Precise Modeling of a Four Wheeled Omni-directional Robot

Hélder P. Oliveira, Armando J. Sousa, A. Paulo Moreira and Paulo J. Costa

Abstract— **Recent applications in Autonomous Robotics are becoming more demanding every day. Modern Mobile Robots use Omni-Directional locomotion in order to achieve additional maneuverability and efficiency. These features are gained at the expense of increased mechanical complexity and increased complexity in control. Demanding applications require precise dynamical model in order to allow for precise locomotion. Dynamical Models are also essential to study limitations of current mechanical configurations and to allow for future enhancements both at controllers level and at mechanical configuration level. The presented work finds a precise dynamical model for a 4 wheeled omni-directional robot and a prototype is used for validation. Simulations and real experimental runs are shown and the adequacy of the model is discussed.**

I. INTRODUCTION

Omni-directional locomotion is a frequent solution in recent autonomous robots. Such robots have advantages over robots with differential locomotion on maneuverability and efficiency. These performance advantages are gained at the expense of mechanical and controller complexity. One of the most frequent solutions is to use some sort of Mecanum Omni-directional wheels [5] and [15]. Omni-directional wheel design is quite delicate and different wheels exhibit very different performances. Wheel construction is often application specific and the presented work uses the wheels shown in figure 1. These wheels are built in-house for several demanding applications. The prototype used in the experiments uses 4 wheels of this kind to achieve omni-directional locomotion. A robot with 4 wheels is expected to have more traction than it's 3 wheeled counterpart.

Fig. 1. 5DPO Omni-directional wheel.

A robot with 3 or more motorized wheels of this kind can have almost independent tangential, normal and angular velocities. Dynamical models for this kind of robots are not very common due to the difficulty in modeling the several internal frictions inside the wheels, making the model somewhat specific to the type of wheel being used [17].

In order to estimate the model for the robot, a prototype was built, as shown in figure 2.

H. P. Oliveira, A. J. Sousa, A. P. Moreira and P. J. Costa are with Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias s/n 4200-465 Porto, Portugal {helder.oliveira,asousa,amoreira,paco}@fe.up.pt

Fig. 2. Omni-directional robot.

Data from experimental runs is taken from overhead camera. The setup is taken from the heritage of the system described in [4] that currently features 25 fps, one centimeter accuracy in position (XX and YY axis) and about 3 degrees of accuracy in the heading of the robot.

Precise dynamical model of a robot is essential to allow good control that, in turn, generates enhanced performance. Dynamical models are a focus of intense research but, as mentioned earlier, results are quite dependent on wheel construction [2][3][9][16][17]. Kinematic behavior is dependent on mechanical configuration and also has been studied before [2][10][11][12][18]. Models are based on linear and non linear dynamical systems and the estimation of parameters has been the subject of continuing research as mentioned in [3][13]. Once the dynamical model is found, its parameters have to be estimated. The most common method for identification of robot parameters are based on the Least Squares method and Instrumental Variables [1].

Autonomous robots are naturally non-linear [8] and the estimation of parameters is complex. Existing estimation methods have to be adapted to the model's structure and noise [6][7][16].

A. Structure

This paper starts by presenting the mechanical of a four wheeled robot. Finding the dynamical model is discussed and then, an initial approach in estimating model parameters for the robot is done. The need for additional accuracy drives the comparative study on relative importance of the estimated parameters. An additional experiment is done for estimating final numerical values for the configuration. Conclusions and future work are also presented.

II. MECHANICAL CONFIGURATION

Figure 3 presents the configuration of the four wheeled robot, as well as all axis and relevant forces and velocities of the robotic system. The wheels are separated by 90 degrees.

Figure 3 show the notation used through-out this paper, detailed as follows:

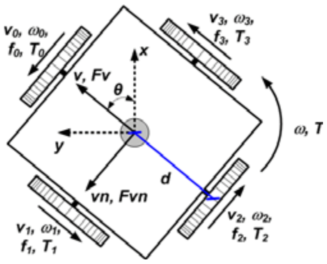

Fig. 3. Four Wheeled Robot.

