

FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO PORTO



Parafoil Control for STRAPLEX

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July 25, 2013

Resumo

Esta dissertação está inserida no projeto STRAPLEX (STRAtospheric PLatform EXperiment), que é um programa da Faculdade da Engenharia da Universidade do Porto em parceria com a Agência Espacial Europeia (ESA). Este projeto é constituído por uma plataforma que permite à comunidade científica enviar experiências para a estratosfera recorrendo a balões de hélio. Devido às condições extremas presentes na estratosfera, o balão rebenta e a plataforma inicia a sua fase de descida estabilizada por um pára-quedas circular. Uma vez que este pára-quedas não permite qualquer tipo de controlo, nesta dissertação sugere-se que seja utilizado um parapente no lugar deste. O objetivo principal desta dissertação é implementar um fiável algoritmo de controlo para este sistema. Este trabalho dá continuidade a uma dissertação desenvolvida anteriormente, que propôs uma estrutura mecânica e de hardware que possibilita a implementação do algoritmo de controlo para a descida da plataforma.

O movimento descendente da plataforma é descrito por um fiável modelo matemático, que inclui os vários movimentos relativos entre os diferentes objetos da plataforma. Este modelo também engloba os distintos tipos de controlo existentes no sistema, possibilitando a implementação de um algoritmo de controlo. Este algoritmo foi concebido para permitir a aterragem da plataforma no local desejado, sob certas condições atmosféricas.

Como suporte ao sistema de controlo, é efetuada uma análise do modelo quanto à sua estabilidade, controlabilidade e observabilidade. No fim, é realizada uma optimização de ganhos de controlo recorrendo a um método de optimização.

Durante o desenvolvimento do trabalho são ilustrados alguns resultados de simulação que foram obtidos recorrendo ao programa MATLAB.

Abstract

This thesis is part of the project STRAPLEX (STRAtospheric PLaTform EXperiment), which results from a partnership between the Faculty of Engineering of the University of Porto and the European Spatial Agency (ESA). This project is constituted by a platform that allows the scientific community to send experiments for educational purpose into the stratosphere, using balloons filled with helium. Due to the extreme conditions in the stratosphere, the balloon bursts and the platform begins to fall back into the atmosphere. This fall is stabilized by a round parachute. Since this type of parachute does not allow any kind of control, in this thesis is suggested the usage of a parafoil, instead of the round parachute. The main goal of this thesis is implementing a reliable control algorithm for this system. This work gives continuity to another one developed before, in which a mechanical structure for the control of the parafoil is proposed. This structure includes some suitable hardware that allows the implementation of the control algorithm.

The downward movement of the platform is described by a reliable mathematical model which includes the various relative movements between each of the elements of the platform. This model also comprises the distinct types of control of the system, allowing the implementation of a control algorithm. Such algorithm was designed to allow the landing of the platform in the desired place, under some constrained atmospheric conditions.

As a support to the control system, an analysis of the stability, controllability and observability of the mathematical model is made. At the end, an optimization of the control gains is developed by using an optimization method.

Along the thesis, some simulation results are shown, which were obtained with the software MATLAB.

Agradecimentos

Ao meu orientador, Professor Sérgio Reis Cunha, pela sua constante motivação, ajuda e dedicação que sempre demonstrou ao longo de todo este trabalho.

Aos meus amigos por todos os momentos, aprendizagens e experiências que me proporcionaram. Em especial aos meus amigos de sempre porque sem dúvida contribuíram para a pessoa que sou hoje. Estou extremamente grato aos meus colegas e amigos Diogo Pernes, Hugo Cruz e Luís Pires pelo companheirismo e amizade ao longo do meu percurso académico. Acho que sem eles, o meu percurso académico não seria traçado da mesma forma.

À minha família por ser sempre um pilar na minha vida. Em especial aos meus pais e irmãos pelo amor e incansável apoio que sempre demonstraram.

À minha namorada, Joana, pela presença, paciência e amor mostrado durante estes últimos 5 anos.

João Luís Granja da Costa

“The true sign of intelligence is not knowledge but imagination”

Albert Einstein

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Abbreviations and Symbols

AR	Aspect Ratio
AHRS	Attitude and Heading Reference System
CD	Convergence Distance
CPU	Central Processing Unit
D	Drag
DOF	Degrees Of Freedom
DPMS	Descent Profile Management System
DTMF	Dual-Tone Multi-Frequency
ESA	European Spacial Agency
GFSK	Gaussian Frequency Shift Keying
GPS	Global Positioning System
GUI	Graphic User Interface
L	Lift
LTI	Linear Time-Invariant
OKID	Observer/Kalman Filter Identification
PD	Proportional and Derivative
PDI	Preparação da Dissertação
RF	Radio Frequency
STRAPLEX	STRAtospheric PLatform EXperiment
SVD	Singular Value Decomposition
TCAS	Traffic Collision Avoidance System

Chapter 1

Introduction

A parafoil is a light flying vehicle and since its invention by Ms. Domina Jalbert in 1960, has been widely used in a variety of windsports and in space and military missions. The parafoil is a very stable paraglider with a large manoeuvrability. For those reasons it can be effectively controlled by an autonomous control system adequately attached to the parafoil. The dynamic model of the parafoil system has been a topic of scientific research in order to reach a reliable and precise control algorithm. This work focuses on the dynamic analysis of the system along with the performance of a control algorithm for a system used in the STRAPLEX. This chapter is split into four sections. In Section 1.1 it is described the motivation of this work. Section 1.2 characterizes in detail the main objectives of this work whereas an outline of this thesis is presented in Section 1.3. Finally, a description of the website support is detailed in Section 1.4.

1.1 Motivation

STRAPLEX project offers students the possibility to send experiments for educational purposes into the stratosphere. A controllable parafoil system was developed by Mario Martins [3] to be incorporated into the project. That work provided improvements in performing a controllable descending flight. Moreover, it opened a new perspective for further enhancements in control algorithms implemented in the STRAPLEX. A precision control algorithm can be designed to allow the missions to land on a desired land point, avoiding undesirable situations like the ones that happened in some past tests. The control algorithm should be performed taking into account the wind presence and the system modes as well as its restrictions about the control action. Along with the author's enthusiasm in the control area, this led the author to accept this exciting challenge. This challenge also requires the development of a mathematical model for the system. This model should describe the system dynamic and it can be a very important tool to understand the system modes and in this way to improve the control algorithm.

1.2 Objectives

A parafoil along with the drone were employed on the STRAPLEX to make the descending phase a controllable flight. The drone had a strong hardware and software system together with a reliable communication architecture which allow the execution of human commands from a ground station. Using a joystick, a user can fully maneuver the system for a desired trajectory. The drone is also able to autonomously maneuver the horizontal and vertical motion of the system.

The main objective of this work is to develop an accurate and steady control algorithm to make the system able to steer itself to a desired trajectory. For that the dynamic motion of the whole system should be previously analysed. A mathematical model should be studied and modelled from the detailed analysis of the system motion, defined by the kinematic and dynamic equations. For an accurate and precision model, a dynamic analysis of each component of the system should be performed taking the constraint forces between them into account. The control algorithm should be developed for the horizontal motion as well as for the vertical motion to allow control in all directions. Several control modes should be designed to perform a redundant control system. By noting that the mathematical model is nonlinear, a linearization method should be applied in order to analyse the stability of the model. Using the linear model, an optimization method can be applied to find the optimal control gains in order to enhance the control system responsiveness. The ultimate target of the control system is the spiral approach strategy algorithm in order to, in certain wind conditions, be possible to make the system autonomously land on the launching point.

1.3 Thesis Outline

This thesis is organized in six chapters. On this one, Chapter 1, it is described the motivation and main objectives of this work. In Chapter 2, the STRAPLEX project is presented as well as its actual state. It is also described a background of the work. Chapter 3 shows and describes the components of the system and the control types. It is also shown the various stages of a desired descending flight. A detailed mathematical model of the system is developed in Chapter 4. Chapter 5 is devoted to the design and implementation of the control algorithm. After the linearization of the nonlinear model, the stability analysis and the tuning of control loops for the control system is performed in Chapter 6. The conclusions and the future work are presented in Chapter 7.

1.4 Website

Using the *Wordpress* blogging tool, a website was created as a support for this thesis. It contains a short description of the work developed as well as of the STRAPLEX project. A short presentation of the mathematical model and the control algorithm is presented together with some simulation results. In Section “Reports”, the weekly reports and the PDI report can be downloaded. In this website, one can also read informations about the author, the final remarks of the work and the software tools used during the development of this work.

Chapter 2

STRAPLEX

This chapter is organized as follows. Section 2.1 presents the STRAPLEX project and explains the several phases of a mission. The actual state of the project as well as the work developed by Mario Martins ([3]) are detailed in Section 2.2. Section 2.3 presents the next step for the STRAPLEX project. Finally, Section 2.4 presents some existent scientific research about parafoil control.

2.1 STRAPLEX Project

STRAPLEX (STRATospheric PLatform EXperiment) is a programme by the University of Porto, Portugal in collaboration with the Education Projects Division of the European Space Agency (ESA) [4]. This programme began in 2005 and offers the scientific community the possibility to make experiences into the stratosphere. It makes it possible to do tests near to the specific space environment of the stratosphere. In this atmospheric layer, near-space conditions can be found, which are interpreted by the scientific community as the near vacuum conditions that are good conditions to make certain experiments. In this concept STRAPLEX project offers the possibility to make following experiments:

- **Experiments related to Stratospheric Balloon design:** Archimedes force, Balloon principles, Ascent velocity, Parachute system, Helium utilisation, etc;
- **Experiments related to the Atmosphere:** Temperature environment, Pressure environment, Atmosphere density, Humidity, sound propagation, pollution, etc;
- **Experiments related to radiation:** Solar radiation flux, solar energy, cosmic ray, etc;
- **Experiments related to tele-detection:** Albedo, colour photography, black and white photography, digital photography, video, data transmission, etc;
- **Experiments related to biology;**
- **Landing systems;**
- **Detachable capsules** (including specific localisation and recovery system).

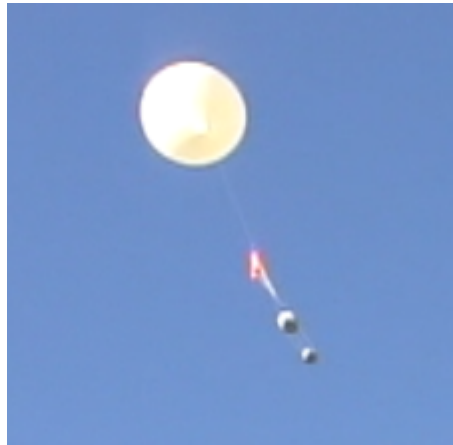


Figure 2.1: Launch of STRAPLEX

The scientific community can then make experiments in a specific environment through STRAPLEX, in a low cost and flexible way. For that, the platform is essentially composed by five main objects: a capsule for payload accommodation, a parachute system, a helium balloon, a transponder and a cutdown system. The four first objects can be viewed in Figure 2.1 which was taken on the last realized launch.

The capsule is the main object of the platform that carries both the scientific experiment and the control system. It reaches a high altitude through means of the balloon filled with helium. Depending on the mass of the experiments, STRAPLEX can reach up to a remarkable 40km altitude. The second system is a round parachute which is activated in the descending phase to stabilize the trajectory of the platform. The cutdown system is used to safely separate the balloon from the platform, when it is no longer necessary. This enables the possibility to, in case of emergency, abort the mission. The transponder is a special device widely used in aviation to increase the air traffic control. This device is installed on the platform to continuously communicate with the air traffic controller or other nearby aircraft equipped with the traffic collision avoidance system (TCAS). The STRAPLEX mission may be split into four distinct phases:

1. Launch;
2. Rising phase;
3. Descending phase;
4. Landing.

In the first phase, the balloon, made of a latex material, is filled with helium until its lower density makes the system go up. Thenceforth the platform attached to the balloon climbs to the stratosphere. During the rising phase, the air pressure decreases which leads the helium to expand until the latex reaches its rupture point, bursting the balloon. Depending on the platform mass, the balloon can burst at 40km altitude. After that, the platform starts the descending phase while the balloon is separated from the platform by the cutdown system. At such high altitude the air mass

is rarefied therefore the resistance force is very low and the platform can reach very high sink rate (approximately 300km/h). Furthermore, the platform can oscillate dangerously, endangering the success of the experiments. With the intention to dampen these oscillations, a round parachute is used and inflated after the balloon bursts. This parachute aides to stabilize the platform, but doesn't allow an effective control of the trajectory. The last phase, landing, is the most critical phase of a mission. It is strongly dependent on the wind conditions and until Mario Martins' s work, there wasn't any kind of control on the landing point. The main goal of that work was to add a control system to avoid unwanted landings. Places like wooden areas and water bodies should be avoided to reduce the recovery operation time and the inherent risks. The solution developed by Mario will later be analysed in Section 2.2.

In all phases of the mission, the capsule is in constant communication with a fixed ground station through a redundant RF communications. Information such as air pressure, humidity, temperature (in and outside the capsule), position, velocity and other relevant informations are measured by the capsule and sent to the ground station. On the ground, the station receives the flight information and stores it for further analysis. Moreover it can send some action for the capsule, e.g. the cutdown command. Through the performed measurements, the capsule can also calculate an estimated landing point. A mobile station receives the coordinates of this point and tracks the platform path in order to recover the platform as soon as it lands.

In Portugal, the STRAPLEX launch is normally carried out in Évora. The moderate climate, low population rate and considerably flat region are good reasons for the success of missions.

2.2 Actual Status

As stated above, the round parachute, which stabilizes the platform in the descending phase, doesn't allow any control of the trajectory. So the landing point is highly influenced by wind effect. The lack of control on these phases already led to some problems on the rescue operation of past missions. In one mission the platform transcended the border with Spain due to a strong west wind. It affected the Spanish traffic and complicated the recover operation. In another mission the platform submerged on Alqueva dam, near Évora. This caused a large waste of time on the rescue operation and some equipment inside the main capsule got damaged.

In order to find a solution for this problem, the engineering student Mário Martins de Sousa oriented by the Professor Sérgio Reis Cunha developed a system based on a controllable parafoil [3]. The control of the parafoil is assured by a special capsule, called Drone, which was built for this purpose. It was built over a strong mechanical structure and contains three servo-actuators attached to the lines of the parafoil. These actuators are able to change the shape of the wing like pilots do in windsports. The actuator's position is managed by a control system supported by a navigation and measurement system. He implemented two types of flight modes: manual and autonomous. In the first one, the user on the ground station maneuvers the parachute using a joystick. The joystick movements are interpreted by the ground station software, named STX center, that sends the respective actions to Drone where the actuators act accordingly. This operation can

be done with the platform in sight or using either the real time video of the capsule or the Google Earth interaction which was implemented on the STX center. In the autonomous mode, the control system directs the platform along a desired path resorting to an algorithm control implemented on the drone's software. This algorithm can be split into four modes:

- Heading;
- Course;
- One point localizer and course;
- Two points localizer.

These four modes of control will be explained in detail in Chapter 5. A series of tests of the parafoil-drone system were conducted to evaluate the functionality of the control system. In manual mode, the control system proved to be a very maneuverable and responsive system. However in autonomous mode, the system sometimes didn't reach the desired point. It was verified that some drawbacks greatly influence the landing point, mainly the wind effect.

The navigation and measurement system inside of drone allow the measurement of relevant information which is used for control loop and further analysis. A GPS receiver and an attitude and heading reference system (AHRS) provide kinematic informations. The first one provides information of the position and velocity on the three axis (North, East and Down). The AHRS system which is composed by a 3-axis accelerometer, magnetometer and a gyro, provides three orientation angles using a Kalman filter. Furthermore, the drone has a set of sensors to continuously track the external and internal parameters, such as temperature, humidity, pressure and power. The internal sensors have a significant role in ensuring the good processing and monitoring of the microcontrollers inside the drone. A hand made pitot tube was built to calculate the airspeed. This method of airspeed measurement employs a differential and an absolute pressure sensor. It relies on the simplified Bernoulli's equation to define the airspeed expression in function of the differential pressure.

All of these informations can be sent to the ground station by a strong communication protocol composed by redundant channels. These redundant channels are based on a GFSK, a 5DPSK and a DTMF modulation, so that if one of the system fails, a total blackout is avoided. Moreover it was implemented a more flexible communication method in the platform which allows a direct communication between the components of the platform and with the ground station. The ground station has the main objective to monitor the platform during the flight. It is essentially composed by a computer which gathers the received flight information and transmits the control action through transceivers. Its interface with the operator is a MATLAB graphic user interface (GUI) called STX center, visible in Figure 2.2.

In the left part of this interface it is possible to see the most important flight information (coordinates, attitude velocity, course, etc), GPS information and some sensor status. The wind forecast, some Drone options and the actuators position are shown in the center part. The right

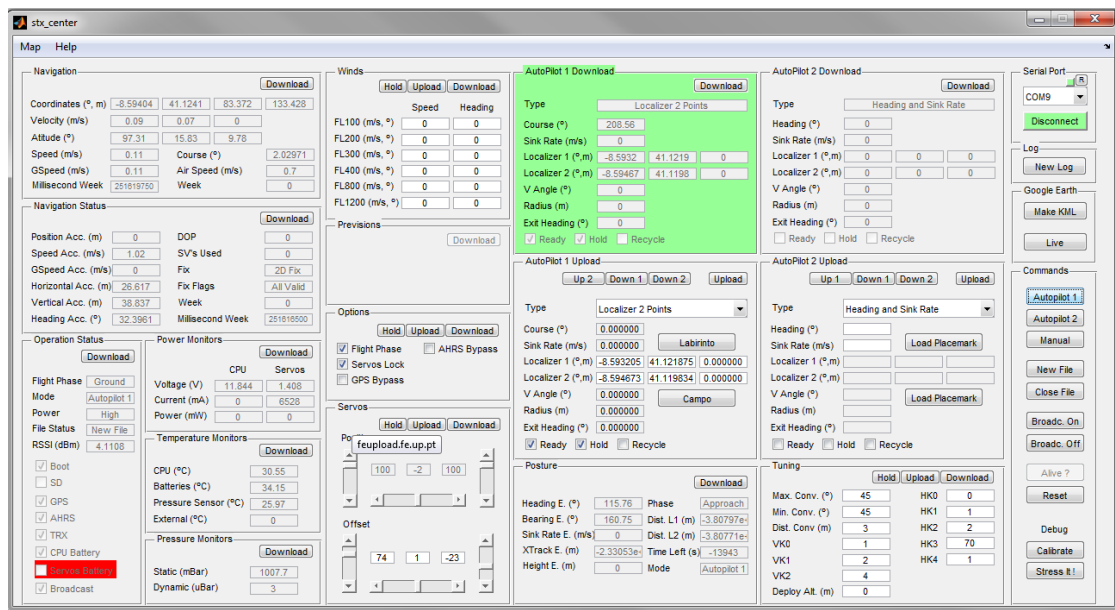


Figure 2.2: STX center

part is responsible for the autonomous control panel where it is possible to download or upload important parameters for the control system (heading/course, landing point, etc). In the right side, there are buttons which allow the user to see the platform trajectory on Google Earth and choose the operating control mode, among other options.

It should be noted that Mario also improved the cutdown system making it more reliable and efficient using a mechanical approach instead of a pyrotechnic method. The cutdown system has the function of separating the helium balloon from the platform. This device has a crucial function on the mission's success because if the balloon doesn't release from the platform, during the descending phase, the balloon, already bursted, will compromise the parafoil control. After some tests, it was proved that the new cutdown system successfully played its function.

2.3 The STRAPLEX Next Step

The parafoil-drone system developed by Mario provided to be a reliable solution with great stability and maneuverability. However during some flight tests the parafoil-drone system didn't reach the default destination because of the wind effects and, on some cases, the double pendulum effect which isn't taken into account in the control system. In order to overcome these issues, and therefore heading towards a desired path, a better control algorithm may be implemented. For example, an algorithm capable of adjusting the internal control gain based on wind effect and some other system features. Moreover a complete control algorithm may be achieved for the descending and landing phase, e.g. through a spiral motion. This trajectory would be chosen from a database depending on the wind direction and the desired landing point which is selected by the user. At the end, a full stratospheric flight should be conducted in order to fully validate the control system.

2.4 State of the Art

Since the appearance of the parafoil, many researches have been carried out proving its great advantages in autonomous flights such as the stability, controllability and maneuverability. Other studies have been performed to estimate the aerodynamic coefficients of a parafoil. Some examples are the articles [5], [6], [7] and [8] that analyse these coefficients for several wing dimensions (area, aspect ratio, among others). [5] and [6] estimated the coefficients testing the parafoil in a wind tunnel. In addition to the aerodynamic coefficients, the stability characteristics were also analysed for various wing settings. In [5], Sanger Burk *et al.* concluded that at angles of attack above about 70° , the wings would not inflate properly and the parafoil could collapse.

John Nikolaides *et al.* presented results of the aerodynamic coefficients and of the parafoil velocity obtained from ascending flights and manned jumps from aircraft tests [8]. It was used a parafoil design for various aspect ratio, wing loadings and trim angles. The parafoil flight was initially modelled in equations of motion to predict the flight performance. Using wind tunnel data, a comparison of predicted and measured flight performances of the parafoil was made and it was concluded that the agreement between the predicted and measured performance was good.

J. Lingard discussed the performance and design of parafoil for the Precision Aerial Delivery System (PADS) [7]. He made a briefly general description of a parafoil and analysed the aerodynamic characteristics of ram-air wings. Theoretical expressions for the aerodynamic coefficients were derived and compared with the existent experimental data for a ram-air wing of aspect ratio 3.0. Analysing the results, it was verified that the lift coefficient (C_L) is proportional to the angle of attack while the drag coefficient (C_D) is a quadratic function of the angle of attack. In relation to the L/D ratio it was verified that it has a peak around an angle of attack of 5° and it improves with the increase of the aspect ratio. Furthermore, the influence of the line length of the parafoil system was investigated on the lift coefficient and on the L/D ratio. J. Lingard also analysed the ram-air parachute flight performance, through the equations of motion in the horizontal and vertical plane. The longitudinal and lateral static stability was studied showing the effect of the trailing edge deflection on the aerodynamic coefficients and on the dynamic of the system. Some conclusions were stated in what concerns the dynamic stability of the parafoil.

A more robust mathematical model of a paraglider system is presented on the articles [9], [10] and [11] which will be described below. The models mainly differ on the number of Degrees of Freedom (DOF), which is defined as the minimum number of generalized coordinates necessary to define the configuration of the system [12]. Some models consider the paraglider-payload system as a rigid body and therefore the dynamic analysis concerns the center of mass of the whole system. Other approach is to consider the relative motion between the paraglider and payload and the constraint forces and moments of the connection lines. The general approach followed on the mathematical model design consists of the determination of the system kinematics and dynamic equations. When the system is considered as a multi-rigid bodies, several frames are considered and the dynamic analysis is made using kinematical equations such as the equation of Coriolis and the Poisson's kinematical equation, presented in Appendix A.1 and A.2, respectively.

Toglia *et al.* presented two different models of decreasing complexity, one with 9 DOF and another with 6 DOF [9]. In the first one, she has taken into account the effect of the payload twisting and therefore the model had 3 DOF for the 3 inertial position of the joint point and 6 DOF to describe the parafoil and payload attitude motion. After an analysis of the kinematics for each body, the system dynamic equations were derived considering the aerodynamic forces, weight forces, reaction forces exerted at the joint point and the apparent force, this one applied on the parafoil. On the other model the relative motion between the two bodies is neglected and the analysis is made on the global center of mass. In this model all forces (total weight force, aerodynamic forces and the apparent force) are applied on the center of mass. On both models, it is used a control applied on the two flaps of the parafoil wing. The deflection of the two flaps was modelled by introducing the symmetrical term δ_s and the asymmetrical term δ_a , where δ_s is equal to the minimum of the two flaps' deflection and δ_a is given by the differential flaps deflection. After setting the kinematics and dynamic equations along with the control equations, it was derived the dynamic equations of motion equation. Using the MATLAB software, the two models were simulated for a free dynamic trajectory and a spiral motion, analysing the flaps' deflection and the payload twist effect. Through the simulation results, the comparison of the two models was investigated and Toglia concluded that the 6 DOF model presented a delay in turning to the spiral motion and had less oscillations due to the lack of presence of relative motion caused by the payload influence.

A more complete and robust mathematical model of a parafoil-payload system was presented in [10] and [11]. The kinematics and dynamic equations were derived in different frames using rotation matrices to switch among frames and the equation of Coriolis to determine the derivative of a vector in moving frames. Accurate and reliable expressions were described for the aerodynamic forces and moments. The equations of motion were presented compactly in a matrix form and the model was numerically integrated using a fourth-order Runge–Kutta algorithm to generate the trajectory of the system from the point of release. Simulations under different flight conditions were performed so that the performance of the controllable parafoil system was evaluated. Furthermore, the response to a control brake deflection was studied for different wing configurations. Slegers *et al.* observed that the parafoil-payload systems can exhibit two modes of directional control: skid and roll steering, depending on the angle of incident and the wing configuration. In [11], Slegers also studied the relative pitching and yawing motion of a payload with respect to a parafoil and it was proposed a proportional-derivative controller to track a desired yaw command. Simulation results demonstrated that the relative yawing motion of the payload resulted in persistent oscillations of the system. These oscillations can be eliminated by reduction of feedback gains, but the resulting tracking performance was poor. It was also shown that a reduced-order linear model was able to predict the original model closed-loop damping of the yaw controller.

A different analysis point of view of a paraglider-payload system was described in [13]. The simplified mathematical model was decoupled in terms of lateral and longitudinal dynamics. The stability analysis was investigated and a control design to solve the path following problem was proposed. Through the dynamic equations of the simplified model, the equilibrium point was

founded for a linear path in the XY plane and a input-output feedback linearizing control is designed using the control flaps deflection as in [9]. In order to test the performance of the control input, two path following tasks (a pure line and a polygonal path) were simulated. In the polygonal path following test, it was verified that the system converged to the reference path for each segment in about 50 s, using only lateral directional control input with acceptable values.

In addition to the mathematical model obtained from the kinematics and dynamics equations, a mathematical model can be found through the identification method. This method consists of obtaining a state-space representation of the system from the observation of data samples. As mentioned in [14], two different type of data can be used: a sequence of impulse responses of a discrete-time LTI system or an input-output data. In a parafoil-payload system, one generally has input-output data provided by sensors installed on the payload system. Based on this type of data, Katayama presented two identification methods, which use the LQ decomposition of data matrices and the SVD of the state matrix. These algebraic procedures were also explained in [14]. An algorithm for each method was proposed based in a detailed description of the respective identification method. The application of the algorithm was illustrated for different types of problems.

An example of an identification method for a commercial powered parafoil vehicle was described by Hur and Valasek [15]. The Observer/Kalman Filter Identification (OKID) methodology was selected for identification of the mathematical model from the observation of flight test data. This method uses Markov parameters to generate a linear state-space model by forming a Hankel matrix. A 8 DOF mathematical model was used to compare with the identified state-space linear model. Using the same initial conditions and inputs, various simulation results of the two models were showed. Based on simulation results, Hur and Valasek concluded that the identification method can identify the dynamic system effectively and accurately.

The parametrical identification problem, which consists of finding the numerical values of variable parameters to better match experimental data, is investigated in [1]. Instead of the OKID identification technique, which is based in just one single criterion, Yakimenko *et al.* studied the multicriteria parametrical identification technique. A set of single criterion identification method used in other articles is firstly presented. A brief explanation of the difference between the parametrical identification is also presented. Figure 2.3 shows the two identification problems.

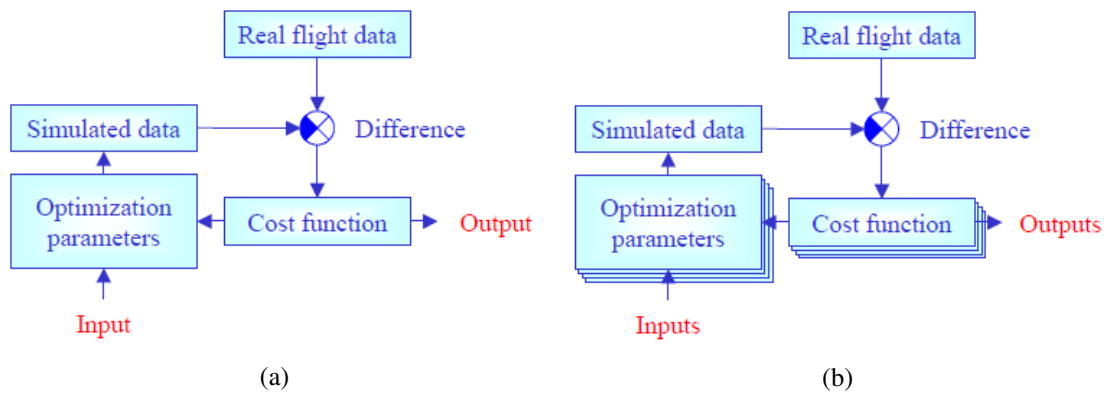


Figure 2.3: Single-criteria identification (a) and multi-criteria identification (b)[1]

Yakimenko *et al.* gathered a set of variable parameters of the model into various distinct groups which will be identified. Applying the PSI method to solve the multicriteria optimization, the multicriteria identification is proposed based on the optimization method. Using the MATLAB software, simulation tests were performed for several distinctive adequacy criteria. A comparison of the trajectory is made between the flight test data and the simulation results of several estimated models. Although not big difference was observed between the several simulation tests and the flight test, the trajectories do not match completely.

The model predictive control strategy for a parafoil-payload system is described in [16]. The model predictive control strategy was described for a simplified 6 DOF model, where the aerodynamic coefficients were estimated using a recursive weighted least-squares estimation. This control strategy consists in a minimization method of an estimated path error. The quadratic cost function was derived in terms of the flight test and control input samples. A control input expression which minimizes the cost function was defined and tested under different desired paths. Three autonomous flight tests showed that the model predictive controller was able to control autonomously the trajectory of a parafoil-payload system.

A parafoil recover system capable of autonomously controlling the descending flight of the system to a recovery area and maneuvering the parafoil to execute a soft landing of a payload in the landing point was disclosed in [17]. The system uses a Descent Profile Management System (DPMS) that can determine important flight parameters (altitude, position of the payload, among others) and the optimal flight path from the released point to the desired recovery area. The control strategy of the trajectory of the system consists in a spiral motion in the direction of a predetermined height above ground. A soft landing of the parafoil is executed by braking to slow the sink rate of the payload. Near to the ground, the parafoil system is released from the payload to prevent the canopy from dragging the payload on the ground.

Chapter 3

System Overview

In this chapter it is presented an overview of the system which will be later modelled in a mathematical model. Firstly the many components of the system are listed and presented in Section 3.1. Each Section 3.2, 3.3, 3.4 and 3.5 describes in detail each component, beginning with the *Parafoil*, followed by the *Drone*, the *Capsule* and at the end the *Transponder*. In each of the previous sections, it is described the physical dimensions of the object and its main features as well as its function in a mission. Section 3.6 presents the control inputs and their features as well as their functionalities. Finally, the several stages of the descending phase are presented and described in Section 3.7.

3.1 System Components

Embedding the parafoil-drone system in the platform, the descending phase of a mission can be divided into two distinct phases: uncontrollable and controllable phase. The first one corresponds to the descending phase where it is used the round parachute. The round parachute doesn't allow any type of control, hence the phase type. The second phase begins when the platform reaches stable air mass. Here it is easier to control the trajectory so that the parachute is released and the parafoil is inflated. A controllable trajectory can now be followed manoeuvring the parafoil. In this phase the platform is composed by four main objects (parafoil, drone, capsule and transponder) as seen in Figure 3.1.

The drone is attached, through a set of lines, to the inferior surface of the wing of the parafoil, also called canopy. The capsule is connected to the drone through three strong lines in a similar manner as the transponder is connected to the capsule. These connections are visible in the right side of Figure 3.1. For reasons of simplification, the three connections will be treated as just one connection.

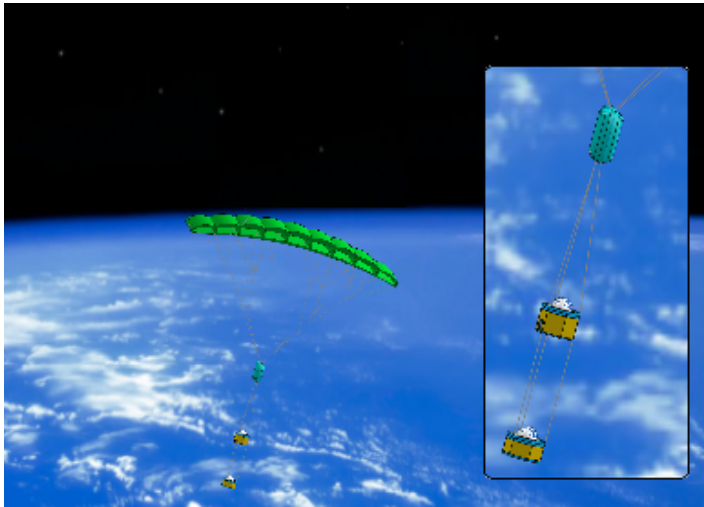


Figure 3.1: STRAPLEX components

3.2 Parafoil

Currently there are many types of parachutes which are employed in windsports and military missions. One that is widely used is the ram-air parachute, also called parafoil. This parachute was designed by Ms. Domina Jalbert in 1960 and proven to be a very stable and manoeuvrable device. It has a rectangular platform with its leading edge opened and the trailing edge closed, so that ram air pressure enters and inflates the canopy, while it is moving through the air. In this work it is used a parafoil which was adapted from a common recreational power kite (Figure 3.2).



Figure 3.2: View of the Parafoil

3.2.1 Dimensions

The parafoil is constituted by a canopy and by a set of lines which connects the canopy with the drone. The canopy is considered to have a fixed shape and it should always be completely inflated during the descending phase. It has an ellipsoidal shape with a plane of symmetry and is made of a thin fabric composed of small sections called cells with no rigid members. The leading edge has cells opening, with 7 centimetres thickness, to allow the air to flow inside while the trailing edge is closed to keep the air inside and maintain the pressure for keeping the canopy inflated. It is very important that the air pressure inside the canopy will be high enough because otherwise there is the danger of the canopy collapsing.

