



**CONTRIBUTIONS ON
REAL OPTIONS AGENCY THEORY**

por

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Nota Biográfica

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Sumário

Os desenvolvimentos na área das Opções Reais frequentemente assumem que uma oportunidade de investimento não apresenta problemas de agência, considerando-se que estas ou são geridas pelos seus detentores ou, caso haja um gestor, este se encontra perfeitamente alinhado com os interesses dos donos da opção. Este pressuposto pode efectivamente ter um grande impacto nas decisões de maximização de valor, nomeadamente, no momento ótimo de investimento. Esta dissertação contribui para a literatura que abrange as questões de agência no âmbito da Teoria das Opções Reais.

A presente dissertação apresenta dois modelos, para oportunidades de investimento exclusivas e oportunidades de investimento partilhadas, incorporando a problemática dos problemas de agência, especificamente quando os accionistas de uma empresa contratam um gestor para supervisionar a opção de investimento e tomar a decisão de investimento. Recorrendo à definição contratual adequada, este modelo incorpora os problemas de agência, criando as condições para o alinhamento de interesses através da construção de um contrato ótimo.

Em ambos os modelos monopolista e duopolista, os accionistas não necessitarão de acompanhar a evolução dos drivers do valor do projeto para garantir o comportamento ótimo do gestor. Um esquema salarial que, de forma adequada, incorpore o custo de oportunidade relativo do gestor e dos accionistas proporciona ex-ante o incentivo que assegura o alinhamento dos interesses. Demonstra-se também que pequenos desvios da combinação salarial ótima motiva grandes desvios das decisões face ao resultado ótimo.

Adicionalmente, os modelos serão aplicados a um caso numérico sendo que, no caso do modelo monopolista, serão também desenvolvidas três extensões relativas existência de impaciência do gestor, à presença de concorrência não declarada e considerando a existência de custos de esforço do gestor.

Abstract

Real options literature commonly assumes that, either the investment opportunity is directly managed by the owners, or the managers are perfectly aligned with them. However, agency conflicts occur and managers reveal interests not totally in line with those of the shareholders. This may have a major impact on the value maximizing decisions, namely, on the optimal timing to invest. This dissertation contributes to the literature that accounts for agency problems on the exercise of real options.

The present dissertation provides a framework for both exclusive and non-exclusive investment opportunity, which comprises the problematic of agency issue, specifically, when the shareholders of a firm need to hire a manager to supervise the investment option and to take the adequate decision. Using a proper contract design, the model can easily encompass agency issues, creating the conditions to interest alignment, through the definition of an optimal contract mix.

In both the cases of monopolistic and duopolistic models, shareholders need not to follow the future evolution of project value drivers in order to guarantee optimal behavior. Therefore, an adequate compensation scheme that correctly embodies the relative opportunity cost of manager and shareholders provides the necessary ex-ante incentive that ensures interest alignment. It is also shown that even small deviations from the optimal compensation scheme may lead to highly sub-optimal decisions. Furthermore, the model is applied using a numerical example and, in the monopolistic investment option, special situations are analyzed, namely considering the presence of impatient managers, non-proprietary/hidden competition real options, and also by considering the existence of effort costs for the manager.

CHAPTER 1

Introduction

Investment decisions are taken with the primary purpose of future compensations over a present investment cost. These compensations, which may not always be a monetary benefit, are supposed to presently overcome the initial investment, incrementing the investors value. Nonetheless, this basic condition is not plainly transposed, from the theoretical context, in which financial theory is developed, to the reality that surrounds our effective decisions since we are simultaneously conditioned by exogenous factors that act, sometimes, in unpredictable ways. We may refer time, space, information and opponents set of possible actions as some of these exogenous dimensions that condition real investments.

Consequently, the existence of such multi-dimensional context surrounding the majority of investment decisions confluges to three fundamental characteristics. The first, concerns the fact that investments cannot be fully reversed to the initial state implying though that the initial investment cost is at least partially sunk. Second, the existence of uncertainty regarding the future, constrains the future cash-flows to a probability estimated result. Third, investment decisions can be delayed for future information gathering so that the investment decision may be more informed.

Classic investment valuation theory, sustained on neo-classic theory principles, often ignored these elements originating imperfect frameworks and leading, most of the times, to sub-optimal investment decisions. In recent decades, literature has improvingly considered that the ability to postpone an investment decision, similar to a call option over an investment, has an implicit value that must be considered in the investment opportunity valuation. This is not the underlying perspective on classic investment theory where investment decisions are taken whenever the present value of cash-flows justifies the investment cost which, therefore, ignores the value of the right but not the obligation to invest.

Real Options framework emerged as a solution, allowing investment valuation theory to embody uncertainty, delaying option, and irreversibility, providing improved solutions to general real life valuation problems and to specific situations - such as research and development or mining investments which, due to their specific characteristics could not be valued by basic approaches without significant loss of quality. Evolution in the last

two decades resulted in several developments, not only in deeper and improved modeling but also in bonding relations to several economic and financial fields such as game theory, industrial economics, environmental economics, public finance or agency theory.

The intersection with game theory is, perhaps, the most important link of real options theory since it allowed, in the simplest manner to extend investment valuation to a non-exclusive investment opportunity context. This field, later called Real Options Game theory, provided not only a theoretical support to study the interactions between utility maximizing players but also, in some cases, a more realistic approach of investment valuation in real life competitive environments.

Nonetheless, most of the theory surrounding real options fails to embody the impact of some economic relations, which may have a significant impact in optimal decisions. Particularly, the common assumptions that an investment opportunity and the subsequent investment decision are fully controlled by the options owners or that, if there is a managerial agent, he is completely aligned with the owner are, usually, too strong and inconsistent with reality. Moreover, investment opportunity owners may maximize their utility by delegating the investment decision to a managerial agent. This brings investment valuation theory to a new level of, whether or not, the dynamics of contract design impact on investment opportunity control and effective implementation decisions.

Agency relation, by definition, consists in a contractual relation between a principal entity and an agent where the latter has legal authority to act for the first. The issue surrounding this kind of contracts consists on the implications of incomplete and asymmetric information on the utility maximizing process of the agent, whom may not act aligned with the principals interests, implying the presence of moral hazard and conflict of interests. Although agency issue is commonly related to the shareholders-manager or equity and debt owners relations, it can be extended to an infinite types of relationship, for instance, a marital relation.

The problematic of agency issues and its inefficiency consequences are essentially related to the misaligning between risk bearing, decision control and utility maximizing, so that decisions that optimally maximize an agents utility may not maximize the value of the principal because their contractual relation do not ensure solidarity on risk bearing. This is mainly the problematic of optimal contract design which in practice may have some implementation difficulties such as legal restriction or operational difficulties - justifying why supervision, sometimes, may be a better practical solution. Although, in theoretical contexts, optimal contract has an invaluable role in the interests alignment between principal and agent.

In recent years, a new breed of works initiated a path where real options theory intersects agency theory. This new branch which, without lose of modesty, we call Real Options Agency Theory presents two paths, that may be common sometimes. The first uses real options framework and methodology to analyze agency relations, further analyzing the options detained by both parts. The second tries to introduce agency problem in investment opportunity valuation tools, recalibrating models. In both fields, contract design theory and control mechanisms are frequently used to minimize these problems.

This dissertation aims to contribute to this field by considering that the control over an investment opportunity and the subsequent investment decision is delegated to a manager whom, by acting in his own interest, may generate sub-optimal decisions for the owners. Consequently, we will drop the usual assumption of absence of agency issues in the case of an exclusive investment opportunity (first essay) and in the non-exclusive situation (second essay). Further, we design a contract that minimizes agency issues and allows to absolute interest alignment without the need of supervision by the owners.

CHAPTER 2

Exclusive Investment Opportunity under Agency Context

1. Introduction

In the recent decades, real options framework has become one of the fundamental foundations of investment evaluation. Laying roots in the options pricing theory of the seminal work of Black and Scholes (1973), it was coined by Myers (1977) in a reference of the growth opportunity on corporate assets. Later, Myers (1984) defended the relevance of real options to strengthen the interaction between finance and strategy in corporate environment, which Kester (1984) reiterated in the same year. Developments rapidly began in capital budgeting area, with groundwork papers such as Brennan and Schwartz (1985) where an investment opportunity was evaluated under a real options framework comprising commodity price uncertainty, and McDonald and Siegel (1986) who considers uncertainty in both projects cash-flows and costs. Other relevant works appeared since then, extending the concept and its applicability, and bridging with various fields. Find two excellent surveys in Dixit and Pindyck (1994) and Trigeorgis (1996).

