
Passive Vibration Control of Beams Via Shunted Piezoelectric Transducing Technologies

Modelling, Simulation and Analysis

by

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“No hill to steep, no ditch to deep...”

Abstract

The work developed is an approach to a very contemporary engineering field, namely active structures. Active structures are a very wide theme, and was explored in this work by studying the capabilities of electro-mechanical energy conversion and dissipation phenomena applied to vibration control field. This kind of phenomena can be achieved by using smart materials, in this case piezoelectric materials. More specifically, this study contemplates the study of two layered beams, comprising elastic and piezoelectric layers, considering also an electrical energy dissipator circuit, shunt. Two types of beams are considered for testing the theory: clamped-free and simply-supported beams.

This work comprehends the development of analytical and numerical models in order to determine the behavior patterns of passive vibration damping using piezoelectric materials and shunt damping designs. Using the Euler-Bernoulli beam's fundamental theory, Hamilton's variational principle and piezoelectric constitutive equations, the analytical model is developed. This model considers several assumptions such as considering mass and stiffness of the piezo as negligible, or neutral axis symmetric on the elastic layer. The analytical model is considered always as an open-circuit case. The numerical model is implemented considering 3D solid elements and is implemented in a FEM software, namely the COMSOL Multiphysics 3.5, and post-processed in Matlab, where electrical DOFs are distinguished considering the two limit electric boundary conditions, closed-circuit and open-circuit models. The difference between EBCs serves to comprehend the electromechanical energy conversion capabilities. The electromechanical phenomenon is evaluated in both models through three different frequency responses types: mechanical actuation (receptance), sensing (voltage per unit force) which represents the electric potential generated on the transducer per unit force applied and the electrical actuation (displacement per unit voltage), where is measured the displacement induced by an unitary electrical potential difference applied on the transducer.

After covering all the mentioned issues, the shunt damping applied to the beams is presented. Here is considered a resonant shunt, which acts similarly to a damped vibration absorber. The shunt is constituted by inductive and resistive elements, and their properties are defined through the electromechanical energy capability for each beam, evaluated through the comparison between the limit EBCs eigenvalues results. Afterwards, the shunt damping is implemented and its damping capabilities are demonstrated for different test cases.

Keywords: passive vibration damping, shunt, electromechanical energy conversion; piezoelectric materials.

Resumo

O presente trabalho apresenta uma abordagem a um campo da engenharia muito em voga, estruturas inteligentes. Estruturas inteligentes são um tema vasto, sendo neste trabalho explorado no âmbito do controlo de vibrações através de fenómenos de conversão de energia mecânica em energia eléctrica através do uso de materiais piezoeléctricos. Para esse fim são consideradas o estudo de vigas compostas por duas camadas, uma delas elástica e a outra piezoeléctrica. Estas vigas são consideradas ligadas a um elemento dissipador de energia eléctrica, um circuito shunt. São consideradas dois tipos de viga, uma encastrada e uma simplesmente apoiada.

Esta dissertação compreende a modelação analítica e numérica do problema, de modo a estabelecer padrões de comportamento de vigas amortecidas passivamente através de materiais piezoeléctricos. A modelação analítica serve-se de vários conceitos essenciais, tais como a teoria de viga de Euler-Bernoulli, o princípio variacional de Hamilton e as leis constitutivas para materiais piezoeléctricos. Este modelo toma em conta várias premissas, como considerar a massa e rigidez do material piezoeléctrico como desprezável, ou o eixo neutro da viga como sendo simétrico em relação à camada elástica. O modelo numérico considera elementos sólidos 3D, e é implementado num software the elementos finitos, nomeadamente o COMSOL Multiphysics 3.5, e é pós-processado em Matlab, onde os graus de liberdade eléctricos são separados consoante as condições de fronteira eléctricas, curto circuito e circuito aberto. A diferença entre ambos os casos serve para perceber as capacidades de conversão de energia mecânica em energia eléctrica. O fenómeno electromecânico é avaliado em ambos os modelos, através de três funções diferentes: actuação mecânica ou receptância, sensorização ou tensão por unidade de força, e actuação eléctrica ou deslocamento por unidade de tensão aplicada no transdutor.

Depois de estudados todos os assuntos anteriormente descritos, é implementado o amortecimento através do uso do shunt nas vigas. O circuito shunt utilizado é do tipo ressonante, e como tal constituído por elementos resistivos e indutivos. As propriedades destes elementos são definidos através da capacidade de conversão de energia mecânica e eléctrica para cada viga, obtida por comparação entre os dois casos limites de condições de fronteira eléctricas. Sucessivamente são introduzidos o shunt nas vigas e demonstrada a sua capacidade de amortecimento para diversos casos.

Palavras-chave: amortecimento passivo de vibrações, shunt, conversão de energia mecânica em eléctrica; materiais piezoeléctricos..

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Chapter 1

Introduction

1.1 Motivation

Precision is a value aimed when working with dynamic structures. Structural and Vibration Control, like many other subjects, was first explored by aerospace and military agencies, financed by defense programs. However, vibration control is spreading among other fields of engineering, as a subject of major interest; automotive industry, railways, sport equipment.

Let us consider a car cornering where the mechanical energy generated by the centrifugal force will be dissipated on its suspension as well as on its chassis (torsional solicitation). The stiffer the chassis, the more energy will be absorbed by the suspension components, which are more easily and freely configured than a chassis. A stiffer chassis will also mean more predictable reactions from the car and less mass transfer rebound, which means more trajectory precision; noise comfort can also be improved by a stiffer chassis. Inner structure damping is also very important specially for localized points of resonance where vibration amplitude can be attenuated, improving its fatigue performance.

Increasing stiffness and damping may be achieved by a better design, better materials (polymeric viscoelastic materials) and even improved manufacturing process. This would be the conventional passive structure configuration approach [[Preumont \(2002\)](#)]. Using actuators and sensors, we pass into active structures, which can lead us to the same level of performance as passive structures with the benefit of being lighter but usually not cheaper. This approach is obtained by using the so called *smart* materials, materials where strain/stress can be induced/produced by different mechanisms like electric fields, magnetic fields, temperature. As these materials are becoming mainstream, combined with electronic component's low cost, an active structural approach is nowadays turning cheaper and cheaper towards the cost of the passive one. It is important also to emphasize the following: a bad structural design will not be compensated with active strategies, it will remain bad in most cases, or at least it will not have the same performance level as flawless design would. So active structures should only be employed when all other approaches have been exhausted.

Passive vibration control via shunted modal piezoelectric transducers (PVC-SMPT) is not considered purely an Active Structure [[Preumont \(2002\)](#)], since properties (stiffness and damping) are varied by extracting mechanical energy through piezoelectric transducers capability of converting it in electrical energy. The electric energy generated on the piezoelectric transducer is dissipated by a shunt electrical circuit, meaning no power input whatsoever, unlike active control solutions. Active structures have commonly piezo-

electric transducing setups which measure the input needed and then compensate it by inducing an electrical charge through the structure. Therefore PVC-SMPT can be named as a semi-active solution, which has been studied in the last decade as an alternative to the active and viscoelastic strategies.

The Piezoelectric transducers can act in three different modes; namely longitudinal, transversal and shear mode. Shunt electronic circuit used for damping purposes, can be inductive, resistive, capacitive or switched [Moheimani (2003)]. The inductive shunt is also known as resonant shunt and operates like a classical mechanically tuned vibration absorber [Hagood and von Flotow (1991)]. The capacitive shunt leads to a frequency dependent stiffness whereas the resistive shunt provides dependent damping [Benjeddou and Ranger (2006)]. Switched shunt circuits makes damping and stiffening only to happen on demand (switch on/off shunt circuits).



Figure 1.1: Carbon fiber chassis as an example for future piezoelectric transducers integration for vibration control.

Piezoelectric materials are easily integrated into structures, these materials are typically available as very thin patches, thereby they can be easily surface bonded or embedded into structures. This second method tends to grow in interest, since composite structures are also becoming the mainstream in structural design, with automotive brands like Toyota, GM, McLaren or Mountain Bike brands promising low priced carbon-fiber products soon. Embedded transducers can benefit from higher damping ratios than a bonded solution, by using shear coupling piezoelectric coefficients [Benjeddou and Ranger (2006)]. An example for future piezoelectric transducers integration for vibration control of carbon fiber chassis is given in figure (1.1).

1.2 High Performance Structures

As mentioned before active structures rely on the use of actuators and sensors to modify its performance in order to meet the needs provided by the surrounding environment. In other words, this concept mimics the ability to adapt itself according to

different conditions, found only in living beings. Animals, plants and others forms of life have evolved in order to meet the conditions of their own habitat, developing disguising camouflages (several insects and reptiles), harsh surfaces (cactus), poisons (snakes and mushrooms), fast DNA mutations (viruses and bacteria). Biologic adaptation mechanisms can take long time periods to develop or can be completely immediate, go no further, the human body has several capabilities of adapting itself for different conditions, like sweating in order to refresh itself when submitted to high temperatures, or the ability that eyes have to adapt for different light conditions by increasing or decreasing the pupils size, etc. This biological inspired engineering is pointed out by many authors, [Vasques (2008), Preumont (2002)], as the way for 21st century engineering, in order to produce high performance structures. Active structures is one of the many categories of structures following this philosophy; high performance structures comprehends several other concepts, like sensory, actuated, controlled, intelligent and adaptive structures see figure (1.2).

Table 1.1: High performance structures.

Category	Sensors	Actuators	Description
Sensory	yes	no	having sensors it allows the monitoring of the actual state of the structure, therefore is often used for structural "health" monitoring purposes
Actuated	no	yes	enable the alteration of the structural or characteristics on demand
Controlled	yes	yes	combines sensing and actuating behaviors through a controller; low degree of structural and electrical integration; the controller is well set apart from the structure
Active	yes	yes	force and displacement based on sensors/actuators; highly integrated sensors/actuators; control integration is weak; used on truss structures.
Intelligent	yes	yes	autonomous structural system; enable auto adaptation to changing environmental conditions; low degree of control integration.
Adaptive	yes	yes	most exclusive controlled structures; both high degree of sensor/actuators and controller integration; capability of altering both mechanical properties and states; have self learning, self adaptive and decision capabilities.

Sensors and actuators are coupled to the host structure by controllers which, if its bandwidth includes vibration modes of the structure, will act dynamically towards the structure. Typically actuators and sensors may have a high degree of integration inside the structure, making a separate modeling a difficult task, leading many times to consider some of their properties as negligible.

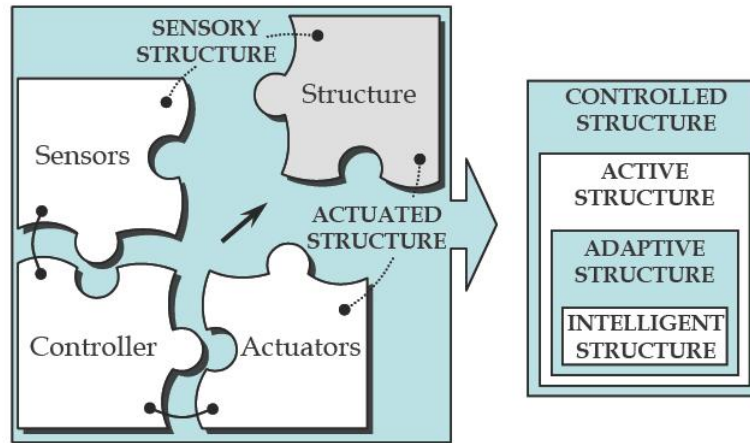


Figure 1.2: Conceptual definition of high performance structures and their constituents, [Vasques (2008)].

1.3 Smart Materials

Classical materials in mechanical engineering, are materials that relate typically strain and stress, elastic constant, or strain and temperature, thermal coefficient. On the other hand, smart materials involve other mechanisms of generating strain, mechanisms such as electric or magnetic fields.

Table 1.2: Examples of physical domains and associated energy conjugated state variables.

Mechanical	Electrical	Magnetic	Thermal	Chemical
stress	electric field	magnetic field	temperature	concentration
strain	electric displacement	magnetic flux	entropy	volumetric flux

The most common smart materials, according to Preumont (2002), are the following:

Shape Memory Alloys (SMA)

This kind of alloys can recover up to 5% strain phase change induced by temperature. They are only suitable for low precision and low frequencies application, and are known for fatigue problems caused by thermal cycling solicitation. SMA is not a class of materials suited for vibration control, except for high frequency applications.

Piezoelectric materials

Piezoelectric materials have a recoverable strain of 0.1% under an electric field. There are two main broad classes of Piezoelectric materials, Piezoceramics and Piezopolymers.

The first ones are used both as actuators and sensors, and can work for a wide range of frequencies, including ultrasonic, while the Piezopolimers are mostly used as sensors, since they have limited control authority due to smaller electromechanical coupling coefficients. This class of smart materials will be more extensively discussed in the further sections, since is the main class covered and utilized in this work.

Magnetostrictive

Magnetostrictive materials have recoverable strain of 0.15% under a magnetic field, its performance is improved when submitted to compression loads, which makes them ideal actuators for load carrying elements; it is also important to mention that these materials have long work life.

Magneto-rheological

Magneto-rheological can be described as fluids carrying micron-sized magnetic particles. By applying a magnetic field, these particles organize themselves in column structures, reacting to minimum shear stress to initiate flow. This class of materials has been used in automotive shock absorbers, for some years now, and have the benefit of giving different damping modes, by using an on/off magnetic field, which is possible by the reversibility and speed of the process of organization/disorganization of the column structures.

1.4 Shunted Piezoelectric Transducers Technologies

This section intends to present a brief overlook at the background of passive vibration control (PVC) with shunted modal piezoelectric transducers (SMPT). PVC with SMPT may be considered as a relatively recent subject, taking us back to the late 1980's [[Crawley and de Luis \(1987\)](#), [Hagood and von Flotow \(1991\)](#)] when it appeared as a derivation from the active vibration control approach.

Piezoelectric material's eminence is nowadays unquestionable and well recognized in the field of vibration control [[Preumont \(2002\)](#), [Vasques and Dias Rodrigues \(2009\)](#)], since these broad of materials have the following particularities: strain related electrical potential and vice-versa, easily connectable to an input/output electrical system, easily surface bonded or embedded into structures.

Piezoelectric materials/transducers can be used as passive energy dissipation devices by using an electrical impedance (shunt). When bonded to a host structure, the piezoelectric transducer will strain along with the structure while it vibrates, generating an electric field. The shunt dissipates the electrical energy thereby dissipating the mechanical vibration energy.

Piezoelectric transducers for passive vibration control schemes were first presented as simple inductor-resistor network coupled as electrical shunt to piezo electrics [[Hagood and von Flotow \(1991\)](#)]. Their performance was very similar to viscoelastic damping treatment. Hagood and Von Flotow performed these first works using Rayleigh-Ritz analytical formulation and, despite the fact that this method leads to overestimated values, they have achieved encouraging correlation between experimental and numerical results.

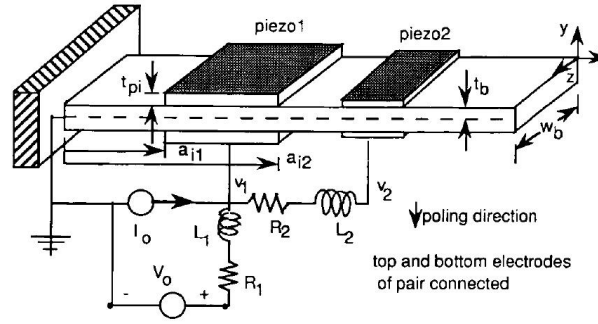


Figure 1.3: Assumed geometry of simple beam showing surface mounted piezoelectric and electronics [Hagood and von Flotow (1991)].

Firstly a single piezoelectric device was used to suppress a single mode only, but other authors have explored further this subject, exploiting concepts like multi-modal suppression [Hollkamp (1994)]. Hollkamp used a single piezoelectric device tuned for various modes. He demonstrated that multi-modal damping could be achieved in his experience, where a two-mode suppression on a clamped-free beam is considered. In order to achieve that goal, parallel L-R-C circuits were added, each one tuned for its mode. Multimodal dampers of this type are able to suppress any number of structural modes, but with handicap of having complex electronics. For instance when an extra branch is added the previous resistive and inductive elements must be retuned to ensure the right performance, meaning that finding the optimal solution may turn into a big puzzle.

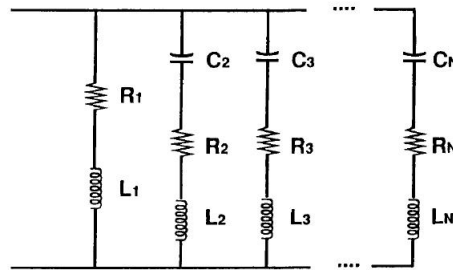


Figure 1.4: Multi-modal shunt circuit [Hollkamp (1994)].

Other techniques were proposed in a survey about innovations on this field made by Moheimani (2003) where is described a technique centered on the use of RL (either parallel, or series) shunt for each mode, and then inserting current blocking LC circuits into each branch. This method also presented difficulties such as rapidly increasing shunt size with the number of modes that are to be shunt damped.

So far have been discussed electric circuit shunt circuits that are realizable with passive components such as resistors, capacitors and inductors, but when the need to shunt damp low frequency modes arise, complications appear as well. For instance, low frequencies require very often large inductances, not physically possible to apply. Such inductive elements are implemented using Gyrator circuits [Moheimani (2003)].

But the passive vibration shunt damping does not rely only on smart electronics, spatially shaped transducers can act as modal filter for the unwanted vibration modes, leading to a more simpler shunt circuit needed [Miu (1993), Schoeftner and Irschik (2009)]. There are two ways to create a distributed transducer. One is to use an array of point transducers, which interact discretely in space, and the other is to use a single, spatially continuous distributed transducer. The first technique mean more electronic components and can suffer from lack of space when several modes are to be damped. The second one relies on a single patch with varying width according to the modes desired to be damped.

Most of these studies have been performed with piezoelectric material bonded to the upper surface of a beam, and most of them considered extension piezoelectric coefficients only (ESD), neglecting shear-mode coefficients, which are typically higher then the extension ones. Benjeddou and Ranger (2006) presented the shear-mode shunted damping (SSD). His work presented quite good results. Using a aluminium beam with sandwiched and surface-bonded piezoceramics patches led him to achieve damping levels twelve times higher than ESD ones and a two times reduction on amplitudes.

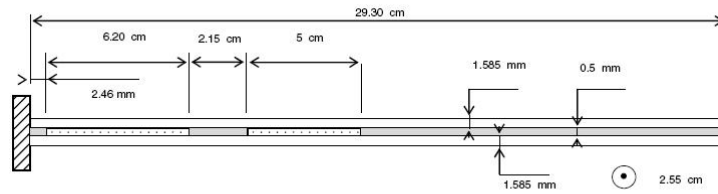


Figure 1.5: SSD model [Benjeddou and Ranger (2006)]

Nowadays are appearing concepts like exact annihilation of vibration modes using shaped control strategies [Schoeftner and Irschik (2009)]. The goal of shape control is to achieve a desired displacement by shaping and actuating the piezoelectric material.

Other concept, which seems to be very trendy nowadays, even for reasons such as energetic sustainability and environmental concerns, is the power harvesting, which consists on producing electrical energy for equipment alimentation from structures vibration [Fleming et al. (2003), Maurini and Porfiri (2004), Lefeuvre et al. (2006)]. Other author have studied power harvesting of beam structures using shoe inserts during human walking activity, with limited success only for low frequencies [Mateu and Moll (2005)].

After this brief overlook at the field of vibration damping, one must say it is undeniable the paramount of piezoelectric shunt technologies methods for future application, whether being applied on buildings, vehicles, tools, etc. Kozek et al. (in press) presented a study of inclusion of piezo-stacks for active vibrations damping in a flexible railway car-body, by introducing bending moments. This study was based on numerical simulation and experimental tests using a 1/10 model. Using this piezoelectric technologies showed an improvement in passenger comfort by 20% to 27%.

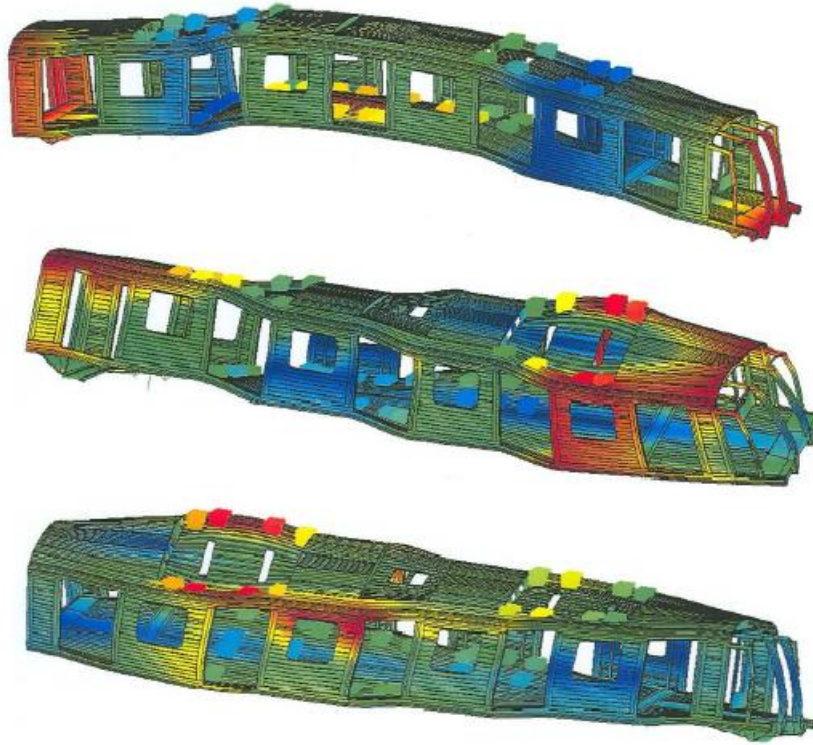


Figure 1.6: Railways car-body first 3 vibration modes [Kozek et al. (in press)].

1.5 Objectives

The main purpose of this text is to get in touch with active structural engineering applied to vibration control, more specifically passive vibration control of beams via shunted piezoelectric transducers technologies.

Other goal will be the definition of the behavior of a two-layered beam, where one of the layers is elastic and the other piezoelectric, using electromechanical models in order to determine mechanical-electrical energy conversion relationship. This mechanical-electrical energy conversion will be the baseline for vibration damping. In order to do that, this work will embrace both analytical and numerical models (finite element models), in order to realize the discrepancy between them. This study will focus on the two-layered beam structure, their shapes; will be considered both uniform and modally shaped configurations, different transducers sizes and locations for two electric boundary conditions (EBCs), Closed-circuit and Open-circuit cases. The difference between the results from both EBCs will give us the idea of the amount of energy conversion capability for each given example.

The vibration damping actually is achieved using a shunt circuit in series with the whole structure, since the electric energy produced in the piezoelectric transducer/electrode is dissipated through it. This work will contemplate a resonant shunt strategy. The shunt parameters are obtained from the EBCs results from each test made. Having in mind the different configurations mentioned previously, shunt performance is also dependent on location, size and shape properties of the transducers. Therefore will made a shunt damping performance evaluation, considering their limitations and practicability.

1.6 Structure of the Dissertation

Considering the aforementioned objectives, this dissertation is organized in six parts which are described as follows:

- Chapter 1 is dedicated to the presentation of the motivation behind this work, along with a background research in order to introduce concepts/terminology and relevant studies made over the last two decades in the field of passive vibration control via shunted strategies. This first Chapter also includes a brief presentation of two inherent concepts, active structures and smart materials, this approach intends to show the foreseen evolution mechanical engineering in the 21th century, which is becoming a more and more multi-disciplinary, rational and even more biologically inspired field of work. The main objectives are also presented in this Chapter.
- Chapter 2 comprehends the formulation of analytical model for a two layered beam, containing two different kinds of materials, namely an elastic material (aluminium, steel, etc), and a piezoelectric material. This formulation is developed having in mind several principles, and assumptions such as Euler-Bernoulli beam's theory, Hamilton's variational theory and piezoelectric constitutive equations. This chapter also contains a more detailed description of piezoelectric materials.
- Chapter 3 leads the formulation made in Chapter 2 further by adding the shunt damping theme, meaning that now the formulation considers now a shunt electrical circuit connected to the host structure. In this chapter are developed electrical flow, electrical potential functions for different electric boundary conditions, different transducers shapes and location.
- In Chapter 4 the formulation made in Chapter 2 and 3 is applied, considering two different approaches: analytical and numerical models. The numerical modeling is base on the finite element theory, which is briefly presented in this chapter, along also with brief presentation of the software in which the FEM model was developed (COMSOL Multiphysics). A comparison between both model is presented for simple elastic beams, and afterward are presented results for mechanical actuation, sensing and electrical actuation elastic beams with piezo layers bonded. In this section are evaluated transducers with different shape, location and size.
- In Chapter 5 the study of behavior of the resonant shunt, added to the models developed in chapter 4, is presented. Here are exploited issues like shunt properties for each transducer configuration and its damping performance.
- Lastly, the dissertation is concluded in Chapter 6, where is presented a summary containing an analysis of the work done focusing significant aspects and also presenting suggestions for experimental implementation and a foresight to what can be applied to the market in terms of passive piezoelectric vibration control is presented.

Chapter 2

Electromechanical Analytical Model

2.1 Introduction

This chapter presents a brief presentation of piezoelectric materials and an analytical model formulation of a two layered beam, in which the top layer is piezoelectric. The displacement field is postulated using Euler-Bernoulli classic beam theory. Afterwards the standard piezoelectric constitutive equations are developed, using assumptions for thin beam's behavior. In order to obtain actuating, sensing equations and boundary conditions, Hamilton's principle based on the virtual work done by electromechanical (internal and external) and inertial forces is applied.

2.2 Piezoelectric Materials.

Piezoelectric materials play an important role on active structural design, since they have the ability to deform when submitted to electric charge (inverse-effect), and to generate electrical charge when submitted to an external force (direct-effect). This effect was first discovered in 1880 by Pierre and Jacques Curie [[Moheimani \(2003\)](#)].

The Piezoelectric effect is anisotropic, meaning its crystalline structures has electric dipoles randomly displaced, thereby there's no electric dipole on the macroscopic level and the response to an externally applied electric field would be canceled within the electric dipoles, without observable deformation effect of the material. In order to benefit from the piezoelectric effect the piezoelectric material is subjected to a process named "poling". Poling consists on heating the piezoelectric material above its "Curie temperature", where the electric dipoles are able to change its orientation. In this stage the material is also submitted to a very strong magnetic field that will define the direction of polarization. Cooling it under the "Curie temperature", the electric dipoles will remain permanently fixed in the polarization direction.

Piezoelectric materials present also Pyroelectric effect, meaning that an electric charge can be generated with temperature and vice versa

The most popular piezoelectric materials are:

- Lead-zirconate-titanate (PZT);
- Polyvinylidene fluoride (PVDF)

PZT transducers are preferred over PVDF materials for actuating purposes, since their piezoelectric coefficients are much higher. For instance the piezoelectric coefficient, e_{31} , is 300 times larger in PZT than in PVDF, see table (2.1). Piezo-ceramics are isotropic materials in the plane, meaning $e_{31} = e_{32}$, while piezopolimers since are polarized under stress are highly anisotropic, having a ratio of 1 to 5 between these same coefficients. In table (2.1) some typical properties of both materials are presented.

Table 2.1: Properties of both PZT and PVDF materials [Preumont (2002)].

Properties	PZT	PVDF	unit
Piezoelectric constants			
d_{33}	300	-25	(m/V)
d_{31}	-150	15	(m/V)
d_{15}	500	—	(m/V)
$e_{31} = d_{31}/s^E$	-7.5	0.025	(C/m ²)
ϵ^T	1800	10	
Young's modulus	50	2.5	(GPa)
Maximum Stress			
Traction	80	200	(MPa)
Compression	600	200	(MPa)
Maximum Strain	Brittle	50%	
Coupling factor k_{33}	0.7	0.1	
Max. Electric Field	2000	5e5	(V/mm)

There are two main designs for piezoelectric actuators/sensors; see for example Physic Instrumente Catalogue [Preumont (2002)]), as presented in figure 2.1:

Stacked design (linear actuator)

The linear Piezo actuators consist on thin ceramic layered stacks, its thickness range can vary from 0.1 to 1mm. Both polarization axis and electric field are normal to the layer, agreeing with the direction of expansion, therefore actuation capability is controlled by e_{33} coefficient.

Linear actuators can be either used for low voltage systems, low voltage piezo (LVPZ, 0.1mm layers, 100V) or high voltage system, high voltage piezo (HVPZ, 1mm layers, 1000V), both present similar strain capabilities, but LVPZ have higher electrical capacitance and require large electrical current, while HVPZ require low current. The maximum expansion can vary from 0.1 to 0.13%.

Laminar design (spatially distributed actuators);

This Piezoelectric design consists on a spatial distributed layers, where the direction of expansion is normal to the present electric field. Thus the coefficient e_{31} is responsible for the correlation between strain and electrical displacement.

Laminar layers can be ceramic or polymeric, the first ones (PZT) have higher actuation capability, as referred before, while polymeric are known for its higher flexibility and wider range of thickness dimensions available. Typically PZT layers have a thickness of approximately 250 μ m.

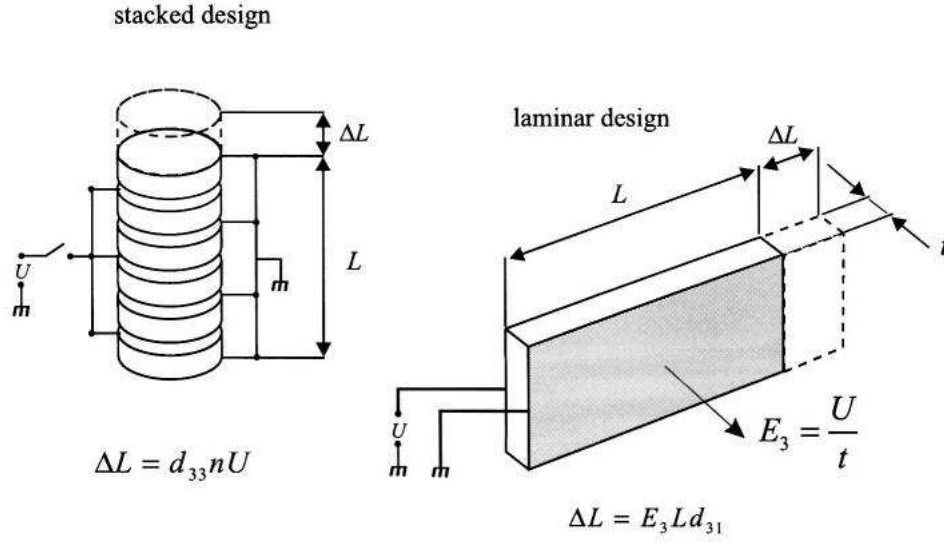


Figure 2.1: Piezoelectric actuators design [Preumont (2002)].

2.2.1 General constitutive equations

For piezoelectric materials, the one-dimensional electromechanical coupling equations can be generically expressed as

$$T = c^E S - e E \quad (2.1)$$

$$D = e S + \epsilon^S E \quad (2.2)$$

where T , S , D and E are, the stress, strain, electric displacement, and electric field respectively; c^E , e and ϵ are Piezoelectric material equivalent Young's modulus, piezoelectric coefficient (relates electric displacement with Strain) and dielectric constant, respectively. The first equation is the baseline for actuating purposes, while the second is for sensing purposes.

A normal orthorhombic crystal piezoelectric material of the class $mm2$, the former equations can be translated into matrix form, yielding:

$$\begin{Bmatrix} T_{xx} \\ T_{yy} \\ T_{zz} \\ T_{yz} \\ T_{zx} \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 \\ c_{12}^E & c_{22}^E & c_{23}^E & 0 & 0 & 0 \\ c_{13}^E & c_{23}^E & c_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}^E \end{bmatrix} \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ S_{yz} \\ S_{zx} \\ S_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}, \quad (2.3)$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ S_{yz} \\ S_{zx} \\ S_{xy} \end{Bmatrix} + \begin{bmatrix} \epsilon_{11}^S & 0 & 0 \\ 0 & \epsilon_{22}^S & 0 \\ 0 & 0 & \epsilon_{33}^S \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix}. \quad (2.4)$$

From the Equation (2.3), one can see that for a stress parallel to the direction of the electric field, for instance, T_{zz} and D_z , the extension observed will be governed by the piezoelectric coefficient e_{33} . It can also be seen that, if a shear stress, for instance, T_{zx} is present, and the present electric field is E_z , the coefficient governing the deformation will be the piezoelectric e_{15} . Both coefficients, e_{33} and e_{15} have higher value than e_{31} , as seen before in Table (2.1), therefore can lead to higher electromechanical coupling models.

2.3 Constitutive Equations for Laminar Transducers

In this work a laminar piezoelectric patch bonded to a thin beam through a strong adhesive will be considered, making possible the consideration of perfect coupling between piezo patch and the beam's surface.

Euler-Bernoulli's theory for beams, is only suitable for beams where its length is much larger than its thickness, $l \gg 20(2h_b)$, where l stands for length and $2h_b$ for thickness. This theory, commonly referred to as simply beam theory, plays an important role in structural analysis because it provides the designer a simple tool. Although more sophisticated tools, such finite element method are now widely divulged, this theory is often used at a pre-design stage because they provide valuable insight into the behavior of structures. Such method is also quite useful when trying to validate a purely computational solution.

A fundamental assumption of this theory is that the cross-section is infinitely rigid in its own plane, meaning no deformations will occur in the plane of the cross-section. Consequently, the in-plane displacement field can be represented simply by two rigid body translations and one rigid rotation. Two additional assumptions deal with out of plane displacements of the section during deformation: the cross-section is assumed to remain plane and normal to the deformed axis of the beam.

Under the action of transverse loads, bending moments, transverse shear forces, axial and transverse shearing stresses will be generated in the beam, inducing the beam to bend, creating a transverse displacement and curvature of the beam axis. The former assumptions are still valid, for pure bending beams, and furthermore, it can be assumed that transverse loading will only cause transverse displacement and curvature. Therefore shear and rotation effects are negligible [Bauchau and Craig (2009)].

Under these assumptions we obtain the following displacement field,

$$w(x, z, t) \equiv w(x, t), \quad u(x, z, t) = -z \frac{dw(x, t)}{dx}, \quad (2.5)$$

where u , w and z mean, respectively, the axial displacement, the transverse displacement, and the distance to the neutral axis of the beam. The axial displacements depends on the rotation, $\frac{dw(x, t)}{dx}$, and the transverse distance between a generic coordinate and the neutral axis.

The normal deformation, ε_{xx} , is obtained using the former equation and derivating it, yielding

$$\varepsilon_{xx} = -z \frac{d^2 w(x, t)}{dx^2} = S_{xx}. \quad (2.6)$$

Furthermore, the related stress, σ_{xx} is given by

$$\sigma_{xx} = -z Y_b \frac{d^2 w(x, t)}{dx^2}, \quad (2.7)$$

where Y_b represents the beam's Young modulus. Having in mind the former assumptions for thin beams and the behavior of piezoelectric materials, it can be made the following considerations:

- it will be only considered deformation on x and z axis, S_{xx} and S_{zz} , neglecting shear deformation components. The deformation S_{yy} , although related to S_{xx} by its Poisson coefficient, it will be neglected based on the small value of the beam's width. The shear deformation components are neglected based on former Euler-Bernoulli assumptions. Therefore we have $S_{yy} = S_{yz} = S_{zx} = S_{xy} = 0$;
- it will also be considered a plain stress tension, $T_{zz} = 0$; this will make the following piezo-electrics coefficients to change, c_{11}^E , e_{31} and ϵ_{33}^S , in order to accommodate that assumption;
- it will only be under consideration stress in the x direction, $T_{xx} \neq 0$; also will be only considered the electric displacement D_z .

The former facts make possible the definition of our constitutive equations, as will also modify piezo-materials constants. Both issues are developed in the following algebraic process:

$$T_{xx} = c_{11}^E S_{xx} + c_{13}^E S_{zz} - e_{31} E_z, \quad (2.8)$$

$$T_{zz} = c_{13}^E S_{xx} + c_{33}^E S_{zz} - e_{33} E_z. \quad (2.9)$$

Since $T_{zz} = 0$, S_{zz} becomes

$$S_{zz} = \frac{e_{33} E_z - c_{13}^E S_{xx}}{c_{33}^E}. \quad (2.10)$$

Now, substituting Equation (2.10) into (2.9), it's obtained T_{xx} ,

$$T_{xx} = S_{xx} \left(c_{11}^E - \frac{(c_{13}^E)^2}{c_{33}^E} \right) + E_z \left(e_{31} - \frac{c_{13}^E e_{33}}{c_{33}^E} \right), \quad (2.11)$$

where the plane-stress one-dimensional piezoelectric Young's modulus and e_{31} piezo-electric coefficient can now be defined as :

$$c_{11}^{E*} = \left(c_{11}^E - \frac{(c_{13}^E)^2}{c_{33}^E} \right), \quad e_{31}^* = \left(e_{31} - \frac{c_{13}^E e_{33}}{c_{33}^E} \right). \quad (2.12)$$

Analogously, it will be made the same algebraic process for electrical displacement, also it shall only be considered electrical flux to occur in the z axis, meaning that electrical displacements, $D_x = D_y = 0$, therefore D_z is defined as

$$D_z = e_{31}S_{xx} + e_{33}S_{zz} + \epsilon_{33}^S E_z, \quad (2.13)$$

or

$$D_z = S_{xx} \left(e_{31} - \frac{c_{13}^E e_{33}}{c_{33}^E} \right) + E_z \left(\epsilon_{33}^S + \frac{e_{33}^2}{c_{33}^E} \right). \quad (2.14)$$

The one-dimensional dielectric constant is now defined as

$$\epsilon_{33}^{S*} = \left(\epsilon_{33}^S + \frac{e_{33}^2}{c_{33}^E} \right). \quad (2.15)$$

Finally, the one-dimensional equivalent electromechanical constitutive equations for beam case with a piezoelectric patch bonded to a Euler-Bernoulli thin beam, can be rewritten as

$$T_{xx} = c_{11}^{E*} S_{xx} - e_{31}^* E_z, \quad (2.16)$$

$$D_z = e_{31}^* S_{xx} + \epsilon_{33}^{S*} E_z. \quad (2.17)$$

2.4 Electric potential and Electric field

According to [IEEE (1988)], the electric field, E_z^p , for linear piezo-electricity depends on the electric potential between electrodes, ϕ_p , and the piezoelectric layer thickness, yielding

$$\phi_p \equiv v(t) = \varphi_p^{top} - \varphi_p^{bottom}, \quad E_z^p = -\frac{v(t)}{2h_p}, \quad (2.18)$$

where $2h_p$ is the piezoelectric layer's thickness, and φ_p^{top} and φ_p^{bottom} represent the electric voltage on the top and bottom electrodes. Having the electrodes connected, the electric potential is null, $\phi_p = 0$; otherwise if the circuit is open, the electric potential will become $\phi_p \neq 0$.

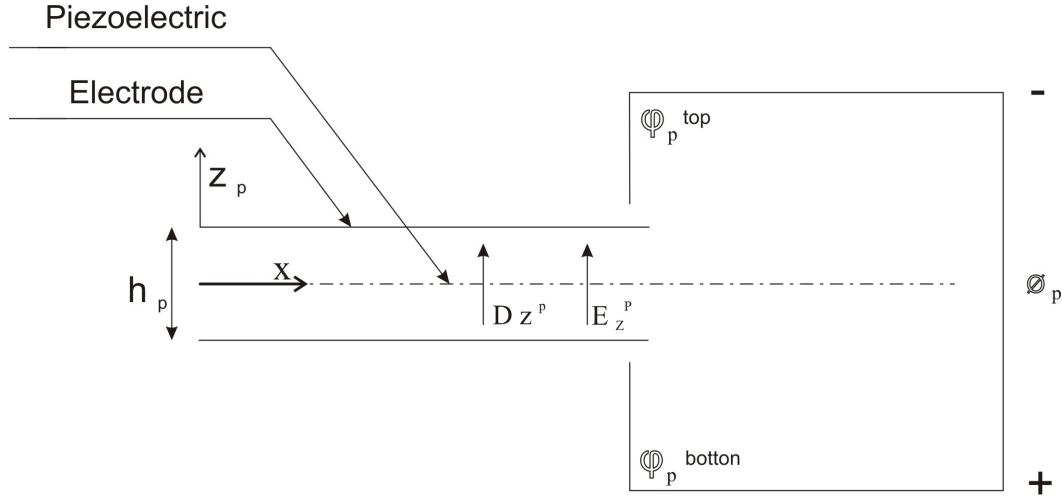


Figure 2.2: Electric field across a piezoelectric layer.

2.5 Variational Formulation

The current section describes a two layered beam transversal motion, as this will be the object of study for the application of passive vibration control with shunted modal piezoelectric transducers (PVC-SMPT). For this purpose it will be used the Hamilton's variational principle applied with Euler-Bernoulli assumptions for thin beams.

Hamilton's principle relies on the determination of dynamics of a physical system considering a variational problem based on the Lagrangian function, which contains all system's physical properties and external forces applied to it. In classical mechanics it can be described as the following sentence:

- The kinetic energy variation, δK , and deformation energy variation, δU , summed to the variation of the work realized by external non conservative forces during a period of time, $[t_1, t_2]$, equals zero.

$$\int_{t_1}^{t_2} \delta(K - U)dt + \int_{t_1}^{t_2} \delta W dt = 0 \quad (2.19)$$

For this study, it will be used an extension of the Hamilton's Variational principle for electromechanical coupling of piezoelectric layer to a beam, where the deformation energy variation, δU , is substituted by the electro-mechanical enthalpy, δH . This new concept will be described in the following sections, along internal stored energy of piezoelectric materials, U . Thus the extended electro-mechanical Hamilton's variational principle is now defined by

$$\int_{t_1}^{t_2} \delta(K - H)dt + \int_{t_1}^{t_2} \delta W dt = 0 \quad (2.20)$$

how

Using Hamilton's variational principle the governing equation for a two layered beam (elastic+piezo-material) as well as the sensing equation for transverse vibration will be obtained, attending the following assumptions:

- Piezoelectric layer's stiffness and mass effect although not negligible are to be considered as such in a latter stage, in order to simplify the solution equation development;
- Piezoelectric layer's thickness is much thinner than the one found in the beam, $h_P \ll h$;
- Arbitrary piezoelectric layer width along the length of the beam.

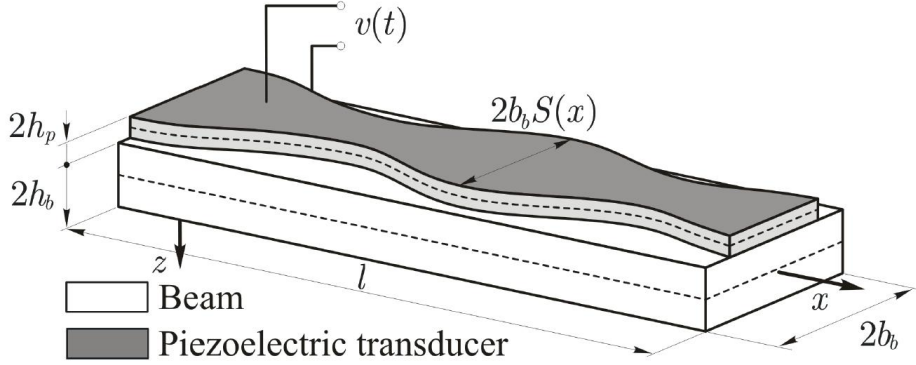


Figure 2.3: Generic beam Structure with an arbitrary spatially shaped distributed piezo electric transducer [Vasques and Dias Rodrigues (2009)]

2.5.1 Virtual work of the internal electro-mechanical forces

Following [IEEE (1988)] standard formulation for linear pieoelectricity, electro-mechanical enthalpy virtual work, H , is given by

$$H = \int_V \frac{1}{2} c_{ijkl}^E S_{ij} S_{kl} - e_{kij} E_k S_{ij} - \frac{1}{2} \epsilon_{ij}^S E_i E_j dV \quad (2.21)$$

Using the constitutive equations, developed before, for our specific problem, H yields

$$H = \int_V \frac{1}{2} c_{11}^{E*} S_{xx}^2 - e_{31}^* S_{xx} E_z - \frac{1}{2} \epsilon_{33}^{S*} E_z^2 dV. \quad (2.22)$$

From the former equation, one can note that electro-mechanical enthalpy can be divided in 3 components, mechanical, H_{uu} , piezoelectric, $H_{u\phi}$ and $H_{\phi u}$, and dielectric, $H_{\phi\phi}$. Although the electro-mechanical enthalpy definition, as stated, considers only the piezoelectric layer, it can be also extended to the host beam layer, by considering only the mechanical virtual term. It is considered a generic layer i , with $i = (p, b)$, where p stands for the piezoelectric layer and b for the host beam. For matters of simplicity, the virtual work of electro-mechanical will be stated in the following way:

$$\delta H = \delta H_{uu}^i - \delta H_{u\phi}^i - \delta H_{\phi u}^i - \delta H_{\phi\phi}^i \quad (2.23)$$

where

$$\delta H_{uu}^p = \int_V \delta S_{xx}^p c_{11}^{E*} S_{xx}^p dV, \quad \delta H_{uu}^b = \int_V \delta S_{xx}^b Y_b S_{xx}^b dV, \quad (2.24)$$

$$\delta H_{u\phi}^p = \int_V \delta S_{xx}^p e_{31}^* E_z dV, \quad (2.25)$$

$$\delta H_{u\phi}^p = \int_V \delta E_z e_{31}^* S_{xx}^b dV, \quad (2.26)$$

$$\delta H_{\phi\phi} = \int_V \delta E_z \epsilon_{33}^* E_z dV. \quad (2.27)$$

In this formulation the axial displacement is considered constant along the piezo's thickness, and equivalent to the axial displacement displayed on the upper surface of the host beam, due to its thickness smaller value when compared to the beam's thickness. The following equations present strain definitions, for both piezo and beam,

$$S_{xx}^p = h_b \frac{d^2 w(x, t)}{dx^2}, \quad (2.28)$$

$$S_{xx}^b = z \frac{d^2 w(x, t)}{dx^2}, \quad (2.29)$$

for $-h_b \leq z \leq h_b$, recalling once more z as the distance between a generic point and the neutral axis.

