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ANALYSIS OF NON-LINEARITIES IN THE PERFORMANCE OF TUNED MASS DAMPERS

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RESUMO

O presente trabalho consiste no estudo da aplicação de sistemas de controlo passivo de vibrações (nomeadamente TMDs) e o seu efeito no comportamento dinâmico de estruturas.

Na primeira parte é feita uma breve introdução teórica aos TMDs, bem como uma breve descrição da sua aplicação prática. Também é descrito o procedimento sugerido quanto a uma sintonização óptima dos mesmos.

De seguida, são descritos diferentes métodos de análise estrutural, baseados em princípios de dinâmica de estruturas, dando particular destaque à Formulação de Espaços de Estado. É feita uma pequena aplicação de simulação numérica deste método a uma estrutura com TMD.

É feita uma análise particular a pontes pedonais tendo em atenção a presença de TMDs. São destacadas particularidades deste tipo de estrutura, nomeadamente a caracterização de forças pedonais actuantes, a determinação de frequências naturais, e o efeito do vão na resposta total da estrutura.

Finalmente, é feito um pequeno estudo de simulação numérica relativamente à influência de factores não lineares no comportamento de um TMD, nomeadamente o efeito do atrito dos amortecedores no funcionamento destes. Estes efeitos são também analisados dentro do contexto da análise de pontes pedonais.

PALAVRAS-CHAVE: Dinâmica de estruturas, sistemas de controlo passivo de vibrações, TMDs, pontes pedonais.

ABSTRACT

The following dissertation consists of a study of passive vibration control systems (namely TMDs), their application and their effect on the dynamic behaviour of structures.

The first part of this work offers a brief theoretical introduction to TMDs, a description of their practical applications and a suggested procedure regarding optimal tuning.

Different methods of dynamic structural analysis are described, with particular emphasis on State-Space representation. Using numerical simulation, a small application of this method is made with structures mounted with TMD.

Particular attention is given to footbridges when considering the presence of TMDs. Particular characteristics of this type of structure are highlighted, including the characterization of pedestrian loads, determination of natural frequencies and the effect of span length in the overall response of the structure.

Finally, a small study is made, using numerical simulation, regarding the influence of non-linear factors in the performance of a TMD, namely the effect of damping friction, in their performance. These effects are also analyzed within the context of footbridge analysis.

KEYWORDS: Structural dynamics, passive vibration control, tuned mass dampers, footbridges.

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1
INTRODUCTION

1.1. OVERVIEW

In the field of Civil Engineering, there have been growing challenges in the design of ambitious structures to the field of structural dynamics. Increasingly taller and more slender structures demand better forms of vibration control against dynamic loads such as strong earthquakes, strong winds, and other harmonic loads. Other structures, such as footbridges of significantly long spans are also subject to vibration problems.

In the interest of mitigating the effect of such loads, vibration control techniques have arisen in the past few decades. One such method is the application of tuned mass dampers, devices that can be mounted in structures, successfully reducing the vibration caused by critical loads.

1.1.1. VIBRATION CONTROL TECHNIQUES

There are three distinct forms of vibration control: passive control, active control and semi-active control.

Devices that rely on passive control can be installed in a certain structure and mitigate the effect of external loads without any external power supply. Such elements tend to be easy to implement and maintain, making them ideal for various types of structures, from tall towers to slender footbridges. Tuned Mass Dampers (TMDs) and Tuned Liquid Dampers (TLDs) are examples of this control technique.

Active systems such as Active Tuned Mass Dampers (ATMDs) rely on an external power supply when functioning, meaning that they are capable of responding differently depending on external factors. Such systems can be effective in distinct situations, making them useful when considering more problematic structures with multiple conditioning fundamental modes, or unpredictable acting loads. While useful, these devices are costly and highly dependent on reliable energy sources and heavy maintenance, limiting their viability.

Semi-active systems consist of passive forms of control where the stiffness and damping may be occasionally re-calibrated for different situations. Such systems seek to achieve the adaptability of active systems, while retaining the economic viability of passive systems, circumventing the disadvantages of both methods.

1.2. Brief Summary of Dissertation

This dissertation will focus primarily on the study of passive control systems, specifically TMDs. Their behaviour will be analysed for different situations, including the consideration of non-linear effects, namely the presence of friction.

The second chapter consists of a more extensive description of a TMD's dynamic behaviour, and how the structure it attempts to control influences the way it is tuned.

The method used to determine the behaviour of a structure with an installed TMD will be covered in chapter 3. This chapter concerns the use of State-Space modelling.

Chapter 4 consists of an analysis of footbridges, and how to control its vibrations using TMDs. The influence of span in a footbridge's behaviour is also covered in this chapter.

The effects of non-linear factors will be covered in chapter 5. The results will be compared to the linear analysis made in previous chapters.

The sixth and final chapter summarises the conclusions found in this dissertation, as well as possible future work.

2

TUNED MASS DAMPERS AND THEIR BEHAVIOUR

2.1. OVERVIEW

Tuned Mass Dampers, also known as TMDs, are a passive vibration control technique. They are devices that can be mounted on structures to greatly reduce the amplitude of mechanical vibrations, without further external energy supply.

TMDs consist of a mass connected to the structure itself through a parallel spring and damper. As depicted below in Figure 2.1, this mass (m2) can act as an additional degree of freedom attached to the primary structure (m1), acting with its own damping and stiffness.

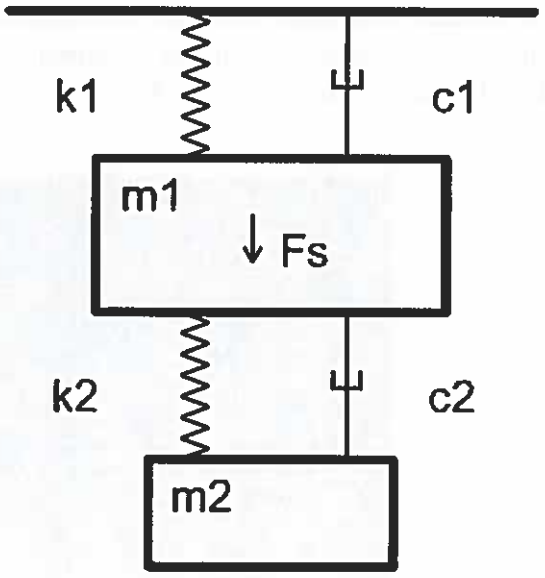


Figure 2.1 – Simplified schematic of a TMD attached to a structure

The natural frequency of this device can be tuned so that the reacting force from these elements acts against the movement of the main structure, buffeting the structure's own movement and reducing its amplitude.

2.2. Practical Application

2.2.1. General Applications

TMDs have found various uses in construction [1]. As was previously mentioned, they are a form of passive vibration control, meaning that they function without the need of any external input or energy source, as such, they are an often viable low-cost, low-maintenance solution when looking to control undesired vibrations in structures.

However, they need to be tuned to a specific fundamental mode of the primary structure, meaning that the number of TMDs to adopt is the same as the number of desired control frequencies. TMDs are preferably used in structures where undesirable vibrations are caused by predictable actions within a small frequency range. Despite this limitation, TMDs have been used in tall structures such as towers and skyscrapers, reducing the dynamic movement caused by external actions such as wind and seismic loads.

A consistently successful use of TMDs is the mitigation of movement caused by human activity, such as in stadiums and bridges, where a large amount of visitors is often observed. Human beings usually move at predictable frequencies, meaning that TMDs are an ideal solution to suppress vibrations caused by such actions.

TMDs have been used frequently in footbridges, as the conditioning fundamental frequency of their individual spans usually exhibit values close to 2 Hz, which is the same as that of walking pedestrians.

2.2.2. Applications in Actual Structures

The CN Tower in Canada, built in 1975, was one of the first large scale structures in which TMDs were implemented. Two devices, weighing 2 tons each, were installed with the goal of reducing wind-induced vibrations. This tower was considered one of the tallest in the world at 553 metres of height, before the Burj Khalifa (829 metres) surpassed it in 2007.

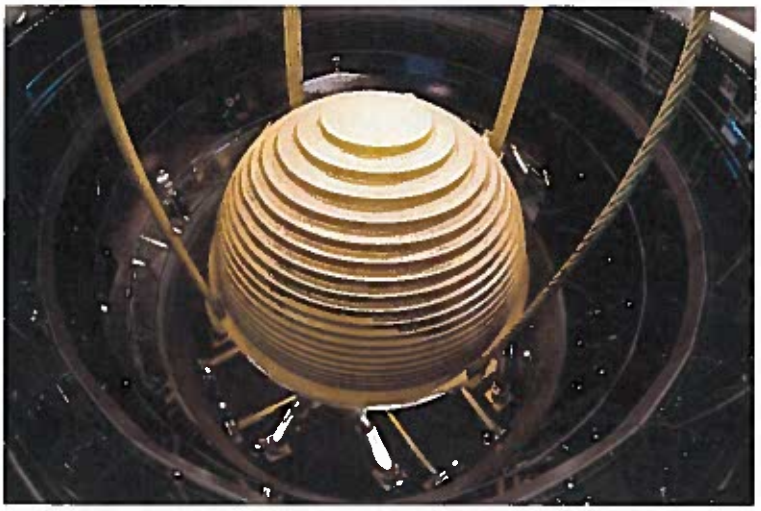


Figure 2.2 – The CN Tower in Toronto

The Taipei 101 (508 metres) in Taiwan is another noteworthy example, possessing a spherical-shaped TMD weighing 650 tons with a 5.5 metre diameter, held in suspension by four steel cables. It is considered the largest and heaviest TMD in the world. It was designed to withstand winds of 60 m/s.



a)



b)

Figure 2.3 – a) The Taipei 101 in Taiwan; b) TMD mounted in Taipei 101

TMDs have also been successfully used in large scale footbridges. One such example is the Pedro e Inês Bridge in Portugal, featuring a span of 110 metres (the bridge's total length is 275 metres). This structure is equipped with TMDs that control both vertical and lateral vibrations.

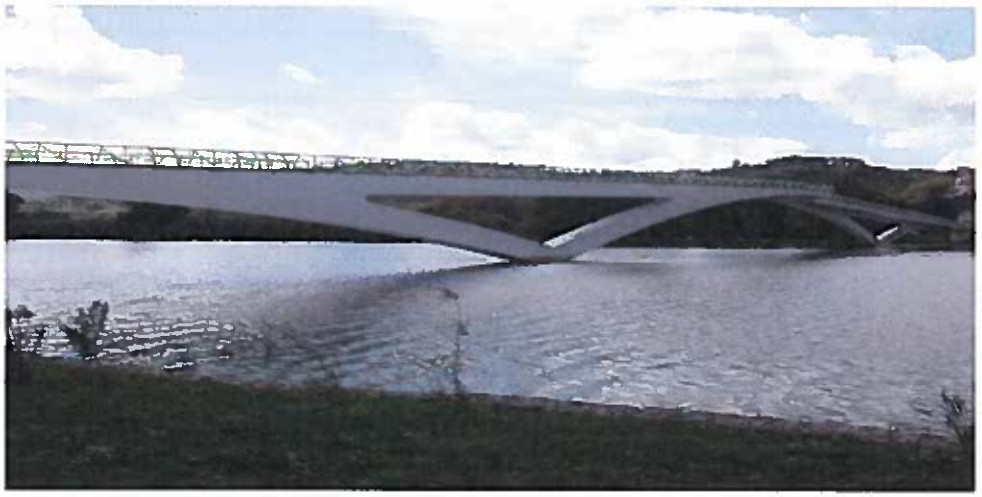


Figure 2.4 – Pedro e Inês Bridge in Coimbra

2.3 TUNING

2.3.1. DYNAMIC PARAMETERS

There are different proposed methods for the tuning a TMD, though most of them use the 2 degree of freedom (DOF) model seen in Figure 2.1.

The parameters used to define a TMD's dynamic characterisation are the mass ($\mu=m_2/m_1$) and frequency ($q_{opt}=\omega_2/\omega_1$) ratios between structure and TMD, as well as the TMD's optimal damping ratio ($\zeta_{2,opt}$).

Once the TMD's mass is defined, its other parameters may be obtained through the following expressions, according to Den Hartog (for undamped structures) [2]:

$$q_{opt} = \frac{1}{(1+\mu)} \tag{2.1}$$

$$\zeta_{2,opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{2.2}$$

Carlos Moutinho [3] suggests some abacuses to facilitate the design of TMDs with mass ratios up to 100% (for damped structures). They can be seen in figures 2.5 and 2.6:

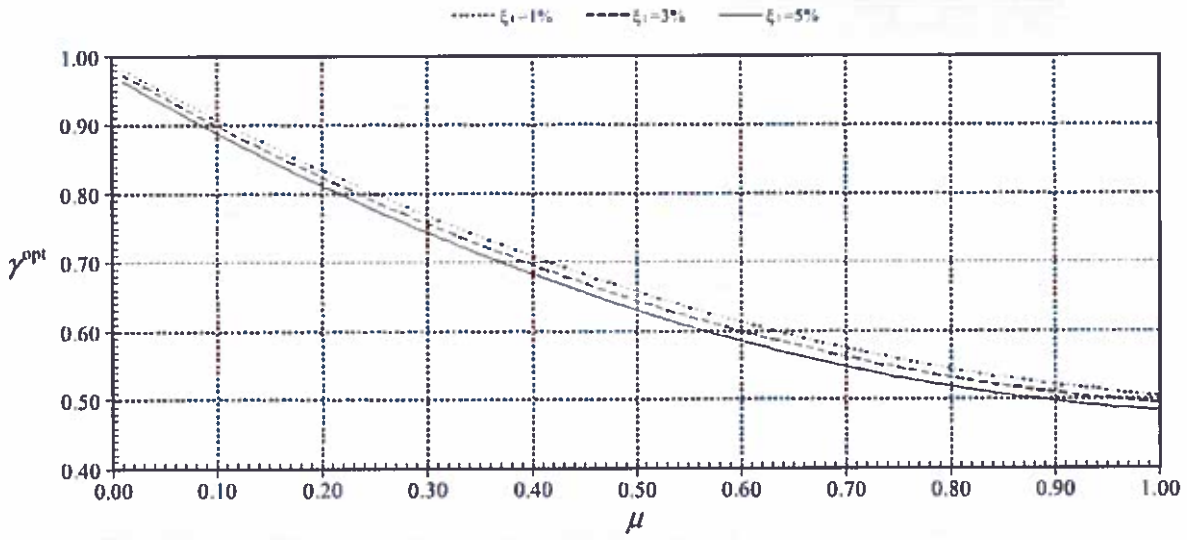


Figure 2.5 – Optimal frequency (q_{opt}) ratios of the TMD for different values of μ and ζ_1 (Moutinho)

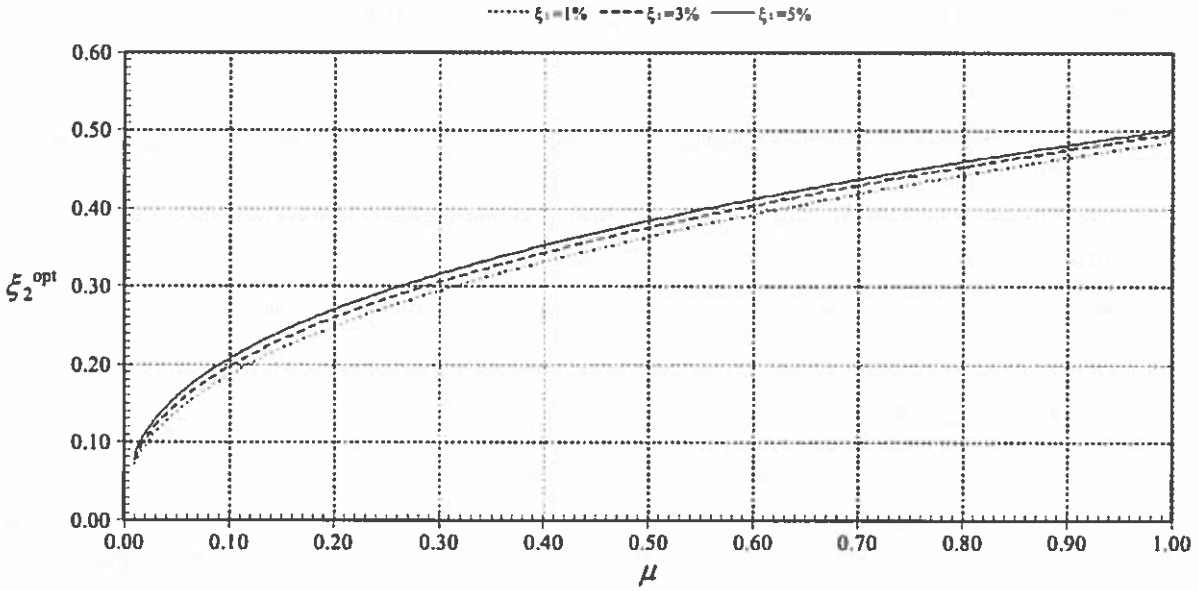


Figure 2.6 – Optimal damping ($\zeta_{2,opt}$) ratios of the TMD for different values of μ and ζ_1 (Moutinho)

For this dissertation, the values used when properly tuning a TMD were chosen based on these graphs. The exact values chosen can be seen in Table 1 below:

Table 1 – Optimum tuning parameters and corresponding effective damping ratios in a 2-DOF system

	ζ_1=0.005		ζ_1=0.01		ζ_1=0.02		ζ_1=0.03	
μ	qopt	$\zeta_{2,opt}$	qopt	$\zeta_{2,opt}$	qopt	$\zeta_{2,opt}$	qopt	$\zeta_{2,opt}$
0.01	0.9892	0.0629	0.9883	0.0636	0.98595	0.0673	0.9836	0.0710
0.02	0.9793	0.0881	0.9782	0.0889	0.97515	0.0929	0.9721	0.0969
0.03	0.9695	0.1071	0.9681	0.1080	0.96455	0.11225	0.9610	0.1165

Once these parameters are chosen, the dynamic properties of the TMD (mass M, natural frequency ω, stiffness K and damping C) are calculated through the following expressions:

$$M_{tmd} = \mu * M \tag{2.3}$$

$$\omega_{tmd} = q_{opt} * \omega \tag{2.4}$$

$$K_{tmd} = \omega_{tmd}^2 * M \tag{2.5}$$

$$C_{tmd} = 2 * \zeta_{2,opt} * M_{tmd} * \omega_{tmd} \tag{2.6}$$

Where M, K, C and ω represent the dynamic properties of the primary system.

While the TMD's contribution to the mitigation of a given structure's vibration is attributed to both stiffness and damping, it is relevant to note that its stiffness provides a larger contribution in the structure's overall response [4], making it imperative that the TMD is properly tuned.

It should also be noted that TMDs may present non-linear performance. Indeed, under certain circumstances, a TMD might not be active, which means that its influence in the movement of the main structure will be minimal. This situation may be represented by considering that the TMD's stiffness (K_{tmd}) is much higher than usual.

This situation, as well as this process, will be examined in more detail further in this dissertation.

