



FEUP Universidade do Porto
Faculdade de Engenharia

ANALYSIS OF CYCLIC AND LONG-TERM EFFECTS IN CONTINUOUS PRECAST RAILWAY BRIDGE DECKS

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by

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Abstract

Important investments have been made worldwide in the modernization and construction of railway lines. Plans for new constructions, in the near future, have also been laid out. Recently, precast concrete has been widely employed in the construction of railway bridges and viaducts. Various prefabricated solutions have been applied, namely: I-shaped or U-shaped precast girders and uni-cellular or bi-cellular precast box girders. As regards the structural scheme in the longitudinal direction, either simple span or continuous decks have been used. In this context, this work aims to contribute to the knowledge about the real performance of these structures, and also to give a contribution in terms of numerical methods for analysis of this type of constructions. Special attention is devoted to continuous bridge decks constructed with precast U-shaped girders, without continuity prestress. This option is justified by the advantages offered by this solution, in terms of economy and ease of construction, and also by the specificities of its structural behaviour, which require special attention in terms of analysis and design. These specificities are: (i) the absence of post-tensioning to resist hogging bending moments above the continuity supports, which requires a detailed analysis of the structural behaviour after cracking; (ii) the adoption of thin webs and slabs, subjected to important cyclic stresses, which implies that their fatigue behaviour must be properly assessed; (iii) the relevance of the construction sequence and of the concrete delayed deformations, in terms of long-term variation of concrete stresses and internal forces. In view of these issues, this thesis comprehends four main parts.

In the first part, a numerical model for the analysis of cracked RC elements subjected to cyclic loading is developed and validated. The adopted analysis strategy involves the explicit consideration of the cyclic bond stress-slip relationship between the tensile reinforcement and the surrounding concrete, and also the relationship between the normal stress at the crack and the crack opening under cyclic loading. The validation, through the comparison with experimental results reported in the literature, reveals that this model provides an accurate description of the non-linear behaviour of the tested elements subjected to cyclic loading.

The second part focuses on the fatigue behaviour of railway bridge decks. Alternative procedures for fatigue analyses are presented, special attention being paid to analyses based on the damage accumulation method. An original methodology for fatigue analysis of girder webs, taking into account the effects of in-plane shear and transverse bending, is developed and validated. The main specificity of this methodology lies in a simple algorithm for calculation of concrete and

stirrup stresses. The comparison between numerical calculations and experimental results reported in the literature reveals that the developed analysis procedures provide results on the safe side, and are suitable to be applied in the design practice. The applicability of the same procedures to problems of shear between web and flanges is also discussed.

The third part is dedicated to the numerical analysis of the long-term behaviour of continuous precast bridge decks, considering the influence of concrete delayed deformations and phased construction. The results of finite-element phased analyses are compared with the experimental results observed in a real viaduct. A Monte Carlo simulation is employed to evaluate the relevance of the uncertainty associated to the quantification of concrete delayed deformations and construction timings.

In the last part, the previously developed and validated numerical methodologies are applied in the analysis of a real railway bridge deck built with precast U-shaped girders, without continuity prestress. The variability of the long-term structure behaviour, owing to concrete delayed deformations and phased construction, is quantified and discussed. Fatigue and ULS analyses of the girder webs and of the top slab, considering in-plane shear and transverse bending, are shown. Stiffness decreases after cracking and residual crack openings are quantified taking into account the influences of traffic (cyclic) loading and concrete shrinkage. Detailed analyses based on the methodologies developed in this thesis are compared with the simplified approaches suggested by the Eurocodes, and conclusions about the relevance of the developed models are drawn.

Resumo

Atualmente estão em curso importantes investimentos na modernização e construção de linhas ferroviárias. Existem também vários projetos para a realização de novas obras num futuro próximo. A pré-fabricação em betão tem sido empregue em larga escala nesses empreendimentos, para a realização de pontes e viadutos. Diversas soluções têm sido utilizadas, nomeadamente vigas pré-fabricadas tipo I e tipo U, e também vigas pré-moldadas em caixão uni-ou bi-celular. No que diz respeito ao esquema estrutural, têm sido empregues quer soluções isostáticas, quer tabuleiros contínuos. Neste contexto, pretende-se com o presente trabalho contribuir para o conhecimento acerca do desempenho real destas estruturas, e sobre as metodologias para análise do seu comportamento estrutural. O trabalho foca, em particular, nos tabuleiros contínuos construídos com vigas tipo U, sem pré-esforço de continuidade. Esta opção é justificada pelas vantagens económicas e construtivas oferecidas por esta solução, e ainda pelas especificidades do seu comportamento estrutural, que requerem cuidados especiais em termos de conceção e análise. Destacam-se as seguintes especificidades: (i) a ausência de pré-esforço de continuidade para resistir a momentos fletores negativos, o que requer uma análise cuidada do comportamento em fase fendilhada; (ii) a adoção de lajes e almas esbeltas, sujeitas a importantes carregamentos cíclicos, tendo portanto relevo o fenómeno de fadiga; (iii) a importância do faseamento construtivo e do comportamento diferido do betão, em termos de distribuição de tensões e esforços a longo prazo. Tendo em conta estes aspetos, esta tese é dividida em quatro partes principais.

Na primeira parte, é desenvolvido e validado um modelo numérico para análise do comportamento cíclico de elementos de betão armado fendilhados. A metodologia de análise adotada envolve a consideração explícita de modelos que descrevem o comportamento cíclico, quer da aderência aço-betão, quer da relação entre a tensão normal a uma fenda e a abertura dessa fenda. A validação, através de resultados experimentais disponível na bibliografia, revela que este modelo permite descrever de modo rigoroso o comportamento não linear dos elementos sujeitos a ensaios cíclicos.

A segunda parte incide sobre a fadiga dos tabuleiros de pontes ferroviárias. São apresentados procedimentos alternativos para a realização de análises de fadiga, sendo dada especial atenção às análises baseadas no método da acumulação de dano. É apresentada e validada uma metodologia original para análises de fadiga das almas das vigas, considerando os efeitos do

esforço transversal no plano e da flexão transversal. A principal especificidade dessa metodologia reside no emprego de um algoritmo simples para cálculo de tensões instaladas nos estribos e no betão. A comparação entre resultados numéricos e experimentais (publicados por outros autores) mostra que a metodologia numérica desenvolvida fornece resultados do lado da segurança e é adequada para ser empregue na prática. É também discutida a aplicabilidade dessa metodologia a problemas de corte entre alma e banzos.

A terceira parte é dedicada à análise numérica do comportamento a longo prazo de tabuleiros contínuos pré-fabricados, considerando a influência do faseamento construtivo e das deformações diferidas do betão. Os resultados de análises faseadas baseadas em modelos de elementos finitos são confrontados com valores observados num viaduto real. É utilizada uma simulação de Monte Carlo para avaliar a importância da variabilidade associada à quantificação das deformações diferidas do betão e dos tempos de construção.

Na última parte, os modelos numéricos previamente desenvolvidos e validados são empregues na análise de um tabuleiro ferroviário real construído com vigas pré-fabricadas tipo U, sem pré-esforço de continuidade. A variabilidade do seu comportamento a longo prazo, considerando as deformações diferidas do betão e o faseamento construtivo, é quantificada e discutida. São levadas a cabo análises de fadiga das almas e da laje superior, considerando o corte no plano e a flexão transversal. É avaliada a redução de rigidez associada à fendilhação, assim como a abertura residual de fendas, tendo em consideração os efeitos do tráfego ferroviário e da retração do betão. As análises detalhadas baseadas nas metodologias desenvolvidas nesta tese são comparadas com as abordagens simplificadas dos Eurocódigos e são retiradas conclusões sobre a relevância dessas metodologias.

Résumé

Actuellement, des investissements importants sont faits dans le monde entier pour la modernisation et la construction de lignes de chemin de fer. Des plans pour de nouvelles constructions, dans un avenir proche, ont également été mis en marche. Les préfabriqués en béton précontraint sont largement utilisés dans la construction de ponts de chemin de fer et de viaducs. Diverses solutions préfabriquées sont appliquées, à savoir, des poutres préfabriquées en I ou en U, ainsi que des poutres caissons préfabriquées uni-cellulaires ou bi-cellulaires. En ce qui concerne le schéma structurel, ces systèmes ont été utilisés pour des ponts isostatiques et hyperstatiques. Dans ce contexte, ce travail va présenter des résultats qui permettront mieux connaître la performance réelle de ces structures et donnera aussi quelques contributions en ce qui concerne les méthodes numériques pour l'analyse de leur comportement structurel. Le travail s'est particulièrement consacré à l'analyse de ponts ayant des tabliers continus construits avec des poutres préfabriqués en U sans précontrainte de continuité. Cette option se justifie par les avantages offerts par cette solution, en ce qui concerne les facteurs économiques et la facilité de construction, autant que par les spécificités de son comportement structurel qui ont besoin d'une attention particulière au niveau de la conception et de l'analyse de son comportement. Les spécificités qui ont été considérées sont les suivantes: (i) l'absence de post-contrainte pour résister aux moments fléchissant sur les appuis nécessite d'une analyse détaillée du comportement structurel après la fissuration du béton; (ii) l'adoption de dalles et de poutres à âmes minces qui seront soumises à des chargements cycliques importants implique que leur comportement en fatigue doit être correctement évalué; (iii) l'importance du procédé de construction et des déformations différées de béton, en ce qui concerne la variation à long terme des contraintes du béton et des efforts internes. Compte tenu de ces questions, cette thèse comprend quatre parties principales.

Dans la première partie, un modèle numérique pour l'analyse des éléments fissurés en béton armé soumis à un chargement cyclique a été développé et validé. La méthodologie adoptée utilise des modèles d'analyse qui décrivent le comportement cyclique de l'adhérence béton-acier et de la relation entre la contrainte perpendiculaire à une fissure et l'ouverture de cette fissure. La validation de ce modèle d'analyse utilisant des résultats expérimentaux disponibles dans la littérature montre qu'il fournit une description précise du comportement non-linéaire des éléments testés soumis à un chargement cyclique.

La deuxième partie se concentre sur le comportement en fatigue des tabliers de ponts de chemin de fer. Les différentes procédures pour l'analyse de la fatigue sont présentées, donnant une attention particulière aux analyses basées sur la méthode d'accumulation des dommages. Une méthodologie originale pour l'analyse de la fatigue de l'âme des poutres en tenant compte des effets de cisaillement dans le plan et de flexion transversale a aussi été développée et validée. La principale spécificité de cette méthode réside dans l'utilisation d'un algorithme simple pour le calcul des contraintes du béton et des étriers. La comparaison entre les calculs numériques et des résultats expérimentaux rapportés dans la littérature montre que cette méthodologie fournit des résultats sur le côté de la sécurité et qu'elle est donc apte à être utilisée dans la pratique du projet. L'applicabilité de cette méthodologie à des problèmes de cisaillement entre l'âme et les membrures des sections est également discutée.

La troisième partie est consacrée à l'analyse numérique du comportement à long terme de ponts préfabriqués à tablier continu, en tenant compte de l'influence des déformations différées du béton et du procédé de construction. Les résultats de l'analyse par éléments finis par étapes sont comparés avec les résultats expérimentaux observés dans un viaduc réel. Une simulation de Monte Carlo est utilisée pour évaluer l'importance de la variabilité associée à la mesure des déformations différées du béton et à la durée des étapes de construction.

Dans la dernière partie, les modèles numériques déjà développés et validés sont utilisés dans l'analyse d'un tablier de pont de chemin de fer réel construit avec des poutres préfabriquées en U, sans précontrainte de continuité. La variabilité du comportement de la structure à long terme, en raison de l'influence des déformations différées du béton et du procédé de construction, est quantifiée et discutée. La fatigue de l'âme des poutres et des dalles est analysée en tenant compte des effets de cisaillement dans le plan et de flexion transversale. La réduction de la rigidité des éléments après la fissuration du béton est analysée, bien comme l'ouverture des fissures, en tenant compte des effets du trafic ferroviaire et du retrait du béton. Les résultats des analyses détaillées basées sur les méthodologies développées dans cette thèse sont aussi comparés avec ceux qui sont obtenus par des méthodes simplifiées proposées par les Eurocodes et des conclusions sur la pertinence des modèles qui ont été développés sont tirées.

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List of symbols and abbreviations

Symbols and abbreviations are defined upon their first appearance in the text. The following list is presented in alphabetic order and does not include some symbols and notations of a secondary order (whose concern is limited to specific applications).

Roman letters

a	Sleeper spacing
a_1	Dimensionless constant used in the bond stress-slip relationship
a_2	Dimensionless constant used in the bond stress-slip relationship
A_c	Cross sectional area of the concrete
$A_{c,eff}$	Effective area of concrete in tension
A_s	Cross sectional area of the steel
$A_{s,sec}$	Secondary reinforcement
A_{sw}/s	Cross-sectional area of stirrups per unit length
ACI209	ACI Committee 209 (1992)
b	Dimensionless constant used in the bond stress-slip relationship
b	Dimensionless constant for calculation of the slip increase under repeated loading
b_0	Distance between the axes of stirrup legs
b_l	Length over which the load applied by each sleeper is assumed to be evenly distributed, in the longitudinal direction
b_m	Length over which the transverse bending moment is assumed to be evenly distributed
b_t	Length over which the load applied by each sleeper is assumed to be evenly distributed, in the transverse direction
b_{triang}	Depth of the triangular diagram of normal stresses if the compression struts
b_{unif}	Depth of the rectangular diagram of normal stresses if the compression struts
b_w	Web thickness
$b_{w,req}$	Minimum required web thickness
B	Axial force applied to one stirrup leg
BCNCL	Basic combination of non-cyclic loading
$B_{l,0}$	Tensile force in the left stirrup leg due to the sole effect of in-plane shear

$B_{r,0}$	Tensile for in the right stirrup leg due to the sole effect of in-plane shear
c	Cover to the reinforcement
C	Vertical component of the compression force in the struts
C_1 and C_2	Fitting parameters to define the time variation of the creep coefficient
CEB	Comité Euro-International du Béton
CEN	Comité Européen de Normalisation
CFM	Continuous Function Model
CIP	Cast in place
CTL	Construction Technology Laboratories
d	Effective depth
d_b	Bar diameter
D	Diameter of the mandrel
D	Damage factor
DAM	Damage Accumulation Method
DESM	Damage Equivalent Stress Method
D_{year}	Damage factor corresponding to one year of traffic
e	Eccentricity of the force applied to the compression struts
$e_{max,fat}$	Upper bound limit for the eccentricity of the compression struts
E_c	Modulus of elasticity of the concrete
$E_{cd,max,i}$	Maximum compressive stress level, to define the fatigue strength of concrete under compression
$E_{cd,min,i}$	Minimum compressive stress level, to define the fatigue strength of concrete under compression
E_{cm}	Mean value of the modulus of elasticity of the concrete
EC2	Eurocode 2 (EN 1992-1-1)
ERRI	European Rail Research Institute
E_s	Modulus of elasticity of the steel
f_c	Concrete cylinder compressive strength
$f_{c,cube}$	Concrete cube compressive strength
f_{cd}	Design value of concrete compressive strength
$f_{cd,fat}$	Fatigue reference strength for concrete under compression (design value)
f_{ck}	Characteristic value of concrete compressive strength
f_{cm}	Mean value of concrete cylinder compressive strength
$f_{cm}(t)$	Mean value of concrete cylinder compressive strength at an age t
$f_{cm,cube}$	Mean value of concrete cube compressive strength
$f_{cm,fat}$	Estimated mean value of the fatigue reference strength for concrete under compression
f_{ctm}	Mean value of axial tensile strength of concrete

<i>fib</i>	The International Federation for Structural Concrete
$f_{p0,1k}$	Characteristic 0.1% proof-stress of prestressing steel
f_y	Yield strength of reinforcement
f_{yd}	Design yield strength of reinforcement
f_{yk}	Characteristic yield strength of reinforcement
F	Applied force
$F_{cw,unit}$	Compression force in the struts, in the direction θ , per unit length
FE	Finite element
FEM	Finite-element model
FL	Fatigue life
FPM	Focal Point Model
G	Permanent action
h	Crack band width
h_0	Notional size of the cross section
h_f	Flange thickness
HSL	High speed line
HSLM	High speed load model
IPQ	Instituto Português da Qualidade
ISO	International Organization for Standardization
k_1	Parameter to define the Wöhler curve for reinforcing and prestressing steel
k_1	Reduction factor to define the design fatigue strength of concrete
k_2	Parameter to define the Wöhler curve for reinforcing and prestressing steel
k_m	Percentage of transverse bending moment that is resisted by the mechanism of eccentricity of the compression struts
k_n	Factor to describe the slip increase under repeated loading
K_B	Stiffness of the bond stress-slip relationship upon unloading from the envelope curve
l_b	Bonded length
LM71	Traffic load model LM71 as defined by the EN 1991-2
L_r	Transmission length for bond stresses
m	Bending moment per unit length
m	Parameter to quantify the tension stiffening effect
$m_{max,l}$	Bending moment that can be resisted by the eccentricity of the compression struts
M	Bending moment
MC90	Model Code 1990
M_y	Bending moment at the onset of yielding
n	Membrane force per unit length

n	Number of random variables
n	Number of steel bars
n_{div}	Number of layers into which the web thickness is divided for fatigue analyses
n_{Ed}	Design value of the compression force in the web, in the transverse direction, per unit length
n	Number of applied stress cycles
N	Axial force
N	Number of load cycles at fatigue failure
N^*	Number of load cycles at fatigue failure at the transition between slopes of the Wöhler curve for reinforcing and prestressing steel
N_{cr}	Cracking load
N_y	Axial force at the onset of yielding
p	Percentile
$\hat{p}$	Estimated percentile
P	Prestress action
PB	Plain bar
PC	Prestressed concrete
PCA	Portland Cement Association
P_i	Axle load
Q	Variable action
$Q_{i,j}$	Load transmitted by the axle load P_i to the sleeper j
RB	Ribbed bar
RC	Reinforced concrete
R_i	Stress ratio, to define the fatigue strength of concrete under compression
s	Slip
s	Parameter to define the time variation of the concrete strength
s_{re}	Slip value considering the influence of repeated loading or long-term loading; slip in the reduced envelope
s_p	Slip at the peak bond stress
s_r	Crack spacing
s_{rm}	Average crack spacing
S	Stress range
S_1 to S_4	Fitting parameters to define the time variation of the shrinkage strain
t	Concrete age
t	Temperature
t_{cont}	Time interval between the prestress release and the casting of the continuity connection
t_T	Temperature adjusted concrete age

TP	Transition pier
TSI	Technical Specifications for Interoperability
TT5	Train type 5, as defined by the EN 1991-2, Annex D
U	Perimeter of reinforcing bars
ULS	Ultimate Limit State
v_f	Applied shear stress due to the transference of forces between web and flanges
v_{freq}	Frequent operating speed
V	Applied shear force
V	Train speed
V_{Ed}	Design value of the applied shear force
w	Crack opening
w_s	Crack opening at the level of the main tension reinforcement
x_a	Distance between the axle load and the sleeper $Q_{i,2}$
y_{na}	Neutral axis depth
z	lever arm
$z_{0.05}$	Upper 0.05 percentage point of the standard normal distribution

Greek letters

α	Factor for quantification of classified railway traffic loads
α_B	Constant to define the frictional bond strength
α_{cw}	Coefficient that accounts for the influence of the axial force on the girder, in the calculation of the compression strength of web struts
β	Percentage of longitudinal force applied to each flange, in problems of shear between web and flanges
$\beta_c(t, t_0)$	Coefficient to describe the development of creep with time after loading
$\beta_{cc}(t_0)$	Coefficient to define the concrete compressive strength at the time t_0
χ	Curvature
χ_m	Average curvature
Δ	Range
$\Delta\sigma_{equ}$	Damage equivalent stress range
ε_c	Normal strain in the concrete
$\varepsilon_{cs}(t)$	Shrinkage strain
$\varepsilon_{cas}(t)$	Autogenous shrinkage strain
$\varepsilon_{cs}(t)$	Drying shrinkage strain
ε_s	Normal strain in the steel
ε_{sI}	Normal strain in the steel in the uncracked phase (state I)
ε_{cI}	Normal strain in the concrete in state I
ε_{sm}	Average steel strain

ε_{sh}	Absolute value of the concrete shrinkage strain
ϕ	Diameter
φ''	Factor to describe the dynamic enhancement due to track defects and vehicle imperfections
φ_0	Notional creep coefficient
$\varphi(t, t_0)$	Creep coefficient
ψ^{cr}	Random variable to describe the variability of the creep coefficient
ψ^{sh}	Random variable to describe the uncertainty in the shrinkage model
γ	Reduction coefficient for definition of the upper bound limit for the eccentricity of the compression struts
$\gamma_{F,fat}$	Partial factor for cyclic actions
$\gamma_{s,fat}$	Partial factor for fatigue verification of reinforcing and prestressing steel
λ	Factor to describe the influence of the type of track maintenance
λ_0 to λ_4	Correction factors for calculation of damage equivalent stress ranges
ν_l	Coefficient that accounts for the influence of shear cracks, in the calculation of the compression strength of web struts
θ	Inclination of the compression struts
θ	Principal direction
ρ	Steel ratio, given by A_s/A_c
σ_c	Normal stress in the concrete
σ_{cI}	Normal stress in the concrete in state I
$\sigma_{cd,max,i}$	Upper stress in a cycle, to define the fatigue strength of concrete under compression
$\sigma_{cd,min,i}$	Upper stress in a cycle, to define the fatigue strength of concrete under compression
σ_{cw}	Normal stress in the concrete struts
σ_s	Normal stress in the steel
σ_{sI}	Normal stress in the steel in state I
σ_{sr}	Steel stress at the crack section
σ_{sw}	Normal stress in the stirrups
τ	Bond stress
$\tau_{b,max}$	Maximum value of cyclically applied bond stress
$\tau_{b,min}$	Minimum value of cyclically applied bond stress
$\tau_{f,i}$	Frictional bond strength
τ_i	Bond stress on the envelope curve from which the unloading starts
τ_{max}	peak bond stress under monotonically increasing load, also known as monotonic bond strength

Chapter 1

Introduction

1.1 Motivation

Important investments are currently being made worldwide in the modernization and expansion of railway lines. In part, these investments have been motivated by the commitment in terms of construction of new high-speed lines. Numerous new lines are currently under construction. China is the world's leader, with an estimated total length of more than 12,000 km in high-speed tracks (for speeds higher than 200 km/h), to be concluded by the end of 2012. Moreover, this state's spending on railroad construction during the present decade is expected to exceed \$700 billion (Ramzy 2010). In Europe, the length of high-speed lines under construction in 2010 amounted to 1,624 km, 210 km and 378 km in Spain, France and Germany, respectively (European Commission 2010). Important initiatives in terms of upgrading and expansion of railway transportations are also being taken by the United States (Grunwald 2010; James 2009), among others. As regards freight traffic, relevant initiatives are also being carried out in order to promote efficient long-distance transport services on national and international corridors (UIC 2011).

In Portugal, various plans for future investments in railway lines have also been laid out. According to the plan approved in 2004 (REFER 2011), the future Portuguese High-Speed Network consists of five routes, and comprehends the Lisbon-Porto axis (292 km) and the Portuguese branch of the Lisbon-Madrid axis (207 km). Important investments are also planned

in terms of construction of conventional goods lines, connecting sea ports to the Portuguese and Spanish hinterland.

Recently, prefabrication has played an important role in the construction of railway bridge decks. In France and Spain, different prefabricated solutions with I-shaped or U-shaped girders have been adopted (Figure 1.1). In France, several viaducts were constructed, in the line LGV Est-Européenne, using precast I-shaped girders (Vavel 2004). The spans were made continuous, but continuity prestress was not employed. In these structures, the span length is, in general, equal to 22.5 m. Each viaduct is divided into a succession of various elementary viaducts separated by expansion joints, so that the length of each continuous deck is approximately equal to 90 m (four continuous spans of 22.5 m). In Spain, U-shaped girders have been widely employed in the construction of bridge decks with one or more box girders (Aparicio 2004; Sobrino 2003). Both isostatic spans and continuous solutions have been employed. Continuity connections have been built with continuity prestress or, alternatively, using only passive reinforcements.

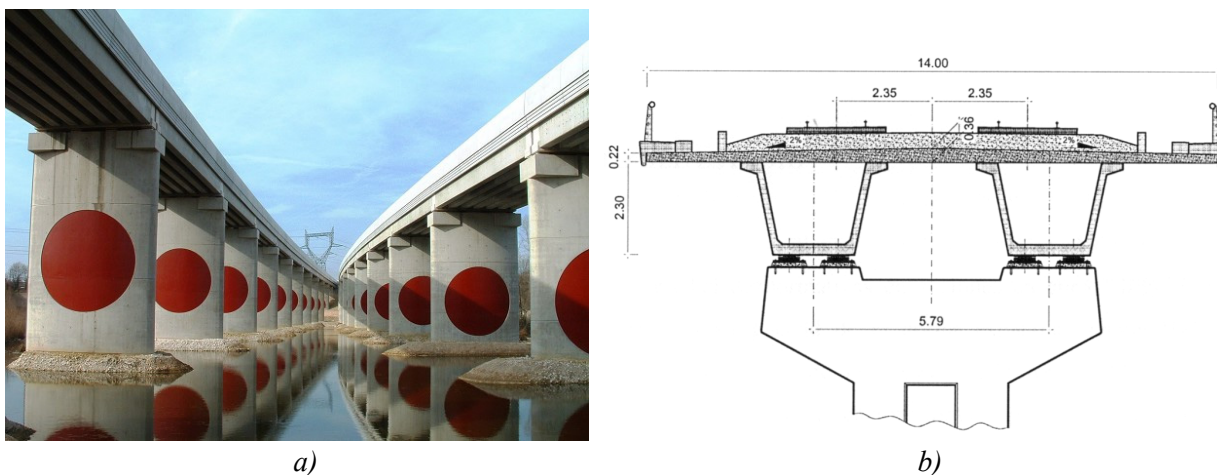


Figure 1.1 –Railway bridge decks made with precast girders:

- a) viaduct in the French line LGV Est-Européenne, built with I-shaped girders (Structurae 2011a);
- b) viaduct in the Spanish line Madrid-Barcelona, made with U-shaped precast girders (Aparicio 2004).

In Italy, more than 90% of new railway viaducts included in the high-speed network were made with simply-supported prestressed concrete decks. Most of these were built with precast elements (Evangelista and Vedova 2004). Precast I-shaped and U-shaped girders, similar to the ones employed in France and in Spain, were also employed. However, other solutions were also adopted. Precast box girders, connected through a narrow strip of cast-in-place concrete, were also employed (Figure 1.2a). In some structures, such as the Piacenza viaduct, very heavy precast elements were used, with a total weight of 970 t per beam (Figure 1.2b). In this case, each span is formed by a single prefabricated element (Evangelista and Vedova 2004; Miotti et al. 2003).

Similar solutions, with very heavy prefabricated elements that form a single span, were also employed in Taiwan (Figure 1.3), with typical spans of 35 m (Fritsch 2003). The full span precast method has also been used in South Korea, with typical spans of 25 m and precast elements with a total weight of ~600 t per beam (Kauffman 2001).



*Figure 1.2 – Precast solutions for railway bridge decks in Italy:
a) deck composed by two precast box girders; b) precast bi-cellular box girder that constitutes a single span, in Piacenza viaduct (Evangelista and Vedova 2004).*



*Figure 1.3 – Railway viaducts in Taiwan, each span being made with a single prefabricated element:
a) construction with erection equipment by VSL (2010);
b) construction with erection equipment designed by Wiecon (2011).*

Precast U-shaped girders have also been used in the construction of high-speed lines in Belgium (Couchard and Detandt 2003; Kaiser 2005). In this case, inclined prefabricated columns, monolithically connected to the girders, were used in order to provide the structure with great horizontal stiffness in the longitudinal direction (Figure 1.4a). Similar solutions have also been adopted in Spain (Figure 1.4b).

In Portugal, precast solutions were also adopted in the design of viaducts for the line linking Lisbon to the Spanish border. In the section from Poceirão to Caia, continuous viaducts made with U-shaped precast girders, with typical spans of 30 m, 35 m and 40 m, were generically adopted (ELOS and LGV 2010). The total length of double-track precast viaducts in the high-speed line amounts to 8.4 km. The length of single-track precast viaducts in the conventional goods line, which goes side by side with the high-speed line along part of the way, reaches a total of 5.1 km.

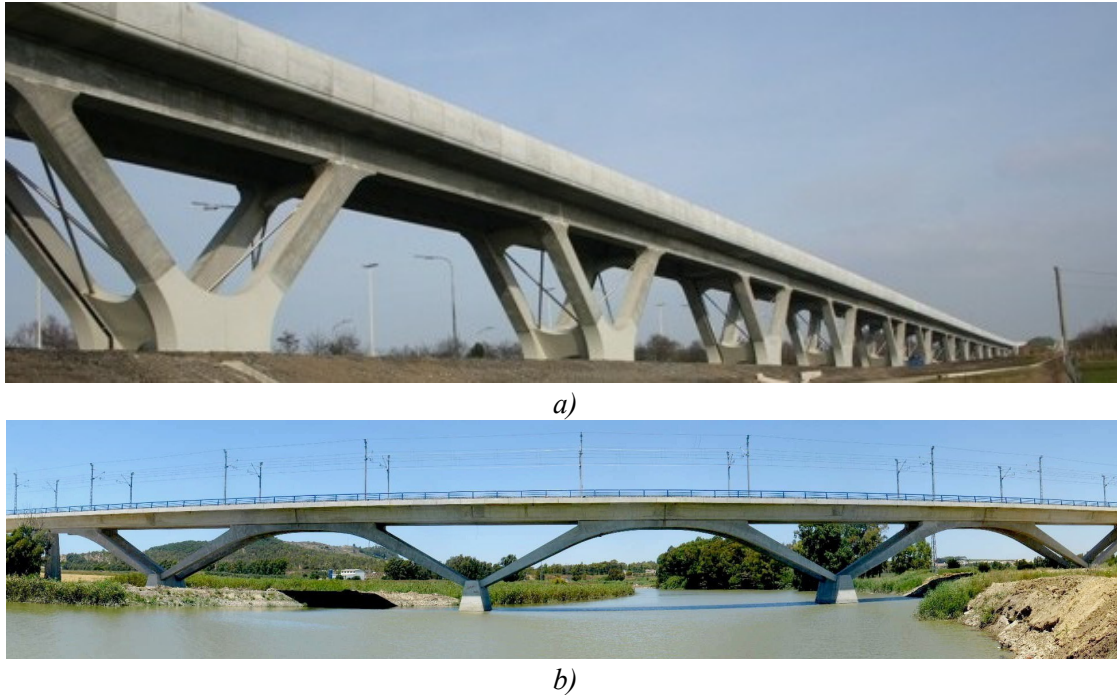


*Figure 1.4 –High-speed railway bridges made with precast U-shaped girders and inclined columns:
a) Battice Viaduct in the line linking Brussels to Cologne (Structurae 2011b);
b) Portal Viaduct in Cadiz (image gently provided by Carlos Fernandez Casado, S.L.).*

In view of the relevance of prefabricated solutions in the construction of modern railway bridge decks, and taking into account the foreseeable investments in the construction of railway lines in the near future, this thesis aims to contribute to the knowledge about the real performance of these structures, and also to give a contribution in terms of numerical methods for analysis of this type of construction.

The relevance of research devoted to the study of typical precast solutions for bridge decks was pointed out by Calavera (fib 2004). This author denotes that most publications regarding case studies and guidance on design and execution of precast bridges generally focus on the most spectacular structures, with very large spans and challenging execution conditions. However, the bulk of the bridge market concerns ordinary bridges with moderate spans. Calavera (fib 2004) also points out that the knowledge about modern precast bridges among authorities and designers is often limited, which motivates several prejudices.

In this work, special attention is also paid to continuous precast bridge decks without continuity prestress. As shown before, this construction technique has recently been employed in France and in Spain.

One of the specificities of this type of structure lies in the absence of continuity prestress in structures whose spans are, in some cases, longer than 30 m. The adoption of continuous precast decks without continuity prestress is strongly motivated by economic and construction factors. When compared to solutions with continuity prestress, the latter is usually less cost-effective and makes the construction procedure more time consuming and complex. However, if continuity prestress is not employed, cracking can occur in the vicinity of continuity supports. Cracking gives rise to modifications of the structural stiffness, and consequently, affects the structure response to the passage of high-speed trains. Therefore, the development of feasible analysis methodologies for calculation of the stiffness decrease in cracked structures subjected to cyclic loading is of interest. Moreover, crack openings must be limited to given threshold values, so that the durability of the structure is not impaired. The development of analysis methodologies for computation of residual crack openings in structures subjected to important cyclic loads is, therefore, needed.

Another specificity of bridge decks built with precast girders has to do with the reduced thickness of the various deck components, which implies that their fatigue behaviour should be analysed in detail. On the one hand, thanks to the adoption of light structures, the traffic (cyclic) loading represents a significant percentage of the total loading. As the importance of cyclic loading increases, fatigue verifications become more critical, when compared to other limit states. On the other hand, precast girders typically have thin webs, subjected to important shear forces, which makes them sensitive to the fatigue phenomenon. Moreover, girder webs are also subjected to important transverse (out-of-plane) bending moments, which must also be taken into account in fatigue analyses. In the case of continuous precast girders without continuity prestress, the occurrence of cracking gives rise to an increased relevance of the fatigue phenomenon, because higher stresses occur in cracked structures. Furthermore, a recent survey carried out by the ERRI Committee D216 (2002) concluded that there is a need for further studies into the application of various existing fatigue analysis procedures for railway bridge decks. Therefore, the development of analysis methodologies for fatigue analysis of railway bridge decks appears to be of interest. Special attention has to be paid to the combined effects of in-plane shear and out-of-plane bending in precast girder webs.

The specificities of bridge decks built with precast elements motivated extensive laboratorial experiments in France. In this country, fatigue tests were carried out at the CERIB (Chefdébien 1998) for evaluation of the fatigue behaviour of the adopted construction procedure, which comprehends continuous bridge decks without continuity prestress, built with precast I-shaped

girders. Continuous beams, made up of two precast girders with spans of 5.35 m, were subjected to cyclic loading, simulating the working life of approximately 100 years on a high-speed TGV line.

Also, precast structures undergo significant stress redistributions, owing to the modification of the structural system during construction and concrete deformations due to creep and shrinkage. The validation of numerical methodologies for calculation of long-term stress redistributions, through the comparison with experimental results obtained in real structures is, at the present time, quite limited. Therefore, the additional validation of long-term analysis procedures, by using monitoring results from the observation of real structures, is of interest.

Both the concrete delayed deformations and the girder ages at the time the continuity is established are subjected to an important variability (i.e., accurate predictions of their actual values can hardly be made at the design stage). Therefore, a significant variability is also associated to the calculated long-term stresses. For that reason, the development of analysis methodologies for calculating the variability of the structure response, taking into account the variability of concrete delayed deformations and construction timings, also appears to be of interest.

1.2 Objectives and main contributions

The main purpose of this thesis consists of the development and validation of numerical methodologies for the analysis of continuous precast railway bridge decks. Special attention is devoted to continuous structures without continuity prestress. New methodologies are proposed for:

- numerical evaluation of the stiffness decrease and residual crack openings at the continuity connection due to cyclic loading;
- fatigue analysis of girder webs subjected to in-plane shear and out-of-plane (transverse) bending.

Numerical methodologies for evaluation of long-term redistributions of stresses in continuous bridge decks built with precast girders are also validated through the comparison with experimental results.

The first part of the present work focuses on the development of a numerical model for analysis of the cyclic behaviour of reinforced and prestressed concrete elements after cracking. Various premises were taken into consideration during the conception of the model. The model should be

able to analyse the response of structures subjected to variable-amplitude load histories. Realistic constitutive models should be used to describe the cyclic behaviour in terms of stress-crack opening relationship and bond stress-slip response. The model should also provide realistic values for both the stiffness decrease and the residual crack openings in structures subjected to cyclic loading. Furthermore, the model should be feasible and robust for application in design practice.

The adopted analysis approach consists of the discretization, through the finite-differences technique, of a reinforced concrete element comprised between the cross section where a crack occurs and the closest cross section where the slip between the tensile reinforcement and the surrounding concrete is null. In the transverse direction, the cross-section is decomposed into horizontal layers. Applied actions can include axial forces and plane bending. The influence of concrete shrinkage is taken into account.

The adopted constitutive models to describe the bond stress-slip behaviour and the normal stress-crack opening relationship consist of modified versions of models reported in the literature. Modifications were introduced in order to solve convergence difficulties, and also to improve the agreement with experimental results.

The developed model was validated through the comparison with experimental results available in the literature for prestressed concrete ties and reinforced concrete beams subjected to cyclic loading.

The second part of this thesis focuses on the fatigue behaviour of reinforced and prestressed concrete railway bridge decks.

Alternative procedures for fatigue analysis of railway bridge decks are presented. The specificities and the field of application of each analysis procedure are presented, aiming at contributing to the knowledge about the fatigue behaviour of railway bridge decks and about methodologies for fatigue analysis of this type of structure. Software for fatigue analyses according to the damage accumulation method (the most detailed analysis procedure among the ones considered in this work) was developed, so that the fatigue damage associated to any traffic scenario can be calculated. A feasible procedure for taking into account the lateral degradation of train axle loads through rails, sleepers and ballast was also implemented.

One of the specific objectives of this work consists of the development of a numerical methodology for fatigue analysis of girder webs, considering the combined effect of in-plane shear and out-of-plane (transverse) bending. This task is considered to be relevant owing to the

limited amount of information available in the literature about this theme, and also owing to the reduced thickness of typical precast girder webs (which makes them sensitive to fatigue). The developed methodology is based on the damage accumulation method, and can, therefore, be used to calculate the fatigue damage caused by any traffic scenario. A specific algorithm for calculation of the stress history applied to the stirrups and to the concrete compression struts was conceived. This methodology was validated through the comparison with experimental results reported by Gaspar (2003).

The third part of the present thesis focuses on the long-term behaviour of continuous bridge decks built with precast girders. The main purpose of this task consists of the validation of numerical methodologies for calculation of long-term redistributions of stresses and bending moments. This validation is carried out by comparing numerical calculations with experimental results observed in a real viaduct. The structure under analysis is the South Access Viaduct to the Lezíria Bridge in Portugal, which has been monitored since the beginning of the construction. This experimental work has been carried out by NewMENSUS (2011) and LABEST (2011). The specific objectives of this task comprehend:

- the evaluation of the adequacy of finite-element viscoelastic phased analyses to predict the real behaviour of this kind of structure;
- the evaluation of the errors associated to simplified analyses based on limited information about the concrete properties and the sequence of construction;
- the evaluation of the adequacy of probabilistic analyses based on Monte Carlo simulations to calculate tolerance intervals that contain the actual results (stresses, strains or bending moments) with a certain confidence level, taking into account the variability of creep, shrinkage and construction timings;
- contribute to the knowledge on the long-term behaviour of this kind of structure.

In the last part of this thesis, the developed and validated analysis methodologies are applied to the analysis of a real precast railway bridge deck, included in the Spanish line linking Alcácer to Valencia. This structure is composed of four continuous spans (interior spans of 33.5 m and end spans of 26.5 m) without continuity prestress. The main purposes of this task are as follows:

- to illustrate the application of the developed and validated numerical methodologies, which allow the detailed analysis of critical aspects of the structural behaviour of this type of structure;

- to contribute to the knowledge about the structural behaviour of this bridge type.

1.3 Organization of the text

This thesis is divided into seven chapters, whose content is briefly summarized in the following paragraphs.

The present **Chapter 1** presents the main motivations for the development of this thesis, describes its main objectives, and outlines the organization of the text.

Chapter 2 is dedicated to the numerical analysis of the cyclic behaviour of reinforced and prestressed elements after cracking. This chapter starts with a review of constitutive models to describe the bond stress-slip relationship and the normal stress-crack opening relationship in structures subjected to cyclic loading. Then, it presents the state of the art in terms of numerical models for analysis of cracked, reinforced and prestressed concrete elements subjected to cyclic loads. After that, the developed numerical methodology is described. Finally, the validation of that methodology is made through the comparison with experimental results reported in the literature.

Chapters 3 and 4 are dedicated to fatigue analyses of railway bridge decks.

Chapter 3 starts with a brief presentation of the fatigue behaviour of concrete, steel and reinforced concrete structures. Then, a critical review of alternative methodologies for fatigue analysis of railway bridge decks is presented. Special attention is paid to bridge decks included in high-speed lines.

Chapter 4 focuses on the fatigue analysis of girder webs subjected to in-plane shear forces and out-of-plane (transverse) bending moments. Firstly, the adequacy of truss models for stress calculations in fatigue analyses of beams subjected to in-plane shear is assessed by comparing numerical calculations with experimental results reported in the literature. Then, the developed procedure for stress calculations considering the combined effect of in-plane shear and out-of-plane bending is described. Then, this procedure is validated through the comparison with experimental results. Even though this chapter focuses on fatigue analyses, available methodologies for ultimate limit state analyses (considering the interaction between in-plane shear and transverse bending) are identified. The applicability of the developed methodology to problems of shear between web and flanges is also discussed.

Chapter 5 is dedicated to the analysis of the long-term behaviour of precast continuous bridge decks, considering the influence of concrete delayed deformations and phased construction. A recently constructed viaduct is adopted as a case study. This chapter starts with the description of

the structure and the monitoring system. Then, the experimental results are compared with the ones calculated by means of finite-element analyses. The results of calibrated analyses – in which the material properties and the sequence of construction are defined taking into account data gathered during the construction – are compared with “blind” analyses – based only on the information that is usually available at the design stage. Finally, the variability of the structure response, due to the uncertainty associated to the quantification of concrete delayed deformations and construction timings, is assessed by means of a probabilistic analysis.

In **Chapter 6**, the previously developed numerical methodologies are applied to the analysis of a real railway bridge deck built with precast U-shaped girders. The structure is described, and the external loads considered in this study are presented. Then, the long-term redistributions of stresses and internal forces are evaluated. Following this, the fatigue behaviour of both the girder webs and the deck slab is analysed and discussed. Finally, the consequences of cracking, owing to the absence of continuity prestress, are discussed.

Chapter 7 presents the main conclusions of this work, and summarizes some future research topics.

Chapter 2

Numerical analysis of the cyclic behaviour of RC elements after cracking

2.1 Introduction

Stiffness is one of the parameters that most affect the dynamic behaviour of railway bridge decks. Even though the maximum mid-span accelerations are not significantly affected by the stiffness of the structure (ERRI D214 1999b), the dynamic displacements and the dynamic enhancement of internal forces depend considerably on the structural stiffness (ERRI D214 1999b). Moreover, the critical speeds at which resonance effects occur also depend on the natural frequencies of the structure which, in turn, are influenced by its stiffness.

Therefore, the accurate prediction of structural stiffness plays an important role in the analysis of railway bridge decks, particularly in the case of structures that are not fully prestressed, in which the admissible cracking frequently causes significant reductions in structural stiffness.

The ERRI Commission D214 (1999a) conducted a survey on the different methods in national codes for calculating bridge stiffness and found different practices for taking into account the stiffening effect of concrete in tensile zones. Based on this survey, this commission identified the following research needs:

- the development of methods for calculation of the stiffness of cracked reinforced concrete in slabs;
- the development of methods for calculation of the stiffness of composite bridge decks in hogging zones over intermediate piers;

- the understanding of the differences in the structural response for short term static loadings and the behaviour at frequencies corresponding to the dynamic response, for different loading amplitudes.

Crack openings are also significantly affected by traffic loading (Rotilio 1998). In the case of railway bridge decks, traffic loads have important values, and consequently, residual crack openings (after removal of live loads) can be significantly affected by the maximum applied live loads. Therefore, cyclic loading should be considered in the calculation of crack openings for durability verifications, particularly in the case of reinforced concrete (RC) structures sensitive to these effects (e.g. continuous railway bridge decks without continuity prestress). However, the Eurocodes fail to do that, as will be shown in Chapter 6. Feasible numerical tools for taking into account the influence of live loads in the calculation of residual crack openings are therefore needed.

The aforementioned reasons were the motivations for the development of a numerical methodology for the analysis of the cyclic behaviour of reinforced concrete members after cracking, which was, in turn, validated through the comparison with experimental results. The methodology was devised so that it could comply with the following principles:

- it should be able to analyse the structural response under the effect of a load history composed by variable-amplitude load cycles;
- realistic constitutive models should be used to describe the concrete cyclic behaviour, with respect to the stress-crack opening and the bond stress-slip relationships; these models should correctly define the non-linear unloading and reloading trajectories of the material behaviour;
- the model should provide realistic values for both the stiffness decrease due to cracking and the residual crack openings due to permanent and variable loads;
- the feasibility and the usability of the model should be a matter of concern, so that this methodology could be used in practice.

The proposed analysis approach consists of the discretization, through the finite differences technique, of a RC element comprised between a cross section where a crack occurs and the closest cross-section where the slip between the tensile reinforcement and the surrounding concrete is null (at the stabilized cracking stage, this cross-section is located halfway between two consecutive cracks). The solution is found iteratively by employing the bisection method. In the transverse direction, the cross-section is decomposed into horizontal layers. The influence of concrete shrinkage is taken into account. This methodology was based on the work of several researchers, identified below, who adopted the same type of model with different purposes. The adopted constitutive models were based on the literature review that is summarized below, too.

Special attention was devoted to the cyclic bond stress-slip behaviour (§2.2) and the cyclic behaviour of concrete in tension (§2.3).

Even though finite-element codes are a powerful tool for the analysis of RC structures, an alternative numerical approach was followed in this work for the following reasons:

- non-linear finite-element analysis software, including realistic constitutive models to describe the concrete cyclic behaviour (strain-crack opening and bond stress-slip relationships), are not usually available for practitioners; it should be noted that the material behaviour upon unloading is usually simulated in finite-element (FE) codes through a linear secant unload through the origin, which is an inadequate assumption for the applications envisaged in this work;
- even in the cases where this type of model is available, its application is not feasible in most practical situations, due to their complexity and amount of time required for modelling and analysis, and also due to convergence difficulties.

Thus, the main purpose of the work presented in this chapter consists in the development of a numerical model for evaluation of the mechanical behaviour of cracked RC members subjected to cyclic axial and bending forces with variable amplitude. This model is expected to be particularly useful for the prediction of the stiffness decrease after cracking, as a function of the applied load level, as well as for the determination of residual crack openings considering the influence of variable loads. This chapter presents:

- the state of the art in terms of constitutive models for the definition of the bond stress-slip relationship and normal stress-crack opening relationship;
- the state of the art in terms of numerical models for the analysis of cracked RC members subjected to cyclic loading;
- the developed numerical methodology;
- the validation of both the developed methodology and the employed constitutive models through the comparison with experimental results.

2.2 Bond between steel and concrete

2.2.1 Bond mechanics

The transference of forces between steel reinforcements and the surrounding concrete (for forces in the direction of the reinforcement axis), in RC or prestressed concrete (PC) structures, is assured by bond stresses. Due to this force transfer, the axial force installed in the reinforcement changes along its length, as does the resultant force in the concrete. Bond stresses are

accompanied by relative displacements between the steel and the concrete (in the same direction). The relative displacement, also called slip, is usually measured with reference to the undisturbed concrete (see Figure 2.1), and consists of two parts, the relative slip at the interface (between steel and concrete) and the shear deformations of the concrete (fib 2000).

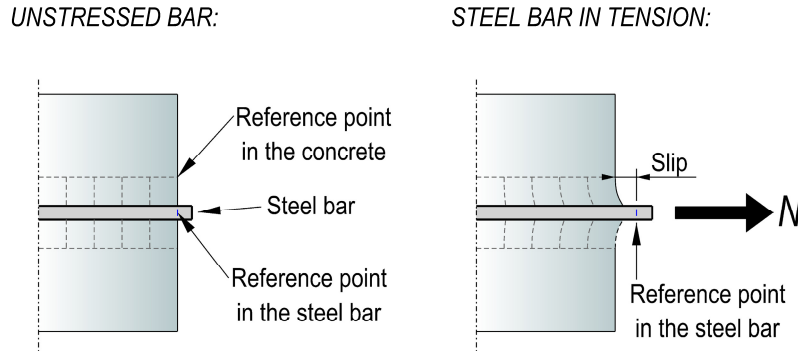


Figure 2.1 – Definition of the relative bar displacement, or slip.

The interaction between the concrete and a ribbed bar subjected to a pull-out force is characterized by the four stages indicated in Figure 2.2 (fib 2000), which represents the local relationship between the bond stress, τ , and the slip, s . The mechanisms of bond resistance for each of the phases are explained below.

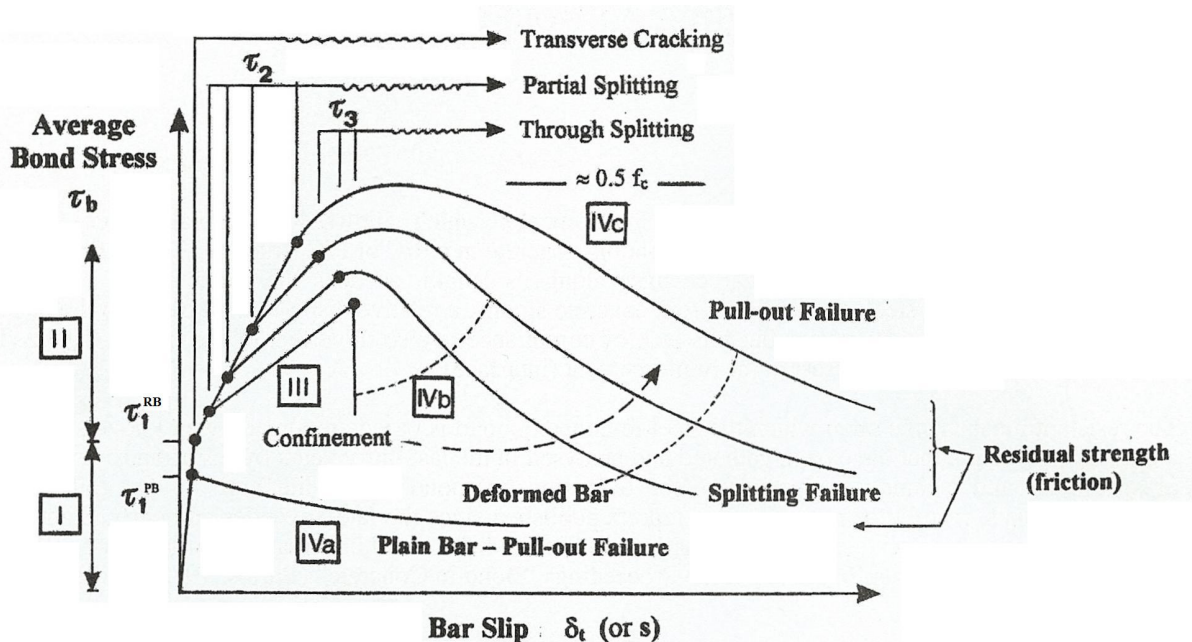


Figure 2.2 – Schematic representation of the local bond stress-slip law. Adapted from fib (2000).

Stage I ($\tau \leq \tau_1 = 0.5 \text{ to } 0.8 f_{ct}$, where f_{ct} represents the axial tensile strength of concrete)

For low stress values, bond is assured by chemical adhesion and micromechanical interaction associated with the microscopically rough steel surface. During this stage, no relative slip at the interface occurs. Thus, slip in stage I is a consequence of the shear deformations in the concrete. There is no cracking and the behaviour is linear-elastic. In the case of ribbed bars (RB), high localized stresses arise close to rib tips.

Stage II ($\tau_1 < \tau \leq \tau_2$)

For higher bond stresses, the chemical adhesion breaks down, transverse micro cracks originate at the rib tips and large bearing stresses occur in the contact between the ribs and the concrete (Figure 2.3). By injecting ink between the bar and the surrounding concrete, Goto (1971) found secondary cracks radiating from each rib (Figure 2.4). He also observed experimental evidence of the separation between the bar and the concrete in the vicinity of the primary crack. When the concrete is separated from the bar, the circumference of the concrete surface, previously in contact with the bar, increases. Hence, circumferential tensile stresses are induced. The emergence of such stresses can also be explained by equilibrium considerations: the resulting inclined compression strut (D in Figure 2.5) is equilibrated by a concrete ring subjected to circumferential tensile stresses (Figure 2.5). However, in stage II, its value is not enough to cause concrete splitting.

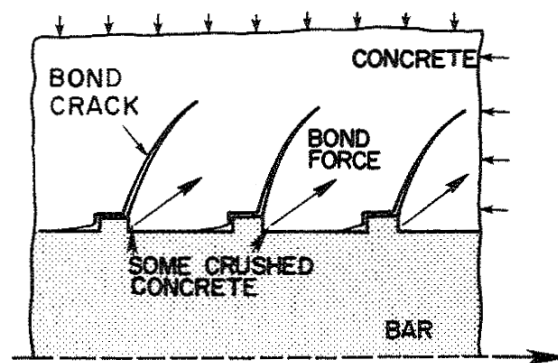


Figure 2.3 – Mechanism of bond resistance for monotonically increasing loading in Stage II (Eligehausen et al. 1983).

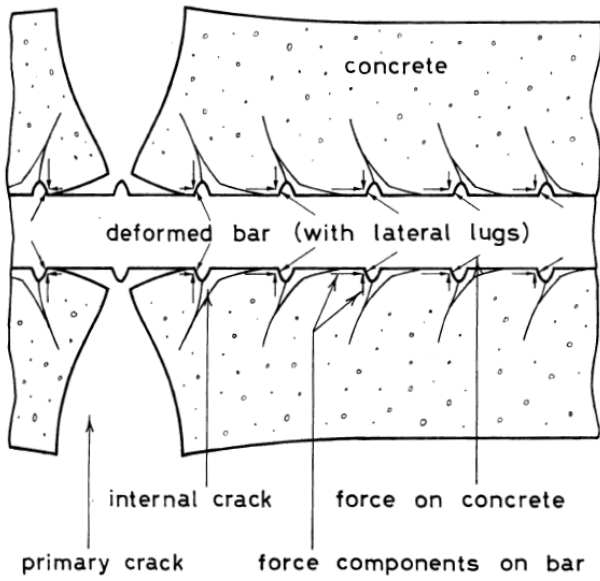
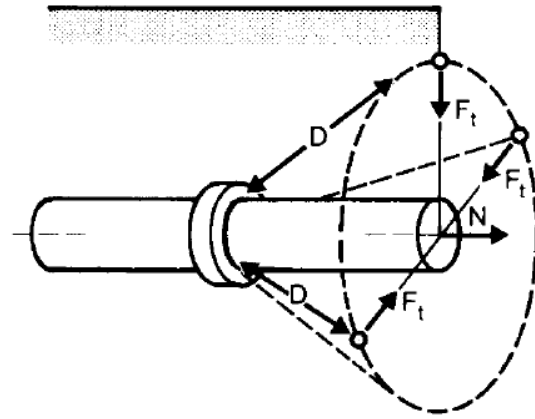


Figure 2.4 – Schematic diagram of the deformation of concrete around reinforcing steel after formation of internal cracks (Goto 1971).



F_t = Radial tension force

D = Compression force

Figure 2.5 – Compression struts and tension ring around a steel bar in tension (Laurencet 1999).

Stage III ($\tau > \tau_2 = 1 \text{ to } 3 \cdot f_{ct}$)

For higher bond stress values, longitudinal splitting cracks spread radially (Figure 2.6), owing to the circumferential tensile stresses. More slip occurs because more local crushing takes place and shear cracks in the concrete keys between ribs are initiated (Figure 2.7).

In the case of light transverse confinement, this stage ends when concrete splitting reaches the outer surface of the concrete member, that is, when through splitting occurs. In this case, this stage is followed by a more or less sudden failure, depending on the transverse confinement (stage IVb in Figure 2.2). However, in relatively-long anchorages with moderate confinement a pull-out failure usually occurs even after through splitting, owing to the simultaneous presence of different stages along the bonded length. In this case, the failure is defined as “splitting-induced pull-out failure” (fib 2000).

In the case of heavy transverse reinforcement or concrete cover, through splitting is prevented by their confining action, and concrete splitting is restricted to a cracked cylinder around the bar.

The end of stage III is usually not reached in RC or PC structures under service loads (fib 2000; Laurencet 1999). Therefore, the behaviour beyond the end of stage III is not of interest within the scope of the present work.

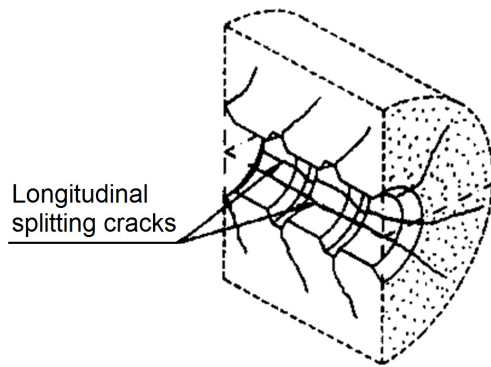


Figure 2.6 – Splitting and transverse cracks. Adapted from fib (2000).

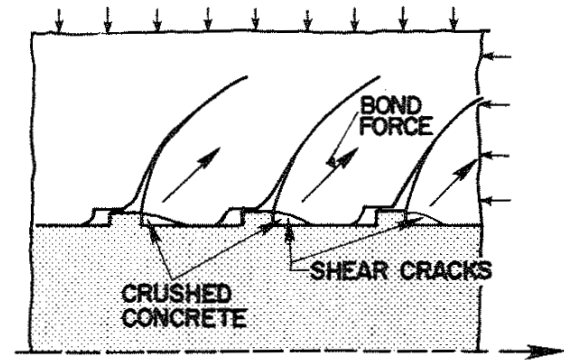


Figure 2.7 - Mechanism of bond resistance for monotonically increasing loading in Stage III (Eligehausen et al. 1983).

Stage IV

Three situations may occur, depending on the degree of confinement:

- In the case of plain bars (PB) this stage immediately follows the breakage of adhesive bond (stage IVa in Figure 2.2). Force transfer strongly depends on the transverse pressure.
- In the case of lightly confined ribbed bars, splitting cracks break out through the whole cover and bar spacing, and the bond tends to fail abruptly (see stage IVb in Figure 2.2). This type of rupture is seldom observed in real structures (Laurencet 1999).

A sufficient amount of transverse reinforcement can assure bond efficiency in spite of concrete splitting. According to fib (2000), bond stress values as high as $1/3$ to $1/2 \cdot f_c$ (where f_c represents the compressive strength of concrete) can be developed at this stage, together with high slip values ($s/d_b > 0.05$, where s represents the bar slip and d_b represents the bar diameter).

After the peak strength, significant bond strength remains even at very large slip values. Gambarova et al. (1996) obtained $\tau/f_c = 0.15$ to 0.30 at slip values $s/d_b = 1/3$, with $d_b = 18$ mm.

- In the case of ribbed bars confined by heavy transverse reinforcement, through splitting does not occur and bond failure is caused by bar pull-out (stage IVc in Figure 2.2). The transfer mechanism changes from rib bearing to friction. Gradual shearing off of the concrete keys cast between each pair of ribs occurs (Eligehausen et al. 1983), as represented in Figure 2.8. The interface is smoothed, leading to a decrease of bond stress.

Cairns (2003) established minimum values for both the quantity of transverse reinforcement and the concrete cover that prevent splitting failures. Different criteria are proposed by the MC90 (CEB 1993), which also takes into account the confining effect of transverse pressure and the interaction between neighbouring bars. Closely spaced bars create a more adverse situation in terms of concrete splitting than a single bar (ACI Committee 408 1966). Furthermore, the decrease of concrete toughness (associated with the increase of the concrete compressive strength) leads to a tendency to more brittle failures and can have major implications in terms of splitting failures, as pointed out by Cairns (2003).

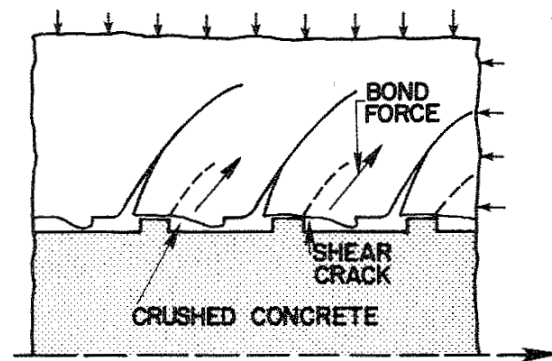


Figure 2.8 - Mechanism of bond resistance for monotonic loading in Stage IV, for highly confined ribbed bars (Eligehausen et al. 1983).

As mentioned before, Stage IV is usually not reached in service. It is usually attained for higher loads, close to the rupture.

2.2.2 End effect

Near the end of a loaded specimen (or near to a primary crack in a tension member), the transference of bond stresses between steel and concrete is impaired by the effect of secondary transverse cracks. These cracks have a conic shape and cause a local punching shear failure if their propagation extends up to the free surface of the concrete (see Figure 2.9).

The end effect becomes evident in the experimental results obtained by Kankam (1997), which are depicted in Figure 2.10. This author tested 200 mm long specimens, reinforced with a 25 mm bar, which was internally instrumented with electrical resistance strain gages. He concluded that, for the same slip, lower bond stresses are obtained near the end of the load specimen. Similar conclusion had been obtained by Nilson (1972).

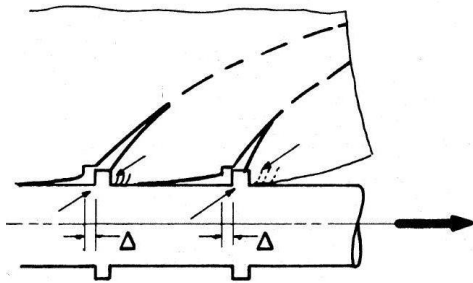


Figure 2.9 – Scheme illustrating the end effect (Viathanatepa et al. 1979).

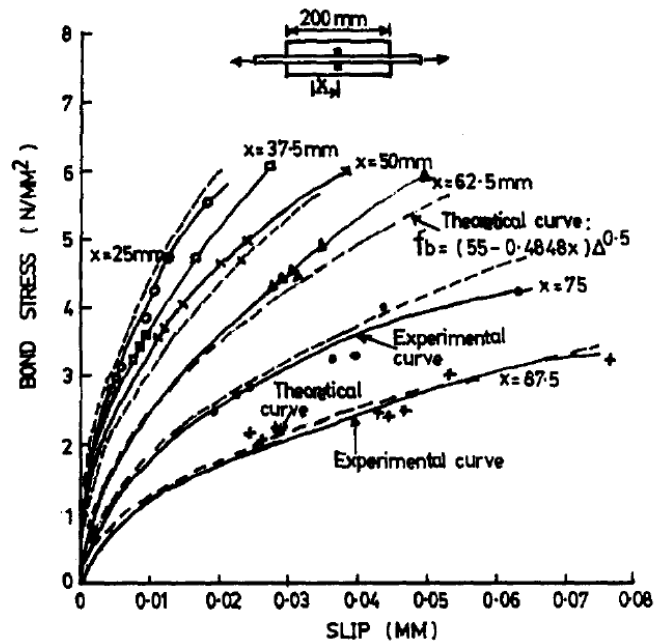


Figure 2.10 - Bond stress-strain relationship at different positions along a bar (Kankam, 1997).

The end effect can be taken into account in analytical calculations by means of a decrease of the bond stress for those parts of the reinforcing bar which are close to a primary crack, as suggested by the MC90. However, this effect is often disregarded in analytical calculations. In fact, its influence on the global response of a RC tie is rather small, when averaged over a longer bond length. According to Ruiz et al. (2007), the influence of this phenomenon can be neglected as long as the bar remains in the elastic domain.

It is also important to remark that the local bond stress-slip relationship can be determined through inverse analyses based on normal force-crack opening relationships observed in RC ties. This was the procedure followed by Farra (1995), who derived the local bond stress-slip relationship that, once applied to all the positions along the bar length (i.e., not considering the end effect) gave rise to the best agreement with the observed crack openings. Therefore, no modification to account for end effects must be considered when bond stress-slip relationships derived this way are considered.

2.2.3 Factors influencing bond performance

The maximum force that can be transmitted by bond is usually defined, in design codes, taking into account the influence of a large variety of factors. The MC90 e.g. considers explicitly the influence of 10 parameters in the calculation of anchorages of reinforcing bars by bond action: bar type, bar shape (straight, bent or hooked), bar diameter, casting position, minimum cover,

confining reinforcement, transverse pressure, number of bars in a bundle, proportion of reinforcement lapped at a section, and concrete strength. Nevertheless, bond performance also depends on additional factors, such as: the bar surface geometry [usually characterized by the relative rib area, defined by the ISO 15630-1 (ISO 2002a)], the state of stress in the concrete, bar corrosion, bar rusting owing to steel exposure to the environment and load history (load repetition, rate, frequency and long-term action). A comprehensive description of the influence of the different factors can be found in fib's state-of-the-art report (fib 2000).

The definition of the ascending branch of the bond stress-slip relationship for numerical evaluation of the service behaviour (deformations and crack openings) is usually quantified as a function of a more restricted set of factors as can be seen in §2.2.5, where different proposals for this relationship are presented. The commonly considered factors are the concrete strength and the casting position (whose influence is accounted for in a simplified manner, by considering two or more bond conditions). Some proposals also take into account the relative rib area of the reinforcements, as explained by Farra (1995). Proposals for taking into account the influence of cyclic loading and long-term effects are also available, as described in §2.2.7 and in §2.2.8. Given that only a limited number of factors are considered in the definition of bond stress-slip laws, the quantification of these relationships is subjected to an important variability.

2.2.4 Tests for evaluation of the bond stress-slip relationship

The most relevant test types and the variants in terms of bonded length are presented in §2.2.4.1 and §2.2.4.2.

2.2.4.1 Form of test

Bond tests may be divided into two major categories, according to their generic form: the transfer type, and the anchorage type.

The **transfer form of test** (Figure 2.11a) consists of applying a tension force to both ends of a steel bar embedded in concrete. It is usually employed in experimental studies to quantify crack openings and stiffness reductions after cracking. In these tests, the force transferred from the reinforcing bar to the concrete can be indirectly evaluated by measuring the distribution of axial strains in the rebar along the bonded length. However, if strains were measured by gages bonded to the bar surface, the bond conditions would be affected by the measurement system. Therefore, an alternative technique to avoid this interference was developed. This technique was pioneered by Mains (1951) and consists of, firstly, obtaining two half bars by longitudinally sawing a steel bar. Then, one of the halves is milled to provide a channel where the strain gages and the wiring are installed. After the installation, the two parts are glued or welded together forming a bar

which has an exterior almost identical to a standard solid reinforcing bar. The variation of bond stress along the length of the bar, during the tests, can be quantified based on the variation of steel strains. A similar methodology was employed by authors such as Bresler et al. (1968), Nilson (1972), Kankam (1997) and Scott and his co-workers (Judge et al. 1989; Judge et al. 1990; Scott 1996; Scott and Gill 1987). The latter managed to install up to 100 gages in a 4 mm × 4 mm channel milled in 12 mm bars.

Even though the internal installation of strain gages avoids the degradation of the bond characteristics around the surface of a bar, it does not avoid the modification of the bar characteristics (namely its cross-section). Recently, a new technique to measure the strain distribution along a reinforcing bar was introduced by Nellen et al. (2002) and Kenel et al. (2005), using optical fibre Bragg grating sensors. These authors equipped a reinforcing bar with an optical fibre with a total of 286 Bragg gratings spaced 10.4 mm apart. The fibre was epoxied into a 1-mm-wide by 1-mm-deep groove at a rib-free side of the reinforcing bar. It was found that the sensors were able to measure large strains and strain gradients with high precision.

The alternative to the transfer form of test is the **anchorage form**. This category comprehends two specific test types: pull-out tests (Figure 2.11b) and beam tests (Figure 2.11c). These tests are more suited to assessment of anchorage strength (Cairns and Plizzari 2003).

Pull-out tests are widely employed because they are economical to manufacture and test and because they clearly represent the concept of anchoring a bar. The two principal objections pointed to these tests are (Cairns and Plizzari 2003):

- the fact that tension in the reinforcing bar is balanced by compression in the concrete (a situation not encountered in practical circumstances, which prevents the development of transverse cracks, and consequently, influences the bond strength);
- the adopted cover to the reinforcement, which is usually larger than that found in real structures, which prevents through splitting and affects the bond strength.

Even though RILEM has proposed a standard setup for pull-out tests (CEN 2005c; Gamski 1973), several different layouts are reported in the literature. It is worth mentioning the ring test, which was devised to study the influence of the normal stress at the interface between the steel bar and the surrounding concrete (fib 2000). In these tests, a pull-out cylindrical concrete specimen is involved by a steel ring, i.e., a steel pipe, where the circumferential strain is measured. Another variant was employed by Gambarova et al. (1996), aiming at quantifying the unfavourable effects of concrete splitting. These authors used presplit specimens, where artificial splitting cracks are induced before starting the test.

The **beam test** (Figure 2.11c) has the advantage of introducing tension both in the bar and in the concrete. Moreover, it provides a better simulation of the bar position in a beam. However, the specimen requires considerable confining reinforcement to avoid shear failure, and so splitting failures are unlikely when small bar diameters are tested. Another shortcoming of this form of test lies in its manufacture, which, according to several authors, raises unnecessary complexity (Cairns and Plizzari 2003). According to fib (2000), the usefulness of this test is limited to the comparison among bars having the same nominal diameter and different deformation patterns (bar surface characteristics). This approach is adopted by the EN 1992-1-1 (CEN 2004), Annex C, which suggests a feasible procedure for acceptance of surface characteristics, based on the results of beam tests, for bars that do not meet the minimum requirements in terms of relative rib area.

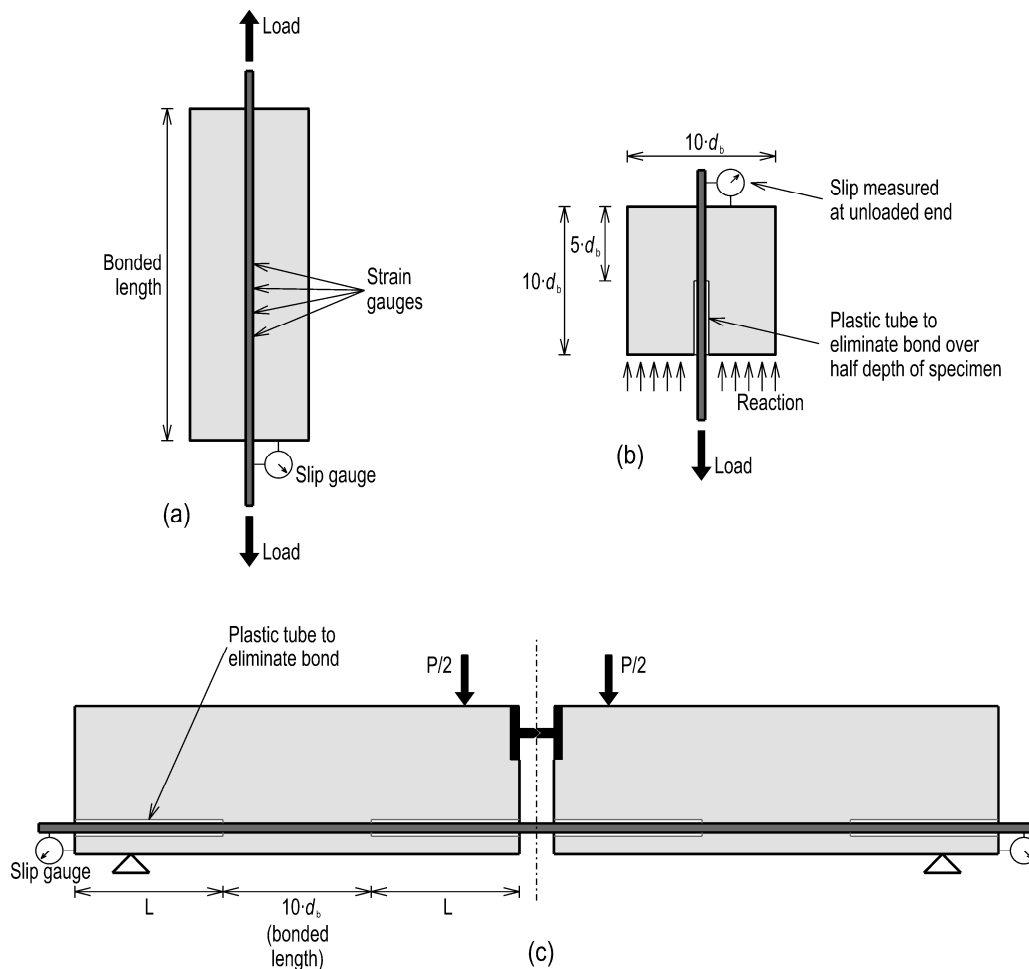


Figure 2.11 – Generic forms of bond test specimens:
(a) transfer type; (b) pull-out; (c) beam.

2.2.4.2 Bonded length

Short bonded lengths are commonly used in tests devised to evaluate the local bond stress-slip relationship (needed for numerical analyses of bond problems). Obtaining the local bond stress-slip relationship requires that both the bond stress and the slip are as uniformly distributed as possible. The most common test type used with this purpose is the pull-out test (despite its aforementioned shortcomings), and the bonded length is usually not larger than $5 \cdot d_b$. Authors such as Oh et al. (2007), Huang et al. (1996), Balázs (1991) and Morita and Kaku (1973) employed even shorter bonded lengths, up to $2 \cdot d_b$. The bond stress is then determined by assuming a uniform distribution along the bonded length. However, Windisch (1985) noted that, even for bonded lengths not larger than $5 \cdot d_b$, the unloaded-end slip is different from the loaded-end slip, i.e., slip varies along the bonded length. Therefore, the loaded-end slip should also be measured and taken into account in the derivation of bond stress-slip relationships. As an alternative, this author suggests that the local bond stress-slip relationship can be determined by using only the unloaded-end slip if the bonded length is not higher than 50 mm. Windisch (1985) also points out that the bonded length must not be too small (e.g. one rib distance), since large scatter was observed in tests with such short bonded-length values.

Long bonded lengths are usually employed to verify that bars can develop an anchorage force equal to the design strength of the bar within a specified bonded length. Long bonded lengths are also employed in the transfer form of test, for experimental evaluation of deformations under service loads. As mentioned before, the internal strain gauging of the reinforcement makes it possible to indirectly evaluate bond stresses along large bonded lengths. This procedure avoids the aforementioned disadvantages of pull-out tests.

The local bond stress-slip relationship can also be determined by means of crack opening-axial force relationships measured in RC ties (this being also a test with long bonded length), even if steel strains are not measured. As mentioned before, this was the procedure followed by Farra (1995).

2.2.5 Bond stress-slip relationship for monotonically increasing loading

The relationship between local bond stress and local slip under monotonic loading follows a non-linear ascending tendency before the peak bond stress is reached. A descending branch occurs for larger slip values, owing to the occurrence of splitting cracks along the bars. When the service behaviour of structures is under analysis, only the ascending branch is relevant, as the peak bond stress typically occurs for slip values around 0.5 to 1 mm, which corresponds to crack openings between 1 and 2 mm. For this reason, only the ascending branch of the relation is discussed in this section. Moreover, only ribbed bars are dealt with.

The most common form of the ascending branch of the bond stress-slip law is given by the following power law (Farra 1995):

$$\tau = \tau_{\max} \cdot \left(\frac{s}{s_p} \right)^b \quad (2.1)$$

where τ is the bond stress, s is the slip, s_p is the slip at the peak bond stress $\tau_{\max}$, and b is a dimensionless constant that defines the shape of the curve. Equation (2.1) may also be written as:

$$\tau = a_1 \cdot f_{cm}^{a_2} \cdot \left(\frac{s}{s_p} \right)^b \quad (2.2)$$

where f_{cm} represents the mean value of concrete cylinder compressive strength and a_1 and a_2 are two dimensionless constants. Equation (2.2) is not independent from the units used to express the values of τ and f_{cm} , as different units imply different values for the parameters a_1 and a_2 . In this work, this equation is always used with τ and f_{cm} in MPa.

Table 2.1 summarizes the proposals of different authors for the parameters that define the bond stress-slip relationship. It can be observed that all the proposals suggest a value $s_p = 1$ mm for ordinary concrete. The exception to this rule is the MC90 proposal for unconfined concrete, according to which the ascending branch of the bond stress-slip relationship ends at $s = 0.6$ mm, owing to the risk of concrete splitting.

Some models reflect the fact that the stiffness of the ascending branch is higher for good bond conditions (the lower the parameter b , the higher the slope of the τ - s law). According to the MC90, good bond conditions may be assumed for:

- bars whose inclination with respect to the horizontal lies in the interval of 45° to 90°;
- bars with an inclination less than 45° to the horizontal which are up to 250 mm from the bottom or at least 300 mm from the top of the concrete layer during concreting.

Figure 2.12 depicts the results provided by the different models presented in Table 2.1, for ordinary concrete with $f_{cm} = 38$ MPa. A significant scatter is observed, which results from the variability of such a complex phenomenon represented as a function of such a limited number of influencing parameters. The observed scatter is also a consequence of different experimental techniques used in the derivation of the presented models. Therefore, the existing models are not capable of accurately predicting the real bond stress-slip relationship for each individual situation. In order to cope with the variability of this phenomenon, the MC90 suggests that the

mean bond stress-slip relationship indicated in Table 2.1 should be associated to a coefficient of variation of approximately 30%.

Table 2.1 - Parameters defining the local bond stress-slip relationship.

Bond stress-slip model		s_p (mm)	a_1	a_2	b
Noakowski (1988)	Good bond conditions	1	0.95	0.66	0.12
	Poor bond conditions		0.8		0.2
MC90 ⁽¹⁾	Good bond conditions	1	2.5	0.5	0.4
	Poor bond conditions		1.25		
König and Bergner (1994)	Ordinary concrete	1	0.29	1	0.3
	High performance concrete		0.16		0.2
Farra (1995)	Good bond conditions	1	0.42	0.95	0.2
	Average bond conditions		0.4		0.3
	Poor bond conditions		0.3		0.35
CEB (1995) ⁽²⁾		1	0.22	1	0.21
Huang et al. (1996)	Ordinary concrete	1	0.45	1	0.4
			0.225		
	High performance concrete	0.5	0.45		0.3
			0.225		

Notes: ⁽¹⁾ f_{ck} should be used instead of f_{cm} in Equation (2.2).

In the case of unconfined concrete (where failure is assumed to be governed by splitting of the concrete) the ascending branch ends at $s = 0.6$ mm.

⁽²⁾ Valid for any bond conditions.

Several authors (Farra 1995; Laurencet 1999) argue that the MC90 approach [based on proposals by Eligehausen et al. (1983)] yields underestimated bond stresses, specially for poor bond conditions or in the case of high strength concrete. Therefore, the actual average of the statistical distribution of bond stresses might not be coincident with the so called mean values indicated in Table 2.1. Laurencet (1999), e.g., suggests that the model proposed by the CEB (1995) should be adopted in analytical studies, for confined concrete, any bond conditions and concrete strength lower than 100 MPa.

In this work, the variability of the bond action is taken into account by carrying out parametric analyses considering alternative values for the bond stress-slip relationship.

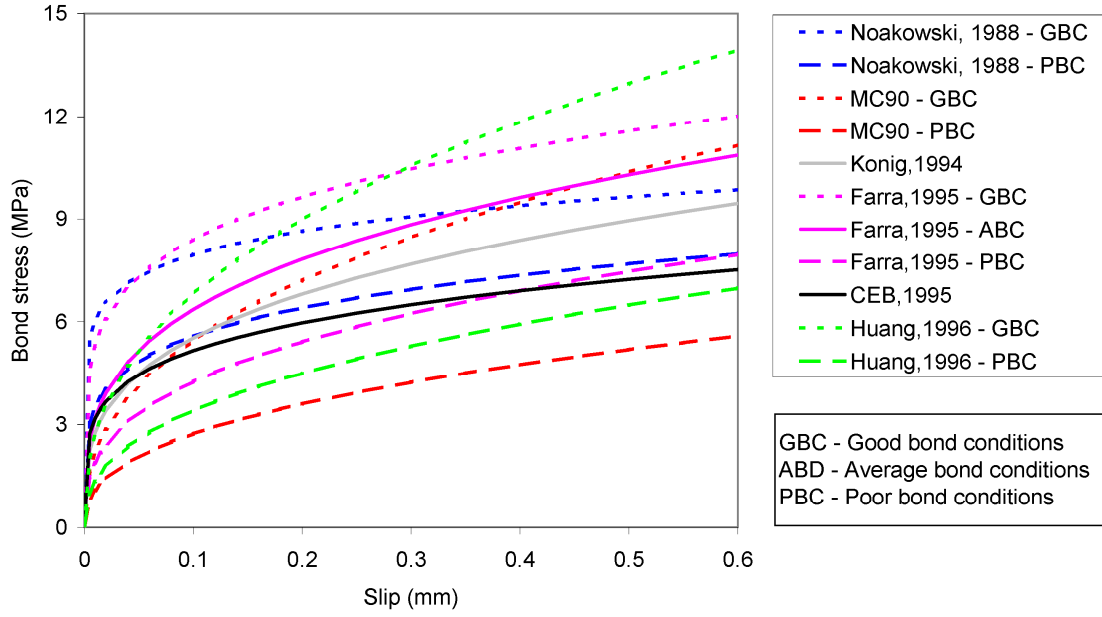


Figure 2.12 - Different proposals for the ascending branch of the bond stress-slip relationship for monotonic loading, and for normal strength concrete ($f_{cm} = 38 \text{ MPa}$).

2.2.6 Calculation of crack width based on the bond stress-slip relationship

The bond stress-slip relationship can be used to predict the mechanical behaviour of a RC tie, in terms of deformations, stress distributions and theoretical crack widths. That can be done by establishing the equilibrium and the compatibility conditions in a small part of a RC tie in the vicinity of a crack, as well as the corresponding boundary conditions.

The example of a RC tie during the crack formation phase is assumed in this analytical derivation, and the tensile force N at the crack is assumed to be totally supported by the reinforcing bar. This force is progressively transmitted from the steel to the surrounding concrete by bond stresses along the transmission length L_r . At the distance L_r from the crack ($x = 0$ in Figure 2.13) the slip equals zero, and the RC tie is in the uncracked phase (state I, with $\varepsilon_s = \varepsilon_c$). The slip increases along the transmission length and reaches the maximum value at the crack section. The crack opening, w , is given by:

$$w = 2 \cdot s_{(x=L_r)} \quad (2.3)$$

By imposing the equilibrium of an infinitesimal element along the transmission length, we have:

$$d\sigma_s(x) \cdot A_s = -d\sigma_c(x) \cdot A_c = n \cdot \pi \cdot d_b \cdot \tau(x) \cdot dx \quad (2.4)$$

where σ_s and σ_c are the normal stresses in the steel and in the concrete, respectively, A_s and A_c are the cross sectional areas of steel and concrete, respectively, d_b is the bar diameter, and n is the number of bars. The slip at a given position x is calculated as the integral (along the length x) of the difference between steel and concrete deformations:

$$s(x) = \int_0^x (\varepsilon_s(x) - \varepsilon_c(x)) dx \quad (2.5)$$

The previous equation can also be written as:

$$\frac{ds(x)}{dx} = \varepsilon_s(x) - \varepsilon_c(x) \quad (2.6)$$

and differentiating gives:

$$\frac{d^2s(x)}{dx^2} = \frac{d\varepsilon_s(x)}{dx} - \frac{d\varepsilon_c(x)}{dx} \quad (2.7)$$

Considering linear-elastic behaviour for both the steel and the concrete, and taking into consideration Equation (2.4), Equation (2.7) can be written as:

$$\frac{d^2s(x)}{dx^2} = \left(\frac{4}{d_b \cdot E_s} + \frac{4 \cdot \rho}{d_b \cdot E_c} \right) \cdot \tau(x) \quad (2.8)$$

where E_s and E_c are the elasticity modulus for steel and concrete, respectively, and ρ is the reinforcement ratio ($\rho = A_s/A_c$). Inserting the bond stress-slip relationship given by Equation (2.1) into Equation (2.8), we have:

$$\frac{d^2s(x)}{dx^2} = \left(\frac{4}{d_b \cdot E_s} + \frac{4 \cdot \rho}{d_b \cdot E_c} \right) \cdot \tau_{\max} \cdot \left(\frac{s(x)}{s_1} \right)^b \quad (2.9)$$

The previous equation can be solved by considering the following boundary conditions: $s_{(x=0)} = s'_{(x=0)} = 0$ [according to Equation (2.6), the first derivative of the slip function equals zero if the steel strain is equal to the concrete strain]. The crack opening is equal to twice the slip at the crack section, and is given by (Farra 1995):

$$w = 2 \cdot \left(\frac{s_p^b \cdot (1+b) \cdot d_b \cdot \sigma_{sr}^2}{8 \cdot \tau_{\max} \cdot E_s \cdot (1 + \rho \cdot E_s / E_c)} \right)^{\frac{1}{1+b}} \quad (2.10)$$

where σ_{sr} represents the steel stress at the crack section.

Integration of Equation (2.8) is harder in the case of unloading, as more complex bond stress-slip relationships are established for that case. It will be shown below (§2.4.2) that, under those circumstances, the solution can be found numerically by employing the finite differences technique.

Moreover, a simple solution of the differential equation can not be obtained for the cases in which the first derivative of the slip at the origin is not equal to zero (i.e., for stages other than the crack formation stage) or if the concrete stress at the crack is different from zero (due to the contact between crack faces). In those cases, the solution can also be found numerically.

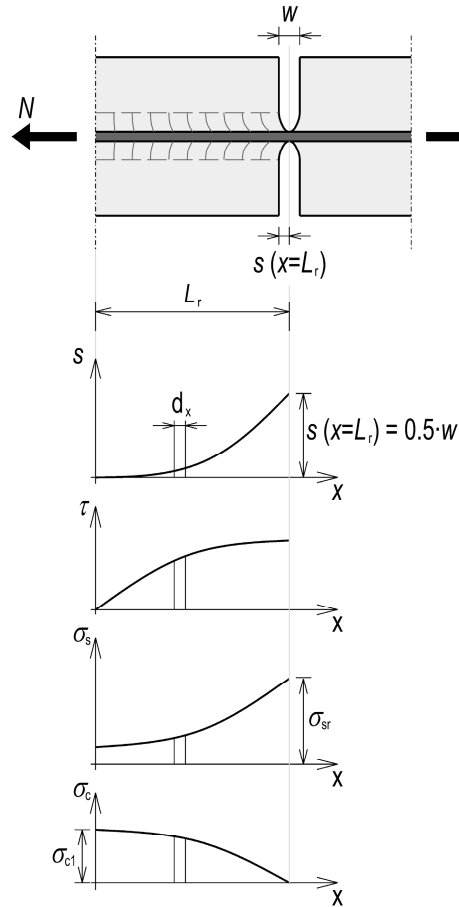


Figure 2.13 - Variation of stresses and deformations along the transmission length. Adapted from Farra (1995). The parameters ε_{sl} , ε_{cl} , σ_{sl} and σ_{cl} represent steel and concrete strains and stresses in state I.

2.2.7 Bond under cyclic loading

Cyclic loadings are usually divided into two categories:

- low-cycle loading, which consists of a few load cycles (less than one hundred) causing high stress amplitudes;
- high-cycle or fatigue loading, which consists of a load history containing many cycles (typically thousands or millions) causing low stress amplitudes.

Low-cycle loading typically arises in seismic events. On the other hand, highway or railway traffic in bridges are typical examples of high-cycle loading. Therefore, only the latter loading category is of interest for this work. The following subsections present the bond behaviour for: reversed or alternating loading (when the relative displacement alternates between positive and negative directions) and for unidirectional repeated loading (when cyclic loading produces relative displacements in one direction only).

2.2.7.1 Bond behaviour under reversed cyclic loading

The behaviour under reversed cyclic loading can be explained by the following phases which were primarily identified by Elgehausen et al. (1983). Different structural responses can be identified depending on the slip level at which the reversal occurs.

If the slip reversal is imposed before shear cracks are initiated in the concrete keys between ribs (Figure 2.14a), a stiff unloading branch is observed (path AF), because only a small part of the slip is caused by inelastic concrete deformations. Then, the friction branch is reached (path GH). This portion of the curve exhibits reduced slope before the bar ribs hit the concrete keys during the backwards movement. Once the point of zero slip is crossed, the stiffness differs little from that of the monotonic envelope on the opposite direction. Unloading from point I, results in a curve (path IKL) similar to the initial unloading curve (path AFH). However, due to previous cracking and crushing of the concrete in the front of the ribs, the point where the bond stresses begin to rise again (point L) will be shifted to the right. The rib will not be bearing fully until the point M is reached. Further loading approximately follows the monotonic envelope.

If unloading occurs after the shear cracks are initiated in the concrete keys (Figure 2.14b), the unloading curve is similar to that of the first case. However, a higher frictional resistance is mobilized (point G), as there is more damage to the concrete. When the loading is reversed, the rib presses against a key whose resistance has been lowered by inclined shear cracks over a part of its length (these shear cracks are induced by the first half-cycle). The splitting cracks created in the first half-cycle close at a higher load than those of the first case (point H). Consequently, splitting cracks in the opposite direction are initiated earlier than in the first case. The

combination of splitting cracks and existing inclined shear cracks along the bar, results in a reduced envelope (path HI) in the negative direction. A new load reversal results in a reduced stiffness and strength (point N) because only the remaining uncrushed concrete between the ribs has to be sheared off.

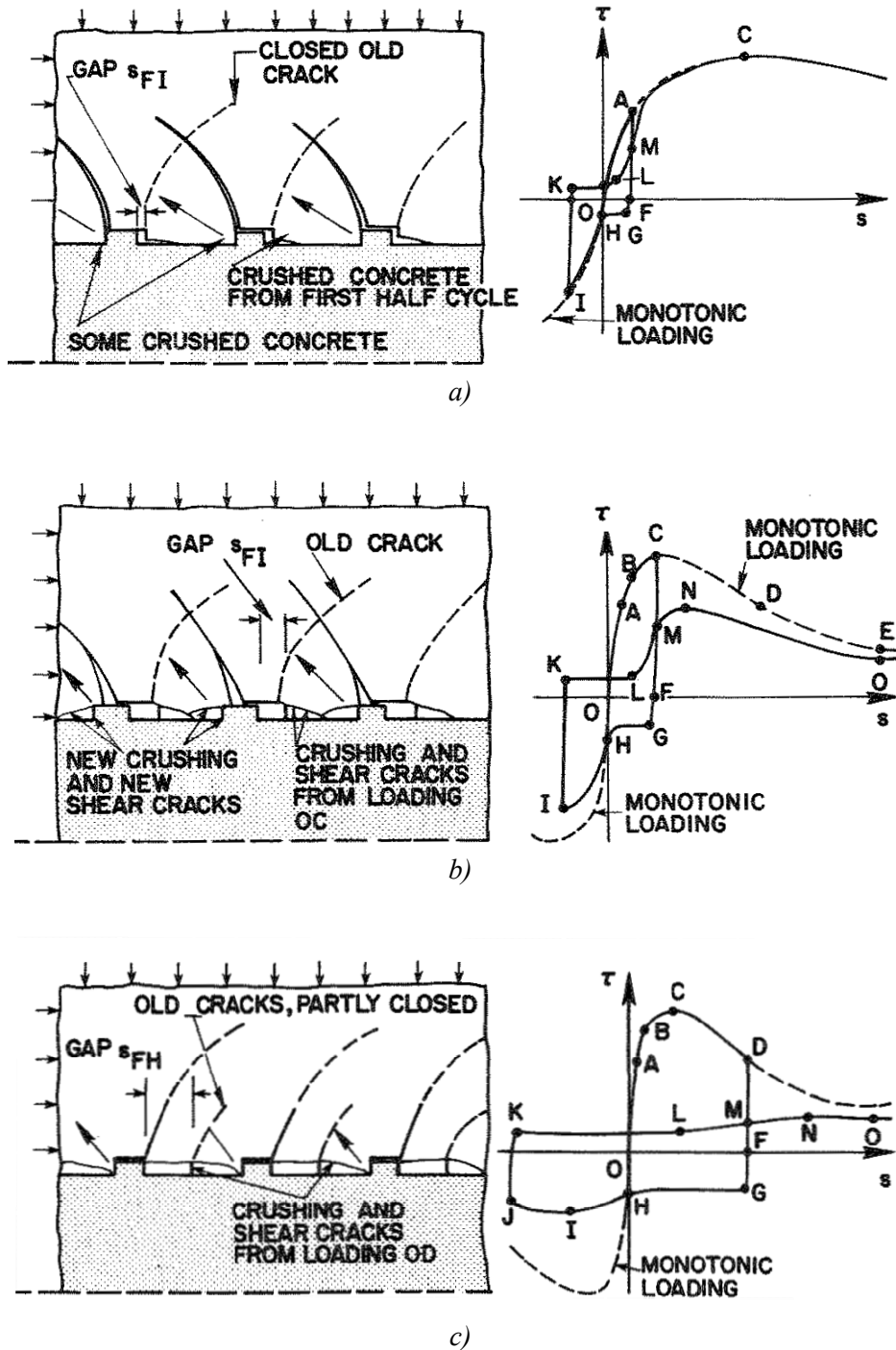


Figure 2.14 - Mechanism of bond resistance under reversed cyclic loading (Eligehausen et al. 1983).

If unloading occurs after the peak bond strength is reached, the bond strength in subsequent load cycles will be significantly reduced (Figure 2.14c). The frictional resistance (point G) will be

larger than the value reached in any of the previous cases, since more damage has occurred before unloading. However, since the concrete is completely sheared, very little force can be transmitted by bond when the direction of loading is reversed (path HIJ). Unloading and reloading in the opposite direction results in very little additional bond capacity beyond that provided by friction since most of the mechanical anchorage has been lost.

2.2.7.2 Bond behaviour under unidirectional cyclic loading

The bond degradation produced by unidirectional loading is less severe than the one caused by reversed loading. This effect can be observed in Figure 2.15, which compares the monotonic bond stress-slip curve against the results of cyclic loading (either without change of slip sign or with sign change). Figure 2.15a, which depicts the results for unidirectional cyclic loading, shows that the monotonic envelope is reached upon reloading if the maximum slip is limited to values less than approximately 0.1 in. (2.54 mm). On the contrary, Figure 2.15b shows that reversed loading with such a maximum slip causes a significant reduction of bond stresses upon reloading.

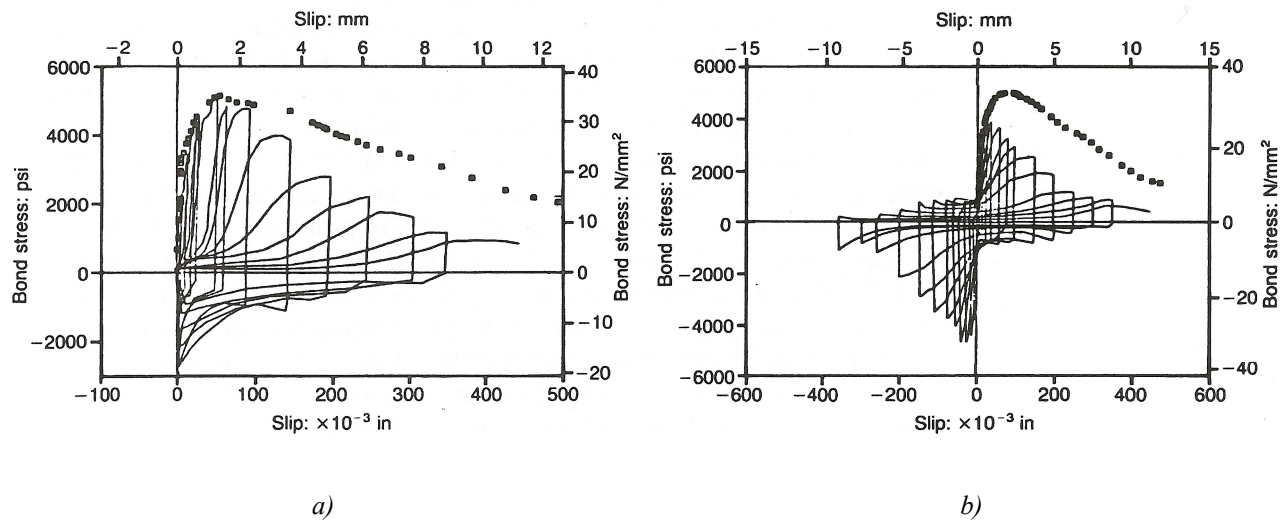


Figure 2.15 - Comparison of monotonic and cyclic bond stress-slip curves: a) unidirectional repeated loading; b) reversed loading. Hawkins' results quoted by the CEB (1996).

Rehm and Eligehausen (1979) found that, if no fatigue failure occurs under the effect of constant amplitude cyclic loading, then previous load cycles do not negatively affect the bond stress-slip relationship near ultimate load levels (under the effect of a short time load) when compared with the monotonic loading behaviour. This conclusion was confirmed by Oh and Kim (2007). These authors concluded that the bond strength and the slip at the peak bond stress are not changed by constant amplitude repeated loading, even for large stress amplitudes and a large number of load cycles (Figure 2.16).

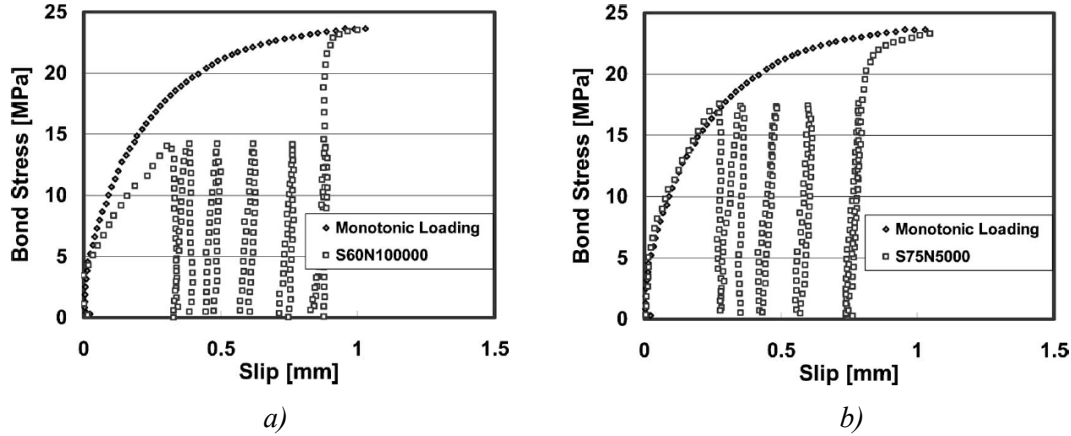


Figure 2.16 - Pull-out test under monotonically increasing loads and after cyclic loading ($d_b=16\text{mm}$; $l_b=2 \cdot d_b$; $f_{cm}=37\text{MPa}$): a) 100,000 cycles with an amplitude equal to 60% of the monotonic bond strength; b) 5,000 cycles with an amplitude equal to 75% of the monotonic bond strength (Oh and Kim 2007).

Fatigue of bond results in increasing slips and (in the case of high-amplitude cyclic loading) failure for a load level lower than the one which would cause bond failure under monotonic loading. Fatigue damage is presumed to be caused by the progressive growth of microcracks and concrete crushing in front of the ribs (CEB 1996; fib 2000).

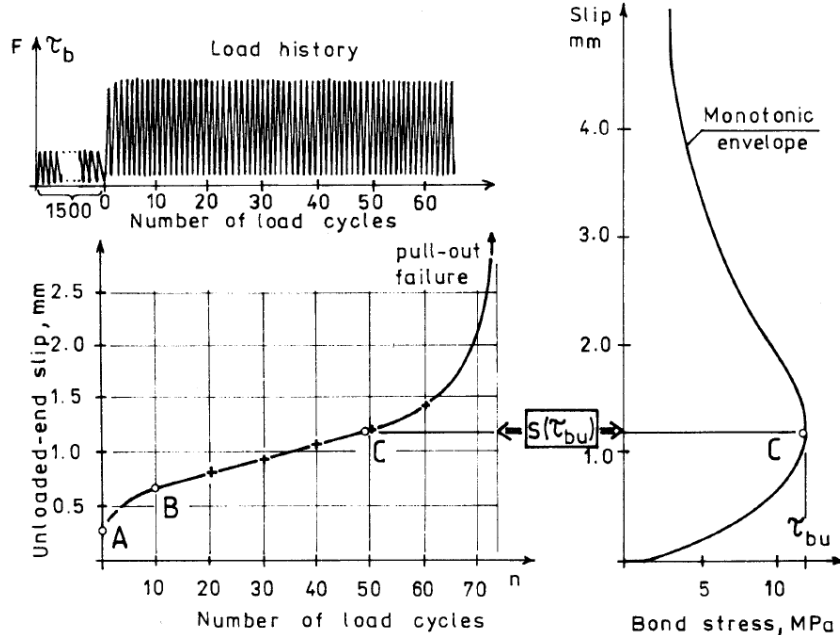


Figure 2.17 - Bond fatigue process under repeated loading: $d_b = 8\text{mm}$; ribbed bar; $l_b = 12 \cdot d_b$; concrete C16 (Balázs 1991).

According to experimental results obtained by Balázs (1991), three different phases can be distinguished when the interface between a reinforcing bar and the concrete is subjected to constant-amplitude cyclic loading. During the initial load cycles (first phase) the slip rate

decreases. Then, in the second phase, the slip increases at a constant pace. Finally, a rapid increase of the slip rate leads to pull-out failure (third phase). Furthermore, Balázs (1991) found that the second phase, where a linear growth of the slip occurs, ends at $s(\tau_{max})$, which represents the slip at the peak bond stress for monotonic loading. In this way, this slip value marks the beginning of the failure branch (Figure 2.17). This behaviour was found to be independent of both the load level and the number of cycles to failure. Therefore, the author proposes that the slip at the peak bond stress, $s(\tau_{bu})$, can be taken as a safe fatigue failure criterion.

The MC90 proposes a methodology for taking into account the influence of repeated loading in the definition of the ascending branch of the bond stress-slip law. According to this document, the slip s_{re} due to repeated loading is given by:

$$s_{re} = s(1 + k_n) \quad (2.11)$$

where s represents the slip under monotonically increasing loading and k_n is a factor given by:

$$k_n = (1 + n)^b - 1 \quad (2.12)$$

where n is the number of load cycles and the coefficient b takes the value 0.107. In this way, the relationship between the bond stress and the maximum slip after n cycles can be obtained by substituting Expression (2.11) into Equation (2.2), which yields:

$$\tau = a_1 \cdot f_{cm}^{a_2} \cdot \left(\frac{s_{re}}{(1 + k_n) \cdot s_p} \right)^b \quad (2.13)$$

where τ and f_{cm} are in MPa. This approach to describe the slip variation as a function of the number of applied load cycles has been followed by various authors (Laurencet 1999; Rotilio 1998; Zanuy et al. 2010).

In Figure 2.18, the factor k_n is plotted against the logarithm of the number of loading cycles. This figure shows that fatigue loading is responsible for an important increase of the slip value. Therefore, the degradation of bond action due to cyclic loading must not be disregarded in the calculation of deformations and crack openings.

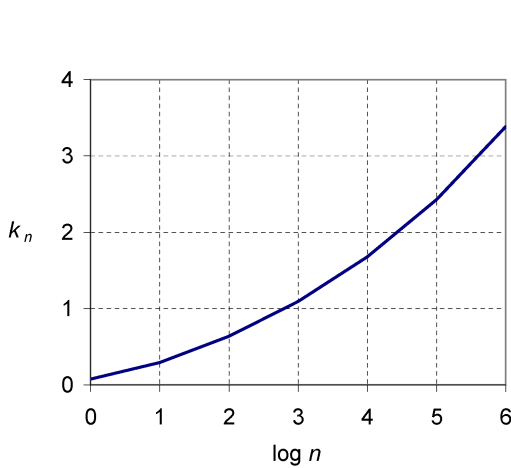


Figure 2.18 - k_n factor according to Equation (2.12).

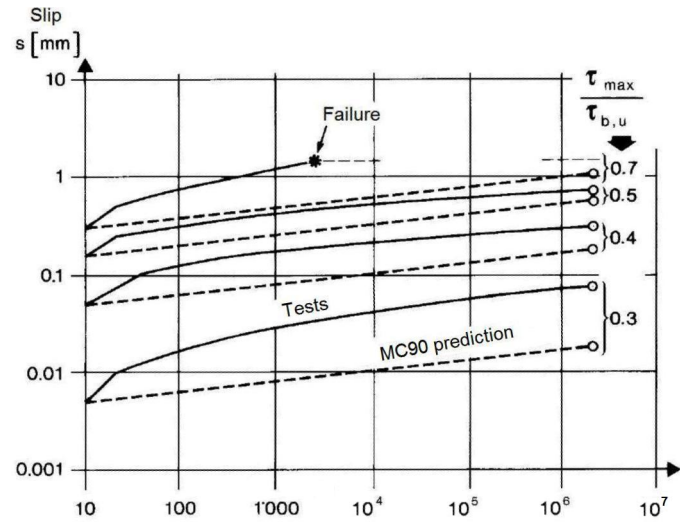


Figure 2.19 - Unloaded-end slip vs. number of load cycles under constant amplitude repeated loading in double logarithmic scale: comparison between Equation (2.11) and experimental results of Koch and Balázs, reported by Balázs (1998). Basic data include $d_b=16\text{mm}$, $l_b=5 \cdot d_b$, $f_{cm}=30\text{MPa}$ and $\tau_{b,min}=0.1 \cdot \tau_{b,max}$.

The MC90 approach to predict the slip increase is based on the pull-out test results by Rehm and Elgehausen (1979), who found that under constant amplitude cyclic loading, the relationship between the slip and the number of load cycles is approximately linear in double logarithmic scale. Similar conclusions (despite slightly different values for the slope of the trend line) were obtained by other authors. Figure 2.19 compares the MC90 Equation (2.11) with the results of pull-out tests under constant-amplitude cyclic loading carried out by Koch and Balázs and reported by Balázs (1998). These results revealed that the MC90 approach underestimates the slip increase (in Figure 2.19, measurements and predictions were fit to start from the same slip at 10 load cycles). These results suggest values for the b coefficient in Equation (2.12) higher than the MC90 proposal. On the contrary, experimental results obtained by Oh et al. (2007) suggest a slightly lower value for that coefficient: 0.098. Nonetheless, both Balázs (1998) and Oh et al. (2007) concluded that the exponent b takes higher values for higher stress amplitudes of the cyclic loading.

The influence of variable stress histories was thoroughly investigated by Balázs (1998). The experimental results obtained by this author clearly indicate that only load levels approaching or exceeding the previous highest level induce considerable slip increase. The contribution of low load levels is almost negligible. Laurencet (1999) suggests that only load cycles with a maximum bond stress higher than 70% of the highest value previously reached produce a significant slip increase. Consequently, only those load cycles must be taken into account in the quantification of the slip increase according to Equations (2.11) and (2.12).

Finally, it should be pointed out that, as well as in the case of monotonically increasing loads, the bond performance under repeated loading exhibits a significant scatter (Edwards and Yannopoulos 1978).

2.2.7.3 Bond stress-slip relationship for cyclic loading

A suitable bond stress-slip relationship is necessary for the numerical analysis of the cyclic behaviour of RC members after cracking. For the present work, only the bond performance under unidirectional repeated loading is relevant, since slip reversals do not take place in the structures under analysis, as shown in Chapter 6. However, most of the cyclic bond models that have been proposed by different authors focus on the behaviour under reversed cyclic loading. Commonly, such bond stress-slip laws are applicable up to large slip values, which are out of the scope of the current work. That is because those models were primarily developed for the study of the dynamic behaviour of certain bar anchorages under high seismic loadings.

Given the limited information available in the literature about models specifically devised to describe the unidirectional cyclic bond behaviour (for slip values observable in real structures under service loads), relevant information provided by models that describe the reversed cyclic bond behaviour is also considered in this section. The applicability of those models for serviceability analyses is discussed below.

Some common features can be identified in the various models available in the literature:

- an envelope curve, which describes the bond stress-slip relationship under monotonically increasing loads;
- a stiff unloading branch, with a stiffness K_B , which describes the behaviour upon unloading from the envelope curve;
- a subsequent horizontal branch characterized by the frictional bond stress, $\tau_{f,i}$.

These features of the cyclic bond performance are consistent with the idealized behaviour described in §2.2.7.1. The frictional bond stress $\tau_{f,i}$ is usually defined as a percentage of the bond stress on the envelope curve from which the unloading starts, τ_i , by means of the constant α_B :

$$\tau_{f,i} = -\alpha_B \cdot \tau_i \quad (2.14)$$

Table 2.2 summarizes the values proposed by different authors for the parameters K_B and $\tau_{f,i}$. It can be observed that those parameters are typically in the range:

$$\tau_{f,i} = -\alpha_B \cdot \tau_i \approx -(0.15 \text{ to } 0.25) \cdot \tau_i$$

$$K_B \approx 200 \text{ to } 500 \text{ MPa/mm}$$
(2.15)

As exposed below, the most relevant differences between the various proposals lie in the behaviour upon reloading and in the definition of a reduced monotonic envelope (which reflects the deterioration of the bond resistance owing to previous load cycles).

Table 2.2 - Parameters that define the bond stress-slip relationship in terms of unloading from the monotonic envelope (see Figure 2.20).

Author	K_B (MPa/mm)	$\tau_{f,i}$
Morita and Kaku (1973)	400	$0.18 \cdot \tau_{\max,i}$
Edwards and Yannopoulos (1978)	200 to 500 ⁽¹⁾	--
Viathanatepa et al. (1979)	slope of the first branch of the monotonic envelope	equal to τ_r (taken from the reduced envelope)
Tassios (1979)	200 to 500 ⁽¹⁾	$0.25 \cdot \tau_{\max,i}$ ⁽²⁾
Ciampi et al. (1981) and Eligehausen et al. (1983)	180	function of τ_r and the peak value of slip reached in either direction
Pochanart and Harmon (1989)	∞	--
Model Code 1990	200	--
Balázs (1991)	400	$<0.7 \cdot \tau_r$

NOTES: τ_i represents the bond stress on the envelope curve from which the unloading starts

τ_r represents the residual bond stress obtained after the concrete between bar ribs is sheared off, under monotonically increasing loading

⁽¹⁾ the higher the slip reached before unloading, the lower the value taken by the parameter K_B

⁽²⁾ lower values are proposed for repeated cycles between fixed slip values

The first analytical model for the bond stress-slip relationship under reversed cyclic loading was proposed by **Morita and Kaku (1973)**, this model being synthesised in Figure 2.21. In this model, both the monotonic envelope and the unloading and reloading paths are composed by straight lines. The model was compared against the results of pull-out and push-out tests (with bonded length equal to twice the pitch of transverse ribs), as shown in Figure 2.22. It is important to note that the results presented in this figure correspond to unidirectional repeated loading, with moderate slip values, which is precisely the type of loading that is of interest for the serviceability analyses to be carried out within the scope of this thesis. The figure shows that

the model proposed by Morita and Kaku (1973) provides a reasonable agreement with the experimental results. In particular, the unloading and reloading paths, despite being composed by straight branches, seem to be suitable to describe the experimental results. Nevertheless, this model does not accurately describe all the features of the cyclic bond performance, as pointed out by Morita and Kaku (1973):

- in the case of cyclic load tests between constant slip limits, the bond deterioration at the peak slip is overestimated, as the number of applied load cycles increases;
- the deterioration of the bond resistance at peak slip, with increasing number of reversed cycles, is not captured by the model (but this issue is not relevant in the case of repeated loading with moderate slip values, as shown in Figure 2.15).