- x, y, θ - Robot's position (x,y) and θ angle to the defined front of robot;
- d $[m]$ - Distance between wheels and center robot;
- v_0, v_1, v_2, v_3 $[m/s]$ - Wheels linear velocity;
- $\omega_0, \omega_1, \omega_2, \omega_3$ $[rad/s]$ - Wheels angular velocity;
- f_0, f_1, f_2, f_3 $[N]$ - Wheels traction force;
- T_0, T_1, T_2, T_3 $[N \cdot m]$ - Wheels traction torque;
- v, vn $[m/s]$ - Robot linear velocity;
- ω $[rad/s]$ - Robot angular velocity;
- F_v, F_{vn} $[N]$ - Robot traction force along v and vn;
- T $[N \cdot m]$ - Robot torque (respects to ω).

III. MODEL

A. Kinematic

The well known kinematic model of an omni-directional robot located at (x, y, θ) can be written as $v_x(t) = dx(t)/dt$, $v_y(t) = dy(t)/dt$ and $\omega(t) = d\theta(t)/dt$ (please refer to figure 3 for notation issues). Equation 1 allows the transformation from linear velocities v_x and v_y on the static axis to linear velocities v and vn on the robot's axis.

$$X_R = \begin{bmatrix} v(t) \\ vn(t) \\ \omega(t) \end{bmatrix} \quad ; \quad X_0 = \begin{bmatrix} v_x(t) \\ v_y(t) \\ \omega(t) \end{bmatrix}$$

$$X_R = \begin{bmatrix} cos(\theta(t)) & sin(\theta(t)) & 0 \\ -sin(\theta(t)) & cos(\theta(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot X_0 \tag{1}$$

The relationship between the wheels velocities and robot velocities are:

$$\begin{bmatrix} v_0(t) \\ v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & d \\ -1 & 0 & d \\ 0 & -1 & d \\ 1 & 0 & d \end{bmatrix} \cdot \begin{bmatrix} v(t) \\ vn(t) \\ \omega(t) \end{bmatrix} \tag{2}$$

It is possible to obtain the equations that determine the robot speeds related with wheels speed but the matrix associated with equation 2 is not square. This is because the system is redundant. It can be found that:

$$v(t) = (1/2) \cdot (v_3(t) - v_1(t)) \tag{3}$$
$$vn(t) = (1/2) \cdot (v_0(t) - v_2(t)) \tag{4}$$
$$\omega(t) = (v_0(t) + v_1(t) + v_2(t) + v_3(t))/(4 \cdot d) \tag{5}$$

B. Dynamic

The dynamical equations relative to the accelerations can be described in the following relations:

$$M \cdot \frac{dv(t)}{dt} = \sum F_v(t) - F_{Bv}(t) - F_{Cv}(t) \tag{6}$$
$$M \cdot \frac{dvn(t)}{dt} = \sum F_{vn}(t) - F_{Bvn}(t) - F_{Cvn}(t) \tag{7}$$
$$J \cdot \frac{d\omega(t)}{dt} = \sum T(t) - T_{B\omega}(t) - T_{C\omega}(t) \tag{8}$$

where the following parameters relate to the robot as follows:

- M $[kg]$ - mass;
- J $[kg \cdot m^2]$ - inertia moment;
- F_{Bv}, F_{Bvn} $[N]$ - viscous friction forces along v and vn;
- $T_{B\omega}$ $[N \cdot m]$ - viscous friction torque with respect to the robot's rotation axis;
- F_{Cv}, F_{Cvn} $[N]$ - Coulomb frictions forces along v and vn;
- $T_{C\omega}$ $[N \cdot m]$ - Coulomb friction torque with respect to robot's rotation axis.

Viscous friction forces are proportional to robot's speed and as such $F_{Bv}(t) = B_v \cdot v(t)$, $F_{Bvn}(t) = B_{vn} \cdot vn(t)$ and $T_{B\omega}(t) = B_\omega \cdot \omega(t)$, where Bv, Bvn $[N/(m/s)]$ are the viscous friction coefficients for directions v and vn and $B\omega$ $[N \cdot m/(rad/s)]$ is the viscous friction coefficient to ω.

The Coulomb friction forces are constant in amplitude $F_{Cv}(t) = C_v \cdot sign(v(t))$, $F_{Cvn}(t) = C_{vn} \cdot sign(vn(t))$ and $T_{C\omega}(t) = C_\omega \cdot sign\ \omega(t)$, where Cv, Cvn $[N]$ are Coulomb friction coefficient for directions v e vn and $C\omega$ $[N \cdot m]$ is the Coulomb friction coefficient for ω.

