce or not.[2]

In the absence of uncertainty and in a context of symmetric Cournot oligopoly with linear demand and costs, a merger is profitable if it comprises a pre-merger market share of at least 80% (Salant et al., 1983; Perry and Porter, 1985). Also, when both firms and the AA are perfectly informed about the merger induced efficiency gains, the antitrust authority usually allows the merger when there are important efficiency gains that would lead to lower prices (Motta and Vasconcelos, 2005; Vasconcelos, 2010).

The present paper is related to two strands of literature on mergers in a Cournot framework: (i) the studies of antitrust authority's merger decisions, where the AA evaluates the welfare effects of mergers and allows for merger remedies under uncertainty, such as Cosnita and Tropeano (2009), Besanko and Spulber (1993), Corchón and Faulí-Oller (2004); and (ii) works that, investigating the impact of uncertainty on the incentives to merge, conclude that the incentives to merge depend on the information structure, such as Gal-Or (1988), Stenbacka (1991), Wong and Tse (1997), Stennek (2003), Qiu and Zhou (2006), Banal-Estañol (2007), Zhou (2008a,b), Amir et al. (2009). Uncertainty can also be seen as an information

[2] See Morgan (2001) for the discussion about the significance of reviewing merger cases with uncertainty.

sharing problem, for instance, firms can have more information about their own costs than the AA. Previous contributions to the literature on information sharing among oligopolists did not consider the possibility of mergers among firms nor AA intervention. In most of the papers, it is assumed that there is market uncertainty because the marginal cost and/or the market demand are unknown to the firms, such as, Novshek and Sonnenschein (1982), Clarke (1983), Sakai (1985), Gal-Or (1985, 1986), Shapiro (1986), Vives (1984, 2002), Li (1985), Sakai and Yamato (1989), Raith (1996), Lagerlöf (2007), Jensen (1992), Elberfeld and O. Nti (2004), among others.

To our knowledge, however, none of the previous papers has addressed the role of efficiency gains which are uncertain for all players in the merger formation process (firms and the AA). The present paper then contributes to fill this gap in the extant literature by assuming that, when firms propose the merger to the AA, all the players (insider, outsiders and AA) are uncertain about the post-merger efficiency gains and therefore they decide by considering the expectations on those gains. Once the merger is consummated, both insider and outsider firms can observe the efficiency gains and compete *à la* Cournot. This is different from Amir et al. (2009)'s paper where, after the merger, only the insider firm observes its cost. Our model is also different from the literature cited before, where it is usually assumed that the merging firms have an informational advantage, knowing more about the merger induced efficiency gains than its rivals and the AA. This scenario of "symmetric" uncertainty is probably more relevant in industries such as electronic communication and ICT markets wherein the pace of technological progress is impressive. Hence, it becomes very difficult for any player in the market (the AA or even the merger partners) to predict what will be the merger specific efficiencies in a context where the competitive snapshot changes very fast and frequently. Further, our analysis is close to Le Pape and Zhao (2013)'s paper. Le Pape and Zhao (2013) analyse Stackelberg mergers' decisions when there is uncertainty on productivity and informational asymmetry between firms. However, here we focus our analysis on both mergers and AA decisions, assuming that the firms compete in a Cournot setting and that all firms have the same degree of uncertainty.

We find that the increase in the degree of uncertainty about merger's efficiency gains benefits firms (in terms of higher expected profits) and consumers (with higher expected consumer surplus). Further, when the degree of uncertainty is high, there is a greater likelihood that firms propose a merger to the AA and the AA accepts it. Therefore, uncertainty enhances both the occurrence of merger proposals and the likelihood that those proposals are cleared by the AA. We also find that, higher degree of uncertainty increases outsider firms' incentives to free-ride on the merger.

The remainder of the paper is organized as follows. In section 4.2 the basic framework of the model is described. Section 4.3 presents the pre-merger equilibrium results. Section 4.4 analyses the equilibrium of the proposed merger formation game. Also, in this section we study merger's free-riding problem. In section 4.5 we present and discuss the obtained numerical results. Finally, section 4.6 offers some concluding remarks. All the proofs are relegated to the appendix.

4.2 Basic Framework

We consider a homogeneous good industry with n firms that compete à la Cournot. The inverse demand function is given by $P = 1 - Q$, where $Q = \sum_{i=1}^{n} q_i$ is the aggregate output and q_i is the quantity produced by firm $i \in \{1, ..., n\}$.

Let k_i denote firm *i*'s capital holdings, where $k_i \in \{1, 2, ..., K\}$. The cost function of a firm i, which owns k_i units of he industry capital and produces q_i units of output, is given by:[3]

$$C(\alpha_j, q_i, k_i) = \frac{\alpha_j K}{k_i} q_i,$$

where K is the total capital in the industry, $\Sigma_i k_i = K$, that is fixed and α_j measures the endogenous efficiency gains, with $\alpha_j \geq 0$.[4]

[3]This is a simplified version of the cost structure proposed by Motta and Vasconcelos (2005) and captures also the specific case studied in Horn and Persson (2001). This cost function is based on the one proposed by Perry and Porter (1985). In their framework firms' marginal cost is linear in output and mergers reduce variable costs.

[4]Efficiency gains may result from firm's combined ability to exploit economies of scale or raise larger

In the next sections we analyse the results before the merger, where the efficiency gains are known with certainty. We then discuss the results obtained after the merger, for both AA and merger decisions, in a context where there is uncertainty about merger induced efficiency gains.

4.3 Pre-Merger equilibrium

Before the merger (BM), we assume that firms are symmetric and that each firm owns one unit of capital, that is, $k_i = 1$. Also, each firm knows, with certainty, its level of efficiency gains and therefore $\alpha_i = \alpha$, where α is the common efficiency level of all firms before the merger.

The equilibrium profits and consumer surplus are then given by:

$$\pi_i^{BM} = \frac{(1-n\alpha)^2}{(n+1)^2}, \text{where } i = 1, .., n. \tag{4.1}$$

$$CS^{BM} = \frac{n^2 (1-n\alpha)^2}{2(n+1)^2}. \tag{4.2}$$

Assumption 4.1. Assume that $\alpha < \frac{1}{n}$.

The previous assumption is imposed in order to exclude the case in which firms in the industry do not produce (are inactive) in the pre-merger equilibrium.

4.4 Merger Analysis

In this section we analyse both the AA's and firms' merger decision, in a setting where all firms in the industry and the AA are uncertain over the merger induced efficiency gains.

Assume that, at the status quo industry, one firm in the industry is randomly selected and has the opportunity to propose, to the AA, a merger involving $m \geq 2$ firms. This firm may

amounts of capital, but also from complementarity between technological or administrative capabilities of firms (Röller et al., 2001).

propose a merger with all or a subset of its rivals. Since each firm operates with a constant marginal cost of production, but the level of its marginal cost is a decreasing function of its capital holdings, the resultant merged firm becomes more efficient than outsiders by having $k_i = m$ units of assets. We assume that m is given exogenously. Hence, the merger brings the capital of merging parties into a single larger firm and, therefore, gives rise to endogenous efficiency gains by decreasing the marginal cost. The level of these potential efficiency gains is captured by the parameter α_i.

When the level of efficiency gains is the same before and after the merger ($\alpha_i = \alpha$), all firms and the AA know with certainty what will be the merger's cost savings. Hence, the higher is the value of α, the stronger the efficiency gains induced by a merger are, and therefore, the higher is the cost reduction after the merger (Motta and Vasconcelos, 2005; Vasconcelos, 2010).

However, in our model, the merger also brings uncertainty about the induced efficiency gains, that is, all firms (insiders and outsiders) and the AA cannot predict precisely the level of the future merger-induced efficiency gains. Thus, we assume that all players are uncertain on what will be the exact value of the efficiency gains level, as measured by $\alpha_i = \alpha_u$. This level of future efficiency gains, α_u, could be higher, lower or equal than the level of efficiency gains before the merger, given by α. Therefore the merger effects on firms costs depends both on the value of α and of the change on the capital holding. If α_u is equal to α then, after the merger, the merged firm knows that it is endowed with the capital of the merging parties and therefore its cost will decrease. In this case, the outsider firm's cost will not change. If α_u is lower than α, the insider firms' cost will decrease due to the increase in capital and also due to the lower value of α. In this case the outsider firms' cost will decrease after the merger. Different results are obtained when α_u is higher than α: if $\alpha_u = k\alpha$, then the insider firm's cost will remain the same as before the merger however, the outsider firms' costs will increase; if $\alpha_u > k\alpha$ both insider and outsider firms' costs increase after the merger; and if $\alpha_u < k\alpha$ then, after the merger, the insider firm's cost decrease and the outsider firms' costs increase.

Assumption 4.2. Let α_u be a random variable distributed over $[0, \frac{1}{2n}]$.

By assuming that $\alpha_u < \frac{1}{2n}$, we are excluding the case where outsiders exit the market after the merger takes place.[5] The expected value of the efficiency gains in the future is denoted by $E(\alpha_u) = \mu$, where $\mu \in [0, \frac{1}{2n}]$, and the variance is denoted by $V(\alpha_u) = \sigma^2 > 0$. The σ^2 represents the degree of uncertainty and captures the efficiency gains fluctuation. The higher is σ^2, the greater is the uncertainty about the merger's efficiency gains.

Firm i has to decide whether or not to merge and the AA has to decide whether or not to accept the proposed merged, both without knowing the actual cost of the merged firm in the future. We assume that firms set output decisions after uncertainty is solved, given that we want to study the effects of uncertainty on the decisions of both merger firms and the AA.

We assume that firms are risk neutral and therefore firms' decisions regarding the merger are based on the expected values of their profits.[6]

We also assume that the AA is risk neutral, and that its decision is based on the expected value of consumer surplus. We consider that the AA's decision is based on expected consumer surplus instead on social welfare and, thus, the AA approves the merger if the expected consumer surplus increases.[7] Thus, the private incentives to merge and the AA decisions are based on expected values.

We model the interactions between the antitrust authority and the merging firms as a

[5]Exit inducing mergers are rare. Relaxing this would complicate our analysis, since we would have to consider not only growth by mergers but also growth by capital accumulation.

[6]Note that if we assume that firms' are risk averse, the utility function of profit for each firm would be given, for instance, by the following constant absolute risk aversion function $u(w) = 1 - \exp(aw)$ where $a > 0$ is the coefficient of absolute risk aversion. Given that the price is P, the costs are c and that the firm chooses an output q to maximize the expected utility of profit: $E(\pi) = \frac{1}{2}(1 - \exp(a(P - c)q)) + \frac{1}{2}(1 - \exp(a(P - 1 - c)q))$. This function is concave in q and in our model, the profit is linear in q.

[7]By assuming that the AA evaluates mergers according to a consumer surplus standard this does not mean that this is always better than the total welfare standard. However, as Lyons (2002) argued, the consumer surplus standard is applied in most antitrust jurisdictions. Other papers also study how the AA should apply the consumer surplus standard when challenging a merger, such as Besanko and Spulber (1993), Neven and Röller (2005), Vasconcelos (2010), Nocke and Whinston (2010), Jovanovic and Wey (2012), among others.

four-stage game, with the following timing:

- **Stage 1:** Firms have to decide whether or not to propose a merger to the AA.
- **Stage 2:** The AA decides whether or not to accept the proposed merger.
- **Stage 3:** Nature chooses α_u and reveals it to all players.
- **Stage 4:** Insider and outsider firms choose quantities competing in the usual Cournot fashion.

In what follows, we will solve the model by following the usual backward induction procedure.

4.4.1 Product Market Competition

The results presented below refer to the Cournot-Nash equilibrium after the merger (AM), where firms know the level of efficiency gains (since uncertainty has been resolved in the previous stage of the game).

In the fourth-stage of the game, firms have already observed the actual value of α_u so, they choose to produce the quantities that maximize their profits. The Cournot equilibrium profits of insider firm (I) and outsider firms (O) and the consumer surplus (CS) are, respectively, given by:

$$\pi_I^{AM} = \frac{\left[\, m - n\alpha_u \left((m-1)(m-n)+1\right)\right]^2}{m^2 (n-m+2)^2} \tag{4.3}$$

$$\pi_O^{AM} = \frac{\left[\, m - n\alpha_u (2m-1)\right]^2}{m^2 (n-m+2)^2} \tag{4.4}$$

$$CS^{AM} = \frac{\left[\, m(n-m+1) - n\alpha_u \left(m(n-m)+1\right)\right]^2}{2m^2 (n-m+2)^2}. \tag{4.5}$$

As we can observe from equations (4.3), (4.4) and (4.5), without uncertainty about the level of efficiency gains, both profits and consumer surplus increase with the level of efficiency gains.

Under uncertainty, outsider firms would exit the market if they expected to produce zero in equilibrium, that is, if $\mu \geq \frac{m}{n(2m-1)}$. If the expected efficiency gains from merger are very high, outsider firms would not be able to make positive expected profits and may exit the market. However, since we assume that $\mu < \frac{1}{2n}$, we exclude this possibility.

The next subsection presents the expected insider and outsider firms' profits, the expected consumer surplus and discusses the effects of uncertainty on these outcomes. These results are based on the results obtained after the uncertainty about the efficiency gains has been resolved.

4.4.2 Expected Profits and Consumer Surplus

The expected profits of both insiders and outsider firms, respectively, $E\left[\pi_I^{AM}\right]$ and $E\left[\pi_O^{AM}\right]$, and the expected consumer surplus, $E\left[CS^{AM}\right]$, are then given by:

$$E\left[\pi_I^{AM}\right] = \frac{\left[m - n\mu\left((m-1)(m-n)+1\right)\right]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2\left[(m-1)(m-n)+1\right]^2}{m^2(n-m+2)^2} \tag{4.6}$$

$$E\left[\pi_O^{AM}\right] = \frac{\left[m - n\mu(2m-1)\right]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2(2m-1)^2}{m^2(n-m+2)^2} \tag{4.7}$$

$$E\left[CS^{AM}\right] = \frac{\left[m(n-m+1) - n\mu\left(m(n-m)+1\right)\right]^2}{2m^2(n-m+2)^2} + \sigma^2\frac{n^2\left[m(n-m)+1\right]^2}{2m^2(n-m+2)^2} \tag{4.8}$$

As we can see, uncertainty affects both the expected profits and the expected consumer surplus.

Lemma 4.1: An increase in the level of efficiency gains' uncertainty benefits both insider and outsider firms but also the consumers, i.e., both $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ are increasing in σ^2. Moreover, uncertainty benefits more the insider firm than the outsider firms, if:

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial\sigma^2} > \frac{\partial E\left[\pi_O^{AM}\right]}{\partial\sigma^2} \iff n > \frac{m(m+1)}{m-1} \equiv \overline{n} \quad \wedge \quad m > 1. \tag{4.9}$$

Proof. See Appendix C.2. □

A higher degree of uncertainty improves firms' expected profits, because profit functions are convex in α_u. Since a firm's profit is convex in its own cost, uncertainty increases the firm's expected profit. Both insider and outsider firms will choose their quantities without knowing the exact value of the costs.[8] The insider firm expects that, after the merger, the cost is going to decrease due to the synergies generated, that is, due to an increase in the capital stock of the merged firm. However, the insider firm does not know how much is this cost saving, since the insider firm's cost decreases in three different scenarios: (i) $\alpha_u < \alpha$, (ii) $\alpha_u = \alpha$ and (iii) $\alpha < \alpha_u < k\alpha$. Since the insider firm expects a lower cost, it knows that, even without uncertainty, the resultant firm is going to produce a large quantity. The uncertainty about how much the cost reduces also affects the outsider firms. Although each outsider firm still has one unit of capital, if the cost saving is very high, we know that, without uncertainty, outsiders will respond by reducing quantity. However, under uncertainty, only if α_u is smaller than α, there will also be significant cost savings for outsider firms and this will increase both quantities and profits of all outsider firms. If this is the case, the expected net effect is an increase in the total quantity and a decrease in the total price (which also happens without uncertainty).

Without uncertainty and cost synergies, mergers usually lead to a sharp reduction of the total output. As consequence, in deterministic models, mergers usually reduce the consumer surplus. However, when there are cost synergies, the reverse can actually occur even without uncertainty. In our model, the higher is the uncertainty about the cost synergies generated by the merger, the larger the value of σ^2 and the greater the expected consumer surplus will be. After the merger, both the insider and outsider firms expect to benefit from cost synergies if α_u is smaller than α and, therefore, expect to increase the quantities produced. However, it may happen that, after the merger, α_u is greater than α and, therefore, the insider firm's cost decreases but the outsider firms' cost increase. In this case, the insider firm still produces more quantity but the outsider firms' will respond by producing a lower quantity. Nevertheless, under uncertainty, both insider and outsider firms do not adjust so sharply their production. Consequently, the net effect is an increase in the total quantity in the market, which affects positively the consumer surplus. Hence, both profits and consumer surplus are increasing functions with respect to the uncertainty parameter, σ^2.

Additionally, under uncertainty, the profits of the insider and outsider firms are affected

[8]This is a similar result as obtained by Zhou (2008a)'s paper.

differently. More precisely, if the number of firms in the market is higher than a threshold $\overline{n}$, uncertainty has a higher effect on insider firm's profits than on outsiders. However, if the number of firms in the market is lower than $\overline{n}$, the reverse occurs. If we solve with respect to m we would get that $\frac{(n-1)-\sqrt{((n-1)^3-4n)}}{3} < m < \frac{(n-1)+\sqrt{((n-1)^3-4n)}}{3}$, that is when the number of insider firms is greater than 50% of the firms in the market usually the impact of uncertainty on outsiders profits is greater than the impact of uncertainty on insider's profit.

Further, the extent of the uncertainty effect on both expected profits and consumer surplus is shown to depend on the number of firms in the market (n) and on the number of firms involved in the merger (m). The higher is the number of firms in the market, the higher the positive impact of uncertainty on consumer surplus and on insider profits is, but the lower the positive impact on outsider profits is. Also, the higher is the number of insider firms, the larger the positive effect of uncertainty on (insider and outsider) profits and on consumer surplus is. However, from numerical simulation, we have seen that as the number of insider firms increases, that is, when insiders involve more than 50% of the firms in the market, uncertainty begins to have a lower but still positive effect on both consumer surplus and insider profits. Hence, in order for the merger to be profitable, the insider firm must compete with a sufficiently high number of outsider firms.[9]

Lemma 4.2: If:

(i) $m = n$ (full merger), both expected consumer surplus and insider profits always decrease with the expected mean over the efficiency gains level;

(ii) $m < n$ (partial merger), the expected outsider profits and the expected consumer surplus decrease with the level of expectations over the efficiency gains, however the expected insider profits increase if those expectations satisfy the following threshold: $\mu > \frac{m}{n[(m-n)(m-1)+1]}$.

Proof. See Appendix C.2. □

[9]For instance, if there are $n = 11$ firms in the market, in order for the merger between two or three firms ($m = 2, 3$) to be profitable, it is necessary to exist at least five outsider firms.

Lemma 4.2 states that, after the merger, the expected outsider firms' profits and the expected consumer surplus decrease with the expected mean over the efficiency gains (μ). If the expectations over the efficiency gains are very high this could mean that, after the merger, outsider firms would have incentives to exit the market since their profits would become negative. Hence, the insider firm would have higher market power, which would negatively affect the consumer surplus. Also, the expected insider firms' profits increase if the expectations over the efficiency gains are sufficiently high and if the merger does not involve all the firms in the market.

Before looking at the results obtained to the AA decision, we assume the following:[10]

Assumption 4.3. Assume that $\mu = \alpha$.

Assumption 4.3 states that the expected efficiency gains level are equal to the benchmark firm's efficiency gains level, that is, the efficiency gains level of both insider and outsider firms at the status quo industry. After the merger, the insider firm expects that its cost is going to reduce, since now only one firm encompasses all capital assets of the merging firms. However, the insider firm does not know how much is this reduction. Therefore, for simplicity, we assume that the expectations on the level of efficiency gains are rational and equal to the efficiency gains before the merger. Hence, the merged firm expects the cost reduction to only depend on the level of capital of the merging parties. This assumption allows us isolate the effects of uncertainty on both AA's and firms' merger decisions. By assuming $\mu = \alpha$ the interpretation of the results is the same as before: the higher is α, the stronger the efficiency gains induced from the merger are.

Recall that we are only considering the scenario in which outsider firms do not exit the market after the merger. Hence, after the merger the market structure consists of $n - m + 1$ firms: $n - m$ outsiders with one unit of capital and one firm with $k = m$ units of capital.[11]

[10]This assumption is not crucial. Actually, we obtain our main results without assuming it, but, in order to present clear expressions we need to impose it. See Appendix C.3 for further information on the results obtained without Assumption 4.3.

[11]However, we do not exclude the case where the merger involves all the firms in the industry. See Appendix

In what follows, both AA and merging firms decisions are made under uncertainty.

4.4.3 Antitrust Authority's Decision

The AA decides whether to allow or block any proposed merger. The AA accepts the merger if the expected consumer surplus after the merger is greater or equal than the consumer surplus before the merger ($E\left[CS^{AM}\right] \geq CS^{BM}$).