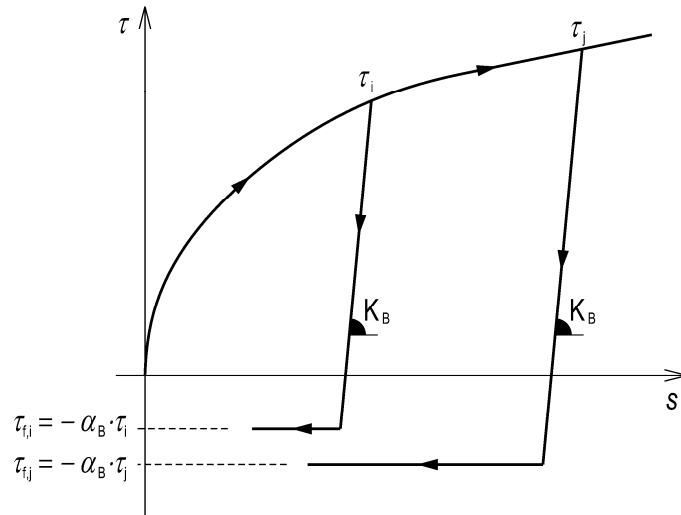


Figure 2.20 - Bond stress-slip relationship: unloading from the monotonic envelope.

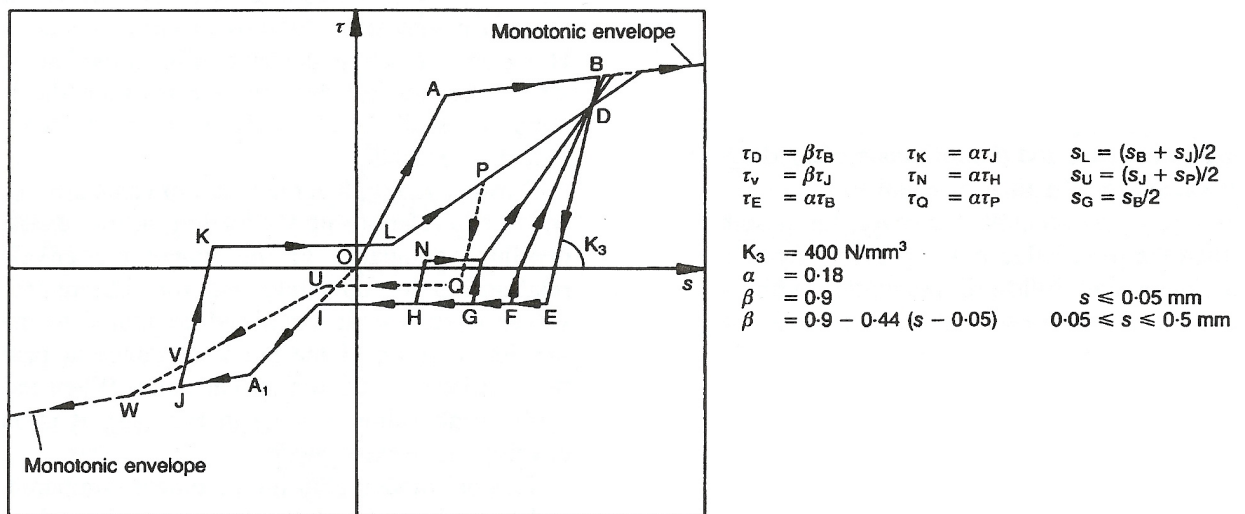


Figure 2.21 - Reversed cyclic bond stress-slip relationship proposed by Morita and Kaku (1973).

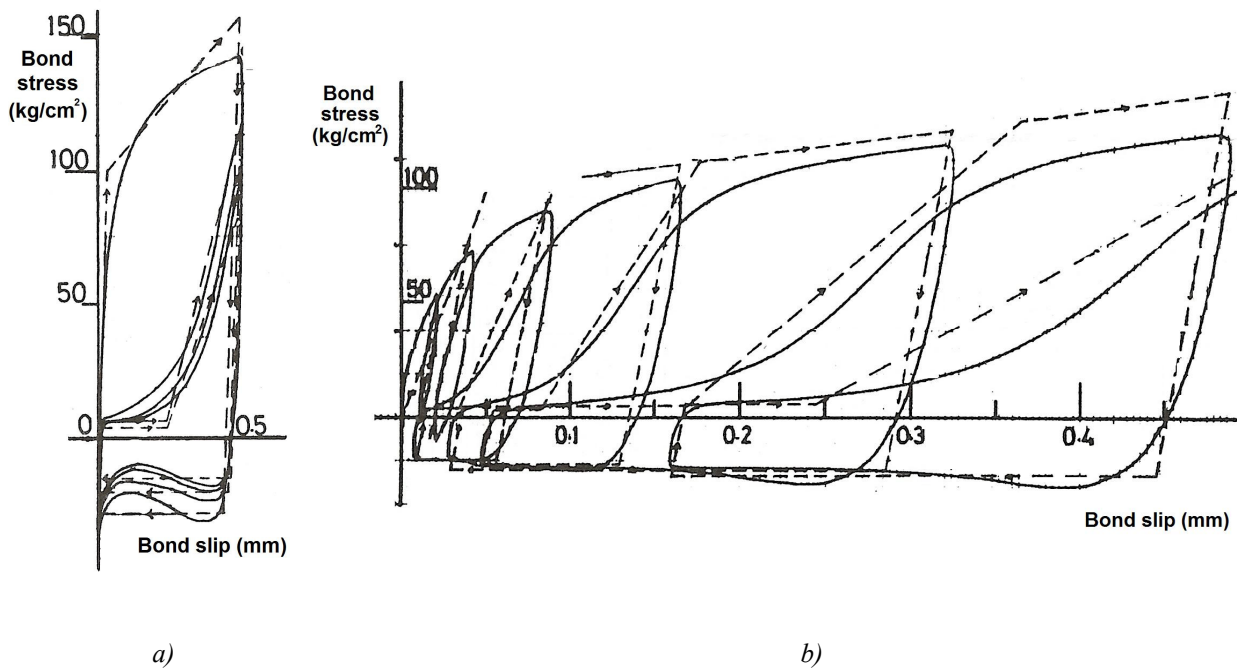


Figure 2.22 - Bond stress-slip curves under different load histories (Morita and Kaku 1973).
Comparison between experimental (continuous line) and analytical results (dashed line):
a) unidirectional repeated loading with constant amplitude cycles
b) unidirectional repeated loading with increasing slip values.

Viathanatepa et al. (1979) proposed a more complete bond stress-slip relationship. In this model, a different monotonic envelope is proposed for the unconfined region where the end effect causes a decrease of the bond strength (see §2.2.2). Moreover, unlike the model proposed by Morita and Kaku (1973), Viathanatepa's model takes into account the modification of the envelope curve owing to previous load cycles. This modification is carried out by reducing the characteristic bond stresses and the slip values of the monotonic envelope curve. The envelope curve only changes when the slip reaches values larger than previous peak slips. The bond stress-slip relationship upon reloading is very similar to the one proposed by Morita and Kaku (1973). However, Viathanatepa et al. (1979) provide more detailed data regarding unloading from intermediate positions of the ascending branches other than the monotonic envelope (see Figure 2.23). This model was a major improvement at the time it was presented. However, as it is emphasized by the CEB (1996), in spite of being rather complicated, it is not general. It requires some 20 parameters, which have no clear physical meaning and must be evaluated from test results. Furthermore, there are significant incoherencies in the definition of the reduced envelopes (CEB 1996).

Figure 2.24 summarizes the model proposed by **Tassios (1979)**. This author presents a distinct approach in terms of deterioration of the frictional bond resistance after repeated cycles between fixed slip values. However, the adopted reloading branch of the bond stress-slip relationship involves important simplifications in comparison with the real structural behaviour. In fact, the

ascending reloading branch seems inappropriate to describe the behaviour observed in Figure 2.22, since the last branch before the monotonic envelope is reached is unrealistically stiff.

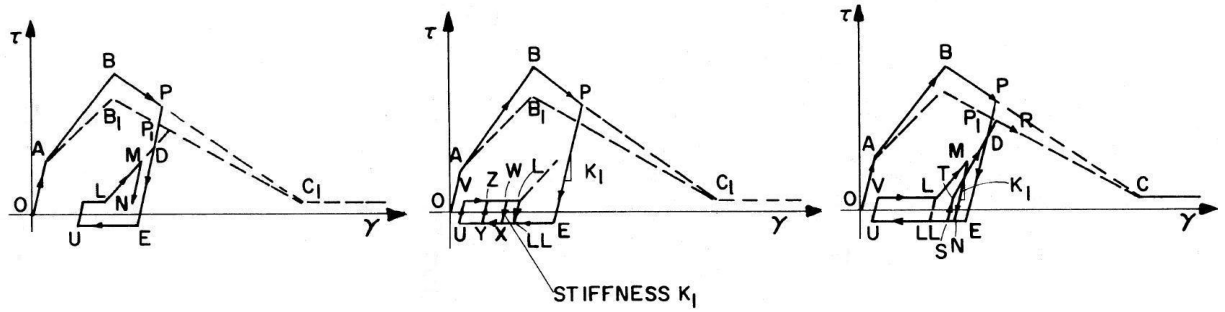


Figure 2.23 - Unloading and reloading inner cycles in the model proposed by Viathanatepa et al. (1979).

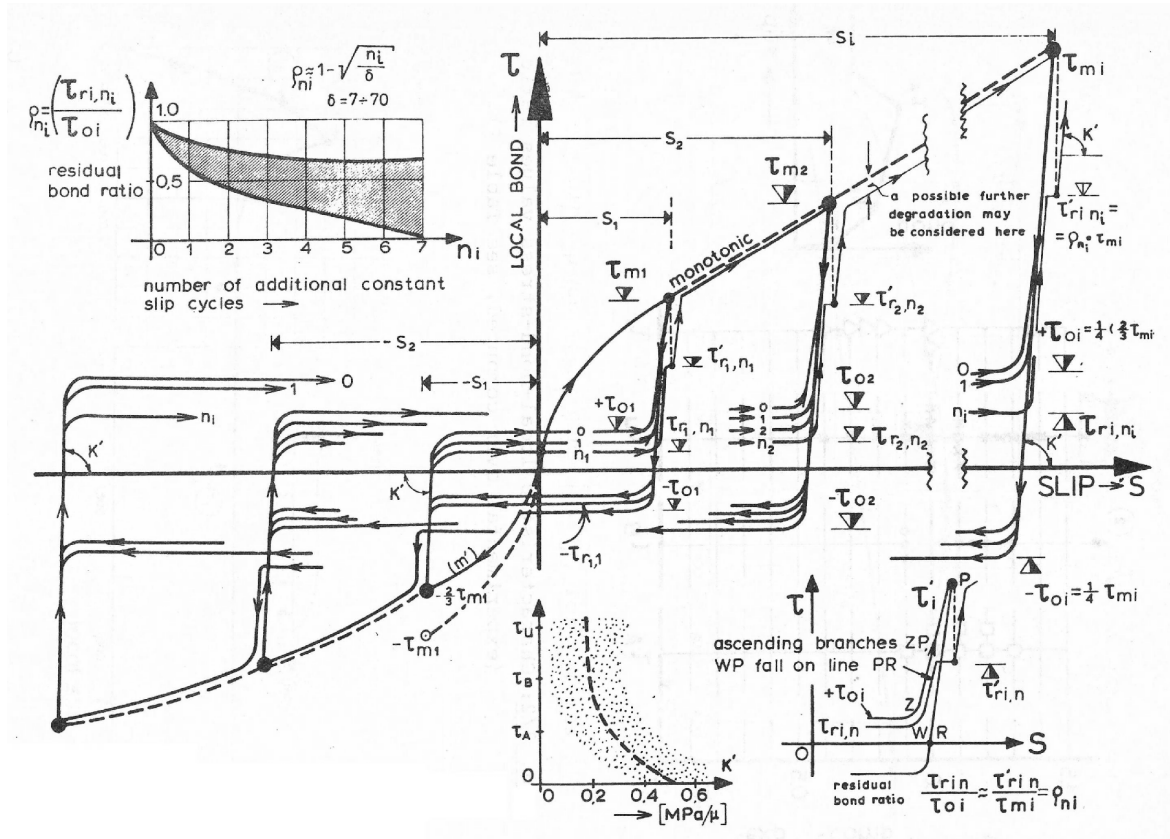


Figure 2.24 - Reversed cyclic bond stress-slip relationship proposed by Tassios (1979).

Compared to the previous model, the model proposed by **Ciampi et al. (1981)** and **Eligehausen et al. (1983)**, which is represented in Figure 2.25, describes the frictional bond resistance in a more rigorous way. Moreover, it considers the reduction of the monotonic envelope due to high slip values reached in previous cycles. However, the reloading branch up to the reduced monotonic envelope has the simplified shape of Tassios' model. For such reason, it is considered inadequate for the current work.

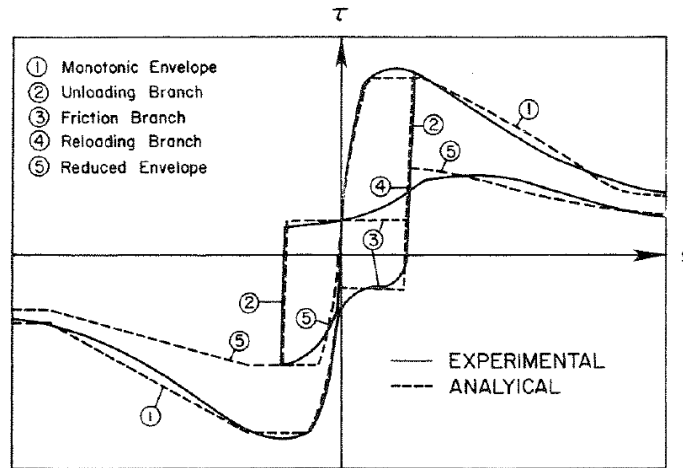


Figure 2.25 - Reversed cyclic bond stress-slip relationship proposed by Ciampi et al. (1981) and Eligehausen et al. (1983).

The unloading and reloading paths of the model proposed by **Pochanart and Harmon (1989)**, which are depicted in Figure 2.26, also seem unable to reproduce the real behaviour under unidirectional repeated loading and moderate slip values.

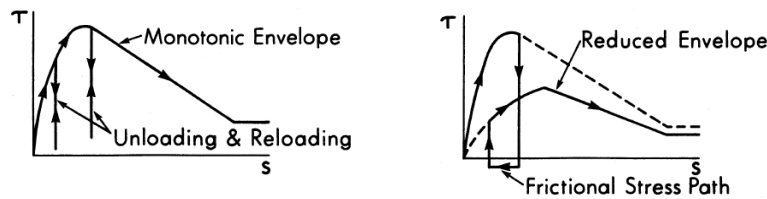


Figure 2.26 - Unloading and reloading paths in the model proposed by Pochanart and Harmon (1989).

Balázs (1991) gives some recommendations for modelling bond fatigue, which are synthesised through the scheme reproduced in Figure 2.27. This author provides indicative ranges of variation for different parameters: peak bond strength after cyclic loading, slip at the peak bond strength after cyclic loading, peak bond strength in the secondary direction and frictional bond stress during unloading. Figure 2.27 and its legend show that if the sign of the bond stress does not change during cyclic loading, then the cyclic envelope practically coincides with the monotonic envelope, that is, the bond strength is not deteriorated. On the contrary, maximum deterioration occurs for symmetrical load reversals.

Similar conclusions were obtained by **Oh and Kim (2007)**. Pull-out tests carried out by these authors (see Figure 2.16) evidenced that both the bond strength and the slip at the peak bond stress are not changed by unidirectional repeated loading if bond fatigue failure does not occur. However, the cyclic loading considered by these authors does not include negative bond stresses. In this way, the slip amplitude due to the cyclic loading is diminutive (the unloading and the reloading paths lie approximately on the stiff branch with slope K_B , represented in Figure 2.20),

which does not reproduce the actual slip variations endured by real structures such as bridge decks.

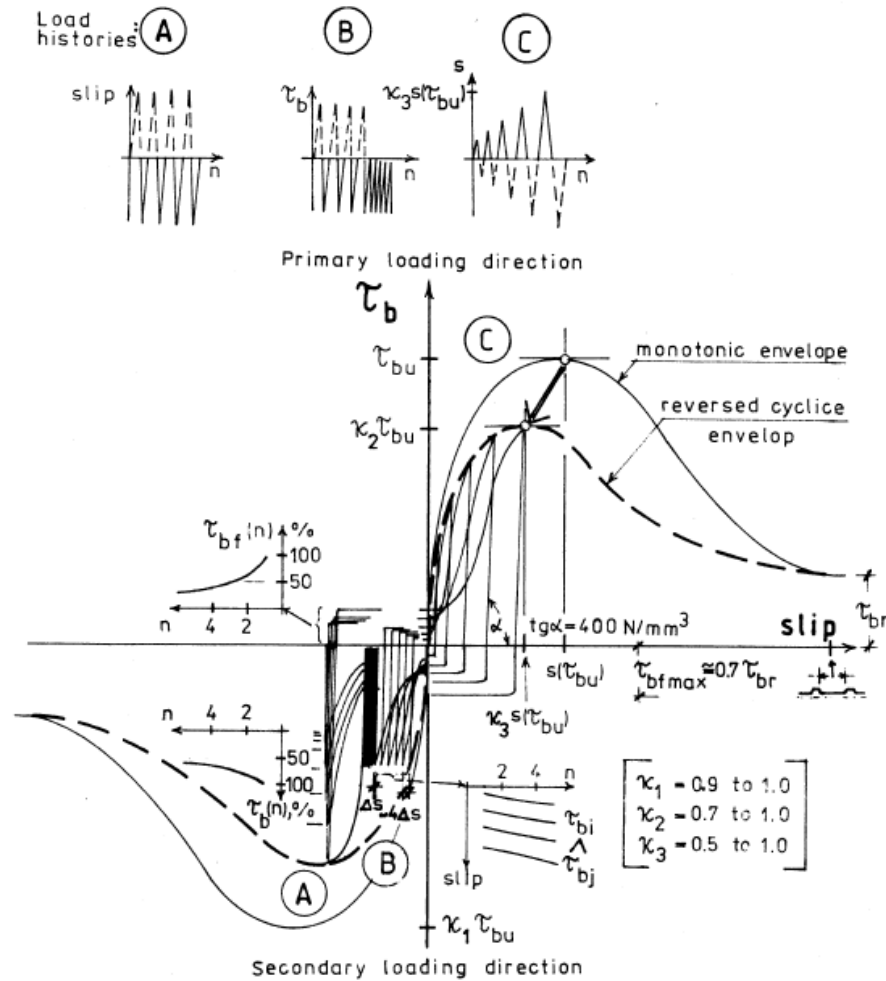


Figure 2.27 - Bond fatigue behaviour under reversed cyclic loading in confined concrete. Continuous lines of load histories indicate the presented parts of the diagrams and dashed lines indicate those which are not presented due to quasi-symmetry. Lower values of coefficients k_2 and k_3 correspond to symmetrical load reversals and their upper values to load reversals limited to the same sign of slip (Balázs 1991).

As regards the bond stress-slip model proposed by the **MC90**, no guidance is given on the unloading and reloading paths of the bond stress-slip relationship besides the simple case without reversal of the stress sign.

The information given above shows that a general model for definition of the local bond stress-slip relationship under repeated loading (describing unloading and reloading paths for moderate slip values compatible with the structural behaviour under service loads) is not available yet. This was precisely the conclusion expressed by Oh and Kim in 2007. Therefore, more research is needed in this field. Following this evidence, a research project was initiated recently at FEUP, aiming at obtaining experimental data that hopefully will contribute to the development of such a

model. Within the scope of that research project, bond performance is evaluated by means of pull-out tests, transfer tests in which the strain variation along the steel bar is measured with optical fibres including arrays of Bragg grating sensors (procedure also followed by others, as shown in §2.2.4), and beam tests. In this thesis, given that those experimental results are not available yet, the bond behaviour is modelled based on the information provided in the literature and synthesised above. The adopted model is presented in §2.4.2.3.

2.2.8 Bond under long-term loading

Similarly to cyclic loads, long-term loads produce an increase in slip and a deterioration of bond action. According to the CEB (1997), several authors (Franke, Rostásy and Kepp and Koch and Balázs) found that the slip vs. duration of load relationship can be approximately represented by a straight line in double logarithmic scale. This approach was followed by the MC90, which suggests that the slip under long-term loading can be determined through a procedure similar to the one employed for taking into account the influence of repeated loading [see Equations (2.11) and (2.12)]. The slip s_{re} after t hours can be expressed as a function of the initial slip s , as shown in Equation (2.11), considering the following k_n factor:

$$k_n = (1 + 10 \cdot t)^{0.08} - 1 \quad (2.16)$$

Equation (2.16) is depicted in Figure 2.28. Figure 2.29 illustrates the relationship between the slip increase (the slip s_{re} after t hours divided by the initial slip s) and the load duration.

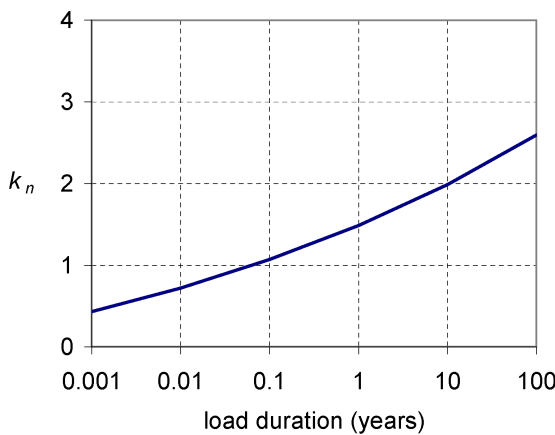


Figure 2.28 - k_n factor for long-term loading, according to Equation (2.16).

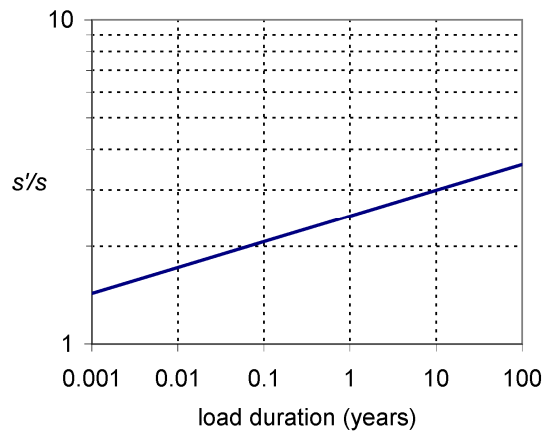


Figure 2.29 - Approximate linear relationship in double logarithmic scale between the slip increase, s_{re}/s and the load duration, calculated according to Equations (2.11) and (2.16).

2.3 *Cyclic behaviour of concrete*

The cyclic behaviour of concrete in the fracture zone plays an important role in the analysis of the cyclic behaviour of RC elements after cracking. Upon unloading, cracks do not close completely, as a consequence of imperfect matching of irregular crack surfaces, particles within the microcracks which have become loose, and irreversible deformations. This behaviour has implications in terms of crack opening and stiffness of concrete members subjected to cyclic loading and must therefore be taken into account in the analyses envisaged in this thesis.

As regards the behaviour of concrete in compression, cyclic loads are responsible for important strength reductions with respect to the monotonic strength, this issue being dealt with in Chapter 3. Moreover, the fatigue degradation of concrete in compression gives rise to increased compressive strains. This process is relevant in the case of concrete stresses higher than 60% of the compressive strength (Zanuy et al. 2009; Zanuy et al. 2007; Zanuy et al. 2011). On the other hand, it can be neglected in serviceability analysis of elements subjected to moderate compressive stresses. According to the RILEM Committee 36 RDL (1984), the variation of the secant modulus of elasticity with increasing number of applied load cycles is not significant for specimens subjected to low stress levels. In this respect, this RILEM Committee considers that low stress levels are the ones that do not give rise to fatigue failures. In the numerical models developed within the scope of this work, to analyse the cyclic behaviour of cracked RC members subjected to cyclic loads, the stiffness decrease for concrete under cyclic loading is neglected. This simplifying hypothesis has been adopted, for example, by Rotilio (1998), assuming that the influence of increasing concrete compressive deformations is considerably less important than cracking and bond-slip effects.

The following sub-sections summarise the main features of the cyclic behaviour of concrete in the fracture zone. Some constitutive models, reported in the literature to describe that behaviour, are also presented. For a comprehensive description of the fatigue performance of concrete subjected to cyclic loading, reference is made to one of the available state of the art reports (CEB 1988; 1996; ERRI D216 2002).

2.3.1 *The tensile behaviour of concrete*

Figure 2.30 illustrates the tensile behaviour of concrete, under the effect of **monotonically increasing deformations**. A test specimen and the variation of axial strain over its length are represented on the left, for three load steps (A, B and C). Those steps are also marked in the stress-deformation curve represented on the right side of the figure.

This stress-deformation relationship is linear almost up to the peak stress. Before this peak is reached, some microcracking occurs (step A), and the relationship becomes non-linear. As microcracking is uniformly distributed at the macro level, a uniform strain distribution over the length of the specimen may be assumed.

Immediately before the peak load, the development of microcracks is concentrated at the weakest part of the specimen (step B). At the macro level, increasing strains occur over the length h of this weak part, thus forming a crack band of width h .

For increasing values of the imposed deformation, the crack-band width diminishes, and the deformation within the crack band increases (step C). After the peak stress, the concrete exhibits a strain-softening behaviour. The resisting tensile stress diminishes as the imposed deformation increases, due to the progressive localized growth of microcracks. Finally, a single macro-crack is formed, and two concrete blocks are separated.

A possible approach for modelling this behaviour is the “fictitious crack model” introduced by Hillerborg et al. (1976), according to which the deformation of the crack band is collected into the crack width, w , of one single “fictitious” crack. In this way, the specimen deformation represented in Figure 2.30 is decomposed into the deformation of the “fictitious” crack and the deformation of the remaining concrete (which is almost linear-elastic up to the peak load). This decomposition is represented on the right side of Figure 2.30.

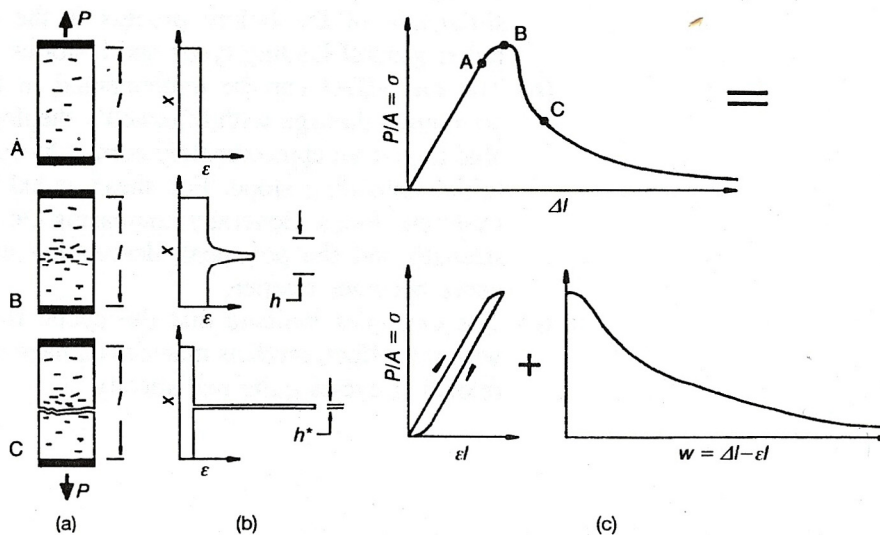


Figure 2.30 - Tensile behaviour of concrete (CEB 1996):

- a) specimen subjected to an imposed deformation;*
- b) variation of axial strain in the specimen, for the increments A, B and C;*
- c) corresponding stress-deformation and stress-crack opening curves.*

If concrete is subjected to **cyclic loading before the peak stress is reached**, the observed deformations increase for an increasing number of load cycles. In the case of constant amplitude loading, the strain increase follows the three-phase diagram depicted in Figure 2.17, thus exhibiting a similar behaviour to the one observed under constant-amplitude bond stresses.

The number of constant-amplitude load cycles, N , that a concrete specimen endures before fatigue failure is a function of the upper stress level, and is usually represented through S-N curves (also known as Wöhler curves). The number of cycles before failure decreases as the upper stress level increases, as shown in Figure 2.31. This figure also indicates that the fatigue strength depends on the lower stress level too (for the same stress range, the number of cycles to failure decreases as the lower stress increases). Furthermore, stress reversals have a more detrimental effect on tensile fatigue than repeated tensile stresses (Figure 2.31).

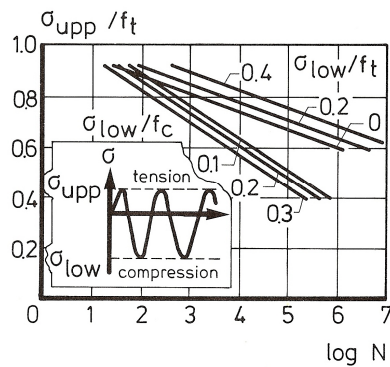


Figure 2.31 - Average S-N curves, for various lower limits, for dry concrete subjected to repeated tension or to alternating tension-compression (Cornelissen 1984).

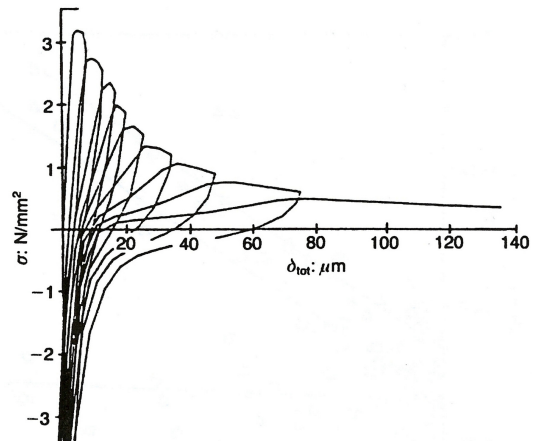


Figure 2.32 - Stress-deformation curve from a cyclic tensile test (Reinhardt et al. 1986).

If concrete is subjected to **cyclic loading after the peak stress is reached**, the $\sigma - \Delta l$ diagram takes the pattern represented in Figure 2.32. It has been found that the envelope curve almost coincide with the $\sigma - \Delta l$ diagram for monotonically increasing loads. Upon unloading, significant residual deformations are observed, which result from imperfect matching of irregular crack surfaces, loose particles within the microcracks and irreversible deformations. Significant hysteresis is observed during unloading-reloading cycles. Moreover, upon reloading, the point from which unloading started is not reached, as a consequence of the damage caused by the unloading-reloading loop.

2.3.2 Constitutive models for the cyclic tensile behaviour of concrete

A large number of experimental results are available for the stress-crack opening relationship in the case of monotonic loading. Numerous models have been proposed to describe this phenomenon. On the contrary, only a few models are available for the case of cyclic loading. These models comprehend an envelope curve (which usually corresponds to the stress-crack opening relationship under monotonically increasing loads) and the unloading and reloading paths.

The simplest models [Rots et al. (1985) - Figure 2.33a] describe the unloading and reloading processes through a straight line that connects the point on the envelope where unloading starts with the origin of the coordinate system. These models are not suitable for the analysis of problems in which residual crack openings are of concern.

Other types of constitutive models (Gylltoft 1984; Reinhardt et al. 1986; Yankelevsky and Reinhardt 1989), which are more refined than the previous one, describe the hysteresis loops through polygonal shapes (Figure 2.33b, c and d).

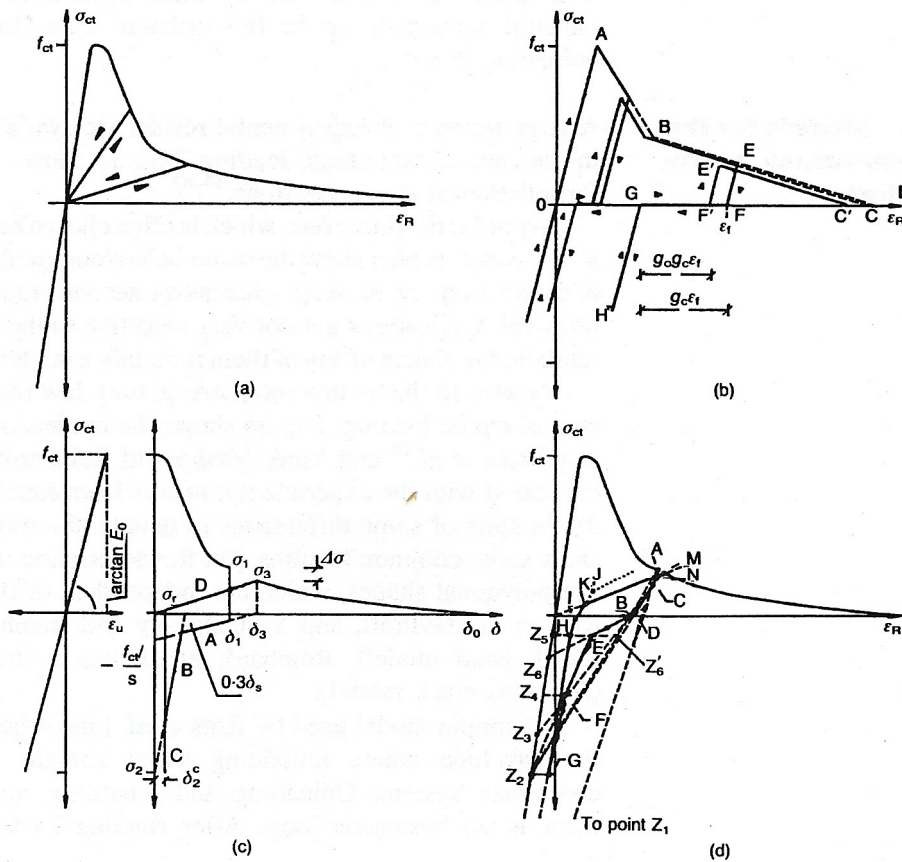


Figure 2.33 - Material models for cyclic loading: a) Rots et al. (1985); b) Gylltoft (1984); c) Reinhardt et al. (1986); Yankelevsky and Reinhardt (1989).

The “focal-point model” (FPM) proposed by Yankelevsky and Reinhardt (1989), which is illustrated in Figure 2.33d, provides a very detailed phenomenological description of the real concrete behaviour. However, unloading and reloading paths are composed by a series of linear branches, thus raising difficulties for an efficient numerical implementation. Moreover, according to this model, the damage that occurs in a loading cycle is independent of the lower stress level in the loop, which is not in accordance with experimental results.

On the contrary, the constitutive model proposed by Hordijk (1991) takes into account the influence of the lower stress level in terms of damage after reloading. Furthermore, this model is more suitable for numerical implementation, since unloading and reloading paths are described through continuous functions. For this reason, this model took the name “continuous-function model” (CFM). Besides describing cycles that start from and return to the envelope curve (which is defined as the stress-crack opening relationship for monotonically increasing loads) the CFM also describes inner loops (those that start from a previous reloading curve). Hordijk (1992) concluded that this model predicts the experimental results very well. This author also found that the CFM leads to a better agreement with experimental results than the FPM.

Given the merits of Hordijk’s CFM, this constitutive model was generically employed in this work, for analyses of the cyclic response of RC beams after cracking. Nevertheless, a modified version of Yankelevsky and Reinhardt’s FPM was adopted to describe the reloading path of the normal stress-crack opening relationship, because this modified model solves convergence difficulties and leads to a better agreement with experimental results in the case of small reloading steps. This model is described below (§2.4.2.4).

2.4 Numerical model for analysis of the cyclic behaviour of cracked RC elements

In this section, the developed model for analysis of the cyclic behaviour of RC beams after cracking is described. This section starts with the presentation of comparable works previously developed by other authors.

2.4.1 Previously developed numerical models

Several analytical methodologies have been proposed by different authors to analyse the behaviour of RC beams, explicitly considering the bond stress-slip relationship without recourse to specific interface finite elements to model the interaction between the reinforcements and the surrounding concrete.

Tassios and Yannopoulos (1981) presented an analytical procedure for analysis of the mechanical behaviour of a reinforced concrete tie after cracking, explicitly considering the bond stress-slip relationship. With the method proposed by these authors, the variation of stresses and strains (both in concrete and steel) along the length of the tie can be determined. The crack opening can also be calculated. This methodology basically consists of dividing the member under analysis (whose length is equal to half the crack spacing) into shorter sub-elements. Then, suitable equilibrium and compatibility conditions are imposed to all the sub-elements. The solution is found iteratively by trying successive values for the slip at the crack section, until all the boundary conditions are satisfied. The Regula Falsi numerical technique is employed in the iterative procedure.

The study presented by Tassios and Yannopoulos (1981) was innovative, but involved some simplifications: the tension-softening behaviour of the concrete was ignored; a simplified bond stress-slip relationship was adopted; the crack spacing had to be known in advance. Moreover, this work was merely analytical, i.e., no experimental validation was presented.

In the late 1990s, Tassios' methodology was further developed at the École Polytechnique Fédérale de Lausanne (EPFL), and was applied to the study of:

- residual crack openings, in reinforced and prestressed concrete ties, taking into account the influence of the variable actions (**Laurencet 1999; Laurencet et al. 1999**);
- residual deformations in reinforced and prestressed bridge-deck girders, also taking into account the influence of the variable actions (**Rotilio 1998**);
- crack openings and deformability of reinforced concrete slabs in steel-concrete composite continuous bridges (**Navarro 2000; Navarro and Lebet 2001**).

At the EPFL, the numerical methodologies developed by the aforementioned authors were validated through the comparison with their own experimental results. These authors concluded that the proposed methods are valid for the purpose for which they were devised.

Laurencet (1999) and Rotilio (1998) employed very similar approaches. As well as in the Tassios and Yannopoulos' methodology, the length of the element under analysis is equal to half the crack spacing. The main difference between their analytical methods lies in the fact that Laurencet (1999) focused on the behaviour of ties subjected to axial forces only, whereas Rotilio (1998) analysed beams subjected to axial forces combined with plane bending.

Given the fact that the main objective of these authors lies in the quantification of residual crack openings and deformations after the variable actions are removed, the analysis comprises a loading phase followed by a unloading one. By using this simplifying assumption, the whole loading history is not explicitly considered. In fact, for structures subjected to high-cycle

loading, the complete reproduction of the loading process cycle by cycle is not feasible due to the large computational effort that would be required. Laurencet (1999) and Rotilio (1998) took into account previously applied load cycles in a simplified manner, by using the reduced bond stress-slip envelope. Figure 2.34 depicts the loading and unloading branches of the bond stress-slip relationship adopted by these authors. The first-loading branch consists of the aforementioned Equation (2.2). In order to take into account the influence of previously applied load cycles, the loading branch for long-term loading follows the reduced envelope [given by Equation (2.13)], which reflects the bond degradation caused by previous cyclic loading. The unloading branch is composed of a stiff descending part (characterized by the slope $K_B = 400 \text{ MPa/mm}$) followed by a horizontal part (defined by the frictional bond stress $\tau_{f,i} = -0.25 \cdot \tau_i$).

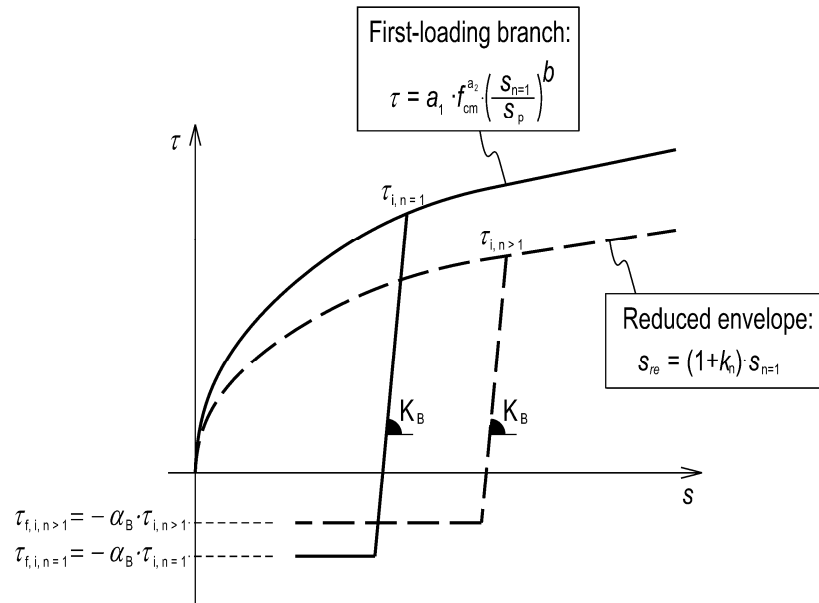


Figure 2.34 - Bond stress-slip relationship. Adapted from Laurencet (1999).

As far as the stress-crack opening relationship is concerned, the models proposed by both Laurencet (1999) and Rotilio (1998) consider the monotonic envelope and the unloading branch of Hordijk's CFM.

Creazza et al. (1993) had already presented a numerical procedure for the analysis of RC elements subjected to bending forces. However, unlike the methodology proposed by Rotilio (1998), the one adopted by Creazza et al. considered simplistic constitutive laws and was valid only for monotonically increasing loading.

The model developed by Navarro (Navarro 2000; Navarro and Lebet 2001) is similar to the Laurencet's one. As mentioned before, Navarro studied steel-concrete composite continuous bridges. In particular, he analysed the mechanical behaviour of the tie formed by the reinforced concrete slab, which is subjected to important tensile forces in the region above the continuity

supports. This author introduced simplifying assumptions such as the adoption of secant unloading towards the origin, in the normal stress-crack opening relationship. However, Navarro took the influence of shrinkage into account. This influence had been disregarded by Laurencet and Rotilio. In this respect, the comparison between experimental results [reported by Gilbert (2004) and Nejadi (2005)] for long-term crack openings and calculated values by Sousa et al. (2008) reveals that the calculated crack openings would be considerably underestimated if shrinkage strains were not taken into account (see Figure 2.35). On the contrary, reasonable numerical estimates are obtained if both the concrete shrinkage strains and the softening behaviour of concrete in the fracture zone are considered in the calculations. Therefore, concrete shrinkage strains are considered in the applications envisaged in this work.

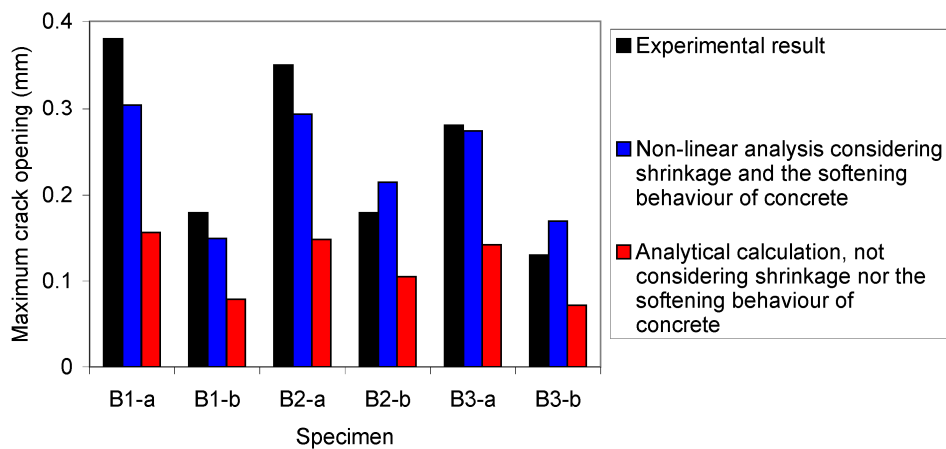


Figure 2.35 – Comparison between experimental results for long-term crack openings by Gilbert (2004) and Nejadi (2005) and calculated values at the level of the main tensile reinforcement (calculations considering the observed maximum crack spacing). Adapted from Sousa et al. (2008).

Recently, Zanuy (Zanuy 2008; Zanuy et al. 2010) has also proposed an analytical methodology for the analysis of RC ties subjected to axial forces. As well as in the afore-mentioned methodologies, the length of the element under analysis is equal to half the crack spacing (which is supposed to be known beforehand). At the crack, the external load is assumed to be carried by the reinforcement, that is, the softening behaviour of concrete is ignored. This author also uses the reduced-envelope concept, in order to reproduce the degradation of the bond stress-slip relationship due to previous load cycles. Unlike the methodologies proposed by Laurencet (1999) and Rotilio (1998), Zanuy's model explicitly considers the reloading path of the bond stress-slip relationship (see Figure 2.36).

A different approach was adopted by Manfredi et al. (1998) and Oliveira et al. (2007) who studied RC beams subjected to monotonic loading by means of a specific finite element that explicitly considers the bond stress-slip relationship. Both authors present the formulation for a finite element that extends between two consecutive potential cracks (this implies that the crack

spacing must be estimated before the analysis is carried out). In both proposals, the cross-section is divided into concrete layers and steel bars.

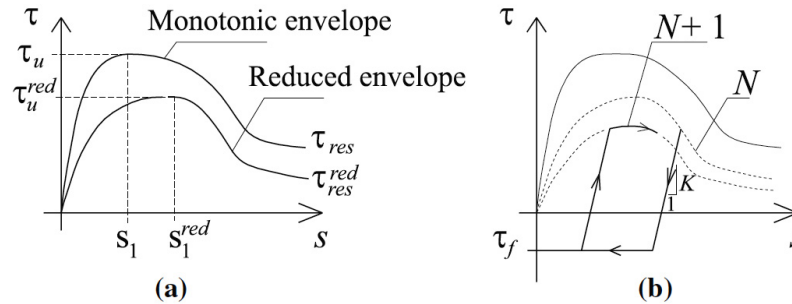


Figure 2.36 - Schematic representation of the bond stress-slip relationship adopted by Zanuy et al. (2010): a) reduced envelope; b) unloading and reloading branches within a load cycle.

In the proposal by Manfredi et al. (1998), each element is subdivided into subintervals with small length, in which the bond stress is assumed to be constant. The solution for each element (strains and stresses along the element length) is found through an iterative procedure, trying successive values for the slip at the crack section, until the convergence criterion is met. That is, within each element, the principles of the models proposed by Tassios, Laurencet, Rotilio, Navarro and Zanuy are followed.

Oliveira et al. (2007) also use an iterative procedure to find the solution. However, in this case, the length of the finite element is decomposed into two domains where the slip function is (in a simplified manner) assumed to vary linearly. In this way, continuous functions are obtained to describe the strain and stress variation along the element length.

A different approach was adopted by Noh et al. (2003). These authors studied cracked RC ties by subdividing an element (whose length is equal to half the crack spacing) into subintervals with small length in which the bond stress is assumed to be constant. An iterative procedure was employed to derive the cyclic N - ε_{sm} relationship for the tie. The difference in these authors' approach lies in the fact that they used the aforementioned N - ε_{sm} relationship to quantify the tension stiffening effect, which is included in their FE analyses by modifying the steel constitutive law.

Similarly, Kwak et al. (2006) proposed an analysis strategy in which the bond stress-slip relationship is used to quantify the tension stiffening effect. This effect is then included in FE analyses by modifying the stress-strain relationship of the reinforcing bars. However, these authors adopted a very simplistic bond stress-slip relationship.

The previous paragraphs show that several authors have employed analytical methodologies for the analysis of RC elements after cracking, considering explicitly the bond stress-slip

relationship between tensile reinforcements and concrete. The analysis typically focuses on an element whose length is equal to half the crack spacing. The most important differences among the various proposals lie in the constitutive laws adopted to model the concrete behaviour in the fracture zone and the bond stress-slip relationship. Furthermore, some models require that a value for crack spacing is assumed beforehand.

2.4.2 Proposed model

This section presents the numerical model developed in the scope of this thesis. This model was devised so that it could be used for calculation of:

- Bending moment-average curvature relationships, in cracked bridge decks subjected to cyclic loading;
- residual crack openings, taking into account the influence of variable actions;
- the consequences of previously applied heavy loads, in terms of mechanical behaviour of cracked RC beams.

The developed model combines various features of the aforementioned proposals by different authors, considering the approaches that are found to be suitable for the problem under analysis. Special attention is dedicated to the constitutive models for the bond stress-slip and for the normal stress-crack opening relationships.

The proposed numerical model analyses the behaviour of a RC element comprised between a cross-section where a crack occurs and the closest cross-section where the slip between the tension reinforcement and the surrounding concrete is null. At the stabilized cracking stage, this cross-section is located halfway between two consecutive cracks.

The solution is found by using the finite differences technique. In the longitudinal direction, the element under analysis is divided into a number of parts of sufficiently small length, Δz ($n-1$ segments and n sections, as represented in Figure 2.37). The total length of the element under analysis is also named the transmission length, L_r , as it is the distance over which the tension force is progressively transmitted from the reinforcement to the surrounding concrete by bond stresses.

In the transverse direction, the cross-section is decomposed into horizontal layers, which must be symmetric across the vertical axis. Within each layer, constant stress and constant strain is assumed.

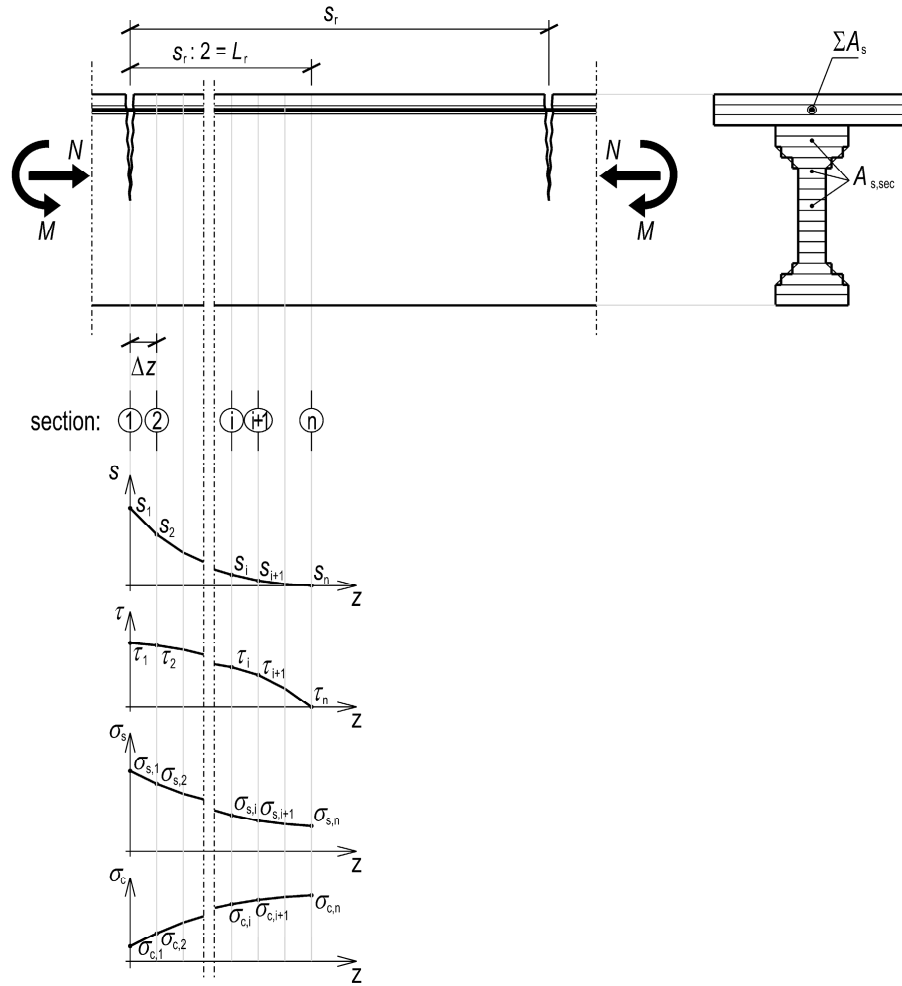


Figure 2.37 - Discretization and distribution of stresses and slip at the level of the main reinforcement.

The influence of cracking in terms of shear deformation is not taken into account in this model, this being the hypothesis followed by the authors quoted in §2.4.1. In this way, the influence of cracking in terms of bending curvature is accurately modelled, but the same can not be said about shear deformations. This effect has been neglected, assuming that the primary reason for the non-linear behaviour after cracking is the non-linear response of the tension chord (Rotilio 1998).

2.4.2.1 Equilibrium and compatibility conditions in the longitudinal direction

The longitudinal distribution of slip, concrete stresses and steel stresses along the transmission length, L_r , follows a linear variation within each segment, as shown in Figure 2.37. The values at the section $i+1$ are derived from those at the previous section i , as follows.

The bond stress τ_i at a given section i is derived from the slip at that same section, s_i , by means of the bond stress-slip law, which is characterized in §2.4.2.3 but can be generically represented as:

$$\tau_i = \tau(s_i) \quad (2.17)$$

The variation of normal stresses in the tension reinforcement between sections i and $i+1$ is determined by:

$$\sigma_{s,i+1} = \sigma_{s,i} - \frac{\tau_i \cdot \Delta z \cdot U}{A_s} \quad (2.18)$$

where U and A_s represent, respectively, the sum of the perimeters and the sum of the cross-sectional areas of all the bars that form the main tension reinforcement. The steel strain is obtained by means of the constitutive law for this material:

$$\varepsilon_{s,i+1} = \varepsilon(\sigma_{s,i+1}) \quad (2.19)$$

The steel is assumed to follow a linear elastic-perfectly plastic behaviour characterized by the modulus of elasticity, E_s , and the yield stress, f_{sy} . The yield stress defines a limit beyond which the model is no longer applicable because such stress levels are out of the scope of the present work.

The variation of the longitudinal force in the concrete surrounding the main reinforcement has the same magnitude and opposite sign as the variation of longitudinal force in the steel. This force is assumed to be distributed over the effective area of concrete in tension, $A_{c,eff}$, which can be quantified according to the prescriptions of the MC90 (Figure 2.38). Thus, the average stress in the effective area of concrete, at the section $i+1$, is given by:

$$\sigma_{c,i+1}^{eff} = \sigma_{c,i}^{eff} + \frac{\tau_i \cdot \Delta z \cdot U}{A_{c,eff}} \quad (2.20)$$

Assuming a linear elastic relationship between the concrete stress and the concrete stress-dependent strain, the concrete deformation in the effective area is given by:

$$\varepsilon_{c,i+1}^{eff} = \frac{\sigma_{c,i+1}^{eff}}{E_c} - \varepsilon_{sh} \quad (2.21)$$

where ε_{sh} represents the absolute value of the concrete shrinkage strain. Concrete creep is not considered because the model was devised for the analysis of cyclic loading effects. The slip at a

given position can be calculated as the integral of the difference between the steel and the concrete deformations, as shown by Equation (2.5). Therefore, the slip value at the section $i+1$ is determined as:

$$s_{i+1} = s_i - (\varepsilon_{s,i+1} + \varepsilon_{s,i}) \cdot \frac{\Delta z}{2} + (\varepsilon_{c,i+1} + \varepsilon_{c,i}) \cdot \frac{\Delta z}{2} \quad (2.22)$$

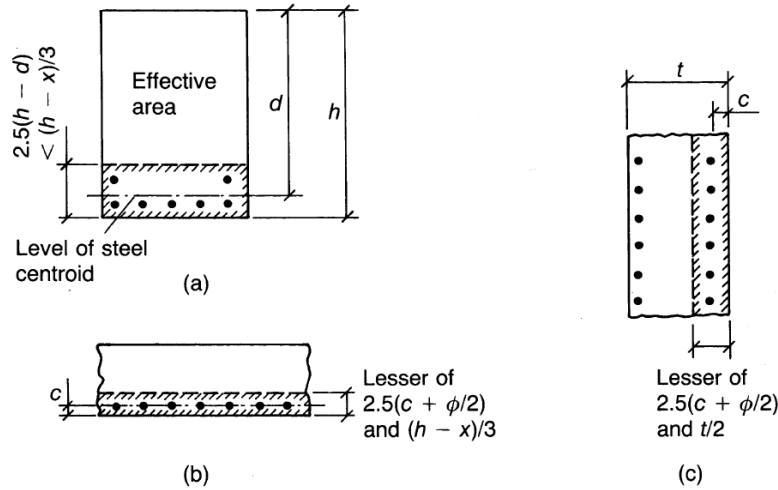


Figure 2.38 – Definition of the effective area of concrete in tension, according to the Model Code 1990, where x represents the neutral axis depth:
a) beam; b) slab; c) member in tension.

2.4.2.2 Cross-sectional analysis

In the previous section, equilibrium and compatibility conditions were established for the tension chord, which comprehends the main tensile reinforcement and the surrounding concrete. The present section deals with the quantification of the strain and stress variations over the various cross-sections.

Different analysis procedures are required for: (a) the crack section; (b) the remaining cross-sections.

At the crack section, the cross-sectional analysis is carried out assuming a linear strain distribution over the depth of the cross-section (Bernoulli Hypothesis). Once the tensile stress at a given concrete fibre reaches the tensile strength f_{ct} during a loading step (that is, once the normal strain reaches the value $f_{ct} / E_c - \varepsilon_{sh}$) the concrete stress becomes governed by the adopted concrete stress-crack opening relationship (see §2.3.2). In this way, the calculation of the concrete stress at a given load step, for a given concrete layer, is based on the previous load history and the crack opening on that layer. The value of the crack opening at a specific concrete

fibre is calculated by assuming a linear variation from zero at the crack tip (see Figure 2.39) up to the value $w_s = 2 \cdot s_l$ at the level of the tension reinforcement (where s_l represents the slip at the first section, that is, the crack section). This hypothesis has already been adopted by authors such as Rotilio (1998). It should be borne in mind that, in the case of the concrete fibres close to the crack tip, w represents, in fact, the magnitude of the *fictitious crack* width, which collects the deformation of the crack band, as explained in §2.3.1.

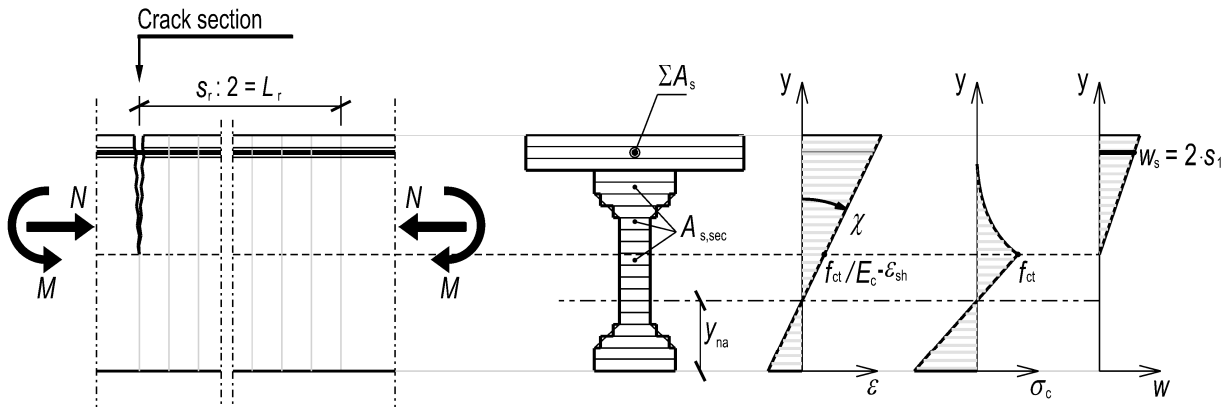


Figure 2.39 - Distribution of concrete stresses and deformations at the crack section.

In the concrete fibres where the tensile strength is not reached, a linear elastic relationship between the concrete stress and the stress-dependent strain is assumed. The non-linear behaviour of concrete in compression is not considered, because this numerical model is devised to analyse the performance of RC members in service.

For a given load step, the solution (average strain and curvature of the cross-section) is found iteratively by means of the Modified Newton Method, considering constant stiffness.

The analysis of the **remaining cross-sections** is carried out in a different manner. As well as in the case of the crack section, a linear strain distribution is assumed to occur over the depth of the cross-section (Figure 2.40).

The main difference between the analysis of these sections and the calculations in the crack section lies in the quantification of the normal stress in the concrete fibres that endure tensile stresses. In this case, this stress is obtained by multiplying the one that corresponds to a linear variation of stresses over the entire cross-section by a coefficient η , as represented in Figure 2.40, so that the influence of concrete cracking can be taken into account. Similar hypotheses have been adopted by other authors (Creazza and Marco 1993; Rotilio 1998) who have analysed the behaviour of RC beams by using comparable analysis approaches.

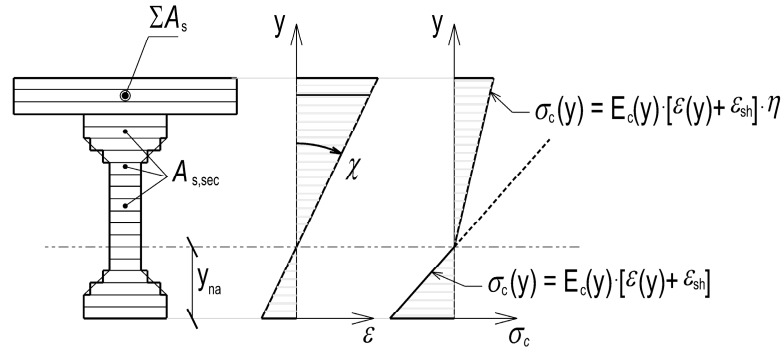


Figure 2.40 - Distribution of concrete stresses and deformations in sections other than the crack section.

Another difference between the analysis of these sections and the analysis of the crack section lies in the problem unknowns. At the crack, the unknowns are the average deformation and the curvature of the cross-section, from which the concrete and the steel stresses can be computed. On the other hand, in the case of the remaining cross sections, the deformation at the level of the main tension reinforcement is known beforehand [as mentioned before, the cross sections are analyzed sequentially, starting from the crack section, and the steel stress at the section $i+1$ is calculated from the one at the previous section by using Equation (2.18)]. Consequently, the unknowns are the curvature and the coefficient η . Once again, the solution is found iteratively by means of the Modified Newton Method.

It is worth mentioning that the concrete stress in the fibres where cracking occurs may take negative (compressive) values. In fact, crack closure is often accompanied by the development of significant compressive stresses, this phenomenon being realistically described by Hordijk's CFM. In the cross sections other than the crack section, the development of compressive stresses in the cracked fibres is reflected by negative η values.

The influence of secondary reinforcements (schematically labelled as $A_{s,sec}$ in Figures 2.39 and 2.40) is also taken into account in the cross-sectional analysis because it may significantly affect the magnitude of the deformation and the crack opening in cracked RC members. The strain (and consequently, the stress) in each layer of secondary reinforcement follows the linear strain distribution of the cross-section. It should be noted that the concrete strain at the level of each secondary reinforcement is not governed by the local bond stress-slip relationship. Instead, the variation of concrete strains along the cross section depth is governed by the CFM or by the parameter η , for crack cross sections or for the remaining ones, respectively. The fact that the bond stress-slip relationship is not explicitly considered at the level of the secondary reinforcements is not relevant as long as the area of the main tensile reinforcement is preponderant.

2.4.2.3 Bond stress-slip relationship

The bond stress-slip law describes the relationship between the local bond stress and the local slip, for uni-directional repeated loading. It comprehends an envelope curve (first loading branch), and unloading and reloading branches.

The first loading branch consists of the envelope curve defined by Equation (2.2). The common values taken by the parameters that define this envelope curve are given in §2.2.5. The shape of unloading and reloading branches is based on the proposal by Morita and Kaku (1973). Even though this is a relatively old model, it was shown in §2.2.7.3 that it seems to be the one that best describes the bond stress-slip history for repeated cyclic loading with small to moderate slip values (see Figure 2.22). The proposal by Viwathanatepa et al. (1979) to describe unloading from intermediate positions of ascending branches other than the monotonic envelope (see Figure 2.23) was also taken into account, since these authors describe events which had not been handled by Morita and Kaku (1973). Unloading and reloading cycles are schematically represented in Figure 2.41a. This figure shows three possible loading-unloading-reloading paths:

O - A - B - C - D - E - F

O - A - B - C - G - H - I - F

O - A - B - C - G - J - K - H - I - F

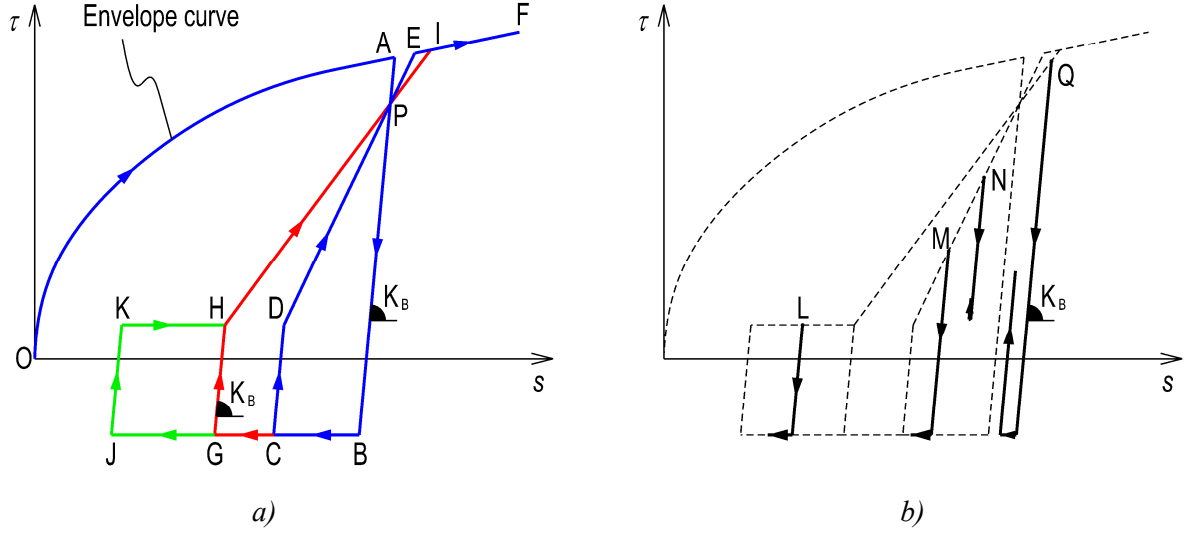
which are defined by the expressions shown in the figure. One can see that unloading and reloading branches are characterized by means of three parameters: K_B , α_B and β_B . Parameters K_B and α_B have already been introduced in §2.2.7.3. Common values taken by these parameters are indicated in Expression (2.15). The parameter β_B defines the position of point P in Figure 2.41a. According to Morita and Kaku (1973) it takes the value:

$$\beta_B = \begin{cases} 0.9 & \text{if } s_A < 0.05 \text{ mm} \\ 0.9 - 4.44 \left(\frac{s_A}{10} - 0.005 \right) & \text{if } 0.05 \text{ mm} \leq s_A < 0.5 \text{ mm} \end{cases} \quad (2.23)$$

where s_A represents the slip of point A in Figure 2.41a, in mm. Figure 2.41b illustrates possible unloading paths from intermediate positions of ascending branches (other than the monotonic envelope).

If the element under analysis is subjected to a large number of load cycles (loading histories composed of several millions load cycles are not unusual), the reproduction of the complete loading history cycle by cycle is not feasible. Rotilio (1998), Laurencet (1999) and Zanuy (2008)

represented the influence of previously applied load cycles by means of the reduced envelope. That approach is also followed in this work. In this way, the influence of the preceding load history is taken into account by considering a reduced envelope which reflects the bond degradation caused by previous cyclic loading. The reduced envelope is given by Equation (2.13), introduced in §2.2.7.2.



$\tau_p = \beta_B \cdot \tau_A$	$s_p = s_A - (1 - \beta_B) \cdot \frac{\tau_A}{K_B}$
$\tau_B = \tau_C = \tau_G = \tau_J = -\alpha_B \cdot \tau_A$	$s_B = s_A - (1 + \alpha_B) \cdot \frac{\tau_A}{K_B}$
	$s_G = \frac{s_A}{2}$
$\tau_D = \tau_H = \tau_K = -\alpha_B \cdot \tau_B$	$s_D = s_C - (1 + \alpha_B) \cdot \frac{\tau_C}{K_B}$

Figure 2.41 - Schematic representation of the adopted bond-slip law.

On the left: loading-unloading-reloading cycles.

On the right: new unloading before the envelope curve is reached.

The reduction of bond stresses owing to the end effect (see §2.2.2) is not considered. This assumption is justified by Ruiz et al. (2007), who suggest that the influence of this phenomenon can be neglected in the analysis of structures in service. Such assumption was also made by other authors (Creazza and Marco 1993; Laurencet et al. 1999; Navarro 2000; Rotilio 1998; Zanuy et al. 2010). Moreover, the reduction of bond stresses due to end effects is implicitly taken into account in the derivation of bond stress-slip laws such as the one presented by Farra (1995), as explained in §2.2.2. Therefore, no modification to account for end effects is considered when such bond stress-slip relationships are employed.

2.4.2.4 Concrete stress-crack opening relationship

In this work, the cyclic behaviour of concrete in tension is generally simulated by using Hordijk's CFM, which provides a realistic description of the concrete behaviour, as mentioned in §2.3.2. However, it was found that the very large stiffness for small reloading steps, in this model, gives rise to convergence difficulties. As justified below (§2.5.1.3), this problem can be solved by adopting a slightly lower stiffness for small reloading steps. Therefore, an alternative reloading branch was adopted, which consists of a modified version of the Focal Point Model (FPM) proposed by Yankelevsky and Reinhardt (1989). In §2.3.2, it was explained that the FPM describes the unloading and reloading paths by means of a series of straight branches. In the proposal by Yankelevsky and Reinhardt (1989), the tangent to the first reloading branch (GH in Figure 2.42) is vertical, i.e., $w_H = w_G$. Thus, the claimed stiffness decrease was implemented by adopting a slightly increased abscissa for the point H. This increase is expressed as a function of the maximum crack opening reached before unloading, w_A , by introducing the dimensionless constant A_{FPM} :

$$w_H = w_G + \frac{w_A}{A_{FPM}} \quad (2.24)$$

In this work, A_{FPM} typically takes a value of 100. It should be noted that this is a minor modification of the FPM proposed by Yankelevsky and Reinhardt. Figure 2.43, which compares three alternative reloading branches (for an illustrative maximum crack opening before unloading of 0.05 mm), shows that the modified version of the FPM is very similar to the original one, which, in turn, is not considerably dissimilar from the reloading branch of the CFM. Nevertheless, as shown below, this modification not only solves convergence difficulties but also leads to a closer agreement with experimental results for small reloading steps.

Finally, it is worth mentioning that the comparison between the FPM and test data, presented by Yankelevsky and Reinhardt (1989), suggests that the tangent to the stress-crack opening relationship, at the start of the reloading branch is actually not vertical and, therefore, the proposed modification is justifiable.

2.4.2.5 Iterative calculation of the slip distribution

The solution is found iteratively, by trying successive values for the slip at the crack cross section, according to the bisection method, seeking the value which guarantees that all the boundary conditions are satisfied. The boundary conditions for a RC tie subjected to axial loading are explained below. It should be noted that this explanation is also applicable to the

tensile chord of RC beams, composed by the main tensile reinforcement and the effective area of concrete in tension.

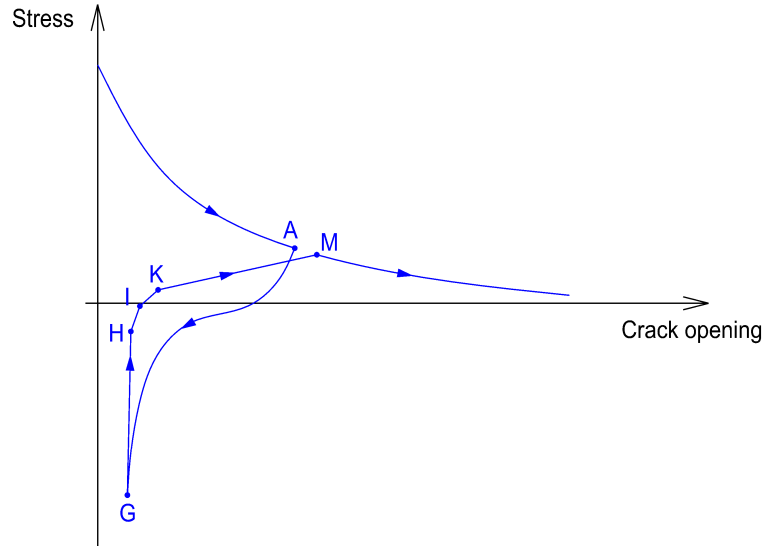


Figure 2.42 – Reloading branch of the normal stress-crack opening relationship according to the modified version of the FPM.

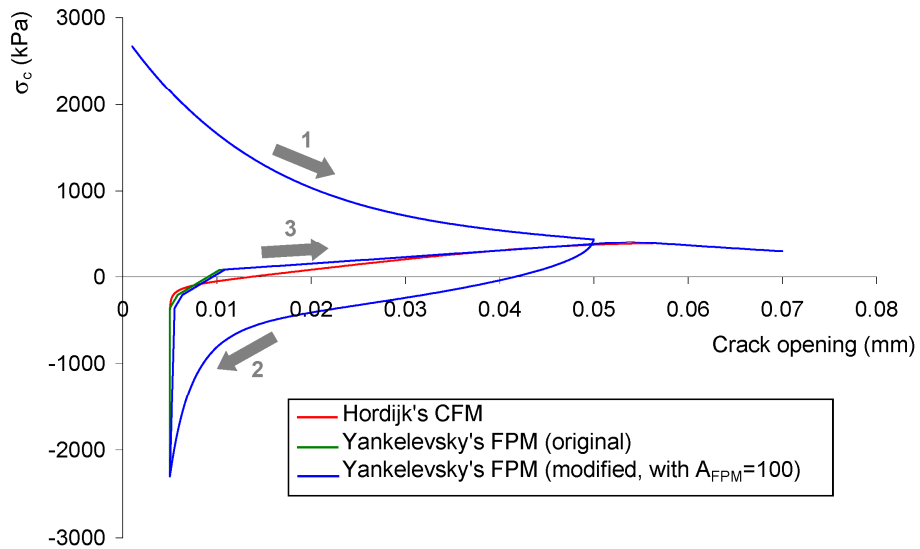


Figure 2.43 – Illustrative comparison of different alternatives for the reloading branch of the normal stress-crack opening relationship (considering $f_{ct} = 2.9 \text{ MPa}$ and $G_F = 0.075 \text{ Nmm/mm}^2$).

At the cross-section located halfway between two adjacent cracks, symmetry considerations impose that the slip value equals zero. The remaining boundary conditions can be understood by observing Figure 2.44.

At the crack formation stage, when the applied load N is lower than or equal to the cracking force N_{cr} (Figure 2.43a), cracking is not extended over the entire length of the tie. Consequently, the transmission length L_r extends between the crack section and the section where the uncracked state (state I) is reached. At the end of the transmission length, the slip equals zero. Moreover, the first derivative of the slip is also zero, since equal strains are observed in the steel and in the concrete [see Equation (2.6)].

Figure 2.44b represents the strain distribution corresponding to a monotonically increasing load with magnitude N , higher than the cracking load N_{cr} . Walraven (1999) comprehensively explains that, at this phase (the stabilized cracking phase), the crack spacing, s_r , theoretically takes values between $L_{r,max}$ and $2 \cdot L_{r,max}$, where $L_{r,max}$ represents the maximum value of the transmission length. $L_{r,max}$ is the length along which the tensile force transmitted to the concrete leads to a tensile stress f_{ct} at the section located halfway between cracks, that is, at the section where the slip equals zero. It should be noted that $L_{r,max}$ decreases as the applied load N increases (increasing slip values lead to higher bond stresses, and consequently, a faster transmission of tensile stresses to the concrete) which justifies the occurrence of new cracks when a RC tie is subjected to increasing tension forces.

The explanation presented in the previous paragraph is also valid in the presence of concrete shrinkage. In that case:

- at the uncracked state, the concrete tensile stress is higher than the one that would occur if shrinkage didn't exist;
- after cracking, increasing shrinkage values lead to increasing slip values, which affects the distribution of bond stresses, normal stresses and crack openings.

In the developed numerical methodology, three different analysis types can be distinguished.

- 1) For increasing loads lower than the cracking load, during the crack formation stage (Figure 2.44a), the solution can be found by seeking the slip value at the crack which guarantees that, at the end of the maximum transmission length, both the slip value and the first derivative of the slip function are null. In this case, the maximum transmission length is also one of the unknowns, i.e., this analysis can be carried out with the purpose of determining the maximum transmission length. Note that the analysis for applied loads lower than the cracking load might be of interest for members in which cracking occurs due to imposed deformations.
- 2) For increasing loads higher than the cracking load, during the crack formation stage, the first derivative of the slip function at the end of the maximum transmission length is no longer null, because the concrete stress is limited to f_{ct} , and consequently, the equality of strains in the steel and in the concrete can not be reached. Therefore, the solution can be

found by iteratively seeking the slip distribution which guarantees that $s = 0$ and $\sigma_c = f_{ct}$ at the end of the maximum transmission length $L_{r,max}$. In this case, $L_{r,max}$ is also one of the unknowns. This analysis type can be used to calculate the average crack spacing s_{rm} during the stabilized cracking phase. Following the suggestions of several authors (CEB 1993; Farra 1995; Jaccoud 1987; Walraven 1999) the end of the crack formation stage can be assumed to occur for an axial load approximately 30% higher than the cracking load. Moreover, the average crack spacing is usually taken as 2/3 of the maximum crack spacing. Therefore, the average crack spacing is given by:

$$s_{rm} = 2 \cdot \left(\frac{2}{3} L_{r,max} \right) \quad (2.25)$$

$L_{r,max}$ being determined for a load 30% higher than the cracking load.

- 3) Once the crack spacing has been determined, the mechanical response due to cyclic loading, during the stabilized cracking phase (Figure 2.44b), can be calculated by seeking the slip distribution which, for each load step, guarantees that the slip vanishes at the end of the transmission length (which is equal to half the crack spacing).

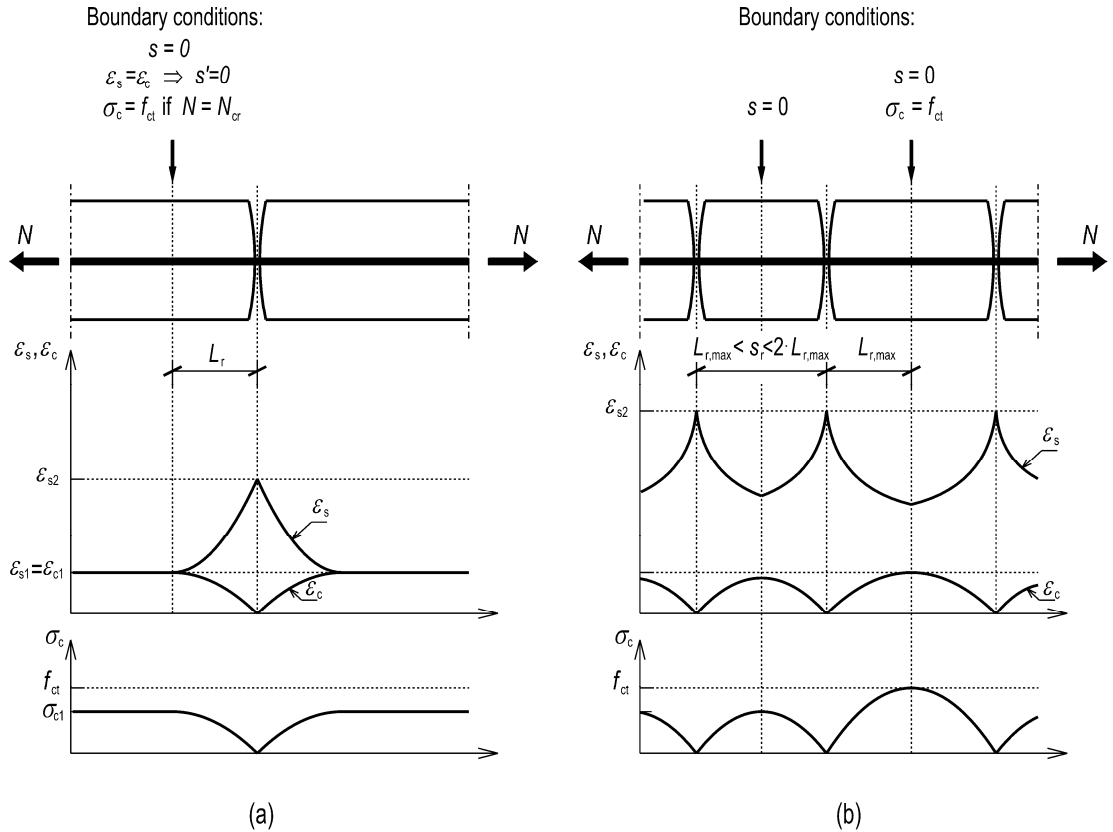


Figure 2.44 – Strain and stress distribution in a tension tie: a) $N \leq N_{cr}$; b) $N > N_{cr}$.

Two different models were implemented.

- The first one was named the Cracked Beam Model, and is suitable for the analysis of beams subjected to constant axial forces (for simulation of prestressing forces, for example) and variable bending moments. In this case, the cross-sectional analysis is performed through the procedure described in §2.4.2.2. The flowchart depicted in Figure 2.45 shows the main calculation steps.
- The second one was named the Cracked Chord Model, and is suitable for the analysis of RC ties subjected to variable histories of axial forces. In this case, the cross section needs not to be decomposed into layers and the section curvature is null.

The employed programming language was Visual Basic for Applications, from Microsoft.

2.4.2.6 Relevant analysis results

The most relevant analysis results, which are obtained for each load step, are indicated in Table 2.3.

Table 2.3 – Analysis results.

scalar values	• crack opening at the level of the main tensile reinforcement, w_s • average curvature, χ_m • average steel strain, ε_{sm}
variation over the depth of the crack section	• crack opening • strain and stress in the various concrete and steel layers
variation over the transmission length	• slip • bond stress • neutral axis depth • curvature • parameter η (see Figure 2.40) • strain and stress in the effective area of concrete and in the main tensile reinforcement

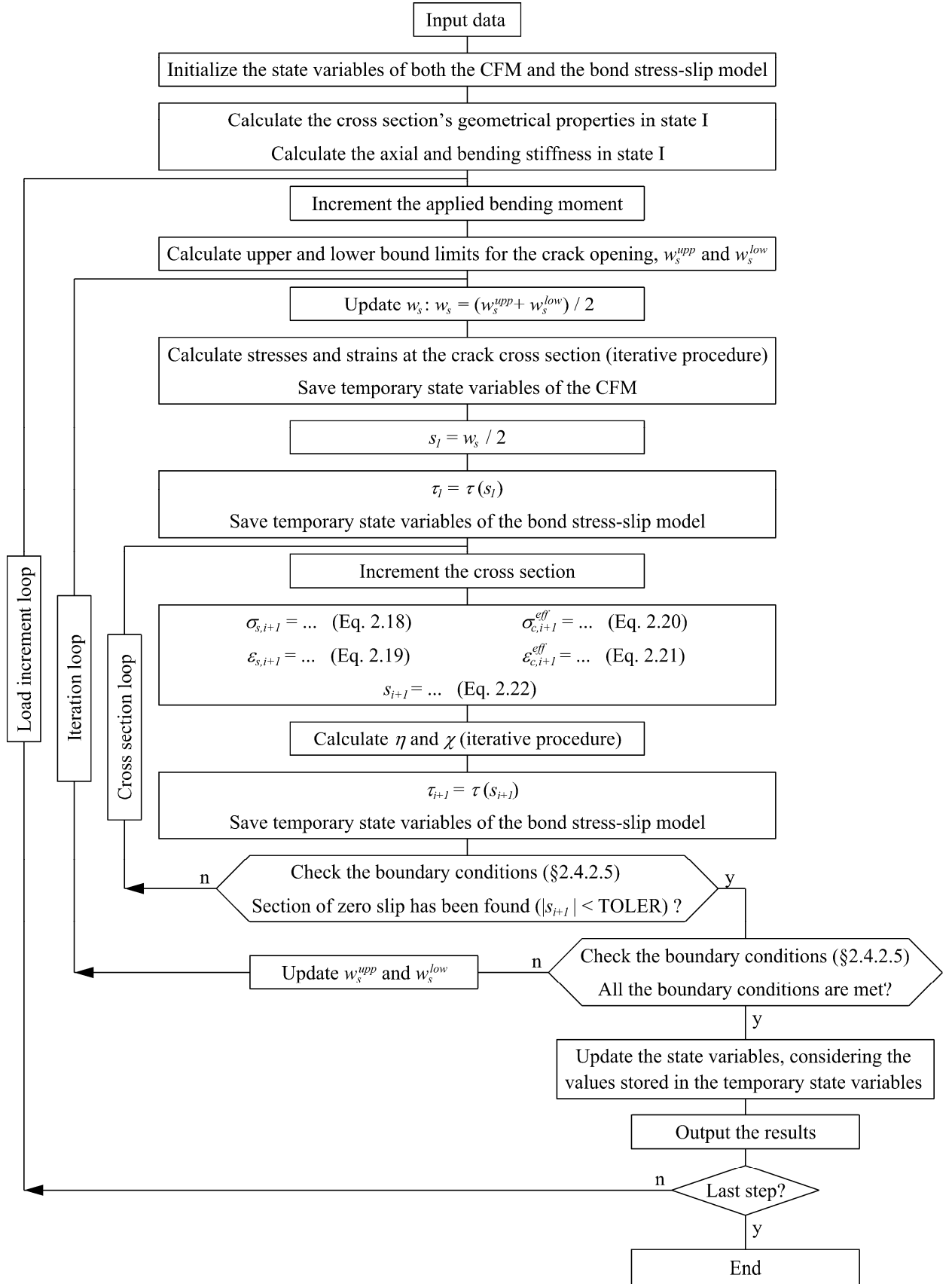


Figure 2.45 – Cracked Beam Model's flowchart.

The mean values for the curvature and for the steel strain are calculated as the average of the individual results obtained for the n cross sections in which the transmission length is decomposed. The relevance of the tension stiffening effect, i.e., the stiffness variation due to the contribution of the concrete between cracks, can be ascertained by means of the parameter m , defined as:

$$m = \frac{\varepsilon_{sm}}{\varepsilon_{s,crack}} \quad (2.26)$$

where $\varepsilon_{s,crack}$ represents the strain in the main tensile reinforcement at the crack section and ε_{sm} is the average strain, in the same reinforcement, along the transmission length. It is well known that, for monotonically increasing loads, the average steel strain is lower than the strain at the crack, which corresponds to $m < 1$. However, upon unloading, the parameter m can take values higher than 1, which means that the non-linear bond-slip and stress-crack opening relationships give rise to average steel strains larger than the value observed at the crack cross section. Values of m higher than 1 correspond to a negative tension stiffening effect, which has important implications in the calculation of bridge-deck deformations, as recognized by Muttoni and Ruiz (2007). These effects can be properly quantified by using the developed model.

2.5 Validation of the proposed numerical model

In this section, the results provided by the developed model are validated through the comparison with experimental results provided by Laurencet (1999) and Neild et al. (2002).

2.5.1 Comparison with experimental results by Laurencet

2.5.1.1 Introduction

In this section, the numerical results provided by the Cracked Chord Model are compared with experimental results obtained by Laurencet (1999), for axially loaded RC ties. The tested specimens are 4.5 m long and the cross section is rectangular with dimensions 0.80 m $\times$ 0.25 m. Measurements of deformation and crack openings were taken along the central 2.6 m. Concrete properties were evaluated in $\phi 150$ mm $\times$ 300 mm cylinders, at 28 and 60 days (Table 2.4). The tests were carried out 7 to 8 weeks after casting the specimens.

Table 2.4 - Concrete properties measured in $\phi 150 \text{ mm} \times 300 \text{ mm}$ cylinders.

	Concrete age	
	28 days	60 days
f_{cm} (MPa)	42.3	47.8
f_{ctm} (MPa)	2.6	2.6
E_{cm} (GPa)	--	37.7

Figure 2.46 presents the experimental results obtained by Laurencet (1999) for one of the tested RC ties (P22). The longitudinal reinforcement of the tie is composed of eight 12 mm rebars. Before the start of the test, the specimen was pre-cracked by means of an imposed deformation, whose average value reached 0.04%. The figure shows the variation of the crack opening in the observed cracks, for two load cycles after pre-cracking.

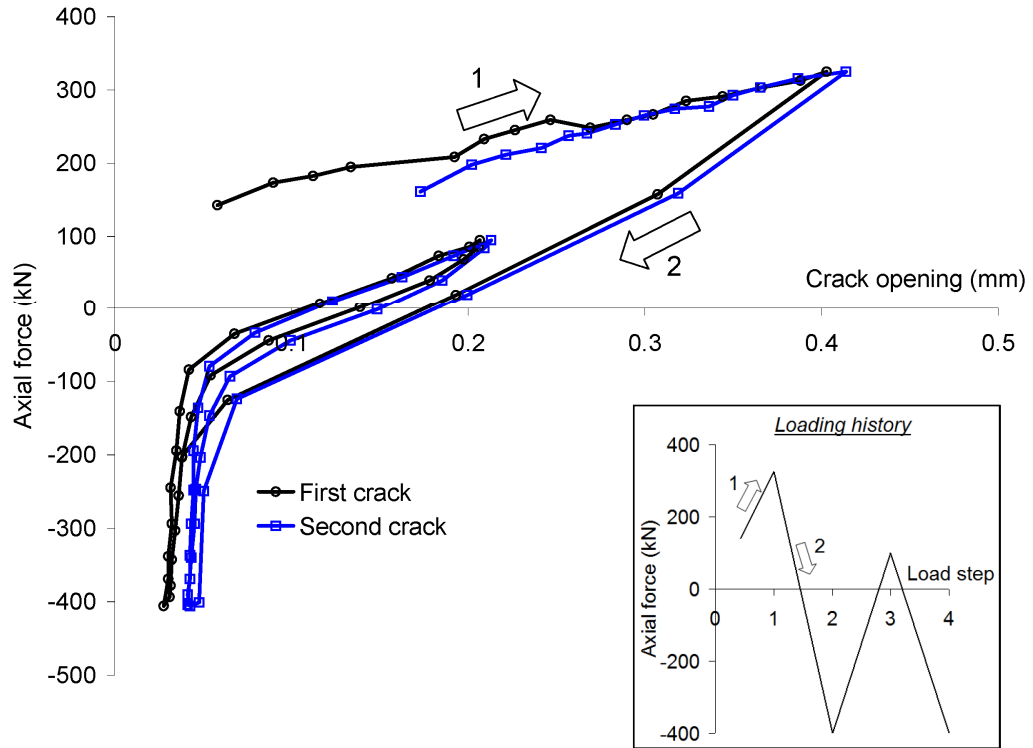


Figure 2.46 - Experimental results by Laurencet (1999) - RC tie P22.

2.5.1.2 Parametric analysis

An accurate calculation of the observed crack opening can only be obtained if the real bond characteristics are known. However, the bond stress-slip relationship can not be precisely established, due to the high variability associated to bond action. Moreover, other concrete properties have non negligible variability, which contributes to the discrepancies between calculated and observed crack openings. Therefore, a parametric analysis is presented in this

section, aiming at evaluating the relevance of the uncertainty associated to the quantification of relevant material parameters.

The variable parameters (or groups of parameters) considered in the parametric analysis are as follows:

- the fracture energy of concrete;
- parameters that establish the concrete strength and stiffness;
- parameters that define the ascending branch of the bond stress-slip relationship;
- parameters that characterize the descending branch of the bond stress-slip relationship.

The upper and lower bound values taken by those parameters (or groups of parameters) are indicated in Table 2.5.

Table 2.5 – Parametric analysis: upper and lower bound values taken by the variable parameters.

Parameter (or group of parameters)	Upper value	Lower value
Concrete Strength and stiffness	$f_{cm} = 48 \text{ MPa}$ $E_{cm} = 38 \text{ GPa}$ $f_{ctm} = 2.6 \text{ MPa}$ (values based on tests at 60 days, see Table 2.4)	$f_{cm} = 38 \text{ MPa}$ $E_{cm} = 33 \text{ GPa}$ $f_{ctm} = 2.6 \text{ MPa}^{(1)}$ (concrete grade C30-37, according to the EN 1992-1-1)
Fracture energy of concrete, $G_F^{(2)}$	$G_F = 1.5 \cdot 0.0942 = 0.1413 \text{ kNm/m}^2$	$G_F = 0.5 \cdot 0.0942 = 0.0471 \text{ kNm/m}^2$
Ascending branch of the bond stress-slip relationship	Stiff ascending branch, according to the proposal by Farra (1995), see Figure 2.47	Soft ascending branch, according to the proposal of the MC90 for poor bond conditions, see Figure 2.47
Unloading branch of the bond stress-slip relationship	Stiff unloading: $K_B = 500 \text{ MPa/mm}$ $\alpha_B = 0.25$	Soft unloading: $K_B = 200 \text{ MPa/mm}$ $\alpha_B = 0.15$

Notes: ⁽¹⁾ According to the EN 1992-1-1, $f_{ctm} = 2.9 \text{ MPa}$ for concrete grade C30/37. In this case, 2.6 MPa was adopted, so that the upper value is not exceeded.

⁽²⁾ 0.0942 kNm/m² is the value of the fracture energy given by the MC1990 for a concrete grade C30 and a maximum aggregate size of 20 mm.

As regards the ascending branch of the bond stress-slip relationship, two extreme curves were chosen from the various proposals presented in §2.2.5. The adopted curves are represented in Figure 2.47, where one can notice the great difference between the two proposals. These

dissimilar curves¹ were chosen aiming at evaluating the impact of the variation of bond-slip characteristics in terms of variability of the results. It is worth mentioning that the MC90 proposal for poor bond conditions provides a lower bound estimate for the bond stresses that will most likely not be reached in this case, not only because the bond conditions are not so unfavourable in the structure under analysis (given the reduced thickness of the specimen), but also because the MC90 proposal often underestimates the real bond stresses (as acknowledged by several authors, see §2.2.5).

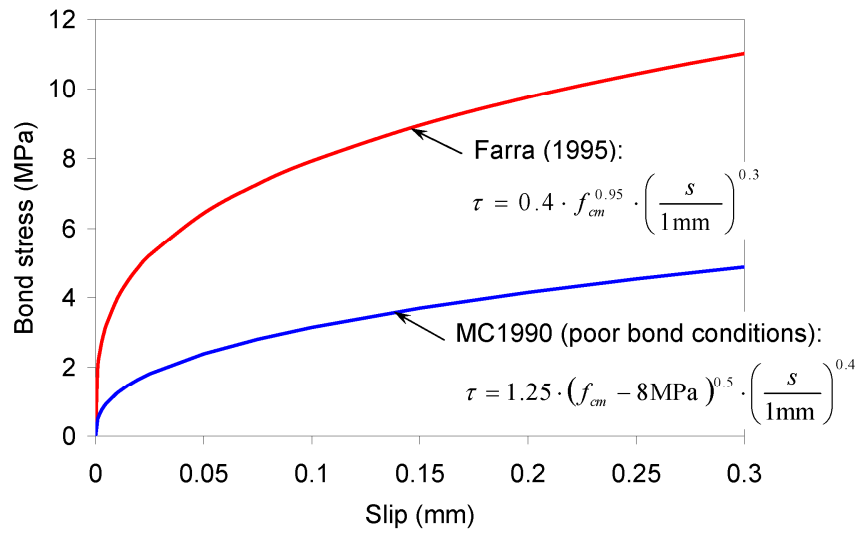


Figure 2.47 – Ascending branch of the bond stress-slip relationships considered in the parametric analysis ($f_{cm} = 48 \text{ MPa}$).

As concerns the descending branch of the bond-stress slip relationship, the upper and lower values taken by the parameters K_B and α_B correspond to the range of variation usually taken by different authors (see §2.2.7.3).

The variability of the stress-crack opening relationship is considered in a simplified manner, by taking different values for the fracture energy of concrete, G_F .

In the parametric analysis, all the possible combinations of the upper and lower bound values for the four parameters (or groups of parameters) were analysed, which results in $2^4 = 16$ different scenarios.

¹ Note that the bond stress-slip relationship also depends on the concrete compressive strength, which was considered not to be accurately known in this case. The curves in Figure 2.47 were calculated for $f_{cm} = 48 \text{ MPa}$, which corresponds to the adopted upper bound value, the lower bound being taken as $f_{cm} = 38 \text{ MPa}$. This fact leads to an even greater difference between the upper and lower bond stress-slip relationships considered in the parametric analysis.

Given the fact that the stabilized cracking phase was not reached during the tests, the calculations were performed considering that the slip between the steel and the surrounding concrete occurs along the maximum transmission length. For each different scenario, the maximum transmission length was calculated under the effect of the maximum load (325 kN), according to the procedure described in §2.4.2.

Figure 2.48 depicts the results of calculations for the load step in which the axial force varies from 325 kN (tension) to -400 kN (compression), that is, the first unloading branch.

Under the effect of the maximum load, a large scatter of results is observed, as the calculated crack width is strongly influenced by the parameters that define the ascending branch of the bond stress-slip relationship. The stiffer bond-slip curve leads to crack openings lower than the observed values, and vice versa. This figure also suggests that the actual bond stress-slip relationship is in-between the considered upper and lower bound curves.

On the contrary, the variability of the results under minimum load is diminutive, the range of calculation results being similar to the difference between the results observed in the two monitored cracks. The figure also shows that the calculated crack opening under minimum load only depends on the value taken by G_F , that is, the normal stress-crack opening relationship is paramount when the tie is subjected to significant compressive forces.

Moreover, Figure 2.48 shows that the relationship between the axial force and the crack opening is highly non-linear, due to the nonlinearity in the unloading branches of both the bond stress-slip relationship and the stress-crack opening model.

Figure 2.49 depicts the calculation results in terms of relationship between the axial force and the average deformation along the transmission length, for the same load step. This figure shows that:

- this relationship is also highly non-linear;
- important residual deformations occur upon unloading, when the applied axial force vanishes;
- the variability of the calculated average deformations under the effect of maximum load is significantly lower than that of the calculated crack openings.

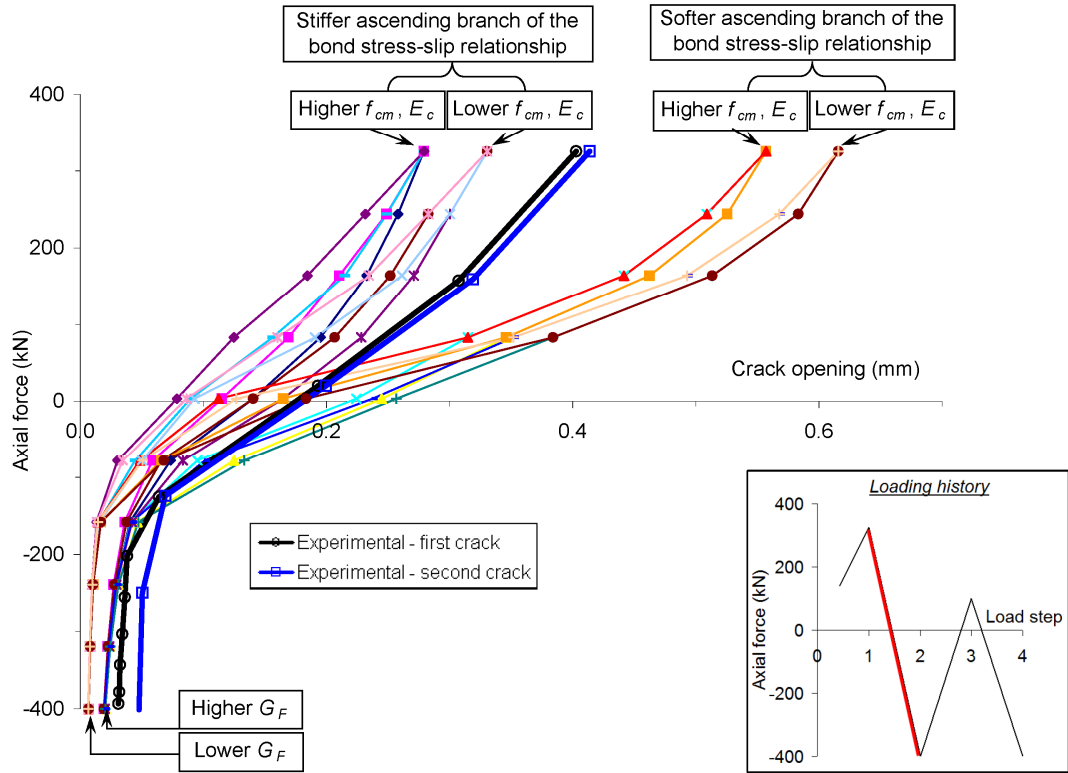


Figure 2.48 – Results of the parametric analysis: crack opening.

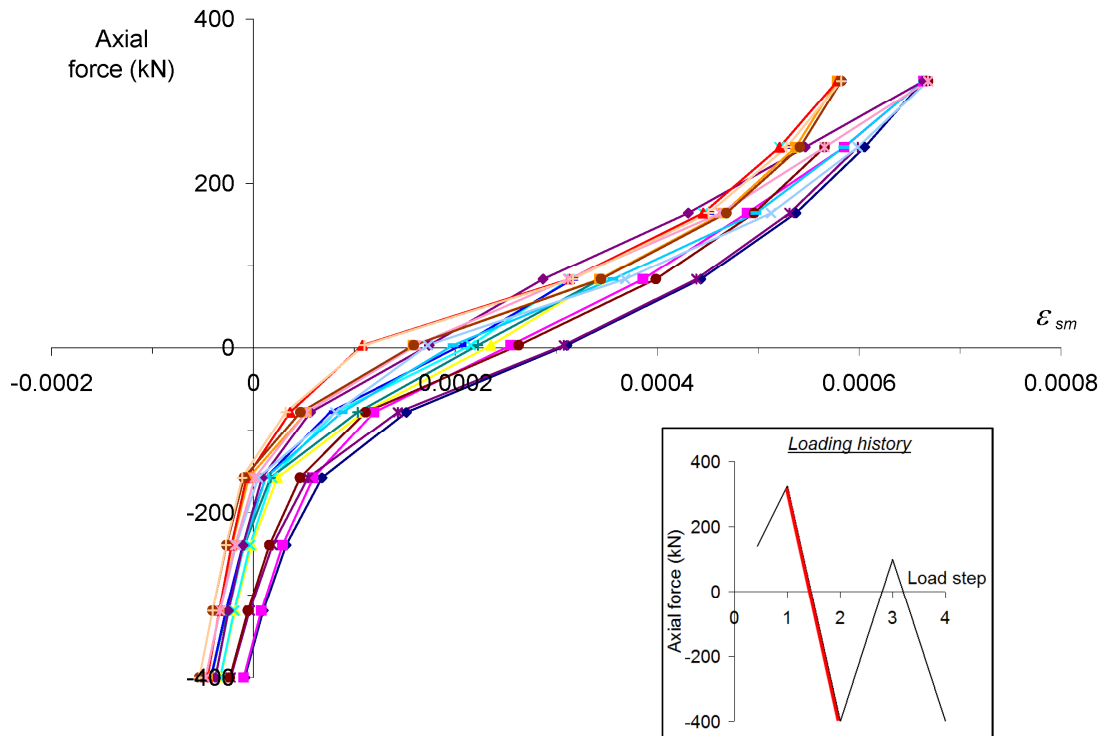


Figure 2.49 – Results of the parametric analysis: average deformation (the colour scheme is the same that was adopted in Figure 2.48).

2.5.1.3 Reloading steps

The capability of the numerical model to reproduce the results observed upon reloading is discussed in this section.

It was shown before that the actual value of the crack opening can only be accurately estimated if an adequate bond stress-slip relationship is adopted (this being particularly true if the RC tie is subjected to large axial forces). Therefore, before the behaviour upon reloading is analysed, a realistic ascending branch of the bond-stress-slip relationship is derived. As mentioned before, plausible bond stress-slip relationships can be obtained by means of inverse analyses, based on observed crack openings (for known axial forces). In §2.2.5, it was shown that the ascending branch of the bond stress-slip relationship usually takes the following configuration:

$$\tau = \tau_{\max} \cdot \left(\frac{s}{s_p} \right)^b \quad (2.27)$$

The coefficient s_p is usually taken as 1 mm, this being the value considered in this study. The coefficient b , which defines the shape of the curve, usually takes values in the range of 0.21 to 0.4 (see Table 2.1). Its influence is illustrated in Figure 2.50.

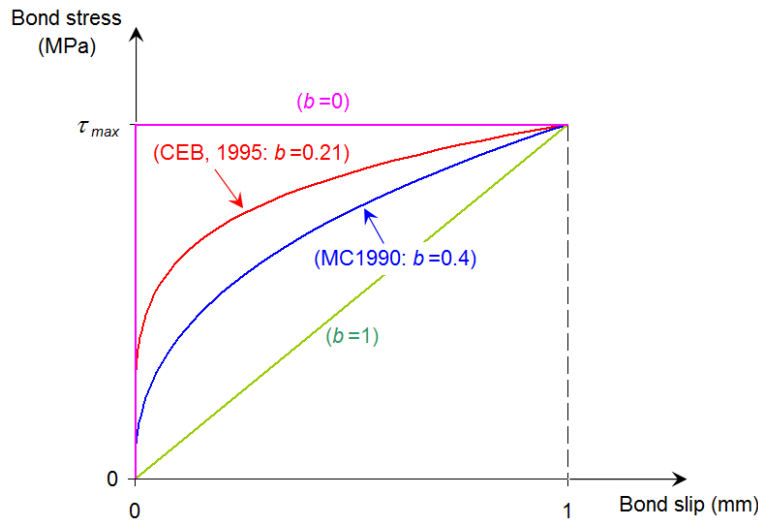


Figure 2.50 - Shape of the ascending branch of the bond stress-slip relationship, depending on the coefficient b .

If the shape of the curve suggested by the MC90 is adopted ($b = 0.4$), and an inverse analysis is conducted in order to determine the bond stress-slip curve that leads to the observed crack opening under maximum load, one can conclude that $\tau_{\max} = 2 \cdot \sqrt{f_{ck}}$ guarantees a satisfactory agreement with the experimental results (considering plausible values for the concrete properties: $f_{cm} = 48$ MPa; $f_{ck} = 40$ MPa; $E_c = 38$ GPa; $f_{ctm} = 2.6$ MPa; $G_F = 0.0942$ kNm/m²). Note that the

MC90 suggests $\tau_{\max} = 2.5 \cdot \sqrt{f_{ck}}$ for good bond conditions and $\tau_{\max} = 1.25 \cdot \sqrt{f_{ck}}$ for other bond conditions. The result of the inverse analysis is in-between and is, therefore, a plausible result.

Figure 2.51 displays the calculated variation of the crack opening during the unloading-reloading cycle, considering the previously derived ascending branch of the bond stress-slip relationship and adopting $K_B = 400$ MPa/mm and $\alpha_B = 0.25$ for definition of the descending branch. It was verified that similar results are achieved if the parameters K_B and α_B take the upper and lower bound values indicated in Table 2.5.

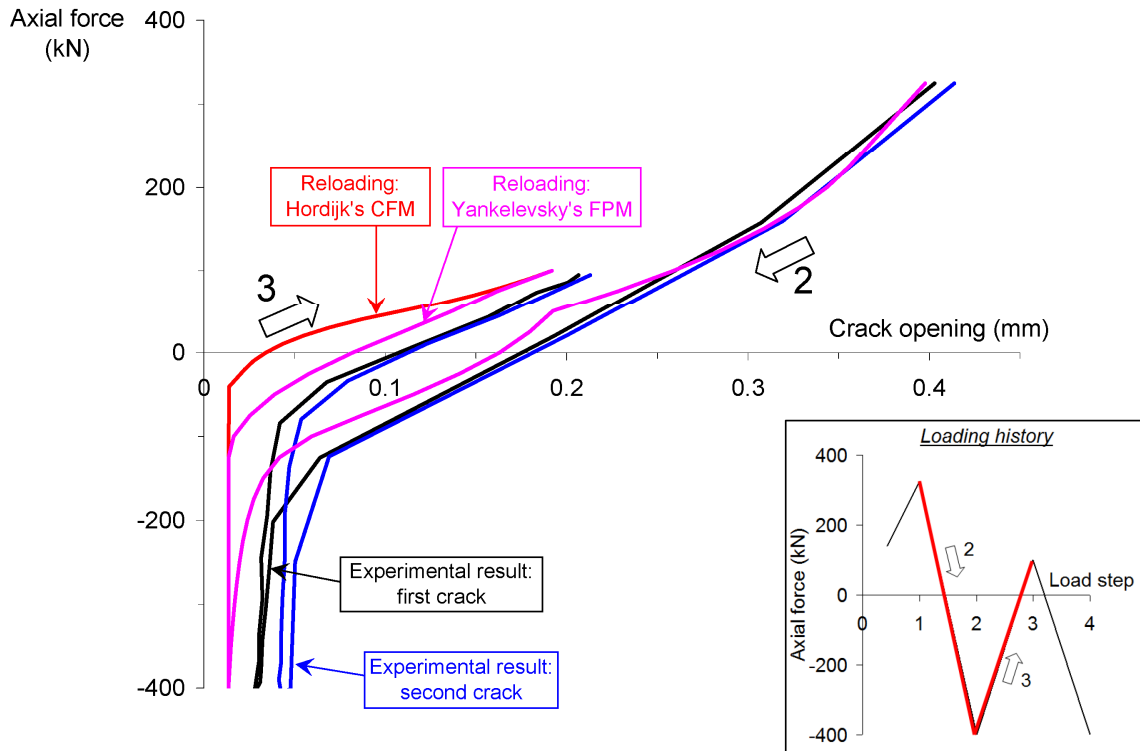


Figure 2.51 – Comparison of experimental results for the unloading-reloading cycle and calculations considering a realistic ascending branch of the bond stress-slip relationship.

In Figure 2.51, one can conclude that the calculated unloading curve closely follows the experimental ones. A discontinuity can be observed in the calculated curve, when the crack opening reaches ~ 0.2 mm. This discontinuity went unnoticed in the results of the parametric analysis presented before, since larger load steps were adopted in those calculations. The reason for this discontinuity can be understood by analysing Figure 2.52. The graph on the left shows the calculated axial force-crack opening relationship, for two different scenarios:

- considering that the normal stress-crack opening relationship is given by the CFM;
- considering that, upon unloading, the normal stress-crack opening relationship is given by a straight line towards the origin (even though this scenario is not realistic, it is considered here with the purpose of evaluating the consequences of this simplification).

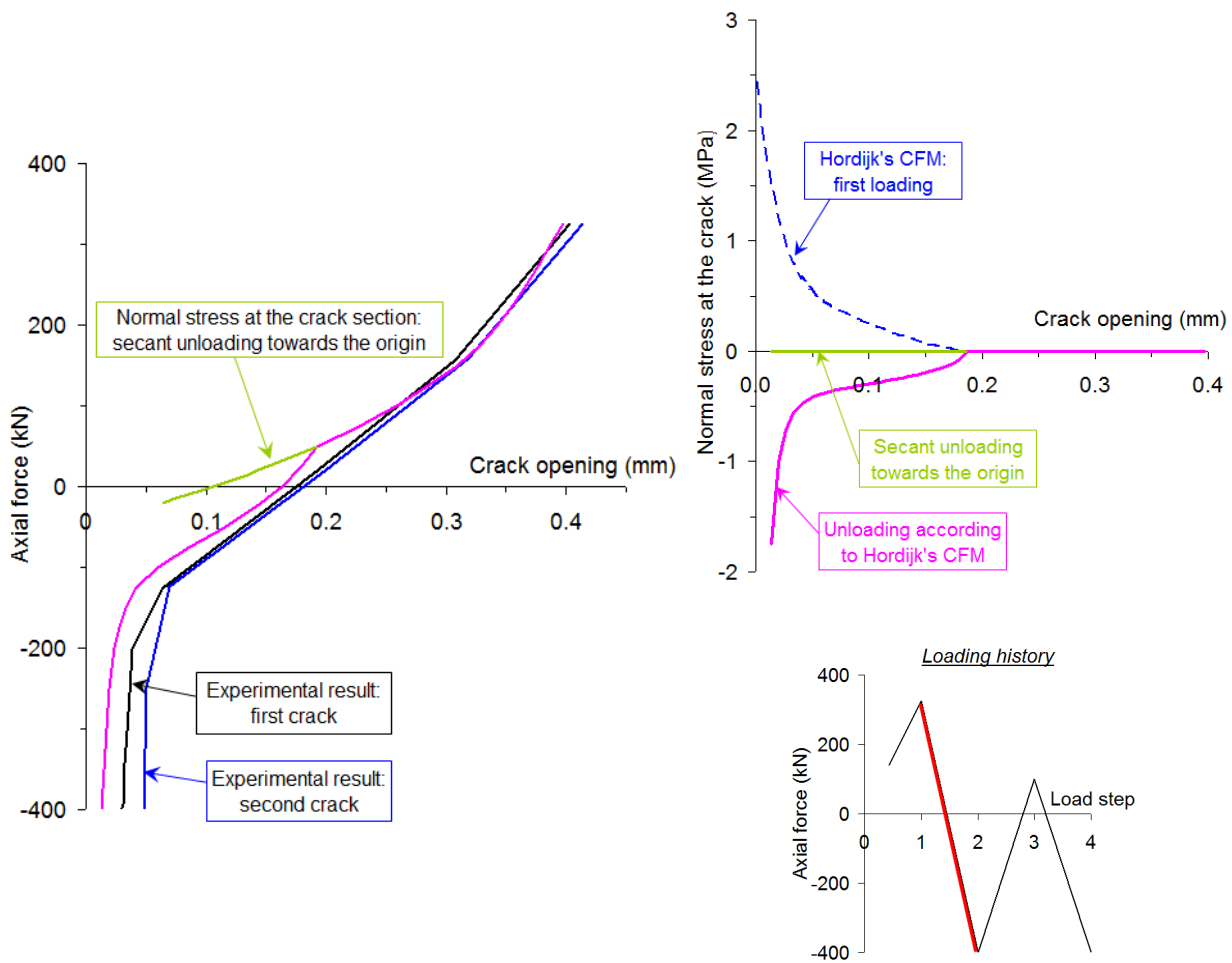


Figure 2.52 - Justification of the discontinuity observed in the calculated results, during unloading. Normal stress-crack opening relationship, crack opening and loading history.

