# COORDINATED CONTROL OF NETWORKED VEHICLE SYSTEMS: SPECIFICATION AND CONTROL SYNTHESIS

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A framework for the specification and design of coordinated control strategies for networked vehicle systems is presented and discussed. The discussion is illustrated with an example of the coordinated operation of two teams of autonomous underwater vehicles executing an oceanographic mission conceivable in a near future.

The mission consists in collecting data to find the local minimum of a given oceanographic scalar field. To do this both teams use a modified version of the simplex optimization algorithm as the basic coordinated control strategy. This basic strategy is further constrained by the exchange of shared data and coordination commands over underwater acoustic communication modems and by the spatial limitations of the underwater acoustic localization system.

The patterns of coordination and control arising in this mission are described. It is shown that they are quite general, and not specific to this application. The strategy is discussed in the more general context of coordination and control of networked vehicles and systems.

Finally, several conclusions are drawn and a brief discussion on future work is included.

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# On detection of the periodic trajectory.

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The problem of detection of periodic trajectories of given system of nonlinear autonomous differential equations is considered. Some variational task with the functional in the form of the norm of Hilbert space for set of functions with one parameter is formulated. The solution is given in the form of modified Picard approximations to the solutions of the system of differential equations. It is used for detection of periodic trajectories.

**Keywords**: periodic motions, nonlinear dynamics.

**Introduction.** In literature, devoted to the periodic motions and their detection in nonlinear dynamic systems, some well-known methods were given. One of them is the famous Poincaré method of the small parameter. For non-autonomous systems the solution is decomposed into the row of degrees of parameter. And periodical motions are detected due to the same rule, using the period of the force function *f*. If the system is autonomous, then the method of the new time is used. In this work the new method, based on the Picard approximations, is used. This method can be used for detection of periodic motions. In order to see, when the trajectory turns back to its initial point, it is needed to have a special instrument (algorithm, criterion etc.). This work offers some iterative procedure, with the help of which it is possible to detect such a motion.

**Formulation of the problem.** Consider the system of nonlinear differential equations $\dot{X} = F(X)$ , $F(X)$ satisfies Lipshitz condition for $X$ in some set $\Omega \subset R^n$. Problem is to check whether the equality $B = X(T,A)$ and $A = X(T,B)$ is true. $X(T,A)$ is the solution of the Cauchy problem with initial data $A$, i.e. $X(0,A) = A$, and $T \in R^1$. $X(T,B)$ is the solution of the Cauchy problem with initial data $B$, i.e. $X(0,B) = B$, and $T \in R^1$. Ability to solve the formulated problem allows us to answer the question about the existence of the period od the trajectory.

**Algorithm** $X_1(s) = A, Y_1(s) = B, s \in [0,1]$; $X_{k+1}(r) = A + T_k \int_0^r F(X_k)ds, \quad Y_{k+1}(r) = B - T_k \int_r^1 F(Y_k)ds$

$$T_k = \begin{cases} T_{k-1}, & \text{if } S(T_{k-1}) > S(T_k); \; T_k^* = ((B-A)^T \cdot \int_0^1 \Phi_k(r)dr) / \int_0^1 \Phi_k^2(r)dr, \quad \text{if } S(T_{k-1}) \le S(T_k) \end{cases}$$

$$\Phi_k(r) = \int_0^r X_k ds + \int_r^1 Y_k ds \quad S(T_k) = \int_0^1 [A - B + T_k\Phi_k(r)]^2 dr \text{ where } A, B, \Phi \in \Omega \subset R^n$$

$S = \int_0^1 [A - B + T_k\Phi_k(r)]^2 dr = 0 \quad (A, B, \Phi \in R^n)$ is true only for $\Phi_k(r) = const = \frac{1}{T_k}(A - B)$. So we could check out closeness $S$ to $0$, $\Phi_k(r)$ to some constant value or $X_k(r) = Y_k(r)$ for any $r \in [0,1]$. Also we use this algorithm for the motion from $B$ to $A$. Thus we have sequencies of time, of B(because S depends on B), and S. So if there is the convergence of all these sequences the trajectory to which point A belongs will be periodic

Statement. The trajectory to which point A belongs will be periodic if
$T_{ki} \to T_i^* \quad \forall i = 1,2 \quad B \to B^* \; S_{ki}(T_{ki}, B) \to S_{ki}(T_i^*, B^*) < \varepsilon \quad \forall \varepsilon > 0 \; \forall i = 1,2$ and $B^*$ is not identical to A

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# COORDINATED CONTROL OF NETWORKED VEHICLES: SPECIFICATION AND CONTROL

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**Abstract.** A framework for the specification and design of coordinated control strategies for networked vehicle systems is presented and discussed. The discussion is illustrated with an example of the coordinated operation of two teams of autonomous underwater vehicles executing an oceanographic mission conceivable in a near future.