In financial theory, optimal investment decisions are taken under the premise of firms market value maximization (Jensen (2001)). Usually, in an all-equity firm, value maximization is not deteriorated by inside determinants when owners have full control of their endogenous variables. When some undesirable factors arise, for instance high ownership dispersion (Berle and Means (1932)) or owners lack of expertise (Shleifer and Vishny (1997)), the shift of control decision to an exogenous entity (agent) is an inevitability which naturally raises the probability of value destruction if there is a misalign of targets between manager and owner. This is the preeminent agency dilemma, formally established and generalized in the seminal paper of Jensen and Meckling (1976), but which first academic observation can be found two hundred years earlier in Smith (1976).

In order to reduce agency conflict, literature proposes internal and external mechanisms. The first category comprises incentive contracts (Jensen and Smith Jr (1985)), insider ownership (Jensen and Meckling (1976)), existence of large investors (Shleifer and Vishny (1986)), board of administration (Fama and Jensen (1983)), the existence of debt and dividend policy, which reduces free cash-flows (Jensen (1986)). External mechanisms encompasses, for instance, managers reputation (Shleifer and Vishny (1997)), managers

market competition (Fama (1980)), output market competition (Hart (1983)), takeovers market (Easterbrook and Fischel (1981)), monitoring by investment professionals (Chung and Jo (1996)) or legal framework (Shleifer and Vishny (1997)). Despite its relevance, these mechanisms have limitations, which several works have shown. We highlight Holmstrom (1982), Shleifer and Vishny (1997), Grossman and Hart (1980), Burkart (1995), Jensen (1993).

Ideally, shareholders would not need to have full information and control over manager if he defines a proper framework of pecuniary incentives, so that risk bearing and risk premium are shared with manager. Nevertheless, as Shleifer and Vishny (1997) notes, incentive contracts can create opportunities for self-dealing under contract negotiation inefficiency which will lead to misappropriation of firm's value to manager.

Investment timing decisions, when studied under a real options approach, usually tend to assume perfect aligning of interests between managers and shareholders, ignoring the impact of agency conflicts. Recently, this issue has kept the attention of some authors, generating bridging papers such as Grenadier and Wang (2005), which examines investment timing decision for a single project, where the owner delegates the investment decision to the manager. Manager behavior will account for asymmetric information and moral hazard, generating sub-optimal decisions that can be corrected through an optimal contract, aligning the incentives of owners and managers, and Nishihara and Shibata (2008) extends this model incorporating a relationship between an audit mechanism and bonus-incentives sensitive to managers deviated actions. Shibata and Nishihara (2010) extends these works by incorporating debt financing on investment expenditure.

Hori and Osano (2010) presents an agency model under a real options framework where managerial compensation is designed endogenously including a contingent claim on firms cash-flows using stock options.

Our primary purpose will be to relax the assumption that managers are perfectly aligned with owners (as assumed in the standard real options literature, e.g. Dixit and Pindyck (1994)) and to provide a perceptive but yet meaningful framework where a principal entity (a shareholder or a group of shareholders) owns an option to invest, but for plausible reasons (i.e., incapacity, opportunity cost or control difficulty) need to hire a manager to supervise the option, to follow market conditions, and to take the investment decision. In order to avoid inadequate actions, a contract structure is defined, using a contingent element based on projects cash-flows, which optimal solution will maximize shareholders value, while transferring decision process to manager.

This work differs from closed related literature, namely Grenadier and Wang (2005) and Hori and Osano (2010), by considering a compensation contract where: (1) the variable component is strictly contingent on the critical value and not through a stock options scheme; (2) the fixed component is continuous over time; but (3) this continuous fixed component differs whether manager is administering the option or running the active project. We distinguish from these authors also by including a pre-exercising management continuous wage.

The rest of the paper is organized as follows. Section 2.1 presents the basic framework where shareholder's optimal investment strategy and manager's optimal solution are derived, considering the incentives contract. Section 2.2 sets the equilibrium solution that aligns the managers interests with those of shareholders. Section 3 presents a comparative statics analysis and a numerical example. Section 4 sets the optimal solutions for some particular situations, namely, for impatient managers, for non-proprietary investment opportunities, and to account for the existence of effort costs for the manager. Section 5 concludes.

2. Model

In this section we derive the model, presenting the assumption, the main steps, and settling the optimal equilibrium for the compensation scheme. This optimal equilibrium will be compared with results one would obtain for a project directly managed by the owners.

2.1. Setup. A firm has an option to invest in a single project. The shareholders decide to hire a manager for running this investment opportunity; the agent will follow the market conditions and take the investment decision. The decision for professional management arises from restrictions that constrict owner's own actions. We assume that shareholders want to maximize their project value, although, they are limited by their own conditions such as lack of specific know-how, equity structure or simply a matter of opportunity cost, which will lead them to hire an agent to manage the option and to take optimal investment timing decision.

Since both stockholders and managers are rational players and utility maximizers, an issue of asymmetric information and control asymmetry (similar to hidden action of Grenadier and Wang (2005)) arises. In a such a context, the owners incapacity in controlling manager's effort and actions, implies that they can't fully control manager engagement. Therefore, stockholders won't be able to identify manager's decisions ex ante neither will be able to reset manager's actions after contractual establishment implicating

that owners must properly design the contract before delegate project's control to manager.

In order to achieve our purpose we will use the standard contingent claims approach, as defined in Dixit and Pindyck (1994). Therefore, we start to define the present value of cash flows as variable $V(t)$ ¹, which follows a geometric Brownian motion (gBm) so that:

$$(2.1) \quad dV(t) = \alpha V(t)dt + \sigma V(t)dz$$

where $V(0) > 0$, dz is the increment of the Wiener process, α is the instantaneous conditional expected relative change in V , also known as drift. $\alpha = r - \delta$ ($r > \delta$), where r is the risk-free rate and δ (> 0) corresponds to the opportunity cost from deferring, and σ is the instantaneous conditional standard deviation. Additionally, shareholders and managers are assumed to be risk neutral players.

Similarly to a call option configuration, shareholders will pay an investment cost K_s (that after spent will be perceived to owners as a sunk cost) and, since manager gets a salary, owners have an additional wage cost.

This wage comprehends two different states. At first, manager will earn an option management fee w_i , corresponding to a continuous fixed wage for managing the idle project, i.e., while he watches market conditions and wait for the appropriate investment moment. While realistic, this is ignored by the related literature. Grenadier and Wang (2005) assumes, implicitly, the manager works for free prior exercising the options, and Hori and Osano (2010) considers a fixed global payment for the manager, which is independent from the time the project remains idle.

After exercising the option, the manager will earn a fixed continuous wage (w_a) plus a value-sharing bonus (ϕV) depending on the value of project cash flows. Note that ϕV is same as the present value of a portion ϕ of each annual cash flow.

We define some assumptions concerning the labor market and manager's inflows, as follows:

Assumption 1: *Owners can't administer directly the option to invest. Also they are unable of properly observe some key value drivers (namely, V , σ , α), so the option becomes useless without a manager.*

Assumption 2: *Managers and options to invest aren't scarce, so that owners can always find a manager for running his projects, and managers can always find another investment opportunity needing to be managed.*

¹For convenience, in the remainder of this paper, we drop the reference to time and simply represent $V(t)$ as V .

Assumption 3: The parameter w_i represents the managers market price for running an idle project (meaning that the owners can't find a less expensive manager). Also, we assume that the fixed wage to manage the active project, w_a , is lower than w_i , so that manager's utility function integrates awaiting value. Note, however, that the lower fixed salary will be compensated by an appropriate value-sharing bonus.

Assumption 4: Manager can only broke contract before option exercise and this only happens if the option becomes worthless. In this case manager will earn a fixed compensation $\frac{w_i}{r}$.

Assumption 5: After establishing the compensation scheme, renegotiations are not allowed.

Assumption 6: For the owners, value-sharing bonus are less expensive than monitoring costs.

Assumption 7: Managers are not wealth enough neither can gather the necessary resources (leverage, for instance) to buy the project.

Shareholders will pay a wage to manager whether he exercises the option or not, so that their option value will have to consider that component. Setting $S(V)$ as shareholders' option value, V_s as theirs optimal exercising value (the trigger), then we have the following ordinary differential equation (o.d.e.):

$$(2.2) \quad \frac{1}{2}\sigma^2V^2S''(V) + (r - \delta)V S'(V) - rS(V) - w_i = 0$$

$S(V)$ must satisfy the appropriate boundary conditions that ensure that shareholders will choose the optimal investment decision. Therefore, we have the following conditions, which the first is the value matching condition, the second is the smooth-pasting condition and the third is an absorption barrier:

$$(2.3) \quad S(V_s) = V_s - K_s - \left(\frac{w_a}{r} + \phi V_s\right)$$

$$(2.4) \quad S'(V_s) = 1 - \phi$$

$$(2.5) \quad S(0) = -\frac{w_i}{r}$$

The last condition arises from Assumption 4, which implies that, since manager is the only player with full information, he will choose to leave the firm receiving a compensation $\frac{w_i}{r}$. Note that if he does so, he won't earn more than the market equilibrium wage, since the additional bonus hypothesis dies, so for self-fulfillment reasons, manager will chase another projects. Additionally, this condition represents the sunk cost that shareholders have for maintaining the option alive.

