Multi-material joints
for high-speed train structures

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Abstract

The work presented in this thesis addresses the design and the simulation of hybrid mechanically fastened joints for the railway industry.

The work includes different levels of detail in the simulation of the joints. An experimental campaign is conducted on single and double lap joints with different geometries and materials. As result of the experimental tests, design charts are proposed to assist the engineers during the preliminary definition of a structure.

To simulate in detail the behavior of a bolted joint, a set of three-dimensional failure criteria and a progressive damage model are proposed. It was shown that the previously developed failure criterion for matrix compression works correctly for the embedded lamina only if the in-situ parameters are correctly computed. For matrix tension it was shown that previous failure criteria are not consistent.

The progressive damage model uses cohesive softening laws which are determined experimentally measuring the R-curves in the longitudinal direction. A methodology to measure the J-Integral around the crack tip using Digital Image Correlation was proposed. It was shown that for compact-tension test the results are virtually the same as those obtained using data reduction schemes based on Finite Element analyses. For compression, the methodology provides a significant improvement on the determination of the R-curve.

The cost of a 3D FE model in terms of computing time is not suitable for the industry where the results must be obtained quickly. Therefore, a simple 2D FE model, using a 2D version of the failure criteria proposed, is used and is applied to a train subcomponent. A good agreement between the numerical predictions and the experimental results was obtained.
Résumé

Le travail présenté dans cette thèse aborde la conception et la simulation des joints hybrides rivetés et boulonnés pour l’industrie ferroviaire.

Le travail concernant la simulation des joints a été conduit à différents niveaux de détail. Une campagne expérimentale est menée pour les joints simples et doubles avec différentes géométries et matériaux. Comme résultats de tests expérimentaux, des tables de conception sont proposées pour assister les ingénieurs durant la première phase de définition de la structure.

Pour simuler en détail le comportement d’un joint boulonné, une série de critères tridimensionnels d’endommagement et une modélisation de propagation de l’endommagement sont proposées. On a montré que le critère d’endommagement précédemment développé pour la compression de la matrice ne fonctionne proprement pour la couche intérieure au laminé que si les paramètres in-situ sont correctement calculés. Pour la traction de la matrice, il a été prouvé que les critères précédemment utilisés ne sont pas cohérents.

Le modèle de propagation de l’endommagement utilise des lois cohésives qui sont déterminées expérimentalement en mesurant les courbes de résistance dans la direction longitudinale. Une méthode qui utilise la corrélation digitale d’images a été proposée pour mesurer l’intégrale J autour de la pointe de la fissure. On a démontré que pour les tests de compact-tension les résultats sont pratiquement équivalents à ceux obtenus en utilisant les méthodes d’analyse basées sur les éléments finis.

Le coût d’un modèle tridimensionnel aux éléments finis en termes de temps de calcul n’est pas approprié pour l’industrie ferroviaire pour laquelle les résultats doivent être obtenus rapidement. En conséquence, il a été proposé un simple modèle 2D qui utilise la version 2D des critères d’endommagement proposés. Une bonne concordance a été trouvée entre les prédictions numériques et les résultats expérimentaux.
Resumo

O trabalho apresentado nesta tese aborda o projecto e simulação de juntas apara-fusadas híbridas para a indústria ferroviária.

O trabalho inclui diferentes níveis de detalhe para simulação das juntas. Realizou-se um programa experimental com juntas de sobreposição simples e dupla usando geometrias e materiais diferentes. Como resultado dos ensaios experimentais, foram propostas regras de projecto para a definição preliminar da estrutura.

Para simular em detalhe o comportamento de uma junta apara-fusada foi proposto um conjunto de critérios de rotura e um modelo de dano progressivo. Foi demonstrado que o critério de rotura da matriz em compressão desenvolvido previamente só funciona correctamente para uma camada embebida se forem utilizados os parâmetros *in-situ*. Para o caso da tracção da matriz, foi demonstrado que os critérios de rotura desenvolvidos previamente não são consistentes.

O modelo de dano progressivo usa leis de dano coesivas que são determinadas experimentalmente através das curvas de resistência na direcção longitudinal. Foi proposta a metodologia de Correlação Digital de Imagem para calcular o integral-*J* na vizinhança da extremidade da fissura. Foi demonstrado que para o ensaio de compact-tension os resultados são semelhantes aos obtidos através de análises em métodos de elementos finitos. Para compressão, a referida metodologia resulta num melhoramento significativo na determinação da curva de resistência.

O elevado custo computacional de um modelo de Elementos Finitos 3D torna-os inviáveis para utilização pela indústria, uma vez que normalmente pretende-se obter resultados rapidamente. Assim, foi implementado um modelo 2D de Elementos Finitos para projecto de um subcomponente de um comboio, usando a versão 2D do critério de rotura proposto. Foi obtido um resultado numérico próximo dos resultados experimentais.
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Acronyms

2D Two-Dimensional.
3D Three-Dimensional.

AFNOR Association française de Normalisation.
BE Bearing.
CC Compact Compression.
CFRP Carbon Fiber Reinforced Plastics.
CI Confidence interval.
CL Cleavage.
CT Compact Tension.
CV Coefficient of variation.
DBJ Double bolted joint.
SBJ Single bolted joint.
DIC Digital Image Correlation.
EADS European Aeronautic Defence and Space Company.
FNLBS First Non-Linearity Bearing Strength.
GFRP Glass Fiber Reinforced Plastics.
IPBS Initial Peak Bearing Strength.
LVDT Linear variable differential transformer.
NT  Net-Tension.

OBS  Offset Bearing Strength.

PT  Pull trough test.

SO  Shear-Out.

STDV  Standard deviation.
List of Symbols

$a, b, c$ variables in Two-Levels Factorial Design (Taguchi Method).

$c_f$ conversion factor (DIC).

d displacement of each pixel (DIC).

$d$ hole diameter of a mechanically fastened joint.

$d_{min}$ minimum diameter of the screw.

e edge distance of a mechanically fastened joint.


$e_{AB}, e_{AC}, e_{BC}$ two-factors effects in Two-Levels Factorial Design (Taguchi Method).

$f$ discrete function of the gray level at each pixel (DIC).

$f_{ub}$ bolt ultimate tensile strength.

$h$ specimen thickness.

$n$ normal at the fracture plane.

$n_1, n_2$ direction cosines.

$n_j$ cubic spline interpolation functions (DIC).

$t$ specimen thickness (bolted joint).

$t_n$ traction vector on the fracture plane.

$u$ unknown displacements (DIC).

$\{u\}$ displacement vector.

$w$ specimen width.
x coordinate of the pixel (DIC).

$A_b$ gross section area of the bolt.

$A, B, C$ main factors in Two-Levels Factorial Design (Taguchi Method).

$A_s$ tensile stress area of the bolt.

$C$ coefficient of correlation (DIC).

$E$ Young’s modulus.

$F_i$ pretension force.

$FI_F$ failure index for fiber tension.

$FI_s$ bolt failure index for shear.

$FI_{st}$ bolt failure index for combined presence of tension and shear.

$FI_t$ bolt failure index for tension.

$FK$ failure index for fiber kinking.

$FMC$ failure index for matrix compression.

$FMT$ failure index for matrix tension.

$J$ J-Integral.

$J_{COH}$ J-Integral calculated on the cohesive zone.

$J_{EXT}$ J-Integral calculated on the external path.

$J_{TIP}$ J-Integral calculated on the crack tip.

$K$ factor used in pretension torque calculation under standard conditions, discrete function (DIC).

$\overline{K}$ average value of $K$ function (DIC).

$K_{sb}$ shear stress concentration factor.

$K_T$ threshold function (DIC).

$K_{tb}$ tensile stress concentration factor.

$L$ support span in three-point bending test.

$M$ surface of reference pattern (DIC).
List of Symbols

$[M]$ matrix of cubic spline interpolation functions (DIC).

$P$ load.

$P_{bp}$ by-pass load.

$P_p$ load reacted at the hole.

$P_u$ load.

$S_b$ bearing stress.

$S_{bp}$ by-pass stress.

$S_L$ longitudinal shear strength.

$S_T$ transversal shear strength.

$T$ pre-tension torque.

$\{T\}$ traction vector.

$W$ strain energy density.

$X_C$ longitudinal compressive strength.

$X_T$ longitudinal tensile strength.

$Y_C$ transverse compressive strength.

$Y_T$ transverse tensile strength.
List of Symbols
Greek Letters

\( \alpha \) angle of fracture plane.
\( \alpha_T \) threshold value (DIC).

\( \beta \) shear factor.

\( \beta_b \) ratio between bearing stress and by-pass stress.

\( \delta \) displacement of the testing machine.

\( \delta_{LVDT} \) displacement of the LVDT.

\( \varepsilon \) strain.

\( \varepsilon_R \) strain at rupture.

\( \{ \varepsilon \} \) strain vector.

\( \eta_L \) slope in the \( \sigma_N-\tau_L \) diagram.

\( \eta_T \) slope in the \( \sigma_N-\tau_T \) diagram.

\( \nu \) Poisson’s ratio.

\( \omega \) angle between the shear stress \( \tau_T \) and \( \tau \).

\( \rho \) density.

\( \sigma \) stress.

\( \sigma_{br} \) bearing stress.

\( \sigma_{ii} \) direct component of the stress tensor.

\( \sigma_N \) component of the traction normal to the fracture plane.

\( \sigma_{rr} \) radial stress.
$|\sigma|$ stress tensor.

$\sigma_{\theta\theta}$ hoop stress.

$\{\sigma\}$ stress vector.

$\sigma_{xy}$ shear stress.

$\sigma_y$ yield stress.

$\tau_{ij}$ shear component of the stress tensor.

$\tau_L$ component of the traction of the fracture plane in L direction.

$\tau_T$ component of the traction of the fracture plane in T direction.

$\Gamma$ contour for the calculation of J Integral in 2D.

$\Omega$ contour for the calculation of J Integral in 3D.
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Chapter 1

Introduction

Composite materials are widely used in the industry because of their superior specific characteristics. They are light and stiff, and composite structures are easy to manufacture and assemble.

Nowadays, however, the use of composites as main load-carrying structures is often limited to the aerospace and automotive industries. While it is possible to find in the railway industry some examples of the use of composites as main load-carrying structures, this is limited to a few cases or to particular applications. In the railway industry, composites are limited to non structural parts (cab, skirts, interiors, etc.). This is justified by the fact that the use of composite requires a know-how that the railway industry does not currently have, and for this reason metals are preferred.

However, the use of composites is a vital issue for the railway industry. In fact, their advantages are numerous, namely:

- the reduction of the weight and, consequently, of the energy necessary for the railway transportation;
- the reduction of the cost of manufacturing: if a correct design is performed it is possible to obtain a simplification of the parts that form a structure and, consequently, a reduction of the time and of the costs required for the assembly of the final structure;
- the reduction of the recurring costs, because composite structures require generally less maintenance when compared with metallic structures.

Comparing the different design solutions of the main structure of a train, namely, the use of only metals, the use of only composites, the use of metals and composites (hybrid solution), it is possible to shown [1] that for a train structure the hybrid solution yields the lowest manufacturing cost. Similarly to other composite structures, one of the main design drivers in hybrid structures is the calculation of the
CHAPTER 1. INTRODUCTION

strength and the behavior of the hybrid joints. In fact, the prediction of the stiffness of hybrid structures is straightforward, and the main challenge in the design of such structures is the prediction of failure, which typically occurs at stress concentrations, such as bolted joints.

Therefore, the main objectives of this work are to investigate the mechanical response of mechanically fastened joints in composite materials and to define design methods for such joints with different levels of refinement, ranging from full three-dimensional Finite Element models to simple analysis models that can provide a solution in a short amount of time.

The thesis is organized as follows.

Chapter 2 presents a literature review on composite bolted joints including both experimental investigations and analysis models.

In Chapter 3, an experimental campaign conducted on bolted joints with different geometries and materials (or material combinations) is presented. These results may form an integral part of the design process.

Chapter 4 presents a new methodology to obtain the material properties that are required for the numerical simulation of fracture of composite joints.

In Chapter 5, a three-dimensional (3D) numerical model to predict failure of composite material is proposed. The numerical model includes, new, fully three-dimensional failure criteria and the associated damage model that represent the propagation of the different failure mechanism.

In Chapter 6, the proposed progressive damage model is used to predict different failure modes in composite bolted joints. The numerical results are compared with experimental data previously obtained.

The computational cost of a three-dimensional analysis is not suitable for an industrial use, where results must be obtained in a short account of time. For this reason, in Chapter 7 a two-dimensional numerical model is used to compute the onset of the damage. The two-dimensional numerical model implement locally a two-dimensional version of the failure criteria presented in Chapter 5. Finally, a real industrial case is studied. A prototype was manufactured and tested and the corresponding experimental results are compared with the numerical predictions.
Chapter 2

Literature Review

2.1 Experimental Analysis

The experimental research on composite bolted joints typically consists in the determination of the geometric and material parameters that affect the joint strength. Generally, two procedures are followed:

- the joint is tested until its final failure varying the parameter under study;
- the stress distribution in the vicinity of the hole is investigated.

2.1.1 Failure Modes

Figure 2.1 shows the classical geometry of the specimen and the geometrical parameters that influence its behavior:

- the width of the specimen \(w\);
- the end distance \(e\);
- the diameter of the hole \(d\);
- the thickness of the specimen \(t\).

Figure 2.1: Geometry of the specimen [2].
The ASTM D 5961/D 5961 M-05 [3] norm reports five common in plane failure modes:

- the lateral or net-tension failure mode;
- the shear-out failure mode;
- the bearing failure mode;
- the tear-out failure mode;
- the cleavage failure mode.

The pull-trough failure mode is an out-of-plane failure mode that must be also considered. All these failure modes identified in ASTM D 5961/D 5961M-05 [3] are shown in Figure 2.2. The joints may also fail in a combination of different failure modes.

![Figure 2.2: Bearing test failure modes for ASTM D 5961/D 5961M-05 [3].](image)

If the tear-out and the cleavage failure modes are considered particular cases of the shear-out failure mode, then the principal in plane failure modes are the net-tension, the shear-out and the bearing planes. These planes are represented in Figure 2.3. The circumferential coordinate direction, $\theta$, and the fibre orientation angle, $\alpha$ are also represented.

![Figure 2.3: Definition of planes, $\theta$ and $\alpha$ [2].](image)

The definitions of these planes allow the definition of the following ultimate stress:
2.1. EXPERIMENTAL ANALYSIS

- the ultimate bearing strength

\[ \sigma_{b(u)} = \frac{P_u}{dt} \]  

(2.1)

- the ultimate tensile strength

\[ \sigma_{n(u)} = \frac{P_u}{(w - d)t} \]  

(2.2)

- the ultimate shear strength

\[ \sigma_{xy(u)} = \frac{P_u}{2et} \]  

(2.3)

where \( P_u \) is the peak (failure) load, \( w \) the width, \( e \) the end distance, \( d \) the hole diameter and \( t \) the specimen thickness (see Figure 2.1).

2.1.2 Influence of the geometry

The importance of width \( w \), end-distance \( e \), bolt diameter \( d \) and laminates thickness \( t \) was highlighted by several authors [4–9]. Hart-Smith [4] highlighted the importance of the \( d/w \) ratio on the joint failure modes. When the diameter is large compared with the width of the specimen net-tension failure modes occurs. The critical value of the \( d/w \) ratio that defines the change in the failure modes from net-tension to bearing depends on the material and on the lay-up used. Kretsis and Matthews [5] showed that the transition from bearing to net-tension reduces the load capacity of the joint as showed in Figure 2.4(a). The authors identified a similar trend in the shear-out and bearing failure modes when \( e/w \) ratio varies, as shown in Figure 2.4(b).

![Figure 2.4](image.png)

Figure 2.4: Variation of bearing strength of \((0/\pm 45)\) laminates [5].

Collings [6] investigated CFRP specimens with different lay-ups and hole-sizes. The author showed that the critical values of \( w/d \) and \( e/d \) depend on the lay-up
used. The author highlighted that no thickness effects on the bearing strength are noticed if a clamping pressure of 22MPa is applied on the washer.

Hart-Smith [4, 7] showed that when the ratio $d/w$ decreases the joint structural efficient increases until an optimum value. Moreover, the author demonstrated that when the load is larger than a critical value delamination occurs near the edge of the hole and this reduces the value of the stress concentration factor. Hart-Smith [4, 7] considered that a small value of the $e/d$ ratio increases the tensile stress concentration factor. He suggested, then, the use a minimum end distance equal to $3d$.

Hodgkinson et al. [9] tested several types of joints in Kevlar fibre-reinforced plastics (KFRP). Like in CFRP and GFRP, small values of $e/d$ ratio caused shear-out failures, large values resulted in tensile failures.

### 2.1.3 Influence of the fiber orientation

Collings [6] tested CFRP laminates with $[0/\pm \alpha]_S$ lay-up. He observed that for these laminates the bearing strength was dependent on the ratio between $0^\circ$ plies and $\pm \alpha^\circ$ plies and, that the effect of these plies was dependent on the type of failure.

Collings [10] studied CFRP balanced laminates with $[0/\pm 45]_S$ and $[90/\pm 45]_S$ lay-ups. It was observed that the presence of $\pm 45^\circ$ plies increases the bearing strength of the laminate until these plies are approximately 75% of the total laminate thickness.

Kretsis and Matthews [5] showed that lay-up influences the strength and failure mode of GFRP joints and that the critical values of $w/d$ and $e/d$ depend on the lay-up used.

Hart-Smith [4, 7] showed that for laminates rich in $0^\circ$ and lacking of $90^\circ$ or with 50% of $0^\circ$ plies and 50% of $\pm 45^\circ$ plies, shear-out failure mode occurs. The author demonstrated that bearing strength is maximum for quasi-isotropic laminates.

### 2.1.4 Influence of lateral constraint

Collings [6] showed that the bearing strength of CFRP joints increases when clamping pressure increases until a maximum of 22MPa. At higher clamping pressures, negligible improvements are noticed. Obviously, clamping pressure has to be limited to avoid failure in the clamped region.
Kretsis and Matthews [5] also demonstrated the beneficial effect of lateral pressure on GFRP joints. They explained that the clamping pressure reduces the propagation of the cracks that develop under the washer.

Matthews et al. [7] considered that the joint strength depends on the through-thickness restrain offered by the fastener. For these reasons a pin loaded specimen gives the lowest bearing strength while bolted joints gives the highest bearing strength.

Hart-Smith [4, 7] concluded that the clamped area has a significant influence on the joint strength. The author noticed that inner laps gives higher values of the bearing strength with respect to the other laps. This occurs because the inner lap is better constrained and because the clamped area is larger.

2.1.5 Influence of the stacking sequence

Quinn and Matthews [11] noticed that the joint strength depends on the stacking sequence used: the through-thickness direct and shear stresses at the hole edge depends on the stacking sequence and these influence the joint strength. Placing 90° plies at the surface of the laminate produces a compressive through thickness direct stress that inhibits the delamination and that increases the joint strength.

Collings [6] also showed that interlaminar shear stress can produce delamination at the hole edge and reduce the joint strength. Large variations in fiber orientation angles in consecutive plies are responsible for higher shear stress, and this fact reduces the strength of the joint.
2.1.6 Influence of the fastener type

Different choices can be made when composite materials have to be joined. Fastener typically used are screws, rivets and bolts. However the correct choice of the fastener is a function of the particular application. Moreover, all fasteners used for metallic joints cannot be used with composites. For this reason it is possible to find very specialized fasteners for particular applications.

Collings [12] noticed that self-tapping screws are easy to install but reduce the joint efficiency. A solution is to use helicoil metallic inserts to avoid thread stripping.

Hart-Smith [4] show that the use of countersunk fasteners reduce the efficiency of the joint if compared with bolts.

Cole et al. [13] highlighted that galvanic corrosion is one of the biggest problem if wrong material are chosen. The authors presented a compatibility chart showing that titanium (and its alloys) is the best choice for CFRP laminates. To reduce the damage during the installation Cole et al. [13] suggested to use fasteners with larger countersunk angles and enlarged footprints.

2.1.7 Influence of metallic inserts

Several authors [14–17] proposed the use of bonded metallic insert to prevent the damage of the hole due to repeated installations or due to the high stress that occurs at the hole edge.

Nilsson [15] compared the strengths of bolted joints in the presence and in the absence of metallic inserts. The authors highlighted that the introduction of the insert reduces the ratios $d/w$ and $d/e$ and this changes the failure mode of the joint. The use of metallic inserts can then either increase or decrease the bearing strength of the joint.

Mirabella [16] performed experimental tests on single-lap CFRP laminates with different insert’s configurations: the single and double straight, the top-hat and the top-hat with tapered ends (See Figure 2.6).
2.1. EXPERIMENTAL ANALYSIS

It was shown that strength and failure mode depends on the material and on the type of the insert used. Moreover, when compared with the specimens without insert, the use of the insert may change the failure mode and the strength of the joints.

2.1.8 Hybrid laminates

The use of carbon-glass hybrid laminates in mechanically fastened joints has been investigated by several authors [4, 18, 19].

Hart-Smith [4] showed that the inclusion of glass fibre softening strips near the loaded hole improved the net-tension strength of carbon fibre-reinforced laminates.

Matthews et al. [18] investigated hybrid carbon/glass laminates with different lay-ups. It was shown that the trend of the strength versus the $w/d$ and $e/d$ ratios is similar to that one exhibited by homogeneous laminates. The maximum strength is achieved when $w/d$ and $e/d$ are greater than 4. For high values of $e/d$ and $w/d$ (bearing failure mode) the full carbon laminate shows the highest values of the strength. For small value of $w/d$, net-tension failure mode occurs and the hybrid laminate exhibit an higher value of the strength when compared with the full carbon joint. This result was confirmed by Hart-Smith [4].

Oh et al. [19] investigated the effects of clamping pressure and stacking sequence on hybrid joints. It was shown that the bearing strength increases with increasing clamping pressures up to 71MPa. It was found also that $[0C/\pm 45G/\pm 45C/90C]_S$ (C:carbon; G:glass) laminates exhibit the higher value of the bearing strength when $\pm 45^\circ$G plies were located between the 90°C and $\pm 45^\circ$C plies.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{Inserts’ geometry [16].}
\end{figure}
2.1.9 Multi fastener joints

In practical applications, several fasteners are used to connect composite laminates. For sufficiently large pitches, the interaction with other bolts is negligible and it is possible to estimate the strength and the failure mode of the joint using single-fastener joint data [6]. If small pitches are used interaction may occur and the strength of the multi-fastener joint is lower than that estimated using single-fastener joint data [8].

Godwin et al. [20] tested multi-bolts joints in GFRP. Lines (parallel to the load) and rows (perpendicular to the load) of bolts were considered. It was found that if the pitch is higher than 6 diameters bearing failure occurs.

When the pitch is equal to 2.5 diameters and the end-distance is 5 diameter the strength is maximized. Net-tension failure (catastrophic) occurs and this corresponds to an increase of 17% of the joint efficiency.

![Figure 2.7: Bolts installed in rows and lines.](image)

Hart-Smith [4,7] highlighted that the improvements in efficiency of multi-fasteners joints is negligible with respect to single-fastener joints. To increase the strength of the joint above the maximum strength of a single-fastener joint, the bearing stress of the multi-fastener joint should be reduced and this is possible only if large \( p_i/d \) are used. The author concluded that using more bolts in line increases negligibly the strength of the joint (two bolt increase the strength of about 10%, eight or ten bolts 25%) and that the strongest joint has only one row of bolts. In practical applications, multirow joints are used for stability in compression.

2.1.10 Stress distribution

A few experimental studies have also been made in order to determine experimentally the stress distribution near a loaded hole.

Hyer and Liu [21] used the photoelastic technique in transparent glass-epoxy plates. The radial and hoop stresses around the hole, the shear stresses in the
shear-out section, the net-section tensile stresses and the compressive stresses in the bearing plane were found.

Using the photoelastic technique, Prabhakaran [22] studied the effect of \( e \) on the stress distribution in a pin-loaded hole. GFRP specimens with \( e/d \) equal to 2, 4 and 6 were considered. It was found that the highest shear stress near the hole occurs in the specimen with the smallest end distance as previously indicated in the work of other authors [4, 5].

### 2.1.11 Fatigue

Many authors considered that a joint designed for a static test satisfies the fatigue design. Hart-Smith [7] showed that composite bolted joints do not show any damage under fatigue load. On the contrary, if interference fits are used, the residual stress may slightly increase due to the relief of stress concentration at the hole edge caused by sub-critical damage. If clearance fits are used, fretting fatigue damage may occur at the bolt-composite contact surface.

Collings [12] showed that after a fatigue test the hole show an elongation that could up to 5% of the hole diameter. The author suggested to use the maximum elongation as a criterion of bolted joints under fatigue load.

Crews [23] also tested CFRP bearing joints after water absorption. It was shown that water can reduce fatigue life up to 40%. The author concluded that care should be taken when designing bolted joints for wet environments.

Smith and Pascoe [24] and Herrington and Sabbaghian [25] showed that increasing the clamping pressure results in an increase of the fatigue life and in a reduction of the hole elongation.

### 2.2 Stress analysis

An accurate stress analysis is required for the prediction of the joint strength. Both classical and numerical methods of stress analysis are present in the literature.

#### 2.2.1 Classical methods of stress analysis

T. de Jong [26] obtained the stress distribution around the hole of a pinned frictionless joint. The analysis was based on the complex function method proposed by Muskhelishvili [27] and his adaptation to orthotropic materials proposed by Lekhnitksii [28]. The authors considered unidirectional, quasi-isotropic and \([0_s/\pm45_s]\) CFRP laminates with different geometries \((w/d = 2.5, 5, \infty)\). The author showed
that the stress distribution at the hole boundary is highly dependent on the lay-up and geometry used.

Zhang and Ueng [29] proposed a way to calculate the direct and the shear stress using Lekhnitksii’s [28] method. Considering infinite dimensions, the stress distributions were found for several lay-ups. Figure 2.8 shows the stress distributions at the hole boundary of a $[0_4/\pm 45]_S$ laminate. It was highlighted that these stress distributions were dependent on the lay-up and on the coefficient of friction.

![Figure 2.8: Stress distributions around a pin-loaded hole in a $[0_4/\pm 45]_S$ laminate for several coefficients of friction [29].](image)

2.2.2 Numerical methods of stress analysis

2.2.2.1 Two-dimensional models

Several authors [30–38] developed 2D models to find the stress distribution around the hole of a loaded laminate. Since these models usually assumed a plane stress state and used the classical lamination theory [39], they cannot account for the effects of stacking sequence or clamping pressure. The contact angle $\phi$ of the pin with the hole boundary was sometimes considered constant [30], or, more realistically, varying non-linearly with the load [31–38, 40].

Naik and Crews [31] studied the effect of clearance in a frictionless pinned laminate. As expected, the contact angle varies with the clearance as shown in Figure 2.9(a) where $c_d$ is the diametral clearance. It can be concluded that increasing the clearance decreases the contact angle. Figure 2.9(b) reports, for a bearing stress of 475 MPa, the distributions of radial and hoop stresses.
2.2. STRESS ANALYSIS

(a) relationship between bearing load and contact angle $\theta_C$

(b) hole boundary stresses

Figure 2.9: Effects of clearance [31].

Crews et al. [34] considered six lay-ups $[0/\pm 45/90]_s$, $[0]$, $[90]$, $[0/90]_s$, $[\pm 45]_s$ and $[0/\pm 45]_s$, assumed a frictionless contact and no clearance. The distributions of the hoop stress, $\sigma_{\theta\theta}$, the radial stress, $\sigma_{rr}$, and the shear stress, $\sigma_{xy}$, were found. As these stress are related with tension, bearing and shear-out failure respectively, the tensile stress concentration factor $K_{tb}$ and the shear stress concentration factor $K_{sb}$, were defined:

$$K_{tb} = \frac{(\sigma_{\theta\theta})_{\text{Max}}}{S_b} \quad (2.4)$$

$$K_{sb} = \frac{(\sigma_{xy})_{\text{Max}}}{S_b} \quad (2.5)$$

where $S_b$ is the bearing stress. Figure 2.10(a) shows that, for a constant value of end distance, $K_{tb}$ increases with decreasing $w/d$ while Figures 2.10(b)-2.10(c), show that both $K_{tb}$ and $K_{sb}$ increase with decreasing $e/d$. 
2.2.2.2 Three-dimensional models

Unfortunately, simple 2D analysis cannot take into account for the through-thickness effects that have an important effect on the joint strength. In order to have better predictions on joints strength, 3D model have developed by several authors [40–47]. The use of three-dimensional models allows to study the influence of the friction, geometry, clearance and through-thickness effect; unfortunately they are computationally expensive and require a severe modeling effort.

2.2.2.3 Multi-fastener joints

The numerical investigations of stress distributions in multi-fastener joints found in the literature were based on two-dimensional finite element models. Both lines [48] (parallel to the load) or rows [49–51] (perpendicular to the load) of fasteners have been considered.

Naik and Crews [51] investigated the stress distribution of a frictionless multi-row pinned joint [51] using an inverse technique previously developed [31]. The coefficient $\beta_b$ was defined as:

$$\beta_b = \frac{S_b}{S_{bp}}$$

(2.6)

where $S_b$ and $S_{bp}$ are the bearing stress and the by-pass stress respectively and are defined as:

$$S_b = \frac{P_b}{d l}$$

(2.7)

$$S_{bp} = \frac{P_{bp}}{(w - d) t}$$

(2.8)
where \( P_b \) and \( P_{bp} \) are respectively the load reacted at the hole and the by-pass load. It was highlighted that, for a constant bearing stress, \( \beta_b \) has a substantial influence on the contact angle and peak stress at the hole boundary: when \( \beta_b \) decreases the contact angle \( \phi \) increases. If a tensile by-pass load is applied, increasing \( S_{bp} \) the hoop stress significantly increases and its peak is located near the contact region. If a compressive by-pass load is applied, increasing \( |S_{np}| \) the peak value of the hoop stress becomes compressive.

2.2.3 Strength prediction methods

The joint strength depends on the definition of the failure: the maximum load supported by the joint, the maximum elongation of the hole [52], the first peak, the first non linearity in the load vs. displacement plot [3] and the load at which crack onset occurs [53].

2.2.3.1 Failure Theories

Several authors [54–57] used the peak local stress on the boundary and a failure theory to predict the laminate strength.

Waszczak and Cruse [54] defined a criterion using the peak stress, the peak strain and the distortional energy. Hart-Smith [4] noticed that the use of these failure theories gives a conservative prediction because the real value of stress at the hole boundary is lower than the elastic one. This is due to the hole size effect (the strength decreases for larger holes). As noticed by Crews [58] and Whitney and Nuismer [59], small holes are interested by high stresses in small regions. The localized damage reduces the stresses and redistribute the load. This effect is less pronounced for large holes.

2.2.3.2 Two-parameter methods

The parameters considered by these methods are the unnotched tensile strength and a characteristic dimension. Two methods are used: the point stress and the average stress method that compare the strength of the unnotched material with the direct stress and the average stress respectively.

Agarwal [60] proposed to modify the average stress criterion. If case of tension failure the stress (normal to the applied load) is averaged over a distance \( a_{0t} \) along the lines AB as shown in Figure 2.11.

Tension failure occurs when the averaged stress over a distance \( a_{0t} \) (assumed equal to 0.229 cm) along \( AB, A'B' \) and \( A''B'' \) reached the laminate tensile strength in the direction tangent to the point \( A, A', A'' \) respectively. The same procedure was used to predict bearing failure (the average stress was calculated over \( a_{0c} \), assumed
equal to 0.622 cm) along $CD$, $C'D'$ and $C''D''$ and shear-out failure (average shear stress along $EF$ assuming $a_{0s}$ equal to 1.143 cm). It was concluded that the accuracy of the method depends on the laminate lay-up.

Wilson and Pipes [61] proposed a point stress criterion to study the influence of $e/d$ and $w/d$ on the shear-out strength. The stress distribution on the shear plane was obtained using a FE model. The characteristic distance was assumed:

$$d_{0s} = \frac{1}{c^*} \left( \frac{R}{R_0} \right)^{m^*}$$

(2.9)

where $R$ is the hole radius, $R_0$ a reference radius and $c^*$ and $m^*$ two parameters experimentally obtained.

For a $[45/0/-45/0_2/-45/0/45/0_2/90]$s laminate, good agreement was found between predicted shear-out strength and experimental results obtained for three fastener sizes.

### 2.2.4 Combined methods

Combined methods use a combination of a characteristic distance [59] and a failure theory and were used by several authors [62–67].

Chang et al. [62] used the Yamada-Sun [68] failure criterion together with a proposed characteristic curve in a 2D FE model.

The Yamada-Sun [68] failure criterion is expressed as:

$$\left( \frac{\sigma_{11}}{X} \right)^2 + \left( \frac{\sigma_{12}}{S_c} \right)^2 = e_f^2$$

(2.10)
where $\sigma_{11}$ and $\sigma_{12}$ are respectively the longitudinal and shear stress in a ply, $X$ is the ply longitudinal strength and $S_c$ the ply shear strength measured from a cross-ply laminate. Failure occurs when $e_f \geq 1$. The characteristic curve represented in Figure 2.12 was defined as:

$$r_c(\theta) = \frac{d}{2} + d_{0t} + (d_{0c} + d_{0u}) \cos \theta$$  \hspace{1cm} (2.11)

d_{0t} and d_{0c} are the characteristic dimensions for tension and compression, respectively. Both parameters were obtained experimentally.

![Figure 2.12: Characteristic curve [62].](image)

Chang et al. [67] used a previous work [62] to account for the non-linear shear stress-shear strain relation in the Yamada-Sun [68] criterion. Hahn and Tsai [69] formula was used:

$$\gamma_{xy} = \left( \frac{1}{G_{xy}} \right) \sigma_{xy} + \alpha_1 \sigma_{xy}^3$$  \hspace{1cm} (2.12)

where $\alpha_1$ is an empirical constant. It was shown that for cross-ply and angle-ply laminates predictions agreed well with experimental data.

### 2.2.5 Progressive damage models

Progressive damage models try to simulate damage initiation and growth using degradation models that are function of the type of damage predicted [70–89].
CHAPTER 2. LITERATURE REVIEW

The progressive damage model developed by Chang and Chang [70] is able to predict tensile and shear-out failures of notched laminates [71]. Four different damage mechanisms were considered: matrix cracking, fibre-matrix shearing, fibre fracture and matrix compressive failure. Matrix cracking was predicted using the Yamada-Sun failure criterion [68]:

\[
\left( \frac{\sigma_{22}}{Y} \right)^2 + \frac{\sigma_{12}^2}{2G_{12}} + \frac{3\alpha_1\sigma_{12}^4}{4S_{2c}^2G_{12}} + \frac{3\alpha_1S_{2c}^4}{4} = e_f^2
\]  

(2.13)

where \( Y \) is the transverse tensile strength of a ply. When at one position in any of the plies in a laminate \( e_f \) is equal to or greater than 1, matrix cracking is assumed to occur in that position and layer. Both fibre-matrix shearing failure and fibre fracture were predicted using the Yamada-Sun [68] criterion:

\[
\left( \frac{\sigma_{11}}{X} \right)^2 + \frac{\sigma_{12}^2}{2G_{12}} + \frac{3\alpha_1\sigma_{12}^4}{4S_{2c}^2G_{12}} + \frac{3\alpha_1S_{2c}^4}{4} = e_f^2
\]  

(2.14)

where \( X \) is the longitudinal tensile strength of a ply. Matrix compressive failure was predicted using the Hashin failure criterion [90]:

\[
\left[ \left( \frac{Y_c}{2S_c} \right)^2 - 1 \right] \frac{\sigma_{22}}{Y_c} + \left( \frac{\sigma_{22}}{2S_c} \right)^2 + \frac{\sigma_{12}^2}{2G_{12}} + \frac{3\alpha_1\sigma_{12}^4}{4S_{2c}^2G_{12}} + \frac{3\alpha_1S_{2c}^4}{4} = e_f^2
\]  

(2.15)

The material properties are modified when damage occurs, and this depends upon the failure mechanisms. When matrix failure occurs is assumed that \( E_{22}, \nu_{21} = 0 \). If fiber failure occurs \( E_{22}, \nu_{21} = 0 \) and the longitudinal modulus \( E_{11} \) and the shear modulus \( G_{12} \) vary following a Weibull distribution:

\[
\frac{E_{11}^d}{E_{11}} = \exp \left[ - \left( \frac{A}{\delta_f^2} \right)^{\beta_f^*} \right]
\]  

(2.16)

\[
\frac{G_{12}^d}{G_{12}} = \exp \left[ - \left( \frac{A}{\delta_f^2} \right)^{\beta_f^*} \right]
\]  

(2.17)

where \( E_{11}^d \) and \( G_{12}^d \) are the reduced moduli, \( \delta_f \) is the fibre failure interaction zone [91], \( A \) is the damage zone predicted by the failure criterion and \( \beta_f^* \) is the shape of the Weibull distribution for the property degradation. Once damage propagated across the laminate, total failure was assumed to occur. It was shown that for a large number of laminate configurations and geometries the error was less than 20%. In other work [74,75], the effect of matrix cracking on the elastic properties was modeled.
2.2. STRESS ANALYSIS

taking into account the analysis of cracked lamina and not assuming simply that $E_{22}, \nu_{21} = 0$.

Lessard and Shokrieh [84] used 2D linear and non-linear models to predict the strength of pin-loaded holes. Matrix tensile and compressive failure, fibre-matrix shearing and fibre tensile failure were considered and predicted using the Hashin [90] failure criterion:

$$\left(\frac{\sigma_{11}}{X_c}\right)^2 > 1, \quad \sigma_{11} < 0 \quad (2.18)$$

where $X_c$ is the longitudinal compressive strength. Once again, when matrix failure occurs it is assumed that $E_{22}, \nu_{21}$ become equal to zero while if fibre-matrix shearing occurs is assumed that $\nu_{12}, \nu_{21}, G_{12}$ are zero. In case of fiber failure all elastic parameters are assumed to be equal to 0. It was shown that for several combinations of $e/d$ and $w/d$ the model was able to predict the strength and the failure modes of a $[0/\pm 45/90]_s$ CFRP laminate. The non-linear model was based on Chang and Lessard [72] work, and resulted in an improved prediction of the bearing failure load.

Chen et al. [41] proposed a 3D model capable of predicting delamination onset based on the Ye [92] delamination criterion. The criterion takes the form:

$$\left(\frac{\sigma_{33}}{Z_{33}}\right)^2 + \left(\frac{\sigma_{31}}{Z_{31}}\right)^2 + \left(\frac{\sigma_{23}}{Z_{23}}\right)^2 \geq 1, \quad \sigma_{33} > 0 \quad (2.19)$$

$$\left(\frac{\sigma_{31}}{Z_{31}}\right)^2 + \left(\frac{\sigma_{23}}{Z_{23}}\right)^2 \geq 1, \quad \sigma_{33} \leq 0 \quad (2.20)$$

For $[0_6/90_6]_s$ and $[90_6/0_6]_s$ laminates it was concluded that increasing friction decreased the delamination onset region at the bearing plane, but enlarged the delamination onset region along the other zones of contact. Different delamination onset regions were obtained for different lay-ups.

2.2.6 Application to multi-fastener joints

Many authors used the strength prediction methods previously described to predict the strength of multi-holes joints [93–95].

Naik and Crews [56, 57] studied the effects of the bearing stress and of bypass stress ratio, $\beta$ (see equations (2.6-2.8)) on a single fastener of a multi-fastener joint. The bearing-by-pass diagram for a $[0/45/90/-45]_2s$ CFRP laminate is shown in Figure 2.13.
where $\circ$ represents measured bearing stress $S_b$ and by-pass stress $S_{np}$ values at which damage occurs and NT, TRB, NC, CRB indicate the failure mode: net-section tension, tension reacted bearing, net-section compression and compression reacted bearing respectively. The authors highlighted that for by-pass loading ($\beta_b = 0$) the specimens failed soon after damage onset, while for bearing loading ($\beta_b = \pm \infty$) the specimens failed at an higher load respect to the damage onset load.

### 2.3 Relaxation in composite joints

The viscoelastic behavior of composites has to be considered during the design of a joint, especially for severe environmental conditions. In fact, creep and relaxation effects are amplified if high temperatures or moisture are present. Viscoelastic effects mainly influence the matrix-dominated properties: for a composite joint, relaxation modifies the value of the preload, and therefore the capacity to support a given load.

#### 2.3.1 Experimental studies

Shivakumar and Crews [96, 97] investigated the clamp-up relaxation in T300/5208 laminates. Three environment’s temperature were investigated: the room-temperature dry (RTD), the room-temperature ambient (RTA, specific humidity SH=0.46%), the elevated temperature (66°C) dry (ETD). A torque of 5.65 Nm was applied to double-shear bolted joints and the specimens were kept at the chosen environment conditions for 100 days. Specimens showed a relaxation of 12% for RTD and EDT and of 14% for RTA conditions.

Horn and Schmitt [98,99] studied the effect of fastener type and torque on clamping relaxation of carbon-fiber single-bolted joint. Titanium bolts were installed with torques of 6.8 and 11.3 Nm. It was shown that after 1000 hours all the specimens
showed a relaxation between 3.8% and 19.3%. To investigate the effect of the bolt’s relaxation on the bearing strength, the joints were tested before and after ageing: it was shown that the influence of relaxation on the bearing strength is negligible.

Thoppul et al. [100] performed a comprehensive study on the effects of various bolt pre-loads, viscoelasticity and external applied static and dynamic loads on bolt load relaxation in carbon/epoxy bolted joints. Four levels of preload were selected: 4200 N, 5050 N, 6700 N and 7850 N that correspond to 12.5%, 15%, 20% and 23.5% of the maximum tensile load. The joint were tested in a 3-point bending test to study the combined effect of shear and bending. After applying an initial preload to the composite bolted joint, the bolt load was monitored for a period of 30 h for the following cases:

- bolt preload in the absence of external beam load;
- bolt preload plus a static applied load of 250 N at the beam midspan;
- bolt preload plus a dynamic load of 250 N amplitude and frequencies of 1 Hz and 5 Hz at the beam midspan.

