

Mechanical Response of Advanced Composites under High Strain Rates

Hannes Körber

Faculdade de Engenharia da Universidade do Porto

Departamento de Engenharia Mecânica



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Abstract

This work presents an investigation of strain rate effects on the elastic, plastic and strength properties of unidirectional carbon-epoxy composites.

The split-Hopkinson pressure bar was used for the high strain rate tests and optimised for composites by means of systematic pulse shaping and direct strain measurements on the specimen, using foil strain gauges or the contactless optical method of digital image correlation. All high strain rate tests were performed under dynamic stress equilibrium and at near constant strain rates. As a result it was possible to obtain both reliable elastic and strength properties from the measured dynamic stress-strain response.

For the carbon-epoxy material system IM7-8552, quasi-static and high strain rate experiments were performed in the longitudinal and transverse compressive direction. A test fixture was developed for the longitudinal compression tests, which allows an interference free strain wave propagation when used for the split-Hopkinson pressure experiment. The strain rate effect on the material response under combined transverse compression and in-plane shear loading was investigated by means of off-axis compression tests. From the latter tests, the quasi-static and dynamic in-plane shear response was determined and the yield strength and failure envelopes for combined transverse compression and in-plane shear loading were established and compared with a state-of-the-art failure criterion. High speed photography was used to study the fracture mode under dynamic loading, and to determine the fracture plane angle for pure transverse compression and for various ratios of transverse compression and in-plane shear. At the strain rates studied in this work, no strain rate effect was observed for the longitudinal compressive modulus, whereas a moderate and consistent increase, with increasing loading rate, was found for the transverse compressive, in-plane shear and off-axis compressive moduli. More significant and again consistent strain rate effects were observed for the longitudinal compressive strength, and for the transverse compressive, in-plane shear and off-axis compressive yield and failure strengths.

Quasi-static off-axis tension tests were performed to investigate the constitutive response and to determine the failure envelope for combined transverse tension and in-plane shear loading. As for the compression tests, the experimental failure envelope was compared with advanced failure criteria.

A simple plasticity model for unidirectional polymer composites is introduced and it is shown how the plasticity model parameters can be determined from the off-axis compression and tension tests performed in this study. The plasticity model and a state-of-the-art failure criteria were implemented into an ABAQUS VUMAT user-material subroutine. By comparing the predicted and experimental stress-strain curves, it is shown that the implemented model and failure criteria can accurately predict the constitutive behaviour of unidirectional polymer composites for both quasi-static and high strain rates.

Resumo

Este trabalho apresenta uma investigação sobre o efeito da taxa de deformação nas propriedades elásticas, plásticas e na rotura de compósitos de carbono-epoxy.

Os ensaios a elevadas taxas de deformação foram realizados numa barra de Hopkinson recorrendo a uma configuração optimizada para materiais compósitos, que incluiu a utilização de extensómetros e técnicas de correlação digital de imagem. Todos os ensaios foram realizados garantindo o equilíbrio dinâmico e uma taxa de deformação aproximadamente constante. Deste modo, foi possível obter resultados fiáveis para as propriedades elásticas e plásticas a partir da relação entre a tensão e a deformação obtida nos ensaios mecânicos.

Foram realizados ensaios de compressão estáticos e dinâmicos para o laminado de carbon-epoxy IM7-8552 nas direcções longitudinal e transversal. Foi desenvolvido um novo sistema de fixação do provete na barra de Hopkinson que garante a ausência de interferência na propagação da onda de deformação no caso dos ensaios de compressão longitudinais. Foram utilizados ensaios em que as fibras não estão alinhadas com a direcção do carregamento para investigar o efeito da taxa de deformação para solicitações multiaxiais. Estes ensaios permitiram obter a tensão de cedência plástica e identificar as condições de rotura do material sob solicitações estáticas e dinâmicas, que foram posteriormente comparadas com as previsões obtidas a partir de critérios de rotura. O ângulo de fractura para os vários estados de tensão foi medido a partir de um sistema de filmagem de alta velocidade. Para as taxas de deformação utilizadas, não foi observado nenhum efeito da taxa de deformação no módulo de elasticidade longitudinal. No entanto, foi observado que aumentando a taxa de deformação há um aumento consistente para os módulos de elasticidade transversal e de corte. Verificou-se também que as tensões de rotura longitudinal em compressão, as tensões de cedência plástica e de rotura transversais em compressão e as tensões de cedência plástica e de rotura em corte aumentam com a taxa de deformação.

Foi desenvolvido um modelo plástico para compósitos unidireccionais e apresentou-se uma metodologia para obter os respectivos parâmetros a partir de ensaios de tracção e compressão em que as fibras não estão alinhadas com a direcção do carregamento. O modelo plástico, juntamente com um critério de rotura avançado, foi implementado numa subrotina do programa de elementos finitos ABAQUS. Através da comparação das relações tensão-deformação obtidas experimentalmente e a partir do modelo numérico demonstrou-se que o modelo prevê com rigor o comportamento mecânico de compósitos unidireccionais em situações de carregamento estático e sob elevadas taxas de deformação.

Résumé

Ce travail présente une étude des effets de la vitesse de déformation sur les propriétés élastiques, plastiques et de rupture des matériaux composites unidirectionnels carbone/époxy.

La technique de compression par barres de Hopkinson a été utilisée pour les essais à grande vitesse de déformation et optimisée pour les composites à l'aide d'impulsions systématique de mise en forme et des mesures de déformation en utilisant des jauges de déformation ou la méthode optique sans contact par corrélation d'image numérique. Tous les essais à grande vitesse de déformation ont été effectués en équilibre dynamique des contraintes et approximativement à des vitesses de déformation constante. En conséquence, on a pu obtenir à la fois des propriétés élastique fiables et des propriétés de rupture à partir de la réponse dynamique contrainte-déformation.

Pour le carbone-époxy IM7-8552, des essais quasi-statique et à grande vitesse de déformation ont été réalisés dans les directions de compression longitudinal et transversal. Un montage a été développé pour les essais de compression longitudinale, ce qui permet la propagation d'une onde de déformation libre d'interférences lorsqu'il est utilisé sur l'essai de compression par barres de Hopkinson. L'effet de la vitesse de déformation sur la réponse du matériau en compression transversale couplé au cisaillement dans le plan a été étudié au moyen d'essais de compression hors-axe. À partir de ces essais, la réponse quasi-statique et dynamique en cisaillement dans le plan a été déterminé et la limite d'élasticité et les enveloppes de rupture pour la compression transversale combinée au cisaillement plan ont été établies et comparées à des critères de rupture dans la littérature. La photographie à grande vitesse a été utilisée pour étudier les modes de rupture en chargement dynamique, et déterminer l'angle du plan de fracture en compression transversale pure et pour différents rapport de compression transversale et cisaillement dans le plan. Pour les vitesses de déformation étudiés dans ce travail, aucun effet de la vitesse de déformation a été observée pour le module de compression longitudinale, alors qu'une augmentation modérée et cohérente, avec l'augmentation du chargement, a été trouvée pour les modules de compression transversale, de cisaillement plan et de compression hors-axe. Des effets de la vitesse de déformation encore plus significatifs ont été observés pour la rupture longitudinale à la compression, aussi que la limite d'élasticité et rupture pour la compression transversale, cisaillement dans le plan et cisaillement hors-axe.

Des essais quasi-statique de traction hors-axe ont été réalisés pour étudier la réponse constitutive et déterminer l'enveloppe de rupture en traction transversale combinée au cisaillement dans le plan. Comme pour les essais de compression, l'enveloppe de rupture expérimental a été comparé à des critères de rupture.

Un modèle de plasticité simple pour les polymères composites unidirectionnels a été in-

troduit. Il est montré comment les paramètres du modèle de plasticité peuvent être déterminés à partir des essais de compression hors-axe et de traction réalisés dans cette étude. Le modèle de plasticité et un critère de rupture ont été mis en œuvre dans ABAQUS en utilisant la sous-routine matérielle VUMAT. En comparant les courbes prévues et expérimentales contrainte-déformation, il est montré que le modèle et le critère de rupture implémentés peuvent prédire avec précision le comportement constitutif des polymères composites unidirectionnels pour les cas quasi-statiques et de grande vitesse de déformation.

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Nomenclature

Latin

a	Pulse shaper cross section
a_0	Initial pulse shaper cross section
a_{66}	Plasticity coefficient of plasticity model in 2D formulation
a_{ij}	Plasticity coefficients of the general yield function
A_{pm}	Master curve power law coefficient
A_{pm}^{dyn}	Master curve power law coefficient for dynamic loading
A_{pm}^{qs}	Master curve power law coefficient for quasi-static loading
A	Cross section
A_0	Bar cross section
A_1	Cross section of bar 1
A_2	Cross section of bar 2
A_b	Bar cross section
A_{s0}	Initial specimen cross section
c	Elastic longitudinal wave speed
c_1	Elastic longitudinal wave speed of bar 1
c_2	Elastic longitudinal wave speed of bar 2
c_A	Elastic longitudinal wave speed of medium A
c_B	Elastic longitudinal wave speed of medium B
c_b	Elastic longitudinal wave speed of the bars
c_s	Elastic longitudinal wave speed of the specimen

d_b	Bar diameter
d_{ps}	Pulse shaper diameter
d_s	Specimen diameter
$d_{striker}$	Striker bar diameter
d_{TC}	Diameter of Tungsten-Carbide insert
$\mathbf{D}^{ep}$	Elastic-plastic tangent modulus
E	Elastic modulus
E_1	Longitudinal modulus
E_{1c}^{dyn}	Dynamic longitudinal compressive modulus
E_{1c}^{qs}	Quasi-static longitudinal compressive modulus
E_{1t}	Longitudinal tensile modulus
E_2	Transverse modulus
E_{2c}	Transverse compressive modulus
E_{2t}	Transverse tensile modulus
E_3	Interlaminar modulus
E_{3c}	Interlaminar compressive modulus
E_{3t}	Interlaminar tensile modulus
E_b	Elastic modulus of bar material
E_{TC}	Elastic modulus of TC-insert
f	Yield function
f_e	Scaling function for elastic properties
f_u	Scaling function for ultimate strength properties
F_1	Load acting at the incident bar-specimen interface
F_2	Load acting at the transmission bar-specimen interface
FI_{FK}	Failure index for fibre compressive failure
FI_{FT}	Failure index for fibre tensile failure
FI_{MC}	Failure index for matrix compressive failure
FI_{MT}	Failure index for matrix tensile failure

G_{12}	In-plane shear modulus
G_{13}	Out-of-plane shear modulus
G_{23}	Transverse shear modulus
G_{Ic}	Mode I interlaminar fracture toughness
G_{Ic}^{mix}	Mode I component of mixed mode I+II fracture toughness
G_{IIc}	Mode II interlaminar fracture toughness
G_{IIc}^{mix}	Mode II component of mixed mode I+II fracture toughness
h	Off-axis parameter function
h_{ps}	Pulse shaper thickness
H_p	Plastic modulus
k	Yield function parameter
K_{ps}	Parameter used during pulse shaping analysis
K	Scaling function coefficient
K_e	Elastic scaling function coefficient
K_u	Ultimate strength scaling function coefficient
l_b	Bar length
l_s	Specimen length
l_{s0}	Initial specimen length
Δl_{SG1}	Distance between incident bar strain gauge and specimen
Δl_{SG2}	Distance between transmission bar strain gauge and specimen
$l_{striker}$	Striker bar length
l_{tab}	Geometric length of oblique tab at centre line
m	Exponent of power law for master curve parameter A
n_f	Scaling function exponent
n_e	Elastic scaling function exponent
n_u	Ultimate strength scaling function exponent
n_{pm}	Master curve power law exponent
n_{ps}	Pulse shaper material response power law exponent

$\mathbf{Q}_e$	Elastic stiffness matrix
$\mathbf{Q}_{ep}$	Elastic-plastic stiffness matrix
$\mathbf{R}$	Rotation matrix
S	σ in ABAQUS notation
S_{11}	σ_{11} in ABAQUS notation
$\bar{S}_{11}$	11-component of the compliance matrix in x-y coordinate system
$\bar{S}_{16}$	16-component of the compliance matrix in x-y coordinate system
S_L	In-plane shear strength
S_L^{dyn}	Dynamic in-plane shear strength
S_L^{qs}	Quasi-static in-plane shear strength
S_L^y	In-plane shear yield strength
S_T	Transverse shear strength
$t_{specimen}$	Specimen thickness
t	Time
t_0	Time at which the loading of the specimen starts
t_1	Time at which pulse shaper ceases to deform for case II shaped pulse
t_2	Time marking the end of a case II shaped pulse
t_s	Transit time of longitudinal wave across the specimen
t_{SG1}	Time record of incident bar strain gauge
$t_{\epsilon_I}^{specimen}$	Time at which the incident wave reaches the specimen
$t_{\epsilon_R}^{specimen}$	Time at which the reflected wave acted on the specimen
$t_{\epsilon_T}^{specimen}$	Time at which the transmitted wave acted on the specimen
t_t	Transit time of longitudinal wave across bar diameter
t_u	Time at ultimate strength
t_{yield}	Time at onset of non-linearity
t^*	Time at which pulse shaper ceases to deform for case I shaped pulse
t^{**}	Time marking the end of a case I shaped pulse
T	Pulse duration of classic incident wave

T_{caseI}	Duration of case I shaped pulse
T_{caseII}	Duration of case II shaped pulse
$\mathbf{T}$	Stress tensor in material coordinate system
$\mathbf{T}^{(\varphi)}$	Stress tensor in misaligned coordinate system φ
$\mathbf{T}^{(\theta_k)}$	Stress tensor in kink-band plane coordinate system θ_k
u	Displacement in longitudinal bar direction
u_1	Displacement of the incident bar-specimen interface
u_2	Displacement of the transmission bar-specimen interface
$\dot{u}_1$	Particle velocity of the incident bar-specimen interface
$\dot{u}_2$	Particle velocity of the transmission bar-specimen interface
u_I	Displacement in longitudinal bar direction during incident wave
u_R	Displacement in longitudinal bar direction during reflected wave
u_T	Displacement in longitudinal bar direction during transmitted wave
u_x	First order displacement gradient of u in x-direction
u_y	First order displacement gradient of u in y-direction
U_p	Particle velocity
U_p^{max}	Maximum particle velocity in striker and incident bar
U_{pI}	Particle velocity in the bar during incident wave
U_{pR}	Particle velocity in the bar during reflected wave
U_{pT}	Particle velocity in the bar during transmission wave
v_x	First order displacement gradient of v in x-direction
v_y	First order displacement gradient of v in y-direction
V_0	Striker bar impact velocity
w	Specimen width
dW^P	Plastic work increment
Δx	Relative displacement in x-direction
X_C	Longitudinal compressive strength
X_c^{dyn}	Dynamic longitudinal compressive strength

X_c^{qs}	Quasi-static longitudinal compressive strength
X_T	Longitudinal tensile strength
Δy	Relative displacement in y-direction
Y_C	Transverse compressive strength
Y_C^y	Transverse compressive yield strength
Y_T	Transverse tensile strength
Z_C	Interlaminar compressive strength
Z_T	Interlaminar tensile strength
Z	Mechanical impedance
Z_0	Characteristic impedance
Z_b	Mechanical impedance of the bar
Z_s	Mechanical impedance of the specimen
Z_{TC}	Mechanical impedance of the Tungsten-Carbide insert

Greek

α	Fracture plane angle with respect to 1-2 material coordinate system
α_0	Fracture plane angle for pure transverse compression
α'	Fracture angle measured from camera image
γ_{12}	Engineering shear strain in 1-2 coordinate system
γ_{xy}	Engineering shear strain in x-y coordinate system
γ_m	Additional fibre rotation
$\gamma_m^{(\theta_k)}$	Additional fibre rotation in kink-band plane coordinate system
γ_{mC}	Additional fibre rotation for pure axial compression
ε	Strain
$\Delta\varepsilon$	Total strain increment vector
ε_{11}	Longitudinal strain
ε_{22}	Transverse strain
$d\varepsilon_{ij}$	Incremental total strain components
$d\varepsilon_{ij}^e$	Incremental elastic strain components
$d\varepsilon_{ij}^p$	Incremental plastic strain components
$\Delta\varepsilon_p$	Plastic strain increment vector
$\bar{\varepsilon}^p$	Effective plastic strain
ε_{eff}^p	Effective plastic strain
$d\bar{\varepsilon}^p$	Effective plastic strain increment
ε_{ps}	Pulse shaper strain
ε_s	Specimen strain
ε_{xx}	Nominal strain in x-direction
ε_{xx}^p	Normal plastic strain component in x-direction
$d\varepsilon_{xx}^p$	Incremental normal plastic strain component in x-direction
ε_{yy}	Nominal strain in y-direction
ε_I	Incident wave
ε_I^{max}	Incident wave amplitude

ϵ_I^{PSA}	Incident wave determined via PSA
ϵ_R	Reflected wave
ϵ_T	Transmitted wave
$\dot{\epsilon}$	Strain rate
$\dot{\epsilon}_{dyn}$	Dynamic strain rate
$\dot{\epsilon}_{qs}$	Quasi-static strain rate
$\dot{\epsilon}_{ps}$	Strain rate in pulse shaper
$\bar{\epsilon}^p$	Effective plastic strain rate
η_1	First order shape function for displacement component v
η_L	Longitudinal friction coefficient
η_L^{dyn}	Dynamic longitudinal friction coefficient
η_L^{qs}	Quasi-static longitudinal friction coefficient
η_T	Transverse friction coefficient
θ	Off-axis angle
θ_0	Initial off-axis angle
$d\theta$	Fibre rotation angle measured for off-axis compression tests
θ_k	Angle defining orientation of kink-band plane
θ_i	Integration scheme identification parameter
Λ	Wave length
$d\lambda$	Incremental plastic multiplier
$\Delta\lambda$	Plastic multiplier
ν	Poisson's ratio
ν_s	Specimen Poisson's ratio
ν_{12}	Major Poisson's ratio in 1-2 plane
ν_{13}	Major Poisson's ratio in 1-3 plane
ν_{23}	Major Poisson's ratio in 2-3 plane
ξ_1	First order shape function for displacement component u
ρ	Density

ρ_1	Density of bar 1
ρ_2	Density of bar 2
ρ_A	Density of medium A
ρ_B	Density of medium B
ρ_s	Density of specimen
ρ_{TC}	Density of Tungsten-Carbide insert
σ	Stress
σ_0	Coefficient for pulse shaper material response power law
σ_{11}	Longitudinal stress
σ_{12}	In-plane shear stress
σ_{22}	Transverse stress
$\Delta\sigma_e$	Elastic strain increment vector
$\bar{\sigma}$	Effective stress
σ_{eff}	Effective stress
$\Delta\sigma_{ep}$	Elastic-plastic strain increment vector
σ_I	Bar stress caused by incident wave
σ_{ij}	Stress tensor components in the 1-2 material coordinate system
σ_{ij}^m	Stress components in misaligned coordinate system (2D)
$\sigma_{ij}^{(\varphi)}$	Stress tensor components in misaligned coordinate system
$\sigma_{ij}^{(\theta_k)}$	Stress tensor components in kink-band plane coordinate system
σ_{max}	Stress amplitude of classic incident wave
σ_n	Normal stress acting on the fracture plane
σ_R	Bar stress caused by reflected wave
σ_s	Specimen stress
σ_T	Bar stress caused by transmitted wave
σ_{xx}	Normal stress in x-direction
τ_{12}	In-plane shear stress
τ_L	Longitudinal shear stress acting on the fracture plane

τ_T	Transverse shear stress acting on the fracture plane
ϕ	Oblique tab angle
φ	Fibre misalignment angle in kink-band plane
φ_0	Initial fibre misalignment angle
φ_C	Fibre misalignment angle for pure axial compression
χ	Power law coefficient for rate dependent master curve parameter A_{pm}

Abbreviations

1D	1-dimensional
2D	2-dimensional
3D	3-dimensional
AF	Auto focus
ASTM	American Society for Testing and Materials
BA	Bars-apart
CAD	Computer aided design
CCD	Charge-coupled device
CFRP	Carbon-fibre-reinforced plastic
CV	Coefficient of variation
DCB	Double cantilever beam
DCF	Dynamic compression fixture
DIC	Digital image correlation
DYN	Dynamic
ELS	End-loaded split
FAA	Federal Aviation Administration
FE	Finite element
FEA	Finite element analysis
FEM	Finite element method
FEUP	Faculdade de Engenharia da Universidade do Porto
FRMM	Fixed-ratio mixed-mode
FRPMCs	Fibre-reinforced polymer-matrix composites

FRPs	Fibre-reinforced plastics
GFRP	Glass-fibre-reinforced plastic
GOM	Gesellschaft für Optische Messtechnik mbH
HBM	Hottinger Baldwin Messtechnik GmbH
ILSS	Interlaminar shear strength
IM	Intermediate modulus
IPS	In-plane shear
LaRC	Langley Research Center
MCV	Mega City Vehicle
MoS2	Molybdenum disulfide
NDT	Non-destructive testing
OAC	Off-axis compression
OAT	Off-axis tension
OFHC	Oxygen-free high purity copper
PC	Personal computer
PEEK	Polyetheretherketon
PSA	Pulse shaping analysis
QS	Quasi-static
RTM	Resin transfer moulding
SG	Strain gauge
SHPB	Split-Hopkinson pressure bar
SHPBA	Split-Hopkinson pressure bar analysis
STDV	Standard deviation
TC	Tungsten-carbide
UD	Unidirectional
UNS	Unified numbering system
UTS	Ultimate tensile strength
VUMAT	User material subroutine for Abaqus/Explicit

WIF	Wedge-insertion fracture
WLCT	Wedge-loaded compact-tension

Chapter 1

Introduction

The ability to withstand dynamic loading is an important design criteria for many structures in aerospace, automotive, marine and civil engineering applications. Classic dynamic load scenarios for aircraft structures are bird impact on the wing leading edge or on the fan blades of aero engines. In the later case it is required that any particles from fractured blades are kept within the engine to avoid further damage of the surrounding wing and fuselage structure. This again results in a highly dynamic load case for the engine housing. In order to provide a maximum level of security for the passengers in the event of a crash-landing, aircraft and helicopter subfloor structures are designed to absorb the impact energy via highly dynamic damage and fracture mechanisms. Crashworthiness of composite structures is also becoming a key design criteria in the automotive industry with the recent introduction of composites extensive car designs such as the BMW Megacity Vehicle (Figure 1.1). The hulls of naval ships, in particular those of mine sweepers, may be subjected to mine blast while the constant pounding of waves in rough seas is another case of dynamic loading. Earthquakes are an example of dynamic loading of civil engineering structures.

It is well known that the mechanical properties of most materials depend on strain rate. For metals, a huge experimental effort was undertaken in this century to investigate the effect of strain rate on the mechanical material properties. As composites continue to replace conventional metallic structures in all of the areas mentioned above, understanding

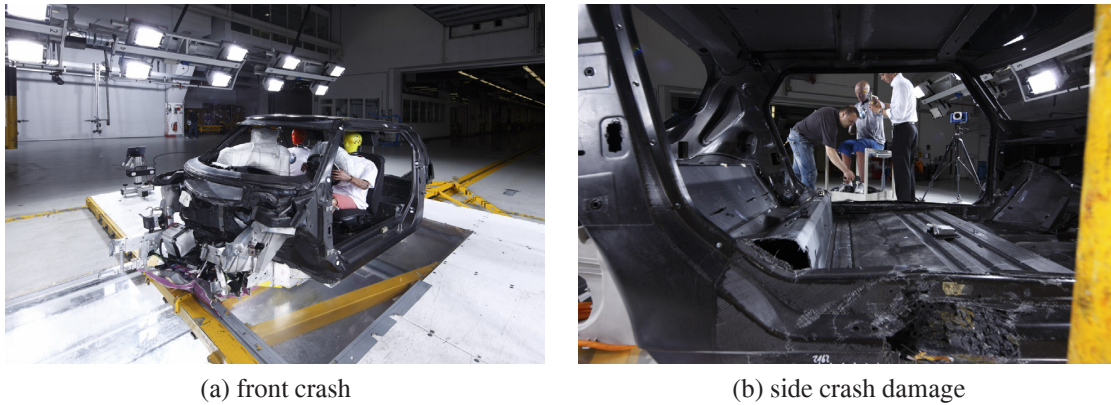


Figure 1.1: BMW Megacity Vehicle (MCV) [1].

the strain rate behaviour of composite material systems has gained significant importance over the past two decades. In comparison to metals, the damage and failure mechanisms of composite materials are not fully understood, in particular for the case of high strain rate loading. Despite this lack of knowledge, composite structures are now increasingly being used for primary aircraft structures with the launch of the Boeing 787 Dreamliner Project in April 2004 being a prominent example (Figure 1.2). Analytical tools for the prediction of the mechanical behaviour of advanced composite structures are still being

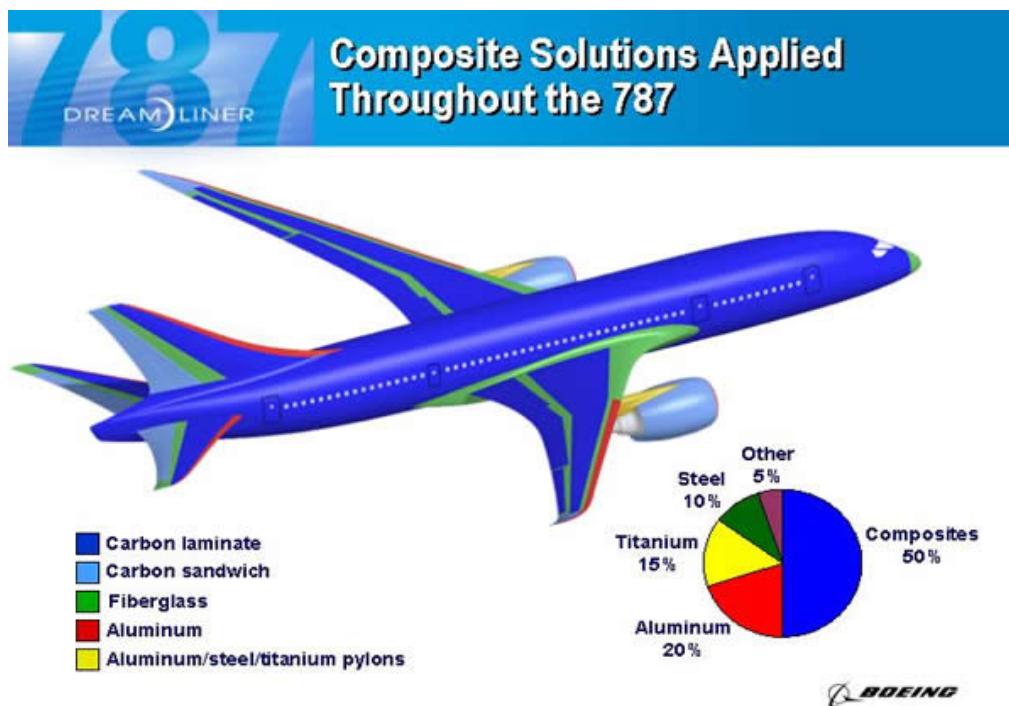


Figure 1.2: Boeing 787 material selection [2].

developed, and manufacturers of primary composite aircraft structures therefore have to deal with the increased cost of physical test programs since passenger safety must be proven to the regulating authorities such as the Federal Aviation Administration (FAA). The objective of the presented work is to investigate in detail the strain rate effect on the elastic, plastic and strength properties of unidirectional carbon-epoxy composites on the basis of an extensive experimental program for both quasi-static and dynamic loading rate regimes, and for uniaxial and multiaxial stress states. In addition, this work intends to lay the foundations for the improved constitutive modelling of the response of unidirectional fibre - polymer matrix composites under high strain rates, which is known to range from linear-elastic to strong nonlinear behaviour, depending on the loading direction with respect to the material coordinate system and on the strain rate. The results obtained in this thesis will provide a solid base for the further development of existing composite constitutive models and failure criteria. On the long run, such work will help to bring down the time and cost penalty of extensive test programs and is a further step toward using the full potential of composites in new applications.

1.1 Layout of Thesis

Chapter 2 provides a thorough literature review of earlier experimental work with respect to dynamic material characterisation of fibre-reinforced polymer matrix composites (FRPMCs). The strain rate effect on the fibre and resin constituent is reviewed, along with the dynamic response of unidirectional composites subjected to in-plane and out-of-plane tensile, compressive and shear loading. In addition, a review of strain rate effects on the interlaminar fracture toughness for mode I, mode II and mixed mode I+II loading is presented.

Chapter 3 contains a detailed description of the dynamic experimental procedure of the split-Hopkinson pressure bar (SHPB), used for the high strain rate experiments. A strong focus lies on the optimisation of this dynamic test method for the use of composite specimens, where relatively small failure strains, high or low failure strengths, and

linear-elastic or nonlinear specimen stress-strain behaviour must all be considered. The principles of the pulse shaping technique are presented and the limitations of the classic SHPB experiment are shown via finite element (FE) simulations for the case of a high strength linear-elastic longitudinal compressive composite specimen. A brief introduction of the digital image correlation (DIC) technique, which is a state-of-the-art optical data reduction method to determine full displacement and strain fields via contactless measurements, is given.

Chapter 4 presents a high strain rate experimental investigation of the longitudinal compressive material response of unidirectional carbon-epoxy, using the optimised SHPB methods developed in the previous Chapter together with new test fixtures designed for the SHPB.

Chapter 5 describes the high strain rate experimental work with respect to the transverse compressive, in-plane shear and combined transverse compressive and in-plane shear response, studied using off-axis compression specimens. The DIC method, introduced in Chapter 3, was used for all quasi-static and dynamic experiments. High speed photography was used to study the fracture modes under dynamic loading, and to determine the fracture plane angle α for pure transverse compression and for various ratios of transverse compression and in-plane shear.

Chapter 6 contains the experimental results from off-axis tension tests, which can be used together with the quasi-static experiments presented in Chapter 5 to analyse the constitutive response of unidirectional carbon-epoxy composites for combined transverse and in-plane shear loading.

Chapter 7 introduces a simple plasticity model for unidirectional composites, defined for the two-dimensional in-plane stress state. It is shown how the plasticity model parameters can be determined from the experiments described in the previous Chapters for both quasi-static and dynamic strain rates. Chapter 7 further introduces a failure criteria for unidirectional composites, defined for the general three-dimensional stress state. The failure criteria was developed in a parallel study at the University of Porto. The experi-

mental observations regarding the strain rate effects on the mechanical material properties of unidirectional carbon-epoxy material systems are summarised and trends are formulated to prepare the experimental data for further analytical and numerical modeling. Implementation details of an Abaqus VUMAT subroutine are presented, which contains the constitutive plasticity model and failure criteria. The model is validated, using available experimental data to show the potential of the constitutive model and failure criteria to accurately predict the quasi-static and high strain rate response of unidirectional carbon-epoxy composites.

Chapter 8 summarises the presented work, provides an overview of the main conclusions, and closes with an outline of anticipated future experimental and numerical modeling work.

Chapter 2

Literature Review

Strain rate studies on a great variety of composite material systems such as unidirectional and quasi-isotropic laminates, woven fabrics or metal-matrix composites can be found in the literature. Because of the unidirectional material system investigated in this study, the following chapter mainly focuses on the review of unidirectional composites with an emphasis on carbon-epoxy material systems.

2.1 Rate Effect on Constituent Properties

To understand the strain rate behaviour of fibre-reinforced composites it is necessary to look not only on the composite itself but also on the individual constituents, the fibre and the matrix. This is particularly important for the development of physically based constitutive models on the micro-mechanical level, where fibre and matrix are treated separately.

2.1.1 Fibre

Only limited experimental data exists regarding the strain rate effect on the mechanical properties of fibres or fibre bundles. Yuanming et al. [3] investigated the mechanical properties of E-glass fibre bundles in tension using a tension split-Hopkinson bar. A significant strain rate effect was found for the modulus, strength and failure strain. In a

follow-up study on kevlar fibre bundles, Wang and Xia [4], reported a significant strain rate behaviour, although not as pronounced as found for E-glass. Recently Zhou et al. [5] performed experimental studies on carbon fibre bundles, using a tension split-Hopkinson bar, and reported that strain rate has no effect on the tensile properties of carbon fibres.

2.1.2 Neat Resin Tension

Gilat et al. [6] investigated the strain rate effect on the tensile response of Cytec Fiberite 977-2 epoxy resin and in a follow-up study [7] further investigated the strain rate behaviour of Shell Chemicals E-862 and Cytec PR-520 epoxy resin, using a conventional hydraulic load frame for quasi-static and medium strain rates and a tension split-Hopkinson bar for high strain rate tests up to about 400s^{-1} . The strain rate effect on the tensile stress-strain response of the resin systems investigated in both studies is shown in Figures 2.1 and 2.2, respectively.

A shift from a ductile to a more brittle stress-strain behaviour with increasing strain rate was observed in all cases, and the failure strain was therefore lower for the dynamic tests. The failure strength was found to increase moderately with increasing strain rate. Gilat et al. [6, 7] further reported that the elastic modulus increased significantly for the high strain rate tests performed on the SHPB, whereas a similar initial elastic response was observed for the quasi-static and medium strain rate tests.

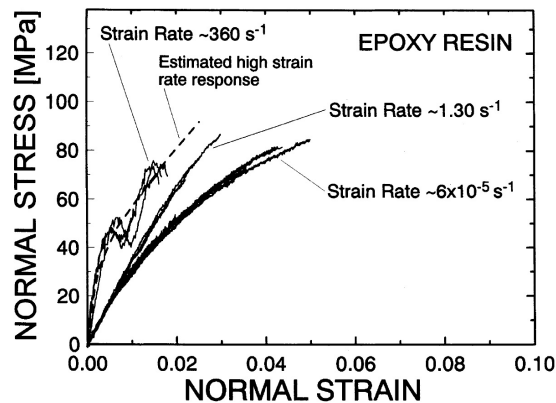


Figure 2.1: Tensile stress-strain response of Cytec Fiberite 977-2 epoxy resin at different strain rates [6].

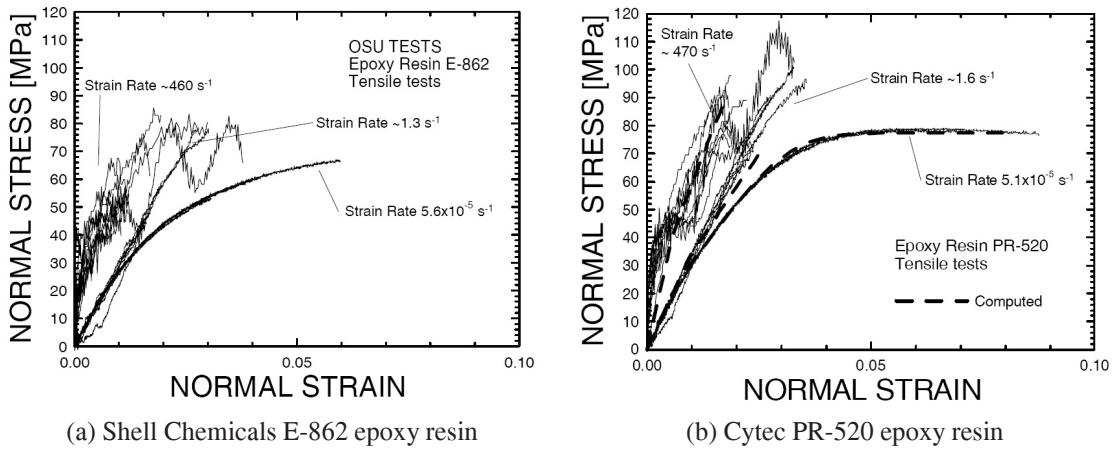


Figure 2.2: Tensile stress-strain response of two epoxy resin systems at different strain rates [7].

It is noted that the results of the SHPB tests reported in [6, 7] must be treated with some reservation since premature failure was observed for the high rate tests if specimens with strain gauges were used, and failure always occurred at the specimen/strain gauge carrier interface. Higher failure strengths were then obtained for the same specimen type if no strain gauges were used. In the latter case however, a comparison of the quasi-static and high strain rate strain response is not possible since it was shown that the specimen strain calculated with the SHPB analysis equation is overpredicted and therefore does not represent the actual specimen strain. In addition to the premature failure caused by the specimen strain gauge, some high strain rate specimens failed outside of the gauge section, which was attributed to hydrostatic components of the stress tensor. A redesign of the SHPB specimen was therefore attempted [7] but satisfactory results were only obtained for the PR-520 resin system.

It appears further that the SHPB specimens were not in a state of dynamic equilibrium up to failure, which is indicated by the oscillations in the high rate stress-strain response shown in Figures 2.1 and 2.2. The initial elastic response reported for the SHPB tests in [6, 7] is therefore not reliable and should be treated with care.

In a recent study, Gerlach et al. [8] performed dynamic experiments up to strain rates of 3800 s^{-1} for Hexcel RTM-6 epoxy specimens, using a tension SHPB. Up to this strain rate, failure was observed to occur in the middle of the specimen gauge section, which

was attributed to the development and use of a novel pulse shaping device, applicable for tension SHPBs. A significant increase was observed for failure strength and modulus, while as the failure strain decreased for increasing loading rates (Figure 2.3).

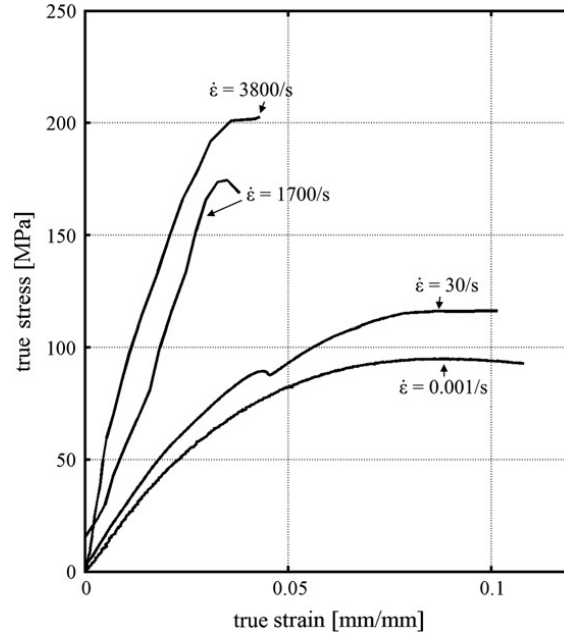


Figure 2.3: Tensile stress-strain response of Hexcel RTM-6 epoxy resin at different strain rates [8].

2.1.3 Neat Resin Compression

The strain rate effect on the compressive properties of neat resins was investigated experimentally by Buckley et al. [9]. Three thermoset systems were studied: Ciba CT-200 (epoxy), 3M PR-500 (epoxy) and Cytec Cycom 5250-4 (bismaleimide). The tests were performed over a strain rate range of $0.001s^{-1}$ to nearly $5000s^{-1}$ using cylindrical specimens with various length to diameter ratios, L/D . For quasi-static tests a conventional screw-driven INSTRON load frame was used, while the medium and high strain rate tests were performed on a hydraulic test machine and the split-Hopkinson pressure bar, respectively. It was shown that the L/D ratio does not affect the test results if the specimen surfaces in contact with the SHPB bar ends are properly lubricated to minimise friction effects. Significant ductility was observed for all three resin systems with quasi-

static failure strains of 90% - 100%. With increasing strain rate, increasing yield stresses and strains were reported. The strain rate effect on the compressive modulus was not reported. When comparing the stress-strain response of the three resin systems for quasi-static, medium and high strain rates in the presented diagrams (Figure 2.4), it appears that this property was only marginally effected. This judgment is however difficult, in particular for the PR-500 resin system, due to some scatter in the initial stress-strain response. In addition to the dynamic compressive stress-strain behaviour, the surface temperature of some high strain rate specimens was monitored via an infrared measurement technique. A temperature increase of up to 30°C was reported and attributed to the adiabatic heating of the specimen at high strain rates.

In a recent study, Gerlach et al. [8] tested Hexcel RTM-6 epoxy resin specimens in compression on a conventional SHPB at strain rates of up to 6000s⁻¹, and reported a significant increase of yield, flow stress and apparent elastic modulus. The apparent

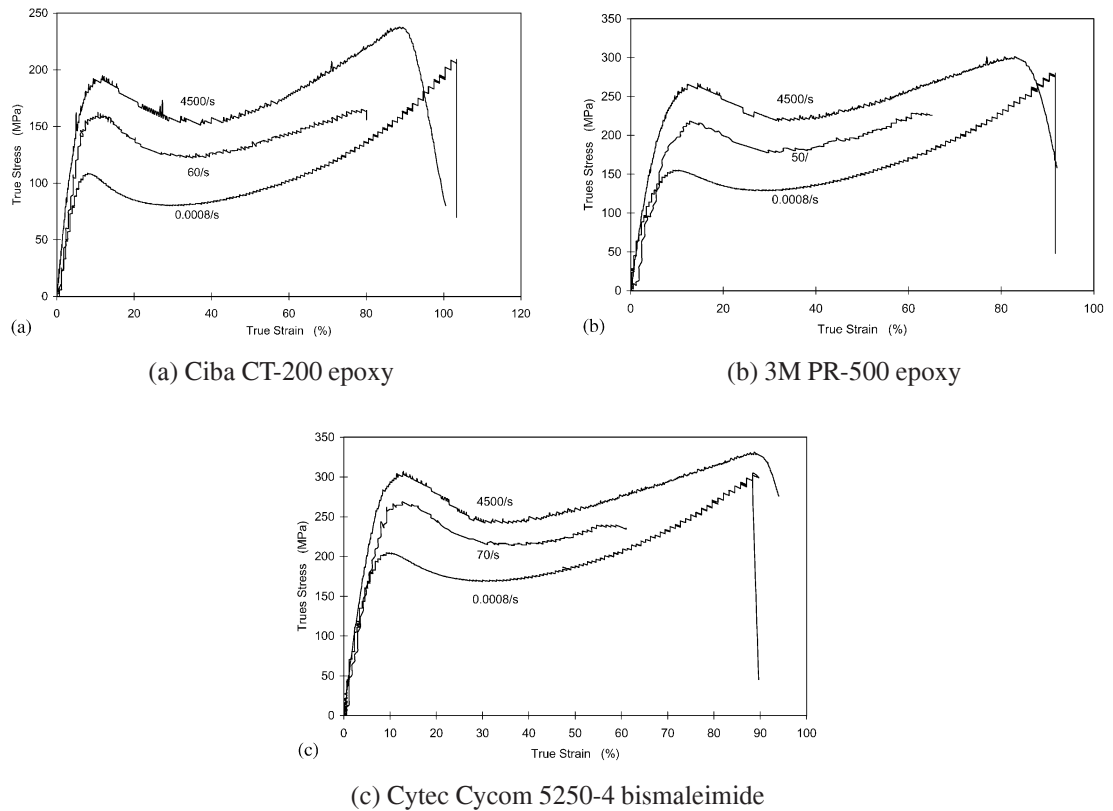


Figure 2.4: Compressive stress-strain response of various thermoset neat resin systems at different strain rates [9].

modulus was defined as secant modulus at 1% strain and used as a pragmatic value for modelling purposes. Gerlach et al. [8] provided a diagram, where the input- and output specimen stress is shown for a SHPB test at a strain rate of 4400s^{-1} , and it appears that the specimen is not in a state of dynamic equilibrium during the initial elastic part of the stress-strain response. The failure strain of the high strain rate specimens was found to be higher than in the quasi-static case (Figure 2.5).

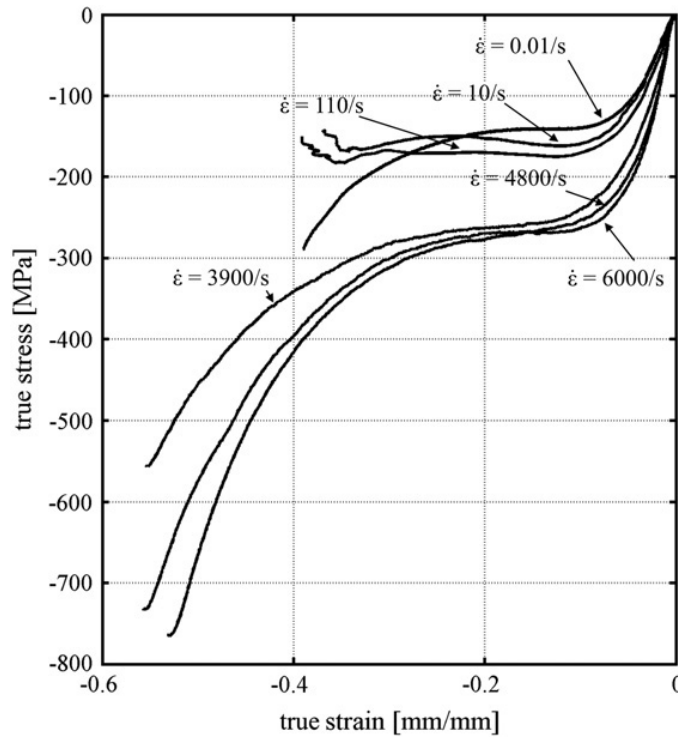


Figure 2.5: Compressive stress-strain response of Hexcel RTM-6 epoxy resin at different strain rates [8].

2.1.4 Neat Resin Shear

The high strain rate shear response of two epoxy resin systems, Cytec PR-520 and Shell Chemicals E-862, was studied by Gilat et al. [7]. The quasi-static and medium strain rate tests were carried out on a bi-axial hydraulic torsion/tension machine and a torsion split-Hopkinson bar was used for the high strain rate experiments. In all cases a thin-walled tube specimen was used. The quasi-static and dynamic shear stress-strain results

are shown in Figure 2.6. For both resin systems, the shear modulus increased and the shear strength significantly increased with increasing strain rate. A ductile material behaviour was observed for all strain rates with a stress plateau at the plastic part of the stress-strain response.

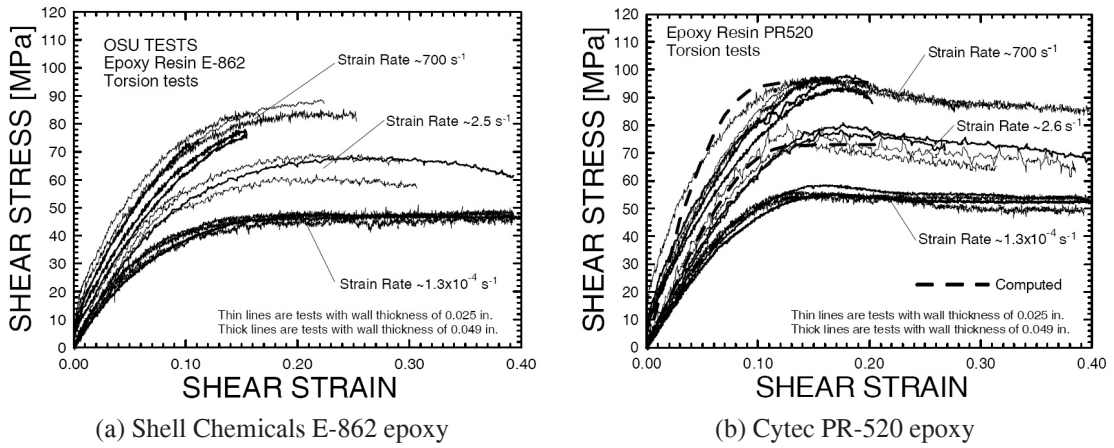


Figure 2.6: Shear stress-strain response of epoxy resin [7].

2.2 Rate Effect on Composite Tensile Properties

2.2.1 In-Plane Tension

Harding and Welsh [10] were the first to successfully study the high strain rate longitudinal tensile behaviour of unidirectional carbon-epoxy laminates, using a tension split-Hopkinson bar apparatus, and found no significant strain rate effects. The same was concluded in a recent study by Taniguchi et al. [11], who performed dynamic experiments up to strain rates of 100 s^{-1} for unidirectional carbon-epoxy specimens made from Toray T700S/2500 prepreg, also using a tension split-Hopkinson bar (Figure 2.7). The results of Harding and Welsh [10] and Taniguchi et al. [11] are consistent with the observations regarding the strain rate effect on carbon fibre bundles reported in Section 2.1.1. While investigating the strain rate effect on the fracture toughness of unidirectional carbon-epoxy laminates (see Section 2.5), Blackman et al. [12] confirmed that the longitudinal

tensile modulus is independent of strain-rate using an ultrasonic non-destructive testing (NDT) method, based on the propagation of Lamb waves in thin composite plates [13]. The material investigated in this study was Fibredux 6376C, by Ciba Composites, UK.

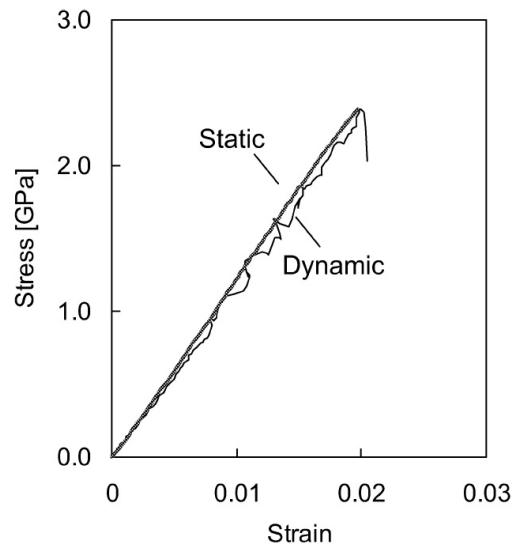


Figure 2.7: Longitudinal tensile stress-strain response of unidirectional carbon-epoxy laminate at different strain rates [11].

In addition to the dynamic material characterisation of Cytec Fiberite 977-2 epoxy resin (see Section 2.1.2), Gilat et al. [6] also investigated the strain rate effect on the transverse tensile properties of the unidirectional carbon-epoxy prepreg system IM7/977-2, using a tension split-Hopkinson bar. It was reported that both transverse tensile modulus and transverse tensile strength increase with increasing strain rate, whereas no strain rate effect was observed for the failure strain. Taniguchi [11] also studied the strain rate effect on the transverse tensile properties and found similar strain rate effects (Figure 2.8).

He further conducted a microscopical study of the fracture surface of the transverse tension specimens and found that the crack propagates along the fibre-matrix interface for both, quasi-static and high strain-rates (Figure 2.9). It is interesting to note that both, Gilat et al. [6] and Taniguchi et al. [11] measured a failure strain of less than 1% for the composite transverse tension tests. The failure strain of the UD composite is therefore significantly lower than the failure strain observed for neat resin tensile tests presented in Section 2.1.2.

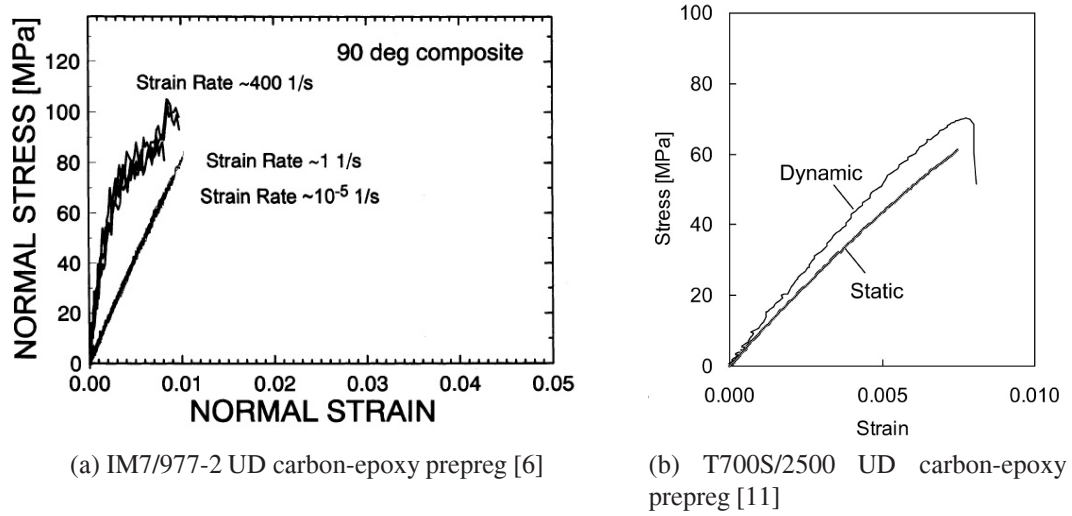


Figure 2.8: Transverse tensile stress-strain response of unidirectional carbon-epoxy laminate at different strain rates.

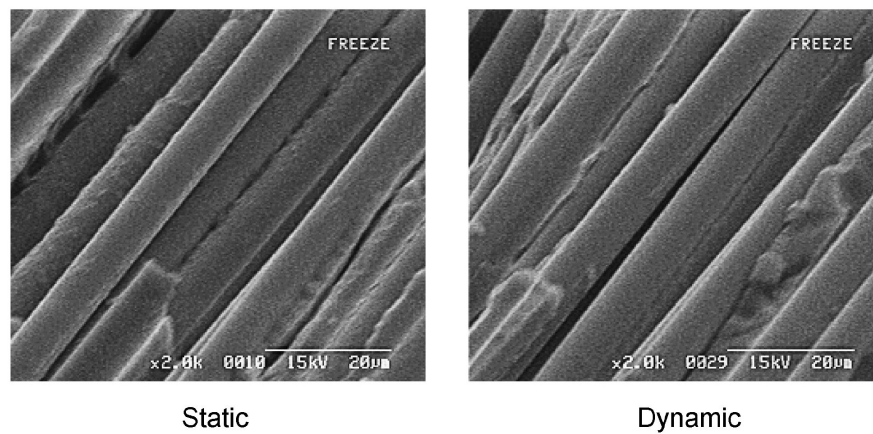


Figure 2.9: Fracture surface of unidirectional carbon-epoxy transverse tension specimen at quasi-static and dynamic strain rates [11].

The difference in the stress-strain behaviour of neat resin and UD composite could be justified by the stress concentrations caused by the fibres, by the triaxial stress state in the resin that results from the curing process in the presence of the fibres and by the fact that failure occurs at the fibre-matrix interface rather than within the resin itself.

2.2.2 Out-of-Plane Tension

The out-of-plane (interlaminar) tensile response of two polymer composite material systems, plain weave E-glass/epoxy and unidirectional AS4/3502 carbon-epoxy, under dy-

dynamic loading was studied by Lifshitz and Leber [14] on a tension split-Hopkinson bar apparatus similar to the one used by Gilat et al. [6, 7]. Custom specimens with non-uniform cross-sections were designed and evaluated via FE analysis. Strain rates of 100s^{-1} - 250s^{-1} were reached in the dynamic tests and the dynamic results were compared with quasi-static data from the literature (GFRP: Gandelsman and Ishai [15], CFRP: Ishai [16]). The interlaminar modulus and strength was found to increase for both material systems under dynamic loading.

It is worth noting that in addition to the pure interlaminar tension tests, combined interlaminar tension-interlaminar shear tests were performed. A valid failure envelope was however only obtained for the E-glass/epoxy composite since the CFRP specimens for combined loading proved to be impossible to manufacture.

2.3 Rate Effect on Composite Compressive Properties

2.3.1 In-Plane Compression

Compared to the dynamic tensile properties, the investigation of the strain rate effect on the in-plane compressive properties has received more attention and therefore a relatively large amount of experimental data exists. This is due to the fact that the longitudinal and transverse compressive response of polymer composites is strongly influenced by the behaviour of the matrix, and can also be attributed to the well established compressive version of the split-Hopkinson bar apparatus.

Kumar et al. [17] studied the dynamic compressive behaviour of a unidirectional E-glass/epoxy composite at various off-axis angles as well as in the longitudinal and transverse direction. The classic SHPB was used in this study and the length of the cylindrical specimens was varied to assure the same dynamic strain rate of approximately 265s^{-1} for all fibre-orientations. The dynamic stress-strain response below 1% strain was not plotted due to the limitations of the classic SHPB technique [17], by which the authors mean the initial non-equilibrium stress state in the dynamic specimen.

Kumar et al. [17] reported a higher longitudinal compressive strength and failure strain with increasing strain rate but found that the longitudinal compressive modulus (extrapolated from the quasi-static and dynamic stress and strains at failure) decreases with increasing strain rate. The latter result is rather unexpected and could not be explained by Kumar et al. [17]. A possible explanation is the overprediction of the specimen strain, calculated via SHPB analysis, if high strength specimens are used (see Section 3.2.2). The dynamic transverse compressive modulus was not reported by Kumar et al. [17]. The transverse compressive and the off-axis strengths were found to increase significantly for high strain rate loading.

As mentioned in Section 2.1.1, it was shown by Yuanming et al. [3] that the mechanical properties of glass fibres are significantly strain rate dependent whereas no strain rate effect was found for carbon fibres [5]. It is therefore likely that the longitudinal compression results obtained by Kumar et al. [17] were influenced by both, matrix and fibres. It is reasonable to assume that the contribution of the fibres to the strain rate effect is significantly lower in the transverse direction.

Hsiao and Daniel [18] used a drop tower and a SHPB [19] for high strain rate experiments on carbon-epoxy laminates made of Hexcel IM6G/3501-6. Longitudinal compression tests at strain rates up to 110s^{-1} were performed on the drop tower, while the rate effect on the transverse compressive properties was investigated at strain rates up to 120s^{-1} on the drop tower and at a strain rate of 1800s^{-1} on the SHPB. The drop tower and corresponding specimen used by Hsiao and Daniel [18] is illustrated in Figure 2.10.

Hsiao et al. [18, 19] found no strain rate effect for the longitudinal compressive modulus but reported a significant increase for longitudinal compressive strength. Due to the linear specimen stress-strain behaviour, the longitudinal failure strain was found to increase as well (Figure 2.11a). The transverse compressive modulus and strength were found to increase with increasing strain rate rate, with a more pronounced rate effect for the transverse compressive strength. No rate effect was observed however for the transverse compressive failure strain (Figure 2.11b).

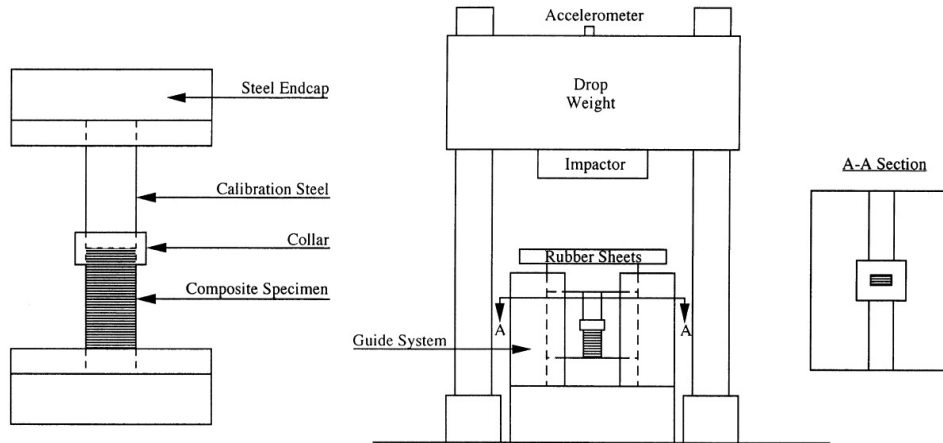
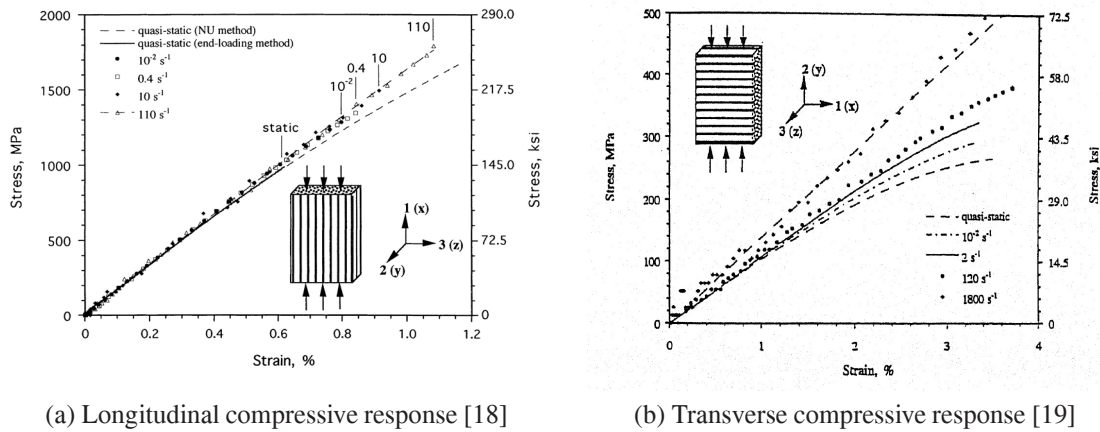


Figure 2.10: Drop tower test setup and specimen used by Hsiao and Daniel [18].



(a) Longitudinal compressive response [18]

(b) Transverse compressive response [19]

Figure 2.11: Longitudinal and transverse compressive stress-strain behaviour of carbon-epoxy at different strain rates [18, 19].

Hosur et al. [20] performed dynamic compression tests on a recovery split-Hopkinson pressure bar and used small cubic specimens with the same dimensions for the longitudinal and transverse directions, and for cross-ply laminates. The investigated material system was Panex 33/DA 4518U carbon-epoxy, supplied by Zoltex Ltd. Dynamic tests at three different strain rates (82s^{-1} , 163s^{-1} and 817s^{-1}) were performed. Hosur et al. [20] reported a more than two-fold increase of the longitudinal compressive modulus but found only a moderate increase for the longitudinal compressive strength (Figure 2.12a). The transverse compressive modulus and strength were also found to increase under dynamic loading, whereas a decrease was observed for the transverse failure strain (Figure

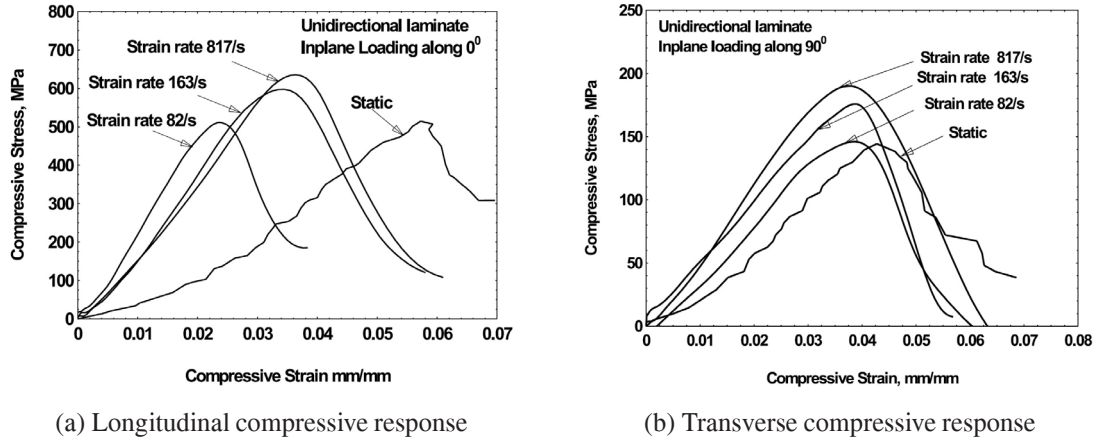


Figure 2.12: Longitudinal and transverse compressive stress-strain behaviour of carbon-epoxy at different strain rates [20].

2.12b).