The relationship between the traction forces and rotation torque of the robot with the traction forces on the wheels, is described by the following equations:

$$\sum F_v(t) = f_3(t) - f_1(t) \tag{9}$$
$$\sum F_{vn}(t) = f_0(t) - f_2(t) \tag{10}$$
$$\sum T(t) = (f_0(t) + f_1(t) + f_2(t) + f_3(t)) \cdot d \tag{11}$$

The traction force on each wheel is estimated by traction torque, which can be determined using the motor current, as described in the following equations:

$$f_j(t) = T_j(t)/r \tag{12}$$
$$T_j(t) = l \cdot K_t \cdot i_j(t) \tag{13}$$

- l - Gearbox reduction;
- r $[m]$ - Wheel radius;
- K_t $[N \cdot m/A]$ - Motor torque constant;
- i_j $[A]$ - Motor current (j=motor number).

C. Motor

The prototype uses brushless motors for the locomotion of the robot. The model for brushless motors is similar to the common DC motors, based on [14].

$$u_j(t) = L \cdot \frac{di_j(t)}{dt} + R \cdot i_j(t) + K_v \cdot \omega_{mj}(t) \tag{14}$$
$$T_{mj}(t) = K_t \cdot i_j(t) \tag{15}$$

- L $[H]$ - Motor inductance;
- R $[\Omega]$ - Motor resistor;
- K_v $[V/(rad/s)]$ - EMF motor constant;
- u_j $[V]$ - Motor voltage (j=motor number);
- ω_{mj} $[rad/s]$ - Motor angular velocity (j=motor number);
- T_{mj} $[N \cdot m]$ - Motor torque (j=motor number).

IV. PARAMETER ESTIMATION

The necessary variables to estimate the model parameters are motor current, robot position and velocity. Currents are measured by the drive electronics, position is measured by using external camera and velocities are estimated from positions.

The parameters that must be identified are the viscous friction coefficients $(Bv, Bvn, B\omega)$, the Coulomb friction coefficients $(Cv, Cvn, C\omega)$ and inertia moment J. The robot mass was measured, and it was 2.34 kg.

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A. Experience 1 - Steady State Velocity

This method permits to identify the viscous friction coefficients $B\omega$ and the Coulomb friction coefficients $C\omega$. The estimation of the coefficient ω was only implemented because inertia moment is unknown, and it is necessary to have an initial estimate of these coefficients. The experimental method relies on applying different voltages to the motors in order to move the robot according his rotation axis - the tests were made for positive velocities. Once reached the steady state, the robot's speed ω and rotation torque T can be measured. The robot speed is constant, so, the acceleration is null, and as such equation 8 can be re-written as follows:

$$\sum T(t) = B_\omega \cdot \omega(t) + C_\omega \tag{16}$$

This linear equation shows that it is possible to test different values of rotation speed and rotation torques in multiple experiences and estimate the parameters.

B. Experience 2 - Null Traction Forces

This method allows for the estimation of the viscous friction coefficients (Bv, Bvn), the Coulomb friction coefficients (Cv, Cvn) and the inertia moment J. The experimental method consists in measuring the robot acceleration and speed when the traction forces were null. The motor connectors were disconnected and with a manual movement starting from a stable position, the robot was pushed through the directions v, vn and rotated according to his rotation axis. During the subsequent deceleration, velocity and acceleration were measured. Because the traction forces were null during the deceleration equations 6, 7, and 8 can be re-written as follows:

$$\begin{aligned} \frac{dv(t)}{dt} &= -\frac{B_v}{M} \cdot v(t) - \frac{C_v}{M} && (17)\\ \frac{dvn(t)}{dt} &= -\frac{B_{vn}}{M} \cdot vn(t) - \frac{C_{vn}}{M} && (18)\\ \frac{d\omega(t)}{dt} &= -\frac{B_\omega}{J} \cdot \omega(t) - \frac{C_\omega}{J} && (19) \end{aligned}$$

These equations are also a linear relation and estimation of all parameters is possible.

The inertia moment J is estimated using the values obtained previously in section IV-A. To do this, equation 19 must be solved in order of J:

$$J = -\frac{\omega(t)}{(d\omega(t)/dt)} \cdot B_\omega - \frac{1}{(d\omega(t)/dt)} \cdot C_\omega \tag{20}$$

C. DC Motor Parameters

The previous electrical motor model (equation 14) includes an electrical pole and a much slower, dominant mechanical pole - thus making inductance L value negligible. To determinate the relevant parameters K_v and R, a constant voltage is applied to the motor. Under steady state condition, the motor's current and the robot's angular velocity are measured. The tests are repeated several times for the same voltage, changing the operation point of the motor, by changing the friction on the motor axis.