It was shown that, depending on the external load, in 30 hours the relaxation is 1.25-4.25%. It was also shown that increasing the preload decreases the relaxation and it is concluded that a sufficiently high preload should be provided. Comparing the bolt relaxation in steel and in composite joint it is possible to conclude that only $\frac{1}{3}$ of the relaxation in composite joints is justified by the viscoelastic behavior of the resin while the remaining $\frac{2}{3}$ of the relaxation is due to other mechanisms (tread slip, plasticity, external load).

### 2.3.2 FE analyses

To develop numerical models able to simulate the viscoelastic behavior of composite joints several authors [96,97,100,101] used the data presented by Kibler and Carter [102] and by Beckwith [103] respectively for epoxy 5208 and for Shell 58-68 epoxy at 75°F (24°C). The lamina properties are calculated from micro-mechanical equations assuming that:

- the fibers have a linear elastic behavior;
- the composite is orthotropic and transversely isotropic;
- the matrix has a linear-viscoelastic behavior that can be described by a power law:

$$D(t) = D_0 + D_1 t^n$$  \hspace{1cm} (2.21)

where

- $D(t)$ is the time dependent isotropic creep compliance of the matrix;
– $D_0$ is the initial elastic compliance of matrix;
– $D_1$ is the creep coefficient of the matrix;
– $t$ is the time;
– $n$ is a dimensionless creep exponent.

- the matrix obeys hygrothermal shift factor rules for time-temperature/moisture superposition;
- the viscoelastic response depends only on the time computed since the application of the load;
- the effective time-dependent compliance, $D(t)$, for the polymer matrix resin is often represented by the empirical power law equation for creep compliance [102].

To account for effects of temperature and moisture by using hygrothermal shift factors, $a_{TH}$, equation (2.21) can be written as [104]:

$$D(t) = D_0 + D_1 \left( \frac{t}{a_{TH}} \right)^n$$

A time-dependent matrix modulus, $E_m(t)$, can be estimated as [102]:

$$E_m(t) \approx \left( \frac{1}{D(t)} \right)$$

The viscoelastic properties of the lamina can be estimated using elastic fiber properties and time-dependent viscoelastic resin properties [105].

Using this procedure, Shivakumar and Crews [96] investigated the effects of viscoelasticity, temperature and moisture on the clamping load of T300/5208 laminates using a 2D FE model. Relaxation of bolt load was predicted for three steady state temperatures (23°C, 66°C and 121°C), for four moisture levels (0%, 0.5%, 1% and 1.5%) and for 1 day, 1 month, 1 year, 20 year exposure durations. The results showed a 8% reduction in bolt load at RTD (23°C and 0% moisture) conditions for the 1 day period and relaxation of 31% for the 20 year duration. It was highlighted that increasing temperature and moisture levels increases the relaxation rate. Moreover it was found that the relaxation due to environmental conditions are additive to the viscoelastic effects. A simple equation was proposed to calculate the time-dependent bolt clamp-up force/preload, $F_t$:

$$F_t = \frac{F_0}{1 + F_1 t^n}$$
where \( F_1 \) and \( n \) are empirical constants obtained fitting the FE results for RTD case and \( F_0 \) represents the initial value of the preload. If it should take into account the effect of the moisture the equation (2.24) should be modified introducing the shift-factor \( a_{TH} \) as:

\[
F_t = \frac{F_0}{1 + F_1(t/a_{TH})^n}
\]  

(2.25)

Equations (2.24)-(2.25) can be used to determine the relaxation only in the case of steady state temperature and moisture conditions.

Shivakumar and Crews [97] extended their previous work to transient temperature and moisture conditions using a small increment time method. A general equation to calculate bolt load relaxation was proposed:

\[
F_t = \frac{\Delta F_{0i}}{1 + F_1 \left( \sum_{j=1}^{N} \frac{t_{j+1}-t_j}{(a_{TH})_j} \right)^n}
\]  

(2.26)

where:

- \( \Delta F_{0i} \) is the elastic clamp-up increment for the \( i \)th time interval;
- \( (a_{TH})_j \) is the shift-factor at \( j^{th} \) time interval;
- \( N \) is the total interval number.

Recently, 3D joint models have been used to predict the effect of viscoelasticity. Gibson [105] used an approximate quasi-static approach to evaluate the bolt relaxation. In this approach, the quasi-elastic solutions were approximated by a series of elastic solutions corresponding to different elastic properties at different times, while the stresses were assumed to be constant during each time increment.

Thoppul et al. [101] compared the relaxation predictions made from an ABAQUS [106] viscoelastic solution with those from the quasi-elastic approach for an unreinforced isotropic epoxy beam. The bolt preload relaxation was predicted for different static and dynamic loads using both viscoelastic and quasi-elastic approaches. Relaxation of about 4.75% in the bolt preload was predicted for a period of 50 h, and the difference between viscoelastic and quasi-elastic predictions was shown to be less than 2%.

### 2.4 Effects of environmental conditions

As previously described the thermal and hygroscopic effects have a significant influence on the behavior of polymer-matrix materials.

Hygrothermal changes of the environment influence the composite behavior mainly in two ways [105]:

1....
Figure 2.14: Stiffness as a function of temperature for a typical polymer matrix material [105].

- when the temperature increases and becomes higher than the glass-transition temperature, \( T_g \), the polymer becomes softer and cannot be used in structural applications. Figure 2.14 shows this effect. The glass-transition temperature in dry conditions is indicated as \( T_{g0} \), while, when the material is fully wet \( M_w \), the wet glass transition temperature is denoted by \( T_{gw} \);

- increase or decrease temperature or moisture cause differential residual hygrothermal stresses and strains.

Therefore, environmental conditions may influence the failure and the strength of both bolted [107–116] or pinned [117–120] composite joints and for these reasons these effects have to be taken into account.

2.5 State of art in the railway industry

Currently, the use of composites is often limited to the aerospace and automotive industries. While it is possible to find in the railway industry some examples of the use of composites, their use is limited to non structural parts (cab, skirts, interiors, etc.). Composites are limited to high-speed trains (secondary structures) and are absent in other kind of trains, such as tramway and metro. This results from the fact that the use of composite requires a know-how that the railways industry does not currently have, and for this reason metals are preferred.

However the use of composites is a vital issue for the railway industry. In fact, their advantages are numerous:

- it allows to reduce the weight and consequently the energy necessary for the transportation;
• it reduces the cost of manufacturing: if the design is correctly performed (integration of functions) a structure manufactured in composites requires less parts and consequently less time to be assembled or disassembled.

• it reduces the recurring costs because composite structures require less maintenance than metal structures.

2.5.1 Comparison of different technologies

In the railway industry, the principal issue related with the use of composites is to find the way to substitute an original structure (that generally is manufactured in aluminum or steel) with another one, manufactured in composites, that gives the same or better performances. Obviously, the new technical solution can only be accepted if a gain in mass and in cost is obtained. Therefore if the composite version of a structure does not ensure these gains in mass and in cost, the solution cannot be accepted. Consequently, it is important to make a comparison between different technologies to evaluate that one that gives the best performance in reduction of weight and cost. Three different carbody-shell can be compared [1]:

• the carbody-shell manufactured completely in aluminum (aluminum carbody-shell, standard);

• the carbody-shell manufactured completely in composites (composite carbody-shell);

• the carbody-shell manufactured using steel and composites (hybrid carbody-shell).

It was shown that the hybrid solution gives the best performances [1]. It can be estimated that an hybrid carbody shell leads to a gain in mass of about 12-24% and a gain in cost of about 20%. Improved performance could be reached by developing enhanced design methods.

2.5.2 Connections in railway industry

Given that a hybrid carbody shell assures the best performance for the next generation of train designs, the most difficult task concerns the design of joints between dissimilar materials (metals and composites). The section of a carbody shell and the main connections are shown in Figure 2.15:

• the connection between the main frame and the floor, A1;

• the connection between the carbody shell and the top floor, A2 (in case of a double deck carbody shell);

• the connection between the main frame and the carbody shell, B;
• the connection between the carbody shell and the roof, C.

Figure 2.16 shows the details of these connections. Figure 2.16(a) shows the current connection between the floor and the main frame. The roof is manufactured in aluminum and is riveted or bolted to the main frame (manufactured in steel). In the future, the floor would be manufactured using glass fiber reinforced composites. Figure 2.16(b) shows the connection of the top floor of a double deck with the carbody shell. Currently, the floor is an aluminum profile, but it could be manufactured using a sandwich structure. Figure 2.16(c) represents the connection between the main frame and the carbody shell. This connection has to be made using two different materials. In fact, the internal panel cannot be toxic, while the external panel must have a good fire resistance [121]. For this reason, the internal panel can be manufactured in phenolic resin, while the external panel can be manufactured in vinylester resin. Figure 2.16(d) shows the connection between the carbody-shell and the roof. Identical considerations as above can be done for the selection of the materials of external and internal panel. The roof could be manufactured in vinylester resins or in epoxy to have better impact performance.

It is possible to find an analogy between the structural connections of Figure 2.15 and the simple connections like single or double lap joint.

To understand how all these materials behave, an experimental campaign is conducted and it is presented in Chapter 3. Different parameters are considered in the experimental work:

• geometrical parameters like \( e, w, d \);
• the different of material used (steel-glass fiber, etc...)

The experimental results will be used to obtain design charts that can be used to estimate in a easy and quick way the strength of a bolted joint.
2.5. STATE OF ART IN THE RAILWAY INDUSTRY

Figure 2.15: Main connections in a carbody shell.

Figure 2.16: Main connections in a carbody shell: details.
Chapter 3

Experimental Work

This chapter describes the experimental work done performed on bolted joints with different materials and joint configurations. The experiments aim to characterize the behavior of the joint when the geometric parameters \((w, e, d)\) vary, and to compare different composite materials for multi-material bolted joints. In the previous chapter (see Section 2.5.2) the typical joints that are used in railway applications were described, and it was shown how it is possible to find an analogy between a structural connection and a simple test configuration. It was found that it is possible to obtain relevant information on the strength of the joint if two types of tests are performed:

- the test for evaluating the bearing response of the composite;
- the test for measuring the fastener pull-trough resistance of the laminate.

The first test was performed using two different joint configurations: the double-lap joint and the single lap-joint. Also, the three point bending test was performed to evaluate the flexural properties of the laminates. The materials used to manufacture the joints are metals and composites that are commonly used in the railway industry.

3.1 Materials used

3.1.1 Metals

The metals used are:

- steel S500MC;
- aluminum 6082 T6.

Both materials are intensively used in the manufacture of trains. The mechanical properties of these materials, reported in Table 3.1, were obtained from the European
norms [122,123]. Figure 3.1 shows the true-stress/true-strain curve of the materials used.

**Table 3.1:** Mechanical properties of metals used

<table>
<thead>
<tr>
<th>material</th>
<th>$E$ (MPa)</th>
<th>$\nu$</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>hardening law</th>
</tr>
</thead>
<tbody>
<tr>
<td>S500MC</td>
<td>210000</td>
<td>0.3</td>
<td>500</td>
<td>7850</td>
<td>$\sigma = 800\varepsilon^{0.745}$ [124]</td>
</tr>
<tr>
<td>6082 T6</td>
<td>70000</td>
<td>0.34</td>
<td>260</td>
<td>2700</td>
<td>$\sigma = 460\varepsilon^{0.11}$</td>
</tr>
</tbody>
</table>

![graph](image)

**Figure 3.1:** True-stress true-strain curve for metals used

### 3.1.2 Composites

The composites used are:

- Fiber Glass-epoxy composite$^1$;
- Fiber Glass-vinylester composite$^2$;
- Fiber Glass-phenolic composite$^2$

The glass-fiber(GF)-epoxy panel was manufactured using Hexply®M9/M10 prepregs with a ply orientation pattern of $[\pm 45/90/\pm 45/0/\pm 45]_s$. The 0-90 plies have an initial thickness (thickness of the prepreg before lamination and curing) of 0.8 mm,

---

$^1$Manufactured by Ecole des Mines de Douai

$^2$Manufactured by Composites Aquitaine, EADS group
3.1. MATERIALS USED

and the ±45 plies have a thickness of 0.6 mm. After curing, the nominal thickness of each ply is 0.5 mm and 0.4 mm for the 0-90 plies and for the ±45 plies respectively. The characteristics of the unidirectional lamina of the GF-epoxy composite are given by the supplier (Hexcel Composites) and are reported in Table 3.2.

The other laminates (i.e. GF-vinylester, GF-phenolic) were manufactured using the technique of resin infusion and they have the quasi-isotropic lay-up reported in Table 3.3. The mechanical properties of these laminates were measured by Composites Aquitaine and are reported in Table 3.4. The material was tested after a heat ageing treatment according to the AFNOR norm [125].

<table>
<thead>
<tr>
<th>ply</th>
<th>type of product</th>
<th>name</th>
<th>supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>13</td>
<td>90° - 600 g/m²</td>
<td>roving UD 600 CHOMARAT</td>
<td>GAZECHIM</td>
</tr>
<tr>
<td>12</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>11</td>
<td>90° - 600 g/m²</td>
<td>roving UD 600 CHOMARAT</td>
<td>GAZECHIM</td>
</tr>
<tr>
<td>10</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>9</td>
<td>0° - 1246 g/m²</td>
<td>UNIE 1200</td>
<td>SELCOM</td>
</tr>
<tr>
<td>8</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>7</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>6</td>
<td>0° - 1246 g/m²</td>
<td>UNIE 1200</td>
<td>SELCOM</td>
</tr>
<tr>
<td>5</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>4</td>
<td>90° - 600 g/m²</td>
<td>roving UD 600 CHOMARAT</td>
<td>GAZECHIM</td>
</tr>
<tr>
<td>3</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td>2</td>
<td>±90° - 600 g/m²</td>
<td>roving UD 600 CHOMARAT</td>
<td>GAZECHIM</td>
</tr>
<tr>
<td>1</td>
<td>±45° - 610 g/m²</td>
<td>EBX 600</td>
<td>SELCOM</td>
</tr>
<tr>
<td></td>
<td>isoftalic poliester gelcoat</td>
<td>GCI S90000 VM10 SAF 1</td>
<td>POLYPROCESS</td>
</tr>
</tbody>
</table>

Table 3.2: Material properties of GF-epoxy UD laminate

<table>
<thead>
<tr>
<th>E (GPa)</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_T (MPa)</td>
<td>85</td>
</tr>
<tr>
<td>ε_R (%)</td>
<td>3.75</td>
</tr>
</tbody>
</table>

Table 3.3: Orientation pattern for GF-vinylester / phenolic composite
Table 3.4: Mechanical properties of GF-vinylester and phenolic UD laminate

<table>
<thead>
<tr>
<th>materials</th>
<th>GF-vinylester&lt;sup&gt;a&lt;/sup&gt;</th>
<th>GF-phenolic&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (MPa)</td>
<td>42830</td>
<td>35200</td>
</tr>
<tr>
<td>$E_2 = E_3$ (MPa)</td>
<td>1530</td>
<td>3000</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>$\nu_{13} = \nu_{23}$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{12}$ (MPa)</td>
<td>2800</td>
<td>3400</td>
</tr>
<tr>
<td>$G_{13} = G_{23}$ (MPa)</td>
<td>2800</td>
<td>3400</td>
</tr>
<tr>
<td>$X_T$ (MPa)</td>
<td>350</td>
<td>355</td>
</tr>
<tr>
<td>$Y_T = Z_T$ (MPa)</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>$X_C$ (MPa)</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>$Y_C = Z_C$ (MPa)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$S_T$ (MPa)</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>$S_L$ (MPa)</td>
<td>22</td>
<td>14</td>
</tr>
<tr>
<td>$\rho$ (kg/m&lt;sup&gt;3&lt;/sup&gt;)</td>
<td>1863</td>
<td>1900</td>
</tr>
</tbody>
</table>

<sup>a</sup> Heat ageing @ 90°C for 24 hours

<sup>b</sup> Heat ageing @ 80°C for 24 hours

3.2 Strain gauges

3.2.1 Selection of strain gauges

The strain gauges used in this work are of the type C2A-13-125LW-350 (see Table 3.5). These general purpose strain gages are produced by Vishay Micro-Measurements. These strain gauges were chosen due to the following reasons:

- Their 350 Ω resistance, associated with a high voltage supply, promotes a better hysteresis effect and the zero load stability;
- The strain gauges already incorporate electrical wiring allowing the remaining electrical cables to be welded without damaging the composite material;
- The grid size is approximately ten times bigger than the fibre diameter, allowing the characterization of the composite material;
- The strain gauges are compensated for aluminum. Taking into account that the tests will be performed at constant temperature, the effect of small temperature variations was neglected.

The strain gauges were connected to the data acquisition system, Spider 80, in quarter-bridge using the three wires technique. To collect the data the software Catman 3.0 was used.
Table 3.5: Strain gages technical data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid resistance (Ω)</td>
<td>350 ± 0.6%</td>
</tr>
<tr>
<td>Temp. Comp. of gage factor (%/100oC)</td>
<td>+1.3 ± 0.2</td>
</tr>
<tr>
<td>Gage factor @ 24oC</td>
<td>2.130 ± 0.5%</td>
</tr>
<tr>
<td>Transverse sensitivity</td>
<td>(0.3 ± 0.2) %</td>
</tr>
</tbody>
</table>

### 3.2.2 Surface preparation

Surface preparation is of crucial importance in strain gauging, because it influences the quality of the adhesion between the test specimen and the strain gauge. The surface preparation will be performed as follows:

- Surface degreasing using HBM-RMS1 cleaning solution (acetone and isopropanol);
- Manually abrasion of the specimen surface with sandpaper n° 400;
- Degreasing using HBM-RMS1 to assure a total removal of all the possible dust and grease that may remain on the specimen surface.

### 3.2.3 Strain gauge bonding

After the cleaning process, and to complete the strain gauging process, it was necessary to execute the steps described beneath:

- Orientation guidelines were drawn upon the specimen surface to aid the correct alignment of the strain gauge;
- The strain gage was finally glued using MBOND 200, which is a cyanoacrylate adhesive.

After these steps, the specimens were ready to be soldered to the data acquisition system wires. The strain gages were installed according to the representation shown in Figure 3.2.

![Figure 3.2: Strain gage installation](image-url)
3.3 Test Plan and Taguchi method

3.3.1 Definition of the geometry

It was shown in the previous chapter that the behavior of the specimens (strength and type of damage) depends on three parameters that are the diameter of the hole, the edge-distance and the width of the specimen. It was shown how the width and the edge distance influence the failure of the joint (2.1.2). In fact, when the ratio \( w/d \) is approximately smaller than 3, the specimen exhibits a net-tension failure while if \( w/d \) is larger the specimens exhibit a bearing failure. In the same way, the ratio \( e/d \) influences the failure of the specimen; if \( e/d \) is smaller than 4 the specimens exhibit shear-out failure while, if \( e/d \) is bigger, bearing occurs.\(^3\) As showed by Camanho et al. [126] the diameter has also an influence on the behavior of the specimen because it influences the characteristics dimensions in tension and in compression. Therefore, it is necessary to study the influence of these three parameters in the mechanical response of the joints manufactured using the material selected. For this purpose, the Taguchi method [127] may be used. If only two levels for each variable are considered the use of a Two-Level Full Factorial Design requires \( 2^3 = 8 \) runs for each material combination. In Table 3.6 the \( 2^3 \) factorial design (plus and minus signs are the two levels for each factor, \( a, b, c \) are the three variables and \( A, B, C \) are the effects of each variable) is reported. For 8 combinations using 3 specimens per test the total number of tests is equal to \( 23 \times 8 \times 3 = 128 \).

Table 3.6: \( 2^3 \) Factorial Design

<table>
<thead>
<tr>
<th>Variable combination</th>
<th>I</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>AB</th>
<th>AC</th>
<th>BC</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>b</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>c</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>abc</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>ab</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ac</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>bc</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(1)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

However, it is reasonable to assume that certain high-order interactions are negligible and to obtain information on the main effects and low order interactions only a fraction of the complete factorial experiment plan is required [128, 129]. It can be

\(^3\)These consideration are rigorous for carbon-fiber composites but they can be generalized for different types of composites
3.3. TEST PLAN AND TAGUCHI METHOD

assumed, for example, that only half of the full-factorial design that corresponds to the above part of Table 3.6 is executed. In this case, using the linear combination of the observations to estimate the main effects A, B and C it is possible to write:

\[ e_A = \frac{1}{2} (a - b - c + abc) \]  
\[ e_B = \frac{1}{2} (-a + b - c + abc) \]  
\[ e_C = \frac{1}{2} (-a - b + c + abc) \]

and the two factors iteration can be written as:

\[ e_{BC} = \frac{1}{2} (a - b - c + abc) \]  
\[ e_{AC} = \frac{1}{2} (-a + b - c + abc) \]  
\[ e_{AB} = \frac{1}{2} (-a - b + c + abc) \]

Thus the following relations can be written: \( e_A = e_{BC} \), \( e_B = e_{AC} \), \( e_C = e_{AB} \); consequently it is impossible to differentiate, for example, between the A and BC. In fact when A, B and C are estimated using equations (3.1)-(3.3), the values of A+BC, B+AC and C+AB are really estimated. However if the two-levels interactions can be neglected also the error in evaluating principal effects can be neglected.

In the case under investigation here, there are three parameters \( d, \frac{e}{d} \) and \( \frac{w}{d} \) and so the two-levels interactions for these parameters are \((d;\frac{e}{d}), (d;\frac{w}{d})\) and \((\frac{w}{d};\frac{e}{d})\). The interaction between \((d;\frac{e}{d})\) can be neglected; in fact it is known that the resistance and the behavior of the joint (shear-out or bearing) depends only on \( \frac{e}{d} \) parameter and so there are not interactions. In the same way, there are no interactions between \((d;\frac{w}{d})\); in this case the resistance and the behavior of the joint (net-tension and bearing) depends only on the \( \frac{w}{d} \) parameter. Also the interaction \((\frac{w}{d};\frac{e}{d})\) can be neglected. Suppose, for example, that a joint has a \( \frac{w}{d} \) lower than 3. In this case if the other parameter, \( \frac{e}{d} \), is equal to 5 or 10 the behavior of the joint does not change because it is a net-tension joint\(^4\). The same thing happens in a joint that has, for example, the \( \frac{e}{d} \) parameter lower than 3; changing the other parameter the behavior of the joint does not change and it will be always a shear-out specimen.

After these considerations, the experimental plan for the tests is designed. For each variable it is necessary to decide two levels, \((\_\_\_\_\_\_\_)\) and \((\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)\):

\[ d_- = 6 \text{ mm} \quad d_+ = 10 \text{ mm} \]  

\(^4\)Rigorously if the parameter \( \frac{e}{d} \) became smaller than 3 the specimen could show a mixed behavior (net-tension / shear-out / cleavage). However, generally, at least one parameters larger than the critical value that differentiates between shear-out / net-tension and bearing behavior.
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.3: Specimens’ geometry

\[(e/d)_- = 2 \quad (e/d)_+ = 4 \quad (w/d)_- = 3 \quad (w/d)_+ = 5\]

The experimental plan is shown in Table 3.7.

Table 3.7: Experimental plan

<table>
<thead>
<tr>
<th>run</th>
<th>D</th>
<th>w/D</th>
<th>e/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

3.3.2 Samples denomination

The specimens are identified in terms of the materials, the geometry and the type of the test. Three specimens were used for each type of test.

3.3.2.1 Three-point bending test (3PB)

The denomination of the specimen is the following:

C-M-I
where M is the material used and I is the index that identify the number of the sample:

- M=E (GF-epoxy), V (GF-vinylester), P (GF-phenolic);
- I=1,2,3

3.3.2.2 Double-lap bolted joint (DBJ)

The denomination of the specimen is the following:

\[ M-J-I \]

where M is the material used, J is the index that identifies the geometry and I is the index that identifies the number of the sample. For our case we have:

- M=E (GF-epoxy), V (GF-vinylester), P (GF-phenolic);
- J=1,2,3,4;
- I=1,2,3

3.3.2.3 Single-lap bolted joint (SBJ)

For SBJ specimens there are two different materials and so the denomination of the specimen is the following:

\[ M1-M2-J-I \]

where M1 and M2 are the material used, J and I have the same meaning and the same values that for BDJ:

- M1=S (steel), A (aluminum), V (GF-vinylester);
- M2=E (GF-epoxy), V, P (GF-phenolic);

3.3.2.4 Pull-Trough test (PT)

For PT specimens the following denomination was used:

\[ PT-M-d-I \]

where M is the material, d is the diameter (in millimeter) of the hole and I is the number of the sample. M and I take the same values that were used in the denomination of DBJ specimen. The d parameter takes the values d=6, 10.
3.4 Specimen preparation

The laminates were cut using a wet cutting tile saw according to the dimensions specified in ASTM norms [3,130]. The dimensions of the specimen (shown in Figure 3.2) were respected with a tolerance of 0.1 mm. The specimens were afterwards drilled and the strain gages were installed as explained in Section 3.2. Figure 3.4 shows the specimens after the installation of the strain gages. Metal tabs (in aluminum) are bonded on the specimen and, at the end, bolts are inserted and preloaded (see Section 3.4.2).

![Figure 3.4: Specimens after the installation of the strain gauges](image)

3.4.1 Hole Drilling

The method for the hole generation consists in drilling the laminate, which is supported by sacrificial frontal and backing plates.

Considering that 2 specimens are drilled in each operation, hard metal drills with a tip angle of 140, for hole generations only, were used. The drills are manufactured by SGS Tool Company and are compliant with the DIN338 standard. The commercial reference of the drills is DIN 338 - Standard Length Drills.

All holes were machined to achieve the tolerance of ±70 µm used in the aerospace applications as reported in [126].

3.4.2 Bolts, characteristics and installation

To avoid plastic deformation at the bolts, high-strength bolts were chosen. The bolts selected are manufactured by UNBRAKO and they have the following characteristics:
3.4. SPECIMEN PREPARATION

- Treads: ANSI B1.13M, ISO 261
- Property class: 12.9-ISO 898/1
- Material: ASTM A574M, DIN ENISO 4762-alloy steel
- Hardness: Rc 38-43
- Tensile Stress: 1300 MPa
- Yield Stress: 1170 MPa

Figure 3.5 shows the bolt’s geometry and Table 3.8 shows the geometric parameters of the bolt selected. The bolts were installed using the general purpose washers having the dimensions shown in Table 3.9.

The pre-load applied to the bolt was defined imposing a 22MPa pressure to the washer. Therefore the values of pretension torque was calculated using:

\[ T = K F_i d_{\text{min}} \]  

(3.10)

where \( K \) is a factor that in standard conditions is equal to 0.2, \( F_i \) is the force applied to the screw and \( d_{\text{min}} \) is the minimum diameter of the screw. Using equation (3.10) the torques are \( T_{d=10} = 10 \text{ N/m} \) and \( T_{d=6} = 2 \text{ N/m} \).

![Figure 3.5: Unbraco bolts](image)

<table>
<thead>
<tr>
<th>D&lt;sup&gt;c&lt;/sup&gt;</th>
<th>pitch</th>
<th>A&lt;sup&gt;a&lt;/sup&gt;</th>
<th>D&lt;sup&gt;a&lt;/sup&gt;</th>
<th>H&lt;sup&gt;a&lt;/sup&gt;</th>
<th>J&lt;sup&gt;c&lt;/sup&gt;</th>
<th>G&lt;sup&gt;b&lt;/sup&gt;</th>
<th>T&lt;sup&gt;b&lt;/sup&gt;</th>
<th>L&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Strength&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>M6</td>
<td>1.0</td>
<td>10.0</td>
<td>6.0</td>
<td>6.0</td>
<td>5.0</td>
<td>3.0</td>
<td>35.0</td>
<td>26.1</td>
<td></td>
</tr>
<tr>
<td>M10</td>
<td>1.5</td>
<td>16.0</td>
<td>10.0</td>
<td>10.0</td>
<td>8.0</td>
<td>4.0</td>
<td>50.0</td>
<td>75.4</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> maximum value

<sup>b</sup> minimum value

<sup>c</sup> nominal value
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Table 3.9: Washers’ geometrical parameters

<table>
<thead>
<tr>
<th>$d_{nom}$ (mm)</th>
<th>$d_{int}$ (mm)</th>
<th>$d_{ext}$ (mm)</th>
<th>$h_{nom}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7.5</td>
<td>11.8</td>
<td>1.4</td>
</tr>
<tr>
<td>10</td>
<td>10.6</td>
<td>19.6</td>
<td>1.9</td>
</tr>
</tbody>
</table>

3.5 Tests

3.5.1 Calculation and Report

3.5.1.1 Statistics

For each series of tests and for each property measured the average value $\bar{x}$, the standard deviation $STDV \ (s_{n-1})$, the coefficient of variation $CV$ and the confidence interval $IC$ at 95% are calculated using the following equations:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$  \hspace{1cm} (3.11)

$$s_{n-1} = \sqrt{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$  \hspace{1cm} (3.12)

$$CV = 100 \times \frac{s_{n-1}}{\bar{x}}$$  \hspace{1cm} (3.13)

$$IC = t_{(n-1;2.5\%)} \frac{s_{n-1}}{\sqrt{n}}$$  \hspace{1cm} (3.14)

where:

- $x_i$ is the measured or derived property;
- $n$ is the number of the specimens
- $t_{(n-1;2.5\%)}$ is the value of the $t$ distribution for a symmetrical 95% confidence interval.

3.5.1.2 Unities of measure

All measured or derived properties, if it is not otherwise specified, are expressed using the following unities of measure:

- displacement in mm
- load in N
3.5. TESTS

- strain in m/m
- stress in MPa

3.5.2 Three Point Bending Test

The 3PB test was performed for two reasons: the first one is to compare the characteristics of the materials without and with heat ageing. The material tested was not aged as explained in section 3.1.2. Therefore, it is necessary to verify if the characteristics of the material are the same as those reported in Table 3.4.

The second reason is that the value of the Young modulus of the laminate can be calculated from the 3PB test. This information will be used for the calculation of the bearing response of the laminate (Section 3.5.3.3).

The tests were performed in INEGI using the INSTRON 4208 testing machine. The testing machine was equipped with a load cell of 100kN.

The experimental set-up is shown in Figure 3.6(a). During the test the temperature was \( t = 23^\circ C \) and under the relative humidity of \( h = 50\% \).

To calculate the elastic moduli the following formula was used [130]:

\[
E_x^L = \frac{PL^3}{4wh^3\delta}
\]  

(3.15)

where \( \delta \) is the displacement measured by the testing machine, \( L \) is the support-span, \( P \) is the load, \( w \) and \( h \) are width and thickness of the specimens\(^5\).

The results are reported in Table 3.10. Figure 3.6(b) shows the load-displacement curve obtained for different materials during the 3PB test.

\(^5\)Rigorously the elastic modulus calculated by equation (3.15) is not the real modulus of the material but the apparent modulus. In [130], in fact, the effects of the shear stresses are neglected. If the effect of the shear is taken into account the displacement in the middle of the specimen can be written as:

\[
E_x^L = \frac{PL^3}{4wh^3\delta} + \frac{3PL}{8whG} = \frac{PL^3}{4wh^3\delta} (1 + \beta)
\]  

(3.16)

Therefore equation (3.15) neglects \( \beta \). However for an high value of the span this approximation is valid.
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Table 3.10: Elastic modulus of the specimens tested

<table>
<thead>
<tr>
<th>specimen</th>
<th>E (MPa)</th>
<th>specimen</th>
<th>E (MPa)</th>
<th>specimen</th>
<th>E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-E-1</td>
<td>19861</td>
<td>C-P-1</td>
<td>17595</td>
<td>C-V-1</td>
<td>17387</td>
</tr>
<tr>
<td>C-E-2</td>
<td>17882</td>
<td>C-P-2</td>
<td>17615</td>
<td>C-V-2</td>
<td>16788</td>
</tr>
<tr>
<td>C-E-3</td>
<td>19663</td>
<td>C-P-3</td>
<td>18085</td>
<td>C-V-3</td>
<td>14644</td>
</tr>
<tr>
<td>Average</td>
<td>19135</td>
<td>Average</td>
<td>17765</td>
<td>Average</td>
<td>16273</td>
</tr>
<tr>
<td>STDV</td>
<td>1090</td>
<td>STDV</td>
<td>277</td>
<td>STDV</td>
<td>1442</td>
</tr>
<tr>
<td>CV (%)</td>
<td>6</td>
<td>CV (%)</td>
<td>2</td>
<td>CV (%)</td>
<td>9</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2708</td>
<td>IC (95%)</td>
<td>689</td>
<td>IC (95%)</td>
<td>3582</td>
</tr>
</tbody>
</table>

Figure 3.6: Three point bending test

3.5.3 Double-lap Bolted Joint

3.5.3.1 Description of the test

This test was done according to the norm ASTM D5961/D5961M-01 [3]. The test was conducted using the MTS-810 testing machine in FEUP’s laboratories. Figure 3.7 shows the experimental set-up used: the specimen was clamped with a special fixture defined in the norm, and two LVDTs\(^6\) were used to obtain the information required to calculate the bearing strain in a more accurate way than that defined in [3] (see Section 3.5.3.3.1). The two LVDTs are also used to measure the eventual misalignment of the specimen.

The testing machine is equipped with a load cell of 100 kN. For each type of configuration of the joint a strain gauge is used. The strain gauges were installed

\(^6\)Linear Variable Differential Transformer.
only in specimen M-J-1 (see Section 3.3.2.2). The speed of the machine in the
displacement controlled test was 2 mm/min.

After each test the damaged specimen was examined and the type of failure
was identified. The type of failure\(^7\) was identified for each specimen according to
Figure 2.2.

At the end of each test the eventual plastic deformation of the bolt was also
verified.

![Figure 3.7: Equipment set-up for DBJ test](image)

### 3.5.3.2 Experimental Data

#### 3.5.3.2.1 Results obtained on specimens E-J-I

**3.5.3.2.1.1 Specimens E-1-I** Figure 3.8 shows the load displacement curve
for the specimens E-1-I. The values of maximum load for each specimen and the
statistics are presented in Table 3.11. Figure 3.9 shows the load-strain curve for
the specimen E-1-1. Finally, Figure 3.10 shows the damaged zone in the specimen
E-1-3. This specimen failed by tear-out and net-tension and showed the presence of
delamination.

\(^7\)SO=Shear-Out, NT=Net-Tension, CL=Cleavage, TO=Tear-Out BE=Bearing
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.8: Load vs. displacement for E-1-I specimens

Figure 3.9: Force vs. strain for E-1-I specimen

Table 3.11: Maximum load for E-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1-1</td>
<td>33695 N</td>
</tr>
<tr>
<td>E-1-2</td>
<td>32129 N</td>
</tr>
<tr>
<td>E-1-3</td>
<td>31408 N</td>
</tr>
<tr>
<td>Average</td>
<td>32411N</td>
</tr>
<tr>
<td>STDV</td>
<td>1169</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2905 N</td>
</tr>
</tbody>
</table>

Figure 3.10: E-1-3 specimen after loading

3.5.3.2.1.2 Specimens E-2-I Figure 3.11 shows the load displacement curve for the specimens E-2-I. The values of maximum load for each specimen and the statistics are presented in Table 3.12. Figure 3.12 shows the load-strain curve for the specimen E-2-1. Finally, Figure 3.13 shows the damaged zone in the specimen E-2-3. This specimen failed by net-tension and showed the presence of delamination.
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3.5.3.2.1.3 Specimens E-3-I  Figure 3.14 shows the load displacement curve for the specimens E-3-I. The values of maximum load for each specimen and the statistics are presented in Table 3.13. Figure 3.15 shows the load-strain curve for the specimen E-3-1. Finally, Figure 3.16 shows the damaged zone in the specimen E-3-3. This specimen failed by tear-out and showed the presence of delamination.

Table 3.12: Maximum load for E-2-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-2-1</td>
<td>23561 N</td>
</tr>
<tr>
<td>E-2-2</td>
<td>23268 N</td>
</tr>
<tr>
<td>E-2-3</td>
<td>24468 N</td>
</tr>
<tr>
<td>Average</td>
<td>23766 N</td>
</tr>
<tr>
<td>STDV</td>
<td>626</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1554 N</td>
</tr>
</tbody>
</table>

Figure 3.11: Load vs. displacement for E-2-I specimens

Figure 3.12: Load vs. strain for E-2-1 specimen

Figure 3.13: E-2-3 specimen after loading
3.5.3.2.1.4 Specimens E-4-I  Figure 3.17 shows the load displacement curve for the specimens E-4-I. The values of maximum load for each specimen and the statistics are presented in Table 3.14. Figure 3.18 shows the load-strain curve for the specimen E-4-1.

It should be noted that that the strain measured is not the bearing strain, but the strain in the point where the strain gage is installed. For this reason, when the load reaches the critical value the deformation measured by the strain gage decreases or it remains constant (between 4000µm/m and 6000µm/m after failure occurs).