2.3.2. Modelling TMDs

2.3.2.1. Single Degree of Freedom Systems

As previously mentioned, TMDs are connected to a single degree of freedom, acting as an additional degree of freedom of the original system. When considering a SDOF with dynamic parameters of M, C and K, the inclusion of a TMD with its own parameters will lead to a system with two degrees of freedom with the following system matrixes:

$$M_t = \begin{bmatrix} M & 0 \\ 0 & M_{tmd} \end{bmatrix} \tag{2.7}$$

$$C_t = \begin{bmatrix} C + C_{tmd} & -C_{tmd} \\ -C_{tmd} & C_{tmd} \end{bmatrix} \tag{2.8}$$

$$K_t = \begin{bmatrix} K + K_{tmd} & -K_{tmd} \\ -K_{tmd} & K_{tmd} \end{bmatrix} \tag{2.9}$$

The new 2-DOF system will have two natural frequencies instead of one, but the maximum amplitude of the response of the new system will be much lower, compared to the original SDOF system.

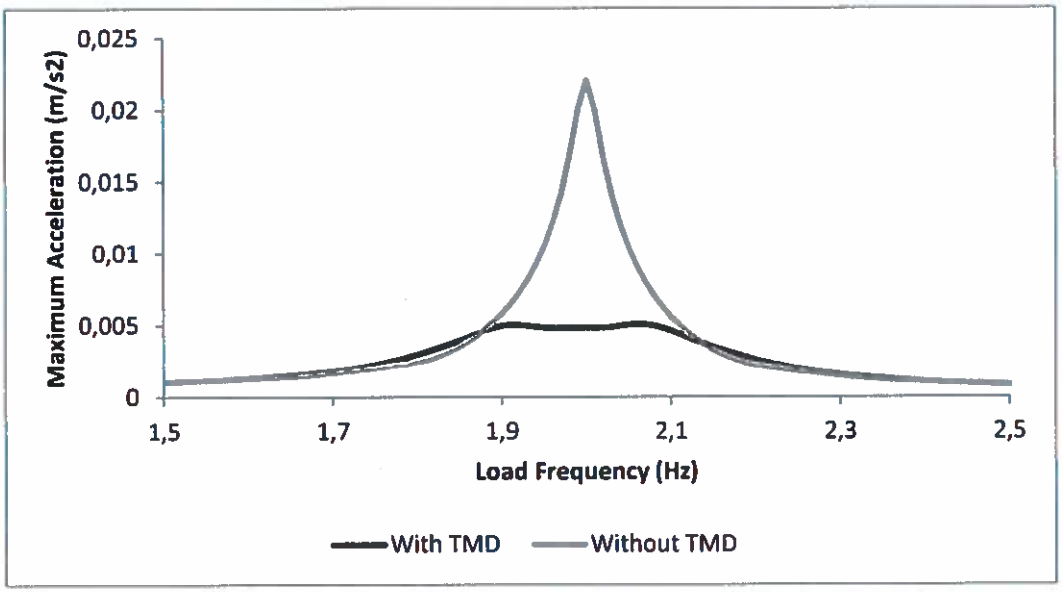


Figure 2.7 – Comparison of the dynamic amplification curves of a structure with and without an installed TMD

The new natural frequencies are still close in value to the original uncontrolled structure's, however, comparing the maximum response of a structure mounted with a properly tuned TMD and without it, it is clear that the TMD is quite effective in controlling its vibration.

2.3.2.2. Multiple Degree of Freedom Systems

The procedure described above can also be applied when considering a MDOF system. Assuming that a TMD is attached to the nth degree of freedom of a system with N degrees of freedom, the parameters of the new system can be defined as:

$$M_t = \begin{bmatrix} M_1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_n & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & M_N & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{tmd} \end{bmatrix} \tag{2.10}$$

$$C_t = \begin{bmatrix} C_{11} & \cdots & C_{1n} & \cdots & C_{1N} & 0 \\ \vdots & \ddots & \cdots & \cdots & \vdots & \vdots \\ C_{n1} & \cdots & C_{nn} + C_{tmd} & \cdots & C_{nN} & -C_{tmd} \\ \vdots & \vdots & \vdots & \ddots & 0 & \vdots \\ C_{N1} & \cdots & C_{Nn} & \cdots & C_{NN} & 0 \\ 0 & \cdots & -C_{tmd} & \cdots & 0 & C_{tmd} \end{bmatrix} \tag{2.11}$$

$$K_t = \begin{bmatrix} K_{11} & \cdots & K_{1n} & \cdots & K_{1N} & 0 \\ \vdots & \ddots & \cdots & \cdots & \vdots & \vdots \\ K_{n1} & \cdots & K_{nn} + K_{tmd} & \cdots & K_{nN} & -K_{tmd} \\ \vdots & \vdots & \vdots & \ddots & 0 & \vdots \\ K_{N1} & \cdots & K_{Nn} & \cdots & K_{NN} & 0 \\ 0 & \cdots & -K_{tmd} & \cdots & 0 & K_{tmd} \end{bmatrix} \tag{2.12}$$

In MDOF systems, due to the possible existence of more than one conditioning fundamental mode, it might be desirable to add more TMDs, associated with other degrees of freedom. For such systems where multiple TMDs are installed over several degrees of freedom, the same method can be applied.

For example, if a structure has two fundamental modes associated with two different degrees of freedom, and two TMDs are mounted on the structure in order to control both modes, then two additional degrees of freedom will be added to the system, each representing a different TMD.

3
TMD LINEAR DYNAMIC ANALYSIS

3.1. INTRODUCTION

In order to analyse the dynamic behaviour of any structure, it is necessary to establish a corresponding structural system, as well as its respective dynamic parameters, namely, its mass, stiffness and damping.

When discussing TMDs, it is often sufficient to consider systems with one or two degrees of freedom (DOFs), as each TMD is usually associated with one fundamental mode, and its associated DOF.

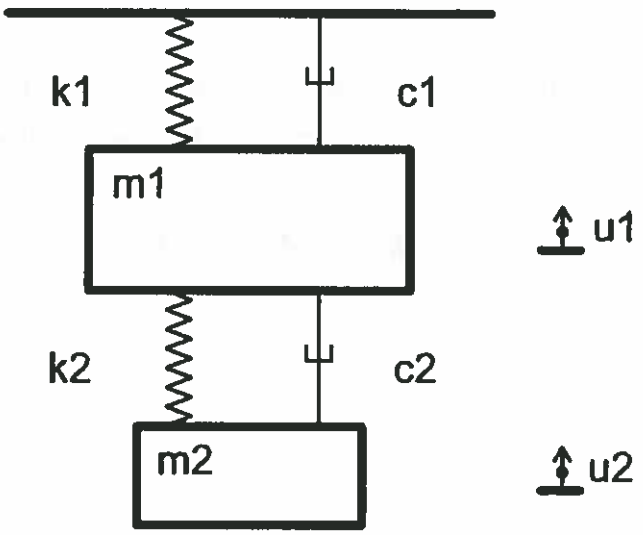


Figure 3.1 – Schematic of a two degree of freedom system

Once the additional mass is connected to a primary structure, idealised as a single degree of freedom (SDOF) system, it acts as a second degree of freedom, with its own mass, damping and stiffness (the latter two being defined when tuning the device).

Most of the theoretical elements shown when making this dynamic analysis are taken from the published works of Anil K. Chopra [5] and Ray Clough [6].

3.2. Dynamic Analysis

3.2.1. Equations of Motion

A common form of representing the motion of a structure with N degrees of freedom subject to external forces is:

$$M\ddot{u}_{(t)} + C\dot{u}_{(t)} + Ku_{(t)} = F_{(t)} \tag{3.1}$$

Where $u_{(t)}, \dot{u}_{(t)}$ and $\ddot{u}_{(t)}$ are Nx1 vectors, representing displacement, velocity and acceleration, respectively. The variables M, C and K are NxN matrixes, respectively representing the mass, damping and stiffness associated with that structure. $F_{(t)}$ is also a Nx1 vector, representing external actions in each DOF.

3.2.2. Natural Frequencies and Vibration Modes

Once the mass and stiffness of a structural system have been defined, that system's natural frequencies can be obtained through the following expression:

$$det(K - \omega^2 M) = 0 \tag{3.2}$$

Resulting in a Nx1 ω vector or, in other words, one natural frequency per DOF.

The system's vibration modes ϕ may then be obtained through the following expression:

$$(K - \omega^2 M) * \phi = 0 \tag{3.3}$$

Resulting in a NxN matrix, or rather, N vectors sized Nx1:

$$\phi = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_N] = \begin{bmatrix} \phi_{11} & \phi_{21} & \cdots & \phi_{N1} \\ \phi_{12} & \phi_{22} & \cdots & \phi_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1N} & \phi_{2N} & \cdots & \phi_{NN} \end{bmatrix} \tag{3.4}$$

The solution may be easily obtained by assigning an unitary value to one of the components of each ϕ_i vector, thus normalizing them immediately.

3.2.3. Damping Matrix

While mass and stiffness matrixes can be obtained through the analysis of a particular structure, the damping matrix must be obtained through other methods. The method chosen in this dissertation was the superposition of modal damping matrixes.

This method consists of calculating a modal damping matrix for each mode i through this expression:

$$c_i = M * \left(\sum_{i=1}^{N} \frac{2\zeta_i \omega_i}{M_i} * \phi_i \phi_i^T\right) * M \tag{3.5}$$

Where ζ_i, ω_i, and M_i are the damping coefficient, natural frequency, and modal mass associated with each mode, respectively. The modal mass is given by:

$$M_i = \phi_i^T M \phi_i \tag{3.6}$$

The dynamic system's real damping matrix C can now be obtained through the sum of all N individual modal damping matrixes:

$$C = \sum_{i=1}^{N} c_i \tag{3.7}$$

3.2.4. Duhamel's Integral

A SDOF system's response to any given force can be obtained through Duhamel's Integral:

$$u(t) = \frac{1}{m*\omega_d} * \int_0^t p_{(\tau)} e^{-\zeta\omega(t-\zeta)} sin(\omega_d(t-\tau))\, d\tau \tag{3.8}$$

Although this expression will not be used in this dissertation, it does provide insight into the inverse relation between the structural system's mass and its displacement when subject to external loads.

In MDOF systems, a modal version of this integral may be expressed, keeping the same inverse relation between mass and displacement.

3.2.5. State-Space Representation

Dynamic systems may also be represented by State-Space theory [7]. This is a formulation that may be applied to a system with any number of DOFs, and uses the matrixes M, C and K, making this a valid approach for different systems of varying complexity.

The method takes into consideration three different variables: state variables, input variables and output variables. State variables reflect the system's response (or "state") at each instant. They are represented by the variable $x_{(t)}$, a 2Nx1 vector formed by the displacement and velocity of each DOF.

$$x_{(t)} = \begin{bmatrix} u_{1(t)} \\ u_{2(t)} \\ \vdots \\ u_{N(t)} \\ \dot{u}_{1(t)} \\ \dot{u}_{2(t)} \\ \vdots \\ \dot{u}_{N(t)} \end{bmatrix} \tag{3.9}$$

Input variables correspond to the external forces being applied to the system in each instant, and are represented by the variable $F_{(t)}$, a Nx1 vector.

$$F_{(t)} = \begin{bmatrix} f_{1(t)} \\ f_{2(t)} \\ \vdots \\ f_{N(t)} \end{bmatrix} \tag{3.10}$$

Output variables are only needed when certain input or state variables are unknown. They can be represented by a Nx1 vector $y_{(t)}$, though they are not needed for this situation.

State-Space representation is also characterised by three matrixes: a State Matrix (A), an Input Matrix (B) and an Output Matrix (that will not be used). Matrixes A and B can be determined directly through the mass, damping and stiffness matrixes:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1} * K & -M^{-1} * C \end{bmatrix} \tag{3.11}$$

$$B = \begin{bmatrix} 0 \\ M^{-1} * I \end{bmatrix} \tag{3.12}$$

Where I is a NxN sized identity matrix.

These two matrixes then allow us to determine the auxiliary matrixes E and G (respectively sized 2Nx2N and 2NxN).

$$E = e^{\Delta t * A} \tag{3.13}$$

$$G = [A^{-1} * (E - J)] * B \tag{3.14}$$

Where J is a 2Nx2N identity matrix. It should be noted that the matrixes E and G are usually constant, because the structural system's dynamic parameters do not change with time.

This method leads to the determination of the system response to an external action through the following expression:

$$x_{(t+\Delta t)} = E * x_{(t)} + G * F_{(t)} \tag{3.15}$$

For each iteration, we can introduce the initial state $x_{(t_i)}$ and acting force $F_{(t_i)}$, along with the E and G matrixes, and determine the new state vector $x_{(t_{i+1})}$. This value can then be used, along with $F_{(t_{i+1})}$, to determine the following iteration $x_{(t_{i+2})}$.

While the state vector only represents displacement and velocity, it is entirely possible to determine the acceleration of the response. Considering the variations of speed in known small intervals of time (Δt), the system's acceleration can be obtained with a simple differential:

$$a_{(t_i)} = \frac{v_{(t_i)} - v_{(t_{i-1})}}{\Delta t} \tag{3.16}$$

Due to its iterative nature, this method is quite versatile, since it allows us to introduce changes in the system's properties at any specific instant of the analysis (for example, a forced displacement, or sudden changes in state or input variables). Furthermore, it is fairly simple to apply it through informatics means, with programs of numerical analysis such as MATLAB.

The results obtained through State-Space representation are quite accurate, provided that the chosen time interval between iterations is sufficiently short (an interval of 0.01 seconds suffices in most cases), and that external forces can be properly characterised.

Finally, while we will only delve into the implementation of non-linear effects further into this dissertation, it is relevant to mention that, due to this method's versatility, it is possible to consider that a structural system's properties might change with time, such as non-linear forms of damping or the sudden appearance of additional degrees of freedom.

3.3. Examples using TMDs

As was previously mentioned, TMDs function as additional DOFs attached to a primary structure, meaning that any system that includes tuned mass dampers is a multiple degree of freedom (MDOF) system. The simplest example when considering TMDs is attaching one device to a SDOF system, turning it into a two DOF system.

Considering, for example, a SDOF system with a mass of 10 ton, a stiffness of 1579 KN/m (meaning a natural frequency of f=2 Hz, or $\omega=4\pi$ rad/s) and a damping coefficient of 1% (C=2.513 KN.s/m), submitted to the following harmonic load:

$$F_{(t)} = 50 * (4\pi * t)\ (KN) \tag{3.17}$$

The resulting resonant response can be seen in Figure 3.2, determined using State-Space Theory through a MATLAB script (found in Appendix A2).

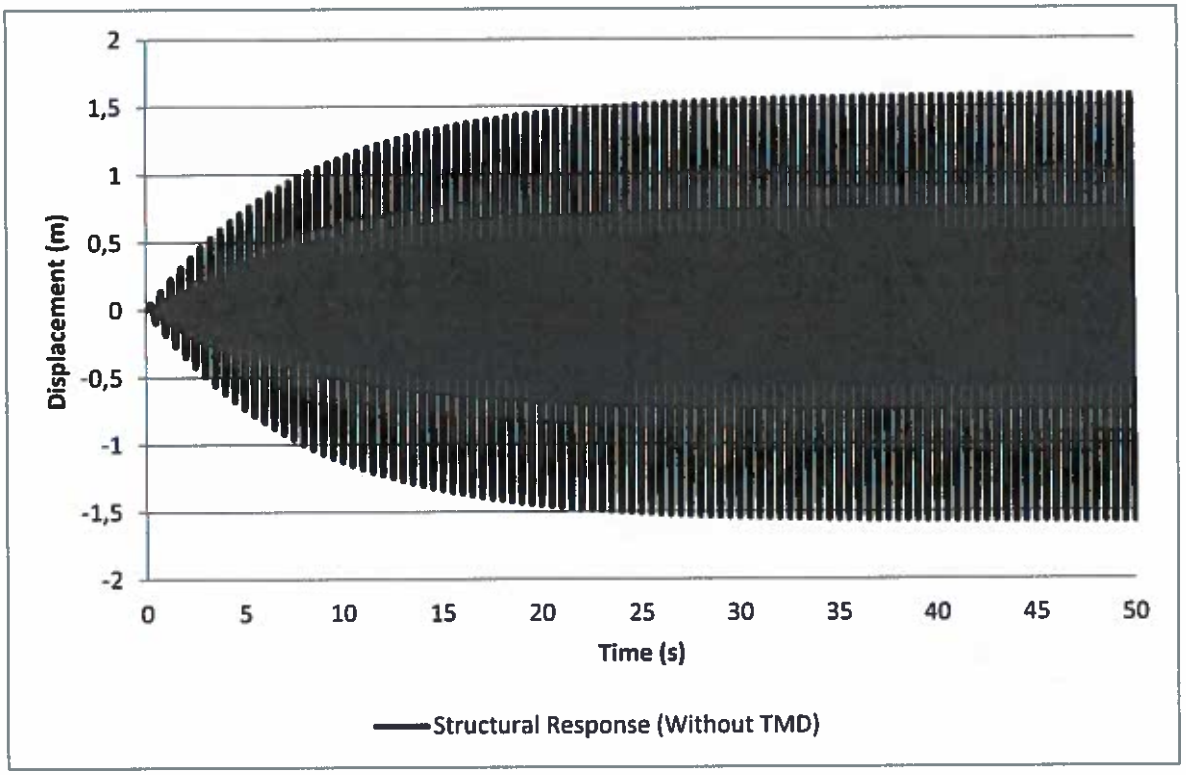


Figure 3.2 – Structural Response of a SDOF system

If we now mount a TMD on this structure using a mass ratio of μ=0.02, it becomes a 2 DOF system with the following dynamic parameters:

$$M_{tmd} = \mu * M \tag{3.18}$$

$$\omega_{tmd} = q_{opt} * \omega \tag{3.19}$$

$$C_{tmd} = 2 * \zeta_{2,opt} * M_{tmd} * \omega_{tmd} \tag{3.20}$$

$$K_{tmd} = \omega_{tmd}^2 * M \tag{3.21}$$

$$M_t = \begin{bmatrix} 10 & 0 \\ 0 & 0.2 \end{bmatrix} (ton) \tag{3.22}$$

$$C_t = \begin{bmatrix} C + C_{tmd} & -C_{tmd} \\ -C_{tmd} & C_{tmd} \end{bmatrix} = \begin{bmatrix} 2.907 & -0.3937 \\ -0.3937 & 0.3937 \end{bmatrix} (KN.s/m) \tag{3.23}$$

$$K_t = \begin{bmatrix} K + K_{tmd} & -K_{tmd} \\ -K_{tmd} & K_{tmd} \end{bmatrix} = \begin{bmatrix} 1609.4 & -30.3 \\ -30.3 & 30.3 \end{bmatrix} (KN/m) \tag{3.24}$$

$$f_t = \begin{bmatrix} 2.1239 \\ 1.8438 \end{bmatrix} (Hz) \tag{3.25}$$

Considering now a harmonic load in resonance with the new system:

$$F_{(t)} = \begin{bmatrix} 50 * sin(2\pi * 2.1239 * t) \\ 0 \end{bmatrix} (KN) \tag{3.26}$$

The new resonant response (in light grey) for the first degree of freedom becomes:

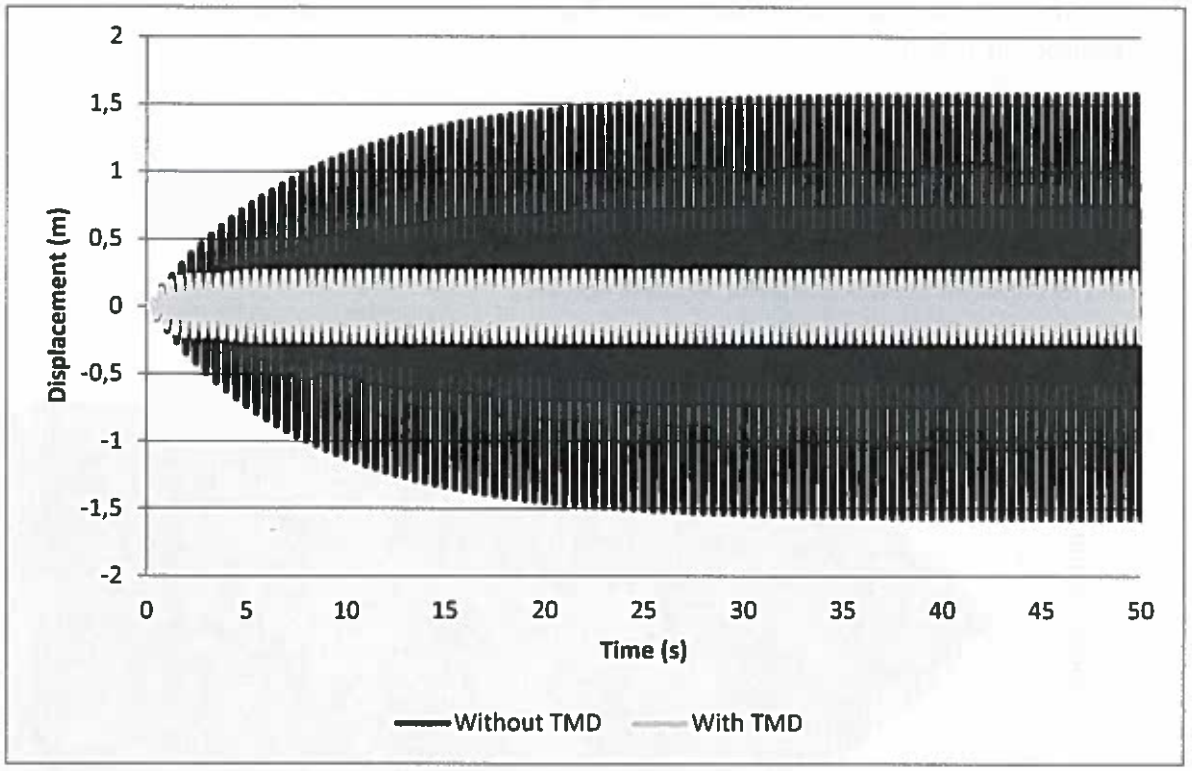


Figure 3.3 – Structural Response of a system with and without TMD

Exhibiting an amplitude roughly five times lower than that of a structure without TMD.