The general solution consists of a homogenous component and a particular solution that, satisfying the last boundary condition, will take the form:

$$(2.6) \quad S(V) = AV^\beta - \frac{w_i}{r}$$

In order to find the owner's option value and the critical value V_s we use the remaining boundary conditions (2.3) and (2.4). Substituting and rearranging we have:

$$(2.7) \quad S(V) = \begin{cases} \left(\frac{V}{V_s}\right)^\beta \frac{1}{\beta-1} \left(K_s - \frac{w_i - w_a}{r}\right) - \frac{w_i}{r} & \text{for } V < V_s \\ (1-\phi)V - K_s - \frac{w_a}{r} & \text{for } V \geq V_s \end{cases}$$

and the trigger:

$$(2.8) \quad V_s = \frac{\beta}{\beta-1} \frac{1}{1-\phi} \left(K_s - \frac{w_i - w_a}{r}\right)$$

where:

$$(2.9) \quad \beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

Equation (2.7) is the value function for shareholders, however shareholders don't actually know their function's value drivers (Assumption 1) and, consequently, the concrete value will depend of manager's choice, which can possibly be misaligned with owner's optimal result. Since owners propose a partial contingent payment, a comparable option is implicitly given to manager though, inversely to shareholders, managers will earn a wage. So we need to estimate manager's value function $M(V)$, and the critical value V_m , by solving the following o.d.e.:

$$(2.10) \quad \frac{1}{2}\sigma^2 V^2 M''(V) + (r-\delta)VM'(V) - rM(V) + w_i = 0$$

respecting boundary conditions:

$$(2.11) \quad M(V_m) = \frac{w_a}{r} + \phi V_m$$

$$(2.12) \quad M'(V_m) = \phi$$

$$(2.13) \quad M(0) = \frac{w_i}{r}$$

As mentioned before, the first two conditions are fundamental to ensure that the optimal decision is taken and the last one is an absorption barrier related to V 's stochastic nature. The general solution of equation (2.10) is:

$$(2.14) \quad M(V) = AV^\beta + \frac{w_i}{r}$$

In which results the following solution for V_m and $M(V)$, where β is given by equation (2.9):

$$(2.15) \quad V_m = \frac{\beta}{\beta - 1} \frac{1}{\phi} \frac{w_i - w_a}{r}$$

and

$$(2.16) \quad M(V) = \begin{cases} \left(\frac{V}{V_m}\right)^\beta \frac{1}{\beta - 1} \frac{w_i - w_a}{r} + \frac{w_i}{r} & \text{for } V < V_m \\ \frac{w_a}{r} + \phi V & \text{for } V \geq V_m \end{cases}$$

2.2. Optimal wage settling. Since w_i is exogenous (Assumption 3), shareholders can only influence manager's decision using w_a and ϕ . By interacting w_a with w_i (with $w_a < w_i$) owners can define manager's opportunity cost for implementing the project, but they can only get manager's to truly align his target with theirs through ϕ so that residual claims are shared between players. In order to determine the optimal ϕ (i.e., ϕ^*), individual optimal decisions must be aligned, so that V_m must equal V_s , which results:

$$(2.17) \quad \phi^* = \frac{K_m}{K_s}$$

where:

$$(2.18) \quad K_m = \frac{w_i - w_a}{r}$$

Equation (2.17) shows that, in the presence of information asymmetry, value-sharing component enforces optimal decisions, which will only depend on the relation between the opportunity cost for the manager (K_m) and the opportunity cost for the shareholders (K_s). Therefore, possibly surprising, shareholders can build an optimal contract scheme ignoring the project key value drivers, and by only knowing K_s , w_i , w_a and r (which is consistent with Assumption 1).

Considering traditional model of investment under contingent claims, as it appears in Dixit and Pindyck (1994), optimal trigger is defined as:

$$(2.19) \quad V^* = \frac{\beta}{\beta - 1} K_s$$

Our model shows that no-agent critical value will be shared between owners and manager. This means that their share of value will equal no-agent critical value excluding the share demanded by agent under his terms. Substituting (2.8) and (2.15) into (2.19) and arranging we have:

$$(2.20) \quad V^* = (1 - \phi)V_s + \phi V_m$$

Additionally, under optimal contract definition (ϕ^*), the agent chooses V_m which equals V_s and V^* . Therefore, shareholders will ensure that manager's critical value is also V^* implying that no value is misappropriated, since manager will only have the sharing-value bonus that owners are willing to give him and that they find acceptable as a reward for manager's loyalty and activity.

3. Comparative Statics and Numerical Example

3.1. Comparative Statics. Recall ϕ^* as presented in equation (2.17), which is a function of w_i , w_a , r , and K_s . Taking the derivatives:

$$(2.21) \quad \frac{\partial \phi^*}{\partial w_i} > 0$$

$$(2.22) \quad \frac{\partial \phi^*}{\partial w_a} < 0$$

$$(2.23) \quad \frac{\partial \phi^*}{\partial r} < 0$$

$$(2.24) \quad \frac{\partial \phi^*}{\partial K} < 0$$

we find that the optimal value-sharing rate (ϕ^*) is positively related with the wage the manager receives prior the project implementation – the higher (lower) w_i , the higher (lower) the opportunity cost for the manager that comes from launching the project, so the higher (lower) the value-sharing that ensures optimal decision –, and negatively related with wage post implementation – the higher (lower) w_a , the lower (higher) the opportunity cost for the manager. ϕ is also negatively related with the interest-rate and the investment cost.

It is particularly relevant to observe the opposite effects that changes in ϕ produces on the optimal triggers V_s and V_m :

$$(2.25) \quad \frac{\partial V_s(\phi)}{\partial \phi} > 0$$

$$(2.26) \quad \frac{\partial V_m(\phi)}{\partial \phi} < 0$$

the higher (lower) the value shared with the manager the higher (lower) the shareholders' trigger, meaning later optimal investment for them, and, on contrary, the higher (lower) the values for ϕ the lower the manager's trigger, meaning sooner optimal investment. This is due to the opposite effect of ϕ on the value of the option to wait and defer the project implementation. Any deviation from critical ϕ creates misaligning between manager and the shareholders and potentially leads to a suboptimal decision taken by the manager, when the shareholders interests are concerned. As we can see from the numerical example below, even a small deviation from ϕ^* has a significant impact on the manager trigger, and, in the final, on the timing for the project implementation.

Additionally, by fixing w_i and w_a ($w_i > w_a$), we find that:

$$(2.27) \quad V_s \rightarrow \frac{\beta}{\beta-1} \left(K_s + \frac{w_a - w_i}{r} \right) < V^* \quad \text{as } \phi \rightarrow 0$$

$$(2.28) \quad V_m \rightarrow +\infty \quad \text{as } \phi \rightarrow 0$$

$$(2.29) \quad V_s \rightarrow +\infty \quad \text{as } \phi \rightarrow 1$$

$$(2.30) \quad V_m \rightarrow \frac{\beta}{\beta-1} \frac{w_i - w_a}{r} \quad \text{as } \phi \rightarrow 1$$

and by fixing ϕ , and remember that $\phi \in (0, 1)$, we see that:

$$(2.31) \quad V_s \rightarrow \frac{\beta}{\beta-1} \frac{1}{1-\phi} K_s > V^* \quad \text{as } w_a \rightarrow w_i$$

$$(2.32) \quad V_m \rightarrow 0 \quad \text{as } w_a \rightarrow w_i$$

Equations (2.27) and (2.28) shows that when there's no variable compensation for the manager, the optimal trigger for the shareholders is lower than the one appearing in the standard real options approach (this is due to the savings in wages that reduces the *net* investment cost). However, if that happens, the manager will never launch the project, since he will not give up the higher fixed salary for managing the option, to just receive lower one for managing the active project. Naturally, in equilibrium, this reduction must be compensated by a positive (and adequate) value-sharing rate. Equations (2.29) and (2.30) indicates that as the value-sharing rate tends to its maximum value, it will be never optimal for the shareholder to sunk the investment cost K_s , and the trigger for the manager tends to its minimum value, where its investment cost corresponds to

$\frac{w_i - w_a}{r}$. Finally, equations (2.31) and (2.32) shows that in absence of wage savings, the shareholders optimal trigger will be higher than that of the project if managed directly by them, and also that for the manager it will be optimal to invest immediately.

3.2. Numerical Example. Let us now present a numerical example. Assume a firm holding the option to invest in a given project. The shareholders decide for professional management, and so they hire a manager for running this investment opportunity. The agent major tasks are to monitor the option's key value-drivers and, at some point in time, take the decision to invest. As we said, the shareholders are interested in designing a contract with the right compensation for the manager, using a contingent element based on projects cash-flows, which guaranties that the option to invest is exercised at the optimal moment, from their point of view. In our model, this is done by defining a critical ϕ , for a given w_a , that aligns both the agent and principle interests.