The connection between the canopy and the drone is essentially supported by two main lines while the lines which are connected to the trailing edge of the canopy on both sides are the brakes. The two main lines have a cascade configuration and they are attached to the canopy in specific points in order to maintain the canopy inflated. Each main line is divided in seven lines that each composed by six smaller lines. The seven lines are grouped and tied in specific points, named canopy rotation points because during the flight, as it was possible to see in previous tests, the canopy has a rotation motion that resolves around these points. The brakes play a more passive role, only activated for control purposes. The functions of the main lines and brakes will be explained in detail in Section 3.6. In order to avoid the interlacing of the main lines, a wooden beam with 34.5 centimetres wide is used. The length and the position of the beam was analysed in [3] in order to decrease the arc of the canopy and insert an anti twist force maintaining the parafoil controllability. Taking into account the lines and wooden beam weights, the total weight of the parafoil is about 700 grams.

The straight line which joins the leading and trailing edge of a canopy is called chord line, which for this canopy is 90 centimetres long. The span is the distance in a straight line between the two sides of the canopy. The span of this canopy is 3.3 meters. The canopy area is calculated as the product of the span times the chord. In this case, the area is equal to 3 square meters. Considering the current mass of the entire system which is about 9 kg, the expected wing loading is 3 kg/m^2 . The wing loading is a very important parameter on the canopy choice since it defines the sink rate.

The main lines are attached under the bottom surface of the canopy so that it has a lateral camber. The height of the arc in the mid point (a_C) is approximately 75 centimetres and it is a parameter with significant influence on the canopy's aerodynamic. Another typical parameter of a canopy is the aspect ratio (AR) which is defined as the ratio of its length to its breadth. This parameter is a typical parameter on the decision of the appropriate canopy for a windsport. The medium value is approximately equal to 4, which is the value of this canopy [18].

Table 3.1 summarizes the main dimensions of the parafoil.

Table 3.1: Parafoil physical features

Parameter	Value	Units
Canopy		
Length (b_C)	3.300	m
Maximum width (c_C)	0.900	m
Minimum width	0.600	m
Maximum thickness (d_C)	0.070	m
Area (S_C)	3.000	m^2
Mass (M_C)	0.700	Kg
Main lines		
Length	3.200	m
Beam		
Length	0.345	m

3.2.2 Features

This parafoil is the main control component of the platform. It proved to have high glide capability and controllability as well as ability to travel large distances with a payload. It also proved to be able to manoeuvre the drone along a predefined path and to approach the target landing point. The realized tests in [3] demonstrated that the parafoil-drone system is able to achieve a forward velocity of 18km/h and a downward velocity of 10km/h. This gives an approximately velocity ratio of 2:1. In the aerospace field, this parameter is normally named glide slope and it is a very important parameter because it defines the glide performance of an aircraft/parafoil. The glide slope is normally calculated by the lift force to drag force ratio. The lift force is defined to be perpendicular to the vector of freestream velocity ¹, in the vertical plane, and it is derived of the differential pressure between the top and the bottom surfaces of the canopy. The drag force is the resistance force which act on the object when it is moving through the air. The direction of this force is the same direction as the vector of freestream velocity's.

The high controllability of the parafoil is essentially obtained because of its light weight and, more importantly, its small mass-to-volume ratio. Indeed, due to small mass-to-volume ratio, it is easier to change the configuration of the canopy and consequently the flight trajectory. The parafoil has the great advantage to control both horizontal and vertical motion. If the right side of the canopy is pulled down, the parafoil will turn to the right. With same analogy, a left curve can be done by pulling down the left side of the canopy. On the other hand the vertical control can be achieved by pulling down the trailing edge. If the trailing edge is deflected, pulling down both brakes, the drag force increases, decreasing the glide slope.

Managing these two types of control, the parafoil is able to follow a desired path. However this is only possible on certain wind conditions. If the wind velocity exceeds the parafoil airspeed, there is no chance of forward progression. In this way, the path control is very dependent on the wind effect. Mário Martins made several flight tests with wind presence and verified that

¹The freestream velocity is the relative velocity of a vehicle with respect to the wind.

with winds stronger than 10 knots, the system is not able to progress forward in relation to the ground [3]. Moreover, it was verified that the parafoil has a natural behaviour to turn into the wind. This feature is very useful because it prevents the parafoil from collapsing. A motion of an object through the air induces forces called aerodynamics forces. These forces will be described mathematically in the mathematical model design. Here follow the descriptions of a few aerodynamic terms that take part in any standard aerodynamic analysis.

3.2.2.1 Angle of Attack (α)

The angle of attack, usually denoted by α , is the vertical angle between the chord line vector and the vector of the freestream velocity. The angle of attack is positive downwards as seen in Figure 3.3, for an airplane. This angle is a key point in what concerns the stability of the parafoil. On the one hand a parafoil flying with a too high attack angle can collapse because the air can not enter the canopy decreasing its inside air pressure. On the other hand with a too negative attack angle the forces applied in the upper surface of the canopy can make the canopy deflate and therefore it can collapse.

3.2.2.2 Side Slip Angle (β)

When lateral motion is considered, there is another angle, named side slip angle (β), which is the angle between the chord line vector and the vector of the freestream velocity in the horizontal plane. A positive angle is shown in Figure 3.3, for an airplane. The same definition applies to a parafoil, considering the chord line as the x-axis.

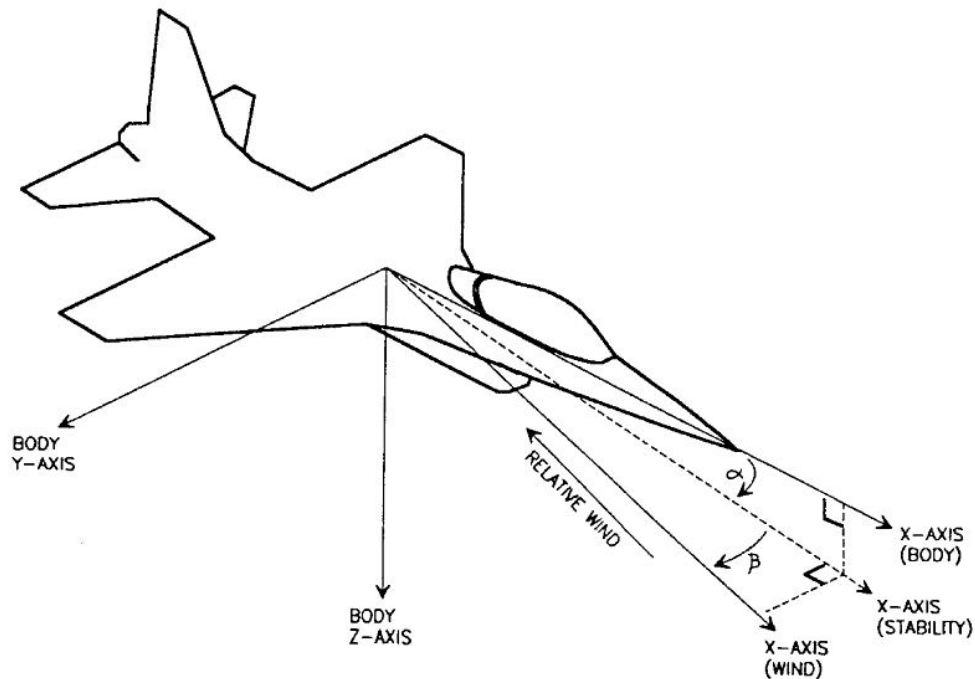


Figure 3.3: Aerodynamic angles (α and β) for an airplane [2]

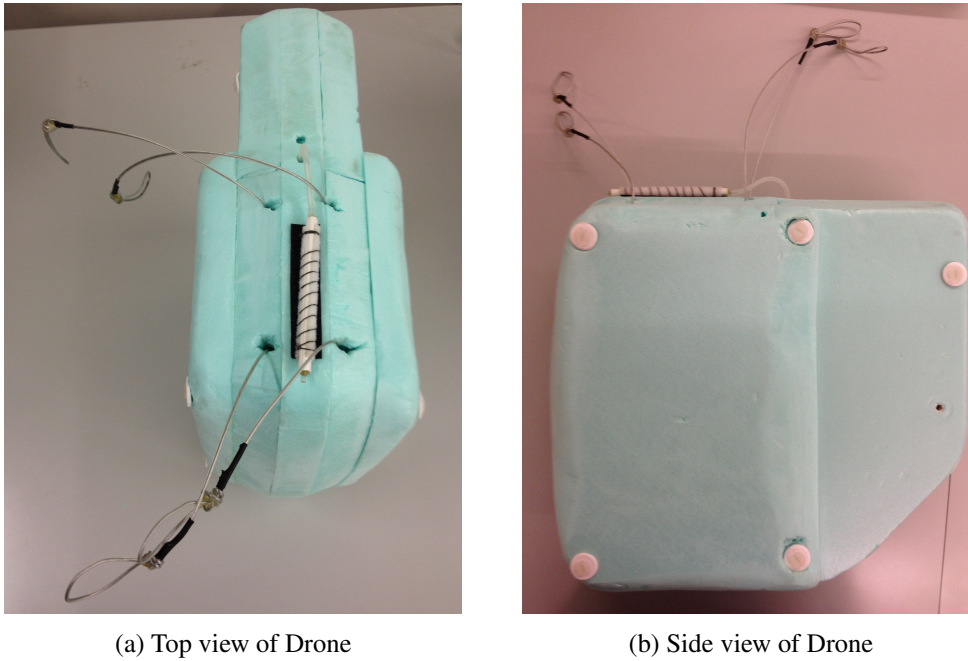


Figure 3.4: View of the Drone

3.3 Drone

The drone was specially developed to perform control and navigation tasks on the platform. This device proved to be able to control a parafoil [3]. Figure 3.4 shows the exterior appearance of the drone, with a top and side view. One can see the four lines which are attached to the main lines and brakes of the parafoil and the pitot tube used to estimate the airspeed.

The shape of the drone was designed in detail to improve the aerodynamic features. Using the Solidworks tool, Mario designed a three-dimensional model of the drone taking into account aspects such as aerodynamics, impact robustness and the position of its center of mass. The tail was designed so that in motion it allows the drone to face the wind just like the parafoil. Moreover, the internal structure was built to support the constraint forces which will be applied in the drone during all the flight phases.

3.3.1 Dimensions

Due to the aerodynamic aspect the drone is larger and taller in the front face. This face has a height of 34 centimetres and a width of 15.5 centimetres while the back face is 20 centimetres high and 9.2 centimetres wide. However, the drone will be approximated to a parallelepiped shape. The center of mass was designed to be approximately 20 centimetres below the upper surface, 17 centimetres from the front part and in the middle of the width. These positions were designed to reduce the effort of the mechanical structure. Two important points are the ones where the main lines leave the control structure. The ratio of the distance between them and the main lines length should be such that the system will be as stable and controllable as possible. For a line length of

3.2 meters, the optimal distance between the points was calculated to be 2 centimetres. Another important feature of the drone is the cover material. Among others, it should dissipate the landing impact and isolate the outside temperature. For that, Styrofoam was used and proved to be an effective cover material. This is a light material so that the drone with all its components weighs about 1770 grams. The most important physical features of the drone are summarized in Table 3.2. The area considered is an estimated area of the front face of the drone which is the face in contact with the air.

Table 3.2: Drone physical features

Parameter	Value	Units
Length (c_D)	0.365	m
Maximum width (b_D)	0.155	m
Minimum width	0.920	m
Maximum height (d_D)	0.340	m
Maximum height	0.200	m
Contact area (S_D)	0.100	m^2
Mass (M_D)	1.770	Kg

3.3.2 Features

The drone has a fundamental role during the descending phase. Besides the transmission tasks with the central ground station, it is responsible for doing important measurements for the control loops. Some of the most important data measured by the drone are its attitude, its position and its linear velocity. To achieve that, the drone uses an AHRS along with a GPS receiver. Through a pitot tube, the drone can also measure the airspeed. However this measurement has one limitation because the pitot tube can only measure in one direction (the drone's direction). A support hardware and software implemented in the drone allow measurements with a rate of 4 measurements per second.

The control actions are performed by servo-actuators. A differential servo-actuator is used to control the main lines with a limited response time. In addition to that, two smaller servo-actuator control the tailing edge of the canopy by the break lines. Each servo-actuator is connected to a microcontroller which along with a CPU processes the control action and drives the servos-actuators using pulse-width modulation.

3.4 Capsule

As it was stated in Chapter 2, the main goal of STRAPLEX is to offer the scientific community the possibility to make experiments into the stratosphere. These experiments are carried inside of the main capsule. In this thesis it will be called capsule indistinctly. This special device was carefully built to assemble a set of electronic equipment that supports the experiments. Figure 3.5 shows the outside view of the capsule.



Figure 3.5: View of the Capsule

The capsule has a hexagonal shape with a semi-hemispherical top surface and an antenna installed on the top, as seen in previous figure. The reason for this top surface, which is covered by Styrofoam, is to ensure the buoyancy of the capsule in water bodies. Furthermore, the capsule has a small window in the front face, where it is installed the video camera.

3.4.1 Dimensions

In this work, the semi-hemispherical top surface and the antenna will be neglected, because its contribution to the capsule motion can be considered negligible. Each face of the hexagon is 20.5 centimetres long and 31.5 centimetres high. The contact area with the air can be approximated to 0.5 square meters. Such as the drone, this capsule is also covered by Styrofoam, so the total weight of the capsule is about 5 kg. The connection between the capsule and the drone is accomplished by three strong lines attached in three eye screws screwed in three specific points. The lines' length can vary but it's usually 3 meters to maximize the stability of the relative motion between these two objects. Table 3.3 presents a summary of the capsule dimensions.

3.4.2 Features

The internal structure of the capsule can be divided in four layers stacked with different functions. The top layer houses a set of antennas which provide communication with the base station and the other objects of the system. Below this layer are the experiments which will be performed in the stratosphere. The third layer is the set of electronic equipment that supports the experiments. It consists of communication devices (receivers and transceivers), modems, data acquisition module

Table 3.3: Capsule physical features

Parameter	Value	Units
Wide	0.410	m
Length	0.410	m
Height	0.315	m
Contact area (S_{CAP})	0.500	m^2
Mass (M_{CAP})	5.000	Kg

and a processing unit. Besides these devices, a video camera is installed on the face which has a window. This camera films the trajectory the capsule makes during the flight phases. The video can then be sent to the ground station, which will be a useful tool for the control manual mode. The communication between the several components of the capsule and with the ground station is based on a strong and redundant communication protocol. This minimizes the probability of a communication failure. The last layer houses the power unit to feed all electronic equipment.

3.5 Transponder

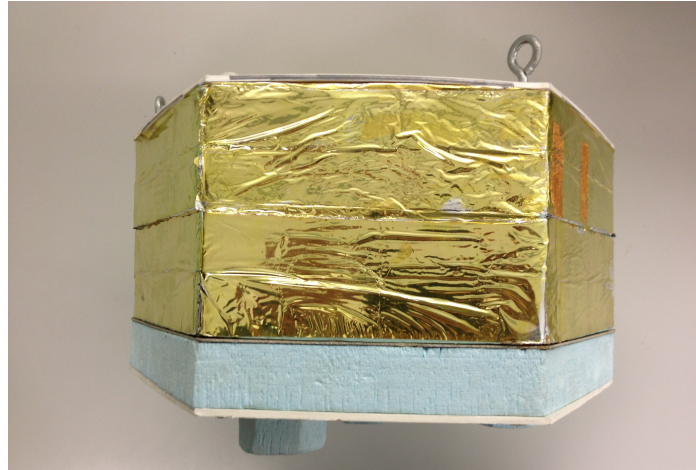


Figure 3.6: View of the Transponder

The last component of the system to analyse is the transponder. In the aerospace industry, it is a special electronic device incorporated in an aircraft that identifies the aircraft for the air traffic controller. Together with an altitude reporting equipment, they are significant elements for the safe operation in the national airspace system [19]. They enhance the air traffic control and collision avoidance system. With this same objective, the platform incorporates one transponder. It is installed into a small capsule (Figure 3.6) which has a structure similar to the main capsule. The transponder weighs 1.735 kilograms and has an approximated contact area of 0.1 square meters. Table 3.4 shows the physical features of the transponder. Such as the connection between the capsule and drone, also the transponder is connected to the capsule by three strong lines attached in three eye screws screwed in three specific points.

Table 3.4: Transponder physical features

Parameter	Value	Units
Wide	0.320	m
Length	0.320	m
Height	0.158	m
Contact area (S_T)	0.100	m^2
Mass (M_T)	1.735	Kg

3.6 Control Input

The majority of scientific research about parafoil control analyses the parafoil-payload system with one type of control, the brakes. They are two lines connected to two flaps located at the tail of the canopy, one on each side. These two lines allow an asymmetric and a symmetric break. When a symmetric break is applied, which corresponds to the two flaps deflected of the same angle, the lift and drag force increase, while the glide slope decreases. This control is very useful to control the glide slope of the system. If one flap is more deflected than other there is an asymmetric break which make the system turn to the side where the flap is more deflected. However, Mário Martins verified that when he applied an asymmetric break action, the parafoil turn rightly accomplish by a glide slope decreasing. This result can be unwanted and therefore a new control was designed using the two main lines. They allow lateral control with low influence in the glide slope.

3.6.1 Main Lines Control

The idea of the main lines control is to use the payload/drone weight shift mechanism to control the parafoil orientation. This method is widely used for human skydivers in windsports. For that a differential servo-actuator can extend or shorten the main lines. The difference length of the main lines induces a lateral displacement of the drone with respect to the canopy. In this way, the weight of drone will be unevenly distributed in the main lines. Moreover as the lines should be always stretched, the canopy will deflate in order to equally distribute the weight of the drone along the main lines again. An example of a right displacement is shown in Figure 3.7, where it was neglected the spacing between the main lines in the drone connection point. It should be noted that it is only presented the lines (grey lines) which connect the wooden beam (brown beam) to the drone.

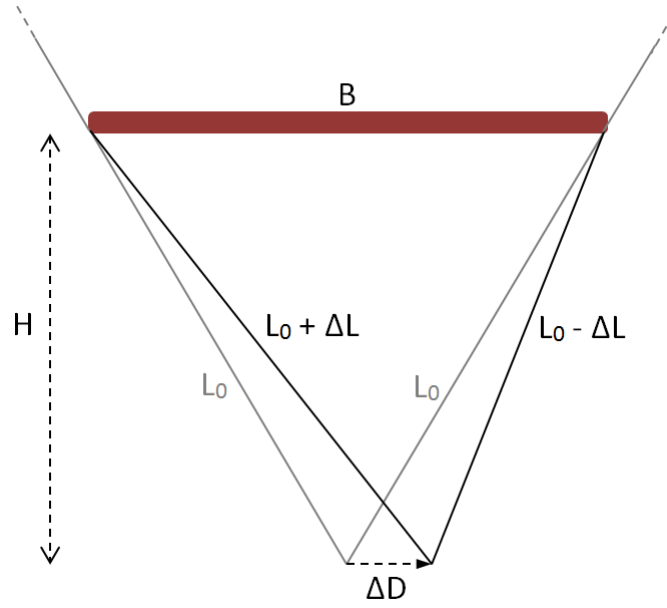


Figure 3.7: Illustration of the main lines control method

B is the beam length and L_0 is the natural line length. It is important to note that this figure is an illustrative exhibition without the right objects dimensions. When a line displacement ΔL is performed (black lines), it induces a lateral displacement ΔD (dashed line) on the drone as seen in Figure 3.7. There is also a vertical displacement on the drone but it is smaller than the lateral displacement and its effect is neglected. Using trigonometric equations, it is possible to write ΔD in function of ΔL , L_0 and B . In effect, taking H as the least distance between drone and the beam:

$$\begin{cases} \left(\frac{B}{2} - \Delta D\right)^2 + H^2 = (L_0 - \Delta L)^2 \\ \left(\frac{B}{2} + \Delta D\right)^2 + H^2 = (L_0 + \Delta L)^2 \end{cases} \quad (3.1)$$

By eliminating the term H , it becomes:

$$\left(\frac{B}{2} + \Delta D\right)^2 - \left(\frac{B}{2} - \Delta D\right)^2 = (L_0 + \Delta L)^2 - (L_0 - \Delta L)^2 \quad (3.2)$$

Solving for ΔD ,

$$\Delta D = \frac{L_0}{\frac{B}{2}} \Delta L \quad (3.3)$$

Being L_0 and B constant parameters, the drone lateral displacement, ΔD , is directly proportional to the main lines displacement, ΔL .

3.7 Flight Stages

As stated above, the descending flight can be divided in two major phases: uncontrollable and controllable phase. The first one starts after the balloon is released and the platform behaves in a very oscillatory manner. In order to stabilize that oscillation, a round parachute is used. This type of parachute is widely used in the aviation industry because it can be easily opened. However this parachute isn't controllable and therefore there isn't any way to control the trajectory. Since the air mass becomes increasingly stable as the platform comes closer to the ground, the downward velocity decreases until reaches a stable value. At this moment, the round parachute is released and the parafoil is inflated. This parachute has the great advantage of controlling the trajectory of the platform, which probably was affected by the wind effect, in the other flight phases. Applying a guidance strategy, it is possible, in certain wind conditions, to guide the platform towards the landing target.

The guidance strategy can be divided in four distinct phases (Figure 3.8). In the first one, the control algorithm takes the system near to the land area following a desired direction. In the second phase, the platform describes a spiral motion in a specific area taking into account the wind effect. The loop number of the spiral should be that the platform reaches a specific altitude (H_3) for the final approach. The third phase executes the final turn to approach the glide slope towards the landing point into the wind. The flight ends with the landing, controlling the glide slope so that the platform lands on the desired point. Near the ground, at H_4 altitude, the brakes are pulled down to the maximum to minimize the impact velocity.

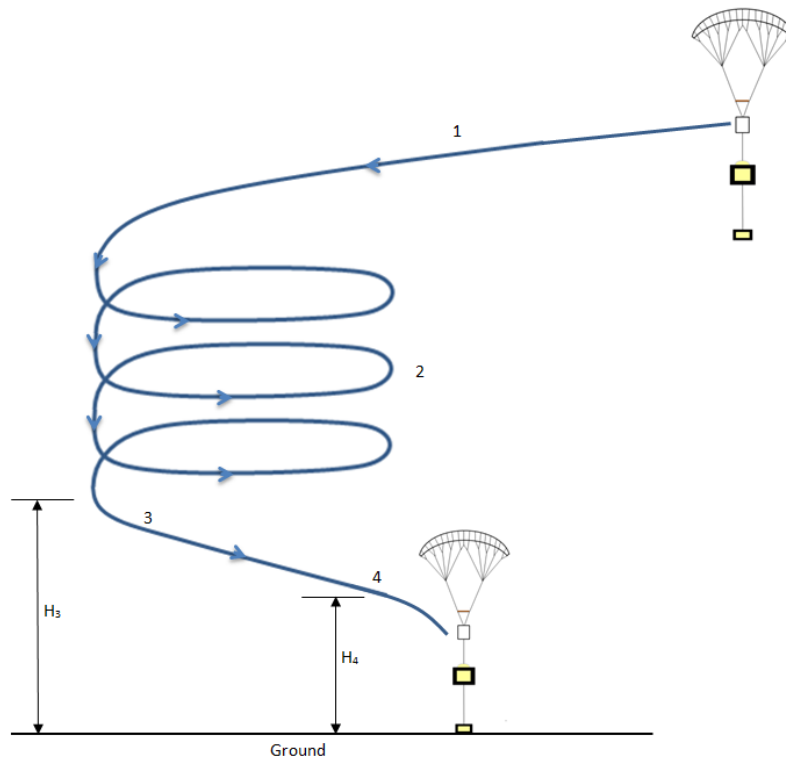


Figure 3.8: Profile of a controllable descending flight

Chapter 4

Mathematical Model

The platform motion in the descending phase will be modelled in a mathematical model. This model is non linear and characterizes the platform dynamic behaviour. Section 4.1 describes in general the dynamic model of the system and presents some assumptions which will be done in the model design. It will also be presented using vector notation as well as the several frames and the respective rotation matrices to switch among frames. The most important points and position vectors are presented in Section 4.2. Section 4.3 analyses the kinematics of each component of the system. After a dynamic analysis of the entire system, made in Section 4.4, the equations of motion are described in Section 4.5. At the end, the simulation results for several situations are shown in Section 4.6.

4.1 Preliminaries

In the motion analysis of an aerospace vehicle, there are some aspects that should be considered. Indeed, in the modern aerospace industry, an accurate simulation of high speed flights over large areas of the Earth surface requires an accurate model of the Earth shape, rotation and gravity as it is referred by Stevens and Lewis in [2]. Another important parameter is the air density which significantly varies with the altitude. Considering a parafoil flight covering a small region of the Earth it is valid to neglect the Earth rotation and to assume a flat Earth. Furthermore, in this work, the variation of gravity with the altitude will be neglected, but, in contrast, the variation of air density with the altitude will be taken into account, because the platform can reach high altitude.

A schematic of the platform composed by four components (parafoil, drone, capsule and transponder) and their respective connection lines along with the control lines is shown in Figure 4.1. It is shown an illustrative representation of the front and side views of the system. In the model design, the system will be treated as a multi-rigid bodies with constraint forces of the connection lines. In this way, the equations of motion will be derived through the kinematic and dynamic analysis of each body, taking into account the constraint forces and the existing relative motion between the bodies.

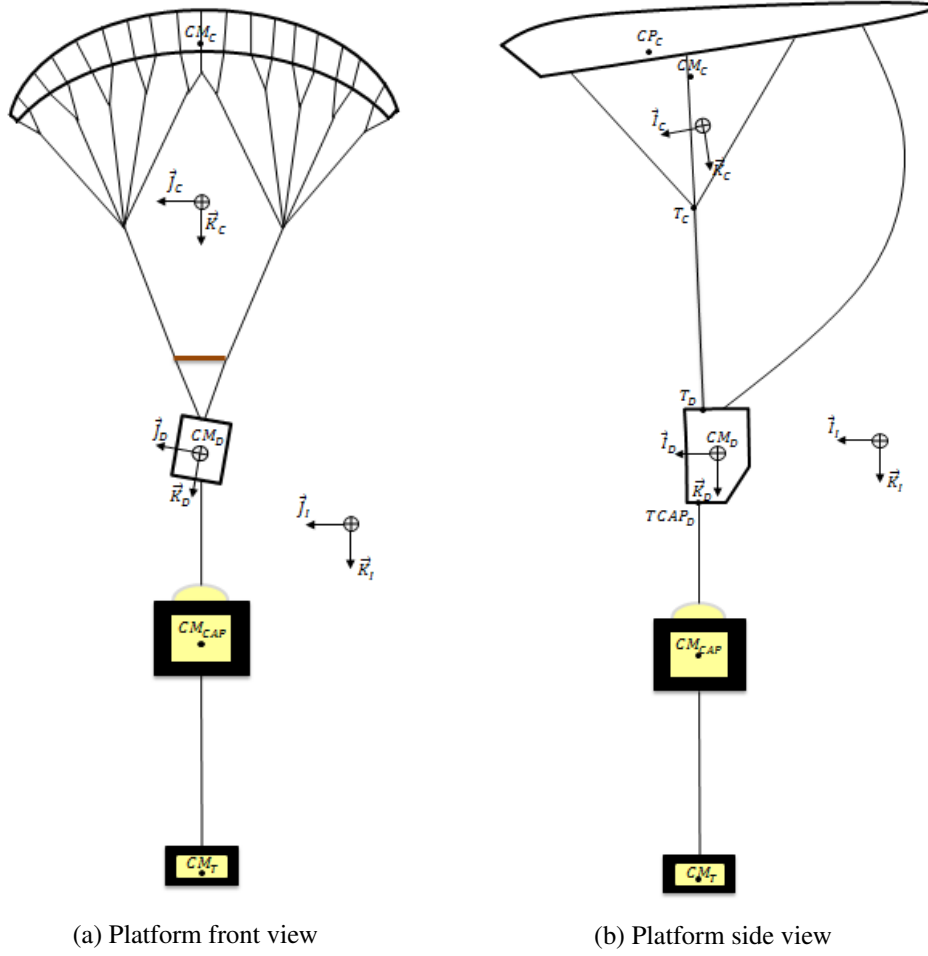


Figure 4.1: Platform schematic

The equations of motion will be organized as a set of first-order differential equations. Solving the equations for the derivatives, symbolized by a dot, the system motion can be symbolically presented as:

$$\dot{X}(t) = f(X(t), U(t)) \quad (4.1)$$

where the *state vector* X is an $(n \times 1)$ column array of the n state variables, the *control vector* U is an $(m \times 1)$ column array of the m control input variables, and f is an array of nonlinear functions.

The *state vector* X contains all the values of the variables that describe the system while the *control vector* U is composed by the control inputs which will be used to perform the control action. In the mechanical field, the state variables for a motion description of a body are normally defined as positions, orientation angles, linear and angular velocities of the body. Where the components of the position and velocity vector are defined respectively as x and u pointing forward, y and v pointing right and z and w pointing down. The three orientation angles are called: roll, pitch and yaw angle while its rates are denoted by: p , q , r respectively. Usually, the orientation angles are represented by the Euler angles that describe a three dimensional orientation of a body and are

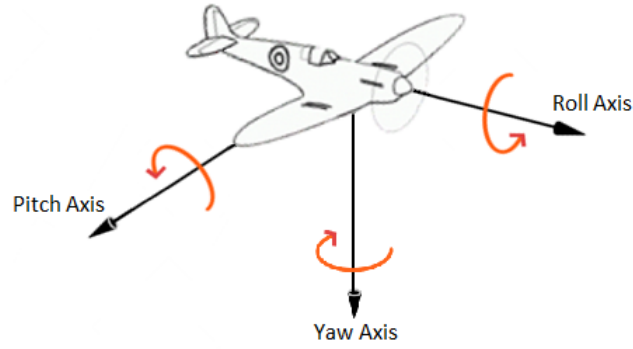


Figure 4.2: Roll, Pitch and Yaw directions for an aircraft

typically denoted as: ϕ , θ and ψ . In the case of an aircraft, defining the longitudinal axis of a plane as the straight line drawn from the tail to the nose, the pitch angle (θ) is the angle between the longitudinal axis and the ground plane. A positive angle means that the nose is higher than the tail. The roll angle (ϕ) means a rotation of the aircraft around the longitudinal axis, where the clockwise orientation is the positive orientation. The horizontal rotation of the aircraft is quantified by the yaw angle (ψ), where a clockwise rotation means a positive angle. The positive directions of these angles can be interpreted as a right-handed rotation around the perpendicular axis of the plane, as shown in Figure 4.2. The same interpretation can be made for a parafoil.

4.1.1 Vector Notation and Frames

The system motion will be described by means of three dimensional vectors which will be expressed in several frames, so it is important to define a clear vector notation such as a description of all the frames which will be used during the model design. Using a similar notation to the one applied in [2], it will be used as follows:

- A right subscript will be used to designate the body under analysis, when only one point is used (C - canopy, D - drone, CAP - capsule and T - transponder). If two points are used, they are represented between “/”, and it means that the left point is referred to another point for a position vector or to a frame for a velocity or acceleration vector. A “/” in a subscript means “with respect to”.
- A left superscript will specify the frame where the vector is expressed.
- A right superscript on a vector will specify the frame in which a derivative is taken .
- The vectorial product between two vectors will be represented by a times “ $\times$ ”.

An example of the notation is:

${}^A\dot{\vec{v}}_{C/I}^B \equiv$ derivative taken in the frame B of the linear velocity vector $\vec{v}$ of the point C with respect to the inertial frame I . The resulting vector is expressed in the frame A .

The frames that will be used on the development of the equations of motion are presented as follows:

- Inertial frame (I) - is the reference frame and it will be considered as the Earth frame using the north, east and down convention.
- Canopy frame (C) - is fixed at the canopy center of mass with its orientation defined by the orientation angles of the canopy. Its axis, $\vec{I}_C$, $\vec{J}_C$ and $\vec{K}_C$, are shown in Figure 4.1 .
- Line frame (L) - is fixed at the canopy center of mass but with the orientation of the set's two main lines.
- Drone frame (D) - is fixed at the drone center of mass with the orientation given by the three Euler angles of the drone. Its axis $\vec{I}_D$, $\vec{J}_D$ and $\vec{K}_D$ can be viewed in Figure 4.1.
- Capsule frame (CAP) - is fixed at the capsule center of mass defined by the orientation angles of the capsule with respect to the drone frame.
- Transponder frame (T) - is fixed at the transponder center of mass defined by the orientation angles of the transponder with respect to the capsule frame.
- Aerodynamic frame (A) - is fixed at the canopy pressure center with its orientation defined by the freestream velocity vector. Its direction is described by the aerodynamic angles α and β , presented in Section 3.2.

4.1.2 Rotation Matrices

A *rotation matrix*, also called a *direction cosine matrix*, is a matrix that transforms the components of a vector expressed in a frame into another frame. For example the rotation matrix T_{A-B} allows the transformation of the components of a vector $\vec{u}$ from frame A to frame B .

$${}^B\vec{u} = T_{A-B} {}^A\vec{u} \quad (4.2)$$

where,

$$T_{A-B} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.3)$$

The frame B is the frame A rotated about the z-axis by a positive angle ψ . Some properties of this matrix are presented in [2] where one can highlight the following:

- Rotation matrices are orthogonal matrices;
- The determinant of a rotation matrix is unity;

- (iii) Successive rotations can be described by the product of the individual rotation matrices.

