FACULDADE DE ECONOMIA DO PORTO

COMPETITIVE EFFECTS OF ASYMMETRIES BETWEEN FIRMS

Joana Patrícia Neves Vaz de Pinho Correia da Silva

Orientador: António Brandão

Co-Orientador: Hélder Vasconcelos

TESE DE DOUTORAMENTO EM ECONOMIA

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Aos meus pais

e a ti, João.

Nota Biográfica

Joana Patrícia Neves Vaz de Pinho Correia da Silva nasceu a 22 de Outubro de 1985, em Vila Nova de Gaia.

Em Setembro de 2003, inscreveu-se na licenciatura em Matemática (Ramo Científico de Matemática Aplicada) na Faculdade de Ciências da Universidade do Porto (FCUP). Nos dois primeiros anos de licenciatura, foi bolseira do programa "Novos Talentos em Matemática" financiado pela Fundação Calouste Gulbenkian. Neste seu primeiro contacto com a investigação científica, desenvolveu dois projectos com docentes da Faculdade de Ciências da Universidade do Porto. Ambos os projectos foram apresentados em palestras no "Seminário Diagonal", realizadas na FCUP. Nos encontros nacionais dos bolseiros do programa, teve a oportunidade de contactar com investigadores de reputado mérito internacional. Concluiu a licenciatura em Julho de 2007, com a média final de 18 valores. Pela conclusão do curso com a mais elevada média do ano, foi-lhe atribuído um prémio da Fundação Eng. António de Almeida.

Em Setembro de 2007, ingressou no programa de doutoramento da Faculdade de Economia do Porto. Em Julho de 2009, terminou a componente escolar do programa, com a média de 17 valores e deu início à elaboração da tese.

No segundo semestre do ano lectivo de 2009/2010, leccionou a disciplina de Macroeconomia II, na Faculdade de Economia do Porto.

O primeiro artigo da presente tese foi apresentado na conferência "UECE Lisbon Meetings 2010 - Game Theory and Applications", organizada pelo ISEG, e no seminário do programa de doutoramento do Porto. O segundo artigo foi apresentado no workshop “Perspectivas da Investigação em Portugal - Economia Industrial”, organizado pelo CEFAGE.

Eles não sabem, nem sonham,
que o sonho comanda a vida.
Que sempre que um homem sonha
o mundo pula e avança
como bola colorida
entre as mãos de uma criança.

António Gedeão, *in* Pedra Filosofal.

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Como nos dizia o grande poeta Fernando Pessoa, “o homem sonha, a obra nasce”. Mas, entre o sonho e a sua concretização, há sempre um longo caminho a percorrer. O caminho é, contudo, menos sinuoso se não o percorremos sozinhos. É de muitos tipos a ajuda que podemos receber. Pode ser a orientação de alguém que, já tendo percorrido trilhos semelhantes, nos consegue desviar dos caminhos sem saída. Ou pode mesmo ser um simples sorriso que, num momento de fraqueza, devolve a esperança ao nosso mundo.

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A todos, o meu obrigado.

Sumário

A maioria dos mercados caracteriza-se por um pequeno número de empresas que concorrem para a obtenção de lucro. É inegável que, por mais insignificantes que sejam, há sempre diferenças entre as empresas concorrentes. Estas diferenças podem concernir os mais variados aspectos. Exemplos típicos são: o custo de produção; o conhecimento das condições do mercado (quando se compara, por exemplo, uma empresa que está há muito tempo instalada no mercado com outra que acaba de entrar); a dimensão, entre outros factores.

Neste contexto, parece redutor considerar, aquando da formulação de um modelo teórico, que as empresas numa dada indústria são completamente idênticas. Apesar de já existirem alguns contributos teóricos que reflectem a existência de assimetrias entre empresas, a maioria deles ainda assume a simetria como pressuposto base. Assim, o principal objectivo da presente tese é contribuir para esta literatura. Os três ensaios que a constituem abordam, ainda que de forma bastante distinta, as consequências da assimetria entre empresas num oligopólio.

No segundo capítulo da tese, é introduzida assimetria de informação no modelo básico de diferenciação horizontal (Hotelling, 1929). Mais especificamente, considera-se que uma empresa está mais informada relativamente às características dos consumidores que outra.

No terceiro capítulo, é analisado em que medida os preços, a procura e os lucros são afectados pela forma como os centros comerciais estão organizados. É dado um ênfase

especial à concorrência entre uma empresa que vende diversos bens (como é o caso de um hipermercado) e um centro comercial constituído por lojas independentes. É ainda averiguado até que ponto a existência de assimetria na forma de retalho é vantajosa, para os consumidores e para as empresas.

No quarto capítulo, considera-se que existem duas empresas (simétricas) instaladas num mercado em crescimento. Assume-se que os custos de entrada não são proibitivos, pelo que, nalgum momento do tempo, acabará por entrar uma nova empresa no mercado. O intuito principal deste capítulo é o de estudar os impactos da existência de assimetria de custos (entre a entrante e as incumbentes) na sustentabilidade da colusão, antes e depois da entrada ocorrer.

Summary

Most markets are characterized by a small number of firms competing for profit. It is undeniable that there are always differences between competitors. These differences may concern various aspects. Typical examples are: the cost of production; the knowledge of market conditions (when comparing, for example, a firm that has long been installed in the market with another that has just entered); the size; among other factors.

In this context, it seems reductive to consider, when formulating a theoretical model, that firms in an industry are completely identical. Although there are some theoretical contributions that reflect the existence of asymmetries between firms, most of them still take symmetry as the basic assumption. The main objective of this thesis is, therefore, to contribute to this literature. The three essays that constitute the thesis discuss, though in a quite distinct way, the consequences of asymmetry between firms in an oligopoly.

In the second chapter of the thesis, it is introduced asymmetric information in the standard model of horizontal differentiation (Hotelling, 1929). More specifically, it is considered that one firm is more informed about the consumers' characteristics than the other.

In the third chapter, it is examined in what sense the prices, the demand and the profits are affected by how shopping centres are organized. It is given a special emphasis to competition between a multi-product firm (like a supermarket) and a shopping mall, consisting of an agglomerate of independent stores. It is also analyzed whether the existence of asymmetry in the mode of retail is advantageous to consumers and to firms.

In the fourth chapter, it is considered that there are two (symmetric) firms installed in a growing market. Since it is assumed that the entry costs are not prohibitive, it will probably enter a new firm in market, at some moment of time. The main purpose of this chapter is to study the impacts of the existence of asymmetric costs (between the entrant and the incumbents) in the sustainability of collusion, before and after entry.

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Chapter 1

Introduction

The present thesis is organized in five chapters. This first chapter gives an overview of the thesis and familiarizes the reader with the addressed issues. Each of the three following chapters is an independent essay. They answer to specific research questions in different economic scenarios. However, all of them consider an oligopoly with some kind of asymmetry across firms. They share the common goal of studying the impacts of firms' asymmetry on the equilibrium outcome. The last chapter concludes the thesis.

In short-run models, the decision of a firm is frequently how much to charge for its products or how many units of output to offer in the market. In chapters 2 and 3, we consider a duopoly in which firms choose prices (*Bertrand competition*). However, if the two firms sold homogeneous products, the (unique) equilibrium price would be equal to the marginal cost of production. In this case, the firms would have zero profits. This is not reasonable, since firms in a small industry have some market power. In 1929, Hotelling contributed to solve this paradox by relaxing the assumption that firms produce homogeneous goods.

More precisely, he considered a duopoly with horizontally differentiated products. By definition, two products are horizontally differentiated when, even if their prices were equal, there would exist consumers preferring one to the other. In “Stability in competition” (1929), Hotelling presented a genial way to model the consumers preferences for such kind of goods. Despite the simplicity in the formulation, his model captures all the essence of horizontal differentiation. In a nutshell, Hotelling considered that consumers’ preferences are spread along a unitary line and each firm produces one good whose characteristics correspond to a point in the line. Obviously, the more different a product is from the consumer’s most preferred, the less utility the good gives to her. To model this, Hotelling assumed that consumers support a transportation cost to purchase the good. Thus, a product whose characteristic is farther from the consumers most preferred implies a higher transportation cost. The transportation costs also have a geographic interpretation. In this case, the firms choose the location of their stores on the Main Street and the consumers’ transportation costs result of time and money spent on the trip to a store.

Hotelling made, however, some simplifications for his setup to be analytically tractable. In particular, he assumed that the two firms were symmetric in all respects (except, eventually, their locations). This assumption definitively simplifies the analysis but is somewhat restrictive. In chapter 2, we contribute to fill this gap, by introducing asymmetric information about consumers’ preferences. More precisely, we assume that one firm is perfectly informed, while the other is uncertain about the consumers’ transportation costs. The characteristics of the goods (i.e. the firms’ locations) are taken as given and the focus of the analysis is on the price-setting. This chapter explores the consequences of this informational asymmetry on the supply side (namely, the impacts on profits) and on the demand side (on consumers’ welfare).

The model presented in chapter 2 is static, since firms set prices once for all. Although the uninformed firm could eventually infer the consumers' transportation costs by observing the price set by the informed firm, it makes no use of this learning, since there is no future interaction. The firm chooses, therefore, its price in a myopic way. It charges the price that maximizes its expected profit. Alternatively, the uninformed firm could make efforts to gather information before choosing its price. It could, for example, ask for the information to a trade association. We escape a bit from what is standard in the literature and propose the informed firm as a possible source of information. As the uninformed firm has no private information to give in return, it may have to propose a monetary transfer in exchange for the information. Such a mechanism can be looked in a suspicious way by an antitrust authority, since communication is usually understood as facilitating collusive agreements. However, in some circumstances, communication allows to decrease inefficiency in the market, which may be advantageous for consumers.

The model of Hotelling (1929) considers that consumers choose where to buy the product only by comparing the delivered price (price plus the transportation cost) of each firm. By doing so, the model analyzes the behaviour of firms in one industry, without taking into consideration the other industries. However, to meet their needs, consumers have to buy several goods. To save time and money, consumers usually try to concentrate their purchases in one single place. Places like department stores or shopping malls allow consumers to buy a variety of products at one single place. As a result, when consumers decide whether to buy one product at a given shopping centre, they are likely to take into account the prices of the other goods at that location too. This motivates the chapter 3, which considers a multi-product version of the model of Hotelling (1929). More concretely, it assumes the existence of a shopping centre at each extreme of the city, where there are available all the goods that consumers want to buy. The two shopping centres may differ regarding their mode of retail. A shopping centre can be a single firm that sells several goods (like a department store or a supermarket) or an agglomeration of several independent shops (like a shopping street or a mall). The third chapter of the thesis aims to analyze how the mode of retail affects competition. More precisely, it studies the impacts on prices, on demand and on profits.

In the forth chapter, unlike the previous ones, we consider a model of repeated interaction. In this case, a firm is aware that its choices in the present influence its profits in the future. In particular, it knows that a significant increase in the volume supplied will probably trigger an increase in the quantity produced by the competitors. The firm must, therefore, compare the short-run benefit with the cost of the future punishment. In this sense, the repeated interaction between firms may facilitate collusion.

Collusive agreements may be of different forms. Firms may, for example, combine prices, decide market shares or avoid the multimarket contact. However, as explicit cartels are forbidden by the antitrust authorities, firms may have to sustain the collusive outcome without meeting to combine decisions (“tacit collusion”). Contrary to what happens in the case of explicit collusion, tacit collusion hampers coordination between firms. If firms do not communicate, they are likely to make wrong decisions that can be understood as a deviation (from the collusive agreement) by the competitors. Thus, the lack of communication may trigger inadequate punishments. There are, however, several factors that facilitate coordination between firms and, therefore, make collusion more likely. Some of these factors are: (i) *the existence of high entry barriers.* On the one hand, if the market entry is easy, there will be more firms in the market, which hampers coordination between firms. On the other hand, an entrant may always adopt an aggressive behaviour and leads to the cartel breakdown. (ii) *the demand growth.* If the demand is growing, the deviations are less attractive. The firms are not willing to give up a high future collusive profit for a small present gain; (iii) *the symmetry between firms* (e.g. in size, in costs of production or in market shares).

In chapter 4, we consider that there are two symmetric incumbents in the market. The demand is assumed to be growing over the time, which increases the probability of a new firm to enter in the market. The production costs of the entrant may be different from those of the incumbents, creating an asymmetry between the entrant and the incumbents. The main goal of this chapter is to determine the impact of such an asymmetry on the likelihood of collusion (before and after the entry).

The mere threat of entry of a third firm in the market affects the sustainability of collusion between incumbents. On the one hand, the incumbents may adopt a more aggressive behaviour so as to postpone (and, if possible, to prevent) the entry of another firm in the market. On the other hand, with the entry, the future profits will decrease, which intensifies the temptation for the incumbents to deviate in the present. Collusion becomes, therefore, harder to sustain. The sustainability of collusion after the entry also depends on how the incumbents react to the entry. They can accommodate the entrant in their collusive agreement ("full collusion") or they can exclude the new firm from it ("partial collusion"). In chapter 4, we analyze both types of collusion. One aspect that, in our model, makes the analysis of full collusion more complex than the analysis of partial collusion is related to the fact that, in the first case, the colluding firms may be asymmetric. Thus, it is necessary to establish an allocation rule of the monopoly profit that somehow incorporates the differences between the firms.

In the fourth chapter, contrary to the previous ones, the firms are assumed to compete in quantities (*Cournot model*). We make this assumption because we are interested in studying the Nestlé-Perrier merger and, in this specific market, it makes more sense to assume that firms compete in quantities (than in prices).

Finally, in chapter 5, there is an overall analysis of the main results of the thesis. We also present some ideas that would be interesting to explore in future research.

Chapter 2

Asymmetric information and exchange of information about product differentiation

* We are grateful to João Correia da Silva, Joana Resende and Inés Macho Stadler for their useful comments and suggestions.

2.1 Introduction

When a firm invests abroad, it faces barriers that make it less competitive than in its home market. These barriers may be of different types: higher delivery costs, existence of tariffs or lack of knowledge about fundamental aspects of the market. We focus our attention on the uncertainty that a firm may have about consumers' characteristics in a foreign market. More concretely, we analyze how an informational advantage of a domestic firm may affect price competition.[1] This issue was already addressed by Moner-Colonques *et al.* (2008) but using a quite different approach from ours.

Based on the work of Hotelling (1929), we model a duopoly in which firms sell horizontally differentiated products. However, we consider that one firm is domestic while the other is foreign. Firms are assumed to differ only in their knowledge about consumers' transportation costs: the domestic firm is perfectly informed, while the foreign firm only knows its prior probability distribution. The differences between our assumptions and those of Moner-Colonques *et al.* (2008) are obvious. They assume that products are homogeneous and that firms compete in quantities. Another significant difference is that, in their model, the domestic firm is also uncertain about demand (but in a lower degree than the foreign firm).

As d'Aspremont *et al.* (1979), we assume that consumers have to support quadratic transportation costs to purchase the good. From a geographical viewpoint, these costs

[1] Even when both firms face uncertainty about some aspect of the market, it is commonly assumed that the domestic firms have an easier time acquiring information. Jiang and Yoneyama (2008) present China as a country where this phenomenon is very common. To be competitive, either Chinese firms or foreign firms must gather information about demand. The authors argue, however, that local firms acquire information at a lower cost than their foreign counterparts.

result from costly travels: the farther is a store from a consumer, the more costly is the product for her. Consumers may support high (physical) transportation costs due to many reasons, namely bad conditions in roadways, expensive fuel or lack of public transports.[2] In a preference-based interpretation of the model, the magnitude of transportation costs is closely related to the degree of product differentiation. The transportation cost is the disutility from purchasing a variety of the good that is different from the favorite.

We study how asymmetric information about consumers' transportation costs affects prices, taking locations as exogenous. This assumption fits very well when firms are stuck with a price for a period of time. This may occur because firms have signed a contract with customers, committing not to change prices over a period of time. Another possible reason is that firms are sluggish to learn about consumers' characteristics (and, therefore, take a while to realize that prices may not be the optimal). We must also recall that, in our model, one firm is domestic while the other is foreign. Thus, it is likely that each firm has chosen the product's characteristics that better fit the tastes of its home market consumers (hence the exogeneity of locations).[3]

When transportation costs are high (low), the domestic firm is aware that charging a higher

[2]Lal and Matutes (1989) provide an additional justification for differences in transportation costs. They consider that some consumers are rich and others are poor. The poor do not support an "opportunity cost for a shopping trip". On the contrary, the rich customers support a positive "cost of time". Following this interpretation, differences in transportation costs can also result from differences in consumers' wealth.

[3]What is commonly assumed in literature is that firms learn about demand before setting prices (e.g. Casado-Izaga, 2000). In these models, uncertainty only affects locations' choices. The main arguments for this assumption are: the high reallocation costs (of building a factory in another place or redesigning the product) and the flexibility of prices. Firms may not change locations but use their knowledge to set optimal prices. Since prices are assumed to be flexible, they are easily adjusted, not being much affected by the existence of uncertainty. Meagher and Zauner (2004) consider two possible scenarios: one in which uncertainty only affects locations and another in which uncertainty also extends to the price setting stage.

price has a low (high) effect on demand. As a result, the domestic firm sets a high (low) price. Owing to its uncertainty, the foreign firm always sets an average price. If transportation costs turn out to be high (low), the foreign firm captures more (less) than half of the market. Thus, despite being less informed, the foreign firm may actually profit more than the rival, when transportation costs are high. In this case, charging a lower price is compensated by a higher demand.

We compare the scenario of asymmetric information with that of perfect information. When transportation costs are high, both firms obtain higher profits with perfect information. When transportation costs are low, asymmetric information enables the domestic firm to profit more and it typically benefits the foreign firm too.

It is commonly assumed in literature that firms ignoring some aspect of the market (e.g. magnitude of demand, consumers' characteristics, rivals' cost function) try to acquire the missing information. In banking, for example, lending entities frequently gather information about borrowers.[4] Market research is a usual means of obtaining information. Agencies responsible for attracting foreign investments and trade associations are other possible sources of information.

In our model, the foreign firm may be interested in acquiring information about consumers' transportation costs, since this increases its expected profit. A market research may be very expensive, discouraging the firm to carry it on. Moreover, agencies generally gather information on various economic sectors. Consumers' preferences for a given product are such specific information that an agency may not have data as accurate as desired by the firm. Only by experimenting the market it may be possible to get this kind of information. In these cases, the foreign firm could directly ask the domestic (and informed)

[4]In a very recent work, Karapetyan and Stacescu (2010) discuss two different ways that banks use in order to be informed about borrowers (information sharing and information acquisition).

firm for information. To best of our knowledge, this is the first work that considers such a means of information acquisition.[5] There are several works dealing with information sharing, namely about demand (e.g. Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984) and Gal-Or (1985)). However, in these models, all firms are uncertain about some parameter of the demand function. Each firm receives a noisy and private signal with information about the unknown parameter. The authors study if firms have incentives to share their private information with the rivals or not.[6] In a different approach, Liu and Serfes (2006) present the "databases with detailed records of consumers' preferences" as a typical example of information that firms sell to others.[7] Karapetyan and Stacescu (2010) analyze the possibility of banks to share information about clients that contracted with them in the past. In our case, the foreign firm has no private information to give in exchange for information about consumers' transportation costs. Thus, the only possibility is the existence of monetary transfers between firms.

Unless issues related to protection of consumers' privacy, the exchange of information is not generally considered to be illegal. Communication between firms may only be looked in a suspicious way by antitrust authorities if it somehow leads to coordination between firms. Direct exchanges of information about prices, quantities or market shares are commonly forbidden since they are understood as attempts to facilitate collusion. We study the welfare effects of communication between firms to anticipate whether it should be allowed by an

[5]This may happen in reality, but firms may keep it in secret to avoid raising the suspicion of antitrust authorities.

[6]There are also several works in literature that address the incentives for information exchange about costs. See, for example, Fried (1984), Friedman (1986) and Gal-Or (1986).

[7]In the model of Liu and Serfes (2006), there are two firms that sell horizontally and vertically differentiated products. Each firm has a database with information on consumers who have purchased its product in the past. The authors analyze the incentives of a firm to directly sell its database to the competitor.

antitrust authority.

Since, in our model, total demand is perfectly inelastic, a change in prices only leads to a transfer of surplus between consumers and firms. As a result, to maximize total surplus is equivalent to minimize aggregate transportation costs. In the presence of asymmetric information, transportation costs are higher (because the indifferent consumer is not in the middle of the city). Thus, it is suboptimal to have one firm less informed than the other.

If there is information exchange, firms' joint profit increases and the aggregate transportation cost decreases. This could give the impression that communication is beneficial both for firms and for consumers. However, we show that the expected value of consumers' surplus decreases when firms communicate. Thus, unless the antitrust authority cares about consumers and producers almost in the same way, it should not allow for communication between firms.[8]

The expected profit of both firms is higher in the case of perfect information. Therefore, in the ex-ante stage, both firms are interested in committing to communicate. When information is hard (verifiable but subject to be omitted), the foreign firm may propose a state-contingent contract that compensates the domestic firm when revealing that transportation costs are low. If the foreign firm holds all the bargaining power, it can extract the surplus of the domestic firm when transportation costs are high. Even supporting this cost, the domestic firm may want to disclose its private information, since its net profits are higher than in the case of asymmetric information. When information is soft (not

[8]Consider that the objective function of the antitrust authority is a weighted average of consumers' and producers' expected surplus. According to our model, communication between firms must be allowed if the weight given to consumers' surplus is smaller than 52%. If the antitrust authority only cares about consumers and domestic firm, communication must be allowed if the weight given to consumers' surplus is smaller that 32%.

verifiable), the domestic firm will always have the incentive to say that transportation costs are high (rendering the message uninformative). Alternatively, the foreign firm could offer a contract providing incentives for the domestic firm to always announce the truth. We find, however, that such an incentive compatible contract does not exist. This impossibility of communication between firms is not an atypical result. Gal-Or (1985), Li (1985) and Kirby (1988), for example, obtained that no information sharing is the unique equilibrium of their models.

Our contribution is innovative in several aspects. To the best of our knowledge, we are the first to introduce uncertainty about consumers' transportation costs (i.e. the degree of product differentiation) in the model of Hotelling (1929). There are several works in the literature that consider uncertainty about demand in this spatial competition model. The most common way is by introducing uncertainty about consumers' location. In the models of Harter (1996), Casado-Izaga (2000) and Meagher and Zauner (2004), firms know that consumers are spread uniformly over a unitary interval, but ignore the actual limits of the interval.[9] In a more recent contribution, Meagher and Zauner (2008) go beyond and consider that firms may not even know the spatial distribution of consumers.[10] Villas-Boas and Schmidt-Mohr (1999) incorporate uncertainty about demand in a different way. They study competition between two banks that provide financing to a continuum

[9] In these three models, firms start by choosing locations and, afterwards, they engage in price competition. Harter (1996) consider that only the firms located inside the city boundaries have positive demand. Casado-Izaga (2000) modifies his model by allowing the firms located outside the city to get a positive demand. Harter (1996) and Casado-Izaga (2000) restrict their analysis to the case in which uncertainty is revealed before firms choose prices. Apart from studying this possible scenario, Meagher and Zauner (2004) also allow uncertainty to extend to the price-setting stage.

[10] The authors give a coastal city as an example in which firms may ignore not only the mean but also the dispersion of the population.

of entrepreneurs. In their model, the uncertainty is with respect to the actual profitability of entrepreneurs' projects. Rhee *et al.* (1992) consider that consumers' preferences are heterogeneous, having a component that firms can observe and another that is unobservable. Finally, in the models of Balvers and Szerb (1996) and Christou and Vettas (2005), firms sell products that are differentiated both horizontally and vertically. There, the uncertainty is about the relative quality of products (i.e. the degree of vertical differentiation).

It is noteworthy that none of the models mentioned above considers the possibility of one firm being more informed than the other(s). This aspect also distinguishes our work from theirs. In our model, only the foreign firm is uncertain about consumers' transportation costs (the domestic firm is perfectly informed). In other words, we assume that there is asymmetric information. In the literature, there are several works that incorporate asymmetric information in the model of Hotelling. However, they commonly assume that production costs are the source of asymmetric information (e.g. Boyer *et al.*, 2003 and Bester, 1998). To the best of our knowledge, only the recent work of Jentzsch *et al.* (2010) addresses the possibility of incomplete information about consumers' transportation costs. These authors use, however, a different basic setup and consider a different process of information exchange.

Several works have already studied the consequences and the feasibility of information sharing.[11] We are, however, innovative in addressing issues related to (a different way of) communication in a duopoly with horizontal differentiation. Vives (1984) also analyzes the duopolists' incentives to exchange information about demand when products are differentiated, but he uses a quite different setup.[12] As far as we know, the only model that

[11] See Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984, 1990), Fried (1984), Gal-Or (1985, 1986), Li (1985), Sakai (1985, 1986, 1989), Shapiro (1986), Kirby (1988), Sakai and Yamato (1989), Ziv (1993) and Raith (1996).

[12] Vives (1990) concludes that the expected total surplus increases with information sharing if the firms

also considers that the uninformed firm may propose a monetary transfer (to the informed firm) in exchange for information is that of Ziv (1993). In his model, firms compete in quantities to sell a homogeneous good and have private information about their production costs. The differences between our model and his are clear. Curiously, he also finds that firms tend to send false information and, to overcome this problem, they may make monetary transfers (rewarding the firm for announcing a “bad” state and penalizing the firm for claiming that the state is “good”).

The remainder of the chapter is structured as follows. Section 2.2 sets up the model. Section 2.3 presents the equilibrium in the case of perfect information and in the case of asymmetric information. Section 2.4 determines the welfare consequences of asymmetric information (for firms and for consumers) and the expected benefits of communication between firms. Section 2.5 obtains possible agreements of information exchange, both when information is verifiable and when information is non-verifiable. Section 2.6 concludes. The Appendix A contains the proofs of most propositions, while the Appendix B computes the profits of the foreign firm if there is communication between firms.

2.2 The model

The model we present is based on Hotelling’s (1929) model of horizontal differentiation. The market is a linear city of unitary length with two firms, 1 and 2, located at the extremes ($x_1 = 0$ and $x_2 = 1$). The products sold by the firms are identical in all respects other than the location where they are sold. Consumers are uniformly distributed on the line and, to buy the product, they incur in a transportation cost that is assumed to be quadratic in

compete in quantities (the opposite is true if firms compete in prices).

distance (d'Aspremont *et al.*, 1979). The utility of a consumer located at $x \in [0,1]$ that buys the product from firm $i \in \{1,2\}$ is defined as the difference between the reservation price for the ideal product, V, and the costs of the purchase (price plus the transportation cost):

$$U_i(x) = V - p_i - t(x - x_i)^2 .$$

The reservation price is assumed to be sufficiently high for the market to be fully covered. The total demand is perfectly inelastic, with each consumer purchasing exactly one unit of the good. Therefore, the consumers' decision is only from which firm to buy the product.

Suppose that firm 1 (also designated as the *domestic* firm) operates in its home market, while firm 2 is a *foreign* firm. We consider that firms have asymmetric information about consumers' transportation costs. Both firms know that these costs are quadratic in distance. However, while firm 1 is fully informed, the rival only knows the prior probability distribution of the parameter t:

$$t = \begin{cases} t_H & \text{with probability} \quad \theta \\ t_L & \text{with probability} \quad 1-\theta \end{cases}, \tag{2.1}$$

where $t_H > t_L > 0$ and $\theta \in \,]0,1[$. The extreme cases, $\theta = 0$ and $\theta = 1$, correspond to the standard model. If $t = 0$, the products sold by the firms would not be differentiated. As it is well known, in this case firms sell their products at the marginal cost and have zero profits.

The timing of the game is the following:

1st stage: Nature chooses t (t_H with probability θ and t_L with probability $1-\theta$). The domestic firm observes the result.

2nd stage: Firms simultaneously choose prices.

Without loss of generality, we assume that both firms have zero marginal costs of production.[13]

2.3 Equilibrium

We start by analyzing the case of perfect information. Then, we introduce asymmetric information as described above. In both scenarios, we determine the price set by each firm and the correspondent profits.

Below, $\Pi_k^{i\theta}$ denotes the profit that firm k obtains in the case of asymmetric information, when $t = t_i$. By Π_k^{ij} we denote the profit of firm k when $t = t_i$ and the foreign firm believes that $t = t_j$, for $k \in \{1, 2\}$ and $i, j \in \{H, L\}$.

2.3.1 Perfect information equilibrium

Suppose that both firms are fully informed about t. In this case, firms charge t for their products and each firm gets half of the market. Firms obtain, therefore, the same profit, which is given by:

$$\Pi_1^{ii} = \Pi_2^{ii} = \frac{t_i}{2} \quad , \quad i \in \{H, L\} \, .$$

2.3.2 Asymmetric information equilibrium

Let us now consider that firms have asymmetric information about the transportation cost parameter, t. Observing the result of nature's choice, t_H or t_L, the domestic firm can choose

[13]Otherwise, the equilibrium prices are simply the sum of the marginal costs with the obtained prices. Thus, the profits remain the same.

the price, p_{1H} or p_{1L}, that maximizes its profit. The foreign firm, knowing only the prior distribution of t, chooses the price, p_2, that maximizes its expected profit.

By definition, the marginal consumer is the consumer that is indifferent between buying the product from firm 1 or from firm 2. The location of such a consumer depends on prices and on t. When $t = t_H$, the marginal consumer, $\tilde{x}_H$, satisfies:

$$U_1\left(\tilde{x}_H\right) = U_2\left(\tilde{x}_H\right) \;\Leftrightarrow\; \tilde{x}_H = \frac{1}{2} + \frac{p_2 - p_{1H}}{2t_H}. \tag{2.2}$$

Analogously, when $t = t_L$, the marginal consumer, $\tilde{x}_L$, is given by:

$$\tilde{x}_L = \frac{1}{2} + \frac{p_2 - p_{1L}}{2t_L}. \tag{2.3}$$

If $0 \leq \tilde{x}_i \leq 1$, for $i \in \{H, L\}$, the expression for the marginal consumer coincides with the demand of firm 1.[14] If $\tilde{x}_i = 0$, firm 2 is monopolistic, while, if $\tilde{x}_i = 1$, it is the firm 1 that is monopolistic.

When $t = t_i$, $i \in \{H, L\}$, the profit function of the domestic firm is:

$$\Pi_1^{i\theta}(p_{1i}) = \begin{cases} p_{1i} & \text{if} \quad p_{1i} < p_2 - t_i \\ p_{1i}\left(\frac{1}{2} + \frac{p_2 - p_{1i}}{2t_i}\right) & \text{if} \quad p_2 - t_i \leq p_{1i} \leq p_2 + t_i \\ 0 & \text{if} \quad p_{1i} > p_2 + t_i \end{cases}$$

and its best-response function is, therefore, given by:[15]

$$p_{1i}\left(p_2\right) = \begin{cases} \frac{p_2 + t_i}{2} & \text{if} \quad 0 \leq p_2 \leq 3t_i \\ p_2 - t_i & \text{if} \quad p_2 > 3t_i \end{cases}.$$

[14] The demand of firm 2 is simply $1 - \tilde{x}_i$.

[15] It is straightforward to verify that the second order conditions are satisfied.

The foreign firm chooses p_2 that maximizes its expected profit: $E\Pi_{2,AI} = \theta\Pi_2^{H\theta} + (1-\theta)\,\Pi_2^{L\theta}$, where:

$$\Pi_2^{i\theta}(p_2) = \begin{cases} p_2 & \text{if } \; p_2 < p_{1i} - t_i \\ p_2\left(\frac{1}{2} + \frac{p_{1i}-p_2}{2t_i}\right) & \text{if } \; p_{1i} - t_i \leq p_2 \leq p_{1i} + t_i \\ 0 & \text{if } \; p_2 > p_{1i} + t_i \end{cases}.$$

Proposition 2.1. *If $0 < \theta < \frac{2}{3}\frac{t_H}{t_H - t_L}$, the unique pure-strategy equilibrium of the game is such that:*

(1) *The prices of the domestic firm are:*

$$p_{1H} = \frac{t_H}{2}\frac{(1-\theta)\,t_H + (1+\theta)t_L}{(1-\theta)t_H + \theta t_L} \;\; ; \;\; p_{1L} = \frac{t_L}{2}\frac{(2-\theta)\,t_H + \theta t_L}{(1-\theta)t_H + \theta t_L} \tag{2.4}$$

and the price of the foreign firm is:

$$p_2 = \frac{t_H t_L}{(1-\theta)t_H + \theta t_L}. \tag{2.5}$$

(2) *Both firms have positive demand:*

$$\tilde{x}_H = \frac{(1-\theta)t_H + (1+\theta)t_L}{4\left[(1-\theta)t_H + \theta t_L\right]} \quad \textit{and} \quad \tilde{x}_L = \frac{(2-\theta)\,t_H + \theta t_L}{4\left[(1-\theta)t_H + \theta t_L\right]}. \tag{2.6}$$

(3) *The profits of domestic firm are:*

$$\begin{cases} \Pi_1^{H\theta} = \frac{t_H}{8}\left[\frac{(1-\theta)t_H + (1+\theta)t_L}{(1-\theta)t_H + \theta t_L}\right]^2 \\ \Pi_1^{L\theta} = \frac{t_L}{8}\left[\frac{(2-\theta)t_H + \theta t_L}{(1-\theta)t_H + \theta t_L}\right]^2 \end{cases}, \tag{2.7}$$

while the profits of the foreign firm are:

$$\begin{cases} \Pi_2^{H\theta} = \frac{t_H t_L}{4}\frac{3(1-\theta)t_H + (3\theta-1)t_L}{\left[(1-\theta)t_H + \theta t_L\right]^2} \\ \Pi_2^{L\theta} = \frac{t_H t_L}{4}\frac{(2-3\theta)t_H + 3\theta t_L}{\left[(1-\theta)t_H + \theta t_L\right]^2} \end{cases}. \tag{2.8}$$

Proof. See the Appendix A. □

Proposition 2.2. *If* $\theta > \frac{2}{3}\frac{t_H}{t_H - t_L}$ *there is no pure-strategy equilibrium of the game.*

Proof. See the Appendix A. □

Henceforward, we restrict the analysis to the cases in which there exists an equilibrium in pure strategies. For this purpose, we make the assumption below.[16]

Assumption 2.1. *The parameters* θ, t_H, t_L *are such that:* $0 < \theta < \frac{2}{3}\frac{t_H}{t_H - t_L}$.

2.4 Consequences of asymmetric information

In this section, we study the impacts of asymmetric information on prices and on welfare, by comparing asymmetric information outcomes with perfect information outcomes. We determine which of these scenarios is more profitable for each of the firms. Finally, we obtain the expected welfare effects of reverting to a perfect information scenario. By doing so, we try to forecast if an antitrust authority would welcome communication between firms.

[16]The assumption is not too restrictive, since it is always satisfied if $t_H \leq 3t_L$ or if $\theta \leq \frac{2}{3}$.

2.4.1 Prices, demand and profits

We start by comparing the prices in the case of asymmetric information with those with perfect information. Recall that t_L and t_H are the prices charged (by both firms) in the case of perfect information.