Consider the values for the parameters presented in Table 1:

Parameter	Value	Description
K_s	\$1,000	Investment cost
r	0.05	Risk-free interest rate
σ	0.20	Instantaneous volatility
δ	0.03	Dividend-yield
w_i	\$5	Fixed wage for managing the idle project
w_a	\$2	Fixed wage for managing the active project
ϕ	—	Value-sharing rate

TABLE 1. The base case parameters.

Based on equations (2.8), (2.15), (2.17), and (2.19) we find the following output values:

Output	Value
ϕ^*	0.06
$V_s = V_m = V^*$	\$2,720.8

TABLE 2. The output values for the parameters presented in Table 1.

The shareholder will offer the manager a value-sharing rate of 0.06, as the price to ensure optimal behavior. In this case, both the owners and the manager share the same trigger, and the project will be launched optimally when V hits \$2,720.8.

Figure 1 shows the impact of different levels of ϕ on the optimal triggers for the shareholders (V_s) and for manager (V_m), and also the optimal trigger for the project if managed directly by the owners (V^*). All the three functions met at $\phi = \phi^*$. While V_s presents a small sensitivity to ϕ , V_m reveals to be highly sensitive to this parameter. The conclusion

is straightforward: if manager behave according his own interest, even small deviations from ϕ^* will imply a significant sub-optimal decision for the shareholders. Establishing a $\phi < \phi^*$ the project will be launched too late, and a $\phi > \phi^*$ the implementation will be taken too soon.

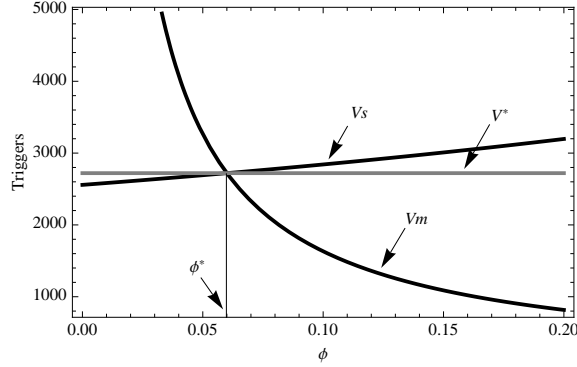


FIGURE 1. The trigger for different levels of ϕ . The other parameters are according Table 1.

Figures 2(a) and 2(b) shows the value functions for the shareholders and for the manager, with the standard appearance. The optimal decision for both will be maintaining the options alive until V hits V_s and V_m , respectively. Note, however, that $S(V)$ presents an odd negative region due to the wage that the owners pays the agent for managing the option. For this reason, there's a region where $M(V)$ dominates $S(V)$, as one can see from Figure 2(c).

Despite our focus on the key parameter ϕ , the compensation scheme presented in this paper consists in a mix between ϕ and w_a . Accordingly, there is a critical pair of ϕ 's and w_a 's (i.e., there is a critical w_a for a given ϕ , or a critical ϕ for a given w_a) that ensures optimal behavior for the manager. Figure 3 pictures this combination: the optimal pairs of ϕ 's and w_a 's correspond to the interception of the two surfaces.

Figure 4 shows the manager value function for different levels of w_a . The higher (lower) the wage for managing the active project, the lower (higher) critical value-sharing rate. So what shareholders should do is to find the optimal scheme trading-off the fixed and variable compensation components.

4. Particular Situations

In this section three particular situations are analyzed. In 4.1 we consider the existence of impatient managers, in 4.2 we account for existence of a non-monopolistic project, and, finally, in 4.3 we extend the model to account for the existence of effort costs for the manager. In all this cases, the impact on compensation is analyzed, and we show the solution for designing the optimal compensation schemes.

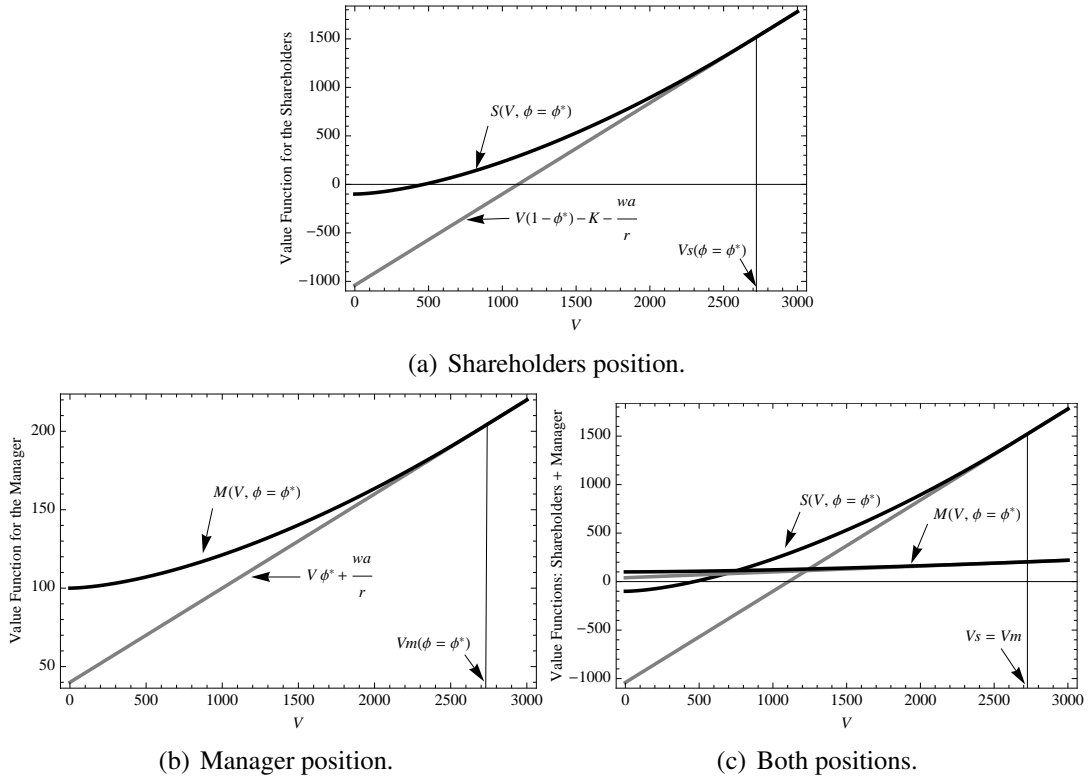


FIGURE 2. The value functions for the shareholders and for manager. The parameters are according Table 1, and $\phi = \phi^* = 0.06$.

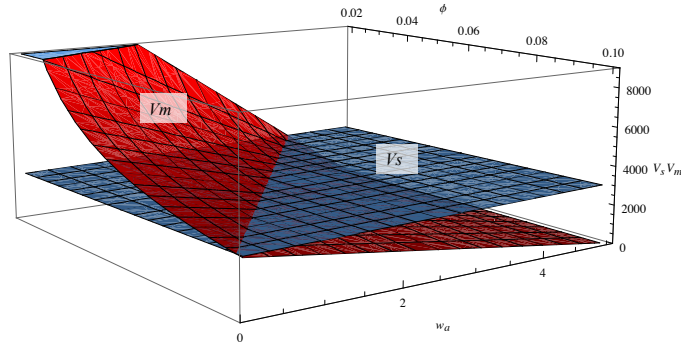


FIGURE 3. The triggers V_s and V_m for different levels of ϕ and w_a . The other parameters are according Table 1.

4.1. Impatient managers. Until now we have assumed that both shareholders and managers value the project cash-flows identically. However, as pointed out by Grenadier and Wang (2005), managers can be more impatient than shareholders. They present several reasons to justify impatient managers: short-term preferences, empire building, greater perquisites consumption and reputation.

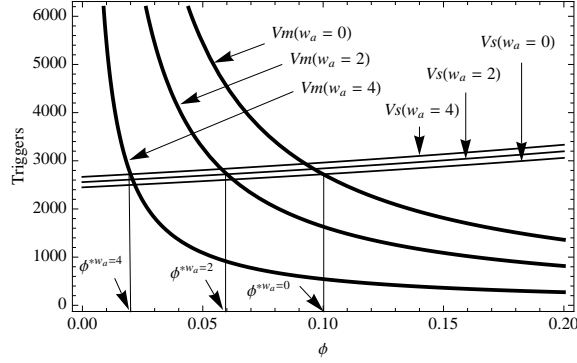


FIGURE 4. The triggers V_s and V_m for different levels of ϕ and w_a . The other parameters are according Table 1.