3.3.1. BLOCKED TMD

It is also possible to emulate a behaviour similar to that of the SDOF system, while using a dynamic system with two degrees of freedom. In order to do so, we only need to significantly increase the rigidity of the TMD by an arbitrarily high amount, and represent that change in the stiffness matrix K. This will simulate a situation in which the TMD is fully attached to the structure, or "blocked", but not functioning.

The variable K_{tmd} will thus be increased here by a factor of one thousand:

$$K_t = \begin{bmatrix} K + 1000 * K_{tmd} & -1000 * K_{tmd} \\ -1000 * K_{tmd} & 1000 * K_{tmd} \end{bmatrix} = \begin{bmatrix} 31849.32 & -30270 \\ -30270 & 30270 \end{bmatrix} (KN/m) \tag{3.27}$$

The matrixes M and C stay the same, but the structure's new natural frequency must be determined:

$$f_t = \begin{bmatrix} 62.534 \\ 1.980 \end{bmatrix} (Hz) \tag{3.28}$$

Taking into account that the frequency of 62.5 Hz is associated with the blocked TMD itself, the soliciting harmonic load in consideration will achieve resonance with the structure with the natural frequency of 1.98 Hz:

$$F_{(t)} = \begin{bmatrix} 50 * sin(3.96\pi * t) \\ 0 \end{bmatrix} (KN) \tag{3.29}$$

The response for the first degree of freedom becomes:

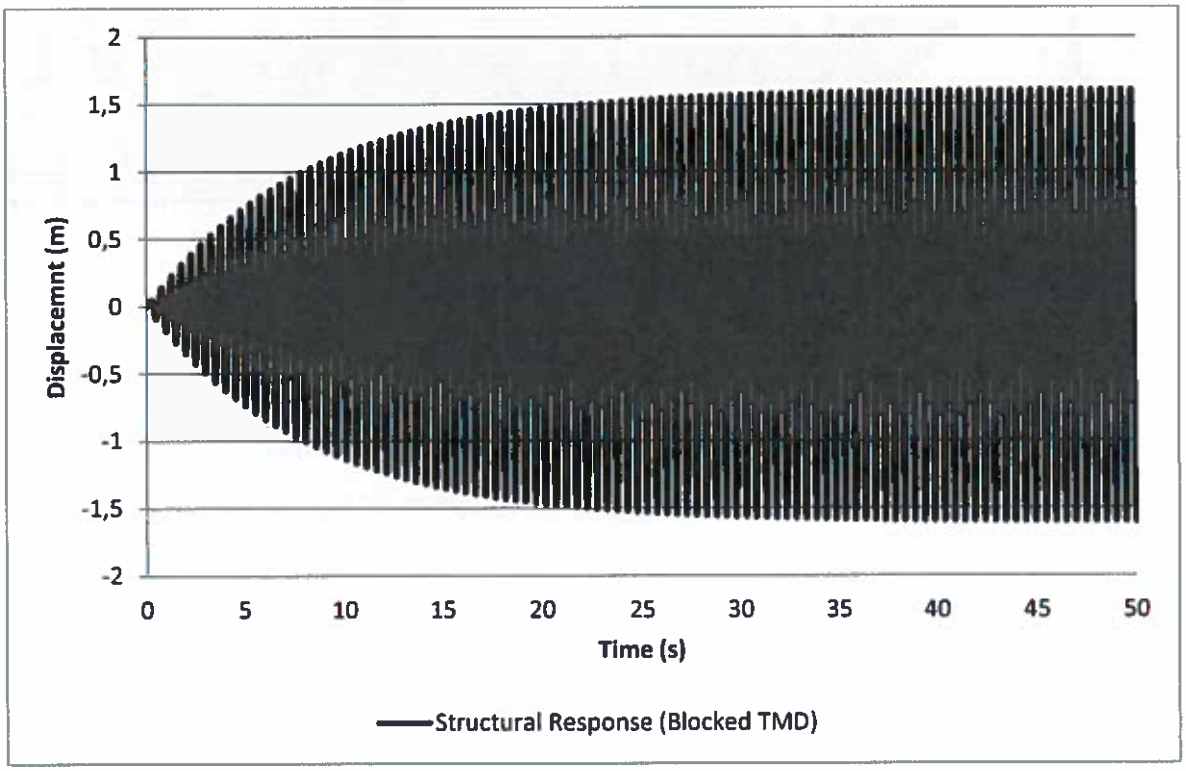


Figure 3.4 – Structural Response of a two DOF system with a blocked TMD

The response seen in Figure 3.4 is nearly identical to the one seen in figure 3.2. The contribution of a blocked TMD to a structure's overall response can be assumed to be negligible.

4
VIBRATION CONTROL IN FOOTBRIDGES

4.1. Overview

Despite the existence of additional methods of vibration control, TMDs remain a viable solution for structures such as footbridges, where conditioning external loads are predictable.

Footbridges are relatively simple systems to work with, and the methods and models described so far can be easily applied when considering the installation of TMDs in such structures.

There are, however, a few particular considerations to take into account when analysing footbridges, such as proper load characterisation, or the effect of span length on the structure's behaviour.

Throughout this chapter, only vertical movement will be analysed. Horizontal vibrations in bridges are also an important concern in their behaviour and design, but it is an issue that falls outside of the scope of this dissertation.

4.2. Dynamic Analysis

4.2.1. Structural Model

When making such a broad analysis, it is important to find models as simplified as possible, knowing that, in most cases, even the most complex structures can be reduced to simpler systems. Simply supported footbridges can be modelled as simple structures, meaning that this is the most adequate system to examine in this study.



Figure 4.1 – Simple Beam, idealised as a SDOF system

The structure in question is a beam, held together by two pinned supports. In most cases, it is sufficient to consider a single vertical degree of freedom in its mid-span (pictured in figure 4.1), as it represents its biggest displacement and the most problematic deformation in the beam.

The modal mass M associated with this DOF is the one concentrated at the middle of the span, that is, half of the beam's entire mass[8]. Considering m as the beam's distributed mass (ton/m), and L as its span, the modal mass is given by:

$$M = m * L/2 \tag{4.1}$$

Assuming that its distributed mass m is 1 ton per metre, we have:

$$M = L/2 \tag{4.2}$$

The stiffness K can be determined through the following expression:

$$K = 48EI/L^3 \tag{4.3}$$

Where E is the Young's modulus of the beam's material, and I is the moment of inertia of its section.

For the purposes of this study, it is considered that the beam's natural frequency associated with this DOF is f=2 Hz, a frequency very close to that of walking pedestrians. This assumption leads to the beam's generalized vertical stiffness in mid span.

$$K = (2\pi f)^2 * M = 16\pi^2 * M \tag{4.4}$$

4.2.2. Load Characterisation

The most common load exciting footbridges is that of walking pedestrians, therefore, the load induced by a walking pedestrian will be the principal conditioning force in consideration throughout the remainder of this chapter. To this end, a function that represents a single average pedestrian walking at normal speed must be defined.

The load function was determined through a simplification of the general expression, considering only a single harmonic [4]:

$$Fp_0(t) = G * \alpha_1 * sin(2 * \pi * fp * t) \tag{4.5}$$

Where fp is the step frequency, G is the pedestrian's weight (often considered 700 N) and α_1 is an adimensional coefficient given by:

$$\alpha_1 = (0.43 * fp - 0.38) \tag{4.6}$$

The load function for a simple beam can be obtained by multiplying function 4.5 by the beam's modal configuration $\Phi(t)$, which is defined not by the beam's length, but by the time taken for a walking pedestrian to cross it. This amount of time, for a walking pedestrian, can be determined from the beam's length, and assigning a velocity of 0.65 m/s to a walking pedestrian, resulting in:

$$tcross \approx 0.65 * L \tag{4.7}$$

$$\Phi(t) = sin(\frac{\pi * t}{tcross}) \tag{4.8}$$

Multiplying $Fp_0(t)$ by $\Phi(t)$, we obtain the load function caused by a single pedestrian, crossing a simply supported bridge:

$$Fp(t) = G * \alpha_1 * sin(2 * \pi * fp * t) * sin(\frac{\pi * t}{tcross}) \tag{4.9}$$

Different walking frequencies were considered for this load. If the natural frequency of a footbridge without TMD is 2 Hz, the system's natural frequencies are slightly different than that when considering the existence of a TMD (2.1239 Hz and 1.8438 Hz in the example seen in subchapter 3.3), even if blocked (1.980 Hz in the example seen in 3.3.1). Appendix A1 shows a function (written in MATLAB) that uses equation 4.9 to create a matrix representative of a pedestrian's walking load according to specific walking frequencies and beam span lengths.

4.2.3. Example

The example seen in subchapter 3.3 is a SDOF system with a specific natural frequency (2 Hz). It has 10 ton of mass, the same as a simple beam with span of 20 metres (following the considerations made with the equations 4.1 and 4.2).

A properly tuned TMD (as seen in subchapter 2.3 of this dissertation) with μ=0.02, mounted in the beam's mid-span, produces a system with two DOFs, with the same dynamic parameters as the ones seen in that example:

$$M_t = \begin{bmatrix} 10 & 0 \\ 0 & 0.2 \end{bmatrix} (ton) \tag{4.10}$$

$$C_t = \begin{bmatrix} 2.907 & -0.3937 \\ -0.3937 & 0.3937 \end{bmatrix} (KN.s/m) \tag{4.11}$$

$$K_t = \begin{bmatrix} 1609.4 & -30.3 \\ -30.3 & 30.3 \end{bmatrix} (KN/m) \tag{4.12}$$

$$f_t = \begin{bmatrix} 2.1239 \\ 1.8438 \end{bmatrix} (Hz) \tag{4.13}$$

The load function of a walking pedestrian is the following (taking into consideration that the average person weighs 700 N, or 0.7 KN):

$$Fp(t) = 0.7 * \alpha_1 * sin(2 * \pi * fp * t) * sin(\frac{\pi * t}{13}) \ (KN) \tag{4.14}$$

Pedestrians usually walk at a frequency close to 2 Hz, with small variations. When analysing the effect of pedestrian action on a footbridge, it is important to set the walking frequency so that it is in resonance with the structure. As seen in the previous calculations, the conditioning frequencies are 1.98 Hz with a blocked TMD, and the values of 2.1239 and 1.8438 Hz when considering an unblocked TMD, even though the beam's natural frequency was 2 Hz at first.

The load function in resonance with a beam mounted with a blocked TMD (fp=1.98 Hz) becomes:

$$Fp(t) = 0.336 * sin(3.96 * \pi * t) * sin(\frac{\pi * t}{13}) \tag{4.15}$$

While the beam with an unblocked TMD has two natural frequencies, it is preferable to choose the higher frequency when characterising the pedestrian's walking speed, as it leads to a higher load amplitude, due to the α_1 parameter. Figure 4.2 shows the maximum registered acceleration of the beam for various different walking frequencies, making this discrepancy clearer.

Taking this factor into consideration, the load function in resonance with a beam mounted by a fully functioning TMD (fp=2.1239) becomes:

$$Fp(t) = 0.3733 * sin(4.2478 * \pi * t) * sin(\frac{\pi * t}{13}) \tag{4.16}$$

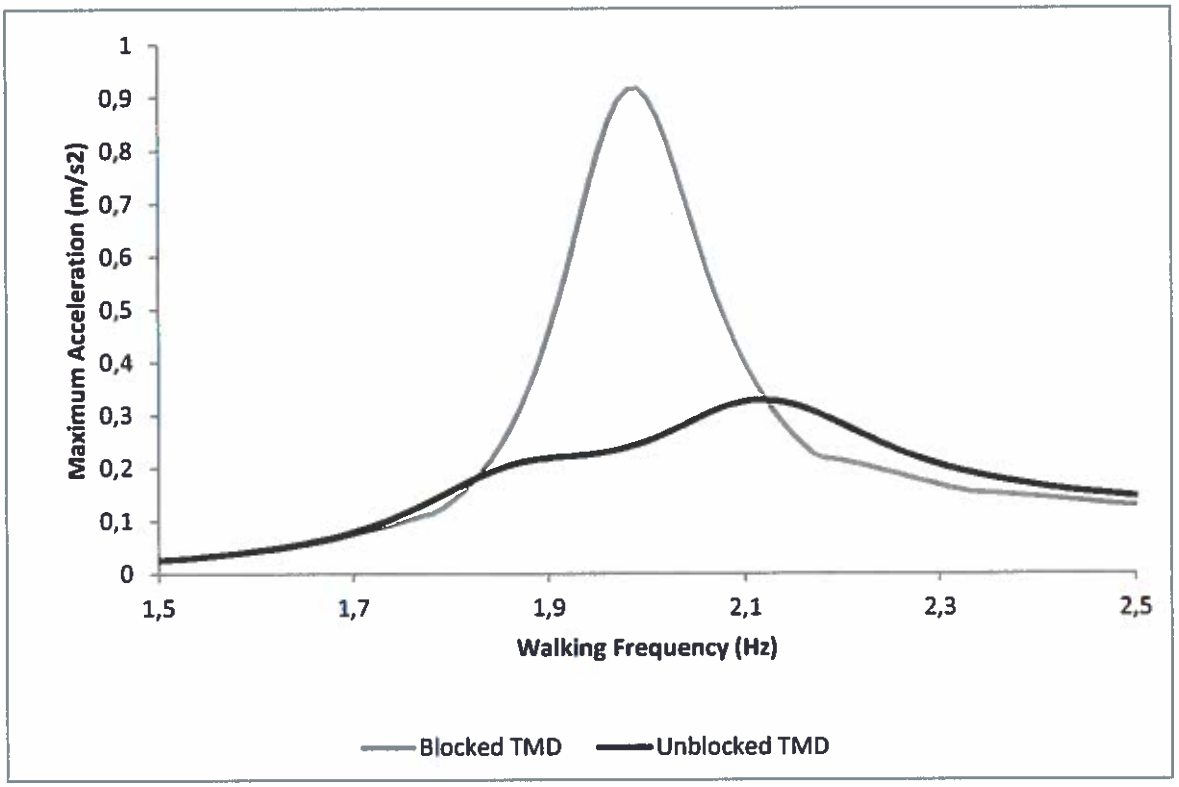


Figure 4.2 – Maximum system response when considering the action of a walking pedestrian

The following graphs compare the response from this 20 metre beam to two different actions: that of a walking pedestrian and a constant harmonic load with the same amplitude, both of them resonant with the structure. That frequency is 1.98 Hz when the TMD is blocked:

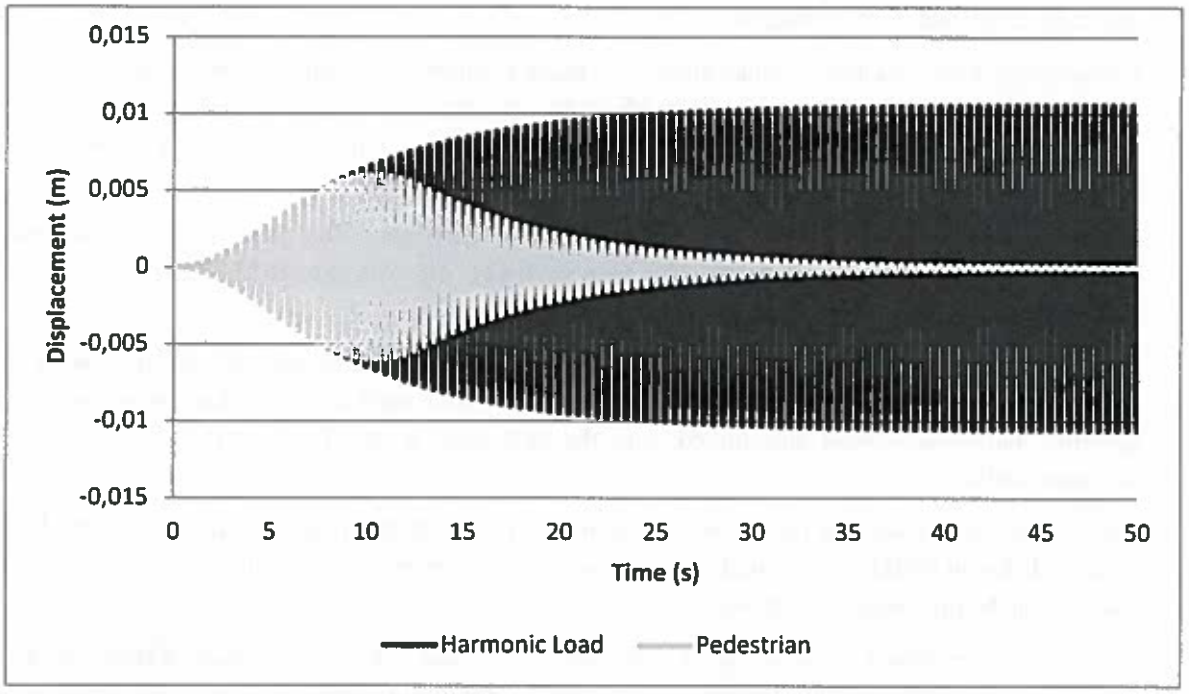


Figure 4.3 – Response of a simple beam to two different loads (Blocked TMD)

