FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO PORTO

Optimization Techniques for the Mixed Urban Rural Solid Waste Collection Problem

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To my grandmother Adelaide

Abstract

Transportation is a key decision area within logistics and the high percentage of costs that it absorbs when comparing with other logistic activities originate a great potential for rationalization and saving.

Multiple decisions are studied and taken when planning transportation activities, which go from mode selection to vehicle routing, and the application of Operation Research analytical methods to support these decisions can play a major role in generating economies. Concerning Vehicle Routing, its real life application as well as its high complexity makes it one of the most popular problems within Operations Research.

The Vehicle Routing Problem (VRP) is a combinatorial optimization problem which consists in designing the optimal set of routes for a fleet of vehicles in order to serve a given set of customers. The Periodic Vehicle Routing Problem (PVRP) is an extension of the classical VRP where customers are visited with different frequencies over a time horizon. The PVRP involves three simultaneous decisions: (1) assign customers to days in a way that frequencies are respected, (2) assign customers to vehicles in each day and (3) route the vehicles through the customers.

This thesis is focused on using hierarchical approaches to model and solve the PVRP. Although hierarchical formulations may not achieve optimal solutions when solved, there are some strong motivations for using them, such as to consider other optimization criteria or to deal with large problems, where computational times increase considerably.

Three types of hierarchical approaches were proposed based on the way decisions are tackled: (1) assign and route together; (2) assign days first - assign vehicles and route second and (3) assign first - route second. Formulations scattered in the literature were organized into those levels.

A real-word solid waste collection system of a municipality in northern Portugal is used as a case study. Mathematical formulations are adapted to its features and the best hierarchical approach to solve the problem is evaluate by taking into consideration a compromise between execution time, total distance travelled and number of performed routes.

Resumo

Os custos de distribuição e transportes constituem uma parcela muito significativa na economia das empresas e possuem um grande potencial para optimização e consequente redução de custos.

Múltiplos estudos têm sido efectuados sobre este tema que vão desde a escolha dos canais de transporte até ao planeamento de rotas para os veículos. A aplicação de ferramentas de Investigação Operacional para suportar estas decisões pode ter um impacto enorme, levando a poupanças consideráveis. O planeamento de rotas de veículos é hoje um dos mais populares problemas abordados em Investigação Operacional, considerando a elevada complexidade do tema e as suas importantes aplicações reais.

O problema clássico de estabelecimento de rotas (VRP) consiste em determinar um conjunto de rotas, com um custo total mínimo, que os veículos terão de realizar para servirem um conjunto de clientes geograficamente dispersos. O problema de estabelecimento de rotas periódico (PVRP) é uma extensão do VRP clássico que elabora um planeamento num horizonte temporal alargado, no qual os clientes têm uma frequência de visita diferenciada. O PVRP é considerado um problema multi-nível que abrange três vertentes: (1) o problema da afectação dos clientes aos diferentes dias do período do horizonte temporal; (2) o problema da afectação dos clientes aos diferentes veículos em cada dia do período e finalmente, (3) o problema do estabelecimento de uma rota para cada veículo em cada dia do período.

Este trabalho usa uma abordagem hierárquica para modelizar e resolver o PVRP. Apesar deste tipo de abordagens poder resultar em soluções não optimizadas, uma vez que apenas garantem a optimização de cada nível hierárquico, existem fortes motivações para a sua consideração em casos com muitos clientes, em que a complexidade é muito elevada ou em casos em que se pretende ter em consideração critérios de optimização diferentes em cada nível.

Foram identificadas e propostas três abordagens hierárquicas diferentes, diferindo nos tipos de problemas tratados em cada nível: (1) Afectação simultânea com o estabelecimento de rotas; (2) Afectação inicial dos clientes aos diferentes dias, seguindo-se a afectação dos clientes aos veículos simultaneamente com o estabelecimento das suas rotas e finalmente (3) Afectação inicial dos clientes aos dias e veículos, seguindo-se do estabelecimento de cada rota.

Um sistema de recolha de resíduos sólidos urbanos indiferenciados de um concelho do norte de Portugal é usado como caso de estudo. Diferentes formulações matemáticas são adaptadas aos parâmetros do caso com vista à optimização dos circuitos dos veículos de recolha. Pretende-se também avaliar qual a melhor abordagem hierárquica para a resolução deste problema em específico, tendo em consideração os tempos de execução, o número de rotas efectuadas e a distância total percorrida pelos veículos.

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Abbreviations and Symbols

ATSP	Asymmetric Travelling Salesman Problem
AVRP	Asymmetric Vehicle Routing Problem
CPS	Chinese Postman Problem
EP	Private Enterprises
FEUP	Faculty of Engineering, University of Porto
INR	National Waste Institute
IR	Recycling Industry
LP	Linear Programming
MA	Environment Ministry
MIP	Mixed Integer Programming
MSW	Municipal Solid Waste
OR	Operations Research
PVRP	Periodic Vehicle Routing Problem
SGRSU	Management Systems of MSW in Portugal
SIG	Geographic Information System
SNIT	Sistema Nacional de Informação Territorial
SPV	Sociedade Ponto Verde
SVRP	Symmetric Vehicle Routing Problem
TSP	Travelling Salesman Problem
VRP	Vehicle Routing Problem

Chapter 1

Introduction

1.1 Motivation

Transportation is a key decision area within logistics. Several studies have established that transportation costs account for a proportion of 11% to 13% of the total production cost of goods [10]. In the public sector, operations like mail delivery and waste collection also generate sizeable expenses. Collection costs, for instance, range between 40% and 60% of a community's solid waste management system expenditures [28]. The high percentage of costs that transportation absorbs when compared with other logistic activities originate a great potential for rationalization and saving.

Generating economies is not the only motivation in this field. Transportation systems also affect the performance of other logistic system indicators such as service quality and environmental impact. A perfect transportation service means that an order is in fact delivered at the agreed time and place and satisfies customer's demand. On the other hand, the environmental impact of transportation is mainly related with exhaust emissions and evaporation/transfer of fuel to the air, water and ground. Climate change (greenhouse effect), over-fertilisation and depletion of the ozone layer are some of the environmental threats faced in the moment [21].

Multiple decisions are studied and taken when planning transportation activities, which go from mode selection and packing to vehicle routing. The use of Operations Research (OR) analytical methods to support these decisions can lead to substantial transportation savings. Concerning Vehicle Routing, the amount of real life applications as well as its high complexity makes it one of the most popular problems within Operations Research.

First proposed by Dantzig and Ramser in 1959 [9], the Vehicle Routing Problem (VRP) is a combinatorial optimization problem which concerns the optimal design of routes to be used by a fleet of vehicles to serve a set of customers. Several objectives can be considered for the VRP, some examples are the minimization of the number of vehicles, the minimization of transportation costs and the balancing of routes. Routes must also satisfy several operational constraints and multiple variations of the VRP with different sets of constraints are presented in the literature. The

VRP is known to be of great complexity and therefore, along with strong formulations and exact decomposition algorithms, numerous heuristics and metaheuristics have also been developed for its resolution.

This thesis concerns the VRP and one of its applications that has already been studied in the literature - Municipal Solid Waste (MSW) collection. In this application, together with the routing problem, several other tactical and strategic decisions must be taken. Figure 1.1 enumerates some of those decisions divided into strategic and tactical ones.

Collection is one of the first steps of a solid waste management system and therefore, when properly planned and implemented, it can generate significant savings in all the waste management system. An efficient collection system can ultimately hold down waste management costs but can also guarantee cleanliness levels and hygiene patterns, besides the above mentioned advantages in customers satisfaction.

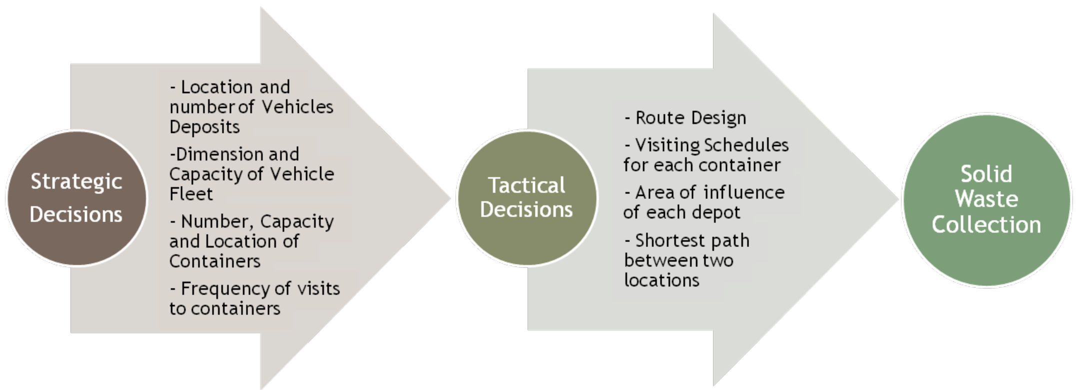

Figure 1.1: Solid Waste Collection Decision Problems

In this project, the literature review on VRP and MSW Collection was done having in mind a specific case study, which concerns the mixed municipal solid waste collection in Ponte de Lima, a Portuguese municipality with rural and urban areas. Currently, the municipality does not have efficient means to make decisions in this area. Routes for mixed waste collection are still hand-made for each month for a period of one week. As the strategic decisions are already taken, the problem is to determine the visiting schedule for each container and the design of routes for the vehicles. In literature this problem is known as the Periodic Vehicle Routing Problem (PVRP), a variant of the VRP. The objectives of the problem are minimizing the number of tours and total distance travelled.

1.2 Objectives and Work Plan

This thesis is focused on using hierarchical approaches to model and solve the PVRP. Two main objectives were defined:

- Collection and division of PVRP formulations into different types of hierarchical approaches. Understanding of the limitations and advantages of each approach.

- Ponte de Lima Waste collection system analysis with focus on its deposition and collection systems. Development of system indicators for the most relevant strategic decisions which affect the problem in study. Adaptation of the hierarchical formulations identified in the previous point in order to take into consideration the case study features. Identification of the best hierarchical approach to solve this problem as being the one with the best compromise between execution time, total distance travelled and number of routes. Resolution of the problem based on the chosen hierarchical approach.

The above mentioned objectives were undertaken during a period of approximately five months, and the work load was divided as presented in figure 1.2. In [17] it is possible to consult a full description of the work carried out in each week.

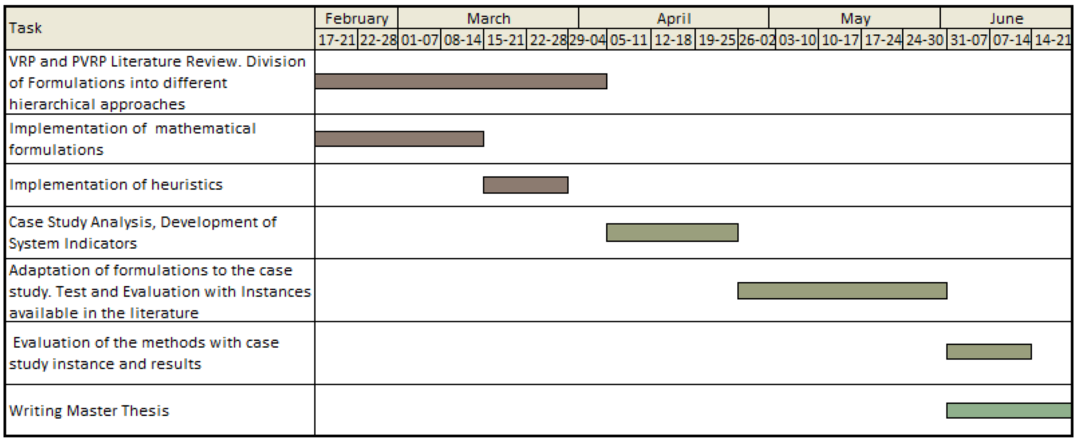

Figure 1.2: Work Plan

1.3 Structure

This dissertation is organized in 6 chapters. Each chapter starts with a small introductory text describing the chapter's intent and to the exception of the present one, ends with a brief summary. The current introduction chapter presents the background motivation for the Vehicle Routing Problem and, more specifically, its application to waste collection. It also enumerates the proposed objectives as well as the work plan for the duration of the project. Finally, it describes the document's organization. The remainder of the document is organized as follows:

Chapter 2, *Case Study Presentation*, provides a description of the Mixed Solid Waste Collection System in Ponte de Lima municipality. A set of system indicators are also presented to evaluate the strategic decisions already taken.

Chapter 3, *Literature Review on Routing Problems*, offers a view on the state-of-art in Operations Research related to transportation problems and, more specifically, to the Vehicle Routing

Problem. It not only includes formulations but also briefly explains solution methods. Then, it presents with more detail mathematical formulations for the Periodic Vehicle Routing Problem, organized by hierarchical approach.

Chapter 4, *Hierarchical Formulations and Solution Methods Applied to the Case Study*, formally describes the Waste Collection Problem in Ponte de Lima and presents the parameters to be considered in the future. Afterwards, mathematical formulations are adapted to the case study features. Four alternative solution methods are proposed, based on different hierarchical approaches and computational experiments are done with instances adapted from the literature. The chapter ends with a conclusion of the best hierarchical approach based on the results from the literature instances.

Chapter 5, *Case Study Results*, tests the four solution methods presented in the previous chapter by using the case study instance and exposes the results for the waste collection problem.

Finally, chapter 6, *Conclusion and Future Work*, infer on the work's achieved objectives and also opens new paths for the work developed.

Chapter 2

Case Study Presentation

This chapter aims to present and describe the case in study. It is intended a plan of routes for the transportation of Mixed Municipal Solid Waste from containers to treatment facilities, taking into consideration the pre-determined frequency of visits for each container, the municipality road network and available resources. For that purpose, it is presented information about the Municipal Solid Waste Management in Ponte de Lima, the geography of the area and the deposition and collection systems. A set of system indicators concerning strategic decisions are also proposed.

2.1 Municipal Solid Waste Management System in Portugal

Solid waste is any substance, object or product with predominantly solid consistency which the holder discards, intends to discard or has the obligation to discard. There are several types of solid waste, classified by the origin. Municipal Solid Waste is the focus of this study. Other types of solid waste include: Industrial Waste, Agriculture Waste, Dangerous Waste and Hospital Waste.

MSW are not only the domestic wastes but also other similar solid wastes according to their nature or composition from origins such as the service sector, commercial, trading or industrial plants and health care units, proving that the daily production does not exceed 1100 liters per producer [1].

2.1.1 Value Chain

An integrated MSW system treats and recovers in different ways waste with different recovery potentialities. After its generation, the waste may be discharged into three different types of containers: selective disposal containers, mixed waste containers and containers for organic matter. After the collection:

- Selective waste is received by sorting facilities, which sort and treat the waste so that the recycling industries can receive it.

- Organic matter is received by a composting plant. Through its treatment, it is possible to produce organic compost which will be used as fertilizer in agriculture.

- Mixed waste is sent to incineration plants or sanitary landfills. Sanitary landfills are facilities for a controlled MSW deposition, avoiding environmental and public health problems. Incineration plants, on the other hand, burn out the waste to produce electrical power. The bottom ash, that is the remaining material after the incineration process, is sorted and sent either for recycling (ferrous metal), civil construction (inert material) or sanitary landfills. Systems without incineration plants send the mixed waste directly to sanitary landfills.

When treatment facilities are distant from the containers, a transfer station may be used, which is a station where the waste is stored and compacted before being transported to treatment facilities. In figure 2.1, adapted from [23], one can see the distribution of the MSW through the different components of the waste collection system.

According to the European Union, and better explained in [1], solid waste must be seen as resources, being sanitary landfills the last option for treatment. Recycling is the best option.

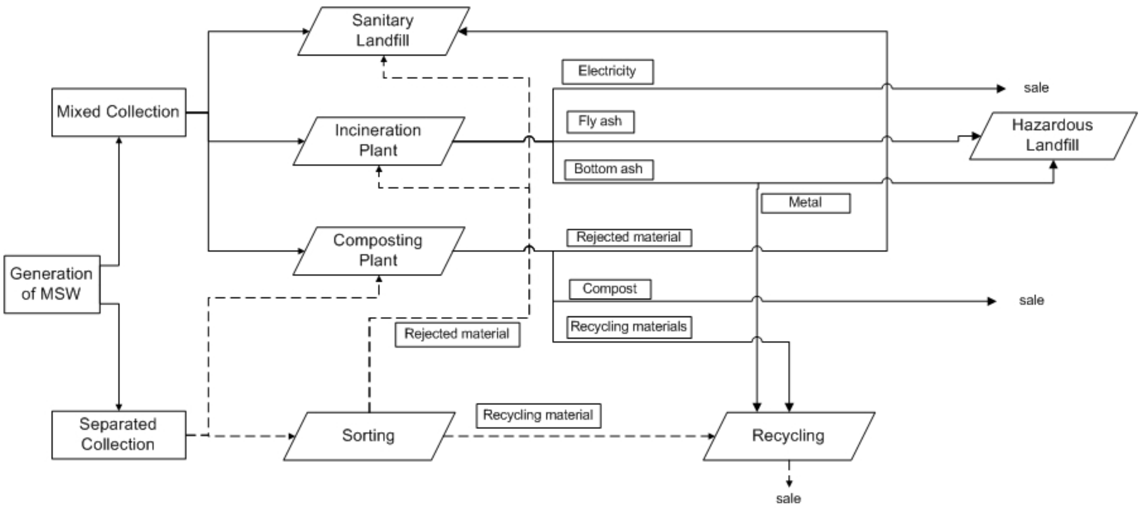

Figure 2.1: Distribution of MSW in Portugal, 2002 (adapted from [23])

2.1.2 Entities Responsible For Mixed Solid Waste Management

Currently, Portugal has an integrate MSW management system. Responsibilities are shared by different entities, namely the municipalities, the MSW Managent Companies (SGRSU) and Sociedade Ponto Verde (SPV).These entities guide their activities according to the legislation and the policies dictated by the Environment Ministry (MA). MSW management activities are controlled and supervised by the National Waste Institute (INR). A short description of the three entities responsible for solid waste management is now presented:

- *Municipalities* define the MSW management system within their jurisdiction. Usually, mixed waste collection is under their responsibilities. They have a contract of delivery and reception of municipal solid waste with one SGRSU. The separate collection is undertaken either by the municipalities, by the SGRSU or by private companies, depending on the municipality.

- *SGRSU* are MSW management systems. These companies are responsible for waste treatment and recovery in a municipality or group of municipalities. In 2009, 34 management systems existed in Portugal but the number is decreasing, as shown in figure 2.2. Each company has its own infrastructures. As far as the recycling process is concerned, their activity consists only in sorting the separate waste. The packaging waste is then transported to the Sociedade Ponto Verde.