In steady state, the inductance L disappears of the equation 14, being rewritten as follows:

$$u_j(t) = R \cdot i_j(t) + K_v \cdot \omega_{mj}(t) \tag{21}$$

As seen in equation 22, by dividing (21) by $i_j(t)$, a linear relation is obtained and thus estimation is possible.

$$\frac{u_j(t)}{i_j(t)} = K_v \cdot \frac{\omega_{mj}(t)}{i_j(t)} + R \tag{22}$$

V. RESULTS

A. Robot Model

By combining previously mentioned equations, it is possible to show that model equations can be rearranged into a variation of the state space that can be described as:

$$\begin{aligned} (dx(t)/dt) &= A \cdot x(t) + B \cdot u(t) + K \cdot sign(x) && (23)\\ x(t) &= [v(t) \;\; vn(t) \;\; w(t)]^T && (24) \end{aligned}$$

This formulation is interesting because it shows exactly which part of the system is non non-linear.

Using equations shown on section III-B and equation 21 the following equations can be derived:

$$\begin{aligned} A &= \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} && (25)\\ A_{11} &= -\frac{2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot M} - \frac{B_v}{M} \\ A_{22} &= -\frac{2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot M} - \frac{B_{vn}}{M} \\ A_{33} &= -\frac{4 \cdot d^2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot J} - \frac{B_w}{J} \end{aligned}$$

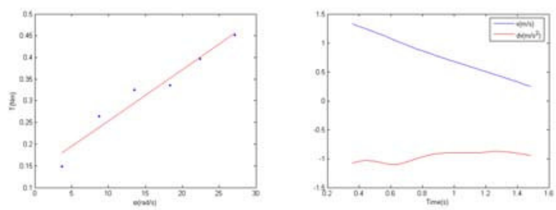
$$B = \frac{l \cdot K_t}{r \cdot R} \cdot \begin{bmatrix} 0 & -1/M & 0 & 1/M \\ 1/M & 0 & -1/M & 0 \\ d/J & d/J & d/J & d/J \end{bmatrix} \tag{26}$$

$$K = \begin{bmatrix} -C_v/M & 0 & 0 \\ 0 & -C_{vn}/M & 0 \\ 0 & 0 & -C_w/J \end{bmatrix} \tag{27}$$

B. Experimental Data for Robot Model

Experience 1 was conducted using an input signal corresponding to a ramped up step. This way wheel sleeping was avoided, that is, wheel - traction problems don't exist.

Shown in Figure 4 are the experimental plots. Due to space constraints only $T vs. \,\omega$ and results of the experiment 2 along the v direction are shown.



(a) Experience 1 - T and ω (b) Experience 2 - Direction v

Fig. 4. Experimental results.

The motor model was presented earlier in equation 14.

Experimental tests to the four motors were made to estimate the value of resistor R and the constant K_v. The numerical value of the torque constant K_t is identical to the EMF motor constant K_v.

Figure 5 plots experimental runs regarding motor 0. Other motors follow similar behavior.

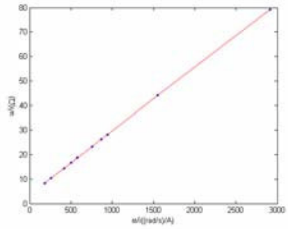


Fig. 5. Experimental tests for motor 0.

C. Numerical Results

Table I presents the experimental results of the friction coefficients and inertial moment.

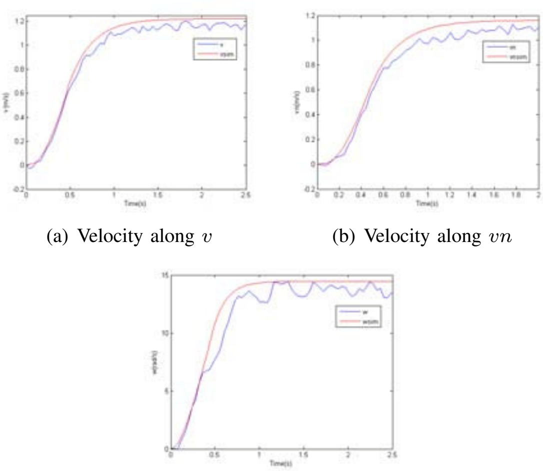
TABLE I

FRICTION COEFFICIENTS AND INERTIA MOMENT.