The two models used to describe the variation of the normal stress upon crack closure are shown on the right-hand side of Figure 2.52. It becomes clear that the discontinuity corresponds to the point in which compressive stresses start to develop at the crack, due to the imperfect match of the two crack faces. From that point onwards (for smaller crack openings), considering secant unloading towards the origin leads to incorrect results, which significantly deviate from the experimental ones.

The discontinuity under analysis is not evident in the experimental results. This is presumably due to the fact that the transition from the phase in which the crack faces are completely separated to the phase in which compressive stresses start to develop is smoother in the test (when compared to the constitutive model adopted in the calculations).

As far as the reloading phase is concerned, two different calculation results are presented in Figure 2.51:

- the results of a calculation in which the reloading phase of the normal stress-crack opening relationship is given by the CFM;
- the results of an alternative calculation in which the Yankelevski's FPM is employed to describe the reloading in the normal stress-crack opening relationship.

As mentioned before, Hordijk's CFM is generally employed in this work to describe the cyclic behaviour of concrete in tension. This model provides a realistic description of the actual concrete behaviour and is suitable for numerical implementation, since each branch between load reversals is described by a single continuous function. However, it was found that this model is responsible for convergence difficulties that occur when the RC tie is subjected to a small increment of tensile force after a significant compressive force has been applied. The reason for that difficulty can be understood by observing Figure 2.53. This figure represents a situation in which the concrete is subjected to significant compressive stresses after unloading (-1.91 MPa). Upon reloading, according to the CFM, an infinitesimal increment (dw) of crack width causes a large variation of normal stress. In this example, if dw is equal to the smallest increment which can be correctly handled by Microsoft Excel 2003 ($10 \cdot 10^{-20}$ m), then the stress increment equals 1.84 MPa. In this way, the smallest force increment for which convergence is possible upon reloading is equal to the force that causes a concrete stress increment of 1.84 MPa. For smaller force increments, convergence is not possible.

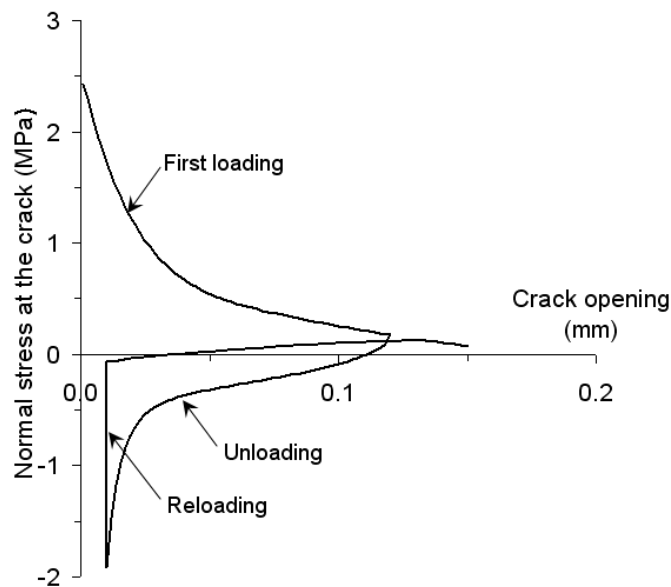


Figure 2.53 – Reloading according to Hordijk's CFM.

As a matter of fact, this convergence difficulty could be solved by using a different iterative procedure. However, it was found that using a different constitutive model to describe the reloading path of the stress-crack width relationship (the Yankelevski's FPM, modified as described in §2.4.2.4) not only resolves the convergence difficulty, but also leads to a closer

agreement with the experimental results (see Figure 2.51). In the calculations according to the FPM, and considering the force increment from -400 kN up to 100 kN, the horizontal offset between the observed crack opening and the numerical result is kept approximately constant (roughly equal to the small difference that exists at the end of the previous unloading phase).

As will be shown below, similar conclusions were obtained in the comparison between numerical results and experimental data obtained by Neild et al. (2002): the modified Yankelevski's FPM solves convergence difficulties and provides a better agreement with the experimental results for small reloading steps.

In order to evaluate the implications of the uncertainty associated to the quantification of the various material parameters, a parametric analysis of the structure response upon reloading was also carried out. All the scenarios introduced in §2.5.1.2 were considered. The results, which are displayed in Figure 2.54, show that the tendency observed in the experimental results is followed by the numerical results for all the scenarios. However, the scatter of the results increases as the axial force increases, like in the case of the unloading stage. Moreover, the observed crack openings under maximum load are also in-between the calculation results considering upper bound and lower bound bond-slip curves.

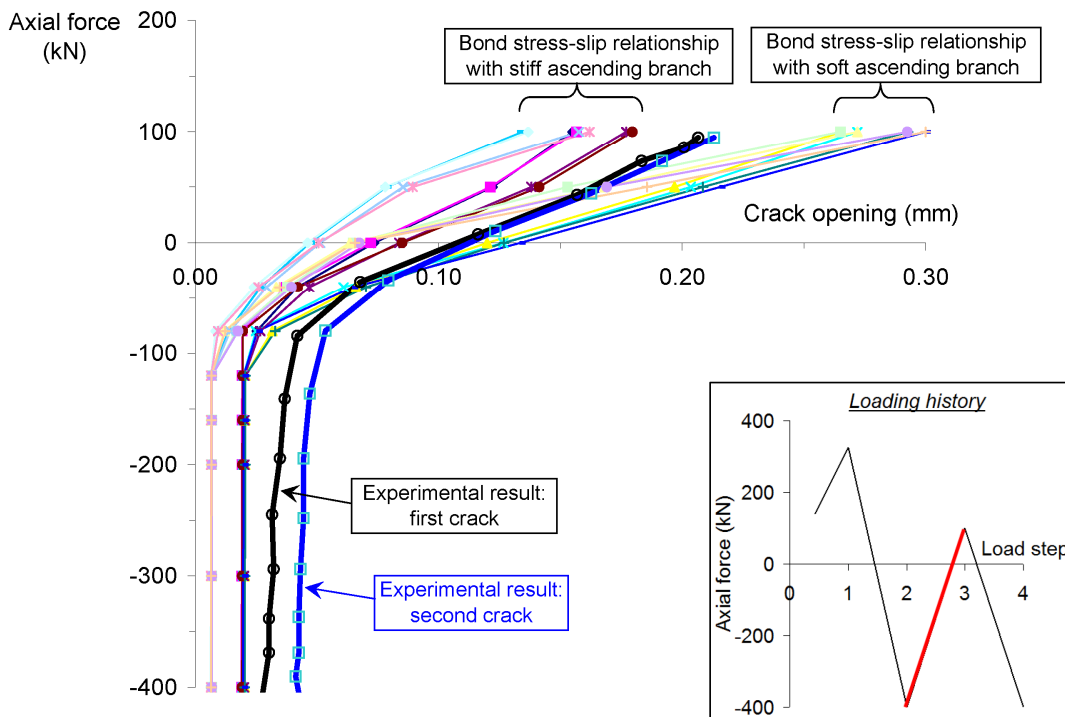


Figure 2.54 - Results of the parametric analysis (reloading according to the Yanklevsky's FPM): crack opening.

2.5.2 Comparison with experimental results by Neild

2.5.2.1 Introduction

In this section, the numerical results provided by the Cracked Beam Model are compared with the experimental data obtained by Neild et al. (2002; 2003). These authors tested two simply supported RC beams, with a span of 2.8 m. The rectangular cross-section of the beams was 20 cm wide and 10.5 cm deep. The tensile reinforcement was composed of three 12 mm rebars.

The tests consisted of applying incremental levels of damage, then removing the damage load and, after that, measuring the bending stiffness of the beam. The damage loading consisted of a point load applied at mid-span. The load corresponding to each damage level increases in steps of 1.5 kN (approximately 10 % of the failure load). That is, the first damage level DL1 consists of a point load of 1.5 kN, DL2 corresponds to a load of 3 kN, and so on. After applying each load level, the load was completely removed and the beams were subjected to three hysteresis loops of small magnitude: ± 40 kg, ± 30 kg and ± 20 kg. Moreover, vibration tests were also carried out after the application and removal of each load level. The small magnitudes of the hysteresis loops were chosen so that the corresponding ranges of curvature are comparable with the ones that occur during the vibration tests. Strain measurements were taken at the mid-span section, at various depths over the cross-section height, by using sensors whose gauge length is approximately equal to 100 mm. According to Neild et al. (2002), high repeatability was observed in the results of the two tested beams.

Figure 2.55 shows the observed moment-rotation relationships for the ± 40 kg load tests, for different damage levels. The rotation was derived from the readings in the strain gages that cross the mid-span section. By observing this figure, one can see that the moment-rotation relationship for each damage level is approximately linear. Consequently, it can be described by the secant stiffness, which is defined as the slope of the straight line that connects the points corresponding to the lower and upper bending moments. The authors concluded that, for each damage level, the three different load tests (± 40 kg, ± 30 kg and ± 20 kg loops) provide similar values for the secant stiffness. Figure 2.57 shows the observed decrease of secant stiffness for increasing levels of damage (this figure depicts the results for the ± 40 kg load tests, which are similar to the ones obtained in the ± 30 kg and ± 20 kg load tests).

The numerical calculations are confined to the central region of the beam, since this was the monitored region. In the experimental work, it was found that one of the cracks was located exactly at mid-span, centred with the strain gages. Furthermore, the observed crack spacing was approximately equal to 80 mm, which corresponds to the stirrup separation distance. In this way,

the length of the element under analysis is equal to 40 mm, which corresponds to the transmission length adjacent to the mid-span crack.

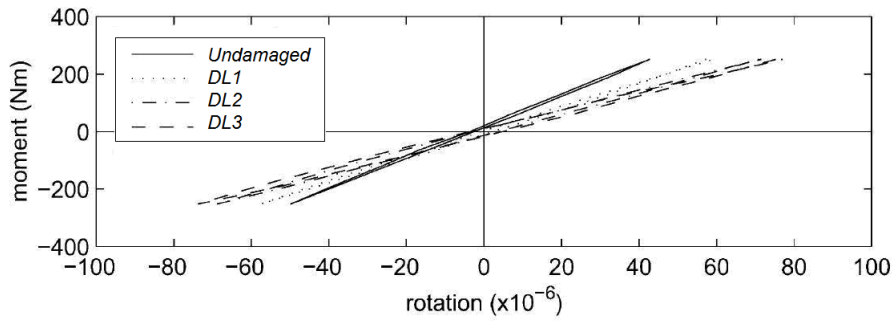


Figure 2.55 - Moment-rotation relationships for the ± 40 kg load tests (Neild et al. 2002).

2.5.2.2 Base calculation

Firstly, the experimental results were compared with calculations in which the definition of the material properties is based on the available information (Neild et al. 2002; 2003). The following values were adopted:

- $f_{cm,cyl} = 42$ MPa (based on the results of cube tests);
- $G_F = 0.08$ kNm/m²; $f_{ctm} = 3$ MPa (plausible values, based on the measured concrete compressive strength);
- $E_c = 30.5$ GPa (based on the bending stiffness, which was experimentally evaluated at the undamaged state);
- ascending branch of the bond stress-slip relationship: model proposed by the CEB (1995), considering a modified value for the parameter τ_{max} ($0.16 \cdot f_{cm}$ instead of $0.22 \cdot f_{cm}$, which is the value suggested by the CEB), so that the calculated crack spacing is coherent with the observed value;
- descending branch of the bond stress-slip relationship: $K_B = 400$ MPa/mm; $\alpha_B = 0.25$ (plausible values, according to the literature review, as shown in §2.2.7.3).

In these calculations, the CFM was used to describe the normal stress-crack opening relationship. Furthermore, the sequence of load steps is similar to the actual load scheme adopted in the experiments. However, minor differences exist between the numerical and the experimental load sequences. In the calculations:

- firstly, the bending moment corresponding to the lower damage level is applied;
- then, the bending moment decreases up to the value corresponding to the sole effect of the beam self weight;

- after that, the bending moment increases again until the bending moment corresponding to the subsequent damage level is reached;

during this reloading phase, the response due to point loads of 30, 40, 100, 200 and 300 kg is also evaluated; the effect of the 30 kg and the 40 kg point loads is to be compared with the experimental results; the effect of the remaining point loads was calculated so that the variation of the structural response for increasing load magnitudes could be assessed;

- the process is repeated until the last damage level is reached, or until reinforcement yielding occurs.

Figure 2.56 represents the calculated moment-average curvature relationship for unloading and reloading cycles corresponding to the damage levels DL2 and DL3. The points corresponding to load increments of 30, 40, 100, 200 and 300 kg are also marked in this figure. These points are used to calculate the secant stiffness for different load increments. For example, the secant stiffness for the 40 kg increment is equal to the slope of the straight line that connects the point corresponding to self weight only and the point corresponding to self weight plus 40 kg.

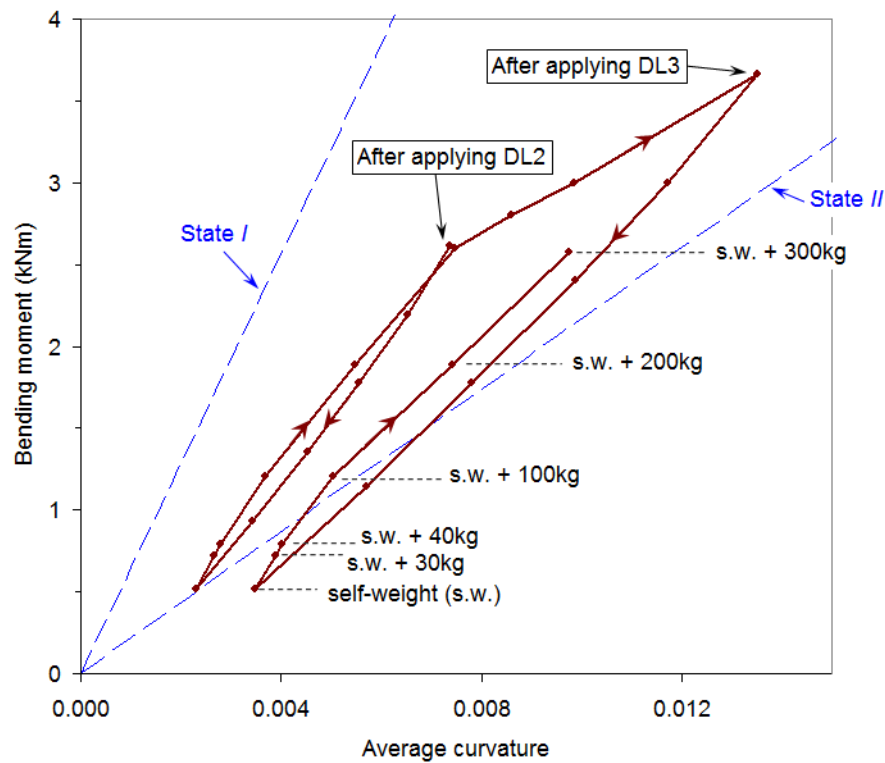


Figure 2.56 - Calculated moment-average curvature relationship: unloading-reloading cycle after applying the damage loads DL2 and DL3.

Figure 2.57 depicts the variation of the secant stiffness, for the different load increments, after the beam is subjected to increasing levels of damage (DL2 to DL8). In this figure, the secant

stiffness is given in percentage of the bending stiffness in state I (uncracked concrete). This figure only represents the results corresponding to the damage levels DL2 to DL8 because:

- when the element under analysis is subjected to the bending moment corresponding to DL1, the solution is not possible;

this impossibility is associated to the fact that this bending moment is close to the cracking moment; the solution of this problem would be possible if lower values were adopted for the tensile strength or/and the fracture energy of concrete;

- damage level DL9 causes steel stresses higher than the yield stress; consequently, the hypothesis used in the numerical calculations are not valid any more (note that the numerical model was devised for the analysis of structures in service and, under those circumstances, yielding of the reinforcement usually does not occur).

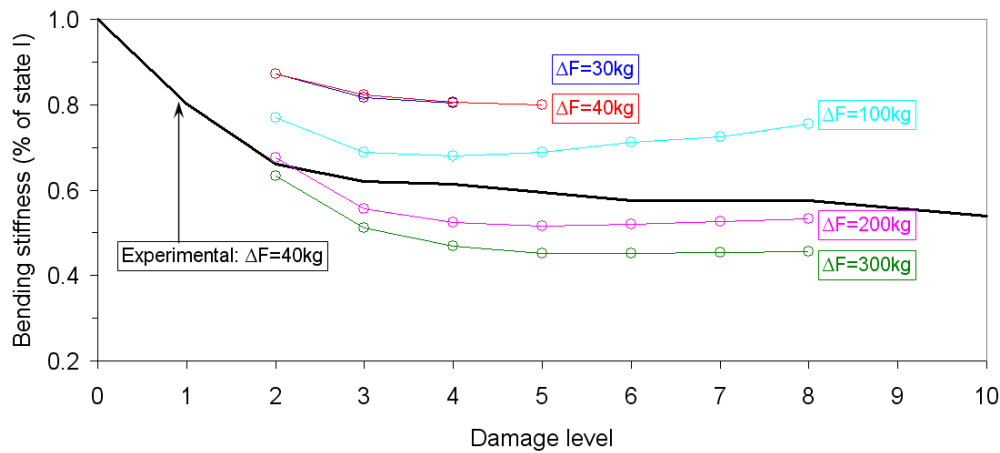


Figure 2.57 - Secant bending stiffness after applying increasing levels of damage load (DL1 to DL10): experimental and calculated values for different load increments ($\Delta F = 30, 40, 100, 200$ and 300 kg).

In the experimental results, one can see that, for damage levels higher than DL3, the stiffness decrease with increasing damage is considerably reduced. This figure also shows that the 30 kg and the 40 kg load increments produce similar numerical results, as well as observed in the experiments. However, the numerically evaluated stiffness decrease is significantly different from that observed in the experimental results (the calculated stiffness is higher than the observed value).

The concavity of the moment-average curvature relationship (displayed in Figure 2.56) is responsible for the decrease of the calculated secant stiffness as the load increment increases from 30 kg to 300 kg. However, the main conclusion taken from Figure 2.57 is that the calculated values considerably deviate from the experimental ones. Moreover, the solution could not be obtained:

- in the case of the 30 kg load increment, for damage levels higher than or equal to DL5;

- in the case of the 40 kg load increment, for damage levels higher than or equal to DL6.

These convergence difficulties were caused by the same reason that did not enable a solution to be obtained for small reloading steps in the previous example (§2.5.1.3). In both cases, the difficulties are due to the fact that the CFM is employed in the calculation of small reloading steps, after the concrete surrounding the tensile reinforcement has been subjected to significant compressive stresses. As mentioned before, this convergence difficulty could be solved by adopting a different solution procedure. However, that approach would be useless since it has been proven that the numerical results for small reloading steps deviate from the experimental ones.

A parametric analysis was conducted in order to assess whether the variation of the material properties adopted in the calculations could justify the difference with respect to the experimental results.

2.5.2.3 Parametric analysis

In the parametric analysis, the stress-crack opening relationship is described by means of the same model that was employed in the base calculation, i.e., the CFM. Table 2.6 presents the upper and lower values taken by the variable parameters. The choice of the variable parameters and their upper and lower bound values follows principles similar to those adopted in the previous parametric analysis (§2.5.1.2). The remaining parameters take the same values that were used in the base calculation.

Table 2.6 – Parametric analysis: upper and lower bound values taken by the variable parameters.

Parameter (or group of parameters)	Upper bound value	Lower bound value
Ascending branch of the bond stress-slip relationship	$a_1 = 0.16$	$a_1 = 1.75$
	$a_2 = 1$	$a_2 = 0.5$
	$b = 0.21$	$b = 0.4$
	$s_l = 1 \text{ mm}$	$s_l = 1 \text{ mm}$
	(1)	(2)
Fracture energy of concrete, G_F	$1.5 \cdot 0.08 = 0.12 \text{ kNm/m}^2$	$0.5 \cdot 0.08 = 0.04 \text{ kNm/m}^2$ ⁽³⁾
K_B	500 MPa/mm	200 MPa/mm
α_B	0.25	0.15

Notes: ⁽¹⁾ Scenario adopted in the base calculation.

⁽²⁾ Scenario corresponding to lower bond stresses than those provided by the upper bound scenario. Note that the transmission length takes the value 0.04 m (half the crack spacing in the stabilized cracking phase) in all the calculations.

⁽³⁾ In the calculations where G_F takes the lower bound value, f_{ct} also takes a reduced value: $f_{ct} = 2 \text{ MPa}$.

Figure 2.58 presents the comparison between the experimental results for the 40 kg load increment and the results of the parametric analysis, considering all the possible combinations of upper and lower bound values for the four variable parameters (or groups of parameters). This figure shows that the variability of the results is diminutive. Therefore, one can conclude that the variation of the material properties does not justify the difference between the numerical and the experimental results. In the following section, the modified Yankelevsky's FPM is employed, in order to evaluate whether the differences between calculations and experiments can be explained by a lower stiffness of the initial ascending branch of the stress-crack opening relationship.

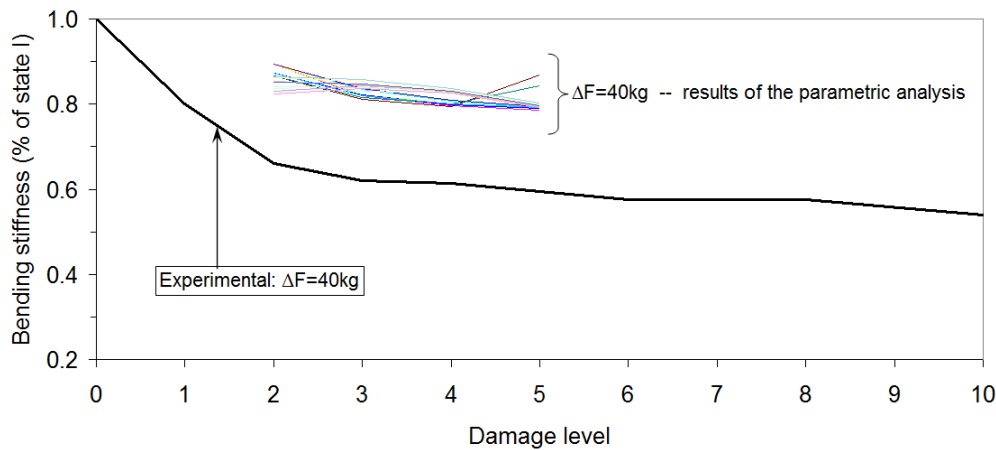


Figure 2.58 - Secant bending stiffness after applying increasing levels of damage load (DL1 to DL10): comparison between experimental results and results of the parametric analysis.

2.5.2.4 Small reloading steps considering the modified Yankelevsky's FPM

Figure 2.59 presents the comparison between the experimental results and the numerical ones, for analyses in which the reloading path of the normal stress-crack opening relationship is simulated according to the modified Yankelevsky's FPM, considering different values for the parameter A_{FPM} : 10^6 , 10^3 and 10^2 . As explained before, these figures display the secant bending stiffness for small reloading steps, after applying increasing levels of damage load (DL1 to DL10). One can see that, for A_{FPM} equal to 10^6 or 10^3 , the numerical results deviate considerably for the experimental ones, even though the difference is smaller than the one that was observed when the reloading path was simulated according to Hordijk's CFM. This difference between experimental and numerical results is due to the unrealistic high stiffness of the reloading path of the normal stress-crack opening relationship. On the other hand, if A_{FPM} takes the value 10^2 , the agreement between the experiments and the calculations is notably improved. Moreover, the numerical results for load increments of 30 kg and 40 kg show similar values in terms of stiffness decrease, which conforms with the experimental observations. Therefore, one can conclude that the adoption of Yankelevsky's FPM with A_{FPM} equal to 10^2 provides a good description of the structural behaviour for small load amplitudes.

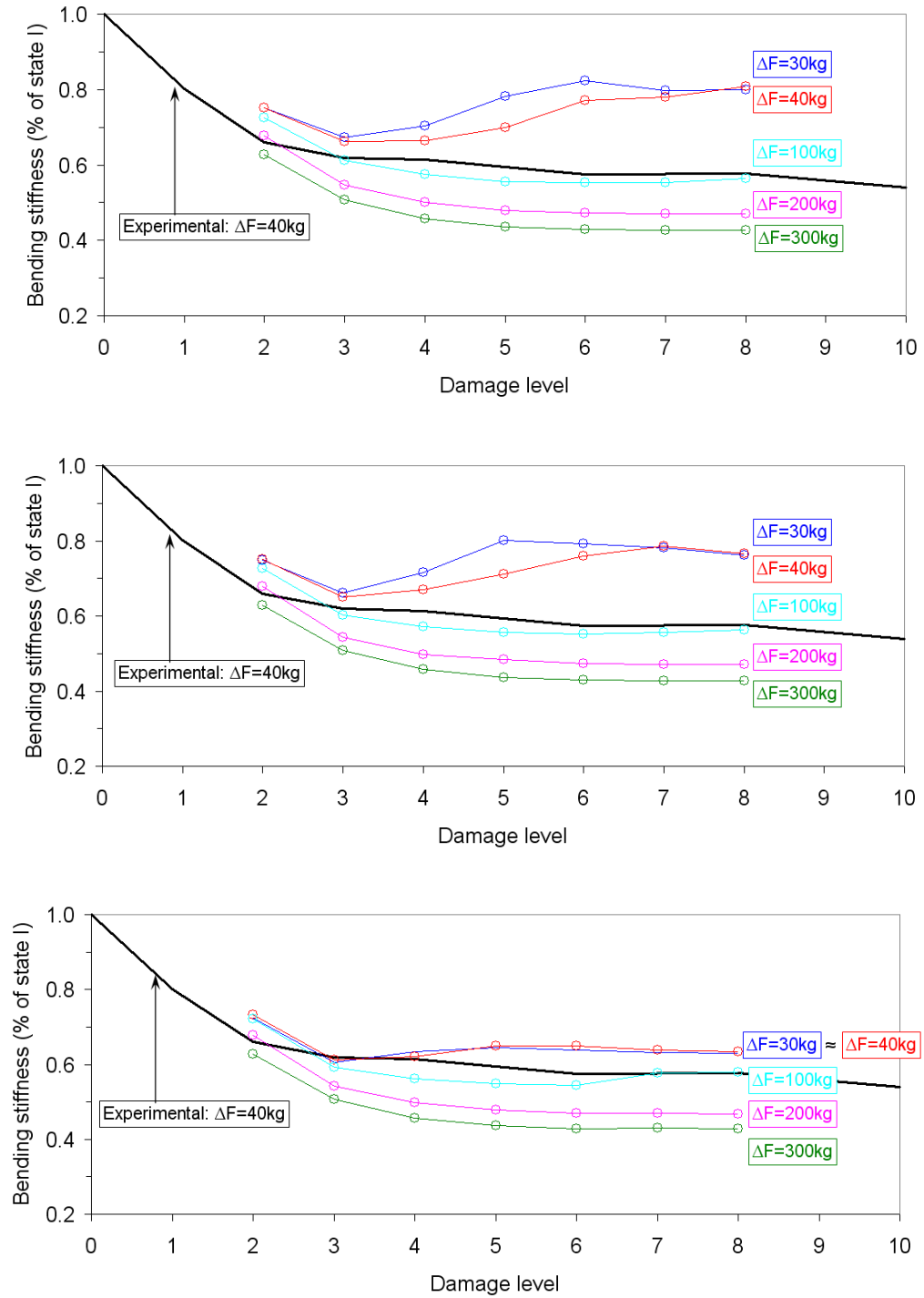


Figure 2.59 – Secant bending stiffness after applying increasing levels of damage (DL1 to DL10).
Experimental results vs. outcome of calculations considering the reloading branch
of the stress-crack opening relationship according to Yankelevsky's FPM.

Top: $A_{FPM} = 10^6$
Centre: $A_{FPM} = 10^3$
Bottom: $A_{FPM} = 10^2$

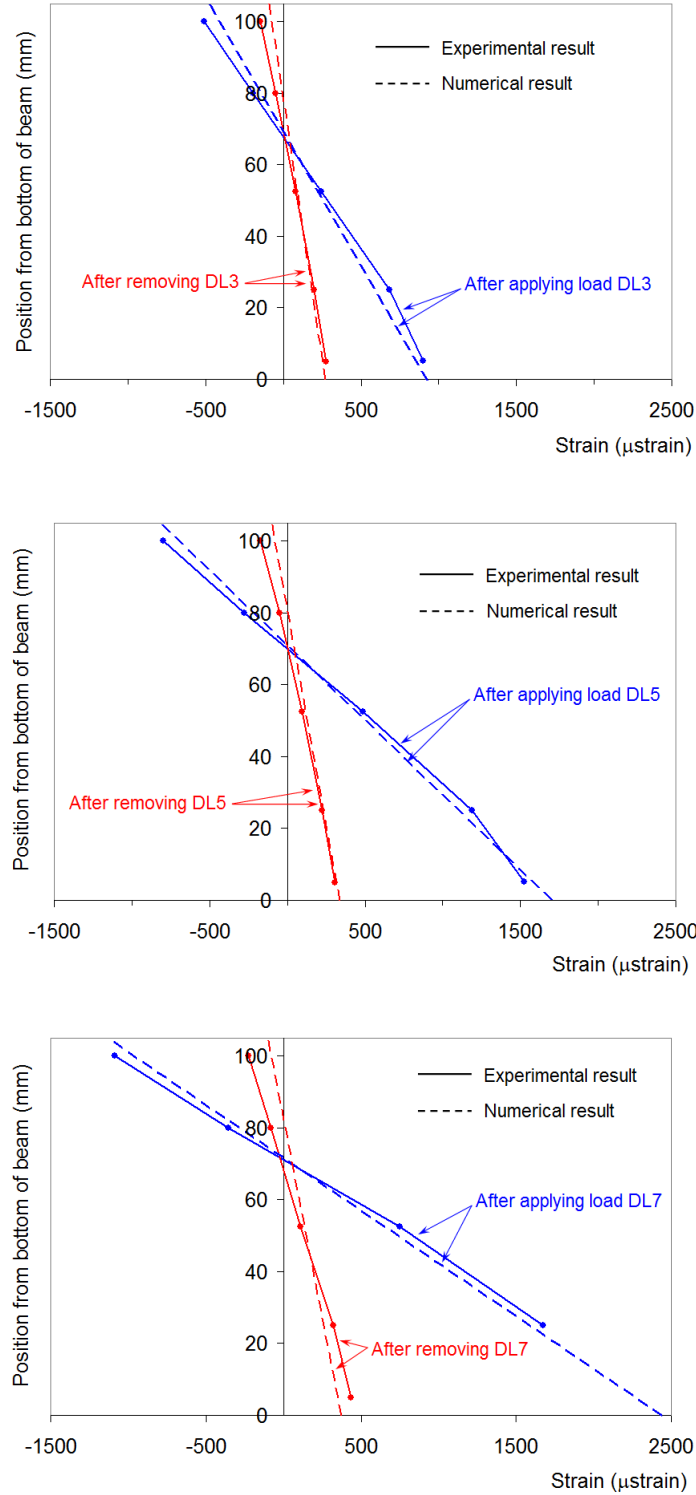


Figure 2.60 - Experimental and numerical results for the strain variation over the cross-section depth, after applying and removing the damage load (reloading path of the stress-crack opening relationship according to the modified Yankelevsky's FPM with A_{FPM} equal to 10^2).

By observing Figure 2.59 one can also see that the calculated stiffness decrease for larger load amplitudes (100, 200 or 300 kg) is almost independent from the value taken by the parameter A_{FPM} .

Figure 2.60, which presents diagrams of normal strains at the mid-span cross section (average values over a distance approximately equal to the crack spacing) after applying and removing the damage load, shows a notable agreement between numerical and experimental results. The relevance of cracking can be understood by analysing Figure 2.61 which presents the comparison between the experimental results and the outcome of simplistic calculations assuming linear elastic behaviour. This figure shows that the actual behaviour significantly deviates from the structural response in state I (assuming linear elastic behaviour).

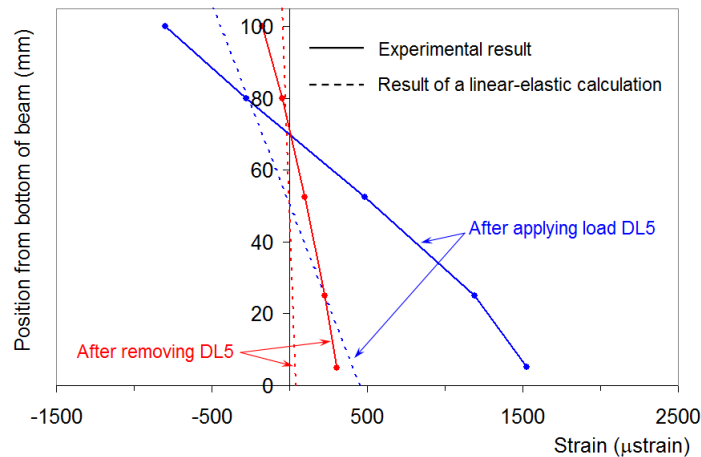


Figure 2.61 - Strain diagrams after applying and removing the damage load DL5. Comparison between the experimental results and the outcome of simplistic calculations assuming linear elastic behaviour.

2.6 Final considerations

A numerical model for analysis of the cyclic behaviour of cracked RC elements was developed and described. The adopted numerical approach involves the explicit consideration of: (i) the cyclic bond stress-slip relationship between the tensile reinforcement and the surrounding concrete; (ii) the relationship between the normal stress at the crack and the crack opening under the effect of cyclic loading.

A state-of-the-art review focusing on the mechanics of bond under monotonic and cyclic loading was carried out. The proposals of different authors for the cyclic bond stress-slip relationship were compared and a constitutive model to be used in the applications envisaged in this work was defined. Research needs, in terms of characterization of the cyclic bond stress-slip relationship, were also identified.

Different constitutive models to describe the stress-crack opening relationship under the effect of cyclic loading were also discriminated. A minor modification was introduced in one of the available constitutive models, so that convergence difficulties could be solved and a better agreement with experimental results for small reloading steps could be obtained.

Then, the proposed numerical methodology for calculation of the cyclic response of cracked RC beams was presented, and the main differences with respect to existing models were pointed out.

The validation of the proposed numerical models, the Cracked Beam Model and the Cracked Chord Model, was performed by using experimental results reported in the literature. This work showed that the numerical models provide an accurate description of the non-linear behaviour of the tested elements under the effect of cyclic loadings. Good agreement between numerical and experimental results was obtained when the proposed constitutive models for both the bond stress-slip law and the normal stress-crack opening relationship were considered.

The fact that a good agreement was obtained between numerical and experimental results for small load amplitudes encourages the use of the proposed analysis methodologies in the interpretation of results of vibration tests of cracked RC members.

Chapter 3

Fatigue analysis of railway bridge decks

3.1 Introduction

While the fatigue behaviour of steel structures has been investigated for more than 150 years, studies into the fatigue response of reinforced and prestressed concrete are comparatively recent. A recent survey among the European Railway Administrations, carried out by the ERRI Committee D216 (2002), aiming at identifying the existing methods for design and assessment of reinforced and prestressed concrete railway bridges, has led to the conclusion that, in the recent past, fatigue verifications were not routinely carried out, and there are some countries where fatigue resistance is not checked at all. Design codes such as the former Portuguese code REBAP (1983) only included some simplistic rules for fatigue verifications. However, fatigue checks have important implications in the design of structures subjected to cyclic loading, such as bridge decks. In fact, it has been shown that numerous existing bridge decks do not comply with the requirements of actual design codes, in terms of fatigue strength (Van Bogaert 2009).

The ERRI Committee D216 (2002) also concluded that there is a need for further research into the application of the various existing fatigue analysis procedures for railway bridge decks. In the case of continuous precast bridge decks without continuity prestress, fatigue may be particularly critical. On the one hand, precast structures are usually composed of thin walls and slabs, which make possible the adoption of light structures where the traffic (cyclic) loading represents a significative percentage of the total loading. As the importance of cyclic loading increases, fatigue verifications become more critical, by comparison with other limit states. On the other hand, in structures without continuity prestress, cracking may occur in the region above the continuity supports. The occurrence of cracking in locations subjected to important cyclic

bending moments and shear forces raises concerns about the fatigue performance of the structure, too.

In view of the relevance of fatigue analyses in the context of precast solutions for railway bridge decks, this section presents the results of a literature review on methodologies for fatigue analysis of these structures. Alternative analysis procedures are presented, special attention being devoted to structures included in high speed lines.

The chapter starts with a brief presentation of the fatigue behaviour of concrete, steel and RC structures. Procedures for quantification of the number of resisting load cycles for steel and concrete are also indicated. A comprehensive description of the fatigue behaviour of reinforced and prestressed concrete structures can be found in one of the available state-of-the-art reports (ACI Committee 215 1997; CEB 1988; ERRI D216 2002; 2003; RILEM Committee 36 RDL 1984).

3.2 Fatigue of reinforcing steel

The fatigue life of reinforcing steel (as well as other metallic materials) can be divided into three phases (ERRI D216 2002):

- the crack initiation phase, characterized by the loss of cohesion followed by micro cracking, which usually takes place in a location where stress concentration occurs;
- a steady crack propagation phase;
- the brittle fracture of the remaining cross section after a phase of unstable crack growth.

The fatigue performance of steel reinforcing bars is influenced by various factors (ACI Committee 215 1997; ERRI D216 2002; 2003; Macgregor et al. 1971; RILEM Committee 36 RDL 1984):

- bar diameter (the fatigue strength of bare steel typically decreases as the bar diameter increases);
- bar curvature (fatigue strength is reduced by the plastic deformation and residual stresses caused by bending);
- stress range or stress amplitude (as long as the total loading does not cause plastic deformation, higher stress ranges are responsible for higher fatigue damage);
- corrosion (evenly distributed surface corrosion has no major effect on fatigue strength, but pitting significantly worsens fatigue behaviour);

- lapped joints [according to ERRI D216/RP3 (2003), laps of sufficient length - more than 20 times the diameter of the bar - have no effect on fatigue behaviour as long as there is no bending in the lap, i.e., failure occurs outside the lap; in contrast, the fatigue strength of welded joints can be reduced by up to ~50%];
- surface characteristics (the shape of the deformations on reinforcing bars affects their fatigue life, because fatigue cracks are initiated at the points of stress concentrations, which, in turn, are significantly influenced by the bar deformations);
- chemical composition and manufacturing method (ACI Committee 215 1997).

Some works have been published on the fatigue behaviour of reinforcing and prestressing steels on the basis of the elastic fracture mechanics theory (Salah el din and Lovegrove 1982; Wang et al. 2009). However, the fatigue life of reinforcing steel bars subjected to axial forces is usually expressed by means of Wöhler curves, which provide the number of permissible load cycles before fatigue failure, N , for a given stress amplitude $\Delta\sigma_s$. The Wöhler curve (also known as S-N curve) for reinforcing and prestressing steel is usually composed of straight lines in double logarithmic scale, as represented in Figure 3.1, and is characterised by the slopes k_1 and k_2 , the number of permissible load cycles at the transition between different slopes, N^* , and the corresponding stress range, $\Delta\sigma_s(N^*)$.

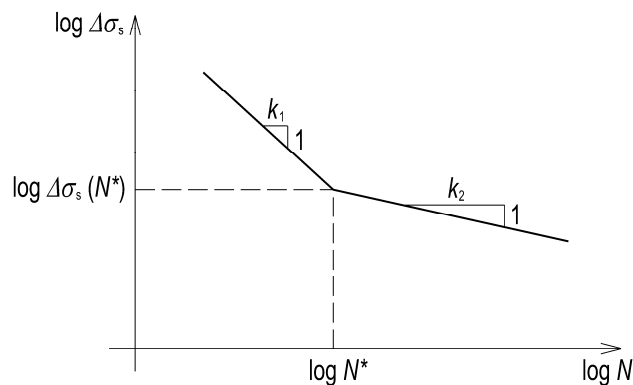


Figure 3.1 - Wöhler curve for reinforcing and prestressing steel.

In the EN 1992-1-1, the parameters that define the characteristic Wöhler curve are quantified taking into account various influencing factors (bar curvature, welded joints, splicing devices and corrosion), as shown in Table 3.1.

Table 3.1 – Parameters that define the characteristic S-N curves for reinforcing steel, according to the EN 1992-1-1.

Type of reinforcement	N^*	k_1	k_2	$\Delta\sigma_{Rsk}(N^*)$ (MPa)
Straight bars	10^6	5	9	162.5
Bent bars	10^6	5	9	$162.5 \cdot (0.35 + 0.026 \cdot D/d_b)$
Welded bars and wire fabrics	10^7	3	5	58.5
Splicing devices	10^7	3	5	35

NOTES: D is the diameter of the mandrel and d_b is the bar diameter.

The fatigue life in existing structures where corrosion has started can be determined by taking $k_2 = 5$ for straight and bent bars.

$\Delta\sigma_{Rsk}(N^*)$ represents the characteristic value of the stress range corresponding to N^* cycles.

The design value of the number of resisting load cycles, N_d , is obtained by dividing the characteristic value of the resisting stress range at N^* cycles by the safety factor $\gamma_{s,fat}$, which, according to the EN 1992-1-1, takes the value 1.15. Accordingly, for a given stress range $\Delta\sigma_s$, N_d is given by:

$$N_d = N^* \cdot \left(\frac{\Delta\sigma_{Rsk}(N^*)/1.15}{\Delta\sigma_s} \right)^k \quad (3.1)$$

where:

$$k = \begin{cases} k_1 & \text{for } \Delta\sigma_s > \Delta\sigma_{Rsk}(N^*)/1.15 \\ k_2 & \text{for } \Delta\sigma_s \leq \Delta\sigma_{Rsk}(N^*)/1.15 \end{cases} \quad (3.2)$$

and the parameters N^* , $\Delta\sigma_{Rsk}(N^*)$, k_1 and k_2 are given in Table 3.1. Equation (3.1) shows that the fatigue strength does not depend on the upper stress level. However, this equation is only valid if the maximum stress in steel exposed to fatigue loads does not exceed its characteristic yield strength.

Table 3.1 is applicable to reinforcing bars that comply with the rules given in the Annex C of the EN 1992-1-1. This annex specifies the fatigue stress range and the corresponding minimum number of load cycles that must be reached in reinforcement suitable for use with the Eurocodes. Fatigue testing shall be in accordance with the EN 10080 (CEN 2005c). The test procedure and the requirements that shall be met by the test piece and equipment are defined by the standard ISO 15630-1 (ISO 2002a).

As regards the comparison between the fatigue strength of reinforcing bars embedded in concrete and the results of in-air fatigue tests, the ACI Committee 215 (1997) notes some contradiction within technical literature. While some researches report that bars embedded in concrete beams had higher fatigue strength than when tested in the air, the opposite conclusion was reached by others. According to the ACI Committee 215 (1997), more recent studies indicate that there should be little difference in the fatigue strength of bars in the air and embedded bars if the geometry of the transverse lugs is adequate to provide a good bond between steel and concrete (a good bond is beneficial to the fatigue strength because it guarantees reduced friction effects). The RILEM committee 36 RDL (1984) points out that the fatigue behaviour of bars embedded in concrete is affected by factors which are not present in in-air fatigue tests of reinforcing bars: friction due to slip between reinforcements and concrete; localization of the fatigue failure at the crack cross section; pressure on transverse ribs; stress gradients due to the curvature of beams; and local curvature peaks at the crack cross sections. While the localization of fatigue failures at the crack cross section might be beneficial (the weak point of the bar will most often not coincide with the crack location), the other influencing factors contribute to lower fatigue strength than that observed in in-air tests. Therefore, the RILEM committee 36 RDL (1984) concludes that the scatter of fatigue test results in beams is widened with respect to the results for bare bars. This effect is taken into account in the S-N curves defined by Equation (3.1) and Table 3.1.

3.3 Fatigue of prestressing steel

Tendons in highly prestressed structural components are usually not susceptible to fatigue because there is little change in the stress levels when the concrete surrounding the prestressing steel is compressed.

Friction between wires, or between wires and the jacket tube, is among the most important factors that reduce the fatigue resistance of prestressing steel (particularly in the case of bent tendons, where the forces resulting from the curvature produce transverse pressure at the contact points between the prestressing steel and the jacket tube). The critical points in terms of fatigue are those located close to a crack, because the tendon stress increases at the section where a crack exists, and friction forces occur as a consequence of relative displacements at the contact surface. The denomination *fretting corrosion* is commonly adopted to represent the reduction of the fatigue resistance resulting from the lateral pressure and slip between prestressing strands and ribs of the steel sheaths.

Tendon couplings are sensitive to fatigue and, for that reason, are usually not located at the points where maximum internal forces occur. Nevertheless, it is important to note that, in the case of unbonded post-tensioned structures, the stress variations are transmitted directly to the anchorages with minor variations along the length of the tendons. Therefore, it is the fatigue

strength of the anchorages (or couplings) that has to be considered, not that of the tendons (RILEM Committee 36 RDL 1984).

According to the EN 1992-1-1, the design value of the number of resisting load cycles for prestressing steels which comply with the rules included in the prEN10138 (CEN 2000a; 2000b; 2000c; 2000d) is given by Equation (3.1), considering the parameters indicated in Table 3.2. Similarly to the restriction imposed in the case of reinforcing steel, the maximum stress shall not exceed the characteristic 0.1% proof-stress, $f_{p0.1k}$. If required, fatigue tests of prestressing steels must follow the requirements and test procedures imposed by the standard ISO 15630-3 (ISO 2002b).

Table 3.2 – Parameters that define the characteristic S-N curves for prestressing steel, according to the EN 1992-1-1.

Type of reinforcement	N^*	k_1	k_2	$\Delta\sigma_{Rsk}(N^*)$ (MPa)
Pre-tensioning	10^6	5	9	185
Post-tensioning:				
single strands in plastic ducts	10^6	5	9	185
straight tendons or curved tendons in plastic ducts	10^6	5	10	150
curved tendons in steel ducts	10^6	5	7	120
splicing devices	10^6	5	5	80

In Table 3.2, one can see that the Wöhler curves for pre-tensioning and single strands in plastic ducts are similar to the one for straight reinforcing bars, the latter exhibiting slightly lower $\Delta\sigma_{Rsk}(N^*)$ values. Tendon curvature, friction between strands and jacket tubes, and tendon couplings are responsible for decreasing fatigue strength.

3.4 Fatigue of concrete

Concrete is a multi-phase, heterogeneous material, which exhibits a complex fatigue behaviour. Concrete fatigue damage is attributed to progressive deterioration of the bond between the coarse aggregate and the cement paste, and to the progression of micro-cracks, whose orientation and length are randomly distributed in concrete. Micro-cracks occur in the cement matrix or between the cement matrix and the aggregate, as a consequence of self-equilibrated stresses (induced by restricted matrix shrinkage) and by applied loads. If the concrete is subjected to important cyclic loading, the resulting local stresses will control the evolution of micro-cracks, and consequently, of the concrete's fatigue performance (Antrim 1965; RILEM Committee 36 RDL 1984). It is important to note that fatigue cracks in concrete have no identifiable surface topography, unlike fatigue cracks in steel, and for that reason, it is difficult to identify fatigue in concrete structures.

The evolution of concrete deformations during a fatigue test is characterized by three different stages (Holmen 1982), as represented in Figure 3.2:

- In the first stage (or crack formation stage), a fast increase of deformation occurs, with a decreasing rate;
- during the second stage, deformations grow at a constant rate;
- during the last cycles (typically the last 10% to 15% of the fatigue life) a rapid increase in deformation occurs, with an increasing rate, leading to the failure of the specimen.

The features of this typical behaviour, composed of three distinct phases, have been used to evaluate fatigue damage in concrete structures, as shown by Cachim (1999) and by the CEB (1988).

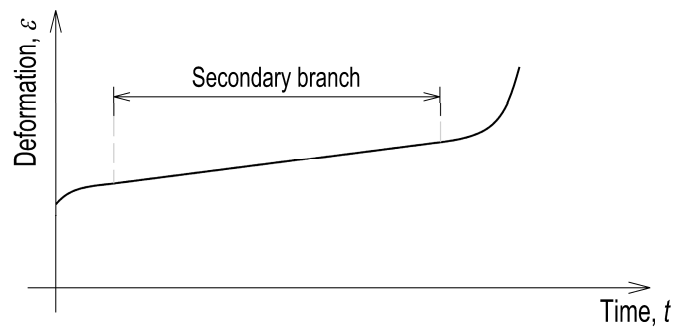


Figure 3.2 - Evolution of deformations during a fatigue test of concrete.

The fatigue behaviour of the concrete is influenced by several factors, including:

- mean stress (unlike steel, concrete fatigue strength depends on the maximum, as well as on the minimum stress in the cycle);
- type of loading (tension or compression);
- loading frequency [a loading frequency of 1 Hz to 15 Hz only has a slight effect if the maximum stress does not exceed 75% of the compressive strength (ACI Committee 215 1997; CEB 1988; ERRI D216 2003; RILEM Committee 36 RDL 1984); at higher stress levels, the fatigue strength decreases as the loading frequency decreases (Holmen 1982; RILEM Committee 36 RDL 1984); the MC90 (CEB 1993) states that its estimates for the concrete fatigue strength are valid for frequencies of loading higher than 0.1 Hz, the fatigue life being reduced for lower frequencies];
- concrete composition, amount of entrained air, mixing and curing conditions [according to the CEB (1988), these variables do not seem to influence the fatigue strength if it is expressed in terms of the corresponding monotonic strength];

- testing conditions (the fatigue life of concrete tested in wet conditions is lower than that of concrete tested in the air);
- confining stresses [confinement seems to prolong the fatigue life considerably (CEB 1988); however, this effect seems to disappear if the stress level is expressed as a function of the monotonic biaxial strength];
- rest periods [rest periods between blocks of cyclic loading seem to increase the fatigue life (RILEM Committee 36 RDL 1984), but contradictory conclusions have also been obtained, as shown by the CEB (1988)].

Wöhler curves for concrete

The fatigue strength of concrete is usually represented through Wöhler curves, which express the number of resisting load cycles as a function of the most determinant influencing factors (the minimum and the maximum stress levels) for constant-amplitude cyclic loading. According to the EN 1992-2 (CEN 2005b), the design value of the number of resisting load cycles for concrete under compression is given by:

$$N_i = 10^{14 \frac{1-E_{cd,max,i}}{\sqrt{1-R_i}}} \quad (3.3)$$

where:

$$\begin{aligned} R_i &= E_{cd,min,i} / E_{cd,max,i} \\ E_{cd,min,i} &= \sigma_{cd,min,i} / f_{cd,fat} \\ E_{cd,max,i} &= \sigma_{cd,max,i} / f_{cd,fat} \end{aligned} \quad (3.4)$$

and $\sigma_{cd,min,i}$ is the lower stress in a cycle, $\sigma_{cd,max,i}$ is the upper stress in a cycle, and $f_{cd,fat}$ is the fatigue reference strength of concrete, which is given by:

$$f_{cd,fat} = k_1 \cdot \beta_{cc}(t_0) \cdot \frac{f_{ck}}{1.5} \cdot \left(1 - \frac{f_{ck}}{250}\right) \quad (3.5)$$

where:

k_1 is a reduction factor, whose recommended value is 0.85; this factor accounts for the actual frequencies of loading, which, in most practical cases, are significantly lower than those applied in experiments (fib 2010);

$\beta_{cc}(t_0)$ is a coefficient to define the compressive strength at the time the cyclic loading starts, t_0 , as given in the EN1992-1-1;

$(1 - f_{ck} / 250)$ is a factor which takes into account the increasing fatigue sensitivity of concrete with increasing compressive strength, with f_{ck} in MPa.

Figure 3.3 depicts the S-N curve for concrete under compression for different minimum stress levels.

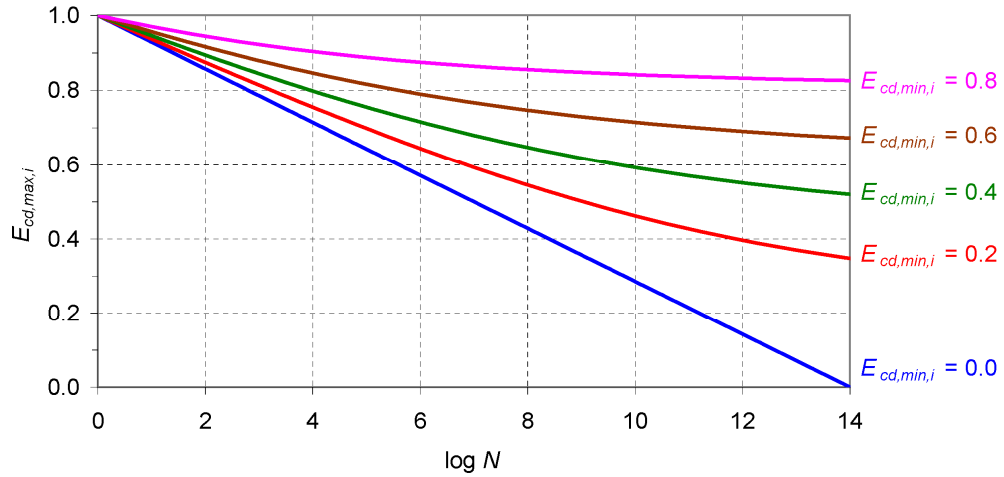


Figure 3.3 - Wöhler curve for concrete under compression, according to the EN 1992-2.

As regards the fatigue behaviour of concrete subjected to tensile stresses, the results of Tepfers and co-workers (Tepfers 1979; Tepfers and Kutti 1979) indicate that the Wöhler curves for tensile and compressive fatigue of concrete are identical. This information is particularly useful for assessing a structure's susceptibility to cracking under repeated loading.

Eccentric compression or tension

In the case of eccentric compression or tension, it was observed by several authors that important stress redistributions occur, according to which the fibres located further inwards from the extreme fibre increasingly participate in carrying the load (Ople and Hulsbos 1966). This effect is more pronounced as the stress gradient increases or, in the case of flexural load, as the depth of the beam decreases. The redistribution of concrete stresses has been analysed by Zanuy et al. (Zanuy 2008; Zanuy et al. 2009; Zanuy et al. 2007) by using numerical models at the cross-section level, taking into account the evolution of concrete deformations with cyclic loading. On the other hand, the MC90 proposes a simplified procedure for taking into account the redistribution of compression stresses, which consists of multiplying the maximum stress in the compression zone by the factor η_c :

$$\eta_c = \frac{1}{1.5 - 0.5 \frac{|\sigma_{c1}|}{|\sigma_{c2}|}} \quad (3.6)$$

where $|\sigma_{c1}|$ is the lowest absolute value of the compressive stress within a distance no longer than 300 mm from the surface, and $|\sigma_{c2}|$ is the highest absolute value of the compressive stress under the same load combination (for the calculation of σ_{c1} and σ_{c2} , a linear elastic behaviour of the concrete under compression must be assumed).

By analysing Equation (3.6), one can conclude that if the neutral axis depth is not larger than 300 mm, then the factor η_c takes the value $2/3$, which corresponds to a strong decrease of the stress amplitude considered in fatigue calculations, and consequently, a drastic increase of the fatigue life. The factor η_c increases up to 1 as the neutral axis depth increases, i.e., as the stress gradient in the compression zone decreases.

Fatigue strength of compression struts in elements subjected to shear

The fatigue strength of compression struts in beams (or beam-like elements) with shear reinforcement is lower than the strength of elements subjected to uniaxial compressive stresses because of the influence of transverse tension. Therefore, design codes such as the MC90, the leaflet UIC 774-1 (UIC 2005), and the Eurocodes² suggest that, in the case of compression struts under the effect of shear forces, the fatigue reference strength $f_{cd,fat}$ must be multiplied by a reduction factor ν . According to the EN 1992-1-1, the reduction factor for fatigue analysis takes the value:

$$\nu = 0.6 \cdot \left(1 - \frac{f_{ck}}{250} \right) \quad (3.7)$$

with f_{ck} in MPa. For $f_{ck} = 50$ MPa, e.g., it yields $\nu = 0.48$. On the other hand, the MC90 suggests that the fatigue reference strength $f_{cd,fat}$ should be reduced in the same way as the ultimate compressive strength of concrete subjected to simultaneously acting compressive and transverse tensile forces, which gives:

² The Eurocodes do not explicitly impose the application of the reduction factor in all the fatigue analysis types, but that reduction must always be considered in the analysis of compression struts subjected to the influence of transverse tension.

$$\nu = \frac{0.6 \left(1 - \frac{f_{ck}}{250} \right)}{0.85 \left(1 - \frac{f_{ck}}{250} \right)} \approx 0.70 \quad (3.8)$$

By analysing Equations (3.5), (3.7) and (3.8), one can see that the proposal by the EN 1992-1-1 is much more conservative than the one suggested by the MC90. However, the latter appears to be more consistent than the former for the reason that, when $f_{cd,fat}$ is multiplied by the reduction factor proposed by the EN 1992-1-1, it turns out that the factor $(1 - f_{ck} / 250)$, which accounts for the increasing fatigue sensitivity of concrete with increasing compressive strength, is considered twice. Therefore, in the applications envisaged in this thesis, the reduction factor for fatigue analyses is given by Equation (3.8), unless stated otherwise.

Scatter of fatigue test results

The scatter of fatigue test results for concrete under compression is higher than the one for reinforcing or prestressing steels subjected to tensile stresses. When the results of different fatigue tests of concrete are represented in Wöhler curves, a large scatter is therefore observed. That variability is, to a great extent, a consequence of the scatter of the fatigue behaviour itself. However, it is also a consequence of different test conditions (no standard fatigue test exists for concrete), and a consequence of the scatter in the ultimate compressive strength of the concrete (which is considerably higher than in the case of steel). As regards this latter aspect, note that the real ultimate strength of a specimen subjected to a fatigue test is not known, and must be inferred from other specimens of the batch. Given the logarithmic relationship between the number of resisting load cycles and the stress level, a small variation in the maximum stress level leads to a great variation in the number of resisting load cycles. For example, a variation from $E_{cd,max,i} = 0.7$ to $E_{cd,max,i} = 0.65$ causes a variation of the fatigue life from 15,849 cycles to 79,433 cycles, according to Equation (3.3), if $E_{cd,min,i} = 0$. This means that a 7% reduction in the stress level might correspond to a 401% increase in the fatigue life of concrete.

Given the large scatter observed in fatigue test results, the design values of the concrete fatigue strength indicated by codes such as the Eurocodes are, in some circumstances, very conservative predictions of the actual fatigue lives. If one considers that the fatigue strength of concrete has received considerably less attention during the design of new structures than the fatigue of steel (ERRI D216 2003), it is not surprising that various existing structures do not comply with the Eurocode requirements for fatigue design. This problem was detected by Van Bogaert (2009), who carried out fatigue analyses of existing bridge decks, and concluded that some fail to

comply with the provisions of the EN 1992-1-1 and the EN 1992-2 in terms of concrete fatigue strength. Given that concrete fatigue failures are seldom observed in real structures, Van Bogaert (2009) suggests that the fatigue provisions included in the Eurocodes may be too conservative, and there is room for further long-term experimental research and improvement of current design codes. The previous observation is also supported by the results of laboratorial fatigue tests. Johansson (2004), who carried out fatigue tests on beams of 2 to 2.5 m spans, concluded that, even though the beams were subjected to high compressive stresses due to bending moments, the fatigue of concrete in compression was never the governing type of failure.

3.5 Fatigue of reinforced and prestressed concrete structures

The fatigue strength of RC or PC structures is closely related to the fatigue properties of their constituent materials, concrete and reinforcing or prestressing steel, and the connection between them. Given that the stress amplitudes in uncracked structures are quite reduced, significant fatigue damage only occurs after cracking, particularly in the cases of steel and of concrete under compression. It is important to note that RC and PC structures can exhibit different modes of failure depending on whether they are subjected to monotonically increasing loading or cyclic loading. In some cases, beams have failed in shear under cyclic loading, although similar beams have failed in bending under static loading (Chang and Kesler 1958). In the following subsections, different types of fatigue failures are briefly presented.

3.5.1 Bending

Cyclic loading gives rise to increasing bending deformations, as a consequence of progressive cracking, decreasing stiffness of the concrete under compression and degradation of the bond connection between the tensile reinforcement and the surrounding concrete. This aspect has previously been dealt with in Chapter 2.

Cyclic loading might also produce premature bending failures, for load levels lower than the ultimate structure capacity.

The fatigue strength of RC structures with reduced reinforcement ratio is conditioned by the fatigue properties of the tensile reinforcement. However, the fatigue strength of reinforcing bars embedded in concrete can be lower than that of bars tested in-air, as mentioned before (§3.2). The fatigue behaviour of prestressing steels is also determined by their interaction with the surrounding concrete, as shown in §3.3.

In the case of structures with high reinforcement ratios, where large concrete stress ranges can occur at the crack cross sections, the fatigue strength can also be conditioned by the fatigue

properties of the concrete in the compression chord. Important stress redistributions occur in the compression zone, as mentioned in §3.4, owing to the transference of concrete stresses from more heavily loaded to less heavily loaded fibres in the cross section.

The fatigue failure of tensile reinforcements occurs without advance warning, i.e., steel fatigue gives rise to brittle fatigue failures. In contrast, the less frequent fatigue failure of concrete compression chords is preceded by major deformations.

3.5.2 Shear in members without shear reinforcement

Figure 3.4 shows the possible modes of failure associated to cyclic shear forces in beams without shear reinforcement. According to the CEB (1988), the shear failure of this type of beam results from the development of a critical shear crack. When this crack reaches a large width, shear transfer across the crack is prevented, and the beam may fail due to fatigue of the compression strut. According to the same reference, it is not possible to predict the number of load cycles that can be resisted after the critical shear crack is formed. In this way, it is advisable not to include this uncertain residual bearing capacity in the design calculations, and consequently, to consider the formation of the shear crack as the failure criterion.

According to RILEM Committee 36 RDL (1984), remarkably low fatigue strengths have been obtained for beams failing in shear, in some researches. Therefore, these types of failure require special attention. The ERRI Commission D216 (2002) concluded that the fatigue behaviour of RC members without shear reinforcement has not been sufficiently studied to date.

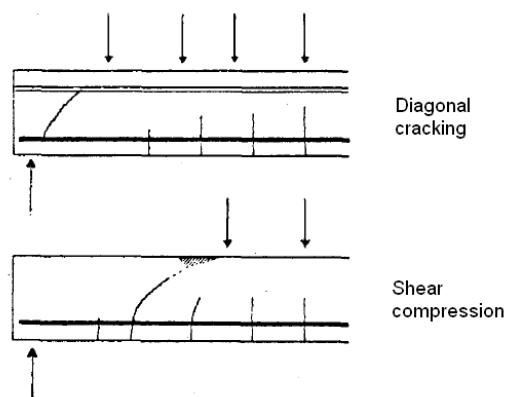


Figure 3.4 - Possible types of shear fatigue failure in beams without shear reinforcement (RILEM Committee 36 RDL 1984).

3.5.3 Shear in members with stirrups

The fatigue strength of beams with shear reinforcement subjected to cyclic shear forces depends on the fatigue properties of both the reinforcement and the web concrete. However, the application of results from fatigue tests on straight bars or concrete prisms is not as straightforward as in the case of fatigue caused by bending moments. The cyclic shear behaviour of beams with stirrups is discussed in detail in Chapter 4.

3.5.4 Shear at the interface between concretes cast at different times

Numerous researches have been devoted to the study of the ultimate shear resistance of the interface between concretes cast at different times (FIP 1982; Hofbeck et al. 1969; Mattock 2001; Mattock and Hawkins 1972). Furthermore, the behaviour of such interfaces under the effect of low-cycle, high-amplitude cyclic loading has also been studied by several researchers (Tassios and Vintzeleou 1987; Vintzeleou 1984; Vintzeleou and Tassios 1987; Walraven 1994). However, the amount of experimental results on the high-cycle fatigue behaviour of this connections is limited (Pruijssers 1988). Given the fact that this issue plays an important role in railway bridges constructed with precast beams, an experimental campaign on this topic was recently initiated at FEUP by Figueira et al. (2010; 2011).

3.6 Alternative approaches for fatigue analyses of RC and PC railway bridge decks

3.6.1 General remarks

In the Eurocodes, three alternative procedures for fatigue verification of reinforced or prestressed concrete structures are identified. In ascending order of accuracy, the available procedures are:

- a simplistic verification by limiting stresses for a given combination of actions;
- the damage equivalent stress method;
- the damage accumulation method;

The damage accumulation method is the most detailed one, acting as a reference for the development of simpler verification procedures. The basic rules for fatigue verification according to these procedures are spread over three documents, the EN 1991-2 (CEN 2003b), the EN 1992-1-1 and the EN 1992-2. In this section, the three alternatives for fatigue analyses are presented. Moreover, some aspects which are not explicitly handled by the Eurocodes are discussed and clarified. This section starts by presenting procedures for calculation of internal forces and stresses. Then, the combinations of actions for fatigue verifications are introduced.

Following this, the three alternatives for fatigue verification are exposed, starting from the more general and detailed one, the damage accumulation method.

3.6.1.1 Calculation of internal forces

The calculation of internal forces due to the passage of real trains can be determined through a transient FE analysis with moving loads. If only the global forces acting on the girders are of interest, then the FE model might consist of a grid of beam elements. On the other hand, if the shear forces and bending moments applied to the deck slab and webs are to be determined, then a FE model with shell elements should be employed. In structures with short spans, the applied loads are significantly affected by the vehicle-structure interaction. On the other hand, in the case of medium or large spans (larger than ~ 30 m), that effect is reduced and the analysis can be carried out considering that the passage of the train is reproduced by a set of moving loads, the axle loads (ERRI D214 1999b). That is the approach adopted in this work. Moreover, the track is not explicitly modelled, i.e., the FE model consists of shell elements that simulate only the deck elements (upper deck slab, webs, bottom slab and support diaphragms). Then, the distribution of forces acting on the deck slab, at each time step, is determined by means of a pre-processing model which simulates the degradation of axle loads through rails, sleepers and ballast.

The time history of forces applied to each sleeper is determined by considering that the set of axle loads is moving at a given constant speed v , whereas the sleepers are located at fixed positions. For each time step, each axle load P_i is distributed (in the direction parallel to the rails) to the four nearest sleepers according to the following equations, proposed by the ERRI Committee D183 (1997):

$$\begin{aligned} Q_{i,1} &= \frac{P_i}{4} \cdot \left(1 - \frac{x_a}{a}\right) & Q_{i,2} &= \frac{P_i}{4} \cdot \left(2 - \frac{x_a}{a}\right) \\ Q_{i,3} &= \frac{P_i}{4} \cdot \left(1 + \frac{x_a}{a}\right) & Q_{i,4} &= \frac{P_i}{4} \cdot \frac{x_a}{a} \end{aligned} \quad (3.9)$$

where a is the sleeper spacing and x_a is the distance between P_i and $Q_{i,2}$, as shown in Figure 3.5. When the axle load is located above a given sleeper, half of the load is received by that sleeper, whereas each of the adjacent sleepers receives one fourth of the load P_i .

The degradation of the forces Q_{ij} through the ballast is determined by assuming a uniform distribution of stresses along the length b_l indicated in Figure 3.5, this hypothesis being suggested in several references (CEN 2003b; ERRI D183 1997; UIC 1994). In the transverse direction, the load applied to the deck slab is assumed to be evenly distributed along the length

b_i , which is determined considering similar hypotheses to those adopted in the longitudinal direction (CEN 2003b; ERRI D183 1997; UIC 1994).

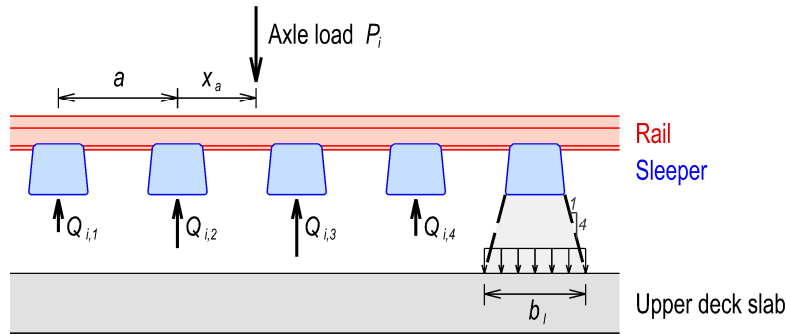


Figure 3.5 – Longitudinal distribution of axle loads due to the influence of rails, sleepers and ballast.

Once the distribution of loads acting on the deck slab has been calculated, the corresponding nodal forces can be obtained. This procedure is repeated for all the time steps. In Figure 3.6, the longitudinal distribution of nodal forces is exemplified, considering three axle loads spaced 1.6 m apart and a mesh size of 0.15 m. In view of the typical sleeper base width of ~ 0.30 m and the common sleeper spacing of ~ 0.60 m, the load transmitted by each sleeper can be assumed to be uniformly distributed over 0.60 m in the longitudinal direction, in FE analyses. In Figure 3.6, the contribution from each axle can be identified by means of the employed colour scheme.

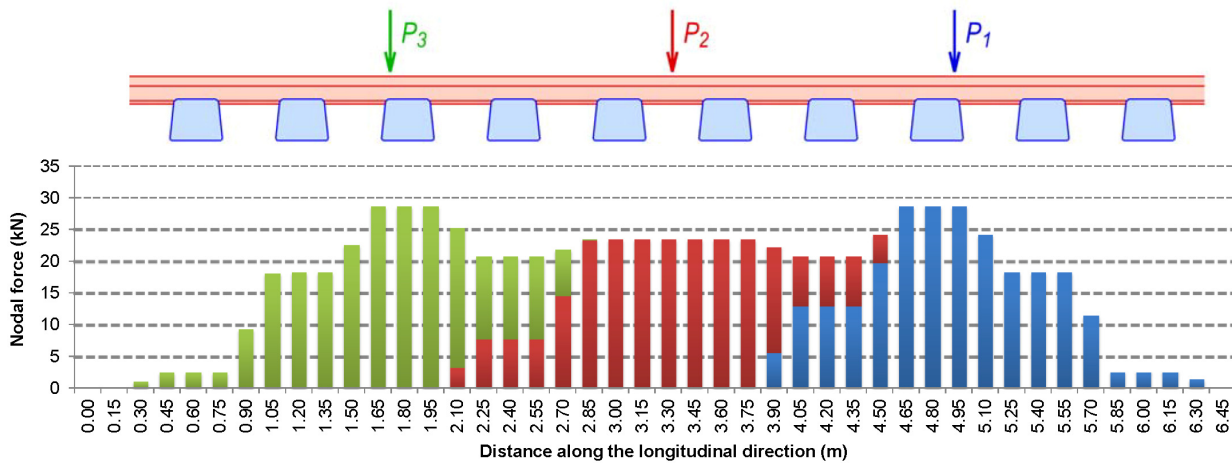


Figure 3.6 – Longitudinal distribution of nodal forces applied to the deck slab, considering three axle loads spaced 1.6 m apart and four-node shell elements with an element size of 0.15 m.

The dynamic effects associated to the railway traffic can be determined by means of a dynamic analysis with moving forces (§3.6.2.1) or, alternatively, by multiplying the static effects of railway traffic by appropriate dynamic factors (§3.6.2.2, §3.6.3 and §3.6.4). The applicability of each of these alternatives is discussed below.

Finally, it is important to remark that some simplifying assumptions, regarding the characterization of traffic actions, can be considered in fatigue analyses, as recognized by the EN 1991-2:

- the eccentricity of vertical traffic loads, in relation to the theoretical position of the track, may be ignored;
- generally, longitudinal actions due to traction and braking, as well as nosing forces, can be disregarded in the fatigue assessment, except in the case of structures supporting tracks in terminal stations.

Centrifugal forces in curved bridges must not be ignored, as they give rise to important asymmetries in the transverse distribution of applied loads. This asymmetry can be quantified as proposed by the UIC (1994).

3.6.1.2 Calculation of stresses

Fatigue verification is based on the control of stress variations caused by cyclic actions. Given the logarithmic relationship between the applied stress ranges and the number of resisting load cycles for steel and concrete, the fatigue life is very sensitive to small variations of the stress amplitude. Therefore, stress calculations for fatigue analyses must be as accurate as possible. The calculation of stresses due to cyclically variable bending moments must comply with the rules listed below.

- It shall be based on the assumption of cracked cross sections. The most unfavourable stresses typically occur at the crack cross section. Ideally, such stresses should be determined by means of models which account for the cyclic response in terms of both the bond-slip and normal stress-crack opening relationships. This alternative was dealt with in detail in Chapter 2. Alternatively, approximate stresses can be determined by ignoring the tensile strength of concrete, and considering the compatibility of strains between the concrete under compression and the reinforcements, this being the typical approach for stress calculations in cracked RC cross-sections (also known as calculations in state II).
- In stress calculations in state II, the ratio of the modulus of elasticity can be taken as 10, as suggested by the MC90. This value, which is slightly higher than the ratio of the modulus of elasticity for short term loading, accounts for some stiffness decrease in the compression chord due to fatigue effects.
- The influence of different bond behaviours of prestressing and reinforcing steels shall be taken into account. That can be done by following the simplified approach proposed by the EN 1992-1-1, which consists of increasing the stress range in the reinforcing steel

calculated under the assumption of perfect bond by a correction factor η (this parameter being quantified in the EN 1992-1-1).

- As mentioned before (§3.4 and §3.5.1), the redistribution of stresses in the compression chord can be quantified by applying the procedure recommended by the MC90.

The calculation of stresses in beams with shear reinforcement subjected to cyclic shear forces is thoroughly discussed in Chapter 4.

3.6.1.3 Combination of actions

In fatigue analyses, the stresses caused by cyclic actions have to be superposed to the ones caused by non-cyclic loads. The most relevant actions to be considered in the fatigue analysis of railway bridge decks are self weight, concrete shrinkage deformations, prestressing forces, temperature variations and traffic actions. The permanent actions (self weight, shrinkage and prestressing) are clearly classified as non-cyclic actions. On the other hand, there is no doubt that railway traffic belongs to the category of cyclic actions. The situation of temperature variations is not so straightforward, as discussed below.

In fatigue verifications according to the Eurocodes, the non-cyclic loads should be considered in the so-called basic combination of non-cyclic loading (BCNCL), which is given by the following expression:

$$\sum_{k \geq 1} G_{k,j} + P + \psi_{1,1} \cdot Q_{k,1} + \sum_{i > 1} \psi_{2,i} \cdot Q_{k,i} \quad (3.10)$$

where $G_{k,j}$ represents the characteristic value of a permanent action, P is the prestress action, $\psi_{1,1} \cdot Q_{k,1}$ represents the frequent value of the base variable action and $\psi_{2,i} \cdot Q_{k,i}$ represents the quasi-permanent value of the remaining variable actions. The effects of cyclic actions, Q_{fat} (internal forces or stresses), should, thus, be added to the ones caused by this combination of non-cyclic actions. The partial factor for cyclic loads, $\gamma_{F,fat}$, is 1.0, as indicated by the EN 1992-1-1.

The role of temperature variations

The role of temperature variations in fatigue analyses is twofold. On the one hand, the daily variation of the temperature gradient over the cross-section depth is itself a cyclic action, responsible for important stresses in continuous structures, as demonstrated by Miller et al. (2004) and Barr et al. (2005). On the other hand, during the calculation of the fatigue damage caused by railway traffic, the effects of temperature variations should be added to the ones

caused by other non-cyclic loading, because the history of applied stresses caused by traffic depends on the state of stress installed on the structure before the vehicles enter the bridge. The latter effect can be taken into account by including the temperature action in the basic combination of non-cyclic loading. As regards the former effect, the calculation of the fatigue damage due to the cyclic temperature variation can only be performed after the action is characterized through an appropriate histogram, which must define the amplitudes of temperature variations and the corresponding number of occurrences. Even though some works on the fatigue effects of temperature variations have been published (Casas and Crespo-Minguillon 1998; Jacob et al. 1987), a procedure for quantification of histograms of temperature variations to be used in fatigue analyses is not available yet. A preliminary proposal for the quantification of such histograms can be found in Sousa et al. (2010b), however, the implications of the simplifications involved in that methodology have not been fully evaluated yet.

The role of the redistribution of stresses due to long-term concrete deformations

In precast continuous bridge decks, internal forces and stresses due to self weight and prestress vary over time, as a result of the sequence of construction and the delayed deformations of concrete. For this reason, the term G_k of the basic combination of non-cyclic loading varies over time. However, it will be shown in Chapter 5 that, thanks to the reduced thickness of the different components of bridge decks constructed with precast girders, a large percentage of long-term redistributions takes place in the first ~2 years after the end of the construction. Therefore, for a fatigue life of 100 years, typically small errors are committed if the fatigue analysis is carried out considering the long-term distribution of stresses in the determination of the basic combination of non-cyclic loading.

It is important to remark that the variability associated to the long-term variation of internal forces is very significant in this type of structure, either in the case of railway bridge decks (as shown in §6.4.2) or in the case of highway viaducts (Sousa and Serra Neves 2006). Therefore, this variability must be considered in the quantification of the characteristic value of permanent actions for fatigue verifications.

3.6.2 Damage accumulation method

The damage accumulation method is a detailed methodology for fatigue analyses, in which the fatigue strength of structural materials is defined by means of S-N curves. This methodology is usually employed in the detailed fatigue assessment of several types of structures, such as steel structures (ECCS 2000). It comprises the following steps:

- definition of the traffic scenario (train types, number of trains per day, and frequent operating speeds);
- calculation of the time variation of internal forces due to the passage of each train;
- calculation of the stress history (at each critical point) caused by the passage of each train;
- identification of the extreme values of the stress history;
- counting of the stress cycles (calculation of the stress range histograms), which is usually carried out through the Rainflow algorithm;
- calculation of the accumulated damage through the Palmgren-Miner rule and the S-N curves of the material;
- calculation of the fatigue life.

The first two steps can be carried out according to two different approaches, depending on whether a dynamic analysis is required or not. These approaches are described in sections §3.6.2.1 and §3.6.2.2, respectively.

Once the time histories of internal forces have been calculated, stress histories can be determined taking into account the principles mentioned in §3.6.1.2.

Then, the stress histories have to be converted into stress histograms, which represent the distribution of stress ranges versus the corresponding number of cycles. To that end, the local peaks of the stress history are firstly identified. After that, a proper cycle counting method must be employed. Several alternative methodologies exist, and the results of the counting process may differ from procedure to procedure. Nevertheless, many authors prefer the Rainflow method because on the one hand, it is based on the mechanics of materials such as steel, and on the other hand, it generally provides higher total damage than other methods and, therefore, it usually gives results on the safe side (ERRI D216 2002). The Eurocodes do not suggest any particular cycle counting method. However, the MC90 does suggest the Rainflow method. Thus, that is the method employed in this work. Its description can be found in publications by Downing and Socie (1982), or Fryba (1996).

Once the stress histograms have been computed, the total damage caused by the traffic scenario can be determined. The fatigue damage can not be directly read from Wöhler curves, as they represent the fatigue life for a constant amplitude cyclic loading. Therefore, a suitable procedure must be employed, so that the influence of different stress ranges is taken into account. The most commonly used procedure is the linear damage-accumulation rule of Palmgren-Miner (Miner 1945). According to this rule, the damage factor, D , is given by:

$$D = \sum_i \frac{n_i}{N_i} \quad (3.11)$$

where n_i is the number of applied cycles for a given stress range and N_i denotes the number of resisting cycles for the same stress range, according to an appropriate Wöhler curve. In the case of concrete, the design value of the number of resisting load cycles can be determined by means of Equation (3.3). The design value of N_i for reinforcing and prestressing steel is given by Equation (3.1). Fatigue failure is reached when $D = 1$.

The Palmgren-Miner rule has been subjected to some criticism, which is justified by the following reasons:

- the order of the load cycles is not taken into account; however, in reality, large stress amplitudes initially generated can cause a crack, which then develops under subsequent small amplitudes, whereas small amplitudes would originate much lower damage if the order was the opposite (ERRI D216 2002; RILEM Committee 36 RDL 1984);
- the assumed linear development of the damage does not correspond to the real development of fatigue damage, which includes three different phases (Shah 1984);
- the application of the Palmgren-Miner rule in the analysis of results of multi-stage fatigue tests has shown that this rule can give either conservative or unsafe results, especially for concrete (CEB 1988; RILEM Committee 36 RDL 1984).

However, despite the well known shortcomings of the Miner's rule, that is the rule adopted in nearly all the damage calculations because of its simplicity, and because it has been shown that more complex rules do not always produce better results (ERRI D216 2002). It is also important to remark that the differences between the results provided by the Miner's rule and the ones obtained in variable-amplitude fatigue tests are not only caused by the inaccuracy of the linear damage-accumulation rule, but also by other factors, namely the scatter in material properties.

The fatigue life, FL , in years is given by:

$$FL = \frac{1}{D_{year}} \quad (3.12)$$

where D_{year} represents the damage factor corresponding to one year of traffic. The fatigue life of new structures must be higher than the intended service life, which is usually taken as 100 years.

Finally, it is worth mentioning that the damage accumulation method works as a reference analysis procedure for the derivation of simplified methodologies for fatigue analyses, such as the damage equivalent stress method.

3.6.2.1 Cases in which dynamic analyses are required

In general, the design of railway bridges in high speed lines (HSLs) requires a dynamic analysis of the passage of trains, so that dynamic effects and resonance phenomena are taken into account. According to the Eurocodes, a dynamic analysis is required whenever the conditions indicated by the EN 1991-2 are met. For example, every continuous bridge deck designed for a maximum line speed at the site greater than 200 km/h must be subjected to a dynamic analysis. This analysis is used to determine the dynamic amplification of internal forces and the maximum vertical acceleration of the deck, which is used to control the possibility of ballast instability. In structures for which dynamic analyses are required, fatigue verifications must be carried out through the damage accumulation method, and the calculation of the history of internal forces caused by the passage of trains must also be determined by means of dynamic analyses.

The definition of the traffic scenario to be used in fatigue analyses is not a straightforward task, because the traffic scenario can hardly be accurately characterized (in particular as regards future traffic). The traffic scenario must define the train types to be considered, the annual traffic volume (tonnage) and the train speeds. The latter factor is particularly critical, because the stress history due to the passage of a train is notably different depending on whether or not its operating speed is close to a resonant speed.

However, determining at the design stage whether resonance will frequently occur is not a simple task. On the one side, the frequent operating speed of a real train (during the entire service life of the structure) is usually an uncertain parameter. On the other side, the actual natural frequencies of the bridge may differ from the values estimated during the design. For these reasons, the resonance speed estimated at the design stage is an approximate value, and allowance should be made for both the range of possible train speeds and the range of the bridge's plausible dynamic characteristics. Different approaches for this problem are found in the literature. According to Muncke (2006), the Deutsche Bahn AG specifies that resonance must be taken into account if the estimated resonance speed is in the range of $0.9 \cdot v_{freq}$ to $1.1 \cdot v_{freq}$, where v_{freq} represents the frequent operating speed. Alternative assumptions adopted for French lines were reported by Picquenot (2009) or by Garnier (2010), as shown in Table 3.3.

Table 3.3 – Traffic scenarios for taking into account resonance effects in fatigue analyses

	Picquenot (2009)	Garnier (2010) ¹
Train type	The trains included in the HSLM-A and estimated real trains	Double TGV
Number of trains per day, per track	75	70
Proportion of traffic that crosses the bridge simultaneously	5%	5%
Train speeds	50% of the trains at estimated frequent operation speeds and 50% of the trains at resonance speeds in the range of 200 to 350 Km/h	50% of the trains at resonance speeds and 50% of the trains at estimated frequent operation speeds ²

NOTES: ¹ Traffic scenario adopted for the French line LGV Rhin-Rhone.

² A less conservative criterion is reported by Hoorpah (2005) and by Datry et al. (2004), according to whom the train speed considered for fatigue analyses in the line TGV Est was equal to the commercial speed of 300 or 350 km/h. In that line, the train type, the traffic volume and the proportion of simultaneous crossings are equal to the ones for the line LGV Rhin-Rhone.

The uncertainty associated to the characteristics of trains to be used in the future is solved by means of high speed load models. In bridges designed for international lines where the European Technical Specifications for Interoperability TSI³ (European Railway Agency 2008a; 2008b) are applicable, the trains included in the load model High Speed Load Model (HSLM) must be considered in fatigue analyses. This load model is described in the EN 1991-2, and is composed of several trains which were devised so that their dynamic effects form an envelope of the ones caused by trains which are allowed to operate in interoperable lines in accordance with the TSI. The HSLM comprises two different train sets: HSLM-A and HSLM-B. Only the HSLM-A needs to be considered in bridges with spans longer than 7 m. This set is composed of 10 different trains with different bogie spacing, so that the resonant speeds of the different trains do not coincide.

In bridges that carry mixed traffic (a combination of high-speed and freight traffic), freight trains should also be included in the traffic scenario. The influence of freight trains is further discussed below.

³ The technical specifications for interoperability define parameters that relate to the infrastructure, rolling stock, operation and traffic management, among others, in order to ensure the interoperability of the trans-European high speed and conventional rail systems, and to allow the safe and uninterrupted movement of trains between countries.

As far as the volume of traffic is concerned, a conservative approach can be adopted by considering an upper bound value of the number of trains per day. This conservative procedure is not too onerous, since the fatigue life is much less sensitive to the volume of traffic (according to the linear damage accumulation rule, the fatigue life is directly proportional to the volume of traffic) than it is to the stress history caused by each train (owing to the logarithmic S-N relationships).

The dynamic analysis of the passage of trains in bridge decks with moderate or large span can be carried out by representing the trains by a series of moving point loads, as mentioned before. According to the findings of the ERRI Committee D214 (1999b), which were adopted by the EN 1991-2, in these analyses, the influence of the vehicle-structure interaction can be indirectly taken into account by increasing the structural damping. On the other hand, the dynamic amplification due to track defects and vehicle imperfections can be accounted for by multiplying the results of dynamic analyses with moving forces by:

$$(1 + \lambda \cdot \varphi'') \quad (3.13)$$

where φ'' is a factor quantified in the EN 1991-2 (Annex C), and λ is a coefficient which takes the values 0.5 or 1.0 for carefully maintained tracks or tracks with standard maintenance, respectively.

In this work, the dynamic analyses are carried out by employing the mode-superposition method. The fact that the applied loads act on a predetermined limited set of nodes makes this analysis quite efficient, because only the uncoupled equations related to the degrees of freedom with non-zero applied nodal forces have to be solved, as explained by Albuquerque (2009). Another important advantage of the mode-superposition method lies in the fact that it is possible to define the damping ratio associated to each mode of vibration. That would not be possible if direct time integration methods (such as the Newmark method) were employed. In this work, the code for mode-superposition analyses implemented by Albuquerque (2009) is employed. The modal analyses for determining modal displacements and internal forces are performed with resource to the finite-element software ANSYS (2009).

The influence of cracking in terms of dynamic behaviour of railway bridge decks, as well as the numerical approaches for the quantification of that influence, were previously discussed in Chapter 2.

Bridges with more than one track

The fatigue analysis of bridges with more than one track should take account of the possibility of simultaneous crossings.

The Eurocodes present a simplified procedure for taking into account the influence of simultaneous crossings for structures in which the simplified method for fatigue verification (the damage accumulation method, which is presented below) is applicable. However, that method is not applicable to bridges that are prone to resonance phenomena, and for those cases, the Eurocodes do not provide any guidance on how the traffic in both tracks should be combined. The following paragraphs introduce a possible approach for taking into account the effect of simultaneous crossings in the fatigue verification of these bridges. The discussion focuses on double track bridges for the reason that this is the most common bridge configuration, but an analogous procedure can be applied to bridges with three or more tracks.

Firstly, it is necessary to define the percentage of traffic that crosses the bridge simultaneously. Indicative values are shown in Table 3.3.

Then, it is necessary to determine the fatigue damage due to both simultaneous and non-simultaneous crossings. The effect of single train passages can be determined by means of dynamic analyses with moving forces. As regards simultaneous crossings, two different situations may occur:

- two tracks with trains normally travelling in opposite directions;
- two tracks with trains normally travelling in the same direction.

As far as the traffic in the same direction is concerned, the ERRI Committee D214 (1999b) recognizes that the likelihood of the dynamic effects of both trains being in phase is significant. Hence, the calculation of resonance effects shall be conservatively carried out considering that the tracks are loaded in phase.

If the tracks carry traffic in opposite directions (which is the most common situation), it is unlikely that both tracks will be loaded in phase (Figueiredo et al. 2007). Consequently, this situation is less detrimental than train passages in the same direction. However, the calculation of the combined effects of traffic in both tracks is more complex in this case. A possible approach for performing such calculation was proposed by Figueiredo et al. (2007). This methodology consists of considering that the location of the point where the trains meet is a random variable x which follows a uniform probability density function $f(x)$ (see Figure 3.7). The stress history associated to each possible value of x is determined by a linear combination of the stress histories of the single passages in both tracks. Then, the fatigue damage for each value of x is

calculated by means of the damage accumulation method. Finally, the total fatigue damage due to simultaneous crossings is obtained by multiplying the corresponding total number of trains by the average of the fatigue damages for each value of x .

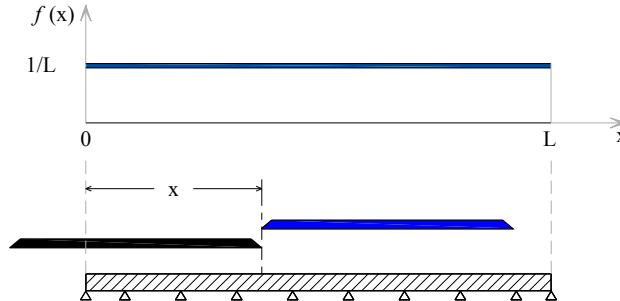


Figure 3.7 - Probability density function of the location of the point where both trains meet (Figueiredo et al. 2007).

3.6.2.2 Cases in which dynamic analyses are not required

If the characteristics of the bridge and the maximum permitted train speed are such that dynamic analyses of the passage of the trains are not required (according to the criteria indicated before), then the fatigue verification according to the damage accumulation method is simpler. This section focuses on the definition of the fatigue loading to be used in those cases.

In bridges where very heavy freight trains are not allowed to operate (the identification of those situations is made below), fatigue verifications can be conducted by adopting one of the three traffic scenarios specified in the EN 1991-2, depending on the envisaged traffic for the structure under analysis:

- *standard traffic* if the structure carries standard traffic;
- *traffic with 250 kN-axles* if the structure carries predominantly heavy freight traffic;
- *light traffic mix* if the structure carries light traffic.

These traffic mixes are characterized in the EN 1991-2, which describes the trains that form each traffic mix and the corresponding frequent operating speeds and volumes of traffic.

The fact that dynamic analyses are not required means that resonance effects are not expectable, and consequently, the history of internal forces caused by the passage of trains can be determined by multiplying the results of a static analysis by appropriate dynamic factors. The history of internal forces due to the static effects of each train can be determined by means of a transient analysis (direct time integration according to the Newmark method, for example),

setting the mass equal to zero. The dynamic factors for fatigue assessment are given in the Annex D of the EN 1991-2.

It was mentioned before that the three traffic scenarios included in the EN 1991-2 are only applicable if very heavy freight trains are not allowed to operate in the structure under analysis. This means that the validity of those scenarios is limited to bridges in which the effects (internal forces and deformations) of real trains are covered by the load models LM71 and SW/0 [these models being defined in the EN 1991-2 and in the UIC leaflets 776-1-R (UIC 2006) and 702-OR (UIC 2003)].

In lines carrying rail traffic heavier than normal, the characteristic values of the load models LM71 and SW/0 must be multiplied by the factor α , which may take values up to 1.46. When multiplied by the factor α , the loads are called *classified vertical loads*. The value of α may be specified in the national annex of the EN 1991-2, or for each individual project. For example, the Portuguese Administration adopted $\alpha = 1.21$ in the HSL Poceirão-Caia (RAVE 2008).

If it is required a value α higher than the unit to simulate the effects of railway traffic, then the traffic scenario for fatigue verifications can not be given by any of the three aforementioned traffic mixes (EN 1991-2). Under such circumstances, an alternative traffic mix should be specified. A possible approach for the definition of alternative traffic scenarios is included in the Danish and in the Swedish assessment codes, as reported by James and Karoumi (2006). This approach is based on the concept of line classification.

The line classification system establishes the characteristics of wagons and locomotives which are allowed to operate in each line. This system consists of a set of line categories (A, B1, B2, C2, C3, C4, etc.), which are specified by the UIC leaflet 700-OR (UIC 2004), by the prEN 15528 (CEN 2006) and by the TSIs (European Railway Agency 2009). Minor differences exist between the categories considered in each of these documents. Each line category is associated to different maximum permitted values for both the mass per axle and the mass per unit length, as shown in Table 3.4. Each Railway Administration must assign a line category to each section of line taking into account the load-carrying capacity of the infrastructure (track, bridges, etc.). Then, a given vehicle (wagon, locomotive, passenger vehicle, etc.) is allowed to operate in that section of line if the bending moments and shear forces caused by that vehicle on a single beam of any span length do not exceed the values calculated from the load model for the corresponding category (the prEN 15528 specifies how this calculation should be carried out). This load model consists of an association of reference freight wagons, depicted in Figure 3.8, whose axle load and mass per unit length are defined in Table 3.4 for each line category. In this way, it is possible to ensure compatibility between the vertical loading characteristics of railway vehicles and the load carrying capacity of the railway infrastructure.

Table 3.4 – Permissible traffic loads (static vertical load) on various line categories according to the UIC Leaflet 700-OR (ERRI D183 1997), the prEN 15528, the TSIs and the Danish assessment code (James and Karoumi 2006).

Permissible mass per unit length	Permissible mass per axle							
	16 t (A)	18 t (B)	20 t (C)	22.5 t (D)	25 t (E)	27.5 t (R)	30 t (S)	33 t (T)
5 t/m (1)	A	B1						
6.4 t/m (2)		B2	C2	D2				
7.2 t/m (3)			C3	D3				
8 t/m (4)			C4	D4	E4	R4	S4	
8.8 t/m (5)					E5	R5	S5	T5
10 t/m (6)							S6	T6
11 t/m (7)								T7

└─ Danish code
└─ UIC Leaflet 700-OR and prEN 15528
└─ TSIs

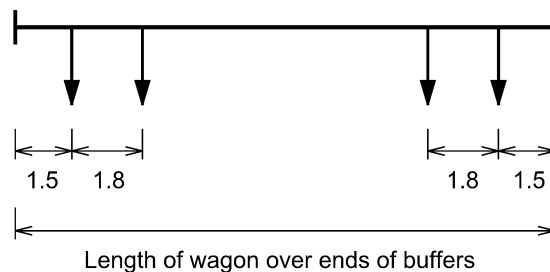


Figure 3.8 - Axle configurations for the reference freight wagons.

For the sake of example, Figure 3.9 shows the classification established by the Portuguese Administration for Railway Lines in the North of Portugal. Here, one can see that the maximum admitted values in terms of axle loads and loads per unit length are 22.5 t and 8 t/m, respectively. If we look at Table 3.5, where the characteristics of the various freight trains included in the EN 1991-2 are shown (in terms of axle loads and loads per unit length), one can see that the masses of vehicles which are allowed to operate in the Portuguese lines are not more severe than those for the trains included in the EN 1991-2.

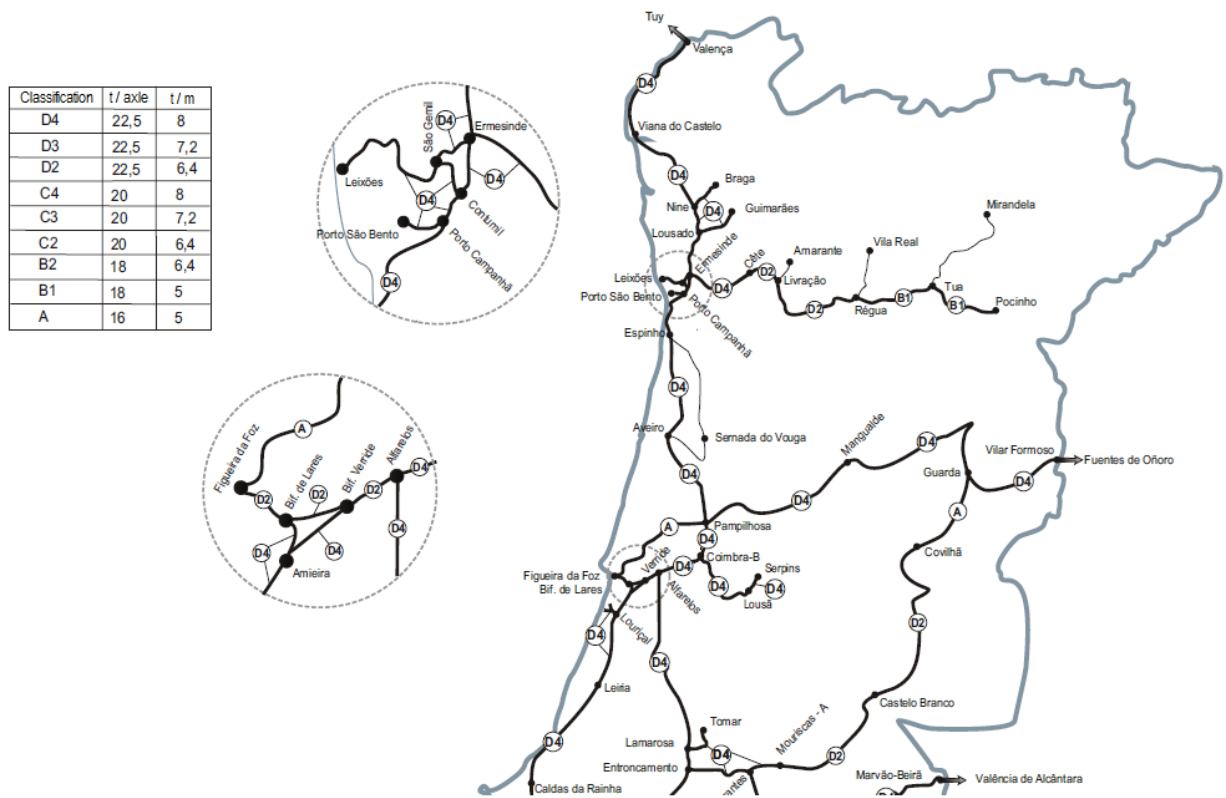


Figure 3.9 - Line categories in the North of Portugal, according to the INTF (2005).

Table 3.5 – Freight train types for fatigue verification, according to the EN 1991-2 (static vertical loads).

Train type	Maximum axle force (kN)	Load per unit length (locomotive) (kN/m)	Load per unit length (wagons) (kN/m)
5	225	80	80
6			43
7			51
8	250	73	46
11			56
12			52

On the other hand, the fatigue trains included in the EN 1991-2 are clearly inadequate in the case of line categories with loads of 8.8 t/m or 27.5 t/axle, or higher. Such line categories have been adopted in some European countries, following the constant demands for increased allowable axle loads on freight wagons, in order to increase the competitiveness of the railway transport sector. In Table 3.4, one can see that at the present time, axle loads of up to 33 t/m are admitted in the Danish code. In these cases, fatigue analyses can be carried out according to the procedure reported by James and Karoumi (2006), which is included in the Danish assessment code, and is similar to the one adopted in Sweden. According to this procedure, the train for fatigue

verifications is composed of 20 reference wagons, whose configuration is the one depicted in Figure 3.8, whose axle loads are the ones displayed in Table 3.4 for each line category, and whose wagon length is defined so that the load per unit length matches the requisite for the line category. It is assumed that the loads from locomotives do not result in larger load effects compared to the load effect from the reference wagon. Empty wagons with the same configuration and length, but with a reduced mass per unit length equal to ~ 10 kN/m, must be considered in the most unfavourable manner, so as to produce maximum efforts. The volume of traffic for calculation of the fatigue damage should be based on records of the tonnage passed in the line under analysis, or on estimates for future traffic.

In the case of very heavy railway traffic, dynamic analyses are not required for the quantification of dynamic enhancements, because resonance effects are not expected, owing to the low train speeds. Instead, dynamic effects can be taken into account by means of the dynamic factors used for fatigue assessment included in the EN 1991-2, annex D.

The discussion presented in the previous section (regarding bridges with more than one track) is also applicable to the cases where a dynamic analysis is not required.

3.6.2.3 Software for fatigue analysis according to the damage accumulation method

The described procedure for fatigue analysis according to the damage accumulation method was implemented in a windows based application, which comprehends various modules to perform the following tasks:

- pre-processing of time histories of nodal forces to simulate the passage of real trains for any traffic scenario;
- post-processing of the results of dynamic analyses of the passage of real trains;
- calculation of stress histories due to cyclic bending moments or shear forces;
- calculation of histograms of applied stresses, by applying the Rainflow counting method;
- calculation of the accumulated damage according to the Palmgren-Miner rule, by using appropriate S-N curves.

The employed programming language was Visual Basic, from Microsoft. A comprehensive description of the developed application can be found elsewhere (Sousa et al. 2010c).

3.6.3 Damage equivalent stress method

The damage equivalent stress method is a simplified method for fatigue verification. Fatigue proving according to this method is based on the quantification of the damage equivalent stress range $\Delta\sigma_{equ}(N^*)$, which represents the constant amplitude stress range that, if repeated N^* times, causes the same fatigue damage to the structure as the total live load in its planned service life. Thus, adequate fatigue resistance exists if $\Delta\sigma_{equ}(N^*)$ does not exceed the fatigue strength for N^* constant-amplitude stress cycles.

The Eurocodes present a feasible procedure for quantification of the damage equivalent stress range, which is based on the multiplication of the maximum stress range due to the load model LM71 (and due to the load model SW/0 when applicable) by the dynamic factor Φ and by the correction factors λ_0 to λ_4 , which take into account the following effects:

- λ_0 - the stress level due to non-cyclic loads (this factor is only used in the fatigue verification of concrete);
- λ_1 - the structural and material behaviour (Wöhler curve), the direction of loading (longitudinal and transverse), the type of loading (bending, shear, torsion) and the load histogram;
- λ_2 - the annual traffic volume;
- λ_3 - the design service life;
- λ_4 - the number of tracks and the influence of simultaneous crossings.

The procedure for quantification of the factors λ_0 to λ_4 is described in the EN 1992-2. This methodology is the result of an extensive numerical research carried out at the Technical University of Munich on behalf of the German Railways, and also sponsored by the UIC/ERRI (ERRI D183 1997). An independent verification of the work developed at the TU Munich was later performed by the Holland Railconsult (ERRI D216 2000). This verification was required by the UIC/ERRI because of this procedure's relevance, as it was supposed to be widely employed by Railways in countries where the Eurocodes are applicable. As a result of this verification, some corrections were recommended in terms of quantification of the factor λ_1 .

This methodology is suitable to be used in the current design practice, although it encompasses two important limitations:

- it is not applicable whenever dynamic analyses of the effect of the trains' passage are required;
- it is not valid for lines carrying railway traffic heavier than normal (if an α factor higher than the unity is required to simulate the real traffic);

In those cases, the fatigue verification must be carried out through the damage accumulation method, considering an adequate traffic scenario, following the procedures described before.

An alternative simplified procedure, for the cases in which the characteristics of railway traffic impose $\alpha > 1$, was adopted by the Spanish code IAPF (Ministerio de Fomento 2007). According to this code, the damage equivalent stress range must be taken as the product of the static effects of the load model LM71, the dynamic factor Φ , the correction factors λ_0 to λ_4 , and the α factor. That is, according to the Spanish code, the α factor should also be considered in the stress calculations for fatigue analyses. By analysing the procedure developed by the ERRI D183 (1997) for derivation of the correction factors λ_0 to λ_4 , one can conclude that, if the S-N curve could be represented by a linear relationship in double logarithmic scale, then the Spanish approach would accurately simulate the EN 1991-2 traffic scenarios with axle loads multiplied by α . This means that this approach would satisfactorily simulate the influence of increased axle loads if the fatigue behaviour of concrete and steel was described by mono-linear S-N curves. For realistic S-N curves, the approach of the Spanish code IAPF provides approximate results, due to the underlying simplifications.

3.6.4 Fatigue proving by limiting stresses

The most simplistic methodology for fatigue verification consists of the comparison between the applied stresses for a given combination of actions and a specified value for the resistant stress. The EN 1992-1-1 provides guidance on the fatigue verification of both reinforcing and prestressing steel, and for concrete under compression, according to this methodology. This approach can be applied in the case of structures which are not sensitive to cyclic actions, so that more laborious analysis methodologies are avoided.

It is important to remark that the method of fatigue proving by limiting stresses is not recommended for the analysis of tensile reinforcements, or of concrete under compression, in structures subjected to important cyclic loading (such as railway bridge decks), because it would provide very conservative and uneconomical results.

Nevertheless, that is the only methodology available in the Eurocodes for the fatigue check of:

- the effects of shear forces in slabs without stirrups;
- the effects of shear at the interfaces between concretes cast at different times.

As regards the shear behaviour of slabs without stirrups, it was shown before that further research is needed. Johansson (2004), who has performed fatigue tests in slab like beams, concluded that the methodology for fatigue proving by limiting stresses included in the MC90 is very conservative. De Backer et al. (2009) also advocate that the shear fatigue rules for members

without stirrups, as proposed by the EN 1992-1-1, might be conservative. Nevertheless, these authors report a case study in which the shear fatigue failure of a RC wall was narrowly avoided. This element was subjected to important cyclic loads because it supported the weight of a steel corbel connected to a crane girder. The shear fatigue strength of the RC wall had not been checked at the design stage. If the fatigue verification had been carried out according to the EN 1992-1-1, it would have become evident that there was insufficient fatigue strength.

3.7 Final considerations

This chapter focuses on the fatigue analysis of railway bridge decks.

Firstly, the fatigue performance of reinforcing steel, prestressing steel and concrete is briefly described. Procedures for quantification of the fatigue strength for these materials, to be used in the applications envisaged in this work, were also indicated.

Then, the main specificities of the fatigue response of reinforced and prestressed structures were introduced.

After that, alternative procedures for fatigue analyses of railway bridge decks were presented: the damage accumulation method, the damage equivalent stress method and the simplified methodology for fatigue proving by limiting stresses.

The limitations of the latter methodology, as well as the implications of those limitations, were presented and discussed.

The specificities and the field of application of the damage equivalent stress method, which is a simplified methodology suitable for application in current design practice, were also shown.

While the application of the damage equivalent stress method is restricted to specific conditions in terms of traffic scenarios, the damage accumulation method can be used to study the effects of any history of applied loads. Therefore, special attention was devoted to the latter methodology. Several issues which are not dealt with in current design codes, such as the Eurocodes, were discussed and clarified. Traffic load models to be used in fatigue analyses of bridges which carry either high speed passenger traffic or very heavy freight traffic were presented. The implications of both temperature variations and long-term stress redistributions due to concrete delayed deformations were discussed. A feasible procedure for taking into account the degradation of axle load effects (owing to the influence of the rails, the sleepers and the ballast) in dynamic analyses with moving loads was presented and implemented in a pre-processing numerical tool for dynamic analyses of the passage of trains.

The described procedure for fatigue analysis according to the damage accumulation method was also implemented in a computer application.

Chapter 4

Fatigue analysis of girder webs subjected to in-plane shear and out-of-plane bending

4.1 Introduction

The main function of girder webs consists of providing adequate capacity to resist longitudinal (in-plane) shear forces. However, these elements are also subjected to other secondary forces. In the case of box girders, important transverse bending moments (m_y , as represented in Figure 4.1) occur, associated to the structure behaviour in the transverse direction, under the effect of both permanent and live loads. Therefore, the most important forces, which must be taken into account in the analysis of girder webs, are in-plane shear and transverse bending (Menn 1986). Other secondary forces, which could affect the stirrup stresses (such as the membrane force in the direction of the stirrups and the twisting moment m_{xy}), typically exhibit negligible values (as can be confirmed by looking at the results presented in Annex 3).

This behaviour is not specific of bridges built with precast U-shaped girders. Generally, it occurs in bridge decks with box girders, namely in structures built with the cantilever method. However, precast elements usually have slender walls, which make them more sensitive to secondary effects in comparison with thicker, cast in situ, elements.

Several methodologies have been proposed for taking into account the combined effects of in-plane shear and transverse bending in ultimate limit state (ULS) analyses. These effects were initially handled in a very simplistic way, by considering that the required cross-sectional area of stirrups should be equal to the maximum between the reinforcement required for in-plane shear

and that required for transverse bending, as proposed by Robinson (Lefaucher 2002). On the other hand, the AFGC (1999) recommends a more conservative approach, which consists of the independent analysis of both effects and the subsequent sum of the required areas of reinforcing steel. These are simplified approaches, and none of them is consistent with the actual structural behaviour. The former might lead to unsafe results. On the other hand, the latter gives rise to:

- excessive amounts of reinforcing steel in the case of moderate concrete stresses;
- incoherent superposition of concrete compressive stresses due to both effects, which might lead to unsafe results in the case of concrete elements subjected to high compressive stresses.

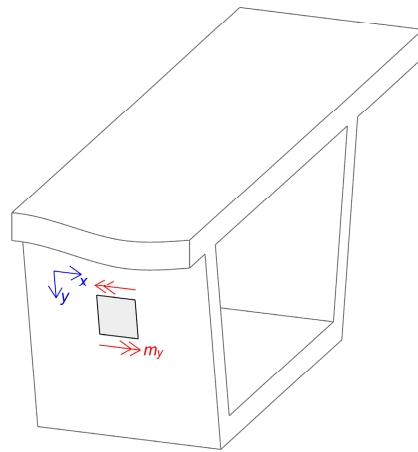


Figure 4.1 – Transverse (out-of-plane) bending moment applied to a girder web.

However, coherent approaches for taking into account the combined effects of in-plane shear and transverse bending in ULS are already available. The methodology proposed by Menn (1986) is based on the experimental observations by Kaufmann and Menn (1976). Alternative approaches were proposed by Lefaucher (2002) and Gaspar (2003). The EN 1992-2 (CEN 2005b) adopted a different methodology, based on the sandwich model.

Considerably less attention has been paid to fatigue analyses. To the author's knowledge, only Gaspar (2003) and Gaspar and Stocchi (2009) have investigated the fatigue behaviour of girder webs under the combined effects of in-plane shear and transverse bending. These authors carried out laboratorial studies and proposed a methodology for fatigue design.

The fatigue analysis of girder webs considering the influence of transverse bending is particularly critical in the case of railway bridge decks built with U-shaped precast girders. On the one hand, cyclic effects associated to railway traffic loads give rise to important fatigue damage. On the other hand, precast girders usually have thin webs; in-plane shear forces give rise to high shear stresses levels in these elements, which make them sensitive to secondary

effects such as transverse bending. Therefore, a new methodology for fatigue analysis of girder webs in railway bridge decks was developed in this thesis, taking into consideration previous achievements by Gaspar (2003) and Gaspar and Stucchi (2009). This methodology was implemented in a Windows based application (Sousa et al. 2010c) and was validated by comparison with experimental results obtained by Gaspar (2003).