We follow Grenadier and Wang (2005), and account the manager's greater impatience by increasing the discount rate from r to $r + \xi$, where ξ captures the direct cost of extending effort. This produces different payoffs valuation for the owners and for the manager.

The optimal value-sharing rate for an impatient manager (ϕ_ξ^*) is as follows (details on derivation appear in Appendix):

$$(2.33) \quad \phi_\xi^* = \frac{r(w_i - w_a)(\beta - 1)\gamma}{rK\beta(\gamma - 1)(r + \xi) - (w_a - w_i)[r(\beta - \gamma) - \beta(\gamma - 1)\xi]}$$

where β is as defined in (2.9), and $\gamma = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r + \xi)}{\sigma^2}}$. It can be shown that $\phi_\xi^* < \phi^*$ for any $\xi > 0$, which means the impatient manager will demand less value-sharing bonus for aligning its interests with the owners'.

The impact of ξ on the optimal value-sharing rate is illustrated in Figure 5.

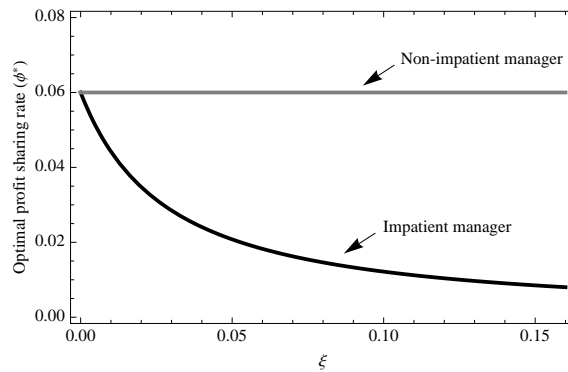


FIGURE 5. The optimal value-sharing rate. The other parameters are according Table 1.

4.2. Non-monopolistic projects. Assume now the investment opportunity is not proprietary, and, by the entrance of a rival, the option to invest can suddenly disappear. This

random catastrophic event can be modeled in a well-known way, by including a parameter λ (> 0) in the o.d.e. that must be followed by the owners value function (where λ is a Poisson rate of arrival)²:

$$(2.34) \quad \frac{1}{2}\sigma^2 V^2 S''(V) + (r - \delta)VS'(V) - (r + \lambda)S(V) - w_i = 0$$

Following the standard procedures, the trigger for the shareholder comes:

$$(2.35) \quad V_s^\lambda = \frac{\eta}{\eta - 1} \frac{1}{1 - \phi} \left(K_s - \frac{w_i - w_a}{r} \right)$$

where:

$$(2.36) \quad \eta = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(r + \lambda)}{\sigma^2}} > 1$$

Since, for any positive λ , $\eta > \beta$ (where β is given by equation 2.9), we have $V_s^\lambda < V_s$. This means that, under the fear of preemption, the shareholders will be interested to invest earlier.

According to Assumption 2 and 4 this catastrophic event don't impact the manager value function nor his optimal trigger, which remains as presented in equation (2.15). Under this setting, the value-sharing rate becomes (see the proof in the Appendix):

$$(2.37) \quad \phi_\lambda^* = \frac{(w_i - w_a)\beta(\eta - 1)(r + \lambda)}{r [(w_a - w_i)(\beta - \eta) + Kr(\beta - 1)\eta] + \lambda [w_a(\beta - \eta) + w_i\beta(\eta - 1) + Kr(\beta - 1)\eta]}$$

It can be shown that $\phi_\lambda^* > \phi^*$, meaning that the owners will need to share more value with the manager in order to align both interest. In fact, and as we can see from Figure 6, the only way the owners have to induce early investment is to give value to managers.

4.3. The inclusion of management effort costs. Supposing manager exerts effort in both states of his value function such that, when he is just managing the option, analyzing market conditions and studying the optimal moment to invest, he spends a continuous effort cost e_i , but when the project turns to be active, he increases his effort cost to e_a , representing all the diligences in order to implement the project and to effectively manage

²At the beginning the o.d.e. takes the form $\frac{1}{2}\sigma^2 V^2 S''(V) + (r - \delta)VS'(V) - rS(V) + \lambda[0 - S(V)] - w_i = 0$, where $\lambda[0 - S(V)]$ captures the expected loss resulting from the entrance of a rival firm, which makes the option value drop from $S(V)$ to 0.

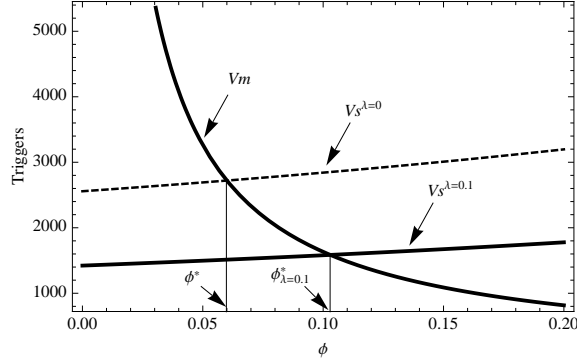


FIGURE 6. The optimal value-sharing rate for non-proprietary option. $\lambda = 0.1$, and the other parameters are according Table 1.

the business³. It is intuitive to think that managing the implemented project demands more effort than managing the option, so we assume that $e_a > e_i$.

Using equation (2.10), we restate o.d.e. considering this setting:

$$(2.38) \quad \frac{1}{2}\sigma^2V^2M''(V) + (r - \delta)VM'(V) - rM(V) + w_i - e_i = 0$$

respecting boundary conditions:

$$(2.39) \quad M(V_m^e) = \frac{w_a}{r} + \phi V_m^e - \frac{e_a}{r}$$

$$(2.40) \quad M(V_m^e) = \phi$$

$$(2.41) \quad M(0) = \frac{w_i - e_i}{r}$$

So that the solution of $M(V)$ and V_m^e (the optimal trigger under the existence of effort costs) are, respectively:

$$(2.42) \quad M(V) = \begin{cases} \left(\frac{V}{V_m}\right)^\beta \frac{1}{\beta - 1} \left[\frac{w_i - w_a}{r} + \frac{e_a - e_i}{r} \right] + \frac{w_i - e_i}{r} & \text{for } V < V_m^e \\ \frac{w_a - e_a}{r} + \phi V & \text{for } V \geq V_m^e \end{cases}$$

and

$$(2.43) \quad V_m^e = \frac{\beta}{\beta - 1} \frac{1}{\phi} \left[\frac{w_i - w_a}{r} + \frac{e_a - e_i}{r} \right]$$

³Grenadier and Wang (2005) also consider positive effort costs but in a different context. The manager, at time zero, incurs in an effort cost in order to increase the likelihood for a higher quality project.

Since shareholders' value function doesn't incorporate manager's effort it remains the same as in equation (2.7). But this occurrence has an implicit problem concerning misaligning of targets and expectations, because now we have a different optimal value-sharing bonus rate (ϕ_e^*), such that when $V_s = V_m^e$:

$$(2.44) \quad \phi_e^* = \frac{\frac{w_i - w_a}{r} + \frac{e_a - e_i}{r}}{K_s}$$

representing:

$$(2.45) \quad E_m = \frac{e_a - e_i}{r}$$

we get:

$$(2.46) \quad \phi_e^* = \frac{K_m + E_m}{K_s} = \phi^* + \frac{E_m}{K_s}$$

which results that $\phi^* < \phi_e^*$.

Note that $\phi^* = \phi_e^*$ if $e_i = e_a$. This way, we show that if an increment of effort costs occurs (by the reasons we presented earlier) and if this increment is ignored, shareholders will give manager a lower value-sharing component and, consequently, a less valuable contract, failing to provide the proper aligning interests.

5. Conclusions

The monopolistic framework presented in this chapter overcomes the common assumption of that, either the investment opportunity is directly managed by the owners, or that managers are perfectly aligned with them. However, agency conflicts occur and managers reveal interests not totally in line with those of the shareholders. This may have a major impact on the value maximizing decisions, namely, on the optimal timing to invest. Despite of this problem, literature that accounts for agency conflicts on the exercise of real options appears to be scarce.

In this context, an optimal contract scheme is proposed (incorporating fixed wages and value-sharing bonus) in order to avoid inadequate actions from the manager. In our model, shareholders need not to follow the future evolution of project value drivers in order to guarantee optimal behavior. It was shown that even small deviations from the optimal compensation scheme may lead to highly suboptimal decisions.

In the end, optimal contracts were also established for special situations, namely, to account for impatient managers, for non-proprietary (or non-monopolistic) real options, and also by considering the existence of effort costs for the manager.