Proposition 2.3. *The prices under asymmetric information are such that:*

$$t_L < p_{1L} < p_2 < p_{1H} < t_H.$$

Proof. See the Appendix A. □

The intuition behind this result is very simple. The transportation cost parameter, t, can be interpreted as a measure of the degree of differentiation between the products sold by the two firms. The higher the consumers' transportation cost, the more differentiated the products are. Then, a high value for t gives market power to firms, which softens price competition. Both firms are aware of this, of course. However, only the domestic firm observes the actual value of t. Thus, when $t = t_H$, the domestic firm can increase its price without losing so much demand. As a result, the foreign firm charges a lower price than the domestic firm, when $t = t_H$. The converse is true when $t = t_L$, that is, when the domestic firm knows that the two products are not so differentiated.

It is also interesting to find out which firm earns more in the presence of asymmetric information.

Proposition 2.4. *When $t = t_L$, the domestic firm takes advantage of its private information, achieving higher profits than the foreign firm:*

$$\Pi_1^{L\theta} \geq \Pi_2^{L\theta}.$$

When $t = t_H$, this may not be the case:

$$\Pi_1^{H\theta} \geq \Pi_2^{H\theta} \quad \textit{if and only if} \quad \frac{t_H}{t_L} \geq \frac{3-\theta}{1-\theta}.$$

Proof. See the Appendix A. □

Curiously, despite being more informed, the domestic firm may not always obtain higher profits than the rival.[17] One can wonder the reason why to be more informed about consumers may not always be an advantage. Recall that, when $t = t_H$, the domestic firm charges more per unit of product. By setting a lower price, the foreign firm captures more demand. Thus, for some parameters of the model, the price gain obtained by the domestic firm may not be enough to offset its demand loss. We can confirm this result in Figure 2.1.

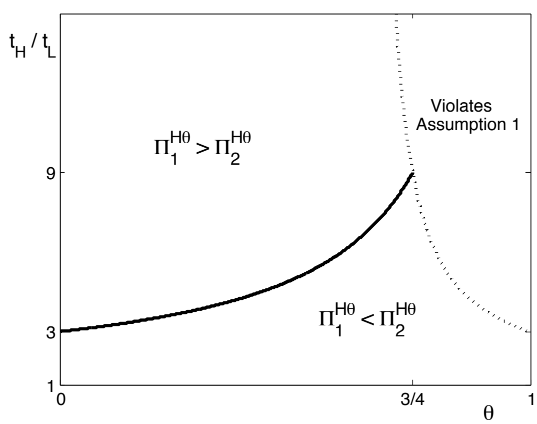

Figure 2.1: Comparison of the two firms' profits in the presence of asymmetric information.

[17]For example, if $t = t_H$ and $t_H < 3t_L$, the foreign firm surely profits more than the domestic firm.

It also seems interesting to determine which scenario (asymmetric or perfect information) is more profitable for each firm. Let us start by analyzing the case of the domestic firm.

Proposition 2.5. *When $t = t_H$, the domestic firm obtains higher profits in the case of perfect information:*

$$\Pi_1^{H\theta} \leq \Pi_1^{HH}.$$

However, when $t = t_L$, the domestic firm earns more in the case of asymmetric information:

$$\Pi_1^{L\theta} \geq \Pi_1^{LL}.$$

Proof. See the Appendix A. $\square$

We can also compare the profits of the foreign firm when it is uncertain about t with the profits when it is perfectly informed.

Proposition 2.6. *When $t = t_H$, the foreign firm obtains higher profits in the case of perfect information than in the case of asymmetric information:*

$$\Pi_2^{H\theta} \leq \Pi_2^{HH}.$$

When $t = t_L$, it depends on the ratio $\frac{t_H}{t_L}$:

$$\begin{cases} \Pi_2^{L\theta} \geq \Pi_2^{LL} & \textit{if} \quad 1 < \frac{t_H}{t_L} \leq \frac{2\theta}{2\theta - 1} \\ \Pi_2^{L\theta} \leq \Pi_2^{LL} & \textit{if} \quad \frac{t_H}{t_L} > \frac{2\theta}{2\theta - 1} \end{cases}.$$

Proof. See the Appendix A. $\square$

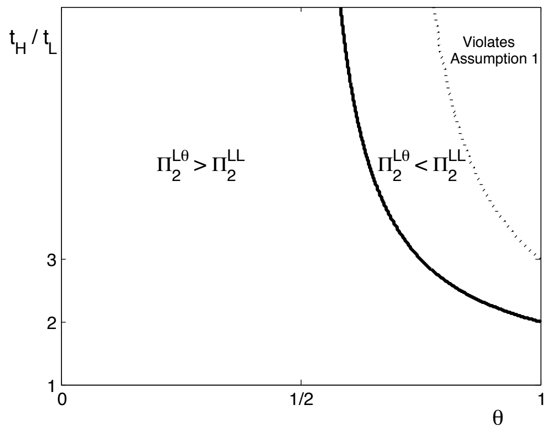

Figure 2.2: Comparison of the foreign firm's profits with and without asymmetric information.

In Figure 2.2, we plot the region of parameters, $\frac{t_H}{t_L}$ and θ, for which the foreign firm has higher profits with asymmetric information than with perfect information.

Combining the results in Propositions 2.5 and 2.6, we conclude that, when $t = t_H$, both firms would be better off with perfect information. On the contrary, if $t = t_L$ and $\theta \leq \frac{1}{2}$, the two firms benefit from the existence of asymmetric information.

2.4.2 Welfare analysis

2.4.2.1 Total surplus

The consumers' reservation price is assumed to be sufficiently high for the market to be fully covered. Total demand is, therefore, perfectly inelastic: each consumer buys exactly one unit of the good. For this reason, a change in prices simply corresponds to a transfer of surplus between consumers and firms.

Thus, to study the consequences of asymmetric information on total surplus, we only need to analyze its impacts on aggregate transportation costs, TC. Given the expression for the marginal consumer, $\tilde{x}$, we have:

$$TC\left(\tilde{x},t\right)=\int_0^{\tilde{x}} tx^2dx+\int_{\tilde{x}}^1 t\left(1-x\right)^2 dx = t\left(\frac{1}{3}-\tilde{x}+\tilde{x}^2\right). \tag{2.9}$$

To maximize total surplus is equivalent to find the minimum of TC. The aggregate transportation cost, TC, achieves its minimum when $\tilde{x}=\frac{1}{2}$. It is straightforward to see that: $\tilde{x}_H<\frac{1}{2}$ and $\tilde{x}_L>\frac{1}{2}$. Therefore, the next corollary follows immediately.

Corollary 2.1. *If firms have asymmetric information, the total surplus is suboptimal.*

2.4.2.2 Consumers' surplus

Let us now analyze whether asymmetric information between firms benefits (or damages) consumers.

Definition 2.1. *The consumers' surplus is defined as:*

$$CS=\tilde{x}\left(V-p_1\right)+\left(1-\tilde{x}\right)\left(V-p_2\right)-TC(\tilde{x},t), \tag{2.10}$$

where p_1 and p_2 are the prices charged by firm 1 and by firm 2, respectively.

When $t=t_L$, perfect information is beneficial for consumers: the transportation costs, TC, are minimal and, furthermore, the prices are lower ($t_L<p_{1L}<p_2$). When $t=t_H$, the effect of asymmetric information on consumers' surplus does not follow immediately.

On the one hand, perfect information leads to the lowest value of TC, which is favorable to consumers. On the other hand, firms charge higher prices ($p_2 < p_{1H} < t_H$), which is harmful for consumers. Thus, we need to find which of these effects (price effect or transportation cost effect) dominates.

Proposition 2.7. *When $t = t_L$, full information is beneficial for consumers. When $t = t_H$, the converse is true: asymmetric information increases the consumers' surplus.*

Proof. See the Appendix A. □

Therefore, when $t = t_H$, the save in the transportation costs with full information is not enough to compensate consumers for paying higher prices. In short, the price-effect dominates, making asymmetric information between firms beneficial for consumers.

2.4.2.3 Expected benefits of communication between firms

Now we attempt to anticipate the reaction of an antitrust authority (AA) regarding communication between firms. To know whether it is desirable for consumers, an AA can compare the consumers' surplus with and without asymmetric information.

It seems reasonable to assume that the antitrust authority is as informed about t as the foreign firm. Therefore, the AA has no way to know ex-ante if the exchange of information is or is not advantageous for consumers.[18] Due to its uncertainty, the AA must base the decision on the expected welfare effect.

[18] As seen before, when $t = t_L$, full information increases the consumers' surplus. On the contrary, when $t = t_H$, consumers are better off without communication.

The consumers' expected benefit from communication between firms is:

$$EB_C = \theta \left[CS_{SI}^H - CS_{AI}^H\right] + (1-\theta)\left[CS_{SI}^L - CS_{AI}^L\right], \tag{2.11}$$

where CS_{SI}^i and CS_{AI}^i are, respectively, the consumers' surplus with perfect information and with asymmetric information, when $t = t_i$, $i \in \{H, L\}$.

Proposition 2.8. *Communication between firms damages the expected value of consumers' surplus (i.e. $EB_C < 0$) .*

Proof. See the Appendix A. □

As a result, the antitrust authority may be reluctant to allow for information sharing between firms. However, we must have in mind that frequently the main objective of AAs is to maximize a weighted sum of consumers' surplus and firms' profits.

The expected benefit of information sharing for the firms is:

$$EB_F = \theta \left[\left(\Pi_1^{HH} + \Pi_2^{HH}\right) - \left(\Pi_1^{H\theta} + \Pi_2^{H\theta}\right)\right] + (1-\theta)\left[\left(\Pi_1^{LL} + \Pi_2^{LL}\right) - \left(\Pi_1^{L\theta} + \Pi_2^{L\theta}\right)\right]. \tag{2.12}$$

Substituting the expressions for profits, we can write:

$$EB_F = \frac{7}{8}\frac{\theta(1-\theta)\left(t_H - t_L\right)^2}{(1-\theta)\,t_H + \theta t_L}. \tag{2.13}$$

Note that EB_F is always greater than zero, meaning that the (expected) joint profit is higher with perfect information.

Let α and $1-\alpha$ be the weights that the antitrust authority gives to EB_C and to EB_F, respectively. The AA should allow for communication if $\alpha EB_C + (1-\alpha)\,EB_F$ is positive. Substituting (2.11) and (2.13) into this weighted sum, we obtain:

$$\alpha EB_C + (1-\alpha)\,EB_F = \frac{1}{16}\frac{\theta\,(1-\theta)\left(t_H - t_L\right)^2}{(1-\theta)\,t_H + \theta t_L}\,(14 - 27\alpha)\,,$$

which is greater than zero if:

$$\alpha \leq \frac{14}{27} \simeq 0.52.$$

Consider now that the antitrust authority is only concerned about consumers and about the domestic firm. The expected benefit of communication for firm 1 is:[19]

$$EB_{F1} = \theta \left[\Pi_1^{HH} - \Pi_1^{H\theta}\right] + (1-\theta)\left[\Pi_1^{LL} - \Pi_1^{L\theta}\right] = \frac{3\theta(1-\theta)\left(t_H - t_L\right)^2}{8\left[(1-\theta)t_H + \theta t_L\right]}.$$

Thus, if β and $1-\beta$ are the weights that the AA gives to consumers' surplus and to domestic firm's profits, it should allow for communication if:

$$\beta EB_C + (1-\beta)\, EB_{F1} > 0 \Leftrightarrow \frac{1}{16}\frac{\theta\,(1-\theta)\,(t_H - t_L)^2}{(1-\theta)\,t_H + \theta t_L}\,(6 - 19\beta) > 0 \Leftrightarrow \beta < \frac{6}{19} \simeq 0,32.$$

As expected, an AA that does not care about the profits made by the foreign firm has to weight even less the consumers' surplus to allow for communication between firms. It is reasonable to conclude that, in these circumstances, the AA is against the exchange of information.

2.5 Exchange of information

When firms want to acquire information about foreign markets, they generally ask foreign investment agencies or trade associations for it or make a market research. We propose another source of information: a domestic (and informed) firm.

According to Vives (2006), "*in general, antitrust authorities, including the European Commission, look with suspicion at information exchanges of individual firms' data, prices*

[19] Notice that we are assuming that the domestic firm reveals its information for free.

and quantities in particular, because they may help monitoring deviations from collusive agreements". However, communication between firms is not always considered to be illegal.

In this section, we obtain possible contracts that the foreign firm can propose to the domestic firm to know the magnitude of transportation costs, t. In contract theory, it is usually assumed that one of the firms has all the bargaining power - the firm that proposes the contract (the *principal*). In a disclosure game, it is legitimate to assume that either the informed party or the uninformed party proposes the contract . For simplicity, we assume that it is the foreign firm that proposes the contract and holds, therefore, all the bargaining power.

Regarding the type of information, we consider three possible scenarios. In the first two scenarios, we assume that information is verifiable. This means that the domestic firm cannot send a false signal. What distinguishes these two scenarios is that: in the first, firms negotiate before the domestic firm to become informed (ex-ante); in the second, the contract is signed after the domestic firm to become informed. Finally, we consider a third scenario in which information is non-verifiable. In this case, the foreign firm must propose a contract that not only induces participation of the domestic firm but also provides incentives for truthful revelation.

2.5.1 Verifiable information

With information being verifiable, the domestic firm is not able to lie. If the foreign firm faced a false message, it could complain in a court of law to punish the rival. This is a very common assumption in the literature (see, for example, Okuno-Fujiwara *et al.* (1990) and Gal-Or (1985)).

2.5.1.1 Ex-ante exchange

We start by analyzing the case in which firms have the possibility of contracting at the beginning of the game, that is, even before the domestic firm to observe the value of t.

As firms are risk neutral, they are indifferent between signing a contract that is contingent on the state of nature or not. Their willingness in accepting the (ex-ante) agreement only depends on their expected profit. For the sake of simplicity, we consider agreements that set a fixed transfer, Ω^v, from the foreign firm to the domestic firm.

The timing of this (disclosure) game is the following:

> *1st stage:* Foreign firm proposes a transfer, Ω^v, in exchange for the information.
>
> *2nd stage:* The domestic firm accepts (or rejects) the contract.
>
> *3rd stage:* Nature chooses t. The domestic firm observes t. If the contract has been accepted, the domestic firm reveals the value of t to the foreign firm.
>
> *4th stage:* Firms simultaneously choose prices.

If the domestic firm accepts the contract, it commits to disclose its information as soon as it is available. The domestic firm will not be able to omit its information, even in the case in which this would be profitable.[20]

The value of receiving information for the foreign firm, Ω_2, is the difference between its expected profit with information and without information. The expected profit with information is:

$$E\Pi_{2,SI} = \theta\frac{t_H}{2} + (1-\theta)\,\frac{t_L}{2},$$

[20]Recall that, without any contingent payment, if $t = t_L$, the domestic firm would prefer not to disclose information.

while, without information, it is:

$$E\Pi_{2,AI} = \frac{1}{2}\frac{t_H t_L}{(1-\theta)\, t_H + \theta t_L}. \tag{2.14}$$

Hence:

$$\Omega_2 = E\Pi_{2,SI} - E\Pi_{2,AI} = \frac{1}{2}\frac{\theta(1-\theta)\,(t_H - t_L)^2}{(1-\theta)\, t_H + \theta t_L}.$$

Similarly, the value of disclosing information for the domestic firm, Ω_1, is equal to:

$$\Omega_1 = E\Pi_{1,SI} - E\Pi_{1,AI} = \frac{3}{8}\frac{\theta(1-\theta)\,(t_H - t_L)^2}{(1-\theta)\, t_H + \theta t_L}. \tag{2.15}$$

We conclude, therefore, that both firms gain, ex-ante, with information disclosure. The domestic firm accepts any monetary transfer $\Omega \in [-\Omega_1; \Omega_2]$ to reveal its private information. As the foreign firm was assumed to hold all the bargaining power, then $\Omega^v = -\Omega_1$.[21] If the domestic firm holds all the bargaining power, then $\Omega^v = \Omega_2$. If both firms have some bargaining power, one possible solution could be obtained by solving the correspondent Nash bargaining game.

It may seem strange to have a contract in which the informed firm pays to reveal its information. This occurs because: (i) the domestic firm gains (in expected value) from disclosing its information; (ii) the foreign firm holds all the bargaining power and captures, therefore, the surplus of the disclosure game. If the domestic firm could unilaterally commit to disclose its information, then our analysis suggests that $\Omega^v = 0$ (because, in that case, the domestic firm would not be willing to pay for something it can credibly do for free).

2.5.1.2 Ex-post exchange

Now, consider that the contract can only be established after the domestic firm has observed the value of t. In this case, the timing of the game is:

[21] In the Appendix B.1, we obtain the expressions for the (actual) profit of the foreign firm, either when $t = t_H$ or $t = t_L$, if firms establish this contract.

1st stage: Nature chooses t. The domestic firm observes t.

2nd stage: The foreign firm proposes a contract (to the domestic firm) offering a contingent monetary transfer, (Ω_H^v, Ω_L^v), in exchange for information about t.

3rd stage: The domestic firm accepts (or rejects) the contract. If the contract is accepted, the domestic firm reveals the value of t to the foreign firm.

4th stage: Firms simultaneously choose prices.

In this case, the rejection of the contract could signal a particular value of t. For simplicity, we assume that, when the contract is rejected, the foreign firm keeps its prior beliefs. This conjecture, called "passive beliefs", is widely used in the literature (Rey and Vergé, 2004; Hart and Tirole, 1990; McAfee and Schwartz, 1994). It implies that the rejection of contract gives no additional information to the foreign firm.

A contract is a pair of transfers, (Ω_H, Ω_L), from the foreign firm to the domestic firm when $t = t_H$ and $t = t_L$, respectively. If the domestic firm rejects the contract, there is no monetary transfer. To be accepted by the domestic firm, the contract must satisfy two participation constraints:

$$\begin{cases} \Omega_L + \Pi_1^{LL} \geq \Pi_1^{L\theta} \\ \Omega_H + \Pi_1^{HH} \geq \Pi_1^{H\theta} \end{cases} \Leftrightarrow \begin{cases} \Omega_L \geq \frac{\theta t_L (t_H - t_L)[(4-3\theta)t_H + 3\theta t_L]}{8[(1-\theta)t_H + \theta t_L]^2} & (PC_L) \\ \Omega_H \geq -\frac{(1-\theta)t_H (t_H - t_L)[3(1-\theta)t_H + (1+3\theta)t_L]}{8[(1-\theta)t_H + \theta t_L]^2} & (PC_H) \end{cases}. \tag{2.16}$$

As we have assumed that the foreign firm has all the bargaining power, (Ω_H, Ω_L) must be the solution of the following problem:

$$\max_{(\Omega_H, \Omega_L)} \quad \theta \left(\Pi_2^{HH} - \Omega_H\right) + (1-\theta) \left(\Pi_2^{LL} - \Omega_L\right). \tag{2.17}$$

As the objective function is obviously decreasing in Ω_H and in Ω_L, the optimal contract is:

$$\begin{cases} \Omega_L^v = \frac{\theta t_L(t_H - t_L)[(4-3\theta)t_H + 3\theta t_L]}{8[(1-\theta)t_H + \theta t_L]^2} > 0 \\ \Omega_H^v = -\frac{(1-\theta)t_H(t_H - t_L)[3(1-\theta)t_H + (1+3\theta)t_L]}{8[(1-\theta)t_H + \theta t_L]^2} < 0 \end{cases} . \tag{2.18}$$

This is the best contract for the foreign firm that ensures the participation of the domestic firm. However, this does not mean that the foreign firm is always willing to offer this contract to the rival. When $t = t_L$, the contract could be so costly that the foreign firm may prefer to remain uninformed. To make its decision, the firm compares the expected profit if it proposes the contract:[22]

$$E\Pi_{2,C} = \theta \left(\Pi_2^{HH} - \Omega_H^v\right) + (1-\theta)\left(\Pi_2^{LL} - \Omega_L^v\right)$$

with the expected profit if it does not propose the contract, $E\Pi_{2,AI}$, given in (2.14).

Proposition 2.9. *The foreign firm is always willing to propose the contract given in (2.18).*

Proof. The foreign firm proposes the contract if:

$$E\Pi_{2,C} \geq E\Pi_{2,AI} \Leftrightarrow \frac{7(1-\theta)\theta\left(t_H - t_L\right)^2}{8\left[(1-\theta)t_H + \theta t_L\right]} \geq 0,$$

which is always true. □

With this contract, (Ω_H^v, Ω_L^v), the domestic firm must share part of its profits with the foreign firm when $t = t_H$. The same occurs in the model of Ziv (1993), who also allows a firm to make a "*direct payment [...] to its competitor when it sends a message*". Concerning the realism of such mechanisms, the author argues that: "*it is not unusual to see very*

[22]In the Appendix B.2, we obtain the expressions for the (actual) profit of the foreign firm, either when $t = t_H$ or $t = t_L$, if firms establish the contract.

complicated transactions that may occur in order to hide an illegal transaction between two firms. Simplifying these transactions may show that the firms are just transferring information through such payments."

This contract may raise suspicions of a collusive behavior between firms (with Ω_H and Ω_L being interpreted as side-payments).[23] To avoid this misunderstanding, the foreign firm could modify the contract in the following way: when $t = t_H$, the domestic firm would have to spend part of its profits on advertising or charity, rather than share the profits with the foreign firm. Formally, such a contract (Ω_H, Ω_L), must also satisfy the domestic firm's participation constraints, given in (2.16), but the objective function is slightly modified:

$$\max_{(\Omega_H,\Omega_L)} \quad \theta\left(\Pi_2^{HH} - max\left\{0, \Omega_H\right\}\right) + (1-\theta)\left(\Pi_2^{LL} - max\left\{0, \Omega_L\right\}\right).$$

It is straightforward to see that the solution of this problem is also given by (2.18). The domestic firm is indifferent between accepting this contract or the previous one. The only change is the receiver of the transfer Ω_H. But the foreign firm is less willing to offer this contract, since it pays the same when $t = t_L$, but receives nothing (instead of Ω_H^v) when $t = t_H$. In the next proposition we obtain the conditions under which the foreign firm is interested in proposing this last contract.

Proposition 2.10. *The foreign firm is willing to propose contract* $(0, \Omega_L^v)$*, with* Ω_L^v *given in* (2.18)*, if:*

$$\frac{t_H}{t_L} \geq \frac{8 - 11\theta + \sqrt{64(1-\theta) + 9\theta^2}}{8\,(1-\theta)}.$$

Proof. See the Appendix A. □

[23] In the cases we are considering, there is no collusion between firms, even when firms agree to communicate. Recall that, when $t = t_L$ and $\theta \leq \frac{1}{2}$, both firms would be better off if they did not communicate.

The region of parameters, $\frac{t_H}{t_L}$ and θ, for which it is profitable (at least in expected value) for the foreign firm to offer this last contract is plotted in Figure 2.3.

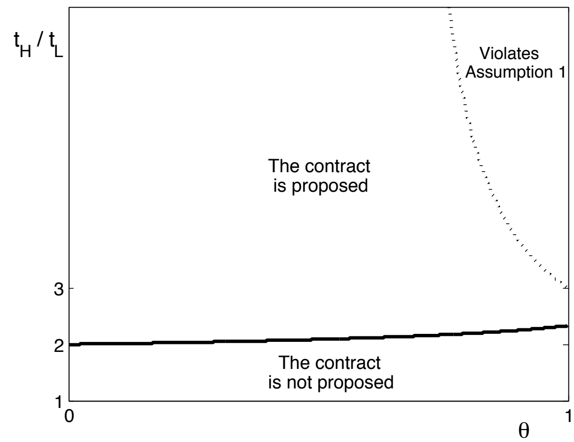

Figure 2.3: Willingness of the foreign firm in proposing the contract $(0, \Omega_L^v)$.

Notice that the foreign firm does not propose this contract if $t_H < 2t_L$.

2.5.2 Unverifiable information

So far, the possibility of a false disclosure of information was ignored. Let us now analyze the case in which the information about t is not verifiable. In this case, the domestic firm is not penalized ex-post if it is caught lying.

Proposition 2.11. *Suppose that the foreign firm always believes in the message it receives from the domestic firm and sets a price accordingly. In this case, the domestic firm always has the incentive to tell that $t = t_H$, regardless of the true value of t.*

Proof. See the Appendix A. □

The domestic firm has the incentive to lie when $t = t_L$, to induce the foreign firm to charge a higher price, which allows itself to set a higher price. Thus, if the domestic firm can send a message without being punished in the case of a false report, it does not represent a reliable source of information.

Let us determine the conditions that a contract, (Ω_H, Ω_L), must satisfy to induce truthful revelation when information is unverifiable.[24] It is necessary to impose two incentive compatibility constraints: the domestic firm must gain by telling the truth either when $t = t_L$ and when $t = t_H$.

Let Π_1^{ij}, for $i, j \in \{H, L\}$, be the profit of firm 1 when $t = t_i$ but the foreign firm thinks that $t = t_j$ and, therefore, charges $p_2 = t_j$. This profit is equal to:[25]

$$\Pi_1^{ij} = \frac{(t_i + t_j)^2}{8t_i}.$$

Using this additional notation, the incentive compatibility constraints can be written as follows:

$$\begin{cases} \Omega_L + \Pi_1^{LL} \geq \Omega_H + \Pi_1^{LH} & (ICC_L) \\ \Omega_H + \Pi_1^{HH} \geq \Omega_L + \Pi_1^{HL} & (ICC_H) \end{cases}.$$

Proposition 2.12. *There is no contract that simultaneously satisfies conditions ICC_L and ICC_H.*

[24]The revelation principle (Myerson, 1979) allows us to restrict the attention to direct mechanisms involving truthful revelation.

[25]The expression is obtained in the proof of proposition 2.11, in the Appendix A.

Proof. A necessary condition to exist a contract satisfying the two incentive compatible constraints is:

$$\Pi_1^{LH} - \Pi_1^{LL} \leq \Pi_1^{HH} - \Pi_1^{HL} \Leftrightarrow \frac{(t_H + t_L)^2}{8t_L} - \frac{t_L}{2} \leq \frac{t_H}{2} - \frac{(t_H + t_L)^2}{8t_H} \Leftrightarrow (t_H - t_L)^2 \leq 0,$$

which is never satisfied. □

As a result, when information is not verifiable, the domestic firm cannot credibly reveal its information. The intuition behind this result is simple. When $t = t_L$, the domestic firm must be compensated from revealing an information that results in a loss of profits ($\Pi_1^{LH} - \Pi_1^{LL}$). This compensation is so high that induces the domestic firm to also announce that $t = t_L$ even when $t = t_H$, in spite of the resulting reduction in profits ($\Pi_1^{HL} - \Pi_1^{HH}$).[26]

To obtain this result we did not assume that it is the foreign firm that has all the bargaining power. We did not even assume that the contract is offered before (or after) the domestic firm observes the value of t. The moment in which the contract is proposed would only change the participation conditions of the domestic firm. If the contract was proposed ex-ante, it should satisfy one (expected) participation constraint: $E\Pi_{1,C} \geq E\Pi_{1,AI}$. If the contract was proposed ex-post, it would have to satisfy the two participation constrains given in (2.16).

2.6 Conclusions

There are several markets in which domestic firms have an informational advantage when competing with foreign firms. Motivated by this, we analyzed the impacts of asymmetric

[26]The two incentive compatibility conditions are incompatible because $\Pi_1^{LH} - \Pi_1^{LL} > \Pi_1^{HH} - \Pi_1^{HL}$.

information about consumers' transportation costs in the model of Hotelling (1929). More precisely, we studied the effects on prices, profits and welfare. We also suggested that a foreign firm could acquire information from a domestic rival. Finally, we anticipated the probable decision of an antitrust authority on allowing or not this type of communication between firms.

We found that the prices under asymmetric information never coincide with the prices charged when both firms have full information. More precisely, when the transportation costs are low (high), the foreign firm sets a higher (lower) price than the rival and, therefore, the domestic firm sells more (less).

In order to evaluate the impacts of asymmetric information on profits, we compared the earnings of the domestic firm with those of its rival. When the transportation costs are low, the domestic firm takes advantage of its private information, achieving higher profits than the rival. However, the domestic firm does not always earn more than the foreign. For instance, if consumers support high transportation costs and the ratio between high and low transportation costs is smaller than three, the foreign firm obtains higher profits.

We also compared the profit of each firm under asymmetric information with that under full information. We noticed that, if the transportation costs are low, the existence of uncertainty can enable both firms to obtain higher profits than in the case of full information. However, when transportation costs are high, asymmetric information hurts the profits of both firms.

To appraise the willingness of the foreign firm in acquiring information, we determined the value of this information for it. We remarked that if the foreign firm offered this amount to the rival in exchange for its information, the domestic firm will always wish to tell that consumers support high transportation costs. Unless there is an external agency involved or the data is verifiable, the domestic firm does not represent a credible source of information. Motivated by this, we tried to design an incentive compatible contract to be proposed by the foreign firm, with which the information would always be truthfully disclosed. We found, however, that such a contract does not exist when information is not verifiable.

In addition, we analyzed the impacts of communication between firms on welfare. Full information yields a Pareto-optimal outcome, with each firm capturing half of the market. Thus, from the social viewpoint, the communication between firms provides an improvement in welfare. In order to isolate the welfare effects on each side of the market, we studied the impacts on the expected consumers' surplus and on expected firms' profits. When the transportation costs are low, the communication between firms is beneficial for them, since they pay less for the product. On the contrary, when the transportation costs are high, their surplus is damaged by communication. Although increasing the expected total surplus, the communication decreases the expected consumers' surplus. That is, firms capture all the increase in social surplus that results from the exchange of information and still extract part of the consumers' surplus. Consequently, an antitrust authority must weight the pros (increase in profits and total surplus) and cons (decrease in consumers' surplus) of allowing communication between rival firms.

Appendices

Appendix A

Proof of Propositions

Proof of Proposition 2.1

Let us search for equilibria in which both firms always have positive demand ($0 < \tilde{x}_L < 1$ and $0 < \tilde{x}_H < 1$). The best-response function of firm 1, when $t = t_i$, is $p_{1i}(p_2) = \frac{p_2+t_i}{2}$. The expression for the expected profit of firm 2 is:

$$E\Pi_2(p_2) = (1-\theta)p_2\left(\frac{1}{2} + \frac{p_{1L}-p_2}{2t_L}\right) + \theta p_2\left(\frac{1}{2} + \frac{p_{1H}-p_2}{2t_H}\right).$$

Thus, its best-response function is:[1]

$$p_2\left(p_{1H}, p_{1L}\right) = \frac{\left(1-\theta\right)p_{1L}t_H + \theta p_{1H}t_L + t_H t_L}{2\left[\left(1-\theta\right)t_H + \theta t_L\right]}.$$

Combining the best-response functions of the two firms and solving the resulting system, we obtain:

$$\begin{cases} p_{1H} = \frac{t_H}{2}\frac{(1-\theta)t_H+(1+\theta)t_L}{(1-\theta)t_H+\theta t_L} \\ \\ p_{1L} = \frac{t_L}{2}\frac{(2-\theta)t_H+\theta t_L}{(1-\theta)t_H+\theta t_L} \\ \\ p_2 = \frac{t_H t_L}{(1-\theta)t_H+\theta t_L} \end{cases}.$$

[1]As $\frac{\partial^2 E\Pi_2}{\partial^2 p_2} = -\frac{(1-\theta)t_H+\theta t_L}{t_H t_L} < 0$, the second order condition is always verified.

Substituting these prices in (2.2) and in (2.3), we find the expressions for the marginal consumer when $t = t_H$ and when $t = t_L$:

$$\tilde{x}_H = \frac{(1-\theta)t_H + (1+\theta)t_L}{4\left[(1-\theta)t_H + \theta t_L\right]} \quad \text{and} \quad \tilde{x}_L = \frac{(2-\theta)\, t_H + \theta t_L}{4\left[(1-\theta)t_H + \theta t_L\right]}.$$

It is straightforward to check that, regardless of the values for θ, t_H and t_L, we always have $0 \leq \tilde{x}_H \leq 1$ and $\tilde{x}_L \geq 0$. However, condition $\tilde{x}_L \leq 1$ is satisfied if and only if:

$$\theta \leq \frac{2}{3}\frac{t_H}{t_H - t_L}.$$

Given the expressions for prices and for demand, the profits of the domestic firm, $\Pi_1^{H\theta}$ and $\Pi_1^{L\theta}$, and of the foreign firm, $\Pi_2^{H\theta}$ and $\Pi_2^{L\theta}$ follow immediately.

□

Proof of Proposition 2.2

As we saw in the proof of Proposition 2.1, if $\theta > \frac{2}{3}\frac{t_H}{t_H - t_L}$, there is no interior equilibrium. It must be the case that either $\tilde{x}_i = 0$ or $\tilde{x}_i = 1$, for some $i \in \{H, L\}$.

(i) Suppose that: $p_{1L} < p_{1H}$.

As the domestic firm charges a lower price when $t = t_L$, its demand is higher in this case: $\tilde{x}_L \geq \tilde{x}_H$. There are no equilibria with $\tilde{x}_H = 1$ (which implies that $\tilde{x}_L \geq 1$). If this were the case, the foreign firm was choosing a price p_2 such that $p_2 > p_{1H} + t_H \geq t_H$ and $p_2 > p_{1L} + t_L \geq t_L$ and it was obtaining a zero (expected) profit. It could choose a lower p_2 and obtain a positive profit. For example, by charging $p_2 = \frac{t_H}{2}$, the foreign firm would have, at least, a positive profit when $t = t_H$ (and, therefore, a positive expected profit). There are also no equilibria such that $\tilde{x}_H = 0$ (since this would imply that $p_{1H} \geq t_H + p_2$, which is incompatible with the domestic firm's best-response function, when $t = t_H$). As a result, the only possibility is to have: $\tilde{x}_L = 1$ and $\tilde{x}_H \in (0, 1)$. In this case, the best-response functions of firm 1 are:

$$p_{1L} = p_2 - t_L \quad \text{and} \quad p_{1H} = \frac{p_2 + t_H}{2}.$$

If $p_2 \neq \frac{p_{1H}+t_H}{2}$, the foreign firm should deviate to $p_2 = \frac{p_{1H}+t_H}{2}$, because this would increase $\Pi_2^{H\theta}$, without decreasing $\Pi_2^{L\theta}$ (which are already null). Combining this condition with the best-response functions of the domestic firm, we obtain:

$$p_{1H} = t_H \quad ; \quad p_{1L} = t_H - t_L \quad \text{and} \quad p_2 = t_H. \tag{A.1}$$

To prove that these prices cannot represent an equilibrium, we only need to verify that the left derivative of $E\Pi_2$ at $p_2 = t_H$ is negative. By slightly decreasing p_2, the foreign firm also achieves a positive demand when $t = t_L$. In this case, its expected profit would be given by:

$$E\Pi_2(p_2) = (1-\theta)p_2\left(\frac{1}{2} + \frac{p_{1L}-p_2}{2t_L}\right) + \theta p_2\left(\frac{1}{2} + \frac{p_{1H}-p_2}{2t_H}\right).$$

Hence,

$$\frac{dE\Pi_2}{dp_2} = (1-\theta)\left(\frac{1}{2} + \frac{p_{1L}-2p_2}{2t_L}\right) + \theta\left(\frac{1}{2} + \frac{p_{1H}-2p_2}{2t_H}\right).$$

Substituting the prices given in $(A.1)$, we obtain:

$$\frac{dE\Pi_2}{dp_2}({t_H}^-) = -(1-\theta)\frac{t_H}{2t_L} < 0,$$

meaning that the foreign firm would increase its profits by slightly decreasing p_2.