The main specificity of the fatigue analysis of girder webs, which is dealt with in this chapter, lies in the calculation of stress histories for a given time variation of shear forces and transverse bending moments. The developed procedure for stress calculations can then be used in fatigue verifications according to the damage accumulation method, which involves the various steps described in Chapter 3:

- definition of the traffic scenario;
- calculation of the history of longitudinal shear forces and transverse bending moments, including dynamic enhancement (dynamic analyses must be carried out if needed and proper models must be used for the distribution of the axle forces through the rails, sleepers and ballast);
- calculation of stress histories for both the stirrups and the concrete, considering the effects of longitudinal shear and transverse bending;
- adoption of suitable S-N curves to describe the fatigue resistance of the materials;
- computation of the fatigue damage (by using the linear damage accumulation rule) and the fatigue life.

The actual structural response of girder webs subjected to in-plane shear and transverse bending is complex and involves several non-linear mechanisms, associated to the non-linear cyclic behaviour of the concrete, the cracks, and the interaction between steel and concrete. Instead of explicitly handling all these non-linear mechanisms, the approach adopted in this work consists of using a simplified algorithm for stress calculations, which is validated through the comparison with experimental results. The algorithm is based on equilibrium conditions and simplifying assumptions for the definition of the concrete stresses' distribution [which, in turn, are based on the experimental observations of Gaspar (2003)].

The simplicity of the developed algorithm makes it suitable for the analysis of variable histories of internal forces, determined through dynamic analyses. Therefore, it can be employed in the calculation of the fatigue damage due to the passage of real trains, taking into account resonance effects.

This chapter comprises two main sections:

- firstly, in §4.2, the procedure for stress calculations considering the sole effect of longitudinal shear is presented and validated through comparisons with experimental results available in the literature;
- then, in §4.3, the developed procedure for stress calculations considering the interaction with transverse bending is described; finally, this procedure is also validated through comparisons with experimental data.

Given that this chapter focuses on fatigue analyses, ULS calculations considering the combined effects of in-plane shear and transverse bending are only briefly addressed. This option is justified by the fact that various methodologies for ULS calculations are already available, as mentioned before. Additional information regarding ULS analyses is given in Annex 1.

4.2 Analysis of the sole effect of in-plane shear

The possible types of shear fatigue failure in beams with shear reinforcement were identified by the RILEM Committee 36 RDL (1984) and are displayed in Figure 4.2. This figure shows two failure modes which depend on the concrete fatigue strength (in the compression chord and in the web, respectively) and two modes associated to steel failures (stirrups and longitudinal tension reinforcement, respectively).

Fatigue failure of the stirrups is the most frequently observed failure mode in fatigue tests. Therefore, this mode has received more attention. This failure mode strongly depends on the fatigue properties of the reinforcement. However, the application of the results from tensile tests with straight bars to shear reinforcement in beams is not as straightforward as the application of those test results to bending reinforcement, because of the following reasons:

- the actual stress history in the stirrups can be very complex, even in the case of constant amplitude loading;
- fatigue failures might occur at the stirrup bends, which might contribute to a reduction of the fatigue strength;
- shear cracks usually cross the stirrups at skew angles, which gives rise to additional stresses due to dowelling and other effects that might impair the fatigue strength.

Consequently, it is much more difficult to determine realistic stress levels acting in shear reinforcement than in bending tensile reinforcement. Fortunately, it has been shown that the stress distribution in the stirrups becomes more uniform (among the various stirrups) as the number of loading cycles increases and the cracking pattern stabilizes (Frey 1982; Ruhnau

1974). Moreover, it has also been shown that the shear fatigue strength is not impaired by the reduced fatigue strength at the stirrup bends, i.e., shear fatigue strength is governed by the characteristics of the straight portion of the stirrups (Frey 1982; Higgins et al. 2007; Higgins et al. 2004; Kreger et al. 1989), because the maximum stirrup stresses do not occur at the bends.

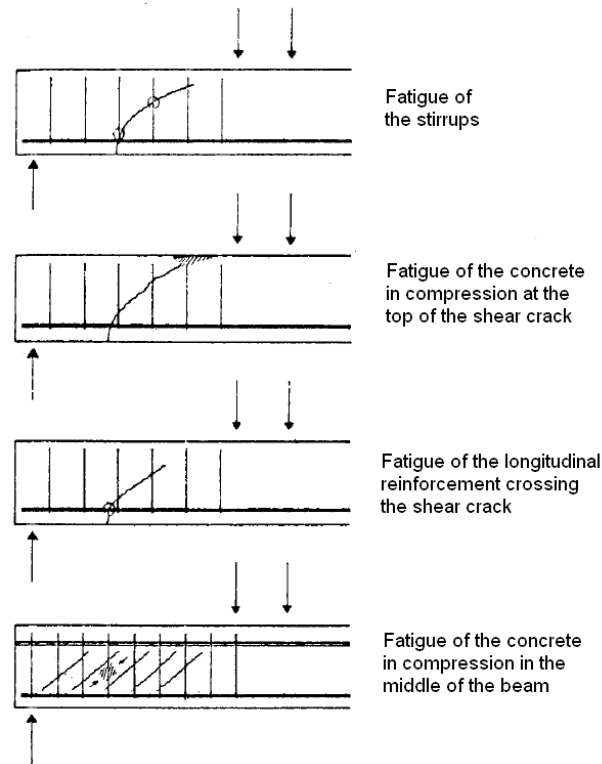


Figure 4.2 - Possible types of shear fatigue failure in beams with shear reinforcement (RILEM Committee 36 RDL 1984).

4.2.1 Methodology for ULS and fatigue analyses

Despite the complexity of the structural behaviour under the effect of cyclic shear forces, stress calculations for fatigue verifications are usually based on variable-inclination truss models (ERRI D216 2002). This procedure was recommended by Frey (1982) and was adopted by the MC90, by the Eurocodes (CEN 2004; 2005b) and by the UIC (2005). A similar methodology is usually employed in ULS analyses (CEN 2004).

The truss model is schematically represented in Figure 4.3, which depicts the forces applied to the truss components, for a beam segment subjected to constant shear force and variable bending moment. In the present work, the stirrups are assumed to be perpendicular to the girder axis and the girder depth is assumed to be constant.

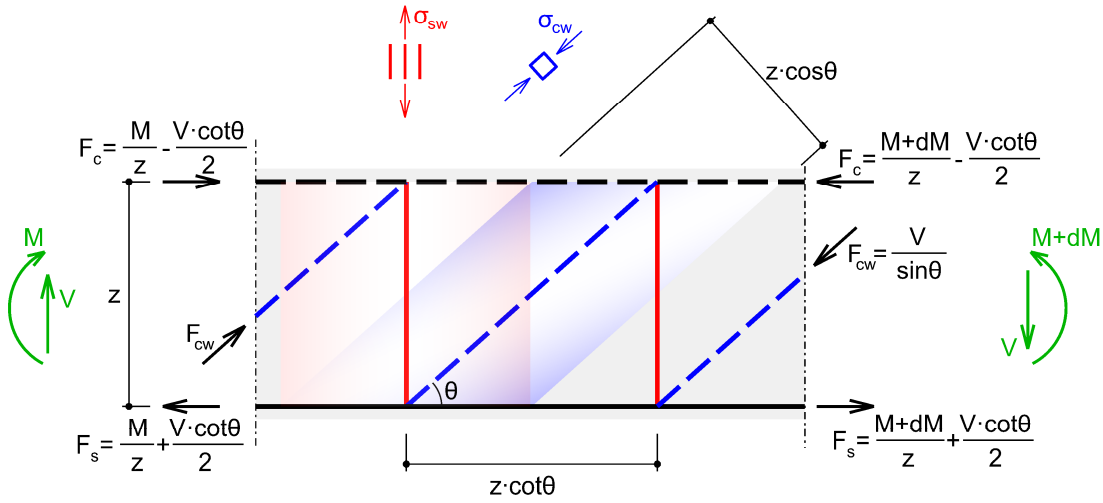


Figure 4.3 – Truss model for calculation of stresses due to shear forces.

The stresses in the compression struts, σ_{cw} , and in the stirrups, σ_{sw} , are determined from equilibrium considerations resulting in the following expressions:

$$\sigma_{cw} = \frac{V}{b_w \cdot z \cdot \sin \theta \cdot \cos \theta} \quad (4.1)$$

$$\sigma_{sw} = \frac{V}{\frac{A_{sw}}{s} \cdot z \cdot \cot \theta} \quad (4.2)$$

where b_w is the web thickness, z is the internal lever arm for longitudinal bending, θ is the inclination of the compression struts with respect to the longitudinal axis of the girder, and A_{sw}/s is the cross-sectional area of stirrups per unit length. The tensile force in the stirrups is balanced by the compression force in the web concrete, in the direction of the stirrups, per unit length, C :

$$C = \frac{V}{z \cdot \cot \theta} \quad (4.3)$$

and the compression force in the struts, in the direction θ , per unit length, is given by:

$$F_{cw,unit} = \frac{V}{z \cdot \sin \theta \cdot \cos \theta} \quad (4.4)$$

the parameters C and $F_{cw,unit}$ being particularly useful for the analysis of girder webs subjected to in-plane shear and out-of-plane bending (§4.3). The inclination of the compression struts to be used in ULS design, θ_{ULS} , should comply with the following limits (CEN 2004):

$$1 \leq \cot \theta_{ULS} \leq 2.5 \quad (4.5)$$

As far as fatigue analyses are concerned, the inclination of the compression struts to be used in the **calculation of stirrup stresses**, θ_{fat} , is given by:

$$\tan(\theta_{fat}) = \sqrt{\tan(\theta_{ULS})} \leq 1 \quad (4.6)$$

According to the previous equation, which is recommended by the EN 1992-1-1, the MC90 and the UIC (2005), stress calculations for fatigue verifications must be based on an inclination θ_{fat} greater than the value adopted in ULS. This is coherent with the actual behaviour of reinforced concrete beams, as the inclination of compression struts decreases as the load increases towards the ULS condition.

In the Eurocodes, no explicit reference is made to the strut inclination to be used in the **calculation of normal stresses in the compression struts**. The UIC leaflet 774-1 (UIC 2005) suggests that both the stirrup stresses and the concrete stresses must be determined by using the angle θ_{fat} given by Equation (4.6). On the contrary, the MC90 states that the stresses in the compression struts must be calculated with θ_{ULS} . Apparently, the origin of the MC90 recommendation lies in the fact that it provides higher concrete stresses, and consequently, more conservative results. In this work, both the stirrup stresses and the stresses in the compression struts are calculated considering the inclination θ_{fat} given by Equation (4.6), as recommended by the UIC Leaflet 774-1, which seems to be consistent with the actual behaviour of concrete structures.

According to the EN 1992-1-1, the design value of the strength of the compression struts, for the purpose of ULS analyses, is given by:

$$\sigma_{cw,max} = \alpha_{cw} \cdot \nu_l \cdot f_{cd} \quad (4.7)$$

where α_{cw} is a coefficient that accounts for the influence of the axial force acting on the girder and ν_l is a reduction factor that accounts for the influence of shear cracks, the values taken by these coefficients being given by the EN 1992-1-1. In ULS, the steel stress must be limited to the design yield strength f_{yd} .

Procedures for calculation of the fatigue strength, for both the stirrups steel and the web concrete under compression, are given in Chapter 3.

4.2.2 Validation through the comparison with experimental results

In this section, the adequacy of truss models for stress calculations within the scope of fatigue analyses is evaluated through the comparison with experimental results reported in the literature. This comparison is undertaken for both reinforced concrete beams (§4.2.2.1) and prestressed concrete members (§4.2.2.2). This confrontation between numerical and experimental results is carried out according to the methodology described in the following paragraphs.

Firstly, the experimental works are briefly described, and relevant conclusions drawn by the authors of those works are summarized, so that the actual fatigue behaviour of beams subjected to cyclic shear forces can be understood.

Then, the inclination of the compression struts to be used in fatigue analyses, θ_{fat} , is determined. Its value is given by Equation (4.6) and depends on the minimum admissible inclination for ULS analyses, θ_{ULS} . The latter is calculated as recommended by the EN 1992-1-1, considering that the design value of the applied shear force is equal to the maximum experimental cyclic shear force multiplied by a partial factor of 1.5. The design value of the concrete compressive strength is taken as the experimentally determined compressive strength divided by a partial factor of 1.5.

After that, the stirrup stresses given by variable-inclination truss models [Equation (4.2)] are compared with the experimental results (observed strains multiplied by the steel modulus of elasticity). The calculations are not only carried out considering the inclination θ_{fat} given by Equation (4.6), but also considering an upper bound value, $\theta_{fat} = 45^\circ$, so that the influence of the adopted inclination can be evaluated.

Following this, calculated fatigue lives are compared with experimental observations. Fatigue lives are determined according to the linear damage accumulation rule in the case of variable amplitude cyclic loading. The fatigue damage calculations are based on experimentally determined S-N curves (for bars similar to ones embedded in the tested beams) when available. Otherwise, the analysis is conducted by considering upper and lower bound S-N curves, adjusted to the results published by diverse authors who tested straight ribbed bars embedded in concrete. These results are shown in Figure 4.4a and were compiled by the ACI Committee 215 (1997). Even though Figure 4.4a includes only about one-third of the total number of S-N curves analysed by the ACI Committee 215 (1997), it includes the ones with highest and lowest fatigue strengths. The upper and lower bound curves, as well as an average S-N curve, which were

adjusted (in the present work) to the experimental results, are shown in Figure 4.4b. These curves are expressed by the following equation:

$$\Delta\sigma = \begin{cases} \Delta\sigma^* \cdot \left(\frac{10^6}{N}\right)^{\frac{1}{k}} & \text{if } N \leq 10^6 \\ \Delta\sigma^* & \text{if } N > 10^6 \end{cases} \quad (4.8)$$

where the parameters $\Delta\sigma^*$ and k take the values indicated in Table 4.1. These curves are also plotted in Figure 4.4c, in double logarithmic scale. The same figure also includes the characteristic S-N curve provided by the EN 1992-1-1 for straight bars embedded in concrete. One can see that the latter is quite similar to the adopted lower bound curve.

Table 4.1 – Parameters of S-N curves based on test results reported by the ACI Committee 215 (1997).

	Lower bound curve	Average curve	Upper bound curve
$\Delta\sigma^*$ (MPa)	161.0	221.0	280.5
k	5.535	4.801	4.951

4.2.2.1 Reinforced concrete members

In this sub-section, the experimental results obtained by Ruhnau (1974), Frey (1982) and Higgins et al. (2004; 2006; 2007) are analysed and compared with numerical predictions.

4.2.2.1.1 Experimental results by Ruhnau

Ruhnau (1974) investigated the influence of repeated loading in terms of stirrup stresses in reinforced concrete beams. Five beams were tested under repeated loading. The same concrete quality and the same amount of reinforcement were used in all the beams. Three different web thicknesses (6 cm, 12 cm and 20 cm) were considered (see Figure 4.5). Moreover, three different loading histories were adopted.

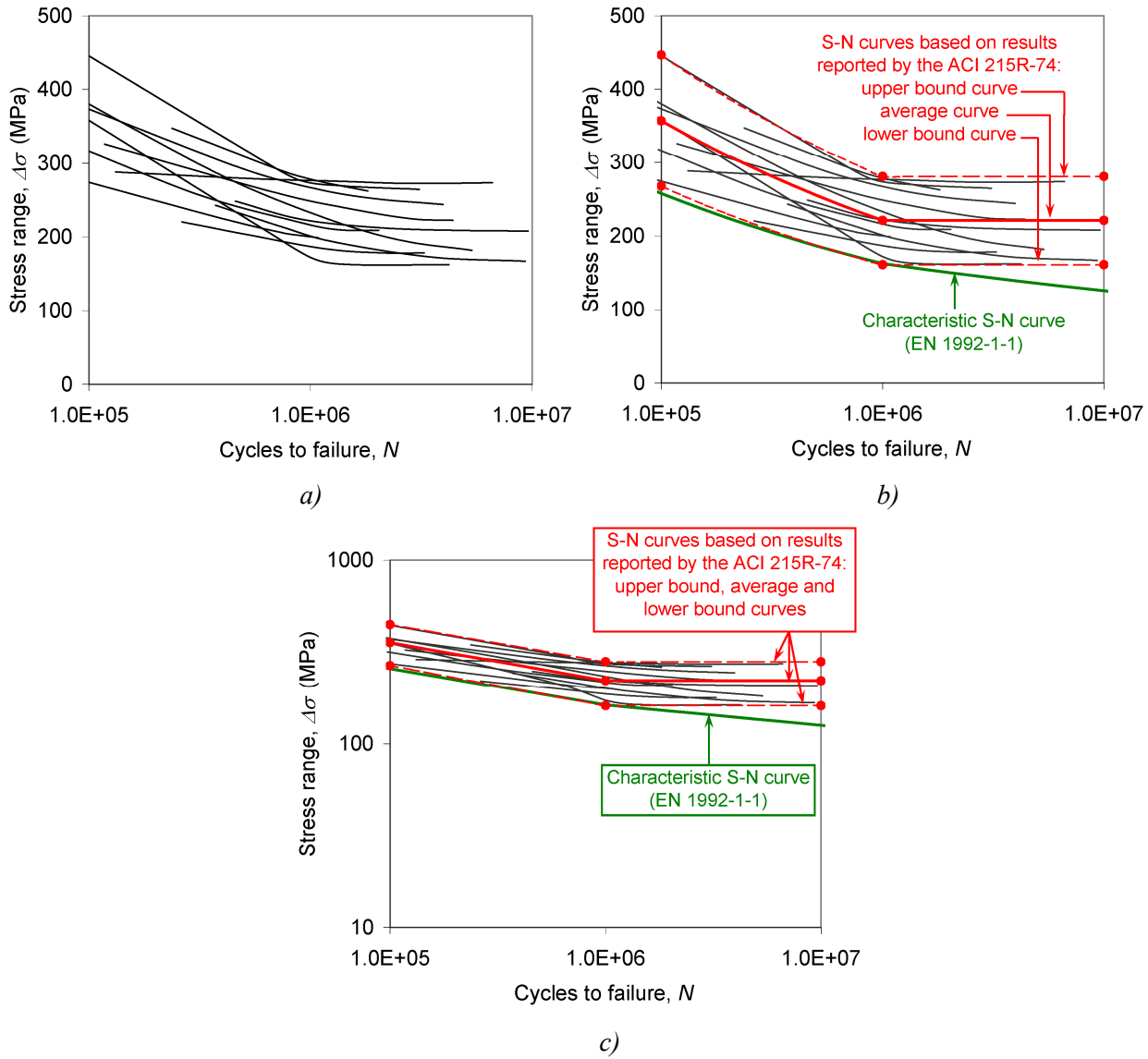


Figure 4.4 – S-N curves for steel bars embedded in concrete:

- a) Results published by various authors and compiled by the ACI Committee 215 (1997);*
- b) S-N curves adjusted to the experimental results and characteristic S-N curves indicated by the EN 1992-1-1 for straight bars;*
- c) S-N curves in double logarithmic scale.*

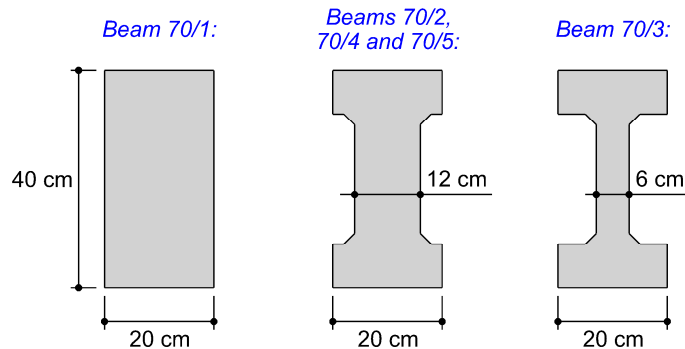


Figure 4.5 – Cross sections of the specimens tested by Ruhnau (1974).

The following data is common to all the tested beams:

- simply supported beams with a span of 4.25 m;
- four-point bending with a shear span of 1.75 m;
- total cross-sectional area of stirrups equal to $5.3 \text{ cm}^2/\text{m}$ ($f_y = 451 \text{ MPa}$);
- cross-sectional area of tension reinforcement equal to 14.9 cm^2 ($f_y = 402 \text{ MPa}$);
- total depth of the cross section equal to 400 mm;
- effective depth of the cross section equal to 370 mm;
- specified concrete compressive strength equal to 31.4 MPa;
- shear span-to-depth ratio equal to 4.38.

Stirrup strains were measured with strain gages with a length of 7.5 cm. In general, two strain gages were placed on each monitored stirrup leg. The total number of strain gages was 16 in two of the tested beams and 32 in the remaining ones.

The loading history consisted of an initial loading and unloading cycle, followed by cyclic loading. The cyclic loading applied to beams 70/1, 70/2 and 70/3 is represented in Figure 4.6. Beam 70/5 was subjected to 10^6 constant amplitude load cycles whereas no cycle was applied to beam 70/4. Finally, the beams were reloaded from zero to failure. The lower load limit within a cycle was set at 10% of the upper load limit.

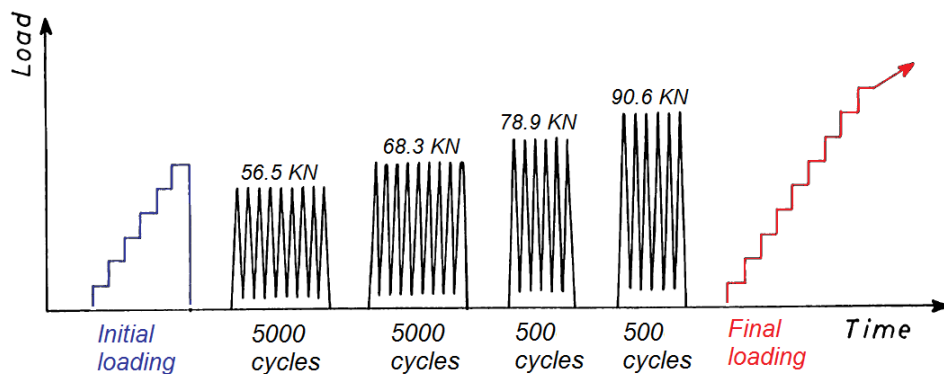


Figure 4.6 - Loading history of beams 70/1, 70/2 and 70/3. Adapted from Ruhnau (1974).

Figure 4.7 presents the stirrup stresses observed in beam 70/3 (the beam having the smallest web thickness). Only this result is displayed here, since similar trends were observed in the remaining tested beams. Nevertheless, the main conclusions taken from the complete experimental campaign are discussed below.

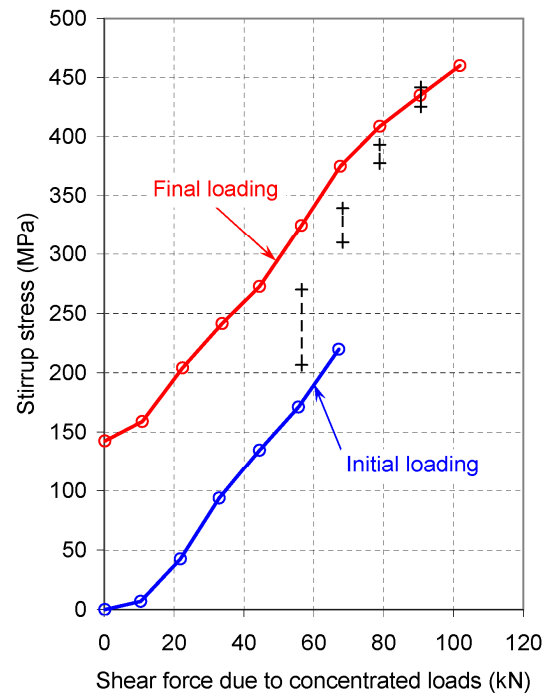


Figure 4.7 - Stirrup stress versus shear force for beam 70/3 (web thickness equal to 6 cm). Measurements taken during cyclic loading, for each different load range, are represented by crosses. For each load range, the crosses represent the highest stress (based on the highest recorded strain value) for the first and for the last load cycle. Adapted from Ruhnau (1974).

Some relevant conclusions could be drawn from the test results:

- Stirrup stresses upon first loading are significantly different from the values observed after even a reduced number of load cycles has been applied, as can be seen in Figure 4.7.
- The scatter of the measured strains is higher when the web thickness is larger. Two reasons justify this observation: firstly, the difference between the actual beam behaviour and a simple truss model increases as the web thickness increases, the actual stirrup stresses exhibiting a larger scatter when the web is thicker; secondly, the inclined-crack spacing increases as the web thickness increases. Consequently, in the case of larger web thicknesses, some gages are not crossed by cracks and low strain values are recorded throughout the test.
- Repeated loads gave rise to new cracks. However, that did not occur in all the tested beams.
- Upon unloading, significant residual stresses were observed in the stirrups (see the stirrup stress for $V = 0$, in the “final loading” curve depicted in Figure 4.7). This is due to bond effects and to the imperfect match of both faces of the cracks (even though Ruhnau does not mention this effect). Consequently, calculation models which assume secant unloading towards the origin of the stress-shear force diagram (such as the truss model) underestimate the stirrup stresses under minimum load.

Even though Ruhnau's work was not devoted to the analysis of the fatigue strength (no fatigue failure was observed), it provided important conclusions regarding the fatigue analysis of beams subjected to cyclic shear forces. Ruhnau (1974) suggested that the outcome of his work (regarding the calculation of stirrup stresses) should be used for the prediction of the fatigue strength of stirrups.

The following paragraphs present the analyses which were performed within the scope of this thesis, aiming at evaluating the adequacy of truss models to predict the experimental results. Table 4.2 presents the upper shear levels applied during cyclic loading and the corresponding measured stirrup stresses (maximum measured value among the monitored stirrups, at the end of cyclic loading).

ULS analyses based on the criterion indicated in §4.2.2, reveal that the minimum admissible inclination of the compression struts, in all the beams except beam 70/3, is:

$$\theta_{ULS} = \arccot 2.5 \approx 22^\circ$$

In the case of beam 70/3 (the one with the narrowest web thickness), the inclination θ_{ULS} takes the value 45° . According to Equation (4.6), the corresponding inclinations to be used in fatigue analyses, θ_{fat} , are 32.44° and 45° , respectively. Table 4.2 presents the calculated stirrup stresses, considering this inclination and also the upper bound value $\theta_{fat} = 45^\circ$. It can be seen that calculations considering $\theta_{fat} = 45^\circ$ provide very conservative estimates for the stirrup stresses. On the contrary, calculations with θ_{fat} given by Equation (4.6) do not provide results on the safe side for all the tested beams. In fact, the stresses measured in beams 70/4 and 70/5 are higher than the calculated values (considering $\theta_{fat} = 32.44^\circ$).

However, it is important to note that the results shown in Table 4.2 refer to the upper stress levels applied in the cycles, and not to the stress range during a cycle [the experimental stress ranges were not disclosed by Ruhnau (1974)]. Upon unloading, significant residual stirrup stresses are observed, as mentioned before. Figure 4.7 shows that the residual stress in beam 70/3, before the final loading is applied, reaches the value 142 MPa. Table 4.3 presents the observed residual stirrup stresses for the remaining tested beams. One can see that, apart from beam 70/1 (the one with largest web thickness) the residual stirrup stresses at the end of cyclic loading lie in the interval of 124 MPa to 142 MPa. In calculations according to conventional truss models, the shear force-stirrup stress relationship is expressed through a straight line which crosses the origin. Therefore, the fact that the calculations of stirrup stresses due to maximum shear levels underestimated the observed values does not imply that truss models underestimate the actual stress ranges due to cyclic loading. Note that the differences between the observed and

the calculated stresses [considering θ_{fat} given by Equation (4.6)], given in Table 4.2, are always lower than the residual stresses (for the corresponding beams) indicated in Table 4.3.

Table 4.2 - Comparison between the experimental results (Ruhnau 1974) and the results of calculations based on the truss model.

Beam	Web thickness (cm)	Shear force ⁽¹⁾ (kN)	Maximum stirrup stress ⁽²⁾ (MPa)	Calculated stirrup stress (MPa)	
				Considering θ_{fat} given by Equation (4.6)	Considering $\theta_{fat} = 45^\circ$
70/1	20	90.6	276	326.3	513.3
70/2	12	90.6	318	326.3	
70/3	6	90.6	440	513.3	
70/4 ⁽³⁾	12	90.6	419	326.3	387.0
70/5	12	68.3	303	246.0	

NOTES:

- ⁽¹⁾ Under the effect of the upper shear level at the end of cyclic loading.
- ⁽²⁾ Experimental result, given by the product of the steel modulus of elasticity by the measured strain.
- ⁽³⁾ This beam was only subjected to two load cycles. First loading was stopped when the tension reinforcement reached the yield point.

Table 4.3 – Residual stirrup stresses (unloaded beams) at the end of cyclic loading, before the final loading is applied. Experimental results reported by Ruhnau (1974).

Beam	Residual stirrup stress ⁽¹⁾ (MPa)
70/1	66
70/2	128
70/3	142
70/4	138
70/5	124

NOTES: ⁽¹⁾ Based on the highest recorded strain value.

Even though no fatigue failure was detected during the tests, it is worth discussing the fatigue behaviour of the tested beams, since a large number of load cycles was applied, and large stirrup stress values were observed.

Given that the fatigue characteristics of the employed steels are not known, the calculated stress ranges are confronted with the S-N curves adjusted to the experimental results compiled by the ACI Committee 215 (see Figure 4.4). The stress histories are calculated with truss models considering the inclination of the compression struts given by Equation (4.6). The following conclusions can be extracted:

- Fatigue calculations according to the damage accumulation method show that the accumulated damage in the stirrups of beams 70/1, 70/2 and 70/3 is lower than the unit (note that these beams were subjected to the loading history described in Figure 4.6, as mentioned before). In the beam where the highest stirrup strains were observed (beam 70/3), the calculated fatigue damage takes the value 0.088, if the fatigue strength is given by the average S-N curve depicted in Figure 4.4b. Therefore no fatigue failure was to be expected.
- Beam 70/5 was subjected to 10^6 constant amplitude load cycles, and no fatigue failure was observed. The calculated stress ranges, considering both the inclination θ_{fat} given by Equation (4.6) and $\theta_{fat} = 45^\circ$ are represented in Figure 4.8. This figure shows that calculations considering $\theta_{fat} = 45^\circ$ provide extremely conservative results, because the number of resisting load cycles for the calculated stress range of 348.3 MPa is considerably lower than the number of applied load cycles. Calculations considering θ_{fat} given by Equation (4.6) yield a stress range which coincides with the average S-N curve. That means that the estimated fatigue life, according to that S-N curve, is precisely equal to the number of applied load cycles. Given that no fatigue failure was observed, the adopted criteria for fatigue verifications appears to provide safe estimates of the actual fatigue life. Note that, if the fatigue analysis was carried out considering the characteristic S-N curve indicated by the EN 1992-1-1 (also represented in Figure 4.8), the obtained fatigue life would be considerably lower than the applied number of load cycles, as shown in Figure 4.8.

The experimental work carried out by Ruhnau does not provide any information about the fatigue strength of the web concrete. The total accumulated fatigue damage in the beam subjected to the highest concrete stresses (beam 70/3) reaches a value of 0.079 in calculations with $\theta_{fat} = 45^\circ$ considering the characteristic S-N curves indicated by the EN 1992-1-1. Therefore, the tested beams are subjected to low compressive stress levels, and consequently, no conclusion can be drawn about their fatigue strength.

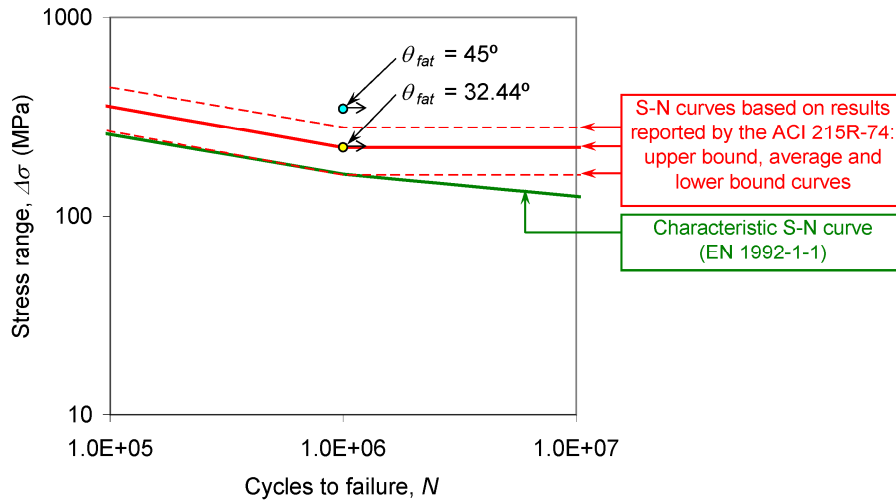


Figure 4.8 – Comparison between calculated stirrup stress ranges [for beam 70/4 tested by Ruhnau (1974)] and S-N curves for straight bars.

4.2.2.1.2 Experimental results by Frey

Frey (1982) investigated the fatigue behaviour of six RC beams under the effect of shear forces. This author presented the results for one of the beams (BI/3), which was found to be representative of the remaining tested beams.

The beam BI/3, represented in Figure 4.9, was simply supported with a span of 5 m, and was subjected to a concentrated load at mid-span. The total depth and the effective depth of the cross section were 680 mm and 620 mm, respectively. Therefore, the shear span-to-depth ratio was equal to 3.68. Different web thicknesses exist on the left side and on the right side of the applied load. The adopted web thicknesses were 100 mm and 150 mm. The cross-sectional area of stirrups was $7.85 \text{ cm}^2/\text{m}$ ($\phi 10 \text{ mm} // 0.20 \text{ m}$) and the characteristic value of the yield strength, f_{yk} , was equal to 460 MPa. The characteristic value of the concrete compressive strength, f_{ck} , was higher than 30 MPa. The beams were subjected to constant amplitude cyclic loading with a load range of 30 to 370 kN, and the stirrup strains were measured by means of glued extensometers with gauge lengths of 100 mm and 300 mm.

Frey (1982) suggested that the truss model is recommended for use in the calculation of the stirrup stresses for fatigue verifications. That suggestion resulted from the following observations:

- the increase of strains with the number of cycles was very high at the start and became smaller once the crack pattern was fully formed; moreover, stirrups with small initial strains after the first few cycles experienced larger strain increases than stirrups which were already subjected to high strains; the strain increase was attributed to the development of the final crack pattern and to the redistribution of internal forces;

- as the number of cycles increased, the growth of the stirrup strain under maximum load ($F = 370$ kN) was higher than the increase of the strain range (associated to the load range $\Delta F = 340$ kN);
- the measured strain variation (under the effect of the force range $\Delta F = 340$ kN) did not exceed the result of calculations based on the truss model.

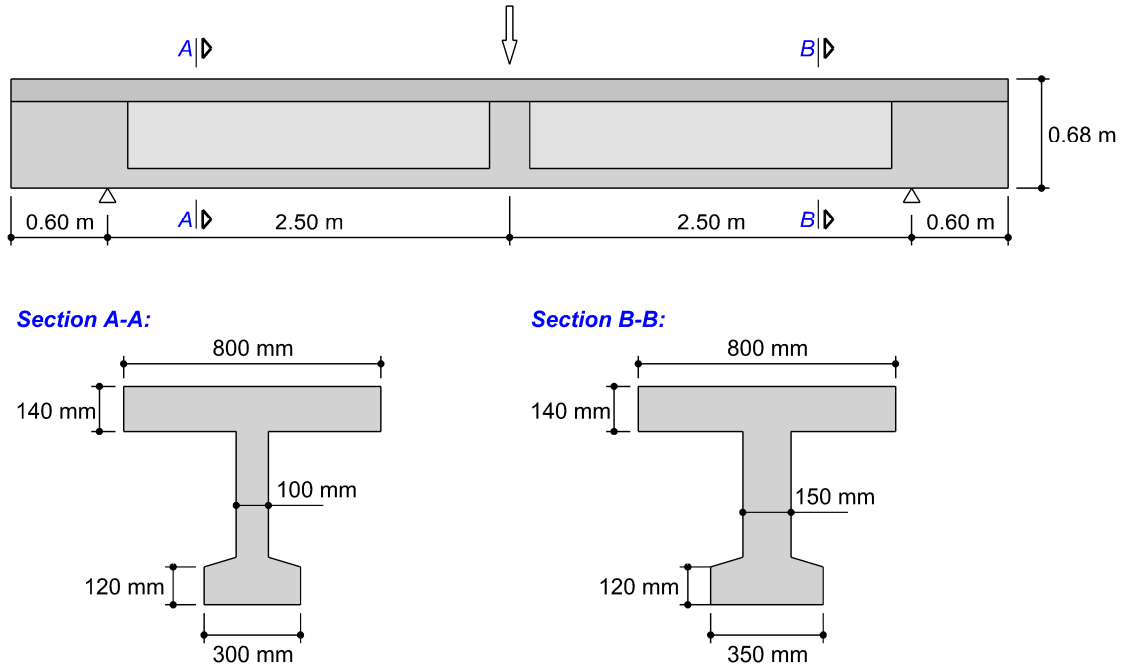


Figure 4.9 – Geometry of the beam BI/3 tested by Frey (1982).

Moreover, this author observed that, from the total of 76 stirrup fatigue failures that were observed in the six tested beams, only two stirrups failed at the lower bend. Those two failures occurred only after two or more stirrups, crossed by the same crack, had previously failed.

The author also concluded that a large number of cycles can be applied to the girder after the first fatigue failure takes place. In the case of the beam BI/3, the first fatigue failure occurred after 2.45 million cycles. However, the beam was subjected to a total of 5.75 million constant amplitude cycles.

The following paragraphs present the analyses performed within the scope of this thesis for evaluation of the adequacy of truss models for fatigue life calculations.

The minimum admissible inclination of the compression struts for ULS analyses, according to the criterion indicated in §4.2.2 is equal to 27.3° . Therefore, the inclination to be used in fatigue analyses, which is given by Equation (4.6), is equal to 35.7° . An alternative upper bound value for the inclination of the compression struts, $\theta_{fat} = 45^\circ$, is also considered in the analysis.

Table 4.4 presents the comparison between the experimental results for beam BI/3 and calculations based on truss models. This table presents stirrup stresses due to both the maximum load level and the cyclic load range.

Table 4.4 - Comparison between experimental results (Frey 1982) and calculations based on truss models.

	Experimental results after $2 \cdot 10^6$ load cycles ⁽¹⁾	Calculations with $\theta_{fat} = 35.7^\circ$ ($\theta_{ULS} = 27.3^\circ$)	Calculations with $\theta_{fat} = 45^\circ$ ($\theta_{ULS} = 45^\circ$)
Maximum stirrup stress under the maximum force $F = 370$ kN	440 MPa	282 MPa	393 MPa
Maximum variation of stirrup stress under the load range $\Delta F = 340$ kN	240 MPa	259 MPa	361 MPa

NOTES: ⁽¹⁾ Results for extensometers with a gauge length of 100 mm; the observed strains are multiplied by a plausible modulus of elasticity of 200 GPa.

Table 4.4 shows that the maximum observed stirrup stress under the maximum load level is higher than the calculated values. But the steel fatigue damage depends on the stress range, and not on the stress under maximum load levels. And, in this case, the maximum observed stress range is lower than the calculation results, either for calculations with $\theta_{ULS} = 27.3^\circ$ or analyses with $\theta_{fat} = 45^\circ$. Therefore, one can conclude that truss models with an inclination of the compression struts given by Equation (4.6) provide conservative estimates for the observed stirrup stress ranges. This conclusion is coherent with the ones drawn in the previous section (Ruhnau's results).

Nevertheless, it is important to note that the employed extensometers have a gauge length of 100 mm. Consequently, higher stirrup stresses may occur at the intersection with a crack. The fatigue life depends on the real maximum stress and not on the measured values. For that reason, the validity of truss models can be assessed by comparing observed and calculated fatigue lives. Given that the real S-N curves for the employed steels are not known, this analysis is conducted by considering the S-N curves adjusted to the results compiled by the ACI Committee 215 (1997). In Figure 4.10, the calculated stirrup stress ranges are represent by points with an abscissa of 2.45 million cycles, i.e., the number of cycles before the first observed fatigue failure. Here one can see that, as expected, calculations with $\theta_{fat} = 45^\circ$ provide extremely conservative results. Calculations considering the inclination θ_{fat} given by Equation (4.6) appear to provide safe estimates for the stirrups' fatigue life because the corresponding point in Figure 4.10 is close to the upper bound S-N curve.

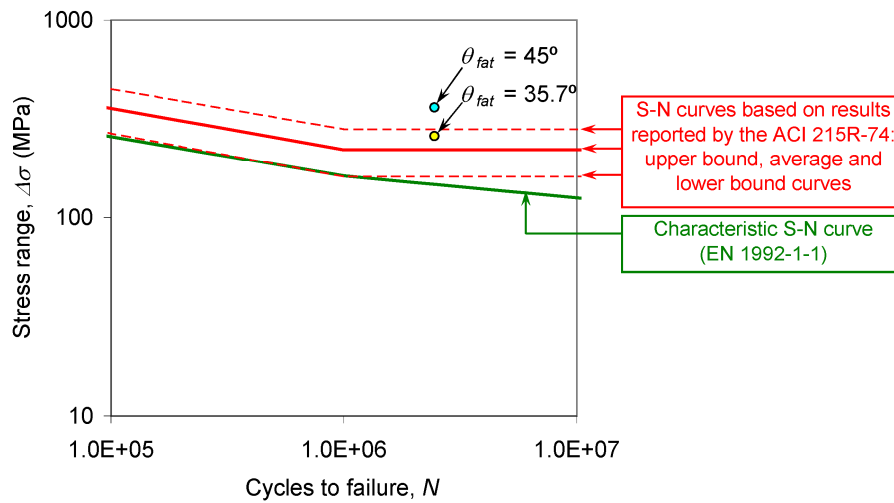


Figure 4.10 - Comparison between calculated stirrup stress ranges [for beam BI/3 tested by Frey (1982)] and S-N curves for straight bars.

As far as the fatigue strength of the compression struts is concerned, no information can be drawn from the experimental results reported by Frey (1982). No compression failure was observed, and the compressive stress level was quite low. Even for a web thickness of 100 mm, the calculated compressive stress under the maximum load, $F = 370$ kN, takes the value 6.88 MPa [considering θ_{fat} given by Equation (4.6)], which is only $\sim 23\%$ of the specified concrete compressive strength (30 MPa).

4.2.2.1.3 Experimental results by Higgins et al.

Higgins et al. (2004; 2006; 2007) carried out field tests and laboratorial experiments to evaluate the high-cycle fatigue behaviour of diagonally cracked RC bridge girders under the effect of shear forces. The relevance of this work lies in the fact that the fatigue behaviour of full-sized RC bridge girders had not previously been investigated, as noted by the authors.

Field tests

Traffic induced strain ranges (at diagonal-crack locations) and crack openings were monitored in 4 bridges for a period of at least 8 days (Higgins et al. 2006). Strain gages with a gauge length of 1.6 mm were installed in the stirrups, at the crack locations, by removing a small portion of concrete cover in order to expose the embedded stirrup. The diagonal-crack opening was also measured near the strain gage location.

Measurements showed that stirrup fatigue failures in the monitored bridges are unlikely, given the low observed stress ranges (measured strain range multiplied by the steel modulus of elasticity). The fatigue check was carried out by converting the observed variable-amplitude

stress history to an equivalent constant-amplitude stress range by using the Miner's rule. The largest calculated value was 18.2 MPa, which led to the conclusion that fatigue fracture is improbable. Moreover, the largest single stress range measured at any location (approximately 83 MPa) was quite low.

The strain measurements were also used to determine an equivalent stress range to be used in laboratorial tests. The authors adopted a number of cycles for the laboratorial tests equal to $2 \cdot 10^6$ and calculated the constant-amplitude stress range that produces a fatigue damage (in the laboratorial tests) equal to the actual fatigue damage in the monitored bridges over a period of 50 years. That value was determined through the Miner's rule of linear damage accumulation, and the calculated stress range was 95 MPa.

Laboratorial tests

Nine beams were subjected to fatigue tests with 2 million load cycles, the load amplitude being chosen so as to produce the aforementioned stirrup stress range of 95 MPa in at least one stirrup, at the intersection with diagonal cracks (Higgins et al. 2007; Higgins et al. 2004).

The adopted cross sections were 1219 mm high, having a T configuration as represented in Figure 4.11. The stirrups were made with ASTM A 615 (ASTM 2003) Grade 300 reinforcing steel, with a nominal diameter of 13 mm and a yield stress of 352 MPa. The main differences between the nine tested specimens were:

- the stirrup spacing, which took the values 10'' (254 mm), 12'' (305 mm) and 18'' (457 mm);
- some of the beams were tested with a T configuration (as represented in Figure 4.11) whereas an inverted-T was adopted in the others (which are identified by the label IT in Table 4.5);
- flexural bar cutoffs existed in some of the tested beams (identified with C in Table 4.5).

Fatigue tests were preceded by the application of loads to impose diagonal cracking (two point loads of ~254 kN spaced 610 mm apart, symmetrical with respect to the mid-span). Then, these loads were removed and fatigue loading was applied. Fatigue loading consisted of two point loads as represented in Figure 4.11. Therefore, the shear span-to-effective depth ratio equals $(7.3 - 0.61)/(2 \cdot 1.143) = 2.93$. The magnitude of the fatigue load varied from 18 to 374 kN, i.e., the shear force range was equal to $(374 - 18)/2 = 178$ kN. During the fatigue test, dead load was also simulated by means of a load applied on the north side (see Figure 4.11). This load could not be kept constant during the test (it varied between 356 kN and 445 kN).

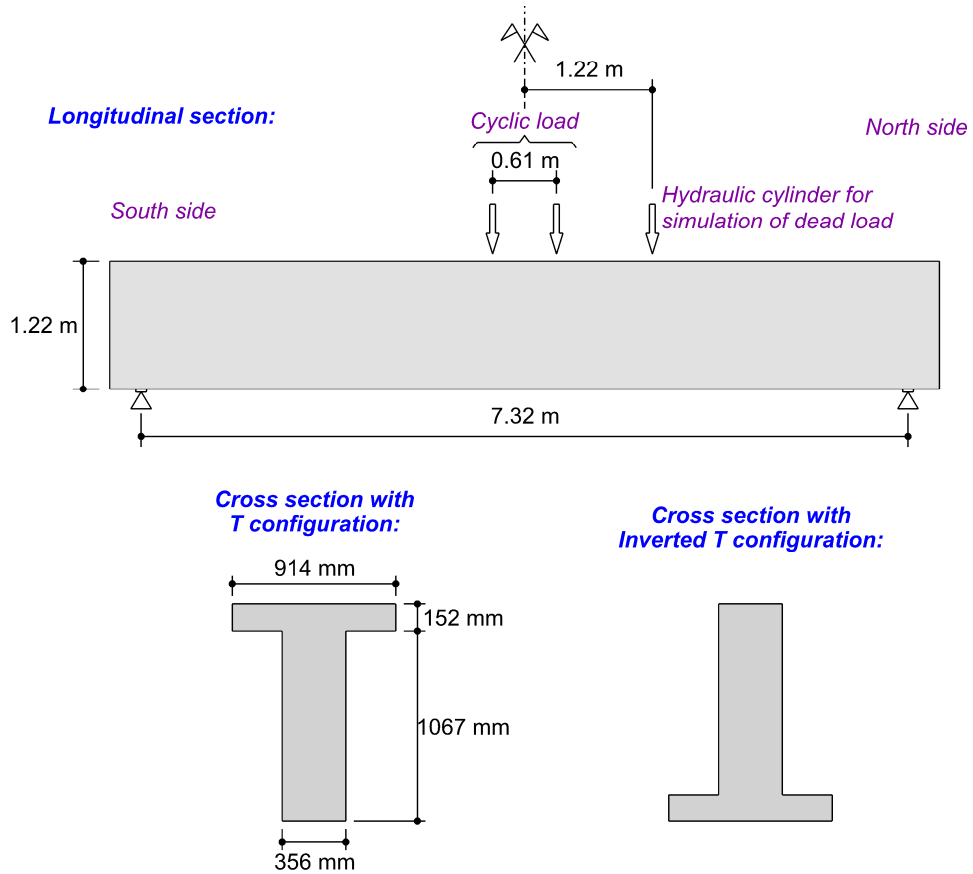


Figure 4.11 – Specimens tested by Higgins et al. (2007; 2004).

During the fatigue tests, stirrup strains were measured by strain gages placed before the concrete was cast, and by strain gages added after cracking, at the intersection between stirrups and diagonal cracks. Table 4.5 presents the maximum observed stirrup stress ranges (measured stirrup strain ranges multiplied by the steel modulus of elasticity) after 2 million cycles. One can see that the observed stress ranges are well above the minimum intended value of 95 MPa in several tested beams.

In the following paragraphs, the analysis conducted within the scope of this thesis is presented. Only the measurements taken on the South side are considered, because the behaviour on the North side is complicated by the dead load cylinder, located close to the North side support. Moreover, fatigue fractures were only observed on the South side.

ULS analyses based on the criterion presented in §4.2.2 reveal that the compression struts are subjected to low stress levels, and consequently, the minimum admissible inclination of the compression struts is:

$$\theta_{ULS} = \arccot 2.5 \approx 22^\circ$$

which corresponds to $\theta_{fat} = 32.44^\circ$. The calculated stress ranges, considering that inclination, are shown in Table 4.5. The results corresponding to an upper bound limit of the inclination, $\theta_{fat} = 45^\circ$, are also shown. One can see that calculations considering $\theta_{fat} = 45^\circ$ always provide results on the safe side, i.e., higher than the maximum observed stress ranges. On the other hand, calculations in which the inclination θ_{fat} is taken as 32.44° do not always yield conservative results. By analysing the situations in which the observed stresses are higher than the model results, a clear pattern cannot be found. This fact puts into evidence the randomness associated to the stress ranges due to cyclic shear forces.

Table 4.5 - Comparison between experimental results (Higgins et al. 2007) and calculations based on truss models.

Specimen n°	Stirrup spacing (inches)	A_{sw}/s (cm ² /m)	Maximum observed stress range after 2 million cycles ⁽¹⁾ (MPa)	Calculated stress range (MPa)	
				$\theta_{fat} = 32.44^\circ$	$\theta_{fat} = 45^\circ$
5 (T)	10	10.47	130	105	165
6 (IT)			150		
7 (IT) (C)			100		
1 (T)	12	8.72	92	126	198
2 (IT)			90		
4 (IT)			148		
8 (IT) (C)			152		
9 (T) (C)			196		
3 (IT)	18	5.82	200 ⁽²⁾	189	297

Notes: ⁽¹⁾ Measurements on the South side only, because the behaviour on the North side is complicated by the effects of the cylinder used to simulate dead load

⁽²⁾ Value observed in the damaged stirrup before fatigue fracture was observed at ~ 1.75 million cycles

As mentioned before, fatigue failures were only observed in beam 3 (IT), which is the one with the widest stirrup spacing. Fractures occurred in two stirrup legs after ~ 1.75 million cycles were applied. The real S-N curves for the employed steels are not known. For that reason, the calculated stress ranges, for that beam, are compared with the number of resisting load cycles given by the S-N curves adjusted to the results reported by the ACI Committee 215 (1997). In Figure 4.12, those S-N curves are confronted with the calculated stresses considering both the inclination of the compression struts given by Equation (4.6) and the upper bound limit, $\theta_{fat} = 45^\circ$. The abscissas of the points which represent those results are equal to $1.75 \cdot 10^6$ cycles, i.e., the real number of resisting load cycles. One can see that the stress calculated with $\theta_{fat} = 45^\circ$

is higher than the stress given by the upper bound S-N curve for $1.75 \cdot 10^6$ load cycles. Therefore, calculations considering this inclination clearly provide conservative results. This conclusion resembles the one that was obtained during the analysis of the results published by Ruhnau (1974) and Frey (Frey 1982). However, it is important to emphasize that this inclination is only being considered in this study with the purpose of evaluating the differences between calculations with that upper bound value and calculations with the inclinations given by Equation (4.6). The main purpose of this study consists in the validation of calculations considering the inclination given by Equation (4.6). By analysing the results of such calculations, one can see that the calculated stress range lies in the interval between the values given by the lower bound S-N curve and the average S-N curve. Therefore, one can not guarantee that, if the real S-N curve of the employed steels was used in the calculations, a safe estimate of the fatigue life would be obtained. Nevertheless, it can be seen that a safe estimate of the fatigue life is obtained in fatigue analyses considering the inclination of the compression struts given by Equation (4.6) and the characteristic S-N curve indicated by the EN 1992-1-1 (see Figure 4.12). In that case, the calculated fatigue life would be equal to $0.47 \cdot 10^6$ cycles, i.e., lower than the real fatigue life ($1.75 \cdot 10^6$ cycles). Given the recognized variability of the phenomenon under analysis, this appears to be a satisfactory result.

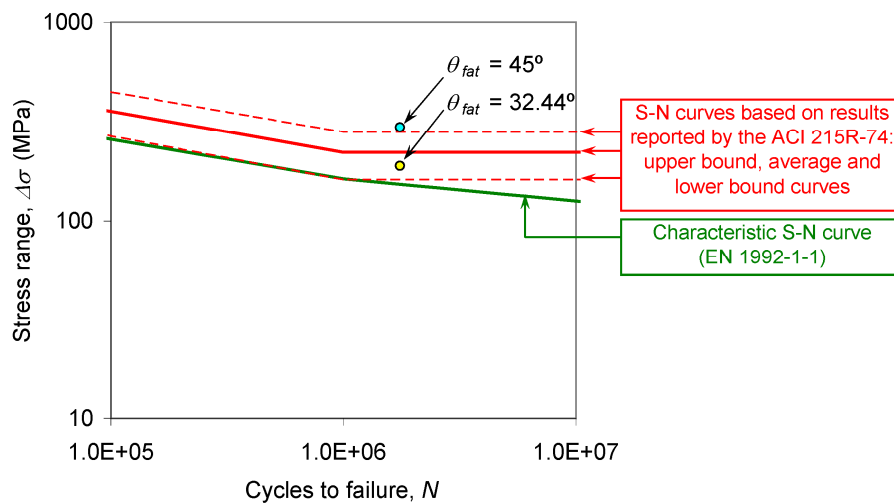


Figure 4.12 - Comparison between calculated stirrup stress ranges [for beam 3 (IT) tested by Higgins et al. (2007)] and S-N curves for straight bars.

It is important to remark that the estimated fatigue lives were calculated under the assumption of constant stress amplitude over the whole stress history. These calculations would not hold true if the actual stress history varied over time. That was not the case, i.e., the experimental results show that the stress range in the critical stirrups was kept approximately constant over time. This fact confirms the feasibility of assuming constant stress ranges over time. Even though stirrup strain ranges did not change significantly during fatigue testing, Higgins et al. (2007) showed that diagonal-crack motions tended to increase under repeated loading. This occurrence was attributed to bond degradation at the stirrup/concrete interface.

The tests carried out by Higgins et al. could not be used to evaluate the fatigue strength of the web concrete, because the compression struts were subjected to low stress levels. The maximum calculated compressive stress, σ_{cw} , given by Equation (4.1), reaches in this case the value 2.26 MPa (on the South side, considering $\theta_{fat} = 32.44^\circ$).

4.2.2.2 Prestressed concrete members

In §4.2.2.1, calculations based on truss models were compared with experimental results for reinforced concrete beams. In the present section, the comparison is extended to prestressed concrete elements.

4.2.2.2.1 Experimental results by Kreger et al.

Kreger et al. (1989) conducted fatigue tests on three full-scale prestressed concrete girders, which were subjected to a large number of constant amplitude load cycles. The 121 cm deep beams, whose cross section is shown in Figure 4.13, had a simply-supported span of 14.6 m. Cyclic loading was applied at quarter points, leading to a shear span-to-depth ratio of 3.0. The stirrup steel had a minimum yield stress of 414 MPa, the total cross-sectional area of stirrups being 4.65 cm²/m.

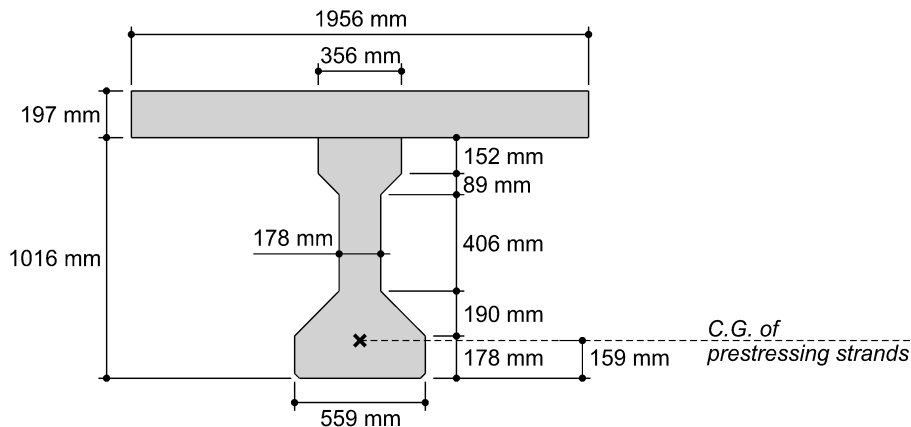


Figure 4.13 – Cross section of the girders tested by Kreger et al. (1989).

Kreger et al. (1989) concluded that:

- Stirrup strains were negligible until diagonal cracking occurred. After the formation of diagonal cracks, the behaviour of the beam appeared to be consistent with the truss type action, with stirrups acting as vertical tension members and concrete as inclined compression struts.
- Diagonal cracks did not completely close when the load was removed.
- The stirrup stresses increased with the number of load cycles.

- The beams had a remarkable shear fatigue resistance after the first stirrup fracture, i.e., a large number of load cycles could be applied after that fracture was detected. The authors concluded that after failure of most of the stirrups, a tied arch mechanism was formed, which made possible the transmission of the applied loads to the supports.

For example, in the case of beam n° 2, the first stirrup fracture took place after 485,000 load cycles, but the beam was subjected to a total of 891,000 cycles. By 750,000 cycles, 9 stirrups had fractured in one of the shear spans, the total number of stirrups in each shear span being 11.

- All the fatigue fractures occurred well away from bends.
- Fretting of reinforcing bars was revealed when fractured stirrups were exposed; fretting was detected by the observation of considerable oxidation. Moreover, measurements of crack openings revealed horizontal displacements at the cracks. The authors concluded that bending of stirrups and fretting effects might have been responsible for reducing the fatigue life of the stirrups.

Kreger et al. (1989) also tested the fatigue behaviour of reinforcing bars in air. Tests were performed for 30 specimens taken from the same heat of steel used for beam stirrups. Figure 4.14 depicts the S-N curve determined by a linear regression to the results of the in-air fatigue tests. The equations representing the 5% and 95% percentiles are also plotted in that figure. By comparing the S-N curves derived by Kreger et al. (1989) and the upper and lower bound S-N curves adjusted to the results compiled by the ACI Committee 215 (1997), one can see that Kreger's curves are enveloped by the latter (see Figure 4.14).

The maximum stress ranges observed during the fatigue tests in different stirrups where fatigue fractures occurred (observed strain ranges multiplied by the steel modulus of elasticity) are also shown in Figure 4.14. The abscissa of each point corresponds to the number of cycles resisted by each stirrup (cycles before inclined cracking occurred were ignored, since stresses were negligible during that stage).

In Figure 4.14, one can see that the experimental stirrup stress ranges are below the regression line. This difference between the fatigue strength of bars tested in the air and the observed stirrup stress ranges can be justified by two main reasons, as pointed out by Kreger et al. (1989): (i) bending of stirrups and fretting effects, which do not occur in in-air tests; (ii) the fact that the strain gages were not installed at the points of maximum stirrup stresses.

The experimental results are now compared with the results of calculations according to the truss model, which were performed within the scope of this thesis. ULS analyses based on the criterion indicated in §4.2.2 reveal that the web concrete is subjected to low compression

stresses, and consequently, the inclination θ_{ULS} takes the value 22° . Accordingly, the inclination to be used in fatigue analysis, θ_{fat} , is equal to 32.44° . The experimental shear force ranges and the calculated stresses are displayed in Table 4.6. One of the conclusions that can be immediately drawn from this table is that the calculated stress ranges are quite high (given that the employed steel had a minimum yield stress of 414 MPa).

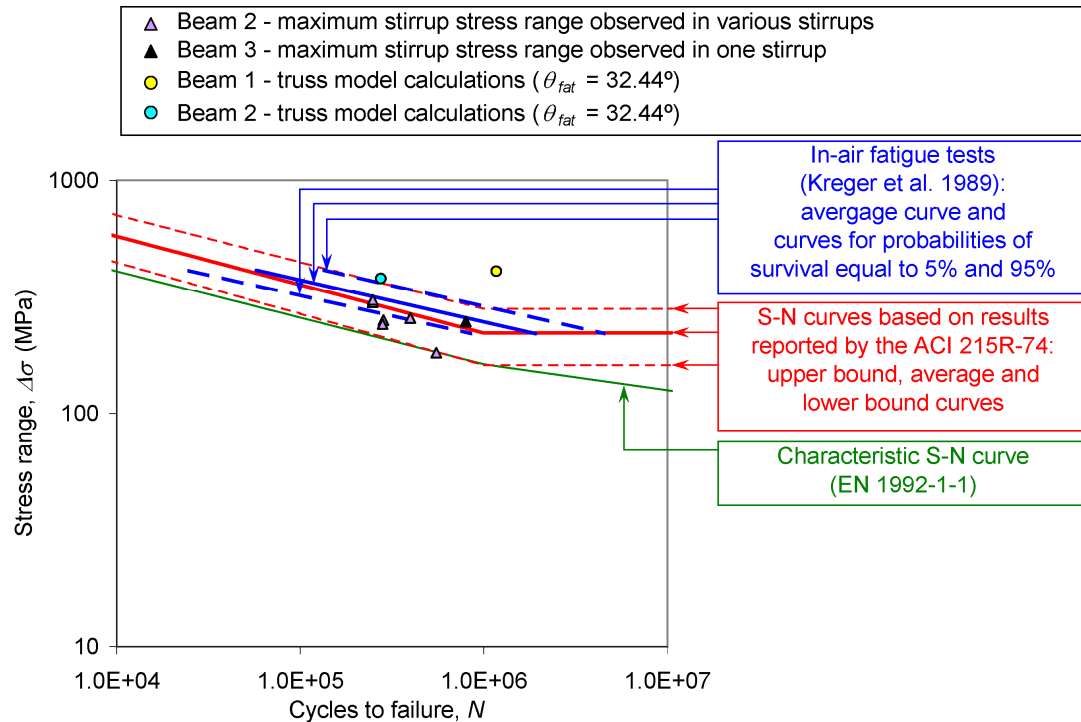


Figure 4.14 - Comparison between stirrup failure data, in-air fatigue tests carried out by Kreger et al. (1989) and calculations based on truss models.

Table 4.6 - Beams tested by Kreger et al. (1989): stirrup stress ranges according to the truss model.

Beam	Shear force range ⁽¹⁾ (kN)	Calculated stirrup stress range (truss model with $\theta_{fat} = 32.44^\circ$) (MPa)
1	311.5	409
2	289.2	380
3 ⁽²⁾	255.9 272.8	336 357

NOTES: ⁽¹⁾ The minimum shear force within a cycle was equal to 44.5 kN.

⁽²⁾ Beam 3 was subjected to 10^6 cycles with a shear force range of 255.9 kN. Then, the shear force range was increased to 272.8 kN. Extensive flexural fatigue damage was detected after 3,133,000 cycles (total number). At this stage, no inclined cracks were visible.

The calculated stress ranges are also plotted in Figure 4.14, these stresses being associated to the following number of cycles:

- In the case of beam 1, inclined cracking was detected after 300,000 cycles. Then, the beam was subjected to 1,170,000 cycles before extensive flexural fatigue damage was observed. No stirrup fracture was detected at this stage. Thus, the calculated stress range (409 MPa) is associated to the number of cycles applied after diagonal cracking was observed.
- In the case of beam 2, inclined cracking was detected at 210,000 cycles (in the South shear span). The first stirrup fracture occurred at 485,000 cycles (in the same shear span). Therefore, the calculated stress range (380 MPa) was associated to the number of cycles that elapsed between diagonal cracking and first fatigue fracture.

The comparison between the calculated stress ranges and the experimental results shows that calculations based on the truss model provide conservative estimates for the fatigue life of the tested beams.

In the case of beam 2, the first stirrup fracture took place 275,000 cycles after inclined cracking appeared. On the other hand, fatigue calculations based on the average regression curve derived from the in-air fatigue tests yield an estimated fatigue life of 91,730 cycles. Even if the calculations were based on the 5%-survival curve, the estimated number of cycles to failure would be lower than the actual beam resistance.

In the case of beam 1, calculations of the fatigue life based on stress values determined through the truss model and in-air S-N curves provide even more conservative results. The calculated stress range (409 MPa) corresponds to an average fatigue life of 60,570 cycles. And, in the reality, no stirrup failure was observed after applying 1,470,000 load cycles.

Nevertheless, it is important to remark that the shear strength of beam 1 might have been amplified by a lifting hook, which also worked as a vertical tension member. The lifting hook was not taken into account in the calculations whose results are presented in Table 4.6. The position of the lifting hook was modified in beams 2 and 3, so that it did not contribute to their shear resistance. Consequently, the most relevant results were obtained with beam 2. Beam 3 does not provide significant information about the stirrups' fatigue strength, too. This is because no inclined cracking was detected before extensive flexural fatigue damage was observed at 3,133,000 cycles. Therefore, the truss mechanism was not activated before that event.

Therefore, Kreger's results indicate that fatigue analyses in which the stress ranges are determined with truss models considering the inclination θ_{fat} given by Equation (4.6) provide results on the safe side. It should be stressed that it was experimentally observed that a tied arch

mechanism contributed to the shear strength of the beams. This mechanism is not taken into account in calculations based on truss models, this being one of the justifications for the conservative results provided by such calculations. Prestressing has also a beneficial effect on the shear strength after inclined cracking. It guarantees lower deformability, and consequently, lower movements at the cracks and increased fatigue life of the stirrups.

As far as the fatigue strength of the compression struts is concerned, no information can be obtained from this experimental research, for the reason that the concrete struts are not subjected to important compressive stresses. Calculations based on the truss model show that the maximum compressive stresses in the struts are approximately equal to 10% of the concrete compressive strength.

4.2.2.2.2 Experimental results by Price and Edwards

Price and Edwards (1971) conducted tests on 17 post-tensioned concrete girders each with a 3.2 m simply supported span. Loading was applied by means of two point loads. Four girders were used to determine the static shear strength. The remaining girders were tested under fatigue loading.

The geometry of the adopted cross section is represented in Figure 4.15. The stirrups were made with 4.75 mm diameter bars, with a single leg, spaced 203 mm apart. The yield strength of the stirrups was 448 MPa and the shear span-to-depth ratio was equal to 3.36.

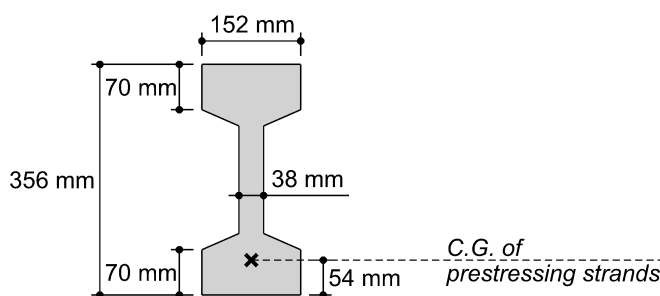


Figure 4.15 – Cross section of the girders tested by Price and Edwards (1971).

Stirrup strains were measured by means of electrical resistance strain gages, which were placed at the position that Price and Edwards considered to be the potential line of cracking: the line joining the load and the support points.

The authors concluded that, after diagonal cracking occurred in the beams, the total force in the stirrups (over the shear span) was approximately equal to half the shear force. That conclusion puts into evidence the relevance of alternative resistance mechanisms, other than the truss action,

namely the tied arch action, with a compression strut connecting the point of load application and the support.

It is important to note that the force carried by the stirrups, indicated in the previous paragraph, was quantified based on readings from the strain gages glued to the stirrups, and therefore, the actual stirrup forces might be slightly higher (the strain gages do not necessarily capture the highest stirrup strain). However, it could not have been much higher because, if one takes into account that the mean experimental static ultimate shear force, $\bar{V}_u$, was equal to 71.3 kN, and consider that the force carried by the stirrups is equal to half that value, then the normal stress in those stirrups is equal to 503 MPa. Therefore, the force carried by the stirrups can not have been much higher than half the applied shear force, because the yield stress of the stirrups is equal to 448 MPa.

The lack of realism of truss models, for calculation of stirrup stresses in the tested beams under the effect of the maximum applied shear force, can be confirmed by looking at the values shown in Table 4.7, which presents the results for two different inclinations of the compression struts. As it will be seen ahead, fatigue calculations based on the truss model also lead to very conservative estimations of the fatigue life.

*Table 4.7 - Beams tested by Price and Edwards (1971).
Calculated stirrup stresses, according to the truss model,
under the effect of the maximum applied shear force.*

θ	Calculated stirrup stress under the effect of the mean experimental static ultimate shear force, $\bar{V}_u = 71.3 \text{ kN}$
22°	1,213 MPa
45°	3,003 MPa

NOTE: z was taken as $0.9 \cdot d = 0.272 \text{ m}$

Table 4.8 summarizes the results of the fatigue tests conducted by Price and Edwards (1971). Several specimens having similar geometric and material characteristics were tested. Three different load levels were considered: 63%, 70% and 77% of the mean static ultimate shear strength. The average fatigue life, its coefficient of variation and the sample size are shown in the table.

The calculated stresses according to the truss model, under the effect of the maximum load level $V_{\max}$, shown in Table 4.8, indicate that the truss model is too conservative in this case. In fact, the calculated stresses are higher than twice the yield strength of the stirrups.

*Table 4.8 - Results of the fatigue tests by Price and Edwards (1971)
and comparison with calculations according to the truss model.*

Maximum load level $\frac{V_{\max}}{\bar{V}_u}$	Minimum load level $\frac{V_{\min}}{\bar{V}_u}$	Mean fatigue life up to first stirrup fracture, $\bar{N}$	Coefficient of variation of N	Sample size	Calculated stress according to the truss model $\theta_{fat} = 32.44^\circ; V = V_{\max}$
0.63	0.25	2,486,700	45.5 %	5	1,202 MPa
0.70	0.25	392,400	36.3 %	7	1,336 MPa
0.77	0.25	90,200	51.6 %	6	1,470 MPa

As regards the fatigue strength of the concrete struts, truss model calculations lead to a maximum compressive stress of 11.73 MPa under the effect of the maximum load level $V_{\max}/\bar{V}_u=77\%$, for an inclination $\theta_{fat} = 32.44^\circ$. This compressive stress corresponds to a low stress level ($\sim 21\%$ of the mean concrete cube compressive strength) and, therefore, no conclusion can be obtained about the fatigue strength of the web concrete. Nevertheless, it is important to note that the final failure of the beams occurred due to crushing of the web concrete, at the points of stress concentrations. However, that failure mode only took place after the fatigue fracture of the stirrups. Hence, the initial criterion of failure was the fatigue fracture of the stirrups.

The fatigue tests led to the following similar conclusions to the ones that were drawn by Kreger et al. (1989):

- stirrup strains were negligible until diagonal cracking occurred;
- diagonal cracks did not completely close when the load was removed;
- the beams had a remarkable shear fatigue resistance after the first stirrup fracture;
- fretting of reinforcing bars was revealed when fractured stirrups were exposed.

Besides those conclusions, Price and Edwards (1971) also observed that:

- the stirrup strains did not significantly increase under repeated loading, whereas the diagonal crack widths increased considerably [similar conclusions were obtained by Higgins et al. (2007)];
- even though the majority of stirrup fractures occurred at the intersection with diagonal cracks, the fatigue fracture took place at the bend of the stirrup in the top flange, in two beams;
- the prior history of repeated loading had no significant effect on the static ultimate shear strength of the beams in subsequent tests to failure.

4.2.3 Concluding remarks on the adequacy of truss models for fatigue analyses

The most relevant conclusions regarding the adequacy of truss models for stress calculations within the scope of fatigue analyses are summarized in the following paragraphs.

The exposed results suggest that truss models can be used to estimate the stirrup stresses in the tested beams, after cyclic loading. After a few load cycles are applied, the stirrup stresses undergo significant stress changes, as concluded by Frey (1982) and Ruhnau (1974). After that, the final crack pattern is reached and only minor stress changes occur. Higgins et al. (2007) found that the stirrup stress ranges remain approximately constant during cyclic loading, too. Truss models proved to be particularly conservative in the case of prestressed elements, because alternative resistance mechanisms play a significant role in that case.

The experimental results show that, upon unloading, significant residual stirrup stresses occur, owing to the non-linear characteristics of both the bond stress-slip relationship and the stress-crack opening relationship. This evidence is consistent with the fact that diagonal cracks do not completely close when the load is removed, as observed by Kreger et al. (1989) and Price and Edwards (1971). This effect is not considered in conventional truss models, which assume secant unloading towards the origin of the shear force-stirrup stress diagram. Therefore, calculations (based on truss models) of stirrup stress ranges are more conservative than calculations of stresses under maximum load levels.

The comparison between experimental and numerical results indicates that truss models, with an inclination of the compression struts, θ_{fat} , given by Equation (4.6), can be employed in the calculation of stirrup stress ranges to be used in fatigue analyses. In this work, it was found that:

- in the case of PC elements, such truss models provide conservative predictions for both the observed stress ranges, and the experimental fatigue lives;
- in the case of non-prestressed members, truss model calculations considering the inclination θ_{fat} given by Equation (4.6) did not always lead to conservative predictions of the experimental stress ranges; nevertheless, such calculations provide reasonable estimates for the real fatigue lives of the tested beams; in fact, the experimental fatigue life was lower than the value calculated considering average S-N curves for one of the beams tested by Higgins et al. (2007) only; the results obtained for this beam do not impair the viability of the adopted analysis methodology because the phenomenon under analysis is associated to an important variability, and calculations based on characteristic S-N curves provided results on the safe side for all the analysed test results;
- calculations considering the upper bound value for the inclination of the compression struts ($\theta_{fat} = 45^\circ$) always provided too conservative results, either in terms of stirrup stress

ranges or in terms of fatigue lives; this conclusion is coherent with the fact that the web concrete, in the beams under analysis, was generally subjected to low compressive stress levels, this situation being in correspondence with a lower inclination of the compression struts.

The experimental results indicate that, in most cases, fatigue failure is governed by the straight portion of the stirrups (Frey 1982; Higgins et al. 2007; Higgins et al. 2004; Kreger et al. 1989). However, Price and Edwards (1971) observed fatigue fractures at the bends. But, even in that case, fatigue analyses based on truss models and S-N curves for the straight portion of the stirrups (i.e., without any decrease of the fatigue strength owing to the influence of bends) led to conservative results.

Moreover, it was concluded that the detrimental effects of fretting and bending in the stirrups, owing to relative shear displacements between the crack faces, need not to be explicitly taken into account, because calculations in which those effects are not explicitly simulated proved to be safe.

The experimental results also showed that a large number of cycles can be applied to the girder after the first stirrup fatigue failure takes place (Frey 1982; Higgins et al. 2007; Kreger et al. 1989; Price and Edwards 1971), however the ultimate capacity is impaired by the fatigue failure of one or more stirrups (Higgins et al. 2007).

In conclusion, the performed comparison between numerical and experimental results validated the adopted analysis methodology, considering the inclination of the compression struts, θ_{fat} , given by Equation (4.6), and S-N curves for straight reinforcing bars. It was found that this methodology provides particularly conservative results in the case of PC beams.

4.3 Analysis of the effects of in-plane shear and out-of-plane bending

In this section, the developed methodology for stress calculations considering the effects of in-plane shear and out-of-plane bending, to be used in fatigue analyses, is presented. The section starts with a brief reference to available models for taking into account this combined effect in ULS analyses.

4.3.1 Methodologies for ULS analysis

As mentioned in the introduction to this chapter, various approaches for ULS analyses of girder webs, considering in-plane shear and transverse bending, are already available. In Annex 1, the ones developed by Menn (1986) and Gaspar (2003) are exposed. Minor modifications were

introduced in the original methodologies, in order to assure consistency with the procedures recommended by the EN 1992-1-1 for ULS shear design of members with shear reinforcement.

In that Annex, one can see that, according to such methodologies, the resistance to transverse bending moments is essentially provided by two mechanisms:

- A lateral shift of the compression strut, without modification of the stirrup stresses. The eccentricity of the compression strut is thus able to withstand part of the applied transverse bending moment. This mechanism was demonstrated by Kaufmann and Menn (1976) and was confirmed by Gaspar (2003). It implies a lateral shift (of equal magnitude) of the plane in which the shear force is applied. This lateral shift has minor implications in terms of the global distribution of forces applied to the structure, namely in the case of bridge decks with thin webs.
- Pure transverse bending of the web, with increasing tensile stresses in the stirrup leg on the flexural tensile side, decreasing tensile stresses in the stirrup leg on the flexural compression side and increasing compression stresses on the flexural compression side.

Annex 1 also presents the validation of such methodologies through the comparison with the experimental results reported by Gaspar (2003).

4.3.1.1 Application to the analysis of deck slabs, considering the influence of shear between web and flanges

Even though the methodologies indicated in Annex 1 are mainly devoted to the analysis of girder webs, they can also be applied to the analysis of another problem in which the combined effects of shear and transverse bending must be taken into account: deck slabs are subjected to important transverse bending moments and, simultaneously, to shear forces caused by the longitudinal variation of the axial force applied to the flange (i.e., the deck) which, in turn, is due to the longitudinal variation of bending moments applied to the girder (see Figure 4.16). The applied shear stress v_f , at the connection between the web and each flange, is given by:

$$v_f = \frac{\beta \cdot V}{z \cdot h_f} \quad (4.9)$$

where β is the percentage of longitudinal force applied to each flange (this parameter takes the value 0.5 in all the applications envisaged in this work), V is the shear force applied to the girder, z is the lever arm of the girder and h_f is the flange thickness.

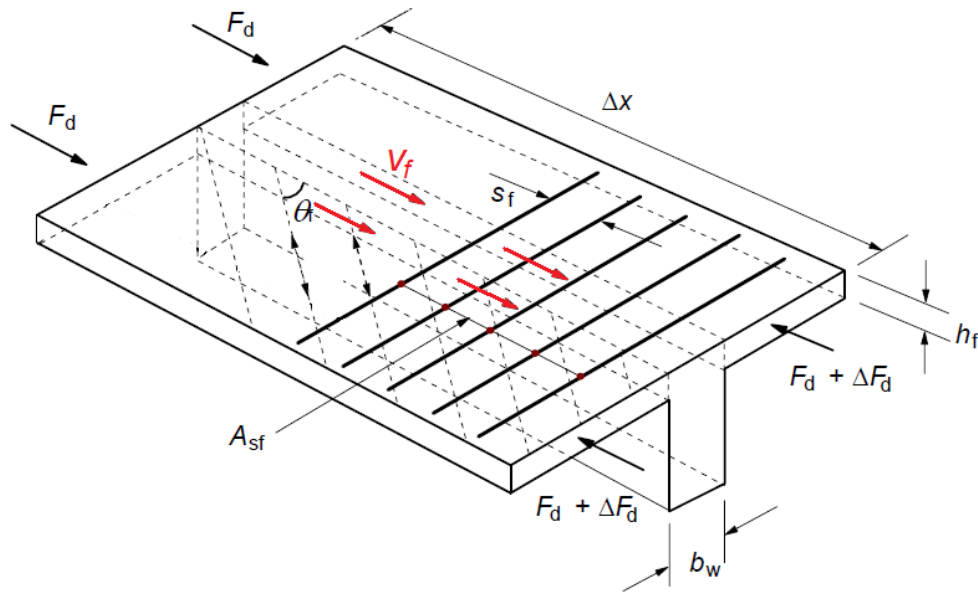


Figure 4.16 – Shear between web and flanges (adapted from the EN 1992-1-1).

By examining the design procedure recommended by the EN 1992-1-1 for ULS analyses considering the sole effect of shear between web and flanges, one can conclude that such analyses can be carried out by employing the truss model (as described in §4.2.1), with the following adaptations:

- the applied shear force must be taken as $\beta \cdot V$ instead of V ;
- the compression strength of the struts must be given by νf_{cd} instead of $\alpha_{cw} \cdot \nu_l f_{cd}$ [Equation (4.7)], where:

$$\nu = 0.6 \left(1 - \frac{f_{ck}}{250} \right)$$

with f_{ck} in MPa;

- the lower bound limit for the inclination of the compression struts θ_f is taken as $\text{arccot}(2)$ or $\text{arccot}(1.25)$, for compression or tension flanges, respectively.

Consequently, if these adaptations are taken into account, the analysis of the effects of transverse bending and shear between web and flanges can be undertaken through the procedures shown in Annex 1. This coherent procedure is, thus, an alternative to the simplistic approach proposed by the EN 1992-1-1. According to this code, the required area of transverse reinforcement is the greater of:

- the area required to resist the sole effect of shear between web and flanges;
- the area required for transverse bending plus half the value required to resist the effect of shear between web and flanges.

Some remarks have, however, to be made:

- If the cross section under analysis is subjected to other important internal forces, namely twisting moments, m_{xy} , alternative analysis methodologies must be employed. Lourenço and Figueiras (1993; 1995) presented an automatic procedure for ULS analysis of shells, considering the influence of membrane and flexural forces. That procedure, according to which the shell thickness is divided into three concrete layers and orthogonal reinforcement layers, can be employed to solve the problem raised in this topic. In this thesis, Lourenço and Figueiras' approach will also be referred to as the three-layer method.
- As mentioned before, the methodologies presented in Annex 1 consider that one of the resistance mechanisms consists of the lateral shift of the plane in which the shear force is applied. While this approach was experimentally tested and validated in girder webs, it was not tested yet (to the author's knowledge) in problems of shear between web and flanges. It should be noted that, in this type of problem, the shear plane must coincide with the one along which the longitudinal forces ΔF_d (indicated in Figure 4.16) are applied. Therefore, such methodologies must be cautiously applied to problems of shear between web and flanges, i.e., not considering the beneficial effect of the eccentricity of compression struts.

In Chapter 6, the results provided by these different approaches are compared.

4.3.2 Proposed methodology for fatigue analysis

As mentioned in the introduction to this chapter, a procedure for stress calculations, to be used in fatigue analyses, has already been presented by Gaspar (2003). In this section, an alternative methodology is exposed, following the underlying principles of Gaspar's approach, i.e., the compatibility of deformations in the web is not explicitly verified, and simplifying assumptions are adopted so that the stress distributions can be evaluated. The main differences between the model proposed by Gaspar (2003) and the one that is proposed in this work are as follows:

- According to Gaspar (2003), the calculation of stirrup stresses due to in-plane shear is carried out through a variable-inclination truss model which is not subjected to the total shear force, i.e., part of the shear force is resisted by alternative mechanisms. In this work, the stirrup stresses are determined through the variable-inclination truss model subjected to the total shear force, for consistency with the methodology proposed by the Eurocodes for fatigue verifications under the sole effect of in-plane shear; such methodology was experimentally validated in §4.2.2.

- In the methodology proposed by Gaspar (2003), a rectangular diagram is adopted to describe the concrete compressive stresses. A certain part of the web thickness is subjected to a uniform stress diagram whereas the remaining part is subjected to zero stresses. In the present work, the compression stresses vary linearly over the web thickness, this assumption being more consistent with the actual behaviour of RC structures under service loads. Both in the proposed model and in the model by Gaspar (2003), tensile stresses in the concrete are not admitted (for coherence with the common assumptions in stress calculations for fatigue design);
- In the model proposed by Gaspar (2003), the calculated stresses depend on two parameters defined by the user: the maximum stress applied to the compression struts and the ratio between the increase of compression stresses in the concrete and in the stirrup leg on the flexural compression side. In the model proposed in this thesis, only one parameter (k_m) is required. A methodology for determination of the k_m values to be considered in fatigue analyses is also proposed.

The developed methodology is described in the following paragraphs.

Firstly, it is important to note that not only do transverse bending moments give rise to different stress histories in the outer and the inner stirrup legs, but these moments also cause different histories of compression stresses in different locations along the web thickness. For this reason, the web thickness is divided into a given number of layers, n_{div} , each of which has a different stress history, and consequently, a different estimated fatigue life. This approach is suggested by the new fib Model Code (fib 2010), which states that, when different parts in the concrete area are exposed to changing maximum and minimum stress levels, each part has to be treated separately, e.g. using the Palmgren-Miner rule. The calculation of the fatigue damage is based on the applied stress at the centre of each layer. In this way, a certain redistribution of compression stresses in the concrete is assumed.

If a girder web is subjected to out-of-plane bending moments, in addition to in-plane shear forces, the transverse bending moment resistance is essentially provided by two mechanisms:

- the eccentricity of the compression strut [part of the transverse bending moment can be resisted through a lateral shift of the compression strut, without modification of the stirrup stresses, as shown by Kaufmann and Menn (1976) and by Gaspar (2003)];
- pure transverse bending of the web, with increasing tensile stresses in the stirrup leg on the flexural tensile side, decreasing tensile stresses in the stirrup leg on the flexural compression side and increasing compression stresses on the flexural compression side.

As mentioned before, the calculation of applied stresses should ideally be based on non-linear analyses with a proper description of the constitutive behaviours under cyclic loading. However, that approach would not be feasible for the design of new structures. Therefore, a simplified approach for stress calculations was developed, considering as premises that it should be as simple as possible, but, at the same time, it should provide a realistic description of the applied stresses, and observe the common assumptions adopted in service analyses of concrete structures: compression stresses vary linearly over the web thickness and tensile stresses in the concrete are not admitted.

Thus, the transverse bending moment resisted by the eccentricity of the compression strut is given by the product of the force C [Equation (4.3)] and the eccentricity of the compression struts, e . The resistance associated to the mechanism of pure transverse bending (the second mechanism mentioned above) is determined by assuming a symmetric variation of the force applied to each stirrup leg, ΔB . It should be noted that the actual distribution of stresses over the length of the stirrup leg on the flexural compression side and in the surrounding concrete, during cyclic loading, is complex, owing to non-linear bond-slip effects and imperfect match of the crack faces (issues dealt with in Chapter 2). However, this assumption is expected to provide realistic values for the stress range in the stirrup leg on the flexural tensile side, which is the most critical one in terms of fatigue damage.

The contribution of each resisting mechanism is defined by the parameter k_m , which represents the percentage of transverse bending moment that is resisted by the first mechanism (the eccentricity of the compression strut). The relationship between the applied bending moment, m , and the contribution of the first mechanism is, thus, given by:

$$k_m \cdot m = C \cdot e \quad (4.10)$$

whereas the contribution of the second mechanism is given by:

$$(1 - k_m) \cdot m = \Delta B \cdot b_0 \quad (4.11)$$

where b_0 stands for the distance between the axes of stirrup legs. The parameter k_m takes values in the interval of 0 to 1. Once the eccentricity of the compression struts is known [its value being determined by solving Equation (4.10) in order to find e], the distribution of compression stresses in the concrete, in the direction θ , can be determined as represented in Figure 4.17. The axial force in each stirrup leg is also schematically represented in that figure (even though the direction of the represented concrete stresses is not coincident with the stirrup axes).

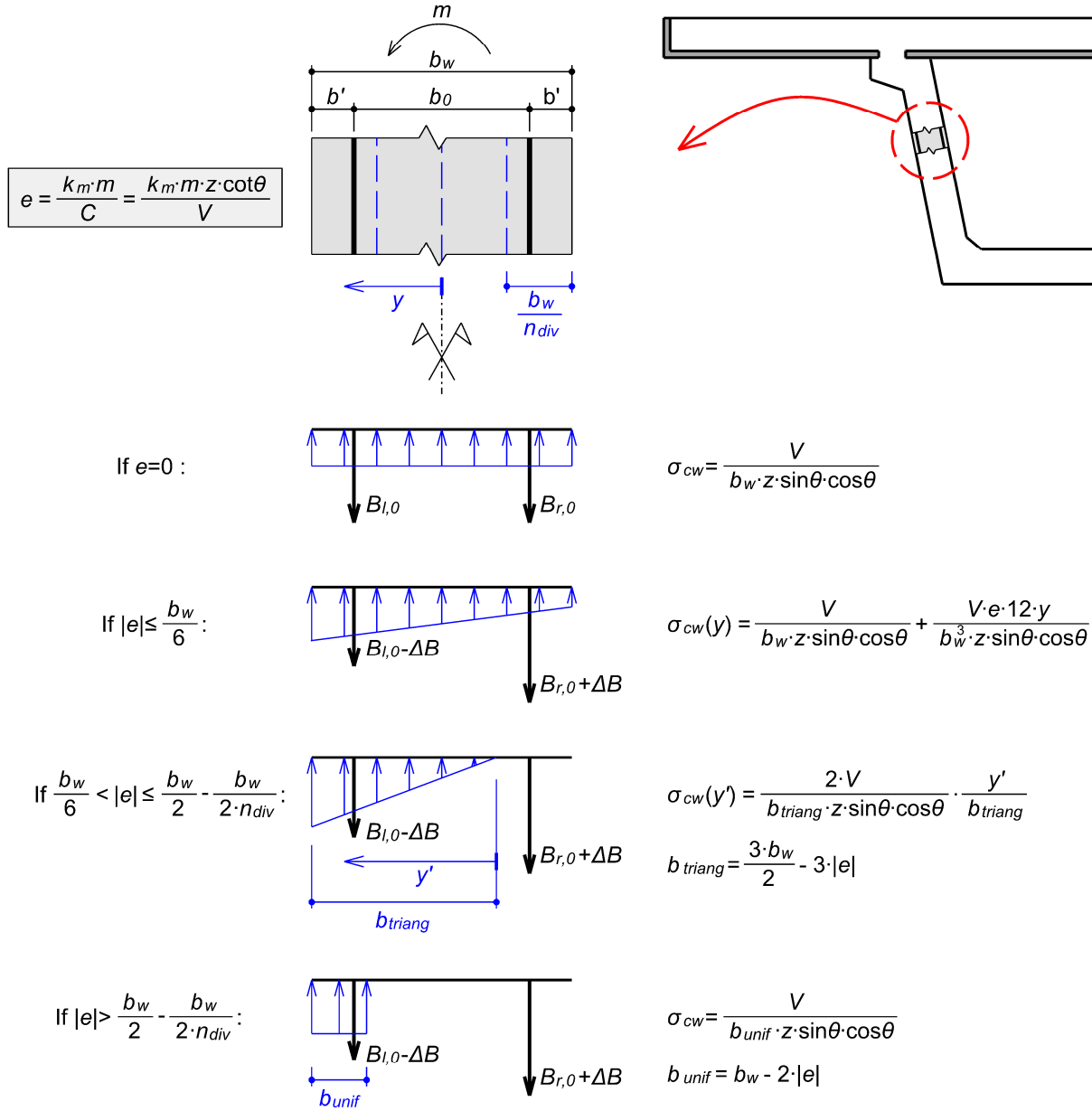


Figure 4.17 – Schematic representation of the distribution of compression stresses in the web struts and forces in the stirrup legs.

If the resultant of the compression stresses lies inside the central core of the web (i.e., if the absolute value of the eccentricity is lower than one sixth of the web thickness) the compression stresses are given by the superposition of the compression force $F_{cw,unit}$ [given by Equation (4.4)], and the corresponding bending moment:

$$F_{cw,unit} \cdot e = \frac{V \cdot e}{z \cdot \sin \theta \cdot \cos \theta} \quad (4.12)$$

On the other hand, if the resultant lies outside the central core, then the distribution of compression stresses is given by a triangular diagram whose base is given by:

$$b_{triang} = 3 \cdot \left(\frac{b_w}{2} - |e| \right) \quad (4.13)$$

with a resultant compression force per unit length equal to $F_{cw,unit}$. As mentioned before, the compression stress is evaluated at the centre of each concrete layer. Therefore, if the eccentricity e is higher than the coordinate y (see Figure 4.17) of the outermost concrete layer, then the stress at the centre of the layer does not provide a good measure of the stress level in the concrete (the calculated stress at this point might even reach zero for large eccentricities of the compression strut). Moreover, the actual distribution of compression stresses is likely to deviate from the triangular diagram as the eccentricity becomes too large, since high stress gradients would occur in that case. Therefore, a uniform diagram of compression stresses is assumed for eccentricities higher than the coordinate of the outermost concrete layer (y , as indicated in Figure 4.17). The depth of this diagram, b_{unif} , is given by:

$$b_{unif} = b_w - 2 \cdot |e| \quad (4.14)$$

If the depth of the compression diagram is inferior to half the width of the concrete layer, the stress at the point where the stresses are evaluated (the centre of the layer), for every concrete layer, becomes zero. In that case, the stress considered in the fatigue calculations for the outermost concrete layer is not zero, but the calculated uniform stress.

The main difficulty in the application of the proposed methodology lies in the fact that the actual value of the parameter k_m is not known. Therefore, fatigue life calculations are performed for different k_m values, so that conservative results are obtained. The range of plausible values for the parameter k_m is determined through the procedure described in the following section.

4.3.2.1 Plausible range of values for the parameter k_m

As mentioned before, the parameter k_m corresponds to the percentage of transverse bending moment that can be resisted through the eccentricity of the compression struts. Therefore, this parameter cannot assume values which would give rise to inadmissible eccentricities of the compression struts (resultant of the compression struts outside the cross section, or eccentricities which correspond to inadmissibly small compression strut widths).

It is also important to take into consideration the conclusions drawn by Gaspar (2003), who carried out fatigue tests in beams subjected to in-plane shear and transverse bending. This author found that the percentage of transverse bending that is equilibrated by the eccentricity of the compression struts depends on the capacity of the web struts to withstand bending. In other words, it can be said that if a given k_m value would give rise to very high concrete compression stresses, than that k_m value is not plausible, i.e., the real contribution of the mechanism of eccentricity of the compression struts will most likely be defined by a lower k_m value.

It is also important to note that the lower the k_m value, the more conservative the calculations of steel fatigue lives, and the less conservative the estimated concrete fatigue lives are.

Thus, the criterion for a conservative definition of the range of k_m values to be considered in fatigue analyses is as follows.

Extreme values of the interval 0 to 1 are initially not considered, because they correspond to the absence of one of the resistance mechanisms. This criterion is consistent with the experimental observations of Gaspar (2003), who found that some variation of the stirrup stresses always occurs, even for small values of applied transverse bending moments. A minimum contribution of 20% from each of the resistance mechanisms is assumed. Thus, the concrete and the steel fatigue lives are calculated for values of k_m ranging from 0.2 to 0.8 with steps of 0.2, i.e., 0.2, 0.4, 0.6 and 0.8.

Values of k_m which give rise to an eccentricity of the compression struts greater than the one corresponding to a uniform distribution of compressive stresses along the width $b_{w,req}/\gamma$ are not admitted. The parameter $b_{w,req}$ represents the minimum required web thickness for ULS verifications, and the reduction factor γ is introduced so that the compression strut's width for fatigue analyses can conservatively be lower than the value required for ULS verifications. That reduction factor shall be greater than the unit, taking, in this work, a value of 1.5. The upper bound limit for the eccentricity of the compression struts, $e_{max,fat}$, is, thus, given by:

$$e_{max,fat} = \frac{b_w}{2} - \frac{b_{w,req}}{2 \cdot \gamma} \quad (4.15)$$

In the case of girders subjected to reduced in-plane shear forces, this criterion controls the upper bound limit, $k_{m,max}$, for the parameter k_m . If $k_{m,max}$ is lower than 0.2, then the fatigue analysis must be carried out for a single k_m value, $k_m = 0$. That means that transverse bending is resisted through a mechanism of simple bending with an internal lever arm equal to b_0 .

Otherwise, the fatigue lives of both the steel and the concrete must be checked for all the admissible k_m values in the range of 0.2 to 0.8, the following exceptions being admitted: (i) $k_m = 0.2$ needs not to be considered if adequate concrete fatigue strength exists for k_m values up to 0.8, i.e., if the web concrete has a large capacity to resist transverse bending; (ii) $k_m = 0.8$ needs not to be considered if a safe amount of stirrups exists, i.e., if adequate steel fatigue strength exists for k_m values as low as 0.2.

In this way, the fatigue strength of both steel and concrete is conservatively checked for a wide range of admissible k_m values: values ranging from 0.2 to 0.6 (or $k_{m,max}$, whichever is the minimum) or in the interval of 0.4 to 0.8, depending on whether or not the mechanism of eccentricity of the compression struts is able to withstand a large percentage of the applied bending moments. In this way, this criterion is coherent with the conclusions drawn by Gaspar (2003), who found that the percentage of transverse bending that is equilibrated by the eccentricity of the compression struts does depend on the capacity of the web struts to withstand bending.

4.3.2.2 Application to the analysis of deck slabs, considering the influence of shear between web and flanges

The methodology for fatigue analysis of girder webs can also be applied to the analysis of deck slabs, considering the interaction between transverse bending and shear due to the connection between web and flanges. Note that, if only the transverse bending was taken into account, then unsafe predications of the deck fatigue life would be obtained, particularly in what concerns the slab cross section adjacent to the girder web (where the highest shear stresses occur).

The fatigue analysis of deck slabs can, thus, be carried out by using the same algorithm that was presented for the analysis of girder webs, with the following adaptations:

- the applied shear force must be taken as $\beta \cdot V$ instead of V (see §4.3.1.1);
- the inclination of the compression struts is given by Equation (4.6) and must comply with the lower bound limits for ULS indicated in §4.3.1.1.

This approach is only valid if other internal forces (namely the twisting bending moment m_{xy}) do not exhibit relevant values in the cross section under analysis. Moreover, fatigue analyses in problems of shear between web and flanges must be cautiously carried out by ignoring the mechanism of eccentricity of the compression struts, for the reasons mentioned in §4.3.1.1. That corresponds to adopting $k_m = 0$.

4.3.3 Validation through the comparison with experimental results by Gaspar

This section presents the comparison between the numerical methodology for fatigue analysis and the experimental results obtained by Gaspar (2003). This author tested four I-shaped beams. Beams 1 and 2 were subjected to monotonically increasing transverse bending moments, in order to evaluate their ultimate capacity. The results of these tests are discussed in Annex 1. Beams 3 and 4 were subjected to cyclic bending moments, so that their fatigue resistance could be assessed. Figure 4.18 shows the cross section of the tested beams and Figure 4.19 presents the test layout. One can see that the shear force was induced by a central point load P , and the transverse bending was applied by means of eccentric forces $F1$ and $F2$, each of which is distributed over the length of 1 m, by using steel spreader beams.

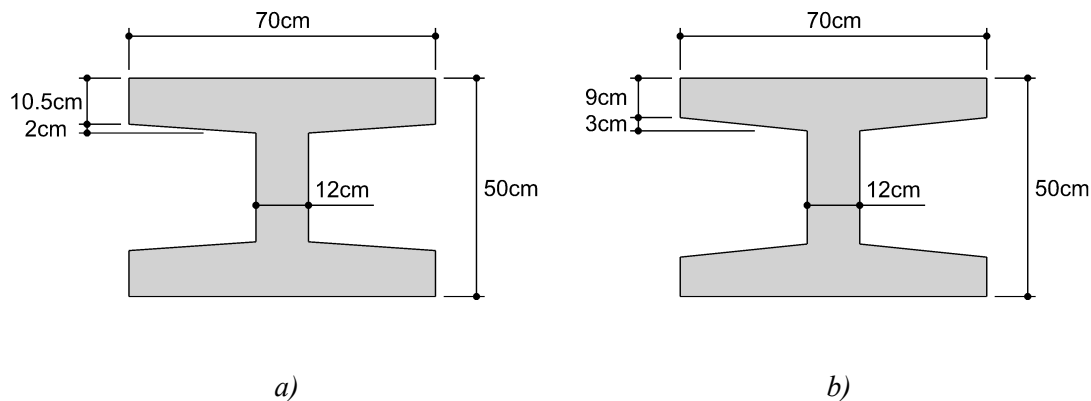


Figure 4.18 – Cross section of the beams tested by Gaspar (2003): a) Beam 1; b) Beams 2 to 4.

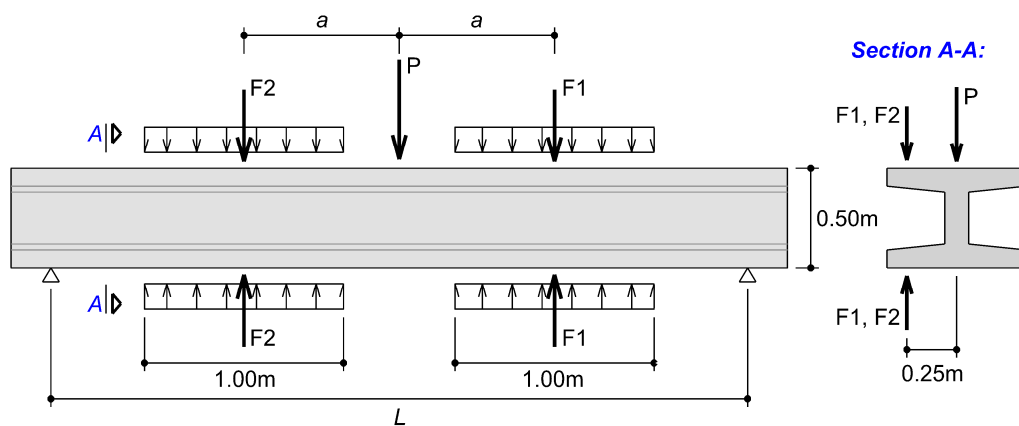


Figure 4.19 – Test layout. Adapted from Gaspar (2003).

Table 4.9 presents relevant characteristics of the tested beams. The cover to the stirrups (from the web surface to the axis of the nearest stirrup) was equal to 1.5 cm.

As regards the monitoring of the results, 40 strain gages were used to measure stirrup strains in each girder. Concrete strains were evaluated with 2 strain rosettes per girder. The applied forces and the global displacements were measured with load cells and LVDTs, respectively.

Table 4.9 – Characteristics of the beams tested by Gaspar (2003).

Beam	Geometry		Stirrups		Concrete	
	L (m)	a (m)	Diameter (mm) and spacing (m)	A_{sw}/s (cm ² /m)	f_{sy} (MPa)	f_c (MPa)
Beam 1	2.36	0.50	$\phi 10//0.10$	15.8	540	18.9
Beam 2	3.50	0.78	$\phi 6.3//0.135$	4.62	630	36.5
Beam 3	3.50	0.78	$\phi 6.3//0.15$	4.16	630	51.7
Beam 4	3.50	0.78	$\phi 6.3//0.24$	2.60	630	52.5

4.3.3.1 Beam 3

The fatigue test of Beam 3 was conducted as follows. Firstly, a point load, $P = 309$ kN, was applied at mid-span in order to induce diagonal cracking in the webs. Then, this load was removed and different load stages were applied. In each stage, the force P was kept constant and transverse bending moments were applied by means of the forces $F1$ and $F2$ represented in Figure 4.19. Force $F2$ is a constant amplitude cyclic load, while $F1$ is a constant force whose value is equal to the average of the lower and upper bound values of $F2$. Table 4.10 presents the magnitude of each load, for each of the three loading stages. The number of cycles applied in each stage is also given in this table. The frequency of the cyclic loading was equal to 3 Hz.

Table 4.10 – Loading stages for Beam 3.

Stage	Constant loads		Cyclic load $F2$ (kN)		
	P (kN)	$F1$ (kN)	Minimum	Maximum	Number of cycles
1	194	20	5	35	18,671
2	66	31	3	44.2	1,021,395
3	66.7	41	9	94.4	1,400,000

After stage 3, the flange of the beam in which the force $F2$ is applied was partially damaged, because a huge force ($F2 = 300$ kN) was accidentally applied during a maintenance operation. Even though additional load cycles were applied later, in the undamaged part of the beam, these

later cycles are not considered here, because of the damage caused by the huge magnitude of the accidental load.

The experimental results are now compared with the model predictions.

Table 4.11 presents the calculated values for the longitudinal shear force and the cyclic transverse bending moment applied in each stage (derived from the forces given in Table 4.10). The shear force is equal to half the force P plus the shear force due to the beam self weight at the distance d from the support. The transverse bending moment, m , is calculated assuming a uniform distribution over half the girder length (1.90 m). This is thought to be a feasible value, since the cyclic load is applied to the beam flange over the length of 1 m (as shown in Figure 4.19) and Gaspar concluded that, after cracking was observed in the girder web, the length that was mobilized to resist the transverse bending moment was significantly higher than the one over which the force was applied. Given the uncertainty associated to the quantification of the length over which the transverse bending moment is distributed (which is herein named b_m), additional calculations are performed, considering an increased value for b_m , equal to two thirds of the girder length ($b_m = 2.53$ m).

For the sake of simplicity, the normal force in the web due to the force $F2$ is conservatively ignored.

Table 4.11 – Internal forces applied to Beam 3.

Stage	In-plane shear force (kN)	Transverse bending moment due to cyclic load $F2$ (kNm/m)			
		(assuming a uniform distribution over 1.90 m)		(assuming a uniform distribution over 2.53 m)	
		Minimum	Maximum	Minimum	Maximum
1	103.0	0.66	4.61	0.50	3.46
2	39.0	0.40	5.82	0.30	4.37
3	39.4	1.18	12.42	0.89	9.33

Figure 4.20 shows the calculated values for the transverse bending moment resistance, as a function of the inclination of the compression struts, for the different test stages identified before, considering the in-plane shear forces shown in Table 4.11 and the parameters displayed in Table 4.12. One can see that the calculated resistances are always higher than the applied bending moments (see Table 4.11). However, important transverse bending moments were applied in Stage 3. The maximum applied bending moment takes the value 12.42 kNm/m if a uniform distribution over 1.90 m is assumed, whereas the bending moment resistance amounts to 13.4 kNm/m (no matter which value is adopted for the inclination of the compression struts).

Table 4.12 – Parameters used in the calculations for Beam 3.

f_{cm} (MPa)	51.7
ν_l	0.6
α_{cw}	1.0
z (m)	0.42

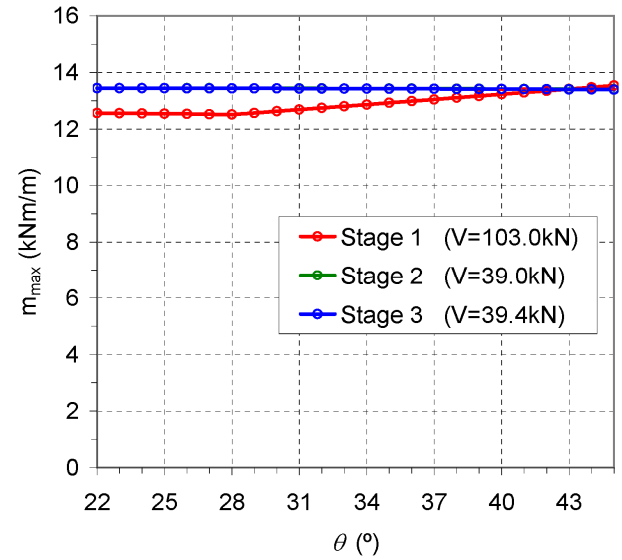


Figure 4.20 – Transverse bending moment resistance in ULS, for Beam 3, according to the proposal by Menn (1986).

ULS calculations also reveal that the compression struts are subjected to low compression stresses due to the sole effect of in-plane shear. Therefore, the lower limit for the inclination of the compression struts is adopted in fatigue analyses [$\theta_{ULS} = 22^\circ$, which corresponds to $\theta_{fat} = 32.4^\circ$, according to Equation (4.6)].

As regards the fatigue strength of the employed materials, in-air fatigue tests were performed by Gaspar (2003) for evaluation of the fatigue characteristics of the employed stirrups. Figure 4.21 depicts the results of the fatigue tests, which are represented by dots. Two different S-N curves are also depicted in this figure:

- the characteristic S-N curve for straight bars, according to the EN 1992-1-1;
- the S-N curve obtained by a curve fitting procedure; this procedure consisted of the calculation of the set of parameters $\Delta\sigma^*$, K_1 , and K_2 (identified in Figure 3.1) that lead to the minimum mean squared error between the S-N curve and the experimental results; N^* (also identified in the same figure) is taken as 10^6 , which is the value suggested by the EN 1992-1-1; the obtained results are shown in Table 4.13.

Figure 4.21 shows that the fatigue strength of the employed steels is considerably higher than the characteristic values given by the EN 1992-1-1. Therefore, the fatigue tests provide valuable information for the comparison between theoretical models and the experimental results obtained in the tested beams. The calculations presented below were carried out considering the S-N curve obtained through the curve fitting procedure.

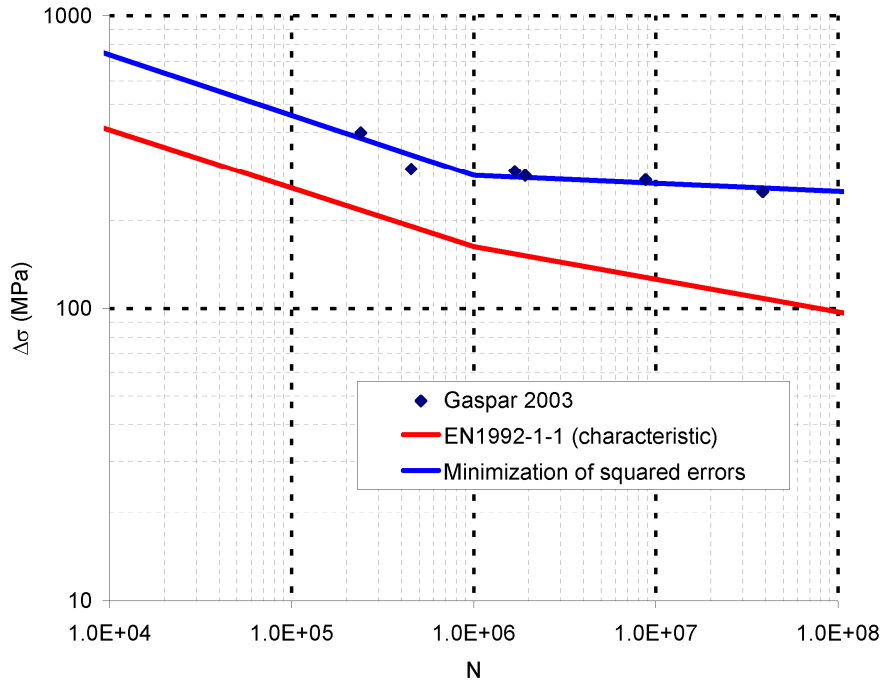


Figure 4.21 – S-N curves for the stirrups employed in Beam 3 and Beam 4.

Table 4.13 – S-N curves for straight reinforcing bars.

	Characteristic curve according to the EN 1992-1-1	Curve fitting to the experimental results reported by Gaspar (2003)
$\Delta\sigma^*$ (MPa)	162.5	284.7
K_1	5	4.85
K_2	9	36.2

As regards the concrete fatigue strength, no experimental data is available. Therefore, Equations (3.3) to (3.5) are used to define the S-N curve for concrete. Equation (3.5) is modified so that an approximate estimate for the mean reference fatigue strength can be obtained:

$$f_{cm,fat} = f_{cm} \cdot \left(1 - \frac{f_{cm}}{250}\right) \cdot \nu = 24.6 \text{ MPa} \quad (4.16)$$

The parameter ν , which accounts for the strength reduction due to tensile forces in the stirrups, takes the value 0.6.