From the properties of the rotation matrix, one can easily find that:

$$T_{B-A} = T_{A-B}^{-1} = T_{A-B}^T \quad (4.4)$$

The non-inertial frames (C , L , D , CAP , T and A) described above can then be obtained from other frame by a three dimensional rotation matrix, where it will be built up as a sequence of plane rotations, as explained in [2]:

- (1) Right-handed rotation about the z-axis (positive ψ)
- (2) Right-handed rotation about the new y-axis (positive θ)
- (3) Right-handed rotation about the new x-axis (positive ϕ)

Where each plane of rotations can be described by the following rotation matrices:

$$T_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \quad (4.5)$$

$$T_\theta = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \quad (4.6)$$

$$T_\psi = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.7)$$

Following this sequence of plane rotations, the orientation of the canopy frame with respect to the inertial frame can be obtained from the rotation matrix T_{I-C} given by:

$$T_{I-C} = T_{\phi_{C/I}} T_{\theta_{C/I}} T_{\psi_{C/I}}$$

$$T_{I-C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_{C/I}) & \sin(\phi_{C/I}) \\ 0 & -\sin(\phi_{C/I}) & \cos(\phi_{C/I}) \end{bmatrix} \begin{bmatrix} \cos(\theta_{C/I}) & 0 & -\sin(\theta_{C/I}) \\ 0 & 1 & 0 \\ \sin(\theta_{C/I}) & 0 & \cos(\theta_{C/I}) \end{bmatrix} \begin{bmatrix} \cos(\psi_{C/I}) & \sin(\psi_{C/I}) & 0 \\ -\sin(\psi_{C/I}) & \cos(\psi_{C/I}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using the common shorthand notation for trigonometric functions ($\sin(\alpha) \equiv s_\alpha$, $\cos(\alpha) \equiv c_\alpha$ and $\tan(\alpha) \equiv t_\alpha$). The transformation matrix T_{I-C} becomes:

$$T_{I-C} = \begin{bmatrix} c_{\theta_{C/I}} c_{\psi_{C/I}} & c_{\theta_{C/I}} s_{\psi_{C/I}} & -s_{\theta_{C/I}} \\ s_{\phi_{C/I}} s_{\theta_{C/I}} c_{\psi_{C/I}} - c_{\phi_{C/I}} s_{\psi_{C/I}} & s_{\phi_{C/I}} s_{\theta_{C/I}} s_{\psi_{C/I}} + c_{\phi_{C/I}} c_{\psi_{C/I}} & s_{\phi_{C/I}} c_{\theta_{C/I}} \\ c_{\phi_{C/I}} s_{\theta_{C/I}} c_{\psi_{C/I}} + s_{\phi_{C/I}} s_{\psi_{C/I}} & c_{\phi_{C/I}} s_{\theta_{C/I}} s_{\psi_{C/I}} - s_{\phi_{C/I}} c_{\psi_{C/I}} & c_{\phi_{C/I}} c_{\theta_{C/I}} \end{bmatrix} \quad (4.8)$$

In the same way, the transformation matrices from the canopy to the line frame, from the line to the drone frame, from the drone to the capsule frame, from the capsule to the transponder frame and from the canopy to the aerodynamic frame are defined in equations (4.9), (4.10), (4.11), (4.12) and (4.13), respectively:

$$T_{C-L} = T_{\theta_{L/C}} \\ T_{C-L} = \begin{bmatrix} c_{\theta_{L/C}} & 0 & -s_{\theta_{L/C}} \\ 0 & 1 & 0 \\ s_{\theta_{L/C}} & 0 & c_{\theta_{L/C}} \end{bmatrix} \quad (4.9)$$

$$T_{L-D} = T_{\phi_{D/L}} T_{\theta_{D/L}} T_{\psi_{D/L}} \\ T_{L-D} = \begin{bmatrix} c_{\theta_{D/L}} c_{\psi_{D/L}} & c_{\theta_{D/L}} s_{\psi_{D/L}} & -s_{\theta_{D/L}} \\ s_{\phi_{D/L}} s_{\theta_{D/L}} c_{\psi_{D/L}} - c_{\phi_{D/L}} s_{\psi_{D/L}} & s_{\phi_{D/L}} s_{\theta_{D/L}} s_{\psi_{D/L}} + c_{\phi_{D/L}} c_{\psi_{D/L}} & s_{\phi_{D/L}} c_{\theta_{D/L}} \\ c_{\phi_{D/L}} s_{\theta_{D/L}} c_{\psi_{D/L}} + s_{\phi_{D/L}} s_{\psi_{D/L}} & c_{\phi_{D/L}} s_{\theta_{D/L}} s_{\psi_{D/L}} - s_{\phi_{D/L}} c_{\psi_{D/L}} & c_{\phi_{D/L}} c_{\theta_{D/L}} \end{bmatrix} \quad (4.10)$$

$$T_{D-CAP} = T_{\phi_{CAP/D}} T_{\theta_{CAP/D}} \\ T_{D-CAP} = \begin{bmatrix} c_{\theta_{CAP/D}} & 0 & -s_{\theta_{CAP/D}} \\ s_{\phi_{CAP/D}} s_{\theta_{CAP/D}} & c_{\phi_{CAP/D}} & s_{\phi_{CAP/D}} c_{\theta_{CAP/D}} \\ c_{\phi_{CAP/D}} s_{\theta_{CAP/D}} & -s_{\phi_{CAP/D}} & c_{\phi_{CAP/D}} c_{\theta_{CAP/D}} \end{bmatrix} \quad (4.11)$$

$$T_{CAP-T} = T_{\phi_{T/CAP}} T_{\theta_{T/CAP}} \\ T_{CAP-T} = \begin{bmatrix} c_{\theta_{T/CAP}} & 0 & -s_{\theta_{T/CAP}} \\ s_{\phi_{T/CAP}} s_{\theta_{T/CAP}} & c_{\phi_{T/CAP}} & s_{\phi_{T/CAP}} c_{\theta_{T/CAP}} \\ c_{\phi_{T/CAP}} s_{\theta_{T/CAP}} & -s_{\phi_{T/CAP}} & c_{\phi_{T/CAP}} c_{\theta_{T/CAP}} \end{bmatrix} \quad (4.12)$$

$$\begin{aligned}
T_{C-A} &= T_\alpha T_\beta \\
T_{C-A} &= \begin{bmatrix} c\alpha & 0 & s\alpha \\ 0 & 1 & 0 \\ -s\alpha & 0 & c\alpha \end{bmatrix} \begin{bmatrix} c\beta & s\beta & 0 \\ -s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
T_{C-A} &= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta & s\alpha \\ -s\beta & c\beta & 0 \\ -s\alpha c\beta & -s\alpha s\beta & c\alpha \end{bmatrix} \tag{4.13}
\end{aligned}$$

4.1.3 State Vector

The number of state variables (n) is usually related to the degrees of freedom (DOF) of the system. The system, which will be analysed here, will be modelled with 14 DOF. Three DOF describe the three inertial position components of the canopy center of mass, three DOF are used to describe the three Euler angles of the canopy and one DOF for the pitch angle of the set's main lines with respect to the canopy frame. There are also included three Euler angles of the drone with respect to the line frame, the roll and pitch angles of the capsule with respect to the drone frame and also the roll and pitch angles of the transponder with respect to the capsule frame. Considering these variables along with their derivatives and also three state variables for the three servo-actuators of the drone, the *state vector* X of this system is composed by 31 state variables:

$$\begin{aligned}
X = [{}^I R_{CM_C/I}, {}^I \Theta_{C/I}, {}^C V_{CM_C/I}, {}^C \omega_{C/I}, {}^L \theta_{L/C}, {}^L q_{L/C}, {}^D \Theta_{D/L}, {}^D \omega_{D/L}, {}^{CAP} \Theta_{CAP/D}, {}^{CAP} \omega_{CAP/D}, \\
{}^T \Theta_{T/CAP}, {}^T \omega_{T/CAP}, \Delta Control]^T \tag{4.14}
\end{aligned}$$

where:

$${}^I R_{CM_C/I} = [x_{CM_C/I}, y_{CM_C/I}, z_{CM_C/I}]^T$$

$${}^I \Theta_{C/I} = [\phi_{C/I}, \theta_{C/I}, \psi_{C/I}]^T$$

$${}^C V_{CM_C/I} = [u_{CM_C/I}, v_{CM_C/I}, w_{CM_C/I}]^T$$

$${}^C \omega_{C/I} = [p_{C/I}, q_{C/I}, r_{C/I}]^T$$

$${}^D \Theta_{D/L} = [\phi_{D/L}, \theta_{D/L}, \psi_{D/L}]^T$$

$${}^D \omega_{D/L} = [p_{D/L}, q_{D/L}, r_{D/L}]^T$$

$${}^{CAP} \Theta_{CAP/D} = [\phi_{CAP/D}, \theta_{CAP/D}]^T$$

$${}^{CAP} \omega_{CAP/D} = [p_{CAP/D}, q_{CAP/D}]^T$$

$${}^T\Theta_{T/CAP} = [\phi_{T/CAP}, \theta_{T/CAP}]^T$$

$${}^T\omega_{T/CAP} = [p_{T/CAP}, q_{T/CAP}]^T$$

$$\Delta Control = [\Delta L, \Delta ServoR, \Delta ServoL]^T$$

Relative to the canopy body, the state vector ${}^I R_{CMC/I}$ describes the three inertial position components of the canopy center of mass, ${}^I \Theta_{C/I}$ defines the orientation of the canopy frame with respect to the inertial frame, ${}^C V_{CMC/I}$ describes the three linear velocity components of the canopy center of mass and ${}^C \omega_{C/I}$ describes the three angular velocity components of the canopy with respect to the inertial frame. The state variables $\theta_{L/C}$ and $q_{L/C}$ are the pitch angle and its rate of the line frame with respect to the canopy frame. To describe the state of the drone body, it is defined the state vector ${}^D \Theta_{D/L}$ which defines the orientation of the drone frame with respect to the line frame and ${}^D \omega_{D/L}$ which is composed by the three angular velocity components of the drone with respect to the line frame. The state vectors ${}^{CAP} \Theta_{CAP/D}$ and ${}^{CAP} \omega_{CAP/D}$ describe the two relative Euler angles of the capsule frame with respect to the drone frame and the capsule relative angular velocity with respect to the drone frame, respectively. In the same way, the state vectors ${}^T \Theta_{T/CAP}$ and ${}^T \omega_{T/CAP}$ describe the two relative Euler angles of the transponder frame with respect to the capsule frame and the transponder relative angular velocity with respect to the capsule frame, respectively.

The state vector $\Delta Control$ regards the state of the three servo-actuators of the drone, where each state variable ΔL , $\Delta ServoR$ and $\Delta ServoL$ is defined as the position of the respective servo-actuator. These positions are controlled by the control system which has finite response time, in other words, the velocity of the servo-actuators is limited. Defining U_L , U_{SR} and U_{SL} as the control input for these servo-actuators, the equations (4.15), (4.16) and (4.17) describe the dynamic of these state variables.

$$\dot{\Delta L} = K_L (U_L - \Delta L) \quad (4.15)$$

$$\dot{\Delta ServoR} = K_{SR} (U_{SR} - \Delta ServoR) \quad (4.16)$$

$$\dot{\Delta ServoL} = K_{SL} (U_{SL} - \Delta ServoL) \quad (4.17)$$

where K_L , K_{SR} and K_{SL} are constants which define the velocity of the servo-actuator. The larger the value is, the faster the system will respond to a control input. One can easily note that in the frequency domain, each equation will originate a real pole situated in the open left-half plane of the $\mathbb{C}$. Moreover, considering these state variables, there is the great advantage of allowing the user to track and quickly analyse the control input value.

4.2 Points and Position Vectors

The most important points and position vectors that will be used in the model will be defined using the notation described in Section 4.1. Some points presented can be viewed in schematic 4.1.

4.2.1 Points

The points that will be used to describe the system are:

- CM_C - canopy center of mass;
- CP_C - canopy pressure center;
- $T1_C$ - tension point on the canopy of the right main line;
- $T2_C$ - tension point on the canopy of the left main line;
- CM_D - drone center of mass;
- CP_D - drone pressure center;
- $T1_D$ - tension point on the drone of the right main line;
- $T2_D$ - tension point on the drone of the left main line;
- $TCAP_D$ - tension point on the drone of the line which connects the capsule to the drone;
- CM_{CAP} - capsule center of mass (it is also the tension point on the capsule);
- CM_T - transponder center of mass (it is also the tension point on the transponder);

4.2.2 Position Vectors

Considering the points presented above, it will be defined the following position vectors expressed in the specific frame.

- ${}^C\vec{R}_{CP_C/CM_C}$ - position vector of the point CP_C with respect to the point CM_C expressed in the canopy frame;
- ${}^C\vec{R}_{T1_C/CM_C}$ - position vector of the point $T1_C$ with respect to the point CM_C expressed in the canopy frame;
- ${}^C\vec{R}_{T2_C/CM_C}$ - position vector of the point $T2_C$ with respect to the point CM_C expressed in the canopy frame;
- ${}^L\vec{R}_{T1_D/T1_C}$ - position vector of the point $T1_D$ with respect to the point $T1_C$ expressed in the line frame;

- ${}^L\vec{R}_{T2_D/T2_C}$ - position vector of the point $T2_D$ with respect to the point $T2_C$ expressed in the line frame;
- ${}^D\vec{R}_{CP_D/CM_D}$ - position vector of the point CP_D with respect to the point CM_D expressed in the drone frame;
- ${}^D\vec{R}_{T1_D/CM_D}$ - position vector of the point $T1_D$ with respect to the point CM_D expressed in the drone frame;
- ${}^D\vec{R}_{T2_D/CM_D}$ - position vector of the point $T2_D$ with respect to the point CM_D expressed in the drone frame;
- ${}^D\vec{R}_{TCAP_D/CM_D}$ - position vector of the point $TCAP_D$ with respect to the point CM_D expressed in the drone frame;
- ${}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D}$ - position vector of the point CM_{CAP} with respect to the point $TCAP_D$ expressed in the capsule frame;
- ${}^T\vec{R}_{CM_T/CM_{CAP}}$ - position vector of the point CM_T with respect to the point CM_{CAP} expressed in the transponder frame;

In addition to these position vectors, it will be used other that can be defined in function of the vectors presented above.

4.3 System Kinematics

In this section, it will be analysed the kinematics of each body of the platform. This analysis will be divided in two categories: rotation and position kinematics. In the first one, the Euler angles, the angular velocity and the angular velocity derivative of each body will be evaluated. On the other hand, in the position kinematics, it will be analysed the position vector, the linear velocity and the linear acceleration of each body. All kinematic expressions will be expressed in the appropriate frame and when it is necessary, the rotation matrices, presented in Section 4.1, will be used under straight parentheses.

4.3.1 Rotation Kinematics

The analysis of the rotation kinematics will be individually made for each body of the platform. It will be defined the Euler angles expression as well as the angular velocity and its derivative expressions of each body. The analysis will start with the canopy, followed by drone and capsule, finishing with the transponder. The expressions will always be determined with respect to the inertial frame and expressed in the inertial frame except for the derivative of the angular velocity which will be expressed with respect to the frame of the body under analysis, for reasons which will be later understood.

4.3.1.1 Canopy

- Euler Angles

The Euler angles of the canopy are defined by the state vector ${}^I\Theta_{C/I} = [\phi_{C/I}; \theta_{C/I}; \psi_{C/I}]$ which described the orientation of the canopy with respect to the inertial frame, expressed in the inertial frame. Rotating the inertial frame to the canopy frame through the rotational matrix T_{I-C} , the Euler angles vector expressed in the canopy frame can be written as:

$${}^C\vec{\Theta}_{C/I} = [T_{I-C}]^I\Theta_{C/I} \quad (4.18)$$

- Angular Velocity

The angular velocity expression is defined as the derivative of the Euler angles vector. However it is desired to express the angular velocity in the canopy frame and the state vector ${}^I\Theta_{C/I}$ is expressed in the inertial frame. So to obtain the canopy angular velocity in function of the derivative of ${}^I\Theta_{C/I}$, it is necessary to multiply the derivative of ${}^I\Theta_{C/I}$ by a specific sequence of the rotational matrices, defined in equations (4.5), (4.6) and (4.7). In the aerospace field, the usual sequence is:

$${}^C\vec{\omega}_{C/I} = \begin{bmatrix} p_{C/I} \\ q_{C/I} \\ r_{C/I} \end{bmatrix} = \begin{bmatrix} \dot{\phi}_{C/I} \\ 0 \\ 0 \end{bmatrix} + T_{\phi_C} \begin{bmatrix} 0 \\ \dot{\theta}_{C/I} \\ 0 \end{bmatrix} + T_{\phi_C} T_{\theta_C} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_{C/I} \end{bmatrix} [2] \quad (4.19)$$

Replacing the rotation matrices T_{ϕ_C} and T_{θ_C} by equations (4.5) and (4.6), the previous equation becomes:

$$\begin{bmatrix} p_{C/I} \\ q_{C/I} \\ r_{C/I} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta_{C/I}) \\ 0 & \cos(\phi_{C/I}) & \sin(\phi_{C/I})\cos(\theta_{C/I}) \\ 0 & -\sin(\phi_{C/I}) & \cos(\phi_{C/I})\cos(\theta_{C/I}) \end{bmatrix} \begin{bmatrix} \dot{\phi}_{C/I} \\ \dot{\theta}_{C/I} \\ \dot{\psi}_{C/I} \end{bmatrix} \quad (4.20)$$

- Derivative of the Angular Velocity

The derivative of the angular velocity for the canopy body, expressed in the canopy frame, is defined in equation (4.21).

$${}^C\dot{\vec{\omega}}_{C/I} = \frac{d}{dt} ({}^C\vec{\omega}_{C/I})^C \quad (4.21)$$

4.3.1.2 Drone

- Euler Angles

The state vector $\Theta_{D/L} = [\phi_{D/L}; \theta_{D/L}; \psi_{D/L}]$ defines the orientation of the drone frame with respect to the line frame. Additionally the orientation of the line frame with respect to the canopy

frame is given by the pitch angle $\theta_{L/C}$. In this way, the orientation of the drone with respect to the inertial frame can be defined in function of $\Theta_{D/L}$, $\theta_{L/C}$ and the canopy Euler angles ${}^C\vec{\Theta}_{C/I}$. The resultant expression expressed in the drone frame is equal to:

$${}^D\vec{\Theta}_{D/I} = [T_{L-D}][T_{C-L}]{}^C\vec{\Theta}_{C/I} + [T_{L-D}]{}^L\vec{\Theta}_{L/C} + {}^D\Theta_{D/L} \quad (4.22)$$

where ${}^L\vec{\Theta}_{L/C}$ is equal to:

$${}^L\vec{\Theta}_{L/C} = \begin{bmatrix} 0 \\ \theta_{L/C} \\ 0 \end{bmatrix} \quad (4.23)$$

- Angular Velocity

The angular velocity of the drone body, expressed in the drone frame, can be obtained deriving the equation (4.22) in the inertial frame.

$${}^D\vec{\omega}_{D/I} = \frac{d}{dt} \left({}^D\vec{\Theta}_{D/I} \right)^I$$

$${}^D\vec{\omega}_{D/I} = \frac{d}{dt} \left([T_{L-D}][T_{C-L}]{}^C\vec{\Theta}_{C/I} \right)^I + \frac{d}{dt} \left([T_{L-D}]{}^L\vec{\Theta}_{L/C} \right)^I + \frac{d}{dt} \left({}^D\vec{\Theta}_{D/L} \right)^I$$

Applying the equation (A.15) of Appendix, the previous equation becomes:

$$\begin{aligned} {}^D\vec{\omega}_{D/I} &= [T_{L-D}][T_{C-L}] \frac{d}{dt} \left({}^C\vec{\Theta}_{C/I} \right)^I + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{\Theta}_{L/C} \right)^I + \frac{d}{dt} \left({}^D\vec{\Theta}_{D/L} \right)^I \Leftrightarrow \\ \Leftrightarrow {}^D\vec{\omega}_{D/I} &= [T_{L-D}][T_{C-L}]{}^C\vec{\omega}_{C/I} + [T_{L-D}]{}^L\vec{\omega}_{L/C} + {}^D\vec{\omega}_{D/L} \end{aligned} \quad (4.24)$$

where ${}^L\vec{\omega}_{L/C}$ is equal to:

$${}^L\vec{\omega}_{L/C} = \begin{bmatrix} 0 \\ q_{L/C} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta}_{L/C} \\ 0 \end{bmatrix} \quad (4.25)$$

and ${}^D\vec{\omega}_{D/L}$ is the angular velocity of the drone frame with respect to the line frame and can be defined in function of the Euler angles rates $\dot{\phi}_{D/L}$, $\dot{\theta}_{D/L}$ and $\dot{\psi}_{D/L}$ in the same way as it was made to obtain the angular velocity expression of the canopy body (4.19).

$${}^D\vec{\omega}_{D/L} = \begin{bmatrix} p_{D/L} \\ q_{D/L} \\ r_{D/L} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta_{D/L}) \\ 0 & \cos(\phi_{D/L}) & \sin(\phi_{D/L})\cos(\theta_{D/L}) \\ 0 & -\sin(\phi_{D/L}) & \cos(\phi_{D/L})\cos(\theta_{D/L}) \end{bmatrix} \begin{bmatrix} \dot{\phi}_{D/L} \\ \dot{\theta}_{D/L} \\ \dot{\psi}_{D/L} \end{bmatrix} \quad (4.26)$$

- Derivative of the Angular Velocity

Deriving the angular velocity expression (4.24) in the drone frame, one obtains:

$$\begin{aligned} {}^D\dot{\vec{\omega}}_{D/I}^D &= {}^D\vec{\omega}_{C/I} \times {}^D\vec{\omega}_{D/I} + [T_{L-D}][T_{C-L}]^C \dot{\vec{\omega}}_{C/I}^C \\ &\quad + {}^D\vec{\omega}_{L/C} \times {}^D\vec{\omega}_{D/L} + [T_{L-D}]^L \dot{\vec{\omega}}_{L/C}^L + {}^D\dot{\vec{\omega}}_{D/L}^D \end{aligned} \quad (4.27)$$

with a derivation provided in Appendix A.4 and ${}^L\dot{\vec{\omega}}_{L/C}^L$ and ${}^D\dot{\vec{\omega}}_{D/L}^D$ are defined in equations (4.28) and (4.29), respectively.

$${}^L\dot{\vec{\omega}}_{L/C}^L = \begin{bmatrix} 0 \\ \dot{q}_{L/C} \\ 0 \end{bmatrix} \quad (4.28)$$

$${}^D\dot{\vec{\omega}}_{D/L}^D = \begin{bmatrix} \dot{p}_{D/L} \\ \dot{q}_{D/L} \\ \dot{r}_{D/L} \end{bmatrix} \quad (4.29)$$

4.3.1.3 Capsule

The analysis of the rotation kinematics of the capsule will be made in a similar manner as it was done for the drone.

- Euler Angles

The orientation of the capsule can be described in function of the Euler angles vectors ${}^D\vec{\Theta}_{D/I}$ and ${}^{CAP}\vec{\Theta}_{CAP/D}$ as follows:

$${}^{CAP}\vec{\Theta}_{CAP/I} = [T_{D-CAP}]^D \vec{\Theta}_{D/I} + {}^{CAP}\vec{\Theta}_{CAP/D} \quad (4.30)$$

where the vector ${}^{CAP}\vec{\Theta}_{CAP/D}$ is given by the state variables $\phi_{CAP/D}$ and $\theta_{CAP/D}$:

$${}^{CAP}\vec{\Theta}_{CAP/D} = \begin{bmatrix} \phi_{CAP/D} \\ \theta_{CAP/D} \\ 0 \end{bmatrix} \quad (4.31)$$

- Angular Velocity

The angular velocity of the capsule, expressed in the capsule frame, can be obtained deriving in the inertial frame the Euler angles vector ${}^{CAP}\vec{\Theta}_{CAP/D}$.

$$\begin{aligned} {}^{CAP}\vec{\omega}_{CAP/I} &= \frac{d}{dt} \left({}^{CAP}\vec{\Theta}_{CAP/I} \right)^I \Leftrightarrow \\ \Leftrightarrow {}^{CAP}\vec{\omega}_{CAP/I} &= \frac{d}{dt} \left([T_{D-CAP}]^D \vec{\Theta}_{D/I} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{\Theta}_{CAP/D} \right)^I \end{aligned}$$

Using the equation (A.15) of Appendix, the capsule angular velocity can be written as:

$$\begin{aligned} {}^{CAP}\vec{\omega}_{CAP/I} &= [T_{D-CAP}] \frac{d}{dt} \left({}^D\vec{\Theta}_{D/I} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{\Theta}_{CAP/D} \right)^I \Leftrightarrow \\ \Leftrightarrow {}^{CAP}\vec{\omega}_{CAP/I} &= [T_{D-CAP}]^D \vec{\omega}_{D/I} + {}^{CAP}\vec{\omega}_{CAP/D} \end{aligned} \quad (4.32)$$

where ${}^{CAP}\vec{\omega}_{CAP/D}$ is obtained from the equation (4.19) for a null yaw angle rate between the capsule and the drone.

$${}^{CAP}\vec{\omega}_{CAP/D} = \begin{bmatrix} p_{CAP/D} \\ q_{CAP/D} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \sin(\theta_{CAP/D}) \\ 0 & \cos(\phi_{CAP/D}) & \sin(\phi_{CAP/D})\cos(\theta_{CAP/D}) \\ 0 & -\sin(\phi_{CAP/D}) & \cos(\phi_{CAP/D})\cos(\theta_{CAP/D}) \end{bmatrix} \begin{bmatrix} \dot{\phi}_{CAP/D} \\ \dot{\theta}_{CAP/D} \\ 0 \end{bmatrix} \quad (4.33)$$

- Derivative of the Angular Velocity

Deriving the capsule angular velocity in the capsule frame, one obtains the following expression:

$${}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} = {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{\omega}_{CAP/I} + [T_{D-CAP}]^D \dot{\vec{\omega}}_{D/I}^D + {}^{CAP}\dot{\vec{\omega}}_{CAP/D}^{CAP} \quad (4.34)$$

with a derivation provided in Appendix A.5 and ${}^{CAP}\dot{\vec{\omega}}_{CAP/D}^{CAP}$ is defined in equation (4.35).

$${}^{CAP}\dot{\vec{\omega}}_{CAP/D}^{CAP} = \begin{bmatrix} \dot{p}_{CAP/D} \\ \dot{q}_{CAP/D} \\ 0 \end{bmatrix} \quad (4.35)$$

4.3.1.4 Transponder

Making the same procedure to obtain the rotation kinematics for the transponder body, one easily derives the Euler angles, angular velocity and the derivative of the angular velocity expressions of the transponder. They are defined in equations (4.36), (4.38) and (4.40), respectively.

- Euler Angles

$${}^T\vec{\Theta}_{T/I} = [T_{CAP-T}] {}^{CAP}\vec{\Theta}_{CAP/I} + {}^T\vec{\Theta}_{T/CAP} \quad (4.36)$$

where the vector ${}^T\vec{\Theta}_{T/CAP}$ is defined by the state variables $\phi_{T/CAP}$ and $\theta_{T/CAP}$ as follows:

$${}^T\vec{\Theta}_{T/CAP} = \begin{bmatrix} \phi_{T/CAP} \\ \theta_{T/CAP} \\ 0 \end{bmatrix} \quad (4.37)$$

- Angular velocity

$$\begin{aligned} {}^T\vec{\omega}_{T/I} &= \frac{d}{dt} \left({}^T\vec{\Theta}_{T/I} \right)^I \Leftrightarrow \\ \Leftrightarrow {}^T\vec{\omega}_{T/I} &= \frac{d}{dt} \left([T_{CAP-T}]^T \vec{\Theta}_{T/I} \right)^I + \frac{d}{dt} \left({}^T\vec{\Theta}_{T/CAP} \right)^I \end{aligned}$$

Using the equation (A.15) of Appendix, the transponder angular velocity becomes:

$$\begin{aligned} {}^T\vec{\omega}_{T/I} &= [T_{CAP-T}] \frac{d}{dt} \left({}^{CAP}\vec{\Theta}_{CAP/I} \right)^I + \frac{d}{dt} \left({}^T\vec{\Theta}_{T/CAP} \right)^I \Leftrightarrow \\ \Leftrightarrow {}^T\vec{\omega}_{T/I} &= [T_{CAP-T}]^{CAP} \vec{\omega}_{CAP/I} + {}^T\vec{\omega}_{T/CAP} \end{aligned} \quad (4.38)$$

where ${}^T\vec{\omega}_{T/CAP}$ is defined in equation (4.39), which was obtained from the equation (4.19) for a null yaw angle rate between the transponder and the capsule.

$${}^T\vec{\omega}_{T/CAP} = \begin{bmatrix} p_{T/CAP} \\ q_{T/CAP} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \sin(\theta_{T/CAP}) \\ 0 & \cos(\phi_{CAP/D}) & \sin(\phi_{T/CAP})\cos(\theta_{T/CAP}) \\ 0 & -\sin(\phi_{T/CAP}) & \cos(\phi_{T/CAP})\cos(\theta_{T/CAP}) \end{bmatrix} \begin{bmatrix} \dot{\phi}_{T/CAP} \\ \dot{\theta}_{T/CAP} \\ 0 \end{bmatrix} \quad (4.39)$$

- Derivative of the Angular Velocity

$${}^T\dot{\vec{\omega}}_{T/I}^T = {}^T\vec{\omega}_{CAP/I} \times {}^T\vec{\omega}_{T/I} + [T_{CAP-T}]^{CAP} \dot{\vec{\omega}}_{CAP/I}^{CAP} + {}^T\dot{\vec{\omega}}_{T/CAP}^T \quad (4.40)$$

with a derivation provided in Appendix A.5 and ${}^T\dot{\vec{\omega}}_{T/CAP}^T$ is equal to:

$${}^T\dot{\vec{\omega}}_{T/CAP}^T = \begin{bmatrix} \dot{p}_{T/CAP} \\ \dot{q}_{T/CAP} \\ 0 \end{bmatrix} \quad (4.41)$$

4.3.2 Position Kinematics

In this sub section it will be analysed the position, linear velocity and linear acceleration of each body of the platform. These three variables describe the translational motion of the body under analysis and, in this work, they will always be determined with respect to the inertial frame and expressed in the frame of the body. The canopy motion will be first analysed followed by the drone and the capsule motion. The position kinematics finishes with the analysis of the transponder motion.

4.3.2.1 Canopy

- Position

The state vector ${}^I R_{CM_C/I} = [x_{CM_C}; y_{CM_C}; z_{CM_C}]$ describes the position of the canopy center of mass with respect to the inertial frame. It is expressed in the inertial frame, but through the rotation matrix T_{I-C} , this vector can be expressed in the canopy frame as follows:

$${}^C \vec{R}_{CM_C/I} = T_{I-C} {}^I \vec{R}_{CM_C/I} \quad (4.42)$$

- Linear Velocity

The linear velocity is defined as the derivative of the position vector in the inertial frame. The linear velocity of the canopy can then be determined as:

$${}^C \vec{V}_{CM_C/I} = \begin{bmatrix} u_{CM_C/I} \\ v_{CM_C/I} \\ w_{CM_C/I} \end{bmatrix} = \frac{d}{dt} \left({}^C \vec{R}_{CM_C/I} \right)^I \quad (4.43)$$

Using the equation (A.15) of Appendix, the equation (4.43) can be written as:

$${}^C \vec{V}_{CM_C/I} = [T_{I-C}] \frac{d}{dt} \left({}^I \vec{R}_{CM_C/I} \right)^I = [T_{I-C}] {}^I \dot{\vec{R}}_{CM_C/I} \quad (4.44)$$

- Linear Acceleration

The linear acceleration is defined as the derivative of the linear velocity in the inertial frame. In this way, the linear acceleration expression for the canopy body is given by:

$${}^C \vec{a}_{CM_C/I} = \frac{d}{dt} \left({}^C \vec{V}_{CM_C/I} \right)^I$$

Applying the equation of Coriolis (A.1), the previous equation becomes:

$$\begin{aligned} {}^C \vec{a}_{CM_C/I} &= {}^C \vec{\omega}_{CM_C/I} \times {}^C \vec{V}_{CM_C/I} + \frac{d}{dt} \left({}^C \vec{V}_{CM_C/I} \right)_C \Leftrightarrow \\ \Leftrightarrow {}^C \vec{a}_{CM_C/I} &= {}^C \dot{\vec{V}}_{CM_C/I}^C + {}^C \vec{\omega}_{CM_C/I} \times {}^C \vec{V}_{CM_C/I} \end{aligned} \quad (4.45)$$

where ${}^C\dot{\vec{V}}_{CM_C/I}^C$ is defined in equation (4.46).

$${}^C\dot{\vec{V}}_{CM_C/I}^C = \begin{bmatrix} \dot{u}_{CM_C/I} \\ \dot{v}_{CM_C/I} \\ \dot{w}_{CM_C/I} \end{bmatrix} \quad (4.46)$$

4.3.2.2 Drone

- Position

The drone position vector will be defined in function of other position vectors, such as it was made for the drone Euler angles vector. So, the drone position vector expressed in the drone frame can be defined as:

$${}^D\vec{R}_{CM_D/I} = [T_{C-D}]^C\vec{R}_{CM_C/I} + [T_{C-D}]^C\vec{R}_{T_C/CM_C} + [T_{L-D}]^L\vec{R}_{T_D/T_C} + {}^D\vec{R}_{CM_D/T_D} \quad (4.47)$$

where ${}^C\vec{R}_{T_C/CM_C} = {}^C\vec{R}_{T_{1C}/CM_C} + {}^C\vec{R}_{T_{2C}/CM_C}$, ${}^L\vec{R}_{T_D/T_C} = {}^L\vec{R}_{T_{1D}/T_{1C}} + {}^L\vec{R}_{T_{2D}/T_{2C}}$ and ${}^D\vec{R}_{CM_D/T_D} = -\left({}^D\vec{R}_{T_{1D}/CM_D} + {}^D\vec{R}_{T_{2D}/CM_D}\right)$

- Linear Velocity

The linear velocity of the drone body is defined as:

$$\begin{aligned} {}^D\vec{V}_{CM_D/I} = & [T_{C-D}]^C\vec{V}_{CM_C/I} + {}^D\vec{\omega}_{C/I} \times \left([T_{C-D}]^C\vec{R}_{T_C/CM_C}\right) + {}^D\vec{\omega}_{L/I} \times \left([T_{L-D}]^L\vec{R}_{T_D/T_C}\right) \\ & + [T_{L-D}]^L\vec{V}_{Control/I} + {}^D\vec{\omega}_{D/I} \times {}^C\vec{R}_{CM_D/T_D} \end{aligned} \quad (4.48)$$

with a derivation provided in Appendix A.7.

- Linear Acceleration

Deriving the drone linear velocity expression, one obtains the following linear acceleration expression:

$$\begin{aligned} {}^D\vec{a}_{CM_D/I} = & [T_{C-D}]^C\dot{\vec{V}}_{CM_C/I}^C + {}^D\vec{\omega}_{C/I} \times {}^D\vec{V}_{CM_C/I} + \left([T_{C-D}]^C\dot{\vec{\omega}}_{C/I}^C + {}^D\vec{\omega}_{C/I} \times {}^D\vec{\omega}_{C/I}\right) \times {}^D\vec{R}_{T_C/CM_C} \\ & + [T_{L-D}]^L\dot{\vec{\omega}}_{L/I}^L \times {}^D\vec{R}_{T_D/T_C} + {}^D\vec{\omega}_{L/I} \times {}^D\vec{\omega}_{L/I} \times {}^D\vec{R}_{T_D/T_C} + 2{}^D\vec{\omega}_{L/I} \times {}^D\vec{V}_{Control/I} \\ & + [T_{L-D}]^L\dot{\vec{V}}_{Control/I}^L + {}^D\dot{\vec{\omega}}_{D/I}^D \times {}^C\vec{R}_{CM_D/T_D} + {}^D\vec{\omega}_{D/I} \times {}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D} \end{aligned} \quad (4.49)$$

with a derivation provided in Appendix A.7.