(ii) Suppose that $p_{1L} = p_{1H}$.

In this case, $\tilde{x}_H = \tilde{x}_L$. Thus, a corner equilibrium would be an absurd, since it would imply that one of the firms always got zero profits (regardless of the value of t).

(iii) Finally, suppose that $p_{1L} > p_{1H}$ or, equivalently, $\tilde{x}_L < \tilde{x}_H$.

As in the case (i), we can exclude the equilibria for which $\tilde{x}_L = \tilde{x}_H = 1$ and those for which $\tilde{x}_H = 0$ or $\tilde{x}_L = 0$ (by the best-response functions of firm 1). Since $\tilde{x}_H > \tilde{x}_L$, the only equilibrium that remains is: $\tilde{x}_H = 1$ and $\tilde{x}_L \in (0,1)$. In this case, the best-response functions of the firm 1 are:

$$p_{1H} = p_2 - t_H \quad \text{and} \quad p_{1L} = \frac{p_2 + t_L}{2}$$

If $p_2 \neq \frac{p_{1L}+t_L}{2}$, the firm 2 should deviate and choose $p_2 = \frac{p_{1L}+t_L}{2}$. By doing so, the foreign firm would increase $\Pi_2^{L\theta}$, without decreasing $\Pi_2^{H\theta}$ (already equal to zero). However, combining this expression for p_2 with the conditions for p_{1L} and p_{1H} determined above, we obtain $p_{1H} = t_L - t_H < 0$, which is an absurd.

□

Proof of Proposition 2.3

Using the expressions (2.4), for prices with asymmetric information, it follows that:

$$p_{1H} < t_H \Leftrightarrow \frac{t_H}{2}\frac{(1-\theta)\,t_H+(1+\theta)t_L}{(1-\theta)t_H+\theta t_L} < t_H \Leftrightarrow t_H > t_L$$

and

$$p_{1L} > t_L \Leftrightarrow \frac{t_L}{2}\frac{(2-\theta)\,t_H+\theta t_L}{(1-\theta)t_H+\theta t_L} > t_L \Leftrightarrow t_H > t_L.$$

Moreover,

$$\begin{aligned} p_{1H} > p_{1L} \quad &\Leftrightarrow \quad \frac{t_H}{2}\frac{(1-\theta)\,t_H+(1+\theta)t_L}{(1-\theta)t_H+\theta t_L} > \frac{t_L}{2}\frac{(2-\theta)\,t_H+\theta t_L}{(1-\theta)t_H+\theta t_L} \\ &\Leftrightarrow \quad (t_H - t_L)\left[(1-\theta)\,t_H+\theta t_L\right] > 0. \end{aligned}$$

Comparing the prices charged by each firm when $t = t_H$, we obtain:

$$p_{1H} > p_2 \Leftrightarrow \frac{t_H}{2}\frac{(1-\theta)\,t_H+(1+\theta)t_L}{(1-\theta)t_H+\theta t_L} > \frac{t_H t_L}{(1-\theta)t_H+\theta t_L} \Leftrightarrow t_H > t_L.$$

When $t = t_L$, the condition

$$p_2 > p_{1L} \Leftrightarrow \frac{t_H t_L}{t_H(1-\theta)+\theta t_L} > \frac{t_L}{2}\frac{(2-\theta)\,t_H+\theta t_L}{(1-\theta)t_H+\theta t_L} \Leftrightarrow t_H > t_L$$

is always true.

□

Proof of Proposition 2.4

When $t = t_L$, the domestic firm obtains higher profits than the rival, since:

$$\Pi_1^{L\theta} \geq \Pi_2^{L\theta} \quad \Leftrightarrow \quad \frac{t_L}{8}\left[\frac{(2-\theta)\,t_H + \theta t_L}{(1-\theta)\,t_H + \theta t_L}\right]^2 \geq \frac{t_H t_L}{4}\frac{(2-3\theta)\,t_H + 3\theta t_L}{\left[(1-\theta)\,t_H + \theta t_L\right]^2}$$

$$\Leftrightarrow \quad \left[(2-\theta)\,t_H + \theta t_L\right]^2 - 2t_H\left[(2-3\theta)\,t_H + 3\theta t_L\right] \geq 0$$

$$\Leftrightarrow \quad \theta\,(t_H - t_L)\left[2t_H + \theta\,(t_H - t_L)\right] \geq 0.$$

The same occurs when $t = t_H$ if:

$$\Pi_1^{H\theta} \geq \Pi_2^{H\theta} \quad \Leftrightarrow \quad \frac{t_H}{8}\left[\frac{(1-\theta)\,t_H + (1+\theta)\,t_L}{(1-\theta)\,t_H + \theta t_L}\right]^2 \geq \frac{t_H t_L}{4}\frac{3\,(1-\theta)\,t_H + (3\theta-1)\,t_L}{\left[(1-\theta)\,t_H + \theta t_L\right]^2}$$

$$\Leftrightarrow \quad \left[(1-\theta)\,t_H + (1+\theta)\,t_L\right]^2 - 2t_L\left[3\,(1-\theta)\,t_H + (3\theta-1)\,t_L\right] \geq 0$$

$$\Leftrightarrow \quad (t_H - t_L)\,(1-\theta)\,(t_H - 3t_L - \theta t_H + \theta t_L) \geq 0$$

$$\Leftrightarrow \quad \frac{t_H}{t_L} \geq \frac{3-\theta}{1-\theta}.$$

$\square$

Proof of Proposition 2.5

When $t = t_H$, the domestic firm always profits more in the case of perfect information since:

$$\Pi_1^{H\theta} \leq \Pi_1^{HH} \quad \Leftrightarrow \quad \frac{t_H}{8}\left[\frac{(1-\theta)\,t_H + (1+\theta)\,t_L}{(1-\theta)\,t_H + \theta t_L}\right]^2 \leq \frac{t_H}{2}$$

$$\Leftrightarrow \quad \left[\frac{(1-\theta)\,t_H + (1+\theta)\,t_L}{(1-\theta)\,t_H + \theta t_L} - 2\right]\left[\frac{(1-\theta)\,t_H + (1+\theta)\,t_L}{(1-\theta)\,t_H + \theta t_L} + 2\right] \leq 0$$

$$\Leftrightarrow \quad \frac{(1-\theta)\,t_H + (1+\theta)\,t_L}{(1-\theta)\,t_H + \theta t_L} - 2 \leq 0$$

$$\Leftrightarrow \quad (t_H - t_L)\,(\theta - 1) \leq 0$$

is always true. The converse happens when $t = t_L$, since:

$$\Pi_1^{L\theta} \geq \Pi_1^{LL} \quad \Leftrightarrow \quad \frac{t_L}{8}\left[\frac{(2-\theta)\,t_H + \theta t_L}{(1-\theta)\,t_H + \theta t_L}\right]^2 \geq \frac{t_L}{2}$$

$$\Leftrightarrow \quad \left[\frac{(2-\theta)\,t_H + \theta t_L}{(1-\theta)\,t_H + \theta t_L} - 2\right]\left[\frac{(2-\theta)\,t_H + \theta t_L}{(1-\theta)\,t_H + \theta t_L} + 2\right] \geq 0$$

$$\Leftrightarrow \quad \theta\,(t_H - t_L) \geq 0$$

is also a universal condition. □

Proof of Proposition 2.6

When $t = t_H$, the foreign firm obtains lower profits with asymmetric information than with perfect information if:

$$\Pi_2^{H\theta} \leq \Pi_2^{HH} \quad \Leftrightarrow \quad \frac{t_H t_L}{4}\frac{3\,(1-\theta)\,t_H + (3\theta - 1)\,t_L}{\left[(1-\theta)\,t_H + \theta t_L\right]^2} \leq \frac{t_H}{2}$$

$$\Leftrightarrow \quad t_L\left[3\,(1-\theta)\,t_H + (3\theta-1)\,t_L\right] \leq 2\left[(1-\theta)\,t_H + \theta t_L\right]^2$$

$$\Leftrightarrow \quad (t_H - t_L)\,(1-\theta)\,(t_L - 2t_H + 2\theta t_H - 2\theta t_L) \leq 0.$$

As $t_H > t_L$ and $\theta \in (0,1)$, the inequality above is equivalent to:

$$(1-2\theta)\,t_L - 2\,(1-\theta)\,t_H \leq 0 \;\Leftrightarrow\; t_H \geq \frac{1-2\theta}{2\,(1-\theta)}t_L\;, \tag{A.2}$$

which is always true, since, $\forall \theta \in (0,1)$ we have that $\frac{1-2\theta}{2(1-\theta)}t_L < t_L$.

When $t = t_L$, the foreigh firm profits more with asymmetric information than with perfect information if:

$$
\begin{aligned}
\Pi_2^{L\theta} \geq \Pi_2^{LL} \quad &\Leftrightarrow \quad \frac{t_L}{4} t_H \frac{(2-3\theta)\, t_H + 3\theta t_L}{[(1-\theta)\, t_H + \theta t_L]^2} \geq \frac{t_L}{2} \\
&\Leftrightarrow \quad t_H \left[(2-3\theta)\, t_H + 3\theta t_L\right] \geq 2 \left[(1-\theta)\, t_H + \theta t_L\right]^2 \\
&\Leftrightarrow \quad \theta \left(t_H - t_L\right) \left(t_H - 2\theta t_H + 2\theta t_L\right) \geq 0 \\
&\Leftrightarrow \quad (1-2\theta)\, t_H + 2\theta t_L \geq 0.
\end{aligned}
$$

The inequality above is equivalent to:

$$
\begin{cases}
t_H \geq -\frac{2\theta}{1-2\theta} t_L & \text{if} \quad \theta < \frac{1}{2} \\
t_L \geq 0 & \text{if} \quad \theta = \frac{1}{2} \\
t_H \leq \frac{2\theta}{2\theta-1} t_L & \text{if} \quad \theta > \frac{1}{2}
\end{cases} .
$$

When $\theta \leq \frac{1}{2}$, the conditions for t_H are verified. When $\theta > \frac{1}{2}$, the interval $\left] t_L, \frac{2\theta}{2\theta-1} t_L \right]$ is non empty and:[2]

$$
\begin{cases}
\Pi_2^{L\theta} \geq \Pi_2^{LL} & \text{if} \quad \frac{t_H}{t_L} \in \left] 1, \frac{2\theta}{2\theta-1} \right] \\
\Pi_2^{L\theta} \leq \Pi_2^{LL} & \text{if} \quad \frac{t_H}{t_L} \in \left] \frac{2\theta}{2\theta-1}, \frac{3-\theta}{1-\theta} \right[
\end{cases} .
$$

□

Proof of Proposition 2.7

When $t = t_L$, the scenario of symmetric information is beneficial for consumers because firms charge a lower price and, as $\tilde{x} = \frac{1}{2}$, the aggregate transportation cost is lower.

[2] Recall that the Assumption 2.1 must be verified.

Let CS^H_{AI} and CS^H_{SI} be the consumers' surplus with asymmetric information and with symmetric information, respectively, when $t = t_H$. Using the definition of consumers' surplus, given in (2.10), we obtain:

$$\begin{cases} CS^H_{AI} = V - p_2 + \tilde{x}_H \left(p_2 - p_{1H}\right) - TC(\tilde{x}_H, t_H) \\ CS^H_{SI} = V - t_H - TC\left(\frac{1}{2}, t_H\right) \end{cases}, \tag{A.3}$$

where:

$$TC(\tilde{x}_H, t_H) = \frac{t_H \left[7\left(1-\theta\right)^2 t_H^2 - 2\left(7\theta^2 - 20\theta + 3\right) t_H t_L + \left(7\theta^2 - 6\theta + 3\right) t_L^2\right]}{48\left[(1-\theta)t_H + \theta t_L\right]^2}$$

and:

$$TC\left(\frac{1}{2}, t_H\right) = \frac{t_H}{12}.$$

Substituting the expressions for TC in $(A.3)$, we obtain:

$$CS^H_{SI} < CS^H_{AI} \quad \Leftrightarrow \quad \frac{1}{16} t_H \left(t_H - t_L\right) \frac{1-\theta}{\left[(1-\theta)t_H + \theta t_L\right]^2} \left(t_L - 13t_H + 13\theta t_H - 13\theta t_L\right) < 0$$

$$\Leftrightarrow \quad t_H > t_L - \frac{12 t_L}{13\left(1-\theta\right)},$$

which is always true.

$\square$

Proof of Proposition 2.8

The expressions for CS^H_{SI} and CS^H_{AI} were already determined in $(A.3)$. Analogously, when $t = t_L$, the consumers' surplus with asymmetric information and with perfect information are given by:

$$\begin{cases} CS^L_{AI} = V - p_2 + \tilde{x}_L \left(p_2 - p_{1L}\right) - TC(\tilde{x}_L, t_L) \\ CS^L_{SI} = V - t_L - TC\left(\frac{1}{2}, t_L\right) \end{cases},$$

where:

$$TC(\tilde{x}_L, t_L) = \frac{t_L \left[\left(7\theta^2 - 8\theta + 4\right) t_H^2 + 2\theta\left(4 - 7\theta^2\right) t_H t_L + 7\theta^2 t_L^2\right]}{48\left[(1-\theta)t_H + \theta t_L\right]^2}$$

and:

$$TC\left(\frac{1}{2}, t_L\right) = \frac{t_L}{12}.$$

Substituting these expressions in the definition of EB_C, given in (2.11), we obtain:

$$EB_C = -\frac{13}{16}\left(t_H - t_L\right)^2 \frac{\theta\left(1-\theta\right)}{\left(1-\theta\right)t_H + \theta t_L},$$

which is always negative.

$\square$

Proof of Proposition 2.10

If the foreign firm proposes contract $(0, \Omega_L^v)$, its expected profit is equal to:

$$E\Pi_{2,C} = \theta \Pi_2^{HH} + (1-\theta)\left(2\Pi_2^{LL} - \Pi_1^{L\theta}\right).$$

The firm prefers this scenario to that of asymmetric information if $E\Pi_{2,C} \geq E\Pi_{2,AI}$, where $E\Pi_{2,AI}$ is given by (2.14). This happens if:

$$E\Pi_{2,C} - E\Pi_{2,AI} \geq 0 \quad \Leftrightarrow \quad \frac{(1-\theta)\theta\left(t_H - t_L\right)\left[4(1-\theta)t_H^2 - (8-11\theta)t_H t_L - 7\theta t_L^2\right]}{8\left((1-\theta)t_H + \theta t_L\right)^2} \geq 0$$

$$\Leftrightarrow \quad 4(1-\theta)\left(\frac{t_H}{t_L}\right)^2 - (8-11\theta)\frac{t_H}{t_L} - 7\theta \geq 0$$

As $t_H > t_L$, the inequality is verified if:

$$\frac{t_H}{t_L} \geq \frac{8 - 11\theta + \sqrt{64(1-\theta) + 9\theta^2}}{8(1-\theta)}.$$

$\square$

Proof of Proposition 2.11

Suppose that $t = t_L$. If the domestic firm announces that $t = t_H$, the foreign firm sets $p_2 = t_H$. Substituting this price on the best-response function of firm 1 when $t = t_L$, we obtain:

$$p_1^{LH} = \frac{t_H + t_L}{2}.$$

By charging this price, the domestic firm has a demand equal to:

$$\tilde{x}_L = \frac{t_H + t_L}{4t_L}.$$

Thus, the profit of the domestic firm is:

$$\Pi_1^{LH} = \frac{(t_H + t_L)^2}{8t_L}.$$

The firm is interested in lying about t if:

$$\Pi_1^{LL} > \Pi_1^{LH} \Leftrightarrow \frac{t_L}{2} > \frac{(t_H + t_L)^2}{8t_L} \Leftrightarrow (t_L - t_H)(3t_L + t_H) > 0 \ ,$$

which is always true.

Similarly, when $t = t_H$ but the foreign firm thinks that $t = t_L$, the profit of the domestic firm is equal to:

$$\Pi_1^{HL} = \frac{(t_H + t_L)^2}{8t_H}.$$

In this case, the domestic firm chooses to tell the truth since:

$$\Pi_1^{HH} > \Pi_1^{HL} \Leftrightarrow \frac{t_H}{2} > \frac{(t_H + t_L)^2}{8t_H} \Leftrightarrow (t_H - t_L)(3t_H + t_L) > 0$$

is always verified. $\square$

Appendix B

Net profits of the foreign firm

B.1 Ex-ante exchange

Consider that firms establish the contract before the domestic firm observes the value of t. As we saw before, if the foreign firm has all the bargaining power, the domestic firm has to pay $|\Omega^v| = \Omega_1$, given in (2.15), to reveal its (future) information.

Let $\Pi^i_{2,C}$, for $i \in \{H, L\}$, denote the profit of firm 2 if the contract is established and Nature chooses $t = t_i$. Therefore:

$$\Pi^H_{2,C} = \Pi^{HH}_2 + \Omega_1 = \frac{\left(4 - \theta - 3\theta^2\right) t_H^2 + 2\theta(3\theta - 1)t_H t_L + 3\theta(1-\theta)t_L^2}{8\left[(1-\theta)t_H + \theta t_L\right]}$$

and

$$\Pi^L_{2,C} = \Pi^{LL}_2 + \Omega_1 = \frac{3(1-\theta)\theta t_H^2 + 2\left(2 - 5\theta + 3\theta^2\right) t_H t_L + \theta(7 - 3\theta)t_L^2}{8\left[(1-\theta)t_H + \theta t_L\right]}.$$

B.2 Ex-post exchange

Consider that firms establish the contract after the domestic firm observes the value of t. Recall that we have assumed that it is the foreign firm has all the bargaining power. Thus, if the domestic firm accepts the contract, it has to pay $|\Omega_H^v|$, given in (2.18), to the rival when it announces that $t = t_H$. However, if the domestic firm reveals that $t = t_L$, it is the foreign firm that pays Ω_L^v to the domestic firm.

By $\Pi_{2,C}^i$, for $i \in \{H, L\}$, we denote the profit of firm 2 if the contract is established and the domestic firms says that $t = t_i$. By making some simple calculations, we obtain:

$$\Pi_{2,C}^H = \Pi_2^{HH} + |\Omega_H^v| = \frac{7(1-\theta)^2 t_H^2 - 2\left(1 - 8\theta + 7\theta^2\right) t_H t_L + \left(7\theta^2 - 2\theta - 1\right) t_L^2}{8\left[(1-\theta)t_H + \theta t_L)\right]^2}\, t_H$$

and

$$\Pi_{2,C}^L = \Pi_2^{LL} - \Omega_L^v = \frac{\left(4 - 12\theta + 7\theta^2\right) t_H^2 + 2\theta(6 - 7\theta) t_H t_L + 7\theta^2 t_L^2}{8\left[(1-\theta)t_H + \theta t_L)\right]^2}\, t_L.$$

Chapter 3

Spatial competition between shopping centers

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3.1 Introduction

Shopping centers have existed for many centuries as galleries, market squares, bazaars or seaport districts. The oldest indoor space where consumers can buy a huge variety of goods is the *Al-Hamidiyah Souq*, in Damascus (Syria), and dates back to the seventh century. Today, shopping centers are responsible for a very significant fraction of consumer spending. In 2005, the sales revenues of all the shopping centers located in the United States totalled \$1.5 trillion (corresponding to an average of \$5000 *per capita*).[1]

One of the reasons why shopping centers are so attractive is that they allow consumers to buy many different kinds of goods without spending much time and money commuting between shops. Therefore, to study competition between shopping centers, one should take into account the demand for many different goods and also the cost of traveling to one or more shopping centers. Most of the existing spatial competition models fail to do so, because they either restrict the analysis to markets with a single good or assume that consumers make all their purchases at the same place (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006). This "one-stop shopping" assumption is very convenient because it allows treating multiple goods as a single bundled good.

We provide a study of competition between shopping centers by extending the standard model of spatial competition (Hotelling, 1929; d'Aspremont *et al.*, 1979) to the case of multiple goods without assuming "one-stop shopping". This extension is straightforward in concept but technically difficult. We consider the existence of two shopping centers located at the extremes of a linear city, selling the same set of goods. Consumers are uniformly spread across the city and buy exactly one unit of each good. They may travel to a shopping

[1] http://www.statemaster.com/graph/lif_sho_mal_est_sal-lifestyle-shopping-malls-estimated-sales.

center and buy all the goods there or travel to both shopping centers and buy each good where it is cheaper.[2]

A shopping center may be either a shopping mall (where each good is sold by an independent firm) or a department store (where a single firm sells all the goods).[3] We solve for the equilibrium prices, market shares and profits in three scenarios of retail organization: (i) competition between a department store and a shopping mall; (ii) competition between two department stores; (iii) competition between two shopping malls.

In the case of competition between a department store and a shopping mall, we find that there may be consumers visiting the two extremes of the city or not, depending on the number of goods that are sold by the shopping centers. If there are up to four goods, all the consumers make their purchases at a single place ("one-stop shopping"). If there are between seven and eleven goods, some consumers are willing to travel to both extremes of the city to buy each good where it is cheaper ("two-stop shopping"). In this case, there is only one good that is cheaper at the shopping mall than at the department store. However, its price is low enough for some consumers to travel there just to buy this good.[4]

Regardless of the number of goods, the price of the bundle is lower at the department store than at the shopping mall. This occurs because the otherwise unrelated goods become complements when they are sold at the same location (and substitutes when they are sold at different extremes of the city). When a shop at the mall considers the possibility of decreasing its price, it only cares about the increase of its own demand and not about the

[2]Consumers are assumed to be fully informed about the prices charged in each extreme of the city.

[3]We rule out bundling strategies (we assume that the price of a bundle of goods is equal to the sum of the price of the individual goods). For an analysis of the bundle pricing problem, see, for example, Hanson and Martin (1990).

[4]If there are five, six or more than eleven goods, there is no price equilibrium in pure strategies.

increase of the demand of the other shops at the mall. In contrast, the department store internalizes this effect, and takes into account that decreasing the price of one good also increases the demand for its other goods. In spite of charging a lower price for the bundle, the department store obtains a higher profit than the shops at the mall taken together.

The scenario in which prices are lower is that of competition between two department stores. In this case, the price charged for the bundle of goods is equal to the price charged in the single-good model (independently of the number of goods). The two department stores obviously capture equal shares of the market and obtain equal profits. These are, unsurprisingly, lower than the profits obtained when competing against a shopping mall.

Finally, in the scenario of competition between two shopping malls, we find that each good is sold at the same price as in the single-good model. The shops behave as if consumers only bought their good. This is the competitive scenario in which prices are higher. The explanation is the same as before: the shops at the mall set the same price as in the single-good model because they do not internalize the positive effect of a price decrease on the other shops at the same mall.

After finding the equilibrium prices in each of the three competitive scenarios, it is straightforward to analyze whether it is more profitable to have a department store offering many products or several independent shops at a mall.[5] We answer this question by considering a two-stage game in which the shopping centers simultaneously choose their modes of retail and then compete in prices. We find that, if the number of goods is lower than five, it is a dominant strategy to be organized as a shopping mall rather than as a department store. Therefore, the competitive scenario that is expected to appear in

[5]Since otherwise unrelated goods become complements when they are sold at the same shopping center, this question is related to the literature on mergers between firms that sell complementary goods. See, for example, Matutes and Regibeau (1992), Economides and Salop (1992) or Bart (2008).

equilibrium is that of competition between two shopping malls. However, if the number of goods is between seven and eleven, there is another equilibrium, which is Pareto-inferior: competition between two department stores.

As explained previously, a department store has stronger incentives to charge lower prices than the independent shops at a mall. If the prices of the rival retailers remained the same, the greater aggressiveness of the department store would be profitable. However, setting lower prices induces the rivals to lower their prices as well. If the number of goods is low, this effect dominates, leading to lower profits for everyone. The reason why both sides win when a department store separates into several independent shops was explained by Innes (2006): "*a multi-product retailer can effectively pre-commit to higher prices by organizing itself as a mall of independent outlets*". If the number of goods is high, it becomes more profitable to compete against a department store by behaving as a department store. But it is still better to compete against a shopping mall by behaving as a shopping mall. This is why the endogenous modes of retail may be two department stores or two shopping malls.

In the literature, the first related result was presented by Edgeworth (1925), who found that it is better, for consumers, to have a single monopolist selling two complementary goods than to have two separate monopolists. More recently, Salant *et al.* (1983) also came up with a similar result, but in a model of Cournot competition. Using a framework that is closer to ours, Bertrand competition with linear demand, Beggs (1994) concluded that separating into several shops at a mall may be desirable or not, depending on the degree of substitutability between the goods sold at the competing shopping centers (either two department stores or two shopping malls may emerge in equilibrium). Innes (2006) studied the effect of entry and concluded that only department stores survive in equilibrium because they compete more aggressively and, therefore, are more effective in deterring entry. Shopping malls would be driven out of the market by department stores because when there is competition between department stores and shopping malls, the former have higher profits.

We also compare the consumers' surplus and the total surplus in the different competitive scenarios. Since all the consumers are assumed to buy exactly one unit of each good, a change in prices simply transfers surplus between consumers and producers. Therefore, total surplus is maximized when consumers shop at the closest shopping center (transportation costs are minimized). This occurs when there are either two department stores or two shopping malls. Unsurprisingly, the consumers' surplus is the highest in the case of competition between two department stores. Competition between two shopping malls is actually the worst scenario for consumers. In spite of having to support higher transportation costs, consumers are better off when there is a department store and a shopping mall than when there are two shopping malls.

Our model is pioneer in extending the spatial competition model (Hotelling, 1929; d'Aspremont *et al.* 1979) to analyze multi-product competition between department stores and shopping malls. To the best of our knowledge, only Lal and Matutes (1989) have presented a multi-product version of the model of Hotelling (1929).[6] They restricted the analysis to the case of competition between two department stores that sell two goods. We have generalized their analysis by allowing a finite number of goods and an alternative mode of retail: the shopping mall.[7]

[6]In the model of Lal and Matutes (1989), there are two types of consumers: the poor and the rich. The poor do not support transportation costs, therefore, they buy each good where it is cheaper ("one-stop shopping" is not assumed). The rich, on the other hand, support transportation costs and, in equilibrium, are not interested in shopping around. Their focus is to study price discrimination across the two segments.

[7]There are other extensions of the spatial competition model that allow for multi-product firms, but in which consumers only buy one of the goods that are available (Laussel, 2006; Giraud-Heraud *et al.*, 2003). Goods available in a shopping center are, in this case, substitutes instead of complements. These models correspond to completely different economic settings.

Klemperer (1992) also proposed a model of a multi-product duopoly in which consumers decide whether to buy all the goods from a single seller or to support an additional cost to buy goods from the two sellers. He found that if the product lines of the sellers are differentiated, some consumers make "two-stop shopping" to benefit from a greater variety of goods. When the product lines are identical, the motive for "two-stop shopping" disappears. Consumers never make "two-stop shopping" to take advantage of price differences (as they do in our model).[8]

Other authors have analyzed multi-product price competition, but did not use the spatial competition model to do so. Moreover, most of them based the analysis on the assumption that consumers make all their purchases at the same shopping center (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006).[9] They support this "one-stop shopping" assumption on the fact that shopping implies time and transportation costs. In our opinion, even with the support of empirical works as the one of Rhee and Bell (2002), who have found that consumers make 94% of their weekly groceries expenditures at the same supermarket, it is of interest to relax the assumption that consumers necessarily make all their purchases at the same place.

The work that is closest to ours is perhaps that of Beggs (1994), who studied a model in

[8]The structure of the model of Klemperer (1992) implies that consumers always buy equal amounts of every good that is sold by the firms - they buy the whole product line. This prevents the model from capturing a dimension of multi-product competition that is crucial to our results, which is the fact that a firm can sell some goods at a low price to attract consumers and take advantage of commuting costs to be able to sell the other goods at a high price. Given the focus of his contribution, this is perfectly acceptable. The main point of his paper is that the sellers may choose the same product line to decrease the level of competition!

[9]An exception is a model proposed by Thill (1992) in which a firm that sells two goods competes against a firm that sells only one of the goods. He concluded that some of the consumers that need to buy both goods end up making "two-stop shopping".

which firms start by choosing their mode of retail and then compete in prices facing a linear demand function. He restricted the analysis to the case of two goods and, as mentioned before, assumed that consumers purchase both goods at the same location ("one-stop shopping" assumption). Smith and Hay (2005) have also studied price competition under alternative modes of retail organization (shopping streets, shopping malls and department stores), but did not consider competition between different modes of retail.

The remainder of the article is organized as follows. In Section 3.2, we setup the model, introduce notation and obtain the demand and the profit functions. In Section 3.3, we present the possible competitive scenarios and find the equilibrium prices in each one. We study the merger game in Section 3.4. Section 3.5 is dedicated to a welfare analysis. Section 3.6 concludes the chapter with some remarks. Most of the proofs are collected in the Appendix C.

3.2 The model

3.2.1 Basic setup

We consider a multi-product version of the model of Hotelling (1929). There is a continuum of consumers uniformly distributed across a linear city, $[0, 1]$. Each consumer buys one unit of each of the products, $i \in \{1, ..., n\} = \mathcal{I}$, which are sold at the extremes of the city ($x = 0$ and $x = 1$). The price of good i at the left extreme (L) is denoted by p_{iL} and the price of good i at the right extreme (R) is denoted by p_{iR}.

The reservation price for each product, V_i, is assumed to be high enough for the market to be fully covered. Thus, the demand is perfectly inelastic and the only decision of consumers is where to buy each product. Each consumer chooses among three possibilities:

(L) to buy all the goods at $x = 0$;

(R) to buy all the goods at $x = 1$;

(LR) to travel to both extremes and buy each good where it is cheaper.

We denote by P_L and by P_R the price that a consumer pays for all the goods at $x = 0$ and at $x = 1$, respectively ($P_L = \sum_{i=1}^{n} p_{iL}$ and $P_R = \sum_{i=1}^{n} p_{iR}$). By P_{LR}, we denote the price that a consumer pays for all the goods if she buys each good where it is cheaper ($P_{LR} = \sum_{i=1}^{n} \min\{p_{iL}, p_{iR}\}$).

To make their decision, consumers take into account not only the prices charged for the products, but also the transportation costs that they must support to acquire them. We assume that the transportation costs are linear in distance. Let $u_L(x)$, $u_R(x)$ and $u_{LR}(x)$ denote the utility attained by an agent located at $x \in [0, 1]$ who chooses to purchase, respectively: (L) all the goods at $x = 0$; (R) all the goods at $x = 1$; (LR) each good where it is cheaper. Then:

$$\begin{aligned}
u_L(x) &= \textstyle\sum_{i=1}^{n} V_i - P_L - tx, \\
u_R(x) &= \textstyle\sum_{i=1}^{n} V_i - P_R - t(1-x), \\
u_{LR}(x) &= \textstyle\sum_{i=1}^{n} V_i - P_{LR} - t.
\end{aligned}$$

It is important to keep in mind that if a consumer travels to both extremes, she supports higher transportation costs than if she had chosen to purchase all the goods at the same location. For this reason, the demand for each product at a certain location is related to the demand for any other product at any location. Products sold at the same location are complementary goods, while products sold at different locations are substitutes.

3.2.2 Demand and profit functions

The consumers that are most likely to purchase a good that is sold at one of the extremes are those who are located closer to that extreme. When all the goods have strictly positive demand at both locations, the consumers near the left extreme are surely buying all the goods at $x = 0$ (their choice is L), while those near the right extreme are surely buying all the goods at $x = 1$ (their choice is R).

Depending on the prices charged for each good at each location, some consumers may find it worthwhile to travel to both extremes of the city, to buy each good where it is cheaper. This occurs if some goods are sufficiently cheaper at $x = 0$ while other goods are sufficiently cheaper at $x = 1$. On the contrary, if the price differences across locations are relatively small, then all the consumers make their purchases at a single location, either at $x = 0$ or at $x = 1$.

These possible demand scenarios are illustrated in Figure 3.1.

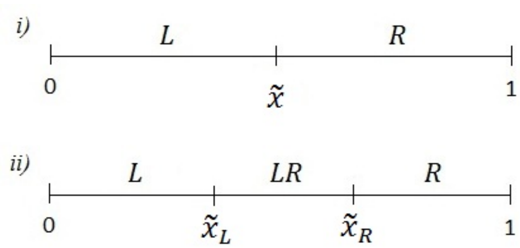


Figure 3.1: Possible demand scenarios.

To obtain the demand for each good at each location, it is useful to find the location of the consumer that is indifferent between each pair of choices (among L, R and LR). Accordingly, we use some additional notation.

By $\tilde{x}_L$, we denote the location of the consumer that is indifferent between L and LR:

$$u_L(\tilde{x}_L) = u_{LR}(\tilde{x}_L) \Leftrightarrow \tilde{x}_L = 1 - \frac{P_L - P_{LR}}{t}. \tag{3.1}$$

We denote by $\tilde{x}_R$ the consumer that is indifferent between R and LR:

$$u_R(\tilde{x}_R) = u_{LR}(\tilde{x}_R) \Leftrightarrow \tilde{x}_R = \frac{P_R - P_{LR}}{t}. \tag{3.2}$$

Finally, we denote by $\tilde{x}$ the consumer that is indifferent between L and R. It is clear from the expression below that $\tilde{x} = \frac{\tilde{x}_L + \tilde{x}_R}{2}$:

$$u_L(\tilde{x}) = u_R(\tilde{x}) \Leftrightarrow \tilde{x} = \frac{1}{2} + \frac{P_R - P_L}{2t}. \tag{3.3}$$

There are consumers traveling to both extremes of the city if $\tilde{x}_L < \tilde{x}_R$, which is equivalent to $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| > t$. Otherwise, all the consumers make their purchases at a single place. It is easy to verify that $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq t$ implies that $0 \leq \tilde{x} \leq 1$. Therefore, in this case, the demand for each good sold at L is $\tilde{x}$ and the demand for each good sold at R is $1 - \tilde{x}$.