Proposition 4.1. The AA will accept the merger if $E\left[CS^{AM}\right] - CS^{BM} > 0$, i.e. iff:

$$\sigma^2 \geq \sigma^2_{AA\{m,n-m,\alpha\}} \equiv \frac{m^2\left(n-m+2\right)^2 n^2 \left(1-n\alpha\right)^2 - \left[m\left(n-m+1\right) - n\alpha\left(m\left(n-m\right)+1\right)\right]^2 \left(n+1\right)^2}{\left(n+1\right)^2 n^2 \left[m\left(n-m\right)+1\right]^2}. \tag{4.10}$$

Proof. See Appendix C.2. □

Proposition 4.1 states that when uncertainty is high, there is a greater likelihood that the AA accepts the merger. Since the consumer surplus before the merger (whose expression is given in (4.2)) does not depend on the uncertainty parameter, the higher is the uncertainty, σ^2, the greater is the expected consumer surplus after the merger. Consequently, when uncertainty is high, the expected consumer surplus variation also increases. Further, when the level of efficiency gains or the number of insider firms increases, $\sigma^2_{AA\{m,n-m,\alpha\}}$ decreases. This means that the interval of uncertainty level above which the AA accepts the merger increases. This happens because the difference between the consumer surplus before and after the merger increases. The reverse occurs when the number of firms increases.

When there are cost synergies, mergers can improve consumer surplus by increasing output. This improvement could actually be higher when there is uncertainty about cost synergies. Also, since those firms that propose the merger to the AA are the ones that usually have more certainty about their future cost savings, this could actually work as a signal to the AA. If this is a good signal, the AA could easily identify the type of firms that will generate greater cost synergies and , therefore, there is a greater likelihood that the merger proposal by this type of firms ends up being accepted by the AA. This being the case, under uncertainty

C.1.4. for further information on full merger results.

there is a greater likelihood that the merger is accepted by the AA.

4.4.4 Merger Decision

In this section, we examine firms incentives to merge. Firms decisions on whether to merge or not result from the comparison between the expected profits of the merged firm with the profits before the merger. Hence, the merger profitability condition depends on the expected profits of the merged firm, that is, firms will propose the merger if $E\left[\pi_I^{AM}\right] \geq m\pi^{BM}$.

Proposition 4.2. If the level of uncertainty is sufficiently high, there is a greater likelihood that firms propose a merger. Firms will propose the merger if they anticipate that it will be profitable, i.e. iff:[12]

$$\sigma^2 \geq \sigma^2_{MP\{m,n-m,\alpha\}} \equiv \frac{(1-n\alpha)^2 m^3 (n-m+2)^2 - [m-n\alpha((m-1)(m-n)+1)]^2 (n+1)^2}{(n+1)^2 n^2 [(m-1)(m-n)+1]^2}. \tag{4.11}$$

Proof. See Appendix C.2. □

From Proposition 4.2, we conclude that uncertainty promotes the expected merger profitability. As the extent of the variance exceeds a certain threshold ($\sigma^2_{MP\{m,n-m,\alpha\}}$) the expected profit of the merged firm becomes larger than the sum of the firm's profits in the benchmark case, and firms that face cost uncertainty choose to merge. Further, when the number of insider firms increases, the region of uncertainty above which the merger is profitable increases ($\frac{\partial \sigma^2_{MP\{m,n-m,\alpha\}}}{\partial m} < 0$), hence the profits after the merger exceed those before the merger and firms have more incentives to propose the merger. In the deterministic model, without efficiency gains, unless the merger involves a sufficient number of insiders, most of the horizontal mergers are unprofitable (Salant et al., 1983). However, in our model, as the uncertainty increases, the expected profit also increases because the gain of the optimal quantity adjustment increases, and therefore it is possible that the expected profit of insider firm (as measured by (4.6)) exceeds the sum of profits of the pre-merger firms (whose ex-

[12]Note that $n > 3$, otherwise insider firm's profit would not depend on the σ^2 parameter and therefore this expression would not exist.

pression is given in (4.1)). Hence, the insider firm has a higher profit both directly from the cost advantage and indirectly from the favourable responses from outsider firms. Since the pre-merger profits are not affected by uncertainty, it is expected that the merger becomes at least more profitable than in the deterministic models.

As the degree of uncertainty becomes larger, firms have more incentives to merge and therefore, we conclude that cost uncertainty is able to induce the firms to merge. This relationship between merger profitability and cost uncertainty is also investigated by Banal-Estañol (2007) and Zhou (2008a,b). The results obtained in these papers are similar to ours however, our framework is different from the adopted by these authors since we assume that all players face the same level of uncertainty. For instance, Banal-Estañol (2007) considers that firms face idiosyncratic uncertainty about costs and that uncertainty generates an informational advantage only to the merging firms, increasing merger profitability. Also, differently from our paper, Zhou (2008a,b) assumes that after the merger, costs are realized and each firm learns its own costs but not the costs of its rivals. Zhou (2008a,b) argues that when costs are uncertain and firms choose quantities before the uncertainty is resolved, a merger is more profitable the greater the uncertainty. Zhou (2008a,b) shows that firm's incentives to merge are enhanced by production rationalization.[13]

4.4.5 Free-riding Problem

In this subsection we study the effects of uncertain efficiency gains on merger's free-riding problem.

In the deterministic models, mergers face a free-riding problem, given that outsider firms are the ones that benefit most from the merger (free-riding problem). In order to assess if there is a free-riding problem in our model, we compare the expected profit of the merged firm with the expected profits of outsiders.

[13] We could extend our analysis in order to allow for differences in the uncertainty level among all the players. We realize that this is an important subject for further research and that this far behind the scope of the present paper.

Proposition 4.3. There is a free-riding problem if $E\left[\pi_I^{AM}\right] < mE\left[\pi_O^{AM}\right]$, i.e., iff:

$$\sigma^2 > \sigma^2_{fr\{m,n-m,\alpha\}} \equiv \frac{m\left(m-1\right)\left[2n\alpha\left(m+n+1\right)-m\right]}{mn^2\left(2m-1\right)^2 - n^2\left[\left(m-1\right)\left(m-n\right)+1\right]^2} - \alpha^2 \tag{4.12}$$

Proof. See Appendix C.2. □

Proposition 4.3 states that, if the uncertainty over merger's efficiency gains is high, outsider firms benefit more from the merger than the insider firms. Hence, an increase in the level uncertainty promotes merger's free-riding problem. After the merger, the merged firm expects to produce more quantity than each outsider firm, due to the synergies generated. The outsider firms' production depends on whether α_u is greater, equal or lower than α. If the level of expected efficiency gains increases and becomes higher than α after the merger, the insider firm still produces more quantity but the outsider firms' will respond by producing a lower quantity. While the profit of the insider firm always increases with the uncertainty, that of an excluded rival also increases as a result of the merger. For high degree of uncertainty, firms would prefer to wait for their rivals to merge and, thus, benefit from the merger.

Lemma 4.3. Keeping the number of firms constant (n), as the number of insider increases (m), the region of uncertainty level below which there is no free-riding decreases, i.e., $\frac{\partial \sigma^2_{fr}}{\partial m} > 0$. Also, keeping the number of insiders constant, an increase in the number of firms in the market, increases the region of uncertainty level below which there is no free-riding, i.e., $\frac{\partial \sigma^2_{fr}}{\partial n} < 0$.

We find that, under uncertainty about cost synergies and keeping the number of firms constant, as the number of insider firms increases, firms have more incentives to deviate and to compete against its rivals as an outsider. Also, keeping the number of insiders constant, as the number of firms in the market increases, outsider firms have less incentives to free-ride on the merger.

4.5 Numerical Application

In what follows, assume that the total quantity of capital available in the industry is equal to four units ($K = 4$) and that this capital is equally distributed amongst four firms ($n = 4$) in the status quo industry structure.

We analyse the results obtained for both firms and AA decisions for three cases:

1) Merger involving two firms ($m = 2$);

2) Merger involving three firms ($m = 3$); and

3) Merger to monopoly ($m = 4$).

Again, we restrict our attention to the case in which, after the merger, outsiders do not exit the market ($\alpha_u < \frac{1}{8}$). Further, we also analyse if there is a free-riding problem for both cases 1 and 2.

The following result sums up the AA decisions for the three merger cases.

Result 4.1.

- The AA accepts the merger involving two firms (case 1) if: $\sigma^2 \geq \sigma^2_{AA\{2,1,1,\alpha\}}$;
- The AA accepts the merger involving three firms (case 2) if: $\sigma^2 \geq \sigma^2_{AA\{3,1,\alpha\}}$;
- The AA accepts the merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{AA\{4,\alpha\}}$.

Proof. See Appendix C.2. □

The following result sums up firms decisions for the three merger cases.

Result 4.2.

- Firms will propose a merger involving two firms (case 1) if: $\sigma^2 \geq \sigma^2_{MP\{2,1,1,\alpha\}}$;
- Firms will propose a merger involving three firms (case 2) if: $\sigma^2 \geq \sigma^2_{MP\{3,1,\alpha\}}$;
- Firms will propose a merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{MP\{4,\alpha\}}$.

Proof. See Appendix C.2. □

The following result sums up the results obtained for the free-riding problem for the first two merger cases.

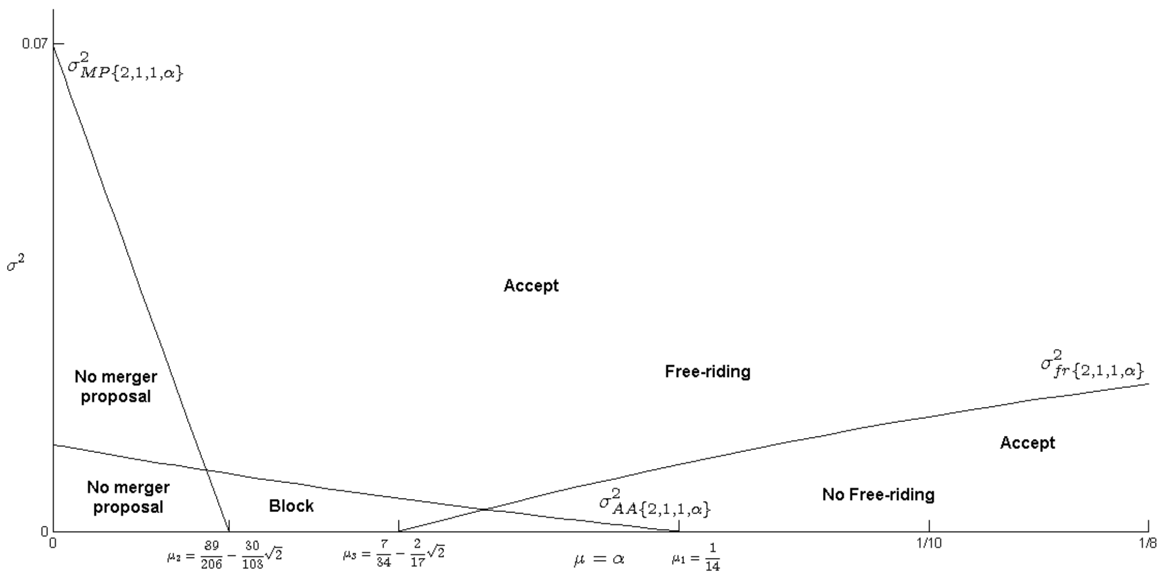
Result 4.3.

- There is a free-riding problem (case 1) if: $\sigma^2 \geq \sigma^2_{fr\{2,1,1,\alpha\}}$;
- There is a free-riding problem (case 2) if: $\sigma^2 \geq \sigma^2_{fr\{3,1,\alpha\}}$.

Proof. See Appendix C.2. □

The next three figures represent graphically the results obtained in Results 4.1, 4.2 and 4.3.

Figure 4.1: AA's decision ($m = 2$)



Analysing Figure 4.1, we illustrate that, when uncertainty and expected mean about the future efficiency gains level are low, $\sigma^2 < \sigma^2_{MP\{2,1,1,\alpha\}}$ and $\mu < \mu_2 = \frac{89}{206} - \frac{30}{103}\sqrt{2}$, any two-firm merger will never be proposed to the AA. As the expected efficiency gains increase, $\sigma^2_{MP\{2,1,1,\alpha\}} < \sigma^2 < \sigma^2_{AA\{2,1,1,\alpha\}}$ and $\mu_2 < \mu < \mu_1 = \frac{1}{14}$, the two firm merger will be proposed to the AA, however it will be blocked since it is expected to reduce the consumer surplus. Further, if both expectations and uncertainty are high, $\sigma^2 > \sigma^2_{AA\{2,1,1,\alpha\}}$, any two firm merger will be proposed and accepted by the AA. Additionally, there exist a free-riding problem when $\sigma^2 > \sigma^2_{fr\{2,1,1,\alpha\}}$, that is, outsider firms earn more from the merger than the insider firms and this could promote the free-riding. Also, if the expected efficiency gains are higher than $\mu_3 = \frac{7}{34} - \frac{2}{17}\sqrt{2}$ and if uncertainty is not very high $\sigma^2 < \sigma^2_{fr\{2,1,1,\alpha\}}$ there is a possibility that the merger is accepted by the AA and there is no free-riding problem.

Figure 4.2: AA's decision ($m = 3$)

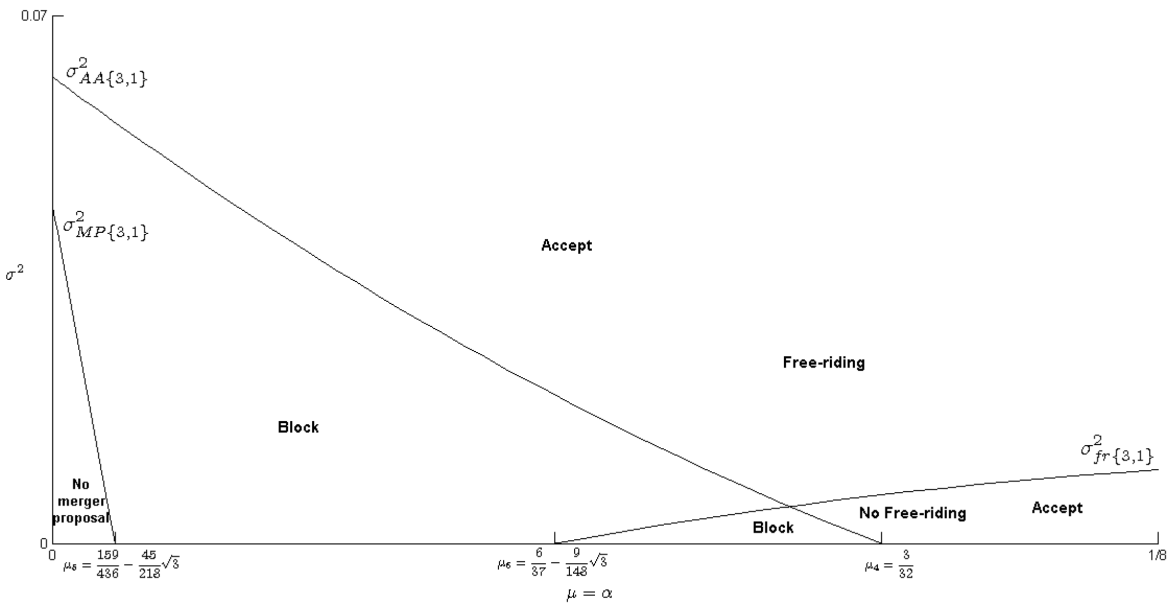
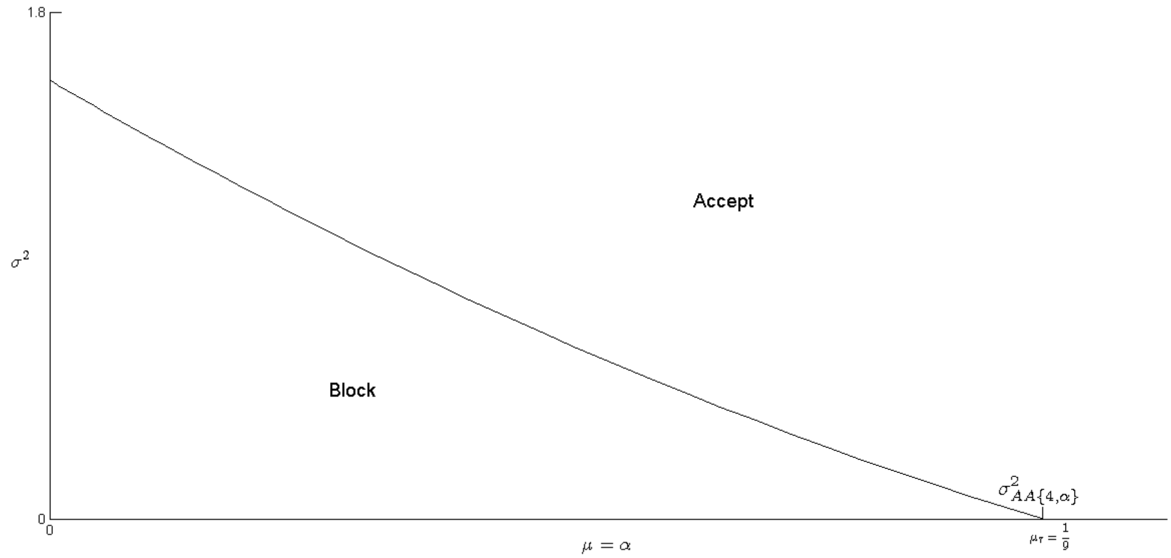


Figure 4.2 summarizes the results obtained when the merger involves three firms. We can observe that when both expected efficiency gains and uncertainty are low, $\sigma^2 < \sigma^2_{MP\{3,1,\alpha\}}$, $\mu < \mu_5 = \frac{159}{436} - \frac{45}{218}\sqrt{3}$, any three-firm merger will never be proposed to the AA. As the expected efficiency gains increase, $\sigma^2_{MP\{3,1,\alpha\}} < \sigma^2 < \sigma^2_{AA\{3,1,\alpha\}}$ and $\mu < \mu_4 = \frac{3}{32}$, any three-firm merger proposal will be proposed and blocked by the AA. However, when both the expected efficiency gains and the uncertainty increase, $\sigma^2 > \sigma^2_{AA\{3,1,\alpha\}}$ and $\mu_4 < \mu < \frac{3}{32}$, the three-firm merger will always be proposed and accepted by the AA. Further, in this case there is also a free-riding problem when $\sigma^2 > \sigma^2_{fr\{3,1,\alpha\}}$, that is, the only outsider firm earns more from the merger than the insider firms and this could also promote merger's free-riding problem. Also, if the expected efficiency gains are higher than $\mu_6 = \frac{6}{37} - \frac{9}{148}\sqrt{3}$ and if uncertainty is not very high $\sigma^2 < \sigma^2_{fr\{3,1,\alpha\}}$ there is a greater likelihood that the merger of three firms is accepted by the AA and outsiders have no incentives to free ride on it.

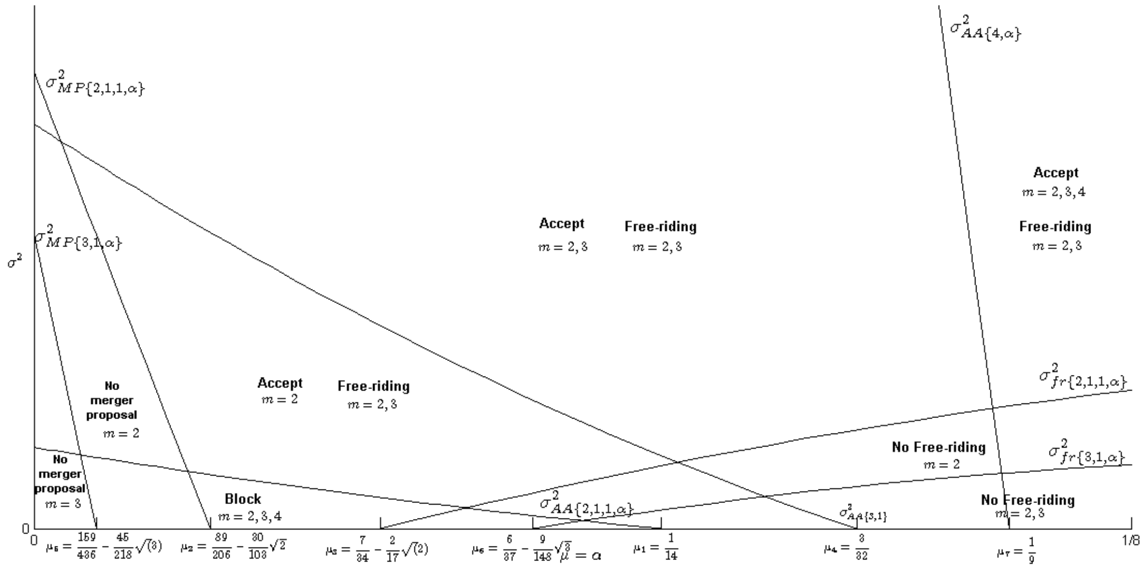
Figure 4.3: AA's decision ($m = 4$)



The results obtained for the full merger case are illustrated in Figure 4.3. Since the $\sigma^2_{MP\{4,\alpha\}}$ is always negative, for any level of uncertainty, the firms have always incentive to propose a merger to monopoly. However, if the expected efficiency gains and uncertainty are low, $\sigma^2 < \sigma^2_{AA\{4,\alpha\}}$ and $\mu < \mu_7 = \frac{1}{9}$, any merger proposal to monopoly will be blocked by the AA. As both uncertainty and expected efficiency gains increase, $\mu > \mu_7 = \frac{1}{9}$, and $\sigma^2 > \sigma^2_{AA\{4,\alpha\}}$, the merger to monopoly will always be accepted by the AA.

Figure 4.4 sums up the results obtained for the three merger cases.