Bing and Sun [21] investigated the strain rate effect on the off-axis compressive strength of small block specimens made from AS4/3501-6 carbon-epoxy. Static and medium rate tests were performed on a MTS hydraulic load frame. For high strain rate tests up to a strain rate of 700s^{-1} , a conventional SHPB was used. From the off-axis strength of the 5° , 11° and 15° specimens, the longitudinal compressive strength was extrapolated in the $\sigma_{11} - \tau_{12}$ diagram and the strength was found to increase linearly with increasing strain rate. Bing and Sun [21] also commented that, based on the results of direct longitudinal compressive tests, the longitudinal compressive modulus is not rate dependent, but no further details were given regarding these tests.

Wiegand [22] investigated the strain rate effect on the transverse and longitudinal compressive properties of unidirectional carbon-epoxy at quasi-static, medium and high strain rates. The quasi-static tests were performed on a conventional screw-driven load frame. For the medium rate tests, a custom hydraulic tester was used. The high rate tests were performed on a conventional SHPB with modified bar ends to attach the specimen.

In case of the transverse compression tests, UTS/RTM-6 dog-boned flat rectangular specimens with a layup of $[0/(90)_8]_s$ were used. The 0° layer was ground off at the gauge section before testing. Strain rates of up to 1000s^{-1} were reported for the transverse compression tests.

For the longitudinal tests, a different material system was used, T700/MTM44 carbon-epoxy, and two different specimen types were cut from a $[(0/90)_3]_s$ cross-ply laminate. A dog-boned flat rectangular specimen was used for the quasi-static and medium rate tests, whereas a straight-sided flat rectangular specimen was used for the high rate tests. The longitudinal tests were performed at strain rates of up to 360s^{-1} .

It was found that both transverse and longitudinal strengths increase with increasing strain rate. The dynamic elastic modulus was not extracted in both cases since it was evident that dynamic equilibrium was only established at the time of failure and not during the initial dynamic elastic response.

Yokoyama and Nakai [23] studied the strain rate effect on the compressive response in the three principal material directions using cubic unidirectional T700/2521 carbon-epoxy specimens. The dynamic tests were performed on a conventional SHPB at strain rates up to 700s^{-1} . It was reported that the longitudinal compressive strength is not strain rate sensitive whereas increases were observed for the transverse and through-thickness directions. It is noted that a separate review of strain rate effects on the out-of-plane compressive response was not performed due to the lack of earlier experimental data.

2.4 Rate Effect on Composite Shear Properties

Even for the quasi-static material characterisation, where well established test standards can be used, the determination of the shear properties is a challenging task. From the variety of existing quasi-static test methods [24, 25, 26], none is found to yield pure shear properties while being easy to perform at the same time, and therefore robust when comparing results between different laboratories. From the commonly used test methods, good agreement exists regarding the accuracy of shear modulus measurements. Huge discrepancies are found however when determining the shear strength, which can be attributed to reasons such as: edge effects, imperfect stress distributions, in-situ effects and perhaps most importantly, the presence of normal stresses.

The investigation of the strain rate effect on the shear response of polymer composites

has received significant attention over the past decades. For dynamic shear testing however, no common standard exists and different approaches have been used in previous experimental studies.

2.4.1 In-Plane Shear

Typical specimens used to study the strain rate effect on the in-plane shear properties are:

1. A $\pm 45^\circ$ cross-ply specimen similar to the quasi-static IPS standard [26] tested in tension.
2. A small rectangular unidirectional off-axis specimen tested in compression.

Dynamic in-plane shear tests utilising the first specimen type were conducted by Staab and Gilat [27], Gilat et al. [6], Taniguchi et al. [11] and Shokrieh and Omid [28].

Staab and Gilat [27] used a tension split-Hopkinson bar to study the response of $\pm 45^\circ$ cross-ply glass/epoxy laminates at axial strain rates of about 1000s^{-1} and found, compared to quasi-static results, an increase for both the strength and the failure strain. The results are presented as axial stress-strain response (Figure 2.13a) and the strain rate effect on the shear modulus was not reported.

Gilat et al. [6] performed similar tests on a tension split-Hopkinson bar for carbon-epoxy specimens at axial strain rates of up to 600s^{-1} . He reported an increase of the initial stiffness at high strain rates but found that the stiffness was unaffected when comparing the quasi-static and medium strain rate tests. The strength increased for the entire range of applied strain rates, whereas the failure strain remained at about the same level. The results were also presented as axial stress-strain response rather than shear stress-shear strain (Figure 2.13b).

Taniguchi et al. [11] investigated $\pm 45^\circ$ carbon/epoxy laminates on a tension split-Hopkinson bar with a custom designed specimen clamping fixture and the results were presented as shear stress-shear strain response (Figure 2.13c). An increase for shear modulus and strength at an applied axial strain rate of 100s^{-1} was reported. The shear failure

strain was found to decrease under dynamic loading.

Shokrieh and Omid [28] studied the strain rate effect on the shear modulus and shear strength of unidirectional glass-epoxy, using $\pm 45^\circ$ IPS tension specimens. The tests were performed on a custom servo-hydraulic testing apparatus up to axial strain rates of 85s^{-1} . It was found that the shear strength increased with increasing strain rate (Figure 2.13d), whereas a moderate decrease was observed for the shear modulus.

It is noted that the considerations and limitations stated for the quasi-static IPS test method [26] should also be applied to the dynamic tests. With this in mind, the measured shear strength should be referred to as *apparent* or *effective* shear strength and results obtained above 5% shear strain should be interpreted with care, because there is significant fibre scissoring at these strain levels and therefore the equations derived for this method are no longer valid.

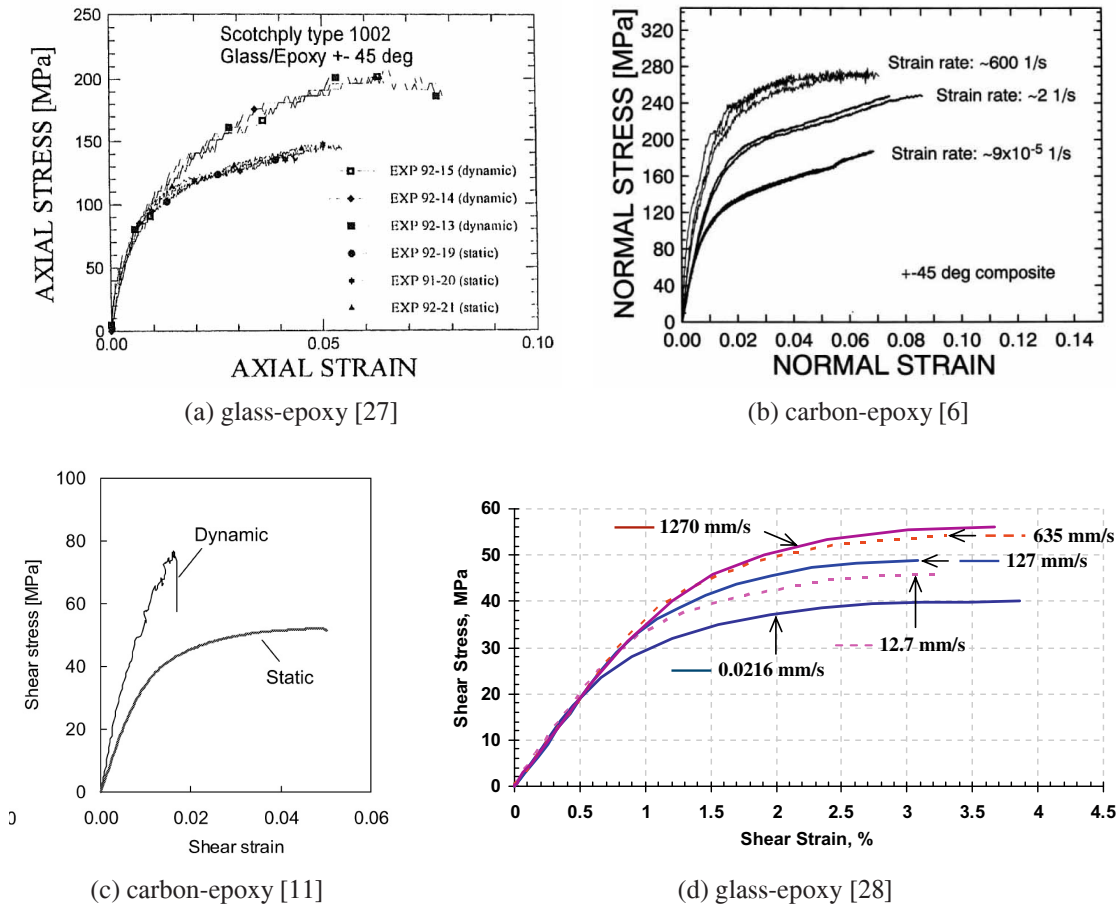


Figure 2.13: Strain rate effect on the stress-strain response of IPS type $\pm 45^\circ$ laminates.

It is further noted that results presented as axial stress-strain response can not be compared directly with those presented as shear stress-shear strain response, since according to the quasi-static IPS test method [26], the shear strain is calculated as the sum of the axial and the transverse strain components. Therefore, the shear strain cannot be determined when only the axial strain component is measured, as done in the classic split-Hopkinson bar experiment where the axial specimen strain is calculated from the bar strain signals via SHPB analysis.

Hsiao et al. [19] performed quasi-static compression tests for carbon-epoxy off-axis specimens and showed that the shear stress-shear strain response is only slightly influenced by the angle-variation. It was concluded that accurate results for the elastic shear properties and for the overall shear stress-strain response can be obtained from off-axis compression tests. The authors then tested 30° and 45° off-axis specimens on a drop tower at axial strain rates of 300s^{-1} and further performed high strain rate tests with a classic SHPB setup at strain rates of up to 1200s^{-1} . The results from the 45° off-axis specimen type is shown in Figure 2.14. A moderate increase of the shear modulus and significant increase of the shear strength was observed at higher strain rates.

For the SHPB tests, the shear modulus was not determined due to the high scatter in the initial shear stress-strain response. It was noted that the shear stress response measured from the off-axis compression specimens is likely to be higher than the true material shear response, due to the presence of normal compressive stresses on the shear plane. Therefore, a similar problem as for the IPS specimen exists, where the shear stress response is thought to be underestimated for shear strains above 1.3% [29].

In a recent study by Tsai and Sun [30], a method to solve the combined-stress problem was presented. Compression tests for glass/epoxy 15° , 30° and 45° off-axis block specimens were performed at quasi-static, medium and high strain rates. The quasi-static and medium rate specimens were tested on a hydraulic test machine. The high strain rate experiments were performed on the classic SHPB at an axial strain rate of 600s^{-1} . The measured axial strength was divided into a transverse normal and shear strength com-

ponent. In the $\sigma_{22} - \tau_{12}$ diagram thus obtained, a value for the shear strength at zero transverse compressive stress was extrapolated, which proved to be lower than the shear strength measured under the presence of normal compressive stress (Figure 2.15). A pronounced strain rate effect was observed for the "extrapolated" shear strength.

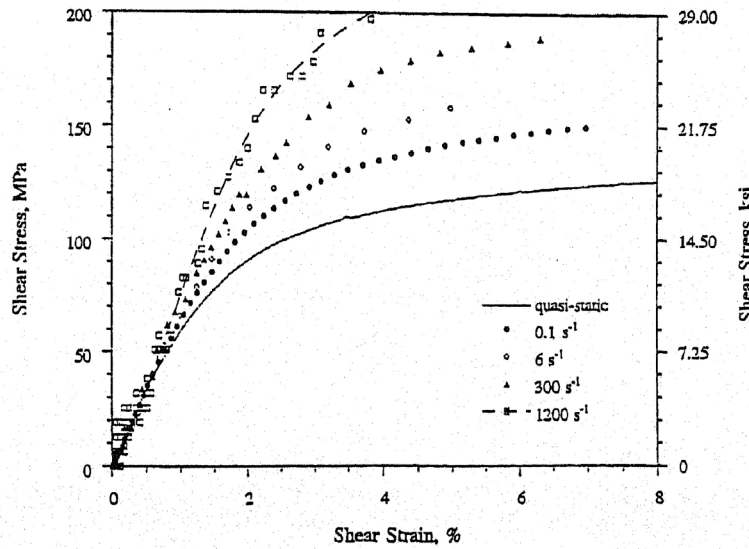


Figure 2.14: Shear stress-strain response of carbon-epoxy at different strain rates obtained from 45° off-axis specimens [19].

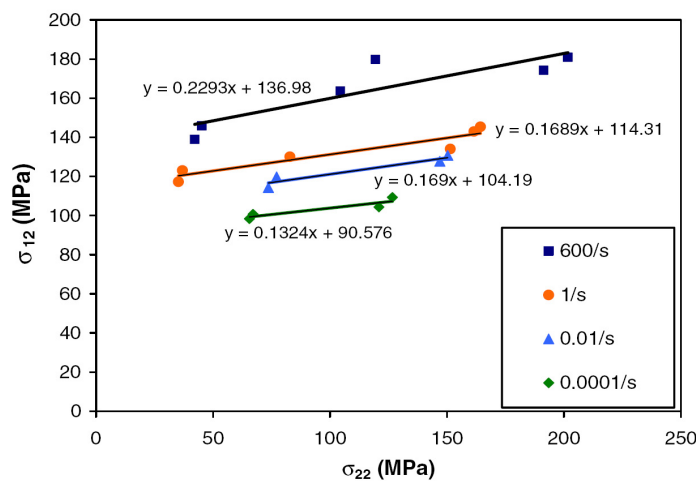


Figure 2.15: Pure shear strength extrapolation from off-axis compression tests in the $\sigma_{22} - \tau_{12}$ diagram [30] ($\sigma_{22} < 0$).

2.4.2 Out-of-Plane Shear

The out-of-plane or interlaminar shear strength (ILSS) of unidirectional carbon-epoxy and carbon-PEEK was investigated by Dong and Harding [31]. For both material systems, $0^\circ/0^\circ$ and $\pm 45^\circ$ laminates were tested, whereas for carbon-PEEK the ILSS response of a $0^\circ/90^\circ$ laminate was determined as well. A single-lap specimen was proposed, which can be tested under dynamic loading on a classic SHPB if some modifications are made to the ends of the incident- and transmission-bar (Figure 2.16). FE analysis was used to obtain an optimal specimen design with an approximately even shear stress distribution along the failure plane and minimal normal stress components at both ends of the lap-joint. Dynamic ILSS at an axial strain rate of 1000s^{-1} was compared with quasi-static strength values using the same specimen type. The results obtained by Dong and Harding are reproduced in Figure 2.17.

For carbon-epoxy, the interlaminar shear strength of both laminates was found to increase

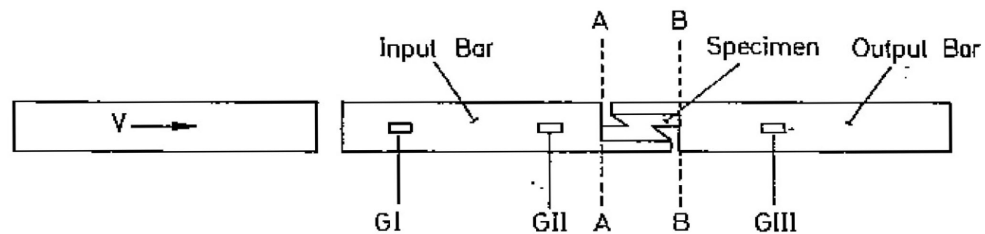


Figure 2.16: Dynamic interlaminar shear strength test setup for SHPB [31].

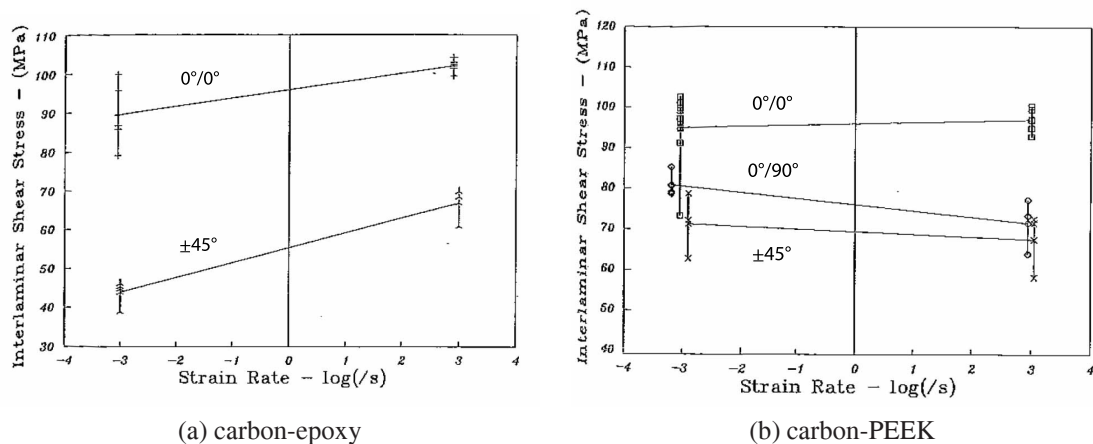


Figure 2.17: Interlaminar shear strength for carbon-epoxy and carbon-PEEK laminates at quasi-static and high strain rates [31].

with increasing strain rate, with a more pronounced rate effect for the $\pm 45^\circ$ laminate. It is noted that the quasi-static ILSS value for the $0^\circ/0^\circ$ carbon-epoxy specimens was twice that of the $\pm 45^\circ$ carbon-epoxy specimens.

For carbon-PEEK, no rate effect was reported for the $0^\circ/0^\circ$ laminate, and a decrease of ILSS with increasing strain rate was observed for the $\pm 45^\circ$ and $0^\circ/90^\circ$ specimen types. In a recent study, Naik et al. [32] investigated the strain rate effect on the interlaminar shear properties for plain-weave carbon-epoxy and plain-weave glass-epoxy composites. Dynamic tests for a strain rate range of 500s^{-1} to 1000s^{-1} were performed on a torsion split-Hopkinson bar, using a thin-walled tube specimen similar to the one used by Gilat et al. [7]. In addition to the thin-walled torsion specimen, single-lap shear tests were performed for the glass-epoxy laminate on a conventional SHPB, using the specimen configuration proposed by Dong and Harding [31]. The interlaminar shear modulus and strength was reported to increase for both material systems under dynamic loading. Naik et al. [32] noted that the similarity of the results for both material systems indicates that the interlaminar shear behaviour is primarily influenced by the matrix, which was the same for both cases.

2.5 Rate Effect on Interlaminar Fracture Toughness

A thorough investigation of strain rate effects on the interlaminar mode I, mode II and mixed mode I+II fracture toughness for unidirectional carbon-epoxy and carbon-PEEK was carried out by Blackman et al. [12, 33, 34].

For mode I loading, a standard DCB specimen was tested at test speeds ranging from $2\text{mm}\cdot\text{min}^{-1}$ (quasi-static) up to $20\text{m}\cdot\text{s}^{-1}$. All quasi-static and intermediate rate tests were performed on a conventional screw driven test machine. The crack length was measured using a traveling microscope mounted in front of the specimen and the crosshead displacement of the test machine was used for measuring the crack opening displacement of the DCB specimen. For test rates above $1.67 \times 10^{-2}\text{m}\cdot\text{s}^{-1}$ and up to $20\text{m}\cdot\text{s}^{-1}$ a servo-hydraulic load frame was used and the specimen was attached to a special rig to account

for dynamic effects in the load chain (Figure 2.18). For the high rate tests, the crack length and crack opening displacement were obtained by evaluation of high speed photography, since it was found that the crosshead displacement significantly underestimates the crack opening displacement.

The equation used to calculate G_{Ic} was modified to enable the calculation of the fracture toughness from the crack opening displacement, crack length and the elastic modulus in fibre direction, rather than from the measured load.

For the carbon-PEEK specimens, stable crack growth was observed for quasi-static test rates whereas for test speeds above $8.3 \times 10^{-5} \text{ m.s}^{-1}$ the crack advanced in a "stick-slip" manner with repeating intervals of stable crack growth followed by rapid crack growth

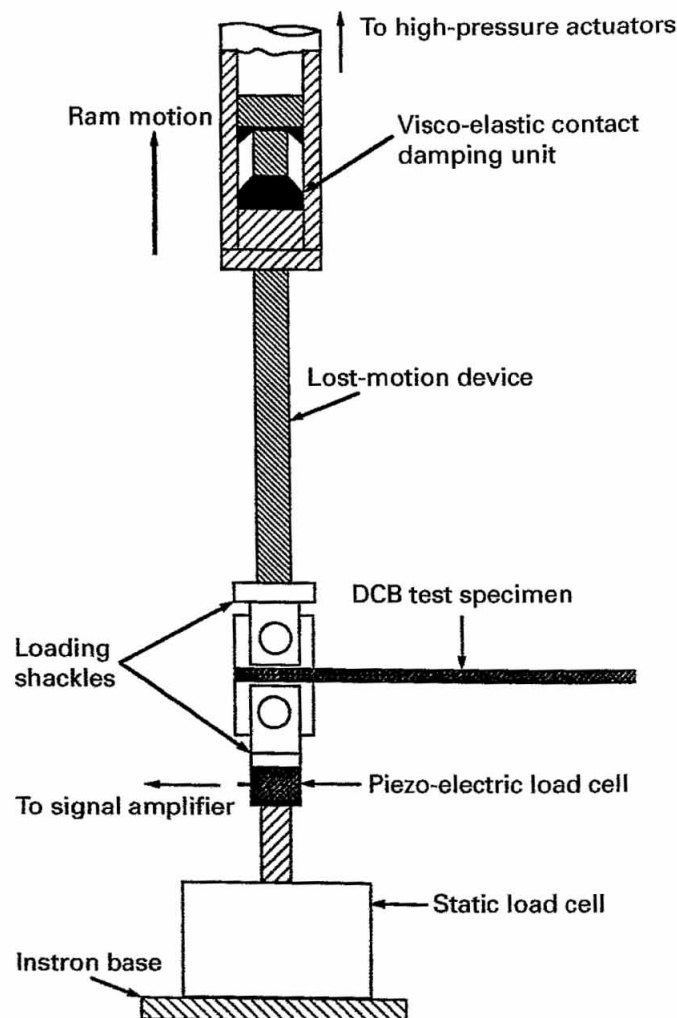


Figure 2.18: DCB high speed test rig [12].

and crack arrest. For the higher test rates, strong oscillations were observed in the load response, which was the reason for deriving a load-independent relation for G_{Ic} .

In the case of carbon-epoxy, relatively stable crack growth was observed over the entire range of test rates with strong oscillations occurring in the load-response for the high speed tests.

The results of the mode I fracture toughness tests for the two unidirectional laminates reported in [12] is reproduced in Figure 2.19, where in case of the carbon-PEEK laminate the G_{Ic} for crack arrest is also shown. A modest reduction of the initiation value of G_{Ic} was found to occur for PEEK-laminate at rates above 5m.s^{-1} whereas the initiation fracture toughness was found to be insensitive to the loading rate in the case of carbon-epoxy. It was noted by the authors that the equation derived for G_{Ic} does not consider dynamic effects. Corrections for dynamic effects, such as the influence of kinetic energy, were therefore investigated in a separate study [33]. It was concluded however that the consideration of such dynamic effects would result in only a small reduction of G_{Ic} and may therefore be neglected.

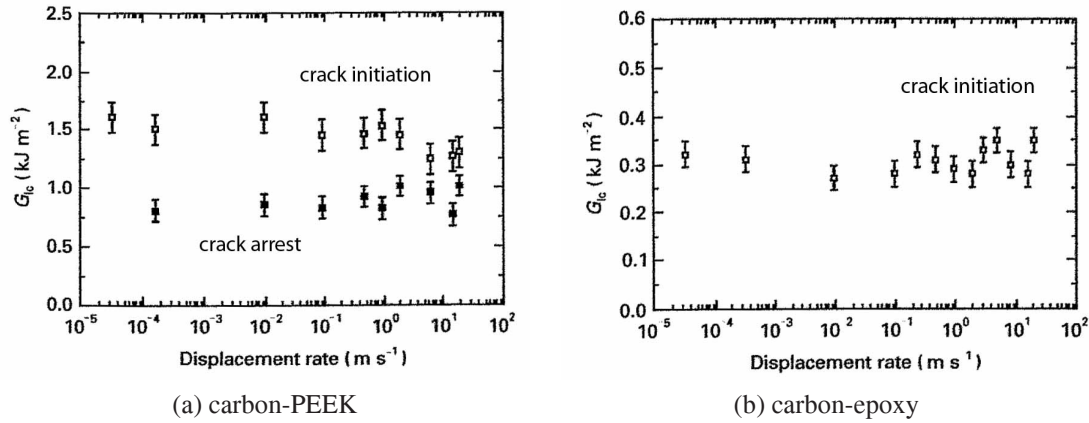


Figure 2.19: Loading rate effect on the mode I interlaminar fracture toughness [12].

The authors also pointed out, that the conflicting results reported in earlier studies [35, 36, 37, 38], where both increasing and decreasing trends were reported, may have been caused by:

1. Deriving the crack opening displacement of the DCB specimen from the test

machine crosshead displacement instead of measuring the true displacement directly on the specimen.

2. Calculating the fracture toughness from the load response, which can be highly obscured by dynamic effects at high test speeds.

The rate effect on mode II and mixed-mode I+II fracture toughness [34] was investigated on a specimen similar to the mode I DCB specimen. This time however, the load was introduced on one cantilever beam only and depending on the direction of the applied load, the specimen was referred to as either end-loaded-split (ELS) mode II specimen or fixed-ratio mixed-mode (FRMM) specimen with a mode I/II ratio of 1.33 (Figure 2.20). The tests were performed on a MTS test machine for quasi-static rates, while a hydraulic load frame, together with the high speed test rig and photography system, was used for the high rate tests. The specimens were loaded at speeds of up to $5\text{ m} \cdot \text{s}^{-1}$.

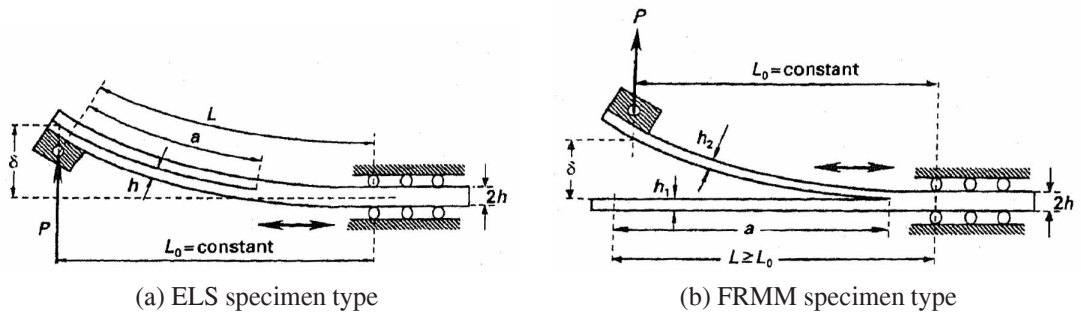


Figure 2.20: Specimens used for mode II and mixed mode I+II interlaminar fracture toughness [34].

As done for mode I, load-independent equations were derived for the mode II fracture toughness G_{IIc} and for the mixed-mode fracture toughness values G_{IIc}^{mix} and G_{IIc}^{mix} .

For mode II loading of carbon-PEEK specimens, unstable crack growth was observed for all but the quasi-static rate. No effect of loading rate on the initiation value of G_{IIc} was observed for the low and intermediate rates while some reduction occurred at the highest rates (Figure 2.21).

In the case of the carbon-epoxy specimen, stable crack growth was observed for low and intermediate rates, while the crack advanced in an unstable manner at higher loading

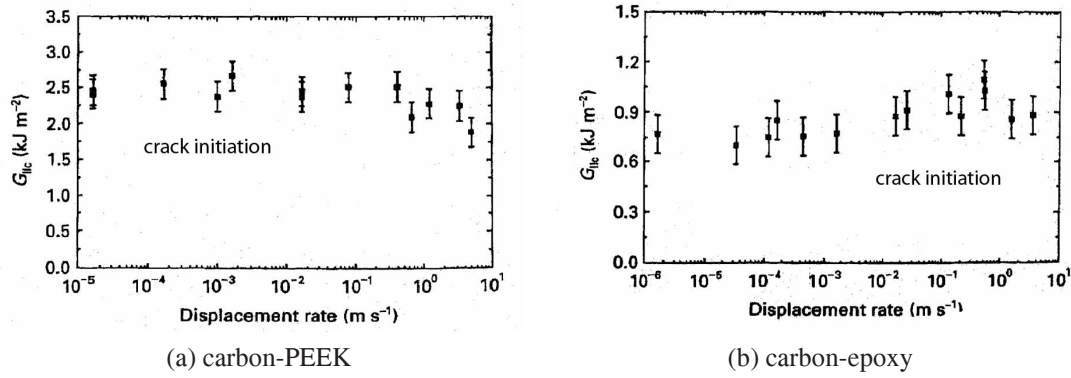


Figure 2.21: Loading rate effect on mode II interlaminar fracture toughness [34].

rates. The initiation value of G_{IIc} was found to be load rate independent for carbon-epoxy over the entire range of test speeds.

The same trends were observed for the FRMM tests with a small reduction of G_{IIc}^{mix} and G_{IIc}^{mix} for PEEK specimens at the highest rates while no rate effect was observed in the case of the carbon-epoxy specimens (Figure 2.22).

Sun and Han [39] adopted a different approach for the investigation of the interlaminar mode I fracture toughness. Using the wedge-insertion fracture (WIF) method, they performed quasi-static and high speed tests with wedge-loaded compact tension (WLCT) specimens made of unidirectional Hexcel S2/8552 glass-epoxy and Cytec Fiberite IM7/977-3 carbon-epoxy composites. The quasi-static tests were performed on a standard MTS load frame at a test speed of $0.6\text{mm}\cdot\text{min}^{-1}$. For the high speed experiment a split-

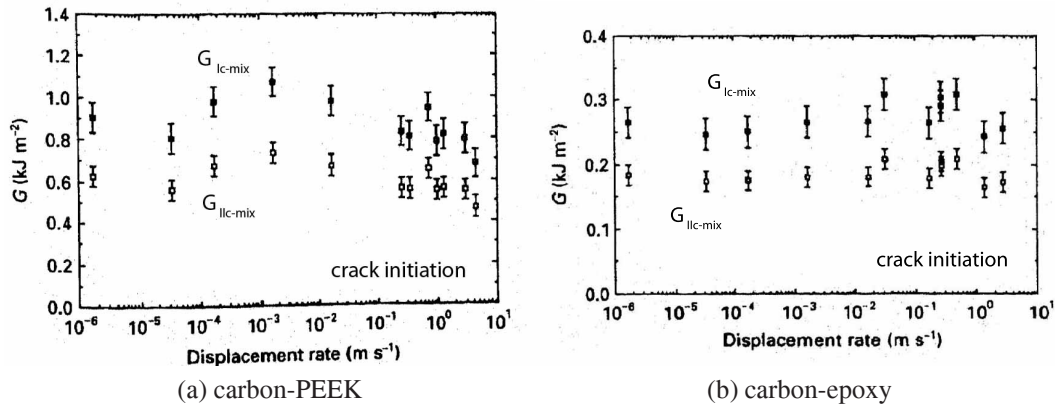


Figure 2.22: Loading rate effect on mixed mode I+II interlaminar fracture toughness [34].

Hopkinson pressure bar with modified incident-bar was used. In both cases crack growth was measured with crack propagation gauges mounted on the side of the specimen. The dynamic test setup and the WLCT specimen used by Sun and Han [39] is illustrated in Figure 2.23. A combined experimental results/finite element analysis approach [40] was used to calculate the static and dynamic initiation and propagation values of the mode I interlaminar fracture toughness. For the glass-epoxy composite, severe fibre-bridging was observed and therefore only the static and dynamic initiation value was considered. As per ASTM D 5528-01 [41], fibre-bridging is considered an artifact of the DCB test for unidirectional composites. Since delaminations in multidirectional laminates generally occur between plies of dissimilar orientation, propagation values of the fracture toughness under the presence of fibre-bridging are considered questionable.

Crack speeds in the range of 450m.s^{-1} to 950m.s^{-1} were observed for the carbon-epoxy specimen under dynamic loading, with the highest crack speeds measured just after crack initiation. For comparison, crack speeds of less than 0.1m.s^{-1} were recorded for the quasi-static tests. The dynamic fracture toughness at crack propagation was found to be relatively constant, despite the variation in crack speed. Moreover, the dynamic fracture

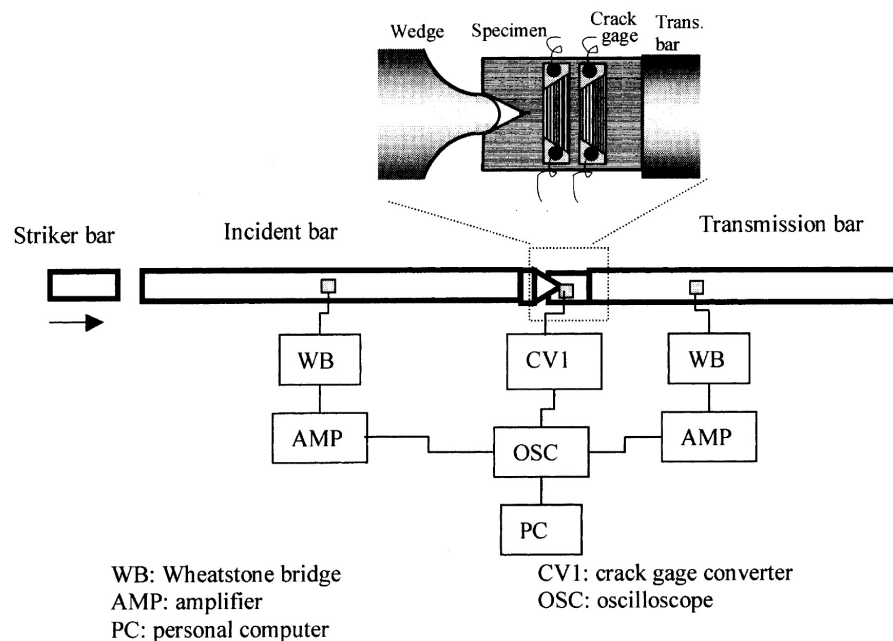


Figure 2.23: Dynamic wedge-insertion fracture (WIF) test configuration [39].

toughness propagation value was approximately equal to both, static and dynamic initiation fracture toughness (Figure 2.24).

It was therefore concluded that the fast crack propagation, resulting from high speed loading of the mode I WLCT specimen, has no effect on the fracture toughness of carbon-epoxy at crack speeds of up to 1000m.s^{-1} .

In the case of the glass-epoxy specimen an increase of initiation fracture toughness was found for dynamic loading. It was noted that this might be partially due to the fibre-bridging observed for this material system, but no further conclusions were presented.

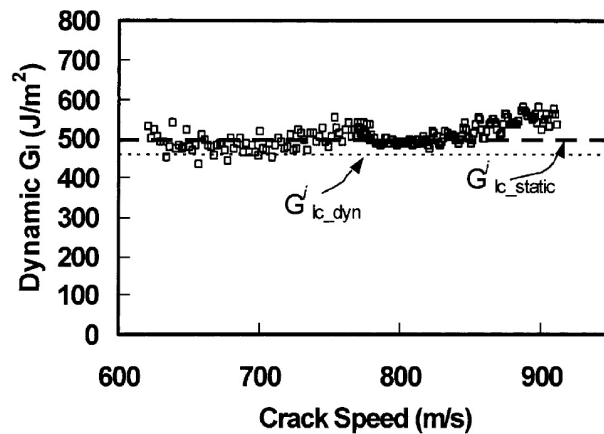


Figure 2.24: Static and dynamic mode I interlaminar fracture toughness for carbon-epoxy [39].

2.6 Comparison of Normalised Literature Results

In Sections 2.1 - 2.5, some previous work in the field of dynamic material characterisation of polymer composites, with an emphasis on carbon-epoxy material systems, was reviewed. The strain rate effect on the in-plane and out-of-plane tensile, compressive and shear behaviour was presented in Section 2.2, 2.3 and 2.4, respectively. The experimental methods and setups were briefly described and selected quasi-static and dynamic stress-strain curves were reproduced from the respective references.

While the detailed review of the previous work provides a good overview of the common

experimental techniques used for dynamic material characterisation of polymer composites, a direct comparison of the presented results is difficult since different material system, experimental methods and specimen types were used.

In this section, the tensile, compressive and shear modulus and strength properties are compared individually by normalising the literature results with respect to the reported quasi-static property. This way it is possible to visualise trends in the strain rate behaviour and figure out whether or not a general consensus exists between the individual studies. The normalised longitudinal, transverse and interlaminar tensile moduli and strengths are shown in Figure 2.25. The individual comparison of the normalised in-plane and interlaminar compressive and shear properties is shown in Figures 2.26 and 2.27, respectively.

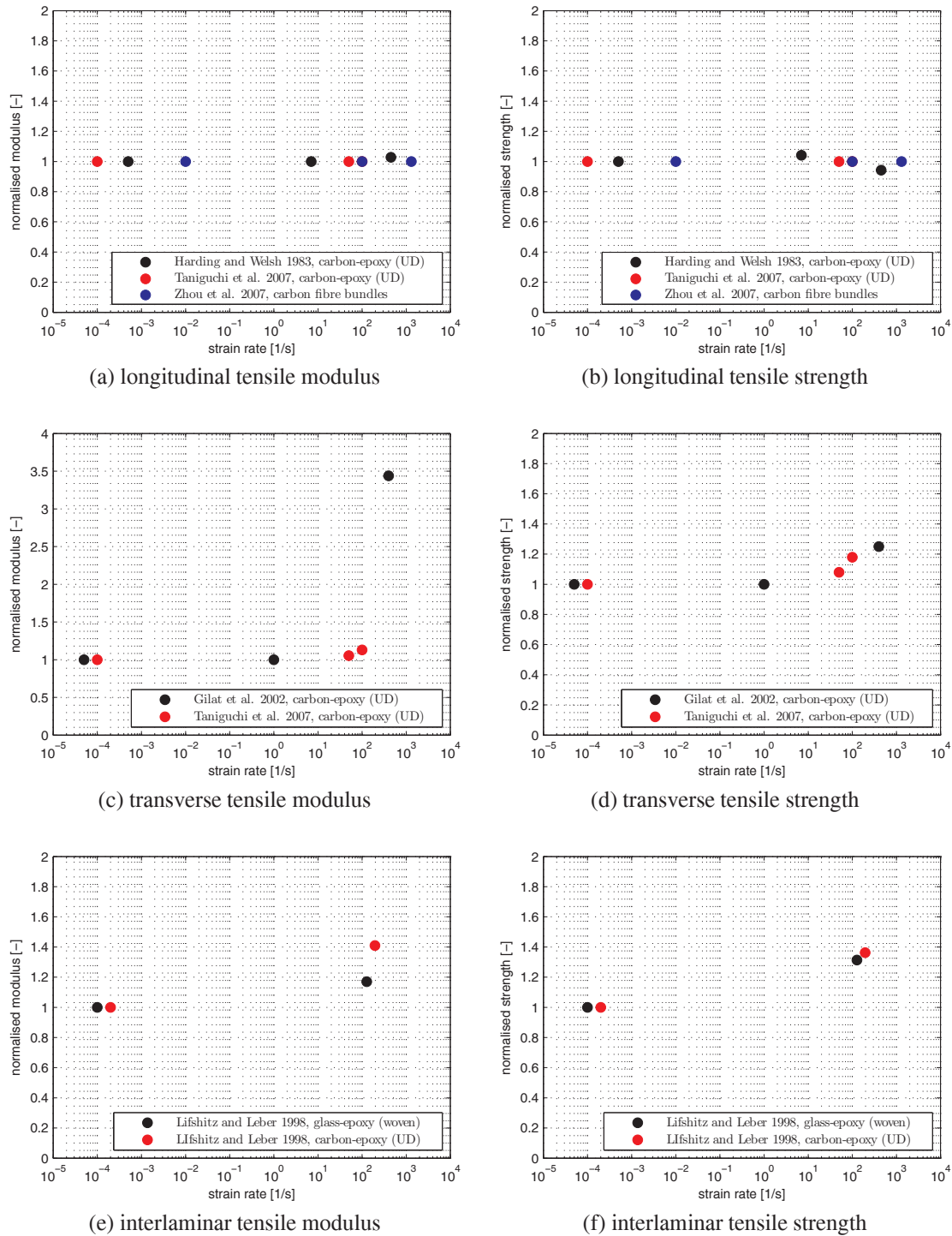
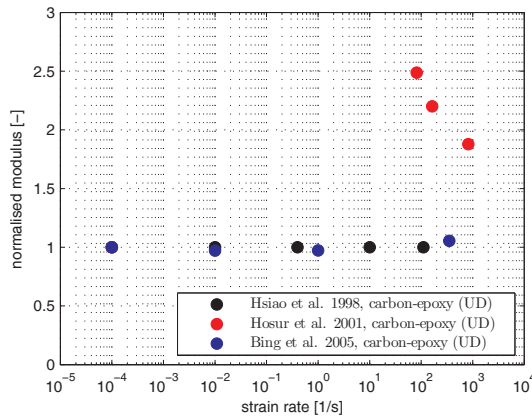
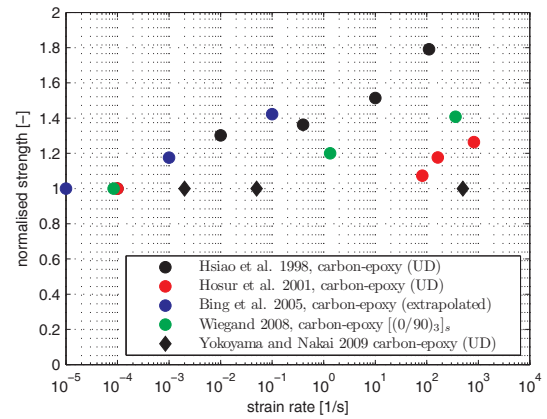


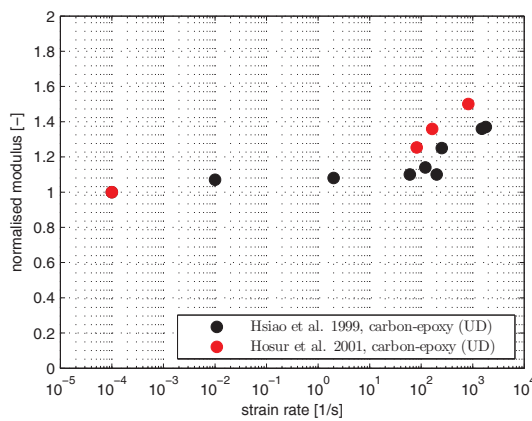
Figure 2.25: Comparison of literature results regarding the strain rate effect on the tensile properties of polymer composites.



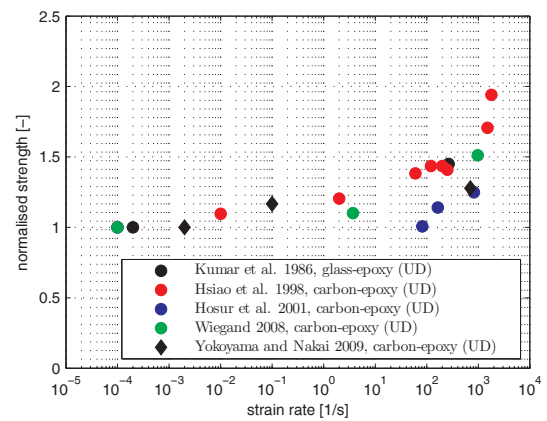
(a) longitudinal compressive modulus



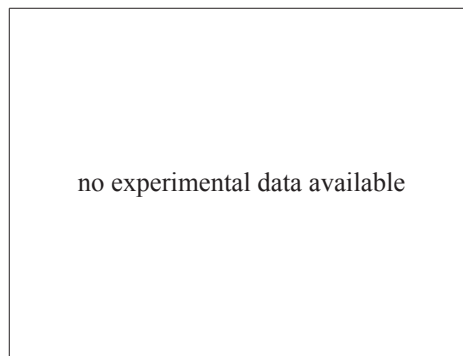
(b) longitudinal compressive strength



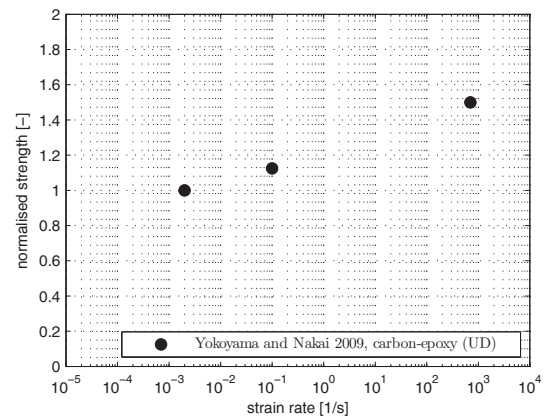
(c) transverse compressive modulus



(d) transverse compressive strength



(e) interlaminar compressive modulus



(f) interlaminar compressive strength

Figure 2.26: Comparison of literature results regarding the strain rate effect on the compressive properties of polymer composites.

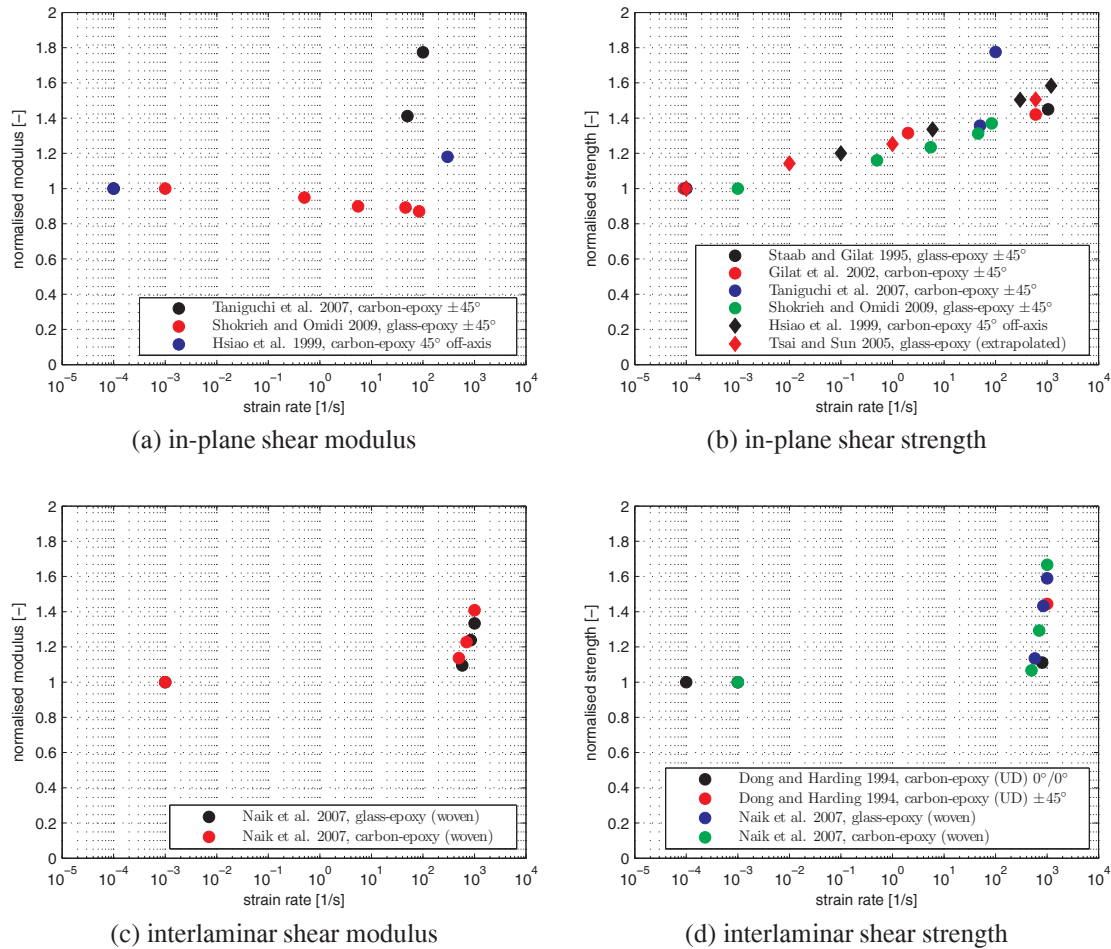


Figure 2.27: Comparison of literature results regarding the strain rate effect on the shear properties of polymer composites.

2.7 Constituent vs. Composite Strain Rate Behaviour

A comparison of the strain rate effect on the stress-strain response of neat epoxy resin and unidirectional carbon-epoxy composite at similar strain rates is shown for tension, compression and shear loading in Figures 2.28-2.30. The diagrams were obtained by digitising selected stress-strain curves from the earlier experimental studies, reviewed in the previous Sections. It is noted that the stress-strain curves for resin compressive loading, in Figure 2.29, were truncated at 20% strain.

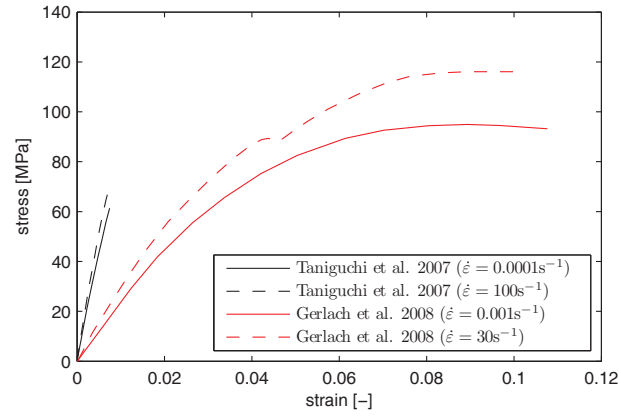


Figure 2.28: Strain rate effect on the transverse tensile stress-strain response of composite - Taniguchi et al. [11], and on the stress-strain response of neat epoxy resin - Gerlach et al. [8].

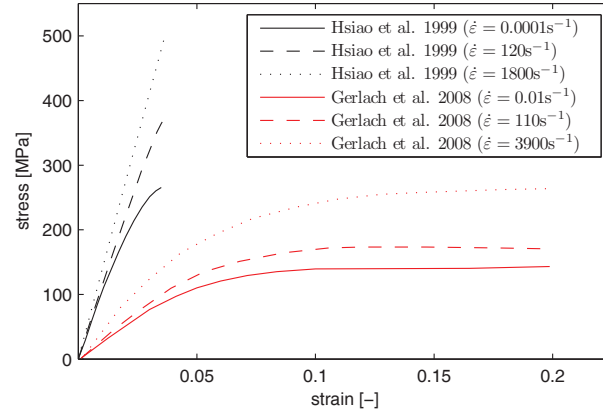


Figure 2.29: Strain rate effect on the transverse compressive stress-strain response of composite - Hsiao et al. [19], and on the stress-strain response of neat epoxy resin - Gerlach et al. [8].

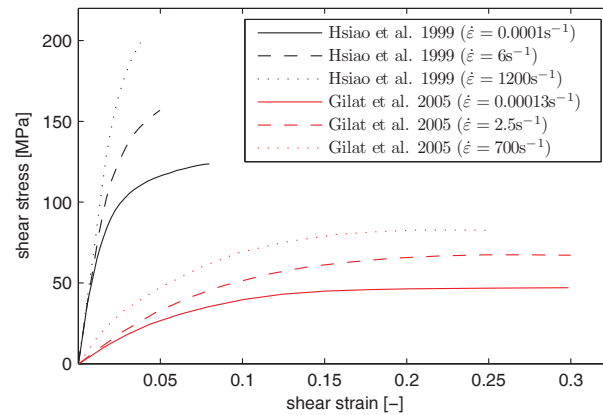


Figure 2.30: Strain rate effect on the in-plane shear stress-strain response of composite - Hsiao et al. [19], and on the shear stress-strain response of neat epoxy resin - Gilat et al. [7].

The difference in the stress-strain behaviour of neat resin and unidirectional composite under quasi-static and dynamic loading can be summarised as follows:

1. The modulus and strength of the composite are significantly higher than that of neat resin, with exception of the tensile strength, which is lower.
2. The failure strain of the composite is significantly lower than that of neat resin in all cases.
3. The stress-strain response of neat resin is ductile for static and dynamic loading in all cases, whereas a trend toward brittle material behaviour at high strain rates can be observed for the composite (it is noted that the quasi-static stress-strain response of the composite under quasi-static tensile loading is already linear).

Despite the significant differences in the constitutive response of neat resin and unidirectional composite, a similar strain rate effect appears to exist for the normalised strength, as shown in Figures 2.31 and 2.32 for tension, compression and shear. This indicates that the rate dependent behaviour of the composite results exclusively from the rate dependent characteristics of the resin.

It is noted that the neat resin compressive strength, used in Figure 2.31b), was obtained from the plateau region of the compressive neat resin stress-strain response, shown in

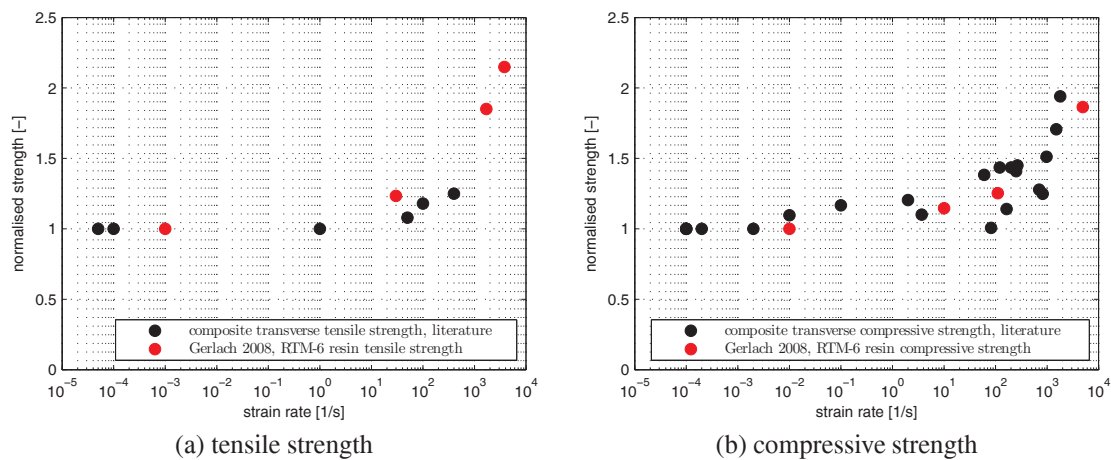


Figure 2.31: Comparison of the strain rate effect on the tensile and compressive strengths properties of composite and resin.

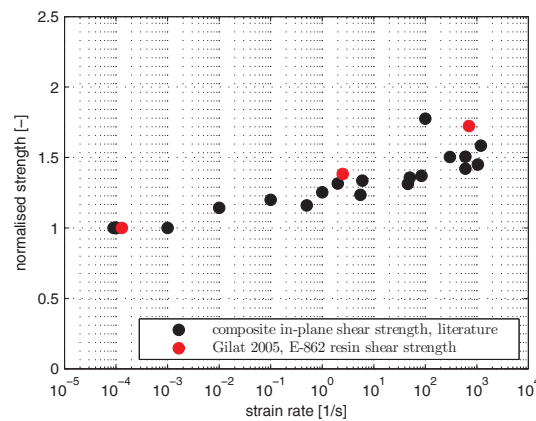


Figure 2.32: Comparison of the strain rate effect on the shear strength of composite and resin.

Figure 2.5, which corresponds to the compressive yield strength reported by Gerlach et al. [8].

The actual failure stresses of the neat resin compressive response, shown in Figure 2.5, are not likely to be reached in a composite due to the high failure strain of about 50%. As noted by Gerlach et al. [8], the compressive resin samples fail under a mixed tension, compression and shear stress state rather than under uniaxial compression.

2.8 Summary and Conclusions

Constituents

The insensitivity of the tensile response of carbon fibres subjected to different strain rates was proven experimentally by Zhou et al. [5] (Section 2.1.1) and indirectly through the tension tests on 0° unidirectional laminates performed by Harding and Welsh [10] and Taniguchi et al. [11] (Section 2.2.1).

Therefore, since the material system to be investigated in this study is a unidirectional carbon-epoxy prepreg (Hexcel IM7-8552), an individual study of the rate effect for the fibre constituent is not necessary.

The strain rate effect on the resin constituent was reviewed in Sections 2.1.2-2.1.4 and further compared with unidirectional composites in Section 2.7, where it was shown that

significant differences exist between the constitutive response of resin and composite for both quasi-static and dynamic loading.

It was however also demonstrated in Section 2.7 that the strain rate effect on the tensile, compressive and shear strength of neat epoxy resin is approximately equal to the strain rate effect on the transverse tensile, transverse compressive and in-plane shear strength of a unidirectional carbon-epoxy laminate. On the other hand it was shown by Taniguchi et al. [11] that the failure mode for a unidirectional composite loaded in transverse tension can be caused by fibre-matrix debonding, rather than by failure of the matrix.

It is therefore concluded that the results obtained from a separate strain rate investigation of neat resin can not be used for the development of a constitutive material model on a meso-mechanical level. It is further noted that the Hexcel 8552 neat resin is a company proprietary product which cannot be purchased.

Composite Tensile Properties

From the review presented in Section 2.2.1 and further from Figure 2.25a and 2.25b in Section 2.6 it is clear that the longitudinal tensile properties of unidirectional carbon-epoxy laminates are not strain rate sensitive.

Moderate rate effects were however reported for the transverse and interlaminar tensile properties (Figure 2.25c-2.25f). The significant increase of the transverse modulus and the change from linear to bi-linear stress-strain behaviour with increasing strain rate (Figure 2.8a), reported by Gilat et al. [6] and other researchers such as Li and Lambros [42], should be treated with care. It is likely that this bi-linear response is an artifact of the post-processing of the SHPB test results and may be caused by errors in the time shifting of the bar strain waves, as shown in a very recent study by Mohr et al. [43]. If most of the specimen deformation occurs during the rise time of the classic near rectangular wave, small time shifting errors may cause significant errors in the calculated dynamic stress-strain response, in particular if the specimen is not in a state of dynamic equilibrium.

A further investigation of the transverse and interlaminar tensile properties of unidirec-

tional composites at very high strain rates above 1000s^{-1} would be necessary to see whether the strain rate effect reported for the tensile response of neat epoxy resin [8] applies also to the tensile response of the composite (see Figure 2.31a). A tension split-Hopkinson bar is not available and therefore a further investigation of the strain rate effect on the tensile properties of unidirectional carbon-epoxy could not be considered in the present study.

Composite Compressive Properties

The compressive properties of unidirectional polymer composites experience a significant strain rate effect (Figure 2.26).

With regard to the longitudinal properties, conflicting results were however reported for the longitudinal modulus (Figure 2.26a) and for the longitudinal strength (Figure 2.26b). Further research is therefore required to clearly determine the strain rate effect on the longitudinal compressive response.

For the transverse compressive properties, the experimental results reported in earlier studies are in better agreement (Figure 2.26c and 2.26d).

Only one earlier study was found regarding the interlaminar compression properties. The strain rate effect on the out-of-plane compressive response appears to be similar to the strain rate effects observed for transverse compression.

For the carbon-epoxy material system IM7-8552 used in the present study, a similar strain rate dependence of the in-plane compressive properties can be expected and dynamic experiments will therefore be carried out for the longitudinal and transverse directions. Out-of-plane dynamic compression tests will not be considered because of the transversely isotropic material behaviour.

Composite Shear Properties

From Section 2.4 and from Figure 2.27 in Section 2.7 it is clear that the shear properties of carbon-epoxy laminates are strain rate sensitive. The two main approaches to study

the shear rate dependence are the tension IPS method, similar to the test standard ASTM D 3518 [26] and the off-axis compression test.

It was shown in various earlier experimental studies that the IPS method can be successfully adopted to the high strain rate environment [6, 11, 27, 28]. However, the main disadvantage of the IPS method is that it is now commonly accepted that the shear strength obtained via the IPS method can not be considered a true material property [26].

The off-axis compression test seems to be better suited to obtain the in-plane shear stress-strain behaviour [19] and the pure in-plane shear strength [30] under dynamic loading since the required end-loaded specimen is ideally suited for the split-Hopkinson pressure bar and can be used without any modifications to the bar ends.

A similar strain rate effect can be expected for the in-plane shear properties of IM7-8552 and dynamic experiments based on the off-axis compression test method will therefore be performed. Improved dynamic in-plane shear experiments are needed to clarify the strain rate effect on the in-plane shear modulus since conflicting results were found in the literature (Figure 2.27a).

Only minor strain rate effects were reported by Dong and Harding [31] for the interlaminar shear strength. Naik et al. [32] observed a rate dependence for both interlaminar shear modulus and strength (Figure 2.27c and 2.27d). It is likely that the strain rate effect on the interlaminar shear response of the unidirectional material system investigated in the present study is similar to the rate effect observed for the in-plane shear response due to the transverse isotropy, and a further investigation of the interlaminar shear response will not be considered.

Interlaminar Fracture Toughness

Based on the review of earlier experimental studies with regard to the strain rate effect on the interlaminar fracture toughness of unidirectional polymer composites (Section 2.5), it is concluded that this material property is not strain rate sensitive. A further investigation is therefore not considered in the present study.

Chapter 3

Experimental Methods

3.1 The Classic SHPB Experiment

The Hopkinson bar technique dates back to the pioneering work of John Hopkinson [44] and Bertram Hopkinson [45, 46] and was therefore named after them.

Davies [47] published a critical review of the Hopkinson bar technique and was able to record the propagation of the elastic wave in the bar for the first time, using a parallel-plate condenser and oscilloscope.

Kolsky [48] modified the classic Hopkinson bar experiment by using two elastic bars instead of one, and therefore introduced the split-Hopkinson pressure bar (SHPB) technique. Due to this major contribution, the SHPB technique is also referred to as Kolsky-bar technique in some publications. Kolsky [48] also laid down the foundations for the classic SHPB analysis, by which the dynamic stress-strain response of the sample, which is placed between the two bars, is calculated from the elastic waves measured in the two bars.

Harding et al. [49] and Lindholm & Yeakley [50] introduced the tension Hopkinson bar experiment and Baker & Yew [51] then contributed the torsion version of the SHPB technique.

3.1.1 Setup and Principles of the Classic SHPB Experiment

A classic SHPB setup, as shown in Figure 3.1, consists of the following components:

1. Three cylindrical bars called striker-, incident- and transmission-bar made of the same material and diameter.

All bar ends must be machined orthogonal with high accuracy to assure an optimal contact between striker- and incident-bar; incident-bar and specimen and between specimen and transmission-bar.

The incident- and transmission-bar should be sufficiently long, with respect to the striker-bar, to allow an interference-free propagation and recording of the bar-strain waves.

2. A bearing- and alignment-fixture with minimal friction between bearings and bars to ensure unhindered propagation of the strain pulses in the bars.
3. A gas gun to fire the striker- onto the incident-bar.
4. Strain gauges mounted on the incident- and transmission-bar to measure the strain pulses. The strain gauges must be mounted sufficiently away from the specimen/bar interfaces to ensure that a uniform strain-state is measured.
5. A sensor measuring the velocity of the striker-bar upon impact on the incident-bar.
6. The specimen, positioned between the incident- and transmission-bar.
7. A high speed amplifier system to amplify the bar strain wave signals. The high speed amplifier generally also completes the Wheatstone-bridge-circuit of the incident- and transmission-bar strain gauges.
8. A data acquisition system to record the bar strain wave signals and striker-bar impact velocity. It generally consists of an oscilloscope, connected to a PC for further data processing.

