guaranteeing strategy, based on a linear-quadratic differential game, is more stable with respect to the measurement noise and requires a lower control effort than other alternatives.

Null controllability in the system modeling quasi-static vibrations of a plate

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In this paper, we study null controllability problems for the system

$$\begin{cases} (1+a^2)\theta_t = \Delta\theta + a\frac{d}{dt}\max\{a\int\limits_\Omega \theta(x,t)\,dx - g,\ 0\} + v(x,t), \\ \qquad\qquad\qquad\qquad\qquad\qquad x\in\Omega\subset R^2,\ t>0 \\ \theta(x,0)=\theta_0(x),\ \text{in } \Omega \\ \theta(x,t)=0,\ \text{in } \Gamma_d\times(0,T) \\ \frac{\partial\theta(x,t)}{\partial\nu} = k(a\int\limits_\Omega \theta(x,t)\,dx - g)\theta(x,t),\ \text{in } \Gamma_c\times(0,T), \end{cases} \tag{7}$$

where Ω is an open bounded domain with boundary $\Gamma=\Gamma_d\cup\Gamma_c$, $k(s)$, $s\in R$, is a given nonnegative differentiable function, and $v(x,t)$ describes the external heat sources.

To explain the interest in studying this problem, we consider a thin, homogeneous isotropic elastic rectangular plate which undergoes a horizontal vibration. Then the displacement $u(x,t)$ and the temperature $\theta(x,t)$ satisfy the equations of *dynamic thermoelasticity*, that, as well as their linearized form, can be found, e. g., in [1]. We consider the case when three sides of the rectangular plate are permanently fixed while the fourth one denoted by Γ_d is free to expand or contract and eventually

may come into contact with a rigid obstacle. Let the constant g be the nominal distance to the obstacle in the reference configuration. We show that for the case of the Signorini's contact conditions for displacement, the linearized equations of thermoelasticity may be decoupled. Thereby, the temperature $\theta(x,t)$ is subject to the differential equation in system (7).

Considering the function $v(x,t)$ as a control applied in the system, we study the null controllability properties of system (7). Our main results are in the following theorem.

Theorem. *For any $\theta_0\in L^2(\Omega)$, there is a control $v\in L^2(\Omega_T)$, $\Omega_T=\Omega\times(0,T)$, that brings the system to the state $\theta(x,T)$ a. e. in Ω. Moreover, if $\theta_0\in H^1(\Omega)$ and $\theta_0(x)=0$ for $x\in \mathrm{cl}\,\Gamma_c\bigcap \mathrm{cl}\,\Gamma_d$, then the solution of system (7) is unique.*

These results extend work reported in [2].

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Motion control of coordinated systems

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In this article, we address the problem of planning and controlling multiple interacting dynamic systems to reach given target sets in a coordinated fashion. There is a significant literature on coordinated control dealing with the problem of motion formation planning and control [1–3]. However, there are requirements for coordinated motion planning rather than for formation keeping [4, 5]. One promising way of expressing these

requirements is by coordinated state constraints. We say that a given set of dynamic systems satisfies coordinated state constraints if some of the constituent dynamic systems satisfy satisfy state constraints which depend on the state variables of other dynamic systems in the considered set. Here, we will consider the state constraints modeled by set-valued maps mapping the state of each system onto constraints for the other systems. The control problem consists in finding a control strategy for each dynamic system enabling it to reach the target at some time within some prescribed time interval while satisfying the coordinated state constraints.

We address these problems using backward reachable set computation and dynamic optimiza- tion techniques [6]. We use reach sets to describe the evolution of a dynamic system, invariant sets to describe the locations where the permanence of an entity within a certain set is ensured, and solvability sets to describe the locations from which a system can evolve to reach a given set. The key observation is that we can represent vehicles, their spatial-temporal evolution, their inter- actions, and their operations in the language of sets using concepts and techniques from dynamic optimization.

An illustration of the proposed a framework for the representation, formal specification, and control synthesis in the context of networked autonomous underwater vehicles is also included, [7], and a framework for the practical implementation of coordinated control strategies at both the planning and control levels is discussed.

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Concavity properties of hamiltonian functions in optimal control problems with infinite horizon

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The paper is devoted to characterizing the impact of technology assimilation on optimization of R&D investment policy for a growing economy. The focus is on the issue of a reasonable balance between the indigenous technology stock and assimilated technology flow. Such statement is closely connected with the problem of optimal allocation of resources [1–2].

The efficiency of utilization of technology depends on an assimilation capacity of an economy to absorb the exogenous technology stock from the global market place. It is assumed in the paper that the assimilation capacity is conditioned by the development of the world market technology stock and the ability to maximize benefits of a learning exercise. Consequently, the assimilation capacity is a function of the level of the indigenous technology stock and the assimilated spillover technology, and the growth rates of these parameters.

Motion Control of Coordinated Systems

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Abstract

In this article, we address the problem of planning and controlling multiple interacting dynamic systems to reach given target sets in a coordinated fashion. There is a significant literature on coordinated control dealing with the problem of motion formation planning and control [7, 5, 6]. However, there are requirements for coordinated motion planning rather than for formation keeping [1, 2]. One promising way of expressing these requirements is by coordinated state constraints. We say that a given set of dynamic systems satisfies coordinated state constraints if some of the constituent dynamic systems satisfy satisfy state constraints which depend on the state variables of other dynamic systems in the considered set. Here, we will consider the state constraints modeled by set-valued maps mapping the state of each system onto constraints for the other systems. The control problem consists in finding a control strategy for each dynamic system enabling it to reach the target at some time within some prescribed time interval while satisfying the coordinated state constraints.

We address these problems using backward reachable set computation and dynamic optimization techniques [3]. We use reach sets to describe the evolution of a dynamic system, invariant sets to describe the locations where the permanence of an entity within a certain set is ensured, and solvability sets to describe the locations from which a system can evolve to reach a given set. The key observation is that we can represent vehicles, their spatial-temporal evolution, their interactions, and their operations in the language of sets using concepts and techniques from dynamic optimization.

An illustration of the proposed a framework for the representation, formal specification, and control synthesis in the context of networked autonomous underwater vehicles is also included, [4], and a framework for the practical implementation of coordinated control strategies at both the planning and control levels is discussed.

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