Figure 4.4: AA's decision ($m = 2, 3, 4$)



We conclude that when the level of uncertainty and expected efficiency gains are high, the firms have incentives to propose any type of merger and this merger has a greater likelihood of being accepted by the AA. When both levels of uncertainty and expected efficiency gains are low, the firms have less incentives to propose the merger. As the expected efficiency gains increase, the firms usually propose the merger in each case, however the AA rejects it for all or some particular cases. Usually in the models without uncertainty about the future synergies generated by the merger, the AA only accepts the merger if these efficiency gains are sufficiently high (Motta and Vasconcelos, 2005; Vasconcelos, 2010). In contrast, the present paper shows that, with uncertainty, the probability of the merger being accepted is higher than without uncertainty and, for some cases, mergers could be accepted even if the expected efficiency gains are intermediate. Finally, as the number of insider firms increases it is more difficult for the merger to be approved by any AA, even with uncertainty, unless the expected efficiency gains are very high.

4.6 Concluding Remarks

In this paper we investigate the effects of mergers in a context wherein, both firms and the AA are uncertain about the level of efficiency gains and, therefore, this uncertainty is going to influence the decision of AA but also firms' incentives to merge.

In the absence of uncertainty and when firms are symmetric, the merger will not be profitable unless 80% of the firms in the industry are part of the merger. However, we find that, under uncertainty and asymmetric firms, even when the merged firm is not composed of at least 80% of the firms in the industry, when the uncertainty increases, the expected profit of the merged firm exceeds insiders' pre-merger profits, and therefore firms have incentives to merge. Moreover, we find that the higher the level of uncertainty in the market regarding merger induced efficiencies, the higher the expected consumer surplus after the merger. This then implies that, given the AAs usually base their merger policy decisions on a consumer welfare standard, higher uncertainty also increases the likelihood that a merger proposal ends up being approved by the AA. Further, we find that a higher degree of uncertainty promotes merger's free-riding problem, by increasing the likelihood that outsider firms benefit more from the merger than the insider firm.

The framework and the assumptions we have assumed are of a particular kind. It would be interesting to extend our model to assess the effects on the decisions of all players, when both firms (insider and outsiders) and the AA face different degrees of uncertainty over the merger's cost savings. Also, it would be interesting to study the case where the AA is a forward looking type and expects that the first merger is followed by a defensive merger. We think that this is a very important subject for further research.

Appendix

Appendix C.1. Model derivations

C.1.1. Pre-Merger equilibrium

Before the merger, firms decide individually their quantity. Knowing that firms are symmetric and that each firm owns one unit of asset, the quantity produced by each firm is given by: $q_i = \frac{1-n\alpha}{(n+1)}$, with $i = 1, .., n$. Therefore, the total quantity and price are respectively given by $Q = n\frac{1-n\alpha}{(n+1)}$ and $P^{BM} = \frac{1+n^2\alpha}{n+1}$. The profit and the consumer surplus are then given by: $\pi_i^{BM} = \frac{(1-n\alpha)^2}{(n+1)^2}$ and $CS^{BM} = \frac{1}{2}\frac{n^2(1-n\alpha)^2}{(n+1)^2}$.

C.1.2. Post-Merger equilibrium

After the merger of m firms, insider firm now owns m units of capital while the outsiders still own 1 unit of capital. Then the merged firm will produce, in equilibrium, $Q_I = \frac{m-n\alpha_u((m-1)(m-n)+1)}{m(n-m+2)}$ and the outsiders will produce $q_o = \frac{m-n\alpha_u(2m-1)}{m(n-m+2)}$.

The equilibrium profits are then given by:
$\pi_I^{AM} = \frac{[\ m-n\alpha_u((m-1)(m-n)+1)]^2}{m^2(n-m+2)^2}$ and $\pi_O^{AM} = \frac{[\ m-n\alpha_u(2m-1)]^2}{m^2(n-m+2)^2}$.

If we study the sign of the derivatives of the profits with respect to α_u, n and m, we get:
$\frac{\partial\pi_I^{AM}}{\partial\alpha_u} > 0$ if $\alpha_u > \frac{m}{n((m-1)(m-n)+1)}$, which is always true.
$\frac{\partial\pi_I^{AM}}{\partial m} > 0$, for all α_u.
$\frac{\partial\pi_I^{AM}}{\partial n} > 0$, always positive if α_u is sufficiently high, that is, if
$\alpha_u > \frac{m}{2n(3m-2)-m(2mn+3)+m^2(m-3)+n^2(m-1)+2}$.
$\frac{\partial\pi_O^{AM}}{\partial\alpha_u} > 0$ if $\alpha_u > \frac{m}{n(2m-1)}$, which is always true.
$\frac{\partial\pi_O^{AM}}{\partial m} > 0$ this is true for $\alpha_u > \frac{m}{n(2m-1)}$ and $\alpha_u < \frac{m^2}{n(2m(m-1)+n+2)}$.
$\frac{\partial\pi_O^{AM}}{\partial n} > 0$, which is always true.

The consumer surplus (CS) is then given by:
$CS^{AM} = \frac{[m(n-m+1)-n\alpha_u(m(n-m)+1)]^2}{2m^2(n-m+2)^2}$.

If we study the sign of the derivatives of the consumer surplus with respect to α_u, n and

m, we get:

$\frac{\partial CS^{AM}}{\partial \alpha_u} > 0$ if $\alpha_u > \frac{m(n-m+1)}{n(m(n-m)+1)}$

$\frac{\partial CS^{AM}}{\partial m} > 0$ if $\frac{m^2}{n(n+2m(m-1)+2)} < \alpha_u < \frac{m(n-m+1)}{n(m(n-m)+1)}$

$\frac{\partial CS^{AM}}{\partial n} > 0$ if $\alpha_u > \frac{m(n-m+1)}{n(m(n-m)+1)} \cup \alpha_u < \frac{m}{m(-2m(n+1)+n(n+4)+m^2-1)+2}$

C.1.3. Expected Insider and Outsider Profits and Expected Consumer Surplus

Applying the expectations on equation (4.3), we obtain equation (4.6):

$$E\left[\pi_I^{AM}\right] = E\left[\frac{m^2 - 2mn\alpha_u\left((m-1)(m-n)+1\right) + n^2\alpha_u^2\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2}\right]$$
$$= \frac{m^2 - 2mnE(\alpha_u)\left((m-1)(m-n)+1\right) + n^2E(\alpha_u^2)\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2}.$$

Knowing that $V(\alpha_u) = E(\alpha_u^2) - E^2(\alpha_u)$ and that $E(\alpha_u) = \mu$ and $V(\alpha_u) = \sigma^2$ we get that:

$$E\left[\pi_I^{AM}\right] = \frac{m^2 - 2mn\mu\left((m-1)(m-n)+1\right) + n^2(\sigma^2+\mu^2)\left((m-1)(m-n)+1\right)^2}{m^2(n-m+2)^2}$$
$$= \frac{\left[m - n\mu\left((m-1)(m-n)+1\right)\right]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2\left[(m-1)(m-n)+1\right]^2}{m^2(n-m+2)^2}.$$

Applying the expectations on equation (4.4), we get equation (4.7):

$$E\left[\pi_O^{AM}\right] = E\left[\frac{\left[m - n\alpha_u(2m-1)\right]^2}{m^2(n-m+2)^2}\right] = E\left[\frac{m^2 - 2mn\alpha_u(2m-1) + n^2\alpha_{iu}^2(2m-1)^2}{m^2(n-m+2)^2}\right]$$
$$= \frac{\left[m - n\mu(2m-1)\right]^2}{m^2(n-m+2)^2} + \sigma^2\frac{n^2(2m-1)^2}{m^2(n-m+2)^2}.$$

Applying the expectations on equation (4.5) we get equation (4.8):

$$E\left[CS^{AM}\right] = E\left[\frac{\left[m(n-m+1) - n\alpha_u\left(m(n-m)+1\right)\right]^2}{2m^2(n-m+2)^2}\right]$$
$$= \frac{\left(m(n-m+1) - n\mu\left(m(n-m)+1\right)\right)^2}{2m^2(n-m+2)^2} + \sigma^2\frac{n^2(m(n-m)+1)^2}{2m^2(n-m+2)^2}.$$

C.1.4. Full Merger Results

The monopoly (M) profit, price and consumer surplus for Stages 3 and 4 are given by:

$$\pi^M = \left(\frac{m - n\alpha_u}{2m}\right)^2$$

$$P^M = \frac{m + n\alpha_u}{2m}$$

$$CS^M = \frac{1}{2}\frac{(m - n\alpha_u)^2}{(2m)^2}$$

$$E\left[\pi^M\right] = \frac{m^2 - 2mn\mu + n^2\left(\sigma^2 + \mu^2\right)}{4m^2}$$

$$E\left[CS^M\right] = \frac{m^2 - 2mn\mu + n^2\left(\sigma^2 + \mu^2\right)}{8m^2}.$$

Appendix C.2. Proofs

C.2.1. Proof of Lemma 4.1

Deriving $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ with respect to the variance, σ^2, we obtain:

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial \sigma^2} = \frac{n^2((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} > 0$$

$$\frac{\partial E\left[\pi_O^{AM}\right]}{\partial \sigma^2} = \frac{n^2(2m-1)^2}{m^2(n-m+2)^2} > 0$$

$$\frac{\partial E\left[CS^{AM}\right]}{\partial \sigma^2} = \frac{n^2(m(n-m)+1)^2}{2m^2(n-m+2)^2} > 0$$

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial \sigma^2} - \frac{\partial E\left[\pi_O^{AM}\right]}{\partial \sigma^2} = \frac{n^2(m-1)[n(m-1)-m(m+1)]}{m^2(n-m+2)}$$

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial \sigma^2} > \frac{\partial E\left[\pi_O^{AM}\right]}{\partial \sigma^2} \iff n > \frac{m(m+1)}{m-1} \equiv \overline{n}.$$

Both $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ are increasing with respect to the variance, σ^2. Thus the profits and the consumer surplus increase as the uncertainty grows.

■

C.2.2. Proof of Lemma 4.2

Differentiating $E\left[\pi_I^{AM}\right]$, $E\left[\pi_O^{AM}\right]$ and $E\left[CS^{AM}\right]$ with respect to μ we find that the derivatives are positive if and only if:

$$\frac{\partial E\left[\pi_I^{AM}\right]}{\partial \mu} = \frac{-2mn((m-1)(m-n)+1)+2n^2\mu((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} > 0 \Leftrightarrow \mu > \frac{m}{n((m-n)(m-1)+1)}.$$

$$\frac{\partial E\left[\pi_O^{AM}\right]}{\partial \mu} = \frac{-2mn(2m-1)+2n^2\mu(2m-1)^2}{m^2(n-m+2)^2} > 0 \Leftrightarrow \mu > \frac{m}{n(2m-1)}.$$

In the region of parameter values wherein the outsiders are active after the merger, $\mu < \frac{m}{n(2m-1)}$, their expected profits always decrease in the level of expected efficiency gains. Otherwise, they will exit the market (and here we assume that all firms are active after the merger takes place).Hence, when the merger does not involve all the firms in the industry ($n > m$), outsiders' profits could increase with the level of expectations if $\mu > \frac{m}{n(2m-1)}$. However, in this region outsiders would exit the market. Since we have excluded this scenario, outsiders' profits always decrease with the μ.

$$\frac{\partial E\left[CS^{AM}\right]}{\partial \mu} = \frac{-2nm(n-m+1)(m(n-m)+1)+2n^2\mu(m(n-m)+1)^2}{2m^2(n-m+2)^2} > 0 \Leftrightarrow \mu > \frac{m(n-m+1)}{n(m(n-m)+1)}.$$

Different results are obtained when the level of expectations is not too high and the merger does not involve all the firms in the industry ($n > m$). Both expected insider profits and expected consumer surplus increase if the expectations satisfy the following thresholds: $\mu > \frac{m}{n((m-n)(m-1)+1)}$ and $\mu > \frac{m(n-m+1)}{n(m(n-m)+1)}$, respectively. From the numerical simulation we know that $\mu > \frac{m}{n((m-n)(m-1)+1)}$ is always negative and since we assume that $\mu > 0$, hence when the merger does not involve all the firms in the industry, the expected insiders' profits increase with the expected efficiency gains level. However, we also know that $\mu > \frac{m(n-m+1)}{n(m(n-m)+1)}$ is always greater than $\frac{1}{2n}$. Hence, when the merger does not involve all the firms in the industry, the expected consumer surplus decreases with the expected efficiency gains level.

Additionally, if the merger involves all the firms in the industry ($n = m$) both expected consumer surplus and expected insider profits increase if $\mu > 1$. Since we assume that

$\mu < \frac{1}{2n} < 1$, therefore both expected consumer surplus and insider profits always decrease with the level of expectations, when the merger is to monopoly.

■

C.2.3. Proof of Proposition 4.1

Equation (4.10) is obtained from the expression for the expected variation of the consumer surplus. It is straightforward to show that $\Delta ECS > 0$ holds for $\sigma^2 > \sigma^2_{\{m,n-m\}}$. Hence, the AA will accept the merger if it anticipates it will enhance expected consumer surplus, i.e., iff: $E\left[CS^{AM}\right] \geq CS^{BM}$. This happens when:

$$\Delta E[CS] = \frac{[m(n-m+1)-n\mu(m(n-m)+1)]^2}{2m^2(n-m+2)^2} + \sigma^2 \frac{n^2[m(n-m)+1]^2}{2m^2(n-m+2)^2} - \frac{1}{2}\frac{n^2(1-n\alpha)^2}{(n+1)^2} \geq 0$$

From Assumption 4.3, we know that $\alpha = \mu$:

$$\frac{[m(n-m+1)-n\alpha(m(n-m)+1)]^2}{2m^2(n-m+2)^2} + \sigma^2 \frac{n^2[m(n-m)+1]^2}{2m^2(n-m+2)^2} - \frac{1}{2}\frac{n^2(1-n\alpha)^2}{(n+1)^2} \geq 0$$

Solving with respect to σ^2, we get:

$$\sigma^2 \geq \frac{m^2(n-m+2)^2 n^2(1-n\alpha)^2 - [m(n-m+1)-n\alpha(m(n-m)+1)]^2(n+1)^2}{(n+1)^2 n^2[m(n-m)+1]^2}.$$

■

Note that:

$$\frac{\partial \sigma^2_{AA\{m,n-m,\alpha\}}}{\partial \alpha} < 0$$
$$\frac{\partial \sigma^2_{AA\{m,n-m,\alpha\}}}{\partial n} < 0$$
$$\frac{\partial \sigma^2_{AA\{m,n-m,\alpha\}}}{\partial m} > 0.$$

C.2.4. Proof of Proposition 4.2

Firms will propose the merger if: $E\left[\pi_I^{AM}\right] \geq m\pi^{BM}$

$$E[MP] \equiv \frac{[m-n\mu((m-1)(m-n)+1)]^2}{m^2(n-m+2)^2} + \sigma^2 \frac{n^2[(m-1)(m-n)+1]^2}{m^2(n-m+2)^2} - m\frac{(1-n\mu)^2}{(n+1)^2} \geq 0$$

Solving with respect to σ^2 and knowing that $\mu = \alpha$ get:

$$\sigma^2 \geq \frac{(1-n\alpha)^2 m^3(n-m+2)^2 - [m-n\alpha((m-1)(m-n)+1)]^2(n+1)^2}{(n+1)^2 n^2[(m-1)(m-n)+1]^2}.$$

■

Note that:

$\frac{\partial\sigma^2_{MP\{m,n-m,\alpha\}}}{\partial\alpha} < 0$

$\frac{\partial\sigma^2_{MP\{m,n-m,\alpha\}}}{\partial n} > 0$

$\frac{\partial\sigma^2_{MP\{m,n-m,\alpha\}}}{\partial m} < 0.$

C.2.5. Proof of Proposition 4.3

There is a free-riding problem when $E\left[\pi_I^{AM}\right] < mE\left[\pi_O^{AM}\right]$, that is,

$$m\left(\frac{[m-n\mu(2m-1)]^2}{m^2(n-m+2)^2}+\sigma^2\frac{n^2(2m-1)^2}{m^2(n-m+2)^2}\right)-\left(\frac{[m-n\mu((m-1)(m-n)+1)]^2}{m^2(n-m+2)^2}+\sigma^2\frac{n^2[(m-1)(m-n)+1]^2}{m^2(n-m+2)^2}\right)>0$$

Solving with respect to σ^2 :

$$\sigma^2 > \frac{m(m-1)[2n\alpha(m+n+1)-m]}{mn^2(2m-1)^2-n^2[(m-1)(m-n)+1]^2}-\alpha^2.$$

■

Note that:

$\frac{\partial\sigma^2_{fr\{m,n-m,\alpha\}}}{\partial\alpha}$ could be greater or lower than zero.

$\frac{\partial\sigma^2_{fr\{m,n-m,\alpha\}}}{\partial n} < 0$

$\frac{\partial\sigma^2_{fr\{m,n-m,\alpha\}}}{\partial m} > 0$

C.2.6. Proof of Result 4.1

Replacing $n = 4$ and $m = 2, 3, 4$, for each merger case, we get the results for the AA's decision in Result 4.1:

- The AA accepts the merger of two firms (case 1) if: $\sigma^2 \geq \sigma^2_{AA\{2,1,1,\alpha\}} \equiv \frac{(14\mu-1)(114\mu-31)}{2500}$;
- The AA accepts the merger of three firms (case 2) if: $\sigma^2 \geq \sigma^2_{AA\{3,1,\alpha\}} \equiv \frac{(32\mu-3)(112\mu-33)}{1600}$;
- The AA accepts the merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{AA\{4,\alpha\}} \equiv \frac{3(9\mu-1)(37\mu-13)}{25}$.

■

C.2.7. Proof of Result 4.2

Replacing $n = 4$ and $m = 2, 3, 4$, for each merger case, we get the results for the merger profitability in Result 4.2:

- Firms will propose a merger of two firms (case 1) if: $\sigma^2 \geq \sigma^2_{MP\{2,1,1,\alpha\}} \equiv \frac{-356\mu+412\mu^2+7}{100}$;
- Firms will propose a merger of three firms (case 2) if: $\sigma^2 \geq \sigma^2_{MP\{3,1,\alpha\}} \equiv \frac{-1272\mu+1744\mu^2+9}{200}$;
- Firms will propose a merger to monopoly (case 3) if: $\sigma^2 \geq \sigma^2_{MP\{4,\alpha\}} \equiv \frac{3(11\mu+1)(7\mu-3)}{25}$.

■

C.2.8. Proof of Result 4.3

Replacing $n = 4$ and $m = 2, 3, 4$, for each merger case, we get the results for the free-riding problem in Result 4.3:

- There is a free-riding problem (case 1) if: $\sigma^2 \geq \sigma^2_{fr\{2,1,1,\alpha\}} \equiv -\mu^2 + \frac{7}{17}\mu - \frac{1}{68}$;
- There is a free-riding problem (case 2) if: $\sigma^2 \geq \sigma^2_{fr\{3,1,\alpha\}} \equiv -\mu^2 + \frac{12}{37}\mu - \frac{9}{592}$.

■

Appendix C.3. Results without Assumption 4.3

By Assumption 4.3 we assume that the expectations on the level of efficiency gains are rational and equal to the efficiency gains before the merger. If we did not consider that $\mu = \alpha$ our results would not change. However, now we would have a three-dimensional analysis of the results, which would be more complex.

- If $\mu < \frac{m}{n(2m-1)}$, outsiders do not exit (NE) the market, hence the AA will accept the merger if $E\left[CS^{AM}\right] \geq CS^{BM}$, that is, if:

$$\Delta CS_{NE} = \frac{m^2(n-m+1)^2 - 2\mu nm(n-m+1)(m(n-m)+1) + n^2\left(\sigma^2+\mu^2\right)(m(n-m)+1)^2}{2m^2(n-m+2)^2} - \frac{n^2(n\alpha-1)^2}{2(n+1)^2} \geq 0;$$

$$\sigma^2 > \frac{-n^2\mu^2(m(n-m)+1)^2(n+1)^2 + 2\mu nm(n-m+1)(m(n-m)+1)(n+1)^2}{n^2(m(n-m)+1)^2(n+1)^2} + \\ + \frac{m^2\left(n^2(n\alpha-1)^2(n-m+2)^2 - (n-m+1)^2(n+1)^2\right)}{n^2(m(n-m)+1)^2(n+1)^2}.$$

$\frac{\partial \Delta CS_{NE}}{\partial \mu} < 0 \Leftrightarrow \mu < \frac{m(n-m+1)}{n(m(n-m)+1)}$;

$\frac{\partial \Delta CS_{NE}}{\partial \sigma^2} = \frac{1}{2} n^2 \frac{(m(m-n)-1)^2}{m^2 (n-m+2)^2}$.

The higher is the σ^2 the greater is the CS^{AM}, the greater the ΔCS_{NE} is.

$\frac{\partial \Delta CS_{NE}}{\partial \alpha} = n^3 \frac{1-n\alpha}{(n+1)^2}$;

$\frac{\partial \Delta CS_{NE}}{\partial \alpha} > 0 \Leftrightarrow \alpha > \frac{1}{n}$, however α must be lower than $\frac{1}{n}$, otherwise firms do not produce before the merger.

- If $\mu > \frac{m}{n(2m-1)}$, then outsider firms will exit the market, hence the AA will accept the merger if $E\left[CS^{AM}\right] \geq CS^{BM}$, that is, if:

$$\Delta CS_E \equiv \frac{(n+1)^2 m^2 - mn\mu + n^2 (\sigma^2 + \mu^2)(n+1)^2 - 4m^2 n^2 (n\alpha - 1)^2}{8(n+1)^2 m^2} \geq 0.$$

$\frac{\partial \Delta CS_E}{\partial \mu} > 0 \Leftrightarrow \mu > \frac{m}{2n(n+1)^2}$;

$\frac{\partial \Delta CS_E}{\partial \alpha} = \frac{-16m^2 n^4 \alpha + 16 m^2 n^3}{2(n+1)^2 8m^2} = n^3 \frac{1-n\alpha}{(n+1)^2}$;

$\frac{\partial \Delta CS_E}{\partial \alpha} > 0 \Leftrightarrow \alpha < \frac{1}{n}$, which is always true.