Figure 2.2: SGRSU in Portugal. Entities with * merged to form Resinorte ([1])

- *Sociedade Ponto Verde* is a private non-profit organization which aims to promote the selective collection and manage the packaging waste in Portugal. According to EU legislation transposed into Portuguese law, economic operators placing packaged goods on the market are responsible for the management and final disposal of the packaging waste. SPV has contracts with these manufacturers and not only guarantees the take back, recovery and recycling of sorted waste but also manages and deals with the final disposal of non-reusable packaging.

It is the responsibility of all the entities involved in the MSW management to organize campaigns to develop awareness and a positive attitude of the citizens as far as the environment, the need for recycling and the active participation are concerned.

A framework of responsibility and management of MSW in Portugal was presented in [23] and can be seen in figure 2.3.

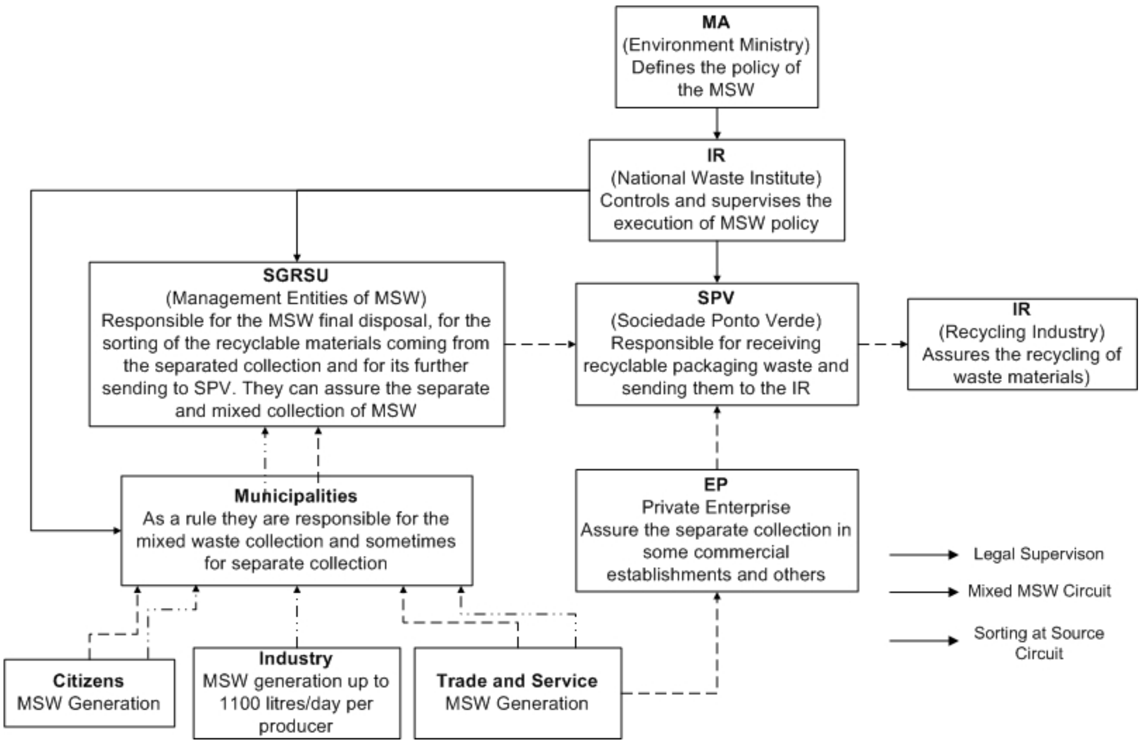

Figure 2.3: Framework of responsibility and management of MSW in Portugal ([23])

2.2 Mixed Municipal Solid Waste Management System in Ponte de Lima

2.2.1 Responsibilities and Value Chain

As stated before, every municipality defines the MSW management system within their jurisdiction. In Ponte de Lima, the mixed waste collection is under the responsibility of the municipality whereas the responsibility for separate collection is undertaken by RESULIMA, a SGRSU with which the municipality also has the contract of delivery and reception. The municipality must deliver the waste collected in places indicated by the company.

RESULIMA is the company responsible for the treatment, recovery and final destination of waste in six municipalities of the Vale do Lima and Baixo Cávado regions. The municipalities served by this company are Arcos de Valdevez, Barcelos, Esposende, Ponte da Barca, Ponte de Lima and Viana do Castelo covering a population of 333,000 inhabitants and a total area of 1778 km^2. In 2008 it treated 114 000 tons of waste. The company is owned by the municipalities that it serves and by Empresa Geral dos Fomentos, with public capitals [3] [15].

Regarding its infrastructures, RESULIMA owns and is responsible for the management of the following infrastructures:

Table 2.1: Resulima Infrastructures

Type	*Number*	*Location*
Sanitary Landfill	1	Vila Fria, Viana do Castelo
Sorting Centre	1	Vila Fria, Viana do Castelo
Transfer Station	1	Oliveira, Arcos de Valdevez
Selective Disposal Containers	2496	
Two collection centres	2	Oliveira, Arcos de Valdevez
Dumps (Already closed)	5	

Mixed Municipal Solid Waste collected by the municipality must be delivered at the Sanitary Landfill or at the transfer station. At this moment, the company does not own any recovery plant for mixed waste and therefore it is directly disposed at the Sanitary Landfill.

2.2.2 Area of Study Description

Ponte de Lima is a municipality located in the north of Portugal, district of Viana do Castelo (figure 2.4) with a total area of 320,26 km^2 and a population of 44 527 inhabitants (2008). The municipality is organized into 51 parishes as represented in figure 2.5. The inhabitants are not uniformly distributed by the area of the municipality being population density higher near the city of Ponte de Lima. Currently, there are five urbanization plans covering the following parishes: Freixo, Ponte de Lima, Correlhã, Refoios do Lima and Fontão and Arcos.

Figure 2.4: Location of the Area in Study in Portugal (picture taken from the municipality SIG [12])

Figure 2.5: Parishes of Ponte de Lima (figure from Resource Center of Ponte de Lima [13])

Table 2.2 presents the area, number of inhabitants and density in all parishes. The most recent information dates from 2001 but one can see on table 2.3 that the overall population of Ponte de Lima has not been increasing significantly in the last years. All parishes have density values ranging from 9 to 270 inhabitants/km^2 with some exceptions in Ponte de Lima parish and nearby. It is also possible to notice that the majority of high density values are located in parishes near the Lima river and one separate density focus is located in the south of the municipality, in Freixo parish. In figure 2.6, the overall population density within the municipality can be observed.

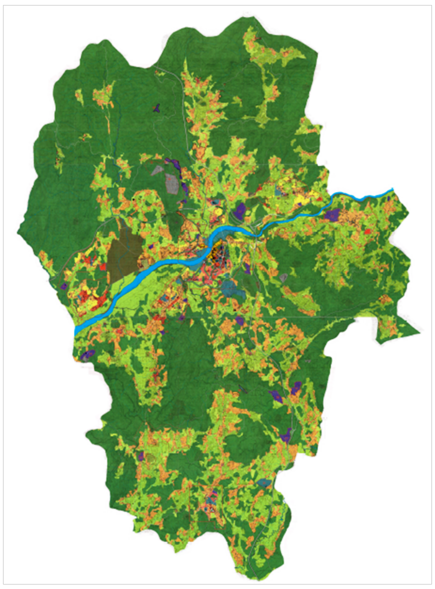

Figure 2.6: Population Density in Ponte de Lima (figure from SNIT [11])

Table 2.2: Population and Geographic Information of Ponte de Lima by Parish

Parish	*Pop (hab)*	*Area (km^2)*	*Pop Density (hab/km^2)*	*Parish*	*Pop (hab)*	*Area (km^2)*	*Pop Density (hab/km^2)*
Anais	1176	7.66	153.5	Gemieira	572	4.40	130.0
Arca	772	1.31	589.3	Gondufe	435	6.08	71.5
Arcos	658	14.43	45.6	Labruja	482	16.73	28.8
Arcozelo	3932	11.87	331.3	Labrujó	153	4.16	36.8
Ardegão	236	3.02	78.1	Mato	285	2.50	114.0
Bárrio	405	5.31	76.3	Moreira do Lima	893	10.16	87.9
Beiral do Lima	767	5.58	137.5	Navió	243	0.90	270.0
Bertiandos	392	2.26	173.5	Poiares	847	5.91	143.3
Boalhosa	215	2.14	100.5	Ponte de Lima	2752	1.41	1951.8
Brandara	479	2.51	190.8	Queijada	328	2.90	113.1
Cabaços	703	5.95	118.2	Reb. Sta Maria	1065	7.20	147.9
Cabração	155	16.43	9.4	Rebordões Souto	1253	7.25	172.8
Calheiros	1047	8.89	117.8	Refóios do Lima	2282	16.40	139.1
Calvelo	744	4.85	153.4	Rendufe	204	2.76	73.9
Cepões	586	3.89	150.6	Ribeira	1841	9.91	185.8
Correlhã	3068	8.04	381.6	Sá	406	2.92	139.0
Estorãos	513	17.10	30.0	Sandiães	423	2.68	157.8
Facha	1482	15.82	93.7	Sta Comba	680	1.66	409.6
Feitosa	828	3.07	269.7	Sta Cruz do Lima	532	2.51	212.0
Fojo Lobal	302	3.10	97.4	Seara	683	4.00	170.8
Fontão	1132	4.72	239.8	Serdedelo	500	6.85	73.0
Fornelos	1535	10.50	146.2	Vilar das Almas	343	5.21	65.8
Freixo	1262	5.44	232.0	Vilar do Monte	113	3.51	32.2
Friastelas	515	4.20	122.6	Vitorino das Donas	1059	4.26	248.6
Gaifar	306	3.01	101.7	Vitorino de Piães	1618	13.51	119.8
Gandra	1141	4.32	264.1				

Table 2.3: Population of Ponte de Lima through the years

Year	1801	1849	1900	1930	1960	1981	1991	2001	2008
Population	13202	29869	33314	36256	42979	43797	43421	44343	44527

Within the municipality region, the area of study covers all parishes but the historic centre of Ponte de Lima parish because collection is carried out door to door in that region. However, it must be taken into consideration when calculating time and capacity occupation of vehicles passing in that area.

2.2.3 Deposition System

Residents of Ponte de Lima municipality deposit their mixed waste in containers strategically located along some roads. 996 containers are non-uniformly distributed through the 51 parishes, the majority with 800 litres of capacity. On table 2.4 one can see the distribution of containers through the parishes.

Recently, in 2009, a new legislation came out which states that in urban areas, containers must be installed at a maximum distance of 100 meters of every residence and at a maximum distance

Table 2.4: Distribution of Containers through Parishes

Parish	*Pop Density (hab/km²)*	*No. Containers*	*Parish*	*Pop Density (hab/km²)*	*No. Containers*
Anais	153.5	24	Gemieira	130.0	15
Arca	589.3	22	Gondufe	71.5	9
Arcos	45.6	20	Labruja	28.8	17
Arcozelo	331.3	85	Labrujó	36.8	6
Ardegão	78.1	10	Mato	114.0	6
Bárrio	76.3	9	Moreira do Lima	87.9	28
Beiral do Lima	137.5	15	Navió	270.0	5
Bertiandos	173.5	16	Poiares	143.3	11
Boalhosa	100.5	8	Ponte de Lima	1951.8	75
Brandara	190.8	13	Queijada	113.1	10
Cabaços	118.2	16	Reb. Sta Maria	147.9	16
Cabração	9.4	8	Rebordões Souto	172.8	22
Calheiros	117.8	21	Refóios do Lima	139.1	48
Calvelo	153.4	16	Rendufe	73.9	8
Cepões	150.6	12	Ribeira	185.8	36
Correlhã	381.6	46	Sá	139.0	12
Estorãos	30.0	17	Sandiães	157.8	7
Facha	93.7	18	Sta Comba	409.6	20
Feitosa	269.7	37	Sta Cruz do Lima	212.0	11
Fojo Lobal	97.4	10	Seara	170.8	12
Fontão	239.8	20	Serdedelo	73.0	12
Fornelos	146.2	23	Vilar das Almas	65.8	10
Freixo	232.0	34	Vilar do Monte	32.2	8
Friastelas	122.6	8	Vitorino das Donas	248.6	25
Gaifar	101.7	9	Vitorino de Piães	119.8	27
Gandra	264.1	19			

of 200 meters in rural areas [14]. This legislation might increase significantly the number of containers in the municipality.

2.2.4 Collection System

The Ponte de Lima Municipality is responsible for the collection of Mixed MSW. For that purpose, it owns five vehicles with capacities of 6.4, 6.8, 11.2, 10.9 and 10.28 Tons. The vehicles are parked in a garage located at Arca parish. 15 workers are hired to guarantee the service, as one driver and two waste collectors operate each truck.

The fleet works during the 5 working days and also on Saturdays. Usually, four trucks depart from the garage each morning at 5h00 and a fifth truck is operated in the afternoon, starting at 14h00. On saturdays, only 3 trucks leave the garage, all at 16h00. The last stop of each tour before the garage is either the transfer station or the sanitary landfill where the waste is weighted and unloaded.

The frequency of visits to the containers vary with their location because of the existence of different filling rates. The number of visits per period for each parish was already set and is presented on table 2.5.

Table 2.5: Frequency of Collection in each Parish

Parish	*Frequency (/Week)*	*Parish*	*Frequency (/Week)*
Anais	2	Gemieira	2
Arca	6	Gondufe	2
Arcos	1	Labruja	1
Arcozelo	6	Labrujó	1
Ardegão	1	Mato	2
Bárrio	2	Moreira do Lima	2
Beiral do Lima	1	Navió	2
Bertiandos	3	Poiares	3
Boalhosa	1	Ponte de Lima	6
Brandara	3	Queijada	2
Cabaços	2	Reb. Sta Maria	4
Cabração	1	Rebordões Souto	2
Calheiros	2	Refóios do Lima	4
Calvelo	2	Rendufe	1
Cepões	3	Ribeira	4
Correlhã	6	Sá	2
Estorãos	2	Sandiães	2
Facha	2	Sta Comba	3
Feitosa	4	Sta Cruz do Lima	2
Fojo Lobal	2	Seara	3
Fontão	3	Serdedelo	1
Fornelos	2	Vilar das Almas	1
Freixo	3	Vilar do Monte	1
Friastelas	2	Vitorino das Donas	3
Gaifar	1	Vitorino de Piães	3
Gandra	2		

2.3 System Indicators

Deposition and collection systems are complex and many factors must be considered in their planning in order to obtain a collection system capable of handling the waste produced in an efficient way while maintaining public health, environment and quality of life patterns.

After the initial planning phase, where variables such as waste types, service area, level of service and SGRSU for waste treatment are identified and addressed, there are some important strategic decisions connected with the logistics of the problem that must be taken, regarding:

- Distribution of containers and their characteristics;
- Frequency of visits per period of time for each container;
- Maximum period of time between successive visits to each container;
- Quantity and characteristics of collection vehicles;
- Distribution of garages for parking the vehicles;
- Number of workers available during each day;

Although it is not the objective of this thesis to address those strategic decisions, but only to guarantee efficient programmes of collection, they affect the performance of the system and it was considered relevant to evaluate some indicators.

2.3.1 Distribution of Containers

When planning the number of containers in a region, at first sight one can say that it is only necessary to take into consideration the population in that region and the waste they produce per day. However, that was only true if the population was located at the exact same place. The distribution of people over the region assume a relevant factor and must be taken into consideration.

In figure 2.7 one can see that in Ponte de Lima municipality 84% of the variation in the number of containers per parish is due to the number of inhabitants. Figure 2.9 results from graph 2.8 by taking out the significantly different parish, and confirms that the population density also affects slightly the distribution of containers.

Notice that the influence of population distribution over the region will increase once the municipality places new containers due to the new legislation. This fact will result in a graph with an increased slope of the linear regression line.

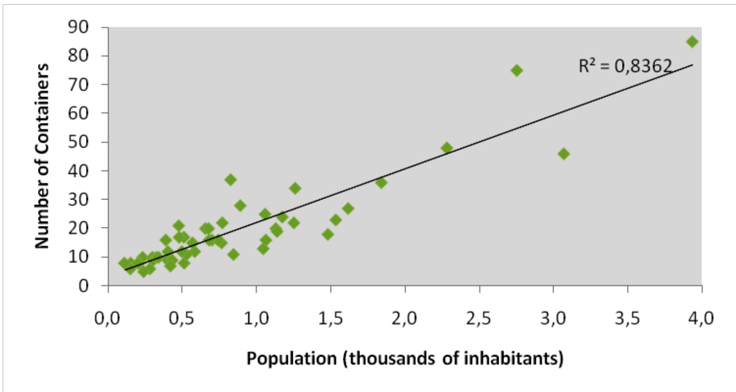

Figure 2.7: Number of containers versus population

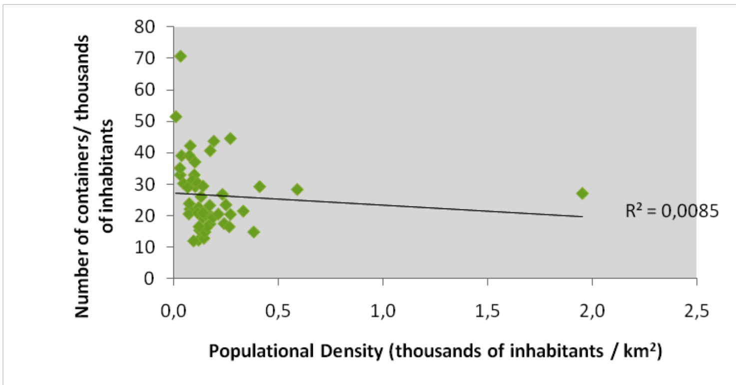

Figure 2.8: Number of containers/thousands inhabitants versus population density

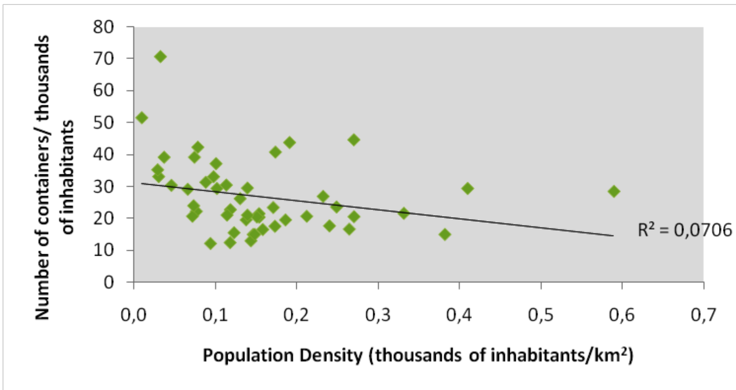

Figure 2.9: Number of containers/thousands inhabitants versus population density without the significantly different parish

2.3.2 Frequency of Visits to Containers

Storage limitation as well as public health, environment and quality of life issues are factors to consider when establishing the minimum collection frequency for each parish. However, the greater the frequency of collection in a community, the more costly will be the collection system and therefore there must be a tight correlation between the number of containers (storage volume) and the frequency of collection.