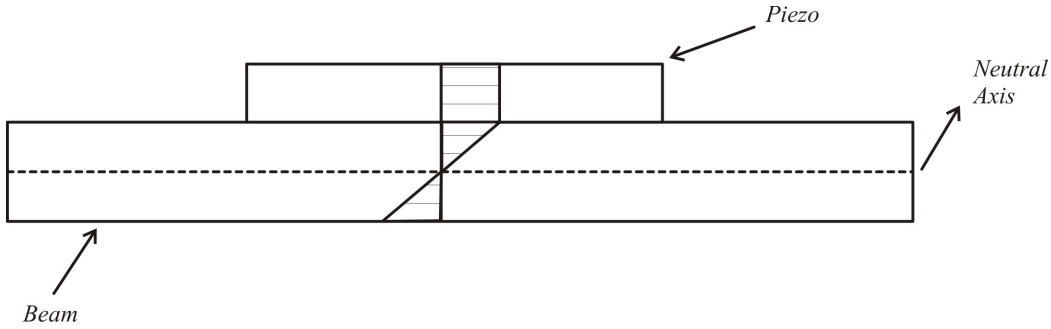


Figure 2.4: Axial displacement distribution.

Some other considerations must be made in order to develop the Equations (2.24)-(2.27). The cross-section area of both piezo and beam yield,

$$A_b = 4b_b h_b, \quad A_p = 4b_p(x) h_p, \quad (2.30)$$

and the second order moment of area are given by

$$I_b = z^2 4b_b h_b, \quad I_p^* = 4b_p(x) h_p h_b^2, \quad (2.31)$$

and that the equivalent young modulus for the piezo layer yields

$$Y_b \equiv c_{11}^{E*}. \quad (2.32)$$

Substituting the former definitions and the electric field equation into Equations (2.24)-(2.27) and integrating with respect to the cross-section, the mechanical terms for both beam and piezo layer become

$$\delta H_{uu}^p = \int_l \delta \frac{d^2 w(x, t)}{dx} Y_p I_p^* \frac{d^2 w(x, t)}{dx} dx, \quad \delta H_{uu}^b = \int_l \delta \frac{d^2 w(x, t)}{dx} Y_b I_b \frac{d^2 w(x, t)}{dx} dx. \quad (2.33)$$

Similarly, the electric terms become

$$\delta H_{u\phi}^p = \int_l \delta \frac{d^2 w(x, t)}{dx} b_p(x) c_E \frac{v(t)}{2h_p} dx, \quad (2.34)$$

$$\delta H_{\phi u}^p = \int_l -\delta \frac{v(t)}{2h_p} e_{31}^* h_b \frac{d^2 w(x, t)}{dx} A_p dx \quad (2.35)$$

$$\delta H_{\phi\phi} = \int_l \delta \frac{v(t)}{2h_p} \epsilon_{33}^{S*} \frac{v(t)}{2h_p} A_p dx, \quad (2.36)$$

where $c_E = 4e_{31}^* h_p h_b$.

2.5.2 Virtual work of inertial forces

The virtual work of the inertial forces in a generic layer i is given by

$$\delta K_p = - \int_V \rho_p \delta w(x, t) \frac{d^2 w(x, t)}{dt^2} dV, \quad (2.37)$$

$$\delta K_b = - \int_V \rho_b \delta w(x, t) \frac{d^2 w(x, t)}{dt^2} dV, \quad (2.38)$$

where ρ_b and ρ_p is the density for either the piezoelectric layer or beam. Let us recall that inertial rotation is neglected due to Euler-Bernoulli considerations. Integrating with respect to the cross-section, the kinetic terms become

$$\delta K_p = - \int_l \rho_p A_p \delta w(x, t) \frac{d^2 w(x, t)}{dt^2} dx, \quad (2.39)$$

$$\delta K_b = - \int_l \rho_b A_b \delta w(x, t) \frac{d^2 w(x, t)}{dt^2} dx. \quad (2.40)$$

2.5.3 Virtual work of external forces

In the determination of the virtual work of the mechanical external forces will be considered the virtual work done by transverse forces and by a prescribed mechanical induced distributed moment

$$\delta W_u = \int_l p_m(x, t) \delta w(x, t) dx + \int_l m_m(x, t) \delta \left[\frac{dw(x, t)}{dx} \right] dx, \quad (2.41)$$

where the second term in the right hand side can be rewritten as

$$\int_l m_m(x, t) \delta \left[\frac{dw(x, t)}{dx} \right] dx = [m_m(x, t) \delta w(x, t)] \Big|_l - \int_l \frac{dm_m(x, t)}{dx} \delta w(x, t) dx. \quad (2.42)$$

2.5.4 Virtual work of the electric charge density in the piezoelectric layer

The virtual work of the electric charge density in the piezoelectric layer is defined by

$$\delta W_\phi = - \int_{A_p^e} \delta \varphi_p \tau_p dA_p^e = - \int_l \delta v(t) \tau_p 2b_p(x) dx, \quad (2.43)$$

where A_p^e denotes the electrode area, τ_p is the applied charge density in the electrode, and $2b_p(x)$ is the piezoelectric arbitrary width. Note that from the definition of the electric potential (2.18), and considering only the applied potential term, one finds that $\varphi_{bottom}^p = 0$ and $\varphi_{top}^p = \phi_p \equiv v(t)$.

2.6 Governing Equations

Having developed in the former section the several components involved in the two layer beam behavior, is time now to proceed with assembling them in the Equation (2.20), and therefore develop the equations that will give us the actuating and sensing equations. This formulation contemplates the inertial and deformation components from the piezoelectric layer, although they will be later neglected, as stated in the former section. Performing the integration in a generic time interval $[t_1, t_2]$, the Equations (2.33)-(2.43) become

$$\begin{aligned} \int_{t_1}^{t_2} \delta H_{uu}^p dt &= \int_{t_1}^{t_2} \left[Y_p I_p^* \frac{d^2 w(x, t)}{dx^2} \frac{d}{dx} \delta w(x, t) - \frac{d}{dx} \left(Y_p I_p^* \frac{d^2 w(x, t)}{dx^2} \right) \delta w(x, t) \right] \Big|_l dt \\ &+ \int_{t_1}^{t_2} \int_0^l \frac{d^2}{dx^2} \left(Y_p I_p^* \frac{d^2 w(x, t)}{dx^2} \right) \delta w(x, t) dx dt, \end{aligned} \quad (2.44)$$

$$\begin{aligned} \int_{t_1}^{t_2} \delta H_{uu}^b dt &= \int_{t_1}^{t_2} \left[Y_b I_b \frac{d^2 w(x, t)}{dx^2} \frac{d}{dx} \delta w(x, t) - \frac{d}{dx} \left(Y_b I_b \frac{d^2 w(x, t)}{dx^2} \right) \delta w(x, t) \right] \Big|_l dt \\ &+ \int_{t_1}^{t_2} \int_0^l \frac{d^2}{dx^2} \left(Y_b I_b \frac{d^2 w(x, t)}{dx^2} \right) \delta w(x, t) dx dt, \end{aligned} \quad (2.45)$$

$$\begin{aligned} \int_{t_1}^{t_2} \delta H_{u\phi}^p dt &= \int_{t_1}^{t_2} \left[c_E \frac{d}{dx} \delta w(x, t) b_p(x) E_z - c_E \delta w(x, t) \frac{db_p(x)}{dx} E_z \right] \Big|_l \\ &+ \int_{t_1}^{t_2} \int_0^l c_E \delta w(x, t) \frac{d^2 b_p(x)}{dx^2} E_z dx dt, \end{aligned} \quad (2.46)$$

where all 3 equations, (2.24) and (2.25) have been integrated in order to x twice.

$$\int_{t_1}^{t_2} \delta H_{\phi u}^p dt = \int_{t_1}^{t_2} \int_l -\delta \frac{v(t)}{2h_p} e_{31}^* h_b \frac{d^2 w(x, t)}{dx} A_p dx dt, \quad (2.47)$$

$$\int_{t_1}^{t_2} \delta H_{\phi \phi} dt = \int_{t_1}^{t_2} \int_l \delta \frac{v(t)}{2h_p} \epsilon_{33}^* \frac{v(t)}{2h_p} A_p dx dt, \quad (2.48)$$

$$\int_{t_1}^{t_2} \delta K_b dt = - \int_{t_1}^{t_2} \int_l \rho_b A_b \delta w(x, t) \frac{d^2 w(x, t)}{dt^2} dx dt, \quad (2.49)$$

$$\int_{t_1}^{t_2} \delta K_p dt = - \int_{t_1}^{t_2} \int_l \rho_p A_p \delta w(x, t) \frac{d^2 w(x, t)}{dt^2} dx dt, \quad (2.50)$$

$$\begin{aligned} \int_{t_1}^{t_2} \delta W_u dt = \int_{t_1}^{t_2} \left[\int_l p_m(x, t) \delta w(x, t) dx - \int_l \frac{dm_m(x, t)}{dx} \delta w(x, t) dx \right] dt, \\ + \int_{t_1}^{t_2} [m_m(x, t) \delta w(x, t)] |_l dt, \end{aligned} \quad (2.51)$$

$$\int_{t_1}^{t_2} \delta W_\phi dt = \int_{t_1}^{t_2} \int_l \delta v(t) \tau_p 2b_p(x) dx dt. \quad (2.52)$$

In order to perform the development of the Hamilton's extended principle (2.20), it is necessary to group the terms related to $\delta w(x, t)$ and $\delta v(t)$, so that variation function δ can be considered arbitrary, and therefore allowing to obtain the actuating, sensing and boundary conditions equations.

$\delta w(x, t)$:

$$\begin{aligned} - \left((\rho_p A_p + \rho_b A_b) \frac{d^2 w(x, t)}{dt^2} \right) - \left(Y_p I_p^* \frac{d^4 w(x, t)}{dx^4} \right) - \left(Y_b I_b \frac{d^4 w(x, t)}{dx^4} \right) - c_E \frac{d^2 b_p(x)}{dx^2} E_z \\ + p_m(x, t) - \frac{dm_m(x, t)}{dx} = 0, \end{aligned} \quad (2.53)$$

for $0 < x < l$.

$\delta v(t)$:

$$\frac{e_{31}^*}{2h_p} h_b \frac{d^2 w(x, t)}{dx^2} A_p - \frac{v(t)}{2h_p} \epsilon_{33}^* \frac{v(t)}{2h_p} A_p + \tau_p 2b_p(x) = 0, \quad (2.54)$$

for $0 < x < l$. Furthermore we also get the concentrated moments at the beam ends,

$$Y_p I_p^* \frac{d^2 w(x, t)}{dx^2} + Y_b I_b \frac{d^2 w(x, t)}{dx^2} = \pm c_E \frac{d}{dx} b_p(x) E_z, \quad (2.55)$$

at $x = (0, l)$ and concentrated forces at the beam ends:

$$\frac{d}{dx} \left(Y_p I_p^* \frac{d^2 w(x, t)}{dx^2} \right) + \frac{d}{dx} \left(Y_b I_b \frac{d^2 w(x, t)}{dx^2} \right) = \pm c_E \frac{db_p(x)}{dx} E_z \pm m_m(x, t) \quad (2.56)$$

at $x = (0, l)$.

2.6.1 Actuating equation

Recalling once more that for simplicity piezoelectric's kinematic and stiffness terms are considered negligible, the actuating equation becomes

$$\left(Y_b I_b \frac{d^4 w(x, t)}{dx^4} \right) + \rho_b A_b \frac{d^2 w(x, t)}{dt^2} = pm(x, t) + p_e(x, t) - \frac{dm_m(x, t)}{dx}, \quad (2.57)$$

for $0 < x < l$. The piezoelectric loading, $p_e(x, t)$, is given by

$$p_e(x, t) = -c_E \frac{d^2 b_p(x)}{dx^2} E_z. \quad (2.58)$$

The two natural boundary conditions are also achieved in the process,

$$M(x, t) = Y_b I_b \frac{d^2 \omega(x, t)}{dx^2} = \pm c_E b_p(x) E_z, \quad (2.59)$$

$$P(x, t) = \frac{d}{dx} \left(Y_b I_b \frac{d^2 \omega(x, t)}{dx^2} \right) = \pm c_E \frac{db_p(x)}{dx} E_z \pm m_m(x, t), \quad (2.60)$$

at $x = (0, l)$.

$M(x, t)$ and $P(x, t)$ are respectively the beam bending moment and shearing force. The piezoelectric actuation for free rotation at the ends of the beam, $\frac{dw(x, t)}{dx} \neq 0$ at $x = (0, l)$, is proportional to the electric field and to the piezo layer width, while if considered a free transverse displacement, $w(x, t) \neq 0$ at $x = (0, l)$, the piezoelectric actuation is equivalent to a concentrated transverse force proportional to the electric field and to the first derivative of the piezo width. The two natural boundary conditions are non-homogeneous, since E_z is only time dependent, therefore it is necessary to modify the approach on boundary conditions, turning them into homogeneous conditions so that modal analysis can be used. Thus mathematically the distributed moment and transverse load can be given by

$$m_{e/m} = M_e(0, t)\delta(x) + M_e(l, t)\delta(x - l), \quad (2.61)$$

$$p_{e/m} = P_e(0, t)\delta(x) + P_e(l, t)\delta(x - l) \quad (2.62)$$

where $\delta(x)$ is a spatial Dirac delta function. Since Equation (2.57) includes the effects of concentrated boundary moments and considering nil mechanical moments are prescribed at the boundaries, i.e. $M_m(0, t) = M_m(l, t) = 0$, the piezoelectric-induced “distributed” moments in equation (2.61), may be alternatively be considered as equivalent “distributed” mechanical moments, i.e. $m_m(x, t) = m_{e/m}(x, t)$. Also an equivalent distributed transverse force $p_m(x, t) = p_{e/m}(x, t)$, can be considered, leading the actuating equation for the electromechanical behavior of a two-layered piezo-elastic adaptive beam with a generic shape of piezoelectric transducer/electrode, to become

$$\left(Y_b I_b \frac{d^4 w(x, t)}{dx^4} \right) + \rho_b A_b \frac{d^2 w(x, t)}{dt^2} - p_e(x, t) - p_{e/m}(x, t) - \frac{dm_{e/m}(x, t)}{dx} = p_m(x, t), \quad (2.63)$$

where the piezoelectrically-induced loading term, $p_e(x, t)$, given by (2.58) and the concentrated ones given by

$$p_{e/m} = P_e(0, t)\delta(x) + P_e(l, t)\delta(x - l) = c_E \left[\frac{db_p(0)}{dx} \delta(x) - \frac{db_p(l)}{dx} \delta(x - l) \right] E_z,$$

$$\frac{dm_{e/m}(x, t)}{dx} = M_e(0, t)\delta'(x) + M_e(l, t)\delta'(x - l) = c_E [b_p(0)\delta'(x) - b_p(l)\delta'(x - l)] E_z. \quad (2.64)$$

The solution of the non-homogeneous actuating equation can be solved now through modal expansion solution, where the response is assumed to be expressed as linear combination of the natural mode shapes,

$$w(x, t) = \sum_{r=1}^n \phi_r(x) \eta_r(t), \quad (2.65)$$

where $\phi_r(x)$ and $\eta_r(t)$ are the r th mode shapes and modal coordinates, respectively, and n is the number of modes considered in the analysis.

2.6.2 Sensing equation

Recalling Equation (2.54), it can be simplified into

$$\tau_p = -e_{31}^* h_b \frac{d^2 w(x, t)}{dx^2} + \frac{v(t)}{2h_p} \epsilon_{33}^{S*}, \quad (2.66)$$

which is the fundamental sensing equation.

In order to obtain the total electrical charge present in the electrode is necessary to integrate (2.66) along the electrode area. Recalling that the electrode shape is not constant, meaning that the width of the electrode is allowed to vary along the length of the beam,

$$q(t) = - \int_{A_e^p} \left[e_{31}^* h_b \frac{d^2 w(x, t)}{dx^2} - \frac{v(t)}{2h_p} \epsilon_{33}^{S*} \right] dA_e^p, \quad (2.67)$$

where

$$dA_e^p = 2b_b S(x), \quad (2.68)$$

and $S(x)$ is a spatial sensitivity function of an arbitrary shaped piezoelectric transducer, comprised in the interval $[-1, 1]$, with the negative values denoting an inverse polarity of the piezoelectric transducer. Substituting (2.68) into (2.67), yields

$$q(t) = -e_{31}^* h_b 2b_b \int_l S(x) \frac{d^2 w(x, t)}{dx^2} dx + \frac{\epsilon_{33}^{S*} b_b}{h_p} \int_l S(x) dx v(t). \quad (2.69)$$

Physically, the former equation shows that the electric charge is proportional to the sum of a weighted integration of the curvature of the beam, recalling the direct piezoelectric effect, meaning that an electrical field is produced by the mechanical deformation, and a weighted integration of the prescribed voltage at the electrodes. As stated before, the sensitivity function, $S(x)$, may take negative values, denoting that the polarity on the piezoelectric transducer has been inverted, see figure (2.5), which can be obtained whether by flipping the piezoelectric patch on the negative polarity zone or by mounting the patch on the opposite side of the beam with the same (positive) direction of polarization.

The electric current, $i(t) = dq(t)/dt$, is given by

$$i(t) = -e_{31}^* h_b 2b_b \int_l S(x) \frac{d^3 w(x, t)}{dx^2 dt} dx + \frac{\epsilon_{33}^{S*} b_b}{h_p} \int_l S(x) dx \frac{v(t)}{dt}. \quad (2.70)$$

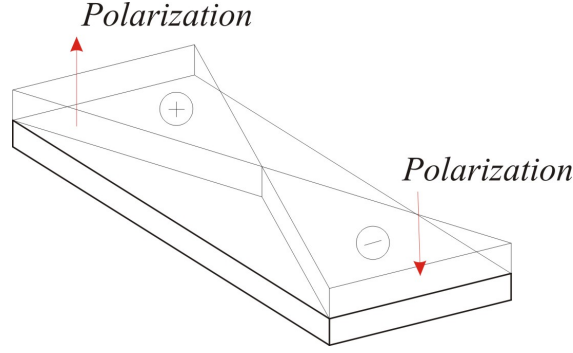


Figure 2.5: Polarization inversion

2.6.3 Single- and multi-modal shaped transducer for spatially filtering

The electric current equation similarly to the motion governing equation can be solved by using modal superposition approach. Thus, the equation (2.70), can be rewritten as

$$i(t) = -e_{31}^* h_b 2b_b \sum_{r=1}^n \left[\int_l S(x) \frac{d^2 \phi_r(x)}{dx^2} dx \right] \frac{d\eta_r(t)}{dt} + \frac{\epsilon_{33}^* b_b}{h_p} \int_l S(x) dx \frac{dv(t)}{dt}. \quad (2.71)$$

The sensitivity function, $S(x)$, is considered to be proportional to a generic sth modal strain distribution, yielding

$$S(x) \propto \alpha_s S(x) = \alpha_s \frac{Y_b I_b}{\omega_s^2} \frac{d^2 \phi_s(x)}{dx^2}, \quad (2.72)$$

being α_s a normalization factor. Substituting (2.72) into (2.71), yields

$$\begin{aligned} i(t) = & -e_{31}^* h_b 2b_b \sum_{r=1}^n \int_l \left[\alpha_s \frac{Y_b I_b}{\omega_s^2} \frac{d^2 \phi_s(x)}{dx^2} \frac{d^2 \phi_r(x)}{dx^2} dx \right] \frac{d\eta_r(t)}{dt} \\ & + \frac{\epsilon_{33}^* b_b}{h_p} \int_l \alpha_s \frac{Y_b I_b}{\omega_s^2} \frac{d^2 \phi_s(x)}{dx^2} dx \frac{dv(t)}{dt}, \end{aligned} \quad (2.73)$$

where if we recall the orthonormality properties, for $\phi_s = \phi_r$, the former equation can be simply reduced to

$$i(t) = -e_{31}^* h_b 2b_b \int_l \alpha_s \frac{d\eta_r(t)}{dt} + C_p^S \frac{dv(t)}{dt}, \quad (2.74)$$

$$C_p^S = \frac{\epsilon_{33}^* b_b}{h_p} \int_l \alpha_s \frac{Y_b I_b}{\omega_s^2} \frac{d^2 \phi_s(x)}{dx^2} dx. \quad (2.75)$$

Equation (2.75) represents the strain-free “modal capacitance”. From Equation (2.74), can be understood the filtering concept achieved by spatially shaped transducers, since the induced current is only proportional to the sth mode contribution to the net vibratory response of the beam, while all other mode’s contributions are automatically filtered. The same way can be seen, in Equation (2.75) that only one mode is actuated by a prescribed voltage.

However, one may want to design a multi-modal transducer, in order to sense/actuate in more than in one single mode. For a specific bandwidth over which the modal transducer should work or specific modes to be sensed/actuated, it can be achieved by generalizing the equation (2.72), as showed in the following equation

$$S(x) \propto \beta \sum_{k=1, k \neq m}^n S_k(x) = \beta \sum_{k=1, k \neq m}^n \left[\alpha_k \frac{Y_b I_b}{\omega_k^2} \frac{d^2 \phi_k(x)}{dx^2} \right], \quad (2.76)$$

where n is the upper mode number of the bandwidth of interest, m are the modes to be filtered, and β is the multi-mode normalization factor. Thus, substituting Equation (2.76) into (2.71), yields

$$\begin{aligned} i(t) = & -e_{31}^* h_b 2b_b \sum_{r=1}^n \left[\int_l \beta \sum_{k=1, k \neq m}^n \left[\alpha_k \frac{Y_b I_b}{\omega_k^2} \frac{d^2 \phi_k(x)}{dx^2} \right] \frac{d^2 \phi_r(x)}{dx^2} dx \right] \frac{d\eta_r(t)}{dt} \\ & + \frac{\epsilon_{33}^* b_b}{h_p} \int_l \beta \sum_{k=1, k \neq m}^n \left[\alpha_k \frac{Y_b I_b}{\omega_k^2} \frac{d^2 \phi_k(x)}{dx^2} \right] dx \frac{dv(t)}{dt}. \end{aligned} \quad (2.77)$$

The previous equation can be generalized in the same way, as the single model was, yielding

$$i(t) = -e_{31}^* h_b 2b_b \beta \sum_{k=1, k \neq m}^n \left[\alpha_k \frac{d\eta_k(t)}{dt} \right] + C_p^S \frac{dv(t)}{dt}. \quad (2.78)$$

Also there will be a generalized multi-modal capacitance, which is given by

$$C_p^S = \frac{\epsilon_{33}^* b_b}{h_p} \int_l \beta \sum_{k=1, k \neq m}^n \left[\alpha_k \frac{Y_b I_b}{\omega_k^2} \frac{d^2 \phi_k(x)}{dx^2} \right] dx. \quad (2.79)$$

Remembering that only the modes included on the bandwidth of interest will be observable, Equation (2.78) can be expressed as

$$\begin{cases} i_k(t) = -e_{31}^* h_b 2b_b \beta \alpha_k \frac{d\eta_k(t)}{dt} + C_{pk}^S \frac{dv(t)}{dt}, & k = r \\ i_k(t) = 0, & k \neq r \end{cases}, \quad (2.80)$$

where the new generalized multi-modal capacitance for the k th mode is given by

$$C_{pk}^S = \frac{\epsilon_{33}^* b_b}{h_p} \int_l \beta \left[\alpha_k \frac{Y_b I_b}{\omega_k^2} \frac{d^2 \phi_k(x)}{dx^2} \right] dx. \quad (2.81)$$

Spatially shaped piezoelectric transducers, as demonstrated in this section, have major interest in alleviating spillover effects, since it makes undesired vibration modes, modes which we do not want to observe in the bandwidth of interest, to be unobservable or controllable.

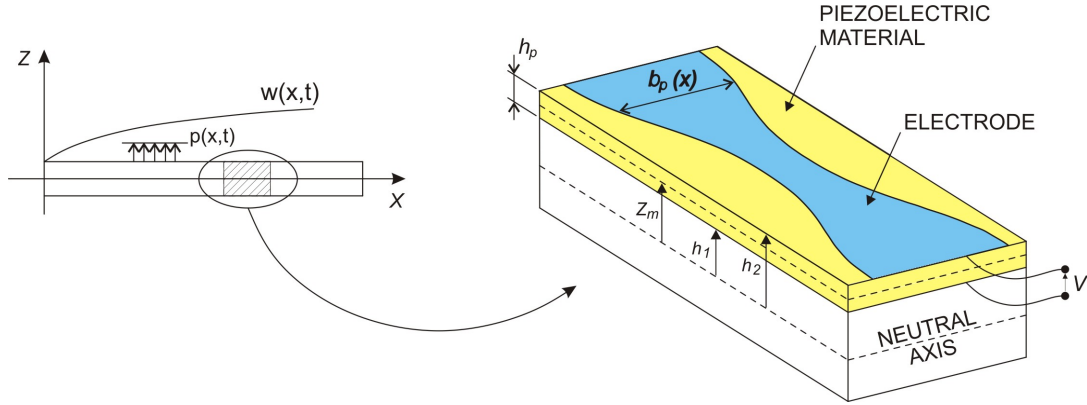


Figure 2.6: Spatially shaped transducer.

2.6.4 Uniform transducer

In the previous section, it was discussed spatially shaped transducers, how they will act only for the modes in which we have interest. The uniform transducer will not perform only for a specific bandwidth of interest, meaning no modal filtering, thus the induced current is proportional to the contribution of all modes to the net vibratory response of the beam,

$$i(t) = -e_{31}^* h_b 2b_b \sum_{r=1}^n \left[\int_l \frac{d^2 \phi_r(x)}{dx^2} dx \right] \frac{d\eta_r(t)}{dt} + C_p^S \frac{dv(t)}{dt}. \quad (2.82)$$

Additionally, regarding the prescribed voltage term, a uniform capacitance determines that all modes are excited by the prescribed voltage, where C_p^S is given by

$$C_p^S = \frac{\epsilon_{33}^* A_p^e}{2h_p}, \quad (2.83)$$

where $A_p^e = 2b_b l$.

2.7 Summary

In this chapter a generic analytical formulation for the study of a two layered beam (elastic+piezoelectric) was presented. As baseline for the given problem, the Euler-Bernoulli beam theory was used, in order to develop the displacement field for our case, considering only axial displacement, which depends on the rotation of the beam, and transverse displacement. Moreover constitutive equations were developed combining the displacement field with the constitutive equations for piezoelectric materials of the crystal class $mm2$. In order to determine actuating and sensing equations, it was used an extension of the Hamilton's variational formulation, where mechanical deformation term is substituted by electro-mechanical enthalpy, although for the host beam is only considered the virtual work of mechanical deformation.

The actuating equation considers not only external mechanical forces, but also electric loading performed by the piezoelectric layer. The electric sensing appears due to piezoelectric direct effect, meaning that an electrical field is generated along with the beam's

deformation. This electric loading is also proportional to the second order derivative of the piezo layer width. Also the two natural boundary conditions, bending moment and shearing forces at $x = (0, l)$ are proportional to the applied electric field and to the width and first order derivative of the piezo layer's width respectively.

As for the sensing equation, it can be seen, that the induced electrical current is given by two different terms, the first one related to the beam's deformation, and the second to inherent capacitance of the piezo.

Both actuating and sensing equation can be further solved using modal expansion. It is shown in this chapter the modal expansion detailed for the sensing equation, in order to discuss spatially shaped transducer/electrodes and their ability to filter unwanted vibration modes, it is also discussed a uniform transducer/electrode behavior, showing that they consider all vibration modes, having no capability of vibration filtering.

Chapter 3

Resonant Shunt Damping for Vibration Control

3.1 Introduction

In this chapter shunt damping is discussed both for single and multi-modal piezoelectric transducers. The main purpose of coupling a shunt circuit to a piezoelectric transducer/electrode is to increase damping performance in the structure in which the piezo is bonded to, by dissipating the electrical energy generated in the piezo layer. The electrical energy generated within the piezo layer is related to the mechanical energy present in the host structure, through the piezoelectric direct effect.

Uniform and spatially shaped (single and multi-modal) transducers are addressed in this chapter, being developed for both cases shunt and electromechanical coupling models. Later frequency response models are developed in order to evaluate damping performance for the cases in study.

It is important to mention that this chapter is greatly based and inspired on the article published by the Co-Advisor of this dissertation, [[Vasques and Dias Rodrigues \(2009\)](#)].

3.2 Piezoelectric Shunt Damping

Piezoelectric shunt damping systems reduce vibrations amplitude through piezoelectric transducer/electrode shunted with an electrical impedance, this impedance design leading to a coupled electrical resonance at the target modal frequencies. However, in real life situation, environmental conditions and structural loading can change the structural resonance frequencies, reducing severely the shunt damping performance, since the electrical impedance remains tuned for the nominal frequencies. Although interesting, this issue will not be discussed in this text, being the motivation of this section the formulation of shunt damping models both for spatially shaped (single and multi-modal) transducers/electrodes and uniform (segmented and non-segmented) transducers. The multi-modal damping is achieved through spatially design of transducers, already discussed in the previous chapter. There are shunt techniques that can act as multi-modal damper, but they require complicated circuits, so this work will contemplate a simple shunt impedance scheme.

3.2.1 Shunted multi-modal piezoelectric transducer

A multi-modal transducer can be said as equivalent to an electrical network containing a capacitor in series with a voltage generator, dynamically dependent on the mechanical induced strain, coupled together to the direct piezoelectric effect, see Equations (2.67) and (2.78), which if connected to an external electrical circuitry can perform as a damping component.

Having in mind the generalized electric current Equation (2.78), which in the Laplace form yields

$$i(s) = -se_{31}^* h_b 2b_b \beta \sum_{k=1, k \neq m}^n [\alpha_k \eta_k(s)] + sC_p^s v(s), \quad (3.1)$$

and recalling that the electrical capacitance, ($C = q/v$), can be seen as electrical impedance inverse, ($Z = v/i$), it is obtained the following, $Z_{oc}(s) = (sC_p^s)^{-1}$, which in fact is the open-circuit electrical impedance and s is the Laplace complex variable. Considering $c_i = e_{31}^* h_b 2b_b$, the previous equation can be presented in a more simpler form

$$i(s) = -sc_i \beta \sum_{k=1, k \neq m}^n [\alpha_k \eta_k(s)] + \frac{1}{Z_{oc}(s)} v(s). \quad (3.2)$$

Adding an electrical circuit in parallel (shunted) with the piezo electric transducer terminals, the voltage across the shunt is given by

$$v(s) = \sum_{k=1, k \neq m}^n v_k(s), \text{ with } v_k(s) = -Z_{sh}(s) i_k(s). \quad (3.3)$$

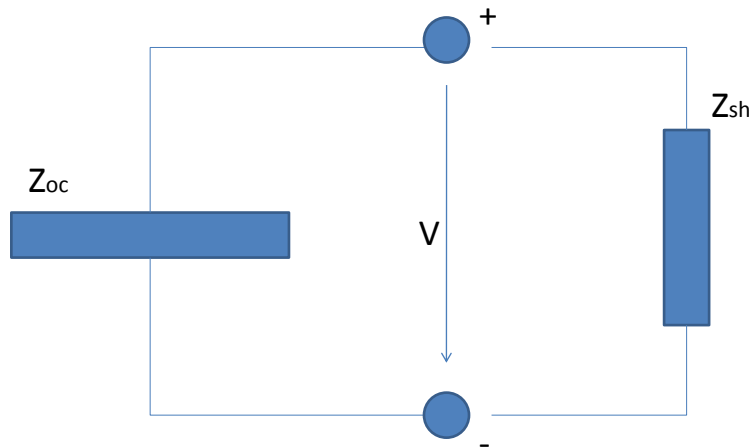


Figure 3.1: Parallel piezoelectric and shunt schematic parallel circuit

The electric current feeding both impedances is the same, and is defined in Equation (3.2). Substituting it into Equation (3.3), the voltage across the shunt yields

$$v(s) = Z_{sh}(s) \left[sc_i \beta \sum_{k=1, k \neq m}^n [\alpha_k \eta_k(s)] - \frac{1}{Z_{oc}(s)} v(s) \right] \quad \text{or} \quad (3.4)$$

$$v_k(s) = Z_{sh}(s) \left[sc_i \beta [\alpha_k \eta_k(s)] - \frac{1}{Z_{oc}(s)} v(s) \right] \quad (3.5)$$

where Equation (3.4) denotes multi-modal voltage function and the Equation (3.5) the single modal voltage function. Doing some algebraic manipulation, the Equation (3.5) becomes

$$v_k(s) = Z_{sh}(s) sc_i \beta [\alpha_k \eta_k(s)] - \frac{Z_{sh}(s)}{Z_{oc}(s)} v(s), \quad (3.6)$$

or

$$v_k(s) = \frac{Z_{sh}(s) Z_{oc}(s)}{Z_{oc}(s) + Z_{sh}(s)} sc_i \beta [\alpha_k \eta_k(s)], \quad (3.7)$$

and recalling that the electrical net impedance for a parallel impedances circuit is

$$Z_{el} = \frac{Z_{sh}(s) Z_{oc}(s)}{Z_{oc}(s) + Z_{sh}(s)}, \quad (3.8)$$

we have, finally,

$$v_k(s) = Z_{el} sc_i \beta [\alpha_k \eta_k(s)]. \quad (3.9)$$

The algebra shown here is only for the single mode shunt voltage, but the idea is extensible for multi-modal voltage shunt, yielding

$$v(s) = Z_{el} sc_i \sum_{k=1, k \neq m}^n \beta [\alpha_k \eta_k(s)]. \quad (3.10)$$

The voltage definition in Equation (3.9) is the voltage definition that should be considered to define the piezoelectric term in the actuating Equation (2.57). Recalling the assumed electric field/voltage defined in Equation (2.18), the piezoelectric loading term in Equation (2.58), in the Laplace form becomes

$$p_e(x, s) = c_v \frac{d^2 b_p(x)}{dx^2} v(s), \quad (3.11)$$

where $c_v = 2e_{31}^* h_b$.

Having reduced the problem of beam's vibration to a parallel impedances electrical network, the damping performance will depend upon the electric boundary conditions (EBC) and on the adequate definition of the shunt impedance, in other words the definition of the resistor and inductance elements. Considering only EBCs for now, two main limit cases can be presented:

- Short-circuit EBC: $Z_{sh}(s) = 0 \Rightarrow Z_{el}(s) = 0, v(s) = 0, p_e(x, s) = 0$;
- Open-circuit EBC: $Z_{sh}(s) = \infty \Rightarrow Z_{el}(s) = Z_{oc}(s), v(s) \neq 0, p_e(x, s) \neq 0$.

3.2.2 Shunted uniform piezoelectric transducer

In this section it will be done a similar development as in the previous section, regarding now the necessary considerations that a uniform width piezoelectric implies. The piezoelectric transducer/electrode Laplace transform of Equation (2.82) yields

$$i(s) = -sc_i \sum_{r=1}^n \left[\int_l \frac{d^2 \phi_r(x)}{dx^2} dx \right] \eta_r(s) + \frac{1}{Z_{oc}(s)} v(s). \quad (3.12)$$

Alternatively, and making a parallel with the multi-modal transducer case, considering that $r = k$ and $m = 0$, the following relationship holds

$$\sum_{k=1, k \neq m}^n (\beta \alpha_k) \eta_k(s) = \sum_{k=1}^n (\beta \alpha_k) \eta_k(s) = \sum_{k=1}^n \left[\int_l \frac{d^2 \phi_k(x)}{dx^2} dx \right] \eta_k(s), \quad (3.13)$$

where

$$\beta \alpha_r = \int_l \frac{d^2 \phi_r(x)}{dx^2} dx, \quad (3.14)$$

which establishes a relationship between multi-modal and uniform actuating equations, therefore allowing the definition of the “uniform capacitance”, (2.83), through the transformation of the “multi-modal capacitance”, Equation (2.81). Using this mathematical abstraction, the multi-modal voltage definition can be transformed to the uniform case, yielding

$$v(s) = Z_{el}(s) sc_i \sum_{k=1}^n \left[\int_l \frac{d^2 \phi_r(x)}{dx^2} dx \right] \eta_r(s) \quad \text{or} \quad v(s) = Z_{el}(s) sc_i \int_l \frac{d^2 \phi_k(x)}{dx^2} dx \eta_k(s), \quad (3.15)$$

where the “uniform capacitance” should be considered for the electrical network impedance Z_{el} definition.

As said before, in the previous section the voltage definition represents the voltage that the uniform piezoelectric transducer is subject to and should be the definition that should be considered to define the piezoelectric loading terms in equation (2.57), but one can not forget the piezoelectric loading term depends upon the second order derivative of the width, and since the width is constant for the uniform non-segmented transducer case, $b_p(x) = b_p$, yields

$$p_e(x, s) = c_v \frac{d^2 b_p(x)}{dx^2} v(s) = 0. \quad (3.16)$$

In other words, piezoelectric loading effects are only introduced by the boundary terms, (2.59) and (2.60), for a non-segmented transducer.

3.3 Electromechanical Model of a Shunted Beam

3.3.1 Shunted multi-modal piezoelectric transducer

Recalling the generic actuating Equation (2.63), and the single and multi-modal sensitivity function defined in the equations (2.72-2.76), together with the assumptions made in

the previous section, is now possible to do the modal projection of the actuating equation. Taking into account orthonormality properties of mode shapes and considering also a viscous damping model with a modal damping ratio ξ_r , the actuating equation in the Laplace domain can now be written as

$$(s^2 + 2\xi_r\omega_r s + \omega_r^2)\eta_r(s) + N_r^e(s) + N_r^{e/m}(s) = N_r^m(s). \quad (3.17)$$

$N_r^m(s)$ denotes the mechanical loading and its modal projection is given by

$$N_r^m(s) = \int_l \phi_r(x) p_m(x, s) dx. \quad (3.18)$$

Furthermore, $N_r^e(s)$ represents the piezoelectric-induced loading term, and expressing it in terms of the applied voltage, yields

$$N_r^e(s) = -c_v \int_l \phi_r(x) \frac{d^2}{dx^2} [b_p(x) [H(x - x_L) - H(x - x_R)]] dx v(s), \quad (3.19)$$

where $H(\cdot)$ is the Heaviside function, which turns the piezoelectric loading term in Equation (2.63) more generic, denoting that the piezoelectric transducer/electrode extends over the host beam from $x = x_L$ to $x = x_R$, with $(x_L, x_R) \in]0, l[$, and therefore the piezoelectric transducer may be a segmented one or may cover the entire beam. This function also contemplates both uniform or varying width transducers/electrodes. If one wants to consider a uniform transducer, just needs to consider that width will respect the following consideration, $b_p(x) = b_p$.

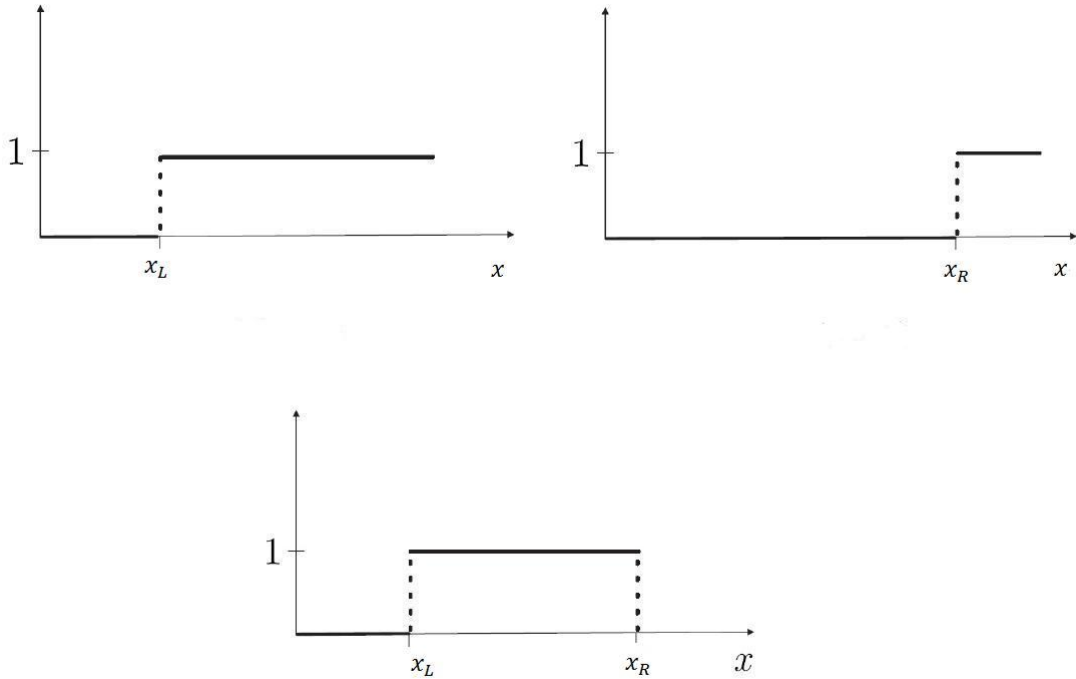


Figure 3.2: Heaviside function

The other term in Equation (3.17), $N_r^{e/m}$ stands for the piezoelectrically-induced concentrated boundary effects, see equation (2.63)

$$N_r^{e/m} = c_v \int_l \phi_r(x) \left[\frac{db_p(0)}{dx} \delta(x) - \frac{db_p(l)}{dx} \delta(x-l) + b_p(l) \delta'(x-l) - b_p(0) \delta'(x) \right] dx v(s), \quad (3.20)$$

$$= c_v \left[\frac{db_p(0)}{dx} \phi_r(0) - \frac{db_p(l)}{dx} \phi_r(l) + b_p(l) \frac{d\phi_r(l)}{dx} - b_p(0) \frac{d\phi_r(0)}{dx} \right] v(s). \quad (3.21)$$

Recalling the multi-modal sensitivity function, for multi-modal shape transducers/electrodes (2.76), and applying into Equation (3.19), considering that the sensor will cover the entire beam, therefore making the Heaviside term not applicable, yields

$$N_r^e(s) = -c_v b_b Y_b I_b \beta \int_l \phi_r(x) \sum_{k=1, k \neq m}^n \left(\frac{\alpha_k}{\omega_k^2} \frac{d^4 \phi_k(x)}{dx^4} \right) dx v(s). \quad (3.22)$$

Making the same exercise for Equation (3.21), the piezoelectrically-induced concentrated boundary yields now

$$\begin{aligned} N_r^{e/m}(s) &= c_v b_b \left[\frac{dS(0)}{dx} \phi_r(0) - \frac{dS(l)}{dx} \phi_r(l) + S(l) \frac{d\phi_r(l)}{dx} - S(0) \frac{d\phi_r(0)}{dx} \right] v(s), \\ &= c_v b_b Y_b I_b \sum_{k=1, k \neq m}^n \left[\frac{\frac{d^3 \phi_k(0)}{dx^3} \phi_r(0) - \frac{d^3 \phi_k(l)}{dx^3} \phi_r(l)}{\frac{d^2 \phi_k(l)}{dx^2} \frac{d\phi_r(l)}{dx} - \frac{d^2 \phi_k(0)}{dx^2} \frac{d\phi_r(0)}{dx}} \right]. \end{aligned} \quad (3.23)$$

From the previous equations it can be determined that for the fully distributed multi-modal transducer case, due to the orthonormality properties of the mode shapes, $N_r^e \neq 0$ only when $k = r$, and that $N_r^{e/m} = 0$ for free, clamped and simply supported ends. In view of this, substituting the shunted electrical voltage in Equation (3.10) into (3.22), yields

$$N_r^e(s) = -c_v c_i b_b Y_b I_b \beta^2 \frac{\alpha_r}{\omega_r^2} \left[\int_l \phi_r(x) \frac{d^4 \phi_r(x)}{dx^4} dx \right] s Z_{el}(s) \sum_{k=1, k \neq m}^n \alpha_k \eta_k(s). \quad (3.24)$$

It can be seen in the previous equation that for the multi-modal transducer case the piezoelectric-induced shunt loading terms do not allow the decoupling of the modal model due to the non-proportional nature of the localised stiffness and damping introduced. However, for weakly coupled modes, in the vicinity of a specific mode r we may assume that the influence of the other modes may be neglected, so that only for $k = r$ the modal contribution is significant therefore allowing the modal decoupling. Thus Equation (3.24) may approximately be written as

$$N_r^e(s) \approx -c_v c_i b_b Y_b I_b \beta^2 \frac{\alpha_r^2}{\omega_r^2} \left[\int_l \phi_r(x) \frac{d^4 \phi_r(x)}{dx^4} dx \right] s Z_{el}(s) \eta_r(s) \quad (3.25)$$

$$= -c_r^e Z_r(s) \eta_r(s), \quad (3.26)$$

where

$$c_r^e = c_v c_i b_b Y_b I_b \beta^2 \alpha_r^2 \quad \text{and} \quad Z_r = s Z_{el}(s). \quad (3.27)$$

The previous Equations developed for shaped modal loadings are represented in figure(3.3).