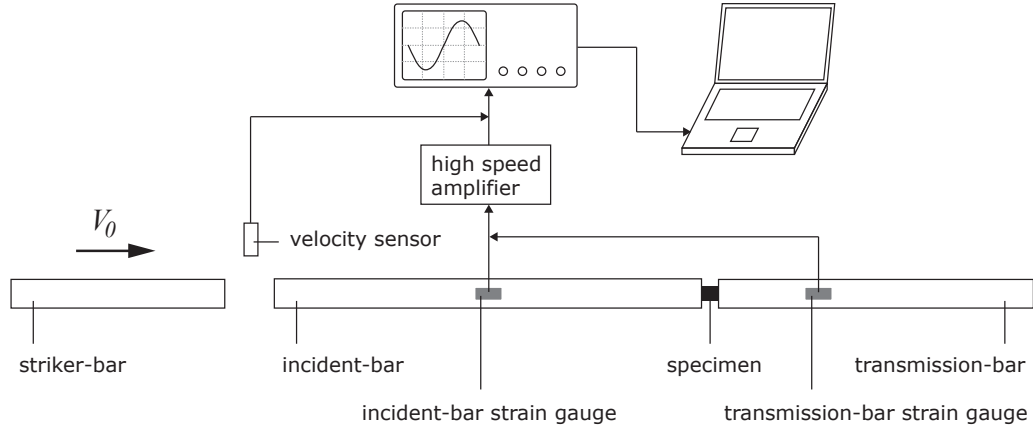


Figure 3.1: Components of classic SHPB Setup (Gas gun and alignment fixture not shown).

The operating principle of the SHPB can be summarised by the following steps (see also Figure 3.2):

1. Creation of incident-wave ϵ_I

The striker-bar is fired from the gas gun and impacts the free end of the incident-bar at a velocity V_0 . A longitudinal elastic compressive strain pulse is created which propagates along the incident-bar toward the incident-bar/specimen interface and is measured by the incident-bar strain gage. The pulse is referred to as *incident-wave* and has the following characteristics:

- The shape is nearly rectangular.
- The geometric length Λ is twice the striker-bar length $l_{striker}$.
- The pulse duration T is proportional to the striker-bar length $l_{striker}$ and the elastic wave velocity of the bar material c_b .

$$T = \frac{2l_{striker}}{c_b} \quad (3.1)$$

- The amplitude ϵ_I^{max} is proportional to the striker-bar velocity V_0 and the elastic wave

velocity c_b .

$$\epsilon_I^{max} = \frac{V_0}{2c_b} \quad (3.2)$$

2. Creation of reflected-wave ϵ_R

Once the incident-wave has reached the incident-bar/specimen interface, a complex reflection occurs due to the mechanical impedance change at the bar/specimen interface. The wave which propagates back toward the free end of the incident-bar as a longitudinal elastic tensile wave is referred to as *reflected-wave* and is again measured by the incident-bar strain gauge.

3. Stress-wave reverberation in specimen

The portion of the wave which is not reflected at the incident-bar/specimen surface is transmitted into the specimen, causing a complex reverberation within the specimen between the surfaces contacting the bars until the dynamic compressive strength of the specimen is reached. Initially the specimen will be non-uniformly loaded but after a certain time, dynamic equilibrium will be obtained. The dynamic equilibrium is a key assumption of the SHPB experiment and will be discussed further in Sections 3.1.2 and 3.2.

4. Creation of transmitted-wave ϵ_T

The portion of the incident-wave which is transmitted through the specimen/transmission-bar surface is referred to as *transmitted-wave* and is measured by the transmission-bar strain gauge. Whereas the incident- and reflected-wave contain high frequency oscillations at the beginning of the pulse duration (Pochhammer Modes, named after Pochhammer [52], who studied the 3D longitudinal stress wave propagation in an infinite cylindrical bar), the shape of the transmitted-wave is relatively smooth which is thought to be the result of damping occurring within the specimen.

From the three bar strain waves, the dynamic mechanical material response of the specimen can be calculated by the SHPB-Analysis presented in Section 3.1.2.

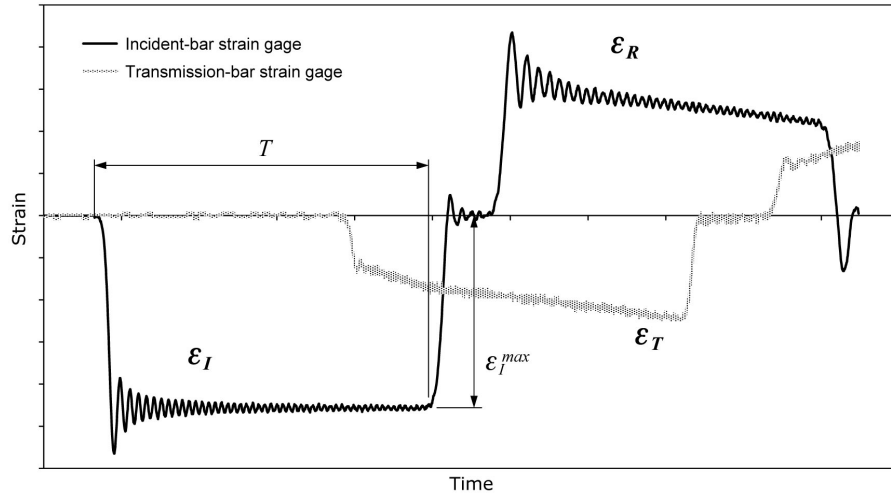


Figure 3.2: Incident-, reflected- and transmitted-wave of classic SHPB experiment (from elastic FE simulation using steel bars and an aluminium specimen).

The wave propagation in the SHPB bars can be illustrated conveniently in the *Lagrange-diagram* shown in Figure 3.3.

At t_0 , the striker-bar impacts the free end of the incident-bar and generates a compressive wave which advances with the elastic wave velocity c_b into the striker- and incident-bar. In the incident-bar, the compressive wave travels undisturbed until it reaches the specimen.

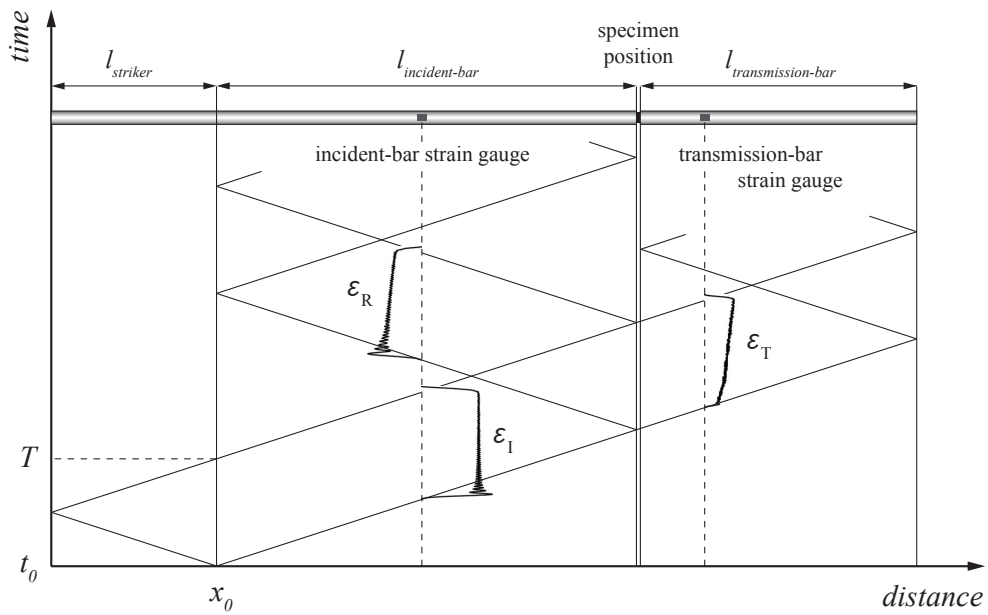


Figure 3.3: Lagrange diagram for FEUP SHPB W18.

In the striker-bar, the compressive wave is reflected as a tensile wave at the free end and travels back toward the incident-bar while canceling the compressive wave. At the time T , the tensile wave front reaches the striker-bar/incident-bar interface and both bars separate. Hence the pulse duration T , defined with Equation (3.1).

The Lagrange-diagram also defines the position of the bar strain gauges for an interference-free recording of the bar strain waves ϵ_I , ϵ_R and ϵ_T . If the incident-bar gauge is positioned too close to the specimen interface, ϵ_I can not fully be recorded due to an interference with ϵ_R . If the gauge is positioned too close to the striker-bar interface, ϵ_R can not fully be recorded due to an interference with the reflection of ϵ_R , occurring at the free end of the incident-bar.

It is noted that an interference-free recording of the bar strain waves is necessary only for the classic one-gauge setup, as used in the present study. Details regarding advanced two-gauge measuring techniques for the SHPB can be found in [53, 54, 55].

The reflection and transmission behaviour of the bar strain waves in a SHPB is strongly influenced by impedance. This physical property determines how an elastic wave is reflected and transmitted if it encounters a boundary.

The *characteristic* or *intrinsic impedance* is a material specific parameter defined as:

$$Z_0 = \rho c \quad (3.3)$$

where ρ is the material density and c is the longitudinal elastic wave velocity:

$$c = \sqrt{E/\rho} \quad (3.4)$$

The *mechanical impedance* is defined as the product of the characteristic impedance Z_0 and the cross-section A through which the wave is passing.

$$Z = AZ_0 = A\rho c \quad (3.5)$$

An example for wave reflection at the interface of two bars having the same diameter but different densities and longitudinal wave velocities is illustrated in Figure 3.4 [56]. Following the notation used in Figure 3.4 the stresses in the elastic bars are calculated as

$$\sigma = \rho c U_p \quad (3.6)$$

where U_p is the particle velocity in the bar section affected by the wave. From Equation (3.6) and the equilibrium condition at the interface between medium A and medium B

$$\sigma_I + \sigma_R = \sigma_T \quad (3.7)$$

the stress-ratios at the interface can be defined as

$$\frac{\sigma_T}{\sigma_I} = \frac{2\rho_{BCB}}{\rho_{BCB} + \rho_{ACA}} \quad (3.8)$$

$$\frac{\sigma_R}{\sigma_I} = \frac{\rho_{BCB} - \rho_{ACA}}{\rho_{BCB} + \rho_{ACA}} \quad (3.9)$$

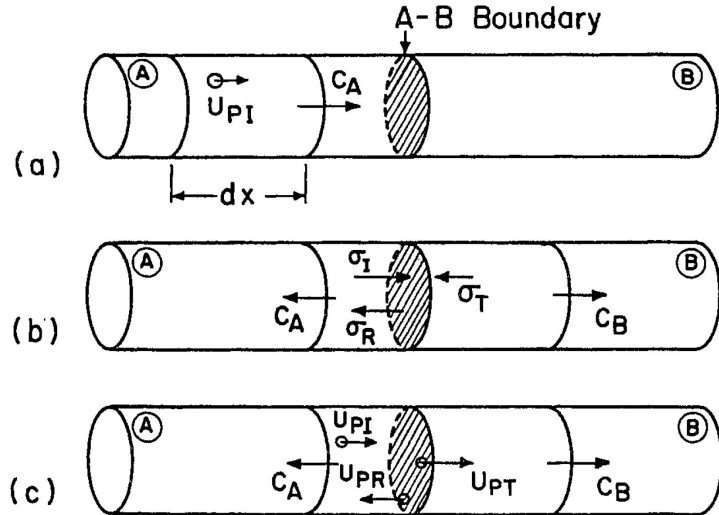


Figure 3.4: Longitudinal wave incident on boundary between two media A and B in normal trajectory: (a) prior to encounter with boundary; (b) forces exerted on boundary (equilibrium condition); (c) particle velocities (continuity). Direction of arrows for reflected wave for case impedance $A >$ impedance B. [56].

The above equations illustrate how the characteristic impedance Z_0 of the material A and B determines the amplitude of the transmitted and reflected pulse.

The reflection and transmission of the incident wave may occur between the two limiting conditions of a free surface ($\rho_{BCB} = 0$) and a rigid surface ($E_B = \infty, c_B = \infty$).

For the free surface, Equations (3.8) and (3.9) become

$$\frac{\sigma_T}{\sigma_I} = 0 \quad \frac{\sigma_R}{\sigma_I} = -1 \quad (3.10)$$

The incident wave is entirely reflected at the boundary and no transmitted wave is created.

If the incident wave is compressive, the reflected wave will be tensile and vice versa.

If the elastic incident wave encounters a rigid surface, the above stress ratio equations become

$$\frac{\sigma_T}{\sigma_I} = \frac{2}{1 + \frac{\rho_{ACA}}{\rho_{BCB}}} \approx 2 \quad \frac{\sigma_R}{\sigma_I} = \frac{1 - \rho_{ACA}/\rho_{BCB}}{1 + \rho_{ACA}/\rho_{BCB}} \approx 1 \quad (3.11)$$

In this case the incident wave is completely reflected as well but the stress direction is maintained. A compressive incident wave results in a compressive reflected wave.

In the more general case, where the incident wave encounters not only a material but also a cross-sectional change as illustrated in Figure 3.5, the equations for the stress ratios between transmitted/incident and reflected/incident wave are of the form

$$\frac{\sigma_T}{\sigma_I} = \frac{2A_1\rho_2c_2}{A_1\rho_1c_1 + A_2\rho_2c_2} \quad (3.12)$$

$$\frac{\sigma_R}{\sigma_I} = \frac{A_2\rho_2c_2 - A_1\rho_1c_1}{A_2\rho_2c_2 + A_1\rho_1c_1} \quad (3.13)$$

and are therefore functions of the mechanical impedance defined with Equation (3.5), where the change in cross-section is considered as well.

Whereas Equations (3.12) and (3.13) provide a good understanding of the impedance influence on the reflection and transmission of an elastic wave at a boundary, they do not provide exact solutions for the split-Hopkinson pressure bar experiment. The above stress-ratio equations are for elastic waves passing from one elastic into another elastic

medium. However, in the SHPB experiment the specimen is generally deformed plastically. Therefore, as an alternative, the explicit Finite Element Analysis (FEA) can be used to study the wave reflection in the SHPB bars and specimen.

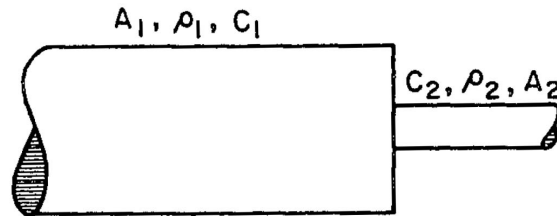


Figure 3.5: Mechanical impedance change in a cylindrical bar [56].

3.1.2 SHPB Analysis

Assumptions and Conditions

The SHPB Analysis, by which the dynamic stress-strain behaviour of the SHPB specimen can be calculated, is based on the *1D wave propagation theory*. However, this theory can only be applied under the following assumptions and conditions:

1. *The wave propagation in the bars is one-dimensional.*

This assumption is satisfied under the following conditions:

- (a) All three bars (striker-, incident- and transmission-bar) are made of homogeneous and isotropic material.
- (b) The bars are uniform in cross-section over the entire length and perfectly aligned along the centre axis.
- (c) The deformation caused by the incident-wave is strictly linear-elastic. This condition depends on the velocity of the striker-bar and the mechanical properties of the bar material.
- (d) The axial strain distribution is equal over the entire diameter of the bars and therefore the strains measured on the surface are representative of the strains

within the bar. This condition is satisfied by choosing a bar-length/bar-diameter ratio $l_b/d_b > 20$).

- (e) The bars are dispersion free. This assumption is not valid for finite diameter bars but may be minimised by selecting a pulse duration which is 10 times larger than the transit time of the longitudinal wave across the bar diameter defined as $t_t = d_b/c_b$. Minimising dispersion in the bars ensures that the three waves measured at the strain gages of the incident- and transmission-bar (significantly away from the specimen and at a different time) are in fact the same waves that have acted on the specimen. In other words: the incident wave does not change its shape when it arrives at the incident-bar/specimen interface *after* being measured at the incident-bar strain gage. Similarly, the reflected and transmitted wave, measured at the bar strain gages *after* they have been created at the bar/specimen interfaces, remain unchanged.

The pulse shaping technique described in Chapter 3.2 has a significant positive effect regarding dispersion in finite diameter bars.

2. *The incident-bar/specimen and specimen/transmission-bar interfaces remain plane at all time.*

This assumption may be satisfied under the following conditions:

- (a) The mechanical impedance of the specimen, Z_s , is lower than the mechanical impedance of the bars Z_b (see Section 3.1.1).
 - (b) The specimen diameter d_s is equal to, or slightly less than, the bar diameter d_b .
3. *The specimen is in dynamic stress equilibrium after an initial 'ringing-up' period.*

This assumption is strongly influenced by the wave velocity of the specimen material c_s . Pulse shaping again has a strong positive effect on the dynamic stress equilibrium condition (see Section 3.2).

4. *Friction and inertia effects in the specimen are minimised.*

Friction at the bar/specimen interfaces and hence a non-uniform deformation of the

specimen, can be minimised by applying a lubricant such as Molybdenum-Disulfide (MoS_2) at the bar/specimen interfaces.

5. *The specimen material is incompressible.*

This assumption is generally met for isotropic metals and polymers satisfying the incompressibility condition:

$$A_s(t)l_s(t) = A_{s0}l_{s0} \quad (3.14)$$

The *1D wave propagation theory* can not be used however for foam materials without additional data acquisition systems such as high-speed cameras.

If the above assumptions and conditions are satisfied, the SHPB analysis can be applied to calculate the dynamic specimen stress-strain response and the strain rate at which the specimen is being deformed. Since the bar strain waves, recorded at the incident-bar and transmission-bar gauges, are measured away from the specimen and at time before and after they have acted on the specimen, they must be shifted first to the time at which they have acted simultaneously on the specimen (Figure 3.6).

A forward time-shift is applied to the incident-wave signal $\epsilon_I(t)$ to determine the time $t_{\epsilon_I}^{specimen}$ at which the incident-wave has reached the specimen.

$$t_{\epsilon_I}^{specimen} = t_{SG1} + \frac{\Delta l_{SG1}}{c_b} \quad (3.15)$$

In Equation (3.15), t_{SG1} is the time corresponding to the signal of the incident-bar strain gauge, c_b is the wave velocity of the bar material and Δl_{SG1} is the geometrical distance between the incident-bar gauge and the specimen.

A backward time-shift is applied to the reflected-wave $\epsilon_R(t)$, which is also measured at the incident-bar strain gauge, and to the transmitted-wave $\epsilon_T(t)$, measured at the transmission-bar strain gauge, to determine the times $t_{\epsilon_R}^{specimen}$ and $t_{\epsilon_T}^{specimen}$ at which the reflected-wave and the transmitted wave have acted on the specimen, respectively.

$$t_{\epsilon_R}^{specimen} = t_{SG1} - \frac{\Delta l_{SG1}}{c_b} \quad (3.16)$$

$$t_{\epsilon_T}^{specimen} = t_{SG2} - \frac{\Delta l_{SG2}}{c_b} \quad (3.17)$$

It is noted that both bar strain gauges record simultaneously, hence $t_{SG1} = t_{SG2}$. In Equation (3.17), Δl_{SG2} is the geometrical distance between the transmission-bar gauge and the specimen.

A time shift example, using the bar strain wave signals shown in Figure 3.2, is presented in Figure 3.6.

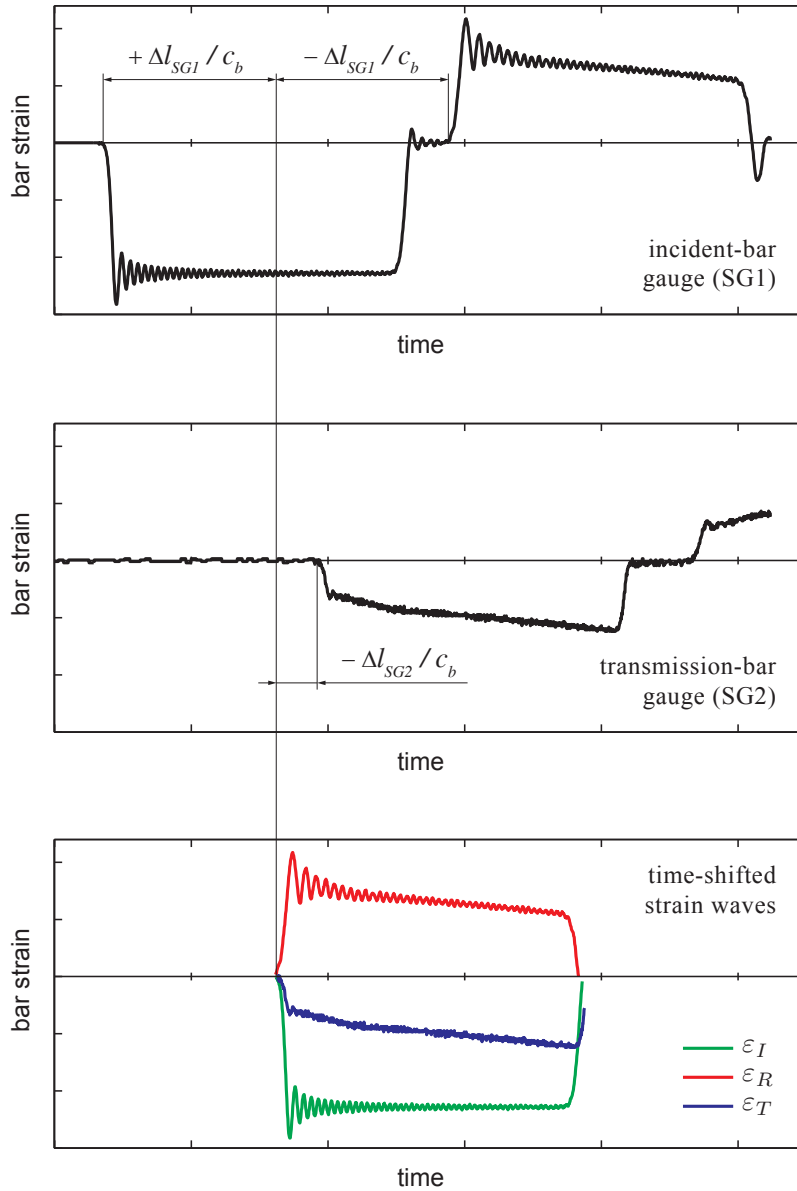


Figure 3.6: Time-shifting of the bar strain waves for the SHPB analysis.

The bar strain waves shown in Figure 3.2 were determined from a finite element simulation with a simple axis-symmetric model of the entire SHPB setup (see Section 3.2.2), using an aluminum specimen with isotropic hardening and not considering specimen failure.

It is noted that in the case of a classic near rectangular incident-wave, which contains high-frequency oscillations at the beginning of the pulse, the assumption of dispersion-free wave propagation (condition 1e at the beginning of this section) is not valid.

Dispersion-correction methods, as proposed by Lifshitz and Leber [57] and by Zhao and Gary [58], must therefore be used during the time-shifting process.

In the present study, dispersion effects were minimised to a point at which they can be neglected, by using shaped incident-pulses. Therefore, dispersion correction methods were not required (see Section 3.2 for further details).

1D Wave Propagation Theory

The *1D wave propagation theory* and hence the SHPB analysis equations is deduced from the 1D wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_b^2} \frac{\partial^2 u}{\partial t^2} \quad (3.18)$$

where u is the displacement in the direction of the bar axis and c_b is the elastic wave propagation velocity of the bar material. In the following, the notation used by Gray III [59] will be used.

The D'Alembert's method provides a solution for the incident- and transmission-bar

$$u = f(x - c_b t) + g(x + c_b t) = u_I + u_R \quad (3.19)$$

$$u = h(x - c_b t) = u_T \quad (3.20)$$

where f , g and h are arbitrary functions [60]. Equation (3.19) is obtained for the incident-bar and Equation (3.20) for the transmission-bar.

The one-dimensional strain is defined as

$$\epsilon = \frac{\partial u}{\partial x} \quad (3.21)$$

Using Equation (3.21) and differentiating Equations (3.19) and (3.20) with respect to the displacement x , yields

$$\epsilon = f' + g' = \epsilon_I + \epsilon_R \quad (3.22)$$

$$\epsilon = h' = \epsilon_T \quad (3.23)$$

Differentiating Equations (3.19) and (3.20) with respect to the time t , yields the particle velocities for the incident-bar and transmission-bar $\dot{u}_1$ and $\dot{u}_2$, respectively (Figure 3.7)

$$\dot{u}_1 = c_b(-f' + g') = c_b(-\epsilon_I + \epsilon_R) \quad (3.24)$$

$$\dot{u}_2 = -c_b\epsilon_T \quad (3.25)$$

The specimen strain rate $\dot{\epsilon}_s$ can be then be defined as

$$\dot{\epsilon}_s = \frac{\dot{u}_1 - \dot{u}_2}{l_s} \quad (3.26)$$

where l_s represents the specimen length.

With Equations (3.24), (3.25) and (3.26), the specimen strain rate $\dot{\epsilon}_s$ is defined as

$$\dot{\epsilon}_s = \frac{c_b}{l_s}(-\epsilon_I + \epsilon_R + \epsilon_T) \quad (3.27)$$

Depending on whether the initial specimen length l_{s0} or the true value $l_s(t)$ is used, the specimen strain rate is referred to as *nominal* or *true* specimen strain rate, respectively. Integration of Equation (3.27) with respect to time yields the specimen strain ϵ_s

$$\epsilon_s = \frac{c_b}{l_s} \int (-\epsilon_I + \epsilon_R + \epsilon_T) dt \quad (3.28)$$

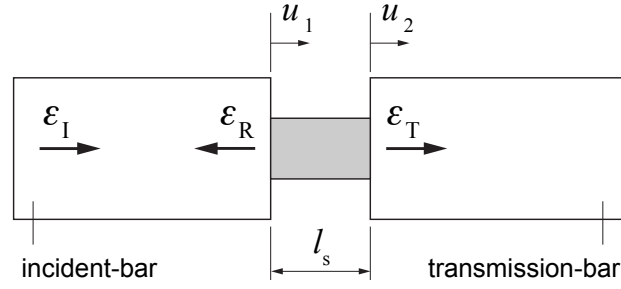


Figure 3.7: Incident-bar/specimen/transmission-bar region (after [59]).

Before calculating the specimen stress σ_s , the forces acting at both specimen end-surfaces must be determined

$$F_1 = A_b E_b (\epsilon_I + \epsilon_R) \quad (3.29)$$

$$F_2 = A_b E_b \epsilon_T \quad (3.30)$$

F_1 is the load acting at the incident-bar/specimen interface and F_2 is the load acting at the transmission-bar/specimen interface. In Equations (3.29) and (3.30), A_b is the cross-section and E_b is the Young's modulus of the SHPB bars. It is assumed here, but must not necessarily be the case, that the incident- and transmission-bar are made from the same material and have the same cross-sectional area.

If the two forces are equal, the specimen is in dynamic equilibrium and therefore

$$\epsilon_T = \epsilon_I + \epsilon_R \quad (3.31)$$

Using this equality, a simplified expression can be derived for the specimen strain rate

$$\dot{\epsilon}_s = 2 \frac{c_b}{l_s} \epsilon_R \quad (3.32)$$

The specimen stress σ_s is obtained by dividing Equations (3.29) and (3.30) by the specimen cross-sectional area A_s . It is assumed that the bar forces F_1 and F_2 , calculated using the strains measured by the bar strain gages, are equal to the forces acting in the bars at the bar/specimen interfaces and are further equal to the forces acting on the specimen.

Again, depending on whether the initial specimen cross-section A_{s0} or the true value $A_s(t)$ is used, the specimen stress is referred to as *nominal* or *true* specimen stress and may be calculated in three different ways.

If Equation (3.30) is used, the specimen stress is obtained via the *1-wave analysis*

$$\sigma_s = \frac{A_b}{A_s} E_b \epsilon_T \quad (3.33)$$

Using Equation (3.29), and hence the two strain waves recorded in the incident-bar, the specimen stress is calculated via the *2-wave analysis*

$$\sigma_s = \frac{A_b}{A_s} E_b (\epsilon_I + \epsilon_R) \quad (3.34)$$

If σ_s is calculated from the average of the two forces, and therefore using all three bar strain wave signals, the *3-wave analysis* is applied:

$$\sigma_s = \frac{A_b E_b}{2A_s} (\epsilon_I + \epsilon_R + \epsilon_T) \quad (3.35)$$

The *1-wave stress* represents the conditions at the specimen/transmission-bar interface and is therefore also referred to as the specimen 'back stress'. The *2-wave stress* represents the conditions at the incident-bar/specimen interface and is therefore called the specimen 'front stress'.

SHPB Analysis Assumptions Revisited

The planar bar/specimen interface assumption

The velocities of the bar/specimen interfaces $\dot{u}_1$ and $\dot{u}_2$ are calculated with Equations (3.24) and (3.25), respectively. Using the interface velocities, the specimen strain rate $\dot{\epsilon}_s$ is defined by Equation (3.26) and subsequently calculated by Equation (3.27). The specimen strain ϵ_s in the loading direction is then obtained by integration of Equation (3.27). In order to accurately determine the specimen strain ϵ_s , it is necessary that the bar/specimen

interfaces remain planar. Otherwise, the specimen strain calculated by the SHPB analysis may be significantly overpredicted.

The planar bar/specimen interface assumption is strongly influenced by the impedance ratio between the SHPB bars and the specimen and further by the specimen geometry as illustrated in Figure 3.8. The bar/specimen interfaces remain planar for equal bar and specimen cross-sections and in the case of an acoustic soft specimen, meaning that the impedance of the specimen is significantly lower than the impedance of the bars (Figure 3.8a).

For a stiff specimen, the impedance difference between bars and specimen may be small, which results in a very low reflected wave signal since the incident wave passes nearly unchanged from the incident-bar into the transmission-bar.

By reducing the specimen cross-section in this case, the mechanical impedance ratio between bar and specimen can be increased and a well balanced wave reflection/transmission behaviour can be achieved. In this case the planar interface assumption may be violated

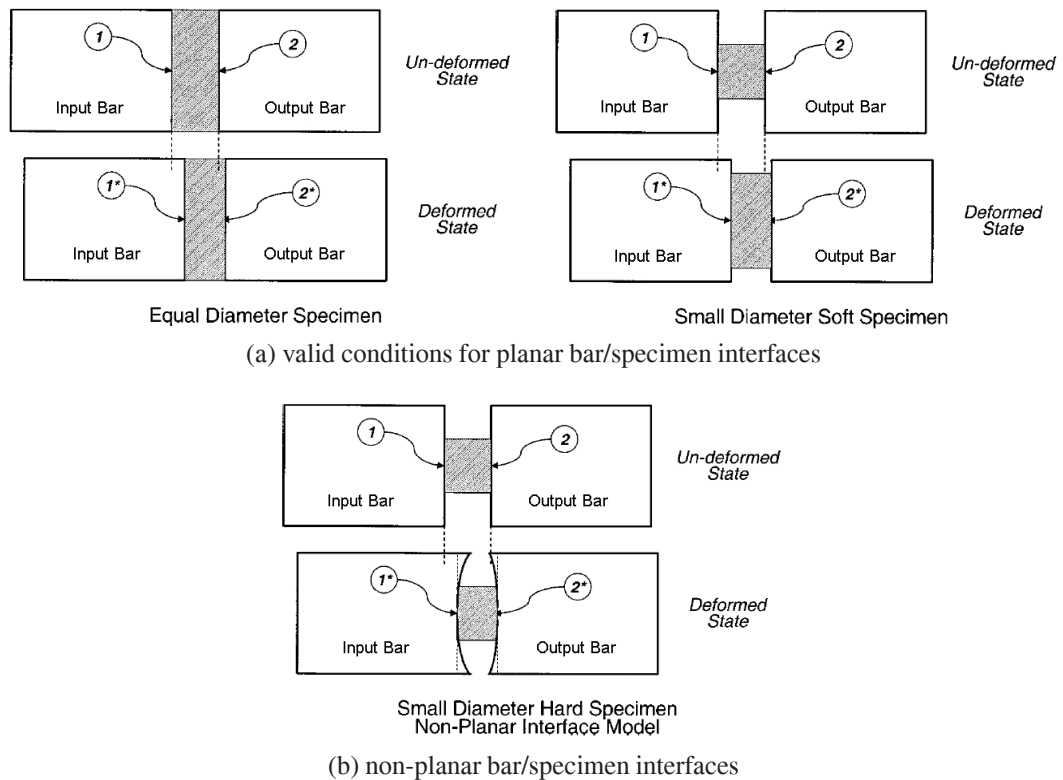


Figure 3.8: Planar and non-planar bar/specimen interface deformation [60].

however, as shown in Figure 3.8b.

Analytical corrections to solve the strain overprediction problem resulting from the punching effect at the bar-ends were recently proposed by Safa and Gary [61].

However, a problem remains for high strength specimens such as ceramics or FRP's loaded in the fibre-direction, where the specimen strength may exceed the strength of the SHPB bar material. In addition to the non-planar interface deformation, the SHPB bars will be plastically deformed and therefore damaged by the indentation of the high strength specimen into the bar end-surfaces.

The indentation problem can be avoided by using two identical high strength and very stiff Tungsten-Carbide (TC) inserts, placed between the bars and the specimen, as illustrated in Figure 3.9. To ensure that the reflection and transmission of the bar strain waves occurs at the specimen end-surfaces and not at the bar/TC-insert interfaces, the mechanical impedance of the TC-insert must be equal to the mechanical impedance of the bars. Using Equation (3.5) and considering the wave propagation velocity c defined by Equation (3.4), the required TC-insert diameter d_{TC} is given by the expression

$$Z_{bar} = Z_{TC} \Rightarrow d_{TC} = d_b \left(\frac{\rho_b E_b}{\rho_{TC} E_{TC}} \right)^{\frac{1}{4}} \quad (3.36)$$

In Equation (3.36), d_b , ρ_b and E_b are the diameter, the material density and the elastic modulus of the SHPB bars while ρ_{TC} and E_{TC} are the material density and elastic modulus of the TC-insert.

Different designs for the TC-inserts have been explored, such as regular or conical cylindric discs [62]. The use of conical inserts is problematic however since an impedance mismatch exists between the bar/TC-insert and TC-insert/specimen interface and the manufacture of the insert is relatively difficult.

To avoid fracture of the brittle insert, it is recommended to apply a lateral support ring which must not be in contact with the bar end in order not to influence the wave-reflection (Figure 3.10).

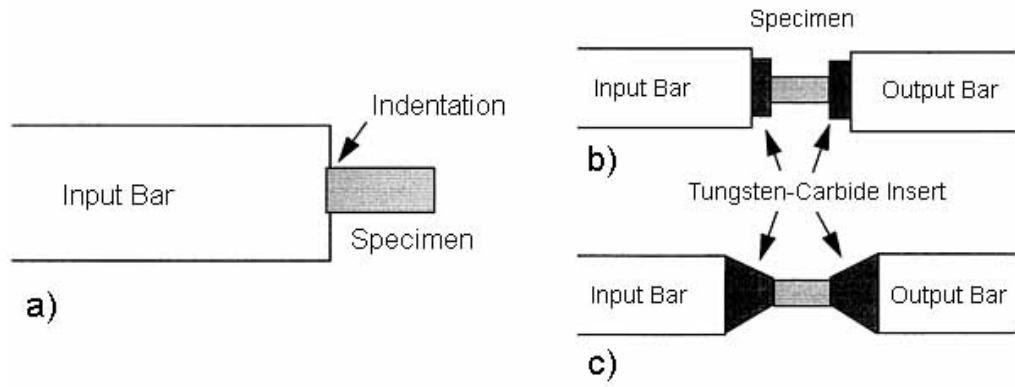


Figure 3.9: Bar-surface indentation using hard specimens [62].

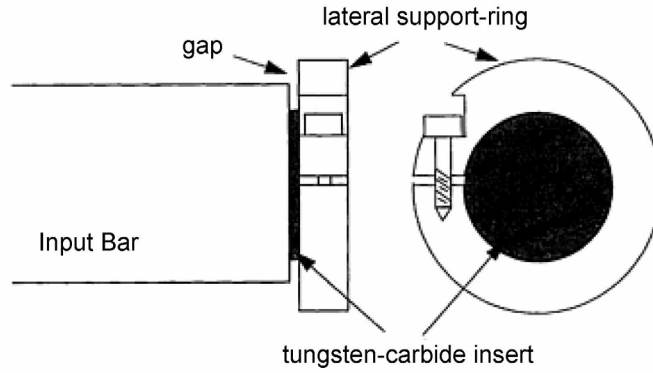


Figure 3.10: Lateral support of TC-insert [62].

The dynamic equilibrium assumption

As noted earlier, this assumption is fundamental for the *1D wave propagation theory*. In a classic SHPB experiment, the equilibrium state is reached after an initial 'ringing-up' period which depends on the specimen length and on the wave propagation velocity in the specimen, c_s .

It is recalled that while as the incident-wave ϵ_I and the reflected-wave ϵ_R contain high frequency oscillations at the beginning (see Figure 3.2), the transmitted wave ϵ_T is smooth which is attributed to dampening effects in the specimen. A smooth specimen stress-time curve can therefore be obtained when using the *1-wave analysis* equation (3.33). However, if the specimen stress is calculated via the *2-wave analysis* equation (3.34), it becomes evident that the specimen is not in dynamic equilibrium at the beginning of the transient loading process.

To determine the length of the initial 'ringing-up' period it is recommended to compare the specimen stress-strain response calculated by the *1-wave analysis*, Equation (3.33), with the stress-strain response determined by the *2-wave analysis*, Equation (3.34).

This comparison is shown for a steel and for a high-purity lead specimen in Figure 3.11. In the case of the steel sample, the dynamic equilibrium is reached after approx. 2% true strain, whilst the lead specimen remains in a non-dynamic equilibrium state.

It should be noted that this equilibrium check method strongly depends on accurate time-shifting and an accurate value of the wave propagation velocity in the bar, c_b . Lifshitz and Leber [57] have shown the effect of a slightly incorrect value of c_b on the dynamic stress-strain curves for various metal specimens.

Gray III [59] stated that "the finite time to achieve stress-state equilibrium demonstrates that the high-rate elastic modulus of a sample cannot be measured by any Hopkinson bar. Because the stress equilibrium does not occur until well over 1% plastic strain, it is impossible to accurately measure the compressive Young's modulus of materials at high strain rate using the SHPB."

This statement is widely accepted nowadays and reported *elastic* properties obtained from SHPB experiments must be treated with care. The dynamic *plastic* material properties can however be obtained with good confidence from the classic SHPB experiment if the duration T of the incident pulse is sufficiently longer than the transit time of the longitudinal wave across the specimen, which is defined as $t_s = l_s/c_s$.

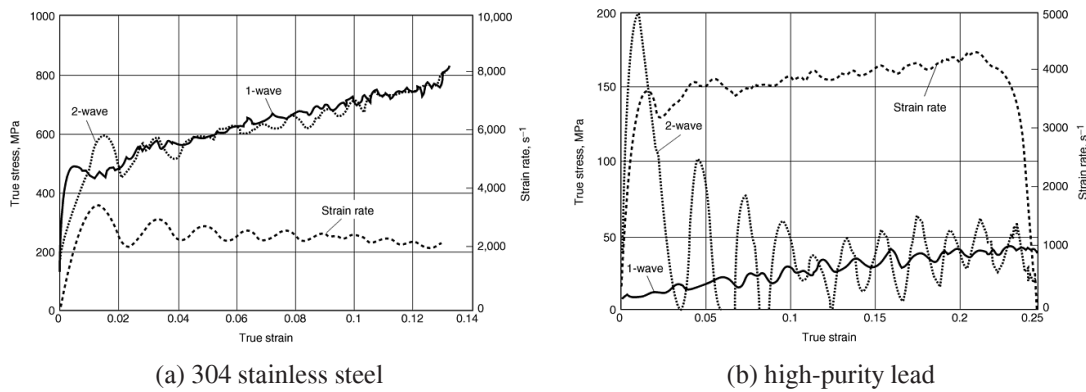


Figure 3.11: Stress equilibrium check for two different materials [59].

FRP's loaded in transverse direction can be considered acoustic soft specimens if steel bars are used, since the impedance of the polymer matrix is much lower than the impedance of the bar material. Due to the low wave velocity c_s in the soft specimen, the dynamic stress-strain response of the sample may exhibit a pronounced sample-size dependency. The time to obtain the dynamic stress equilibrium increases for higher specimen length-specimen diameter ratios l_s/d_s as illustrated in Figure 3.12 and hence a small l_s/d_s ratio is recommended for soft materials [63].

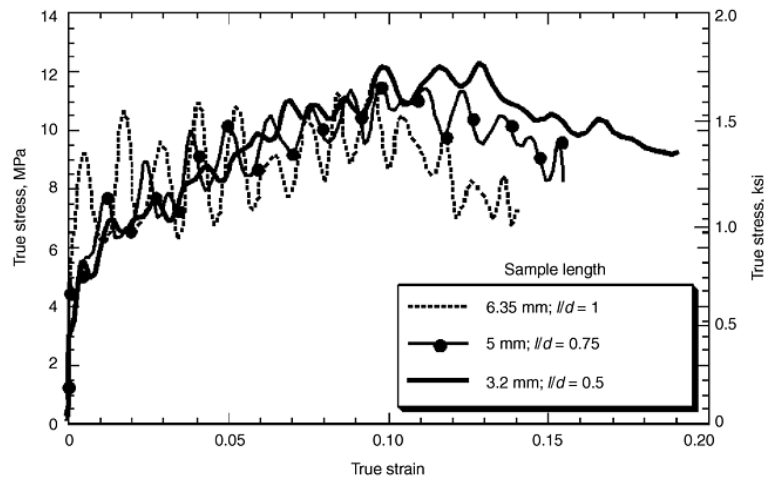


Figure 3.12: Stress-strain response of 6.35mm diam Adiprene L100 samples as a function of sample length at high strain rate (2500s^{-1}) [63].

It is noted that the pulse shaping technique introduced in Section 3.2 provides a method of eliminating the high frequency oscillations in the incident- and reflected-wave signal, which in turn also eliminates the 'ringing-up' time and therefore dynamic stress equilibrium is established very early in the transient loading. It is then possible to measure the dynamic elastic properties of the specimen with good confidence.

Friction and Inertia Effects

From the previous section it is clear that the 'ringing-up' time has a significant influence on the accuracy of the dynamic stress-strain response in a classic SHPB experiment. The time interval required to equalise the forces at both specimen ends is attributed to the longitudinal inertia effects caused by the rapid particle accelerations in the specimen.

Davis and Hunter [64] proposed the following correction expression for the errors caused by inertia effects:

$$\sigma_s^C(t) = \sigma_s^M(t) + \rho_s \left[\frac{l_s^2}{6} - v_s \frac{d_s^2}{8} \right] \left(\frac{\partial^2 \epsilon(t)}{\partial t^2} \right) \quad (3.37)$$

where the subscripts *C* and *M* denote the *corrected* and the *measured* specimen stress, respectively. The correction term will be zero if either the strain rate is constant or the term in square brackets is zero. The latter leads to the condition

$$\frac{l_s}{d_s} = \sqrt{v_s \frac{3}{4}} \quad (3.38)$$

For a specimen Poisson's ratio of $v_s = 0.333$, the above criteria then results in an optimal ratio of $l_s/d_s = 0.5$.

To minimise errors due to friction effects at the bar/specimen interfaces, the test standard ASTM E9-09 [65] recommends a specimen length-specimen diameter ratio of

$$1.5 < \frac{l_s}{d_s} < 2.0 \quad (3.39)$$

It is apparent that the conditions for minimising errors due to inertia and friction effects can not be satisfied simultaneously. Gray III [59] therefore suggests a compromise of

$$0.5 < \frac{l_s}{d_s} < 1.0 \quad (3.40)$$

and the use of lubricants, such as Molybdenum-Disulfide, at the bar/specimen interfaces. If a constant strain rate is achieved in the SHPB experiment, Equation (3.37) is of lesser importance.

It will be shown in the following section that the achievement of constant strain rates in the SHPB experiment significantly depends on the incident-wave shape with respect to the specimen stress-strain behaviour. For FRP specimens, the specimen stress-strain behaviour may span the entire range of ductile plasticity with large failure strains to brittle linear-elastic behaviour with small failure strains.

3.2 Pulse Shaping

In the classic SHPB experiment, a near rectangular incident wave is generated by a direct impact of the striker-bar on the free end of the incident-bar. This pulse shape is ideally suited for metals or other materials experiencing large plastic strains since it imposes a nominally uniform strain rate during plastic deformation of the specimen. If the duration of the incident pulse is sufficiently long, the non-equilibrium state at the begin, where the specimen is deformed elastically, can be ignored if only the plastic mechanical material properties are of interest, and when considering that the elastic strain component is significantly smaller than the plastic strain. The classic SHPB experiment has therefore been used successfully to study the material behaviour of ductile metal specimens.

For brittle materials such as ceramics or FRP's loaded in the longitudinal direction, the failure strain is significantly lower than the failure strain of ductile metals and the stress-strain response is mainly linear-elastic up to failure. It will be demonstrated in Section 3.2.2 that if a linear-elastic specimen is loaded with a classic near rectangular incident-wave, failure may occur well before the specimen reaches dynamic stress equilibrium and the strain rate during the test is far from being constant. It is therefore not possible to perform a reliable dynamic material characterisation for brittle and linear-elastic specimens using the classic SHPB setup.

Nemat-Nasser et al. [66] suggested the use of a ramp shaped incident-wave, which can be obtained by placing a *pulse shaper* in the form of a small disc made from a very ductile metal, such as copper or aluminium, between the striker- and incident-bar.

A carefully designed ramp shaped incident-wave generates a constant reflected wave ϵ_R in the case of linear-elastic specimen behaviour, which according to Equation (3.32) then yields a constant specimen strain rate. In addition, a shaped incident-wave contains very little or no high frequency oscillations, which minimises the dispersion effect and the dynamic equilibrium in the specimen is reached very early.

On the negative side, pulse shaping reduces the achievable strain rate. This disadvantage is outweighed by the advantages of this method. In fact, Gama et al. [60] recommend the

use of shaped incident-waves for every SHPB experiment, irrespective of the specimen material behaviour.

3.2.1 Pulse Shaping Analysis

It is quite difficult, if not impossible, to find the optimal pulse shape by experimental trial and error. Therefore, the pulse shaping analysis (PSA) proposed by Nemat-Nasser et al. [66] was adopted in this work.

In this section, a phenomenological overview of the pulse shaping technique is presented. A detailed analytical description of the implemented PSA can be found in Appendix A.

The PSA is based on the isochoric rate-independent axi-symmetric plastic deformation of the pulse shaper (the elastic deformation is neglected) and on the propagation of elastic waves in the striker-bar. To create a desired pulse shape, a variety of parameters must be brought into accordance:

- The compressive stress-strain response of the pulse shaper material.
- The pulse shaper diameter d_{ps} and thickness h_{ps} .
- The elastic modulus E_b and material density ρ_b and hence the wave propagation velocity c_b of the striker-bar.
- The striker-bar length $l_{striker}$ and diameter $d_{striker}$.
- The striker-bar velocity V_0 .

A test must be performed to obtain the compressive stress-strain response of the pulse shaper material. For the PSA, the plastic flow of the pulse shaper is then governed by an empirically established power-law relation between the axial true stress and the corresponding engineering strain.

$$\sigma = \sigma_0 \epsilon^{n_{ps}} \quad (3.41)$$

This experimental curve-fitting is shown for oxygen-free high-purity copper (OFHC) in Figure 3.13, where $\sigma_0 = 570\text{MPa}$ and $n_{ps} = 1/5$. It is noted that this stress-strain response was determined from a SHPB experiment using an OFHC specimen and already applying the pulse shaping technique [66].

Three important conditions govern the deformation of the pulse shaper and have a direct influence on the wave shape:

Condition 1 postulates that the cross-section of the pulse shaper $a(t)$ must always be smaller or equal than the cross-section of striker- and incident-bar:

$$a(t) \leq A_b \quad (3.42)$$

Condition 2 concerns the particle velocity U_p in the striker- and incident-bar, which can never be greater than in the case of a direct impact without pulse shaper:

$$U_p \leq \frac{V_0}{2} \quad (3.43)$$

Condition 3 is the time needed for the elastic wave to travel back and forth in the striker-bar, from the moment of first contact with the pulse shaper until it reaches the pulse shaper again, after being reflected at the free end of the striker-bar. This time period corresponds to the pulse duration T of the classic incident wave and was defined with Equation (3.1).

While *Condition 1* must always be satisfied, two possible pulse shapes can be obtained, depending on *Condition 2* and *Condition 3*.

If the particle velocity in both striker- and incident-bar has reached the maximum of $U_p^{max} = V_0/2$ before T is reached, the pulse shaper ceases to deform and remains rigid. The created incident-wave then has a trapezoidal shape with a maximum strain level equal to that of the classic near rectangular incident-wave, defined with Equation (3.2). Unloading from the maximum strain level always occurs at T and therefore depends on

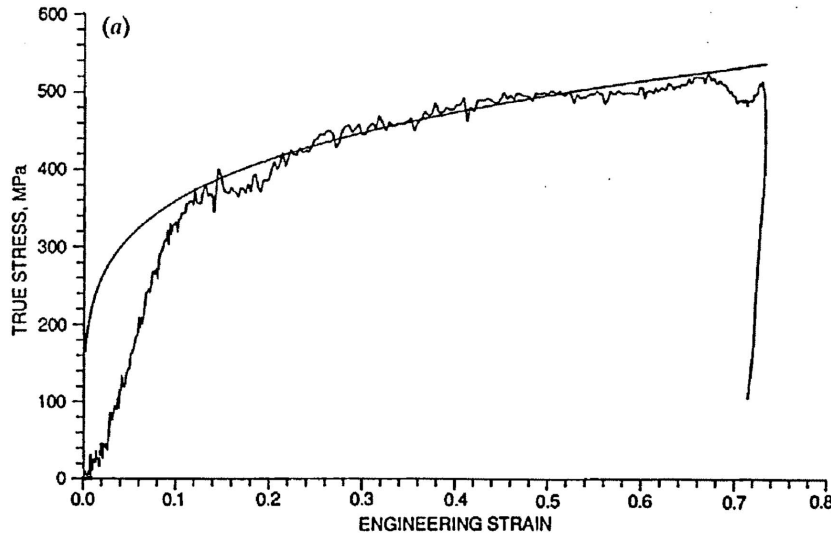


Figure 3.13: Measured and fitted high strain rate compressive stress-strain response of an OFHC pulse shaper [66].

the length of the striker-bar.

If T is reached before the maximum particle velocity U_p^{max} , the pulse shaper is still deformed but from T onwards this deformation is influenced by the returning initial wave-front which has passed back and forth through the striker-bar. If all parameters are properly set, a triangular shaped pulse and thus the ramp shaped incident-wave required for a linear-elastic specimen, can be created. In the latter case, the maximum strain level is slightly less than that of the classic near rectangular or trapezoidal shaped pulse at the same striker-bar impact velocity V_0 .

Experimental and PSA predictions of shaped incident-waves are shown in Figure 3.14. The tests were carried out with a SHPB setup consisting of $\varnothing 18\text{mm}$ steel striker-, incident- and transmission-bars, with lengths of 0.5m, 2.6m and 1.7m, respectively. Since it was only necessary to measure the incident-wave at the incident-bar strain gauge, the transmission-bar was not used.

For the trapezoidal shaped incident-wave shown in Figure 3.14a, a copper pulse shaper with a diameter $d_{ps} = 4\text{mm}$ and a thickness of $h_{ps} = 0.5\text{mm}$ was used. The impact velocity of the striker-bar was $V_0 = 8\text{ms}^{-1}$. The experimental and predicted shaped incident-wave are compared with a classic near rectangular incident-wave obtained from a direct

impact at a striker-bar velocity of $V_0 = 8.3\text{ms}^{-1}$. In addition, the theoretical rectangular pulse determined from Equations (3.1) and (3.2) is shown.

For the triangular shaped incident-wave shown in Figure 3.14b, a copper pulse shaper with a diameter of $d_{ps} = 5\text{mm}$ and a thickness of $h_{ps} = 1.5\text{mm}$ was used. The impact velocity of the striker-bar was $V_0 = 9.7\text{ms}^{-1}$ and the classic experimental incident-wave was obtained from a direct impact test at approximately the same impact velocity.

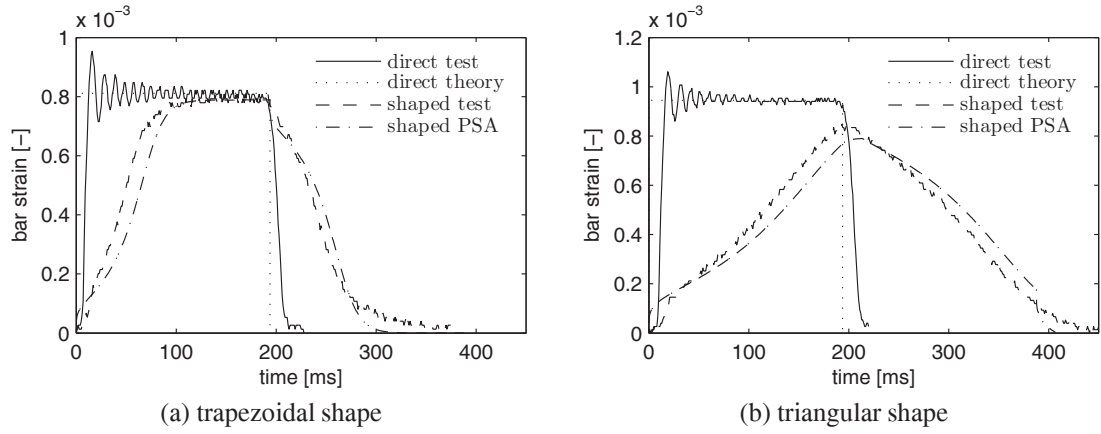


Figure 3.14: Comparison of classic and shaped incident-waves.

Two important observations can be made from the examples shown in Figure 3.14:

1. The high-frequency oscillations at the beginning of the classic incident-waves are eliminated in the case of the shaped pulses.
2. The pulse duration of a shaped pulse is always longer than that of the classic near rectangular pulse.

Observation 1 is important since the reduction of the high-frequency oscillations significantly reduces the dispersion effect and therefore greatly simplifies the SHPB analysis. The absence of these oscillations in the incident-wave, and subsequently in the reflected wave, further means that the dynamic equilibrium state in the specimen will be reached earlier than in the classic SHPB experiment.

Observation 2 is important since the longer pulse duration of the shaped incident-wave must be considered during the setup of the SHPB experiment to further guarantee an

interference-free measurement of the bar strain waves ϵ_I , ϵ_R and ϵ_T . The classic relation for the pulse duration T , given by Equation (3.1), is no longer valid if shaped incident pulses are used. However, it is seen in Figure 3.14 that the PSA can provide a good estimate of the pulse duration of the shaped incident-wave.

It is noted that the parameters σ_0 and n_{ps} , needed in Equation (3.41) for the curve-fitting of the compressive stress-strain response of the pulse shaper material, were those reported by Nemat-Nasser et al. [66] for oxygen-free high-purity copper (OFHC). In the present work, a high-residual phosphorus copper alloy with international specification UNS C12200 (not annealed) was used instead. It is likely that the above parameters may be slightly different. Despite this, the predicted incident-wave shapes shown in Figure 3.14 correlate reasonable well with the shaped incident-waves measured in the experiment. The power-law parameters can therefore be used to pre-determine shaped incident-waves of two distinct shapes at various durations and bar strain amplitudes.

3.2.2 Evaluation of Shaped Incident-Waves by Means of Finite Element Simulations

The difficulty of the SHPB experiment is to know in advance how and at which strain rate the specimen will be deformed during the experiment. Only after applying the SHPB analysis to the three bar strain waves ϵ_I , ϵ_R and ϵ_T , recorded during the test, is it possible to have an inside look into the deformation event of the specimen. The preparation of the experiment is further complicated if shaped incident-waves are to be used and the specimen behaviour is different from that of the classic ductile large strain to failure metal specimen type, for which the classic SHPB setup is well suited.

Explicit finite element analysis provides a powerful tool to study the influence of various incident-wave shapes, obtained via PSA, on the specimen deformation and can be used to identify problem areas in order to eliminate trial and error in the setup of the experiment. The modeling strategy presented here was used to develop and prepare the SHPB experiments presented in Chapters 4 and 5.

Finite Element Model

A simple two-dimensional axis-symmetric FE model was built with the commercial FE code ABAQUS/Explicit (Version 6.7). It resembles a true-scale SHPB setup consisting of $\varnothing 18\text{mm}$ steel striker-, incident- and transmission-bars, with lengths of 0.5m, 2.6m and 1.7m, respectively. A fictive cylindric specimen with a diameter of 4mm and a length of 8mm is used. In addition, two TC-inserts with a diameter of 12mm and a thickness of 5mm were placed between the bars and the specimen (see Section 3.1.2) since the FE model presented here is used to demonstrate the importance of using a ramp shaped incident-wave for a linear-elastic high strength sample, such as a FRP specimen loaded in longitudinal direction.

Reduced integration axis-symmetric solid elements (ABAQUS element type C4AXR) were selected for the mesh and symmetry boundary conditions were applied at the center axis. A zero-friction contact and linear-elastic material behaviour was defined for all model parts. The material properties are given in Tables 3.1 and 3.2.

The three bar strain waves ε_I , ε_R and ε_T needed for the SHPB analysis were generated from the nominal strain component NE11 of an element corresponding to the position of the incident- and transmission-bar strain gauge. Two simulations will be explained in further detail:

A classic SHPB test with a near rectangular incident-wave, as obtained from a direct impact of the striker- on the incident-bar, was simulated by specifying a constant velocity boundary condition for the striker-bar node set.

In the case of a shaped incident-wave, the striker-bar was not used and the incident-wave was applied as a distributed surface load to the free end of the incident-bar.

Figure 3.15 shows the FE mesh in the proximity of the specimen. In Figure 3.16, the entire FE model is shown together with the propagation of the incident-wave for the direct impact simulation (Figure 3.16b,c) and for the case of a shaped incident-wave (Figure 3.16d). For clarity, the mesh was mirrored at the centre-axis and scaled by a factor of 2 in the y-direction.

Table 3.1: Isotropic elastic material properties used for FE simulation.

Material	Model Part	Modulus E [MPa]	Poisson's Ratio ν [-]	Density ρ [kg/m ³]
Steel	SHPB-bars	206000	0.29	7850
Tigra T10MG	TC-insert	585000	0.22	14450

Table 3.2: Orthotropic elastic material properties of Hexply IM7-8552 used for FE simulation [67].

Modulus E [MPa]		Poisson's Ratio ν [-]		Shear Modulus G [MPa]	
E_1^a	171420	ν_{12}^a	0.32	G_{12}	5290
E_2^a	9080	ν_{13}^c	0.32	G_{13}^c	5290
E_3^c	9080	ν_{23}^b	0.5	G_{23}^c	3974
Density ρ [kg/m ³]		1590			

^a Obtained from longitudinal (E_1) and transverse tensile tests (E_2).
^b Assumed value.
^c Calculated assuming transverse isotropy.

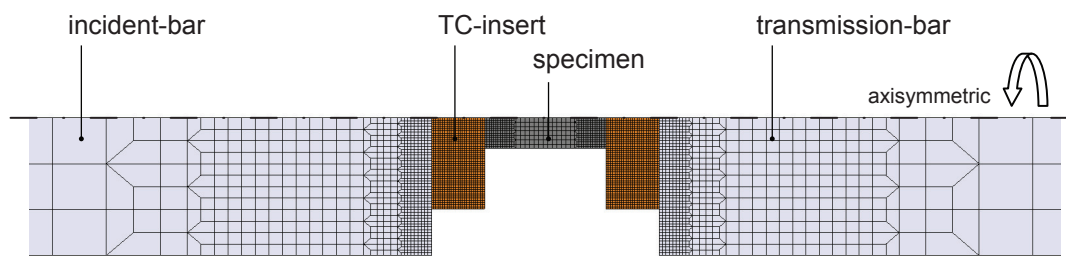


Figure 3.15: FE mesh in the proximity of the specimen.

Linear-Elastic Specimen and Classic Incident-Wave

Figure 3.17 summarises the SHPB analysis results obtained from the simulation with a linear-elastic specimen loaded by a classic near rectangular incident-wave. The impact

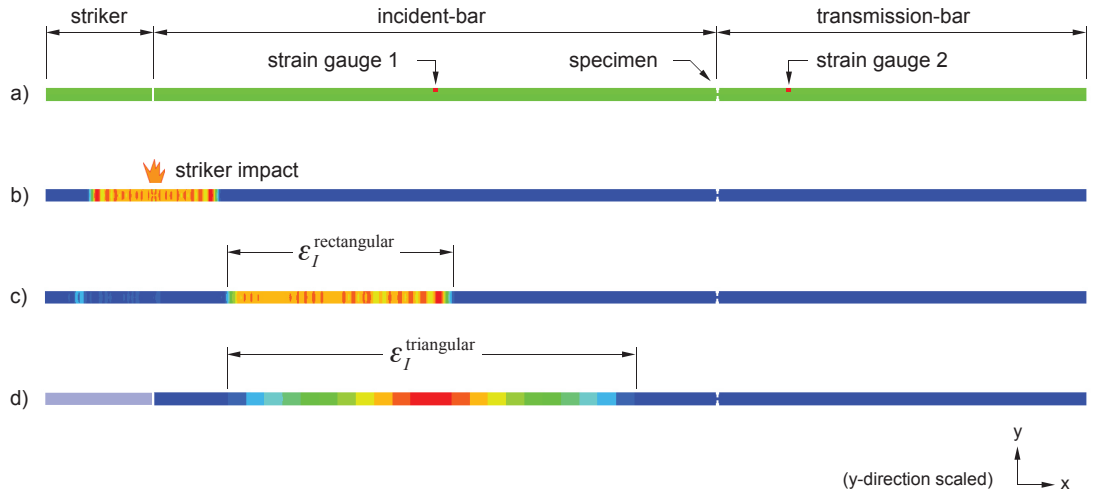


Figure 3.16: Full scale FE model of SHPB setup (a) and propagation of classic rectangular (b,c) and ramp shaped incident-wave (d).

velocity of the striker-bar was $V_0 = 5.5\text{ms}^{-1}$, which generates an incident-wave with a duration of approximately $200\mu\text{s}$ and an amplitude of 0.0535% strain.

Diagram a) shows the three bar strain waves as recorded by the strain gauges on the incident-bar (SG1) and on the transmission-bar (SG2).

Diagram b) shows the shifted bar strain waves as required for the SHPB analysis.

Diagram c) shows the velocities $\dot{u}_1$ and $\dot{u}_2$ of the incident-bar/specimen and transmission-bar/specimen interface, calculated by Equations (3.24) and (3.25), respectively.

Diagram d) shows the forces F_1 and F_2 , acting on the specimen at the bar/specimen interfaces. F_1 and F_2 are calculated from Equations (3.29) and (3.30), respectively.

Diagram e) compares the specimen strain calculated via SHPB analysis, Equation (3.28), with the strain obtained directly from the specimen (simulated specimen strain gauge).

Diagram f) compares the 2-wave and the 1-wave specimen stress, determined with Equations (3.34) and (3.33), respectively, and further shows the stress obtained directly from the specimen.