4.3.2.3 Capsule

- Position

In a similar manner as it was made for the drone position vector, the capsule position vector will also be defined as a function of other position vectors. This vector expressed in the capsule frame is defined in equation (4.50).

$${}^{CAP}\vec{R}_{CM_{CAP}/I} = [T_{D-CAP}]^D \vec{R}_{CM_D/I} + [T_{D-CAP}]^D \vec{R}_{TCAP_D/CM_D} + {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \quad (4.50)$$

- Linear Velocity

Deriving the equation (4.50) in the inertial frame, the capsule linear velocity becomes:

$${}^{CAP}\vec{V}_{CM_{CAP}/I} = [T_{D-CAP}]^D \vec{V}_{CM_D/I} + {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} + {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \quad (4.51)$$

with a derivation provided in Appendix A.8.

- Linear Acceleration

The capsule linear acceleration expression can be written as:

$$\begin{aligned} {}^{CAP}\vec{a}_{CM_{CAP}/I} = & [T_{D-CAP}]^D \vec{a}_{CM_D/I} + [T_{D-CAP}]^D \dot{\vec{\omega}}_{D/I}^D \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} \\ & + {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} + {}^{CAP}\dot{\vec{\omega}}_{CAP/I}^CAP \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \\ & + {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \end{aligned} \quad (4.52)$$

with a derivation provided in Appendix A.8.

4.3.2.4 Transponder

The transponder is attached to the capsule in a similar manner as the capsule is attached to the drone. So, one can previously note that the two bodies pairs (capsule-drone and transponder-capsule) are similar therefore it is predictable that the transponder position kinematics will be similar to the capsule position kinematics. Thus, the transponder position kinematics will be determined in the same way as it was made for the capsule body.

- Position

The transponder position vector expressed in the transponder frame can be written as:

$${}^T\vec{R}_{CM_T/I} = [T_{CAP-T}]^{CAP}\vec{R}_{CAP/I} + {}^T\vec{R}_{CM_T/CM_{CAP}} \quad (4.53)$$

- Linear Velocity

The transponder linear velocity is defined in equation (4.54) with its derivation provided in Appendix A.9.

$${}^T\vec{V}_{CM_T/I} = [T_{CAP-T}]^{CAP}\vec{V}_{CM_{CAP}/I} + {}^T\vec{\omega}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} \quad (4.54)$$

- Linear Acceleration

The linear acceleration expression for the transponder body is defined in equation (4.55) with its derivation provided in Appendix A.9.

$${}^T\vec{a}_{CM_T/I} = [T_{CAP-T}]^{CAP}\vec{a}_{CM_{CAP}/I} + {}^T\dot{\vec{\omega}}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} + {}^T\vec{\omega}_{T/I} \times {}^T\vec{\omega}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} \quad (4.55)$$

4.4 System Dynamics

In this section, the translation and rotation dynamic equations of each body of the platform will be analysed. These equations describe the dynamic behaviour of a body. The translation dynamic equation of a body B (4.56) is provided by equating the time derivative of linear momentum with the total forces applied in B . Whereas the rotation dynamic equation of the body B (4.57) is provided by equating the time derivative of angular momentum with the total moments applied in B .

$$\begin{aligned} \sum \vec{F}_B &= \frac{d}{dt} \left(M_B {}^B\vec{V}_{B/I} \right)^I \Leftrightarrow \\ \Leftrightarrow \sum \vec{F}_B &= M_B {}^B\vec{a}_{B/I} \end{aligned} \quad (4.56)$$

where M_B is the mass of the body B .

$$\sum \vec{M}_B = \frac{d}{dt} \left([I_B]^B \vec{\omega}_{B/I} \right)^I \quad (4.57)$$

where $[I_B]$ is the inertial matrix of the body B .

Applying the equation of Coriolis (A.1), the previous equation can be written as:

$$\sum \vec{M}_B = {}^B\vec{\omega}_{B/I} \times ([I_B]^B \vec{\omega}_{B/I}) + \frac{d}{dt} ([I_B]^B \vec{\omega}_{B/I})^B$$

As the inertial matrix is a constant matrix, its derivative is null and the rotation dynamic equation of the body B becomes:

$$\begin{aligned} \sum \vec{M}_B &= {}^B\vec{\omega}_{B/I} \times ([I_B]^B \vec{\omega}_{B/I}) + [I_B] \frac{d}{dt} ({}^B\vec{\omega}_{B/I})^B \Leftrightarrow \\ \Leftrightarrow \sum \vec{M}_B &= {}^B\vec{\omega}_{B/I} \times ([I_B]^B \vec{\omega}_{B/I}) + [I_B]^B \dot{\vec{\omega}}_{B/I}^B \end{aligned} \quad (4.58)$$

It should be noted that the derivative of the angular velocity ${}^B\dot{\vec{\omega}}_{B/I}$ is taken in the B frame. Hence having analysed the derivatives of the angular velocities in the frame of the body under analysis.

In order to perform the equations (4.56) and (4.58) for the entire system, the forces and the moments applied in each body of the system will be previously analysed.

4.4.1 Forces

The forces applied in the system will be evaluated separately for each body. Moreover, these force expressions will be defined in the frame of the body under analysis.

4.4.1.1 Canopy

In the canopy body there are 5 forces applied: weight force ${}^C\vec{F}_{w_C}$, aerodynamic force ${}^C\vec{F}_{A_C}$, apparent force ${}^C\vec{F}_{app}$ and the constraint forces ${}^C\vec{T}_{1_{D/C}}$ and ${}^C\vec{T}_{2_{D/C}}$.

I. Weight Force

The weight force originates from the gravitational acceleration g that is considered invariant with the altitude of the body. This force, for the canopy body, expressed in the canopy frame can be written as:

$$\begin{aligned} {}^C\vec{F}_{w_C} &= M_C [T_{I-C}] \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \Leftrightarrow \\ \Leftrightarrow {}^C\vec{F}_{w_C} &= M_C g \begin{bmatrix} -\sin(\theta_{C/I}) \\ \sin(\phi_{C/I}) \cos(\theta_{C/I}) \\ \cos(\phi_{C/I}) \cos(\theta_{C/I}) \end{bmatrix} \end{aligned} \quad (4.59)$$

where M_C is the mass of the canopy.

II. Aerodynamic Force

In the aerodynamic field when a body immersed in a fluid moves through it, the relative motion between them induces a force on the body. This force is named aerodynamic force and has the

opposite direction of the body motion. It is a function of the magnitude of the density of the air ρ , the airspeed of the body V_A , the body's contact area S and dimensionless aerodynamic coefficients. In the aerospace field these coefficients are: Lift coefficient (C_L), Drag coefficient (C_D) and Sideforce coefficient (C_Y) whose expressions for a parafoil will be modelled as in [10]:

$$C_D = C_{D0} + C_{D\alpha^2} \alpha^2 \quad (4.60)$$

$$C_L = C_{L0} + C_{L\alpha} \alpha \quad (4.61)$$

$$C_Y = C_{Y\beta} \beta \quad (4.62)$$

where the angle α and β are the aerodynamic angles previously referred.

The aerodynamic force of the canopy expressed in the canopy frame can then be written as:

$${}^C \vec{F}_{Ac} = -\frac{1}{2} \rho V_{C/A}^2 S_C [T_{A-C}] \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \quad (4.63)$$

where S_C is the contact area of the canopy and $V_{C/A} = \sqrt{u_{C/A}^2 + v_{C/A}^2 + w_{C/A}^2}$ where $u_{C/A}$, $v_{C/A}$ and $w_{C/A}$ are the components of the canopy freestream ${}^C \vec{V}_{C/A}$ that is defined as:

$${}^C \vec{V}_{C/A} = {}^C \vec{V}_{CM_C/I} - [T_{I-C}]^I \vec{V}_{A/I} \quad (4.64)$$

where ${}^I \vec{V}_{A/I}$ is the wind velocity expressed in the inertial frame.

The aerodynamic angles then become $\alpha = \text{atan}\left(\frac{w_{C/A}}{u_{C/A}}\right)$ and $\beta = \text{asin}\left(\frac{v_{C/A}}{V_{C/A}}\right)$.

III. Apparent force

In addition to the aerodynamic force, when a body moves through a fluid there is an additional force due to the accelerating fluid. This action can be modelled as an apparent mass and inertial which for a lightly loaded flight vehicle, such as a parafoil, can have a large effect on the flight dynamics. Lissaman and Brown [20] calculated the forces and moment forces expression from the apparent mass and inertial, relating the kinetic energy of the fluid to the resultant force and moment force. The importance of these force and moment force is higher, the smaller the wing loading therefore it should be considered on the canopy dynamics. The apparent mass contribution expressed in the canopy frame can be written as:

$${}^C \vec{F}_{app} = -{}^C \vec{\omega}_{C/I} \times \left([M_{app}] {}^C \vec{V}_{C/A} \right) - [M_{app}] {}^C \dot{\vec{V}}_{CM_C/I} - [M_{app}] \left({}^C \vec{\omega}_{C/I} \times \left([T_{I-C}]^I \vec{V}_{A/I} \right) \right) \quad (4.65)$$

with a derivation provided in Appendix A.10 and the apparent mass M_{app} is a matrix of dimension 3×3 :

$$M_{app} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \quad (4.66)$$

where the terms A , B , C can be calculated using the following formulas given by Lissaman and Brown:

$$\begin{aligned} k_A &= 0.848 \frac{\pi}{4} & A &= k_A \rho d_C^2 b_C \left(1 + \frac{8}{3} a_C^3\right) \\ k_B &= 0.339 \frac{\pi}{4} & B &= k_B \rho c_C [d_C^2 + 2a_C^2 (1 - d_C^2)] \\ k_C &= \frac{AR}{1 + AR} \frac{\pi}{4} & C &= k_C \rho c_C^2 b_C \sqrt{1 + 2a_C^2 (1 - d_C^2)} \end{aligned}$$

The term AR is the aspect ratio of the canopy and the parameters a_C , b_C , c_C and d_C are the dimensions of the canopy defined in Section 3.2.

IV. Constraint Forces

The constraint forces ${}^C\vec{T}_{1D/C}$ and ${}^C\vec{T}_{2D/C}$ are the pulling forces from the two main lines which in the canopy side have the same directions of the position vectors ${}^L\vec{R}_{T1D/T1C}$ and ${}^L\vec{R}_{T2D/T2C}$ expressed in the canopy frame, respectively.

Through the forces presented above, the translation dynamic equation (4.56) for the canopy body can then be written as:

$$M_C {}^C\vec{a}_{CM_C/I} = {}^C\vec{F}_{wC} + {}^C\vec{F}_{Ac} + {}^C\vec{F}_{app} + {}^C\vec{T}_{1D/C} + {}^C\vec{T}_{2D/C} \quad (4.67)$$

4.4.1.2 Drone

Analysing the forces applied in the drone body, it should be considered 5 forces: weight force ${}^D\vec{F}_{wD}$, aerodynamic force ${}^D\vec{F}_{Ad}$, and the constraint forces ${}^D\vec{T}_{1C/D}$, ${}^D\vec{T}_{2C/D}$ and ${}^D\vec{T}_{CAP/D}$.

I. Weight Force

The weight force of drone expressed in its frame can be written as:

$${}^D\vec{F}_{wD} = M_D g [T_{C-D}] \begin{bmatrix} -\sin(\theta_{C/I}) \\ \sin(\phi_{C/I}) \cos(\theta_{C/I}) \\ \cos(\phi_{C/I}) \cos(\theta_{C/I}) \end{bmatrix} \quad (4.68)$$

where M_D is the mass of the drone.

II. Aerodynamic Force

The derivation of the aerodynamic force expression of the drone body will be performed considering the drone shape as a cube so that the aerodynamic coefficient in three directions is equal to C_{D_D} . In this way, the aerodynamic force expressed in the drone frame can be defined as:

$${}^D\vec{F}_{A_D} = -\frac{1}{2}\rho V_{D/A} S_D C_{D_D} \begin{bmatrix} u_{D/A} \\ v_{D/A} \\ w_{D/A} \end{bmatrix} \quad (4.69)$$

where S_D is the contact area of the drone, $V_{D/A} = \sqrt{u_{D/A}^2 + v_{D/A}^2 + w_{D/A}^2}$ and $u_{D/A}$, $v_{D/A}$ and $w_{D/A}$ are the components of the drone freestream ${}^D\vec{V}_{D/A}$ that is defined as:

$${}^D\vec{V}_{D/A} = {}^D\vec{V}_{CM_D/I} - [T_{C-D}][T_{I-C}]^T \vec{V}_{A/I} \quad (4.70)$$

III. Constraint Forces

In the drone body, there are three constraint forces: ${}^C\vec{T}_{1_{C/D}}$, ${}^C\vec{T}_{2_{C/D}}$ and ${}^D\vec{T}_{CAP/D}$. The first ones have the same magnitude as ${}^C\vec{T}_{1_{D/C}}$, ${}^C\vec{T}_{2_{D/C}}$ and they expressed in the drone frame can be written as:

$${}^D\vec{T}_{1_{C/D}} = -[T_{C-D}]^C \vec{T}_{1_{D/C}} \quad (4.71)$$

$${}^D\vec{T}_{2_{C/D}} = -[T_{C-D}]^C \vec{T}_{2_{D/C}} \quad (4.72)$$

The third force is the pulling force from the line which connects the capsule to the drone. The direction of this force in the drone side is equal to the direction of the vector ${}^{CAP}\vec{R}_{CM_{CAP}/T_{CAP_D}}$ expressed in the drone frame.

Taking the forces previously analysed, the translation dynamic equation of the drone body is equal to:

$$M_D {}^D\vec{a}_{CM_D/I} = {}^D\vec{F}_{w_D} + {}^D\vec{F}_{A_D} + {}^D\vec{T}_{1_{C/D}} + {}^D\vec{T}_{2_{C/D}} + {}^D\vec{T}_{CAP/D} \quad (4.73)$$

4.4.1.3 Capsule

Analysing the capsule body, there are 4 forces applied: weight force ${}^{CAP}\vec{F}_{w_{CAP}}$, aerodynamic force ${}^{CAP}\vec{F}_{A_{CAP}}$, and the constraint forces ${}^{CAP}\vec{T}_{D/CAP}$ and ${}^{CAP}\vec{T}_{T/CAP}$.

I. Weight Force

The weight force of the capsule expressed in its frame can be defined as:

$${}^{CAP}\vec{F}_{w_{CAP}} = M_{CAP}g[T_{D-CAP}][T_{C-D}] \begin{bmatrix} -\sin(\theta_{C/I}) \\ \sin(\phi_{C/I})\cos(\theta_{C/I}) \\ \cos(\phi_{C/I})\cos(\theta_{C/I}) \end{bmatrix} \quad (4.74)$$

where M_{CAP} is the mass of the capsule.

II. Aerodynamic Force

The aerodynamic force expression of the capsule body is defined in a similar manner as it was made for the drone body. Expressing in the capsule frame, it becomes:

$${}^{CAP}\vec{F}_{A_{CAP}} = -\frac{1}{2}\rho V_{CAP/A} S_{CAP} C_{D_{CAP}} \begin{bmatrix} u_{CAP/A} \\ v_{CAP/A} \\ w_{CAP/A} \end{bmatrix} \quad (4.75)$$

where S_{CAP} is the contact area of the capsule, $C_{D_{CAP}}$ is the drag coefficient of the capsule, $V_{CAP/A} = \sqrt{u_{CAP/A}^2 + v_{CAP/A}^2 + w_{CAP/A}^2}$ and $u_{CAP/A}$, $v_{CAP/A}$ and $w_{CAP/A}$ are the components of the capsule freestream ${}^{CAP}\vec{V}_{CAP/A}$ that is defined as:

$${}^{CAP}\vec{V}_{CAP/A} = {}^{CAP}\vec{V}_{CM_{CAP}/I} - [T_{D-CAP}][T_{C-D}][T_{I-C}]^T \vec{V}_{A/I} \quad (4.76)$$

III. Constraint Forces

The capsule is attached to the drone and to the transponder by two lines as seen in schematic 4.1, therefore it is clear to consider two constraint forces: ${}^{CAP}\vec{T}_{D/CAP}$ and ${}^{CAP}\vec{T}_{T/CAP}$. The first one has the same magnitude as ${}^D\vec{T}_{CAP/D}$ and it expressed in the capsule frame can be written as:

$${}^{CAP}\vec{T}_{D/CAP} = -[T_{D-CAP}]^T {}^D\vec{T}_{CAP/D} \quad (4.77)$$

The second force is the pulling force from the line which connects the capsule to the transponder. The direction of this force on the capsule side is given by the direction of the vector ${}^T\vec{R}_{CM_T/CM_{CAP}}$ expressed in the capsule frame.

The translation dynamic equation of the capsule body is defined in equation (4.78).

$$M_{CAP} {}^{CAP}\vec{a}_{CM_{CAP}/I} = {}^{CAP}\vec{F}_{w_{CAP}} + {}^{CAP}\vec{F}_{A_{CAP}} + {}^{CAP}\vec{T}_{D/CAP} + {}^{CAP}\vec{T}_{T/CAP} \quad (4.78)$$

4.4.1.4 Transponder

In the transponder body only 3 forces are applied: weight force ${}^T\vec{F}_{w_T}$, aerodynamic force ${}^T\vec{F}_{A_T}$, and the constraint force ${}^T\vec{T}_{CAP/T}$.

I. Weight Force

The weight force of the transponder expressed in its frame is defined as:

$${}^T\vec{F}_{w_T} = M_T g [T_{CAP-T}] [T_{D-CAP}] [T_{C-D}] \begin{bmatrix} -\sin(\theta_{C/I}) \\ \sin(\phi_{C/I}) \cos(\theta_{C/I}) \\ \cos(\phi_{C/I}) \cos(\theta_{C/I}) \end{bmatrix} \quad (4.79)$$

where M_T is the mass of the transponder.

II. Aerodynamic Force

Applying the equation (4.69) for the transponder body, the aerodynamic force expression can be written as:

$${}^T\vec{F}_{A_T} = -\frac{1}{2} \rho V_{T/A} S_T C_{D_T} \begin{bmatrix} u_{T/A} \\ v_{T/A} \\ w_{T/A} \end{bmatrix} \quad (4.80)$$

where S_T is the contact area of the transponder, C_{D_T} is the drag coefficient of the transponder, $V_{T/A} = \sqrt{u_{T/A}^2 + v_{T/A}^2 + w_{T/A}^2}$ and $u_{T/A}$, $v_{T/A}$ and $w_{T/A}$ are the components of the transponder freestream ${}^T\vec{V}_{T/A}$ that is defined as:

$${}^T\vec{V}_{T/A} = {}^T\vec{V}_{CM_T/I} - [T_{CAP-T}] [T_{D-CAP}] [T_{C-D}] [T_{I-C}]^I \vec{V}_{A/I} \quad (4.81)$$

III. Constraint Force

In the transponder there is only the constraint forces ${}^T\vec{T}_{CAP/T}$ which has the same magnitude as ${}^{CAP}\vec{T}_{T/CAP}$. This force expressed in the transponder frame can be written as:

$${}^T\vec{T}_{CAP/T} = -[T_{CAP-T}] {}^{CAP}\vec{T}_{T/CAP} \quad (4.82)$$

Applying the translation dynamic equation (4.56) for the transponder body it becomes:

$$M_T {}^T\vec{a}_{CM_T/I} = {}^T\vec{F}_{w_T} + {}^T\vec{F}_{A_T} + {}^T\vec{T}_{CAP/T} \quad (4.83)$$

4.4.2 Moments

In the analysis of the rotation dynamic of the system, it should be noted that two bodies, capsule and transponder, are free to rotate in the z-axis of its frame. So, the contribution of these rotations in the system motion can be neglected and therefore it will only be considered the rotation dynamic of the canopy and the drone body.

4.4.2.1 Canopy

In the canopy body there are 5 moments applied in its center of mass: aerodynamic moment ${}^C\vec{M}_{Ac}$, apparent moment ${}^C\vec{M}_{app}$ and the moment forces ${}^C\vec{F}_{Ac}$, ${}^C\vec{T}_{1D/C}$ and ${}^C\vec{T}_{2D/C}$.

I. Aerodynamic Moment

Such as the aerodynamic force exists when a body moves through a fluid, a rotational motion of the body also induces an opposed rotational motion caused by the fluid. This opposed rotational motion is measured by the aerodynamic moment which is defined in terms of magnitude of the density of the fluid ρ , the airspeed of the body V_A , the contact area of the body S and the dimensionless aerodynamic coefficients: rolling moment coefficient (C_l), pitching moment coefficient (C_m) and yawing moment coefficient (C_n). The expressions of these coefficients for a canopy will be modelled in a similar manner as in [10]:

$$C_{lc} = \frac{b_c^2}{2V_{Ac}} C_{lpc} p_{C/A} \quad (4.84)$$

$$C_{mc} = \frac{c_c^2}{2V_{Ac}} C_{mqc} q_{C/A} \quad (4.85)$$

$$C_{nc} = \frac{b_c^2}{2V_{Ac}} C_{nrc} r_{C/A} + b_c C_{n\beta_c} \beta \quad (4.86)$$

where $p_{C/A}$, $q_{C/A}$ and $r_{C/A}$ are the components of the aerodynamic angular velocity ${}^C\vec{\omega}_{C/A}$ which is equal to ${}^C\vec{\omega}_{C/I}$ and C_{lpc} , C_{mqc} , C_{nrc} and $C_{n\beta_c}$ are constant values.

A special attention should be given to the parameter $C_{n\beta_c}$. This parameter measures the natural behaviour of the canopy to turn into wind direction. Taking the aerodynamic coefficients previously defined, the aerodynamic moment can be written as:

$${}^C\vec{M}_{Ac} = -\frac{1}{2}\rho V_{C/A}^2 S_C \begin{bmatrix} C_{lc} \\ C_{mc} \\ C_{nc} \end{bmatrix} \quad (4.87)$$

where S_C is the contact area of the canopy and $V_{C/A}$ is the magnitude of the vector defined in equation (4.64).

II. Apparent Moment

The apparent moment has the same origin of the apparent force, previously explained. Considering the apparent inertial I_{app} , the apparent moment of the canopy expressed in its frame is defined as:

$$\begin{aligned} {}^C\vec{M}_{app} &= -\frac{d}{dt} ([I_{app}]^C \vec{\omega}_{C/A})^I \Leftrightarrow \\ \Leftrightarrow {}^C\vec{M}_{app} &= -\frac{d}{dt} ([I_{app}]^C \vec{\omega}_{C/I})^I \end{aligned}$$

Applying the equation of Coriolis (A.1), the apparent moment becomes:

$$\begin{aligned} {}^C\vec{M}_{app} &= -({}^C\vec{\omega}_{C/I} \times ([I_{app}]^C \vec{\omega}_{C/I})) - \frac{d}{dt} ([I_{app}]^C \vec{\omega}_{C/I})^C \Leftrightarrow \\ \Leftrightarrow {}^C\vec{M}_{app} &= -[I_{app}]^C \dot{\vec{\omega}}_{C/I} - ({}^C\vec{\omega}_{C/I} \times ([I_{app}]^C \vec{\omega}_{C/I})) \end{aligned} \quad (4.88)$$

where I_{app} is the inertial matrix given by:

$$I_{app} = \begin{bmatrix} I_A & 0 & 0 \\ 0 & I_B & 0 \\ 0 & 0 & I_C \end{bmatrix} \quad (4.89)$$

with the terms I_A , I_B , I_C computed by the following formulas given by Lissaman and Brown [20]:

$$\begin{aligned} k_A^* &= 0.055 \frac{AR}{1+AR} & I_A &= k_A^* \rho c_C^2 b_C^3 \\ k_B^* &= 0.0308 \frac{AR}{1+AR} & I_B &= k_B^* \rho c_C^4 b_C [1 + \frac{\pi}{6} (1+AR) AR a_C^2 d_C^2] \\ k_C^* &= 0.0555 & I_C &= k_C^* \rho d_C^2 b_C^3 (1 + 8a_C^2) \end{aligned}$$

III. Moment of Force

The moment of force is defined as the vectorial product between the force and the vector from the point where the moment of force is calculated to the point where the force is applied. In this way the moment of force ${}^C\vec{M}_{F_{Ac}}$, ${}^C\vec{M}_{T1_{D/C}}$ and ${}^C\vec{M}_{T2_{D/C}}$ are defined in equation (4.90), (4.91) and (4.92), respectively.

$${}^C\vec{M}_{F_{Ac}} = {}^C\vec{R}_{CP_C/CM_C} \times {}^C\vec{F}_{Ac} \quad (4.90)$$

$${}^C\vec{M}T1_{D/C} = {}^C\vec{R}_{T1_C/CM_C} \times {}^C\vec{T}1_{D/C} \quad (4.91)$$

$${}^C\vec{M}T2_{D/C} = {}^C\vec{R}_{T2_C/CM_C} \times {}^C\vec{T}2_{D/C} \quad (4.92)$$

Taking the moments analysed above, the rotation dynamic expression of the canopy body can then be written as:

$$[I_C]{}^C\dot{\vec{\omega}}_{C/I} + {}^C\vec{\omega}_{C/I} \times ([I_C]{}^C\vec{\omega}_{C/I}) = {}^C\vec{M}_{Ac} + {}^C\vec{M}_{app} + {}^C\vec{M}_{F_{Ac}} + {}^C\vec{M}T1_{D/C} + {}^C\vec{M}T2_{D/C} \quad (4.93)$$

where I_C is the canopy moment of inertial which is given by the following diagonal matrix, defined in [9]:

$$I_C = \frac{M_C}{12} \begin{bmatrix} b_C^2 + d_C^2 & 0 & 0 \\ 0 & c_C^2 + d_C^2 & 0 \\ 0 & 0 & b_C^2 + c_C^2 \end{bmatrix} \quad (4.94)$$

4.4.2.2 Drone

In the drone body 4 moments should be considered: aerodynamic moment ${}^D\vec{M}_{Ad}$ and the moment of forces ${}^D\vec{T}1_{C/D}$, ${}^D\vec{T}2_{C/D}$ and ${}^D\vec{T}_{CAP/D}$.

I. Aerodynamic Moment

The aerodynamic moment of the drone is defined in a similar manner as it was made for the aerodynamic moment of the canopy described in equation (4.87).

$${}^D\vec{M}_{Ad} = -\frac{1}{2}\rho V_{D/A}^2 S_D \begin{bmatrix} C_{l_D} \\ C_{m_D} \\ C_{n_D} \end{bmatrix} \quad (4.95)$$

where S_D is the contact area of the drone, $V_{D/A}$ is the magnitude of the drone freestream defined in equation (4.70) and C_{l_D} , C_{m_D} and C_{n_D} are dimensionless aerodynamic coefficients given by:

$$C_{l_D} = \frac{b_D^2}{2V_{Ad}} C_{lpD} p_{D/A} \quad (4.96)$$

$$C_{m_D} = \frac{c_D^2}{2V_{Ad}} C_{mqD} q_{D/A} \quad (4.97)$$

$$C_{n_D} = \frac{b_D^2}{2V_{Ad}} C_{nrD} r_{D/A} \quad (4.98)$$

where $p_{D/A}$, $q_{D/A}$ and $r_{D/A}$ are the components of the aerodynamic angular velocity ${}^D\vec{\omega}_{D/A}$ which is equal to ${}^D\vec{\omega}_{D/I}$ and C_{lp_D} , C_{mq_D} and C_{nr_D} are constant values.

II. Moment of Force

Using the same definition of moment of force, previously set out, the moments of force ${}^D\vec{M}T1_{C/D}$, ${}^D\vec{M}T2_{C/D}$ and ${}^D\vec{M}T_{CAP/D}$ are defined in equations (4.99), (4.100) and (4.101) respectively.

$${}^D\vec{M}T1_{C/D} = {}^D\vec{R}_{CM_D/T1_D} \times {}^D\vec{T}1_{C/D} \quad (4.99)$$

$${}^D\vec{M}T2_{C/D} = {}^D\vec{R}_{CM_D/T2_D} \times {}^D\vec{T}2_{C/D} \quad (4.100)$$

$${}^D\vec{M}T_{CAP/D} = {}^D\vec{R}_{CM_D/TCAP_D} \times {}^D\vec{T}_{CAP/D} \quad (4.101)$$

The rotation dynamic expression of the drone body can then be determined as:

$$[I_D] {}^D\ddot{\vec{\omega}}_{D/I} + {}^D\vec{\omega}_{D/I} \times ([I_D] {}^D\vec{\omega}_{D/I}) = {}^D\vec{M}_{Ad} + {}^D\vec{M}T1_{C/D} + {}^D\vec{M}T2_{C/D} + {}^D\vec{M}T_{CAP/D} \quad (4.102)$$

where I_D is the drone moment of inertial which is given by the following diagonal matrix, defined in [9]:

$$I_D = \frac{M_D}{12} \begin{bmatrix} b_D^2 + d_D^2 & 0 & 0 \\ 0 & c_D^2 + d_D^2 & 0 \\ 0 & 0 & b_D^2 + c_D^2 \end{bmatrix} \quad (4.103)$$

4.5 Equations of Motion

In Sections 4.3 and 4.4 it was analysed the kinematics and dynamics equations of the entire system. Theses equations together with the equations (4.15), (4.16), (4.17) represent the equations of motion of the system. A list of these equations is presented below:

- ${}^I\ddot{\vec{R}}_{CM_C/I} = [T_{I-C}]^T {}^C\vec{V}_{CM_C/I}$
- $\begin{bmatrix} \dot{\phi}_{C/I} \\ \dot{\theta}_{C/I} \\ \dot{\psi}_{C/I} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta_{C/I}) \\ 0 & \cos(\phi_{C/I}) & \sin(\phi_{C/I})\cos(\theta_{C/I}) \\ 0 & -\sin(\phi_{C/I}) & \cos(\phi_{C/I})\cos(\theta_{C/I}) \end{bmatrix}^{-1} \begin{bmatrix} p_{C/I} \\ q_{C/I} \\ r_{C/I} \end{bmatrix}$

- $M_C^C \ddot{a}_{CM_C/I} = {}^C\vec{F}_{w_C} + {}^C\vec{F}_{A_C} + {}^C\vec{F}_{app} + {}^C\vec{T}1_{D/C} + {}^C\vec{T}2_{D/C}$
- $[I_C]^C \dot{\vec{\omega}}_{C/I}^C + {}^C\vec{\omega}_{C/I} \times ([I_C]^C \vec{\omega}_{C/I}) = {}^C\vec{M}_{A_C} + {}^C\vec{M}_{app} + {}^C\vec{M}F_{A_C} + {}^C\vec{M}T1_{D/C} + {}^C\vec{M}T2_{D/C}$
- $\dot{\theta}_{L/C} = q_{L/C}$
- $$\begin{bmatrix} \dot{\phi}_{D/L} \\ \dot{\theta}_{D/L} \\ \dot{\psi}_{D/L} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta_{D/L}) \\ 0 & \cos(\phi_{D/L}) & \sin(\phi_{D/L})\cos(\theta_{D/L}) \\ 0 & -\sin(\phi_{D/L}) & \cos(\phi_{D/L})\cos(\theta_{D/L}) \end{bmatrix}^{-1} \begin{bmatrix} p_{D/L} \\ q_{D/L} \\ r_{D/L} \end{bmatrix}$$
- $M_D^D \ddot{a}_{DM_D/I} = {}^D\vec{F}_{w_D} + {}^D\vec{F}_{A_D} + {}^D\vec{T}1_{C/D} + {}^D\vec{T}2_{C/D} + {}^D\vec{T}_{CAP/D}$
- $[I_D]^D \dot{\vec{\omega}}_{D/I}^D + {}^D\vec{\omega}_{D/I} \times ([I_D]^D \vec{\omega}_{D/I}) = {}^D\vec{M}_{A_D} + {}^D\vec{M}T1_{C/D} + {}^D\vec{M}T2_{C/D} + {}^D\vec{M}T_{CAP/D}$
- $$\begin{bmatrix} \dot{\phi}_{CAP/D} \\ \dot{\theta}_{CAP/D} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \sin(\theta_{CAP/D}) \\ 0 & \cos(\phi_{CAP/D}) & \sin(\phi_{CAP/D})\cos(\theta_{CAP/D}) \\ 0 & -\sin(\phi_{CAP/D}) & \cos(\phi_{CAP/D})\cos(\theta_{CAP/D}) \end{bmatrix}^{-1} \begin{bmatrix} p_{CAP/D} \\ q_{CAP/D} \\ 0 \end{bmatrix}$$
- $M_{CAP}^{CAP} \ddot{a}_{CM_{CAP}/I} = {}^{CAP}\vec{F}_{w_{CAP}} + {}^{CAP}\vec{F}_{A_{CAP}} + {}^{CAP}\vec{T}_{D/CAP} + {}^{CAP}\vec{T}_{T/CAP}$
- $$\begin{bmatrix} \dot{\phi}_{T/CAP} \\ \dot{\theta}_{T/CAP} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \sin(\theta_{T/CAP}) \\ 0 & \cos(\phi_{T/CAP}) & \sin(\phi_{T/CAP})\cos(\theta_{T/CAP}) \\ 0 & -\sin(\phi_{T/CAP}) & \cos(\phi_{T/CAP})\cos(\theta_{T/CAP}) \end{bmatrix}^{-1} \begin{bmatrix} p_{T/CAP} \\ q_{T/CAP} \\ 0 \end{bmatrix}$$
- $M_T^T \ddot{a}_{CM_T/I} = {}^T\vec{F}_{w_T} + {}^T\vec{F}_{A_T} + {}^T\vec{T}_{CAP/T}$
- $\dot{\Delta}L = K_L(U_L - \Delta L)$
- $\dot{\Delta}ServoR = K_{SR}(U_{SR} - \Delta ServoR)$
- $\dot{\Delta}ServoL = K_{SL}(U_{SL} - \Delta ServoL)$

These equations of motion can be expressed compactly in matrix form as:

$$M \begin{bmatrix} \dot{X} \\ F_C \end{bmatrix} = N \quad (4.104)$$

where X is the state vector defined in equation (4.14) and F_C are a set of the constraint forces $F_C = [T1_{D/C} \ T2_{D/C} \ T_{CAP/D} \ T_{T/CAP}]^T$. The matrix M has dimension 35×35 and is composed by blocks of type M_{ij} with $i = 1, 2, \dots, 15$ and $j = 1, 2, \dots, 15$. N is a column matrix composed by N_i blocks with $i = 1, 2, \dots, 15$. The blocks M_{ij} and N_i are derived from the equations of motion and are defined in Appendix A.11. Although an algebraic manipulation of the equations of motion can eliminate the constraint forces, it is easier to find their values during the numerical solution along with the model states. But it should be emphasized that they aren't state variables.