Finally, Figure 3.19 shows the damaged zone in the specimen E-4-3. This specimen failed by bearing and showed the presence of delamination.
Table 3.14: Maximum load for E-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>first load</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-4-1</td>
<td>34416 N</td>
<td>37954 N</td>
</tr>
<tr>
<td>E-4-2</td>
<td>37037 N</td>
<td>38527 N</td>
</tr>
<tr>
<td>E-4-3</td>
<td>37333 N</td>
<td>37333 N</td>
</tr>
<tr>
<td>Average</td>
<td>36262 N</td>
<td>37938 N</td>
</tr>
<tr>
<td>STDV</td>
<td>1606</td>
<td>597</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>3988 N</td>
<td>1483 N</td>
</tr>
</tbody>
</table>

3.5.3.2.2 Results obtained on specimens V-J-I

3.5.3.2.2.1 Specimens V-1-I Figure 3.20 shows the load displacement curve for the specimens V-1-I. The values of maximum load for each specimen and the statistics are presented in Table 3.15. Figure 3.21 shows the load-strain curve for the specimen V-1-1. Finally, Figure 3.22 shows the damaged zone in the specimen V-1-2. This specimen failed by shear-out and cleavage and showed the presence of delamination.
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.20: Load vs. displacement for V-1-I specimens

Figure 3.21: Load vs. strain for V-1-I specimen

Table 3.15: Maximum load for V-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-1-1</td>
<td>21995N</td>
</tr>
<tr>
<td>V-1-2</td>
<td>20843 N</td>
</tr>
<tr>
<td>V-1-3</td>
<td>21136 N</td>
</tr>
<tr>
<td>Average</td>
<td>21325 N</td>
</tr>
<tr>
<td>STDV</td>
<td>599</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1487 N</td>
</tr>
</tbody>
</table>

Figure 3.22: V-1-2 specimen after loading

3.5.3.2.2 Specimens V-2-I  Figure 3.23 shows the load displacement curve for the specimens V-2-I. The values of maximum load for each specimen and the statistics are presented in Table 3.16. Figure 3.24 shows the load-strain curve for the specimen V-2-1. Finally, Figure 3.25 shows the damaged zone in the specimen V-2-3. This specimen failed by shear-out, bearing and net-tension and showed the presence of delamination.
3.5. TESTS

Figure 3.23: Load vs. displacement for V-2-I specimens

Figure 3.24: Load vs. strain for V-2-1 specimen

Table 3.16: Maximum load for V-2-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-2-1</td>
<td>14134 N</td>
</tr>
<tr>
<td>V-2-2</td>
<td>14524 N</td>
</tr>
<tr>
<td>V-2-3</td>
<td>13703 N</td>
</tr>
<tr>
<td>Average</td>
<td>14120 N</td>
</tr>
<tr>
<td>STDV</td>
<td>411</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1020 N</td>
</tr>
</tbody>
</table>

3.5.3.2.2.3 Specimens V-3-I  Figure 3.26 shows the load displacement curve for the specimens V-3-I. The values of maximum load for each specimen and the statistics are presented in Table 3.17. Figure 3.27 shows the load-strain curve for the specimen V-3-1. Finally, Figure 3.28 shows the damaged zone in the specimen V-3-2. This specimen failed by shear-out and cleavage and showed the presence of delamination.
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.26: Load vs. displacement for V-3-I specimens

Figure 3.27: Load vs. strain for V-3-1 specimen

Table 3.17: Maximum load for V-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-3-1</td>
<td>14252 N</td>
</tr>
<tr>
<td>V-3-2</td>
<td>15431 N</td>
</tr>
<tr>
<td>V-3-3</td>
<td>14945 N</td>
</tr>
<tr>
<td>Average</td>
<td>14876 N</td>
</tr>
<tr>
<td>STDV</td>
<td>593</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1472 N</td>
</tr>
</tbody>
</table>

Figure 3.28: V-3-2 specimen after loading

3.5.3.2.2.4 Specimens V-4-I Figure 3.29 shows the load displacement curve for the specimens V-4-I. The values of maximum load for each specimen and the statistics are presented in Table 3.18. Figure 3.30 shows the load-strain curve for the specimen V-4-1. Finally, Figure 3.31 shows the damaged zone in the specimen V-4-1. This specimen failed by bearing and shear-out and showed the presence of delamination.
3.5. TESTS

Figure 3.29: Load vs. displacement for V-4-I specimens

Figure 3.30: Load vs. strain for V-4-1 specimen

Table 3.18: Maximum load for V-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>first load</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-4-1</td>
<td>28779 N</td>
<td>31284 N</td>
</tr>
<tr>
<td>V-4-2</td>
<td>27862 N</td>
<td>31184 N</td>
</tr>
<tr>
<td>V-4-3</td>
<td>26748 N</td>
<td>32377 N</td>
</tr>
</tbody>
</table>

| Average  | 27796 N    | 31615 N      |
| STDV     | 1017       | 662          |
| CV (%)   | 4          | 2            |
| IC (95%) | 2527 N     | 1644 N       |

Figure 3.31: V-4-1 specimen after loading

3.5.3.2.3 Results obtained on specimens P-J-I

3.5.3.2.3.1 Specimens P-1-I Figure 3.32 shows the load displacement curve for the specimens P-1-I. The values of maximum load for each specimen and the statistics are presented in Table 3.19. Figure 3.33 shows the load-strain curve for the specimen P-1-1. Finally, Figure 3.34 shows the damaged zone in the specimen P-1-1. This specimen failed by bearing, shear-out and cleavage and showed the presence of delamination.
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.32: Load vs. displacement for P-1-I specimens

Figure 3.33: Load vs. strain for P-1-I specimen

Table 3.19: Maximum load for P-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-1-1</td>
<td>23357 N</td>
</tr>
<tr>
<td>P-1-2</td>
<td>23468 N</td>
</tr>
<tr>
<td>P-1-3</td>
<td>24592 N</td>
</tr>
<tr>
<td>Average</td>
<td>23806 N</td>
</tr>
<tr>
<td>STDV</td>
<td>683</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1697 N</td>
</tr>
</tbody>
</table>

Figure 3.34: P-1-1 specimen after loading

3.5.3.2.3.2 Specimens P-2-I Figure 3.35 shows the load displacement curve for the specimens P-2-I. The values of maximum load for each specimen and the statistics are presented in Table 3.20. Figure 3.36 shows the load-strain curve for the specimen P-2-1. Finally, Figure 3.37 shows the damaged zone in the specimen P-2-2. This specimen failed by bearing and net tension and showed the presence of delamination.
3.5. TESTS

Figure 3.35: Load vs. displacement for P-2-I specimens

Figure 3.36: Load vs. strain for P-2-I specimen

Table 3.20: Maximum load for P-2-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-2-1</td>
<td>17028 N</td>
</tr>
<tr>
<td>P-2-2</td>
<td>17242 N</td>
</tr>
<tr>
<td>P-2-3</td>
<td>17618 N</td>
</tr>
<tr>
<td>Average</td>
<td>17296 N</td>
</tr>
<tr>
<td>STDV</td>
<td>299</td>
</tr>
<tr>
<td>CV (%)</td>
<td>2</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>742 N</td>
</tr>
</tbody>
</table>

3.5.3.2.3.3 Specimens P-3-I

Figure 3.38 shows the load displacement curve for the specimens P-3-I. The values of maximum load for each specimen and the statistics are presented in Table 3.21. Figure 3.39 shows the load-strain curve for the specimen P-3-1. Finally, Figure 3.40 shows the damaged zone in the specimen P-3-3. This specimen failed by bearing and cleavage and showed the presence of delamination.
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.38: Load vs. displacement for P-3-I specimens

Figure 3.39: Load vs. strain for P-3-1 specimen

Table 3.21: Maximum load for P-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-3-1</td>
<td>14893 N</td>
</tr>
<tr>
<td>P-3-2</td>
<td>15759 N</td>
</tr>
<tr>
<td>P-3-3</td>
<td>16114 N</td>
</tr>
<tr>
<td>Average</td>
<td>15589 N</td>
</tr>
<tr>
<td>STDV</td>
<td>628</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1560 N</td>
</tr>
</tbody>
</table>

Figure 3.40: P-3-3 specimen after loading

3.5.3.2.3.4 Specimens P-4-I Figure 3.41 shows the load displacement curve for the specimens P-4-I. The values of maximum load for each specimen and the statistics are presented in Table 3.22. Figure 3.42 shows the load-strain curve for the specimen P-4-1. Finally, Figure 3.43 shows the damaged zone in the specimen P-4-3. This specimen failed by bearing and showed the presence of delamination.
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Figure 3.41: Load vs. displacement for P-4-I specimens

Figure 3.42: Load vs. strain for P-4-I specimen

Table 3.22: Maximum load for P-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>first load</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-4-1</td>
<td>33360</td>
<td>33360</td>
</tr>
<tr>
<td>P-4-2</td>
<td>30276</td>
<td>31884</td>
</tr>
<tr>
<td>P-4-3</td>
<td>31080</td>
<td>33377</td>
</tr>
<tr>
<td>Average</td>
<td>31572</td>
<td>32874</td>
</tr>
<tr>
<td>STDV</td>
<td>1600</td>
<td>857</td>
</tr>
<tr>
<td>CV (%)</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>3974</td>
<td>2129</td>
</tr>
</tbody>
</table>

Figure 3.43: P-4-3 specimen after loading

3.5.3.2.4 Summary for DBJ specimens Table 3.23 presents the statistics of the maximum load measured for each specimen type. The values obtained are similar for GF-vinylestyer and GF-phenolic specimens, probably for the reason that these two material were be manufactured using the same technique (resin infusion) and because the same lay-up is used.
Table 3.23: Maximum load for DBJ specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>Type(^a)</th>
<th>Average (N)</th>
<th>STDV</th>
<th>CV (%)</th>
<th>IC @ 95% (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1-I</td>
<td>TO, NT</td>
<td>32411</td>
<td>1169</td>
<td>4</td>
<td>2905</td>
</tr>
<tr>
<td>E-2-I</td>
<td>NT</td>
<td>23766</td>
<td>626</td>
<td>3</td>
<td>1554</td>
</tr>
<tr>
<td>E-3-I</td>
<td>TO</td>
<td>18935</td>
<td>1238</td>
<td>7</td>
<td>3076</td>
</tr>
<tr>
<td>E-4-I</td>
<td>BE</td>
<td>36262</td>
<td>1606</td>
<td>4</td>
<td>3988</td>
</tr>
<tr>
<td>V-1-I</td>
<td>SO, CL</td>
<td>21325</td>
<td>599</td>
<td>3</td>
<td>1487</td>
</tr>
<tr>
<td>V-2-I</td>
<td>SO, BE, NT</td>
<td>14120</td>
<td>411</td>
<td>3</td>
<td>1020</td>
</tr>
<tr>
<td>V-3-I</td>
<td>SO, CL</td>
<td>14876</td>
<td>593</td>
<td>4</td>
<td>1472</td>
</tr>
<tr>
<td>V-4-I</td>
<td>BE, SO</td>
<td>27796</td>
<td>1017</td>
<td>4</td>
<td>2527</td>
</tr>
<tr>
<td>P-1-I</td>
<td>BE, SO, CL</td>
<td>23806</td>
<td>683</td>
<td>3</td>
<td>1697</td>
</tr>
<tr>
<td>P-2-I</td>
<td>BE, NT</td>
<td>17296</td>
<td>299</td>
<td>2</td>
<td>742</td>
</tr>
<tr>
<td>P-3-I</td>
<td>BE, CL</td>
<td>15589</td>
<td>628</td>
<td>4</td>
<td>1560</td>
</tr>
<tr>
<td>P-4-I</td>
<td>BE</td>
<td>31572</td>
<td>1600</td>
<td>5</td>
<td>3974</td>
</tr>
</tbody>
</table>

\(^a\) SO=Shear-Out, NT=Net-Tension, CL=Cleavage, TO=Tear-Out, BE=Bearing

3.5.3.3 Analysis of bearing response (DBJ)

3.5.3.3.1 Correct calculation of \(\varepsilon_{br}^i\) To analyze the bearing response of the specimens it is necessary to calculate the value of the bearing stress \((\sigma_{br}^i)\) and the bearing strain \((\varepsilon_{br}^i)\) for each specimen. The value of these properties can be calculated as:

\[
\sigma_{br}^i = \frac{P_i}{dh} \quad (3.17)
\]

\[
\varepsilon_{br}^i = \frac{d_i - d}{d} \quad (3.18)
\]

where:

- \(\sigma_{br}^i\) is the bearing stress;
- \(\varepsilon_{br}^i\) is the bearing strain;
- \(P_i\) is the load;
- \(h\) is the thickness of the specimen;
- \(d\) and \(d_i\) are respectively the diameter of the hole before and during the loading.

The suggested expression [3] to evaluate the bearing strain in equation (3.18) is (see Figure 3.44):
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\[ \varepsilon^{br}_i = \frac{v - v_0}{d} = \frac{\delta_{LVDT}}{d} \]  \hspace{1cm} (3.19)

where \( \delta_{LVDT} \) is the displacement registered by the LVDT. However as showed by Camanho et al. [126] equations (3.18) and (3.19) are not completely equivalent. In fact the ASTM norm [3] neglects the deformation of the specimen. The correct equation that consider also the deformation of the specimen is:

\[ \varepsilon^{br}_i = \frac{\delta_{LVDT} - \frac{P_L}{\text{wo} E^L_x}}{d} \]  \hspace{1cm} (3.20)

where \( E^L_x \) is the Young modulus in the direction of the load \( P_i \) applied to the bolt. Note that the value of \( E^L_x \) has not be calculated but measured during the 3PB test.

![Diagram](image_url)

**Figure 3.44:** Correct calculation of bearing strain

3.5.3.3.2 Effective bearing strain  In the following plots of the bearing stress as a function of the bearing strain, the later is not calculated directly using (3.20). Rather, the effective bearing strain is calculated using equation (3.20) applying a correction factor to evaluate the effective origin of the plot.

For this purpose the Bearing Chord Stiffness in the essentially linear part of the bearing stress/bearing strain curve was calculated as \( E^{br} = \Delta \sigma^{br} / \Delta \varepsilon^{br} \). The effective origin is found by intersecting the chord stiffness line with the bearing strain axis.

3.5.3.3 Derived Properties  The properties evaluated after the bearing test are:

- the Offset Bearing Strength at 2%, defined in [3] as the strength calculated as the intersection between the bearing stress-bearing strain curve and the offset of the chord line (offset=2%);
• the Initial Peak Bearing Strength: this is the strength at the initial peak;
• the First Non Linearity Bearing Strength: the strength at the first non-linearity in the curve.

Tables 3.25-3.27 show that these three values are practically the same and this is due to the particular material. It should be noted that the Ultimate Strength (defined as the maximum value of the strength) is not reported; this because the material shows an initial peak and after that the load does not increase but rather remains constant or decreases.

These properties depend also on the thickness of the specimens near the hole that are reported in Table 3.24.

**Table 3.24:** Thickness of bearing specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>h (mm)</th>
<th>Specimen</th>
<th>h (mm)</th>
<th>Specimen</th>
<th>h (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-4-1</td>
<td>6.45</td>
<td>P-4-1</td>
<td>6.85</td>
<td>V-4-1</td>
<td>6.65</td>
</tr>
<tr>
<td>E-4-2</td>
<td>6.45</td>
<td>P-4-2</td>
<td>6.80</td>
<td>V-4-2</td>
<td>6.50</td>
</tr>
<tr>
<td>E-4-3</td>
<td>6.45</td>
<td>P-4-3</td>
<td>6.80</td>
<td>V-4-3</td>
<td>6.80</td>
</tr>
</tbody>
</table>

3.5.3.3.4 Experimental results for bearing specimens

3.5.3.3.4.1 Specimens E-4-I The curves of the bearing stress as a function of the bearing strain are shown in Figure 3.45. Table 3.25 presents the bearing strengths.

**Table 3.25:** Bearing response for E-4-I specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-4-1</td>
<td>562</td>
<td>564</td>
<td>562</td>
</tr>
<tr>
<td>E-4-2</td>
<td>566</td>
<td>574</td>
<td>524</td>
</tr>
<tr>
<td>E-4-3</td>
<td>567</td>
<td>579</td>
<td>573</td>
</tr>
</tbody>
</table>

Average 565.278 572.556 552.947
STDV 2.544 7.379 25.794
CV (%) 0.450 1.289 4.665
IC (95%) 6.320 18.330 64.077

\(^a\) Offset Bearing Strength
\(^b\) Initial Peak Bearing Strength
\(^c\) First Non-Linearity Bearing Strength

**Figure 3.45:** Bearing stress vs. bearing strain for E-4-I specimens
3.5. TESTS

3.5.3.3.4.2 Specimens P-4-I  The curves of the bearing stress as a function of the bearing strain are shown in Figure 3.46. Table 3.26 presents the bearing strengths.

![Figure 3.46: Bearing stress vs. bearing strain for P-4-I specimens](image)

Table 3.26: Bearing response for P-4-I specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-4-1</td>
<td>441</td>
<td>498</td>
<td>435</td>
</tr>
<tr>
<td>P-4-2</td>
<td>436</td>
<td>469</td>
<td>446</td>
</tr>
<tr>
<td>P-4-3</td>
<td>463</td>
<td>488</td>
<td>453</td>
</tr>
<tr>
<td>Average</td>
<td>446.507</td>
<td>485.010</td>
<td>444.542</td>
</tr>
<tr>
<td>STDV</td>
<td>14.656</td>
<td>14.734</td>
<td>9.214</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3.282</td>
<td>3.038</td>
<td>2.073</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>36.408</td>
<td>36.600</td>
<td>22.890</td>
</tr>
</tbody>
</table>

\(^a\) Offset Bearing Strength  
\(^b\) Initial Peak Bearing Strength  
\(^c\) First Non-Linearity Bearing Strength

3.5.3.3.4.3 Specimens V-4-I  The curves of the bearing stress as a function of the bearing strain are shown in Figure 3.47. Table 3.27 presents the bearing strengths.

![Figure 3.47: Bearing stress vs. bearing strain for V-4-I specimens](image)

Table 3.27: Bearing response for V-4-I specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-4-1</td>
<td>398</td>
<td>444</td>
<td>402</td>
</tr>
<tr>
<td>V-4-2</td>
<td>389</td>
<td>390</td>
<td>390</td>
</tr>
<tr>
<td>V-4-3</td>
<td>370</td>
<td>396</td>
<td>396</td>
</tr>
<tr>
<td>Average</td>
<td>385.531</td>
<td>409.918</td>
<td>395.982</td>
</tr>
<tr>
<td>STDV</td>
<td>14.062</td>
<td>29.370</td>
<td>6.038</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3.647</td>
<td>7.165</td>
<td>1.525</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>34.932</td>
<td>72.959</td>
<td>14.999</td>
</tr>
</tbody>
</table>

\(^a\) Offset Bearing Strength  
\(^b\) Initial Peak Bearing Strength  
\(^c\) First Non-Linearity Bearing Strength

3.5.3.4 Discussion on results for DBJ

In the previous sections, it was shown that specimens with the same geometry but different materials exhibit different failure modes. To compare these results it should
be noted that the laminates tested are all quasi-isotropic but they have different lay-ups and thickness and they were manufactured using different techniques. For these reasons, to compare the results the \( P/h \) parameter instead that the load \( P \) should be used.

In fact, the different failure modes (net-tension, shear-out, bearing) are related with different stresses defined for each failure mode as the net-tension stress, the shear-out stress and the bearing stress (see equations (2.1-2.3)). These stress are proportional to the \( P/h \) ratio and for this reason this parameter it is used to compare the behavior of different laminates.

Figures 3.48-3.51 show the load-displacement curves and the histograms of the \((P/h)_{Max}\) ratio for different specimens. The dashed rectangle in the histogram represents the 95% interval of confidence.

Figure 3.48: Comparison of results for specimens #1 (DBJ)

Figure 3.49: Comparison of results for specimens #2 (DBJ)
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(a) load vs. displacement curves

(b) $(P/h)_{Max}$ for different materials

Figure 3.50: Comparison of results for specimens #3 (DBJ)

(a) load vs. displacement curves

(b) $(P/h)_{Max}$ for different materials

Figure 3.51: Comparison of results for specimens #4 (DBJ)

It is observed that the GF-epoxy specimens exhibit the highest strength. However, these specimens are characterized by a catastrophic failure while the other specimens are characterized by a progressive failure after the peak load.

The specimens manufactured with GF-phenolic and GF-vinylester show the same behavior after the peak load and this is due to the fact that both were manufactured using the vacuum resin infusion technique.

Regarding the strength of the joint, the GF-epoxy specimens show the best performances while the GF-vinylester show the worst.

3.5.4 Single-lap Bolted Joint

3.5.4.1 Description of the test

The test was effectuated according to the norm ASTM D5961/D5961M-01 [3].
The test was conducted using the INSTRON-4208 test machine in the laboratories of INEGI. The experimental set-up used is shown in Figure 3.52.

The test machine is equipped with a 100kN load cell. For each type of configuration a strain gauge was used. The strain gauges were installed only in the specimens M1-M2-J-1 (see Section 3.3.2.3). The speed of the machine (displacement controlled test) was 2mm/min. The temperature of the room was 23°C and the relative humidity was 50% for all the duration of the tests.

After each test the damaged specimen was examined and the type of failure was identified. It was observed that the bolts did not suffer any plastic deformation.

![Equipment set-up for SBJ test](image)

**Figure 3.52:** Equipment set-up for SBJ test

### 3.5.4.2 Specimens P-A-J-I

#### 3.5.4.2.1 Specimens P-A-1-I

Figure 3.53 shows the force-displacement curve for the specimens P-A-1-I. In Table 3.28 the value of the maximum load for each specimen and the statistics are reported. Figure 3.54 shows the force-strain curve for the specimen P-A-1-1. Figure 3.55 shows the damaged zones in the specimen P-A-1-2.

The composite is damaged and the aluminum deformed plastically, due to the effect of bending moment. Before complete failure, it is possible to note that some interlaminar cracks appear in the composite; these cracks are responsible for the reduction of the stiffness of the specimen. These consideration are valid for all the
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specimens tested.

![Graph](image1)

**Figure 3.53:** Load vs. displacement for P-A-1-I specimens

![Graph](image2)

**Figure 3.54:** Load vs. strain for P-A-1-I specimen

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-A-1-1</td>
<td>19424</td>
</tr>
<tr>
<td>P-A-1-2</td>
<td>20876</td>
</tr>
<tr>
<td>P-A-1-3</td>
<td>20884</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>20395</strong></td>
</tr>
<tr>
<td>STDV</td>
<td>841</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2088</td>
</tr>
</tbody>
</table>

**Table 3.28:** Maximum load for P-A-1-I specimens

![Image](image3)

(a) side view

(b) top view

**Figure 3.55:** P-A-1-2 specimen after loading

3.5.4.2.2 Specimens P-A-2-I Figure 3.56 shows the force-displacement curve for the specimens P-A-2-I. In Table 3.29 the value of the maximum load for each specimen and the statistics are reported. Figure 3.57 shows the force-strain curve for the specimen P-A-2-1. Figure 3.58 shows the damaged zones in the specimen P-A-2-2.
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Table 3.29: Maximum load for P-A-2-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-A-2-1</td>
<td>11340</td>
</tr>
<tr>
<td>P-A-2-2</td>
<td>12456</td>
</tr>
<tr>
<td>P-A-2-3</td>
<td>12148</td>
</tr>
<tr>
<td>Average</td>
<td>11981</td>
</tr>
<tr>
<td>STDV</td>
<td>576</td>
</tr>
<tr>
<td>CV (%)</td>
<td>5</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1432</td>
</tr>
</tbody>
</table>

3.5.4.2.3 Specimens P-A-3-I Figure 3.59 shows the force-displacement curve for the specimens P-A-3-I. In Table 3.30 the value of the maximum load for each specimen and the statistics are reported. Figure 3.60 shows the force-strain curve for the specimen P-A-3-1. Figure 3.61 shows the damaged zones in the specimen P-A-3-3.
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Figure 3.59: Load vs. displacement for P-A-3-I specimens

Figure 3.60: Load vs. strain for P-A-3-1 specimen

Table 3.30: Maximum load for P-A-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-A-3-1</td>
<td>12540</td>
</tr>
<tr>
<td>P-A-3-2</td>
<td>12788</td>
</tr>
<tr>
<td>P-A-3-3</td>
<td>13132</td>
</tr>
<tr>
<td>Average</td>
<td>12820</td>
</tr>
<tr>
<td>STDV</td>
<td>297</td>
</tr>
<tr>
<td>CV (%)</td>
<td>2</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>739</td>
</tr>
</tbody>
</table>

Figure 3.61: P-A-3-3 specimen after loading

3.5.4.2.4 Specimens P-A-4-I Figure 3.62 shows the force-displacement curve for the specimens P-A-4-I. In Table 3.31 the value of the maximum load for each specimen and the statistics are reported. Figure 3.63 shows the force-strain curve for the specimen P-A-4-1. Figure 3.64 shows the damaged zones in the specimen P-A-4-2.
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Figure 3.62: Load vs. displacement for P-A-4-I specimens

Figure 3.63: Load vs. strain for P-A-4-I specimen

Table 3.31: Maximum load for P-A-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-A-4-1</td>
<td>34012</td>
</tr>
<tr>
<td>P-A-4-2</td>
<td>33236</td>
</tr>
<tr>
<td>P-A-4-3</td>
<td>33404</td>
</tr>
<tr>
<td>Average</td>
<td>33551</td>
</tr>
<tr>
<td>STDV</td>
<td>408</td>
</tr>
<tr>
<td>CV (%)</td>
<td>1</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1014</td>
</tr>
</tbody>
</table>

Figure 3.64: P-A-4-2 specimen after loading

3.5.4.3 Specimens P-S-J-I

3.5.4.3.1 Specimens P-S-1-I Figure 3.65 shows the force-displacement curve for the specimens P-S-1-I. In Table 3.32 the value of the maximum load for each specimen and the statistics are reported. Figure 3.66 shows the force-strain curve for the specimen P-S-1-1. Figure 3.67 shows the damaged zones in the specimen P-S-1-2.
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Figure 3.65: Load vs. displacement for P-S-1-I specimens

Figure 3.66: Load vs. strain for P-S-1-1 specimen

Table 3.32: Maximum load for P-S-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-S-1-1</td>
<td>23708</td>
</tr>
<tr>
<td>P-S-1-2</td>
<td>22592</td>
</tr>
<tr>
<td>P-S-1-3</td>
<td>21412</td>
</tr>
<tr>
<td>Average</td>
<td>22571</td>
</tr>
<tr>
<td>STDV</td>
<td>1148</td>
</tr>
<tr>
<td>CV (%)</td>
<td>5</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2852</td>
</tr>
</tbody>
</table>

Figure 3.67: P-S-1-2 specimen after loading

3.5.4.3.2 Specimens P-S-2-I Figure 3.68 shows the force-displacement curve for the specimens P-S-2-I. In Table 3.33 the value of the maximum load for each specimen and the statistics are reported. Figure 3.69 shows the force-strain curve for the specimen P-S-2-1. Figure 3.70 shows the damaged zones in the specimen P-S-2-2.
3.5.4.3.3 Specimens P-S-3-I  Figure 3.71 shows the force-displacement curve for the specimens P-S-3-I. In Table 3.34 the value of the maximum load for each specimen and the statistics are reported. Figure 3.72 shows the force-strain curve for the specimen P-S-3-1. Figure 3.73 shows the damaged zones in the specimen P-S-3-2.
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Figure 3.71: Load vs. displacement for P-S-3-I specimens

Figure 3.72: Load vs. strain for P-S-3-1 specimen

Table 3.34: Maximum load for P-S-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-S-3-1</td>
<td>13180</td>
</tr>
<tr>
<td>P-S-3-2</td>
<td>13960</td>
</tr>
<tr>
<td>P-S-3-3</td>
<td>14232</td>
</tr>
<tr>
<td>Average</td>
<td>13791</td>
</tr>
<tr>
<td>STDV</td>
<td>546</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1356</td>
</tr>
</tbody>
</table>

Figure 3.73: P-S-3-2 specimen after loading

3.5.4.3.4 Specimens P-S-4-I  Figure 3.74 shows the force-displacement curve for the specimens P-S-4-I. In Table 3.35 the value of the maximum load for each specimen and the statistics are reported. Figure 3.75 shows the force-strain curve for the specimen P-S-4-1. Figure 3.76 shows the damaged zones in the specimen P-S-4-2.
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3.5.4.4 Specimens V-A-J-I

3.5.4.4.1 Specimens V-A-1-I  Figure 3.77 shows the force-displacement curve for the specimens V-A-1-I. In Table 3.36 the value of the maximum load for each specimen and the statistics are reported. Figure 3.78 shows the force-strain curve for the specimen V-A-1-1. Figure 3.79 shows the damaged zones in the specimen V-A-1-3.
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Figure 3.77: Load vs. displacement for V-A-1-I specimens

Figure 3.78: Load vs. strain for V-A-1-I specimen

Table 3.36: Maximum load for V-A-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-A-1-1</td>
<td>18212</td>
</tr>
<tr>
<td>V-A-1-2</td>
<td>19964</td>
</tr>
<tr>
<td>V-A-1-3</td>
<td>19708</td>
</tr>
<tr>
<td>Average</td>
<td>19295</td>
</tr>
<tr>
<td>STDV</td>
<td>946</td>
</tr>
<tr>
<td>CV (%)</td>
<td>5</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2351</td>
</tr>
</tbody>
</table>

Figure 3.79: V-A-1-3 specimen after loading

3.5.4.4.2 Specimens V-A-2-I Figure 3.80 shows the force-displacement curve for the specimens V-A-2-I. In Table 3.37 the value of the maximum load for each specimen and the statistics are reported. Figure 3.81 shows the force-strain curve for the specimen V-A-2-1. Figure 3.82 shows the damaged zones in the specimen V-A-2-1.
3.5.4.4.3 Specimens V-A-3-I  Figure 3.83 shows the force-displacement curve for the specimens V-A-3-I. In Table 3.38 the value of the maximum load for each specimen and the statistics are reported. Figure 3.84 shows the force-strain curve for the specimen V-A-3-1. Figure 3.85 shows the damaged zones in the specimen V-A-3-3.
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Figure 3.83: Load vs. displacement for V-A-3-I specimens

Figure 3.84: Load vs. strain for V-A-3-I specimen

Table 3.38: Maximum load for V-A-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-A-3-1</td>
<td>13052</td>
</tr>
<tr>
<td>V-A-3-2</td>
<td>12292</td>
</tr>
<tr>
<td>V-A-3-3</td>
<td>12872</td>
</tr>
<tr>
<td>Average</td>
<td>12739</td>
</tr>
<tr>
<td>STDV</td>
<td>397</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>987</td>
</tr>
</tbody>
</table>

Figure 3.85: V-A-3-3 specimen after loading

3.5.4.4.4 Specimens V-A-4-I Figure 3.86 shows the force-displacement curve for the specimens V-A-4-I. In Table 3.39 the value of the maximum load for each specimen and the statistics are reported. Figure 3.87 shows the force-strain curve for the specimen V-A-4-1. Figure 3.88 shows the damaged zones in the specimen V-A-4-2.
Table 3.39: Maximum load for V-A-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-A-4-1</td>
<td>33416</td>
</tr>
<tr>
<td>V-A-4-2</td>
<td>33968</td>
</tr>
<tr>
<td>V-A-4-3</td>
<td>31320</td>
</tr>
<tr>
<td>Average</td>
<td>32901</td>
</tr>
<tr>
<td>STDV</td>
<td>1397</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>3470</td>
</tr>
</tbody>
</table>

3.5.4.5 Specimens V-S-J-I

3.5.4.5.1 Specimens V-S-1-I Figure 3.89 shows the force-displacement curve for the specimens V-S-1-I. In Table 3.40 the value of the maximum load for each specimen and the statistics are reported. Figure 3.90 shows the force-strain curve for the specimen V-S-1-1. Figure 3.91 shows the damaged zones in the specimen V-S-1-2.
3.5. TESTS

Figure 3.89: Load vs. displacement for V-S-1-I specimens

Figure 3.90: Load vs. strain for V-S-1-I specimen

Table 3.40: Maximum load for V-S-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-S-1-1</td>
<td>20296</td>
</tr>
<tr>
<td>V-S-1-2</td>
<td>22156</td>
</tr>
<tr>
<td>V-S-1-3</td>
<td>19384</td>
</tr>
<tr>
<td>Average</td>
<td>20612</td>
</tr>
<tr>
<td>STDV</td>
<td>1413</td>
</tr>
<tr>
<td>CV (%)</td>
<td>7</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>3509</td>
</tr>
</tbody>
</table>

Figure 3.91: V-S-1-2 specimen after loading

3.5.4.5.2 Specimens V-S-2-I Figure 3.92 shows the force-displacement curve for the specimens V-S-2-I. In Table 3.41 the value of the maximum load for each specimen and the statistics are reported. Figure 3.93 shows the force-strain curve for the specimen V-S-2-1. Figure 3.94 shows the damaged zones in the specimen V-S-2-2.
3.5.4.5.3 Specimens V-S-3-I  Figure 3.95 shows the force-displacement curve for the specimens V-S-3-I. In Table 3.42 the value of the maximum load for each specimen and the statistics are reported. Figure 3.96 shows the force-strain curve for the specimen V-S-3-1. Figure 3.97 shows the damaged zones in the specimen V-S-3-3.
3.5. TESTS

Figure 3.95: Load vs. displacement for V-S-3-I specimens

Figure 3.96: Load vs. strain for V-S-3-1 specimen

Table 3.42: Maximum load for V-S-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-S-3-1</td>
<td>14408</td>
</tr>
<tr>
<td>V-S-3-2</td>
<td>14384</td>
</tr>
<tr>
<td>V-S-3-3</td>
<td>12568</td>
</tr>
<tr>
<td>Average</td>
<td>13787</td>
</tr>
<tr>
<td>STDV</td>
<td>1055</td>
</tr>
<tr>
<td>CV (%)</td>
<td>8</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2622</td>
</tr>
</tbody>
</table>

Figure 3.97: V-S-3-3 specimen after loading

3.5.4.5.4 Specimens V-S-4-I Figure 3.98 shows the force-displacement curve for the specimens V-S-4-I. In Table 3.43 the value of the maximum load for each specimen and the statistics are reported. Figure 3.99 shows the force-strain curve for the specimen V-S-4-1. Figure 3.100 shows the damaged zones in the specimen V-S-4-3.
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Figure 3.98: Load vs. displacement for V-S-4-I specimens

Figure 3.99: Load vs. strain for V-S-4-I specimen

Table 3.43: Maximum load for V-S-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-S-4-1</td>
<td>35844</td>
</tr>
<tr>
<td>V-S-4-2</td>
<td>34732</td>
</tr>
<tr>
<td>V-S-4-3</td>
<td>35876</td>
</tr>
<tr>
<td>Average</td>
<td>35484</td>
</tr>
<tr>
<td>STDV</td>
<td>651</td>
</tr>
<tr>
<td>CV (%)</td>
<td>2</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>1618</td>
</tr>
</tbody>
</table>

(a) side view

(b) top view

Figure 3.100: V-S-4-3 specimen after loading

3.5.4.6 Specimens V-P-J-I

3.5.4.6.1 Specimens V-P-1-I  Figure 3.101 shows the force-displacement curve for the specimens V-P-1-I. In Table 3.44 the value of the maximum load for each specimen and the statistics are reported. Figure 3.102 shows the force-strain curve for the specimen V-P-1-1. Note that in this case there are two curves: one is the strain detected by the strain gage installed on vinylester, the other is detected by the strain gage installed on phenolic. Figure 3.103 shows the damaged zones in the specimen V-P-1-3 and V-P-1-2 respectively.
3.5. TESTS

Figure 3.101: Load vs. displacement for V-P-1-I specimens

Figure 3.102: Load vs. strain for V-P-1-I specimen

Table 3.44: Maximum load for V-P-1-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-P-1-1</td>
<td>18324</td>
</tr>
<tr>
<td>V-P-1-2</td>
<td>20292</td>
</tr>
<tr>
<td>V-P-1-3</td>
<td>20204</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>19607</strong></td>
</tr>
<tr>
<td>STDV</td>
<td>1112</td>
</tr>
<tr>
<td>CV (%)</td>
<td>6</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2762</td>
</tr>
</tbody>
</table>

Figure 3.103: V-P-1-I specimens after loading

3.5.4.6.2 Specimens V-P-2-I Figure 3.104 shows the force-displacement curve for the specimens V-P-2-I. In Table 3.45 the value of the maximum load for each specimen and the statistics are reported. Figure 3.105 shows the force-strain curve for the specimen V-P-2-1. Figure 3.106 shows the damaged zones in the specimen V-P-2-1.
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Figure 3.104: Load vs. displacement for V-P-2-I specimens

Table 3.45: Maximum load for V-P-2-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-P-2-1</td>
<td>12100</td>
</tr>
<tr>
<td>V-P-2-2</td>
<td>11376</td>
</tr>
<tr>
<td>V-P-2-3</td>
<td>11752</td>
</tr>
<tr>
<td>Average</td>
<td>11743</td>
</tr>
<tr>
<td>STDV</td>
<td>362</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>899</td>
</tr>
</tbody>
</table>

Figure 3.105: Load vs. strain for V-P-2-1 specimen

Figure 3.106: V-P-2-1 specimen after loading

3.5.4.6.3 Specimens V-P-3-I Figure 3.107 shows the force-displacement curve for the specimens V-P-3-I. In Table 3.46 the value of the maximum load for each specimen and the statistics are reported. Figure 3.108 shows the force-strain curve for the specimen V-P-3-1. Figure 3.109 shows the damaged zones in the specimen V-P-3-2.
3.5. TESTS

Figure 3.107: Load vs. displacement for V-P-3-I specimens

Table 3.46: Maximum load for V-P-3-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-P-3-1</td>
<td>13000</td>
</tr>
<tr>
<td>V-P-3-2</td>
<td>13712</td>
</tr>
<tr>
<td>V-P-3-3</td>
<td>10616</td>
</tr>
<tr>
<td>Average</td>
<td>12443</td>
</tr>
<tr>
<td>STDV</td>
<td>1622</td>
</tr>
<tr>
<td>CV (%)</td>
<td>13</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>4028</td>
</tr>
</tbody>
</table>

Figure 3.108: Load vs. strain for V-P-3-I specimen

3.5.4.6.4 Specimens V-P-4-I Figure 3.110 shows the force-displacement curve for the specimens V-P-4-I. In Table 3.47 the value of the maximum load for each specimen and the statistics are reported. Figure 3.111 shows the force-strain curve for the specimen V-P-4-1. Figure 3.112 shows the damaged zones in the specimen V-P-4-1.
CHAPTER 3. EXPERIMENTAL WORK

Figure 3.110: Load vs. displacement for V-P-4-I specimens

Figure 3.111: Load vs. strain for V-P-4-I specimen

Table 3.47: Maximum load for V-P-4-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-P-4-1</td>
<td>33936</td>
</tr>
<tr>
<td>V-P-4-2</td>
<td>33020</td>
</tr>
<tr>
<td>V-P-4-3</td>
<td>31916</td>
</tr>
<tr>
<td>Average</td>
<td>32957</td>
</tr>
<tr>
<td>STDV</td>
<td>1011</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2513</td>
</tr>
</tbody>
</table>

3.5.4.7 Analysis of bearing response (SBJ)

The analysis of bearing response is performed using the same procedure applied in the analysis of the DBJ bearing specimen.

The calculation of the bearing strain is effectuated as described in 3.5.3.3.1. In analogy with equation (3.19) the following formula to evaluate the bearing strain is used:

\[
\varepsilon_{br}^{i} = \frac{\delta_{LVDT} - \frac{P_{L0}}{whE_{x,M1}} - \frac{P_{L0}}{whE_{x,M2}}}{d}
\]  

(3.21)

In this case there are two materials to consider, M1 and M2 (see Figure 3.113). It should be noted that when the second material is a metal as steel the correc-
tion $-PL_0/whE_{x,M2}^L$ can be neglected. However when $M2$ is aluminum or another composite this correction must be used.

![Figure 3.113: Bearing correction for SBJ](image)

3.5.4.7.1 Specimens P-A-4-I Figure 3.114 shows the curves of the bearing stress as a function of the effective bearing strain. Table 3.48 reports the derived properties with statistics. These properties are illustrated in Section 3.5.3.3.3.

![Figure 3.114: Bearing stress vs. bearing strain for P-A-4-I specimens](image)

**Table 3.48: Bearing response for P-A-4-I specimens**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS$^a$</th>
<th>IPBS$^b$</th>
<th>FNLBS$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-S-4-1</td>
<td>419</td>
<td>587</td>
<td>320</td>
</tr>
<tr>
<td>P-S-4-2</td>
<td>427</td>
<td>522</td>
<td>310</td>
</tr>
<tr>
<td>P-S-4-3</td>
<td>468</td>
<td>570</td>
<td>350</td>
</tr>
<tr>
<td>Average</td>
<td>438</td>
<td>560</td>
<td>327</td>
</tr>
<tr>
<td>STDV</td>
<td>26.531</td>
<td>33.710</td>
<td>20.817</td>
</tr>
<tr>
<td>CV (%)</td>
<td>6.054</td>
<td>6.023</td>
<td>6.372</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>65.907</td>
<td>83.739</td>
<td>51.711</td>
</tr>
</tbody>
</table>

$^a$ Offset Bearing Strength  
$^b$ Initial Peak Bearing Strength  
$^c$ First Non-Linearity Bearing Strength

3.5.4.7.2 Specimens P-S-4-I Figure 3.115 shows the curves of the bearing stress as a function of the effective bearing strain. Table 3.49 reports the derived properties with statistics.
3.5.4.7.3 Specimens V-A-4-I  Figure 3.116 shows the curves of the bearing stress as a function of the effective bearing strain. Table 3.50 reports the derived properties with statistics.

![Figure 3.116: Bearing stress vs. bearing strain for V-A-4-I specimens](image)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-A-4-1</td>
<td>290</td>
<td>499</td>
<td>278</td>
</tr>
<tr>
<td>V-A-4-2</td>
<td>320</td>
<td>518</td>
<td>259</td>
</tr>
<tr>
<td>V-A-4-3</td>
<td>377</td>
<td>474</td>
<td>298</td>
</tr>
<tr>
<td>Average</td>
<td>329</td>
<td>497</td>
<td>278</td>
</tr>
<tr>
<td>STDV</td>
<td>44.110</td>
<td>22.068</td>
<td>19.502</td>
</tr>
<tr>
<td>CV (%)</td>
<td>13.410</td>
<td>4.440</td>
<td>7.007</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>109.576</td>
<td>54.820</td>
<td>48.446</td>
</tr>
</tbody>
</table>

\(^a\) Offset Bearing Strength  
\(^b\) Initial Peak Bearing Strength  
\(^c\) First Non-Linearity Bearing Strength

3.5.4.7.4 Specimens V-S-4-I Figure 3.117 shows the curves of the bearing stress as a function of the effective bearing strain. Table 3.51 reports the derived properties with statistics.

![Figure 3.117: Bearing stress vs. bearing strain for V-S-4-I specimens](image)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-S-4-1</td>
<td>290</td>
<td>499</td>
<td>278</td>
</tr>
<tr>
<td>V-S-4-2</td>
<td>320</td>
<td>518</td>
<td>259</td>
</tr>
<tr>
<td>V-S-4-3</td>
<td>377</td>
<td>474</td>
<td>298</td>
</tr>
<tr>
<td>Average</td>
<td>329</td>
<td>497</td>
<td>278</td>
</tr>
<tr>
<td>STDV</td>
<td>44.110</td>
<td>22.068</td>
<td>19.502</td>
</tr>
<tr>
<td>CV (%)</td>
<td>13.410</td>
<td>4.440</td>
<td>7.007</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>109.576</td>
<td>54.820</td>
<td>48.446</td>
</tr>
</tbody>
</table>

\(^a\) Offset Bearing Strength  
\(^b\) Initial Peak Bearing Strength  
\(^c\) First Non-Linearity Bearing Strength

3.5.4.7.3 Specimens P-S-4-I  Table 3.49 shows the derived properties with statistics.

![Figure 3.115: Bearing stress vs. bearing strain for P-S-4-I specimens](image)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-S-4-1</td>
<td>419</td>
<td>587</td>
<td>320</td>
</tr>
<tr>
<td>P-S-4-2</td>
<td>427</td>
<td>522</td>
<td>310</td>
</tr>
<tr>
<td>P-S-4-3</td>
<td>468</td>
<td>570</td>
<td>350</td>
</tr>
<tr>
<td>Average</td>
<td>438</td>
<td>560</td>
<td>327</td>
</tr>
<tr>
<td>STDV</td>
<td>26.531</td>
<td>33.710</td>
<td>20.817</td>
</tr>
<tr>
<td>CV (%)</td>
<td>6.054</td>
<td>6.023</td>
<td>6.372</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>65.907</td>
<td>83.739</td>
<td>51.711</td>
</tr>
</tbody>
</table>

\(^a\) Offset Bearing Strength  
\(^b\) Initial Peak Bearing Strength  
\(^c\) First Non-Linearity Bearing Strength

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-S-4-1</td>
<td>419</td>
<td>587</td>
<td>320</td>
</tr>
<tr>
<td>P-S-4-2</td>
<td>427</td>
<td>522</td>
<td>310</td>
</tr>
<tr>
<td>P-S-4-3</td>
<td>468</td>
<td>570</td>
<td>350</td>
</tr>
<tr>
<td>Average</td>
<td>438</td>
<td>560</td>
<td>327</td>
</tr>
<tr>
<td>STDV</td>
<td>26.531</td>
<td>33.710</td>
<td>20.817</td>
</tr>
<tr>
<td>CV (%)</td>
<td>6.054</td>
<td>6.023</td>
<td>6.372</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>65.907</td>
<td>83.739</td>
<td>51.711</td>
</tr>
</tbody>
</table>
3.5. TESTS

3.5.4.7.5 Specimens V-P-4-I  Figure 3.118 shows the curves of the bearing stress as a function of the effective bearing strain. Table 3.52 reports the derived properties with statistics.

Table 3.52: Bearing response for V-P-4-I specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>OBS(^a) 2%</th>
<th>IPBS(^b)</th>
<th>FNLBS(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-P-4-1</td>
<td>330</td>
<td>512</td>
<td>317</td>
</tr>
<tr>
<td>V-P-4-2</td>
<td>352</td>
<td>498</td>
<td>322</td>
</tr>
<tr>
<td>V-P-4-3</td>
<td>365</td>
<td>483</td>
<td>320</td>
</tr>
<tr>
<td>Average</td>
<td>349</td>
<td>498</td>
<td>320</td>
</tr>
<tr>
<td>STDV</td>
<td>17.692</td>
<td>14.503</td>
<td>2.517</td>
</tr>
<tr>
<td>CV (%)</td>
<td>5.069</td>
<td>2.914</td>
<td>0.787</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>43.949</td>
<td>36.027</td>
<td>6.252</td>
</tr>
</tbody>
</table>

\(^a\) Offset Bearing Strength  
\(^b\) Initial Peak Bearing Strength  
\(^c\) First Non-Linearity Bearing Strength

3.5.4.8 Discussion on results for SBJ  
To compare different SBJ specimens the load \(P\) is considered and not the \(P/h\) ratio because the vinylester and the phenolic laminates have approximately the same thickness.