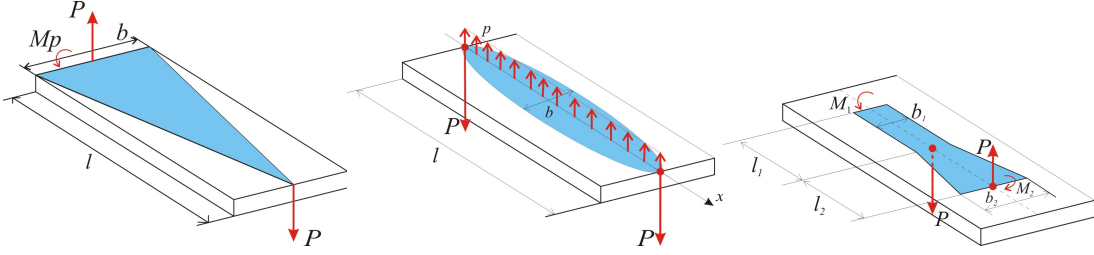


Figure 3.3: Shape transducers loading influence.

3.3.2 Shunted uniform piezoelectric transducer

According to the assumption that the width of the piezoelectric transducer/electrode does not change along the length of the beam, and recalling the governing equations of a generic spatially shaped adaptive beam presented in Section 2, some modifications and considerations should be made in order to derive the electromechanical model of a uniformly shaped adaptive beam. Thus, considering the modal projection defined in Equation (2.65) and substituting it into Equation (2.63), taking into consideration the orthonormality properties of the mode shapes, the equation of motion in the Laplace domain for the r th mode, considering also a proportional viscous damping model with a modal damping ratio ξ_r , is formally identical to the one derived for the multi-modal case, yielding

$$(s^2 + 2\xi_r\omega_r s + \omega_r^2)\eta_r(s) + N_r^e(s) + N_r^{e/m}(s) = N_r^m(s). \quad (3.28)$$

In this case, though, the loading term $N_r^e(s)$ deserves some attention; in this view of the fact that the piezoelectric patch is width independent, i.e. $b_p(x) = b_p$, it is null in the case of a transducer fully covering the length of the beam, and non-null otherwise. Thus, the general expression in Equation (3.19) accommodating those assumptions becomes

$$N_r^e = -c_v b_p \left[\frac{d\phi_r(x_L)}{dx} - \frac{d\phi_r(x_R)}{dx} \right] v(s). \quad (3.29)$$

It is worthy to point out that the previous term corresponds to the application of two voltage dependent concentrated moments at $x = (x_L, x_R)$, see figure (3.4), and that this later loading term must only be applied in the case of a segmented piezoelectric patch. As can be attested by examining Equation (3.21), the expression is identical to the second boundary loading term if $b_p(l) = b_p(0) = b_p$ and if $x_L = 0$ and $x_R = l$. Thus considering the definitions in Equation (2.64), and confirming the aforementioned discussion, for the uniform case the piezoelectrically-induced concentrated boundary loading terms

$$N_r^{e/m}(s) = -c_v b_p \left[\frac{d\phi_r(0)}{dx} - \frac{d\phi_r(l)}{dx} \right] v(s). \quad (3.30)$$

Substituting the shunted electrical voltage for the uniform case given in Equation (3.9) into both Equations (3.29) and (3.30), the shunted voltage loading terms become

$$N_r^e(x_L, x_R, s) = -b_p c_v c_i \left[\frac{d\phi_r(x_L)}{dx} - \frac{d\phi_r(x_R)}{dx} \right] Z_r(s) \sum_{k=1}^n \left[\frac{d\phi_k(x)}{dx} \Big|_{x_L}^{x_R} \eta_k(s) \right], \quad (3.31)$$

$$N_r^{e/m}(s) = -b_p c_v c_i \left[\frac{d\phi_r(0)}{dx} - \frac{d\phi_r(l)}{dx} \right] Z_r(s) \sum_{k=1}^n \left[\frac{d\phi_k(x)}{dx} \Big|_{x_L}^{x_R} \eta_k(s) \right]. \quad (3.32)$$

It can be seen in the previous equations that for the uniform transducer case the piezoelectric-shunt loading terms also do not allow the decoupling of the modal model due to the non-proportional nature of the stiffness and damping introduced. Again, similarly to what was assumed for the multi-modal case, for weakly coupled modes, the other modes may be neglected so that only for $k = r$ the modal contribution is significant, therefore allowing the modal decoupling. Thus, Equations (3.31) and (3.32) may approximately be written as

$$\begin{aligned} N_r^e(x_L, x_R, s) &\approx -b_p c_v c_i \left[\frac{d\phi_r(x_L)}{dx} - \frac{d\phi_r(x_R)}{dx} \right] Z_r(s) \frac{d\phi_r(x)}{dx} \Big|_{x_L}^{x_R} \eta_r(s) \\ &= c_r^e(x_L, x_R) Z_r(s) \eta_r(s), \end{aligned} \quad (3.33)$$

$$\begin{aligned} N_r^{e/m}(s) &\approx -b_p c_v c_i \left[\frac{d\phi_r(0)}{dx} - \frac{d\phi_r(l)}{dx} \right] Z_r(s) \frac{d\phi_r(x)}{dx} \Big|_0^l \eta_r(s) \\ &= c_r^{e/m} Z_r(s) \eta_r(s), \end{aligned} \quad (3.34)$$

where

$$c_r^e(x_L, x_R) = b_p c_v c_i \left[\frac{d\phi_r(x_L)}{dx} - \frac{d\phi_r(x_R)}{dx} \right]^2 \quad \text{and} \quad c_r^{e/m} = b_p c_v c_i \left[\frac{d\phi_r(l)}{dx} - \frac{d\phi_r(0)}{dx} \right]^2 \quad (3.35)$$

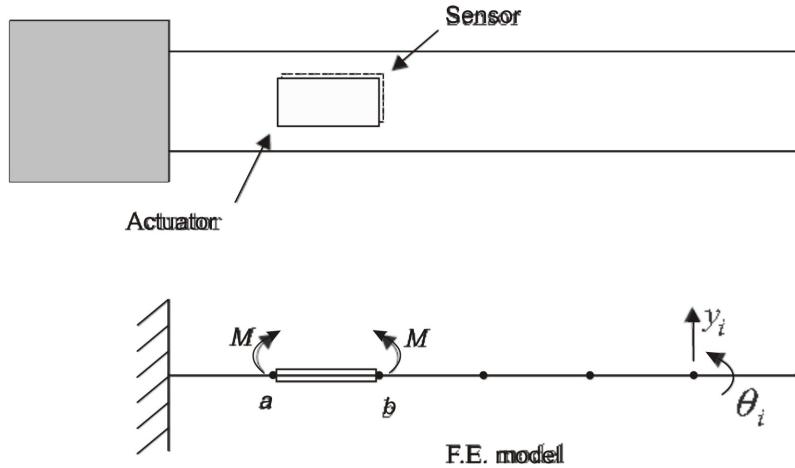


Figure 3.4: Segmented uniform transducer.

3.4 Frequency Response Model

The adaptive beam model has considered generic distributed mechanical loading term $p_m(x, t)$. In general, that term might consider the loading effects of both distributed

and/or concentrated transverse forces. For the purpose of defining the frequency response model, let us focus here on the single application of a concentrated force $F(t)$ applied at $x = x_F$, which can be defined in the Laplace domain as

$$p_m(x, s) = \delta(x - x_F)F(s). \quad (3.36)$$

Substituting the previous definition into the r th modal equation of motion defined in Equation (3.17) and considering the multi-modal transducer/electrode shape case, i.e. $N_r^{e/m}(s) = 0$, yields

$$[s^2 + 2\xi_r\omega_r s + \omega_r^2 - c_r^e Z_r(s)] \eta_r(s) = F(s)\phi_r(x_F). \quad (3.37)$$

Considering the modal projection of the spatial transverse displacement in equation (2.65), defining $s = j\omega$, the generic displacement-based (receptance) frequency response function (FRF) in the frequency domain can be readily obtained from Equation (3.37) by dividing $\eta_r(s)$ by $F(s)$, yielding

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \sum_{r=1}^{\infty} \frac{\phi_r(x_F)\phi_r(x)}{(\omega_r^2 - \omega^2) + j(2\xi_r\omega_r\omega) - c_r^e Z_r(j\omega)}, \quad (3.38)$$

where W^* and F^* are the complex displacement and disturbance force amplitudes. The previous equation can still hold for the case of uniform piezoelectric transducers by performing the adequate correspondence of c_r^e in Equation (3.38) to the definitions in Equations (3.35), substituting it by the first definition, $c_r^e = c_r^e(x_L, x_R)$, for the case of a segmented uniform transducer patch or by the second one, $c_r^e = c_r^{e/m}$, for the case of full length piezoelectric patches being used.

3.5 Resonant Shunt Damping

In order to demonstrate the application of the theory presented in the previous sections, some examples will be considered, using for that matter an adaptive beam with uniform or arbitrarily shaped, single-or multi-modal piezoelectric transducers/electrodes under a pre-determined shunted vibration control strategy. Different boundary mechanical conditions and segmented configurations of both uniform and shaped configurations are also considered. The main purpose here is to perform a qualitative comparison study of damping performance between uniform shaped and modally shaped transducers/electrodes.

Recalling the definition of the net electrical impedance in Equation (3.8), let us consider that the passive electrical network (shunt) is defined as containing a resistor element, R , in series with an inductor, L , and tuned to a single mode and connected in series to the piezoelectric transducer. That said, the shunt impedance is given by

$$Z_{sh}(s) = R + sL, \quad (3.39)$$

which leads the net electrical impedance to become the following

$$Z_{el}(s) = \frac{R + sL}{s^2 C_p^s L + s C_p^s R + 1}. \quad (3.40)$$

Relating the previous net electrical impedance with the second definition, for the r th mode, in Equation (3.27), and considering $s = j\omega$, it is obtained the “modal net impedance” in the frequency domain

$$Z_r(j\omega) = \frac{j\omega R - \omega^2 L}{-\omega^2 C_p^s L + j\omega C_p^s R + 1}. \quad (3.41)$$

Substituting the previous Equation into the frequency response model in Equation (3.38), and doing some algebraic manipulation we have the r th mode frequency model

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \frac{A_r(x_F, x) \left(\left[-\omega^2 L + \frac{1}{C_p^s} \right] + j\omega R \right)}{\left[\omega^4 L - \omega^2 A_1 + \frac{\omega_R^2}{C_p^s} \right] + j \left[\omega^3 A_2 + \omega A_3 \right]}, \quad (3.42)$$

where

$$A_1 = \omega_r^2 L + 2\omega_r R \xi_r + \frac{1}{C_p^s} - c_r^e \frac{L}{C_p^s}, \quad (3.43)$$

and

$$A_2 = -2\omega_r L \xi_r - R, \quad (3.44)$$

and

$$A_3 = \frac{2\omega_r \xi_r}{C_p^s} + R\omega_r^2 - c_r^e \frac{R}{C_p^s}. \quad (3.45)$$

The coefficient $A_r(x_F, x) = \phi_r(x_F)\phi_r(x)$ is the modal constant which depends upon the spatial coordinate where the concentrated excitation force is applied and the spatial coordinate of the measured response. The Equation (3.42) can be simplified by performing a few non-dimensionalizations relative to the r th mode mechanical short-circuit natural frequency $\omega_r^E = \omega_r$,

$$\omega_e^S = \frac{1}{\sqrt{C_p^s L}}, \quad \delta_r = \frac{\omega_e^S}{\omega_r^E}, \quad \rho_r = RC_p^s \omega_r^E, \quad \gamma_r = \frac{\omega}{\omega_r^E} \quad (3.46)$$

so that we get

$$\frac{\bar{W}_r(j\omega)}{\bar{W}_r^{st}(0)} = \frac{\delta_r^2 - \gamma_r^2 + j\gamma_r \delta_r^2 \rho_r}{(\delta_r^2 + j\rho_r \delta_r^2 \gamma_r - \gamma_r^2)(1 + 2j\xi_r \gamma_r - \gamma_r^2) - \chi_r^2(j\rho_r \delta_r^2 \gamma_r - \gamma_r^2)}, \quad (3.47)$$

where χ_r^2 is a generalized electromechanical coupling coefficient given by

$$\chi_r^2 = \delta_r^2 c_r^e L. \quad (3.48)$$

The previous coefficient is of paramount importance to appropriately design adaptive (or smart) piezoelectric structural systems for vibration and structural acoustics control. In fact, it is a parameter analogous to the *electromechanical coupling factor* for the material [IEEE (1988)], quantifying the electromechanical energy conversion in the material, which for the piezoelectric transducer operating in the 3-1 (thickness-extension/bending) mode is given by

$$k_{31} = \frac{e_{31}}{\sqrt{\epsilon_{31}^T Y_p}}. \quad (3.49)$$

However, the former coefficient, χ_r , is not only a function of the piezoelectric material characteristics, as the later k_{31} is, but is also a function of the relative stiffness of the piezoelectric transducer and the hosting mechanical structure. In physical terms, it relates the amount of strain energy that is stored in the piezoelectric material compared to the total strain energy stored by the adaptive structure, weighed by the electromechanical energy conversion efficiency of the piezoelectric material being used. Therefore, a higher value of k_{31} indicates larger energy conversion and, hence, better electromechanical coupling in the material. However, important aspects in the analysis and design are not only the piezoelectric material properties but also the relationship between the placement of the piezoelectric material and the modal response of the system. In fact, the same piezoelectric material will act differently for different vibration modes of the host structure, and this phenomenon is mainly caused by factors such as location, shape and geometry of the transducer as it relates to the host structure. Unlike k_{31} , the generalized coefficient χ_r considers all these factors, making a more embracing modal coupling parameter, accounting the influence of both material and structural electromechanical coupling aspects in design. Only a combination of high material and structural (relative stiffness) coupling will produce a high value of χ_r , denoting a large coupling between the piezoelectric transducer and the host structure system.

Examining the Equation (3.47), the rationale for the definition of χ_r defined in Equation (3.48) shows the dependability of resonant shunt design (inductive-resistive), on the following three non-dimensional parameters:

- The generalized modal electromechanical coupling coefficient, χ_r ;
- The non-dimensional modal dissipation tuning design parameter, $\rho_r = RC_p^s \omega_r^E$;
- The non-dimensional modal tuning ratio of the free-strain electrical to short-circuit mechanical natural frequency, $\delta_r = \omega_e^S / \omega_r^E$, with $\omega_e^S = (C_p^s L)^{-1/2}$.

Looking deeper into these parameters, one can become aware, that the former, χ_r , is defined by the piezoelectric material properties and structural coupling, which as already stated, is the result of the chosen specific location, shape and geometry of the piezoelectric transducer. As for the other two parameters, they vary according to the chosen values of the shunt resistance, R , and inductance, L .

Focusing in the last two parameters, many authors have developed several techniques for finding the optimal tuning of a single-mode resonant shunt circuit [Hagood and von Flotow (1991), Tsai and Wang (1999)], i.e. defining the optimal values for both resistance, R , and inductance, L , belonging on the shunt impedance, $Z_{sh}(s)$, in Equation (3.40), that would yield the “best” damping performance of the shunted system. Among these, a method making the analogy with the damped vibration absorber and a based on a FRF approach, where the tuning parameter are considered optimal for passive shunt damping when they minimize the peaks of the FRF, has been widely employed in the context of the so-called resonant circuit shunting to define the optimal parameters [Hagood and von Flotow (1991)]. Furthermore, in that method the structural system damping is neglected in the analysis, i.e. $\xi_r = 0$. It is well-known, though, that for small values of damping the optimal parameters of a damped vibration absorber will not be too much affected making, for practical reasons and simplicity of analysis, the methodology satisfactorily valid for damped structures. Thus following the FRF-based approach proposed by [Hagood and von Flotow (1991)], the optimal resonant shunted piezoelectric non-dimensional modal frequency tuning ratio, δ_r^{opt} , is given by

$$\delta_r^{opt} = \sqrt{1 + \chi_r^2} \implies L_r^{opt} = \frac{1}{C_p^s (\omega_r^E)^2 (1 + \chi_r^2)} \quad (3.50)$$

and the non-dimensional modal dissipation tuning design parameter, ρ_r^{opt} , is given by

$$\rho_r^{opt} = \sqrt{2} \frac{\chi_r}{1 + \chi_r^2} \implies R_r^{opt} = \frac{\sqrt{2}}{C_p^s \omega_r^E} \left(\frac{\chi_r}{1 + \chi_r^2} \right). \quad (3.51)$$

Electromechanical coupling in piezoelectric devices gives rise to the fact that the properties of the material are also a function of the mechanical and electrical boundary conditions. An important parameter when identifying the mechanical parameters of a given piezoelectric adaptive structure is the electrical boundary condition that exists between the two opposing electrodes. We may consider that the electrodes are short-circuited, and in that case it results in a zero electrical field across the electrodes, i.e. $E = 0$, but it does allow electrical charge to flow from the positive pole to the negative one. On opposition, we may consider that the electrical terminals are open such that no charge can flow between the electrical terminals, i.e. $D = 0$. In view of this, it is improper to refer to mechanical parameters, such as the natural frequency, without specifying the electrical boundary condition. For that purpose the superscripts $(\cdot)^E$ and $(\cdot)^D$ are usually adopted to denote parameters evaluated under two aforementioned electric boundary conditions. This gives rise to a stiffness modification of the structural system resulting in two distinct values for the natural frequencies of a certain mode r , denoted here by ω_r^E and ω_r^D . This latter aspect can be used to alternatively define χ_r in terms of the relative frequency variation between the two electrical boundary conditions.

Substituting the optimal tuning ratio and dissipation parameters, δ_r^{opt} and ρ_r^{opt} , given in Equations (3.50) and (3.51), the magnitude of the damped relative displacement FRF at the modal frequency $\omega = \omega_r^E$, i.e. with $\gamma_r = 1$, is given by

$$\frac{|\bar{W}_r(j\omega_r^E)|}{\bar{W}_r^{st}(0)} = \sqrt{\frac{\chi_r^2 + 2}{2\chi_r^4 - 4\sqrt{2}\xi_r\chi_r^3 + (1 + 4\xi_r^2)\chi_r^2 - 4\sqrt{2}\xi_r\chi_r + 8\xi_r^2}}. \quad (3.52)$$

From the previous equation one can easily comprehend the limit cases of χ_r , meaning complete absence of piezoelectric transducer damping effect or on the other hand this damping effect will be at the maximum level possible, i.e. for $\chi_r \rightarrow (0, 1)$, yields

$$\frac{|\bar{W}_r(j\omega_r^E)|}{\bar{W}_r^{st}(0)} = \begin{cases} \frac{1}{2\xi_r} & \chi_r \rightarrow 0 \\ \left(\frac{3}{3 + 12\xi_r^2 - 8\sqrt{2}\xi_r} \right)^{\frac{1}{2}} & \chi_r \rightarrow 1 \end{cases}. \quad (3.53)$$

So far, only issues such as the influence of piezoelectric materials parameters on the electromechanical energy conversion, or the definition of optimal electric circuit parameters have been presented. However the main purpose of this work is to compare damping efficiency between uniform transducers and shaped ones, both using a certain piezoelectric material and the same pre-determined optimal passive circuitry. The main difference between both cases will be the value of the generalized electromechanical coefficient χ_r , since in this work, has been neglected the piezoelectric's stiffness and mass effects. Therefore, one conclude that the higher the value of χ_r , the better and more efficient the resonant shunt strategy shunt is.

In order to demonstrate the theory presented before, a few examples considering an adaptive beam with uniform or arbitrarily shaped, single and multi-modal, piezoelectric transducers/electrodes under a pre-determined shunted vibration control strategy, are

considered in what follows and the values of χ_r for each case are qualitatively compared. In addition, different boundary mechanical conditions and segmented configurations of both uniform and shaped configurations are considered.

3.5.1 Adaptive beam with a uniform piezoelectric transducer

Considering a two-layered adaptive beam with a uniform piezoelectric transducer full-covering the beam. Under those circumstances, considering the definition of $c_r^{e/m}$ in Equation (3.35), the generalized modal electromechanical coupling coefficient, is given by

$$\chi_r^2 = \frac{1}{(\omega_r^E)^2 C_p^s} c_r^{e/m} = \left[\frac{\left[\frac{d\phi_r(l)}{dx} - \frac{d\phi_r(0)}{dx} \right]^2}{\int_l \frac{d^2\phi_r(x)}{dx^2} \frac{d^2\phi_r(x)}{dx^2} dx} \right] \left(\frac{3h_p h_b}{lh_b^2} \right) \left(\frac{Y_p}{Y_b} \right) \left(\frac{k_{31}^2}{1 - k_{31}^2} \right). \quad (3.54)$$

3.5.2 Adaptive beam with a uniform segmented piezoelectric transducer

In this second example, the two layered adaptive beam is also assumed to have a uniform segmented piezoelectric transducer, see figure (3.5), bonded to the beam. Under those circumstances, considering the definition of $c_r^e(x_L, x_R)$ in Equation (3.35), the generalized modal electromechanical coupling coefficient is given by

$$\chi_r^2(x_L, x_R) = \frac{1}{(\omega_r^E)^2 C_p^s} c_r^e(x_L, x_R) = \left[\frac{\left[\frac{d\phi_r(x_R)}{dx} - \frac{d\phi_r(x_L)}{dx} \right]^2}{\int_l \frac{d^2\phi_r(x)}{dx^2} \frac{d^2\phi_r(x)}{dx^2} dx} \right] \left(\frac{3h_p h_b}{lh_b^2} \right) \left(\frac{Y_p}{Y_b} \right) \left(\frac{k_{31}^2}{1 - k_{31}^2} \right). \quad (3.55)$$

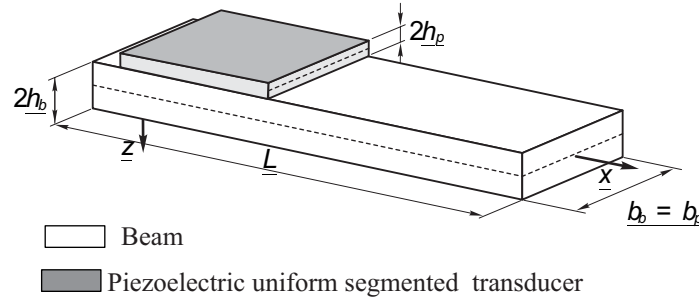


Figure 3.5: Uniform segmented piezoelectric transducer.

3.5.3 Adaptive beam with a single-mode piezoelectric transducer

According to Equation (2.72), the shape of the piezoelectric transducer/electrode may be defined in terms of the normalized generic sth mode shape, $\phi_r(x)$, so that $k = s$. Accordingly, considering the expression of the expression of c_r^e in Equation (3.27), χ_r is only non-zero when $r = s$, yielding

$$\chi_r^2 = \frac{1}{(\omega_r^E)^2 C_p^s} c_r^e = \left[\left(\frac{\omega_s^E}{\omega_r^E} \right) \frac{\alpha_r^2}{\int_l \alpha_s \frac{d^2\phi_s(x)}{dx^2} dx} \right] \left(\frac{3h_p h_b}{h_b^2} \right) \left(\frac{Y_p}{Y_b} \right) \left(\frac{k_{31}^2}{1 - k_{31}^2} \right). \quad (3.56)$$

3.5.4 Adaptive beam with a multi-mode piezoelectric transducer

Again, considering a multi-mode configuration, according to Equation (2.72), the shape of the piezoelectric transducer/electrode may be defined in terms of the an arbitrary number of mass normalized mode shapes k , $\phi_k(x)$, so that $k = 1 \dots n$ with $k \neq m$. Accordingly, considering the expression c_r^e in Equation (3.27), $\chi_r(x)$ is only non-zero when $r = k$, yielding

$$\chi_r^2 = \frac{1}{(\omega_r^E)^2 C_p^s} c_r^e = \left[\frac{\beta \alpha_r^2}{\sum \int_l \left(\frac{\omega_r^E}{\omega_k^E} \right)^2 \alpha_k \frac{d^2 \phi_k(x)}{dx^2} dx} \right] \left(\frac{3h_p h_b}{h_b^2} \right) \left(\frac{Y_p}{Y_b} \right) \left(\frac{k_{31}^2}{1 - k_{31}^2} \right). \quad (3.57)$$

3.5.5 Performance comparison of the uniform and multi-modal transducers

In the previous sections the generalized electromechanical coupling factors for different choices of the shape of the transducer/electrode were derived. Since the main objective of this text is to infer the qualitative advantages of using a modally shaped transducer/electrode combined with a resonant shunt damping strategy, the derived uniform and single-mode generalized electromechanical coupling coefficients are compared. For a valid comparison, piezoelectric/elastic materials, boundary conditions and dimensions are kept the same, being the only difference between both trials, the shape of the piezoelectric transducer/electrode. Thus, dividing Equation (3.56) by (3.54), the r th mode modal/uniform generalized coupling coefficient ratio is given by

$$\frac{\chi_r^{modal}}{\chi_r^{uniform}} = l \alpha_r \frac{\int_l \frac{d^2 \phi_r(x)}{dx^2} \frac{d^2 \phi_r(x)}{dx^2} dx}{\left[\frac{d\phi_r(x_R)}{dx} - \frac{d\phi_r(x_L)}{dx} \right]^2 \int_l \frac{d^2 \phi_r(x)}{dx^2} dx}. \quad (3.58)$$

Another ratio which is used to assess the damping efficiency between the single-mode and uniform transducer/electrode configurations is the ratio of the magnitude of the undamped relative displacement FRF at the modal frequency $\omega = \omega_r^E$, so that from Equation (3.52) we get

$$\frac{\bar{W}_r^{modal}}{\bar{W}_r^{uniform}} = \sqrt{\frac{\left(\frac{\chi_r^2 + 2}{2\chi_r^4 + \chi_r^2} \right)^{modal}}{\left(\frac{\chi_r^2 + 2}{2\chi_r^4 + \chi_r^2} \right)^{uniform}}}. \quad (3.59)$$

3.6 Summary

This chapter contains the development of sensing equation, considering electrical current and electrical potential in order to establish an impedance model that allow us to study the integration of electrical circuit shunt. Both electrical boundary conditions are studied as upper and lower limit of the capability of inducing the piezoelectric effect. The resonant shunt lies between these limits and is tuned for optimal value for maximum energy dissipation, for different cases, like different shape transducers, uniform segmented or not and modal transducers.

Chapter 4

Verification and Validation

4.1 Introduction

This chapter deals with the validation of mechanical actuation, sensing and electrical actuation behaviors of smart beams, both with analytical and numerical methods.

The numerical method is presented in the first section, where is described the software in which it was implemented, then is presented the formulation considering a piezoelectric problem, meaning are developed actuation sensing equation for both EBC limit cases. The numerical method is validated, in the second section, by comparing numerical results for clamped-free and simply-supported bare beams with the results performed by the analytical method. This validation served also to create a mesh pattern which would be utilized later for smart beams.

In the third and fourth sections presents the verification and validation of this passive vibration damping theory, developed in the dissertation. In the third section are considered uniform segmented and full covering transducers, different locations and different sizes, while the fourth section is dedicated to modally shaped transducers.

4.1.1 Finite element method validation

4.1.1.1 MEMS module

In this section is validated the finite element method by comparison with analytical models. The validation is first developed for beams without piezoelectric transducers, considering only simple elastic beam. Two main cases are approached, clamped-free and simply supported beam boundary conditions are studied, using static, eigenfrequency and a frequency response analysis. It will be also formulated the finite element model for beams considering the use of segmented uniform piezoelectric transducers not connected to any external electric circuit, in order to evaluate the performance of the beam's motion when coupled to this kind of materials.

As mentioned before the software in which is developed the finite element method is the COMSOL Multiphysics, that will be briefly explained in this section. This software as its name suggests offers several possibilities of combined analysis, therefore the finite element analysis can contemplate phenomena other than pure structural analysis. The COMSOL Multiphysics have AC/DC, acoustic, chemical, fluid and heat and MEMS modules, each one with several options. The one that comes to interest in this work is

the MEMS module, being the one containing the option for working with piezoelectric materials.

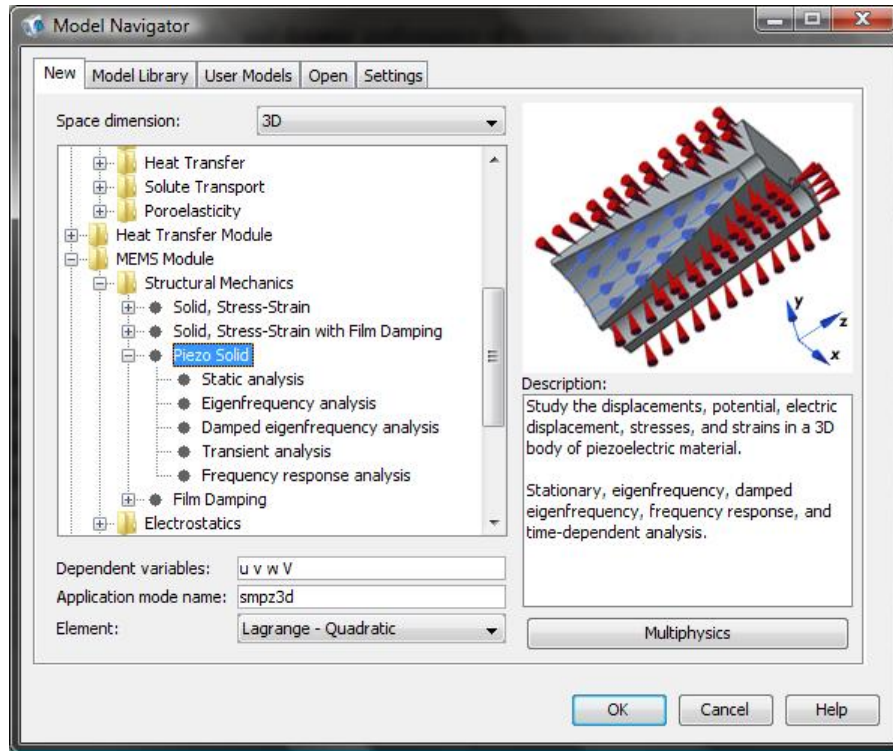


Figure 4.1: MEMS module.

The term MEMS stands for MicroElectricMechanical Systems, regarding that we are working with tiny devices with electrical phenomenon related to some mechanical motion. MEMS technology exploits the existing microelectronic infrastructures to create complex machines on a micrometer scale. Extensive application for these devices exist in both commercial and industrial systems. Integrated silicon pressure sensors, accelerometers and motion detectors are known for several years of application in the automotive and industrial fields.

MEMS devices involve multiple areas of physics, the most basic device will only contain electrical and mechanical phenomena, but is also common that the coupling of mechanical and electric elements involve thermal and electromechanical effects, therefore adding a third and fourth physical phenomenon.

The majority of MEMS devices are manufactured using lithography-based micro fabrication, a technology refined by the microelectronics industry for highly integrated circuits. However lithography fabrication methods, have their limitations on geometrical structures in MEMS devices. Micro-fabrication is based on planar technology where the components are typically flat. From the modeling point of view a flat structure presents some challenges, specifically in mesh generation and in finding numerical solutions. In order to obtain accurate solutions, the shaped triangle (2D) or tetrahedron (3D) mesh should be as regular as possible. In flat structures one can achieve regularity by decreasing the mesh size to accommodate the short distances, but doing so increases memory requirements. This issue can be attenuated by using meshing tools such as mesh rescaling, mesh mapping and mesh extrusion.

The strategy used for meshing in this work was the use of hexahedric 3D elements

for the elastic beam validation. This method is also applied for the elastic beam with uniform piezoelectric transducer, while beams containing modally shaped transducers, use extruded triangles. Due to the smaller value of thickness, both for the beam and the piezoelectric transducer, quadratic elements have been used in order to avoid locking phenomena. The beam considered is a aluminium beam with following geometrical and mechanical properties.

Table 4.1: Beam's geometrical and mechanical properties.

Y_b	70	GPa
ν	0.3	
ρ_b	2710	kg/m^3
l	0.3	m
b_b	0.03	m
h_b	0.002	m

4.1.1.2 Finite element model

The finite element model (FEM) used by the COMSOL Multiphysics is by default based on Weak Forms, and despite being a feature of a solution technique and not a modeling process, it is important to refer its implementation. Using weak forms brings the benefit of obtaining the exact Jacobian necessary for fast convergence of strongly non-linear problems, despite the mesh modeling process chosen.

As referred above two types of geometry are used for our mesh elements, hexahedric for uniform shapes, and extruded triangles for more complicated geometries. Both elements belong to the Lagrangian second order (quadratic) finite element family. Therefore hexahedric have 20 nodes and extruded triangles have 15 nodes. When considering only an elastic beam each node will have 3 degrees of freedom (DOFs), namely the displacement components in the directions x , y and z . Since our major objective considers the use of piezoelectric material, we need to add a fourth degree of freedom, the electrical potential, $\phi_p \equiv v(t)$. When dealing with smart beams, the nodes from the elastic component will have the electrical DOFs null obviously.

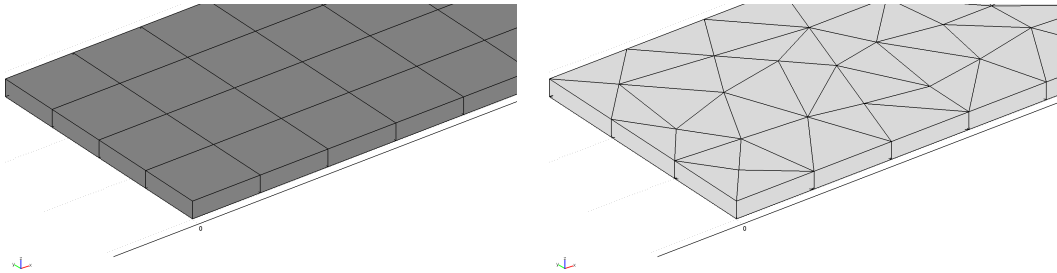


Table 4.2: Hexahedric and Extruded triangles elements

Having in mind both actuation Equation (2.63) and sensing Equation (2.70) developed in chapter 2, one must perform the same development in a FEM model. That can

be achieved by assembling the mass matrix, the mechanical stiffness matrix, the piezo-electric matrix and dielectric matrix. Beyond the matrices assembling, it is also needed to build acceleration, displacement, electrical potential, mechanical loading and electrical loading vectors. FEM actuating and sensing Equations are given as

$$[M_{uu}] \{\ddot{u}\} + [K_{uu}] \{u\} + [K_{u\phi}^A] \{\phi_A\} = \{F\}, \quad (4.1)$$

where ϕ_A represent only the electric degrees of freedom that perform influence on the beam's motion (actuation DOFs). The former equation can also be represented as

$$[M_{uu}] \begin{Bmatrix} \ddot{u}_1 \\ \ddot{v}_1 \\ \ddot{w}_1 \\ \ddot{u}_2 \\ \ddot{v}_2 \\ \vdots \\ \ddot{w}_n \end{Bmatrix} + [K_{uu}] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ \vdots \\ w_n \end{Bmatrix} + [K_{u\phi}] \begin{Bmatrix} \phi_{A1} \\ \phi_{A2} \\ \phi_{A3} \\ \phi_{A4} \\ \phi_{A5} \\ \vdots \\ \phi_{An} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ \vdots \\ F_n \end{Bmatrix}. \quad (4.2)$$

On the other hand, the sensing equation only comprehends the electric degrees of freedom with capability of detecting the smart beam's motion, ϕ_S , and is given by an induced voltage

$$[K_{\phi u}] \{u\} + [K_{\phi\phi}] \{\phi_S\} = \{Q\}, \quad (4.3)$$

or

$$[K_{\phi u}] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ \vdots \\ w_n \end{Bmatrix} + [K_{\phi\phi}] \begin{Bmatrix} \phi_{S1} \\ \phi_{S2} \\ \phi_{S3} \\ \phi_{S4} \\ \phi_{S5} \\ \vdots \\ \phi_{Sn} \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ \vdots \\ Q_n \end{Bmatrix}. \quad (4.4)$$

Recalling Equation (2.23), one can easily observe the prevailing similarity between both analytical and numeric models, where the influence done by both stiffness and mass matrices, $[M_{uu}]$ and $[K_{uu}]$, can be considered equivalent to the variational component δH_{uu} ; nonetheless the finite element considers the piezo transducer's mass and stiffness whereas the variational principle does not. The same can be said for the piezoelectric and dielectric components, where the $[K_{u\phi}]$ is related to $\delta H_{u\phi}$, $[K_{\phi u}]$ is related to $\delta H_{\phi u}$ and $[K_{\phi\phi}]$ is related to $\delta H_{\phi\phi}$.

Considering the FEM actuation Equation (4.1) and sensing Equation (4.3), and considering that there is no electrical loading, $Q = 0$, we can proceed on defining the actuation equation for both EBC considered, OC and CC. By considering no electrical loading, the sensing equation yields now

$$\{\phi_S\} = -[K_{\phi\phi}^S]^{-1} [K_{\phi u}^S] \{u\}. \quad (4.5)$$

Substituting the Equation (4.5) in Equation (4.1) and fixing $\phi_A = 0$, it is obtained the actuation equation for closed circuit,

$$[M_{uu}] \ddot{u} + [K_{uu} - K_{u\phi}^S K_{\phi\phi}^{S-1} K_{\phi u}^S] \{u\} = \{F\}, \quad (4.6)$$

or more simply

$$[M_{uu}] \{\ddot{u}\} + [K_{uu}^*] \{u\} = \{F\}. \quad (4.7)$$

The open circuit Equation is similar to previous one, differing only by considering $\phi_A \neq 0$, in other words are added the actuation electrical degrees of freedom, which belong to the top surface of the piezoelectric material, yielding

$$\{\phi_A\} = - [K_{\phi\phi}^A]^{-1} [K_{\phi u}^A] \{u\}, \quad (4.8)$$

$$[M_{uu}] \{\ddot{u}\} + [K_{uu}^* - K_{u\phi}^A K_{\phi\phi}^{A-1} K_{\phi u}^A] \{u\} = \{F\}. \quad (4.9)$$

It is of major paramount to point out that different EBCs mean also different piezoelectric and dielectric matrices. Considering the lower electrode from the transducer as grounded for both cases, while the upper electrode will be considered as grounded for closed circuit, and will be considered to have a zero/charge symmetry for an open circuit case. Therefore the piezoelectric and dielectric matrices will consider only the inner electrical degrees of freedom of the piezoelectric transducer for a closed circuit, whereas the open circuit considers also the electric degrees of freedom on the upper electrode. One can conclude, that although closed-circuit has no electrical potential variation between electrodes, there is still present an electric potential variation along the inner electrical degrees of freedom, therefore suggesting that a closed-circuit model will also produce the piezoelectric effect, obviously not as significant as in the open-circuit model.

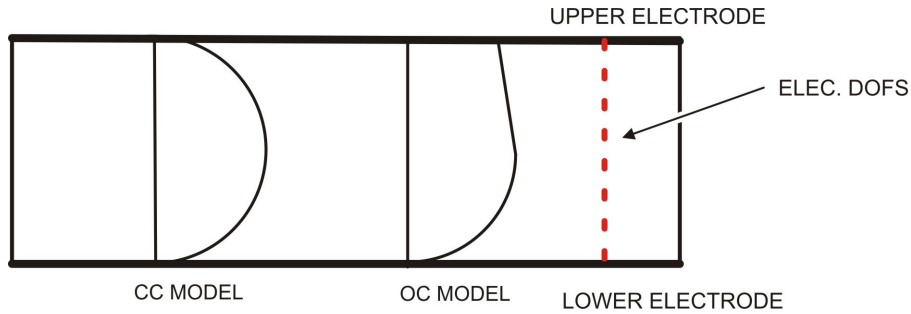


Figure 4.2: Electrical potential variation, CC and OC.

Including the shunt circuit on finite element is done by considering electrical flux as a function of the shunt impedance,

$$\{\dot{Q}\} = \left[\frac{1}{-Z_{sh}} \right] \{\dot{\phi}\}. \quad (4.10)$$

and recalling the Equation (4.3), the shunt integration equation yields now

$$\left[\frac{1}{-Z_{sh}} \right] \{\dot{\phi}\} = [K_{\phi u}] \{\dot{u}\} + [K_{\phi\phi}] \{\dot{\phi}\}. \quad (4.11)$$

Developing the former equation in Laplace form, it yields

$$- [K_{\phi u}] \{u(s)\} s = s \left[\left(\frac{1}{-Z_{sh}} \right) [I] + [K_{\phi\phi}] \right] \{\phi(s)\}, \quad (4.12)$$

or more simply

$$\{\phi_{Shunt}\} \equiv \{\phi(s)\} = \left[\left(\frac{1}{-Z_{sh}} \right) [I] + [K_{\phi\phi}] \right]^{-1} [K_{\phi u}] \{u\}, \quad (4.13)$$

where it is obtained the electrical charge vector under the influence of the parallel shunt circuit, $\{\phi_{Shunt}\}$. Substituting it in Equation (4.1), are obtained the equation for actuation the shunt circuit,

$$[M_{uu}] \{\ddot{u}\} + [K_{uu}] \{u\} + [K_{u\phi}] \{\phi_{Shunt}\} = \{F\}. \quad (4.14)$$

In order to estimate the capability of generating electrical charge from the beam's motion is also developed the FEM voltage per unit of load equivalent FRF function, considering again that there is no exterior electrical charge applied to the piezo patch. Recalling the sensing Equation (4.3) and considering the Open Circuit case we obtain,

$$\{\phi_S\} = -[K_{\phi\phi}]^{-1} [K_{\phi u}] \{u\}, \quad (4.15)$$

where the displacement field, $\{u\}$, is given by the Equation (4.9). For matters of evaluation it is not considered an equi-potential surface on the upper electrode, and therefore the FEM model give us an array of values for the surface. These values depend on the their location and on the mesh discretization. It is also considered the evaluation of electrical potential peaks values, in order to observe the deviation from the average value which is obtained by considering an equi-potential surface.

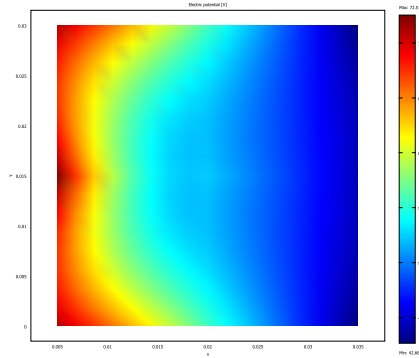


Figure 4.3: Electrical potential distribution across a piezoelectric transducer.

The image presented at figure 4.3 shows the distribution of electrical potential in a piezoelectric transducer located near the fixed tip of a clamped-free beam. One can observe higher values of electrical potential closer to the fixed tip, where the deformation motion is more significant. It is also important to mention that the previous image shows a tendency and was produced in COMSOL without post-processing.

4.2 Elastic Beams Analytical and FEM Models

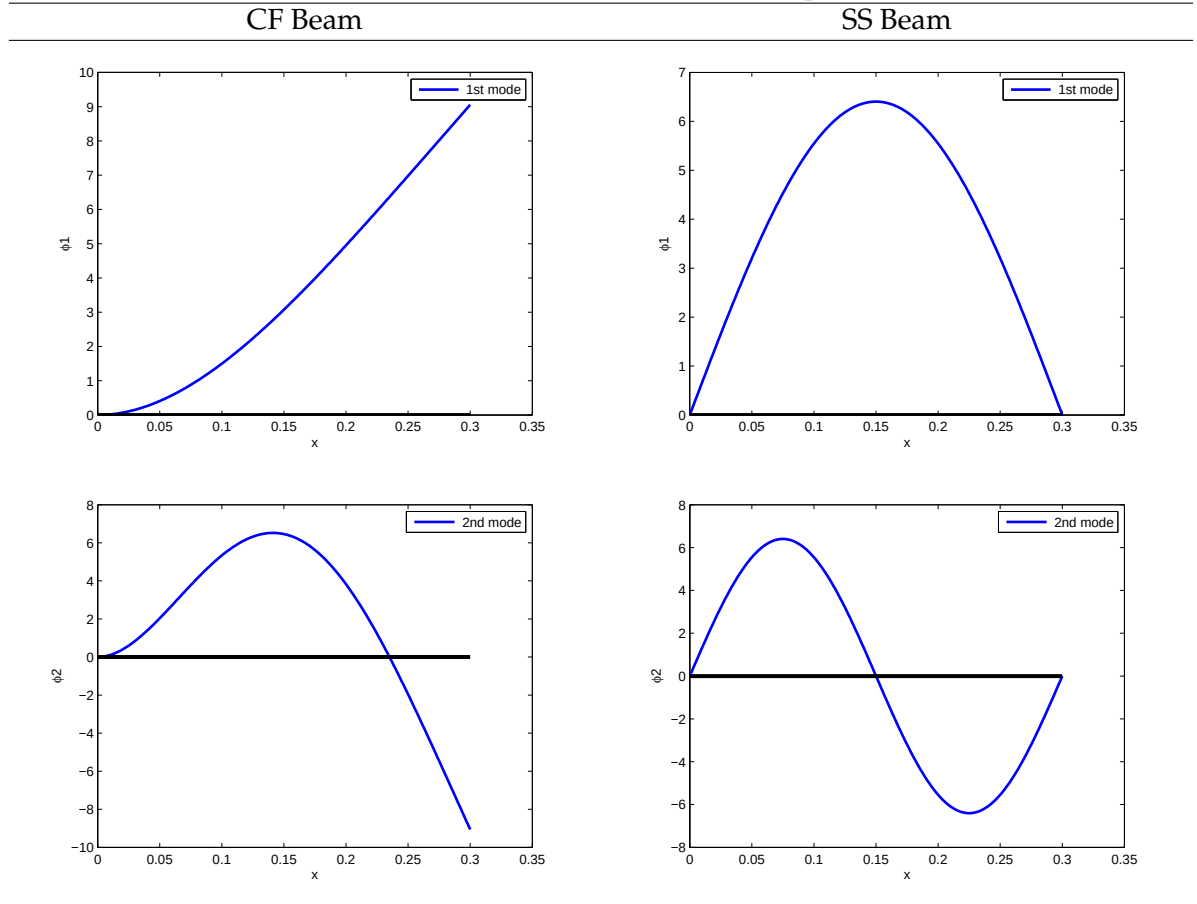
The clamped-free (*CF*) case is largely studied for piezoelectric transducers implementation, for its simplicity of experimental setup assembling, and also for its predictable motion when submitted to a generic force. The simply-supported (*SS*) case shows more

complexity since symmetric or anti-symmetric mode-shapes can be easily canceled by a wrong choice of the excitation location, however mathematically its modeling is easier than the *CF* case. These differences make it interesting to explore the implementation of passive vibration damping for both cases. In the following tables are presented a brief description of both models properties.