By fixing the number of containers in each parish, it is expected that the greater the number of containers per thousands inhabitants, the lower the frequency. However, in the graph of figure 2.10 one can see that this correlation does not exist. Interestingly, in the graphs of figures 2.11 and 2.12 one can see the dependency of the collection frequency with the population density. In fact, the nearer the parish is from the center of the municipality, the higher the population density and more politically influent citizens exist, which explains the regression. This was already observed by Francis et al in [18], where instances are created based on the traditional city model of figure 2.13. In figure 2.14, the similarities with the Ponte de Lima municipality can be observed.

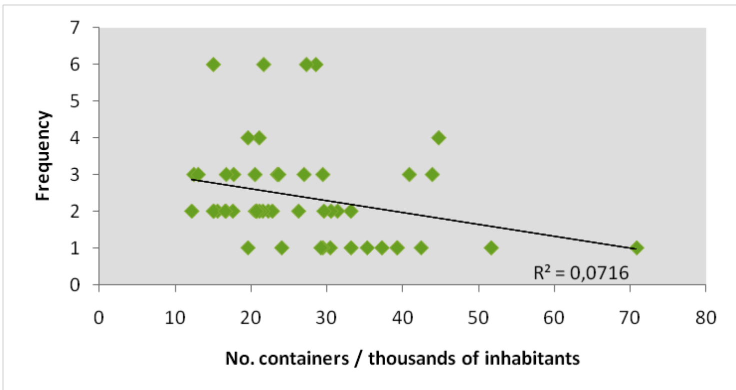

Figure 2.10: Collection frequency versus Number of containers /thousands inhabitants

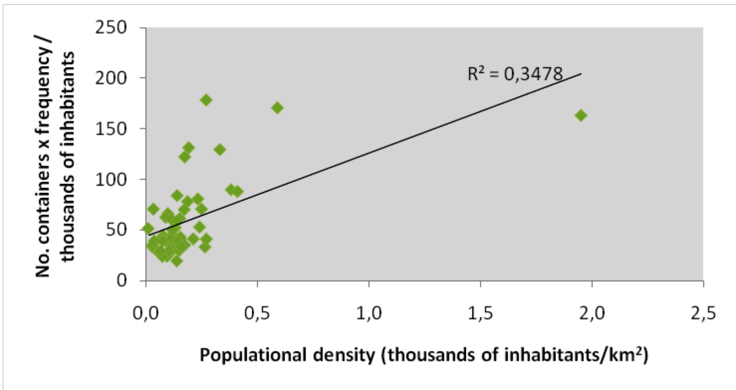

Figure 2.11: Number of containers x Frequency / thousands inhabitants versus population density

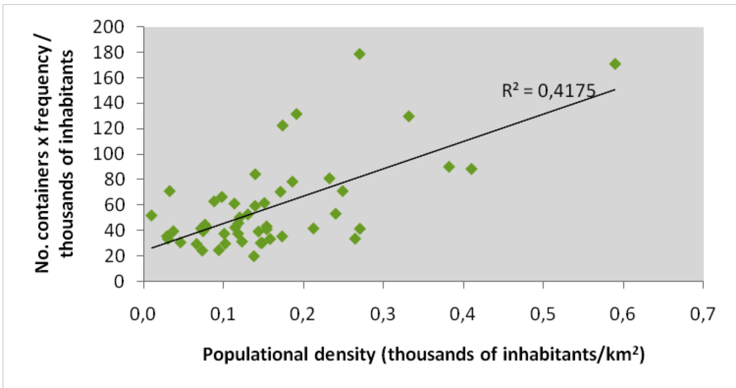

Figure 2.12: Number of containers x Frequency / thousands inhabitants versus population density without the significantly different parish

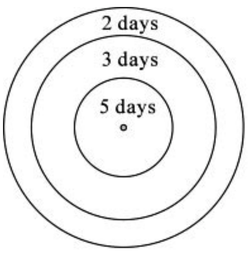

Figure 2.13: Traditional City Model [18]

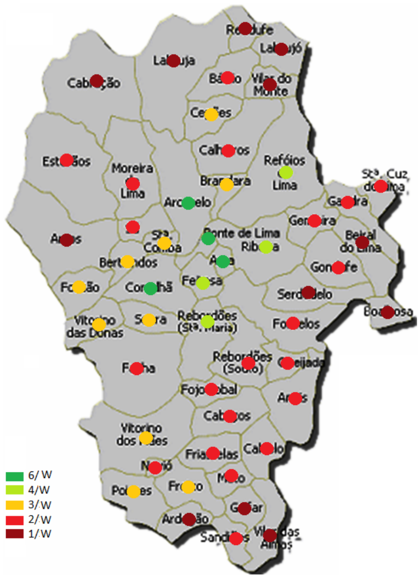

Figure 2.14: Similarities between Ponte de Lima and the Traditional City Model

2.3.3 Waste Production Rates

The rate of Municipal Solid Waste production has been increasing, both in absolute and in per capita values. Increasing population levels, rapid economic growth and higher community living standards may be reasons for that fact. The graph of figure 2.15 shows the increase in per capita values of solid waste both in Ponte de Lima and Portugal. The percentage of waste deposited in selective containers has been increasing as well and consequently, the rate of deposited mixed municipal solid waste deposited has been increasing in a reduced rate (figure 2.16).

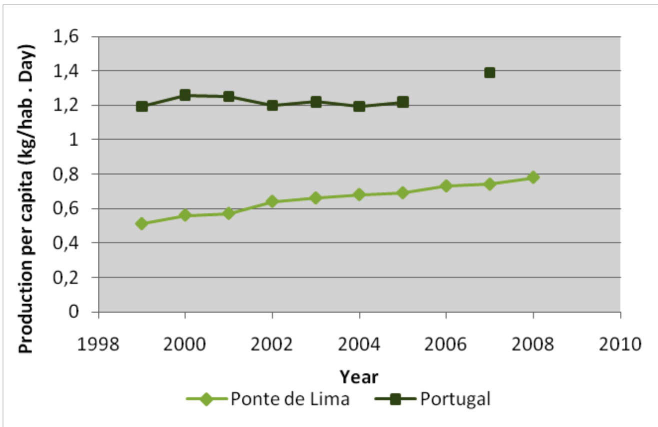

Figure 2.15: MSW production per capita in Ponte de Lima and Portugal

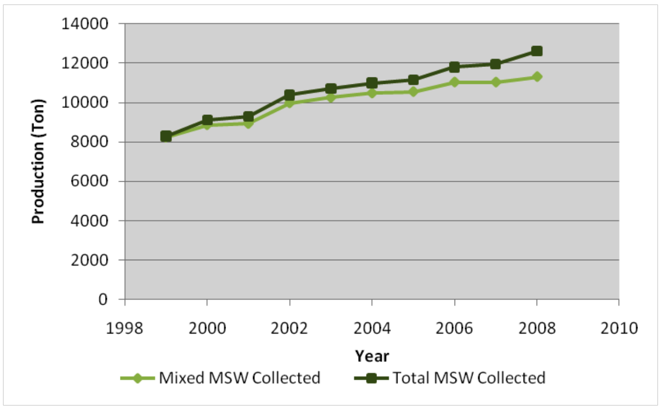

Figure 2.16: Mixed versus total MSW production in Ponte de Lima

2.4 Summary

In this chapter the relevant information of mixed solid waste management is presented and analysed in detail. The objective of the project concerning the case study is the design of a periodic plan of routes for mixed waste collection in Ponte de Lima municipality in order to minimize operational costs by minimizing the number of tours and distance travelled. The plan must take into consideration the frequency of visits, the location of the containers and the treatment facilities, the road network and the resources available. All the information for that purpose was also presented in detail, divided into deposition and collection systems. To conclude, as the plan must be easily adapted, a set of system indicators were presented regarding the number and distribution of containers and its frequency of visits.

Chapter 3

Literature Review on Routing Problems

This chapter aims to present a literature review in transportation problems, and more specifically, in the Vehicle Routing Problem. Mathematical Formulations and solution methods to this problem and to some of its variants are presented and explained. A greater focus is put in formulations based on hierarchical approaches.

3.1 Transportation and Operations Research

As already stated, transportation is a key decision area within logistics, not only because of the percentage of the total costs that it absorbs, but also because of its role in the performance of logistic systems. Therefore, multiple decisions are studied within this area and OR analytical analysis and solution approaches play an important role in this domain. Proof of that is the study realised by Psaraftis, which identified in the literature six families of transportation models: (1) *Vehicle Routing*; (2) *Shortest Path*; (3) *Traffic Assignment*; (4) *Fleet Management*; (5) *Air Traffic Control* and (6) *Facility Location Models* [16].

Routing problems have been receiving extensive treatment in the literature because of its great complexity and consequently, difficulty in generating good or optimal solutions. Additionally, the potential for improvements is high and there are numerous real life applications.

In this type of problems, the road network is generally described through a graph consisting of vertices connected by edges. At this point, it is common to distinguish between (1) *node-covering-problems*, in which customers are at the vertices and the edges correspond to the costs of travelling between the vertices and (2) *edge-covering-problems*, where customers are along the edges which represent road sections [22]. Graphs might be undirected if there is no distinction in the way edges are traversed or directed, if there are differences of costs in both directions of the edges or even the existence of one direction only.

In figure 3.1 one can see the most addressed problems regarding the two classes of routing problems mentioned above.

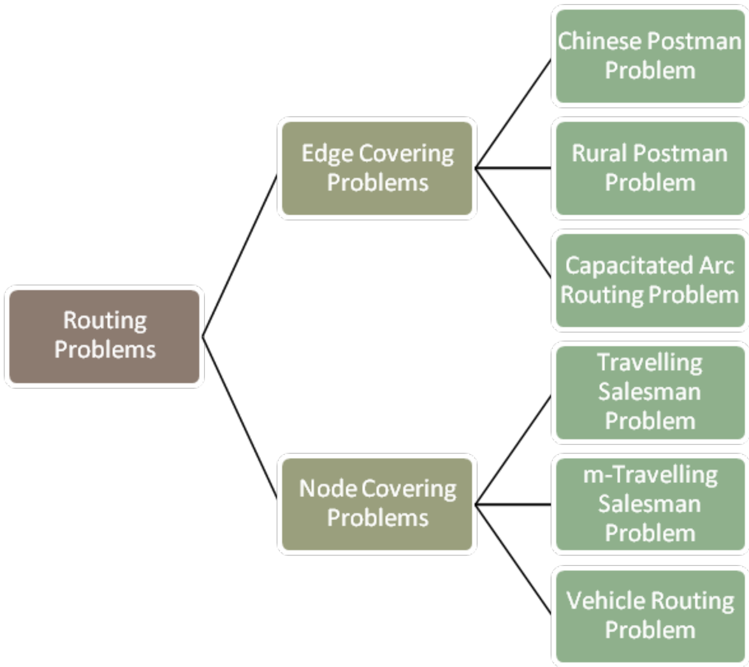

Figure 3.1: Types of Routing Problems

The Chinese Postman Problem was first defined by Guan in 1962 and consists of determining the shortest circuit to traverse every edge of a graph at least once [10]. However, in several arc routing contexts, it is not necessary to traverse all edges of a graph but only a subset of required edges. The Rural Postman Problem was introduced by Orloff in 1974 and consists of determining a minimum cost circuit through a subset of edges that includes all these required edges [10]. When the problem involves the design of multiple tours, the Capacitated Arc Routing Problem is more appropriate. It aims to determine a least cost traversal of all edges of the graph, by using the available vehicles in such a way that each vehicle starts and ends at the same location and the total demand of customers associated with each vehicle route does not exceed its capacity. This problem was introduced by Golden and Wong in 1981 [10].

Concerning node-covering-problems, the most fundamental and best known one is the Travelling Salesman Problem. It consists of determining the minimum distance route that begins at a given node of a network, visits all the members of a specific set of vertices at least once, and returns to the initial vertex. Its mathematical version was introduced in the 1800s by Hamilton and Kirkmanin [27] [22]. The m-travelling salesman problem, on the other hand, involves the design of a pre-specified number of distinct tours that collectively visit all the demanded points at least once [22]. When there are constraints of vehicle capacity or maximum-distance types, there is the Vehicle Routing Problem which will be further explained on section 3.2.

During this project it was decided to model the problem of the case study as a Vehicle Routing Problem. The area in study has rural areas where containers are various streets apart and the magnitude of the distances between them assume more importance than the exact path. By not considering the complete road network, the complexity of the problem decreases significantly.

Edge-covering-problems would be more appropriate in problems where the waste collection is done door-to-door or in a totally urban area, where containers exist in almost every street.

3.2 Vehicle Routing Problem

First proposed by Dantzig and Ramser in 1959, the Vehicle Routing Problem is a combinatorial optimization problem which concerns the optimal design of routes in order that a set of vehicles, initially at a common location known as depot, can visit a set of customers and deliver or collect discrete quantities of goods. An example of the VRP problem is represented in figure 3.2.

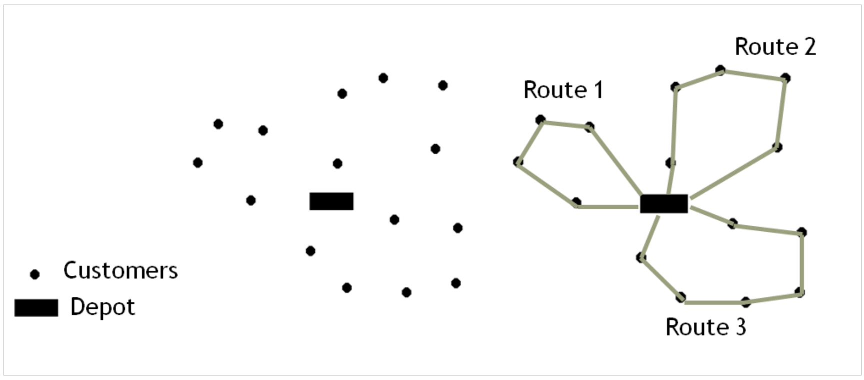

Figure 3.2: VRP exemplification

Formal Description of the VRP Problem Let $G = (V,A)$ be a complete directed graph where $V = \{v_0, v_1, ..., v_n\}$ is the vertex set and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the arc set. Vertex v_0 correspond to the depot where m vehicles with capacities $Q_1, Q_2 ... Q_m$ are available. Vertices $V \backslash \{v_0\}$, correspond to the places to visit having each customer i a demand q_i to be delivered or collected and a service time of d_i. A non-negative cost c_{ij} and a travelling time t_{ij} is associated with each arc $(i, j) \in A$ which represent the cost and time of travelling from vertex i to vertex j. These costs and times are independent of the vehicles and there is no incompatibility between customers and vehicles. If cost values satisfy $c_{ij} = c_{ji}$ for all $i, j \in V$, then the problem is said to be symmetric (SVRP). Otherwise, it is called Asymmetric VRP (AVRP).

VRP calls for the determination of m routes, each performed by a single vehicle. The routes start and end at the depot, the requirements of every customer are fulfilled by exactly one vehicle, all the operational constraints are satisfied and the global transportation costs are minimized. The operational constraints concern the vehicle capacity and maximum duration of the routes.

3.2.1 Mathematical Models

Three different basic modelling approaches have been proposed for the Vehicle Routing Problem: (1) *Vehicle Flow Formulations*; (2) *Commodity Flow Formulations* and (3) *Set Partitioning Problems* [31].

Vehicle Flow Formulations use integer variables associated with each arc of the graph to count the number of times the arc is traversed by a vehicle. They are particularly suited for cases in which the cost of the solution can be expressed as the sum of the costs associated with the arcs and when the most relevant constraints concern the direct transition between the customers. However, it cannot handle some practical issues, for instance when the cost of a solution depends on the overall vertex sequence or on the type of vehicle assigned to a route. This family of formulations is the most frequently used.

In *Commodity Flow Formulations*, additional integer variables are associated with the arcs or edges representing the flow of the commodities along the paths travelled by the vehicles. Models of this type have only recently been used for the formulation of VRP.

Finally, in *Set Partitioning Problems* every feasible circuit has a binary variable associated with it and the objective is the determination of a collection of circuits with minimum costs such that each customer is served exactly once. The main advantage is that it allows for extremely general route costs. Moreover, restrictions concerning the feasibility of a single circuit are not needed.

In this literature review, only *Vehicle Flow Formulations* will be presented. This type of formulations have greater flexibility in incorporating additional features and are therefore, more suitable to model more constrained versions of the VRP, which will be the case in section 3.3. Moreover, the case study problem aims to minimize the total distance travelled and the number of vehicles and both can be addressed with this type of formulations. Only asymmetric models will be considered due to reasons explained later in this section.

The first formulation consists of equations (3.1 - 3.8) and is based on the one originally presented by Golden et al [24]. It uses 3-index-binary decision variables x_{ijk} that equals 1 if vehicle k visits customer j immediately after visiting customer i and 0 otherwise. The notation is summarized in Appendix A.

$$minimize \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{m} c_{ij} x_{ijk} \tag{3.1}$$

Subject to

$$\sum_{i=0}^{n} \sum_{k=1}^{m} x_{ijk} = 1 \qquad j = 1, ..., n \tag{3.2}$$

$$\sum_{i=0}^{n} x_{ihk} - \sum_{j=0}^{n} x_{hjk} = 0 \quad k = 1, ..., m; h = 0, ..., n \tag{3.3}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} q_i x_{ijk} \leq Q_k \qquad k = 1, ..., m \tag{3.4}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} (t_{ij} + d_i) x_{ijk} \leq D_k \qquad k = 1, ..., m \tag{3.5}$$

$$\sum_{j=1}^{n} x_{0jk} \leq 1 \qquad k = 1, ..., m \tag{3.6}$$

$$\sum_{v_i \in S}\sum_{v_j \in S} x_{ijk} \leq |S| - 1 \quad \forall S \subseteq V_c; |S| \geq 2; k = 1, ..., m \tag{3.7}$$

$$x_{ijk} \in \{0, 1\} \qquad i, j = 1, ..., n; k = 1, ..., m \tag{3.8}$$

Equation (3.1) is the objective function which is minimizing the total costs, represented by the sum of all costs of the arcs traversed by vehicles to attend customers. Equations (3.2) to (3.8) are the constraints of the problem and will now be explained. Constraints (3.2) impose that each customer is only visited once while constraints (3.3) ensure that when a vehicle arrives at a customer it also leaves the customer. Limits on vehicle capacity and route duration are imposed through constraints (3.4) and (3.5). Constraints (3.6) specify that each vehicle is used at most once every day. Constraints (3.7) are standard subtour elimination constraints that, together with constraints (3.2), impose the connectivity of the routes. Finally, constraints (3.8) guarantee that the variables are binary.