It is convenient to denote the vector of prices of all the goods at both locations by $\mathbf{p} \in \mathbb{R}_+^{2n}$ and to consider the following sets:

$$S_1 = \left\{ \mathbf{p} \in \mathbb{R}_+^{2n} : \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq t \right\};$$
$$S_2 = \left\{ \mathbf{p} \in \mathbb{R}_+^{2n} : \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| > t \right\}.$$

If there are consumers that travel to both extremes, the demand for a good depends on whether this good is cheaper at L or at R. Denoting by $\mathcal{I}_L$ and $\mathcal{I}_R$ the sets of goods that are strictly cheaper at L and R, respectively, we can write the expressions for the indifferent consumers as follows:

$$\tilde{x}_L = 1 - \frac{1}{t} \sum_{i \in \mathcal{I}_R} (p_{iL} - p_{iR}) \tag{3.4}$$

and

$$\tilde{x}_R = \frac{1}{t} \sum_{i \in \mathcal{I}_L} (p_{iR} - p_{iL}). \tag{3.5}$$

The demand for a good $i \in \mathcal{I}_L$ at L is $min\{\tilde{x}_R, 1\}$, while its demand at R is $max\{0, 1 - \tilde{x}_R\}$. If $i \in \mathcal{I}_R$, its demand at L is $max\{0, \tilde{x}_L\}$ and its demand at R is $min\{1 - \tilde{x}_L, 1\}$. In case of a tie ($p_{iL} = p_{iR}$), each consumer that travels to both extremes may either buy good i at L or at R. Any tie-breaking assumption leads to the same results. We can assume, for example, that half of the consumers buys good i at L and the other half buys it at R.

The demand for good i at $x = 0$ is:

$$q_{iL} = \begin{cases} \tilde{x} & \text{if} \quad \mathbf{p} \in S_1 \\ \min\{\tilde{x}_R, 1\} & \text{if} \quad \mathbf{p} \in S_2 \ \wedge \ p_{iL} < p_{iR} \\ \frac{1}{2}\left(\min\{\tilde{x}_R, 1\} + \max\{0, \tilde{x}_L\}\right) & \text{if} \quad \mathbf{p} \in S_2 \ \wedge \ p_{iL} = p_{iR} \\ \max\{0, \tilde{x}_L\} & \text{if} \quad \mathbf{p} \in S_2 \ \wedge \ p_{iL} > p_{iR} \end{cases},$$

while the demand for the same good at $x = 1$ is $q_{iR} = 1 - q_{iL}$.

Without loss of generality, the marginal cost of producing one unit of each of the goods is assumed to be zero. Under this assumption, the profits coincide with the sales revenues. This simplification does not affect any of the results in the paper.[10]

The profit that results from selling good i at $x = 0$ is:

$$\Pi_{iL} = \begin{cases} p_{iL}\left(\frac{1}{2} + \frac{P_R - P_L}{2t}\right) & \text{if} \quad \mathbf{p} \in S_1 \\ p_{iL}\min\left\{\frac{P_R - P_{LR}}{t}, 1\right\} & \text{if} \quad \mathbf{p} \in S_2 \ \wedge \ p_{iL} < p_{iR} \\ \frac{p_{iL}}{2}\left(\min\left\{\frac{P_R - P_{LR}}{t}, 1\right\} + \max\left\{0, 1 - \frac{P_L - P_{LR}}{t}\right\}\right) & \text{if} \quad \mathbf{p} \in S_2 \ \wedge \ p_{iL} = p_{iR} \\ p_{iL}\max\left\{0, 1 - \frac{P_L - P_{LR}}{t}\right\} & \text{if} \quad \mathbf{p} \in S_2 \ \wedge \ p_{iL} > p_{iR} \end{cases}.$$

[10]To obtain the equilibrium prices for the case in which the marginal costs are different from zero (being equal across locations), simply add the marginal costs to the equilibrium prices that we obtain.

By symmetry, the profit that results from selling good i at the right extreme of the city is:

$$\Pi_{iR} = \begin{cases} p_{iR}\left(\frac{1}{2}+\frac{P_L-P_R}{2t}\right) & \text{if} \quad \mathbf{p}\in S_1 \\ p_{iR}\min\left\{\frac{P_L-P_{LR}}{t},1\right\} & \text{if} \quad \mathbf{p}\in S_2 \ \wedge \ p_{iR}<p_{iL} \\ \frac{p_{iR}}{2}\left(\min\left\{\frac{P_L-P_{LR}}{t},1\right\}+\max\left\{0,1-\frac{P_R-P_{LR}}{t}\right\}\right) & \text{if} \quad \mathbf{p}\in S_2 \ \wedge \ p_{iR}=p_{iL} \\ p_{iR}\max\left\{0,1-\frac{P_R-P_{LR}}{t}\right\} & \text{if} \quad \mathbf{p}\in S_2 \ \wedge \ p_{iR}>p_{iL} \end{cases}.$$

3.2.3 Modes of retail and price-setting behavior

On the supply side, we consider two different modes of retail: department stores and shopping malls. A department store is a multi-product firm that sells the n goods at the same location. For example, a department store at $x=0$ sells goods $\{iL\}_{i\in\mathcal{I}}$, seeking to maximize its profit, $\Pi_L=\sum_{i=1}^{n}\Pi_{iL}$. A shopping mall is a group of single-product firms that sell each of the n goods at the same location. For example, a shopping mall at $x=1$ is composed by n firms, each firm selling one good, iR, with the objective of maximizing its individual profit, Π_{iR}. We exclude the possibility of coordinated behavior among shops at a mall. Each shop chooses how much to charge for the product it sells, taking the remaining prices as given.

3.2.3.1 Profit maximization by a department store

With prices that induce one-stop shopping (which are such that $\mathbf{p}\in S_1$), the profit of a department store is:

$$\Pi_L = P_L\left(\frac{1}{2}+\frac{P_R-P_L}{2t}\right).$$

In this domain, profit maximization only requires that:

$$P_L = \frac{P_R}{2} + \frac{t}{2}. \tag{3.6}$$

It is always possible for the department store to set prices that add up to this P_L and that induce one-stop shopping. For example, by setting $p_{iL} = \frac{P_L}{P_R} p_{iR}, \forall i \in \mathcal{I}$.

With prices that induce two-stop shopping (which are such that $\mathbf{p} \in S_2$), the profit of the department store is given by:

$$\Pi_L = \frac{1}{t} \sum_{i \in \mathcal{I}_L} p_{iL} \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}) + \sum_{i \in \mathcal{I}_R} p_{iL} \left[1 - \frac{1}{t} \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR}) \right]. \tag{3.7}$$

For any $i \in \mathcal{I}_L$, the first-order condition is:

$$\frac{\partial \Pi_L}{\partial p_{iL}} = 0 \;\Leftrightarrow\; \sum_{i \in \mathcal{I}_L} p_{iL} = \frac{1}{2} \sum_{i \in \mathcal{I}_L} p_{iR}. \tag{3.8}$$

While for any $i \in \mathcal{I}_R$, the first-order condition is:

$$\frac{\partial \Pi_L}{\partial p_{iL}} = 0 \;\Leftrightarrow\; \sum_{i \in \mathcal{I}_R} p_{iL} = \frac{1}{2} \sum_{i \in \mathcal{I}_R} p_{iR} + \frac{t}{2}. \tag{3.9}$$

The prices that satisfy these first-order conditions induce two-stop shopping (that is, imply that $\mathbf{p} \in S_2$) if and only if:

$$\sum_{i \in \mathcal{I}_L} p_{iR} - \sum_{i \in \mathcal{I}_R} p_{iR} > t. \tag{3.10}$$

This is actually a necessary and sufficient condition for the department store to be interested in inducing two-stop shopping.

Lemma 3.1. *A department store located at $x = 0$ prefers two-stop shopping if and only if:* $\sum_{i \in \mathcal{I}_L} p_{iR} - \sum_{i \in \mathcal{I}_R} p_{iR} > t$.

Proof. Observe that if the prices at the right extreme do not satisfy condition (3.10), then, to induce two-stop shopping, the department store would have to sell the bundle of goods $\mathcal{I}_L$ at a lower price than the bundle $\mathcal{I}_R$:

$$\begin{cases} \sum_{i\in\mathcal{I}_L} p_{iR} - \sum_{i\in\mathcal{I}_R} p_{iR} \leq t \\ \sum_{i\in\mathcal{I}_L} (p_{iR} - p_{iL}) + \sum_{i\in\mathcal{I}_R} (p_{iL} - p_{iR}) > t \end{cases} \Rightarrow \sum_{i\in\mathcal{I}_L} p_{iL} < \sum_{i\in\mathcal{I}_R} p_{iL}.$$

But this implies that, for any given P_L, the one-stop shopping scenario is more profitable than the two-stop shopping scenario:

$$\sum_{i\in\mathcal{I}_L} p_{iL} < \sum_{i\in\mathcal{I}_R} p_{iL} \;\Leftrightarrow\; \left(\tilde{x}_R - \frac{\tilde{x}_R}{2} - \tilde{x}_L\right)\sum_{i\in\mathcal{I}_L} p_{iL} + \left(\tilde{x}_L - \frac{\tilde{x}_L}{2} - \tilde{x}_R\right)\sum_{i\in\mathcal{I}_R} p_{iL} < 0 \;\Leftrightarrow$$

$$\Leftrightarrow\; \tilde{x}_R \sum_{i\in\mathcal{I}_L} p_{iL} + \tilde{x}_L \sum_{i\in\mathcal{I}_R} p_{iL} < \tilde{x} P_L. \tag{3.11}$$

□

Observe that adding (3.8) and (3.9), we obtain (3.6). This means that the department store actually charges the same price for the bundle of n goods regardless of whether it induces two-stop shopping or not.

Lemma 3.2. *A department store located at $x = 0$ always sets the price of the bundle of n goods to: $P_L = \frac{P_R}{2} + \frac{t}{2}$.*

When a department store induces two-stop shopping, only one of the goods is more expensive there that at the other shopping center. All the other goods are cheaper.

Lemma 3.3. *When inducing two-stop shopping, a department store located at $x = 0$ typically chooses the set $\mathcal{I}_R$ to contain a single element, $j \in argmin_{i\in\mathcal{I}} p_{iR}$. The only exception is when there is more than one good i with $p_{iR} = 0$. In this case, $\mathcal{I}_R$ can be any non-empty subset of those goods.*

Proof. See the Appendix C. □

To understand this result, it may be useful to look at two-stop shopping as a situation in which there is competition in markets for two bundled goods: $\mathcal{I}_L$ and $\mathcal{I}_R$. The department store faces the problem of choosing the partition of $\mathcal{I}$ that defines the two bundles. With this in mind, the previous result reflects the fact that the department store prefers to face prices at the shopping mall that are very asymmetric. The partition that is chosen maximizes the price of $\mathcal{I}_L$ at the shopping mall and minimizes the price of $\mathcal{I}_R$ at the shopping mall. The department store, then, sells $\mathcal{I}_L$ at a lower price and $\mathcal{I}_R$ at a higher price. In the end, two-stop shopping is profitable if the department store is able to sell $\mathcal{I}_L$ (the bundle for which it has a high demand) at a higher price than $\mathcal{I}_R$ (the bundle for which it has a low demand).

The profit-maximizing behavior of a department store is summarized in Proposition 3.1.

Proposition 3.1. *Let $j \in argmin_{i\in\mathcal{I}} p_{iR}$. If $\sum_{i\neq j} p_{iR} - p_{jR} \leq t$, a department store located at $x = 0$ induces one-stop shopping, setting prices for the individual goods that are such that: $\sum_{i\neq j}(p_{iR} - p_{iL}) + p_{jL} - p_{jR} \leq t$ and $P_L = \frac{P_R}{2} + \frac{t}{2}$. Otherwise, the department store induces two-stop shopping, setting $\sum_{i\neq j} p_{iL} = \frac{1}{2}\sum_{i\neq j} p_{iR}$ (with $p_{iL} < p_{iR}, \forall i \neq j$) and $p_{jL} = \frac{1}{2}p_{jR} + \frac{t}{2}$.*

3.2.3.2 Profit maximization by the shops at the mall

In this subsection, we consider the profit-maximization problem of an individual shop located at the right extreme of the city.

To study the behavior of Π_{iR} as a function of p_{iR}, it is convenient to define a partition of the domain of p_{iR} that separates the cases in which: (D_1) all consumers buy good i at R;

(D_2) there is two-stop shopping with $i \in \mathcal{I}_R$; (D_3) there is one-stop shopping; (D_2) there is two-stop shopping with $i \in \mathcal{I}_L$; (D_5) no consumer buys good i at R:

$$\begin{aligned}
D_1 &= [0\,,\ -t + p_{iL} + s_{Ri}]\,; \\
D_2 &= \left]-t + p_{iL} + s_{Ri}\,,\ -t + p_{iL} + s_{Li} + s_{Ri}\right[\,; \\
D_3 &= [-t + p_{iL} + s_{Li} + s_{Ri}\,,\ t + p_{iL} - s_{Li} - s_{Ri}]\,; \\
D_4 &= \left]t + p_{iL} - s_{Li} - s_{Ri}\,,\ t + p_{iL} - s_{Li}\right[\,; \\
D_5 &= \left[t + p_{iL} - s_{Li}\,,\ +\infty\right[\,,
\end{aligned}$$

where $s_{Li} = \sum_{j \in I_L \setminus \{i\}} (p_{jR} - p_{jL})$ and $s_{Ri} = \sum_{j \in I_R \setminus \{i\}} (p_{jL} - p_{jR})$. The partition above is valid as long as $s_{Li} + s_{Ri} \leq t$, which implies that there may be one-stop shopping or two-stop shopping depending on the value of p_{iR}. Otherwise, D_3 becomes empty and the transition between D_2 and D_4 occurs at $p_{iR} = p_{iL}$.

Accordingly, the demand for good iR, as a function of p_{iR}, is:

$$q_{iR} = \begin{cases} 1, & p_{iR} \in D_1 \\ \frac{1}{t} \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR})\,, & p_{iR} \in D_2 \\ \frac{1}{2} + \frac{1}{2t} P_L - \frac{1}{2t} P_R, & p_{iR} \in D_3 \\ 1 - \frac{1}{t} \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL})\,, & p_{iR} \in D_4 \\ 0, & p_{iR} \in D_5 \end{cases} . \tag{3.12}$$

The demand is linear in each branch. Its derivative is initially zero (in D_1), then it is $-\frac{1}{t}$ (in D_2), changes to $-\frac{1}{2t}$ (in D_3), becomes $-\frac{1}{t}$ again (in D_4) and, finally, vanishes (in D_5).

Accordingly, the profit function is concave in each branch. It starts at zero (for $p_{iR} = 0$) and ends at zero (for $p_{iR} \in D_5$).

If D_3 is not empty ($s_{Li} + s_{Ri} \leq t$), the demand and the profit are globally continuous. Otherwise ($s_{Li} + s_{Ri} > t$), they jump downwards at the transition between D_2 and D_4.

In this subsection, we will not provide a complete characterization of the price-setting behavior of the shops at a mall. We only write, for future reference, the profit function in the relevant branches (D_2, D_3 and D_4) and the corresponding first-order conditions.

In D_2, the profit of the shop is:

$$\Pi_{iR} = \frac{1}{t} p_{iR} \sum_{j \in \mathcal{I}_R} \left(p_{jL} - p_{jR}\right), \tag{3.13}$$

which leads to the following first-order condition:

$$p_{iR} = \sum_{j \in \mathcal{I}_R} \left(p_{jL} - p_{jR}\right) \;\Leftrightarrow\; p_{iR} = \frac{1}{2} \sum_{j \in \mathcal{I}_R} p_{jL} - \frac{1}{2} \sum_{j \in \mathcal{I}_R \setminus \{i\}} p_{jR}. \tag{3.14}$$

In D_3, the profit of the shop is:

$$\Pi_{iR} = p_{iR} \left(\frac{1}{2} + \frac{P_L - P_R}{2t} \right). \tag{3.15}$$

The corresponding first-order condition is:

$$p_{iR} = P_L - P_R + t \;\Leftrightarrow\; p_{iR} = \frac{P_L}{2} - \frac{1}{2} \sum_{j \in \mathcal{I} \setminus \{i\}} p_{jR} + \frac{t}{2}. \tag{3.16}$$

Finally, in D_4, the profit of the shop is:

$$\Pi_{iR} = \frac{p_{iR}}{t} \left[t - \sum_{j \in \mathcal{I}_L} \left(p_{jR} - p_{jL}\right) \right], \tag{3.17}$$

and the first-order condition is given by:

$$p_{iR} = t - \sum_{j \in \mathcal{I}_L} \left(p_{jR} - p_{jL}\right) \;\Leftrightarrow\; p_{iR} = \frac{t}{2} + \frac{1}{2} \sum_{j \in \mathcal{I}_L} p_{jL} - \frac{1}{2} \sum_{j \in \mathcal{I}_L \setminus \{i\}} p_{jR}. \tag{3.18}$$

3.3 Competitive scenarios

In this section, we study the following competitive scenarios:

- department store at $x = 0$ and a shopping mall at $x = 1$;
- two department stores, one at $x = 0$ and another at $x = 1$;
- two shopping malls, one at $x = 0$ and another at $x = 1$.

3.3.1 Competition between a department store and a shopping mall

We start by considering the case in which there is a department store located at $x = 0$ and a shopping mall located at $x = 1$. The department store chooses the prices of the n goods with the objective of maximizing its total profit ($\sum_{i=1}^{n} \Pi_{iL}$), while each of the shops at the mall seeks to maximize its individual profit (Π_{iR}).

3.3.1.1 Equilibria with one-stop shopping

In an equilibrium with one-stop shopping, the first-order conditions for profit-maximization by the shops at the mall (3.16) imply that:

$$p_{iR} = P_L - P_R + t \;\Rightarrow\; P_R = np_{iR} = nP_L - nP_R + nt \;\Rightarrow\; P_R = \frac{n}{n+1}\left(P_L + t\right). \quad (3.19)$$

Combining this condition with the first-order condition for profit-maximization by the department store (3.6), we obtain the candidate equilibrium prices:

$$\begin{cases} P_L = \frac{P_R}{2} + \frac{t}{2} \\ P_R = \frac{n}{n+1}\left(P_L + t\right) \end{cases} \Rightarrow \begin{cases} P_L = \frac{2n+1}{n+2}t \\ P_R = \frac{3n}{n+2}t \quad \text{with} \quad p_{iR} = \frac{3}{n+2}t,\ \forall i \in \mathcal{I} \end{cases}. \quad (3.20)$$

By Lemmas 3.1 and 3.3, we know that the department store prefers to deviate and set prices that induce two-stop shopping if and only if:

$$\sum_{i\in\mathcal{I}_L} p_{iR} - \sum_{i\in\mathcal{I}_R} p_{iR} > t \Leftrightarrow \frac{3(n-2)}{n+2}t > t \Leftrightarrow n > 4.$$

This is why the above situation (3.20) can only be an equilibrium for $n \leq 4$.

To avoid that the shops at the mall deviate to a situation with two-stop shopping, we need to impose the following condition on the individual prices at the department store:[11]

$$\sum_{i=1}^{n} \left| p_{iL} - \frac{3}{n+2}t \right| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t.$$

Proposition 3.2. *In the case of competition between a department store and a shopping mall, there is an equilibrium with one-stop shopping if and only if* $n \leq 4$. *It is such that:*

(1) It is cheaper to buy the n *goods at the department store than at the shopping mall:*

$$\begin{cases} P_L = \sum_{i=1}^{n} p_{iL} = \frac{2n+1}{n+2}t, \quad \textit{with} \quad \sum_{i=1}^{n} \left| p_{iL} - \frac{3}{n+2}t \right| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t \\ P_R = \sum_{i=1}^{n} p_{iR} = \frac{3n}{n+2}t, \quad \textit{with} \quad p_{iR} = \frac{3}{n+2}t, \ \forall i \in \mathcal{I} \end{cases}.$$

(2) The demand is greater at the department store:

$$\begin{cases} q_{iL} = \frac{2n+1}{2n+4} \\ q_{iR} = \frac{3}{2n+4} \end{cases}.$$

(3) The department store earns more profits than the shops at the mall taken together:

$$\begin{cases} \Pi_L = \sum_{i=1}^{n} \Pi_{iL} = \frac{(2n+1)^2}{2(n+2)^2}t \\ \Pi_R = \sum_{i=1}^{n} \Pi_{iR} = \frac{9n}{2(n+2)^2}t \end{cases}.$$

[11] This is shown in the proof of Proposition 3.2. See the Appendix C.

Proof. See the Appendix C. □

The department store does not care about how much to charge for each individual good because all its customers buy the entire bundle of goods. What matters for the department store is the price of the bundle.[12]

The department store charges a lower price for the bundle of n goods because, when compared with the shops at the mall, it has an additional incentive to set low prices. By decreasing the price of one good (for example, the price of books), the department store increases the demand for all the goods that are sold there (books, groceries, etc.). At the shopping mall, the bookshop, when choosing the price to set for books, only takes into account the effect on its own demand, ignoring the effect of the price of books on the demand for groceries and for the remaining goods.

As a result of setting lower prices, the department store captures more than half of the market. It does not capture the whole market because the customers that are closer to the shopping mall weight the price advantage of the department store against the proximity advantage of the shopping mall. In equilibrium, the shopping mall retains the consumers that are sufficiently close.

Comparing the joint profit at each extreme of the city, we find that the department store earns more than the shops at the mall taken together.

3.3.1.2 Equilibria with two-stop shopping

In an equilibrium with two-stop shopping, the department store must be selling $n-1$ goods at a lower price than the shopping mall and a single good at a higher price than the shopping

[12] The indeterminacy of individual prices at the department store does not extend to the demand.

mall (see Lemma 3.3).

Adding the first-order conditions for the profit-maximization by the $n-1$ shops at the mall that sell goods in $\mathcal{I}_L$, given by (3.18), we obtain the following reaction function:

$$\sum_{i\in\mathcal{I}_L} p_{iR} = (n-1)t - (n-1)\sum_{i\in\mathcal{I}_L}(p_{iR}-p_{iL}) \;\Leftrightarrow\; \sum_{i\in\mathcal{I}_L} p_{iR} = \frac{n-1}{n}\left(t+\sum_{i\in\mathcal{I}_L} p_{iL}\right).$$

Combining it with the first-order condition for profit-maximization by the department store (3.8), we obtain:

$$\begin{cases} \sum_{i\in\mathcal{I}_L} p_{iR} = \frac{2n-2}{n+1}t, \quad \text{with} \quad p_{iR} = \frac{2}{n+1}t, \forall i \in \mathcal{I}_L \\ \sum_{i\in\mathcal{I}_L} p_{iL} = \frac{n-1}{n+1}t \end{cases}.$$

For the single shop at the mall that sells a good $i \in \mathcal{I}_R$, the first-order condition for profit-maximization is (3.14):

$$p_{iR} = \frac{p_{iL}}{2}.$$

While the corresponding first-order condition for profit-maximization by the department store is (3.9):

$$p_{iL} = \frac{p_{iR}}{2} + \frac{t}{2}.$$

Combining the two conditions, we obtain:

$$p_{iR} = \frac{t}{3} \quad \text{and} \quad p_{iL} = \frac{2t}{3}.$$

In spite of charging a higher price, the department store has a greater demand because many of its customers do not find it profitable to make two-stop shopping:

$$q_{iR} = \frac{1}{3} \quad \text{and} \quad q_{iL} = \frac{2}{3}.$$

The department store is only interested in setting prices that induce two-stop shopping if condition (3.10) is satisfied. Substituting the expressions for the candidate equilibrium prices at $x = 1$, this condition becomes:

$$\frac{2n-2}{n+1}t - \frac{t}{3} > t \;\Leftrightarrow\; n > 5.$$

We must also verify that the shop at the mall that is setting the low price does not deviate to a price that induces one-stop shopping. From, the first-order condition (3.16), we find the following candidate deviation (which induces one-stop shopping for $n \leq 8$):

$$p_{iR} = \frac{P_L}{2} - \frac{1}{2}\sum_{j\in\mathcal{I}\setminus\{i\}} p_{jR} + \frac{t}{2} = \frac{n-1}{2(n+1)}t + \frac{t}{3} - \frac{n-1}{n+1}t + \frac{t}{2} = \frac{n+4}{3(n+1)}t.$$

The corresponding profit is:

$$\Pi_{iR} = p_{iR}\,(1-\tilde{x}) = \frac{(n+4)^2}{18(n+1)^2}t.$$

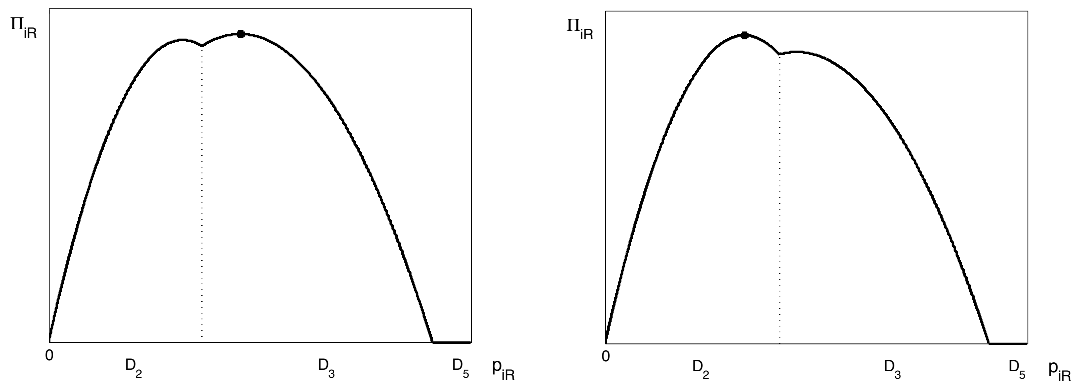
It is higher than the equilibrium profit when:

$$\frac{(n+4)^2}{18(n+1)^2}t > \frac{1}{9}t \;\Leftrightarrow\; (n+4)^2 > 2(n+1)^2 \;\Leftrightarrow\; n < 6.243.$$

The profit function of this shop (with all the other prices at their equilibrium levels), for $n = 6$ and $n = 7$, is shown in Figure 3.2.

Finally, we need to verify that the shops at the mall that are setting higher prices than the department store do not deviate to a price that induces one-stop shopping. In the Appendix C we show that they deviate from the candidate equilibrium when $n \geq 12$.

As illustrated in Figure 3.3, for $n \geq 12$, it is preferable for these shops to decrease prices with the objective of capturing the customers that make two-stop shopping.



(a) $n = 6$. (b) $n = 7$.

Figure 3.2: Profit function of the shop that sells good $i \in \mathcal{I}_R$ at the mall, with the remaining prices as in the equilibrium with two-stop shopping.

Proposition 3.3. *In the case of competition between a department store and a shopping mall, there is an equilibrium with two-stop shopping if and only if* $7 \leq n \leq 11$. *It is such that:*

(1) It is cheaper to buy $n-1$ *of the* n *goods at the department store than at the shopping mall:* $\#\mathcal{I}_L = n-1$ *and* $\#\mathcal{I}_R = 1$.

(2) The prices of the goods that are cheaper at the department store, $i \in \mathcal{I}_L$*, are such that:*

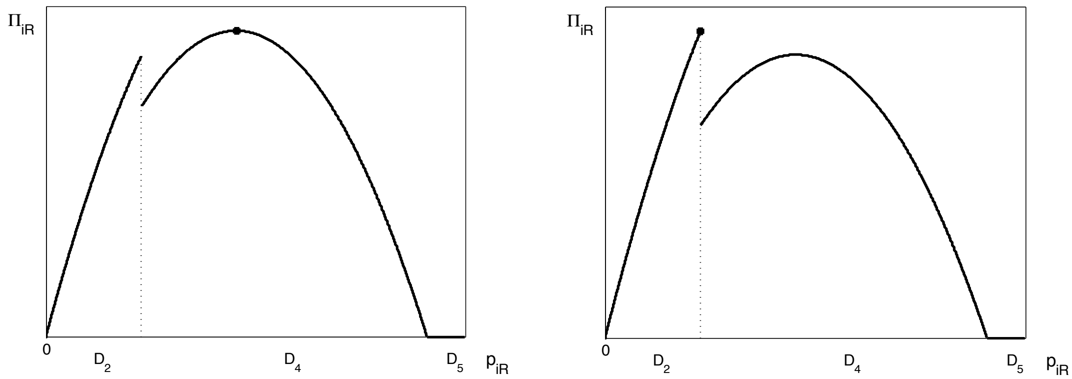
$$\begin{cases} \sum_{i\in\mathcal{I}_L} p_{iL} = \frac{n-1}{n+1}t, \quad \textit{with} \quad p_{iL} \leq \frac{12}{(n+1)^2}t, \forall i \in \mathcal{I}_L \\ \sum_{i\in\mathcal{I}_L} p_{iR} = \frac{2(n-1)}{n+1}t, \quad \textit{with} \quad p_{iR} = \frac{2}{n+1}t, \forall i \in \mathcal{I}_L \end{cases},$$

and the corresponding demands are:

$$q_{iL} = \frac{n-1}{n+1} \quad \textit{and} \quad q_{iR} = \frac{2}{n+1}.$$

(3) The prices of the only good that is cheaper at the shopping mall, $i \in \mathcal{I}_R$*, are:*

$$p_{iL} = \frac{2}{3}t \quad \textit{and} \quad p_{iR} = \frac{1}{3}t,$$



(a) $n = 10$.

(b) $n = 12$.

Figure 3.3: Profit function of a shop that sells good $i \in \mathcal{I}_L$ at the mall, with the remaining prices as in the equilibrium with two-stop shopping.

and the demands are:

$$q_{iL} = \frac{2}{3} \quad \textit{and} \quad q_{iR} = \frac{1}{3}.$$

(4) The department store earns more profits than the shops at the mall taken together:

$$\begin{cases} \Pi_L = \frac{(n-1)^2}{(n+1)^2}t + \frac{4t}{9} \\ \Pi_R = \frac{4(n-1)}{(n+1)^2}t + \frac{t}{9} \end{cases}.$$

Curiously, independently of the number of goods, the consumers that buy all the products at the department store are those located at $\left[0, \frac{2}{3}\right[$. For a consumer located at $x \in \left]\frac{2}{3}, 1\right]$ who visits the department store to buy the goods $i \in \mathcal{I}_L$, the extra transportation cost of also visiting the shopping mall is smaller than $\frac{t}{3}$. However, the difference in the price of good $i \in \mathcal{I}_R$ is: $p_{iL} - p_{iR} = \frac{t}{3}$. Thus, for all these consumers, it is worthwhile to buy the good $i \in \mathcal{I}_R$ at the shopping mall. As the number of goods increases, there are more consumers willing to make their purchases at both extremes of the city.

As in the equilibria with one-stop shopping, we conclude that the department store has a higher profit than all the shops at the mall taken together.

Since there only exists equilibrium with one-stop shopping when $n \leq 4$ and with two-stop shopping when $7 \leq n \leq 11$, we conclude that there is no equilibrium (in pure strategies) when the number of goods is $5 \leq n \leq 6$ or $n \geq 12$.

3.3.2 Competition between two department stores

Now, we consider the case in which there are two department stores, one at each extreme of the city. Each department store chooses the price to charge for each of the n products, with the objective of maximizing its profit, taking as given the prices set by the other department store.[13]

Proposition 3.4. *In the case of competition between two department stores:*

(1) The price of the bundle is equal to the transportation cost parameter:

$$P_L = P_R = t, \quad \textit{with} \quad \sum_{i=1}^{n} |p_{iL} - p_{iR}| \leq t.$$

(2) Consumers make all their purchases at the closest department store:

$$q_{iL} = q_{iR} = \frac{1}{2}.$$

(3) The resulting profits are also independent of the number of goods:

$$\Pi_L = \Pi_R = \frac{t}{2}.$$

[13] This is an extension of the case analyzed by Lal and Matutes (1989), where both department stores sell only two products ($n = 2$).

Proof. See the Appendix C. □

In equilibrium, the department stores charge the same price for the bundle of n goods (there is, once more, some indeterminacy regarding the split of the bill between the goods). It is, then, obvious that no consumer is willing to travel to both extremes of the city. They all buy the n goods at the department store that is closer.

What may be surprising is that the margin (difference between price and marginal cost) with n goods is the same as in the standard Hotelling model, in which a single good is sold. The reason why the margin is not greater with n goods is related to the fact the reservation utility of the customer is not relevant for the pricing decisions of the firms (as long as it is high enough, as is typically assumed). With one-stop shopping, the n goods are equivalent to a single bundled good. Therefore, even if customers attribute a higher utility to the n goods than to a single good, the margin remains constant and equal to the transportation cost parameter.[14]

3.3.3 Competition between two shopping malls

In the case of competition between two shopping malls (one at each extreme of the city), the shops that sell the same good at different locations are direct competitors. However, their demand also depends on the prices of the other goods. This interdependence across shops selling different goods exists because, when deciding where to buy each good, consumers take into account not only the price but also the transportation costs that they have to support. A shop benefits from having low prices for the goods sold at the same location

[14]The same occurs in the case of Bertrand competition with homogeneous products. Independently of the number of products that firms sell, their equilibrium margin is always null.

(since this attracts customers to its location); and high prices for the goods sold at the other location (since this repels customers from the other location). But since this externality has no influence on the pricing decisions of the shops, the equilibrium of the model replicates that of the single-product model.

Proposition 3.5. *In the case of competition between two shopping malls:*

(1) The price of each good is equal to the transportation cost parameter:

$$p_{iL} = p_{iR} = t, \ \forall i \in \mathcal{I}.$$

(2) Consumers make all their purchases at the closest shopping mall:

$$q_{iL} = q_{iR} = \frac{1}{2}, \ \forall i \in \mathcal{I}.$$

(3) The profit of each firm is also independent of the number of goods:

$$\Pi_{iL} = \Pi_{iR} = \frac{t}{2}, \ \forall i \in \mathcal{I}.$$

Proof. See the Appendix C. □

The joint profit of the n shops located at each shopping mall is greater than the profits obtained in any of the alternative scenarios that we have considered.

3.4 Endogenous modes of retail

Until now, we have assumed that the organization of each shopping center was exogenous. In this section, we analyze whether it is more profitable for a shopping center to be organized as a department store or as a shopping mall.

More precisely, we consider a two-stage game in which the shopping centers start by simultaneously deciding their modes of retail (department store or shopping mall) and then compete in prices. The resulting payoffs are, depending on the chosen mode of retail, the profits of the department store or the joint profits of the shops at a mall, in the equilibrium of the corresponding scenario.[15]

For $n \leq 4$, the profits of the shopping centers in each competitive scenario are shown in Table 3.1. We conclude that shopping malls have higher profits than department stores, regardless of whether they compete against a department store or a shopping mall.

	Department Store	**Shopping Mall**
Department Store	$\left(\frac{1}{2}t\ ,\ \frac{1}{2}t\right)$	$\left(\frac{(2n+1)^2}{2(n+2)^2}t\ ,\ \frac{9n}{2(n+2)^2}t\right)$
Shopping Mall	$\left(\frac{9n}{2(n+2)^2}t\ ,\ \frac{(2n+1)^2}{2(n+2)^2}t\right)$	$\left(\frac{n}{2}t\ ,\ \frac{n}{2}t\right)$

Table 3.1: Profits of the competing shopping centers, for $n \leq 4$.