Table 4.3: Beam's eigenfrequency and mode shapes

<i>BC</i>	<i>Freq Eq.</i>	<i>Roots ($\beta; l$)</i>	<i>Mode Shapes</i>	<i>Nodes</i>
<i>CF</i>	$\cos\beta l$ $*\cosh\beta l = -1$	1.8751	$\phi_n = C_n(\cosh\beta_n x - \cos\beta_n x - \alpha_n(\sinh\beta_n x - \sin\beta_n x))$ $C_n = \frac{1}{\sqrt{\rho A \int \phi_n^2}}$ $\alpha_n = \frac{\sinh\beta_n l - \sin\beta_n l}{\cosh\beta_n l + \cos\beta_n l}$	0.774 0.5/0.868 0.356/0.644/0.906
		4.6941		
		7.8548		
<i>SS</i>	$\sin\beta l = 0$	$\left(\frac{(2n-1)\pi}{2}\right)$	$\phi_n = C_n \sin\beta_n x$ $C_n = \sqrt{\frac{2}{\rho A l}}$	0.5 0.333/0.667 0.356/0.644/0.906
		π		
		2π		
		3π		
		4π		
		5π		

Table 4.4: Beam's 1st and 2nd mode shapes



Ahead are presented the finite element results here for eigenfrequencies, static analysis and frequency response analysis, and a comparison to the analytical results.

4.2.1 Clamped-Free beam (CF)

The *CF* FEM model consists of simple beam, with the left tip fixed and the right one free, the force is considered as being an unitary load distributed along the cross-section at the free tip. The mesh used in the beam is constituted by 26 elements, all of them hexahedral from the Lagrangian-quadratic element family. Overall the model has 1215 degrees of freedom.

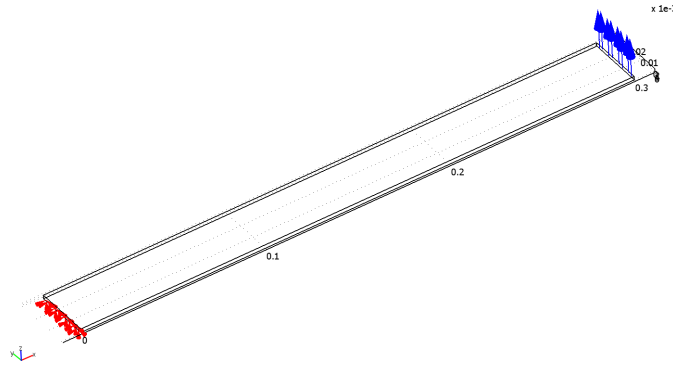


Figure 4.4: Clamped-Free Beam

Comparison between FEM and analytical results:

- Eigenfrequency values

Table 4.5: CF Beam Eigenvalues.

(Hz)	analytical	numerical	deviation (%)
ω_1	18.24	18.57	1.78
ω_2	114.37	116.91	2.14
ω_3	320.20	331.30	3.35
ω_4	627.40	661.42	5.14

- Static analysis

$$\text{max.displacement} = \frac{Fl^3}{3EI} \quad (4.16)$$

The maximum displacement occurs on the free tip of the beam, here is considered a unitary load.

Table 4.6: Maximum displacement

	analytical	numerical	deviation (%)
max.displacement(m)	0.0064	0.0063	1.56

- Frequency response analysis

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \sum_{r=1}^{\infty} \frac{\phi_r(x)\phi_r(x_F)}{(\omega_r^2 - \omega^2) + j(2\xi_r\omega_r\omega)} \quad (4.17)$$

For a frequency response analysis is considered a direct function, meaning that the loading and measuring points are the same and once more is considered a unitary load on the free tip of the beam. Let us recall that Equation (4.17) is the same as the FRF Equation (3.38), presented in the former chapter, differing only by not contemplating the presence of a piezoelectric transducer. It is considered both for analytical and numerical a viscous-damping value of 0.2% in the analytical method and an equivalent loss factor damping of 0.4% for the FEM model.

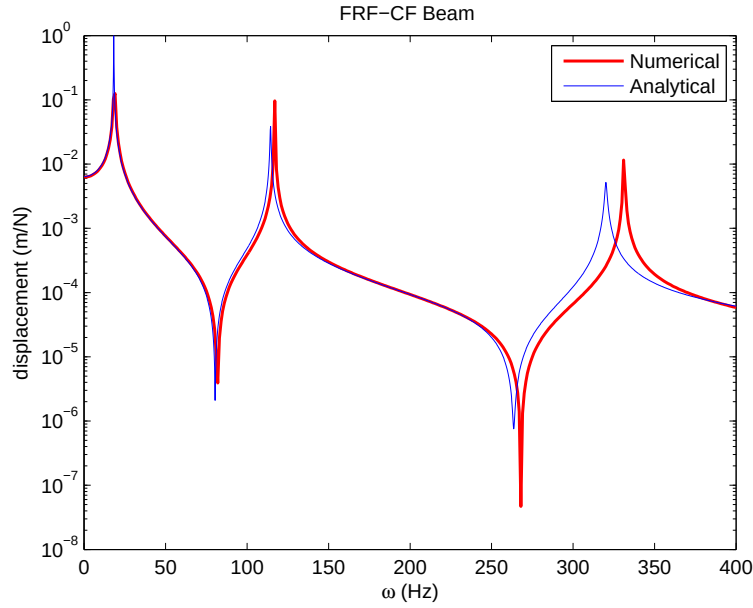


Figure 4.5: CF bare beam FRF.

4.2.2 Simply-Supported beam (SS)

For the simply supported beam is followed the same strategy for the finite element method validation. The mesh is similar presenting also 26 hexahedral elements and 1215 degrees of freedom.

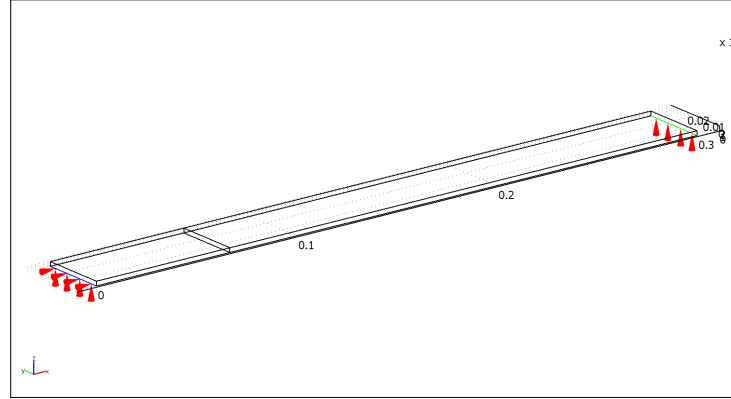


Figure 4.6: SS beam

- Eigenfrequency values

Table 4.7: SS Beam Eigenvalues

(Hz)	analytical	numerical	deviation (%)
ω_1	51.21	51.40	0.37
ω_2	204.90	207.65	1.32
ω_3	460.92	474.89	2.94
ω_4	819.2	862.38	4.98

- Static analysis

For the static analysis the load is applied at the $\frac{2}{9}l$ cross-section of the beam. In the FEM software the unitary load is substituted by an equivalent distributed pressure along the cross-section, the same way it was made for the CF beam.

$$\text{max.displacement} = \frac{Fa}{3EI} \left(\frac{l^2 - a^2}{3} \right)^{\frac{3}{2}}, \quad a = \frac{2}{9}l \quad (4.18)$$

Table 4.8: Maximum displacement

	analytical	numerical	error (%)
maximum.displacement(m)	1.92e-4	1.97e-4	2.54

- Frequency response analysis

The frequency response function is the same as presented in Equation (4.17), and like in the CF test it is a direct function, where the measuring point and loading point are the same. The loading point is the same mentioned for the static analysis, it was chosen in order to observe all mode shapes, for instance if the load was located at mid length of the beam, the second mode resonance would be canceled out due to the anti-symmetry nature of this mode shape.

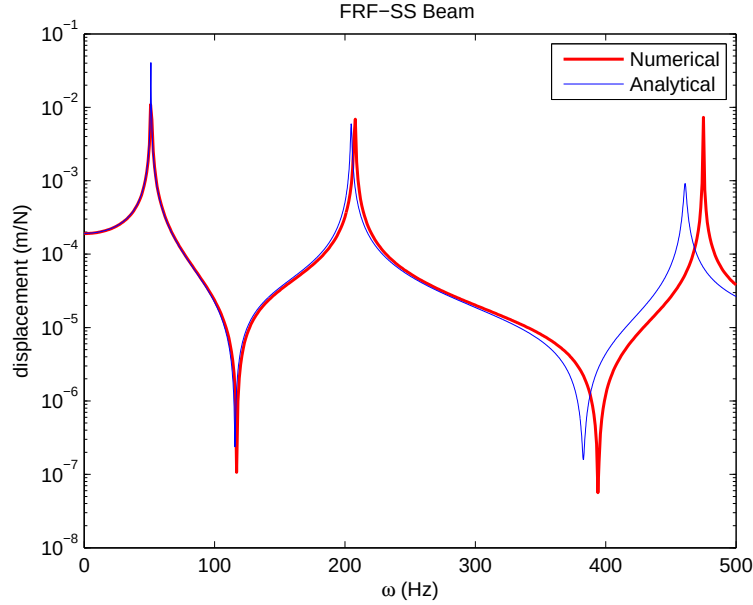


Figure 4.7: SS bare beam FRF.

The FEM results can achieve better results with a more refined mesh, but motivated for long times of post-processing, the mesh was decreased in order to make it practicable. Despite this not being the best discretization, the deviation from the FEM model to the analytical model is reasonable, except for the 4th mode, where a 5% deviation is quite significant. Therefore FRF graphics will only show the 3 first resonances.

4.3 Smart Beam: Uniform Transducers

After a brief validation of the finite element method, it is time now to take this study one step further, meaning by that, piezoelectric patches are now considered attached to our CF and SS beams. In this section are developed considerations about the influence of these elements on the beams behavior, being developed the relations between force applied and displacement (receptance, FRF), voltage induced by an applied force, and finally the relation between displacement and a voltage applied. It is expected to observe some changes in resonances and amplitude values. Another matter that one must have in mind, is the fact that the analytical formulation developed so far does not consider the mass and stiffness effects of the piezoelectric patch, while the FEM model for its nature covers all of the mentioned issues, meaning by that a disagreement between analytical and numerical results will exist. Despite this expectable disagreement, the phenomeno-

logical behavior must be similar for both models, the difference being smaller as the $\frac{h_b}{h_p}$ ratio increases.

In order to make a simpler and fast approach it is considered for now uniform and segmented piezoelectric transducers, although generic expressions are presented so that they can easily be applied later for more complex cases, like modally shaped piezoelectric transducers.

4.3.1 Actuation Behavior: Displacement per Unit of Force

The expression “actuation” regards the typical frequency response function, Equation (3.38), applied to a smart beam. This expression considers the presence of a piezoelectric patch bonded to the host structure. The piezoelectric layer follows the motion of bending while is applied a generic load, and recalling that piezoelectric materials when deformed generate electric charge, it is expectable that this electro-mechanical energy conversion will attenuate in a certain way the action of the load on the beam, meaning by that eigenvalues will rise, or in other words the simple presence of a piezoelectric layer can improve the stiffness of the beam, depending also on the electric boundary conditions. In order to quantify the actuation capability of piezoelectric transducers is needed to make a comparison between two specific electric boundary conditions (EBC), Open-Circuit (OC) and Closed-Circuit (CC). A closed-circuit model implies that there is no difference of electrical potential between the upper and lower electrodes (analytical model) of the piezo layer, and despite existing electrical charge flux, the piezo layer will not induce any action over the beam beyond the one motivated by the mass and stiffness of the piezo layer itself. On the contrary, the open-circuit presents no electrical charge flux, and implies a voltage difference between the electrodes, this phenomenon adds forces and/or moments, depending on the shape of the layer, that will act in opposition to the load applied on the structure.

The analytical formulation presented in this work does not comprehend the mass and stiffness of the piezoelectric material, leaving only the inner capacitance of the piezo to contribute on the FRF. Therefore the CC’s FRF is the same as the ones shown before for the aluminium simple beams. The influence of the piezo can only be measured analytically for the OC circuit model, where the capacitance is no longer null. The FEM approach allow us to test these different electric boundary conditions, and by that way estimate the influence of the piezoelectric effect and the influence of mass and stiffness of the piezo layer.

As mentioned before, the piezo patch used to validate these phenomena, has a constant width, $b_p(x) = b_p$, and it is segmented, leading the piezoelectric effect to induce two concentrated moments on its borders. The piezoelectric material chosen for all the following analysis is the PXE-5.

Table 4.9: PXE-5 properties.

elastic matrix	$c_{11} = c_{22}$	$c_{12} = c_{21}$	$c_{13} = c_{31}$	c_{33}	$c_{44} = c_{55} = c_{66}$
Nm^{-2}	9.4511e10	4.3229e10	4.1332e10	8.0348e10	2.564e10
piezoelectric matrix	e_{11}	e_{33}	e_{33}	e_{33}	e_{33}
Cm^{-2}	-8.9531	-8.9531	22.4058	13.2051	13.2051
dielectric matrix	ϵ_{11}	ϵ_{22}	ϵ_{33}		
ϵ/ϵ_0^*	1032.3	1032.3	400		
density					
Kg/m^3	7800				
ECC**	K_{31}	K_{33}			
	0.38	0.75			

* $\epsilon_0 = 8.85\text{e-}12 \text{ F/m}$

**ECC is the electromechanical coupling coefficient of the piezoelectric material.

The analytical displacement per unit of force equation (receptance) can be written as

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \sum_{r=1}^{\infty} \frac{\phi_r(x_F)\phi_r(x)}{(\omega_r^2 - \omega^2) + j(2\xi_r\omega_r\omega) - b_p c_v c_i \left[\frac{d\phi_r(x_R)}{dx} - \frac{d\phi_r(x_L)}{dx} \right] Z_r}, \quad (4.19)$$

for uniform patches, where x_R and x_L represent its borders. One can easily adapt it to a full covering piezo by matching x_R with l and x_L with 0. The coefficient Z_r considering an open-circuit model is the inverse of the uniform capacitance of the piezo, therefore can be expressed as the following Equation,

$$Z_r = \frac{2h_p}{A_e \epsilon_{33}^*}. \quad (4.20)$$

The former Equation (4.19) was developed taking into account the decoupling of mode shapes used in the formulation in Chapter 3. However, uniform transducers, in reality, are excited by all the mode shapes presents in the beam's motion. Therefore, one can consider a different approach, by not performing the decoupling of the mode shapes, and develop a new and more accurate FRF. This solution oblige one to use a matrix form equation defined as

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \{\phi_r\}^T [(-\omega^2 + 2j\xi_r\omega_r\omega + \omega) [I] - c_v c_i b_p Z_r [M_{rk}]]^{-1} \{\phi_r\}, \quad (4.21)$$

where the generic term M_{rk} of the coupling matrix is defined by

$$M_{rk} = \left[\frac{d\phi_r(x)}{dx} \Big|_{x_L} \frac{d\phi_k(x)}{dx} \Big|_{x_R} \right]. \quad (4.22)$$

For the modally shaped transducers, the displacement per unit of force function is clearly decoupled, but it is more complex than Equation (4.19). The shaped transducers implies the consideration of an also shaped capacitance function, Equation (2.81), and a shaped loading term, Equation (3.22), yielding

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \sum_{r=1}^{\infty} \frac{\phi_r(x_F)\phi_r(x)}{(\omega_r^2 - \omega^2) + j(2\xi_r\omega_r\omega) - c_r^e Z_r}, \quad (4.23)$$

where $c_r^e Z_r$ yields

$$c_r^e Z_r = b_p c_v c_i Y_b I_b \beta^2 \alpha_r^2 \left[\int_l \phi_r(x) \frac{d\phi_r^4(x)}{dx^4} dx \right] \left[\frac{\epsilon_{33}^* b_b}{h_p} \beta \int_l \frac{Y_b I_b}{\omega_r^2} \frac{d\phi_r^2(x)}{dx^2} dx \right]^{-1}. \quad (4.24)$$

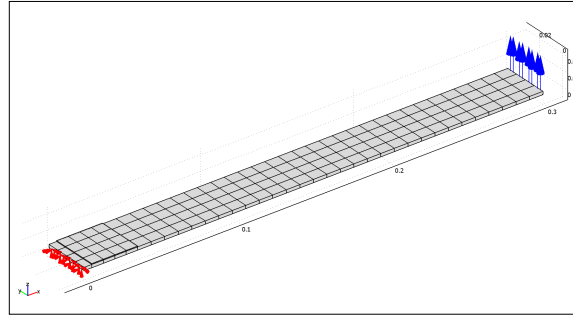


Figure 4.8: Smart beam mesh sample (test one).

4.3.1.1 CF beam actuation results

In order to evaluate the behavior of a CF beam with a piezoelectric patch bonded to it, are considered 3 different patches sizes, two of them are segmented and are placed in 2 different position; it is also considered a full covering piezo. This allows us not only to understand how a piezo can affect a beam, but how other issues like location or shape, will influence the beam's behavior.

Table 4.10: Piezo's geometry

test	1	2	3	4	5
length (l)	0.03	0.030	0.060	0.060	0.3
width ($2b_p$)	0.03	0.03	0.03	0.03	0.03
thickness ($2h_p$)	0.0005	0.0005	0.0005	0.0005	0.0005
location*	0.005	0.135	0.005	0.120	—

* distance to the left beam's extremity.

- Test 1

The first test considers the first piezo configuration, shown in table (4.10).

Table 4.11: Analytical eigenfrequency OC values for a 30mm piezo located 5 mm away from the clamped tip.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	18.24	19.6	19.6	6.94%
ω_2	114.37	119.1	118.8	4.13%
ω_3	320.20	326.8	326.8	2.02%
disp ($\omega = 0$)	0.0064	0.0056	0.0058	12.5%

* decoupled mode shapes; ** coupled mode shapes; *** deviation evaluates the piezoelectric effect considering decoupled mode shapes.

As can be attested, both in the previous table and in the following figures, one can consider modal decoupling for uniform transducers, since their eigenvalues are very close to the ones presented by the coupled modes model, this happens as well as for their FRF functions, which are very similar.

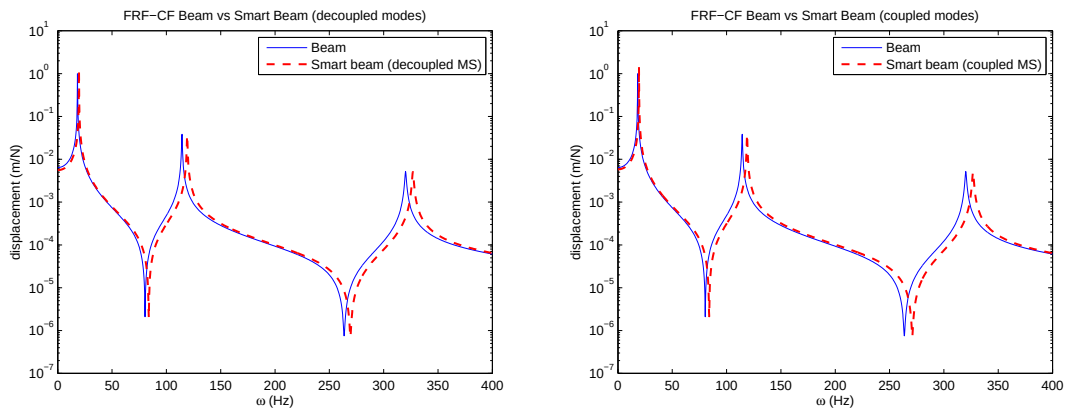


Figure 4.9: Analytical receptance function. Decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the first test has a mesh containing 30 hexahedral elements, yielding 1440 degrees of freedom.

Table 4.12: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	18.57	20.09	20.18	7.56%	0.45%
ω_2	116.91	123.54	123.98	5.4%	0.36%
ω_3	331.30	342.45	343.49	3.25%	0.3%
disp ($\omega = 0$)	0.0063	0.0055	0.0055	12.6%	0%

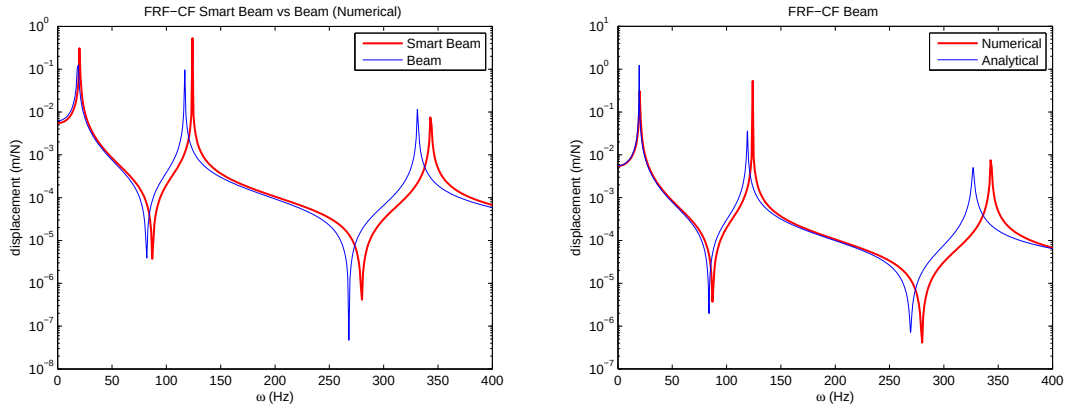


Figure 4.10: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

One can already conclude that the analytical model overestimates the piezoelectric effect, looking to the numbers presented by the numerical model, one can see that the biggest influence on the beam's eigenfrequency and static displacement is induced by the mass and stiffness of the piezoelectric patch. This disagreement was already expected, since the analytical formulation does not consider those properties.

- Test 2

The second test considers the same piezo, now located at the beam's mid length.

Table 4.13: Analytical eigenfrequency OC values for a 30mm piezo located 135 mm away from the clamped tip.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	18.24	18.4	18.4	0.87%
ω_2	114.37	119.4	119	4.21%
ω_3	320.20	320.2	320.2	0%
disp ($\omega = 0$)	0.0064	0.00628	0.00623	1.88%

* decoupled mode shapes; ** coupled mode shapes; *** deviation evaluates the piezoelectric effect considering decoupled mode shapes.

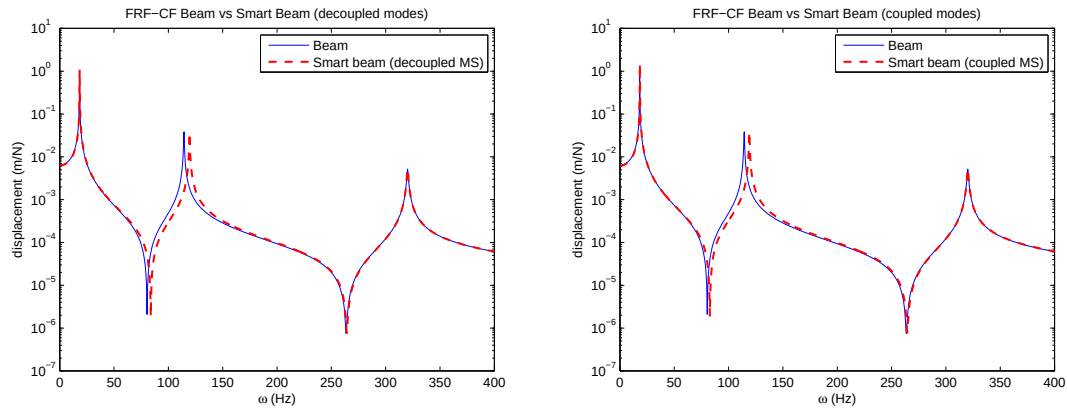


Figure 4.11: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 32 hexahedral elements, yielding 1530 degrees of freedom

Table 4.14: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	18.57	18.46	18.48	-0.59%	0.11%
ω_2	116.91	115.84	116.05	-0.92%	0.2%
ω_3	331.30	329.70	329.72	-0.48%	0.01%
disp ($\omega = 0$)	0.0063	0.00603	0.00602	4.30%	0.17%

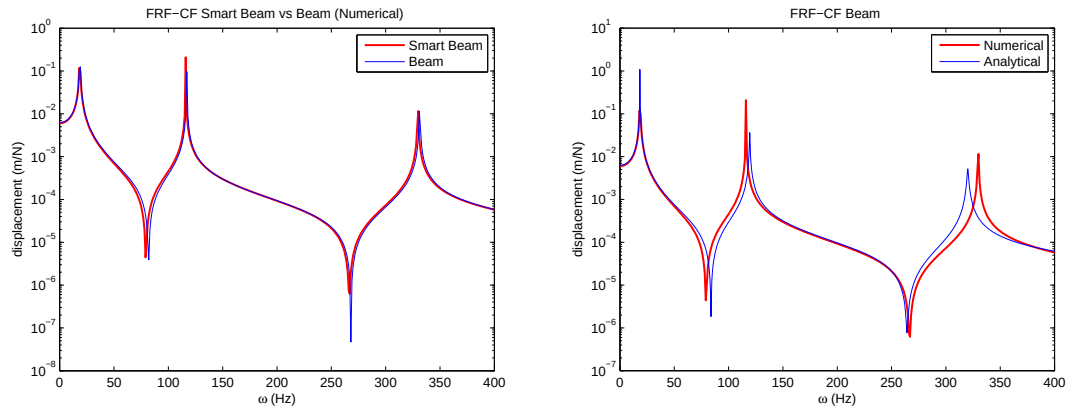


Figure 4.12: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

Comparing this test with the first one can see, that the same piezo patch located in different areas, acts very differently on the beam's behavior. The second test shows less piezoelectric influence, having smaller eigenvalues frequencies both on the analytical and numerical models than the first test, and since we are making these tests on a clamped-free beam it seems rather natural that a piezo located near the clamped tip has more influence than one located far away. Looking now exclusively to the numerical data, and

recalling that most of the influence comes from the mass and stiffness of the piezoelectric material, one can see that the second configuration actually decreases the eigenfrequencies values, which one can consider logical, since the mass added by the piezo is far from the clamped tip.

- Test 3

On this third test, we get the same location configuration as the one in the first test, but now with a piezo with the double size of the first one. One can observe that this larger piezo has a greater influence than the piezo used on the first test.

Table 4.15: Analytical eigenfrequency OC values for a 60mm piezo located 5 mm away from the clamped tip.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	18.24	20.4	20.2	10.6%
ω_2	114.37	118.6	118.4	3.57%
ω_3	320.20	321.4	321.4	0.37%
disp ($\omega = 0$)	0.0064	0.0051	0.0054	20.3%

* decoupled mode shapes; ** coupled mode shapes.*** deviation evaluates the piezoelectric effect considering decoupled mode shapes.

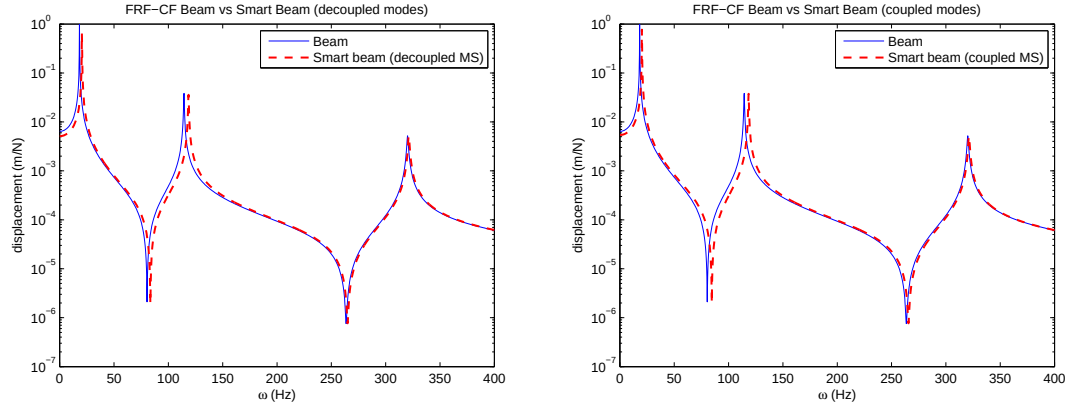


Figure 4.13: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 38 hexahedral elements, yielding 1800 degrees of freedom

Table 4.16: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	18.57	21.61	21.78	14.01%	0.80%
ω_2	116.91	124.25	125	6.1%	0.40%
ω_3	331.30	330.83	331.71	-0.14%	0.30%
disp ($\omega = 0$)	0.0063	0.0049	0.0048	22%	2.04%

Like in the first test, this test produces a rise in every single value of eigenfrequency. Both analytical and numerical model show now bigger eigenvalues, but looking only to the numerical results, one can see that for the third resonance, the bigger piezo's mass has the opposite effect, decreasing the Closed-circuit eigenvalue. Despite that, and turning into an Open-circuit, this third eigenvalue rises above the original eigenvalue.

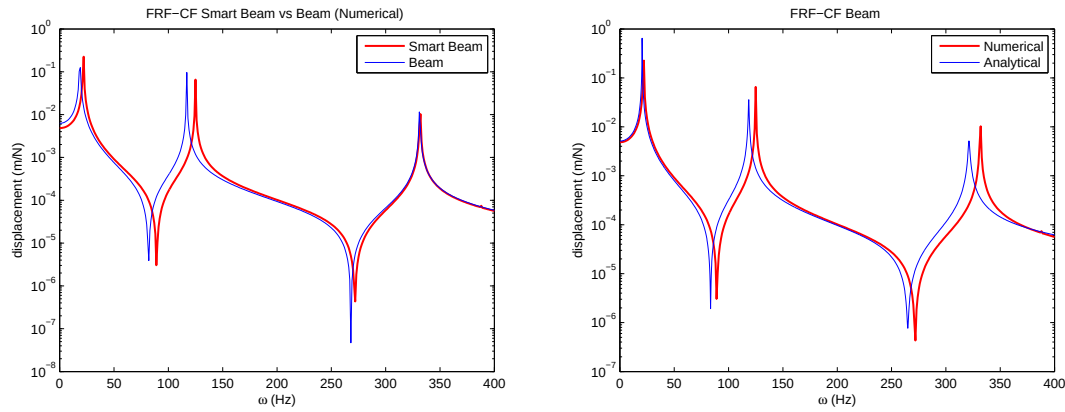


Figure 4.14: Numerical receptance FRF function comparison between a beam and a smart beam on the left side, on the right side is the comparison between numerical and analytical decoupled FRFs functions.

- Test 4u

On this fourth test, the mid-length configuration is recovered, but now with the same piezoelectric patch size from the third test. Comparing the analytical numbers from this test to the ones obtained in the second test, one can see that they share the same phenomenon, having both smaller eigenvalues than the tests with same piezo's sizes but located near the clamped tip. Although having a larger on this location seems to produce a bigger piezoelectric effect.

Table 4.17: Analytical eigenfrequency OC values for a 60mm piezo located 120 mm away from the clamped tip.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	18.24	18.6	18.6	2%
ω_2	114.37	123.8	122.8	6,89%
ω_3	320.20	320.2	320.2	0.02%
disp ($\omega = 0$)	0.0064	0.0061	0.0061	4.68%

* decoupled mode shapes; ** coupled mode shapes.*** deviation evaluates the piezoelectric effect considering decoupled mode shapes.

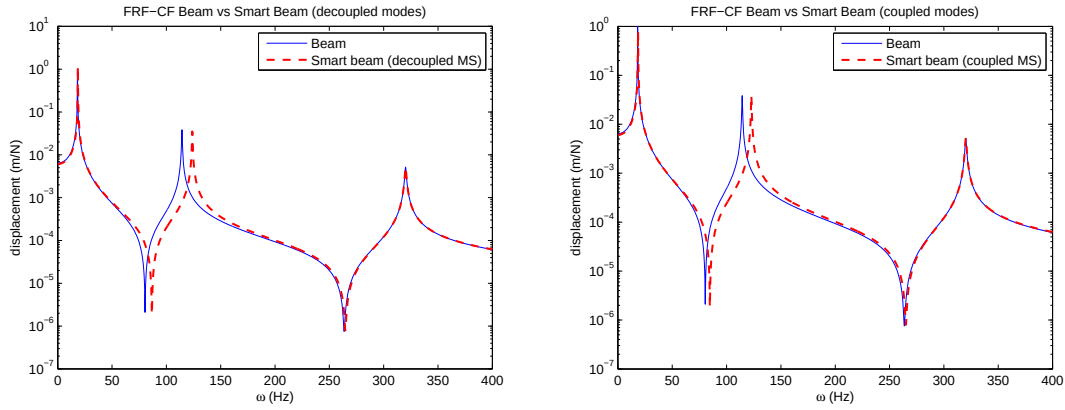


Figure 4.15: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 36 hexahedral elements, yielding 1710 degrees of freedom.

Table 4.18: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	18.57	18.34	18.35	1.24%	0.05%
ω_2	116.91	116.72	117.2	-0.163%	0.41%
ω_3	331.30	327.25	327.42	-1.22%	0.052%
disp ($\omega = 0$)	0.0063	0.0058	0.0057	7.98%	1.72%

Once more this location and considering exclusively the numerical data, shows a decrease in eigenvalues due to the mass influence, and the piezoelectric effect is not enough to overcome this issue, therefore eigenvalues overall decrease with this location.

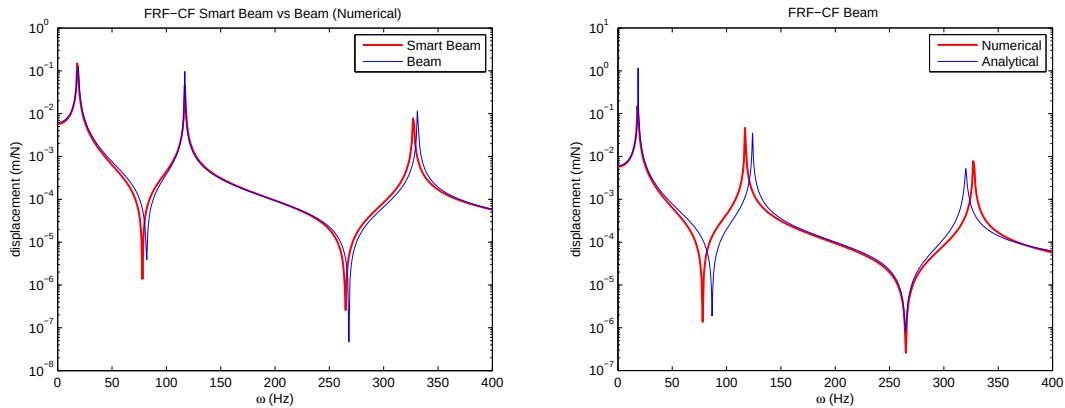


Figure 4.16: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

- Test 5

This test considers a full covering piezoelectric transducer.

Table 4.19: Analytical eigenfrequency OC values for a full covering piezo.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	18.24	20.6	20.4	11.45%
ω_2	114.37	119.4	119.2	4.21%
ω_3	320.20	324.8	324.8	1.42%
disp ($\omega = 0$)	0.0064	0.0050	0.0048	20%

* decoupled mode shapes; ** coupled mode shapes.*** deviation evaluates the piezoelectric effect considering decoupled mode shapes.

Looking at the analytical data, it is observable that all eigenvalues rise for this configuration, this can be explained by the larger capacitance this piezo has compared to the segmented ones, recalling that the capacitance considers the piezoelectric surface area.

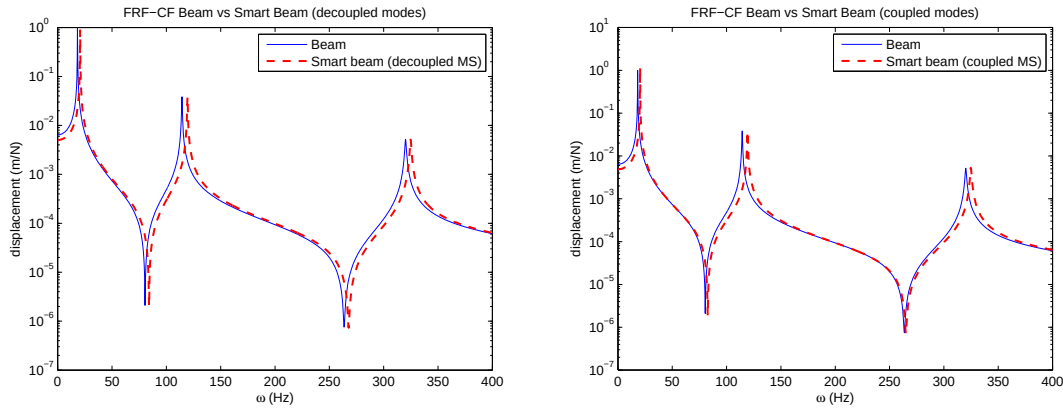


Figure 4.17: Analytical receptance function, decoupled mode shapes (left), right coupled mode shapes (right).

The FEM model for the second test has a mesh with 60 hexahedral elements, yielding 2740 degrees of freedom.

Table 4.20: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	18.57	20	20	7.16%	0%
ω_2	116.91	123	123	5%	0%
ω_3	331.30	347	350	4.5%	0.86%
disp ($\omega = 0$)	0.0063	0.0031	0.0030	50%	3.22%

The numerical data also shows the same tendency of increasing all eigenvalues, and decreasing by a large margin the static displacement. But as mentioned in all other 4 cases, the main influence comes from the piezoelectric transducer's mass, for instance looking for the Closed-circuit case, one can see a decrease of 50% on the static displacement. Looking to the Open-circuit data, one can observe a residual piezoelectric effect.

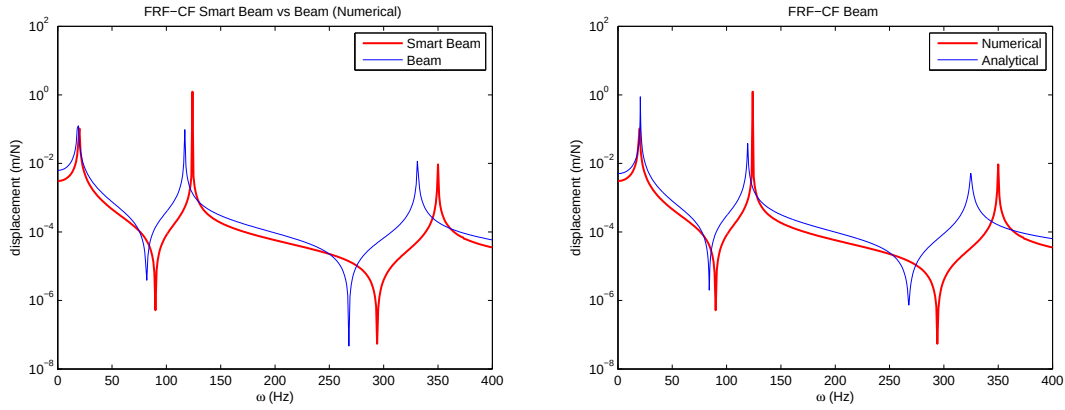


Figure 4.18: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

From all these tests, it is reasonable to conclude that analytical models typically overestimate the piezoelectric effect, despite this issue, the analytical model and numeric show analogous behaviors, having more influence on the beam's motion for segmented beam located close to the clamped tip and for the full covering piezo. In all numeric evaluations it was observed the prevalence of the mass effect over the piezoelectric one.

4.3.1.2 SS beam actuation results

Analogously to the behavior CF beam are made the same tests with the five configurations already described. The SS beam deserves more attention when it comes to chose the loading point, while in the CF beam, it was chosen without any doubt the free extremity. In order not to cancel vibration modes the load was applied at $\frac{2}{9}l$.

- Test 1

This first test uses the smallest piezo transducer mentioned in table (4.10), located 5mm away from the left tip of the beam. As can be seen in the following table this configuration does not increase much the eigenvalues neither decrease much the static displacement.

Table 4.21: Analytical eigenfrequency OC values for a 30mm piezo located 5mm away from the left tip of the beam.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	51.21	51.4	51.21	0.37%
ω_2	204.90	206.4	206	0.73%
ω_3	460.92	467.6	465.8	1.4%
disp ($\omega = 0$)	1.92e-4	1.91e-4	1.90e-4	0.52%

* decoupled mode shapes; ** coupled mode shapes; *** deviation evaluates the piezoelectric effect considering decoupled mode shapes.

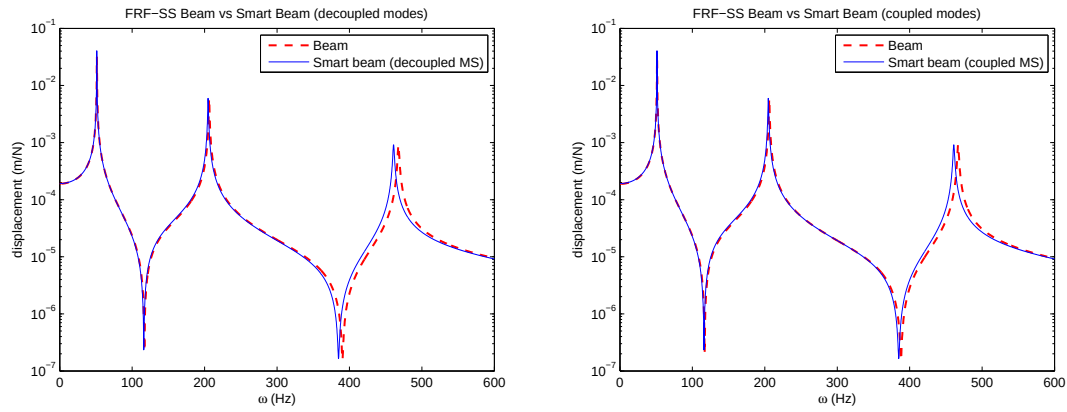


Figure 4.19: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 34 hexahedral elements, yielding 1620 degrees of freedom. Having in mind now the numerical results one can observe that, this configuration in fact decreases the eigenvalues, behavior motivated by the location of the mass of the piezo, since once more the piezoelectric effect is very reduced.

Table 4.22: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	51.40	51.30	51.31	-0.2%	0.02%
ω_2	207.65	206.7	207	-0.45%	0.15%
ω_3	474.89	472.30	473	-0.55%	0.15%
disp ($\omega = 0$)	1.97e-4	1.88e-4	1.88e-4	4.56%	0%

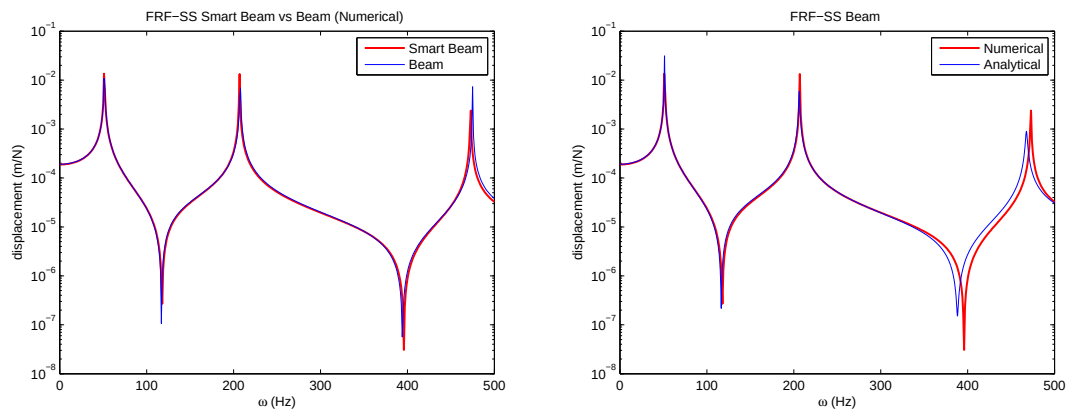


Figure 4.20: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

- Test 2

The second simply-supported beam test, moves the piezoelectric patch to the mid length of the beam, and looking at the numerical data several conclusions can be made. The

first one is that this configuration increases more the eigenvalues than the test one did, the second conclusion is that this location does not affect the second resonance eigenvalue at all. This can be explained for the symmetric loading provided by the transducer acting over a anti-symmetric mode shape. Recalling that a segmented transducer induces two concentrated moments, on its borders, and having in mind the anti-symmetry of the second mode shape, one can easily understand that both moments will cancel each other.