$\frac{\partial \Delta CS_E}{\partial \sigma^2} = \frac{n^2 (n+1)^2}{8(n+1)^2 m^2} > 0.$

The higher is the σ^2 the greater the CS^M is and, thus, the greater is ΔCS_E. Hence, with high uncertainty on the level of efficiency gains, the greater is the likelihood that the AA accepts the merger to monopoly.

- If $\mu < \frac{m}{n(2m-1)}$, outsiders do not leave the market. Hence, firms will propose the merger if it is profitable, i.e., $E\left[\pi_I^{AM}\right] \geq m\pi^{BM}$:

$$\frac{m^2 - 2mn\mu((m-1)(m-n)+1) + n^2\left(\sigma^2+\mu^2\right)((m-1)(m-n)+1)^2}{m^2 (n-m+2)^2} \geq m\frac{(1-n\alpha)^2}{(n+1)^2}.$$

$$MP_{NE} \equiv \frac{m^2 - 2mn\mu((m-1)(m-n)+1) + n^2\left(\sigma^2+\mu^2\right)((m-1)(m-n)+1)^2}{m^2 (n-m+2)^2} - m\frac{(1-n\alpha)^2}{(n+1)^2} \geq 0.$$

$\frac{\partial MP_{NE}}{\partial \mu} = \frac{-2mn((m-1)(m-n)+1) + 2n^2 \mu((m-1)(m-n)+1)^2}{m^2 (n-m+2)^2}$.

$\frac{\partial MP_{NE}}{\partial \mu} > 0 \Leftrightarrow \mu > \frac{m}{n((m-n)(m-1)+1)}$ (always true).

$\frac{\partial MP_{NE}}{\partial \alpha} = 2nm\frac{1-n\alpha}{(n+1)^2}$.

$\frac{\partial MP_{NE}}{\partial \alpha} > 0 \Leftrightarrow \alpha < \frac{1}{n}$ (always true).

$\frac{\partial MP_1}{\partial \sigma^2} = \frac{n^2((m-1)(m-n)+1)^2}{m^2(n-m+2)^2} > 0$.

- If instead, $\mu > \frac{m}{n(2m-1)}$, outsider firms exit the market. Hence, firms will propose the merger to monopoly if it is profitable:

$$MP_E \equiv \frac{m^2 - 2mn\mu + n^2\left(\sigma^2 + \mu^2\right)}{4m^2} - m\frac{(1-n\alpha)^2}{(n+1)^2} \geq 0.$$

$\frac{\partial MP_E}{\partial \mu} = \frac{-2mn+2n^2}{4m^2}$.

$\frac{\partial MP_E}{\partial \mu} > 0 \Leftrightarrow \mu > \frac{m}{n}$.

$\frac{\partial MP_E}{\partial \alpha} = 2mn\frac{1-n\alpha}{(n+1)^2}$.

$\frac{\partial MP_E}{\partial \alpha} > 0 \Leftrightarrow \alpha > \frac{1}{n}$ this is not true since $\alpha < \frac{1}{n}$.

$\frac{\partial MP_E}{\partial \sigma^2} = \frac{n^2}{4m^2} > 0$.

Chapter 5

Does Vertical Integration Promote Downstream Incomplete Collusion? An Evaluation of Static and Dynamic Stability

This paper is published in Journal of Industry Competition and Trade, February 2014[1]

5.1 Introduction

The theoretical and empirical literature on vertical integration has gained a great contribution from researchers on industrial economics over many years. It has been very important for economists to understand the main determinants of vertical integration, to identify the type of transactions that are mediated within firms through vertical integration or conducted through

[1]This chapter benefited a lot from comments of anonymous reviewers from JICT and from the seminar participants at the 8th Seminar Day of the Doctoral Programme in Economics in Faculty of Economics of University of Porto and at the UECE Lisbon Meetings: Game Theory and Application in Lisbon, 2012. All of the comments greatly contributed to improving the final version of this chapter. Also, we would like to thank Professors António Brandão and Sofia Castro Gothen for their helpful comments and advices.

the market. In this paper we focus on the coordinated effects of vertical integration that have been ignored for several years. Chicago scholars emphasized the pro-competitive effects of vertical integration (Stigler, 1963, 1971). However, with the introduction of game-theory tools, post-Chicago scholars claimed that vertical integration could also have anti-competitive effects (Riordan, 2008; Riordan and Salop, 1995; Salop and Scheffman, 1987). Therefore, the analysis of the anticompetitive effects of vertical integration has been a debated topic of research in Industrial Organization. Sometimes, vertical integration is used as a strategy to eliminate the rivals from the market, allowing the achievement of greater efficiency and higher profits. Also, occasionally, firms adopt this type of organization in order to create barriers to entry, to increase market power and to facilitate collusion. Antitrust authorities have remained concerned that vertical integration might facilitate collusion at upstream and downstream levels (Riordan, 2008). Vertical integration might facilitate collusion by supporting the punishment and monitoring mechanisms and also by allowing agreement between firms. Vertical integration facilitates collusion if, after the merger, upstream or downstream firms are able to coordinate in a more effectively way than if they were operating separately. Further, vertical integration may increase the degree of symmetry between firms and the level of market transparency, which in turn makes it easier for firms to collude.

The main contribution of this paper is to highlight the importance of considering, simultaneously, two types of firms' strategies: vertical integration and collusion, in a context where some firms have no incentives to collude (incomplete collusion). This analysis is useful to antitrust authorities in order to more properly evaluate situations where vertical integration is used to enforce collusion, even when there are fringe firms involved and also to quantify its impacts on social welfare. When evaluating a vertical merger, usually, antitrust authorities focus on the coordinated effects, on the impacts on competition and on market foreclosure, which in turn, reduce social welfare (Ordover et al., 1990; Hart and Tirole, 1990). However, here we show that vertical merger and incomplete collusion can actually promote social welfare. Similar to Salinger (1988) we conclude that vertical integration does not necessarily result in market foreclosure of the unintegrated firms and that the vertical merger can eliminate the double marginalization problem, by decreasing the price of the final good.

There are a few real examples of situations where this question is important. For instance, in 2011, the UK's Office of Fair Trading (OFT) issued an infringement decision against the following supermarkets: Arla, Asda, Dairy Crest, McLelland (prior to its acquisition by Groupe Lactalis), Safeway (prior to its acquisition by Morrisons), Sainsbury's, Tesco, The Cheese Company and Wiseman. OFT found that the parties had infringed the Competition Act 1998 by coordinating increases in the final prices of certain dairy products (cheese, milk, butter) in 2002 and/or 2003.[2] Similarly, German antitrust authority reported that German retail gasoline firms such as Aral (BP), Esso (EXXonMobil), Shell, Total, Orlen, OMV, Agip (Eni), Avia and Westfalen were operating a cartel and coordinating their retail fuel prices (Bundeskartellamt, 2011). Moreover, in the report, German antitrust authority concluded that there was a "tremendous price transparency" in the market. However, German antitrust authority did not take into account the possibility that there was a casual relation between this emergence of the cartel and the vertical relationships in this sector. These are two important examples of downstream incomplete collusion, in industries characterized by significant vertical relations.

Further, in 2008, the European Commission (EC) has approved the proposed acquisition of Tele Atlas, a navigable digital map manufacturer, by TomTom, a retailer of portable navigation devices (EC, 2008). The EC was first concerned on whether the vertical merger would lead to a decrease of competition or increase of other retailers' costs and, consequently, harm consumers. However, EC did not take into account that the merged firm could increase retailers' ability to collude through sales of navigable digital maps.

In 2001, the Department of Justice (DOJ) challenged Premdor's acquisition of Masonite from International Paper Company. The DOJ was concerned that the vertical merger between Masonite, a manufacturer of molded doorskins and Premdor, a retailer of molded doors, would facilitate collusive pricing in the upstream market but also in the downstream market (DOJ, 2001).

In 1999, the U.S. Federal Trade Commission (FTC) was concerned that the merger between Barnes & Noble, the largest book retailer, and Ingram, the largest wholesaler, would

[2] See OFT (2011) for more details.

raise other retailers' costs and lead to foreclosure of this retailers from access to an important upstream suplier (FTC, 1999). FCT used these arguments to decline the acquisition, however there are other arguments, namely that the vertical merger could also increase coordination between retailers if, for instance, Barnes & Noble manipulated the wholesale prices charged to its' competitiors.

As we can see from the above examples, the link between vertical integration and downstream collusion is very important in real cases and the study of its effects using theoretical models will significantly contribute to understand the behavior of the distinct decision makers (firms, antitrust authorities, consumers, among others).

We could not find any academic literature that considered simultaneously both incomplete collusion and vertical integration. The related literature analyses either collusion and vertical integration or only incomplete collusion, separately. Therefore, our contribution is twofold. Firstly, our paper adds to the literature that studies how vertical integration facilitates downstream and upstream collusion. Chen (2001), Nocke and White (2007); Nocke and Whinston (2010) and Normann (2009) analyzed this question in relation to upstream collusion while, Chen and Riordan (2007) and Mendi et al. (2011) investigate the effects on downstream collusion. However, the referred works assumed that all the firms in the industry accept the collusive agreement.

Secondly, our paper adds to the literature on the sustainability of incomplete collusion. Despite the limited literature, there are some studies of incomplete cartels available such as: Selten (1973), D'Aspremont et al. (1983), Donsimoni et al. (1986), Martin (1993), Shaffer (1995), Escrihuela-Villar (2004, 2008). These works differ not only on the type of framework (dynamic or static) but also on the type of competition (price or quantities) and the type of game (simultaneous or sequential). However, in these studies, it is not assumed that the industry has a vertically integrated structure.

In this paper we combine the two issues to study how vertical integration affects downstream collusion when there are fringe downstream firms. In the static framework our paper is closely related to D'Aspremont et al. (1983) that studies the necessary internal and external conditions for static stability. More recently, Escrihuela-Villar (2008) studied how the number of firms in the cartel affects the possibility that its members can sustain a collusive agreement. Following D'Aspremont et al. (1983), Martin (1993), Thoron (1998), Escrihuela-Villar (2008) we also endogeneized the cartel formation by analyzing the number of firms that are willing to accept the collusive agreement. Also, our paper is close to Shaffer (1995)'s who analyzed the size and uniqueness of the stable cartel when the fringe is Cournot and the cartel behaves as a Stackelberg leader. Moreover, like Escrihuela-Villar (2009*b*) we assume that, when there is collusion, fringe firms choose the output that maximizes their profit, taking cartel firms' output as given. In the dynamic framework, our paper is closely related to Martin (1993) that analyzed the conditions for dynamic cartel stability when collusion is enforced with two alternative punishment strategies: a trigger strategy (Friedman, 1971) or a stick-and-carrot strategy (Abreu, 1986). Martin (1993) found that either a trigger strategy or a stick-and-carrot strategy sustain cartel joint profit maximization in the presence of a fringe. Furthermore, Escrihuela-Villar (2004) analyzed the effects of the cartel size on the sustainability of a collusive agreement and concluded that, with both trigger and stick-and-carrot strategies, collusion is easier to sustain the larger the cartel is.

In order to analyze the effects of vertical integration on downstream incomplete collusion, we construct three models: a model without vertical integration, a model where there is vertical integration with a cartel downstream firm and, finally, a model where there is vertical integration with a fringe downstream firm. In particular, we study each model alone and then we compare the results between these models. Moreover, we provide a numerical example to illustrate the results of these three theoretical models.

We find that, in general, a vertical merger with a cartel or a fringe downstream firm enforces collusion. The main reason is that the vertical merger increases the difference between collusive and NE payoffs. Additionally, the vertical merger with a fringe firm always promotes collusion because it also decreases the difference between deviation and collusive payoffs, and hence cartel firms have fewer incentives to deviate from the collusive agreement. However, for low downstream market concentration the opposite occurs when the vertical merger is with a cartel firm. In this case, the vertical merger increases the difference between deviation and collusive payoffs and therefore hinders collusion. Further, the vertical merger with a cartel or a fringe firm increase upstream competition and therefore the final result is an increase in the total wholesale quantity. Finally, a welfare analysis shows that social welfare can increase with the vertical mergers due to the partial elimination of double marginalization.

The remainder of the paper is organized as follows: Section 5.2 introduces the common assumptions of three models and analyzes the general static and dynamic stability conditions for the industry without vertical integration (Section 5.2.1), with vertical integration with a downstream cartel firm (Section 5.2.2) and with vertical integration with a fringe downstream firm (Section 5.2.3). Section 5.3 presents the welfare analysis and Section 5.4 discusses the results of both stability conditions obtained for the three models. These two sections are also based on the numerical simulation results. Finally, Section 5.5 concludes.

5.2 The Baseline Model

We consider an industry with two identical upstream firms ($N = 2$) and M symmetric downstream firms.[3] The upstream firms produce a homogeneous intermediate good at a constant marginal cost c and sell this good to the downstream firms. Downstream firms have the same technology for transforming one unit of the input into one unit of the output (production function with fixed proportions) and do so at a constant marginal cost w per unit, where w is also the input price. By assumption, we normalize all the other downstream costs to zero.

The upstream firms compete in quantities and make simultaneous and public offers to the downstream firms. For simplicity, we assume that the upstream firms simultaneously offer to the downstream firms a linear contract, where each downstream firm has to pay to the upstream firm the linear price w for each input quantity ordered. From the point of view of downstream firms, the market-clearing input price, w, is determined by equating the total amount of output supplied by the upstream firms with the demand of the downstream firms,

[3] Differently from Bos and Harrington (2010) we assume that, before and after the merger, the firms in the incomplete cartel are symmetric in order to focus on the vertical relationship between upstream and downstream firms and to isolate the collusive anticompetitive effect of vertical integration.

that is, w is the inverse demand function from downstream firms to upstream firms. Also, we assume that all downstream firms accept the contract. Then, downstream firms place their quantity orders, make their payments to upstream firms and compete in quantities to supply the downstream market.

The interaction between upstream and downstream firms is repeated forever and we assume that there is complete information. We analyze the stationary collusive equilibrium in which some downstream firms form an incomplete cartel. That is, there are F $(0 < F < M)$ firms that form the fringe and they have no incentives to collude and the remaining K $(0 < K < M)$ firms have incentives to collude. To the best of our knowledge in all the cartel and fringe literature [for instance Selten (1973); D'Aspremont et al. (1983); Donsimoni (1985); Donsimoni et al. (1986); Martin (1993); Shaffer (1995); Thoron (1998); Lofaro (1999); Konishi and Lin (1999); Posada (2001); Escrihuela-Villar (2009*b*,*a*)] it is assumed that the cartel behaves as a leader with respect to the fringe. Some of the literature cited above compared the Cournot and Stackelberg cases and concluded that when all firms simultaneously compete in quantities, the cartel is not stable.[4] The cartel is generally not profitable and hardly stable because cartel firms have incentives to deviate from the agreement due to the fact that fringe firms have incentives to free ride, by increasing their output.[5] However, when cartel acts as a leader usually it has relatively higher profits than in the simultaneous game and also leadership is more effective in enforcing an agreement and reducing the free-riding problem. Therefore, and following the result found by Shaffer (1995), that an "endogenous sequence of play between a stable cartel and a Cournot fringe will assign a leader's role to the cartel and a follower's role to the fringe" (Proposition 7 of Shaffer, 1995), we assume that the cartel behaves as a Stackelberg leader. Hence, fringe firms choose the output that maximizes their profit, taking the output of cartel firms as given.

[4]For the three models, we did the computations assuming that both cartel and fringe firms simultaneously choose quantities and we confirm that the cartel is not statically stable. Also, for models 1 and 2 and for all simulation cases there is no collusion under the simultaneous choice.

[5]This is similar to the Salant et al. (1983) horizontal mergers' paradox where with simultaneous game mergers are generally not profitable.

The inverse demand for the final consumption good is given by the linear function $P(Q) = \max(0, a - bQ)$, where $Q = Q_K + Q_F$ is the output produced by cartel and fringe firms, P is the output price. We assume that $a > c$.

The timing of the game is the following:

Stage 1. The upstream firms decide simultaneously the wholesale quantities.

Stage 2. The downstream firms set quantities in the retail market.

The game is solved by backward induction.

In order to assess if vertical integration promotes downstream collusion when there are fringe downstream firms, we compare the results obtained for three situations: without vertical integration (model 1), with vertical integration with a cartel downstream firm (model 2) and with vertical integration with a fringe downstream firm (model 3). For each model we analyze first the static and then the dynamic stability of the cartel. Following D'Aspremont et al. (1983) a cartel is statically stable if i) a member of the cartel earns at least as great a profit as it would have earned if it joins the fringe (internal stability condition - ISC) and ii) a fringe firm earns at least as great a profit by staying in the fringe as it would if it joins the cartel (external stability condition - ESC). Therefore, the cartel is statically stable if it is both internally and externally stable.

Moreover, we examine the dynamic stability of the cartel. We assume that downstream cartel firms compete repeatedly in an infinite number of periods (Time is discrete: $t = 1, 2, ...$) and seek to maximize the present discounted value of its profits, using a discount factor $\delta \in [0, 1]$. We follow the standard description of strategies as it is found in Escrihuela-Villar (2009*b*) and we assume that cartel firms adopt grim trigger strategies (Friedman, 1971).[6] Under these strategies the cartel firms stick to the collusive agreement until there is a defection, in which revert forever to the static Nash Equilibrium. Each cartel downstream firm produces the output $q_{k_j}^C$ and after deviating each cartel firm produces $q_{k_j}^{NE}$.

Regarding fringe firms, the optimal response consists of maximizing their current period's payoff, acting as Cournot quantity-setters. A fringe firm produces q_{f_l} if all downstream cartel

[6]More severe strategies have been studied by Abreu (1986), but focusing on the complete cartels case.

members accept the collusive agreement and $q_{f_l}^{NE}$ if cartel members have reverted to Cournot behavior. Under both cases, fringe downstream firms maximize their profits, given the outputs of other firms. However, when a downstream cartel firm deviates from the collusive agreement, fringe firms react to that deviation, maximizing their profits, taking into account the output of the deviating firm. We denote the profit function of a cartel downstream firm and of a fringe downstream firm, respectively, as π_k and π_f. As shown by Friedman (1971), cartel firms colluding in each period can be sustained as a Subgame Perfect Nash Equilibrium (SPNE) of the repeated game with the strategy profile defined above if and only if the Incentive Compatibility Constraint (ICC) is satisfied: $\frac{1}{1-\delta}\pi_k^C \geq \pi_k^d + \frac{\delta}{1-\delta}\pi_k^{NE}$, where π^d, π^C and π^{NE} are, respectively, the cartel downstream deviation, collusion and punishment (Cournot-Nash) profits.

For each model (without vertical integration, vertical integration with a cartel firm, vertical integration with a fringe firm) we find the optimal quantities and the corresponding profits in the three cases a) collusion in the downstream market, obtaining π^C b) Cournot competition in the downstream market, obtaining π^{NE} and c) deviation from the collusion by one firm, obtaining π^d.

In subsections 5.2.1, 5.2.2 and 5.2.3 we first present the three models in a general framework, that is, for any K and F downstream firms. Then, for each model, we provide a numerical simulation in order to improve the analysis of the effects of vertical integration in the dynamic stability of collusion. Here we assume a limited number of cartel and fringe firms ($K \in]0, 12]$ and $F \in]0, 10]$) because these represent the most significant real world cases of industries where the firms collude and it is difficult to have real cases with more than 12 firms in the cartel. Furthermore, some cartel and fringe firms numbers are removed from the analysis because they do not meet the stability conditions.

5.2.1 The Model without Vertical Integration

5.2.1.1 Collusive Results

We first analyze the downstream firms' decisions for any possible decision of the upstream firms. Then, considering the optimal decision of the downstream firms, we analyze the upstream firms' decisions.

Downstream firms determine the quantities that maximize their profits having as retail price P and as cost w (unit input price). The profit function of one representative downstream fringe (for instance, firm 1) is given by:

$$\pi_{f_1}^{D} = (a - b\sum_{l=1}^{F} q_{f_l} - b\sum_{j=1}^{K} q_{k_j} - w)q_{f_1}\text{, with } l = 1,..,F \text{ and } j = 1,..,K$$

where q_{k_j} and q_{f_l} represent, respectively, the outputs produced by each cartel downstream firm and each fringe downstream firm. By symmetry, all the fringe firms choose the same output and, therefore, the best reply function of each fringe firm is given by $q_f = \frac{a-w-bQ_K}{b(F+1)}$.

Acting as a Stackelberg leader against the downstream Cournot fringe firms, the cartel produces the monopoly output, $Q_K = \frac{a-w}{2b}$, and each member of the cartel produces $q_{k_j} = \frac{a-w}{2bK}$, with $j = 1,..,K$. Substituting Q_K into the best reply function of the fringe firms we obtain $q_{f_l} = \frac{a-w}{2b(F+1)}$, with $l = 1,..,F$. The total output in the downstream market and the price are given by $Q\left(w\right) = \frac{(a-w)(2F+1)}{2b(F+1)}$ and $P\left(w\right) = \frac{a+w(2F+1)}{2(F+1)}$, respectively. The total quantity of the downstream market is also the quantity bought from upstream firms. Then, the inverse demand function for the upstream firms is given by $w\left(Q\right) = \frac{a(2F+1)-2bQ(F+1)}{2F+1}$.