The number of subtour elimination constraints in (3.7) grows exponentially with n (exponential cardinality) and turns practically impossible to solve directly the linear programming relaxation of the problem. In fact, S represents any subset of V and therefore, in a problem with n customers there are $(2^n - n)$ constraints in (3.7). One possible way to overcome this fact is to gradually add to the formulation the inequalities whenever they are violated. Alternatively, the family of constraints (3.9) - (3.10) are equivalent to (3.7) and have polynomial cardinality.

$$u_{ik} - u_{jk} + Q_k x_{ijk} \leq Q_k - q_j \quad \forall i, j \in Vc; i \neq j; q_i + q_j <= Q_k; k = 1, ..., m \tag{3.9}$$

$$q_i \leq u_{ik} \leq Q_k \qquad \forall i \in Vc; k = 1, ...m \tag{3.10}$$

In this family of constraints, $u_{ik} \in V_c$ is an additional variable representing the load of vehicle k after visiting customer i. They were proposed for the TSP by Miller, Tucker and Zemlin and were later extended to the AVRP. They impose both capacity and connectivity requirements and consequently, when using them, equations (3.4) turn redundant. The linear programming relaxation of the alternative formulation (3.1 - 3.3), (3.5-3.6), (3.9-3.10) and (3.8) is much weaker than formulation (3.1 - 3.8). Only asymmetric formulations of VRP were considered in order to use this family of subtour elimination constraints.

Fisher and Jaikumar formulated the VRP as a linear assignment problem instead. The solution of this formulation is a feasible assignment, in terms of capacity, of customers to vehicles [30]. The decision variables are x_{ik} and equal one if customer i is visited by vehicle k and 0 otherwise.

$$minimize \sum_{i=1}^{n} \sum_{k=1}^{m} c_{ik} x_{ik} \tag{3.11}$$

Subject to

$$\sum_{k=1}^{m} x_{ik} = 1 \qquad i = 1...n \tag{3.12}$$

$$\sum_{i=1}^{n} q_i x_{ik} \leq Q_k \qquad k = 1...m \tag{3.13}$$

$$x_{ik} \in \{0,1\} \quad i = 1,...,n; k = 1,...,m \tag{3.14}$$

Equation (3.11) represents an estimative of the total distance travelled. For that, c_{ik} was defined as a measure of the distance contribution of customer i to the route followed by vehicle k if customer i were to be delivered by that vehicle. Constraints (3.12) ensure that each customer is assigned to a vehicle whereas constraints (3.13) ensure that the vehicle capacity is not exceeded. Finally, constraints (3.14) guarantee the integrality of the variables.

To estimate the contribution of customers to routes, c_{ik}, the authors generated m "seed points" (one for each vehicle) and let c_{ik} equal the extra distance travelled when customer i is inserted into the route in which vehicle k travels out from the depot to its seed point and back again [5].

After obtaining the solution, the delivery sequence for each vehicle can be determined by applying an heuristic or by optimally solving the MIP formulation of TSP for each vehicle. Notice that as this is an hierarchical approach, it may not obtain an optimal solution for the aggregate problem. It only guarantees the optimal solution of each part of the approach.

3.2.2 Solution Methods

The most intuitive way of solving combinatorial problems is to enumerate and evaluate all possible solutions and choose the one with the lowest cost or with the highest profit. However, for only a few customers, the number of possible combinations becomes excessively high which turns this method impractical. The VRP has been extensively studied since the early sixties and many approximate and exact approaches were already presented. Exact methods obtain and guarantee an optimal solution but sometimes in excessively high execution times whereas approximate methods (known as heuristics) do not guarantee an optimal solution but obtain a possibly good solution within a limited amount of time. Hence, when choosing the best approach to solve a VRP problem, one must evaluate the trade off between efficiency and effectiveness of the method in obtaining solutions for the problem.

Exact methods are based on different formulations for the problem and in exact algorithms, such as linear programming and branch and bound. However, some programs cannot be solved directly, even for moderate size VRP's, and lower bounds are usually computed by using cutting planes and column generation techniques. To strengthen the relaxations, a variety of valid inequalities especially developed for the VRP are also presented in the literature [6] [7]. The largest VRP which can be consistently solved by the most effective exact algorithms proposed so far contains about 50 customers [31], whereas larger instances may be solved only in particular cases.

Concerning approximate methods, several families of heuristics have been proposed for the VRP which can be broadly classified into two main classes: *classical heuristics*, developed mostly between 1960 and 1990, and *metaheuristics* whose growth has occurred in the last decade.

Classical VRP heuristics can be broadly classified into two categories: (1) *constructive heuristics* and (2) *improvement heuristics*. *Constructive heuristics* gradually build a feasible solution while keeping an eye on solution cost whereas *improvement heuristics* attempt to upgrade any feasible solution by performing a sequence of edge or vertex exchanges within or between vehicle routes. Most standard procedures include a construction and improvement heuristics which perform a relatively limited exploration of the search space, stopping the search whenever a better solution is not found. Consequently, they sometimes become trapped in local minimums. However, these procedures typically produce good solutions within modest computing times. Moreover, they are easy to implement and most of them can be easily extended to account for the diversity of constraints encountered in real life contexts.

Metaheuristics, on the other hand, perform a more thorough search of the solution space and are less likely to end in local minimums. These methods typically combine sophisticated neighbourhood search rules, memory structures and recombinations of solutions. The quality of solutions produced by these methods is much higher than that obtained by classical heuristics but the price to pay is once more increased computing times. Moreover, the procedures are usually context dependent and require finely tuned parameters which turn their extension to other situations difficult. Interestingly, the most well known metaheuristics are based in natural phenomenons such as genetic algorithms, which are based in the process of natural evolution or ant colony systems, which are based on the behaviour of ants seeking a path between their colony and a source of food.

Table 3.1 presents the most successful and well-known heuristics to solve the VRP. [25]

Table 3.1: Well known examples of classical heuristics and metaheuristics

Classical Heuristics	*Metaheuristics*
Savings: Clarke and Wright (1964)	Taburouse:Gendreau,Hertez and Laporte (1994)
Sweep: Gillet and Miller (1974)	FIND: Renaud, Boctor and Laporte (1996)
GAP: Fisher and Jaikumar (1981)	Granular Tabu Search: Toth and Vigo (1998)

By far, the best known approach to solve the VRP is the *savings heuristic* from Clark and Wright, also known by the authors' names. It has stood out over the years as being flexible enough to handle a wide range of practical constraints, with relatively fast computational times and capable of generating solutions that are near the optimum [20]. The heuristic will now be presented, by following the approach of [22].

Savings Heuristic Clarke and Wright heuristic is based on the concept of *saving* – an estimate of the cost reduction obtained by serving two customers sequentially in the same route, rather than in two separate ones. If customers i and j are served individually in two routes, the associated *saving* is defined as presented in equation (3.15) and illustrated in figure 3.3. If s_{ij} is positive, then serving i and j consecutively in a route is profitable. The larger s_{ij} is, the more desirable is to combine i and j in the same tour. However, i and j cannot be combined if, by doing so, the resulting tour violates one or more constraints of the VRP.

$$s_{ij} = c_{i0} + c_{0j} - c_{ij} \tag{3.15}$$

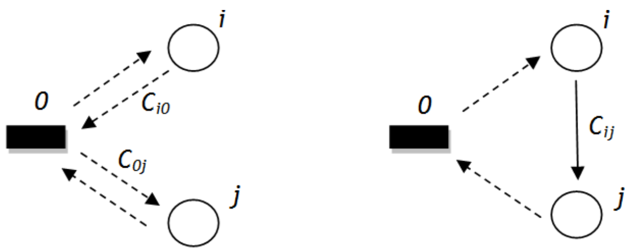

Figure 3.3: Concept of Saving in Clark and Wright heuristic

The algorithm considers all customer pairs and creates a *savings list* with the savings values of all pairs of clients i and j ranked in descending order of magnitude. It then starts processing the *savings list* with the topmost entry and decide for:

- New route creation: if neither i or j have been assigned to a route, a new route is created including both i and j.
- Route Extension: If one of the customers has already been included in one route in such a way that is possible to add the arc (i, j), the route is extended to one more customer.
- Merging of routes: if both customers i and j have already been included in two different routes, in such a way that it is possible to merge them by the arc (i, j), the two routes are merged.
- Doing nothing: if none of the preceding forms of insertion are valid or if constraints are violated, the saving is discarded.

If at the end of processing the *savings list* customers exist that do not belong to a route, a new route is created to serve each customer.

Generally, only non-negative *savings* are considered but if the number of vehicles is to be minimized, then negative saving values may also be taken into account. The algorithm above described regards the parallel version because more than one route may be active at the same time. However, it may be easily implemented sequentially.

The Clark and Wright heuristic can be programmed to run very efficiently and, since it involves very simple manipulation of the data set, it can be used with large problems. An additional advantage of this heuristic is that it can handle a wide range of practical constraints by just running more check routines before inserting a customer into a route.

Due to its greedy nature, one drawback of the original Clark and Wright algorithm is that it tends to produce good routes at the beginning, but less interesting routes towards the end. In order to control the greediness of the algorithm, generalized savings of the form $s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij}$ were proposed. The larger the λ, the more emphasis is put on the distance between the two vertices to be connected. Reports stated that values 0.4 and 1.0 yield good solutions.

A number of authors have also tried to optimize the merger of routes through the use of a matching algorithm. Although it yields improvements to the standard algorithm, this happens at the expense of much higher computational times [31].

3.3 Variants of the Vehicle Routing Problem

The classical Vehicle Routing Problem only accounts with the capacity of vehicles and maximum duration of routes as operational constraints. However, organisations have other specific needs. In order to be more closer to practical distribution problems, the VRP has a family of extensions in which specific additional constraints are associated with the problem. In particular, these variants are characterized by the existence of time intervals to visit each customer, multiple depots, multiple trips to be performed by the vehicles, just to name some.

In many real life applications, the vehicle routing problems are inherently periodic, in the sense that the customers are not served on a daily basis, but are characterized in terms of some sort of periodicity of the demand. This periodicity imposes a strong relationship between decisions that have to be taken during different days and therefore it is not possible to solve the problem on a single day basis and then to replicate the solution over time. Examples of applications with periodic deliveries are: courier services, elevator maintenance and repair, vending machines replenishment, delivery of interlibrary loan material and collection of waste.

The Periodic Vehicle Routing Problem is the variant of the VRP that generalizes the classical vehicle routing problem by extending the planning period from a single day to multiple days. This particular variant will be reviewed more thoroughly in the next section.

3.3.1 Periodic Vehicle Routing Problem

The Periodic Vehicle Routing Problem was introduced in 1974 by Beltrami and Bodin as a generalisation of the classical VRP [19]. It combines the problematic of the standard Vehicle Routing Problem with that of planning customer visits over a given time period (Figure 3.4).

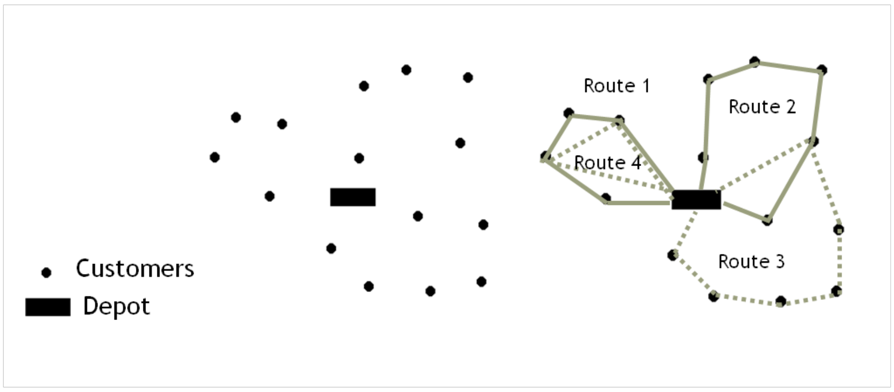

Figure 3.4: PVRP exemplification

PVRP works with a planning horizon of t days. Each customer, besides his demand q_i and service duration d_i, must additionally specify service frequency e_i and a set C_i of allowable combinations of visits days, known as *schedules*. In this way, the assignment problem is reduced to select a visit combination for each customer. For example, if $e_i = 2$ and $C_i = \{\{1,3\},\{2,4\},\{3,5\}\}$, then customer i must be visited twice and these visits should take place on days 1 and 3, 2 and 4 or on days 3 and 5. To specify which days belong to each schedule there are the constants a_{rl} that are equal to one if day l belongs to the visit combination r.

The overall objective of the PVRP is to find a set of tours for each vehicle that minimizes total travel cost while satisfying operational constraints. The operational constraints concern vehicle capacity, duration of routes and service requirements.

3.3.1.1 Mathematical Models

From the above definition, it can be seen that PVRP involves the following three simultaneous decisions:

1. Selecting a schedule from a candidate set of schedules for each customer, resulting in an assignment of customers to the days of the period;

2. Assign a set of customers to be visited by each vehicle on each day;

3. Route the vehicles for each day of the planning period.

Russel and Igo called it an “Assignment Routing Problem” and mentioned the difficulties of choosing a schedule for each node together with solving the routing problem. They also stated that the problem is in picking a valid day combination for a specific service frequency.

In literature, different approaches to this problem originated different formulations, based on the number of decisions tackled simultaneously. The following relevant ones for this project are presented:

- Cordeau, Gendreau and Laporte, based on the one from Christofides and Beasley, formulate the PVRP as a routing problem with a selection decision involved. The three decisions are tackled at the same time, resulting in one mathematical model [8].
- Tan and Beasley, based on the VRP formulation from Fisher and Jaikumar, presented a hierarchical formulation in which at a first stage customers are assigned to schedules - (1) and afterwards a VRP is solved for each day - (2) and (3) [30].
- Mourgaya and Vanderbeck also opted by a hierarchical approach. Decisions (1) and (2) are tackled at a first stage, which they called the "tactical panning" stage, while decision (3) is addressed in a second stage which is reduced to a TSP for each day and vehicle [26].
- Tan and Beasley also presented a formulation for this last approach by considering only the assignment of days in their previous formulations. [30].

The following pages will go through each one of the formulations, presenting the model and, if necessary, details of the constraints. The notation is summarized in Appendix A.

Assign and route together (Cordeau, Gendreau and Laporte) This formulation uses 4-index-binary variables x_{ijkl} that equals 1 if and only if vehicle k visits customer j immediately after visiting customer i during day l ($i \neq j$) as well as the 2-index-binary variables y_{ir} which equals to 1 if and only if schedule $r \in C_i$ is assigned to customer i.

$$minimize \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m}\sum_{l=1}^{t} c_{ij}x_{ijkl} \tag{3.16}$$

Subject to

$$\sum_{r\in C_i} y_{ir} = 1 \qquad i = 1,...,n \tag{3.17}$$

$$\sum_{j=0}^{n}\sum_{k=1}^{m} x_{ijkl} - \sum_{r\in C_i} a_{rl}y_{ir} = 0 \qquad i = 1,...,n; l = 1,...,t \tag{3.18}$$

$$\sum_{i=0}^{n} x_{ihkl} - \sum_{j=0}^{n} x_{hjkl} = 0 \qquad k = 1,...,m; h = 0,...,n; l = 1,...,t \tag{3.19}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} q_i x_{ijkl} \leq Q_k \qquad k = 1,...,m; l = 1,...,t \tag{3.20}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} (t_{ij}+d_i)x_{ijkl} \leq D_k \qquad k = 1,...,m; l = 1,...,t \tag{3.21}$$

$$\sum_{j=1}^{n} x_{0jkl} \leq 1 \qquad k = 1,...,m; l = 1,...,t \tag{3.22}$$

$$\sum_{v_i\in S}\sum_{v_j\in S} x_{ijkl} \leq |S|-1 \qquad \forall S \subseteq V_c; |S| \geq 2; k = 1,...,m; l = 1,...,t \tag{3.23}$$

$$x_{ijkl} \in \{0,1\} \qquad i,j = 1,...,n; k = 1,...,m; l = 1,...,t \tag{3.24}$$

$$y_{ir} \in \{0,1\} \qquad i = 1,...,n; r \in C_i \tag{3.25}$$

Constraints (3.17) mean that one feasible visit combination must be assigned to each customer while constraints (3.18) guarantee that each customer is visited only on the days corresponding to the assigned combination. The remainder of the constraints are similar to the ones in the corresponding VRP formulation.

An alternative family of constraints for subtour elimination equivalent to (3.23) may also be obtained by considering the following equations:

$$u_{ikl} - u_{jkl} + Q_k x_{ijkl} \leq Q_k - q_j \quad i,j = 1,...,n; i \neq j; q_i + q_j \leq Q_k; k = 1,...,m; l = 1,...,t \quad (3.26)$$

$$q_i \leq u_{ikl} \leq Q_k \qquad i = 1,...,n; k = 1,...m; l = 1,...,t \quad (3.27)$$

Assign days first - Assign vehicles and route second (Tan and Beasley) Regarding the first stage of the formulation - assignment of days, only the 2-index-binary variables y_{ir} above described are used.

$$minimize \sum_{i=1}^{n} \sum_{l=1}^{t} c_{il} y_{ir} \quad (3.28)$$

Subject to

$$\sum_{r \in C_i} y_{ir} = 1 \qquad i = 1,...,n \quad (3.29)$$

$$\sum_{i=1}^{n} \sum_{r \in C_{ik}} q_i a_{rl} y_{ir} \leq \sum_{k=1}^{m} Q_k \qquad l = 1,...,t \quad (3.30)$$

$$y_{ir} \in \{0,1\} \quad i,j = 1,...,n; r \in C_i \quad (3.31)$$

The only constraint worth the explanation is (3.30), which ensures that the total delivered on any day does not exceed the total vehicle capacity.

Parameters c_{il} are supposed to be derived in some way in order to measure the distance contribution of customer i to any route involving customer i on day l.

It is worth to notice that this formulation does not guarantee that the maximum duration is not exceeded, since it is dependent of the sequence of customers on routes. Furthermore, it might occur that the only solutions to the next level split the demand of a customer into more than one vehicle.

