Bank-Branch Location and Sizing under Economies of Scale

by

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Dedication

To my parents,
and to the memory of my grandparents.

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Abstract

The bank-branch restructuring problem seeks to locate bank-branches, maintaining or closing existing branches, in order to provide the service required by its clients, at minimum total cost.

This problem has not been the subject of much study, nonetheless it is a common problem that banks are faced with from time to time.

We address this nonlinear problem, since economies of scale exist, by formulating it as a mixed binary, integer linear model. The model obtained can be solved by a ready-available software, such as CPLEX. However, the dimensionality problem soon becomes the issue and thus, we also propose a heuristic to solve, approximately, this problem.

We have developed a local search heuristic to solve the bank-branch location and sizing problem with concave cost functions. In order to do so, we have modelled this problem as a mixed integer and binary programming model. The heuristic is based on the solution to a related linear integer programming problem. This solution is subsequently improved by iteratively applying drop and swap operations. The computational experiments performed show the effectiveness and efficiency of the proposed heuristic.

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Chapter 1

Introduction

As a consequence of the integration in the European Common Market, Europe became an area where the free movement of goods, persons, services, and capital is ensured. With this new open market a promising "invasion" of foreign companies leads Portuguese banks to take measures, in order to be competitive regarding foreign banks. In the early 90's, the Portuguese banking sector was rather similar to its European partners, due to the return to private initiative and management. There was also a growth in the number of banks and in bank-branch networks. Despite that, it has been only in the late 90's that merges have started taking place. In 1997 there were 62 banks operating in Portugal. In 1998, this number has increased slightly since seven new banks have arisen in the market and four banks have withdrawn from it. The banks exiting the market were merged into Portuguese existing bank institutions. In 2000, seven more banks exit the Portuguese market merging into existing banks. Several motivations can be found behind these merges: the possibility to decrease the risk; the increase of the market share; the belief that the government protects better large dimension institutions; and finally an increase in the capacity to compete in a global market (Pacheco, 2003).

The redesign of the branch network becomes essential to those merging banks, not only to optimize the branch network, but also due to the technological changes in the way services are provided. It should also be noticed that new services have become available at bank-branches. Although the closure of bank-branches can lead to a loss on the volume of deposits, which is one of the most important issues along with credit volume (Boufounou, 1995), it must be considered, as keeping unnecessary branches can be too expensive. Furthermore, new cheaper ways of providing some of the services have arisen, such as ATMs (Automatic Teller Machines), internet banking, and telephone banking.

Restructures may also occur in other situations such as in acquisitions or the entrance of a new bank in the market. When a bank starts operating in a foreign country, it usually opens just one or two branches. Then, as the number of clients increases and the bank brand achieves recognition, the branch network needs to be expanded. The number of branches to open and their specific location becomes crucial, and these decisions depend on the bank objectives in terms of growth, market position, and client segments to be reached. In some sense, new banks are in advantage since the opening of branches is usually considered as a statement of power. Furthermore, a new bank can plan more carefully and freely the location strategy, than existing banks after an acquisition or merge process, since the latter ones have to take into account the existing branches. Another reason for rethinking a branch network is that locations that use to be excellent may, some years latter, no longer be interesting due to social changes. The reverse may also happen.

The above reasons show the importance of the number of branches on a bank-branch network and, above all, the importance of their location. The redesign of a branch network may include closing existing branches, opening new branches, or resizing existing branches, as will be explained later.

1.1 The Problem of Bank-Branch Network Restructure

Having introduced the context in which our problem arises, a short description of the bank-branch restructuring problem, that will be addressed in this work, is presented next. Since we wanted to make this study as close to reality as possible, we have used information regarding the Portuguese operational banking characteristics and the Portuguese territorial ordainment. Therefore, the subject of our study is a typical bank operating in the Portuguese territory.

Portugal is divided into major political areas, *Districts*, which are subdivided into smaller areas named *Counties*, that can be regarded as different markets where a bank operates. Each market is further divided into regions of demand, named *Parishes.* In this study, a single district will be considered. Therefore, only counties and parishes will be mentioned. We assume that all clients of a parish are located at its geographical centre. The same applies to branches thus, there can only exist a single branch per parish. A bank is restructuring its branch network, in order to find the number of branches needed and their best location such that the client demands are satisfied. Therefore, we need to know the demand of each client, i.e. the amount of service units the client needs, and the coverage capacity of each branch, that is, its service capacity. Branches may have different sizes and different coverage capacities. Clients are considered different and thus, they have different coverage needs regarding bank services. As in a real situation, we consider that clients are the ones choosing which branch or branches they will be banking at. There is a minimum and a maximum (ideal) number of service units needed, associated with each client. This is due to the recognition that a bank aims to not only to guarantee that clients are provided with a minimum service, but also provide different services and service quality amongst the clients they have. Therefore, a bank must provide the minimum coverage, nevertheless it is considered to be incurring in a penalty cost, whenever it cannot provide the service quality desired by its clients, i.e. when they are not maximally covered.

A branch restructure involves costs and a bank is a financial institution seeking profit. Therefore, it is logical that the bank wishes to make this restructure at a minimum cost. We consider the existence of three different kinds of decisions to be taken, regarding banks branches: closing an existing branch that is no longer considered desirable; opening a new branch to be able to provide service or better service to a certain area; upgrade or downgrade the size of an existing branch due to clients change in number or type; or maintaining an existing branch as it is. On one hand, whenever the bank reaches the conclusion that a certain client has service needs exceeding the existing branches capacity, it considers the possibility of opening a branch in a parish nearby. On the other hand, if the coverage is exceeding, by far, the demand of a client, or if there is another branch that could be opened and operated instead, at a lower cost, then the bank may consider closing one or more branches. Finally, there can also be a resizing of the bank-branch. It may happen that in a certain area, the need for service units has changed, increased or decreased. Therefore, if the costs involved are reduced, the branch may continue to operate but with a different size. Opening a branch increases the total costs by an amount related to the immediate expenses such as refurbishment, security, and equipment costs. As before, closing a branch has a cost associated, that will be accounted for. To each operating branch we associate an operating cost.

We think that the analysis of this problem cannot be complete without a restructure in human resources, since costs associated with the employees account for a large amount of operating costs. Therefore, in order to open a branch, a bank must hire employees to operate it. As a consequence, the total costs will go up as there is the need to train new employees and also due to their salaries. If a branch is being closed, employees are no longer needed and they may have to be dismissed. Hence, the total cost will, on one hand, increase due to the compensations that must be paid, and on the other hand decrease, since their salaries will no longer have to be paid.

From a practical point of view, the major contribution of this MSc thesis is the study of the restructuring of a bank-branch network, which although recognized as a problem faced by banks, has not been the subject of much study. Furthermore, in the few studies found, the human resources issue has always been left out. We propose a mixed binary and integer linear formulation to this problem that can be solved by off-the-shelf available software. In our case, we have used CPLEX. However, the formulation obtained has an enlarged number of variables since the original problem is not linear and therefore, only small size instances can be solved. In addition we have developed a heuristic of a local search nature and thus it involves two main steps. In the first step, a related linear programming problem is iteratively solved with updated cost functions. At the end of this step we have found a feasible solution which becomes the starting point of step 2. In step 2, the solution is improved by performing drop and swap operations.

1.2 Related Problems

In this section, we will be referring to other theoretical problems, closely related to but a simplification of the one we have just described. The purpose here is to compare them with ours in order to point-out the main similarities, as well as, the main differences.

The bank-branch network restructure problem is closely related to the location problem. In order to make the comparison between the two problems easier, we will use "branch" rather than the usually used "facility". General location problems deal with a set of clients I, and a set of branches J. A branch cost f_j is associated to the opening of branch j in J, and a demand w_i to be satisfied is associated to each client i in I. Furthermore, there are service costs c_{ij} to be paid for, if client i is served by branch j.

Several different location problems have been addressed in the literature. The variations of the location problem, arise from the different assumptions made. A brief discussion is provided bellow.

In the *K-median problem* the location of at most k-branches must be found. The objective is that the sum of the distances between each client i and its nearest opened branch j, is minimum. In the *P-centre problem* the location of p-branches opened must be found, but in here, we want to minimize the maximum distance between a client i and its nearest opened branch j. For both problems, the distance between a client $i \in I$ and a branch $j \in J$ can be seen as a service cost c_{ij}, and the opening costs are set to $f_j = 0$.

The location problem is known as the *fixed-charge facility location problem* if a nonzero opening cost is associated to each (facility) branch. For this type of problems there are no restrictions on the number of branches to be opened. The objective is to open the necessary branches in order to supply all clients with the required service (or demand) at minimum cost. The cost, which is to be minimized, consists of two components, namely: the branch opening costs and the service providing costs. This cost is obtained by adding up the sum of opening costs with the sum of the servicing costs. This problem can be further divided into *capacitated* and *uncapacitated*, depending on whether the ability to supply clients is limited or not.

Another type of problems closely related to the bank-branch network restructure is the covering problem. In the covering problem the objective is to find the smallest number of branches in order to cover each client by at least one branch. Amongst the covering problems we can distinguish the classical covering problem, the *multicovering problem* also known as generalized set covering problem (Hall & Hochbaum, 1992), and the *multifacility multicovering problem* also known as multilevel location set covering problem (Church & Gerrard, 2003) or even unbounded multi-cover problem (Xiaoming & Slyke, 1984). In the multicovering problem each client i needs w_i service covers. We also have servicing costs c_{ij}, associated to each branch $j \in J$. The objective of this problem is to cover all demands at a minimum cost. The difference between this problem and all the others is that the previous problems consider that each client only needs a single cover unit, $w_i = 1, \forall i \in I$. The multifacility multicovering problem differs from the multicovering problem because it allows to open more than one branch in the same site.

As we can see, all these problems have similarities with the one we are addressing, but none of them covers all the issues we are dealing with. In our branch network restructuring problem we want to locate branches at a minimum cost, such as all problems previously described. Besides that, we want to allocate clients to branches, similarly to the multicovering problem. Different branch sizes can be seen as multiple facilities. Furthermore, we also consider closing and resizing existing branches. Human resources are also considered in our problem. Regarding the total cost, which is also to be minimized, we consider branch opening costs and servicing costs, as well as branch closing, branch resizing costs, and branch operating costs. Regarding the employees we have firing and hiring costs. In addition, we also consider penalty costs. These costs are associated with the difference between maximum coverage required and coverage provided. From the problems in the literature only the fixed-charge and the multicovering location problems have branch and servicing costs. All the other problems only consider servicing costs. In branch network restructuring problems we have branches which are limited in the number of cover units they can give, such as in capacitated location problems. We assume that each client needs a certain number of covers (service units). The only problem that deals with this need for several cover units is the multicovering problem. None of the former problems deals with employees, or penalty costs. Therefore, the problem we are addressing is an extension of the above since it is a mixture of the above problems with several additional features.

We want to locate branches, and allocate clients to branches, in order to cover clients and their needs for banking services, at a minimum cost. In order to do so, we may open new branches, close or resize existing branches. Clients may be allocated to several branches. Costs with branches include opening, closing, operating, servicing and penalties. Firing and hiring are the costs related to employees, which is an extra feature we consider. The quality of the service provided is also considered. This is handled by what we called a minimum and ideal coverage. The minimum coverage requested by a client is understood as the minimum service that must be provided. The ideal coverage works as service quality that we want to have in the service provided but for some reason it is thought better not to give. Thus, the difference between the coverage provided (that must be at least the minimum coverage) and the ideal coverage is penalized as if it where a cost.

1.3 Outline

This thesis is organized as follows. In chapter 2 we present a review of the literature. We focus on three main subjects: bank-branch restructures; an overview of the bank-branch location problem; and local search algorithms. Chapter 3 introduces a detailed description of the addressed problem, the notation followed and the corresponding mathematical model. The data collected and the description of the generated data is presented in chapter 4, along with a description of the problems generated to test the heuristic developed. In chapter 5, the solution methodology developed to solve our problem is described and implemented. Computational results are also provided and discussed. In chapter 6, we give a summary of the work presented and draw the main conclusions. Directions and suggestions for future research are also given in this chapter.

Chapter 2

Literature Review

2.1 Introduction

The problem of bank-branch location has long been present in the financial world, nonetheless it has not been the subject of much academic study. Thus, not much literature has been devoted to it. Although some attempts to solve this problem can be found, usually either they look at the mathematics behind it or at the data, but never both. In addition, the inclusion of the employees has never been an issue, thus neglecting an important part of the problem. The literature review is divided into three main topics. Firstly, we introduce the context of bank-branch restructures and the situations where they have arise; secondly, we present the solution methods already attempted for the bank-branch location problem; and thirdly, we give a brief account of local search algorithms and their applications, since we propose a local search based heuristic.

2.2 Bank-Branch Restructures

The choice of the problem addressed in this work was mainly motivated by merges that have been taking place between well known banks in the past few years. These merges bring about issues such as how clients communicate with their "new" bank and how this "new" bank manages the large number of branches resulting from the merge. Therefore, we have started to look for information regarding merges and in particular, regarding branch network restructures that usually take place after merges.

Avery *et al.* (1999) have studied the association between merges and modifications in the branch network, measured by changes in the number of bank-branches per capita. The authors used a complex data set made of mergers, acquisitions, failures, liquidations over time, and the specific location of bank branches. Furthermore, they have included economic and demographic data such as the county-level income and population estimates, among others. They also have classified branches geographically, using the zip-codes corresponding to their location in the USA. Several statistical and data analysis techniques were used in their study. The major conclusion they have been able to draw was that branch network restructuring is effectively related to merger activity. On one hand, a reduction of the number of branches is strongly associated to a merge[1] if the merging institutions have branches within the same zip-code area. On the other hand, bank-branch networks seem to be unaffected if merges occur within the same market but not in the same zip-code area.

Cabral and Majure (1992) have performed a study on the evolution of branch restructures in Portugal. According to them, between 1985 and 1991 the number of branches has grown substantially, and in the following four years it has almost duplicated. Their study looked at two issues: the high growth rate and the differences between the two banks that had grown the most. They have used a theoretical model that explained concurrency between banks, based on the number of branches. Their model consists on a partial adjustment model and it includes information about the location of branches (rural or urban areas). They have considered 18 markets, corresponding to the 18 districts[2] in Portugal and the information used was relative to 1989 and 1990. They have concluded that some banks have increased the number of branches in order to fight back the increase on the branch network of their competitors. Furthermore, they also have concluded that the larger the bank the more probable was this sort of action.

We put forward the following question: Is it possible to have a branch restructure

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[1] The authors claim that the conclusions are to be robust across time and valid for both rural and urban areas.

[2] A District is the largest division in the Portuguese territory ordainment.

in situations other than merges? In our research, we have found a work in (Nordlb, 2002), where the author discusses a bank restructure, that recently took place in a Lithuanian bank. The bank wanted to restructure its branch network in order to adequate distribution of services and products to its clients. The restructure was planed taking into consideration aspects such as client needs, the intention to cover the entire territory, competition between banks, among others. Three different types of branches were considered: business centres, branches, and sub-branches (small branches). The territory was divided into four main regions, north, south, east, and west. The bank was mainly concerned with its clients, and wanted branches to pay them more attention. Therefore, they would not close any branch in the restructure; instead, they would give them new roles. Business centres were associated with corporate and retail clients; all other clients were to be served by branches and sub-branches.

Another issue we have looked at, is the impact of new technologies on the way banking is performed nowadays. Banking service has been changing in the past few years. For example, some years ago all banking activities were taking place at bank-branches and each bank had no more than a few branches spread out in Portugal. As time went by, the banks wanted to reach more and more clients and thus, they have started spreading their branch-network all over the country. This branch expansion lasted several years until new technologies were developed. In 1985 ATMs were introduced in Portugal and started changing the "Portuguese way of banking" (Sibs, 2004). Two decades ago, all banking transactions were handled in "physical" branches, while nowadays the number of transactions done in ATMs is high and increasing, as can be seen in Table 2.1.

The modern way of life does not allow for an active relationship between clients and bank-branches, especially concerning their different timetables. Banks soon realize it and have started taking measures to provide their clients with alternative ways of banking. In 1994 the first telephone-banking was launched in Portugal, offering 24-hour services over the telephone (BusinessWeek, 2004) and (SummitReports, 2004). With the increasing number of internet users, banks launched the internet-banking service. This service has been used by a growing number of bank clients due to its versatility. More recently, the ultimate banking service was created: the

Year	No of ATMs	Total of Transactions	Withdraws	Inquiries	Payments	Other Transactions
1999	6.831	395.452.832	227.784.693	123.270.256	36.906.555	7.491.328
2000	7.864	457.165.526	250.981.991	136.550.041	60.405.800	9.227.694
2001	8.482	527.417.947	275.778.935	153.075.437	85.875.094	12.688.481
2002	9.032	588.806.749	296.905.441	171.770.535	105.125.585	15.005.188
2003	9.521	631.912.059	314.766.155	187.465.715	114.378.182	15.302.007

Source: Website of Marktest - www.marktest.pt

Table 2.1: Evolution on the use of ATMs.

mobile-banking service (Silva, 2004). This means that a client may have access to its bank account literally wherever he may be. The technology revolution that has been taking place all over the world in the past few years, has greatly affected the way bank-branches operate, and forced them to assume a different role in banking (Howland, 2000).

Along with the new technologies, already mentioned, there is a whole new kind of branches, other than the traditional ones. Banks start to allocate branches in new sites, trying to capture as much attention as possible in every place where a potential client may be. Lately, the preference has been shopping-centers and supermarkets (Howland, 2000), but there are banks that see further and started opening branches in university premises, near their potential future clients, offering them advantages of many kinds in order to attract them[3]. Bank clients are being divided into segments (Mols, 1999) and new kinds of branches and branch services are being designed to target the specific segments. An example of a Canadian bank that has decided to open some branches specifically to offer investment information, as well as, related products and services, can be found in Howland (2000). In this example, it can be seen that even the decoration, colors and design, were chosen to match the preferences, of the segment of clients the branch has been opened for.