Figure 3.122 shows the load-displacement curves for different materials combinations. The curve related to the double bolted joint and the curves related to
the single bolted joints are shown for each type of specimen and for each material. Figure 3.123 shows the histograms of the strength for different single-bolted joint. The dashed rectangle represents the confidence interval at 95%.

The DBJ configuration exhibits the highest value of the failure load for all types of specimens, except for the bearing specimen (specimen #4, see Figures 3.122(g)-3.122(h) and Figures 3.123(g)-3.123(h)). Surprisingly, for this type of specimen the failure load of the DBJ is the lowest. This result obtained for multi-material joint is not in agreement to the previous work presented in literature for other material (carbon-epoxy) [126]. However, it may be possible to explain this behavior taking into account that two effects occurs a single bolted joint is loaded:

- the first is that bending effects reduce the contact between the bolt and the surfaces of the hole when increasing the load as described in Figure 3.119. This create a stress concentration on the edge of the hole and reduce the strength of the joint;

- the second is that bending effects change the failure modes from in plane failure mode to in-plane/pull-trough failure modes. This increase the load capacity of the specimen.

![Figure 3.119: Load case of a bolted joint](image)

![Figure 3.120: Load vs. displacement for V-P-4-I specimens](image)

![Figure 3.121: Aluminium-vinylester connection after being loaded](image)
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(a) P-J-1-I specimens (#1)
(b) V-J-1-I specimens (#2)
(c) P-J-2-I specimens (#3)
(d) V-J-2-I specimens (#4)
(e) P-J-3-I specimens (#5)
(f) V-J-3-I specimens (#6)
(g) P-J-4-I specimens (#7)
(h) V-J-4-I specimens (#8)

Figure 3.122: Comparison of results for SBJ
CHAPTER 3. EXPERIMENTAL WORK

(a) P-J-1-I specimens (#1)
(b) V-J-1-I specimens (#2)
(c) P-J-2-I specimens (#3)
(d) V-J-2-I specimens (#4)
(e) P-J-3-I specimens (#5)
(f) V-J-3-I specimens (#6)
(g) P-J-4-I specimens (#7)
(h) V-J-4-I specimens (#8)

Figure 3.123: Strength for SBJ specimens
Figures 3.120 and 3.121 clarify this second effect. Figure 3.120 shows the load-displacement curve for the V-A-4-I specimen. After the linear region, in the point A of the curve, delamination occurs and it reduces the stiffness of the specimen. The point B is the peak of the curve where the bearing became predominant. Figure 3.121 shows the joint with the delamination. It should be noted the deformation of the specimen due to the bending moment. It may be noted also the bearing and the penetration of the washer into the material. This penetration is responsible for the increase of the load carried by the joint.

3.5.4.9 Design charts

In Section 3.5.4.8 a discussion on the relative performance of each material was presented. However, it is more useful summarize the results using design charts that can easily relate the strength of the joint to the geometric parameters $d$, $w$ and $e$. The experimental points were fitted using a smoothing spline imposing that when $w/d \geq 5$ and $e/d \geq 4$ the strength is equal to the maximum the bearing strength. These fitting functions allow to generate for each material combination the contour plots of the $P/dh$ as function of $w/d$ and $e/d$. The variable $P/dh$ is used as normalized load and has the same meaning of the bearing stress. It should be pointed out that the charts presented are affected by some limitations:

- the experimental points used, were generated for diameters of the bolts of 6mm and 10mm. The curves then, can be used for all the diameter between 6mm and 10mm, that are practically those used in industry. For diameter out of the range the estimation may not be sufficiently accurate.

- it is assumed that the flexural compliances of the two joined parts of a SBJ are comparable. If one of the two parts has a much higher compliance, the joints does not exhibit high displacements out-of-plane when subjected to external load; in this case, the DBJ chart should be used.

- as showed by Camanho et al. [126] the diameter influences the characteristic distance in compression and in tension. In the present charts this effect is not taken into account and it is considered negligible for the range of diameters under investigation (6mm and 10mm).

Figures 3.124-3.131 show the design charts for the following cases: DBJ Glass-Epoxy, DBJ Glass-Vinylester, DBJ Glass-Phenolic, SBJ Glass-Vinylester/Aluminum, SBJ Glass-Vinylester/Steel, SBJ Glass-Phenolic/Aluminum, SBJ Glass-Phenolic/Steel, SBJ Glass-Vinylester/Glass-Phenolic.

For all the materials (or material combinations), with the exception of the Glass-Epoxy, when $w/d$ or $e/d$ increase, the strength of the joint increases. Only for Glass-Epoxy joint when $w/d$ decreases the strength increases. This is due to the fact that Glass-Epoxy were manufactured using prepregs and these exhibit a much higher strength if tension failure occurs. However the failure should be always the bearing because a net-tension failure produces an instable failure.
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Figure 3.124: DBJ Glass-Epoxy design chart

Figure 3.125: DBJ Glass-Vinylester design chart
3.5. TESTS

Figure 3.126: DBJ Glass-Phenolic design chart

Figure 3.127: SBJ Glass-Vinylester/Aluminum design chart
Figure 3.128: SBJ Glass-Vinylester/Steel design chart

Figure 3.129: SBJ Glass-Phenolic/Aluminum design chart
3.5. TESTS

Figure 3.130: SBJ Glass-Phenolic/Steel design chart

Figure 3.131: SBJ Glass-Vinylester/Glass-Phenolic design chart
3.5.5 Pull-Trough Test

3.5.5.1 Description of the test

The pull-through is performed following the norm Standard Test Method for Measuring the Fastener Pull-Through Resistance of a Fiber-Reinforced Polymer Matrix Composite [131].

This test method contains two procedures, A and B. Since procedure A is more complex and has inherent problems associated with the flexural stiffness of the specimen to test, the specimens procedure B is used.

The test was conducted using the INSTRON-4208 testing machine in the laboratories of INEGI. Figure 3.132 shows the experimental set-up used. The testing machine was equipped with a load cell of 100kN. The speed of the machine (displacement controlled test) if 2mm/min. The temperature of the room was 23°C and the relative humidity was 50% for all the duration of the tests.

![Figure 3.132: Equipment set-up for PT test](image)

After each test the damaged specimen was examined and the type of failure was identified.

The test results depend on Clearance Hole parameter \( C_b \). This is the diameter of the plate that it is used in procedure \( B \) of the test. In the tests performed \( C_b = 30\text{mm} \). For each specimen are reported the dimensions, the ratio of the Clearance Hole Diameter \( C_b \) to Fastener Hole Diameter \( d \), and the ratio of the Fastener Hole Diameter to the thickness of the specimen \( (h) \). Specimens are squares with a length of 120mm.
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3.5.5.2 Derived properties

The curve of the load as a function of the displacement for a pull-through test is used to identify three important characteristics that are:

- the Initial Sub-Critical Failure Force: the force at the first sub-critical failure of the specimen;
- the Initial Sub-Critical Failure Displacement: the displacement at the first sub-critical failure of the specimen;
- the Failure Force: the maximum force attained in the test.

The specimen shows a first failure mode (generally delamination) at a low load. After delamination, the specimen is able to support increasing loads. This point is identified in the curve by a changing of the linearity or by a peak.

3.5.5.3 GF-epoxy specimens

Table 3.53 reports the dimensions of the specimens and the geometric parameters of the equipment used. Figure 3.133 shows the plots of the force/load curves for epoxy specimens. Plot 3.133(a) shows the curves for PT-E-6-I while plot 3.133(b) show the curves for PT-E-10-I. In Table 3.54-3.55 the derived properties for the specimen tested are reported.

It should be noted that some specimen exhibited a bolt failure.

Table 3.53: GF-E specimens’ dimensions

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Diameter (mm)</th>
<th>Cb/D</th>
<th>h (mm)</th>
<th>D/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-E-6-1</td>
<td>6</td>
<td>5</td>
<td>6.45</td>
<td>0.930</td>
</tr>
<tr>
<td>PT-E-6-2</td>
<td>6</td>
<td>5</td>
<td>6.30</td>
<td>0.952</td>
</tr>
<tr>
<td>PT-E-6-3</td>
<td>6</td>
<td>5</td>
<td>6.40</td>
<td>0.938</td>
</tr>
<tr>
<td>PT-E-10-1</td>
<td>10</td>
<td>3</td>
<td>6.50</td>
<td>1.538</td>
</tr>
<tr>
<td>PT-E-10-2</td>
<td>10</td>
<td>3</td>
<td>6.50</td>
<td>1.538</td>
</tr>
<tr>
<td>PT-E-10-3</td>
<td>10</td>
<td>3</td>
<td>6.40</td>
<td>1.563</td>
</tr>
</tbody>
</table>

8The bolts used for these tests were not those described in 3.4.2 but general purpose bolts (D933 8.8)
Table 3.54: Pull-trough test, results for PT-E-6-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>In. Sub-crit. failure load (N)</th>
<th>failure load (N)</th>
<th>In. Sub-crit. failure displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-E-6-1</td>
<td>12184</td>
<td>17272</td>
<td>1.344</td>
</tr>
<tr>
<td>PT-E-6-2</td>
<td>12004</td>
<td>16596</td>
<td>1.447</td>
</tr>
<tr>
<td>PT-E-6-3</td>
<td>12312</td>
<td>17068</td>
<td>1.258</td>
</tr>
<tr>
<td>Average</td>
<td>12167</td>
<td>16979</td>
<td>1.350</td>
</tr>
<tr>
<td>STDV</td>
<td>155</td>
<td>347</td>
<td>0.095</td>
</tr>
<tr>
<td>CV (%)</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>384</td>
<td>861</td>
<td>0.236</td>
</tr>
</tbody>
</table>

Table 3.55: Pull-trough test, results for PT-E-10-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>In. Sub-crit. failure load (N)</th>
<th>failure load (N)</th>
<th>In. Sub-crit. failure displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-E-10-1</td>
<td>19140</td>
<td>32460</td>
<td>1.462</td>
</tr>
<tr>
<td>PT-E-10-2</td>
<td>20532</td>
<td>32724</td>
<td>1.477</td>
</tr>
<tr>
<td>PT-E-10-3</td>
<td>20584</td>
<td>34888</td>
<td>1.829</td>
</tr>
<tr>
<td>Average</td>
<td>20085</td>
<td>33357</td>
<td>1.590</td>
</tr>
<tr>
<td>STDV</td>
<td>819</td>
<td>1332</td>
<td>0.208</td>
</tr>
<tr>
<td>CV (%)</td>
<td>4</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>2035</td>
<td>3309</td>
<td>0.516</td>
</tr>
</tbody>
</table>

Figure 3.133: Pull-trough test; load vs. displacement for GF-epoxy specimens
3.5. TESTS

3.5.5.4 GF-phenolic specimens

Table 3.56 reports the dimensions of the specimens and the geometric parameters of the equipment used. Figure 3.134 shows the plots of the force/load curves for epoxy specimens. Plot 3.134(a) shows the curves for PT-E-6-I while plot 3.134(b) show the curves for PT-E-10-I. In Table 3.57-3.58 the derived properties for the specimen tested are reported.

Table 3.56: GF-P specimens’ dimensions

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Diameter (mm)</th>
<th>Cb/D</th>
<th>h (mm)</th>
<th>D/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-P-6-1</td>
<td>6</td>
<td>5</td>
<td>6.80</td>
<td>0.882</td>
</tr>
<tr>
<td>PT-P-6-2</td>
<td>6</td>
<td>5</td>
<td>6.80</td>
<td>0.882</td>
</tr>
<tr>
<td>PT-P-6-3</td>
<td>6</td>
<td>5</td>
<td>6.80</td>
<td>0.882</td>
</tr>
<tr>
<td>PT-P-10-1</td>
<td>10</td>
<td>3</td>
<td>6.60</td>
<td>1.515</td>
</tr>
<tr>
<td>PT-P-10-2</td>
<td>10</td>
<td>3</td>
<td>6.80</td>
<td>1.471</td>
</tr>
<tr>
<td>PT-P-10-3</td>
<td>10</td>
<td>3</td>
<td>6.50</td>
<td>1.538</td>
</tr>
</tbody>
</table>

Table 3.57: Pull-trough test, results for PT-P-6-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>In. Sub-crit. failure load (N)</th>
<th>failure load (N)</th>
<th>In. Sub-crit. failure displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-P-6-1</td>
<td>12340</td>
<td>17348</td>
<td>1.228</td>
</tr>
<tr>
<td>PT-P-6-2</td>
<td>11752</td>
<td>17236</td>
<td>1.301</td>
</tr>
<tr>
<td>PT-P-6-3</td>
<td>12384</td>
<td>17132</td>
<td>1.360</td>
</tr>
<tr>
<td>Average</td>
<td>12158</td>
<td>17238</td>
<td>1.296</td>
</tr>
<tr>
<td>STDV</td>
<td>352</td>
<td>108</td>
<td>0.066</td>
</tr>
<tr>
<td>CV (%)</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>876</td>
<td>268</td>
<td>0.164</td>
</tr>
</tbody>
</table>
Table 3.58: Pull-trough test, results for PT-P-10-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>In. Sub-crit. failure load (N)</th>
<th>failure load (N)</th>
<th>In. Sub-crit. failure displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-P-10-1</td>
<td>19140</td>
<td>32460</td>
<td>1.462</td>
</tr>
<tr>
<td>PT-P-10-2</td>
<td>18528</td>
<td>32780</td>
<td>1.403</td>
</tr>
<tr>
<td>PT-P-10-3</td>
<td>17976</td>
<td>31608</td>
<td>1.492</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>18548</strong></td>
<td><strong>32282</strong></td>
<td><strong>1.452</strong></td>
</tr>
<tr>
<td><strong>STDV</strong></td>
<td><strong>582</strong></td>
<td><strong>605</strong></td>
<td>0.045</td>
</tr>
<tr>
<td><strong>CV (%)</strong></td>
<td><strong>3</strong></td>
<td><strong>2</strong></td>
<td>3</td>
</tr>
<tr>
<td><strong>IC (95%)</strong></td>
<td><strong>1446</strong></td>
<td><strong>1504</strong></td>
<td>0.112</td>
</tr>
</tbody>
</table>

Figure 3.134: Pull-trough test; load vs. displacement for GF-phenolic specimens

3.5.5.5 GF-vinylester specimens

Table 3.59 reports the dimensions of the specimens and the geometric parameters of the equipment used. Figure 3.135 shows the plots of the force/load curves for epoxy specimens. Plot 3.135(a) shows the curves for PT-E-6-I while plot 3.135(b) show the curves for PT-E-10-I. In Table 3.60-3.61 the derived properties for the specimen tested are reported.
### Table 3.59: GF-V specimens’ dimensions

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Diameter (mm)</th>
<th>Cb/D</th>
<th>h (mm)</th>
<th>D/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-V-6-1</td>
<td>6</td>
<td>5</td>
<td>7.00</td>
<td>0.857</td>
</tr>
<tr>
<td>PT-V-6-2</td>
<td>6</td>
<td>5</td>
<td>7.15</td>
<td>0.839</td>
</tr>
<tr>
<td>PT-V-6-3</td>
<td>6</td>
<td>5</td>
<td>6.80</td>
<td>0.882</td>
</tr>
<tr>
<td>PT-V-10-1</td>
<td>10</td>
<td>3</td>
<td>7.00</td>
<td>1.429</td>
</tr>
<tr>
<td>PT-V-10-2</td>
<td>10</td>
<td>3</td>
<td>6.70</td>
<td>1.493</td>
</tr>
<tr>
<td>PT-V-10-3</td>
<td>10</td>
<td>3</td>
<td>7.20</td>
<td>1.389</td>
</tr>
</tbody>
</table>

### Table 3.60: Pull-trough test, results for PT-V-6-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>In. Sub-crit. failure load (N)</th>
<th>failure load (N)</th>
<th>In. Sub-crit. failure displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-V-6-1</td>
<td>11200</td>
<td>17184</td>
<td>1.127</td>
</tr>
<tr>
<td>PT-V-6-2</td>
<td>11404</td>
<td>17624</td>
<td>1.315</td>
</tr>
<tr>
<td>PT-V-6-3</td>
<td>11340</td>
<td>17204</td>
<td>1.463</td>
</tr>
<tr>
<td>Average</td>
<td>11314</td>
<td>17337</td>
<td>1.302</td>
</tr>
<tr>
<td>STDV</td>
<td>104</td>
<td>248</td>
<td>0.168</td>
</tr>
<tr>
<td>CV (%)</td>
<td>1</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>259</td>
<td>617.213</td>
<td>0.418</td>
</tr>
</tbody>
</table>

### Table 3.61: Pull-trough test, results for PT-V-10-I specimens

<table>
<thead>
<tr>
<th>specimen</th>
<th>In. Sub-crit. failure load (N)</th>
<th>failure load (N)</th>
<th>In. Sub-crit. failure displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT-V-10-1</td>
<td>18448</td>
<td>34172</td>
<td>1.447</td>
</tr>
<tr>
<td>PT-V-10-2</td>
<td>18920</td>
<td>32864</td>
<td>1.440</td>
</tr>
<tr>
<td>PT-V-10-3</td>
<td>18876</td>
<td>39692</td>
<td>1.477</td>
</tr>
<tr>
<td>Average</td>
<td>18748</td>
<td>35576</td>
<td>1.455</td>
</tr>
<tr>
<td>STDV</td>
<td>260</td>
<td>3624</td>
<td>0.020</td>
</tr>
<tr>
<td>CV (%)</td>
<td>1</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>IC (95%)</td>
<td>647</td>
<td>9002</td>
<td>0.049</td>
</tr>
</tbody>
</table>
3.5.5.6 Damaged zones

Figure 3.136 shows the photos of a specimen after being tested. Both sides of the specimen are visible. The bottom side, Figure 3.136(b), shows delamination near the hole. As explained before, delamination occurs at the begin of the test when the load is relatively low (initial sub-critical failure). If the load increases, the intralaminar fracture in the material starts and, at the end, the specimen is totally penetrated by the washer.

(a) intralaminar fracture  
(b) delamination at sub-critical failure

Figure 3.136: Pull-trough specimens after loading

3.5.5.7 Comparison of the PT test results

Figure 3.137 shows the failure loads of the specimens tested. As expected, increasing the diameter increases both the sub-initial critical failure load and the failure load.
of the specimen. The results show that the material exhibit slight differences in the values of the two failure loads considered. This is in contrast with the previous tests on bolted joints where epoxy specimens had better performance. It should be also noted that increasing the diameter also increases the ratio between the initial sub-critical failure load to the ultimate failure load. This means that larger holes exhibit an initial sub-critical failure load that is relatively lower when compared to the ultimate failure load.

**Figure 3.137:** Comparison of PT tests failure loads

### 3.6 Conclusions

In this chapter a comprehensive experimental study on hybrid joints was presented. Two types of test were considered: in plane shear test and pull-through tests. All the tests were conducted considering specimens with different geometries and with different materials or material combinations. It was shown that the geometry affects the failure mode and that materials with the same geometries may show a different failure mode.

The main differences in the failure load of the in-plane shear test occur in the epoxy specimens that always show a value of the failure load higher than that of the other specimens. This was attributed to the fact that epoxy-specimens were manufactured using prepregs while the other specimens (phenolic and vinylester) were manufactured using resin infusion.

The results of the in-plane shear tests are presented as design charts that can be used during the conception of a hybrid structure (see section 3.5.4.9).

If pull-trough is considered, the results are approximatively the same for the three materials, both in terms of the failure load and of the initial sub-critical failure load.
Chapter 4
CT-CC experimental tests

A simple empirical design methodology for multi-material bolted joints was presented in the previous chapter. However, a purely empirical design methodology require a high number of experimental tests in bolted joints. An improved design methodology, consists in performing a few experimental tests to characterize the basic material properties, and use these properties in analysis models. This chapter addresses the measurement of two properties that are required to simulate the fracture of composite bolted joints: the fracture toughness associated to fiber-dominated failure mechanism. For this purpose, the measurement of the crack resistance curve is essential. In fact, crack resistance curves not only characterize the fracture toughness of the material but it also allows to identify the parameters of softening laws used in the numerical simulation of failure in composite bolted joints [132–134]. These parameters are used in the numerical 3D model presented in Chapter 5.

The test is performed following the procedure described by Pinho et al. [135] and using the Digital Image Correlation (DIC) to compute in a fast way the crack resistance curve of the material. The proposed method is based on the identification of the crack tip location using Digital Image Correlation and the calculation of the J-integral directly from the test data using a simple expression derived for cross-ply composite laminates.

The material tested are carbon IM7/8552 and epoxy-glass fiber (presented in Chapter 3) respectively.

4.1 DIC technique

4.1.1 Introduction

The digital image correlation, developed by Sutton et al. [136,137] allows to compare digital images of the body at the undeformed state (reference state) and deformed state (state of the body at time t). Each image is described using a discrete function that represents the gray level at each pixel. At time $t^*$, this function can be expressed
as:

\[ f^* (x^*) = f (x + d (x)) \]  (4.1)

where \( f \) is the discrete function at the reference state, \( f^* \) is the function at the deformed state, \( x \) ans \( x^* \) are the vector of the coordinate of the points at the reference state and at the deformed state, \( d (x) \) is the displacement of each point. The determination of the displacements, \( d (x) \), is obtained minimizing the coefficient of correlation \( C \) defined as:

\[
C = 1 - \frac{\int_{\Delta M} f (x) f (x + d (x)) \, dx}{\sqrt{\int_{\Delta M} f^2 (x) \, dx \int_{\Delta M} f^2 (x + d (x)) \, dx}}
\]  (4.2)

where \( \Delta M \) is the surface of the reference pattern. The displacements are expressed as:

\[ d (P) = \sum_j n_j (P) u_j \]  (4.3)

where, \( u_j \) are the unknown displacements, \( P \) is a point of the pattern, and \( n_j \) are the functions chosed. \( d (P) \) is found minimizing equation (4.2).

Since the displacements \( u_j \) are unknown, equations (4.3) is written \( j \) times. In this case the problem admits a solution.

Finally, equation (4.3) can be written in a compact way as:

\[ d = [M] u \]  (4.4)

where \( d \) is the vector of displacements that minimizes equation (4.2), \( [M] \) is the matrix of functions \( n_j \) and \( u \) is the vector of unknown displacements. The problem is solved merely inverting the matrix \( [M] \):

\[ u = [M]^{-1} d \]  (4.5)

The functions \( n_j \) are generally bilinear (\( \{ n_j \} = \{ x, y, xy, 1 \} \)) and a cubic-spline interpolation is used to find the displacement field [138].

The Figure 4.1 shows a pattern \( ABCD \) centered in \( P \) and the deformed pattern \( A^*B^*C^*D^* \) centered in \( P^* \).
4.1. DIC TECHNIQUE

The displacement of a point of the pattern can be written as:

\[ d_x(x, y) = u_1 \tilde{x} + u_2 \tilde{y} + u_3 \tilde{x} \tilde{y} + u_4 \]  \hspace{1cm} (4.6)

\[ d_y(x, y) = u_5 \tilde{x} + u_6 \tilde{y} + u_7 \tilde{x} \tilde{y} + u_8 \]  \hspace{1cm} (4.7)

where \( \tilde{x} = \frac{x - x_A}{L} \) and \( \tilde{y} = \frac{y - y_A}{L} \) are the homogenized coordinate on the pattern and \( L \) is the length of the side of the pattern.

In the equations (4.6) there are 8 displacements unknown, for this reason it is written for each point of the undeformed and deformed pattern (\( ABCD, A^*B^*C^*D^* \)). The matrix \([M]\) in Equation 4.5 is then \( 8 \times 8 \).

It should be observed that the technique of digital image correlations uses continuous functions and for this reason it is complicated obtain good results in the case of a fracture in the material. For this reason a numerical algorithm [139] is used to evaluate the crack length in the case of discontinuous displacements.

4.1.2 Crack detection algorithm

The algorithm used should be able to detect a crack into the material. The following assumptions are made:

- \( M \) and \( N \) are two points in the reference image;
- \( d(x) \) is the displacement applied at the image;
- \( M^* \) and \( N^* \) are the two points in the deformed image and are separated by a geometric or material discontinuity (see Figure 4.2).
The measure of the variation of distance between these two points, $M\bowtie N$, in presence of a discontinuity can be expressed as:

$$M\bowtie N = \|M^*N^* - MN\| \quad (4.8)$$

Generally, $\forall (P, P^*)$, $O^*P^* = OP + d(P)$. Equation (4.8) can be re-written as:

$$M\bowtie N = \|d(N) - d(M)\| \quad (4.9)$$

If the discontinuity between two points is defined in this way a criterion that allows to detect a discontinuity into a pattern $ABCD$ should be written as (see Figure 4.3):
4.1. DIC Technique

Figure 4.3: Different position of the discontinuity respect to the pattern

\[ K(P) = \max (A \cup C; B \cup D) \]
\[ \max (\|d(C) - d(A)\|; \|d(D) - d(B)\|) \]  

Equation (4.10)

Figure 4.3 shows that this criterion is able to detect a crack into the material independently of the direction that assumes this crack into the pattern. It should be noticed that this criterion is able to detect the crack only in the case of brittle fracture.

Equation (4.10) provides a measure of the discontinuity inside the pattern. To discriminate if a pattern is damaged or undamaged a threshold should be applied to this function. If is assumed that the threshold applied is proportional to the arithmetic average of function \( K(P) \) overall the image, the threshold function \( K_T(P) \) can be defined as:

\[ K(P) \geq \alpha \overline{K} \Rightarrow K_T(P) = 1 \]  
\[ K(P) < \alpha \overline{K} \Rightarrow K_T(P) = 0 \]  
\[ K(P) = \text{NaN} \Rightarrow K_T(P) = 0 \]

where \( \overline{K} \) is the average of \( K(P) \) overall the image, \( \alpha \) is a threshold value that can be chosen by the operator and NaN indicate that for that pattern the information is not available. In this work a low value of \( \alpha \), \( \alpha = 2 \), was chosen to do not lose the information in proximity of the crack tip. Figure 4.4 shows the relation between the crack length and time for different values of \( \alpha \).
Using equations (4.11-4.13), the function $K_T(P)$ takes the values:

- $K_T(P) = 0$ in the patterns where the material is undamaged;

- $K_T(P) = -1$ if no information are available. Typically this happens in the pattern where the material is completely damaged and no information is possible obtain using the digital image correlation;

- $K_T(P) = 1$ in the patterns where a discontinuity is present but the material is not completely damaged. This happens at the crack tip.

Figure 4.5 shows the $K_T$ function computed for a CT carbon specimen. It is noted that the function takes the value $K_T = 0$ where the material is undamaged, $K_T = -1$ where the material is completely fractured and $K_T = 1$ at the crack tip where damage occurs.
The information that has to be extracted from the DIC system is the crack length that is computed as the distance from the crack point in the deformed image and the crack tip in the undeformed image. Considering that patterns of $15 \times 15$ pixels are used, spatial resolution of the method is $\pm 7$ pixels. Considering that the conversion factor assumed the average value of $c_f \approx 0.016 \frac{\text{mm}}{\text{pixel}}$ the crack length was obtained with an approximation of $\pm 0.10$ mm. The Matlab code that implements this algorithm is shown in Appendix A.

### 4.1.3 J-Integral computation

#### 4.1.3.1 J-Integral of a composite laminate

This section presents a procedure to calculate the J-integral for the CT and CC specimens using the Digital Image Correlation (DIC). Kang et al. [140] used a similar way to evaluate the J-Integral for cracks in metallic foils. The procedure is extended here for the case of a composite laminate.

Figure 4.6 shows the contour at the crack tip used for the calculation of the J-integral in 2D case.
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(a) Contour around crack tip
(b) AA detail

Figure 4.6: Contour for the calculation of the J-Integral in 2D case

The 2D J-Integral for an elastic material can be written as:

\[ J = \int_{\Gamma} \left( Wdy - T_i \frac{\partial u_i}{\partial x} ds \right) \] (4.14)

where \( \Gamma \) is an anticlockwise contour around the crack tip, \( W \) is the strain energy density, \( T_i \) and \( u_i \) are, respectively, the components of the traction and displacement vectors on \( \Gamma \). \( s \) is the curvilinear coordinate on \( \Gamma \).

From the equilibrium of the triangular element \( dx dy ds \) shown in Figure 4.6(b), the following relations are derived:

\[ n_1 = \cos \alpha = \frac{dy}{ds} \] (4.15)
\[ n_2 = \sin \alpha = \frac{-dx}{ds} \] (4.16)

and

\[ \{T\} = [\sigma] \{n\} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} = \begin{bmatrix} \sigma_{xx} \cos \alpha & \sigma_{xy} \sin \alpha \\ \sigma_{xy} \cos \alpha & \sigma_{yy} \sin \alpha \end{bmatrix} \] (4.17)

where \( n_1 \) and \( n_2 \) are the directions cosines of the outer normal vector of the arc element \( ds \) as reported in Figure 4.6(b).

The strain energy density can be written as:

\[ W = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} = \frac{1}{2} \{\sigma\} \{\varepsilon\}^T = \frac{1}{2} (\sigma_{xx} \varepsilon_{xx} + \sigma_{yy} \varepsilon_{yy} + \sigma_{xy} \varepsilon_{xy}) \] (4.19)

where \( \{\sigma\} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}^T \) and \( \{\varepsilon\} = \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}\} \).

Equation (4.14) allows to evaluate the value of the J-Integral in the 2D case. In the case of a laminate the equation (4.14) should be extended to the 3D case and the thickness of the laminate must be taken into account.
4.1. DIC TECHNIQUE

Figure 4.7 shows the contour for the calculation of the J-integral in the case of a body with thickness $t$.

![Figure 4.7: Contour for the calculation of J-integral in 3D case](image)

The surface surround the crack is equal to $\Omega = \Omega_0 + \Omega_1 + \Omega_2$. As $n_1 = 0$ and $T_i = 0$ on $\Omega_1$ and $\Omega_2$, the J-integral is written as:

$$J = \frac{1}{t} \int_0^t \int_{\Omega_0} \left( Wdx_2dx_3 - T_i \frac{\partial u_i}{\partial x_1} dsdx_3 \right)$$  (4.20)

If the laminate is a cross-ply only $0^\circ$ and $90^\circ$ laminae are present. If the thicknesses of $0^\circ$ and $90^\circ$ laminae are $t^0$ and $t^{90}$ respectively, equation (4.20) can be re-expressed as:

$$J = \frac{1}{t} \int_{\Omega_0} \left( \int_0^{t^0} (W^0 dx_2 - T^0_i \frac{\partial u_i}{\partial x_1} ds) dx_3 + \int_0^{t^{90}} (W^{90} dx_2 - T^{90}_i \frac{\partial u_i}{\partial x_1} ds) dx_3 \right)$$  (4.21)

Equation (4.21) implicitly assumes that strains (and displacements) through the thickness are constant. Moreover, assuming that no bending occurs (the laminate is balanced), substituting equations (4.18-4.19) in equation (4.21) results in:

$$J = \frac{1}{t} \int_{\Omega_0} \left( \int_0^{t^0} W^0 n_1 dx_3 - \frac{\partial u_i}{\partial x_1} \int_0^{t^0} T^0_i dx_3 + \int_0^{t^{90}} W^{90} n_1 dz - \frac{\partial u_i}{\partial x_1} \int_0^{t^{90}} T^{90}_i dz \right) ds =$$

$$= \frac{1}{t} \int_{\Omega_0} \left( \frac{1}{2} \left( \int_0^{t^0} [\sigma^0] dz + \int_0^{t^{90}} [\sigma^{90}] dx_3 \right) n_1 \{\varepsilon\}^T ds + \ldots \right)$$

$$\ldots - \left( \int_0^{t^0} [\sigma^0] dz + \int_0^{t^{90}} [\sigma^{90}] dx_3 \right) \left\{n_1 \left\{ \frac{\partial u}{\partial x_1} \right\}^T \right\} ds =$$

$$= \frac{1}{t} \int_{\Omega_0} \left( \frac{1}{2} \{\sigma^0\} t_0 + \{\sigma^{90}\} t^{90} \right) n_1 \{\varepsilon\}^T - \left( [\sigma^0] t_0 + [\sigma^{90}] t^{90} \right) \left\{n_1 \left\{ \frac{\partial u}{\partial x_1} \right\}^T \right\} ds$$  (4.22)
If the thickness \( t^0 = t^{90} = t/2 \) equation (4.22) can be simplified as:

\[
J = \int_{\Omega_0} \left( \frac{1}{2} \left\{ \sigma^0 \right\} + \frac{1}{2} \left\{ \sigma^{90} \right\} \right) n_1 \left\{ \varepsilon \right\}^T - \left( \frac{1}{2} \left[ \sigma^0 \right] + \frac{1}{2} \left[ \sigma^{90} \right] \right) \left\{ n \right\} \left( \frac{\partial u}{\partial x_1} \right)^T \right) ds
\]

\[
= \int_{\Omega_0} \left( \frac{1}{2} \left\{ \bar{\sigma} \right\} n_1 \left\{ \varepsilon \right\}^T - \left[ \bar{\sigma} \right] \left\{ n \right\} \left( \frac{\partial u}{\partial x_1} \right)^T \right) ds
\]

(4.23)

where \( \bar{\sigma} \) are the averaged values of the stresses (\( \bar{\sigma} = \sigma^0 + \sigma^{90} \)).

Therefore, the J-Integral for the laminate can be calculated using the averaged stress through the laminate in equation (4.14). Using the Digital Image Correlation, only the strains are obtained and the averaged value of the stress can be obtained averaging the values of the elastic matrix of the material (\( \left\{ \bar{\sigma} \right\} = \bar{C} \left\{ \varepsilon \right\} \) where \( \bar{C} = \frac{1}{2} \left( [C^0] + [C^{90}] \right) \)).

Details of the algorithm used to calculate the J-Integral are presented in Section 4.1.3.3.

4.1.3.2 J-Integral for quasi-brittle materials

Figure 4.8 shows the contour for the calculation of J-Integral in the presence of a cohesive zone. For the path-independence of the J-Integral the following conservation integral is defined:

\[
I = \oint_{\Gamma + C_0 + C_+ + C_-} \left( W n_1 - \frac{\partial u}{\partial x_1} \cdot t \right) ds = 0
\]

(4.24)

\( I \) is equal to zero because the complete path is closed and the limited region is simply connected.

Equation (4.24) can be extended as:
\[ I = \int_\Gamma (W_{n_1} \frac{\partial u}{\partial x_1} \cdot t) \, ds + \int_{C_0} (W_{n_1} \frac{\partial u}{\partial x_1} \cdot t) \, ds + \int_{C_+} (-\frac{\partial u^+}{\partial x_1} \cdot t^+) \, ds + \int_{C_-} (-\frac{\partial u^-}{\partial x_1} \cdot t^-) \, ds \] 

(4.25)

since \( n_1 = 0 \) on \( C_+ \) and \( C_- \). The two last integrals in equation 4.25 represent the contribution of the cohesive zone. Since \( t^+ = -t^- = t \) from equilibrium considerations, and \( dx = dx_1 \):

\[ I_{COH} = \int_{C_-} \left( t^- \cdot \frac{\partial u^+}{\partial x_1} - \frac{\partial u^-}{\partial x_1} \right) \, dx_1 = -\int_C \left( t \cdot \frac{\partial \|u\|}{\partial x_1} \right) \, dx_1 \] 

(4.26)

where \( \|u\| \) is the crack opening. From Rice [141]:

\[ I_{COH} = -\int_C \frac{\partial}{\partial x_1} \left( \int_0^{\|u\|} t \cdot d\|u\| \right) \, dx_1 = \int_0^{\|u_T\|} t \cdot d\|u\| \] 

(4.27)

where \( \|u_T\| \) is the crack opening at the beginning of the FPZ.

Therefore, equation (4.25) can be re-written as:

\[ \int_\Gamma (W_{n_1} \frac{\partial u}{\partial x_1} \cdot t) \, ds + \int_0^{\|u_T\|} t \cdot d\|u\| - \int_\Gamma (W_{n_1} \frac{\partial u}{\partial x_1} \cdot t) \, ds = 0 \] 

(4.28)

\[ \int_\Gamma (W_{n_1} \frac{\partial u}{\partial x_1} \cdot t) \, ds + \int_0^{\|u_T\|} t \cdot d\|u\| = \int_\Gamma (W_{n_1} \frac{\partial u}{\partial x_1} \cdot t) \, ds \] 

(4.29)

In equation (4.29) the first integral represents the J-Integral at the crack tip, \( J_{TIP} \), the second, as explained before is the J-Integral at the cohesive zone, \( J_{COH} \), while the third integral is the J-Integral calculated on the external path, \( J_{EXT} \).

Equation (4.29) can be re-written as:

\[ J_{EXT} = J_{COH} + J_{TIP} \] 

(4.30)

During the development of the R-curve:

\[ J_{TIP} + \int_0^{\|u_T\|} t \cdot d\|u\| = J_{EXT} \] 

(4.31)

During steady-state propagation:

\[ J_{TIP} + \int_0^{\|u_T\|} t \cdot d\|u\| = J_{EXT} \] 

(4.32)
4.1.3.3 Algorithm implemented in Matlab

Using the digital Image correlation the following parameters can be obtained:

- the matrices of the displacements on the plane $[u_1], [u_2]$
- the matrices of the strains $[\varepsilon_{11}], [\varepsilon_{22}], [\varepsilon_{12}]$

This information will be used for the calculation of the J-integral and are stored as matrices $m \times n$ where $m$ and $n$ are the dimension, in pixels, of the processed part of the original image. Therefore, displacements and strain are $m \times n$ matrices.

Figure 4.9 shows the rectangular contour used and the boundary of the images. This contour contains a finite number of pixels at therefore only discrete quantities of the relevant parameters are determined along this pattern.

![Figure 4.9: Contour used in Matlab algorithm](image)

The calculation of the J integral follows the following steps:

- Average stresses, $\{\sigma\}$. The average stresses are computed from the transformed stiffness matrices of the $0^\circ$ and $90^\circ$ plies, $[C^0]$ and $[C^{90}]$ respectively, as $\{\sigma\} = \frac{1}{2} \left( [C^0] + [C^{90}] \right) \{\varepsilon\}$.

- Differentials $dx_1$, $dx_2$ and $dS$. The differentials $dx_1$ and $dx_2$ are taken as the differences between the centers of adjoining subsets, measured along the corresponding axes. The differential $dS$ is the Euclidian norm of $dx_1$ and $dx_2$.

- Vectors normal to the contour, $\{n\}$. These vectors are directly defined by the simple contour sub-divisions shown in Figure 4.9, taking the following forms: $\{1, 0, 0\}^T$ on $C_3$, $\{0, 1, 0\}^T$ on $C_4$, $\{-1, 0, 0\}^T$ on $C_1$, and $\{0, -1, 0\}^T$ on $C_2$. 

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- Derivative of the displacement field, \( \left\{ \frac{\partial u}{\partial x_1} \right\} \). This vector is calculated using the central difference method applied in three adjoining subsets:

\[
\left\{ \frac{\partial u}{\partial x_1} \right\} \approx \left\{ \frac{\Delta u}{\Delta x_1} \right\} = \left\{ \frac{u^{i+1} - u^{i-1}}{2\Delta x_1} \right\}
\]  \hspace{1cm} (4.33)

The algorithm to calculate the J-Integral of a composite laminate is implemented in Matlab code and is reported in Appendix B. The Algorithms for the calculation of the crack length and the J-Integral are used in a interactive Matlab code that allows an easy and fast calculation of the crack R-curve. Figure 4.10 shows the main window of the developed software.

![Figure 4.10: Main window of Matlab software developed](image)

4.2 Experimental Work

4.2.1 Experimental Setup

4.2.1.1 Preparation of the specimens

The specimen were manufactured as described in Pinho et al. [135]. Figure 4.11 shows the geometry for CT and CC specimens.
For the carbon specimens (IM78552) the lay-up was $[0^\circ/90^\circ]_8$, for a total thickness of $h=4$ mm. For the glass-specimens the lay-up was $[0^\circ/90^\circ]_2$, for a total thickness of $h=6$ mm.

To use the Digital Image Correlation, the specimens where painted with a speckle of the average dimension of 0.1 mm. Pinho et al. [135] noticed that paint peeled in compression specimens and this effect did not allow to have data for the region near the crack tip. To avoid this effect the surface of the specimens was treated as in the case of strain gauges are used (see 3.2.2). The manual abrasion of the specimen surface and the degreasing using acetone and isopropanol permit a perfect adhesion between the paint and the specimen. No peeling effects were observed in the vicinity of the crack tip during the tests.

4.2.1.2 Specimens nomenclature

The denomination adopted for the specimens is the following:

\[ J-M-I \]

where J identifies the type of test (CT or CC) where M is the material used, and I is the index that identifies the number of the sample:

- J=CT, CC;
- M=C (carbon), G (glass);
- I=1,2,3.

4.2.1.3 Testing machine set-up

The test was conducted using the MTS 312.31 testing machine in FEUP’s laboratories. The capacity of the testing machine is 250 kN. Tests were performed using...
4.2. EXPERIMENTAL WORK

4.2.1.4 Photomechanical set-up and measurement parameters

The digital image correlation ARAMIS software by GOM (http://www.gom.com/) was used in this work. This measurement system is equipped with an 8-bit Baumer Optronic FWX20 camera (resolution of 1624×1236 pixels, pixel size of 4.4µm and sensor format of 1/1.8”) coupled with a Schneider-Kreuznach Componar-S 50mm f/2.8 lenses. For mobility and adaptability, the support of the cameras was mounted on a Foba ALFAE tripod, which was positioned facing the testing machine.