Table 4.23: Analytical eigenfrequency OC values for a 30mm piezo located 135mm away from the left tip of the beam.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	51.21	53.4	51.6	4.1%
ω_2	204.90	204.90	204.9	0%
ω_3	460.92	480.2	477	4.2%
disp ($\omega = 0$)	1.92e-4	1.82e-4	1.83e-4	5.2%

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

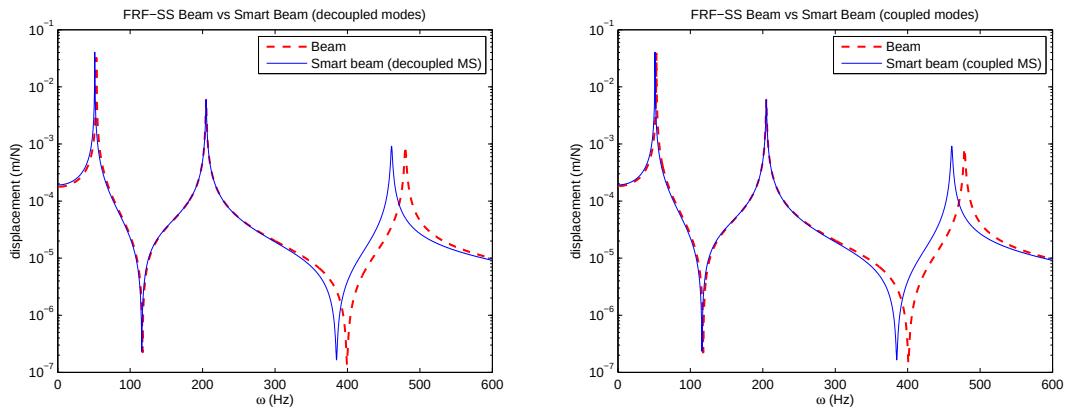


Figure 4.21: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 32 hexahedral elements, yielding 1530 degrees of freedom. The numerical results have a more complicated interpretation, the first eigenfrequency is reduced, motivated by the piezo's mass. The second resonance increases, unlike what is expected, as explained before for the analytical results. However the deviation on the second mode has the lesser deviation of all 3 cases, giving some coherence to the results. This increase can be motivated by the piezo mass, or for the simple mesh discretization. For the third resonance there is a major increase over the original value, most of it motivated by mass and stiffness from the transducer.

Table 4.24: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	51.38	50.80	50.88	-1.13%	0.16%
ω_2	207.64	207.80	207.80	0.08%	0%
ω_3	474.89	480.52	481.83	1.17%	0.27%
disp ($\omega = 0$)	1.97e-4	1.79e-4	1.78e-4	9.14%	0.55%

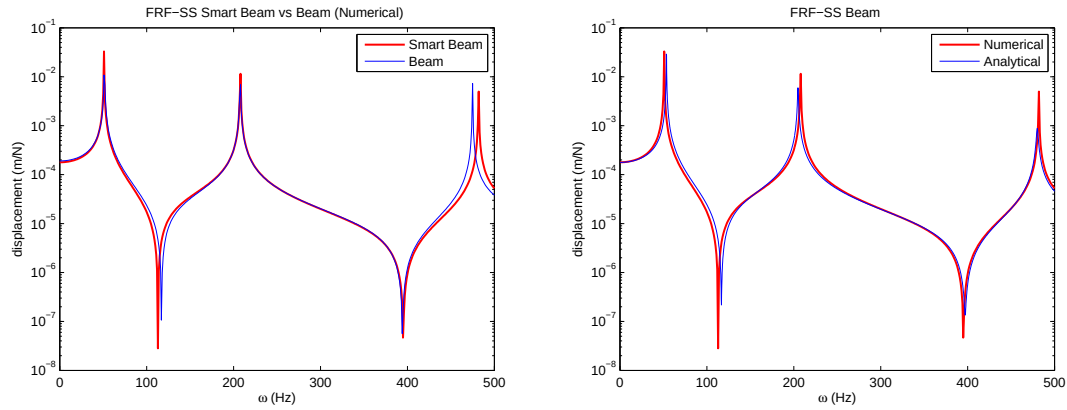


Figure 4.22: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

- Test 3

The third case has a 60mm piezo located at 5mm away from the left tip. Like in the first test this configuration has no major influence on the rise of eigenvalues, but having a piezo with twice the size of the first, the increase ratio is more noticeable, mainly for higher frequencies.

Table 4.25: Analytical eigenfrequency OC values for a 60mm piezo located 5mm away from the left tip of the beam.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	51.21	51.8	51.6	0.8%
ω_2	204.90	212.2	210.2	3.4%
ω_3	460.92	485.2	480.2	5%
disp ($\omega = 0$)	1.92e-4	1.72e-4	1.71e-4	10%

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

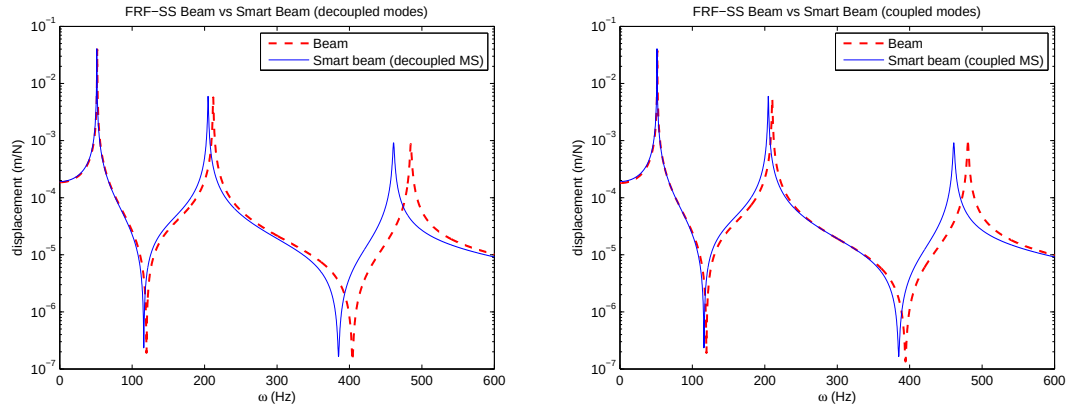


Figure 4.23: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 36 hexahedral elements, yielding 1710 degrees of freedom.

Table 4.26: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	51.38	51.11	51.14	-0.46%	0.06%
ω_2	207.64	206.7	207.19	-0.45%	0.24%
ω_3	474.89	478.50	480.50	0.75%	0.42%
disp ($\omega = 0$)	1.97e-4	1.72e-4	1.71e-4	12.6%	0.6%

The numerical results show a slight cutback on the first and second resonance, whereas the third resonance grows. This decrease is once more motivated by the piezo location, and how its mass and stiffness affect the beam.

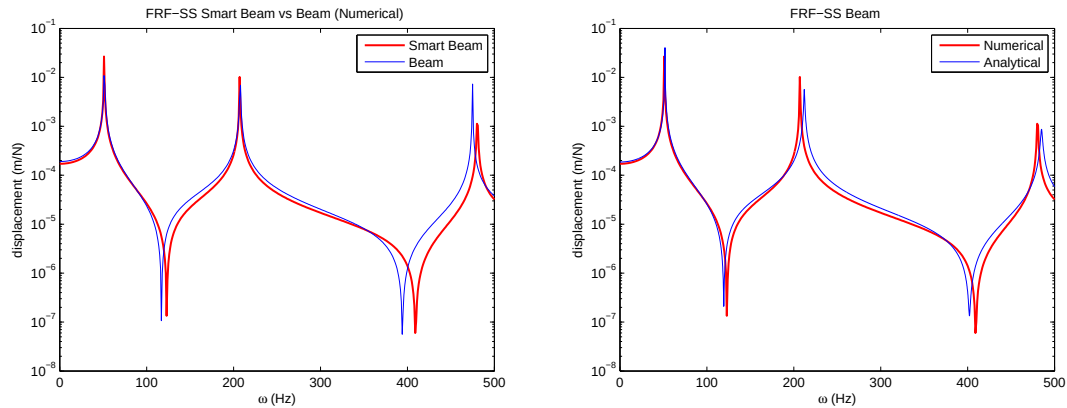


Figure 4.24: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

- Test 4

Having now the 60mm transducer with same location as the one used in second test, we see again no piezoelectric effect on the anti-symmetric resonance, namely the second one. As for the other resonances we observe once more that a larger transducer has more capability of inducing the piezoelectric effect, its larger area means a bigger capacitance, therefore explaining its greater capability for electromechanical energy conversion.

Table 4.27: Analytical eigenfrequency OC values for a 60mm piezo located 120mm away from the left tip of the beam.

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	51.21	55.6	54.8	6.6%
ω_2	204.90	204.90	204.90	0%
ω_3	460.92	491.4	488	6.7%
disp ($\omega = 0$)	1.92e-4	1.67e-4	1.76e-4	13%

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

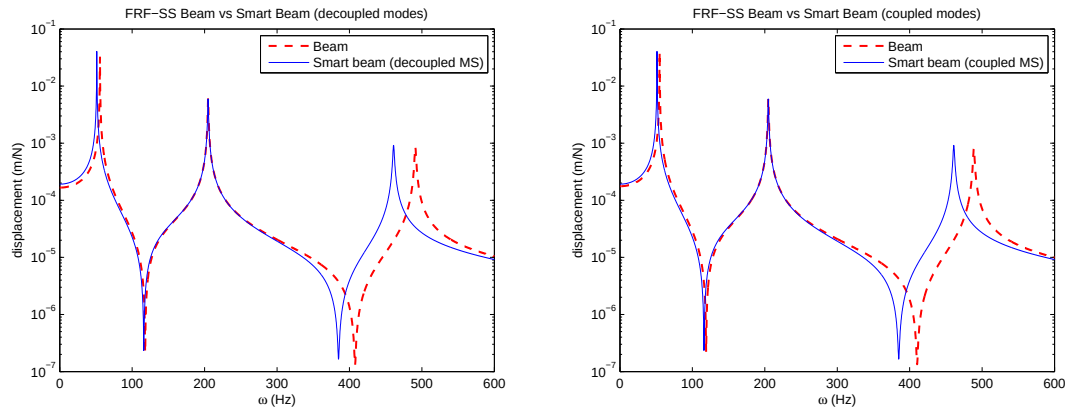


Figure 4.25: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 36 hexahedral elements, yielding 1710 degrees of freedom.

Table 4.28: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	51.38	50.67	50.90	-1.4%	0.45%
ω_2	207.64	206.20	206.25	-0.7%	0.02%
ω_3	474.89	481.93	484.60	1.46%	0.6%
disp ($\omega = 0$)	1.97e-4	1.67e-4	1.66e-4	15.22%	0.6%

The numerical values resemble the ones obtained in the previous test, with the two first resonances decreasing in Closed-circuit case. Since this piezo is located at mid-length, one would expect the second resonance to remain unchanged, but that does not occur. The second resonance has a slight change motivated mainly by the mass effect, since the piezoelectric effect in this mode is obviously smaller than the one occurring for the first and third resonances.

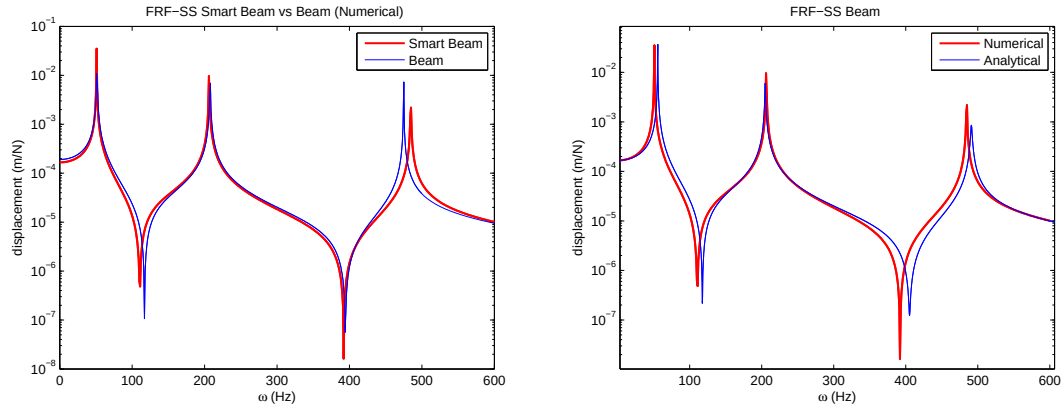


Figure 4.26: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

- Test 5

This last actuation test considers a full covering transducer on a simply-supported beam. Shown in the following table are the analytical values, which are very optimistic. Since is considered a non-segmented transducer, the capacitance is larger than the ones from segmented transducers, and therefore can be explained the higher values of analytical piezoelectric effect. Being a non-segmented transducers means also that will have no influence on the second resonance, since this configuration, like the second and fourth tests, is symmetric.

Table 4.29: Analytical eigenfrequency OC values for full covering piezo .

(Hz)	Beam	Smart Beam OC*	Smart Beam OC**	Deviation***
ω_1	51.21	60.2	59.6	15%
ω_2	204.90	204.90	204.90	0%
ω_3	460.92	470.2	470	2%
disp ($\omega = 0$)	1.92e-4	1.5e-4	1.45e-4	22%

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

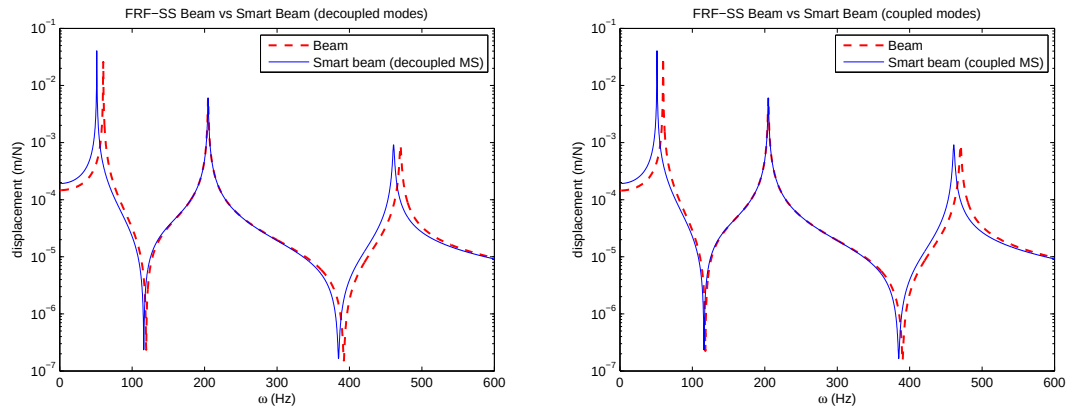


Figure 4.27: Analytical receptance function, decoupled mode shapes (left), coupled mode shapes (right).

The FEM model for the second test has a mesh with 60 hexahedral elements, yielding 2790 degrees of freedom.

Table 4.30: Numeric eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	Smart Beam CC	Smart Beam OC	M and K effect	Piezo. effect
ω_1	51.38	54.27	55	5.33%	1.33%
ω_2	207.64	218	223	4.8%	2.24%
ω_3	474.89	497	507	4.45%	2%
disp ($\omega = 0$)	1.97e-4	8.9e-5	8.5e-5	55%	4.5%

The numerical values show how great influence of the full covering transducer on the beam's behavior, the mass influence has increased, but the piezoelectric effect has also increased as well. It would be expectable to see small differences for the second resonance, for the symmetry of the transducer, but the mass of the transducer is very high when compared to the beam's mass, and cannot be neglected. For instance this aluminium beam weighs approximately 0.0488kg and the non-segmented transducer weighs about 0.0351kg, which is nearly a 72% increase of mass.

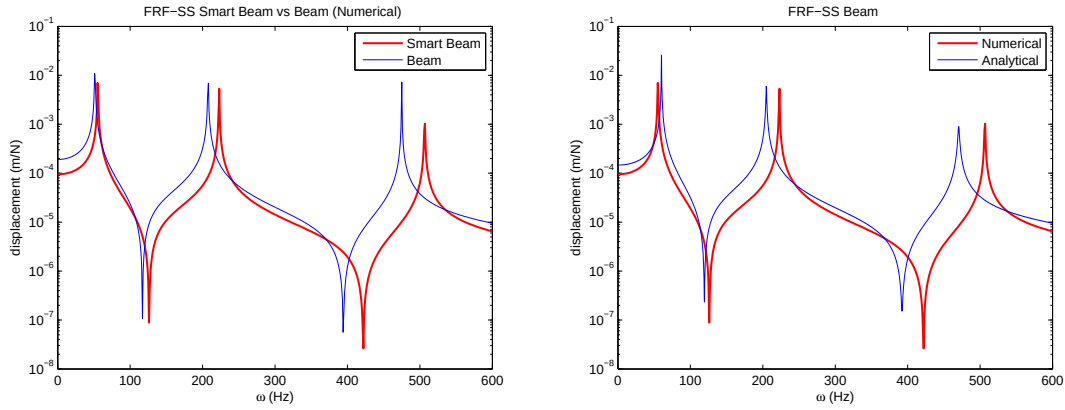


Figure 4.28: Numerical receptance function comparison between a beam and a smart beam (left), comparison between numerical and analytical decoupled FRFs functions (right).

4.3.1.3 Mechanical actuation (receptance) overview

The vast amount of tests give us the following information:

- the analytical model typically considers an higher value of piezoelectric effect than the numeric model;
- uniform transducers can be implemented using modal decoupling analytical formulation, for every case here presented, decoupled and coupled models did not have great disagreement;
- for the analytical model, bigger transducers mean always bigger piezoelectric effect, since this model only counts the piezoelectric capacitance, which is obtained of the piezo's surface area;
- different locations mean different piezoelectric effect on the beam, for clamped-free beams, the more close the transducer is to the clamped tip, the better piezoelectric performance it will induce, while for the simply-supported beams the mid-length located transducer will not produce any piezoelectric effect for anti-symmetric mode shapes;
- the numerical model shows that in reality, the mass and stiffness affects more the beam's behavior than the BC and the piezoelectric effects, which are residual in all cases;
- subjects as location are also of major importance considering the numerical model, as observed for clamped-free beams using segmented transducers located far from the clamped tip, the eigenvalues would decrease due to mass and stiffness effects, and since the piezoelectric effect was not big enough to counter this phenomenon, we ended up with similar problem of placing a weight on a bare beam, making its bending behavior more prominent;
- although the disagreements in results, between numerical and analytical models, they share the same phenomenology, mainly for lower resonances.

4.3.2 Sensing Behavior: Voltage per Unit of Force

The sensing behavior is considered to be the voltage induced along the piezo, by the bending motion motivated from a force applied to a beam. Analytically and recalling the definition of electric current for modal transducers and uniform transducer in Equations (2.78) and (2.82) respectively, we can proceed in developing the sensing equation by taking one important consideration: electrical current must be null, i.e. $i(t) = 0$. Once more we end up working on the OC model. The development of the sensing function will be here presented for uniform transducers only. If $i(t) = 0$, the Equation (2.82) yields now

$$C_p^s \frac{dv(t)}{dt} = e_{31}^* h_b 2b_b \sum_{r=1}^n \left[\int_l \frac{d^2 \phi_r(x)}{dx^2} dx \right] \frac{d\eta_r(t)}{dt}. \quad (4.25)$$

Integrating the previous Equation in order to time it is obtained the relationship between electrical potential, $v(t)$, and generalized modal coordinate,

$$v(t) = \frac{e_{31}^* h_b 2b_b}{C_p^s} \sum_{r=1}^n \left[\int_l \frac{d^2 \phi_r(x)}{dx^2} dx \right] \eta_r. \quad (4.26)$$

Since the deviation between coupled and decoupled mode shapes is not significant we opted for developing the voltage per unit of load FRF function considering decoupled mode shapes, yielding

$$\frac{\bar{V}(x, x_F, j\omega)}{\bar{F}} = \sum_{r=1}^{\infty} \frac{e_{31}^* h_b 2b_b}{C_p^s} \frac{\phi_r(x_F)}{(\omega_r^2 - \omega^2) + j(2\xi_r \omega_r \omega) - c_r^e Z_r} \frac{d\phi_r(x)}{dx} \Big|_{x_L}^{x_R}, \quad (4.27)$$

where $x_R = l$, and $x_L = 0$, for full covering piezoelectric transducers. Ahead are presented the results for the same 5 tests performed in the actuation section, both for clamped-free and simply supported beam.

4.3.2.1 CF beam sensing results

In this section is followed the same procedure, starting with smaller transducer in the first test and ending with a full covering piezo in fifth test, thereby this results are respective to the same cases presented before.

- Test 1

This first test is relative to the 30mm transducer located 5mm away from the clamped tip. One can see that numerical and analytical model share the same phenomenology, but as for electrical potential value is obvious the big gap between analytical and numerical results (Equi-Potential surface). This goes along with the already mentioned conclusion that our analytical model overestimates the piezoelectric effect. The electrical potential peak is more close to the one obtained analytically.

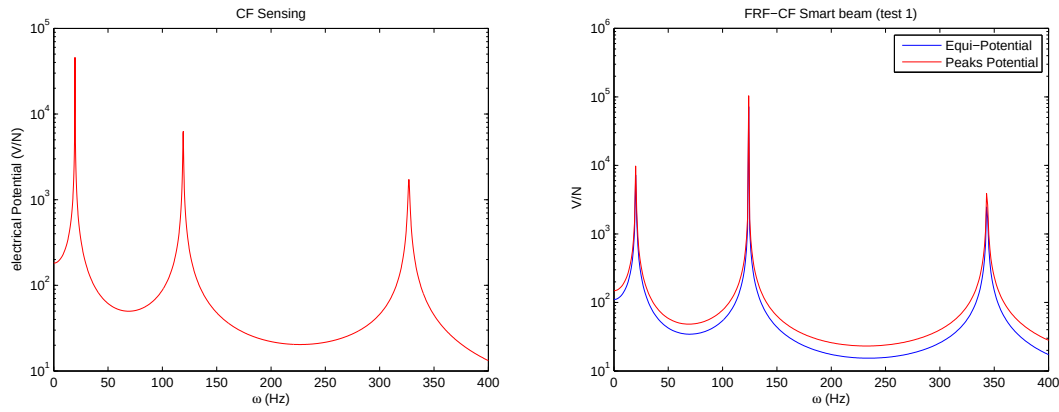


Figure 4.29: Voltage per unit of load; analytical (left) and numerical (right).

Table 4.31: Voltage per unit of load for zero frequency: analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	181.5	110.4	148.4

- Test 2

For a 30mm piezo located the voltage per unit of force frequency response already show anti-resonances. The behavior from both analytical and numerical voltage per unit of force frequency responses is very similar, but the gap between electrical potential values is the same as in the first test. The modification of location from the tip to mid length, shows lesser capability of sensing, with smaller values of electrical potential being detected. This second location suffers less from mechanical deformation than the first one, so there is also less mechanical energy available to convert.

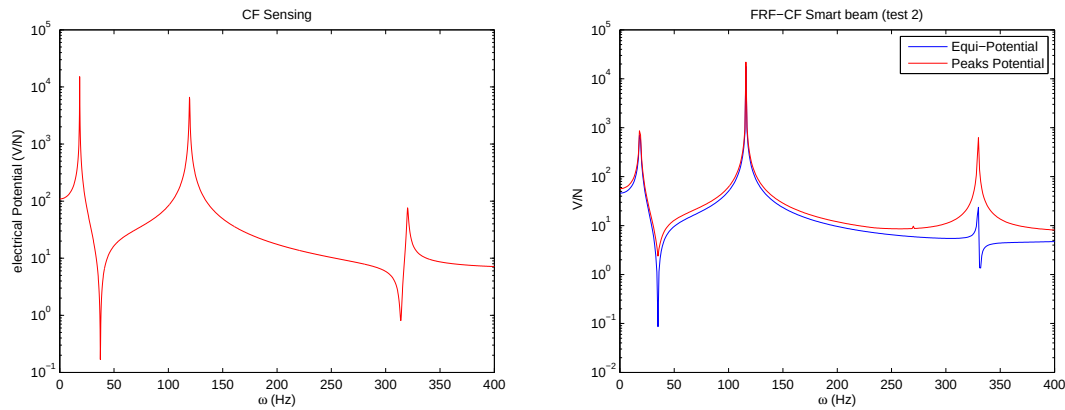


Figure 4.30: Voltage per unit of load (left), analytical and numerical (right).

Table 4.32: Voltage per unit of load for zero frequency: analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	109.2	46.01	56.82

- Test 3

Having the same location as the first test, with a transducer with twice the size, the behavior is very similar to the first test, although there's slight decrease in the electrical potential values.

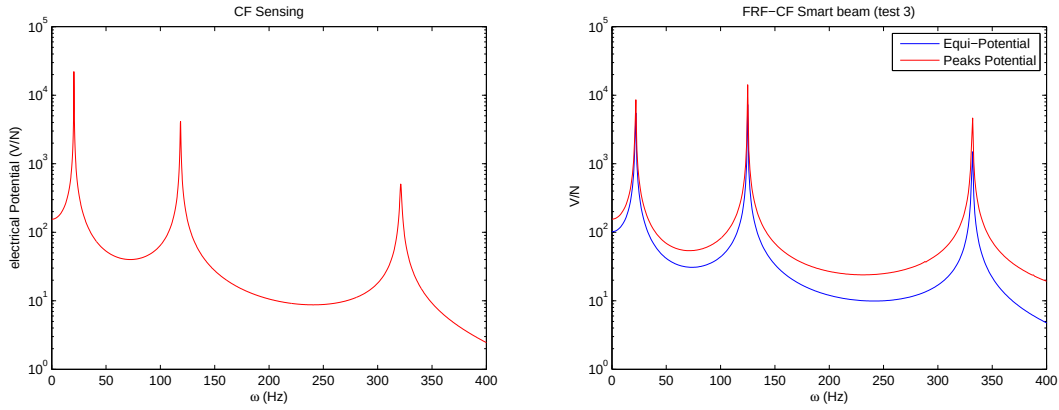


Figure 4.31: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.33: Voltage per unit of load for zero frequency: analytical and numerical.

	Voltage analytical	Equi-potential	Peaks Potential
$\omega = 0$	154.9	101.9	155.5

- Test 4

The same piezo as in test 3, now located at mid length, shows the same decrease in relation to test 3 that we have seen before, from test 1 to test 2. The motivation of this phenomenon has already been explained.

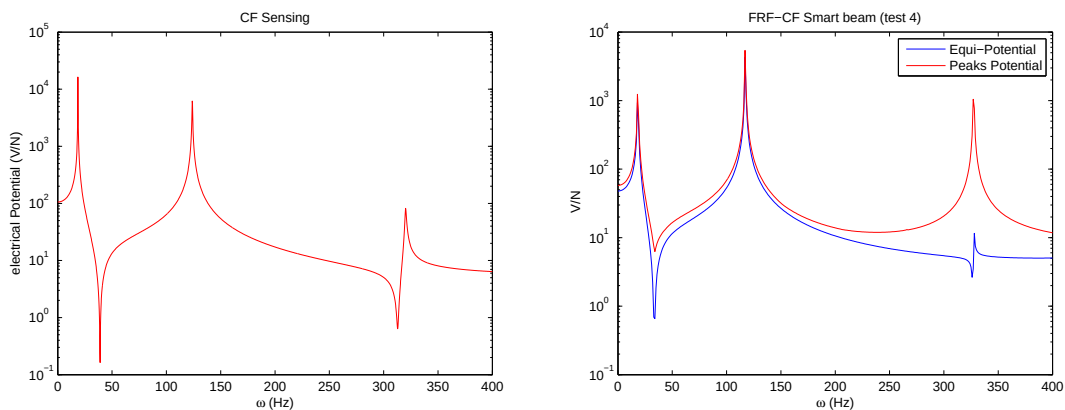


Figure 4.32: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.34: Voltage per unit of load for zero frequency: analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	105.7	47.82	57.91

- Test 5

For the full covering transducer we see a great disagreement in both, behavior and electrical potential. For instance the analytical model does not consider anti-resonances, whereas the numeric model does consider, this may be motivated for not considering negative electrical potential values in the analytical model. As for the electrical potential for a static loading, one can observe that the analytical model shows a value 7 times bigger than the Equi-potential.

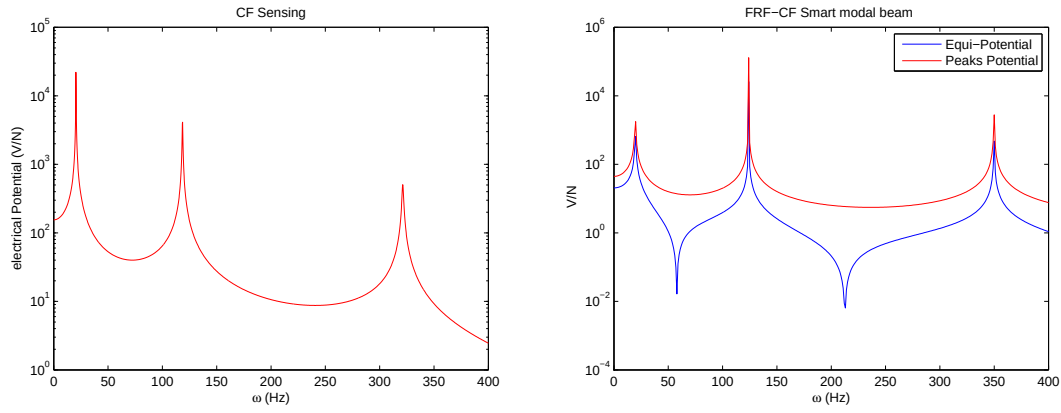


Figure 4.33: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.35: Voltage per unit of load for zero frequency: analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	151.6	21.2	59.2

4.3.2.2 SS beam results

- Test 1

Considering now the 30mm piezo located 5mm away of the left tip of the beam for a simply-supported beam, one can easily see the same behavior happening for both numerical and analytical models. Also electrical potential disagreement is not so high as presented for the test 1 in a clamped-free beam.

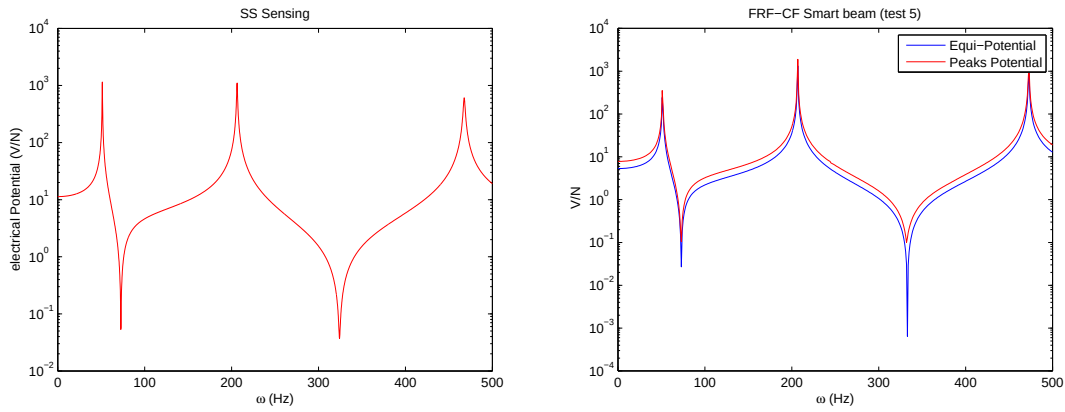


Figure 4.34: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.36: Voltage per unit of load for zero frequency, analytical and numerical.

	Voltage analytical	Equi-potential	Peaks Potential
$\omega = 0$	11.32	5.33	7.83

- Test 2

Moving the transducer to the mid length of beam one can observe that analytically we have no observable resonance for the second mode as expected, although the numerical method shows a resonance, resonance that can be explained for Poisson effect, which is not considered in the analytical model. Is of major paramount to have in mind that the analytical model is a 1D model, and the numeric a 3D model, and if consider a 3D beam being stretched in on direction, the beam will be compressed in the transversal direction. This compression is also capable of generating the piezoelectric effect. In the analytical model we have deduced a linear equivalent piezoelectric coefficient e_{31}^* , that only counts with influence from the of the e_{31} and e_{33} piezoelectric coefficients and the elastic c_{13} coefficient. The 3D model uses also the e_{32} piezoelectric coefficient, and having the same value as e_{31} , this coefficient can not be neglected.

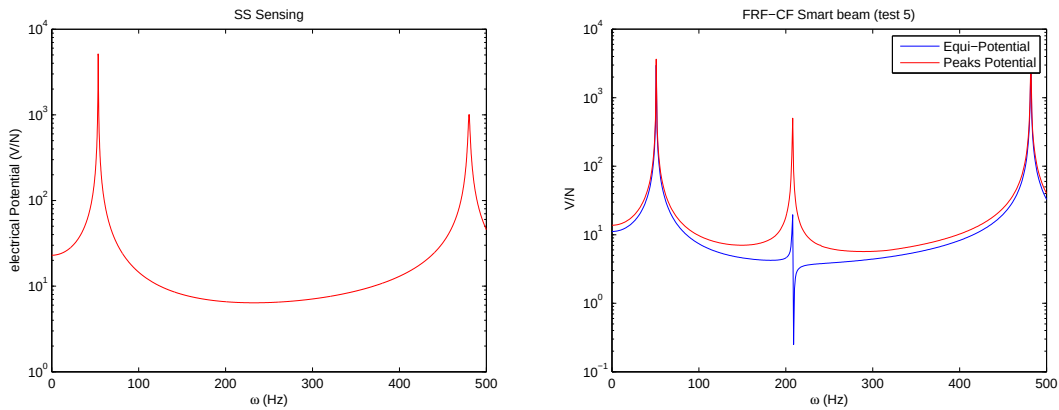


Figure 4.35: Voltage per unit of load, analytical (left) and numerical (right).

The electrical potential values are “reasonably” close.

Table 4.37: Voltage per unit of load for zero frequency: analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	22.95	11.16	13.77

• Test 3

In this test we have a 60mm piezo located near the left tip of beam. Both analytical and numerical behavior are the same, with the usual disagreement in electrical potential values. Comparing with the test one, one can see that a larger transducer generates higher values of electrical potential.

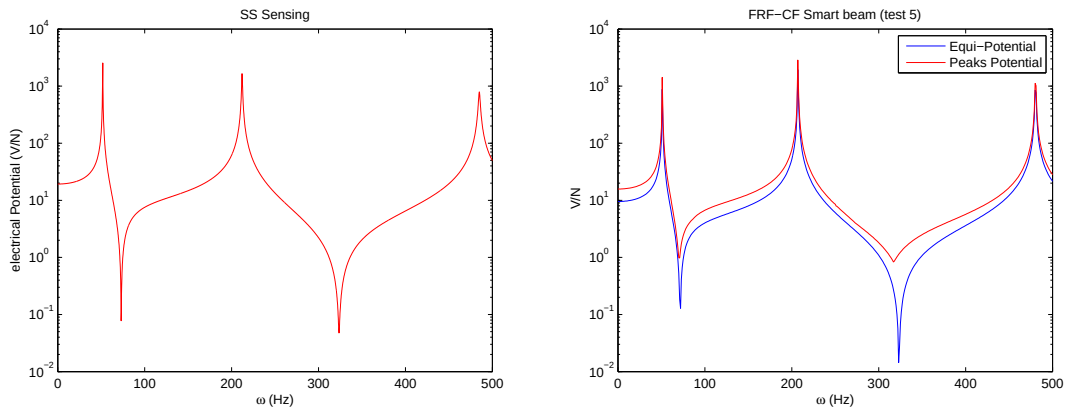


Figure 4.36: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.38: Voltage per unit of load for zero frequency analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	19.28	9.53	15.76

• Test 4

What has been commented in test 2 is applicable to test 4, the behavior shows the same anomaly for the numeric model. Once more we can state that the larger piezo produces higher values of electrical potential, by comparing this test to test 2.

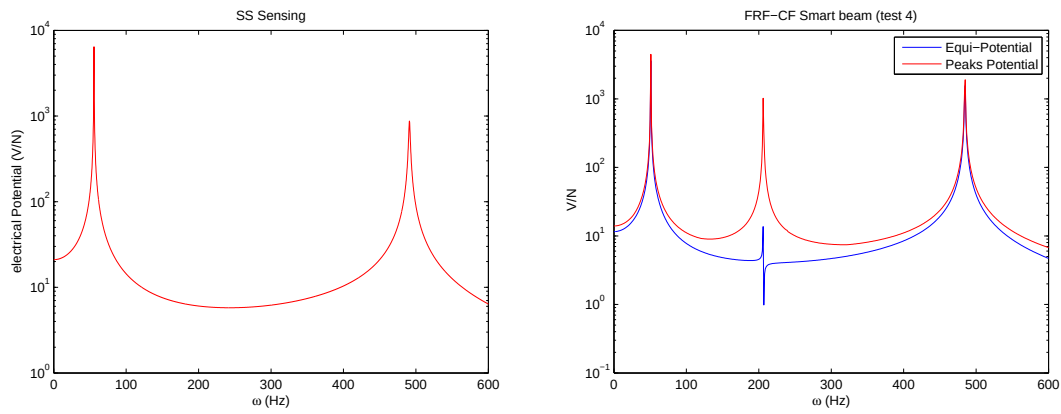


Figure 4.37: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.39: Voltage per unit of load for zero frequency: analytical and numerical.

	Voltage analytical	Equi-potential	Peaks Potential
$\omega = 0$	21.4	11.57	14

- Test 5

The full covering transducer model shows a similar anomaly to the one found in tests 2 and 4 for the second mode shape, although in this case the analytical model also considers a resonance.

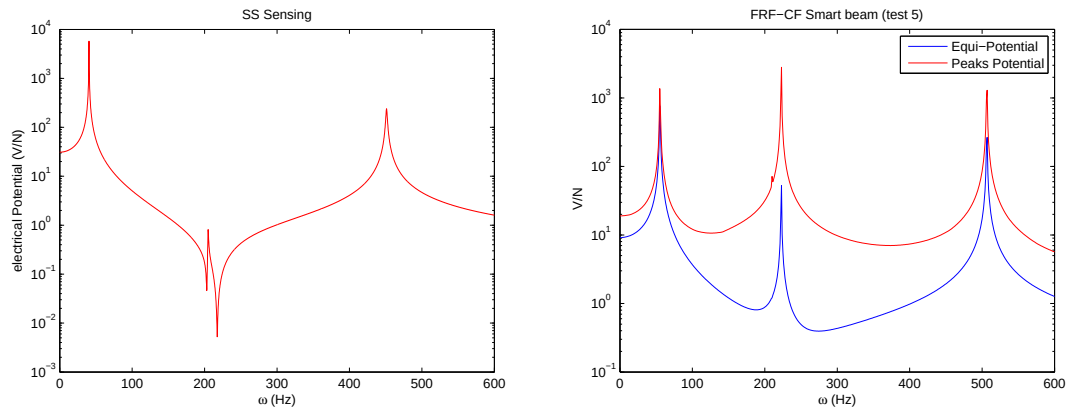


Figure 4.38: Voltage per unit of load, analytical (left) and numerical (right).

Table 4.40: Voltage per unit of load for zero frequency: analytical and numerical.

	Voltage analytical	Equi-potential	Peaks Potential
$\omega = 0$	31.04	9.17	19

4.3.3 Electrical Actuation: Displacement per Unit of Voltage

In this section we consider a prescribed voltage on the upper electrode of the piezoelectric transducer and how it influences on the beam. Recalling that a piezoelectric uniform

segmented transducer will induce two concentrated moments on its borders is expectable that the will deform as a consequence. Recalling the Equation (3.28) and neglecting both the piezoelectric induced loading term $N_r^e(s)$ and the mechanical loading term $N_r^m(s)$, it is obtained the motion equation for a prescribed voltage load,

$$(s^2 + 2\xi_r\omega_r s + \omega_r^2) + N_r^{e/m}(s) = 0, \quad (4.28)$$

or

$$(s^2 + 2\xi_r\omega_r s + \omega_r^2) - c_v b_p \left[\frac{d\phi_r(x_R)}{dx} - \frac{d\phi_r(x_L)}{dx} \right] v(s) = 0. \quad (4.29)$$

Recalling that

$$\bar{X} = \sum_{r=1}^n \phi_r \eta_r, \quad (4.30)$$

the dynamic function

$$\frac{\bar{X}(x, x_F, j\omega)}{\bar{V}} = \sum_{r=1}^{\infty} c_v b_p \frac{\phi_r(x_F)}{(\omega_r^2 - \omega^2) + j(2\xi_r\omega_r\omega) - c_r^e Z_r} \frac{d\phi_r(x)}{dx} \Big|_{x_R}^{x_L}. \quad (4.31)$$

Ahead are presented the results for the same uniform transducers already studied, but now is removed the mechanical loading and give the system a electrical loading in the form of an electrical potential of 1V applied on the upper electrode of the surface.

4.3.3.1 CF beam results

- Test 1

Applying 1V to the 30mm transducer shows that for the analytical model the eigenvalues will decrease, whereas the numerical shows the opposite, this issue is motivated by the transducer mass and stiffness considered in FEM model. The displacement per unit of voltage frequency response behavior is very similar, and the static displacement is lesser than in the original beam.

Table 4.41: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	18.24	16	20
ω_2	114.37	109	124
ω_3	320.20	313.2	343
disp ($\omega = 0$)	0.0064	5.3e-6	1.034e-6

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

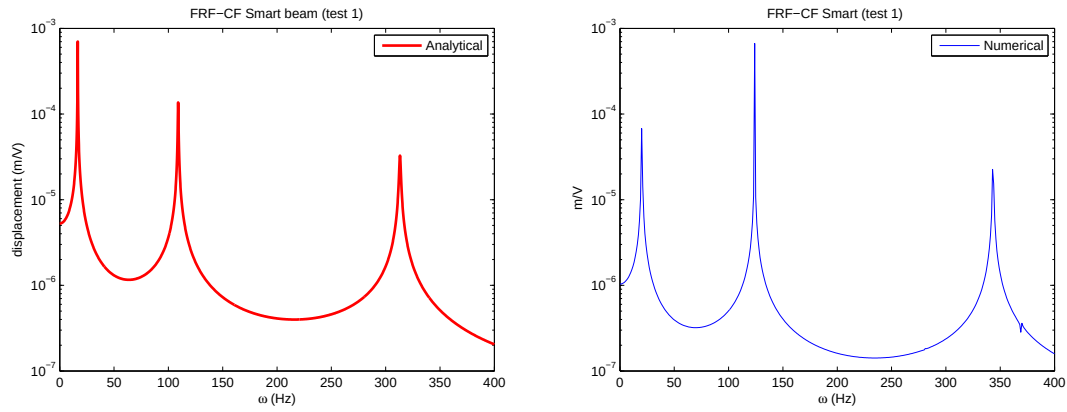


Figure 4.39: Displacement per unit of Voltage: analytical (left) and numeric (right).

- Test 2

Considering the same piezo at mid length, we see a lesser decrease in eigenvalues for the analytical model and an also lesser increase in the eigenvalues in the numerical model.

Table 4.42: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	18.24	18	19
ω_2	114.37	108.4	116
ω_3	320.20	320.2	329/353
disp ($\omega = 0$)	0.0064	2.5e-6	4.4e-7

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

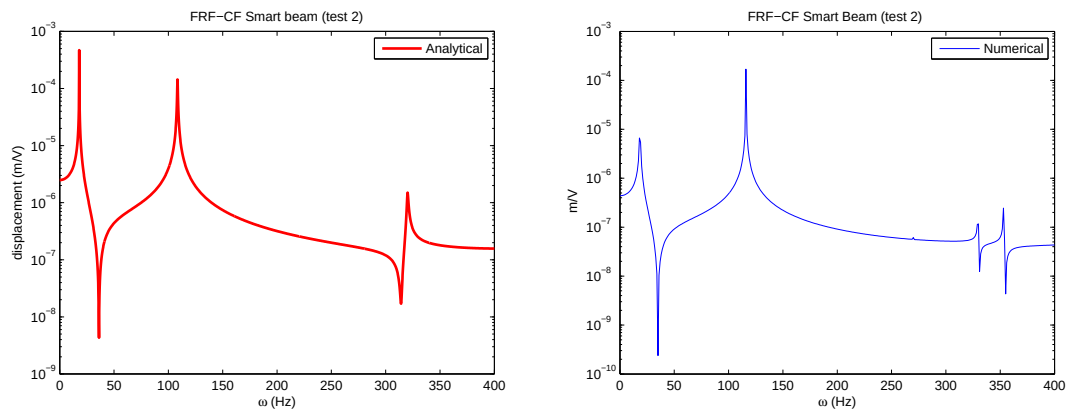


Figure 4.40: Displacement per unit of Voltage: analytical (left) and numeric (right).

The displacement per unit of voltage frequency response behavior is very similar, apart from a fourth resonance close to the third one in the numerical model. This fourth resonance cannot be seen in the analytical model, since it only covers the bending motion of the beam, and this resonance occurs when the beam twists, in other words we are in presence of a torsion resonance.