Despite the development of new technologies and the consequent change in banking services, banking at a branch is still preferred by bank clients (Nadler, 2002). In a study presented by Avkiran (1999), the major conclusion drawn is that bank employees still have an important role to play in banking. Therefore, although clients like to have access to automated services, they are still using branches and

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[3] This fact has been noticed lately in Portugal.

becoming more demanding concerning the employees conduct. Therefore, Avkiran suggests that it should be made an adequate choice on the number of employees to serve clients in bank-branches, and that they should be provided with specific training in order to have a professional and a responsive attitude towards clients. Bank clients feel frustrated if they have to stand in a queue, when they can see half the employees "free" to help them. Avkiran also suggests that there should be a clear separation between tellers and employees associated with back office operations. If an employee is not supposed to serve clients then it should be out of site. Human resources also suffer with branch restructures, and other strategic movements. In Portugal banks have been reducing the number of employees (mainly by early retirement) and at the same time increasing productivity by choosing the employees with "better" education and performance.

Another question that might be asked is whether bank-branches may disappear in the near future. Anne Howland (2000) answers that question, by saying:

> "... it's not time to give up on the traditional bank-branch just yet. After all, it's that familiar sign at street level, and the staff within the branch, that are often a clients first, and perhaps most comfortable, introduction to the bank, serving as a point of entry to the growing number and diversity of banking products, services and delivery channels."

2.3 The Bank-Branch Location Problem

In the area of bank-branch location, particularly the one devoted to the operational research point-of-view, the literature is scarce. Wang *et al.* (2003) and Miliotis, Dimopoulou, and Giannikos (2002) focused on the mathematical formulation of the model, based on facility location and set-covering models. Wang *et al.* (2003), studied a budget constrained facility relocation problem where they considered both opening new facilities and closing existing facilities. Their objective is to minimize the total weighted travel distance for clients, subject to a budget constraint and a constraint on the total number of open(ed) facilities. Three heuristics have been developed to solve the problem: a Greedy-Interchange (GI) Heuristic, a Tabu Search (TS) heuristic, and a Lagrangean Relaxation (LR). The GI heuristic consisted of two phases. In phase one, the authors open or close facilities assuring that the constraint on the total number of opened facilities is satisfied. This provides them with an initial feasible solution. The second phase consists of an interchange procedure that performs a swap between a pair of facilities until there are no feasible swaps or no feasible swap improves the objective function. The pair of facilities to be considered consists of an existing facility and of a nonexisting one. The TS heuristic uses the first phase and a modified version of the second phase of the heuristic. In the modified second phase, the best feasible swap is implemented regardless of the impact on the objective function value. The algorithm continues by repeating swaps until there has not been an improvement on the objective function for k consecutive swaps. In the LR heuristic there are three phases. The first two correspond to the GI heuristic. In the third phase, the problem is relaxed with respect to the budget constraints and to the constraint stating that every client is assigned to exactly one location. The solution to the relaxed problem is then embedded into a branch-and-bound algorithm to solve the problem exactly. The authors tested the heuristics on 270 problems with 459 clients and 84 facility sites. These problems have been generated based on a real problem of locating bank-branches in a large-scale township in the New York State. The computational time requirements of these heuristics have been compared with that of CPLEX (ILOG, 2004). The authors have found the LR approach, which is the most time consuming amongst the three, to be 28 times faster than the CPLEX.

Miliotis *et al.* (2002), presented a methodology for determining the optimal location of bank-branches. In their approach they address the problem by sequentially solving two related problems. Initially they solve the problem of finding the minimum number of branches needed, subject to providing clients with their minimum coverage requirements. Then, given the number of branches to be located, they determine the exact branch locations in order to maximize the total coverage provided (to clients). They partitioned the demand space according to two levels of characteristics: inter-regional and intra-regional. Both these partitions were made using Geographical Information System (GIS) techniques. In the first level, they have partitioned the demand space into regions according to their economic and demographic characteristics and rank them with respect to their needs for bank-branches coverage. In the second level, each region is further partitioned into sub-regions having different needs for banking services. The problem is then solved in two stages. In the first stage, the minimum number of branches needed to be located in a certain area is obtained by solving a classical covering problem. By doing so, it is implicitly assumed that clients are serviced by a branch if it is no further away than a critical distance limit and also that branches can serve all clients it can reach. In the second stage, they determine the exact location for branches, in order to maximally cover the clients. The problem is modelled as a maximal coverage location problem where the objective is to maximize the number of demand points that are covered, subject to having the number of branches found in the first stage. The solution procedure just described was applied to a specific case of a Greek bank, and was solved by using LINGO (Lindo, 2004). The results reported are not compared to any other methodology and no other results are reported.

Melachrinoudis and Min (2001) address the problem of bank-branch location-allocation where it is considered the existence of three levels of bank services: Automatic Teller Machines (ATMs), bank-branch offices, and main branches. The authors developed a Chance-Constrained (CC) Goal-Programming (GP) model. The CC allows for the inclusion of uncertainty and risk associated with parameters and constraints, while the GP deals with diverse and conflicting goals. The uncertainties are resolved by using estimates and minimum risk targets. The authors solve a specific problem, faced in some town in the USA, using LINGO. Although the results are not compared to any other methodology or results reported, they have been compared to the existing situation. The authors have found that three of the four branches coincide with the current location.

Another interesting problem, closely related to the bank-branch location was presented by Aleskerov, Ersel, and Yolalan (2003). The authors studied the problem of allocating personnel among branches of a large-scale commercial bank. The available personnel is allocated in order to reduce inefficiencies resulting from the gap between available and needed human resources. They developed a normative two-stage method that assumes branches to be homogeneous in terms of workload pattern. Nevertheless, a modification to address the case of heterogeneous branches has also been addressed. The method seeks, in a first stage, to rank branches in terms of performance criteria pre-defined by bank management. This rank uses the Condorcet practical rule (Nanson, 1882), that says that each branch must be sorted according to each individual criterion separately. The dominating branch becomes first in the final rank ordering and is excluded from the branches list. For the remaining branches, the concept of dominating branch is broadened to cover branches that are ranked as "first or second", "first, second or third", and so on. In the second stage, the shortage of personnel is given by the ratio of requested to existing personnel. For branches equally ranked, priority is given to the one with the highest level of personnel demand. This method was applied to real data from a bank in Turkey.

A different kind of approach has been presented by Boufounou (1995). He developed a model for a Greek bank, to support management decision-making in establishing branch goals, evaluating performance, and planning new locations. The volume of deposits has been identified as the key feature in the location of new branches and in measuring branch performance. Therefore, models have been developed to estimate the power of a branch to attract deposits having into consideration the characteristics of the area where the branch is, or may be, located. Data relating to the location characteristics such as, trade characteristics, bank competitive situation, and internal branch characteristics, was collected and analysed using principal components analysis. Boufounou stated that the total volume of deposits was mainly determined by the branch age, number of employees, and the professional and commercial characteristics of the location. The presence of competitor banks is also identified as an important feature.

Another possible approach to solve the bank-branch problem is provided by a GIS model, see for example Morrison and O'Brien (2001) and Willer (1990). Morrison and O'Brien discussed a GIS-based spatial interaction model designed to help banks taking decisions about the closure of some of their branches. They focused on the experience of a particular bank and its needs to model the impact of closing some branches on the remaining parts of the network. The authors developed a systematic method for site evaluation to assist in the rationalization of the branch-network. The model consists of four stages: estimation of the probability of a branch being visited, by using the standard retail Huff model[4] (Huff, 1964); generation of the expected distribution of clients, which is based on the number of clients living in a particular area and the probability that they make transactions at different locations; computation of the expected number of transactions at a given branch, which is simply the summation of the number of expected consumers at a given branch; and estimation of the impact of removing one or more branches from a certain branch-network. The authors were able to conclude that closing branches in areas of lower socioeconomic status places a heavier burden on the population, in terms of access to other branches, than if an alternative set of branches had been closed.

A spatial decision support system (SDSS) that helps to solve bank location problems has been presented by Willer (1990). The prototype consists of a GIS engine, an analysis module, a system interface, and a user interface. The GIS engine is the central part of the system, and was chosen to manage data and to display cartographic information. The implemented analysis module consists of a location-allocation model previously developed by Densham (1990) which in turn is based on a vertex substitution heuristic by Teitz and Bart (1968). In order to maintain compatibilities between the different software packages used, a system interface was also developed. Finally, a user interface has been created to facilitate the usage of the prototype. The analysis module was applied to the location problem using the decision-makers preliminary set of objectives. These objectives determined the initial set of alternative solutions, which are then evaluated by the decision-makers. This process is iterative and will stop when the decision-makers are satisfied with the definition of the problem and are able to select a solution for it.

Next we present a section devoted to the local search algorithms and a brief account of the state of the art.

2.4 Local Search Algorithms

Local search algorithms, also called local search heuristics or even neighborhood heuristics, are used to solve a large variety of problems such as location problems.

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[4] The Huff model was developed to estimate the probability of clients living at location x visiting a branch in location y.

Local search heuristics have a basic structure with three components:

- A method to obtain an initial solution;
- A method to iteratively improve the initial solution;
- A stop criterion.

All three phases are very important since all have an impact on the quality of the final solution to be obtained and on the computational time required to obtain it. For example, if the initial solution is very poor it might not be possible to obtain a good final solution, at least in a reasonable amount of time. Regarding the improvement method, if a very simple method is used then, probably the solution obtained is poor. On the other hand, a sophisticated method may be computationally prohibitive. The stop criterion has to be carefully chosen otherwise the method may be cycling. In the next sections, we discuss some of the techniques used for each of the three phases.

2.4.1 Initial Solution

There are several ways of reaching an initial solution, and the chosen one will depend mainly on the problem and on the authors choice. Some authors, start with an initial solution that includes all facilities, and then at a later stage some are eliminated by, for example a drop approach (Feldman et al., 1966). Other authors, choose the initial solution as an arbitrary set of facilities. For example, Korupolu and Plaxton (2000) developed a local search heuristic to solve uncapacitated k-median problems, where the initial solution considered is an arbitrary set of $(1 + \alpha)k$ facilities, with α being a sufficiently large constant. Arya *et al.* (2001) and (2004) also solve the k-median problem, but instead they choose an arbitrary set of exactly k facilities as an initial solution. Wang *et al.* (2003) give a local search heuristic to solve a budget constrained location problem. In this problem they consider the existence of q facilities and a desired number of p facilities. Therefore, depending on the relation between p and q, they include or exclude facilities in order to have an initial solution with p facilities. Another approach to find an initial solution is to solve a problem related to the original one. A linear relaxation of the k-median problem is the method used by Charikar and Guha (1999). They allow the variables related to the assignment and to the choice of the location to be a centre, to take positive continuous values rather than integers. They also find an initial solution to a facility location problem by sorting all facilities in increasing order of a cost combining opening and service costs. Facilities are then chosen to be part of the initial solution by increasing cost value. Khumawala and Kelly (1974) solved a linear warehouse location problem in order to find an initial solution for the location of warehouses with concave costs. The linear model is obtained by ignoring the variable warehousing costs and considering only the warehouse fixed costs. This last kind of solutions, although not found for the original problem, can constitute a good initial solution (Khumawala & Kelly, 1974).

2.4.2 Neighborhood Definition and Stop Criterion

After determining the initial solution X the next step is to define the operation or operations that allow for changing the solution X into another solution X'. The set of all solutions that can be reached from solution X by a simple operation is called the neighborhood of the solution $N(X)$. The best solution of the neighborhood is known as a local optimum for the problem. The operations should be defined in order to move from solution X to the local optimum as quicker as possible or at least to a solution X' as closer as possible to the referred local optimum. The most basic operations defined in local search heuristics are *add*, *drop*, and *swap* operations. Let us use the terminology of facility location problems in order to better explain these operations. In the add operation, the idea is to open one further facility which is to be added to the existing solution, while in the drop operation the idea is quite the opposite, i.e. we want to choose from the current solution one facility to be closed. Finally, a swap is an interchange between facilities, where one open facility is going to be closed and thus, removed from the current solution, while at the same time a non-opened facility is going to be opened and thus, appended to the current solution.

The operations to be iteratively used in order to improve the solution, depend mainly on the technique used to find the initial solution. As we have seen earlier, Feldman *et al.* (1966) used in the initial solution all warehouses therefore, the operation defined by them was the drop. They tried two different approaches, dropping a warehouse that represented the largest saving and dropping the smallest warehouse regardless of the savings. The results for both approaches were similar. A drop is attempted until every single warehouse has been tested. Wang *et al.* (2003) have defined the initial solution for the budget constrained location problem, to be a set of exactly q facilities, which was the number desired by the management thus, they used a swap operation. Amongst the possible swaps that improve the objective function value and lie within the budget limit, it is chosen the one leading to the best objective function value. This swap procedure is repeated until no more such swaps exist. Slightly different operations can be defined based on the three already mentioned. Arya *et al.* (2004) analysed local search with single swaps and with multiple swaps, to solve the k-median problem. The single swap, is an interchange between an existing facility that is being closed and a non-existing facility which is being opened. The single swap is performed, over an arbitrary set of k facilities, if the swap improves the solution, i.e. if the total service cost decreases. The multiple swap operation is a simultaneous interchange of $p > 1$ facilities, that is, p open facilities are being closed and p non-open facilities are being opened and included in the current solution . They also have studied the uncapacitated facility location, for which facility dropping, adding, and swapping are tried. This last approach was also followed by Korupolu *et al.* (2000), for the uncapacitated k-median problem and for the capacitated facility location with splittable demands. Charikar and Guha (1999) consider the addition of a single facility and the drop of several facilities, in order to solve the location problem. The gain associated to a facility i is the difference between the current service and facility costs, and the costs obtained by including i in the solution. The method is to randomly choose a facility from the current solution and test whether its gain is positive. If that is the case, then the facility is included in the solution, and if necessary, one or more facilities are removed from the solution. If any facility leaves the solution, all its clients will be reassigned to the new facility. Add-swap, 2-swap, and permutation have been considered by Ghosh (2003) to solve the uncapacitated facility location problem. In the add-swap operation, the local search algorithm starts by opening (adding) a facility in one of the sites considered in the problem. Then, the algorithm moves forward with swaps (interchanging one opened facility with a non-opened one), until no more swap moves improve the solution. The algorithm repeats the add and the swap phases, alternating them until no improvement can be obtained for the objective function value. The 2-swap approach allows for the three basic operations, adding, dropping, and swapping facilities. Finally, the permutation operation is performed. This later operation differs from the others, since in this case, neighborhood solutions differ in exactly two locations.

2.4.3 Results Reported

The results reported in the literature, regarding local search algorithms, are of two different types, theoretical results and computational results. The theoretical results are proofs, for the achieved *local optimality gap*, by means of mathematical demonstration of Theorems and Lemmas. The local optimality gap is obtained by computing the ratio between a local optimum and a global optimum. Computational results are based on applications of the algorithms to practical problems, both using real and generated problems.

Let us start by the theoretical results obtained by some authors. Charikar and Guha (1999) achieved a 3% gap for the Uncapacitated Facility Location Problem (UFL) while Korupolu *et al.* (2000) have proved that a local search procedure for the UFL, that allows adding, dropping, and swapping operations, has a local optimality gap of 5%. They have proved the same result for the k-median problem. Their results have been improved by Arya *et al.* (2004), who were able to prove a local optimality gap of 3% for the UFL. They have also provided a $3+2/p$ local optimality gap for the k-median problem, when multiple swaps are allowed.

Feldman *et al.* (1966) have tested their heuristic on 12 warehouse location problem instances which have been previously formulated by Kuehn and Hamburger (1963). As only minor improvements were obtained they have decided to test the heuristic on a larger problem instance they constructed. For this problem, which has 49 warehouse sites and 200 clients, two different cost functions (linear and concave) were considered. Each of these versions was solved by a drop heuristic and an add heuristic. Both heuristics performed well in the case of the linear cost function, but the drop approach was significantly better when a concave cost function was being considered. Ghosh (2003) tested three different local search heuristics on 180 generated UFL problems having 100 facilities and 100 clients. He was able to conclude that the solution quality was independent of the heuristic used. Nevertheless, the computational time requirements vary. The 2-swap local search heuristic being the quickest, especially when compared with the permutation operation. Wang *et al.* (2003) have tested their heuristics on 270 problems with 459 customers and 84 potential facility sites. The results obtained were compared to an optimum solution, obtained by using CPLEX.

2.5 Conclusion

In this chapter we have reviewed the literature on problems related to the bank-branch restructure problem. The works surveyed are the most important regarding the work being presented in here and allow for a global vision of the work that has been done on this type of problems. Next, we provide a detailed description of the problem to be addressed.

Chapter 3

Problem Description

In previous chapters we have introduced the context in which the addressed problem arises. In this chapter, a detailed description of this problem is provided, as well as, the assumptions that were made in order to define it. Then, the notation used is introduced followed by the mathematical formulation.

3.1 Introduction

Banks operating in Portugal seem to be reaching some stability concerning the number of branches. It is more than evident that sometimes small adjustments have to be made, due to market research results, changes in the distribution of the population and/or companies, or simply in order to achieve a better efficiency level. However, such small adjustments are easy to deal with. For example, if a branch is closed, employees are redistributed and clients transferred to other branches. This occurrence does not represent damage to the banks image. Nevertheless, it brings several monetary costs and savings incurred. If the opposite decision is taken, that is, if a branch is opened, costs are mainly due to branch and employees expenses although saving may also happen due to service costs . The real problem arises when adjustments concerning a whole bank-branch network are deemed necessary. In this case, the number of branches to be opened, closed, or resized, may become too large to manage without a structured and specialized decision making approach. In order to help making such decisions, we have studied and formulated this specific problem and then developed a solution approach. To start with we began by deciding which issues should be included. In order to do so, a thorough research was conducted, and finally we have decided to include: branches, clients, sites, covers, and employees. These are the key factors we want to study.