Proposition 3.6. *If $n \leq 4$, it is a dominant strategy for a shopping center to organize itself as a shopping mall.*

[15]We do not consider the possibility of some shops merging and others remaining separated. Opening the possibility of partial mergers can change the outcome of the game, as suggested by the contributions of Salant, Switzer and Reynolds (1983) and Kamien and Zang (1990).

For $7 \leq n \leq 11$, the payoff matrix is given by Table 3.2. In this case: to compete against a department store, it is better to be organized as a department store; while to compete against a shopping mall it is better to be organized as a shopping mall.

	Department Store	**Shopping Mall**
Department Store	$\left(\frac{1}{2}t\ ,\ \frac{1}{2}t\right)$	$\left(\frac{(n-1)^2}{(n+1)^2}t + \frac{4}{9}t\ ,\ \frac{4(n-1)}{(n+1)^2}t + \frac{1}{9}t\right)$
Shopping Mall	$\left(\frac{4(n-1)}{(n+1)^2}t + \frac{1}{9}t\ ,\ \frac{(n-1)^2}{(n+1)^2}t + \frac{4}{9}t\right)$	$\left(\frac{n}{2}t\ ,\ \frac{n}{2}t\right)$

Table 3.2: Profits of the competing shopping centers, for $7 \leq n \leq 11$.

Proposition 3.7. *If* $7 \leq n \leq 11$, *there are two equilibria: competition between two shopping malls and competition between two department stores.*

Of course, the shopping centers would prefer to coordinate on the equilibrium with two shopping malls. Therefore, we should expect to have shopping malls at both extremes of the city.

3.5 Welfare analysis

3.5.1 Social welfare

In this model, total demand is perfectly inelastic (each consumer buys one unit of each good that is available in the market, independently of the prices of the goods). Therefore, a change in prices only leads to a transfer of surplus between consumers and firms. The total surplus remains unaltered. In this context, the maximization of total surplus is equivalent to the minimization of the total transportation costs incurred by consumers.

It is clear that the total transportation costs are minimized when each consumer shops at the closest store. This occurs in the case of competition between two department stores and in the case of competition between two shopping malls. When there is a department store competing with a shopping mall, the indifferent consumer is no longer located at the middle of the city. If $n \leq 4$, there are more consumers shopping at the department store than at the shopping mall ($\tilde{x} > \frac{1}{2}$). If $7 \leq n \leq 11$, there are consumers who shop at both extremes of the city. The total transportation costs are even higher in such a situation. Thus, the existence of different modes of retail diminishes the total surplus.

3.5.2 Consumers' surplus

Let CS_{DD}, CS_{MM} and CS_{DM} denote the consumers' surplus in each of the three scenarios: (DD) competition between two department stores; (MM) competition between two shopping malls; and (DM) competition between a department store and a shopping mall.

When the mode of retail is the same in both extremes of the city, the price of the bundle of goods is the same at both shopping centers. As a result, the indifferent consumer is located at the middle of the city and the total transportation cost is minimized. We have:

$$CS_{DD} = \tilde{x}\left(V - t\right) + \tilde{x}\left(V - t\right) - t\int_0^{\tilde{x}} x\ dx - t\int_{\tilde{x}}^1 \left(1 - x\right)\ dx = V - \frac{5}{4}t;$$

$$CS_{MM} = \tilde{x}\left(V - nt\right) + \tilde{x}\left(V - nt\right) - t\int_0^{\tilde{x}} x\ dx - t\int_{\tilde{x}}^1 \left(1 - x\right)\ dx = V - \frac{4n+1}{4}t,$$

where $V = \sum_{i \in \mathcal{I}} V_i$. When a department store competes with a shopping mall and the number of goods is less than or equal to 4 (equilibrium with one-stop shopping), the consumers' surplus is:

$$\begin{aligned} CS_{DM} &= \int_0^{\tilde{x}} (V - P_L - tx)\ dx + \int_{\tilde{x}}^1 [V - P_R - t(1-x)]\ dx \\ &= V - \frac{10n^2 + 28n + 7}{4(n+2)^2} t. \end{aligned}$$

When $7 \leq n \leq 11$ (equilibrium with two-stop shopping), it is given by:

$$\begin{aligned} CS_{DM} &= \int_0^{\tilde{x}_L} (V - P_L - tx)\ dx + \int_{\tilde{x}_L}^{\tilde{x}_R} (V - P_{LR} - t)\ dx + \int_{\tilde{x}_R}^1 [V - P_R - t(1-x)]\ dx \\ &= V - \frac{19n^2 + 20n - 17}{9(n+1)^2} t. \end{aligned}$$

From the point of view of consumers, competition between department stores is the most favorable scenario. Prices are lower than in the other scenarios, and transportation costs are minimized. It is not so straightforward to compare the case of competition between two shopping malls (lower transportation costs) with the case of competition between a shopping mall and a department store (lower prices). We find that the price effect dominates. Consumers prefer competition between a shopping mall and a department store rather than competition between two shopping malls.

Proposition 3.8. *Comparing the consumer' surplus in the three competitive scenarios, for* $2 \leq n \leq 4$ *and for* $7 \leq n \leq 11$*, we obtain:*

$$CS_{DD} > CS_{DM} > CS_{MM}.$$

It is somewhat surprising that the lower the number of independent stores in the market, the higher the consumers' surplus. This result contradicts the typical intuition, according to which as the number of firms in the market increases, competition becomes stronger, leading to lower prices. This is not the case, since the price for the bundle of goods is cheaper when there are only two department stores.

3.6 Conclusions

We have developed a multi-product version of the model of Hotelling (1929) to study competition between shopping centers that can be organized as department stores or as shopping malls. In particular, we analyzed how the modes of retail affect prices, market shares and profits, and which retail structures are more likely to emerge endogenously.

Comparing the competitive behavior of a department store with that of a shopping mall, we found (as in previous works) that the department store competes more aggressively. This occurs because a department store, when choosing prices, takes into account that a price drop in one good increases the demand for all its goods. In contrast, a shop at a mall only takes into account its individual demand when choosing the price to charge for its good.

When a department store competes with a shopping mall, the bundle of goods is cheaper at the department store than at the shopping mall. Nevertheless, the demand-effect more than compensates the price-effect and the department store obtains higher profits than the shops at the mall taken together. In spite of having higher profits, the department store has incentives to separate itself into a shopping mall. If the shops at each extreme decide whether to organize themselves as a shopping mall or as a department store, the competitive scenario that is expected to emerge is competition between two shopping malls.[16]

The most distinctive feature of our work, with respect to the existing literature, is that we do not restrict consumers to make all their purchases in a single place. Still, we found that when the mode of retail is the same in the two shopping centers, no consumer finds it worthwhile to visit both places ("one-stop shopping" holds in equilibrium). However, if there is one department store competing with one shopping mall, there may be consumers traveling to both extremes of the city to buy each product where it is cheaper (this occurs if the number of goods for sale is between seven and eleven).

To understand why the "one-stop shopping" assumption may be too strong, suppose that there are nine different goods for sale at a shopping mall, each and every one at a price equal to 1. With the transportation cost being equal to 5, the best response of a department store would be to price the bundle at $P_L = 7$. With "one-stop shopping", the department store would have a demand equal to 0.7 and a profit of 4.9. But the department store can also induce "two-stop shopping", by charging a price of 3 for one of the goods and a price of 4 for the remaining eight goods. The demand for the expensive good would decrease to 0.6, but the demand for the other goods would increase to 0.8. As a result, the profit would increase to 5.

If the number of goods is five, six or higher than eleven, there is no equilibrium in pure strategies for the case of competition between a department store and a shopping mall. Therefore, a possible extension of this work is to allow for mixed strategies in the price competition stage. This could be compatible with having informed consumers. The shops would set prices simultaneously and irreversibly, and then consumers would observe

[16]This is actually the competitive scenario that consumers desire the least. Having two shopping malls or two department stores is equally optimal in terms of total surplus. However, consumers are better off in the case of competition between two department stores (since prices are lower).

the prices and decide their purchases. An alternative would be to consider uninformed consumers. They would have to travel to a shopping center to observe the prices that were actually set, and then decide whether to travel also to the other shopping center or not. We leave this extension for future work.

Appendices

Appendix C

Proof of Propositions and Lemmas

Proof of Lemma 3.3.

Inspection of the profit function (3.7) shows that Π_L depends positively on the price of the bundle $\mathcal{I}_L$ at the right extreme ($\sum_{i\in\mathcal{I}_L} p_{iR}$) with coefficient $\frac{1}{t}\sum_{i\in\mathcal{I}_L} p_{iL}$, and on the price of the bundle $\mathcal{I}_R$ at the right extreme ($\sum_{i\in\mathcal{I}_R} p_{iR}$) with coefficient $\frac{1}{t}\sum_{i\in\mathcal{I}_R} p_{iL}$. From condition (3.11), the latter coefficient is lower than the former whenever two-stop shopping is preferred.

Observe that the choice of $\mathcal{I}_L$ and $\mathcal{I}_R$ only impacts the attainable profit through the distribution of P_R between $\sum_{i\in\mathcal{I}_L} p_{iR}$ and $\sum_{i\in\mathcal{I}_R} p_{iR}$.

Thus, it is in the interest of the department store to choose the bundle $\mathcal{I}_R$ that minimizes $\sum_{i\in\mathcal{I}_R} p_{iR}$ (and the bundle $\mathcal{I}_L$ that maximizes $\sum_{i\in\mathcal{I}_L} p_{iR}$). This is achieved when $\mathcal{I}_R$ only contains the good that is the cheapest at R.

The set $\mathcal{I}_R$ can only contain more goods if these have null price at R.

$\square$

Proof of Proposition 3.2.

To finish the proof, we must verify that $p_{iR} = \frac{3}{n+2}t$ maximizes the profit of the shop at the mall that sells good iR. So far, we only know that it is a maximum in D_3, as long as $\sum_{i=1}^{n} |p_{iL} - \frac{3}{n+2}t| \leq t$ (which implies that it is inside the domain).

With $P_L = \frac{2n+1}{n+2}t$ and $p_{jR} = \frac{3}{n+2}t, \forall j \neq i$, the profit function has a local maximum in D_3, attained at $p^*_{iR} = \frac{3}{n+2}t$. The derivative of the demand with respect to price is $-\frac{1}{2t}$ in D_3 and $-\frac{1}{t}$ in D_4. Therefore, if a marginal price increase is not profitable, it is also not profitable to make a greater price increase to D_4, where the demand is more price-sensitive.

The maximum is either $p^*_{iR} = \frac{3}{n+2}t$ or attained in D_2. In D_2, the demand for good iR is $q_{iR} = \frac{1}{t}\left(s_{Ri} + p_{iL} - p_{iR}\right)$ and the corresponding profit is $\Pi_{iR} = \frac{1}{t}\left(s_{Ri}p_{iR} + p_{iL}p_{iR} - p^2_{iR}\right)$. The first-order condition is satisfied at $p^{**}_{iR} = \frac{s_{Ri}}{2} + \frac{p_{iL}}{2}$, implying that:

$$\Pi_{iR}\left(p^{**}_{iR}\right) = \frac{\left(s_{Ri} + p_{iL}\right)^2}{4t}.$$

We are sure that $p^*_{iR} = \frac{3}{n+2}t$ is a global maximizer if:

$$\Pi_{iR}\left(p^{**}_{iR}\right) \leq \Pi_{iR}\left(p^{*}_{iR}\right) \;\Leftrightarrow\; \frac{\left(s_{Ri} + p_{iL}\right)^2}{4t} \leq \frac{9}{2(n+2)^2}t \;\Leftrightarrow\; s_{Ri} + p_{iL} \leq \frac{\sqrt{18}}{n+2}t.$$

Otherwise, the alternative maximizer, $p^{**}_{iR} = \frac{s_{Ri}}{2} + \frac{p_{iL}}{2}$, upsets our equilibrium if it belongs to the domain, D_2. This is occurs if:

$$\begin{cases} \frac{s_{Ri}}{2} + \frac{p_{iL}}{2} \leq -t + p_{iL} + s_{Li} + s_{Ri} \\ \frac{s_{Ri}}{2} + \frac{p_{iL}}{2} \geq -t + p_{iL} + s_{Ri} \end{cases}. \tag{C.1}$$

In (the candidate) equilibrium, we have $P_R - P_L = \frac{n-1}{n+2}t$. As a result:

$$\sum_{j \in I \setminus \{i\}} \left(p_{jR} - p_{jL}\right) + \frac{3}{n+2}t - p_{iL} = \frac{n-1}{n+2}t \;\Leftrightarrow\; s_{Li} = \frac{n-4}{n+2}t + s_{Ri} + p_{iL}. \tag{C.2}$$

Substituting (C.2) in (C.1), we find that p_{iR}^{**} is in the domain D_2 when:

$$s_{Ri} + p_{iL} \in \left[\frac{4}{n+2}t, 2t\right].$$

We surely have $s_{Ri} + p_{iL} \leq 2t$, because:

$$s_{Li} \leq t \;\Leftrightarrow\; \frac{n-4}{n+2}t + s_{Ri} + p_{iL} \leq t \;\Leftrightarrow\; s_{Ri} + p_{iL} \leq \frac{6}{n+2}t.$$

Then, if $s_{Ri} + p_{iL} > \frac{\sqrt{18}}{n+2}t$, we have p_{iR}^{**} in D_2 and it upsets our candidate equilibrium, p_{iR}^{*}. As a result, our candidate is an equilibrium if and only if, $\forall i \in \mathcal{I}$:

$$s_{Ri} + p_{iL} \leq \frac{\sqrt{18}}{n+2}t \;\Leftrightarrow\; s_{Li} \leq \frac{n+\sqrt{18}-4}{n+2}t.$$

If $\mathcal{I}_R$ is non-empty, the conditions that bind are those for $i \in \mathcal{I}_R$ (because s_{Li} is maximal), which can be written as:

$$\sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| \leq \frac{n+\sqrt{18}-4}{n+2}t.$$

If $\mathcal{I}_R$ is empty, then all the above conditions are surely satisfied because, in this case:

$$\sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| = \sum_{i \in \mathcal{I}} (p_{iR} - p_{iL}) = P_R - P_L = \frac{n-1}{n+2}t < \frac{n+\sqrt{18}-4}{n+2}t.$$

To obtain a more elegant condition, notice that (in the candidate equilibrium):

$$P_R - P_{LR} - P_L + P_{LR} = \frac{n-1}{n+2}t \;\Leftrightarrow\; \sum_{i \in \mathcal{I}_R} |p_{iL} - p_{iR}| = \sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| - \frac{n-1}{n+2}t \;\Leftrightarrow$$

$$\Leftrightarrow\; \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| = 2\sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| - \frac{n-1}{n+2}t.$$

Therefore, the equilibrium condition can be written as:

$$\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq \frac{2\left(n+\sqrt{18}-4\right)}{n+2}t - \frac{n-1}{n+2}t = \frac{n+6\sqrt{2}-7}{n+2}t. \quad \square$$

Proof of Proposition 3.3.

To finish the proof, we need to check that the shops at the mall that are selling goods at a higher price than the department store do not wish to deviate as long as $7 \leq n \leq 11$. The candidate equilibrium maximizes their profits in the domain D_4, but we must guarantee that the shops do not prefer to choose prices in D_3 (if it exists) or in D_2.

It is convenient to start with some preliminary calculations. In the candidate equilibrium, for the shop that sells good $i \in \mathcal{I}_L$, we have:

$$s_{Li} = \sum_{j \in \mathcal{I}_L \setminus \{i\}} (p_{jR} - p_{jL}) = \frac{n-3}{n+1} t + p_{iL};$$

$$s_{Ri} = \sum_{j \in \mathcal{I}_R \setminus \{i\}} (p_{jL} - p_{jR}) = \frac{t}{3}.$$

We know that D_3 is empty if and only if:

$$s_{Li} + s_{Ri} > t \;\Leftrightarrow\; \frac{n-3}{n+1} t + p_{iL} + \frac{t}{3} > t \;\Leftrightarrow\; p_{iL} > \frac{11-n}{3(n+1)} t.$$

For $n \geq 12$, the domain D_3 is surely empty (for $n = 11$ it is either empty or a singleton).

Let us start by considering the case in which D_3 is empty and study whether a deviation to D_2 is profitable. From (3.14), the interior maximizer in D_2 is:

$$p_{iR}^{**} = \frac{p_{iL}}{2} + \frac{t}{6}. \tag{C.3}$$

This alternative only belongs to D_2 if $p_{iR}^{**} < p_{iL}$. Equivalently, if:

$$\frac{p_{iL}}{2} + \frac{t}{6} < p_{iL} \;\Leftrightarrow\; p_{iL} > \frac{t}{3}. \tag{C.4}$$

But since $p_{iL} < \frac{2}{n+1} t$, condition (C.4) cannot hold for $n \geq 7$. This means that the optimal choice in D_2 is at the frontier: $p_{iR} = p_{iL}$ (infinitesimally close, but lower than p_{iL}). The resulting profit is

$\Pi_{iR} = \frac{p_{iL}}{3}$. We conclude that the shop gains with this deviation if:

$$\frac{p_{iL}}{3} > \frac{4}{(n+1)^2}t \;\Leftrightarrow\; p_{iL} > \frac{12}{(n+1)^2}t.$$

It is straightforward to confirm that $p_{iL} > \frac{12}{(n+1)^2}t$ implies that $p_{iL} > \frac{11-n}{3(n+1)}t$, which was a prerequisite (for D_3 to be empty).

Notice that if $p_{iL} \leq \frac{12}{(n+1)^2}t, \forall i \in \mathcal{I}_L$, then:

$$\sum_{i\in\mathcal{I}_L} p_{iL} \leq \frac{12(n-1)}{(n+1)^2}t \;\Leftrightarrow\; \frac{n-1}{n+1}t \leq \frac{12(n-1)}{(n+1)^2}t \;\Leftrightarrow\; n \leq 11.$$

For $n \geq 12$, the candidate is not an equilibrium because at least one of the shops deviates.

Now suppose that D_3 is not empty. We must have $p_{iL} \leq \frac{11-n}{3(n+1)}t$ and $p_{iL} < \frac{2}{n+1}t$. The first condition can only hold for $n \leq 11$, while the second is implied by the first. In this case, we must consider deviations to D_3 and also to D_2.

The interior maximum in D_2, given by (C.3), is again outside the domain because:

$$p_{iL} \leq \frac{11-n}{3(n+1)}t \;\Rightarrow\; \frac{p_{iL}}{2} + \frac{t}{6} > -t + p_{iL} + s_{Li} + s_{Ri}.$$

The candidate deviation is, therefore, at the frontier. But if it is profitable to increase the price until the frontier of D_2 it is surely profitable to keep increasing the price after entering D_3 because the price-sensitivity of demand is lower in D_3. It is enough to consider deviations in D_3.

From (3.16), the candidate deviation is:

$$p_{iR} = \frac{P_L}{2} - \frac{1}{2}\sum_{j\in\mathcal{I}\setminus\{i\}} p_{jR} + \frac{t}{2} = \frac{n+13}{6(n+1)}t.$$

This deviation is outside D_3, because:

$$\frac{n+13}{6(n+1)}t \leq t + p_{iL} - s_{Li} - s_{Ri} \;\Leftrightarrow\; \frac{n+13}{6(n+1)}t \leq \frac{11-n}{3(n+1)}t \;\Leftrightarrow\; n \leq 3,$$

which is false.

Therefore, the maximum in D_3 is at the frontier with D_4. Global continuity of the profit function (satisfied as long as D_3 is not empty) and concavity in D_4 implies that the local maximum at D_4 is actually the global maximum.

□

Proof of Proposition 3.4.

We start by showing that an equilibrium with two-stop shopping is not possible.

With $n = 2$, one of the goods must be cheaper at $x = 0$ and the other must be cheaper at $x = 1$. Combining the first-order conditions, (3.8) and (3.9), we obtain, for $i \in \mathcal{I}_L$:

$$\begin{cases} p_{iL} = \frac{p_{iR}}{2} \\ p_{iR} = \frac{p_{iL}}{2} + \frac{t}{2} \end{cases} \Leftrightarrow \begin{cases} p_{iL} = \frac{1}{3}t \\ p_{iR} = \frac{2}{3}t. \end{cases}.$$

For $j \in \mathcal{I}_R$, we obtain $p_{jL} = \frac{2}{3}t$ and $p_{jR} = \frac{1}{3}t$. This is an equilibrium, but no consumer gains by shopping at both extremes (only the consumer in the middle, $x = \frac{1}{2}$, is indifferent between one-stop shopping and two-stop shopping).

For $n > 2$, since both department stores wish to have a single good that is more expensive than at the other extreme, the only possibility is that prices are symmetric for all the goods except two. Being symmetric, they must be null, otherwise the department stores would gain by undercutting each other. Lemma 3.3 confirms that there can only exist two goods with strictly positive prices (one at each extreme). But this means that the department store at $x = 0$ is offering the goods in $\mathcal{I}_L$ for free and, therefore, would prefer to deviate and induce one-stop shopping.

With one-stop shopping, the first-order conditions for the profit-maximization problems of the department stores imply that:

$$P_L = \frac{P_R}{2} + \frac{t}{2} \quad \text{and} \quad P_R = \frac{P_L}{2} + \frac{t}{2},$$

yielding:

$$P_L = P_R = t.$$

It is straightforward to verify that they do not have incentives to deviate (3.10).

The equilibrium demand and profits follow immediately.

$\square$

Lemma C.1. *When there are two shopping malls in the city, no consumer shops at both extremes of the city (in equilibrium).*

Proof of Lemma C.1.

By way of contradiction, suppose that the vector of prices that maximize the profits of the shops is such that $\mathbf{p} \in S_2$. More precisely, that $0 < \tilde{x}_L < \tilde{x} < \tilde{x}_R < 1$.

(**i**) There cannot be any $i \in \mathcal{I}$ for which $p_{iL} = p_{iR} > 0$. If that was the case, the shop selling good iL could infinitesimally reduce its price and conquer all consumers at $x \in [\tilde{x}_L, \tilde{x}_R]$. The shop selling good iR would have the same incentive to decrease its price.

It cannot also be the case that $p_{iL} = p_{iR} = 0$ for some $i \in \mathcal{I}$. In such a situation, both shops would obtain a null profit. However, the shop selling good i at $x = 0$, could choose $p_{iL} > 0$ and profit $\Pi_{iL} = p_{iL}\tilde{x}_L > 0$. The same argument applies to the shop selling good iR.

(**ii**) Since $p_{iL} \neq p_{iR}$, $\forall i \in \mathcal{I}$, we have $\mathcal{I}_L \cup \mathcal{I}_R = \mathcal{I}$. Thus, if the cardinality of $\mathcal{I}_L$ is k, the cardinality of $\mathcal{I}_R$ is $n - k$.

The profit function of the shop that sells the good iL is:

$$\Pi_{iL} = \begin{cases} p_{iL}\,\tilde{x}_R, & i \in \mathcal{I}_L \\ p_{iL}\,\tilde{x}_L, & i \in \mathcal{I}_R \end{cases} ,$$

while the profit function of the shop selling the good iR is:

$$\Pi_{iR} = \begin{cases} p_{iR}\left(1 - \tilde{x}_R\right), & i \in \mathcal{I}_L \\ p_{iR}\left(1 - \tilde{x}_L\right), & i \in \mathcal{I}_R \end{cases}.$$

If $i \in \mathcal{I}_L$, the first-order conditions are:

$$\begin{cases} \frac{\partial \Pi_{iL}}{\partial p_{iL}} = 0 \\ \frac{\partial \Pi_{iR}}{\partial p_{iR}} = 0 \end{cases} \Leftrightarrow \begin{cases} p_{iL} = P_R - P_{LR} \\ p_{iR} = t - P_R + P_{LR} \end{cases} \Rightarrow p_{iR} = t - p_{iL}.$$

The expressions above imply that $\forall i, j \in \mathcal{I}_L$: $p_{iL} = p_{jL}$ and $p_{iR} = p_{jR}$. Moreover:

$$p_{iL} = \sum_{j \in \mathcal{I}_L} \left(p_{jR} - p_{jL}\right) = k\left(t - 2p_{iL}\right) \Leftrightarrow p_{iL} = \frac{k}{2k+1}t$$

and

$$p_{iR} = \frac{k+1}{2k+1}t.$$

Analogously, if $i \in \mathcal{I}_R$, then:

$$p_{iL} = \frac{n-k+1}{2n-2k+1}t \quad \text{and} \quad p_{iR} = \frac{n-k}{2n-2k+1}t.$$

The expressions for the marginal consumers, $\tilde{x}_L$ and $\tilde{x}_R$, follow immediately:

$$\tilde{x}_L = \frac{n-k+1}{2n-2k+1} \quad \text{and} \quad \tilde{x}_R = \frac{k}{2k+1}.$$

It is straightforward to see that $\tilde{x}_L > \tilde{x}_R$. Contradiction.

□

Proof of Proposition 3.5.

By Lemma C.1 (in this Appendix), there is no equilibrium with prices in S_2. Therefore, we must seek prices satisfying the condition $\sum_{i=1}^{n} |p_{iL} - p_{iR}| \leq t$.

As obtained in (3.19), the first-order conditions of the n shops at the malls imply that:

$$P_L = \frac{n}{n+1}\left(t + P_R\right) \quad \text{and} \quad P_R = \frac{n}{n+1}\left(t + P_L\right).$$

Therefore:

$$P_L = P_R = nt.$$

Using (3.16), we obtain the individual prices:

$$p_{iL} = p_{iR} = t, \forall i \in \mathcal{I}.$$

To complete the proof, we must verify that these local maxima are global maxima. We need to check if each shop chooses the price t, when the remainders charge t for their products. Without loss of generality, we consider the shop that sells good iR.

Substituting $p_{jL} = p_{jR} = t, \forall j \neq i$ and $p_{iL} = t$ in the demand for good iR, given in (3.12), we obtain:

$$q_{iR} = \begin{cases} 1 - \frac{p_{iR}}{2t}, & p_{iR} \in [0, 2t] \\ 0, & p_{iR} \in]2t, +\infty[\end{cases}.$$

and

$$\Pi_{iR} = \begin{cases} p_{iR}\left(1 - \frac{p_{iR}}{2t}\right), & p_{iR} \in [0, 2t] \\ 0, & p_{iR} \in]2t, +\infty[\end{cases},$$

The profit function is globally concave and continuous. Therefore, the local maximum is also the global maximum.

The equilibrium demand and profits follow immediately.

□

Chapter 4

Asymmetric collusion with growing demand

* We are grateful to João Correia da Silva for his useful comments and suggestions.

4.1 Introduction

Until 1992, the three major suppliers of bottled water in France were Source Perrier SA (hereafter, simply Perrier), BSN and Nestlé, with market shares of 36%, 23% and 17%, respectively. The remaining market share was owned by many small firms. In 1992, Nestlé notified the European Commission (EC) of its public bid to take over Perrier. This would not be accepted by the Commission for providing the group Nestlé-Perrier with a dominant position in the market. In anticipation of this reaction, Nestlé had established a parallel agreement with BSN in which Nestlé committed to sell Volvic (the major still mineral source of Perrier) to BSN if the merger were approved. The Commission rejected this proposal, arguing that the two new groups, Nestlé-Perrier and BSN-Volvic, would be very similar (with respect to market share, size, production costs and technology), which would facilitate their future coordination.[1] Moreover, the Commission claimed that, in these circumstances, the entry of a new firm in the market would be almost impossible.[2] To overcome this, Nestlé modified the original proposal, committing itself to sell some brands and capacity of water to a competitor (with no connections to Nestlé or to BSN), such that this competitor would have, at least, 3000 million litres of capacity per year. These conditions would allow the new firm to become an active competitor in the market. The

[1]Commission Decision of 22 July 1992, Case n. IV/M.190 - Nestlé/Perrier.

[2]According to the EC, there were considerable barriers to entry in the French bottled water market. The existing brands have invested in advertising for a long time, which created a high consumer fidelity. Moreover, the number of available sources in France is almost inexistent and there are several legal requirements that must be fulfilled for a source water to be labelled as ‘mineral water’. For all these reasons, “the establishment of a new brand would require heavy investment and could take a long time”. Even a foreign company with reputation in the market could not, in practice, compete with the national firms. The main impediment to imports of bottled water is the high transportation cost for a “low-value/ high-volume” product.

demand growth may also encourage market entry. As we can read in recitals 44 and 50 of the Commission decision, the demand for bottled water grew in 1989 and in 1990 and it was expected "a realistic growth rate of 5%" over the following years. However, a potential entrant is aware that, to become active in the bottled water market, it has not only to acquire the rights to explore one source but also to invest in advertising to convince consumers to buy its product.[3]

The main objective of this chapter is to characterize collusion sustainability in markets where demand is growing and may trigger entry by new firms whose size (and, thus, efficiency) may be different from that of the incumbent firms. In particular, following Vasconcelos (2008), we consider a model with two incumbents and one potential entrant. To become active in the market, the entrant has to support a (fixed) entry cost. The three firms produce a homogeneous good, whose demand is growing in a deterministic way. We extend the model of Vasconcelos by considering that firms may differ in the quantity of capital they own. We assume that a firm with more capital produces the same quantity of output at a lower cost (Perry and Porter, 1985). The total capital available to the industry is fixed over the time. Our goal is to adapt the model of Vasconcelos (2008) to better fit the French bottled water industry, after the Nestlé-Perrier merger subject to the European Commission constraints.[4] Here, the incumbents are the groups Nestlé-Perrier and BSN-Volvic, having approximately the same capacity of water (capital). For obvious reasons, the total capacity of water is limited.[5]

We consider the basic setting of an infinitely repeated game, in which firms choose the quantity to produce in each period. The marginal costs of production depend (positively)

[3]See the recital 33 of the Commission decision.

[4]As Compte *et al.* (2002), we exclude the small local producers from the analysis.

[5]We are assuming that all the sources in France are already discovered. Moreover, we do not consider the possibility of a substantial improvement in transports that could make imports profitable.

on the quantity produced and (negatively) on the stock of capital owned by the firm. If the entry costs are not prohibitive, there is one first phase in which the two incumbents are alone in the market and a second phase in which they have to compete with a third firm. We study the incentives for collusion in each of these phases. Notice that the forecast of the entry of a new firm has implications on the incentives of the two incumbents in sustaining collusion, even in the phase in which the entrant is not active. We start by considering that collusion is all-inclusive, that is, it extends to all the active firms. As Patinkin (1947), we consider that, in the collusive scenario, the individual quantities are chosen in order to maximize the joint profit (or, equivalently, to minimize the total cost to supply the aggregate demand). The difficulty concerns the allocation of the monopoly profit when the entrant is already established in the market. If the three firms have equal shares of capital, they naturally divide up their joint profit in equal parts. The doubt arises when their shares of capital are different. In this case, we consider two possibilities: (i) the rule given by the solution of the correspondent Nash bargaining problem; (ii) the rule that each firm receives the profit corresponding to the quantity it produces under joint profit maximization. Interestingly, according to the second rule, the firm's output quota is proportional to its capital. For this reason, we designate this rule by Proportional rule.[6] We compare the individual profits with the two allocation rules and study the implications of choosing one or the other on the incentives for collusion.

We find that the two rules only give the same allocation of the monopoly profit, when the three firms are symmetric. When firms are asymmetric, the Proportional rule is preferred by the firm(s) owning more capital, while the Nash bargaining rule is preferred by the firm(s) with less capital. Notice that, according to the Nash bargaining rule, a firm may receive a

[6]In this setting, the Proportional rule conforms with the joint profit maximization outcome. For some motivation for this rule, see Bos and Harrington (2010) and the references cited therein.

profit that is different from that correspondent to the quantity it produces (determined so as to maximize the joint profit). In this case, there must exist side-payments between firms.

If firms adopt the Nash bargaining rule, the incentives for collusion do not qualitatively depend on how the industry capital is split between the incumbents and the entrant. If firms give little value to future profits (i.e. the discount factor is low) and: (i) the demand grows slowly, collusion is not sustainable either before or after the entry of the third firm; (ii) the demand growth is moderate, collusion could be sustainable after the entry, but the cartel breaks down before the entry; (iii) the demand grows extremely fast, collusion can be sustained either before or after the entry. When the discount factor is high, collusion between the three firms is always sustainable. Moreover, a faster demand growth increases the possibilities of collusion between the two incumbents before the entry. These results are very similar to those obtained by Vasconcelos (2008) for the case in which the three firms have equal shares of the capital.

Interestingly, if the monopoly profit is allocated according to the Nash bargaining rule, the higher the share of capital owned by a firm, the higher the incentives for the firm to disrupt the collusive agreement. This finding is exactly the opposite from that commonly found in the literature.[7] This difference may result from the assumption of this particular allocation rule. In fact, if firms adopt the Proportional rule (like in Vasconcelos, 2005), the firm with a lower share of the industry capital has higher incentives to deviate. However, with this rule, the incentives for collusion depend on how the industry capital is distributed between the incumbents and the entrant. In particular, a large discrepancy in firms' capacities may hinder collusion. Independently of the rule used by firms to allocate output, the overall message is that asymmetries hurt collusion. Our results suggest, however, that the mapping between firms' assets and their incentives to disrupt the collusive agreement depends very

[7] See, for example, Motta (2004) or Vasconcelos (2005).

much on the rule chosen by firms to allocate the collusive output.

We also analyze the scenario of *partial collusion*, in which the incumbents do not include the entrant in the collusive agreement. In this case, after the entry, the cartel behaves like a Stackelberg leader, choosing its quantity before the entrant. In the framework of Vasconcelos (2008), such a collusive scenario was never sustainable. In our model, if the firms have a reasonable discount factor, collusion before the entry is always sustainable. After the entry, collusion is also possible, but only for few values of the discount factor and demand growth.

The Nestlé-Perrier merger has been repeatedly discussed in literature. Motta (1999) makes an analysis of the EC merger policy and refers to this case as being the first in which the concept of joint dominance prevents the Commission to allow for the (initial) proposal of Nestlé to acquire Perrier.[8] Compte *et al.* (2002) also study this case but from a theorical point of view. They study the impacts of asymmetric capacity constraints on collusion in a general setting and apply the results to the case of Nestlé-Perrier. Regarding this case, they analyze the conditions for collusion in four different scenarios: before the merger; after the proposed merger with transfer of Volvic; after the merger without the transfer of Volvic and after the merger with transfer of Volvic and divestiture. Curiously, they find that the scenario proposed by the Commission (merger with transfer of Volvic and divestiture) is more conducive to collusion than the initially proposed by Nestlé (merger with transfer of Volvic). The differences between their work and ours are evident. First, they consider price competition with capacity constraints, while we consider quantity competition with asymmetric production costs. Second, in their model the demand is stable over the time, while demand is growing in our model. Our assumption is more realistic

[8]Until then, the Commission was mainly concerned with the single firm dominance that may arise from a merger.

concerning the particular case of Nestlé-Perrier.[9] Finally, they find that the division rule of the collusive profit that is most favorable to collusion is the one in which a firm receive a share proportional to its capacity. In our model, collusion may not be sustainable in equilibrium with the Proportional rule. Thus, at least for these cases, the Nash bargaining rule is preferable.