Diagram g) shows the nominal specimen strain rate as obtained from Equation (3.27).

Diagram h) eventually shows the dynamic specimen stress-strain response, determined in three different ways:

The first curve is the stress-strain response as obtained entirely by SHPB analysis from

Equations (3.33) and (3.28).

The second curve uses the SHPB analysis stress together with the strain measured directly on the specimen (simulated specimen strain gauge).

The third curve uses both stress and strain from the specimen. It is noted that this is only possible for a FE simulation but not for a real experiment since the specimen stress must be determined via SHPB analysis in the latter case.

While keeping in mind that the deformation of the specimen is strictly elastic (specimen failure is not modeled), a number of observations can be made from the results presented in Figure 3.17:

Interface Velocity Diagram:

Three distinct regimes can be identified. From $0 < t < 100\mu\text{s}$, both interfaces deform at different speeds. From $100\mu\text{s} < t < 200\mu\text{s}$, the two interfaces deform at the same constant velocity of approximately 2.75m.s^{-1} , which is the maximum particle velocity in the bars defined by Equation (3.43). If the deformation at both sides of the specimen is occurring at the same velocity however, the specimen is not deformed. From $200\mu\text{s} < t < 300\mu\text{s}$, the velocity of both interfaces is a mirror image of the first regime.

Bar Forces Diagram:

During the first $50\mu\text{s}$, significant oscillations can be observed for F_1 , which is the force acting at the incident-bar/specimen interface, while a rather smooth curve is obtained for F_2 . It must therefore be concluded that the specimen is not in dynamic equilibrium during this initial $50\mu\text{s}$.

Specimen Strain Diagram:

During the first $50\mu\text{s}$, where the specimen is not in dynamic equilibrium, the specimen strain reaches about 85% of its maximum strain level. From $100\mu\text{s} < t < 200\mu\text{s}$ the spec-

imen strain remains approximately constant. After $t = 200\mu\text{s}$ the specimen is unloaded. The constant strain period was already concluded from the *interface velocity diagram*. It is further observed that the specimen strain calculated via SHPB analysis is significantly overpredicted.

Specimen Stress Diagram:

The dynamic non-equilibrium during the first $50\mu\text{s}$ can also be observed in the specimen stress diagram. Whereas a smooth curve is obtained from the *I-wave* stress equation (3.33), the high frequency oscillations observed for F_1 are reflected in the stress curve calculated via the *2-wave* stress equation (3.34).

In contrast to the specimen strain, the specimen stress is well predicted by the SHPB analysis.

Specimen Strain Rate Diagram:

During the time period in which the specimen is deformed ($0 < t < 100\mu\text{s}$), the strain rate is not constant.

After a sharp rise to a maximum value of approx. 600s^{-1} at $t = 20\mu\text{s}$, the strain rate decreases and is zero between $100\mu\text{s} < t < 200\mu\text{s}$. The period of zero strain rate is then followed by a negative peak at $t = 220\mu\text{s}$.

This strain rate behaviour for an elastic specimen loaded by the classic rectangular incident-wave was already mentioned by Nemat-Nasser et al. [66] and is the motivation for finding a different incident-wave shape which is able to produce a constant strain rate in the specimen.

Dynamic Stress-Strain Response Diagram:

In *diagram e)* it was observed that the specimen strain is overpredicted by the SHPBA whereas it was seen in *diagram f)* that the specimen stress is sufficiently well determined. If the stress-strain response is therefore obtained entirely from the SHPBA results, the

specimen strength may be correctly predicted but wrong conclusions will be drawn for the dynamic modulus. A much better dynamic stress-strain response can be obtained if the specimen stress is measured directly on the specimen, as evident when comparing the SHPBA/specimen and the FEM/specimen stress-strain curves in *diagram h*).

It is noted however that for a real experiment the stress-strain curve may still be incorrect due to the observed non-equilibrium state (*diagram d,f*)) and the non-constant specimen strain rate (*diagram g*)).

It is further noted that the classic near rectangular incident wave is insufficiently used in the case of linear-elastic specimen behaviour, since the specimen is only loaded during the first half of the pulse duration.

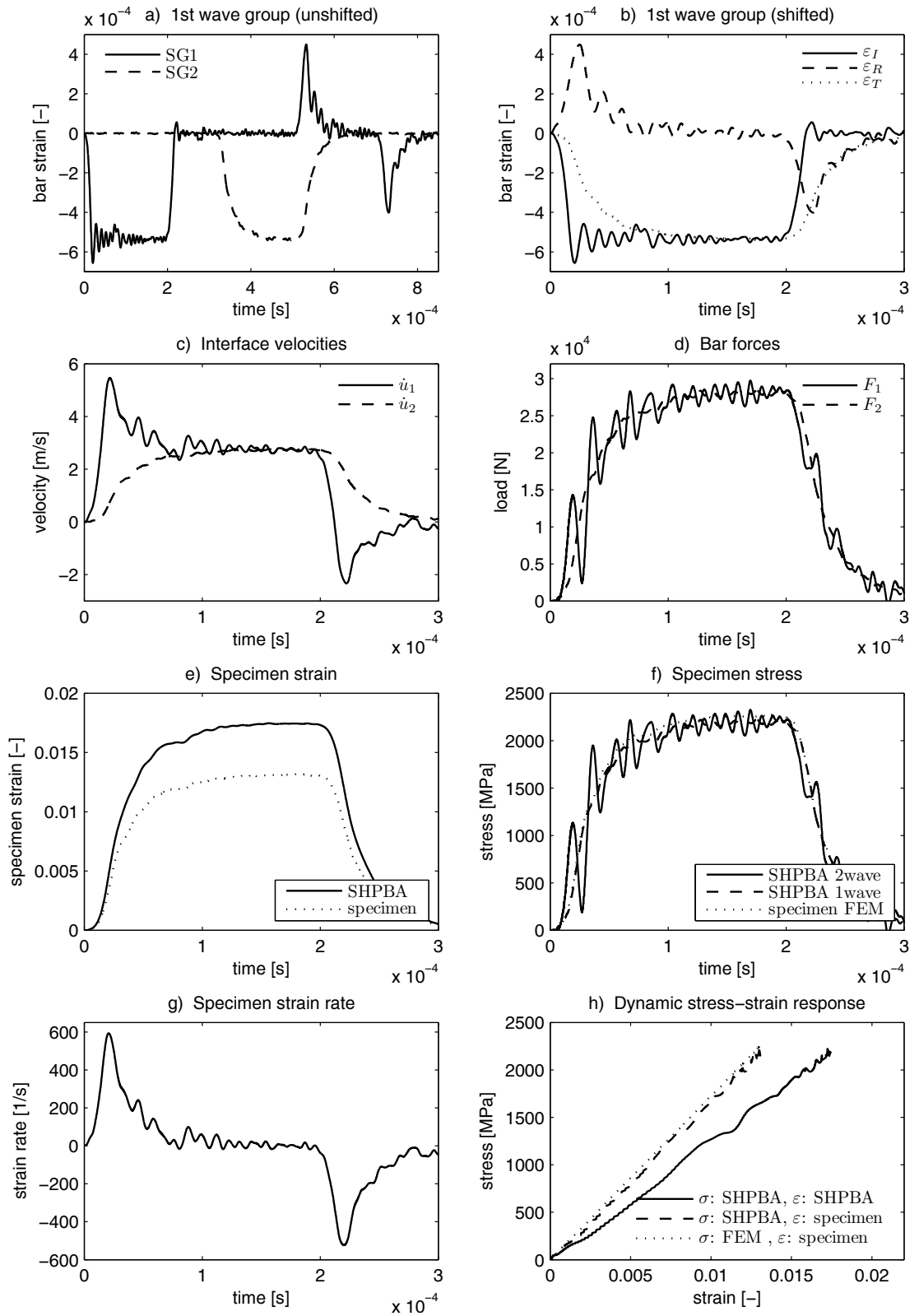


Figure 3.17: SHPBA for linear-elastic specimen and classic incident-wave.

Linear-Elastic Specimen and Ramp Shaped Incident-Wave

In the next example, an ideal ramp shaped incident wave is used. Via PSA it was determined that the striker-bar must have an impact velocity of $V_0 = 6\text{ms}^{-1}$ in order to produce the same incident wave strain amplitude as in the previous case. The diameter and thickness of the pulse shaper would need to be $d_{ps} = 1\text{mm}$ and $h_{ps} = 0.8\text{mm}$, respectively. The duration of the shaped incident wave is approximately twice the duration of the classic near rectangular wave, while using the same striker-bar. The SHPBA results of the simulated SHPB experiment are summarised in Figure 3.18 in the same order as explained for the previous example. It is noted that *diagram a)* of Figure 3.18 also shows the shaped incident wave as determined via PSA.

To see whether the different pulse-shape has the desired effect of producing a constant-strain rate test, one has to look at the strain rate equation. Eliminating ϵ_R in Equation (3.27) using Equation (3.31) yields:

$$\dot{\epsilon} = 2 \frac{c_b}{l_s} (\epsilon_T - \epsilon_I) \quad (3.44)$$

From Equation (3.44) it is clear that the specimen is loaded at a constant strain rate if the difference between the transmitted wave ϵ_T and the incident wave ϵ_I is constant. The same conclusion can be drawn from Equation (3.32) for a constant reflected wave ϵ_R . In Figure 3.18b it is seen that these conditions are achieved with the chosen ramp shaped incident wave and the linear-elastic specimen is now deformed at a constant strain rate of approximately 80s^{-1} (*diagram g*)).

In addition it is seen that the high-frequency oscillations in the incident and reflected wave signal have disappeared¹. The bar forces F_1 and F_2 (*diagram d*)) are identical, which in turn leads to identical specimen stress curves (*diagram f*)), regardless of whether the specimen stress is calculated via the 2-wave stress equation (3.34) or the 1-wave stress

¹It should be noted that for the present FE simulation, the ramp shaped incident wave was applied directly at the free end of the incident bar via an amplitude controlled distributed surface load and will therefore be oscillation-free by definition. From Figure 3.14b it is clear however that this is also the case for a real shaped incident wave.

equation (3.33). From this it is concluded that the specimen is always in dynamic equilibrium. It is further observed from *diagram f)* that the specimen stress is correctly determined by the SHPBA equations, when comparing the SHPBA stress-time with the FEM stress-time response, obtained directly from an element in the specimen mesh.

Other than in the example of the classic near rectangular incident wave, presented in the previous section, where the specimen deformation occurred mainly at the beginning of the incident wave and then remained at a constant strain level, it is seen from *diagram e)* in Figure 3.18 that in the case of a ramp shaped incident wave, the specimen strain increases linear to the maximum value, which occurs at the time corresponding to the maximum strain amplitude of the incident wave. It is therefore concluded that the incident wave is used in an optimal way this time.

An issue that remains is the overprediction of the specimen strain by the SHPBA, as seen in *diagram e)*, where the SHPBA specimen strain-time response is compared with the specimen strain measured directly on the specimen.

In summary the following conclusions can be drawn:

- A ramp shaped incident wave must be used in the case of linear-elastic specimen behaviour to ensure a constant specimen strain rate.
- By using a shaped incident wave, the high-frequency oscillations can be eliminated and dynamic stress equilibrium can be achieved instantly.
- The specimen stress is well determined via the SHPB analysis.
- The specimen strain however is overpredicted by the SHPBA and in order to obtain a correct dynamic stress-strain response, the specimen strain should be measured directly on the specimen.

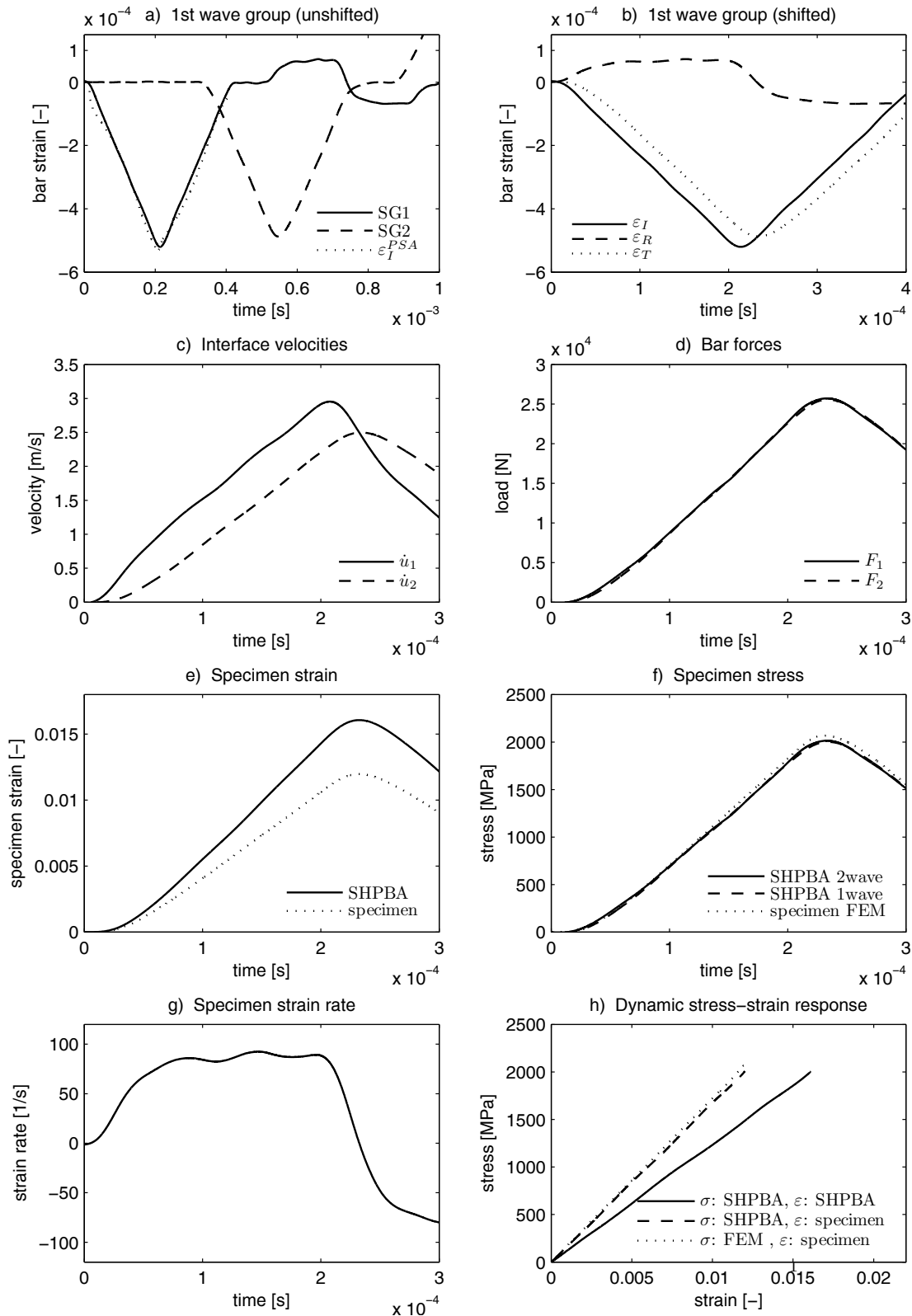


Figure 3.18: SHPBA for linear-elastic specimen and ramp shaped incident-wave.

3.3 Digital Image Correlation

The conventional and well established strain gauge method is widely used to measure the strain of small coupons up to large structures subjected to mechanical or thermal loading. This method is however cumbersome during the setup and the strain gauge must be in contact with the measured object. It is generally accepted that large strain measurements with foil strain gauges must be corrected due to the transverse sensitivity of the measuring grid and non-linearity errors [68], yet there is no consensus regarding the non-linearity issue [69]. In addition the strain range which can be measured with standard foil strain gauges is generally limited to about 5% strain.

Aside from the well established pointwise strain gauge technique, a number of contactless full-field optical methods have recently been proposed. They can be classified into interferometric techniques, such as holography, speckle and moire interferometry, and non-interferometric techniques, such as grid methods and digital image correlation (DIC) [70]. Compared with the DIC method, interferometric methods have a higher performance in terms of spatial resolution but must generally be conducted on vibration isolated optical platforms and require the use of laser equipment. The DIC method has less stringent setup requirements and can be applied from micro to large structures outside of the laboratory environment. Therefore the DIC method is increasingly used in the areas of experimental solid mechanics and structural testing.

Some of the main advantages of the DIC method over traditional techniques such as strain gauging are:

- Contact-less measuring technique.
- Full displacement and strain field instead of pointwise data.
- Strain measurements possible within a range of 0.01% to several 100%.
- No corrections necessary since true strain is measured.
- Camera images reveal the deformation and failure mechanisms.

- Fast specimen turn around and fast specimen preparation.
- In-plane (2D) and out-of-plane (3D) measurements possible.
- Displacement and strain field data can be used to verify FE simulations.
- Measuring object size may range from microscopic to large structure scale.
- Depending on the digital camera type, measurements can be performed from static to ultra high-speed loading rates.

The 2D DIC technique for measuring in-plane displacement and strain was used for the majority of experiments presented in this work. An overview and some details of this advanced displacement and strain measuring technique will therefore be given in this section. A more detailed discussion of the DIC technique is provided by Pan et al. [70] and Sutton et al. [71].

DIC is an optical method based on digital image processing and numerical computing [70]. A CCD camera captures the deformation of the specimen at a user defined frame rate. The digital images are then post-processed by a DIC software package, which compares the undeformed reference state with the deformed states.

Figure 3.19 shows a typical 2D DIC experimental setup. White light sources are required to illuminate the specimen surface. The CCD camera must be perpendicular to the specimen surface and area of interest and the digital camera images are exported to a PC for post-processing via the DIC software.

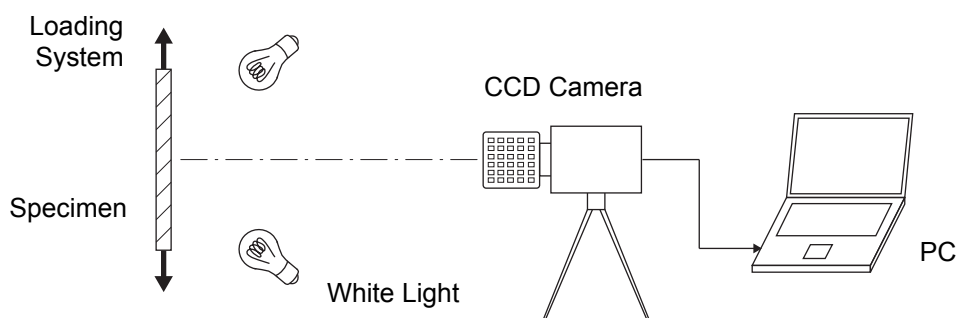


Figure 3.19: Typical 2D DIC experimental setup (after [70]).

The specimen must be prepared for the DIC measurement if the natural texture of the specimen surface does not have a random gray intensity distribution, required for this technique. Aerosol spray painting is generally used to generate a random black-on-white speckle pattern, which must deform together with the specimen and therefore acts as a carrier of the deformation information. The speckle size is chosen in accordance with the camera resolution, measuring object size, and the required displacement and strain field resolution. Figures 3.20 and 3.21 show examples for quasi-static/high-resolution and dynamic/low-resolution CCD camera images used in this work.

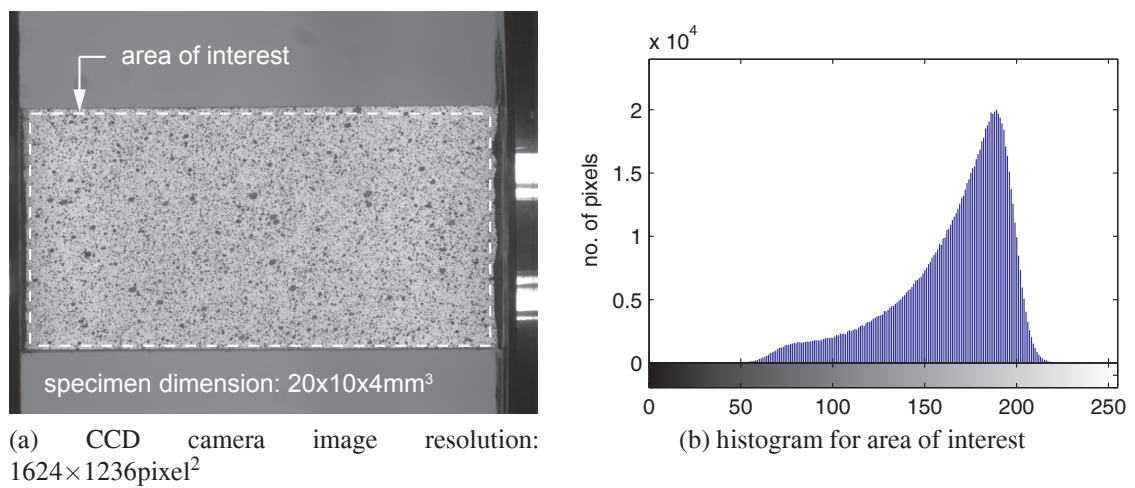


Figure 3.20: Example of high-resolution CCD image and gray-scale distribution of measuring area.

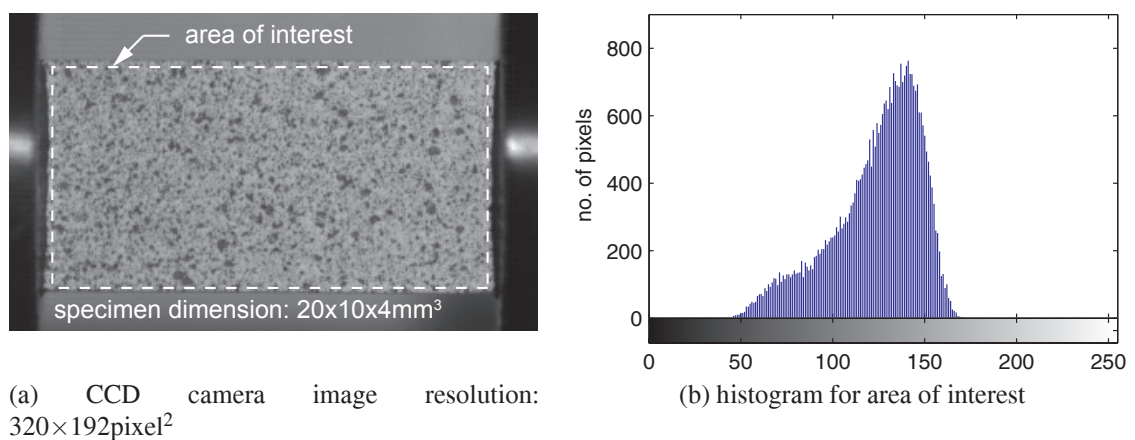


Figure 3.21: Example of low-resolution CCD image and gray-scale distribution of measuring area.

The gray intensity distribution (histogram) can be adjusted by the intensity of the light source and the aperture of the camera lens. It should be well balanced in the midrange of the gray-scale to allow correct processing via the DIC software.

To calculate the specimen deformation and strain field with the DIC software, the area of interest must be divided into evenly spaced pixel subsets or facets. Each facet contains a user defined horizontal and vertical number of pixels (facet size). The facets may overlap each other in the horizontal and vertical direction by a user defined value of pixels, which is called the facet step. As shown in Figure 3.22, each facet then corresponds to a value in the displacement field.

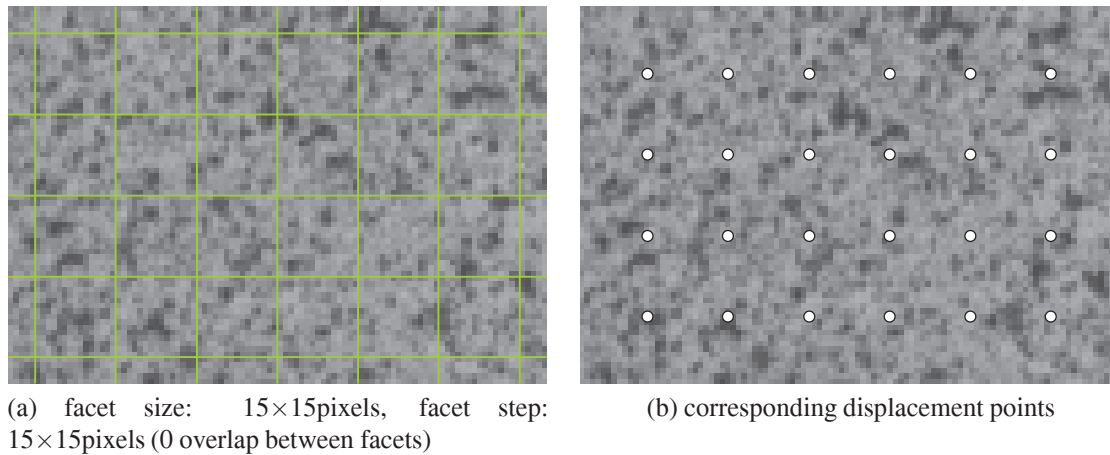


Figure 3.22: Typical facet size and corresponding displacement field points.

The principle of 2D DIC is the tracking of the same pixels between two images recorded before and after deformation [70], as illustrated in Figure 3.23. Facets or subsets are used since a subset of pixels contains a wider range of gray scale levels, which distinguishes itself from other subsets. Each facet can therefore be uniquely identified in the deformed state.

Pan et al. [70] provide a number of different correlation criteria which are used to evaluate the similarity between the reference and deformed subset. As shown in Figure 3.23, the difference between the position of the reference subset centre and the deformed subset centre then defines the displacement vector for the respective facet.

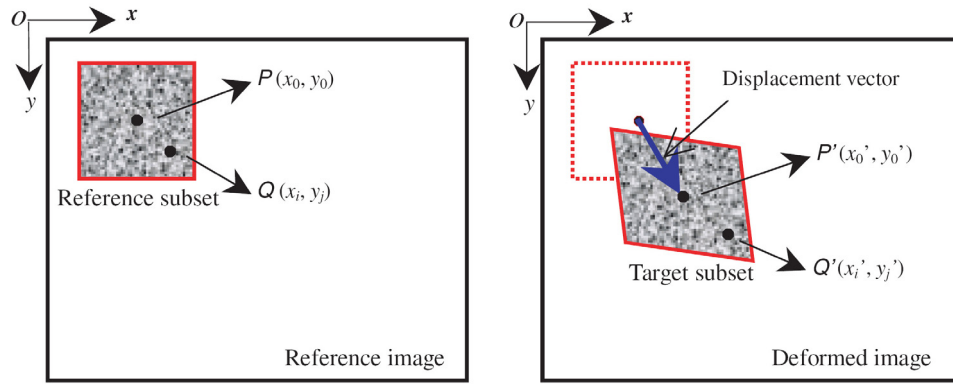


Figure 3.23: Reference subset before deformation and deformed subset [70].

First order shape functions are commonly used to evaluate the shape change of the deformed subset and allow the determination of translation, rotation, shear, normal strains and their combinations. According to the shape functions, any point $Q(x_i, y_j)$ around the subset center point $P(x_0, y_0)$ can be mapped to the point $Q'(x'_i, y'_j)$ in the deformed subset.

$$\begin{aligned} x'_i &= x_i + \xi_1(x_i, y_i) \\ y'_j &= y_j + \eta_1(x_i, y_i) \end{aligned} \quad (i, j = -M : M) \quad (3.45)$$

In Equation (3.45), M is defined by the reference facet size of $(2M+1) \times (2M+1)$ pixels. The first order shape functions ξ_1 and η_1 are defined as

$$\begin{aligned} \xi_1(x_i, y_i) &= u + u_x \Delta x + u_y \Delta y \\ \eta_1(x_i, y_i) &= v + v_x \Delta x + v_y \Delta y \end{aligned} \quad (3.46)$$

where $\Delta x = x_i - x_0$, $\Delta y = y_i - y_0$, u, v are the x- and y-directional displacement component of the reference subset centre point, and u_x, u_y, v_x, v_y are the first order displacement gradients of the reference subset.

Once the deformation field is determined, the strain field can be calculated using again a subset of points in the displacement field. This subset of displacement points is called the computation size. The smallest possible computation size is 3×3 , which may be used to determine very local strain information, provided that the spatial resolution of the area

of interest was sufficiently high and hence a high resolution displacement field could be determined.

A detailed description of the deformation and strain field calculation procedures and a detailed summary of possible DIC error sources is provided by Pan et al. [70]. Another valuable source of information regarding the basic concepts, theory and applications of image correlation is the book of Sutton et al. [72].

It is noted that the GOM image correlation software package ARAMIS was used in this work [73].

Chapter 4

Experiments: Longitudinal Compression

4.1 Introduction

It is widely accepted that the longitudinal *tensile* properties of UD carbon-epoxy composites are not strain rate sensitive, as demonstrated in an early study by Harding and Welsh [10] and confirmed in a recent study by Zhou et al. [5]. For the longitudinal *compressive* properties however, a variety of results were reported by different authors. Hsiao and Daniel [18] used a falling weight impact tower and thick composite specimens with bonded steel end caps. They reported an increase of longitudinal compressive strength at higher strain rates but found no rate effects for the longitudinal compressive modulus of elasticity. Hosur et al. [20] tested cubic compression samples using a recovery split-Hopkinson pressure bar and observed a significant increase for the modulus and a relatively small increase for strength under dynamic loading. Bing and Sun [21] proposed an extrapolation method to obtain the longitudinal compressive strength over a wide range of strain rates from off-axis compression test specimens and reported a linear increase of the longitudinal compressive strength with increasing strain rate. Yokoyama and Nakai [23] tested cubic rectangular specimens with a conventional split-Hopkinson

pressure bar (SHPB) and reported an increase for the initial compressive modulus but concluded that the compressive strength is not strain rate sensitive. Wiegand [22] studied the high strain rate compressive behaviour in fibre direction using flat rectangular specimens, made from a cross-ply laminate and tested in a SHPB with clamped slotted end caps, and observed a linear increase of the compressive strength over a wide range of strain rates.

The above examples show that the effect of strain rate on the longitudinal compressive behaviour of carbon-epoxy composites is still subject of debate. In particular the rate effect on the longitudinal compressive modulus needs to be further investigated. From those authors that used a SHPB for the dynamic tests, Hosur et al. [20] and Yokoyama and Nakai [23] reported an increase of the modulus with increasing strain rate whereas Bing and Sun [21] found no strain rate effect. Wiegand [22] excluded the modulus from his study since dynamic equilibrium was established late in the experiment.

The objective of the present section is to identify the effects of strain rate on the longitudinal compressive modulus and on the longitudinal compressive strength of carbon-epoxy composites using a SHPB for the dynamic tests. Significant attention is given to the reliability of the SHPB experiment by addressing potential issues such as dispersion correction, dynamic stress equilibrium and pulse shaping to obtain constant strain rate experiments which allow a direct comparison of the dynamic results with the quasi-static reference case. If dynamic equilibrium is achieved early in the test, the entire dynamic specimen stress strain curve can be used and the dynamic longitudinal compressive modulus can be measured with confidence.

4.2 Design of Dynamic Experiment

The SHPB experiment used here was developed using the finite element (FE) program ABAQUS to evaluate different incident pulse shapes ϵ_I and the corresponding reflected and transmitted wave signals ϵ_R and ϵ_T , respectively. As pointed out by Nemant-Nasser et al. [66] a monotonically increasing ramp-like incident pulse is best suited to obtain

a constant strain-rate SHPB test if the specimen behaviour is linear-elastic up to failure. Since it is difficult to obtain the required incident pulse shape by trial and error, the pulse shaping analysis proposed by Nemat-Nasser et al. [66] was used to determine the parameters for the ramp-like incident pulse.

Due to the higher strength of the longitudinal carbon-epoxy specimen compared to the strength of the bar material of the SHPB, Tungsten-Carbide (TC) inserts with a diameter of 12mm and a thickness of 5mm were placed on both sides of the specimen. The high strength and very stiff TC-inserts reduce the stress concentrations at the specimen ends and avoid indentation of the bar-ends [62]. To ensure that the reflection and transmission of the bar strain waves occurs at the specimen end-surfaces and not at the bar/TC-inserts, the mechanical impedance of the TC-inserts must be equal to the mechanical impedance of the 18mm steel bars. The mechanical impedance Z is defined as $Z = A\rho c$, where A is the cross-section perpendicular to the direction of the longitudinal wave, ρ is the material density and c is the longitudinal wave velocity.

A specimen fixture, referred to as Dynamic Compression Fixture (DCF), was designed to align and stabilise the relatively thin rectangular specimen. The final design of the dynamic experiment is shown in Figure 4.1.

The specimen gauge length is the entire specimen length since the friction between DCF plate and specimen surface is insufficient to transfer the load into the specimen via shear.

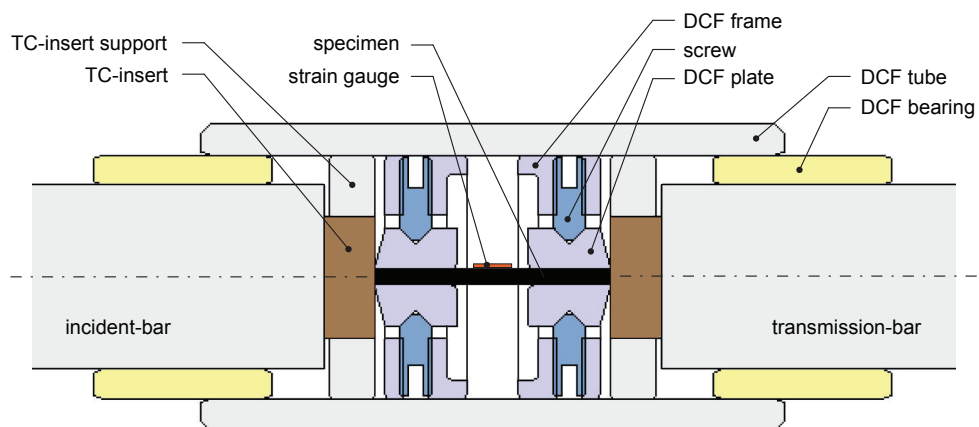


Figure 4.1: SHPB test setup with dynamic compression fixture (DCF).

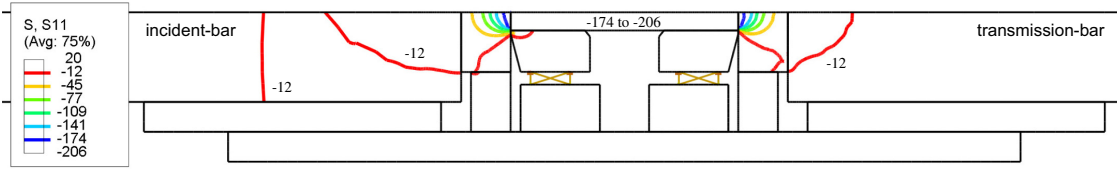
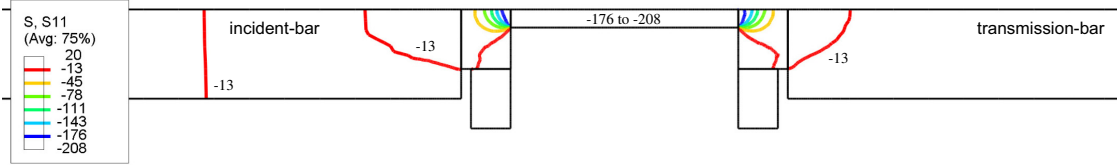
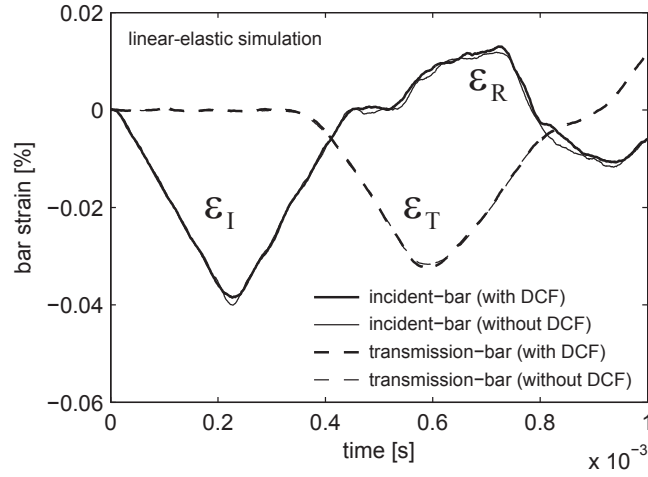
Therefore the load introduction in the experiment can be seen as a supported end-loading method. The relatively low amount of transverse pre-compression, applied to the specimen due to the DCF plate and a *finger-tight* tightening of the screws, is considered to not affect the experimental results and was consistently maintained for the quasi-static and dynamic tests.

To allow an undisturbed passing of the bar strain waves, a gap was left between the SHPB bars and the TC-insert support. Also, the DCF plates were chamfered, leaving only a minimal contact surface at the TC-insert/specimen-interface (Figure 4.1). With the specimen fixture, additional parts are introduced into the SHPB setup, and it must be ensured that those additional parts do not influence the bar waves and subsequently the dynamic specimen stress response, which is calculated from these waves via SHPB analysis.

Using a simple linear-elastic axis-symmetric model (ABAQUS element type CAX4R), two FE simulations were performed to verify that the bar waves are not influenced by the parts of the DCF. For the first simulation (Figure 4.2a) all parts of the DCF were included while in the second simulation (Figure 4.2b) the DCF was removed.

A comparison of the two simulations, using an ideal triangular pulse as obtained from a 500mm long striker-bar fired at a velocity of 5m.s^{-1} and an appropriate pulse shaper, is shown in Figure 4.2c where it is seen that the bar-strain response is very similar in both cases. It is important to note that the comparison of the bar strain wave response with and without DCF parts, shown in Figure 4.2c, can only be done with the FE model and should be understood as a way of checking the functionality of the designed DCF fixture. The actual rectangular specimen is too thin and can not be tested without being supported by a fixture.

It is further noted that the bar strain wave response shown in Figure 4.2c resulted from a linear-elastic simulation, without considering specimen failure. It will be shown later that in a real SHPB experiment, the amplitude of the transmitted wave is lower due to the failure of the specimen.

(a) simulation with DCF, arrival of incident wave ϵ_I at the specimen, $S11 = \sigma_{11}$ (MPa)(b) simulation without DCF, arrival of incident wave ϵ_I at the specimen, $S11 = \sigma_{11}$ (MPa)

(c) comparison of predicted bar strain waves

Figure 4.2: FE simulations of SHPB test using an axis-symmetric 2-dimensional model and predicted bar strain waves.

In the FE model, the striker-bar and the pulse shaper were not included. Instead, the incident pulse ϵ_I was applied directly at the impact end of the incident-bar via a distributed surface pressure. A zero-friction contact was defined between all parts, except for the contact between TC-insert and TC-insert support, where a fixed connection was assumed. The assumption of a zero-friction contact is justified since the purpose of the FE simulations is to check the influence of the DCF parts on the wave propagation rather than modelling realistic friction behaviour at the interfaces. For the screw connection between the DCF plate and the DCF frame, connector elements were used (see Figure 4.2a). The

cross section of the specimen in the axis-symmetric FE model corresponds to the cross section of the rectangular specimen. The elastic material properties and densities of all parts used in the simulation are summarised in Table 4.1. The orthotropic elastic mechanical properties of the composite specimen were taken from an earlier quasi-static material characterisation program and are presented in Table 4.2 [74].

When analysing the simulation results, a small relative motion was observed between the DCF-frame/DCF-plate unit and the specimen at the transmission bar end, which can be contributed to inertia effects.

Table 4.1: Elastic mechanical properties of isotropic materials used for FE simulation.

Material	Model Component	Modulus E [MPa]	Poisson's Ratio ν [-]	Density ρ [kg/m ³]
Steel	SHPB-bars TC-insert support DCF tube DCF plates	206000	0.29	7850
Aluminum	DCF frame	70000	0.33	2820
Tungsten-Carbide	TC-insert	585000	0.22	14450
Tigra T10MG				
Nylon 6	DCF-bearing	2600	0.39	1130

Table 4.2: Orthotropic elastic mechanical properties of Hexply IM7-8552 used for FE simulation [74].

Modulus E [MPa]		Poisson's Ratio ν [-]		Shear Modulus G [MPa]	
E_1^a	171420	ν_{12}^a	0.32	G_{12}	5290
E_2^a	9080	ν_{13}^c	0.32	G_{13}^c	5290
E_3^c	9080	ν_{23}^b	0.5	G_{23}^c	3974
Density ρ [kg/m ³]		1590			

^a Obtained from longitudinal (E_1) and transverse tensile tests (E_2).
^b Assumed value.
^c Calculated assuming transverse isotropy.

In Section 4.4.2 it will be shown that failure occurred at either the incident-bar side, the transmission-bar side or at both interfaces. Therefore it was concluded that the observed relative motion does not affect the experimental results.

The axis-symmetric FE model was also used to find a suitable specimen cross section, with respect to the available SHPB setup ($\varnothing 18\text{mm}$ steel bars). As shown in Figure 4.2c, a specimen cross section of $A_S = 10.5\text{mm}^2$ was found to generate both a strong reflected and a strong transmitted bar strain wave signal. A well balanced reflection/transmission behaviour at the bar/specimen interface is necessary to calculate reliable dynamic stress-strain curves via SHPB analysis. The specimen width and thickness were then set to 7mm and 1.5mm, respectively to obtain the flat and rectangular specimen used in the present study.

A specimen length of 23mm was chosen as a result of the fabricable size of the DCF parts and the need to place a linear strain gauge of the type HBM 1-LY11-1.5/350 onto the unsupported specimen section (Figure 4.1). The dimensions of the test specimen were therefore defined within the framework of the SHPB experiment and then also used for the quasi-static experiment to maintain consistency.

4.3 Experimental Setup

4.3.1 Material, Specimen and Quasi-Static Test Setup

The intermediate-modulus fibre and toughened epoxy resin material system HexPly IM7-8552 is commonly used in the aerospace industry for primary structure components. In accordance with the pre-preg curing cycle, a 12-ply unidirectional plate was manufactured on a SATIM hot press. From this panel, specimens with nominal dimensions of $23 \times 7 \times 1.5\text{mm}^3$ (length $\times$ width $\times$ thickness) were cut on a water-cooled diamond saw. After cutting, the surface parallelism tolerances of all opposing surfaces were found to be within 0.02mm and the loading surfaces were of good quality. A further treatment of the loading surfaces was therefore not performed.

Rather than using cubic or cylindric specimens, which are common for SHPB experiments [20, 21, 23], flat rectangular specimens, similar to those of the well accepted quasi-static test standard ASTM D 3410 [75] were chosen. It is noted that in the ASTM D 3410 quasi-static test method the load is introduced into the specimen via shear at the clamped ends. For the present study it was decided to use end-loaded specimens since this is the load introduction method best suited for SHPB experiments.

All specimens were equipped with one linear strain gauge of the type HBM 1-LY11-1.5/350 at the specimen centre (see Figure 4.1). The strain gauge wires were extended to allow the mounting of the strain gauge terminal on the outside of the DCF. Figure 4.3 shows the DCF-specimen unit and the quasi-static test setup, where load adapters were used at the top and bottom to replace the incident- and transmission-bar. Friction between the TC-inserts and the specimen end-surface (Figure 4.1) was reduced by applying a thin layer of lubricant. The quasi-static tests were performed on a MTS-810 servo-hydraulic test machine. For data acquisition, the load cell of the test machine and the specimen strain gauge were connected to a HBM Spider-8 data acquisition system.

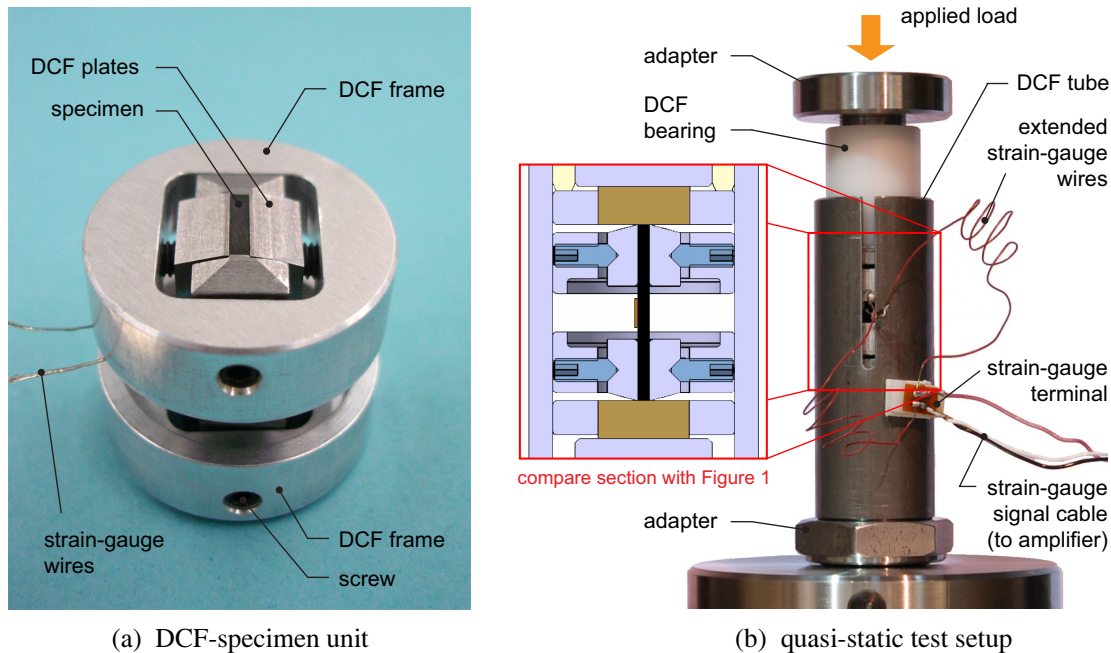


Figure 4.3: DCF-specimen unit (a) and quasi-static test setup (b).

4.3.2 Dynamic Experimental Setup

Dynamic tests at strain rates between 63s^{-1} and 118s^{-1} were carried out on a SHPB setup, as shown in Figure 4.4. The SHPB consisted of $\varnothing 18\text{mm}$ steel striker-, incident- and transmission-bars with lengths of 0.5m, 2.6m and 1.7m, respectively. The incident-bar strain gauges are positioned in the middle of the incident-bar and hence at a distance of 1.35m away from the specimen interface. On the transmission-bar, the strain gauges are positioned 0.3m away from the specimen interface.

The bar strain gauges are operated in bending compensation mode. An independent measurement of bending waves, which might be necessary when testing off-axis compression specimens due to the occurring extension-shear coupling effect [21], was not considered necessary in the present study.

The impact-velocity V_0 of the striker-bar was about 6.1m.s^{-1} . For all tests, copper pulse shapers with a diameter of 4mm and a thickness of 1mm were used. The pulse shapers were manufactured from a 1mm copper plate on a sheet metal stamping machine.

Friction between the TC-inserts and the specimen end-surface (Figure 4.1) was reduced by applying a thin layer of lubricant. The SHPB bar strain signals were amplified by PICO CM015 10V bridge signal conditioner modules. For the specimen strain gauge, a FYLDE FE-H379-TA high speed transducer amplifier was used. The amplified bar and strain gauge signals were recorded with a TEKTRONIX TDS3014B oscilloscope and exported to a PC for further data processing. The DCF setup for SHPB testing is shown in Figure 4.5a and a typical specimen strain gauge signal and bar strain response, with a ramped incident-pulse ϵ_I , is shown in Figure 4.5b.

It is noted that the SHPB theory relation for the incident pulse duration $T = 2l_{striker}/c_B$, where $l_{striker}$ is the striker-bar length and c_B is the elastic wave velocity of the bar material, is not valid for shaped incident pulses. The duration of the shaped incident pulse is always longer than the duration of a classic rectangular pulse and depends on the dimensions of the pulse shaper. It can however be determined by the pulse shaping analysis proposed by Nemat-Nasser et al. [66] (Chapter 3).

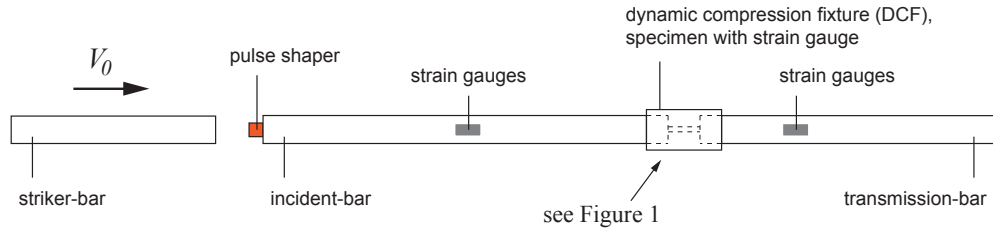
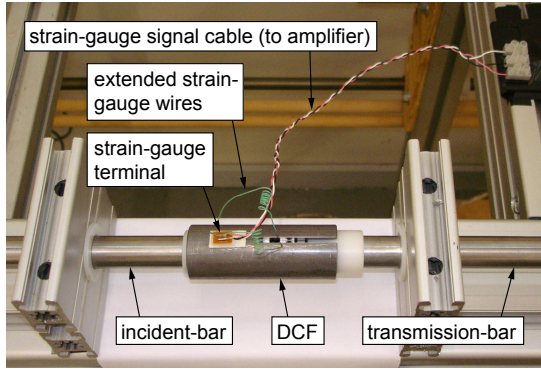
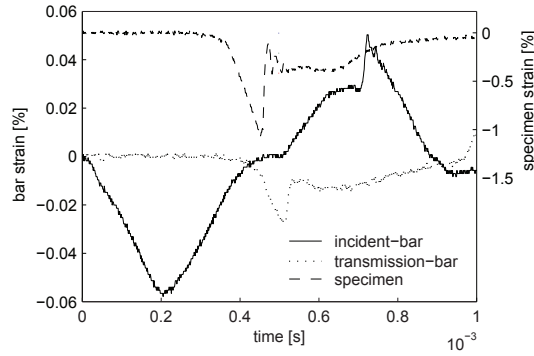


Figure 4.4: Split-Hopkinson pressure bar configuration.



(a) DCF setup for SHPB



(b) typical gauge signals

Figure 4.5: Dynamic test setup (a) and bar strain waves with ramp shaped incident pulse and specimen strain gauge signal (b).

4.4 Experimental Results

4.4.1 Quasi-Static Experimental Results

Five quasi-static compression tests were performed. To check the functionality of the dynamic compression fixture, the first specimen was loaded and unloaded at a displacement rate of $0.1\text{mm}\cdot\text{min}^{-1}$ to stress levels of 350MPa, 500MPa and 700MPa, before being loaded until failure. All further specimens were loaded until failure at a displacement rate of $0.5\text{mm}\cdot\text{min}^{-1}$ which corresponds to a quasi-static strain rate of $\dot{\epsilon}_{qs} = 3.6 \times 10^{-4}\text{s}^{-1}$, considering the nominal specimen length of 23mm. Due to a non-functioning strain gauge for specimen No. 4, only the failure load was recorded in this case. Figure 4.6 shows the longitudinal compressive stress-strain response for all quasi-static specimens, where the end points of each curve correspond to the failure load. For clarity in Figure 4.6, the stress-strain data was truncated after the maximum load.

The quasi-static longitudinal compressive strength X_C^{qs} was obtained by dividing the fail-

ure load by the specimen cross-section. Following the test standard ASTM D 3410 [75], the quasi-static longitudinal compressive modulus of elasticity E_{1C}^{qs} was obtained from the specimen stress-strain curve between a strain range of $1000\mu\epsilon$ and $3000\mu\epsilon$. The quasi-static experimental results are summarised in Table 4.3.

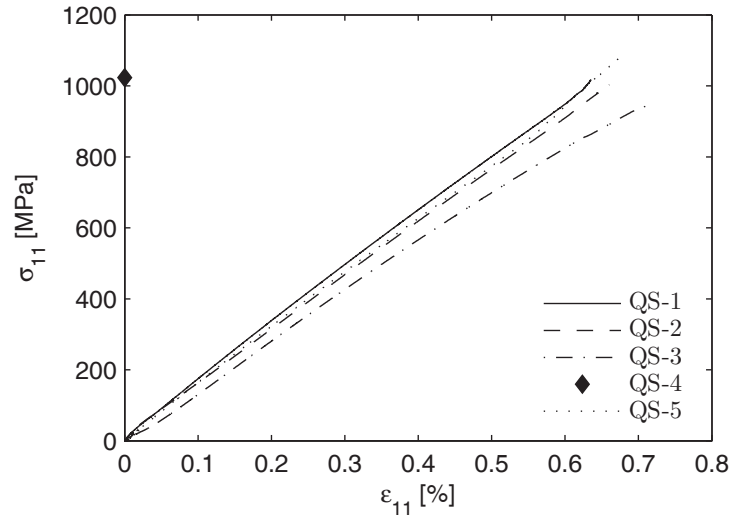


Figure 4.6: Quasi-static longitudinal compressive stress-strain response.

Table 4.3: Quasi-static experimental results.

Test	Strain Rate $\dot{\epsilon}$ [s^{-1}]	Modulus E_{1C}^{qs} [MPa]	Strength X_C^{qs} [MPa]
QS-1	7.2×10^{-5}	160992	1018
QS-2	3.6×10^{-4}	152485	1002
QS-3	3.6×10^{-4}	148167	946
QS-4	3.6×10^{-4}	-	1023
QS-5	3.6×10^{-4}	156301	1093
Mean	-	154486	1017
STDV	-	5464	53
CV (%)	-	3.5	5.2

All quasi-static specimens failed at either the upper or the bottom end-surface in a through-thickness fibre shear type mode as shown in Figure 4.7. It is likely that the observed failure mode is a boundary induced phenomenon since the average quasi-static longitudinal compressive strength of $X_C^{qs} = 1017\text{MPa}$ (Table 4.3) is lower than the quasi-static longitudinal compressive strength reported for a similar material system in [21]. A similar observation was made by Hsiao and Daniel [18] when comparing the quasi-static longitudinal compressive strength obtained from a pure end-loading method with the strength value obtained from a combined shear/end loading method.

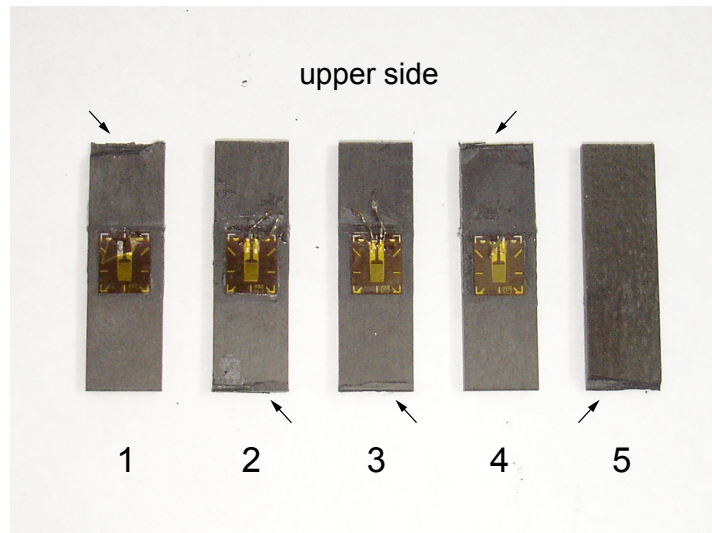


Figure 4.7: Quasi-static specimen failure mode (arrows indicate failure position).

4.4.2 Dynamic Experimental Results

Five dynamic tests were performed. Classic SHPB analysis was used to calculate the loads F_1 (Equation (3.29)) and F_2 (Equation (3.30)), acting at the incident-bar and transmission-bar side of the specimen, respectively. The specimen strain $\epsilon_S(t)$, the specimen strain rate $\dot{\epsilon}_S(t)$ and the specimen stress $\sigma_S(t)$ were determined with Equations (3.28), (3.27) and (3.35), respectively.

It is noted that for the results presented in this section, the specimen strain and the specimen strain rate were obtained directly from a strain gauge mounted on the specimen. The

specimen strain and strain rate as calculated via the SHPB analysis equations (3.28) and (3.27), respectively, are only used for comparison.

Since the bar strain waves ϵ_I , ϵ_R and ϵ_T are measured away from the specimen and at times before and after they have acted on the specimen, dispersion correction methods, as proposed by Lifshitz and Leber [57] or Zhao and Gary [58], are generally required to correct the bar waves before applying the SHPB analysis. By using shaped incident pulses, as in the present study, the dispersion effects can be minimised since the high frequency oscillations, usually present at the beginning of a classic incident pulse, are avoided. As shown by Ninan et al. [76], a bars-apart (BA) test with uncoupled incident- and transmission-bars, can be used to evaluate the amount of dispersion still present in the SHPB setup. Figure 4.8 shows the incident- (sign inverted) and reflected-wave signal from a BA-test performed with the present SHPB configuration. The shape, duration and amplitude of the two wave signals are nearly identical and therefore it is concluded that dispersion effects can be neglected in the present study.

A post-processing example of the bar strain waves via SHPB analysis is presented for a representative specimen in Figure 4.9. The bar strain waves must be shifted first to the time corresponding to the moment at which they have acted on the specimen (Figure 4.9a). A comparison of the loads F_1 and F_2 , calculated with Equations (3.29) and (3.30),

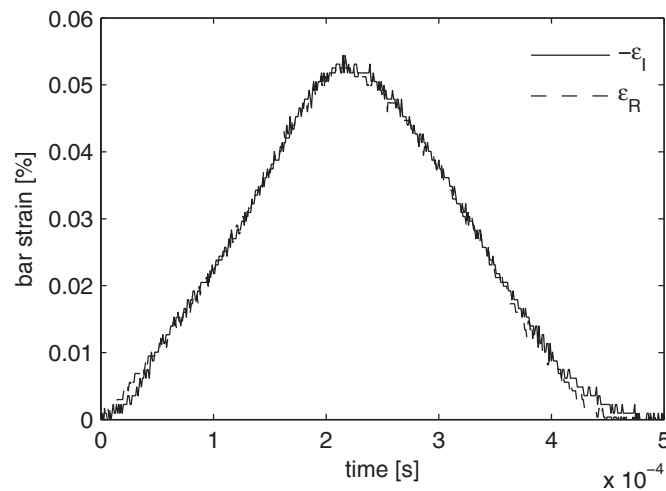


Figure 4.8: Incident- and reflected bar waves of bars-apart (BA) test with present SHPB configuration.

respectively, is shown in Figure 4.9b. It is seen that the two load-time responses are very similar and therefore it is concluded that dynamic equilibrium is established early in the experiment. As a result it is possible to obtain the dynamic longitudinal compressive chord modulus of elasticity from the dynamic stress-strain response. Figure 4.9c shows the specimen strain calculated with equation (3.28) and also the specimen strain obtained directly from the specimen strain gauge. It is seen that the specimen strain gauge response correlates very well with the load-time response shown in Figure 4.9b, whereas the specimen strain calculated with equation (3.28) is significantly overpredicted. Figure 4.9d shows the strain rate-time response as obtained from equation (3.27) and the strain rate determined from the slope of the specimen strain gauge signal (Figure 4.9c). Again the strain rate-time response obtained from the specimen strain gauge signal correlates very well with the loading history of the specimen shown in Figure 4.9b and proves that the specimen was loaded at a near constant strain rate, which should be a fundamental condition for any material characterisation test. Since the specimen strain calculated with equation (3.28) is overpredicted, the strain rate obtained from the SHPB analysis equation (3.27) is also overpredicted and therefore does not represent the actual specimen strain rate.

Figure 4.10 shows the dynamic specimen stress-strain response of the specimen also used for Figure 4.9. In Figure 4.10, the stress-strain response calculated entirely via SHPB analysis is compared with the stress-strain response where the specimen strain is obtained directly from the specimen strain gauge. For both curves, the specimen stress was calculated using the 3-wave stress equation (3.35). From Figures 4.9c and 4.9d it is clear that the stress-strain curve with the strain measured from the specimen strain gauge is the correct result and is therefore used for further data reduction.

The longitudinal compressive stress-strain response of all dynamic tests is presented in Figure 4.11. The noise in the curves shown in Figure 4.11a is caused by digitalisation of the bar and specimen strain gauge signals in the oscilloscope. A further data treatment was therefore necessary to eliminate this noise. The stress-strain curves shown in Figure

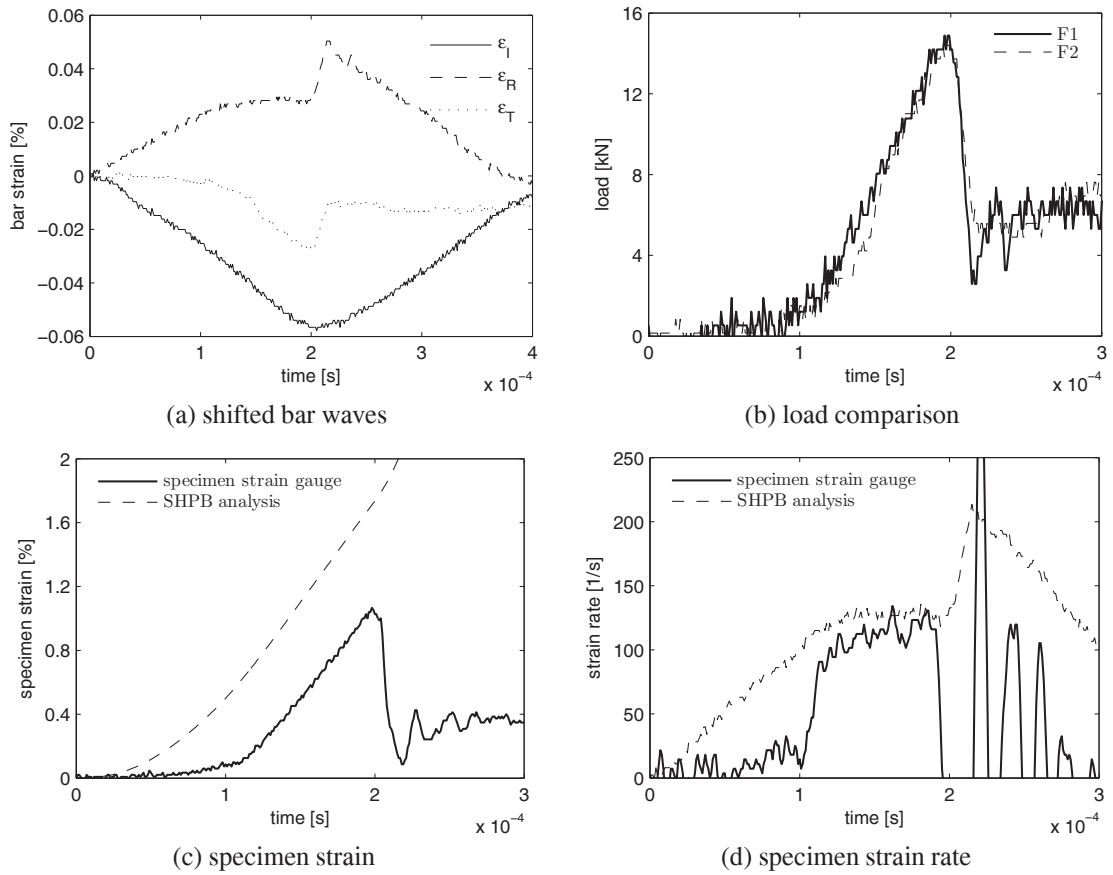


Figure 4.9: SHPB analysis results for a representative dynamic specimen.