In the following sections, we present a detailed description of the problem we have considered and discuss the decisions taken regarding the selected parameters and their values.

3.2 The Detailed Problem

Firstly, we define the geographical sites considered in the problem. We have followed the political territorial division used in Portugal, where the territory is divided into districts. Each district is divided into sub-areas, named counties and, each of them is further divided into parishes. We chose the district of Porto to collect data and further information because we are more sensitive to what happens in this district, and also because it is the district for which we are able to gather more and better quality information. Although there are several districts, in this work we consider one district only. Thus, only counties and parishes are used to define possible locations. Parishes are seen as being a unique client located at its geographical centre. Therefore, no more than one branch is to be located at each parish.

Whenever a branch is opened, the bank is faced with three main cost types: building acquisition/renting costs and the corresponding refurbishment; costs with furniture and equipment, such as computers, desks, counters, ATMs, air-conditioning, among others; and finally, the extremely important expenses with the security system, such as safes, video-cameras, monitors, VCRs, digital recorders, and other surveillance equipment. We have learned from professionals in the area, that nowadays in Portugal banks tend to rent buildings rather than acquire them. Buying implies one lump sum payment, while renting implies monthly payments. Hence, rents cannot be integrated into the opening costs, which are considered to be paid only once. *Opening costs* h_{ij} are therefore, mainly due to branch refurbishment, equipment, and acquisition of the security system. In order to get an estimate for these costs, some specialized companies (KLSecurity, 2004; ATMMachine, 2004; Diebold, 2004; BrownSafe, 2004) and bank employees were consulted. Regarding the refurbishment, we first had to find out which branch sizes are more commonly used. We learned that in Portugal, some banks began to standardize their branch sizes. The most common being middle size branches whose premises are for about 7-10 employees. Although smaller and larger branches can also be found, usually there are three main sizes: small branches, having approximately 4-6 employees; medium size branches having roughly 7-10; and larger branches having between 11-15 employees. Branch areas obviously vary depending on the number of employees, and on the availability of the real estate market at the time. In this work, we consider that a medium size branch, has an area of about 100 square meters. Refurbishment costs obviously depend on the branch area. To estimate refurbishing costs, we have made a list of needed items, such as floors, counters, paints, among others, which has allowed us to come up with a cost per square meter.

Let us consider the opening of a branch. Employees represent an important role in a branch network restructure, and imply costs other than their salary. For simplicity, in this problem we consider all employees to be of the same category. After thoroughly thinking of employees cost implications, and collecting data from bank employees, we got to the conclusion that besides their salary, other costs such as office material, equipment (desks and computers) and training, must be considered. Since we have associated the working equipment with the cost of opening the branch, we then consider that the *cost of hiring* T an employee is mainly due to training expenses. Training usually takes no less than 2, and no more than 3 weeks[1]. Thus, training costs are related to travelling expenses and the loss of productivity during the training period. When an employee needs to be trained outside his working area, the bank usually pays up for the travelling expenses. In addition, when an employee is in training, his productivity is not the same as if he was working at the bank full-time, and this also constitutes a cost for the bank. In summary, opening costs are obtained by adding up the costs incurred with refurbishment, equipment, and security systems.

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[1] Every year, bank employees are presented with two or more training sessions, which are provided by external entities. But, when it comes to the initial training, this seems not to be the case, as it is usually provided within the bank facilities.

Consider now branches already opened, regardless of being an existent branch or a newly opened one. In this case an *operating cost* f_{ij} is incurred despite the service provided. We consider that operating costs consist of three distinct components, namely: administrative costs, costs with the employees, and renting costs. Administrative costs include electricity, water, expenses with office supplies, and other similar costs. Since branches have different sizes, it is expected that administrative costs are distinct. But, obviously they are not proportional to branch sizes, as economies of scale often exist, see Figure 3.1. Costs with employees include expenses such as salaries, taxes, insurances, and holiday subsidy. For the sake of simplicity, we term them "salary" costs s_j, although all referred costs are included. We chose the average salary reported by the Portuguese Bank Association (APB, 2003) as a base value. Also, we consider that salaries vary across counties to reflect different living costs, although this is not very usual in Portugal[2]. Finally, we have estimated the *renting costs* e_{ij} by collecting data from a real estate agency. We were told that in some counties, the values varied significantly from parish to parish, and even from street to street, while in other counties this seemed not to be the case. In Table A.2 and Table A.1, see appendix A, we have summarized the information regarding

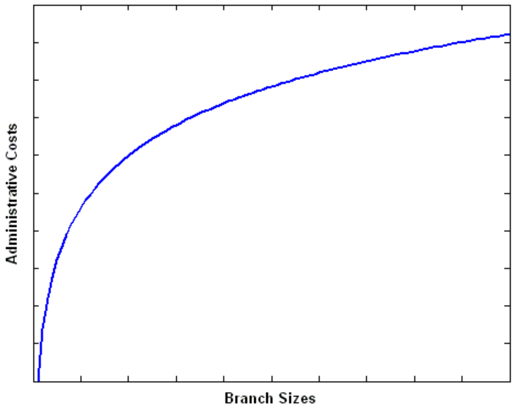

Figure 3.1: Branch sizes versus administrative costs.

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[2] These differences are usual in foreign countries where banks and other companies add a substantial subsidy to the monthly salary to compensate the employee for living in a very expensive city. This happens, for example, in London.

acquisition prices. These prices are generally used to compute rental prices by using a 0.6% conversion rate[3]. Hence, renting costs are proportional to the size of branch premises. The restructure of a bank-branch network involves strategic decisions, which are not made very often. Therefore, the problem we have in hands should be analysed over a time period comprising a few years. However, the fast changing technology does not allow for considering a very long time period. Taking into account both issues, long term decisions as well as the fast technology development, we have decided to use a 10 years period. Operating costs, however, involve mainly monthly costs subject to inflation, that are to be paid over the 120 months period. Therefore, these values must be adjusted both to expected inflation over the considered period and to the fact that they are considered now although only paid for in the future. For simplicity, we assume that these two factors are similar and thus need not to be accounted for.

We have already discussed how to handle opening costs and operating costs. Recall from section 1.1, that we have also considered the possibility of closing an unnecessary branch. Therefore, we must also address the cost involved in such a decision, *closing costs* g_{ij}. At first it seems that closing a branch can only bring cost savings, since the operational costs would no longer exist. Nevertheless, such a decision should be carefully considered as it may lead to a loss of clients and image deterioration. Unfortunately, these issues are not easily translated into a monetary cost. Thus, we have decided to interview an expert in accounting, who was able to identify key factors associated with costs incurred when closing a branch. Nevertheless, such information was of no good since we were not able to collect the specific data needed. In order to overcome the difficulties associated with finding values for closing costs, we have decided to follow the work by Wang *et al.* (2003), where closing costs are considered to be 3 to 4 times smaller than opening costs. However, this was not enough since in that work the authors do not consider the implications in terms of employees. Therefore, we also consider a *firing cost* (compensation) CMP_j that must be paid to dismissed employees. In Portugal, a dismissed employee has the right to receive one monthly salary, for each worked year. To compute this value we have averaged out the values reported in (APB, 2003) for the number of years

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[3] The rule usually used by real estate traders.

worked by each employee.

Having addressed the main cost components associated with the possible decisions, we move forward to analyse the decisions, interactions and implications involved in this problem. It should be noticed that most of the considerations that are presented next, are based on the previously mentioned interviews.

Before we begin, we must make notice that in Portugal, employees can usually be transferred from one location to another, if this new location belongs to the same district. Recall that in this problem we are considering all counties and parishes as belonging to the same and only district. Therefore, if a branch is closed that does not mean that all employees will be fired, since they can be transferred from closing branches to opening ones. Thus, the number of employees to be fired or to be hired depends only on the difference between the initial total number of existing employees, and on the total number of employees needed to operate the branch network after the restructure is performed. Decisions on transferring employees do not imply costs and may be taken a posteriori.

It is known that banks depend a great deal on the number of deposits their clients make. If a bank has no money in its safes, it can not lend it, and can not do what they are supposed to do, which is "make more money"[4]. Following this reasoning, we can imagine that a certain location, which can represent more volume of deposits (such as, for example, parish Foz in Porto) is more appealing to the bank than some others (for example, parish Sé in Porto). To include this information into our problem, each client has a number associated, that represents its banking potential. Therefore, the more interesting a client is to the bank, the higher is that number, which we called *ideal coverage* $\overline{W}_{ij}$. However, since not all client's potential banking may be captured, we also consider the minimum effort to be done to attract the client, which we call *minimum coverage needed* $\underline{W}_{ij}$. Therefore, we must always satisfy the minimum coverage, aiming to fulfill the satisfaction of the ideal coverage. To do so, we introduce a *penalty* cost whenever a client is ideally under covered, although the necessary minimum coverage is provided. This penalty, will ensure that the ideal coverage will be satisfied unless servicing costs are larger than the penalty. The value of such a penalty is related to the average service cost when only

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[4] Of course this a simplistic way of putting it, but it is only to be able to illustrate the idea that follows.

minimum required service is enforced.

Each branch has associated a nonnegative integer capacity that represents the branch service capacity, i.e. the number of service units (covers) it can provide clients with. This capacity is obviously dependent on the branch size.

Common sense tell us that unless there is a huge motivation for a client to choose a certain bank, the client tends to choose the closest bank to be its bank service provider (Boufounou, 1995). Therefore, the bank has to be ascertain that its image is located almost everywhere a potential client may be (Howland, 2000), so that clients do not change to another competitor. This attitude is actually taken by small to medium banks. In our problem, this is used to define which branches can serve which clients. Further details are given in the following section.

A thorough study of the problem lead us to identify the described issues as the most important to be accounted for. In the next section we present the notation used, which has already been introduced in a "soft" way, and then we present the mathematical programming model that was developed to represent this problem.

3.3 Notation

In this section, we describe the notation to be used to model our problem.

As we have said before, we will be considering one single district which consists of a set of n counties $j \in C$. Each county j is divided into m_j parishes $i \in D_j$, $(i = 1, ...m_j)$, where a branch of size $k \in K$, $(k = 1, 2, 3)$ can be located. Let B_j with $j \in C$, be the set of existing branches at county j.

In order to better represent the model we also define D as the set of all possible parishes $D = \bigcup\limits_{j \in C} D_j$, and $B = \bigcup\limits_{j \in C} B_j$ the set of parishes where a branch already exists. There are "two" types of branches, namely the potential $D_j \backslash B_j$, and the existing B_j branches, with $j \in C$. A potential branch is located in every parish where currently no branch is operating. Existing branches B_j, with $j \in C$, can be further divided into closable CB_j and non closable NCB_j branches ($B_j = CB_j \cup NCB_j$). As we consider more than one branch size, for existing branches we also must have information on branch size $k(i, j)$. The capacity of each branch is considered to be proportional to the branch size. More specifically a large branch can provide up to three covers to the same client; a medium branch can provide up to two covers to the same client; and a small branch can provide just one cover to the same client.

A distance standard DS, up to which a client may use a branch is established. A binary matrix summarizing which branches can provide banking services for which clients is constructed, based on the distance and on the geographical locations of both branches and clients. Each element a_{ij}^{lm} is set to 1 if branch in parish i of county j, is no further away from client in parish m of county l, than the DS; and set to 0 otherwise.

We have the number of employees ϵ_{ij}^{k} for each existing branch thus, we also have the total number of existing employees E. Each potential branch has a number of employees ϵ_{ij}^{k} needed to operate it, that depends on branch size and location. Employees have salaries s_j dependent on the county they work on. Hired employees imply a hiring cost, termed training cost T_j, while fired employees imply a firing cost, termed compensation CMP_j. The firing cost is proportional to the average number of years AY worked at the bank.

The problem to be solved is one of determining which branches must be opened with which size and which branches must be closed, subject to providing the clients with at least, the minimum service they require. These decisions are to be made at minimum cost. Therefore, it follows the costs representation and notation.

Costs are associated with branches, employees, and willingness for targeting clients potential, that is, coverage provided. Branch costs are divided into opening, closing, and operating costs, which have been mentioned before.

Opening branches imply four types of costs, namely: rent e_{ij}, refurbishment R, equipment EQ, and security SCT costs. A refurbishment is usually needed to transform the rented space into a bank-branch. Therefore, it is a function of the branch size, i.e. branch area in square meters, σ_{ij}^{k}. Equipment costs EQ, are related to all kinds of equipment employees have to operate within their daily work at the bank. As such, this cost depends on the number of employees ϵ_{ij}^{k}. Finally, security costs SCT, are considered to be similar to all branches. Therefore, the opening cost h_{ij}^{k} is a nonlinear function of $\sigma_{ij}^{k}, e_{ij}, \epsilon_{ij}^{k}$ and SCT.

The closing cost g_{ij}^{k}, as we have mentioned before, is calculated as a percentage of the opening costs (Wang et al., 2003) and thus also a nonlinear function of $\sigma_{ij}^{k}, e_{ij}^{k}, \epsilon_{ij}^{k}$

and SCT.

Opening costs h_{ij}^k and closing costs g_{ij}^k are paid once, whenever a decision on opening and closing a branch respectively, is made.

Operating costs f_{ij}^k are paid for every branch that is operated, regardless of being an existing branch or a newly opened branch. This cost is made of three components, namely: administrative costs α_{ij}^k, salaries s_j, and rent e_{ij}. It should be noticed, that the total values for all three components vary with location, size, and number of employees.

Operating branches are limited in respect to the service they can provide each client with, as well as, to the total service they can provide to all clients. For example, suppose that in parish i and county j we are operating a branch of size k. Although, this branch can only provide up to k units of service to any single client, it may provide up to αk if several clients are being served by it, see Figure 3.2. In the example given, $k = 3$ thus no client can get more than 3 units of service.

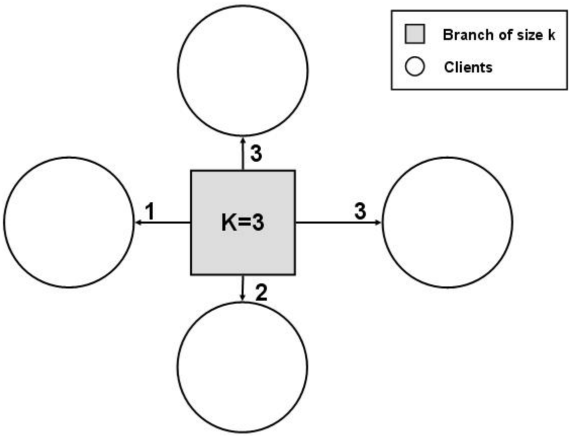

Figure 3.2: Coverage provided by a branch of size k to its clients.

Nevertheless, in total this branch is providing 9 units which is possible as long as $\alpha k \geq 9$, that is $\alpha \geq 3$. The difference between the ideal service $\overline{W}_{lm}$, to be provided to a client and the actual service the client is provided with, is used to penalize the solution obtained. Such penalty P_{lm}, is proportional to the above mentioned difference and depends on the client's buying potential, i.e. the renting value of the area where the client is located. However, all clients must be provided with at least the minimum service $\underline{W}_{lm}$, required by them. Furthermore, there is a cost for the service provided v_{ij}^{lm}, which depends on the branch location, branch size, and amount of service provided.

The decision variables identified for this problem are therefore,

- $y_{ij}^{k} = \begin{cases} 1, & \text{if a branch of size } k \text{ is closed in parish } i \text{ of county } j, \\ & \text{where } j \in C, i \in CB_j, k \in K, \\ 0, & \text{otherwise.} \end{cases}$

- $z_{ij}^{k} = \begin{cases} 1, & \text{if a branch of size } k \text{ is opened in parish } i \text{ of county } j, \\ & \text{where } j \in C, i \in D_j \backslash NCB_j, k \in K, \\ 0, & \text{otherwise.} \end{cases}$

- $x_{ij}^{k} = \begin{cases} 1, & \text{if a branch of size } k \text{ is operating in parish } i \text{ of county } j, \\ & \text{where } j \in C, i \in D_j, k \in K, \\ 0, & \text{otherwise.} \end{cases}$

It should be noticed that a branch can be opened at a location where a branch already exists, provided that the existing branch is allowed to be closed, that is, if $j \in C$ and $i \in D_j \backslash NCB_j$. Therefore, we have to close the existing branch if we want to open there another branch with a different size.

- q_{ij}^{lm}, number of service units provided by branch in parish i of county j to client in parish l of county m.
- $he_j \geq 0$, number of employees hired in county j, where $j \in C$.
- $fe_j \geq 0$, number of employees fired in county j, where $j \in C$.