In a very recent contribution, Olczak (2009) also analyzes the Nestlé-Perrier merger. Curiously, the incorporation of demand uncertainty in the model of Compte *et al.* (2002) leads to opposite predictions. Olczak finds that the scenario accepted by the European Commission is more favorable to consumers than that in which the Nestlé acquires Perrier without any constraint.

However, the paper that is closer to ours is that of Vasconcelos (2008).[10] He considers that the three firms are symmetric, which does not apply to the French market of bottled water after the (approved) merger. Despite the groups Nestlé-Perrier and BSN-Volvic have similar capacities, the same is not true regarding the potential entrant. As a result, we extend his model by allowing for asymmetries in the stocks of capital. We also explore in more depth the possibility of partial collusion, since it is possible in our context but it was not with his assumptions. Finally, another distinction between the two papers is that we restrict the analysis to the case of perfect collusion, while Vasconcelos (2008) derives the maximal degree of collusion that can be sustained in equilibrium. We make this simplification because the asymmetry between firms complicates the expressions for profits. As a result, it would be very hard to derive the maximal degree of collusion.

[9]Contrary to our article, Compte *et. al* characterize optimal penal codes, while we assume that all firms obtain the Cournot profit in the punishment phase (trigger strategies).

[10]The model of Capuano (2002) also analyzes the impacts of demand growth in the sustainability of collusion. We even borrow from him the analytical expression for the aggregate demand. However, we follow more closely the methodology of Vasconcelos (2008).

In the model of Vasconcelos (2008), the three firms are symmetric and, therefore, split the collusive profit equally among firms. In our case, it is necessary to define how the collusive profit is distributed between firms with different stocks of capital. We make the division rule endogenous, by determining the Nash bargaining solution. In the model of Osborne and Pitchik (1983), the two heterogeneous firms also split the monopoly profit according to their bargaining power. Like us, the authors find that the smallest firm is the one that benefits more from such a kind of division rule (since its profit per unit of capacity is higher than that obtained by the biggest firm). The authors assume that both firms produce at a constant unit cost up to their capacities, while we assume that the firm with more capital has a lower unit cost and that unit costs are increasing. They consider that if firms have different cost functions, side-payments between the firms may be feasible. In his model of price competition, Harrington (1991) assumes that firms are asymmetric with respect to their cost functions.[11] The author claims, however, that it is not reasonable to assume the existence of side-payments. He also argues that to consider firms maximizing joint profits is an *ad hoc* assumption. Instead of specifying an optimal collusive price, the author considers that firms choose prices and market shares according to the Nash bargaining solution. These assumptions are quite different from ours. We obtain the (individual) quantities that maximize the joint profit and derive the Nash bargaining solution to divide it among firms. In our opinion, both assumptions are legitimate.

There are several papers in the literature analyzing the impacts of capacity constraints on the ability to collusion. Brock and Scheinkman (1985) is an earlier contribution dealing with this issue. These authors assume that the N firms are symmetric regarding their capacities

[11] Miklós-Thal (2009) characterizes the optimal collusion in the presence of cost asymmetry, without restricting to any class of strategies. Curiously, she finds that, if side-payments are allowed, cost asymmetry facilitates collusion.

and study the impact of changing the (individual) capacity on the critical discount factor. Fabra (2006) also considers that firms are symmetric with respect to their capacities. In her model, firms support zero marginal costs up to their capacities and an infinite cost above the capacity. We assume that production costs are not constant and that they depend on the share of capital owned by the firm. More precisely, we use a simplified version of the cost function considered by Vasconcelos (2005).[12]

The remainder of the chapter is organized as follows. Section 4.2 sets up the basic model. Section 4.3 determines conditions for full collusion to be sustainable after and before the entry of the third firm. In Section 4.4, we analyze the sustainability of collusion, if the incumbents do not include the entrant in their collusive agreement (partial collusion). In section 4.5, we present the results obtained from numerical simulation. Section 4.6 concludes. In the Appendix D, we derive the expressions for profits in the different competitive scenarios. The Appendix E presents the equilibrium when the entry costs are prohibitively high. Finally, the Appendix F contains the proofs of most propositions.

4.2 The model

We consider a model of quantity competition between two incumbents (firm 1 and firm 2) and one potential entrant (firm 3). The three firms produce homogeneous goods.

[12]The differences between the model of Vasconcelos (2005) and ours are several. Vasconcelos (2005) only looks for equilibria in which firms get a share of the market that equals their share in the industry capital, for all equilibrium paths. We do not restrict our attention to any specific kind of equilibria. Moreover, the demand in his model is stable. Finally, he considers simple penal codes strategies, which punish the deviant in a more severe way than the trigger strategies that we consider.

Following Capuano (2002), we assume that the demand is a linear function with deterministic growth, measured by a parameter $\mu > 1$. More precisely, in period t, the (aggregate) demand is given by:

$$Q_t = \mu^t - p_t, \tag{4.1}$$

where p_t denotes the price in period t. The inverse demand is, therefore, given by: $p_t = \mu^t - Q_t$.

As in the model of Perry and Porter (1985), a firm has to own a fraction of the industry capital to produce units of output. The stock of capital available for the industry is fixed and normalized to one. We consider that the two incumbents are symmetric, owning equal shares of capital, that is, $k_1 = k_2 = k$. Obviously, there only exits the possibility of a new firm to enter in the market if $k < \frac{1}{2}$.

The cost of the firm i, owning a share k_i of the industry capital, to produce q_{it} units of output is given by:

$$C(q_{it}, k_i) = \frac{q_{it}^2}{2k_i}.$$

Thus, the cost function depends negatively on the stock of capital and the marginal production costs are increasing. Moreover, there are no fixed production costs.[13]

To enter in the market, the firm 3 has to invest in capital. The entry cost is assumed to be fixed, $F > 0$. In particular, it does not depend on the amount of capital acquired. As a result, it is on the interest of an entrant to get all the available capital in the market, that is, $k_3 = 1 - 2k$. The entry occurs when the present value of the firm is maximal.

Firms play an infinitely repeated game. In each period t, for $t \in \{1, 2, ...\}$, the active firms simultaneously choose the quantity to produce. Thus, the firm's payoff is the discounted

[13]This is a simplified version of the cost function considered by Vasconcelos (2005).

sum of its profit in each period. We assume that the three firms have the same discount factor, δ. For technical reasons, we restrict the variation of the demand growth parameter, μ, such that: $\mu^2\delta < 1$.

In the first period, each (active) firm produces the quantity established by the collusive agreement. The firms keep producing the collusive quantities as long as there is no defections. If one firm disrupts the collusive agreement, all the firms start producing the Nash equilibrium quantity in all the following periods. In short, the firms use *trigger strategies*.

In the competitive path, the firm i chooses the quantity, q_{it}, that maximizes its own profit:

$$\Pi_{it}^c(q_{it}) = (\mu^t - Q_t)q_{it} - \frac{q_{it}^2}{2k_i}.$$

In the case of competition between the two incumbents, the Cournot equilibrium profit of each firm is equal to:[14]

$$\Pi_{1t}^c(2) = \Pi_{2t}^c(2) = \frac{k(1+2k)}{2(1+3k)^2}\mu^{2t} \equiv \alpha_2\mu^{2t}. \tag{4.2}$$

If the firm 3 has already entered in the market, the equilibrium profit of the incumbent i, for $i \in \{1, 2\}$, is equal to:

$$\Pi_{it}^c(3) = \frac{2k(1+2k)(1-k)^2}{(3+3k-8k^2)^2}\mu^{2t} \equiv \alpha_{3i}\mu^{2t}, \tag{4.3}$$

while the equilibrium profit of the entrant is:

$$\Pi_{3t}^c(3) = \frac{(1+k)^2\,(3-10k+8k^2)}{2\,(3+3k-8k^2)^2}\mu^{2t} \equiv \alpha_{33}\mu^{2t}. \tag{4.4}$$

If the three firms are active, the incumbent i profits morethan the entrant if its share in the industry capital is higher than that of the entrant, that is, if $k \geq \frac{1}{3}$. This occurs because:

$$\Pi_{it}^c(3) \geq \Pi_{3t}^c(3) \Leftrightarrow -3+8k+9k^2-18k^3 \geq 0 \Leftrightarrow \left(k-\frac{1}{3}\right)\left(9+3k-18k^2\right) \geq 0$$

[14]See the Appendix D for details.

and $9 + 3k - 18k^2$ is always greater than zero for $k \in (0, 1/2)$. Moreover, as it would be expected, the equilibrium profit of the incumbents increases with k, while the equilibrium profit of the entrant decreases with k.

4.3 Full collusion

4.3.1 Collusive quantities

Suppose that the entry costs are not prohibitive, that is, the firm 3 enters the market at some moment of time. The greater the value of F, the later the entry. Moreover, the entrant also takes into consideration the demand growth to decide when to enter in the market.

We focus the attention on the ability of firms to sustain perfect collusion. That is, we assume that the cartel chooses the quantity to be produced by each firm that maximizes the joint profit. Thus, before the entry, the cartel chooses (q_{1t}, q_{2t}) such to maximize:

$$\Pi_t^m(q_{1t}, q_{2t}) = \left[\mu^t - (q_{1t} + q_{2t})\right](q_{1t} + q_{2t}) - \left(\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k}\right).$$

It is easy to show that the individual collusive output and the cartel profit in period t are given by:[15]

$$q_{1t}^m(2) = q_{2t}^m(2) = \frac{k}{1+4k}\mu^t$$

and

$$\Pi_t^m(2) = \frac{k}{1+4k}\mu^{2t}.$$

As the two incumbents are symmetric, they divide the monopoly profit in equal parts. Thus, the individual collusive output is:

$$\Pi_{1t}^m(2) = \Pi_{2t}^m(2) = \frac{k}{2(1+4k)}\mu^{2t} \equiv \beta_2\mu^{2t}. \tag{4.5}$$

[15] See the Appendix D for details.

In this section, we consider that the incumbents accommodate the entry of the new firm. More precisely, they include the entrant in their collusive agreement. Thus, after the entry, the cartel chooses (q_{1t}, q_{2t}, q_{3t}) such to maximize:

$$\Pi_t^m(q_{1t}, q_{2t}, q_{3t}) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right](q_{1t} + q_{2t} + q_{3t}) - \left[\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k} + \frac{{q_{3t}}^2}{2(1-2k)}\right].$$

In this case, the aggregate output is equal to $Q_t^m(3) = \frac{\mu^t}{3}$ and the firm i produces $q_{it}^m(3) = k_i Q_t^m(3)$. The correspondent joint profit is:

$$\Pi_t^m(3) = \frac{\mu^{2t}}{6}.$$

Now, we need to specify how this profit is split between the three firms. Notice that, in this case, the three firms probably differ in the stock of capital they own $\left(\text{unless } k = \frac{1}{3}\right)$. Thus, to consider that the three firms divide the monopoly profit in equal parts may not be reasonable.

4.3.2 Collusive agreements

When the colluding firms are asymmetric, there are several admissible rules to divide the collusive (aggregate) profit. We consider two quite natural rules:

$a)$ the *Nash bargaining rule*: each firm gets a share correspondent to the Nash bargaining solution;

$b)$ the *Proportional rule*: a rule that each firm receives the profit corresponding to the quantity it produces under joint profit maximization, $q_{it}^m(3)$.

Let us start by determining the individual profits, if firms adopt the Nash bargaining rule. In this case, let $\Pi_{it}^{m,N}(3)$, for $i \in \{1,2,3\}$, denote the collusive profit of firm i in period t.

Proposition 4.1. *If firms allocate the monopoly profit according to the Nash bargaining rule, the individual profit of the incumbent i, for $i \in \{1,2\}$, is:*

$$\Pi_{it}^{m,N}(3) = \frac{k\left(21 - 6k - 51k^2 + 32k^3\right)}{9\left(3 + 3k - 8k^2\right)^2}\mu^{2t} \equiv \beta_{3i}\mu^{2t}, \tag{4.6}$$

while the collusive profit of the entrant is:

$$\Pi_{3t}^{m,N}(3) = \frac{27 - 30k - 93k^2 + 60k^3 + 64k^4}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t} \equiv \beta_{33}\mu^{2t}. \tag{4.7}$$

Proof. Let λ_{it}, for $i \in \{1,2,3\}$, be the share of firm i in the monopoly profit of period t, that is, $\Pi_{it}^m(3) = \lambda_{it}\Pi_t^m(3)$. These weights must solve the following problem:

$$max_{\lambda_{1t},\lambda_{2t},\lambda_{3t}} \; \left[\lambda_{1t}\Pi_t^m(3) - \Pi_{1t}^c(3)\right]\left[\lambda_{2t}\Pi_t^m(3) - \Pi_{2t}^c(3)\right]\left[\lambda_{3t}\Pi_t^m(3) - \Pi_{3t}^c(3))\right]$$

$$\text{s.t.} \quad \lambda_{1t} + \lambda_{2t} + \lambda_{3t} = 1 \quad \text{and} \quad \lambda_{it}\Pi_t^m(3) - \Pi_{it}^c(3) \geq 0.$$

Substituting $\lambda_{3t} = 1 - \lambda_{1t} - \lambda_{2t}$ into the objective function, the correspondent first-order conditions are:

$$\begin{cases} \left[\left[1 - \lambda_{1t} - \lambda_{2t}\right]\Pi_t^m(3) - \Pi_{3t}^c(3)\right] - \left[\lambda_{1t}\Pi_t^m(3) - \Pi_{1t}^c(3)\right] = 0 \\ \left[\left[1 - \lambda_{1t} - \lambda_{2t}\right]\Pi_t^m(3) - \Pi_{3t}^c(3)\right] - \left[\lambda_{2t}\Pi_t^m(3) - \Pi_{2t}^c(3)\right] = 0 \end{cases}$$

As $\Pi_{1t}^c(3) = \Pi_{2t}^c(3)$, it follows that $\lambda_{1t} = \lambda_{2t} \equiv \lambda_t$, where:

$$\lambda_t = \frac{\Pi_t^m(3) + \Pi_{1t}^c(3) - \Pi_{3t}^c(3)}{3\Pi_t^m(3)}. \tag{4.8}$$

Substituting the expressions for profits, we conclude that λ_t is constant over the time:

$$\lambda_t = \frac{1 + 6\alpha_{31} - 6\alpha_{33}}{3} \equiv \lambda\,,\ \forall t.$$

As a result, the collusive profit of the incumbent i, for $i \in \{1,2\}$, is:

$$\Pi_{it}^m(3) = \frac{1 + 6\alpha_{3i} - 6\alpha_{33}}{18}\mu^{2t} = \frac{k\left(21 - 6k - 51k^2 + 32k^3\right)}{9\left(3 + 3k - 8k^2\right)^2}\mu^{2t},$$

while:

$$\Pi_{3t}^{m}(3) = \frac{1 - 12\alpha_{3i} + 12\alpha_{33}}{18}\mu^{2t} = \frac{27 - 30k - 93k^2 + 60k^3 + 64k^4}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t}.$$

□

If firms adopt the Proportional rule, the collusive profit of firm i in period t is given by:

$$\Pi_{it}^{m,P}(3) = \left(\mu^t - \frac{\mu^t}{3}\right)\frac{k_i\mu^t}{3} - \frac{1}{2k_i}\left(\frac{k_i\mu^t}{3}\right)^2 = k_i\frac{\mu^{2t}}{6} = k_i\Pi_t^m(3). \tag{4.9}$$

In this case, the share of each firm in the monopoly profit is equal to its share in the industry capital.

The firm 1 has higher profits with the Nash bargaining rule than with the Proportional rule if:[16]

$$\Pi_{1t}^{m,N}(3) \geq \Pi_{1t}^{m,P}(3) \quad \Leftrightarrow \quad k\left(15 - 66k + 15k^2 + 208k^3 - 192k^4\right) \geq 0$$

$$\Leftrightarrow \quad -k\left(k - \frac{1}{3}\right)\left(45 - 63k - 144k^2 + 192k^3\right) \geq 0$$

The polynomial $45 - 63k - 144k^2 + 192k^3$ is always positive, for $k \in (0, 1/2)$. Thus, the incumbents prefer the Nash bargaining rule if $k < \frac{1}{3}$. Otherwise, they are better off with the Proportional rule.

Let us now analyze how the allocation rule affects the profit of the entrant. This firm has a higher share of the monopoly profit with the Nash bargaining rule than with the Proportional rule if:

$$\Pi_{3t}^{m,N}(3) \geq \Pi_{3t}^{m,P}(3) \Leftrightarrow 2k\left(k - \frac{1}{3}\right)\left(45 - 63k - 144k^2 + 192k^3\right) \geq 0$$

[16]The same results apply to the firm 2, since the incumbents are symmetric.

Consequently, the entrant prefers the Nash bargaining rule if $k > \frac{1}{3}$, and prefers the Proportional rule otherwise.

When $k = \frac{1}{3}$, the three firms own equal shares in the industry capital. As a result, they have the same bargaining power and split the monopoly profit in equal parts. This is why the two rules coincide in this particular case.

When $k \neq \frac{1}{3}$, we found that the Proportional rule is preferred by the large firm(s), whereas a division of profits based on the Nash bargaining solution would be preferred by the smallest firm(s).[17] To clarify the intuition of this result, let us analyze in more detail the Nash bargaining rule. With this rule, the firms divide up the monopoly profit according to their relative bargaining power. The bargaining power of one firm is measured by its payoff if the bargain is not reached, the called *threat*. In our model, the firms' threats correspond to the Cournot profits. As we saw before, the higher the share of capital owned by one firm, the higher its Cournot profit. Thus, a larger firm has more bargaining power. However, the three firms can equally well break the agreement and force the reversion to the threat-point. As a result, they equally divide the excess of the joint profit over the sum of the Cournot profits. Notice that, using the equality (4.8) and that $\Pi_{1t}^c(3) = \Pi_{2t}^c(3)$, we can write the collusive profit of the incumbent 1 as follows:

$$\begin{aligned}\Pi_{1t}^{m,N}(3) &= \lambda_t \Pi_t^m(3) = \frac{\Pi_t^m(3) + \Pi_{1t}^c(3) - \Pi_{3t}^c(3)}{3} \\ &= \Pi_{1t}^c(3) + \frac{\Pi_t^m(3) - [\Pi_{1t}^c(3) + \Pi_{1t}^c(3) + \Pi_{3t}^c(3)]}{3}.\end{aligned}$$

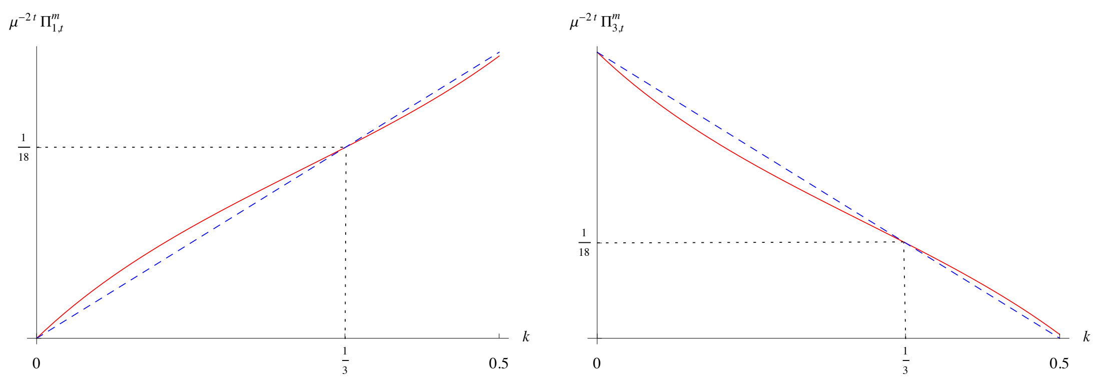
By analogy, we could write the collusive profit of the entrant as the sum of its Cournot profit with the third part of the excess of the monopoly profit over the sum of the Cournot profit

[17] In a quite different setting, Osborne and Pitchik (1983) also found that when the firms use the Nash bargaining solution to divide up the monopoly profit, "the balance of forces is in favor to the small firm" (p. 60).

of the three firms. Thus, it is clear that a larger firm receives a higher share of the monopoly profit. However, this is the firm for which the difference between the collusive profit and the Cournot profit is lower.

With the Proportional rule, each firm receives the profit corresponding to the quantity it produces under joint profit maximization. As a larger firm is more efficient, it produces a higher share of the cartel's output. Thus, when the monopoly profit is allocated according to the output quota, the larger firms are the most benefited.

In Figure 4.1, we represent the firms' collusive profit when it is adopted the Nash bargaining rule or the Proportional rule.



(a) Incumbent.

(b) Entrant.

Figure 4.1: Collusive profits with the Nash bargaining rule (solid line) and with the Proportional rule (dashed line).

As, the firm i produces the quantity $q_{it}^m(3)$ regardless the adopted rule to allocate the monopoly profit, if firms choose the Nash bargaining rule, there must exist side-payments between firms. More precisely, when $k < \frac{1}{3}$, the incumbents must receive a higher profit than that corresponding to the quantity they produce. Thus, the entrant has to share part of its profits with the incumbents. That is, the entrant must pay the following amount to each incumbent i, for $i \in \{1, 2\}$:

$$SP_{3\to i} = \Pi_{it}^{m,N} - \Pi_{it}^{m,P} = \frac{k(1-3k)\left(15 - 21k - 48k^2 + 64k^3\right)}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t}.$$

Conversely, when $k > \frac{1}{3}$, each incumbent i has to pay the following amount to the entrant:

$$SP_{i\to 3} = \frac{\Pi_{3t}^{m,N} - \Pi_{3t}^{m,P}}{2} = \frac{k(3k-1)\left(15 - 21k - 48k^2 + 64k^3\right)}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t} = -SP_{3\to i}.$$

4.3.3 Sustainability of collusion after the entry

In this section, we consider that the incumbents include the entrant in their collusive agreement (*full collusion*). Moreover, we start by assuming that firms allocate the monopoly profit according to the Nash bargaining rule.

4.3.3.1 Optimal entry period

Suppose that the firm 3 enters in the market along the collusive path. Consider also that firms have a high enough discount factor for collusion to be sustainable after the entry. As a result, the entrant receives the collusive profit in all the periods that follow its entry. Thus, if the entry occurs in period t, the present discounted value of the profit of firm 3 is:[18]

$$V^m(t) = \sum_{s=t}^{\infty} \Pi_{3s}^{m,N}(3)\delta^s - \delta^t F = \beta_{33}\sum_{s=t}^{\infty}\mu^{2s}\delta^s - \delta^t F = \beta_{33}\frac{\left(\mu^2\delta\right)^t}{1-\mu^2\delta} - \delta^t F,$$

where β_{33} is given by (4.7). The optimal entry period is that for which V^m is maximum. If t was a continuous variable, the optimal entry period must verify the following first-order

[18] We are assuming that, if the firm supports the entry cost in period t, it already makes the profit of this period.

condition:

$$\beta_{33}\frac{\mu^{2t}\delta^t}{1-\mu^2\delta}ln(\mu^2\delta) - F\delta^t ln(\delta) = 0.$$

Solving this equation in order to t, we obtain:

$$t_1(\mu, \delta, F, k) = \frac{1}{2ln(\mu)}ln\left[\frac{ln(\delta)}{ln(\mu^2\delta)}\frac{F(1-\mu^2\delta)}{\beta_{33}}\right]. \tag{4.10}$$

To ensure that $t_1(\mu, \delta, F, k) > 1$, it is necessary that:

$$F \geq \frac{ln(\mu^2\delta)}{ln(\delta)}\frac{\mu^2\beta_{33}}{1-\mu^2\delta}.$$

However, in our model, the time is discrete and the expression obtained for t_1 may not be an integer. If this happens, the firm 3 must compare the value of V^m in the largest previous integer of t_1 with the value of V^m in the smallest following integer of t_1. The optimal discrete entry time is, therefore, given by:

$$\tilde{t}_1 = \begin{cases} \lceil t_1 \rceil & \text{if} \quad V^m(\lceil t_1 \rceil) > V^m(\lfloor t_1 \rfloor) \\ \lfloor t_1 \rfloor & \text{if} \quad V^m(\lceil t_1 \rceil) \leq V^m(\lfloor t_1 \rfloor) \end{cases}, \tag{4.11}$$

where:

$$\lceil t \rceil = \max\{n \in \mathbb{N} : n \leq t\} \quad \text{and} \quad \lfloor t \rfloor = \min\{n \in \mathbb{N} : n \geq t\}.$$

It is straightforward to see that the higher the share of firm 3 in the industry capital, the earlier is its entry in the market. This is a very natural result, since a higher share of capital corresponds to a higher collusive profit.

4.3.3.2 Critical discount factor

Let us write the incentive compatibility constraint (hereafter, ICC) that must be satisfied for collusion to be sustainable after the entry. We have already determined the profit of each firm in the collusive and in the punishment paths. Thus, it is only missing to compute the deviating profits to write the ICC.

If the firm i, for $i \in \{1,2,3\}$, decides to deviate in period t, it assumes that the rivals are producing the collusive output and chooses the quantity, $q_{it}^{d}(3)$, that maximizes its individual profit:

$$\Pi_{it}^{d}\left(q_{it}\right)=\left[\mu^{t}-\left(q_{it}+\frac{k_{j}}{3}\mu^{t}+\frac{1-k_{i}-k_{j}}{3}\mu^{t}\right)\right]q_{it}-\frac{q_{it}^{2}}{2k_{i}},$$

for $j \in \{1,2,3\}$ and $j \neq i$. It is straightforward to obtain the deviating profit of firm i:[19]

$$\Pi_{it}^{d}(3)=\frac{k_{i}\left(2+k_{i}\right)^{2}}{18(1+2k_{i})}\mu^{2t}.$$

Thus, if the deviating firm is the incumbent i, for $i \in \{1,2\}$, its profit in period t is equal to:

$$\Pi_{it}^{d}(3)=\frac{k\left(2+k\right)^{2}}{18(1+2k)}\mu^{2t}\equiv\gamma_{3i}\mu^{2t}, \tag{4.12}$$

If the deviating firm is the entrant, we obtain:

$$\Pi_{3t}^{d}(3)=\frac{(3-2k)^{2}(1-2k)}{18(3-4k)}\mu^{2t}\equiv\gamma_{33}\mu^{2t}. \tag{4.13}$$

In each period $t \geq t_1$ that follows the entry, the firm i, for $i \in \{1,2\}$, prefers to be in collusion than to deviate, if the following incentive compatibility constraint holds:

$$\sum_{s=t}^{\infty}\Pi_{is}^{m,N}(3)\delta^{s-t} \quad \geq \quad \Pi_{it}^{d}(3)+\sum_{s=t+1}^{\infty}\Pi_{is}^{c}(3)\delta^{s-t} \Leftrightarrow$$

$$\Leftrightarrow \left(\beta_{3i}-\alpha_{3i}\right)\delta^{-t}\sum_{s=t+1}^{\infty}\left(\mu^{2}\delta\right)^{s} \quad \geq \quad \left(\gamma_{3i}-\beta_{3i}\right)\mu^{2t} \Leftrightarrow \mu^{2}\delta \geq \frac{\gamma_{3i}-\beta_{3i}}{\gamma_{3i}-\alpha_{3i}} \equiv \mu^{2}\tilde{\delta}_{i}. \tag{4.14}$$

[19]For details, see the Appendix D.

Substituting the expressions for α_{3i}, β_{3i} and γ_{3i} and with some simple algebra, we obtain:

$$\mu^2\tilde{\delta}_1(k) = \mu^2\tilde{\delta}_2(k) = \frac{-6 + 36k + 51k^2 - 190k^3 - 103k^4 + 208k^5 + 64k^6}{k\left(36 + 33k - 186k^2 - 119k^3 + 208k^4 + 64k^5\right)}$$

and

$$\mu^2\tilde{\delta}_3(k) = \frac{45 - 240k + 309k^2 + 414k^3 - 1348k^4 + 1088k^5 - 256k^6}{54 - 270k + 342k^2 + 390k^3 - 1332k^4 + 1088k^5 - 256k^6}.$$

It is straightforward to verify that, $\tilde{\delta}_1\left(\frac{1}{3}\right) = \tilde{\delta}_2\left(\frac{1}{3}\right) = \tilde{\delta}_1\left(\frac{1}{3}\right)$. It is also possible to check that $\frac{d\tilde{\delta}_1(k)}{dk} > 0$, for $k \in \left(0, \frac{1}{2}\right)$, which means that the higher the value of k, the more difficult is for the incumbents to comply with the collusive agreement. On the contrary, in the considered domain, we have that $\frac{d\tilde{\delta}_3(k)}{dk} < 0$, meaning that a higher value for k makes collusion more appealing to the entrant.

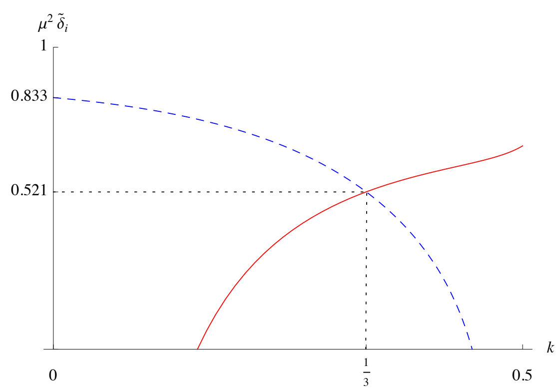


Figure 4.2: Critical (adjusted) discount factor for the incumbents (solid line) and for the entrant (dashed line), if firms divide the monopoly profit according to the Nash bargaining rule.

Moreover, if $\mu^2\delta < \frac{49}{94} \approx 0.521$, there is no possibility of collusion, regardless of the value of k. If $\mu^2\delta \geq \frac{45}{54} \approx 0.833$, perfect collusion is always sustainable.

As the three firms have the same discount factor, the critical discount factor is given by:

$$\tilde{\delta}(k) = \max\left\{\tilde{\delta}_1(k), \tilde{\delta}_3(k)\right\} = \begin{cases} \tilde{\delta}_3(k) & \text{if} \quad k < \frac{1}{3} \\ \tilde{\delta}_1(k) & \text{if} \quad k \geq \frac{1}{3} \end{cases}. \tag{4.15}$$

There are several works in the literature showing that the existence of asymmetry in capacities makes collusion less likely.[20] Our findings reinforce this result, since the minimum value for the critical discount factor is obtained when firms have equal shares of capital, $k = \frac{1}{3}$.

What is actually a surprising result of our model is that the firm owning the biggest share of capital is the one for which the ICC is binding. For $k < \frac{1}{3}$, it is the entrant that has more incentives to defect; while for $k > \frac{1}{3}$ the incumbents have more incentives to disrupt the collusive agreement than the entrant. This finding is exactly the opposite of that in the model of Vasconcelos (2005).[21] However, he considers that, in the collusive scenario, firms obtain the profit corresponding to quantity they produce (Proportional rule). Let us analyze whether a change in the allocation of the monopoly profit might be responsible for this difference in results.

Suppose, therefore, that firms adopt the Proportional rule. Let us explore the effects of this assumption on the critical discount value. By changing the allocation rule, the Cournot profits remain the same. Moreover, as the deviating profit of one firm was determined by assuming that the other firms were producing the quantities that maximize the joint profit (which do not depend on how the monopoly profit is divided between firms). Thus, the values of α_{3i} and of γ_{3i} do not change. Obviously, the same does not apply to β_{3i}, which is now equal to $\frac{k_i}{6}$. Substituting these values in the condition (4.14), we obtain the critical

[20] See, for example, Lambson (1995), Davidson and Deneckere (1984), Compte *et al.* (2002).

[21] Vasconcelos (2005) assumes that the demand is stable. This is a limit case of our model, in which $\mu \to 1$. Such an assumption would naturally change the magnitude of $\tilde{\delta}$, but not the nature of the results.

discount factor for the incumbents:

$$\mu^2\hat{\delta}_1(k) = \mu^2\hat{\delta}_2(k) = \frac{\left(3 - 11k^2 + 8k^3\right)^2}{k\left(36 + 33k - 186k^2 - 119k^3 + 208k^4 + 64k^5\right)},$$

and for the entrant:

$$\mu^2\hat{\delta}_3(k) = \frac{k\left(3 + 3k - 8k^2\right)^2}{27 - 81k + 9k^2 + 213k^3 - 240k^4 + 64k^5}.$$

It is straightforward to check that $\tilde{\delta}_i\left(\frac{1}{3}\right) = \hat{\delta}_i\left(\frac{1}{3}\right)$, $\forall i \in \{1,2,3\}$. This was expected because, when the firms have equal shares of capital, the Nash bargaining rule and the Proportional rule coincide.

As the three firms have the same discount factor, the critical discount factor is given by:

$$\hat{\delta}(k) = \max\left\{\hat{\delta}_1(k), \hat{\delta}_3(k)\right\} = \begin{cases} \hat{\delta}_1(k) & \text{if} \quad k < \frac{1}{3} \\ \hat{\delta}_3(k) & \text{if} \quad k \geq \frac{1}{3} \end{cases}.$$

Proposition 4.2. *If* $k \in (0, 0.199) \cup (0.436, 1/2)$ *and firms adopt the Proportional rule to allocate the monopoly profit, perfect collusion is not sustainable after the entry.*

Proof. See the Appendix F. □

In Figure 4.3, we represent the critical (adjusted) discount value for the incumbents and for the entrant, if they adopt the Proportional rule.

For $k < 0.199$, the incumbents have no incentives to comply with the collusive agreement. In this case, the incumbents are considerably smaller than the entrant (that owns more than half of the stock of industry capital). As a result, if the monopoly profit is allocated according to the Proportional rule, the incumbents get a small share of the monopoly profit

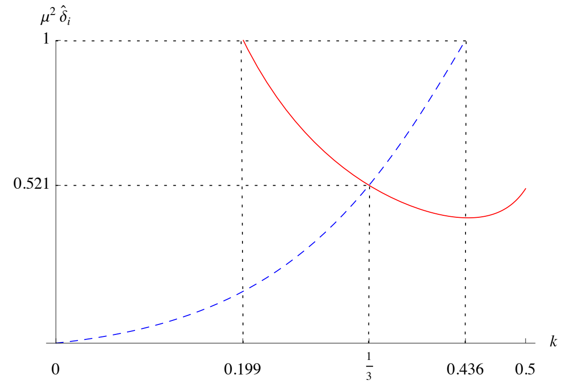


Figure 4.3: Critical (adjusted) discount factor for the incumbents (solid line) and for the entrant (dashed line), if firms adopt the Proportional rule to divide the monopoly profit.

and may prefer to deviate.[22] If $k > 0.436$, there is also an imbalance in the distribution of the industry capital, but now in favor of the incumbents. Each incumbent has almost half the industry capital, so that the entrant has, at best, 12.8% of the industry capital. Now, for the reasons mentioned above, it is the entrant that is not willing to collude.

Furthermore, with the Proportional rule, the binding ICC is that of the small firm(s). This is the result found by Vasconcelos (2005) and it is exactly the converse of that we have obtained with the Nash bargaining rule. This finding alerts us to the importance of the allocation rule of the monopoly profit in the firms' incentives to sustain collusion.