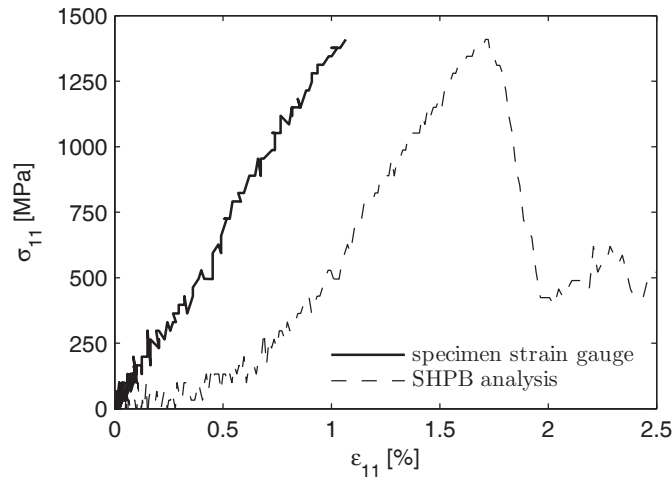


Figure 4.10: Dynamic longitudinal compressive stress-strain response for a representative dynamic specimen.

4.11a were replaced by quadratic trend lines and a zero-shift along the stress-axis was performed (omitting the x^0 -term of the trend line equation). The final dynamic stress-strain curves are shown in Figure 4.11b. From this diagram the dynamic longitudinal

compressive strength X_C^{dyn} was obtained as the maximum of each curve. The dynamic longitudinal compressive chord modulus of elasticity E_{1C}^{dyn} was calculated between the strain range of $1000\mu\epsilon$ and $3000\mu\epsilon$ as recommended by the test standard ASTM D 3410 [75], applicable in quasi-static cases. The dynamic test results are summarised in Table 4.4.

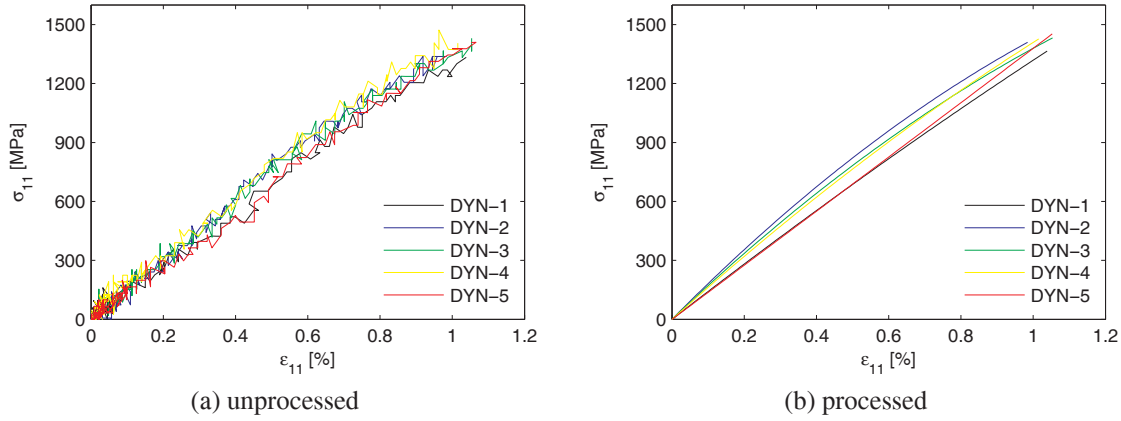


Figure 4.11: Dynamic longitudinal compressive stress-strain response.

The failure mode for the dynamic specimens is shown in Figure 4.12. All specimens failed at the end-surfaces. In particular the failure mode of dynamic specimen 1 and 4 appears to be very similar to the failure mode observed for the quasi-static case (Figure 4.7).

Table 4.4: Dynamic experimental results.

Test	Strain Rate $\dot{\epsilon}$ [s^{-1}]	Modulus E_{1C}^{dyn} [MPa]	Strength X_C^{dyn} [MPa]
DYN-1	118	138350	1365
DYN-2	63	168590	1409
DYN-3	77	160250	1432
DYN-4	97	155320	1427
DYN-5	110	137550	1453
Mean	93	152012	1417
STDV	23	13688	33
CV (%)	25.0	9.0	2.3

The inclination of the fracture surface for the dynamic specimen 2, 3 and 5 indicates a kink-band type failure mode. It is important to note however that a clear identification of the dynamic failure mode from the recovered specimen is difficult in the classic SHPB experiment. Although it was observed that all dynamic specimens failed at the passing of the first wave group, as shown for a representative specimen in Figure 4.9, the initial failure mode may be obscured since the reflected wave ϵ_R and the transmitted wave ϵ_T return and reload the specimen after being reflected at the free ends of the incident- and transmission-bar. The fact that failure occurred at either the incident-bar side, the transmission-bar side or at both interfaces, as for dynamic specimen 3, proves that dynamic force equilibrium was established in the experiment.

It was noted earlier that the measured quasi-static longitudinal compressive strength X_C^{qs} is likely to be lower than the actual quasi-static strength value since failure always occurred at the specimen end-surfaces. A similar conclusion can be drawn for the dynamic longitudinal compressive strength X_C^{dyn} reported in Table 4.4. Despite this limitation, the trend of the strain rate effect on the longitudinal compressive strength was captured by the present method since failure mode and position were consistent for both strain rate regimes.

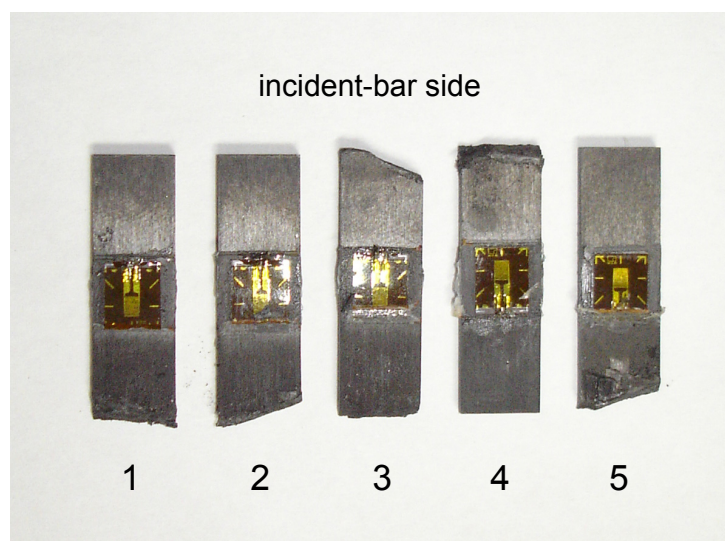


Figure 4.12: Dynamic specimen failure mode.

The longitudinal compressive stress-strain responses of both load regimes, quasi-static and dynamic, are compared in Figure 4.13 where an increase of about 40% is observed for the longitudinal compressive strength while the longitudinal compressive modulus is not strain rate sensitive.

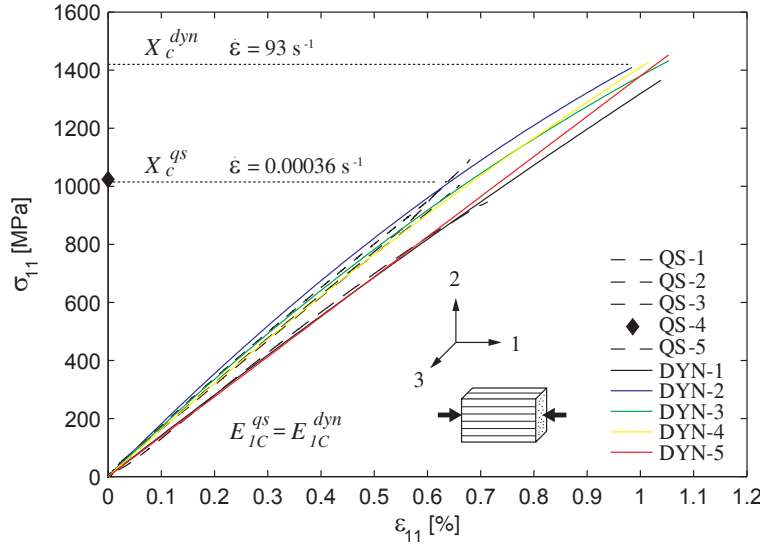


Figure 4.13: Comparison of quasi-static and dynamic longitudinal compressive stress-strain response.

4.5 Conclusion

A clear identification of the effect of strain rate on the longitudinal compressive properties of unidirectional carbon-epoxy composites was presented in this section. Significant attention was given to the reliability of the SHPB experiment by addressing potential issues such as dispersion correction, dynamic stress equilibrium and pulse shaping. A ramp shaped incident-pulse was used since it is then possible to test a specimen with linear stress-strain behaviour up to failure at a constant strain rate. The pulse shaping analysis proposed by Nemat-Nasser et al. [66] was adopted to obtain the required parameters for the ramp shaped incident-pulse. Via pulse shaping, the high frequency oscillations at the beginning of the incident pulse are avoided and hence dispersion effects are minimised.

Careful pulse shaping and proof of dynamic stress equilibrium established early in the transient loading process gives credibility to the dynamic elastic modulus measurements. It was demonstrated that a better dynamic stress-strain response can be obtained when using a specimen strain gauge, compared to the stress-strain response calculated entirely from the bar-waves via SHPB analysis.

The new fixture required to align and stabilise the flat rectangular specimen used in this study was designed via explicit finite element analysis to not influence the bar strain waves and subsequently the results of the SHPB analysis. The numerical predictions were later confirmed by the dynamic experiments performed with the SHPB.

Quasi-static tests at a strain rate of $3.6 \times 10^{-4} \text{s}^{-1}$ and dynamic tests at strain rates up to 118s^{-1} were performed with the developed test setup, and the following conclusions were drawn:

- The longitudinal compressive chord modulus of elasticity is not strain rate sensitive up to the strain rates considered in this study.
- The longitudinal compressive strength increased by about 40%.

The results presented in this section correlate well with those of Hsiao and Daniel [18] and with the observations of Bing and Sun [21] and Wiegand [22], regarding the dynamic longitudinal compressive strength. It is noted that Hsiao and Daniel reported a higher strength increase at a similar strain rate level. This may have been caused by using a different material system, compared to the one selected in the present study, and a different dynamic test setup (Drop Tower).

Chapter 5

Experiments: Transverse Compression, In-Plane Shear and Combined Loading

5.1 Introduction

The strain rate effect on the transverse compression, in-plane shear and combined transverse compression and in-plane shear behaviour of unidirectional carbon-epoxy was investigated using the same experimental setup and is therefore presented together in this chapter.

The study of the strain rate effect on these properties has drawn significant attention by the international research community and with advances in experimental techniques it will continue to do so in the future.

Vogler and Kyriakides [77] studied the nonlinear behaviour of unidirectional AS4/PEEK in shear and transverse compression. A custom biaxial loading fixture was designed to perform biaxial tests under three loading history regimes: transverse compression in the presence of constant shear stresses, the shear response under the presence of constant transverse compression and proportional loading of transverse compression and shear. In addition, quasi-static and medium rate tests were performed for pure transverse compression, pure in-plane shear and neat PEEK resin. For the later experiments it was observed

that the increase of the transverse and of the in-plane shear strength was similar to the increase of the neat resin strength.

Hsiao et al. [19] performed high strain rate tests for the carbon-epoxy material system IM6G/3501-6 on a drop tower and split-Hopkinson pressure bar (SHPB) and reported results for dynamic transverse compressive modulus, strength and ultimate strain as well as in-plane shear modulus and strength. It was stated that it was not possible to monitor the failure process and the use of the Recovery SHPB, invented by Nemat-Nasser et al. [66], was suggested.

Hosur et al. [20] performed high strain rate compression tests on a Recovery SHPB for carbon-epoxy in the longitudinal and transverse material directions and further studied the dynamic compressive response of cross-ply laminates. Compared to the trends found by Hsiao et al. [19], Hosur et al. [20] reported a different strain rate behaviour for the unidirectional compression tests.

Tsai and Sun [30] performed SHPB experiments with 15° , 30° and 45° off-axis compression specimens to study the dynamic in-plane shear strength and failure strain of unidirectional glass-epoxy S2/8552. The pure in-plane shear strength at $\sigma_{22} = 0$ was extrapolated from the off-axis test data in the combined transverse compression/in-plane shear stress space.

In a recent study, Kawai and Saito [78] performed quasi-static and medium rate tension and compression tests on a servo-hydraulic testing machine at elevated temperature using tabbed off-axis carbon-epoxy specimens. A detailed discussion of the failure modes and fracture surface angles was given and modifications to well-known composite failure criteria, such as Tsai-Wu [79], Hoffmann [80], Tsai-Hill [81] and Hashin-Rotem [82], were proposed to consider the influence of transverse compression on the in-plane shear strength of unidirectional laminae.

Lee et al. [83] performed an optical study of loading-rate effects on the fracture behaviour of carbon-epoxy T800/3900-2 and reported an increase of the material fracture toughness for dynamic loading. The fracture toughness decreased with increasing de-

degrees of anisotropy for both quasi-static and dynamic loading.

In this chapter, the mechanical properties of the unidirectional carbon-epoxy material system IM7-8552 which are most effected by the visco-plastic response of the polymer matrix, namely the compressive and in-plane shear properties are investigated. A detailed knowledge of the strain rate effects on the properties in the main material directions and on the material behaviour under combined loading is required to validate and further develop existing composite constitutive models and failure criteria. High strain rate experiments were performed on the SHPB from which the complete dynamic elastic-plastic specimen stress-strain response can be obtained. Digital image correlation (DIC) techniques were used for the quasi-static and high strain rate experiments to obtain the in-plane strain field over the entire specimen surface. By using a high speed camera for the high strain rate experiment, fundamental assumptions of the SHPB technique were verified. In addition, the specimen failure was monitored and the angle of the fracture plane was identified. The relevance of DIC for SHPB experiments was demonstrated in a recent study by Gilat et al. [84]. A comprehensive review of the digital image correlation technique was given in Chapter 3.

5.2 Material and Experimental Procedures

5.2.1 Material and Test Specimens

The unidirectional carbon-epoxy prepreg system HexPly® IM7-8552 was selected for the present study. In accordance with the prepreg curing cycle, 32-ply unidirectional plates were manufactured on a SATIM hot press. From these panels, off-axis compression specimens with fibre orientation angles $\theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$ and transverse compression specimens ($\theta = 90^\circ$) were cut on a water-cooled diamond saw. All specimens have the same nominal dimensions of $20 \times 10 \times 4\text{mm}^3$ and are in accordance with the end-loading compression test standard ASTM D 695 [85]. An in-plane aspect ratio of 2 was chosen to obtain a larger homogeneous strain field not affected by edge effects

at the specimen end-surfaces. Prior to the tests with the DIC measurement system, a number of quasi-static transverse compression tests with back-to-back linear foil strain gauges were performed with a previous version of the quasi-static test setup, to evaluate the influence of bending within the relatively long specimen (Figure 5.1).

As shown in Figure 5.1b, the *percent bending value* was found to be within the limits of $\pm 10\%$ as defined by the test standard ASTM D 3410 [75]. Hence the DIC strain measurement from only one side was considered valid for all successive tests. End-loaded compression specimens were selected since complex test fixtures are not required and therefore the specimen is well suited for SHPB experiments. After cutting, the surface parallelism tolerances of all opposing surfaces were found to be within 0.02mm and the loading surfaces were of good quality. A further treatment of the loading surfaces was therefore not performed.

The specimens were prepared for DIC measurement by applying a random black-on-white speckle pattern to the specimen surface using aerosol spray painting. A fine speckle pattern was applied to the quasi-static specimens whereas a slightly coarser pattern was used for the dynamic specimens, considering the different image resolutions of the respective cameras.

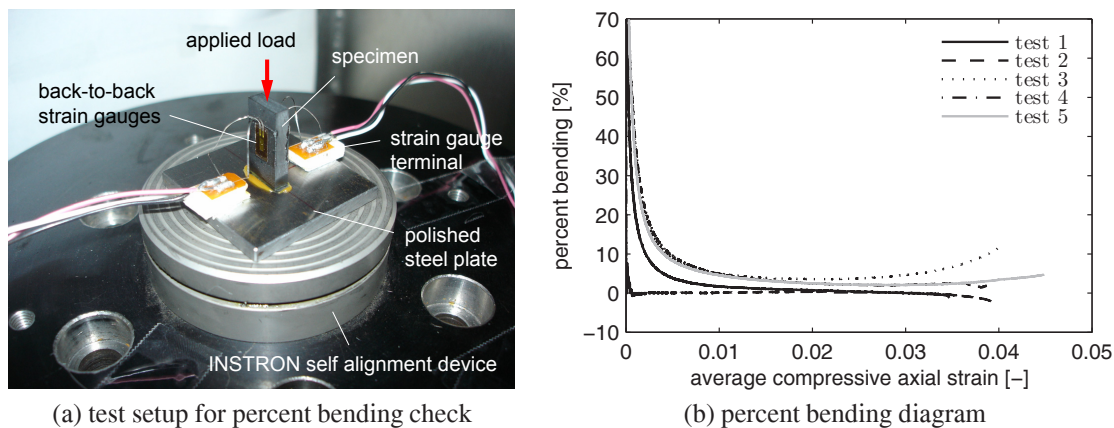


Figure 5.1: Evaluation of percent bending using back-to-back linear strain gauges.

5.2.2 Quasi-Static Experimental Setup

Quasi-static tests were carried out on a standard INSTRON 4208 load frame at a constant displacement rate of $0.5\text{mm}\cdot\text{min}^{-1}$. This corresponds to a quasi-static strain rate of $\dot{\epsilon}_{qs} \approx 4 \times 10^{-4}\text{s}^{-1}$, considering the nominal specimen length of 20mm. The quasi-static test setup is shown in Figure 5.2 with a self-aligning loading setup similar to that used by Tsai and Sun [30]. Polished tungsten-carbide (TC) inserts were used to avoid damage of the loading faces due to the high compression strength of the carbon fibre. Friction between TC-inserts and specimen end-surfaces was minimised by a thin layer of Molybdenum-Disulfide (MoS_2).

The GOM ARAMIS[®] software and measurement system (version 6.02) was used to obtain the quasi-static specimen strain field and consisted of an 8-bit Baumer Optronic FWX20 camera with a resolution of $1624 \times 1236 \text{ pixel}^2$, coupled with a Nikon AF Micro-Nikkor 200mm f/4D IF-ED lens and a 50mm extension tube. The camera was positioned at a distance of 1m away from the specimen surface. Two standard halogen lamps on either side of the camera guaranteed an even illumination of the specimen surface. The acquisition rate of the camera was set to 1 frame per second (fps) with a shutter speed of 40ms and an aperture of $f/11$.

5.2.3 Dynamic Experimental Setup

High strain rate tests, at strain rates between 90s^{-1} and 350s^{-1} were carried out with a classic SHPB setup as shown in Figure 5.3, consisting of $\varnothing 16\text{mm}$ steel striker-, incident- and transmission-bars with length 0.6m, 2.6m and 1.6m, respectively¹. For the 45° off-axis compression tests, a striker-bar with a length of 0.8m was used due to the higher strain-to-failure observed for this specimen type. The incident-bar strain-gauges were positioned in the middle of the bar at a distance of 1.3m away from the specimen interface. On the transmission-bar, the strain-gauges were positioned 0.3m away from the

¹The strain rates are average values of the maximum strain rate and the strain rate at ultimate failure for a respective specimen type. The lowest average of 90s^{-1} was measured for the 15° off-axis specimen, whilst the higher value of 350s^{-1} was measured for the 60° off-axis specimen (see Table 5.3)

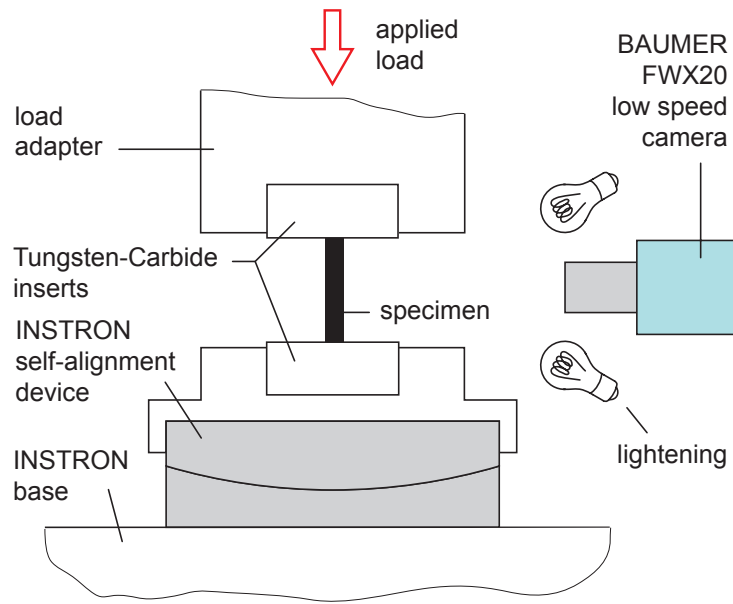


Figure 5.2: Quasi-static compression test setup.

specimen interface. A FYLDE FE-H379-TA high speed transducer amplifier with a gain setting of $1000\times$ was used to amplify the bar-gauge signals which were then recorded with a TEKTRONIX TDS3014B oscilloscope.

A PHOTRON SA5 high speed camera, coupled with a SIGMA 105mm f2.8 EX DG Macro lens was used for the SHPB experiments and positioned at 0.5m away from the specimen surface. Two camera settings were used for the dynamic experiments. To obtain the in-plane strain field from the deformation of the speckle-patterned specimen front surface, a frame-rate of 100000fps with an image resolution of $320 \times 192 \text{ pixel}^2$ and a shutter speed of $9.8\mu\text{s}$ was selected. Additional tests were then carried out for the 45° , 60° and 75° off-axis compression and for the transverse compression specimen types to measure the angle of the fracture plane. For this latter tests, the specimen was filmed from the side. It is noted that for the PHOTRON SA5 camera, the maximum frame rate depends on the area of interest. Due to the smaller area of interest for the fracture angle tests, the image size could be reduced to a resolution of $320 \times 72 \text{ pixel}^2$, which resulted in a higher frame rate of 232500fps and a shutter speed of $4.3\mu\text{s}$. In the dynamic case the shutter speed (or exposure time) is by default approximately equal to the inverse of the

camera frame rate. For both camera configurations the lens aperture was set to $f/2.8$. Two units of the DEDOLIGHT 400D daylight lightning system were positioned at either side of the camera to guarantee an even illumination of the specimen. To avoid reflections, the shiny outer-surfaces of the bars adjacent to the specimen were covered with matt black insulation tape as shown in Figure 5.4. Friction between the specimen and the bar-end surfaces was reduced by applying a thin layer of MoS_2 paste.

All SHPB experiments were performed with shaped incident pulses using copper pulse shapers (copper alloy UNS C12200 / DIN 2.0090, not annealed). The dimensions of the pulse shaper, for the incident pulse shape best suited for the respective specimen stress-strain response, were determined by the pulse shaping analysis proposed by Nemat-Nasser et al. [66] (see Section 3.2).

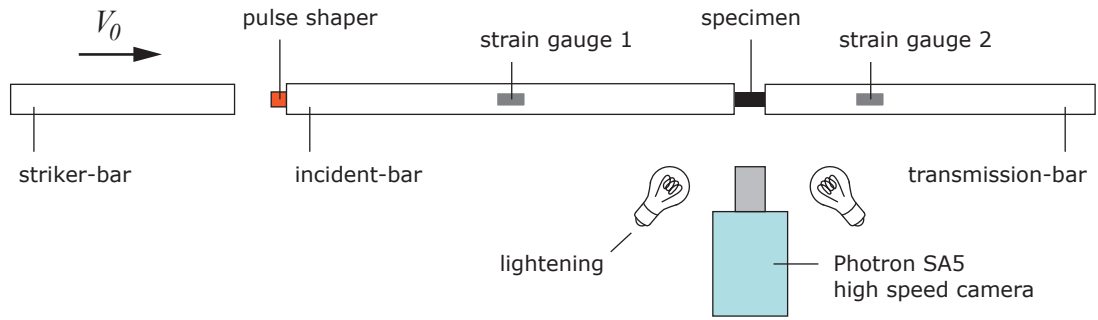


Figure 5.3: Split-Hopkinson pressure bar test setup.

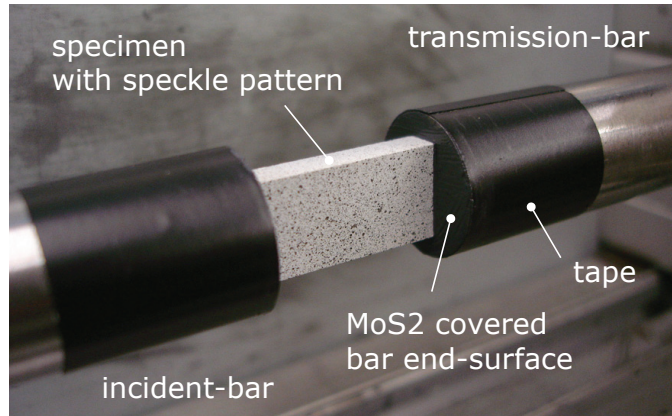


Figure 5.4: Specimen setup for SHPB.

5.3 Data Reduction Methods

5.3.1 Transverse Compression and Off-Axis Properties

For the quasi-static tests the axial stress σ_{xx} was calculated by dividing the load measured from the load-cell of the testing machine by the specimen cross-section, whereas for the dynamic test, the axial stress was obtained by SHPB analysis (see Chapter 3).

The in-plane strain field $\{\epsilon_{xx}, \epsilon_{yy}, \gamma_{xy}\}^T$ in the loading coordinate system was obtained from the DIC software ARAMIS for an area slightly smaller than the total specimen surface to reduce DIC calculation errors at the boundaries. The quasi-static and dynamic input parameters for the ARAMIS software and the resulting resolutions are given in Table 5.1. The conversion factor relates the camera pixel size to the specimen dimensions and is defined by the magnification of the optical system. The facet size, facet step and computation size were chosen to obtain suitable displacement and strain resolutions, in a compromise with the image resolution of the camera. The displacement and strain resolutions are the standard deviations of the noisy signal obtained by processing a set of images taken before applying any deformation.

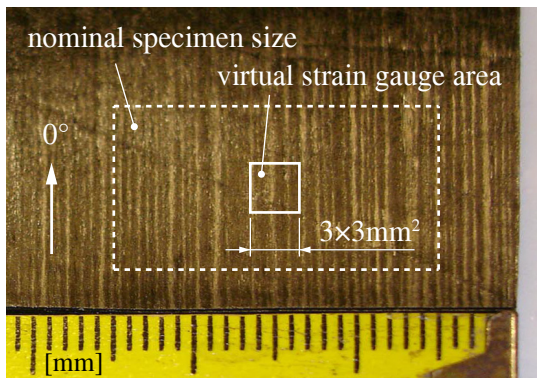
Table 5.1: ARAMIS input parameters and resolutions.

	quasi-static	dynamic
conversion factor	0.013 mm/pixel	0.074 mm/pixel
facet size	15×15 pixel ²	10×10 pixel ²
facet step	15×15 pixel ²	5×5 pixel ²
computation size	5×5 facets ²	3×3 facets ²
displacement resolution	2×10^{-2} pixel	1×10^{-2} pixel
strain field resolution	0.02 [%]	0.04 [%]

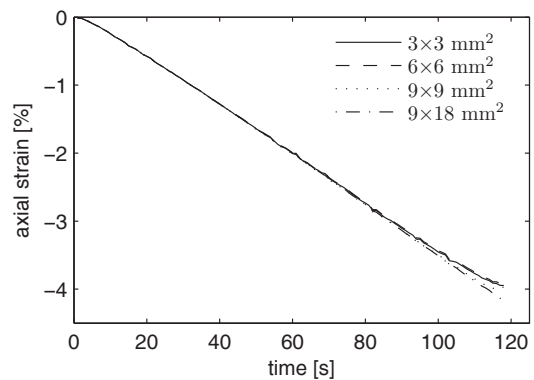
For the stress-strain curves presented in this chapter it was necessary to average the strain components over a virtual strain gauge. Since homogeneous strain fields are expected to occur at the specimen centre, virtual gauge areas of 3×3 mm² for the quasi-static case

and $6 \times 6 \text{ mm}^2$ for the dynamic case were chosen. The area sizes represent characteristic grid sizes of conventional foil strain gauges. The greater area for the dynamic case was selected due to the lower camera resolution and different facet size, facet step and computation size as compared to the quasi-static case. The quasi-static virtual strain gauge area of $3 \times 3 \text{ mm}^2$ can represent the true average behaviour of the UD prepreg system IM7-8552 as shown in Figure 5.5, where the tow count structure of the material is shown together with the nominal dimensions of the test specimen and the virtual strain gauge area. Figure 5.5 also shows that the measured strain is independent of the virtual strain gauge area.

From the quasi-static and dynamic axial stress-strain responses, modulus of elasticity, yield strength, ultimate strength and ultimate strain were determined. The yield criteria was defined as 0.02% plastic axial strain.



(a) IM7-8552 12K tow count structure before curing



(b) different virtual strain gauge area sizes for quasi-static transverse compression test

Figure 5.5: IM7-8552 12K tow count structure and evaluation of virtual strain gauge area size.

5.3.2 In-Plane Shear Properties

The in-plane shear response was obtained from the off-axis tests by transformation of the applied stress and strain from the global coordinate system, where xx coincides with the loading direction, into the material coordinate system, where the 1-direction coincides

with the fibre and the 2-direction coincides with the in-plane matrix direction:

$$\begin{aligned}\sigma_{22} &= \sigma_{xx} \sin^2 \beta \\ \tau_{12} &= -\sigma_{xx} \sin \beta \cos \beta\end{aligned}\tag{5.1}$$

$$\begin{aligned}\epsilon_{22} &= \epsilon_{xx} \sin^2 \beta + \epsilon_{yy} \cos^2 \beta - 0.5 \gamma_{xy} \sin 2\beta \\ \gamma_{12} &= -\epsilon_{xx} \sin 2\beta + \epsilon_{yy} \sin 2\beta + \gamma_{xy} \cos 2\beta\end{aligned}\tag{5.2}$$

The transformation angle β consists of the initial off-axis angle θ_0 and the additional fibre-rotation angle $d\theta$ occurring during the test, which is also provided by the ARAMIS software (shear angle). σ_{22} and ϵ_{22} represent the transverse components of the stress and strain tensors, respectively, while τ_{12} and γ_{12} represent the shear components of the stress and strain tensors in the material coordinate system. From the $\tau_{12} - \gamma_{12}$ shear stress-strain response, the shear modulus and shear yield strength were determined. The shear yield criteria was defined as 0.02% plastic shear strain.

The pure shear strength was determined by an extrapolation method proposed by Tsai and Sun [30]. Equation (5.1) was applied to the axial strength measured for the 15° and 30° off-axis specimens with in-plane shear failure modes, while considering the additional fibre-rotation angle $d\theta$. The pure in-plane shear strength can then be extrapolated in the combined transverse-compression/in-plane shear diagram at $\sigma_{22} = 0$.

5.3.3 Fracture Plane Angle

With the high speed camera used for the SHPB experiment it is possible to measure the fracture plane angle for the transverse compression and off-axis compression specimens with transverse compression dominated failure modes. For the transverse compression specimen, the fracture angle appearing on the specimen side surface is equal to the angle of the fracture plane. For the off-axis specimens however the filmed angle does not represent the fracture plane angle. A CAD model of one part of the failed specimen was

used to determine the angle of the fracture plane in this case. The method is illustrated in Figure 5.6, where the angle α' in the upper left image is the fracture angle as seen through the camera. The specimen is simply rotated by the off-axis angle θ and from the projection of the rotated specimen, the actual fracture plane angle α can be measured.

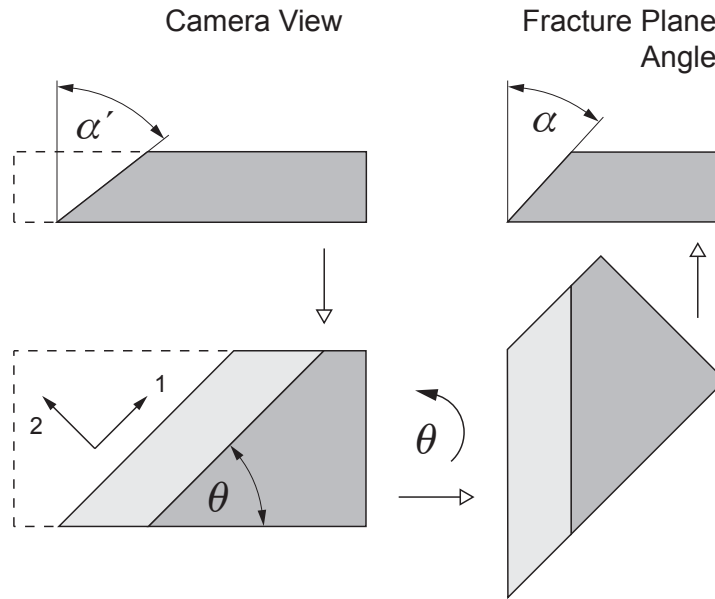


Figure 5.6: Determination of fracture plane angle α for dynamic off-axis tests.

5.3.4 SHPB Data Reduction

The specimen strain ϵ_s , specimen strain rate $\dot{\epsilon}_s$ and specimen stress σ_s were calculated from the SHPBA Equations (3.28), (3.27) and (3.33), respectively, which were presented in Chapter 3.

It is noted that for the dynamic stress-strain relations presented in this chapter, the specimen strain was obtained from the DIC software ARAMIS since the strain calculated with Equation (3.28) was found to be overpredicted. This is demonstrated for selected tests in Figure 5.7, where coincident strains were only observed for the transverse compression specimen type.

An alternative and perhaps more accurate method for determining the specimen strain

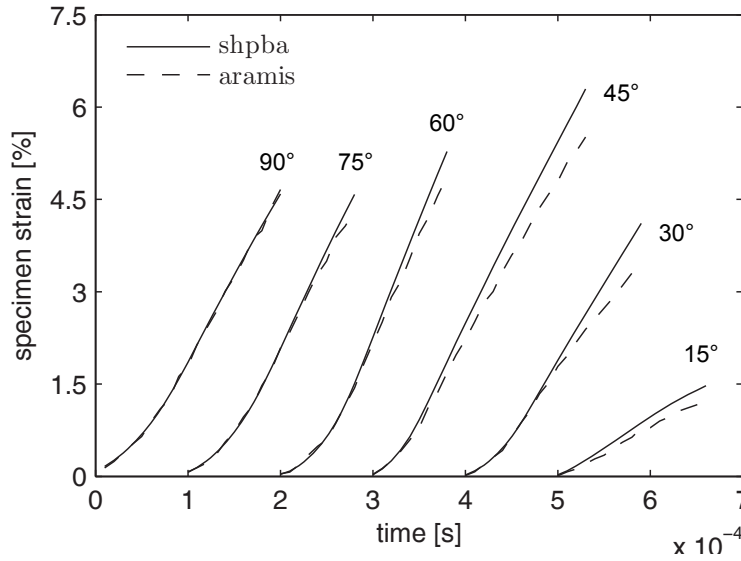


Figure 5.7: SHPB analysis specimen strain overprediction.

rate is the derivative of the true specimen strain, measured with the ARAMIS software, with respect to time, $\dot{\epsilon}_s = d\epsilon_{xx}/dt$. However, the slight noise in the true specimen strain obtained from ARAMIS, which is hardly noticable in Figure 5.7, is greatly amplified by the differentiation and therefore the accuracy of this method is reduced. For all dynamic tests presented in this chapter, the strain rate obtained from SHPBA using Equation (3.27) was compared with the above derivative of the true specimen strain and only a small difference was found. The calculation of the specimen strain rate from Equation (3.27) is considered valid, since changes in the mechanical response of the specimen are only expected to occur if the magnitude of the applied strain rate changes.

The calculation of the specimen stress from Equation (3.33) is valid under the following conditions [60].

The wave propagation is elastic and one-dimensional, which is generally satisfied by keeping the incident-pulse below the yield strength of the bar material and by selection of an appropriate bar material and geometry. In addition, the bars must be dispersion-free, since the bar-strain waves, recorded at the gauges of incident- and transmission-bar, are measured away from the specimen and at times before and after they have acted on the specimen. This condition can not be satisfied for SHPBs in general but the dispersion

errors, mainly caused by high-frequency oscillations usually present at the beginning of classic near rectangular incident-pulses, can be minimised by using shaped incident pulses as in the present study. As explained in Chapter 4, a bars-apart (BA) test with uncoupled incident- and transmission-bars was used to evaluate the amount of dispersion still present in the SHPB setup. Figure 5.8 shows the amplified incident (inverted) and reflected pulse signal from a BA-test performed with the present SHPB configuration using a 0.6m long striker bar fired at a velocity of $V_0=8\text{ms}^{-1}$ and a copper pulse shaper with a diameter of 4.3mm and a thickness of 0.5mm. The shape, duration and amplitude of the two pulses are nearly identical and therefore it is concluded that dispersion effects can be neglected.

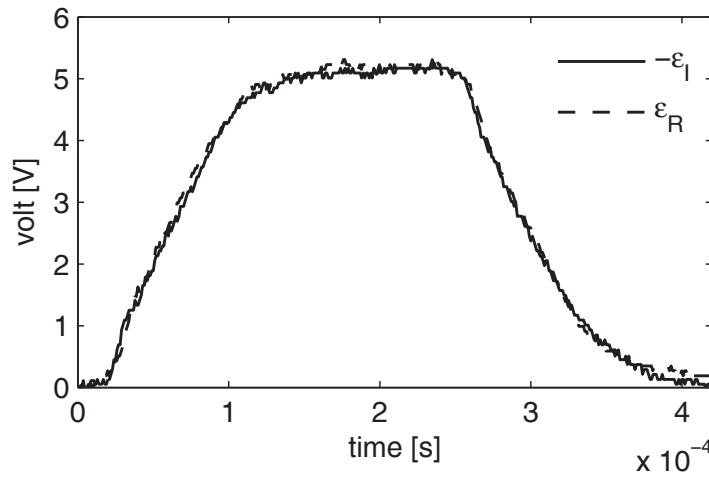


Figure 5.8: Shaped pulses from BA-test with present SHPB configuration.

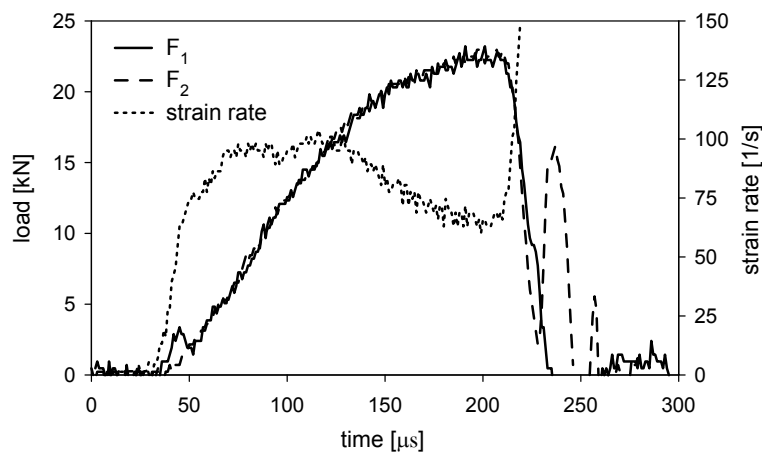
In addition, *the specimen should be in dynamic equilibrium*, meaning that the loads acting on the incident- and transmission-bar ends of the specimen are balanced. This can be verified by a comparison of the load F_1 , acting at the incident-bar/specimen interface and calculated from the incident- and reflected waves with Equation (3.29), with the load F_2 acting at the transmission-bar/specimen interface and calculated from the transmitted wave using Equation (3.30).

Due to the use of shaped incident-pulses, the specimen is gradually loaded and dynamic equilibrium is established very early. The specimen stress can therefore be calculated with Equation (3.33), which uses only the transmitted-wave, and the high strain rate

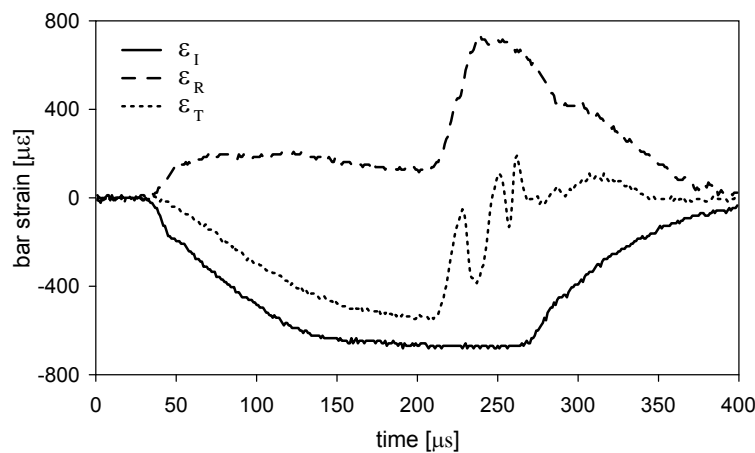
elastic properties can be obtained with confidence. Figure 5.9a shows the dynamic equilibrium for a representative 15° off-axis compression test.

Figure 5.9a also contains the strain-rate history calculated with Equation (3.27). Although it is not an assumption required for the SHPB analysis, a constant strain rate should be used for all material characterisation tests. To perform a constant strain rate SHPB test, the incident-pulse shape should be matched to the load-time history of the specimen, represented by the shape of the transmitted-pulse [62, 66]. Figure 5.9b shows the bar strain waves for the 15° off-axis test and it is seen that this match is well accomplished. The specimen is therefore loaded at a near constant strain rate.

To ensure that the measured stress-strain response represents the actual specimen be-



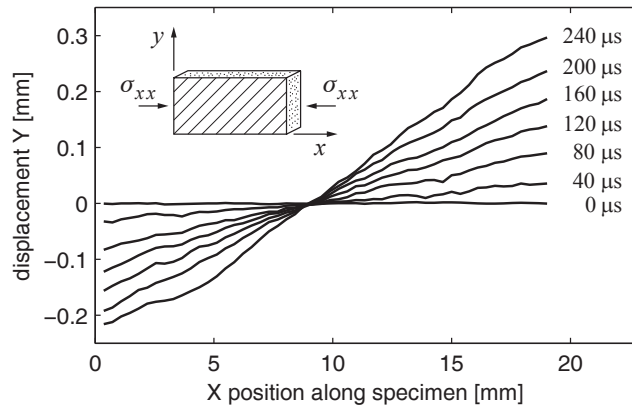
(a) load balance and strain rate



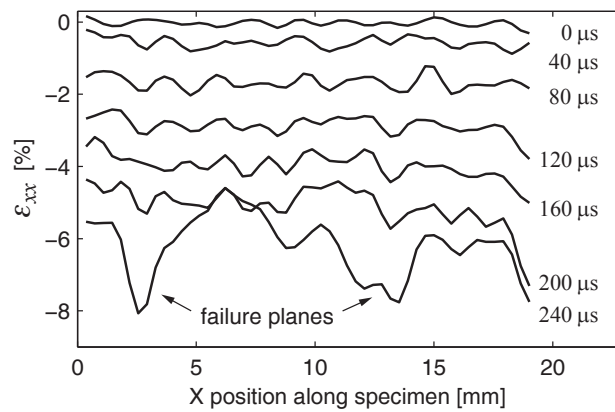
(b) bar strains

Figure 5.9: SHPB analysis results for a 15° off-axis compression specimen.

haviour, the specimen deformation and the specimen strain must be uniform. These verifications are particularly important for the off-axis compression tests due to the extension-shear coupling effect typical for this specimen. With the high speed camera and DIC software ARAMIS these assumptions can easily be verified. Figure 5.10 shows the Y-displacement and the axial compressive strain component ϵ_{xx} for a 45° off-axis compression specimen measured over the entire specimen length as a function of time. It is seen that the specimen rotation is symmetrical and thus the extension-shear coupling is not restricted. It is further seen that the axial strain distribution is uniform up to failure. The waviness in Figure 5.10b is not caused by signal noise but it represents the tow count structure of the prepreg system shown in Figure 5.5.



(a) displacement Y



(b) axial strain

Figure 5.10: Uniform specimen deformation of dynamic 45° off-axis compression test (see also Figure 5.17).

5.4 Experimental Results

5.4.1 Quasi-Static Experimental Results

For each off-axis angle, three specimens were tested to keep the experimental efforts within reasonable limits while still allowing a statistical treatment of the data. More tests were however performed for the 15° specimen type, since two failure modes were observed in this case. From a total of 12 tested 15° specimens, 9 failed in kink-band mode at either top or bottom end-surface and 3 failed in an in-plane shear dominated mode. Figure 5.11 shows a representative quasi-static 15° specimen with kink-band failure mode. The extension-shear coupling effect is clearly visible by the superimposed fibre rotation angle (ARAMIS shear angle).

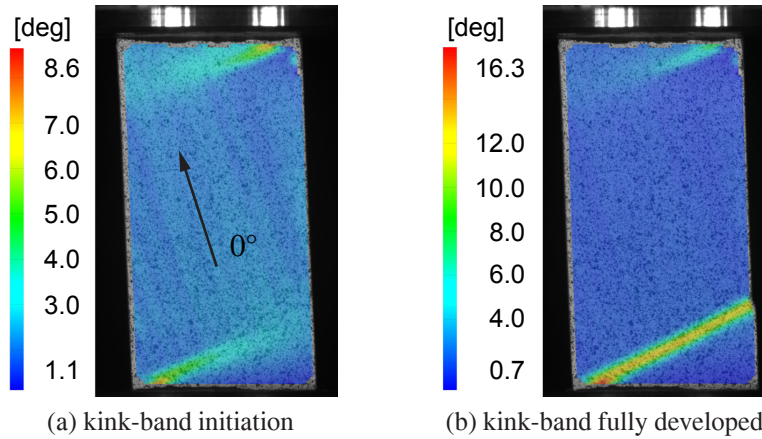


Figure 5.11: Kink-band failure mode of quasi-static 15° off-axis specimen (superimposed shear angle).

Kawai and Saito [78] also observed these two failure modes for 15° off-axis compression tests, and reported significant differences for the stress-strain behaviour with respect to the failure mode. Kawai and Saito [78] found that the strength of the 15° specimen failing in the kink-band mode was higher than the strength of the specimen failing in an in-plane shear dominated failure mode. Also, the plastic flow of the in-plane shear failure specimen occurred at a much lower stress level. This different stress-strain behaviour was not confirmed with the present 15° off-axis tests.

For both 15° and 30° specimens, a stick-slip behaviour at the beginning of the quasi-static test was identified while analysing the camera images. This indicates that the friction between the specimen end-surfaces and the TC-inserts of the loading fixture was still too high and prevented to some extent the full development of the extension-shear deformation. From the quasi-static stress-strain response of the 15° specimens it was concluded that the stick-slip behaviour and the different failure mode had no effect on the measured axial modulus and ultimate strength. The friction behaviour however had an effect on the yield strength since the linear region of the stress-strain curve is extended for some specimens. For this reason the yield strength was not determined for the 15° and 30° specimen types.

The failure mode of all quasi-static specimen types is shown in Figure 5.12. As observed by Kawai and Saito [78], the fracture surface for the 15° (in-plane shear failure mode) and for the 30° off-axis compression specimens is slightly inclined with regard to the through-thickness direction. This indicates a small influence of transverse compression stresses acting on the fracture surface.

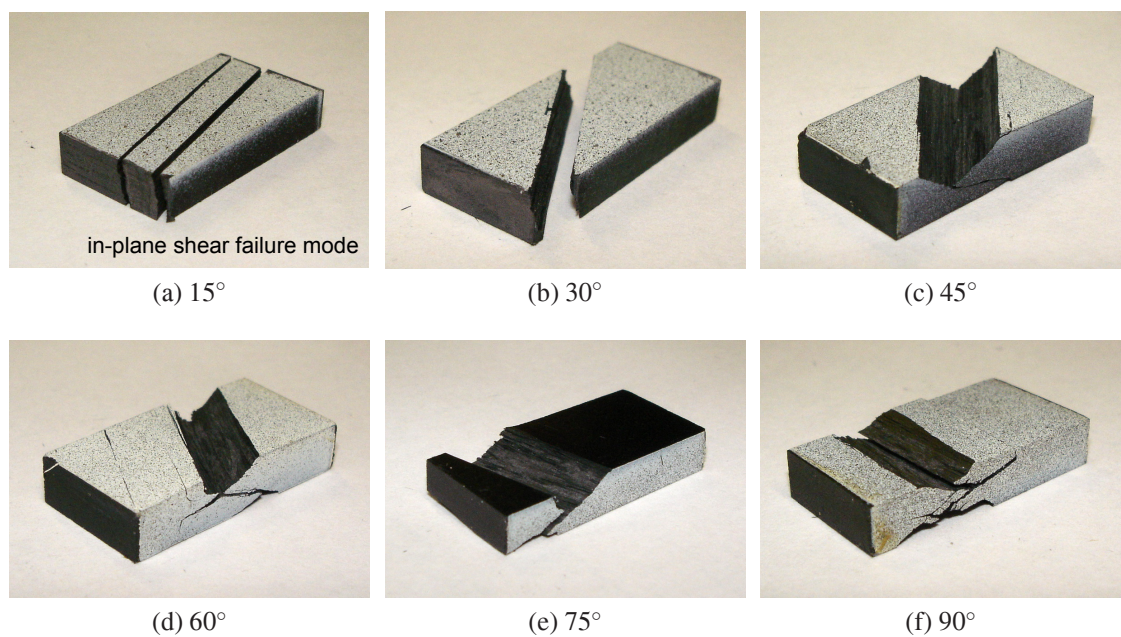


Figure 5.12: Quasi-static failure modes.

The failure mode for the 45° , 60° , 75° and 90° specimen types is transverse-compression dominated and failure occurred either in the middle or initiated at the loading surfaces. This was not considered to be critical since the failure position had no influence on the ultimate strength.

Figures 5.13 and 5.14 show the deformation with superimposed axial compressive strain for a 45° off-axis and for a transverse compression specimen, respectively. The time t_u represents the last image captured before ultimate failure.

The quasi-static off-axis and transverse compression properties are summarised in Table 5.2. The quasi-static in-plane shear properties will be presented together with the dynamic shear data in Section 5.5.

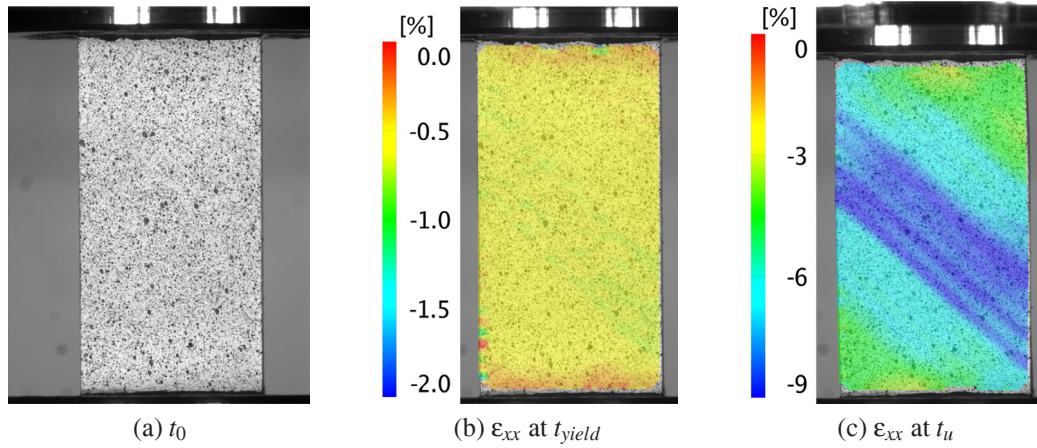


Figure 5.13: Quasi-static 45° off-axis compression test.

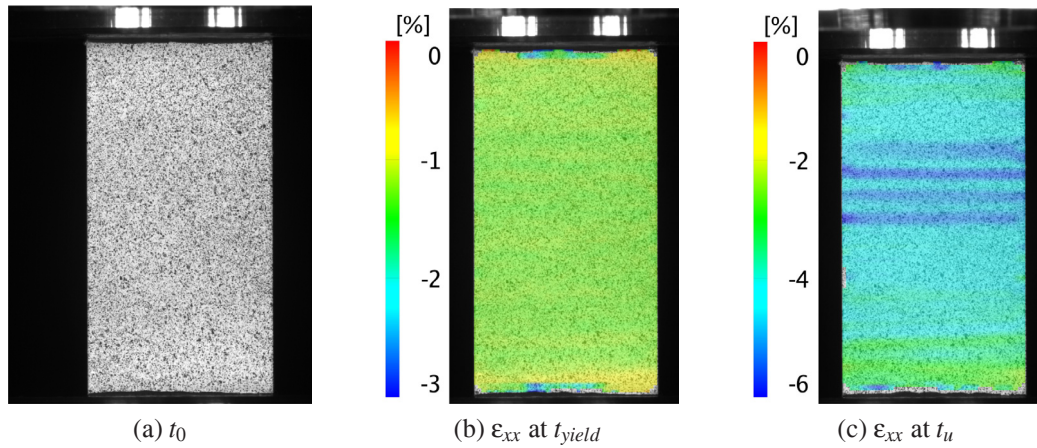


Figure 5.14: Quasi-static transverse compression test.

Table 5.2: Quasi-static off-axis and transverse compression properties.

Fibre Angle		No. of Tests	Modulus [MPa]	Yield [MPa] Strength	Ult. [MPa] Strength	Ult. [%] Strain
15°	Mean	12	55284	-	399 (1.9°) ^b	1.27
	STDV		2277	-	12	0.16
	CV [%]		4.1	-	3.0	12.4
30°	Mean	3	21691	-	266 (2.3°) ^b	3.68
	STDV		988	-	3	0.28
	CV [%]		4.6	-	1.2	7.6
45°	Mean	3	13084	70 (0.2°) ^a	254 (1.3°) ^b	8.38
	STDV		1050	5	10	1.15
	CV [%]		8.0	7.0	3.8	13.7
60°	Mean	3	9790	88 (≈ 0°) ^a	263 (≈ 0°) ^b	6.16
	STDV		324	4	5	1.79
	CV [%]		3.3	4.7	1.7	29.1
75°	Mean	3	8818	95 (≈ 0°) ^a	252 (≈ 0°) ^b	4.20
	STDV		213	11	7	0.59
	CV [%]		2.4	11.1	2.8	13.9
90°	Mean	3	8930	104	255	4.26
	STDV		301	5	3	0.32
	CV [%]		3.3	4.4	1.0	7.4
Properties for off-axis angles 15° to 75° are in axial direction, Strain Rate $\dot{\epsilon} \approx 4 \times 10^{-4}\text{s}^{-1}$ for all quasi-static tests, ^a Fibre-rotation at yield strength, ^b Fibre-rotation at ultimate strength						

5.4.2 Dynamic Experimental Results

Five dynamic tests were performed for the 15° and four specimens were tested for the 30° off-axis specimen type. For the 45°, 60°, 75° and 90° specimen types, three specimens were tested for each fibre orientation angle. More specimens were tested for the 15° and 30° types due to the mixed failure modes observed for the quasi-static 15° off-axis test and since those two specimen types were used to obtain the pure in-plane shear strength via the extrapolation method explained in Section 5.3.2. Significant fibre-rotation was observed for the 15°, 30° and 45° off-axis specimen types. A representative deformation

and axial compressive strain field for a dynamic 15° off-axis test is shown in Figure 5.15. Both, axial compressive strain distribution and fibre-rotation are uniform until failure, which occurs by initiation of a kink-band, immediately followed by multiple in-plane shear failure along the fibre-direction.

The 30° off-axis compression specimens all failed in an in-plane shear dominated failure mode and no kink-band initiation was observed for this specimen type. Deformation, failure mode and axial compressive strain field for a 30° off-axis specimen is shown in Figure 5.16.

The 45° , 60° and 75° off-axis compression and the transverse compression specimen types failed in a transverse compression dominated failure mode. Representative deformation and axial compressive strain fields are shown in Figures 5.17 to 5.20.

The dynamic off-axis and transverse compression properties are summarised in Table 5.3. As explained for the quasi-static case in Section 5.4.1, the dynamic yield strength for the 15° and 30° off-axis specimen types was not determined.

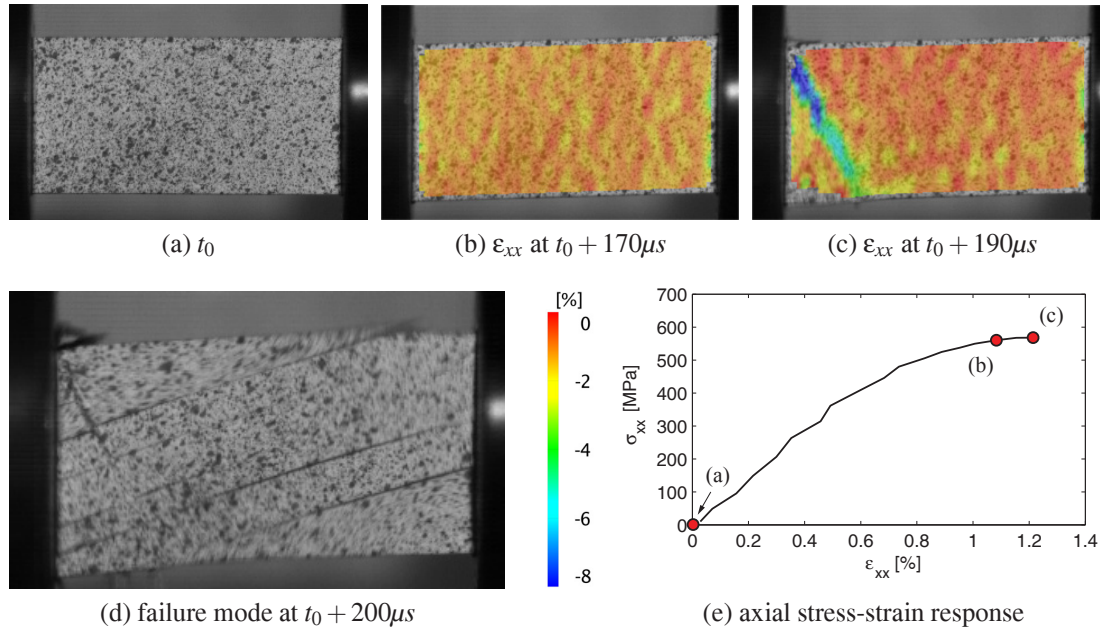


Figure 5.15: Dynamic 15° off-axis compression test.

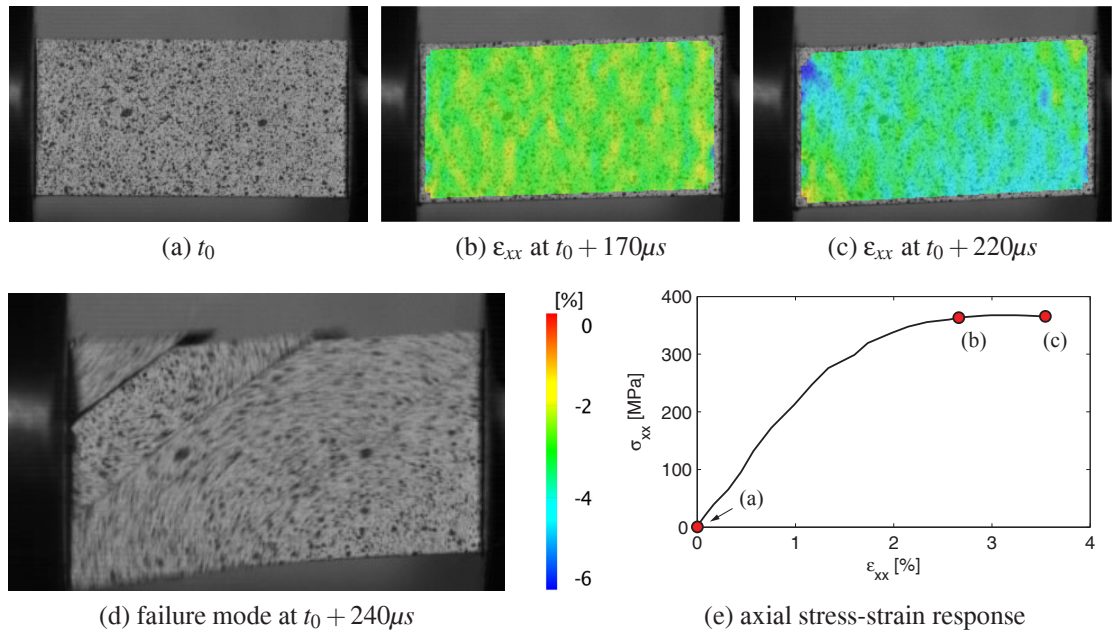


Figure 5.16: Dynamic 30° off-axis compression test.

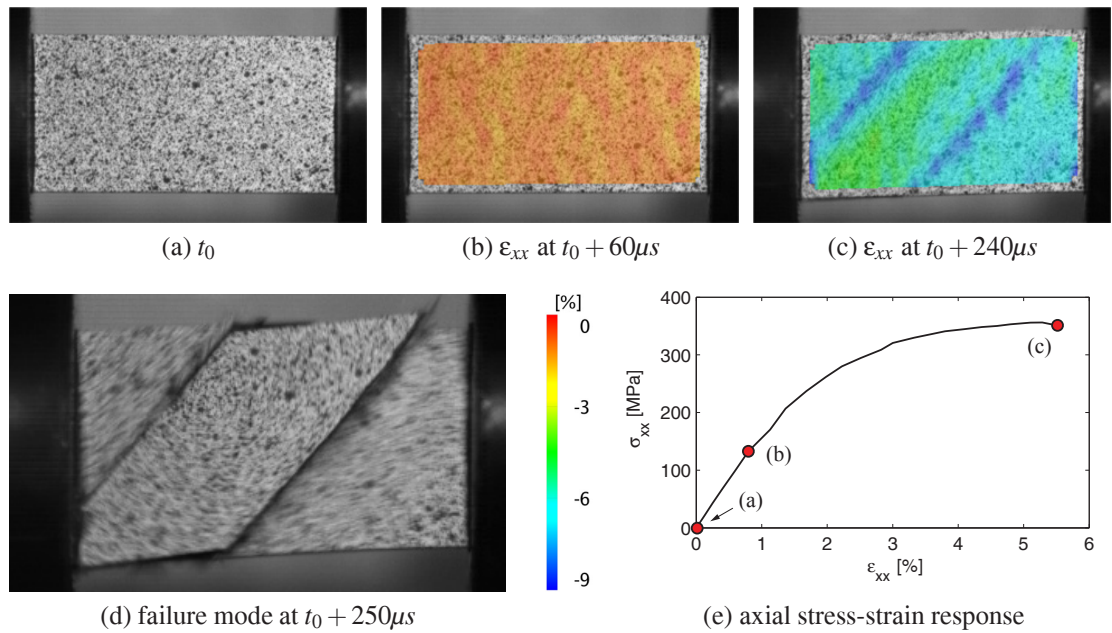


Figure 5.17: Dynamic 45° off-axis compression test.