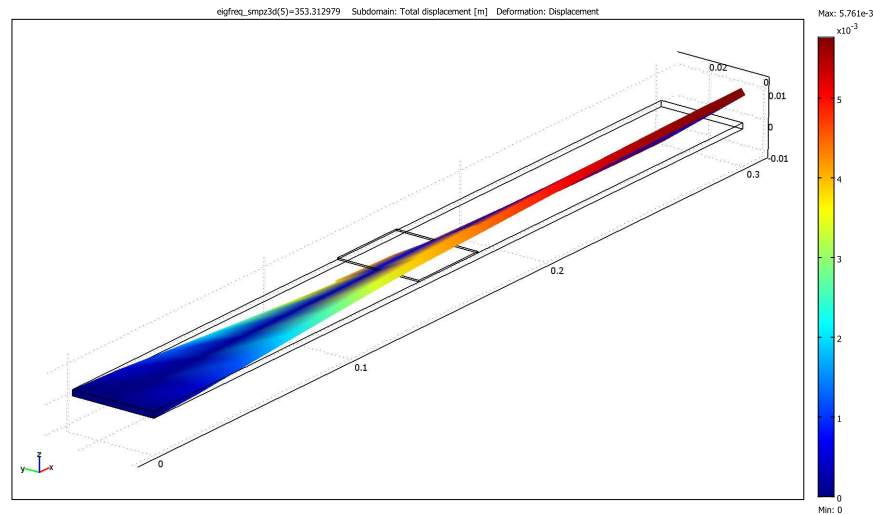


Figure 4.41: Torsion resonance for the frequency of 353 Hz

- Test 3

This test shows also a decrease for analytical eigenvalues and an increase for numerical eigenvalues, having a very similar behavior as the first transducer did, however being this transducer bigger, the decrease and increase ratios are larger. This test shows agreement in displacement per unit of voltage frequency response behavior between analytical and numerical models, but like in the third test we have a fourth resonance of torsion nature.

Table 4.43: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	18.24	15.4	22
ω_2	114.37	109.8	125
ω_3	320.20	319	332/380
disp ($\omega = 0$)	0.0064	1.1e-5	1.87e-6

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

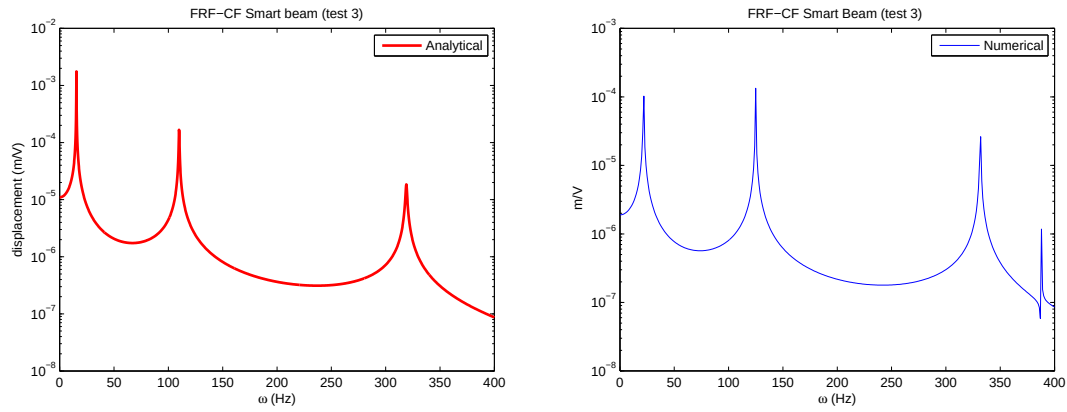


Figure 4.42: Displacement per unit of Voltage: analytical (left) and numeric (right).

- Test 4

This test shows the same behavior as the second one, also with a torsion resonance near the third resonance.

Table 4.44: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	18.24	17.6	18
ω_2	114.37	102.2	117
ω_3	320.20	320.2	328/350
disp ($\omega = 0$)	0.0064	6.08e-6	9.2e-7

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

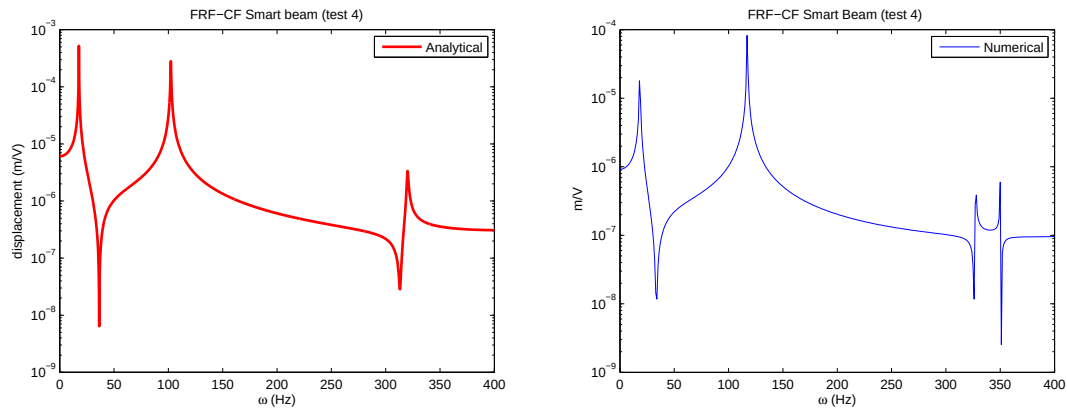


Figure 4.43: Displacement per unit of Voltage: analytical (left) and numeric (right).

- Test 5

For the full covering transducer both analytical and numerical values rise, and are very similar. One must not forget that the analytical model overestimates the piezoelectric effect, and as shown for mechanical actuation, the bigger the transducer the bigger the

piezoelectric effect is considered in this model. Both displacement per unit of voltage frequency response show the same behavior.

Table 4.45: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	18.24	20.4	20
ω_2	114.37	119.2	124
ω_3	320.20	324.4	349
disp ($\omega = 0$)	0.0064	1.38e-5	6.57e-9

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

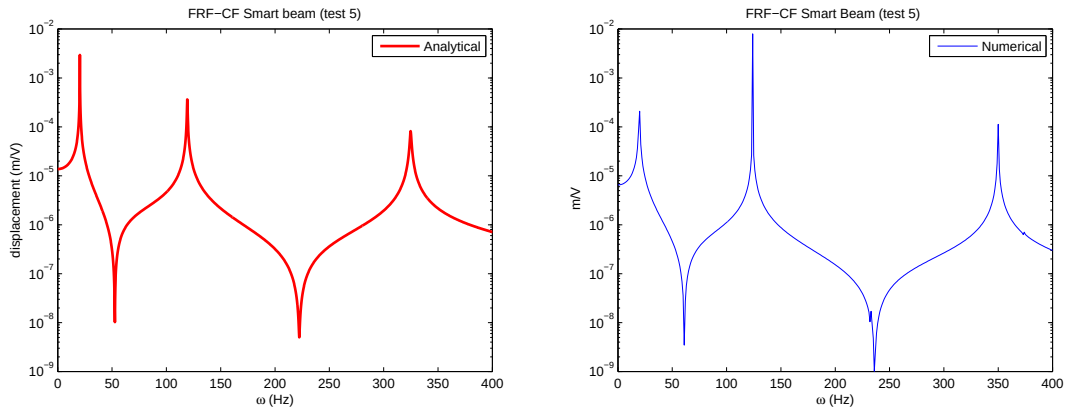


Figure 4.44: Displacement per unit of Voltage: analytical (left) and numeric (right).

4.3.3.2 SS beam results

Is considered the same exercise, now, for simply supported beams. As occurred for the clamped-free beams, the phenomenology is similar for both analytical and numerical models, despite having also some eigenvalues and amplitude deviation.

- Test 1

For the first test, with the smaller transducer, one can observe very similar frequency response figures and eigenvalues very close for the analytical and numerical models.

Table 4.46: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	51.21	51.2	51
ω_2	204.90	206	207
ω_3	460.92	465.8	473
disp ($\omega = 0$)	1.92e-4	1.20e-9	5.12e-8

* decoupled mode shapes; ** c, specially for modally shaped transducers which have higher χ_r for the mode in which they are designed for. coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

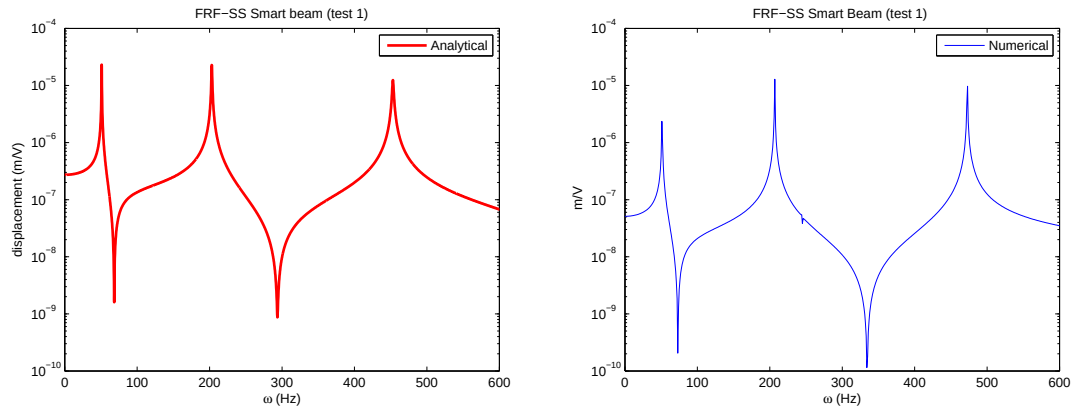


Figure 4.45: Displacement per unit of Voltage: analytical (left) and numeric (right).

- Test 2

Using the same transducer, now located at mid length of the beam, the deviation between analytical and numeric models is slightly higher. The bigger deviation happens for the static displacement, where the analytical shows a much smaller value. Being a symmetric transducer, one can see that both solution present a much smaller amplitude for the second resonance which corresponds to an anti-symmetric mode shape. One can also observe in the numeric figure a torsion resonance near the second bending resonance.

Table 4.47: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	51.21	53	51
ω_2	204.90	207	208
ω_3	460.92	474	482
disp ($\omega = 0$)	1.92e-4	2.22e-10	1.06e-7

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

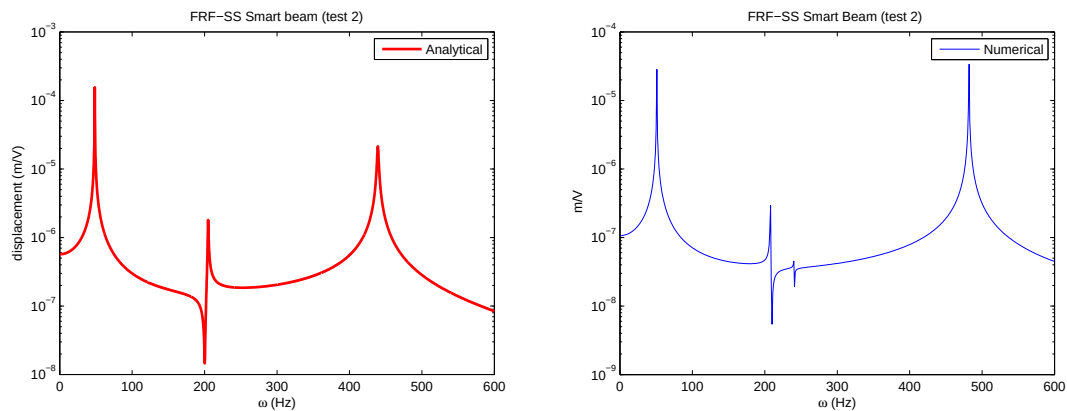


Figure 4.46: Displacement per unit of Voltage: analytical (left) and numeric (right).

- Test 3

In this test is used the 60 mm transducer, for the same location as in the first test, and one can see analytical eigenvalues increasing compared to the first test. As for the numeric model, only the third resonance shows a significant increase.

Table 4.48: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	51.21	51.6	51
ω_2	204.90	210	207
ω_3	460.92	479.5	480
disp ($\omega = 0$)	1.92e-4	7.82e-11	1.76e-7

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

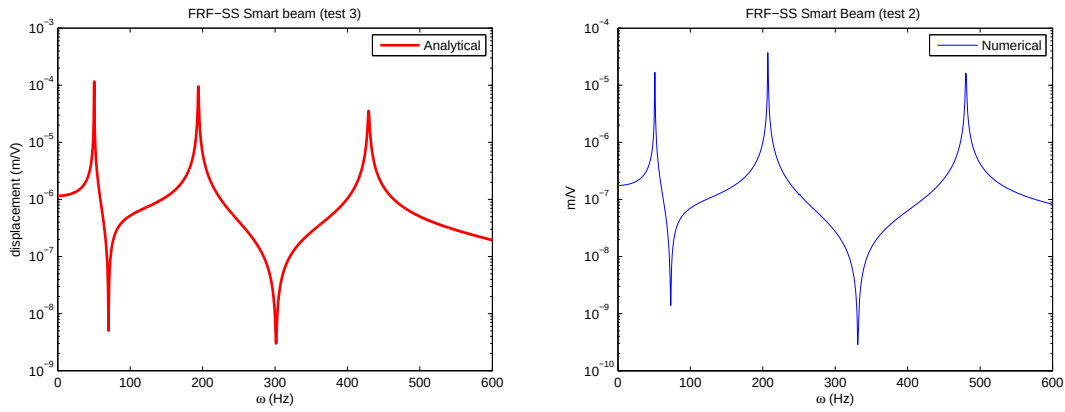


Figure 4.47: Displacement per unit of Voltage: analytical (left) and numeric (right).

• Test 4

This test is very similar to the second one, eigenvalues do not differ much from those obtained in the second test. The frequency response graphics show the same phenomena as seen in the second test.

Table 4.49: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	51.21	54.8	51
ω_2	204.90	202	208
ω_3	460.92	487	485
disp ($\omega = 0$)	1.92e-4	1.6e-11	2.3e-7

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

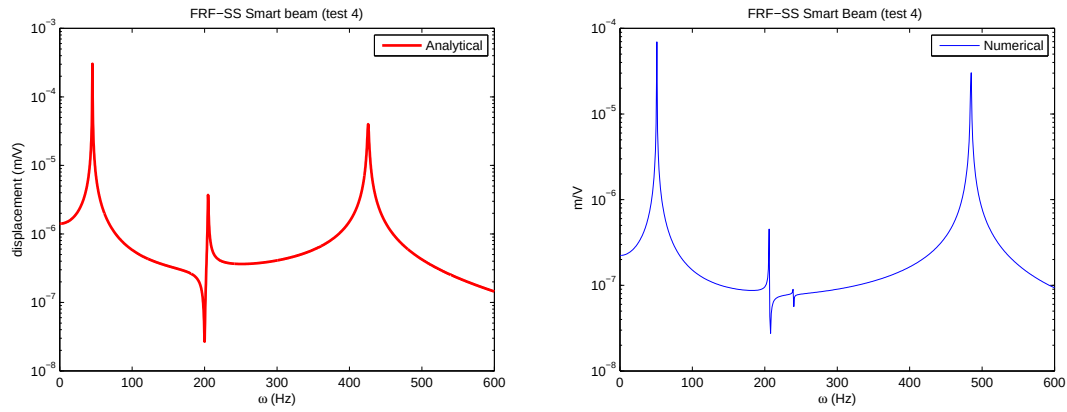


Figure 4.48: Displacement per unit of Voltage: analytical (left) and numeric (right).

- Test 5

For a full covering transducer, one can see the same behavior as observe in the second and fourth tests, although the eigenvalues present a bigger deviation for the numerical model.

Table 4.50: Eigenfrequency OC values.

(Hz)	Beam	Analytical	Numerical
ω_1	51.21	60.20	55
ω_2	204.90	205.2	223
ω_3	460.92	469.4	507
disp ($\omega = 0$)	1.92e-4	2.4e-6	8.4e-7

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

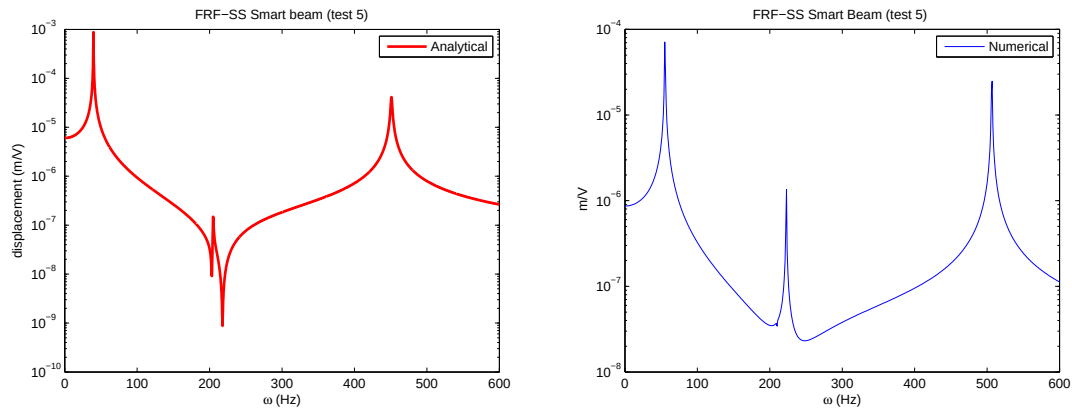


Figure 4.49: Displacement per unit of Voltage: analytical (left) and numeric (right).

4.4 Smart Beam: Modal Transducers

After studying the behavior of uniform transducers, it is important to study modally shaped transducers. This kind of transducers are shaped according to determined vi-

bration mode shape, expecting them to behave distinctly from the uniform transducers, specially in the frequency bandwidth containing the respective resonance. Ahead are displayed the results for the same fields, as done with uniform transducers.

4.4.1 Actuation behavior : displacement per unit of force

Unlike uniform transducers, modally shaped transducers considers modal decoupling, since every shape is proportional to the second mode shape derivative of a certain mode shape. Recalling Equations (2.75) and (3.27), the displacement per unit of load Equation yields now

$$\frac{\bar{W}(x, x_F, j\omega)}{\bar{F}} = \sum_{r=1}^{\infty} \frac{\phi_r(x_F)\phi_r(x)}{(\omega_r^2 - \omega^2) + j(2\xi_r\omega_r\omega) - b_p c_v c_i \alpha_r^2 Z_r}, \quad (4.32)$$

where, Z_r is the modal impedance for the mode that we are analyzing. The shapes of the piezo are also proportional to the second mode shape derivative and then normalized with respect to the beam's width. Here are presented the first 2 modal transducers for both CF and SS beams with analytical formulation. FEM results are only presented for the first mode for both for CF and SS beams. The lack of more numeric results is caused by the difficulties presented in meshing very narrow areas with 3D solid elements, which are the only available in the MEMS module. This issue assumed to very important on the second modally shaped transducer for the SS beam, where one can count 3 very narrow areas, two on the its extremities, and a third one at mid length.

4.4.1.1 CF results

The following table shows the shapes configurations for the two first mode, as can be seen, the second modal configuration presents a nodal point. This means that this transducer can be "divided" in two different sections where the polarization is opposite from on side to the other. As an exercise it was made an analytical comparison between two cases for the second transducer, one where we did not invert the polarity and the other with polarization inversion. Firstly are shown the analytical models, regarding always that by this formulation it is underlying an open-circuit case. Secondly are shown the FEM results both for OC and CC cases.

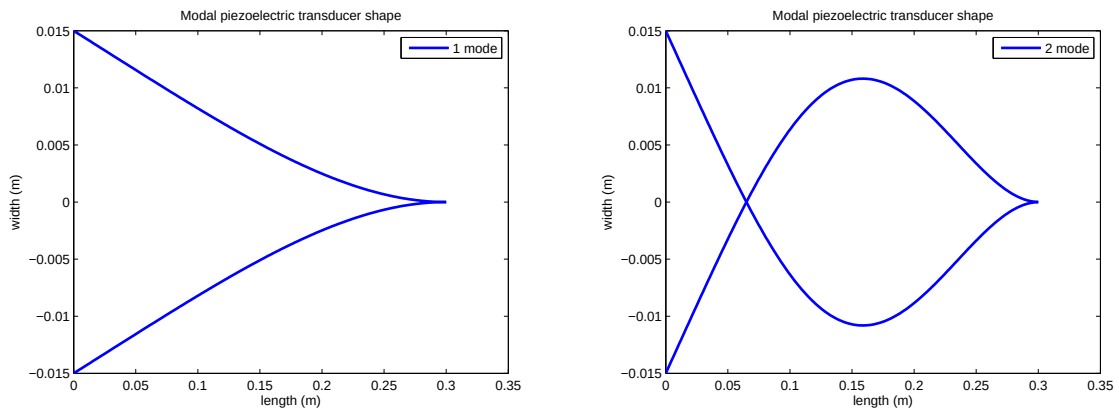


Table 4.51: CF modal shapes

Table 4.52: Analytical eigenfrequency OC values (Matlab).

(Hz)	Beam	1 st mode	2 nd mode	2 nd mode inv. polarization
ω_1	18.2444	22	79.6	79.6
ω_2	114.3368	115	138	138
ω_3	320.1486	320.2	329.4	329.4
disp ($\omega = 0$)	0.0064	0.0044	4.6e-4	9.2-e4

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

One can observe from the table above that modally shaped transducers increase eigenvalues. For the first mode, the transducer affects only the first mode like expected, but for the second modally shaped transducer we see great increases in all resonances, both for the test with inverted polarization and non inverted polarization. This phenomena was not expected, the expectable would be the changing only to happen for the second resonance. This issue cannot be ignored, probably means that modally shape theory must be implemented with more programming sophistication.

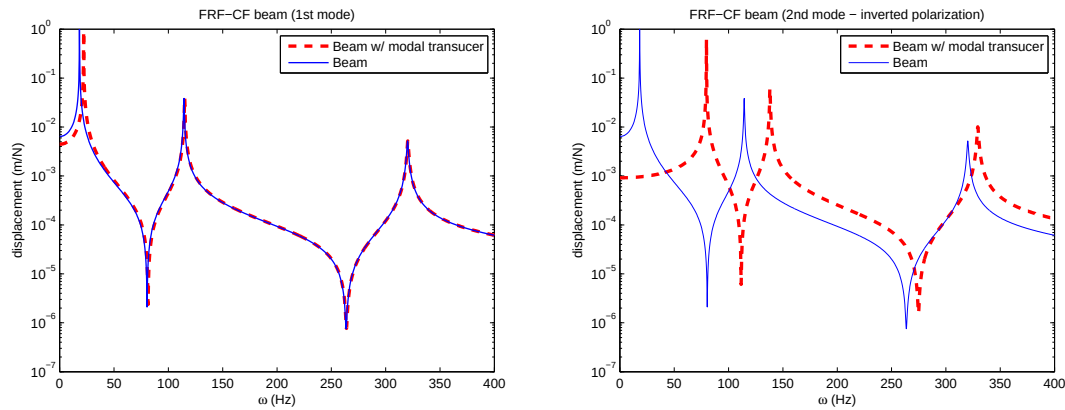


Figure 4.50: CF Beams with modally shaped transducers FRFs.

The FEM model for the first modally shaped transducer has a mesh with 58 extruded triangles elements, yielding 1368 degrees of freedom. The FEM has been developed only for the first modally shaped transducer, and one can realize that the results are unexpected, as the analytical were. All eigenvalues rise and the FRF behavior is similar to a full covering uniform transducer.

Table 4.53: Numeric Eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	1 st mode CC	1 st mode OC
ω_1	18.57	24	24
ω_2	116.91	132	134
ω_3	331.30	375	378
disp ($\omega = 0$)	0.0063	0.0038	0.0036

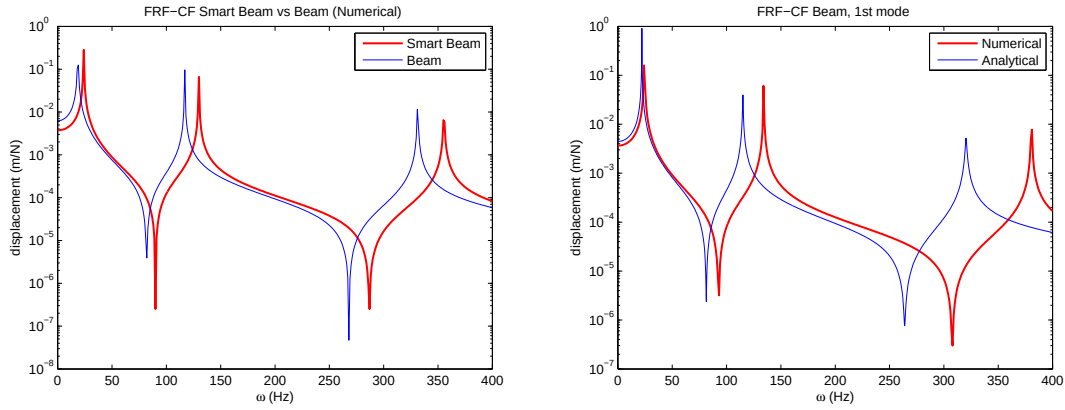


Figure 4.51: OC smart beam's FRF (Numerical) and Numerical vs Analytical

One can conclude that modally shaped transducers have more to it than the theory presented in this dissertation, and as the numerical and analytical behavior totally disagree, one would prefer to use numeric models to evaluate this kind of transducers.

4.4.1.2 SS results

As performed for the CF beam, here are presented the analytical results for the first two mode shape transducers and the numerical result for the first mode shape.

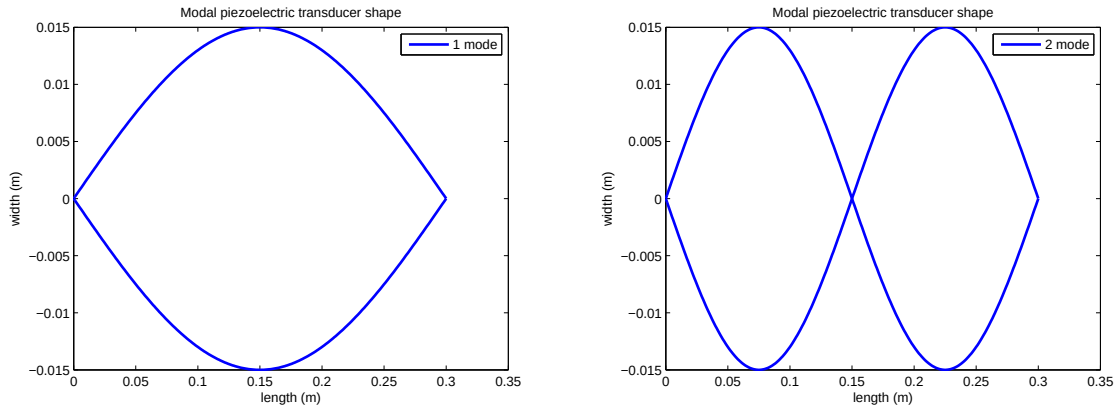


Table 4.54: SS modal shapes

Table 4.55: Analytical eigenfrequency OC values (Matlab).

(Hz)	Beam	1 st mode	2 nd mode	2 nd mode inv. polarization
ω_1	51.21	61.8	148	148
ω_2	204.90	207.8	247.4	247.4
ω_3	460.92	462.4	481.4	481.4
disp ($\omega = 0$)	1.92e-4	1.39e-4	4.012e-5	9.44e-4

* decoupled mode shapes; ** coupled mode shapes; *** this compares the beam's value with the smart beam considering coupled mode shapes.

The disagreement with the expectations for the clamped-free modally shaped trans-

ducers, remains for the simply-supported beam. For the first modally shape transducer the analytical result seems to be what we have expected, but for the second modally shaped transducer we see a major deviation on the resonances other than the second one.

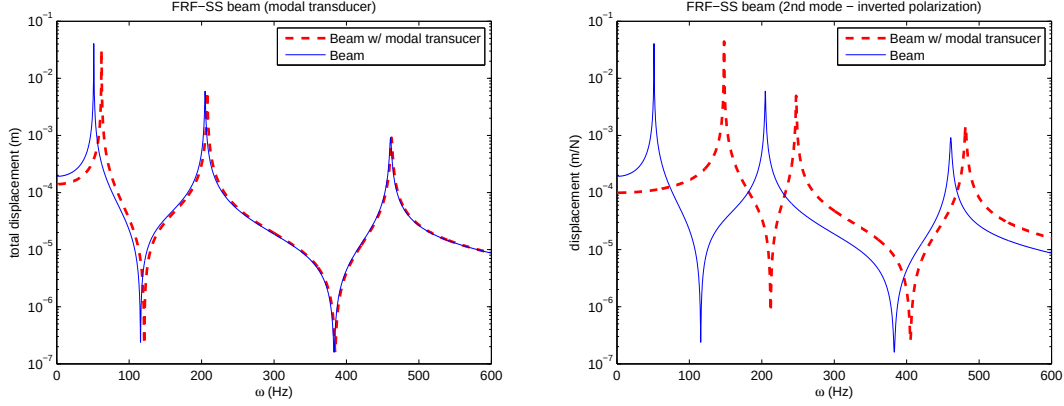


Figure 4.52: CF Beams with modally shaped transducers FRFs.

The FEM model for the first modally shaped transducer has a mesh with 58 extruded triangles elements, yielding 1368 degrees of freedom.

Table 4.56: Numeric Eigenfrequency values (COMSOL Multiphysics).

(Hz)	Beam	1 st mode CC	1 st mode OC
ω_1	51.38	55	56
ω_2	207.64	229	234
ω_3	474.89	592	—
disp ($\omega = 0$)	1.97e-4	1.1e-4	1.04e-4

Looking now to the numeric results for first modally shaped transducer we see that all eigenvalues rise, although for the first resonance there is present some similarity, the second and third resonances are moved for much higher frequencies. Once more, we must state that the analytical model does not comprehend well the shaped transducers problem. Although working rather similar in the first mode, the higher modes include nodes and more complex shapes that are not represented by this theory. As for the numeric values they must be understood as more reliable, as they consider mass and stiffness from the transducer, considers the Poisson effect, and considers also all piezoelectric coefficients in all directions.

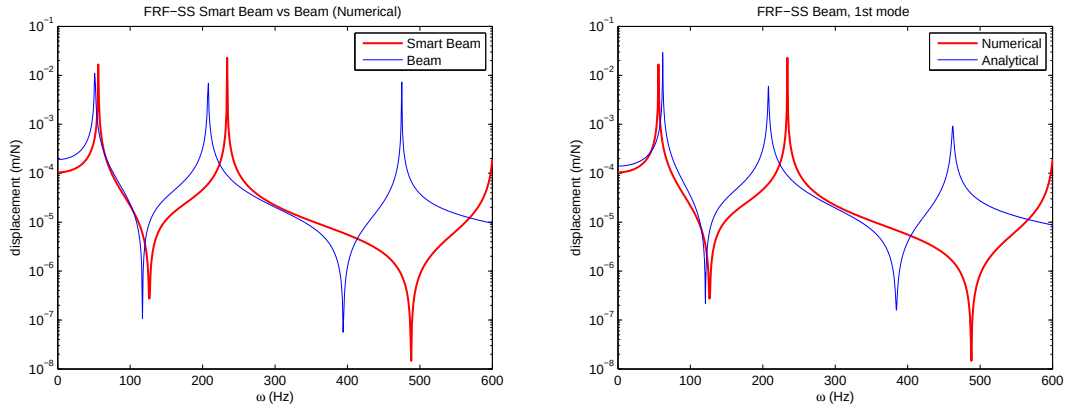


Figure 4.53: OC smart beam's FRF (Numerical) and Numerical vs Analytical

4.4.2 Sensing behavior: voltage per unit of force

The sensing behavior frequency response is very similar to the one presented for uniform transducers, although now is considered a modal capacitance and the mode shape respective to the mode shape in question. Therefore it is expected that sensing frequency response show only the a single resonance for a single modally shaped transducer, filtering all other resonances. Recalling the Equation (2.74) it is obtained he following FRF expression,

$$\frac{\bar{V}(x, x_F, j\omega)}{\bar{F}} = \frac{e_{31}^* h_b 2b_b}{C_p^s} \frac{\phi_r(x_F)}{(\omega_r^2 - \omega^2) + j(2\xi_r \omega_r \omega) - c_r^e Z_r} \frac{d\phi_r(x)}{dx} \Big|_{x_L}. \quad (4.33)$$

Since we have unexpected results for the mechanical actuation for modally shaped transducers, it was decided to study the sensing behavior for the first mode only, for both cases, simply supported and clamped free beams.

4.4.2.1 CF beam sensing results

- First modal transducer

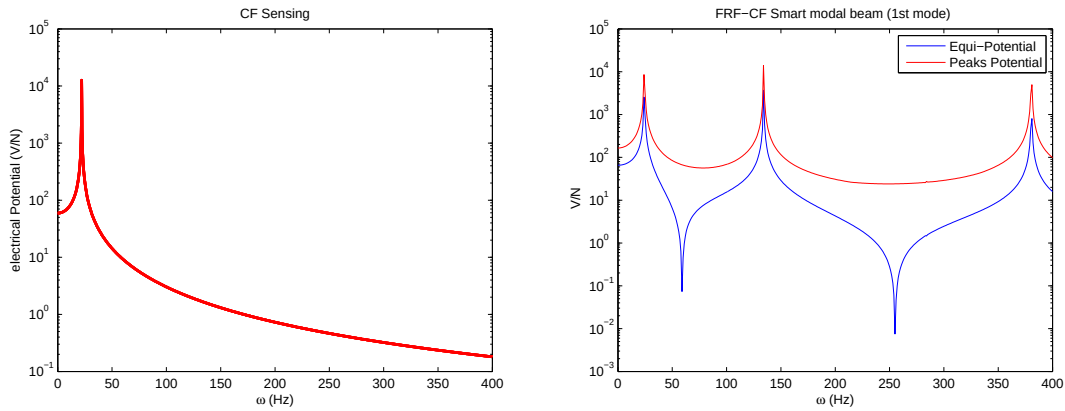


Figure 4.54: Voltage per unit of load, analytical and numerical.

Table 4.57: Voltage per unit of load, analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	60.32	65.46	165
eigenvalues (Hz)	22	24/134/378	24/134/378

The analytical results seems very reasonable, one can see the filtering behavior, having only the first mode present on the FRF figure, but the numerical model indicates the opposite, it is not possible to see any filtering phenomenon. The same happens for the simply-supported beam as can be attested ahead. Having disagreements both for mechanical actuation and sensing behaviors, one must conclude that shaped transducers are more complex than what we had expected, and that analytical and numeric models must be rethought and accompanied by experimental tests.

4.4.2.2 SS beam sensing results

- First modal transducer

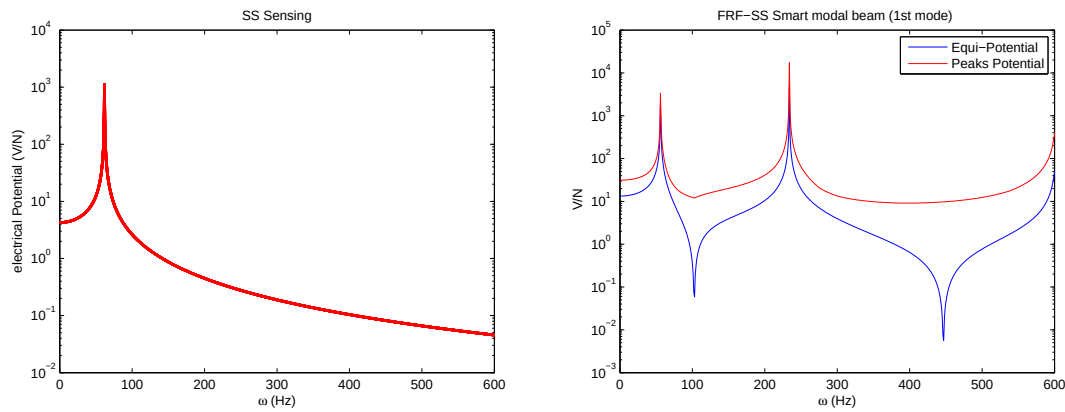


Figure 4.55: Voltage per unit of load, analytical and numerical.

Table 4.58: Voltage per unit of load, analytical and numerical.

Voltage	analytical	Equi-potential	Peaks Potential
$\omega = 0$	4.26	13.3	31.21
eigenvalues (Hz)	61.8	56/234	56/234

4.5 Summary

This Chapter comprehends several models for validation of the formulation done in the previous chapters. Firstly was validated FEM models, than was tested the influence of the presence of a piezoelectric patch on a beam.

The analytical model shows higher piezoelectric effect than the numeric model. One can see through the numerical models that the most influence that a piezoelectric transducer has on the beam comes from the consideration of the mass and stiffness of it. Despite the previous considerations, the phenomenological piezoelectric effect acts the

same way both for numerical and analytical models when considering uniform transducers, despite amplitude and eigenvalues disagreement. Issues such as location and size proved to be of major importance, different locations or sizes can improve or not stiffness of the beam, these aspects occurred both for analytical and numeric models, although the mass influence considered on the numerical model lead many times to a decrease on eigenvalues.

As for shaped transducers, this theory proved a big disagreement with the numeric model. In the analytical model, although mechanical actuation for the first mode appeared reasonable, the second mode for both types of beams tested, presented a large disagreement to what we'd expect to observe. One can conclude that the analytical implementation was not well done. Considering the sensing behavior for the first mode showed exactly what we have been expecting, since one can only observe the first resonance, while all others are filtered. When it comes to comparing it to the few results obtained by the numeric model, and as already mentioned, there is a big phenomenological disagreement with the analytical model; one can not observe any filtering in numeric models, this issue may be related to a bad mesh discretization, and bad element chosen. The 3D elements worked fairly well for uniform transducers, where the mesh was very regular, while when using shaped transducers this did not happen. Shaped transducers have many narrow areas, and the curvature they have makes difficult the agreement between the mesh from the beam and mesh from the transducer. It is also important to mention that the reduced thickness of the beam and piezo asks for shell elements, which are not available in the MEMS module of the software.

Chapter 5

Application and Analysis

5.1 Introduction

After studying the behavior of piezoelectric and elastic beam, it is time to introduce the resonant shunt therefore entering the passive damping field. A resonant shunt during the formulation as been described as an impedance, this impedance is made out from a resistive and an inductive element. In this section will be performed the analysis for uniform transducers only, due to the lack of confidence on the modally shaped models. In this chapter will only be considered the analytical formulation, justified for the lack of time to perform a decent numeric shunt model.

5.2 Generalized Electromechanical Coupling Coefficient

The generalized electromechanical coupling coefficient, χ_r , developed in Chapter 3, can be measured in a easier way than the one presented before in Equations (3.54)-(3.57), by simply comparing the eigenfrequency values obtained for the two EBC studied before. The difference between both cases, closed circuit and open circuit, will give us a valid quantification of the the analytical generalized electromechanical coupling coefficient, regarding less time expended on programming implementation and adjacent errors that may occur. Since this study considers only analytical results, the frequency for closed circuit are the same as the ones presented for the beam with no piezoelectric transducer attached to it. This is coherent with the analytical theory, since it does not take into account the transducer mass and stiffness, and as observed in the previous chapter, these issues have great influence on the eigenfrequency values. Since the objective underlying this project is to understand the passive vibration damping phenomenon, using only an approximation seems quite reasonable for this study. The coefficient χ_r is calculated as the following Equation shows,

$$\chi_r^2 = \frac{\omega_R^{OC} - \omega_R^{CC}}{\omega_R^{OC}}. \quad (5.1)$$

The generalized electromechanical coupling coefficient, as mentioned before, measures, as its name suggests, the amount of mechanical energy that can be converted to the electrical form. Therefore a higher value of χ_r will mean an also higher capability for vibration damping. In this section are presented the values of χ_r obtained for analytical uniform and modal transducers, studied before. The values of inductance and resistive elements are developed from the χ_r , regarding the Equations (3.50) and (3.51).

For uniform transducers, it will be evaluated the damping for the 3 first modes, while for modally shaped will only considered the shape respective mode.

5.2.1 Clamped free beam with uniform transducers.

For the clamped free uniform transducers will be considered the following 3 configurations, leaving behind the location influence, since that parameter is already evaluated in the previous chapter. Here are considered two segmented transducers near the clamped tip and a full covering transducer.

Table 5.1: Piezo's geometry for shunt damping (CF beam)

Test	1	2	3
length (l)	0.030	0.060	0.3
width ($2b_p$)	0.03	0.03	0.03
thickness ($2h_p$)	0.0005	0.0005	0.0005
location*	0.005	0.005	— — —

* distance to the left beam's extremity.

Considering the first resonance, for the 3 different configurations, are calculated the shunt parameters, that are shown on the following table.

Table 5.2: First mode shunt parameters

	Test 1	Test 2	Test 3
ω_R^{OC}	19.6	20.4	20.6
ω_R^{CC}	18.24	18.24	18.24
χ_r^2	0.0694	0.1059	0.1146
R ($K\Omega$)	172.48	103.02	21.55
L (H)	4039	1952	393

One can easily observe that different piezoelectric transducer mean different shunt parameters, analogously to their Open-Circuit resonance eigenfrequency, which are also different. For the first mode it is observable that the bigger the transducer, the better χ_r^2 it develops. As the generalized electromechanical conversion coefficient grows, the smaller values of inductance and resistance are needed. It is also important to emphasize that all 3 tests show the need of extremely big inductances, impracticable to apply with a common inductive element, therefore all cases must use a digital inductance generator.

Table 5.3: Second mode shunt features

	Test 1	Test 2	Test 3
ω_R^{OC}	119.1	118.6	119.4
ω_R^{CC}	114.37	114.37	114.37
χ_r	0.0397	0.0357	0.0421
R ($K\Omega$)	21.416	10.187	2.230
L (H)	105.78	53	10.69

Shunting the second mode seems to be easier than shunting the first mode, the inductance and resistance values, although still high, are much lower now. For instance

considering the first piezo tuned for the first mode and tuned for the second mode, the inductance value is 38 times bigger in the first case.

Other conclusions can be taken from this second example, unlike the previous mode shunt damping the piezoelectric size does not mean a better generalized electromechanical coupling coefficient. Although the first and second transducers are located at the same place, the second is twice the size of the first one and develops a smaller second resonance, thereby an also smaller coupling coefficient. The third piezo, which is a full covering piezo continues to show the highest coupling coefficient and therefore smaller inductance and resistance values.

Table 5.4: Third mode shunt features

	Test 1	Test 2	Test 3
ω_R^{OC}	326.8	321.4	324.8
ω_R^{CC}	320.2	320.2	320.2
χ_r	0.0202	0.0037	0.0142
$R (K\Omega)$	0.567	1.215	0.474
$L(H)$	1.40	6.98	1.40

By tuning all 3 transducers, now for the third resonance, one can observe a tendency, the higher the resonance the smaller the coupling coefficient, and also is smaller the inductance and resistance values. This phenomenon can be attested by comparing all 3 mode shunt tuning models, the first one demands higher values than the second one, while the second one demands higher values than the third shunt tuning.

For the 3 mode shunting are already present reasonable inductance values.

Another conclusion that one can take from this exercise, is that no matter the location of the transducer, all transducers can be tuned for diverse modes, although some locations may present better coupling coefficients.

5.2.2 Simply-supported beam with uniform transducers.

For the simply-supported beam are made the same tests, now considering the segmented piezo located at the beam's mid length. It is also considered a full covering transducer.

Table 5.5: Piezo's geometry for shunt damping (SS beam)

test	1	2	3
length (l)	0.030	0.060	0.3
width ($2b_p$)	0.03	0.03	0.03
thickness ($2h_p$)	0.0005	0.0005	0.0005
location*	0.135/0.005	0.120/0.005	— — —

* distance to the left beam's extremity, first value for symmetric mode shapes and second one for anti-symmetric mode shapes.

Considering the first resonance, for the 3 different configurations, are calculated the shunt parameters, that are shown on the following table.

Table 5.6: First mode shunt parameters

	Test 1	Test 2	Test 3
ω_R^{OC}	53.4	55.6	60
ω_R^{CC}	51.21	51.21	51.21
χ_r^2	0.041	0.074	0.1465
$R (K\Omega)$	49017	32810	8440
$L(H)$	531	256	48

The previous table shows some similarities with the table (5.2), since the coupling coefficient grows the same way, from the smaller transducer to bigger one. One can observe that for the nature of natural boundary conditions of simply supported beams, eigenvalues are higher than the ones of a clamped-free beam, and therefore the piezoelectric effect although being the same, means bigger difference between the beam's eigenvalues and the smart beam's eigenvalues. This issue is of major paramount because one can easily observe that inductance values for the first resonance of a clamped-free beam are much bigger than the ones need for shunting the first resonance for a simply supported beam. One can say that a simply-supported beam is therefore more viable.

The following table shows the same shunt parameters, but for the second mode shape, and for this reason, the piezo configuration cannot be the same. In the previous examples all the transducers had location symmetry, this implies no capability of acting on anti-symmetric mode shapes, such as the second mode shape. In the previous chapter this configuration shown that the Open-Circuit models had the same eigenfrequency as the beam for the second mode shape. Having this in mind, we have opted to locate the transducers 5mm away from the left tip of the beam. Also is not performed a full covering piezo test, since this configuration suffers from the same problem of symmetry.

Table 5.7: Second mode shunt parameters

	Test 1	Test 2
ω_R^{OC}	206	212.2
ω_R^{CC}	204.90	204.90
χ_r^2	0.0053	0.034
$R (K\Omega)$	4.533	5.648
$L(H)$	34	16.8

What as already been concluded for the clamped-free beam, the higher the frequencies we want to damp, the smaller values of inductance are demanded. One can observe the same phenomenon going on for the Simply-Supported beam.

Shunting the third resonance, made possible returning to same three configurations, shown for the first resonance. The shunt parameters for the third mode are presented ahead.

Table 5.8: Third mode shunt parameters

	Test 1	Test 2	Test 3
ω_R^{OC}	480.2	491.4	470.4
ω_R^{CC}	460.92	460.92	460.92
χ_r^2	0.0401	0.0620	0.020
$R (K\Omega)$	5.393	3.218	0.397
$L(H)$	6.57	5.22	0.67

5.3 Shunt Damping

In this section is evaluated the damping performance for all the aforementioned cases in the previous section, where shunt parameters were developed. It is time now to understand how these parameters reflect on the beam's behavior. Inductance and resistance values do not say nothing for themselves, since different piezo may need different inductances and/or resistance to achieve the same damping performance. That serves to remind us that, although the parameters developed before are important, issues like shape, location, capacitance cannot be forgotten. Like the previous section, this one is divided in two sub-sections, for clamped-free and simply supported beams respectively.