An auxiliary variable was defined to make the model more readable:

- unprovided service units for client in parish i and county j, $r_{lm} = \overline{W}_{lm} - \sum\limits_{j \in C} \sum\limits_{i \in D_j} q_{ij}^{lm}$

3.4 Mathematical Model

Having defined, in the previous section, the data and notation used we now present the mixed integer mathematical model for the problem herein described. It should be noticed that all cost components are for the 10 years period.

$$\min Cost = \sum_{j\in C}\sum_{i\in D_j}\sum_{k\in K} f_{ij}^{k}(x) + \sum_{j\in C}\sum_{i\in CB_j}\sum_{k\in K} g_{ij}^{k}(y) + \sum_{j\in C}\sum_{i\in D_j\setminus NCB_j}\sum_{k\in K} h_{ij}^{k}(z) + \sum_{j\in C} T_j \cdot he_j + \sum_{j\in C} CMP_j \cdot fe_j + \sum_{m\in C}\sum_{l\in D_m} P_{lm}\cdot r_{lm} + \sum_{j\in C}\sum_{i\in D_j}\sum_{m\in C}\sum_{l\in D_m} q_{ij}^{lm}\cdot v_{ij}^{lm}. \quad (3.1)$$

subject to:

$$x_{ij}^{k_i} = 1, \quad \forall j\in C, \forall i\in NCB_j, k_i = k(i,j), \quad (3.2)$$

$$x_{ij}^{k\neq k_i} = 0, \quad \forall j\in C, \forall i\in NCB_j, \forall k\neq k_i \in K, \forall k_i = k(i,j), \quad (3.3)$$

$$x_{ij}^{k_i} = 1 - y_{ij}^{k_i}, \quad \forall j\in C, \forall i\in CB_j, k_i = k(i,j), \quad (3.4)$$

$$x_{ij}^{k\neq k_i} = z_{ij}^{k\neq k_i}, \quad \forall j\in C, \forall i\in CB_j, \forall k\neq k_i \in K, \forall k_i = k(i,j), \quad (3.5)$$

$$x_{ij}^{k} = z_{ij}^{k}, \quad \forall j\in C, \forall i\in D_j\setminus B_j, \forall k\in K, \quad (3.6)$$

$$\sum_{k\in K} z_{ij}^{k} \leq 1, \quad \forall j\in C, \forall i\in D_j\setminus B_j, \quad (3.7)$$

$$\sum_{k\in K} x_{ij}^{k} \leq 1, \quad \forall j\in C, \forall i\in D_j\setminus B_j, \quad (3.8)$$

$$\sum_{j\in C}\sum_{i\in D_j} q_{ij}^{lm} \geq \underline{W}_{lm}, \quad \forall m\in C, \forall l\in D_m, \quad (3.9)$$

$$\sum_{j\in C}\sum_{i\in D_j} q_{ij}^{lm} + r_{lm} = \overline{W}_{lm}, \quad \forall m\in C, \forall l\in D_m, \quad (3.10)$$

$$q_{ij}^{lm} \leq a_{ij}^{lm} \cdot \sum_{k \in K} k \cdot x_{ij}^{k}, \forall j, m \in C, \forall i \in D_j, \forall l \in D_m, \tag{3.11}$$

$$\sum_{m \in C} \sum_{l \in D_m} q_{ij}^{lm} \leq \alpha \cdot a_{ij}^{lm} \sum_{k \in K} k \cdot x_{ij}^{k}, \forall j \in C, \forall i \in D_j, \tag{3.12}$$

$$\sum_{j \in C} \sum_{i \in D_j} \sum_{k \in K} \epsilon_{ij}^{k}(x) \cdot x_{ij}^{k} = E + \sum_{j \in C} he_j - \sum_{j \in C} fe_j, \tag{3.13}$$

$$\sum_{i \in CB_j} \sum_{k \in K} \epsilon_{ij}^{k}(y) \cdot y_{ij}^{k} - fe_j \geq 0, \quad \forall j \in C, \tag{3.14}$$

$$\sum_{i \in D_j \backslash B_j} \sum_{k \in K} \epsilon_{ij}^{k}(z) \cdot z_{ij}^{k} - he_j \geq 0, \quad \forall j \in C, \tag{3.15}$$

$$he_j, fe_j, r_{lm}, q_{ij}^{lm} \geq 0, \text{integer} \tag{3.16}$$

$$x_{ij}^{k}, y_{ij}^{k}, z_{ij}^{k} \in \{0,1\}. \tag{3.17}$$

As we can see, the objective function[5] (3.1), minimizes the total cost incurred when restructuring a bank-branch network. This cost is divided into four categories: costs related to branches, which include operating costs, closing costs, and opening costs; costs related to the employees, which include hiring and firing costs; a penalty cost when the ideal coverage is not completely provided; and finally we included a service cost. This way, the solution to the mathematical model guarantees that the minimum coverage is provided, at the same time that ideal coverage is attempted. It should be noticed that the objective function is concave as it is given by the sum of linear and concave components (operating costs and service costs). The functions f_{ij}^{k}, g_{ij}^{k}, and h_{ij}^{k}, which have been fully described although not mathematically described, are non linearly dependent on several factors.

All nonclosable branches must be operating for the existing branch size (3.2) and (3.3). Constraint (3.4) guarantees that an existing branch operates unless it is closed. All the other sizes for the location of the existing branches, will operate only if opened, equation (3.5). Furthermore, a branch can only be opened there if the existing one is closed. In constraint (3.6) it is stated that potential branches will be operating if they are opened. Equations (3.7) and (3.8) guarantee that whenever a

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[5] In this work, we will use the expression "objective function" and "cost function" indifferently.

branch can be opened no more than one size is opened, and that only one branch size is being operated, respectively.

Constraints (3.9) and (3.10) are related to service received, which is bounded from below by a minimum guaranteed coverage and from above by an ideal coverage, respectively. On the other hand, constraints (3.11) and (3.12) are related to the service provided. A branch may give service to a client only if the DS is respected and the service provided is bounded by the branch capacity to one client, equation (3.11), and to a set of clients, equation (3.12).

Constraint (3.13), ensures the existence of the required number of employees to operate the branches, after the branch network is restructured. Equations (3.14) and (3.15) guarantee that the needed employees are made available through firing and hiring, respectively.

Finally, constraints (3.16) and (3.17), state the integer and binary nature of the variables, respectively.

In this chapter, after describing in detail the problem to be considered, we have made an exhaustive enumeration of all the parameters and variables used to define the mathematical model. Furthermore, we have explained the reasons behind our choices, and the notation associated with the definition of the model. In the next chapter we will refer to the generation of test problems and to the data that has been collected.

Chapter 4

Problems Generation

In order to use the mathematical model obtained in section 3.4, we need to find data for the parameters previously defined. Two possibilities exist, gathering real applications data or generating our own data. In our case, given the inexistence of available data, we have generated the data needed. Whenever possible, this is done by using bank related data that we were able to collect mainly from the report issued by the Portuguese Bank Association (APB, 2003).

4.1 Data Collecting

Once the mathematical formulation for our problem was defined, we started searching for data. This process proved to be very time consuming, especially having in consideration what was achieved. A lot of effort and time was spent searching for data on the internet and in all sorts of libraries. At the end, we were confronted with the inexistence of specific data, in particular when related to small areas such as parishes. Therefore, we decided to generate the needed data. Decisions on parameters have been made, either by following the approaches of other authors or by using the information that we were able to collect from some interviews. To help us understand banks and their operating characteristics we have interviewed bank employees and managers, and we have used our experience as clients. Furthermore, we have also interviewed other specialists, such as accountants and real estate traders, to obtain insights on the parameter values and on how parameters affect one another.

As previously mentioned, the main source used for bank data has been the report of the Portuguese Bank Association (APB, 2003). The list of banks that appear in the aforementioned report is large, but it can be divided in two broad categories: on the one hand, we have the oldest and largest banks. Banks that have been operating in a steady state for a long time. On the other hand, there is a group of banks which have few offices and mainly concentrate in one or two locations. Furthermore, most of these branches have only been operating for a few years. Since we are looking at the restructuring problem it makes no sense to regard the later group of banks, as they would not provide reliable data values. Thus, the group under consideration is about 25% of the banks operating in Portugal, and is used as a reference to get some of the values herein used.

4.2 Generated Data

Next, we introduce and discuss the parameters that have been generated. Explanations clarifying and justifying our choices are also given, whenever deemed necessary.

- *Existing employees (E)*

  The number of existing employees is given by the sum of the number of employees over the existing branches;

- *Average number of working years at a bank (AY)*

  Based on the numbers provided by the Portuguese Bank Association report (APB, 2003) we use 9 as the average number of working years at a bank;

- *Ideal coverage ($\overline{W}_{lm}$)*

  The ideal coverage value, has been found in the same manner as in (Miliotis et al., 2002). In their work, Miliotis *et al.* have divided cities into three types, according to the number of inhabitants. Given the data used, we thought that five city types was more appropriate. Therefore, the ideal coverage value is allowed to vary between one and five service units;

- *Minimum coverage ($\underline{W}_{lm}$)*

As we have said before, we impose a mandatory lower limit of service to be provided to each client, i.e. the number of covers to be satisfied. This parameter is randomly chosen between [1;$\overline{W}_{lm}$], since 1 is the minimum and $\overline{W}_{lm}$ is the maximum service a client may require;

In our problem we are minimizing the total cost which is made of seven components. Next, we will describe in detail each of these components.

- *Opening costs (h_{ij}^{k})*

  As we have mentioned before, opening costs h_{ij}^{k} include costs with refurbishment R, equipment EQ, and security systems SCT. In order to get estimates for these costs, we have consulted some specialized companies (KLSecurity, 2004; ATMMachine, 2004; Diebold, 2004; BrownSafe, 2004) and bank employees.

  Refurbishment costs depend mainly on branch size although economies of scale usually exist. Thus, they can be written as $c_R + R \cdot \sigma_{ij}^{k}$.

  Equipment costs depend on the number of employees, and again they are not entirely proportional to such number. For example, if the bank buys a single computer, its price has usually no reductions, but if the number of computers bought is larger then, a reduction is usually obtained. This reduction may be proportional to the number of computers bought, until an acceptable reduction rate limit is reached. Besides reductions, there is always some per unit savings in terms of transportation and installation, and some equipment is shared among employees, such as printers, fax-machine, or Xerox machines. Thus, equipment cost is given by $c_{EQ} + EQ \cdot \epsilon_{ij}^{k}$.

  Expenses with security systems SCT may be considered equal for every branch since all require the same items in very similar sizes and numbers.

  We can have a large number of different cost combinations because each cost component can take one of several different values within a specified interval. For example, the area occupied by some branch i, σ_{ij}^{k}, must be adequate to accommodate all its employees. But, even for branches with the same size k the number of employees working in it may vary because it is related for instance, with the amount of back-office work of the branch and the daily clients servicing rate. Besides that, the area occupied by a branch includes not only the space needed to accommodate the employees, but also rooms for meetings, WCs, small wardrobes, among others. Thus, the branch size is not directly proportional to the number of employees, and usually increases at a smaller rate when comparing with the increase of the size k of the branch. Therefore, the function representing the opening costs $h_{ij}^{k}(z)$, is a concave function.

As we have seen opening costs are concave and depend only on the branch size. Recall that we are considering only three possible sizes thus, in reality only three possible values of opening costs matter. Therefore, by defining different variables for each branch size we are able to use a linear programming model. This can be seen in Figure 4.1. Although a concave cost function is given, only

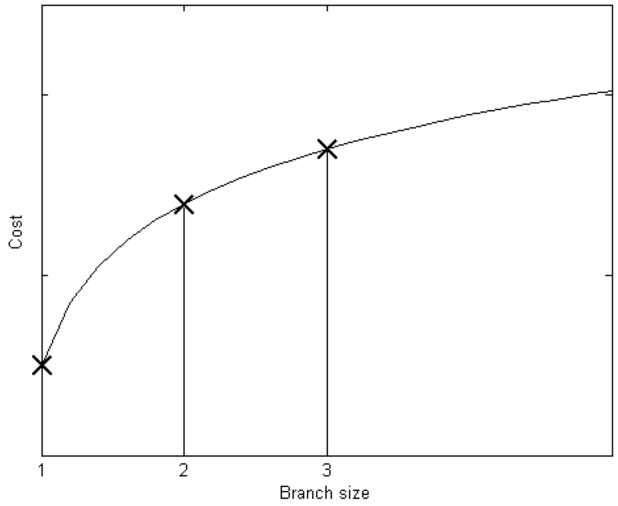

Figure 4.1: Opening cost versus branch size.

the three points indicated by a cross are of interest. Let C_1, C_2, and C_3 be the cost value associated with those points. If x^1, x^2, and x^3 are binary variables associated with branch size 1, 2, and 3, respectively we can write that the cost is given by $C_1x^1 + C_2x^2 + C_3x^3$, as long as, we make sure that only one of the three variables can have value 1. Furthermore, the cost we obtain in this way is the exact one and not an approximation.

The opening cost is then given by,

$$\begin{aligned} h_{ij}^{k}(z) &= R \cdot \sigma_{ij}^{k}(z) + EQ \cdot \epsilon_{ij}^{k}(z) + SCT \cdot z \\ &= c_r \cdot z_{ij} + r \cdot \sigma_{ij}^{k} \cdot z_{ij}^{k} + eq_{ij}^{k} \cdot \varepsilon_{ij}^{k} \cdot z_{ij}^{k} + SCT \cdot z_{ij}^{k}. \end{aligned} \tag{4.1}$$

- *Closing costs(*g_{ij}^{k}*)*

  Closing costs are calculated as a percentage of the opening costs. This percentage is randomly drawn from the interval [25;33].

- *Operating costs(*f_{ij}^{k}*)*

  Operating costs include three different types of costs: administrative costs α_{ij}^{k}, renting costs e_{ij}, and salaries s_j.

  Since branches have different sizes, and may have a different number of employees, it is expected that the administrative costs vary. These costs should not be proportional to branch size, since economies of scale exist and often lead to concave costs. Administrative costs depend mainly on the size of the branch σ_{ij}^{k} but, again not at a proportional rate. For example, if a branch occupies twice the area occupied by another branch, it is not expected electricity and water bills, or general office supplies to be twice as much. Specially, since the last ones, typically, have reductions when bought in large quantities. Therefore, these costs are also concave. To have an initial reference value, for administrative costs, we have used the data given in (APB, 2003). The average administrative cost was used to infer the interval for the medium size branch, and the others were obtained following a concave function.

  We have learned that the administrative costs reported in (APB, 2003) include *service costs* as well. And we have been told that service costs account for roughly 50% of the reported administrative costs.

  Expenses with the rent depend on the number of square meters occupied by the branch and on its location, parish i and county j. Therefore, rents are proportional to the area of the branch $e_{ij} \cdot \sigma_{ij}^k$. Renting costs were obtained at the interviews with real estate traders[1], see appendix A. As branches can be of three different sizes, three different areas occupied by them must also be considered. Since the specific ideal size may not be available on the market, we have associated each branch size with a different interval of values. Branch areas are then uniformly randomly generated in [80;100], [100;120] and [120;140] for small, medium, and large size branches, respectively.

The salary we have considered in this work is actually not the salary itself, because it also includes: taxes, insurances, holiday subsidy and other similar expenses. The average salary in (APB, 2003) was chosen to represent the value used for the salary parameter. Recall that salaries have small variations between counties.

Therefore, operating costs are obtained by using the following expression:

$$\begin{aligned} f_{ij}^k(x) &= \alpha_{ij}^k(x) + s_j^n \cdot \epsilon_{ij}^k(x) + \sigma_{ij}^k(x) \cdot e_{ij} \\ &= \alpha_{ij}^k \cdot x_{ij}^k + s_j^n \cdot \epsilon_{ij}^k \cdot x_{ij}^k + \sigma_{ij}^k \cdot e_{ij} \cdot x_{ij}^k. \end{aligned} \tag{4.2}$$

- *Hiring costs (T_j)*

  Hiring costs depend on three expenses: travelling expenses, meals, and loss of productivity. As training is, most of the times performed outside the working area, the bank usually pays for the travelling expenses. We have considered 15 days as the necessary period for training an employee, which corresponds to the number of working days in three weeks. Banks may pay a value per km (*ppkm*) or a train/bus ticket. In this problem we have decided to use the first option, which makes travelling expenses proportional to the number of travelled kms. Therefore, we need to know the number of kms travelled by the employees, in order to attend the training. The average distance between a parish and all the other parishes is then calculated (*avgdist*), since training can

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[1] It should be noticed that the figures in tables A.2 and A.1, appendix A, refer to acquisition prices in the district of Porto. Renting prices are then computed as have been mentioned in section 3.2.

be provided in any of the operating branches. We have also considered that the bank pays for 2 meals (pm) a day. The other component that contributes to the hiring cost (T) is the loss of productivity, which is converted into a monetary value given by 50% of the monthly salary (s_j). The *hiring cost* can thus be represented by:

$$T_j = 15 \cdot 2 \cdot avgdist \cdot ppkm + \frac{s_j}{2} + pm. \tag{4.3}$$

- *Firing costs (CMP_j)*

  The compensations to be paid to each fired employee, are calculated based on the monthly average salary[2] and average working years, as follows:

  $$CMP_j = \frac{s_j}{14} \cdot AY. \tag{4.4}$$

- *Penalty (P_{ij})*

  As we have mentioned previously, one of the objectives of a bank, as a financial institution, is to raise the highest volume of deposits (Boufounou, 1995). Thus, it is very different to loose clients with high banking potential, in comparison to the same loss if the client has a low banking potential. A client has a high or low banking potential if he lives or works in a parish with high or low socioeconomic standards, respectively. Therefore, we associate each client with a measure of his banking potential. To calculate the banking potential we use each parish's renting prices. We assume that high average income is related to high renting prices. Therefore, high banking potential clients have associated a higher penalty, in order to guarantee that they are entirely, or almost entirely satisfied.

  In order to obtain a value for the P_{ij} parameter, we have solved several problems without the penalty component. For each run we have computed and

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[2] Recall that a working year in Portugal comprises 14 months.

stored the total cost value and the total number of covers provided. These allowed for the computation of the cost per coverage provided ($CPCov$). Then, we easily obtain the average cost per coverage provided, which is calculated as follows:

$$CPCov = \frac{TotalCost}{Total\ no.\ of\ covers}. \tag{4.5}$$

The penalty value, is based on this value, and varies according to client banking potential. Thus, the penalty cost per unit of unprovided coverage for each client, is given by:

$$P_{ij} = \frac{e_{ij} \cdot CPCov}{avgrent}, \tag{4.6}$$

where

$$avgrent = \frac{\max\{e_{ij}\} - \min\{e_{ij}\}}{3}. \tag{4.7}$$

This way, the bank is highly penalized if it does not provide all the service units needed by medium or high banking potential clients.