In the set-up, the optical system was positioned with regard to the surface of the specimen, mounted into the testing machine. A laser pointer was used to guarantee a correct alignment. The working distance (defined between the specimen’s surface and the support of the cameras) was set in the range of 200 mm. The lens was adjusted to be in focus with regard to the surface of interest, setting the lens aperture to f/2.8 in order to minimize the depth of field. The aperture of the lens was then slightly closed (f/11) in order to improve the depth of field during testing. The shutter time was set to 50 ms, according to the cross-head displacement rate during testing (2 mm.min⁻¹) and the size of the camera unit cells (4.4 µm). The light source was finally adjusted in order to guarantee an even illumination of the specimen’s surface and to avoid over-exposition (i.e., the saturation of pixels over the field of view).
Table 4.1: Parameters used in the ARAMIS-GOM software

<table>
<thead>
<tr>
<th>Project parameter - Facet</th>
<th>Conversion factor</th>
<th>0.180 mm/pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facet size</td>
<td>30 pixel</td>
<td></td>
</tr>
<tr>
<td>Step size</td>
<td>30 pixel</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Project parameter - Strain</th>
<th>Computation size</th>
<th>5 macro-pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Validity code</td>
<td>55%</td>
<td></td>
</tr>
</tbody>
</table>

| Strain computation method  | Total            |               |

| Image recording            | Acquisition frequency | 1 Hz           |

Table 4.1 reports the parameters used in ARAMIS-GOM software. For the region of interest was set about $20 \times 20 \text{mm}^2$, defining a conversion factor of about $0.185 \text{ mm.pixels}^{-1}$. In the digital image correlation method, the displacement field is measured by analysing the geometrical deformation of the images of the surface of interest, recorded before and after loading. For this purpose, the initial (undeformed) image is mapped by square facets (correlation windows), within which an independent measurement of the displacement is calculated. Therefore, the facet size, on the plane of the object, will characterise the displacement spatial resolution. The facet step (i.e., the distance between adjacent facets) can also be set either for controlling the total number of measuring points over the region of interest or for enhancing the spatial resolution by slightly overlapping adjacent facets. Typically, a great facet size will improve the precision of the measurements but also will degrade the spatial resolution [142]. Thus, a compromise must be found according to the application to be handled. In this work, a facet size of $30 \times 30 \text{ pixel}^2$ was chosen, attending to the size of the region of interest, the optical system (magnification) and the quality of the granulate (average speckle size) obtained by the spray paint. The facet step was also set to $30 \times 30 \text{ pixel}^2$ (Table 4.1), in order to avoid statistically correlated measurements. The in-plane displacements were then numerically differentiated in order to determine the strain field need for the material characterization problem.

4.2.2 Experimental results

To evaluate the fracture toughness of the fiber CT and CC specimens are used and two techniques are used to calculate the fracture toughness. The first technique [135] requires the use FEM to evaluate the J-integral. The J-integral is calculated for a FE model representing a specimen with unitary thickness. The fracture toughness of the specimen is obtained as:
4.2. EXPERIMENTAL WORK

\[ G = J_{FEM} \left( \frac{P}{h} \right)^2 \]  \hspace{1cm} (4.34)

where \( J_{FEM} \) is the J-integral calculated from the FEM, \( P \) is the load during the experimental test and \( h \) is the thickness of the composite.

The second technique, here proposed, consists in the calculation of the J-Integral using directly the data that come out from Aramis, as explained in Section 4.1.3.3.

4.2.2.1 Calculation of J-integral using FEM

To evaluate the J-integral directly from the FEM a parametric numerical model was defined using Abaqus 6.7 [106]. The Python [143] script of the parametric model is reported in Appendix C. This allows to automatically generate and run several CT-CC FE models for different crack lengths. The function \( J_{FEM} \) of equation (4.34) is calculated from the FE model and it can be used to compute the energy release rate \( G \) of the laminate when at a given value of the crack length. The energy release rate of the fiber is shown by Pinho et al. [135] to be equal to twice this value.

![FEM model of CT/CC specimen](image)

**Figure 4.13:** FEM model of CT/CC specimen

The J-Integral can be fitted by a function of the crack length \( a \):

\[ JJ = \left( p_0 + p_1 a^{p_2} \right) \frac{p_3 + p_4 a}{p_5 - p_6 a} \]  \hspace{1cm} (4.35)

where \( p_i \) are the best-fitting parameters. In Table 4.2 the parameters for the Glass Fiber and IM7/8552 tested are reported. It should be noted that these parameters are function of the geometry of the specimen.

**Table 4.2:** Parameters for the J-integral fitting (thickness \( h = 4\text{mm} \))

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( p_0 )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
<th>( p_4 )</th>
<th>( p_5 )</th>
<th>( p_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM78552</td>
<td>-1.870e-6</td>
<td>-6.984e-21</td>
<td>9.044</td>
<td>-4.996</td>
<td>1.517</td>
<td>-4.269e+1</td>
<td>-8.874e-1</td>
</tr>
<tr>
<td>GF</td>
<td>-8.775e-7</td>
<td>-2.571e-21</td>
<td>9.145</td>
<td>-7.312</td>
<td>1.004</td>
<td>-4.584e+1</td>
<td>-9.556e-1</td>
</tr>
</tbody>
</table>
Figure 4.14 reports the fitting function and the numerical point from FEM. A good level of approximation is observed.

![Figure 4.14: J-Integral for CT/CC specimens: numerical values and fitting functions.](image)

4.2.2.2 CT-C-I specimens

A typical load-displacement relation obtained in the CT tests is shown in Figure 4.15.

![Figure 4.15: Load-displacement in a CT-C-I test specimen](image)

The load was measured using the 100kN load cell, and the displacement was measured using the linear variable differential transformer (LVDT) connected to the hydraulic actuator of the test machine.

Figure 4.15 shows that the load-displacement relation is linear up to approximately 80% of the peak load, and that crack propagation occurs in discrete jumps.
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Figure 4.16 shows the R-curve measured from the FEM post-processing of the test results method proposed by Pinho et al. [135] and that obtained by post-processing the displacement and strain fields measured by the DIC system.

![Figure 4.16: R-curves extracted from a CT-C-I specimen using FEM and DIC](image)

Figure 4.16: R-curves extracted from all CT-C specimens using DIC, and corresponding mean R-curve. Each symbol corresponds to one CT test

Figure 4.17 shows a good correlation between the FEM and DIC data reduction methods. This means that the fracture process zone that bridges the crack has a minor effect on the displacement and strain fields in the regions where the Finite Element model computes the J-integral.

Figure 4.17 shows the R-curves obtained from the three CT tests. Figure 4.17 also shows the mean value of the fracture process zone, 3.4mm, and the mean values
of the initial fracture toughness and that corresponding to steady-state crack propagation, 97.8J/m² and 133.3J/m² respectively. These values are slightly lower than the mean values of the fracture toughness for initiation and steady-state propagation, 113.8J/m² and 146.7J/m² respectively, measured by Pinho [144] for the same material system.

4.2.2.3 CC-C-I specimens

A typical load-displacement relation obtained from a CC test is shown in Figure 4.18.

Figure 4.18 shows the R-curve measured from the FEM post-processing of the test results obtained by the method proposed by Pinho et al. and the R-curve obtained using the DIC data reduction procedure proposed here.

Figure 4.19 shows that the FEM-based solution yields unrealistically high values of the fracture toughness during the propagation of fiber kink bands. For 15mm of kink-band propagation the fracture toughness computed using the FEM is approximately twice that calculated using the DIC system. The reason for this fact is that the FEM-based calculation of the J-integral does not account for the contact and load transfer across the band of the kinked fibers. These effects clearly affect the displacement and strain fields along the contours of the J-integral computed using FEM. On the other hand, the DIC-based method uses the actual displacement and strain fields on the surface of the specimen, thus providing a more accurate R-curve. However, the contact stresses that are transferred along the kink band still pollute the data obtained using DIC because they introduce one additional term on the LHS of equation (4.29).

Delamination associated with the propagation of the kink band from the initial notch was also observed in the CC-C tests. The presence of delamination renders
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4.2.2.4 CT/CC-G-I specimens

Unfortunately, no results are available for the glass specimen tested. In fact, during the test several problems arose.

First of all, a straight crack propagation was not observed during the test. In fact, indentation in the proximity of the holes, large delaminations and crack propagation in the direction perpendicular to the main crack were observed.

All these phenomena do not allow the use of FEM or of DIC to calculate the R-curve. Figure 4.20 shows a CC-G-I specimen during the loading and the associated function \( K_T \) defined in equation (4.11).

It can be noticed that the use of DIC and the crack detection algorithm enable the identification of complicated crack paths. However, the J-Integral computed is wrong because using DIC the out-of-plane displacements are considered zero and obviously this is not the case. It means that the strains calculated by DIC are affected by this error. Moreover, the other failure mechanism that appears during the test (in addition at the propagation of the main crack) are not negligible.
4.2.2.5 Conclusion on CT and CC tests

The comparison between the R-curves obtained in CT specimens using the FE-based post-processing and the DIC-based method indicates that the results are virtually the same and that the method proposed here is a valid alternative to measure R-curves associated with longitudinal tensile failure mechanisms in composite materials. The initial value of the fracture toughness associated with longitudinal tensile failure mechanisms in IM7/8552 is 98.7 J/m$^2$, and it raised up to 133.7 J/m$^2$ for steady state propagation. The mean value of the associated cohesive zone is 3.4 mm.

It is concluded that the FE-based data reduction method is inadequate for the measurement of R-curves in CC specimens because it severely overpredicts the fracture toughness. The DIC-based method is an improvement over FE-based data reduction methods because it is based on the actual displacement field on a predefined contour. However, the contribution of the contact tractions to the J-integral still needs to be quantified and used in the data reduction method to improve the accuracy of the data. The initial value of the fracture toughness associated with longitudinal compressive failure mechanisms in IM7-8552 measured using DIC was 47.5 J/m$^2$; the measured fracture toughness increased up to 315 J/m$^2$ for 15 mm of propagation of a kink-band. However, the values computed for the fracture toughness using the CC specimen do not account for the energy dissipated by the delamination that accompanied the propagation of the kink-band. In addition, the delamination does not allow the accurate measurement of the tip of the kink-band. Therefore, it is concluded that these difficulties render the CC test method unsuitable to generate R-curves and that an alternative test method should be developed.

The same considerations apply for Glass fiber specimens. In addition, these specimens showed additional failure mechanisms that render the use of the CT/CC specimens unsuitable to measure the fracture toughness of fiber-dominated failure mechanism. In this case a new test method should be developed.
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4.2.2.6 Determination of the parameters of the softening law

The CT and CC tests were performed to compute the parameters of the softening law of the progressive damage model. For each damage mode the softening laws are used to represent the evolution of the damage in terms of the dissipated energy (see Figure 4.21).

![Figure 4.21: Softening law used in the progressive damage model.](image)

Dávila et al. [132] proposed a methodology to compute the parameter of the exponential law proposed by Maimí et al. [133, 134]:

\[
X_{PO} = (1 - n) X_c f(m, n) \tag{4.36}
\]

\[
G^L_c = G_c [1 - (1 - m) f(m, n)] \tag{4.37}
\]

\[
G^L_{c} = G_c (1 - m) f(m, n) \tag{4.38}
\]

with:

\[
f(m, n) = 1 - \frac{1 - n}{n} \frac{m}{1 - m} \tag{4.39}
\]

\[
m = G_1/G_c \tag{4.40}
\]

\[
n = 1 - \frac{2}{3} \gamma \frac{1 - m}{l_p} \frac{EG_c}{X_c^2} \tag{4.41}
\]

where \(X_c\) is the ultimate failure strength, \(X_{PO}\) is the pull-out or transition strength, \(l_pz\) is the length of process zone (LPZ), \(\gamma\) is the parameter for the models for the LPZ (\(\gamma = 0.884\) for Rice model), \(G_c\), \(G_1\), \(G^L_c\) and \(G^E_c\) are respectively the energy release
rate, the initiation value of the energy release rate, the linear and the exponential partitions.

Using equations (4.36-4.41) and considering that for IM7/8552 carbon specimen the following values were measured: $G_c = 133.8$, $X_c = 2326.2$, $l_p = 3.4$, $E = 90.65E3$, $G_1 = 98.7$, the parameters computed for the longitudinal cohesive law in tension are:

$$X_{PO}^+ = 161.0\text{MPa}, \ G_{cL}^+ = 119.9\text{J/m}^2, \ G_{cE}^+ = 23.8\text{J/m}^2.$$

For the longitudinal cohesive law in compression no parameters are available and an inverse identification technique was used; the parameters obtained were:

$$X_{PO}^- = 600\text{MPa}, \ G_{cL}^- = 72.6\text{J/m}^2, \ G_{cE}^- = 77.4\text{J/m}^2.$$
Chapter 5

3D Numerical Model

Having defined a new experimental technique to measure the fracture toughness associated to fibre-dominated failure mechanism, this chapter describes the 3D numerical model developed to predict the strength of composite bolted joints. As previously mentioned, the strength of a mechanically fastened joint depends on the pressure applied by the bolt. In fact, a pressure of 22MPa on the washer is suggested in the literature for carbon fiber joints to maximize the strength [6] while for glass this value increases up to 70MPa [19]. The application of a lateral pressure has a beneficial effect on the strength of the composite (obviously until the pressure is lower than a critical value). For this reason the failure criteria used in the numerical model has to consider 3D stress states.

The objective of this chapter is to develop a new 3D numerical model that is applicable to composite bolted joints.

5.1 Matrix Failure Criteria for 3D stress states

The failure criteria proposed in this chapter can be thought as an extension of the LARC03 criteria to 3D stress states [145]. It should be noticed that the effect of the lateral pressure $\sigma_{33}$ has an influence mainly on the matrix-dominated failure modes (matrix failure and fiber kinking). The influence of $\sigma_{33}$ on the fiber tensile failure mode was taken as negligible.

For a general stress state, the failure criteria is a function of the traction vector acting on the fracture plane as shown in Figure 5.1. Defining $\alpha$ as the angle between the fracture plane and the through-the-thickness direction, the failure index depends on the stresses acting on the fracture surface and therefore on $\alpha$.

The stress tensor is denoted as $T$:

$$T = \begin{bmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_{22} & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_{33} \end{bmatrix}$$ (5.1)
and the unit normal vector to the fracture plane is \( n = \begin{bmatrix} 0 & \cos(\alpha) & \sin(\alpha) \end{bmatrix}^T \) the vector of the traction stress \( t_n \) acting on the fracture plane is obtained as:

\[
t_n = T \cdot n = \begin{bmatrix}
\cos(\alpha) \tau_{12} + \sin(\alpha) \tau_{13} \\
\cos(\alpha) \sigma_{22} + \sin(\alpha) \tau_{23} \\
\cos(\alpha) \tau_{23} + \sin(\alpha) \sigma_{33}
\end{bmatrix}
\] (5.2)

The orientation of the fracture plane is function of the components of the traction vector that maximizes a failure index for a particular value of \( \alpha \). These components are the normal stress and the two shear stresses (longitudinal and transverse) as shown in Figure 5.1.

The component in \( N \) direction is obtained as

\[
\sigma_N = t_n \cdot \begin{bmatrix} 0 \\ \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}^T = \cos(\alpha)^2 \sigma_{22} + 2 \cos(\alpha) \sin(\alpha) \tau_{23} + \sin(\alpha)^2 \sigma_{33} \] (5.3)

while the component in \( T \) direction, \( \tau_T \), is given by

\[
\tau_T = t_n \cdot \begin{bmatrix} 0 \\ -\sin(\alpha) \\ \cos(\alpha) \end{bmatrix}^T =
\]

\[
= -\sin(\alpha) \cos(\alpha) \sigma_{22} - \sin(\alpha)^2 \tau_{23} + \cos(\alpha)^2 \tau_{23} + \cos(\alpha) \sin(\alpha) \sigma_{33}
\] (5.4)

Finally, the component in \( L \) direction, \( \tau_L \), is obtained as

\[
\tau_L = t_n \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T = \cos(\alpha) \tau_{12} + \sin(\alpha) \tau_{13}
\] (5.5)
The failure criteria developed here depend on the sign of the normal stress. There is a failure criterion for matrix compression if $\sigma_N > 0$, and failure criterion for matrix tension when $\sigma_N < 0$.

5.1.1 **Failure criteria for matrix compression**

5.1.1.1 **Puck criterion**

Puck et al. [146–149] performed matrix compression tests and noticed that a modified Mohr-Coulomb criterion can be used to predict the orientation of the fracture plane. For pure transverse compression loading ($\sigma_{22} \neq 0$ and $\sigma_{ij} = 0$) the maximum shear stress occurs for $\alpha = \pm \pi/4$. However, experimentally, it was observed that, generally, for fiber reinforced plastics the fracture angle is equal to $\alpha_0 = 53 \pm 2^\circ$. The difference between the two angles was associated to the presence of the compressive stress.

Puck and Shürmann’s criterion [148] is expressed as:

$$\quad FMC = \left( \frac{\tau_L}{S_{L}^{is} - \eta_L \sigma_N} \right)^2 + \left( \frac{\tau_T}{S_{T}^{is} - \eta_T \sigma_N} \right)^2 \quad \text{(5.6)}$$

where $S_{L}^{is}$ is the longitudinal shear strength, $S_{T}^{is}$ is the in-situ transversal shear strength, and $\eta_L$ and $\eta_T$ are the two slopes in the $\sigma_N-\tau$ diagram when $\sigma_N = 0$ and are defined as:

$$\eta_L = -\left. \frac{\partial \tau_L}{\partial \sigma_N} \right|_{\sigma_N=0} \quad \eta_T = -\left. \frac{\partial \tau_T}{\partial \sigma_N} \right|_{\sigma_N=0} \quad \text{(5.7)}$$

It should be noted that in all the formulation presented here the in-situ strengths are used. Equation (5.6) can be used to evaluate in a consistent way the different parameters that define the failure criteria.

5.1.1.1.1 **Hypothesis 1**: $\sigma_{22} = -Y_C^{is}$ If a only compression $\sigma_{22} = -Y_C^{is}$ is applied in the transverse direction (2) the material should fail with a fracture angle equal to $\alpha = \alpha_0$.

If the loading stress is used in equation (5.6) the failure index is expressed as:

$$\quad FMC = \frac{\sin(\alpha)^2 \cos(\alpha)^2 Y_C^{is}}{(S_T^{is} + \eta_T \cos(\alpha)^2 Y_C^{is})^2} \quad \text{(5.8)}$$

while the derivative of this function with respect to $\alpha$ yields:
\[
\frac{\partial F_{MC}}{\partial \alpha} = 2 \sin(\alpha) \cos(\alpha)^3 Y_{is}^2 - 2 \frac{\sin(\alpha)^3 \cos(\alpha) Y_{\nu}^2}{(S_T^s + \eta_T \cos(\alpha)^2 Y_{\nu}^2)^2} + 4 \frac{\sin(\alpha)^3 \cos(\alpha)^3 Y_{is}^2 \eta_T}{(S_T^s + \eta_T \cos(\alpha)^2 Y_{\nu}^2)^3}
\]

(5.9)

The fracture should occur at the plane \( \alpha = \alpha_0 \) or \( \alpha = \pi - \alpha_0 \). For this value of the angle \( \alpha \) it should be \( F_{MC} = 1 \) and \( \frac{\partial F_{MC}}{\partial \alpha} = 0 \). Substituting \( \alpha = \alpha_0 \) in equations (5.8) and (5.9), the transversal shear strength and the compressive transversal strength can be obtained solving this system of equations:

\[
\begin{align*}
F_{MC}\big|_{\alpha=\alpha_0} & = 1 \\
\frac{\partial F_{MC}}{\partial \alpha}\big|_{\alpha=\alpha_0} & = 0
\end{align*}
\]

(5.10)

Assuming the following relationship for the friction coefficients [149]

\[
\frac{\eta_L}{\eta_T} = \frac{S_L^s}{S_T^s}
\]

(5.11)

and considering that the in-plane shear strength and the longitudinal friction coefficient are material parameters that can be measured, the system of equation (5.10) can be solved obtaining the following values for \( Y_{is} \) and \( S_T^s \):

\[
Y_{is} = -\frac{S_L^s}{\eta_L \cos(\alpha_0)^2} \left( 2 \cos(\alpha_0)^2 - 1 \right)
\]

(5.12)

\[
S_T^s = 1 \sqrt{\frac{1}{2} \left( \frac{2 \sin(\alpha_0)^2 - 1}{S_L^s} \right)} \frac{S_L^s}{\eta_L}
\]

(5.13)

If these definitions are used the failure index is equal to 1, the first derivative is equal to zero and the second derivative reads:

\[
\left. \frac{\partial^2 F_{MC}}{\partial \alpha^2} \right|_{\alpha=\alpha_0} = \frac{2}{\cos(\alpha_0)^2 \left( \cos(\alpha_0)^2 - 1 \right)}
\]

(5.14)

which is negative as required. This assures that for \( \alpha = \alpha_0 \) the maximum of the function is found. If the conditions expressed by equations (5.12) and (5.13) are not satisfied the function will take the value 1 for \( \alpha = \alpha_0 \) but the maximum of the function will be somewhere else. This will result in errors if the failure angle is found maximizing numerically the equation (5.8).

For example, for carbon IM7/8552 (\( S_L^s = 130.2 \), \( \eta_L = 0.5 \)) the failure index as a function of the angle \( \alpha \) is shown in Figure 5.2. It should be noticed that the maximum is correctly calculated for \( \alpha = \alpha_0 \).
Figure 5.2: $FMC$ as a function of $\alpha$ for IM-78552 ($\sigma_{22} = -Y^{is}_C$)

Hypothesis 2: $\tau_{12} = S^{is}_L$ If only the shear $\tau_{12} = S^{is}_L$ is applied the fracture plane must be defined by $\alpha = 0$.

In this case the failure index of equation (5.6) assumes simply the value

$$FMC = \cos(\alpha)^2$$

(5.15)

while the derivative of this function assumes the value

$$\frac{\partial FMC}{\partial \alpha} = -2 \cos(\alpha) \sin(\alpha)$$

(5.16)

As expected, the failure index and the derivative assume the value of 1 and 0 respectively when $\alpha = 0$. The second derivative of this function takes a negative value ($\partial^2 FMC/\partial \alpha^2 = -2$) and this assures that the point found is a maximum for the failure index function. Figure 5.3 shows the failure index as a function of the angle $\alpha$. The fracture angle is well predicted.
Figure 5.3: \( FMC \) as a function of \( \alpha (\tau_{12} = S_{L}^{is}) \)

Figure 5.4: \( FMC \) as a function of \( \alpha (\tau_{23} = Y_{p}^{is}) \)
5.1. MATRIX FAILURE CRITERIA FOR 3D STRESS STATES

5.1.1.1.3 **Hypothesis 3:** \( \tau_{32} \neq 0 \) Supposing that only \( \tau_{23} \neq 0 \) the failure index is:

\[
F_{MC} = \frac{(-\sin(\alpha)^2 \tau_{23} + \cos(\alpha)^2 \tau_{23})^2}{(S_{T}^{is} - 2 \eta T \cos(\alpha) \sin(\alpha) \tau_{23})^2}
\] (5.17)

If \( \tau_{23} \) reaches its critical value \( (\tau_{23} = Y_{T}^{is}) \) the fracture plane occurs at an angle of \( \alpha = 45^\circ \). However, in this case equation (5.17) does not take the value of 1 but 0 as shown in Figure 5.4. This means that this type of fracture cannot be computed using Puck’s criterion for compression.

5.1.1.1.4 **Conclusion** Puck’s failure criterion describe very well the fracture of the matrix if \( Y_{C}^{is} \) and \( S_{T}^{is} \) are defined according to equations (5.12) and (5.13). If only \( \tau_{23} \) is different from zero the criterion cannot be used to predict the fracture angle. For this reason, Puck’s criterion should be only used when \( \sigma_{N} < 0 \) and not when \( \sigma_{N} = 0 \).

5.1.2 Failure Criteria for Matrix Tension

5.1.2.1 Quadratic Interaction Criterion

The quadratic interaction criterion proposed by Pinho [150] assumes a quadratic relationship between the components of the stress on the fracture plane. The criterion is given by the following expression

\[
F_{MT} = \left( \frac{\sigma_{N}}{Y_{T}^{is}} \right)^2 + \left( \frac{\tau_{T}}{S_{T}^{is}} \right)^2 + \left( \frac{\tau_{L}}{S_{L}^{is}} \right)^2
\] (5.18)

5.1.2.1.1 **Hypothesis:** \( \sigma_{22} = Y_{T}^{is} \) Consider that a stress \( \sigma_{22} = Y_{T}^{is} \) is applied to the material. The fracture plane should be \( \alpha = 0^\circ \). Under these hypotheses the failure index assumes the value \( F_{MT} = 1 \). However, if the fracture angle is not given, but rather searched by maximizing the failure index as a function \( \alpha \), as normally done, the failure index reads:

\[
F_{MT} = \cos(\alpha)^4 + \sin(\alpha)^2 \cos(\alpha)^2 k^2
\] (5.19)

where \( k = Y_{T}^{is}/S_{T}^{is} \).

Figure 5.5 shows the \( FMT \) function as a function of \( k \).
The derivative of equation (5.19) is:

$$\frac{\partial F_{MT}}{\partial \alpha} = -4 \cos^3(\alpha) \sin(\alpha) + 2 \sin(\alpha) \cos(\alpha)^3 k^2 - 2 \sin(\alpha)^3 \cos(\alpha) k^2$$  \hspace{1cm} (5.20)

while the second derivative reads:

$$\frac{\partial^2 F_{MT}}{\partial \alpha^2} = -4 + 2 k^2$$  \hspace{1cm} (5.21)

that should be negative for a maximum of the function given by equation (5.19). Consequently in the case of a unidirectional compression the criterion results in correct prediction only if:

$$k \leq \frac{\sqrt{2}}{2} \Rightarrow Y_{T}^{is} \leq \sqrt{2} S_{T}^{is}$$  \hspace{1cm} (5.22)

For example, in the case of IM7/8552 carbon the $k$ assumes the value $k_{IM78552} = \frac{Y_{T,IM78552}^{is}}{S_{T,IM78552}^{is}} = 160.2/74.7 = 2.1$. It should be noted that the value of $S_{T}^{is}$ is calculated using equation (5.10). The failure index $FMT$ for IM7/8552 is reported in Figure 5.6.
This means that under pure transverse tension, the failure criterion will predict that the failure occurs when $\sigma_{22} < Y^t_{T}$, and the corresponding fracture plane will be different than $\alpha = 0$. This is obviously wrong.

### 5.1.2.2 Puck criterion

Puck proposed an elliptical interaction criterion that can be written as [151]:

\[
FMT = \left( \frac{\tau}{S^{is}} \right)^2 + c_1 \frac{\sigma_N}{Y^{Ts}_{T}} + c_2 \left( \frac{\sigma_N}{Y^{Ts}_{T}} \right)^2
\]  

(5.23)

where $\tau$ is the shear on the fracture plane, $\sigma_N$ is the normal stress and $c_1$ and $c_2$ are parameters that can be determined imposing appropriate boundary conditions. $S^{is}$ is the shear strength on the fracture plane. The shear on the fracture plane is given as

\[
\tau = \sqrt{\tau_L^2 + \tau_T^2}
\]  

(5.24)

and the angle $\omega$ between the shear stress $\tau_T$ and $\tau$ is equal to:

\[
\omega = \arctan \left( \frac{\tau_L}{\tau_T} \right)
\]  

(5.25)
5.1.2.2.1 Determination of $S^{is}$ If only $\tau_T$ and $\tau_L$ are present and $\sigma_N$ is equal to zero the failure criteria for tension (equation (5.23)) and for compression (equation (5.6)) should give the same results. If not the two criteria are not consistent. Therefore, $FMC = FMT$ when $\sigma_N = 0$.

Under this hypothesis the following expression for $S^{is}$ is derived:

$$S^{is} = \frac{1}{\sqrt{(\sin(\omega)/S^{is}_L)^2 + (\cos(\omega)/S^{is}_T)^2}}$$  \hspace{1cm} (5.26)

Obviously, $S^{is}$ depends on $S^{is}_L$, $S^{is}_T$ and $\omega$.

5.1.2.2.2 Determination of $c_1$ and $c_2$ To evaluate $c_1$ and $c_2$, assume that a unidirectional tensile stress $\sigma_{22} = Y^{is}_T$ is applied in the transverse direction. The failure index is equal to 1 and the angle of fracture $\alpha$ is equal to 0. Imposing these conditions the following condition is obtained:

$$c_1 + c_2 = 1$$  \hspace{1cm} (5.27)

Taking the derivative of the failure index of equation (5.23)

$$\frac{\partial FMT}{\partial \sigma_N} = \frac{2\tau}{S^{is}_T^2} \frac{\partial \tau}{\partial \sigma_N} + \frac{c_1}{Y^{is}_T} + \frac{2c_2 \sigma_N}{Y^{is}_T^2}$$  \hspace{1cm} (5.28)

and supposing that $\sigma_N = 0$ equation (5.28) results in:

$$\frac{\partial FMT}{\partial \sigma_N} \bigg|_{\sigma_N=0} = -\frac{2\tau \eta}{S^{is}_T^2} + \frac{c_1}{Y^{is}_T}$$  \hspace{1cm} (5.29)

where $\eta = -\frac{\partial \tau}{\partial \sigma_N} \bigg|_{\sigma_N=0}$.

The expression of $\eta$ can be expressed in function of $\eta_L$ and $\eta_T$. In fact, considering the following equation:

$$\tau^2 = \tau^2_L + \tau^2_T$$  \hspace{1cm} (5.30)

and differentiating in $\frac{\partial}{\partial \sigma_N}$ it results in:

$$2\tau \frac{\partial \tau}{\partial \sigma_N} = 2\tau_L \frac{\partial \tau_L}{\partial \sigma_N} + 2\tau_T \frac{\partial \tau_T}{\partial \sigma_N}$$  \hspace{1cm} (5.31)

then remembering the definition of the friction coefficients (equation (5.7)), under the hypothesis of $\sigma_N = 0$, equation (5.31) yields:

$$\eta = \eta_L \frac{\tau_L}{\tau} + \eta_T \frac{\tau_T}{\tau}$$  \hspace{1cm} (5.32)

To calculate the coefficient $c_1$, consider the following stress state: $\sigma_N = 0$, $\tau_T = 0$ and $\tau_L = S^{is}_L$. In this case the matrix should fail and the derivative of equation (5.29) should be equal to zero because the failure index is a maximum at that point. Under these hypotheses $\eta|_{\tau_T=0} = \eta_L$ and equation (5.29) can be re-written as:
\[
\frac{\partial F_{MT}}{\partial \sigma_N} = -2\eta S^T_{L} \sigma_N + \frac{c_1 Y^{is}_T}{S^T_{L}} = 0 \quad \Rightarrow \quad c_1 = \frac{2\eta Y^{is}_T}{S^T_{L}} \quad (5.33)
\]

Substituting the \( c_1 \) in equation (5.27) \( c_2 \) is obtained

\[
c_1 + c_2 = 1 \quad \Rightarrow \quad c_2 = 1 - \frac{2\eta Y^{is}_T}{S^T_{L}} \quad (5.34)
\]

The failure index of equation (5.23) assumes that the following expression:

\[
F_{MT} = \left( \frac{\tau}{S^T_{is}} \right)^2 + 2\eta S^T_{L} \sigma_N \frac{S^T_{is}}{S^T_{L}} + \left( 1 - \frac{2\eta Y^{is}_T}{S^T_{L}} \right) \left( \frac{\sigma_N}{Y^{is}_T} \right)^2 \quad (5.35)
\]

5.1.2.2.3 Hypothesis: \( \sigma_{22} = Y^{is}_T \) If \( \sigma_{22} = Y^{is}_T \) with \( \sigma_{ij} = 0 \) the material should fail by transverse matrix cracking and the angle should \( \alpha = 0 \). The failure index under this hypothesis is \( F_{MT} = 1 \) and the first derivative is equal to \( \partial F_{MT}/\partial \alpha = 0 \).

However the second derivative reaches:

\[
\frac{\partial^2 F_{MT}}{\partial \alpha^2} \bigg|_{\alpha=0} = \frac{2Y^{is}_T S^T_{T}}{S^T_{is} S^T_{L}} - \frac{4\eta S^T_{L} Y^{is}_T}{S^T_{is} S^T_{L}} - \frac{4(S^T_{is} - 2\eta S^T_{L} Y^{is}_T)}{S^T_{is} S^T_{L}} \quad (5.36)
\]

that should be negative if the fracture angle \( \alpha \) corresponds to the maximum of the function \( F_{MT} \). The second derivative is negative when:

\[
\left( -\eta S^T_{is} + \sqrt{\eta^2 S^T_{is}^2 + 2S^T_{L}^2} \right) S^T_{is} < Y^{is}_T < \left( -\eta S^T_{is} - \sqrt{\eta^2 S^T_{is}^2 + 2S^T_{L}^2} \right) S^T_{is} \quad (5.37)
\]

For example, for carbon IM7/8552 equation (5.37) can be written as

\[
-129 < Y^{is}_T < 86 \quad [\text{MPa}] \quad (5.38)
\]

that is not respected. This means that in this case, the point for \( \alpha = 0 \) will not be a maximum for the function \( F_{MT} \), as clearly shown in the Figure 5.7 that represents \( F_{MT} \) as a function of \( \alpha \).
5.1.2.3 Conclusion

The 3D interaction criteria previously proposed cannot be used to predict the failure of the matrix in general conditions. In fact, both the quadratic and the elliptical criteria, for the case of a simple transversal compression work if some condition are satisfied. These conditions are not realistic for most technical composites. It should be noted that the equations (5.22) and (5.37) assure the applicability of the failure criteria only in the case of pure transversal tension. Otherwise the applicability of the criterion used depends on the stress state applied.

5.1.2.4 Proposed Criterion

It was previously demonstrated that the quadratic interaction criteria of equations (5.18)-(5.23) cannot be used to predict the failure of the material under general stress states. Therefore, a new failure criterion is proposed in this chapter.

First of all it should be noticed that, experimentally, when a tensile stress is applied to a brittle material the fracture plane is always perpendicular to the maximum principal stress.

Puck noticed that the presence of the shear stresses reduces the strength of the matrix. However, the orientation of the fracture plane is defined by the maximum principle stress, and not by the shear stress [151].

A criterion that uses the maximum principal stress on the transverse plane (23) and considers that fracture occurs when the principal stress reaches the value of the transversal strength $Y_{T}^{t}$ could be expressed as:

Figure 5.7: Puck criterion: $FMT$ as a function of $\alpha$ for IM7/8552 ($\sigma_{22} = Y_{T}^{t}$)
5.1. MATRIX FAILURE CRITERIA FOR 3D STRESS STATES

\[
FMT = \left( \frac{\tau_L}{S_L^{is}} \right)^2 + \left( \frac{\sigma_I}{Y_T^{is}} \right)^2 \tag{5.39}
\]

where \( \sigma_I \) is the maximum principal stress on the transversal plane (23) and \( \tau_L \) is the shear in direction 1 that acts on the surface perpendicular to \( \sigma_I \). Obviously \( \sigma_I \geq 0 \).

The principal stress \( \sigma_I \) is given as:

\[
\sigma_I = \frac{\sigma_{22} + \sigma_{33}}{2} + \sqrt{\left( \frac{\sigma_{22} - \sigma_{33}}{2} \right)^2 + \tau_{23}^2} \tag{5.40}
\]

The angle between the direction of the highest principal stress and the 22 axis is given by

\[
\alpha_I = \frac{1}{2} \arctan \left( \frac{2\tau_{23}}{\sigma_{22} - \sigma_{33}} \right) \tag{5.41}
\]

The shear stress \( \tau_L \) that acts on the fracture plane is given as:

\[
\tau_L = \cos (\alpha_I) \tau_{12} + \sin (\alpha_I) \tau_{13} \tag{5.42}
\]

It should be noted that the criterion proposed yields accurate predictions for the several load cases that were previously considered. In fact:

- if \( \sigma_{22} = Y_T^{is} \) and \( \sigma_{ij} = 0 \) the failure index takes the value \( FMT = 1 \) and the fracture plane is \( \alpha_I = 0 \);
- if \( \sigma_{33} = Y_T^{is} \) and \( \sigma_{ij} = 0 \) the failure index takes the value \( FMT = 1 \) and the fracture plane is \( \alpha_I = \pi/2 \);
- if \( \tau_{23} \neq 0 \) and \( \sigma_{ij} = 0 \) the failure index takes the value \( FMT = 1 \) when \( \tau_{23} = Y_T^{is} \) and the fracture plane is \( \alpha_I = \pi/4 \).

However, the criterion defined is only applicable if the principal stress is predominant with respect to the shear \( \tau_L \).

Consider for example the following stresses state: \( \tau_{12} = S_L^{is} \), \( 0 < \sigma_{33} < Y_T^{is} \) and \( \sigma_{ij} = 0 \). Under this hypothesis the material should fail. However, if the criterion of equation (5.39) is used the fracture plane is given by \( \alpha = \pi/2 \). Using equation (5.5) results in \( \tau_L = 0 \) and consequently \( FMT < 1 \), that is obviously wrong.

Consequently the criterion proposed here is written as:

\[
FMT = \left( \frac{\tau_L}{S_L^{is}} \right)^2 + \left( \frac{\sigma_N}{Y_T^{is}} \right)^2 \tag{5.43}
\]

where \( \sigma_N \) and \( \tau_L \) are defined in equations (5.3) and (5.5) respectively and are determined maximizing the failure index \( FMT \) respect to \( \alpha \).

If \( \tau_L \) is not predominant the criteria of equations (5.43) and (5.39) give the same results. Different stresses states are considered in the following to show that the criterion proposed can be effectively used to predict failure of matrix when \( \sigma_N \geq 0 \).
5.1.2.4.1 Hypothesis 1: $\sigma_{22} = Y^i_T$ Consider that a stress $\sigma_{22} = Y^i_T$ is applied to the material. The function $FMT$ assumes the value:

$$FMT = \cos(\alpha)^4$$

Figure 5.8 shows the failure index $FMT$ as a function of $\alpha$. The predicted fracture plane is correctly computed as $\alpha = 0$ and the failure index takes correctly the value of $FMT = 1$.

![Figure 5.8: $FMT$ as a function of $\alpha$ ($\sigma_{22} = Y^i_T$)](image)

5.1.2.4.2 Hypothesis 2: $\sigma_{33} = Y^i_T$ Consider that a stress $\sigma_{33} = Y^i_T$ is applied to the material. The function $FMT$ assumes the value:

$$FMT = \sin(\alpha)^4$$

Figure 5.9 shows the failure index $FMT$ as a function of $\alpha$. The predicted fracture plane is correctly computed as $\alpha = \pi/2$. The failure index takes correctly the value $FMT = 1$ at the fracture plane.
5.1. MATRIX FAILURE CRITERIA FOR 3D STRESS STATES

5.1.2.4.3 Hypothesis 3: $\tau_{23} = Y_T^{is}$ Consider that a stress $\tau_{23} = Y_T^{is}$ is applied to the material. The function $FMT$ assumes the value:

$$FMT = 4\cos(\alpha)^2\sin(\alpha)^2$$ (5.46)

Figure 5.10 shows the failure index $FMT$ as a function of $\alpha$. The predicted fracture plane is correctly computed as $\alpha = \pm \pi/4$ and the failure index takes correctly the value of $FMT = 1$.

5.1.2.4.4 Hypothesis 4: $\tau_{12} = S_L^{is}$ Consider that a stress $\tau_{12} = S_L^{is}$ is applied to the material. The function $FMT$ assumes the value:

$$FMT = \cos(\alpha)^2$$ (5.47)

Figure 5.11 shows the failure index $FMT$ as a function of $\alpha$. The predicted fracture plane is correctly computed as $\alpha = 0$ and the failure index takes correctly the value of $FMT = 1$.

5.1.2.4.5 Hypothesis 5: $\tau_{13} = S_L^{is}$ Consider that a stress $\tau_{13} = S_L^{is}$ is applied to the material. The function $FMT$ assumes the value:

$$FMT = \sin(\alpha)^2$$ (5.48)

Figure 5.12 shows the failure index $FMT$ as a function of $\alpha$. The predicted fracture plane is correctly computed as $\alpha = \pi/2$ and the failure index takes correctly the value of $FMT = 1$. 
Figure 5.10: \(FMT\) as a function of \(\alpha\) \((\tau_{23} = Y_{T}^{is})\)

Figure 5.11: \(FMT\) as a function of \(\alpha\) \((\tau_{12} = S_{L}^{is})\)
5.1. MATRIX FAILURE CRITERIA FOR 3D STRESS STATES

5.1.2.4.6 Summary

The criterion defined provides accurate predictions for the different load cases analyses. In fact:

- if $\sigma_{22} = Y_T^{is}$ and $\sigma_{ij} = 0$ the failure index takes the value $FMT = 1$ and the fracture plane is $\alpha = 0$;
- if $\sigma_{33} = Y_T^{is}$ and $\sigma_{ij} = 0$ the failure index takes the value $FMT = 1$ and the fracture plane is $\alpha = \pi/2$;
- if $\tau_{23} \neq 0$ and $\sigma_{ij} = 0$ the failure index takes the value $FMT = 1$ when $\tau_{23} = Y_T^{is}$ and the fracture plane is $\alpha_I = \pi/4$;
- if $\tau_{12} = S_L^{is}$ and $\sigma_{ij} = 0$ the failure index takes the value $FMT = 1$ and the fracture plane is $\alpha = 0$;
- if $\tau_{13} = S_L^{is}$ and $\sigma_{ij} = 0$ the failure index takes the value $FMT = 1$ and the fracture plane is $\alpha = \pi/2$.

In the case that only $\tau_{23}$ is different from zero, Puck’s criterion for compression cannot predict the fracture angle (see Section 5.1.1.1.3). However experimental evidence shows that if the only $\tau_{23}$ is present, the fracture plane is oriented at $45^\circ$.

This criterion can detect the right fracture plane and so it should be used when $\sigma_N \geq 0$, while the Puck’s criterion for compression should be used only in the case of $\sigma_N < 0$. 