5.3.1 Clamped-free beam with uniform transducers.

All three cases performed rather similarly for all modes in question, all presented a behavior similar to a damped mass absorber, presenting two resonance peaks standing on the left and right sides of the original resonance, and a anti-resonance located at the eigenfrequency where before we had the original eigenfrequency.

5.3.1.1 First mode tuning

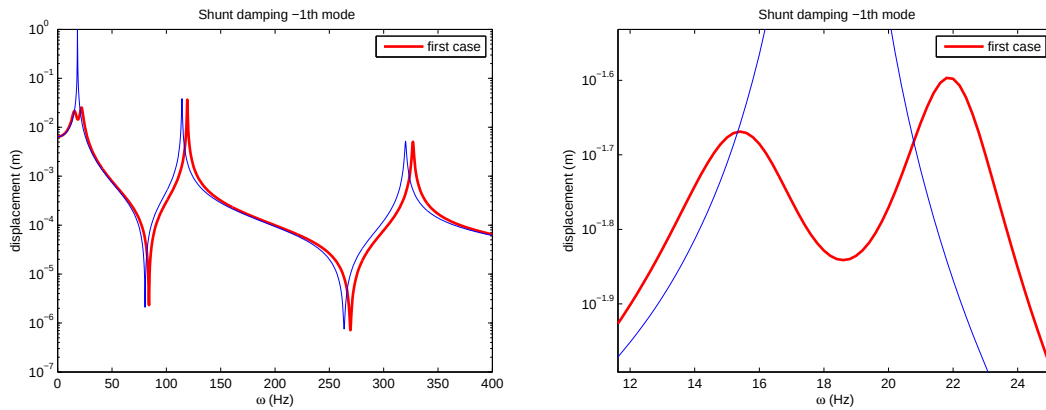


Figure 5.1: Test one result, piezo with 30mm of length, located 5mm way from the clamped tip.

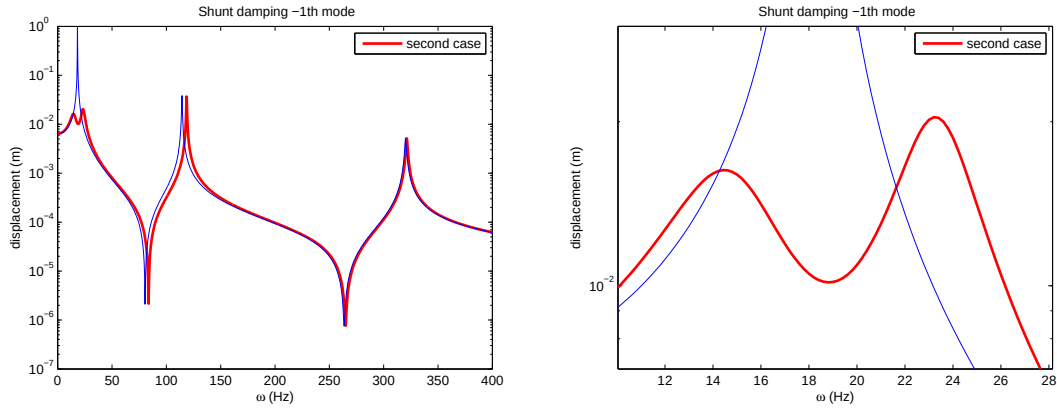


Figure 5.2: Test two result, piezo with 60mm of length, located 5mm way from the clamped tip.

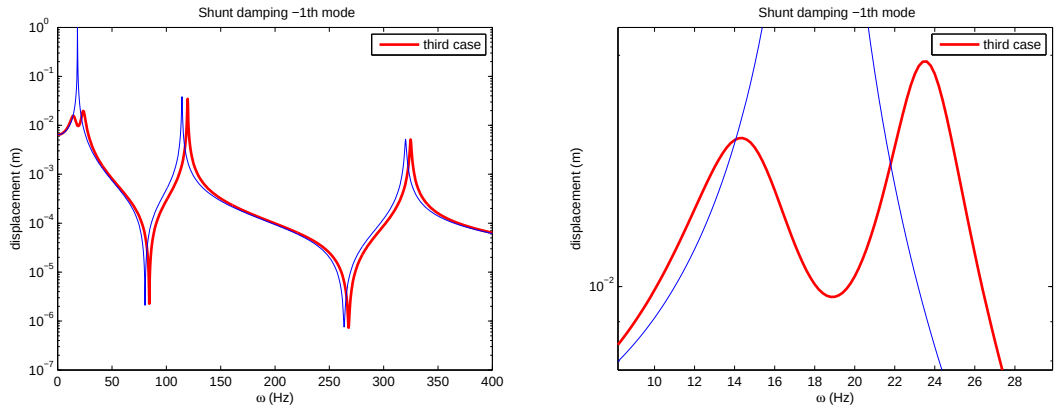


Figure 5.3: Test three result, full covering piezo.

Table 5.9: First mode damping performance

	Test 1	Test 2	Test 3
left resonance (Hz)	15.4	14.4	14.4
amplitude (m)	0.021	0.016	0.015
anti-resonance (Hz)	18.6	18.8	19
amplitude (m)	0.014	0.01	0.009
original amplitude (m)	0.992	0.992	0.992
amplitude decrease	98.5%	99%	99,1%
right resonance (Hz)	22	23.2	23.6
amplitude (m)	0.025	0.020	0.020

One can observe that the first transducer as the smaller interval between the resonances, it has also the lesser amplitude decrease, and looking at table (5.2) one can see that the first test had the higher inductance and resistance values. Anyway, all three cases present very high decrease values, very close to 100%.

5.3.1.2 Second mode tuning

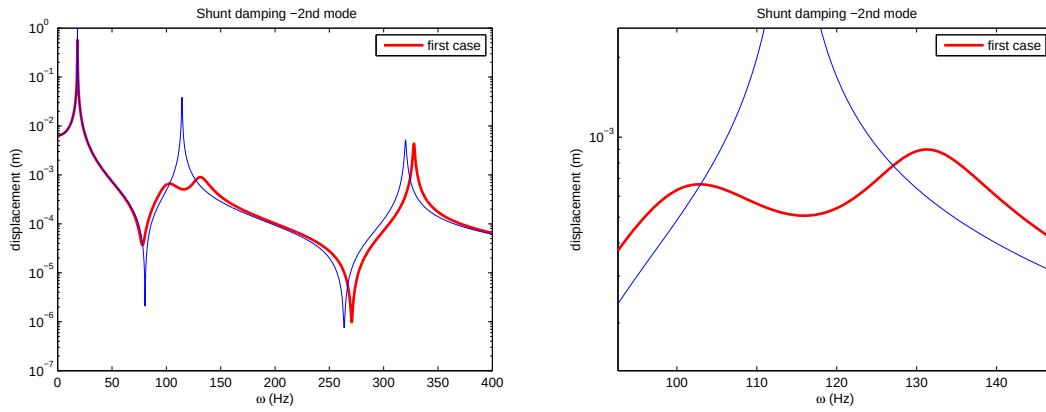


Figure 5.4: Test one result, piezo with 30mm of length, located 5mm way from the clamped tip.

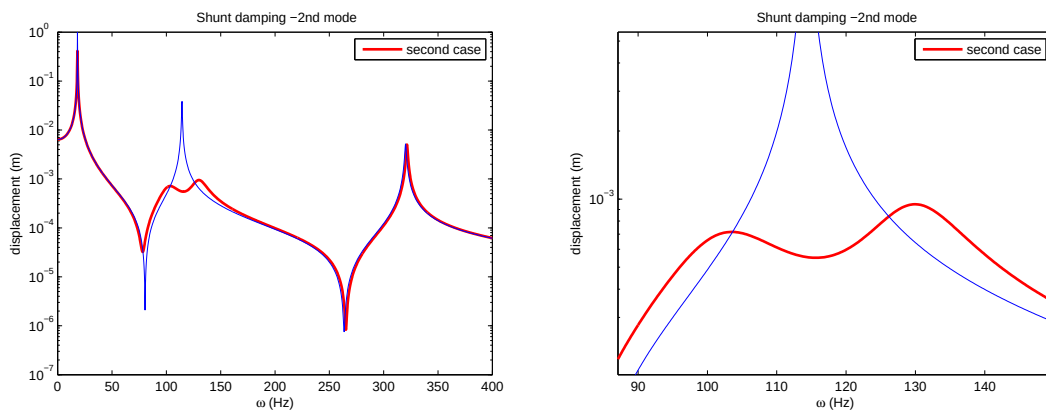


Figure 5.5: Test two result, piezo with 60mm of length, located 5mm way from the clamped tip.

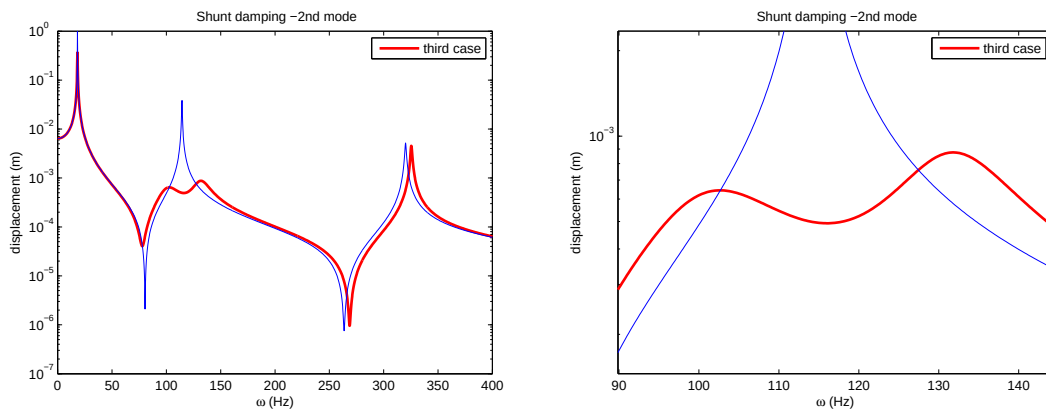


Figure 5.6: Test three result, full covering piezo.

Table 5.10: Second mode damping performance

	Test 1	Test 2	Test 3
left resonance (Hz)	102.2	103.2	102.4
amplitude (m)	6.6e-4	7e-4	6.4e-4
anti-resonance (Hz)	116.4	116.2	116.2
amplitude (m)	5.1e-4	5.5e-4	4.9e-4
original amplitude (m)	0.0341	0.0341	0.0341
amplitude decrease	98.5%	98.4%	98.6%
right resonance (Hz)	131	130	131.6
amplitude (m)	9e-4	9.5e-4	8.7e-4

In this case one can see that the full covering transducer has the best amplitude decrease, with the smaller values of inductance and resistance needed. It is also interesting to compare the first and second test, where now the smaller transducer has better performance.

5.3.1.3 Third mode tuning

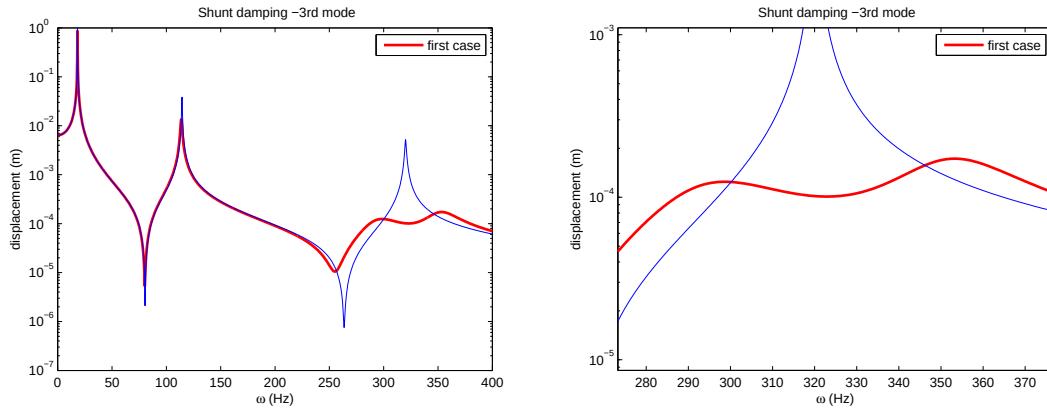


Figure 5.7: Test one result, piezo with 30mm of length, located 5mm way from the clamped tip.

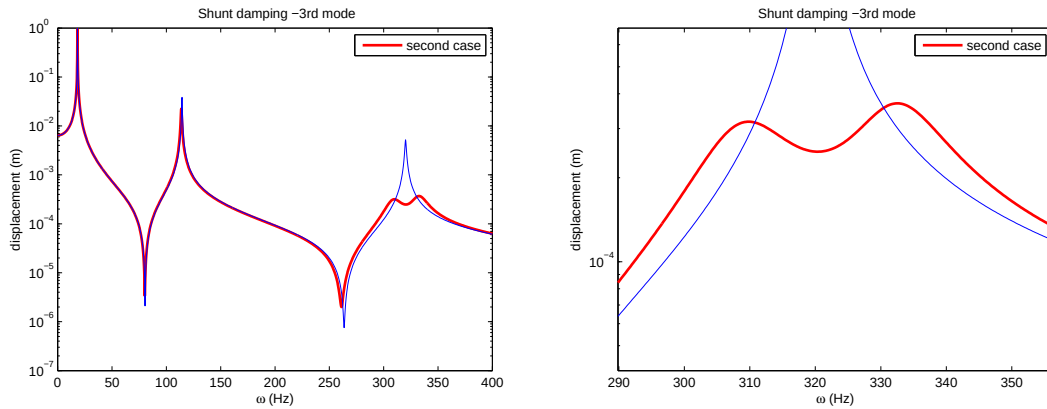


Figure 5.8: Test two result, piezo with 60mm of length, located 5mm way from the clamped tip.

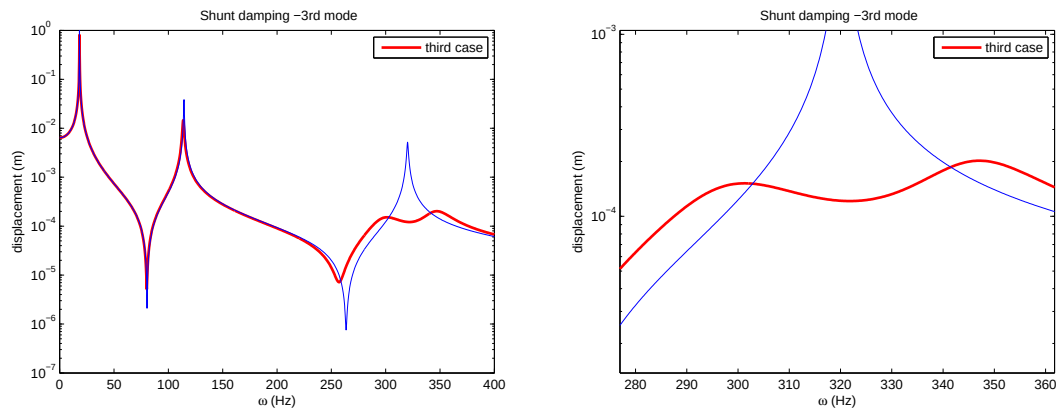


Figure 5.9: Test three result, full covering piezo.

Table 5.11: Third mode damping performance

	Test 1	Test 2	Test 3
left resonance (Hz)	296.8	309.6	300.2
amplitude (m)	1.2e-4	3.2e-4	1.5e-4
anti-resonance (Hz)	323.6	320.8	321.2
amplitude (m)	1.01e-4	2.48e-4	1.2e-4
original amplitude (m)	5.2e-3	5.2e-3	5.2e-3
amplitude decrease	98.6%	94.5%	97.7%
right resonance (Hz)	353	333.2	347
amplitude (m)	1.73e-4	3.6e-4	2e-4

In this third case the best performance is achieved with the smaller transducer, which for this resonance is also the one with lesser demand for resistance and inductance values. It is also the test show a bigger interval between the resonances, and their amplitudes are also the smaller from the set presented here.

5.3.2 Simply-supported beam with uniform transducers.

5.3.2.1 First mode tuning

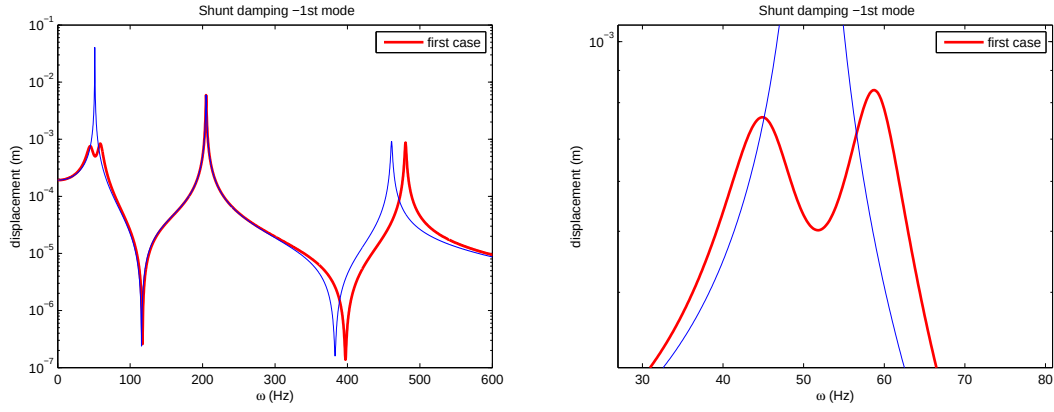


Figure 5.10: Test one result, piezo with 30mm of length, located at mid-length of the beam.

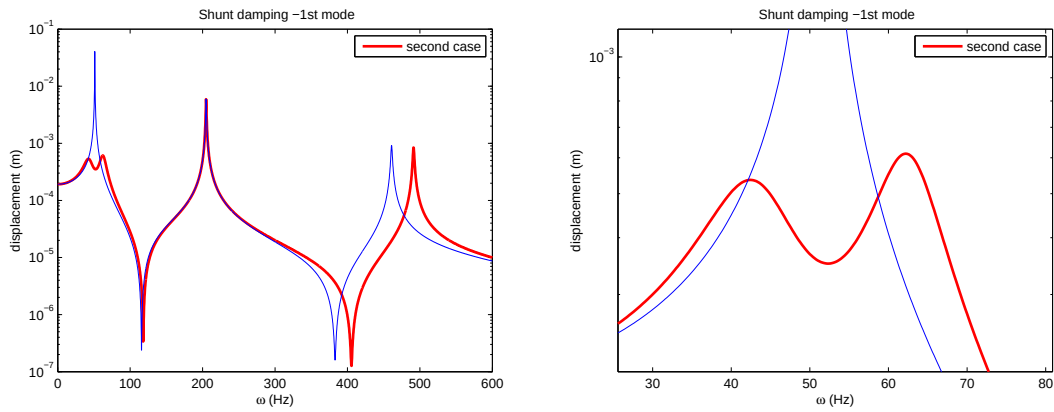


Figure 5.11: Test two result, piezo with 60mm of length, located at mid-length of the beam.

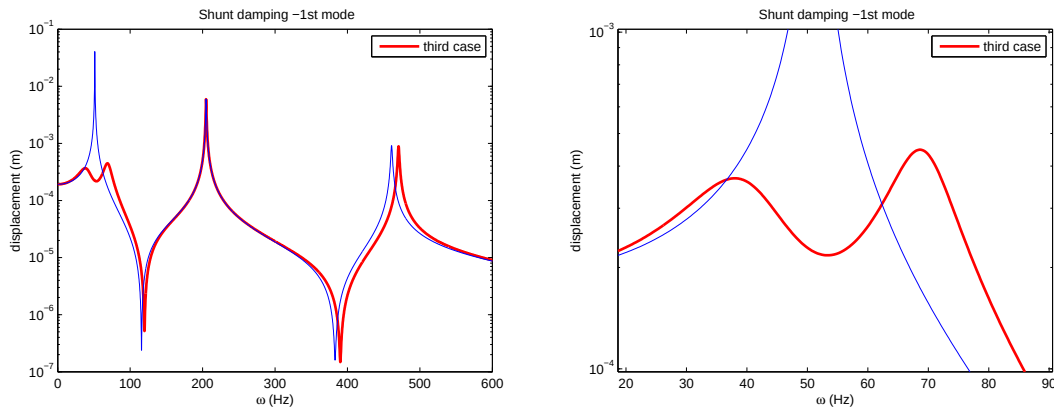


Figure 5.12: Test three result, full covering piezo.

Table 5.12: First mode damping performance

	Test 1	Test 2	Test 3
left resonance (Hz)	44.8	42.6	38
amplitude (m)	7.6e-4	5.4e-4	3.7e-4
anti-resonance (Hz)	51.6	52.4	53.4
amplitude (m)	5e-4	3.5e-4	2.2e-4
original amplitude (m)	0.041	0.041	0.041
amplitude decrease	98,8%	99%	99.1%
right resonance (Hz)	58.8	62.4	68.8
amplitude (m)	8.4e-4	6.1e-4	4.5e-4

Similarly to the first mode damping for clamped-free beam, the smaller transducer has lesser amplitude decrease, although all 3 transducers present very similar values of decrease. It is important to mention that a bigger transducer has less inductance and resistance values needed and better damping performance.

5.3.2.2 Second mode tuning

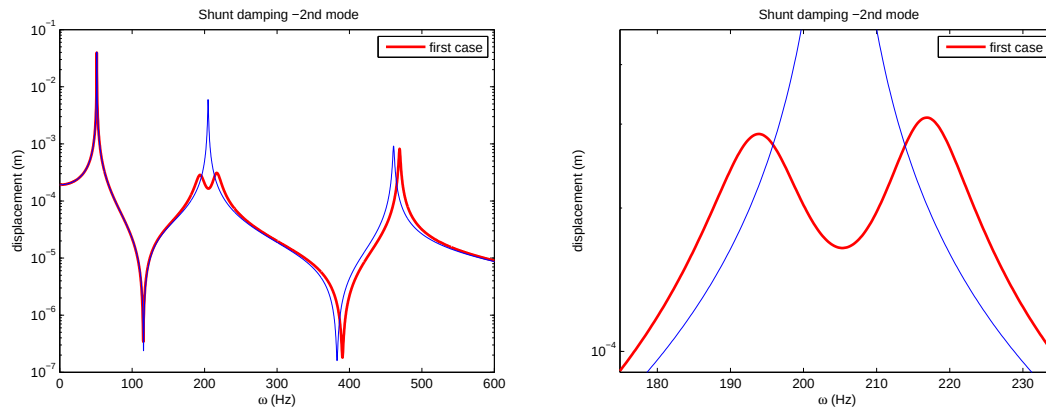


Figure 5.13: Test one result, piezo with 30mm of length, located 5mm away from the left tip of the beam.

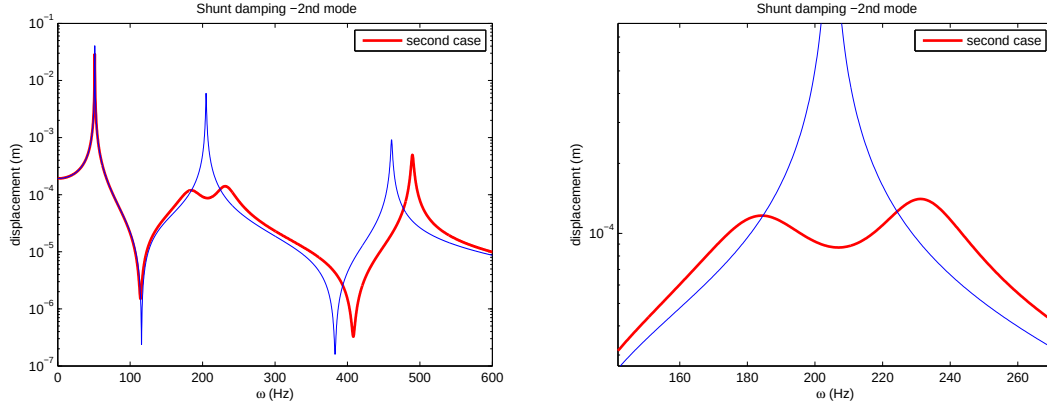


Figure 5.14: Test two result, piezo with 60mm of length, located 5mm away from the left tip of the beam.

Table 5.13: Second mode damping performance

	Test 1	Test 2
left resonance (Hz)	193.6	184
amplitude (m)	2.8e-4	1.2e-4
anti-resonance (Hz)	205.4	207.2
amplitude (m)	1.6e-4	8.7e-5
original amplitude (m)	0.006	0.006
amplitude decrease	97.3%	98.6%
right resonance (Hz)	217	231.4
amplitude (m)	3.1e-4	1.4e-4

In this test, once more the bigger piezo, with 60mm of length, has a better damping performance, and corresponds to the transducer requiring smaller values of inductance.

5.3.2.3 Third mode tuning

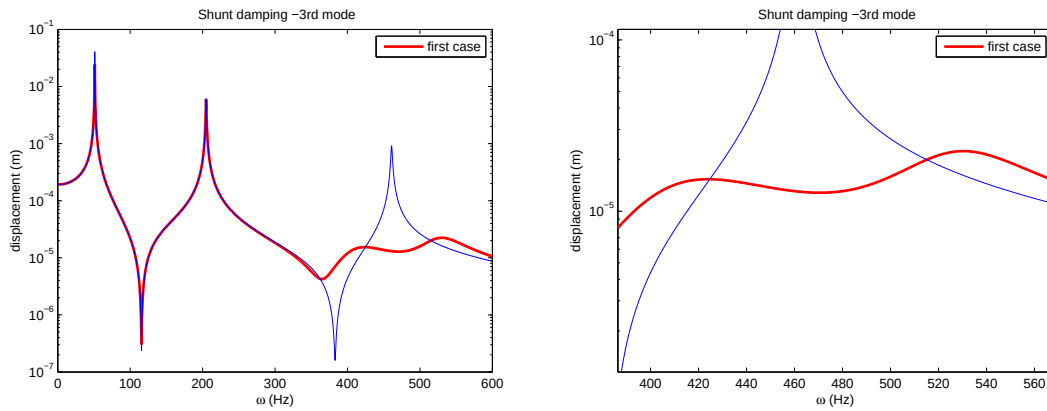


Figure 5.15: Test one result, piezo with 30mm of length located at mid-length of the beam.

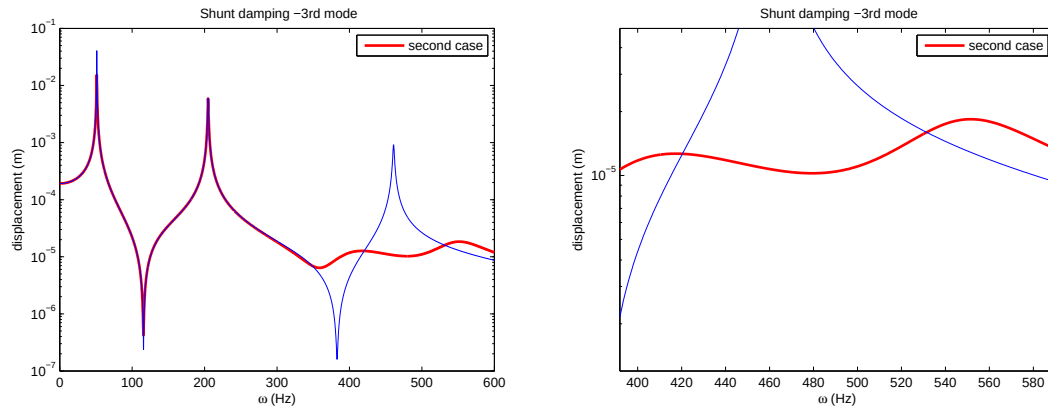


Figure 5.16: Test two result, piezo with 60mm of length located at mid-length of the beam.

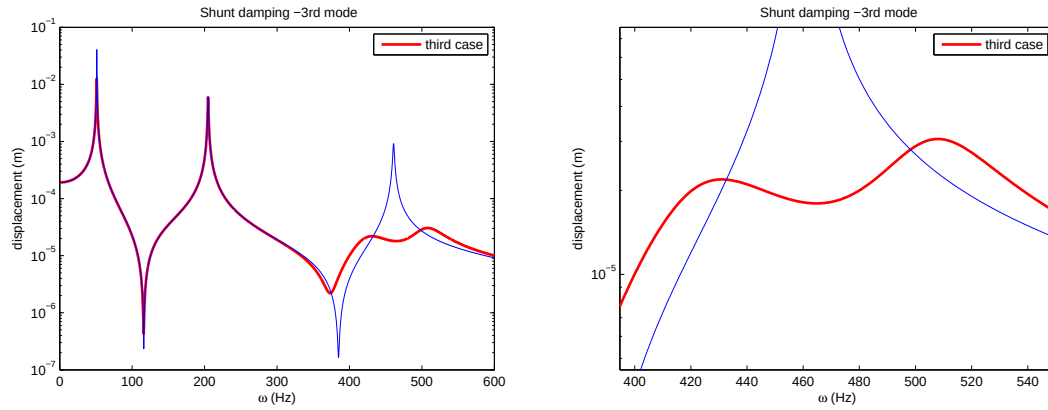


Figure 5.17: Test three result, full covering piezo.

Table 5.14: Third mode damping performance

	Test 1	Test 2	Test 3
left resonance (Hz)	419	411.4	427.8
amplitude (m)	1.5e-5	1.25e-5	2.2e-5
anti-resonance (Hz)	476.6	485.6	465.6
amplitude (m)	1.3e-5	1e-5	1.8e-5
original amplitude (m)	9.1e-4	9.1e-4	9.1e-4
amplitude decrease	98.6%	98.9%	98%
right resonance (Hz)	530.8	550	508.4
amplitude (m)	2.24e-5	1.84e-5	3.06e-5

As one can observe the better performance is achieved with 60 mm transducer, despite the full covering transducer has smaller inductance requirements than the second test.

5.4 Summary

In this chapter is considered the inclusion of a resonant shunt. Including a resonant shunt was developed only for analytical model, models which for their higher electromechanical coupling coefficient, they produce inductance and resistive values higher than the numerical would show. For that matter the results presented here are only phenomenological and representative of what can be achieve by a passive shunt vibration damping. For every case the the shunt damping means significant amplitude decrease. Higher frequencies have lesser inductance requirements, and having original amplitudes smaller than the ones for lower frequencies, is easier to be damped. Different sizes and locations mean different inductance and resistance values and different damping performance. Not always the biggest transducer with smaller inductance values will cope better than a smaller one with higher inductance values and vice-versa.

Chapter 6

Conclusion

6.1 Introduction

In this chapter are analyzed the significant aspects about this work, and are made some considerations about the results obtained in order to create some conclusions and also to suggest some improving, such as experimental implementation, and how this technologies can make it into the market. Therefore will be briefly discussed the feasibility, reliability, efficiency and cost of the techniques studied. This chapter contains an overall insight of the work in the overview section, conclusions made in section with the same name, the new and most significant contributions and suggestions for further research's.

6.2 Overview of the Dissertation

In general the development of passive vibration control via shunt technologies implies the following 5 main aspects :

- Appropriate electromechanical analytical and numeric model formulation for several distinct cases, uniform, segmented and full covering transducers, and modally shaped transducers; The analytical model is developed having in mind the beam's Euler theory, and an extension of the Hamilton's variational principle. During the analytical development are made several assumptions in order to simplify the solution formulation. The Numeric model is also developed considering 3D solid elements, and is implemented in a Multiphysics software and post-processed in Matlab, where electrical DOFs are distinguished for different EBCs;
- Three different behavior studies for the two main limit EBCs, mechanical actuation, sensing, and electrical actuation, for one to quantify and qualify the electromechanical phenomenon;
- Shunt tuning consistent with the model in study, regarding factors as shape, material, location, boundary conditions and excitation of the structure;
- Several tests, numerical and analytical, are made, in order to embrace all the theory developed and all issues addressed, such as location, size or shape of transducers. Mechanical actuation, sensing and electrical actuation is tested both for closed-circuit and open-circuit cases. Test regarding shunt technologies are also developed, where are obtained the ideal shunt parameters, and obtained the FRFs of damped beams ;

- During all this dissertation is made a critical analysis of the results obtained, and are made some important conclusions about the analytical model performance compared to the numeric model.

6.3 Conclusions

The conclusions to be taken from this dissertation may be collected in 4 main groups, contemplating all 5 previous chapters.

Chapter 1

- The “proof of concept” that high performance structures incorporating piezoelectric technologies have a wide potential to solve structural vibration related problems has been done;
- There is a valid reason for the growing interest by the industrial market on these technologies, due to more everyday demanding clients;
- The piezoelectric effect can be taken one step forward, and not only dissipate vibration, but can also be a energy storing method;

Chapter 2

- Simpler models like Euler-Bernoulli give us a decent phenomenological overview of the piezoelectric effect on beams, although with over rated values;
- Hamilton’s principle can give us a faster formulation of the problem when compared to the more traditional, equilibrium approach using Newton’s second law.

Chapter 3 and Chapter 4

- Shape, location and materials can make the beam’s performance to be completely different, from filtering vibrations modes to stiffening them;
- Although not considering mass and stiffness from the piezoelectric materials in the analytical formulation, these factors make great influence on the beam’s behavior;
- Analytical and Numerical models show similar physical behavior, but different piezoelectric effects, for uniform transducers. Shaped transducer analytical model is not well implemented and has large discrepancy to the results obtained on the FEM model. The FEM model is not the most suitable for the smaller thickness objects we worked on, since FEM results are also quite disappointing.

Chapter 5

- Shunt damping can reduce greatly the vibration amplitudes, creating an anti-resonance located at the same frequency value where before existed a resonance;
- The Shunt damping creates also two resonances, located before and after their original resonance, their magnitude is smaller than the one from the original magnitude and is mainly controlled by the resistor element;
- The behavior of a passive vibration via shunted technologies are similar to the ones made by a more common damped vibration absorber system;

- The values of the χ_r can be obtained by two different ways, the first one and more complicated is done through algebraic manipulation, since it's implementation can imply on the occurrence of errors and time wasting calculus, one can perform a second and easier way with interesting values, simply by obtaining the χ_r through the comparison of the OC resonance frequency and the CC resonance frequency;
- The damping is obtained by the resistive element, and the "mass-stiffness" is obtained by the inductance and capacitance;
- The resonant shunt, utilized in this work, implies very large inductance elements, therefore this process has some "feasibility" issues. This problem is more often to occur for lower frequency. In order to outline this problem one must consider the implementation of digitally synthesized inductance, for example Gyrator circuits.

6.4 New and Most Significant Contributions

This work has brought to me the knowledge of how multidisciplinary mechanical engineering is, learning concepts as active structures or smart materials, concepts not that mentioned along this mechanical mastership. The relevance of these concepts is nowadays undeniable, in times where resources must be preserved and streamlined, and precision and safety are one key factor for that to be achieved. These new concepts can and must turn our future constructions more safe, auto sufficient, reliable and more rational.

6.5 Suggestions for further Research

For further research some suggestions come easily to my mind. For a start an experimental implementation of the formulation developed should be done. Another suggestion lies on a more embracing analytical formulation, where shear effects and piezoelectric mass and stiffness effects are accounted for. Additionally, when h_b tends to be of same order of magnitude than h_p , symmetry is not valid anymore and the decoupling of bending and membrane behaviors is nor more possible. In these cases, the inclusion of membrane stress-strain behavior into the analytical model is necessary for reasonable accuracy and model representativeness.

Another issue that can be suggested is, how can one put this technologies on the market, what cost must be taken, where to applied them, considering real life cycles, where can be thought manners for auto-adjustment of shunt technologies for different temperatures, since this can be one important factor for the success of implementation of these technologies.

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Appendix

MATLAB Codes

```
close all; clc; clear all;

%%
% João Miguel Gonçalves Ribeiro
% Em01095
% Passive vibration control
% via shunted Piezoelectric
% transducing technologies

%% Clamped-free beam

%% Beam's geometric and mechanical properties %
E=70e9;rho=2710;bb=0.030/2;hb=0.002/2;A=2*bb*2*hb;
l=0.3;I=(2*bb*(2*hb)^3)/12;
% disp(' ')
% l=input('Comprimento L da viga em /m ? ');

%% mode shapes
modos=5;
% disp(' ')
% modos=input('número de modos naturais ? ');
% if modos<5
%     modos=5;
% else
%     modos=modos;
% end

%% square roots
betha=zeros(1,modos);
betha(1:5)=[1.8751; 4.6941; 7.8548; 10.9955; 14.1372];
for r=6:modos
    betha(r)=(r-1/2)*pi;
end
alpha=zeros(1,modos);
alpha(1:3)=[0.7341; 1.0185; 0.9992];
for r=4:modos
    alpha(r)=1;
end

%% frequency bandwidth

wi=0:0.2:400;
w_rad = wi*2*pi;

%% eigenfrequency values
```

```

C1=sqrt(E*I/(rho*A*l^4));
wr=betha.^2*C1;           % rad/s
wr_Hz=wr/(2*pi);         % Hertz
for i=1:length(wr)
    wr_quad(i,:)=wr(i)^2;
end
%% modal damping

for r=1:length(betha)
    qsi(r)=0.002;
end

%% application location

x=l;

FRF=zeros(length(wi));

for r=1:modos
    ms2=@(y)(cosh(betha(r)/l*y)-alpha(r)*sinh(betha(r)/l*y)-(cos(betha(r)/l*y)-alpha(r)*sin(betha(r)/l*y))...
        .*(cosh(betha(r)/l*y)-alpha(r)*sinh(betha(r)/l*y)-(cos(betha(r)/l*y)-alpha(r)*sin(betha(r)/l*y)));
    int_ms2=quadgk(ms2,0,l);
    Cn(r)=1/(sqrt(rho*A)*sqrt(int_ms2));
end

for r=1:modos
    ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r).x/l)-sin(betha(r).x/l));
    FRF(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*qsi(r)*wr(r)-w_rad));
end

FRF_total=zeros(1,length(wi));
for r=1:length(betha)
    FRF_total(1,:)=FRF_total(1,:)+FRF(r,:);
end

%% plot
figure(1)
plot(wi,(abs(FRF_total)),'b')
set(gca,'yscale','log')
xlabel('\omega (Hz)')

```

```
ylabel('total displacement (m)')
legend('\xi=0.2%')
title('FRF-CF Beam')
%%
%% Piezoelectric mechanical and electric properties

                                %PXE-5 %
% valores da matriz constitutiva elástica
c11=9.4511e10; c12=4.3229e10; c13=4.1322e10; c33=8.0348e10; c44=2.4
5641e10;

% valores da matriz constitutiva piezoelectrica
e31=-8.9531; e33=22.4058; e24=13.2051; e15=e24;

% valores da matriz constitutiva dielectrica
dielec11=0.9136e-8; dielec22=dielec11; dielec33=0.3540e-8;
permitividade=8.85e-12;
drel11=dielec11/permitividade; drel33=dielec33/permitividade;
% densidade
rho_piezo=7800;

c11x=(c11-((c13^2)/c33));
e31x=(e31-(c13*e33)/c33);
dielecc33x=(dielec33+((e33^2)/c33));

%% Piezo shape -- Uniform tranducers

bp=0.03/2;hp=0.0005/2;

% segmented tranducer
xR=0.035;
xL=0.005;
%
% xR=0.165;
% xL=0.135;
%
% xR=0.065;
% xL=0.005;
% %
% xR=0.180;
% xL=0.120;

% Continuum transducer
% xR=l*0.99;
```

```
% xL=0.001;
```

```
Aelec=2*bp*(xR-xL);
cv=2*e31x*hb;
ci=e31x*hb*2*bb;
Zr=(2*hp/(Aelec*dielecc33x)) % uniform transducer capacitance inverse
Cpp=1/Zr;
%% decoupled OC FRF
```

```
FRF_Piezo1=zeros(length(wi));
cv=2*e31x*hb;
ci=e31x*hb*2*bb;
```

```
for r=1:length(betha)
    Zr=(2*hp/(Aelec*dielecc33x));
    ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r).  
*x/l)-sin(betha(r).*x/l));
    dms=((Cn.*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha  
(r)/l-alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha  
(r)/l)))...
    -(Cn.*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)*betha(r)  
/l-alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)*betha(r)  
/l))))).^2;
    FRF_Piezo1(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*psi(r)  
*wr(r)*w_rad)+(cv*ci*bp*Zr*dms(r)));
```

```
end
FRF_Piezo_total1=zeros(1,length(wi));
for r=1:length(betha)
    FRF_Piezo_total1(1,:)=FRF_Piezo_total1(1,:)+FRF_Piezo1(r,:);
end
```

```
figure(2)
plot(wi,(abs(FRF_total)),'b')
set(gca,'yscale','log')
hold on
plot(wi,(abs(FRF_Piezo_total1)),'--','linewidth',1.5,'Color','r')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('displacement (m/N)')
legend('Beam', 'Smart beam (decoupled MS)')
title('FRF-CF Beam vs Smart Beam (decoupled modes)')
```

```
%% coupled OC FRF
```

```
matRigid= diag(wr_quad); % stiffness matrix
```

```
% Mode shapes
```

```
for i=1:modos
```

```
    vecFM(i,:)=Cn(i)*(cosh(betha(i)*x/l)-cos(betha(i)*x/l)-alpha(i).*(  
    (sinh(betha(i).*x/l)-sin(betha(i).*x/l))));
```

```
end
```

```
%Mrk1=cv*ci*bp*Zr*(transpose(dmr))*dmk;
```

```
for r=1:modos
```

```
    dmr(r)=Cn(r)*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha(r)/l-  
    alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha(r)/l)...  
    -Cn(r)*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)*betha(r)/l-  
    alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)*betha(r)/l));
```

```
    dmk=-dmr;
```

```
end
```

```
Mrk=-cv*ci*bp*Zr*(transpose(dmr))*dmk;
```

```
ModShapes=vecFM;
```

```
% Damping matrix
```

```
matAmortecimento=eye(modos,modos);
```

```
matAmortecimento=matAmortecimento*(2*1j*qs(i));
```

```
for i=1:modos
```

```
    matAmortecimento(i,i)=matAmortecimento(i,i)*wr(i);
```

```
end
```

```
matQSI=matAmortecimento;
```

```
% Inertial Matrix
```

```
matInercia=eye(modos,modos);
```

```
for k=1:length(w_rad)
```

```
    matInercia_k=matInercia*(w_rad(k)^2);
```

```
    matQSI_k=matQSI*w_rad(k);
```

```
    matDIN=-matInercia_k+matQSI_k+matRigid+Mrk;
```

```
    aux1=matDIN\ModShapes;
```

```
    FRF_Piezo(k,:)=transpose(ModShapes)*aux1;
```

```
end
```

```

figure(3)
plot(wi, (abs(FRF_total)), 'b')
set(gca, 'yscale', 'log')
hold on
plot(wi, (abs(FRF_Piezo)), '--', 'linewidth', 1.5, 'Color', 'r')
set(gca, 'yscale', 'log')
xlabel('\omega (Hz)')
ylabel('displacement (m/N)')
legend('Beam', 'Smart beam (coupled MS)')
title('FRF-CF Beam vs Smart Beam (coupled modes)')

```

```

%% voltage per unit of load and displacement per unit of voltage
%% decoupled

```

```

sensing=zeros(1, length(wi));
sensing2=zeros(1, length(wi));

```

```

ct=e31x*hb*2*bb*Zr;
for r=1:length(betha)
    Zr=(2*hp/(Aelec*dielecc33x))
    ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r). $\sqrt{x/l}$ )-sin(betha(r). $\sqrt{x/l}$ ));
    dmt=((Cn.*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha $\sqrt{(r)/l}$ -alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha $\sqrt{(r)/l}$ )))...
        -(Cn.*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)*betha(r) $\sqrt{/l}$ -alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)*betha(r) $\sqrt{/l}$ )))));

```

```

    dms=((Cn.*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha $\sqrt{(r)/l}$ -alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha $\sqrt{(r)/l}$ )))...
        -(Cn.*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)*betha(r) $\sqrt{/l}$ -alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)*betha(r) $\sqrt{/l}$ ))))).^2;

```

```

    SENSING(r,:)=ct*((Cn(r)*ms)*dmt(r))./(((wr(r).^2)-w_rad.^2)+1j* $\sqrt{(2*qs_i(r)*wr(r)*w\_rad)}$ );
    SENSING2(r,:)=ct*((Cn(r)*ms)*dmt(r))./(((wr(r).^2)-w_rad.^2)+1j* $\sqrt{(2*qs_i(r)*wr(r)*w\_rad)}$ )+(cv*ci*bp*Zr*dms(r)));

```

```

    sensing(1,:)=sensing(1,:)+SENSING(r,:);
    sensing2(1,:)=sensing2(1,:)+SENSING2(r,:);

```

```

end

```

```

for r=1:length(betha)
    ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r).  

*x/l)-sin(betha(r).*x/l));
    dmt=((Cn.*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha  

(r)/l-alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha  

(r)/l)))...
    -(Cn.*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)*betha(r)  

/l-alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)*betha(r)  

/l))) );
    EA(r,:)=bp*cv*((Cn(r)*ms)*dmt(r))./(((wr(r).^2)-w_rad.^2)+1j*(2*qs  

i(r)*wr(r)*w_rad)+(cv*ci*bp*Zr*dms(r)));
    EACT(1,:)=EA(1,:)+EA(r,:);
end
figure(4)
plot(wi,abs(sensing2),'r')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('electrical Potential (V/N)')
title('CF Sensing')

%% coupled
x_med=xR;
for i=1:modos
    vecFMmed(i,:)=Cn(i)*(cosh(betha(i)*xR/l)-cos(betha(i)*xR/l)-alpha  

(i).*(sinh(betha(i).*xR/l)-sin(betha(i).*xR/l)));
end

ModShapes_med=vecFMmed;

for k=1:length(w_rad)

    matInercia_k=matInercia*(w_rad(k)^2);
    matQSI_k=matQSI*w_rad(k);
    matDIN=-matInercia_k+matQSI_k+matRigid-Mrk;

% derivada
for r=1:modos
    dmX(r,:)=Cn(r)*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)  

*betha(r)/l-alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)  

*betha(r)/l))...
    -Cn(r)*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha(r)  

/l-alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha(r)  

/l));
end