- *Service costs* (v_{ij}^{lm})

  Finally, service costs are considered to be half the administrative costs:

$$\overline{v}_{ij}^{lm}(x) \quad = \quad \frac{\alpha_{ij}^{\overline{k}}(x)}{2}, \tag{4.8}$$

where $\overline{k}$ is the medium size branch. We have already seen that administrative costs are not directly proportional to the branch size k. The same happens with the service costs. Therefore, we consider that clients far away from the branch represent higher servicing costs. This way, servicing costs depend not only on the administrative costs and branch location, but also on the location of the client.

$$v_{ij}^{lm}(x) \quad = \quad \log(\overline{v}_{ij}^{lm}(x)), \tag{4.9}$$

4.3 Problems Generation

In this section, we present a list of parameters considered in the computational experiments. We would like to stress that all the values bellow serve merely as a guide, since the model and solution procedure do not depend on them. For most of the parameters used, the final value is computed using randomly generated values. Hence, the values given in here are one possible set of values and together they define one problem instance. Make notice that whenever a value is referred to as being generated in an interval, we actually mean to say that the value has been randomly generated and uniformly distributed in such interval.

- the number of counties n is generated in [5; 15][3];
- the number of parishes m is generated in [15; 50] and must be larger than or equal to n;
  - the number of parishes in each county $|D_j|$ is given by a random distribution of the m parishes by the n counties;
- we randomly choose a percentage between [0.06; 0.46][4] to the number of existing parishes $|B|$;
- the set of potential branches $D_j \backslash B_j$ is given by the set of all branches except those already opened;
- the set of nonclosable branches NCB_j is given by randomly drawing 10% of the existing branches B_j;

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[3] We have used the number of counties existent in the district of Porto, as a reference.

[4] The interval of values where we generate the number of existing branches, reflects the minimum and maximum value for the largest banks in the district of Porto.

- closable branches CB_j are given by $B_j \backslash NCB_j$;
- a pair of coordinates (x_{ij}, y_{ij}) for each parish centre is randomly generated within a square [0; 100];
- the euclidean distance d_{ij}^{lm} between parishes is computed as $d_{ij}^{lm} = \sqrt{(x_{ij} - x_{lm})^2 + (y_{ij} - y_{lm})^2}$;
- the distance standard DS has been set to 50;
- the cover matrix a_{ij}^{lm} is a binary matrix and is obtained using the standard distance DS and the distance matrix d_{ij}^{lm}. If a_{ij}^{lm} is less than the standard distance, then the branch covers that client and thus, a_{ij}^{lm} is set to 1 (one), otherwise a_{ij}^{lm} is set to 0 (zero).
- operating costs f_{ij}^{k} are uniformly and randomly generated, based on the following parameters values:
    - administrative costs α_{ij}^{k} are randomly generated according to Table 4.1;
    - renting costs e_{ij} are randomly generated in [4.5; 19.5]€[5], and the total rent per branch, is proportional to the branch size;
    - the annual salary s_j is randomly generated in [47072; 52026]€;
- opening costs h_{ij}^{k} depend on the sum of the following parameters:
    - the cost to be paid for every remodelled square meter, $R = 188$€ ;
    - the average value for equipment costs, $EQ = 1700$€ per employee;
    - security systems, $SCT = 50200$€;
    - branch areas σ_{ij}^{k} depend on branch sizes, and are randomly generated within the intervals given in Table 4.2;
- the number of employees ϵ_{ij}^{k} in each branch is randomly drawn in the intervals given in Table 4.3.

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[5] Acquisition values can be consulted in Table A.2 and Table A.1.

Branch Size	**Minimum**	**Maximum**
1	125000	140000
2	140000	150000
3	150000	152500

Table 4.1: Administrative Costs.

Branch size	**Minimum**	**Maximum**
1	80	100
2	100	120
3	120	140

Table 4.2: Allowed branch areas, in square meters.

- closing costs g_{ij}^k are a percentage, between [0.25; 0.33], of the opening costs;
- hiring costs (training) T_j depend on the following parameters:
    - the price per km, $ppkm = 1$€, which has been found to be a common figure;
    - average distance $avgdist$, which is obtained from the coordinates;
    - meals price, $pm = 5$ €each;
- firing costs (compensations) CMP_j depend on the monthly salary, and on the average years working at the bank $AY = 9$;
- the penalty value P_{ij} is computed as in equation 4.6;
- the ideal coverage $\overline{W}_{lm}$ is randomly generated in the interval [1,5];
- the minimum coverage $\underline{W}_{ij}$ is randomly generated in [1;$\overline{W}_{lm}$];

Branch Size	**Minimum**	**Maximum**
1	4	6
2	7	10
3	11	15

Table 4.3: Number of Employees.

- the time period, $NY = 10$ years;
- cost for servicing clients v_{ij}^k is obtained as in equation (4.8).

A data collecting was effectuated, although we had no access to all the desirable values. For that reason, we interviewed some professionals and used also common sense, in order to find some of the values we needed. In the next chapter, we will present the methodology used to solve the problem previously modelled in section 3.4.

Chapter 5

Solution Methodology

In this chapter, we present the local search heuristic that was developed to solve the problem we are addressing. We first discuss on the procedure used to find an initial feasible solution, and then on how this solution can be improved. We also give an example, with which we illustrate the usage of the method developed. Finally, we present the computational results obtained.

5.1 Introduction

Once the problem has been defined, the following step is to choose a method to solve it. There are two different kinds of methods that can be used, exact and approximate methods. Exact methods seek an optimal solution, while approximate methods search for a good solution. Finding an optimal solution for combinatorial problems can be prohibitive since exact solution methods explicitly or implicitly enumerate all feasible solutions. When the problem instance being solved is small, the method is usually well behaved. However, for large size instances, the number of possible solutions increases exponentially and enumerating them all would be very time consuming if not impossible. This is the main reason for using approximate methods. Although for these methods no guarantees exist that the solution found is a global optima, they usually are able to find a good solution more quickly and consuming less computer resources (Ghosh, 2003). Furthermore, this type of methods can be used to solve large size instances, which cannot be solved by exact methods, at least in a reasonable amount of time. Therefore, approximate methods can be used, either if we are not seeking an optimal solution or if the problem is too complex to be solved by exact methods.

5.2 Solution Approaches

The first part of our work was to develop a Mixed Integer Programming (MIP) model that, while representing the problem we have in hands, could be solved by CPLEX. In order to do so, we have defined three binary variables for each branch location and each branch size. These variables indicate whether a branch is to be operated, has been newly opened, or has been closed. Overall we have $3 \times k \times m$ binary variables, where m is the total number of possible locations (all parishes in all counties), k is the number of different branch sizes considered, and n is the number of counties. In order to decide how service is provided we have defined m^2 integer variables, since service may be provided from any branch location to any client.

Obviously, we refrain ourselves from considering every cost and decision dependent on the branch size since that would lead us to an unmanageable problem. For example, if service costs are considered dependent on the branch size, which we think more realistic, the number of service variables would be $k \times m^2$ rather than m^2.

The model given in section 3.4, has been set-up in a format compatible to CPLEX and solved by using CPLEX. However, given that CPLEX works with matrices derived from the mathematical model, we are faced with a dimensionality problem. The number of variables in each problem instance is given by $3 \times m \times k$ for x_{ij}^k, y_{ij}^k, and z_{ij}^k, by $2 \times n$ for he_j and fe_j, by m for r_{lm}, and by m^2 for q_{ij}^{lm}. For each row, that is, for each constraint in the model, each of these variables has to be instantiated with a value. Therefore, the matrix size grows rapidly with problem size, leading to very large computer memory requirements. For many of the problems experimented, CPLEX could not solve the problem due to the lack memory. A better computer would not be the solution, since as the number of variables grows very quickly it would allow for a marginal increase in problem size. In order to be able to solve larger size problem instances, which realistically banks are faced with, we have developed a heuristic based on local search. This heuristic is the subject of the next section.

5.3 Local Search Methods

A feature of many combinatorial problems is that while there may be several optimal solutions, there are many more that are, in some sense, only local optima. In our case, since we are minimizing a concave function over a convex feasible region (defined by the linear constraints) a local optima may not be a global optima (Fontes et al., 2003). A local search algorithm requires the definition of "local" and also of "search". Furthermore, it requires a starting point, i.e. an initial feasible solution.

Let us introduce the concept of neighborhood to make the idea of local optima more clear. A neighborhood of a solution X, $N(X)$ is a set of solutions that can be reached from X by a simple operation. A solution X is said to be a local optimum if no better solution exists in the specified neighborhood $N(X)$.

Local search methods vary according to:

1. definition of neighborhood (local optimum),
2. search technique for a local optimum (operation), and
3. initial solution technique.

The major drawback of local search algorithms is that they might get trapped into a local optimum. An obvious way to escape from getting trapped into a local optimum, is to run the algorithm with different initial solutions. This sort of approach, however, has the disadvantage of having no guaranteed improvement and also of being time consuming (Fontes, 2000).

In our work, we have decided to use two different kinds of neighborhood. A solution X' belongs to the neighborhood of a solution X, if X' is defined by excluding a branch i from solution X. If solution X'' differs from solution X by only one branch then, X'' is also a neighbor solution of X. Both operations can only take place if no feasibility is violated and if the total cost decreases with the correspondent operation.

Thus, after finding the initial solution we improve it by using a 2-op heuristic, which is further described in section 5.4.2.

5.4 Description of the Local Search Heuristic

The local search heuristic we have developed consists of two phases.

In the first phase, we obtain an initial solution by solving a related programming linear problem, which is further explained in the next section. The second phase, consists on improving the initial solution by applying drop and swap operations. In the drop operation some branches are to be dropped and the clients served by them are either associated to other branches or provided with no more than the minimum service required. Recall that each client service has associated two integer numbers: one representing the minimum service required and another representing the ideal service coverage. This operation is applied in steps 1, 2, and 3. The swap operation is divided in two: the swap between branches with the same size; and the swap between sizes of a single branch. The swap operation is applied in steps 4, 5, and 6. A schematic representation of the method we have just described is given in Figure 5.1.

In the next two sections, we discuss how to find an initial feasible solution and how to improve the feasible solution in hand, with the two operations, drop and swap, that we have developed.

5.4.1 Initial Solution - Linearization

Many different methodologies can be followed to find an initial solution. For example, some authors start by having all branches opened (Butenko et al., 2003), while others choose an arbitrary subset of those (Korupolu et al., 2000; Arya et al., 2004).

In our case, an initial feasible solution is found by solving a linear programming problem. For this problem the objective function is to cover all demand locations at a minimum service cost.

Let q_{ij}^{lm} be the number of covers provided by branch i located in county j, to a client located in parish l in county m. Let $k_{\max}$ be the maximum branch size allowed, assumed to be the same for every location. And let and k_{ij}^{e} be the size of the existent branch at county j and parish i. The problem to be solved is then,

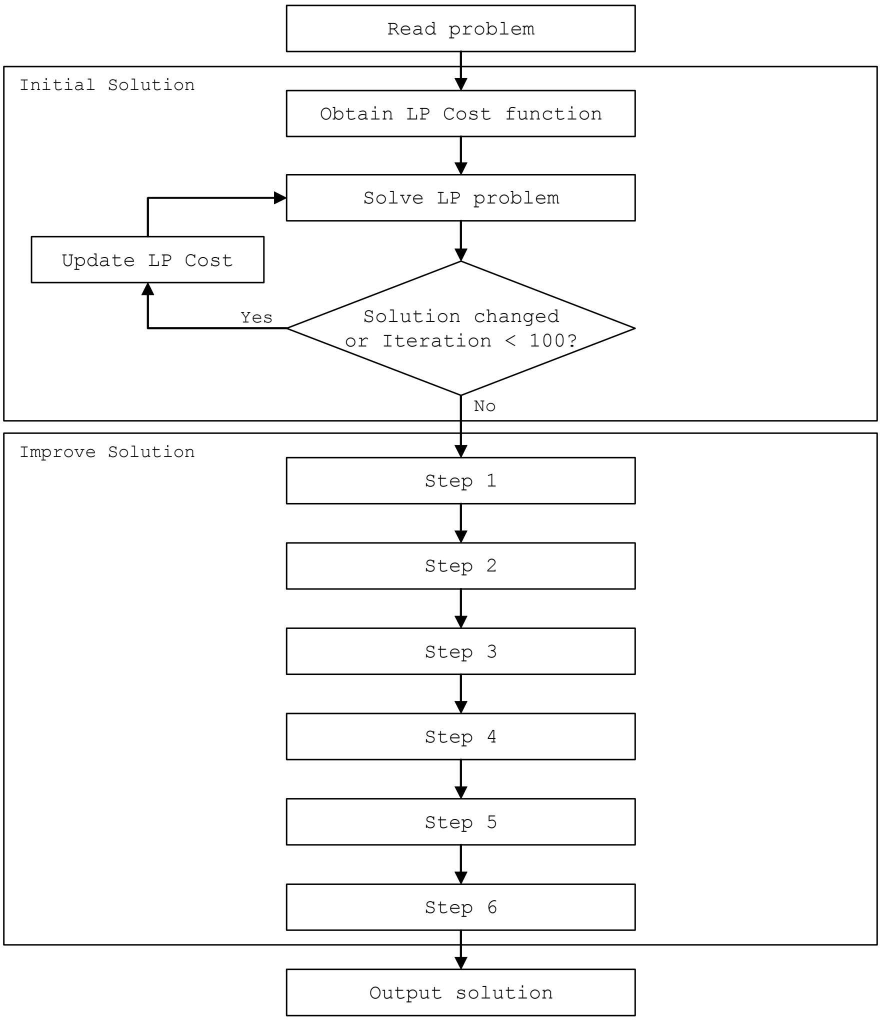

Figure 5.1: Representation of local search heuristic.

$$\min \sum_{j\in C}\sum_{i\in D_j}\sum_{m\in C}\sum_{l\in D_m}\phi_{ij}^{lm} \tag{5.1}$$

subject to:

$$\sum_{j\in C}\sum_{i\in D_j} q_{ij}^{lm} \geq \underline{W}_{lm}, \quad \forall m\in C, \forall l\in D_m, \tag{5.2}$$

$$\sum_{j\in C}\sum_{i\in D_j} q_{ij}^{lm} \leq \overline{W}_{lm}, \quad \forall m\in C, \forall l\in D_m, \tag{5.3}$$

$$\sum_{m\in C}\sum_{l\in D_m} q_{ij}^{lm} \leq k_{\max}\cdot\alpha, \quad \forall j\in C, \forall i\in D_j\backslash NCB_j, \tag{5.4}$$

$$\sum_{m\in C}\sum_{l\in D_m} q_{ij}^{lm} \leq k_{ij}^{e}\cdot\alpha, \quad \forall j\in C, \forall i\in NCB_j, \tag{5.5}$$

$$q_{ij}^{lm} \leq k_{ij}^{e}\cdot a_{ij}^{lm}, \quad \forall j,m\in C, \forall l\in D_m, \forall i\in NCB_j, \tag{5.6}$$

$$q_{ij}^{lm} \leq k_{\max}\cdot a_{ij}^{lm}, \quad \forall j,m\in C, \forall l\in D_m, \forall i\in D_j\backslash NCB_j, \tag{5.7}$$

$$q_{ij}^{lm} \geq 0 \quad \text{integer}\,. \tag{5.8}$$

Where we define,

$$\phi_{ij}^{lm} = \begin{cases} \overline{h}_{ij} + \overline{v}_{ij}^{lm}\cdot q_{ij}^{lm}, & \text{if } q_{ij}^{lm} > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{5.9}$$

$$\overline{v}_{ij}^{lm} = \sum_{m\in C}\sum_{l\in D_m}(v_{ij}^{lm} - P_{lm}). \tag{5.10}$$

$$\overline{h}_{ij} = h_{ij} + f_{ij}. \tag{5.11}$$

Upper (ideal) and lower (minimum) limits are imposed to the number of covers every demand location is given, which is guaranteed by equations (5.2) and (5.3), respectively. The total number of covers provided by each potential branch must be at most α times the maximum branch capacity if the branch is to be opened, equation (5.4); otherwise if it is to remain opened, the number of covers given must be at most α times the existing branch capacity (5.5). Equations (5.6) and (5.7) impose similar constraints, but now to the covers that can be provided to a single client. Constraint (5.8) states the integer nature of the variables.

The objective function considered includes the original service costs v_{ij}, the penalty costs P_{lm}, as well as, the opening h_{ij} and operating f_{ij} costs. As we have mentioned before, there are three different types of branches: existing and closable, existing but nonclosable, and potential branches[1]. Potential branches have associated two kinds of costs, opening costs and operating costs, while closable branches and nonclosable branches have associated closing costs and operating costs, respectively.

The idea is to successively solve LP problems with updated cost functions. The cost functions are updated by using the information of the solution of the previous iteration. This approach is based on the work of Kim and Pardalos (1999). They have used this technique to solve the fixed-charge network flow problems. (Fixed-charge problems have cost functions that are made of a fixed component and a linear variable component.) This method is called the dynamic slope scaling procedure and its objective is to find a linear factor that can represent the variable and fixed costs at the same time. In order to find the linear factor a slope and an initial value have to be determined. The slope of the line connecting the origin and $(q_{ij}^{lm}, \phi_{ij}^{lm}(q))$ is given by the value of the varying costs $\overline{v}_{ij}^{lm}$. In this work we have used a modified version of their method. For further details on the dynamic slope scaling procedure refer to Kim and Pardalos (1999) and Kim and Pardalos (2000).

In our case we can also identify the two correspondent costs: the fixed operational cost; and the varying servicing cost. In order to guide the linear problem towards a solution where it opens the minimum necessary number of branches we have decided to include opening costs for the branches not yet opened. This way, we expect the linear model not to open unnecessary branches. We included neither costs incurred by hiring and firing employees, nor costs incurred by closing branches. For more details on the objective function see appendix B.

An initial solution is obtained by solving the above linear programming model.

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[1] Potential branches consist of all possible locations where a branch can be opened.

Let T be the iteration identification.

