Interactive Educational Software for Dynamic Systems Control

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Abstract - This paper describes a tool for interactive learning that can be used as a teaching aid in control systems theory. It can be used by students for problem solving and individual learning. It can also be a powerful tool for speeding up the development cycle for robot control systems. As there is no need to do any special programming or debugging, the students can focus on the control items. The developed system is ready to use and allows testing different control methods. Usually, in control theory classes, students face a system defined only as a mathematical equation. The developed application allows them to control a realistic system with almost real world dynamics. A friendly appearance based on 3D graphics captivates the student's attention and gives instant feedback on the controller performance. As a case study, an inverted pendulum is controlled while placed on an omnidirectional vehicle. The inverted pendulum poses a classical control problem, a nonlinear and unstable system, typical of a large number of devices such as Segway Human Transporter and Rocket navigation. Omnidirectional vehicles allow movements in every direction and they are widely used in robotics where the extra mobility is an important advantage.

Index Terms - Control, Education, Virtual Laboratories.

INTRODUCTION

Virtual laboratories present advantages like low cost, friendly user, interactivity, allow simultaneous multiple users and support individual learning [1]. The objective of this project is to develop a virtual laboratory that allow students to learn and test control systems interacting with a 3D graphical interface based on the open source GLScene. Graphics provide one of the most natural means of communicating with a computer, since our highly 2D and 3D pattern recognition abilities allow us to perceive and process pictorial data rapidly and efficiently. Interactive computer graphics thus permits extensive, high-bandwidth user-computer interaction. This significantly enhances ability to understand data, to perceive trends and to visualize real or imaginary objects [2].

The developed Virtual Laboratory world behavior and graphics are based on open source platforms. Dynamic engines like Newton Dynamics, Beyond Virtual, YADE-Yet Another Dynamic Engine and ODE-Open Dynamics Engine are powerful tools that allow programmers to create a physic world composed by objects connected through joints and simulate them. The simulation dynamics is based on ODE, an articulated rigid body dynamics engine, which includes forces and collision treatment. As an example, the presented application supports the simulation of an inverted pendulum placed on an omnidirectional vehicle, shown in a friendly 3D interface.

User can interact with the application testing different behaviour for each control method such as state space, PID and manual control. Noise addition in position and angle, packet loss as a real network and force saturation limits approach this simulator to reality.

This paper is organized as follows: Initially, the inverted pendulum system and some applications are described. Then, system modeling equations where a State-Space approach is used is described. The next subsection explains how feedback controller is made. The next section presents how students can interact with the simulated world in a real time, where results are shown in the graphical interface. Finally, last section rounds up with conclusions and future work.

INVERTED PENDULUM OVERVIEW

An inverted pendulum is a pendulum having its center of gravity located above its pivot point. An inverted pendulum is therefore inherently unstable. When its center of gravity is directly above the pivot point, it may remain static. If the pivot is static and the center of gravity is slightly displaced from the vertical position, the pendulum will not return to its original position but will tend to find a new equilibrium position such that its center of gravity is at its lowest possible position. A feedback control system that positions the pivot may be used to balance an inverted pendulum [3]. Some applications of the inverted pendulum are presented in the next subsections.

I. Segway

An inverted pendulum is present in devices such as the unicycle and the Segway Human Transporter, illustrated in Figure 1. In these devices, the pivot of the pendulum is the axle of a wheel or pair of wheels. In a unicycle the wheel is powered by the rider. In a Segway Human Transporter, the wheel is powered by an electric motor. Motion of the wheel, or pair of wheel, is controlled so that the pendulum is dynamically balanced [4].

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There is also another version of Segway, the Segway Robotic Mobility Platform (RMP), that is a modified version of the Segway Human Transporter (HT) designed to provide scientists and engineers a mobile base to use in robotics research [5].

II. Rocket Navigation

The inverted pendulum control model is similar to the rocket launch [6]. It was believed that, in flight, the rocket would ‘hang’ from the engine like a pendulum hanging from a pivot. The weight of the fuel tank would keep the rocket flying straight up as long as the fuel lasted. However, this belief is incorrect, such a rocket will never fly in a straight line and will always turn and crash into the ground soon after launch. This is what happened to Goddard’s rocket [7]. Nowadays, Rockets use Inertial Navigation to solve this problem. Inertial navigation employs accelerometers and gyroscopes to determine speed and position. [8].