Through an inversion matrix, the equation (4.104) can be written in function of the derivatives of the state vector and the constraint forces.

$$\begin{bmatrix} \dot{X} \\ F_C \end{bmatrix} = M^{-1}N \quad (4.105)$$

The previous equation is of the type $\dot{X} = f(X, U)$ where the set of nonlinear functions f is given by the product of matrix $M^{-1}N$. Thus it is determined the mathematical model of the system.

4.6 Simulation Results

The nonlinear model described in equation (4.105) should be numerically integrated within a limited time to generate the trajectory described by the system, under initial conditions. A numerical solution method is the Runge-Kutta method, which is described by Stevens and Lewis [2]. In the MATLAB environment, there is a set of functions to solve differential equations and one of them is the *ode45* which integrates the system of differential equation $\dot{X}(t) = f(X(t), U(t))$ from time T_0 to T_{FINAL} with initial conditions X_0 . This function uses the fourth order of the Runge-Kutta algorithm and it will be used to simulate the nonlinear model.

The physical parameters of the four components of the system were presented in Chapter 3 while the position vectors and the aerodynamic coefficients are provided in Tables 4.1 and 4.2, respectively. The aerodynamic coefficients were defined somewhat similar to those used in [10] and tuned comparing the simulations results with the existent tests results provided by Mario Martins in [3].

Table 4.1: Position vectors

Position vector	$X(cm)$	$Y(cm)$	$Z(cm)$
${}^C\vec{R}_{CP_C/CM_C}$	15.0	0.00	-15.0
${}^C\vec{R}_{T1_C/CM_C}$	10.0	-85.0	60.0
${}^C\vec{R}_{T2_C/CM_C}$	10.0	85.0	60.0
${}^L\vec{R}_{T1_D/T1_C}$	-8.00	85.0	255
${}^L\vec{R}_{T2_D/T2_C}$	-8.00	-85.0	255
${}^D\vec{R}_{CP_D/CM_D}$	-3.00	0.00	-2.00
${}^D\vec{R}_{T1_D/CM_D}$	7.00	-2.25	-17.5
${}^D\vec{R}_{T2_D/CM_D}$	7.00	2.25	-17.5
${}^D\vec{R}_{TCAP_D/CM_D}$	4.75	0.00	14.5
${}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D}$	0.00	0.00	300
${}^T\vec{R}_{CM_T/CM_{CAP}}$	0.00	0.00	500

Table 4.2: Aerodynamic coefficients

Parameter	Value
C_{D0}	0.150
$C_{D\alpha^2}$	1.000
$C_{Y\beta}$	0.064
C_{L0}	0.250
$C_{L\alpha}$	2.300
C_{lp_C}	0.755
C_{mq_C}	0.842
C_{nr_C}	0.090
$C_{n\beta_C}$	-0.200
C_{D_D}	0.900
C_{lp_D}	0.655
C_{mq_D}	3.500
C_{nr_D}	0.200
$C_{D_{CAP}}$	1.000
C_{D_T}	1.000

An uncontrolled flight was simulated using a MATLAB code for an interval range of $[0, 15]$ seconds. The simulation starts from 100 meters above sea level with a forward velocity of 3.3 m/s and a 1 m/s downward velocity. Relative to the Euler angles, $\theta_{C/I}$ is initialized with 6.5 degrees, $\theta_{L/C}$ with -9 degrees, $\theta_{D/L}$ with 20 degrees, $\theta_{CAP/D}$ with -7 degrees and $\theta_{T/CAP}$ with 5 degrees. The other state variables are initially nulls. Figure 4.3 shows the three dimensional trajectory of the system. For a better representation, 4 points are used to represent the parafoil system, three points for the drone and one point for the capsule and transponder. The canopy is represented by three collinear points (PL_C , CM_C and PR_C) as seen in the closest view. The other point of the parafoil represents the rotation point of the canopy (R_C). The other three points from top to bottom of the closest view represented the canopy-drone tension point (TC_D), drone center of mass (CMC_D), and drone-capsule tension point ($TCAP_D$). The capsule and transponder are represented by their center of mass.

A more detailed description of the trajectory of each body is shown in Figure 4.4, 4.5, 4.6 and 4.7. It is also shown the variation of the Euler angles as well as the linear and angular velocity of each body. All these parameters are expressed in the inertial frame. Initially, as seen in any figure, the system motion presents a short and damped oscillation. This effect is due to the pendulum effect of the drone, capsule and transponder. After approximately ten seconds, the system stabilizes and has a stable flight.

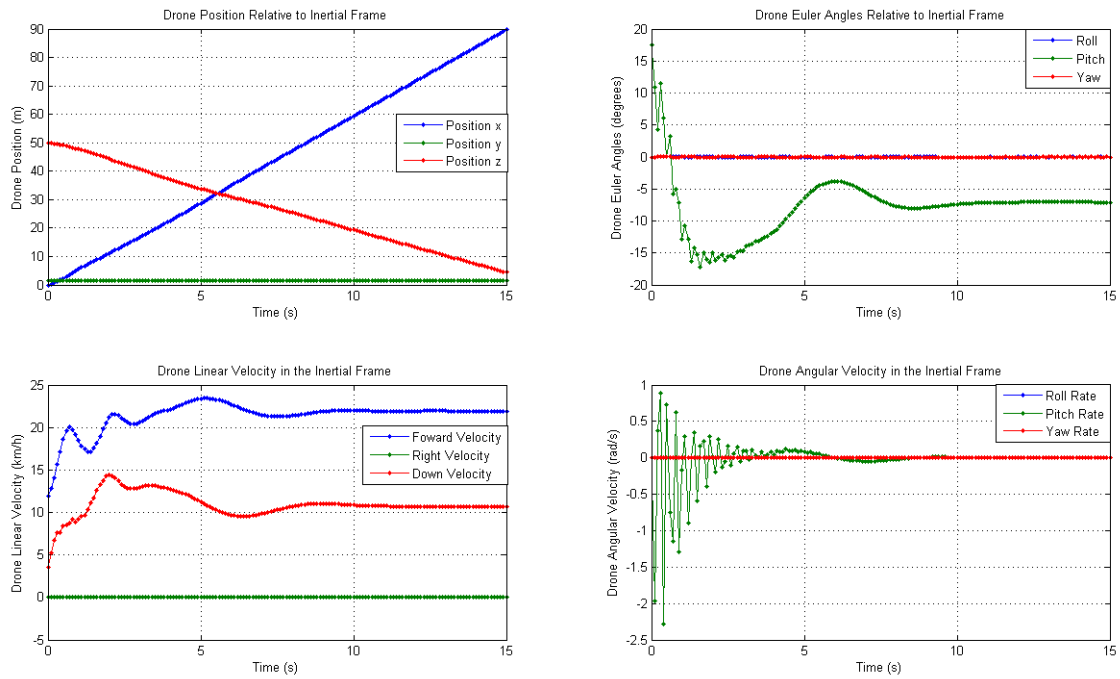


Figure 4.5: Drone kinematics in a simulation of an uncontrolled flight

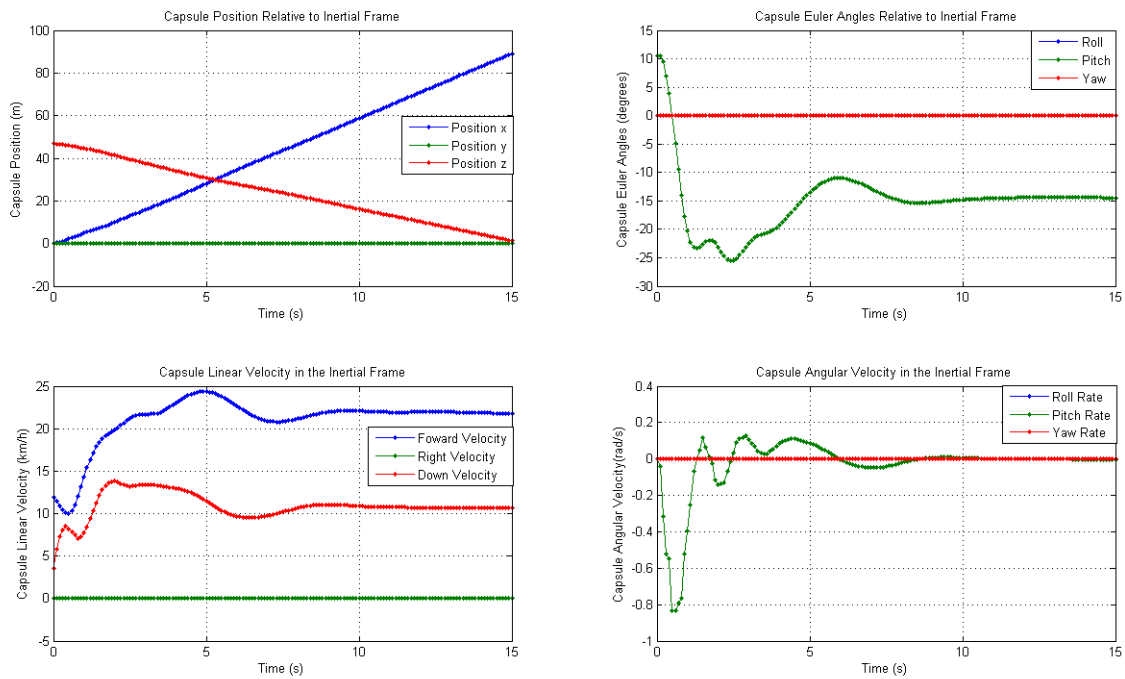


Figure 4.6: Capsule kinematics in a simulation of an uncontrolled flight

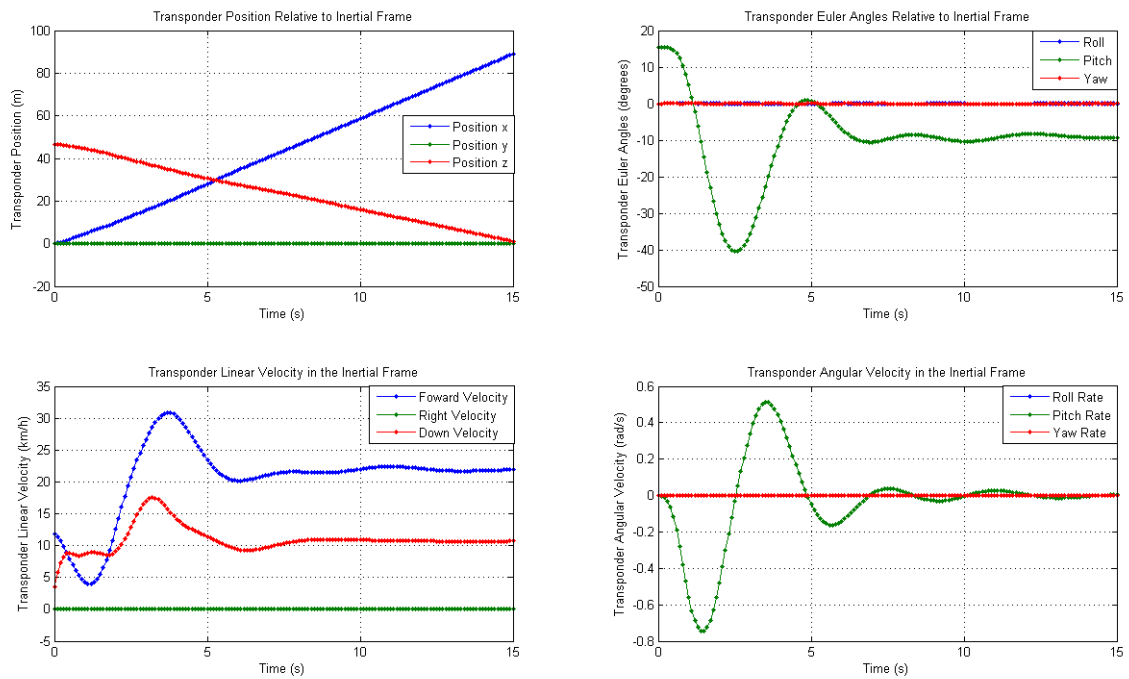


Figure 4.7: Transponder kinematics in a simulation of an uncontrolled flight

In aviation as well as in the paragliding sport, a special motion where the pilot should be careful is the spiral motion. Indeed, if the paraglider motion enters in a spiral, it can be very difficult for the pilot to release the paraglider from the recursive motion. In this way, it is important to analyse in detail this motion. For that, it was simulated the model for a control to the right during 10 seconds. 5 seconds after the action of the control, it was verified that the parafoil goes out of the spiral motion, without any kind of control. This behaviour is very desirable because the parafoil tries alone to follow a stable motion, avoiding irrecoverable motions. The simulation results are illustrated in Figures 4.8 and 4.9.

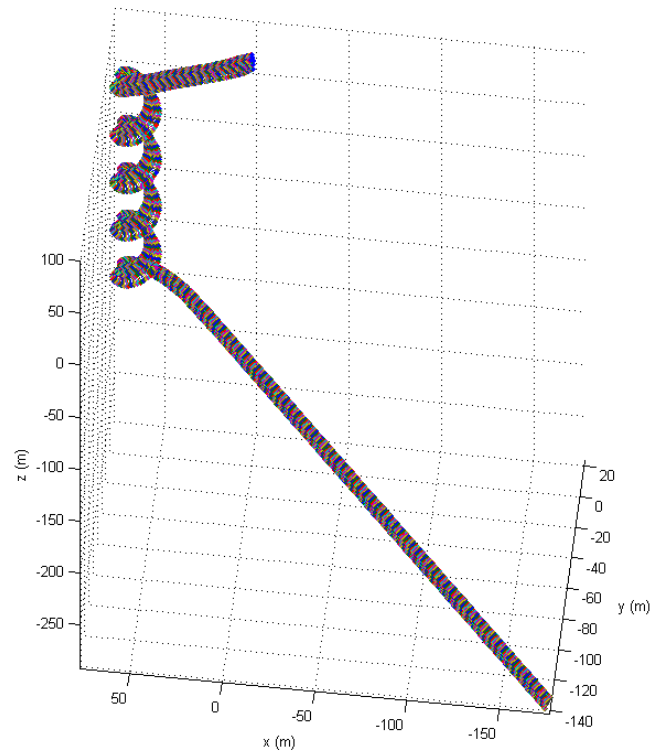


Figure 4.8: Three dimensional trajectory of the platform in a simulation of a spiral motion

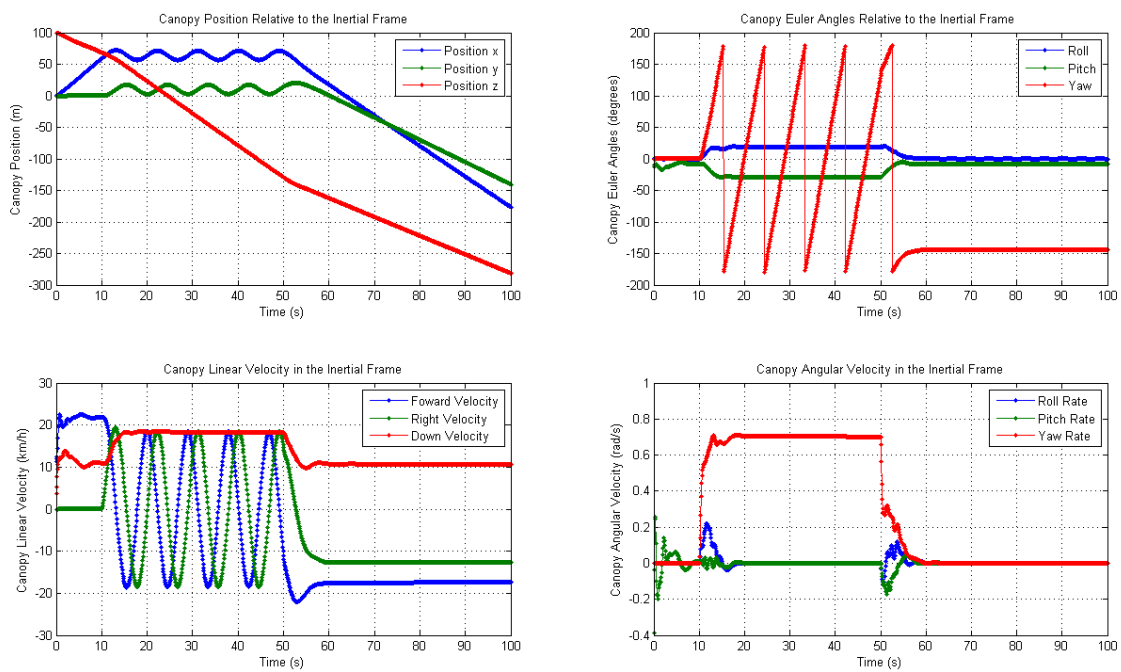


Figure 4.9: Canopy kinematics in a simulation of a spiral motion

One objective of this work is to analyse the behaviour of the platform with the parafoil at a very high altitude. If the parafoil is able to stabilize the system and mainly control its trajectory, it can replace the round paraglider. In this case, it is possible to remove the round paraglider decreasing the platform weight and its complexity. For a launching at 35 km, it is verified an initial exponential increasing of the downward velocity of the canopy, followed by an increasing forward velocity (Figure 4.11). This increase is accomplished by an altitude drop, as seen in Figure 4.11. However, the platform quickly (approximately after 50 seconds) reaches a stable trajectory and the velocity decreases as the system descends. The altitude drop is expected because the resistance of the air is very slow due to the rarefied air at this altitude.

Analysing the Euler angles of the canopy, it should be noted that the canopy reaches a high negative pitch angle (-70 degrees). Although this value can be dangerous, the canopy easily stabilizes in a stable pitch angle. With this result and not considering other possible oscillations at this altitude, it is possible to conclude that the parafoil can stabilize the platform at high altitude and therefore it can replace the round paraglider.

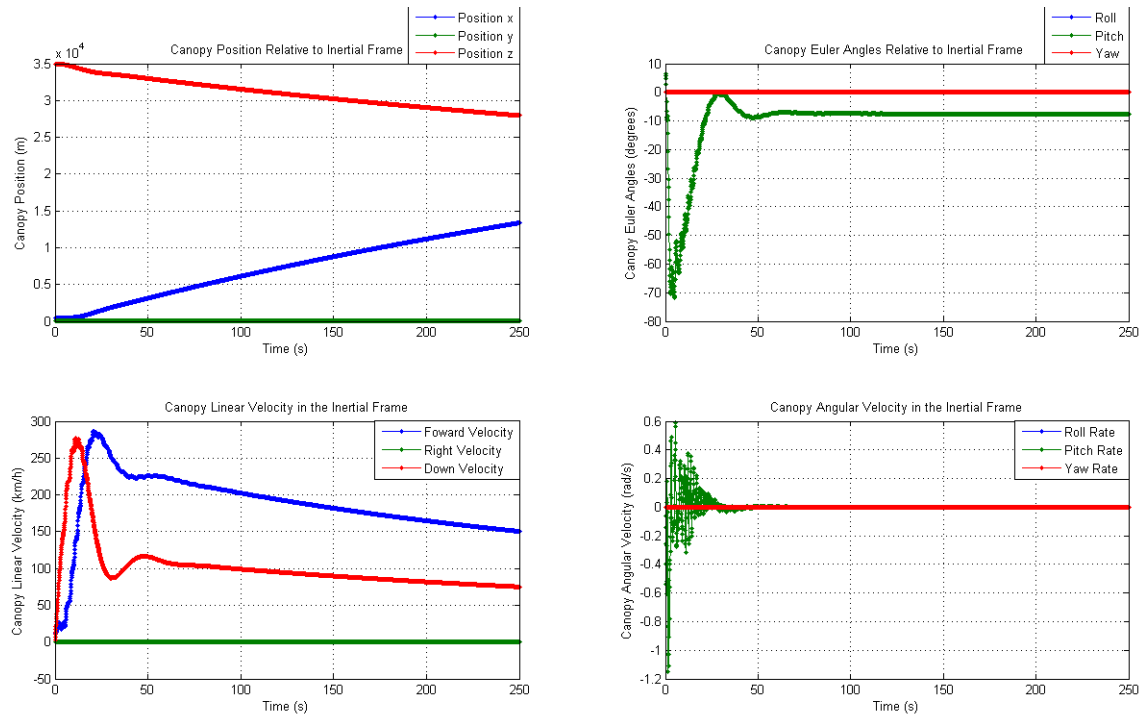


Figure 4.10: Canopy kinematics of a launching at 35 km

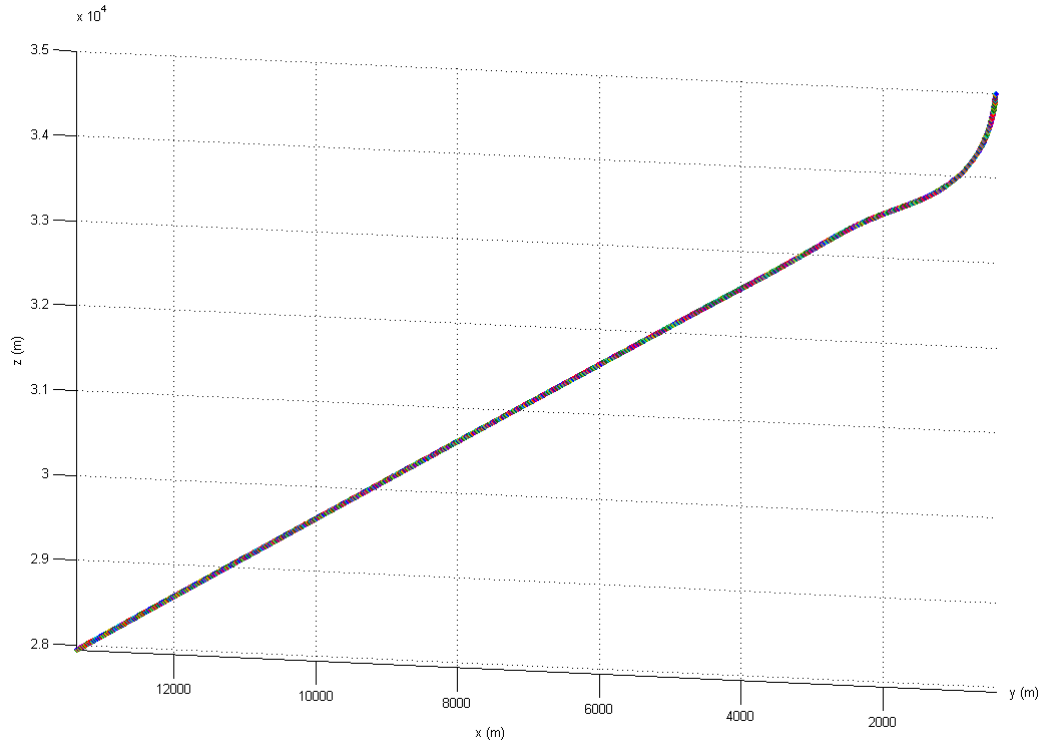


Figure 4.11: Three dimensional trajectory of the platform launched at a high altitude

4.6.1 Experimental and Simulation Results Comparison

A tuning of the aerodynamic coefficients and of some important position vectors of the model was made comparing the simulation results with the existent experimental results. The system which was used in the flight tests is only composed by the parafoil and drone, therefore slight modifications were made to the mathematical model. Although the wind effect was neglected, because the measures of this parameter aren't very reliable, the comparison of the results is a good procedure to understand some behaviours of the dynamic system. For the aerodynamic coefficients defined in Table 4.2, both results are presented in Figure 4.12. It should be noted that the results presented correspond to the drone kinematics and the linear velocity is presented rotated with course, which in this case is equal to yaw angle, so that the right velocity should always be zero.

Observing the Euler angles, such as measuring as simulating, it is possible to see that the yaw angle has 3 oscillations during 8 seconds. Moreover, the oscillation about the roll axis is similar in both cases, with approximately 2 oscillations in 8 seconds. However there are some differences on the magnitude of the angles between the two results. These differences aren't surprising because the wind effect has a significant effect and it isn't taken into account in the simulation results. Analysing the position and the experimental linear velocity, it is clear that at 6 seconds, a strange event happens leading to an increasing velocity and an altitude drop. Neglecting this event, the non linear model can predict the similar behaviours and the modes of the system. In addition to the probable wind presence, it should be noted that the experimental results are also affected by noise as seen in the right velocity.

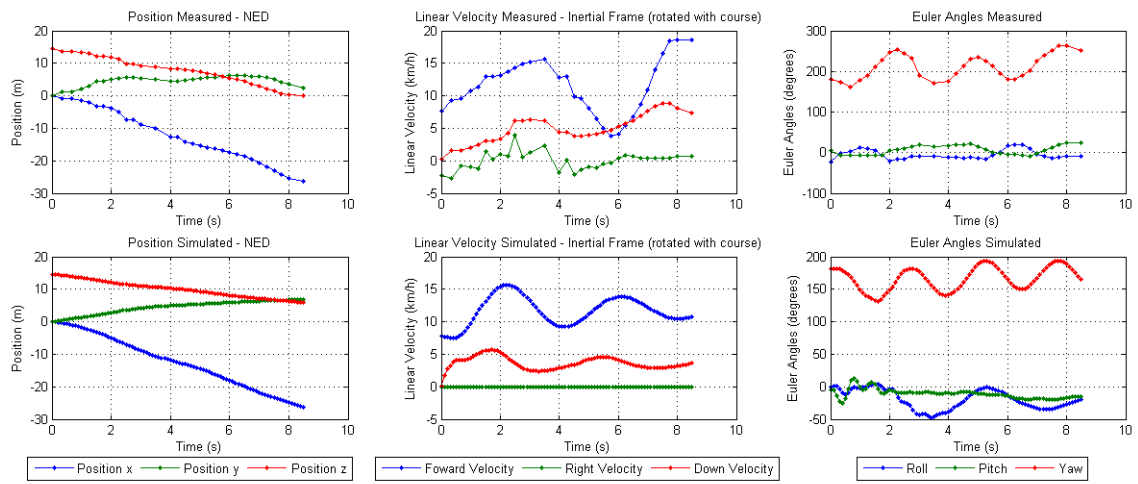


Figure 4.12: Experimental and simulation results comparison

Chapter 5

Control Algorithm

The control algorithm performed in this work will be analysed in this chapter. The main goal of the control algorithm is to manage the system to follow a specific path. For that it is used two control inputs: main lines and symmetric brakes. The main lines allow changing the orientation of the system while the brakes, in the symmetric mode, allow changing the glide slope of the trajectory. In this way, the control algorithm can be divided in *Horizontal Control* and *Vertical Control*. The first one is controlled by the main lines while the brakes control the vertical component. Sections 5.1 and 5.2 describe these two types of control, respectively. At the end, a new approach for the landing phase is designed in Section 5.3, basing on the guidance strategy in spiral mode previously presented in Section 3.7.

5.1 Horizontal Control

The main goal of the horizontal control system is to command the system to follow a specific path controlling the length of the main lines. A displacement of these lines, made by the differential servo-actuator, makes the canopy rotate about the roll axis. On the other hand, a non-zero roll angle induces a yaw motion in the canopy, making the system turn for the deflected side of the canopy. For example, if the right line is pulled down, the roll angle of the canopy increases which induces the system to turn to the right side.

The path forward can be defined by three ways: a specified direction or a specified direction and one point or two points. Taking into account possible wind presence during the flight, three types of strategy control were defined: *Heading*, *Course*, *One Point Localizer* and *Course or Two Points Localizer*. These types of control are performed as a feedback control and the controller used is a PD (Proportional and Derivative) controller. This controller acts in the system with a proportional gain and a derivative gain on the error and error variation, respectively.

5.1.1 Heading

The heading control allows managing the system toward a direction with a specified heading, where heading is the angle between the direction that the chord line of the canopy is pointing and

the North, i.e. it is equal to the yaw angle. The control errors of this control system are the heading error (H_{Error}) and heading error of the variation (HV_{Error}), defined in equations (5.1) and (5.2), respectively.

$$H_{Error} = \psi_C - H_0 \quad (5.1)$$

$$HV_{Error} = \dot{\psi}_C \quad (5.2)$$

where H_0 is the desired heading of the specific path and its variation is null.

The PD controller for this control system can then be written as:

$$H_U = -HK_1 H_{Error} - HK_2 HV_{Error} \quad (5.3)$$

where HK_1 and HK_2 are constants which determine the control responsiveness.

5.1.2 Course

In the presence of wind, the heading control, presented above, can be inefficient because it doesn't take into account the wind effect. Indeed, in the presence of wind, the system can experience a side shift caused by the wind and may describe a non-desired trajectory. To avoid this situation, the course control is performed where course refers to the angle between the direction of the path which the canopy describes and the North. In other words, the course is the angle between the inertial velocity components $u_{CM_C/I}$ and $v_{CM_C/I}$. In a null wind condition, the heading and course angles are equals. The controller of the course control system is similar to the one used in the heading control system, replacing the heading errors by the course error (C_{Error}) and course error of the variation (CV_{Error}). These two errors are defined in equations (5.4) and (5.5).

$$C_{Error} = \arctan\left(\frac{v_{CM_C/I}}{u_{CM_C/I}}\right) - C_0 \quad (5.4)$$

$$CV_{Error} = \dot{\psi}_C \quad (5.5)$$

where C_0 is the desired course of the specific path and its variation is null. It should be noted that the velocity components $^I u_{CM_C/I}$ and $^I v_{CM_C/I}$ are expressed in the inertial frame. Due to the lack of available data about the variation of the linear velocity, the error CV_{Error} is approximately to the

error HV_{Error} described in equation (5.2). This assumption can be considered valid because the variation of the course angle is similar to the variation of the heading angle.

Considering the course errors, the controller of the course control system is defined as:

$$H_U = -HK_1C_{Error} - HK_2CV_{Error} \quad (5.6)$$

5.1.3 One Point Localizer and Course or Two Points Localizer

A more complete control strategy uses a specified course and a localizer or if it is more desired, two localizers. These two types of control are equivalents, both define a specific path which passes through a point. So only one controller will be analysed which can be applied in both control systems. Because it is easier, *One Point Localizer and Course* method will be analysed. This method is one of the most used navigation methods (Figure 5.1). This type of control system guides the system through a desired path in the direction of the localizer.

To develop an accurate controller, three different errors are considered: cross track error (CT_{Error}), bearing error (B_{Error}) and bearing error of variation (BV_{Error}), where cross track error is the distance between the actual point and its projection in desired path as seen in Figure 5.1. Through trigonometric equations, the expression (5.7) of this error can be easily determined. In order to avoid a great overshoot on the control response and to ensure a shorter path toward the target, this control system has two stages: approach and following phase. The first one corresponds to a cross track error higher than $\frac{CD}{5}$, where CD is the value of the Convergence Distance, given by the user. In this phase, the system follows to a path at an angle α defined in equation (5.8).

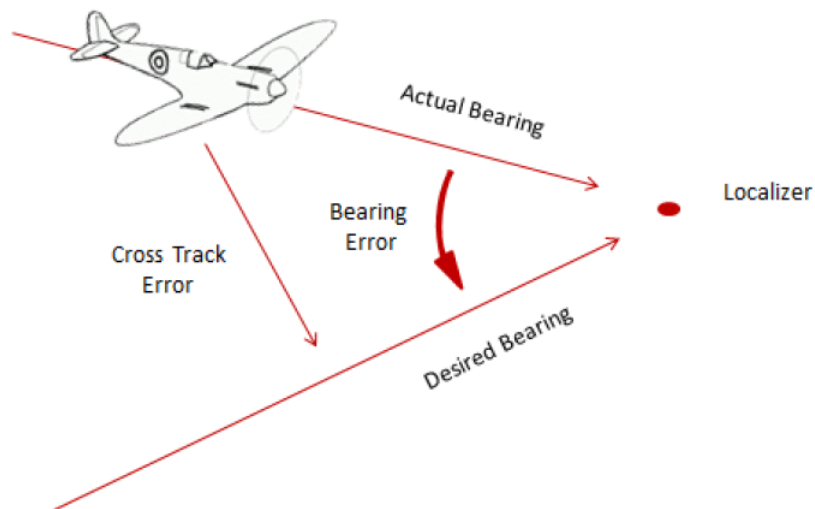


Figure 5.1: One point localizer and course [3]

$$CT_{Error} = (y_{CM_C/I} - y_0) \cos(C_0) - (x_{CM_C/I} - x_0) \sin(C_0) \quad (5.7)$$

where x_0 and y_0 are the positions (x,y) of the localizer and C_0 is the desired course of the specific path.

$$\alpha = \arctan \left(\frac{\min(|CT_{Error}|, CD)}{CD} \right) \text{sign}(CT_{Error}) \quad (5.8)$$

where sign is the mathematical function that extracts the sign of a real number.

The error B_{Error} is defined as the angle between the ideal path and the actual path of the system (Figure 5.1). Relative to the angle α it can be calculated as:

$$B_{Error} = \arctan \left(\frac{I_{v_{CM_C/I}}}{I_{u_{CM_C/I}}} \right) - (C_0 - \alpha) \quad (5.9)$$

The error BV_{Error} is equal to the course error variation CV_{Error} (5.5). The control expression of this phase is then given by:

$$H_U = -HK_1 B_{Error} - HK_2 BV_{Error} \quad (5.10)$$

When the cross track error is under the threshold value $\left(\frac{CD}{5}\right)$, the phase switches to the following phase. The main goal of this phase is to ensure that the system heads the desired path toward the target. The control expression of this phase can be written as:

$$H_U = -HK_1 B_{Error} - HK_2 BV_{Error} - HK_3 CT_{Error} \quad (5.11)$$

where CT_{Error} is defined in equation (5.7) and B_{Error} and BV_{Error} are equal to C_{Error} and CV_{Error} , respectively (equations (5.4) and (5.5)).

An example of the *One Point Localizer and Course* method is illustrated in Figure 5.2. The localizer point chosen is $(x = 100, y = 200)$ and the desired course is equal to 90 degrees. Initially, the platform follows a course of 45 degrees and only when it approaches the line $x = 100$, the platform turns to the course of 90 degrees. After some time, it reaches the localizer, as seen in figure. In this example it is visible the two distinct phases of this control algorithm.