Appendix

Proof of equation (2.33): The value function for the shareholders stands as in equation (2.7). The value function for the impatient manager must satisfy the o.d.e.: $\frac{1}{2}\sigma^2x^2M''(V) + (r - \delta)xM'(V) + w_i = (r + \xi)M(V)$, see Grenadier and Wang (2005) for the arguments. Following standard procedures the trigger comes $V_m^\xi = \frac{\gamma}{\gamma-1} \frac{1}{\phi} \frac{w_i - w_a}{r + \xi}$, where $\gamma = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(-\frac{1}{2} + \frac{\alpha}{\sigma^2})^2 + \frac{2(r+\xi)}{\sigma^2}}$, while the trigger for the owners remains: $V_s = \frac{\beta}{\beta-1} \frac{1}{1-\phi} (K_s + \frac{w_a - w_i}{r})$. Both triggers met for $\phi = \phi_\xi^*$: $V_m^\xi(\phi_\xi^*) = V_s(\phi_\xi^*)$. Solve for ϕ_ξ^* .

Proof of equation (2.37): The new trigger for the owners appears in equation (2.35), the triggers for the manager remains as in (2.15). Both triggers met for $\phi = \phi_\lambda^*$: $V_m(\phi_\lambda^*) = V_s^\lambda(\phi_\lambda^*)$. Solve for ϕ_λ^* .

CHAPTER 3

Duopolistic Investment Opportunity under Agency Context

1. Introduction

Real options theory emerged in capital budgeting field as an answer to the limitations of classic approaches, such as the Net Present Value, embodying growth opportunities until then ignored. Firstly alluded by Myers (1977) which considered growth opportunities on corporate assets as a necessary component of the firms market value, it was founded on the original work of Black and Scholes (1973), but rapidly evolved and was expanded to diverse areas. Nonetheless, differently from financial options, real-world investment decisions concerns non-exclusive opportunities, usually taken in a competitive environment (Zingales (2000)). Consequently, the necessary interaction of real options theory, industrial organization and game theory emerged, encompassing the problematic of multiplayer investment opportunities.

A diverse and complex set of models has flown, presenting different characteristics, approaches and applications to general cases and also specific industries, such as research and development (Reiss (1998), Garlappi (2004), Weeds (2002)), real estate development (Grenadier (1996)), financial sector Baba (2001), agriculture (Odening et al. (2007)), airplane production sector (Shackleton et al. (2004)) and much others, showing that the applicability of real options game theory is far from being exhausted. We highlight Azevedo and Paxson (2011) which provide a complete review and supply a complete set of characteristics to categorize the type of Real Options Game models.

The founding work on real options game theory was developed by Smets (1993). Although it was originally focused on international finance, presenting a framework where firms can expand their profit flows through investment, it was further generalized and simplified on Dixit and Pindyck (1994), where a duopolistic real options game is presented in a new market context. Preemptive actions, in consequence of symmetric strategies between homogeneous players, leads to an earlier entry trigger for the leading firm and a later trigger for the follower, comparatively to the exclusive real option scenario.

Several developments came after, where we highlight Thijssen et al. (2002), which proposes the use of symmetric mixed strategies in a stochastic environment in order to

deal with the problem of investment coordination failure¹. This solution avoids the necessity of previous assumptions regarding the investment decisions and firms roles. Huisman et al. (2003) provides some developments, namely the case of asymmetric firms (lower investment costs for one of the players), the impact of different possible technologies and the effect that new information have on the reduction of overall uncertainty. Nonetheless, this field is not exclusive of duopolistic interaction as seen, for example, on Bouis et al. (2009) which provides a multiple firms framework, generalizing to an n-dimensional market a three-firm oligopolistic model. Asymmetrical market shares and variable market size hypotheses were further developed by Pereira and Rodrigues (2010). Nonetheless, in all these papers it is ignored that investment decisions are often taken by managers that may not be fully aligned with owners' value maximization target.

Value maximization goal has been considered, for long, the prevailing long-term goal of firms. As Jensen (2001) explains, this means that all management decisions should be taken with a purpose of increasing the value of all financial claims on a firm. Although, this has been a source of debate, it has been accepted in the last 200 years as the more suitable solution. Nonetheless, the attribution of decision taking role to an agent, without the appropriate and effective mechanisms of incentives and control, may lead to suboptimal decisions and value misappropriation. Some circumstances of necessary shift of the decision role are, for example, high ownership dispersion (Berle and Means (1932)) or owners lack of expertise (Shleifer and Vishny (1997)). Although we can trace agency issues to Smith (1976)², the formalization of agency theory and agency issues is acknowledged to Jensen and Meckling (1976). Agency risk arises when in a contractual relation between utility maximizing parties, the principal delegates decision authority in a context of asymmetrical information, which may lead to agency costs if there is interest misaligning between the parties.

Financial literature proposes internal and external mechanisms in order to mitigate agency risk and thus reduce agency costs. The first branch contains incentive contracts (Jensen and Smith Jr (1985)), insider ownership (Jensen and Meckling (1976)), existence of large investors (Shleifer and Vishny (1986)), board of administration (Fama and Jensen (1983)), the existence of debt and dividend policy, which reduces free cash-flows (Jensen (1986)). External mechanisms encompasses, for instance, managers reputation (Shleifer and Vishny (1997)), managers market competition (Fama (1980)), output market competition (Hart (1983)), takeovers market (Easterbrook and Fischel (1981)), monitoring by

¹Although this is an important step stone, for simplicity sake we rely on Dixit and Pindyck (1994) results and assumptions in this paper.

²"The directors of companies, being managers of others people money, cannot be expected to watch over it with the same vigilance with which they watch over their own."

investment professionals (Chung and Jo (1996)) or legal framework (Shleifer and Vishny (1997)). Despite its relevance, these mechanisms have limitations, which several works have shown. We highlight Holmstrom (1982), Shleifer and Vishny (1997), Grossman and Hart (1980), Burkart (1995), Jensen (1993).

Investment timing decisions, when studied under a real options approach, usually tend to assume perfect aligning of interests between managers and shareholders or full control of investment decisions by the owners, ignoring though the impact of agency conflicts. This has been, for long time, a usual assumption which we can track to Myers (1977) who explicitly assumes that firms managers act in the shareholders interests.

Considering a firm as a nexus of relations bonded by a contractual structure (Jensen and Meckling (1976)) and not conditioned by inside inefficiencies³ and outside inefficiencies⁴, it is theoretically possible to design an optimal contract so that the risk bearing and risk premium are shared between the principal and the agent, eliminating value maximization deviations and misappropriations.

The context in which we present our work differs from closed related literature, since we analyze the agency problem under a duopolistic investment timing context. Consequently, we do not find similar works on literature that approach the agency relation in non-exclusive investment opportunities. Nonetheless, we observe some rapid developments on real options agency theory in the past few years, concerning the problematic of agency relations in investment timing decisions contexts.

We highlight the major advances of Grenadier and Wang (2005), which examines investment timing decision for a single project, where the owner delegates the investment decision to the manager. Managerial behavior accounts for asymmetric information and moral hazard, generating sub-optimal decisions that can be corrected through an optimal contract, aligning the incentives of owners and managers. Nishihara and Shibata (2008) extends this model incorporating a relationship between an audit mechanism and bonus-incentives sensitive to managers deviated actions. Shibata and Nishihara (2010) extends these papers by incorporating debt financing on investment expenditure.

Moreover, Hori and Osano (2010) presents an agency model under a real options framework where managerial compensation is designed endogenously including a contingent claim on firms cash-flows using stock options. Kanagaretnam and Sarkar (2011), in order to mitigate the underinvestment problem of Mauer and Ott (2000), considers an agency compensation with a fixed component and a equity share aligning the interests of manager with both bondholders and shareholders.

³Self-dealing contract negotiation as noted by Shleifer and Vishny (1997).

⁴Legal distortions, for example.

We aim to relax the assumption that managers are perfectly aligned with owners (as assumed in the standard real options literature, e.g. Dixit and Pindyck (1994)) and frequently ignored in non-exclusive investment opportunities. Moreover, we provide a perceptible but yet meaningful framework where two firms share an option to enter in a new market, but for plausible reasons (i.e., incapacity, opportunity cost or control difficulty) both need to hire a manager entity to supervise the option, to follow market conditions, to perceive the rivals preemptive behavior and, consequently, to take the investment decision. Although, since managers are utility maximizers, some interests misaligning may lead to suboptimal investment timing decision, which demands ex-ante optimal contract design to mitigate the risk and eliminate eventual costs.

The rest of the paper is organized as follows. Section 2 presents the competition framework under a necessary agency relationship established in both firms, where subsection 2.1 concerns the results for the follower firm and subsection 2.2 concerns the solution for the leader firm. Section 3 presents the impact of agency contract definition on the resulting equilibria of both firms, where subsection 3.1 sets the equilibrium solution that aligns the managers interests with those of both firms shareholders, and section 3.2 analyzes the ex-ante incentives of principal and agents regarding the consequences of contract design. Section 4 presents a comparative statics analysis and a numerical example.