As mentioned before, fatigue analyses must be carried out for a set of plausible values for the parameter k_m . The upper bound limit for this parameter is determined by the condition that the eccentricity of the compression struts should not exceed the upper limit given by Equation (4.15). If Equation (4.10) is solved in order to find the value of k_m , considering the eccentricity

$e_{max,fat}$ given by Equation (4.15) and the force C given by Equation (4.3), then the upper bound limit for the parameter k_m can be obtained:

$$(k_m)_{max} = \frac{V \cdot e_{max,fat}}{z \cdot \cot \theta_{fat} \cdot m_{max}} \quad (4.17)$$

where m_{max} represents the maximum transverse bending moment applied in each stage (values given in Table 4.11). Note that a closed form solution for $(k_m)_{max}$ can be obtained in this case because the applied shear force takes a constant value in each load stage. The obtained results for each load stage and for each b_m value are shown in Table 4.14. This table also displays the minimum required web thickness $b_{w,req}$ (calculated with design values for both the applied shear force and the concrete compressive strength) and the eccentricity $e_{max,fat}$ for each load stage.

Table 4.14 – Upper bound limit for the parameter k_m

Stage	$b_{w,req}$ (m)	$e_{max,fat}$ (m)	$(k_m)_{max}$	
			$b_m = 1.90 \text{ m}$	$b_m = 2.53 \text{ m}$
1	0.0606	0.0398	1.34 ⁽¹⁾	1.79 ⁽¹⁾
2	0.0229	0.0524	0.53	0.71
3	0.0232	0.0523	0.25	0.33

Notes: ⁽¹⁾ As mentioned in §4.3.2.1, k_m values higher than 0.8 are not admitted

Following the criteria introduced in §4.3.2.1, for each load stage, fatigue analyses are carried out for all the k_m values in the range of 0.2 to 0.8, with steps of 0.2, excluding values higher than $(k_m)_{max}$. Figures 4.22 and 4.23 display the calculated stress ranges, for the three test stages.

By observing these figures, some conclusions can be drawn about the fatigue damage in the stirrups, at the end of each stage:

- The accumulated damage during the stages 1 and 2 always takes negligible values. The highest value occurs at the end of stage 2, for calculations with $k_m = 0.2$ and bending moment evenly distributed over 1.90 m (as expected, Figures 4.22 and 4.23 confirm that these are the most unfavourable values for the parameters k_m and b_m , in terms of calculations of steel fatigue damage). In this case, the accumulated fatigue damage, D , at the end of the second stage takes a reduced value, $D = 0.00058$ (noting that fatigue failure is assumed to occur for $D = 1$).
- Figures 4.22 and 4.23 also show that the accumulated damage at the end of the third stage is greater than the unit, i.e., the number of load cycles given by the S-N curve for the

calculated stress ranges is lower than the actual number of applied load cycles. This result is obtained no matter which value is taken by the parameter b_m .

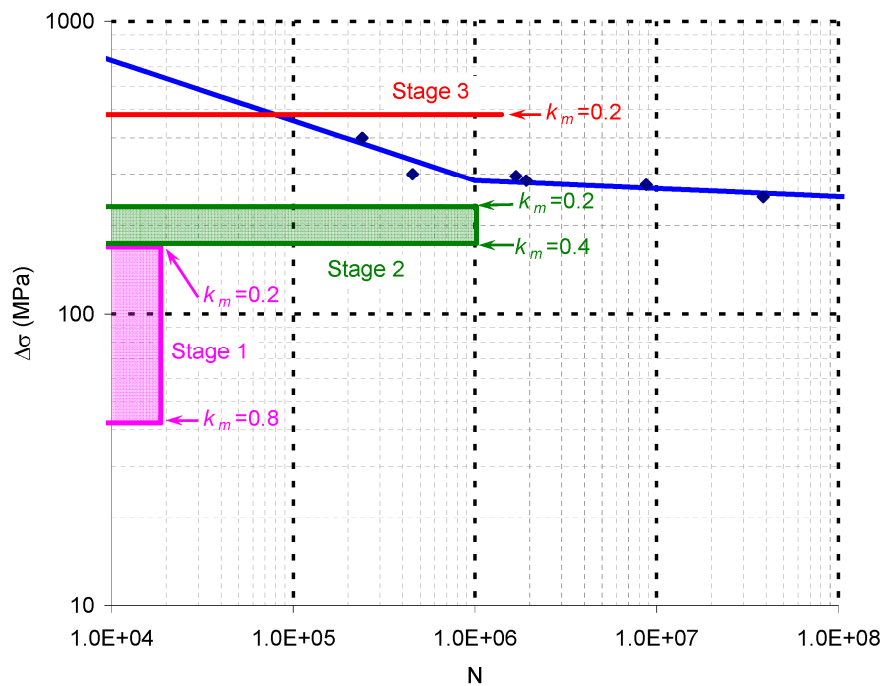


Figure 4.22 – Calculated stirrup stress ranges, considering that the transverse bending moment is evenly distributed over 1.90 m.

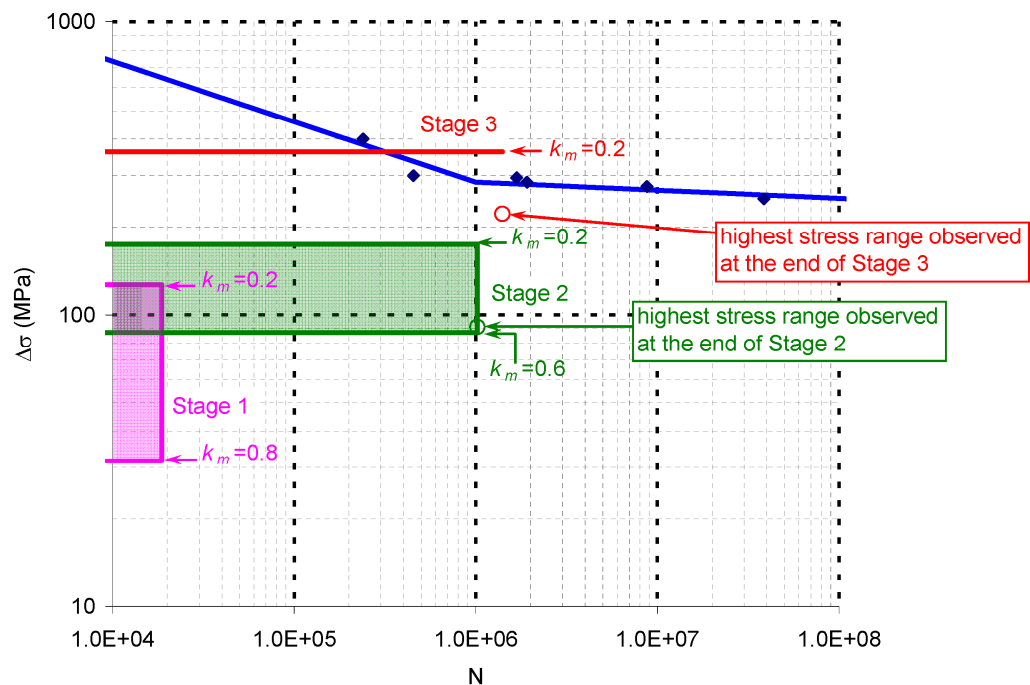


Figure 4.23 – Calculated stirrup stress ranges, considering that the transverse bending moment is evenly distributed over 2.53 m.

The aforementioned results show that the calculation method provides conservative estimates for the fatigue life of the tested beam 3. The calculation results indicate that fatigue fractures should

have occurred before the end of Stage 3, but no fatigue failure was detected in the reality. Moreover, in the calculations with $b_m = 1.90$ m, the steel stress under the effect of the maximum bending moment reaches the value 673.9 MPa, a value higher than the yield stress of the stirrups (630 MPa). This result also indicates that the proposed calculation method provides conservative estimates for the steel stresses (the actual fatigue life would be even lower than the estimates based on the S-N curve, if the steel actually experienced plastic strains).

As regards the fatigue strength of the web concrete, the calculated damage at the end of Stage 3 is well below 1, for both calculation scenarios (with different values for the length over which the bending moment is distributed). In the tests, concrete fatigue failure was not detected too.

The highest stress ranges observed at the end of stages 2 and 3 (measured strain ranges times the measured steel modulus of elasticity), which are plotted in Figure 4.23, also indicate that the proposed numerical methodology provides conservative predictions of the stress ranges. The observed stress range at the end of Stage 3 is lower than the calculated values. As regards the Stage 2, the model also provides conservative results in terms of stress ranges, since the highest observed stress range is close to the lowest calculated value (for the different scenarios in terms of k_m and b_m values). It is important to note that the strain gages were not located at the intersection with the cracks, and consequently, the actual highest stress ranges might be higher than the measured values. Nevertheless, the fact that no fatigue failure took place during Stage 3 confirms that the proposed numerical model provides conservative estimates for the stirrups fatigue life.

As regards the uncertainty associated to the quantification of the parameter k_m , Figures 4.22 and 4.23 show that the range of plausible values for this parameter decreases as the ratio between the maximum transverse bending moment and the in-plane shear force increases. In the case of Stage 3, this uncertainty does not exist and, according to the proposed criteria for quantification of k_m , the fatigue analysis must be carried out for a definite value, $k_m = 0.2$, because unrealistic values for the eccentricity of the compression struts are obtained when the k_m exceeds the upper bound values shown in Table 4.14. Therefore, the weakness of the proposed methodology, which lies in the inaptitude to quantify a single value for the parameter k_m , for some load cases, does not affect calculations with important transverse bending moments.

On the other hand, the uncertainty associated to the quantification of the stress range for Stage 1 is quite high, as shown in Figures 4.22 and 4.23. Nevertheless, it is important to note that, in this case, the stress range is merely due to the variation of the transverse bending moment, since the in-plane shear force is constant in each stage. In the case of real precast girders, in railway bridges, the applied longitudinal shear force undergoes important cyclic variations and, therefore, the uncertainty associated to the quantification of stress ranges in the stirrups is

decreased, for the reason that the calculation of stress ranges due to in-plane shear is not affected by the parameter k_m . Chapter 6 will show that, despite the weakness associated the quantification of k_m , the proposed methodology is a feasible procedure to carry out conservative fatigue analyses, to be employed in the design of new structures.

4.3.3.2 Beam 4

The main characteristics of Beam 4 are presented in Figures 4.18 and 4.19, and in Table 4.9. The main feature of this beam lies in the reduced cross-sectional area of shear reinforcement ($A_{sw}/s = 2.60 \text{ cm}^2/\text{m}$). The test involved the following stages:

- firstly, diagonal cracking was induced through a point load applied at mid span ($P = 202 \text{ kN}$);
- then, the previously applied force P was removed and the first stage of cyclic loading was applied. This stage consisted of a constant force P applied at mid span, a constant amplitude cyclic load $F2$ and a constant load $F1$.
- after that, a new stage of cyclic loading was applied, with constant loads P and $F1$ and cyclic load $F2$.
- finally, the beam was loaded monotonically up to failure; this stage is not considered here because several fatigue failures were observed during the second stage of cyclic loading.

The magnitude of the applied loads and the number of cycles applied in each stage are described in Table 4.15. Several stirrup fatigue fractures were observed during Stage 2. The first one took place after 724,800 cycles, the second one was detected after 757,697 cycles and the third one occurred after 1,176,575 cycles. Fatigue fractures always took place in the straight portion of the stirrups, far from the bends.

Table 4.15 – Loading stages for Beam 4.

Stage	Constant loads		Cyclic load $F2$ (kN)		
	P (kN)	$F1$ (kN)	Minimum	Maximum	Number of cycles
1	40	22	4	40	1,028,000
2	40	0	4	80	1,598,667

The experimental results are now compared with the outcomes of the proposed numerical model. The analysis approach is the same that was followed in the case of Beam 3. Given the uncertainty associated to the quantification of the length over which the transverse bending moment can be assumed to be evenly distributed, b_m , two different scenarios are adopted,

considering $b_m = 1.90$ m (half the beam length) and $b_m = 2.53$ m (two thirds of the beam length). The corresponding transverse bending moments are quantified in Table 4.16.

Table 4.16 – Internal forces applied to Beam 4.

Stage	Longitudinal shear force (kN)	Transverse bending moment due to cyclic load F_2 (kNm/m)			
		(assuming a uniform distribution over 1.90 m)		(assuming a uniform distribution over 2.53 m)	
		Minimum	Maximum	Minimum	Maximum
1	26.0	0.53	5.26	0.40	3.95
2	26.0	0.53	10.53	0.40	7.91

As regards the ultimate capacity of the tested beam, according to the methodology proposed by Menn (1986), the bending moment resistance, for a shear force of 26.0 kN, is practically independent from the inclination of the compression struts and takes the value 8.5 kNm/m (basic data used in the calculations is shown in Table 4.17). By comparing this result with the applied bending moment, one can conclude that the cyclic bending moment applied during Stage 2 reached important values. For $b_m = 1.90$ m, the maximum applied bending moment is even higher than the calculated resistance.

Table 4.17 – Parameters used in the calculations for Beam 4.

f_{cm} (MPa)	52.5
ν_l	0.6
α_{cw}	1.0
z (m)	0.42

Given the small in-plane shear force applied to this beam, lower bound values are adopted for the inclination of the compression struts: $\theta_{ULS} = 22^\circ$ and $\theta_{fat} = 32.4^\circ$. The corresponding upper bound limits for the eccentricity of the compression struts, for fatigue analyses, are shown in Table 4.18, these values being determined through the procedure detailed in §4.3.3.1. The small values obtained for the parameter k_m , particularly in the case of Stage 2, show that only a small portion of the applied bending moment can be resisted through the eccentricity of the compression strut. Therefore, according to the proposed criteria (§4.3.2.1), a single k_m value is admitted for Stage 2, $k_m = 0.2$, whereas two different values must be considered for Stage 1, $k_m = 0.2$ and $k_m = 0.4$.

The comparison between fatigue analyses and experimental results is carried out by describing the fatigue strength of the stirrups through the S-N curve adjusted to the fatigue test results (as shown in Figure 4.21). The definition of the concrete fatigue strength is based on the same

hypotheses that were adopted for Beam 3. In this case, the parameter $f_{cm,fat}$ takes the value 24.9 MPa.

Table 4.18 – Upper bound limit for the parameter k_m .

Stage	$b_{w,req}$ (m)	$e_{max,fat}$ (m)	$(k_m)_{max}$	
			$b_m = 1.90$ m	$b_m = 2.53$ m
1	0.0150	0.0550	0.41	0.55
2	0.0150	0.0550	0.21	0.27

The estimated stress ranges, for Stages 1 and 2 and for the different values adopted for the parameters k_m and b_m , are shown in Figures 4.24 and 4.25. In this graphical representation, the calculated stress ranges are associated to the following number of load cycles:

- the maximum number of load cycles applied during Stage 1 (1,028,000 cycles, as shown in Table 4.15), since no fatigue failure was observed during this stage;
- the maximum number of load cycles before the first fatigue fracture occurred in Stage 2 (724,800 cycles).

In this way, the proposed calculation method provides conservative results if the aforementioned number of cycles is greater than the abscissa of the S-N curve (for the corresponding calculated stress range).

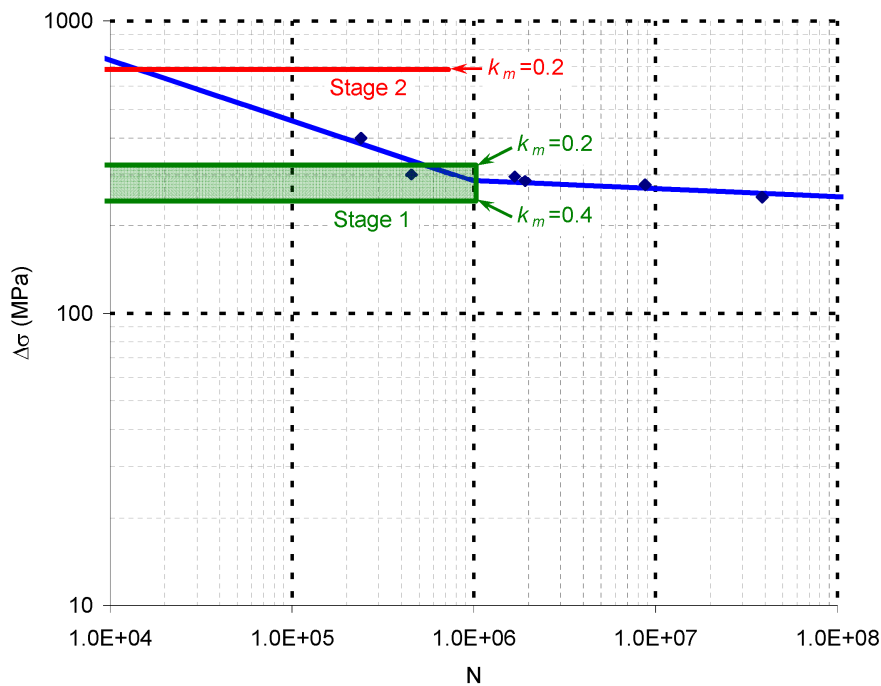


Figure 4.24 – Calculated stirrup stress ranges, considering that the transverse bending moment is evenly distributed over 1.90 m.

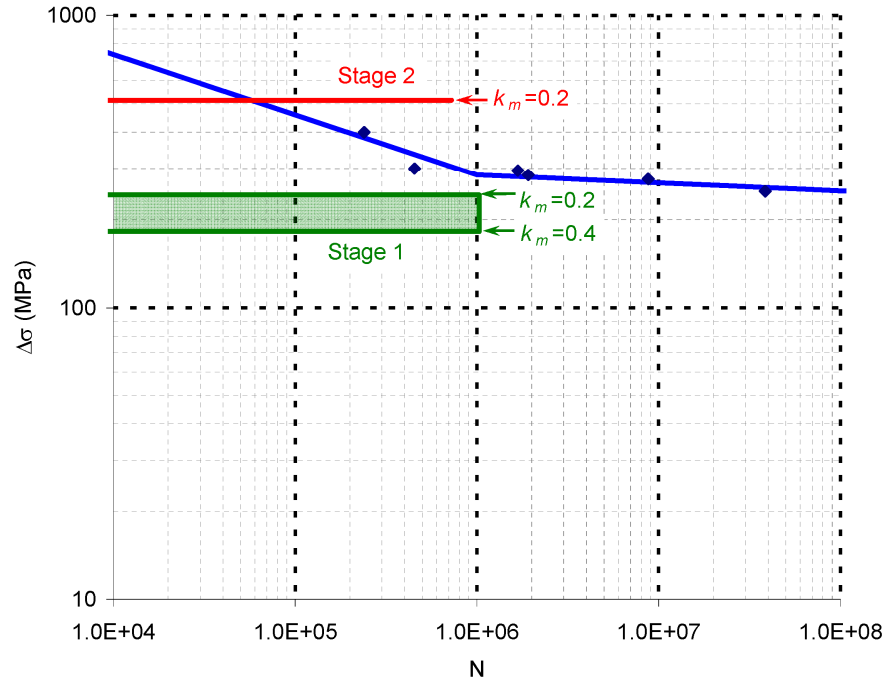


Figure 4.25 – Calculated stirrup stress ranges, considering that the transverse bending moment is evenly distributed over 2.53 m.

It thus becomes clear that the proposed calculation method provides conservative results for Beam 4. No matter which value is adopted for the length over which the bending moment is assumed to be evenly distributed (1.90 m or 2.53 m), the estimated fatigue life during Stage 2 is clearly lower than the number of applied load cycles before the first fatigue fracture occurred. Obviously, the estimated fatigue life decreases as b_m decreases.

It is important to note that the calculated steel stress under the effect of the maximum bending moment, in Stage 2, reaches values higher than the yield stress of the employed steel ($f_{sy} = 630$ MPa):

- 871 MPa for $b_m = 1.90$ m;
- 692 MPa for $b_m = 2.53$ m.

These results also confirm that the proposed analysis methodology overestimates the actual stress ranges (thus providing conservative results). This is because the employed S-N curve is based on laboratorial tests within the elastic domain, and consequently, lower fatigue lives would occur if the applied stresses actually exceeded the yield stress of the stirrups.

Finally, it is important to remark that, in the case of Beam 4, the uncertainty associated to the quantification of the stress range in the stirrups is quite reduced (a single k_m value is admitted in Stage 2 and two values are admitted in the case of Stage 1), because the eccentricity of the compression strut can only withstand a small fraction of the applied bending moment.

4.4 Final considerations

This Chapter focuses on stress calculations for fatigue analyses of girder webs. It comprehends two main sections. The first one is devoted to the analysis of the sole effect of in-plane shear. In the second one, the influence of transverse bending is also taken into account. Even though this chapter focuses on fatigue analyses, available methodologies for ULS calculations considering the interaction between in-plane shear and transverse bending are also briefly addressed (further developments on this theme can be found in Annex 1). Experimental results reported in the literature are used in the context of this chapter for evaluation of the applicability of truss models for fatigue analyses of beams subjected to in-plane shear, and also for validation of the proposed models for analysis of the combined effects of in-plane shear and transverse bending.

The comparison between the outcome of the truss model and experimental results for beams subjected to in-plane shear confirmed that this type of model can be used in design practice for calculation of stirrup stress ranges, in the context of fatigue analyses. The inclination of the compression struts for fatigue calculations shall be determined through Equation (4.6), as a function of the inclination considered in ULS analyses. The fatigue strength of the stirrups shall be quantified based on S-N curves for straight reinforcing bars, i.e., not considering the strength reduction at the bends. Truss models proved to be particularly conservative in the case of prestressed elements, because alternative resistance mechanisms play a significant role in that case.

An innovative procedure for stress calculations considering the effects of in-plane shear and transverse bending was presented. This numerical methodology can be used in fatigue analyses of girder webs in railway bridges according to the Damage Accumulation Method. The proposed methodology allows the fatigue verification of both the stirrups and the web concrete. The simplicity of the algorithm for stress calculations makes it feasible to calculate stress histories due to the passage of trains, the histories of internal forces (in-plane shear and transverse bending) being calculated by means of dynamic FE analyses. Given the complexity of the cyclic behaviour of cracked girder webs subjected to in-plane shear and transverse bending, the calculated stresses on both the steel stirrups and the web concrete depends on a parameter k_m which defines the relative contribution of two resistance mechanisms. Conservative fatigue analyses are obtained if fatigue verifications are carried out considering not a single k_m value, but the plausible range of k_m values. That range is determined through the procedure indicated in this chapter.

The comparison between the output of the proposed methodology and the experimental results reported by Gaspar (2003) showed that the analysis methodology provides conservative estimates for the fatigue life of the tested beams. As regards the uncertainty associated to the

quantification of the parameter k_m , it was shown that the range of plausible values for this parameter decreases as the ratio between the maximum transverse bending moment and the in-plane shear force increases. If the girder web under analysis is subjected to important transverse bending moments, then the interval of plausible values exhibits a small amplitude, i.e., the uncertainty associated to the quantification of k_m is considerably reduced in the presence of important transverse bending moments. In Chapter 6, where a real precast railway bridge deck is analysed, it will be shown that, despite the weakness associated the quantification of k_m , the proposed methodology is a feasible procedure to carry out conservative fatigue analyses, to be employed in the design of new structures.

The applicability of the proposed methodology to problems of shear between web and flanges was also discussed. It was recommended that, before it is proven that the mechanism of eccentricity of the compression struts can be considered in this type of problem, a conservative approach should be followed, i.e., the beneficial effects of that mechanism should not be considered, both in ULS and in fatigue analyses.

Despite the fact that the presented numerical methodologies were primarily devised for the analysis of girder webs, it was shown that they can also be applied in the analysis of deck slabs, considering the interaction between in-plane shear forces (resulting from the transference of forces between webs and flanges) and transverse bending.

Chapter 5

FE analysis of the long-term behaviour of precast continuous bridge decks

5.1 Introduction

It is well known that precast continuous bridge decks undergo important long-term stress changes, as a consequence of phased construction and concrete delayed deformations (creep and shrinkage). Several methods have been proposed to evaluate the long-term behaviour of this kind of structures. Early studies by Mattock (1961) gave rise to the Portland Cement Association (PCA) method (Freyermuth 1969), the Construction Technology Laboratories (CTL) method (Osterle et al. 1989), the software RMCalc (McDonagh and Hinkley 2003) and a numerical methodology which includes the influence of concrete cracking (Mirmiran et al. 2001). Both the PCA and the CTL methods have been largely employed by practitioners to predict restraint moments (Hastak et al. 2003), i.e., bending moments caused by concrete delayed deformations and phased construction. However, the application of those methods is restricted to the most common bridge deck typology, in which the continuity is achieved by means of a continuity diaphragm and a RC slab. They can not be employed in the analysis of structures constructed with different techniques, for example if the slab is post-tensioned. That difficulty was overtaken by other authors by using numerical methodologies for the time dependent analysis of concrete frames (Ghali et al. 2002; Ma et al. 1998), or general-purpose FE software (Kwak and Seo 2000; Marí 2000).

The validation of the aforementioned calculation methods has traditionally been carried out by comparing numerical calculations against the results of laboratorial experiments with reduced-

scale specimens, either under indoor controlled conditions (Mattock 1961; Newhouse et al. 2008; Peterman and Ramirez 1998; Valdés 1997) or under outdoor environmental conditions (Miller et al. 2004). However, even though laboratorial experiments provide valuable information, they do not accurately reproduce the scale, the environmental conditions, the construction procedures and the loading that occur in real structures. In fact, laboratorial tests are usually carried out on reduced scale models, which often lead to the simplification of the actual geometric shape of real structures. Furthermore, the real environmental effects (seasonal and daily variations of temperature, relative humidity, rainfall, etc.) are not reproduced in laboratory, which may influence the delayed deformations of concrete. Moreover, traffic loading can only be reproduced in a simplified manner in laboratorial experiments. Therefore, the calculation methods should ideally be validated by comparison with results observed in real structures (Barr et al. 2005; Cruz and Wisniewski 2004), but such field data are scarce due to the difficulty in monitoring full-size bridges over long periods of time.

The recent construction of a major bridge in Portugal - the Lezíria Bridge - which comprises a 9.16 km viaduct made up of precast beams, offered the opportunity to observe the long-term behaviour of a real structure and to compare measurements against the results of numerical calculations (Sousa et al. 2012a; Sousa et al. 2012b). A permanent monitoring system was installed in this bridge (a work carried out by NewMENSUS and LABEST) and measurements were taken since the beginning of the construction. The data collected in the first 32 months of observation is used in this thesis with the following purposes:

- to evaluate the adequacy of FE analyses, considering creep, shrinkage, cracking, relaxation and the construction sequencing, to calculate the real behaviour of this kind of structure;
- to assess the consequences of carrying out simplified analyses based on limited information about concrete properties and construction sequence (i.e., considering only the information usually available at the design stage);
- to evaluate the adequacy of probabilistic analyses based on Monte Carlo simulations to calculate tolerance intervals that contain the actual results with a certain confidence level, taking into account the variability of creep, shrinkage, and construction timings;
- to contribute to a better understanding of the long-term behaviour of this kind of structure;

It is important to note that, even though the Lezíria viaduct carries highway traffic, it comprehends geometric shapes and construction sequences that are similar to those found in railway bridges and viaducts. For that reason, the conclusions to be taken through the analysis of this structure are generically applicable to continuous precast railway bridge decks.

This chapter starts with the presentation of the Lezíria viaduct and the monitoring system. Then, the implemented strategy for the numerical analysis is exposed. Afterwards, the most relevant results of the deterministic analyses are shown, discussed and compared with experimental observations. Finally, the probabilistic analysis is presented and the most relevant results are discussed.

5.2 Case study – the precast approach viaduct to the Lezíria Bridge

5.2.1 Structure description

The south approach viaduct to the Lezíria Bridge is a 9,160 m structure whose deck is made up of precast beams. It is divided into 22 elementary viaducts, separated by expansion joints, with lengths ranging from 250 m to 530 m. This work focuses on the V14S viaduct, which is 468 m long and is composed by 13 continuous spans with a span length of 36 m (Figure 5.1).

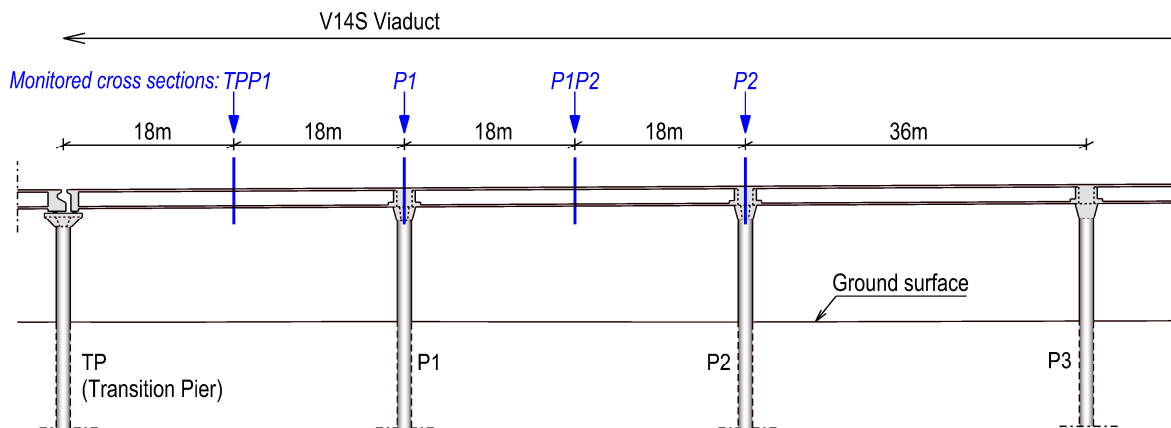


Figure 5.1 - Longitudinal section of the south approach viaduct to the Lezíria Bridge (partial depiction) and monitored cross sections.

The cross section of the viaduct is 29.95 m wide and is composed by 4 precast U-shaped girders (1.75 m high) and a 0.25 m thick slab (see Figures 5.2 and 5.3). The girders are spaced 7.50 m from centreline to centreline and are prestressed by straight prestressing strands located at the bottom slab. The web thickness takes values of 0.15 m (in the mid-span region) and 0.20 m (for distances to the supports lower than ~7 m). The deck slab is composed of precast concrete planks (whose thickness varies between 0.08 m and 0.10 m) monolithically connected to the remaining cast-in-place (CIP) portion of the slab.

Girder continuity is provided by a CIP diaphragm, reinforcing steel bars and straight post-tensioning cables in the CIP slab. The continuity diaphragm, which is 2.30 m thick, is monolithically connected to the CIP piers.

The slab is cast in separate phases. Firstly, a portion of slab above the piers is cast simultaneously with the diaphragms. This slab strip extends approximately 6 m for each side of the pier axis. After this concrete has gained the required strength, the continuity post-tensioning is applied. The mid-span portion of the slab is cast at a later stage.

The foundations are provided by circular CIP piles, with a diameter of 1.5 m (as the piers), crossing alluviums with variable constitution, and reaching the maximum depth of 47 m. Above the ground surface, the pier-pile cross section is kept constant up to the cap (Figure 5.1). The pier cap provides the necessary room for positioning the precast girders, avoiding the necessity of provisory support systems.

Additional details about the structure geometry, the layout of ordinary and prestressing reinforcements, and about the employed materials can be found elsewhere (Sousa et al. 2009).

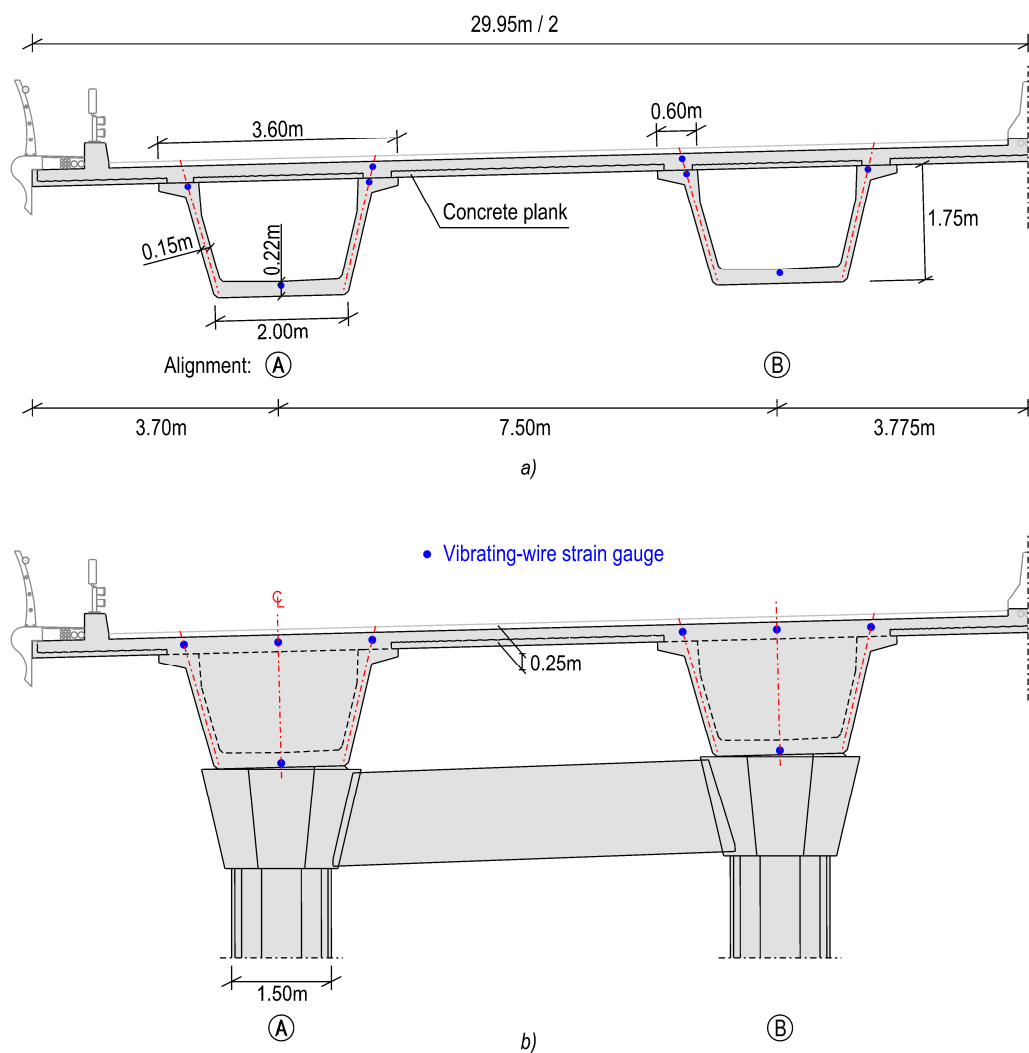


Figure 5.2 - Cross section of the viaduct and position of vibrating-wire strain gages:
a) at mid-span; b) above the pier axis.



Figure 5.3 – Images of the construction, during the erection of precast girders and before placing the precast planks.

This viaduct was constructed in 10 months, beginning in February 2006 (Figure 5.4).

	2006											
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Precast beams												
Piles												
Piers (above the ground)												
Erection of precast beams												
Erection of precast planks												
Slab casting (above piers)												
Slab casting (mid-span)												

Figure 5.4 - Schematic representation of the construction sequence.

5.2.2 Monitoring system

Vibrating-wire strain gages and temperature sensors were installed in two mid-span cross sections and two cross sections above the pier axes (cross sections TPP1, P1, P1P2 and P2 identified in Figure 5.1). In each cross section, measurements were taken in the beams located in both the alignments A and B, as shown in Figure 5.2. The exact location of each strain gage is identified in Figure 5.5. One can see that the sensors were installed both in the girder and in the CIP slab. In one of the girders (in the first span, alignment B), measurements were taken continuously since the start of concreting. When it was not possible to measure continuously since casting, reference measurements were taken before the start of concreting. Concrete temperature was measured by means of the internal thermistor of the strain gages. The strain values captured by the vibrating-wire strain gages were corrected by eliminating the effect of the

free thermal deformation of the wire and of the concrete. For this purpose, the thermal dilation coefficient took the value $11 \cdot 10^{-6}/^{\circ}\text{C}$ in the case of the wire (given by the manufacturer) and $8 \cdot 10^{-6}/^{\circ}\text{C}$ in the case of the concrete (experimentally evaluated in climatic chamber tests). Additional details about the compensation of temperature effects in the structure under analysis are given in the paper by Sousa et al. (2012b).

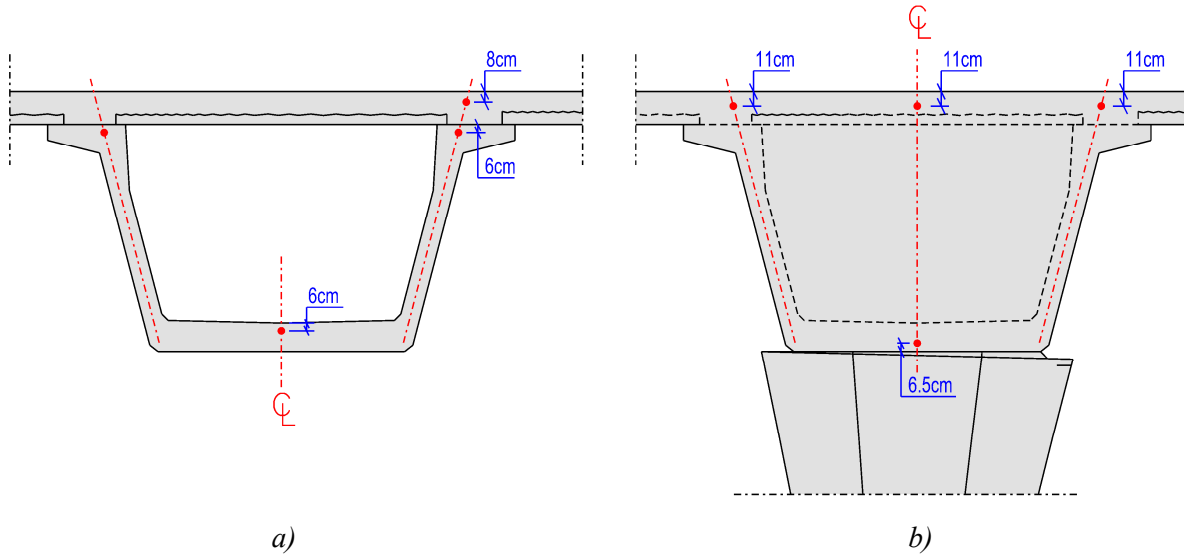


Figure 5.5 – Position of vibrating-wire strain gages: a) at mid-span; b) above the pier axis.

Shrinkage was measured in four $15 \times 15 \times 55 \text{ cm}^3$ prisms, which have two $15 \times 55 \text{ cm}^2$ faces exposed to drying. Two prisms were made of girder concrete and the other two were cast with slab concrete. The prisms were subjected to the same curing procedure that was applied to the structural elements cast with the same concrete. That procedure consisted of steam curing in the case of the precast beams (before the release of pre-tensioning strands) and intermittent water spraying in the case of the CIP slab. After pouring the concrete of the second-span deck slab, half of the prisms were kept in interior environment (inside the box girder) whereas the others were kept in exterior environment sheltered from rain.

In addition to concrete strain and temperature, other parameters were observed. Temperature and relative humidity of the environment were measured, both inside and outside of the box girder. Linear variable differential transformers (LVDT) were employed to measure the relative horizontal displacement at the expansion joints and durability sensors were installed in selected piers and precast girders. Further details about the monitoring and data acquisition systems can be found elsewhere (Sousa et al. 2011b).

5.3 FE analysis strategy

Obtaining a correct prediction of the structural behaviour requires appropriate analysis methods and an accurate modelling of the material properties.

Three-dimensional FE models with brick elements allow a detailed analysis of complex structures, but, on the other hand, their use for modelling large and complex structures may become impractical. In this way, two distinct approaches are employed in this work.

The long-term structural response considering the construction sequencing and the time variation of the material properties was analysed with a 2D finite-element model, by using the general-purpose finite-element code DIANA (Witte 2005). Structural discretization was carried out using beam elements, in accordance with the Timoshenko-Mindlin theory. Shear deformation is taken into account, which implies that the plane initially normal to the beam axis remains plane after the deformation, but not necessary normal to the axis. Normal stress varies linearly over the cross-section depth whereas the shear strain is forced to be constant. The elements are numerically integrated along the beam axis and within the cross section. Longitudinally, each beam element has three nodes and is based on an isoparametric formulation. In this analysis, the beam elements are approximately 1 m long. Only one alignment of girders is modelled, as the loading and slab width over each alignment are approximately equal (see Figure 5.2). In this model, both geometry and loading are symmetric across the vertical axis of the girder. The adopted mesh is schematically represented in Figure 5.6, which shows a detail of the connection between the girder and the pier. In this figure, one can see that the longitudinal axis of the finite element that represents the girder is located at the level of the girder's bottom fibre. The offset between that axis and the cross section's geometrical centre is automatically considered through the formulation of the adopted finite element (it should be noted that the cross section's geometrical centre changes when the slab is added to the model). Rigid elements were employed to connect the top of the pier to the precast girders' extremities, so that the actual distance between the girders' temporary supports is accurately simulated. The temporary supports are simulated by imposing that the vertical displacement of the node belonging to the rigid element is equal to the vertical displacement of the girder's extremity, for every time or load increment. Compatibility of horizontal displacements has also to be established in one of the temporary supports. The temporary connections are removed at the end of the construction. The soil-pile interaction is simulated by means of elastic supports in the horizontal direction (see Figure 5.6), whose stiffness was quantified according to geotechnical studies carried out at the design stage of the structure (COBA et al. 2005).

Another FE model, a three-dimensional one, with eight-node solid brick elements (with eight Gauss points), was used to analyze the local behaviour of the connection between the deck and

the piers. This is because the sensors employed to evaluate the strains in the deck, in the region above the continuity supports, were positioned in a zone of strong geometric discontinuity. As previously stated, they were placed in the section coinciding with the pile axis. Therefore, a 3D FE model was used to analyze the behaviour of this continuity connection (Figures 5.7 and 5.8). With this model, it was possible to compute the strains in the position of the sensors due to a given load case, so that the numerical calculations could be compared with the observed results. Moreover, the results of the 3D model were employed to derive an “effective cross section” for the continuity diaphragm to be used in the FE analysis with beam elements. The effective cross section is herein defined as the section with such a geometric shape that, when the Bernoulli’s hypothesis of plane sections after deformation is employed:

- the strain in the position of the bottom sensor is equal to the one calculated in the 3D model, for the same load case;
- the strain in the position of the top sensors is equal to the average of the values calculated with the 3D model in those locations, for the same load case.

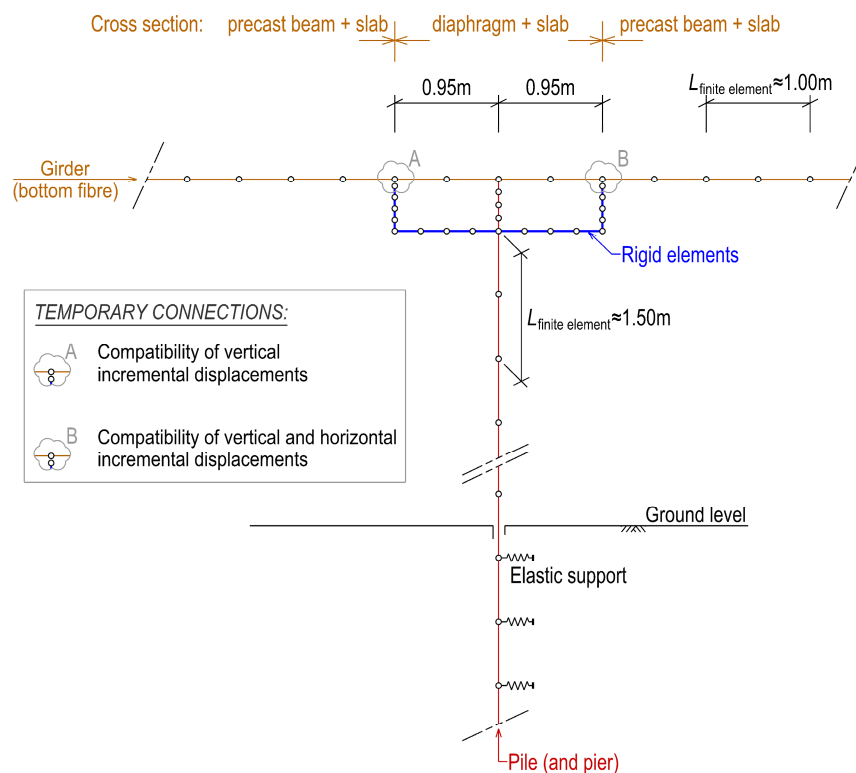


Figure 5.6 – Schematic representation of the 2D FE model with focus on the connection between the deck and the pier.

A comprehensive description of the 3D finite-element analysis and the determination of the effective cross section can be found in Sousa et al. (2009). The FE software DIANA (Witte 2005) was employed in this analysis.

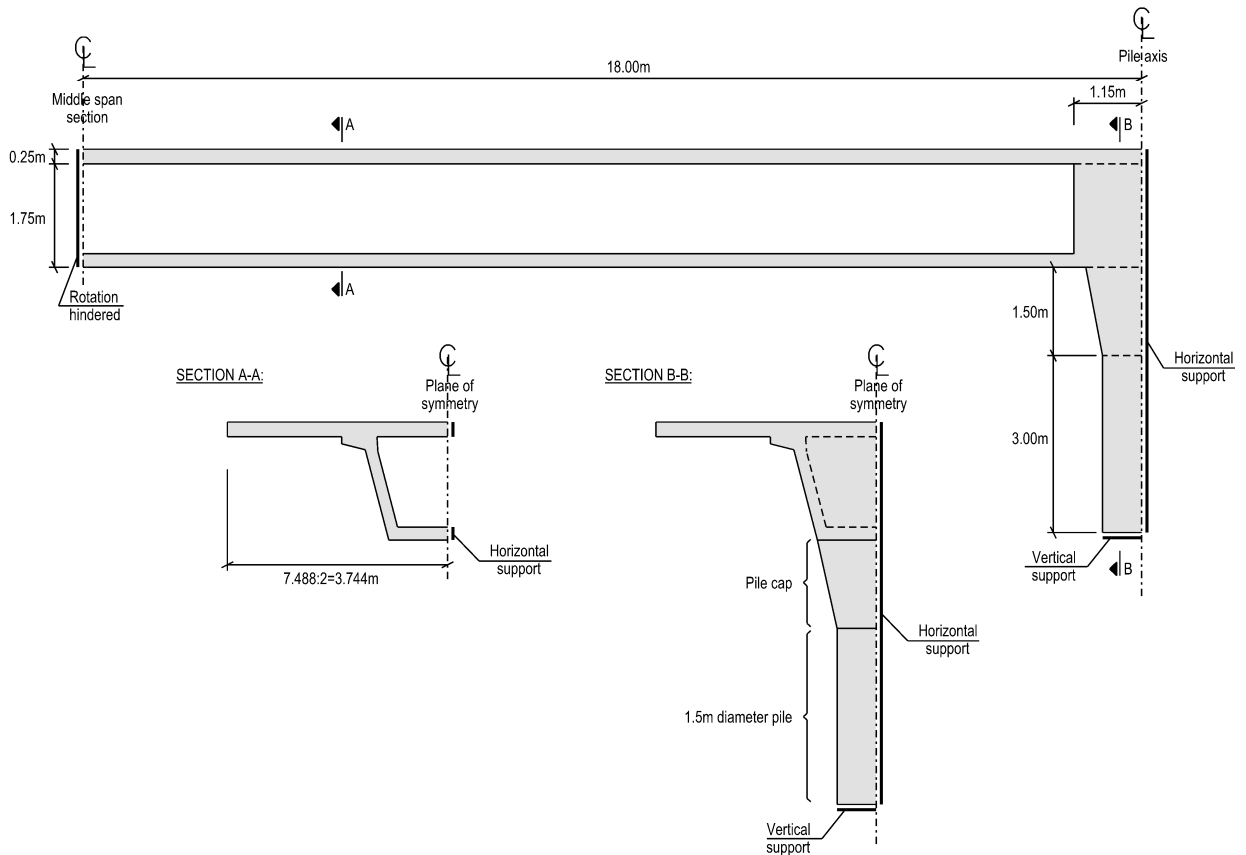


Figure 5.7 – Geometry of the 3D FE model.



Figure 5.8 – Mesh of the 3D FE model.

5.3.1 Phased analysis

A phased analysis was performed, considering the 2D model, so that the influence of staged construction could be taken into account. The FE model is changed at every new stage, by adding new elements or modifying the connection to the supports, or by changing the beam cross section (which enables the activation of the precast girder and the slab at different times).

The sequencing adopted in the analysis follows the chronology observed during the construction, which was recorded for future reference. Events occurring in close dates were considered to occur simultaneously, in the cases in which that simplification does not affect significantly the long-term structure response. For example, all the piles were considered to be cast at the same time, and all the piers (above the ground) were considered to be cast in two separate dates only. This simplification has minor implications, because the piers are more than 2 months old when the first continuity connection is cast.

Table 5.1 describes the sequence of construction considered in the FE analysis. Post-tensioning operations are not indicated in the table. They are carried out, in general, 5 days after the corresponding continuity connection is cast.

As regards the concrete age at the time the elements are activated for the first time, two cases must be distinguished. Precast girders are activated at the time of prestressing release, which is assumed to occur 1.7 days after they are cast, this value being justified below (§5.3.4.1). The other concrete elements are activated 1 day after casting. They are not activated sooner because the present work does not focus on the early age behaviour. Nevertheless, their self-weight is applied in the numerical model at the time the elements are cast.

5.3.2 Loading

The structure self weight was determined taking into account the average reinforcement ratio in each element and considering a specific weight of 24 kN/m^3 for the concrete and 77 kN/m^3 for the steel.

The dead weight of non-structural elements was taken as the average of the load acting on the flange of the exterior girder (22.875 kN/m, due to sidewalk, handrail, safety barrier, kerb and bituminous surfacing) and the load applied to the flange of the interior girder (18.440 kN/m, due to bituminous surfacing and Jersey barrier). This simplification is considered admissible given the small difference between the load values directly applied to each girder flange.

5.3.3 Reinforcement modelling

The reinforcement, both ordinary and prestressed, was modelled using embedded reinforcement elements, whose deformation is calculated from the displacement field of the concrete finite-element in which they are embedded. Both instantaneous and time-dependent prestress losses are automatically computed. Steel relaxation is computed by employing a user supplied subroutine which was developed to that effect (Sousa and Serra Neves 2009a), because the DIANA FE package does not include prestress relaxation models. The modulus of elasticity of reinforcing

steel and prestressing steel took the values 200 GPa and 195 GPa, respectively. According to the results of the quality control procedures (TACE 2007), the tendon stress before release of the pre-tensioning strands took the value 1372 MPa.

Table 5.1 - Construction sequence considered in the FE analysis.

Event	Time (days) since the start of the construction
Prestress release for beams of span 6	0
Casting of piles	60
Prestress release for beams of spans 7 and 13	93
Casting of piers (in general)	138
Prestress release for beams of spans 3, 4, 5, 9, 10 and 11	148
Casting of pier P1	155
Prestress release for beams of spans 2, 8 and 12	169
Erection of precast beams (spans 2 to 13)	174
Prestress release for beams of span 1	178
Erection of precast beams (span 1)	211
Casting of the continuity connection above P2 and P3	232
Casting of the continuity connection above P4 and P5	234
Casting of the continuity connection above P6 and P7	238
Casting of the continuity connection above P8 and P9	239
Casting of the continuity connection above P10 and remaining slab in span 4	244
Casting of the continuity connection above P11 and remaining slab in span 5	245
Casting of the continuity connection above P12 and remaining slab in span 6	246
Casting of the continuity connection above P1 and remaining slab in span 7	247
Casting of the slab (mid-span region) in spans 8 and 9	248
Casting of the slab (mid-span region) in spans 3 and 10	251
Casting of the slab (mid-span region) in spans 2, 11 and 12	253
Casting of the slab (mid-span region) in span 13	263
Casting of the slab (mid-span region) in span 1	300
Removal of temporary supports	329
Pavement, side walks and Jersey barrier	441

Notes: Span 1 extends from TP to P1 (see Figure 5.1), span 2 extends from P1 to P2, and so on.

Casting of the continuity connection consists of pouring the concrete in the diaphragm and in the slab strip which extends approximately 6 m for each side of the pier axis.

5.3.4 Concrete modelling

Obtaining a correct prediction of the structural behaviour requires an accurate modelling of concrete deformations. In this work, a smeared crack model with strain decomposition was adopted, which allows the consideration of concrete creep, shrinkage and cracking. In each point, the total strain is given by the sum of a concrete strain component and a crack strain.

The so-called concrete strain includes stress independent strains (shrinkage and thermal deformations) and stress dependent strains. The viscoelastic behaviour of concrete owing to creep and ageing effects is simulated through the Kelvin Chain model. Within the scope of this work, a user supplied subroutine was developed, so that the time variation of concrete creep and modulus of elasticity can be modelled according to the EN 1992-1-1 proposals. The user supplied subroutine also includes correction factors, so that the constitutive model can be adjusted to experimental data (Sousa and Serra Neves 2009b). In the compressive domain, a linear viscoelastic behaviour is assumed (suitable for service analyses).

Cracking is reproduced through a smeared fixed-crack approach. In the area surrounding the reinforcing bars, tension stiffening is taken into account by modifying the concrete stress-strain relationship according to the proposal of Figueiras (1983). Outside that area, crack formation and growth is simulated through a linear tension softening model. After cracking, a constant shear retention factor is adopted, with a value of 0.1. A comprehensive description of this constitutive model can be found elsewhere (Borst 1991; Witte and Kikstra 2005).

For each loading or time increment, the solution is obtained iteratively with the Newton-Rapson Method. A converged solution was obtained when the out of balance energy norm ratio reaches a value lower than 0.0001.

The evaluation of the concrete properties was based on:

- the results of compression tests of 150 mm cubes, which were carried out during the construction, at different ages;
- the measurements taken in 4 shrinkage prisms;
- the results of the retro-analysis of a simply-supported precast girder, whose deformation was measured since the start of concrete pouring.

The expressions proposed by the EN 1992-1-1 were adopted to describe the time variation of the concrete properties (modulus of elasticity, creep and shrinkage). Whenever possible, the quantification of the parameters involved in those expressions was based on the aforementioned experimental data.

5.3.4.1 Evolution of concrete compressive strength

The time variation of the concrete compressive strength provides valuable information since it correlates to the evolution of the concrete modulus of elasticity. According to the EN 1992-1-1, the compressive strength at a given age, $f_{cm}(t)$, can be expressed as:

$$f_{cm}(t) = \exp\left\{s\left(1 - \sqrt{28/t}\right)\right\} \cdot f_{cm} \quad (5.1)$$

where t represents the concrete age in days, s is a coefficient that characterizes the evolution of the concrete strength and f_{cm} is the mean value of the concrete compressive strength, at the age of 28 days. The parameters f_{cm} and s were determined by a curve fitting procedure, which consisted of minimizing the mean square error between the cube test results (at different ages) and the value given by Equation (5.1). Figure 5.9 represents the results obtained for the slab concrete batches in the region where monitoring was carried out. In this figure, one can see that the s parameter obtained for the slab concrete is approximately equal to the value suggested by the EN 1992-1-1 for concretes made with rapidly hardening cements ($s = 0.20$). This is consistent with the fact that this concrete was made with cement CEM 42.5 R (a small amount of cement CEM 32.5 R was also employed).

In the case of the precast girders, steam curing was employed in order to increase the strength gain in the first hours after casting (the compression test cubes were subjected to similar curing conditions). In this way, the prestressing release could be carried out approximately 12 hours after casting. The influence of the curing temperature on the maturity of concrete may be taken into account by modifying the age at loading according to the equation (EN 1992-1-1, MC90):

$$t_T = \sum_{i=1}^n \Delta t_i \cdot e^{\left(13.65 - \frac{4000}{273 + T(\Delta t_i)}\right)} \quad (5.2)$$

where t_T is the temperature adjusted concrete age which replaces t in the corresponding equations, Δt_i is the number of days where the temperature $T(\Delta t_i)$ prevails in °C. The recorded temperatures in the girder which was continuously monitored since the concrete pouring were employed in the calculation of the temperature adjusted concrete age at the time of prestressing release. The results indicate that, 12 hours after casting, the equivalent age of the girder concrete is approximately equal to 1.7 days. Consequently, this adjusted age was taken as the concrete age at the time of prestressing release, for the purposes of calculation of the time variation of concrete strength, elasticity modulus and creep deformation. Figure 5.10 presents the results of the curve-fitting procedure applied to the girders where monitoring was carried out.

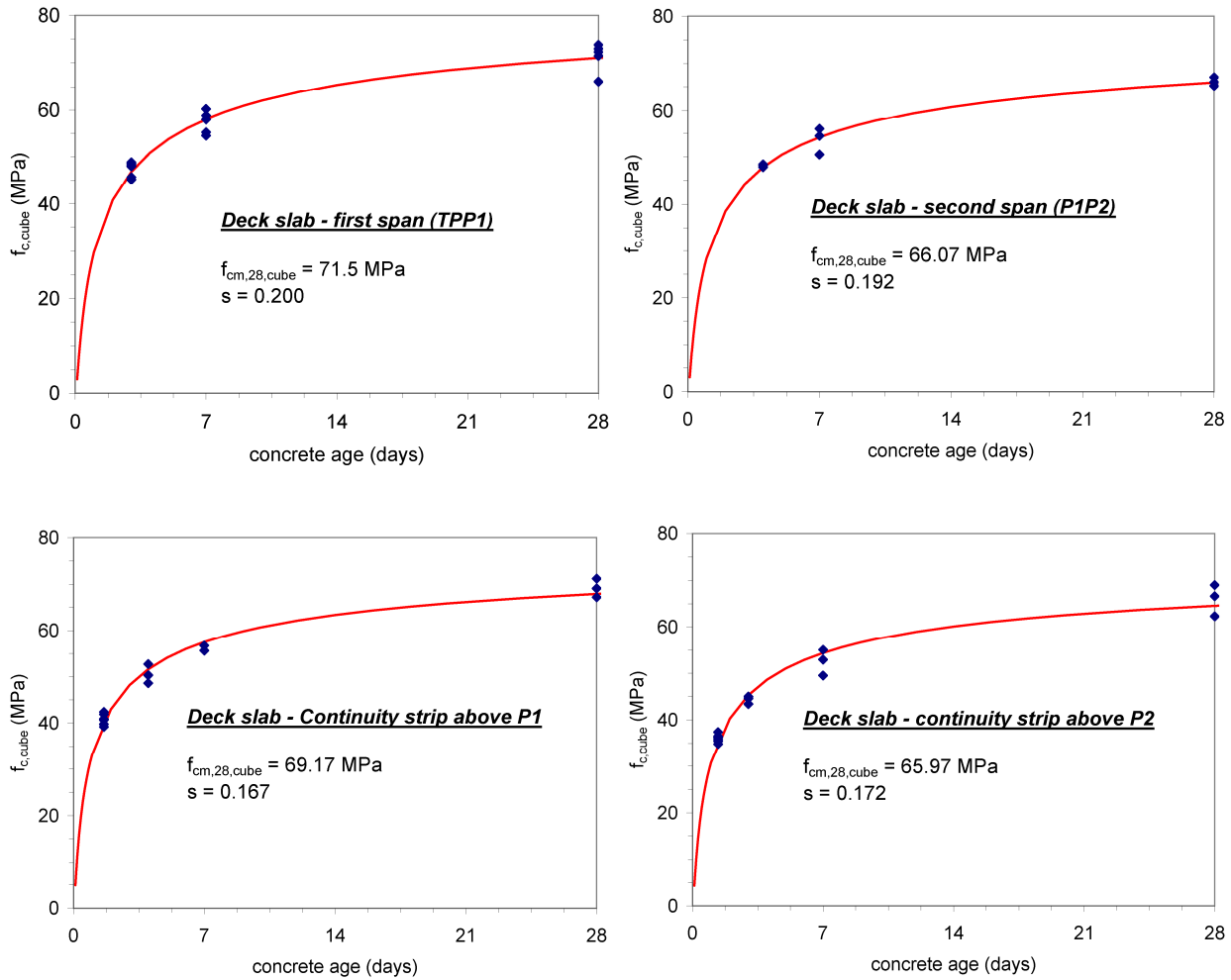


Figure 5.9 - Evolution of the slab concrete compressive strength: experimental results and curve fitting.

A similar procedure was applied to the concrete employed in the construction of piles and piers.

Because the cylinder strength was not experimentally evaluated, it was taken as 82% of the observed cube strength. This relation corresponds to the average value of the ratio between the cylinder strength and the cube strength for the same class of concrete, in the EN 1992-1-1. It is adopted in this work as there is no simple relation between the strengths obtained with cylinders and cube specimens (Neville 1995). The values considered in the analysis can be found in Table 5.2.

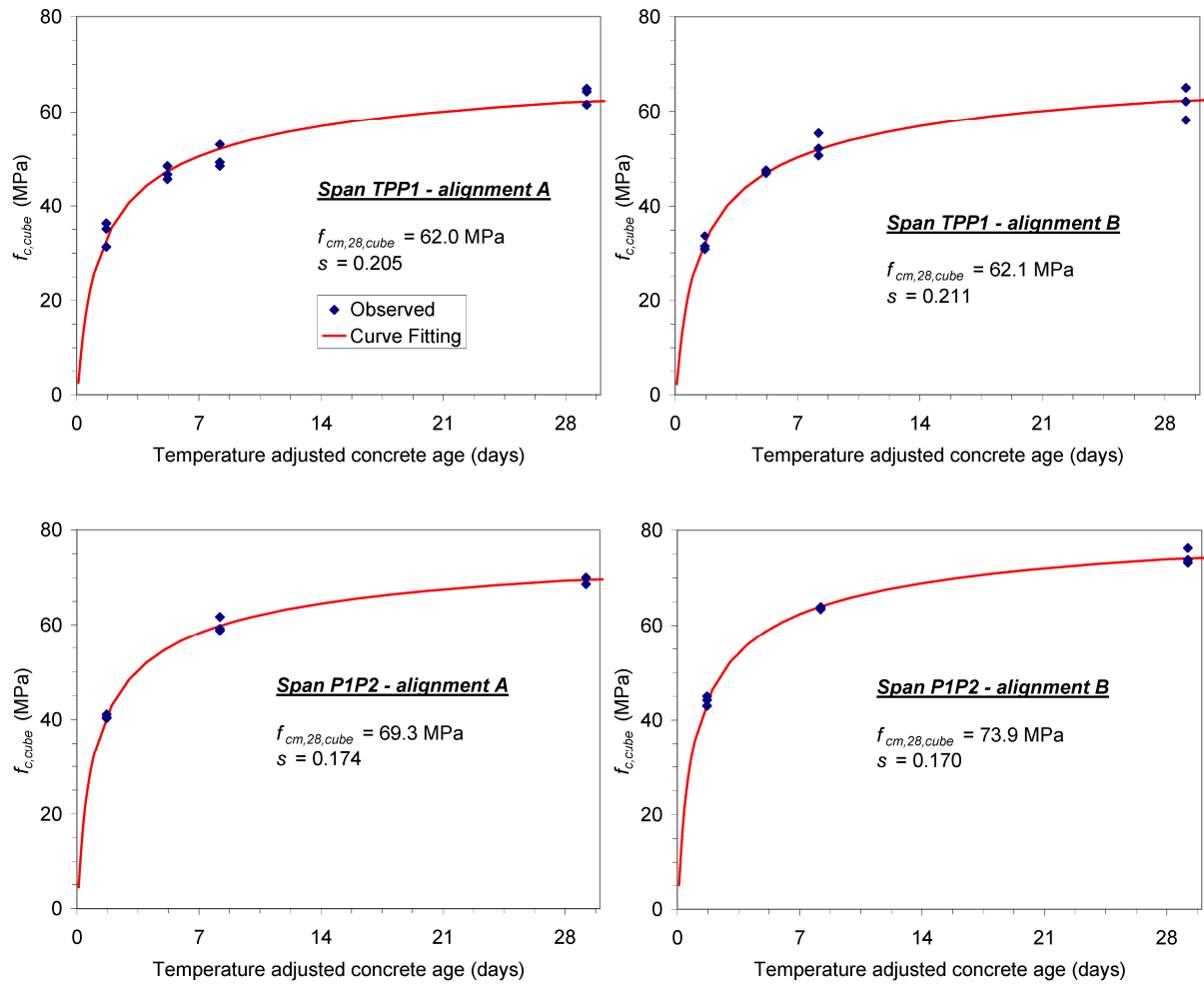


Figure 5.10 - Evolution of the girder concrete compressive strength: experimental results and curve fitting.

Table 5.2 - Concrete properties based on cube compression tests.

	$f_{cm,cube}$ (MPa)	$f_{cm,cyl}$ (MPa)	E_c (MPa)	s
Piles	60.47	49.59	37344	0.350
Piers (above the ground)	46.65	38.25	34547	0.250
Precast beams ^(a)	66.82	54.79	38479	0.190
CIP slab (above the piers) ^(a)	67.57	55.41	38609	0.170
CIP slab (mid span) ^(a)	68.78	56.40	38815	0.196

Note: ^(a) Average of the values obtained for the first and the second spans (the region where monitoring was carried out).

5.3.4.2 Modulus of elasticity

The determination of the tangent modulus of elasticity, E_c , was based on the observed concrete compressive strength, by means of the EN 1992-1-1 equation:

$$E_c = 1.05 \cdot 22000 \cdot (f_{cm,cyl}/10)^{0.3} \quad (E_c \text{ and } f_{cm,cyl} \text{ in MPa}) \quad (5.3)$$

where $f_{cm,cyl}$ represents the mean value of the concrete cylinder strength at the age of 28 days (see Table 5.2). The variation of the modulus of elasticity with time correlates to the time variation of the compressive strength (determined above), and is given by the following equation according to the EN 1992-1-1:

$$E_c(t) = \left\{ \exp \left[s \left(1 - \sqrt{28/t} \right) \right] \right\}^{0.3} \cdot E_c \quad (5.4)$$

where E_c and s take the values indicated in Table 5.2.

5.3.4.3 Shrinkage

The time variation of the shrinkage strain, $\varepsilon_{cs}(t)$, was obtained by fitting the results of the four shrinkage prisms to the following function (Figures 5.11 and 5.12):

$$\varepsilon_{cs}(t) = \frac{t}{t + S_1} \cdot S_2 + S_3 \cdot \sin \frac{t \cdot 2\pi}{365} + S_4 \cdot \cos \frac{t \cdot 2\pi}{365} \quad (5.5)$$

where t is the time (in days) after drying begins, S_1 and S_2 are fitting parameters that characterize the development of shrinkage, and S_3 and S_4 are parameters that describe the yearly (seasonal) shrinkage variation. Given that both the girder and the slab are exposed to the interior and the exterior environments, the shrinkage strain was taken as a weighted average of the values observed in the interior and the exterior prisms. Given that it is not possible to accurately quantify the influence of each environment (Santos 2007), the following assumptions were made: in the case of the girder, a 50% contribution from each environment was assumed; in the case of the slab, which has a larger surface exposed to the exterior environment, the contribution from the exterior environment was taken as 75%. The consequences of adopting different contribution percentages are evaluated below.

It is important to note that, for each type of concrete, only one prism was kept in exterior atmosphere and another in interior environment. Therefore, the variability of the phenomenon could not be quantified.

The evolution of the shrinkage strain also depends on the cross-sectional shape. Some shrinkage models express this dependency through the notional size, h_0 , which is defined as twice the ratio between the cross-sectional area and the perimeter exposed to drying. The notional size of the diaphragm is considerably different from the notional size of the shrinkage prisms. Therefore, the influence of the cross-sectional shape was taken into account in the estimation of the diaphragm shrinkage. The adopted procedure is indicated in §5.3.4.3.1. On the contrary, in the cases of the precast girders and the slab, their notional size (quantified in Table 5.3) is not very dissimilar from the size of the shrinkage prisms. No correction to account for different notional sizes was therefore applied in the estimation of the slab shrinkage and of the girder shrinkage. The option to not consider the influence of the cross-sectional shape is also justified by the relevance of autogenous shrinkage, particularly in the case of the precast girder. According to the EN 1992-1-1 approach, the cross-sectional shape affects the development of drying shrinkage, but not the time variation of the autogenous shrinkage. And, in the case of the precast girder, very important shrinkage strains were observed in the first few days after casting (Figure 5.12), which puts into evidence the relevance of autogenous shrinkage.

It is also important to bear in mind that vibrating-wire strain gages do not accurately capture the strain variation in the first few hours after casting, due to the stiffness of the metallic casing (Azenha et al. 2009). Moreover, the fact that the thermal dilation coefficient of concrete decreases in the first hours after casting (Viviani et al. 2007) contributes to inaccurate compensation of temperature effects when this compensation is made considering the thermal dilation coefficient of mature concrete. Therefore, the measured strain (whose results are exposed in this work) is not the real strain (actually experienced by the concrete) after compensation of thermal deformations. Instead, it is the strain captured by the vibrating-wire strain gages, after compensation of the free thermal deformation by means of a constant thermal dilation coefficient of $8 \cdot 10^{-6}$ (which is valid for mature concrete and was experimentally determined, as mentioned above). However, the differences between the captured strain and the real strain are not problematic in the present work, which is not devoted to the analysis of the early age concrete behaviour, for the following reasons:

- These differences are only significant in the first hours after casting, as noted by Azenha et al. (2009).
- In the case of the precast girders, a retro-analysis procedure (summarized below) was employed in the identification of the creep and shrinkage deformations experienced by the concrete in the first 40 days after casting. It should be remarked that, in the cases of

the shrinkage prisms, the precast girders and the CIP slab, the strain measurements were set to zero at the instant of time immediately before the concrete was poured.

- In the case of the continuity diaphragms, important positive strain variations ($\sim 100 \mu\epsilon$) were recorded in the first few hours after casting. Given that the effects which explain such strain variations are not considered in the adopted numerical models, the strains measured in the diaphragm were set to zero at the instant of time before applying the continuity post-tensioning (5 days after casting). In this way, the subsequent measured strain variations can be compared with the calculated values, because the calculated stress is approximately zero at the time that the measured strains were zeroed out.

The observed strains in one of the precast beams, during the first period of time in which it behaves as a simply supported structure, were used to obtain additional information regarding the delayed concrete deformation. In fact, if it is assumed that the structure is subjected to known forces during this stage, then a retro-analysis can be performed in order to calculate the creep coefficient $\phi(t, t_0)$ and the shrinkage $\epsilon_{cs}(t)$ (two unknowns) that correspond to the observed strains on the top and the bottom flanges of the mid-span cross section (two conditions). In this calculation, the age-adjusted elasticity modulus method (Ghali et al. 2002) was applied to describe the concrete stress-strain relationship. This problem has a single solution if a given value is assumed for the ageing coefficient (it took a plausible value, 0.5, as it doesn't significantly influence the results) and the concrete modulus of elasticity [given by Equation (5.4) and Table 5.2]. Figure 5.13 depicts the time variation of the observed strains and the outcome of the retro-analysis. Shrinkage calculated through the retro-analysis is also plotted in Figure 5.12, where it can be observed that the results of this analysis are not significantly different from those measured in the shrinkage prisms.

In the FE analysis, the result of the retro-analysis was used to describe the shrinkage development in the first 40 days after casting (before continuity is established). Afterwards, the shrinkage variation is based on the results of the shrinkage prisms. A comprehensive description of the retro-analysis and the curve-fitting procedure can be found elsewhere (Sousa et al. 2009).

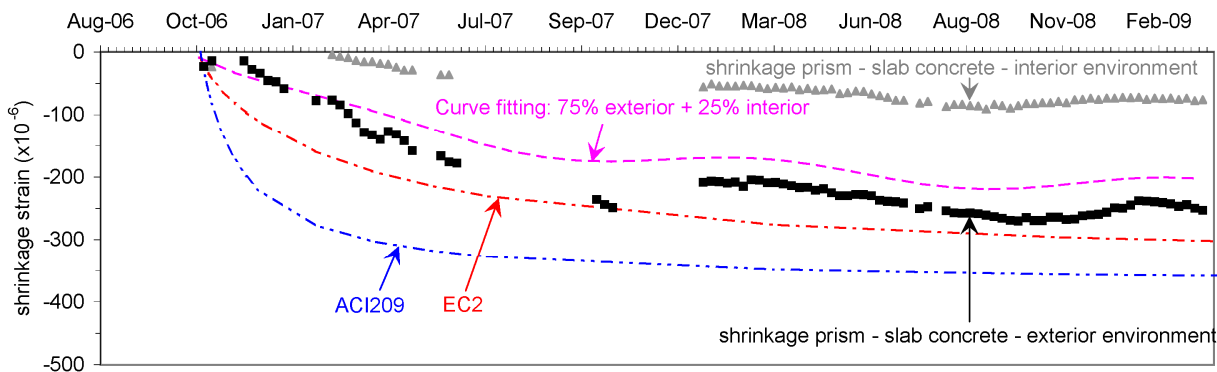


Figure 5.11 - Shrinkage strain of the slab concrete.

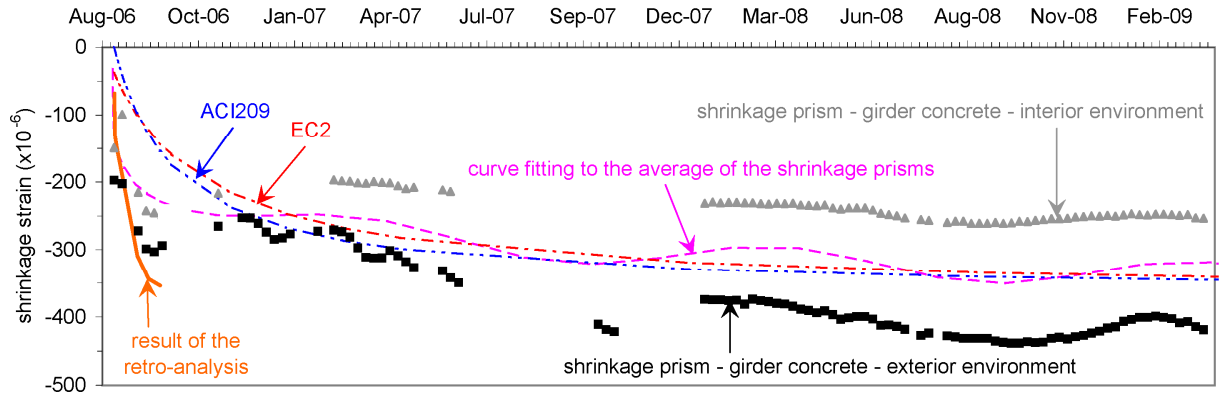


Figure 5.12 - Shrinkage strain of the girder concrete.

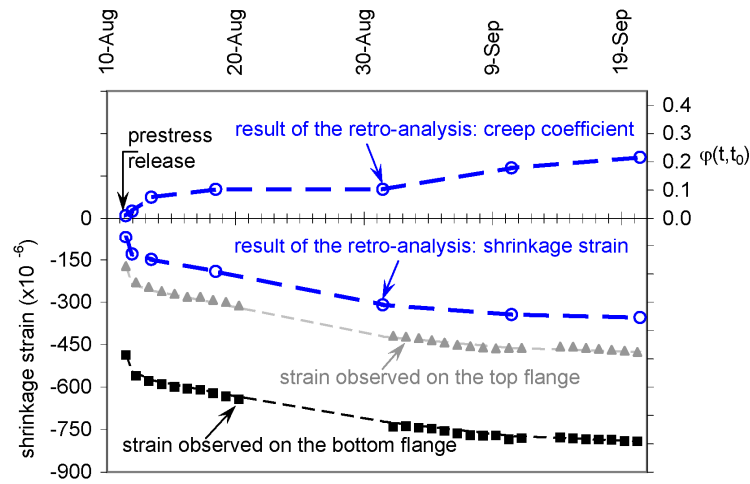


Figure 5.13 - Observed strains and results of the retro-analysis.

5.3.4.3.1 Shrinkage of the diaphragm concrete

The notional size of the diaphragm cross section (which took the value 791 mm, based on the geometrical characteristics of the diaphragm) is significantly higher than that of the shrinkage prisms (150 mm). Consequently, shrinkage will develop slowly in the diaphragm. According to the EN 1992-1-1, the shrinkage of a cross section whose notional size equals 791 mm can be obtained from that of an 150 mm-notional-size cross section by multiplying the parameters S_1 and S_2 [in Equation (5.5)] by 12.1 and 0.757, respectively. This was the strategy employed in this work to derive the shrinkage curve of the diaphragm cross section from that of the shrinkage prisms.

5.3.4.4 Creep

Direct measurements of concrete creep are not available. For that reason, the results of the retro-analysis were also used to derive the time variation of the creep coefficient for the girder concrete. Thus, the time variation of the creep coefficient, $\varphi(t, t_0)$, is given by (Santos 2001):

$$\varphi(t, t_0) = C_1 \cdot \varphi_0 \cdot [\beta_c(t, t_0)]^{C_2} \quad (5.6)$$

where φ_0 is the notional creep coefficient and $\beta_c(t, t_0)$ is a coefficient to describe the development of creep with time after loading. C_1 and C_2 were introduced in Equation (5.6) so that the creep model could be adjusted to the results of the retro-analysis depicted in Figure 5.13. The parameters C_1 and C_2 were calculated by minimizing the squared error between the results of the retro-analysis and the values provided by Equation (5.6). The obtained results are shown in Table 5.3.

Given that creep of the slab concrete and of the diaphragm concrete was not measured, the creep deformations of these elements were quantified according to the EN 1992-1-1 provisions. The values adopted for the various parameters used in the calculation of the creep coefficient [which is given by Equation (5.6)] are identified in Table 5.3. The value adopted for the relative air humidity (80%) corresponds to the average of (Sousa et al. 2012): (i) the average observed values in the exterior environment (~70%); (ii) the average observed values inside the box girder (~90%).

Table 5.3 - Data for calculation of the creep coefficient according to Equation (5.6).

	Girders	CIP slab	Diaphragms
C_1	0.368	1	1
C_2	1.370	1	1
f_{cm} (MPa)	54.79	55.41 or 56.40 ⁽¹⁾	55.41
h_0 (mm)	182 or 202 ⁽²⁾	250	791
Relative humidity (%)	80	80	80
Cement type	Rapid-hardening		

Notes: ⁽¹⁾ 55.41 MPa along the length in which continuity post-tensioning is applied and 56.40 MPa in the mid-span region (see Table 5.2).

⁽²⁾ 182 mm in the mid-span region (where the web thickness is equal to 150 mm) and 202 mm in the region close to the supports (where the web thickness is equal to 200 mm).

5.4 Deterministic analysis results

5.4.1 Comparison between numerical and experimental results

Figures 5.14 and 5.15 present measured and calculated strains for a mid-span cross section and for one of the cross sections above the continuity supports. The experimental results were taken at 6 a.m. for the reason that small temperature differences occur at this time.

As regards the results for the mid-span cross section, Figure 5.14 reveals a very good agreement between experimental and numerical results in the first 40 days after casting, as a consequence of the determination of concrete creep and shrinkage through a retro-analysis procedure, during this period of time. Nevertheless, a good agreement is also observed after this period, both in the subsequent construction phases and after the closure of the construction. Construction events such as the erection of precast planks and the casting of the slab cause important strain variations, which are clearly reproduced both in the experimental and in the numerical results. Important time variation of strains is also observed: in the case of the girder, the most relevant time-dependent strains occur in the first weeks after casting; in the case of the slab, its strain evolution is mainly governed by its shrinkage.

Still regarding to the slab shrinkage, it was mentioned before that such deformation was estimated by assuming a 75% and a 25% contribution of the exterior and the interior shrinkage prisms, respectively. Figure 5.16 shows that if a more uniform contribution from each environment was assumed (e.g. 50% for each environment) a slightly poorer agreement with the experimental results occurs. However, the differences between these two calculation scenarios are small.

In the cross sections above the supports, significantly lower strain values were observed, as can be seen in Figure 5.15. For a better understanding of the consequences of shrinkage, the observed results are compared with two calculation scenarios: one considering the diaphragm shrinkage as described before (“base calculation”); other considering that the shrinkage-strain development in the diaphragm is equal to the one adopted for the deck slab. By observing Figure 5.15 it can be concluded that the aforementioned scenarios represent an envelope of the long-term development of the strains in this cross section. The diaphragm shrinkage is, in fact, less than the one in the slab. However, the procedure adopted to express the dependency between the shrinkage strain and the notional size of the diaphragm cross section leads to an underestimation of the diaphragm shrinkage. Other researchers (Santos 2007) have also concluded that the EN 1992-1-1 fails to predict the relation between the shrinkage strain development and the notional size of the cross section. Nevertheless, it was concluded that even though the two scenarios lead to significantly different strain evolutions, those scenarios generate almost

coincident results in terms of stresses (either at mid-span or in the continuity diaphragm) and restraint bending moments above the supports (Sousa et al. 2009). In this way, the uncertainty involved in the prediction of the diaphragm shrinkage does not affect the most relevant results in terms of practical design: the stresses and the internal forces.

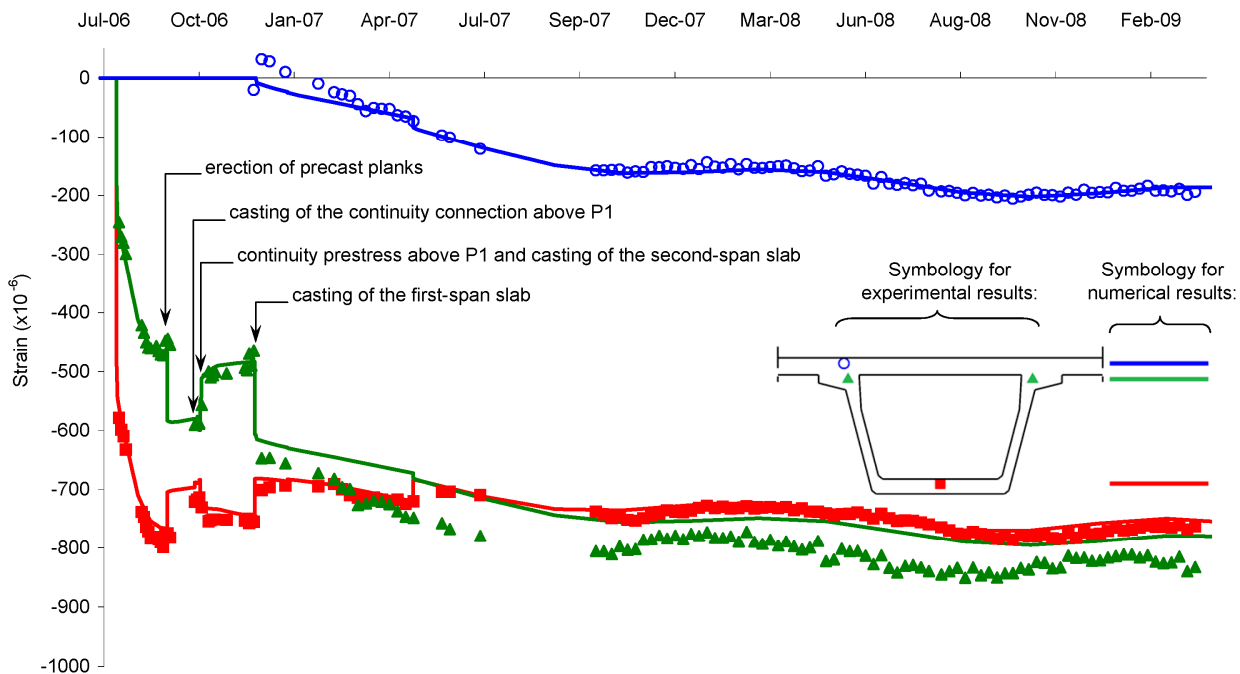


Figure 5.14 - Concrete deformation in the mid-span cross section (first span; alignment B). Values observed at 6a.m. When more than one measurement was taken at the same cross-section depth, the figure shows the average measurement.

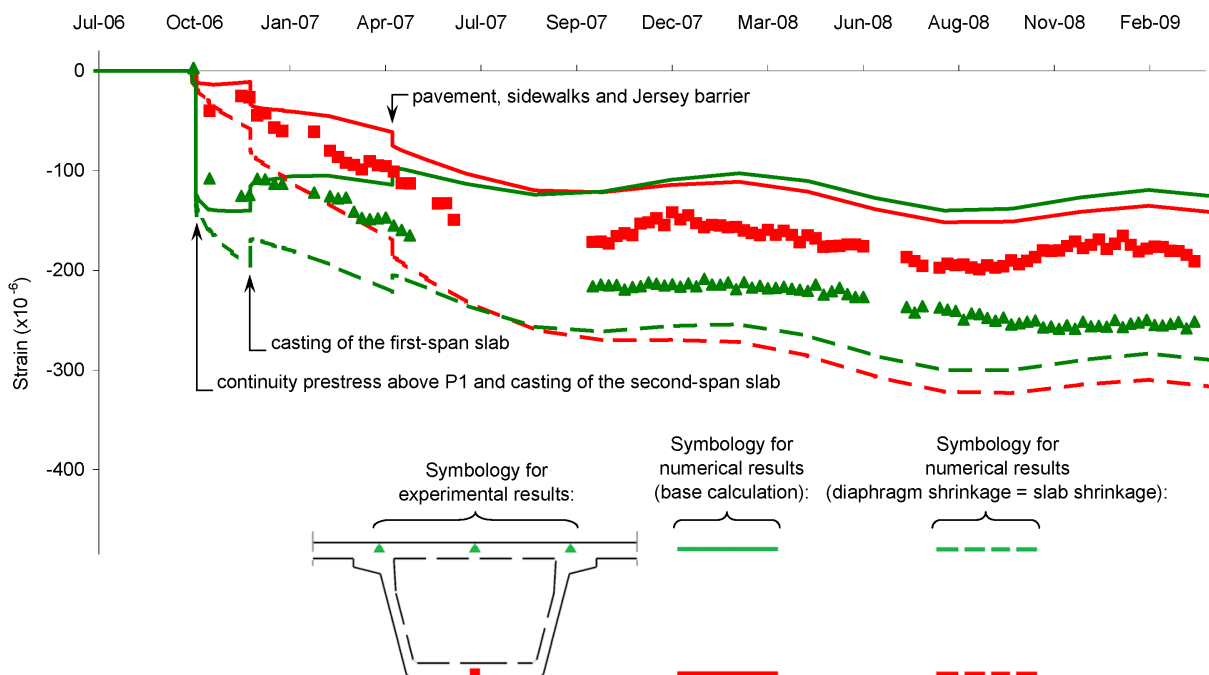


Figure 5.15 - Concrete deformation in the cross section above P1 (alignment A). Values taken at 6a.m. When more than one measurement was taken at the same cross-section depth, the figure shows the average measurement.

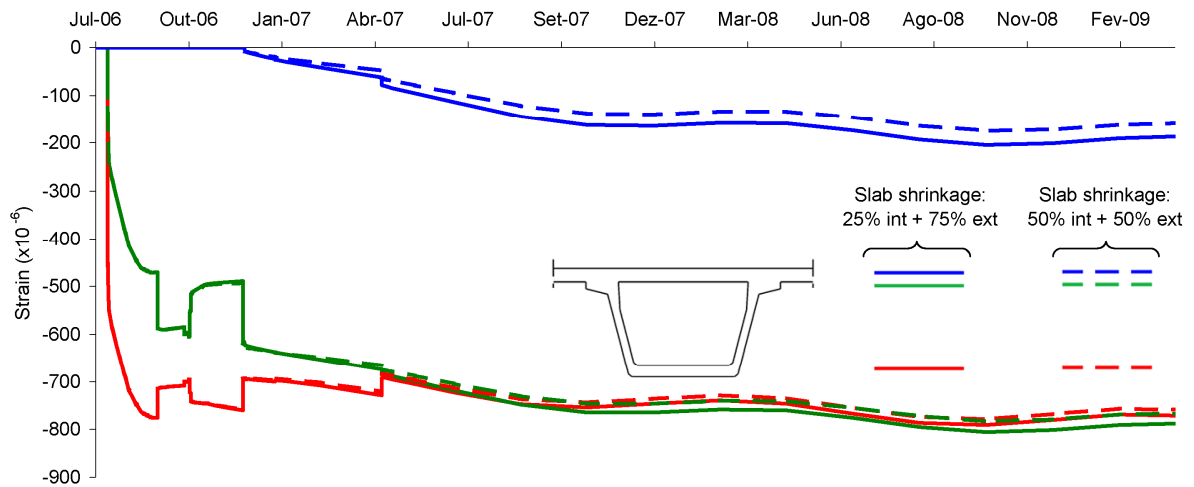


Figure 5.16 - Calculated strain ($\cdot 10^{-6}$) in the cross section TPP1 considering different values for the slab shrinkage.

As regards the variability of the observed strains, similar strain values were measured in the alignments A and B (identified in Figure 5.2), which validates the option of modelling only one girder alignment. The closest prediction of strains in the mid-span cross sections was obtained in the first span, alignment B, because the retro-analysis was based on strains observed in this girder. Nevertheless, the variability of the observed strains in the monitored cross sections was not significant. The conclusions presented before are also applicable to the remaining cross sections.

It is worth mentioning that cracking was only observed in the piers which are closer to the expansion joints, as a consequence of the longitudinal displacement imposed to the top of the piers, caused by the long-term contraction of the deck. However, parametric analyses showed that cracking in the piers does not significantly affect the results in the monitored cross sections, due to the high lateral flexibility of these elements. Parametric analyses also showed that the results are not significantly influenced by the stiffness of elastic supports that simulate the horizontal support of the piles (due to the soil-pile interaction). It was found that similar concrete strains are obtained in the deck, for two different geotechnical scenarios, based on lower and upper bound values for the soil stiffness, which were recommended by the geotechnical studies (COBA et al. 2005).

5.4.2 Calculations based on the ACI 209 and the EN 1992-1-1 creep and shrinkage models

In the previous section, it was concluded that the employed analysis strategy provides a good agreement with the experimental results. However, the analysis was based on data not often

available at the design stage. At such time, only limited information is available concerning the material properties and the construction sequence. Moreover, the concrete delayed deformation is estimated according to the provisions of design codes. Therefore, the results obtained through the calibrated numerical analysis (the “base calculation” identified in §5.4.1) are now compared with the results of calculations based on the data that is usually available at the design stage, so that the errors due to simplified design assumptions can be ascertained. The following simplified calculations are considered:

- Two different analyses were performed, wherein the quantification of creep and shrinkage deformations is based on the information that is usually available at the design stage (the EN 1992-1-1 was followed in one of the calculations and the ACI Committee 209 (1992) was considered in the other). In these analyses, the sequence of construction is the real one, i.e., the same that was used in the “base calculation”. In this way, the differences are only due to the concrete material properties.
- Two additional analyses were made, in which a simplified construction sequence is adopted. In these analyses, it is considered that all the girders were cast at the same time, all the diaphragms and continuity strips of the slab were cast simultaneously at a later stage, and the mid-span portions of the slab were cast all together later on (Table 5.4). These calculations are identified with $t_{cont} = 15$ days and $t_{cont} = 90$ days, which means that the period of time between prestress release and the casting of the continuity diaphragms is 15 and 90 days, respectively. Both analyses were carried out considering the material properties as defined in the EN 1992-1-1.

Table 5.4 – Simplified sequence of construction considered in the FE analyses.

Event	Time (days) since the start of the construction
Casting of piles and piers	0
Prestressing release	90
Erection of precast planks; casting of continuity connections	$90 + t_{cont}$
Continuity post-tensioning	$90 + t_{cont} + 5$
Casting of the remaining slab	$90 + t_{cont} + 5 + 14$
Removal of temporary supports	$90 + t_{cont} + 5 + 14 + 14$
Pavement, side walks and Jersey barrier	$90 + t_{cont} + 5 + 14 + 14 + 1$

The parameters used for predictions according to these models are shown in Table 5.5. Shrinkage estimates are depicted in Figures 5.11 and 5.12. It should be remarked that correction factors for concrete composition were not considered in the ACI creep and shrinkage models

because concrete mix is usually unknown at the design stage and these correction factors are not relevant in normal cases, as noted by the ACI Committee 209 (1992).

Table 5.5 - Data for creep and shrinkage calculations based on the EN 1992-1-1 and the ACI 209.

	Girders	CIP slab	Diaphragms
f_{cm} (MPa)	48	38	38
h_0 (mm)	^(a)	250	791
Relative humidity (%)	80	80	80
Curing time (days)	^(b)	1	1
Cement type	Rapid-hardening		
Initial curing	steam	moist	moist

Notes: ^(a) h_0 takes the values 182 mm or 202 mm depending on the web thickness, which equals 150 mm (in the mid-span region) or 200 mm (close to the supports)

^(b) drying begins at the time of prestress release

Figure 5.11 shows that both the ACI209 and the EN 1992-1-1 (which is identified in the figure through the acronym EC2) overestimate the slab shrinkage, the largest differences being detected in the case of the ACI209. On the other hand, these codes provide a good prediction of the girder shrinkage after the initial four months (Figure 5.12), even though important differences are found in the initial months. These results are consistent with the commonly acknowledged fact that the coefficient of variation of shrinkage values for short-term results is much higher than for long-term ones (Tsubaki 1988; CEB 1993; Fernández-Gómez and Landsberger 2007). Furthermore, differences between observed values and code predictions were to be expected because the employed concretes contain superplasticizers and pozzolanic materials, whose effects can not be accurately reproduced by codes because of the great variety of additives and different combinations used (Bazant 2001).

Given the inaccuracies in girder shrinkage predictions, it is not surprising that calculations based on the EN 1992-1-1 and ACI209 lead to strain evolutions that are different from the observed values in the initial months after casting. However, from a practical point of view, the designer is usually more interested in obtaining accurate predictions of stresses and restraint bending moments rather than in predicting the strain variation. And the simplified calculations provide good estimates of the applied stresses and forces, as will be shown below. Note that it is herein assumed that the “base calculation” provides realistic results in terms of stresses and internal forces as it leads to a good agreement with the observed strains and realistic material properties were employed in that calculation.

Figure 5.17 presents the evolution of the restraint bending moment in the section above the first continuity support. This graph exclusively presents the bending moments due to the concrete delayed deformation, that is, it does not include the sudden variation of bending moment due to applied loading. It can be seen in the figure that the calculation based on the ACI209 predicts a faster development of the hogging bending moment in the first weeks after casting the deck slab, which is coherent with the fact that the ACI209 predicts higher slab shrinkage in this period (see Figure 5.11). This is because a negative restraint moment develops when the slab concrete tends to shrink more than the girder.

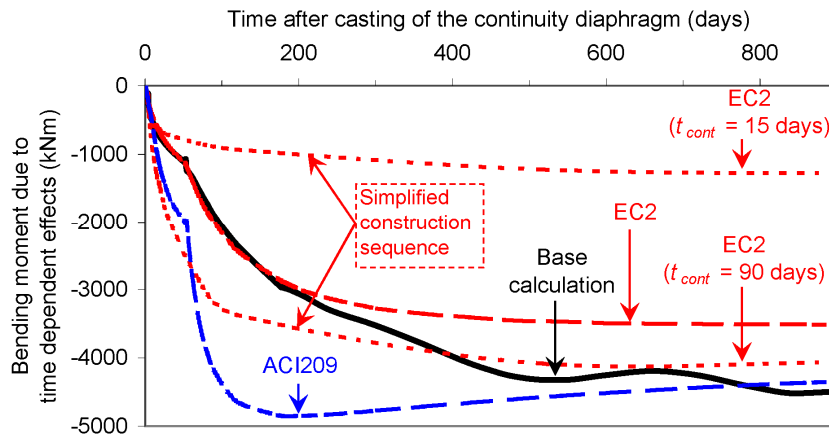


Figure 5.17 - Restraint bending moment at the cross section above P1.

Figure 5.17 also shows that adopting a simplified sequence of construction leads to approximate results if the period of time between pre-tensioning and slab casting takes realistic values. The actual time intervals were 113 days and 80 days for the first and the second span, respectively. This justifies the fact that the simplified analysis with $t_{cont} = 90$ days leads to a close agreement with the “base calculation”, whereas important errors occur if $t_{cont} = 15$ days. These conclusions are in accordance with the results of parametric analyses carried out by other authors (McDonagh and Hinkley 2003; Miller et al. 2004), which showed that the age of the girders at the time the continuity is established is one of the most important factors that affect the long-term behaviour of this kind of structure.

Figure 5.17 also reveals that, at the end of the 900-day period of time, the results of the simplified calculations (apart from the one with $t_{cont} = 15$ days) do not differ significantly from the results of the “base calculation”. Similar conclusions can be withdrawn by analyzing the stress results in Figure 5.18. This figure presents the stresses at the end of the observation period (i.e., 900 days after casting the continuity diaphragm) for the two critical sections. When the results of the simplified calculations are compared with the “base calculation”, it becomes evident that the most significant differences occur on the top of the precast girder, in the mid-span cross section. This is a consequence of the fact that this is the location where, in this type of composite cross section, higher stresses are caused by the differential shrinkage between the slab

and the girder. In this way, this point is the most sensible one to variations in the long-term behaviour of the slab or the girder. However, the differences found at this location are not relevant because this point usually does not govern the design. The differences (between the results of the simplified analyses and the reference results of the “base calculation”) found in the remaining locations focused in Figure 5.18 are negligible from a practical point of view, except when $t_{cont} = 15$ days, which resembles the conclusions taken about the restraint bending moment.

It should also be remarked that the seasonal variation of shrinkage strains [visible in Figures 5.11 and 5.12, and included in Equation (5.5)] gives rise to seasonal variations of restraint bending moments (see the results of the “base calculation” in Figure 5.17) as well as of concrete stresses. However, these seasonal variations are negligible from a practical point of view, owing to their reduced magnitude.

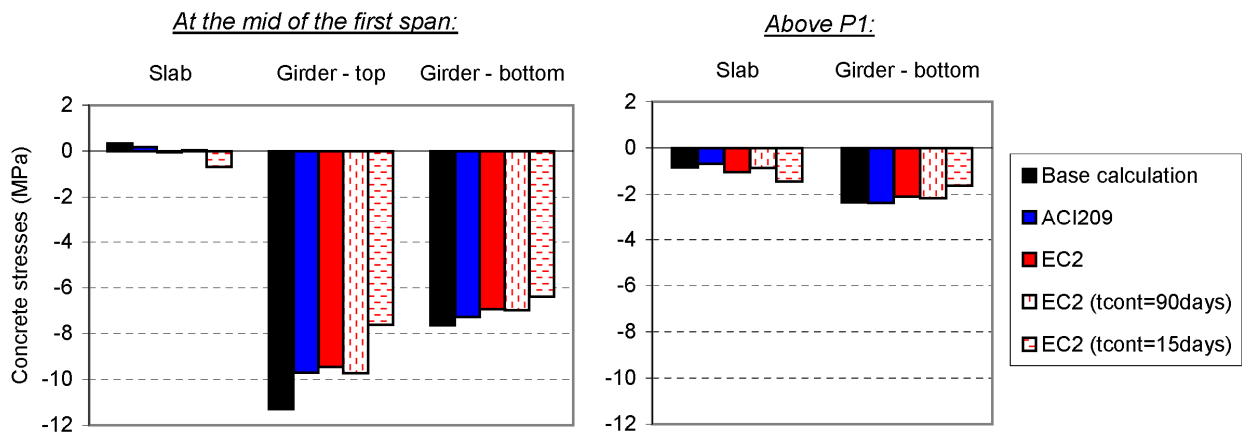


Figure 5.18 - Stresses at the end of the observation period: cross sections at the mid of the first span and above P1.

To conclude, it is worth mentioning that the fact that by the end of the first year the most significant portion of the stress redistribution has already occurred has important implications in terms of fatigue verifications. Fatigue strength depends not only on the value of cyclic actions but also on the stresses caused by permanent loads. If the latter vary over time, then the fatigue damage due to a given cyclic load also varies over time, which makes the fatigue analyses more complex. But, if the stresses due to permanent loads almost stabilize after a short period of time, then the fatigue damage that occurs in the remaining period of time (which is, consequently, the most relevant one) can be determined by assuming constant values for the stresses caused by permanent loads.

5.5 Probabilistic analysis

Parametric analyses of the long-term behaviour of the south approach viaduct to the Lezíria Bridge showed that the calculated restraint moments strongly depend on the calculation

hypotheses, namely the creep and shrinkage estimates and the girder ages at the time that the continuity connection is cast. In other words, the variability of the calculation results is important and, therefore, a single deterministic calculation might lead to results significantly different from the actual ones.

The influence of creep and shrinkage variability in continuous bridges made with precast girders has been evaluated by some authors (Clark and Sugie 1997; Valdés 1997) by means of parametric analyses, in which the structure response is calculated for different sets of creep and shrinkage values. However, the probability associated with the occurrence of each of the obtained results can not be quantified with this type of approach.

Various authors (Bazant and Kim 1989; Oh and Yang 2000; Pan et al. 2011; Santos 2001; Sousa and Serra Neves 2006; Takács 2002; Tsubaki 1988) have highlighted the advantages of probabilistic approaches to describe the properties of concrete, in studies of the delayed behaviour of concrete bridges. With this type of analysis, the expected range of variation of the results (namely stresses and internal forces) can be determined, taking into account the statistical distribution of the variable parameters that most affect the response variability. This approach was therefore followed in this work. Plausible statistical distributions are assumed for creep and shrinkage deformations, and for the girder ages at the time that the continuity is established. Then, a Monte Carlo simulation is implemented in order to evaluate the variability of the structural response, in terms of long-term distributions of stresses and restraint bending moments due to permanent actions. Note that the objective of the work presented in this section is not to carry out a complete probabilistic analysis, but only to ascertain the variability of the structural response in service limit states due to the variability of both the creep and shrinkage deformations and the girder ages at the time the continuity is established.

5.5.1 Monte Carlo simulation method

The Monte Carlo simulation technique is based on the concept of random sampling in order to artificially simulate a large number of experiments and to observe the results. According to this method, the statistical distribution of the structural response can be determined by performing a large number of individual calculations (or “runs”) in which the random variables take random values that follow the underlying variables’ distribution. The Monte Carlo simulation of the structural behaviour involves the following stages (Haldar and Mahadevan 2000; Henriques 1998; Melchers 1999):

- choice of the random variables;
- quantification of the statistical properties for all the random variables;

- random generation of n values for each of the random variables, according to their statistical distributions;
- deterministic evaluation of the structural response for each set of realizations of all the random variables;
- statistical analysis of the sample, composed of the set of n observations for each output variable (internal force, stress or deformation), in order to estimate its probability distribution;
- evaluation of the simulation accuracy.

The mean value and the variability of each output variable (result) can be found by determining the sample's cumulative distribution function $F(x)$, which gives the probability that the output variable will be found at a value less than or equal to x . The cumulative distribution function can be used to identify percentiles for the statistical distribution of the results, the percentile being defined as the value of the output variable below which a certain percent of the observations fall.

The number of simulations required to achieve a certain level of accuracy increases as the probability of occurrence decreases and the number of random basic variables increases. In the case of ultimate limit state analyses, associated to very low probabilities of failure (typically $\sim 10^{-4}$), a large number of computer runs is required, this being an important limitation of this methodology, since it might lead to very time consuming calculations if each individual run requires considerable computational effort. On the other hand, serviceability limit states are associated to higher probabilities of failure [typically between 5% and 10%, as can be concluded from the work of Wisniewsky (2007)], and consequently, a lower number of computer runs is required in this case.

The accuracy of the Monte Carlo analysis can be evaluated through the confidence interval of the estimated probability of failure, for a given confidence level (Haldar and Mahadevan 2000; Montgomery and Runger 2011) or by plotting the variation of both the calculation result and its estimated standard deviation as a function of the number of samples n , as suggested by (Melchers 1999).

The required number of simulations can be greatly reduced if improved sampling methods, such as the Latin Hypercube method, are adopted. That alternative was not followed in this work, because the probabilistic approach is applied to serviceability analyses only, and therefore, very low probabilities of occurrence are not of interest. Moreover, only a limited number of Monte Carlo simulations are to be carried out.

5.5.2 Material properties and construction sequence

Each computer run is performed by employing the 2D finite-element model described in §5.3, considering the geometry, loading and reinforcement behaviour indicated in that section. The concrete behaviour (modulus of elasticity, creep and shrinkage) is simulated according to the models included in the EN 1992-1-1, considering the parameters indicated in Table 5.5. The adopted sequence of construction is shown in Table 5.4.

5.5.3 Basic random variables

An important issue in the Monte Carlo simulation is the identification of the critical parameters which should be considered basic random variables and the definition of their statistical properties. Long-term serviceability analyses of phased constructions are usually carried out by taking as random variables the parameters related to the delayed concrete deformations, i.e., creep and shrinkage deformations (Bazant and Liu 1985; Madsen and Bazant 1983; Yang 2007). The uncertainty of creep and shrinkage deformations can be dealt with by considering, separately, the uncertainty in the influencing parameters (such as environment temperature, relative humidity and parameters associated to the concrete composition) and the model uncertainty, also taking into account the uncertainty inherent to the microscopic creep and shrinkage mechanisms (Bazant and Liu 1985; Madsen and Bazant 1983; Pan et al. 2011; Sousa and Serra Neves 2006; Takács 2002; Yang 2007). Alternatively, the uncertainty associated to the quantification of creep and shrinkage deformations can be estimated without explicit distinction of external and internal uncertainty factors (Wisniewski 2007), this being the approach followed in this work. Three types of basic random variables are considered:

- the parameter Ψ^{cr} , which is introduced in the EN 1992-1-1 creep formulae as shown by Equation (5.7), so that the creep variability is taken into account:

$$\phi(t, t_0) = \Psi^{cr} \cdot \phi_0 \cdot \beta_c(t, t_0) \quad (5.7)$$

where $\phi(t, t_0)$ is the creep coefficient, ϕ_0 is the notional creep coefficient, and $\beta_c(t, t_0)$ is a coefficient to describe the development of creep with time; both ϕ_0 and $\beta_c(t, t_0)$ are quantified according to the EN 1992-1-1;

- the parameter Ψ^{sh} , which represents the uncertainty in the shrinkage model, as shown by Equation (5.8):

$$\varepsilon_{sh}(t) = \Psi^{sh} [\varepsilon_{cas}(t) + \varepsilon_{cds}(t, t_s)] \quad (5.8)$$

where the calculation of the shrinkage strain $\varepsilon_{sh}(t)$ is based on the autogenous shrinkage strain $\varepsilon_{cas}(t)$ and the drying shrinkage strain $\varepsilon_{cds}(t, t_0)$, which are given by the EN 1992-1-1;

- the time step t_{cont} identified in Table 5.4, which corresponds to the time elapsed between the prestress release and the casting of the continuity connection; as shown before (§5.4.2), the girder ages at the time that the continuity is established significantly affect the long-term redistributions of stresses and bending moments; moreover, those ages are usually not known at the design stage of new structures; this time interval should therefore be considered as a random variable.

Tables 5.6 and 5.7 show the adopted statistical properties for the basic random variables. A normal distribution was assumed for the creep and shrinkage deformations, with coefficients of variation of 20% and 35%, respectively, as suggested by the MC90. These values were determined based on an extensive data bank of test results.

It is assumed that all the variables included in Tables 5.6 and 5.7 are independent from each other.

The independency between girder and slab properties is obvious, as their concrete compositions, casting procedures and curing conditions are distinct. The independency between the diaphragm creep and shrinkage properties and the slab ones is not so obvious. However, a simplified approach is followed in this work, which consists of assuming that the long-term deformations of the slab and the diaphragms are independent from each other. As noted before, these elements exhibit significantly different geometries, which are responsible for considerably different time variations of creep and shrinkage deformations, and this difference might not be well predicted by the code formulae. Nevertheless, it is important to note that the diaphragm delayed deformations are not paramount, because they do not affect significantly the global structure response, as shown in §5.4.

The independency between creep and shrinkage deformations is a simplifying hypothesis, which has been adopted by several authors (Bazant and Kim 1989; Oh and Yang 2000; Pan et al. 2011; Takács 2002; Wisniewski 2007), even though methodologies to handle the correlation between creep and shrinkage models are described in the literature (Xi and Bazant 1989).

The absence of correlation between the concrete properties and the time step t_{cont} is logical, because these parameters are very different in nature. The variable t_{cont} is assigned a uniform distribution with the upper and lower limits given in Table 5.7. Feasible values are adopted for these limits, based on the experience of local precast manufacturers.

Table 5.6 – Statistical properties of basic random variables with normal distribution.

Basic random variable with normal distribution	Mean	Coefficient of variation (%)
Creep uncertainty for the girder concrete, Ψ_{girder}^{cr}	1	20
Creep uncertainty for the slab concrete, Ψ_{slab}^{cr}	1	20
Creep uncertainty for the diaphragm concrete, $\Psi_{diaphragm}^{cr}$	1	20
Shrinkage uncertainty for the girder concrete, Ψ_{girder}^{sh}	1	35
Shrinkage uncertainty for the slab concrete, Ψ_{slab}^{sh}	1	35
Shrinkage uncertainty for the diaphragm concrete, $\Psi_{diaphragm}^{sh}$	1	35

Table 5.7 – Statistical properties of basic random variables with uniform distribution.

Basic random variable with uniform distribution	Lower limit	Upper limit
time step t_{cont}	30 days	150 days

5.5.4 Results and discussion

Figures 5.19 to 5.22 show some representative results of a Monte Carlo simulation with 100 computer runs. The variability of each result (concrete stress or bending moment in a specific location, at a certain instant of time) can be estimated by means of the statistical distribution of the Monte Carlo simulation results. The results can be analysed by calculating percentiles of that statistical distribution. Figures 5.19 and 5.20 show the time variation of the calculated percentiles, corresponding to probabilities of exceedance of 5%, 10%, 90% and 95%.

Figure 5.19 shows the time variation of the restraint bending moment at the cross section above P1. In this figure, the results of the Monte Carlo simulation are compared with the previously presented results (“base calculation” and parametric analysis based on the EN 1992-1-1 and the ACI209). In §5.4, it was concluded that the calculated time variation of the restraint moment is strongly influenced by the calculation hypotheses (creep and shrinkage models and construction sequence). Therefore, it is not surprising that the results of the probabilistic analysis show a large statistical dispersion. If one looks at the interval bounded by the estimated 5% and 95% percentiles, one can see that its amplitude is quite large, reaching a value of ~8100 kNm 900 days after casting the continuity diaphragm. This amplitude is almost twice the maximum restraint moment reached in the “base calculation”. A short time after the continuity is established (some weeks) the dispersion of results is already quite large. These results confirm that deterministic calculations of the long-term structure behaviour might lead to results significantly different from the actual ones. The figure also shows that the 10% and 90%

percentiles approximately bound the results of all the parametric analyses. The calculation labelled with *EC2* ($t_{cont}=15days$), an extreme scenario in terms of girder ages, leads to values close to the 90% percentile, whereas the calculation based on the *ACI209* provides values similar to the 10% percentile, specially in the first months after the continuity is established. As regards the “base calculation” (calibrated calculation which provides a realistic estimate of the structure behaviour, as shown before), its results are bounded by the 10% and 90% percentiles. The only exception happens in the very first weeks after the continuity connection is made, owing to different construction sequences adopted in the “base calculation” (the real one) and in the Monte Carlo simulation (a simplified one).

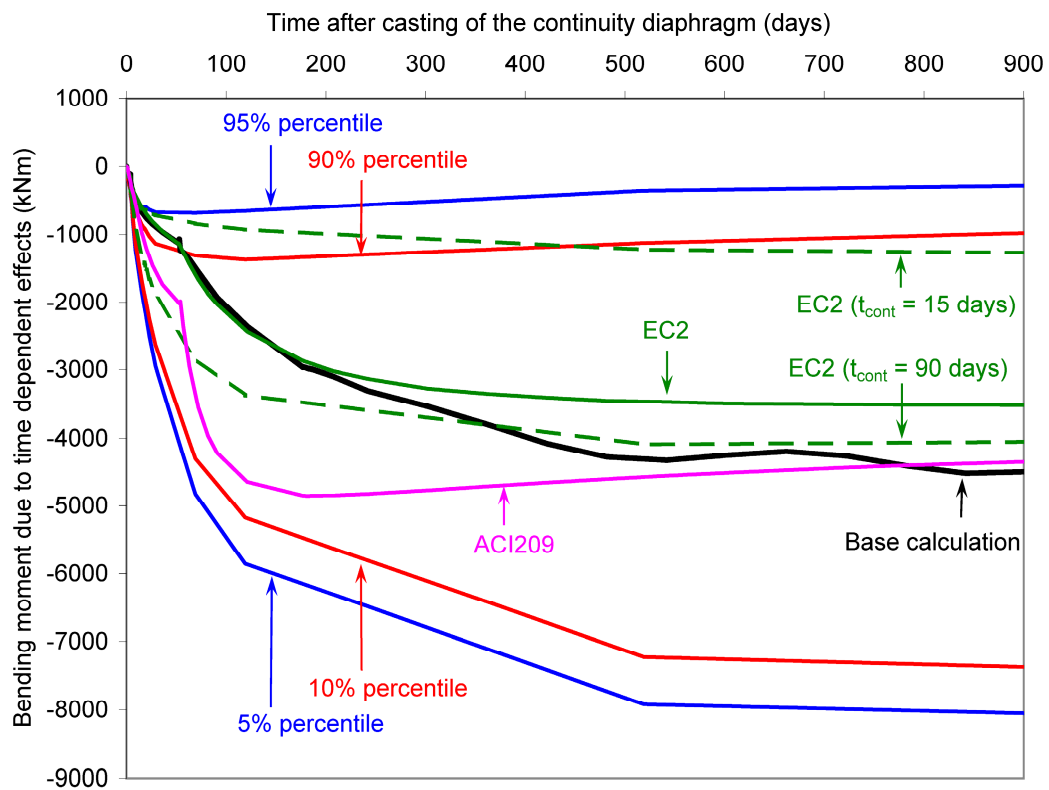


Figure 5.19 - Restraint bending moment at the cross section above P1.
Comparison between parametric and probabilistic approaches.