The same procedure was made when considering that there is a TMD in normal operation mounted on the beam. Both external loads were determined taking into account the system's new natural frequency of 2.1239 Hz:

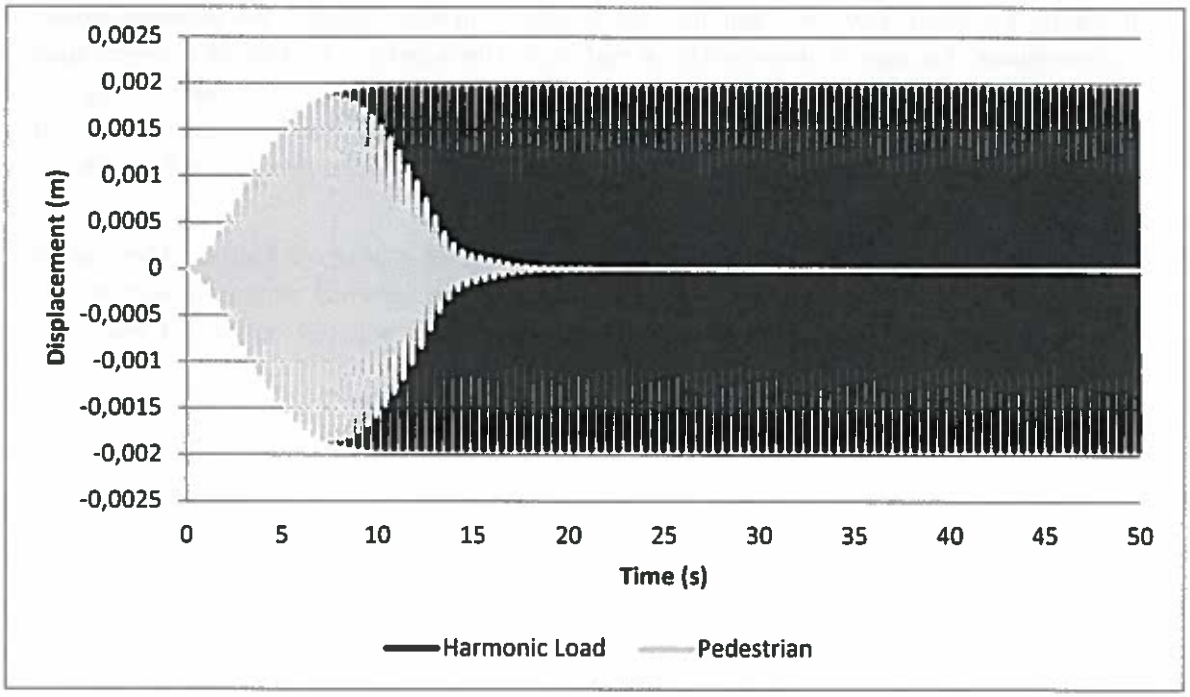


Figure 4.4 – Response of a simple beam to two different loads (Unblocked TMD)

4.3. Influence of Beam's Span

The span (L) of a beam directly influences its mass and stiffness, as seen in equations 4.2 and 4.3 and, when considering footbridges, it is also important to note that pedestrians' crossing time is lengthened, affecting their load on the structure.

Considering these variables, a small study was made regarding the influence of the beam's span on the system response to a walking pedestrian. Much like the previous example, the mass of the bridge was set to 1 ton/m, and its natural frequency was defined as 2 Hz, a problematic value when considering pedestrian loads, as was previously seen.

Different spans were set from 10 to 50 metres. For each case, the mass and load function were calculated, and a test similar to the one seen in figure 4.3 was run in MATLAB. The structure's maximum acceleration was measured for each test.

When considering TMDs, the same process was done, but also taking into account that the structure's natural frequency is different, depending on the TMD. For each test, the changes to the structure's dynamic parameters were determined, and the pedestrian's step frequency was defined as to be resonant with it.

The results can be seen in figures 4.5 through 4.8. Different damping coefficients for the bridge, as well as different TMDs (as defined by their mass ratio μ, meaning that their mass also varies with the mass of the beam), were considered.

Despite the lengthened crossing times from the pedestrians, longer spans lead to reduced maximum vibrations, due to their increased mass. Furthermore, vibration control is comparatively more effective for longer spans.

The graphs obtained through this procedure should be useful when making preliminary dynamic analyses of future bridges. They also provide a pondered estimate of the sort of influence that TMDs have on bridges of different spans.

It should be noted, however, that the results shown in these graphs are idealised under specific circumstances that may be improbable in real cases (for example, bridges of a longer spans do not often exhibit a natural frequency of 2 Hz, even if that would be the worst case for any footbridge). However, the values seen in these graphs can be normalised according to the specific situation in analysis. The subchapter below provides one such procedure when dealing with distributed masses different than 1 ton/m.

Finally, these tests were run when considering the action of a single pedestrian. There are other load forms to consider when designing bridges (such as the heightened actions caused by a crowd of people), but these require entirely different considerations, and fall outside of the scope of this analysis.

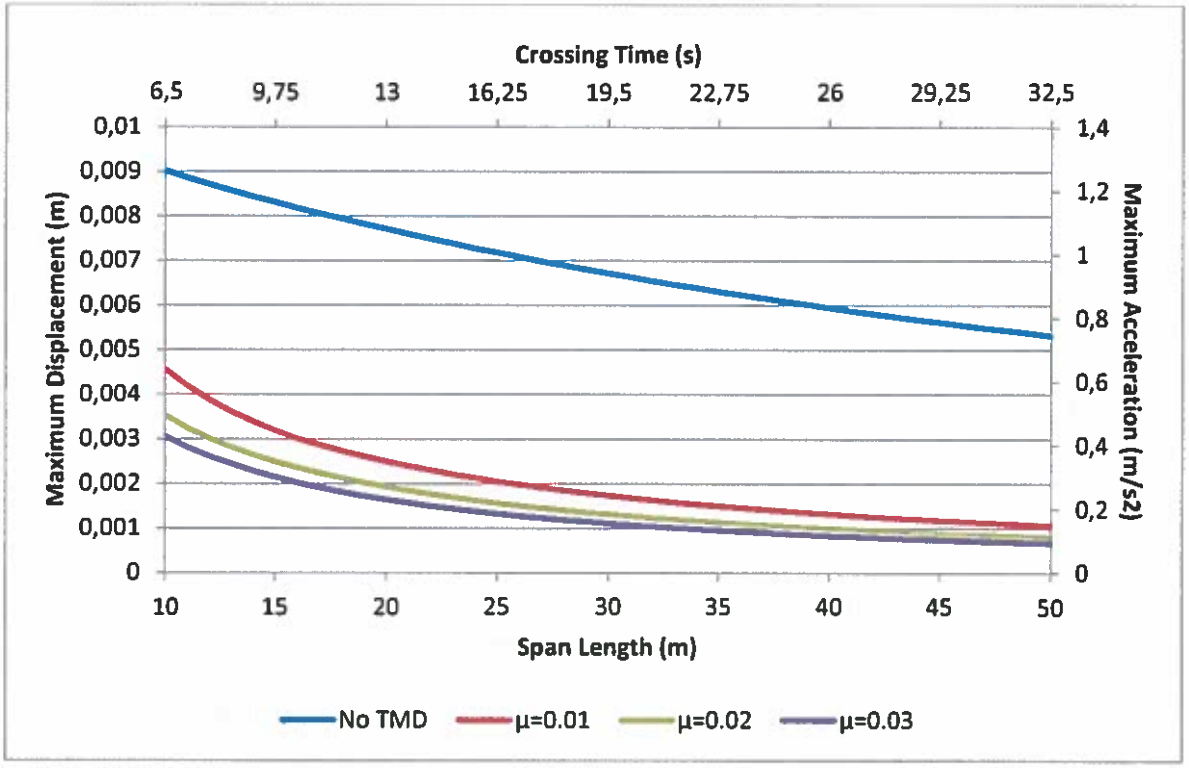


Figure 4.5 – Maximum Accelerations of a simple beam (ζ_1=0.5%)

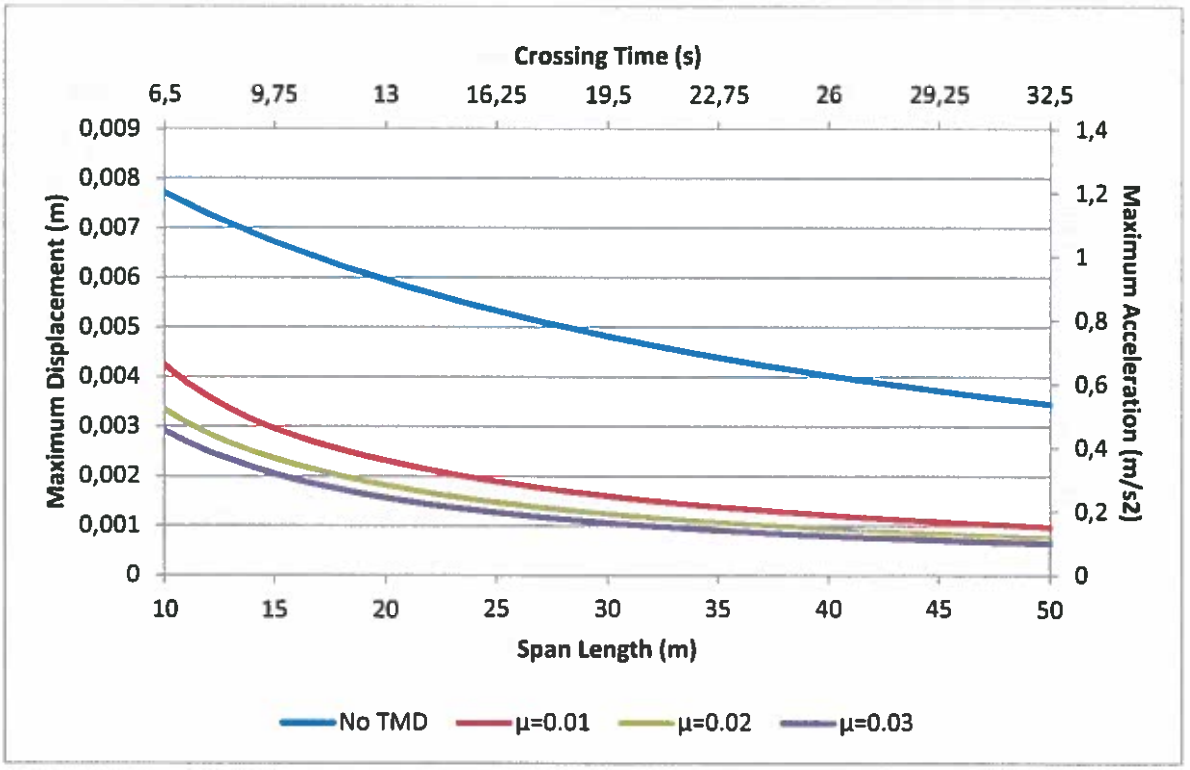


Figure 4.6 – Maximum Accelerations of a simple beam (ζ_1=1%)

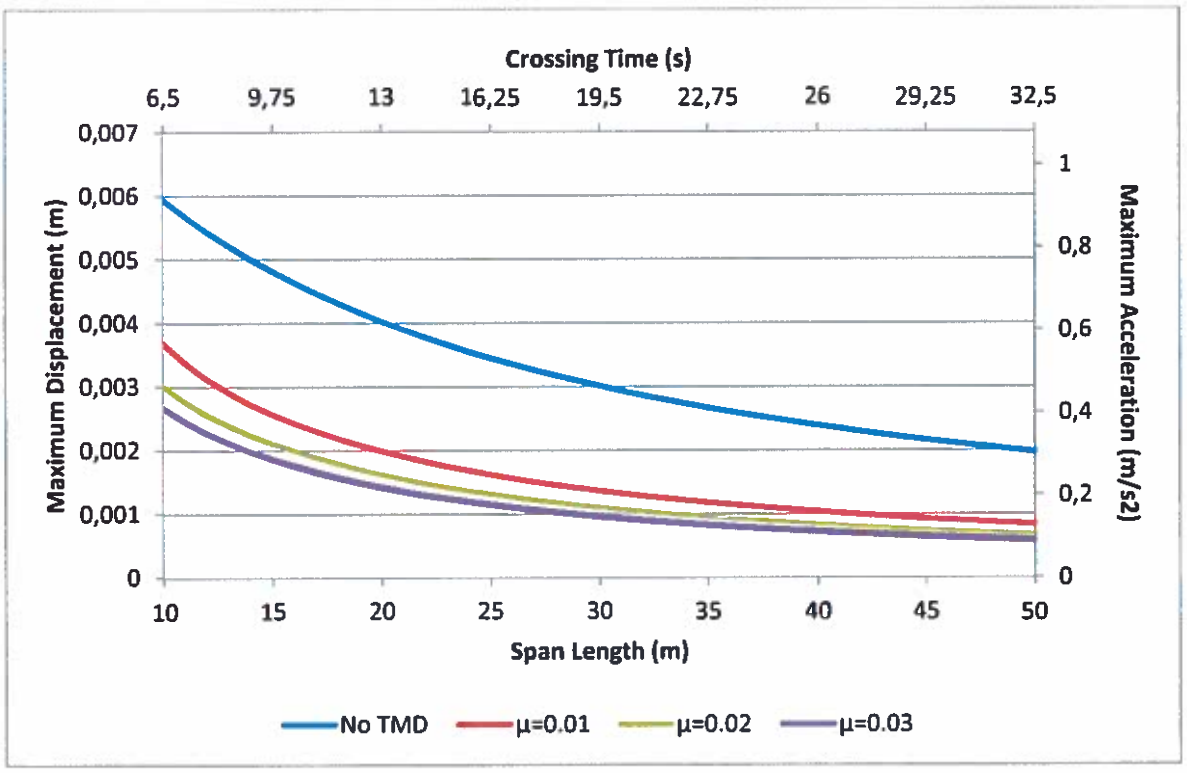


Figure 4.7 – Maximum Accelerations of a simple beam (ζ_1=2%)

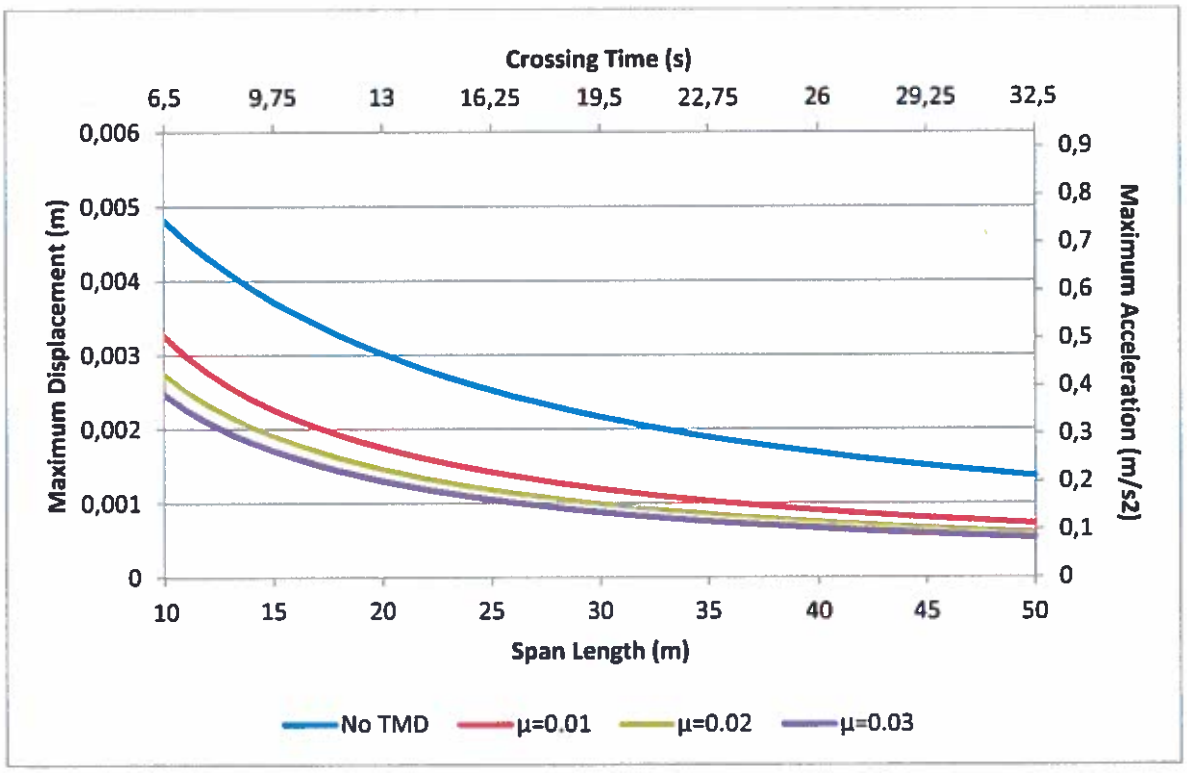


Figure 4.8 – Maximum Accelerations of a simple beam (ζ_1=3%)

4.3.1. VARIATIONS IN MASS

The previous graphs were made with the consideration that the bridge in study had a mass of 1 ton/m. This should not hamper their utility, however, as Duhamel's integral shows that a structural system's displacement – and by extension its acceleration – is inversely proportional to its mass.

$$u(t) = \frac{1}{m*\omega_d} * \int_0^t p_{(\tau)} e^{-\zeta\omega(t-\zeta)} sin(\omega_d(t-\tau))\, d\tau \tag{4.17}$$

In other words, if two different systems have identical natural frequencies and damping, but different mass, the ratio of their respective responses to any given action can be given by the inverse ratio of their masses.

$$\frac{u_1}{u_2} = \frac{m_2}{m_1} \tag{4.18}$$

This means that the previous graphs may be used for footbridges with a distributed mass different from 1 ton/m, provided that the system response is corrected by equation 4.18.

4.3.1.1. Example

Here is the test run for a beam with a span of 30 metres, weighing 1 ton/m, damping coefficient of 1%, without TMD:

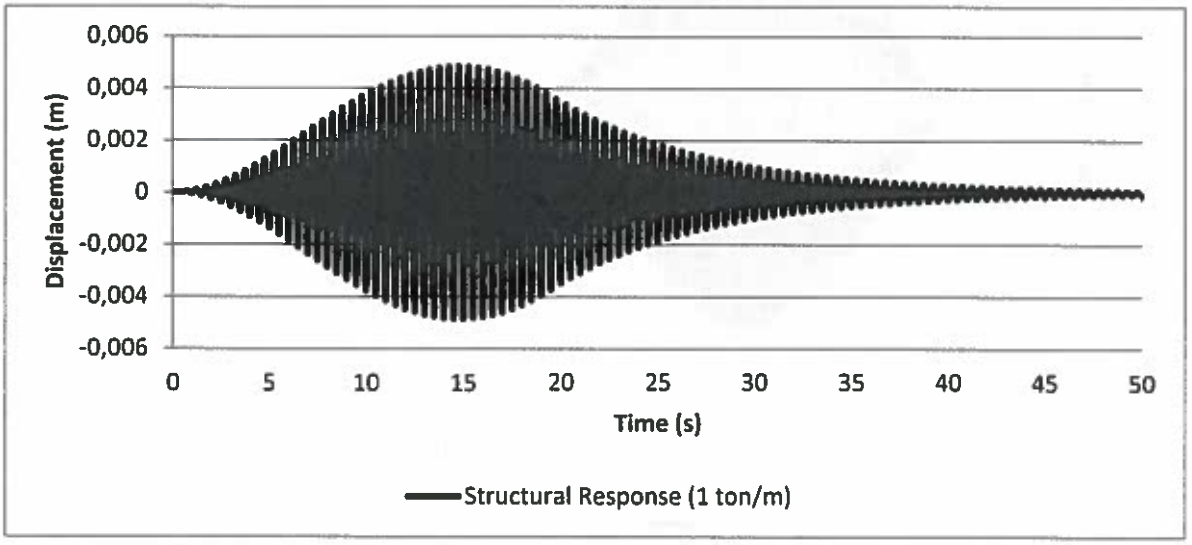


Figure 4.9 – Structural Response considering 1 ton/m

Figure 4.10 shows the same test, but considering that the beam weighs 2 ton/m. As expected, the measured displacement (Maximum 2.43 mm) is half of that of the one measured in Figure 4.9 (Maximum: 4.86 mm).