2. Competition Framework

In this section, we will present the problem of agency interaction between shareholders and a management entity considering that the opportunity to invest is simultaneously shared with another symmetric firm. We consider that shareholders lack the ability, knowledge or time to take the appropriate optimal choices in search of equity value maximization. So, similarly to our monopoly agency solution, each firm will need to outsource their optimal investment decisions, delegating power to an agent. Firms owners will negotiate with each respective manager and design a contract that pays a management fee w_i while the project is idle and, after exercising the option to invest, a mix of timely continuous fixed wage w_a plus a value-sharing bonus fee ϕ . As we will see, and similarly to the monopolistic case, both firms' agency contracts are arranged initially, without further intervention of the shareholders on the investment decisions. Furthermore, if the proper contract is accorded, shareholders guarantee managers optimal behavior, and interests alignment.

Both firms are risk-neutral and fully aware of the rivals behavior. They can produce a unit of output after entering the market, which is totally absorbed by consumers due to

their infinitely elastic demand nature. The price of each of these units behaves stochastically over time, according to equation (3.1), which follows the Dixit and Pindyck (1994) Chapter 9 notation:

$$(3.1) \quad P_t = Y_t D(Q_t)$$

and

$$(3.2) \quad dY_t = \alpha Y_t dt + \sigma Y_t dz$$

The market value is represented by Y_t , consisting on an exogenous shock following a geometric Brownian motion according to equation (3.2). D_t is the inverse demand function - non-stochastic decreasing function - that depends on the market supply output (Q_t) which has three states, the first relating the absence of players ($Q_t=0$), the second concerning the monopoly state ($Q_t=1$) and the last one representing the duopoly state ($Q_t=2$). For simplification we will further consider Y_t as Y . In relation to equation (3.2), dz is the increment of the Wiener process, α is the instantaneous conditional expected relative change in V , also known as drift. $\alpha = r - \delta$ ($r > \delta$), where r is the risk-free rate and $\delta (> 0)$ represents the opportunity cost from deferring, and σ is the instantaneous conditional standard deviation.

We will further present a leader-follower framework, extending the duopolistic model of Dixit and Pindyck (1994), which provides a simplifying but meaningful adaption of Smets (1993). Initially, only one of the firms invests a fixed cost K_s , earning a temporarily monopoly pay-off, and after the second firm achieves its optimal exercising moment, also investing K_s , both firms will equally share the market (the firms are assumed to be symmetric ex-post). Nonetheless, both Dixit and Pindyck (1994) and Smets (1993) consider that investment timing decisions in duopoly context are free of agency issues, which we will further embody.

In order to provide the adequate competitive incentives in a symmetrical strategies environment, we need to ensure that, while managing the investment option, both agents are in equal positions and have the same incentives. This leads to the following assumptions:

Assumption 1: *Owners can't administer directly the option to invest. Also they are unable of properly observe some key value drivers (namely, Y , σ , α), so the option becomes useless without a manager.*

Assumption 2: *We assume the presence of ex-ante perfect information between firms which, additionally to symmetrical characteristic of firms, implies equal composition of fixed wage w_a and value-sharing factor ϕ .*

Assumption 3: *Managers do not have time restrictions. We assume managers as an abstract entity so that there are no lifetime restrictions.*

Assumption 4: *Managers and options to invest aren't scarce, so that owners can always find a manager for running his projects, and managers can always find another investment opportunity needing to be managed.*

Assumption 5: *The parameter w_i represents the managers market price for running an idle project (meaning that the owners can't find a less expensive manager). Also, we assume that the fixed wage to manage the active project, w_a , is lower than w_i , so that manager's utility function integrates awaiting value. Note, however, that the lower fixed salary will be compensated by an appropriate value-sharing bonus.*

Assumption 6: *Manager can only broke contract before the option exercise and this only happens if the option becomes worthless. In this case manager will earn a fixed compensation $\frac{w_i}{r}$.*

Assumption 7: *For the owners, value-sharing bonus are less expensive than monitoring costs.*

Assumption 8: *Managers are not wealth enough neither can gather the necessary resources (leverage, for instance) to buy the project.*

Assumption 9: *Both firms and managers are risk-neutral.*

Assumption 10: *Firms are assumed to be all-equity.*

2.1. Follower Firm. Solving this problem backwardly, therefore, assuming that the leader already invested, we will find firstly the optimal decision for the follower firm, specifically for both the follower firm shareholders and the follower manager. During the period which is not yet optimal to invest, the follower shareholders' value function $S^F(Y)$ must satisfy the following ordinary differential equation:

$$(3.3) \quad \frac{1}{2}\sigma^2 Y^2 S^{F''}(Y) + (r - \delta)Y S^{F'}(Y) - rS^F(Y) - w_i = 0$$

solved by a second-order Cauchy-Euler equation that, after embodying the absorption barrier stated on equation (3.7), results on the following general equation:

$$(3.4) \quad S^F(Y) = BY^{\beta_1} - \frac{w_i}{r}$$

subject to the boundary conditions:

$$(3.5) \quad S^F(Y_s^F) = (1 - \phi) \frac{Y_s^F D(2)}{r - \alpha} - \frac{w_a}{r} - K_s$$

$$(3.6) \quad S^{F'}(Y_s^F) = (1 - \phi) \frac{D(2)}{r - \alpha}$$

$$(3.7) \quad S^F(0) = -\frac{w_i}{r}$$

where equation (3.5) and equation (3.6) concerns, respectively, the value matching condition and the smooth pasting condition at the optimal investment trigger Y_s^F .

Therefore, accounting conditions (3.5) and (3.6), we have the value function for the shareholders of the follower firm $S^F(Y)$:

$$(3.8) \quad S^F(Y) = \begin{cases} \left(\frac{Y}{Y_s^F} \right)^{\beta_1} \frac{1}{\beta_1 - 1} \left(K_s - \frac{w_i - w_a}{r} \right) - \frac{w_i}{r} & \text{for } Y < Y_s^F \\ (1 - \phi) \frac{D(2)Y}{r - \alpha} - K_s - \frac{w_a}{r} & \text{for } Y \geq Y_s^F \end{cases}$$

being the optimal investment trigger Y_s^F :

$$(3.9) \quad Y_s^F = \frac{\beta_1}{\beta_1 - 1} \frac{1}{1 - \phi} \frac{\delta}{D(2)} \left(K_s - \frac{w_i - w_a}{r} \right)$$

where β_1 is the positive root of the fundamental quadratic equation:

$$(3.10) \quad \frac{1}{2} \sigma^2 \beta_1 (\beta_1 - 1) + (r - \delta) \beta_1 - r = 0$$

so that

$$(3.11) \quad \beta_1 = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} > 1$$

Similarly to the proprietary option scenario presented earlier, the effective optimal investment moment will be choose by the manager. Therefore, we need to estimate his value function $M^F(Y)$ and preferred investment trigger Y_m^F , solving the following o.d.e.:

$$(3.12) \quad \frac{1}{2} \sigma^2 Y^2 M^{F''}(Y) + (r - \delta) Y M^{F'}(Y) - r M^F(Y) + w_i = 0$$

The general solution of equation (3.12) aggregates an homogenous component and a particular solution that, satisfying the absorption barrier stated in equation (3.16), will take the form:

$$(3.13) \quad M^F(Y) = CY^{\beta_1} + \frac{w_i}{r}$$

conditioned by the restrictions:

$$(3.14) \quad M^F(Y_m^F) = \phi \frac{Y_m^F D(2)}{r - \alpha} + \frac{w_a}{r}$$

$$(3.15) \quad M^{F'}(Y_m^F) = \phi \frac{D(2)}{r - \alpha}$$

$$(3.16) \quad M^F(0) = \frac{w_i}{r}$$

The first two conditions are, respectively, the value matching and the smooth pasting conditions, which aim to ensure optimal decisions. As noted, the latter restriction is an absorption barrier related to Y 's stochastic characteristic.

After some arithmetics concerning the remaining boundary conditions (equation (3.14) and (3.15)), we obtain the specific value function for the manager entity of the follower company ($M^F(Y)$) and its optimal investment trigger (Y_m^F):

$$(3.17) \quad M^F(Y) = \begin{cases} \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \frac{1}{\beta_1 - 1} \left(\frac{w_i - w_a}{r}\right) + \frac{w_i}{r} & \text{for } Y < Y_m^F \\ \phi \frac{D(2)Y}{\delta} + \frac{w_a}{r} & \text{for } Y \geq Y_m^F \end{cases}$$

$$(3.18) \quad Y_m^F = \frac{\beta_1}{\beta_1 - 1} \frac{1}{\phi} \frac{\delta}{D(2)} \frac{w_i - w_a}{r}$$

2.2. Leader Firm. Using backward calculation, we will now derive the payoff for the leader firm, assuming that the follower firm will behave optimally. After entering the market, paying the sunk cost K_s , the manager of the leader firm has no further decisions to make. Nonetheless, the payoff of both the principal and agent of the leading company will be affected by their rivals' subsequent actions.

