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INTERNAL FRACTURES IN SPHERES DUE TO STRESS WAVE FOCUSING

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Abstract—The locations and times of occurrence of internal fractures in Perspex spheres subjected to localized explosive loading are investigated. An analysis of stress wave reflection from free boundaries based on the method of geometrical acoustics is found to give predictions which are in good agreement with results obtained from high speed photographs.

NOTATION

- t time
- $p(t)$ pressure function
- $\mathbf{u}$ displacement vector
- $\mathbf{v}$ particle velocity vector
- V jump in particle velocity at the wave front
- c_L dilatational wave speed
- c_T distorsional wave speed
- c_0 wave velocity of incident wave
- c_1 wave velocity of reflected P-wave
- c_2 wave velocity of reflected S-wave
- E Young's modulus
- x cartesian coordinates, in general
- K subscript and superscript referring to the wave front
- R_K, S_K principal radii of curvature of the wave front
- R_S, S_S principal radii of curvature of the boundary
- ϕ scalar potential for irrotational strain
- ψ vector potential for distorsional strain
- λ Lame's constant
- μ Lame's constant
- ν Poisson's ratio
- ρ density
- $\tau(x)$ wave function
- ξ unit vector normal to the wave front
- $\zeta^{(1)}$ unit vector normal to the boundary
- $\zeta^{(2)}, \zeta^{(3)}$ unit vectors tangent to the boundary
- σ_{ij} stress tensor
- θ_0 angle of incidence
- θ_1 angle of reflection of the P-wave
- θ_2 angle of reflection of the S-wave
- Γ boundary surface
- ∇ gradient operator $\left[=\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)\right]$
- ∇^2 Laplacian operator $\left(=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}\right)$.

INTRODUCTION

The propagation of stress discontinuities in an elastic medium has been the subject of a numbe of investigations for sometime, and an early contribution in this field is that due t Friedlander[1]. The relevant theoretical concepts of the topic are also outlined by Keller[2] and some applications of the ray theory and wave front analysis to the problem of stress wav propagation through cylindrical and spherical inclusions embedded in an elastic medium can b found in [3–7]. A comprehensive analysis on the propagation of a plane shock wave fror through a lens-shaped elastic body has recently been reported by Ting and Herrmann[8]. Thei predictions of the displacement at the rear surface of the body were in very good agreemer with the experimental results. At much the same time, Lovell *et al.*[9] have studied the interna fracturing behaviour of a Perspex sphere subjected to localised explosive shock loading at on