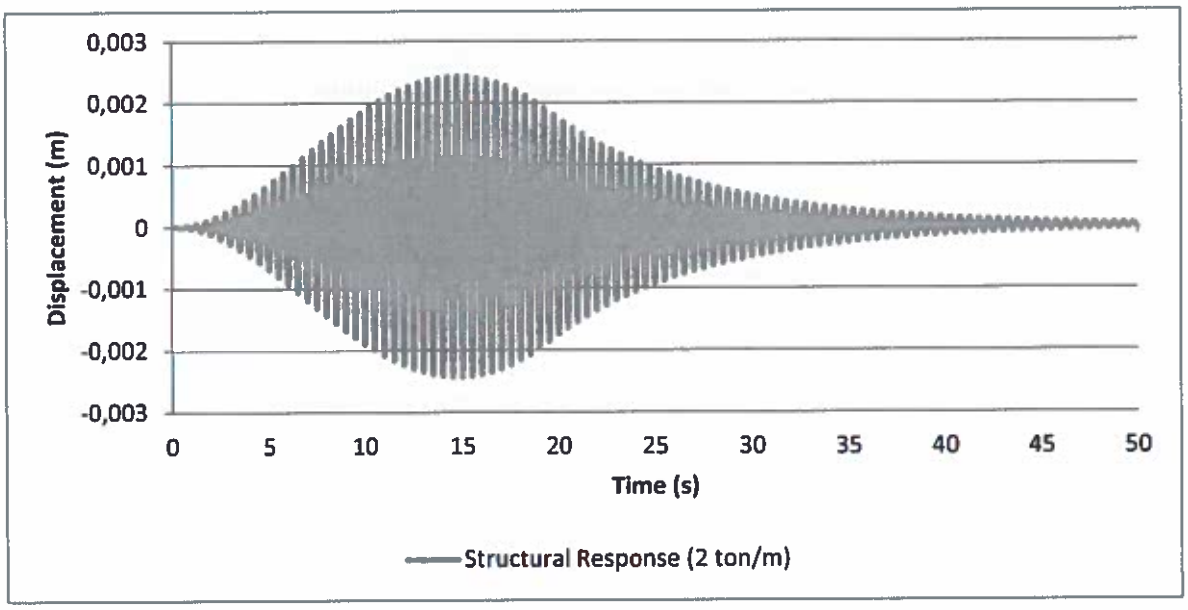


Figure 4.10 – Structural Response considering 2 ton/m

This holds true for systems with TMDs. Figure 4.11 shows a similar comparison between 1 and 2 ton/m, for a simple beam with a 30 metre span, a damping coefficient 1%, and mounted with a properly tuned TMD of μ=0.02.

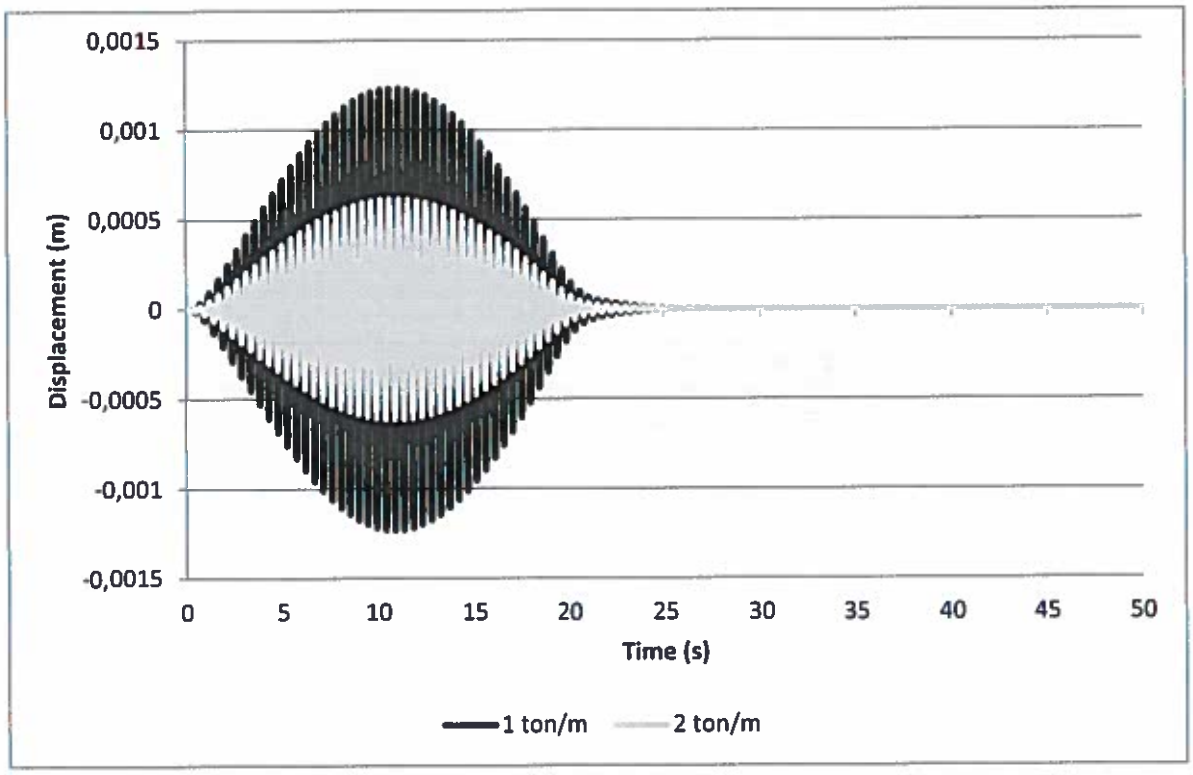


Figure 4.11 – Comparison of two different values of mass

Taking this into consideration, the graphs represented in figures 4.5 through 4.8 remain accurate for bridges with natural frequencies near 2 Hz, as differences in mass are easy to correct.

5

ANALYSIS OF NON-LINEAR EFFECTS

5.1. OVERVIEW

Every consideration taken with TMDs in this dissertation assumes that they are either "blocked" (or disabled), or "unblocked" (fully operational) from the beginning to the end of the system response.

In practical situations, however, TMDs are not permanently unblocked. They only start working after a certain threshold of movement has been reached. In other words, one must take into consideration that a TMD is either blocked or unblocked at different instants, instead of the binary problem that was considered thus far.

TMDs may have several non-linearities, such as friction problems, gaps and non-linear viscous damping. This chapter focuses on non-linear effects arising from friction in the sliding surfaces of guiding axes.

5.2. FRICTION IN TMDS

5.2.1. UNBLOCKING ACCELERATION

The reason why TMDs take some time to unblock is the presence of friction in the connection of the inertial mass to the primary structure (as depicted in Figure 5.1). This friction must be overcome to free the TMD from the main structure, so that it might work towards its intended use.

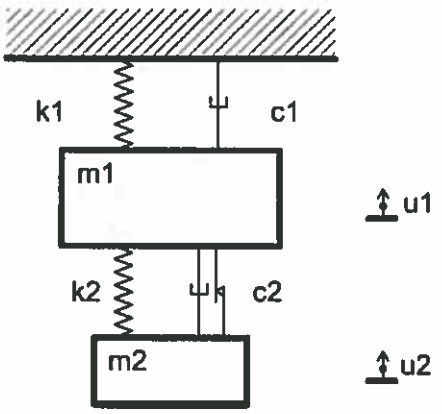


Figure 5.1 – Simplified schematic of a TMD, including a friction element

Since force is directly proportional to acceleration, this inertial force can be translated as an equivalent acceleration that is necessary to unblock the TMD – an "Unblocking Acceleration". It will turn on once it reaches this acceleration or higher, but it will remain turned off for lower values.

This threshold will be known as "Unblocking Acceleration" for the remainder of this dissertation.

It should be noted, however, that this threshold depends entirely on the structure, the TMD, and their execution on-site. While it is entirely possible to determine a TMD's unblocking acceleration with empirical methods, there is currently no way to accurately predict its value before the structure itself is built and the device is installed.

One study [9] notes that this value can range between 0.15 to 0.30 m/s^2 for a single case.

5.2.2. Blocking Acceleration

Other than the existence of static friction, one must also consider the existence of kinetic friction, that is, the inertial force required for an object to cease movement. Much like with unblocking acceleration, this friction translates as a lower threshold of acceleration that the TMD can reach before it blocks again – a "Blocking Acceleration".

As with unblocking acceleration, this value is impossible to predict analytically. The only certainty is that, since kinetic friction is always lower than static friction, the blocking acceleration will always be lower than unblocking acceleration.

5.2.3. Example

Considering the idea that a TMD only starts functioning at a certain point during the analysis, one must adjust the methods presented thus far to accommodate this concept.

As mentioned in Chapter 3, State-Space representation allows for change with each iteration. In this case, one must make a distinction between "Blocked" and "Unblocked" states, and construct different stiffness matrixes for each, leading to two distinct State Matrixes (A).

The trigger to this change is acceleration. The system's acceleration is checked in the current iteration, and compared to the threshold of assumed unblocking acceleration. The TMD turns on when it surpasses unblocking acceleration, and it turns off when it lowers below blocking acceleration.

This can be done in MATLAB by assigning a binary value called "signal" to the method, indicating whether the TMD is currently functioning or not. This allows the program to recognize when to unblock, when to stay unblocked, and when to finally block again. The MATLAB function written for this purpose can be found in Appendix A3.

Taking the previous example seen in subchapter 4.2.3 of a 20 metre beam subject to a walking pedestrian (fp=1,98 Hz), mounted by a properly tuned TMD of μ=0.02, and considering that it is always active from start to finish, we see the following response:

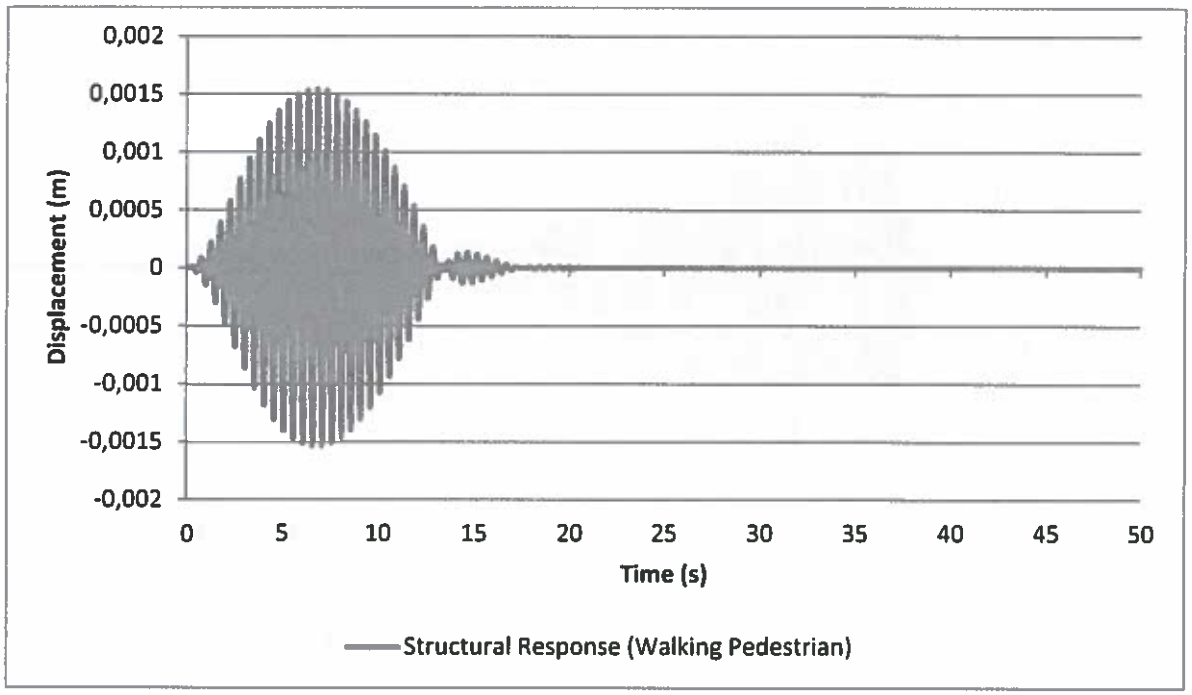


Figure 5.2 – Structural Response of a simple beam to a walking pedestrian (functioning TMD)

When introducing non-linear effects, two different tests were run. First, setting the unblocking acceleration as 0.15 m/s^2, and a second test setting that value as 0.30 m/s^2. The blocking acceleration was set as 80% of the unblocking acceleration, for both cases (though it is an entirely arbitrary value). The resulting responses can be seen in Figures 5.3 and 5.4, as well as a binary signal, representing the period of time where the TMD is unblocked:

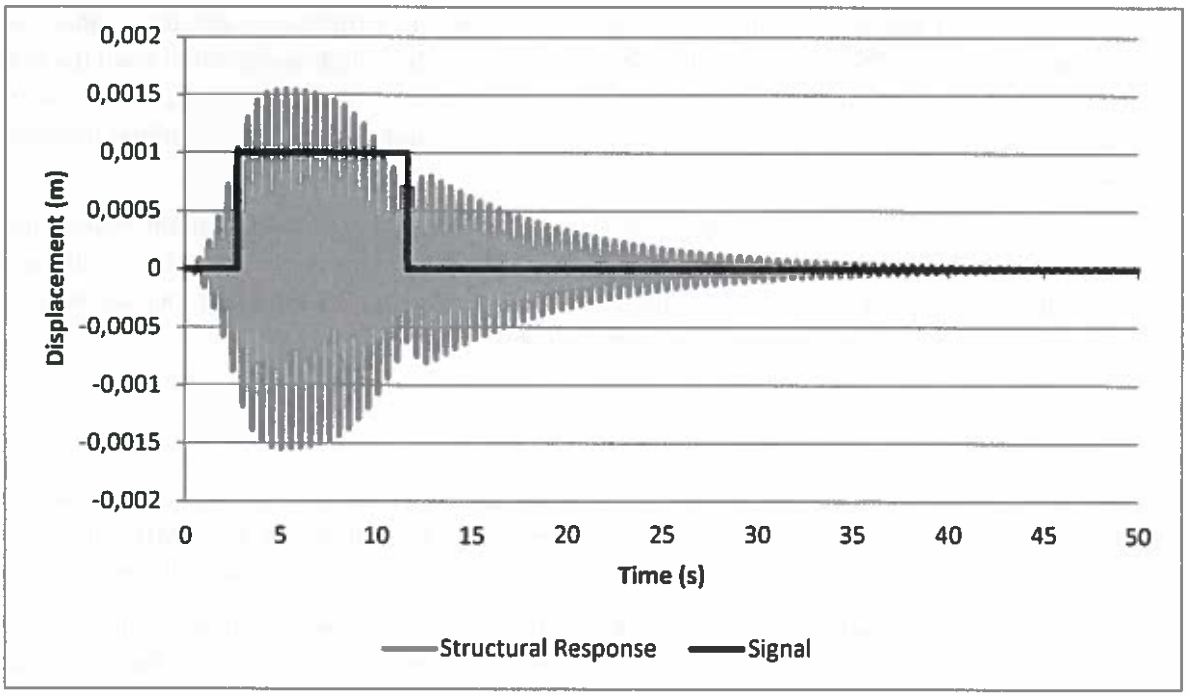


Figure 5.3 – Structural Response of a simple beam (Unbl.Accel.=0.15 m/s2)

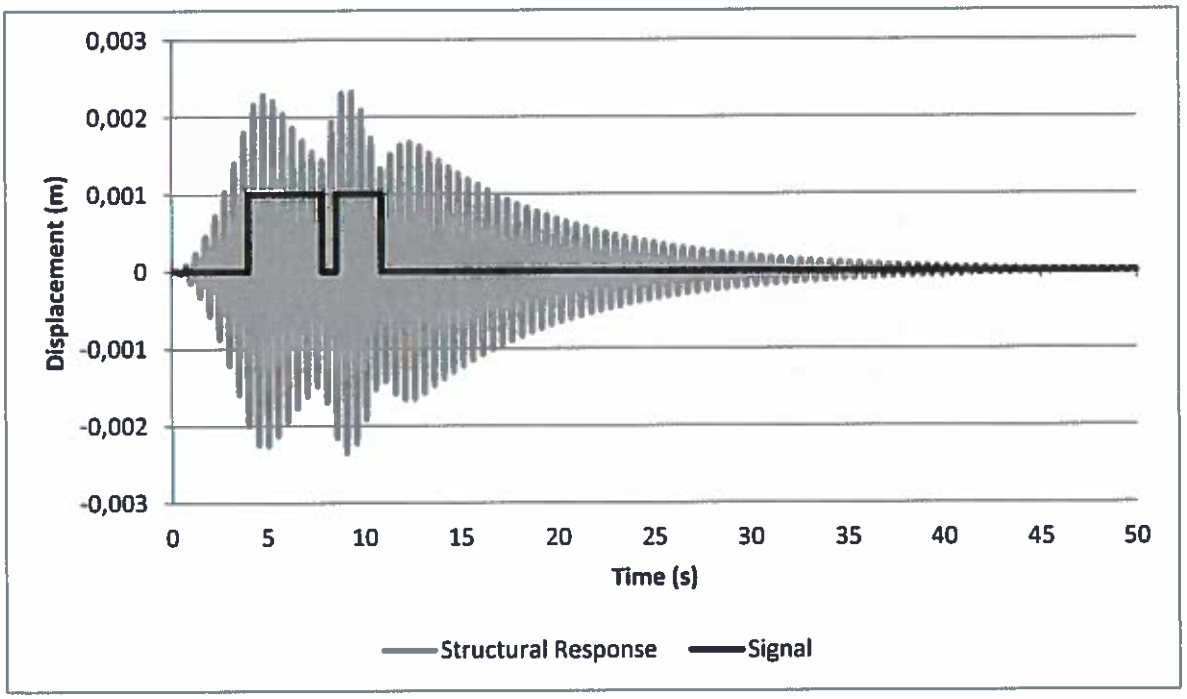


Figure 5.4 – Structural Response of a simple beam (Unbl.Accel.=0.30 m/s2)

The response seen in Figure 5.3 (Unbl.Accel.=0.15 m/s2) does not differ much from the original linear analysis in Figure 5.2, both cases showing similar values for their maximum displacement (approximately 1.55 mm).