Figure 5.20 shows the simulation results in terms of concrete stresses at the middle of the first span. These results are plotted as a function of the time after the end of the construction, i.e., after the temporary supports are removed. Three different graphs are presented which depict the stresses on the points where the strains are measured: on the girder bottom flange, on the girder upper chord and on the slab (see Figure 5.2). The Y-axis scale is kept constant over the three graphs, so that the results can be easily compared. This figure shows that the largest variability occurs in the upper chord of the precast girder. This conclusion is consistent with the results of the parametric analyses, as it was shown before that this is the most sensitive point (among the ones under analysis) to the variability of the basic variables. In fact, important long-term redistributions of forces occur between the girder upper chord and the slab. The stress in the

girder upper chord is particularly sensitive to this effect because its cross-section area is significantly smaller than the slab's one.

Figure 5.20 also presents the results of the “base calculation”, which is expected to provide a close estimate for the actual applied stresses. It should be noted that, in this figure, the results of the “base calculation” only start after the non-structural dead loads (pavement, side-walks and Jersey barrier) were actually applied (112 days after the end of the construction). In the probabilistic analysis, these loads were applied shortly after the end of the construction, because this is a “blind” calculation with no information about the real construction times. Therefore, Figure 5.20 presents comparable results, i.e., stresses after all the loads are applied. This figure shows that, as expected, the results of the “base calculation” fall within the envelopes formed the 10% and the 90% percentiles. The girder upper chord is also the location where the results of the “base calculation” are closest to one of the percentiles, and consequently, more distant from the median of the probabilistic analysis results.

The time variation of stresses and restraint moments was also evaluated for the remaining monitored cross sections. It was concluded that the results of the “base calculation” are always bounded by 10% and the 90% percentiles, apart from a small difference in terms of restraint bending moment in the first weeks after casting (an exception which has already been justified).

This example reveals the usefulness of the Monte Carlo simulation for the evaluation of the long-term behaviour of continuous precast girder bridges. With this approach, the variability of the structure response can be quantified. That variability is not negligible and, in this case, the realistic results of the “base calculation” are well bounded by the 10% and the 90% percentiles of the probabilistic analysis results.

As mentioned before, the accuracy of the calculated percentiles varies with the number of computer runs considered in the Monte Carlo simulation. That accuracy can be qualitatively assessed by plotting the simulation results as a function of the number of computer runs, as recommended by Melchers (1999), the required number of computer runs being the value beyond which the calculated percentile do not vary (significantly) if the number of computer runs is increased. Figures 5.21 and 5.22, which depict the variation of the calculated percentiles with the number of computer runs, show that 100 runs provide approximate estimates for the 10% and 90% percentiles. The accuracy of the calculation can be quantitatively assessed through the confidence interval of the estimated percentile. The 90% confidence interval is given by (Haldar and Mahadevan 2000; Montgomery and Runger 2011):

$$\hat{p} - z_{0.05} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{0.05} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad (5.9)$$

where $\hat{p}$ is the estimated percentile (in the sample), n is the sample size (the number of runs), $z_{0.05}$ is the upper 0.05 percentage point of the standard normal distribution and p is the actual percentile of population that is delimited by the calculated result. In this case, the 90% confidence interval takes the value:

$$\hat{p} - 0.049 \leq p \leq \hat{p} + 0.049$$

i.e., we can say with 90% confidence that the result (concrete stress or bending moment) that was estimated to correspond to a 10% chance of being exceeded corresponds, in reality, to a probability of being exceeded between 5.1% and 14.9%. This accuracy is thought to be sufficient for taking into account the results variability owing to creep and shrinkage uncertainty, in service analyses, in the context of the present work.

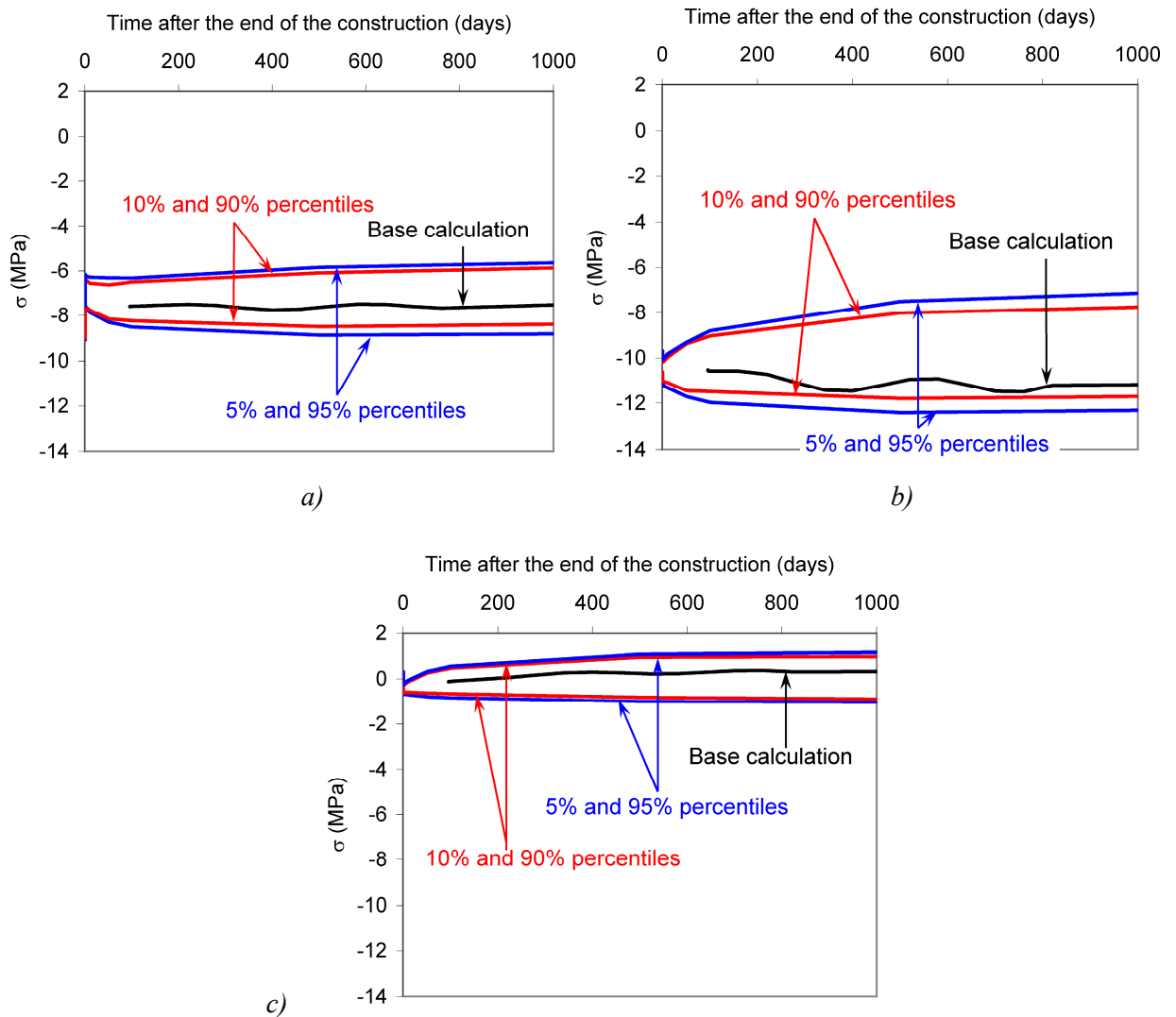


Figure 5.20 - Stresses in the mid-span cross section of the first span. Comparison of the calibrated “base calculation” and the probabilistic analysis. Results at the sensor locations: a) girder bottom flange; b) girder upper chord; c) slab.

The long-term distributions of concrete stresses determined with the Monte Carlo simulation were found to follow a normal distribution. The verification of the normality was carried out according to the procedure proposed by Aivazian (1970), based on the asymmetry and flatness coefficients of the normal distribution.

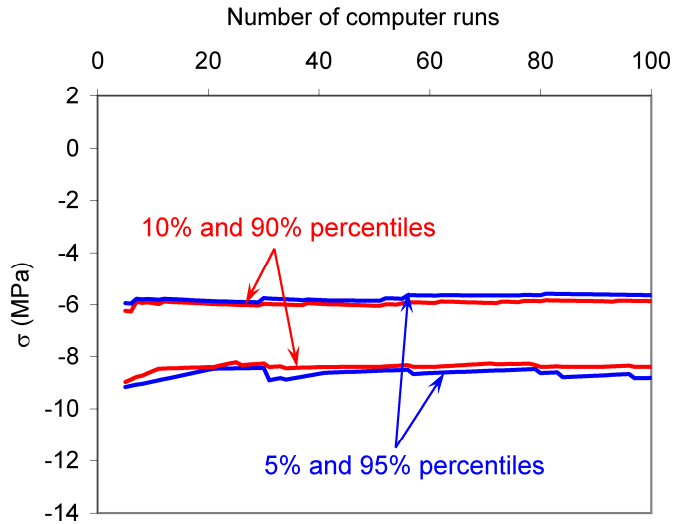


Figure 5.21 – Stresses at the location of the bottom flange sensor in the mid-span cross section of the first span. Evolution of the calculated percentiles with the number of computer runs.

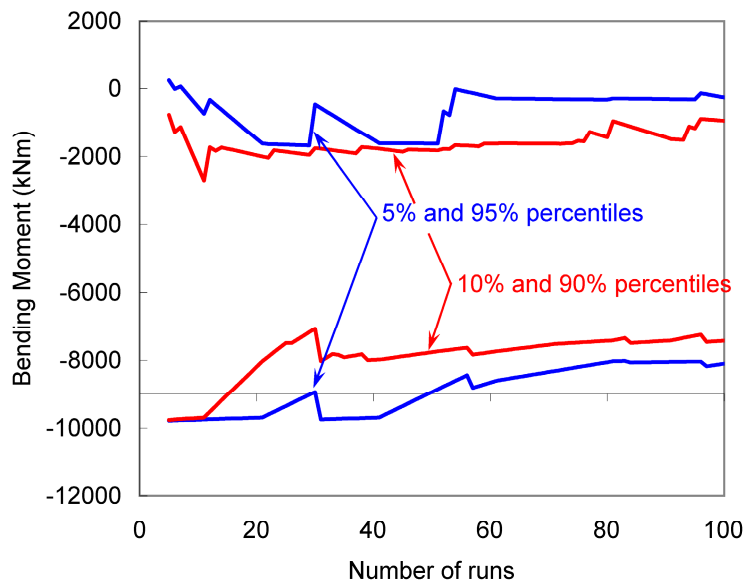


Figure 5.22 - Restraint bending moment at the cross section above P1, 1,000 days after the end of the construction. Evolution of the calculated percentiles with the number of computer runs.

5.6 Final considerations

A numerical methodology for prediction of time-dependent effects in precast continuous bridges was validated through the comparison with experimental results observed in a real structure. The

employed analysis strategy gives a good prediction of the structural response if the material properties are accurately modelled and the actual sequence of construction is considered.

Even though the design codes do not accurately predict the actual development of the concrete delayed deformations, calculations carried out according to these codes may lead to a good estimate of the final restraint moments if realistic values for the girder age at the time that the continuity is established are adopted in the analysis.

In this structure, the difference between actual shrinkage values and code predictions decrease as the observation period increases. This is consistent with the commonly acknowledged fact that shrinkage variability is higher for short-term tests.

In the structure under analysis, seasonal shrinkage variations were responsible for slight seasonal variations of the structural response. The obtained results indicate that this effect needs not to be taken into account in design calculations because it does not originate relevant stresses or deformations.

The actual time variation of strains in the diaphragms was not precisely predicted. But it was shown that the uncertainty involved in the prediction of diaphragm strains does not affect the most relevant results in terms of practical design: stresses and internal forces.

Given that the time variation of stresses and bending moments in this type of structure strongly depends on creep and shrinkage deformations and construction timings (which are variables with a strong statistical dispersion), a Monte Carlo simulation of the long-term behaviour was carried out. This simulation showed that the calculation of both restraint bending moments and concrete stresses in critical sections is significantly affected by the uncertainty associated to the quantification of creep and shrinkage deformations and construction timings. It was also shown that the stresses and bending moments determined through the “base calculation” (which are expected to be realistic estimates of the real stresses and moments, owing to model calibrations based on experimental data) are well bounded by the 10% and the 90% percentiles of the results of the Monte Carlo simulation (remember that only the data that is usually available at the design stage was considered in the probabilistic analysis).

The Monte Carlo simulation appears to be a feasible approach for taking into account the variability of the structural response, in this type of structure, due to the uncertainty associated to the quantification of creep and shrinkage deformations and construction timings.

Chapter 6

Numerical analysis of a real precast railway bridge deck

6.1 Introduction

In this section, the previously developed and validated numerical tools are applied in the analysis of a real precast railway bridge deck, which is composed of four continuous spans with interior and end spans of 33.5 m and 26.5 m, respectively. This structure is included in the Spanish Railway Line “Eje Ferroviario Alta Velocidad del Levante” that connects Alcácer to Valencia, and is located at the kilometric point 607+950.

One of the specificities of this structure, which justifies its thorough analysis, lies in the fact that no continuity prestress was employed, i.e., only non-prestressed reinforcement is used in the continuity connection. The adoption of continuous precast decks without continuity prestress is strongly motivated by economic and construction factors: in comparison with solutions with continuity prestress, the latter is usually less cost-effective, and its construction is more time consuming and complex. However, this is still a controversial solution, because prestressed solutions are usually employed in railway bridge decks with such span lengths. As shown in Chapter 1, this construction typology has been employed in small-span highway and railway bridges, but the knowledge about the real performance of this bridge type in medium span railway decks is quite limited.

Another specificity of this construction type has to do with the reduced thickness of the various deck components. The cross section shape is optimized so that the girder weight is minimized. Therefore, thin webs are employed, which leads to high shear stresses. Moreover, cracking occurs in the webs, due to the combined effects of in-plane forces and out-of-plane bending and

also due to the absence of continuity prestress. Consequently, the fatigue performance of the webs must be carefully analysed, taking into account the influence of transverse bending moments. Research devoted to this problem is quite limited, which justifies its analysis within the scope of this work.

Moreover, thin slabs are also prone to fatigue damage under the effect of railway traffic loading. Deck slabs are not only subjected to important cyclic out-of plane forces, but also to important in-plane shear effects due to the transference of forces between flanges and webs. Therefore, the developed methodologies for quantification of local effects due to traffic actions and for fatigue verifications are employed in the analysis of the deck slab.

Also, precast structures undergo significant stress redistributions, caused by the concrete delayed deformations and the modification of the structural system during the construction. Given that these factors have significant variability (i.e., accurate predictions of their actual values can hardly be made at the design stage), significant variability is also associated to the calculated long-term stresses. The consequences of this variability are also addressed in this chapter.

In short, one can say that this construction type offers important advantages (namely lower construction cost and reduction of construction time), but its structural behaviour is complex and has not been fully investigated yet. The main purposes of the work presented in this chapter are twofold: (i) to illustrate the application of various numerical tools that were developed and validated in this thesis, which allow the detailed analysis of critical aspects of these constructions' structural behaviour; (ii) to contribute to the knowledge about the structural behaviour of this bridge type.

The presentation of this case study is organized as follows: firstly, the structure is described and the external loads considered in this study are presented; then, the long-term stress redistribution due to creep and shrinkage is addressed; after that, the fatigue behaviour of both the girder webs and the deck slab, under the combined effects of in-plane shear forces and out-of-plane bending, is analysed and discussed; finally, the consequences of cracking, owing to the absence of continuity prestress, are evaluated.

6.2 Structure description

The bridge deck under analysis is composed of a 2.10 m deep U-shaped precast girder and a 8.96 m wide cast-in-place slab, forming a box-girder deck as depicted in Figure 6.1. The geometry of the precast girder is shown in Figure 6.2, and the reinforcement adopted in this bridge (both ordinary and prestressed reinforcements) is presented in Annex 2. The deck slab, which is 0.30 m thick, is made with 0.06 m thick precast planks, so that the use of formworks

and scaffolding is avoided. These precast planks are monolithically connected to the cast-in-place concrete by means of steel lattice girders, as shown in Figure A2.3 of Annex 2.

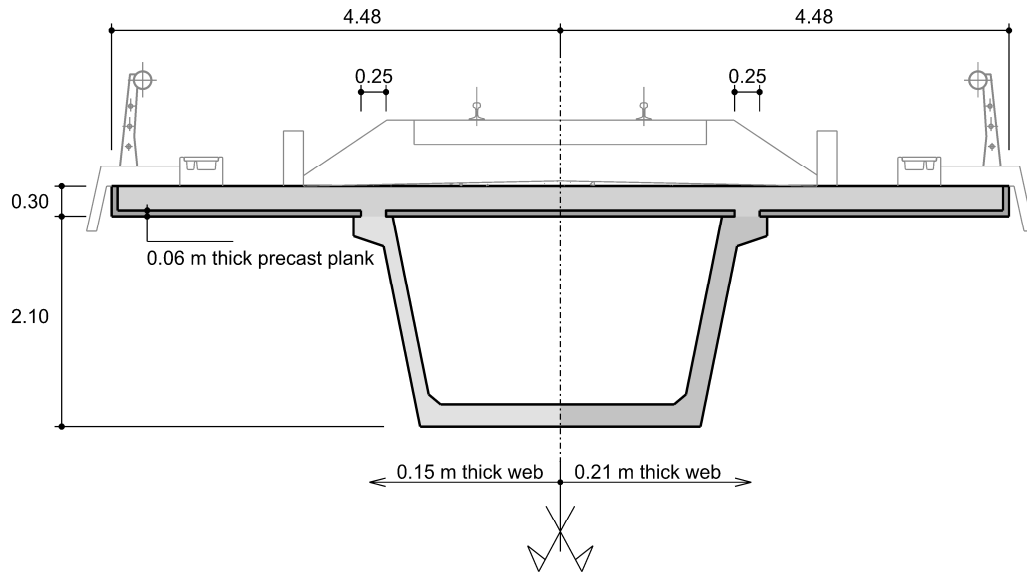


Figure 6.1 - Geometry of the cross section (dimensions in m).

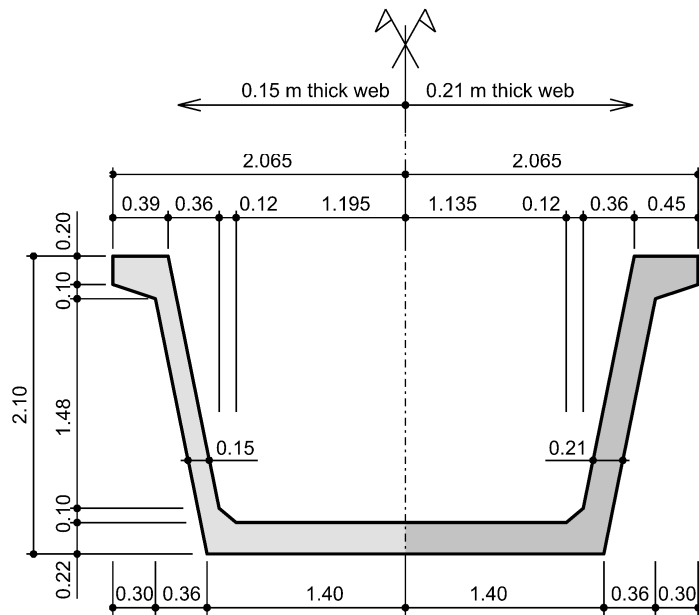


Figure 6.2 - Girder cross section (dimensions in m).

Reduced web thicknesses were adopted, with different values near the continuity supports and in the mid-span region, as represented in Figures 6.2 and 6.3.

The transmission of forces from the deck girder to piers and abutments is made by guided pot bearings. Two bearing pads exist in each support alignment, so that the deck is fixed in torsion.

In the transverse direction, relative displacements between the girder and the infrastructure (piers and abutments) are not permitted, i.e., all the piers and abutments carry transverse horizontal forces. On the contrary, sliding is allowed in the longitudinal direction. In this direction, the deck is fixed to one of the abutments, so that the longitudinal forces are mainly resisted by this abutment (minor horizontal forces are also transmitted to the piers and the other abutment as a consequence of friction effects).

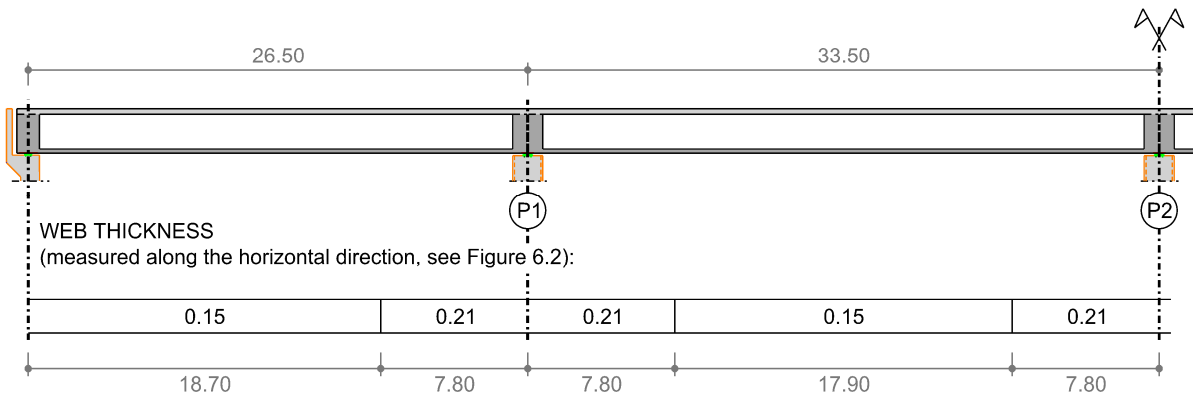


Figure 6.3 - Longitudinal section (dimensions in m).

The continuity between spans is established by a diaphragm, longitudinal reinforcing bars in the deck slab and, at the lower level, five short reinforcing bars whose extremities are fixed to the concrete by means of steel anchor plates (see Figure A2.5 of Annex 2).

The construction of the deck involves the following main stages:

- placement of permanent bearing pads;
- girder erection (at this phase, the girders rest on temporary supports, which are placed close to the permanent ones);
- erection of precast slab planks and placement of the slab reinforcement;
- execution of the continuity connection between girders;
- removal of temporary supports;
- casting of the slab.

The slab is cast in three different phases, which are identified in Figure 6.4. Firstly, a central 5 m wide slab strip is cast in the region over the continuity supports. Then, the remaining central strip is cast. Finally, the lateral strips are cast. The pouring of phases 2 and 3 does not begin until the concrete of the previous phase reaches 80% of the 28-day compressive strength.

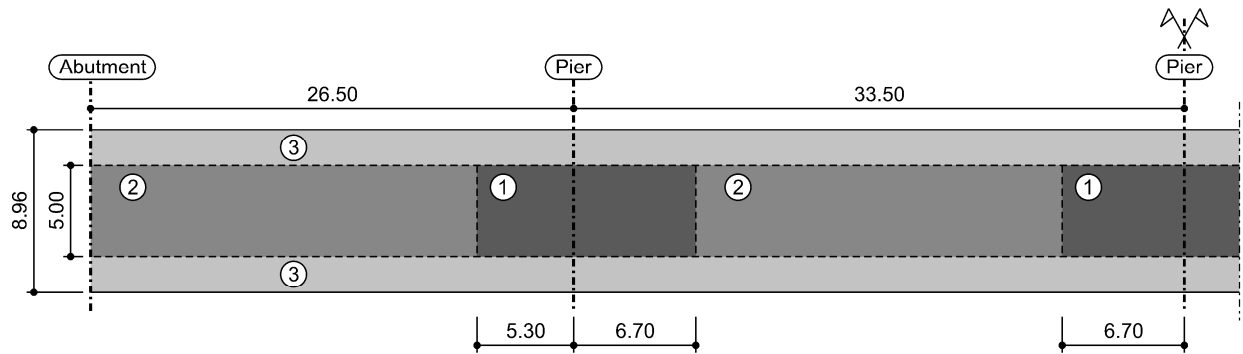


Figure 6.4 - Sequence of construction of the slab (top view).

The strength classes for both the concrete and the steel adopted in this structure are indicated in Table 6.1. This table also presents the values adopted for the concrete cover to the reinforcement.

Table 6.1 – Material properties and concrete cover to the reinforcement.

Concrete	
Precast girders	C50/60 [EN 206-1 (CEN 2005d)]
Slab	C30/37 [EN 206-1 (CEN 2005d)]
Reinforcing steel	B500S [UNE 36068 (AEN/CTN 36 2011)]
Prestressing steel	Y1860S7-15.2 ($A_p = 140 \text{ mm}^2$) [prEN 10183-3 (CEN 2000c)]
Cover to the reinforcement	
precast girders and planks	2 cm
Cast-in-place slab	2.5 cm

This structure carries a single railway ballasted track, whose gauge is equal to 1.668 m (the gauge in Iberian lines). This track carries freight traffic and is part of a railway line which also includes two additional tracks for high speed traffic.

6.3 Loading

This section presents the loads that were considered to be relevant in the context of the analyses developed in this work. Given that this work focuses on the deck behaviour, only the loads that are responsible for significant stresses in this substructure (in normal design situations) were taken into account. Thus, besides the permanent loads (self weight and prestressing), imposed loads due to temperature gradients and railway traffic were considered in the analyses.

6.3.1 Self weight

The structure self weight was calculated considering that the specific weight of reinforced concrete is 25 kN/m^3 . The self weight of the remaining non-structural materials is quantified in Table 6.2.

As regards the mass of the ballast, it is important to remark that its value is significantly influenced by the material type, by the material condition (dry or saturated, clean or dirty, intact or crushed) and by the actual thickness of the ballast layer, which may be modified by track lifting operations. Therefore, upper and lower bound values shall be considered during the design of these structures. Guidance on the quantification of those values is given in various documents (IPQ 2009a; Ministerio de Fomento 2007; Sustainable Bridges WP4 2007). The mass of the ballast not only affects the magnitude of internal forces due to permanent loads, but it also influences the dynamic response of the structure. Nevertheless, it has been shown that the maximum dynamic enhancement of forces does not depend on the value of that mass. Only the resonance speeds and the maximum deck accelerations are significantly affected (ERRI D214 1999b). All the analyses presented in this chapter were carried out considering the ballast average mass, because the influence of the variability associated to the quantification of the ballast mass is not under analysis in this work. Details about the quantification of this mass can be found in (Sousa et al. 2011a).

Table 6.2 – Non-structural permanent loads.

Description		Total load per linear meter (kN/m)
Ballast		48.33
2 rails UIC 60		1.20
monoblock concrete sleepers with track fastenings		4.80
waterproofing coating and protection		4.69
sidewalks	parapets and kerbs	10.00
	ballast barrier	5.00
	walls and covering for service ducts	6.30

6.3.2 Prestressing

The maximum applied stress during tensioning was taken as 1425 MPa , this value being given by the design specifications for this structure.

The prestress force at a given time is determined by deducting the value of prestress losses from the initial stress. The immediate losses due to anchorage slip were determined by adopting plausible values for the length of the precast bed (100 m) and the wedge draw-in (6 mm). The remaining prestress losses (immediate losses due to elastic concrete deformation and long-term losses owing to steel relaxation and concrete creep and shrinkage) were automatically determined through the FE calculations using DIANA. As mentioned before, a specific user-supplied subroutine was developed to perform the calculation of relaxation losses (Sousa and Serra Neves 2009a).

6.3.3 Temperature gradients

Normally, uniform temperature changes do not lead to significant stresses in bridge girders, unless the axial deformation of the deck is significantly restrained. For that reason, uniform temperature variations were not considered in this study.

On the other hand, temperature gradients (also known as temperature differences) due to daily temperature variations cause important bending moments in hyperstatic bridge decks. The quantification of the vertical temperature difference for different types of bridge decks can be carried out according to the EN 1991-1-5 (CEN 2003a). Two situations must be considered:

- $\Delta T_{M,heat}$ with the top surface warmer than the bottom one;
- $\Delta T_{M,cool}$ with the top surface cooler than the bottom one.

In this work, vertical temperature differences were simulated by assuming a linear temperature variation along the cross-section depth. Even though this is a simplification (the actual non-linear temperature distribution is not explicitly considered), it is valid for practical purposes, as recognized by the EN 1991-1-5 (CEN 2003a; IPQ 2009b). This code gives the following characteristic values for the temperature differences in concrete bridge decks (difference between temperatures in the top and bottom surfaces):

- $\Delta T_{M,heat} = 9^{\circ}\text{C}$
- $\Delta T_{M,cool} = -5^{\circ}\text{C}$

The aforementioned values take into account the influence of ballast in reducing the temperature gradient due to heating on the top surface. The calculation of internal forces in the deck girders (in FE analyses with beam elements) is thus carried out by considering temperature differences of 9°C and -5°C between the upper slab surface and the bottom of the precast girder. Such temperature profile is not adequate for calculation of the local effects (namely transverse bending moments) in the deck slab and in the girder webs caused by temperature differences across the deck slab thickness. For that reason, those local effects are determined by considering (in FE

analyses with shell elements) temperature differences of 9°C and -5°C between the upper and lower slab surfaces. This is a simplistic assumption, which is expected to provide conservative estimates of the local effects in the slab and in the girder webs. This assumption is adopted in this work because further studies regarding the characterization of the real temperature distribution in this type of structure are not to be conducted within the scope of this thesis.

In the case of concrete box girders, important temperature differences also occur between the outer and the inner web surfaces. According to the NP EN 1991-1-5, the characteristic value of this temperature difference equals 5 °C (exterior surface warmer) if the cantilever slab span is larger than the cross-section depth (like in the structure under analysis).

6.3.4 Railway traffic

As mentioned before, this structure carries freight traffic. Therefore, all the fatigue verifications included in this chapter were carried out considering the heavy traffic mix of real trains defined in the EN 1991-2 (Annex D). This traffic mix includes four trains whose operating speeds and number of passages per day are defined in the EN 1991-2.

The calculation of plausible extents of cracking was carried out considering the heaviest real train included in the heavy traffic mix, the train type 5 with 25 t axles and a total load per unit length of 80 kN/m.

Ultimate limit state (ULS) analyses are also carried out in this chapter, particularly for the study of the web and the slab under the combined effects of longitudinal shear and transverse bending. In those analyses, the load models LM71, SW/0 and SW/2, defined in the EN 1991-2, are also considered when they are more unfavourable than real trains. The factor α , which defines the classified vertical loads, took the value 1.21, as defined by the Spanish code IAPF (2007).

Even though the structure under analysis carries freight traffic, high speed traffic is considered in §6.6, so that the influence of cracking in terms of dynamic behaviour under the passage of high speed trains can be evaluated. For that purpose, traffic loading is represented by the trains included in the load model HSLM-A (defined in the EN 1991-2), which has to be considered in lines where European high speed interoperability criteria are applicable.

The distribution of axle loads through rails, sleepers and ballast was taken into account in FE analyses for calculation of local effects in the deck slab and in the girder webs. The distribution of axle loads by sleepers (in the longitudinal direction), as well as the lateral degradation of the load applied to each sleeper, were defined according to the model presented in Chapter 3, considering 0.30 m wide and 2.60 m long monoblock sleepers.

6.4 Long-term redistribution of stresses and internal forces

Given the relevance of the construction sequence and the long-term concrete deformations, a time-dependent non-linear analysis was carried out to calculate the time variation of stresses and deformations due to permanent actions. A Monte Carlo simulation was implemented so that the influence of the uncertainty associated to the quantification of concrete creep and shrinkage deformations and the construction schedule could be evaluated. The DIANA software was employed and the analysis methodology was the same adopted and validated in Chapter 5.

6.4.1 Numerical model and material properties

Phased long-term analyses were performed with a FE model in which the structure is modelled with beam finite elements in accordance to the Timoshenko-Mindlin theory. These elements are fully integrated along the beam axis and in its cross section. Only half of the bridge length is modelled owing to the symmetry of both the structure geometry and the loading, the length of the finite elements being approximately equal to 1 m.

The concrete behaviour is simulated through a fixed smeared crack model with strain decomposition, as described in Chapter 5. This model allows creep, shrinkage and cracking effects to be handled simultaneously. The tensile behaviour of concrete was modelled by using the tension stiffening model proposed by Figueiras (1983). The adopted concrete properties and the parameters for estimation of creep and shrinkage deformations are shown in Table 6.3. Strength and stiffness characteristics were estimated according to the EN 1992-1-1, for the strength classes indicated in Table 6.1. Both the concrete creep and shrinkage deformations and the time variation of the elasticity modulus are also quantified according to the models given by the EN 1992-1-1. As mentioned before, a user-supplied subroutine was developed so that these material models could be considered in DIANA analyses (Sousa and Serra Neves 2009b).

The longitudinal reinforcement, both ordinary and prestressed, was modelled using embedded reinforcement elements, whose deformation is calculated from the displacement field of the concrete finite element in which they are embedded. The modulus of elasticity was taken as 200 GPa and 195 GPa for ordinary and prestressed reinforcements, respectively. Pre-tensioning was modelled by means of an initial stress applied to the tendons. No initial stress was considered over the length in which the strands are debonded from the concrete. A linear variation of the initial tendon stress was assumed over the transmission length, from zero to the maximum value. For the sake of simplicity, this transmission length was considered to be equal to the length of one finite element. Relaxation losses were computed by employing the developed user-supplied subroutine (Sousa and Serra Neves 2009a), considering low relaxation strands (relaxation class 2 according to the EN 1992-1-1).

The sequence of construction is taken into account by means of phased analyses, in which the finite element model is changed at every new stage, by including new elements, adding new parts to the cross section or modifying the connection to the supports. The cross section is composed of three different parts (each of which is activated at a different time) so that the actual sequence of construction can be simulated: firstly, only the precast girder exists; then a 5 m wide slab strip is cast; the remaining slab portion is cast at a later stage.

Table 6.3 - Material properties for time-dependent analyses.

	Girder concrete	Slab concrete
f_{cm} (MPa)	58	38
E_{cm} (GPa)	37	33
f_{ctm} (MPa)	4.1	2.9
h_0 (mm)	^(a)	300
Relative humidity (%)	75	75
Curing time (days)	^(b)	1
Cement type	Rapid-hardening	Rapid-hardening

NOTES:

- ^(a) h_0 takes the values 192 mm or 227 mm depending on the web thickness, which equals 150 mm (in the mid-span region) or 210mm (close to the supports);
- ^(b) drying begins at the time of prestress release.

The adopted sequence of construction is shown in Table 6.4. A new phase is initiated whenever the FE model is changed. The variable Δt_I , which represents the period of time between the prestress release and the erection of precast planks, was introduced in the time scale so that different construction schedules could be simulated by changing this variable.

Plausible values were taken for the increments of time between construction events. The girder concrete age at the time of prestress release takes the value 3.5 days, so that the concrete strength at that time is 70% of the 28-day strength (the value indicated in the design specifications), given that the time variation of concrete strength is defined by the EN 1992-1-1 model for concrete made with rapid-hardening cement. The time interval between different slab pouring operations is 7 days, which allows the concrete to attain 80% of its 28-day strength. The slab elements are activated one day after they are cast (they are not activated sooner because the early age behaviour is not of interest in this study).

The activation of reinforcing and prestressing steels occurs simultaneously with the activation of the concrete element in which they are embedded.

Table 6.4 – Sequence of construction considered in the FE analyses.

Phase	Elements added in each phase	Time (days) at the end of the event	Event	Applied load
1	Precast girders Temporary supports	0	Prestress release	Girder self weight and prestress
		Δt_I	Increment of time (Δt_I)	--
		Δt_I	Erection of precast planks	Planks self weight
		$\Delta t_I + 7$	Increment of time (7 days)	--
2	Continuity diaphragm Final supports ^(a)	$\Delta t_I + 7$	--	Diaphragm self weight
		$\Delta t_I + 7$	Casting of the slab (part 1)	Self weight of the slab (part 1)
		$\Delta t_I + 8$	Increment of time (1 day)	--
3	Slab (part 1)	$\Delta t_I + 14$	Increment of time (6 days)	--
		$\Delta t_I + 14$	Casting of the slab (part 2)	Self weight of the slab (part 2)
		$\Delta t_I + 15$	Increment of time (1 day)	--
4	Slab (part 2)	$\Delta t_I + 21$	Increment of time (6 days)	--
		$\Delta t_I + 21$	Casting of the slab (part 3)	Self weight of the slab (part 3)
		$\Delta t_I + 22$	Increment of time (1 day)	--
5	Slab (part 3)	$\Delta t_I + 52$	Increment of time (30 days)	--
		$\Delta t_I + 52$	Track and side-walks	Non-structural permanent loads
		$\Delta t_I + 20,052$	Increment of time (20,000 days)	--

NOTES: ^(a) The temporary supports are removed when the final ones are added.

6.4.2 Variability of the long-term structural response

The influence of the statistical variation of creep, shrinkage and construction timings can be evaluated by means of Monte Carlo simulations, as shown in Chapter 5. Therefore, the probabilistic analysis of the long-term behaviour of this railway bridge deck is carried out considering the same independent variables that were adopted in the analysis of the approach viaduct to the Lezíria Bridge, as shown in Tables 6.5 and 6.6. The statistical distribution of the independent variables that represent creep and shrinkage uncertainty follows the same principles that were adopted in Chapter 5. As regards the construction sequence, the most important parameter in terms of long-term distribution of concrete stresses and internal forces is the girder age at the time that the continuity connection is established. This is because this age determines

the percentage of the girder creep and shrinkage deformation that has already taken place before the slab is cast, and consequently, the magnitude of long-term stress redistributions strongly depends on this age. The uncertainty associated to the quantification of this age can be taken into account by taking as independent variable the time step Δt_I , introduced in Table 6.4. In this work, that variable is assumed to follow a uniform distribution. Its upper and lower bound values are indicated in Table 6.6.

Three different Monte Carlo simulations are carried out, so that the influence of cracking and also the influence of the lower bound limit for the variable Δt_I can be evaluated:

- in the first simulation, cracking is not considered and the lower limit for Δt_I takes the value of 7 days;
- a different simulation is made with the same statistical distribution for Δt_I , but considering the influence of cracking;
- in the third simulation, Δt_I is increased to 30 days. The purpose of this one consists of evaluating the benefits of considering an increased lower time limit, in terms of reduction of the results variability. In fact, the designer might impose a minimum value for the girder age at the time that the slab is cast, so that a minimum percentage of creep and shrinkage deformations takes place in the girder before the continuity connection is made, and by doing so, the designer reduces the uncertainty associated to the construction sequence timings.

Table 6.5 – Statistical properties of basic random variables with normal distribution.

Basic random variable with normal distribution	Mean	Coefficient of variation (%)
Creep uncertainty for the girder concrete, Ψ_{girder}^{cr}	1	20
Creep uncertainty for the slab concrete, Ψ_{slab}^{cr}	1	20
Creep uncertainty for the diaphragm concrete, $\Psi_{diaphragm}^{cr}$	1	20
Shrinkage uncertainty for the girder concrete, Ψ_{girder}^{sh}	1	35
Shrinkage uncertainty for the slab concrete, Ψ_{slab}^{sh}	1	35
Shrinkage uncertainty for the diaphragm concrete, $\Psi_{diaphragm}^{sh}$	1	35

Table 6.6 - Statistical properties of basic random variables with uniform distribution.

Basic random variable with uniform distribution	Lower limit	Upper limit
time step Δt_I	7 or 30 days ⁽¹⁾	150 days

NOTES: ⁽¹⁾ The influence of the lower bound limit is evaluated by carrying out Monte Carlo simulations in which this parameter takes different values (7 or 30 days).

Given that the influence of cracking is to be taken into account, then live loads must be considered in the analyses, because these loads give rise to cracking in locations which would be uncracked if live loads were not applied. This is particularly important in the case of railway bridges due to the large magnitude of traffic loads. Therefore, the characteristic values of live loads (traffic loads and temperature variations) were applied at two different times:

- immediately after applying the non-structural dead loads, i.e., at the beginning of the service life;
- two years later, i.e., after a significant part of the long-term stress redistributions takes place, because new cracks might arise due to increased tensile stresses in some locations.

At these times, live loads are applied and removed, so that cracking due to these actions is induced, but no live load is permanently applied during the course of time. Live loads are not applied in the analysis without cracking effects, because in this case these loads would not cause any modification of the long-term calculation results.

In these analyses, railway traffic is represented by the heaviest train included in the heavy traffic mix defined in the EN 1991-2 (the train type 5, including the corresponding dynamic enhancement), and not design load models such as the LM71 or the SW/0, so that realistic cracking levels, in service, are obtained.

Figures 6.5 and 6.6 display representative results of the three different Monte Carlo simulations, with 100 computer runs each. Figure 6.5 shows the time variation of the stress at the bottom fibre of the mid-span cross section of the second span, whereas Figure 6.6 gives the time variation of the bending moment at the cross section above P2 (see Figure 6.3). These figures depict the estimated 10% and 90% percentiles of the structure response, determined through the cumulative distribution function of the simulation results for each point in time.

By comparing the results of the three simulations, one can see that the redistribution of bending moments due to cracking leads to lower hogging moments at the support and lower compressive stresses at the bottom fibre of the mid-span cross section. Furthermore, cracking leads to a

significant reduction of the interval bounded by the 10% and the 90% percentiles, for both the stresses and the bending moments. By comparing the results of the simulations with different lower bound values for the independent variable Δt_I , one can realize that, as expected, the variability of the results decreases significantly as the lower limit for Δt_I increases. Figure 6.6 clearly shows that the variation of the lower limit for Δt_I affects mainly the lower bound value of the statistical distribution of the hogging moment. This is because younger girders lead to lower hogging moments as discussed in Chapter 5.

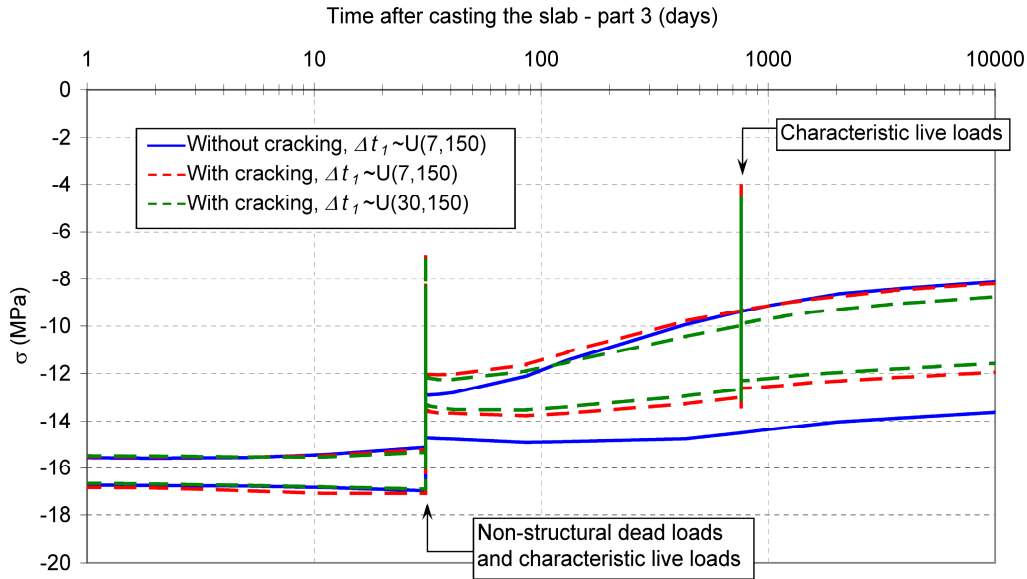


Figure 6.5 - Results of the probabilistic analysis: stresses at the bottom surface of the mid-span cross section (second span) corresponding to the 10% and the 90% percentiles.

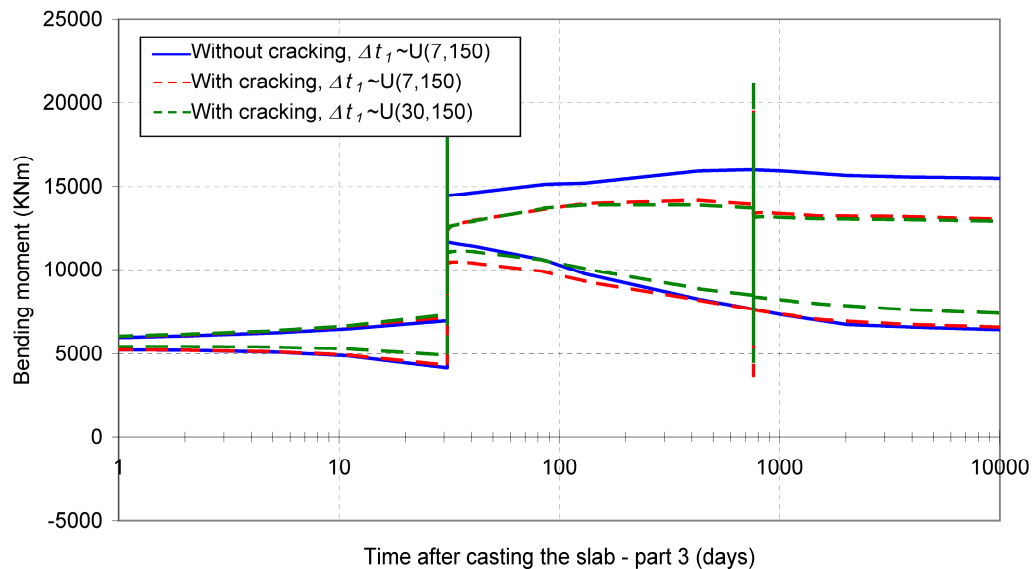


Figure 6.6 - Results of the probabilistic analysis: bending moments on the cross-section above the third support corresponding to the 10% and the 90% percentiles.

Figure 6.5 also provides important information regarding the decompression check at the bottom fibre of the mid-span cross section. This figure shows that this fibre remains compressed when the structure is subjected to live loads [minimum compression stress of -4.04 MPa when $\Delta t_I \sim U(30, 150)$], even if the variability of the long-term stress redistributions is taken into account through the 10% and the 90% percentiles of the calculated statistical distribution of concrete stresses. However, it is important to note that significantly lower compressive stresses would be obtained⁴ if railway traffic was represented by load model LM71, and not by the adopted freight train.

It is also interesting to note that the variability of the long-term behaviour is, in this case, more important than the variability of the ballast load (which is admittedly important in the context of railway bridge decks). In Figure 6.5, the difference between the 10% and the 90% percentiles at 10,000 days reaches 2.8 MPa in the case of the Monte Carlo simulation in which $\Delta t_I \sim U(30, 150)$. On the other hand, simple calculations (whose presentation is not justifiable in this context) show that the stress due to the upper bound of the characteristic ballast load is only 1.14 MPa higher than the stress due to the lower one.

Figures 6.5 and 6.6 also show that the most important part of the long-term redistributions takes place in the first few years after the end of the construction. In Figure 6.5, if we look at the results of the Monte Carlo simulation with $\Delta t_I \sim U(30, 150)$, we can see that the lower compressive stress decreases 2.91 MPa in the first two years after the end of the construction, due to the concrete delayed deformations and relaxation losses, whereas in the following 25 years ($\sim 9,125$ days) the compressive stress only decreases 1.16 MPa.

The results of the Monte Carlo simulation can also be represented in terms of diagrams of bending moments and shear forces for different percentiles of the sample distribution, as shown in Figure 6.7. This figure reveals that the variability of bending moments is significantly higher than the variability of shear forces. This is because small variations in the support reactions (due to long-term effects) give rise to important variations of bending moments, but reduced variations of shear forces.

The results presented in this section and in Chapter 5 show that the construction typology under analysis is sensitive to the uncertainty associated to the quantification of creep and shrinkage deformations and construction timings. Moreover, it is shown that the statistical distribution of

⁴ The calculation of stresses due to the load model LM71 and the corresponding dynamic factors is straightforward, since it does not depend on the concrete delayed deformations or on the sequence of construction. Simple calculations show that if the load model LM71 was considered in the characteristic combination of live loads (instead of real trains), the applied compressive stress on the point under analysis would be 4.49 MPa lower.

the structure response (in terms of stresses or internal forces) can be quantified through a Monte Carlo simulation and can be expressed by percentiles of the statistical distribution of each response variable, e.g. the 10% and the 90% percentiles. Even though the variability of the structural response is usually not relevant for ULS design (since the structure ductility is usually large enough so that creep and shrinkage effects can be redistributed through cracking and yielding), some examples of the usefulness of this probabilistic approach can be identified:

- The probabilistic approach provides quantitative information about the variability of the structure response.
- The quantitative information about the variability of stresses and bending moments due to permanent actions can be used in serviceability limit state analyses (e.g. decompression checks and verification of maximum crack widths), so that these checks are not based on deterministic analyses of highly variable phenomena. Instead, safer designs would be obtained if these checks were carried out considering the statistical distribution of the structure response (by means of the 10% and 90% percentiles e.g.). At the present time, such probabilistic approach is not required by design codes such as the Eurocodes.
- The uncertainty associated to the quantification of the long-term stress redistributions is also relevant in terms of fatigue design, because the value of the applied bending moments due to permanent actions strongly affects the fatigue life of concrete. In Sousa et al. (2010a), it is shown that the fatigue strength of the bottom slab of the precast structure under analysis, which is subjected to important compression forces in the vicinity of the supports, is sensitive to the magnitude of the hogging moment applied to the continuity connection. Therefore, fatigue analyses based on deterministic evaluations of bending moments due to permanent actions are not advisable in this type of construction. Instead, a prudent approach for fatigue analyses of elements subjected to important compression stresses (such as the girder bottom slab in the vicinity of the continuity supports) should be based on probabilistic analyses for calculation of the characteristic effects of permanent actions (thus taking into account the variability of creep deformations, shrinkage strains and girder ages at the time that the continuity is established).

It should be noted that if the influence of the variation of creep, shrinkage and construction timings was evaluated by means of parametric analyses (considering all the possible combinations of upper and lower bound values for these variables), significantly poorer information would be obtained. Figure 6.8 presents the envelope results of a parametric analysis considering the same model, the same loading, the same construction sequence and the same loading sequence that were adopted in the probabilistic analysis. Upper and lower bound values for creep and shrinkage deformations are taken as the 10% and the 90% percentiles of the statistical distribution assumed for those variables in the probabilistic analyses. Minimum and

maximum values for Δt_I are taken as 7 days and 150 days, respectively. By comparing Figure 6.8 with Figure 6.7, one can see that the range defined by the upper and the lower bound values of the parametric analysis results is considerably larger than the range defined by the 10% and the 90% percentiles of the probabilistic analysis. Moreover, unlike the probabilistic analysis, such a parametric analysis does not provide any information about the probability of occurrence of the envelope results depicted in Figure 6.8.

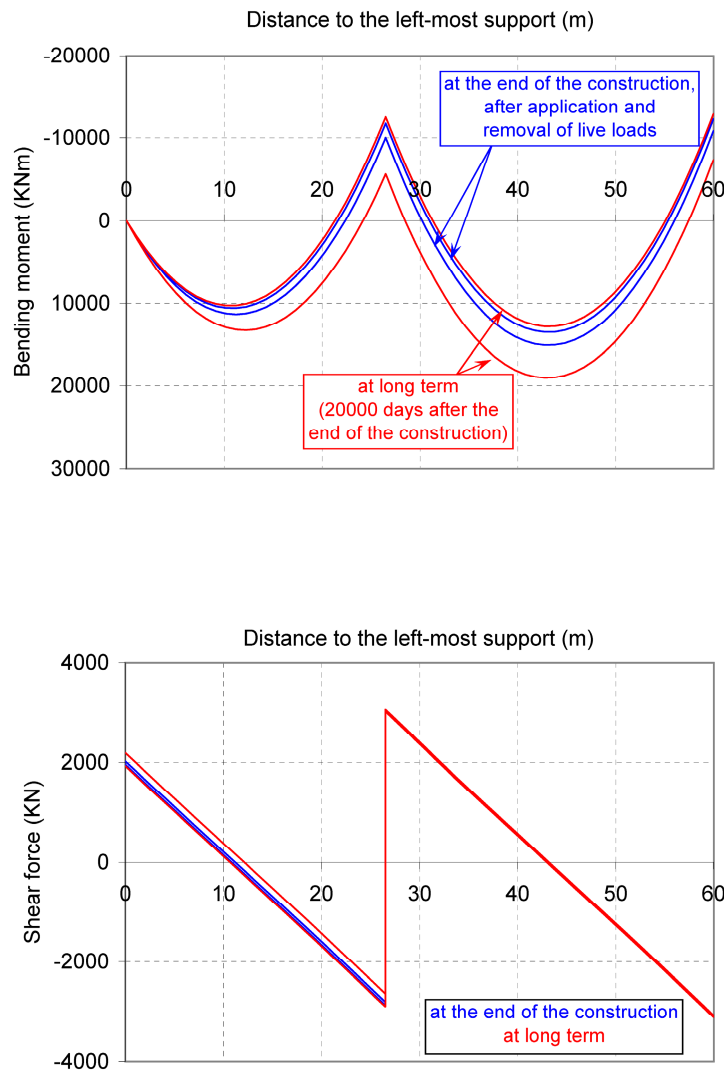


Figure 6.7 – Results of the Monte Carlo simulation with cracking influence and $\Delta t_I \sim U(30, 150)$: bending moment diagrams (not including the isostatic effect of prestress) and shear force diagrams corresponding to the 10% and the 90% percentiles.

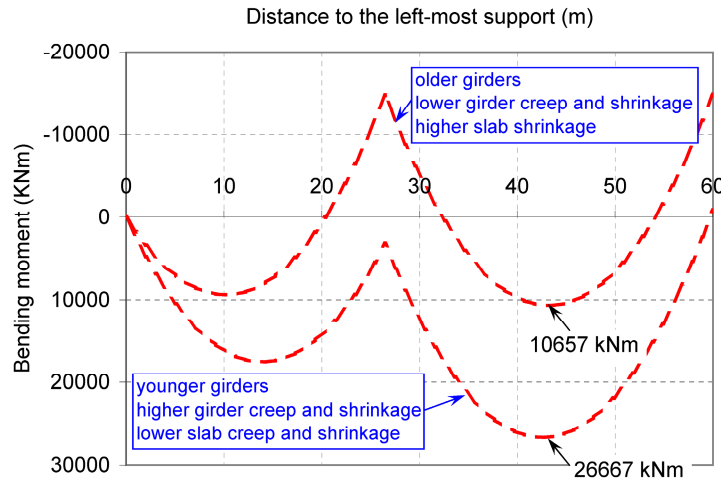


Figure 6.8 - Results of the parametric analysis considering cracking influence: long-term envelope of bending moments in the first and second spans, not including the isostatic effect of prestress.

6.5 Fatigue and ULS analysis of the girder web and the deck slab

In this section, the developed numerical methodologies for taking into account the combined effects of longitudinal shear and transverse bending are applied. Fatigue and ULS analyses are carried out for both the girder webs and the deck slab.

This section starts with the presentation of the calculated transverse bending moments and shear forces, due to permanent actions, temperature effects and railway traffic. Given that this work focuses on the application of the developed methods for fatigue analyses and not on the calculation of the applied internal forces, the calculation of internal forces is briefly summarized, and the relevant results are presented for future reference.

All the relevant combinations of permanent loads, temperature effects and traffic loads are taken into account, both in fatigue analyses and in ULS verifications. The combinations of actions are quantified according to the provisions of the Eurocodes. The results are shown in diagrams where the variation of the internal forces and the analysis results over one of the spans are shown (the second span, between P1 and P2, as represented in Figure 6.3).

6.5.1 Calculation of internal forces

The internal forces installed in the deck slab and in the girder webs are determined through FE analyses in which the structure is modelled with four-node shell elements, the size of each element side being approximately equal to 30 cm. The FE package ANSYS (2009) is employed, and the different structural components (top and bottom slabs, webs and support diaphragms) are simulated through shell elements, the plane of each element being positioned at the mid surface of each concrete plate. The cross section of the FE model is represented by a dashed line in

Figure 6.9, where one can see that the web is simulated through a straight panel. No deviation of the plane of the finite elements is considered at the top of the web, where the web thickness is locally enlarged. The modal superposition method is employed in the calculation of the dynamic effects of the real trains' passage. The influence of calculation parameters such as modal damping ratios and number of modes of vibration considered in the calculations is briefly discussed in this section.

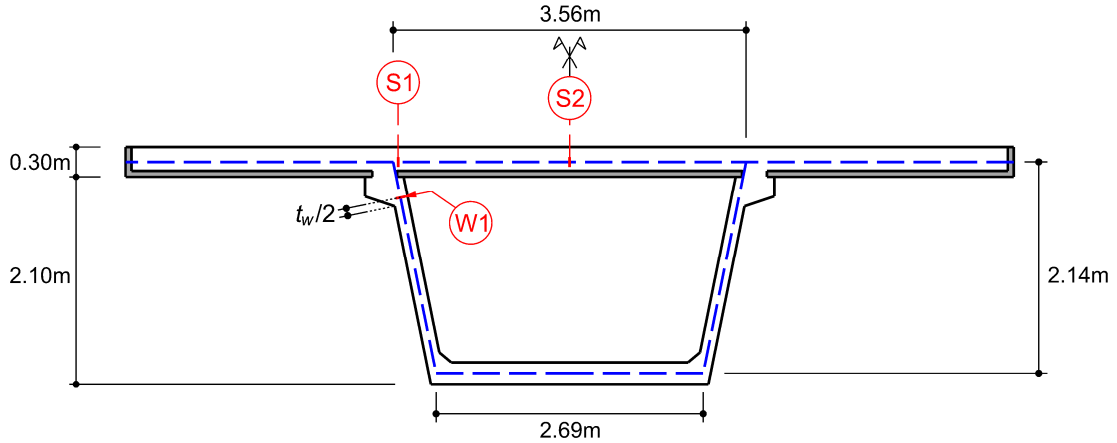


Figure 6.9 – Cross sections for output of the results.

The construction sequence is not simulated in this FE analysis. Therefore, the out-of-plane bending moments due to the structure self weight are calculated by considering that the corresponding loads are applied to the finished structure. This is a simplified procedure, which does not take into account the fact that, before the third part of the slab has been cast and the concrete has hardened, the structure exhibits a different configuration. However, this procedure is acceptable in this context, because:

- The construction phase which is responsible for the highest transverse bending moments is the third phase of slab construction, which corresponds to casting the slab wings (see Figure 6.4). As shown before, this construction phase is carried out only after the slab is monolithically connected to the girder webs. Therefore, the adopted analysis procedure (not considering the construction sequence) provides approximate estimates for the out-of-plane moments due to this load case.
- The internal forces due to permanent loads vary with time, due to concrete creep deformations. As a consequence of such variations, the real distribution of internal forces deviates from the initial distribution (calculated through a phased analysis) and gets closer to the distribution calculated with the adopted simplified procedure.
- A significant percentage of dead loads is formed by the non-structural permanent loads, and these are applied after the structure is finalized, i.e., the internal forces caused by these loads are not affected by phased construction.

Figure 6.9 depicts the relevant locations for ULS and fatigue analysis of the deck slab and the web. The cross sections S1 and S2, in the slab, correspond to the locations where the highest cyclic bending moments (due to traffic loads) occur. The cross section W1 is the critical location for analysis of the girder webs, because the highest cyclic transverse bending moments (in the web) occur at the top extremity. The cross section W1 is located at half the current web thickness from the location where the local enlargement of the web starts, this distance being based on the simplified rules given by the EN 1992-1-1.

In the present section, only the diagrams of in-plane shear forces and transverse bending moments are shown. The diagrams for the remaining flexural and membrane forces, in the cross sections S1, S2 and W1, along the second span, are shown in Annex 3.

This section also presents the results of a simplistic calculation of the transverse bending moments due to static load cases (permanent loads, temperature effects and traffic load models LM71 and SW/2) by means of a finite-element model with beam elements. One meter wide beam elements are employed to model the structure cross section through a 2D frame (the beam axes are represented by the dashed line in Figure 6.9), this structure being simply supported at the intersection of the bottom slab and the webs. Two different models were considered, one for each web thickness. The comparison of the results provided by the beam FEM and the shell FEM shows the relevance of the restraint introduced by diaphragms and supports, and the influence of the web thickness variation.

6.5.1.1 Permanent loads and temperature differences

The transverse bending moments in the locations S1, S2 and W1, due to permanent loads and temperature effects, are shown in Figure 6.10. One can see that the most important differences between the results provided by the beam and the shell models occur in the case of the temperature difference in the deck slab. As expected, the variation of the web thickness gives rise to important modifications of the transverse bending moments applied to the web (as the web stiffness increases, the location W1 becomes subjected to significantly higher transverse bending moments).

6.5.1.2 Railway traffic actions

Figure 6.11 presents the envelopes of transverse bending moments due to the passage of one real train, the train type 5 (TT5) identified in the Annex D of the EN 1991-2 (the train with the highest weight per unit length), the train speed being equal to 80 km/h (the value recommended by the EN 1991-2). In this figure, the static effects of this train's passage (determined through a transient analysis with structural mass taken as zero) are compared with the results of calculations in which the dynamic effects are taken into account.

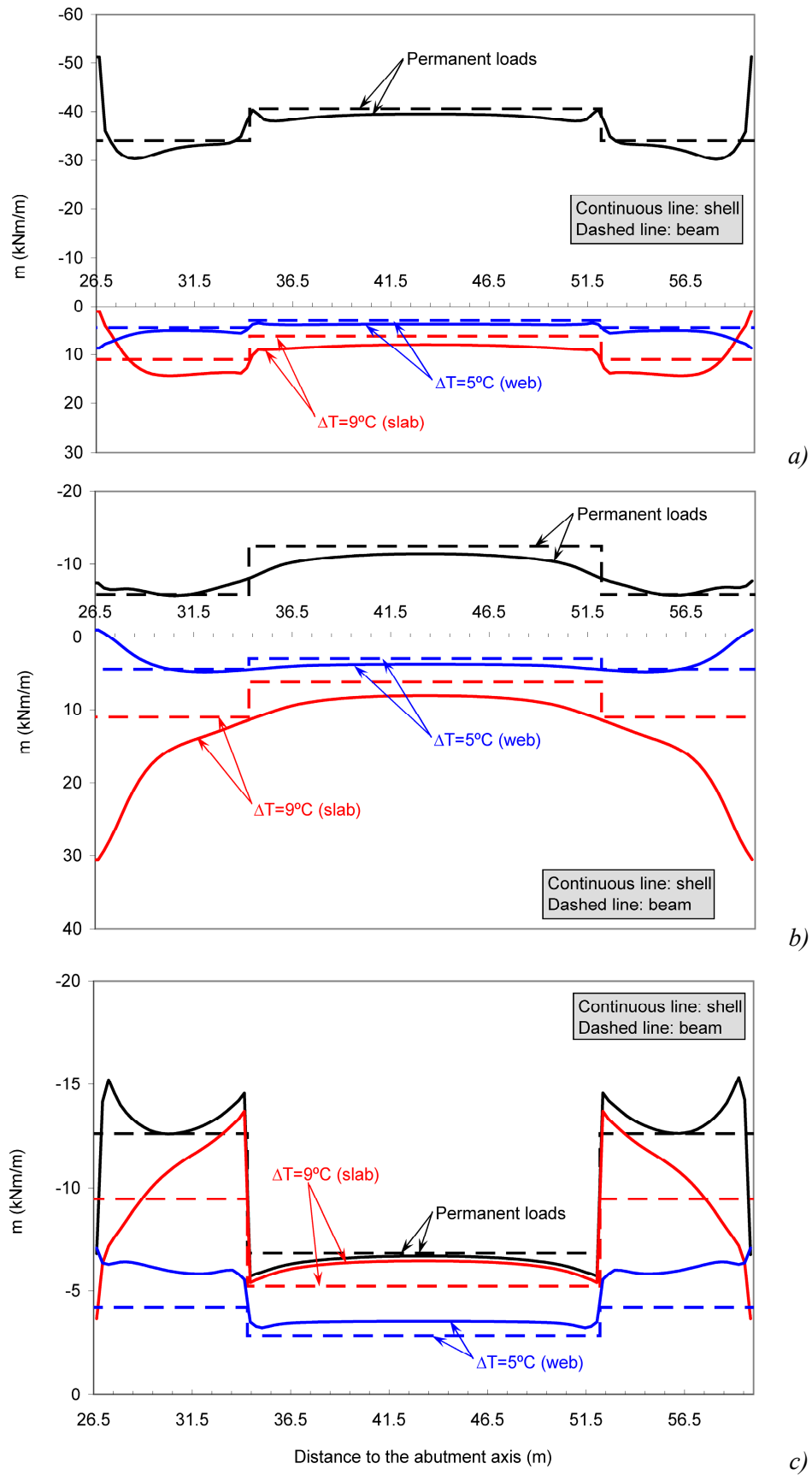


Figure 6.10 – Diagrams of transverse bending moments due to permanent loads and temperature differences, for the second span: a) cross section S1; b) cross section S2; c) cross section W1.

As mentioned before, dynamic analyses were carried out through the modal superposition method, and a parametric analysis was performed so that the influence of the adopted modal damping ratio and the assumed number of modes of vibration could be ascertained. The adopted values for these parameters are shown in the figure legend. The modal damping ratio took, in general, the value $\xi = 1\%$, which is the value recommended by the EN 1991-2. The influence of this parameter was assessed by performing additional analyses considering different (higher) modal damping ratios. This is because it has been shown that 1% is a conservative assumption for precast railway viaducts with span lengths similar to the ones in the viaduct under analysis (Malveiro et al. 2011a; Malveiro et al. 2011b). As regards the total number of modes of vibration considered in the modal superposition method, three different analyses were performed, in which the upper natural frequency takes the value 120 Hz (450 modes of vibration), 60 Hz (181 modes of vibration) and 30 Hz (88 modes of vibration). The results of the parametric analysis clearly show that an analysis in which the upper natural frequency is equal to 30 Hz is clearly inadequate. In this case, not even the static effects of the train's passage are captured by the modal superposition method. On the other hand, an upper natural frequency of 60 Hz leads to small errors with respect to the more accurate results obtained with an upper frequency of 120 Hz. As regards the modal damping ratio, Figure 6.11 shows that this parameter plays a minor role in this case, since the moderate value of the train speed does not give rise to important resonance effects.

In Figure 6.12, the envelopes of bending moments due to the train type 5 (identified by the label TT5) are compared with the envelopes of the traffic load models LM71 and SW/2. All the load models are affected by dynamic coefficients, partial factors γ , and factor α , as defined by the EN 1991-2 (see Table 6.7), so that the most unfavourable traffic load model for ULS analysis can be identified. These figures show that the load model LM71 is responsible for the highest bending moments in all the cross sections under analysis. Therefore, only this load model is considered in ULS analyses. Real trains such as train type 5 are to be considered in fatigue analyses. As regards the type of analysis for calculation of bending moments due to real trains, Figure 6.12 shows that the dynamic and the static analysis multiplied by the dynamic coefficient yield approximate results.

Figure 6.12 also illustrates the results of simplified analyses with beam FE models. In this case, the bending moments due to load model LM71 were determined by considering that the point load of 250 kN is evenly distributed along a length of 1.6 m (the distance in the longitudinal direction between successive point loads). One can see that, for this unicellular box girder supporting a single track, approximate predictions of the maximum bending moments can be obtained with the simplified model. As expected, the biggest differences take place in the region close to the continuity diaphragm, where the simplified model is too conservative (namely in terms of bending moments in the web).

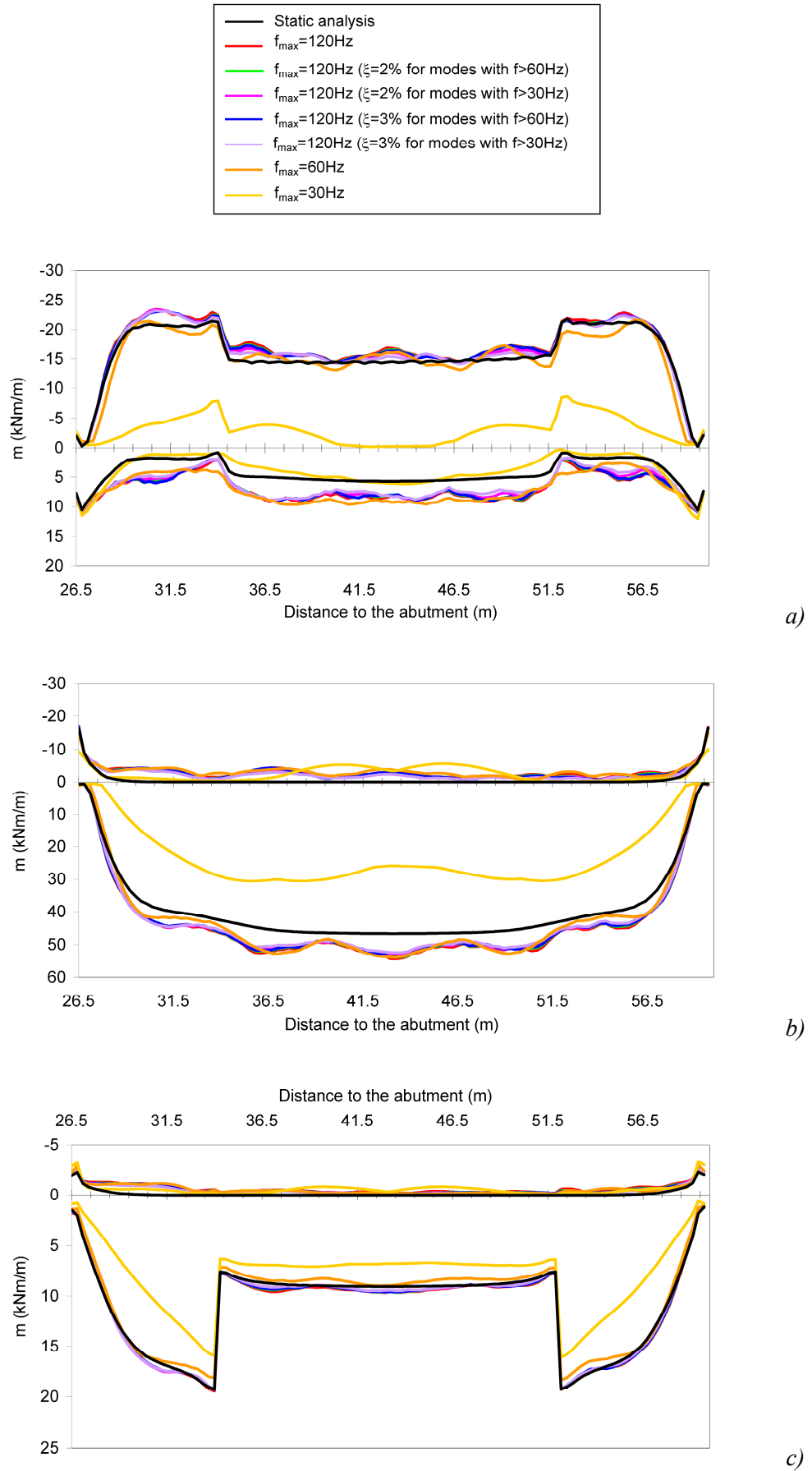


Figure 6.11 – Envelopes of transverse bending moments due to the passage of the fatigue train 5, for the second span: a) cross section S1; b) cross section S2; c) cross section W1.

Table 6.7 - Dynamic coefficients and partial factors γ for traffic load models LM71 and SW/2, and train type 5, as defined by the EN 1991-2 and the EN 1990 (Annex A2).

Traffic load model	Dynamic coefficient	Partial factor γ for ULS	Factor α
LM71	1.43 ⁽²⁾	1.45	1.21
SW/2	1.43	1.20	--
Train type 5 (static analysis)	1.48 ⁽³⁾	1.45	--
Train type 5 (dynamic analysis) ⁽¹⁾	1.38 ⁽⁴⁾	1.45	--

NOTES: ⁽¹⁾ Dynamic analysis with an upper natural frequency of 120 Hz and $\xi = 1\%$

⁽²⁾ $\Phi_3 = 1.43$ (for tracks with standard maintenance)

⁽³⁾ $(1+\phi'+\phi'') = 1.48$ according to the Annex C of the EN 1991-2

⁽⁴⁾ $(1+\phi'') = 1.38$ according to the Annex C of the EN 1991-2

In Figure 6.12, one can also see that the envelopes of bending moments due to the load model LM71 exhibit a wave-shaped form. This is due to the fact that those envelopes were determined by considering 5 train positions along each half span. Smoother envelopes would be achieved if more load positions were considered, but the curves represented in Figure 6.12 are valid for design purposes.

6.5.1.3 Combinations of actions

Once the different load cases have been analysed, the various combinations of actions can be determined. In this section, only the envelopes of transverse bending moments for characteristic combinations of actions are shown, because these values are to be compared with the cracking moment in order to evaluate whether stress calculations for fatigue analyses must be carried out on the basis of a cracked section. By comparing the envelopes (Figure 6.13) and the cracking moments (Table 6.8), one can see that sections S1, S2 and W1 must be considered to be cracked for the purpose of fatigue analyses. Even though the cracking moment of the web is not exceeded in the vicinity of the continuity supports, the fatigue analysis of those regions also needs to be carried out under the assumption of cracked sections, due to the influence of the remaining force components. That is, if the principal tensile stress was calculated and compared with the tensile strength, it would be concluded that those cross sections are cracked (see §6.6). It is important to remark that the calculation of cracking moments was based on the axial concrete tensile strength, not the flexural tensile strength, for the reason that it has been shown that the flexural tensile strength leads to overestimated cracking moments for loads applied when important self equilibrated stresses (due to differential shrinkage) are installed in the cross section under analysis (Azenha et al. 2011). Even lower cracking bending moments would be obtained if the strength reduction due to cyclic loading was taken into account.

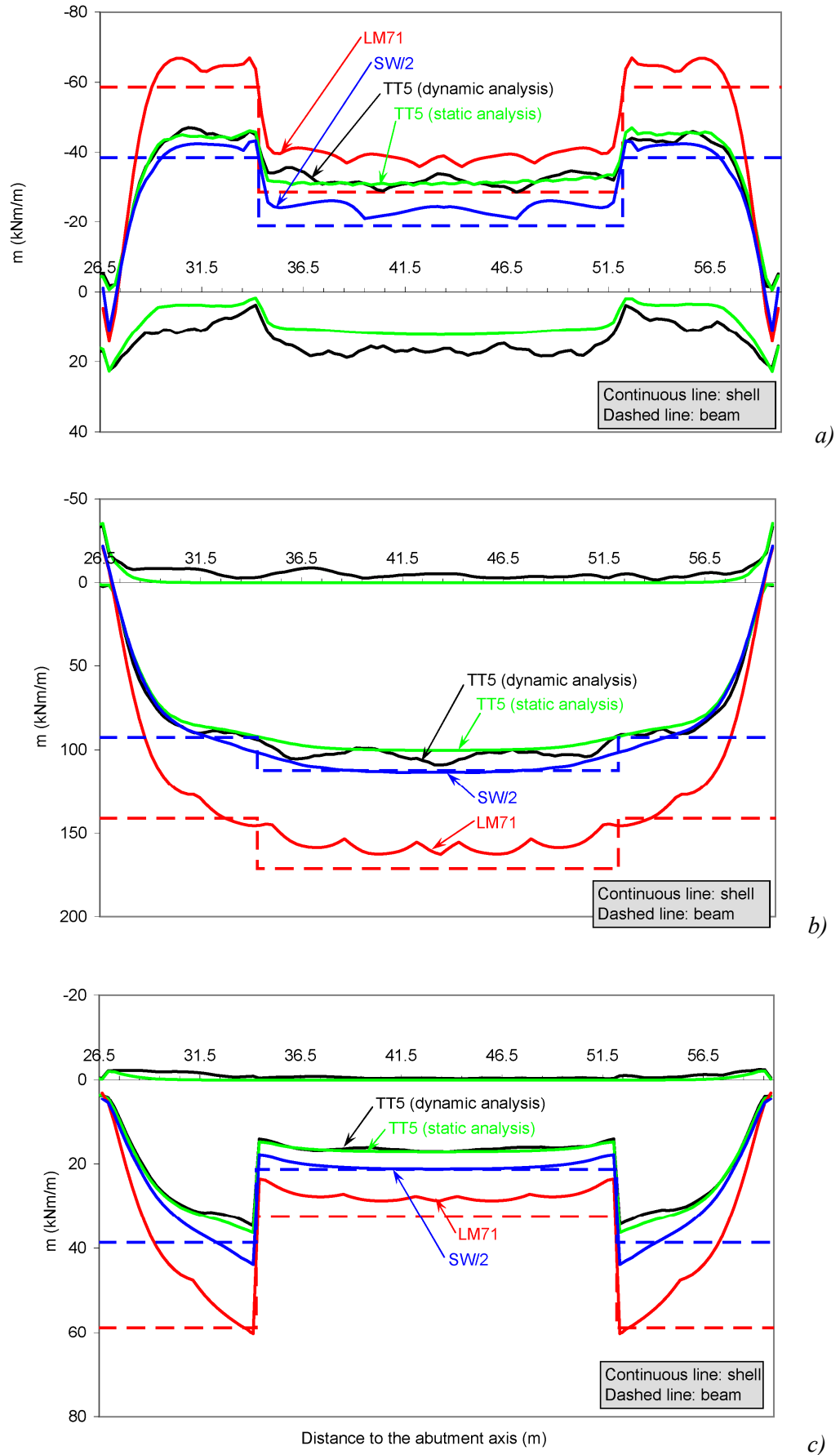


Figure 6.12 – Design value of bending moments in ULS (dynamic coefficients and partial factors γ included) due to rail traffic loads, for the second span: a) cross section S1; b) S2; c) W1.

Given the results shown in the previous sections, it is not surprising that the finite element model with beam elements provides good estimates of the maximum applied bending moments, as shown in Figure 6.13. However, these models are too conservative in the region close to the continuity diaphragms.

Table 6.8 – Cracking moments based on the mean value of the axial concrete tensile strength.

	f_{ctm} (MPa)	M_{cr} (kNm/m)
Slab	2.90	43.5
0.21 m thick web	4.10	29.0
0.15 m thick web	4.10	14.8

6.5.2 Fatigue and ULS analysis of the girder web

This section focuses on the analysis of the girder web, under the combined effects of longitudinal shear and transverse bending. The analyses are carried out for half the length of the second span.

The necessary diagrams of applied shear forces for the various load cases are shown in Figure 6.14. In this figure, the traffic effects are represented by the load model LM71, which is determinant for ULS verifications.

In the fatigue analyses, the effects of cyclic loading (i.e., the internal forces due to the passage of trains) are superposed to the ones due to non-cyclic loadings. Following the suggestion by the Eurocodes, the various non-cyclic loadings are combined in such a way as to obtain the so called basic combinations of non-cyclic loadings (BCNCL). Temperature differences are also included in the combination of non-cyclic loadings, because temperature gradients affect the state of stress installed in the girder webs before the trains enter the bridge, which, in turn, affects the histograms of stress ranges due to the passage of the trains. The diagrams of basic combinations of non-cyclic loadings are represented in Figure 6.15. One can see that, unlike the shear force, the transverse bending moment in BCNCL strongly depends on the temperature action.

The traffic scenario for fatigue verifications is the heavy traffic mix specified by the EN 1991-2, with a traffic volume of $25 \cdot 10^6$ t/year, and the required fatigue life is equal to 100 years. The fatigue strength of the stirrups is given by the S-N curves proposed by the EN 1992-1-1 for straight reinforcing bars [Equation (3.1)]. The S-N curves adopted for concrete under compression are those defined in the EN 1992-2 [Equation (3.3)]. The strength reduction factor ν for fatigue analyses is taken as 0.7, following the discussion presented in §3.4.

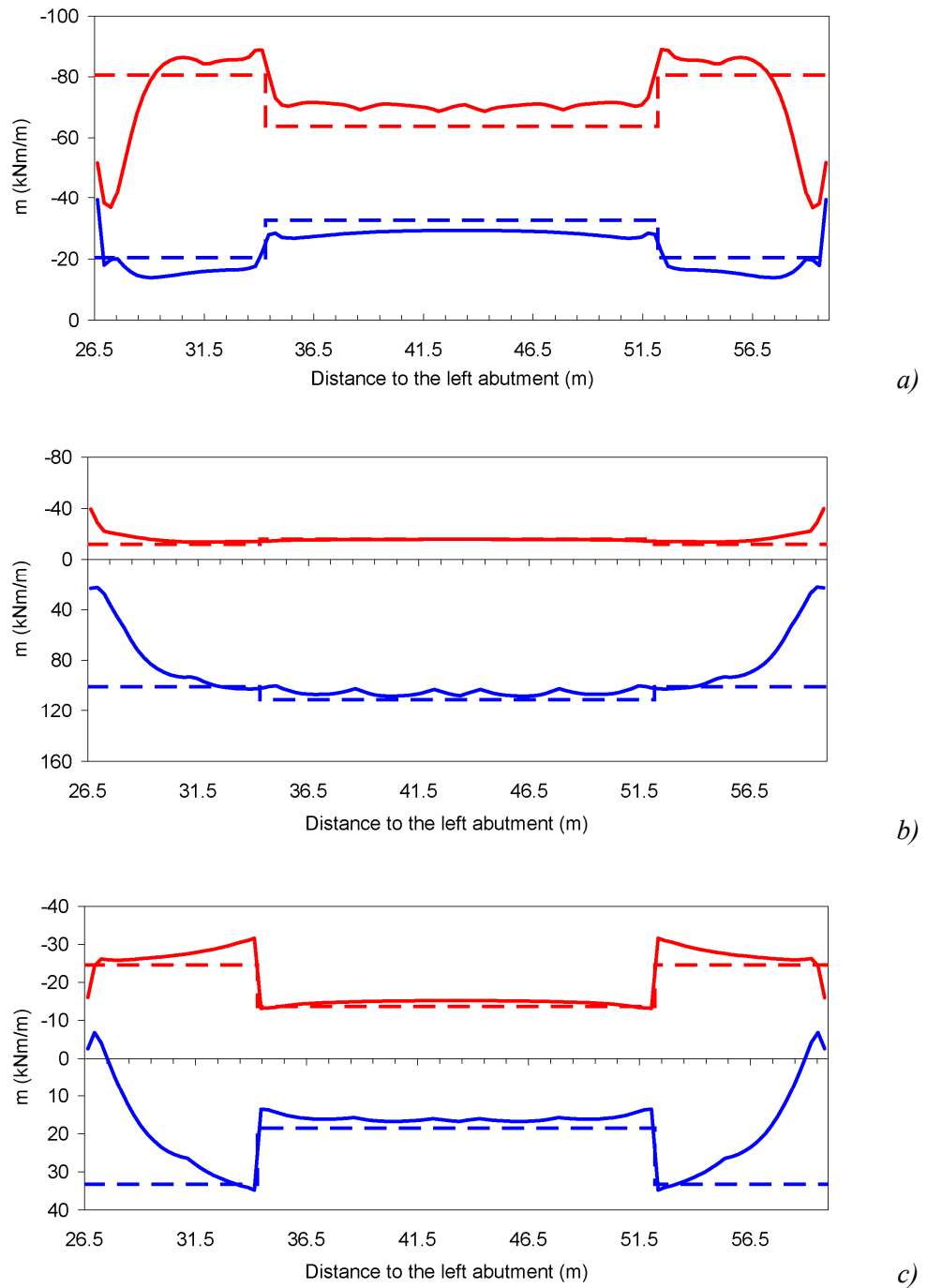


Figure 6.13 – Envelopes of transverse bending moments for characteristic combinations of actions, along the second span, at the following critical sections: a) S1; b) S2; c) W1. Continuous lines and dashed lines for analyses with shell finite elements and beam finite elements, respectively.

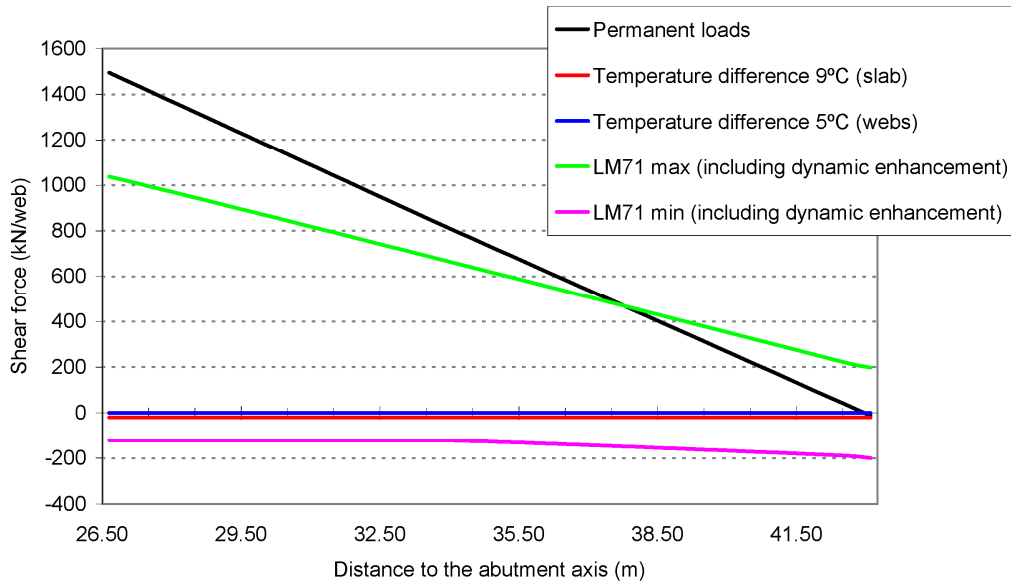


Figure 6.14 – Diagrams of longitudinal shear forces.

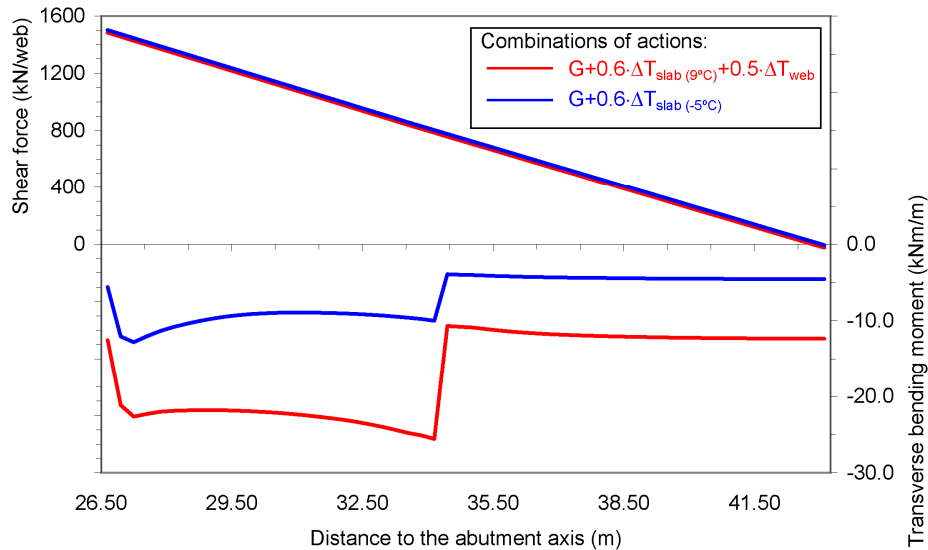


Figure 6.15 – Diagrams of basic combinations of non-cyclic loading, for shear forces and transverse bending moments at the location W1 (identified in Figure 6.9).

The fatigue analysis involves the various steps identified before. The first one consists of calculating the histories of shear forces and transverse bending moments due to the passage of the different trains within the adopted traffic scenario. Figure 6.16 shows these histories for one of the cross sections under analysis. Given that the operating speed for these trains (shown in Table 6.9) does not give rise to important resonance effects, the static effects of the trains' passage (shown in Figure 6.16) were determined through transient analyses in which the mass matrix is taken as zero. The dynamic effects are taken into account through the dynamic enhancement specified in the EN 1991-2, Annex D, and shown in Table 6.9.

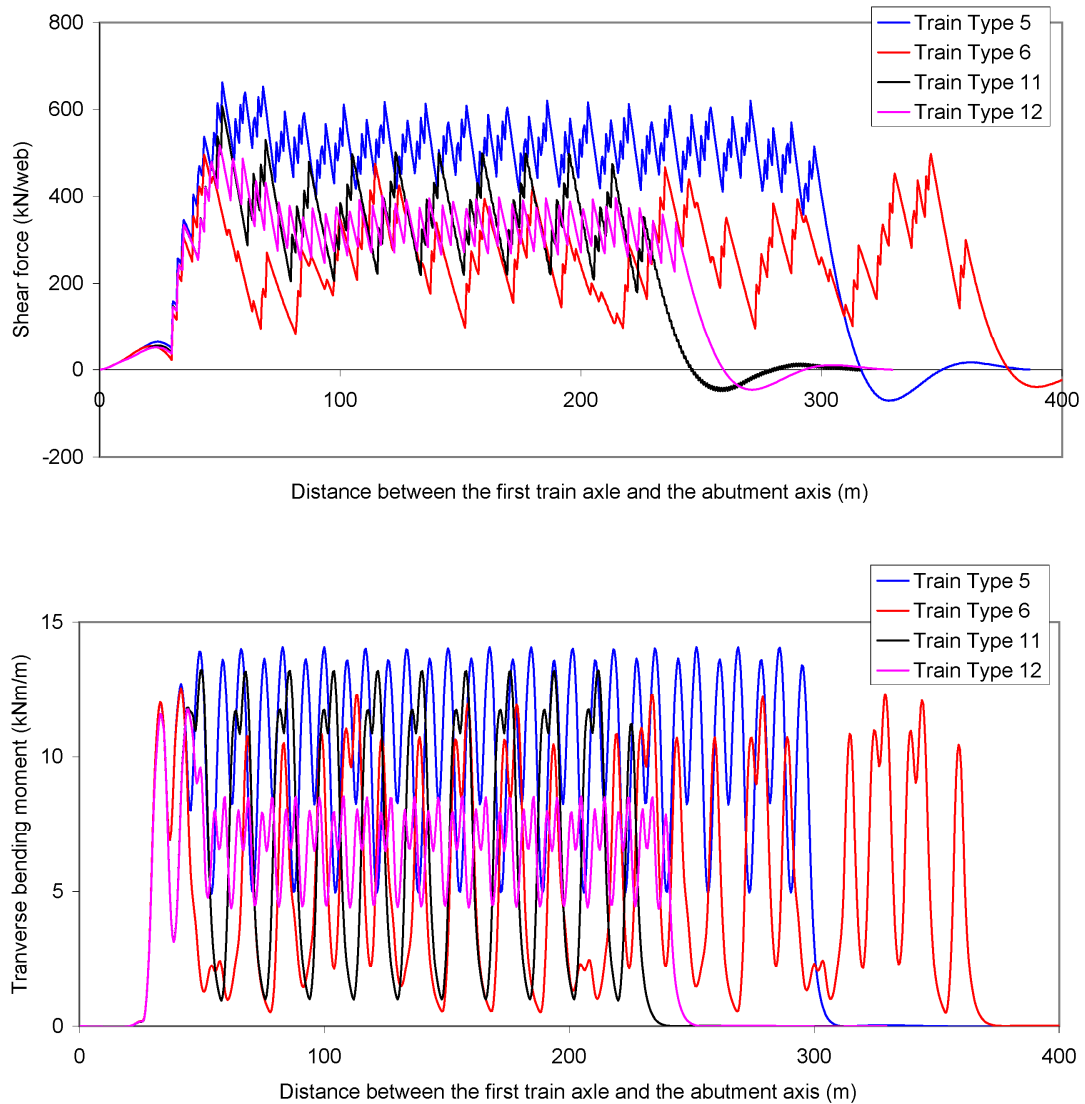


Figure 6.16 – Histories of shear forces and transverse bending moments at the location W1, at the cross section 30.5 m apart from the abutment axis, not including the dynamic enhancement.

Table 6.9 – Dynamic enhancement of shear forces and transverse bending moments for fatigue analyses.

Train type (EN 1991-2, Annex D)	Operating speed (km/h)	Dynamic enhancement $1 + 0.5 \cdot \varphi' + 0.25 \cdot \varphi''$	
		For shear forces	For transverse bending moments
5	80	1.057	1.125
6	100	1.073	1.150
11	120	1.091	1.176
12	100	1.073	1.150

These histories of internal forces due to the passage of trains are superposed to the non-cyclic loads, whose values, for the various combinations of actions, are shown in Figure 6.15.

Once the histories of in-plane shear forces and transverse bending moments are determined, the stress histories are computed through the algorithm presented in §4.3.2. The web thickness was divided into four concrete layers. The cover to the stirrups, with respect to the stirrup axis, is in this case equal to 2.8 cm. The inclination of the compression struts to be used in fatigue analyses is given by Equation (4.6), and depends on the minimum inclination admissible in ULS verifications, which, in turn, depends on the web thickness, on the concrete compressive strength and on the maximum applied in-plane shear forces. Such minimum admissible inclination can be quantified by employing Equations (4.1), (4.5) and (4.7). The calculated inclinations are depicted in Figure 6.17. This figure shows that the highest inclination for fatigue analyses takes the value $\theta_{FAT} = 36^\circ$, and occurs in the region close to the continuity support. It is important to note that the inclination calculated at the face of the support ($\theta_{FAT} = 36^\circ$) must be adopted for all the cross sections whose distance to the support axis is shorter than the effective depth of the cross section, d (EN 1992-1-1). The inclination to be used for cross sections whose distance to the support axis is greater than 2.5 m is $\theta_{FAT} = 32^\circ$, and corresponds to the minimum admissible inclination for fatigue analyses (see Figure 6.17). These are the values adopted in the fatigue analyses of the girder web, either in calculations in which transverse bending is disregarded, or in the ones where that effect is taken into account.

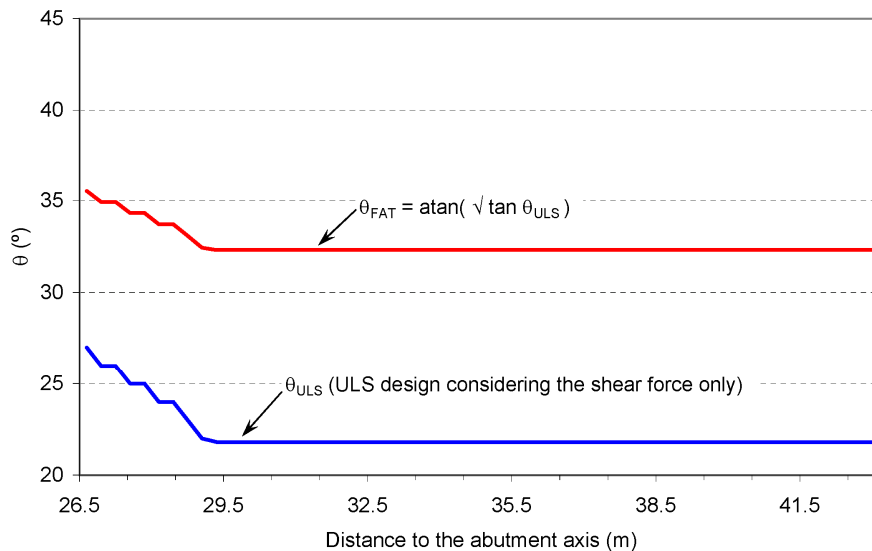


Figure 6.17 – Inclination of the compression struts: results of the ULS design considering the sole effect of the shear force and the corresponding inclination for fatigue design.

Once the stress histories are calculated, a cycle counting algorithm is applied, so that the stress histories can be converted into stress histograms. This step is performed by employing the Rainflow algorithm as described by Fryba (1986). After that, the annual fatigue damage due to the adopted traffic scenario can be computed by applying the Palmgren-Miner rule for linear damage accumulation. The fatigue life is given by the inverse of the annual fatigue damage.

Table 6.10 and Figure 6.18 present the results of fictitious fatigue analyses not considering the effects of transverse bending. These results are only shown for illustrative purposes, so that the relevance of transverse bending can be evaluated through the comparison of analyses in which this effect is, or is not, considered.

Fatigue analyses are carried out for four cross sections, whose distance to the support axis is 2 m, 4 m, 6 m and 10 m, respectively. The first one is at a distance from the support approximately equal to the effective depth of the cross section, this being the critical cross section for quantification of the required area of shear reinforcement in ULS analyses considering the sole effect of in-plane shear (EN 1992-1-1). The remaining cross sections are chosen so as to be representative of the different situations in terms of maximum applied forces and web thicknesses.

Table 6.10 presents the calculated fatigue lives for the four cross sections, considering the real web thickness and the real area of shear reinforcement. This table shows that calculations considering the provided area of stirrups yield fatigue lives well beyond 100 years. The calculated fatigue life for the compression struts is well beyond the required minimum value of 100 years, too.

Table 6.10 – Fatigue life of the web stirrups under the effect of the Heavy Traffic Mix, not considering the influence of transverse bending moments (considering the real web thickness and the real cross-sectional area of shear reinforcement).

Distance to the abutment axis (m)	A_{sw}/s (cm ² /m/web)	Fatigue life (years)
28.5	31.4	11,558
30.5	31.4	108,264
32.5	22.6	12,794
36.5	15.8	3,831

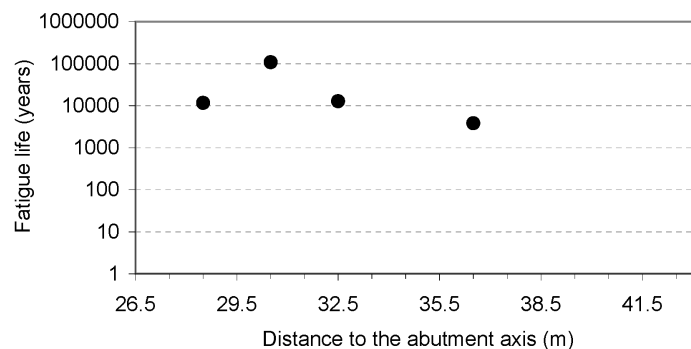


Figure 6.18 - Fatigue life of the web stirrups under the effect of the Heavy Traffic Mix, not considering the influence of transverse bending moments (considering the real web thickness and the real cross-sectional area of shear reinforcement).

The results of more realistic analyses, taking into account the influence of transverse bending, are presented below.

The histories of concrete and steel stresses, for one of the trains and one of the cross sections under analysis, are shown in Figure 6.19 (only the peaks of each stress cycle are shown, since only these values are used in the calculation of stress histograms). The corresponding histograms of steel stresses are shown in Figure 6.20. The results are shown for two extreme k_m values (0.2 and 0.8) and also for calculations in which transverse bending is not considered.

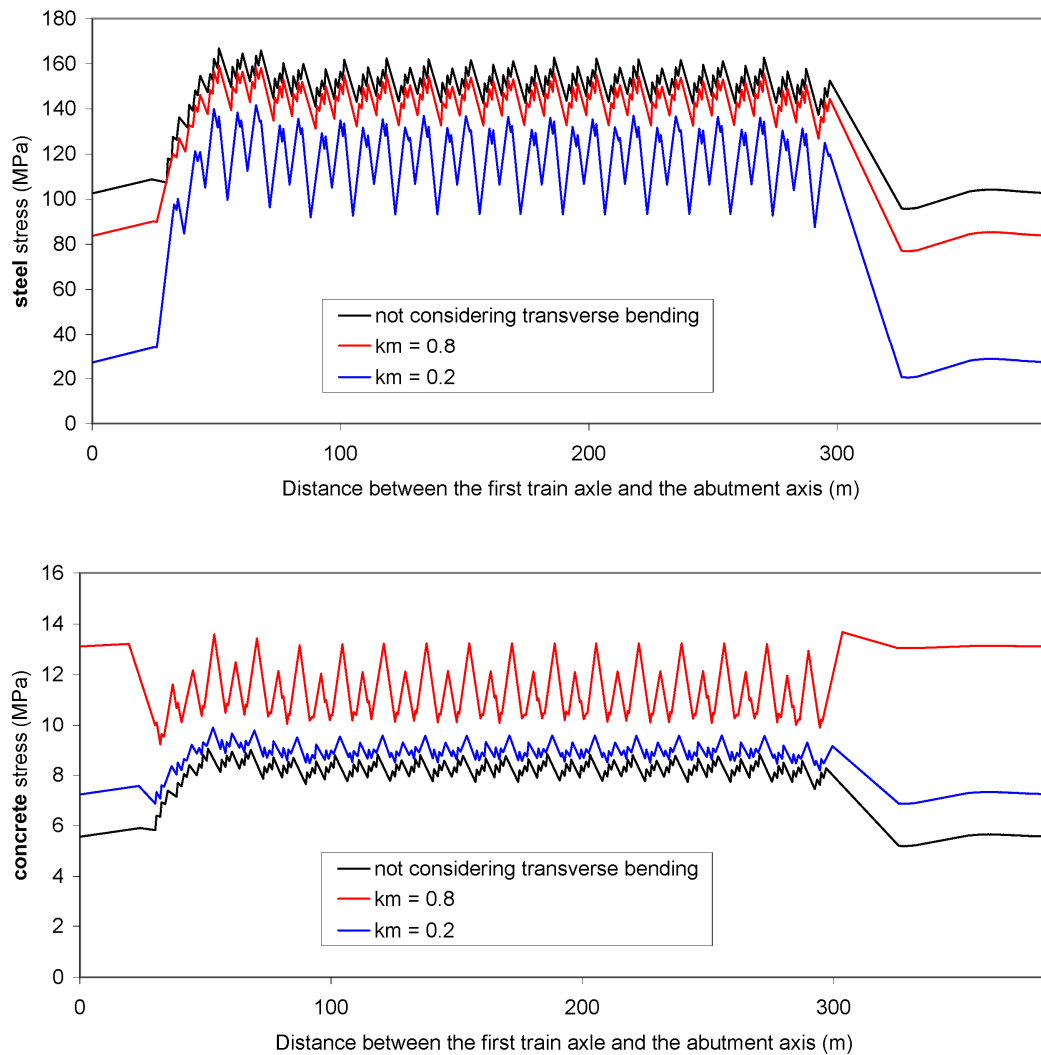


Figure 6.19 – Stress histories in the outer stirrup leg and the outer concrete layer of the web, at the location W1 of the cross-section situated 30.5 m apart the abutment axis. Stresses due to the train type 5 and the most unfavourable basic combination of non-cyclic loads ($G + 0.6 \cdot \Delta T_{slab} (9^{\circ}\text{C}) + 0.5 \cdot \Delta T_{web}$). Results of analyses considering the real web thickness and the real cross-sectional area of shear reinforcement

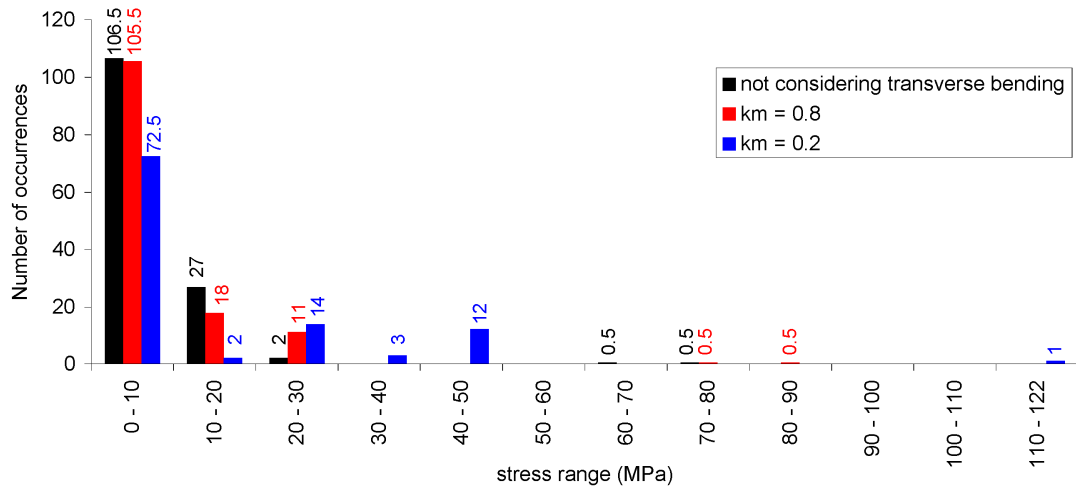


Figure 6.20 – Histograms of stress cycles for the histories of steel stresses represented in Figure 6.19.

By analysing the stress histograms shown in Figure 6.20, one can see that the majority of applied stress cycles lies in the range of low steel stresses, and consequently, the fatigue damage caused by these stress cycles is negligible. In this case, only the two greatest half cycles, which correspond to the entrance and leaving of the train, are responsible for significant fatigue damage (see Table 6.11, where the total damage caused by a single train passage is compared with the damage due to the two greatest half cycles).

Table 6.11 – Fatigue damage due to a single passage of the train type 5 for the histories of steel stresses represented in Figure 6.19.

	Total damage due to a single train passage	Damage due to the two greatest half cycles
Not considering transverse bending	$1.4473 \cdot 10^{-9}$	$1.4471 \cdot 10^{-9}$
$k_m = 0.8$	$5.6619 \cdot 10^{-9}$	$5.6612 \cdot 10^{-9}$
$k_m = 0.2$	$2.0139 \cdot 10^{-7}$	$2.0110 \cdot 10^{-7}$

Figure 6.21 summarizes the results of ULS and fatigue analyses by presenting the required areas of shear reinforcement. The results of analyses in which transverse bending is not considered are compared with analyses considering the combined effects of in-plane shear and transverse bending, so that the influence of transverse bending can be ascertained. The results depicted in this figure correspond to the most unfavourable combination of actions. The results of ULS verifications are represented by lines and shown for the entire length of the half span under analysis. ULS calculations were performed for cross sections with steps of 0.30 m along the

longitudinal direction of the girder. On the other hand, the results of fatigue analyses based on the damage accumulation method (DAM) are represented by dots in the same figure.

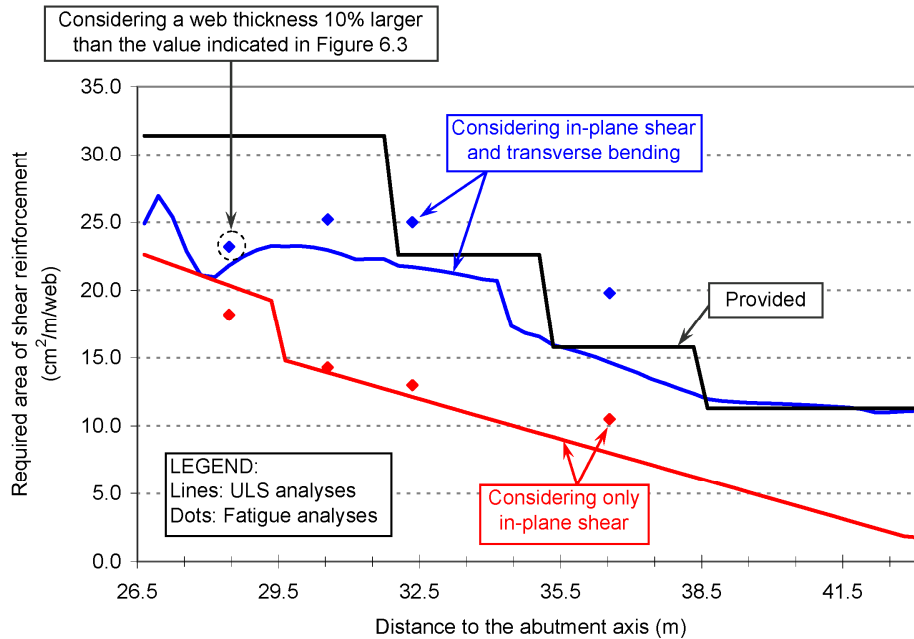


Figure 6.21 - Required area of shear reinforcement for ULS and fatigue analyses. Comparison between analyses in which transverse bending is not considered and analyses considering the effects of in-plane shear and transverse bending.

ULS analyses considering the effects of in-plane shear and transverse bending are carried out according to the procedure described in Annex 1, based on the proposal by Menn (1986). It should be noted that, while the calculation of the cross sectional area of shear reinforcement required for ULS verifications is direct, the area required for fatigue verifications is determined iteratively (by assuming tentative values until the area which guarantees a fatigue life of 100 years is found).

The calculated fatigue lives of the web stirrups, considering the real web thicknesses (indicated in Figure 6.3) and the cross sectional area of shear reinforcement required for ULS analyses taking into account the effects of shear and transverse bending (quantified in Figure 6.21), are shown in Figure 6.22 (top). In that figure, the dots represent the fatigue life of the stirrups for different k_m values in the range of 0.2 to 0.8. The adopted symbology identifies the cases in which the calculated fatigue life is lower than 100 years. If a given k_m value is not admissible in a specific cross section (for the reason that the upper bound limit for the eccentricity of the compression struts is exceeded), the fatigue life of the stirrups is not quantified for that k_m value. Therefore, by analysing this graphical representation and taking into account the criteria for definition of the critical k_m value presented in §4.3.2.1, one can evaluate whether the adopted web thickness and cross sectional area of stirrups guarantee that the intended fatigue life (100 years) is reached. The results shown in Figure 6.22 (top) indicate that, in three of the cross

sections under analysis, k_m values higher than 0.4 are not admissible (otherwise, the eccentricity $e_{max,fat}$ would be exceeded). The same figure, which refers to calculations in which the area of shear reinforcement is taken as the area required for ULS verifications, also shows that the fatigue life of the stirrups is lower than 100 years. As regards the fatigue strength of the web concrete, the figure also shows that the adopted web thickness (indicated in Figure 6.3) is not sufficient in the case of the cross section at a distance d from the support. Therefore, the area of transverse reinforcement required for fatigue verifications was quantified by considering an increased web thickness in this cross section (a 10% increase with respect to the web thickness indicated in Figure 6.3 was found to be largely sufficient). The required areas of shear reinforcement are displayed in Figure 6.21, and the calculated fatigue lives of the stirrups for the various admissible k_m values are shown in Figure 6.22 (bottom).