The second level of the formulation concerns the VRP, which has already been modelled in section 3.2.1.

Assign first - route second (Mourgaya and Vanderbeck) The first hierarchical level, known as tactical planning phase, is formulated in terms of decision variables x_{ikl}, which equal 1 if customer i is visited in period l by vehicle k, and also in terms of variable y_{ir}, already described in the previous formulation. An auxiliary decision variable z_{ijl} is also used, which equals one if customers i and j are in the same cluster in period l. The formulations takes the form:

$$minimize \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{t} c_{ij} z_{ijl} \tag{3.32}$$

Subject to

$$\sum_{r \in C_i} y_{ir} \geq 1 \qquad i = 1,...,n \tag{3.33}$$

$$\sum_{k=1}^{m} x_{ikl} - \sum_{r \in C_i} a_{rl} y_{ir} \geq 0 \qquad i = 1,...,n; l = 1,...,t \tag{3.34}$$

$$x_{ikl} + x_{jkl} - z_{ijl} \leq 1 \quad i,j = 1,...,n; k = 1,...,m; l = 1,...,t \tag{3.35}$$

$$\sum_{i=1}^{n} q_i x_{ikl} \leq Q_k \qquad k = 1,...,m; l = 1,...,t \tag{3.36}$$

$$x_{ikl} \in \{0,1\} \quad i,j = 1,...,n; k = 1,...,m; l = 1,...,t \tag{3.37}$$

$$y_{ir} \in \{0,1\} \qquad i = 1,...,n; r \in C_i \tag{3.38}$$

$$z_{ijl} \in \{0,1\} \qquad 1, j = 1,...,n; l = 1,...,t \tag{3.39}$$

Constraints (3.35) are the only ones that are new and they arise by the linearisation of the objective function which otherwise would be represented by $\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{m} \sum_{l=1}^{t} c_{ij} x_{ikl} x_{jkl}$. In this way, the need for an auxiliary decision variable becomes clear as well. The only drawback of the formulation is its weak Linear Programming (LP) relaxation.

To restrict the search space, the authors additionally suggest that the adjacency list of each customer only includes its close neighbours. They limited the neighbourhood to those customers for which $c_{ij} \leq \frac{max_{ij} c_{ij}}{2}$.

The second level of the formulation is a TSP, which is explained and formulated individually in section 3.4.

Assign first - route second (Tan and Beasley) This formulation uses the same decision variables as the other formulation of the same authors above presented. The only difference concerning the parameters are the constants c_{ikl}, which are a measure of the distance contribution of customer i to the route followed by vehicle k on day l.

$$minimize \sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{l=1}^{t} c_{ikl} x_{ikl} \tag{3.40}$$

Subject to

$$\sum_{r \in C_i} y_{ir} = 1 \qquad i = 1,...,n \tag{3.41}$$

$$\sum_{k=1}^{m} x_{ikl} - \sum_{r \in C_i} a_{rl} y_{ir} = 0 \quad i = 1,...,n; l = 1,...,t \tag{3.42}$$

$$\sum_{i=1}^{n} q_i x_{ikl} \leq Q_k \quad k = 1,...,m; l = 1,...,t \tag{3.43}$$

$$x_{ikl} \in \{0,1\} \quad i,j=1,...,n;k=1,...,m;l=1,...,t \tag{3.44}$$

$$y_{ir} \in \{0,1\} \qquad i,j=1,...,n;r \in C_i \tag{3.45}$$

It is important to notice that hierarchical approach models may not achieve optimal solutions or even feasible ones. For instance, in the last approach, when solving the TSPs, there might be no feasible sequencing of customers that allow to meet operational constraints such as bounded travel times. However, there are some strong motivations for using a hierarchical approach to the PVRP such as (1) to account for other optimization criteria such as optimizing workload balancing and regionalisation or (2) to deal with larger problems where computational times increase considerably.

Workload balancing between vehicles can be of great importance not only to ensure a form of fairness between the different drivers but to achieve "robust solutions", where routes are not filled-up and can accommodate extra demands. Moreover, the workload can be a surrogate measure of some operational constraints such as bounded travel time. Regarding regionalisation, it is important to assign drivers to routes concentrated in an area so that they became familiar with that region. In fact, drivers tend to dislike routes that have been optimized for length and spread over quite different areas.

3.3.1.2 Solution Methods

For a given number of customers, PVRP instances are typically harder to solve than VRP instances: not only one is confronted with the extra complexity of having to schedule the visits, but the number of vertices to include in routes is the number of customers multiplied by the number of visits to those customers. Therefore, heuristic methods are often used to solve this problem and, more recently, metaheuristics applied to the PVRP have been emerging.

With some exceptions, all PVRP approaches were also considered as a multilevel combinatorial optimization problem where in the first level an allowable day is assigned to each customer and, in the second level, the VRP problem for each day is solved. Interestingly, Beltrami and Bodin developed an approach in which the routes are firstly developed and only after assigned to days of the period.

In [24], a description of the history of solution approaches to the PVRP can be found.

3.4 Travelling Salesman Problem

The Travelling Salesman Problem is the most fundamental and best known problem of all the node-covering-problems and one of the most intensively studied ones in computational mathematics and operations research.

Given a collection of customers and the cost of travelling between each pair of them, the TSP consists of determining the minimum cost route that visits all of the customers and returns to the starting point. In the standard version, the travel costs are symmetric in the sense that travelling from customer i to customer j costs just as much as travelling from j to i. However, one will consider the asymmetric version, ATSP, to follow the same line as the rest of the chapter and due to the same reasons.

The ATSP is defined as a directed graph $G = (V,A)$, where $V = \{1,...,n\}$ is the vertex set corresponding to the customers, $A = \{(i, j) : i, j \in V\}$ is the arc set, and a non-symmetric cost matrix (c_{ij}) is defined on A. Many of ATSP formulations consist of an assignment problem with integrality and subtour elimination constraints. They use binary x_{ij} variables equal to 1 if and only if arc (i, j) belongs to the solution. The basic model is as follows [2]:

$$minimize \sum_{i=0}^{n}\sum_{j=0}^{n} c_{ij}x_{ij} \tag{3.46}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1,...,n \tag{3.47}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1,...,n \tag{3.48}$$

$$x_{ij} \in \{0,1\} \qquad i, j = 1,...,n \tag{3.49}$$

$$\sum_{v_i \in S}\sum_{v_j \in S} x_{ij} \leq |S| - 1 \quad \forall S \subseteq 2,...,n; 2 \leq |S| \geq n-1 \tag{3.50}$$

Constraints (3.47) and (3.48) ensure that each vertex is incident to one outgoing arc and one incoming arc. Constraint (3.50) is the subtour elimination constraint.

Similarly to the VRP, a family of constraints equivalent to (3.50) and having polynomial cardinality may be obtained by considering the following constraints:

$$u_i - u_j + (n-1)x_{ij} \leq n-2 \quad i, j = 2,...,n \tag{3.51}$$

$$1 \leq u_i \leq n-1 \qquad i = 2,...,n \tag{3.52}$$

where $u_i \in V_c$ define the order in which each customer i is visited on a tour. These subtour elimination constraints are known as MTZ - Miller, Tucker and Zemlin and were the origin of the ones for the VRP.

The simplicity of the statement of the problem is deceptive - the TSP is one of the most intensely studied problems and yet no effective solution method is known for the general case and its complexity is still unknown.

3.5 Summary

By reading this chapter, the reader obtained the background knowledge to understand the remainder of this thesis, concerning the VRP and PVRP descriptions, formulations and solution algorithms.

A greater focus was put on mathematical formulations, more specifically, formulations based on hierarchical approaches. Although they may not obtain optimal solutions or even feasible ones when solved, there are some strong motivations for using them, such as to account for other optimization criteria or to deal with large problems, where computational times increase considerably.

This chapter also presented alternative ways to model the computationally hard constraints for subtour elimination, in order to decrease the complexity of the models. However, one may have in mind that the linear programming relaxation of the models becomes much weaker.

Solution methods were reviewed and the savings heuristic, from Clark and Wright was explained. This heuristic is by far the best known approach because of its simplicity, flexibility to handle a wide range of practical constraints, relatively low computational times and capability of generating good solutions. In this thesis, solutions were obtained using an optimization software and only this specific heuristic was implemented.

Chapter 4

Hierarchical Formulations and Solution Methods Applied to the Case Study

This chapter aims to model and present different solution methods to solve the waste collection problem of Ponte de Lima Municipality. In the same line of the literature review, it exposes multiple hierarchical formulations adapted to the parameters and objectives of the case study. The solution methods considered try to explore the potential of these formulations in order to conclude the best compromise for the waste collection problem. Computational experiments realized on instances from the literature are described.

4.1 Problem Description

The problem concerns the Mixed Municipal Solid Waste Collection in Ponte de Lima Municipality. All strategic decisions are already taken and known in advance such as number and location of containers, frequency of visits, staff number and working hours and location of the treatment facilities.

The problem is therefore reduced to a tactical level and three simultaneous decisions are involved: (1) Select a periodic visit plan for each container; (2) Assign a group of containers to be visited by each vehicle on each day; (3) Design the routes to be travelled by each vehicle on each day of the planning period.

The objective of the problem is to minimize the operational cost of the collection system taking into consideration the following characteristics:

- Routes must start and end in the garage where the trucks are parked;
- The last stop before the garage must be one of the available vehicle discharging stations (transfer stations or sanitary landfills);

- Containers must be totally emptied with the pre-determined frequency and the distance between consecutive visits must be balanced in each schedule;
- The quantity of waste collected by each vehicle on each tour must not exceed its capacity;
- The number of routes in each day must not exceed the available resources (number of vehicles, drivers and waste collectors).
- Each tour must take less than a determined number of hours, in order to respect the employees' working hours.

Operational costs are related to the distance travelled and to the number of vehicles that leave the garage in each day.

4.2 Case Study Parameters

When choosing which information should be taken into consideration in a model, one is choosing the level of complexity and detail of it. It is a compromise between resemblance to reality and effort to achieve a solution.

The parameters that were taken into consideration in the formulations are now going to be presented. Notation and units are summarised in Appendix B.

Planning Horizon In Ponte de Lima, containers are periodically emptied, but not on a daily basis. The exact visiting days are not fixed but the frequency of visits per period. At this moment, the lowest frequency for a container is one visit in a week, which suggests a collection plan of one week. However, if the lowest frequency turns less than once in a week, this period is not possible any more. Parameter t defines the planning horizon.

Road Network In order to simplify and reduce the dimension of the problem, containers are grouped by parishes. This is possible because the service frequency is the same for all containers in the same parish.

This grouping strategy was considered the best approach because the distances between parishes are superior to the distances between containers inside each parish, which suggests that those containers must all be visited on the same day and by the same vehicle. Moreover, plans are currently made in terms of parishes.

One drawback is that in this way it is not possible to define the order in which containers are visited inside a parish as well as the roads that trucks must follow, once the road network is not totally represented in the graph. However, this information was not considered to be relevant because it can be easily obtained by applying a simpler problem for each day, truck and location combination (For example, CPS - Chinese Postman Problem). Furthermore, even if a problem is not solved downstream, the variation in the distance is not significant when considering the driver's knowledge of the road network and their capability to decide.

The problem is going to be defined in a complete directed graph $G = (V,A)$, where $V = \{v_0, v_1, ..., v_{n+p}\}$ is the vertex set and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the arc set. Vertex v_0 correspond to the garage whereas vertices $\{v_{n+1}, ..., v_{n+p}\}$ correspond to the p available vehicle discharging stations. Finally, vertices $\{v_1, ..., v_n\}$ correspond to the parishes.

Each arc (v_i, v_j) which connects locations i and j has a distance d_{ij} associated with it. The distances are not euclidean but are obtained by taking into consideration the Ponte de Lima road network. It was considered that $d_{ij} = d_{ji}$ because at this level of abstraction it is not relevant to take into consideration possible variations due to one way streets.

Customer Demands Each parish is characterized by the total waste to be collected in each visit, q_i and the service time st_i.

Being the daily waste produced per capita in Ponte de Lima equal to w_capita and the number of inhabitants of parish i equal to $n\_inhabitants_i$, a measure of the total waste deposited at the time of collection in parish i can be computed as shown in equation 4.1. If the space between successive visits is balanced, the total waste to be collected becomes closer to the reality.

$$q_i = \frac{w\_capita \times t \times n\_inhabitants_i}{e_i} \tag{4.1}$$

Concerning service duration, it can be measured as the total time spent driving between containers plus the total time spent unloading those containers. For that purpose, three additional parameters were defined: number of containers in parish i, nc_i, medium driving time between containers of parish i, $t\_bcontainers_i$, and medium time of unloading a container, t_unload. The service duration of parish i is then computed as shown in equation 4.2.

$$st_i = (nc_i - 1) \times t\_bcontainers_i + nc_i \times t\_unload \tag{4.2}$$

Available Resources A set K of m vehicles with capacities $Q_1, Q_2, ..., Q_m$ is considered to exist. However, in each day of the planning period, a vehicle is only operational if the crew to operate it is available. Hence, sets $K_l \in K$ were created to indicate which vehicles are available in each day l. Vehicles must be considered in K_l as many times as the number of times they leave the depot during day l.

A maximum time duration of T is defined for the routes in order to respect employees' working hours. To check the duration of a route it is also necessary to know the driving time between parishes, which can be computed as presented in equation 4.3. Parameter vel was defined, representing the medium speed of the vehicles between parishes.

$$t_{ij} = \frac{d_{ij} \times 60}{vel} \tag{4.3}$$

Schedules All possible schedules shall be included in set C. The space between consecutive visits must be as much balanced as possible.

Each parish must specify its service frequency, e_i, and a subset C_i of allowable schedules of C.

4.3 Mathematical Formulations

The case study problem is going to be modelled as a Periodic Vehicle Routing Problem. By considering the parameters presented above, it is possible to conclude that the PVRP formulations described in the literature cannot be applied directly. Two major differences are observed: (1) all vehicles must be emptied in one discharging station before returning to the depot and (2) the number of available vehicles vary with the day of the planning period.

As already covered in the literature review, the existence of different approaches to this problem originated different formulations, based on the number of decisions tackled simultaneously.

In this section, a variety of hierarchical formulations will be presented adapting the features of the case study. All approaches are based on the models described in the literature review. Therefore, only the differences will be stated, and the reader is invited to read chapter 3 as well as the notation for this section, in appendix B. The formulations are the following:

- *Assign and Route together approach*: One formulation is presented which was adapted from the PVRP model from Cordeau, Gendreau and Laporte.
- *Assign days first - assign vehicles and route second approach*: Two formulations were considered. The first one does not have a direct correspondence to one of the literature models, but was based on the assign first-route second formulation from Mourgaya and Vanderbeck. The second formulation, on the other hand, was adapted from Tan and Beasley's.
- *Assign first - route second approach*: Two formulations are proposed based on the two existing ones in the literature review (the first from Mourgaya and Vanderbeck and the second from Tan and Beasley).

4.3.1 Assign and Route Together Approach

The resulting formulation for the PVRP has also the 4-index decision variables x_{ijkl} and the 2-index decision variables y_{ir}. As additional variables it also includes u_{ikl}, which was also introduced in the literature for enabling an alternative way for subtour elimination.

$$minimize \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{l=1}^{t}\sum_{k\in K_l} d_{ij}x_{ijkl} \tag{4.4}$$

Subject to

$$\sum_{r\in C_i} y_{ir} = 1 \quad i = 1,...,n \tag{4.5}$$

$$\sum_{j=0}^{n+p}\sum_{k\in K_l} x_{ijkl} - \sum_{r\in C_i} a_{rl}y_{ir} = 0 \qquad i=1,...,n; l=1,...,t \tag{4.6}$$

$$\sum_{i=0}^{n+p} x_{ipkl} - \sum_{j=0}^{n+p} x_{pjkl} = 0 \qquad p=0,...,n; l=1,...,t; k\in K_l \tag{4.7}$$

$$\sum_{j=1}^{n+p} x_{0jkl} \le 1 \qquad l=1,...,t; k\in K_l \tag{4.8}$$

$$\sum_{i=0}^{n+p}\sum_{j=0}^{n+p} (t_{ij}+st_i)x_{ijkl} \le D_k \qquad l=1,...,t; k\in K_l \tag{4.9}$$

$$\sum_{i=n+1}^{n+p} x_{i0kl} = \sum_{j=1}^{n+p} x_{0jkl} \qquad l=1,...,t; k\in K_l \tag{4.10}$$

$$u_{ikl} - u_{jkl} + Q_k x_{ijkl} \le Q_k - q_j \qquad i,j=1,...,n+p; d_i+d_j <= Q_k; l=1,...,t; k\in K_l \tag{4.11}$$

$$q_i \le u_{ikl} \le Q_k \qquad i=1,...,n; l=1,...,t; k\in K_l \tag{4.12}$$

$$x_{ijkl} \in \{0,1\} \qquad i,j=1,...,n; l=1,...,t; k\in K_l \tag{4.13}$$

$$y_{ir} \in \{0,1\} \qquad i=1,...,n; r\in C_i \tag{4.14}$$

$$u_{ikl} \in \{0,1\} \qquad i=1,...,n; l=1,...,t; k\in K_l \tag{4.15}$$

The main difference between this formulation and the formulation described in the literature is that the vehicles available in each day l are identified in set K_l. Constraints (4.10) were also added to the formulation to guarantee that the last stop of each tour before the depot is a discharging station.

4.3.2 Assign Days First - Assign Vehicles and Route Second Approach

First level - Day assignment Two formulations are presented for day assignment. The first one was developed by adapting the day and vehicle assignment formulation from Mourgaya and Vanderbeck.

$$minimize \sum_{i=2}^{n}\sum_{j=1}^{i-1}\sum_{l=1}^{t} d_{ij}z_{ijl} + \sum_{l=1}^{t}\sum_{k\in K_l} P w_{kl} \tag{4.16}$$

Subject to

$$\sum_{r\in C_i} y_{ir} \ge 1 \qquad i=1,...,n \tag{4.17}$$

$$\sum_{r\in C_i} a_{rl}y_{ir} + \sum_{r\in C_j} a_{rl}y_{jr} - z_{ijl} \le 1 \qquad i=2,...,n; j=1,...,(i-1); l=1,...,t; \tag{4.18}$$

$$\sum_{i=1}^{n}\sum_{r\in C_i} (q_i a_{rl} y_{ir}) \le S \sum_{k\in K_l} Q_k w_{kl} \qquad l=1,...,t \tag{4.19}$$

$$z_{ijl} \in \{0,1\} \qquad i=2,...,n; j=1,...,(i-1); l=1,...,t \tag{4.20}$$

$$y_{ir} \in \{0,1\} \qquad i=1,...,n; r\in C_i \tag{4.21}$$

$$w_{kl} \in \{0,1\} \qquad l=1,...,t; k\in K_l \tag{4.22}$$

This formulation does not have the decision variable x. As vehicles are not being assigned, the decision variable would become x_{il} which is equal to $\sum_{r\in C_i} a_{rl} y_{ir}$. Constraints (4.19) ensure that the total vehicle capacity for each day is not exceeded.A slack parameter S ranging from 0 to 1 was included in the constraints in order to guarantee that only a percentage of the total capacity of the vehicles is utilized. The first reason is to avoid that the deman d of a customer is split into more than one vehicle, which would lead to infeasible solutions in the next level. The second reason is due to the fact that the parameter q_i only represents a forecast of total waste to be collected and with the slack S one is accounting with possible fluctuations - "robust solutions". Moreover, reducing the total load of each vehicle can help, in some cases, to avoid that the total route time is not exceeded, which is not guaranteed in this level.