Hence, the upstream firms determine the quantities that maximize their profits, where the wholesale price is $w\left(Q\right)$ and the cost is c. The individual profit function for an upstream firm is given by:

$$\pi_i^U = (w-c)q_i^U = (\tfrac{a(2F+1)-2bQ(F+1)}{2F+1} - c)q_i^U \text{ with } i = 1,2 \text{ and } Q = q_1^U + q_2^U$$

By symmetry $q_1^U = q_2^U$, then the quantity that maximizes the individual profit of the upstream firm is $q_i^U = \frac{A(2F+1)}{6(F+1)}$ and the total output is $Q = \frac{A(2F+1)}{3(F+1)}$, where $A = \frac{a-c}{b}$.

Considering the above results, the equilibrium upstream and downstream prices are $w(c) = \frac{a+2c}{3}$ and $P(c) = \frac{a(2+F)+c(2F+1)}{3(F+1)}$.[7] The equilibrium downstream cartel and fringe quantities are $q_{k_j} = \frac{A}{3K}$ and $q_{f_l} = \frac{A}{3(F+1)}$, respectively.

The profits are the following:

- $\pi_i^U = \frac{A^2 b(2F+1)}{18(F+1)}$, for upstream firms;
- $\pi_{k_j}^D(K,F) = \frac{A^2 b}{9K(F+1)}$, for downstream cartel firms; and
- $\pi_{f_l}^D(K) = \frac{A^2 b}{9(F+1)^2}$, for downstream fringe firms.

Before the vertical merger, both upstream firms are setting the wholesale price (w) above their marginal costs (c) because they have market power. Also, downstream firms that will buy from upstream firms will then set the retail price (P) above their marginal costs (w). Therefore, there is a double marginalization problem because the input is being market up above marginal cost twice $(P > w > c)$, which harms social welfare.

Following D'Aspremont et al. (1983) in order to prove that the cartel is statically stable we have to verify both internal and external stability conditions.

∘ **Internal Stability Condition (ISC)**: a member of the cartel earns at least as great a profit as it would have earned if it joins the fringe: $\pi_k(K,F) \geq \pi_f(F+1)$. For this particular case, we get: $\frac{A^2 b}{9K(F+1)} \geq \frac{A^2 b}{9(F+2)^2} \Leftrightarrow K \leq \frac{(F+2)^2}{F+1}$.

∘ **External Stability Condition (ESC)**: a fringe firm earns at least as great a profit by staying in the fringe as it would if it join the cartel: $\pi_k(K+1, F-1) \leq \pi_f(F)$. For this particular case, we get: $\frac{A^2 b}{9F(K+1)} \leq \frac{A^2 b}{9(F+1)^2} \Leftrightarrow K \geq \frac{(F+1)^2}{F} - 1$.

Proposition 5.1. [Martin, 1993][8] Without vertical integration, downstream collusion is statically stable if and only if:

$$\frac{(F+1)^2}{F} - 1 \leq K \leq \frac{(F+2)^2}{F+1}$$

As we can see, without vertical integration the profits are the same for both upstream

[7] As we can see, the equilibrium input price is a function of the upstream firms' cost and then we always have that the price charged by the upstream firms is always greater than their costs, $w > c$.

[8] This is the same result as in Martin (1993)'s book.

firms. [9] Further, following Thoron (1998), in a symmetric framework, the definition of stable cartel is in fact the definition of stable cartel size. Therefore, we conclude that the cartel of K firms is stable if and only if, for all $F \geq 1$ fringe firms, there are $F+2$ or $F+3$ cartel firms.[10] Then, the cartel should have, respectively, $50\% - 75\%$ or $50\% - 80\%$ firms in the industry.[11]

To verify that the cartel is dynamically stable we also need to obtain the Cournot and the deviation results.

5.2.1.2 Cournot Equilibrium

The punishment profit is the downstream profit obtained under the static Nash-Cournot Equilibrium. The profits for downstream firms are given by:[12]

$$\pi^D_{j=l} = \frac{4A^2b}{9(F+K+1)^2},\ j = 1,..,K \text{ and } l = 1,..,F.$$

The Cournot Equilibrium results are, somehow, representative of the no collusive results. Comparing the Cournot profit with the collusive profit of a cartel firm, it is possible to see that the collusive profit is higher than the Cournot profit for $K > F+1$, which is compatible with

[9]Comparing the cartel's individual profit with fringe's individual profit we conclude that the first is higher than the latter if and only if the number of cartel firms is lower than $F+1$. However, as we show above, this is incompatible with the static stability conditions. Therefore, although the difference is small, the individual profit of a cartel firm is lower than the individual profit of a fringe firm. This result is the same in the three models. However, the comparison of cartel's and fringe's profits is carried out on the endogenous cartel formation's literature which is far beyond the scope of this paper. For further details on endogenous cartel formation see Selten (1973), Donsimoni et al. (1986), Escrihuela-Villar (2008), Escrihuela-Villar (2009*b*), Bos and Harrington (2010), among others.

[10]This conclusion is obtained from the simulation results with $K \in 3,..,12$ and $F \in 1,..,10$. We consider that this simulation represents the most significant cases of collusion, as it is difficult to have real cases with more than 12 firms in the cartel. For more details see Appendix D.1.1.

[11]This result is similar to Salant et al. (1983) that indicate that mergers are only likely to occur when 80% of the firms in a market are included in the merger.

[12]For a detailed description of the results with Cournot competition in the downstream level see Appendix D.1, and Appendix D.2.2.

the static stability conditions. Then, the cartel firms have always incentives to form a cartel. We also verify that the individual profit of a fringe firm is higher than a cartel firm. Then, fringe firms have incentives to free-riding and prefer that their rivals fom the cartel in order to have higher profit.[13] Moreover, when we move from Cournot to the collusive scenario, the profitability of the cartel under collusion depends on two opposite forces: on the one hand the cartel firms produce lower quantity and thereby, increase its profits. On the other hand, the fringe firms react by increasing their quantity which reduces the profitability of the cartel firms. In this case the second effect dominates the first, which explains why the profit of cartel firm, under collusion, is lower than the profit of a fringe firm.

5.2.1.3 Deviation Results

Consider that one firm from the cartel, firm D_{k_1} for example, deviates while the others downstream cartel firms maintain their quantities and the fringe firms react to the deviation. In this case the downstream profits are:[14]

- $\pi_{k_1}^{d} = \frac{A^2 b(K+1)^2}{36K^2(F+1)}$, for the deviation downstream cartel firm;
- $\pi_{k_j}^{D} = \frac{A^2 b(K+1)}{18K^2(F+1)}$, for the $j = 2, ..., K$ downstream cartel firms; and
- $\pi_{f_l}^{D} = \frac{A^2 b(K+1)^2}{36K^2(F+1)^2}$, for the $l = 1, .., F$ downstream fringe firms.

By deviating from the collusive agreement, the deviating cartel downstream firm gains higher profit than if it was in collusion, in part due to the high deviation quantity. The remaining cartel downstream firms continue to produce the same collusive quantity, however, fringe firms, by reacting to the deviation, reduce their quantity. Moreover, both cartel and fringe firms earn a lower profit, than if there was collusion.

To study the dynamic stability of collusion we replace the deviation, collusion and punishment profits in the ICC and get the critical discount factor for collusion at downstream level with fringe downstream firms, when there is no vertical integration. The result is sum-

[13]This result is similar to Salant et al. (1983) horizontal mergers' paradox already known in the literature and it is present in the three models of the paper.

[14]For a detailed description of the results with deviation from the cartel see Appendix D.1.2.

marized in Proposition 5.2.

Proposition 5.2. Without vertical integration, downstream collusion is supported as a SPNE if and only if:

$$\delta \geq \frac{(K-1)^2(F+K+1)^2}{(F+K+1)^2(K+1)^2-16K^2(F+1)} \equiv \delta^*_{withoutVI}$$

Table 5.1 displays the values of K and F (with $K \in]0,12]$ and $F \in]0,10]$), obtained by numeric simulation, that satisfy both static (ESC and ISC) and dynamic stability conditions.

Table 5.1: Static and Dynamic Stability Results - Model 1

K	F	$\delta^*_{withoutVI}$	K	F	$\delta^*_{withoutVI}$
3	**1**	0.892857	**8**	**6**	0.997106
4	**1**	0.835052	**9**	**6**	0.991288
4	**2**	0.964989	**9**	**7**	0.998057
5	**2**	0.927536	**10**	**7**	0.993940
5	**3**	0.984802	**10**	**8**	0.998634
6	**3**	0.963020	**11**	**8**	0.995619
6	**4**	0.992129	**11**	**9**	0.999003
7	**4**	0.978852	**12**	**9**	0.996732
7	**5**	0.995419	**12**	**10**	0.999251
8	**5**	0.986848	...	...	...

With a constant number of cartel firms, if F increases, then the region above which collusion is sustainable (δ) decreases which imply that $\delta^*_{withoutVI}$ is higher. Therefore, an increase in the number of fringe firms hinders collusion.

$$\frac{\partial\delta}{\partial F} = \frac{-16K^2(K-1)^2(F-K+1)(F+K+1)}{((F+K+1)^2(K+1)^2-16K^2(F+1))^2} > 0$$

Also, in a constant the number of fringe firms, if K increases, then the region above which collusion is sustainable (δ) increases which implies that $\delta^*_{withoutVI}$ is lower. Therefore, an increase in the number of cartel firms enforces collusion.

$$\frac{\partial \delta}{\partial K} = \frac{4(K-1)(F-K+1)(F+K+1)\left(\left(2K^2+1\right)(2F+1)+K\left(F^2-K^2-3\right)+(F-K)^2\right)}{\left((F+K+1)^2(K+1)^2-16K^2(F+1)\right)^2} < 0$$

By comparing these results with those obtained with full collusion $(F = 0)$ we conclude that when a cartel faces outsider competition it has a limited ability to charge a price above competition levels. Hence, the collusive prices, under incomplete collusion are lower than under complete collusion. Also, this explains why when the cartel is incomplete, collusion is harder to sustain the higher the number of fringe firms is.

5.2.2 The Model of Vertical Integration with a cartel member

Here we assume that there is vertical integration between one downstream cartel firm (D_{k_1}, for instance) and one upstream firm (U_1, for instance). The total profit for the vertically integrated firm now depends on both upstream and downstream activities.

5.2.2.1 Collusive Results

We assume, as in the model without vertical integration, that the cartel firms behave as a Stackelberg leader and the fringe firms as followers. Additionally, we assume that although firm D_{k_1} is vertically integrated with firm U_1, it has autonomy from the parent firm regarding the quantity decision. Therefore, firm D_{k_1} is allowed to collude with some rivals considering only the downstream profits. Here we are considering a specific type of vertical integration where the upstream firm is the parent firm that maximizes the total profit while the downstream firm decides considering only the downstream profits. This assumption captures, for example, what happens when there is functional separation between upstream and downstream.[15] With functional separation the integrated downstream firm's operations are set apart, legally, organizationally and/or physically from its upstream firm.[16]

[15] For instance, in UK telecommunications industry, the downstream firm Openreach is run and managed separately from the economic group British Telecom (Whalley and Curwen, 2008). Openreach is an example of functional separation.

[16] We have chosen this type of vertical separation for several reasons. This type of vertical separation allows the elimination of the double marginalization problem. Also, it decreases integrated upstream firm's incentives

Further, due to the type of upstream-downstream's relationship we assume that only downstream firms will be involved in the punishment strategies.[17]

Under the above assumptions we obtain for the downstream market the same results as in the model without vertical integration. The difference from the previous model is due to the different decision process of the parent firm U_1. At Stage 1, firm 1 maximizes the total profit that includes the profits from the upstream and downstream activities. Hence, the profit function of firm U_1 is:

$$\pi_1^{VI^K} = (w-c)q_1^U + (a - bQ - w)q_{k_1}$$

Firm U_2 has the same profit function as in the model without vertical integration. Both upstream firms decide, simultaneously, the upstream quantity that maximizes each individual profits.[18] Therefore, the equilibrium profits are given by:

- $\pi_1^{VI^K} = \frac{A^2 b(2F+1)[K(2F+1)(K+2FK+2)-1]}{2(F+1)B^2}$, for the integrated firm, where $A = \frac{a-c}{b}$ and $B = 3K + 6FK - 1$;
- $\pi_2^U = \frac{A^2 b(2F+1)(K+2FK-1)^2}{2(F+1)B^2}$, for the non-integrated upstream firm;
- $\pi_{k_j}^D = \frac{A^2 bK(2F+1)^2}{(F+1)B^2}$, for $j = 1, .., K$ downstream cartel firms; and
- $\pi_{f_l}^D = \frac{A^2 bK^2(2F+1)^2}{(F+1)^2 B^2}$, for $l = 1, .., F$ downstream fringe firms.

From the numerical results[19] we verify that there is an incentive for vertical integration because the total profit of D_{k_1} and U_1 is higher than the independent upstream. The incentive to vertically integrate is also due to the fact that the total profit of D_{k_1} and U_1 is higher than the sum of the profits of D_{k_1} and U_1 if there was no vertical integration. Although the profit

to discriminate in favor of its downstream firm, it decreases the incentives to use the upstream price to raise rivals' costs or to foreclosure the rivals from the market. Further, functional vertical separation restores firm's incentives to compete fiercely in the downstream market and does not create asymmetries in this market. Hence, with this type of vertical separation we can better isolate the collusive anticompetitive effect of the vertical merger.

[17]This is different from Nocke and White (2007)'s paper where the authors analyzed a simplified one-stage game with both upstream and downstream firms involved in the punishment strategies.

[18]For a more detailed explanation of the derivations see Appendix D.1.3.

[19]See Appendix D.2.3.

of the independent upstream firm is lower than in the case without vertical integration, the profits for the remaining downstream firms (cartel and fringe) are now higher. Also, when there is vertical integration with a cartel firm, both wholesale and retail prices decrease and the total quantity increase. Then, we conclude that the vertical merger with a cartel firm decreases the double marginalization problem.

To verify that the cartel is statically stable we analyze both internal and external stability conditions (D'Aspremont et al., 1983) and the conclusions are summarized by Proposition 5.3.

Proposition 5.3. When U_1 and D_{k_1} are vertically integrated, downstream collusion is statically stable if and only if:

∘ **Internal Stability Condition:**[20]

$$\frac{K(2F+1)^2}{(F+1)B^2} \geq \frac{(2F+3)^2(K-1)^2}{(F+2)^2(6F-9K-6FK+10)^2}$$

∘ **External Stability Condition:**

$$\frac{(K+1)(2F-1)^2}{F[3(K+1)(2F-1)-1]^2} \leq \frac{K^2(2F+1)^2}{(F+1)^2B^2}$$

Using the results from the numerical simulation[21] we conclude that for this model, collusion is sustainable if and only if there are $F+2$ or $F+3$ cartel firms and if $K > 3$. This is the same result as in the model without vertical integration, however, in this case, there is a lower bound for the number of cartel firms. If the number of firms in the cartel is small, for instance $K = 3$, their profits attract fringe firms. Consequently, fringe firms find it desirable to join the cartel, making the cartel statically unstable. However, with more firms in the cartel, the individual profit obtained by a fringe firm when it stays out of the cartel is higher than the profit that it would get if it joins the cartel, which ensures the external static stability. Hence,

[20]Relatively to the ISC, here we have two situations: both downstream integrated and non-integrated cartel firms can deviate to the fringe if their profits is not as great as they would have if they join the fringe. However, due to the type of vertical relationship in this model, the vertical merger does not lead to foreclosure nor create asymmetries in downstream market. Hence, both integrated and non-integrated cartel firms have the same profit and therefore we only have one ISC.

[21]See Appendix D.2.1.

we have to impose the lower bound above otherwise the ESC would not be satisfied and the cartel would not be statically stable.

5.2.2.2 Cournot equilibrium

When all downstream firms decide their quantities à Cournot we obtain the following profits in equilibrium:[22]

- $\pi_1^{VI^K} = \frac{A^2 b[(F+K)\left((F+K+1)^2(F+K)^2-4\right)+4(F+K)^2(F+K+1)]}{(F+K+1)E^2}$, for the integrated firm, where $E = 3(F+K+1)(F+K) - 2$;

- $\pi_2^U = \frac{A^2 b(F+K)(F+K-1)^2(F+K+2)^2}{E^2(F+K+1)}$, for the non-integrated upstream firm; and

- $\pi_{f_l=k_j}^D = \frac{4A^2 b(F+K)^2}{E^2}$, for $j = 1,..,K$ and $l = 1,..,F$ downstream firms.

The Cournot results also represent the results without collusion. Then, under Cournot the profits of the vertical merger but also of downstream cartel and fringe firms are now lower than under collusion due to the increase of Cournot quantity and the decrease of retail price. Comparing the Cournot results with those obtained without vertical integration we conclude that the cartel downstream profits are now higher than before. Also, under Cournot, both retail and wholesale prices are lower than in the first model.

5.2.2.3 Deviation Results

Regarding the deviation from the agreement in the downstream market we analyze two different cases: 1) when the deviating firm is the firm D_{k_1} that is vertically integrated; 2) when the deviating firm is one of the non-integrated colluding firms, for instance firm D_{k_2}. As before we assume that the fringe firms react to the deviation.

For the first case the profits are the following:[23]

- $\pi_1^{VI^K} = \frac{A^2 bH[(-4K+8FK^2+5K^2-1)(8FK^2+7K^2+1)+4KH(K+1)^2]}{4K(F+1)G^2}$, for the integrated firm, where $H = 3K + 4FK - 1$ and $G = -8K + 24FK^2 + 17K^2 - 1$.

[22] Appendix D.1.4. presents the detailed description of the results with Cournot competition in the downstream market.

[23] Appendix D.1.5 presents the detailed description of the results with deviation from the agreement for both cases.

- $\pi_2^U = \frac{A^2bH(-4K+8FK^2+5K^2-1)^2}{4K(F+1)G^2}$, for the non-integrated upstream firm;
- $\pi_{k_1}^d = \frac{A^2b(K+1)^2H^2}{(F+1)G^2}$, for the deviation downstream cartel firm;
- $\pi_{k_j}^D = \frac{2A^2b(K+1)H^2}{(F+1)G^2}$, for the $j = 2, .., K$ downstream cartel firms; and
- $\pi_{f_l}^D = \frac{A^2b(K+1)^2H^2}{(F+1)^2G^2}$, for the $l = 1, .., F$ downstream fringe firms.

By deviating from the collusive agreement, the integrated cartel downstream firm earns higher profit than if it was in collusion, therefore it has incentives to deviate from the agreement. Also, the total profit of the vertical integrated firm and the profit of the independent upstream firm are higher than under collusion or without vertical integration. The profits of the independent cartel firms and of fringe firms are lower than if they were in collusion.

Thereafter we present the constraint on the discount factor that ensures the dynamic stability of collusion. When U_1 and D_{k_1} are vertically integrated, downstream collusion is dynamically stable if and only if:

$$\delta \geq \frac{E^2[(K+1)^2H^2B^2-K(2F+1)^2G^2]}{B^2[(K+1)^2H^2E^2-4(F+K)^2(F+1)G^2]} \equiv \delta_{D_{k_1}}$$

Under the second case D_{k_2} deviates from the collusive agreement, assuming that D_{k_j}, with $j = 1, .., K \setminus \{2\}$ produce the collusive quantities. The profits are the following:

- $\pi_1^{VIF} = A^2b\frac{2K(K+1)H^2+H\left(-2K+4FK^2+3K^2-1\right)\left(4FK^2+3K^2+1\right)}{4K(F+1)I^2}$ for the integrated firm, where $I = -4K + 12FK^2 + 9K^2 - 1$;
- $\pi_2^U = \frac{A^2bH(-2K+4FK^2+3K^2-1)^2}{4K(F+1)I^2}$, for the non-integrated upstream firm;
- $\pi_{k_2}^d = \frac{A^2bH^2(K+1)^2}{4(F+1)I^2}$, for the deviation downstream cartel firm;
- $\pi_{k_j}^D = \frac{A^2b(K+1)H^2}{2(F+1)I^2}$, for the $j = 1, .., K \setminus \{2\}$ downstream cartel firms; and
- $\pi_{f_l}^D = \frac{Ab^2H^2(K+1)^2}{4(F+1)^2I^2}$, for $l = 1, .., F$ downstream fringe firms.

By deviating from the collusive agreement, the non-integrated cartel downstream firm obtains higher profit. However, also the vertical integrated firm's profit and the independent upstream firm's profits are higher. The remaining cartel firms' profits and the fringe firms' profits are lower.

Subsequently we present the constraint on the discount factor that ensures dynamic stability of collusion, in the second case. When U_1 and D_{k_1} are vertically integrated, downstream collusion is dynamically stable if and only if:

$$\delta \geq \frac{E^2[(K+1)^2H^2B^2-4K(2F+1)^2I^2]}{B^2[(K+1)^2H^2E^2-16(F+K)^2(F+1)I^2]} \equiv \delta_{D_{k_2}}$$

Proposition 5.4 summarizes the necessary conditions of dynamic stability of the cartel when there is vertical integration with a cartel firm.