![Figure 5.12: FMT as a function of $\alpha$ ($\tau_{13} = S_L^{is}$)](attachment://image.png)
CHAPTER 5. 3D NUMERICAL MODEL

5.1.2.5 Failure envelopes for matrix failure

Figures 5.13-5.17 show the failure envelopes for matrix failure when Puck’s criterion is used for compression, $\sigma_N < 0$, and the proposed criterion is used for tension $\sigma_N \geq 0$, for an IM7/8552 carbon laminate. The parameter used to the definition of the material are reported in Table 5.1

Table 5.1: Mechanical properties of IM7/8552

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_T^{is}$ (MPa)</th>
<th>$Y_C^{is}$ (MPa)</th>
<th>$S_L^{is}$ (MPa)</th>
<th>$\eta_L$</th>
<th>$\alpha_0$ (°)</th>
<th>$G_{12}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2323.5</td>
<td>1200.1</td>
<td>160.2</td>
<td>198</td>
<td>130.2</td>
<td>0.5</td>
<td>53</td>
<td>5290.0</td>
</tr>
</tbody>
</table>

Figure 5.13(a) shows the shear $\tau_{12}$ as a function of $\sigma_{22}$. As a 2D case is considered the two criteria takes simply the form of the Puck criterion for 2D, when $\sigma_{22} < 0$, and Hashin criterion, when $\sigma_{22} \geq 0$. As expected, when $\sigma_{22}$ becomes compressive the apparent shear strength increases.

Figure 5.13(b) shows the shear $\tau_{13}$ as a function of $\sigma_{22}$. As $\tau_{13}$ and $\sigma_{22}$ act on different planes no interaction between them is noticed and the value of $\tau_{13}$ remains always lower than $S_L^{is}$.

Figure 5.13: Shear stresses as a function of $\sigma_{22}$

---

It should be noted that in-situ strength are used; therefore, an embedded lamina is considered.
Figure 5.14: \( \tau_{12} \) as a function of \( \sigma_{22} \) for several value of the transverse stress \( \sigma_{33} \)

Figure 5.14 shows the shear \( \tau_{12} \) as a function of \( \sigma_{22} \) when a transverse stress, \( \sigma_{33} \), is applied. The red envelopes are obtained when \( \sigma_{33} \) is positive and the blue ones when it is negative. If \( \sigma_{33} > 0 \) the envelope remain constant when \( \sigma_{22} > 0 \). This can be explained with the fact that there is no interaction between \( \sigma_{22} \) and \( \sigma_{33} \) when both are positive. In this case the fracture plane is calculated using the principal stress on the transverse plane (23). If \( \sigma_{22} < 0 \), an application of a positive \( \sigma_{33} \) decreases the strength. In the case that a compressive \( \sigma_{33} \) is applied the reduction of strength is noticed in tension (\( \sigma_{22} > 0 \)) although in compression (\( \sigma_{22} < 0 \)) the compressive \( \sigma_{33} \) is beneficial.

Figures 5.15-5.17 show the failures envelopes of \( \sigma_{33} \) as a function of \( \sigma_{22} \) when the shear stresses (respectively \( \tau_{12}, \tau_{13}, \tau_{23} \)) vary.

As expected, when the shear stresses (\( \tau_{12}, \tau_{13}, \tau_{23} \)) are equal to zero the failure envelope intercept the two axis in the points \((Y_T, 0), (0, Y_T), (-Y_C, 0)\) and \((0, -Y_C)\). Moreover, it should pointed out that when \( \sigma_{22} \) and \( \sigma_{33} \) are both positive the failure envelope remains correctly inside the region defined by \( \sigma_{22} \leq 0 \) and \( \sigma_{33} \leq 0 \). Finally, it should be observed that the failure envelopes are open in the region characterized by hydrostatic pressure (\( \sigma_{22} = \sigma_{33} = -p \)).

In Figures 5.15-5.16 the shear stresses (\( \tau_{12}, \tau_{13} \)) vary between 0 and \( S^L \), while in Figure 5.17 \( \tau_{23} \) varies between 0 and \( S^T \). This for the reason that outside these values the material should fail. However, it should be noticed the maximum value of \( \tau_{23} \) for \( \sigma_N \geq 0 \) is \( Y^{is}_T \). This is not represented in Figure 5.17 because in the case of compression it does not make any sense.
Figure 5.15: $\sigma_{33}$ as a function of $\sigma_{22}$ for different values of $\tau_{12}$

Figure 5.16: $\sigma_{33}$ as a function of $\sigma_{22}$ for different values of $\tau_{13}$
5.2 Fiber Failure Criteria for 3D stress states

5.2.1 Fiber tension failure ($\sigma_{11} > 0$)

The failure index for failure in compression as suggested by Dávila et al. [145] can be calculated simply as:

$$FI_F = \frac{\varepsilon_{11}}{\varepsilon_T}$$

(5.49)

In this case it is simply assumed that transversal stress has a negligible influence on the longitudinal strength of the fibers.

5.2.2 Fiber kinking failure ($\sigma_{11} \leq 0$)

The 3D stress state has a relevant influence on on fiber kinking. Two hypotheses are normally made:

- the first hypothesis, from Rosen, considers that kink band are the final result of micro-buckling of the fibers [152];

- the second hypothesis, from Argon, assumes that kink bands are triggered by localized matrix failure in the vicinity of misaligned fibers [153].
Schultheisz and Waas [154] concluded that in technical composites kinking is originated by local micro-structural defects that could trigger the kinking phenomenon.

The model presented here is based on Argon’s approach [153] and on the subsequent developments by Dávila et al. [145] and Pinho et al. [150].

Figure 5.18 shows the different coordinate systems and planes that are used in this model. The coordinate system 123 is aligned with the material axes of the composite: 1 represents the fiber direction, 2 and 3 the transverse directions.

In Figure 5.18(a) the kinking plane is highlighted. The coordinate system relative to the kinking plane is $1^{(\theta)}2^{(\theta)}3^{(\theta)}$ and it is obtained by 123 rotating itself around the axis 1 of an angle $\theta$.

Figure 5.18(b) show the fibers on the kinking plane. The kinking-angle is $\varphi$ and the respective coordinate system is $1^{(\varphi)}2^{(\varphi)}3^{(\varphi)}$ that can be obtained by $1^{(\theta)}2^{(\theta)}3^{(\theta)}$ after a rotation of an angle $\varphi$ around the axis $3^{(\theta)}$. Then it is possible use the failure criteria of equations (5.6) and (5.43) in the misalignment coordinate system $1^{(\varphi)}2^{(\varphi)}3^{(\varphi)}$ to check for failure.

Figure 5.18(c) shows the fracture angle with respect to the misalignment frame, and identified by the angle $\alpha$.

Kinking failure can then be predicted using the matrix failure index in the misalignment frame. This require a double rotation of the stress tensor.

First of all the angle $\theta$ should be calculated. Generally, the kinking plane is triggered by the defects in the composite and so it means that $\theta$ is function of the local defects of the material. However, if we make the hypothesis that the influence of the defects can be neglected the angle $\theta$ can be taken as a function of the stresses and it can be calculated by the maximum principal stress that acts on the transverse plane 23. In fact, if $\tau_{23}^{(\theta)} \neq 0$ this will result in a movement perpendicular to the kinking-plane contradicting the evidence that fibers kink on this plane. For this reason, using the Mohr’s circle theory the angle $\theta$ can be obtained as:

$$\tan(2\theta) = \frac{2\tau_{23}}{\sigma_{22} - \sigma_{33}}$$

(5.50)
5.2. FIBER FAILURE CRITERIA FOR 3D STRESS STATES

Using the stress tensor defined in equation (5.1), if a rotation of $\theta$ (counterclockwise) is applied the stress tensor in the new coordinate system results:

$$T^{(\theta)} = R(\theta) \cdot T \cdot R(\theta)^T$$ (5.51)

where $R(\theta)$ is the rotation matrix defined as:

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$ (5.52)

In the $\theta$ coordinate system stresses assume the values:

$$\sigma_{11}^{(\theta)} = \sigma_{11}$$ (5.53)

$$\sigma_{22}^{(\theta)} = \cos(\theta)^2 \sigma_{22} + 2 \cos(\theta) \sin(\theta) \tau_{23} + \sigma_{33} \sin(\theta)^2$$ (5.54)

$$\sigma_{33}^{(\theta)} = \sin(\theta)^2 \sigma_{22} - 2 \cos(\theta) \sin(\theta) \tau_{23} + \sigma_{33} \cos(\theta)^2$$ (5.55)

$$\tau_{12}^{(\theta)} = \cos(\theta) \tau_{12} + \sin(\theta) \tau_{13}$$ (5.56)

$$\tau_{23}^{(\theta)} = -\sin(\theta) \cos(\theta) \sigma_{22} + \cos(\theta) \sin(\theta) \sigma_{33} + \left(\cos(\theta)^2 - \sin(\theta)^2\right) \tau_{23}$$ (5.57)

$$\tau_{13}^{(\theta)} = -\sin(\theta) \tau_{12} + \cos(\theta) \tau_{13}$$ (5.58)

Having found the fiber-kinking plane, the stresses should be rotated in the misalignment frame. The stresses on this coordinate system can be written as:

$$T^{(\varphi)} = R(\varphi) \cdot T^{(\theta)} \cdot R(\varphi)^T$$ (5.59)

where $R(\varphi)$ is the rotation matrix defined as:

$$R(\varphi) = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & 0 \\ -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (5.60)

Finally the stresses in the misalignment frame can be expressed as:

$$\sigma_{11}^{(\varphi)} = \cos(\varphi)^2 \sigma_{11}^{(\theta)} + 2 \cos(\varphi) \sin(\varphi) \tau_{12}^{(\theta)} + \sigma_{22}^{(\theta)} \sin(\varphi)^2$$ (5.61)

$$\sigma_{22}^{(\varphi)} = \sin(\varphi)^2 \sigma_{22}^{(\theta)} - 2 \cos(\varphi) \sin(\varphi) \tau_{12}^{(\theta)} + \sigma_{22}^{(\theta)} \cos(\varphi)^2$$ (5.62)

$$\sigma_{33}^{(\varphi)} = \sigma_{33}^{(\theta)}$$ (5.63)

$$\tau_{12}^{(\varphi)} = -\sin(\varphi) \cos(\varphi) \left(\sigma_{11}^{(\theta)} - \sigma_{22}^{(\theta)}\right) + \left(\cos(\varphi)^2 - \sin(\varphi)^2\right) \tau_{12}^{(\theta)}$$ (5.64)

$$\tau_{23}^{(\varphi)} = -\sin(\varphi) \tau_{13}^{(\theta)} + \cos(\varphi) \tau_{23}^{(\theta)}$$ (5.65)

$$\tau_{13}^{(\varphi)} = \cos(\varphi) \tau_{13}^{(\theta)} + \sin(\varphi) \tau_{23}^{(\theta)}$$ (5.66)
Then, matrix failure can be predicted using equations (5.6, 5.43). The components of the traction on the fracture plane can be expressed as:

\[
\sigma_N^{(\phi)} = \cos(\alpha)^2 \sigma_{22}^{(\phi)} + 2 \cos(\alpha) \sin(\alpha) \tau_{23}^{(\phi)} + \sin(\alpha)^2 \sigma_{33}^{(\phi)}
\]

\[
\tau_T^{(\phi)} = -\sin(\alpha) \cos(\alpha) \left( \sigma_{22}^{(\phi)} - \sigma_{33}^{(\phi)} \right) + (\cos(\alpha)^2 - \sin(\alpha)^2) \tau_{23}^{(\phi)}
\]

\[
\tau_L^{(\phi)} = \cos(\alpha) \tau_{12}^{(\phi)} + \sin(\alpha) \tau_{13}^{(\phi)}
\]

If \( \sigma_N^{(\phi)} < 0 \) the failure index becomes:

\[
FK_{MC} = \left( \frac{\tau_L^{(\phi)}}{S_{L}^{is} - \eta_L \sigma_N^{(\phi)}} \right)^2 + \left( \frac{\tau_T^{(\phi)}}{S_{T}^{is} - \eta_T \sigma_N^{(\phi)}} \right)^2
\]

while if \( \sigma_N^{(\phi)} \geq 0 \) the failure index reads:

\[
FK_{MT} = \left( \frac{\tau_L^{(\phi)}}{S_{L}^{is}} \right)^2 + \left( \frac{\sigma_N^{(\phi)}}{Y_{T}^{is}} \right)^2
\]

The failure index for fiber kinking will assume the value

\[
FK = \max \{ \max \{FK_{MC}\}, \max \{FK_{MT}\} \}
\]

where \( \max \{FK_{MC}\} \) and \( \max \{FK_{MT}\} \) are the maximum values of equations (5.70) and (5.71) with respect to \( \alpha \).

### 5.2.2.1 Determination of the angle \( \phi \)

Dávila et al. [145] used a combination of Argon’s approach and LaRC02/03 failure criterion to calculate the angle \( \phi \) in the 2D case. The stresses in the misalignment coordinate frame of Figure 5.19 are:

\[
\sigma_{11}^{(m)} = \cos(\phi)^2 \sigma_{11} + \sin(\phi)^2 \sigma_{22} + 2 \sin(\phi) \cos(\phi) |\tau_{12}| \]

\[
\sigma_{22}^{(m)} = \sin(\phi)^2 \sigma_{11} + \cos(\phi)^2 \sigma_{22} - 2 \sin(\phi) \cos(\phi) |\tau_{12}| \]

\[
\tau_{12}^{(m)} = -\sin(\phi) \cos(\phi) \sigma_{11} + \sin(\phi) \cos(\phi) \sigma_{22} + (\cos(\phi)^2 - \sin(\phi)^2) |\tau_{12}| \]

![Figure 5.19: Stresses in the misalignment frame (2D)](image-url)
5.2. FIBER FAILURE CRITERIA FOR 3D STRESS STATES

Supposing that the material fails under axial compression the stresses take the values \( \sigma_{11} = -X_C, \sigma_{22} = \tau_{12} = 0 \). Substituting these values, equations (5.73) can be rewritten

\[
\begin{align*}
\sigma_{11}^{(m)} &= -\cos(\varphi_C)^2 X_C \\
\sigma_{22}^{(m)} &= -\sin(\varphi_C)^2 X_C \\
\tau_{12}^{(m)} &= \sin(\varphi_C) \cos(\varphi_C) X_C
\end{align*}
\]

where \( \varphi_C \) is the angle at the failure when a pure axial compression is applied.

Substituting these stress in the LaRC02/03 failure criterion it becomes:

\[
S_{is}^L = X_C \left( \sin(\varphi_C) \cos(\varphi_C) - \eta_L \sin(\varphi_C)^2 \right)
\]

Equation (5.79) can be solved for \( \varphi_C \) obtaining

\[
\varphi_C = \arctan \left( \frac{1 - \sqrt{1 - 4 \left( \frac{S_{is}^L}{X_C} + \eta_L \right) \frac{S_{is}^L}{X_C}}}{2 \left( \frac{S_{is}^L}{X_C} + \eta_L \right)} \right)
\]

The total misalignment \( \varphi \) can be thought as the sum of an initial constant misalignment angle \( \varphi_0 \) (that represents the manufacture defects and imperfections in the materials) and a \( \gamma_m \) angle that is originated by the shear loading applied and depends on the shear constitutive law \( \tau = f(\gamma_m) \).

At failure under axial compression the additional angle \( \gamma_m \) becomes:

\[
\gamma_mC = f^{-1} \left( \tau_{12}^{(m)} \right) = f^{-1}(\sin(\varphi_C) \cos(\varphi_C) X_C)
\]

For a material that exhibits linear behavior in shear, \( \tau_{12}^{(m)} = G_{12} \gamma_m \) equation (5.81) can be solved in close form obtaining

\[
\gamma_mC = \frac{\sin(2\varphi_C) X_C}{2G_{12}} \approx \frac{\varphi_C X_C}{G_{12}}
\]

considering small angle approximation.

The initial misalignment angle can then obtained as

\[
\varphi_0 = \varphi_C - \gamma_mC
\]

For a generic loading the strain \( \gamma_m \) is obtained solving the equation \( f(\gamma_m) = \tau_{12}^{(m)} \) that can be written as:

\[
f(\gamma_m) = -\sin(\varphi) \cos(\varphi) (\sigma_{11} - \sigma_{22}) + 2 \left( \cos(\varphi)^2 - \sin(\varphi)^2 \right) |\tau_{12}|
\]
Solving equation (5.84) for $\gamma_m$ the angle $\varphi$ becomes:

$$\varphi = \text{sgn}\{\tau_{12}\}(\varphi_0 + \gamma_m) \quad (5.85)$$

Assuming small angle approximation equation (5.84) results

$$f(\gamma_m) = (\varphi_0 + \gamma_m)(-\sigma_{11} + \sigma_{22}) + |\tau_{12}| \quad (5.86)$$

that for a material that exhibits linear behavior can be solved as:

$$\gamma_m = \frac{-\varphi_0 G_{12} + |\tau_{12}|}{G_{12} + \sigma_{11} - \sigma_{22} - \varphi_0} \quad (5.87)$$

Pinho et al. [150] suggested to apply the equations (5.84-5.87) considering the stresses that acts on the kinking plane. Equation (5.84) becomes:

$$f(\gamma_m) = -\sin(\varphi)\cos(\varphi)\left(\sigma^{(\theta)}_{11} - \sigma^{(\theta)}_{22}\right) + 2(\cos(\varphi)^2 - \sin(\varphi)^2)|\tau^{(\theta)}_{12}| \quad (5.88)$$

Finally for linear shear behavior equation (5.88) results:

$$\gamma_m = \frac{-\varphi_0 G_{12} + |\tau^{(\theta)}_{12}|}{G_{12} + \sigma^{(\theta)}_{11} - \sigma^{(\theta)}_{22} - \varphi_0} \quad (5.89)$$

### 5.2.2.2 Failure envelopes for fiber failure

Figures 5.20-5.23 show the failure envelopes for an IM7/8552 carbon laminate. The parameter used to the definition of the material are reported in Table 5.1.

Figure 5.20 shows the $\sigma_{11}$-$\sigma_{22}$ failure envelope where are highlighted the regions interested by different failure phenomena.
5.2. FIBER FAILURE CRITERIA FOR 3D STRESS STATES

In the region characterized by kinking for matrix compression, the LaRC03 failure criterion predicts an increase of the axial compressive strength and this is confirmed by experimental results [155]. The criterion proposed here show an increase of the compressive strength only in a small region of the envelope but there is not an increasing of the compressive strength if a biaxial compression is applied. This seems to contradict the experimental evidence but it is not the case. First of all, it should be highlighted that the fact that the compressive stress does not increase can be explained remembering the definition of the angle \( \theta \) that identify the kink band plane. When \( \sigma_{22} \) becomes smaller than zero (\( \sigma_{33} = 0 \)), the angle of the principal stress changes from 0° to 90°. If a compression in 2 direction is applied this will not influence the fiber kinking because the fiber will kink in the perpendicular plane. It is consistent with the model defined having a constant value of the compressive strength when \( \sigma_{22} < 0 \). However, if the loading condition or the manufacture of the material can justify a predisposition of the material to kink with an angle \( \theta = 0 \) (in 2 direction) the proposed failure criteria can detect the increasing of compressive strength in the biaxial compression region.

Figure 5.21 shows the \( \sigma_{11}-\sigma_{22} \) failure envelope when a transverse stress is applied.

\[ \text{Figure 5.20: } \sigma_{11}-\sigma_{22} \text{ failure envelope for IM7/8552 carbon} \]

\[ \text{\footnotetext{2Near the point that identifies the switching of failure from kinking for matrix tension to kinking for matrix compression}} \]
In this case, as both $\sigma_{22}$ and $\sigma_{33}$ are compressive the increase of the strength in the biaxial compression region is shown. It should be noticed that when a transverse compression is applied ($\sigma_{33} < 0$) the strength of the material decreases in tension ($\sigma_{22} > 0$) and increases in compression ($\sigma_{22} < 0$). When a transverse tension is applied the compressive strength decreases ($\sigma_{22} < 0$) but no effects can be noticed in the tensile strength ($\sigma_{22} > 0$) and this is due to the fact that in tension ($\sigma_N > 0, \tau_L = 0$) the failure index is function of the principal stress on the transversal plane (23), it means that there is not interaction between $\sigma_{22}$ and $\sigma_{33}$ because the failure depends only on the maximum value of $\sigma_{22}$ or $\sigma_{33}$. In this case in fact $\sigma_N = \max \{\sigma_{22}, \sigma_{33}\}$.

Figure 5.22 shows the $\sigma_{11}$-$\tau_{12}$ failure envelope. For the symmetry the envelopes $\sigma_{11}$-$\tau_{12}$ and $\sigma_{11}$-$\tau_{13}$ should be the same. It should be noticed that when only $\sigma_{11}$ and $\tau_{12}$ (or $\tau_{13}$) are applied, the principal stresses on the transverse plane are zero and for these reason $\theta$ assume infinite values. Under these conditions, the kinking-plane is triggered by the defects of the material and theoretically infinite solutions are possible because $\theta$ can take all the values between 0 and $\pi$. Figure 5.22 shows the two envelopes for $\theta = 0$ and $\theta = \pi/2$ that represent the boundaries of the relevant region. As the envelope for $\theta = 0$ gives the most conservatives results this should be used.

Figure 5.21 shows the $\sigma_{11}$-$\tau_{12}$ failure envelope when a transverse stress $\sigma_{33}$ is applied. When $\sigma_{33} < 0$ the kinking plane is calculated by $\theta = 0$ while is $\sigma_{33} > 0$, $\theta = \pi/2$. For the symmetry the envelopes $\sigma_{11}$-$\tau_{12}$ and $\sigma_{11}$-$\tau_{13}$ should be the same.
\[ \theta = \pi. \] An increasing in transverse compression or tension results in a reduction of the strength of the material.

### 5.3 Verification Problems

#### 5.3.1 Example 1: Relationship between \( Y^i_C \), \( \eta_L \) and \( S^i_L \)

In Section 5.1.1.1 the following relationship between \( Y^i_C \), \( \eta_L \) and \( S^i_L \) was found:

\[
Y^i_C = -\frac{S^i_L (2 \cos (\alpha_0)^2 - 1)}{\eta_L \cos (\alpha_0)^2} \quad (5.90)
\]

This relationship is written for the in-situ properties but has to be respected also in the case of a unidirectional lamina.

It is shown here that equation (5.90) is respected for the technical composites commonly used. Table 5.2 reports the material parameters of AS4/55A, E-Glass/LY556/HT907/DY063 and T800/3900-2 unidirectional laminae.
Figure 5.23: $\sigma_{11}$-$\tau_{12}$ failure envelope for different values of transverse stress $\sigma_{33}$ for IM7/8552 carbon

Table 5.2: Material properties of investigated composites

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>reference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_T$ (MPa)</td>
<td>–</td>
<td>1140</td>
<td>–</td>
</tr>
<tr>
<td>$X_C$ (MPa)</td>
<td>–</td>
<td>570</td>
<td>–</td>
</tr>
<tr>
<td>$Y_T$ (MPa)</td>
<td>27.0</td>
<td>35</td>
<td>48.8</td>
</tr>
<tr>
<td>$Y_C$ (MPa)</td>
<td>91.8</td>
<td>130</td>
<td>201.7</td>
</tr>
<tr>
<td>$S_L$ (MPa)</td>
<td>66.0</td>
<td>72</td>
<td>100.9</td>
</tr>
<tr>
<td>$G_{12}$ (MPa)</td>
<td>6600</td>
<td>5830</td>
<td>5500</td>
</tr>
<tr>
<td>$\alpha_0$ ($^\circ$)</td>
<td>53</td>
<td>53</td>
<td>53</td>
</tr>
</tbody>
</table>

Figure 5.24 shows the $\tau_{12}$ as a function of $\sigma_{22}$ for the materials of Table 5.2. For each curve the tangent is calculated for $\sigma_N \to 0$ and remember the definition of $\eta_L$ (equation (5.7)) is possible to calculate its value for each material as the angular coefficient of the tangent.

Table 5.3 reports for each material the measured $S_L$, $Y_C$, $\eta_L$ and the value of $Y_C$ that can be calculated using equation (5.90). The predicted values of $Y_C$ are in good agreement with the experimental results.
5.3. VERIFICATION PROBLEMS

![Graph showing τ_{12} as a function of σ_{22} for different materials and correspondent values of Y_C^s, η_L and S_L^s](image)

**Figure 5.24:** τ_{12} as a function of σ_{22} for different materials and correspondent values of Y_C^s, η_L and S_L^s

<table>
<thead>
<tr>
<th>Material</th>
<th>S_L</th>
<th>η_L</th>
<th>Measured Y_C</th>
<th>Calculated Y_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS4/55A</td>
<td>51.3</td>
<td>0.43</td>
<td>91.8</td>
<td>90.2</td>
</tr>
<tr>
<td>E-Glass/LY556/HT907/DY063</td>
<td>66.0</td>
<td>0.41</td>
<td>130.0</td>
<td>123.0</td>
</tr>
<tr>
<td>T800/3900-2</td>
<td>100.9</td>
<td>0.36</td>
<td>201.7</td>
<td>212.0</td>
</tr>
</tbody>
</table>

**Table 5.3:** Material properties of investigated composites

5.3.2 Example 2: σ_{22}-τ_{12} failure envelopes

Figure 5.25 show the σ_{22}-τ_{12} failure envelopes for four different materials: AS4/55A, E-Glass/LY556/HT907/DY063, T800/3900-2 and IM7/8552. The mechanical parameters are reported in Tables 5.2 and 5.4. For carbon IM7/8552 static and dynamic results are reported. It can be noticed that the proposed model fits well the experimental results for static and dynamic case.
5.3.3 Example 3: Compression of thick lamina

Koerber et al. [156] presented compression test based on ASTM Standard D 695 of end-loaded thick specimens. The specimens are manufactured using IM7/8552 carbon-epoxy. The material parameters are shown in Table 5.4.

Table 5.4: Material properties of IM78552

<table>
<thead>
<tr>
<th>test type</th>
<th>static</th>
<th>dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_T$ (MPa)</td>
<td>2323.5</td>
<td>–</td>
</tr>
<tr>
<td>$X_C$ (MPa)</td>
<td>1017.5</td>
<td>1428.0</td>
</tr>
<tr>
<td>$Y_T$ (MPa)</td>
<td>160.2</td>
<td>–</td>
</tr>
<tr>
<td>$S_L$ (MPa)</td>
<td>89.6</td>
<td>122.3</td>
</tr>
<tr>
<td>$\eta_L$</td>
<td>0.38</td>
<td>0.41</td>
</tr>
<tr>
<td>$G_{12}$ (MPa)</td>
<td>5290</td>
<td>5290</td>
</tr>
<tr>
<td>$\alpha_0$ (°)</td>
<td>55.3</td>
<td>56.3</td>
</tr>
</tbody>
</table>
5.3. VERIFICATION PROBLEMS

Two types of test are performed:

- transverse compression;
- off-axis compression at 0°, 15°, 30°, 45°, 60°, 75°.

The test are conducted statically and dynamically. Experimental data and numerical results are reported in Figure 5.26. Good agreement between experimental results and the proposed model is observed.

![Figure 5.26: Off-axis compression test of IM7/8552 carbon lamina](image)

5.3.4 Example 4: $\sigma_{11}$-$\sigma_{22}$ failure envelopes

Hinton et al. [157] report biaxial test of E-Glass/MY750 epoxy lamina. Table 5.5 shows the material properties of E-Glass/MY750 epoxy.

**Table 5.5: Material properties of E-Glass/MY750 epoxy**

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_T$ (MPa)</th>
<th>$Y_C$ (MPa)</th>
<th>$S_L$ (MPa)</th>
<th>$G_{12}$ (MPa)</th>
<th>$\alpha_0$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1280</td>
<td>800</td>
<td>40</td>
<td>145</td>
<td>73</td>
<td>5830</td>
<td>53</td>
</tr>
</tbody>
</table>

The experimental data and the numerical results are shown in Figure 5.27. It should be noted that some data are not taken into accounts to validate the numerical model because they are affected by experimental errors due to the presence of buckling. These points are highlighted in Figure 5.27. For the other points good agreement between experiments and the proposed model is observed.
5.3.5 Example 5: $\sigma_{11}-\tau_{12}$ failure envelopes

Hinton et al. [157] present experimental test of T300/914C. The mechanical parameter of the material are shown in Table 5.6.

Figure 5.28 show the failure envelope generated by the proposed failure criteria and the experimental data.

The model reports the two extreme solutions for $\theta = 0$ and $\theta = \pi$. The dispersion presented by the data could be an effect of the different angles of the kinking plane. The data present a high dispersion and some of them are clearly affected by experimental error (as those that present a shear strength higher than $S_L^p$). For this reason, it could be concluded that the comparison between experiments and the proposed model is acceptable.

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_T$ (MPa)</th>
<th>$Y_C$ (MPa)</th>
<th>$S_L$ (MPa)</th>
<th>$G_{12}$ (MPa)</th>
<th>$\alpha_0$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>900</td>
<td>27</td>
<td>200</td>
<td>80</td>
<td>5500</td>
<td>53</td>
</tr>
</tbody>
</table>
5.3. VERIFICATION PROBLEMS

5.3.6 Example 6: Influence of hydrostatic pressure

As discussed previously, the proposed model predicts higher value of the strength when a hydrostatic pressure is applied to the composite.

However, to predict the correct value of the strength under high hydrostatic pressure is a formidable task. In fact, hydrostatic pressure [155] has a very important influence on the mechanical parameter of the composites. If a hydrostatic pressure is applied:

- the elastic and shear moduli increase. Depending on the resin used the elastic and shear moduli are linear or bilinear functions of the hydrostatic pressure. The bilinear behavior is considered an effect of the hydrostatic pressure that shifts the sub-zero secondary glass transition temperature;

- some resins exhibit yielding when the applied pressure is greater than 200 MPa. As the yielding phenomenon is pressure-dependent a pressure-dependent yield criterion should be used to predict this behavior. Pae [155] proposed for the resin a criterion that consider the second invariant of the deviatoric stress tensor, $J_2$, a material constant $\alpha_m$, and the first invariant of the stress tensor $I_1$. Successively, this theory was extended to the anisotropic materials;

- the composites exhibit a plastic behavior.

Hine et al. [158] presented experimental results of a E-glass/MY750 lamina tested in compression and in shear under hydrostatic confinement. The mechanical properties of the material are shown in Table 5.5.
Figures 5.29-5.30 show the compressive strength and the shear strength as a function of the superimposed hydrostatic pressure of thick lamina tested under hydrostatic pressure.

![Figure 5.29: Compressive strength as function of the hydrostatic pressure](image1)

![Figure 5.30: Shear strength as function of the hydrostatic pressure](image2)

It can be concluded that the proposed criterion can tackle the increase in the strength of the composites under high values of the hydrostatic pressure. However, the correlation between predictions and experimental data appears to be acceptable up
to a pressure of 200 MPa. This pressure should represent the pressure that shifts the sub-zero secondary glass temperature at the room temperature at which the test are conducted. After this pressure, some material parameters change for the reasons explained before, and as these effects are not taken into account in the proposed failure criteria, the experimental data and the predicted model are different for hydrostatic pressure higher than 200MPa.

5.3.7 Conclusions
The proposed failure criteria are able to consider the effect of a three-axial stress state on a composites. The model is completely defined when these parameters are known: $X_T$, $X_C$, $\alpha_0$, $Y_C$, $\eta_L$, $S_L$ and $G_{12}$. It was shown that $Y_C$ is function of $\alpha_0$, $S_L$ and $\eta_L$. As experimentally the value of $Y_C$ is very difficult to measure, equation (5.90) gives a very easy method to calculate $Y_C$.

The model can predict the experimental results in different load conditions and a good agreement is shown. If high value of hydrostatic pressure are applied the elastic properties of the material change and this is not taken into account.

The introduction of the non-linear behavior and the hydrostatic pressure effect should be subjects of further developments.

5.4 Damage Model

Maimí et al. [133, 134] proposed a constitutive damage model for laminated composites that has its foundation in irreversible thermodynamics, and that uses the LaRC04 criteria as damage activation functions.

In this work the same damage model is used but with different damage activation functions. Changes made concern the introduction of the softening law for fiber compression and the extension to the 3D stress state. The details will be explained in the following points.

5.4.1 Complementary free energy and damage operators
The scalar function proposed for the complementary free energy density [133] is:

$$G = \frac{\sigma_{11}^2}{2(1 - d_1)} E_1 + \frac{1}{2E_2} \left( \frac{\sigma_{22}^2}{(1 - d_2)} + \frac{\sigma_{33}^2}{(1 - d_3)} \right) - \frac{\nu_{12}}{E_1} \sigma_{11} (\sigma_{22} + \sigma_{33}) +$$
$$- \frac{\nu_{23}}{E_2} \sigma_{22}\sigma_{33} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{2(1 - d_6)} G_{12} + (\alpha_{11} \sigma_{11} + \alpha_{22} (\sigma_{22} + \sigma_{33})) \Delta T +$$
$$+ (\beta_{11} \sigma_{11} + \beta_{22} (\sigma_{22} + \sigma_{33})) \Delta M$$

(5.91)

where:
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\begin{itemize}
\item $E_1, E_2, \nu_{12}$ and $G_{12}$ are the in-plane elastic orthotropic properties of the uni-directional lamina
\item $d_1, d_2$ and $d_6$ are the damage variable. The damage variable $d_1$ is associated with longitudinal (fiber) failure, whereas $d_2$ is the damage variable associated with transverse matrix cracking and $d_6$ is a damage variable influenced by longitudinal and transverse cracks
\item $\alpha_{11}$ and $\alpha_{22}$ are the coefficients of thermal expansion in the longitudinal and transverse directions
\item $\beta_{11}$ and $\beta_{22}$ are the coefficients of hygroscopic expansion in the longitudinal and transverse directions
\item $\Delta T$ and $\Delta M$ are the differences of temperature and moisture content with respect to the corresponding reference values.
\end{itemize}

Using this definition of the free energy it can be shown that the strain tensor is equal to the derivative of the complementary free energy density with respect to the stress tensor:

$$\varepsilon = \frac{\partial G}{\partial \sigma} = H : \sigma + \alpha \Delta T + \beta \Delta M$$  

(5.92)

where the compliance tensor $H$ can be written as:

$$H = \begin{bmatrix}
1 & -\nu_{12} & 0 & 0 \\
\frac{1}{(1-d_1) E_1} & \frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\
-\frac{\nu_{12}}{E_1} & \frac{1}{(1-d_2) E_2} & \frac{\nu_{23}}{E_2} & 0 \\
0 & -\frac{\nu_{23}}{E_2} & \frac{1}{(1-d_3) E_2} & 0 \\
0 & 0 & 0 & \frac{1}{(1-d_6) G_{12}} \\
0 & 0 & 0 & 0 & \frac{1}{(1-d_6) G_{12}}
\end{bmatrix}$$  

(5.93)

To determine the active damage variables, it is necessary to define the longitudinal and transverse damage modes as follows:

$$d_1 = d_1 \langle \sigma_{11} \rangle + d_1 \langle -\sigma_{11} \rangle$$
$$d_2 = d_2 \langle \sigma_{22} \rangle + d_2 \langle -\sigma_{22} \rangle$$
$$d_3 = d_2 \langle \sigma_{33} \rangle + d_2 \langle -\sigma_{33} \rangle$$

(5.94)

where $\langle x \rangle$ is the McCauley operator defined as $\langle x \rangle := (x + |x|)/2$. 

5.4. DAMAGE MODEL

5.4.2 Damage activation functions

The four damage activation functions, $F_N$, associated with damage in the longitudinal ($N = 1+ , 1-$) and transverse ($N = 2+ , 2-$) directions, are defined as:

\[
F_{1+} = \phi_{1+} - r_{1+} \leq 0 \quad ; \quad F_{1-} = \phi_{1-} - r_{1-} \leq 0
\]
\[
F_{2+} = \phi_{2+} - r_{2+} \leq 0 \quad ; \quad F_{2-} = \phi_{2-} - r_{2-} \leq 0
\]

(5.95)

where the loading functions $\phi_N$ ($N = 1+, 1-, 2+, 2-$) depend on the strain tensor and material constants (elastic and strength properties). The elastic domain thresholds $r_N$ ($N = 1+, 1-, 2+, 2-$) take an initial value of 1 when the material is undamaged, and they increase with damage. The elastic domain thresholds are internal variables of the constitutive model, and are related to the damage variables $d_M$ ($M = 1+, 1-, 2+, 2-, 6$) by the damage evolution laws.

The criterion for fiber tension failure (see equation 5.49) is:

\[
\phi_{1+} = \frac{E_1}{X_T} \varepsilon_{11}
\]

(5.96)

where the effective stress tensor $\tilde{\sigma}$ is computed as $\tilde{\sigma} = H_0^{-1} : \varepsilon$. $H_0$ is the undamaged compliance tensor obtained from equation (5.93) using $d_1 = d_2 = d_6 = 0$.

The damage activation function used to predict damage under longitudinal compression ($\tilde{\sigma}_{11} < 0$) is established to be equal to

\[
\phi_{1-} = \sqrt{FK}
\]

(5.97)

where $FK$ is defined in equation (5.72).

The damage activation function used to predict damage under transversal tension ($\sigma_N > 0$) is (see equation (5.43)):

\[
\phi_{2+} = \sqrt{FMT} = \sqrt{\left(\frac{\tau_T}{S_T^{1s}}\right)^2 + \left(\frac{\sigma_N}{Y_T^{1s}}\right)^2}
\]

(5.98)

The damage activation function used to predict damage under transversal compression ($\sigma_N \leq 0$) is (see equation (5.6)):

\[
\phi_{2-} = \sqrt{FMC} = \sqrt{\left(\frac{\tau_T}{S_T^{1s} - \eta_T \sigma_N}\right)^2 + \left(\frac{\tau_L}{S_L^{1s} - \eta_L \sigma_N}\right)^2}
\]

(5.99)

5.4.3 Damage evolution

The evolution of the threshold values $r_N$ is mathematically expressed by the Kuhn-Tucker conditions:

\[
\dot{r}_N \geq 0 \quad ; \quad F_N \leq 0 \quad ; \quad \dot{r}_N F_N = 0
\]

(5.100)
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The damage activation functions, equations (5.95), must always be non-positive and if the gradient is positive, there is damage evolution; the consistency condition to satisfy is:

\[ \dot{F}_N = \dot{\phi}_N - \dot{r}_N = 0 \]  

(5.101)

The evolution of the elastic domain can be represented by the following equations [133]:

- **transverse loading**:

  Tension loading: \( \dot{r}_2^+ = \dot{\phi}_2^+ \) and \( \dot{r}_2^- = 0 \)

  Compression loading: \( \dot{r}_2^- = \dot{\phi}_2^- \) and \( \dot{r}_2^+ = \begin{cases} \dot{\phi}_2^- & \text{if } r_2^+ \leq r_2^- \\ 0 & \text{if } r_2^+ > r_2^- \end{cases} \)

  The integration of the previous expressions results in:

  \[
  r_2^+ = \max \left\{ 1, \max_{s=0,t} \left\{ \phi_{2-}^s \right\}, \max_{s=0,t} \left\{ \phi_{2+}^s \right\} \right\} \\
  r_2^- = \max \left\{ 1, \max_{s=0,t} \left\{ \phi_{2-}^s \right\} \right\} 
  \]  

(5.102)

- **longitudinal loading**

  Tension loading: \( \dot{r}_1^+ = \dot{\phi}_1^+ \) and \( \dot{r}_1^- = 0 \)

  Compression loading: \( \dot{r}_1^- = \dot{\phi}_1^- \) and \( \dot{r}_1^+ = \begin{cases} \dot{\phi}_1^- & \text{if } r_1^+ \leq r_1^- \\ 0 & \text{if } r_1^+ > r_1^- \end{cases} \)

  The integration of the previous expressions results in:

  \[
  r_1^+ = \max \left\{ 1, \max_{s=0,t} \left\{ \phi_{1+}^s \right\}, \max_{s=0,t} \left\{ \phi_{1-}^s \right\} \right\} \\
  r_1^- = \max \left\{ 1, \max_{s=0,t} \left\{ \phi_{1-}^s \right\} \right\} 
  \]  

(5.104)

5.4.4 Dissipation

The rate of energy dissipation per unit volume resulting from the evolution of damage is given by:
5.4. DAMAGE MODEL

\[ \Xi = \partial G / \partial d_1 \dot{d}_1 + \partial G / \partial d_2 \dot{d}_2 + \partial G / \partial d_6 \dot{d}_6 = Y_1 \dot{d}_1 + Y_2 \dot{d}_2 + Y_6 \dot{d}_6 \geq 0 \] (5.105)

The form of the complementary free energy assures that the thermodynamic forces \( (Y_M) \) conjugated to their respective damage variables \( (d_M) \) are always positive:

\[
\begin{align*}
Y_1 &= \frac{\partial G}{\partial d_1} = \frac{\sigma^2_{11}}{2(1-d_1)^2E_1} \geq 0 \\
Y_2 &= \frac{\partial G}{\partial d_2} = \frac{\sigma^2_{22}}{2(1-d_2)^2E_2} \geq 0 \\
Y_6 &= \frac{\partial G}{\partial d_6} = \frac{\sigma^2_{12}}{2(1-d_6)^2G_{12}} \geq 0
\end{align*}
\] (5.106)

Therefore, the condition of positive evolution of damage variables \( (\dot{d}_M \geq 0) \) is a sufficient condition for the fulfillment of the second law of thermodynamics.

5.4.5 Damage evolution laws

To define the constitutive model, the damage variables \( d_N \) must be related to the internal variables \( r_N \). In particular, when material is undamaged the internal variables \( r_N \) take the initial value of 1, and \( d_N(r_N = 1) = 0 \).