The mission consists in collecting data to find the local minimum of a given oceanographic scalar field. To do this, both teams use a modified version of the simplex optimization algorithm as the basic coordinated control strategy. This strategy is constrained by the exchange of shared data and coordination commands over underwater acoustic communication modems, and by the spatial limitations of the underwater acoustic localization system.

The patterns of coordination and control arising in this mission are described, and the intra and inter team coordinated control problems are formulated in the context of weak invariance. Finally, the main conclusions are presented.

## § 1. Introduction

This article addresses the problem of specification and design of coordinated control strategies for networked vehicle systems. New control design challenges emerge from the fact that the interaction among the controlled dynamic systems whose behavior has to be prescribed has to be taken into account. Here, we focus on oceanographic and environmental field studies based on multiple autonomous underwater vehicles (AUVs) in order to address the issues concerning complex patterns of coordination and control.

In [2, 3], we proposed a framework for the representation, formal specification, and control synthesis for networked vehicles. This framework is based on simple concepts from set theory and dynamic optimization, [1].

We use reach sets to describe the evolution of a dynamic system, invariant sets to describe the locations where the permanence of an entity within a certain set is

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ensured, and solvability sets to describe the locations from which a system can evolve to reach a given set. The key observation is that we can represent vehicles, their spatial-temporal evolution, their interactions, and their operations in the language of sets using concepts and techniques from dynamic optimization.

The paper is organized as follows. In the next section, we discuss an oceanographic mission involving autonomous underwater vehicles to provide a concrete reference for our approach. Then, an instance of the general coordinated control problem is formulated in section 3, and conditions for intra and inter team coordinated control in the context of weak invariance are outlined in section 4. Finally, in the last section, some conclusions are drawn.

## § 2. An Oceanographic Mission

Let us consider an oceanographic mission involving the motion coordination of two teams of $AUV$s in order to find and locate the local minimum of a scalar field, say water salinity, pertinent to the description of a given oceanographic phenomenon.

While one of the teams, referred to by $LPS$, provides a Local Positioning System service to the other team, denoted by $S$, this one samples the scalar field and seeks one point of minimum.

The Local Positioning System works as follows. Each $LPS$ vehicle has a $GPS$ receiver and an acoustic transponder – the vehicle is required to operate at the surface to receive the $GPS$ signal providing time and position. The transponder emits regularly, with a known frequency, an acoustic ping encoding the name of the emitter as well as the time and the location of emission. Each AUV from the $S$ team is also equipped with an acoustic system. This system detects the arrival of acoustic pings, and decodes them to extract the position and name of the emitter and the time when the acoustic ping was emitted. This information, together with the time of arrival of the ping, is used to compute the distance between the AUV and the emitter. Given the fact that depth data is easily available, the determination of the absolute position requires at least three $LPS$ vehicles to form a simplex. Due to attenuation, the $LPS$ service is only available within a neighborhood $P(t)$ of the $LPS$ team.

The search service works as follows. The $S$ team implements a version of the simplex optimization algorithm to find the local minimum of the scalar field. Each vehicle has a suite of oceanographic sensors and a low-bandwidth underwater acoustic communication system in order to implement the search strategy in coordination with the rest of the team, and with the $LPS$ team.

In terms of motion coordination, the $S$ team assumes the role of the leader. The $LPS$ team controls the motions of its vehicles in order to keep all of the vehicles from the $S$ team inside $P(t)$. The coordination results from information exchanged between the two teams.

## § 3. Formulation of the Coordinated Control Problem

Two types of motion coordination are involved in the operation scenario described in the previous section: 1) intra-team for both the $LPS$ and the $S$ vehicles ensuring that their services are delivered, and 2) inter-team coordination. An initial allocation of vehicles to teams is done at the planning stage that follows the mission specification.

Each one of the $n_l$ vehicles in the $LPS$ team is equipped with a GPS receiver, a transponder, a radio, and an acoustic modem with ranges of, respectively, $r_t$, $r_r$, and $r_a$. The $n_s$ vehicles of the $S$ team has a Conductivity Temperature Depth (CTD) sensor, an acoustic modem, and a navigation acoustic system, being $r_a$, the range of the acoustic modem. We denote the $(x, y, z)$ position of the $j^{th}$ vehicle from the $S$ $(LPS)$ team by $X_{S,j}(t)$ $(X_{LPS,j}(t))$, $j = 1, \ldots, n_s(t)$ $(j = 1, \ldots, n_l(t))$.