Proposition 5.4. Under vertical integration with a cartel firm, collusion is dynamically stable if and only if:

$$\delta \geq \max(\delta_{D_{k_1}}, \delta_{D_{k_2}}) \equiv \delta^*_{withVI^K}$$

From numeric simulation, we verify that $\delta_{D_{k_1}}$ is always higher than $\delta_{D_{k_2}}$. Although the difference between the cartel and the Nash equilibrium profits is the same for both cases, the difference between the deviation and the cartel profits is higher when the integrated cartel firm deviates from the agreement. Hence, the integrated firm has lower incentives to collude and higher incentives to deviate from the agreement and therefore collusion is harder to sustain in this case. The opposite occurs for the non-integrated firm. The main reason which explains this result is that the integrated upstream firm takes into account the profit of its downstream firm when deciding the quantities. Then, if it is the integrated downstream cartel firm deviating from the collusive agreement, this directly affects the profit function of the integrated upstream firm which, in turn, influences upstream quantities' decisions that will affect downstream collusion. However, if it is a non-integrated downstream cartel firm that deviates from the agreement, this does not directly affect the profit function of the integrated upstream firm. Hence, comparing the cutoffs obtained for both situations we conclude that $\delta^*_{withVI^K} = \delta_{D_{k_1}}$.

The values of the critical discount factors above which collusion is sustainable and the results for the static stability conditions, for different numbers of F and K are shown on Table 5.2:

Table 5.2: Static and Dynamic Stability Results - Model 2

K	F	$\delta^*_{withVI^K}$	K	F	$\delta^*_{withVI^K}$
3	**1**	0.882065*	**8**	**6**	0.997111
4	**1**	0.831454	**9**	**6**	0.991340
4	**2**	0.963883	**9**	**7**	0.998063
5	**2**	0.927391	**10**	**7**	0.993976
5	**3**	0.984628	**10**	**8**	0.998638
6	**3**	0.963152	**11**	**8**	0.995644
6	**4**	0.992100	**11**	**9**	0.999007
7	**4**	0.978966	**12**	**9**	0.996750
7	**5**	0.995419	**12**	**10**	0.999253
8	**5**	0.986926	...	...	...

*For this value the ESC is not satisfied.

Analyzing Table 5.2 we conclude that, in order for collusion to be sustainable both statically and dynamically, we have to impose a lower bound for the number of cartel firms ($K > 3$). As we can see, for $K = 3$ and $F = 1$ the external stability condition (ESC) is not satisfied and therefore we do not have static stability. Once again, collusion is easier to sustain the higher the number of cartel firms is and the lower the number of fringe firms is.

Comparing the results obtained by Proposition 5.4 with those obtained with Proposition 5.2 (see Table 5.1) we conclude that, for some level of market concentration, the vertical merger with a cartel firm promotes collusion. However, due to the merger, collusion is less easily sustained when the concentration in the market is low enough.

5.2.3 The Model of Vertical Integration with a fringe firm

Here we assume that there is vertical integration between one downstream fringe firm (D_{f_1}, for instance) and one upstream firm (U_1, for instance).

5.2.3.1 Collusive Results

Again, we assume that although firm D_{f_1} is vertically integrated with firm U_1, it has autonomy from the parent firm regarding the quantity decision.

The profit function of firm U_1 is:

$$\pi_1^{VIF} = (w - c)q_1^U + (a - bQ_K - bQ_F - w)q_{f_1}$$

Firm U_2 has the same profit function as in the model without vertical integration. Both upstream firms decide, simultaneously, the upstream quantity that maximizes each individual profits.[24] Therefore, the equilibrium profits are given by:

- $\pi_1^{VIF} = A^2 b \frac{2(F+1)(2F+1)^2 + F(2F+1)(2F+3)\left(3F+2F^2+2\right)}{2(F+1)J^2}$, for the integrated firm, where $J = 9F + 6F^2 + 2$;
- $\pi_2^U = \frac{a^2 b F^2 (2F+1)(2F+3)^2}{2(F+1)J^2}$, for the non-integrated upstream firm;
- $\pi_{k_j}^D = \frac{A^2 b (F+1)(2F+1)^2}{KJ^2}$, for $j = 1, .., K$ downstream cartel firms; and
- $\pi_{f_l}^D = \frac{A^2 b (2F+1)^2}{J^2}$, for $l = 1, .., F$ downstream fringe firms.

Looking at the numerical results[25] we conclude that the profit of the vertically integrated firm is always higher than the profit of the independent upstream firm U_2. Once again, there is an incentive for firms to vertically integrate because the total profit of the vertically integrated firm is higher than the sum of the profits of D_{f_1} and U_1 without vertical integration. The profits of the fringe and cartel firms are higher than in the case without vertical integration. That is, when there is vertical integration with a fringe firm, cartel firms have higher incentives to collude and as we can see from the simulation, their profits are higher than

[24] For a more detailed explanation of the derivations see Appendix D.1.6.

[25] See Appendix D.2.4.

the Cournot case or than the case where there is no vertical integration. Also, the vertical merger with fringe firm reduces both wholesale and retail prices and increases the total quantity in the market. Hence, as in the previous model, the vertical merger reduces the double marginalization problem.

Once again to verify that the cartel is statically stable we have to ensure both internal and external stability conditions:

∘ **Internal Stability Condition:**

$$K \leq \frac{(2F+1)^2(F+1)\left(21F+6F^2+17\right)^2}{(2F+3)^2 J^2}$$

∘ **External Stability Condition**:

$$K \geq \frac{F(2F-1)^2 J^2}{(-3F+6F^2-1)^2(2F+1)^2} - 1$$

Proposition 5.5. When U_1 and D_{f_1} are vertically integrated, downstream collusion is statically stable if and only if:

$$\frac{F(2F+1)^2 J^2}{(-3F+6F^2-1)^2(2F+1)^2} - 1 \leq K \leq \frac{(2F+1)^2(F+1)\left(21F+6F^2+17\right)^2}{(2F+3)^2 J^2}$$

Using the results from the numerical simulation[26] we conclude that collusion is sustainable if and only if there are $F+2$ or $F+3$ cartel firms and if $F > 1$. This is the same result as in the model without vertical integration, however here there is a lower bound for the number of fringe firms. If there is only one fringe firm ($F = 1$), this firm finds it desirable to join the cartel, making the cartel statically unstable. However, with more firms in the fringe, the individual profit obtained by a fringe firm when it stays out of the cartel is higher than the profit that it would get if it joins the cartel. Hence, fringe firms have no incentives to join the cartel and therefore the external static stability condition is ensured. Then, in order to satisfy the ESC and ensure the static stability of the cartel we impose the lower bound above.

[26] See Appendix D.2.1.

5.2.3.2 Cournot equilibrium

The Cournot results are the same as in the previous model because at the downstream market all firms have the same behavior.

5.2.3.3 Deviation Results

Considering that, for instance firm D_{k_1} deviates from the collusive agreement and, as before, the fringe firms react to the deviation, the profits are the following:[27]

- $\pi_1^{VIF} = A^2 b \frac{4K(F+1)(K+1)^2H^2 + (K+2FK-1)(5K+4FK+1)\left(14FK^2+8F^2K^2-2FK+7K^2+1\right)H}{4KL^2(F+1)}$, for the integrated firm, with $L = -8K + 42FK^2 + 24F^2K^2 - 6FK + 17K^2 - 1$;
- $\pi_2^U = A^2 b \frac{H(K+2FK-1)^2(5K+4FK+1)^2}{4KL^2(F+1)}$, for the non-integrated upstream firm;
- $\pi_{k_1}^d = A^2 b \frac{(F+1)(K+1)^2H^2}{L^2}$, for the deviation downstream cartel firm;
- $\pi_{k_j}^D = 2A^2 b \frac{(F+1)(K+1)H^2}{L^2}$, for the $j = 2, .., K$ downstream cartel firms; and
- $\pi_{f_l}^D = A^2 b \frac{H^2(K+1)^2}{L^2}$, for the $l = 1, .., F$ downstream fringe firms.

By deviating from the collusive agreement, the cartel firm earns higher profit. However, if the firm does not deviate it will earn lower profit than if there were no vertical integration. Then the incentives to deviate from the collusive agreement are lower. The constraint described by Proposition 5.6 ensure that collusion is dynamically stable.

Proposition 5.6. When U_1 and D_{f_1} are vertically integrated, downstream collusion is dynamically stable if and only if:

$$\delta \geq \frac{E^2\left[(F+1)(K+1)^2H^2KJ^2 - (F+1)(2F+1)^2L^2\right]}{KJ^2\left[(F+1)(K+1)^2H^2E^2 - 4(F+K)^2L^2\right]} \equiv \delta^*_{withVIF}$$

Table 5.3 shows the values of the critical discount factors above which collusion is sustainable and the results of the static stability conditions, for different number of F and K firms:

[27] Appendix D.1.7. presents the detailed description of the results with deviation.

Table 5.3: Static and Dynamic Stability Results - Model 3

K	**F**	$\delta^*_{withVI^F}$	**K**	**F**	$\delta^*_{withVI^F}$
3	**1**	0.744721*	**8**	**6**	0.996443
4	**1**	0.724597*	**9**	**6**	0.990245
4	**2**	0.940123	**9**	**7**	0.997678
5	**2**	0.901072	**10**	**7**	0.993323
5	**3**	0.977686	**10**	**8**	0.998401
6	**3**	0.953961	**11**	**8**	0.995231
6	**4**	0.989391	**11**	**9**	0.998853
7	**4**	0.974993	**12**	**9**	0.996476
7	**5**	0.994154	**12**	**10**	0.999149
8	**5**	0.984944	...	...	...

* For these values the ESC is not satisfied.

From the results in Table 5.3 we conclude the cartel is not statically stable for $F = 1$ because the external stability condition is not satisfied. Then, only when there are $F+2$ or $F+3$ and when $F > 1$ collusion is stable. Again, collusion is more difficult to sustain the lower the number of cartel firms is and the higher the number of fringe firms is. Further, comparing the results obtained with Proposition 5.6 with those obtained in Proposition 5.2 (see Table 5.1) it is possible to conclude that vertical merger with a fringe firm always promotes collusion.

5.3 Welfare analysis

In this section we discuss the social welfare implications of the three models.

When there is vertical integration we can have two main effects. On one hand, we have the collusive effects, that is, vertical integration increases the incentives for downstream firms to collude. With high incentives to collude, downstream firms will increase the retail price (P) and decrease the total quantity (Q), which harms the social welfare. On the other hand, vertical integration has efficiency effects: with the vertical merger the total retail quantity increases and the retail price decreases, when comparing with the situation without the vertical merger (model 1).[28] Also, the vertical merger increases upstream competition which decreases the wholesale price (w). Further, we find that the vertical integration (models 2 and 3) reduces the problem of double marginalization, but it does not solve it because upstream firms are still setting the input price above their marginal cost. The reduction of the double marginalization problem could also be obtained with other vertical restraints (for instance, non-linear prices, royalties, resale price maintenance, among others) or even without collusion (for instance, under Cournot). We find that the reduction of the double marginalization problem triggered by the vertical merger is higher in the presence of collusion than without it. Also, in this paper we assume that there are only two upstream firms. We believe that the double marginalization problem could be totally eliminated if we increase the number of upstream firms.

We conclude that the positive effects more than compensate the negative effects which explains why there is an improvement in welfare when we move from model 1 to models 2 and 3.

Table 5.4 displays the differences of social welfare (SW) between models 1 and 2 and between models 1 and 3, decomposing it into the variation of consumer surplus (CS) and producer surplus (PS).

[28] In the comparison of prices and quantities for the three models, we assume that $a = 1$, $b = 1$ and $c = 0.5$. We verify that changes in the values of the parameters a, b and c do not modify the discussion of the results among the three models nor the variation of social welfare. More details on the calculations can be provided upon request to the authors.

Table 5.4: Welfare analysis

		Model 1 vs Model 2			Model 1 vs Model 3		
K	**F**	△ PS %	△ CS %	△ SW %	△ PS %	△CS %	△ SW %
3	**1**	-	-	-	-	-	-
4	**1**	-0.08%	5.80%	1.88%	-	-	-
4	**2**	-0.46%	3.42%	1.03%	-0.63%	4.60%	1.38%
5	**2**	-0.36%	2.72%	0.82%	-0.63%	4.60%	1.38%
5	**3**	-0.40%	1.93%	0.56%	-0.50%	2.42%	0.70%
6	**3**	-0.33%	1.61%	0.47%	-0.50%	2.42%	0.70%
6	**4**	-0.32%	1.25%	0.35%	-0.38%	1.50%	0.42%
7	**4**	-0.27%	1.07%	0.30%	-0.38%	1.50%	0.42%
7	**5**	-0.25%	0.87%	0.24%	-0.29%	1.02%	0.28%
8	**5**	-0.22%	0.76%	0.21%	-0.29%	1.02%	0.28%
8	**6**	-0.20%	0.64%	0.18%	-0.23%	0.74%	0.20%
9	**6**	-0.18%	0.57%	0.16%	-0.23%	0.74%	0.20%
9	**7**	-0.17%	0.50%	0.13%	-0.19%	0.56%	0.15%
10	**7**	-0.15%	0.45%	0.12%	-0.19%	0.56%	0.15%
10	**8**	-0.14%	0.39%	0.11%	-0.15%	0.44%	0.12%
11	**8**	-0.13%	0.36%	0.10%	-0.15%	0.44%	0.12%
11	**9**	-0.12%	0.32%	0.09%	-0.13%	0.35%	0.09%
12	**9**	-0.11%	0.29%	0.08%	-0.13%	0.35%	0.09%
12	**10**	-0.10%	0.27%	0.07%	-0.11%	0.29%	0.08%

As we can observe from Table 5.4, under incomplete collusion, the social welfare increases when we move from an industry with no vertical integration to an industry where there is vertical integration either with a cartel or a fringe downstream firm. This is also true when we compare the results obtained under Cournot. With vertical integration, consumers see their welfare improving because the total quantity in the market increases. Also, firms have lower profits due to the reduction of both retail and wholesale prices. The increase in the consumer surplus is higher than the decrease in the producer surplus, hence the net effect is that vertical integration increases social welfare. These efficiency effects are similar in both models 2 and 3, however the vertical merger with a fringe firm increases more the total quantity in the market than the vertical merger with a cartel firm. This explains why the increase of the social welfare is higher in model 3 than in model 2.

5.4 Discussion of Results

In order to verify if vertical integration promotes collusion, we compare the results of the dynamic and static stability conditions for the three models.[29]

From the numerical results we conclude that, for the first model, the participation conditions are verified if the number of cartel firms are $F+2$ or $F+3$, that is the cartel should have respectively, $50\% - 75\%$ or $50\% - 80\%$ firms in the industry. However, for models 2 and 3 we have to introduce additional assumptions ($K > 3$ and $F > 1$, respectively) otherwise, the external stability conditions (ESC) are not satisfied and therefore the cartel would become statically unstable.

As mentioned in section 5.3, we identify two effects of the vertical merger: the efficiency effect and the collusive effect. Under trigger strategies, the non-cooperative collusion is stable because the difference between deviation and collusive payoffs is large enough to outweigh, in present discounted value, the difference between collusive and NE payoffs. For a downstream cartel firm, vertical integration increases the difference between collusive and NE payoffs so tends to increase cartel stability. Therefore, vertical integration has a collusive effect in the downstream market.[30] However, this collusive effect is distinct when we analyse a merger with a fringe firm or a merger with a cartel firm.

[29] See Appendix D.2.1.

[30] The collusive effect obtained in this paper is different from Chen (2001) 's because we have different model's specifications. Chen (2001) argues that the vertical merger softens price competition in the final market and changes rival's incentive in selecting input suppliers.

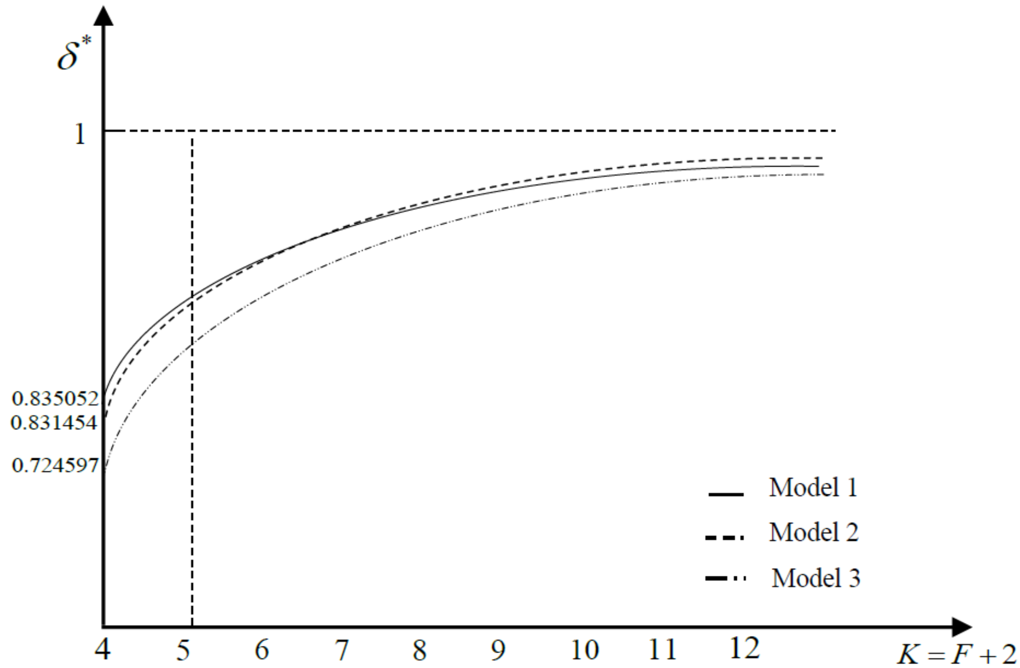


Figure 5.1: Model's critical discount values for K=F+2

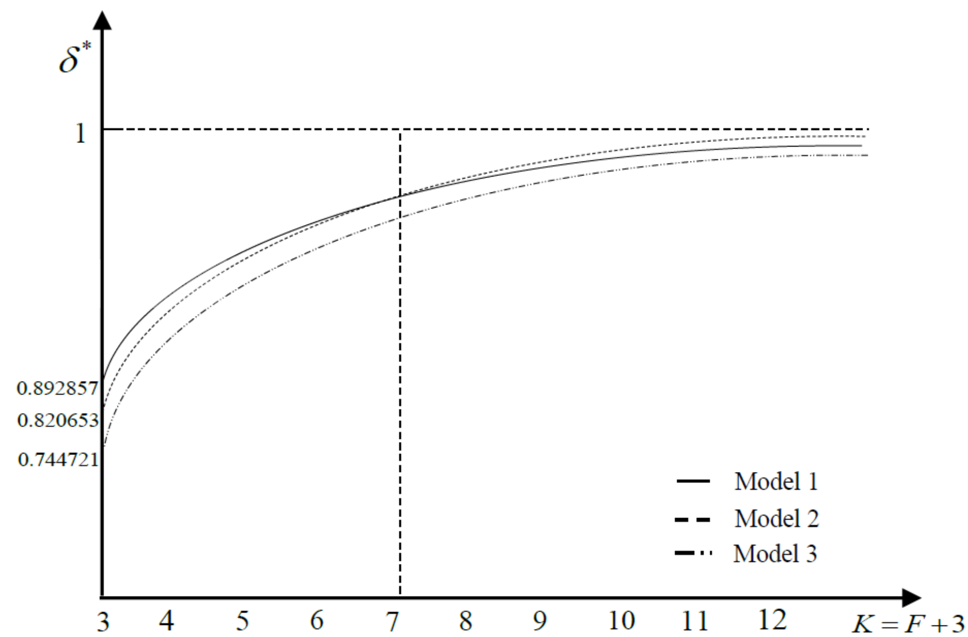


Figure 5.2: Model's critical discount values for K=F+3

After analyzing the critical discount values (Figures 5.1 and 5.2), we conclude that vertical integration always promotes collusion when the upstream firm merges with a downstream fringe firm. This is justified because vertical integration with a fringe firm reinforces the incentives for the cartel firms not to deviate from the collusive agreement. As the vertical integrated firm is now in a privileged position, cartel firms take this as a threat and therefore react defensively against this competitive firm. Furthermore, there is an incentive for firms to vertically integrate because the profit of the vertical integrated firm is higher than the sum of profits of a D_f and an U firms, without vertical integration. Also, although the vertical merger increases the difference between the collusive and NE payoffs, it also decreases the difference between deviation and collusive payoffs. Since the second difference is higher than the first, the vertical merger with a fringe firm decreases the critical discount factor (δ^*) and therefore increases the cartel stability, for any level of downstream market concentration.

Differently, it is possible to find situations where vertical integration with a cartel firm hinders collusion. This depends on the number of firms in the downstream market. Although there is an incentive to vertically integrate due to the possibility of getting higher profits, this could make collusion more difficult to sustain. For example, for $K = 4$ and $F = 2$, vertical integration either with a fringe or a cartel firm promotes collusion. Also, for this case, vertical integration with a fringe firm incentives more the collusion than vertical integration with a cartel firm. But for $K = 8$ and $F = 5$ vertical integration increases cartel sustainability only when it is with a fringe firm. One reason that explains why collusion is more difficult to sustain when there is vertical integration with a cartel firm, is the low level of market concentration. With a high number of firms in the market, the vertically integrated firm continues to have higher profit than its rivals, however cartel individual profits will be lower because they have to share the cartel total profit with a high number of firms and therefore they will earn more from deviating. Also, although the vertical merger increases the difference between the collusive and the NE payoffs, it also increases the difference between deviation and collusive payoffs. Thus, since the second difference is higher than the first, the vertical merger with a cartel firm increases the critical discount factor (δ^*) and makes collusion harder to sustain.