However, the case seen in Figure 5.4 (Unbl.Accel.=0.30 m/s2) shows a visible difference, with the structure reaching a maximum displacement of 2.23 mm twice. The change in unblocking acceleration has significantly affected this simply supported beam.

When the TMD reaches the unblocked state, the "blocked" structure is already resonant with the walking pedestrian. This means that the TMD needs to handle a high acceleration from the moment it is turned on, possibly higher than any value that it would register if it had been unblocked from the beginning. The higher the threshold of unblocking acceleration, the greater the initial movement that the TMD must control.

It is clear that the unblocking acceleration has a great influence on the maximum displacement of a structure, making it a potentially conditioning variable. The existence of blocking acceleration may cause the existence of further spikes (depending on the duration of the load), but seems to have no significant effect on the structure's maximum response.

5.2.4. Comparison of Excitation Frequency

The previous example was made when considering an excitation of 1.98 Hz, which is in resonance with a blocked TMD. In theory, the initial vibrations that occur before the TMD's unblocking are greater, but it is relevant to make a more detailed comparison between the two solicitation frequencies.

A small study was made considering a simply supported beam with a natural frequency of 2 Hz, a damping coefficient of ζ=1%, and mounted by a properly tuned TMD of μ=0.02. Different unblocking accelerations were set from 0 to 1.5 m/s^2 (in intervals of 0.1). A test was run in MATLAB for each

one of these values, measuring the structure's maximum acceleration for each test. This analysis was made for two different span lengths: 20 metres and 50 metres.

Blocking acceleration was not considered, not only because there are no accurate estimates that can be made regarding its value, but also because, as seen in subchapter 5.2.3, it has no significant influence in the structure's maximum response. This value was set as 0.001% of the unblocking acceleration (making it effectively non-existent).

The results can be seen in Figures 5.5 and 5.6:

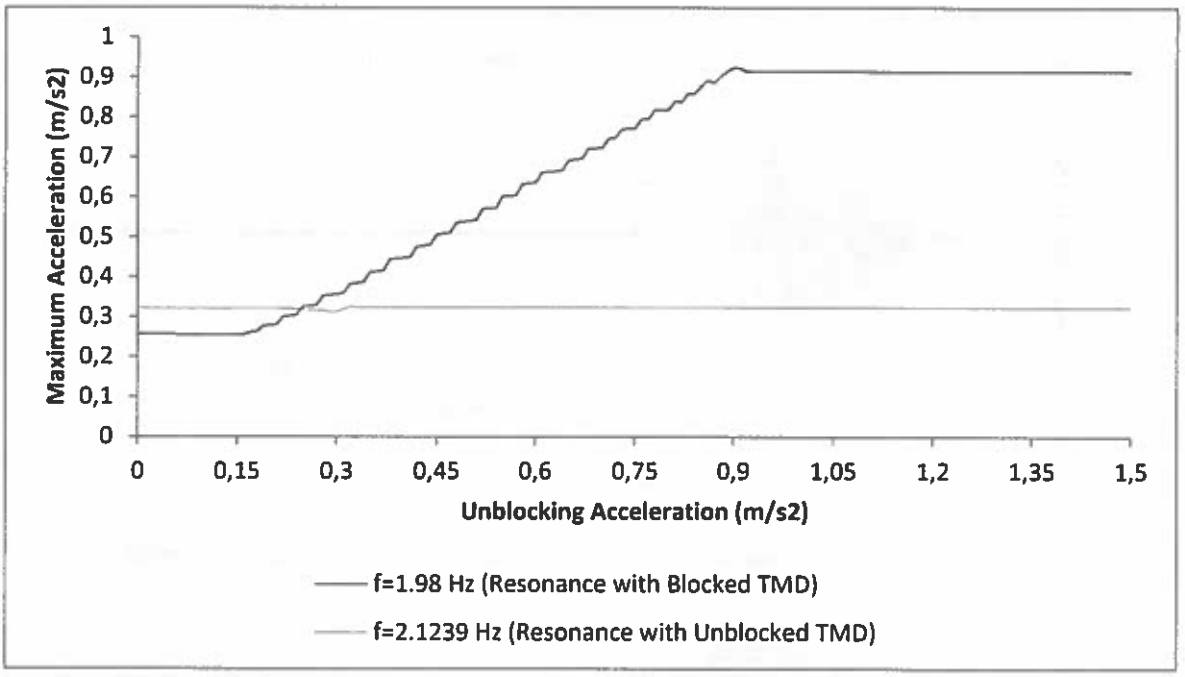


Figure 5.5 – Comparison between solicitation frequencies (Span length of 20 m)

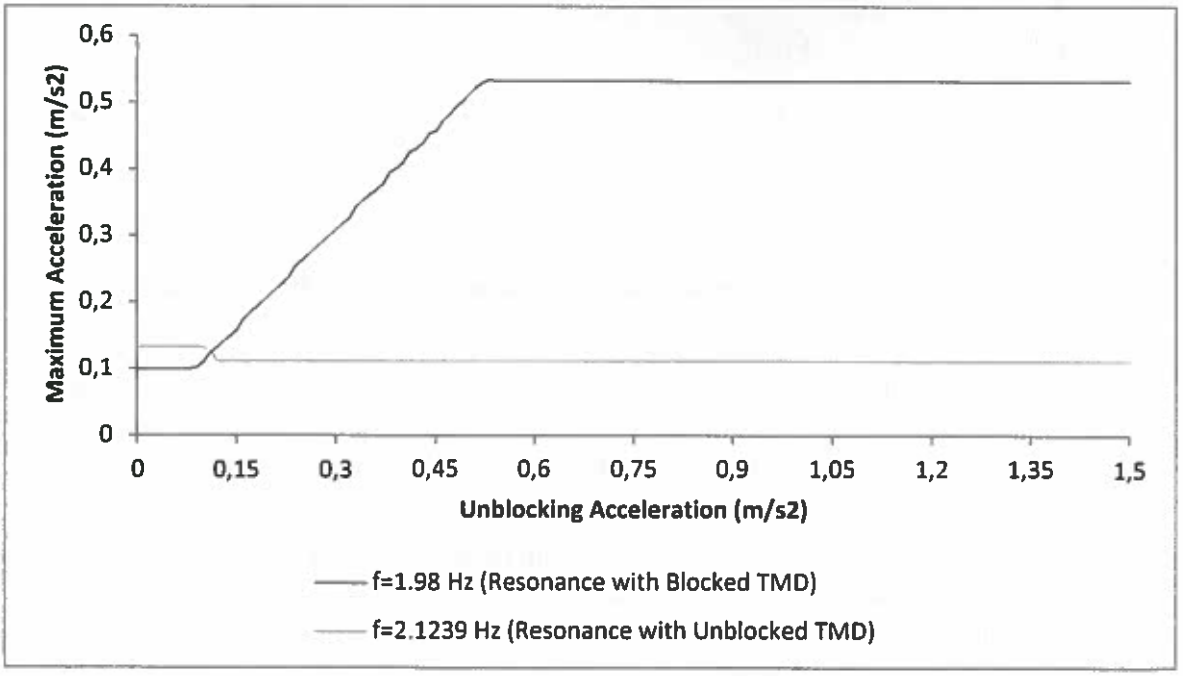


Figure 5.6 – Comparison between solicitation frequencies (Span length of 50 m)

For both graphs, the behaviour of the dark grey lines is expected. When the unblocking acceleration is sufficiently low, the TMD unblocks early, and its influence on the structural response is similar to the one it would have if it was permanently unblocked, as can be seen by the initial straight line.

If the unblocking acceleration is higher than a certain threshold (≈0.20 m/s2 in Figure 5.5), then the structure's response will exhibit a higher maximum acceleration (as previously seen in Figure 5.4). There is a direct linear correlation between unblocking acceleration and maximum registered acceleration, represented by an erratic line. However, if the unblocking acceleration is higher than a second threshold (≈0.90 m/s2 in Figure 5.5), then the TMD never unlocks, as the external load is not sufficient to do so, hence the constant straight line.

Figures 5.7 and 5.8 represent this shift for the case where the beam is 20 metres long:

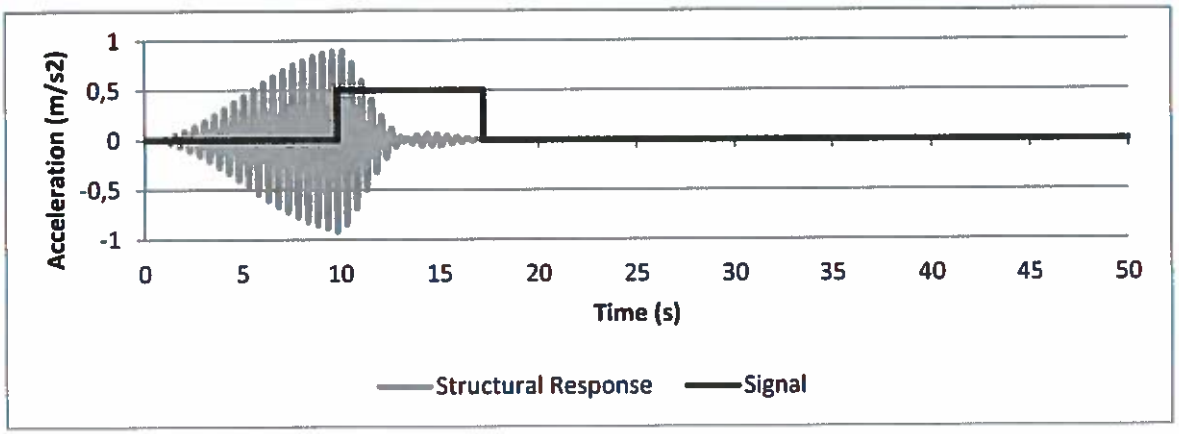


Figure 5.7 – Structural Response of a simple beam (Unbl.Accel.=0.90 m/s2)

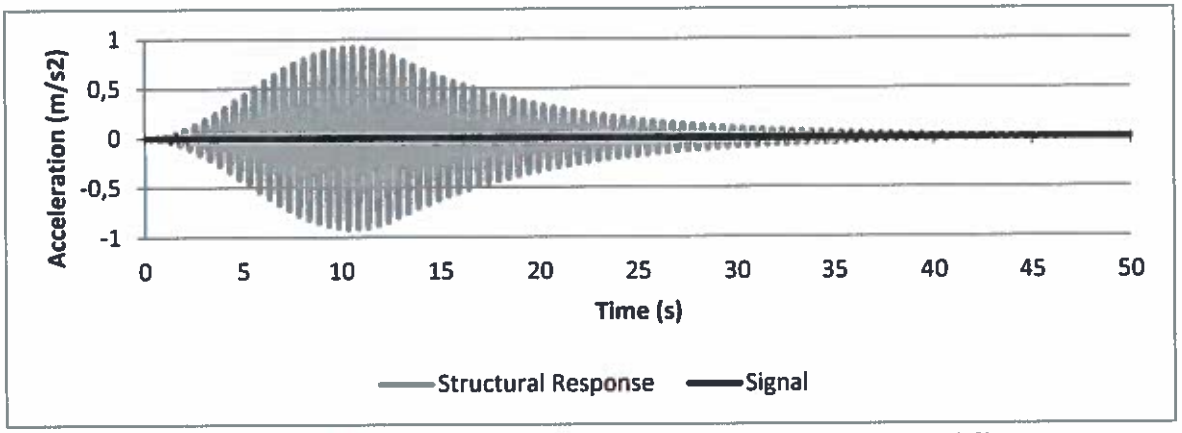


Figure 5.8 – Structural Response of a simple beam (Unbl.Accel.=0.95 m/s2)

The behaviour of the light grey lines is less clear, but it follows the same patterns. The initial behaviour for low values of unblocking acceleration is the same as that of the dark grey lines', the main difference being that the registered values are higher (because the external load is in resonance with the behaviour of an unblocked TMD).

If the unblocking acceleration is higher than a certain threshold (≈0.30 m/s2 in Figure 5.5), then the structural response never reaches the point where the TMD unlocks, as the external load is not resonant with the beam when its TMD is still unlocked. This shift is more pronounced in Figure 5.6, but it is present in Figure 5.5 as well.

5.2.4.1. Conditioning Excitation Frequency

It can be seen that, for most values of unblocking acceleration, the frequency of 1.98 Hz is the more conditioning solicitation. However, for smaller values, the conditioning frequency is actually 2.12 Hz, likely because the TMD is turned on early in those situations.

While it would be relevant to consider the most conditioning soliciting frequency for every situation, it is impossible to know which one to use without doing a similar study for every combination of parameters (span length, damping and TMD weight).

It is, however, acceptable to consider the solicitation of 1.98 Hz as conditioning for most situations. The cases where this isn't true (that is, when the unblocking acceleration is too low), the difference between responses is small enough as to not be significant.

5.4. Effect on Maximum Acceleration

Taking into account that the structure is already moving when the TMD's signal is turned on, it becomes apparent that the unblocking acceleration noticeably affects the maximum accelerations measured in the previous chapter. As such, a study was made to re-evaluate the results of the small study made in subchapter 4.3.

As before, a simply supported beam subject to a walking pedestrian was considered. The mass of the bridge was set to 1 ton/m, and its natural frequency was defined as 2 Hz. Different spans were set from 10 to 50 metres. For each case, the mass and load function were calculated, and a test was run in MATLAB, the structure's maximum acceleration being measured for each test.

The pedestrian's walking rate was determined for the structure with a blocked TMD, for each test (1.98 Hz in the previous examples). As established in subchapter 5.2.4, this is the conditioning solicitation for most situations.

Furthermore, the existence of blocking acceleration was not considered, seeing as it does not significantly affect the structure's maximum response (as seen in figure 5.4).

The results can be seen in figures 5.9 through 5.20. Different damping coefficients for the bridge, TMDs (as defined by their mass ratio μ) and potential values for unblocking acceleration were considered, as well as a comparison with a permanently blocked TMD and a permanently unblocked TMD. The resulting graphs were approximated with polynomial trend lines in Excel.

Much like the analysis made in subchapter 4.3, the values seen in these graphs are idealised according to specific parameters, and can be normalised for other cases in similar ways. However, the practical utility of these graphs is less relevant, as unblocking acceleration cannot be determined with accuracy, but it does serve as a guideline of what sort of changes to expect, when taking unblocking acceleration into consideration in multiple cases.

It is relevant to note that unblocking acceleration seems to have a greater effect in bridges with longer spans. In fact, the action of a single pedestrian in footbridges of long span and high damping coefficients may not even reach the point where a TMD unblocks.

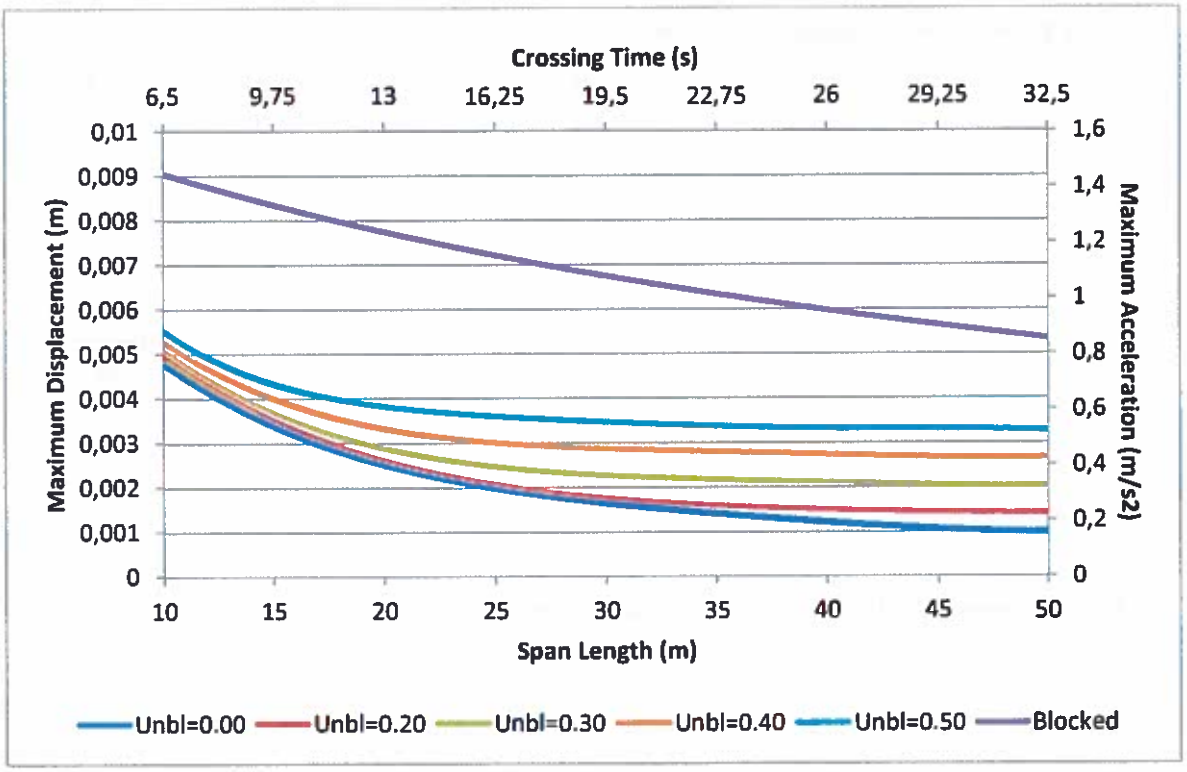


Figure 5.9 – Maximum Accelerations (ζ_1=0.5%, μ=0.01)

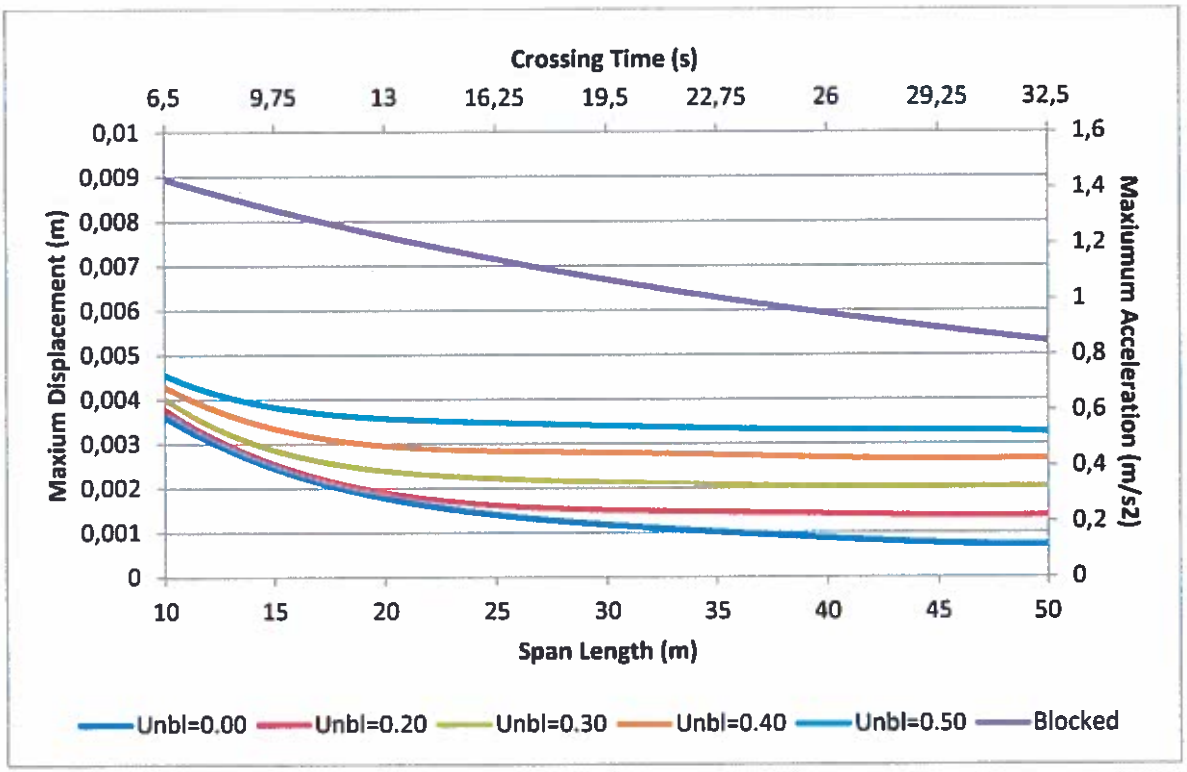


Figure 5.10 – Maximum Accelerations (ζ_1=0.5%, μ=0.02)