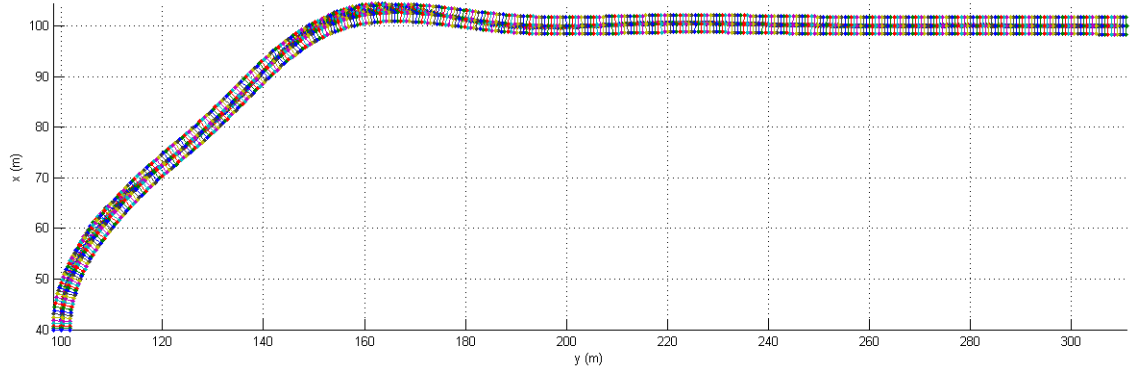


Figure 5.2: Three dimensional trajectory in a simulation of an horizontal controlled flight

5.2 Vertical Control

The main goal of the vertical control system is to handle the system to a specific glide slope. For that, two types of errors are used: altitude error (A_{Error}) and glide slope error (GS_{Error}). The altitude error is the distance between the actual point and its perpendicular projection in the desired path. Through a trigonometric manipulation, this error can be calculated using the equation (5.12). Such as in the *One Point Localizer and Course* control system, this control has two stages: approach and following phase. The first one is activated when the altitude error is higher than $\frac{CD}{5}$. On the other hand, the following phase is activated. The approach phase controls the system to follow a path at an angle β defined in equation (5.13).

$$A_{Error} = (y_{CM_C/I} - y_0) \cos(\theta) \cos(C_0) + (y_{CM_C/I} - y_0) \cos(\theta) \sin(C_0) - (z_{CM_C/I} - z_0) \sin(\theta) \quad (5.12)$$

where x_0 , y_0 and z_0 are the positions (x,y,z) of the localizer, C_0 is the desired course of the path and θ is the angle of the path slope.

$$\beta = \arctan\left(\frac{\min(|A_{Error}|, CD)}{CD}\right) \text{sign}(A_{Error}) \quad (5.13)$$

The glide slope error can then be written as:

$$GS_{Error} = \frac{c_{w_{CM_C/I}}}{c_{u_{CM_C/I}}} - (GS_0 - \tan(\beta)) \quad (5.14)$$

where GS_0 is the desired glide slope. It should be noted that $c_{w_{CM_C/I}}$ and $c_{u_{CM_C/I}}$ are expressed in the canopy frame.

The controller expression of this phase is a proportional controller with gain VK_1 :

$$V_U = -VK_1 GS_{Error} \quad (5.15)$$

When the altitude error is under the threshold value $\left(\frac{CD}{5}\right)$, the stage switches to the following phase. The main goal of this phase is to ensure that the system heads the desired glide slope toward the target. The controller expression of this phase can be written as:

$$V_U = -VK_1 GS_{Error} - VK_2 A_{Error} \quad (5.16)$$

where A_{Error} and GS_{Error} are defined in equations (5.12) and (5.17), respectively, and VK_1 and VK_2 are constants which determine the control responsiveness.

$$GS_{Error} = \frac{C_{w_{CMC/I}}}{C_{u_{CMC/I}}} - GS_0 \quad (5.17)$$

An example of this control is presented in Figure 5.3. One can see that the trajectory can be spilt into two distinct parts. At the beginning, the control system limits the glide slope of the trajectory. When the system reaches a certain altitude (around 90 meters), the control system switches to the next phase (following phase). After that, the platform goes down with a smooth glide slope in the direction of the localizer.

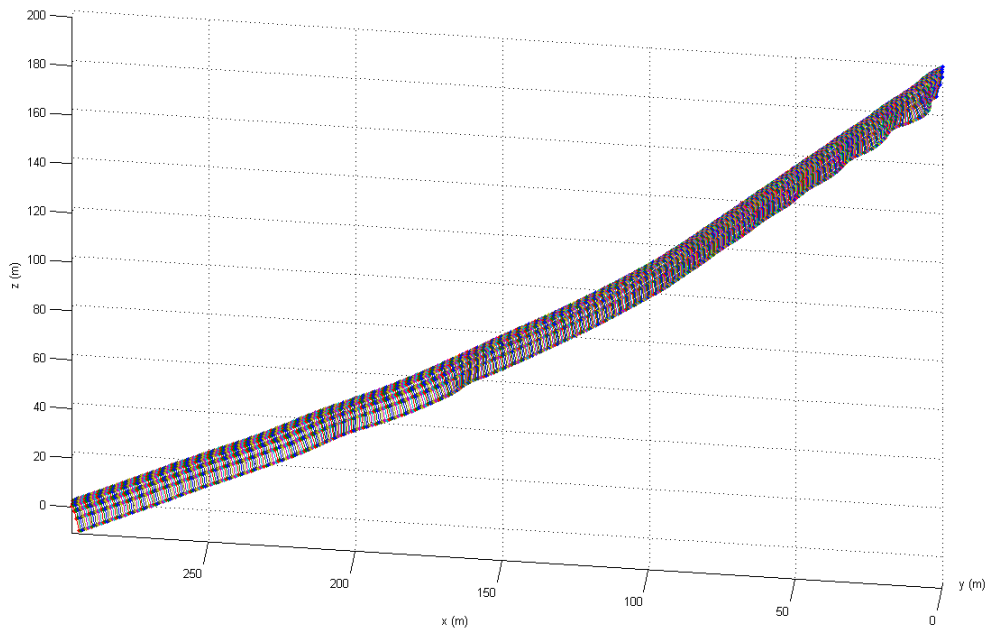


Figure 5.3: Three dimensional trajectory in a simulation of a vertical controlled flight

5.3 Spiral Mode

In Section 3.7 it was presented a desired controllable flight. After the *Heading* or *Course* control is applied to guide the system through a desired direction, the system should perform a spiral motion until the land phase. This section will analyse the *Spiral Mode* control which will be designed to control the spiral motion as well as the land phase of the flight. Before the explanation of this algorithm, a detailed description of the guidance strategy in spiral mode should be done. The main goal of this control system is to ensure that the system lands safely on a desired place. However due to the probable constant wind presence in the troposphere layer, a straight path can be undesirable. Indeed, the system may not be able to overcome the wind effect. A possible solution is the spiral motion. Dividing the spiral motion in four different branches (like a square), the system can easily correct some wind perturbation by scaling the branches length. On the other hand, the system can fly in a slide spiral motion affected by the wind presence.

With these assumptions in mind, a control algorithm was designed. The control strategy is divided in seven distinct phases. In the first one (*approximation phase*) the *One Point Localizer and Course* control system is used to manage the system to a desired point, taking into account the wind presence. The localizer point is chosen according to the region where the system is. An example is showed in Figure 5.4, where the system flies over the green region. In this example, the control should manage the system for the right branch (Branch 2).

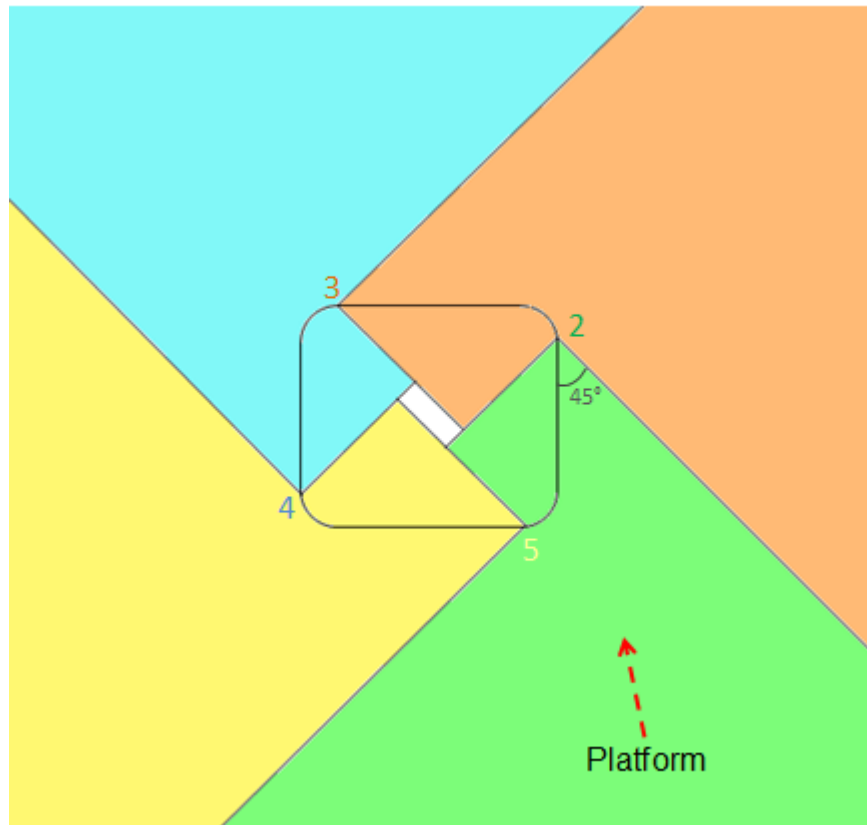


Figure 5.4: Spiral region decision

The four next phases correspond to each branch of the spiral. In each branch, it is used the *One Point Localizer and Course* control system to guide the system through the respective branch until it reaches a safe distance from the next phase. This distance should be enough to allow a turn of 90 degrees. After phase 5 is concluded phase 2 restarts. Phase 6 corresponds to the final stage of the spiral motion and starts when the system is in phase 5 at a certain altitude. This altitude is continuously computed during the spiral motion for an accurate landing. In phase 6, it is activated both *One Point Localizer and Course* and *Vertical Control* systems in direction to the desired land point. This phase is previously computed so that the system lands into the wind. The spiral control system finishes with phase 7 where the brakes are pulled down in order to reduce the impact velocity. The block diagram (Figure 5.7) shows the spiral control algorithm designed in this work.

An example of this control method is shown in Figures 5.5 and 5.6, where the wind presence is taken into account. The wind direction is from the north to the south with 0.5 m/s of magnitude. As seen in Figure 5.5, the system describes a slide spiral motion in the direction of the desired landing point ($x = 200, y = 100, z = 0$), which was initially defined. The system describes 3 spirals, with a branch length of approximately 100 meters, until reaches the final approach. Due to the wind presence, each spiral has a branch longer than the other in order to approach the landing point. After completing 3 spirals, the system faces the wind in the final phases. After heading the desired direction, the vertical control system manages the glide slope of the trajectory. When the platform is near to the ground the brakes are pulled down to the maximum, decreasing the velocity as seen in Figure 5.6. It should be noted that the yaw angle was limited in the range between -180° to 180° .

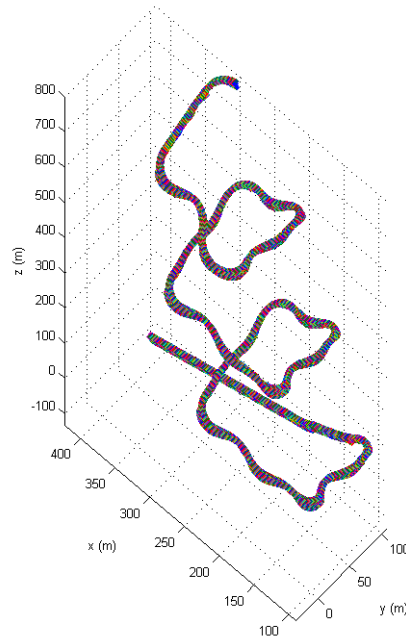


Figure 5.5: Three dimensional trajectory in a simulation of a spiral controlled flight

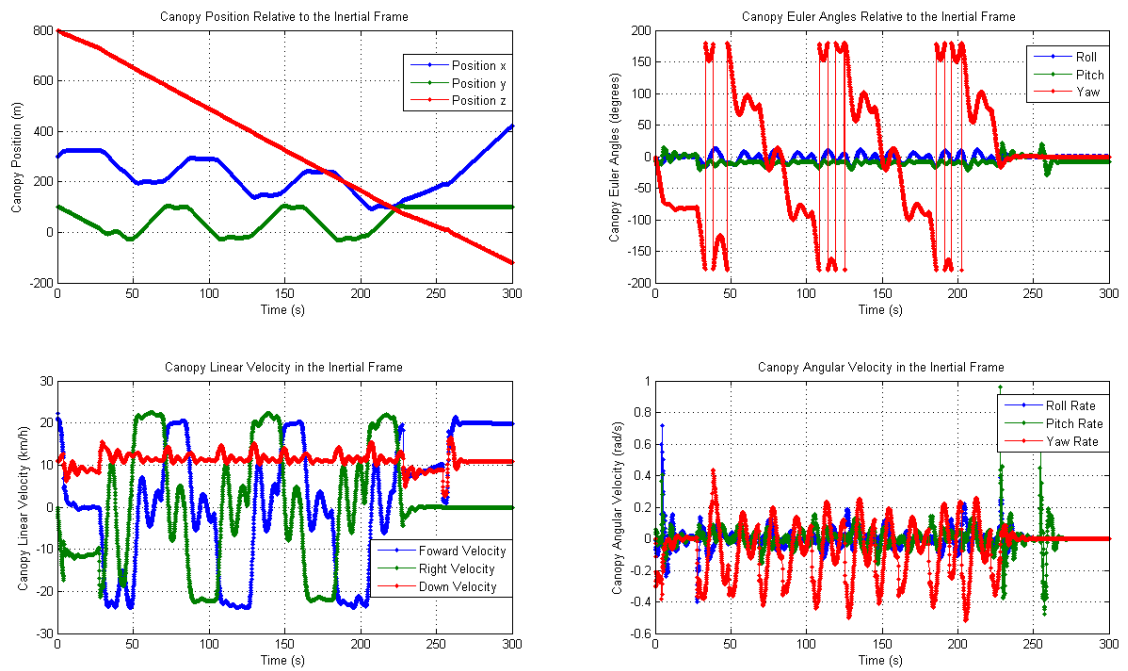


Figure 5.6: Canopy kinematics in a simulation of a spiral controlled flight

Chapter 6

Stability Analysis

When designing a control system, one of the most important questions which should be in mind is whether the system is stable, because an unstable control system is usually dangerous and unwanted. Before the stability analysis, the non linear model is firstly linearised by the Lyapunov's linearization method in Section 6.1. In this section, a comparison of the linear model with the nonlinear model is made resorting to the simulation results. The stability of the linear model is analysed in Section 6.2. For the purposes of this work, it is studied the internal stability, controllability and observability. The optimization of the control gains to enhance the control system responsiveness is developed in Section 6.3. Section 6.4 presents the Graphic User Interface (GUI) specially developed in this work.

6.1 Linear Model

The equations of motion described in Chapter 4 are nonlinear differential equations which can be expressed compactly in matrix form as:

$$\dot{X} = f(X, U) \quad (6.1)$$

where X and U are the state and input vector, respectively. X is a vector of n state variables while U has dimension p , number of control inputs. As it was stated in Chapter 4, $n=35$ and $p=2$.

A linearization method allows defining the derivative of the state vector $\dot{X}$ as a linear combination of the state vector themselves and the input vector of the form:

$$\dot{X} = AX + BU \quad (6.2)$$

where A is a square matrix of dimension $n \times n$ and it is usually called *state matrix*. B is the *input matrix* of dimension $n \times p$.

The equation (6.2) along with the equation (6.3) defines completely a linear system. This representation is called *space state* and it is a very important tool in the control theory.

$$Y = CX + DU \quad (6.3)$$

where Y is the output vector of dimension q , C is the *output matrix* of dimension $q \times n$ and D is the *feedthrough matrix* of dimension $q \times p$.

The output variables which will be considered in this work are the variables which the drone can measure.

$$Y = \begin{bmatrix} I_{x_{CM_D/I}}, & I_{y_{CM_D/I}}, & I_{z_{CM_D/I}}, & I_{u_{CM_D/I}}, & I_{v_{CM_D/I}}, & I_{w_{CM_D/I}}, & I_{\phi_{D/I}}, & I_{\theta_{D/I}}, & I_{\psi_{D/I}} \end{bmatrix} \quad (6.4)$$

6.1.1 Lyapunov's Linearization Method

One linearization method is the Lyapunov's linearization method developed by Lyapunov to apply on the stability theory. His method is a formalization of the intuition that a nonlinear system should behave similarly to its linearized approximation for small range motions [21]. The linearization method should be performed around an equilibrium point ¹ $[x^*, u^*]$. On a small range region, the equation (6.1) can be approximated to:

$$\dot{x} = \left(\frac{\partial f}{\partial x} \right)_{(x=x^*, u=u^*)} x + \left(\frac{\partial f}{\partial u} \right)_{(x=x^*, u=u^*)} u \quad (6.5)$$

Comparing the equation (6.5) with (6.2), one can note that both are equivalent, taking:

$$A = \left(\frac{\partial f}{\partial x} \right)_{(x=x^*, u=u^*)} \quad (6.6)$$

$$B = \left(\frac{\partial f}{\partial u} \right)_{(x=x^*, u=u^*)} \quad (6.7)$$

So, the linearization of a nonlinear model consists in deriving the n differential equations f for the n state variables to obtain the matrix A while B is obtained deriving the n differential equations f in order to the q input variables. Where the derivatives should be performed around an equilibrium point. The same procedure should be made in the output equation to obtain the linear equation (6.3).

Using the Lyapunov's linearization method, the linear model was developed around the equilibrium point whose non-zero components are shown in Table 6.1. This equilibrium point was chosen observing the simulation results illustrated in Chapter 4. The values x and z can take any number in the meter unit except z which must be negative.

¹An equilibrium point is a solution of the system equations in which the derivative of the state vector in this point is null.

Table 6.1: Equilibrium point

Parameter	X	Z	u	w	$\theta_{C/I}$	$\theta_{L/C}$	$\theta_{D/L}$	$\theta_{CAP/D}$	$\theta_{T/CAP}$
Value	x	z	22 m/s	11 m/s	-8.1°	-5.3°	6.1°	-7.1°	5.4°

X , Z , u and w are the positions and linear velocity components of the canopy, respectively.

Using a MATLAB code specifically developed to apply the Lyapunov's linearization method, the nonlinear model as well as the output equation were linearised. The matrices of the space state A , B , C and D which were obtained by the linearization method are illustrated in Appendix A.12.

6.1.2 Linear and Nonlinear Model Comparison

A comparison of the simulation results of two models is performed to evaluate and validate the linear model. For the same initial conditions, the results obtained are:

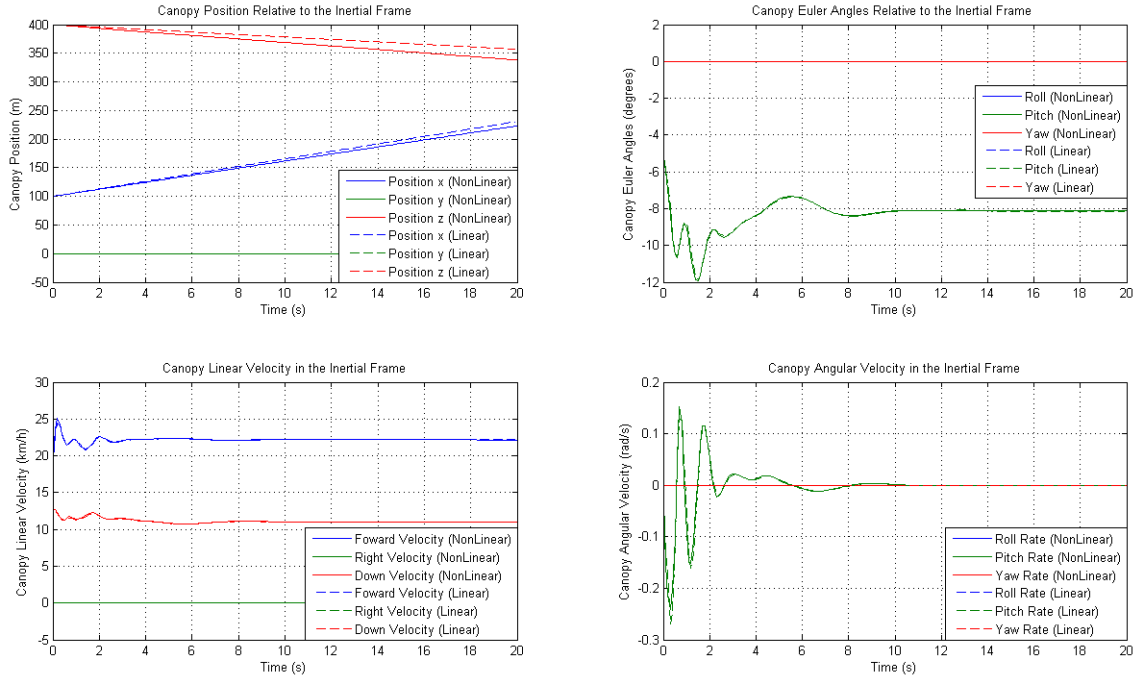


Figure 6.1: Simulation results of the linear and nonlinear model

Analysing the dynamic of the canopy body, one can observe that both results are very similar. In effect, there is an overlap of the graphics, except to the positions x and z . These differences exist because the positions x and z of the equilibrium point are undefined. However, since these variables don't affect the system dynamic, the differences observed aren't significant. In addition to these variables, the position y and yaw angle of the canopy don't affect the system dynamic either and can take any value in the equilibrium point. These conclusions are very important because these variables haven't got an equilibrium value but they are always stable. In the control theory, this means that the matrix A will have at least 4 eigenvalues at the origin.

6.2 Stability Analysis

6.2.1 Stability

Qualitatively, a system is said as stable if starting the system somewhere near its desired operating point implies that it will stay around the point from then onwards [21]. The Lyapunov stability theory allows to easily analyse the stability of a system analysing the state matrix A . According to the Lyapunov theorem, a linearized system or a matrix A is strictly stable, if all eigenvalues of A are strictly in the left-half complex plane [21]. Thus, calculating the eigenvalues λ of the matrix A , presented in Appendix A.12, one obtains:

$$\begin{aligned} \lambda = [& 0, 0, 0, 0, -0.26 \pm 1.53j, -0.36 \pm 1.64j, -0.40 \pm 0.93j, -0.62, -0.74 \pm 3.85j, \dots \\ & -0.91 \pm 5.34j, -1.16 \pm 22.98j, -1.17 \pm 4.45j, -1.32 \pm 30.72j, -1.88 \pm 6.53j, \dots \\ & -2.03, -3.91 \pm 5.50j, -5.0, -5.0, -5.0, -15.30 \pm 10.47j]^T \end{aligned}$$

As it was expected, one can observe that there are four eigenvalues equals to zero. These eigenvalues corresponds to the state variables which can take any value without affecting the system dynamic. As it was stated before, these variables are three positions and yaw angle of the canopy body. Neglecting the eigenvalues of these state variables, one can note that the other eigenvalues of A are in the left-half complex plane (part real less than zero), so the system is stable.

6.2.2 Controllability

As explained in [22], a system or the pair (A, B) is said to be controllable if for any initial state $x(0) = x_0$ and any final state x_f , there exists an input sequence that transfers the system from x_0 to x_f in a finite time. Mathematically, the controllability of a system can be checked through the controllability matrix which is given by:

$$\mathcal{C}(A, B) = \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix} \quad (6.8)$$

where n is the number of state variables.

According to Chi-Tsong Chen [22], a necessary and sufficient condition for the system to be controlled is that $\text{rank}(\mathcal{C}(A, B)) = n$. If $\text{rank}(\mathcal{C}(A, B)) < n$, there exists $n - \text{rank}(\mathcal{C}(A, B))$ uncontrollable state variables and $\text{rank}(\mathcal{C}(A, B))$ controllable state variables and the system is said to be uncontrollable. Using the MATLAB software, it was verified that 29 state variables are controllable ($\text{rank}(\mathcal{C}(A, B)) = 29$) and therefore the system isn't fully controllable. Indeed, the position x and position z aren't controllable.

6.2.3 Observability

A similar analysis can be done for the concept of the observability which is dual to that of controllability. Observability studies the possibility of determining the state vector from the output. For that it is necessary to consider the state equations (6.2), (6.3). A system or the pair (A, C) is said to be observable if any unknown initial state x_0 can be determined over a finite time $t_1 > 0$ from the knowledge of the input u and the output y over the time t_1 [22]. In a similar manner for the concept of the controllability, the verification of observability of a system can be performed through the observability matrix given by:

$$\mathcal{O}(A, C) = \begin{bmatrix} C \\ CA \\ \vdots \\ C^{n-1}A \end{bmatrix} \quad (6.9)$$

According to Chi-Tsong Chen, a necessary and sufficient condition for the system to be observable is that $\text{rank}(\mathcal{O}(A, C)) = n$. If $\text{rank}(\mathcal{O}(A, C)) < n$, there exists $n - \text{rank}(\mathcal{O}(A, C))$ unobservable state variables and $\text{rank}(\mathcal{O}(A, C))$ observable state variables and the system is said to be unobservable. Using the MATLAB software, it was verified that all state variables are observable ($\text{rank}(\mathcal{O}(A, C)) = 31$) and therefore the system is observable.

6.3 Optimization of the Control Gains

The control vector U of each control system developed in Chapter 5, can be written in the following state feedback form:

$$U = -KX \quad (6.10)$$

where K is a vector of dimension $p \times n$ and dependent of the control gains (HK_1, HK_2, HK_3, VK_1 or VK_2).

Replacing U in equation (6.2) by the expression in (6.10), it becomes:

$$\dot{X} = (A - BK)X \quad (6.11)$$

So, the closed-loop control can modify the controllable eigenvalues of the state matrix. An usual procedure to determine the optimal K is to place the desired closed-loop eigenvalues of the system by computing K . The desired closed-loop eigenvalues are normally chosen moving away the slowest controllable eigenvalues of the origin. A very used method is the pole placement technique which determines the optimal vector K for a set of desired closed-loop eigenvalues of the system. However, this technique isn't applicable in the control system developed here, because the control system isn't able to handle all state vectors. In effect, the control vector K is limited for each control system.

Besides the pole placement technique, the optimal vector K can be found through a optimization method. Knowing the non-zero elements of the vector K , through the linearization method, it is possible to find the optimal vector which minimizes a cost function. The cost function is defined by the difference between the real part of the eigenvalues of the matrix A and the real part of the desired closed-loop eigenvalues. Computing this method for each control system, the optimal control vector K was determined. The vectors are described in Appendix A.13, where KH_1 is the optimal vector of the *Heading Control*, KH_2 is the optimal vector of the *Course Control*, KH_{3a} is the optimal vector of the approach phase of the *One Point Localizer and Course Control*, KH_{3b} is the optimal vector of the following phase of the *One Point Localizer and Course Control*, KV_a is the optimal vector of the approach phase of the *Vertical Control*, KV_b is the optimal vector of the following phase of the *Vertical Control*. Through these optimal control vectors, it is now possible to determine the optimal control gains of the controllers.

6.4 Graphic User Interface

The objective of an easy and quick simulation of the model led to the development of a Graphic User Interface (GUI). This interface (Figure 6.2) was done on the GUI development environment of the MATLAB software and allow the user to simulate several types of models and choose the desired control algorithm, among others.

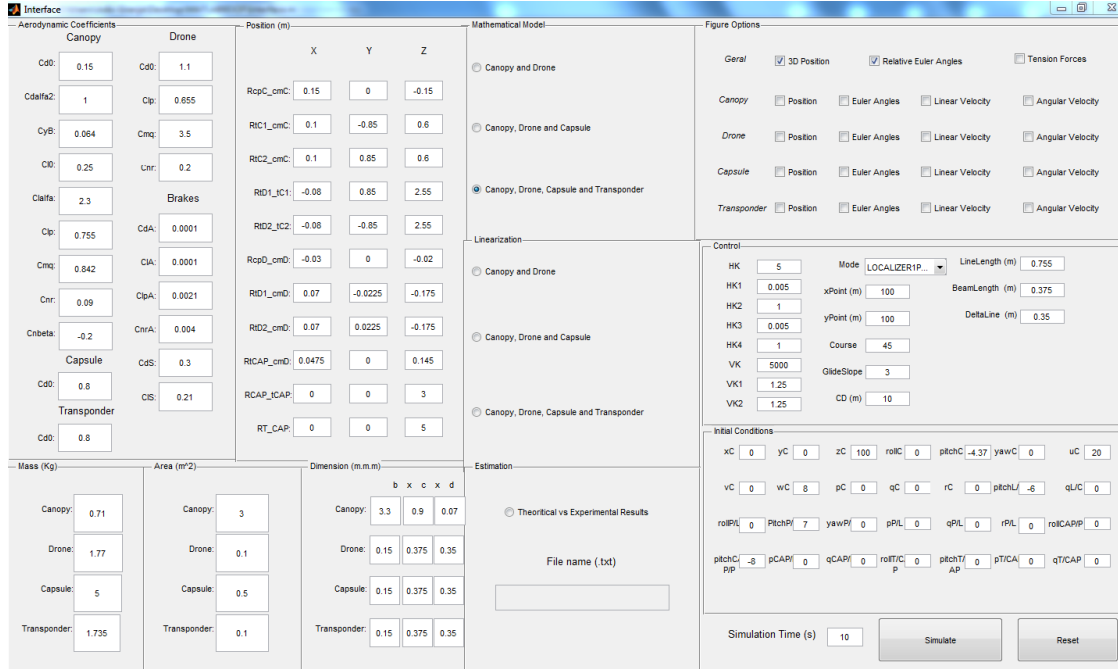


Figure 6.2: Graphic User Interface

This interface can be divided in three sub-windows. The first one, on the left side, is composed by five panels where the user can introduce data of the system which will be used in the simulation. On the left upper side it is possible to change the aerodynamic coefficients of the four

components of the platform as well as the brakes'. It should be noted that these parameters have some restrictions on their value, since these should be realistic values. The panel *Position* provides the possibility to set different configurations of the system, with respect to the lines' length and position of the most important points of the bodies. The mass, area and dimension of the bodies can also be chosen in the three lower left corner panels.

On the center of the screen, the second sub-window shows the different simulations which can be performed: *Mathematical Model*, *Linearization* and *Estimation*. In the first one it is simulated the mathematical model of the selected bodies and the data introduced on the other sub-windows. In the *Linearization* panel, the linearization method and the stability analysis is made. Finally, the comparison of the theoretical with the experimental results can be done in the *Estimation* panel. In this panel, the user should introduce the file name, which contains the experimental results, ending in ".txt".

On the right side of the GUI, the third sub-window is divided in three panels. On the right upper corner, several types of figures can be selected for a better analysis of the simulation results. The control mode as well as its parameters can be chosen on the second panel. In this way, each control system responsiveness can be analysed changing the respective control gains. Finally, the initial conditions of the mathematical model, if necessary, can be changed on the right lower corner.

The *Simulate* button along with the *Simulation Time*, which should be introduced in seconds, allow the simulation of the model. If it is desired, the default values of the parameters of the model can be reselected through the *Reset* button.

Chapter 7

Final Remarks

The final conclusions as well as the next steps in the framework of the parafoil control for the STRAPLEX will be presented. Section 7.1 presents some general conclusions of the work developed. It is highlighted the flight behaviour modelled by the mathematical model and the control algorithm specially developed for a specific purpose. In Section 7.2, it is presented a few important steps forward in the STRAPLEX.

7.1 Conclusions

A new and high fidelity mathematical model was developed for the STRAPLEX in order to model the nonlinear dynamic motion of a descending flight. Through an analysis of the kinematics and dynamics of the system, a nonlinear model with 31 state variables was derived. During the development of this model, the expressions of the position, Euler angles, linear and angular velocity, linear and angular acceleration of the bodies were defined resorting to kinematical equations. In addition to these equations, the dynamic equations analysing the forces and moments applied in each body of the platform (canopy, drone, capsule and transponder) were derived. The aerodynamic forces and moments as well as the apparent force and moment for the canopy body were studied and modelled to make the model the most realistic possible. In addition to these forces, it was taken into account the constraint forces and the relative motion between the bodies.

The features of the system were determined through direct measurements (e.g. weight, physical dimensions, among others) and through indirect measurements (e.g. aerodynamic coefficients) obtained from an analysis of the existent experimental results and of the results of the scientific paper. The model was numerically integrated for several realistic flight conditions using a code specially developed in the MATLAB environment. The simulation results demonstrated that a stable flight is quickly achieved even when the platform is launched at high altitude. In this situation, the platform reaches high downward and forward velocities, making a striking total velocity of around 300km/h . However the parafoil proved to be able to stabilize the platform motion and after reaching a peak, the velocity decreases to a stable value. This last result is very important

once the parafoil can replace the paraglider, reducing the complexity of the STRAPLEX. Moreover the control system can be activated as soon as the platform reaches a stable motion. Another important conclusion was achieved when the platform enters in a spiral motion. As it is desired, the platform can get out of this motion without any kind of control.

One of main objectives of this work was to design a robust control algorithm to land the platform in a specific landing point taking into account the wind effect. An algorithm based in a spiral motion approach was designed using the horizontal and vertical control system. A virtual flight was simulated with certain wind conditions and for a chosen landing point. The control system is able to guide the platform to the landing point with an error considerably reduced and controlling the wind effect. The simulation results proved that the followed approach in the design of the control algorithm is an innovation and reliable work with capacity to correct wind effect.

To investigate the stability of the system, the nonlinear model was firstly linearized through the Lyapunov's linearization method. Studying the eigenvalues of the state matrix A , it was verified that the model or the system is stable and fully observable and has 29 controllable states. As it isn't intended to fully control the two uncontrollable states, it is concluded that the system can follow any descending trajectory and all state variables of the model can be determined knowing the flight measurements done by the drone during the flight. The optimal control gains to place the desired closed-loop eigenvalues were found using an optimization method. In order to perform an easy and quick analysis of the simulation results, a GUI was developed. The user can change the features of the system and choose the control mode as well as the landing point and the arrival angle, among others.