5.4.6 Computational Model

Using Bažant’s crack band model [159], the damage evolution laws for each damage variable take the values [134]:

Bažant’s crack band model [159] assures the correct response of the finite element model regularizing the dissipated energy:

\[ g_M = \frac{G_M}{l^*}, \ M = 1+, 1-, 2+, 2-, 6 \] (5.107)

where \( G_M \) is the fracture toughness, \( g_M \) is the energy dissipated per unit volume, and \( l^* \) is the characteristic length of the finite element.

\[
\begin{align*}
   d_{1+} &= 1 - \frac{1}{r_{1+}} \exp \{ A_{1+} (1 - r_{1+}) \} \\
   d_{1-} &= 1 - \frac{1}{r_{1-}} \exp \{ A_{1-} (1 - r_{1-}) \} \\
   d_{2+} &= 1 - \frac{1}{r_{2+}} \exp \{ A_{2+} (1 - r_{2+}) \} \\
   d_{2-} &= 1 - \frac{1}{r_{2-}} \exp \{ A_{2-} (1 - r_{2-}) \} \\
   d_{6} (r_{2+}) &= 1 - \frac{1}{r_{2+}} \exp \{ A_{6} (1 - r_{2+}) \}
\end{align*}
\] (5.108)

The energy dissipated per unit volume for uniaxial stress conditions is obtained by integrating the rate of dissipation, equation (5.105):

\[ g_M = \int_0^\infty Y_M \dot{d}_M dt = \int_1^\infty \frac{\partial G}{\partial d_M} \frac{\partial d_M}{\partial r_M} dr_M, \ M = 1+, 1-, 2+, 2-, 6 \] (5.109)
Applying the crack band model, equation (5.107):

\[ \int_{1}^{\infty} \frac{\partial G}{\partial d} \frac{\partial d_{M}}{\partial r_{M}} \, dr_{M} = \frac{G_{M}}{l^{*}}, \quad M = 1+, 1-, 2+, 2-, 6 \] (5.110)

Using equations (5.107) and (5.109) and substituting in (5.110), it is possible to calculate the damage law parameters \( A_{M} \) that assure that the dissipated energy computed by the numerical model is independent of mesh refinement. It is possible to obtain the following analytical closed form solutions:

\[ A_{1+} = \frac{2l^{*}X_{T}^{2}}{2E_{1}G_{1+} - l^{*}X_{T}^{2}} \] (5.111)

\[ A_{1-} = \frac{2l^{*}X_{C}^{2}}{2E_{1}G_{1-} - l^{*}X_{C}^{2}} \] (5.112)

\[ A_{2+} = \frac{2l^{*}Y_{T}^{2}}{2E_{2}G_{2+} - l^{*}Y_{T}^{2}} \] (5.113)

\[ A_{2-} = \frac{2l^{*}Y_{C}^{2}}{2E_{2}G_{2-} - l^{*}Y_{C}^{2}} \] (5.114)

\[ A_{6} = \frac{2l^{*}S_{L}^{2}}{2G_{12}G_{6} - l^{*}S_{L}^{2}} \] (5.115)
Chapter 6

Numerical Results

A 3D numerical model was presented in Chapter 5, while in Chapter 4 a methodology to measure the resistance curves in the longitudinal failure of composites was proposed. The R-curves allow the calculation of the parameters of the softening laws used in the progressive damage model, and describe the damage in function of the dissipated energy for each failure mode. The softening law parameters are calculated from empirical data in Section 4.2.2.6 for IM7/8552 carbon in tension and are computed using an inverse technique method for compression.

In this chapter, the numerical results will be compared with experimental data on IM7/8552 double shear bolted joints presented by Camanho et al. [160].

The tests were performed on double shear bolted joints having three different geometries that result in different failure modes: bearing, shear-out and tension failure modes. The geometries of the specimens are presented in Table 6.1.

<table>
<thead>
<tr>
<th>specimen</th>
<th>d (mm)</th>
<th>w (mm)</th>
<th>w/d</th>
<th>e (mm)</th>
<th>e/d</th>
<th>d_w (mm)</th>
<th>h (mm)</th>
</tr>
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<td>7</td>
<td>42</td>
<td>6</td>
<td>21</td>
<td>3</td>
<td>12.5</td>
<td>3</td>
</tr>
<tr>
<td>DBJT 2</td>
<td>6</td>
<td>12</td>
<td>2</td>
<td>24</td>
<td>4</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>DBJS 3</td>
<td>6</td>
<td>36</td>
<td>6</td>
<td>9</td>
<td>1.5</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

1 Double Shear Bolted Joint Bearing
2 Double Shear Bolted Joint Tension
3 Double Shear Bolted Joint Shear-Out
4 diameter of the washer
5 thickness

Three different Finite Element models of the double shear bolted joints, shown in Figure 6.1, were created using a Python parametric model. The numerical results are compared with experimental data in terms of both the stresses and the damaged zones.
Figure 6.1: FE models

(a) bearing

(b) tension

(c) shear
6.1 FE model

6.1.1 Python scripts

Using Python script, a parametric model of the double bolted joint was developed. The scripts that generate the models, written in Python, enable the generation of several FEM models simply running these script in the Abaqus CAE utility. It should be noticed that the following parameters can be easily changed:

1. The materials of the joint. The materials are managed using Python dictionaries [143]. The user can introduce other materials just adding other dictionaries.

2. The geometry of the joint. The following parameters can be set:
   (a) The diameter of the bolt $d_b$
   (b) The clearance $c$ between the diameter of the bolt $d_b$ and the diameter of the hole $d$
   (c) The width of the joint $w$
   (d) The end-distance $e$
   (e) The length of the joint $L$
   (f) The type and the geometry of the bolt (screw, nut and washer)
   (g) The thickness of each ply and the thickness of the cohesive elements (if cohesive elements are requested)
   (h) The orientation of the fiber for each lamina

3. The loads applied during the steps (and in particularly the thermal load during the thermo-mechanic step, the pressure on the washer, and the displacement applied to the joint)

4. Other numerical parameters that influence the analysis (time step, time increment, mass scaling, etc...)

6.1.2 Details of the models

6.1.2.1 Geometry of the model

Figure 6.2 shows the isometric view of the DBJM. As explained before, all the geometric parameters can be set in a input file and are generated automatically in Abaqus CAE. It can be observed that different parts are modeled: the bolt (screw, nut and washers) and the composite laminate.

First of all, it is possible to observe that the laminate it is not modeled taking advantage of the symmetry. Even if the plane $xy$ is a symmetry plane the joint was entirely modeled. This for two reasons: first, if a damage model is used, damage can
occur in one side of the joint and the symmetry is lost; second, if a plane of symmetry is used, on the symmetry plane the cohesive elements have to have a thickness that is half of that of the other cohesive elements. In an explicit analysis this means that the critical time increment is reduced and so, CPU time increases. With cohesive elements the use of a plane of symmetry can result in an higher computational cost.

6.1.2.2 TIE constraints

The use of cohesive elements and a damage material model results in high computational times. To mitigate this problem, it is possible to use the damage model only in the parts of the joint where damage occurs. In the other regions, not affected by damage propagation, the composite is simulated as a linear elastic material. For this reason, the composite laminate consists of two different regions that can be easily identified in Figure 6.4. The region around the hole is modeled using cohesive elements and damage model for bricks. In the second region only elastic elements are present.

The dimensions of the elements in these two regions are quite different. As explained before in Section 5.4.6, in order to guarantee that the numerical solution is independent of the discretization, the elements must have a size lower then a critical value [159]. Obviously this is not necessary for the elastic elements. For these reasons the two regions are modeled with different elements sizes that are shown in Figure 6.4.

To connect these two region TIE constraints are used on the two surfaces with non-conforming meshes. If a TIE constraint is applied nodes are tied only where
the surfaces are close to one another and this connection is kept during the entire analysis. Generally one surface in the constraint is designated to be the slave surface; the other surface is the master surface. Figure 6.3 shows a magnified view of the connection between this two region. The red surface is that one where TIE constraint are applied.

![Isometric view of DBJ model, detail](image)

**Figure 6.3:** Isometric view of DBJ model, detail

![DBJ model, mesh](image)

**Figure 6.4:** DBJ model, mesh

### 6.1.2.3 Loading

To reproduce the experimental tests for a Double Bolted Joint the simulation is divided in three load steps:
1. first step: thermo-mechanical step. In this step a thermal load, $\Delta T < 0$ is applied to the composite laminate. This correspond to the thermal load occurring during the curing process. This thermal load generates residual stresses that have influence on the damage of the composite.

2. second step: bolt pre-loading. The pre-load is generated applying two opposite forces to the screw and the nut. This two forces are designated with $P$ in Figure 6.5(a). During this load step the surface of contact between the bolt and the nut is characterized by:
   
   (a) frictionless tangential behavior. The friction on the surface highlighted in Figure 6.5(b) is equal to $f = 0$.
   (b) hard contact normal behavior, that avoids interpenetration between the screw and the nut.
   (c) the separation after contact is allowed.

3. third step: loading of the joint. In this step the $u$ displacements are applied to the joint. During this step the contact surface is defined as:
   
   (a) rough tangential behavior. The friction on the surface between screw and nut is equal to $f = \infty$.
   (b) hard contact normal behavior, that avoids interpenetration between the screw and the nut.
   (c) the separation after contact is not allowed.

![Figure 6.5: DBJ model, loading configuration](image)

It should be noted that using this technique the behavior of the bolt is simulated in a very efficient way. Many authors simulate the presence of a bolt simply introducing a pressure on the area of contact between washers and laminate. However, the simple use of a pressure is not suitable to study bolted joint for different reasons. First of all, if a progressive damage model is used, as the damage occurs in the
proximity of the hole the pressure on the contact area has to decrease because if the material is damaged the stiffness of the bolted joint reduces and also the bolt load is reduced. Therefore, the use of a constant pressure on the washers is not correct. In the same way, in the case of single bolted joint, the application of the load generally causes a deformation of the joint in the direction normal to the laminate. In this condition the heads of the bolt contribute with the bottom surface to the load carried by the joint. Therefore the pressure applied on the washers has to increase.

6.1.2.4 Elements used and typical size

Three different type of elements are used:

- 8-node brick elements, with elastic material;
- 8-node brick elements, with VUMAT constitutive model;
- 8-node brick elements, with VUMAT constitutive model for cohesive elements. The explicit formulation for cohesive elements is described in the work of González et al. [161]

6.1.2.5 Contacts

In 6.1.2.3 the different contact definitions will be explained for the contact pair nut/screw. For all other surfaces the general explicit contact has been used. The general contact algorithm has to be chosen because it allows to consider the contact of a body with itself and the failure of the elements. The friction is set to be equal to $f = 0.3$.

6.2 Comparison between FEM and experiments

6.2.1 Bearing

Figure 6.6 shows the relation between the bearing stress and the corrected displacement for the FE model and for the experimental tests. The bearing stress (see Section 2.1.1) is defined as:

$$\sigma^b = \frac{P}{dh} \quad (6.1)$$

where $P$ is the load, $d$ is the diameter of the hole, $h$ is the thickness of the specimen. The displacement is corrected to take into account the compliance of the testing machine. If $\delta_{meas}$ is the measured displacement and $C$ is the compliance of the machine, the corrected displacement $\delta$ can be computed as:

$$\delta = \delta_{meas} - CP \quad (6.2)$$
176  

CHAPTER 6. NUMERICAL RESULTS

It should be noted that the first-peak load is predicted accurately but after that damage occurs the experimental and the numerical curves show a difference that is attributable mainly to two reasons:

- the proposed failure criteria are fully 3D while the damage model used is quasi-3D. This means that the failure onset is accurately predicted but not the evolution of the damage in conditions where the 3D stress states prevail;

- the damage evolution in bearing failure mode depends primarily to the softening law for fiber in compression. This was not found experimentally but using an inverse identification technique. The parameters used may differ from the real parameters.

Figure 6.7 shows the contour plot of the damage index of the fiber, $d_1$, in function of the bearing stress $\sigma^b$. The damaged zones correspond to those observed experimentally (see Figure 6.8) and are located uniformly in the neighborhood of the hole.
6.2. COMPARISON BETWEEN FEM AND EXPERIMENTS

Figure 6.7: Contour plots of the damage index of the fiber in function of the bearing stress for the bearing specimen

Figure 6.8: Bearing specimen after loading
6.2.2 Net-Tension

Figure 6.9 shows the relation between the bearing stress and the corrected displacement for the FE model and for the experimental tests of the tension specimen.

![Graph showing the relation between bearing stress and corrected displacement](image)

**Figure 6.9:** Tensile stress as function of the corrected displacement

Figure 6.11 shows the contour plot of the damage index of the fiber, $d_1$, in function of the bearing stress $\sigma^b$. As expected for tension failure (see Figure 6.10), the main damaged zones are located along the net-section plane of the specimen.

![Tension specimen after loading](image)

**Figure 6.10:** Tension specimen after loading
6.2. COMPARISON BETWEEN FEM AND EXPERIMENTS

Figure 6.11: Contour plots of the damage index of the fiber in function of the bearing stress for the tension specimen
6.2.3 Shear-Out

Figure 6.12 shows the relation between the bearing stress and the corrected displacement for the FE model and for the experimental tests of the shear specimen.

Figure 6.14 shows the contour plot of the damage index of the fiber, \( d_1 \), in function of the bearing stress \( \sigma_b \). It should be noted that when damage is developed, the fracture is located on the shear plane (see Figure 6.14(h)).

![Figure 6.12: Shear stress as function of the corrected displacement](image1)

![Figure 6.13: Shear specimen after loading](image2)
6.2. COMPARISON BETWEEN FEM AND EXPERIMENTS

Figure 6.14: Contour plots of the damage index of the fiber in function of the bearing stress for the shear specimen
6.3 Conclusions

The first-peak load for each specimen is reported in Table 6.2, and the mean value is compared with the numerical predictions. Good agreement is found in the prediction of the first peak load.

<table>
<thead>
<tr>
<th>Specimen Ref.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>mean</th>
<th>numerical</th>
<th>error (%)</th>
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<td>15950</td>
<td>16080</td>
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<td>16120</td>
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<td>16120</td>
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</tr>
<tr>
<td>DTB</td>
<td>10136</td>
<td>9356</td>
<td>9428</td>
<td>9344</td>
<td>9560</td>
<td>9564.8</td>
<td>9348</td>
<td>2.3</td>
</tr>
<tr>
<td>DSB</td>
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<td>13140</td>
<td>12780</td>
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<td>13180</td>
<td>12746</td>
<td>13610</td>
<td>6.3</td>
</tr>
</tbody>
</table>

It can be concluded that 3D numerical simulations can predict with accuracy the value of the first peak load. However the computational cost is not suitable for industrial applications because each model takes about 3 days to run using a cluster of 32 cpus. Therefore a simplified two-dimensional method will be presented in the following chapter.
Chapter 7

2D Numerical Model

7.1 Introduction

3D failure models provide a good prediction for the behavior of a composite bolted joints, from damage onset to failure of the joint. However, the use of such type of analysis is not applicable in an industrial environment where results should be obtained quickly. For this reason, a two-dimensional (2D) model is proposed here to provide fast predictions of the strength of composite bolted joints. Experimental tests of a subcomponent are performed, and the results are used to verify the accuracy of the model.

In Section 2.5 the connections that can be manufactured using a combination of different materials (metals and composites) in train structures were identified.

Here, one of these connections (connection C in Figure 2.16) is analyzed. An industrial design method is proposed and experimental test are performed. Numerical simulations are compared with experimental results. A quick-design method is discussed in the last part of the chapter and a methodology to predict the joint strength in a fast way is proposed.

7.2 Experimental Analysis

7.2.1 Subcomponent Design

The design of the industrial sub-component is shown in Figure 7.1. It represents the connection between the side of the carbody shell (in steel, 1 in Figure 7.1) and the omega beam of the roof (in glass-fiber, 2 in Figure 7.1).

The lay-up of the laminate is [(0/±45)/0]s, where 0° corresponds to the longitudinal direction of the omega beam. The nominal thickness of the composite beams, in the neighboring of the holes is equal to 11.8mm. The mechanical properties of the unidirectional lamina are shown in Table 7.1.

The connection is made using the 5mm UNBRAKO bolts (3 in Figure 7.1)
already used in experimental test described in Chapter 3. Section 3.4.2 provides the mechanical parameters of these bolts. It should be highlighted that, as explained before, the torque used to install the bolts is very low and therefore bolts work in shear and not in traction. This means that for this kind of application the use of bolts or rivets is indifferent.

Table 7.1: UD mechanical properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
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<td>$E_1$ (MPa)</td>
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</tr>
<tr>
<td>$E_2 = E_3$ (MPa)</td>
<td>3000</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\nu_{13} = \nu_{23}$</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{12}$ (MPa)</td>
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<td>$G_{13} = G_{23}$ (MPa)</td>
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<tr>
<td>$X_T$ (MPa)</td>
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</tr>
<tr>
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<td>$S_T$ (MPa)</td>
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<tr>
<td>$S_L$ (MPa)</td>
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</tbody>
</table>

7.2.2 Experimental Setup

To study the behavior of the joint, when a bearing failure mode is present, the sub-component is tested as shown in Figure 7.2. As the steel panel is very flexible, the load has to be applied using a hinge in order to guarantee a predominant shear stress field on the plane of the bolts.

Figure 7.3 shows the experimental setup of the Instron machine used to test the sub-component.

Figure 7.4 shows the solution used to connect the component to the test fixture. On the top, the component is connected to the main structure of the machine as shown in Figure 7.4(a). Bolts are used in conjunction with a drilled steel-plate to prevent damage of the composite during loading.
7.2. EXPERIMENTAL ANALYSIS

Figure 7.4(b) shows the connection on the bottom of the sub-component. An hinge was designed to connect the component to the cylinder of the servo-hydraulic machine. The hinge allows testing the sub-component in the presence of misalignment of the load rig.

7.2.3 Experimental Results

To reduce the value of the peak-load during the test, only a few bolts were used to connect the two parts of the sub-component. For safety reasons, it was decided to use only 2 bolts during the tests. The tests were performed in displacement control and a speed of 1mm/min was used.

Figure 7.5 shows a representative load-displacement curve obtained during the experimental test.

Figure 7.5 compare the load-displacement curve with the elastic load-displacement curve (tangent to the curve). The first non-linearity point is for $P_{nl} \approx 15000$N and the peak-load is $P_{max} \approx 25800$N.
When the load $P$ reach the critical value $P_{nl}$ the composite start to fail (elastic limit). The failure mode is bearing as shown in Figure 7.6(a). At the same load, also the steel panel exhibits bearing failure in proximity of the bolts as reported in Figure 7.6(b). Finally, when the load reach the peak-load $P_{max}$ the bolts fail (see Figure 7.6(c)) because of the combination of shear and traction applied.

Generally, in design, the elastic limit is considered as the limit condition and not the ultimate failure of the bolt. The methodology described in the following will enable the prediction of the load at which the composite start to fail (i.e. the value of $P_{nl}$).
7.3 Quick Design Tool and Analysis Results

7.3.1 Generalities

Camargo and Lambert [126] proposed a methodology that enables the prediction of the elastic limit and of the ultimate joint failure load in a short amount of time. This methodology uses the Complex Variable Theory of Elasticity (CVTE) to obtain analytical solutions for the stress distribution around the hole avoiding the use of the Finite Element Method. Using the Classical Lamination Theory the stress distribution is calculated for each ply.

The prediction of the elastic limit is based on the LaRC03 failure criteria while the prediction of the joint failure is based on the use of a characteristic curve [162], applying the failure criteria on the characteristic curve, using local and averaged stresses. Obviously, the characteristic distances in compression and in tension should be measured.

The stress distribution around the hole, considering plane stress, is based on the use of the Airy stress function $F$:

\begin{align*}
\sigma_{xx} & = \frac{\partial^2 F}{\partial y^2} \\
\sigma_{yy} & = \frac{\partial^2 F}{\partial x^2} \\
\sigma_{xy} & = -\frac{\partial^2 F}{\partial x \partial y}
\end{align*}

The general expression for the function $F$ depends upon the root of the characteristic equation and is defined as:

\[ F = 2\Re \{ F_1(Z_1) + F_2(Z_2) \} \] (7.4)

where $F_1(Z_1)$ and $F_2(Z_2)$ are analytical functions of the complex variables $Z_1 = x + R_1y$ and $Z_2 = x + R_2y$ respectively.

The methodology used to define the function $F_i(Z_i)$ is based on the work done by Garbo and Ogonowski [163]. The stress distribution is considered as obtained by the superposition of the in-plane bearing and by-pass loads as shown in Figure 7.7.
Therefore, the stress distribution of a laminate with a loaded hole is obtaining superimposing two stress distributions: that one a cosinusoidal distribution of pressure on the contact zone of the hole (in-plane bearing) and the stress distribution of a laminate with an open hole remotely loaded.

The resulting stress distribution is a good approximation of the real stress state when the ratio $w/d \geq 4$ and $e/d \geq 3$, i.e. when bearing failure mode is present. It should be noted that the stress distribution is calculated under the hypothesis that no bending is present in the mid-plane of the laminate and when the pressure can be assumed cosinusoidal (this happens when the laminate is quasi-isotropic and the clearance is small).

However, some problems arise in industrial application. For example, in some parts the geometrical conditions $w/d \geq 4$ and $e/d \geq 3$ are not respected and in this case the stress distribution around the hole could not be sufficiently approximated. The 2D method proposed here try to solve this problem and presents several advantages that are fundamental in an industrial environment, namely:

- the method is fast;
- the method is sufficiently realistic;
- different failure criteria can be implemented (for composite parts, metal parts, bolts and rivets);
- the method uses automatically the results of the FE analysis and this simplify the work of the engineers.

A bolt is simply represented by a kinematic constraint of different nodes that are as many as the parts connected together. For example, for a simple connections of a two panels, two nodes will be part of the same kinematic constraint.

For each node is possible to consider a path of nodes that lie around the coupled node. Figure 7.8 show the coupled node (in red) and the nodes (in blue) that lie in a path around the coupled node (contour nodes). After the analysis has been performed, it is possible postprocess the data and obtain the nodal loads applied to each node. In particular $F_{x,bolt}$, $F_{y,bolt}$ and $F_{x,i}$, $F_{y,i}$ are obtained for the coupled
node and the contour nodes respectively. $x$ and $y$ lie on the plane identified by the contour nodes, $z$ is parallel to the axis of the bolt.

**Figure 7.8:** Coupled node and contour nodes considered for the submodelling.

For each coupled node, a submodel can be defined and solved to obtain a good approximation of the stress distribution around the hole. Figure 7.9 shows the submodel generated by the algorithm for each coupled node. At each of the contour nodes of Figure 7.8 corresponds a node on the contour. At this node the nodal loads $F_x^i$ and $F_y^i$ are applied. At the edge of the hole an equivalent cosinusoidal pressure is applied.

**Figure 7.9:** Loads and pressure applied to the sub-model

Solving this model the real stress distribution is found and the Failure Indexes of the materials and of the bolts can be evaluated. In the case of a composite the
Failure Index is computed using the set of failure criteria presented in Chapter 5 but limited at the 2D case while for metals the failure criterion is simply $FI = \sigma_{eq}/\sigma_y$ where $\sigma_{eq}$ is the von-Mises stress. The Failure Indexes for the connected parts are then locally calculated for each element. Failure Indexes for bolts (shear, traction, combined shear/traction) are defined following NF EN 1993-1-8 norm [164]. The failure index for shear, $FI_s$, is:

$$FI_s = P_s/P_{sR}$$  \hspace{1cm} (7.5)

with

$$P_{sR} = \alpha_v f_{ub} A_b/\gamma_{M2}$$  \hspace{1cm} (7.6)

where $P_s$ is the shear load acting on the bolt, $P_{sR}$ is the shear load at rupture, $A_b$ is the gross section area of the bolt, $\alpha_v$ is a parameter that depends on the class of the bolt and $f_{ub}$ is the ultimate tensile strength of the bolt. For tension the failure index $FI_t$ is calculated as:

$$FI_t = P_t/P_{tR}$$  \hspace{1cm} (7.7)

with

$$P_{tR} = k_2 f_{ub} A_s/\gamma_{M2}$$  \hspace{1cm} (7.8)

where $P_t$ is the tensile load acting on the bolt, $P_{tR}$ is the tensile load at rupture, $A_s$ is the tensile stress area of the bolt and $k_2$ is a parameter that is equal to 0.63 for countersunk bolts and 0.9 for the others. The failure index for the combining presence of shear and traction, $FI_{st}$, is calculated as:

$$FI_{st} = FI_s + \frac{FI_t}{1.4}$$  \hspace{1cm} (7.9)

### 7.3.2 Industrial application

The methodology explained in 7.3.1 is applied to a real industrial case and in particularly at the prototype designed and tested as discussed in 7.2.

The commercial software ANSYSv10 is used to define the FE model of the prototype. Figure 7.10 shows the FE model of the prototype where the loads and displacement applied and the coupled nodes are shown. As only two components are linked together the each kinematic constraint consist of only 2 nodes: 4334-20708 and 4329-20711 are the two couples of nodes that represent the bolts. Between the two components contact elements are used to avoid element interpenetration.
A non-linear static analysis is conducted and for each step the algorithm is run. For each coupled node the algorithm is run and the distribution of the Failure Index in the material is obtained.

Figure 7.12 shows the Failure Indexes at the critical element as function of the applied load. The material starts to fail in proximity of the bolt 4329-20711. Steel and composite have approximately the same failure index and the load at which failure begins is equal to 15000 N. This finding is in agreement with the experimental results.
Figure 7.12: Failure Indexes as function of the applied load
Chapter 8
Conclusions and future work

8.1 Conclusions

In this thesis, an investigation of composite bolted joints was presented at different levels of detail.

A preliminary experimental campaign was performed to investigate the influence of the geometric parameters on single-shear or double-shear bolted joints with different materials or material combinations. The results were used to present design charts that can be used by engineers during the preliminary design of a composite joint. Two types of test were considered: in plane shear test and pull-through tests. All the tests were conducted considering specimens with different geometries and with different materials or material combinations. It was shown that the geometry affects the failure mode, and that materials with the same geometries may show a different failure mode.

The main differences in the failure load of the in plane shear test concerns with the epoxy specimens that show always a value of the failure load that is higher than that shown by other specimens. This is due to the fact that epoxy-specimens were manufactured using prepregs while the other specimens (phenolic and vinylester) were manufactured using resin infusion.

If pull-trough is considered the results are almost the same for the three materials, considering both the failure load and the initial sub-critical failure load. As expected, it was found that increasing the diameter, increases both the sub-initial critical failure load and the failure load of the specimen. This means that larger holes exhibit an initial sub-critical failure load that is relatively lower if compared to the ultimate failure load. The values of the sub-critical load and of the failure load are virtually the same for the different material tested.

After this preliminary study a 3D set of failure criteria to take into account the clamping pressure applied to the bolt was developed.

It was shown that, for matrix compression, Puck’s failure criterion can be used...
for the unidirectional laminate and for the embedded lamina, but only if the in-situ parameters are correctly computed. If not, the model does not work properly. In particular it was shown that $Y_C$ and $S_T$ are in-situ parameters that can be calculated knowing $\eta_L$, $\alpha_0$ and $S_T^p$.

For matrix-tension, it was shown that Puck’s elliptical failure criterion and Pinho’s quadratic criterion cannot be used for composite materials. The new failure criterion proposed for matrix-tension allows to obtain a consistent definition of the failure index for a general 3D stress state. It was also shown that the failure criterion proposed is applicable for $\sigma_n \geq 0$, and that it can be used consistently together with Puck criterion for compression. As the matrix failure has a critical influence on the fiber kinking, the failure criteria for fiber kinking were also modified.

The predictions of the fully 3D failure criteria were compared with experimental results and good agreement is generally observed. It was shown also that if a hydrostatic compression is present, up to a critical value, the difference between the experimental data and the predicted values is acceptable.

The failure criteria proposed were used in a progressive damage model. Even if the failure criteria is fully 3D the progressive damage model is only quasi-3D.

The progressive damage model uses softening laws that define the evolution of the damage in terms of the dissipated energy. To compute the parameters of the softening laws, an experimental campaign was performed using CT and CC specimen for both IM7/8552 and glass fiber.

Using the digital image correlation (DIC) a new methodology was proposed to compute the J-integral around the crack tip and to measure the crack length.

The comparison between the R-curves obtained in CT specimens using the FE-based post-processing and the DIC-based method indicates that the results are virtually the same and that the proposed method is a valid alternative to measure R-curves associated with longitudinal tensile failure mechanisms in composite materials. The initial value of the fracture toughness associated with longitudinal tensile failure mechanisms in IM7/8552 is 98.7J/m$^2$, and it raised up to 133.7J/m$^2$ for steady state propagation. The mean value of the associated cohesive zone is 3.4mm.

It is concluded that the FE-based data reduction method is inadequate for the measurement of R-curves in CC specimens because it severely overpredicts the fracture toughness. The DIC-based method is an improvement over FE-based data reduction methods because it is based on the actual displacement field on a pre-defined contour. However, the contribution of the contact tractions to the J-integral still needs to be quantified and used in the data reduction method to improve the accuracy of the data. The initial value of the fracture toughness associated with longitudinal compressive failure mechanisms in IM7/8552 measured using DIC was 47.5J/m$^2$; the measured fracture toughness increased up to 315J/m$^2$ for 15mm of propagation of a kink-band. However, the values computed for the fracture toughness using the CC specimen do not account for the energy dissipated by the de-
lamination that accompanied the propagation of the kink-band. In addition, the delamination does not allow the accurate measurement of the tip of the kink-band. Therefore, it is concluded that these difficulties render the CC test method unsuitable to generate R-curves and that an alternative test method or test procedure should be developed.

The same considerations apply for Glass fiber specimens. In addition, these specimens showed additional failure mechanism that render the use of the CT/CC specimens unsuitable to measure the fracture toughness of fiber-dominated failure mechanisms. In this case, a new test method should be proposed.

Having defined the relevant parameters for the cohesive laws, a progressive damage model was used to simulate failure in IM7/8552 double-shear bolted joints.

Using a parametric Python script, three FE models were defined: bearing, tension, and shear models. It was concluded that the 3D numerical simulations can predict with accuracy both the value of the first peak load and the damage zones.

However, the computational cost of a 3D numerical simulation is not suitable for industrial applications because each model takes about 3 days to run using a cluster of 32 cpus. Therefore, a simplified two-dimensional method was developed.

The two-dimensional numerical model computes the damage of the composite using a two-dimensional version of the previously proposed failure criteria. Since this FE model is generally applied to the entire structure, or to a sub-assembly, the model also considers the onset of the damage in the other connected parts, or in the bolts. If the connected parts are metallic, the failure index is simply calculated using the von-Mises stress. The failure indexes for the bolts (shear, traction, combined shear/traction) are defined following the NF EN 1993-1-8 norm. To validate the model, a mock-up of a real train connection was manufactured and the experimental data was compared with experimental results. It was concluded that the two-dimensional model proposed can accurately predict the onset of the damage in the connected parts of the sub-assembly.

8.2 Future work

The following topics will be the subject of further studies:

- a fully three-dimensional progressive damage model will be defined to be consistent with the three-dimensional failure criteria already proposed;
- a methodology to evaluate the R-curve in compression will be developed as no methodologies to evaluate the energy release rate in compression currently available;
- a methodology to homogenize the cohesive laws on the laminate thickness will be proposed. This will allow the use of one brick/shell element for the entire
thickness of the laminate and will enable the use of the progressive damage model at the laminate level, thus reducing the computational cost.
Bibliography


Appendix A

Crack detection algorithm

1 clear all
2
3 % Algorithm to find crack length
4 % G. Catalanotti, 2009
5
6
7
8 % load the data
9 string_path=strcat(dir,'\',specimen,'\rawdata.mat');
10 eval(['load -mat ' ' string_path ']);
11 string_path=strcat(dir,'\',specimen,'\misc.mat');
12 eval(['load -mat ' ' string_path ']);
13 string_path=strcat(dir,'\',specimen,'\matdata.mat');
14 eval(['load -mat ' ' string_path ']);
15 dir_res=[dir ' ' 'Mat_Res' ' ' specimen ' ' specimen];
16
17 % displacement of the image
18 stages = length(aramis2D.displ);
19
20 % find size of image
21 Size=size(aramis2D.displ(1,1).X);
22 m=Size(1,1);
23 n=Size(1,2);
24
25 % preallocated variables
26 K_all= struct;
27 length_windows=zeros(stages,1);
28 length_J_windows=0;
29
30 % for loop for all stages of the test
31 for J = 1:stages
32
33 % load displacement along x for stage J
34 displ_x = aramis2D.displ(1,J).X;
35
36 % load displacement along y for stage J
37 displ_y = aramis2D.displ(1,J).Y;
38
39 % find the dimensions of the matrix
40 Size=size(aramis2D.displ(1,J).X);
41 m=Size(1,1);
42 n=Size(1,2);
% find the matrix of the displacements
displ = (displ_x.^2+displ_y.^2).^0.5;

% zeros
n_zeros=zeros(m,1);
m_zeros=zeros(1,n+1);

% the algorithm is implemented here
% this variable are defined concatenating the displacement
% values with a empty row and a empty colon
% if displacement matrix has dimension m*n these
% matrices have dimensions (m+1)*(n+1)
displ_A= vertcat(horzcat(displ,n_zeros),m_zeros)./4;
displ_B= vertcat(horzcat(n_zeros,displ),m_zeros)./4;
displ_C= vertcat(m_zeros,horzcat(displ,n_zeros))./4;
displ_D= vertcat(m_zeros,horzcat(n_zeros,displ))./4;

matr_transf=ones(m+1,n+1);
matr_transf(1:m+1, 1)=2;
matr_transf(1:m+1, n+1)=2;
matr_transf(1, 1:n+1)=2;
matr_transf(m+1, 1:n+1)=2;
matr_transf(1, 1)=4;
matr_transf(1, n+1)=4;
matr_transf(m+1, 1)=4;
matr_transf(m+1, n+1)=4;

% grid_values is the matrix that contains the values
% of the displacements for the vertex of the patterns
grid_values=(displ_A+displ_B+displ_C+displ_D).*matr_transf;

% modulus of displacement for vertex of the patterns
displ_CA=abs((displ_C-displ_A));
displ_DB=abs((displ_D-displ_B));

% compute matrix K (criterion)
K=max(displ_CA,displ_DB);

% threshold
alpha=2.0;

K=K;
% apply criterion
K_T = -1 if value of K for the pattern is NaN
% in this case the material is completely damaged
K_NaN=isnan(K);
K(isnan(K))=-1;
% K_T equal to zero if the value of K is less
% the average for the image. In case the image
% material is undamaged
K_(isnan(K))=0;
avg_K=sum(sum(K))/sum(sum(K_NaN));
K(K<alpha*avg_K&(K>0)) = 0;
K(K>=alpha*avg_K) = 1;
% delete solutions for the edge of the image
dedge=5;
K(1:edge,:) = 0;
K(m-edge:m,:) = 0;
K(:,1:edge) = 0;
surf(K)
set(gca, 'CLim', [-1.0, 1.0]);
colorbar
zlim([-1 +1])
xlabel(['stage = ',num2str(J)])
ylabel('K_T')
% eval(['print -dpng ' dir_res '-' num2str(J,'%03.4g')])
M(J) = getframe;
[row, col] = find(K==0);
if isempty(col);
    length_J_windows=0;
else
    length_J_windows=n-col(1);
end
K_all(1,J).K= K;
% compute the crack length (unity of measure is in this
% case the length of the size of the pattern= 15mm)
length_windows(J)= length_J_windows;
if length_J_windows>0.95*n
    break
end
photo_number=(1:1:J)';
crack=zeros(J,1);
% apply conversion factor to obtain the crack length in mm
pixel_window=15;
for I = 1:J
    crack(I,1)=(length_windows(I)-... length_windows(1))*pixel_window*... converfactor+a0;
end
filename_movie=[specimen '.avi'];
% movie2avi(M, filename_movie)
% find time of the stages.
% this is useful for the correlation
% of the data from the MTS to find the
% function load=load(crack)
time_photo=photo_number.*deltaT;
plot(time_photo,crack,'-o');
xlabel('time [s]');
ylabel('crack length [mm]');
% save data
string_path=strcat(dir,'\',specimen,'\Aramis.mat');
save(string_path, 'time_photo', 'crack')
Appendix B

J-Integral algorithm

1  % J-integral calculation algorithm
2  % G. Catalanotti, 2009
3  clear all
4
5  Ex=90650;
6  Ey=90650;
7  v_xy=0.32;
8  Gxy=5290;
9
10  % name of the project
11  specimens=['CT_C_1'; 'CT_C_2'; 'CT_C_3'; ];
12  % name of the directory
13  dir='F:\PhD\CT-CC\Fracture';
14
15  for JJ=1:3,
16    specimen=specimens(JJ,:);
17    % load the data
18    string_path=strcat(dir,'\',specimen,'\rawdata.mat');
19    eval(['load -mat ' ' ' string_path ' ' ]);
20    string_path=strcat(dir,'\',specimen,'\misc.mat');
21    eval(['load -mat ' ' ' string_path ' ' ]);  
22    string_path=strcat(dir,'\',specimen,'\matdata.mat');
23    eval(['load -mat ' ' ' string_path ' ' ]);  
24    string_path=strcat(dir,'\',specimen,'\Aramis.mat');
25    eval(['load -mat ' ' ' string_path ' ' ]);  
26  end
27
28  % displacement of the image
29  stages = length(aramis2D.displ);
30  % find size of image
31  Size=size(aramis2D.displ(1,1).X);
32  m=Size(1,1);
33  n=Size(1,2);
34  % contour for J integral
35  ji_vect=zeros(stages,1);
36  % homogeneize matrix
37  SIZES=zeros(stages,2);
38  for J = 1:stages
39    SIZES(J,:)=size(aramis2D.displ(1,J).X);
40    end
41  M=min(SIZES(:,1)) ; N=min(SIZES(:,2)) ;
42  % four loop for all stages of the test
43  for J = 2:stages
44    ...
APPENDIX B. J-INTEGERAL ALGORITHM

% load displacement along x for stage J
disp1_x = aramis2D.displ(1,J).X;

% load displacement along y for stage J
disp1_y = aramis2D.displ(1,J).Y;

% find the dimensions of the matrix
Size=size(aramis2D.displ(1,J).X);
m=Size(1,1);
n=Size(1,2);

% find the matrix of the displacements
disp1= (disp1_x.^2+disp1_y.^2).^0.5;

% zeros
n_zeros=zeros(m,1);
m_zeros=zeros(1,n+1);

% compute the J integral directly from ARAMIS
delt=2;
N_=N-delt;
M_=M-delt;

% coordinate of top-left vertex of the image
vm=int8((m-M_)/2);
vn=int8((n-N_)/2);

% contour is the matrix that has the value 1
% in the points of the contours
% used for the calculation of the J integral
top=zeros(m,n);
top(vm,vn+1:vn+N_)=ones(1,N_);
top_left=zeros(m,n);
top_left(vm,vn)=1;
top_right=zeros(m,n);
top_right(vm,vn+N_+1)=1;

bot=zeros(m,n);
bot(vm+M_,vn+1:vn+N_)=ones(1,N_);
bot_left=zeros(m,n);
bot_left(vm+M_,vn)=1;
bot_right=zeros(m,n);
bot_right(vm+M_,vn+N_+1)=1;

right=zeros(m,n);
right(vm+1:M_-1+vm,vn)=ones(M_-1,1);
left=zeros(m,n);
left(vm+1:vm,vn+N_+1)=ones(M_-1,1);

top_left=top+bot_left+bot_right+...
top_left+top_right+right+left;

Dx=top-bot-1/2*bot_left-1/2*bot_right+...
+1/2*top_left+1/2*top_right;
Ds=Dx*converfactor;

Dy=left-right+1/2*bot_left-1/2*bot_right+...
+1/2*top_left+1/2*top_right;
Dy=Dy*converfactor;

eps_x = aramis2D.strain(1,J).X;
eps_y = aramis2D.strain(1,J).Y;
eps_xy = aramis2D.strain(1,J).S*100;
\[ \sigma_x = \varepsilon_x \cdot E_y \cdot \tau_{xy} \cdot E_x \cdot \varepsilon_y; \]
\[ \sigma_y = \varepsilon_y \cdot E_y \cdot \tau_{xy} \cdot E_x \cdot \varepsilon_x; \]
\[ \tau_{xy} = G_{xy} \cdot \varepsilon_{xy}; \]
\% \quad \text{compute derivatives}
\[ (d_{\text{displ}_x}, FY) = \text{gradient}(\text{displ}_x); \]
\[ (d_{\text{displ}_y}, FY) = \text{gradient}(\text{displ}_y); \]
\[ D_x = D_y = \text{Inf}; \]
\[ d_{\text{displ}_x}_x = -\frac{d_{\text{displ}_x}}{\text{converfactor}}; \]
\[ d_{\text{displ}_y}_y = \frac{d_{\text{displ}_y}}{\text{converfactor}}; \]
\% \quad \text{compute Ds}
\[ D = (D_x^2 + D_y^2)^{0.5}; \]
\% \quad \text{compute directions cosines}
\[ n_1 = \frac{D_y}{D}; \]
\[ n_2 = -\frac{D_x}{D}; \]
\% \quad \text{compute tractions}
\[ T_1 = \sigma_x \cdot n_1 + \tau_{xy} \cdot n_2; \]
\[ T_2 = \tau_{xy} \cdot n_1 + \sigma_y \cdot n_2; \]
\[ t_{\text{du}_x}_x = T_1 \cdot d_{\text{displ}_x}_x \cdot D; \]
\[ t_{\text{du}_y}_y = T_2 \cdot d_{\text{displ}_y}_y \cdot D; \]
\[ j_i = \text{Wdy} - t_{\text{du}_x}_x \cdot D; \]
\[ j_i = j_i \cdot \text{contour}; \]
\[ j_i = (\text{sum} \cdot (\text{sum}(j_i))); \]
\[ j_i = \text{vector}(J, 1) = j_i; \]
end

\% \quad j_i \text{ vector refers to the laminate as the thickness of zero degree laminae is}
\% \quad \text{half of the total thickness, to find the } j_i \text{-integral for zero-degree laminae}
\% \quad \text{this value has to be multiply by } 2
\% \quad j_i \text{zero} = j_i \text{vector} \cdot 2.0;
incr\_crack = \text{crack} - \text{ones(size}(\text{crack})) \cdot \text{crack}(1, 1);
eval(['\text{\text{'crack, } specimen' = crack'])
eval(['\text{\text{'G, specimen' = j_i\text{zero}')])
eval(['\text{\text{'incr\_crack, } specimen' = incr\_crack'])
end
plot(incr\_crack\_CT\_C\_4, G\_CT\_C\_4, 'ro', ...
    incr\_crack\_CT\_C\_2, G\_CT\_C\_2, 'bs', ...
    incr\_crack\_CT\_C\_3, G\_CT\_C\_3, 'ks')
Appendix C