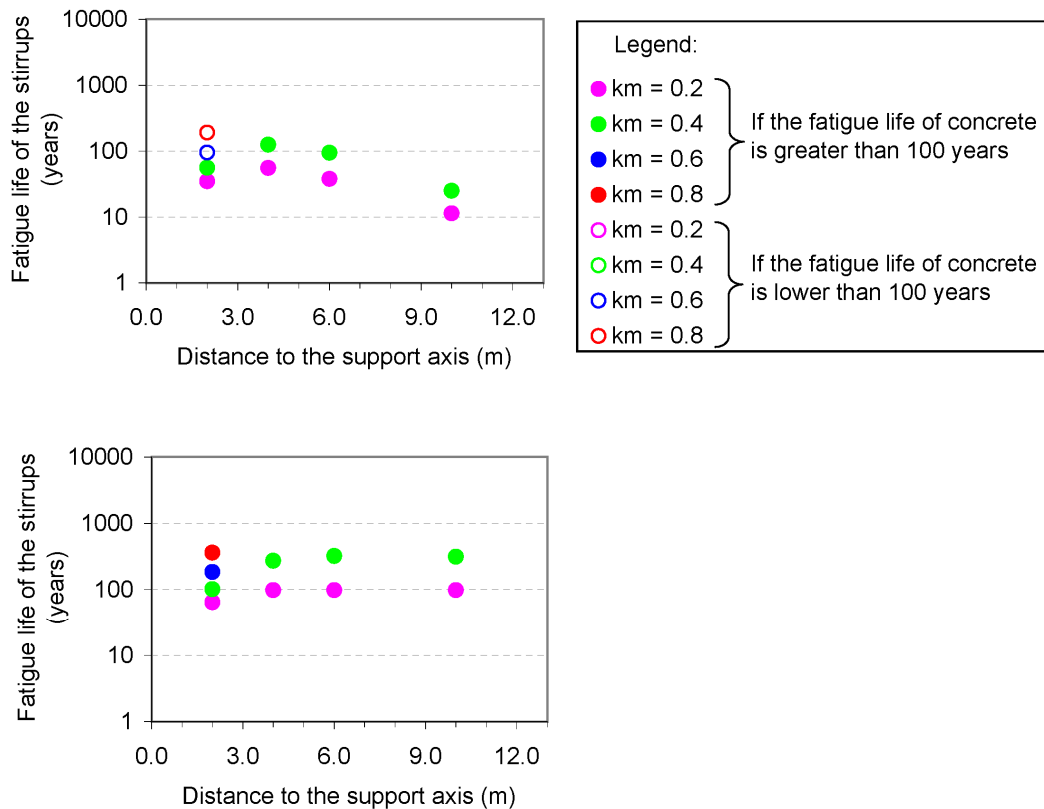


Figure 6.22 – Fatigue lives of the stirrups for different k_m values, under the effect of the heavy traffic mix of railway traffic. Results considering the area of transverse reinforcement required for ULS verifications (top) and considering the area which guarantees a fatigue life of 100 years (bottom).

The results presented in Figure 6.21 show that transverse bending can not be ignored in the design of this type of structure, because the required areas of transverse reinforcements are considerably larger when this effect is taken into account, when compared with calculations considering the sole effect of in-plane shear. This conclusion is valid for both ULS and fatigue verifications. This figure also shows that fatigue plays an important role in this type of structures, because fatigue verifications are, in general, more detrimental than ULS analyses (apart from one

of the cross sections under analysis, for calculations in which the influence of transverse bending is disregarded).

The work presented in the current section shows that the methodology proposed in §4.3.2 is an effective tool to be used at the design stage of this type of structure, for taking into account the influence of fatigue effects and transverse bending.

It is important to remark that current design codes such as the Eurocodes do not explicitly address fatigue verifications of girder webs considering the influence of shear and transverse bending. Given the relevance of such verifications (as demonstrated through the results presented in this section), it seems advisable to explicitly address this issue in future revisions of the Eurocodes. In Figure 6.21, where the area of shear reinforcement provided in this structure is also represented, one can see that the real area is generally higher than the value required for ULS verifications, both for analyses considering the sole effect of in-plane shear and for calculations considering transverse bending too. However, the application of the novel methodology for fatigue analysis developed in this thesis suggests the adoption of increased amounts of shear reinforcements in two of the analysed cross sections, and a slightly higher web thickness in the region close to the continuity support.

Finally, it is also worth pointing out that, despite the relevance of the phenomenon under analysis (particularly in the case of box-girder bridges built with thin webs), very few research works have been devoted to this theme, as mentioned in Chapter 4. The knowledge about this issue is still limited. Therefore, a conservative approach for these fatigue analyses was developed within the scope of the present thesis. The confrontation with experimental results revealed that the adopted analysis approach provides results on the safe side, as shown in Chapter 4.

6.5.3 Fatigue and ULS analysis of the deck slab

This section presents the fatigue and ULS analysis of the deck slab, for two different cross sections:

- the slab cross section adjacent to the girder web (section S1, as represented in Figure 6.9), which is subjected to the highest cyclic hogging bending moments;
- the mid-span cross section (section S2, as shown in the same figure), where the highest sagging bending moments are applied.

In the former, the interaction of transverse bending with shear between webs and flanges has to be taken into account, since that is the location where the highest in-plane shear stresses occur. As regards the latter, only transverse bending needs to be considered. This is because the in-plane shear force (and consequently, the tensile force in the transverse reinforcement) decreases

as the distance to the web increases, and vanishes at mid-span. This fact can be confirmed by looking at the diagram of n_{xy} forces in the section S2, in Annex 3, Figure A3.5.

The analysis is carried out for half the length of the second (interior) span, so that the influence of the longitudinal variation of internal forces can be evaluated.

6.5.3.1 Slab section at the intersection with the girder web

Figure 6.23 presents the area of transverse reinforcement required for ULS verifications, for the slab section at the intersection with the girder web (section S1), according to three different analysis procedures:

- the procedure based on Menn's proposal (1986), with the modifications indicated in Annex 1 (considering that transverse bending is withstood by two resistance mechanisms, i.e., the eccentricity of compression struts and pure transverse bending);
- the simplistic procedure given by the EN 1992-1-1 for taking into account the combined effects of in-plane shear and transverse bending (procedure described in §4.3.1.1);
- considering the sole effect of transverse bending (this procedure is not strictly valid; these results are shown for comparative purposes only, so that the increase of required transverse reinforcement owing to longitudinal shear can be quantified).

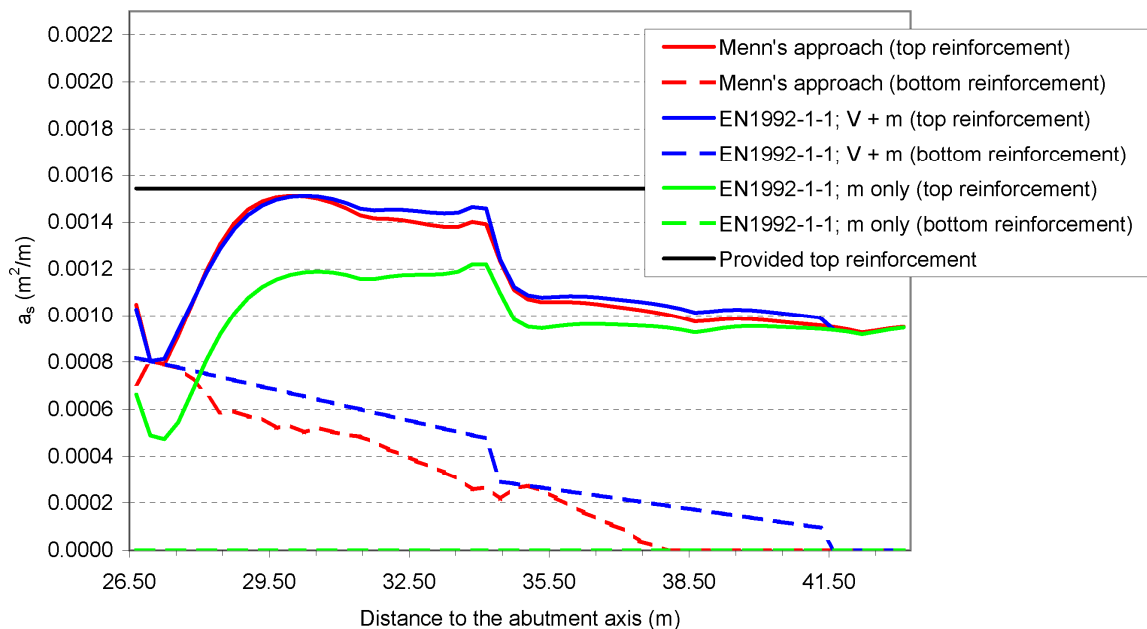


Figure 6.23 – Cross-sectional area of transverse reinforcement required in ULS.

Figure 6.23 displays the required areas for both the top and the bottom reinforcements (the latter being required to resist the effects of in-plane shear). These results correspond to the envelope of the values obtained for all the relevant combinations of the following actions: permanent loads,

G , temperature differences in the top slab, $\Delta T_{+9^\circ\text{C}}(\text{slab})$ and $\Delta T_{-5^\circ\text{C}}(\text{slab})$, temperature difference across the web thickness, $\Delta T_{5^\circ\text{C}}(\text{web})$, and load model LM71. The internal forces due to each load case are shown in Annex 3. The combinations of actions were quantified according to the EN 1990 (CEN 2002). The cover to the reinforcement, with respect to the reinforcement axis, was taken as 5 cm and the parameter β [used in Equation (4.9)] took the value 0.5.

By comparing the results of calculations in which shear is, or is not, taken into account, one can see that its influence is clearly non negligible, particularly in the case of cross sections near the continuity support. It is also interesting to note that the very simplistic procedure suggested by the EN 1992-1-1 yields very similar results to the ones obtained through the method based on Menn's approach (1986). The highest value for the required area of transverse reinforcement occurs at the cross section located ~ 3 m apart from the continuity support, the required area being, in this case, inferior to the provided one, as shown in Figure 6.23.

The results depicted in Figure 6.23 were calculated considering the effects of in-plane shear and transverse bending only. However, the section S1 is subjected to other non negligible forces, as shown in Annex 3 (Figures A3.3 and A3.4). Therefore, the three-layer method developed by Lourenço and Figueiras (1993), was employed to calculate the required areas of reinforcement, considering the influence of all the flexural forces (m_x , m_y and m_{xy}) and all the membrane forces (n_x , n_y and n_{xy}). Figure 6.24 shows the results for the calculated area of transverse top reinforcement, for the most unfavourable combination of actions (the one that gave rise to the highest required areas in Figure 6.23):

$$\gamma_G \cdot G + \gamma_{LM71} \cdot \alpha \cdot \Phi_3 \cdot LM71 + \gamma_{\Delta T} \cdot \psi_{0,\Delta T} \cdot \Delta T_{-5^\circ\text{C}}(\text{slab}) \quad (6.1)$$

where the partial factors γ_G , γ_{LM71} , and $\gamma_{\Delta T}$ take the values 1.35, 1.45 and 1.5, respectively, the factor α is equal to 1.21, the factor $\psi_{0,\Delta T}$ is equal to 0.6 and the dynamic factor Φ_3 takes the values 1.43 or 1.074 for forces associated to the local structure behaviour (flexural forces) or forces related to the global structure behaviour (membrane forces), respectively.

In Figure 6.24, the influence of m_{xy} is evaluated by comparing the results of calculations:

- considering the upper bound envelope of the moment m_{xy} due to the load case LM71. (positive m_{xy} values);
- considering the lower bound envelope of the moment m_{xy} due to the load case LM71. (negative m_{xy} values);
- disregarding the moment m_{xy} , i.e., considering $m_{xy} = 0$ for all the load cases.

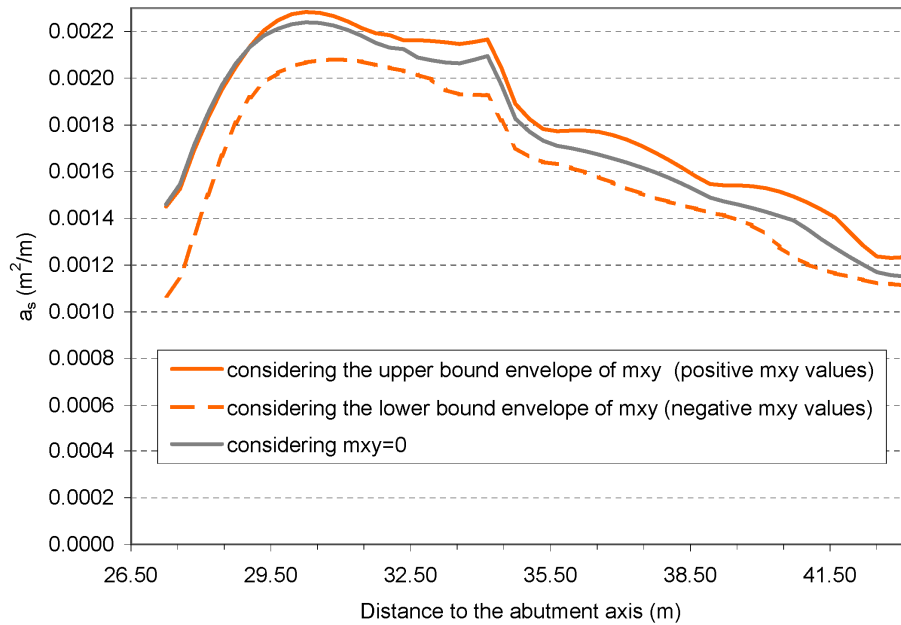


Figure 6.24 – Cross sectional area of transverse top reinforcement required in ULS, considering the effects of n_x , n_y , n_{xy} , m_x , m_y and m_{xy} . Calculations according to the algorithm of Lourenço and Figueiras (1993), for different values of m_{xy} .

This figure shows that the area of transverse reinforcement required by calculations in which the twisting moment m_{xy} is disregarded is approximately equal to the area required by calculations in which that flexural force is taken into account, particularly in the locations where the highest required areas are obtained (near the continuity support). This is justified by the fact that railway traffic loads applied to the slab, in the vicinity of the supports, give rise to negative m_{xy} values. Therefore, in that region, diminutive positive m_{xy} values are obtained (see Annex 3, Figure A3.3). Slightly higher negative m_{xy} values occur, but the tangential stresses in the upper slab layer due to negative m_{xy} values oppose the effects of in-plane shear forces n_{xy} (see the conventions for positive forces in Annex 3, Figure A3.1), and consequently, negative m_{xy} values have not a detrimental effect in terms of required areas of transverse top reinforcement.

Figure 6.25 presents the results of additional calculations according to the three-layer method, in which various flexural and membrane forces are progressively disregarded. In the last calculation hypothesis (considering $m_{xy}=0$, $n_y=0$, $m_x=0$, $n_x=0$), only in-plane shear (n_{xy}) and transverse bending (m_y) are considered [the same forces that are considered in the calculations based on the algorithm proposed by Menn (1986)]. This figure reveals that all the internal forces apart from n_{xy} and m_y can, in practical terms, be disregarded in the quantification of the required area of transverse reinforcement, particularly in the case of the most unfavourable locations (near the support). Among the various flexural and membrane forces whose relevance is evaluated in that figure (n_y , m_x , and n_x), the force n_y (transverse tension) is the most important one, and its significance increases as the distance to the mid-span section decreases.

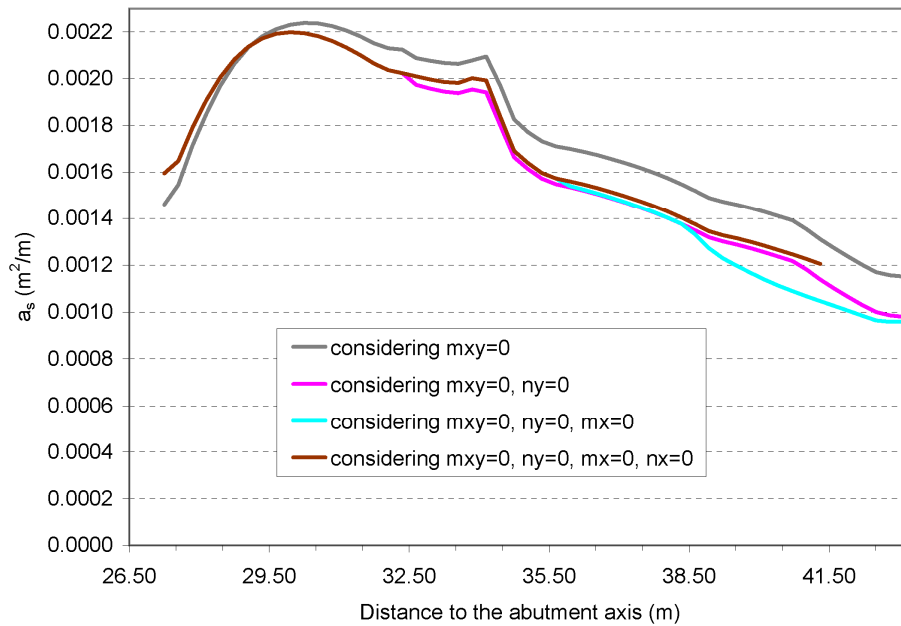


Figure 6.25 – Cross sectional area of transverse top reinforcement required in ULS, considering the effects of n_x , n_y , n_{xy} , m_x , m_y and m_{xy} . Calculations according to the algorithm of Lourenço and Figueiras (1993). Results of calculations in which one or more force component is disregarded.

By comparing the results depicted in Figures 6.23 and 6.24, one can see that the required area of transverse top reinforcement in calculations according to the algorithm developed by Lourenço and Figueiras (1983) is considerably higher than the values obtained either in analyses based on the method proposed by Menn (1986) or calculations following the EN 1992-1-1 approach. As discussed in §4.3.1.1, the applicability of the methodology proposed by Menn (1986) to problems of shear between web and flanges is questionable. Therefore, it was suggested in that section that, in the case of problems of shear between web and flanges, the methodology proposed by Menn (1986) should not be directly applied in the calculation of the combined effects of shear and transverse bending. Instead, the eccentricity of the compression struts should be cautiously taken as zero, because the possibility of considering such eccentricity in this type of problem has (to the author's knowledge) not been proven yet. The results of calculations based on Menn's approach considering an eccentricity of the compression struts equal to zero are shown in Figure 6.26. That figure also includes the previously shown results of calculations based on Menn's approach (considering the eccentricity of the compression struts) and calculations based on the three-layer method. One can see that the results of calculations based on Menn's approach are considerably more conservative and closer to the results of the three-layer method if the eccentricity of the compression struts is not considered. And consequently, both the three-layer method and the calculations not considering the struts eccentricity are considerably more severe than the simplistic procedure recommended by the EN 1992-1-1 (whose results are shown in Figure 6.23). Therefore, it seems advisable to conduct further analyses of the validity of the simplistic procedure recommended by the EN 1992-1-1.

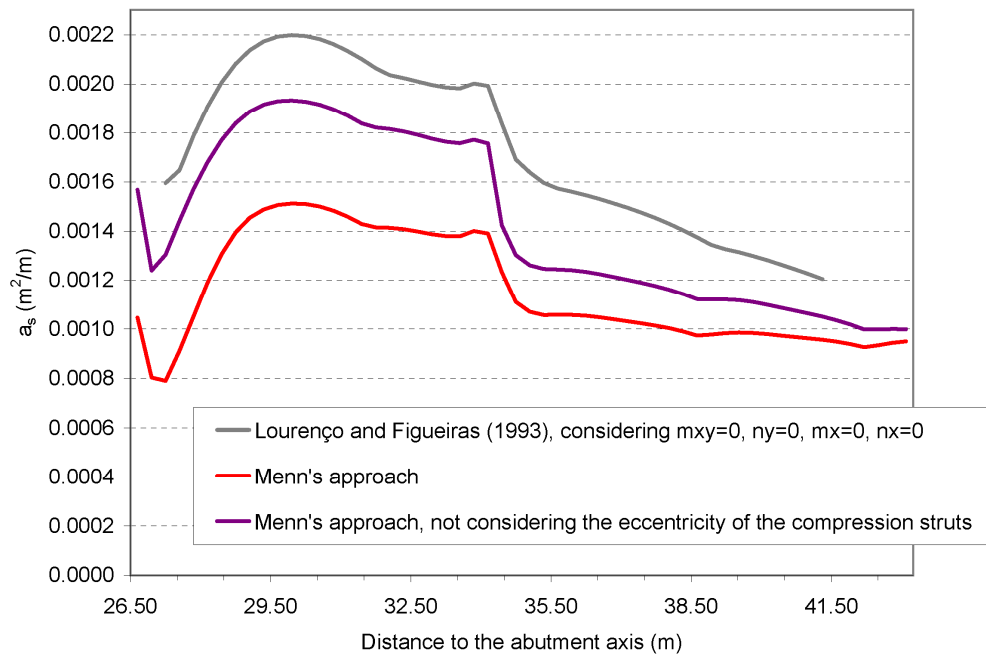


Figure 6.26 – Comparison of three distinct approaches for taking into account the combined effects of in-plane shear and transverse bending

As regards the differences between the results of the three-layer method and the outcome of calculations based on Menn's approach not considering the struts eccentricity, the main reason for those differences lies on the different hypothesis adopted for the distribution of concrete stresses on the flexural compression side. In the three-layer method, a single compression strut is considered on the flexural compression side. Contrarily, in the Menn's approach, two compression struts exist: one which carries the effects of in-plane shear and another (with a design strength of f_{cd}) which resists the compression force due to transverse bending. The lever arm associated to transverse bending is higher in calculations based on the Menn's approach and, consequently, this approach requires a lower area of transverse top reinforcement.

In the next paragraphs, the results of fatigue analyses based on the DAM are briefly shown and discussed.

The results of ULS analyses revealed that the most important forces, in terms of calculation of the force applied to the transverse top reinforcement, are transverse bending (m_y) and in-plane shear (n_{xy}). Therefore, fatigue analyses can be conducted according to the algorithm introduced in §4.3.2. Moreover, as mentioned in §4.3.2.2, these analyses should be cautiously carried out by ignoring the mechanism of eccentricity of the compression struts, i.e., by considering $k_m = 0$. Nevertheless, the results of calculations in which that mechanism is considered are also shown below, so that the differences can be evaluated.

Figure 6.27 presents the calculated fatigue lives, for four representative cross sections at different distances from the support. The traffic scenario corresponds to the heavy traffic mix specified by the EN 1991-2. Table 6.12 presents the parameters considered in the fatigue analyses:

- the adopted area of transverse reinforcement, which is approximately equal to the area required by ULS analyses based on the EN 1992-1-1 approach;
- the inclination of the concrete struts θ_{FAT} and the upper bound limit for the eccentricity of the force applied to these struts $e_{max,fat}$, which are determined through the criteria presented in §4.3.2 (it should be noted that $e_{max,fat}$ is only used in calculations in which the mechanism of eccentricity of the compression struts is admitted).

Table 6.12 –Parameters for fatigue analyses according to the DAM.

Distance to the abutment axis (m)	θ_{FAT} (°)	a_s (required by ULS analyses based on the EN 1992-1-1) (cm ² /m)	$e_{max,fat}$ (m)
28.5	42	13.1	0.098
30.5	42	15.0	0.105
32.5	42	14.1	0.111
36.5	35	10.5	0.118

In Figure 6.27, the results of analyses in which the mechanism of eccentricity of the compression struts is ignored are represented by black circles. One can see that the calculated fatigue lives are greater than 100 years, except in the case of the cross section farthest from the support. Even in that case, the calculated fatigue life is only slightly lower than 100 years. Therefore, fatigue is not critical in this case.

Figure 6.27 also shows the results of analyses in which the mechanism of eccentricity of the compression struts is admitted. The calculated steel fatigue lives are only shown for admissible k_m values, i.e., for values which guarantee that the eccentricity of the compression struts is not higher than $e_{max,fat}$. In the same figure, one can see that this condition plays an important role in diminishing the uncertainty associated to the quantification of k_m . In the cross section farthest from the continuity support, for example, k_m values higher than 0.2 are not admissible. The upper bound limit for the k_m value decreases as the applied shear force decreases, because lower shear forces imply a reduction in the percentage of transverse bending that can be supported by the eccentricity of the compression struts.

If the mechanism of eccentricity of the compression struts is considered in the calculations, the obtained fatigue lives are well beyond 100 years, as shown in Figure 6.27. This figure reveals that the concrete fatigue life is always higher than 100 years. By applying the criteria indicated in §4.3.2.1, one concludes that the steel fatigue life should be quantified considering $k_m = 0.2$.

Therefore, considering the mechanism of eccentricity of the compression struts leads to higher calculated steel fatigue lives. However, before it is proven that the struts eccentricity can be considered in problems of shear between web and flanges, it seems advisable to prudently ignore that mechanism, as recommended in §4.3.2.2.

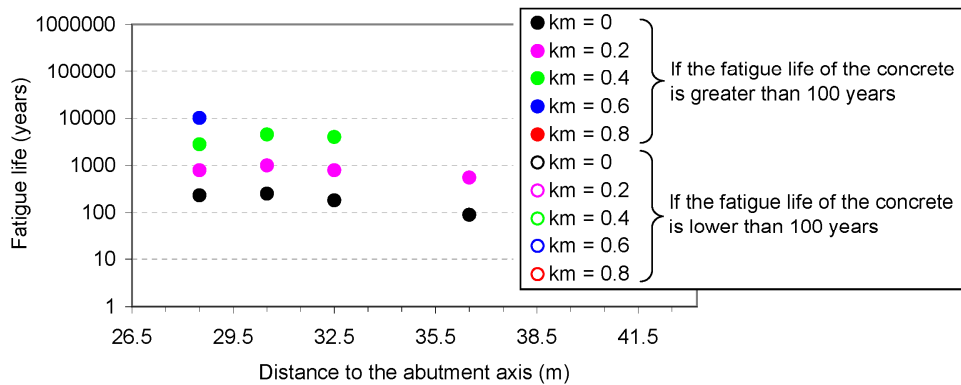
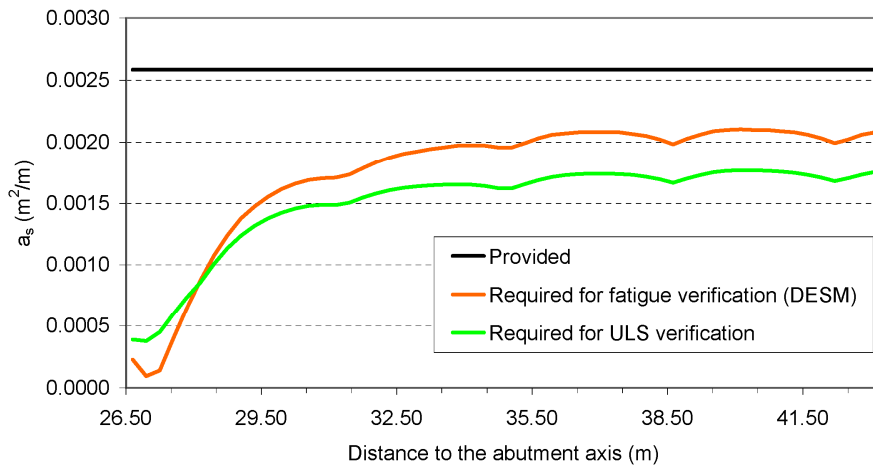


Figure 6.27 – Steel fatigue lives for the Heavy Traffic Scenario, determined through the DAM, considering the area of transverse reinforcement required for ULS verification.

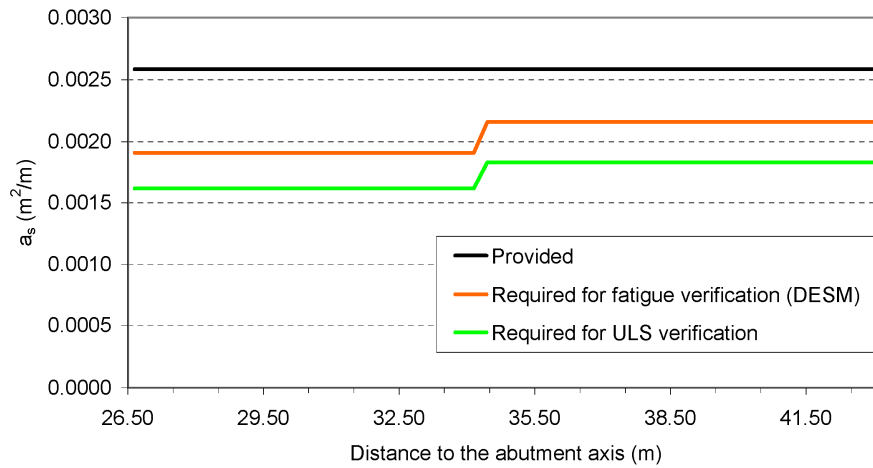
6.5.3.2 Mid-span section of the slab

The calculation of the required transverse reinforcement, in terms of both ULS verifications and fatigue verifications according to the DESM, is straightforward, since the interaction of transverse bending with shear between web and flanges is not to be taken into account. The results of such calculations are depicted in Figure 6.28a. In this figure, one can see that the fatigue verification is more severe than the ULS one. Furthermore, the amount of transverse reinforcement required by these verifications is lower than the value actually provided in this structure, as represented in the same figure.

Figure 6.28b, which depicts the results of calculations in which transverse bending moments are determined through the simplified FE analysis with a 2D plane frame, shows that this model is sufficiently accurate for design purposes. In fact, the maximum required area of transverse reinforcement is roughly equal to the value determined through the more detailed FE analysis with shell elements.



a) Results based on bending moments determined through shell FE models.



b) Results of simplified calculations in which transverse bending moments are determined with beam models.

Figure 6.28 – Cross-sectional area of transverse bottom reinforcement required for ULS verification and fatigue verification through the DESM.

The validity of the simplified fatigue calculations (through the DESM) can be evaluated by comparing their results with the outcome of more detailed fatigue calculations (based on the DAM). Figure 6.29 shows the calculated steel fatigue lives for two different scenarios:

- considering the real (provided) area of transverse reinforcement;
- considering the area of transverse reinforcement required by the fatigue analysis according to the DESM.

These results confirm the validity of the DESM for this analysis, since fatigue lives roughly equal to 100 years are obtained for calculations considering the area of transverse reinforcement estimated through the DESM. As expected, the area of transverse reinforcement provided in this structure leads to considerably longer steel fatigue lives.

As regards the concrete fatigue strength, the calculated fatigue lives are well beyond 100 years.

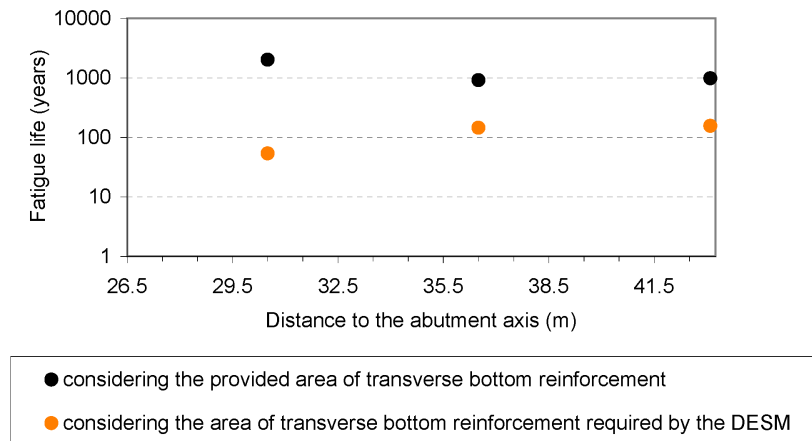


Figure 6.29 – Steel fatigue lives for the heavy traffic scenario, determined through the DAM.

6.6 Influence of cracking at the continuity connection

In this section, the developed Cracked Beam Model is employed in the analysis of the structure behaviour under the influence of cyclic bending moments. The analysis focuses on the continuity connection between spans, where transverse cracks occur under the influence of hogging bending moments.

Cracking at the continuity connection is not only due to the applied hogging moments, but it is also influenced by tensile stresses (on the slab) caused by differential shrinkage between the slab and the precast girder, and also between the slab concrete and the slab longitudinal reinforcement. These effects were taken into account in the probabilistic analysis presented in §6.4.2. Therefore, the probability of cracking occurrence (which is herein defined as the quotient between the number of computer runs in which cracking occurs at a specific integration point and the total number of computer runs) can be estimated by analysing the results of the Monte Carlo simulation. Figure 6.30 depicts the variation of this probability along the length of the second span, for integration points located in the mid-plane of the deck slab, after live loads were applied for the second time (two years after the end of the construction). As expected, the extent of cracking exhibits important variability, since the tensile stresses in the slab are influenced by concrete creep and shrinkage deformations and also by construction timings, which are variable parameters. In cross sections whose distance to the continuity supports is shorter than ~ 4.1 m, the estimated probability of cracking is higher than 50%, and for distances shorter than ~ 2.9 m, this value reaches 100%. Therefore, cracking effects must be taken into account.

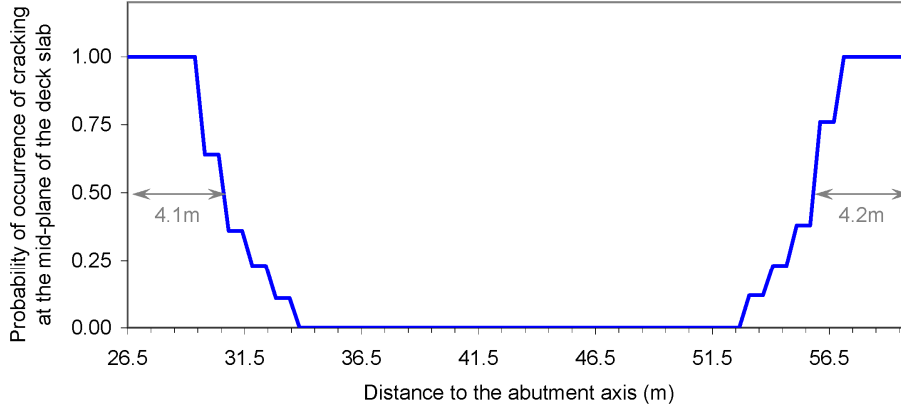


Figure 6.30 - Probability of occurrence of cracking at the mid-plane of the deck slab.

In the following sub-sections, the influence of cracking in terms of cyclic behaviour of the bridge girder in the vicinity of the support is firstly discussed (§6.6.1). Given the uncertainty associated to the quantification of different parameters used in the analysis of the cyclic behaviour after cracking, a parametric analysis is also conducted (§6.6.2). Finally, the influence of cracking in terms of dynamic behaviour of the structure is evaluated (§6.6.3).

6.6.1 Local analysis of the cyclic behaviour after cracking

Before the effects of cyclic loading can be evaluated, the crack spacing in the slab (in the region above the continuity supports) must be quantified. This quantification can be carried out by analysing the longitudinal behaviour of the deck slab under increasing tensile forces. The calculated average deformation of the deck slab under increasing tensile load, considering the maximum distance between cracks, is shown in Figure 6.31. Given that the tensile stress in the concrete varies between zero (at the crack) and f_{ct} (at the cross section where the slip between steel and concrete vanishes) for all the load steps, the result under monotonically increasing load is the same that would be obtained by simple hand calculations considering an average retention of concrete tensile stresses of $0.6 \cdot f_{ct}$:

$$\varepsilon_{sm} = \frac{N}{A_s} - 0.6 \cdot \frac{f_{ct}}{E_s \cdot \rho_s} \quad (6.2)$$

where ε_{sm} is the average deformation, A_s is the area of tensile reinforcement, ρ_s is the reinforcement ratio with respect to the effective area of concrete and E_s is the steel modulus of elasticity. For the unloading stage, such a closed-form solution can not be obtained, and numerical methods need to be used. For the sake of example, the tie response upon unloading (from a hypothetical maximum load of 12700 kN), calculated with the Cracked Chord Model, is also shown in Figure 6.12.

As regards the basic data, the following values are considered in the calculations presented in §6.6, unless stated otherwise: $f_{ct} = f_{ctm} = 2.9$ MPa; $G_F = 0.1$ kNm/m²; $K_B = 400$ MPa/mm; $\alpha_B = 0.25$; the ascending branch of the bond stress-slip relationship is based on the proposal by Farra (1995) for average bond conditions ($a_1 = 0.4$, $a_2 = 0.95$ and $b = 0.3$); the slab width is taken as the effective width according to the suggestion of the EN 1992-1-1, which, in this case amounts to 6.10 m; the structure geometry and reinforcement are shown in §6.2 and in Annex 2; the main tensile reinforcement is taken as the reinforcement inside the effective slab width; the effective area of concrete around the main tensile reinforcement is equal to the effective slab width multiplied by the slab height; for long-term analyses, k_n is taken as 3.5, and a reasonable value is assumed for the long-term shrinkage strain ($40 \cdot 10^{-5}$); for the definition of the cross section's geometry above the continuity support, the continuity diaphragm is not taken into account, i.e., the calculations are carried out considering the geometry outside the diaphragm, this option being motivated by the reduced extent of the continuity diaphragm; secondary reinforcements in the web and in the compression chord are also considered in the analysis.

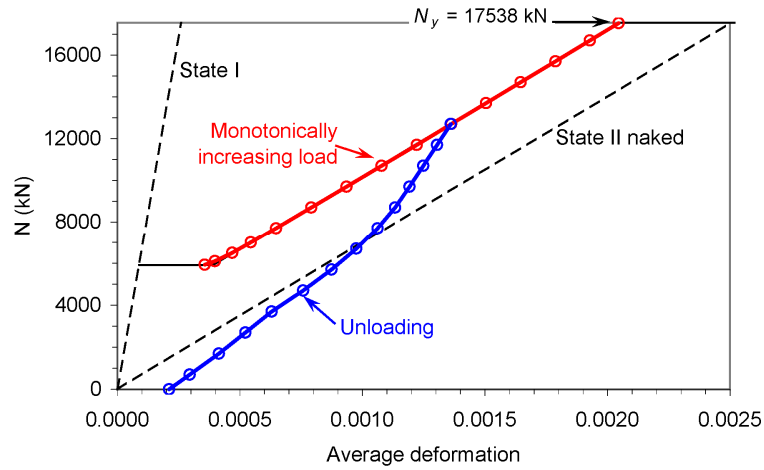


Figure 6.31 – Axial force-average deformation relationship for the deck slab, considering the maximum distance between cracks: monotonically increasing load and unload from $N = 12700$ kN.

The analysis of the slab behaviour under increasing axial loads also provides the value of the transmission length $L_{r,max}$, which, in turn, makes the determination of the average crack spacing s_{rm} possible. As mentioned in Chapter 2, the average crack spacing is taken as 2/3 of the maximum crack spacing, the latter being equal to twice the maximum transmission length at the end of the crack formation stage. Following the suggestions of several authors (CEB 1993; Farra 1995; Jaccoud 1987; Walraven 1999), the end of the crack formation stage is assumed to occur for an axial load approximately 30% higher than the cracking load. Considering the progression of the transmission length shown in Figure 6.32, the average crack spacing takes the value:

$$s_{rm} = 2 \cdot \left(\frac{2}{3} L_{r,max} \right) = \frac{4}{3} \cdot 0.193 \text{ m} = 0.257 \text{ m}$$

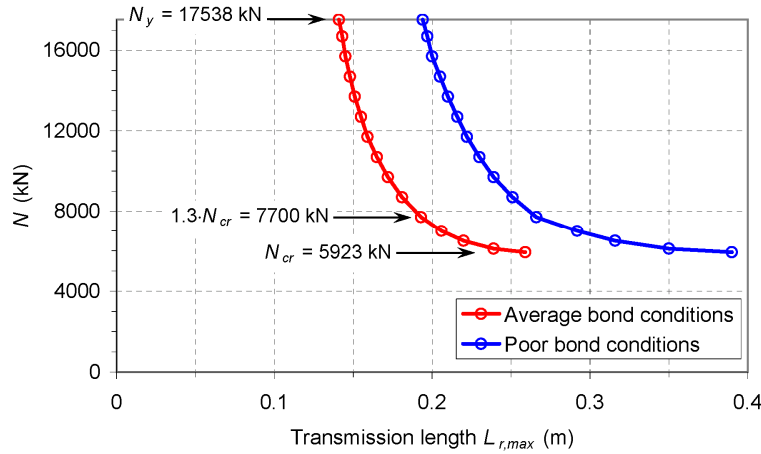


Figure 6.32 – Calculation of the transmission length for different bond conditions according to the proposals by Farra (1995).

Once the crack spacing is determined, the cyclic behaviour of the girder can be evaluated. Four different cross sections are analysed: the cross section above the support P2, and the cross sections at distances of 1 m, 2 m and 3 m from the support axis. The former is a reinforced concrete section with no prestress. On the other hand, in the remaining cross sections, significant pre-tensioning forces are installed at the level of the girder's bottom flange. Note that, in reality, the analysis is not carried out for a single cross section. Instead, the region under analysis is the beam segment delimited by the crack cross section and the cross section where the slip between steel and reinforcement is null. The term “cross section under analysis” is here used with the purpose of simplifying the language.

The analysis is conducted considering the bending moment diagrams represented in Figure 6.33: bending moments due to permanent loads (10% and 90% percentiles and median value, which corresponds to a cumulative distribution of 50%) were determined through the probabilistic analysis in which cracking is taken into account, and $\Delta t_I \sim U(30, 150)$; traffic loading is represented by a typical freight train (the fatigue train type 5, which is the heaviest freight train included in Annex D of the EN 1991-2); temperature differences of +9°C (top surface warmer) and -5°C are also considered.

The mechanical response of the four cross sections under analysis to the cyclic loads caused by the freight train can be evaluated through the Cracked Beam Model. Figure 6.34 depicts the calculated bending moment-average curvature relationship, for load cycles which consist of the superposition of the most plausible value of the bending moment due to permanent actions and the bending moments due to railway traffic, i.e.:

- the maximum bending moment in each cycle is equal to the sum of the median of the moment due to permanent actions and the maximum hogging moment due to train type 5;

- the minimum value of each cycle is given by the median of permanent actions plus the maximum sagging moment due to the same train.

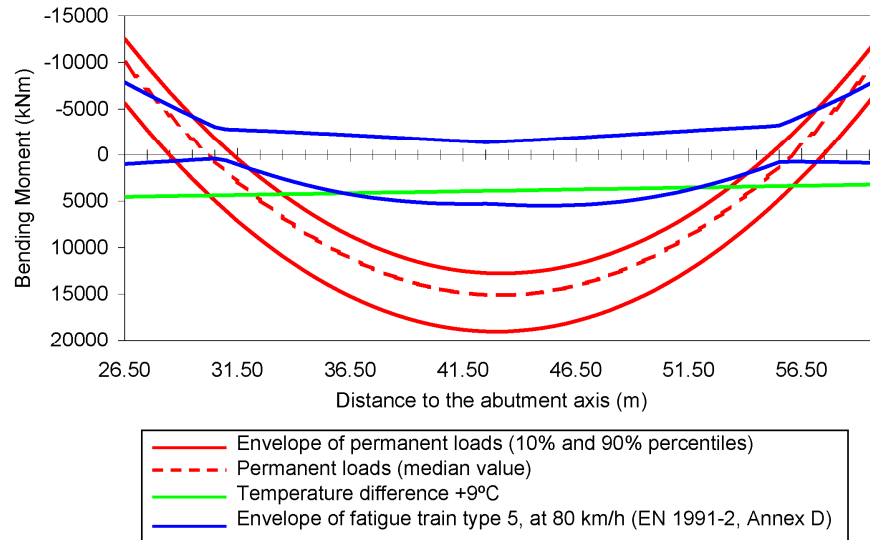


Figure 6.33 – Diagrams of bending moments for the second span of the viaduct.

In order to evaluate the increase of average deformation as the number of applied load cycles increases, as well as the influence of shrinkage, three different analysis results are shown in Figure 6.34:

- the results for the first load cycle;
- the results after a large number of load cycles is applied (i.e., considering the degradation of bond action through the parameter $k_n = 3.5$), but not considering the influence of shrinkage;
- the results obtained if both the degradation of bond action due to cyclic loading and shrinkage are considered in the analysis.

The moment-average curvature relationships for state I and state II naked, as well as the MC90 estimates for monotonically increasing bending moments (not including the influence of shrinkage), are also represented in Figure 6.34, which facilitates the understanding of:

- the increase of average curvature due to cracking (through the comparison of the outcome of the Cracked Beam Model and the results for state I);
- the influence of tension stiffening (by comparing the results of the Cracked Beam Model and the calculations for state II naked);
- the differences between the response for monotonically increasing loads and cyclic loads.

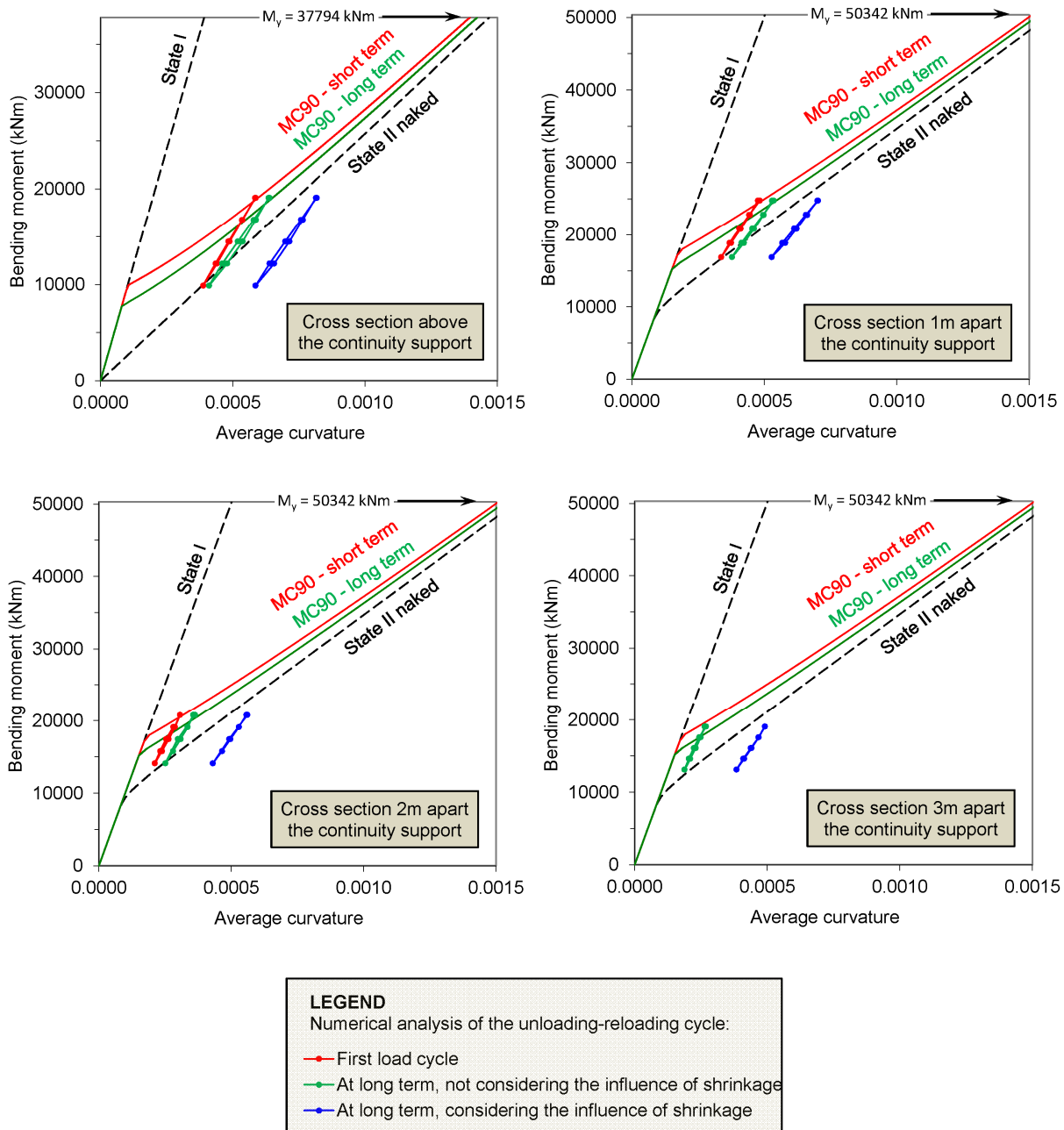


Figure 6.34 – Moment-curvature diagrams for the four cross sections under analysis.

The bending moment includes the isostatic effect of prestress and is referred to the centroid of the cross section in State I.

The comparison between the MC90 estimates and the results of the Cracked Beam Model shows that the MC90 provides good estimates for the average curvature under maximum load. Note that if shrinkage strains are also taken into account in calculations according to the MC90, good agreement is obtained with the average deformation under maximum load provided by the Cracked Beam Model (see Figure 6.35). However, the mechanical behaviour during unloading and reloading cycles can only be evaluated if more refined analysis methodologies, such as the Cracked Beam Model, are employed.

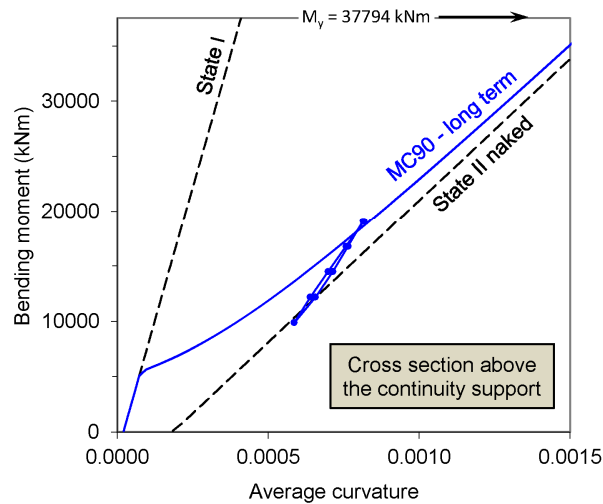


Figure 6.35 – Moment curvature diagram for the cross section above the continuity support. Comparison with MC90 estimates, considering the influence of shrinkage.

By observing the results of the Cracked Beam Model represented in Figure 6.34, one can see that small hysteresis occurs during the unloading and reloading cycles, and therefore, the moment-average curvature relationship under cyclic loading can be approximately represented by a straight line that connects the points of maximum and minimum load. The slope of that line (the quotient between the variation of bending moment and the variation of average curvature) expresses the bending stiffness for cyclic loading after cracking. The resulting bending stiffnesses (in percentage of the bending stiffness in state I) are shown in Table 6.13, where one can see that important stiffness decrease occurs as the number of applied load cycles increases. Moreover, shrinkage is also responsible for important stiffness decreases in the prestressed cross sections. Note that the results for the first load cycle are not presented for the cross section 3 m apart from the continuity support, because convergence was not possible due to the proximity between the maximum applied bending moment and the cracking bending moment.

Table 6.13 – Bending stiffness (secant value, considering the maximum and minimum bending moments caused by the real freight train), in percentage of the stiffness in state I.

	Above the continuity support	1 m apart from the support	2 m apart from the support	3 m apart from the support
First load cycle	49%	57%	70%	--
At long term, not considering the influence of shrinkage	42%	52%	63%	78%
At long term, considering the influence of shrinkage	42%	46%	53%	55%

For a better understanding of both the influence of the bond degradation due to cyclic loading and the influence of shrinkage, and also for a better comprehension of the hysteretic behaviour upon unloading, Figures 6.36 and 6.37 depict the variation of the most relevant analysis results along the transmission length, for the cross section located above the continuity support. The following analysis results are considered: normal stress in the main tensile reinforcement, normal stress in the effective area of concrete that surrounds the tensile reinforcement, bond stress, bond slip, neutral axis depth, η parameter (introduced in §2.4.2.2) and curvature of the cross section. The results are presented for four load decrements, of equal magnitude, between the maximum and the minimum load.

By comparing the results for the first load cycle (column on the left) and after a large number of load cycles has been applied (column on the centre), one can see that the bond stress is significantly reduced on the long term. Consequently, the normal stresses in the effective area of concrete are also reduced, and the longitudinal variation of steel stresses is also lower. The long-term decrease of the values taken by the η parameter is also a consequence of lower tensile stresses in the concrete. Still looking at the long-term diagrams of bond stresses, one can see that upon unloading, the frictional bond stress branch (identified in §2.2.7.3) is progressively reached along the transmission length, which justifies the hysteresis (non-linear response) observed in the moment-average curvature diagrams. Moreover, as the bond stress reaches negative values (under decreasing values of the applied bending moment), compressive stresses occur in the effective area of concrete (even though the main tensile reinforcement is subjected to large tensile stresses). Compressive concrete stresses are also observed at the crack cross section, when the beam is subjected to the minimum bending moment, during the first load cycle, this effect being a consequence of the non-linear stress-crack opening relationship at the crack. The hysteretic behaviour is also evident in the bond slip diagrams, for analyses considering the previous application of a large number of load cycles. As mentioned before, the figures show the results for four decrements of bending moment with equal magnitude, but the bond slip diagrams reveal that the magnitude of the slip decrement progressively increases, which leads to a slightly non-linear moment-curvature relationship. The main cause of non-linear behaviour is, in this case, the progressive reaching of the frictional bond strength along the transmission length. In the first load cycle, the frictional bond strength is only locally reached after the last load decrement, which justifies the fact that the moment-average curvature relationship for this cycle is practically linear. The reaching of the frictional bond strength is also responsible for the stiffness decrease upon unloading after a large number of load cycles has been applied.

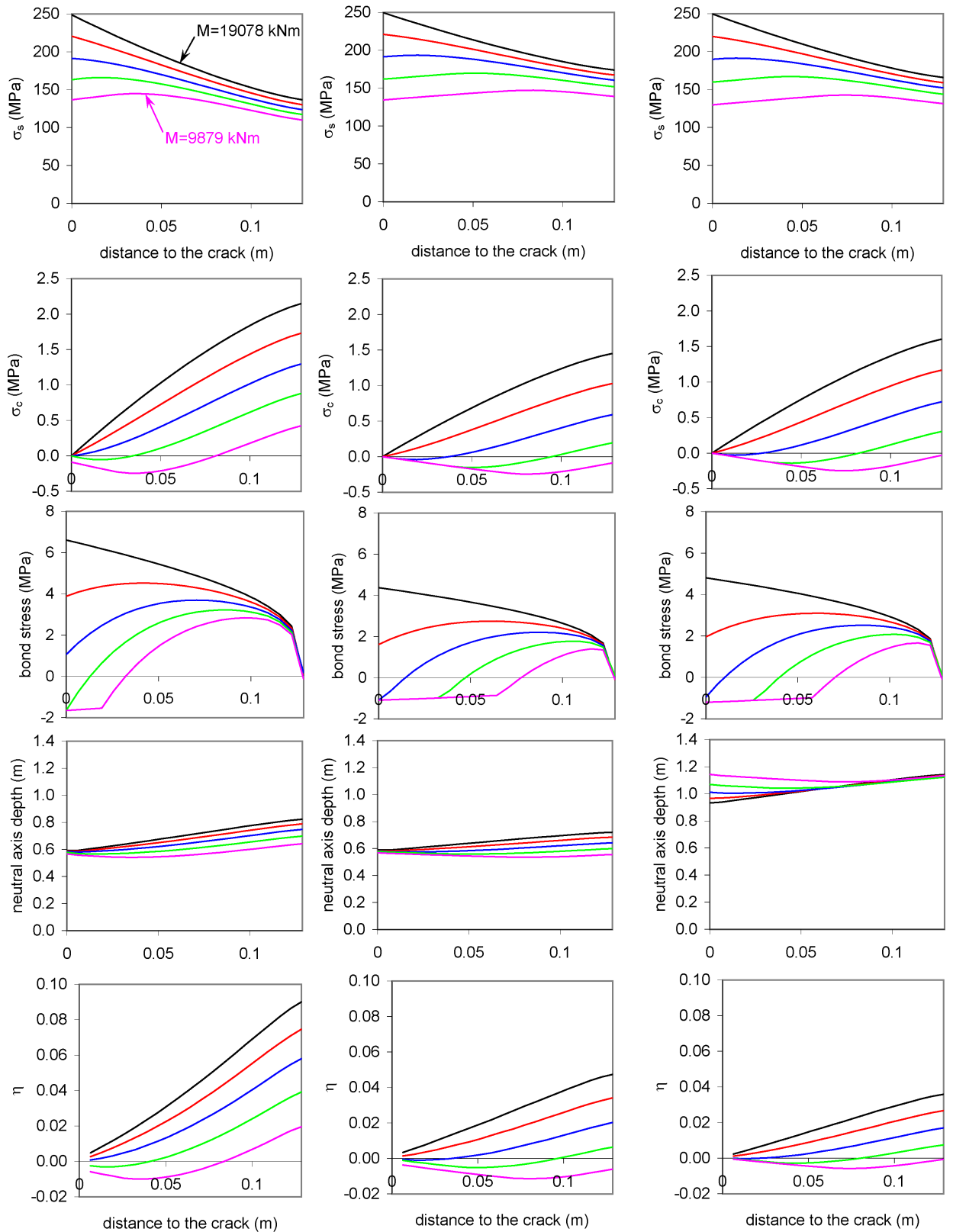


Figure 6.36 – Structural response upon unloading, as a function of the distance to the crack (crack at the cross section above the continuity support): normal stress in the main tensile reinforcement, normal stress in the effective area of concrete, bond stress, neutral axis depth and parameter η .

Left: short term behaviour; Centre and Right: long-term behaviour, not considering and considering the influence of concrete shrinkage, respectively.

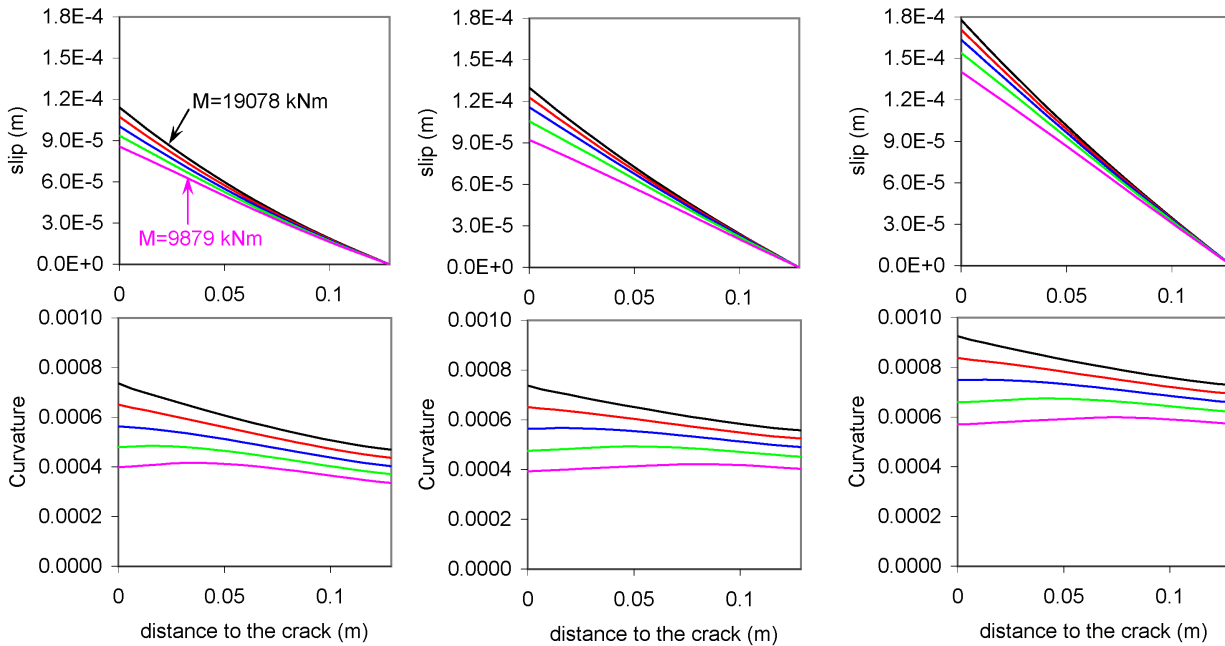


Figure 6.37 – Structural response upon unloading, as a function of the distance to the crack (crack at the cross section above the continuity support): slip between main tensile reinforcement and surrounding concrete, and curvature. Left: short term behaviour; Centre and Right: long-term behaviour, not considering and considering the influence of concrete shrinkage, respectively.

As regards the influence of shrinkage, one can see that the tensile stress in the reinforcement remains almost unchanged if shrinkage deformations are taken into account. However, the shortening of the concrete fibres under compression significantly increases, and consequently, the average curvature is also amplified. For the same reason, the neutral axis depth undergoes a considerable increase. Concrete shrinkage is also responsible for an important rise of the slip between concrete and tensile reinforcement (the maximum slip of 0.13 mm increases to 0.18 mm due to shrinkage). Nevertheless, only a small increase of bond stress under maximum load occurs (from 4.4 MPa to 4.8 MPa), due to the exponentiation in the bond stress-slip relationship. The fact that shrinkage leads to increased crack openings is also the cause of lower tensile stresses along the web, and consequently, lower values of the parameter η .

6.6.2 Parametric analysis of the local behaviour after cracking

Given the uncertainty associated to the quantification of some parameters used in calculations through the Cracked Beam Model, a parametric analysis was conducted aiming at evaluating the influence of those parameters in terms of stiffness decrease under cyclic loading and crack openings. The parametric analysis was carried out for the cross section above the continuity support and for the cross section located 1 m apart from the first one. As mentioned before, the

latter is subjected to the influence of forces due to pre-tensioning, as well as the influence of secondary reinforcements along the web.

The parametric analysis focuses on the structural response upon unloading after a large number of load cycles has been applied. The variable parameters are as follows:

- bending moment installed in the structure before traffic loading is applied; the upper and lower bound limits considered in the parametric analysis consist of the 10% and 90% percentiles of the permanent actions added to the frequent value of the temperature difference;
- fracture energy of concrete;
- tensile strength of concrete;
- unloading branch of the normal stress-crack opening relationship; secant unloading towards the origin was considered as an alternative to the Hordijk's CFM, even though it is well known that this alternative unloading branch provides poorer predictions than the Hordijk's CFM; it is considered here for evaluation of the errors associated to the use of that simplified model;
- descending branch of the bond stress-slip relationship;
- ascending branch of the bond stress-slip relationship;
- secondary reinforcement in the girder web and in the bottom flange (the influence of the secondary reinforcement was only evaluated in the case of the cross section 1 m apart from the support axis, because no secondary reinforcement exists in the other cross section).

The parametric analysis is devoted to the study of the structure response after a large number of load cycles has been applied, and taking into consideration the influence of shrinkage, because these are the critical conditions for which higher crack openings and stiffness decreases occur. The different scenarios considered in the parametric analysis are identified in Figures 6.38 and 6.39, and the results are shown in the same figures. Results for the first load cycle and results after the application of a large number of load cycles (not considering the influence of shrinkage) are also presented in the figures, so that they can be easily compared with the results for the remaining scenarios. Note that the scenarios are characterized through the differences with respect to the base analysis presented in §6.6.1.

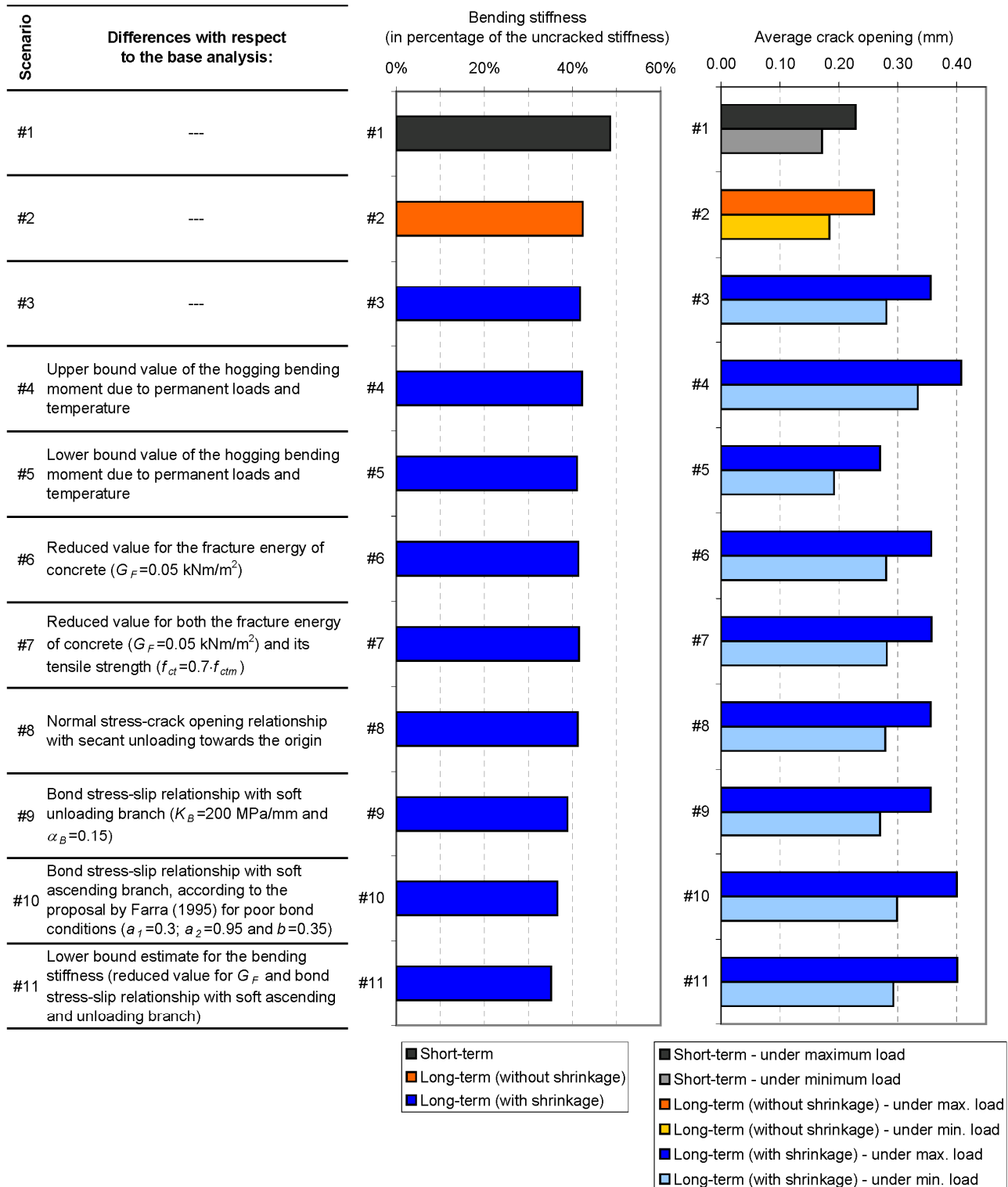


Figure 6.38 – Scenarios and results of the parametric analysis for the cross section above the support axis.

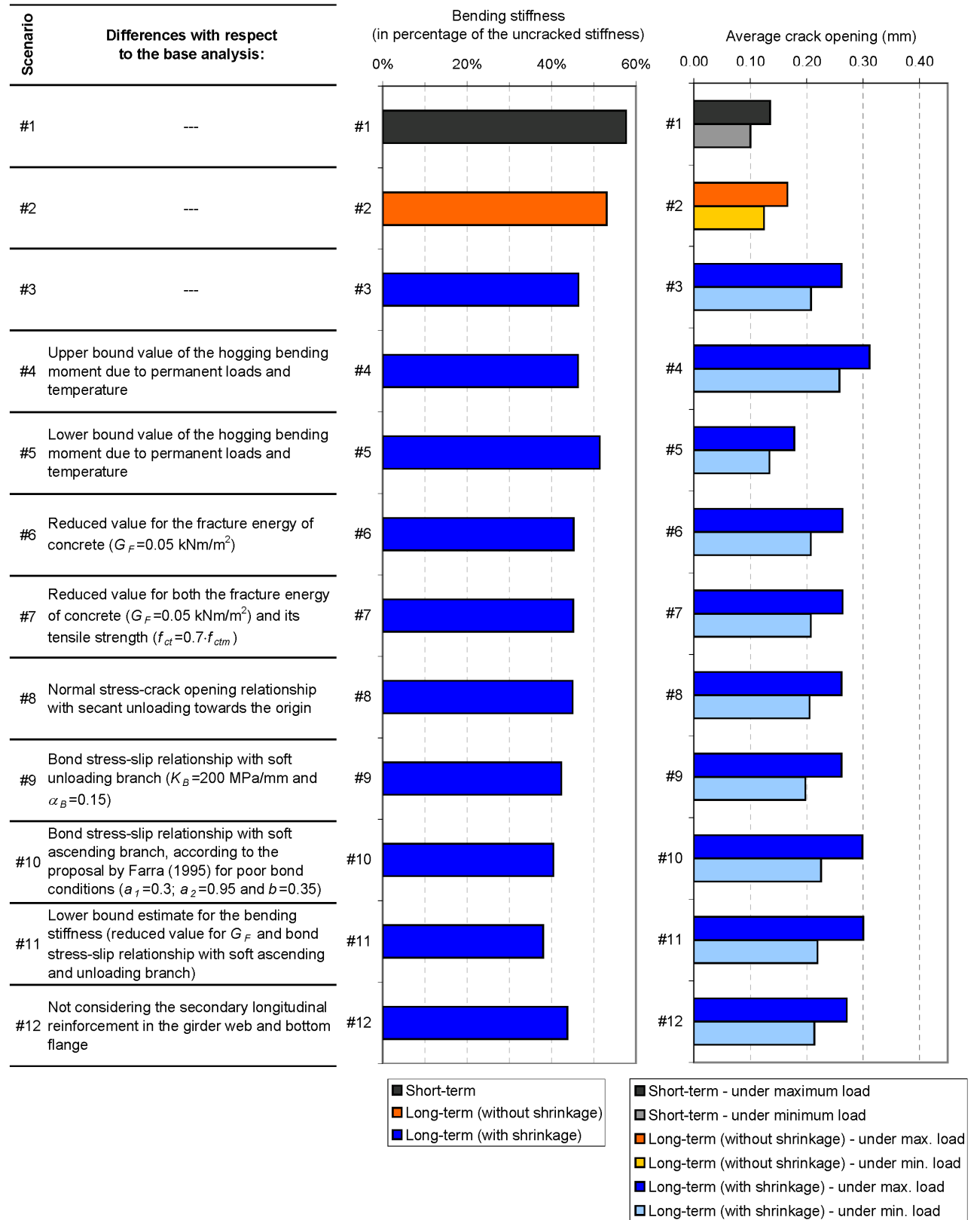


Figure 6.39 – Scenarios and results of the parametric analysis for the cross section 1 m apart the support axis.

In Figure 6.38, it is interesting to note that the bending stiffness is almost independent from all the parameters considered in the parametric analysis, except those related to the bond stress-slip relationship. The degradation of bond action due to the number of applied load cycles is responsible for the differences between scenarios #1 and #2. Considering poorer bond conditions in the definition of the bond-slip relationship (in terms of either the ascending branch or the descending branch of this law) leads to lower bending stiffness upon unloading.

Another important conclusion lies in the fact that the variability of the calculated bending stiffness is quite reduced (upper and lower limits of 42% and 35% of the stiffness in state I, the lower bound value corresponding to a very conservative estimate of the bond conditions). Therefore, one can say that the error associated to the quantification of the stiffness decrease after cracking through this methodology is quite low.

As regards the calculation of the average crack opening (results also shown in Figure 6.38), it is important to note that the residual crack opening is significantly affected by the value of the maximum live load. In fact, the minimum bending moment is approximately equal to half the maximum value, but the crack opening under minimum load roughly amounts to 3/4 of the value of the opening under maximum load. As far as the variability of the calculation results is concerned, one can see that, unlike bending stiffness, crack opening is significantly affected by shrinkage and by the maximum reached bending moment.

The results obtained for the cross section 1 m apart from the support axis provide similar conclusions, but the following differences can be detected:

- in this case, the bending stiffness upon unloading depends on the magnitude of the bending moment due to permanent loads and temperature; this is because the lower bending moments considered in scenario #5 lead to the contact between the crack faces upon unloading, i.e., upon unloading, compressive stresses develop at the crack for lower bending moments, and consequently, the bending stiffness upon unloading increases when the bending moment due to permanent loads decreases;
- by comparing the results for scenarios #2 and #3, we can see that shrinkage leads to lower bending stiffness upon unloading; this stiffness decrease is also due to the contact between the crack faces upon unloading (this contact occurs in the absence of shrinkage, but that does not happen after the concrete shrinks, due to the shrinkage-induced increase of crack width).

Figure 6.39 also shows that the secondary reinforcement does not play a significant role in this case. This is because the quantity of web reinforcement is negligible when compared with the total amount of main tensile reinforcement in the deck slab.

In Figures 6.38 and 6.39, the calculated average crack openings were shown and discussed. However, for verification of the proper durability of the structure, the maximum crack widths (based on conservative estimates for the bond conditions, and taking into account the integral of relative strains along the transmission length $L_{r,max}$) are of interest. Maximum crack widths can be determined through the proposed Cracked Beam Model, which provides more realistic predictions than the procedure proposed by the Eurocodes (EN 1992-1-1 and EN 1992-2). In fact, the Eurocodes state that the maximum crack width in RC members, in quasi-permanent combinations of actions, must be limited to 0.3 mm (for common exposure classes). However, in the case of railway bridge decks, the influence of traffic loading is not taken into account in the calculation of maximum crack widths, because the quasi-permanent value of traffic loading is zero [as stated in the EN 1990, Annex A2 (CEN 2005a)]. Moreover, the procedure recommended by the Eurocodes for calculation of maximum crack widths does not take into account the influence of shrinkage strains.

In the following paragraphs, the results provided by the procedure recommended by the Eurocodes are compared with the outcomes of the Cracked Beam Model. The calculations are carried out for the cross section above the continuity support, which is the most unfavourable one, as shown before.

According to the EN 1992-1-1, the maximum crack spacing, $s_{r,max}$, takes the value 0.507 m, and the maximum crack width in quasi-permanent combination of actions is equal to 0.295 mm. In this calculation, the bending moment due to permanent actions is conservatively estimated considering the 10% percentile, and only the reinforcing bars within the effective slab width of 6.10 m are taken into account⁵. Therefore, the limit value of 0.3 mm is not exceeded.

The results of calculations according to the Cracked Beam Model are shown in Figure 6.40⁶. As shown in the figure, different scenarios in terms of bond conditions and bending moment due to permanent actions were considered. The figure shows the maximum value for the crack width under maximum load and for the residual crack width after the traffic load is removed, the latter being of interest for durability checks. The calculated maximum crack spacing (twice the

⁵ In the transverse direction, the crack width in the slab increases as the distance to the girder web decreases. The effective slab width approach provides conservative estimates for the maximum crack width at the intersection with the web, as could be confirmed through the non-linear analysis of the structure with shell finite elements. The detailed presentation of this FEM analysis is thought to be dispensable in this context.

⁶ Note that, even though the calculations were carried out considering the geometry outside the continuity diaphragm (as mentioned before), parametric analyses (whose results are not shown here) revealed that similar maximum crack openings are obtained if the gross section of the continuity diaphragm (with significantly increased web thickness) is adopted in the calculations.

maximum transmission length needed to develop, in the effective area of concrete, a tensile stress equal to f_{ctm} , when the structure is subjected to the maximum bending moment) is also presented.

By observing the results for poor bond conditions, one can see that even though the maximum crack spacing calculated with the Cracked Beam Model is slightly lower than the value given by the EN 1992-1-1, the former leads to residual crack widths equal to almost twice the value given by the latter. The influence of shrinkage strains and traffic loading are, therefore, very important and ought not to be ignored. According to the results of the Cracked Beam Model, the maximum residual crack opening is expected to lie in the interval of 0.4 mm to 0.6 mm (the result strongly depends on the ascending branch of the bond stress-slip relationship and, to a minor extension, on the bending moment due to permanent actions). Values close to the upper bound value (~ 0.4 mm) are more plausible, for the reason that good bond conditions exist in the deck slab (see §2.2.5), and consequently, Farra's proposal for PBC provides a very conservative estimate of the bond properties.

As a conclusion, one can say that even though the cross section under analysis complies with the Eurocodes provisions for serviceability limit states of cracking, it would be advisable to guarantee lower residual crack openings. A residual crack opening of ~ 0.3 mm could be reached if one of the following measures was adopted:

- uneven distribution of tensile reinforcement along the slab width, with higher reinforcement ratio inside the effective slab width and lower quantity of reinforcement outside that region;
- different construction sequence, so that a significant portion of the slab is cast before the continuity connection between spans is established – this hypothesis would give rise to lower hogging bending moments at the supports, and consequently, smaller crack openings in the slab (even though higher pre-tensioning forces would be required owing to larger sagging bending moments at mid-span);
- smaller bar spacing, in order to achieve lower crack spacing.

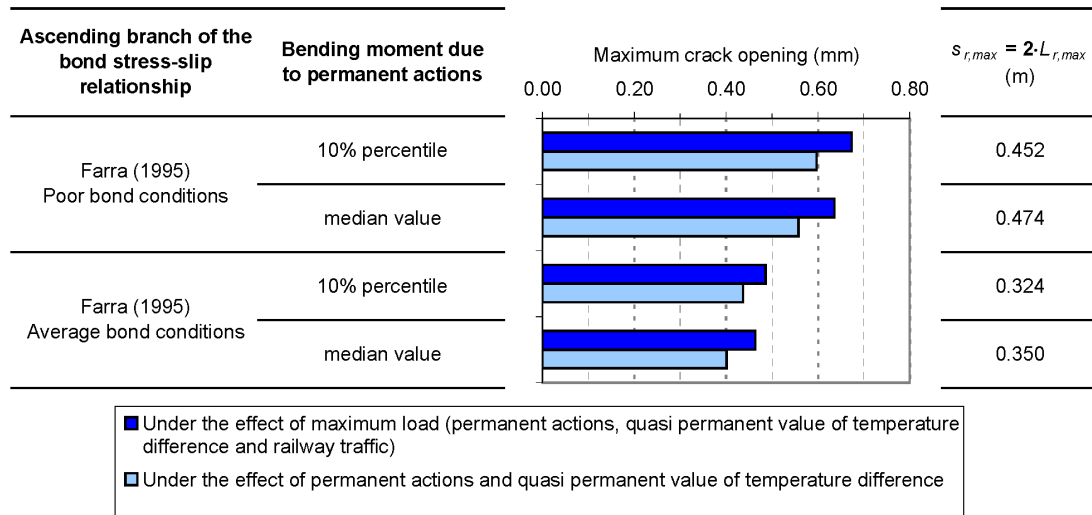


Figure 6.40 – Maximum crack opening at the cross section above the continuity support.
 Results of calculations according to the Cracked Beam Model.

6.6.3 Global analysis of the dynamic behaviour after cracking

The results shown before revealed that the flexural response of the structure after cracking, under the effect of cyclic loads due to railway traffic, can be approximately represented by a linear bending moment-average curvature relationship with a reduced stiffness with respect to the stiffness in state I. The stiffness decrease due to cracking might be particularly relevant in the case of structures that carry high speed traffic, because the dynamic response of the structure strongly depends on its stiffness. Therefore, even though the bridge deck under analysis was designed to carry freight traffic and not high speed passenger traffic, some relevant results of the dynamic analysis of the structure are presented here aiming at exemplifying the consequences of cracking in terms of dynamic response to the passage of high speed trains. For that purpose, the results of the dynamic analysis of an uncracked structure are compared with those obtained for a structure in which the response to live loads (considering the influence of cracking) is determined considering the stiffness decrease depicted in Figure 6.41. The stiffness reduction is considered in the region where the probability of cracking is higher than 50% (see Figure 6.30), and the reduced stiffness takes the values shown in Table 6.13 for the structure response after a large number of load cycles has been applied, and considering the influence of shrinkage.

In Figure 6.42, the calculated vertical displacements due to a uniformly distributed load of 100 kN/m are shown for 5 different calculation hypotheses, so that the influence of shear deformation and the influence of cracking can be evaluated.

- The 3 analyses in which the stiffness decrease due to cracking is not considered and shear deformation is taken into account provide very similar results. In one of these analyses, the structure is modelled with shell finite elements. In the remaining ones, Timoshenko beam finite elements are used - the cross section is modelled with the total slab width in

one of the cases, and in the other one, the effective slab width of 6.10 m (based on the EN 1992-1-1 provisions) is considered over a length of ~ 5 m to each side of the continuity supports. As expected, adopting the effective slab width leads to a closer agreement with the results of the shell finite element model.

- The analysis in which shear deformation is not taken into account leads to vertical displacements considerably lower than the remaining analyses. This result puts into evidence the importance of shear deformation in this kind of structure, as a consequence of the reduced web thicknesses.
- The analysis in which cracking is taken into account (by decreasing the bending stiffness as represented in Figure 6.41) leads to an important increase of vertical displacements, particularly in the interior spans.

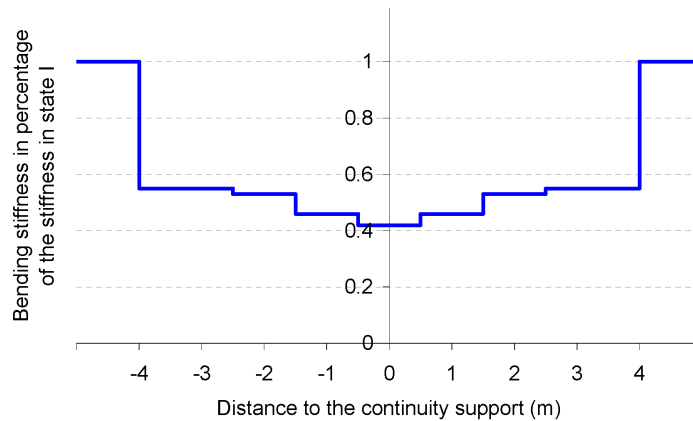


Figure 6.41 – Bending stiffness in the region affected by cracking.
Values adopted in the dynamic analysis of the passage of the HSLM trains.

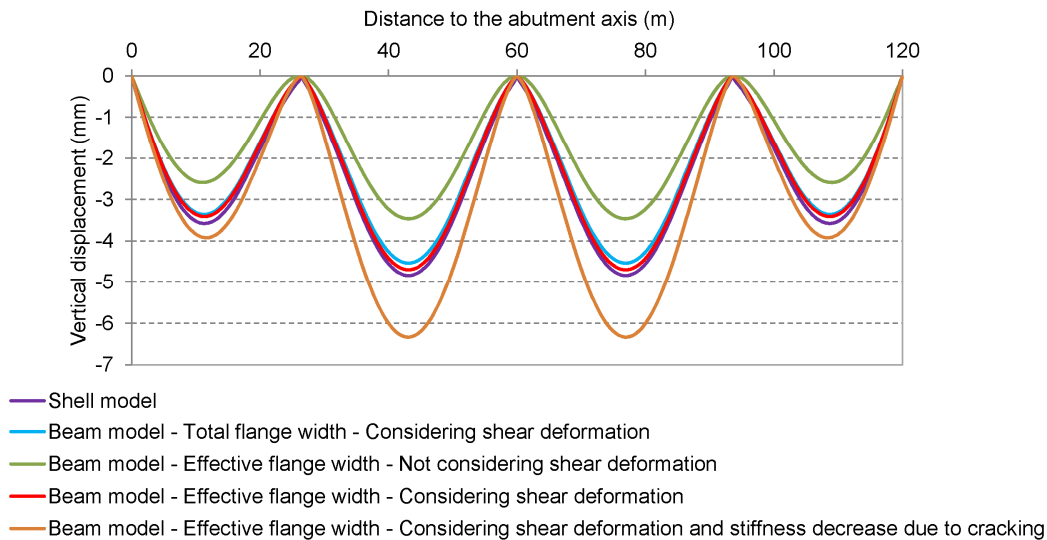


Figure 6.42 – Vertical deflection due to a vertical load of 100 kN/m.
Results provided by different finite-element models.

The stiffness decrease due to cracking is also responsible for the decrease of frequencies of vibration and the modification of mode shapes, as shown in Figure 6.43, where the first four frequencies and modes of vibration for the structure under analysis are shown. The decrease in the natural frequencies, for these modes, varies between 2.9% (1st mode) and 9.4% (4th mode).

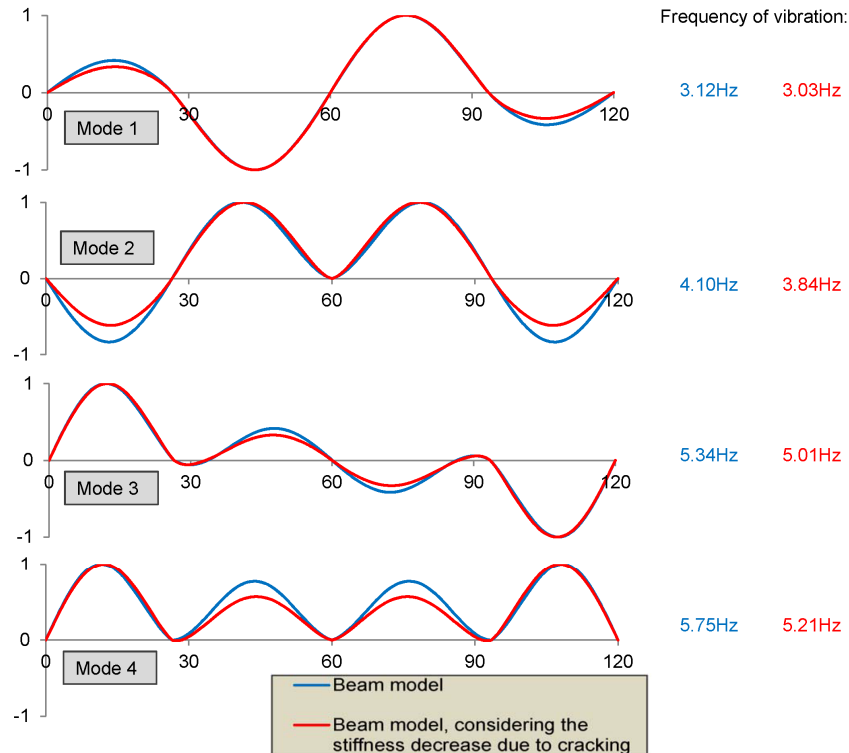


Figure 6.43 – Influence of cracking in terms of modes of vibration and natural frequencies. Comparison of analysis results considering, or not considering, the stiffness decrease due to cracking.

The influence of cracking in terms of response to the passage of high speed passenger trains can be seen in Figures 6.44, 6.45 and 6.46, where the vertical accelerations and the bending moments due to the trains included in the load model HSLM-A (EN 1991-2) are shown. These figures show the envelopes of the results as a function of the train speed, for the cross section at the mid of the second span and for the central continuity support. The envelopes of vertical accelerations are also shown for the cross section at the centre of the second span. The results, either considering the influence of cracking or not considering that influence, are determined through a dynamic analysis with mobile forces, according to the modal superposition method, considering modes of vibration with frequencies lower than 30 Hz, the time increment being equal to 0.001 s. As regards modal damping ratios, it is well known that their values have a strong influence on the dynamic structural behaviour, but are very difficult to predict at the design stage (Magalhães et al. 2010). In this study, the modal damping ratio takes the value 1%, as suggested by the EN 1991-2 (the evaluation of the damping influence is out of the present work's scope). The structure is modelled with Timoshenko beam elements, taking shear deformation into account.

As regards the train speeds, values between 150 km/h and 420 km/h are considered with increments of 5 km/h.

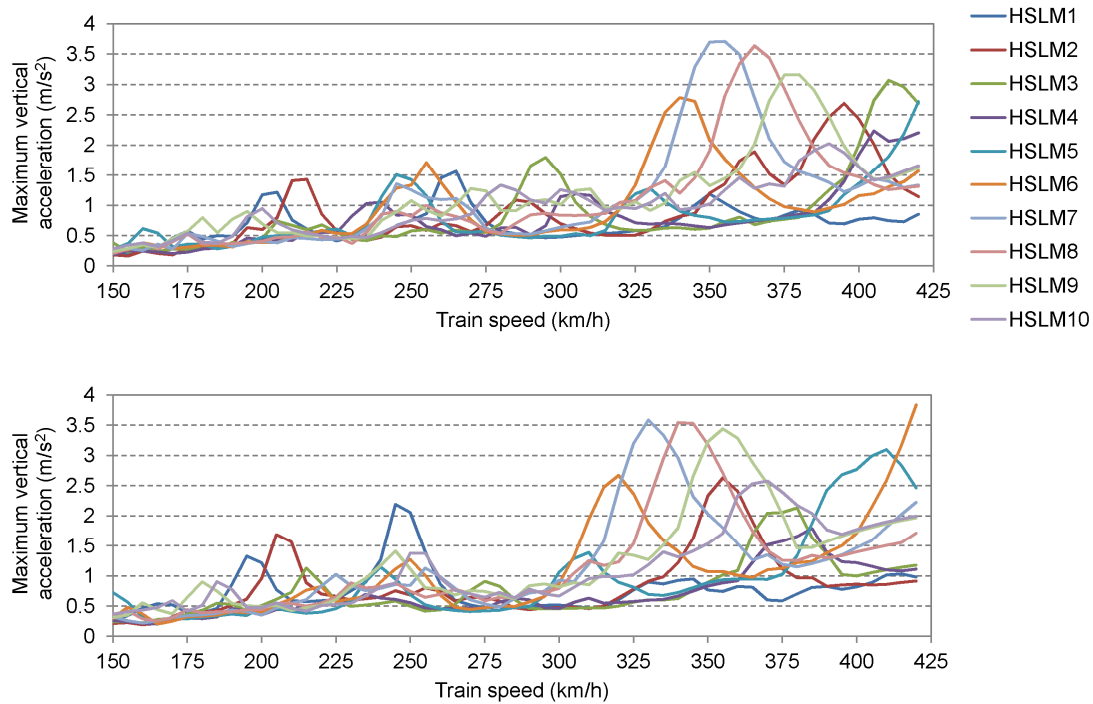


Figure 6.44 – Maximum vertical acceleration at the centre of the second span, as a function of train speed. Top: not considering the influence of cracking. Bottom: considering the influence of cracking.

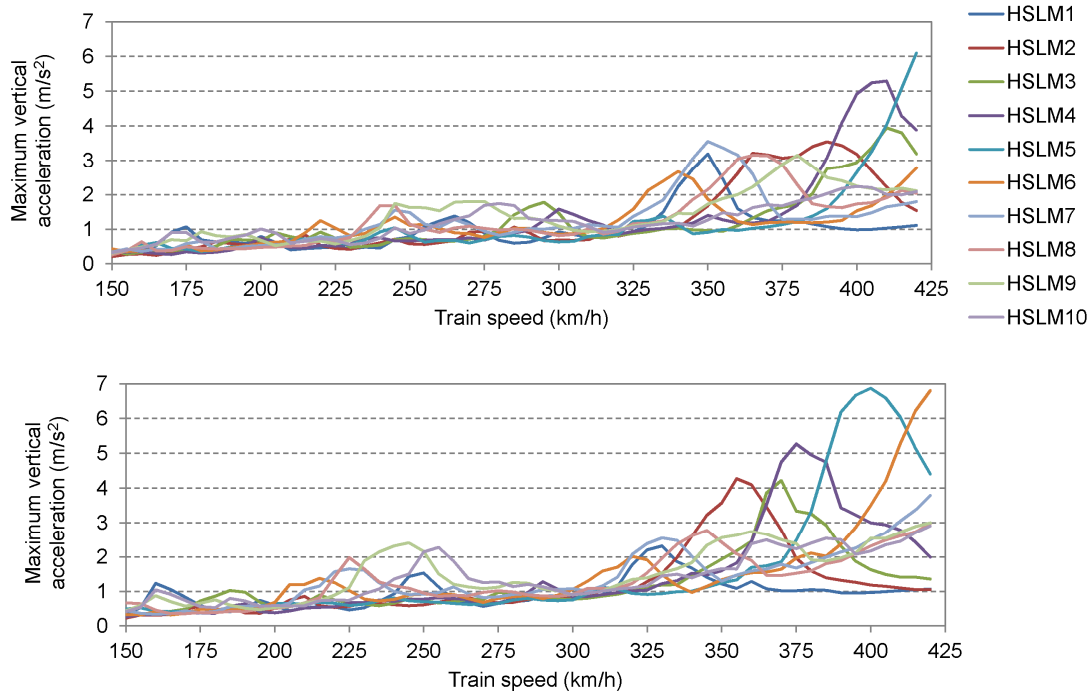


Figure 6.45 – Maximum vertical acceleration at the centre of the end span, as a function of train speed. Top: not considering the influence of cracking. Bottom: considering the influence of cracking.

The results (either the accelerations or the bending moments) show that the stiffness decrease due to cracking gives rise to decreasing resonance speeds, as a consequence of decreasing frequencies of vibration. In the span between P1 and P2, the largest resonance effects, which occur for train speeds greater than 300 km/h, arise when the period of time between the passage of successive bogies equals the natural period of vibration of the 2nd mode, due to the symmetry of this mode of vibration (see Figures 6.44 and 6.46). In the case of the end span (see Figure 6.45, bottom), the most unfavourable resonance effects take place when the period of time between passages of successive HSLM 5 bogies is equal to the natural period of vibration of the 3rd mode. In Figure 6.44, one can see that even though the resonance speeds decrease with cracking, the maximum vertical accelerations remain approximately unchanged. This evidence conforms with the expected structural response (ERRI D214 1999b): the maximum displacements increase as the stiffness decreases, but that does not change the maximum vertical accelerations. However, if we look at the accelerations envelope for train HSLM 5 for train speeds lower than 420 km/h, in Figure 6.45, one can see that the maximum acceleration in analyses considering the influence of cracking ($\sim 7 \text{ m/s}^2$) is greater than the value obtained when that influence is not considered in the analyses ($\sim 6 \text{ m/s}^2$). This is a consequence of decreasing resonance speeds. In the case of the analyses not considering the influence of cracking, the acceleration peak for train HSLM 5 lies outside the range of train speeds considered in the calculations. Given that the maximum permitted vertical acceleration in ballasted tracks is equal to 3.5 m/s^2 (EN 1990, Annex A2), the results in Figure 6.45 show that this criterion would set limits to the maximum permitted train speeds, if the structure under analysis was included in a high speed passenger line.

The stiffness decrease due to cracking also gives rise to the redistribution of bending moments, with decreasing bending moments at the continuity supports, as shown in Figure 6.46. In this figure, the envelopes of bending moments due to the load model LM71 and the fatigue train type 5 at 80 km/h (EN 1991-2, Annex D) are also included, which shows that:

- even though high speed passenger trains are considerably lighter than freight trains, the bending moments due to HSLM at resonance are higher than the values due to the freight train type 5;
- the load model LM71 is more adverse than the HSLM in terms of hogging moments at the supports and sagging moments at mid-span. However, the opposite happens for bending moments with reverse sign – the circulation at resonance is particularly critical for the estimation of maximum sagging bending moments at the supports.

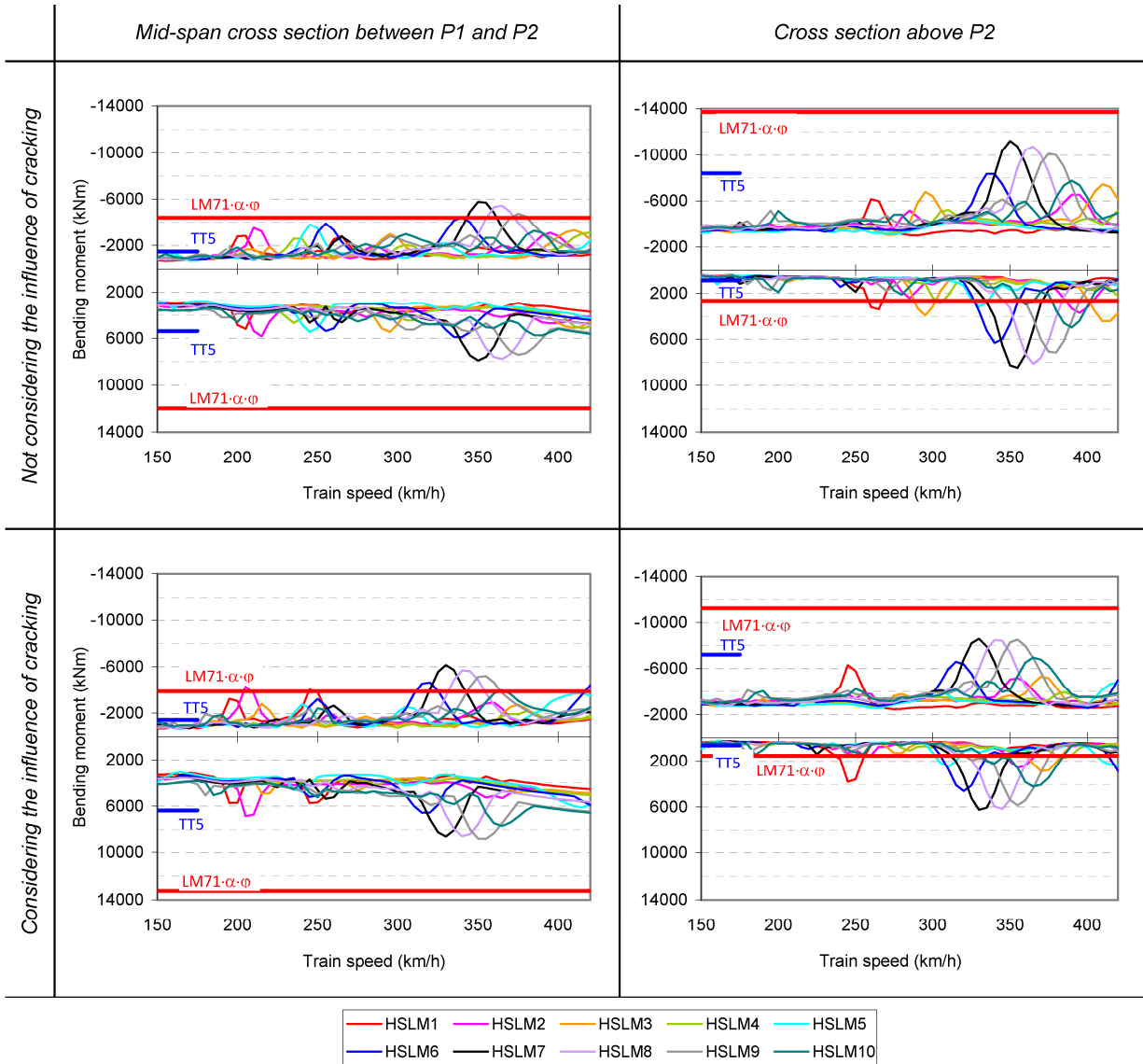


Figure 6.46 – Envelopes of bending moments at the mid-span cross section between P1 and P2 and at the cross section above P2.

6.7 Final considerations

In this Chapter, the previously developed and validated analysis procedures were applied in the study of a real continuous precast bridge deck without continuity prestress.

The variability of the long-term structure behaviour was evaluated by means of a Monte Carlo simulation, taking as independent variables the creep and shrinkage deformations and the girder ages at the time that the continuity connection is made. It was concluded that the structure response is sensitive to the uncertainty associated to the quantification of those variables. It was also shown that this analysis approach provides quantitative information about the variability of the structure response. With this information, the variability of long-term effects can be taken

into account both in serviceability limit state analyses and in verifications of the fatigue strength of concrete elements subjected to important cyclic compressive forces.

The results of the Monte Carlo simulation also revealed that the variability of bending moments is significantly higher than the variability of shear forces. Moreover, it was shown that a significant portion of long-term redistributions takes place in the first two years after the end of the construction, as a consequence of the reduced thickness of both the precast girder walls and the deck slab (which, in turn, gives rise to high rates of creep and shrinkage development in the first months after casting). Therefore, it seems advisable to consider the long-term distribution of internal forces (due to permanent actions) in fatigue verifications, for practical purposes.

The developed methodologies for fatigue analyses considering the effects of in-plane shear and transverse bending were applied in the study of both the girder webs and the deck slab. ULS analyses were also carried out, by using available methodologies for taking into account the combined effects of shear and out-of-plane bending.

It was shown that fatigue verifications are, in general, more detrimental than ULS verifications. Moreover, it was demonstrated that transverse bending can not be ignored in the design of this type of structure, because the required areas of transverse reinforcements are considerably larger when this effect is taken into account, when compared to calculations considering the sole effect of in-plane shear. The proposed methodology for taking into account the influence of fatigue effects and transverse bending proved to be an effective tool to be used at the design stage of this type of structure.

The developed methodologies for taking into account the effects of in-plane shear and transverse bending were also applied to the analysis of the deck slab. The results of ULS and fatigue analyses were shown and discussed. It was shown that, even though the deck slab is subjected to non-negligible twisting bending moments m_{xy} , this flexural force has minor implications in terms of required areas of transverse slab reinforcement. It was shown that fatigue and ULS analyses of the transverse slab reinforcement can be, for practical purposes, carried out considering only the effects of in-plane shear and transverse bending. The results of calculations in which the mechanism of eccentricity of the compression struts is, or is not, taken into account were presented and discussed. It was shown that ignoring that mechanism leads to results on the safe side. As suggested in Chapter 4, that mechanism should not be considered before it is demonstrated that its beneficial effects can be safely used in problems of shear between web and flanges. It was shown that ULS analyses based on the simplistic procedure suggested by the EN 1992-1-1 provide lower required areas of transverse reinforcement than more detailed calculations in which the mechanism of eccentricity of the compression struts is not considered.

Therefore, it was suggested that further studies into the adequacy of the EN 1992-1-1 approach should be conducted in the future.

Regarding the calculation of local bending moments in the slab and in the girder web, the implications of the number of modes of vibration considered in dynamic analyses through the modal superposition method were exposed. It was also shown that, for this unicellular box girder supporting a single track, approximate predictions of the maximum transverse bending moments can be obtained through simplified FE analyses with a 2D plane frame. However, these models are too conservative in the region close to the continuity diaphragms.

The consequences of cracking in the continuity connection were also evaluated. The mechanical response of cross sections where cracking occurs, under the influence of cyclic loading, was studied by means of the Cracked Beam Model. It was shown that small hysteresis occurs during the unloading and reloading cycles, and therefore, the moment-average curvature relationship under cyclic loading can be approximately represented by a straight line that connects the points corresponding to maximum and minimum loads. The slope of that line expresses the bending stiffness for cyclic loading after cracking, and can be used in linear dynamic analyses to calculate the structure response under the passage of high speed trains.

The influence of different parameters used in calculations through the Cracked Beam Model, and the way they affect both the calculated stiffness decrease and the residual crack openings, were evaluated through a parametric analysis. This study showed that the variability associated to the quantification of the stiffness decrease after cracking is much lower than that related to the quantification of residual crack openings. Nevertheless, the proposed methodology for evaluation of crack openings is more realistic than the one included in design codes such as the EN 1992-1-1, for the reason that the latter does not explicitly take into account the influence of shrinkage and the consequences of traffic loading. The results obtained for this bridge deck suggest that the proposed Cracked Beam Model is also more conservative than the procedure given by the EN 1992-1-1. For the reasons mentioned above, it is advisable to carry out the calculation of maximum residual crack openings, in this type of structure, according to the procedure followed in §6.6.2.

Some relevant results of linear dynamic analysis, in which the influence of cracking was considered by means of the reduced bending stiffness for cyclic loading, were also shown and briefly discussed. It was shown that, in this type of structure, the stiffness decrease due to cracking is responsible for:

- increasing vertical displacements, particularly in the interior spans;
- decreasing frequencies of vibration and modification of mode shapes;

- decreasing resonance speeds during the passage of high speed trains;
- increasing vertical accelerations under the passage of high speed trains, due to resonance peaks which would lie outside the considered range of train speeds if cracking was ignored;
- an important redistribution of bending moments, with decreasing bending moments at the continuity supports.

The exposed results also put in evidence the relevance of shear deformation in this type of structure and the importance of resonance effects in terms of quantification of maximum sagging bending moments at the continuity supports.

Finally, some comments are worth being made, regarding the serviceability and the ultimate capacity of the structure under analysis, as well as the adequacy of its geometry and adopted areas of reinforcements. As pointed out in the introduction of this chapter, this work presents the analysis of an exiting (recently built) structure, but the purpose of the work does not consist in the numerical assessment of its condition. Instead, the main objectives of this work are:

- to exemplify the application of the developed numerical procedures;
- to test their applicability;
- to illustrate the practical consequences of their application.

On the one hand, it is worth remarking that this chapter does not present an exhaustive verification of all the serviceability and ultimate limit states. On the other hand, the fact that the application of the developed methodologies suggests (in some situations) the adoption of higher areas of longitudinal and transverse reinforcements does not mean that the structure design was not properly carried out according to existing design codes. In fact, it was shown that:

- ULS analyses of the girder webs, taking into account the effects of in-plane shear and transverse bending, and considering the real web thickness, showed that the real area of shear reinforcement (provided in this structure) is higher than the amount required by the calculations.
- Fatigue analyses of the girder webs, according to the methodology developed in the present thesis, suggested the adoption of increased areas of shear reinforcement or slightly higher web thicknesses in some of the analysed cross sections. Nevertheless, it is important to remark that, on the one hand, the developed methodology was devised so that it could be employed in the analyses of new structures, and the confrontation with experimental results showed that it provides results on the safe side. And, on the other hand, at the present time, the specificities of fatigue analyses considering the effects of in-plane shear and transverse bending are not addressed in design codes such as the

Eurocodes. Given the relevance of such effects (as demonstrated through the results presented in this chapter), it seems advisable to explicitly address this issue in future revisions of the Eurocodes.

- As regards the top slab, it was shown that refined ULS analysis methodologies require, in certain locations, areas of transverse reinforcements higher than the amounts provided in this structure. However, the provided areas comply with the provisions of the EN 1992-1-1, which presents a simplistic procedure for calculation of transverse slab reinforcements.
- The provided areas of transverse reinforcement in the top slab guarantee adequate fatigue strength.
- As regards the quantification of maximum crack widths in the deck slab above the continuity supports, it was shown that, according to the developed analysis procedure, maximum residual crack openings of 0.3 mm could only be obtained if higher areas of longitudinal reinforcement were adopted (or, alternatively, if the construction procedure was modified). However, it was shown that the analysed cross sections comply with the EN 1992-1-1 provisions for serviceability limit states of cracking. It was shown that, while the simplified approaches proposed by the EN 1992-1-1 for taking into account the effects of cyclic loading and concrete shrinkage do not accurately simulate the real structural behaviour, the Cracked Beam Model, developed in this thesis, provides a more realistic description of the structure response.
- It was shown that the structure under analysis is not able to carry high speed passenger traffic, for the reason that the maximum admissible vertical deck accelerations would be exceeded. However, this particular structure is included in a conventional freight line, and therefore, in this case, vertical deck accelerations for such high speeds need not to be checked.

The work presented in this thesis will hopefully contribute for further improvements of the Eurocodes, so that they can be more coherent with the actual behaviour of railway bridge decks, in general, and of the type of structure under analysis in this thesis, in particular.

Chapter 7

Conclusions and future research

7.1 Conclusions

This thesis focuses on the development and validation of numerical methodologies for evaluation of the service behaviour of continuous railway bridge decks constructed with precast girders. Special attention is paid to solutions without continuity prestress. Three different specificities of the structural behaviour of this type of construction are considered: *(i)* influence of cyclic loading in terms of stiffness decrease and residual crack openings in cracked RC or PC members; *(ii)* fatigue strength, special attention being given to girder webs subjected to the combined effects of in-plane shear and out-of-plane bending; *(iii)* influence of concrete creep and shrinkage and phased construction in terms of long-term stress redistributions.

In the first part of the present work, a numerical model for the analysis of the cyclic behaviour of cracked RC elements was developed. The adopted numerical approach involves the explicit consideration of both the cyclic bond stress-slip relationship between the tensile reinforcement and the surrounding concrete, and the relationship between the normal stress at the crack and the crack opening under the effect of cyclic loading.

A comprehensive state-of-the-art review focusing on the mechanics of bond under monotonic and cyclic loading was carried out. The proposals of different authors for the cyclic bond stress-

slip relationship were compared, and a constitutive model to be used in the applications envisaged in this work was defined. Research needs in terms of characterization of the cyclic bond stress-slip relationship were also identified.

Different constitutive models to describe the stress-crack opening relationship under the effect of cyclic loading were also identified. A minor modification was introduced in one of the available constitutive models, so that convergence difficulties could be solved, and a better agreement with experimental results for small reloading steps could be obtained.

New numerical models for calculation of the cyclic response of cracked RC members were presented: the Cracked Beam Model and the Cracked Chord Model. The differences between these models and the ones presented by different authors were pointed out.

The validation of the proposed numerical models was performed by using experimental results reported in the literature. This work showed that the developed models provide an accurate description of the non-linear behaviour of the tested elements under the effect of cyclic loading. Good agreement between numerical and experimental results was obtained when the proposed constitutive models for both the bond stress-slip relationship and the normal stress-crack opening relationship were considered.

The second part of this work focuses on the fatigue behaviour of railway bridge decks. Three alternative procedures for fatigue analysis of railway bridge decks were presented: the damage accumulation method, the damage equivalent stress method and the simplified methodology for fatigue proving by limiting stresses. The limitations and field of application of each procedure were identified. Several issues which are not dealt with in current design codes (such as the Eurocodes) were discussed and clarified. A feasible procedure for taking into account the degradation of axle load effects (owing to the influence of the rails, the sleepers and the ballast) in dynamic analyses with moving loads was presented and implemented in a pre-processing numerical tool for dynamic analysis of the passage of trains. A windows application for fatigue analysis through the damage accumulation method was also implemented.

A numerical methodology for taking into account the effects of in-plane shear and transverse bending, for fatigue analyses of railway bridge girder webs, was developed and validated through the comparison with experimental results.

In this methodology, truss models are used for calculation of the stresses caused by in-plane shear. The validity of this approach was checked by comparing the outcome of truss-model calculations with experimental results reported in the literature. It was found that safe estimates

for the stirrup fatigue lives are obtained if the stirrup stress ranges are calculated considering the EN 1992-1-1 proposal for the inclination of the compression struts, and the fatigue strength of the stirrups is given by the characteristic S-N curves for straight reinforcing bars. Truss models proved to be particularly conservative in the case of prestressed elements, because alternative resistant mechanisms play a significant role in that case.

An original procedure for stress calculations considering the combined effects of in-plane shear and transverse bending was developed. This numerical methodology can be used in fatigue analyses of girder webs in railway bridges according to the Damage Accumulation Method. The proposed methodology allows the fatigue verification of both the stirrups and the web concrete. The simplicity of the algorithm for stress calculations makes it feasible to calculate stress histories due to the passage of trains when the histories of internal forces (in-plane shear and transverse bending) are calculated by means of dynamic FE analyses. Given the complexity of the cyclic behaviour of cracked girder webs subjected to in-plane shear and transverse bending, the calculated stresses for both the steel stirrups and the web concrete depend on a parameter, k_m , which defines the relative contribution of two resisting mechanisms. Conservative fatigue analyses are obtained if the fatigue analysis is verified, not for a single k_m value, but for the plausible range of k_m values. A procedure for determination of the range of plausible k_m values was also indicated.

The comparison between the output of the proposed methodology and the experimental results reported in the literature showed that the developed analysis methodology provides conservative estimates for the fatigue life of the tested beams.

Numerical procedures for calculation of the ULS capacity of girder webs considering the interaction between in-plane shear and transverse bending were also presented, in Annex 1, and validated through the comparison with experimental results. It was shown that the adopted numerical models provide conservative estimates for the ultimate capacity of the tested beams. The conservative results are, in part, a consequence of the reduced shear span-to-depth ratio of the tested beams. Moreover, the fact that the tested beams were not subjected to cyclic in-plane shear forces (only to cyclic transverse bending moments) also contributes to actual stirrup stresses lower than the predictions of truss models. Parametric analyses were carried out to evaluate the influence of various parameters involved in ULS analyses. It was shown that the normal force (related to the transverse deck behaviour) does not play a significant role in ULS calculations, and conservative estimates are obtained if this force is ignored.

The applicability of the proposed methodology to problems of shear between web and flanges was also discussed. It was recommended that, before it is proven that the mechanism of eccentricity of the compression struts can be considered in this type of problem, a conservative

approach should be followed, i.e., the beneficial effects of that mechanism should not be considered, both in ULS and in fatigue analyses.

The third part of the present thesis was dedicated to the long-term analysis of continuous precast bridge decks, considering the influence of concrete delayed deformations and phased construction. A numerical methodology for long-term analyses was presented and validated through the comparison with experimental results observed in a real structure. It was found that the employed analysis strategy gives a good prediction of the structural response, if the material properties are accurately modelled, and the actual sequence of construction is considered.

Even though the design codes do not accurately predict the actual development of the concrete delayed deformations, it was found that, in the case of the structure under analysis, good estimates are obtained for long-term restraint bending moments when creep and shrinkage are modelled based on the EN 1992-1-1, if realistic values for the girder ages at the time that the continuity is established are adopted in the analysis.

A feasible procedure for taking into account seasonal shrinkage variations in long-term analyses was presented. Nevertheless, it was concluded that seasonal shrinkage variations need not to be taken into account in design calculations, because they do not originate relevant stresses or deformations.

Even though the actual time variation of strains in the diaphragms could not be accurately calculated, it was shown that the uncertainty involved in the prediction of diaphragm strains does not affect the most relevant results in terms of practical design: the stresses and the internal forces.