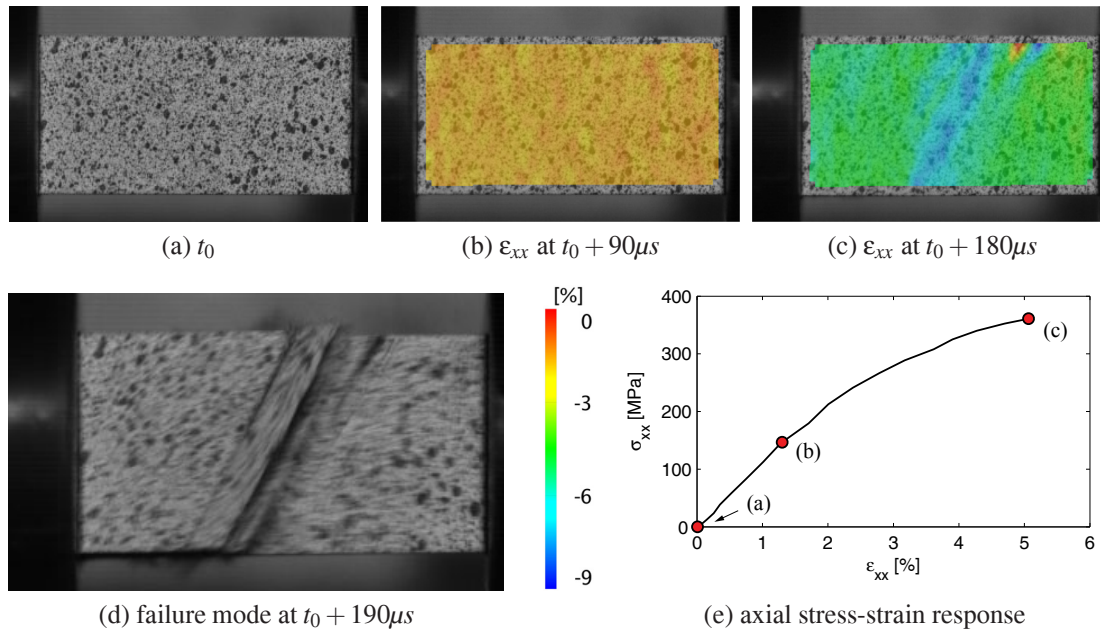


Figure 5.18: Dynamic 60° off-axis compression test.

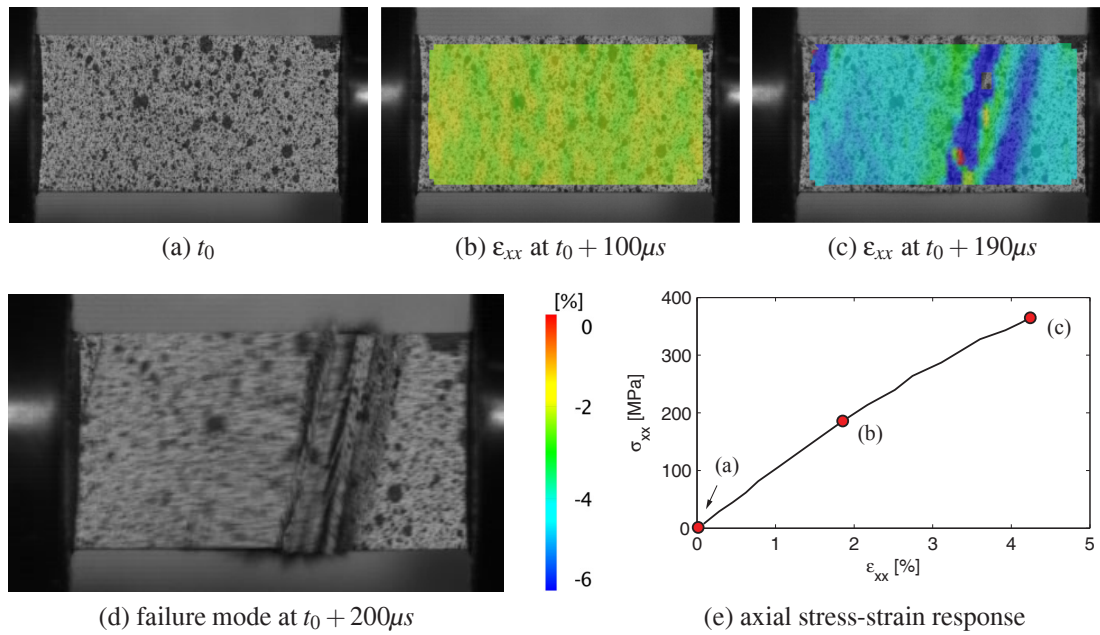


Figure 5.19: Dynamic 75° off-axis compression test.

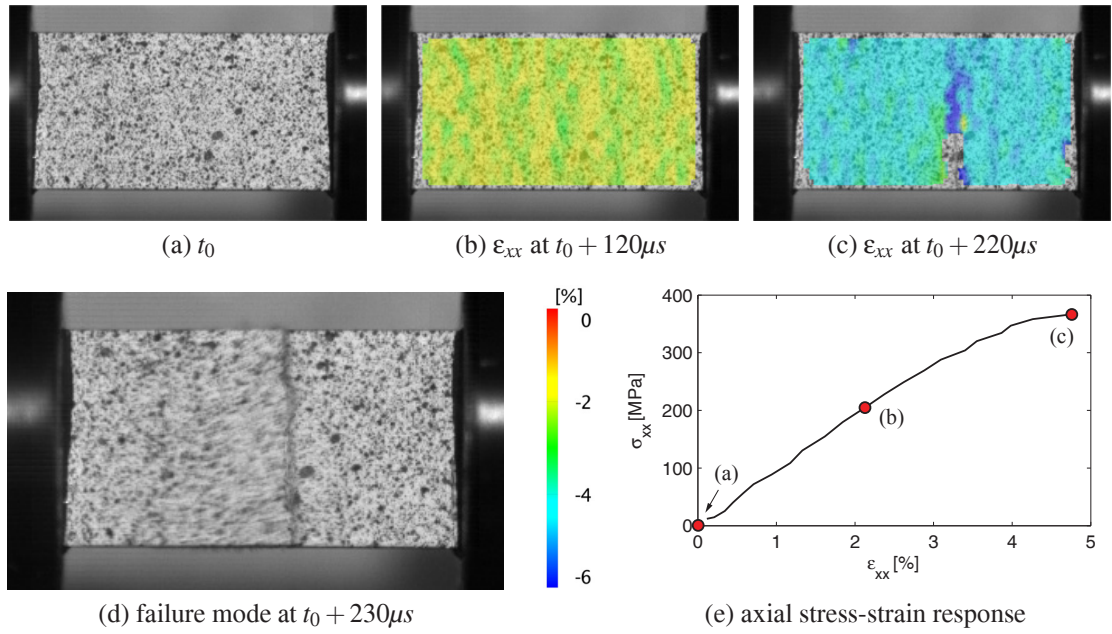


Figure 5.20: Dynamic transverse compression test.

Table 5.3: Dynamic off-axis and transverse compression properties

Fibre Angle	No. of Tests	Strain Rate ^a [s ⁻¹]	Modulus [MPa]	Yield [MPa] Strength	Ult. [MPa] Strength	Ult. [%] Strain		
15°	5	-	122	64	73927	-	549 (1.9°) ^c	1.27
	STDV	-	16	7	1656	-	15	0.09
	CV [%]	-	13.2	11.7	2.2	-	2.7	7.4
30°	4	-	246	220	23964	-	370 (2.1°) ^c	3.14
	STDV	-	14	22	1498	-	3	0.4
	CV [%]	-	5.6	10.0	6.3	-	0.8	12.7
45°	3	280	321	270	15848	129 (0.3°) ^b	354 (1.4°) ^c	6.00
	STDV	15	3	8	987	16	3	0.56
	CV [%]	5.4	0.8	2.8	6.2	12.5	0.7	9.3
60°	3	331	367	340	11729	156 (0.2°) ^b	365 (0.4°) ^c	4.81
	STDV	23	16	15	263	14	5	0.29
	CV [%]	7.0	4.3	4.3	2.2	9.2	1.4	6.0
75°	3	305	317	276	9998	185 (≈ 0°) ^b	363 (≈ 0°) ^c	4.38
	STDV	15	5	2	350	41	3	0.17
	CV [%]	4.9	1.6	0.6	3.5	22.3	0.9	3.9
90°	3	271	276	227	10019	190	371	4.58
	STDV	12	16	20	207	14	9	0.24
	CV [%]	4.5	5.6	8.8	2.1	7.2	2.5	5.2

Properties for off-axis angles 15° to 75° are in axial direction,

^a yield = $\dot{\epsilon}$ at yield strength, *max* = maximum $\dot{\epsilon}$, *failure* = $\dot{\epsilon}$ at ultimate strength,

^b Fibre-rotation at yield strength, ^c Fibre-rotation at ultimate strength

Properties for off-axis angles 15° to 75° are in axial direction,

^a *yield* = $\dot{\epsilon}$ at yield strength, *max* = maximum $\dot{\epsilon}$, *failure* = $\dot{\epsilon}$ at ultimate strength,

^b Fibre-rotation at yield strength, ^c Fibre-rotation at ultimate strength

The transverse dominated failure mode for the 45° , 60° , 75° and 90° specimens is also clearly visible from the additional tests performed with the second camera configuration, to measure the fracture surface angle α . Four additional specimens were tested for each of the above specimen types. From these tests, only those with a crack appearing away from the end-surfaces of the specimen were considered valid. For the 45° off-axis specimens, the crack appeared mostly on the opposite side and therefore only one valid result was obtained for this specimen type. The crack evolution for the valid fracture plane angle specimens is shown in Figures 5.21 - 5.28. For the off-axis specimen types, the data reduction procedure explained in Section 5.3.3 was used to determine the actual fracture plane angle α from the angle α' seen in the digital images.

Two valid tests were performed for the transverse compression specimen. However, considering the approximately equal stress-strain behaviour of the transverse compression and the 75° off-axis compression test, a total of 5 valid tests can be used to determine the dynamic transverse compression fracture angle α_0 , which was found to be 56.4° . The results for all valid tests are summarised in Table 5.4.

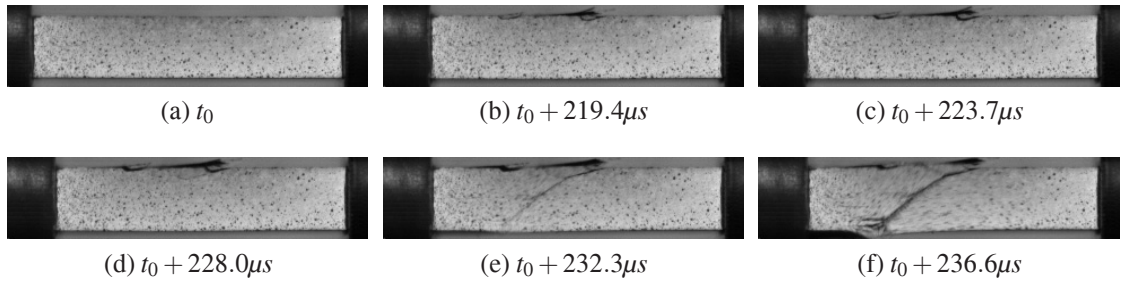


Figure 5.21: Crack evolution for dynamic 45° off-axis compression specimen 1.

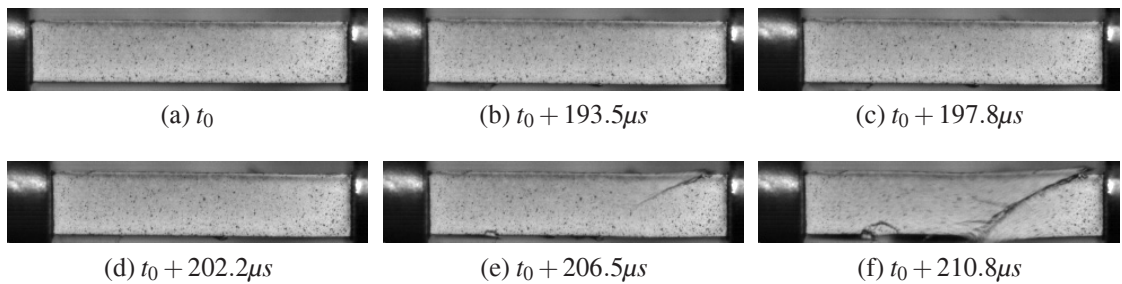


Figure 5.22: Crack evolution for dynamic 60° off-axis compression specimen 1.

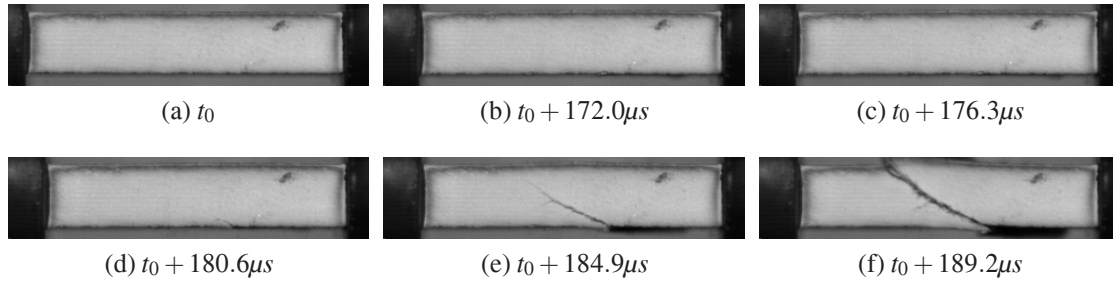


Figure 5.23: Crack evolution for dynamic 60° off-axis compression specimen 2.

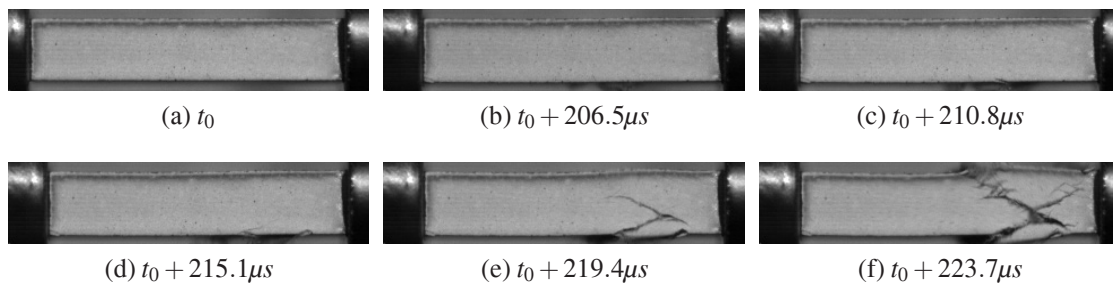


Figure 5.24: Crack evolution for dynamic 75° off-axis compression specimen 1.

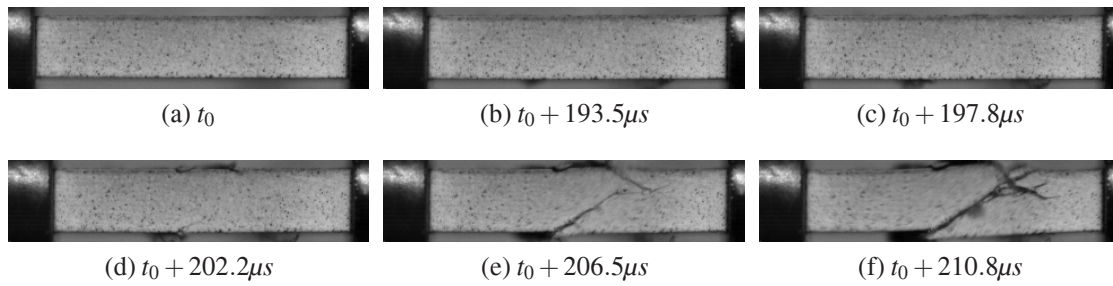


Figure 5.25: Crack evolution for dynamic 75° off-axis compression specimen 2.

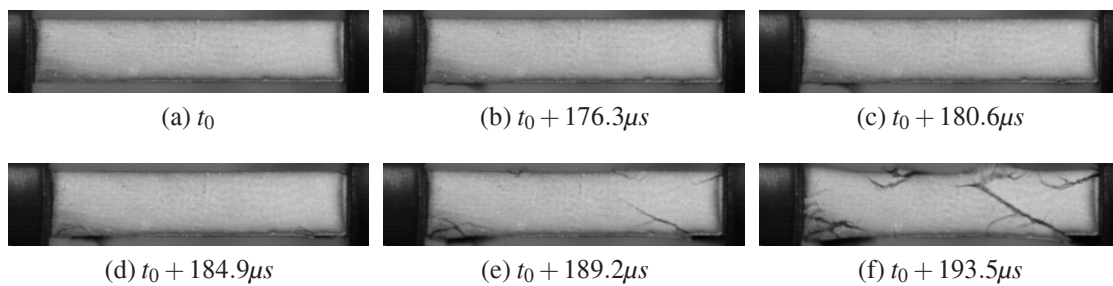


Figure 5.26: Crack evolution for dynamic 75° off-axis compression specimen 3.

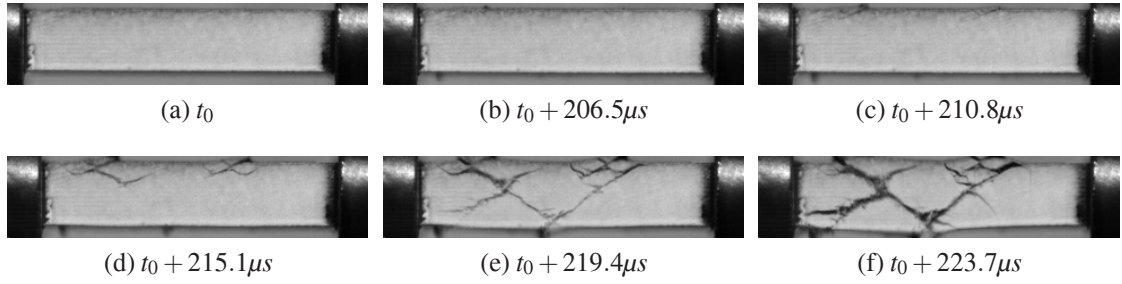


Figure 5.27: Crack evolution for dynamic transverse compression specimen 1.

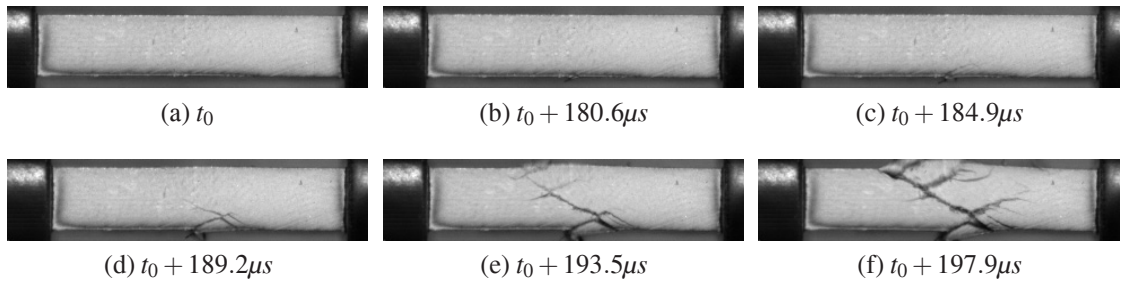


Figure 5.28: Crack evolution for dynamic transverse compression specimen 2.

Table 5.4: Dynamic fracture plane angle.

Specimen Type	No. of valid Tests	α' [°]	α [°]
45°	1	50	40.1
60°	1	60	56.3
	2	57	53.1
75°	1	60	59.1
	2	55	54.0
	3	57	56.1
90°	1		55
	2		58

5.5 Discussion

Figure 5.29 shows a comparison of the quasi-static and dynamic axial compressive stress-strain response with respect to each specimen type, whereas in Figure 5.30 the stress-

strain responses are compared with respect to the strain rate regime. In all cases an increase of the axial compressive modulus of elasticity is observed, yet it is not as pronounced as the increase of the axial compressive strength. The ultimate axial strain decreases with increasing strain rate for the 30° , 45° and 60° off-axis specimen types whereas no significant rate effect is observed for the 15° , 75° off-axis and transverse compression specimens. It is noted that all quasi-static 15° off-axis compression specimens are shown in Figure 5.29, including kink-band and in-plane shear failure modes.

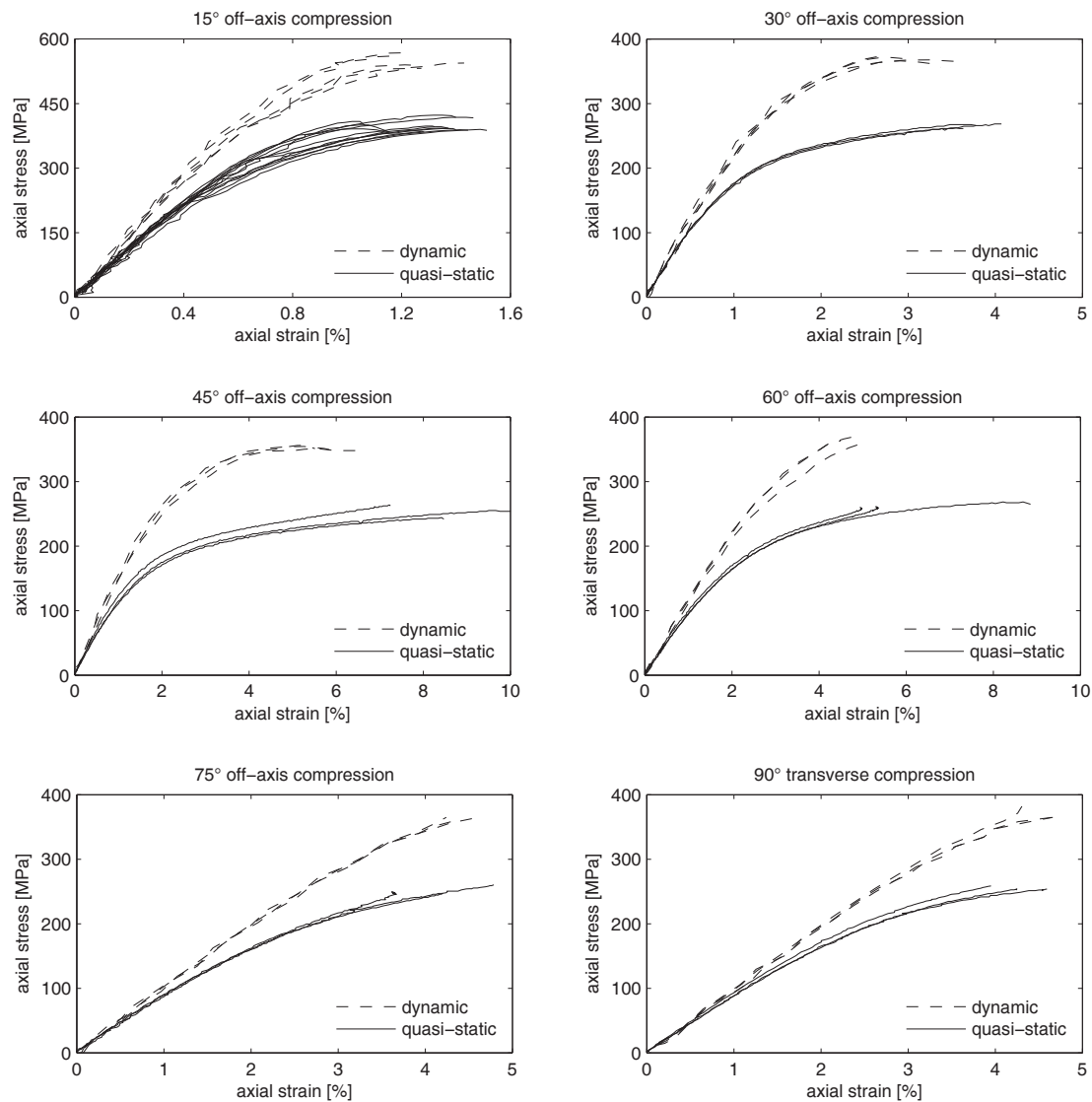


Figure 5.29: Quasi-static and dynamic axial stress-strain responses from off-axis and transverse compression tests (see Table 5.3 for average dynamic strain rates).

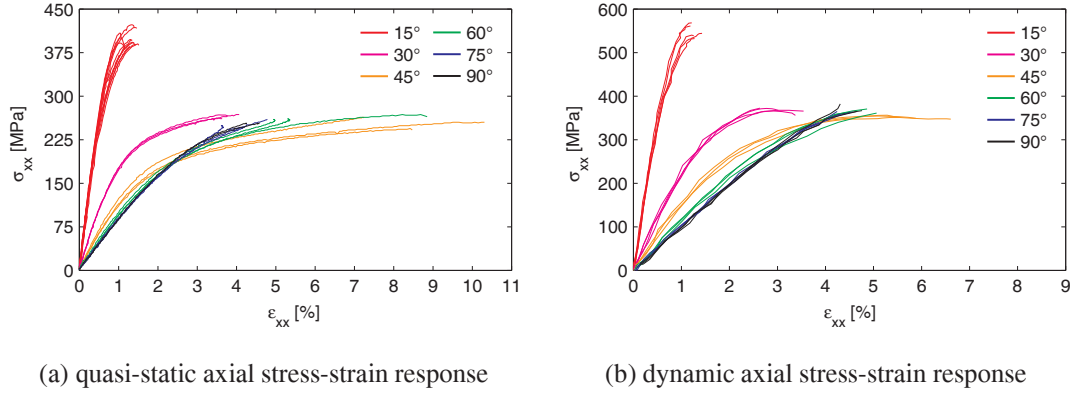


Figure 5.30: Quasi-static and dynamic comparison of axial stress-strain response for all specimen types.

5.5.1 Transverse Compression Properties

With respect to the transverse compression properties, a moderate increase of 12% was observed for the elastic modulus. The yield strength increased significantly by about 83% and the transverse compressive strength increased by 45%. This correlates well with the trends reported by Hsiao et al. [19], who reported a modulus increase of 10-15% and a strength increase of 41-45% for the carbon-epoxy system IM6G/3501-6 at a strain rate of $120\text{-}250\text{s}^{-1}$.

5.5.2 In-Plane Shear Properties

The in-plane shear curves, obtained from Equations (5.1) and (5.2) for representative quasi-static and dynamic 15° , 30° , 45° and 60° off-axis compression specimens are shown in Figure 5.31a. For both strain rate regimes, it is seen that the initial part of the shear stress-strain curve is similar with respect to the different off-axis specimen types. The ratio of transverse-compression and in-plane shear stresses therefore seems to have little effect on the in-plane shear stress-strain response of the above specimen types, whereas the in-plane shear strength is significantly effected.

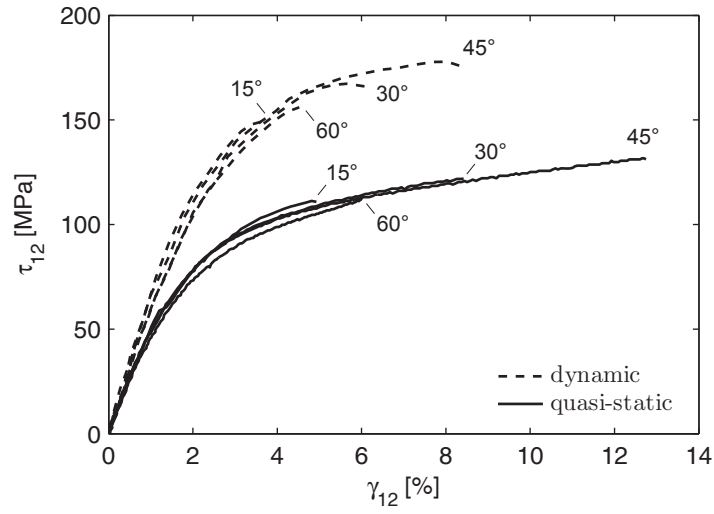
The quasi-static and dynamic shear modulus G_{12} and the in-plane yield strength S_y^L were determined from the $\tau_{12} - \gamma_{12}$ curve of the 45° off-axis compression specimen because

of the stick-slip behaviour observed for the 15° and 30° off-axis specimens. This decision is supported by the observation of Vogler and Kyriakides [77], who performed pure shear and biaxial transverse compression/in-plane shear experiments for AS4/PEEK on a custom fixture with various fixed biaxial loading ratios. They observed that the shear stress-strain response of a test corresponding to a 45° off-axis test is very close to that of pure shear loading.

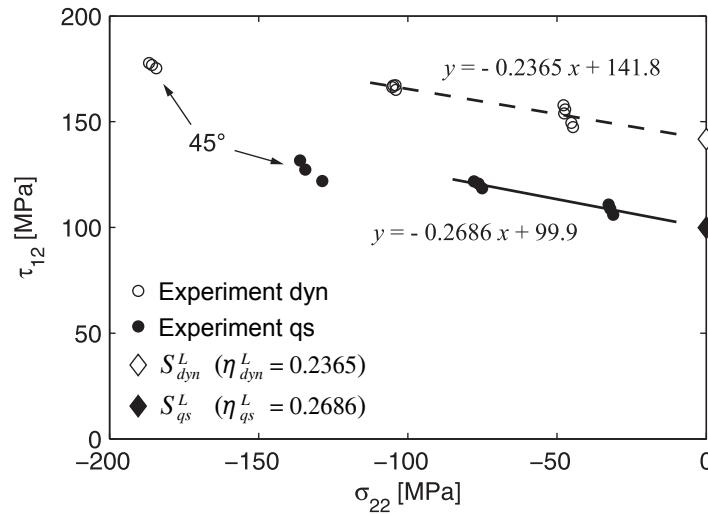
Regarding the elastic in-plane shear properties obtained in the present study, an increase of 25% was observed for the in-plane shear modulus, whereas the in-plane shear yield strength increased by 88% when comparing both strain rate regimes.

Due to the dependency of the in-plane shear strength on the biaxial stress state, the shear strength cannot be obtained from the shear stress-strain curves shown in Figure 5.31a. Instead, the pure shear strength was determined with the extrapolation method proposed by Tsai and Sun [30] from the 15° and 30° off-axis specimens, which failed in an in-plane shear dominated failure mode. Figure 5.31b shows the failure strength of the quasi-static and dynamic 15° and 30° off-axis specimens in the combined transverse compression/in-plane shear stress diagram. An increase of 42% was observed for the pure in-plane shear strength in the present study. A quantification of the rate effect on the ultimate shear strain was not performed, due to the dependency of the failure strength on the ratio of biaxial loading. It is noted that Tsai and Sun [30] also included the 45° off-axis specimen when extrapolating the pure shear strength. However, it was shown in Sections 5.4.1 and 5.4.2 that the failure mode for the 45° off-axis specimen type is transverse compression dominated and therefore this specimen type was excluded here.

The quasi-static and dynamic in-plane shear properties are summarised in Table 5.5. The strain rate effects for shear modulus and pure shear strength are again similar to the trends observed by Hsiao et al. [19], who reported an increase of 18% and 50% for the shear modulus and ultimate shear strength, respectively, using 45° off-axis specimens tested at a strain rate of 300s^{-1} on a drop tower. Tsai and Sun [30] observed an increase of 50% for the extrapolated shear strength of S2/8552 glass-epoxy at a strain rate of 600s^{-1} .



(a) IPS response from off-axis test



(b) pure IPS strength extrapolation

Figure 5.31: In-plane shear (IPS) response and extrapolation of pure IPS strength.

Table 5.5: Quasi-static and dynamic in-plane shear properties.

	Modulus G_{12} [MPa]	Yield Strength S_L^y [MPa]	Ultimate Strength S_L [MPa]
Quasi-Static	5068	29.7	99.9
Dynamic	6345	55.7	141.8

5.5.3 Combined Transverse Compression and In-Plane Shear

Figure 5.32 compares the strain rate effect observed for the off-axis compression properties with those observed for the transverse compression and in-plane shear properties. Uniform trends were found for modulus, yield strength and ultimate strength. As an average value for all specimen types, the modulus increases by 20% (some scatter is noted for the 15° and 30° tests, though). The yield strength increases by an average value of 85% and the least scatter was found for the ultimate strength, with an average increase of 40%. A similar observation was made by Vogler and Kyriakides [77] when comparing the strain rate effect on the transverse compressive and in-plane shear strength of neat PEEK resin.

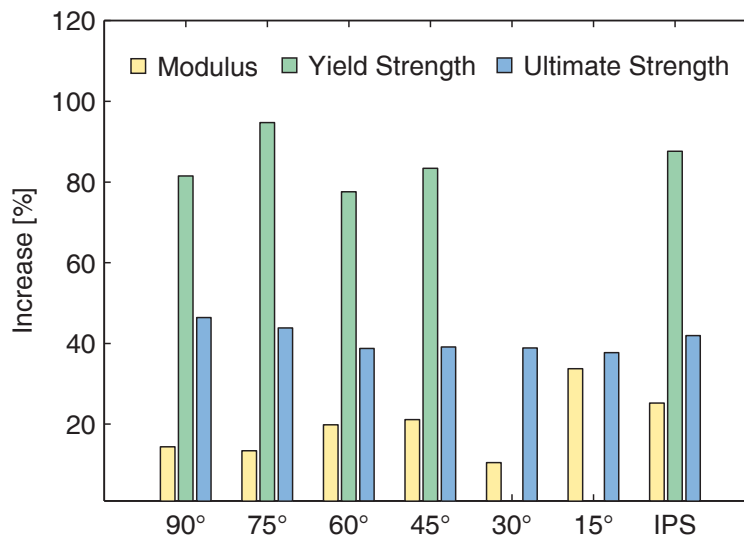
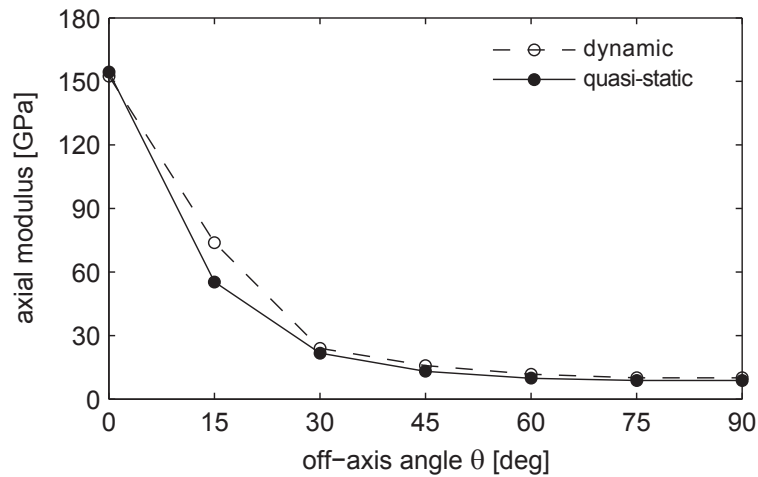


Figure 5.32: Strain rate effect on elastic modulus, yield and failure strength.

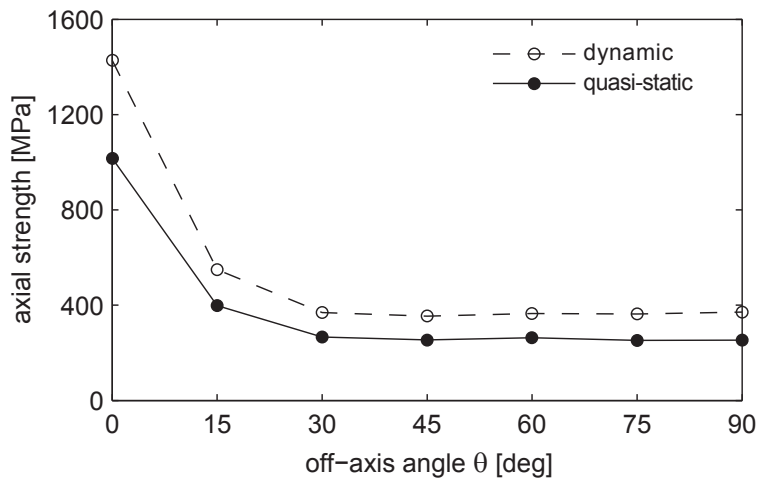
Figure 5.33 shows the quasi-static and dynamic axial modulus and ultimate strengths as a function of the off-axis angle. For completeness, the longitudinal compressive modulus and strength were included in these diagrams. The longitudinal compressive experiments were performed with a different test setup but for similar strain rates as the tests discussed here. Details of the longitudinal compression test setup can be found in Chapter 4.

Whilst a small modulus increase was observed for all off-axis and transverse compression tests, no rate effect was observed for the longitudinal compressive modulus. Considering

the high ratio between longitudinal and transverse compressive moduli, the rate effect on the elastic modulus for the off-axis and transverse compression specimens is hardly noticeable at the strain rates considered in this study. By contrast, the increase observed for the axial off-axis and transverse strengths was also observed for the longitudinal compressive strength.



(a) modulus of elasticity



(b) ultimate strength

Figure 5.33: Compressive modulus and ultimate strength vs. off-axis angle θ .

Quasi-static and dynamic failure envelopes for combined transverse compression and in-plane shear loading are shown in Figure 5.34. The experimental envelopes were obtained by applying Equation (5.1) to the ultimate strengths measured for the transverse and off-axis compression specimens.

The experimental results are presented together with the Puck criterion for matrix failure under transverse compression [86]. The Puck failure criterion for matrix cracking generalises Mohr's criterion by imposing a quadratic interaction between the shear stresses that act on a possible fracture plane:

$$\left(\frac{\tau_T}{S_T - \eta_T \sigma_n} \right)^2 + \left(\frac{\tau_L}{S_L - \eta_L \sigma_n} \right)^2 = 1 \quad (5.3)$$

Similarly to Mohr's original idea, the Puck failure criterion for matrix failure under transverse compression accounts for the effect of transverse compression on the matrix failure conditions by increasing the shear strengths by the product between the normal stress acting on the fracture plane and the corresponding friction coefficient.

The stresses τ_T , τ_L and σ_n used in Equation (5.3) act on the fracture plane, as shown in Figure 5.35, and are defined for the two-dimensional in-plane stress state as functions of the in-plane stresses σ_{22} , τ_{12} and the angle of the fracture plane α :

$$\begin{aligned} \sigma_n &= \sigma_{22} \cos^2 \alpha \\ \tau_T &= -\sigma_{22} \sin \alpha \cos \alpha \\ \tau_L &= \tau_{12} \cos \alpha \end{aligned} \quad (5.4)$$

S_T and S_L are the respectively transverse and longitudinal shear strengths, and η_T and η_L represent the respectively transverse and longitudinal friction coefficients. For the predicted failure envelopes shown in Figure 5.34, the longitudinal friction and strength parameters η_L and S_L were obtained via the extrapolation method shown in Figure 5.31b. The transverse friction and strength parameters can be calculated in the following way [87]:

$$\eta_T = -\frac{1}{\tan 2\alpha_0} \quad (5.5)$$

$$S_T = Y_C \cos \alpha_0 \left(\sin \alpha_0 + \frac{\cos \alpha_0}{\tan 2\alpha_0} \right) \quad (5.6)$$

where Y_C and α_0 are respectively the strength and fracture angle for pure transverse com-

pression. The fracture angle α_0 , for pure transverse compression, can be calculated by using Equations (5.5) and (5.6) together with Puck's assumption for the ratio between the longitudinal and transverse shear friction and strength parameters of a transversely isotropic material:

$$\frac{\eta_L}{S_L} = \frac{\eta_T}{S_T} \Rightarrow \eta_L = -\frac{S_L \cos 2\alpha_0}{Y_C \cos^2 \alpha_0} \quad (5.7)$$

The calculated fracture angle is $\alpha_0 = 52^\circ$ for both quasi-static and dynamic data sets which is close to the average angle of 56.5° observed in the SHPB experiments (Table 5.4). It should be noted that the fracture angle is calculated using the heuristic relation represented in Equation (5.7). A similar difference between the fracture angle calculated using the Mohr-Coulomb criteria and the corresponding experimental data was observed by Daniel et al. [88].

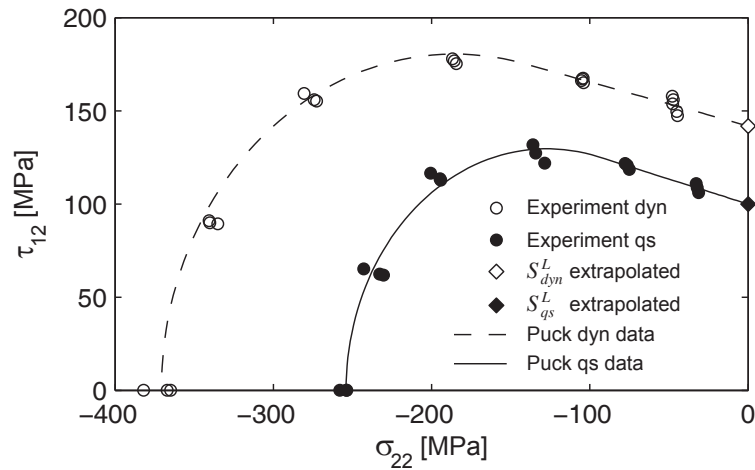


Figure 5.34: Quasi-static and dynamic failure envelopes for combined transverse compression and in-plane shear loading.

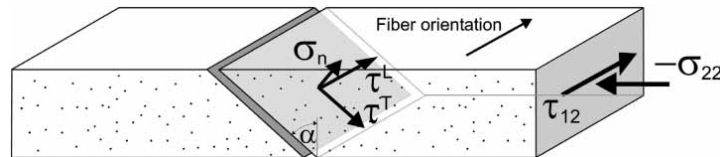


Figure 5.35: Stresses acting on the fracture plane of a unidirectional polymer composite [87].

The predicted failure envelopes shown in Figure 5.34 are functions of the fracture surface angle α and are found by searching for the angle which maximises Equation (5.3) within a range of possible angles ($0 < \alpha < \alpha_0$) [87]. A comparison between the fracture angle α obtained from the SHPB experiments and the predicted fracture surface angle [89] is shown in Figure 5.36.

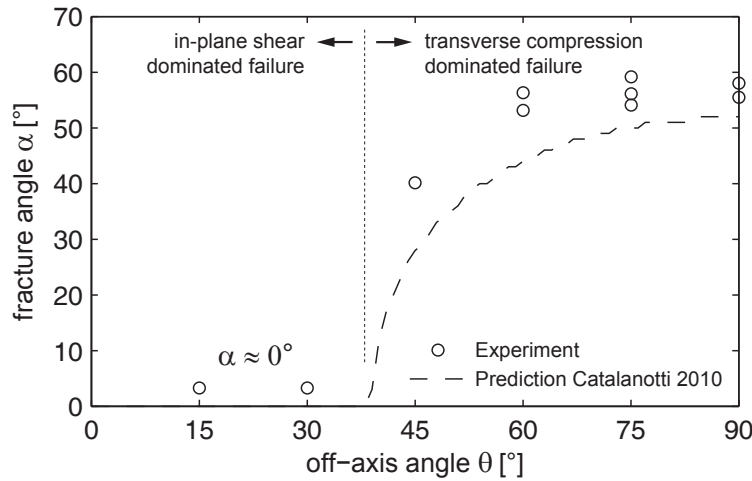


Figure 5.36: Dynamic fracture plane angle.

Since the high speed camera was not available for the quasi-static tests, the quasi-static transverse fracture angle was not determined. Measuring the orientation of the initial fracture plane from the retrieved quasi-static specimen is not possible since those specimens failed with multiple bifurcating cracks as shown in Figure 5.12. However, Wiegand [22] performed fracture surface angle tests with high speed photography for transverse compressive specimens made of UTS/RTM6 carbon-epoxy at quasi-static, medium and high strain rates and concluded that the inclination of the fracture plane does not change with increasing loading rate. The same was concluded by Vural and Ravichandran [90], who performed quasi-static to high strain rate transverse compression tests for glass-epoxy S2-8552 (same resin as in the present study) and further reported that confining pressure (hydrostatic pressure) has no effect on the fracture surface angle for the range of confining pressures and strain rates investigated in the respective study.

It is reasonable to assume that the influence of longitudinal compressive stresses σ_{11} can

be neglected when deriving the failure envelope for combined transverse compression and in-plane shear loading shown in Figure 5.34. The stress-strain behaviour in longitudinal compression for the present material system is linear-elastic up to failure and the longitudinal compressive strength X_C is significantly higher than the transverse compressive strength Y_C and the in-plane shear strength S_L . The value of σ_{11} is small and can be neglected so long as the failure mode is matrix dominated and the fracture plane is parallel to the fibre direction. In the case of some 15° off-axis compression specimens, two different failure modes were observed: fibre-kinking (fibre failure due to failure of the supporting matrix) and in-plane shear failure (matrix failure mode). It is however seen in Figure 5.29, that the failure load and the stress-strain response is similar, despite the two failure modes.

The above assumption is further supported by the biaxial longitudinal and shear loading data provided by Soden et al. [91], which indicates that the in-plane shear strength is not significantly influenced by the presence of longitudinal compressive stresses for high ratios of X_C/σ_{11} .

From the quasi-static and dynamic stress-strain responses (Figure 5.29), it was also possible to obtain the yield envelopes for combined transverse compression and in-plane shear loading. The yield criteria was defined as 0.02% plastic axial strain for the off-axis compression and transverse compression specimens. The in-plane shear yield strength was determined at 0.02% shear strain using the in-plane shear stress-strain response of the 45° off-axis specimen type. The yield envelopes are shown together with the failure envelopes in Figure 5.37. In the quasi-static case the material yields earlier than in the dynamic case, which can be attributed to the stiffening of the axial stress-strain response under dynamic loading due to the visco-plastic response of the polymer matrix (see Figure 5.29).

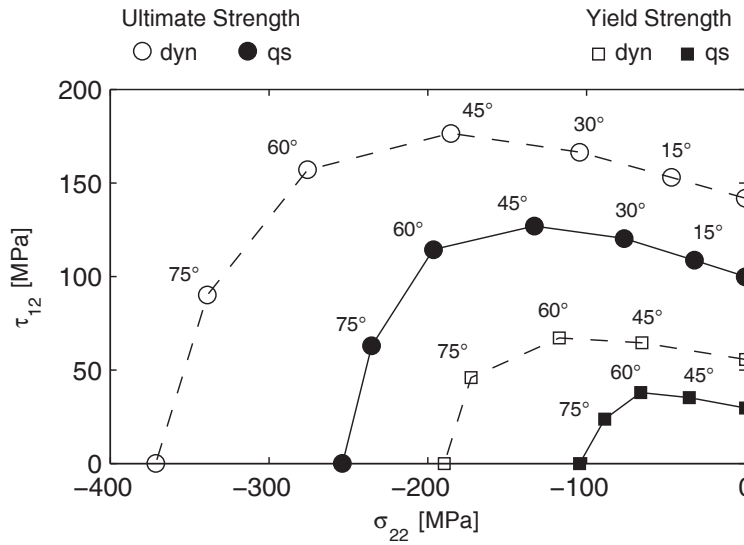


Figure 5.37: Experimental quasi-static and dynamic yield and failure envelopes.

5.6 Conclusions

In this Chapter, 15°, 30°, 45°, 60° and 75° off-axis and transverse compression tests with end-loaded rectangular unidirectional carbon-epoxy specimens were performed under quasi-static and under high strain rates.

It was demonstrated that end-loaded specimens are well suited to determine the off-axis and the transverse compressive properties of unidirectional polymer composites. This is particularly useful for high strain rate tests on the SHPB since the wave propagation for straight-ended bars is well understood, while modified bar-ends may significantly influence the bar-waves and subsequently the dynamic specimen stress-strain response, calculated from the bar waves via SHPB analysis.

The pulse shaping analysis proposed by Nemat-Nasser et al. [66] was used to systematically obtain suited incident-pulses for the respective specimen stress-strain behaviour. SHPB experiments with the specimen in dynamic equilibrium and loaded at near constant strain rates were performed for all of the above specimen types. As a result, both elastic and strength properties were obtained from the dynamic tests with confidence.

For the high strain rate tests a PHOTRON SA5 high speed camera, set to a suitable combination between frame rate and spatial resolution was used, thus allowing a continuous

recording during the entire dynamic test.

The in-plane strain field of the specimen was obtained for the quasi-static and dynamic tests from the ARAMIS digital image correlation (DIC) software. Fundamental SHPB analysis assumptions, such as uniform specimen deformation and strain distribution were validated. Moreover, it was demonstrated that the specimen strain obtained from SHPB analysis was over-predicted for all specimen types except for the transverse compression test. With the aid of digital image correlation, true specimen strain could be obtained and recorded beyond the strain limit of currently available standard foil strain gauges. Contactless measurement techniques such as DIC are also best suited for SHPB experiments, since it was shown by Gilat et al. [6] that foil strain gauges, placed on neat resin SHPB specimens, may cause premature failure.

Significant fibre-rotation was observed for the quasi-static and dynamic 15°, 30° and 45° off-axis compression specimens, which was taken into account during the data reduction for those specimen types. The additional rotation-angle was obtained directly from the DIC software.

The pure IPS strength was obtained with the extrapolation method proposed by Tsai and Sun [30]. The yield in-plane shear strength and the in-plane shear modulus were obtained from the shear-stress and shear-strain components of the 45° off-axis tests.

With the high frame rate and sufficient resolution of the high speed camera, it was possible to visualise the initiation and propagation of the failure process and measure the fracture angle α for the 45°, 60°, 75° off-axis and transverse compression specimen types.

It was found that the Puck failure criterion for matrix compressive failure [86] provides excellent strength predictions for both quasi-static and dynamic loading. In addition, the fracture angles were accurately predicted. It was observed that the dynamic experimental failure envelope is consistently larger than the quasi-static one.

Due to the early dynamic equilibrium and near constant strain rates in the SHPB experiment, the dynamic yield envelope for combined transverse compression and in-plane shear loading could also be determined and compared with the quasi-static yield envelope.

lope.

For the strain rates studied in the present work, the rate effect on the transverse compressive and in-plane-shear behaviour of the carbon-epoxy material system IM7-8552 can be summarised as follows:

Transverse Compression

- The modulus of elasticity increased by a moderate 12%.
- The yield strength increased significantly by about 83%.
- The failure strength increased by 45%.
- The observed rate effect on the transverse compressive failure strain is insignificant and therefore it is concluded that this property is not strain rate sensitive for the strain rates considered in this work.

In-Plane Shear

- The in-plane shear modulus, obtained from the shear stress-strain component of the 45° off-axis compression tests, increased by 25%.
- The yield strength, obtained from the same shear stress-strain curves, increased by 88%.
- The pure in-plane shear strength, extrapolated in the combined $\sigma_{22} - \tau_{12}$ stress-diagram from the failure strength of 15° and 30° specimens which failed in an in-plane-shear dominated failure mode, increased by 42%.
- The strain rate effect on the in-plane shear failure strain was not determined due to the dependency of the apparent failure strength and strain on the ratio of biaxial loading.

Combined Transverse Compression and In-Plane Shear

- As an average for all specimen types subjected to combined loading, the axial modulus increased by 20%. Some scatter was however observed for the 15° and 30° tests.
- A relatively uniform increase was observed for the yield strength, with an average value of 85%.
- The least scatter, when comparing the individual specimen types, was found for the rate effect on the failure strength, with an average increase of 40%.

It is understood that the two strain rate regimes considered in this study do not allow a complete determination of the rate effect on the material properties of the present material system. Experimental evidence suggests that a shift from approximately linear to exponential strain rate behaviour occurs between 100s^{-1} and 1000s^{-1} , with significant increases for modulus and strength above 1000s^{-1} [8, 19, 22].

Chapter 6

Experiments: Combined Transverse Tension and In-Plane Shear

6.1 Introduction

To fully understand the in-plane constitutive response of the carbon-epoxy material system chosen in the present work, it is also necessary to perform tension tests with varying ratios of transverse tension and in-plane shear. Off-axis tension (OAT) tests are used here to study the nonlinearity occurring under tensile loading and to establish the quasi-static failure envelope for combined transverse tension and in-plane shear.

In the previous chapter it was shown that the extension-shear coupling effect and subsequent fibre rotation is not restrained for off-axis compression tests, when using end-loaded specimens and minimising friction at the specimen end surfaces.

For off-axis tension tests however, the specimen must be clamped and extension-shear coupling leads to significant stress concentrations near the clamping regions and to a non-uniform strain field in that region. Due to the stress concentrations, the failure load may be lower than the actual material strength and the failure envelope for combined tension and shear will be underestimated.

Sun and Berreth [92] addressed this issue and performed off-axis tension tests using tabs

made of fiberglass knit embedded in a silicon rubber matrix. The compliant rubber matrix allowed shear deformation to occur and better results were obtained for the in-plane shear properties. Due to the silicon matrix however, the adhesion of the tab to the specimen was of poor quality.

In a follow-up study, Sun and Chung [93] proposed an oblique angle tab, made of conventional aluminum alloy and showed that the stress concentration can be significantly reduced, leading to a more uniform strain field in the specimen. The proposed oblique angle tab was successfully used by other researchers for quasi-static [94] and dynamic [11] off-axis tension tests, and was also adopted in the present study.

6.2 Oblique Angle Tab Design

The off-axis tension specimen with oblique tabs is schematically shown in Figure 6.1.

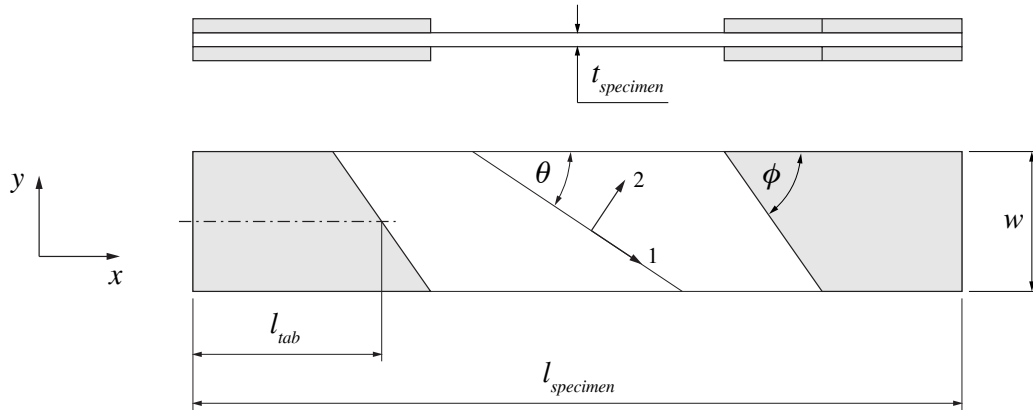


Figure 6.1: Off-axis tension specimen with oblique tab.

According to Sun and Chung [93], the oblique tab angle ϕ can be determined from the relation

$$\cot \phi = -\frac{\bar{S}_{16}}{\bar{S}_{11}} \quad (6.1)$$

where $\bar{S}_{11}$ and $\bar{S}_{16}$ are components of the compliance matrix in the loading coordinate system x - y , defined in terms of the in-plane elastic material constants E_1 , E_2 , G_{12} and ν_{12}

and the off-axis angle θ as [95]:

$$\begin{aligned}\bar{S}_{11} &= \frac{\cos^4 \theta}{E_1} + \frac{\sin^4 \theta}{E_2} + \frac{1}{4} \left(\frac{1}{G_{12}} - 2 \frac{\nu_{12}}{E_1} \right) \sin^2 2\theta \\ \bar{S}_{16} &= - \left(\frac{2}{E_2} + 2 \frac{\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) \sin^3 \theta \cos^3 \theta + \left(\frac{2}{E_1} + 2 \frac{\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) \cos^3 \theta \sin \theta\end{aligned}\quad (6.2)$$

Using the in-plane elastic material constants from an earlier quasi-static material characterisation of IM7-8552 [67], namely $E_1 = 171420\text{MPa}$, $E_2 = 9080\text{MPa}$, $G_{12} = 5290\text{MPa}$ and $\nu_{12} = 0.32$, the oblique tab angle ϕ can be plotted as a function of the off-axis angle θ as shown in Figure 6.2.

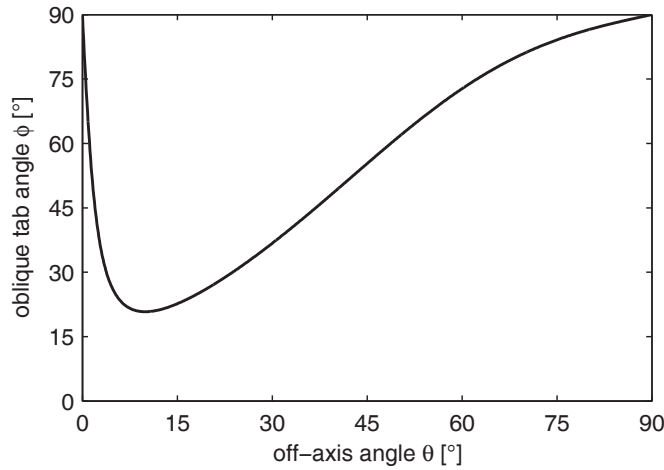


Figure 6.2: Oblique tab angle ϕ as a function of the off-axis angle θ .

6.3 Experimental Setup

Off-axis tension specimens with 15° , 30° , 45° , 60° and 75° off-axis angles were cut from a 12-ply unidirectional panel, which was manufactured in the same way as described in Section 5.2.1. Three specimens were made for each off-axis angle, with nominal dimensions of $250 \times 20 \times 1.5\text{mm}^3$. The tabs were manufactured from a 1.5mm aluminum sheet and bonded to the specimen using Araldite 2011 epoxy adhesive. The dimension l_{tab} (Figure 6.1) was maintained constant while the oblique angle ϕ was varied according

Table 6.1: Angle configuration for off-axis tension specimens.

off-axis angle	θ [°]	15	30	45	60	75
oblique tab angle	ϕ [°]	23	37	55	73	84

to the off-axis specimen type (Table 6.1).

As shown in Figure 6.3, all specimens were prepared for digital image correlation measurements by applying a random black-on-white speckle pattern to the specimen surface using aerosol spray painting.

The DIC data acquisition system (Figure 6.4), used to measure the in-plane strain field at the specimen centre, was the same as the one used for the quasi-static off-axis compression tests described in Section 5.2.2. The images were captured at a frame rate of 1fps and the off-axis tension specimens were tested at a constant displacement rate of $2\text{mm}\cdot\text{min}^{-1}$, using the same INSTRON test machine as for the compression tests.



(a) untreated



(b) with speckle pattern

Figure 6.3: Off-axis tension specimen before and after preparation for digital image correlation.



Figure 6.4: Off-axis tension test setup with low speed DIC data acquisition system.

The axial tensile specimen stress σ_{xx} was determined by dividing the load obtained from the load cell of the test machine by the specimen cross section.

6.4 Experimental Results

As shown in Figure 6.5 for two representative tests, the off-axis tension specimens failed predominantly in the middle of the gauge section. The anticipated goal of reducing the stress concentrations at the clamping regions, by using oblique angle tabs, was therefore successfully accomplished.

The axial stress-strain response of all tested off-axis tension specimens is shown in Figure 6.6a. As done for the compression tests, described in Chapter 5, the axial strain ϵ_{xx} was determined as the average of the in-plane strain field component ϵ_{xx} , with respect to a virtual strain gauge area of $3 \times 3 \text{ mm}^2$ at the specimen centre.

A systematic error occurred during the DIC data acquisition, which was later identified to been caused by rigid body and realignment movements of the load chain at the beginning of the tests. The resulting irregularity in the measured strain field is most pronounced for the 15° off-axis tension specimen. As a result, a reliable determination of the axial modulus and yield strength was not possible for the off-axis tension tests. The axial

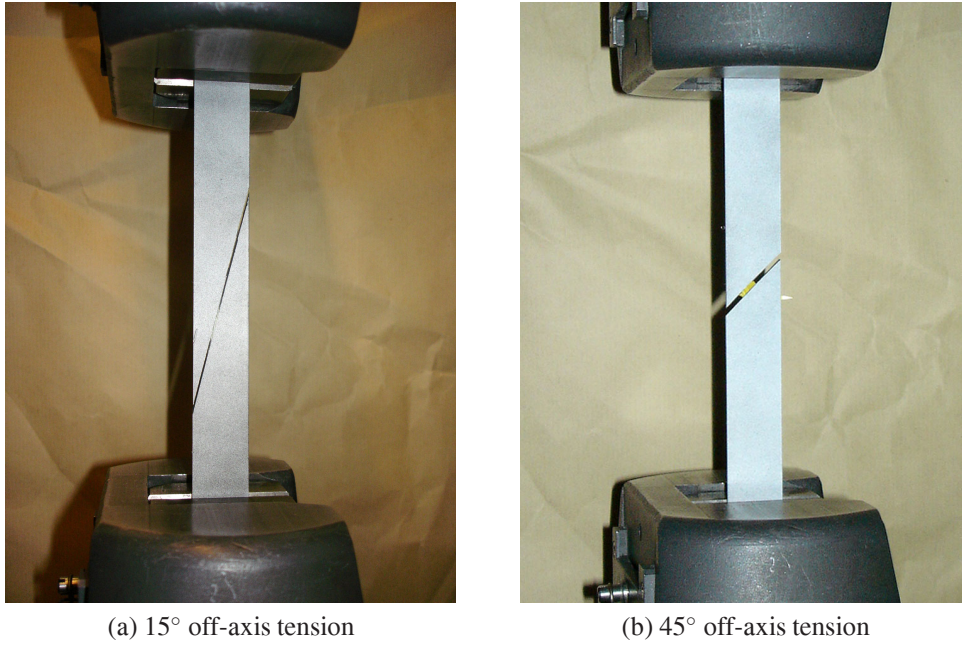


Figure 6.5: Examples of failed off-axis tension specimens.

ultimate strength could be measured however, since the irregularity did not appear in the stress-time response (Figure 6.6b).

From the axial ultimate strength, determined as the maximum of the stress-time response shown in Figure 6.6b and summarised in Table 6.2, the failure envelope for combined transverse tension and in-plane shear was constructed, by calculating the transverse tension and in-plane shear stress component at failure with Equation (5.1).

Together with the quasi-static off-axis compression, transverse compression and in-plane shear data from Chapter 5, and further with the transverse tension test data from [67]

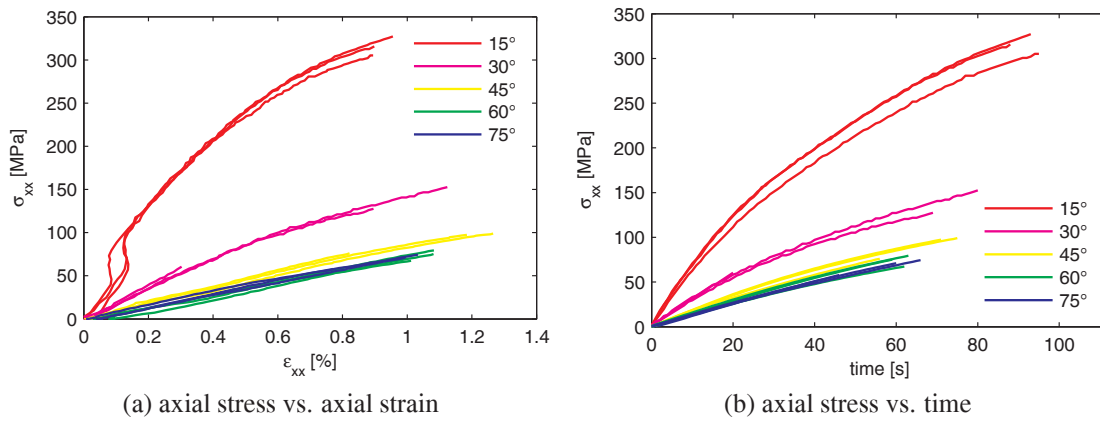


Figure 6.6: Axial stress-strain and axial stress-time response.

Table 6.2: Quasi-static off-axis tension test results.