**INVERTED PENDULUM CONTROL**

Feedback control may be defined as the use of difference signals, determined by comparing the actual values of system variables to their desired values, as a mean of controlling a system. An everyday example of a feedback control system is an automobile speed control, which uses the difference between the actual and the desired speed to vary the fuel flow rate [9]. To achieve the systems feedback control it is necessary to specify its characteristics, usually known as system model represented by differential equations. In order to use linear control methods, those equations must be linearized. A way to represent these equations is the state space form, as a set of state variables such as cart speed and position and pendulum angular speed and position. A closed loop diagram system is shown in Figure 2.

The $K$ values are the feedback gain vector obtained in the following subsection II.

I. System Modeling

Controlling the inverted pendulum is a classical control problem because the pendulum dynamics is both nonlinear and unstable [10]. The dynamic equations of motion for the control system are linearized, although the simulated system is realistic and based on the Dynamics Engine. Controlling the 2D inverted pendulum (placed on an omnidirectional vehicle) is similar to control two one-directional inverted pendulums placed on carts moving along right angle axles. The final force can be found by the sum of each individual force. By this way, a first approach of one-directional inverted pendulum control is presented.

The inverted pendulum system consists of two moving parts:
- The Pendulum
- The cart

The dynamic equations of motion for the system are linearized assuming that pendulum does not move more than a few degrees away from the vertical. One important issue to emphasize is that cart wheels dynamics is ignored, in other words, their mass and moment of inertia are considered null. The actuator and sensor dynamics are also despised.

The modeled system is shown in Figure 3.
The constants and variables for this case study are defined as follows:

- $M$ mass of the cart 3 kg
- $m$ mass of the pendulum 1.57 kg
- $b$ friction of the cart 0.1 Nm/s
- $l$ length to pendulum center of mass 2 m
- $I$ inertia of the pendulum 8.37 kgm$^2$
- $F$ force applied to the cart
- $N$ reaction force applied to the cart
- $x$ cart position coordinate
- $y$ gravitation direction
- $\theta$ pendulum angle from vertical
- $g$ gravity acceleration 9.8 m/s$^2$

The pendulum moment of inertia is shown in (1).

$$I = \frac{1}{2}ml^2$$

The linearized system equations can be represented in State-Space form, as shown in the next equations [11]:

$$\begin{bmatrix}
\dot{x} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
x \\
\theta
\end{bmatrix} + \begin{bmatrix}
0 & B
\end{bmatrix} u$$

$$Y = C \begin{bmatrix}
x \\
\dot{x} \\
\theta \\
\dot{\theta}
\end{bmatrix}$$

Where $x$, $\dot{x}$, $\theta$ and $\dot{\theta}$ are the output and input signals. The simplified $A$, $B$ and $C$ matrices are presented in (4), (5) and (6).

$$A = \begin{bmatrix}
0 & -\frac{1}{l} & \frac{mgl^2}{\lambda} & 0 \\
0 & -\frac{b}{\lambda} & 0 & 0 \\
0 & -\frac{mlb}{\lambda} & 0 & 1 \\
0 & 0 & -\frac{mgl(M + m)}{\lambda} & 0
\end{bmatrix}$$ (4)

$$B = \begin{bmatrix}
\frac{l + ml^2}{\lambda} \\
\frac{b}{\lambda} \\
\frac{ml}{\lambda}
\end{bmatrix}$$ (5)

$$C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}$$ (6)

The controller objective is to compute the applied force that stabilizes the pendulum in the vertical for a desired position. The force applied to the cart can be given by the state feedback vector presented in (7).

$$F = -[k_1 \ k_2 \ k_3 \ k_4] [x \ \dot{x} \ \theta \ \dot{\theta}]$$

$$K = [0 \ 0 \ 0 \ 1] \cdot Q^{-1} \cdot \alpha(A)$$ (8)

Where $\alpha(s)$ is the characteristic equation, presented in (9) and $Q$ is the controllability matrix, given by (10).

$$\alpha(s) = (s - p_1) \cdot (s - p_2) \cdot (s - p_3) \cdot (s - p_4)$$ (9)

$$Q = \begin{bmatrix}
B & AB & A^2B & A^3B
\end{bmatrix}$$ (10)

The $p_1$, $p_2$, $p_3$ and $p_4$ values are the desired closed loop poles that define the system dynamics.