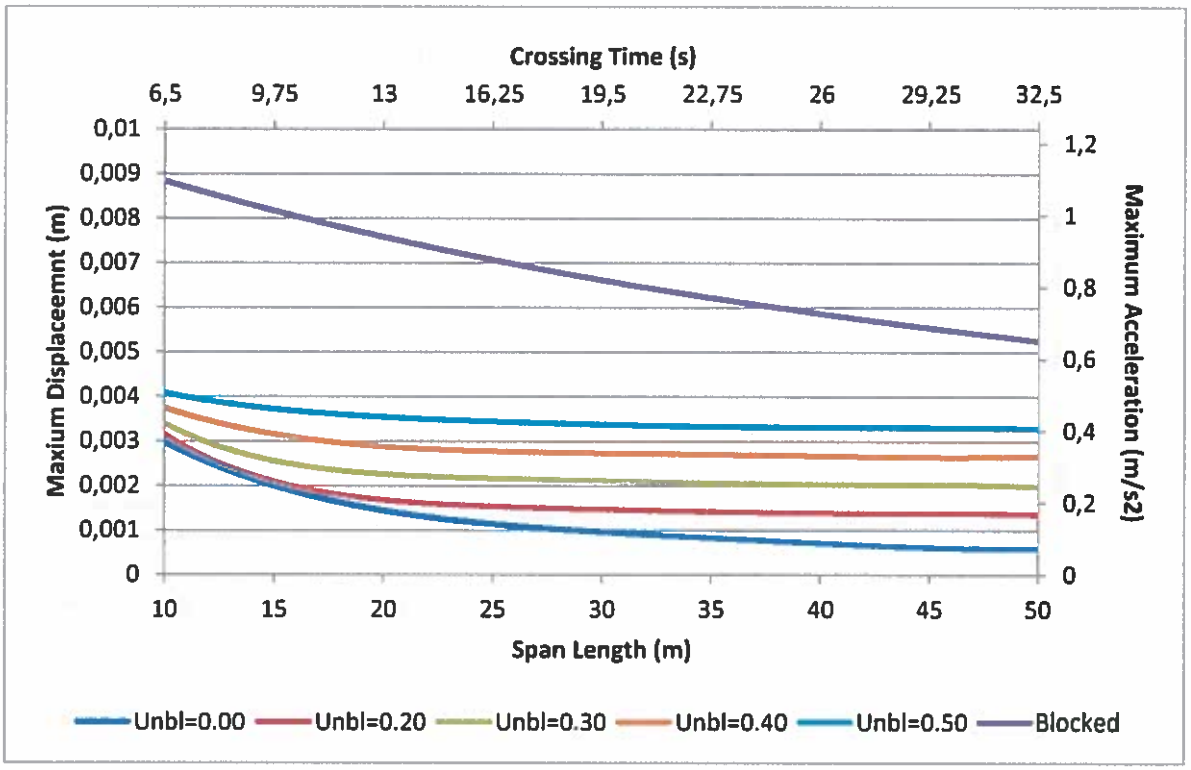


Figure 5.11 – Maximum Accelerations (ζ_1=0.5%, μ=0.03)

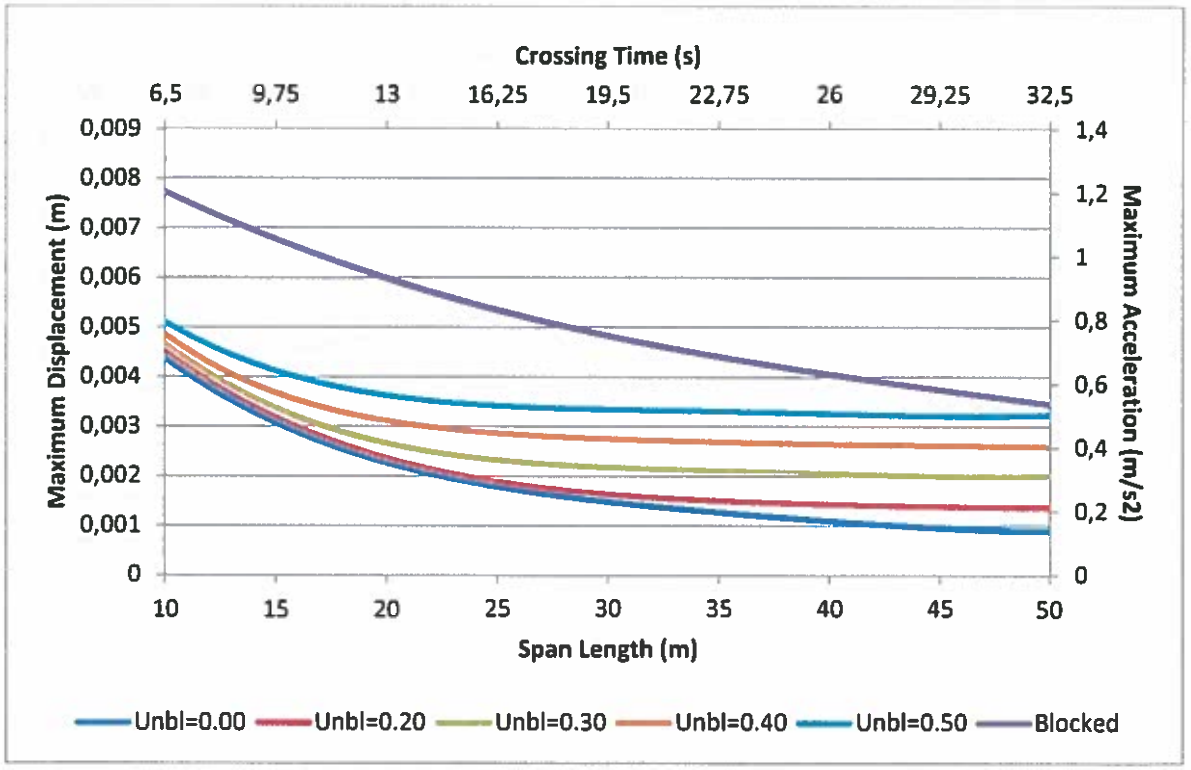


Figure 5.12 – Maximum Accelerations (ζ_1=1%, μ=0.01)

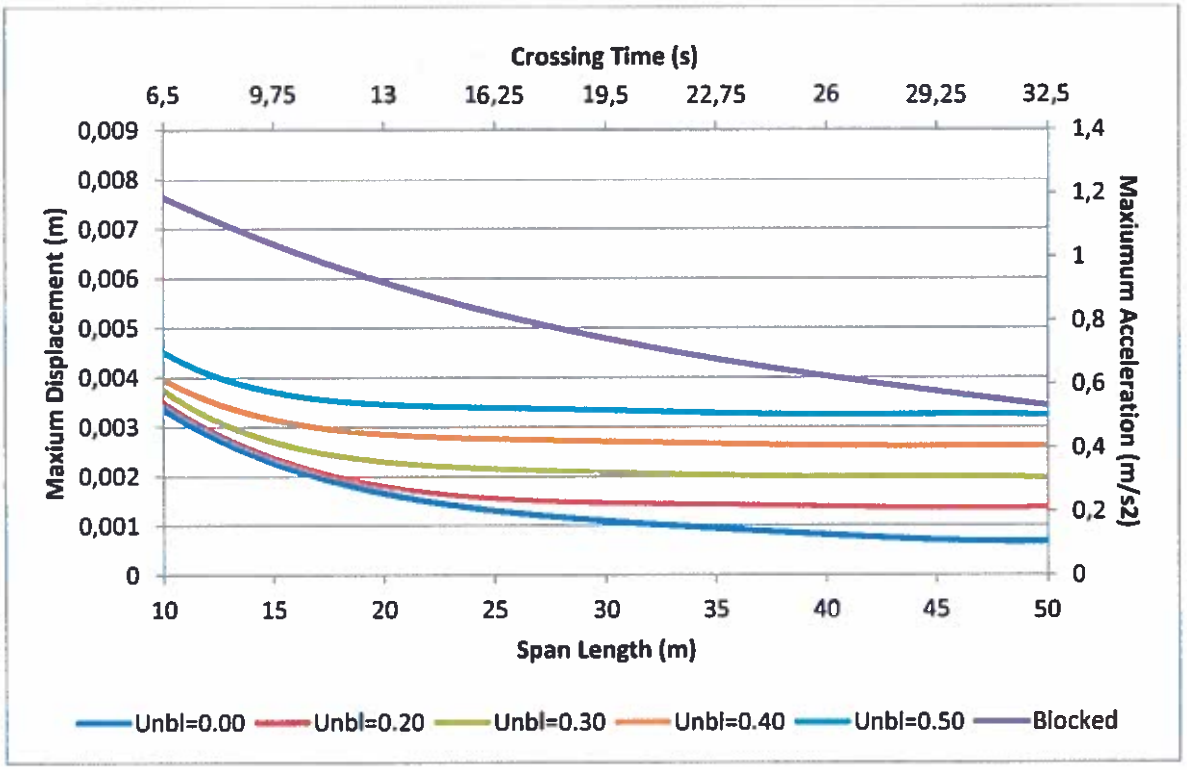


Figure 5.13 – Maximum Accelerations (ζ_1=1%, μ=0.02)

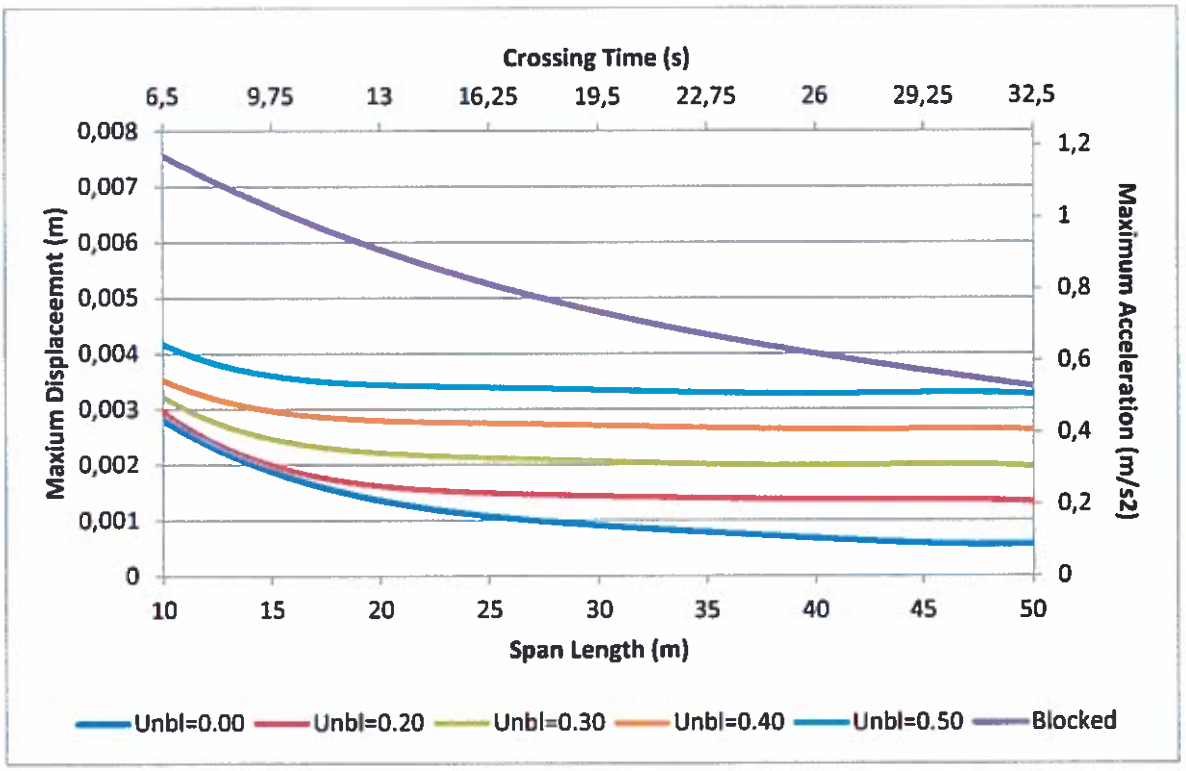


Figure 5.14 – Maximum Accelerations (ζ_1=1%, μ=0.03)

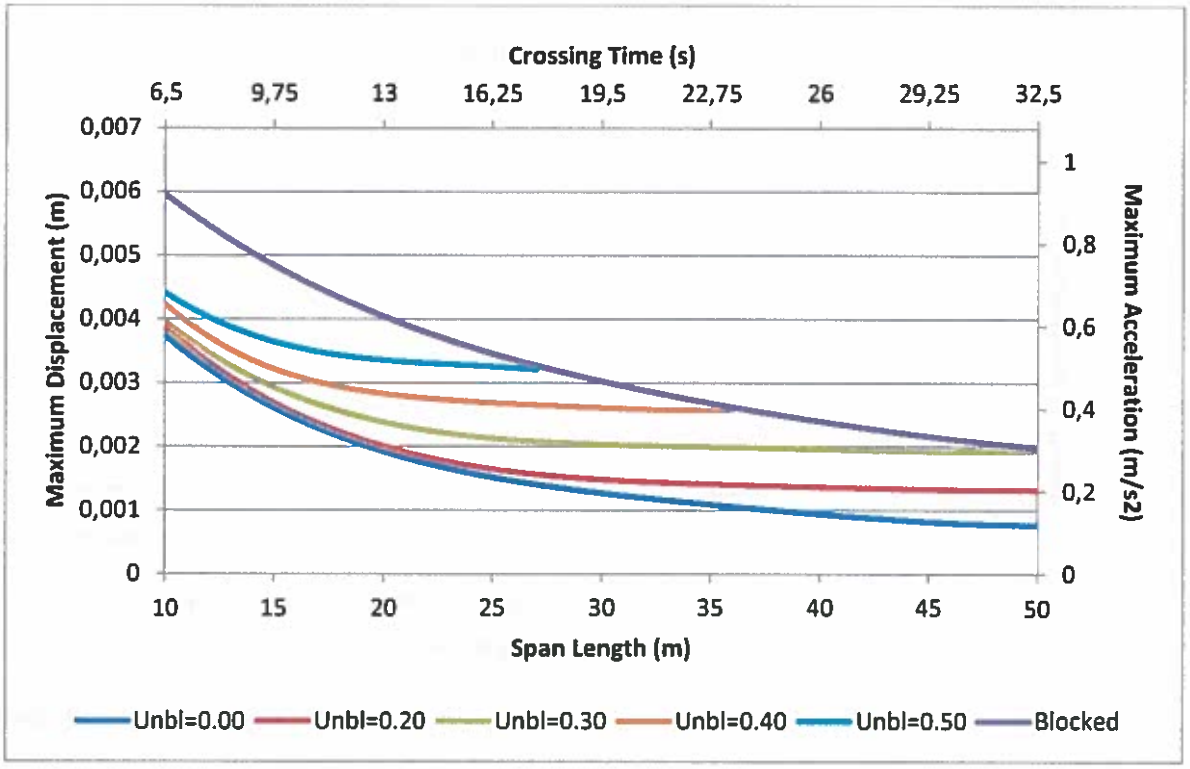


Figure 5.15 – Maximum Accelerations (ζ_1=2%, μ=0.01)

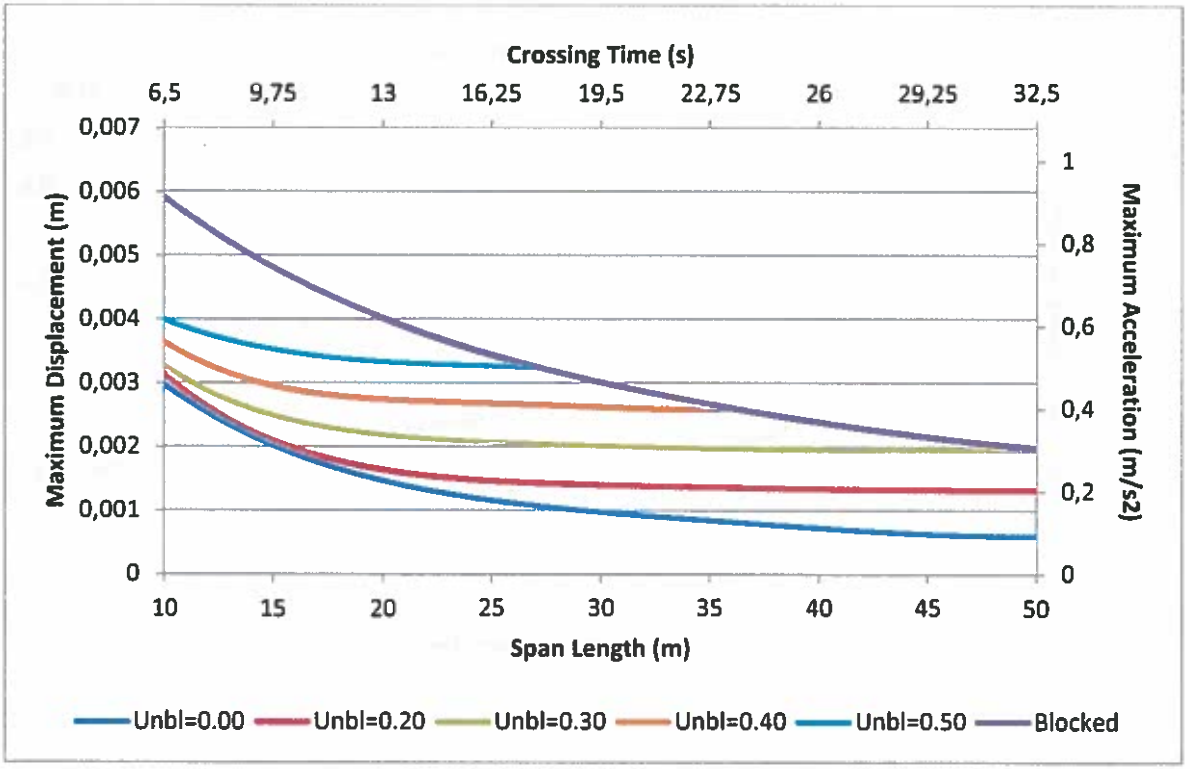


Figure 5.16 – Maximum Accelerations (ζ_1=2%, μ=0.02)