Given that the time variation of stresses and bending moments in this type of structure strongly depends on creep and shrinkage deformations and construction timings (which are variables with a strong statistical dispersion), a Monte Carlo simulation of the long-term behaviour was carried out. This simulation showed that the calculation of both restraint bending moments and concrete stresses in critical sections is significantly affected by the uncertainty associated to the quantification of creep and shrinkage deformations and construction timings. It was also shown that, in the case of the structure under analysis, the 10% and the 90% percentiles of a blind Monte Carlo simulation (only based on data usually available at the design stage) provide an envelope for the results of a calibrated deterministic analysis. It was found that the adopted strategy for probabilistic analysis appears to be a feasible approach for taking into account the variability of the structural response in this type of structure due to the uncertainty associated to the quantification of creep and shrinkage deformations and construction timings.

In the last part of this work, the previously developed and validated analysis procedures were applied in the study of a real continuous precast railway bridge deck without continuity prestress.

The Monte Carlo simulation of the long-term structure behaviour, taking as independent variables the creep and shrinkage deformations and the girder ages at the time that the continuity connection is made, showed that the structure response is sensitive to the uncertainty associated to the quantification of the aforementioned variables. It was also shown that this probabilistic approach provides quantitative information about the variability of the structure response. With this information, the variability of long-term effects can be taken into account both in serviceability limit state analyses and in verifications of the fatigue strength of concrete elements subjected to important cyclic compressive forces.

The results of the Monte Carlo simulation also revealed that the variability of bending moments is significantly higher than the variability of shear forces. Moreover, it was shown that a significant portion of long-term redistributions takes place in the first two years after the end of the construction, as a consequence of the reduced thickness of both the precast girder walls and the deck slab (which, in turn, gives rise to high rates of creep and shrinkage development in the first months after casting). Therefore, it seems advisable to consider the long-term distribution of internal forces (due to permanent actions) in fatigue verifications.

The developed methodologies for fatigue analyses considering the effects of in-plane shear and transverse bending were applied in the study of both the girder webs and the deck slab. ULS analyses were also carried out, by using available methodologies for taking into account the combined effects of shear and out-of-plane bending.

It was shown that fatigue verifications are, in general, more detrimental than ULS verifications. Moreover, it was demonstrated that transverse bending can not be ignored in the design of this type of structure, because the required areas of transverse reinforcements are considerably larger when this effect is taken into account, when compared to calculations considering the sole effect of in-plane shear. The proposed methodology for taking into account the influence of fatigue effects and transverse bending proved to be an effective tool to be used at the design stage of this type of structure.

The developed methodologies for taking into account the effects of in-plane shear and transverse bending were also applied to the analysis of the deck slab. The results of ULS and fatigue analyses were shown and discussed. It was shown that, even though the deck slab is subjected to non-negligible twisting bending moments m_{xy} , this flexural force has minor implications in terms of required areas of transverse slab reinforcement. It was shown that fatigue and ULS analyses of

the transverse slab reinforcement can be, for practical purposes, carried out considering only the effects of in-plane shear and transverse bending. The results of calculations in which the mechanism of eccentricity of the compression struts is, or is not, taken into account were presented and discussed. It was shown that ignoring that mechanism leads to results on the safe side. As suggested in Chapter 4, that mechanism should not be considered before it is demonstrated that its beneficial effects can be safely used in problems of shear between web and flanges. It was shown that ULS analyses based on the simplistic procedure suggested by the EN 1992-1-1 provide lower required areas of transverse reinforcement than more detailed calculations in which the mechanism of eccentricity of the compression struts is not considered. Therefore, it was suggested that further studies into the adequacy of the EN 1992-1-1 approach should be conducted in the future.

The consequences of cracking in the continuity connection were also evaluated by using the developed numerical model, the Cracked Beam Model. It was shown that small hysteresis occurs during the unloading and reloading cycles, and therefore, the moment-average curvature relationship under cyclic loading can be approximately represented by a straight line that connects the points corresponding to maximum and minimum loads. The slope of that line expresses the bending stiffness for cyclic loading after cracking, and can be used in linear dynamic analyses to calculate the structure response under the passage of high speed trains.

The influence of different parameters needed in calculations according to the Cracked Beam Model and the way they affect both the calculated stiffness decrease and the residual crack openings were evaluated through a parametric analysis. This study showed that the variability associated to the quantification of the stiffness decrease after cracking is much lower than that related to the quantification of residual crack openings. Nevertheless, the proposed methodology for evaluation of crack openings is more realistic than the one included in design codes, such as the EN 1992-1-1, for the reason that the latter does not explicitly take into account the influence of shrinkage and the consequences of traffic loading. The results obtained for this bridge deck suggest that the proposed Cracked Beam Model is also more conservative than the procedure given by the EN 1992-1-1. For the reasons mentioned above, it is advisable to carry out the calculation of maximum residual crack openings in this type of structure according to the procedure recommended in this work.

Regarding the influence of cracking in terms of dynamic response of the structure to the passage of high speed trains, it was shown that in this type of structure, the stiffness decrease due to cracking is responsible for increasing vertical displacements, decreasing frequencies of vibration, modification of mode shapes, decreasing resonance speeds and significant redistributions of bending moments. The relevance of shear deformations and the consequences of resonance

effects in terms of quantification of maximum sagging bending moments at the continuity supports were also pointed out.

7.2 Future research

Various numerical tools for evaluation of the service behaviour of railway bridge decks constructed with precast beams were developed and validated within the scope of this thesis. However, additional developments are needed so that the procedures for assessment and design of this type of structure can be improved. Moreover, the developed numerical tools can also be applied in the analysis of other structure types. Some research topics, in line with the developed work, are presented in the following paragraphs.

- **Influence of cracking in terms of local behaviour of bridge deck slabs**

The developed numerical tool for analysis of the cyclic behaviour of cracked RC or PC members was, in this thesis, applied for the analysis of the mechanical behaviour of deck girders, i.e., the analysis of the cyclic behaviour after cracking focused on the global deck behaviour. In Chapter 6, it was shown that cracking also occurs in the deck slab, as a consequence of local bending moments applied to this element.

Preliminary analyses revealed that slab cracking plays an important role in the case of double-track railway bridge decks whose cross section is formed by two precast U-shaped girders. In these structures, the bending moments and shear forces installed in each girder, caused by traffic loads on one of the tracks, strongly depend on the stiffness of the slab element that connects the two girders. Therefore, the dynamic response of the structure to the passage of high speed trains in one of the tracks is affected by that stiffness.

Furthermore, in the case of double-track railway bridge decks built with uni-cellular box girders, slab cracking caused by transverse bending moments also plays an important role. In these structures, the slab spans are larger than the ones found in typical bridge decks built with light precast, U-shaped or I-shaped, girders. Consequently, the influence of cracking should be taken into account during the dynamic analysis of those uni-cellular bridge decks.

The developed numerical models can be used in these analyses of the local behaviour of deck slabs. Further calibrations should be carried out through the comparison with experimental results considering histories of bending moments with sign reversals (a situation which typically occurs in deck slabs).

- **Experimental work for calibration of bond stress-slip models**

Further calibration of bond stress slip models should be carried out, considering the influence of cyclic loading. This calibration can be undertaken by means of laboratorial experiments in which cracked RC elements are subjected to cyclic loading, and the longitudinal variation of steel strains is measured. Optical fibres with arrays of Bragg Grating Sensors can be used for those measurements. This technique offers an important advantage: steel strains can be measured in closely spaced locations with minor interference with the structural behaviour.

- **Numerical and experimental evaluation of the cyclic response of cracked RC elements for different load levels**

The developed numerical model for analysis of the cyclic behaviour of cracked RC elements is based on constitutive models which take into account the influence of the load level. It was shown that the model results agree positively with experimental observations for either low or high load levels. Therefore, this model can be used for interpretation of the results of vibration tests of cracked RC members (tests in which structures are subjected to reduced load levels). Moreover, the model can be used to estimate the differences in terms of structural stiffness of cracked RC or PC elements when the response under low load levels (vibration tests, for example) is compared with the response under high load levels (traffic loading, for example).

Laboratorial tests in beams subjected to different load levels need to be carried out so that further experimental data can be obtained for calibration and validation of this numerical model.

- **Calculation of residual crack openings in highway bridge decks**

The developed numerical model for analysis of the cyclic behaviour of cracked RC elements can be used in the analysis of other types of structures subjected to important cyclic loads for calculation of the influence of cyclic loading in terms of residual crack openings. Special attention needs to be paid to highway bridge deck slabs, where cracking can occur owing to transverse bending. The developed numerical model can be used to establish guidelines for taking into account the influence of cyclic loading during the evaluation of maximum residual crack openings.

- **Influence of cracking in terms of shear deformations**

In Chapter 6, it was shown that in this type of structure, shear deformations play an important role because of the high shear stresses installed in the girder webs. The influence of cracking in terms of bending stiffness decrease was evaluated in this thesis. However, cracking also affects shear deformations. Numerical methods for evaluation of

the influence of cracking in terms of shear deformability increase should, therefore, be implemented.

- **Calibration of the model for fatigue analysis of girder webs through the comparison with finite-element analysis**

The developed model for stress calculations, to be used in the fatigue analyses of girder webs, was validated in this thesis through the comparison with experimental results. The developed methodology is suitable for the analysis of complex histories of applied forces, because it requires reduced computational effort, and is not impaired by convergence difficulties. However, it does not provide a single stress result. Instead, the outcome of the calculations is a range of plausible stress values, owing to the uncertainty associated to the quantification of the parameter k_m . Therefore, the web thicknesses and quantities of shear reinforcements required to provide adequate fatigue lives are, in some cases, overestimated.

The results provided by the developed methodology should be compared with the outcomes of finite-element analyses considering appropriate constitutive models to describe the non-linear cyclic behaviour of the materials. This comparison should provide additional bases for the definition of narrower values for the range of plausible k_m values, i.e., to reduce the range of calculated stress results. Moreover, finite-element analyses can also be used to evaluate the magnitude of bending moment redistributions, caused by web cracking.

- **Finite element analysis of problems of shear between web and flanges**

The developed methodology for fatigue analyses considering the effects of in-plane shear and transverse bending were also applied in this thesis to problems of shear between web and flanges. A conservative approach was adopted, which consisted of disregarding the beneficial effects of the eccentricity of the compression struts. This approach was adopted for the reason that, to the author's knowledge, the possibility to consider that resistance mechanism in such type of problem was not demonstrated yet. Further improvements can be made by employing finite-element analyses to evaluate the plausible range of k_m values to be considered in these cases.

- **Fatigue damage due to daily temperature variations**

The relevance of temperature variations in terms of fatigue damage in bridge decks needs to be clarified. Procedures for characterization of the temperature action (number of cycles and corresponding amplitudes of temperature variations) should be established. This task can be based on records of temperature variations measured in real structures subjected to permanent monitoring.

- **Fatigue analyses of double track railway bridge decks**

The case study presented in this thesis consists of a single track structure. Double-track decks, whose cross section is composed of two precast U-shaped girders, have been frequently employed in new railway lines. The fatigue analysis of these structures is more laborious owing to the possibility of simultaneous crossings in both tracks. A possible approach to handle this problem was summarized in Chapter 3. The developed methodologies should be applied for the analysis of these double-track structures, in order to identify the most critical spots in terms of fatigue strength, and also to establish guidelines on the fatigue analysis of these structures.

- **Monitoring of railway bridge decks built with precast U-shaped girders**

Experimental data regarding the behaviour of real railway bridge decks built with U-shaped girders is still scarce. Such experimental information is important for validation of numerical models and design assumptions, and also to confirm that the requirements in terms of service behaviour are met. The foreseeable construction of new railway lines in Portugal offers the opportunity to observe the real behaviour of these structures. This task should comprehend long-term measurements (concrete deformations, concrete temperatures and relative displacements of expansion joints) and short-term monitoring campaigns (evaluation of the structural behaviour during the passage of real trains and determination of frequencies of vibration, modes of vibration and modal damping ratios). The temperature distribution across the cross-section depth should be characterized in detail. Special attention should also be paid to the experimental evaluation of the influence of cracking.

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Annex 1

Ultimate capacity of girder webs considering the effects of in-plane shear and out-of-plane bending

This Annex presents two different analysis procedures for calculation of the ultimate capacity of girders webs subjected to the combined effects of in-plane (longitudinal) shear forces and out-of-plane (transverse) bending moments. The first procedure is based in the method developed by Gaspar (2003). The other one is based on the method originally presented by Menn (1986). The main differences between the procedures presented here and the original ones are as follows:

- the procedures presented here are consistent with the method included in the EN 1992-1-1 for shear design of members with shear reinforcement;
- the membrane force in the web, in the transverse direction, is taken into account.

Both calculation procedures consist of two different stages:

- in Stage 1, the resistance to transverse bending moments is provided by the eccentricity of the compression strut;
- if the applied bending moment is higher than the one that can be resisted in Stage 1, then additional bending moments can be resisted through the variation of the tensile stress in the stirrups and the variation of the compressive force in the web concrete (Stage 2).

In Stage 1, there is no difference between the calculation procedures proposed by Gaspar (2003) and Menn (1986). In Stage 2, these authors followed different hypothesis for calculation of the applied stresses. The stirrups are assumed to be perpendicular to the girder axis and the girder is assumed to have constant depth.

Stage 1

If the web is subjected to the sole effect of in-plane shear, the tensile force in each stirrup leg per unit length is equal to (see Figure A1.1):

$$B_{l,0} = B_{r,0} = \frac{V_{Ed}}{z \cdot \cot \theta \cdot 2} \quad (A1.1)$$

where $B_{l,0}$ and $B_{r,0}$ are the tensile forces per unit length in the left and in the right stirrup legs, respectively, V_{Ed} is the design value of the applied shear force, z is the lever arm and θ is the inclination of the compression struts. The angle θ must lie in the interval $45^\circ \geq \theta \geq 21.8^\circ$ ($1 \leq \cot \theta \leq 2.5$). The tensile force in the stirrups is balanced by the vertical component of the compression force in the struts, C :

$$C = \frac{V_{Ed}}{z \cdot \cot \theta} \quad (A1.2)$$

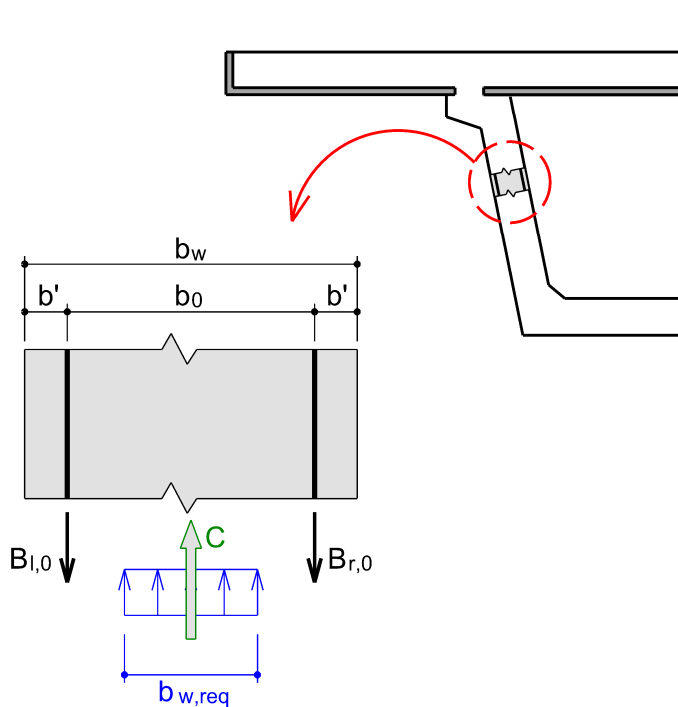


Figure A1.1 – Forces due to the sole effect of in-plane shear.

The minimum required web thickness, $b_{w,req}$, depends on the compression strength of the struts and is given by:

$$b_{w,req} = \frac{V_{Ed} \cdot (\cot \theta + \tan \theta)}{\alpha_{cw} \cdot \nu_1 \cdot f_{cd} \cdot z} \quad (A1.3)$$

where α_{cw} and ν_1 are coefficients that account for the influence of the axial force on the girder and of the shear cracks, these coefficients being defined in the EN 1992-1-1. If the minimum required web thickness is lower than the actual web thickness, b_w , the excess capacity can be used to resist transverse bending. In the presence of transverse bending, the resultant compression force on the web shifts towards the flexural compression side of the web, without changes in the stirrup forces. The compression force in the web, in the transverse direction, n_{Ed} , is equilibrated by the increase of compression stresses in the concrete (whose resultant equals ΔC_1 , as represented in Figure A1.2). In this Annex, positive bending moments are assumed to cause increasing compression stresses on the left side of the web. The bending moment resistance is, thus, given by:

$$m_{max,1} = (C + \Delta C_1) \cdot e_{max} = \left(\frac{V_{Ed}}{z \cdot \cot \theta} + n_{Ed} \right) \cdot \frac{b_w - b_{w,req}}{2} \quad (A1.4)$$

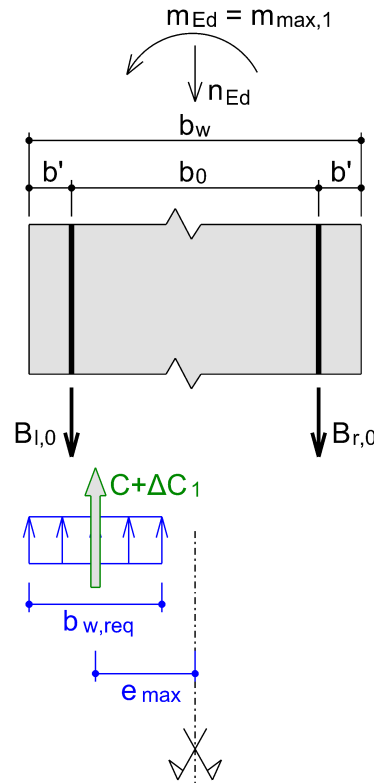


Figure A1.2 – Bending moment resistance due to the eccentricity of the compression strut.

Stage 2

If the applied bending moment is higher than the value that can be resisted by the eccentricity of the compression struts, $m_{max,1}$, then the additional resistance is provided by increasing tensile stresses in the stirrup leg on the flexural tensile side and increasing compression stresses on both the other stirrup leg and the concrete on the flexural compression side. As mentioned before, different hypotheses for calculation of the applied stresses were adopted by Gaspar (2003) and by Menn (1986).

Analysis procedure based on the method proposed by Gaspar

This author assumes that the position of the resultant concrete compressive force remains unchanged with respect to the position in Stage 1 (see Figure A1.3). The calculation of applied stresses depends on the relationship between the increase of compression force in the concrete in the direction of the web axis, ΔC_2 , and the decrease of tensile force in the left stirrup leg, ΔB_l . This relationship is established through the dimensionless parameter β_{c2} :

$$\beta_{c2} = \frac{\Delta C_2}{\Delta B_r} = \frac{\Delta C_2}{\Delta B_l + \Delta C_2} \quad (A1.5)$$

where ΔB_r stands for the increase of tensile stresses in the right stirrup leg. The parameter β_{c2} takes values in the interval $0 \leq \beta_{c2} \leq 1$. Gaspar (2003) suggests $\beta_{c2} = 0.5$.

The resultant tensile force on the right stirrup leg must not be higher than its tensile strength:

$$\Delta B_r \leq \left(\frac{A_{sw}}{s} \right)_{one\ leg} \cdot f_{yd} - B_{r,0} \quad (A1.6)$$

where $(A_{sw}/s)_{one\ leg}$ is the cross-sectional area of one stirrup leg per unit length and f_{yd} is the design yield strength of the stirrups. The resistant bending moment is, thus, given by:

$$m_{max,2} = (C + \Delta C_1 + \Delta C_2) \cdot e_{max} + (\Delta B_r + \Delta B_l) \cdot \frac{b_o}{2} \quad (A1.7)$$

where b_o is the distance between the stirrup axes, as represented in Figure A1.1.

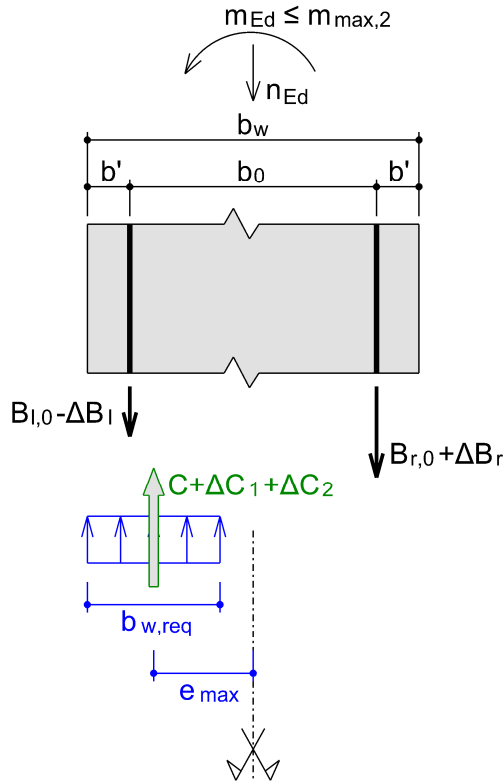


Figure A1.3 – Bending moment resistance in Stage 2, according to the proposal by Gaspar (2003).

The additional compression forces ΔC_1 and ΔC_2 , which are applied in the direction parallel to the stirrup axes, lead to the rotation of the principal direction and the increase of the principal stress applied to the web concrete. According to Gaspar (2003), the magnitude of the forces ΔC_1 and ΔC_2 must be limited so that the resultant compression stress (in the direction of these forces) is not higher than $0.85 \cdot f_{cd}$. However, in the practical applications envisaged in this work, the applied stress is always considerably lower than this limit value. An alternative to the procedure suggested by Gaspar, for evaluation of the relevance of forces ΔC_1 and ΔC_2 , can be based on the calculation of the modification of the principal direction and the increase of the principal compression stress applied to the web concrete. These calculations are carried out as shown in Figure A1.4. The applications envisaged within the scope of the present work showed that the increase of principal compression stress (owing to ΔC_1 and ΔC_2) is diminutive.

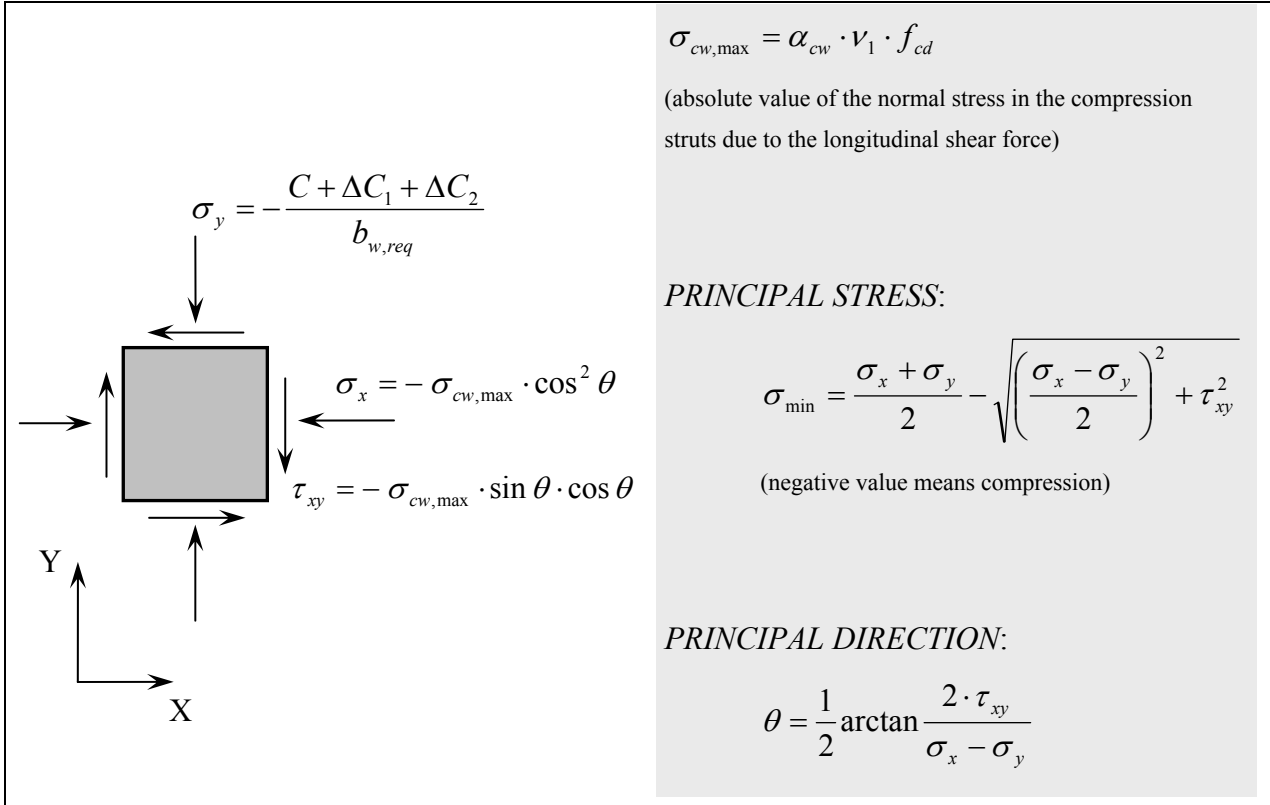


Figure A1.4 – Calculation of the principal directions and the principal compression stress in the web concrete, due to the forces ΔC_1 and ΔC_2 .

Analysis procedure based on the method proposed by Menn

Menn (1986) assumes that the left stirrup leg never experiences compression stresses. Two different cases are distinguished in this author's proposal:

- the case in which in-plane shear predominates;
- the situation in which transverse bending predominates.

If shear predominates, i.e., if the stress in the stirrups due to the sole effect of in-plane shear is higher than (or equal to) half its yield stress, then the bending moment resistance in Stage 2 is reached without any additional increase of compression stresses in the concrete, i.e., only the stirrup stresses are changed (see Figure A1.5). The bending moment resistance is, thus, given by:

$$m_{max,2a} = (C + \Delta C_1) \cdot e_{max} + (\Delta B_r) \cdot b_o \quad (A1.8)$$

where the variation of the tensile force in the right stirrup leg is given by:

$$\Delta B_r = \left(\frac{A_{sw}}{s} \right)_{one\ leg} \cdot f_{yd} - B_{r,0} \quad (A1.9)$$

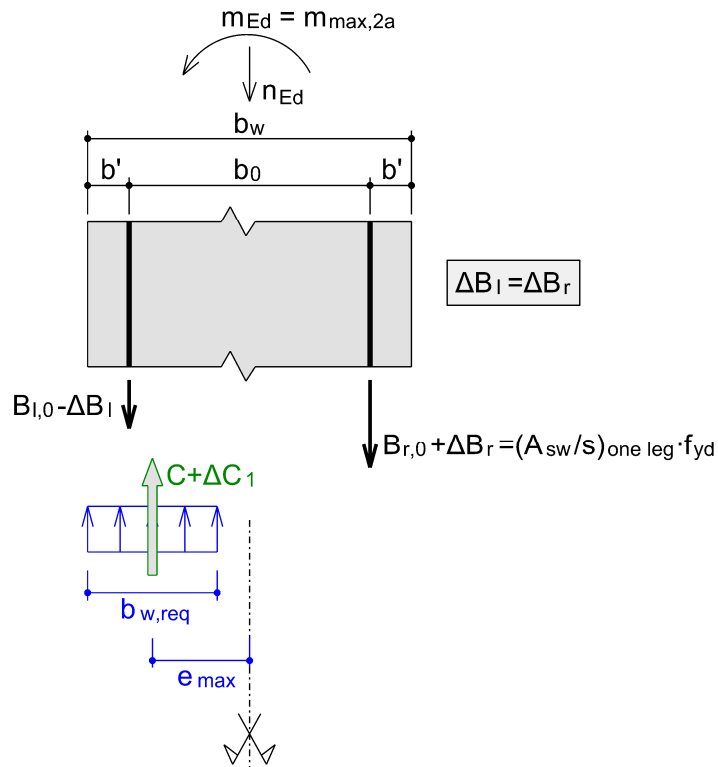


Figure A1.5 - Bending moment resistance in Stage 2, according to the model by Menn (1986), for the case in which shear predominates.

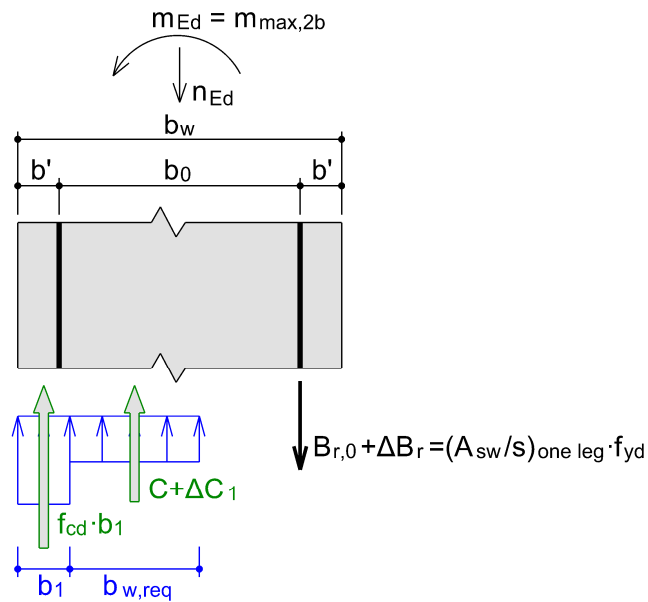


Figure A1.6 - Bending moment resistance in Stage 2, according to the model by Menn (1986), for the case in which transverse bending predominates.

If transverse bending predominates, the normal force in the left stirrup leg vanishes. The additional compression force is resisted by a concrete layer, whose width is equal to b_l , subjected

to the stress f_{cd} (see Figure A1.6). The compression strut (whose breath depends on the magnitude of the longitudinal shear force, as explained before) must be shifted back towards the centreline of the web, as represented in Figure A1.6.

The bending moment resistance is, thus, given by:

$$m_{\max,2b} = f_{cd} \cdot b_1 \cdot \frac{b_w - b_1}{2} + (C + \Delta C_1) \cdot \frac{b_w - 2 \cdot b_1 - b_{w,req}}{2} + \left(\frac{A_{sw}}{s} \right)_{one\ leg} \cdot f_{yd} \cdot \frac{b_0}{2} \quad (A1.10)$$

where:

$$b_1 = \frac{\left(\frac{A_{sw}}{s} \right)_{one\ leg} \cdot f_{yd} - C}{f_{cd}} \quad (A1.11)$$

It is worth mentioning that Equation (A1.10) must not be applied in the case of girder webs with high percentage of stirrups subjected to important transverse bending moments and reduced longitudinal shear forces. In this case, compression stresses in the left stirrup leg should be admitted.

Validation through the comparison with experimental results

In this section, the results provided by the numerical methodologies for ULS analyses are compared with the experimental results obtained by Gaspar (2003). The experimental campaign conducted by this author is presented in §4.3.3. Two beams were subjected to monotonically increasing transverse bending moments. Beam 1 had a higher ratio of shear reinforcement and lower concrete compressive strength, so that the compressive strength of the web concrete could influence the girder capacity. On the other hand, the resistance of Beam 2 was expected to be governed by the tensile strength of the stirrups.

Beam 1

The main characteristics of Beam 1 are described in §4.3.3. The test consisted of applying a force $P \cong 708$ kN at mid span, which was kept approximately constant for the remainder of the test. Thus, the maximum applied shear force is ~ 354 kN. Then, monotonically increasing transverse bending moments were applied through the forces $F1$ and $F2$. According to Gaspar (2003), the ultimate limit state was reached for $F1 \cong F2 \cong 130.5$ kN. When this force was

reached, strong discontinuities were observed in the measured stirrup strains. Nevertheless, higher forces could be applied, and the maximum value reached by the forces F was 178.7 kN.

Given the dimensions of the beam length (2.8 m) and the length over which the load is applied (2.0 m), a uniform distribution of bending moment over the entire beam length is assumed. Therefore, the transverse bending moment applied to the web in ultimate limit state takes the value:

$$m = \frac{130.5 \cdot 0.25}{1.40} = 23.3 \text{ kNm/m}$$

The corresponding compression force in the web due to the forces $F1$ and $F2$ amounts to:

$$n = \frac{130.5}{1.40} = 93.2 \text{ kN/m}$$

The experimental results are now compared with the numerical predictions given by the models described before. Table A1.1 shows the parameters considered in the calculations. The first evidence of the calculations is that the web concrete is subjected to very important compressive stresses. In fact, the web is 0.12 m thick, a value significantly lower than the minimum required thicknesses given by Equation (A1.3), considering the average cylinder compressive strength:

- 0.152 m for $\theta = 45^\circ$
- 0.219 m for $\theta = 22^\circ$

these values being determined considering the real applied shear force (not multiplied by the partial factor for actions), the average cylinder compressive strength and the remaining parameters indicated in Table A1.1.

Table A1.1 – Parameters used in the calculations for Beam 1

f_{cm} (MPa)	18.9
ν_l	0.6
α_{cw}	1.0
z (m)	0.41

These values show that Equation (A1.3) provides conservative estimates for the strength of the compression struts, i.e., the actual strength is greater than the value considered in the calculations. Consequently, a parametric analysis is carried out, considering increasing values for

the strength of the web concrete. Additional parametric analyses, whose results are presented in the following sections, were implemented for evaluation of the influence of:

- the parameter β_c involved in the calculations according to the model proposed by Gaspar (2003);
- the normal force in the web;
- the tensile force in the stirrups due to the sole effect of the longitudinal shear force.

Note that, one of the specificities of Beam 1 lies in the fact that its shear span-to-depth ratio (2.36) takes a small value, considerably lower than the one for the remaining tested beams. Consequently, part of the shear force applied to Beam 1 might migrate directly to the support without giving rise to tensile forces in the stirrups, thus making the interpretation of the test results more difficult. Therefore, the magnitude of the tensile force in the stirrups due to in-plane shear is one of the aspects evaluated by means of parametric analyses.

Influence of the strength of the compression struts

It was shown before that if the strength of the compression struts is given by:

$$\sigma_{cw,max} = \alpha_{cw} \cdot \nu_l \cdot f_{cm} \quad (A1.12)$$

and the parameters α_{cw} , ν_l and f_{cm} take the values shown in Table A1.1, very conservative estimates are obtained for the ultimate capacity of the tested beam. Consequently, a parametric analysis is carried out considering increasing values for that strength. Figure A1.7 shows the calculated bending moment resistances, for increasing values of the strength of the compression struts, considering values 30%, 40%, 50% and 60% higher than the outcome of Equation (A1.12).

Figure A1.7 displays the calculation results for Stages 1 and 2 described in Annex 1, the resistance in Stage 1 being given by the sole effect of the eccentricity of the compression strut. In the calculations based on the model proposed by Gaspar (2003), the parameter β_c takes the value 0.5, the influence of this parameter being discussed in the next section. The calculation results are presented for the range of possible values for the inclination of the compression struts θ , in steps of 1°. For example, for a strength increase of 30%, θ varies between 39° and 44°. This is the range of values for which the beam is able to resist the sole effect of in-plane shear. Outside this range:

- for higher θ values, the cross-sectional area of stirrups is lower than the required value; consequently, for $\theta = 44^\circ$, the bending resistance is nearly equal to the resistance in Stage

1, i.e., the resistance is due to the eccentricity of the compression strut because the stirrups are not able to provide additional bending moment resistance;

- for lower θ values, the actual web thickness is smaller than the value required in terms of resistance to the sole effect of the longitudinal shear force.

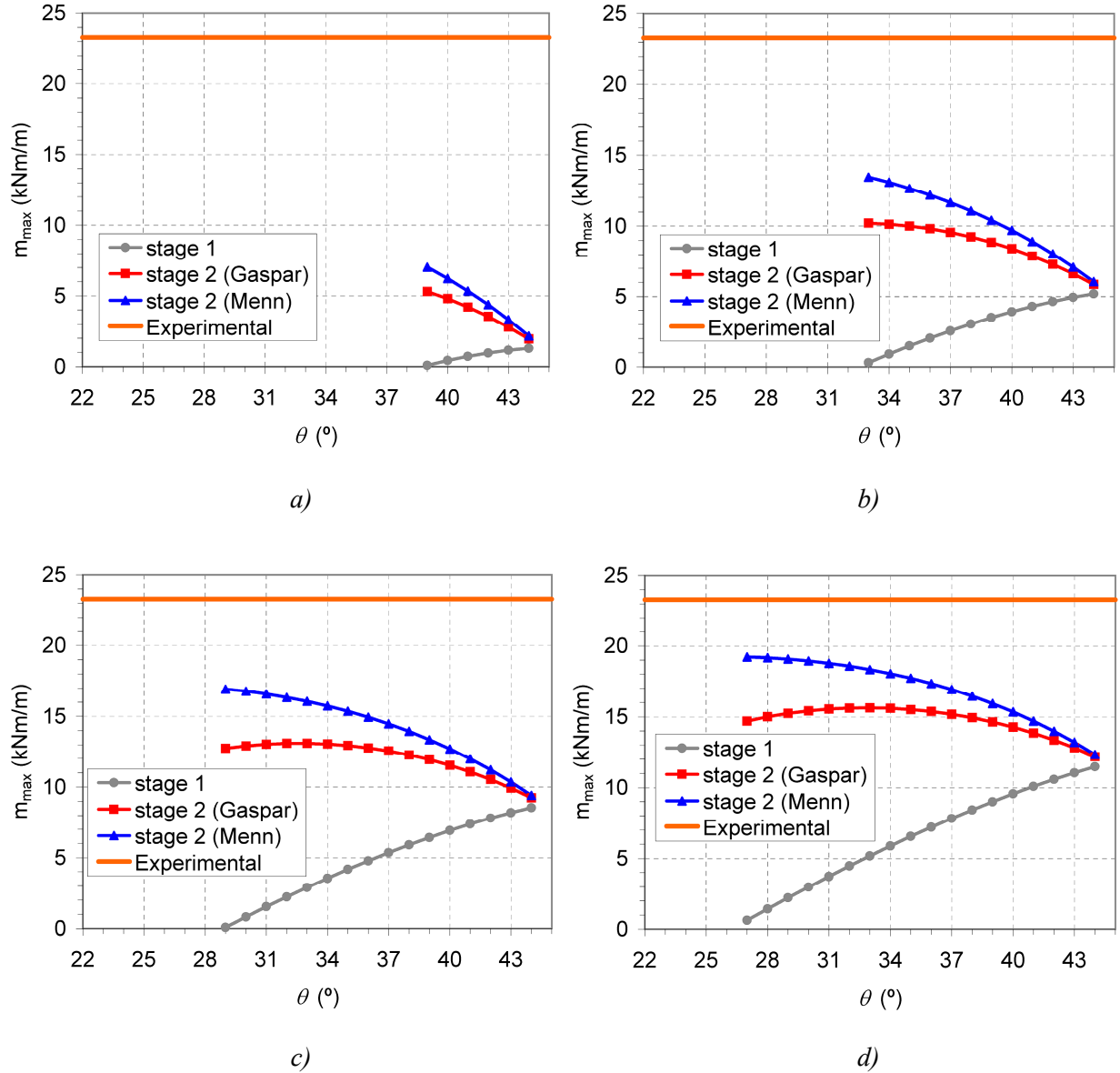


Figure A1.7 – Calculated values for the transverse bending moment resistance, according to the models based on the proposals by Gaspar (2003) with $\beta_c = 0.5$ and Menn (1986). Results for increasing values of the strength of the compression struts, with respect to the outcome of Equation (A1.12):

a) 30% increase; b) 40% increase; c) 50% increase; d) 60% increase.

Figure A1.7 shows that, even for high values of the strength increase, the calculated bending moment resistance is still lower than the experimental one. These results indicate that the proposed calculation models provide conservative results for this beam.

Influence of the parameter β_c

Figure A1.8 shows the influence of the parameter β_c involved in the calculations according to the model proposed by Gaspar (2003). In these calculations, the strength of the compression struts takes a value 50% higher than the one given by Equation (A1.12). One can see that the calculated bending moment resistances increase as β_c decreases. This is a consequence of the fact that the web is subjected to important compressive forces. Higher values of β_c correspond to higher percentages of the bending moment resisted by compression forces in the concrete and tensile forces in the stirrup on the flexural tensile side. In this mechanism, the internal lever arm is lower than the one in the case that the bending moment resistance is assured by variation of the stirrup stresses only, and therefore, the bending moment resistance decreases as β_c increases.

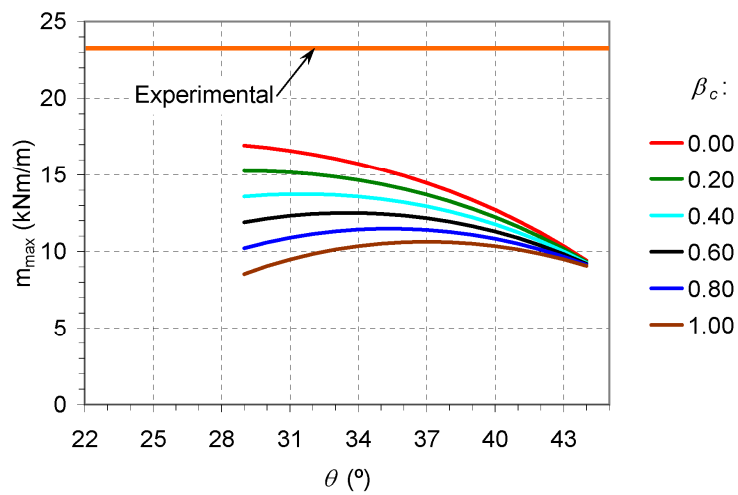


Figure A1.8 – Calculated values for the transverse bending moment resistance, according to the model proposed by Gaspar (2003), for different values of β_c , considering that $\sigma_{cw,max}$ is 50% higher than the value given by Equation (A1.12).

Figure A1.8 also shows that the influence of the parameter β_c decreases as the inclination θ increases, for the reason that, for higher θ values, the bending moment resistance is mainly provided by the eccentricity of the compression strut (resistance in Stage 1, which is not influenced by the parameter β_c).

Note that, for $\beta_c = 0$, the calculated bending moments according to the proposals of Gaspar (2003) and Menn (1986) coincide. According to both models, if the shear force predominates and $\beta_c = 0$, the bending moment resistance is provided by the symmetric variation of the tensile stress in the stirrup legs.

Influence of the normal force

In this section, the influence of the normal force related to the transverse behaviour (the one that, in this case, is caused by the forces $F1$ and $F2$) is evaluated. The calculation results are also compared with the web resistances in simple bending and composed bending, i.e., the results of calculations in which in-plane shear is ignored.

Figure A1.9 shows the calculated bending moment resistances according to the proposals by Menn (1986) and Gaspar (2003). In these calculations, the strength of the compression struts also takes a value 50% higher than the one given by Equation (A1.12). This figure shows that the influence of the normal force is not relevant and conservative estimates are obtained if this force is ignored. If a lower strength of the compression struts was adopted in the calculations, the influence of the normal force would be even smaller. Therefore, the normal force can be omitted in the calculations.

On the other hand, the in-plane shear force is responsible for an important decrease of the bending moment resistance, as shown in Figure A1.9, where the results of calculations in simple bending and composed bending (disregarding the in-plane shear force) are also plotted.

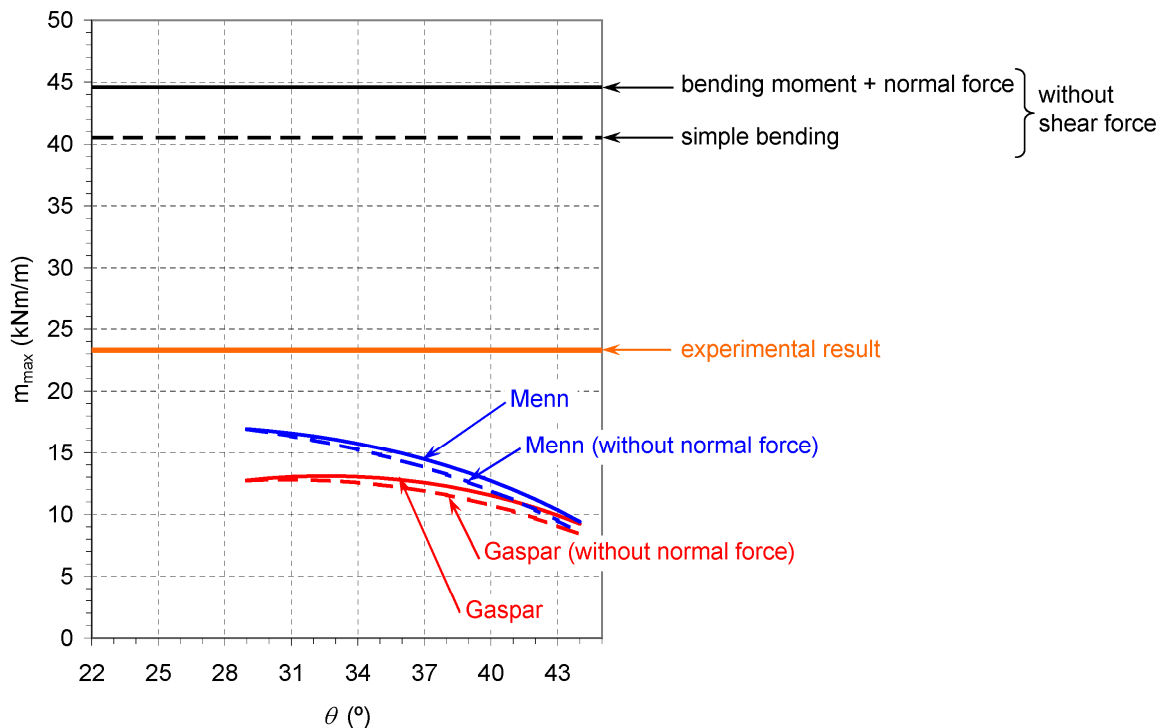


Figure A1.9 – Calculated values for the transverse bending moment resistance [$\beta_c = 0.5$ and $\sigma_{cw,max}$ 50% higher than the value given by Equation (A1.12)]. Influence of the transverse membrane force and the in-plane shear force.

Influence of the stirrup stress due to the sole effect of in-plane shear

In the previous sections, it was shown that the numerical predictions stood clearly below the experimental result if the strength of the compression struts was increased up to a value 60% higher than the average estimate given by Equation (A1.12). The high experimental result can be explained by other effect, which has already been mentioned before: the stirrups stress due to the longitudinal shear force might be lower than the value given by the truss model. In fact, part of the shear force might be transmitted to the support through a direct compression strut, as a consequence of the reduced shear span-to-depth ratio.

Thus, the following figures present the calculated bending moment resistances on the assumption that the stirrup stress due to the longitudinal shear force is:

- 15% lower than the value given by the truss model (results plotted in Figure A1.10);
- 30% lower than the value given by the truss model (results plotted in Figure A1.11).

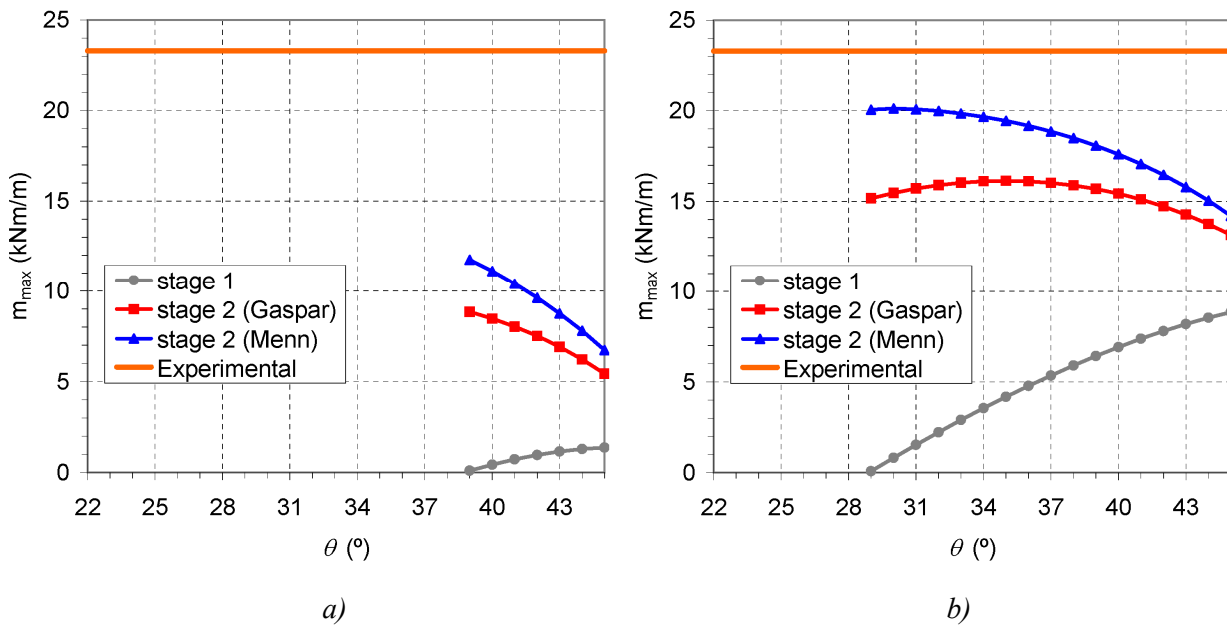


Figure A1.10 – Calculated values for the transverse bending moment resistance, according to the proposals by Gaspar (2003) with $\beta_c = 0.5$ and Menn (1986) considering a 15% decrease of the stirrup stresses due to in-plane shear:

- $\sigma_{cw,max}$ 30% higher than the value given by Equation (A1.12);
- $\sigma_{cw,max}$ 50% higher than the value given by Equation (A1.12).

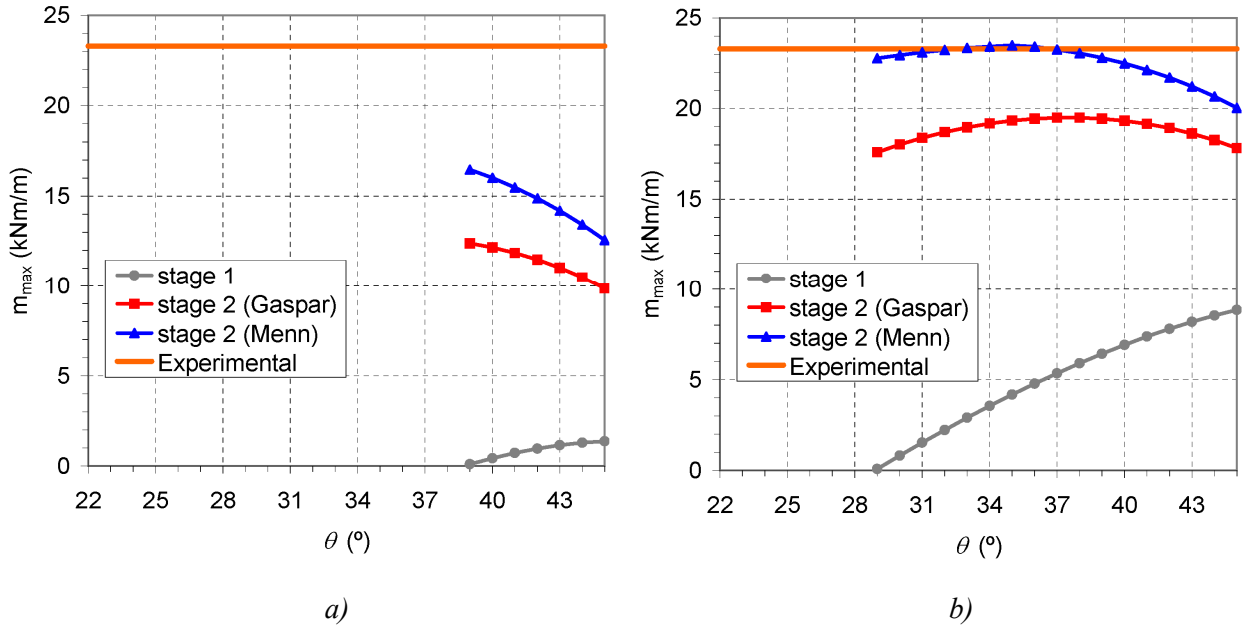


Figure A1.11 – Calculated values for the resistant transverse bending moment, according to the models based on (Gaspar 2003) with $\beta_c = 0.5$ and (Menn 1986) considering a 30% decrease of the stirrup stresses due to in-plane shear:
a) $\sigma_{cw, \max}$ 30% higher than the value given by Equation (A1.12);
b) $\sigma_{cw, \max}$ 50% higher than the value given by Equation (A1.12).

These figures show that the reduction of the stirrup stresses due to the shear force plays an important role in terms of bending moment resistance. Nevertheless, the previous figures show that the experimental bending resistance is only reached if important variations of both the compression strength of the struts and the stirrup stresses due to in-plane shear are adopted in the calculations (50% increase and 30% decrease, respectively). Therefore, one can conclude that the calculation models provide conservative estimates for the resistance of Beam 1. But it is important to note that the conservative results are, in part, a consequence of the reduced shear span-to-depth ratio of the tested beam. In a real structure, the shear span takes, in general, higher values and this effect becomes less relevant. This effect is also expected to play a less important role in the remaining tested beams, since the shear span-to-depth ratio is higher in those cases.

Moreover, the fact that this beam was not subjected to cyclic in-plane shear forces also contributes to actual stirrup stresses lower than the predictions of truss models, as concluded in §4.2.2. In the case of real railway bridges, the shear forces undergo important cyclic variations, and consequently, a better agreement between the model estimates and the actual stirrup stresses due to in-plane shear is expected.

Beam 2

The main geometrical and material characteristics of Beam 2 are shown in §4.3.3. This beam has a lower ratio of shear reinforcement and a higher shear span-to-depth ratio in comparison with Beam 1. The loading sequence was as follows:

- firstly, a point load P was applied in order to induce diagonal cracking in the web;
- then, this load was removed;
- after that, a point load $P = 392$ kN was applied and kept approximately constant during the remainder of the test;
- finally, increasing values of the forces $F1$ and $F2$ were applied until rupture.

The ULS was reached when the forces $F1$ and $F2$ took the value 155 kN. Thus, the transverse bending moment and the transverse normal force in ULS are equal to:

$$m = \frac{155 \cdot 0.25}{1.90} = 20.4 \text{ kNm/m}$$

$$n = \frac{155}{1.90} = 81.6 \text{ kN/m}$$

The previous values were calculated under the assumption that the entire beam length contributes to the bending moment resistance, because Gaspar (2003) observed that, after cracking, all the beam length is mobilized to resist this force.

Figure A1.12 presents the calculated bending moment resistances, considering the mean values of the material resistances and the parameters shown in Table A1.2. In these calculations, the parameter β_c took the value 0.5, the influence of this parameter being analysed in the next section. This figure shows that the calculated resistances are considerably lower than the experimental result. For inclinations θ greater than $\sim 31^\circ$, the cross-sectional area of the stirrups becomes lower than the value required to resist the sole effect of in-plane shear. Therefore, the no solution exists for values of θ greater than that limit value.

In the following sections, the differences between experimental and numerical results are further discussed and the influence of different calculation parameters is examined by means of parametric analyses.

Table A1.2 – Parameters used in the calculations for Beam 2.

f_{cm} (MPa)	36.5
ν_l	0.6
α_{cw}	1.0
z (m)	0.42
V (kN)	196

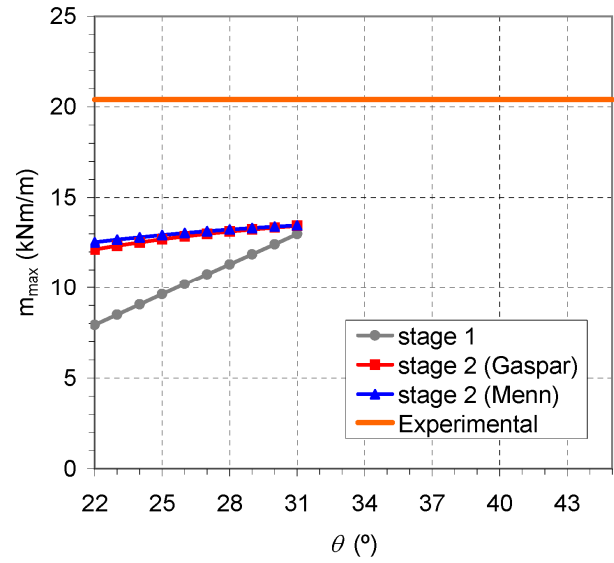


Figure A1.12 – Calculated bending moment resistances according to the proposals by Gaspar (2003) with $\beta_c = 0.5$ and Menn (1986).

Influence of the parameter β_c

Figure A1.13 shows the influence of the parameter β_c , for calculations in which the material resistances are defined by their mean values. This figure shows that, in this case, the influence of the parameter β_c is not relevant (this influence is much less relevant than in the case of Beam 1). This is because the minimum required web thickness given by Equation (A1.3) is significantly lower than the actual web thickness. Therefore, the lever arm for the cases in which the compressive force is materialized by the concrete or, alternatively, by the stirrups, is similar. Thus, the parameter β_c plays a minor role in this case.

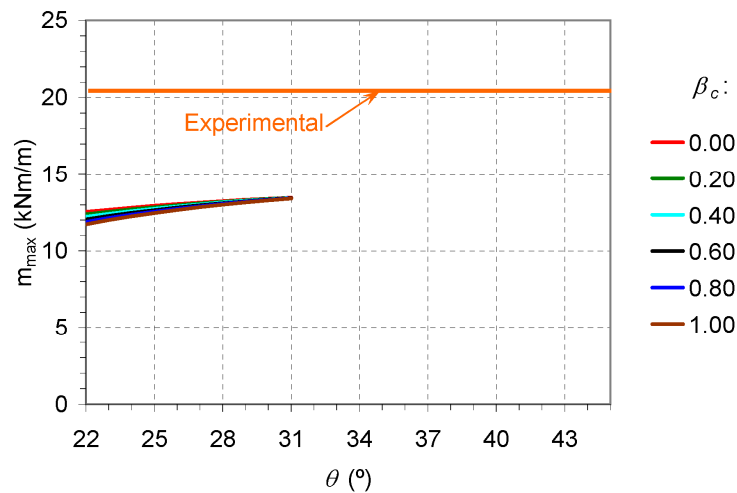


Figure A1.13 – Bending moment resistances according to the model based on Gaspar (2003). Influence of the parameter β_c .

Given that the longitudinal shear force predominates over the transverse bending, the model proposed by Menn (1986) yields the same results than the one recommended by Gaspar (2003) for $\beta_c = 0$. The reason for this similitude has already been indicated before. Given the small relevance of the parameter β_c , the proposals by Gaspar (2003) and Menn (1986) yield similar results, no matter which value of β_c is adopted. Like in the case of Beam 1, higher values of β_c give rise to more conservative predictions, i.e., lower calculated bending resistances.

Influence of the normal force

Figure A1.14 depicts the results of calculations in which both the normal force (related to the transverse behaviour) and the in-plane shear force are taken into account or, alternatively, are not considered. One can see that, in this case, the normal force plays a more important role than in the case of Beam 1. This is because the ratio between the normal force and the compression force C defined by Equation (4.3) is higher in the case of Beam 2.

The calculations based on the proposals of Gaspar (2003) and Menn (1986) yield similar results, as mentioned and justified in the previous section.

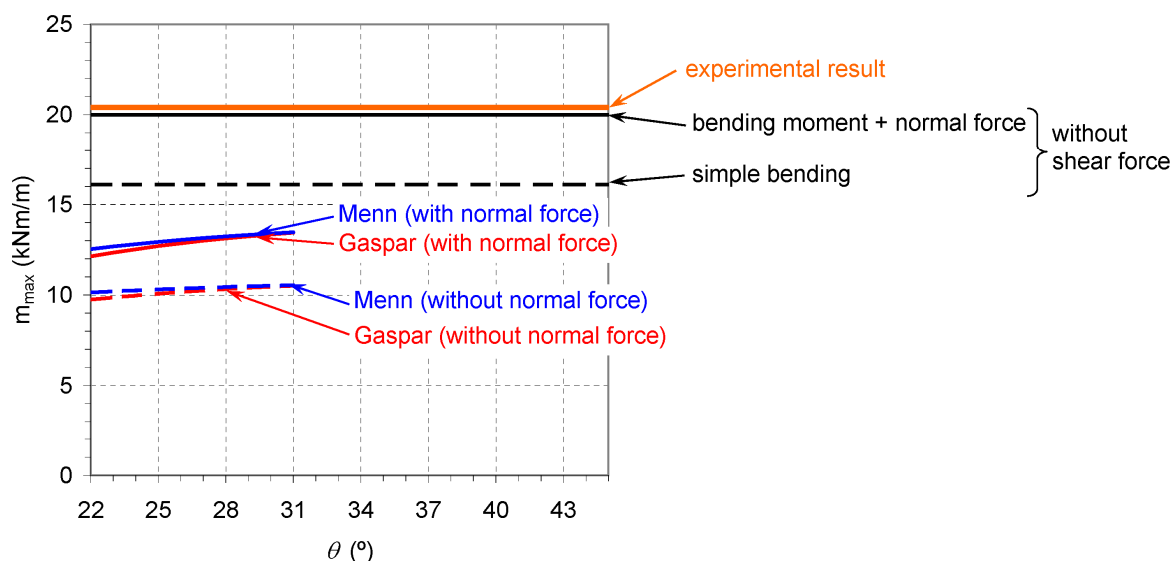


*Figure A1.14 – Calculated values for the bending moment resistance ($\beta_c = 0.5$).
Influence of the transverse normal force and the longitudinal shear force.*

Figure A1.14 also shows the results of calculations in which the in-plane shear force is disregarded. While this type of calculation provided results clearly higher than the experimental one for Beam 1 (i.e., the experimental results showed that in-plane shear is responsible for an important decrease of the transverse bending moment resistance for Beam 1), in the case of Beam 2, the experimental result is similar to the bending moment resistance in simple bending (disregarding the shear force).

Given that important stirrup stresses are obtained under the sole effect of in-plane shear (64.8% of f_{yd} for $\theta = 22^\circ$ and 96.3% of f_{yd} for $\theta = 31^\circ$, according to the truss model), and the cover to the stirrups is quite small, the shear force must be responsible for a non negligible reduction of the transverse bending moment resistance. Therefore, the fact that the experimental bending resistance is even larger than the calculated resistance in composed bending can only be justified:

- by the fact the peak strength of the employed steels is significantly larger than the yield stress considered in the analyses [in-air tests of the employed steels, performed by Gaspar (2003), showed that important post-yield strain hardening occurs];
- by the direct transmission of part of the applied force to the supports without giving rise to tensile stresses in the stirrups.

The former effect is usually disregarded in design calculations. The latter is analysed below.

Influence of the strength of the compression struts

Figure A1.15 presents the calculated bending moment resistances for increasing values of the strength of the compression struts:

- resistance given by Equation (A1.12);
- resistance 30% higher than the value given by Equation (A1.12);
- resistance 60% higher than the value given by Equation (A1.12).

Only the model proposed by Menn (1986) is considered in this analysis, since both models provide similar results.

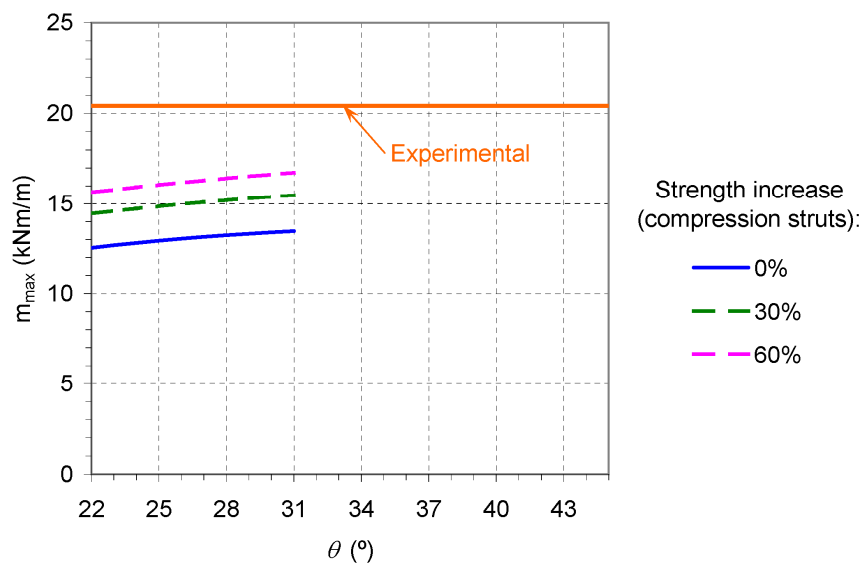


Figure A1.15 – Calculated values for the bending moment resistance according to the proposal by Menn (1986), for increasing values of the strength of the compression struts.

Figure A1.15 shows that the influence of the strength of the compression struts is significantly less important than in the case of Beam 1. This is because the ratio between the applied compression stress due to the sole effect of in-plane shear and the strength of the struts is considerably lower in the case of Beam 2.

In Figure A1.15, one can see that even if the adopted strength of the compression struts is 60% higher than the value given by Equation (A1.12), the bending moment resistance is significantly lower than the experimental result. Therefore, the obtained experimental results can not be explained by the variation of this parameter.

Influence of the percentage of the shear force resisted by the web

As mentioned before, part of the shear force can be transmitted to the supports through alternative mechanisms that complement the truss behaviour. In this particular case, given the important dimensions of the beam flanges (see Figure 4.18) an important part of the shear force can be resisted by the flanges themselves. Therefore, the previously presented results were compared with the outcome of calculations in which only a given percentage (70% in this case) of the total shear force is resisted by the truss mechanism. The results are depicted in Figure A1.16, where one can see that even if the strength of the compression struts is increased by 30% and the shear force resisted by the truss mechanism is decreased by 30%, the calculation results are still lower than the experimental one.

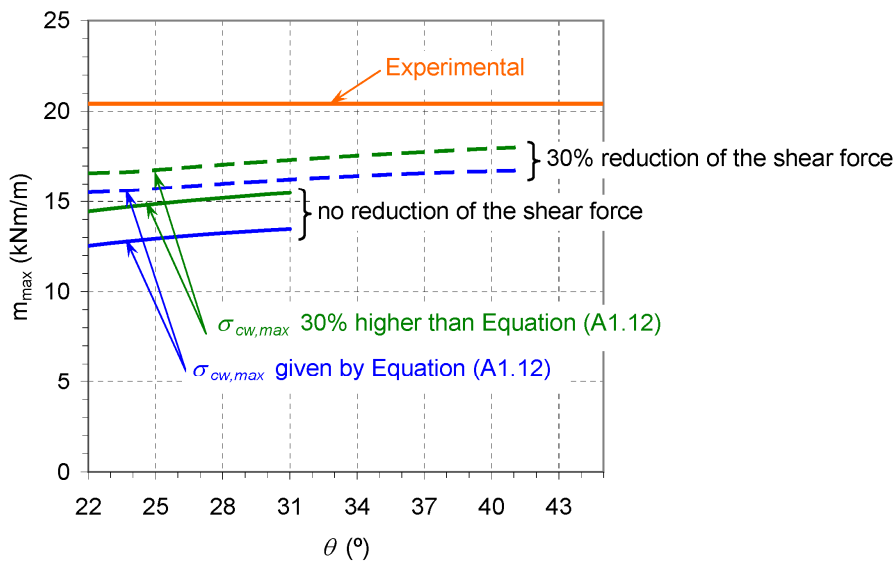


Figure A1.16 – Calculated values for the bending moment resistance according to the proposal by Menn (1986), for increasing values of the strength of the compression struts and decreasing values of the shear force resisted by the truss mechanism.

Final considerations

Two alternative methodologies for calculation of the ultimate capacity of girder webs considering the effects of in-plane shear and transverse bending were described. The transverse membrane force is also considered. These analysis methodologies are based on the methods developed by Menn (1986) and by Gaspar (2003). The original procedures presented by those authors were adapted so that the calculations are consistent with the EN 1992-1-1 approach for ULS analyses of beams subjected to in-plane shear.

The comparison between numerical and experimental results for the ultimate capacity of webs subjected to the combined effects of in-plane shear and transverse bending revealed that the adopted numerical models provide conservative estimates for the ultimate capacity of the beams tested by Gaspar (2003). The conservative results are, in part, a consequence of the reduced shear span-to-depth ratio of the tested beams (particularly in the case of Beam 1). Moreover, the fact that the tested beams were not subjected to cyclic in-plane shear forces also contributes to actual stirrup stresses lower than the predictions of truss models. Parametric analyses were carried out to evaluate the influence of various parameters involved in ULS analyses. It was shown that the compression force (related to the transverse deck behaviour) does not play a significant role in ULS calculations, and conservative estimates are obtained if this force is ignored.

Annex 2

Reinforcement layout in the structure considered in Chapter 6

In this Annex, the reinforcement (both ordinary and prestressed reinforcement) adopted in the precast bridge deck dealt with in Chapter 6 is graphically presented.

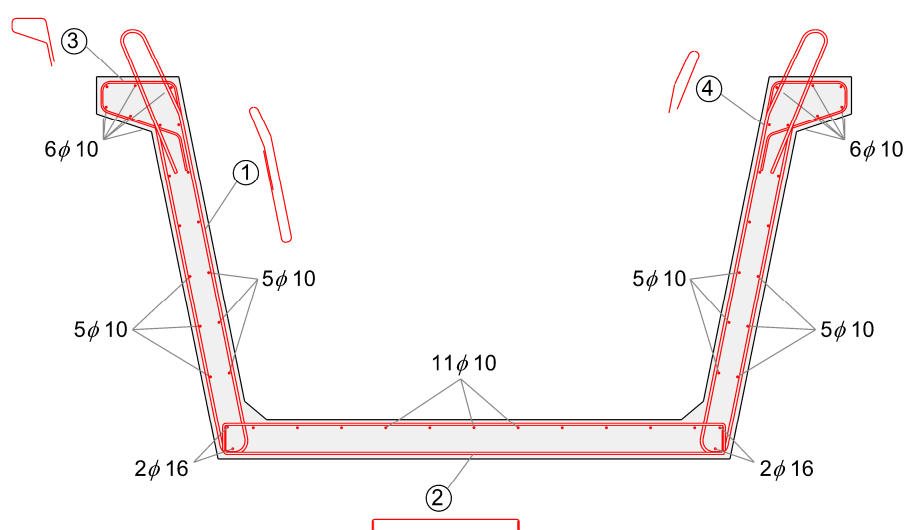


*Figure A2.1 - Girder reinforcements
(reinforcements (1) to (4) are defined in Figure A2.6).*

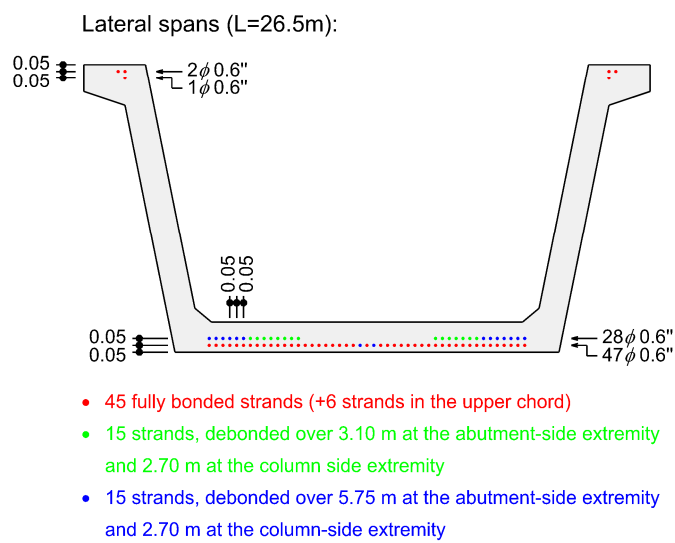
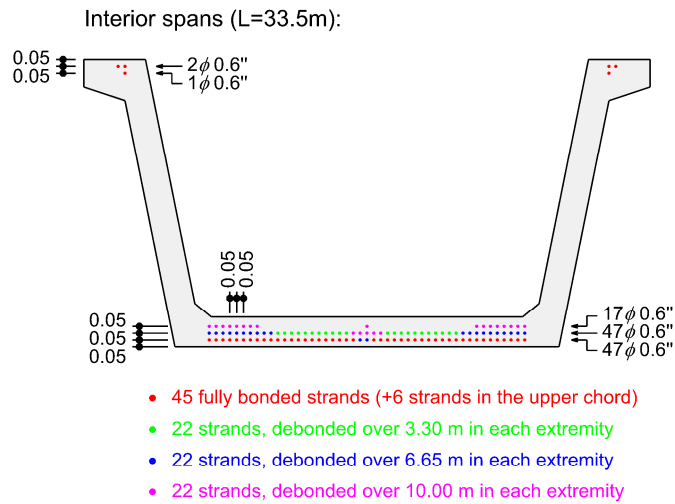


Figure A2.2 - Prestressing layout (dimension in m).

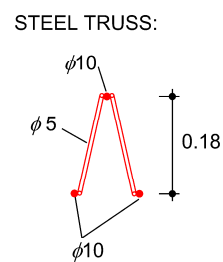
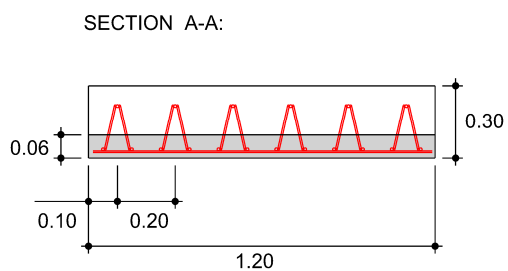
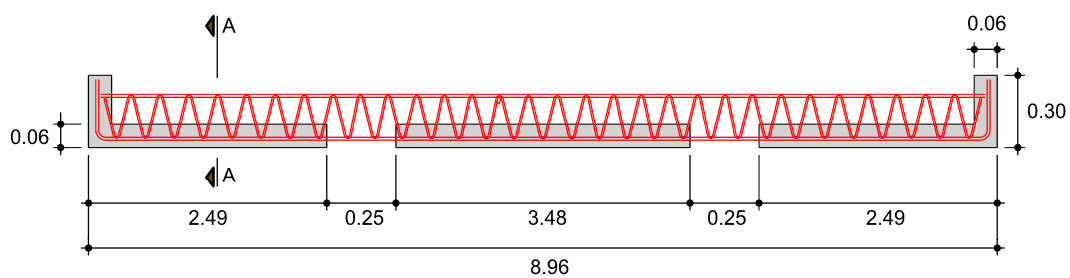


Figure A2.3 – Precast plank reinforcements (dimensions in m).

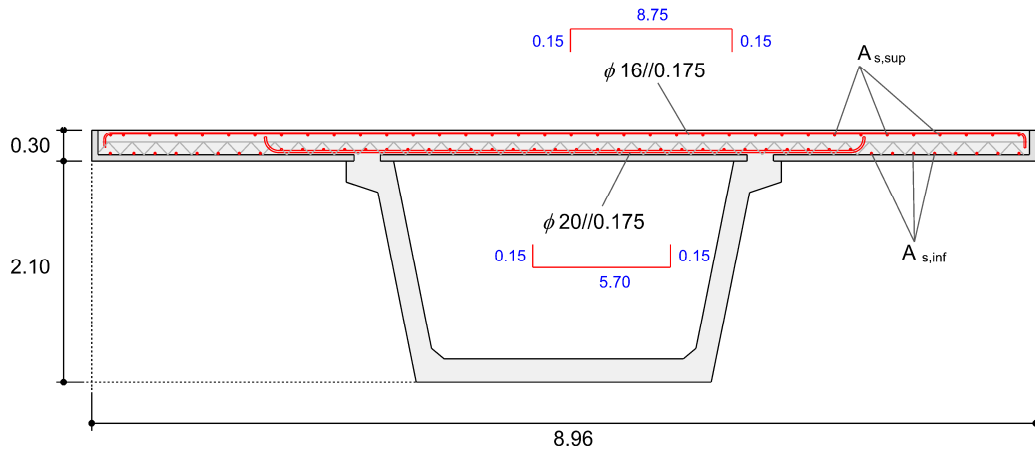
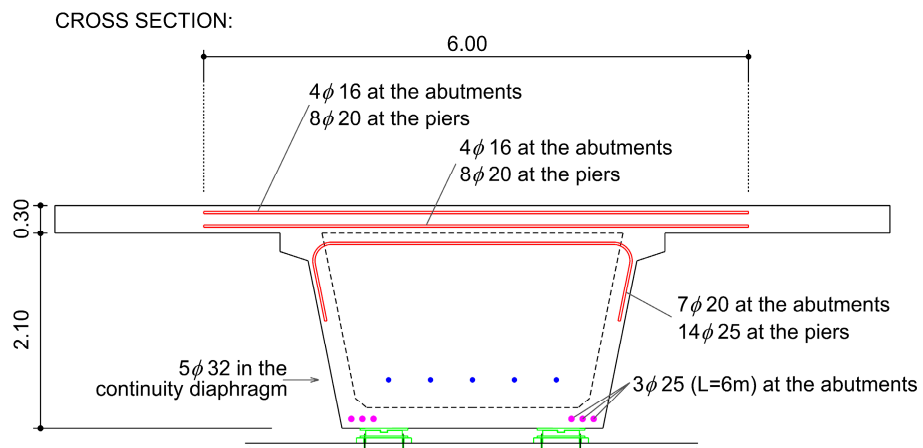


Figure A2.4 - Slab reinforcements (dimensions in m).
Reinforcements $A_{s,sup}$ and $A_{s,inf}$ are defined in Figure A2.6.



LONGITUDINAL SECTION:

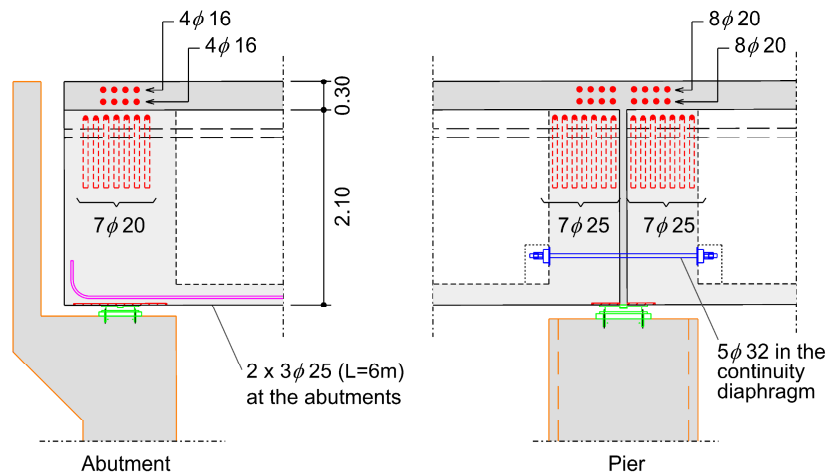
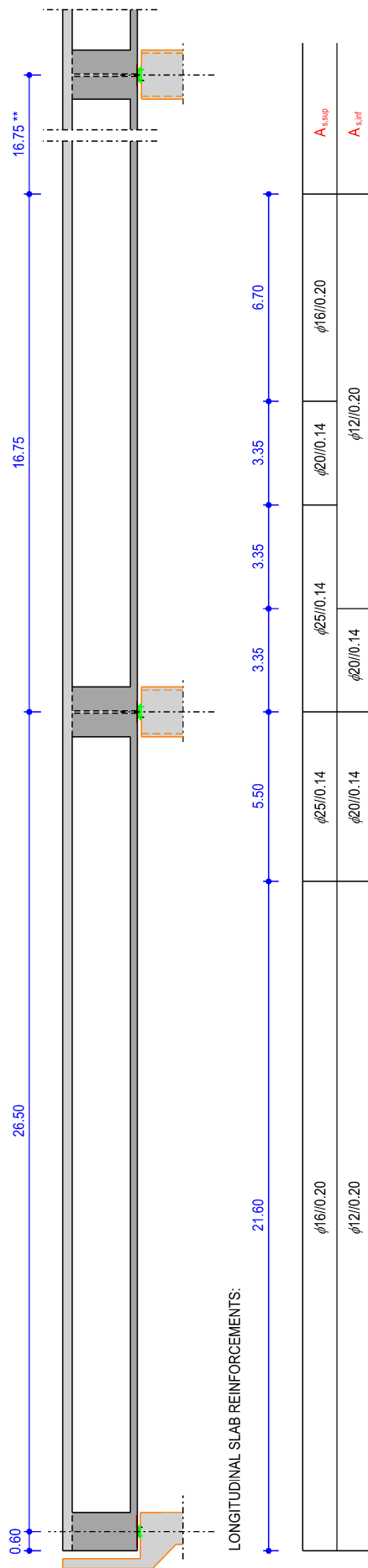


Figure A2.5 - Additional diaphragm reinforcements.



TRANSVERSAL GIRDER REINFORCEMENTS:

1.03	3.60	2.80	2.80	2.80	2.80	2.80	2.80	2.80	2.80	1.60	2.80	1.03	1.02	4.20	3.40	3.40	4.70
$\phi 16/0.10 + \phi 16/0.20 *$	$\phi 12/0.20 + \phi 10/0.20$	$\phi 12/0.20$	$\phi 10/0.20$	$\phi 10/0.20 + \phi 8/0.20$	$\phi 10/0.20$	$\phi 16/0.20$	$\phi 10/0.20$	$2\phi 12/0.20$	$\phi 12/0.10 + \phi 16/0.20 *$	$\phi 16/0.10 + \phi 16/0.20 *$	$\phi 16/0.20 + \phi 12/0.20$	$2\phi 12/0.20$	$\phi 16/0.10 + \phi 16/0.20$	$\phi 16/0.20 + \phi 10/0.20$	$2\phi 10/0.20$	$2\phi 10/0.20$	$\phi 12/0.20$
$\phi 16/0.20 + \phi 10/0.20$	$\phi 16/0.20 + \phi 10/0.20$	$\phi 16/0.20$	$\phi 10/0.20 + \phi 8/0.20$	$2\phi 12/0.20$	$2\phi 12/0.20$	$\phi 16/0.20$	$\phi 16/0.20 + \phi 10/0.20$	$2\phi 12/0.20$	$\phi 16/0.10 + \phi 16/0.20$	$\phi 16/0.10 + \phi 16/0.20$	$\phi 16/0.20 + \phi 10/0.20$	$\phi 16/0.20 + \phi 10/0.20$	$\phi 16/0.20 + \phi 10/0.20$	$\phi 10/0.20$	$\phi 16/0.20 + \phi 8/0.20$	$2\phi 10/0.20$	$\phi 12/0.20$
$\phi 10/0.10$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 8/0.20$	$\phi 8/0.20$	$\phi 12/0.20$
---	---	$\phi 10/0.20$	$\phi 12/0.20$	$\phi 10/0.20$	$\phi 10/0.20$	$\phi 8/0.20$	$\phi 10/0.20$	$\phi 20/0.20$	---	---	---	$\phi 16/0.20$	$\phi 10/0.20$	$\phi 8/0.20$	$\phi 8/0.20$	$\phi 12/0.20$	$\phi 12/0.20$
①	②	③	④														

* not projecting from the precast girder
 ** in the interior span, the reinforcement layout is symmetric with respect to the mid-span

Figure A2.6

Annex 3

Diagrams of internal forces for the structure considered in Chapter 6

This Annex presents the diagrams of flexural and membrane forces for the second span of the structure considered in Chapter 6, along the cross-sections S1, S2 and W1 (identified in Figure 6.9). The adopted convention for positive membrane forces (n_x , n_y and n_{xy}) and flexural forces (m_x , m_y and m_{xy}) is indicated in Figures A3.1 and A3.2. Figures A3.3 to A3.8 present the calculated forces for the following load cases:

- *Permanent loads* (self weight and non-structural dead loads);
- $dT + 9^{\circ}\text{C}$ (*slab*), which corresponds to a temperature difference across the thickness of the top slab, the top surface being 9°C warmer than the bottom one;
- $dT + 5^{\circ}\text{C}$ (*web*), which corresponds to a temperature difference across the web thickness, with a warmer exterior surface;
- *LM71*, which represents the load model LM71 (EN 1991-2); the results shown in the figures are not multiplied by the factor α or by the dynamic factor;
- *TT5*, which corresponds to the train type 5, introduced in the Annex D of the EN 1991-2.

The diagrams for negative temperature differences in the slab, $dT - 5^{\circ}\text{C}$ (*slab*), are not shown because they are directly proportional to the diagrams for positive temperature differences.

Two curves are shown in each graph for the traffic loads *LM71* and *TT5*. Those curves correspond to the top and bottom envelopes due to such live loads.

The forces were calculated by employing:

- the FE model with beam elements described in §6.4.1;
- the FE model with shell elements introduced in §6.5.1.

The membrane forces n_x due to permanent loads were calculated by using the beam model, and taking into account the construction sequence. Average values were adopted for creep and shrinkage deformations and also for the girder ages at the time the continuity is established. The n_x forces due to permanent loads, presented in Figures A3.3, A3.5 and A3.7, are long-term values (corresponding to the last step of the phased analysis, after the 20,000-day increment of time). The isostatic effect of pretensioning is also included in the results, which justifies the discontinuities visible in the diagrams.

The shear forces n_{xy} (for all the load cases) in the cross sections S1 and W1 were determined as a function of the vertical shear force, V , in the FE model with beam elements, for consistency with the assumptions commonly adopted in ULS analyses according to the EN 1992-1-1. The shear force n_{xy} in the web (cross section W1) is thus given by:

$$n_{xy} = \frac{V}{2 \cdot z \cdot \cos \alpha} \quad (A3.1)$$

where z represents the internal lever arm (2.14 m) and α is the inclination of the web with respect to the vertical direction (11.434°). The shear force n_{xy} in the cross section S1 is given by:

$$n_{xy} = \frac{\beta \cdot V}{2 \cdot z} \quad (A3.2)$$

where the parameter β , which was introduced in §4.3.1.1, takes the value 0.5.

The remaining membrane forces, as well as the flexural forces, were calculated by employing the FE model with shell elements. The envelopes of forces due to the load model LM71 were calculated by considering eight distinct positions of the concentrated forces along the structure's second span. This fact justifies the wave-shaped appearance of the diagrams of flexural forces due to this load case.

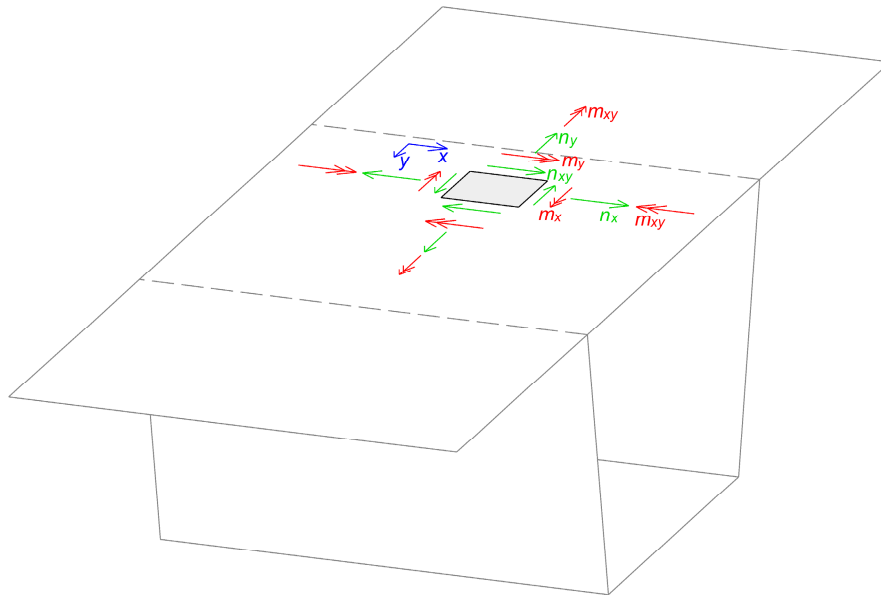


Figure A3.1 – Flexural and membrane forces in a slab element (valid for the cross sections S1 and S2).

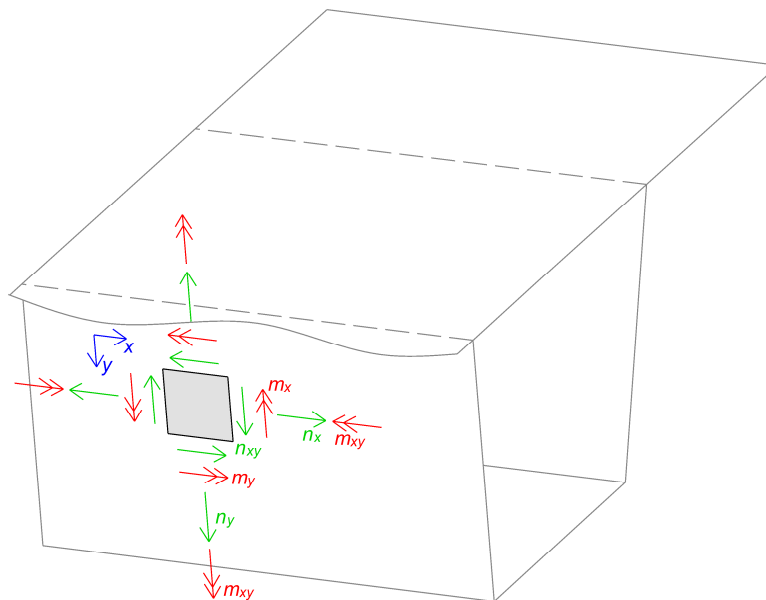


Figure A3.2 – Flexural and membrane forces in a web element (valid for the cross section W1).

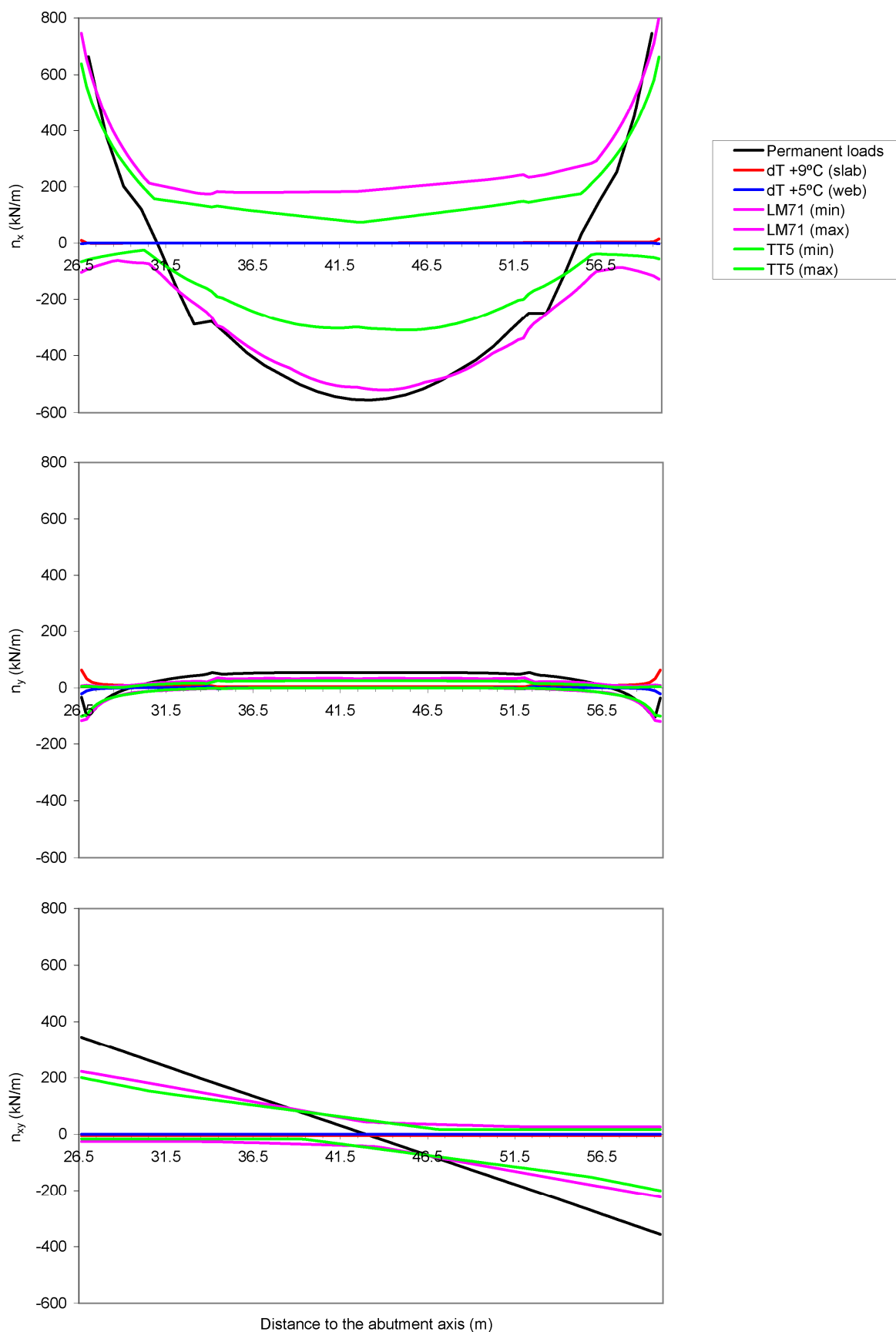


Figure A3.3 – Diagrams of membrane forces at the cross section S1.

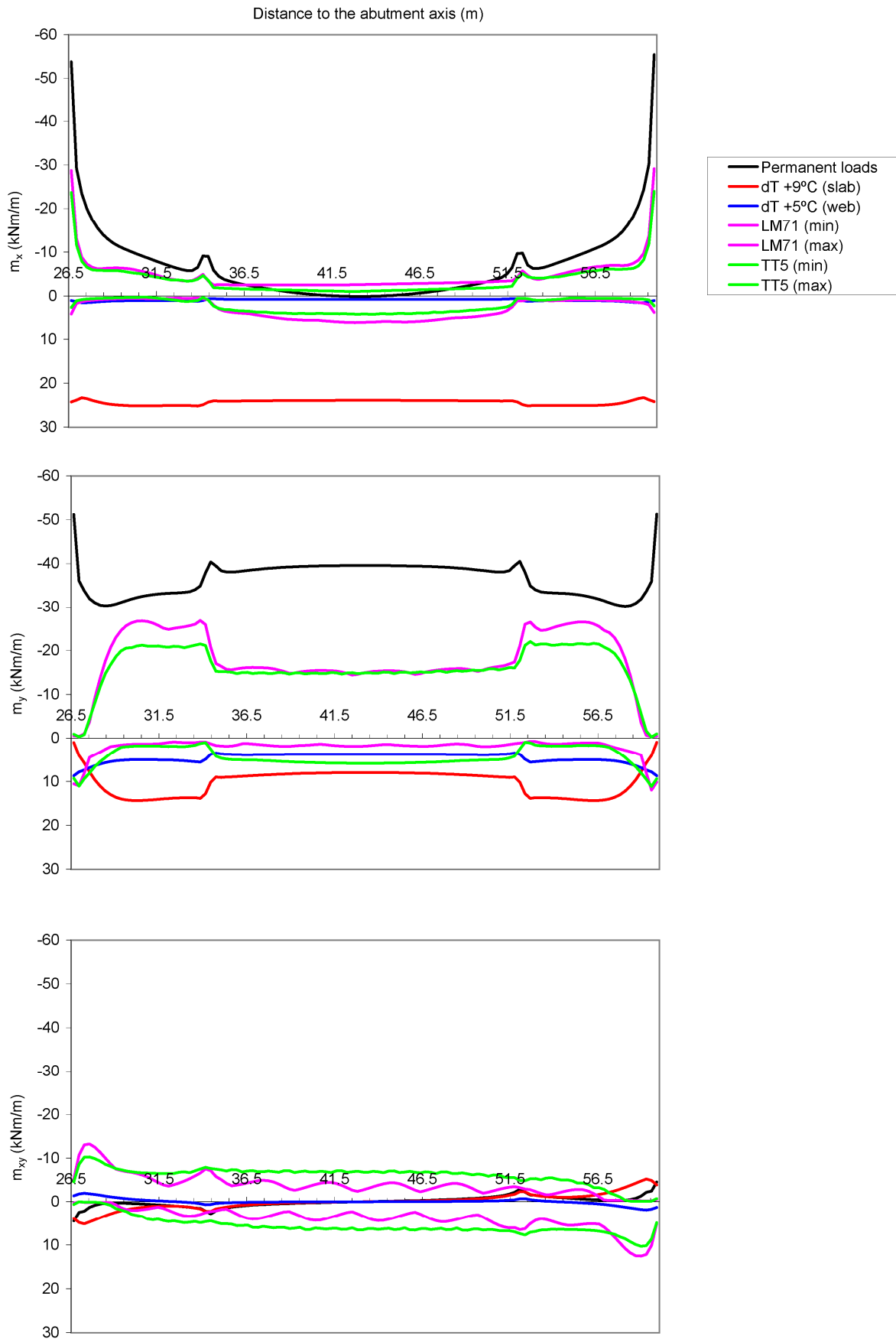


Figure A3.4 – Diagrams of flexural forces at the cross section S1.

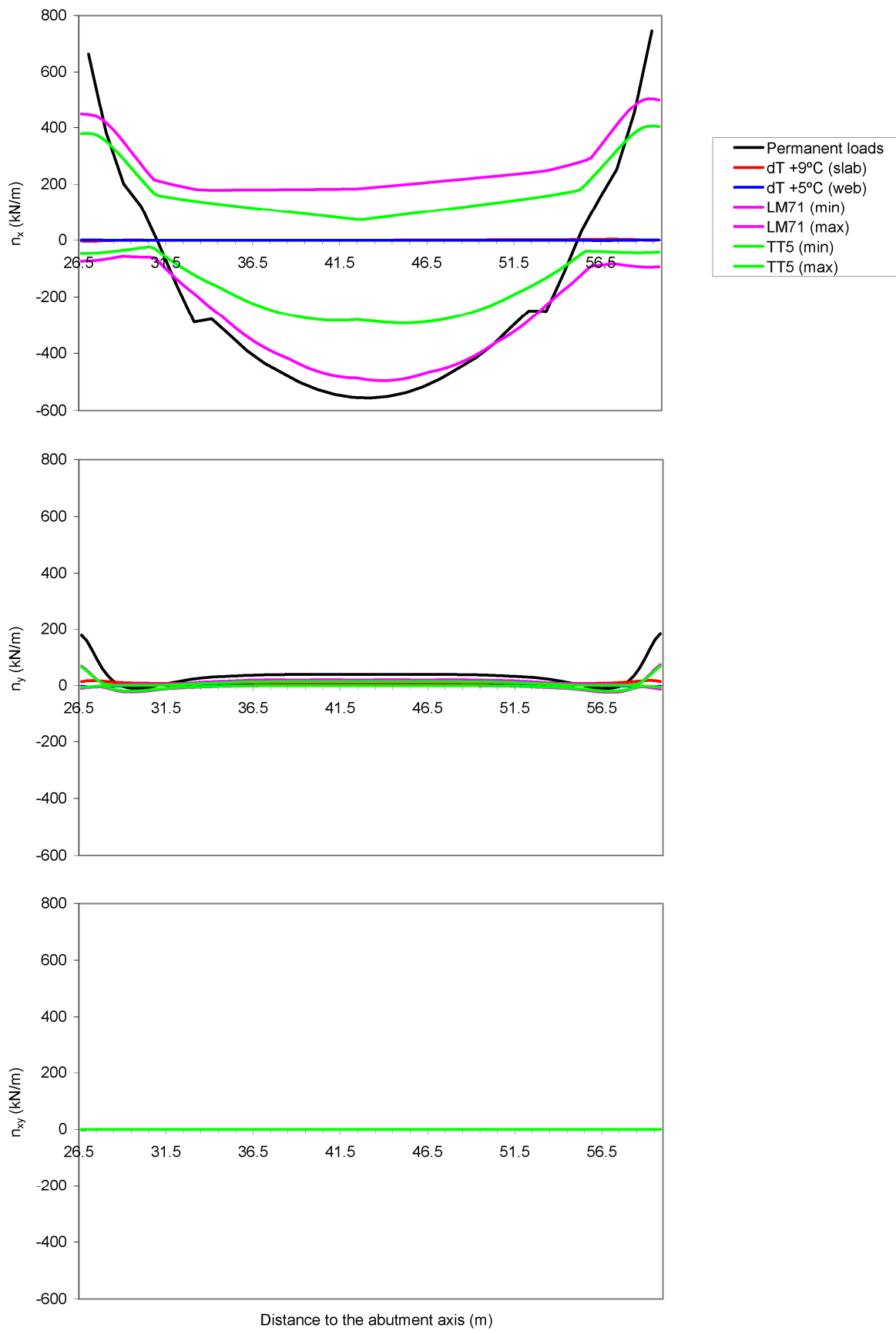


Figure A3.5 – Diagrams of membrane forces at the cross section S2.

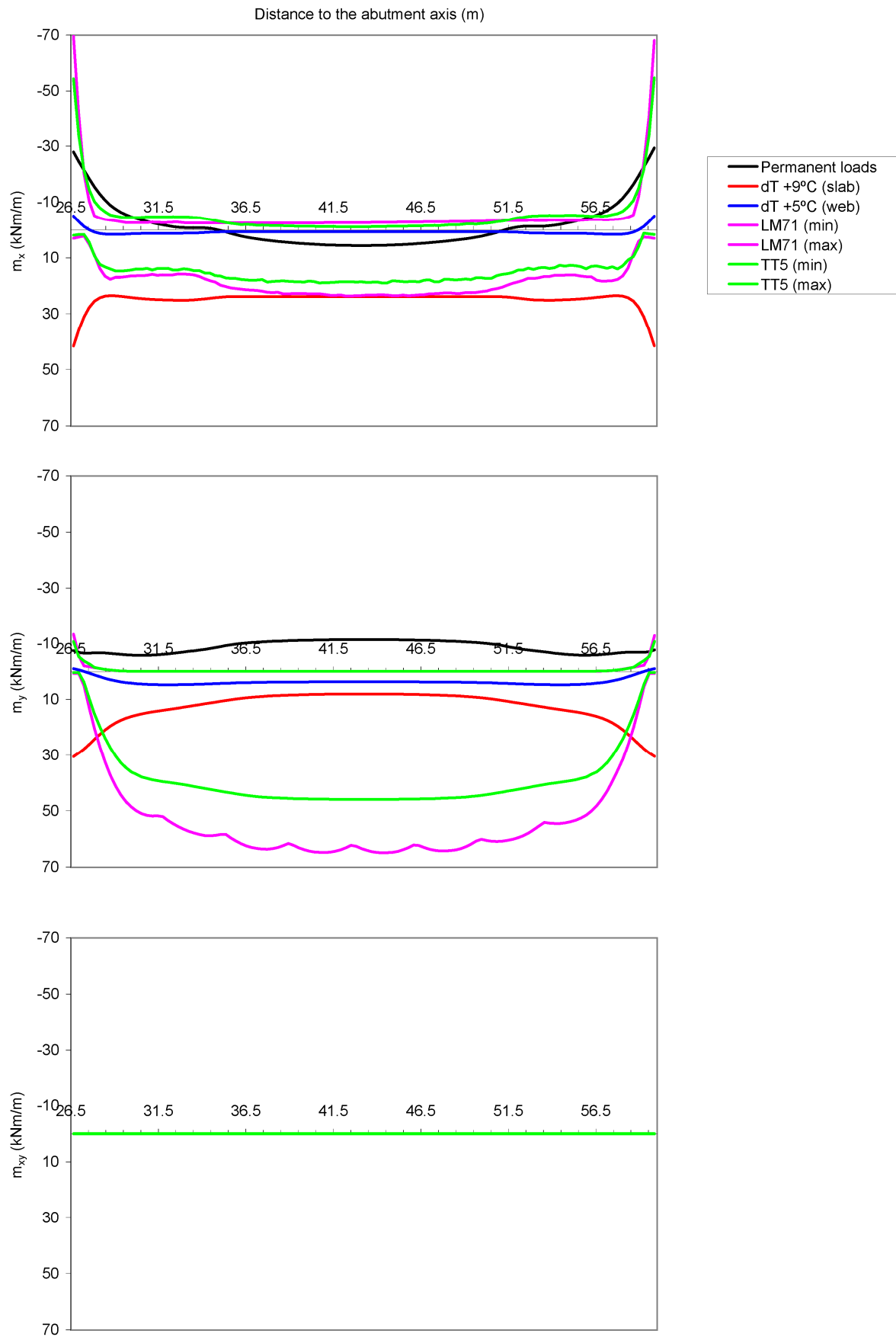


Figure A3.6 – Diagrams of flexural forces at the cross section S2.

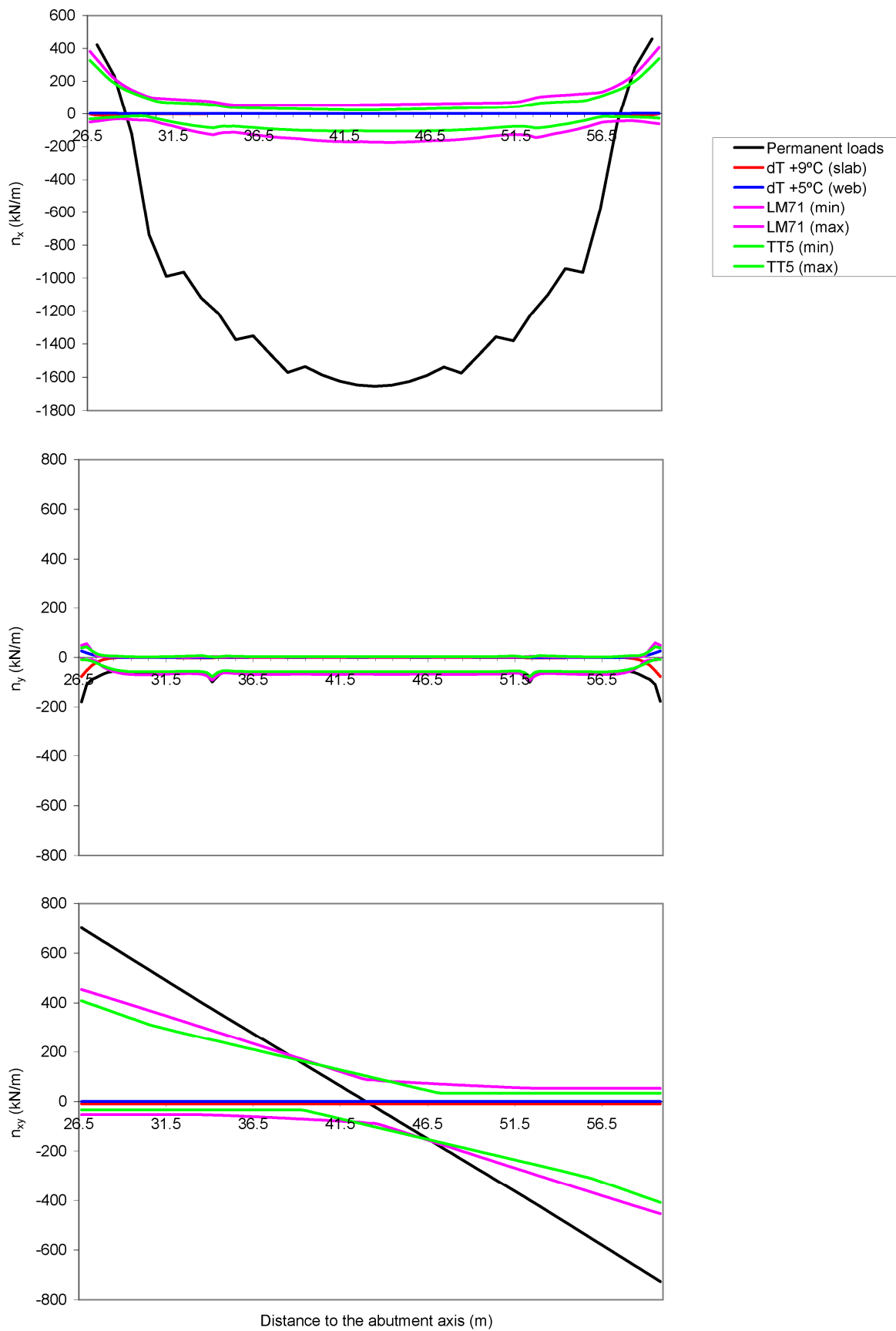


Figure A3.7 – Diagrams of membrane forces at the cross section W1.

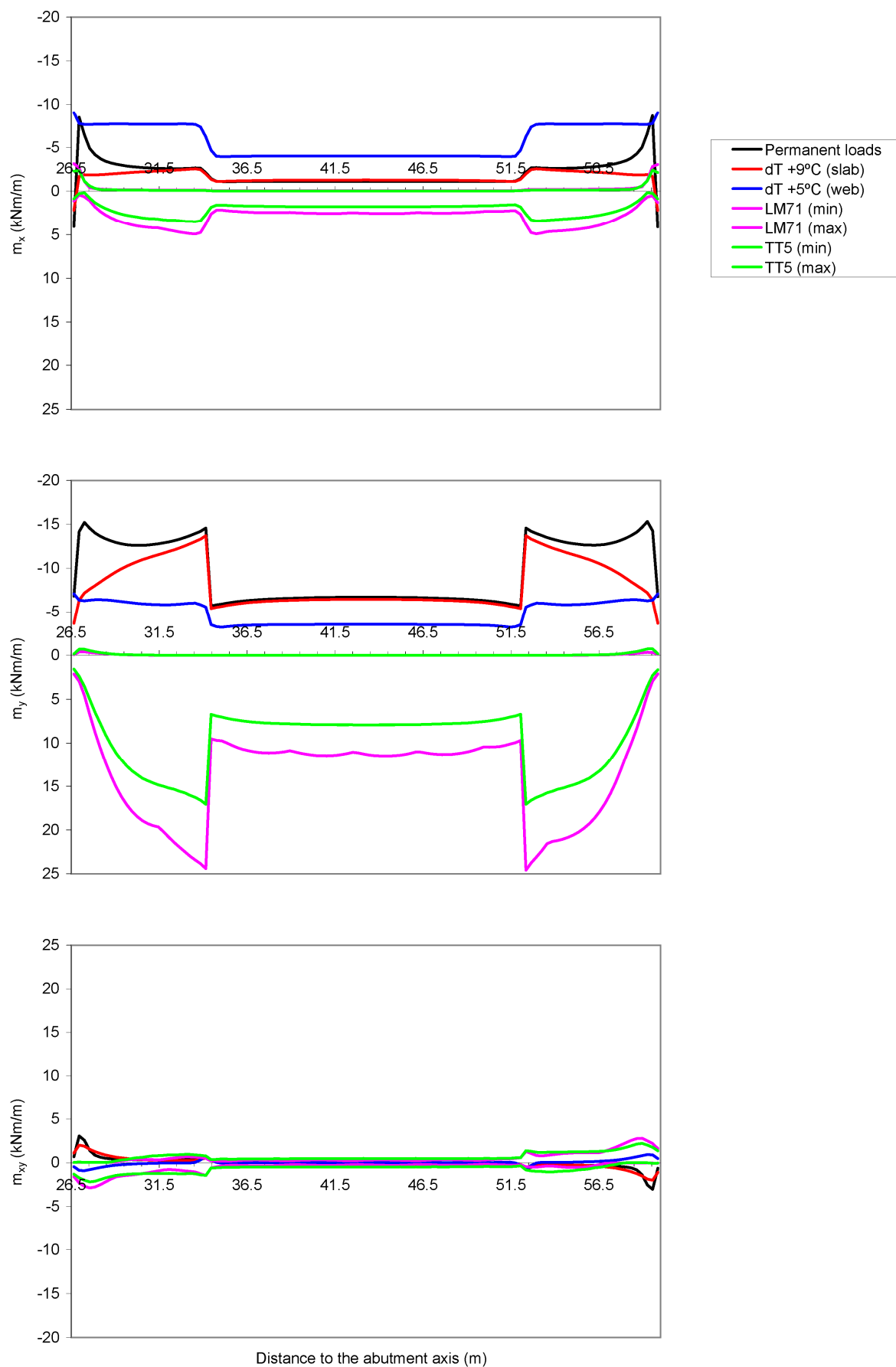


Figure A3.8 – Diagrams of flexural forces at the cross section W1.