For $T = 0$ the linear programming model that is being solved disregards the fixed costs from the objective function[2], that is,

$$f = \overline{v} \cdot q. \tag{5.12}$$

The solution obtained is subsequently improved by an iterative procedure, which updates the cost functions as follows:

For $T > 0$

$$\overline{v}^{T+1} = \begin{cases} \overline{v}^T + \dfrac{\overline{h}^T}{q^T \cdot \varphi^T}, & \text{if} \quad q^T > 0 \\ \overline{v}^R, & \text{if} \quad q^T = 0, \end{cases} \tag{5.13}$$

where φ^T is the number of demand locations serviced by branch i in county j, at iteration T and q^T the total number of covers provided by branch i in county j at iteration T, i.e. $q^T = \sum_{m \in C} \sum_{l \in D_m} q_{ij}^{lm}$. R is the index of the last iteration where $q^T > 0$. In the update step, for every $q^T > 0$, we do a distribution of the fixed costs over the demand locations receiving service units from that branch. This distribution is proportional to the number of service units given to each demand location. The size of the branch is not considered in this linear model thus, all the used parameter values concern the average cost for the considered sizes.

At each iteration the real costs are computed. Recall that the total costs are made of four major components: costs with branches, which include operating costs, closing costs, and opening costs; costs with employees, hiring and firing costs; penalties for not providing the ideal coverage; and finally servicing costs.

The update procedure stops whenever one of the following two conditions happens: if two consecutive solutions are exactly the same, which means that no more

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[2] For ease of notation the indices have been disregarded.

improvement can be achieved; or if the maximum number of iterations allowed is achieved. In our first experiments we allowed the algorithm to perform up to 2000 iterations. But, the results obtained soon indicate that the best solution was generally obtained within the first iterations, thus the number of iterations has been set to 100.

The initial solution for our heuristic is provided by the best solution obtained with this procedure.

5.4.2 Improving the Initial Solution

After finding an initial feasible solution, as explained in the previous section, we apply the local search operations defined in section 5.4 to improve it further. In order to do so, we consider the following operations:

- removal of branches regardless of being existent or newly opened ones;
- the downsizing of branches, that is, to substitute a branch of a specific location by another of smaller size at the same location;
- the swap of a branch at a specific location by a branch at some other location.

We do not consider adding branches in this heuristic since typically the initial solution completely satisfies the ideal coverage. We allow the swap of branches, in the same location, but with different sizes.

Our heuristic consists of 6 steps performed consecutively, each of which is described in detail below.

Consider a solution obtained as given in section 5.4.1. Let O be the set of operating branches serving at least one client and let $\overline{O}$ be its complement, i.e. $D \backslash O = \overline{O}$.

Step 1

This step consists of dropping branches that serve only one client, if no infeasibility results from this operation, see Figure 5.2. To start with we identify the subset of branches O_1 serving only one client. Let cl_k be the client served by branch k

in O_1. If the number of service units being given to client cl_k by branch k can be given by the other branches in $O \backslash \{k\}$, then branch k is disregarded and the service units it was providing are redistributed to the other branches; otherwise, the total number of covers that client cl_k can receive from branches in $O \backslash \{k\}$ (EC) may be above or bellow its minimum required. In the latter case, branch k cannot be disregarded because it would violate the feasibility of the solution. Nevertheless, in the former case, branch k can be excluded from the solution, although, penalty costs are incurred.

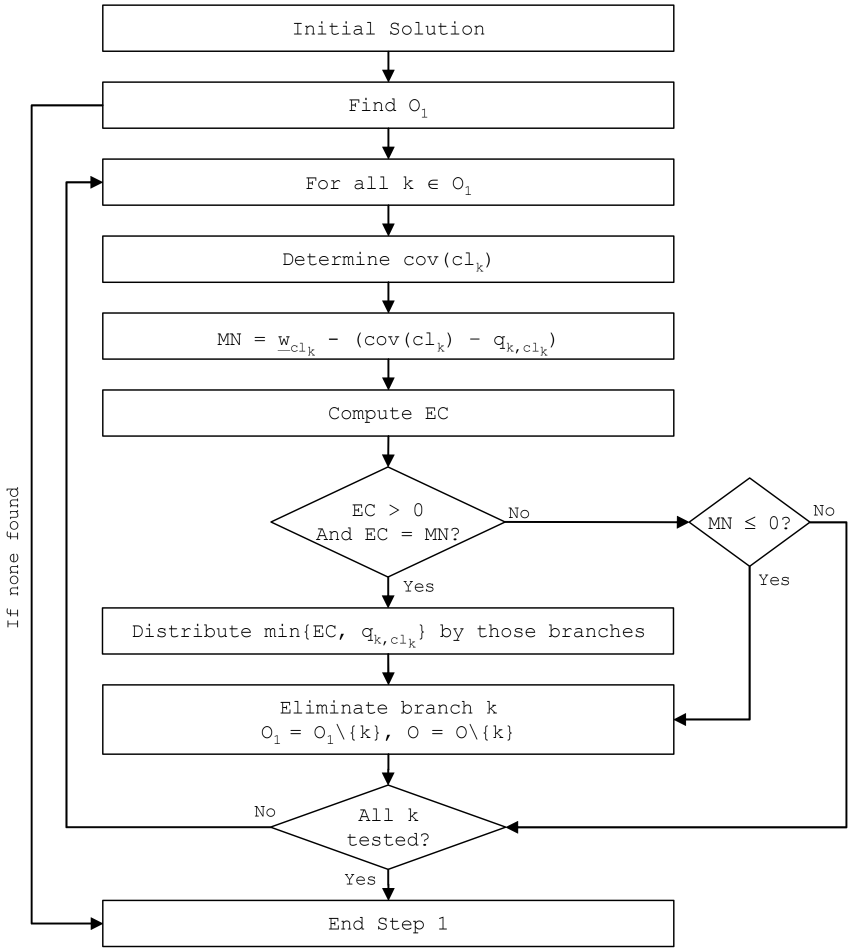


Figure 5.2: Flow diagram for step 1 of the heuristic.

To compute the cost gain obtained in each step all components of the original cost function must be included, namely: costs incurred with penalties; employees; opening, closing, or operating branches; and servicing costs. It should be noticed that the cost gain must be computed in relation to the originally existing branch-network. Furthermore, the cost functions to be used are the original ones and not the ones used in the LP.

In step 1, the cost variation is most likely positive since we are dropping a branch and the operating cost component, which is saved, is far more expensive than any other.

To compute the cost gain care must be taken since it uses the original cost functions and also the original branch-network configuration. Thus, when moving from a solution x to a solution x', if a branch is closed:

- we definitely save the operating cost;
- we may pay the closing cost if it originally existed;
- we may save the opening cost if it originally did not exist;
- additional penalty cost may be incurred if clients loose coverage;
- a variation in the service costs occurs although we do not know, without computing it if it is positive or negative;
- we may incur on or save costs with employees since a reduction/ increase relative to the previous iteration may result either in costs or savings, depending on the comparison to the original number of employees. For example, assume that in solution x we have 50 employees and this number comes down to 45 in solution x'. This results in a saving if the original number of employees was bellow 45 as we have avoided to recruit 5 new employees. On the other hand, if the original number was above 50 the cost increases since we have to pay compensations to 5 more employees. For an original number of employees between 45 and 50 a loss or a gain could happen depending on the relation between hiring and firing costs.

Step 2

In this step, we consider dropping the remaining branches servicing only one client. See Figure 5.3. In order to do so all operating branches are searched for, in order to identify the ones that may cover cl_k. Even if this is accomplished at the cost of providing less coverage for the clients they are currently serving, as long as, they receive their minimum requirements. For the ones satisfying that condition, if among the clients we count a total number of extra service units (PEC) at least equal to

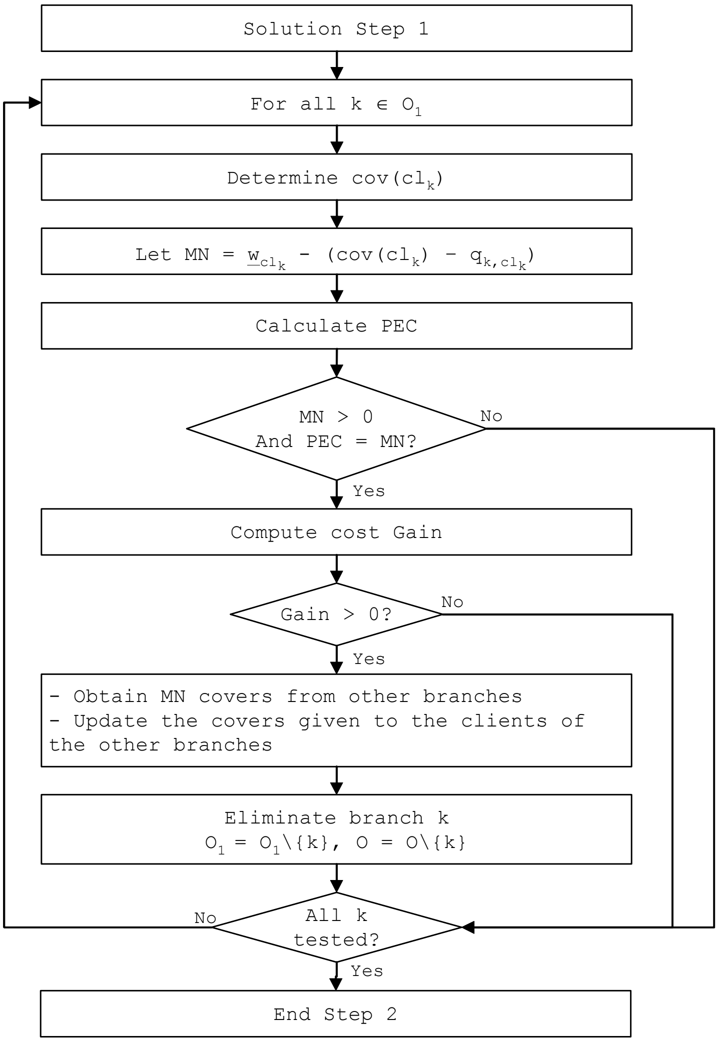


Figure 5.3: Flow diagram for step 2 of the heuristic.

the number of service units client cl_k needs, then branch k is disregarded if there is a positive gain with this operation. Recall that in the computation of the gain, we have to account for the penalty cost for not providing all the former service units to the clients of the branches now providing services to cl_k. This procedure is repeated until all branches servicing only one client are analysed.

Step 3

In this step we try to eliminate branches which are not using all service capacity. Let O_2 denote the set of such branches, provided that they can be closed. For each branch $k \in O_2$ we start by determining if all its clients are already minimally satisfied by other operating branches. If this is the case, then branch k is "dropped" but only if the total cost variation with this operation is positive. This step is illustrated in Figure 5.4.

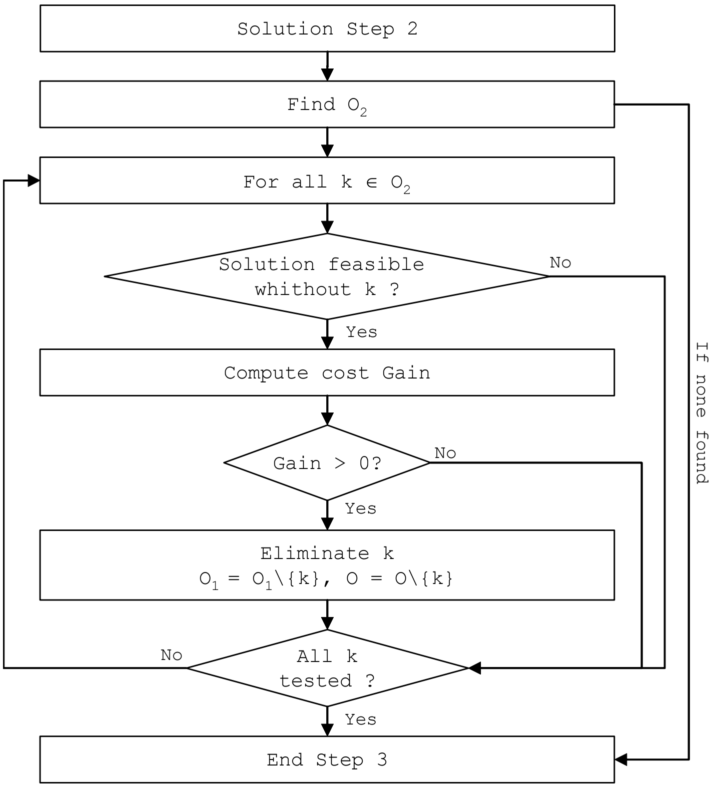


Figure 5.4: Flow diagram for step 3 of the heuristic.

Step 4

Step 1 to 3 consider dropping branches only. In step 4 we analyse the possibility of downsizing for each branch that has remained opened, see Figure 5.5.

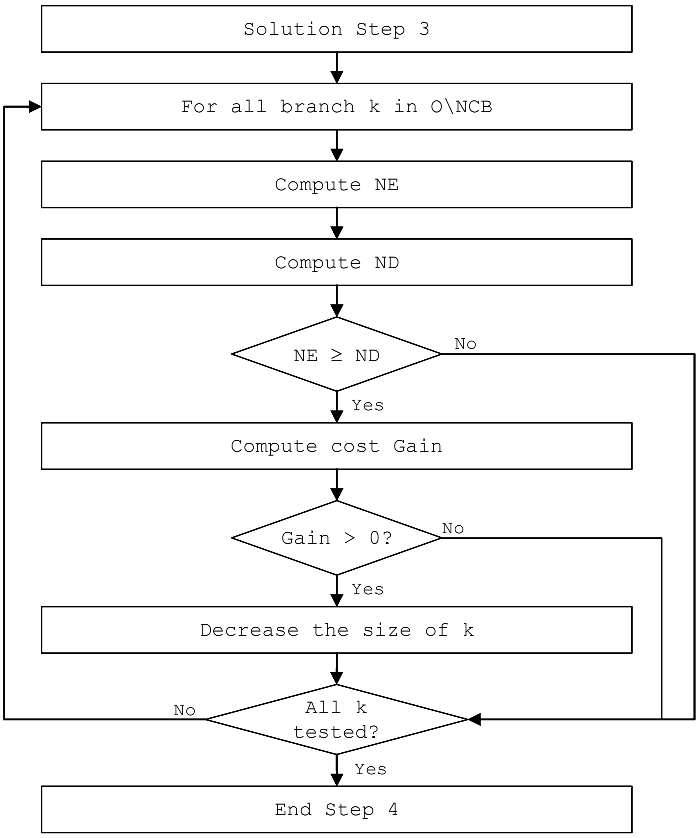


Figure 5.5: Flow diagram for step 4 of the heuristic.

To accomplish the downsizing (swap by smaller size), for each branch k in $O \backslash NCB$ we start by computing NE as the total number of service units being given to all its clients cl_k, beyond the minimum required. If this number is larger than the number of service units to be removed ND, due to a downsize of branch k and if the cost gain is positive, then this swap is performed.

Step 5

This step is similar to the above one but now a different approach for the downsizing of branches is tried. Let O_3 denote the set of branches which are not providing all their servicing capacity. For each branch $k \in O_3 \backslash NCB$ we determine SUR as the number of service units to be removed due to a downsize of branch k, and we try to distribute these service units by branches in $O_3 \backslash k$. Since the difference between the operational costs is always larger than the difference between servicing costs, and since no coverage loss exists (no penalties have to be accounted for), the downsize is performed immediately without having to calculate and test the gain. See Figure 5.6.

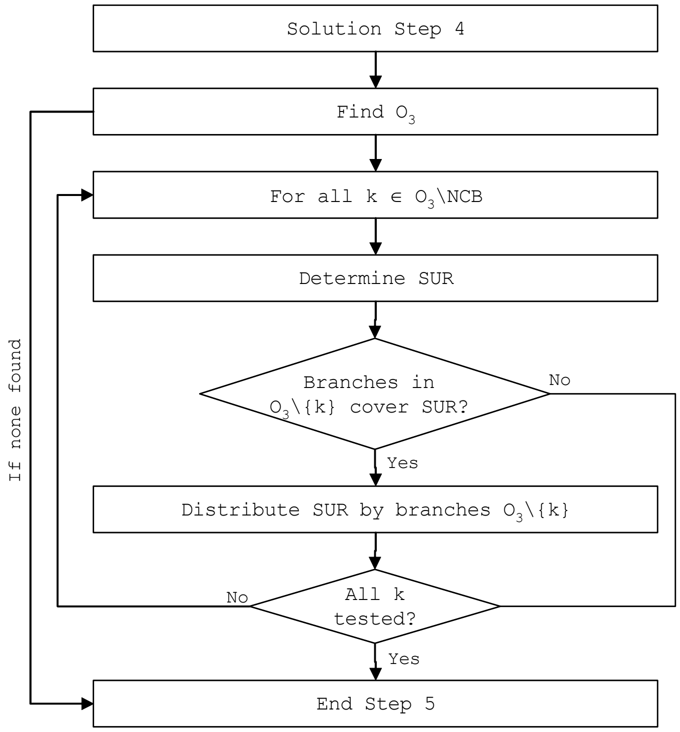


Figure 5.6: Flow diagram for step 5 of the heuristic.

Step 6

In the final step we try to swap branches, i.e. branch locations. We start by sorting the set of operating branches O, by descending order of operational costs. For each $k \in O \backslash NCB$ the closed set $\overline{O}$ is searched for, in order to find all the branches that can service all cl_k. Let O_4 denote the set of such branches. For each branch $j \in O_4$ we compute the cost gain obtained by swapping branch k for branch j. The branch swap with the largest positive gain is selected to be performed for the swap. This step is performed until no more swaps improve the cost function. Step 6 is illustrated in Figure 5.7.