There are two distinct types of constraints to be satisfied by the vehicles in order to perform a mission as a team: 1) *Structural constraints* - required to coordinate their operations and maintain the integrity of the team, 2) *Service constraints* - in order to provide services. Since the violation of the former implies the collapse of the team and the violation of the latter only degrades the way the service is delivered, structural constraints have precedence over the service constraints.

Next we provide a simplified formulation of the above constraints.

LPS team. At time $t$, the positioning service provided is available at all locations $X$ such that there are at least three vehicles from the $LPS$-team within distance $r_t$, i.e.,

$$P(t) = \{X \in \Re 3 : \exists i, j, k \in \text{LPS} \quad k \neq i \wedge i \neq j \wedge j \neq k \wedge d(X_{LPS,j}, X) \leq r_t$$
$$\wedge\, d(X_{LPS,i}, X) \leq r_t \wedge d(X_{LPS,k}, X) \leq r_t\}$$

where $d(X, Y) = \|X - Y\|_2$ is the distance function. Furthermore the *Service Constraints* dictate that they have to form a simplex in the plane, i.e.,

$$\exists i, j, k \in LPS \text{ with } j \neq i, i \neq k, k \neq j \text{ such that:}$$
$$d(X_{LPS,i}, X_{LPS,k}) < d(X_{LPS,i}, X_{LPS,j}) + d(X_{LPS,j}, X_{LPS,k}).$$

Finally, in order to be able to coordinate their motions and to follow the $S$ team, it is required that the $LPS$ vehicles have to form a communication network where every two distinct vehicles should be able to communicate between them. This *Structural Constraint* can be expressed in terms of the connectedness of a certain graph $T$ (see [2]). Each $LPS$ vehicle is a vertex of the graph and an edge between two vertices emerges whenever the distance between the corresponding vehicles is less than the radio communication range $r_r$.



S-team The navigation procedure to be implemented by these vehicles in the search of the minimum of the considered scalar field consists in a version of the simplex algorithm. This requires permanent communication among the vehicles and, therefore, as with the $LPS$-team, the *Structural Constraints* are expressed as the connectedness of the corresponding graph. The *Service Constraints* are easily formulated via this algorithm which involves both spatial and logical relations which depend on the data gathered by sampling the scalar field. In order to simplify the description, let us consider a scalar field $f(x)$ evolving in the horizontal plane (in $\Re^2$) with a unique local minimum in the region of interest. Let $i = 1$ and consider $A_i$, $B_i$, and $C_i$, forming a simplex such that $f(A_i) \leq f(B_i) \leq f(C_i)$. Then, the *Algorithm* consists in repeating the following computations until the minimum is found:

1. Compute the midpoint $z_i$ of the segment joining $A_i$ and $B_i$ and let $\bar{v} = z_i - C_i$, the vector joining the points $C_i$ and $z_i$.
2. Let $y_i = z_i + \bar{v}$ and $C_{i+1} = \max_f\{B_i, y_i\}$.
3. If $C_{i+1} = B_i$, let $A_{i+1} = \min_f\{A_i, y_i\}$, $B_{i+1} = \max_f\{A_i, y_i\}$, $i = i + 1$ and go to step 1, otherwise:
4. Take some $\alpha \leq 1$ (possibly more than one value), let $\bar{v} = \alpha\bar{v}$. Goto step 2.

By $\max_f(\min_f)\{A, B\}$ it is meant either $A$ or $B$ depending on whether $f(A) \geq f(B)$ $((f(A) \leq f(B))$ or the opposite holds. One simple criterion to stop the algorithm could be the first iteration for which the maximum distance between any pair of the three points is below a given value.

Notice that this procedure determines points where data should be collected in an appropriate order and not necessarily by the same vehicle. The actual path that each of one or more vehicles travel in order to get the data should be the one for which the efficiency of the resources usage is maximized and, therefore, depends strongly on physical features such as the vehicle dynamics and actuation profile, features of the phenomena under observation, currents and environmental perturbations.

This algorithm is scalable with respect to the number of vehicles used to implement it. It is not difficult to imagine the implementation of this algorithm with simultaneous values for $\alpha$ (some possibly greater than 1) and several vehicles sampling the scalar field at the corresponding points. Then, the obtained data is compared and the $\alpha$ with minimum value of $f$ is selected.