[22]Notice that, in this case, the Cournot profit of the incumbents is not significantly higher than their collusive profit.

4.3.4 Sustainability of collusion before the entry

4.3.4.1 Optimal entry time

Consider that the entry occurs when the incumbents are in the punishment path. If firms play as Cournot oligopolists in all stages that follow the entry, the present discount value of the entrant's profits, if it enters in period t, is:

$$V^c(t) = \sum_{s=t}^{\infty} \Pi^c_{3s}(3)\delta^s - \delta^t F = \alpha_{33}\frac{(\mu^2\delta)^t}{1-\mu^2\delta} - \delta^t F,$$

where α_{33} is given by (4.4). If t was a continuous variable, the maximum value for V^c would be achieved at:[23]

$$t_2(\mu, \delta, F, k) = \frac{1}{2ln(\mu)} ln\left[\frac{ln(\delta)}{ln(\mu^2\delta)}\frac{F(1-\mu^2\delta)}{\alpha_{33}}\right]. \tag{4.16}$$

Notice that the entry is later if it occurs along the punishment path than along the collusive path, that is, $t_2 > t_1$. This occurs because the entrant has more profits if it is in collusion with the incumbents than if it is competing with them (i.e. $\beta_{33} > \alpha_{33}$).

To ensure that the expression obtained for t_2 represents a value greater than 1, the fixed entry cost, F, must be sufficiently high:

$$F \geq \frac{\mu^2\alpha_{33}ln(\mu^2\delta)}{ln(\delta)(1-\mu^2\delta)}.$$

If the expression (4.16) is non integer, the optimal (discrete) entry time of firm 3 is given by:

$$\tilde{t}_2 = \begin{cases} \lceil t_2 \rceil & \text{if} \quad V^c(\lceil t_2 \rceil) > V^c(\lfloor t_2 \rfloor) \\ \lfloor t_2 \rfloor & \text{if} \quad V^c(\lceil t_2 \rceil) \leq V^c(\lfloor t_2 \rfloor) \end{cases}.$$

[23] To obtain the expression for t_2, we followed the same steps as those to obtain t_1.

4.3.4.2 Critical discount factor

Now, we determine the minimum discount factor for collusion to be sustainable (by the incumbents) before the entry. Below, we assume that, after the entry, firms divide the monopoly profit according to the Nash bargaining rule. By doing so, we are ensuring that there always exist a value for δ such that collusion is sustainable after the entry, regardless of the value of k. Moreover, we consider that $\delta(k) > \tilde{\delta}(k),\ \forall k \in (0, 1/2)$, where $\tilde{\delta}(k)$ is given by (4.15).

Consider a period $t \in \left\{0, 1, ..., \tilde{t}_1 - 1\right\}$, where $\tilde{t}_1$ is given by (4.11). The incumbent i, for $i \in \{1, 2\}$, is willing to collude with the other incumbent before the entry of firm 3 if the following incentive compatibility constraint is satisfied:

$$\sum_{s=t}^{\tilde{t}_1-1} \delta^{s-t}\Pi_{is}^{m}(2) + \sum_{s=\tilde{t}_1}^{\infty} \delta^{s-t}\Pi_{is}^{m,N}(3) \geq \Pi_{it}^{d}(2) + \sum_{s=t+1}^{t+\tilde{t}_2-1} \delta^{s-t}\Pi_{is}^{c}(2) + \sum_{s=t+\tilde{t}_2}^{\infty} \delta^{s-t}\Pi_{is}^{c}(3). \quad (4.17)$$

Notice that we have not yet determined the collusive profit of the incumbent i, $\Pi_{is}^{m}(2)$, nor its deviating profit, $\Pi_{is}^{d}(2)$, before the entry. By maximizing the incumbents' joint profit and dividing the monopoly profit in two equal parts, we obtain:[24,25]

$$\Pi_{it}^{m}(2) = \frac{k}{2(1+4k)}\mu^{2t} \equiv \alpha_2\mu^{2t}. \quad (4.18)$$

If the incumbent i deviates in period t, while the other incumbent is producing the collusive output, it obtains the following profit:[26]

$$\Pi_{it}^{d}(2) = \frac{k(1+3k)^2}{2(1+2k)(1+4k)^2}\mu^{2t} \equiv \gamma_2\mu^{2t}. \quad (4.19)$$

[24] For details, see the Appendix D.

[25] As the two incumbents own equal shares of capital, it is indifferent if they divide the monopoly profit according to the Nash bargaining rule or to the Proportional rule. In both cases, each incumbent gets half of the joint profit.

[26] For details, see the Appendix D.

Substituting the expressions for profits, the ICC (4.17) can be rewritten as follows:

$$\beta_2 \sum_{s=t}^{\tilde{t}_1 - 1} (\mu^2\delta)^s + \beta_{3i} \sum_{s=\tilde{t}_1}^{\infty} (\mu^2\delta)^s \geq \gamma_2(\mu^2\delta)^t + \alpha_2 \sum_{s=t+1}^{t+\tilde{t}_2-1} (\mu^2\delta)^s + \alpha_{3i} \sum_{s=t+\tilde{t}_2}^{\infty} (\mu^2\delta)^s,$$

which is equivalent to:

$$\beta_2 \frac{(\mu^2\delta)^t - (\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} + \beta_{3i} \frac{(\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} \geq \gamma_2(\mu^2\delta)^t + \alpha_2 \frac{(\mu^2\delta)^{t+1} - (\mu^2\delta)^{\tilde{t}_2+t}}{1-\mu^2\delta} + \alpha_{3i} \frac{(\mu^2\delta)^{\tilde{t}_2+t}}{1-\mu^2\delta} \tag{4.20}$$

Lemma 4.1. *If the incentive compatibility constraint* (4.20) *is satisfied for* $t = \tilde{t}_1 - 1$*, then it is satisfied for all* $t \in \{0, 1, ..., \tilde{t}_1 - 1\}$.

Proof. See the Appendix F. □

The Lemma states that if the incentive compatibility constraint is satisfied for the period that immediately precedes the entry of firm 3, it is verified for all the previous periods. Substituting $t = \tilde{t}_1 - 1$ in the inequality (4.20), we obtain that:

$$\beta_2 \frac{(\mu^2\delta)^{\tilde{t}_1-1} - (\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} + \beta_{3i} \frac{(\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} \geq \gamma_2(\mu^2\delta)^{\tilde{t}_1-1} \quad + \quad \alpha_2 \frac{(\mu^2\delta)^{\tilde{t}_1} - (\mu^2\delta)^{\tilde{t}_2+\tilde{t}_1-1}}{1-\mu^2\delta} + \quad + \quad \alpha_{3i} \frac{(\mu^2\delta)^{\tilde{t}_2+\tilde{t}_1-1}}{1-\mu^2\delta} \tag{4.21}$$

Now, we find a sufficient condition for collusion to not be an equilibrium. In other words, we look for values of δ such that the condition (4.21) does not hold.

Proposition 4.3. *(No collusion) Given* $\mu > 1$ *and* $k \in (0, 1/2)$*, the incumbent* i *is not willing to collude if the discount factor,* δ*, satisfies the inequality below:*

$$(\mu^2\delta)^{t_2-1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) < 0. \tag{4.22}$$

Proof. See the Appendix F. □

It may happen that firms are patient so that collusion would be sustainable (by the three firms) after the entry, $\delta(k) > \tilde{\delta}(k)$, but not the sufficient for collusion to be sustainable (by the two incumbents) before the entry. In the next proposition, we present a sufficient condition for perfect collusion to be sustainable before the entry.

Proposition 4.4. *(Perfect collusion) Given $\mu > 1$ and $k \in (0, 1/2)$, the incumbent i is willing to (perfectly) collude before the entry if the discount factor, δ, satisfies the inequality below:*

$$(\mu^2\delta)^{t_2+1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0. \tag{4.23}$$

Proof. See the Appendix F. □

4.4 Partial collusion

Up to this moment, we have considered that collusion was all-inclusive. More precisely, we have assumed that if the incumbents were colluding, they would include the entrant in a more inclusive agreement. Consider now that the incumbents form a cartel, but they do not include the entrant in their agreement. The entrant chooses, therefore, the output level that maximizes its individual profit in each period.

We assume that the cartel acts as a Stackelberg quantity leader, while the entrant is a follower, playing *a la* Cournot. Thus, in each period, there is the following two-stage game:

1^{st} *stage:* The incumbents choose the quantities that maximize their joint profit;

2^{nd} *stage:* The entrant observes the quantity chosen by each incumbent and chooses the quantity that maximizes its individual profit.

The equilibrium profit of the incumbent i, for $i \in \{1, 2\}$, in period t is given by:[27]

$$\Pi_{it}^{pm}(3) = \frac{2k(1-k)^2}{(3-4k)\,(3+4k-8k^2)}\mu^{2t} \equiv \zeta_{3i}\mu^{2t}, \tag{4.24}$$

while the profit of the entrant is:

$$\Pi_{3t}^{pm}(3) = \frac{(1-2k)\,(3-4k^2)^2}{2(3-4k)\,(3+4k-8k^2)^2}\mu^{2t} \equiv \zeta_{33}\mu^{2t}. \tag{4.25}$$

Proposition 4.5. *If $k \in (0, k^*)$, for $k^* \approx 0.342$, the entrant profits more than each incumbent. If $k \in (k^*, 1/2)$, each incumbent profits more than the entrant.*

Proof. See the Appendix F. □

In Figure 4.4, it is represented the individual profit of each firm in the scenario of partial collusion.

Curiously, if the three firms have equal shares of capital, that is, $k = \frac{1}{3} < k^*$, the entrant profits more than each incumbent. There exists *second-mover advantage*. This is a surprising result, since that, in the Stackelberg model, the leader uses to have advantage in playing first.

[27] See the Appendix D for details.

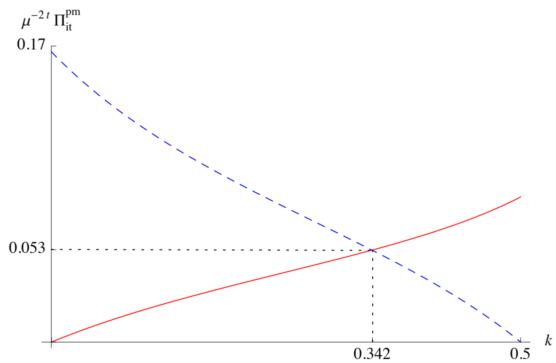


Figure 4.4: Adjusted profit of each incumbent (solid line) and of the entrant (dashed line), in the scenario of partial collusion.

4.4.1 Sustainability of collusion after the entry

4.4.1.1 Optimal entry time

As in the scenario of full collusion, the firm 3 enters in the market when its present discounted value is maximal. In this case, it is given by:

$$V^{pm}(t) = \zeta_{33}\frac{\left(\mu^2\delta\right)^t}{1-\mu^2\delta} - \delta^t F.$$

Following the same steps as in the case of full collusion, we find that the optimal (discrete) entry time is given by:

$$\tilde{t}_3 = \begin{cases} \lceil t_3 \rceil & \text{if} \quad V^{pm}(\lceil t_3 \rceil) > V^{pm}(\lfloor t_3 \rfloor) \\ \lfloor t_3 \rfloor & \text{if} \quad V^{pm}(\lceil t_3 \rceil) \leq V^{pm}(\lfloor t_3 \rfloor) \end{cases},$$

where

$$t_3(\mu,\delta,F,k) = \frac{1}{2ln(\mu)} ln\left[\frac{ln(\delta)}{ln(\mu^2\delta)}\frac{F(1-\mu^2\delta)}{\zeta_{33}}\right]. \tag{4.26}$$

To ensure that $t_3(\mu, \delta, F, k) \geq 1$, the entry cost, F, must be sufficiently high:

$$F \geq \frac{ln(\mu^2\delta)}{ln(\delta)} \frac{\mu^2\zeta_{33}}{1-\mu^2\delta}.$$

4.4.1.2 Critical discount factor

Let us write the incentive compatibility constraint that must be satisfied for each incumbent to be willing to collude with the other incumbent after the entry of firm 3. To do so, we need the expression for the deviating profit. If the incumbent i, $i \in \{1, 2\}$, decides to deviate in period t, its profit is equal to:[28]

$$\Pi_{it}^{pd}(3) = \frac{2k(1-k)^2 \left(3+3k-8k^2\right)^2}{(1+2k)(3-4k)^2 \left(3+4k-8k^2\right)^2} \mu^{2t} \equiv \eta_{3i}\mu^{2t}. \tag{4.27}$$

As a result, the incumbent i, for $i \in \{1, 2\}$, is willing to (partially) collude after the entry if the following ICC is satisfied:

$$\sum_{s=t}^{\infty} \Pi_{is}^{pm}(3)\delta^{s-t} \geq \Pi_{it}^{pd}(3) + \sum_{s=t+1}^{\infty} \Pi_{is}^{c}(3)\delta^{s-t},$$

where $\Pi_{is}^{c}(3)$ is the Cournot profit of one incumbent, if the three firms are active in the market. The expression for $\Pi_{is}^{c}(3)$ is given in (4.3). Substituting the expressions for profits in the last inequality, we obtain:

$$\mu^2\delta \geq \frac{\eta_{3i} - \zeta_{3i}}{\eta_{3i} - \alpha_{3i}} \equiv \mu^2\bar{\delta}_i. \tag{4.28}$$

It follows that $\bar{\delta}_1 = \bar{\delta}_2 \equiv \bar{\delta}$, which was expected, since the incumbents are symmetric. After some algebric manipulation, we can write the critical (adjusted) discount factor as follows:

$$\mu^2\bar{\delta}(k) = \frac{\left(3+3k-8k^2\right)^2}{18+36k-79k^2-96k^3+128k^4}. \tag{4.29}$$

[28] For details, see the Appendix D.

Deriving this expression in order to k, we obtain that:

$$\frac{d(\mu^2\bar{\delta})}{dk}=\frac{2k\left(3+8k^2\right)\left(3+3k-8k^2\right)}{\left(18+36k-79k^2-96k^3+128k^4\right)^2}.$$

As $3+3k-8k^2$ is positive, for all $k\in(0,1/2)$, the critical discount factor, $\bar{\delta}$, is (strictly) increasing in k. Thus, a high value of k enhances the possibilities of collusion (after the entry). Notice that, if the incumbents are very small (when compared with the entrant), the difference between their collusive profit and their Cournot profit is not very high. This provides an incentive for the incumbents to break the collusive agreement, in order to receive the deviating profit. The only chance for collusion to be sustainable is if the incumbents greatly value their future profits. Notice that when $k\rightarrow 0$, we have that $\bar{\delta}\rightarrow 1/2$. When $k\rightarrow 1/2$, we have that $\mu^2\bar{\delta}\rightarrow 0.51$. Thus, if the incumbents own high shares of capital, collusion is almost certain (for reasonable values of δ).

4.4.2 Sustainability of collusion before the entry

The incumbents are willing collude before the entry if the following incentive compatibility constraint:[29]

$$\sum_{s=t}^{\tilde{t}_3-1}\delta^{s-t}\Pi_{is}^{m}(2)+\sum_{s=\tilde{t}_3}^{\infty}\delta^{s-t}\Pi_{is}^{pm}(3)\geq\Pi_{it}^{d}(2)+\sum_{s=t+1}^{t+\tilde{t}_2-1}\delta^{s-t}\Pi_{is}^{c}(2)+\sum_{s=t+\tilde{t}_2}^{\infty}\delta^{s-t}\Pi_{is}^{c}(3)\quad(4.30)$$

is satisfied for all periods t, $t\in\left\{0,1,...,\tilde{t}_3-1\right\}$. Substituting the expressions for profits and using the fact that if the ICC is verified for $t=\tilde{t}_3-1$, then it is verified for all $t\in\left\{0,1,...,\tilde{t}_3-1\right\}$, we can write it as:

$$\left(\mu^2\delta\right)^{\tilde{t}_2}\left(\alpha_2-\alpha_{3i}\right)+\mu^2\delta\left(-\beta_2+\zeta_{3i}+\gamma_2-\alpha_2\right)+\left(\beta_2-\gamma_2\right)\geq 0.$$

[29] If the firm 3 enters in the market along the punishment phase (i.e. the incumbents are competing *a la* Cournot), the optimal entry time is given by the expression (4.16).

Similarly to the case of full collusion, it is possible to show that the incumbents are not willing to collude in any period before the entry if:

$$(\mu^2\delta)^{t_2-1}(\alpha_2-\alpha_{3i})+\mu^2\delta(-\beta_2+\zeta_{3i}+\gamma_2-\alpha_2)+(\beta_2-\gamma_2)<0. \tag{4.31}$$

On the contrary, if the inequality above is satisfied:

$$(\mu^2\delta)^{t_2+1}(\alpha_2-\alpha_{3i})+\mu^2\delta(-\beta_2+\zeta_{3i}+\gamma_2-\alpha_2)+(\beta_2-\gamma_2)\geq 0, \tag{4.32}$$

the incumbents are willing to collude in all periods before the entry.

4.5 Numerical examples

In this section, we make the graphical representations of sufficient conditions for which no collusion can be sustainable before the entry of firm 3 and for which perfect collusion can be sustainable before the entry. We start by analyzing the case in which the incumbents include the entrant in their agreement (full collusion) and, at the end of the Section, we consider the scenario of partial collusion.

4.5.1 Full collusion

We restrict the attention to the cases in which collusion can be sustained after the entry (by the two incumbents and by the entrant). Recall that the critical discount factor depends on how the three firms divide the monopoly profit. The admissible region of parameters (μ,δ) is, therefore, such that:

- $\mu^2\tilde{\delta}_3(k)\leq\mu^2\delta(k)<1$ if $k<\frac{1}{3}$ and $\mu^2\tilde{\delta}_1(k)\leq\mu^2\delta(k)<1$ if $k\geq\frac{1}{3}$, if firms adopt the Nash bargaining rule;

- $\mu^2\hat{\delta}_1(k) \leq \mu^2\delta(k) < 1$ if $k < \frac{1}{3}$ and $\mu^2\hat{\delta}_3(k) \leq \mu^2\delta(k) < 1$ if $k \geq \frac{1}{3}$, if firms adopt the Proportional rule.

In the figures below, these admissible regions for the parameters correspond to the areas in between the two dashed lines. More precisely, the dotted line represents the critical (adjusted) discount factor in each scenario, while the dashed line corresponds the condition $\mu^2\delta = 1$.

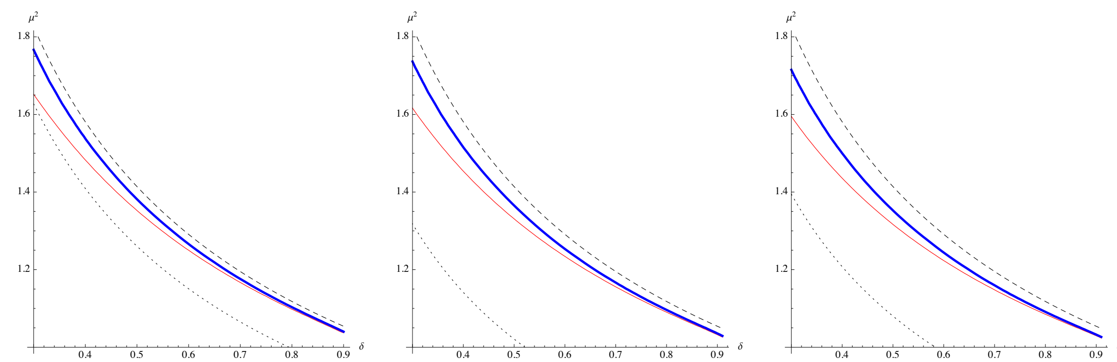
The thick line represents the sufficient condition identified in the proof of Proposition 4.4. For the pairs (δ, μ^2) above this line, perfect collusion can be sustained before (and after) the entry. The other solid line represents the sufficient condition identified in the proof of Proposition 4.3. For the pairs (δ, μ^2) below this line, collusion is not sustainable before the entry.

To build the graphics we need to assign a value to the (fixed) entry cost, F, and to the share of capital owned by each incumbent, k. With regard to F, we allow for two possibilities: one in which F is low and another in which F is relatively high. We consider that F is low if the entry occurs up to the 20$^{\text{th}}$ period and relatively high if the entry occurs between the 20$^{\text{th}}$ and the 60$^{\text{th}}$ periods.[30] Regarding k, we consider three possibilities: (i) $k = 0.1$, corresponding to the case in which the incumbents are small (when compared with the entrant); (ii) $k = 1/3$, corresponding to the case in which the incumbents and the entrant are identical; (iii) $k = 0.4$, corresponding to the case in which the incumbents are large (when compared with the entrant).

[30]Notice that the limits for F depend on μ, δ and k. For simplicity, we set reasonable values for $\mu(= 1.2)$, $\delta(= 0.6)$ and determine, for each value of k, the correspondent value for F.

4.5.1.1 Nash bargaining rule

Let us start by considering that the (fixed) entry cost is low. The graphics obtained are presented in Figure 4.5.



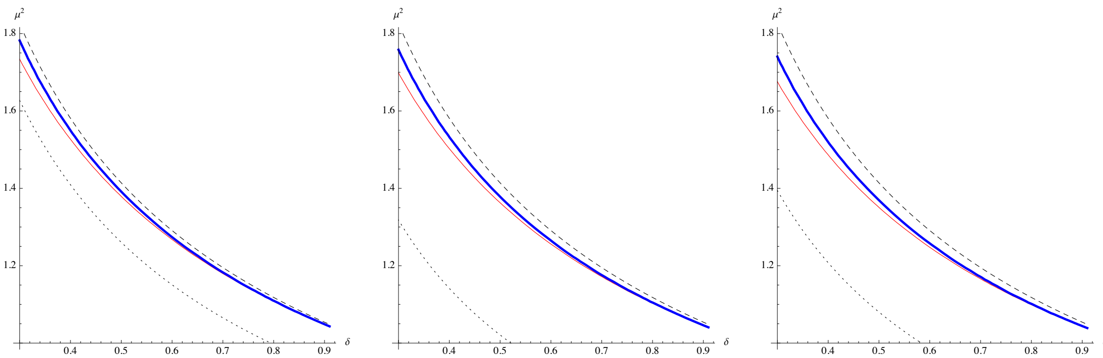
(a) $k = 0.1$; $F = 1.9$. (b) $k = 1/3$; $F = 0.8$. (c) $k = 0.4$; $F = 0.5$.

Figure 4.5: Sustainability of collusion with the Nash bargaining rule, when F is low.

Summarizing, for the pairs of parameters (μ^2, δ):

- below the dotted line, no collusion can be sustainable neither before nor after the entry;
- in the area in between the dotted line and the thin solid line, collusion would be sustainable after the entry, but it is not before the entry;
- in the area in between the thick solid line and the dashed line, full collusion is sustainable before and after the entry;
- in the area in between the two solid lines, collusion can be both sustainable after the entry as not. For some pairs, it may even exist the two types of equilibria.

In Figure 4.6, we present the graphics obtained if the entry cost is relatively high.

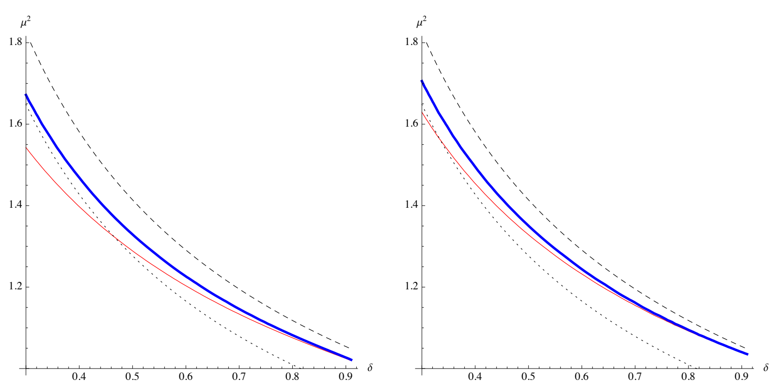


(a) $k = 0.1$; $F = 5.7$. (b) $k = 1/3$; $F = 2.5$. (c) $k = 0.4$; $F = 1.6$.

Figure 4.6: Sustainability of collusion with the Nash bargaining rule, when F is relatively high.

4.5.1.2 Proportional rule

As we saw in Proposition 4.2, if $k = 0.1 < 0.199$ and firms adopt the Proportional rule, (full) collusion is not sustainable after the entry. Moreover, if $k = \frac{1}{3}$, the three firms have equal shares of capital. As a result, the Nash Bargaining rule and the Proportional rule coincide. Thus, it only makes sense to consider the case $k = 0.4$.



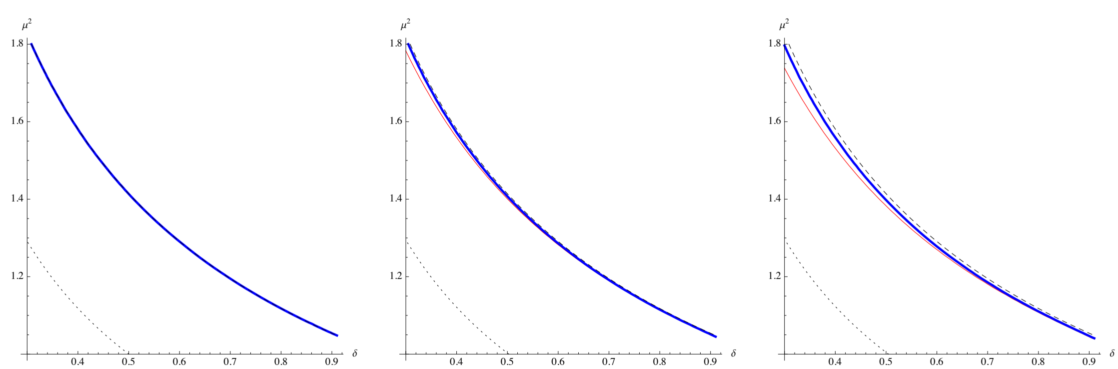
(a) $F = 0.5$. (b) $F = 1.6$.

Figure 4.7: Sustainability of collusion with the Proportional rule, when $k = 0.4$.

Comparing these figures with the correspondents if firms adopt the Nash bargaining rule, the major difference concerns the position of the dotted lines. Graphically, we confirm that the Proportional rule establishes a higher critical discount factor for collusion to be sustainable after the entry, i.e., collusion is less likely to occur.

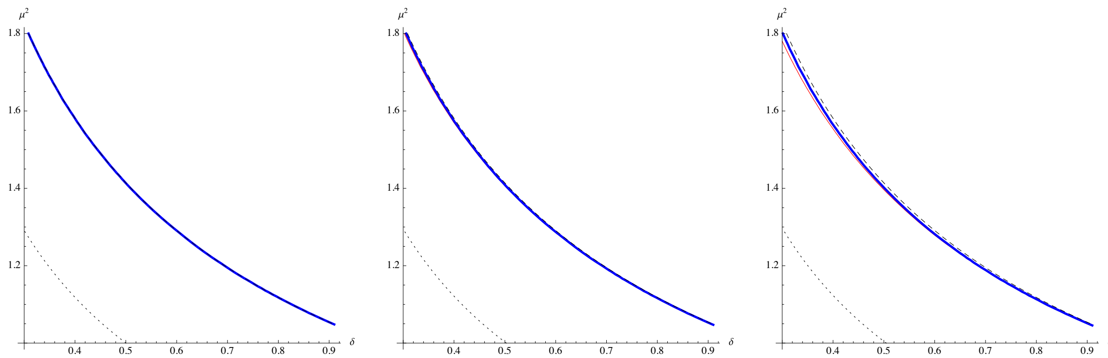
4.5.2 Partial collusion

Now, we consider that the incumbents do not include the entrant in their collusive agreement. As in the case of full collusion, in the graphics below the admissible region for parameters (μ^2, δ) is divided in sub-regions, according to the sustainability of collusion before and after the entry. More precisely, the thin solid line is the boundary of the region defined by condition (4.31), while the thick solid line is the boundary of the region defined by (4.32). Finally the dotted line corresponds to the minimum value of $\mu^2\delta$, for which collusion is sustainable after the entry. The analytical expression for this curve is given in (4.29). In Figure 4.8, the (fixed) entry cost is low while, in Figure 4.9, this cost is relatively high.



(a) $k = 0.1$; $F = 1.8$. (b) $k = 1/3$; $F = 0.8$. (c) $k = 0.4$; $F = 0.5$.

Figure 4.8: Sustainability of partial collusion, when F is low.



(a) $k = 0.1$; $F = 5.6$. (b) $k = 1/3$; $F = 2.5$. (c) $k = 0.4$; $F = 1.6$.

Figure 4.9: Sustainability of partial collusion, when F is relatively high.

When the incumbents are very small ($k = 0.1$), the two solid lines and the line $\mu^2\delta = 1$ are almost overlapped. Although there exists a considerable number of combinations (μ^2, δ) for which collusion would be sustainable after the entry, collusion is almost impossible before the entry. The same applies when the three firms own equal shares of capital. When the incumbents are considerably large, there only are few combinations (μ^2, δ) for which collusion is sustainable before and after the entry. Graphically, we conclude that by increasing the magnitude of F (i.e. by delaying the entry of firm 3), the two solid lines become closer to each other. That is, the region of indeterminacy decreases.

Comparing the graphics obtained for the scenario of full collusion with those of partial collusion, we conclude that partial collusion is more difficult to sustain before the entry than full collusion. Curiously, the incumbents are more willing to collude before the entry if they know that, after the entry, the firm 3 will join the cartel.

4.6 Conclusions

Inspired by the case of Nestlé-Perrier merger, we analyzed how asymmetry in the stock of capital owned by the incumbent firms and the entrant may affect the sustainability of collusion. Curiously, the market entry of a new firm changes the incentives for collusion even before the entry. On the one hand, the prospect of the entry reduces the expected cost of a deviation, without changing its short-run benefit. On the other hand, the incumbents are aware that by disrupting the collusive agreement they may delay the entry.

After the entry, the new firm can either be included in the collusive agreement or excluded from it. In the case of full collusion, the existence of asymmetry between firms makes the distribution of the monopoly profit nontrivial. We considered that firms could choose the Nash bargaining rule or the Proportional rule. We found that, regardless the allocation rule, the overall message is that asymmetries hurt collusion. Our results, however, suggest that the mapping between firms' size and their corresponding incentives to abide by the collusive agreement depend very much on the rule chosen by the collusion partners to allocate the collusive profit. In particular, if firms adopt the Proportional rule, collusion is not sustainable after the entry if the asymmetry across firms is noteworthy. This happens because the share of the monopoly profit allocated to the smaller firm(s) may not be enough to prevent the firm(s) from deviating. On the contrary, if firms divide the monopoly profit according to the Nash bargaining rule, collusion is always sustainable after the entry, regardless of how the industry capital is distributed among firms. This occurs because the Nash bargaining rule demands a large firm to share part of its profits with the small firm(s). The small firm has no longer incentives to deviate. With this rule, it would be the large firm that would have more incentives to deviate. However, if the firms are sufficiently patient, collusion is always sustainable, despite the side-payments.

Either when the collusive agreement is all-inclusive or it is not, collusion is easier to sustain after the entry of the new firm than before. This is even more evident when the incumbents decide not to include the entrant in their collusive agreement (partial collusion). Actually, if the incumbents are very small (when compared with the entrant), collusion before the entry is almost impossible.

In our model, the share of capital owned by each incumbent is exogenously given. As the entry cost is assumed not to depend on the capital acquired (and the industry capital is limited), the share of capital of the entrant is automatically determined. This is a strong assumption but fits very well to the French industry of bottled water, after the Nestlé-Perrier merger. It would, however, be interesting to introduce an initial stage, in which the incumbents choose their capacities, as in the models of Benoit and Krisnha (1987) and Knittel and Lepore (2010). We leave this to future work since it is out of the scope of this chapter.