Moreover, with the merger the collusive and the deviation profit of each cartel firm increase in both models: when the merger is with a fringe firm, cartel firms have higher collusive profits than when the merger is with cartel firms. Also, when the merger is with a fringe firm, cartel firms have less incentives to deviate from the collusive agreement than when the merger is with a cartel firm, because the cartel deviation profit in the third model is lower than in the second model. This explains why the vertical merger with a fringe firm increases more the incentives to collude than the merger with a cartel firm.

Furthermore, in both cases, we identify the efficiency effects of the vertical merger. The vertical merger with a cartel or a fringe downstream firm makes the upstream market asymmetric and this asymmetry is higher in model 3 than in model 2. Hence, with the vertical merger the upstream competition increases and therefore the final result is that the total wholesale quantity increases. This increase in the total quantity is smaller the larger is the number of downstream firms. Also, the increase of the wholesale quantity is higher in model 3 than in model 2. With higher wholesale quantity, both wholesale and retail prices decrease, which in turn, increase the quantity produced by the downstream firms and affects the stability of collusion. Thus, the increase in upstream competition also affects downstream collusion. Then, similar to Chen (2001)'s model, the vertical merger generates efficiency gains because the merged firm improves the production of its inputs and also intensifies price competition in the final market. Also, the vertical merger is efficiency increasing because, as we mentioned in the previous section, it reduces the double marginalization problem and this reduction is higher when there is collusion than if there was no collusion.

Summarizing, the vertical merger with a fringe firm always promotes downstream collusion while if the merger is with a cartel firm we can have the two situations for different levels of market concentration. For low level of market concentration, the vertical merger with a cartel firm increases the difference between the collusive and NE profits. This increase however decreases as the number of downstream firms increase and consequently vertical integration with a cartel firm hinders collusion.

We compare models 1 and 2 to evaluate the difference between our results and those obtained if there was full collusion ($F = 0$). We conclude that when an incomplete cartel faces outsider competition it has a limited ability to charge a price above competition levels. Hence, when the cartel is not all-inclusive the collusive prices are lower than under complete collusion and therefore, incomplete collusion is harder to sustain than complete collusion. Also, this explains why when the cartel is incomplete, the sustainability of collusion decreases, the higher the number of fringe firms is. Moreover, we conclude that, under full collusion, a vertical merger with a cartel firm always promotes collusion for low level of market concentration ($K \leq 5$). However, with a high number of cartel firms, the vertical merger with a cartel firm also hinders collusion. This results are similar to those obtained with incomplete collusion.

5.5 Concluding Remarks

This paper explores the effect of vertical integration on the incentives to collude, when there is incomplete collusion in the downstream market. A central result of the paper is that in general by acquiring a downstream firm (a cartel or a fringe firm), the vertically integrated firm promotes collusion at downstream level. The vertical merger increases the cartel firms' incentives to collude and decreases the deviation incentives from the collusive agreement.

However, for low downstream market concentration, the opposite result is found when the vertical merger is with a cartel downstream firm. The larger the number of cartel firms is the lower the profit that each cartel firm earns and therefore the higher the incentive to disrupt the collusive agreement is. Hence, vertical integration with a cartel firm hinders downstream collusion for high number of cartel firms in downstream market. The results we found are very important to understand the type of strategies that firms follow in markets where there is incomplete collusion and vertical integration. Further, the combination of these two types of strategies does not have the same effects in markets with different level of concentration.

In spite of collusion, we verify that vertical integration increases social welfare due to the partial elimination of double marginalization. These results could be important for antitrust authorities that are concerned with the effects of such mergers on downstream collusion.

The framework and the assumptions we have assumed are of a particular kind. Further research on the analysis of the relationship between vertical merger and incomplete collusion should consider what happens if there is vertical integration with more than one downstream firm. Also, it would be interesting to study the case where the downstream firm also takes into account the profits of the upstream firm, relaxing the assumption that it has an independent behavior from the upstream firm. We think that these are very important and useful subjects for further research.

Appendix

Appendix D.1. Equilibrium Profits

Appendix D.1.1. Cournot profits for non-integrated equilibrium - Model 1

Under this case all downstream firms decide individually their quantity. Knowing that the firms are symmetric, the quantity produced by each downstream firm is given by: $q_{k_j} = q_{f_l} = \frac{a-w}{b(F+K+1)}$, with $j = 1, .., K$ and $l = 1, ..., F$. Therefore, the inverse demand function to upstream firms is $w = a - \frac{b(F+K+1)Q}{F+K}$. At stage 1, the upstream firms decide the quantity, maximizing their profits functions: $\pi_i^U = (w - c)q_i^U = (\frac{a(F+K)-b(F+K+1)(q_i^U+q_2^U)}{F+K} - c)q_i^U$, with $i = 1, 2$. The equilibrium results are:

$q_i^U = \frac{A(F+K)}{3(F+K+1)}$, with $A = \frac{a-c}{b}$

$q_{k_j}^D = q_{f_l}^D = \frac{2A}{3(F+K+1)}$.

$P = \frac{a(3+F+K)+2c(F+K)}{3(F+K+1)}$ $\qquad w = \frac{a+2c}{3}$

$\pi_i^U = \frac{A^2 b(F+K)}{9(F+K+1)}$ $\qquad \pi_{k_j}^D = \pi_{fl}^D = \frac{4A^2 b}{9(F+K+1)^2}$.

Appendix D.1.2. Deviation profits for non-integrated firms - Model 1

Firm D_{k_1} deviates from the collusive agreement knowing that firms D_{k_j}, with $j = 2, .., K$, produce the collusive quantities and fringe firms D_{f_l}, with $l = 1, .., F$, react to the deviation.

Then residual demand for the deviating firm is given by: $P = a - bq_{k_1}^d - (K-1)\frac{a-w}{2K} - F\frac{a-w-bQ_K}{(F+1)}$ and the deviation quantity is $q_{k_1}^{deviation} = \frac{(a-w)(K+1)}{4bK}$.

At stage 1 the upstream firms decide the quantity, knowing that $w = a - \frac{4b(F+1)Q}{H}$, where $H = 3K + 4FK - 1$. Then the upstream firms will produce ,in equilibrium, $q_i^U = \frac{AH}{12K(F+1)}$, with $i = 1, 2$. The equilibrium prices are given by $w = \frac{a+2c}{3}$ and $P = \frac{a(3K+2FK+1)+cH}{6K(F+1)}$ and the downstream quantities are $q_{k_1}^{deviation} = \frac{A(K+1)}{6K}$, $q_{k_j} = \frac{A}{3K}$, with $j = 2, .., K$ and $q_{f_l} = \frac{A(K+1)}{6K(F+1)}$, with $l = 1, .., F$. The equilibrium profits are:

$\pi_i^U = \frac{A^2 bH}{36K(F+1)}$ $\qquad \pi_{k_1}^{deviation} = \frac{A^2 b(K+1)^2}{36K^2(F+1)}$

$\pi_{k_j}^D = \frac{A^2 b(K+1)}{18K^2(F+1)}$, with $j = 2, .., K$ $\qquad \pi_{fl}^D = \frac{A^2 b(K+1)^2}{36K^2(F+1)^2}$, with $l = 1, .., F$.

Appendix D.1.3. Collusion profits under Vertical Integration with a Cartel downstream firm - Model 2

At Stage 1, upstream firm 1, vertically integrated, maximizes the total profit that includes the profits from the upstream and downstream activities. Upstream firm 2 maximizes its own profit. Hence, the profit functions of firms U_1 and U_2 are:

$$\pi_1^{VI^K} = (w-c)q_1^U + (a - bQ - w)q_{k_1}$$
$$\pi_2^U = (w-c)q_2^U$$

where $w = \frac{a(2F+1)-2bQ(F+1)}{2F+1}$, $q_{k_1} = \frac{a-w}{2bK}$ and $Q = q_1^U + q_2^U$. Then the upstream firms will produce, in equilibrium, $q_1^U = \frac{A(2F+1)(K(2F+1)+1)}{2B(F+1)}$ and $q_2^U = \frac{A(2F+1)(K(2F+1)-1)}{2B(F+1)}$, where $A = \frac{a-c}{b}$ and $B = 3K + 6FK - 1$. By substitution we obtain the equilibrium prices $w = \frac{a(K(2F+1)-1)+2Kc(2F+1)}{B}$ and $P = \frac{a\left((F+1)B-K(2F+1)^2\right)+K(2F+1)^2c}{B(F+1)}$ and the equilibrium quantities for the downstream cartel and fringe firms $q_{k_1} = \frac{A(2F+1)}{B}$ and $q_{f_l} = \frac{AK(2F+1)}{B(F+1)}$. The equilibrium profits are:

$$\pi_1^{VI^K} = \frac{A^2b(2F+1)[K(2F+1)(K+2FK+2)-1]}{2(F+1)B^2} \qquad \pi_2^U = \frac{A^2b(2F+1)(K+2FK-1)^2}{2(F+1)B^2}$$
$$\pi_{k_j}^D = \frac{A^2bK(2F+1)^2}{(F+1)B^2}, \text{ for } j = 2,..,K \qquad \pi_{f_l}^D = \frac{A^2bK^2(2F+1)^2}{(F+1)^2B^2}, \text{ for } l = 1,..,F.$$

Appendix D.1.4. Cournot profits under Vertical Integration with a Cartel or Fringe downstream firm - Models 2 and 3

Assuming that the downstream integrated behaves as a subsidiary firm that decides its quantity in an independent way from the parent upstream firm, the first stage the results are the same as in the model without vertical integration. The profit functions for upstream firms are:

$$\pi_1^{VI^K} = (w-c)q_1^U + (a - b(q_1^U + q_2^U) - w)q_{k_1}$$
$$\pi_2^U = (w-c)q_2^U$$

which lead to the following results:

$q_1^U = \frac{A(F+K)[(F+K+1)(F+K)+2]}{(F+K+1)E}$, where $E = 3(F+K+1)(F+K)-2)$

$q_2^U = \frac{A(F+K)(F+K-1)(F+K+2)}{(F+K+1)E}$

$w = \frac{a[(F+K+1)(F+K)-2]+2c(F+K)(F+K+1)}{E}$ $\quad P = \frac{a[(F+K)^2+3(F+K)-2]+2c(F+K)^2}{E}$

$q_{k_j} = q_{f_k} = \frac{2A(F+K)}{E}$, with $j = 1,..,K$ and $l = 1,..,F$.

$\pi_2^U = \frac{A^2b(F+K)(F+K-1)(F+K+2)[(F+K+1)(F+K)-2]}{E^2(F+K+1)}$

$\pi_{k_j}^D = \pi_{f_l}^D = \frac{4A^2b(F+K)^2}{E^2}$, with $j = 1,..,K$ and $l = 1,..,F$.

$\pi_1^{VI} = \frac{A^2b(F+K)[(F+K+1)^2(F+K)^2-4]}{(F+K+1)E^2} + \frac{4A^2b(F+K)^2}{E^2}$

Appendix D.1.5. Deviation profits under Vertical Integration with a Cartel downstream firm - Model 2

First Case: D_{k_1}, that is vertically integrated, deviates from the collusive agreement knowing that D_{k_j}, for $j = 2,..,K$ produce the collusive quantities and that fringe firms D_{f_l} react to the deviation.

The residual demand for firm D_{k_1} is: $P = a - bq_{k_1}^{deviation} - (K-1)b\frac{a-w}{2bK} - bF\frac{a-w-bQ_K}{b(F+1)}$ and the quantity is $q_{k_1}^{deviation} = \frac{(K+1)(a-w)}{4bK}$. The upstream firms maximize their profits, given by

$$\pi_1^{VI} = (w-c)q_1^U + (a - b(q_1^U + q_2^U) - w)q_{k_1}^{deviation}$$
$$\pi_2^U = (w-c)q_2^U$$

knowing that $w = a - \frac{4bK(F+1)Q}{H}$.

Then the equilibrium quantities, prices and profits are given by:

$q_1^U = \frac{AH(8FK^2+7K^2+1)}{4K(F+1)G}$, where $G = -8K + 24FK^2 + 17K^2 - 1$

$q_2^U = \frac{AH(-4K+8FK^2+5K^2-1)}{4K(F+1)G}$

$w = \frac{a(8FK^2+5K^2-4K-1)+4cKH}{G}$ $\quad P = \frac{a(8F^2K^2+17FK^2+8K^2-F-2K-2)+cH^2}{(F+1)G}$

$q_{k_1}^{deviation} = \frac{AH(K+1)}{G}$ $\quad q_{f_l} = \frac{AH(K+1)}{G(F+1)}$, with $l = 1,..,F$ $\quad q_{k_j} = \frac{2AH}{G}$, with $j = 2,..,K$

$\pi_2^U = \frac{A^2b(-4K+8FK^2+5K^2-1)^2H}{4KG^2(F+1)}$ $\quad \pi_{k_1}^{deviation} = \frac{A^2b(K+1)^2H^2}{(F+1)G^2}$

$\pi_{k_j}^D = \frac{2A^2b(K+1)H^2}{(F+1)G^2}$ $\quad \pi_{f_l}^D = \frac{A^2b(K+1)^2H^2}{(F+1)^2G^2}$

$\pi_1^{VI^K} = \frac{A^2b(-4K+8FK^2+5K^2-1)(8FK^2+7K^2+1)H}{4KG^2(F+1)} + \frac{A^2b(K+1)^2H^2}{(F+1)G^2}$

Second Case: D_{k_2} deviates from the collusive agreement, knowing that D_{k_j}, with $j = 1, .., K \setminus \{2\}$ produce the collusive quantities and the fringe downstream firms react to that deviation.

The residual demand for firm D_{k_2} is given by $P = a - bq_{k_2}^{deviation} - (K-1) * \frac{a-w}{2K} - bF\frac{a-w-bQ_K}{(F+1)b}$ and the deviation quantity is $q_{k_2}^{deviation} = \frac{(a-w)(K+1)}{4bK}$.

The upstream firms maximize their profits, knowing that $w = a - \frac{4K(F+1)Qb}{H}$. Then the equilibrium quantities, prices and profits are given by:

$q_1^U = \frac{AH(4FK^2+3K^2+1)}{4K(F+1)I}$, where $I = -4K+12FK^2+9K^2-1$ $\quad q_2^U = \frac{A(-2K+4FK^2+3K^2-1)H}{4K(F+1)I}$

$w = \frac{a(4FK^2+3K^2-2K-1)+2cKH}{I}$ $\quad P = \frac{a(8F^2K^2+18FK^2+9K^2-2F-2K-3)+cH^2}{2(F+1)I}$

$q_{k_2}^{deviation} = \frac{AH(K+1)}{2I}$ $\quad q_{f_l} = \frac{AH(K+1)}{2I(F+1)}$ $\quad q_{k_j} = \frac{AH}{I}$

$\pi_2^U = \frac{A^2bH(-2K+4FK^2+3K^2-1)^2}{4KI^2(F+1)}$

$\pi_{k_2}^{deviation} = \frac{A^2b(K+1)^2H^2}{4(F+1)I^2}$ $\quad \pi_{k_j}^D = \frac{A^2b(K+1)H^2}{2(F+1)I^2}$, with $j = 1, .., K\setminus\{2\}$

$\pi_{f_l}^D = \frac{A^2b(K+1)^2H^2}{4(F+1)^2I^2}$, with $l = 1, .., F$

$\pi_1^{VI^K} = \frac{A^2b(K+1)H^2}{2(F+1)I^2} + \frac{A^2(-2K+4FK^2+3K^2-1)(4FK^2+3K^2+1)H}{4KI^2(F+1)}$

Appendix D.1.6. Collusion profits under Vertical Integration with a Cartel downstream firm - Model 3

At Stage 1, uptream firm 1, vertically integrated, maximizes the total profit that includes the profits from the upstream and downstream activities. Upstream firm 2 maximizes its own profit. Hence, the profit functions of firms U_1 and U_2 are:

$$\pi_1^{VI^F} = (w-c)q_1^U + (a - bQ_K - bQ_F - w)q_{f_1}$$
$$\pi_2^U = (w-c)q_2^U$$

The equilibrium upstream quantities are given by: $q_1^U = \frac{A(2F+1)\left(3F+2F^2+2\right)}{2(F+1)J}$, where $J = 9F + 6F^2 + 2$ and $q_2^U = \frac{AF(2F+3)(2F+1)}{2(F+1)J}$. By substitution we obtain the equilibrium prices $w = \frac{2c(2F^2+3F+1)+a(2F^2+3F)}{J}$ and $P = \frac{a(2F^2+5F+1)+c(4F^2+4F+1)}{J}$ and the equilibrium quantities for the downstream cartel and fringe firms $q_{k_j} = \frac{A(F+1)(2F+1)}{KJ}$ and $q_{f_l} = \frac{A(2F+1)}{J}$. The equilibrium profits are:

$\pi_1^{VI^F} = A^2b\frac{2(F+1)(2F+1)^2+F(2F+1)(2F+3)\left(3F+2F^2+2\right)}{2(F+1)J^2}$ $\quad \pi_2^U = \frac{a^2bF^2(2F+1)(2F+3)^2}{2(F+1)J^2}$

$\pi^D_{k_j} = \frac{A^2b(F+1)(2F+1)^2}{KJ^2}$, for $j = 1, .., K$ $\qquad$ $\pi^D_{f_l} = \frac{A^2b(2F+1)^2}{J^2}$, for $l = 2, .., F$

Appendix D.1.7. Deviation profits under Vertical Integration with a Fringe downstream firm - Model 3

Firm D_{k_1} deviates from the collusive agreement knowing that D_{k_j}, for $j = 2, .., K$ produce the collusive quantities and that fringe firms D_{f_l} react to the deviation. The residual demand for firm D_{k_1} is given by $P = a - bq^{deviation}_{k_1} - (K-1)b\frac{a-w}{2bK} - bF\frac{a-w-bQ_K}{(F+1)b}$. Therefore, the quantity for this firm is $q^{deviation}_{k_1} = \frac{(a-w)(K+1)}{4bK}$ and the quantity for the fringe firms is $q_{f_l} = \frac{(a-w)(K+1)}{4b(F+1)K}$, with $l = 1, .., F$. The upstream firms maximize their profits:

$$\pi^{VI}_1 = (w-c)q^U_1 + (a - b(q^U_1 + q^U_2) - w)q_{f_1} \text{ and } \pi^U_2 = (w-c)q^U_2,$$

knowing that $w = a - \frac{4K(F+1)Qb}{H}$. Then the equilibrium quantities, prices and profits are given by:

$q^U_1 = \frac{AH\left(14FK^2+8F^2K^2-2FK+7K^2+1\right)}{4K(F+1)K}$, where $L = -8K + 42FK^2 + 24F^2K^2 - 6FK + 17K^2 - 1$

$q^U_2 = \frac{AH(5K+4FK+1)(K+2FK-1)}{4K(F+1)L}$

$w = \frac{a(K+2FK-1)(5K+4FK+1)+4cK(F+1)H}{L}$ $\qquad$ $P = \frac{2a(4F^2K^2+9FK^2+4K^2-K-1)+cH^2}{L}$

$q^{deviation}_{k_1} = \frac{AH(F+1)(K+1)}{L}$ $\qquad$ $q_{f_l} = \frac{AH(K+1)}{L}$, with $l = 1, .., F$

$q_{k_j} = \frac{2AH(F+1)}{L}$, with $J = 2, .., K$

$\pi^U_2 = \frac{A^2bH(K+2FK-1)^2(5K+4FK+1)^2}{4KL^2(F+1)}$ $\qquad$ $\pi^{deviation}_{k_1} = \frac{A^2bH^2(K+1)^2(F+1)}{L^2}$

$\pi^D_{k_j} = \frac{2A^2bH^2(F+1)(K+1)}{L^2}$, with $j = 2, .., K$ $\qquad$ $\pi^D_f = \frac{A^2bH^2(K+1)^2}{L^2}$, with $l = 1, .., F$

$\pi^{VI^F}_1 = \frac{A^2bH^2(K+1)^2}{L^2} + \frac{A^2bH(K+2FK-1)(5K+4FK+1)(14FK^2+8F^2K^2-2FK+7K^2+1)}{4KL^2(F+1)}$

Appendix D.2. Numerical Simulation

Appendix D.2.1. Results from Static and Dynamic Stability[31]

		Static Stability			Dynamic Stability			
		Model 1	Model 2	Model 3	Model 1	Model 2		Model 3
K	F				$\delta^*_{withoutVI}$	$\delta^*_{D_{k_1}}$	$\delta^*_{D_{k_2}}$	$\delta^*_{withVIF}$
3	1	ESC	No ESC	No ESC	0.892857	0.882065	0.860994	0.744721
		ISC	ISC	ISC				
4	1	ESC	ESC	No ESC	0.835052	0.831454	0.812044	0.724597
		ISC	ISC	ISC				
4	2	ESC	ESC	ESC	0.964989	0.963883	0.960915	0.940123
		ISC	ISC	ISC				
5	2	ESC	ESC	ESC	0.927536	0.927391	0.922718	0.901072
		ISC	ISC	ISC				
5	3	ESC	ESC	ESC	0.984802	0.984628	0.983868	0.977686
		ISC	ISC	ISC				
6	3	ESC	ESC	ESC	0.963020	0.963152	0.961592	0.953961
		ISC	ISC	ISC				
6	4	ESC	ESC	ESC	0.992129	0.992100	0.991830	0.989391
		ISC	ISC	ISC				
7	4	ESC	ESC	ESC	0.978852	0.978966	0.978320	0.974993
		ISC	ISC	ISC				
7	5	ESC	ESC	ESC	0.9954188	0.9954187	0.995301	0.994154
		ISC	ISC	ISC				
8	5	ESC	ESC	ESC	0.986848	0.986926	0.986615	0.984944
		ISC	ISC	ISC				
8	6	ESC	ESC	ESC	0.997106	0.997111	0.997052	0.996443
		ISC	ISC	ISC				
9	6	ESC	ESC	ESC	0.991288	0.991340	0.991174	0.990245
		ISC	ISC	ISC				
9	7	ESC	ESC	ESC	0.998057	0.998063	0.998030	0.997678
		ISC	ISC	ISC				
10	7	ESC	ESC	ESC	0.993940	0.993976	0.993879	0.993323
		ISC	ISC	ISC				
10	8	ESC	ESC	ESC	0.998634	0.998638	0.998619	0.998401
		ISC	ISC	ISC				
11	8	ESC	ESC	ESC	0.995619	0.995644	0.995584	0.995231
		ISC	ISC	ISC				
11	9	ESC	ESC	ESC	0.999003	0.999007	0.998994	0.998853
		ISC	ISC	ISC				
12	9	ESC	ESC	ESC	0.996732	0.996750	0.996711	0.996476
		ISC	ISC	ISC				
12	10	ESC	ESC	ESC	0.999251	0.999253	0.999245	0.999149
		ISC	ISC	ISC				

[31] Note that ESC - External Stability Condition and ISC - Internal Stability Condition.