Parameters	Estimated values
$J(kg \cdot m^2)$	0.016
$B_v(N/(m/s))$	0.477
$B_{vn}(N/(m/s))$	0.600
$B_\omega N \cdot m/(rad/s)$	0.011
$C_v(N)$	1.873
$C_{vn}(N)$	2.219
$C_\omega(N \cdot m)$	0.135

Found parameters from data from all motors are $K_v = 0.0259\ V/(rad/s)$ and $R = 3.7007\ \Omega$.

To validate the model were made some experimental test on using a step voltage with a initial acceleration ramp, see figure 6.



(a) Velocity along v

(b) Velocity along vn

(c) Angular velocity ω

Fig. 6. Experimental runs for initial model validation.

To evaluate the performance of the model, the real values of the velocities were compared to the estimated ones. The following numeric parameters are used to evaluate the adequacy of the model:

$$e_{max} = max\,|v(k) - v_{sim}(k)| \tag{28}$$

$$\bar{e} = \frac{1}{N} \cdot \sum_{i=1}^{N} e_i \tag{29}$$

$$\sigma = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} (e_i - \bar{e})^2} \tag{30}$$

The analysis of the errors of the experimental run shown in figure 6, in accordance to equations 28, 29 and 30 are analyzed later in table IV.

D. Sensitivity Analysis

To understand which model parameters have more influence on the robot's dynamics, a comparison was made between the matrices of the model.

The model equation 23 is a sum of fractions. Analyzing the contribution of each parcel and of the variable portion within each fraction, a sensitivity analysis is performed, one estimated parameter at a time.

1) Matrix A, robot moving along v direction;

$$\left(\frac{2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot M}\right) = \frac{K_{a1}}{R} = 3.6676$$
$$(B_v/M) = K_{a2} \cdot B_v = 0.2041$$

2) Matrices B and K, robot moving along v direction with constant voltage motor equal to $6V$;

$$\left(\frac{l \cdot K_t}{r \cdot R \cdot M}\right) \cdot 12 = \frac{K_b}{R} \cdot 12 = 5.5227$$
$$(C_v/M) = K_k \cdot C_v = 0.7879$$

The same kind of analysis could be taken further by analyzing other velocities (vn and ω). Conclusions reaffirm that motor parameters have more influence in the dynamics than friction coefficients. This means that it is very important to have an accurate estimation of the motor parameters. Some additional experiences were designed to improve accuracy. The method used previously does not offer sufficient accuracy to the estimation of R. This parameter R is not a physical parameter and includes a portion of the non-linearity of the H bridge powering the circuit that, in turn, feeds 3 rapidly switching phases of the brushless motors used. In conclusion, additional accuracy in estimating R is needed.

E. Experience 3 - Parameter Estimation Improvement

The parameter improving experience was made using a step voltage with an initial acceleration ramp.

As seen in section V-A the model was defined by the equation 23 and the Least Squares method can be used to improve the estimation. The system model equation can be rewritten as:

$$y = \theta_1 \cdot x_1 + \theta_2 \cdot x_2 + \theta_3 \cdot x_3 \tag{31}$$

Where $x_1 = x(t)$, $x_2 = u(t)$, $x_3 = 1$ and $y = dx(t)/dt$. The parameters θ are estimated using:

$$\theta = \left(x^T \cdot x\right)^{-1} \cdot x^T \cdot y \tag{32}$$

$$x = [x_1(1) \ldots x_1(n)\ x_2(1) \ldots x_2(n)\ x_3(1) \ldots x_3(n)]^T \tag{33}$$

Estimated parameters can be skewed and for this reason instrumental variables are used to minimize the error, with vector of states defined as

$$z = [\overline{x_1}(1) \dots \overline{x_1}(n)\ x_2(1) \dots x_2(n)\ x_3(1) \dots x_3(n)]^T \quad (34)$$

The parameters θ are now calculated by:

$$\theta = \left(z^T \cdot x\right)^{-1} \cdot z^T \cdot y \quad (35)$$

The numerical value of R for each motor was estimated for each motor and then averaged to find R=4.3111 Ω. The results are present on followings tables. Table II shows values estimated by the experiment mentioned in this section.

TABLE II
PARAMETERS ESTIMATED USING THE METHOD 3.

Parameters	Values
$J(kg \cdot m^2)$	0.0288
$B_v(N/(m/s))$	0.5181
$B_{vn}(N/(m/s))$	0.7518
$B_\omega N \cdot m/(rad/s)$	0.0165
$C_\omega(N \cdot m)$	0.1411

The final values for friction and inertial coefficients are averaged with results from all 3 experimental methods and the numerical values found are presented in Table III.