**INTERACTIVE REAL-TIME SIMULATION**

Real-Time Simulation is a powerful tool that allows analysis and improvement of systems behavior in response to actions and events, reducing human errors, accidents and system damages. Some time ago, simulation tools were mainly text based, nowadays, computer graphics development allows to make great scenarios like 3D rendering, camera positioning and freedom textures captivating student’s attention. It has also been well established, within the literature about agents and robotics, that simulation can be a powerful tool for speeding up the development cycle for robot control systems [13]. The difficulties of operating and testing robots, whether it is from limited debugging tools or the need to constantly change batteries, make simulation an appealing tool to speed up the development cycle [14].

The presented system aims to help students in control engineering courses. In the developed applications there are no commands but buttons instead with a friendly 3D graphical interface. The graphical based interactive interface, allows users to choose the closed loop poles. Assuming that simulation time step is much faster than dynamics, it is possible to assure that it is a continuous time model. A time step of 20 ms is enough to validate this approach.

In next subsections, the interactive graphic applications are described.

I. Inverted Pendulum Placed on a Cart

The developed application, presented in Figure 4, allows users to interact with the simulated system. A graphical time evolution shows the cart position, speed and pendulum angle and a zoom feature is also implemented in order to highlight...
some details. A top and a frontal view, where user can place the camera everywhere, show the 3D real world. User can also change the setpoint, add noise to the sensors measure, test nonlinearities and saturation.

The applied force is represented in the graphical interface by an arrow. A detailed screen shot, presented in Figure 5, shows the inverted pendulum placed on a cart in its initial position with its texture of scenario and wheels.

A dynamic controller, based on a fourth order Bessel prototype [12] with a time settling of 10 seconds and a setpoint of 10 m, presented in Figure 6, is implemented where $a=0.40$, $b=0.51$, $c=-0.55$ and $d=0.17$, with the feedback gain vector, represented by:

$$K = [-0.26 -1.46 75.80 41.03].$$

As result, at 50 seconds, the cart set point is changed from 0 to 10 meters. A force is applied to the cart in order to balance the pendulum only a few degrees from the vertical. Then an opposite force is applied in order to set the cart position at 10 meters and the pendulum angle to vertical. At 56 seconds, the cart arrived to its desired position and pendulums angle is stabilizing.

II. Inverted Pendulum Placed on an Omnidirectional Vehicle

The wheeled omnidirectional robots have a minimum of 3 special wheels, which allow the robot to move in every direction. The movement of these robots does not have the restraints of the differential robots, presenting the disadvantage of a more complex control. Their applications nowadays are mainly in robotics soccer [15] and wheelchairs for disabled [16]. An example of the control of an inverted pendulum placed on an omnidirectional vehicle is shown in Figure 7.
In the developed application the typical three wheel [17] and four wheel [18] omnidirectional vehicles were used, as shown in Figure 8 and Figure 9, where robots are following the ball while controlling its inverted pendulum. The applied forces are represented in the graphical interface by arrows.

FIGURE 7
2D INVERTED PENDULUM SCREEN SHOT

FIGURE 8
INVERTED PENDULUM PLACED ON A 3 WHEEL OMNIDIRECTIONAL VEHICLE

FIGURE 9
INVERTED PENDULUM PLACED ON A 4 WHEEL OMNIDIRECTIONAL VEHICLE

CONCLUSIONS AND FUTURE WORK

This paper has introduced an interactive learning tool for automatic control courses, where it is desired to control an inverted pendulum placed on a cart and on an omnidirectional vehicle. The developed systems allow students to focus on the control theory, helping them to improve their skills. The 3D visualization, textures and animation effects are the most important advantage to students for a better understanding and enthusiasm in the physical systems study. The graphic, where are shown position, angles, velocities and applied force, presents real time data in a way that can be easily decoded by students.

As a future work, several educational experiments can be achieved with this tool, such as Humanoid robot control, helicopter flight control, anthropomorphic robot arm control and car traction and brake control systems.
REFERENCES