Appendices

Appendix D

Profits

D.1 Two firms

D.1.1 Cournot competition

In the case of competition between the two incumbents, each firm chooses the quantity that maximizes its own profit (taking the quantity produced by the rival firm as given). In period t, the firm i,$i \in \{1, 2\}$, chooses q_{it} that maximizes:

$$\Pi_{it}(q_{it}, q_{jt}) = p_t q_{it} - C(q_{it}, k) = \left[\mu^t - (q_{it} + q_{jt})\right] q_{it} - \frac{q_{it}^2}{2k},$$

for $j \in \{1, 2\}, j \neq i$. The associated first-oder condition (henceforward, FOC) is:

$$-2q_{it} - \frac{q_{it}}{k} - q_{jt} + \mu^t = 0. \tag{D.1}$$

By analogy, the FOC of the maximization problem of firm j is:

$$-2q_{jt} - \frac{q_{jt}}{k} - q_{it} + \mu^t = 0. \tag{D.2}$$

Combining (D.1) and (D.2), we obtain the equilibrium output of each firm in period t:

$$q_{1t}^{c}(2) = q_{2t}^{c}(2) = \frac{k}{1+3k}\mu^{t}.$$

The correspondent individual profit is equal to:

$$\Pi_{1t}^{c}(2) = \Pi_{2t}^{c}(2) = \frac{k(1+2k)}{2(1+3k)^{2}}\mu^{2t}.$$

D.1.2 Collusion

If the two incumbents decide to form a cartel, their joint share of the industry capital is equal to $2k$. In period t, firms produce the quantity, $q_{1t}^{m}(2)$ and $q_{2t}^{m}(2)$, that maximize their joint profit:

$$\Pi_{t}^{m}(q_{1t}, q_{2t}) = \left[\mu^{t} - (q_{1t} + q_{2t})\right](q_{1t} + q_{2t}) - \left(\frac{{q_{1t}}^{2}}{2k} + \frac{{q_{2t}}^{2}}{2k}\right),$$

The correspondent first-order conditions are:

$$\begin{cases} \mu^{t} - 2q_{1t} - 2q_{2t} - \frac{q_{1t}}{k} = 0 \\ \mu^{t} - 2q_{1t} - 2q_{2t} - \frac{q_{2t}}{k} = 0 \end{cases}.$$

Solving this system, we obtain that:

$$q_{1t}^{m}(2) = q_{2t}^{m}(2) = \frac{k}{1+4k}\mu^{t}.$$

Substituting these quantities in the expression of joint profit, we obtain that:

$$\Pi_{t}^{m}(2) = \frac{k}{1+4k}\mu^{2t}.$$

As the two incumbents are symmetric, they divide the monopoly profit in equal parts:

$$\Pi_{1t}^{m}(2) = \Pi_{2t}^{m}(2) = \frac{k}{2(1+4k)}\mu^{2t}.$$

D.1.3 Deviation

If the incumbent i, $i \in \{1, 2\}$, decides to deviate in period t, it produces the quantity, $q_{it}^d(2)$, that maximizes the following function:

$$\Pi_{it}^d(q_{it}) = \left[\mu^t - \left(q_{it} + \frac{k}{1+4k}\mu^t\right)\right] q_{it} - \frac{q_{it}^2}{2k}.$$

The associated first-order condition is:

$$-2q_{it} - \frac{q_{it}}{k} + \mu^t - \frac{k}{1+4k}\mu^t = 0,$$

whose solution is

$$q_{it}^d(2) = \frac{k(1+3k)}{1+6k+8k^2}\mu^t.$$

Substituting this quantity in $\Pi_{it}^d(q_{it})$, we obtain the deviating profit of firm i:

$$\Pi_{it}^d(2) = \frac{k(1+3k)^2}{2(1+2k)(1+4k)^2}\mu^{2t}.$$

D.2 Three firms

D.2.1 Cournot competition

If the firm 3 is active in the market and the three firms are in competition, the firm 1 chooses $q_{1t}^c(3)$ that maximizes the following function:

$$\Pi_{1t}(q_{1t}, q_{2t}, q_{3t}) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right] q_{1t} - \frac{q_{1t}^2}{2k}.$$

The correspondent first-order condition is:

$$-2q_{1t} - \frac{q_{1t}}{k} - q_{2t} - q_{3t} + \mu^t = 0. \tag{D.3}$$

By analogy, the FOC correspondent to the profit maximization problem of firm 2 is:

$$-q_{1t} - 2q_{2t} - \frac{q_{2t}}{k} - q_{3t} + \mu^t = 0. \tag{D.4}$$

The profit function of the firm 3 is given by:

$$\Pi_{3t}(3) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right] q_{3t} - \frac{q_{3t}^2}{2(1-2k)}.$$

Thus, the associated FOC is:

$$-q_{1t} - q_{2t} - 2q_{3t} - \frac{q_{3t}}{1-2k} + \mu^t = 0. \tag{D.5}$$

Combining $(D.3)$, $(D.4)$ and $(D.5)$, we obtain the individual quantities in equilibrium:

$$q_{1t}^c(3) = q_{2t}^c(3) = \frac{2(1-k)k}{3+3k-8k^2}\mu^t \quad \text{and} \quad q_{3t}^c(3) = \frac{(1+k)(1-2k)}{3+3k-8k^2}\mu^t. \tag{D.6}$$

The correspondent (individual) profits are:

$$\Pi_{1t}^c(3) = \Pi_{2t}^c(3) = \frac{2k(1+2k)(1-k)^2}{(3+3k-8k^2)^2}\mu^{2t} \quad \text{and} \quad \Pi_{3t}^c(3) = \frac{(1+k)^2\left(3-10k+8k^2\right)}{2\left(3+3k-8k^2\right)^2}\mu^{2t}.$$

D.2.2 Collusion

If the three firms are in collusion in period t, the firms produce the quantities $(q_{1t}^m, q_{2t}^m, q_{3t}^m)$ that maximize their joint profit:

$$\Pi_t^m(q_{1t}, q_{2t}, q_{3t}) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right](q_{1t} + q_{2t} + q_{3t}) - \left(\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k} + \frac{{q_{3t}}^2}{2(1-2k)}\right),$$

The correspondent first-order conditions are:

$$\begin{cases} -2q_{1t} - \frac{q_{1t}}{k} - 2q_{2t} - 2q_{3t} + \mu^t = 0 \\ -2q_{1t} - 2q_{2t} - \frac{q_{2t}}{k} - 2q_{3t} + \mu^t = 0 \\ -2q_{1t} - 2q_{2t} - 2q_{3t} - \frac{q_{3t}}{1-2k} + \mu^t = 0 \end{cases}.$$

Solving this system, we obtain that:

$$q_{1t}^{m}(3) = q_{2t}^{m}(3) = \frac{k}{3}\mu^{t} \quad \text{and} \quad q_{3t}^{m}(3) = \frac{1-2k}{3}\mu^{t}.$$

Substituting them in the expression for Π_t^m, we obtain that:

$$\Pi_t^m(3) = \frac{\mu^{2t}}{6}.$$

D.2.3 Deviation

Consider that the three firms are colluding and the firm i, $i \in \{1,2,3\}$, decides to deviate in period t. In this case, it produces the quantity that maximizes the following function:

$$\Pi_{it}^{d}\left(q_{it}\right) = \left[\mu^{t} - \left(q_{it} + \frac{k_j}{3}\mu^{t} + \frac{1-k_i-k_j}{3}\mu^{t}\right)\right] q_{it} - \frac{q_{it}^{2}}{2k_i},$$

for $j \in \{1,2,3\}$ and $j \neq i$. The associated first-order condition is:

$$-2q_{it} + \mu^{t} - \frac{q_{it}}{k_i} - \frac{1-k_i-k_j}{3}\mu^{t} - \frac{k_j}{3}\mu^{t} = 0.$$

Solving this equation in order to q_{it}, we obtain:

$$q_{it}^{d}(3) = \frac{k_i\left(2+k_i\right)}{3\left(1+2k_i\right)}\mu^{t}.$$

The correspondent profit is:

$$\Pi_{it}^{d}(3) = \frac{k_i\left(2+k_i\right)^{2}}{18(1+2k_i)}\mu^{2t}.$$

D.3 Partial collusion

D.3.1 Collusion

Consider that the incumbents form a cartel, but they do not include the entrant in their agreement. Moreover, assume that the entrant becomes a follower, while the cartel behaves as a Stackelberg leader. Therefore, the entrant chooses the quantity, $q_{3t}^{pm}(3)$, that maximizes its individual profit:

$$\Pi_{3t}^{pm}(q_{1t},q_{2t},q_{3t}) = [\mu^t - (q_{1t}+q_{2t}+q_{3t})]q_{3t} - \frac{q_{3t}^2}{2(1-2k)}.$$

Thus, its best-response function to the incumbents' output is given by:

$$q_{3t}(q_{1t},q_{2t}) = \frac{1-2k}{3-4k}\left(\mu^t - q_{1t} - q_{2t}\right). \tag{D.7}$$

As the cartel acts as a Stackelberg leader, it chooses the quantities $q_{1t}^{pm}(3)$ and $q_{2t}^{pm}(3)$ that maximize the incumbents' joint profit, given the best-response function of the firm 3. That is, the cartel maximizes the following function:

$$\begin{aligned}\Pi_t^{pm}(q_{1t},q_{2t}) &= \left[\mu^t - q_{1t} - q_{2t} - \frac{1-2k}{3-4k}\left(\mu^t - q_{1t} - q_{2t}\right)\right](q_{1t}+q_{2t}) - \left(\frac{q_{1t}^2}{2k}+\frac{q_{2t}^2}{2k}\right)\\ &= \frac{2(1-k)}{3-4k}\left(\mu^t - q_{1t} - q_{2t}\right)(q_{1t}+q_{2t}) - \left(\frac{q_{1t}^2}{2k}+\frac{q_{2t}^2}{2k}\right).\end{aligned}$$

The correspondent first-order conditions are:

$$\begin{cases} -q_{1t} - \frac{q_{1t}}{k} - q_{2t} + \left(-1+\frac{1-2k}{3-4k}\right)(q_{1t}+q_{2t}) + \mu^t - \frac{1-2k}{3-4k}\left(-q_{1t}-q_{2t}+\mu^t\right) = 0 \\ -q_{1t} - q_{2t} - \frac{q_{2t}}{k} + \left(-1+\frac{1-2k}{3-4k}\right)(q_{1t}+q_{2t}) + \mu^t - \frac{1-2k}{3-4k}\left(-q_{1t}-q_{2t}+\mu^t\right) = 0 \end{cases}.$$

Solving the system, we obtain that:

$$q_{1t}^{pm}(3) = q_{2t}^{pm}(3) = \frac{2k(1-k)}{3+4k-8k^2}\mu^t.$$

Substituting these quantities in the best-response function of the entrant, we obtain:

$$q_{3t}^{pm}(3) = \frac{(1-2k)\left(3-4k^2\right)}{(3-4k)\left(3+4k-8k^2\right)}\mu^t$$

As a result, the profit of the incumbent i, for $i \in \{1,2\}$, in period t is given by:

$$\Pi_{it}^{pm}(3) = \frac{\Pi_t^{pm}(q_{1t}^{pm},q_{2t}^{pm})}{2} = \frac{2k(1-k)^2}{9-40k^2+32k^3}\mu^{2t},$$

while the profit of the entrant is:

$$\Pi_{3t}^{pm}(3) = \frac{(1-2k)\left(3-4k^2\right)^2}{2(3-4k)\left(3+4k-8k^2\right)^2}\mu^{2t}.$$

D.3.2 Deviation

If the incumbent i, for $i \in \{1,2\}$, deviates in period t, it produces the quantity, $q_{it}^{pd}(3)$, that maximizes the following function:

$$\Pi_{it}^{pd}(q_{it}) = \left[\mu^t - \left(q_{it} + \frac{2k(1-k)}{3+4k-8k^2}\mu^t + \frac{(1-2k)\left(3-4k^2\right)}{(3-4k)\left(3+4k-8k^2\right)}\mu^t\right)\right] q_{it} - \frac{q_{it}^2}{2k}.$$

The first-order condition of this maximization problem is:

$$-2q_{it} - \frac{q_{it}}{k} + \mu^t - \frac{2(1-k)k\mu^t}{3+4k-8k^2} - \frac{(1-2k)\left(3-4k^2\right)\mu^t}{(3-4k)\left(3+4k-8k^2\right)} = 0.$$

Solving the equation in order to q_{it}, we obtain that:

$$q_{it}^{pd}(3) = \frac{2k(1-k)\left(-3-3k+8k^2\right)}{(1+2k)(3-4k)\left(-3-4k+8k^2\right)}\mu^t.$$

The correspondent profit is:

$$\Pi_{it}^{pd}(3) = \frac{2k(1-k)^2\left(3+3k-8k^2\right)^2}{(1+2k)(3-4k)^2\left(3+4k-8k^2\right)^2}\mu^{2t}.$$

Appendix E

Prohibitive entry costs

Consider that the (fixed) entry cost, F, is prohibitively high. Despite the market is growing, firm 3 never considers it profitable to enter in the market.

Perfect collusion is sustained by the two incumbent firms, in period t, if the following incentive compatibility constraint is satisfied:

$$\sum_{s=t}^{\infty} \Pi_{is}^{m}(2)\delta^{s-t} \quad \geq \quad \Pi_{it}^{d}(2) + \sum_{s=t+1}^{\infty} \Pi_{is}^{c}(2)\delta^{s-t}$$

Substituting the expressions for profits, we come to the following inequality:

$$(\beta_2 - \alpha_2)\,\delta^{-t} \sum_{s=t+1}^{\infty} (\mu^2\delta)^s \geq (\gamma_2 - \beta_2)\,\mu^{2t} \Leftrightarrow (\mu^2\delta)^{-t}\frac{(\mu^2\delta)^{t+1}}{1-\mu^2\delta} \geq \frac{\gamma_2 - \beta_2}{\beta_2 - \alpha_2}$$

Therefore, collusion is sustainable if the adjusted discount factor, $\mu^2\delta$, is sufficiently high:

$$\mu^2\delta \geq \frac{\gamma_2 - \beta_2}{\gamma_2 - \alpha_2} = \frac{(1+3k)^2\left(1+5k+2k^2-9k^3\right)}{1+11k+40k^2+42k^3-44k^4-81k^5} \equiv \mu^2\hat{\delta}. \tag{E.1}$$

The critical (adjusted) discount factor, $\mu^2\hat{\delta}$, is smaller than 1 (since $\beta_2 \geq \alpha_2$).

Appendix F

Proof of Propositions

Proof of Proposition 4.2

Perfect collusion is not sustainable if the (adjusted) discount factor of some firm is greater than one. We analyze, in separate, the the critical value of the incumbents and of the entrant:

(i) The critical (adjusted) discount factor of the firm 1 (analogous to firm 2) is greater than one if:

$$\mu^2\hat{\delta}_1(k) > 1 \Leftrightarrow \frac{\gamma_{31} - \frac{k}{6}}{\gamma_{31} - \alpha_{31}} > 1 \Leftrightarrow \frac{k\left(-3 + 18k - 3k^2 - 72k^3 + 64k^4\right)}{6\left(3 + 3k - 8k^2\right)^2} < 0.$$

For $k \in \left(0, \frac{1}{2}\right)$, the last inequality is equivalent to:

$$f(k) = -3 + 18k - 3k^2 - 72k^3 + 64k^4 < 0.$$

The first-order derivative of f is given by:

$$f'(k) = 18 - 6k - 216k^2 + 256k^3,$$

whose roots are:

$$k_1 = \frac{3-\sqrt{393}}{64} < 0 \ \vee \ k_2 = \frac{3+\sqrt{393}}{64} \in \left(0, \frac{1}{2}\right) \ \vee \ k_3 = \frac{3}{4} > \frac{1}{2}.$$

Thus, f' is positive for $k \in (0, k_2)$ and it is negative for $k \in (k_2, 1/2)$. Therefore, f is (strictly) increasing in $(0, k_2)$ and it (strictly) decreasing in $(k_2, 1/2)$. Moreover,

$$f(0) < 0 \quad ; \quad f(k_2) > 0 \quad \text{and} \quad f\left(\frac{1}{2}\right) > 0.$$

As f is continuous, by the intermediate value theorem, there exists $k^* \in (0, k_2)$ such that $f(k^*) = 0$. As f is increasing in this domain, k^* is the unique root of f in $(0, k_2)$. Finally, as $f(k_2) > f(1/2) > 0$ and f is (strictly) decreasing in $(k_2, 1/2)$, we conclude f has no roots in this interval. As a result,

$$f(k) < 0 \ \wedge \ k \in \left(0, \frac{1}{2}\right) \Leftrightarrow k \in (0, k^*).$$

Using, for example, the bisection method we can find that $k^* \approx 0.199$.

(ii) Consider now the case of the entrant:

$$\mu^2 \hat{\delta}_3(k) > 1 \Leftrightarrow \frac{\gamma_{33} - \frac{1-2k}{6}}{\gamma_{33} - \alpha_{33}} > 1 \Leftrightarrow \frac{2k(1-2k)\left(3 - 6k - 9k^2 + 16k^3\right)}{3\left(3 + 3k - 8k^2\right)^2} < 0.$$

For $k \in \left(0, \frac{1}{2}\right)$, the last inequality is equivalent to:

$$g(k) = 3 - 6k - 9k^2 + 16k^3 < 0.$$

As g is continuous, $g(0) > 0$ and $g(1/2) < 0$, there is $k^{**} \in (0, 1/2)$, such that $g(k^{**}) = 0$. Let us now prove that k^{**} is unique. The first-order derivative of g is:

$$g'(k) = -6 - 18k + 48k^2,$$

whose roots are:

$$k_4 = \frac{3-\sqrt{41}}{16} < 0 \quad \vee \quad k_5 = \frac{3+\sqrt{41}}{16} > \frac{1}{2}.$$

Therefore, the function g' is negative for $k \in \left(0, \frac{1}{2}\right)$, which implies that g is (strictly) decreasing in this domain. As a result, k^{**} is unique and

$$g(k) < 0 \ \wedge \ k \in \left(0, \frac{1}{2}\right) \Leftrightarrow k \in \left(k^{**}, \frac{1}{2}\right).$$

Once again, using the bisection method, we find that $k^{**} \approx 0.436$. $\square$

Proof of Lemma 4.1

Dividing the inequality (4.20) by δ^t, we obtain that:

$$\beta_2 \frac{(\mu^2\delta)^t - (\mu^2\delta)^{\tilde{t}_1}}{\delta^t(1-\mu^2\delta)} + \beta_{3i} \frac{\mu^{2\tilde{t}_1}\delta^{\tilde{t}_1 - t}}{1-\mu^2\delta} \geq \gamma_2\mu^{2t} + \alpha_2 \frac{(\mu^2\delta)^{t+1} - (\mu^2\delta)^{\tilde{t}_2+t}}{\delta^t(1-\mu^2\delta)} + \alpha_{3i} \frac{\mu^{2(\tilde{t}_2+t)}\delta^{\tilde{t}_2}}{1-\mu^2\delta} \quad \text{(F.1)}$$

Evaluating the last inequality at $t = \tilde{t}_1 - 1$, the ICC can be written as follows:

$$(\gamma_2 - \beta_2)\mu^{2(\tilde{t}_1 - 1)} \leq \beta_{3i} \frac{\mu^{2\tilde{t}_1}\delta}{1-\mu^2\delta} - \alpha_2 \frac{(\mu^2\delta)^{\tilde{t}_1} - (\mu^2\delta)^{\tilde{t}_1 - 1 + \tilde{t}_2}}{\delta^{\tilde{t}_1 - 1}(1-\mu^2\delta)} - \alpha_{3i} \frac{\mu^{2(\tilde{t}_1 - 1 + \tilde{t}_2)}\delta^{\tilde{t}_2}}{1-\mu^2\delta}$$

Multiplying both sides by $\delta^{\tilde{t}_1}(1-\mu^2\delta)$ and after some rearranging, we obtain:

$$(\mu^2\delta)^{\tilde{t}_1 - 1}\left[\gamma_2 - \beta_2 + (\beta_2 - \gamma_2 - \beta_{3i} + \alpha_2)\mu^2\delta + (\alpha_{3i} - \alpha_2)(\mu^2\delta)^{\tilde{t}_2}\right] \leq 0.$$

As F is assumed to be sufficiently high to guarantee that $\tilde{t}_1 > 1$ and $\mu^2\delta < 1$, a sufficient condition for the last ICC to be satisfied is:

$$A(\mu, \delta, k) \equiv \gamma_2 - \beta_2 + (\beta_2 - \gamma_2 - \beta_{3i} + \alpha_2)\mu^2\delta + (\alpha_{3i} - \alpha_2)(\mu^2\delta)^{\tilde{t}_2} \leq 0. \quad \text{(F.2)}$$

Consider now a period $t = \tilde{t}_1 - \tau$ for $1 \leq \tau \leq \tilde{t}_1$. Substituting in $F.1$ and rearranging the terms of the inequality, it becomes:

$$\gamma_2\mu^{2(\tilde{t}_1 - \tau)} \quad - \quad \beta_2 \frac{(\mu^2\delta)^{\tilde{t}_1 - \tau} - (\mu^2\delta)^{\tilde{t}_1}}{\delta^{\tilde{t}_1 - \tau}(1-\mu^2\delta)} \leq$$

$$\beta_{3i} \frac{\mu^{2\tilde{t}_1}\delta^{\tau}}{1-\mu^2\delta} - \alpha_2 \frac{(\mu^2\delta)^{\tilde{t}_1 - \tau + 1} - (\mu^2\delta)^{\tilde{t}_1 - \tau + \tilde{t}_2}}{\delta^{\tilde{t}_1 - \tau}(1-\mu^2\delta)} - \alpha_{3i} \frac{\mu^{2(\tilde{t}_1 - \tau + \tilde{t}_2)}\delta^{\tilde{t}_2}}{1-\mu^2\delta}.$$

Multiplying both sides of the inequality by $\delta^{\tilde{t}_1-\tau}(1-\mu^2\delta)$, we obtain:

$$\begin{aligned}\gamma_2(\mu^2\delta)^{\tilde{t}_1-\tau}(1-\mu^2\delta) \quad - \quad & \beta_2(\mu^2\delta)^{\tilde{t}_1-\tau}+\beta_2(\mu^2\delta)^{\tilde{t}_1} \leq \\ & \beta_{3i}(\mu^2\delta)^{\tilde{t}_1}-\alpha_2(\mu^2\delta)^{\tilde{t}_1-\tau+1}+\alpha_2(\mu^2\delta)^{\tilde{t}_1-\tau+\tilde{t}_2}-\alpha_{3i}(\mu^2\delta)^{\tilde{t}_1-\tau+\tilde{t}_2}\end{aligned}$$

$$\Leftrightarrow (\mu^2\delta)^{\tilde{t}_1-\tau}\left[\gamma_2-\beta_2+(\alpha_2-\gamma_2)\mu^2\delta+(\beta_2-\beta_{3i})(\mu^2\delta)^{\tau}+(\alpha_{3i}-\alpha_2)\mu^{\tilde{t}_2}\right]\leq 0$$

Notice that we can write the last inequality as follows:

$$A(\mu,\delta,k)-(\beta_2-\beta_{3i})\mu^2\delta\left[1-(\mu^2\delta)^{\tau-1}\right]\leq 0. \tag{F.3}$$

Therefore, if condition $(F.2)$ holds and $\beta_2-\beta_{3i}>0$, the condition $(F.3)$ is, *a fortiori*, verified (recall that $\mu^2\delta<1$ and $\tau\geq 1$, implying that $1-(\mu^2\delta)^{\tau-1}>1$). Let us show that $\beta_2-\beta_{3i}>0$. The expressions for β_2 and for β_{3i} are given in (4.5) and (4.6), respectively. Therefore,

$$\beta_2-\beta_{3i}=\frac{k\left(39+6k-201k^2-88k^3+320k^4\right)}{18(1+4k)\left(3+3k-8k^2\right)^2}.$$

As $k>0$, the inequality above is equivalent to:

$$p(k)\equiv 39+6k-201k^2-88k^3+320k^4>0.$$

The first-order derivative of p is given by:

$$p'(k)=6-402k-264k^2+1280k^3.$$

Moreover, as p' is continuous and:

$$\left.\begin{array}{l}p'(-0.5)<0\\ p'(-0.4)>0\end{array}\right\}\quad\Rightarrow\quad \exists k_1\in(-0.5,-0.4):p'(k_1)=0$$

$$\left.\begin{array}{l}p'(0)<0\\ p'(0.1)>0\end{array}\right\}\quad\Rightarrow\quad \exists k_2\in(0,0.1):p'(k_2)=0$$

$$\left.\begin{array}{l} p'(0.6) < 0 \\ p'(0.7) > 0 \end{array}\right\} \quad \Rightarrow \quad \exists k_3 \in (0.6, 0.7) : p'(k_3) = 0.$$

As p' is a polynomial of third degree, its only zeros are k_1, k_2 and k_3. We conclude, therefore, that p is increasing in $(0, k_2)$ and decreasing in $(k_2, 0.5)$. Thus, the minimum of p in the interval $(0, 0.5)$ must be achieved at one limit of this interval. As $p(0) > p(0.5) > 0$, we conclude that $p(k) > 0$, $\forall k \in (0, 0.5)$. This ends the proof. □

Proof of Proposition 4.3

Multiplying both sides of the inequality (4.21) by $(1 - \mu^2\delta)(\mu^2\delta)^{1-\tilde{t}_1}$ and rearranging the terms, this ICC can be rewritten as follows:

$$(\mu^2\delta)^{\tilde{t}_2}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0. \tag{F.4}$$

Let us now look at the incentive compatibility constraint (4.17). It is straightforward to see that the ICC is more easily satisfied the smaller the value of $\tilde{t}_2$. When $\tilde{t}_2$ decreases, the right hand side of the inequality decreases, while the left hand side does not change. Recall, however, that $\tilde{t}_2$ was defined as being an integer. In order to avoid integer problems and because we are looking for a sufficient condition for collusion not be sustainable, we can focus on the extreme value of $\tilde{t}_2$ that most facilitates collusion, that is, $\tilde{t}_2 = t_2 - 1$.

Thus, for any δ that verifies the following inequality:

$$(\mu^2\delta)^{t_2-1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) < 0,$$

where $i \in \{1, 2\}$, collusion is not sustainable before the entry. □

Proof of Proposition 4.4

To avoid the integer problems of the inequality $(F.4)$, we may substitute $\tilde{t}_2$ for an integer. Looking at the incentive compatibility constraint (4.17), we conclude that a higher value for $\tilde{t}_2$ hinders collusion, since it shortens the punishment phase. As we are looking for a sufficient condition for collusion to be sustainable, we can focus on the value of $\tilde{t}_2$ that most hurt collusion. Thus, substituting $\tilde{t}_2$ for $t_2 + 1$ in the inequality (4.21), we come to the following inequality:

$$(\mu^2\delta)^{t_2+1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0.$$

□

Proof of Proposition 4.5

Let us compare the profit of each incumbent with the profit of the entrant when the incumbents are colluding and the entrant is competing *a la* Cournot:

$$\Pi_{it}^{pm}(3) \geq \Pi_{3t}^{pm}(3) \Leftrightarrow -\frac{9 - 30k - 16k^2 + 100k^3 - 64k^4}{2(3-4k)\left(3+4k-8k^2\right)^2} \geq 0$$

Restricting to the domain $(0, 0.5)$, the last inequality is equivalent to:

$$p(k) \equiv 9 - 30k - 16k^2 + 100k^3 - 64k^4 \leq 0.$$

The first-order derivative of p is given by:

$$p'(k) = -30 - 32k + 300k^2 - 256k^3.$$

As p' is continuous and:

$$\left.\begin{array}{r} p'(-0.3) > 0 \\ p'(-0.2) < 0 \end{array}\right\} \quad \Rightarrow \quad \exists k_1 \in (-0.3, -0.2) : p'(k_1) = 0$$

$$\left.\begin{array}{r} p'(0.5) < 0 \\ p'(0.6) > 0 \end{array}\right\} \quad \Rightarrow \quad \exists k_2 \in (0.5, 0.6) : p'(k_2) = 0$$

$$\left.\begin{array}{r} p'(0.8) > 0 \\ p'(0.9) < 0 \end{array}\right\} \quad \Rightarrow \quad \exists k_3 \in (0.8, 0.9) : p'(k_3) = 0.$$

As p' is a polynomial of third degree, its only zeros are k_1, k_2 and k_3. Thus, p' has no zeros in the interval $(0, 0.5)$, which means that p is (strictly) decreasing in this interval. Therefore, the polynomial p can have, at most, one zero in the interval $(0, 0.5)$. Moreover, as:

$$p(0.3415) > 0 \quad \text{and} \quad p(0.3416) < 0$$

we conclude that there exists $k^* \in (0.3415, 0.3416)$ such that $p(k^*) = 0$. Thus, if $k \in (k^*, 0.5)$, then $p(k) \leq 0$. $\square$

Chapter 5

Conclusions

This last chapter presents the main results of the thesis as well as some ideas that would be interesting to explore in future research.

The three essays that constitute this thesis study how the existence of (a certain kind of) asymmetry between oligopolists affects the equilibrium outcomes. In spite of this common goal, the theoretical approach adopted in each chapter is quite distinct. More specifically, the basic model is different from chapter to chapter. Therefore, it would not be reasonable to draw general conclusions of the thesis, without contextualizing them in the corresponding chapter.

In the second chapter, we considered a market with two firms that are asymmetrically informed about consumers' transportation costs. In the case of perfect information, the two firms would obtain equal profits, since they would set the same price and they would have equal market shares. With asymmetric information, the prices of the two firms never coincide. In particular, when the transportation costs are low, the informed firm chooses a lower price than the uninformed firm. However, its demand is higher. We concluded that the demand-effect more than compensates the price-effect and, therefore, the informed firm obtains a higher profit than the rival. When transportation costs are high, the informed firm chooses a higher price than the competitor and has, therefore, a lower demand. In this case, the dominant effect (price or demand) is not so immediate. More precisely, it depends on the probability of consumers to support high transportation costs. Namely, if this probability is very high, the demand-effect dominates. That is, the uninformed firm, even practicing a lower price, obtains a higher profit than the competitor.

In this setting, the existence of asymmetric information between firms hinders the social welfare. However, if transportation costs are low, consumers benefit from the existence of asymmetry between firms. Conversely, if transportation costs are high, consumers would be better if both firms were perfectly informed. A curious finding is that, in expected value, consumers benefit from the existence of asymmetry between firms. Thus, the firms are those that support (in expected value) the inefficiency resulting from imperfect information.

Given these conclusions, we examined the possibility of firms to develop a mechanism of information sharing. In particular, we studied the possibility of the uninformed firm to propose a monetary transfer (to the informed firm) in exchange for its information. When the transportation costs are high, it could be advantageous for both firms to remain asymmetrically informed. However, once questioned about the information, the informed firm has no way to omit the information. Not to reveal information would be, by itself, information. We could have considered that, if perfect information was harmful for both firms, they could set the prices of the asymmetric information equilibrium (i.e. the uninformed firm would choose its price as it ignored the information). This would be a kind of collusion between firms. However, if the firms were willing to combine prices, they could do better than this. They could, for example, choose the prices that maximize their joint profit. Thus, we assumed that, once informed about the magnitude of the transportation costs, the uninformed firm chooses the (equilibrium) price of perfect information.

In expected value, the informed firm gains by disclosing its private information. If the information is verifiable and the uninformed firm holds all the bargaining power (because it is proposing the contract), it may extract all the rival' s surplus of the disclosure game. More precisely, if the contract is proposed ex-ante (i.e. before the informed firm observing the magnitude of the transportation costs) and transportation costs turn out to be low, the informed firm would be better of if it had not committed to reveal its (future) information. If the contract is proposed ex-post, the informed firm only accepts it if the contract compensates it when it reveals that the transportation costs are low. On the other hand, the informed firm does not mind of sharing part of its profits with the rival, when it announces that transportation costs are high.

If the information is unverifiable, the informed firm always has the incentive to announce that the transportation costs are high. The informed firm is tempted to lie (when the transportation costs are low) to induce the competitor to set a higher price. To overcome this, an incentive compatible contract should monetarily compensate the informed firm when it announced that transportation costs were low. However, in this case, the firm would start to prefer to always announce that transportation costs were low (to receive the monetary transfer). We concluded, therefore, that there is no incentive compatible contract.

In future research, it would be interesting to make the firms' locations endogenous. We could consider, for example, that the firms choose their locations in a firts stage (prior to the price setting). In this case, the uninformed firm is uncertain about the consumers' transportation costs when making its location choice. It locates, therefore, at the point that maximizes its expected profit. In a second stage, the firms would choose prices. Notice that the uninformed firm may try to infer the magnitude of the transportation costs by observing the location of the rival. Aware of this, the informed firm may decide to always locate at the same point, regardless of the magnitude of the transportation costs (pooling equilibrium). By doing so, the informed firm does not reveal any information through its location. The informed firm may, however, use its location to signal the magnitude of the transportation costs. In this case, it has to choose one location when the transportation costs are high and another when the transportation costs are low (separating equilibrium). As a result, it would be necessary to analyze whether the informed firm is not always interested in signalling that the transportation costs are high (as it happens in our model).

In the third chapter of the thesis, we considered a model in which consumers simultaneously choose where to buy several goods. We assumed that all the products are available in the two shopping centres of the city. The possible asymmetry between shopping centers concerns their mode of retail. More precisely, we considered that a shopping centre can be a multi-product firm (in our terminology, a department store) or an aggregate of independent firms (a shopping mall). We studied three possible competitive scenarios: (i) competition between a department store and a shopping mall; (ii) competition between two department stores; (iii) and competition between two shopping malls.

The results are richer in the case of competition between a department store and a shopping mall. In this competitive scenario, if there are up to four goods in the market, no consumer visits both extremes of the city (i.e. the "one-stop shopping" condition holds). If there are between seven and eleven goods, there are consumers travelling to both shopping centers and buying each product where it is cheaper (i.e. there is "two-stop shopping"). This occurs because one product is more expensive at the department store (than at the shopping mall), but all the remainder are cheaper there. In equilibrium, the bundle of goods is always cheaper at the department store. However, the demand-effect more than compensates the price-effect and the department store profits more than the stores at the mall taken together.

If there are five, six or more than eleven goods in the market, there is no equilibrium in pure strategies. Given these conclusions, it would be interesting to allow for mixed strategies in the price competition stage in a future work.

When the mode of retail is the same in the two extremes of the city, there always exists a unique equilibrium with "one-stop shopping", regardless of the number of goods. As the price of bundle is the same in both extremes, each consumer makes all his purchases at the nearest shopping center. The shopping centers have, therefore, equal profits. However, the bundle is considerably cheaper when there are two department stores than two shopping malls. The department stores compete more aggressively because they take into account that the price of one good affects the demand of all goods they sell.

The scenario that provides a higher surplus to consumers is that of competition between two department stores. However, if the mode of retail is made endogenous (i.e. if there is a stage prior to the price setting, in which the shopping centers may choose whether to be organized as department stores or as shopping malls), we should expect to have shopping malls at both extremes of the city.

In the second chapter of the thesis, we concluded that asymmetry between firms is (in expected value) advantegeous for consumers; while the firms prefer (in expected value) to be symmetric. In the third chapter, however, we found that symmetry was desired both by consumers both by firms (although they preferred different symmetric scenarios). This warns us that, to decide whether symmetry is desirable or not, it is not enough to say what side of the market (demand or supply) we are considering.

In the fourth chapter of the thesis, we considered a model where the demand is growing over the time. We assumed that there are two incumbents and one potential entrant in the market. The firms can be asymmetric with respect to the stock of capital they own. More precisely, we considered that the two incumbents are identical and only the entrant may differ from the rivals. The main objective of this chapter was to analyze how the asymmetry among firms affects the sustainability of collusion (before and after the entry).

If the firms are identical, they divide up the monopoly profit in equal parts, along the collusive path. However, the allocation of the monopoly profit is not so immediate if the firms are asymmetric. We considered two possible rules for firms to divide up the monopoly profit: the Proportional rule and the Nash bargaining rule. Obviously, the share of the monopoly profit received by each firm depends on the chosen rule. However, what is interesting is that the choice of the rule also affects the sustainability of collusion. We concluded, in particular, that if firms adopt the Proportional rule, collusion may never be sustainable. With this rule, each firm receives a share of the monopoly profit that is equal to its share in the industry capital. Therefore, the small firms receive low collusive profits. For these firms, the difference between the collusive profit and the Cournot profit is not very high. Consequently, their incentives to disrupt the collusive agreement are so strong that can lead to the cartel breakdown. On the contrary, if firms adopt the Nash bargaining rule and they are patient enough, collusion may always be sustainable in equilibrium. With the Nash bargaining rule, the small firms receive a side-payment (from the large firms), in addition to the profit resulting from the quantity they produce. Thus, they no longer have incentives to deviate. In this case, we could fear that the large firms preferred to deviate. We found, however, that if the discount factor is sufficiently high, collusion may always be sustainable after the entry.

Another interesting result of this chapter concerns the feasibility of partial collusion after the entry, which was not possible in the original version of the model (Vasconcelos, 2008). Notice that, even if we had restricted the analysis to the symmetric case, there would exist a significant difference between the model of Vasconcelos (2008) and ours. In our model, if firms were symmetric, they would have the same cost function, but the marginal cost function would depend on the quantity produced. In the model of Vasconcelos (2008), firms have constant marginal costs of production. Thus, the expressions for profits are different in the two models. It would be interesting to analyze the model of Vasconcelos (2008) considering our cost function. Only then, we could conclude whether the difference in results (with regard to partial collusion) derives from the asymmetry between firms or from the difference in the cost functions.

In future work, it would be interesting to analyze what would happen if the demand, rather than to be growing, was shrinking (converging to zero, eventually) over the time. We could start by considering that there were three asymmetric firms in the market. To analyze the sustainability of collusion, it would be necessary to see whether the less efficient firm would want to get out of market at some moment of time. It would also be interesting to study the incentives of the most efficient firms (i.e. those with a larger stock of capital) to disrupt the collusive agreement, in order to force the exit of a (less efficient) rival.

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Sempre chegamos ao sítio aonde nos esperam.

José Saramago