TABLE III
PARAMETERS OF DYNAMICAL MODELS.

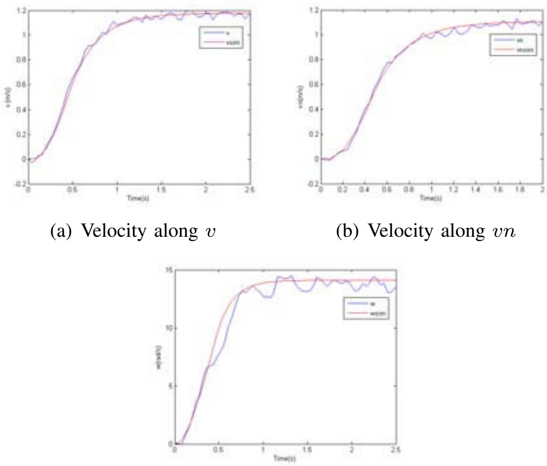
Parameters	Final values
$d\ (m)$	0.089
$r\ (m)$	0.0325
l	5
$K_v\ (V/(rad/s))$	0.0259
$R\ (\Omega)$	4.3111
$M(kg)$	2.34
$J\ (kg \cdot m^2)$	0.0228
$B_v\ (N/(m/s))$	0.4978
$B_{vn}\ (N/(m/s))$	0.6763
$B_\omega\ (N \cdot m/(rad/s))$	0.0141
$C_v\ (N)$	1.8738
$C_{vn}\ (N)$	2.2198
$C_\omega\ (N \cdot m)$	0.1385

F. Model Validation Experiences

The models were validated with experimental tests on using a step voltage with an initial acceleration ramp. Due to space constraints in this document, figure 7 show plots for some of the runs only. Other runs confirm the global validity of the model as simulation follows reality closely.

The analysis of the errors of the experimental run shown in figure 7, in accordance to equations 28, 29 and 30 are presented in table IV.

Figure 8 shows the fit of the error of the experimental run along the v axis, shown in figure 6(a) (with the parameter estimated initially) when compared to the fit of the errors for the final estimated parameters, as shown previously in figure 7(a). Clearly, the mean of the fits is closer to zero in the final parameters thus producing adequate model performance. Other plots follow similar behaviors, collected results available in table IV.



(a) Velocity along v

(b) Velocity along vn

(c) Angular velocity ω

Fig. 7. Experimental runs for final model.

TABLE IV
ERROR ANALYSIS FOR INITIAL AND FINAL ESTIMATED PARAMETERS.

	Initial			Final		
Vels.	e_{max}	$\bar{e}$	σ	e_{max}	$\bar{e}$	σ
v	0.096	-0.052	0.026	0.061	0.008	0.024
vn	0.135	-0.070	0.034	0.068	-0.009	0.026
w	3.345	-0.919	0.852	2.504	-0.452	0.732

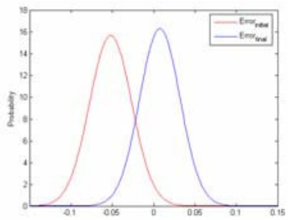


Fig. 8. Comparison error fits for initial and final estimated parameters.

VI. CONCLUSIONS AND FUTURE WORK

A. Conclusions

This paper presents a model for a mobile omni-directional robot with 4 wheels. The derived model is non-linear but maintains some similarities with linear state space equations. Friction coefficients are most likely dependent on robot and wheels construction and also on the weight of the robot. The model is derived assuming no wheel slip as in most service robotic applications.

The test ground is smooth and carpeted. Experience data was gathered by overhead camera capable of determining position and orientation of the robot with good accuracy.

Experiences were made to estimate the parameters of the model for the prototype. The accuracy of the presented model is discussed and the need for additional experiences is proved. The initial estimation method used two experiences to find all parameters but a third experience is needed to improve the accuracy of the most important model parameters. Sensitivity analysis shows that the most important model parameters concern motor constants.

The exactness of the found model was discussed with experimental runs. The estimated model was shown to be adequate for the used prototype.

B. Future Work

The work presented is part of a larger study. Future work will include further tests with different prototypes including prototypes with suspension and with a different number of wheels. Another issue is finding the limits of slippage. The model can also be enlarged to including the limits for slippage and movement with controlled slip for studying traction problems. This study will enable effective full comparison of robots with different number of wheels.

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