Despite the complexity of the platform motion during the descending flight, the designed model is successfully able to predict its motion and modes. Furthermore it is an important basis for the control system which was efficiently developed and simulated or for other kind of control which can be designed in future developments.

7.2 Future Work

Since the mathematical model developed is the first formal treatment of the platform motion in the STRAPLEX, it is an important basis for further development in what concerns to the control algorithm and the platform motion analysis. A few important steps are outlined below:

- (1) More reliable airspeed data can be measured through a more robust measurement device, instead of the pitot tube. The angular velocity of the drone can also be estimated using differential samples of the drone Euler angles.
- (2) A multi-criteria identification method based on flight tests data can be performed to find the numerical values of some parameters of the system (e.g. aerodynamic coefficients) to better match flight data.
- (3) A identification method for the system can be investigated to obtain a state-space representation of the system. A comparison of this representation with the mathematical model

obtained in this work can be analysed to determine the fundamental modes of the system motion.

- (4) The control strategies developed in this work can be tested in the STRAPLEX. The flight tests should be made under different atmospheric conditions in order to validate the control system.
- (5) A better and more accurate optimization method can be investigated to optimize the control gains. A simulation and flight tests of each control system with the optimal gains can be studied and compared with other flight tests where nonoptimal control gains are used.

Appendix A

Appendix

A.1 Equation of Coriolis

During the study of the wind currents, the French engineer and mathematician Gaspard de Coriolis demonstrated that the Earth rotation has an effect in atmospheric motions. This effect, named Coriolis effect, explains the different orientation of the wind in the Northern Hemisphere and Southern Hemisphere. Based in the Coriolis effect, Gaspard defined the equation of Coriolis (A.1) to determine a vector derivative in moving frames. The vector w is fixed in a frame A which is rotating with respect to the reference frame R with an angular velocity $\vec{\omega}_{A/R}$.

$${}^A\dot{w}^R = {}^A\dot{w}^A + {}^A\vec{\omega}_{A/R} \times {}^A\vec{w} \quad (\text{A.1})$$

The derivation of the equation of Coriolis can be found in [2]. From this result some conclusions can be drawn:

- Calculating a vector derivative in different frames only requires the angular velocity vector between the two frames;
- The angular velocity derivative is the same in either frame, ${}^A\dot{\vec{\omega}}_{A/R}^R = {}^A\dot{\vec{\omega}}_{A/R}^A$. This is made evident because the vectorial product of the same vector is null.

A.2 Poisson's Kinematical Equation

The Poisson's kinematical equation or also called strapdown equation (A.2) allows calculating the derivative of a rotation matrix. The derivation of this equation can be found in [2] and it concludes that for a rotation matrix T_{R-A} , which transforms a vector w from the frame R to the frame A , and for an instantaneous angular velocity vector $\vec{\omega}_{A/R}$, the derivative of the rotation matrix in the frame A is given by:

$$\frac{d}{dt} ([T_{R-A}])^A = -{}^A\vec{\omega}_{A/R} \times [T_{R-A}] \quad (\text{A.2})$$

This equation is a useful tool in developing equations of motion, mainly to calculate the derivative of a vector. Taking a vector $\vec{w}$ fixed in the frame R , this vector can be expressed in the frame A from the rotation matrix T_{R-A} :

$${}^A\vec{w} = [T_{R-A}]^R\vec{w} \quad (\text{A.3})$$

The derivative of the vector ${}^A\vec{w}$ in the frame A can be written as:

$$\frac{d}{dt} ({}^A\vec{w})^A = \frac{d}{dt} ([T_{R-A}]^R\vec{w})^A \quad (\text{A.4})$$

Applying the product rule, the previous equation is equal to:

$${}^A\dot{\vec{w}}^A = \frac{d}{dt} ([T_{R-A}])^A {}^R\vec{w} + [T_{R-A}] \frac{d}{dt} ({}^R\vec{w})^R \quad (\text{A.5})$$

Using the Poisson's kinematical equation (A.2):

$${}^A\dot{\vec{w}}^A = (-{}^A\vec{\omega}_{A/R} \times T_{R-A})^R\vec{w} + [T_{R-A}]^R\dot{\vec{w}}^R \quad (\text{A.6})$$

Simplifying, it becomes:

$${}^A\dot{\vec{w}}^A = -{}^A\vec{\omega}_{A/R} \times {}^A\vec{w} + {}^A\dot{\vec{w}}^R \quad (\text{A.7})$$

It should be noted that this equation is the equation of Coriolis presented in Appendix A.1.

A.3 Derivative of a Vector

The derivative of a vector $\vec{w}$ expressed in a frame A is equal to the derivative of the vector expressed in other frame B , multiplied by the rotation matrix T_{B-A} . The proof follows, where the derivative of the vector is taken in the inertial frame I :

$$\frac{d}{dt} ({}^A\vec{w})^I \quad (\text{A.8})$$

Applying the equation of Coriolis (A.1), the equation (A.8) can be written as:

$$\begin{aligned} {}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + \frac{d}{dt} ({}^A\vec{w})^A \Leftrightarrow \\ \Leftrightarrow {}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + \frac{d}{dt} ([T_{B-A}]^B\vec{w})^A \end{aligned} \quad (\text{A.9})$$

Through the Poisson's equation (A.2), it is possible to simplify the second term of the right side.

$$\begin{aligned}
{}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + {}^A\vec{\omega}_{B/A} \times {}^A\vec{w} + [T_{B-A}] \frac{d}{dt} ({}^B\vec{w})^B \Leftrightarrow \\
\Leftrightarrow {}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + {}^A\vec{\omega}_{B/A} \times {}^A\vec{w} + [T_{B-A}] \left({}^A\vec{\omega}_{I/B} \times {}^B\vec{w} + \frac{d}{dt} ({}^B\vec{w})^I \right)
\end{aligned} \quad (\text{A.10})$$

Knowing that the angular velocity ${}^A\vec{\omega}_{B/A}$ is equal to:

$${}^A\vec{\omega}_{B/A} = {}^A\vec{\omega}_{B/I} - {}^A\vec{\omega}_{A/I} \quad (\text{A.11})$$

The equation (A.10) becomes:

$${}^A\dot{\vec{w}}^I = {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + ({}^A\vec{\omega}_{B/I} - {}^A\vec{\omega}_{A/I}) \times {}^A\vec{w} + [T_{B-A}] (-{}^A\vec{\omega}_{B/I} \times {}^B\vec{w} + {}^B\dot{\vec{w}}^I) \quad (\text{A.12})$$

Using the commutative and distributive property of the vectorial product,

$$\begin{aligned}
{}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + {}^A\vec{\omega}_{B/I} \times {}^A\vec{w} - {}^A\vec{\omega}_{A/I} \times {}^A\vec{w} + [T_{B-A}] ({}^B\vec{w} \times {}^A\vec{\omega}_{B/I} + {}^B\dot{\vec{w}}^I) \Leftrightarrow \\
\Leftrightarrow {}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{B/I} \times {}^A\vec{w} + [T_{B-A}] ({}^B\vec{w} \times {}^A\vec{\omega}_{B/I} + {}^B\dot{\vec{w}}^I)
\end{aligned} \quad (\text{A.13})$$

Using the matrix distributive property and once again the commutative property of the vectorial product, the previous equation becomes:

$$\begin{aligned}
{}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{B/I} \times {}^A\vec{w} + {}^A\vec{w} \times {}^A\vec{\omega}_{B/I} + [T_{B-A}] {}^B\dot{\vec{w}}^I \Leftrightarrow \\
\Leftrightarrow {}^A\dot{\vec{w}}^I &= {}^A\vec{\omega}_{B/I} \times {}^A\vec{w} - {}^A\vec{\omega}_{B/I} \times {}^A\vec{w} + [T_{B-A}] {}^B\dot{\vec{w}}^I
\end{aligned} \quad (\text{A.14})$$

Finally, the derivative of the vector $\vec{w}$ expressed in the frame A can be written as:

$${}^A\dot{\vec{w}}^I = [T_{B-A}] {}^B\dot{\vec{w}}^I \quad (\text{A.15})$$

A.4 Drone Rotation Kinematics

- Derivative of the Angular Velocity

The derivative of the angular velocity of the drone body expressed in the drone frame is given by:

$${}^D\dot{\vec{\omega}}_{D/I}^D = \frac{d}{dt} ({}^D\vec{\omega}_{D/I})^D \quad (\text{A.16})$$

Using the drone angular velocity expression defined in equation (4.24) and applying the equation (A.15),

$$\begin{aligned}
{}^D\dot{\vec{\omega}}_{D/I}^D &= \frac{d}{dt} ([T_{L-D}][T_{C-L}]^C \vec{\omega}_{C/I})^D + \frac{d}{dt} ([T_{L-D}]^L \vec{\omega}_{L/C})^D + \frac{d}{dt} ({}^D\vec{\omega}_{D/L})^D \Leftrightarrow \\
&\Leftrightarrow {}^D\dot{\vec{\omega}}_{D/I}^D = [T_{L-D}][T_{C-L}] \frac{d}{dt} ({}^C\vec{\omega}_{C/I})^D + [T_{L-D}] \frac{d}{dt} ({}^L\vec{\omega}_{L/C})^D + \frac{d}{dt} ({}^D\vec{\omega}_{D/L})^D
\end{aligned} \quad (A.17)$$

Applying the equation of Coriolis (A.1), the previous equation becomes:

$$\begin{aligned}
{}^D\dot{\vec{\omega}}_{D/I}^D &= [T_{L-D}][T_{C-L}] \left({}^D\vec{\omega}_{C/D} \times {}^C\vec{\omega}_{C/I} + \frac{d}{dt} ({}^C\vec{\omega}_{C/I})^C \right) \\
&\quad + [T_{L-D}] \left({}^D\vec{\omega}_{L/D} \times {}^L\vec{\omega}_{L/C} + \frac{d}{dt} ({}^L\vec{\omega}_{L/C})^L \right) + {}^D\dot{\vec{\omega}}_{D/L}^D
\end{aligned} \quad (A.18)$$

Finally, the derivative angular velocity expression for the drone body can be written as:

$$\begin{aligned}
{}^D\dot{\vec{\omega}}_{D/I}^D &= {}^D\vec{\omega}_{C/I} \times {}^D\vec{\omega}_{D/I} + [T_{L-D}][T_{C-L}]^C \dot{\vec{\omega}}_{C/I}^C \\
&\quad + {}^D\vec{\omega}_{L/C} \times {}^D\vec{\omega}_{D/L} + [T_{L-D}]^L \dot{\vec{\omega}}_{L/C}^L + {}^D\dot{\vec{\omega}}_{D/L}^D
\end{aligned} \quad (A.19)$$

A.5 Capsule Rotation Kinematics

- Derivative of the Angular Velocity

The derivative of the angular velocity of the capsule body is obtained deriving the angular velocity, defined in equation (4.32), in the capsule frame:

$$\begin{aligned}
{}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} &= \frac{d}{dt} ({}^{CAP}\vec{\omega}_{CAP/I})^{CAP} \Leftrightarrow \\
&\Leftrightarrow {}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} = \frac{d}{dt} ([T_{D-CAP}]^D \vec{\omega}_{D/I})^{CAP} + \frac{d}{dt} ({}^{CAP}\vec{\omega}_{CAP/D})^{CAP}
\end{aligned} \quad (A.20)$$

Applying the equation (A.15), the previous equation becomes:

$${}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} = [T_{D-CAP}] \frac{d}{dt} ({}^D\vec{\omega}_{D/I})^{CAP} + \frac{d}{dt} ({}^{CAP}\vec{\omega}_{CAP/D})^{CAP} \quad (A.21)$$

Applying the equation of Coriolis (A.1),

$${}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} = [T_{D-CAP}] ({}^{CAP}\vec{\omega}_{D/CAP} \times {}^D\vec{\omega}_{D/I}) + [T_{D-CAP}] \frac{d}{dt} ({}^D\vec{\omega}_{D/I})^D + {}^{CAP}\dot{\vec{\omega}}_{CAP/D}^{CAP} \quad (A.22)$$

Finally, the derivative of the capsule angular velocity can be written as:

$${}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} = {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{\omega}_{CAP/I} + [T_{D-CAP}]^D \dot{\vec{\omega}}_{D/I}^D + {}^{CAP}\dot{\vec{\omega}}_{CAP/D}^{CAP} \quad (A.23)$$

A.6 Transponder Rotation Kinematics

- Derivative of the Angular Velocity

The derivative of the angular velocity of the transponder body is obtained deriving the angular velocity, defined in equation (4.38), in the transponder frame:

$$\begin{aligned} {}^T\dot{\vec{\omega}}_{T/I}^T &= \frac{d}{dt} ({}^T\vec{\omega}_{T/I})^T \Leftrightarrow \\ \Leftrightarrow {}^T\dot{\vec{\omega}}_{T/I}^T &= \frac{d}{dt} ([T_{CAP-T}]^{CAP} \vec{\omega}_{CAP/I})^T + \frac{d}{dt} ({}^T\vec{\omega}_{T/CAP})^T \end{aligned} \quad (A.24)$$

Applying the equation (A.15), the previous equation becomes:

$${}^T\dot{\vec{\omega}}_{T/I}^T = [T_{CAP-T}] \frac{d}{dt} ({}^{CAP}\vec{\omega}_{CAP/I})^T + \frac{d}{dt} ({}^T\vec{\omega}_{T/CAP})^T \quad (A.25)$$

Applying the equation of Coriolis (A.1),

$${}^T\dot{\vec{\omega}}_{T/I}^T = [T_{CAP-T}] ({}^T\vec{\omega}_{CAP/T} \times {}^{CAP}\vec{\omega}_{CAP/I}) + [T_{CAP-T}] \frac{d}{dt} ({}^{CAP}\vec{\omega}_{CAP/I})^{CAP} + {}^T\dot{\vec{\omega}}_{T/CAP}^T \quad (A.26)$$

Finally, the derivative of the transponder angular velocity is given by:

$${}^T\dot{\vec{\omega}}_{T/I}^T = {}^T\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{\omega}_{T/I} + [T_{CAP-T}]^{CAP} \dot{\vec{\omega}}_{CAP/I}^{CAP} + {}^T\dot{\vec{\omega}}_{T/CAP}^T \quad (A.27)$$

A.7 Drone Position Kinematics

- Linear Velocity

The drone linear velocity is defined as:

$${}^D\vec{V}_{CM_D/I} = \frac{d}{dt} ({}^D\vec{R}_{CM_D/I})^I \quad (A.28)$$

Using the expression of the drone position vector. defined in equation (4.47), the equation (A.28) can be written as:

$${}^D\vec{V}_{CM_D/I} = \frac{d}{dt} \left([T_{C-D}]^C \vec{R}_{CM_C/I} \right)^I + \frac{d}{dt} \left([T_{C-D}]^C \vec{R}_{T_C/CM_C} \right)^I + \frac{d}{dt} \left([T_{L-D}]^L \vec{R}_{T_D/T_C} \right)^I + \frac{d}{dt} \left({}^D\vec{R}_{CM_D/T_D} \right)^I \quad (\text{A.29})$$

Applying the equation (A.8), the previous equation can be developed to:

$${}^D\vec{V}_{CM_D/I} = [T_{C-D}] \frac{d}{dt} \left({}^C\vec{R}_{CM_C/I} \right)^I + [T_{C-D}] \frac{d}{dt} \left({}^C\vec{R}_{T_C/CM_C} \right)^I + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{R}_{T_D/T_C} \right)^I + \frac{d}{dt} \left({}^D\vec{R}_{CM_D/T_D} \right)^I \quad (\text{A.30})$$

Applying the equation of Coriolis (A.1),

$$\begin{aligned} {}^D\vec{V}_{CM_D/I} &= [T_{C-D}]^C \vec{V}_{CM_C/I} + [T_{C-D}] \left({}^D\vec{\omega}_{C/I} \times {}^C\vec{R}_{T_C/CM_C} + \frac{d}{dt} \left({}^C\vec{R}_{T_C/CM_C} \right)^C \right) \\ &\quad + [T_{L-D}] \left({}^D\vec{\omega}_{L/I} \times {}^L\vec{R}_{T_D/T_C} + \frac{d}{dt} \left({}^L\vec{R}_{T_D/T_C} \right)^L \right) + {}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D} \\ &\quad + \frac{d}{dt} \left({}^D\vec{R}_{CM_D/T_D} \right)^D \end{aligned} \quad (\text{A.31})$$

The vectors ${}^C\vec{R}_{T_C/CM_C}$ and ${}^D\vec{R}_{CM_D/T_D}$ are always constant, therefore their derivatives are null. So, the equation (A.31) becomes:

$$\begin{aligned} {}^D\vec{V}_{CM_D/I} &= [T_{C-D}]^C \vec{V}_{CM_C/I} + {}^D\vec{\omega}_{C/I} \times \left([T_{C-D}]^C \vec{R}_{T_C/CM_C} \right) + [T_{L-D}] \left({}^D\vec{\omega}_{L/I} \times {}^L\vec{R}_{T_D/T_C} \right) \\ &\quad + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{R}_{T_D/T_C} \right)^L + {}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D} \end{aligned} \quad (\text{A.32})$$

In Chapter 3 it was demonstrated that a displacement of the main lines induces a horizontal displacement in the drone relative to the canopy, therefore the term $\frac{d}{dt} \left({}^L\vec{R}_{T_D/T_C} \right)^L$ isn't null. Naming the horizontal displacement rate as ${}^L\vec{V}_{Control/I}$, the linear velocity of the drone becomes:

$$\begin{aligned} {}^D\vec{V}_{CM_D/I} &= [T_{C-D}]^C \vec{V}_{CM_C/I} + {}^D\vec{\omega}_{C/I} \times \left([T_{C-D}]^C \vec{R}_{T_C/CM_C} \right) + {}^D\vec{\omega}_{L/I} \times \left([T_{L-D}]^L \vec{R}_{T_D/T_C} \right) \\ &\quad + [T_{L-D}]^L \vec{V}_{Control/I} + {}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D} \end{aligned} \quad (\text{A.33})$$

- Linear Acceleration

By definition, the linear acceleration is equal to the derivative of the linear velocity, so the linear acceleration of the drone becomes:

$${}^D\vec{a}_{CM_D/I} = \frac{d}{dt} \left({}^D\vec{V}_{CM_D/I} \right)^I \quad (\text{A.34})$$

Using the expression of the drone linear velocity, defined in equation (A.33), the linear acceleration of the drone is given by:

$$\begin{aligned} {}^D\vec{a}_{CM_D/I} = & \frac{d}{dt} \left([T_{C-D}]^C \vec{V}_{CM_C/I} \right)^I + \frac{d}{dt} \left({}^D\vec{\omega}_{C/I} \times \left([T_{C-D}]^D \vec{R}_{T_C/CM_C} \right) \right)^I \\ & + \frac{d}{dt} \left({}^D\vec{\omega}_{L/I} \times \left([T_{L-D}]^L \vec{R}_{T_D/T_C} \right) \right)^I + \frac{d}{dt} \left([T_{L-D}]^L \vec{V}_{Control/I} \right)^I \\ & + \frac{d}{dt} \left({}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D} \right)^I \end{aligned} \quad (\text{A.35})$$

Using the equation (A.15) and applying the product rule, the previous equation becomes:

$$\begin{aligned} {}^D\vec{a}_{CM_D/I} = & [T_{C-D}] \frac{d}{dt} \left({}^C\vec{V}_{CM_C/I} \right)^I + \frac{d}{dt} \left({}^D\vec{\omega}_{C/I} \right)^I \times \left({}^D\vec{R}_{T_C/CM_C} \right) + {}^D\vec{\omega}_{C/I} \times \frac{d}{dt} \left([T_{C-D}]^C \vec{R}_{T_C/CM_C} \right)^I \\ & + \frac{d}{dt} \left({}^D\vec{\omega}_{L/I} \right)^I \times \left({}^D\vec{R}_{T_D/T_C} \right) + {}^D\vec{\omega}_{L/I} \times \frac{d}{dt} \left([T_{L-D}]^L \vec{R}_{T_D/T_C} \right)^I + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{V}_{Control/I} \right)^I \\ & + \frac{d}{dt} \left({}^D\vec{\omega}_{D/I} \right)^I \times {}^D\vec{R}_{CM_D/T_D} + {}^D\vec{\omega}_{D/I} \times \frac{d}{dt} \left({}^D\vec{R}_{CM_D/T_D} \right)^I \end{aligned} \quad (\text{A.36})$$

Through the conclusion stated in Appendix A.1, the angular velocity derivative is the same in either frame, so:

$$\begin{aligned} {}^D\vec{a}_{CM_D/I} = & [T_{C-D}] \frac{d}{dt} \left({}^C\vec{V}_{CM_C/I} \right)^I + \frac{d}{dt} \left({}^D\vec{\omega}_{C/I} \right)^C \times \left({}^D\vec{R}_{T_C/CM_C} \right) + {}^D\vec{\omega}_{C/I} \times \frac{d}{dt} \left([T_{C-D}]^C \vec{R}_{T_C/CM_C} \right)^I \\ & + \frac{d}{dt} \left({}^D\vec{\omega}_{L/I} \right)^L \times \left({}^D\vec{R}_{T_D/T_C} \right) + {}^D\vec{\omega}_{L/I} \times \frac{d}{dt} \left([T_{L-D}]^L \vec{R}_{T_D/T_C} \right)^I + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{V}_{Control/I} \right)^I \\ & + \frac{d}{dt} \left({}^D\vec{\omega}_{D/I} \right)^D \times {}^D\vec{R}_{CM_D/T_D} + {}^D\vec{\omega}_{D/I} \times \frac{d}{dt} \left({}^D\vec{R}_{CM_D/T_D} \right)^I \end{aligned} \quad (\text{A.37})$$

Once again applying the equation (A.15), the equation of Coriolis (A.1), the Poisson's equation (A.2) and some results obtained in Appendix A.7, the linear acceleration of drone can be written as:

$$\begin{aligned}
{}^D\vec{a}_{CM_D/I} = & [T_{C-D}] \left({}^D\vec{\omega}_{C/I} \times {}^C\vec{V}_{CM_C/I} + \frac{d}{dt} \left({}^C\vec{V}_{CM_C/I} \right)^C \right) + [T_{C-D}] \frac{d}{dt} \left({}^C\vec{\omega}_{C/I} \right)^C \times \left({}^D\vec{R}_{T_C/CM_C} \right) \\
& + {}^D\vec{\omega}_{C/I} \times {}^D\vec{\omega}_{C/I} \times \left([T_{C-D}]^C\vec{R}_{T_C/CM_C} \right) + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{\omega}_{L/I} \right)^L \times \left({}^D\vec{R}_{T_D/T_C} \right) \\
& + {}^D\vec{\omega}_{L/I} \times \left({}^D\vec{\omega}_{L/I} \times [T_{L-D}]^L\vec{R}_{T_D/T_C} + [T_{L-D}]^L\vec{V}_{Control/I} \right) + [T_{L-D}] \left({}^D\vec{\omega}_{L/I} \times {}^L\vec{V}_{Control/I} \right) \\
& + [T_{L-D}] \frac{d}{dt} \left({}^L\vec{V}_{Control/I} \right)^L + \frac{d}{dt} \left({}^D\vec{\omega}_{D/I} \right)^D \times {}^D\vec{R}_{CM_D/T_D} + {}^D\vec{\omega}_{D/I} \times {}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D}
\end{aligned} \tag{A.38}$$

Simplifying, the drone linear acceleration can be written as:

$$\begin{aligned}
{}^D\vec{a}_{CM_D/I} = & [T_{C-D}]^C\dot{\vec{V}}_{CM_C/I}^C + {}^D\vec{\omega}_{C/I} \times {}^D\vec{V}_{CM_C/I} + [T_{C-D}]^C\dot{\vec{\omega}}_{C/I}^C \times {}^D\vec{R}_{T_C/CM_C} \\
& + {}^D\vec{\omega}_{C/I} \times {}^D\vec{\omega}_{C/I} \times {}^D\vec{R}_{T_C/CM_C} + [T_{L-D}]^L\dot{\vec{\omega}}_{L/I}^L \times {}^D\vec{R}_{T_D/T_C} + {}^D\vec{\omega}_{L/I} \times {}^D\vec{\omega}_{L/I} \times {}^D\vec{R}_{T_D/T_C} \\
& + 2 {}^D\vec{\omega}_{L/I} \times {}^D\vec{V}_{Control/I} + [T_{L-D}]^L\dot{\vec{V}}_{Control/I}^L + {}^D\dot{\vec{\omega}}_{D/I}^D \times {}^D\vec{R}_{CM_D/T_D} \\
& + {}^D\vec{\omega}_{D/I} \times {}^D\vec{\omega}_{D/I} \times {}^D\vec{R}_{CM_D/T_D}
\end{aligned} \tag{A.39}$$

A.8 Capsule Position Kinematics

- Linear Velocity

The capsule linear velocity can be determined deriving the capsule position vector.

$${}^{CAP}\vec{V}_{CM_{CAP}/I} = \frac{d}{dt} \left({}^{CAP}\vec{R}_{CM_{CAP}/I} \right)^I \tag{A.40}$$

Replacing the capsule position vector by the expression defined in equation (4.50),

$${}^{CAP}\vec{V}_{CM_{CAP}/I} = \frac{d}{dt} \left([T_{D-CAP}]^D\vec{R}_{CM_D/I} \right)^I + \frac{d}{dt} \left([T_{D-CAP}]^D\vec{R}_{TCAP_D/CM_D} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \right)^I \tag{A.41}$$

Using the equation (A.15) and the equation of Coriolis (A.1), the previous equation becomes:

$$\begin{aligned}
{}^{CAP}\vec{V}_{CM_{CAP}/I} = & [T_{D-CAP}] \frac{d}{dt} \left({}^D\vec{R}_{CM_D/I} \right)^I + [T_{D-CAP}] \frac{d}{dt} \left({}^D\vec{R}_{TCAP_D/CM_D} \right)^I \\
& + {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} + \frac{d}{dt} \left({}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \right)^{CAP}
\end{aligned} \tag{A.42}$$

Applying again the equation of Coriolis (A.1) and knowing that ${}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D}$ is constant in the capsule frame (therefore its derivative is null), the capsule linear velocity is given by:

$$\begin{aligned}
{}^{CAP}\vec{V}_{CM_{CAP}/I} &= [T_{D-CAP}]^D \vec{V}_{CM_D/I} + {}^{CAP}\vec{\omega}_{D/I} \times \left([T_{D-CAP}]^D \vec{R}_{TCAP_D/CM_D} \right) \\
&\quad + [T_{D-CAP}] \frac{d}{dt} \left({}^D\vec{R}_{TCAP_D/CM_D} \right)^D + {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D}
\end{aligned} \tag{A.43}$$

The position vector ${}^D\vec{R}_{TCAP_D/CM_D}$ is also constant in the drone frame, so its derivative is null. Finally, the capsule linear velocity can then be written as:

$${}^{CAP}\vec{V}_{CM_{CAP}/I} = [T_{D-CAP}]^D \vec{V}_{CM_D/I} + {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} + {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \tag{A.44}$$

- Linear Acceleration

In the derivation of the capsule linear acceleration, it will be followed a similar manner used to derived the drone linear acceleration expression. The capsule linear acceleration is defined as:

$${}^{CAP}\vec{a}_{CM_{CAP}/I} = \frac{d}{dt} \left({}^{CAP}\vec{V}_{CM_{CAP}/I} \right)^I \tag{A.45}$$

Using the capsule linear velocity expression, defined in equation (A.44), the capsule linear acceleration can be written as:

$$\begin{aligned}
{}^{CAP}\vec{a}_{CM_{CAP}/I} &= \frac{d}{dt} \left([T_{D-CAP}]^D \vec{V}_{CM_D/I} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} \right)^I \\
&\quad + \frac{d}{dt} \left({}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \right)^I
\end{aligned} \tag{A.46}$$

Applying the equation (A.15) and the product rule, the previous equation becomes:

$$\begin{aligned}
{}^{CAP}\vec{a}_{CM_{CAP}/I} &= [T_{D-CAP}] \frac{d}{dt} \left({}^D\vec{V}_{CM_D/I} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{\omega}_{D/I} \right)^I \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} \\
&\quad + {}^{CAP}\vec{\omega}_{D/I} \times \frac{d}{dt} \left({}^{CAP}\vec{R}_{TCAP_D/CM_D} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{\omega}_{CAP/I} \right)^I \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \\
&\quad + {}^{CAP}\vec{\omega}_{CAP/I} \times \frac{d}{dt} \left({}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \right)^I
\end{aligned} \tag{A.47}$$

Applying now some conclusions stated in Section A.1, the equation (A.47) can be written as:

$$\begin{aligned}
{}^{CAP}\vec{a}_{CM_{CAP}/I} &= [T_{D-CAP}]^D \vec{a}_{CM_D/I} + \frac{d}{dt} \left([T_{D-CAP}]^D \vec{\omega}_{D/I} \right)^D \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} \\
&\quad + {}^{CAP}\vec{\omega}_{D/I} \times \frac{d}{dt} \left({}^{CAP}\vec{R}_{TCAP_D/CM_D} \right)^I + \frac{d}{dt} \left({}^{CAP}\vec{\omega}_{CAP/I} \right)^{CAP} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \\
&\quad + {}^{CAP}\vec{\omega}_{CAP/I} \times \frac{d}{dt} \left({}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \right)^I
\end{aligned} \tag{A.48}$$

Using some results derived in Appendix A.8 and applying again the equation (A.15), the capsule linear acceleration becomes:

$$\begin{aligned}
 {}^{CAP}\vec{a}_{CM_{CAP}/I} &= [T_{D-CAP}]^D \vec{a}_{CM_D/I} + [T_{D-CAP}]^D \dot{\vec{\omega}}_{D/I}^D \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} \\
 &\quad + {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{\omega}_{D/I} \times {}^{CAP}\vec{R}_{TCAP_D/CM_D} + {}^{CAP}\dot{\vec{\omega}}_{CAP/I}^{CAP} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D} \\
 &\quad + {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{\omega}_{CAP/I} \times {}^{CAP}\vec{R}_{CM_{CAP}/TCAP_D}
 \end{aligned} \tag{A.49}$$

A.9 Transponder Position Kinematics

- Linear Velocity

The linear velocity of the transponder body is defined as the derivative of the transponder position vector, defined in equation (4.53).

$$\begin{aligned}
 {}^T\vec{V}_{CM_T/I} &= \frac{d}{dt} \left({}^T\vec{R}_{CM_T/I} \right)^I \Leftrightarrow \\
 \Leftrightarrow {}^T\vec{V}_{CM_T/I} &= \frac{d}{dt} \left({}^T\vec{R}_{CM_{CAP}/I} \right)^I + \frac{d}{dt} \left({}^T\vec{R}_{CM_T/CM_{CAP}} \right)^I \Leftrightarrow \\
 \Leftrightarrow {}^T\vec{V}_{CM_T/I} &= \frac{d}{dt} \left([T_{CAP-T}]^{CAP} \vec{R}_{CM_{CAP}/I} \right)^I + \frac{d}{dt} \left({}^T\vec{R}_{CM_T/CM_{CAP}} \right)^I
 \end{aligned} \tag{A.50}$$

Applying the equation (A.15) and the equation of Coriolis (A.1), the previous equation becomes:

$${}^T\vec{V}_{CM_T/I} = [T_{CAP-T}] \frac{d}{dt} \left({}^{CAP}\vec{R}_{CM_{CAP}/I} \right)^I + {}^T\vec{\omega}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} + \frac{d}{dt} \left({}^T\vec{R}_{CM_T/CM_{CAP}} \right)^T \tag{A.51}$$

Since, in the transponder frame, the transponder is fixed with respect to the capsule, the term $\frac{d}{dt} \left({}^T\vec{R}_{CM_T/CM_{CAP}} \right)^T$ is null. So, the linear velocity of the transponder body can be written as:

$${}^T\vec{V}_{CM_T/I} = [T_{CAP-T}]^{CAP} \vec{V}_{CM_{CAP}/I} + {}^T\vec{\omega}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} \tag{A.52}$$

- Linear Acceleration

Deriving the equation (A.52), one obtains the linear acceleration of the transponder body:

$$\begin{aligned}
 {}^T\vec{a}_{CM_T/I} &= \frac{d}{dt} \left({}^T\vec{V}_{CM_T/I} \right)^I \Leftrightarrow \\
 \Leftrightarrow {}^T\vec{a}_{CM_T/I} &= \frac{d}{dt} \left([T_{CAP-T}]^{CAP} \vec{V}_{CM_{CAP}/I} \right)_I + \frac{d}{dt} \left({}^T\vec{\omega}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} \right)^I
 \end{aligned} \tag{A.53}$$

Applying the equation (A.15) and using the product rule, the previous equation becomes:

$$\begin{aligned} {}^T\vec{a}_{CM_T/I} = & [T_{CAP-T}] \frac{d}{dt} \left({}^{CAP}\vec{V}_{CM_{CAP}/I} \right)_I + \frac{d}{dt} \left({}^T\vec{\omega}_{T/I} \right)^I \times {}^T\vec{R}_{CM_T/CM_{CAP}} \\ & + {}^T\vec{\omega}_{T/I} \times \frac{d}{dt} \left({}^T\vec{R}_{CM_T/CM_{CAP}} \right)^I \end{aligned} \quad (A.54)$$

Using some results of Appendix A.9 and conclusions stated in Appendix A.1, the transponder linear acceleration is defined as:

$${}^T\vec{a}_{CM_T/I} = [T_{CAP-T}]^{CAP}\vec{a}_{CM_{CAP}/I} + {}^T\dot{\vec{\omega}}_{T/I}^T \times {}^T\vec{R}_{CM_T/CM_{CAP}} + {}^T\vec{\omega}_{T/I} \times {}^T\vec{\omega}_{T/I} \times {}^T\vec{R}_{CM_T/CM_{CAP}} \quad (A.55)$$

A.10 Apparent Force

The apparent force such as any force is equal to the time derivative of the linear momentum, where in this case the mass is the apparent mass M_{app} .

$${}^C\vec{F}_{app} = - \frac{d}{dt} \left([M_{app}] {}^C\vec{V}_{C/A} \right)^I \quad (A.56)$$

Applying the equation of Coriolis (A.1),

$${}^C\vec{F}_{app} = - \left({}^C\vec{\omega}_{C/I} \times \left([M_{app}] {}^C\vec{V}_{C/A} \right) + \frac{d}{dt} \left([M_{app}] {}^C\vec{V}_{C/A} \right)^C \right) \quad (A.57)$$