Fibre angle θ	No. of tests	Axial ultimate strength [MPa]	
15°	3	Mean	316
		STDV	11
		CV [%]	3.5
30°	3 ^a	Mean	140
		STDV	18
		CV [%]	12.7
45°	3	Mean	91
		STDV	13
		CV [%]	14.1
60°	3	Mean	74
		STDV	6
		CV [%]	8.2
75°	3	Mean	68
		STDV	9
		CV [%]	13.0

^a one of three tested 30° off-axis tension specimens failed prematurely and was therefore excluded from the statistical data treatment

(average transverse tensile strength $Y_T = 62.3\text{MPa}$), the complete failure envelope in the $\sigma_{22} - \tau_{12}$ stress space can be constructed as shown in Figure 6.7. For combined transverse compression and in-plane shear, the experimental failure envelope is compared with the Puck matrix compression failure criterion [86] (see Section 5.5.3 for further details). For combined transverse tension and in-plane shear, the experimental failure envelope is compared with the quadratic interaction criterion, proposed by Hashin [96]

$$\left(\frac{\tau_{12}}{S_L}\right)^2 + \left(\frac{\sigma_{22}}{Y_T}\right)^2 = 1 \quad (6.3)$$

and with the failure criterion for transverse tension proposed by Dávila et al. [87], which is defined as

$$(1 - g)\frac{\sigma_{22}}{Y_T} + g\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\tau_{12}}{S_L}\right)^2 = 1 \quad (6.4)$$

in linear and quadratic terms of the stress components acting on the fracture plane. The

parameter g in Equation (6.4) is defined as the ratio of the interlaminar mode I and mode II fracture toughnesses G_{Ic} and G_{IIc} , respectively.

$$g = \frac{G_{Ic}}{G_{IIc}} \quad (6.5)$$

Using the values $G_{Ic} = 0.2774 \text{kJm}^{-2}$ and $G_{IIc} = 0.7879 \text{kJm}^{-2}$, determined for IM7-8552 in the quasi-static material characterisation program [67], the parameter g takes a value of 0.3521. It is noted that for $g = 1$, Equation (6.4) is equal to the quadratic interaction criterion defined with Equation (6.3).

It is seen in Figure 6.7, that the quadratic Hashin criterion [96] tends to overestimate the failure stress for the 15° and 30° off-axis tension specimens, whereas a better correlation between predicted and experimental failure envelope is achieved with the criterion proposed by Dávila et al. [87].

Figure 6.7 also shows the experimental yield strength envelope, constructed from the off-axis compressive and transverse compressive yield strengths reported in Table 5.2, and

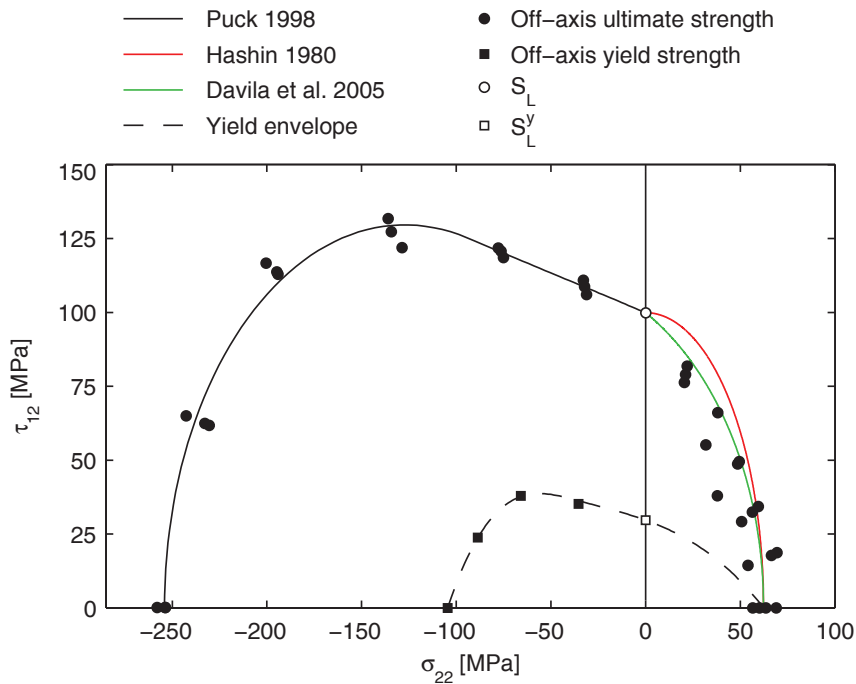


Figure 6.7: Quasi-static failure and yield envelopes in the $\sigma_{22} - \tau_{12}$ stress space.

the quasi-static in-plane shear yield strength reported in Table 5.5. As mentioned above, it was not possible to determine the off-axis tension yield strength due to the systematic error in the DIC strain field measurement. Considering however the linear transverse tension stress-strain response reported in [67], the yield envelope for combined transverse tension and in-plane shear can be approximated by fitting the experimental data as shown in Figure 6.7.

6.5 Conclusions

Off-axis tension tests were performed to determine the constitutive response and the failure envelope of IM7-8552 for combined transverse tension and in-plane shear under quasi-static loading.

The off-axis tension tests can be used together with the quasi-static compression tests described in Chapter 5, to evaluate the material response in the $\sigma_{22} - \tau_{12}$ stress space.

Unfortunately, elastic modulus and yield strength measurements were not possible for the off-axis tension tests due to a systematic error in the DIC strain field data acquisition. The quasi-static off-axis tensile strength was however determined with good accuracy and was used to construct the experimental failure envelope for combined transverse tension and in-plane shear.

By using oblique angle tabs, the stress concentrations at the clamping regions, resulting from extension-shear coupling, could be reduced and premature failure was avoided. The off-axis specimens then predominantly failed in the middle of the gauge section.

The experimental failure envelope for transverse tension and in-plane shear was compared with the Hashin [96] criterion and with the matrix tension criterion proposed by Dávila et al. [87]. A better correlation between experimental and predicted failure envelope was obtained with the criterion proposed by Dávila et al. [87].

Chapter 7

Numerical Modelling

7.1 Plasticity Model

The plasticity model developed by Sun and Chen [97] is used in this work to describe the non-linear constitutive behaviour of the unidirectional carbon-epoxy IM7-8552. The plasticity model is based on a quadratic yield function $f(\sigma_{ij})$, defined in terms of the stress components σ_{ij} in the principal material directions for the general 3-dimensional case as:

$$\begin{aligned} 2f(\sigma_{ij}) &= a_{11}\sigma_{11}^2 + a_{22}\sigma_{22}^2 + a_{33}\sigma_{33}^2 \\ &\quad + 2a_{12}\sigma_{11}\sigma_{22} + 2a_{13}\sigma_{11}\sigma_{33} + 2a_{23}\sigma_{22}\sigma_{33} \\ &\quad + 2a_{44}\sigma_{23}^2 + 2a_{55}\sigma_{13}^2 + 2a_{66}\sigma_{12}^2 \\ &= k \end{aligned} \tag{7.1}$$

The anisotropy of the plasticity is governed by the coefficients a_{ij} . By applying the Hill-type yield function for orthotropic materials, the number of independent coefficients can be reduced as follows:

$$\begin{aligned} a_{12} &= a_{33} - 0.5(a_{11} + a_{22} + a_{33}) \\ a_{13} &= a_{22} - 0.5(a_{11} + a_{22} + a_{33}) \\ a_{23} &= a_{11} - 0.5(a_{11} + a_{22} + a_{33}) \end{aligned} \tag{7.2}$$

The incremental plastic strain components $d\varepsilon_{ij}^p$ are defined in terms of the yield function f and the plastic multiplier $d\lambda$ through the associated flow rule:

$$d\varepsilon_{ij}^p = \frac{\partial f}{\partial \sigma_{ij}} d\lambda \quad (7.3)$$

The incremental plastic work is defined as:

$$dW^p = \sigma_{ij} d\varepsilon_{ij}^p = 2f d\lambda \quad (7.4)$$

and the effective stress is defined as:

$$\bar{\sigma} = \sqrt{3f} \quad (7.5)$$

The plastic work increment can be defined in terms of the effective stress $\bar{\sigma}$ and the effective plastic strain increment $d\bar{\varepsilon}^p$ such that

$$dW^p = \bar{\sigma} d\bar{\varepsilon}^p \quad (7.6)$$

By substitution of Equations (7.4) and (7.5) into Equation (7.6), the effective plastic strain increment can be written as

$$d\bar{\varepsilon}^p = \frac{2}{3} \bar{\sigma} d\lambda \quad (7.7)$$

from which the plastic multiplier $d\lambda$ can be obtained as

$$d\lambda = \frac{3}{2} \left(\frac{d\bar{\varepsilon}^p}{\bar{\sigma}} \right) \left(\frac{d\bar{\sigma}}{\bar{\sigma}} \right) \quad (7.8)$$

The scalar k in Equation (7.1) is found by comparing the plastic yield function (7.1) and Equation (7.5)

$$k = \frac{2}{3} \bar{\sigma}^2 \quad (7.9)$$

Following a general practice in plasticity, the incremental strain $d\varepsilon_{ij}$ can be decomposed

into an elastic strain component $d\epsilon_{ij}^e$ and a plastic strain component $d\epsilon_{ij}^p$, such that

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p \quad (7.10)$$

Sun and Chen [97] further assume that the longitudinal plastic strain increment $d\epsilon_{11}^p$ can be neglected since most unidirectional FRPs behave linearly in longitudinal tension and compression, as shown for both quasi-static and dynamic loading in Chapter 4.

$$d\epsilon_{11}^p = 0 \quad (7.11)$$

The assumption (7.11) can be used together with the associated flow rule (7.3) to determine the plastic coefficients a_{11} , a_{12} and a_{13} , which are found to be:

$$a_{11} = a_{12} = a_{13} = 0 \quad (7.12)$$

Substitution of (7.12) into the condition for the plasticity coefficients of the Hill-type yield function (7.2) leads to:

$$a_{22} = a_{33}, \quad a_{23} = -a_{22} \quad (7.13)$$

Assuming also that $a_{22} = 1$ [97], the general yield function (7.1) reduces for the case of an in-plane stress state to:

$$2f = \sigma_{22}^2 + 2a_{66}\sigma_{12}^2 \quad (7.14)$$

The in-plane plastic strain increments are then found by applying the associated flow rule (7.3) to the in-plane yield function (7.14), and are defined as

$$\begin{Bmatrix} d\epsilon_{11}^p \\ d\epsilon_{22}^p \\ d\gamma_{12}^p \end{Bmatrix} = \begin{Bmatrix} 0 \\ \sigma_{22} \\ 2a_{66}\sigma_{12} \end{Bmatrix} d\lambda \quad (7.15)$$

where γ_{12} denotes engineering shear strain.

The out-of-plane plastic strain increment can be calculated as

$$d\varepsilon_{33}^p = a_{23}\sigma_{22}d\lambda \quad (7.16)$$

For the case of plane stress, the effective stress is then given by

$$\bar{\sigma} = \left[\frac{3}{2}(\sigma_{22}^2 + 2a_{66}\sigma_{12}^2) \right]^{1/2} \quad (7.17)$$

and the effective plastic strain increment is defined as

$$d\bar{\varepsilon}^p = \left[\frac{2}{3}(\sigma_{22}^2 + 2a_{66}\sigma_{12}^2) \right]^{1/2} d\lambda \quad (7.18)$$

The incremental plastic stress-strain relation only depends on the values of the plasticity coefficient a_{66} and the plastic multiplier $d\lambda$.

7.1.1 Deriving the Plasticity Model Parameters

The plasticity coefficient a_{66} and the relation between the effective plastic stress $\bar{\sigma}$ and the effective plastic strain $\bar{\varepsilon}^p$ can be obtained from off-axis tests, as illustrated for off-axis compression in Figure 7.1.

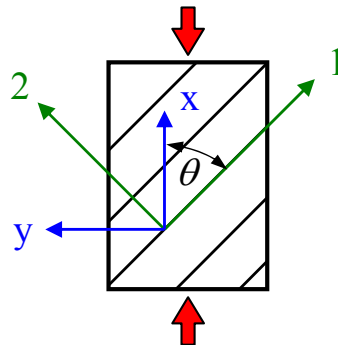


Figure 7.1: Off-axis compression test coordinate systems.

The x -axis corresponds to the loading direction and the 1-axis corresponds to the fibre direction of the unidimensional test specimen. The in-plane stress components in the material coordinate system can be expressed as

$$\begin{aligned}\sigma_{11} &= \sigma_{xx} \cos^2 \theta \\ \sigma_{22} &= \sigma_{xx} \sin^2 \theta \\ \sigma_{12} &= -\sigma_{xx} \sin \theta \cos \theta\end{aligned}\tag{7.19}$$

where σ_{xx} is the applied stress in the loading direction.

By substitution of Equation (7.19) into Equations (7.17) and (7.18), the effective stress $\bar{\sigma}$ and the effective plastic strain increment $d\bar{\epsilon}^p$ can be formulated in terms of the measured axial stress σ_{xx} such that

$$\bar{\sigma} = h(\theta) \sigma_{xx}\tag{7.20}$$

$$d\bar{\epsilon}^p = \frac{2}{3} h(\theta) \sigma_{xx} d\lambda\tag{7.21}$$

where the function $h(\theta)$ depends on the plasticity coefficient a_{66} and is defined as

$$h(\theta) = \left[\frac{3}{2} (\sin^4 \theta + 2a_{66} \sin^2 \theta \cos^2 \theta) \right]^{1/2}\tag{7.22}$$

According to standard transformation from the material to the loading coordinate system, the plastic strain increment $d\epsilon_{xx}^p$ in the loading direction can be expressed through the plastic strain increments in the material system as

$$d\epsilon_{xx}^p = \cos^2 \theta d\epsilon_{11}^p + \sin^2 \theta d\epsilon_{22}^p - \frac{1}{2} \sin 2\theta d\gamma_{12}^p\tag{7.23}$$

Using Equations (7.15) and (7.19), Equation (7.23) can be rewritten as

$$d\epsilon_x^p = [\sin^4 \theta + 2a_{66} \sin^2 \theta \cos^2 \theta] \sigma_{xx} d\lambda = \frac{2}{3} h^2(\theta) \sigma_{xx} d\lambda\tag{7.24}$$

A comparison of Equations (7.21) and (7.24) yields the relation

$$d\bar{\epsilon}^p = d\epsilon_{xx}^p / h(\theta) \quad (7.25)$$

which can be integrated to obtain the effective plastic strain $\bar{\epsilon}^p$

$$\bar{\epsilon}^p = \epsilon_{xx}^p / h(\theta) \quad (7.26)$$

With Equations (7.20) and (7.26), the effective stress-effective plastic strain curves from off-axis specimens with different off-axis angles can be expressed in terms of the measured axial stress σ_{xx} and the measured axial plastic strain ϵ_{xx}^p .

The model proposed by Sun and Chen [97] is applicable to the respective composite material system if a value for the plasticity coefficient a_{66} can be found such that the $\bar{\sigma}$ - $\bar{\epsilon}^p$ curves for the different off-axis specimen types collapse into one master effective stress-effective plastic strain curve, which can then be approximated by the power law

$$\bar{\epsilon}^p = A_{pm} \bar{\sigma}_{pm}^{n_{pm}} \quad (7.27)$$

by choosing an appropriate parameter set for A_{pm} and n_{pm} .

The parameter a_{66} is calculated, using a MATLAB script and the experimental data obtained from the transverse and off-axis compression experiments.

Sun and Chen [97] noted that the effective stress-effective plastic strain curve is independent of the parameter a_{66} in the case of transverse loading, where $\theta = 90^\circ$. The effective stress-effective plastic strain curve from a test where the loading axis is equal to the transverse direction of the material, can therefore yield the master curve, provided that plasticity is observed for the transverse test.

Figure 7.2 shows the collapsed effective stress-effective plastic strain curves, obtained with Equations (7.20) and (7.26) from the quasi-static and dynamic axial stress-axial strain responses of the off-axis compression (OAC) and transverse compression tests

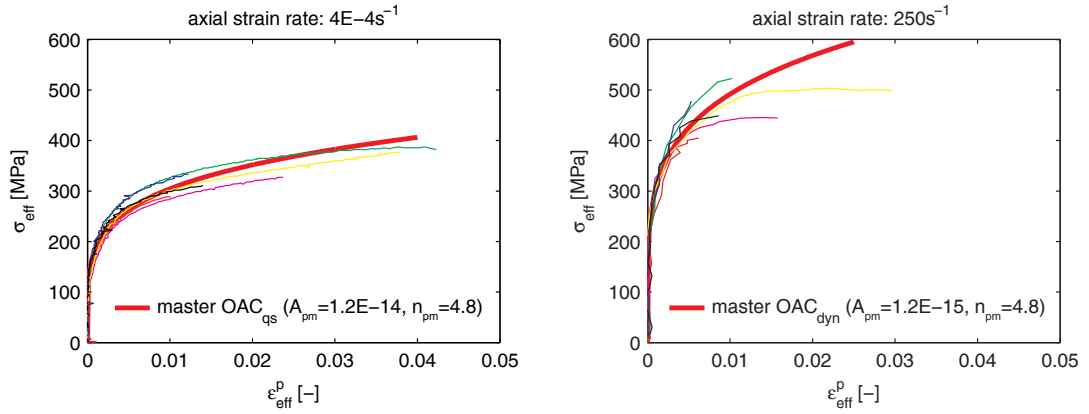


Figure 7.2: Collapsed experimental $\bar{\sigma} - \bar{\epsilon}^p$ curves for two strain rate regimes ($a_{66} = 2.2$).

shown in Figures 5.29 and 5.30. It is noted that the master curves were obtained using the actual fibre orientation angle, as explained in Chapter 5.

It is seen in Figure 7.2 that the same plasticity coefficient $a_{66} = 2.2$ can be used for both strain rate regimes. Also, the master curve parameter n_{pm} does not depend on the rate of loading. Only the master curve parameter A_{pm} is found to be rate dependent.

Considering rate effects in a subsequent development of the original plasticity model [98, 99, 100], a power law was proposed to describe the rate dependency of the master curve parameter A_{pm} as a function of the effective plastic strain rate $\bar{\epsilon}^p$. For convenience, the measured axial strain rate is used here instead of the effective plastic strain rate, since the difference between the measured axial strain rate and the effective plastic strain rate can be neglected when considering the magnitude variation between the quasi-static and dynamic strain rate regime. The rate dependency of the master curve parameter A is therefore defined as

$$A_{pm} = \chi(\dot{\epsilon})^m \quad (7.28)$$

In Figure 7.3, the master curve parameter A_{pm} is plotted as a function of the average axial strain rate, and it is shown how the power law parameters χ and m in Equation (7.28) can be determined from the linear relation in the $\log A_{pm} - \log \dot{\epsilon}$ diagram.

Figure 7.4 shows the collapsed effective stress-effective plastic strain curves, obtained with Equations (7.20) and (7.26) from the quasi-static off-axis tension (OAT) tests de-

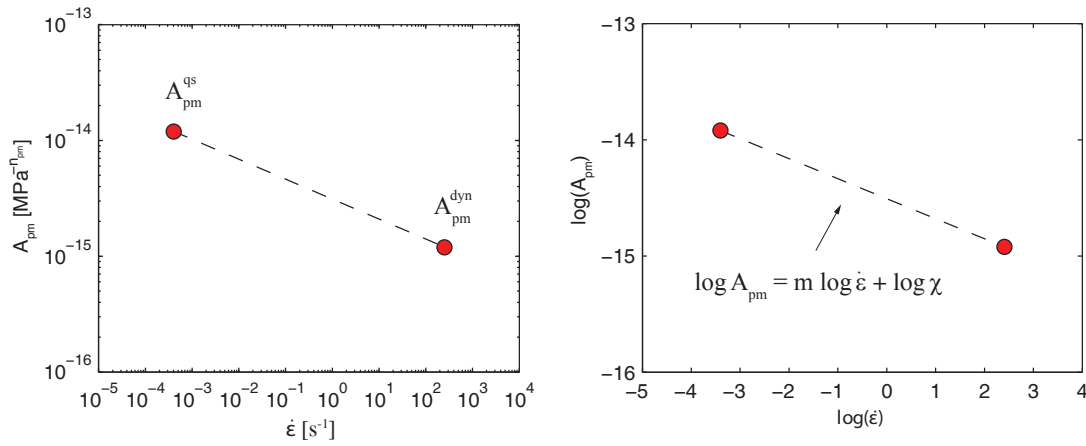


Figure 7.3: Rate dependency of master curve parameter A_{pm} for OAC data set.

scribed in Chapter 6. For comparison the quasi-static master curve from the OAC tests (Figure 7.2) is also shown.

It is found that the value of the plasticity parameter a_{66} is equal to 2.2, as observed already for both quasi-static and dynamic OAC tests. It is further seen that the OAT master curve is lower than the OAC master curve, but of similar shape, which can be attributed to the effect of hydrostatic pressure on the yield strength of polymers.

It is noted that Figure 7.4 corresponds to a complete in-plane plasticity characterisation of the unidirectional carbon-epoxy material system IM7-8552.

Since dynamic off-axis tension test data was not available, some assumptions are made

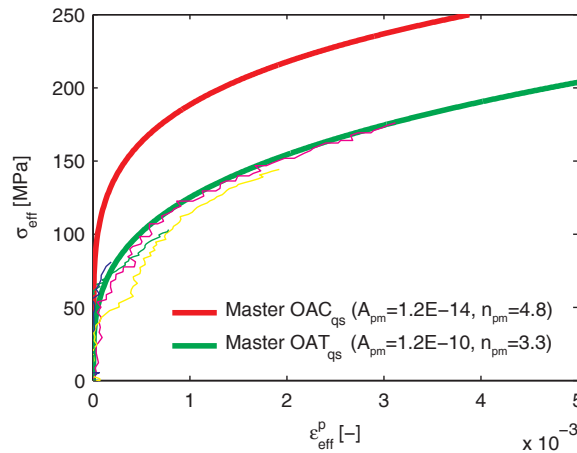


Figure 7.4: Collapsed experimental $\bar{\sigma} - \bar{\epsilon}^p$ curves from quasi-static off-axis tension tests ($a_{66} = 2.2$).

to establish the rate dependency of the tensile master curve.

For the OAC tests it was found that only the master curve parameter A_{pm} is rate dependent. It is therefore assumed that the same holds for off-axis tension. It is further assumed that the parameter m in Equation (7.28) is equal for both OAC and OAT tests. Based on these assumptions it is possible to determine a value for A_{pm} at a higher strain rate and find the missing parameter χ in the $\log A_{pm} - \log \dot{\epsilon}$ diagram shown in Figure 7.5.

The plasticity parameter sets for off-axis compression and off-axis tension are summarised in Table 7.1. A comparison of the compressive and tensile master curves, plotted for two strain rate regimes, is shown in Figure 7.6.

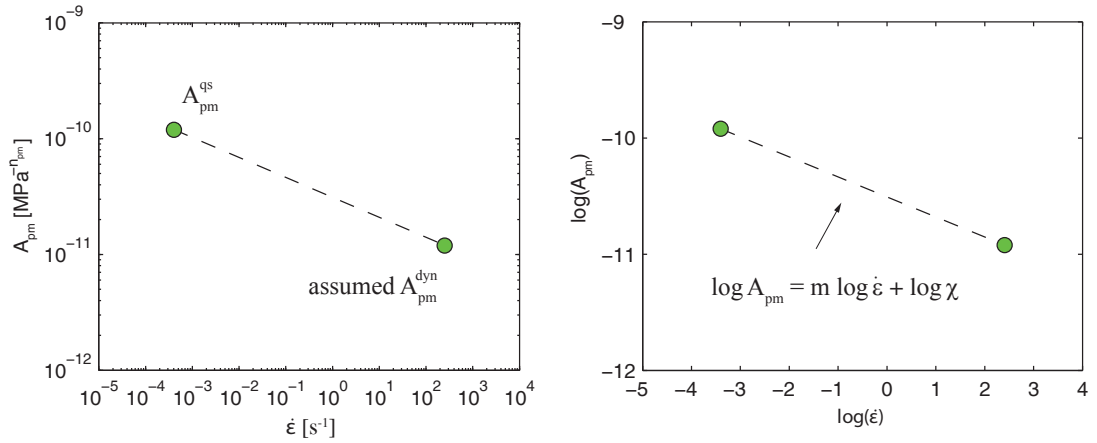


Figure 7.5: Rate dependency of master curve parameter A_{pm} for OAT data set.

Table 7.1: Plasticity model parameters from off-axis compression (OAC) and off-axis tension (OAT) tests.

	OAC	OAT
a_{66}	2.2	2.2
n_{pm}	4.8	3.3
m	-0.172	-0.172
χ	3.1e-15	3.1e-11

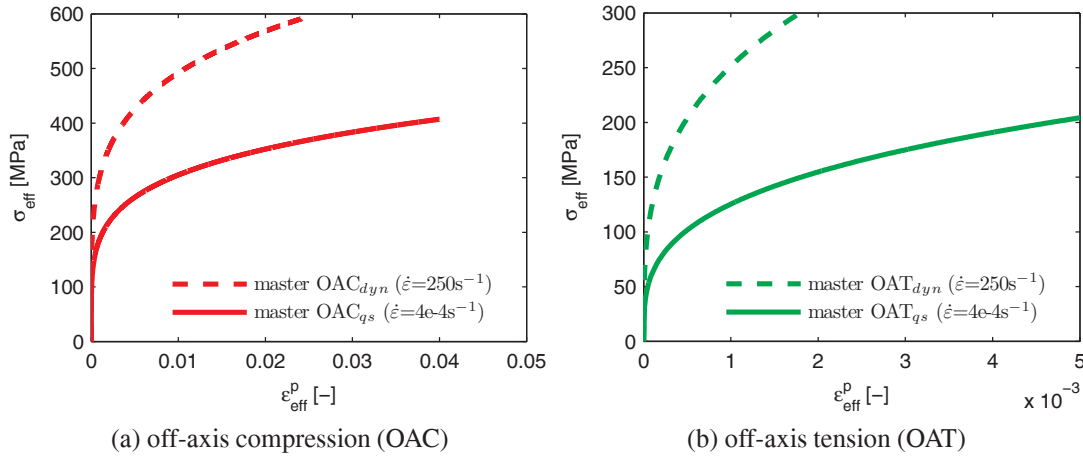


Figure 7.6: Compressive and tensile master curves plotted for two strain rate regimes.

7.2 Failure Criteria for Unidirectional Composites

The failure criteria used in the present work was developed in a parallel study at the University of Porto and a detailed description can be found in [89]. The criteria was developed for the general three-dimensional stress state and the failure index equations for the respective failure modes are therefore presented in the general form. The failure criteria was then implemented for an in-plane stress state, since the constitutive plasticity model, described in the previous Section, was only implemented for an in-plane stress state at the time of writing this thesis. It is noted that the failure criterion for matrix compression was already introduced for the in-plane stress state in Section 5.5.3, and is repeated in the present Section for clarity reasons.

7.2.1 Matrix Failure

In a unidirectional laminae, matrix failure occurs along a fracture plane which is parallel to the fibre direction and inclined at an angle α , with respect to the through-thickness direction as shown in Figure 5.35.

For the general three-dimensional stress state the stresses σ_n , τ_T and τ_L , which act on the

fracture plane as shown in Figure 5.35, are defined as

$$\begin{aligned}\sigma_n &= \cos^2 \alpha \sigma_{22} + 2 \cos \alpha \sin \alpha \sigma_{23} + \sin^2 \alpha \sigma_{33} \\ \tau_T &= -\sin \alpha \cos \alpha \sigma_{22} + (\cos^2 \alpha - \sin^2 \alpha) \sigma_{23} + \sin \alpha \cos \alpha \sigma_{33} \\ \tau_L &= \cos \alpha \sigma_{12} + \sin \alpha \sigma_{13}\end{aligned}\quad (7.29)$$

Matrix Compression

If the stress component σ_n is negative, the failure index for matrix compressive failure FI_{MC} is defined according to the formulation proposed by Puck and Schürmann [86]

$$FI_{MC} = \left(\frac{\tau_T}{S_T - \eta_T \sigma_n} \right)^2 + \left(\frac{\tau_L}{S_L - \eta_L \sigma_n} \right)^2 \quad (7.30)$$

The transverse shear strength S_T and the transverse friction coefficient η_T in Equation (7.30) are calculated according to the formulations proposed by Dávila et al. [87]

$$S_T = Y_C \cos \alpha_0 \left(\sin \alpha_0 + \frac{\cos \alpha_0}{\tan 2\alpha_0} \right) \quad (7.31)$$

$$\eta_T = \frac{-1}{\tan 2\alpha_0} \quad (7.32)$$

where Y_C is the transverse compressive strength and α_0 is the fracture plane angle for pure transverse compression, which according to [86] should take a value of $\alpha_0 = 53^\circ \pm 2^\circ$.

The longitudinal shear strength S_L and the longitudinal friction coefficient η_L can be obtained from experiments as described in Section 5.5.2. As noted by Dávila et al. [87], η_L can also be calculated from the heuristic relation between the longitudinal and transverse friction coefficients and shear strengths [86]

$$\frac{\eta_L}{S_L} = \frac{\eta_T}{S_T} \Rightarrow \eta_L = -\frac{S_L \cos 2\alpha_0}{Y_C \cos^2 \alpha_0} \quad (7.33)$$

Matrix Tension

If the stress component σ_n is positive, the failure index for matrix tensile failure FI_{MT} is defined as [89]

$$FI_{MT} = \left(\frac{\tau_L}{S_L} \right)^2 + \left(\frac{\sigma_n}{Y_T} \right)^2 \quad (7.34)$$

where Y_T is the transverse tensile strength of the unidirectional laminae.

7.2.2 Fibre Failure

Fibre Tension

A non-interacting failure criterion similar to the LaRC03 criteria proposed by Dávila et al. [87] was implemented for fibre tension. It is activated in the case of positive longitudinal stresses ($\sigma_{11} > 0$) and defined as

$$FI_{FT} = \frac{\sigma_{11}}{X_T} \quad (7.35)$$

where X_T is the longitudinal tensile strength. The influence of the shear stress components σ_{12} and σ_{13} as proposed by Hashin [96] is neglected since only limited experimental evidence exists to support this interaction assumption [91].

Fibre Compression

Failure due to longitudinal compression is assumed to occur with the formation of a kink-band. The presented failure criteria is based on the approach of Argon [101] and on the later developments by Dávila et al. [87], Pinho et al. [102] and Catalanotti [89].

The kink-band is triggered by an initial fibre misalignment which introduces shear stresses acting on the fibres. These shear stresses promote a fibre rotation, resulting in a further shear stress increase. Eventually this instability leads to the formation of a kink-band as a result of failure in the fibre-surrounding matrix. The plane in which fibre-kinking occurs for the generalised three-dimensional stress state is shown in Figure 7.7.

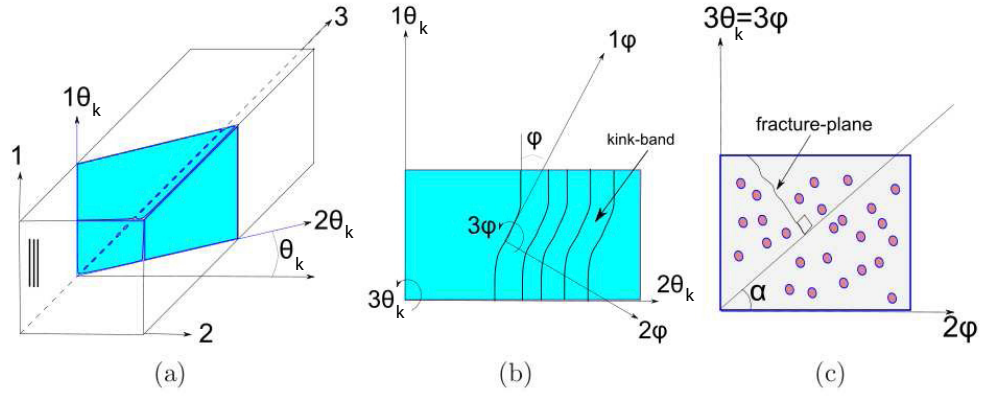


Figure 7.7: Fibre kinking plane [89].

The fibre kinking failure criterion evaluates the stress state in the misaligned fibre coordinate system $1^{(\varphi)} - 2^{(\varphi)} - 3^{(\varphi)}$ shown in Figure 7.7b and 7.7c.

The stresses in the misaligned coordinate system are calculated from the stresses acting in the $1 - 2 - 3$ material coordinate system by two consecutive rotations:

θ_k -rotation

The angle θ_k , defining the orientation of the kink band plane, is considered to be related to local defects in the material. Catalanotti [89] proposed that θ_k can be approximated by calculating the maximum principal stress which acts on the $2 - 3$ plane. Using Mohr's stress circle theory, the angle θ_k can be obtained as

$$\tan(2\theta_k) = \frac{2\sigma_{23}}{\sigma_{22} - \sigma_{33}} \quad (7.36)$$

The stress tensor $\mathbf{T}$ for the general 3d stress state is defined in the $1 - 2 - 3$ material coordinate system as

$$\mathbf{T} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad (7.37)$$

The stress tensor $\mathbf{T}^{(\theta_k)}$ in the $1^{(\theta_k)} - 2^{(\theta_k)} - 3^{(\theta_k)}$ coordinate system (Figure 7.7a and 7.7b) is calculated as

$$\mathbf{T}^{(\theta_k)} = \mathbf{R}(\theta_k) \cdot \mathbf{T} \cdot \mathbf{R}(\theta_k)^T \quad (7.38)$$

where $\mathbf{R}(\theta_k)$ is the rotation matrix

$$\mathbf{R}(\theta_k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_k & \sin \theta_k \\ 0 & -\sin \theta_k & \cos \theta_k \end{bmatrix} \quad (7.39)$$

Hence the stresses in the θ_k coordinate system are

$$\begin{aligned} \sigma_{11}^{\theta_k} &= \sigma_{11} \\ \sigma_{22}^{\theta_k} &= \cos^2(\theta_k) \sigma_{22} + 2 \cos \theta_k \sin \theta_k \sigma_{23} + \sin^2(\theta_k) \sigma_{33} \\ \sigma_{33}^{\theta_k} &= \sin^2(\theta_k) \sigma_{22} + 2 \cos \theta_k \sin \theta_k \sigma_{23} + \cos^2(\theta_k) \sigma_{33} \\ \sigma_{12}^{\theta_k} &= \cos(\theta_k) \sigma_{12} + \sin(\theta_k) \sigma_{13} \\ \sigma_{23}^{\theta_k} &= -\sin(\theta_k) \cos(\theta_k) \sigma_{22} + \cos(\theta_k) \sin(\theta_k) \sigma_{33} + 2 (\cos^2(\theta_k) - \sin^2(\theta_k)) \sigma_{23} \\ \sigma_{13}^{\theta_k} &= \sin(\theta_k) \sigma_{12} + \cos(\theta_k) \sigma_{13} \end{aligned} \quad (7.40)$$

φ -rotation

The determination of the fibre misalignment angle φ is based on the two-dimensional approach proposed by Dávila et al. [87]. To be used for the presented generalised three-dimensional fibre kinking failure criterion, φ is subsequently defined with the components of the stress tensor $\mathbf{T}^{(\theta_k)}$.

According to Dávila et al. [87], the fibre misalignment can be idealised as a local region of fibre waviness, as shown in Figure 7.8, where the stresses in the 2d misaligned coordinate system m are defined as

$$\begin{aligned}
\sigma_{11}^m &= \cos^2 \varphi \sigma_{11} + \sin^2 \varphi \sigma_{22} + 2 \sin \varphi \cos \varphi |\sigma_{12}| \\
\sigma_{22}^m &= \sin^2 \varphi \sigma_{11} + \cos^2 \varphi \sigma_{22} - 2 \sin \varphi \cos \varphi |\sigma_{12}| \\
\sigma_{12}^m &= -\sin \varphi \cos \varphi \sigma_{11} + \sin \varphi \cos \varphi \sigma_{22} + (\cos^2 \varphi - \sin^2 \varphi) |\sigma_{12}|
\end{aligned} \tag{7.41}$$

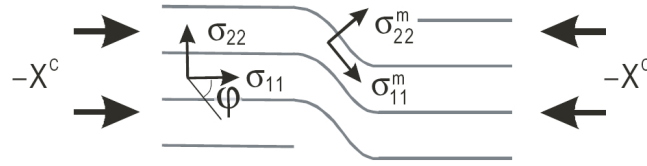


Figure 7.8: Fibre misalignment idealised as local waviness [87].

Assuming a state of pure axial compression where $\sigma_{11} = -X_C$ and $\sigma_{22} = \sigma_{12} = 0$, Equation (7.41) becomes

$$\begin{aligned}
\sigma_{11,C}^m &= -\cos^2 \varphi_C X_C \\
\sigma_{22,C}^m &= -\sin^2 \varphi_C X_C \\
\sigma_{12,C}^m &= \sin \varphi_C \cos \varphi_C X_C
\end{aligned} \tag{7.42}$$

where the angle φ_C is the total misalignment angle for pure axial compression.

Substitution of $\sigma_{22,C}^m$ and $\sigma_{12,C}^m$ of Equation (7.42) into the LaRC03#1 failure criterion of [87], and considering that the fibre kinking failure mode is dominated by in-plane shear stresses rather than transverse compression, the fracture angle is equal to 0° and $\tau_{eff}^T = 0$.

For this case the LaRC03#1 failure criterion [87] becomes

$$\tau_{eff}^L = X_C (\sin \varphi_C \cos \varphi_C - \eta_L \sin^2 \varphi_C) = S_L \tag{7.43}$$

Solving Equation (7.43) leads to the quadratic equation

$$\tan^2 \varphi_C \left(\frac{S_L}{X_C} + \eta_L \right) - \tan \varphi_C + \left(\frac{S_L}{X_C} \right) \tag{7.44}$$

where the smaller root then gives the total misalignment angle for pure axial compression

$$\varphi_C = \tan^{-1} \left(\frac{1 - \sqrt{1 - 4 \left(\frac{S_L}{X_C} + \eta_L \right) \frac{S_L}{X_C}}}{2 \left(\frac{S_L}{X_C} + \eta_L \right)} \right) \quad (7.45)$$

For a general stress state, the angle φ is the sum of the initial misalignment angle φ_0 and the additional fibre rotation γ_m caused by the shear stresses acting on the fibre.

Considering still the pure axial compression stress state and the constitutive shear law

$$\sigma_{12,C}^m = G_{12} \gamma_{mC}$$

$$\gamma_m = \gamma_{mC} = \frac{\sigma_{12,C}^m}{G_{12}} = \frac{X_C \sin \varphi_C \cos \varphi_C}{G_{12}} = \frac{\sin(2\varphi_C) X_C}{2G_{12}} \quad (7.46)$$

When further considering small angle approximations

$$\gamma_{mC} \approx \frac{X_C}{G_{12}} \varphi_C \quad (7.47)$$

The initial misalignment angle φ_0 is then defined as

$$\varphi_0 = \varphi_C - \gamma_{mC} \quad (7.48)$$

For the generic load case the additional fibre rotation angle γ_m is obtained by solving the shear stress constitutive law

$$\gamma_m = \frac{\sigma_{12}^m}{G_{12}} = \frac{\sin \varphi \cos \varphi (\sigma_{22} - \sigma_{11}) + (\cos^2 \varphi - \sin^2 \varphi) |\sigma_{12}|}{G_{12}} \quad (7.49)$$

Using small angle approximation, Equation (7.49) becomes

$$\gamma_m = \frac{\varphi(\sigma_{22} - \sigma_{11}) + |\sigma_{12}|}{G_{12}} \quad (7.50)$$

Substitution of $\varphi = \varphi_0 + \gamma_m$ into 7.50 and rearranging the equation for γ_m yields

$$\gamma_m = \frac{\varphi_0 \sigma_{12} + |\sigma_{12}|}{G_{12} + \sigma_{11} - \sigma_{22}} - \varphi_0 \quad (7.51)$$

Returning now to the general 3d stress state, the additional fibre rotation angle γ_m must be defined with the components of the stress tensor $\mathbf{T}^{(\theta_k)}$ [89]

$$\gamma_m^{(\theta_k)} = \frac{\varphi_0 G_{12} + |\sigma_{12}^{(\theta_k)}|}{G_{12} + \sigma_{11}^{(\theta_k)} - \sigma_{22}^{(\theta_k)}} - \varphi_0 \quad (7.52)$$

And thus the fibre misalignment angle φ becomes

$$\varphi = \text{sgn}[\sigma_{12}^{(\theta_k)}] \left(\varphi_0 + \gamma_m^{(\theta_k)} \right) \quad (7.53)$$

where $\text{sgn}[\dots]$ extracts the signal of the stress component $\sigma_{12}^{(\theta_k)}$.

The stress tensor $\mathbf{T}^{(\varphi)}$ in the misaligned coordinate system can now be obtained from the stress tensor $\mathbf{T}^{(\theta_k)}$ by a further rotation around the fibre misalignment angle φ

$$\mathbf{T}^{(\varphi)} = \mathbf{R}(\varphi) \cdot \mathbf{T}^{(\theta_k)} \cdot \mathbf{R}(\varphi)^T \quad (7.54)$$

where the rotation matrix $\mathbf{R}(\varphi)$ is defined as

$$\mathbf{R}(\varphi) = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7.55)$$

The stresses in the misalignment frame $1^{(\varphi)} - 2^{(\varphi)} - 3^{(\varphi)}$ are then defined as

$$\begin{aligned}
 \sigma_{11}^{(\varphi)} &= \cos^2 \varphi \sigma_{11}^{(\theta_k)} + 2 \cos \varphi \sin \varphi \sigma_{12}^{(\theta_k)} + \sin^2 \varphi \sigma_{22}^{(\theta_k)} \\
 \sigma_{22}^{(\varphi)} &= \sin^2 \varphi \sigma_{11}^{(\theta_k)} - 2 \cos \varphi \sin \varphi \sigma_{12}^{(\theta_k)} + \cos^2 \varphi \sigma_{22}^{(\theta_k)} \\
 \sigma_{33}^{(\varphi)} &= \sigma_{33}^{(\theta_k)} \\
 \sigma_{12}^{(\varphi)} &= -\sin \varphi \cos \varphi (\sigma_{11}^{(\theta_k)} - \sigma_{22}^{(\theta_k)}) + 2 (\cos^2 \varphi - \sin^2 \varphi) \sigma_{12}^{(\theta_k)} \\
 \sigma_{23}^{(\varphi)} &= -\sin \varphi \sigma_{13}^{(\theta_k)} + \cos \varphi \sigma_{23}^{(\theta_k)} \\
 \sigma_{13}^{(\varphi)} &= \cos \varphi \sigma_{13}^{(\theta_k)} + \sin \varphi \sigma_{23}^{(\theta_k)}
 \end{aligned} \tag{7.56}$$

As mentioned at the beginning of this section, the kink band failure mode under fibre compression is the result of failure in the fibre-surrounding matrix. Therefore it is again necessary to evaluate the matrix compression and matrix tension failure criteria introduced with Equations (7.30) and (7.34), respectively.

The tractions on the fracture plane, Equation (7.29), are now defined in terms of the stress tensor $\mathbf{T}^{(\varphi)}$ however.

$$\begin{aligned}
 \sigma_n^{(\varphi)} &= \cos^2 \alpha \sigma_{22}^{(\varphi)} + 2 \cos \alpha \sin \alpha \sigma_{23}^{(\varphi)} + \sin^2 \alpha \sigma_{33}^{(\varphi)} \\
 \tau_T^{(\varphi)} &= -\sin \alpha \cos \alpha \sigma_{22}^{(\varphi)} + (\cos^2 \alpha - \sin^2 \alpha) \sigma_{23}^{(\varphi)} + \sin \alpha \cos \alpha \sigma_{33}^{(\varphi)} \\
 \tau_L^{(\varphi)} &= \cos \alpha \sigma_{12}^{(\varphi)} + \sin \alpha \sigma_{13}^{(\varphi)}
 \end{aligned} \tag{7.57}$$

If $\sigma_n^{(\varphi)} < 0$ the Failure Index is

$$FI_{MC,K} = \left(\frac{\tau_T^{(\varphi)}}{S_T - \eta_T \sigma_n^{(\varphi)}} \right)^2 + \left(\frac{\tau_L^{(\varphi)}}{S_L - \eta_L \sigma_n^{(\varphi)}} \right)^2 \tag{7.58}$$

while for $\sigma_n^{(\varphi)} > 0$ the Failure Index is

$$FI_{MT,K} = \left(\frac{\tau_L^{(\varphi)}}{S_L} \right)^2 + \left(\frac{\sigma_n^{(\varphi)}}{Y_T} \right)^2 \tag{7.59}$$

The failure index for fibre kinking will assume the value

$$FI_{FK} = \max \{ \max [FI_{MC,K}], \max [FI_{MT,K}] \} \quad (7.60)$$

where $\max [FI_{MC,K}]$ and $\max [FI_{MT,K}]$ are defined by the angle α which maximises $FI_{MC,K}$ and $FI_{MT,K}$.

7.3 Strain Rate Dependency

The strain rate effect on the mechanical behaviour of carbon-epoxy composites is evident from the review presented in Chapter 2, and from the experimental data presented in Chapters 4 and 5. Table 7.2 gives a qualitative overview of the strain rate dependency of the mechanical material properties of a UD carbon-epoxy composite.

The strain rate dependency of the elastic and strength properties can be introduced into the constitutive equations and into the failure criteria by fitting the experimental data using suitable scaling functions. Wiegand [22] used a simple function of the form

$$f(\dot{\epsilon}) = 1 + \sqrt{K\dot{\epsilon}} \quad (7.61)$$

where the rate dependency of the elastic and strength properties depends on an appropriate selection of the parameter K . This convenient curve fitting approach was adopted in the present study.

Figure 7.9 shows the scaling functions used for the strain rate dependent in-plane elastic properties E_{2t} , E_{2c} and G_{12} , and for the strain rate dependent in-plane strength properties Y_T , Y_C and S_L . The normalised experimental data shown in Figure 7.9 combines the data presented in Figures 2.25, 2.26 and 2.27 with the results from a strain rate study on RTM-6 resin performed by Gerlach et al. [8] and, in case of the in-plane shear strength, with the strain rate study on neat resin performed by Gilat et al. [7]. Figure 7.9 further contains the data from the strain rate study on IM7-8552, presented in Chapter 5 [103].

It seems appropriate to introduce a scaling function of the form

$$f(\dot{\epsilon}) = 1 + (K\dot{\epsilon})^{\frac{1}{n_f}} \quad (7.62)$$

and therefore be able to choose a function $f_e(\dot{\epsilon})$ to describe the strain rate effect on the in-plane elastic and a function $f_u(\dot{\epsilon})$ to describe the strain rate effect on the in-plane strength properties of unidirectional carbon-epoxy composites.

Considering the above scaling function approach (Equation (7.62)), the strain rate effect on the in-plane elastic and strength properties of a unidirectional carbon-epoxy, such as IM7-8552, can be defined as

$$\begin{aligned} E_1(\dot{\epsilon}) &= E_1^0 = \text{constant} \\ E_2(\dot{\epsilon}) &= E_2^0 f_e(\dot{\epsilon}) \\ G_{12}(\dot{\epsilon}) &= G_{12}^0 f_e(\dot{\epsilon}) \\ X_T(\dot{\epsilon}) &= X_T^0 = \text{constant} \\ X_C(\dot{\epsilon}) &= X_C^0 f_u(\dot{\epsilon}) \\ Y_T(\dot{\epsilon}) &= Y_T^0 f_u(\dot{\epsilon}) \\ Y_C(\dot{\epsilon}) &= Y_C^0 f_u(\dot{\epsilon}) \\ S_L(\dot{\epsilon}) &= S_L^0 f_u(\dot{\epsilon}) \end{aligned} \quad (7.63)$$

where the superscript ⁰ refers to the respective quasi-static property and the scaling functions f_e and f_u for the elastic and strength properties of IM7-8552 are defined with Equation (7.64).

It is noted that it is common practice to not distinguish between tension and compression in case of the longitudinal and transverse moduli E_1 and E_2 , respectively, but calculate the respective average from the tensile and compressive values. In general, a somewhat lower value is observed for both the longitudinal and the transverse compressive modulus compared to the measured tensile property.

Table 7.2: Qualitative overview of strain rate effects on the in-plane and out-of-plane properties of unidirectional carbon-epoxy composites.

Property	Description	$\dot{\epsilon}$ -effect	Comments
E_{1t}	longitudinal tensile modulus	NO	Figure 2.25a
E_{1c}	longitudinal compressive modulus	NO	Section 4.5
E_{2t}	transverse tensile modulus	YES	Figure 2.25c
E_{2c}	transverse compressive modulus	YES	Section 5.5.1
G_{12}	in-plane shear modulus	YES	Section 5.5.2
ν_{12}	major in-plane Poisson's ratio	NO	assumption, no experimental evidence
E_{3t}	interlaminar tensile modulus	YES	Figure 2.25e, transverse isotropy: $E_{3t} = E_{2t}$
E_{3c}	interlaminar compressive modulus	YES	Figure 2.26e, transverse isotropy: $E_{3c} = E_{2c}$
G_{13}	interlaminar shear modulus	YES	Figure 2.27c, transverse isotropy: $G_{13} = G_{12}$
G_{23}	transverse shear modulus	YES	dependent property: $G_{23} = 0.5E_2/(1 + \nu_{23})$
ν_{13}	out-of-plane Poisson's ratio 13	NO	transverse isotropy: $\nu_{13} = \nu_{12}$
ν_{23}	out-of-plane Poisson's ratio 23	NO	assumption, no experimental evidence
X_T	longitudinal tensile strength	NO	Figure 2.25b
X_C	longitudinal compressive strength	YES	Section 4.5
Y_T	transverse tensile strength	YES	Figure 2.25d
Y_C	transverse compressive strength	YES	Section 5.5.1
S_L	in-plane shear strength	YES	Section 5.5.2
Z_T	interlaminar tensile strength	YES	Figure 2.25f, transverse isotropy: $Z_T = Y_T$
Z_C	interlaminar compressive strength	YES	Figure 2.26f, transverse isotropy: $Z_C = Y_C$
S_T	transverse tensile strength	YES	Equation (5.6)
Y_C^y	transverse compressive yield strength	YES	Figure 5.32
S_L^y	in-plane shear yield strength	YES	Figure 5.32

$$\begin{aligned}
 f_e(\dot{\epsilon}) &= 1 + (K_e \dot{\epsilon})^{\frac{1}{n_e}} \quad , \quad K_e = 1.60 \times 10^{-4} \quad , \quad n_e = 2 \\
 f_u(\dot{\epsilon}) &= 1 + (K_u \dot{\epsilon})^{\frac{1}{n_u}} \quad , \quad K_u = 1.13 \times 10^{-4} \quad , \quad n_u = 4
 \end{aligned}
 \tag{7.64}$$

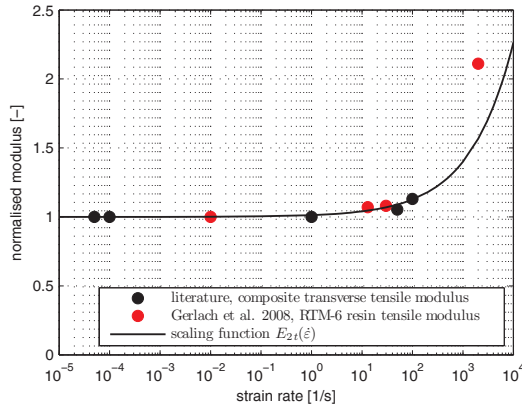
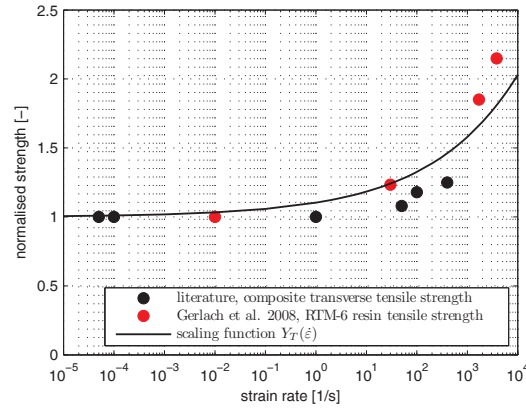
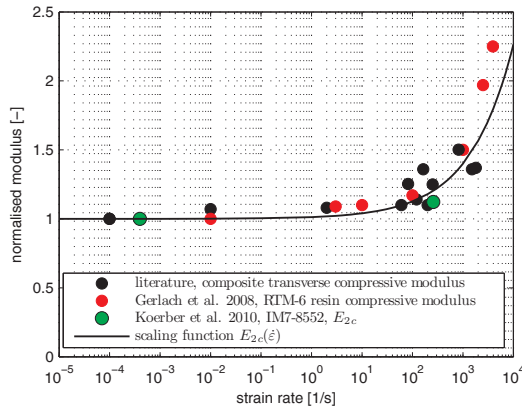
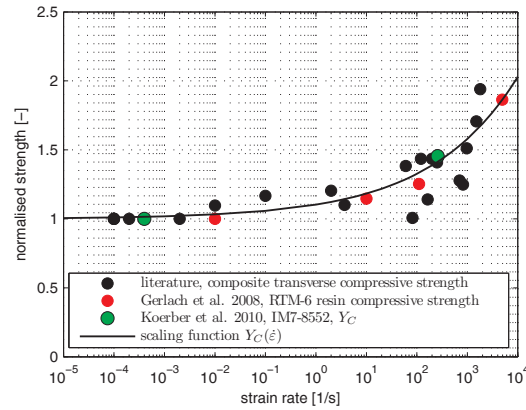
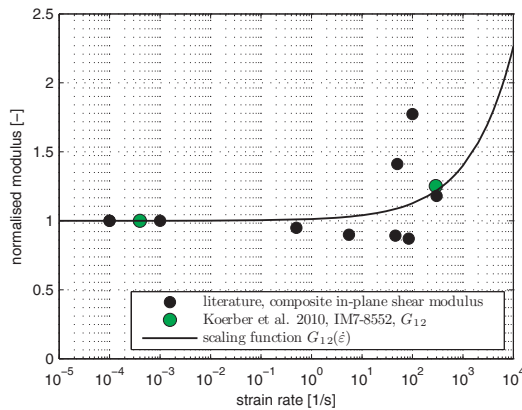
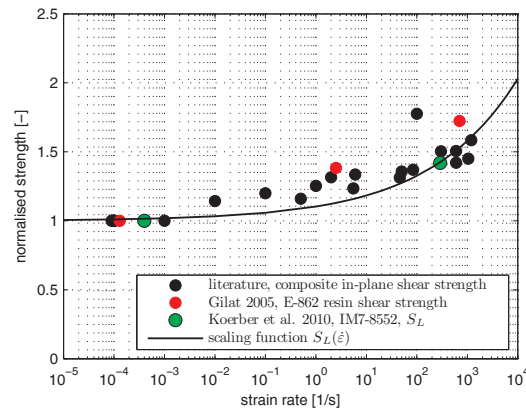
(a) transverse tensile modulus E_{2t} (b) transverse tensile strength Y_T (c) transverse compressive modulus E_{2c} (d) transverse compressive strength Y_C (e) in-plane shear modulus G_{12} (f) in-plane shear strength S_L

Figure 7.9: Strain rate effect on the in-plane moduli and strengths of carbon-epoxy composites and neat epoxy resin.

It appears from Figure 7.9 that one function $f_e(\dot{\epsilon})$ is sufficient to fit the strain rate behaviour of the transverse tensile modulus E_{2t} , the transverse compressive modulus E_{2c} and the in-plane shear modulus G_{12} . Similarly, the function $f_u(\dot{\epsilon})$ can reasonable well describe the strain rate effect on the transverse tensile strength Y_T , the transverse compressive strength Y_C and the in-plane shear strength S_L .

The elastic scaling function $f_e(\dot{\epsilon})$ accounts for the moderate rate dependency of the in-plane moduli up to strain rates of $\dot{\epsilon} = 10^2 \text{s}^{-1}$, while then experiencing a strong rate effect if the strain rate is further increased. The strength scaling function $f_u(\dot{\epsilon})$ accounts for the earlier rate dependency of the in-plane strength properties and the more gradual increase at the higher strain rates.

For the carbon-epoxy material system IM7-8552 used in the present work, a similar rate effect was observed for the transverse compressive strength Y_C , the in-plane shear strength S_L [103] and for the longitudinal compressive strength X_C (Section 4.5) [104]. It is therefore reasonable to also use the scaling function $f_u(\dot{\epsilon})$ to describe the rate dependency of the longitudinal compressive strength X_C (Figure 7.10).

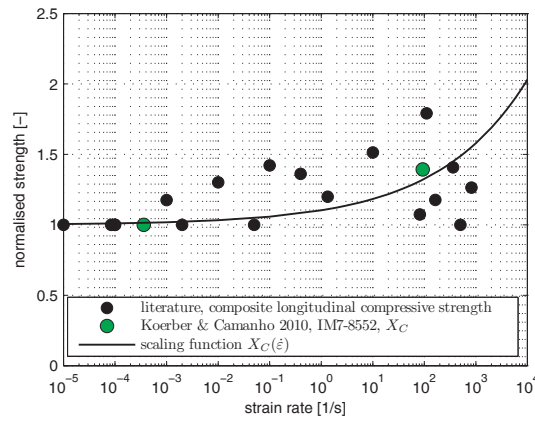


Figure 7.10: Strain rate effect on the longitudinal strength of carbon-epoxy composites.

7.4 Model Implementation into an ABAQUS VUMAT Subroutine

Following standard procedures for time discretisation schemes, the stress tensor $\boldsymbol{\sigma}_{n+1}$ at the current time step $t = n + 1$ can be calculated according to the equation

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_n + [(1 - \theta_i) \mathbf{D}_n^{ep} + \theta_i \mathbf{D}_{n+1}^{ep}] : \Delta \boldsymbol{\varepsilon} \quad (7.65)$$

where $(\cdot) : (\cdot)$ denotes a double contraction, $\boldsymbol{\sigma}_n$ is the stress tensor at the previous time step, $\mathbf{D}_n^{ep}$ is the elasto-plastic tangent modulus of the previous time step, $\mathbf{D}_{n+1}^{ep}$ is the elasto-plastic tangent modulus of the current time step, $\Delta \boldsymbol{\varepsilon}$ is the current strain increment tensor and θ_i is a parameter governing the integration scheme type.

If $\theta_i = 1$, a backward Euler or fully implicit integration scheme is used, while for $\theta_i = 0$ the integration scheme is forward Euler or fully explicit.

To simplify the implementation process, a forward Euler or fully explicit integration scheme was used for the plastic material model and therefore Equation (7.65) becomes

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_n + \mathbf{D}_n^{ep} : \Delta \boldsymbol{\varepsilon} \quad (7.66)$$

where the stress tensor of the current time step only depends on the stress tensor of the previous time step and on the elasto-plastic tangent modulus of the previous time step.

7.4.1 Implementation of the Plasticity Model and Yield Check

In order to distinguish elastic and plastic steps, the effective plastic strain $\bar{\varepsilon}^p$ is used for the yield check procedure. The effective plastic strain $\bar{\varepsilon}^p$ is an internal state variable of the constitutive model and must be initialised in the Abaqus input file with a value of 0.0. As shown in Figure 7.12, a YIELD parameter is defined for each element at the beginning of the current step, corresponding to the value of the $\bar{\varepsilon}^p$ state variable from the previous

step. In the case of the first increment, the YIELD parameter is equal to 0.0 since the initial value of the state variable is used.

With Equation (7.17), the effective stress of the current step $\bar{\sigma}_{n+1}$ is calculated, using the stress components of the previous step and thus following the forward Euler integration scheme.

$$\bar{\sigma}_{n+1} = \left[\frac{3}{2} (\sigma_{22,n}^2 + 2a_{66}\sigma_{12,n}^2) \right]^{1/2} \quad (7.67)$$

Equation (7.27) then defines the effective plastic strain of the current step as

$$\bar{\epsilon}_{n+1}^p = A_{pm} \bar{\sigma}_{n+1}^{n_{pm}} \quad (7.68)$$

and the yield check at the current time step can be carried out according to the inequality condition

$$\bar{\epsilon}_{n+1}^p \leq YIELD \quad (7.69)$$

If Equation (7.69) is *true*, the current step is elastic and the elastic strain increment $\Delta\boldsymbol{\sigma}_e$ is calculated as

$$\Delta\boldsymbol{\sigma}_e = \boldsymbol{Q}_e \Delta\boldsymbol{\epsilon} \quad (7.70)$$

where $\boldsymbol{Q}_e$ is the elastic stiffness tensor, defined in matrix notation as [105]:

$$\boldsymbol{Q}_e = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (7.71)$$

and $\Delta\boldsymbol{\epsilon}$ is the applied total strain increment tensor of the current step.

It is noted that the elastic step is only calculated in two cases:

1. Initial Step

For the initial step the YIELD parameter corresponds to the initial value of the $\bar{\epsilon}^p$ state variable (0.0). Further $\bar{\epsilon}_{n+1}^p = \bar{\sigma}_{n+1} = 0$ since the components of the stress tensor of the previous step (*stressOld*) are equal to 0.

Therefore it is guaranteed that the first step is elastic, which is a requirement for any Abaqus VUMAT subroutine.

2. Unloading/Reloading

In the case of unloading and reloading at a stress state below the already reached maximum stress state, the current effective plastic strain $\bar{\epsilon}_{n+1}^p$ is smaller than the value stored in the $\bar{\epsilon}^p$ state variable.

Therefore it is guaranteed that new plasticity can only occur if the current stress state exceeds the maximum stress state already reached in the analysis.

If Equation (7.69) is *false*, the YIELD parameter is updated with the current value of the effective plastic strain $\bar{\epsilon}_{n+1}^p$ and the *Plastic* subroutine, shown in (Figure 7.13), is called. Following Sun and Yoon [106] and considering the forward Euler integration scheme, the plastic multiplier $\Delta\lambda$ is a scalar defined as:

$$\Delta\lambda = \frac{\left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \mathbf{Q}_e \Delta \boldsymbol{\epsilon}}{\frac{4}{9} \bar{\sigma}^2 H_p + \left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \mathbf{Q}_e \frac{\partial f}{\partial \boldsymbol{\sigma}}} \quad (7.72)$$

where the partial derivative of the two-dimensional yield function, Equation (7.14), with respect to the plane stress components is given by

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} = \left\{ \begin{array}{c} 0 \\ \sigma_{22,n} \\ 2a_{66}\sigma_{12,n} \end{array} \right\} \quad (7.73)$$

using the stress components from the previous step.