Parametric Model for J-Integral calculation

Filename: MAIN.py

```python
# parametric model to calculate J-Integral in CT specimen
# G. Catalanotti, 2009

import os

# name of working directory
os.chdir(r'D:/results_JI/IM78552/CC_T=0')

# maximum and minimum value of effective crack length
a_eff_max=45.0
a_eff_min=15.0

# increment in the crack length
INCR=.25

incr=(a_eff_max-a_eff_min)/INCR+1.0
incr=int(incr)

for incr_ in range(incr):
    a_eff=a_eff_min+incr_*INCR
    print a_eff
    execfile('main_.py')
    mdb.Job(name=string_name, model=string_name)
    mdb.jobs[string_name].submit(consistencyChecking=OFF)
    mdb.jobs[string_name].waitForCompletion()
    execfile('read_odb.py')
```

Filename: main_.py

```python
# CT_CC model
# G. Catalanotti, 2009
# symmetric model

L
------------------------
|                      |
|                      |
|  dl (d)             |
```
APPENDIX C. PARAMETRIC MODEL FOR J-INTEGRAL CALCULATION

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>d2</td>
<td></td>
<td>a</td>
</tr>
</tbody>
</table>

H=30.0
L=65.0
D=12.5
d1=7.5+12.5/2.0
d2=10.0+12.5/2.0

string_material='IM78552'
thickness_plies=32*[0.125]
angle_plies=8*[0, 90, 90, 0]
size_el=0.5

# type 'CT' or 'CC'
type_specimen='CC'

executeOnCaeStartup()

mdb.Model(name='Model-A')

s = mdb.models['Model-A'].ConstrainedSketch(name='__profile__', sheetSize=200.0)
g, v, d, c = s.geometry, s.vertices, s.dimensions, s.constraints
s.setPrimaryObject(option=STANDALONE)
s.Line(point1=(0.0, 0.0), point2=(L-a, 0.0))
s.Line(point1=(L-a, 0.0), point2=(L-a, H))
s.Line(point1=(L-a, H), point2=(0.0, H))
s.Line(point1=(0.0, H), point2=(~a, H))
s.Line(point1=(~a, H), point2=(~a, 0.0))
s.Line(point1=(~a, 0.0), point2=(0.0, 0.0))
s.CircleByCenterPerimeter(center=(~(a-d1), d2), point1=(~(a-d1), d2+d/2.0))
p = mdb.models['Model-A'].Part(name='Part-1', dimensionality=THREE_D, type=DEFORMABLE_BODY)
p.BaseShell(sketch=s)
s.unsetPrimaryObject()
p = mdb.models['Model-A'].parts['Part-1']
del mdb.models['Model-A'].sketches['__profile__']

# finish geometry

execfile('library_materials.py')
mdb.models['Model-A'].Material(name=string_material, description='')
exec('MatId='+string_material)
mdb.models['Model-A'].materials[string_material].Elastic(type=ENGINEERING_CONSTANTS,
221
table=((MatId[’E1’], MatId[’E2’],MatId[’E2’], MatId[’v12’], MatId[’v12’], MatId[’v23’],
MatId[’G12’], MatId[’G23’], MatId[’G23’]), ))

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mdb.models[’Model-A’].materials[string_material].Density(table=((MatId[’density’], ), ))
mdb.models[’Model-A’].materials[string_material].Conductivity(table=((1.0, ), ))
mdb.models[’Model-A’].materials[string_material].SpecificHeat(table=((1.0, ), ))
mdb.models[’Model-A’].materials[string_material].Expansion(type=ORTHOTROPIC, table=((MatId[’alpha1’],
MatId[’alpha2’], MatId[’alpha2’]), ))

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85
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87

# definition of section
Layup=[]

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89
90
91
92
93

for i in range(1,len(thickness_plies)+1):
exec("sectionLayer"+str(i)+"=section.SectionLayer(material=’"+string_material+"’,
thickness="+str(thickness_plies[i-1])+
", orientAngle="+str(angle_plies[i-1])+",
numIntPts=3, plyName=’ply_"+str(i)+" ’) " )
exec("Layup.append(sectionLayer"+str(i)+")"
)

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95
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mdb.models[’Model-A’].CompositeShellSection(name=’Section-1’, preIntegrate=OFF,
poissonDefinition=DEFAULT, thicknessModulus=None, temperature=GRADIENT,
useDensity=OFF, nodalThicknessField=’’, integrationRule=SIMPSON, layup=tuple(Layup))

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101
102

# assign sections
p = mdb.models[’Model-A’].parts[’Part-1’]
f = p.faces

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104
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region = regionToolset.Region(faces=f)
p.SectionAssignment(region=region, sectionName=’Section-1’, offset=0.0)

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# mesh
p.seedPart(size=size_el, deviationFactor=0.1)
p = mdb.models[’Model-A’].parts[’Part-1’]
f = p.faces
pickedRegions = f
p.setMeshControls(regions=pickedRegions, elemShape=QUAD, algorithm=MEDIAL_AXIS)
p = mdb.models[’Model-A’].parts[’Part-1’]

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elemType1 = mesh.ElemType(elemCode=S8R, elemLibrary=STANDARD)
elemType2 = mesh.ElemType(elemCode=STRI65, elemLibrary=STANDARD)

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119
120
121
122

p = mdb.models[’Model-A’].parts[’Part-1’]
f = p.faces
faces = f
pickedRegions =(faces, )
p.setElementType(regions=pickedRegions, elemTypes=(elemType1, elemType2))

123
124

p.generateMesh()

125
126
127
128
129
130

# assembly
a = mdb.models[’Model-A’].rootAssembly
a.DatumCsysByDefault(CARTESIAN)
p = mdb.models[’Model-A’].parts[’Part-1’]
a.Instance(name=’Part-1-1’, part=p, dependent=ON)

131
132
133

# steps
mdb.models[’Model-A’].StaticStep(name=’thermo_mechanic’, previous=’Initial’)

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137
138
139
140

# symmetry
a = mdb.models[’Model-A’].rootAssembly
e1 = a.instances[’Part-1-1’].edges
edges1 = e1.getSequenceFromMask(mask=(’[#4 ]’, ), )
region = regionToolset.Region(edges=edges1)
mdb.models[’Model-A’].YsymmBC(name=’BC-1’, createStepName=’thermo_mechanic’,


region

# constrain displacement on x
a1 = mdb.models['Model-A'].rootAssembly
v1 = a1.instances['Part-1-1'].vertices
verts1 = v1.getSequenceFromMask(mask=('[#1 ]', ),
region = regionToolset.Region(vertices=verts1)
mdb.models['Model-A'].DisplacementBC(name='BC-3', createStepName='thermo_mechanic',
region=region, u1=0.0, u2=UNSET, u3=UNSET, ur1=UNSET, ur2=UNSET, ur3=UNSET,
amplitude=UNSET, fixed=OFF, distributionType=UNIFORM, fieldName='',
localCsys=NONE)

# static
if type_specimen=='CT':
    fact=1.0
else:
    fact=-1.0

a1 = mdb.models['Model-A'].rootAssembly
v1 = a1.instances['Part-1-1'].vertices
verts1 = v1.getSequenceFromMask(mask=('[#1 ]', ),
region = regionToolset.Region(vertices=verts1)
mdb.models['Model-A'].ConcentratedForce(name='Load-1',
createStepName='thermo_mechanic', region=region, cf2=fact, localCsys=None)

a = mdb.models['Model-A'].rootAssembly
v1 = a.instances['Part-1-1'].vertices
verts1 = v1.getSequenceFromMask(mask=('[#4 ]', ),
crackFront = regionToolset.Region(vertices=verts1)
a = mdb.models['Model-A'].rootAssembly
v1 = a.instances['Part-1-1'].vertices
v11 = a.instances['Part-1-1'].vertices
a.engineeringFeatures.ContourIntegral(name='Crack', symmetric=ON,
crackFront=crackFront, crackTip=crackTip,
extensionDirectionMethod=Q_VECTORS, qVectors=((v1[1], v11[2]),
midNodePosition=0.5, collapsedElementAtTip=NONE)
mdb.models['Model-A'].fieldOutputRequests['F-Output-1'].setValues(variables=(
    'S', 'E', 'U', 'JK'))
mdb.models['Model-A'].historyOutputRequests['H-Output-1'].setValues(variables=(
    'ALLAE', 'ALLCD', 'ALLMD', 'ALLLE', 'ALLFD', 'ALLIE', 'ALLID', 'ALLKE',
    'ALLKL', 'ALLPF', 'ALLQB', 'ALLSE', 'ALLSD', 'ALLVD', 'ALLWK', 'ETOTAL',
    'DBS', 'DBT', 'DBSF'))
mdb.models['Model-A'].HistoryOutputRequest(name='H-Output-2',
createStepName='thermo_mechanic', contourIntegral='Crack',
sectionPoints=DEFAULT, rebar=EXCLUDE, numberOfContours=25)

a_eff_s=str(a_eff)
ind=a_eff_s.index('.')
a_eff_s_1=a_eff_s[0:ind]
a_eff_s_2=a_eff_s[ind+1::]
a_eff_s_2_=''
for i in range(0,3):
    try:
        a_eff_s_2[a_eff_s_2.index('')]
    except IndexError:
        a_eff_s_2_+='0'}
a_eff_s=a_eff_s_1+'d'+a_eff_s_2

string_name=type_specimen+'_'+string_material+'_'+a_eff_s

mdb.models.changeKey(fromName='Model-A', toName=string_name)

#---> go to odb

FILENAME: library_materials.py

IN78552={'type':'composite',
'density':1.59e-9,
'E1':171420.,
'E2':9080.,
'G12':5290,
'v12':0.32,
'alp1':-5.5E-6,
'alp2':25.8E-6,
'XT': 2323.5,
'XP0':232.3,
'XC':1201,
'YT':160.2,
'Y0':199.8,
'ALPHA0':0.925,
'SL': 130.2,
'SLud': 92.3,
'GIC_F1': 31.5,
'GIC_FE': 50.0,
'GIC_M': 0.2774,
'GIC_M': 0.7879,
'GIC_PC': 106.3,
'GIC_M': 1.3092,
'beta1': 0.000,
'beta2': 0.005,
'DM': 0.0,
'Eta_viscous': 0.001,
'v23': 0.52,
'D23': 420000,  # check it
'GIC': 0.969,
'GIIC': 1.719,
'S3_o': 120.0,
'S1_o': 120.0,
'ETA': 1.2,
'Em': 120000.0,
'POISm': 0.3,
'del_ele': 0.9,
'Em': 90000,
'vm': 0.3,
'density_m': 1.59e-9,
'alp_m': 25.8E-6}

FILENAME: read_odb.py

import os

dir_path=os.getcwd()

odb = session.openOdb(name=dir_path+'\'+string_name+'.odb')

assembly = odb.rootAssembly

step=odb.steps

skey=step.keys()[0]

J_integral_contours=step[skey].historyRegions['ElementSet PIBATCH'].historyOutputs

J_integral_keys=J_integral_contours.keys()
import numpy as n
import matplotlib.pyplot as plt
import cPickle
import os
dir_path=os.getcwd()

files_dir=os.listdir(dir_path)

files_JI=[]

for file_name_ in files_dir:
    if file_name_[-3::]=='.JI':
        files_JI.append(file_name_)

print files_JI

CL0=[] #
JI0=[] #
JI_all0=[]

for file_name_ in files_JI:
    FILE = open(file_name_, 'r')
    JI_ = cPickle.load(FILE)
    JI_all0.append(JI_)

ind1=file_name_[:ind1].find('.,')
ind2=file_name_[:ind1].find('_')
str_crack=file_name_[:,ind1+ind2:].find('d')

crack=float(str_crack['ind_d']+'.'+str_crack[ind+ind_d::])

CL0.append(crack)

JI_orig=[]

while JI_[[-1]<0.0:
    JI_.remove(JI_[[-1])

JI0.append(JI_[[-1])

print '----------------'
print 'crack length='
print crack
print 'J-integral value='
print JI_[[-1]
print 'contours JI'
print JI_orig
print 'modified contours JI'
print JI_
FILE.close()

# graphs
plt.plot(CL0, JI0, '-o')
plt.ylabel('Energy Release')
plt.show()

#
data_ = []
data_.append(CL0)
data_.append(JI0)
data = n.array(data_)
data = n.transpose(data)
n.savetxt('JI_data.txt', data, fmt='%12.6G')

# import cPickle
FILE = open('CL_JI', 'w')
cPickle.dump(CL0, FILE)
cPickle.dump(JI0, FILE)
FILE.close()
Appendix D
2D Model for Industrial Application

/* 2D failure model for industrial application
* G. Catalanotti 2010

/NDPR
/OUT, output.txt
/COM, --------------- ANALYSIS RESULTS ---------------
/COM,
/COM, | node | F_Index | Shear | Tension | Combined | L Pres |
/COM,
/OUT
/UIS, MSGPOP,3
!UIS, ABORT, OFF

*SET, ALL
/UIS, ABORT, OFF

*PARSAY, ALL,'savedparameter', 'txt', ' !initialize parameters file
/CLEAR !clear database
/RESUME ! resume original job
/ALLSEL, ALL

/INPUT, 'zinput', 'txt', 0 ! import input data

! find all the coupled nodes
*GET, MAX_coupled_number, CP, , MAX
/NSEL, S, CP, 1, MAX_coupled_number, 1
*GET, highest_node, NODE, NUM, MAX, , ,
*DIM, all_nodes, ARRAY, highest_node, , ,
*GET, all_nodes, NODE, , NSEL, , , 2

! number of selected nodes
*GET, n_number, NNODE, COUNT, , ,
*DIM, coupled_nodes, ARRAY, n_number, , ,

ff = 0
*DO, I, 1, highest_node, 1
  *IF, all_nodes(I, 1, 1), EQ, 1, THEN
    ff = ff + 1
  coupled_nodes(ff, 1, 1) = I
  *ENDIF
*ENDDO
*DIM, EL_NUMBER, ARRAY, n_number, , ,
APPENDIX D. 2D MODEL FOR INDUSTRIAL APPLICATION

*DIM,TYPE_NUMBER,ARRAY,n_number,, , ,
*DIM,CS_NUMBER,ARRAY,n_number,, , ,
*DIM,REALC_NUMBER,ARRAY,n_number,, , ,
*DIM,MAT_NUMBER,ARRAY,n_number,, , ,
*DIM,SEC_NUMBER,ARRAY,n_number,, , ,
!
*DO,I,1,n_number
  NSEL,S, , , coupled_nodes(I,1,1)
  ESLN,S
  ESEL,R,ENAM,,element_used
  *GET,highest_elem,ELEM,,NUM,MAX, , , ,
  ALLSEL,ALL
  *
  *GET,TN,ELEM,highest_elem,ATTR,TYPE
  *GET,CSN,ELEM,highest_elem,ATTR,ESYS
  *GET,RN,ELEM,highest_elem,ATTR,REAL
  *GET,MN,ELEM,highest_elem,ATTR,MAT
  *
  EL_NUMBER(I,1,1)=highest_elem
  TYPE_NUMBER(I,1,1)=TN
  CS_NUMBER(I,1,1)=CSN
  REALC_NUMBER(I,1,1)=RN
  MAT_NUMBER(I,1,1)=MN
  *
  *ENDDO

!* get highest defined section in the model
*GET,highest_section, SECP, NUM, MAX
*DO, I, 1, n_number, 1
  *DO, J, 1,highest_section
    ESEL,S, , , EL_NUMBER(I,1,1)
    ESL,J,SEC,,J
    *GET,highest_elem,ELEM,,NUM,MAX, , , ,
    *IF,highest_elem,EQ,0,THEN
      SEC_NUMBER(I,1,1)=J
    *ENDIF
  *ENDIF
  *ENDDO
*ENDDO
!
PI = 3.14159

*DIM,DIAMETERS,TABLE,n_number,1,1,,
*TREAD,DIAMETERS,'input_diameter','txt',' ',1,
*DIM,FURS,TABLE,n_number,1,1,,
*TREAD,FURS,'input_fur','txt',' ',1,
!
get jobname
*GET,jobname,ACTIVE, ,JOBNAM  ! get current jobname
*DO, coupled_nodeI, 1, n_number, 1
  PARSALL,'savedparameter','txt',
coupled_node=coupled_nodes(coupled_nodeI,1,1)
diameter=DIAMETERS(coupled_node)
fur=FURS(coupled_node)
ALLSEL,ALL
CSYS,0
allsel,all
*GET,highest_node,NODE,,NUM,MAX, , ,
*DIM,all_nodes,ARRAY,highest_node,, , ,
*GET,highest_elem,ELEM,,NUM,MAX,,
*DIM,all_elem,ARRAY,highest_elem,,
*DIM,all_elem2,ARRAY,highest_elem,,
*DIM,all_elem3,ARRAY,highest_elem,,
*DIM,before_nodes,ARRAY,highest_node,,
*DIM,after_nodes,ARRAY,highest_node,,

! select coupled node
NSEL,S,,coupled_node

*GET,high_elem_4,ELEM,,NUM,MAX,,
*GET,elem_area,ELEM,high_elem_4,AREA

dim=0.5*sqrt(elem_area)

half_side=INT(2*diameter/dim)

*IF,half_side,EQ,0,then
  half_side=1
*ENDIF

! select coupled node
NSEL,S,,coupled_node

*DO,I,1,half_side,1
  ! selected node in array
  VGET,before_nodes,NODE,,NSEL,,2
  ! select elements attached to coupled node
  ESLN,S
  ! select nodes attached to elements
  NSEL,S
  VGET,after_nodes,NODE,,NSEL,,2
*ENDDO

*DIM,contour,ARRAY,highest_node,,
*VOPER,contour,after_nodes,SUB,before_nodes,,
*VSCFUN,contour_n,SUM,contour
contour_n=contour_n/2

*DIM,contour_nodes,ARRAY,contour_n,,
NSEL,NONE

K=0

*DO,I,1,highest_node,1
  *IF,contour(I,1,1),EQ,2,THEN
    NSEL,A,,I
    K=K+1
  ENDIF
*ENDO

*DIM,contour_x,ARRAY,contour_n,,
*DIM,contour_y,ARRAY,contour_n,,
*DIM,contour_z,ARRAY,contour_n,,

*DO,I,1,contour_n,1
  *GET,xcoord,NODE,contour_nodes(I,1,1),LOC,X
  *GET,ycoord,NODE,contour_nodes(I,1,1),LOC,Y
  *GET,zcoord,NODE,contour_nodes(I,1,1),LOC,Z
APPENDIX D. 2D MODEL FOR INDUSTRIAL APPLICATION

174 contour_x(I,1,1)=xcoord
175 contour_y(I,1,1)=ycoord
176 contour_z(I,1,1)=zcoord
177 !
178 *SET,xcoord
179 *SET,ycoord
180 *SET,zcoord
181 *ENDDO
182
183 !!!!! sort nodes
184
185 NSEL,S,,coupled_node !select coupled node
186 ESEL,S !select attached node
187 ESEL,R,ENAME,,element_used
188
189 *GET,elem_sys_n,ELEM,,NUM,MAX,.,.,.
190 *GET,elem_sys,ELEM,elem_sys_n,ATTR,ESYS ! get element coordinate system
191
192 *GET,max_coord_sys,CDSY,,NUM,MAX
193 *IF,max_coord_sys,LT,10,THEN
194 max_coord_sys=10
195 *ENDIF
196
197 max_coord_sys=max_coord_sys+1
198 ! define coordinate system concord to the nodes
199 CSYS,0,
200 DSYS,0,
201
202 *GET,center_x,NODE,coupled_node,LOC,X
203 *GET,center_y,NODE,coupled_node,LOC,Y
204 *GET,center_z,NODE,coupled_node,LOC,Z
205
206 CSYS,0,
207 DSYS,0,
208 CLOCAL, max_coord_sys,0, center_x,center_y, center_z
209
210 CSYS,0,
211 DSYS,0,
212 CLOCAL, max_coord_sys+1,1, center_x,center_y, center_z
213
214 CSYS,max_coord_sys+1,
215 DSYS,max_coord_sys+1 ,
216
217 *DIM,contour_angle,ARRAY,contour_n,,, , ,
218 *DO, I, 1, contour_n, 1
219  *GET,angle_coord,NODE,contour_nodes(I,1,1),LOC,Y
220  *IF,angle_coord,LT,0,THEN
221  contour_angle(I,1,1)=angle_coord+360
222  *ELSE
223  contour_angle(I,1,1)=angle_coord
224  *ENDIF
225  *SET,angle_coord
226 *ENDDO
227
228 !sort_nodes
229 *DIM,key_points,ARRAY,contour_n,,, , ,
230 *DIM,key_points_x,ARRAY,contour_n,,, , ,
231 *DIM,key_points_y,ARRAY,contour_n,,, , ,
232 *DIM,key_points_z,ARRAY,contour_n,,, , ,
233
234 *VSCFUN,index_min_angle,LMIN,contour_angle
235 *VSCFUN,index_max_angle,LMAX,contour_angle
*IF,(360-contour_angle(index_max_angle,1,1)),LT,contour_angle(index_min_angle,1,1),THEN
  contour_angle(index_max_angle,1,1)=contour_angle(index_max_angle,1,1)-360
ENDIF

*DO, I, 1, contour_n, 1
  *VSCFUN,index_min_angle,LMIN,contour_angle
  key_points(I,1,1)=index_min_angle !contour_nodes(index_min_angle,1,1)
  key_points_x(I,1,1)=contour_x(index_min_angle,1,1)
  key_points_y(I,1,1)=contour_y(index_min_angle,1,1)
  key_points_z(I,1,1)=contour_z(index_min_angle,1,1)
  contour_angle(index_min_angle,1,1)=361
*ENDDO

CSYS,0
!
*GET,center_x,NODE,coupled_node,LOC,X
*GET,center_y,NODE,coupled_node,LOC,Y
*GET,center_z,NODE,coupled_node,LOC,Z

DSYS,0
section=SEC_number(coupled_nodeI,1,1)

*IF,section,NE,0,THEN
  FLAG='C'
  ! numero layer
  *GET, num_layer, SHEL, section, PROP, NLAY
  ! thickness
  *GET, tot_thick, SHEL, section, PROP, TTHK
  !
  *DIM,thick_layer,ARRAY,num_layer,...,,
  *DIM,ori_layer,ARRAY,num_layer,...,,
  *DIM,mat_layer,ARRAY,num_layer,...,,

  *DO, I,1,num_layer,1
    *GET, thick_layer_i, SHEL, section,LAYD, I, THIC
    *GET, ori_layer_i, SHEL, section,LAYD, I, ANGL
    thick_layer(I,1,1)=thick_layer_i
    ori_layer(I,1,1)=ori_layer_i
    mat_layer(I,1,1)=mat_layer_i
  *ENDDO
ELSE
  FLAG='H'
ENDIF
!
/POST1
INRES,ALL
FILE,'rst','
SET,LIST,999
SET,...,...,setnumb
ALLSEL,ALL
*
NSEL,S,...,coupled_node
*
RSYS, max_coord_sys
FSYS,RSYS,ALL
*
*GET,FX,FSUM,,ITEM,FX
*GET,FY,FSUM,,ITEM,FY
*GET,FZ,FSUM,,ITEM,FZ
*
*GET,ux_node,NODE,coupled_node,U,Z
FINISH
ALLSEL,ALL
APPENDIX D. 2D MODEL FOR INDUSTRIAL APPLICATION

```plaintext
/PREP7
EDELE,ALL
NDELE,ALL

*GET,max_elem_type,ETYP,1,NUM,MAX
*DG,I,1,max_elem_type,1
*IF,I,NE,TYPE_NUMBER(coupled_nodeI,1,1),THEN
ETDEL,I
*ENDIF
*ENDDO

ALLSEL,ALL

*SET,AFTER_NODES
*SET,ALL_ELEM
*SET,ALL_NODES
*SET,BEFORE_NODES
*SET,CONTOUR
*SET,ALL_ELEM2
*SET,ALL_ELEM3

FINISH
PARSAV,ALL,'savedparameter','.txt','

*SET,ALL

CDWRITE,DB,,'cdb',,'',''
/CLEAR

/INPUT,'savedparameter','.txt',0
/Prep7

rad=diameter/2
rad_bolt=1.7*rad

meshsize=(rad_bolt-rad)/10

*DO, I, 1, contour_n, 1
  K,I,contour_x(I,1,1),contour_y(I,1,1),contour_z(I,1,1),
*ENDDO

*DIM,key_points_all,ARRAY,contour_n+1,1,1, ,
*DO, I, 1, contour_n, 1
  key_points_all(I,1,1)=key_points(I,1,1)
*ENDDO
key_points_all(contour_n+1,1,1)= key_points(1,1,1)

!* b spline
*DO, J, 0, 7, 1
  FLST,3,half_side+1,3
  *DO, I, 1, half_side+1, 1
    FITEM,3, key_points_all(I+J*half_side,1,1)
  *ENDDO
BSPLIN, ,P51X
*ENDDO

K,1000,center_x,center_y,center_z,
!
```
CSPK,11,0,1000,key_points(1,1,1),key_points(2,1,1),1,1,
! cylindrical
CSPK,12,1,1000,key_points(1,1,1),key_points(2,1,1),1,1,
DSYS,12
CSYS,12
*GET,k_y1,KP,key_points(1,1,1),LOC,Y
*GET,k_y2,KP,key_points(1+contour_n/4,1,1),LOC,Y
*GET,k_y3,KP,key_points(1+2*contour_n/4,1,1),LOC,Y
*GET,k_y4,KP,key_points(1+3*contour_n/4,1,1),LOC,Y
K,1001,rad,k_y1,0
K,1002,rad,k_y2,0
K,1003,rad,k_y3,0
K,1004,rad,k_y4,0
*GET,k_y1a,KP,key_points(1+contour_n/8,1,1),LOC,Y
*GET,k_y2a,KP,key_points(1+contour_n/4+contour_n/8,1,1),LOC,Y
*GET,k_y3a,KP,key_points(1+2*contour_n/4+contour_n/8,1,1),LOC,Y
*GET,k_y4a,KP,key_points(1+3*contour_n/4+contour_n/8,1,1),LOC,Y
K,1005,rad,k_y1a,0
K,1006,rad,k_y2a,0
K,1007,rad,k_y3a,0
K,1008,rad,k_y4a,0
LARC, 1001, 1002, 1005
LARC, 1002, 1003, 1006
LARC, 1003, 1004, 1007
LARC, 1004, 1001, 1008
K,2001,rad_bolt,k_y1,0
K,2002,rad_bolt,k_y2,0
K,2003,rad_bolt,k_y3,0
K,2004,rad_bolt,k_y4,0
K,2005,rad_bolt,k_y1a,0
K,2006,rad_bolt,k_y2a,0
K,2007,rad_bolt,k_y3a,0
K,2008,rad_bolt,k_y4a,0
LARC, 2001, 2002, 2005
DSYS,0
CSYS,0
LSTR, 2001, key_points(1,1,1)
LSTR, 2002, key_points(1+contour_n/4,1,1)
LSTR, 2003, key_points(1+contour_n/2,1,1)
LSTR, 2004, key_points(1+3*contour_n/4,1,1)
LSTR, 1001, 2001
LSTR, 1002, 2002
LSTR, 1003, 2003
LSTR, 1004, 2004
LCCAT,1,2
LCCAT,3,4
LCCAT,5,6
LCCAT,7,8
APPENDIX D. 2D MODEL FOR INDUSTRIAL APPLICATION

```
434 AL, 1, 2, 18, 13, 17
435 AL, 18, 3, 4, 19, 14
436 AL, 19, 5, 6, 20, 15
437 AL, 20, 7, 8, 17, 16
438 AL, 22, 14, 23, 10
439 AL, 23, 15, 24, 11
440 AL, 24, 16, 21, 12
441 AL, 21, 13, 22, 9
442 ratio=5
443 LELSE, 21, meshsize, , , ratio, , , , 1
444 LELSE, 22, meshsize, , , ratio, , , , 1
445 LELSE, 23, meshsize, , , ratio, , , , 1
446 LELSE, 24, meshsize, , , ratio, , , , 1
447 ALSEL, ALL
448 Fboltx=FX
449 Fboltz=FY
450 Fboltz=abs(FZ)
451 pres_max=Fboltz/2/rad
452 beta=atan2(Fbolty,Fboltx)/PI*180
453 CDREAD, DB,, 'cdb',,, ,
454 TYPE, TYPE_NUMBER(coupled_nodeI,1,1)
455 MAT, MAT_NUMBER(coupled_nodeI,1,1)
456 REAL, , REALC_NUMBER(coupled_nodeI,1,1)
457 *IF, SEC_NUMBER(coupled_nodeI,1,1), NE, O, THEN
458 SECNUM, SEC_NUMBER(coupled_nodeI,1,1)
459 *ELSE
460 SECNUM, ,
461 *ENDIF
462 ESYS, max_coord_sys
463 CSYS,max_coord_sys+1,
464 K, 1200, rad*2, beta, ,
465 !*
466 CSKP,max_coord_sys+2,1,1000,1200,1003,1,1,
467 ! store results for all layer
468 KEYOPT, TYPE_NUMBER(coupled_nodeI,1,1),8,1
469 LCAT, 1, 2
470 LCAT, 3, 4
471 LCAT, 5, 6
472 LCAT, 7, 8
473 ndiv0=10
474 LELSE, 9, , ndiv0, , , , , 1
475 LELSE, 10, , ndiv0, , , , , 1
476 LELSE, 11, , ndiv0, , , , , 1
477 LELSE, 12, , ndiv0, , , , , 1
478 LELSE, 13, , ndiv0, , , , , 1
479 LELSE, 14, , ndiv0, , , , , 1
480 LELSE, 15, , ndiv0, , , , , 1
481 LELSE, 16, , ndiv0, , , , , 1
482 siz1=2*PI*rad/4/10*2
483 LELSE, 17, siz1, , , , , , , 1
484 LELSE, 18, siz1, , , , , , , 1
```
LESIZE,19,siz1, , , , , , ,1
LESIZE,20,siz1, , , , , , ,1

MSHKEY,1
AMESH,ALL

CSYS, max_coord_sys+2,
NSEL,S,LOC,X,rad
NSEL,R,LOC,Y,-90,+90

! pressure on hole's edge
*DEL,_FNCNAME
*DEL,_FNCMTID
*DEL,_FNCC SYS
*SET,_FNCNAME,'COSIN'
*DIM,_FNCC SYS,1
*SET,_FNCC SYS(1),pres_max
*DIM,%_FNCNAME%,TABLE,6,3,1,...,%_FNCC SYS%
!
! Begin of equation: pres_max*cos({Y})
*SET,%_FNCNAME%(0,0,1), 0.0, -999
*SET,%_FNCNAME%(2,0,1), 0.0
*SET,%_FNCNAME%(3,0,1), %_FNC_C1(1)%
*SET,%_FNCNAME%(4,0,1), 0.0
*SET,%_FNCNAME%(5,0,1), 0.0
*SET,%_FNCNAME%(6,0,1), 0.0
*SET,%_FNCNAME%(0,1,1), 1.0, -1, 10, 1, 3, 0, 0
*SET,%_FNCNAME%(0,2,1), 0.0, -2, 0, 1, 17, 3, -1
*SET,%_FNCNAME%(0,3,1), 1, 99, 0, 1, -2, 0, 0
! End of equation: pres_max*cos({Y})

SF,ALL,PRES, %COSIN%

ALLSEL,ALL

FINISH
/SOL

!* Antype,0
DELTIM,1,1,1
OUTRES,ERASE
OUTRES,NSOL,LAST
OUTRES,RSol,LAST
OUTRES,NLOA,LAST
OUTRES,STRS,LAST

submodelname=strcat('node_',CHRVAL(coupled_node))

!* select exterior lines
FLST,5,8,4,ORDE,2
ITEM,5,1
ITEM,5,-8
LSEL,S, , ,P81X
!* select attached nodes
NSLL,S,1
/PREP7
!* write NODE file
WWRITE, submodelname,'NODE'
allsel,all
FINISH

SAVE,submodelname,'db',
PARSAV,ALL,'savedparameter','txt','

!*
/CLEAR,START
/INPUT,'savedparameter','txt',,,0
RESUME,jobname,'db'
ALLSEL,ALL

/INPUT,'savedparameter','txt',,,0
/POST1
INRES,ALL
FILE,jobname,'rst',','
SET,LAST
CBDOF,submodelname,'NODE',' ',submodelname,'CBDO',' ',0,0
PARSAV,ALL,'savedparameter','txt','
!*
FINISH
/CLEAR

/INPUT,'savedparameter','txt',,,0
RESUME,submodelname,'db',,0,0

/INPUT,'savedparameter','txt',,,0
/SOL

/INPUT,submodelname,'CBDO',,,0
DCUM,ADD

!*
FINISH
command=strcat('/FILNAME,',submodelname)
SAVE,submodelname,'db','
PARSAV,ALL,'savedparameter','txt','
/clear
/FILNAME,submodelname,0
RESUME,submodelname,'db',,0,0
/SOLU
!*
SOLVE
!*
FINISH
!*
/POST1
SET,LAST
!*
! SEC_number(coupled_nodeI,1,1)
! FLAG='C'
! num_layer
! tot_thick
! thick_layer
! ori_layer
! mat_layer
! thick_layer_i
! mat_layer_i
! ori_layer_i
! thick_layer(I,1,1)
! ori_layer(I,1,1)
! mat_layer(I,1,1)
!*
! FLAG='H'
!*
*GET,High_elem_sub,ELEM,,NUM,MAX,,
*DIM,F_MATRIX,ARRAY,High_elem_sub,1,1,,
*DIM,F_FIBER,ARRAY,High_elem_sub,1,1,,
! select areas near the bolts
FLST,5,4,5,ORDE,2
ITEM,5,5
ITEM,5,-8
ASEL,S, , ,P51X
ESLA,S

*DIM,dam Elem,ARRAY,High Elem Sub,1,1, , ,
*VGET,dam Elem,ELEM, ,ESEL, , ,2

*DIM,dam Elem2,ARRAY,High Elem Sub,1,1, , ,
*VGET,dam Elem2,ELEM, ,ESEL, , ,2

ASL, S

! define F ETABLE (full of zeros)
ETABLE,F,U,X

*DIM,Mises_vect,ARRAY,High Elem Sub,1,1, , ,

*IF,SEC_number(coupled_nodeI,1,1),EQ,0,THEN
num_layer_ALL=1
*ENDIF

*DO,LAYER_Imo,1,num_layer_ALL,1
  
  RSYS,SOLU !RSYS, max_coord_sys
  !
  ETABLE,S11,S,X
  ETABLE,S22,S,Y
  ETABLE,S12,S,XY
  
  mat_string=strcat('material_',CHRVAL(mat_layer(LAYER_Imo,1,1)))
  /INPUT,mat_string,'txt',,, 0 ! import material parameters
  !
  ST_is=0.5*(2.0*sin(alpha0)**2-1.0)*SL_is/(sqrt(1.d0-sin(alpha0)**2)*sin(alpha0)*eta_L)
  YC_is=-SL_is*(2.0*cos(alpha0)**2-1.0)/(ETA_L*cos(alpha0)**2)
  eta_T=eta_L*ST_is/SL_is

  *DIM,S11vect,ARRAY,High Elem Sub,1,1, , ,
  *DIM,S22vect,ARRAY,High Elem Sub,1,1, , ,
  *DIM,S12vect,ARRAY,High Elem Sub,1,1, , ,
  *VGET,S11vect,ELEM, ,ETAB,S11, , ,2
  *VGET,S22vect,ELEM, ,ETAB,S22, , ,2
  *VGET,S12vect,ELEM, ,ETAB,S12, , ,2

  *DO,1,1,High Elem Sub,1
  
  s11=S11vect(1,1,1)
  s22=S22vect(1,1,1)
  s12=S12vect(1,1,1)

FMC=O
FMT=O
APPENDIX D. 2D MODEL FOR INDUSTRIAL APPLICATION

\[ \alpha = \alpha_{deg} \times \frac{\pi}{180.0} \]

\[ \sigma_N = \frac{s_{22}}{2.0} + \frac{s_{22}}{2.0} \cos(2.0 \times \alpha) \]

\[ \tau_T = -\sin(\alpha) \times \cos(\alpha) \times s_{22} \]

\[ \tau_L = s_{12} \cos(\alpha) \]

\[ F_{M_C} = \frac{\tau_T}{ST_{is} - \eta_T \times \sigma_N} + \frac{\tau_L}{SL_{is} - \eta_L \times \sigma_N} \]

\[ F_{M_T} = \frac{\sigma_N}{YT_{is}} + \frac{\tau_L}{SL_{is}} \]

\[ F_{IIFT} = \frac{s_{11}}{XT} \]

\[ \theta = \frac{\pi}{2} \]

\[ cc = \text{ABS}(SL_{is}/XC) \]

\[ aa = cc + \eta_L \]

\[ \text{IF} \left( 1.0 - 4.0 \times aa \times cc \right) \leq 0, \text{THEN} \]

\[ F_{MC_{CAULEY}} = 0.0 \]

\[ \text{ELSE} \]

\[ F_{MC_{CAULEY}} = 1.0 - 4.0 \times aa \times cc \]

\[ \text{ENDIF} \]

\[ \text{sq} = \sqrt{F_{MC_{CAULEY}}} \]

\[ \phi_{C} = \text{atan} \left( \frac{1.0 - \text{sq}}{2.0 \times aa} \right) \]

\[ \gamma_{mc} = 2.0 \times \phi_{C} \times XC / 2.0 / G_{12} \]

\[ \phi_0 = \phi_{C} - \gamma_{mc} \]

\[ s_{11} \theta = s_{11} \]

\[ s_{22} \theta = s_{22} \cos(\theta) \]

\[ s_{33} \theta = s_{22} \sin(\theta) \]

\[ s_{12} \theta = s_{12} \]

\[ s_{32} \theta = \sin(\theta) \times \cos(\theta) \]

\[ s_{31} \theta = -\sin(\theta) \times \cos(\theta) \times s_{22} \]

\[ s_{13} \theta = -\sin(\theta) \times s_{12} \]

\[ \gamma_{m} = (\phi_0 \times \theta) + G_{12} + \text{abs}(s_{12} \theta) \]

\[ \text{IF} \left( s_{12} \theta, \geq 0 \right) \text{THEN} \]

\[ \phi_0 = \phi_0 \times \theta + \gamma_{m} \]

\[ \text{ELSE} \]

\[ \phi_0 = \phi_0 \times \theta \]
s11PHI = cos(PHI) ** 2 * s11THETA + 2.0 * cos(PHI) * sin(PHI) * s12THETA + sin(PHI) ** 2 * s12THETA = s12PHI = sin(PHI) * cos(PHI) * s11THETA
s12PHI = sin(PHI) * cos(PHI) * s11THETA
s13PHI = cos(PHI) * s13THETA + sin(PHI) * s23THETA
s33PHI = s33THETA

FMCi = 0
FMTi = 0

*DO, alpha_deg, 0, 180, 30
alpha = alpha_deg * PI / 180.0

sigma_N = (s22PHI + s33PHI) / 2.0 + (s22PHI - s33PHI) / 2.0 * cos(2.0 * alpha) + s23PHI * sin(2.0 * alpha)
tau_T = sin(alpha) * cos(alpha) * s22PHI - sin(alpha) ** 2 * s23PHI + cos(alpha) ** 2 * s23PHI
tau_L = s12PHI * cos(alpha) + s13PHI * sin(alpha)

*IF, sigma_N, LT, 0, THEN
FM_C = (tau_T / (ST_is - eta_T * sigma_N)) ** 2 + (tau_L / (SL_is - eta_L * sigma_N)) ** 2
*IF, FM_C, GT, FMC, THEN
FM_Ci = FM_C
*ENDIF

*ENDIF

*IF, sigma_N, GE, 0, THEN
FM_T = (sigma_N / YT_is) ** 2 * (tau_L / (SL_is - eta_L * sigma_N)) ** 2
*IF, FM_T, GT, FMT, THEN
FMTi = FM_T
*ENDIF

*ENDIF

*ENDDO

FK_MAX = max(FMCi, FMTi)

*ENDIF

*IF, FIfT, GE, FK_MAX, then
FF_MAX = FIfT
*else
FF_MAX = FK_MAX
*ENDIF

F_FIBER(I,1,1) = FF_MAX

*ENDDO

*VSCFUN, F_MATRIX_MAX, MAX, F_MATRIX
*VSCFUN, F_FIBER_MAX, MAX, F_FIBER

ETABLE, F_MATRIX, U, X
ETABLE, F_FIBER, U, X

*VPUT, F_MATRIX, ELEM, , ETAB, F_MATRIX, , , 2
*VPUT, F_FIBER, ELEM, , ETAB, F_FIBER, , , 2

SMAX, Fnew, F_MATRIX, F_FIBER, 1, 1.
APPENDIX D. 2D MODEL FOR INDUSTRIAL APPLICATION

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ELSE
mat_string=strcat('material_',CHRVAL(MAT_NUMBER(coupled_nodeI,1,1)))
/INPUT,mat_string,'txt',,, 0 ! import material parameters
ETABLE,Mises,S,EQV
SMULT,Fnew,Mises, ,1/yield_stress,1,
ENDIF
SMAX,F,Fnew,F,1,1,
ENDDO
DIM,F,ARRAY,High_elem_sub,1,1, , 
VOET,F,ELEM, ,ETAB,F, ,2
VOPER,F,F,MULT,dam_elem, , 
VSCFUN,F_MAX,MAX,F
VPUT,F,ELEM, ,ETAB,F, , ,2
FINISH
!* Fshear=Fboltxy/0.6/fur/PI/diameter**2*4*1.25
\ Ftension=Fboltz/0.6/fur/PI/diameter**2*4*1.25
Fcomb=Fshear+Ftension/1.4
alph=1
Flpres=Fboltxy/2.5/fur/alph/diameter/tot_thick*1.25
! /OUT,output,txt,,APPEND
*WRITE,coupled_node, F_MAX, Fshear, Ftension, Fcomb, Flpres
(F11.0,' ',F11.6,' ',F11.6,' ',F11.6,' ',F11.6,' ',F11.6)
/OUT
!
PARSAV,ALL,'savedparameter','txt',
clear
/INPUT,'savedparameter','txt',,, 0
/FILNAME,jobname,0
RESUME
/INPUT,'savedparameter','txt',,, 0
ENDDO
GOPR