Replacing ${}^C\vec{V}_{C/A}$ by the equation (4.64) and by noting that the apparent mass is a constant matrix, the previous equation becomes:

$$\begin{aligned} {}^C\vec{F}_{app} = & - {}^C\vec{\omega}_{C/I} \times \left([M_{app}] {}^C\vec{V}_{C/A} \right) - [M_{app}] \frac{d}{dt} \left({}^C\vec{V}_{CM_C/I} - [T_{I-C}]^I \vec{V}_{A/I} \right)^C \Leftrightarrow \\ \Leftrightarrow {}^C\vec{F}_{app} = & - {}^C\vec{\omega}_{C/I} \times \left([M_{app}] {}^C\vec{V}_{C/A} \right) - [M_{app}] \frac{d}{dt} \left({}^C\vec{V}_{CM_C/I} \right)^C + [M_{app}] \frac{d}{dt} \left([T_{I-C}]^I \vec{V}_{A/I} \right)^C \end{aligned} \quad (A.58)$$

Applying the Poisson's equation (A.2) and assuming a constant wind velocity,

$$\begin{aligned}
{}^C\vec{F}_{app} &= -{}^C\vec{\omega}_{C/I} \times \left([M_{app}] {}^C\vec{V}_{C/A} \right) - [M_{app}] {}^C\dot{\vec{V}}_{CM_C/I} + [M_{app}] \left({}^C\vec{\omega}_{I/C} \times \left([T_{I-C}] {}^I\vec{V}_{A/I} \right) \right) \\
&\quad + [M_{app}] \left([T_{I-C}] \frac{d}{dt} \left({}^I\vec{V}_{A/I} \right)^I \right) \Leftrightarrow \\
\Leftrightarrow {}^C\vec{F}_{app} &= -{}^C\vec{\omega}_{C/I} \times \left([M_{app}] {}^C\vec{V}_{C/A} \right) - [M_{app}] {}^C\dot{\vec{V}}_{CM_C/I} - [M_{app}] \left({}^C\vec{\omega}_{C/I} \times \left([T_{I-C}] {}^I\vec{V}_{A/I} \right) \right)
\end{aligned} \tag{A.59}$$

A.11 Matrices M and N of the Nonlinear Model

Considering:

$$\vec{R}_{T1} = [0; \cos(\text{atan2}({}^L\vec{R}_{T1_D/T1_C}(3), {}^L\vec{R}_{T1_D/T1_C}(2))), \sin(\text{atan2}({}^L\vec{R}_{T1_D/T1_C}(3), {}^L\vec{R}_{T1_D/T1_C}(2)))];$$

$$\vec{R}_{T2} = [0; \cos(\text{atan2}({}^L\vec{R}_{T2_D/T2_C}(3), {}^L\vec{R}_{T2_D/T2_C}(2))), \sin(\text{atan2}({}^L\vec{R}_{T2_D/T2_C}(3), {}^L\vec{R}_{T2_D/T2_C}(2)))];$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$T_{AnglesC} = \begin{bmatrix} 1 & 0 & -\sin(\theta_{C/I}) \\ 0 & \cos(\phi_{C/I}) & \sin(\phi_{C/I})\cos(\theta_{C/I}) \\ 0 & -\sin(\phi_{C/I}) & \cos(\phi_{C/I})\cos(\theta_{C/I}) \end{bmatrix}^{-1};$$

$$T_{AnglesD/L} = \begin{bmatrix} 1 & 0 & -\sin(\theta_{D/L}) \\ 0 & \cos(\phi_{D/L}) & \sin(\phi_{D/L})\cos(\theta_{D/L}) \\ 0 & -\sin(\phi_{D/L}) & \cos(\phi_{D/L})\cos(\theta_{D/L}) \end{bmatrix}^{-1};$$

$$T_{AnglesCAP/D} = \begin{bmatrix} 1 & 0 & \sin(\theta_{CAP/D}) \\ 0 & \cos(\phi_{CAP/D}) & \sin(\phi_{CAP/D})\cos(\theta_{CAP/D}) \\ 0 & -\sin(\phi_{CAP/D}) & \cos(\phi_{CAP/D})\cos(\theta_{CAP/D}) \end{bmatrix}^{-1};$$

$$T_{AnglesT/CAP} = \begin{bmatrix} 1 & 0 & \sin(\theta_{T/CAP}) \\ 0 & \cos(\phi_{T/CAP}) & \sin(\phi_{T/CAP})\cos(\theta_{T/CAP}) \\ 0 & -\sin(\phi_{T/CAP}) & \cos(\phi_{T/CAP})\cos(\theta_{T/CAP}) \end{bmatrix}^{-1};$$

And using the common convention for the vector cross product of two vectors ${}^A\vec{R} = [R_x \ R_y \ R_z]^T$ and $\vec{F} = [F_x \ F_y \ F_z]^T$,

$${}^A SR\vec{F} = \begin{bmatrix} 0 & -R_z & R_y \\ R_z & 0 & -R_x \\ -R_y & R_x & 0 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix};$$

The blocks of the matrices M and N are defined as follows:

A.11.1 Matrix M

$$M_{1,0} = [I_3];$$

$$M_{2,1} = [I_3];$$

$$M_{3,2} = M_D [I_3] + [M_{APP}];$$

$$M_{3,15} = -[T_{C-L}]^T \vec{R}_{T1};$$

$$M_{3,16} = -[T_{C-L}]^T \vec{R}_{T2};$$

$$M_{4,3} = [I_C] + [I_{APP}];$$

$$M_{4,15} = -{}^C SR_{T1C/CM_C} \left([T_{C-L}]^T \vec{R}_{T1} \right);$$

$$M_{4,16} = -{}^C SR_{T2C/CM_C} \left([T_{C-L}]^T \vec{R}_{T2} \right);$$

$$M_{5,4} = 1;$$

$$M_{6,6} = [I_3];$$

$$M_{7,2} = M_D [T_{C-D}];$$

$$M_{7,3} = -M_D \left({}^D SR_{T_C/CM_C} + {}^D SR_{T_D/T_C} + {}^D SR_{CM_D/T_D} \right) [T_{C-D}];$$

$$M_{7,5} = -M_D \left({}^D SR_{T_D/T_C} + {}^D SR_{CM_D/T_D} \right) [T_{L-D}][0; 1; 0];$$

$$M_{7,7} = -M_D {}^D SR_{CM_D/T_D};$$

$$M_{7,12} = M_D \left([T_{L-D}][0; -K_L; 0] + 2 {}^D S\omega_L [T_{L-D}][0; 1; 0] \right);$$

$$M_{7,15} = [T_{L-D}] \vec{R}_{T1};$$

$$M_{7,16} = [T_{L-D}] \vec{R}_{T2};$$

$$M_{7,17} = -[T_{D-CAP}]^T [0; 0; 1];$$

$$M_{8,3} = [I_D][T_{C-D}];$$

$$M_{8,5} = [I_D][T_{L-D}][0; 1; 0];$$

$$M_{8,7} = [I_D];$$

$$M_{8,15} = {}^D SR_{T1D/CM_D} \left([T_{L-D}] \vec{R}_{T1} \right);$$

$$M_{8,16} = {}^D SR_{T2D/CM_D} \left([T_{L-D}] \vec{R}_{T2} \right);$$

$$M_{8,17} = -{}^D SR_{TCAP_D/CM_D} ([T_{D-CAP}][0; 0; 1]);$$

$$M_{9,8} = [1 \ 0; 0 \ 1; 0 \ 0];$$

$$M_{10,2} = M_{CAP} [T_{D-CAP}] [T_{C-D}];$$

$$M_{10,3} = -M_{CAP} [T_{D-CAP}] \left({}^D SR_{T_C/CM_C} + {}^D SR_{T_D/T_C} + {}^D SR_{CM_D/T_D} \right) [T_{C-D}] \\ - M_{CAP} \left({}^{CAP} SR_{TCAP_D/CM_D} [T_{D-CAP}] [T_{C-D}] + {}^{CAP} SR_{CM_{CAP}/TCAP_D} [T_{D-CAP}] [T_{C-D}] \right);$$

$$\begin{aligned}
M_{10,5} &= -M_{CAP}[T_{D-CAP}] \left({}^D SR_{T_D/T_C} + {}^D SR_{CM_D/T_D} \right) [T_{L-D}][0; 1; 0] \\
&\quad - M_{CAP} {}^{CAP} SR_{TCAP_D/CM_D}[T_{D-CAP}][T_{L-D}][0; 1; 0] \\
&\quad - M_{CAP} {}^{CAP} SR_{CM_{CAP}/TCAP_D}[T_{D-CAP}][T_{L-D}][0; 1; 0]; \\
M_{10,7} &= -M_{CAP} \left([T_{D-CAP}] {}^D SR_{CM_D/T_D} + {}^{CAP} SR_{TCAP_D/CM_D}[T_{D-CAP}] + {}^{CAP} SR_{CM_{CAP}/TCAP_D}[T_{D-CAP}] \right); \\
M_{10,9} &= -M_{CAP} {}^{CAP} SR_{CM_{CAP}/TCAP_D}[1 \ 0; 0 \ 1; 0 \ 0]; \\
M_{10,12} &= M_{CAP}[T_{D-CAP}] \left([T_{L-D}][0; -K_L; 0] + 2 {}^D S\omega_L[T_{L-D}][0; 1; 0] \right); \\
M_{10,17} &= [0; 0; 1]; \\
M_{10,18} &= -[T_{CAP-T}]^T [0; 0; 1]; \\
M_{11,10} &= [1 \ 0; 0 \ 1; 0 \ 0]; \\
M_{12,2} &= M_T[T_{CAP-T}][T_{D-CAP}][T_{C-D}]; \\
M_{12,3} &= -M_T[T_{CAP-T}][T_{D-CAP}] \left({}^D SR_{T_C/CM_C} + {}^D SR_{T_D/T_C} + {}^D SR_{CM_D/T_D} \right) [T_{C-D}] \\
&\quad - M_T[T_{CAP-T}] \left({}^{CAP} SR_{TCAP_D/CM_D}[T_{D-CAP}][T_{C-D}] + {}^{CAP} SR_{CM_{CAP}/TCAP_D}[T_{D-CAP}][T_{C-D}] \right) \\
&\quad - M_T^T SR_{CM_T/CM_{CAP}}[T_{CAP-T}][T_{D-CAP}][T_{C-D}]; \\
M_{12,5} &= -M_T[T_{CAP-T}][T_{D-CAP}] \left({}^D SR_{T_D/T_C} + {}^D SR_{CM_D/T_D} \right) [T_{L-D}][0; 1; 0] \\
&\quad - M_T[T_{CAP-T}] {}^{CAP} SR_{TCAP_D/CM_D}[T_{D-CAP}][T_{L-D}][0; 1; 0] \\
&\quad - M_T[T_{CAP-T}] {}^{CAP} SR_{CM_{CAP}/TCAP_D}[T_{D-CAP}][T_{L-D}][0; 1; 0] \\
&\quad - M_T^T SR_{CM_T/CM_{CAP}}[T_{CAP-T}][T_{D-CAP}][T_{L-D}][0; 1; 0]; \\
M_{12,7} &= -M_T[T_{CAP-T}] \left([T_{D-CAP}] {}^D SR_{CM_D/T_D} + {}^{CAP} SR_{TCAP_D/CM_D}[T_{D-CAP}] \right) \\
&\quad - M_T \left([T_{CAP-T}] {}^{CAP} SR_{CM_{CAP}/TCAP_D}[T_{D-CAP}] + {}^T SR_{CM_T/CM_{CAP}}[T_{CAP-T}][T_{D-CAP}] \right); \\
M_{12,9} &= -M_T \left([T_{CAP-T}] {}^{CAP} SR_{CM_{CAP}/TCAP_D} + {}^T SR_{CM_T/CM_{CAP}}[T_{CAP-T}] \right) [1 \ 0; 0 \ 1; 0 \ 0]; \\
M_{12,11} &= -M_T^T SR_{CM_T/CM_{CAP}}[1 \ 0; 0 \ 1; 0 \ 0]; \\
M_{12,12} &= M_T[T_{CAP-T}][T_{D-CAP}] \left([T_{L-D}][0; -K_L; 0] + 2 {}^D S\omega_L[T_{L-D}][0; 1; 0] \right); \\
M_{12,18} &= [0; 0; 1]; \\
M_{13,12} &= 1; \\
M_{14,13} &= 1; \\
M_{15,14} &= 1; \\
\end{aligned}$$

The remaining blocks are zero matrices.

A.11.2 Matrix N

For reasons of simplification, the vectors with respect to the inertial frame will be represented without “/I” in the right subscript.

$$N_1 = [T_{I-C}]^T \vec{V}_{CMC};$$

$$N_2 = T_{AnglesC}^C \vec{\omega}_{CMC};$$

$$\begin{aligned}
N_3 &= -M_C^C S\omega_C^C \vec{V}_{CMC} - [M_{APP}]^C S\omega_C [T_{I-C}] \vec{V}_{A/I} - {}^C S\omega_C [M_{APP}] \vec{V}_{C/A} + {}^C \vec{F}_{WC} + {}^C \vec{F}_{AC}; \\
N_4 &= {}^C SR_{CP_C/CM_C} {}^C \vec{F}_{AC} + {}^C \vec{M}_{AC} - {}^C S\omega_C ([I_C] + [I_{APP}]) {}^C \vec{\omega}_{CMC}; \\
N_5 &= {}^L q_{L/C}; \\
N_6 &= T_{AnglesD/L} {}^D \vec{\omega}_{D/L}; \\
N_7 &= {}^D \vec{F}_{WD} + {}^D \vec{F}_{AD} - M_D ({}^D S\omega_C ([T_{C-D}] {}^C \vec{V}_{CMC}) + {}^D S\omega_C {}^D S\omega_C {}^D \vec{R}_{T_C/CM_C} + {}^D S\omega_L {}^D S\omega_L {}^D \vec{R}_{T_D/T_C}) \\
&\quad - M_D ({}^D S\omega_D {}^D S\omega_D {}^D \vec{R}_{CM_D/T_D} + {}^D SR_{T_D/T_C} [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C) + {}^D SR_{CM_D/T_D} [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C)) \\
&\quad - M_D {}^D SR_{CM_D/T_D} ({}^D S\omega_D {}^D \vec{\omega}_L); \\
N_8 &= [I_D] ([T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C) + {}^D S\omega_D {}^D \vec{\omega}_L) - {}^D S\omega_D [I_D] {}^D \vec{\omega}_D + {}^D SR_{CP_D/CM_D} {}^D \vec{F}_{AD} + {}^D \vec{M}_{AD}; \\
N_9 &= T_{AnglesCAP/D} {}^{CAP} \vec{\omega}_{CAP/D}; \\
N_{10} &= {}^{CAP} \vec{F}_{WCAP} + {}^{CAP} \vec{F}_{ACAP} - M_{CAP} [T_{D-CAP}] ({}^D S\omega_C ([T_{C-D}] {}^C \vec{V}_{CMC}) + {}^D S\omega_C ({}^D S\omega_C {}^D \vec{R}_{T_C/CM_C})) \\
&\quad - M_{CAP} [T_{D-CAP}] ({}^D S\omega_L ({}^D S\omega_L {}^D \vec{R}_{T_D/T_C}) + {}^D S\omega_D ({}^D S\omega_D {}^D \vec{R}_{CM_D/T_D}) + {}^D SR_{T_D/T_C} [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C)) \\
&\quad - M_{CAP} [T_{D-CAP}] ({}^D SR_{CM_D/T_D} [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C) + {}^D SR_{CM_D/T_D} ({}^D S\omega_D {}^D \vec{\omega}_L)) \\
&\quad - M_{CAP} ({}^{CAP} SR_{TCAP_D/CM_D} [T_{D-CAP}] ([T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C) + {}^D S\omega_D {}^D \vec{\omega}_L)) \\
&\quad - M_{CAP} ({}^{CAP} S\omega_D ({}^{CAP} S\omega_D {}^{CAP} \vec{R}_{TCAP_D/CM_D}) + {}^{CAP} SR_{CM_{CAP}/TCAP_D} ([T_{D-CAP}] ([T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C))) \\
&\quad - M_{CAP} ({}^{CAP} SR_{CM_{CAP}/TCAP_D} ([T_{D-CAP}] ({}^D S\omega_D {}^D \vec{\omega}_L) + {}^{CAP} S\omega_{CAP} {}^{CAP} \vec{\omega}_D)) \\
&\quad - M_{CAP} ({}^{CAP} S\omega_{CAP} ({}^{CAP} S\omega_{CAP} {}^{CAP} \vec{R}_{CM_{CAP}/TCAP_D})); \\
N_{11} &= T_{AnglesT/CAP} {}^T \vec{\omega}_{T/CAP}; \\
N_{12} &= {}^T \vec{F}_{WT} + {}^T \vec{F}_{AT} - M_T [T_{CAP-T}] ((T_{D-CAP}) ({}^D S\omega_C ([T_{C-D}] {}^C \vec{V}_{CMC}) + {}^D S\omega_C ({}^D S\omega_C {}^D \vec{R}_{T_C/CM_C})) \\
&\quad - M_T [T_{CAP-T}] [T_{D-CAP}] ({}^D S\omega_L ({}^D S\omega_L {}^D \vec{R}_{T_D/T_C}) + {}^D S\omega_D ({}^D S\omega_D {}^D \vec{R}_{CM_D/T_D})) \\
&\quad - M_T [T_{CAP-T}] [T_{D-CAP}] ({}^D SR_{T_D/T_C} [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C) + {}^D SR_{CM_D/T_D} [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C)) \\
&\quad - M_T [T_{CAP-T}] [T_{D-CAP}] ({}^D SR_{CM_D/T_D} {}^D S\omega_D {}^D \vec{\omega}_L) \\
&\quad - M_T [T_{CAP-T}] ({}^{CAP} SR_{TCAP_D/CM_D} ([T_{D-CAP}] ([T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C) + {}^D S\omega_D {}^D \vec{\omega}_L))) \\
&\quad - M_T [T_{CAP-T}] ({}^{CAP} S\omega_D {}^{CAP} S\omega_D {}^{CAP} \vec{R}_{TCAP_D/CM_D} + {}^{CAP} SR_{CM_{CAP}/TCAP_D} ([T_{D-CAP}] [T_{L-D}] ({}^L S\omega_L {}^L \vec{\omega}_C))) \\
&\quad - M_T [T_{CAP-T}] ({}^{CAP} SR_{CM_{CAP}/TCAP_D} ([T_{D-CAP}] ({}^D S\omega_D {}^D \vec{\omega}_L) + {}^{CAP} S\omega_{CAP} {}^{CAP} \vec{\omega}_D)) \\
&\quad - M_T [T_{CAP-T}] ({}^{CAP} S\omega_{CAP} ({}^{CAP} S\omega_{CAP} {}^{CAP} \vec{R}_{CM_{CAP}/TCAP_D})) \\
&\quad - M_T ({}^T SR_{CM_T/CM_{CAP}}) ([T_{CAP-T}] ([T_{D-CAP}] ([T_{L-D}] ({}^L \vec{\omega}_L {}^L \vec{\omega}_C) + {}^D S\omega_D {}^D \vec{\omega}_L) + {}^{CAP} S\omega_{CAP} {}^{CAP} \vec{\omega}_D)) \\
&\quad - M_T ({}^T SR_{CM_T/CM_{CAP}} ({}^T S\omega_T {}^T \vec{\omega}_{CAP}) + {}^T S\omega_T ({}^T S\omega_T {}^T \vec{R}_{CM_T/CM_{CAP}})); \\
N_{13} &= K_L (Hu - \Delta L); \\
N_{14} &= K_{SR} (Vu - \Delta ServoR); \\
N_{15} &= K_{SL} (Vu - \Delta ServoL);
\end{aligned}$$

A.12 Matrices A, B, C and D of the Linear Model

The elements of the matrices A, B, C and D will be written in scientific notation with one decimal place. Moreover, some matrices are split into sub-matrices.

A.12.1 Matrix A

The matrix A is composed by five sub-matrices as follows:

$$A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \end{bmatrix}$$

where the sub-matrices A_i , with $i = 1, 2, \dots, 5$, are given by:

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 3.1 & 0 & 9.9 \times 10^{-1} & 0 & 1.1 \times 10^{-1} \\ 0 & 0 & 0 & -3.7 & 0 & 6.1 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & -6.1 & 0 & -1.1 \times 10^{-1} & 0 & 9.9 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -9.3 & 0 & -5.6 & 0 & 2.3 \times 10^1 \\ 0 & 0 & 0 & 7.5 & 0 & 0 & 0 & -4.7 & 0 \\ 0 & 0 & 0 & 0 & -5.5 \times 10^{-1} & 0 & -1.5 & 0 & -4.1 \\ 0 & 0 & 0 & -5.1 \times 10^{-1} & 0 & 0 & 0 & -1.8 & 0 \\ 0 & 0 & 0 & 0 & 1.6 & 0 & 5.8 & 0 & -2.2 \times 10^1 \\ 0 & 0 & 0 & -1.2 \times 10^{-1} & 0 & 0 & 0 & 7.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.2 & 0 & -5.0 & 0 & 1.8 \times 10^1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.4 & 0 & 0 & 0 & -4.1 & 0 \\ 0 & 0 & 0 & 0 & 6.0 \times 10^{-1} & 0 & -6.4 \times 10^{-1} & 0 & 3.4 \\ 0 & 0 & 0 & -9.1 \times 10^{-1} & 0 & 0 & 0 & -9.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.1 & 0 & 0 & 0 & 5.8 & 0 \\ 0 & 0 & 0 & 0 & -1.8 \times 10^{-2} & 0 & -1.4 \times 10^{-1} & 0 & 4.5 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7.6 \times 10^{-2} & 0 & 0 & 0 & -2.6 \times 10^{-1} & 0 \\ 0 & 0 & 0 & 0 & 8.6 \times 10^{-3} & 0 & 5.9 \times 10^{-2} & 0 & -1.7 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 1.1 \times 10^{-1} & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & -1.4 \times 10^1 & 0 & 8.0 \times 10^1 & 1.7 \times 10^{-1} & 0 \\ 2.5 & 0 & -4.8 & 0 & 0 & -2.6 \times 10^1 \\ 0 & -6.0 \times 10^{-1} & 0 & 4.4 \times 10^1 & -2.6 \times 10^{-1} & 0 \\ -7.0 & 0 & 7.3 \times 10^{-1} & 0 & 0 & 4.3 \times 10^1 \\ 0 & -2.2 \times 10^1 & 0 & 2.2 \times 10^2 & 1.5 & 0 \\ 3.5 & 0 & -6.2 & 0 & 0 & 1.0 \times 10^1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.0 \times 10^1 & 0 & -3.1 \times 10^2 & -2.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6.5 \times 10^1 & 0 & 2.6 & 0 & 0 & -7.4 \times 10^2 \\ 0 & -9.0 & 0 & 8.6 \times 10^1 & 2.8 \times 10^{-1} & 0 \\ 2.3 \times 10^1 & 0 & 7.4 & 0 & 0 & -2.1 \times 10^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -6.2 \times 10^1 & 0 & -3.2 & 0 & 0 & 7.3 \times 10^2 \\ 0 & -7.9 \times 10^{-1} & 0 & 3.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4.6 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2.7 & 0 & 2.0 \times 10^{-1} & 0 & 0 & -2.9 \times 10^1 \\ 0 & 1.2 & 0 & -1.5 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 7.2 \times 10^{-1} & 0 & 0 & 1.0 \times 10^{-1} & 0 & 0 \\ 0 & -2.4 \times 10^{-1} & 3.2 \times 10^{-3} & 0 & 2.2 \times 10^{-3} & 4.7 \\ -1.1 & 0 & 0 & -1.6 \times 10^{-1} & 0 & 0 \\ 0 & 4.0 \times 10^{-1} & -5.2 \times 10^{-3} & 0 & -3.5 \times 10^{-3} & -7.6 \\ 6.5 & 0 & 0 & 9.2 \times 10^{-1} & 0 & 0 \\ 0 & 9.4 \times 10^{-2} & -1.2 \times 10^{-3} & 0 & -8.4 \times 10^{-4} & -1.8 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3.9 \times 10^1 & 0 & 0 & -1.1 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 1.1 \times 10^{-1} & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ -4.6 \times 10^2 & 8.0 & -8.3 \times 10^{-2} & 0 & 2.0 \times 10^{-2} & 5.2 \times 10^2 \\ 0 & 0 & 0 & -1.1 & 0 & 0 \\ 0 & -3.4 \times 10^1 & -1.1 \times 10^{-1} & 0 & -8.5 \times 10^{-2} & -1.7 \times 10^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4.9 & -3.4 \times 10^{-1} & 0 & -5.7 \times 10^{-2} & -5.3 \times 10^2 \\ 4.1 \times 10^2 & 0 & 0 & 4.7 \times 10^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.9 & 3.1 \times 10^{-1} & 0 & 5.0 \times 10^{-2} & 2.5 \times 10^1 \\ -3.1 \times 10^{-4} & 0 & 0 & 5.6 \times 10^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.6 \times 10^{-1} & 0 & 9.1 \times 10^{-2} & 0 & 9.1 \times 10^{-2} & 0 & 2.3 \times 10^{-2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4.0 \times 10^{-1} & 0 & -1.4 \times 10^{-1} & 0 & -1.4 \times 10^{-1} & 0 & -3.6 \times 10^{-2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.4 & 0 & 8.3 \times 10^{-1} & 0 & 8.3 \times 10^{-1} & 0 & 2.1 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.1 & 0 & -1.1 & 0 & -1.1 & 0 & -2.8 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.4 \times 10^2 & 0 & 1.3 & 0 & 1.3 & 0 & 3.4 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & -4.2 \times 10^{-1} & 0 & 1.4 & 0 & 0 & 0 \\ -2.8 \times 10^2 & 0 & -1.9 & 0 & 3.1 \times 10^{-1} & 0 & -2.8 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 0 & 2.8 \times 10^{-1} & 0 & -4.7 & 0 & -2.4 \times 10^{-1} & 0 \\ 3.0 \times 10^1 & 0 & 5.9 \times 10^{-1} & 0 & -4.6 & 0 & -4.1 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 4.1 \times 10^{-3} & -4.1 \times 10^{-3} \\ 5.8 \times 10^1 & 4.5 \times 10^{-2} & -4.5 \times 10^{-2} \\ 0 & 9.2 \times 10^{-4} & -9.2 \times 10^{-4} \\ 9.0 \times 10^1 & 2.3 \times 10^{-3} & -2.3 \times 10^{-3} \\ 0 & -4.1 \times 10^{-3} & 4.1 \times 10^{-3} \\ 5.0 \times 10^1 & 3.4 \times 10^{-1} & -3.4 \times 10^{-1} \\ 0 & 0 & 0 \\ 0 & 3.4 \times 10^{-3} & -3.4 \times 10^{-3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1.0 \times 10^3 & -3.0 \times 10^{-1} & 3.0 \times 10^{-1} \\ 0 & 7.9 \times 10^{-4} & -7.9 \times 10^{-4} \\ -2.9 \times 10^2 & -4.9 \times 10^{-1} & 4.9 \times 10^{-1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9.6 \times 10^2 & 3.0 \times 10^{-1} & -3.0 \times 10^{-1} \\ 0 & -1.5 \times 10^{-4} & 1.5 \times 10^{-4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -3.8 \times 10^1 & -1.1 \times 10^{-2} & 1.1 \times 10^{-2} \\ 0 & 6.8 \times 10^{-5} & -6.8 \times 10^{-5} \\ -5.0 & 0 & 0 \\ 0 & -5.0 & 0 \\ 0 & 0 & -5.0 \end{bmatrix}$$

A.12.2 Matrix B

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 2.8 & 0 \\ 0 & 0 \\ -4.7 & 0 \\ 0 & 0 \\ -1.1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.5 \times 10^1 & 0 \\ 0 & 0 \\ -5.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 2.1 \times 10^1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1.0 & 0 \\ 0 & 0 \\ 5.0 & 0 \\ 0 & 5.0 \\ 0 & 5.0 \end{bmatrix}$$

A.12.3 Matrix C

The matrix C is composed by two sub-matrices as follows:

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

where the sub-matrices are given by:

$$C_1 = \begin{bmatrix} 5.0 \times 10^{-1} & 0 & 0 & 0 & 0 & 0 \\ 5.0 \times 10^{-1} & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & -3.3 & 0 & 0 & -3.7 & 0 \\ 3.3 & 0 & 8.0 \times 10^{-2} & 3.1 & 0 & -6.2 \\ 0 & -8.0 \times 10^{-2} & 0 & 0 & 6.2 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & -1.1 \times 10^{-1} \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.1 \times 10^{-1} & 0 & 1.0 \\ 0 & 0 & 0 & 0 & -3.3 & 0 \\ 0 & 0 & 0 & 3.3 & 0 & 8.0 \times 10^{-2} \\ 0 & 0 & 0 & 0 & -4.6 \times 10^{-1} & 0 \\ 2.7 & 0 & 2.5 \times 10^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.7 & 0 & 2.5 \times 10^{-1} \\ 0 & -1.8 \times 10^{-1} & 0 & 0 & 0 & 0 \\ 1.8 \times 10^{-1} & 0 & 6.0 \times 10^{-2} & 0 & 0 & 0 \\ 0 & -5.0 \times 10^{-2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.8 \times 10^{-1} & 0 \\ 0 & 0 & 0 & 1.8 \times 10^{-1} & 0 & 6.0 \times 10^{-2} \\ 0 & 0 & 0 & 0 & -7.0 \times 10^{-2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.4 & 0 & 0 & -5.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.0 & 0 & -5.0 \times 10^{-2} \\ 0 & 1.0 & 0 \\ 5.0 \times 10^{-2} & 0 & 1.0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1.0 & 0 & 4.0 \times 10^{-2} \\ 0 & 1.0 & 0 \\ -4.0 \times 10^{-2} & 0 & 1.0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T$$

A.12.4 Matrix D

$$D = \begin{bmatrix} 0 & 0 & 0 & 5.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

	$\begin{bmatrix} 0 & 0 \\ 2.2 \times 10^{-1} & 0 \\ 0 & 0 \\ -2.0 \times 10^{-1} & 0 \\ 0 & 0 \\ 5.6 \times 10^{-1} & 0 \\ 0 & 0 \\ 9.1 \times 10^{-2} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 3.6 \times 10^{-1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$		$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 5.1 \\ 0 & 0 \\ 0 & 2.3 \times 10^{-1} \\ 0 & 0 \\ 0 & -7.0 \times 10^{-1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$		$\begin{bmatrix} 0 & 2.2 \times 10^{-1} \\ 0 & 2.2 \times 10^{-1} \\ 0 & -8.9 \times 10^{-1} \\ 0 & 0 \\ 0 & 5.1 \\ 0 & 0 \\ 0 & 2.3 \times 10^{-1} \\ 0 & 0 \\ 0 & -7.0 \times 10^{-1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$
$KH_{3b} =$		$KV_a =$		$KV_b =$	

References

- [1] Oleg A. Yakimenko and Roman B. Statnikov. Multicriteria parametrical identification of the parafoil-load delivery system. *18th AIAA Aerodynamic Decelerator Systems Technology Conference and Seminar*, 2005.
- [2] Brian L. Stevens and Frank L. Lewis. *Aircraft Control and Simulation*. John Wiley & Sons, Inc, Second edition, 2003.
- [3] Mário Martins de Sousa. Autonomous paraglider for straplex. Master's thesis, Faculdade de Engenharia da Universidade do Porto, July 2012.
- [4] L. Rolo G. Pont and M. Duarte. *STRAPLEX-UM-1-8*. ESA Education Projects Division, June 2006.
- [5] Jr. Sanger M. Burk and George M. Ware. Static aerodynamic characteristics of three ram-air-inflated low-aspect-ratio fabric wings. *Technical Note D-4182*, NASA, September 1967.
- [6] Jr. Sanger M. Burk. Wind-tunnel investigation of aerodynamic characteristics of a 1/2-scale model of an ejection seat with a rigid-wing recovery system. *Technical Note D-5922*, NASA, August 1970.
- [7] J. Stephen Lingar. Ram-air parachute design. *13th AIAA Aerodynamic Decelerator Systems Technology Conference*, May 1995.
- [8] John D. Nicolaides and Michael A. Tragarz. Parafoil flight performance. Technical report, University of Notre Dame, June 1971.
- [9] Chiara Toglia and Marilena Vendittelli. Modeling and motion analysis of autonomous paragliders. Technical report, Università di Roma “La Sapienza”, May 2010.
- [10] Nathan J. Slegers. Effects of canopy-payload relative motion on control of autonomous parafoils. *Journal of Guidance, Control, and Dynamics* Vol. 33, No. 1, January-February 2010.
- [11] Nathan Slegers and Mark Costello. Aspects of control for a parafoil and payload system. *Journal of Guidance, Control, and Dynamics* Vol. 26, No. 6, November–December 2003.
- [12] James W. Kamman. Me 459 dynamics of machinery - degrees of freedom of mechanical systems. Mechanical & Aeronautical Engineering - Western Michigan University, available on Ph.D. James Kamman's Homepage at <http://homepages.wmich.edu/~kamman/index.htm>. Accessed in May, 2013.
- [13] Marilena Vendittelli Chiara Toglia and Leonardo Lanari. Path following for an autonomous paraglider. *49th IEEE Conference on Decision and Control*, December 2010.

- [14] Tohru Katayama. *Subspace Methods for System Identification (Communications and Control Engineering)*. Springer, 2005.
- [15] Gi-Bong Hur and John Valasek. System identification of powered parafoil-vehicle from flight test data. *AIAA atmospheric flight mechanics conference and exhibit*, August 2003.
- [16] Nathan Slegers and Mark Costello. Model predictive control of a parafoil and payload system. *Journal of Guidance, Control, and Dynamics* Vol. 28, No. 4, July-August 2005.
- [17] Jr. Leland M. Nicolai, William R. Ramsey and Douglas J. Robinson. Autonomous payload recovery system. (US 6,808,144 B1), October 2004.
- [18] Inc. Coastal Wind Sports. Choosing your first traction kite. <http://www.coastalwindsports.com/ChoosingAKite.html>. Accessed in June, 2013.
- [19] Federal Aviation Administration. Altitude reporting equipment and transponder system maintenance and inspection practices. *Advisory Circulars, No. 43-6C*, September 2012.
- [20] P. Lissaman and G. Brown. Apparent mass effects on parafoil dynamics. *AIAA Paper 93-1236*, May 1993.
- [21] Slotine JJE Li W. *Applied Nonlinear Control*. Prentice Hall, 1st edition, 1991.
- [22] Chi-Tsong Chen. *Linear System Theory and Design*. Oxford University Press, third edition, 1999.