The last change was the introduction of the binary decision variables w_{kl} which assume value 1 if vehicle k goes out of the depot in day l. These variables were used to penalize in the objective function vehicles leaving the depot in each day. In fact, the objective function was favouring routes associated with a compact cluster of customers which was leading to solutions that used all vehicles available in each day.

Regarding Tan and Beasley formulation, few changes were carried out. The only difference remains in the equation that guarantees that the total vehicle capacity for each day is not exceeded. The slack parameter S was also included.

It was also necessary to determine a way of calculating parameters c_{il}. The most logical solution would be to consider the two nearest customers j and h attended in the same day l and compute the additional distance of inserting i in the middle of both:

$$c_{il} = d_{ij} + d_{ih} - d_{jh} \tag{4.23}$$

However, it would turn c_{il} into a decision variable and the problem would become non linear. Therefore, instead of c_{il}, a global contribution d_i of customer i was considered, which was determined considering the same equation (4.23) but being j and h the two nearest customers with the same frequency.

Second level - Vehicle assignment and Routing (VRP) Regarding the second level, it consists of a VRP formulation, adapted from the one presented in the literature review in the same way as the PVRP. A VRP needs to be solved for each day of the planning period and customers assigned to each day l are presented in the set V_l. The formulation is, for each day l:

$$minimize \sum_{i\in V_l} \sum_{j\in V_l} \sum_{l=1}^{t} \sum_{k\in K_l} d_{ij} x_{ijk} \tag{4.24}$$

Subject to

$$\sum_{i\in V_l} \sum_{k\in K_l} x_{ijk} = 1 \quad j \in Vc_l \tag{4.25}$$

$$\sum_{i \in V_l} x_{ihk} - \sum_{i \in V_l} x_{pjk} = 0 \qquad i \in V_l; k \in K_l \tag{4.26}$$

$$\sum_{j \in V_l \setminus \{v_0\}} x_{0jk} \leq 1 \qquad k \in K_l \tag{4.27}$$

$$\sum_{i \in V_l} \sum_{j \in V_l} (t_{ij} + st_i) x_{ijk} \leq D_k \qquad k \in K_l \tag{4.28}$$

$$\sum_{i \in Vp_l} x_{i0k} = \sum_{j \in V_l} x_{0jk} \qquad k \in K_l \tag{4.29}$$

$$u_{ik} - u_{jk} + Q_k x_{ijk} \leq Q_k - q_j \qquad i, j \in V_l \setminus \{v_0\}; d_i + d_j <= Q_k; k \in K_l \tag{4.30}$$

$$q_i \leq u_{ik} \leq Q_k \qquad i \in Vc_l; k \in K_l \tag{4.31}$$

$$x_{ijk} \in \{0, 1\} \qquad i, j \in V_l; k \in K_l \tag{4.32}$$

$$u_{ikl} \in \{0, 1\} \qquad i \in V_l; k \in K_l \tag{4.33}$$

4.3.3 Assign First - Route Second Approach

First level - Day and Vehicle assignment Two formulations are presented for day and vehicle assignment. The first is an adaptation from the existing one from Mourgaya and Vanderbeck.

$$minimize \sum_{i=2}^{n} \sum_{j=1}^{i-1} \sum_{l=1}^{t} d_{ij} z_{ijl} + \sum_{l=1}^{t} \sum_{k \in K_l} P w_{kl} \tag{4.34}$$

Subject to

$$\sum_{r \in C_i} y_{ir} \geq 1 \qquad i = 1, ..., n \tag{4.35}$$

$$\sum_{k \in K_l} x_{ikl} - \sum_{r \in C_i} a_{rl} y_{ir} \geq 0 \qquad i = 1, ..., n; l = 1, ..., t \tag{4.36}$$

$$x_{ikl} + x_{jkl} - z_{ijl} \leq 1 \qquad i = 2, ..., n; j = 1, ..., (i-1); l = 1, ..., t; k \in K_l \tag{4.37}$$

$$\sum_{i=1}^{n} (q_i x_{ikl}) \leq Q_k \qquad l = 1, ..., t; k \in K_l \tag{4.38}$$

$$\sum_{i=1}^{n} x_{ikl} - n w_{kl} \leq 0 \qquad l = 1, ..., t; k \in K_l \tag{4.39}$$

$$z_{ijl} \in \{0, 1\} \qquad i = 2, ..., n; j = 1, ..., (i-1); l = 1, ..., t \tag{4.40}$$

$$y_{ir} \in \{0, 1\} \qquad i = 1, ..., n; r \in C_i \tag{4.41}$$

$$w_{kl} \in \{0, 1\} \qquad l = 1, ..., t; k \in K_l \tag{4.42}$$

$$x_{ikl} \in \{0, 1\} \qquad i = 1, ..., t; l = 1, ..., t; k \in K_l \tag{4.43}$$

In this formulation, the decision variables w were also introduced for the minimisation of the number of routes performed in each day. However, its value is defined by constraint (4.42).

Formulation from Tan and Beasley is also similar to the one presented in the literature. Instead of c_{ikl}, the d_i parameter is also used and is calculated in the same way as presented in (4.23).

Second level - Routing(TSP) Regarding the second level, it consists of a TSP formulation, also adapted from the one presented in the literature:

$$minimize \sum_{i=0}^{n+p} \sum_{j=0}^{n+p} c_{ij} x_{ij} \tag{4.44}$$

Subject to

$$\sum_{j=0}^{n+p} x_{ij} = 1 \quad i = 0, ..., n \tag{4.45}$$

$$\sum_{i=0}^{n+p} x_{ij} = 1 \quad j = 0, ..., n \tag{4.46}$$

$$\sum_{i=n+1}^{n+p} x_{i0} = 1 \tag{4.47}$$

$$\sum_{i=0}^{n+p} \sum_{j=n+1}^{n+p} x_{ij} = 1 \tag{4.48}$$

$$\sum_{i=0}^{n+p} \sum_{j=0}^{n+p} (st_i + t_{ij}) x_{ij} \leq D \tag{4.49}$$

$$u_i - uj + nx_{ij} \leq n - 1 \quad i, j = 1, ..., n \tag{4.50}$$

$$1 \leq u_i \leq n \quad i = 1, ..., n \tag{4.51}$$

$$x_{ij} \in \{0, 1\} \quad i, j = 1, ..., n \tag{4.52}$$

The subtour elimination constraints (4.50-4.51) were slightly changed to account with the depot. An additional constraint (4.47) was also included to make sure that the last stop before the depot is a discharging station.

4.4 Solution Methods

The more decisions are tackled at the same time, the greater the number of decision variables and constraints and, consequently, the greater the complexity of the model. On the other hand, by considering all variables at the same time better results are obtained. When choosing the best formulation for a specific case, one is choosing the best compromise between efficiency and effectiveness. In order to compare the different hierarchical formulations and decide upon the best compromise, the experiments included the solution approaches presented in table 4.1.

IBM ILOG CPLEX 12.1 was the optimization engine used to solve the Mathematical Programming problems. The execution time was limited to 15 minutes in the first levels and 5 minutes in the second levels because the previously described formulations have weak linear programming relaxations and therefore, it would take a long time to prove optimality. The second level has a smaller limit because the problem is run multiple times in each problem. It was used Microsoft Visual Studio 2008, C++ environment, to integrate the two levels of the hierarchical approaches. A C++ interface for CPLEX was used for that purpose.

Preliminary tests proved that VRP and PVRP formulations did not achieve any integer solution within the pre-established execution time. Consequently, an adapted Clarke and Wright Heuristic was used instead in methods 1 and 2. The PVRP formulation was not used at all because heuristics for this problem are based on multi-level approaches.

Table 4.1: Solution Approaches

No.	*Hierarchical Approach*	*Formulation*	Solution Method
1	Assign Days First Assign Vehicles and Route Second	Adapted from: Mourgaya and Vanderbeck	1^{st} level: Optimization Software (15min) 2^{nd} level: Clarke and Wright Heuristic
2	Assign Days First Assign Vehicles and Route Second	Adapted from: Tan and Beasley	1^{st} level: Optimization Software (15min) 2^{nd} level: Clarke and Wright Heuristic
3	Assign First Route Second	Adapted from: Mourgaya and Vanderbeck	1^{st} level: Optimization Software (15min) 2^{nd} level: optimization software (5 min)
4	Assign First Route Second	Adapted from: Tan and Beasley	1^{st} level: Optimization Software (15min) 2^{nd} level: optimization software (5 min)

Adapted Clarke and Wright Heuristic Three important differences can be found in the algorithm when comparing to the one explained in the literature review: (1) Savings are computed in a different form; (2) Each route is associated with a vehicle and if no vehicles are available no more routes are possible to be created; (3) Each route is associated with a discharging station, which is reassessed whenever a new customer is introduced in the last position and is taken into consideration when checking the route length and time duration.

Savings were changed because the last stop of every route is a discharging station. The new form of computing the *savings* is presented in equation (4.53) and illustrated in figure 4.1, where $h*$ and h represents the nearest discharging stations from parish i and j respectively.

$$s_{ij} = d_{ih} + d_{h0} + d_{0j} - d_{ij} \tag{4.53}$$

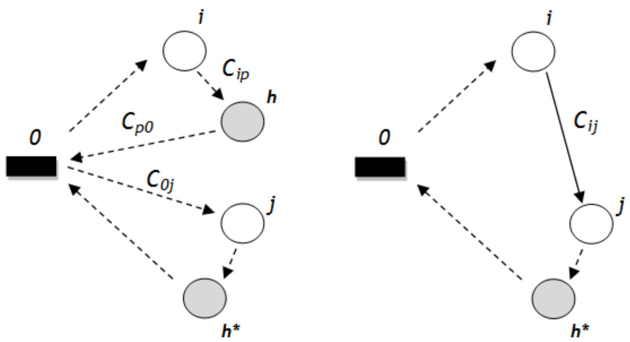

Figure 4.1: Adapted concept of Saving in Clark and Wright heuristic

A step by step description of the algorithm follows:

Step 1 Compute for all i and $j \in V_l : s_{ij} = d_{ip} + d_{p0} + d_{0j} - d_{ij}$; Sort all the savings s_{ij} and list them in descending order of magnitude (”savings list”).

Step 2 Starting from the top of the savings list, proceed with the following if, in any case, maximum duration of routes and vehicles capacity are not exceeded:

Step 2.1 If neither i or j have already been assigned to a route and if there are vehicles available to assign: Find the nearest discharging station $h \in Vd$ from parish j; Create route $(0 \rightarrow i \rightarrow j \rightarrow h \rightarrow 0)$; Assign available vehicle $k \in K_l$ with higher capacity.

Step 2.2 If i or j have already been included in a route R

Step 2.2.1 If $R = (0 \rightarrow ... \rightarrow i \rightarrow h \rightarrow 0)$: Find the nearest discharging station $h* \in Vd$ from parish j; Remove h; Include j and $h*$ in R so that $R = (0 \rightarrow ... \rightarrow i \rightarrow j \rightarrow h* \rightarrow 0)$

Step 2.2.2 If $R = (0 \rightarrow j \rightarrow ... \rightarrow h \rightarrow 0)$: Include i in R so that: $R = (0 \rightarrow i \rightarrow j \rightarrow ... \rightarrow h \rightarrow 0)$

Step 2.3 If both i and j have already been included in two different routes R_i and R_j

Step 2.3.1 If $R_i = (0 \rightarrow i \rightarrow ... \rightarrow h \rightarrow 0)$ and $R_j = (0 \rightarrow j \rightarrow ... \rightarrow h* \rightarrow 0)$: Invert route R_i; Remove h; Remove vehicles assigned to both routes; Merge both routes into a route R so that $R = (0 \rightarrow ... \rightarrow i \rightarrow j \rightarrow ... \rightarrow h* \rightarrow 0)$; Assign available vehicle $k \in K_l$ with higher capacity.

Step 2.3.2 If $R_i = (0 \rightarrow ... \rightarrow i \rightarrow h \rightarrow 0)$ and $R_j = (0 \rightarrow j \rightarrow ... \rightarrow h* \rightarrow 0)$: Remove h; Remove vehicles assigned to both routes; Merge both routes into a route R so that $R = (0 \rightarrow ... \rightarrow i \rightarrow j \rightarrow ... \rightarrow h* \rightarrow 0)$; Assign available vehicle $k \in K_l$ with higher capacity.

Step 2.3.3 If $R_i = (0 \longrightarrow i \longrightarrow ... \rightarrow h \rightarrow 0)$ and $R_j = (0 \rightarrow ... \rightarrow j \rightarrow h* \rightarrow 0)$: Remove h and $h*$; Remove vehicles assigned to both routes; Invert route R_j; Merge both routes into a route R so that $R = (0 \rightarrow ... \rightarrow i \rightarrow j \rightarrow ... \rightarrow h** \rightarrow 0)$; Compute $h** \in Vd$; Assign available vehicle $k \in K_l$ with higher capacity.

Step 3 For each $i \in V_l$ that was not included in a route in the last step: Compute the increased distances in introducing i at the beginning and at the end of each existing route

Step 3.1 If i fits in one of the positions without exceeding vehicle capacity and maximum route duration: introduce i in the place where the increased distance is smaller; If i that place is the last position: Remove h; Find the nearest discharging station $h*$ from parish i; Include $h*$.

Step 3.1 If i do not fit in any of the positions and if there are vehicles available: Find the nearest discharging station h from parish i; Create route $R = (0 \rightarrow i \rightarrow h \rightarrow 0)$; Assign available vehicle $k \in K_l$ with higher capacity.

Step 4 If there are $i \in V_l$ that were not included in any route: Create an additional route; Insert all the remaining i;

4.4.1 Computational Experiments with Instances from the Literature

Solution methods were tested on a classical set of 32 PVRP instances introduced by various authors and available in [4]. Their characteristics are summarized in table C.1 of Appendix C.

In order to turn the instances compatible with the developed formulations, the following assumptions were made: (1) All vehicles are available in all days and (2) Only one discharging station exists, located at the depot. In this way, total distance can be compared with instance's best known solutions. Preliminary tests were also realized in order to define $S = 0,95$ and $P = 10 \times max(d_{ij})$.

The tests were done in a personal computer with a AMD Turiom(tm) 64×2 running at 2.20 GHz, with 3 GB of random access memory. Both Visual Studio 2008 and CPLEX runned in 32 bits.

Table 4.2 shows an analysis carried out to the results obtained in each approach. The full results, as well as detailed solutions for one instance may be consulted in appendix C.

Table 4.2: Analysis of Results from Instances

Inst	*n*	*mxt*	*Approach 1*			*Approach 2*			*Approach 3*			*Approach 4*			Best
			GapTD	*NR*	*ET*	*GapTD*	*NR*	*ET*	*GapTD*	*NR*	*ET*	*GapTD*	*NR*	*ET*	NR
p01	51	6													5
p02	50	15	23.1%	14	900	31.8%	15	0	54.1%	14	906	70.7%	14	1	14
p03	50	5							27.4%	5	919	82.2%	5	0	5
p04	75	10							113.2%	10	903	87.3%	10	2	10
p05	75	30	16.5%	28	902	17.1%	28	1				88.8%	26	3	27
p06	75	10	37.9%	10	900				25.6%	10	905	87.1%	10	1	10
p07	100	8	16.7%	8	902	36.5%	8	2	113.1%	8	905	104.5%	8	8	8
p08	100	25	21.2%	24	903	25.7%	22	6	117.6%	25	907	113.9%	22	270	20
p09	100	8	55.6%	8	901	206.2%	12	1	51.3%	8	1245	104.5%	8	8	8
p10	100	20	44.2%	18	903	54.0%	18	4	139.6%	20	909	114.7%	18	4	16
p11	139	20													15
p12	163	15	47.0%	14	905	44.1%	14	17	80.7%	15	2421	66.8%	13	1576	12
p13	417	63				42.8%	62	41				189.2%	63	23	60
p14	20	8	6.3%	8	3	8.9%	8	0	4.3%	6	901	38.1%	6	1	8
p15	38	8	37.3%	8	244	26.6%	9	0	8.5%	8	901	66.7%	8	23	8
p16	56	8	68.4%	12	901	106.6%	11	1	64.0%	8	916	67.8%	8	307	8
p17	40	16	11.0%	14	901	9.8%	13	1	55.7%	14	901	66.4%	13	5	12
p18	76	16	13.5%	16	901	18.1%	16	2	102.9%	16	903	111.7%	16	11	14
p19	112	16	68.3%	18	904	65.8%	19	7	121.4%	16	913	124.2%	16	29	14
p20	184	16	208.4%	20	924	241.1%	20	29	155.9%	16	1639	153.4%	16	1855	20
p21	60	24	9.0%	22	901	12.9%	22	2	90.6%	24	906	83.1%	20	3	18
p22	114	24	8.8%	24	905	18.9%	23	8	125.8%	24	911	117.1%	23	98	20
p23	168	24	30.6%	24	919	37.0%	24	29	154.6%	24	947	130.5%	24	527	22
p24	51	18	22.1%	17	901	43.8%	16	1	55.0%	17	903	62.3%	15	2	14
p25	51	18	23.5%	16	901	42.4%	18	1	40.0%	15	902	56.3%	18	3	15
p26	51	18	21.0%	18	901	33.3%	18	0	54.4%	17	901	54.2%	18	3	16
p27	102	36	30.4%	35	905	49.4%	35	3	25.5%	36	906	99.8%	34	2	29
p28	102	36	32.9%	37	904	43.1%	36	3	83.3%	36	907	80.3%	34	2	30
p29	102	36	31.2%	37	902	39.6%	39	3	83.5%	36	909	101.8%	35	3	32
p30	153	54	82.1%	57	909	85.4%	54	10				103.0%	51	5	42
p31	153	54	84.8%	60	910	79.7%	55	9				87.2%	49	5	46
p32	153	54	68.1%	60	911	74.1%	58	8				102.2%	54	5	48
AVR			35.1%	23	844	41.9%	25	7	77.7%	16	1016	93.9%	22	159	20
STD			23.2%			24.8%			45.5%			31.2%			

Before analysing the performance of hierarchical formulations, some comments must be made to the results presented in the appendix. Firstly, it can be seen that a set of instances did not obtained results in some methods because they failed to achieve a feasible solution in one of the two levels. In each case, the reason is one of the following:

- No integer solution is found within the pre-defined maximum execution time;
- Total demand of customers is superior than 95% of total capacity of vehicles (applied to the first level of methods 1 and 2)
- There is no possible way of fitting all customers into routes without splitting the demand of a customer into more than one vehicle (applied to the second level of methods 1 and 2).