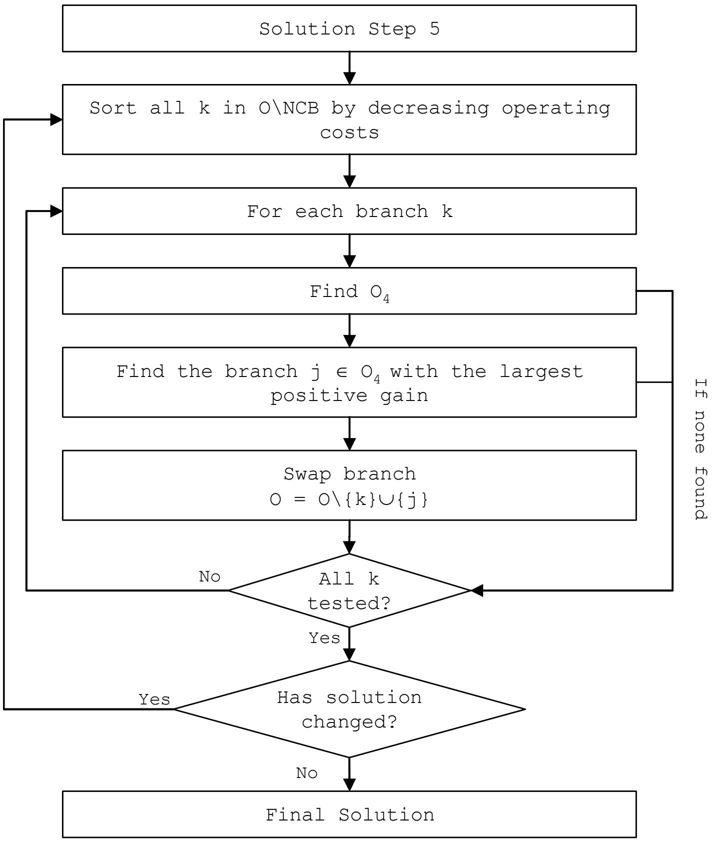


Figure 5.7: Flow diagram for step 6 of the heuristic.

5.4.3 An Illustrative Example

An illustrative example of the solution found in each step is given in this section.

A small problem instance, comprising 10 parishes randomly distributed in 4 counties, has been generated. The current situation of the bank-branch network is represented in Figure 5.8.

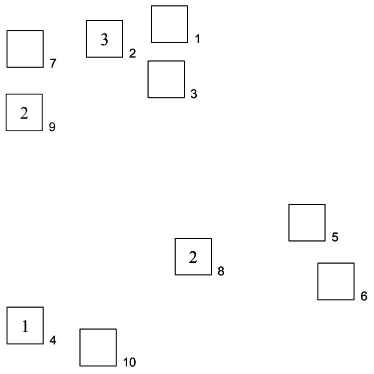


Figure 5.8: Current situation of the bank-branch network.

$\boxed{3}_1$ Branch of size 3, located in parish 1.

$\boxed{\ }_5$ Potential branch located in parish 5.

Branches are represented by squares and two figures are associated to each branch: one inside the square, which is the branch size, and an outside one, to the right of the square, which is the branch identification. For the sake of simplicity, we have numbered each branch with the parish identification. Initially, there are four open branches that can give up to 24 service units. Branch 8 is a nonclosable branch. The client identification number and the minimum and the ideal number of service units required by each client are given in Table 5.1.

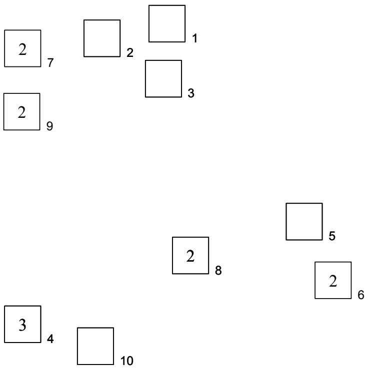
Recall that only one client exists for each parish.

The solution provided by CPLEX has five operating branches, whose total service capacity is 33 units, see Figure 5.9.

Nevertheless, only 30 units of service are being provided. It should be noticed that two branches have been opened and one branch has been closed. Furthermore,

Client	Minimum	Ideal
1	2	4
2	1	2
3	2	3
4	2	4
5	3	4
6	2	4
7	1	2
8	3	4
9	1	5
10	2	3
Total	19	35

Table 5.1: Number of minimum and ideal covers required by the 10 clients.



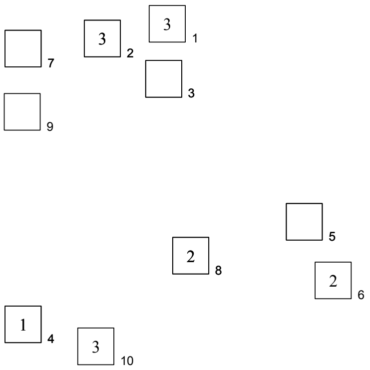
Branch, Client	No. Services
q4,4	3
q4,10	3
q6,5	2
q6,6	2
q6,8	2
q7,1	2
q7,2	1
q7,3	1
q7,7	2
q8,5	2
q8,6	2
q8,8	2
q9,1	2
q9,2	1
q9,3	2
q9,9	1
Total	30

Figure 5.9: Optimal solution provided by CPLEX.

from the three branches that remained open, two have kept their size, while the other has been changed to a larger size.

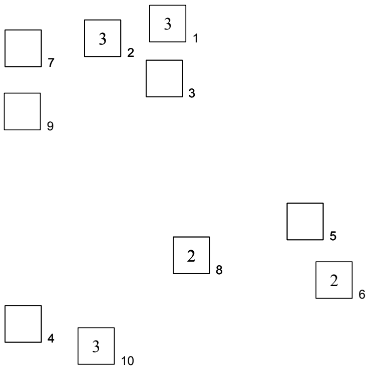
In the first phase of the developed heuristic an initial solution is found by solving the linear problem (LP) associated with the original mixed-integer problem. The initial solution provided by the LP is presented in Figure 5.10. The cost of this solution is about 10% higher than the cost of the CPLEX solution. Let us define the error between two solutions x and $\overline{x}$ as $E = \dfrac{(x - \overline{x})}{\overline{x}} \times 100$.

There are six open branches, which are giving 35 units of service to the clients. As we can see in the table of Figure 5.10, only branch 4 is servicing a single client, which is itself. Next, we illustrate the first step of the local search heuristic, which tries to eliminate this branch. In this step, all branches that can provide service to client 4 are identified. These branches are analysed sequentially until a branch not



Branch, Client	No. Services
q1,1	3
q1,3	2
q1,7	2
q1,9	2
q2,1	1
q2,2	2
q2,3	1
q2,9	3
q4,4	1
q6,5	2
q6,6	2
q8,5	2
q8,6	2
q8,8	2
q10,4	3
q10,8	2
q10,10	3
Total	35

Figure 5.10: Solution provided by the LP (error 10.1%).

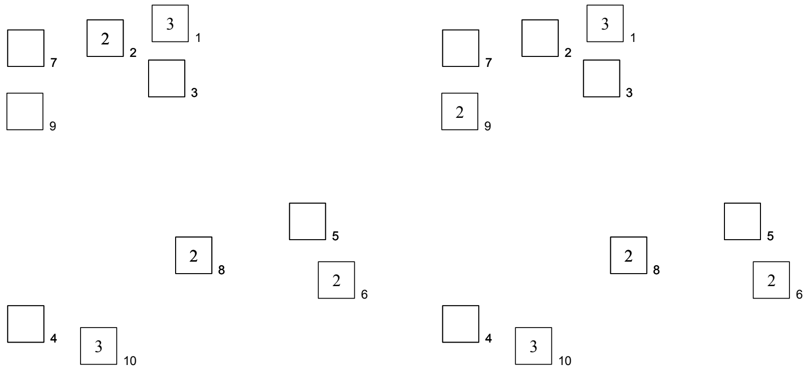


Branch, Client	No. Services
q1,1	3
q1,3	2
q1,7	2
q1,9	2
q2,1	1
q2,2	2
q2,3	1
q2,9	3
q6,5	2
q6,6	2
q8,5	2
q8,6	2
q8,8	2
q10,4	3
q10,8	2
q10,10	3
Total	34

Figure 5.11: Solution obtained at the end of step 1 (10 % error)

providing all its service capacity is found. In this particular case no such branch is found, and since client 4 is already receiving more than the minimum required, which is two service units, branch 4 is immediately disregarded. Figure 5.11 presents the solution after the first step, which is around 8% higher than the CPLEX solution.

The cost variation has three positive components that represent real savings, operating, employees, and service costs and two negative components, associated with penalties and closing costs. Details of the solution are provided in the Table of Figure 5.11. Figure 5.12, illustrate the solutions relative to branches and sizes, at the end of step 4 and step 6, respectively (3.3% and 2.6% error).



a) Step 4 (3.3% error) b) Step 6 (2.6 % error)

Figure 5.12: Solution obtained after step 4 and step 6, respectivelly.

Branch, Client	No. Services
q1,1	3
q1,3	2
q1,7	2
q1,9	2
q6,5	2
q6,6	2
q8,5	2
q8,6	2
q8,8	2
q9,1	1
q9,2	2
q9,3	1
q9,9	2
q10,4	3
q10,8	2
q10,10	3
Total	33

Client	Ideal	Given
1	4	4
2	2	2
3	3	3
4	4	3
5	4	4
6	4	4
7	2	2
8	4	4
9	5	4
10	3	3
Total	35	33

Table 5.2: Coverage provided by the solution of the heuristic.

(For this problem, no improvements have been obtained with steps 2, 3, and 5.). The heuristic is operating one more branch than the current branch-network. However, three branches have been newly opened and two branches have been closed. The service being provided is above the minimum but below the ideal service, i.e. 33 units of service are being provided, see Table 5.2. The final solution obtained with the heuristic, has a cost which is 2.6% above the CPLEX solution. But the CPLEX solution, however, provides "worst" service since in this solution only 30 units of service are provided, while in the heuristic solution 33 units of service are being provided. A summary of the quality of the solution, as well as, the variation on the number of employees is given in Table 5.3.

	No. Employees	Error %
LP Solution	28	10.072
Step 1	24	8.024
Step 4	19	3.323
Step 6	19	2.564

Table 5.3: Summary of the quality and number of employees of the solution.

5.5 Computational Experiments

The local search heuristic we propose has been implemented in Visual C++ 6.0. In order to test its performance computational experiments were carried out on a 1.8-GHz Pentium4 with 256 MB of RAM. We have also implemented in CPLEX the MIP model given in chapter 4.

The results obtained from the heuristic are compared with the results obtained from the CPLEX, since the later provides optimal solutions.

Several computational experiments have been performed in order to find out not only the quality of the results obtained but also the impact of changing some factors.

To test the quality of the solution obtained through the use of our heuristic we have generated several problems of several sizes and solved them both with the heuristic and with the CPLEX. The solution quality has been measured by the percentage optimality error E. Let $\overline{x}$ be the optimal solution value and x the heuristic solution value, then the percentage optimality error is given by,

$$E = \frac{(x - \overline{x})}{\overline{x}} \times 100. \tag{5.14}$$

In Table 5.4 we report, both for CPLEX and Heuristic on the variation of the of employees E; the percentage ratio Q between covers provided and ideal coverage required; the number of operating branches B; and the computational time required to solve the problem, in CPU seconds. Overall fifty problems have been solved and the average error has been found to be about 6%.

In this set of problems, the smaller error is 1.32% and has been found for a problem with 18 parishes and 7 counties, while the largest is 13.22% and has been found for a problem with 28 parishes and 7 counties. In terms of computational time, the heuristic has a good performance, even in those cases for which the CPLEX takes

			CPLEX				Heuristic				
Problem	**m**	**n**	**E**	**Q %**	**B**	**Time**	**E**	**Q %**	**B**	**Time**	**Error %**
1	16	8	-6	100	6	1	1	98	6	4	11.20
2	17	9	-5	100	7	3	2	100	7	5	4.27
3	18	7	-8	100	6	2	-3	98	6	1	6.91
4	18	7	-7	98	6	5	-2	100	6	3	1.32
5	21	9	-3	100	7	2	-1	98	7	5	5.44
6	21	10	-8	100	8	10	-7	98	8	2	3.34
7	22	7	26	94	9	2	38	99	8	1	9.45
8	23	9	4	99	9	1	7	97	8	4	7.59
9	24	8	17	99	10	1	22	95	10	11	11.54
10	26	9	22	99	11	2	26	95	11	3	10.74
11	27	7	15	99	11	2	27	99	10	3	8.14
12	28	7	41	100	9	3	42	92	9	12	13.22
13	28	12	-5	98	10	13	-3	100	11	5	3.71
14	29	11	8	97	9	3	14	100	9	3	2.72
15	30	5	-12	99	10	31	-2	100	10	5	2.60
16	30	6	34	98	11	3	41	100	10	3	2.03
17	31	6	62	95	11	3	70	98	11	17	5.59
18	31	10	54	95	12	6	64	96	12	21	9.41
19	32	5	13	95	11	5	24	99	11	8	5.85
20	34	7	61	98	12	7	68	100	12	6	3.00
21	34	8	52	98	13	3	61	100	12	8	3.44
22	36	7	91	96	14	4	103	99	14	8	6.00
23	36	10	5	98	12	8	10	97	12	14	7.47
24	38	10	54	94	14	5	66	98	15	16	6.36
25	39	9	57	95	13	7	75	100	13	8	8.68
26	41	5	92	99	14	8	105	98	13	9	9.97
27	41	5	103	96	14	4	118	97	13	16	7.08
28	41	6	58	100	15	8	71	98	15	8	10.80
29	41	11	39	99	18	6	59	100	16	10	8.67
30	41	12	63	92	15	7	78	97	16	8	6.28
31	42	9	2	97	15	6	17	100	15	10	5.96
32	43	11	1	98	17	7	10	100	16	15	2.08
33	44	9	8	93	14	6	29	100	15	30	6.16
34	45	7	115	99	17	10	119	97	16	21	4.80
35	46	10	66	93	16	9	84	98	16	19	6.35
36	46	10	77	95	16	9	95	99	17	23	6.74
37	48	7	120	99	21	9	128	98	19	24	6.89
38	48	7	133	97	18	8	141	97	18	19	5.85
39	48	10	70	98	18	11	79	99	17	18	3.51
40	49	7	71	95	18	12	89	99	19	14	6.92
41	49	11	69	93	15	9	74	95	15	25	7.87
42	49	11	58	99	17	13	68	100	17	23	3.32
43	54	10	130	99	21	21	141	99	20	18	4.14
44	55	8	92	100	20	462	105	99	18	15	6.62
45	55	12	27	97	20	26	39	99	20	41	3.75
46	57	7	94	98	19	65	117	100	20	20	9.10
47	57	11	126	99	20	46	137	99	20	114	4.69
48	60	8	0	100	22	172	8	100	21	39	2.69
49	61	7	105	98	21	461	111	100	21	53	1.70
50	64	5	140	97	25	342	159	99	24	65	5.65
Average			48	97	14	37.38	58	98	14	16.66	**6.15**

Table 5.4: Quality of the solutions.

M	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
15	99.13	99.13	99.13	99.05	99.05	97.21
20	99.79	99.94	99.94	99.81	99.83	96.86
25	99.29	99.14	99.14	98.53	98.33	93.53
35	98.42	98.42	98.42	97.86	97.86	93.51
40	99.62	99.43	99.43	99.43	98.94	95.64
45	99.12	99.12	99.12	99.05	98.71	96.12

Table 5.5: Average cost variation on each step.

a long time. Actually, on average the heuristic needs less than half the time to solve the problems. The heuristic solution is usually more expensive but it should be noticed that it provides more service than the CPLEX solution. This can also be observed regarding the number of employees, which is typically higher (more hired or less fired) than on the CPLEX solution. Regarding the number of operating branches, on average it is the same for both the heuristic and the CPLEX solution, although it has been found equal or larger for the heuristic, in more than half the problems.

5.6 Steps Performance

We have also analysed the effectiveness of the steps involved in the heuristic by looking at partial results on 30 problems. In 100% of the cases step 1 and step 6 result in an improvement of the current solution. By contrast, step 3 has never achieved any improvement. Steps 2, 4, and 5 allow for improving the current solution in about 50%, 83% and 67% of the problems, respectively. These results are summarized in Table 5.5. The cost variation is computed regarding the initial feasible solution obtained, as given in section 5.4.1. In this table we report on six size instances, which comport to 30 problems instances.

5.7 Limit of CPLEX and of the Heuristic

We have tested CPLEX in order to find out the size of the largest problem that it would be able to solve. CPLEX cannot solve problems having more than 65 to 70 parishes. As we have already mentioned in section 5.2, CPLEX works with matrices derived from the mathematical model, and for problems having 70 parishes the matrix becomes too large. Recall that the number of variables depends mainly on the number of parishes and is given by $3 \times m \times k + 2 \times n + m + m^2$, where m is the number of parishes, n is the number of counties, and k is the number of possible branch sizes. For example, in a problem with 70 parishes and 15 counties, the number of variables reaches 5630. As the number of columns is given by the number of parishes, and the number of rows is roughly the same, such a problem requires a matrix whose size is 5630 × 5630, which is stored into a file of 125Mb.

The problem size impact on the heuristic has also been tested, and it has been observed that it can solve problems up to 150 or 160 parishes. Problems of these sizes are solved in less than 500 CPU seconds. This value has been obtained as an average of six problems solved. For those problems, no quality results can be reported since the CPLEX can no longer solve them. Furthermore, the dimensionality problem we are faced with from the heuristic is due to CPLEX. This happens, since our LP problem is solved using the CPLEX and again the limits are imposed by the matrix representation.

5.8 Number of Counties

Some experiments have been performed in order to check the influence of the number of counties. The results obtained consider problems having M = 25, 28, 32, 37, and 41 parishes, and up to M counties. The optimality error obtained for these problems seems to have no pattern at all. See Figure 5.13. The same conclusion applies to the computational time, see Figure 5.14. Therefore, the variation on the number of counties does not seem to affect either the ability to solve or the time needed to solve the problem.