The plastic modulus H_p of the current step is defined as the derivate of the effective stress, Equation (7.27), with respect to the effective plastic strain and using the current effective plastic strain from Equation (7.68)

$$H_{p,n+1} = \frac{d\bar{\sigma}}{d\bar{\epsilon}^p} = \frac{1}{n_{pm}A_{pm}} \left(\frac{\bar{\epsilon}_{n+1}^p}{A_{pm}} \right)^{\frac{1-n_{pm}}{n_{pm}}} \quad (7.74)$$

Once the plastic multiplier $\Delta\lambda$ is calculated, the plastic strain increment vector $\Delta\epsilon_p$ can be obtained according to the associated flow rule, Equation (7.3), as

$$\Delta\epsilon_p = \frac{\partial f}{\partial \sigma} \Delta\lambda \quad (7.75)$$

According to [106], the elasto-plastic stiffness matrix $\mathbf{Q}_{ep}$ can be obtained as

$$\mathbf{Q}_{ep} = \mathbf{Q}_e - \mathbf{Q}_e \frac{\frac{\partial f}{\partial \sigma} \left(\frac{\partial f}{\partial \sigma} \right)^T \mathbf{Q}_e}{\frac{4}{9} \bar{\sigma}^2 H_p + \left(\frac{\partial f}{\partial \sigma} \right)^T \mathbf{Q}_e \frac{\partial f}{\partial \sigma}} \quad (7.76)$$

and the elasto-plastic stress increment $\Delta\sigma_{ep}$ can be calculated as

$$\Delta\sigma_{ep} = \mathbf{Q}_{ep} \Delta\epsilon \quad (7.77)$$

Depending on the Yield condition (7.69), the current stress tensor σ_{n+1} is then calculated from either the elastic stress increment tensor $\Delta\sigma_e$ or from the elasto-plastic stress increment tensor $\Delta\sigma_{ep}$

$$\sigma_{n+1} = \sigma_n + \begin{cases} \Delta\sigma_e & \text{if elastic step} \\ \Delta\sigma_{ep} & \text{if plastic step} \end{cases} \quad (7.78)$$

The current plastic, elastic and total strain components and selected parameters of the plasticity model are stored as state variables as shown in Figure 7.12.

It is noted that for the implemented plasticity model [97], a yield strength, as required for

most of the classical plasticity models, is not needed. The nonlinear constitutive response is instead controlled by the power law defined for the effective stress - effective plastic strain response, Equation (7.27), and which can be determined from experimental data as described in Section 7.1.1. For continuous loading, the material always yields as long as Equation (7.69) is true.

This gradually appearing nonlinearity is justified by the fact that unidirectional fibre composites exhibit no defined yield point as shown in Figure 5.29. Due to the power law, the amount of plastic strain is however very small for low stress levels and the calculated stress-strain response is linear until the effective plastic strain reaches a considerable amount (Figure 7.6).

7.4.2 Implementation of the Failure Criteria

The stress-based failure criteria are evaluated on the basis of the stress tensor of the current step $\boldsymbol{\sigma}_{n+1}$ (or *stressNew* in Abaqus VUMAT notation). The theory and the relevant equations for the failure criteria were presented in Section 7.2, while the implementation framework is given by the flowcharts in Figures 7.14, 7.15 and 7.16.

It is again noted that the failure criteria introduced in Section 7.2 was defined for the general three-dimensional case [89]. Since the constitutive model is currently only implemented for in-plane loading, the failure criteria also reduce to a two-dimensional version. This simplifies the calculation of the matrix compression failure index FI_{MC} , defined with Equation (7.30).

As shown by Dávila et al. [87], FI_{MC} is found by searching for the angle α which maximises Equation (7.30) within the range of $0^\circ \leq \alpha \leq \alpha_0$. To reduce computational cost, the angle search can be done with a reduced number of angles α for the two-dimensional case. Figure 7.11 shows that for both, quasi-static loading ($\dot{\epsilon}_{qs} = 4e - 4s^{-1}$) and high strain rate loading ($\dot{\epsilon}_{dyn} = 250s^{-1}$), a selection of 4 angles ($\alpha = 0^\circ, 30^\circ, 45^\circ, \alpha_0$) is sufficient to approximate the predicted failure envelope under combined transverse compression and in-plane shear loading.

The two diagrams presented in Figure 7.11 were generated by substituting Equation (7.29) into Equation (7.30), setting FI_{MC} equal to 1 and solving for σ_{12} . The in-plane shear stress σ_{12} then becomes a function of the transverse compressive stress σ_{22} and the fracture angle α .

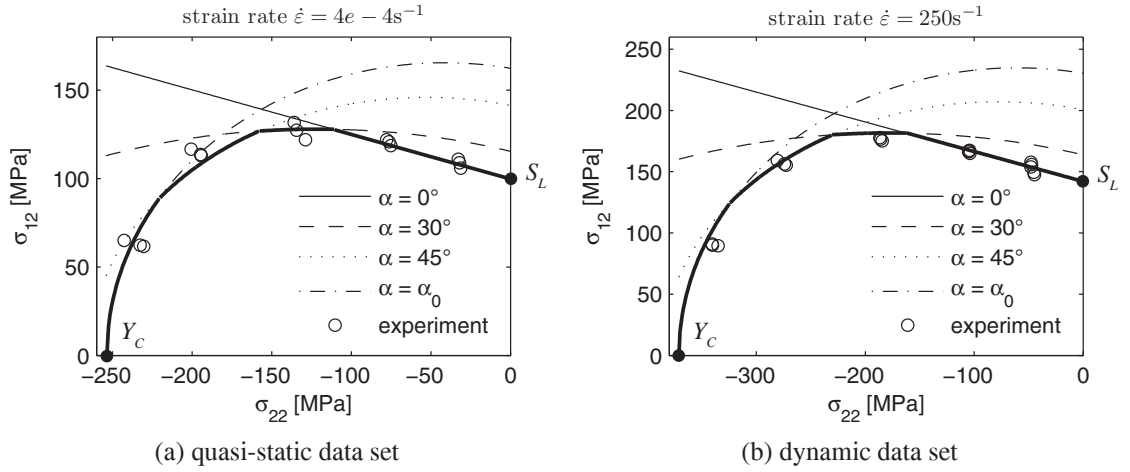


Figure 7.11: Angle selection to search for the maximum of FI_{MC} .

It is noted that for the general three-dimensional stress state, the angle search range must be extended to $0^\circ \leq \alpha \leq 180^\circ$ since, depending on the stress state, a fracture angle of $\alpha > \alpha_0$ may occur. To reduce computational cost for the general 3d case, more sophisticated search algorithms such as the *Extended Golden Section Search*, proposed by Wiegand [107], may be used.

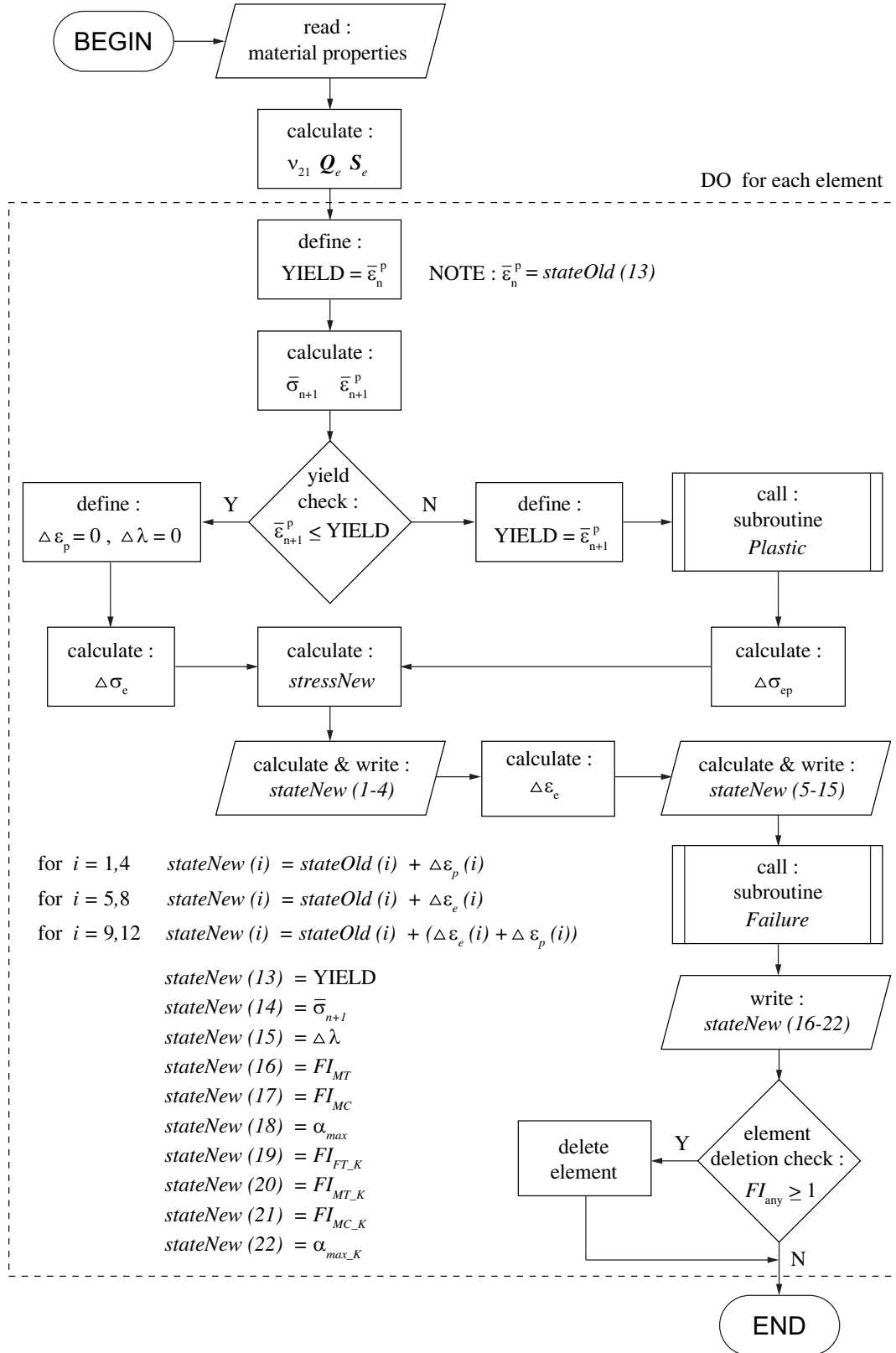


Figure 7.12: Flowchart of Main VUMAT Subroutine.

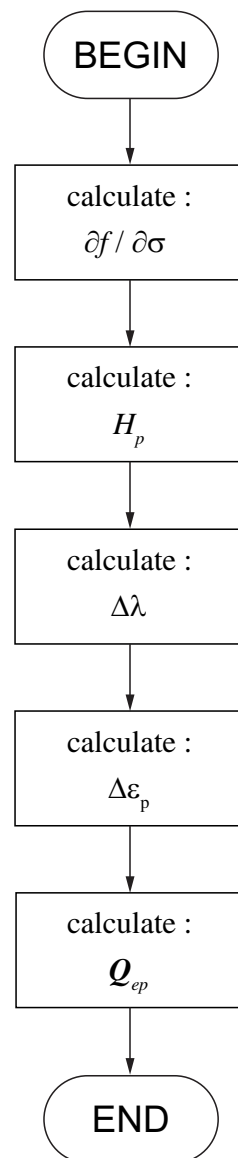
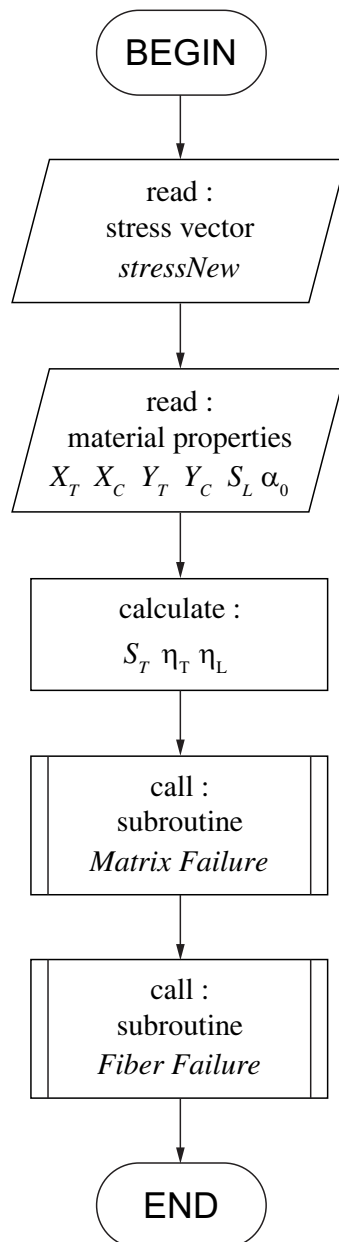
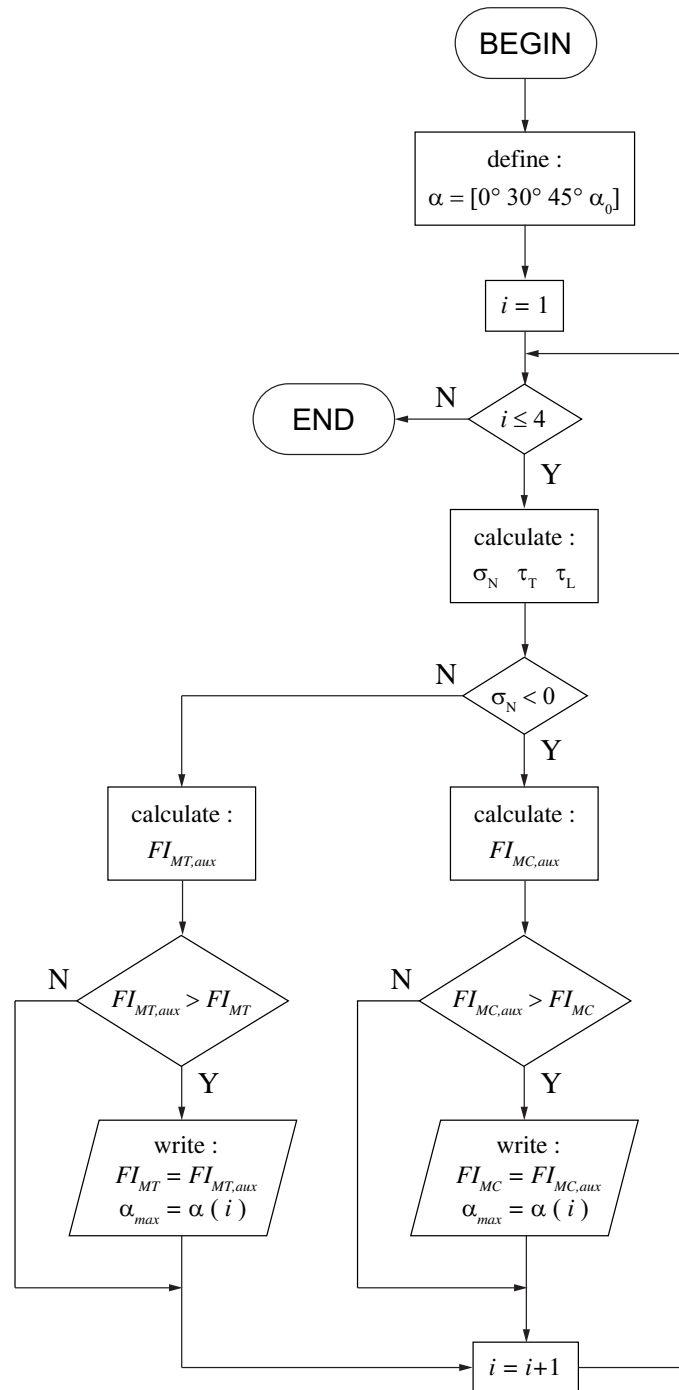
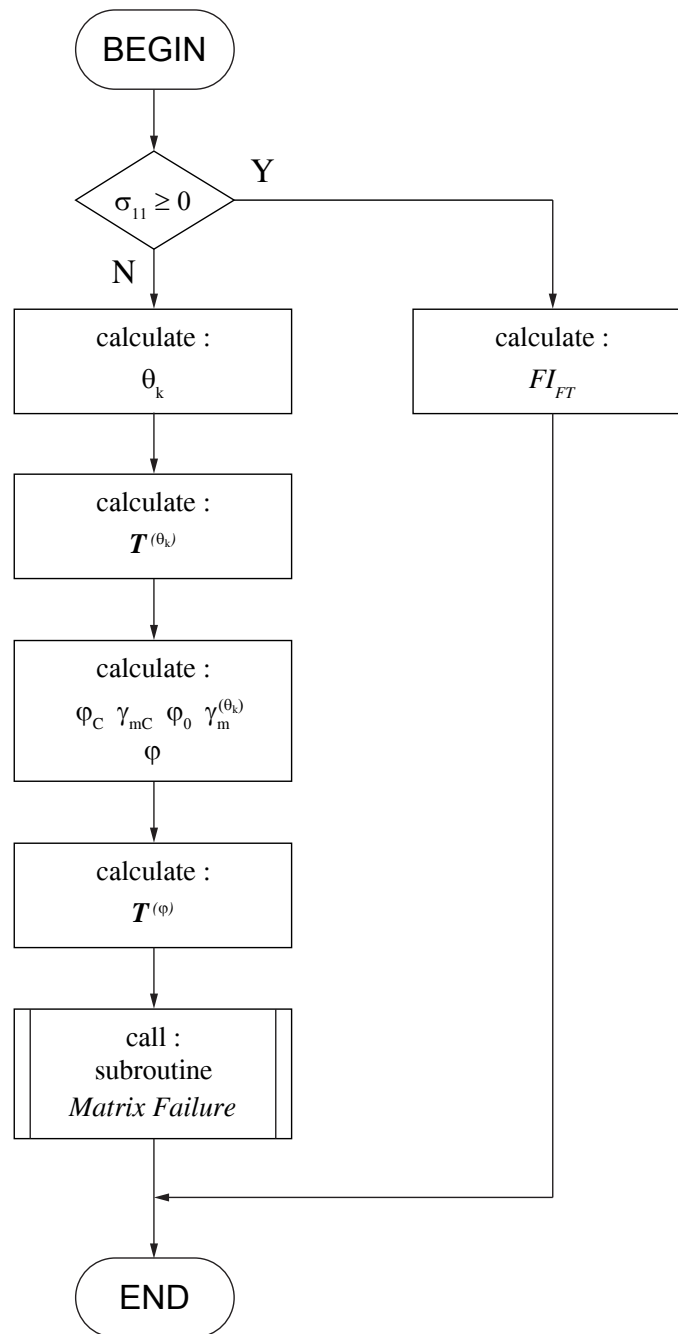


Figure 7.13: Flowchart of *Plastic* Subroutine.

Figure 7.14: Flowchart of *Failure* Subroutine.

Figure 7.15: Flowchart of *Matrix Failure* Subroutine.

Figure 7.16: Flowchart of *Fibre Failure* Subroutine.

7.5 Model Validation

At the time of writing, the VUMAT subroutine did not include the rate-dependency discussed in Sections 7.1.1 and 7.3. The rate-dependent properties were instead specified directly in the input deck. This preliminary version of the VUMAT subroutine was validated by testing a single element (Abaqus element type S4R) in various loading conditions and comparing the predicted stress-strain response with experimental data.

The purpose of the presented simulations is to show the capability of this relatively simple plasticity model to accurately predict the plastic response of the unidirectional carbon-epoxy material system IM7-8552 at quasi-static and dynamic strain rates. Future work is however required to further develop the VUMAT subroutine and the aspects which need to be addressed are summarised in Chapter 8.

The parameters used for the validation simulations are shown in Table 7.3. Regarding the elastic parameters listed in Table 7.3, the longitudinal modulus E_1 was determined as the average of the longitudinal tensile modulus $E_{1t} = 171420\text{MPa}$, reported in [67], and the quasi-static longitudinal compressive modulus $E_{1c} = 154486\text{MPa}$ listed in Table 4.3. As mentioned in Section 7.3, no strain rate effects were found for this property.

Similarly, the quasi-static transverse modulus E_2 was determined as the average of the transverse tensile modulus $E_{2t} = 9080\text{MPa}$ reported in [67] and the quasi-static transverse compressive modulus $E_{2c} = 8930\text{MPa}$ listed in Table 5.2. The transverse compressive modulus for the dynamic simulations was calculated with Equation (7.63).

The quasi-static in-plane shear modulus G_{12} was determined as the average of the shear modulus measured via the ASTM standard [26] and reported in [67], $G_{12}^{IPS} = 5290\text{MPa}$, and the quasi-static in-plane shear modulus obtained from the 45° off-axis tests $G_{12}^{OAC} = 5068\text{MPa}$ (Table 5.5). The dynamic in-plane shear modulus was obtained from Equation (7.63).

The major in-plane Poisson's ratio ν_{12} is assumed to be independent of the rate of loading, due to the lack of dynamic experimental data for this property.

Regarding the plasticity model parameters listed in Table 7.3, the parameter a_{66} was

found to be constant, as noted in Section 7.1.1. The master curve parameter n_{pm} was found to be different for compression and tension, but independent of strain rate. The master curve parameter A_{pm} was calculated with Equation (7.28), using the parameters m and χ specified in Table 7.1.

With respect to the parameters required for the failure criteria, the fracture plane angle for pure transverse compression α_0 is rate independent and was determined as described in Section 5.5.3.

The longitudinal tensile strength X_T was taken from [67] and is not rate dependent. The quasi-static longitudinal compressive strength X_C was taken from Table 4.3 and scaled for the dynamic simulation, using Equation (7.63).

The transverse tensile strength Y_T was taken from [67] and scaled with Equation (7.63). The quasi-static transverse compressive strength Y_C and the quasi-static in-plane shear strength S_L were taken from Tables 5.2 and 5.5, respectively, and scaled with Equation (7.63) for dynamic loading.

Table 7.3: Properties used for VUMAT single element simulation.

			compression ($\dot{\epsilon} = 4e - 4s^{-1}$)	compression ($\dot{\epsilon} = 250s^{-1}$)	tension ($\dot{\epsilon} = 4e - 4s^{-1}$)
elastic response	E_1	[MPa]	162953	162953	162953
	E_2	[MPa]	9003	10804	9003
	G_{12}	[MPa]	5179	6215	5179
	ν_{12}	[-]	0.32	0.32	0.32
plastic response	a_{66}		2.2	2.2	2.2
	n_{pm}		4.8	4.8	3.3
	A_{pm}		1.2e-14	1.2e-15	1.2e-11
failure criteria	α_0	[°]	52	52	52
	X_T	[MPa]	2326	2326	2326
	X_C	[MPa]	1017	1434	1017
	Y_T	[MPa]	62	88	62
	Y_C	[MPa]	255	360	255
	S_L	[MPa]	100	141	100

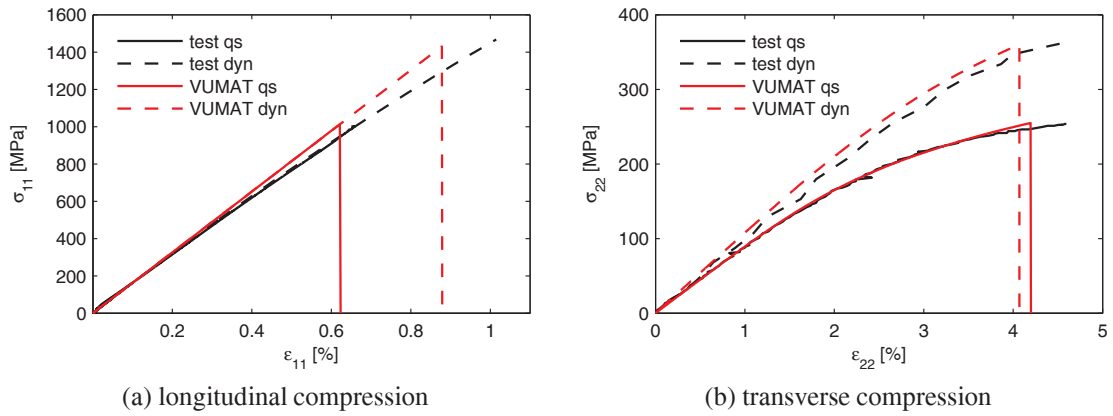


Figure 7.17: Validation of numerical model for longitudinal and transverse compression under quasi-static and high strain rate loading.

A comparison between the predicted stress-strain response, obtained from the single element simulations, using the parameters specified in Table 7.3, and the experimentally measured stress-strain response is shown for a variety of in-plane loading directions in Figures 7.17 - 7.20.

Figure 7.17 shows the predicted and experimental stress-strain curves for quasi-static and dynamic longitudinal compressive (Figure 7.17a) and transverse compressive (Figure 7.17b) loading. The experimental longitudinal and transverse compressive stress-strain curves were taken from Figures 4.13 and 5.29, respectively. It is seen that for both quasi-static and high strain rate loading, the implemented constitutive model accurately predicts the linear-elastic longitudinal compressive stress-strain response and the non-linear stress-strain behaviour for transverse compression. The stiffening of the transverse compressive stress-strain curve at high strain rate loading is well captured and the failure modes are correctly predicted for both strain rate regimes, with fibre-kinking in the case of longitudinal compression and compressive matrix failure in the case of transverse compression.

Figure 7.18 compares the predicted and experimental axial stress-strain curves for the quasi-static and dynamic 15°, 30°, 45°, 60° and 75° off-axis compression tests shown in Figure 5.29. With respect to the overall axial stress-strain response and ultimate strength levels, the correlation between simulation and experiments is very good for all off-axis

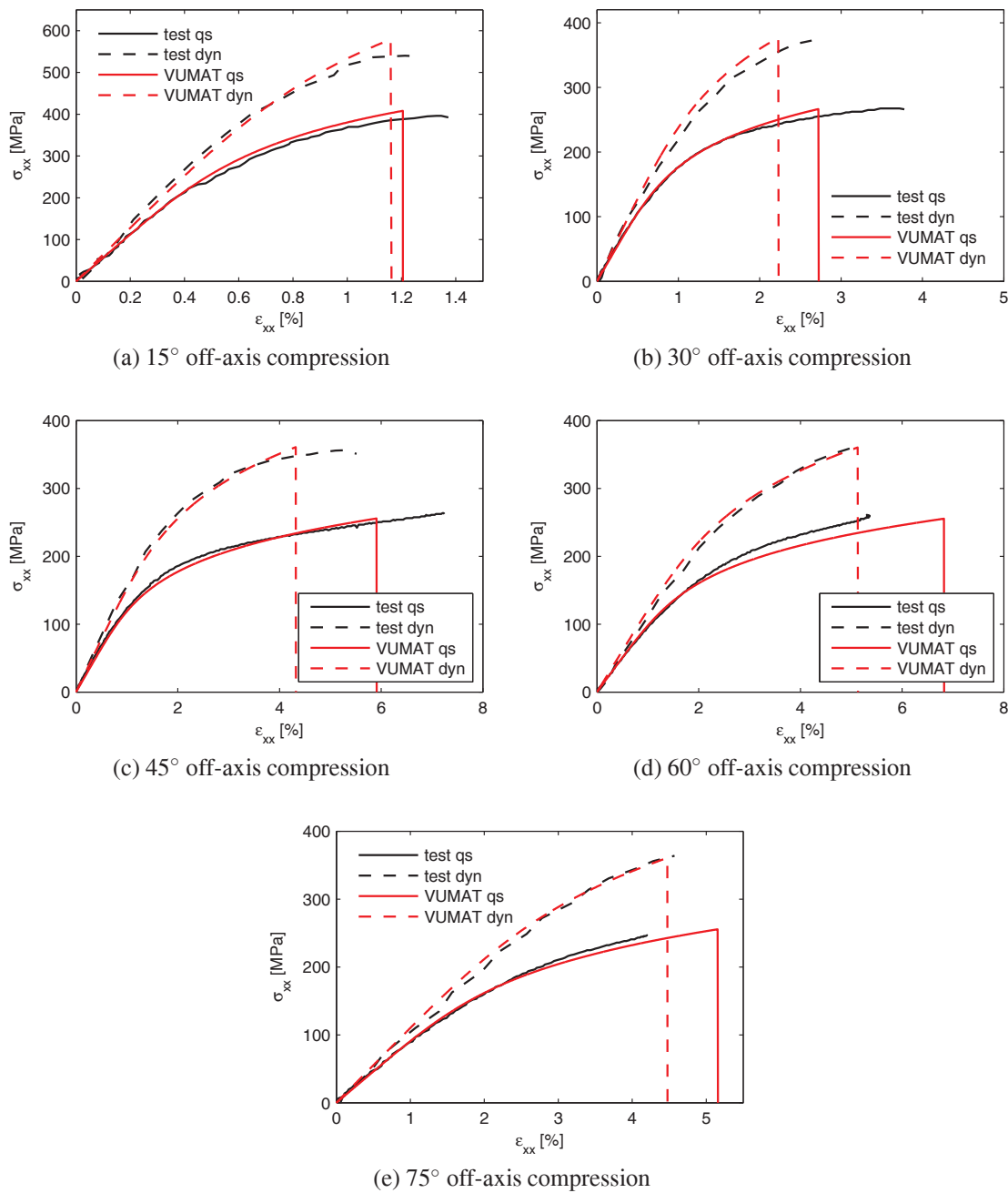


Figure 7.18: Validation of numerical model for combined transverse compression and in-plane shear under quasi-static and high strain rate loading.

angles and both strain rate regimes. The discrepancy between the predicted and measured ultimate strain in some cases is an expected result, since a small variation of the predicted strength causes a significant variation of the predicted ultimate strain, due to the low gradient of the axial stress-strain response at high levels of plastic strain.

It is noted that the predicted failure mode for all test cases shown in Figure 7.18 was

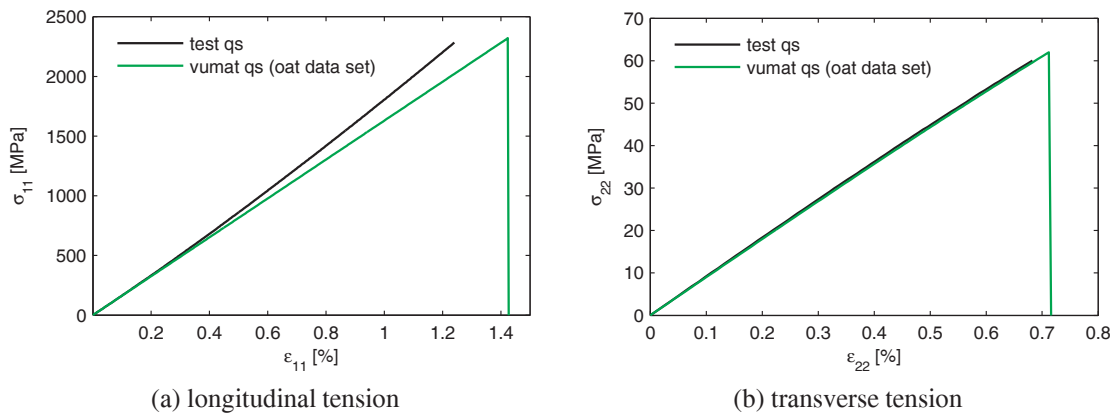


Figure 7.19: Validation of numerical model for longitudinal and transverse tension under quasi-static loading.

compressive matrix failure, with different fracture plane angles α , depending on the ratio of transverse compression and in-plane shear loading (compare Figure 7.11), whereas for some quasi-static 15° off-axis compression tests, a distinct kink-band failure mode was observed as well (Figure 5.11).

Figure 7.19 shows the predicted and experimental quasi-static longitudinal tensile (Figure 7.19a) and transverse tensile (Figure 7.19b) stress-strain response. The experimental data was taken from an earlier quasi-static material characterisation for IM7-8552 [67]. It is seen that the predicted stress-strain response is linear for both longitudinal and transverse tension. In the case of longitudinal tension, the implemented constitutive model is linear-elastic only, whereas for transverse tension, the non-linearity which would occur at higher stress levels as a result of the master curve specified for tensile loading (Figure 7.4), is cut off by the matrix tension failure criterion. For both cases, the implemented failure criteria accurately predicts the ultimate strength and failure mode.

The continuously increasing stiffness observed for the experimental longitudinal stress-strain response shown in Figure 7.19a, can be attributed to the increasing alignment of the initially imperfectly aligned carbon fibre tows in the prepreg.

Figure 7.20 compares the predicted and the experimental quasi-static axial stress-strain response of the off-axis tension tests described in Chapter 6. As noted in Chapter 6, the irregularity at the beginning of the axial stress-strain curve of the 15° off-axis tension

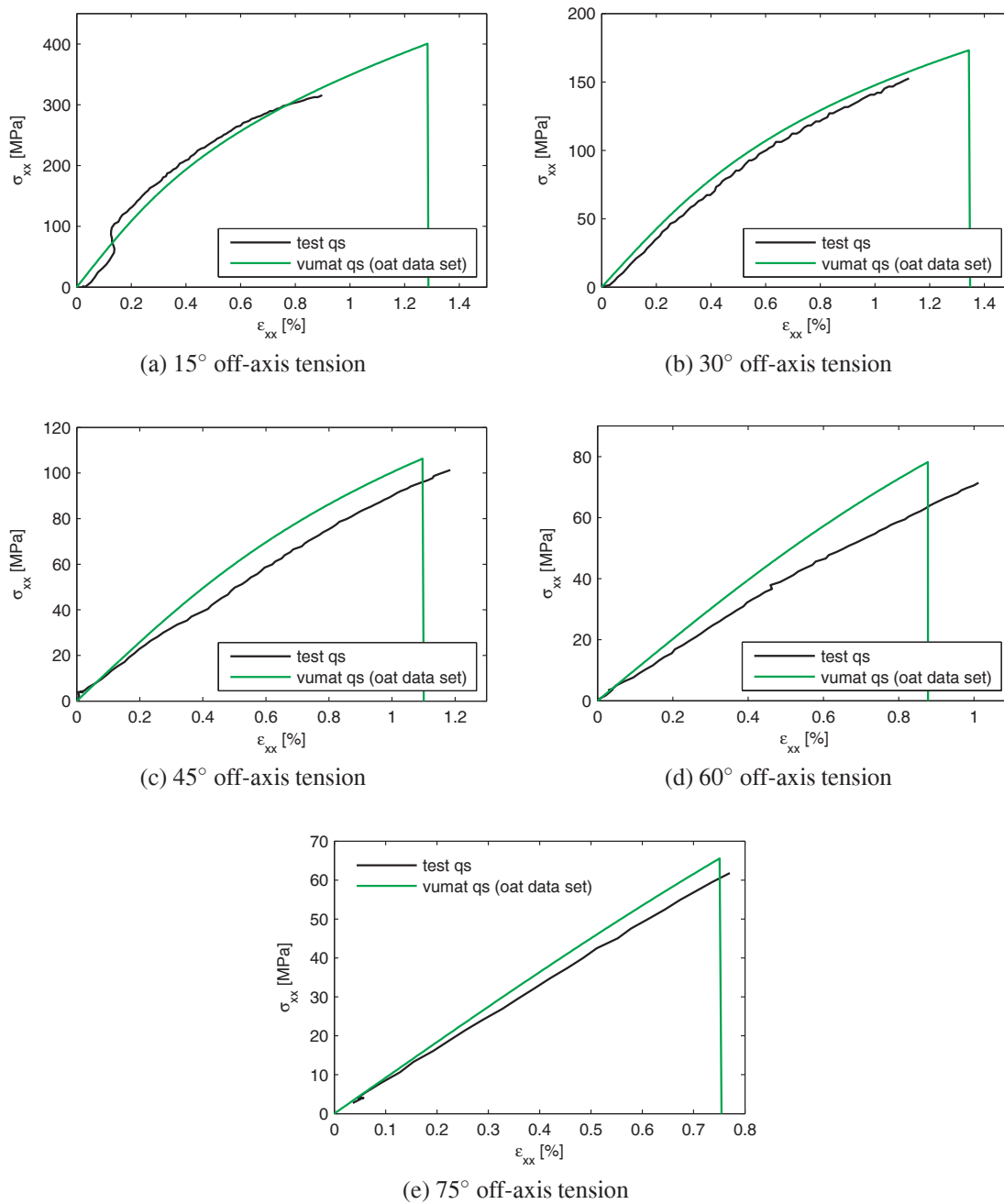


Figure 7.20: Validation of numerical model for combined transverse tension and in-plane shear under quasi-static loading.

test was caused by the data acquisition system and does not represent the actual specimen stress-strain behaviour. Nevertheless, the test data is of sufficient quality to be used for the validation of the numerical model under combined in-plane shear and transverse tensile loading.

The constitutive model predicts the stronger nonlinear behaviour for the 15° and 30° off-

axis tension tests as well as the decreasing nonlinearity with increasing off-axis angle. For the 75° off-axis tension test, the predicted axial stress-strain response is approximately linear due to the low ultimate strength of this specimen type.

The predicted failure mode for all test cases shown in Figure 7.20 was matrix tension. As seen in Figures 7.20a and 7.20b, the failure criterion defined with Equation (7.34) overpredicts the ultimate strength for the 15° and 30° off-axis tension specimen type, which can be attributed to the quadratic interaction formulation used in Equation (7.34). As shown in Chapter 6, the failure envelope for combined transverse tension and in-plane shear loading is better approximated by an earlier version of the matrix tension criterion [87], which also contains a linear interaction term.

7.6 Summary and Conclusions

A simple plasticity model, proposed by Sun and Chen [97] was chosen to describe the nonlinear stress-strain behaviour of the UD carbon-epoxy material system IM7-8552. The constitutive formulations of the plasticity model were presented in Section 7.1. The model relies on the formulation of a master curve, which describes the effective stress-effective plastic strain response and can be formulated in a rate-dependent form.

The model parameters can be determined from off-axis compression and off-axis tension tests, as shown in Section 7.1.1. It was found that the main plasticity model parameter a_{66} is constant for quasi-static and high strain rate compressive loading, as well as for quasi-static tensile loading. An evaluation of the plasticity parameters for high strain rate tensile loading was not possible since dynamic tension tests were not performed in the present study. It was further observed that the quasi-static compressive and tensile master curves were of very similar shape, with yielding occurring at a higher stress level in the case of compression, which was attributed to the fact that the epoxy resin is a pressure-dependent material.

It is noted that the quasi-static compressive and tensile master curves shown in Figure 7.4 correspond to a complete in-plane plasticity characterisation of IM7-8552.

In Section 7.2, the latest version of the phenomenological and physical based failure criteria for unidirectional fibre composites, developed at the University of Porto in a parallel study [89], was briefly described for the general three-dimensional case.

Section 7.3 gave a qualitative overview of the rate dependency of the in-plane and out-of-plane mechanical material properties of IM7-8552, and a simple curve fitting approach was presented to describe the rate dependency. It was found that respectively one trend line formulation was sufficient to describe the rate sensitivity of the in-plane elastic moduli and the rate sensitivity of the in-plane strengths.

In Section 7.4 it was shown how the plasticity model and the failure criteria were implemented into an Abaqus VUMAT subroutine for an in-plane two-dimensional stress state. A forward Euler, or fully explicit, integration scheme was chosen for the implementation of the plasticity model.

In Section 7.5, the VUMAT subroutine was validated via single-element simulations. The subroutine was tested in all principle in-plane material directions for compressive and tensile loading, further for combined transverse compressive and in-plane shear loading as well as combined transverse tensile and in-plane shear loading. For the compressive test cases, validation simulations were performed at two strain rate regimes. The simulated stress-strain responses were compared with available experimental data and it is concluded that the relatively simple plasticity model, in combination with the advanced failure criteria, accurately predicts the constitutive response of unidirectional carbon-epoxy IM7-8552 with linear-elastic behaviour in fibre direction and transverse tension, and non-linear behaviour for combined loading and transverse compression.

It is noted that a preliminary version of the VUMAT subroutine was used at the time of writing this thesis, which could not automatically account for the rate-dependency of the plastic response, the rate-dependency of the elastic material properties and the rate-dependency of the strength properties required for the failure criteria. The aspects which need to be considered and addressed in the future development of the subroutine will be presented in the following and concluding Chapter 8.

Chapter 8

Summary and Conclusion

To obtain reliable high strain rate data for the carbon-epoxy material system IM7-8552 investigated in the present study, a quasi-static and high strain rate characterisation of the in-plane matrix dominated compressive and shear properties, namely longitudinal compression, transverse compression, in-plane shear and combined transverse compression and in-plane shear, was performed.

The carbon-epoxy prepreg IM7-8552 was chosen in this study, since a considerable data base of quasi-static mechanical material properties existed from a previous material characterisation program, which could be used as a reference for the quasi-static and high strain rate tests presented in this work.

From the review of the earlier experimental studies, presented in Chapter 2, it was apparent that the split-Hopkinson pressure bar (SHPB) is a very usefull experimental apparatus to determine the high strain rate material response in the strain rate range of $100\text{s}^{-1} < \dot{\epsilon} < 1000\text{s}^{-1}$ and was therefore chosen for the dynamic experiments presented in this work.

The setup and principles of the classic SHPB experiment were introduced in Chapter 3, along with the classic SHPB analysis procedure, which was used subsequently for the analysis of the high strain rate experiments. The assumptions and conditions of the SHPB analysis were studied in detail and relevant issues, which must be considered when test-

ing polymer composite specimens were identified.

Pulse shaping was considered to be of great importance due to the very different stress-strain behaviour of the polymer composite specimen with regard to the different material directions. An overview of the pulse shaping technique for SHPB experiments was presented in Chapter 3, whereas a detailed description of the pulse shaping analysis (PSA) can be found in Appendix A.

To highlight the importance of pulse shaping, particularly for the dynamic longitudinal compression tests (see Chapter 4), finite element simulations of a SHPB test with a longitudinal compression specimen were performed and described in Chapter 3, using shaped and classic incident waves. From the analysis of these simulations the following conclusions were drawn:

- A ramp shaped incident wave must be used in the case of linear-elastic specimen behaviour to ensure a constant specimen strain rate.
- By using a shaped incident wave, the high-frequency oscillations in the incident- and reflected wave can be eliminated and dynamic stress equilibrium can be achieved instantly.
- The specimen stress is well determined by the SHPB analysis.
- The specimen strain however is overpredicted by the SHPB analysis and to obtain a correct dynamic stress-strain response, the specimen strain should be measured directly on the specimen.

Chapter 3 further contains a comprehensive overview of the digital image correlation (DIC) technique, which was used to obtain the in-plane strain field for the experiments described in Chapters 5 and 6.

A clear identification of the effect of strain rate on the longitudinal compressive properties of unidirectional carbon-epoxy composites was presented in Chapter 4. Significant

attention was given to the reliability of the SHPB experiment by addressing potential issues such as dispersion correction, dynamic stress equilibrium and pulse shaping.

Ramp shaped incident-pulses were used since a specimen with linear stress-strain behaviour can then be tested at a constant strain rate. Careful pulse shaping and proof of dynamic stress equilibrium established early in the transient loading process gives credibility to the dynamic elastic modulus measurements.

It was demonstrated that a better dynamic stress-strain response can be obtained when using a specimen strain gauge, compared to the stress-strain response calculated entirely from the bar-waves via SHPB analysis.

The fixture required to align and stabilise the flat rectangular specimen was designed by explicit finite element analysis to not influence the bar strain waves and subsequently the results of the SHPB analysis. The numerical predictions were later confirmed by the dynamic experiments performed with the SHPB.

Quasi-static tests at a strain rate of $3.6 \times 10^{-4} \text{s}^{-1}$ and dynamic tests at strain rates up to 118s^{-1} were performed with the developed test setup, and the following conclusions were drawn:

- The longitudinal compressive chord modulus of elasticity is not strain rate sensitive up to the strain rates considered in this study.
- The longitudinal compressive strength increased by about 40%.

In Chapter 5, off-axis and transverse compression tests with end-loaded rectangular unidirectional carbon-epoxy specimens were performed for quasi-static and high strain rates. It was demonstrated that end-loaded specimens are well suited to determine the off-axis and the transverse compression properties of unidirectional polymer composites. This is particularly useful for high strain rate tests on the SHPB since the wave propagation for straight-ended bars is well understood, while modified bar-ends may significantly influence the bar strain waves and hence the dynamic specimen stress-strain response, calculated from the bar strain waves via SHPB analysis.

Pulse shaping was used to systematically obtain suited incident-pulses for the respective specimen stress-strain behaviour. SHPB experiments with the specimen in dynamic equilibrium and loaded at near constant strain rates were performed for all specimen types. As a result, both elastic and strength properties were obtained from the dynamic tests with confidence.

For the high strain rate tests a PHOTRON SA5 high speed camera, set to a suitable combination of frame rate and spatial resolution was used, thus allowing a continuous recording during the dynamic test.

The in-plane strain field was obtained over the entire specimen surface for both quasi-static and dynamic tests from the ARAMIS digital image correlation (DIC) software. Fundamental SHPB analysis assumptions, such as uniform specimen deformation and strain distribution were validated. Moreover, it was demonstrated that the specimen strain obtained from SHPB analysis was over-predicted for all specimen types except for the transverse compression test. With the aid of digital image correlation, true specimen strain could be obtained and recorded beyond the strain limit of currently available standard foil strain gauges.

Significant fibre-rotation was observed for the quasi-static and dynamic 15°, 30° and 45° off-axis compression specimens, which was taken into account during the data reduction for those specimen types. The additional rotation-angle was obtained directly from the DIC software.

The pure in-plane shear strength was determined with the extrapolation method proposed by Tsai and Sun [30]. The yield in-plane shear strength and the in-plane shear modulus were obtained from the shear stress-strain response of the 45° off-axis tests.

With the high frame rate and sufficient resolution of the high speed camera, it was possible to capture the initiation and propagation of the failure process and measure the fracture angle α for the 45°, 60°, 75° off-axis and transverse compression specimen types.

It was found that the Puck failure criterion for matrix compressive failure [86] provides excellent strength predictions for both quasi-static and dynamic loading. In addition, the

fracture angles were accurately predicted. It was observed that the dynamic experimental failure envelope is consistently larger than the quasi-static one.

Due to the early dynamic equilibrium and near constant strain rates in the SHPB experiment, the dynamic yield envelope for combined transverse compression and in-plane shear loading was determined and compared with the quasi-static yield envelope.

For the strain rates studied in the present work, the rate effect on the transverse compressive and in-plane-shear behaviour of the carbon-epoxy material system IM7-8552 can be summarised as follows:

Transverse Compression

- The modulus of elasticity increased by a moderate 12%.
- The yield strength increased significantly by about 83%.
- The failure strength increased by 45%.
- The observed rate effect on the transverse compressive failure strain is insignificant and therefore it is concluded that this property is not strain rate sensitive for the strain rates considered in this work.

In-Plane Shear

- The in-plane shear modulus, obtained from the shear stress-strain component of the 45° off-axis compression tests, increased by 25%.
- The yield strength, obtained from the same shear stress-strain curves, increased by 88%.
- The pure in-plane shear strength, extrapolated in the combined $\sigma_{22} - \tau_{12}$ stress-diagram from the failure strength of 15° and 30° specimens which failed in an in-plane-shear dominated failure mode, increased by 42%.

- The strain rate effect on the in-plane shear failure strain was not determined due to the dependency of the apparent failure strength and strain on the ratio of biaxial loading.

Combined Transverse Compression and In-Plane Shear

- As an average for all specimen types subjected to combined loading, the axial modulus increased by 20%. Some scatter was however observed for the 15° and 30° tests.
- A relatively uniform increase was observed for the yield strength, with an average value of 85%.
- The least scatter, when comparing the individual specimen types, was found for the rate effect on the failure strength, with an average increase of 40%.

In Chapter 6, quasi-static off-axis tension tests were performed to determine the constitutive response and the failure envelope of IM7-8552 for combined transverse tension and in-plane shear.

The off-axis tension tests can be used together with the quasi-static compression tests described in Chapter 5, to evaluate the material response in the $\sigma_{22} - \tau_{12}$ stress space.

Unfortunately, modulus and yield strength measurements were not possible for the off-axis tension tests due to a systematic error in the DIC strain field data acquisition. The quasi-static off-axis tensile strength was however determined with good accuracy and was used to construct the experimental failure envelope for combined transverse tension and in-plane shear.

By using oblique angle tabs, the stress concentrations at the clamping regions, resulting from extension-shear coupling, could be reduced and premature failure was avoided. The off-axis specimens then predominantly failed in the middle of the gauge section.

The experimental failure envelope for transverse tension and in-plane shear was compared with the Hashin [96] criterion and with the matrix tension criterion proposed by Dávila et al. [87]. A better correlation between experimental and predicted failure envelope

lope was obtained with the criterion proposed by Dávila et al. [87].

Chapter 7 introduced a simple plasticity model, proposed by Sun and Chen [97], to describe the nonlinear stress-strain behaviour of the unidirectional carbon-epoxy material system IM7-8552. The model relies on the formulation of a master curve, which describes the effective stress-effective plastic strain response and can be formulated in a rate-dependent form.

The model parameters can be determined from off-axis compression and off-axis tension tests. It was found that the main plasticity model parameter a_{66} is constant for quasi-static and high strain rate compressive loading as well as for quasi-static tensile loading. An evaluation of the plasticity parameters for high strain rate tensile loading was not possible since dynamic tension tests were not performed in the present study. It was further observed that the quasi-static compressive and tensile master curves are of similar shape, with yielding occurring at a higher stress level in the case of compression, which was attributed to hydrostatic pressure effects.

A simple curve fitting approach was presented to describe the rate dependency of the mechanical material properties. It was found that respectively one trend line formulation was sufficient to describe the rate sensitivity of the in-plane elastic moduli and the rate sensitivity of the in-plane strengths.

It was shown how the plasticity model and a failure criteria were implemented into an Abaqus VUMAT subroutine for an in-plane two-dimensional stress state. A forward Euler, or fully explicit, integration scheme was chosen for the implementation of the plasticity model.

The VUMAT subroutine was validated via single-element simulations. The subroutine was tested in all principle in-plane material directions for compressive and tensile loading, further for combined transverse compressive and in-plane shear loading as well as combined transverse tensile and in-plane shear loading. For the compressive test cases, validation simulations were performed at two strain rate regimes. The simulated stress-

strain responses were compared with available experimental data and it was concluded that the relatively simple plasticity model, in combination with the advanced failure criteria, accurately predicts the constitutive response of unidirectional carbon-epoxy IM7-8552, with linear-elastic behaviour in fibre direction and transverse tension, and non-linear behaviour for combined loading and transverse compression.

It is noted that the version of the VUMAT subroutine does not automatically account for the rate-dependency of the plastic response, the rate-dependency of the elastic material properties and the rate-dependency of the strength properties required for the failure criteria. The aspects which need to be considered and addressed in the future development of the subroutine are presented in the following Section.

8.1 Future Work

8.1.1 Future Experimental Work

The experiments presented in this work were performed at two strain rate regimes: quasi-static tests at approximately $4\text{e-}4\text{s}^{-1}$ and high strain rate tests, using the SHPB apparatus, at approximately 250s^{-1} . To establish sound trends regarding the strain rate effect on the mechanical properties of unidirectional carbon-epoxy composites, data from a third strain rate regime would be required.

From Figure 7.9 it is concluded that the most valuable information can be obtained from dynamic tests at strain rates above 1000s^{-1} , since both modulus and strength properties are likely to increase significantly at this strain rate compared to the trends observed from the quasi-static range to strain rates of about 10 to 100s^{-1} . Currently no reliable SHPB test data exists however for unidirectional carbon-epoxy composites at strain rates above 1000s^{-1} , in particular for the elastic properties. Although some earlier experimental studies reported results at such high strain rates, the results are questionable as explained in Section 2.8.

The trend lines presented in Figure 7.9 were therefore established by also using high

strain rate experimental results from a neat resin study performed by Gerlach et al. [8], who paid significant attention to the reliability of his experiments by addressing the important issue of pulse shaping and provided sufficient proof of dynamic stress equilibrium.

Since a tension SHPB was not available in the present study, a dynamic material characterisation could not be considered. High strain rate transverse tension and off-axis tension test would allow a further validation of the implemented plasticity model and failure criteria for high strain rate tensile loading.

8.1.2 Future Numerical Modelling Work

The main emphasis of the future work lies in the further implementation and development of the plasticity model and failure criteria.

The issues that still need to be addressed include:

- Automatic rate dependency of the plastic response.
- Automatic rate dependency of the elastic properties.
- Automatic rate dependency of the strength properties used in the failure criteria.
- Dependency of the plastic model parameters on the hydrostatic pressure (currently a different parameter set must be selected for tension and compression, compare Table 7.1).
- Incorporation of a damage model, such as the one proposed by Maimí et al. [108].
- Generalisation of the plasticity model for the three-dimensional stress state.
- Implementation of an implicit integration scheme for the plasticity model to improve the robustness of the predicted plastic response.

It is noted that a simpler plasticity model, such as isotropic plasticity, may also be evaluated to increase computational efficiency, considering that the elastic-plastic stress-strain response of the unidirectional carbon-epoxy IM7-8552 was successfully determined in Chapter 7 for various off-axis loading directions and in the direction of transverse compression by using only one master curve definition.

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Appendix A

Pulse Shaping Analysis - Analytical Approach

The type of the shaped incident wave determined by the Pulse Shaping Analysis (PSA), proposed by Nemat-Nasser et al. [66], depends on the conditions, given by the Equations (3.1) and (3.43) in Chapter 3.

The trapezoidal shape, denoted here as case I, is obtained if the pulse shaper ceases to deform before the time T , which corresponds to the pulse duration of a classic incident wave and defined with Equation (3.1), is reached.

The pulse shaper strain ϵ_{ps} , during the interval $0 \leq t \leq T$, is obtained by solving the differential equation:

$$\dot{\epsilon}_{ps} = \frac{V_0}{h_{ps}} \left(1 - K_{ps} \frac{\epsilon_{ps}^{n_{ps}}}{1 - \epsilon_{ps}} \right) , \quad 0 \leq t \leq T \quad (\text{A.1})$$

where V_0 is the striker-bar impact velocity, h_{ps} is the pulse shaper thickness and n_{ps} is the exponent of the power-law relation, established for the stress-strain response of the pulse shaper with Equation (3.41). The parameter K_{ps} is defined as

$$K_{ps} = \frac{a_0 \sigma_0}{A_0 \sigma_{max}} \quad (\text{A.2})$$

where a_0 and A_0 are respectively the initial pulse shaper cross-section and the cross-section of the SHPB bars (assumed constant), σ_0 is the coefficient of the pulse shaper material response power-law and σ_{max} is the maximum achievable bar stress, obtained by multiplying the maximum achievable bar strain ϵ_I^{max} , Equation (3.2), by the elastic modulus E_b of the bar material.

The strain in the incident bar, during the interval $0 \leq t \leq T$, is defined as [66]

$$\epsilon_b(t) = \frac{a_0 \sigma_0}{A_0 E_b} \frac{\epsilon_{ps}^{n_{ps}}(t)}{1 - \epsilon_{ps}(t)}, \quad 0 \leq t \leq T \quad (A.3)$$

The time t^* at which the pulse shaper ceases to deform is defined by the condition

$$\left. \frac{d\epsilon_{ps}}{dt} \right|_{t^*} = 0 \quad (A.4)$$

In the case of a trapezoidal shaped incident wave, the strain in the incident bar at T must be equal to the theoretical maximal bar strain ϵ_I^{max} , since $t^* < T$.

$$\epsilon_b(T) = \epsilon_I^{max} \quad (A.5)$$

The above condition is used during the pulse shaping analysis to evaluate whether a case I (trapezoidal) or a case II (ramped / triangular) shaped incident wave must be calculated. If Equation (A.5) is true, and hence a case I incident wave is determined, the strain in the incident bar, during the interval $T \leq t \leq t^{**}$, is given as

$$\epsilon_b(t) = \epsilon_I^{max} - \epsilon_b(t - T), \quad T \leq t \leq t^{**} \quad (A.6)$$

At t^{**} , the strain in the incident bar is zero and the time t^{**} is therefore equal to the duration of the case I shaped incident wave

$$T_{caseI} = t^{**} \quad (A.7)$$

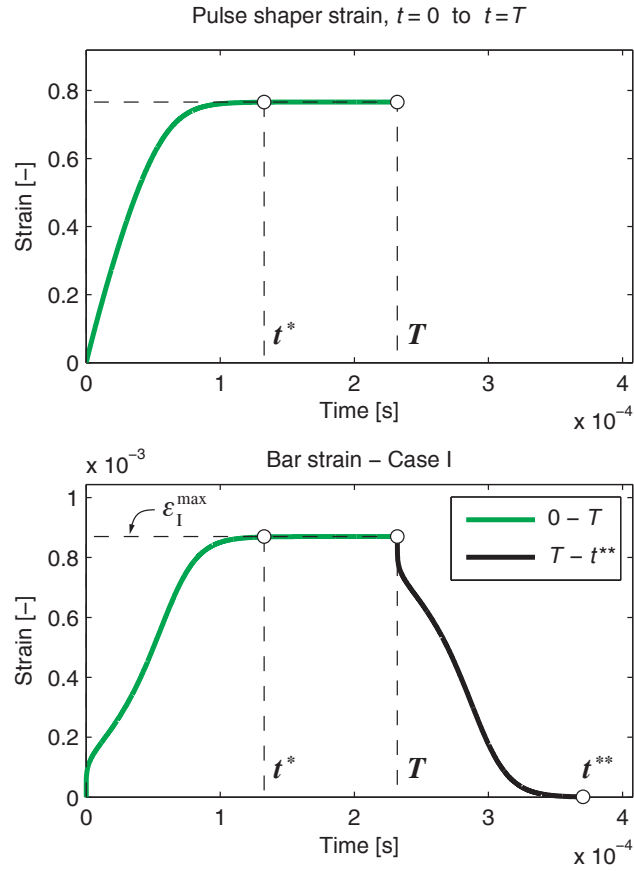


Figure A.1: Strain-time response of pulse shaper and incident wave for pulse shaping analysis case I (trapezoidal shape).

The strain-time response of the pulse shaper and the corresponding strain-time curve for a case I shaped incident wave is shown in Figure A.1.

If a pulse shaper with a higher thickness h_{ps} then in the previous example is used, while keeping all the other relevant parameters constant ($l_{striker}, V_0, d_{ps}$), the rise time of the incident wave can be increased to the point where condition (A.5) is no longer valid.

In other words, the maximum particle velocity $U_p^{max} = V_0/2$ in the bars, Equation (3.43), has not been reached at the time T and the pulse shaper continues to deform. From T onwards, this deformation is however influenced by the returning initial wave front, which has passed back and forth through the striker bar. The resulting incident wave has a ramped / triangular shape, denoted here as case II (Figure A.2).

As for the case I, the pulse shaper strain ϵ_{ps} , during the interval $0 \leq t \leq T$, is determined by solving the differential equation (A.1) and the strain in the incident bar ϵ_b is calculated

with Equation (A.3).

During the time interval $T \leq t \leq t_1$, the pulse shaper strain is determined by solving the delayed differential equation

$$\dot{\epsilon}_{ps}(t) = \frac{V_0}{h_{ps}} \left(1 - K_{ps} \frac{\epsilon_{ps}^{n_{ps}}(t)}{1 - \epsilon_{ps}(t)} - K_{ps} \frac{\epsilon_{ps}^{n_{ps}}(t-T)}{1 - \epsilon_{ps}(t-T)} \right) , \quad T \leq t \leq t_1 \quad (\text{A.8})$$

At t_1 , the pulse shaper ceases to deform and the strain in the incident bar decreases. Using $\dot{\epsilon}_{ps}(t_1) = 0$, the condition for the time t_1 is determined from Equation (A.8) as

$$\frac{\epsilon_{ps}^{n_{ps}}(t_1)}{1 - \epsilon_{ps}^{n_{ps}}(t_1)} = \frac{1}{K_{ps}} - \frac{\epsilon_{ps}^{n_{ps}}(t_1 - T)}{1 - \epsilon_{ps}^{n_{ps}}(t_1 - T)} \quad (\text{A.9})$$

Using the pulse shaper strain calculated by solving Equation (A.8), the incident bar strain

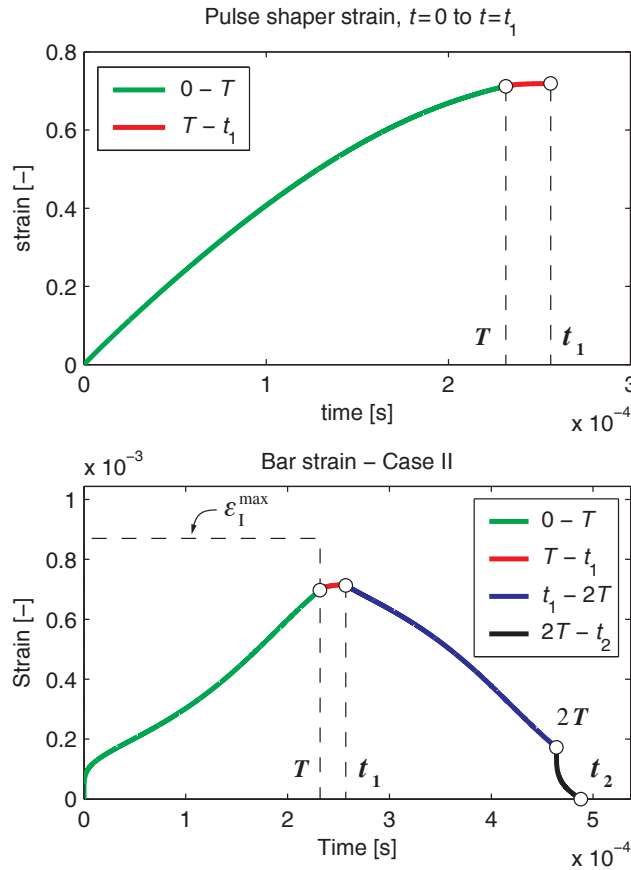


Figure A.2: Strain-time response of pulse shaper and incident wave for pulse shaping analysis case II (ramped / triangular shape).

for the interval $T \leq t \leq t_1$ is defined as

$$\varepsilon_b(t) = \frac{a_0 \sigma_0}{A_0 E_b} \frac{\varepsilon_{ps}^{n_{ps}}(t)}{1 - \varepsilon_{ps}(t)} \quad , \quad T \leq t \leq t_1 \quad (\text{A.10})$$

For the interval $t_1 \leq t \leq 2T$, the incident bar strain ε_b is defined as

$$\varepsilon_b(t) = \varepsilon_I^{max} - \varepsilon_b(t - T) \quad , \quad t_1 \leq t \leq 2T \quad (\text{A.11})$$

and for the subsequent interval $2T \leq t \leq t_2$ as

$$\varepsilon_b(t) = \varepsilon_I^{max} - \varepsilon_b(t - T) - \varepsilon_b(t - 2T) \quad , \quad 2T \leq t \leq t_2 \quad (\text{A.12})$$

The time t_2 marks the instance at which the strain in the incident bar is again zero and therefore corresponds to the duration of the case II shaped incident wave.

$$T_{caseII} = t_2 \quad (\text{A.13})$$

Figure A.3 illustrates how the shape of the incident wave is changed when the pulse shaper thickness h_{ps} is increased while keeping all other parameters constant.

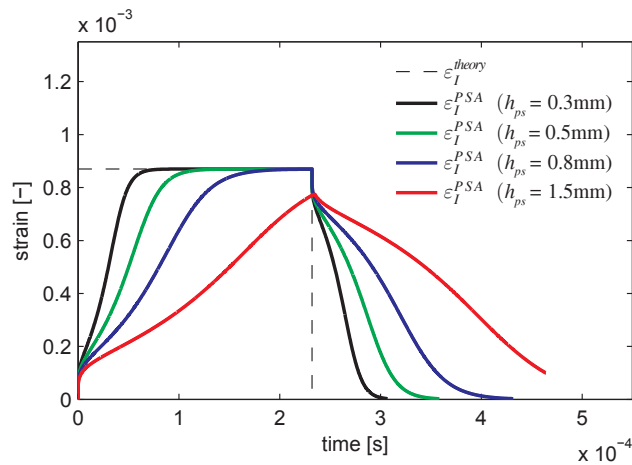


Figure A.3: Change of incident wave shape by using pulse shapers with identical diameter d_{ps} but varying thickness h_{ps} .

It is noted that the differential equation (A.1) and the delayed differential equation (A.8) were solved numerically by using the pre-defined MATLAB subroutines *ode45* and *dde23*, respectively.