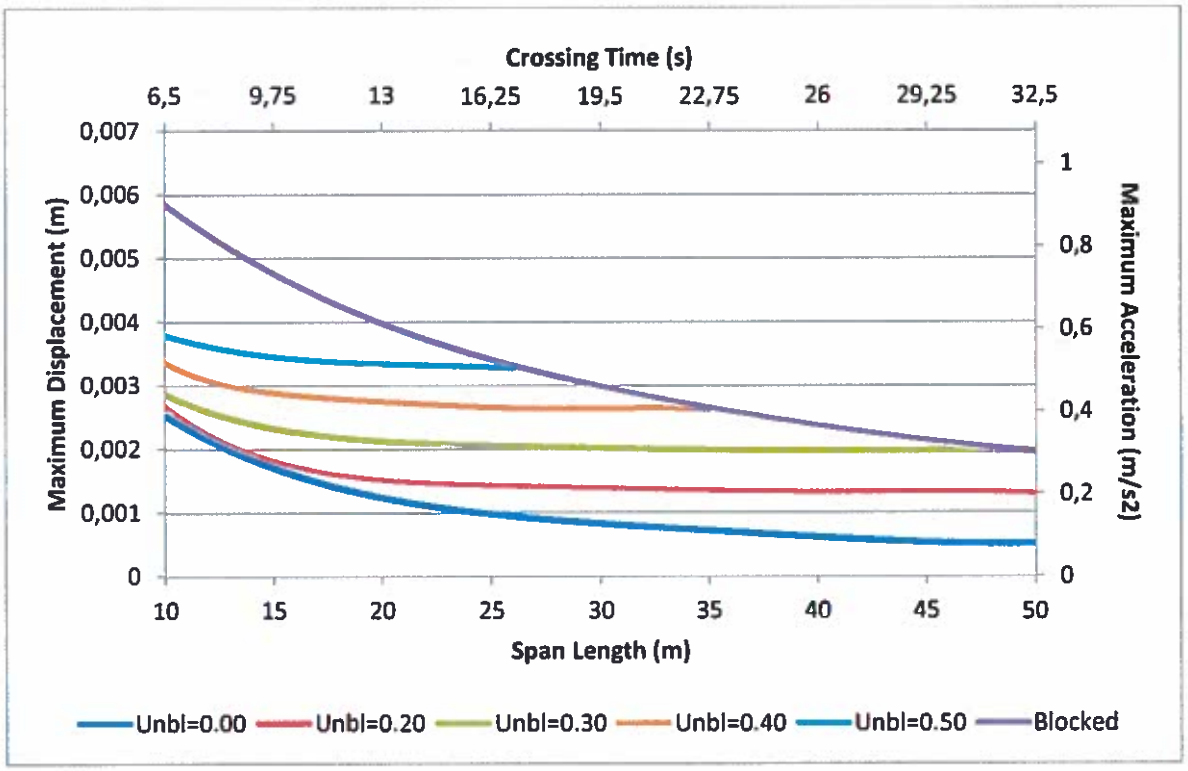


Figure 5.17 – Maximum Accelerations (ζ_1=2%, μ=0.03)

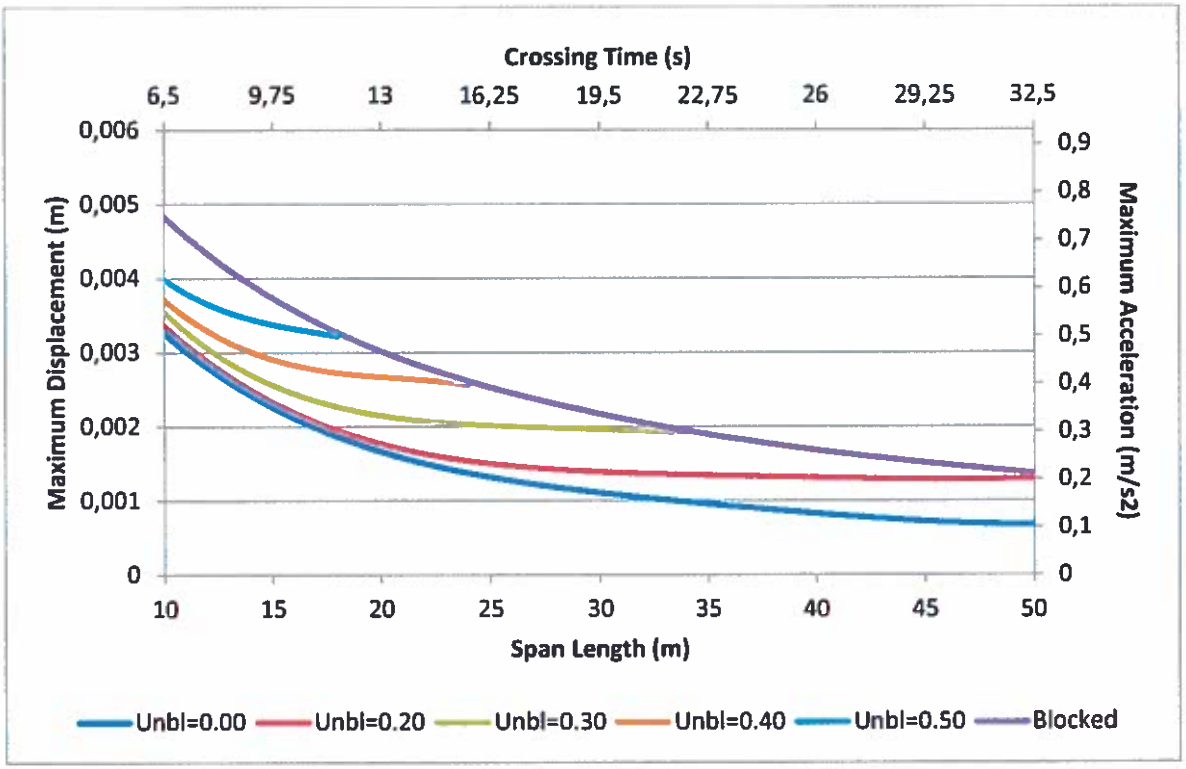


Figure 5.18 – Maximum Accelerations (ζ_1=3%, μ=0.01)

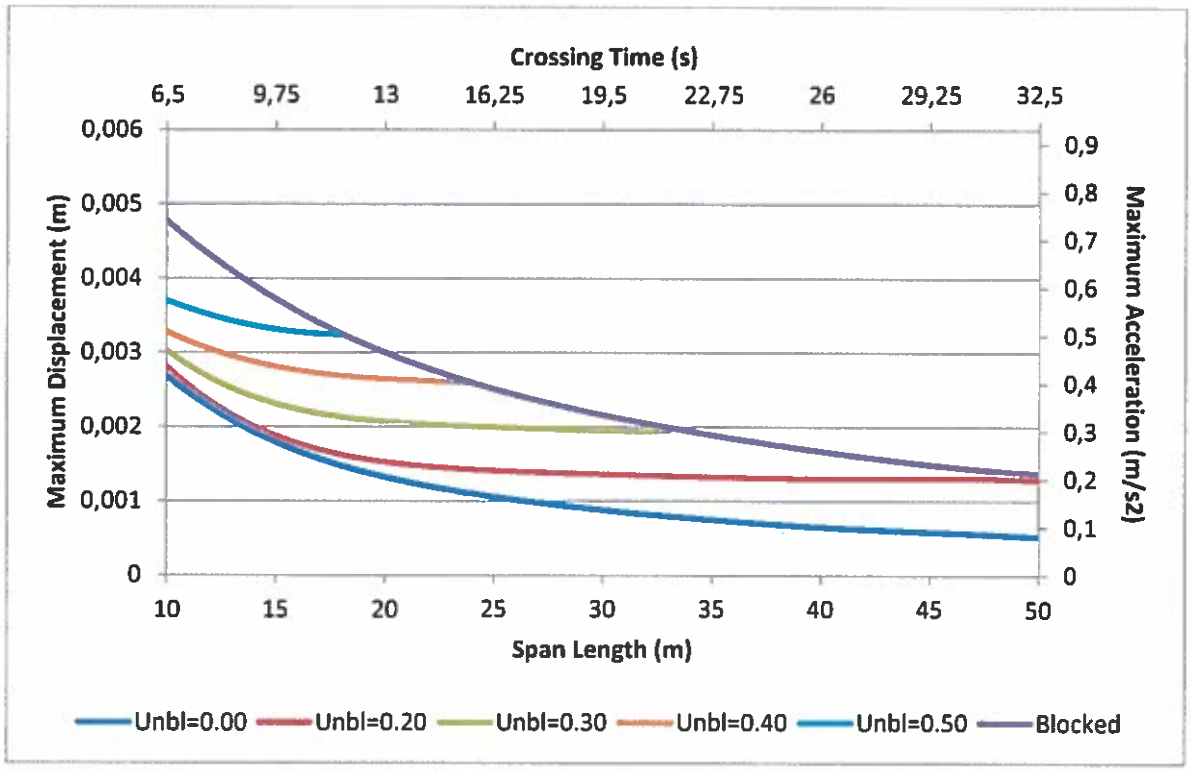


Figure 5.19 – Maximum Accelerations (ζ_1=3%, μ=0.02)

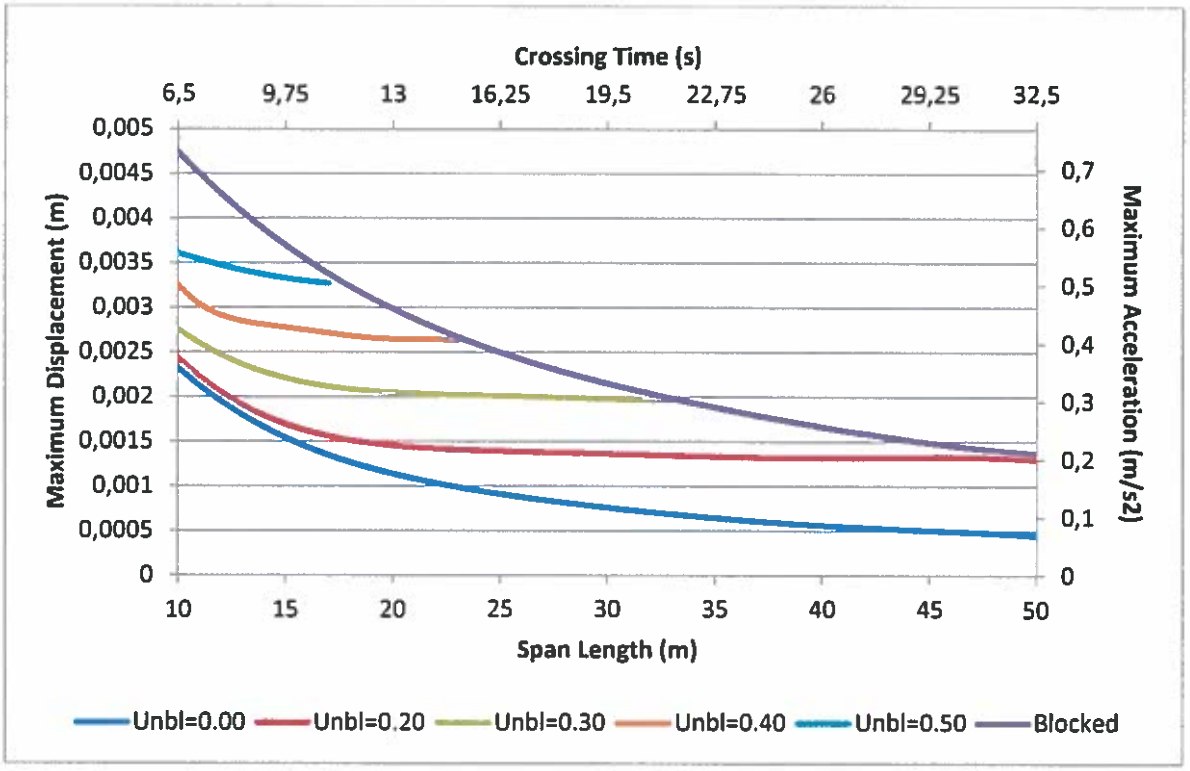


Figure 5.20 – Maximum Accelerations (ζ_1=3%, μ=0.03)

6
FINAL REMARKS

6.1. CONCLUSIONS

In the field of structural engineering, many challenges remain. Despite the advances made in the last few decades, even traditional techniques of vibration control still have room for improvement, even in common structures such as footbridges.

The study of non-linear effects revealed the influence that unseen factors such as damper friction have on a TMD's performance. Even if this work simply analysed their perceived effects, it is a decisive step forward in increasing the accuracy with which the performance of TMDs and the behaviour of structures can be predicted.

It is clear that that variables such as unblocking acceleration significantly influence the performance of a TMD, even if it is, for now, a mostly unknown value.

The studies made regarding the influence of span length in a bridge's behaviour should also be useful in optimising future choices in vibration control, especially as the graphs are sufficiently versatile as to be applied to the design of any type of footbridge, regardless of materials or construction method.

6.2. FUTURE WORK

This work only touches on a few forms of unseen non-linear aspects in TMDs and vibration control. The effects of non-linearities should be measured more proficiently, very little is known regarding specific values.

Knowing more common values of unblocking acceleration could lead to more optimal design of TMDs. As the graphs of chapter 5 showed, there are significant differences in overall performance from the device, depending on the value of unblocking acceleration.

Furthermore, it might be worthwhile to pursue the possibility of making further studies on other types of non-linearities that were not explored here, such as non-viscous forms of damping.

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APPENDIX
MATLAB Scripts

Appendix A1 – Load Characterisation of Walking Pedestrian

```
function [F] = Fwalk(G,fp,V,dt,tin,tfin)
% G - Pedestrian weight (≈0.7 KN)
% fp - Walking Frequency (1.5-2.5 Hz)
% V - Span (m)
% dt - Time Interval (s)
% tin - Initial Moment (s)
% tfin - Final Moment
%
% Provides a 2xnt matrix (F), representative of a walking pedestrian on a
% simply supported beam.
a=0.43*fp-0.38;
tcross=0.65*V;
T=tin:dt:tcross;
nt=length(T);
if T(1,nt)~=tcross
    T(1,nt+1)=tcross;
end
f=zeros(2,length(T));c=0;
for i=T
    c=c+1;
    fi=G*a*sin(2*pi()*1*fp*i)*sin(pi()*i/tcross);
    f(2:size(f,1),c)=fi;
    f(1,c)=i;
end
lf=size(f,2);hf=size(f,1);
tmid=f(1,lf); T2=tmid:dt:tfin; lt=size(T2,2);
F=zeros(hf,lf+lt-1);
F(1:hf,1:lf)=f; F(1,lf+1:size(F,2))=T2(1,2:lt);
end
```

Appendix A2 – Determining structural response (including acceleration)

```
function [X] = StrRes(M,C,K,x,F,a,dt)
% M - Mass Matrix (ton)
% C - Damping Matrix (KN.s/m)
% K - Stiffness Matrix (KN/m)
% x - State Variable - Initial Displacement (m) and Velocity (m/s)
% F - Input Variables - Matrix
%   Time Vector(s) + External Load Vector (KN)
% a - Initial Acceleration (m/s^2)
% dt - Time Interval (s)
%
% Provides the dynamic response for a system with N degrees of freedom,
% utilising State-Space formulation.
N=size(M,1);n=size(F,2);
A=[zeros(N),eye(N);-M\K,-M\C];
B=[zeros(N);M\eye(N)];
E=expm(dt*A);
G=(A\(E-eye(2*N)))*B;
T=F(1,:);X=zeros(3*N+1,n);
c=1;
xi=x;
for i=1:1:n
    if i==1 & F(2:(N+1),1)==zeros(N,1)
    else
        c=c+1;
        fi=F(2:(N+1),i);
        xi=E*xi+G*fi;
        X(1:(2*N),c)=xi;
    end
end
X(1:(2*N),1)=x;
X(3*N+1,:)=T;

for i=2:1:length(X)
    for j=1:1:N
        X(2*N+j,i)=(X(N+j,i)-X(N+j,i-1))/dt;
    end
end
X(2*N+1:3*N,1)=a;
end
```

Appendix A3 – Determining structural response (including unblocking acceleration)

```
function [X,Z] = NonLin(Mt,Ct,Kt,x,F,a,dt,g,unbl,fp)
% Mt - Mass Matrix (ton)
% Ct - Damping Matrix (KN.s/m)
% Kt - Stiffness Matrix (KN/m)
% x - State Variable - Initial Displacement (m) and Velocity (m/s)
% F - Input Variables - Matrix
%   Time Vector(s) + External Load Vector (KN)
% a - Initial Acceleration (m/s^2)
% dt - Time Interval (s)
% g - Degree of Freedom associated with a TMD ("1" in 2-DOF systems)
% unbl - Unblocking Acceleration (m/s^2)
% fp - Excitation Frequency (Hz)
%
% Provides the dynamic response for a system with N degrees of freedom,
% utilising State-Space formulation.
%   This function must take into account the existence of a TMD, associated
%   to one of the N degrees of freedom of the original structure.
%   The TMD will start functioning once the degree of freedom "g" achieves
%   an acceleration of "unbl".

N=size(Mt,1);n=size(F,2);
K=Kt;K(g,g)=K(g,g)-K(N,N)+(K(N,N)*1000);
K(N,N)=K(N,N)*1000;K(g,N)=K(g,N)*1000;K(N,g)=K(N,g)*1000;
A=[zeros(N),eye(N);-Mt\K,-Mt\Ct]; At=[zeros(N),eye(N);-Mt\Kt,-Mt\Ct];
B=[zeros(N);Mt\eye(N)];
E=expm(dt*A); Et=expm(dt*At);
G=(A\(E-eye(2*N)))*B; Gt=(At\(Et-eye(2*N)))*B;
T=F(1,:);X=zeros(3*N+1,n);Z=zeros(3*N+1,n);
xi=x;t=1/fp;r=round(t/dt);
bl=0.80*unbl; % Blocking acceleration (80% as an arbitrary value)
for i=1:1:n
    if i==1
        X(1:(2*N),i)=x;X((2*N+1):(3*N),i)=a;X(3*N+1,i)=T(1,i);
        Z(1:N,i)=x(1:N);Z((N+1):(2*N),i)=a;Z(3*N+1,i)=T(1,i);
        for j=1:1:N
            if a(j,1)>=unbl
                Z(2*N+j,i)=1;
            end
        end
    else
        fi=F(2:(N+1),i);
        if Z(2*N+g,i-1)==0
            if i>=3 & Z(2*N+g,i-2)==1
                xi(N,1)=xi(g,1);xi(2*N,1)=xi(N+g,1);
                X(3*N,i)=X(2*N+g,i);
            end
            xi=E*xi+G*fi;
        else
            xi=Et*xi+Gt*fi;
        end
        X(1:(2*N),i)=xi;
        for j=1:1:N
            X(2*N+j,i)=(X(N+j,i)-X(N+j,i-1))/dt;
            if i<=r
                utemp=X(j,1:i);atemp=X(2*N+j,1:i);
```

```
            else
                utemp=X(j,(i-r):i);atemp=X(2*N+j,(i-r):i);
            end
            umax=max(abs(utemp));amax=max(abs(atemp));
            Z(j,i)=umax;Z(N+j,i)=amax;
            if Z(2*N+g,i-1)==0
                if amax>=unbl
                    Z(2*N+j,i)=1;
                end
            else
                if amax>=bl
                    Z(2*N+j,i)=1;
                end
            end
        end
        X(3*N+1,i)=T(1,i);Z(3*N+1,i)=T(1,i);
    end
end
end
```