These reasons are not applied to instances p01 and p11 which didn't achieve results with neither of the methods. The reason is connected to the way instances were constructed.

Another comment worth of being stated is related to the *AR* column in the tables of solution approaches 1 and 2 (tables C.2 and C.3). These approaches have implemented in the second level the Clarke and Wright heuristic which is a heuristic that produces good routes at the beginning but less interesting ones towards the end. As the number of possible routes was limited by the existing vehicles in the adapted heuristic, the majority of instances failed to achieve a solution. The program was changed in order to permit one vehicle more and column *AR* shows the maximum load of that vehicle during the period.

Focusing now on table 4.2, formulations will be evaluated taken into consideration the resulting (1) *Execution time* (2) *total distance* travelled and (3) *number of routes*.

By analysing *execution times*, one can see that methods 2 and 4 achieve solutions before the time limit, which means that all levels are being optimized. Regarding methods 1 and 3, the first level does not achieve the optimal solution within the time limit and consequently, by increasing that time, both methods have potential of achieving better results. However, it is important to have in mind that both methods have weak linear relaxations and therefore, they might already be in the optimal solution after 900 seconds without being capable of proving it.

Regarding *total distance*, method 1 outperforms all other methods with an average gap of 35.1%. Averages were computed by taken out the most different points, being in this situation instance p20. Both formulations from assign days first - assign vehicles and route second approach perform significantly better.

By looking into the *number of routes*, the two last methods use less routes than the other two. However, it might not be because of the common hierarchical approach but because of the chosen heuristic in methods 1 and 2. Those methods have implemented in the second level a very greedy heuristic which, as already mentioned, produces good routes at the beginning but less interesting ones towards the end.

When analysing which hierarchical approach has the best performance, the approach *assign days first - assign vehicles and route second* seams the most promising option. In fact, distances are considerably lower and the number of tours might decrease if in the second level another method is used: a less greedy heuristic or a improvement heuristic. However, the number of tours should not be devalued against the total distance travelled because employee's salaries and vehicle's maintenance are important sources of costs.

Concerning formulations, Mourgaya and Vanderbeck based formulations outperform in all aspects and have potential to obtain better results with higher execution times. Another advantage of these formulations is that, because of the way the objective function is constructed, it tends to form clusters, which is also appreciated by the drivers. The one drawback of these formulations are the high execution times.

4.5 Summary

PVRP formulations described in the literature review could not be applied directly because of some particular characteristics of the waste collection problem. In this chapter, hierarchical formulations were adapted and afterwards tested with a set of instances available in the literature in order to evaluate the best compromise for the case study.

Two different hierarchical approaches were considered: (1) Assign First - Route second and (2) assign days first - assign vehicles and route second. Within each approach, two different formulations were presented, the first ones based on Mourgaya and Vanderbeck and the second based on Tan and Beasley.

Computational Experiments showed that assigning customers to days first and then solving as many VRPs as the number of days in the period is the most promising hierarchical approach. Total distance of solutions are considerably lower and, although being the number of tours slightly higher, it has potential to improve, if other solutions methods are applied.

Concerning formulations, Mourgaya and Vanderbeck based formulations outperform in all aspects and even have potential to obtain better results with higher limit execution times. Another advantage of these formulations is that, because of the way the objective function is constructed, it tends to form clusters, which is also appreciated by the drivers. The one drawback of these formulations are the high execution times.

The formulation with most potential of achieving better results is therefore the one adapted from Mourgaya and Vanderbeck which assigns customers to days first and afterwards assign vehicles and designs the routes.

Chapter 5

Case Study Results

This chapter aims to present optimized plans of routes for the transportation of Mixed Municipal Solid Waste from containers to treatment facilities in Ponte de Lima Municipality. Four plans are presented, corresponding to the results from the four methods developed and implemented in section 4. A comparison is made in order to conclude which of the four methods is the best hierarchical approach for this problem.

5.1 Case Study Instance

Based on the information acquired from Ponte de Lima Municipality and presented in Chapter 2, an instance was prepared with the case study values so that the problem could be solved by the methods. However, some information is not implicitly presented and will be covered in this section.

Road Network Distances d_{ij} $i, j \in V$ were obtained by using Google Maps Software, which has a shortest path algorithm. The matrix with all the distances will not be presented in this document due to its dimension.

Customer Demands and Available Resources The information from the tachographs was not obtained on time and therefore, it was not possible to assign values to parameters: $t\_bcontainers_i$, t_unload and vel. Consequently, the service time of each parish st_i and the driving time between parishes t_{ij} could not be computed and results were obtained without checking maximum duration of routes.

Schedules A list of possible schedules for each frequency were generated, always balancing the space between visits. All parishes with the same frequency can be visited by the same schedules.

- Frequency of 1 visit per week: {Monday}, {Tuesday}, {Wednesday}, {Thursday}, {Friday}, {Saturday}.
- Frequency of 2 visits per week: {Monday, Thursday}, {Monday, Friday}, {Tuesday, Friday}, {Tuesday, Saturday}, {Wednesday, Saturday}
- Frequency of 3 visits per week: {Monday, Wednesday, Friday}, {Monday, Wednesday, Saturday}, {Monday, Thursday, Saturday}, {Tuesday, Thursday, Saturday}
- Frequency of 4 visits per week: {Monday, Wednesday, Friday, Saturday}, {Monday, Tuesday, Thursday, Saturday}, {Monday, Wednesday, Thursday, Saturday}
- Frequency of 6 visits per week: {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

5.2 Results and Comparison with the current solution

The plan of routes obtained with each method can be consulted in Appendix D, in tables D.1, D.2, D.3 and D.4. Execution times limited to 15 minutes in the first level and 5 minutes in the second level. The slack variable S was set on 0,95 and the penalty P on $10/timesmax(d_{ij}$.

A summary of the results containing Execution Times, Total Distance travelled and Number of Routes is presented in table 5.1.

Table 5.1: Summary of the Solutions for the Case Study

Method	*ET*	*TD*	*NR*
1	904	1874.20	26
2	2	1896.90	27
3	902	2168.90	27
4	901	1897.40	21

As expected, assigning customers to days first and then solving as many VRPs as the number of days in the period generated results with lower distances but increased number of vehicles. However, the reduction of the distance is not significant when compared with the number of routes less that the solution of method 4 has. Therefore, for the specific problem of the case study, and by using the solution methods presented in section 4.4, the best compromise between number of routes and total distance travelled belongs to the fourth method, from the *Assign first - Route second* approach and formulation adapted from Tan and Beasley.

Table 5.2 presents the total distance and number of routes of the current solution and its comparison with the best solution found.

All the plans of routes achieved with the methods improved the current solution. Method 4 decreased in 21% the total distance travelled and also reduced the number of routes done in 6. Furthermore, these solutions guarantee that all parishes are visited with the pre-determined frequency and that the visits are balanced over the period.

Table 5.2: Comparison Between Current and New Solution for the Case Study

Solution	*TD*	*NR*
Current Solution	2389	26
New Solution	1897	21
Gap	20.6%	5

5.3 Summary

An instance was prepared with the case study information and the four methods presented in section 4 were applied. The plan with the best compromise between number of routes and total distance travelled was obtained with the fourth method, from the *Assign first - Route second* approach and formulation adapted from Tan and Beasley.

When comparing the new plans with the current solution, it was verified an improvement in total distance travelled and number of routes. Regarding method 4, that improvement is 21% for total distance and 6 for number of routes. Furthermore, the new plans guarantee that all parishes are visited with the pre-determined frequency and that the visits are uniformly distributed over the period.

Chapter 6

Conclusions and Future Work

This chapter's intent is to infer on the work's achieved objectives. It will also open new paths for the work developed.

6.1 Work's Assessment

When assessing the work carried out during the thesis, two major topics must be discussed: the classification of the Periodic Vehicle Routing Problem formulations by type of hierarchical approach and the results obtained for the case study.

The existing literature about the Vehicle Routing Problem defines the PVRP as a multi-decision problem and mentions the difficulties of choosing a schedule for each customer together with solving the routing problem. Hierarchical approaches, with decisions tackled at different stages, have already been developed. However, it was not found any document containing a clear classification of formulations by type of hierarchical approach, or even a comparison of the performance of formulations by the way decisions are tackled.

Although hierarchical formulations may not achieve optimal solutions when solved, there are some strong motivations for using them, such as to account with different optimization criteria in each level or to deal with large problems, where computational times increase considerably.

In this thesis, different formulations from the literature were put together and divided by hierarchical approach. Three types of hierarchical approaches were considered: (1) Assign and route together; (2) Assign days first - assign vehicles and route second and (3) Assign first - route second. The advantages and drawbacks of each type were discussed and computational experiments based on instances available in the literature enabled a first comparison taken into consideration execution time, number of routes and total distance travelled.

Assigning days first and then solving as many VRPs as the number of days in the period was considered the most promising approach. Assign and route together has impossible execution times and this one outperforms in total distance the assign first - route second approach. Concerning the number of vehicles, it was not possible to evaluate objectively which approach performs better.

Looking now into the case study, a profound knowledge and sensitivity for the problem were acquired during the modelling process, resulting in a set of formulations adapted to the features of the waste collection problem in Ponte de Lima. Moreover, the adapted formulations are sufficiently generalised to be applied to other waste collection problems.

Information provided by the municipality was not enough to enable the development of a total instance and therefore, results were obtained without checking maximum duration of routes. The four methods were applied to the case study instance developed and four improved plans of routes were obtained for the municipality. Those plans also guarantee that all parishes are visited with the pre-determined frequency and that the space between consecutive visits to a container are balanced.

6.2 Future Work

Due to the reduced existing time, not all hierarchical formulations in the literature were covered and computational experiments were also limited. In the following items, a set of changes and further analysis proposals are presented:

- To achieve more fair results between the approaches, another heuristic, less greedy, should be implemented in the second level of the *assign days first - assign vehicles and route second* approach. Other alternative would be the introduction of an improvement heuristic.
- For Mourgaya and Vanderbeck formulation, it would be interesting to trace the cost of each solution identified against time, in order to choose the best maximum execution time.
- When choosing the best approach for a problem, it is important to have information of the performance of the approaches with an increasing number of customers, vehicles, period and frequencies. For that, new instances should be generated, varying only one parameter at a time.

Decision Support Systems are computer systems that tie together models, data, analysis tools, and presentation tools into a single integrated package. These systems are intended for repeat use, either by executives themselves or by their analytic staff [29]. Although this thesis only contemplated a static solution for the waste collection problem in Ponte de Lima, the next step should be the development of a Decision Support System and its integration with the geographic information system of the municipality. The success of the utilization of OR techniques is due to the development of computer systems, from both the hardware and the software points of view, and to the increasing integration of information systems into the productive and commercial processes.

The adapted formulations have also potential of improvement, such as:

- Test for time limit in the first level of assign days first - assign vehicles and route second
- Introduce constraints for equilibrate work load between the routes;
- Deeper sensitivity analysis to the slack parameter S;
- A more careful study of the problem as a multi-criteria problem, taking into consideration not only the reduction of costs but also factors such as the satisfaction of clients and drivers.
- It would also be interesting to try to include strategic decisions into the formulations, such as location and dimension of depots and containers.

Methods should also be tested in different scenarios in order to guarantee its suitability to future changes in the municipality. Based on the system indicators presented in section 2, the author ends her thesis by suggesting the utilization of the population parameter as an independent variable to generate new scenarios. In [18] a good approach to the generation of scenarios is also presented.

Appendix A

Basic Notation

In this first appendix, it is possible to consult the notation used to represent the parameters of the models presented in section 3 - *literature review*.

A.1 Sets

The problems are defined on a direct graph G = (V,A), where:

V	vertices, corresponding to locations $\|V\| = n+1$, $V = \{v_0, v_1, ..., v_n\}$
Vc	places to visit, known as customers $\|V_c\| = n$, $V_c = V \backslash \{v_0\}$
A	arcs $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$
K	Vehicles $\|K\| = m$
C	Schedules
C_i	Allowable schedules for each customer i

A.2 Indices

k	vehicles
i, j, h	vertices
l	days
r	visit combinations

A.3 Parameters

m	number of vehicles
n	number of customers
Q_k	capacity of vehicle k
D_k	maximum duration of routes for vehicle k
c_{ij}	cost of travelling between vertices i and j
t_{ij}	travelling time between vertices i and j
d_i	duration of the service for vertex i
q_i	duration of the service for vertex i

c_{ik} measure (derived in some fashion way) of the cost contribution of customer i to the route followed by vehicle k if customer i were to be delivered to by vehicle k

t planning horizon (period)

e_i service frequency for vertex i

a_{rl} Binary constant equals to one if day l belongs to the visit combination r

c_{il} measure (derived in some fashion way) of the cost contribution of customer i to any route involving customer i on day l

c_{ikl} measure (derived in some fashion way) of the contribution of customer i to the route followed by vehicle k on day l

Appendix B

Case Study Notation

In this second appendix, it is possible to consult the notation used to represent the parameters of the models presented in section 4 - *Models and Solution Methods Applied to the Case Study*.

B.1 Sets

The problems are defined on a direct graph G = (V,A), where:

V	locations $\|V\| = n+p+1$, $V = \{v_0, v_1, ..., v_{n+p}\}$
Vc	parishes $\|V_c\| = n$, $V_c = \{v_1, v_1, ..., v_n\}$
Vd	vehicle discharging stations $\|V_p\| = p$, $V_d = \{v_{n+1}, v_{n+2}, ..., v_{n+p}\}$
A	arcs $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$
K	Vehicles, $\|K\| = m$
K_l	Vehicles available on day l, $K_l \subseteq K$, $\|K_l\| = m_l\|$
C	Schedules
C_i	Allowable schedules for each parish i

B.2 Indices

k	vehicles
i, j, h	vertices
l	days
r	visit combinations

B.3 Parameters from the Instance

t	planning horizon (days)
n	number of parishes
p	number of vehicle discharging stations
m_l	number of vehicles available on day l
d_{ij}	distance between locations i and j (km)

q_i	total waste to be collected in parish i (Kg)
st_i	service duration in parish i (minutes)
nc_i	number of containers in parish i
w_capita	MSW production per capita in Ponte de Lima (kg/inh $\times$ day)
$n\_inhabitants_i$	Population of parish i
e_i	service frequency for vertex i
t_unload	medium time of unloading a container (minutes)
$t\_bcontainersi$	duration of going from one container to another inside parish i (minutes)
t_{ij}	time duration of going from locations i to j (minutes)
Q_k	capacity of vehicle k (Kg)
T	maximum duration of routes (minutes)
vel	velocity of vehicles between parishes (km/h)
a_{rl}	Binary constant equals to one if day l belongs to the visit combination r
d_i	distance contribution of customer i to any route involving the customer (km)

B.4 Parameters from the formulations

P	Penalty of using each vehicle
S	Total Vehicle capacity Slack

B.5 Results

NR	Number of routes
TD	Total distance (km)
ET	Execution Time (seconds)
AR	Additional Route
$GapTD$	Gap between TD of the approach solution and TD of Best Known Solution (in %)
AVR	Average
STD	Standard Deviation
D	Route distance (km)
q	Load of the Route (kg)

Appendix C

Extended Results of Computational Experiments

In chapter 4 four solution approaches to solve the case study were defined, which were validated through computational experiments done to a set of 32 PVRP Instances. This appendix contains the characteristics of the instances as well as their individual results in the four approaches.

C.1 Characteristics and Best Known Solutions of the Instances

The characteristics of the 23 instances from computational experiments are summarized in table C.1. In all cases, the maximum duration of routes is $T = \infty$.

Table C.1: Characteristics and Best Known Solution of Instances

Instance	*n*	*m*	*t*	*Q*	*TD*	*NR*	*Instance*	*n*	*m*	*t*	*Q*	*TD*	*NR*
p01	51	3	2	160			p17	40	4	4	20	1597.75	12
p02	50	3	5	160	1322.87	14	p18	76	4	4	30	3147.24	14
p03	50	1	5	160	524.61	5	p19	112	4	4	40	4834.34	14
p04	75	5	2	140	835.43	10	p20	184	4	4	60	8367.40	20
p05	75	6	5	140	2027.99	27	p21	60	6	4	20	2184.04	18
p06	75	1	10	140	836.37	10	p22	114	6	4	30	4271.11	20
p07	100	4	2	200	826.14	8	p23	168	6	4	40	6602.59	22
p08	100	5	5	200	2034.15	20	p24	51	3	6	20	3687.46	14
p09	100	1	8	200	826.14	8	p25	51	3	6	20	3777.15	15
p10	100	4	5	200	1595.84	16	p26	51	3	6	20	3795.33	16
p11	139	4	5	235	779.29	15	p27	102	6	6	20	21956.46	29
p12	163	3	5	140	1195.88	12	p28	102	6	6	20	22934.71	30
p13	417	9	7	2000	3511.62	60	p29	102	6	6	20	22909.36	32
p14	20	2	4	20	954.81	8	p30	153	9	6	20	75016.58	42
p15	38	2	4	30	1862.63	8	p31	153	9	6	20	78179.89	46
p16	56	2	4	40	2875.24	8	p32	153	9	6	20	80479.20	48

C.2 Results of the Instances

In this section, it is possible to consult the results of the instances obtained in every approaches in tables C.2, C.3, C.4 and C.5.