<u>LPS-S coordination</u>. The coordinated control of the $LPS$ and $S$ teams requires the consideration of additional constraints which ensure that coordination requirements are met. These are:

1. The vehicles from the $S$ team should remain inside the set $P(t)$, i.e., if $C_S(t)$ is the convex closure of the locations of the members of the $S$ team, then, we should have $C_S(t) \subset P(t)$, $\forall t$.

2. The vehicles in both teams should be able to communicate among themselves. If $r_a$ is the maximum range of communication, then the constraint $\min_{i=1\ldots n_s(t), j=1\ldots n_l(t)} \{d(X_{LPS,j}(t), X_{S,i}(t))\} \leq r_a$, $\forall t$, defines $D_{LPS}(t)$ and $D_S(t)$ as the sets of locations for vehicles of the teams, respectively, $S$ and $LPS$ from which they can communicate with the other team at time $t$. Then, this coordination requirement can be expressed by $D_{LPS}(t) \cap D_S(t) \neq \emptyset$, $\forall t$.

## § 4. Coordination and Control

Now, we will discuss very briefly how the intra-team and inter-team coordination conditions are formulated as nested problems of invariance [2, 3] in the framework of value functions as presented in [4, 5].

For the $LPS$ *intra-team invariance*, consider the $n_l(t)$ vehicles from the $LPS$ team, being the dynamics of the $i^{th}$ vehicle given by:

$$\dot{x}_{LPS,i}(t) = f_{LPS,i}(x_{LPS,i}(t), u_{LPS,i}(t)), \quad u_{LPS,i}(t) \in U_{LPS,i}(t), \quad x_{LPS,i}(t) \in \Re^k,$$

for $i = 1, \ldots, n_l(t)$. The service and structural constraints that the team has to satisfy can be expressed by

$$\varphi_i(t, x_{LPS}(t)) \leq 1, \quad \varphi_i^0(x_{LPS}(0)) \leq 1, \quad i = 1, \ldots, n_l(t).$$

Then, the problem can be stated as follows:

Find the largest invariant set $\mathcal{WI}_{LPS}(t)$ such that, if the initial position of the system lies in this set, it is always possible to find a control $u_{LPS}(t)$ that prevents the state of the system to violate the service and structural constraints.

We put $\phi_k(t_0, \tau, x_{LPS}) := \max\left\{ \max_{t_0 \leq t \leq \tau} \{\varphi_k(t, x_{LPS}(t))\}, \varphi_k^0(x_{LPS}(t_0))\right\}$ and consider the value function $V(\tau, z) = \min_{u_{LPS}(t_0,\tau)} \left\{ \min_{k=1,\ldots,l} \{\phi_k(t_0, \tau, x_{LPS})\} | x_{LPS}(\tau) = z \right\}$, then $\mathcal{WI}_{LPS}(t) = \{z : V(t, z) \leq 1\}$.

Analogously, we can derive the $S$ *intra-team invariance* conditions and obtain the corresponding $S$-team invariant set, $\mathcal{WI}_S(t)$.



Now, to see how this approach is amenable to deal with the *inter-team invariance* problem, consider $J$ to be the map that associates the position of the $LPS$ vehicles with the set $P(t)$, and define a function $\tilde{d}_A(B)$ of two sets, $A$, $B$, by $\tilde{d}_A(B) = max\{d_A(x) : x \in B\}$ where $d_A(\cdot)$ is the usual Euclidian distance. Given the fact the $LPS$ team has to follow the $S$-team that plays the role of leader, the inter-team constraint can be expressed by $\varphi_{LPS}^S(t, x_{LPS}(t)) \leq 1$ where $\varphi_{LPS}^S(t, z) = \tilde{d}_{\mathcal{WI}_S}(J(z)) + 1$.

## § 5. Conclusions

The potentially rich behavior of networked vehicles and systems results from the way vehicles, controllers, service providers, and devices are connected, and from the way connections among these agents evolve with time. In order to address these issues, we represent specifications as logical statements and use set-theoretic constructs that are amenable to mathematical manipulation at the design stage. Then, we define invariants that the implementation is required to satisfy.

In what concerns design, we formulate the coordination and control problems in the setting of dynamic optimization. In this setting complex requirements can be expressed as the disjunction of joint-state constraints and relative motion coordination, in terms of invariance, of level sets of value functions, and of reachability. In doing this, we are able to derive conditions under which the invariants will be true, and synthesize controllers that ensure invariance.

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