```

```
matDIN2=-matInercia_k+matQSI_k+matRigid;
aux3=inv(matDIN)*dmX;

X_V(k)=(cv*bp*transpose((ModShapes)))*aux3;
```

```
end
```

```
figure(5)
plot(wi,abs(X_V), 'linewidth',2, 'color', 'r')
set(gca, 'yscale', 'log')
xlabel('\omega (Hz)')
ylabel('displacement (m/V)')
legend( 'Analytical')
title('FRF-CF Smart beam (test 5)')
```

```
%% shunt
```

```
% coupling factor
```

```
K31=0.38;
```

```
% generalized electromechanical coupling coefficient
```

```
auxgecc=(3*hp*(hb)/l*hb^2)*(c11x/E)*((K31^2)/(1-K31^2));
```

```
for r=1:modos
```

```
Ngecc=(Cn(r)*(alpha(r)*((betha(r)*cos((betha(r)*xR)/l))/l - (betha(r)*
*cosh((betha(r)*xR)/l))/l)...
+ (betha(r)*sin((betha(r)*xR)/l))/l + (betha(r)*sinh((betha(r)*xR)/
l))/l)...
- (Cn(r)*(alpha(r)*((betha(r)*cos((betha(r)*xL)/l))/l - (betha(r)*
*cosh((betha(r)*xL)/l))/l)...
+ (betha(r)*sin((betha(r)*xL)/l))/l + (betha(r)*sinh((betha(r)*xL)/
l))/l)).^2;
```

```
Dgecc=@(x)(Cn(r)*((betha(r)^2*cos((betha(r)*x)/l))/l^2 - alpha(r)*
((betha(r)^2*sin((betha(r)*x)/l))/l^2 ...
+ (betha(r)^2*sinh((betha(r)*x)/l))/l^2) + (betha(r)^2*cosh((betha
(r)*x)/l))/l^2)).^2;
```

```
intDgecc=quadgk(Dgecc,0,l);
```

```
GECC(r,:)=(326.8-320.2)/326.8;
```

```
GECC(r,:)=0.1818
```

```
DOPT(r,:)=sqrt(1+GECC(r));
```

```
RHOPT(r,:)=sqrt(2)*(sqrt(GECC(r))/(1+GECC(r)));
```

```
L(r,:)=Zr/(((wr(r).^2)*(1+GECC(r))));
```

```
R0(r,:)=((Zr*sqrt(2))/((wr(r))))*(sqrt(GECC(r))/(1+GECC(r)));
end
```

```
indx=1;
```

```
Zel=(1j*w_rad*R0(indx)-(w_rad.^2)*L(indx));
```

```
Zel2=Zel./((-Cpp*L(indx)*w_rad.^2)+(1j*Cpp*R0(indx).*w_rad)+1);
ZelR=(1j*w_rad*R0(indx))./((1j*Cpp*R0(indx).*w_rad)+1);
```

```
for r=1:length(betha)
```

```
    ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r).  
*x/l)-sin(betha(r).*x/l));  
    dms=((Cn.*(sinh(betha(r)*xL/l)*betha(r)/l+sin(betha(r)*xL/l)*betha  
(r)/l-alpha(r)*(cosh(betha(r)*xL/l)*betha(r)/l-cos(betha(r)*xL/l)*betha  
(r)/l))).  
    -(Cn.*(sinh(betha(r)*xR/l)*betha(r)/l+sin(betha(r)*xR/l)*betha(r)  
/l-alpha(r)*(cosh(betha(r)*xR/l)*betha(r)/l-cos(betha(r)*xR/l)*betha(r)  
/l))).^2;  
    FRESSIONANTSHUNT(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*qs  
(r)*wr(r)*w_rad)+(1*dms(r)*cv*ci*Zel2*bp));  
    FRESISTIVESHUNT(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*qs  
(r)*wr(r)*w_rad)+(1*dms(r)*cv*ci*ZelR*bp));
```

```
end
```

```
FRFSHUNT(1,:)=zeros(1,length(wi));
```

```
for r=1:length(betha)
```

```
    FRFSHUNT(1,:)=FRFSHUNT(1,:)+FRESSIONANTSHUNT(r,:);
```

```
end
```

```
FRFRESISTIVE(1,:)=zeros(1,length(wi));
```

```
for r=1:length(betha)
```

```
    FRFRESISTIVE(1,:)=FRFRESISTIVE(1,:)+FRESISTIVESHUNT(r,:);
```

```
end
```

```
% RESSONANT SHUNT
```

```
figure(6)
```

```
plot(wi,(abs(FRFSHUNT)), 'linewidth',2, 'color', 'r')
```

```
set(gca, 'yscale', 'log')
```

```
hold on
```

```

plot(wi, (abs(FRF_total)), 'b')
set(gca, 'yscale', 'log')
xlabel('\omega (Hz)')
ylabel('displacement (m)')
legend('third case')
title(' Shunt damping -3rd mode')

%% Piezo shape -- modal tranducers

cv=2*e31x*hb;
ci=e31x*hb*2*bb;
cvnode=-cv;
cinode=-ci;

%% modal shape
mode_index=1;
%%

CN=Cn(mode_index);
BETHA=betha(mode_index);
ALPHA=alpha(mode_index);

msmodal= CN*(cosh(BETHA*x/l)-cos(BETHA*x/l)-ALPHA*(sinh(BETHA*x/l)-sin(
(BETHA*x/l))));
% 1st diff
dmsmodal=CN*(ALPHA*((BETHA*cos((BETHA*x)/l))/l - (BETHA*cosh((BETHA*x)/
l))/l)...
+ (BETHA*sin((BETHA*x)/l))/l + (BETHA*sinh((BETHA*x)/l))/l);
% 2nd diff
ddmsmodal=CN*((BETHA^2*cos((BETHA*x)/l))/l^2 - ALPHA*((BETHA^2*sin(
(BETHA*x)/l))/l^2 ...
+ (BETHA^2*sinh((BETHA*x)/l))/l^2) + (BETHA^2*cosh((BETHA*x)/l))/
l^2);
% aux normalization calc.
auxNorm=CN*((BETHA^2*cos((BETHA*0)/l))/l^2 - ALPHA*((BETHA^2*sin(
(BETHA*0)/l))/l^2 ...
+ (BETHA^2*sinh((BETHA*0)/l))/l^2) + (BETHA^2*cosh((BETHA*0)/l))/
l^2);

B=bb/auxNorm; % normalization factor
x1=0:0.001:0.3;
shape0=CN*(cosh(BETHA*x1/l)-cos(BETHA*x1/l)-ALPHA*(sinh(BETHA*x1/l)-sin(
(BETHA*x1/l))));
orgshape=0*x1;
shape1=CN*(ALPHA*((BETHA*cos((BETHA*x1)/l))/l - (BETHA*cosh((BETHA*x1)/

```

```

/l))/l)...
    + (BETHA*sin((BETHA*x1)/l))/l + (BETHA*sinh((BETHA*x1)/l))/l);
shape=CN*((BETHA^2*cos((BETHA*x1)/l))/l^2-ALPHA*((BETHA^2*sin(
((BETHA*x1)/l))/l^2 ...
    + (BETHA^2*sinh((BETHA*x1)/l))/l^2) + (BETHA^2*cosh((BETHA*x1)/l))/
/l^2)*B;
figure(7)
plot(x1,shape0,'linewidth',2)
hold on
plot(x1,orgshape,'linewidth',3,'color','k')
legend('1st mode')

figure(8)
plot(x1,shape1)

figure(9)
plot(x1,shape,'linewidth',2)
hold on
plot(x1,-shape,'linewidth',2)
xlabel('length (m)')
ylabel('width (m)')
legend('4 mode')
title('Modal piezoelectric transducer shape')

%% modal capacitance

AUXCPS=@(z)CN*((BETHA^2*cos((BETHA*z)/l))/l^2 - ALPHA*((BETHA^2*sin(
((BETHA*z)/l))/l^2 ...
    + (BETHA^2*sinh((BETHA*z)/l))/l^2) + (BETHA^2*cosh((BETHA*z)/l))/
/l^2);
int=quadgk(AUXCPS,0,l);

Cps=(dielecc33x*bb/hp)*(E*I/wr(mode_index)^2)*int;
Zrm=1/Cps;
alphabeta=int^1;

for r=1:modos
    ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r).
*x/l)-sin(betha(r).*x/l));

    der=CN*(ALPHA*((BETHA*cos((BETHA*x)/l))/l ...
        -(BETHA*cosh((BETHA*x)/l))/l)...
    + (BETHA*sin((BETHA*x)/l))/l + (BETHA*sinh((BETHA*x)/l))/l);

```

```
FRFPiezoM(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*psi(r)*wr(r)*w_rad)...
+(cv*ci*bb*Zrm*alphanbeta));
```

```
ms1= cosh(betha(r)*(0.064)/l)-cos(betha(r)*(0.064)/l)-alpha(r).*(sinh(betha(r).*(0.064)/l)-sin(betha(r).*(0.064)/l));
```

```
FRFPiezoMn(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*psi(r)*wr(r)*w_rad)...
+(cv*ci*bb*Zrm*alphanbeta));
```

```
ms2= cosh(betha(r)*(x-0.064)/l)-cos(betha(r)*(x-0.064)/l)-alpha(r).*(sinh(betha(r).*(x-0.064)/l)-sin(betha(r).*(x-0.064)/l));
```

```
FRFPiezoMnn(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*psi(r)*wr(r)*w_rad)...
+(cvnode*cinode*bb*Zrm*alphanbeta));
```

```
%% sensing behavior
```

```
WR=wr(mode_index)
```

```
SENSINGMODAL(r,:)=(Zrm*e31x*hb*2*bb/auxNorm)*der*(CN*msmodal)./(((WR.^2)-w_rad.^2)+1j*(2*psi(r)*WR*w_rad)...
+(cv*ci*bb*Zrm*alphanbeta));
```

```
end
```

```
FRF_MODAL=zeros(1,length(wi));
```

```
for r=1:length(betha)
```

```
FRF_MODAL(1,:)=FRF_MODAL(1,:)+FRFPiezoM(r,:);
```

```
end
```

```
figure(10)
```

```
plot(wi,abs(FRF_MODAL),'--','color','r','linewidth',2.5)
```

```
set(gca,'yscale','log')
```

```
hold on
```

```
plot(wi,abs(FRF_total),'b','linewidth',1)
```

```
set(gca,'yscale','log')
```

```
xlabel('\omega (Hz)')
```

```
ylabel('displacement (m/N)')
```

```
legend('Beam w/ modal transucer', 'Beam')
```

```
title('FRF-CF beam (1st mode)')
```

```
FRF_MODALn=zeros(1,length(wi));
```

```
for r=1:length(betha)
```

```
FRF_MODALn(1,:)=FRF_MODALn(1,:)+FRFPiezoMn(r,:)+FRFPiezoMnn(r,:);
```

end

```
figure(11)
plot(wi,abs(FRF_MODALn),'--','color','r','linewidth',2.5)
set(gca,'yscale','log')
hold on
plot(wi,abs(FRF_total),'b')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('displacement (m/N)')
legend('Beam w/ modal transucer', 'Beam')
title('FRF-CF beam (2nd mode - inverted polarization)')
```

```
figure(12)
plot(wi,abs(SENSINGMODAL),'r','linewidth',2)
set(gca,'yscale','log')
hold on
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('electrical Potential (V/N)')
% legend( 'Voltage per unit of Force')
title('CF Sensing')
```

%% SHUNT

% generalized electromechanical coupling coefficient

```
auxgecc=(3*hp*(hb)/l*hb^2)*(c11x/E)*((K31^2)/(1-K31^2));
```

```
Dgeccmodal=@(x)CN*((BETHA^2*cos((BETHA*x)/l))/l^2 - ALPHA*((BETHA^2*sin((BETHA*x)/l))/l^2 ...
```

```
+ (BETHA^2*sinh((BETHA*x)/l))/l^2) + (BETHA^2*cosh((BETHA*x)/l))/l^2);
```

```
intDgeccmodal=quadgk(Dgeccmodal,0,l);
```

```
GECCmodal=((auxNorm/(B*bb*intDgecc))*auxgecc);
```

```
DOPTmodal=sqrt(1+GECCmodal);
```

```
RHOPTmodal=sqrt(2)*(sqrt(GECCmodal)/(1+GECCmodal));
```

```
GECCmodal=(1*(22-18)/22);
```

```
% GECCmodal=0.0694
```

```
modeindex=mode_index;
```

```
LM=Zr/(((wr(modeindex).^2)*(1+GECCmodal)));
```

```
ROM=abs(((Zr*sqrt(2))/((wr(modeindex))))*(sqrt(GECCmodal)/(1+GECCmodal)));
```

```
Zelmodal=(1j*w_rad.*ROM-(w_rad.^2).*LM);
```

```
Zel2modal=Zelmodal./((-Cpp*LM*w_rad.^2)+(1j*Cpp*ROM.*w_rad)+1);
```

```
for r=1:modos
```

```
ms= cosh(betha(r)*x/l)-cos(betha(r)*x/l)-alpha(r).*(sinh(betha(r).*x/l)↵  
-sin(betha(r).*x/l));
```

```
FRFPiezoMSHUNT(r,:)=(Cn(r)*ms)^2./(((wr(r).^2)-w_rad.^2)+1j*(2*qsi(r)↵  
*wr(r)*w_rad)...  
+(cv*ci*bb*Zel2modal*alphabetha));
```

```
end
```

```
FRFSHUNTM(1,:)=zeros(1,length(wi));
```

```
for r=1:length(betha)
```

```
FRFSHUNTM(1,:)=FRFSHUNTM(1,:)+FRFPiezoMSHUNT(r,:);
```

```
end
```

```
FRFRESISTIVE(1,:)=zeros(1,length(wi));
```

```
for r=1:length(betha)
```

```
FRFRESISTIVE(1,:)=FRFRESISTIVE(1,:)+FRESISTIVESHUNT(r,:);
```

```
end
```

```
% RESSONANT SHUNT
```

```
figure(13)
```

```
plot(wi,(abs(FRFSHUNTM)), 'linewidth',3, 'color', 'r')
```

```
set(gca, 'yscale', 'log')
```

```
hold on
```

```
plot(wi,(abs(FRF_total)), 'b')
```

```
set(gca, 'yscale', 'log')
```

```
xlabel('\omega (Hz)')
```

```
ylabel('displacement (m)')
```

```
legend( 'second modal shape - CF beam')
```

```
title(' Shunt damping -2nd mode')
```

```
close all; clc; clear all;

%%
% João Miguel Gonçalves Ribeiro
% Em01095
% Passive vibration control
% via shunted Piezoelectric
% transducing technologies

%% Simply-supported beam

%% propriedades mecânicas e geométricas da viga %
E=70e9;rho=2710;bb=0.030/2;hb=0.002/2;A=2*bb*2*hb;
l=0.3;I=(2*bb*(2*hb)^3)/12;
% disp(' ')
% l=input('Comprimento L da viga em /m ? ');
%% modos naturais
modos=5;
% disp(' ')
% modos=input('número de modos naturais ? ');
% if modos<5
%     modos=5;
% else
%     modos=modos;
% end
%% gama de frequências
wi=0:0.2:600;
w_rad = wi*2*pi;
wr=zeros(1,length(w_rad));
%% ponto de aplicação da carga e de medição para FRF directa

x=(2/9)*l;

%% amortecimento modal

for r=1:length(modos)
    qsi(r)=0.002;
end
%% frequências de ressonância e FRF viga simples

FRF=zeros(length(wi));
C1=sqrt(E*I/(rho*A*l^4));
Cn=(sqrt(2/(rho*A*l)));
for r=1:modos
    wr=((r*pi)^2)*C1;                % rad/s
```

```

    wr_Hz=wr/(2*pi);                                % Hertz
    wr_quad=wr^2;
    ms=sin(r*pi/l*x);
    FRF(r,:)=(Cn*ms)^2./((wr^2-w_rad.^2)+1j*(2*qsi*wr*w_rad));
    vec_wr(r,:)=wr;                                %vector de frecuencias naturais
    vec_wr2(r,:)=wr^2;
end

FRF_total=zeros(1,length(wi));
for r=1:modos
    FRF_total(1,:)=FRF_total(1,:)+FRF(r,:);
end

figure(1)
plot(wi,(abs(FRF_total)),'b')
set(gca,'yscale','log')

%% geometria do piezo -- uniforme
bp=0.03/2;hp=0.0005/2;

%% propriedades mecânicas do material piezoelectrico PXE-5 %

% valores da matriz constitutiva elástica
c11=9.4511e10; c12=4.3229e10; c13=4.1322e10; c33=8.0348e10; c44=2.4
5641e10;

% valores da matriz constitutiva piezoelectrica
e31=-8.9531; e33=22.4058; e24=13.2051; e15=e24;

% valores da matriz constitutiva dielectrica
dielec11=0.9136e-8; dielec22=dielec11; dielec33=0.3540e-8;
permitividade=8.85e-12;
drel11=dielec11/permitividade; drel33=dielec33/permitividade;
% densidade
rho_piezo=7800;

c11x=(c11-((c13^2)/c33));
e31x=(e31-(c13*e33)/c33);
dielecc33x=(dielec33+((e33^2)/c33));

%% Piezo shape -- Uniform tranducers

bp=0.03/2;hp=0.0005/2;

```

```
% Piezo segmentado
```

```
% xR=0.035;
% xL=0.005;
% xR=0.065*0.99;
% xL=0.005*0.99;
% xR=0.165*0.99;
% xL=0.135*0.99;
xR=0.180*0.99;
xL=0.120*0.99;
```

```
% Piezo contínuo
```

```
  xL=0.001;
  xR=l*0.99;
```

```
Aelec=2*bp*(xL-xR);
cv=2*e31x*hb;
ci=e31x*hb*2*bb;
```

```
%% sem acoplamento
```

```
FRF_piezo=zeros(modos,length(wi));
for r=1:modos
    ms=sin(r*pi/l*x); % formas naturais
    Cre(r,:)=((Cn*(pi*r/l)*cos(r*pi/l*xR)-Cn*(pi*r/l)*cos(r*pi/l*xL)))^2;
    wr=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4)); % frequências de ressonância
    Zr=(2*hp/(Aelec*dielecc33x));
    c_piezo=ci*cv*bp*Zr;
    FRF_piezo(r,:)=(Cn*ms)^2./((wr^2-w_rad.^2)+(1j*(2*qli*wr*w_rad))-(Cre(r)*bp*ci*cv*Zr));
```

```
end
```

```
FRF_piezo_total=zeros(1,length(wi));
for r=1:modos
    FRF_piezo_total(1,:)=FRF_piezo_total(1,:)+FRF_piezo(r,:);
end
```

```
figure(2)
plot(wi,(abs(FRF_piezo_total)),'--','linewidth',1.5,'Color','r')
set(gca,'yscale','log')
hold on
plot(wi,(abs(FRF_total)),'b')
set(gca,'yscale','log')
```

```
xlabel('\omega (Hz)')
ylabel('displacement (m/N)')
legend('Beam', 'Smart beam (decoupled MS)')
title('FRF-SS Beam vs Smart Beam (decoupled modes)')

%% com acoplamento
%% matriz de acoplamento
for r=1:modos
    dmr(r)=Cn*cos(r*pi*xL/l)*(r*pi/l)-Cn*cos(r*pi*xR/l)*(r*pi/l);
    dmK=-dmr;
end
MrK=-cv*ci*bp*Zr*(transpose(dmr))*dmK;

%% matriz rigidez

matRigid= diag(vec_wr2);

%% Vectores de contribuição modal

for i=1:modos
    ms=sin(i*pi/l*x);
    vecFM(i,:)=Cn*ms;
end

ModShapes=vecFM;

%% matriz de amortecimento

matAmortecimento=eye(modos,modos);
matAmortecimento=matAmortecimento*(2*1j*qsI);
for i=1:modos
    matAmortecimento(i,i)=matAmortecimento(i,i)*vec_wr(i);
end
matQSI=matAmortecimento;

%% matriz de inercia
matInercia=eye(modos,modos);

%% matriz dinâmica
X_VV=zeros(1,length(wi));

for r=1:modos
```

```
dmX(r,:)=Cn*cos(r*pi*xL/l)*(r*pi/l)-Cn*cos(r*pi*xR/l)*(r*pi/l);
```

```
end
```

```
for k=1:length(w_rad)
```

```
    matInercia_k=matInercia*(w_rad(k)^2);
    matQSI_k=matQSI*w_rad(k);
    matDIN=-matInercia_k+matQSI_k+matRigid-Mrk;
    aux1=matDIN\ModShapes;
    FRF_Piezo(k,:)=transpose(ModShapes)*aux1;
```

```
    aux3=inv(matDIN)*dmX;
    X_V(k,:)=(cv*bp*transpose((ModShapes)))*aux3;
```

```
end
```

```
figure(3)
plot(wi,(abs(FRF_Piezo)),'--','linewidth',1.5,'Color','r')
set(gca,'yscale','log')
hold on
plot(wi,(abs(FRF_total)),'b')
xlabel('\omega (Hz)')
ylabel('displacement (m/N)')
legend('Beam', 'Smart beam (coupled MS)')
title('FRF-SS Beam vs Smart Beam (coupled modes)')
```

```
figure(5)
plot(wi,abs(X_V),'linewidth',2,'color','r')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('displacement (m/V)')
legend('Analytical')
title('FRF-SS Smart beam (test 5)')
```

```
%% sensorização decoupled
```

```
sensing=zeros(1,length(wi));
sensing2=zeros(1,length(wi));
```

```
ct=e31x*hb*2*bb*Zr;
```

```
for r=1:modos
```

```
    Zr=(2*hp/(Aelec*dielecc33x))
    ms=sin(r*pi/l*x);
```

```

dmt(r,:)=-(Cn*(pi*r/l)*cos(r*pi/l*xR)-Cn*(pi*r/l)*cos(r*pi/l*xL))
wr=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4));
Cre(r,:)=((Cn*(pi*r/l)*cos(r*pi/l*xR)-Cn*(pi*r/l)*cos(r*pi/l*xL)))^2;

```

```

SENSING2(r,:)=ct*((Cn*ms))*dmt(r)./((wr^2-w_rad.^2)+(1j*sqrt(2*psi*wr*w_rad))-(Cre(r)*bp*ci*cv*Zr));

```

```

sensing2(1,:)=sensing2(1,:)+SENSING2(r,:);

```

```

end

```

```

figure(80)
plot(wi,abs(sensing2),'r')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('electrical Potential (V/N)')
title('SS Sensing')

```

```

%% Resonant shunt

```

```

%% shunt

```

```

% coupling factor

```

```

K31=0.38;

```

```

Cpp=1/Zr;

```

```

% generalized electromechanical coupling coefficient

```

```

auxgecc=(3*hp*(hb)/l*hb^2)*(c11x/E)*((K31^2)/(1-K31^2));

```

```

for r=1:modos

```

```

Ngecc=-((Cn*pi^2*r^2*sin((pi*r*xR)/l))/l^2 ...

```

```

-(Cn*pi^2*r^2*sin((pi*r*xL)/l))/l^2).^2;

```

```

wr=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4));

```

```

Dgecc=@(x)((pi*Cn*r*cos((pi*r*x)/l))/l).^2;

```

```

intDgecc=quadgk(Dgecc,0,l);

```

```

% GECC(r,:)=((Ngecc/intDgecc)*auxgecc);

```

```

GECC(r,:)=(61.8-51.21)/51.21;

```

```

DOPT(r,:)=sqrt(1+GECC(r));

```

```

RHOPT(r,:)=sqrt(2)*(sqrt(GECC(r))/(1+GECC(r)));

```

```

L(r,:)=Zr/(((wr^2)*(1+GECC(r))));

```

```

R0(r,:)=((Zr*sqrt(2))/((wr)))*(sqrt(GECC(r))/(1+GECC(r)));

```

end

indx=1;

Zel=(1j*w_rad*R0(indx)-(w_rad.^2)*L(indx));

Zel2=Zel./((-Cpp*L(indx)*w_rad.^2)+(1j*Cpp*R0(indx).*w_rad)+1);

ZelR=(1j*w_rad*R0(indx))./((1j*Cpp*R0(indx).*w_rad)+1);

for r=1:modos

ms=sin(r*pi/l*x);

% formas naturais

Cre(r,:)=((Cn*(pi*r/l)*cos(r*pi/l*xR)-Cn*(pi*r/l)*cos(r*pi/l*xL)))^2;

wr=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4)); % frequências de

ressonância

FRESSONANTSHUNT(r,:)=(Cn*ms)^2./((wr.^2-w_rad.^2)+1j*(2*psi*wr*w_rad)-(Cre(r)*bp*ci*cv*Zel2));

FRESISTIVESHUNT(r,:)=(Cn*ms)^2./((wr.^2-w_rad.^2)+(1j*(2*psi*wr*w_rad)-(Cre(r)*bp*ci*cv*Zel2)));

end

FRFSHUNT=zeros(1,length(wi));

for r=1:modos

FRFSHUNT(1,:)=FRFSHUNT(1,:)+FRESSONANTSHUNT(r,:);

end

% RESSONANT SHUNT

figure(6)

plot(wi,(abs(FRFSHUNT)), 'linewidth',2, 'color', 'r')

set(gca, 'yscale', 'log')

hold on

plot(wi,(abs(FRF_total)), 'b')

set(gca, 'yscale', 'log')

xlabel('\omega (Hz)')

ylabel('displacement (m)')

legend('third case')

title(' Shunt damping -3rd mode')

%% Piezo shape -- modal tranducers

%% electromechanical coupling factor

cv=2*e31x*hb;

```
ci=e31x*hb*2*bb;
```

```
% modal shape
```

```
mode_index=1;
```

```
msmodal=Cn*sin(mode_index*pi/l*x);
```

```
% 1st diff
```

```
dmsmodal=(pi*Cn*mode_index*cos((pi*mode_index*x)/l))/l;
```

```
% 2nd diff
```

```
ddmsmodal=-(Cn*pi^2*mode_index^2*sin((pi*mode_index*x)/l))/l^2;
```

```
% aux normalization calc.
```

```
x1=0:0.001:0.3;
```

```
auxNorm=-(max(abs(Cn*pi^2*mode_index^2*sin((pi*mode_index*l/2)/l))/l^2));
```

```
% l/2 (1st, 3rd, 5th mode) l/4(2nd mode) l/8(4th mode)
```

```
B=bb/auxNorm; % normalization factor
```

```
x1=0:0.001:0.3;
```

```
shape=-(Cn*pi^2*mode_index^2*sin((pi*mode_index*x1)/l/2))/l^2*B;
```

```
figure(8)
```

```
plot(x1,shape,'linewidth',2)
```

```
hold on
```

```
plot(x1,-shape,'linewidth',2)
```

```
xlabel('length (m)')
```

```
ylabel('width (m)')
```

```
legend ('1 mode')
```

```
title('Modal piezoelectric transducer shape')
```

```
shape0=Cn*sin(mode_index*pi/l*x1);
```

```
orgshape=0*x1;
```

```
figure(9)
```

```
plot(x1,shape0,'linewidth',2)
```

```
hold on
```

```
plot(x1,orgshape,'linewidth',3,'color','k')
```

```
legend (' 3nd mode' )
```

```
%% modal capacitance
```

```
AUXCPS=@(z)-(Cn*pi^2*mode_index^2*sin((pi*mode_index*z)/l))/l^2;
```

```
int=quadgk(AUXCPS,0,l);
```

```
Cps=(dielecc33x*bb/hp)*(E*I/vec_wr(mode_index)^2)*int;
```

```
Zrm=1/Cps;
```

```

alphabeta=int^1;
cvnode=-cv;
cinode=-ci;

FRFMpiezo=zeros(modos,length(wi));
for r=1:modos
    ms=sin(r*pi/l*x); % formas naturais
    Cre(r,:)=((Cn*(pi*r/l)*cos(r*pi/l*xR)-Cn*(pi*r/l)*cos(r*pi/l*xL)))^2;
    wr=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4)); % frequências de ressonância
    FRFMpiezo(r,:)=(Cn*ms)^2./((wr^2-w_rad.^2)+(1j*(2*psi*wr*w_rad)))+(alphabeta*bp*ci*cv*Zrm);
    ms1=sin(r*pi/l*0.15);
    ms2=sin(r*pi/l*(x-0.15));
    FRFMpiezo1(r,:)=(Cn*ms1)^2./((wr^2-w_rad.^2)+(1j*(2*psi*wr*w_rad)))+(alphabeta*bp*ci*cv*Zrm);
    FRFMpiezo2(r,:)=(Cn*ms2)^2./((wr^2-w_rad.^2)+(1j*(2*psi*wr*w_rad)))+(alphabeta*bp*cinode*cvnode*Zrm);
end

FRFMpiezot=zeros(1,length(wi));
for r=1:modos
    FRFMpiezot(1,:)=FRFMpiezot(1,:)+FRFMpiezo(r,:);
end
FRFMpiezot2=zeros(1,length(wi));
for r=1:modos
    FRFMpiezot2(1,:)=FRFMpiezot2(1,:)+FRFMpiezo1(r,:)+FRFMpiezo2(r,:);
end

figure(10)
plot(wi,(abs(FRFMpiezot)),'--','color','r','linewidth',2)
set(gca,'yscale','log')
hold on
plot(wi,(abs(FRF_total)),'b')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('total displacement (m)')
legend('Beam w/ modal transucer', 'Beam')
title('FRF-SS beam (modal transducer)')

figure(12)
plot(wi,(abs(FRFMpiezot2)),'--','color','r','linewidth',2)
set(gca,'yscale','log')
hold on

```

```

plot(wi, (abs(FRF_total)), 'b')
set(gca, 'yscale', 'log')
xlabel('\omega (Hz)')
ylabel('displacement (m/N)')
legend('Beam w/ modal transducer', 'Beam')
title('FRF-SS beam (2nd mode - inverted polarization)')
%% sensing behavior
for r=1:modos
wr(r, :)=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4))
der=(pi*Cn*mode_index*cos((pi*mode_index*xR)/l))/l ...
    -(pi*Cn*mode_index*cos((pi*mode_index*xL)/l))/l;
msmod=sin(mode_index*pi/l*x);
SENSINGMODAL(r, :)=(Zrm*e31x*hb*2*bb/auxNorm)*der*(Cn*msmod)./((321^2-
w_rad.^2)...
    +(1j*(2*qsi*321*w_rad))+(alphabeta*bp*ci*cv*Zrm));
end

```

```

figure(14)
plot(wi, abs(SENSINGMODAL), 'r', 'linewidth', 2)
set(gca, 'yscale', 'log')
hold on
xlabel('\omega (Hz)')
ylabel('electrical Potential (V/N)')
% legend( 'Voltage per unit of Force')
title('SS Sensing')

```

```
%% SHUNT
```

```
% generalized electromechanical coupling coefficient
auxgecc=(3*hp*(hb)/l*hb^2)*(c11x/E)*((K31^2)/(1-K31^2));
```

```

Dgeccmodal=@(x)-(Cn*pi^2*mode_index^2*sin((pi*mode_index*x)/l))/l^2;
intDgeccmodal=quadgk(Dgeccmodal, 0, l);
GECCmodal=(auxNorm/(B*bb*intDgecc))*auxgecc;
DOPTmodal=sqrt(1+GECCmodal);
RHOPTmodal=sqrt(2)*(sqrt(GECCmodal)/(1+GECCmodal));

```

```

modeindex=mode_index
GECCmodal=(61.8-51.21)/61.8;

```

```

LM=abs((1/((Cps*(vec_wr(modeindex).^2)*(1+GECCmodal))))))
ROM=((Zrm*sqrt(2))/((vec_wr(modeindex))))*(sqrt(GECCmodal)/

```

```
(1+GECCmodal));
```

```
% LM=46;
```

```
% ROM=9.53e3;
```

```
Zelmodal=(1j*w_rad.*ROM-(w_rad.^2).*LM);
```

```
Zel2modal=Zelmodal./((-Cpp*LM*w_rad.^2)+(1j*Cpp*ROM.*w_rad)+1);
```

```
ZelRmodal=(1j*w_rad*R0(indx))./((1j*Cpp*R0(indx).*w_rad)+1);
```

```
for r=1:modos
```

```
ms=sin(r*pi/l*x);
```

```
% formas naturais
```

```
Cre(r,:)=((Cn*(pi*r/l)*cos(r*pi/l*xR)-Cn*(pi*r/l)*cos(r*pi/l*xL)))^2;
```

```
wr=((r*pi)^2).*sqrt((E*I)/(rho*A*l^4)); % frequências de ressonância
```

```
FRFPiezoMSHUNT(r,:)=(Cn*ms)^2./((wr^2-w_rad.^2)+(1j*(2*psi*wr*w_rad)...  
+(cv*ci*bb*Zel2modal*alphabeta));
```

```
end
```

```
FRFSHUNTM(1,:)=zeros(1,length(wi));
```

```
for r=1:modos
```

```
FRFSHUNTM(1,:)=FRFSHUNTM(1,:)+FRFPiezoMSHUNT(r,:);
```

```
end
```

```
% RESSONANT SHUNT
```

```
figure(11)
```

```
plot(wi,(abs(FRFSHUNTM)), 'linewidth',3, 'color','r')
```

```
set(gca, 'yscale', 'log')
```

```
hold on
```

```
plot(wi,(abs(FRF_total)), 'b')
```

```
set(gca, 'yscale', 'log')
```

```
xlabel('\omega (Hz)')
```

```
ylabel('displacement (m)')
```

```
legend( 'first modal shape - SS beam')
```

```
title(' Shunt damping -1st mode')
```

```

save viga fem
%clc;close all;clear all;
%% load FEM struct
load viga;
fem.xmesh=mesheextend(fem);
[Null]=femstatic(fem,'out',{'Null'},'solcomp',{'u','v','w'});

%% mesh dofs and coord details considering bound cond (through 'Null')
nodesc = xmeshinfo(fem,'out','nodes','solcomp',{'u','v','w'},'null',↵
Null);
nodes = xmeshinfo(fem,'out','nodes','solcomp',{'u','v','w'});

figure(1)
meshplot(fem);
%% matrix assembling

% no constraints
K = assemble(fem,'out','K'); % STIFFNESS MATRIX
E = assemble(fem,'out','E'); % MASS MATRIX
L = assemble(fem,'out','L'); % MASS MATRIX

cdofs=[];
for in = 1:size(Null,1)
    if isempty(find(Null(in,:)==1))==1;
        cdofs(end+1)=in;
    end
end
Lc=L; Lc(cdofs)=[];

Kc=K; Kc(:,cdofs)=[];Kc(cdofs,:)=[];
Ec=E; Ec(:,cdofs)=[];Ec(cdofs,:)=[];
%% dofs & coords
Coords=nodesc.coords; % nodes coords
Dofs=nodesc.dofs; % dofs

%% model geometry

%lmax=max(abs(Coords(1,:))); %CFbeam
lmax=max(abs((2/9)*Coords(1,:))); %SSbeam
width=max(abs(Coords(2,:)));
hb=0.002;

%% frequency bandwidth

```

```

wii=0:1:600; %cf
%wii=0:1:600; %ss
w_rad=wii*(2*pi);

%% FRF

% eigenfrequency=sqrt(eig(full(Kc),full(Ec)))/(2*pi);
%% ELASTIC BEAM
%displacement per unit of force
for i =1:length(wii)
    Kdin=-((w_rad(i)^2)*Ec)+Kc;
    displacement=Kdin\Lc;
    nds=linspace(1,1,length(Coords)); nds=cumsum(nds);
    measurepoint=find(Coords(1,:)>lmax*.99 & Coords(1,:)↵
<lmax*1.01 );
    nds=nds(measurepoint);
    measurepoint2=find(Coords(3,measurepoint)>hb*.99 & Coords↵
(3,measurepoint)<hb*1.01 );
    nds=nds(measurepoint2);
    measurepoint3=find(Coords(2,nds)>width*.99 & Coords(2,nds)↵
<width*1.01 );
    nds=nds(measurepoint3);
    Coords(:,nds);
    displ1(i)=displacement(Dofs(3,nds));
end

figure(35)
plot(wii,abs(displ1))
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel('total displacement (m)')
legend('Beam', 'Beam+piezo (decoupled MS)')
title('FRF-CF Beam + Piezo (decoupled modes)')

```

```
save viga fem
%clc;close all;clear all;
%%
% João Miguel Gonçalves Ribeiro
% Em01095
% Passive vibration control
% via shunted Piezoelectric
% transducing technologies

%% Post processing fem program
%% load FEM struct

load viga;
fem.xmesh=mesheXtend(fem);
[Null]=femstatic(fem,'out',{'Null'},'solcomp',{'V','u','v','w'});

%% mesh dofs and coord details considering bound cond (through 'Null')
nodesc = xmeshinfo(fem,'out','nodes','solcomp',{'V','u','v','w'},'null',Null);
nodes = xmeshinfo(fem,'out','nodes','solcomp',{'V','u','v','w'});

figure(1)
meshplot(fem);
%% matrix assembling

K = assemble(fem,'out','K'); % STIFFNESS MATRIX
E = assemble(fem,'out','E'); % MASS MATRIX
L = assemble(fem,'out','L'); % MASS MATRIX

% constraints removal
cdofs=[];
for in = 1:size(Null,1)
    if isempty(find(Null(in,:)==1))==1;
        cdofs(end+1)=in;
    end
end

Lc=L; Lc(cdofs)=[];
Kc=K; Kc(:,cdofs)=[];Kc(cdofs,:)=[];
Ec=E; Ec(:,cdofs)=[];Ec(cdofs,:)=[];
%% dofs & coords
Coords=nodesc.coords; % nodes coords
Dofs=nodesc.dofs; % dofs
```

```
%% frequency bandwidth
```

```
w=0:1:400; %cf
%w=0:1:600; %ss
w_rad=w*(2*pi);
```

```
%% geometry analysis
```

```
width=max(abs(Coords(2,:)));
hpu=max(abs(Coords(3,:)));
hb=0.002;
hpl=hb+min(abs(Coords(3,:))); %CF
%lmax=max(abs(Coords(1,:))); %SS
lmax=max(abs((2/9)*Coords(1,:)));
disp(lmax);
```

```
%% SMART BEAM - EBC DOFs grouping
```

```
ele_elecUP=find(Coords(3,*)>hpu*.99 & Coords(3,*)<hpu*1.01 );
ele_elecLO=find(Coords(3,*)>hb*.99 & Coords(3,*)<hb*1.01 );
ele_elecMID=find(Coords(3,*)>hb+((hpu-hpl)/2)*.99 & Coords(3,*)<hb+((hpu-hpl)/2)*1.01 );
```

```
%
```

```
% Upper electrode DOFs
```

```
for i=1:length(ele_elecUP)
    AUXX=ele_elecUP(i);
    Dofs_UP=Dofs(1,AUXX);
    if Dofs_UP~=0
        KhipphiUP(i,i)=Kc(Dofs_UP,Dofs_UP);
        kc_aux = Kc(:,Dofs_UP); kc_aux(Dofs(1,find(Dofs(1,:)~=0))) = [];
        KuphiA(:,i)=kc_aux;
    end
end
```

```
end
```

```
KphiuA=KuphiA.';
```

```
%% Lower electrode DOFs
```

```
for i=1:length(ele_elecLO)
    aux2=ele_elecLO(i);
    Dofs_LO=Dofs(1,aux2);
    if Dofs_LO~=0
        KhipphiLO(i,i)=Kc(Dofs_LO,Dofs_LO);
        kc_aux = Kc(:,Dofs_LO); kc_aux(Dofs(1,:)) = [];
        KuphiS(:,i)=kc_aux;
```

```

    else
        KhipphiLO=0;
        KuphiS=0;
    end
end
KphiuS=KuphiS.';

%% Mid DOFs

for i=1:length(ele_elecMID)
    aux3=ele_elecMID(i);
    Dofs_MID=Dofs(1,aux3);
    if Dofs_MID~=0
        KhipphiMID(i,i)=Kc(Dofs_MID,Dofs_MID);
        kc_aux = Kc(:,Dofs_MID);
        kc_aux(Dofs(1,find(Dofs(1,:)~=0))) = [];
        KuphiMID(:,i)=kc_aux;
    else
        KhipphiMID=0;
        KuphiMID=0;
    end
end
KphiuMID=KuphiMID.';
KhipphiMID_i=KhipphiMID^-1;

%% electric DOFs elimination from mechanical matrices
Dofs_aux=find(Dofs(1,:)~=0);
Kc(Dofs(1,Dofs_aux),:) = []; Ec(Dofs(1,Dofs_aux),:) = [];
Kc(:,Dofs(1,Dofs_aux)) = []; Ec(:,Dofs(1,Dofs_aux)) = [];
Lc(Dofs(1,Dofs_aux),:) = [];

%% MEMORY CLEANING
clear K
clear E
clear L
clear Null

%% electric boundary conditions
EBC='OC'
switch EBC
    case 'CC'

        % closed circuit stiffness
%
        Kuuast=Kc

```

```

Kuuast=Kc-KuphiMID*KhipphiMID_i*KphiuMID;
% eigenfrequency=sqrt(eig(full(Kuuast),full(Ec)))/(2*pi);
for i =1:length(w)
    Kdin=(-((w_rad(i)^2)*Ec)+Kuuast);

    % measure point
    nds=linspace(1,1,length(Coords)); nds=cumsum(nds);
    measurepoint=find(Coords(1,:)>lmax*.99 & Coords(1,:)↵
<lmax*1.01 );
    nds=nds(measurepoint);
    measurepoint2=find(Coords(3,measurepoint)>hb*.99 & Coords↵
(3,measurepoint)<hb*1.01 );
    nds=nds(measurepoint2);
    measurepoint3=find(Coords(2,nds)>width*.99 & Coords(2,nds)↵
<width*1.01 );
    nds=nds(measurepoint3);
    Coords(:,nds);
    % displacement per unit of load
    displacement=Kdin\Lc;
    displ(i)=displacement(199); %continuum CF
    %Tdispl(i)=displacement(Dofs(4,nds));
    if rem(i,50)==0
        disp([num2str(i), '/', num2str(length(w))])
    end

end

figure(20)
plot(w,abs(displ))
set(gca, 'yscale', 'log')

case 'OC'

% measure point
nds=linspace(1,1,length(Coords)); nds=cumsum(nds);
measurepoint=find(Coords(1,:)>lmax*.99 & Coords(1,:)↵
<lmax*1.01 );
nds=nds(measurepoint);
measurepoint2=find(Coords(3,measurepoint)>hb*.99 & Coords(3,↵
measurepoint)<hb*1.01 );
nds=nds(measurepoint2);
measurepoint3=find(Coords(2,nds)>width*.99 & Coords(2,nds)↵
<width*1.01 );
nds=nds(measurepoint3);
Coords(:,nds);

for i =1:length(w)

```

```

KUPHI=KuphiMID+KuphiA;
KPHIPHI=KhipphiMID+KhipphiUP;
KPHIPHII=KPHIPHI^-1;
KPHIU=KUPHI.';
KOC=KUPHI*KPHIPHII*KPHIU;
Kuuast=(Kc-KuphiMID*KhipphiMID_i*KphiuMID-KOC);
Kdin=-((w_rad(i)^2)*Ec)+Kuuast;
eigenfrequency=sqrt(eig(full(Kuuast),full(Ec)))/(2*pi);
% displacement per unit of load
displacement=Kdin\Lc;
displ(i)=displacement(Dofs(4,nds));
%   displ(i)=displacement(199); % continuum piezo cf

% voltage per unit of load

VF=inv(KhipphiUP)*KphiuA*displacement;
    if rem(i,50)==0
        disp([num2str(i), '/', num2str(length(w))])
    end
VFm(i,:)=mean(VF);
VFp(i,:)=max(abs(VF));

    phiv=ones(length(KPHIPHI));
    EA=KuphiA*phiv;
    displacementEA=Kdin\EA;
    displEA(i)=displacementEA(Dofs(4,nds));
%   displEA(i)=displacementEA(199);

end

figure(20)
plot(w,abs(displ))
set(gca,'yscale','log')
figure(21)
plot(w,abs(VFm),w,abs(VFp),'r')
set(gca,'yscale','log')
xlabel('\omega (Hz)')
ylabel(' V/N')
legend('Equi-Potential', 'Peaks Potential')
title('FRF-SS Smart modal beam ')
figure(22)
plot(w,abs(displEA))
set(gca,'yscale','log')

```

```
xlabel('\omega (Hz)')
ylabel(' m/V')
legend('Numerical')
title('FRF-SS Smart Beam (test 5) ')
```

```
case 'shunt'
    KPHIU=KphiuMID+KphiuA;
    KPHIPHI=KhipphiMID+KhipphiUP;

    R=1; L=200;
    Kshunt=zeros(length(KPHIPHI),length(KPHIPHI));
    w=0:1:500;
    for i=1:length(w)
        for u=1:length(KPHIPHI)

            Kshunt(i,i)=1/(R+(1j*w(i)*L));
            K_AUX=Kshunt+KPHIPHI;
            KSHUNT=(K_AUX^-1)*KPHIU;
        end
    end

otherwise

end
```