Table C.2: Results of the Instances in Solution Approach 1

Instance	*TD*	*NR*	*ET*	*AR*	*Instance*	*TD*	*NR*	*ET*	*AR*
p01					p17	1773.63	14	901.00	
p02	1628.12	14	900.00		p18	3571.64	16	901.00	
p03	No Solution in level 1				p19	8137.66	18	904.00	53.00
p04	No Solution in level 1				p20	25805.67	20	924	82.00
p05	2362.07	28	902.00		p21	2380.17	22	901.00	
p06	No Solution in level 1				p22	4645.82	24	905.00	
p07	964.41	8	902.00		p23	8620.17	24	919.00	
p08	2465.29	24	903.00		p24	4502.93	17	901.00	
p09	1285.63	8	901.00		p25	4665.50	16	901.00	
p10	2301.56	18	903.00		p26	4592.91	18	901.00	
p11					p27	28630.34	35	905.00	3.00
p12	1758.30	14	905.00		p28	30479.86	37	904.00	10.00
p13	Out of Memory				p29	30060.66	37	902.00	6.00
p14	1014.83	8	3.00		p30	136606.41	57	909.00	74.00
p15	2557.92	8	244.00		p31	144460.92	60	910.00	90.00
p16	4842.7	12	901.00	22.00	p32	135272.8	60	911.00	102.00

Table C.3: Results of the Instances in Solution Approach 2

Instance	*TD*	*NR*	*ET*	*AR*	*Instance*	*TD*	*NR*	*ET*	*AR*
p01					p17	1,753.60	13	1.00	
p02	1,743.10	15	0.00		p18	3,717.05	16	2.00	8.00
p03	No Solution in level 1				p19	8,017.41	19	7.00	56.00
p04	No Solution in level 1				p20	28,542.46	20	29.00	103.00
p05	2,375.78	28	1.00	30.00	p21	2,465.60	22	2.00	5.00
p06	No Solution in level 1				p22	5,078.45	23	8.00	
p07	1,127.88	8	2.00		p23	9,043.81	24	29.00	72.00
p08	2,556.74	22	6.00		p24	5,303.16	16	1.00	
p09	2,529.62	12	1.00	86.00	p25	5,378.26	18	1.00	5.00
p10	2,458.33	18	4.00		p26	5,057.60	18	0.00	5.00
p11					p27	32,813.50	35	3.00	28.00
p12	1,722.85	14	17.00	14.00	p28	32,820.44	36	3.00	24.00
p13	5,015.38	62	41.00		p29	31,980.84	39	3.00	5.00
p14	1,039.34	8	0.00		p30	139,081.19	54	10.00	123.00
p15	2,357.30	9	0.00	18.00	p31	140,508.28	55	9.00	120.00
p16	5,940.16	11	1.00	32.00	p32	140,074.06	58	8.00	128.00

C.3 Detailed Results of one Instance

This last section contains detailed information about the results obtained for instance p02 in all the approaches experimented, in tables C.6, C.7, C.8, C.9 and C.10.

Table C.4: Results of the Instances in Solution Approach 3

Instance	*TD*	*NR*	*ET*	*Instance*	*TD*	*NR*	*ET*
p01				p17	2488.32	14	901.00
p02	2037.95	14	906.00	p18	6384.86	16	903.00
p03	668.32	5	919.00	p19	10705.39	16	913.00
p04	1780.92	10	903.00	p20	21413.75	16	1639.00
p05	No Solution in level 1			p21	4163.32	24	906.00
p06	1050.54	10	905.00	p22	9645.14	24	911.00
p07	1760.69	8	905.00	p23	16810.06	24	947.00
p08	4426.31	25	907.00	p24	5716.85	17	903.00
p09	1250.26	8	1245.00	p25	5286.42	15	902.00
p10	3824.28	20	909.00	p26	5860.79	17	901.00
p11				p27	27548.10	36	906.00
p12	2161.31	15	2421.00	p28	42033.63	36	907.00
p13	Out of Memory			p29	42033.63	36	909.00
p14	995.42	6	901.00	p30	No Solution in level 1		
p15	2021.47	8	901.00	p31	No Solution in level 1		
p16	4714.77	8	916.00	p32	No Solution in level 1		

Table C.5: Results of the Instances in Solution Approach 4

Instance	*TD*	*NR*	*ET*	*Instance*	*TD*	*NR*	*ET*
Instance	*TD*	*NR*	*ET*				
p01				p17	2658.05	13	5.00
p02	2258.09	14	1.00	p18	6662.41	16	11.00
p03	955.93	5	0.00	p19	10836.38	16	29.00
p04	1564.71	10	2.00	p20	21200.30	16	1854.52
p05	3828.74	26	3.00	p21	3998.59	20	3.00
p06	1564.71	10	1.00	p22	9273.72	23	98.00
p07	1689.57	8	8.00	p23	15217.68	24	527.00
p08	4351.63	22	270.00	p24	5985.93	15	2.00
p09	1689.57	8	8.00	p25	5902.57	18	3.00
p10	3425.58	18	4.00	p26	5850.73	18	3.00
p11				p27	43874.75	34	2.00
p12	1995.05	13	1576.00	p28	41346.66	34	2.00
p13	10156.37	63	23.00	p29	46231.08	35	3.00
p14	1318.23	6	1.00	p30	152317.83	51	5.00
p15	3105.79	8	23.00	p31	146370.78	49	5.00
p16	4823.60	8	307.00	p32	162695.01	54	5.00

Table C.6: Extended Best Known Results of Instance p02

l	*k*	*D*	*q*	*Route*
1	1	97.87	158	0 2 20 29 34 49 5 12 0
1	2	113.23	160	0 47 17 44 42 40 41 25 18 0
2	1	105.73	157	0 8 31 28 3 36 35 20 2 32 0
2	2	82.92	155	0 6 14 25 13 41 18 0
2	3	103.3	157	0 12 15 45 33 39 30 34 9 38 0
3	1	79.93	137	0 47 4 41 19 42 44 37 12 0
3	2	96.30	157	0 18 25 24 43 7 23 48 27 0
3	3	100.79	157	0 11 2 20 16 34 49 5 0
4	1	109.72	157	0 12 33 39 10 30 34 50 9 38 46 0
4	2	105.66	158	0 32 2 20 35 3 28 31 26 8 0
4	3	82.92	155	0 18 41 13 25 14 6 0
5	1	122.67	159	0 48 23 7 43 25 41 18 0
5	2	16.12	29	0 12 0
5	3	105.71	156	0 27 1 22 2 20 21 34 16 11 0

Table C.7: Extended Results of Instance p02 with Solution Approach 1

l	*k*	*D*	*q*	*Route*
1	2	118.55	153	0 23 18 25 41 19 17 12 51 0
1	3	131.57	148	0 2 16 34 21 29 20 36 28 22 1 51 0
2	3	109.16	141	0 38 9 50 34 30 39 33 10 49 51 0
2	2	106.92	154	0 37 44 15 41 13 25 18 51 0
2	1	147.52	158	0 7 32 20 11 2 5 12 51 0
3	3	106.78	154	0 8 31 28 3 35 20 2 16 51 0
3	1	136.41	153	0 18 42 41 25 43 23 48 51 0
3	2	116.03	131	0 12 47 14 6 27 34 51 0
4	3	129.81	145	0 18 41 40 13 25 7 51 0
4	2	113.83	157	0 46 38 49 9 34 30 39 33 45 44 51 0
4	1	83.61	139	0 32 20 2 11 5 12 51 0
5	2	85.30	144	0 12 47 42 41 4 18 51 0
5	3	136.04	158	0 8 26 31 3 35 20 2 34 51 0
5	1	107.39	117	0 6 24 25 14 43 48 27 51 0

Table C.8: Extended Results of Instance p02 with Solution Approach 2

l	*k*	*D*	*q*	*Route*
1	3	114.25	151	0 9 16 34 29 20 3 2 11 51 0
1	2	166.65	156	0 49 44 42 41 25 43 7 26 48 51 0
1	1	63.69	97	0 12 18 6 32 51 0
2	2	144.82	153	0 5 33 17 41 40 13 25 14 51 0
2	1	140.36	160	0 27 8 31 28 36 20 21 34 30 10 46 51 0
2	3	95.60	140	0 47 18 1 22 2 12 51 0
3	3	155.58	159	0 41 32 3 35 20 2 12 51 0
3	2	131.80	134	0 44 38 49 39 34 9 16 11 51 0
3	1	147.18	160	0 18 42 25 23 43 7 48 6 51 0
4	3	104.83	143	0 34 30 33 5 12 47 51 0
4	2	87.29	121	0 27 8 31 28 20 2 51 0
4	1	79.33	140	0 18 41 13 25 14 51 0
5	3	153.30	159	0 37 45 39 34 50 35 20 2 38 51 0
5	1	108.85	140	0 18 4 19 41 25 24 23 51 0
5	2	49.56	39	0 15 12 51 0

Table C.9: Extended Results of Instance p02 with Solution Approach 3

l	*k*	*D*	*q*	*Route*
1	1	134.12	97	0 48 14 4 19 17 37 15 11 51 0
1	2	147.59	158	0 7 25 13 18 44 45 5 51 0
1	3	147.99	159	0 12 41 30 34 20 2 51 0
2	1	150.03	121	0 9 2 22 3 31 26 24 25 51 0
2	2	181.11	153	0 20 35 43 18 41 12 51 0
2	3	126.23	145	32 38 34 50 16 28 8 27 6 51 0
3	1	143.12	158	0 49 10 30 39 33 44 42 41 13 51 0
3	2	97.27	160	0 11 5 47 18 14 23 48 51 0
3	3	160.38	160	0 25 7 2 20 34 12 51 0
4	1	134.27	152	0 38 16 21 29 2 20 35 36 3 31 51 0
4	2	140.92	160	0 46 12 18 41 40 25 8 51 0
4	3	159.26	111	0 9 34 32 1 28 43 6 27 51 0
5	1	157.18	158	0 12 39 2 23 25 18 51 0
5	2	158.49	160	0 47 41 42 33 49 34 20 51 0

Table C.10: Extended Results of Instance p02 with Solution Approach 4

l	*k*	*D*	*q*	*Route*
1	1	257.01	147	0 24 43 26 22 32 29 50 9 10 39 15 37 42 40 4 46 51 0
1	3	112.81	159	0 12 47 18 14 6 20 51 0
2	1	140.15	134	0 2 34 41 13 25 51 0
2	2	179.09	157	0 48 8 31 36 11 12 44 19 41 51 0
2	3	190.97	153	0 45 38 16 21 20 35 3 23 25 51 0
3	1	131.29	151	0 27 1 28 2 34 49 18 51 0
3	2	143.24	154	0 32 34 5 47 41 25 6 51 0
3	3	183.81	131	0 7 43 14 13 42 39 30 9 51 0
4	1	141.57	154	0 12 18 17 33 20 2 51 0
4	2	160.25	158	0 18 41 34 2 31 8 51 0
4	3	126.49	117	0 27 20 48 25 12 51 0
5	1	166.05	146	0 44 49 38 11 16 35 3 28 23 51 0
5	2	168.87	145	0 7 2 30 5 12 41 51 0
5	3	156.51	146	0 25 18 33 34 20 51 0

Appendix D

Optimized Plans of Routes for Solid Waste Collection

This appendix contains the optimized plans of routes for Mixed Municipal Solid Waste collection in Ponte de Lima Municipality. The parishes were assigned to the indexes respecting the order presented in the tables of chapter 2. Index 52 corresponds to the transfer station in Arcos de Valdevez and index 53 to the sanitary landfill in Viana do Castelo.

Table D.1: Plan of routes obtained with Solution Approach 1

l	*k*	*D*	*q*	*Route*
1	5	81.7	9843.60	0 19 37 23 34 33 51 53 0
1	3	89.0	8800.74	0 16 42 4 6 15 49 52 0
1	4	74.7	8892.78	0 22 36 18 46 53 0
1	2	51.7	4816.50	0 35 39 52 0
1	1	45.2	5426.07	0 2 41 26 52 0
2	5	113.6	10719.50	0 9 1 38 8 21 50 53 0
2	3	68.2	10326.40	0 16 19 44 32 4 10 52 0
2	4	82.0	9314.76	0 28 45 2 35 13 39 52 0
2	2	45.1	3492.45	0 41 27 52 0
2	1	47.2	3589.56	0 7 52 0
3	5	100.6	10972.30	0 2 51 31 24 20 14 25 43 5 53 0
3	3	76.0	10540.10	0 35 16 46 11 23 34 53 0
3	4	73.5	6321.12	0 47 4 15 52 0
4	5	83.2	9574.50	0 4 6 29 40 30 13 52 0
4	3	80.5	9661.08	0 35 44 21 8 42 12 17 52 0
4	4	61.7	8481.72	0 19 16 18 50 53 0
4	2	60.5	6173.31	0 41 2 10 39 52 0
4	1	52.9	6261.84	0 22 26 52 0
5	5	92.3	9946.56	0 36 38 1 48 34 33 53 0
5	3	84.0	10621.30	0 51 37 16 46 35 52 0
5	4	82.6	10168.10	0 3 32 4 15 28 52 0
5	2	54.4	3185.52	0 2 45 27 52 0
6	5	90.8	9891.96	0 19 11 23 24 20 14 31 43 53 0
6	3	76.9	9506.64	0 2 50 21 8 44 4 10 52 0
6	4	60.8	8842.86	0 35 16 13 39 52 0
6	*	45.1	2153.97	0 41 52 0

Table D.2: Plan of routes obtained with Solution Approach 2

l	*k*	*D*	*q*	*Route*
1	5	81.2	10826.40	0 35 19 37 20 24 23 27 52 0
1	3	87.2	8799.57	0 41 2 36 14 11 33 34 53 0
1	4	77.7	9760.92	0 4 6 15 49 13 39 52 0
1	2	65.7	6477.90	0 44 21 8 42 32 52 0
1	1	67.8	5857.80	0 16 46 50 10 52 0
2	5	80.9	10932.50	0 22 38 1 43 31 53 0
2	3	65.9	10531.60	0 35 16 18 51 53 0
2	4	45.0	1847.04	0 2 45 52 0
2	2	64.0	6754.80	0 4 28 26 52 0
3	5	75.9	6651.84	0 19 16 23 34 53 0
3	3	98.5	9583.08	0 17 21 4 30 13 52 0
3	4	52.9	5563.74	0 35 10 39 52 0
3	2	45.1	2756.13	0 2 41 52 0
4	5	81.8	10027.70	0 19 37 20 24 14 36 35 52 0
4	4	68.0	9908.34	0 44 8 3 32 4 52 0
4	1	60.7	3584.10	0 15 39 52 0
4	3	86.5	10450.40	0 2 50 46 16 11 51 33 53 0
4	2	47.4	5743.53	0 41 7 52 0
5	5	89.3	10183.70	0 1 48 25 5 23 34 53 0
5	3	84.1	9605.70	0 4 6 29 40 13 10 52 0
5	2	51.3	3948.36	0 2 47 9 52 0
5	4	91.2	9302.28	0 18 16 21 42 12 52 0
5	1	46.0	6154.98	0 35 27 26 52 0
5	*	53.1	3591.90	0 22 52 0
6	3	62.5	10013.30	0 2 35 16 46 50 41 52 0
6	5	96.5	10998.80	0 19 51 43 31 38 8 44 45 52 0
6	4	70.7	7668.96	0 4 15 39 28 52 0

Table D.3: Plan of routes obtained with Solution Approach 3

l	*k*	*D*	*q*	*route*
1	1	80.9	4359.4	0 20 31 14 45 52 0
1	2	57.1	5736.9	0 4 26 52 0
1	3	82.3	10831.1	0 2 35 44 39 19 16 43 53 0
1	4	91.7	9596.8	0 8 46 41 22 48 33 53 0
1	5	103.1	8701.7	0 13 15 21 50 36 23 53 0
2	1	55.1	5992.0	0 18 51 53 0
2	2	54.7	3067.0	0 4 52 0
2	3	132.9	7307.1	0 47 28 40 6 12 34 53 0
2	4	89.4	8487.2	0 10 42 16 1 11 53 0
2	5	74.4	8668.9	0 2 37 32 35 27 52 0
3	1	64.8	5404.6	0 19 21 39 52 0
3	2	85.6	5260.3	0 49 3 50 53 0
3	3	91.1	8000.9	0 41 30 44 8 16 46 53 0
3	4	79.1	9439.6	0 2 4 13 38 24 53 0
3	5	95.1	9723.5	0 35 7 15 23 5 53 0
4	2	53.1	3591.9	0 22 52 0
4	3	70.5	6135.5	0 35 33 31 25 34 53 0
4	4	71.2	8594.8	0 2 4 29 26 52 0
4	5	83.9	10075.3	0 39 10 16 51 14 52 0
5	1	78.8	5858.6	0 13 8 11 23 53 0
5	2	64.9	6202.6	0 21 19 18 52 0
5	3	106.7	9495.7	0 27 9 1 36 15 46 50 53 0
5	4	77.7	9020.7	0 2 16 44 42 4 6 52 0
5	5	97.0	9331.6	0 35 32 20 43 41 45 52 0
6	3	80.7	10098.3	0 35 41 10 51 24 34 53 0
6	4	77.2	9395.9	0 2 38 37 19 17 52 0
6	5	69.9	9147.9	0 16 4 39 28 52 0

Table D.4: Plan of routes obtained with Solution Approach 4

l	*k*	*D*	*q*	*route*
1	1	71.0	6364.0	0 21 22 9 52 0
1	3	96.0	10595.5	0 39 15 42 44 19 46 11 34 53 0
1	4	88.5	9439.6	0 4 50 1 23 53 0
1	5	85.5	10352.6	0 2 35 16 10 41 14 33 53 0
2	2	68.7	6750.1	0 36 16 7 52 0
2	3	140.5	10827.2	0 28 13 30 12 8 38 31 51 53 0
2	4	88.5	10168.1	0 2 17 18 37 24 53 0
2	5	94.5	10118.2	0 4 35 26 45 43 53 0
3	3	82.9	10558.1	0 2 21 32 4 6 10 27 52 0
3	4	86.6	10264.0	0 47 39 35 19 20 25 53 0
3	5	93.2	10897.8	0 41 44 29 16 46 23 53 0
4	3	88.2	10707.1	0 13 8 3 19 51 33 34 53 0
4	4	97.0	10126.0	0 4 15 1 14 50 53 0
4	5	60.4	10887.7	0 2 35 16 22 41 52 0
5	2	66.1	6530.2	0 16 38 24 53 0
5	3	85.6	10895.8	0 4 42 21 18 11 53 0
5	4	95.1	10158.7	0 35 37 48 39 45 52 0
5	5	117.8	10757.8	0 2 36 43 5 23 46 44 49 26 52 0
6	3	112.3	10244.5	0 2 8 32 4 15 40 27 31 53 0
6	4	93.5	9838.9	0 35 39 10 6 19 20 50 53 0
6	5	85.5	11043.7	0 28 41 13 16 51 34 53 0

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