5.9 Number of Parishes

Similarly to the variation of the number of counties, we have also studied the impact of the variation on the number of parishes. Some of the results obtained are represented in Figure 5.15. In this graph we plot the error for three sets of seven problems each, one with N = 10 counties, other with N = 15 counties, and another

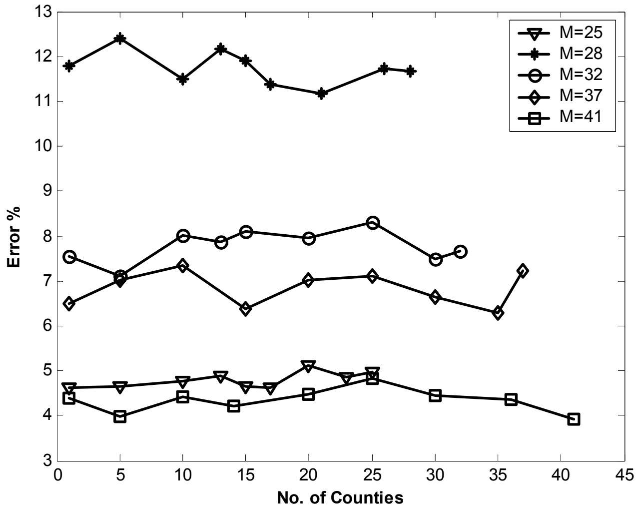


Figure 5.13: Average error versus number of counties.

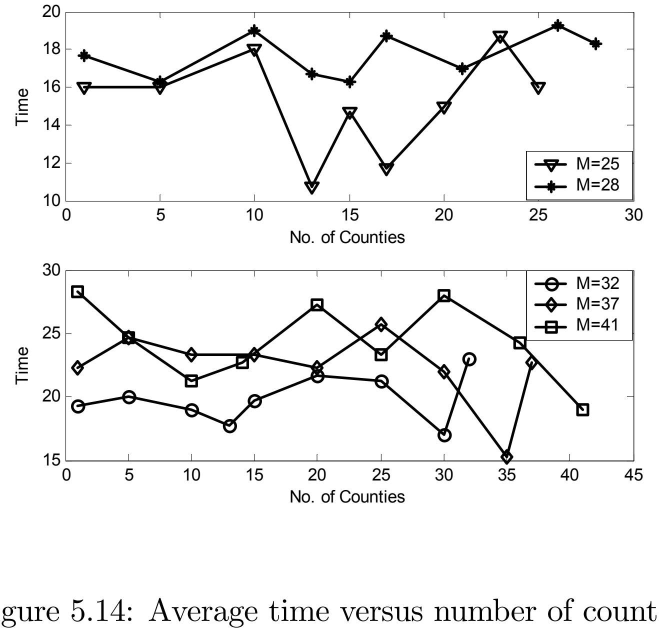


Figure 5.14: Average time versus number of counties.

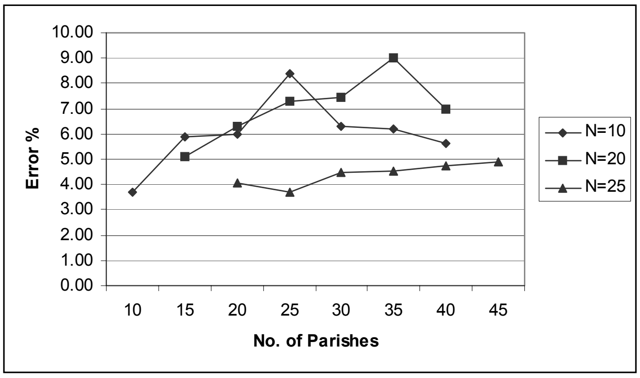


Figure 5.15: Average error for varying number of parishes.

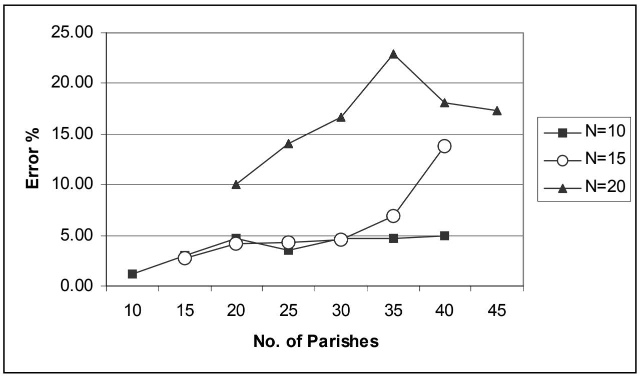


Figure 5.16: Average computational time for varying number of parishes.

with N = 20 counties as the key. As it can be seen, the error increases with the increase of the number of parishes, for a fixed number of counties.

An increasing tendency is also present for the computational time, see Figure 5.16.

5.10 Distance Standard

The distance standard is a parameter that defines the maximum distance between clients and branches that can be used to provide service to them. This distance standard can be viewed as the distance a client is willing to travel in order to make

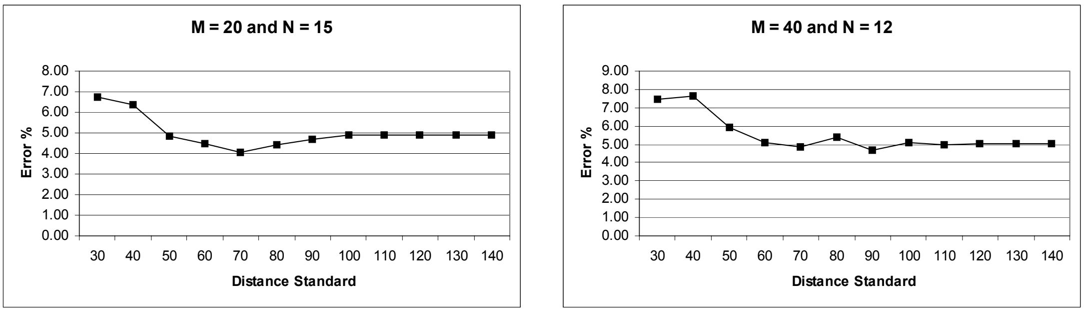


Figure 5.17: Average error variation with the distances standard.

banking operations. In Figure 5.17, we present the results obtained for two problem sizes. We have started at a distance standard of 30 because we have realized that with lower values it was very difficult to generate possible problems. Recall that the client requirements vary in the interval [1;5]. As the distance standard decreases the number of branches that can cover a client decreases as well, and it may happen that only one branch can serve a client. If this is the case, and if the client requires more than 3 service units, the problem is impossible.

From the graphs in Figure 5.17, it can be seen that the error decreases as the distance standard increases.

If we look closer at the graphs in Figure 5.17, we can see that after a certain value the error seems to stabilize. In order to study it further we have zoomed in by studying the error behavior for distance standard values between 80 and 140, see Figure 5.18. As it can be seen, the error stabilizes for larger distance standard values (100 on the left-hand side graph and 120 on the right-hand side).

A similar analysis has also been done for computational time. In Figure 5.19, we give the computational time behavior for increasing distance standard considering the same problems of Figure 5.17 and 5.18. Again, we have zoomed in for distance standard values between 80 and 140, the results obtained are given in Figure 5.20.

An increasing time tendency may be observed, which is probably due to the larger number of branches that has to be tested in order to find the cheaper ones. For example, in the extreme case where every branch covers every client, in the sixth step of the heuristic a cost variation for every potential branch (closed branch) has to be calculated before performing a swap of branches. On the other hand, if the

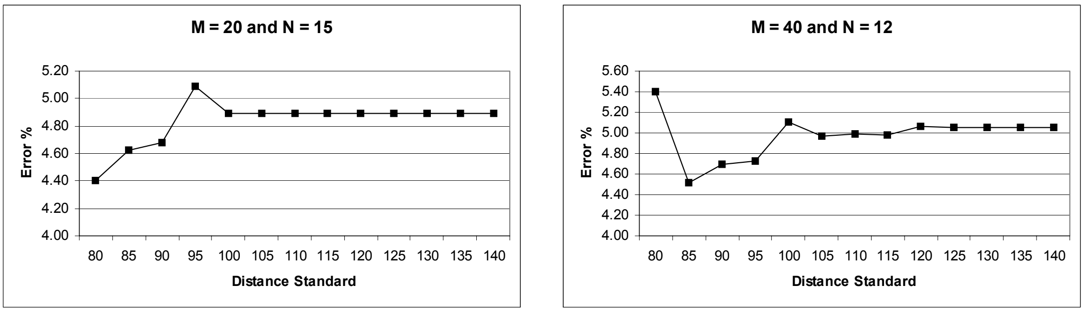


Figure 5.18: Zoomed in average error.

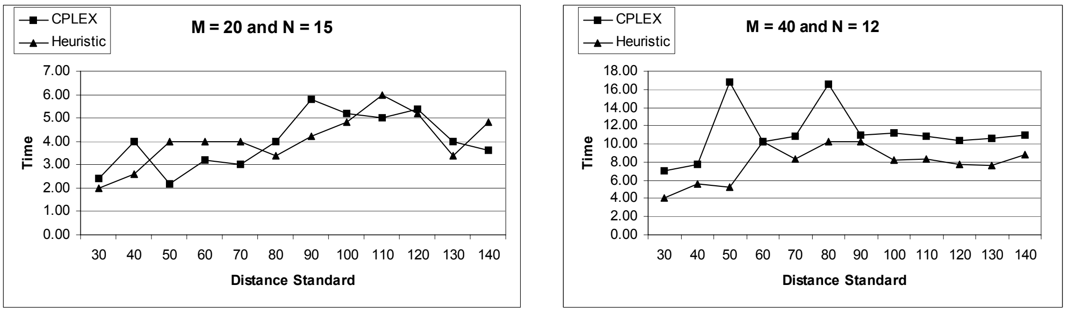


Figure 5.19: Average computational time variation with the distance standard.

distance standard is small, then the number of clients covered by each branch is also small and thus, there are few branches to be tested before performing the swap.

Thus, we may conclude that an increase in the distance standard seems to affect the optimality error by decreasing it and also the computational time needed to solve the problem, by increasing it.

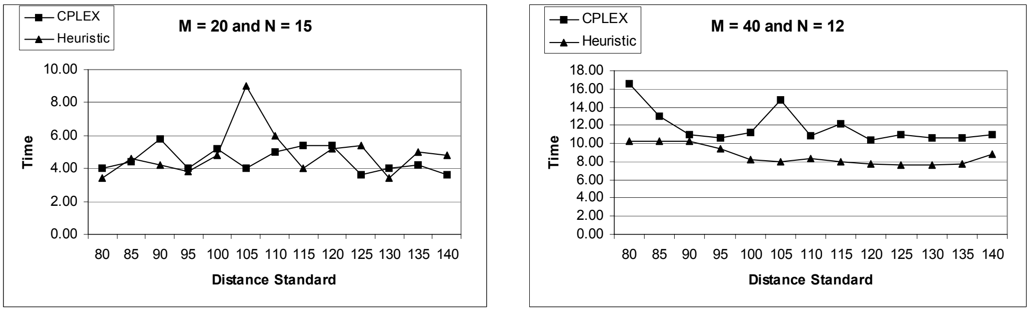


Figure 5.20: Zoomed in average computational time.

Chapter 6

Conclusions

This chapter summarises the work developed and highlights the main contributions achieved. Finally, some potential avenues of future research are suggested.

6.1 Summary

In this thesis, we have addressed the bank-branch restructuring problem. This problem is often faced by banks, however very few studies have been reported in this area. The restructure of a bank-branch network is very complex and incorporates many different components. Although we have just included some of them, we have presented a problem that deals with an issue never included before, that is employees.

We have addressed this problem as a bank-branch location and sizing problem and have modelled it as a mixed binary integer linear model. The objective is to find the location and size of the bank-branches that should be operating in order to satisfy a certain service coverage required by the clients, at a minimum cost. Since the original problem is not linear, the number of variables used to model it is large and therefore, only small instances can be optimally solved.

This model has been solved using CPLEX. The solution obtained has been used to measure the quality of a heuristic solution we have also developed.

A local search heuristic to solve the bank-branch location and sizing problem with concave cost functions has been developed. The heuristic has two phases. In the first phase, we solve a related linear integer programming problem which is determined by dropping all variables except the ones associated with providing service. This related problem is iteratively solved using CPLEX. At each iteration the cost function is updated, based on the information regarding the solution of the previous iteration. This is repeated until no changes are reported for two consecutive solutions. The solution obtained at the end of this iterative procedure is the initial solution of our local search heuristic. In the second phase, we improve upon the initial solution by applying drop and swap operations. The drop operation consists on dropping branches that provide service to one single client, provided that the client is at least minimally covered by other operating branches. The swap operation is divided into two steps. In the first step, a branch of size k is swapped by a branch located at the same site and having size $k-1$, if a cost reduction is achieved. In the second step, an open branch is interchanged with a closed branch, of the same size, if the value of the cost function decreases. From the results reported it can be seen that our heuristic is able of solving large size problems that cannot be addressed by CPLEX. Therefore, the quality of the heuristic cannot be measured in these cases.

6.2 Suggestions for Future Research

Based on the results obtained and reported in the previous chapter and on the experience resulting from this research, we are able to make some suggestions for further research.

The computational performance of the heuristic developed can be improved. Some suggestions to do so are given bellow.

One of the aspects to be looked at is the steps used in the local search. In the previous chapter, we have seen that at least one of them, step 3, was not needed since it has never decreased the value of the cost function. Besides that, step 2 and step 5 need to be improved or even taken off the heuristic because they are not used very often.

Another type of improvement that can be attempted is related to incorporate further information regarding the problem itself. For instance, it would be interesting to consider interaction between districts, and to distinguish between rural and urban areas, such as Cabral and Majure (1992) and Avery *et al.* (1999) have done. It is known that in rural areas bank clients are willing to travel a larger distance comparing to clients living in urban areas. Therefore, instead of a single distance standard, two or more would have to be applied.

Competition between banks is also a very interesting issue to include in this problem since, as we have already discussed in section 2.2, the behavior of the bank depends mainly on its dimension and on the growth adopted policies, this last one very influenced by the competitors attitudes.

In a bank there are different types of employees: managers, tellers, credit evaluators, regional managers, to mention but a few. All these employees have different roles and costs and are obviously not homogeneous as we have considered. Also, different types of branches, with different needs for each employees type, could be included in the problem.

Although the tendency seems to be the one of closing branches (Morrison & O'Brien, 2001; Fernandes, 1998; APB, 2003), that is an attitude that cannot be taken without proper thinking. A bank is a financial institution thus, depends a lot on the image perceived by its clients. If a bank starts closing a large number of its branches, its current and potential clients would think that the bank had financial problems. Furthermore, the bank would have to deal with unions, which generally try to confront employers to prevent massive firing of personnel. These are the major reasons for restricting the number of branches to be closed. Limits on the number of fired and hired employees should also be imposed since the average number of employees per bank-branch has been decreasing these past few years (APB, 2003). These limits have to be carefully thought, since in some situations they do not make any sense. For example, if an expansion on the branch network is to be taking place, the bank will need to hire a large number of employees to operate those new branches.

As it can be seen, the issues and data that can be incorporated in this problem are countless and each brings in additional complexity but also accuracy. In future approaches, we intend to study several of them and the possible changes they may bring to the results.

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Appendix A

Real Estate Data

We have summarized, in the following tables, information about acquisition prices, for each county. The county of Porto is presented in a different table due to the huge differences found from parish to parish.

Porto	€	
Parish	min	max
Foz do Douro	2000	3250
Paranhos (Antas)	1500	2500
Santo Ildefonso	1500	2000
Other parishes	1000	2000

Table A.1: Acquisition prices, per square meter, in the county of Porto.

	€	
County	minimum	maximum
Amarante	750	1100
Baião	750	1100
Felgueiras	900	1500
Gondomar	1000	2000
Lousada	750	1100
Maia	1000	2000
Marco de Canaveses	750	1100
Matosinhos	1000	2000
Paços de Ferreira	1000	1500
Paredes	750	1100
Penafiel	900	1500
Porto	-	-
Póvoa de Varzim	1000	2000
Santo Tirso	750	1100
Trofa	750	1100
Valongo	1000	2000
Vila do Conde	1000	2000
Vila Nova de Gaia	1000	2000

Table A.2: Acquisition prices, per square meter, in the district of Porto.

Appendix B

Linear Programming Model Objective Function

In this appendix, a description of the objective function, for the linear programming model of section 5.4, is given.

In the objective function we include operating, opening, penalty, and service costs.

Opening Costs

$$\sum_{i,j} h_{ij}, \text{ for all branches not yet opened.} \tag{B.1}$$

Operating Costs

$$\sum_{i,j} f_{ij}, \text{ for all branches being operated.} \tag{B.2}$$

Penalty Costs

$$\sum_{l,m} P_{lm} \cdot (\overline{W}_{lm} - \sum_{i,j} q_{ij}^{lm}) = \sum_{l,m} P_{lm} \cdot \overline{W}_{lm} - \sum_{i,j} \sum_{l,m} P_{lm} \cdot q_{ij}^{lm} \tag{B.3}$$

Service Costs

$$\sum_{i,j} \sum_{l,m} v_{ij}^{lm} \cdot q_{ij}^{lm}. \tag{B.4}$$

Objective Function

$$\phi_{ij} = \begin{cases} h_{ij} + f_{ij} + \sum\limits_{l,m} P_{lm} \cdot \overline{W}_{lm} - \\ \quad - \sum\limits_{l,m} P_{lm} \cdot q_{ij}^{lm} + \sum\limits_{l,m} v_{ij}^{lm} \cdot q_{ij}^{lm}, & \text{if } \sum\limits_{l,m} q_{ij}^{lm} > 0 \\ \\ 0, & \text{otherwise,} \end{cases} \tag{B.5}$$

as $\sum\limits_{l,m} P_{lm} \cdot \overline{W}_{lm}$ is a constant it can be disregarded, and therefore equation (B.5) can be simplified to

$$\phi_{ij} = \begin{cases} h_{ij} + f_{ij} + \sum\limits_{l,m} \overline{v}_{ij}^{lm} \cdot q_{ij}^{lm}, & \text{if } \sum\limits_{l,m} q_{ij}^{lm} > 0 \\ \\ 0, & \text{otherwise,} \end{cases} \tag{B.6}$$

where $\overline{v}_{ij}^{lm} = v_{ij}^{lm} - P_{lm}$ and $\overline{h}_{ij} = h_{ij} + f_{ij}$.