Appendix D.2.2. Results for Model 1[32]

a) Collusive Results							
K	F	U qt	K qt	F qt	U profit	K profit	F profit
3	1	0.25000	0.11111	0.16667	0.08333	0.01852	0.02778
4	1	0.25000	0.08333	0.16667	0.08333	0.01389	0.02778
4	2	0.27778	0.08333	0.11111	0.09259	0.00926	0.01235
5	2	0.27778	0.06667	0.11111	0.09259	0.00741	0.01235
5	3	0.29167	0.06667	0.08333	0.09722	0.00556	0.00694
6	3	0.29167	0.05556	0.08333	0.09722	0.00463	0.00694
6	4	0.30000	0.05556	0.06667	0.10000	0.00370	0.00444
7	4	0.30000	0.04762	0.06667	0.10000	0.00317	0.00444
7	5	0.30556	0.04762	0.05556	0.10185	0.00265	0.00309
8	5	0.30556	0.04167	0.05556	0.10185	0.00231	0.00309
8	6	0.30952	0.04167	0.04762	0.10317	0.00198	0.00227
9	6	0.30952	0.03704	0.04762	0.10317	0.00176	0.00227
9	7	0.31250	0.03704	0.04167	0.10417	0.00154	0.00174
10	7	0.31250	0.03333	0.04167	0.10417	0.00139	0.00174
10	8	0.31481	0.03333	0.03704	0.10494	0.00123	0.00137
11	8	0.31481	0.03030	0.03704	0.10494	0.00112	0.00137
11	9	0.31667	0.03030	0.03333	0.10556	0.00101	0.00111
12	9	0.31667	0.02778	0.03333	0.10556	0.00093	0.00111
12	10	0.31818	0.02778	0.03030	0.10606	0.00084	0.00092

b) Cournot Results					
K	F	U qt	K qt=F qt	U profit	K=F profit
3	1	0.26667	0.13333	0.08889	0.01778
4	1	0.27778	0.11111	0.09259	0.01235
4	2	0.28571	0.09524	0.09524	0.00907
5	2	0.29167	0.08333	0.09722	0.00694
5	3	0.29630	0.07407	0.09877	0.00549
6	3	0.30000	0.06667	0.10000	0.00444
6	4	0.30303	0.06061	0.10101	0.00367
7	4	0.30556	0.05556	0.10185	0.00309
7	5	0.30769	0.05128	0.10256	0.00263
8	5	0.30952	0.04762	0.10317	0.00227
8	6	0.31111	0.04444	0.10370	0.00198
9	6	0.31250	0.04167	0.10417	0.00174
9	7	0.31373	0.03922	0.10458	0.00154
10	7	0.31481	0.03704	0.10494	0.00137
10	8	0.31579	0.03509	0.10526	0.00123
11	8	0.31667	0.03333	0.10556	0.00111
11	9	0.31746	0.03175	0.10582	0.00101
12	9	0.31818	0.03030	0.10606	0.00092
12	10	0.31884	0.02899	0.10628	0.00084

[32] Note that the quantities are multiplied by $A = \frac{a-c}{b}$ and the profits by $A^2 b$.

c) Deviating Results									
K	**F**	**U qt**	**K1 qt dev**	**K qt**	**F qt**	**U profit**	**K profit**	**K dev profit**	**F profit**
3	1	0.27778	0.22222	0.11111	0.11111	0.09259	0.00617	0.02469	0.01235
4	1	0.28125	0.20833	0.08333	0.10417	0.09375	0.00347	0.02170	0.01085
4	2	0.29861	0.20833	0.08333	0.06944	0.09954	0.00347	0.01447	0.00482
5	2	0.30000	0.20000	0.06667	0.06667	0.10000	0.00222	0.01333	0.00444
5	3	0.30833	0.20000	0.06667	0.05000	0.10278	0.00222	0.01000	0.00250
6	3	0.30903	0.19444	0.05556	0.04861	0.10301	0.00154	0.00945	0.00236
6	4	0.31389	0.19444	0.05556	0.03889	0.10463	0.00154	0.00756	0.00151
7	4	0.31429	0.19048	0.04762	0.03810	0.10476	0.00113	0.00726	0.00145
7	5	0.31746	0.19048	0.04762	0.03175	0.10582	0.00113	0.00605	0.00101
8	5	0.31771	0.18750	0.04167	0.03125	0.10590	0.00087	0.00586	0.00098
8	6	0.31994	0.18750	0.04167	0.02679	0.10665	0.00087	0.00502	0.00072
9	6	0.32011	0.18519	0.03704	0.02646	0.10670	0.00069	0.00490	0.00070
9	7	0.32176	0.18519	0.03704	0.02315	0.10725	0.00069	0.00429	0.00054
10	7	0.32188	0.18333	0.03333	0.02292	0.10729	0.00056	0.00420	0.00053
10	8	0.32315	0.18333	0.03333	0.02037	0.10772	0.00056	0.00373	0.00041
11	8	0.32323	0.18182	0.03030	0.02020	0.10774	0.00046	0.00367	0.00041
11	9	0.32424	0.18182	0.03030	0.01818	0.10808	0.00046	0.00331	0.00033
12	9	0.32431	0.18056	0.02778	0.01806	0.10810	0.00039	0.00326	0.00033
12	10	0.32513	0.18056	0.02778	0.01641	0.10838	0.00039	0.00296	0.00027

Appendix D.2.3. Results for Model 2[33]

a) Collusive Results									
K	**F**	**U qt**	**K1 qt dev**	**K qt**	**F qt**	**U profit**	**K profit**	**K dev profit**	**F profit**
4	1	0.235714	0.2357143	0.085714	0.1714286	0.102245	0.074082	0.014694	0.029388
4	2	0.26836	0.26836	0.08475	0.11299	0.10509	0.08642	0.00958	0.01277
5	2	0.27027	0.27027	0.06757	0.11261	0.10257	0.08766	0.00761	0.01268
5	3	0.28606	0.28606	0.06731	0.08413	0.10468	0.09352	0.00566	0.00708
6	3	0.28700	0.28700	0.05600	0.08400	0.10343	0.09414	0.00470	0.00706
6	4	0.29627	0.29627	0.05590	0.06708	0.10496	0.09753	0.00375	0.00450
7	4	0.29681	0.29681	0.04787	0.06702	0.10425	0.09788	0.00321	0.00449
7	5	0.30290	0.30290	0.04783	0.05580	0.10539	0.10009	0.00267	0.00311
8	5	0.30323	0.30323	0.04183	0.05577	0.10495	0.10031	0.00233	0.00311
8	6	0.30753	0.30753	0.04180	0.04777	0.10583	0.10185	0.00200	0.00228
9	6	0.30776	0.30776	0.03714	0.04776	0.10553	0.10200	0.00177	0.00228
9	7	0.31095	0.31095	0.03713	0.04177	0.10623	0.10314	0.00155	0.00174
10	7	0.31111	0.31111	0.03341	0.04176	0.10602	0.10324	0.00140	0.00174
10	8	0.31358	0.31358	0.03340	0.03711	0.10659	0.10412	0.00124	0.00138
11	8	0.31369	0.31369	0.03036	0.03710	0.10644	0.10419	0.00113	0.00138
11	9	0.31565	0.31565	0.03035	0.03339	0.10690	0.10488	0.00101	0.00111
12	9	0.31574	0.31574	0.02782	0.03338	0.10679	0.10494	0.00093	0.00111
12	10	0.31734	0.31734	0.02781	0.03034	0.10718	0.10550	0.00084	0.00092

[33] Note that the quantities are multiplied by $A = \frac{a-c}{b}$ and the profits by $A^2 b$.

b) Cournot Results							
K	**F**	**U1 -qt**	**U2 qt**	**K qt=F qt**	**VI - profit**	**U profit**	**K=F profit**
4	1	0.30303	0.26515	0.11364	0.09965	0.08437	0.01291
4	2	0.30415	0.27650	0.09677	0.10045	0.08919	0.00937
5	2	0.30572	0.28464	0.08434	0.10123	0.09259	0.00711
5	3	0.30737	0.29076	0.07477	0.10194	0.09511	0.00559
6	3	0.30896	0.29552	0.06716	0.10258	0.09704	0.00451
6	4	0.31042	0.29933	0.06098	0.10314	0.09856	0.00372
7	4	0.31176	0.30245	0.05584	0.10364	0.09979	0.00312
7	5	0.31297	0.30505	0.05150	0.10409	0.10081	0.00265
8	5	0.31408	0.30725	0.04779	0.10449	0.10166	0.00228
8	6	0.31507	0.30913	0.04459	0.10485	0.10239	0.00199
9	6	0.31598	0.31076	0.04178	0.10518	0.10301	0.00175
9	7	0.31681	0.31218	0.03931	0.10547	0.10355	0.00155
10	7	0.31756	0.31344	0.03712	0.10574	0.10402	0.00138
10	8	0.31826	0.31456	0.03516	0.10598	0.10444	0.00124
11	8	0.31889	0.31555	0.03339	0.10620	0.10481	0.00112
11	9	0.31948	0.31645	0.03180	0.10641	0.10515	0.00101
12	9	0.32002	0.31726	0.03035	0.10660	0.10545	0.00092
12	10	0.32052	0.31800	0.02902	0.10677	0.10572	0.00084

c_1) Deviating Results											
K	**F**	**U1 qt**	**U2 qt**	**K1 qt deviation**	**K qt**	**F qt**	**VI - profit**	**U2 profit**	**K profit**	**K dev profit**	**F profit**
4	1	0.326394	0.258678	0.04334	0.08668	0.10835	0.12354	0.07931	0.00939	0.02348	0.01174
4	2	0.328265	0.283784	0.04270	0.08540	0.07117	0.11918	0.08990	0.00608	0.01519	0.00506
5	2	0.327273	0.286364	0.03409	0.06818	0.06818	0.11808	0.09112	0.00465	0.01395	0.00465
5	3	0.328663	0.298168	0.03388	0.06777	0.05082	0.11628	0.09611	0.00344	0.01033	0.00258
6	3	0.328226	0.299429	0.02821	0.05642	0.04937	0.11576	0.09671	0.00279	0.00975	0.00244
6	4	0.329197	0.306235	0.02812	0.05623	0.03936	0.11480	0.09959	0.00221	0.00775	0.00155
7	4	0.328968	0.306945	0.02409	0.04818	0.03854	0.11452	0.09993	0.00186	0.00743	0.00149
7	5	0.32967	0.311355	0.02404	0.04808	0.03205	0.11394	0.10179	0.00154	0.00616	0.00103
8	5	0.329536	0.311794	0.02103	0.04205	0.03154	0.11377	0.10200	0.00133	0.00597	0.00099
8	6	0.330065	0.314878	0.02100	0.04200	0.02700	0.11338	0.10330	0.00113	0.00510	0.00073
9	6	0.32998	0.315169	0.01866	0.03732	0.02666	0.11327	0.10344	0.00099	0.00497	0.00071
9	7	0.33039	0.317444	0.01864	0.03729	0.02330	0.11300	0.10440	0.00087	0.00434	0.00054
10	7	0.330333	0.317646	0.01678	0.03355	0.02307	0.11292	0.10449	0.00077	0.00426	0.00053
10	8	0.330661	0.319392	0.01676	0.03353	0.02049	0.11272	0.10523	0.00069	0.00378	0.00042
11	8	0.33062	0.319538	0.01524	0.03048	0.02032	0.11266	0.10530	0.00062	0.00372	0.00041
11	9	0.330888	0.32092	0.01523	0.03046	0.01827	0.11251	0.10588	0.00056	0.00334	0.00033
12	9	0.330859	0.321029	0.01396	0.02792	0.01815	0.11247	0.10593	0.00051	0.00329	0.00033
12	10	0.331081	0.322149	0.013952	0.027905	0.016489	0.1123405	0.1064	0.00046	0.002991	0.000272

c_2) Deviating Results											
K	F	U1 qt	U2 qt	K2 qt deviation	K qt	F qt	VI - profit	U2 profit	K profit	K dev profit	F profit
4	1	0.29888	0.27243	0.21160	0.08464	0.10580	0.10546	0.08796	0.00895	0.02239	0.01119
4	2	0.31030	0.29277	0.21037	0.08415	0.07012	0.10731	0.09568	0.00590	0.01475	0.00492
5	2	0.30896	0.29552	0.20149	0.06716	0.06716	0.10596	0.09704	0.00451	0.01353	0.00451
5	3	0.31504	0.30498	0.20109	0.06703	0.05027	0.10724	0.10056	0.00337	0.01011	0.00253
6	3	0.31445	0.30632	0.19530	0.05580	0.04882	0.10662	0.10121	0.00272	0.00954	0.00238
6	4	0.31822	0.31172	0.19512	0.05575	0.03902	0.10752	0.10319	0.00218	0.00761	0.00152
7	4	0.31792	0.31247	0.19103	0.04776	0.03821	0.10719	0.10355	0.00182	0.00730	0.00146
7	5	0.32049	0.31594	0.19093	0.04773	0.03182	0.10784	0.10481	0.00152	0.00608	0.00101
8	5	0.32032	0.31640	0.18789	0.04175	0.03131	0.10764	0.10503	0.00131	0.00588	0.00098
8	6	0.32218	0.31882	0.18783	0.04174	0.02683	0.10814	0.10590	0.00112	0.00504	0.00072
9	6	0.32207	0.31912	0.18547	0.03709	0.02650	0.10801	0.10605	0.00098	0.00491	0.00070
9	7	0.32348	0.32090	0.18543	0.03709	0.02318	0.10840	0.10668	0.00086	0.00430	0.00054
10	7	0.32340	0.32111	0.18355	0.03337	0.02294	0.10831	0.10678	0.00077	0.00421	0.00053
10	8	0.32451	0.32247	0.18353	0.03337	0.02039	0.10862	0.10726	0.00068	0.00374	0.00042
11	8	0.32446	0.32262	0.18199	0.03033	0.02022	0.10856	0.10734	0.00061	0.00368	0.00041
11	9	0.32535	0.32369	0.18197	0.03033	0.01820	0.10882	0.10771	0.00055	0.00331	0.00033
12	9	0.32531	0.32380	0.18070	0.02780	0.01807	0.10877	0.10777	0.00050	0.00327	0.00033
12	10	0.32604	0.32467	0.18068	0.02780	0.01643	0.10898	0.10807	0.00046	0.00297	0.00027

Appendix D.2.4. Results for Model 3[34]

a) Collusive Results									
K	F	U1 -qt	U2 qt	K qt	F qt	VI - profit	U2 profit	K profit	F profit
4	2	0.30303	0.26515	0.08523	0.11364	0.10933	0.08437	0.00968	0.01291
5	2	0.30303	0.27027	0.06818	0.11364	0.10933	0.08437	0.00775	0.01291
5	3	0.30572	0.28606	0.06747	0.08434	0.10656	0.09259	0.00569	0.00711
6	3	0.30572	0.28700	0.05622	0.08434	0.10656	0.09259	0.00474	0.00711
6	4	0.30896	0.29627	0.05597	0.06716	0.10596	0.09704	0.00376	0.00451
7	4	0.30896	0.29681	0.04797	0.06716	0.10596	0.09704	0.00322	0.00451
7	5	0.31176	0.30290	0.04786	0.05584	0.10598	0.09979	0.00267	0.00312
8	5	0.31176	0.30323	0.04188	0.05584	0.10598	0.09979	0.00234	0.00312
8	6	0.31408	0.30753	0.04182	0.04779	0.10621	0.10166	0.00200	0.00228
9	6	0.31408	0.30776	0.03717	0.04779	0.10621	0.10166	0.00178	0.00228
9	7	0.31598	0.31095	0.03714	0.04178	0.10649	0.10301	0.00155	0.00175
10	7	0.31598	0.31111	0.03343	0.04178	0.10649	0.10301	0.00140	0.00175
10	8	0.31756	0.31358	0.03341	0.03712	0.10677	0.10402	0.00124	0.00138
11	8	0.31756	0.31369	0.03037	0.03712	0.10677	0.10402	0.00113	0.00138
11	9	0.31889	0.31565	0.03036	0.03339	0.10704	0.10481	0.00101	0.00112
12	9	0.31889	0.31574	0.02783	0.03339	0.10704	0.10481	0.00093	0.00112
12	10	0.32002	0.31734	0.02782	0.03035	0.10729	0.10545	0.00084	0.00092

[34] Note that the quantities are multiplied by $A = \frac{a-c}{b}$ and the profits by A^2b.

b) Cournot Results

K	F	U1 -qt	U2 qt	K qt=F qt	VI - profit	U profit	K=F profit
4	2	0.30415	0.27650	0.09677	0.10045	0.08919	0.00937
5	2	0.30572	0.28464	0.08434	0.10123	0.09259	0.00711
5	3	0.30737	0.29076	0.07477	0.10194	0.09511	0.00559
6	3	0.30896	0.29552	0.06716	0.10258	0.09704	0.00451
6	4	0.31042	0.29933	0.06098	0.10314	0.09856	0.00372
7	4	0.31176	0.30245	0.05584	0.10364	0.09979	0.00312
7	5	0.31297	0.30505	0.05150	0.10409	0.10081	0.00265
8	5	0.31408	0.30725	0.04779	0.10449	0.10166	0.00228
8	6	0.31507	0.30913	0.04459	0.10485	0.10239	0.00199
9	6	0.31598	0.31076	0.04178	0.10518	0.10301	0.00175
9	7	0.31681	0.31218	0.03931	0.10547	0.10355	0.00155
10	7	0.31756	0.31344	0.03712	0.10574	0.10402	0.00138
10	8	0.31826	0.31456	0.03516	0.10598	0.10444	0.00124
11	8	0.31889	0.31555	0.03339	0.10620	0.10481	0.00112
11	9	0.31948	0.31645	0.03180	0.10641	0.10515	0.00101
12	9	0.32002	0.31726	0.03035	0.10660	0.10545	0.00092
12	10	0.32052	0.31800	0.02902	0.10677	0.10572	0.00084

c) Deviating Results

K	F	U1 qt	U2 qt	K1 qt deviation	K qt	F qt	VI - profit	U2 profit	K profit	K dev profit	F profit
4	2	0.30833	0.29375	0.21003	0.08401	0.14002	0.10601	0.09632	0.00588	0.01470	0.00490
5	2	0.30896	0.29552	0.20149	0.06716	0.13433	0.10596	0.09704	0.00451	0.01353	0.00451
5	3	0.31335	0.30582	0.20081	0.06694	0.10041	0.10612	0.10111	0.00336	0.01008	0.00252
6	3	0.31377	0.30666	0.19519	0.05577	0.09760	0.10617	0.10143	0.00272	0.00952	0.00238
6	4	0.31692	0.31237	0.19491	0.05569	0.07797	0.10665	0.10362	0.00217	0.00760	0.00152
7	4	0.31719	0.31283	0.19092	0.04773	0.07637	0.10670	0.10379	0.00182	0.00729	0.00146
7	5	0.31948	0.31645	0.19078	0.04769	0.06359	0.10717	0.10515	0.00152	0.00607	0.00101
8	5	0.31966	0.31673	0.18779	0.04173	0.06260	0.10721	0.10525	0.00131	0.00588	0.00098
8	6	0.32138	0.31922	0.18771	0.04171	0.05363	0.10760	0.10617	0.00112	0.00503	0.00072
9	6	0.32151	0.31941	0.18539	0.03708	0.05297	0.10764	0.10624	0.00098	0.00491	0.00070
9	7	0.32283	0.32122	0.18534	0.03707	0.04633	0.10797	0.10690	0.00086	0.00429	0.00054
10	7	0.32293	0.32135	0.18348	0.03336	0.04587	0.10799	0.10694	0.00077	0.00421	0.00053
10	8	0.32398	0.32273	0.18345	0.03335	0.04077	0.10827	0.10744	0.00068	0.00374	0.00042
11	8	0.32405	0.32282	0.18193	0.03032	0.04043	0.10829	0.10747	0.00061	0.00368	0.00041
11	9	0.32490	0.32391	0.18191	0.03032	0.03638	0.10852	0.10786	0.00055	0.00331	0.00033
12	9	0.32496	0.32398	0.18065	0.02779	0.03613	0.10854	0.10788	0.00050	0.00326	0.00033
12	10	0.32567	0.32486	0.18063	0.02779	0.03284	0.10873	0.10820	0.00046	0.00297	0.00027

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