We will firstly solve the leader firm shareholders' problem. Therefore, we must solve the following o.d.e. to achieve their value function $S^L(Y)$:

$$(3.19) \quad \frac{1}{2}\sigma^2 Y^2 S^{L''}(Y) + (r - \delta)Y S^{L'}(Y) - rS^L(Y) + (1 - \phi)D(1)Y - w_a = 0$$

The last two terms, which is the non-homogeneous component, represents the cash-flow that shareholders earn during the monopolistic period. The general solution of this o.d.e. is given by the following equation:

$$(3.20) \quad S^L(Y) = GY^{\beta_1} + (1 - \phi) \frac{YD(2)}{r - \alpha} - \frac{w_a}{r}$$

The unknown constant G is calculated using only a boundary restriction, which is the value matching condition at the optimal trigger of the follower shareholders, since the firms are symmetric ex-post, both value functions must meet at $Y_s^L = Y_s^F$. So, at Y_s^F , the leader shareholders value during their monopoly state must equal the simultaneous investment value. This boundary condition is represented on equation (3.21), where β_1 is as previously stated on equation (3.11):

$$(3.21) \quad S^L(Y_s^F) = (1 - \phi) \frac{Y_s^F D(2)}{r - \alpha} - \frac{w_a}{r}$$

This yields the following value function for the leader firm shareholders:

$$(3.22) \quad S^L(Y) = \begin{cases} (1 - \phi) \frac{YD(1)}{r - \alpha} + \left(\frac{Y}{Y_s^F} \right)^{\beta_1} \frac{\beta_1}{\beta_1 - 1} \left(1 - \frac{D(1)}{D(2)} \right) \left(K_s - \frac{w_i - w_a}{r} \right) - \frac{w_a}{r} & \text{for } Y < Y_s^F \\ (1 - \phi) \frac{YD(2)}{r - \alpha} - \frac{w_a}{r} & \text{for } Y \geq Y_s^F \end{cases}$$

The optimal trigger $Y_s^L < Y_s^F$ for the leader shareholder must be such that it will be indifferent for both firms' shareholders to become a leader or a follower. This happens when the value for the leader shareholder excluding the sunk cost K_s equals the value for the follower shareholders. Therefore, through this rent equalization principle we obtain equation (0.64) on Appendix, allowing the derivation of Y_s^L . This equation allow us to prove the existence of a root, other than and strictly below the root Y_s^F :

LEMMA 2.1. *There exists a unique point $Y_s^L \in (0, Y_s^F)$ such that:*

$$(3.23) \quad S^L(Y_s^L) - K_s = S^F(Y_s^L)$$

$$(3.24) \quad S^L(Y) - K_s < S^F(Y), \text{ for } Y < Y_s^L$$

$$(3.25) \quad S^L(Y) - K_s \geq S^F(Y), \text{ for } Y > Y_s^L$$

Proof 2.1: See appendix.

This results lead us to the following expression concerning the stopping time of the leader shareholders:

$$(3.26) \quad T_s^L = \inf\{t \geq 0 : Y \in [Y_s^L, Y_s^F]\}$$

The shareholders framework is important in order to identify the firm's optimal decisions to implement. Nevertheless, the prevailing investment decisions are taken by the manager, and so we will now present the leader manager framework, using the same backwards calculation presented earlier in this section. In order to achieve that, we must solve the following o.d.e:

$$(3.27) \quad \frac{1}{2}\sigma^2 Y^2 M^{L''}(Y) + (r - \delta)Y M^{L'}(Y) - rM^L(Y) + \phi D(1)Y + w_a = 0$$

Symmetrically to the shareholders, the last two terms concerns the cash-flow earned by the manager after entering the market but before the rival's entry. The general solution of the o.d.e is given by the following equation:

$$(3.28) \quad M^L(Y) = HY^{\beta_1} + \phi \frac{YD(1)}{r - \alpha} + \frac{w_a}{r}$$

In order to find the solution we will use the following value matching condition at the follower manager's entry point (Y_m^F):

$$(3.29) \quad M^L(Y_m^F) = \phi \frac{Y_m^F D(2)}{r - \alpha} + \frac{w_a}{r}$$

This results in the following value function for the leader company manager:

$$(3.30) \quad M^L(Y) = \begin{cases} \phi \frac{YD(1)}{r - \alpha} + \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \frac{\beta_1}{\beta_1 - 1} \left(1 - \frac{D(1)}{D(2)}\right) \left(\frac{w_i - w_a}{r}\right) + \frac{w_a}{r} & \text{for } Y < Y_m^F \\ \phi \frac{YD(2)}{r - \alpha} + \frac{w_a}{r} & \text{for } Y \geq Y_m^F \end{cases}$$

Manager of the leading company will choose its optimal trigger $Y_m^L < Y_m^F$, at the moment that it is indifferent for both managers to become leader or follower. The expression that yields Y_m^L is represented in equation (0.70), from the Appendix. We must define and prove that this point exists and it is unique. Therefore:

LEMMA 2.2. *There exists a unique point $Y_m^L \in (0, Y_m^F)$ such that:*

$$(3.31) \quad M^L(Y_m^L) - K_s = S^F(Y_s^L)$$

$$(3.32) \quad M^L(Y) < M^F(Y), \text{ for } Y < Y_m^L$$

$$(3.33) \quad M^L(Y) \geq M^F(Y), \text{ for } Y > Y_m^L$$

Proof 2.2: See appendix.

The stopping time of the leader manager will be:

$$(3.34) \quad T_m^L = \inf\{t \geq 0 : Y \in [Y_m^L, Y_m^F]\}$$

3. Contract Design and Agency Issues

This section explains the relationship between agency problems and contract design, presenting the impact of agency contract definition on the equilibria of both of the follower and leader equity holders and their respective managers. As mentioned earlier, this compensation scheme is defined by combining the three components - the exogenous idle project's fixed wage (w_i), the active project's fixed wage (w_a) and value-sharing component (ϕ) - which, if adequate, can assure aligned managerial behavior. This section will also present the impact of the contractual framework in agency costs mitigation.

3.1. Optimal Contract Solution. In order to align shareholders and manager's interests at the follower firm, an optimal mix of w_a and ϕ must be provided. This ensures that the agent chooses Y_s^j ($j \in \{L, F\}$ where L stands for the Leader and F for the Follower) as its respective optimal investment trigger. Equaling both players' optimal triggers:

$$(3.35) \quad Y_s^F = Y_m^F$$

Shareholders and managers of the follower firm get the following optimal combination⁵:

$$(3.36) \quad \phi^* = \frac{K_m}{K_s}$$

where:

$$(3.37) \quad K_m = \frac{w_i - w_a}{r}$$

Similarly to the follower firm optimal contract, the leader shareholders also ensure optimal behavior of its manager, through an equal compensation package, mixing the fixed wage w_a and the value-sharing component ϕ concerning the relative opportunity cost

⁵Similar to the monopolistic solution approached earlier (see Chapter (2)).

between the leader firm's players, therefore guaranteeing that manager takes the optimal decision, investing at Y_s^L .

Therefore, considering $\Omega(Y, \phi)$ as the aggregate value function of the leader shareholders ($S^L(Y)$ as stated on Equation (3.22)) and the leader manager ($M^L(Y)$ as stated on Equation (3.30)), restricted by the leader managers prevailing entry triggers Y_m^L and Y_m^F , both dependent variables of ϕ :

$$(3.38) \quad \Omega(Y, \phi) = \begin{cases} \frac{YD(1)}{r - \alpha} + \frac{\beta_1}{\beta_1 - 1} \left(1 - \frac{D(1)}{D(2)}\right) \left[\left(\frac{Y}{Y_s^F(\phi)}\right)^{\beta_1} K_s - \left(\left(\frac{Y}{Y_s^F(\phi)}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F(\phi)}\right)^{\beta_1} \right) \frac{w_i - w_a}{r} \right] & \text{for } Y < Y_m^F \\ \frac{YD(2)}{r - \alpha} & \text{for } Y \geq Y_m^F \end{cases}$$

Consequently, we have:

LEMMA 3.1. *The optimal contract mix $\phi^*(w_a, w_i, K_s)$, presented on equation (3.36) guarantees that manager chooses the leader shareholders' optimal trigger:*

$$(3.39) \quad Y_s^L|_{\phi=\phi^*} = Y_m^L|_{\phi=\phi^*}$$

Implied from:

$$(3.40) \quad \arg \max_{\phi \in (0,1)} \Omega(Y, \phi) = \frac{K_m}{K_s}$$

Proof 3.1: See appendix.

These results show, first of all, that shareholders of both the leader and follower firm can determine an optimal combination of fixed wage and a value-sharing component ensuring that managers optimally choose their entry triggers, avoiding value misappropriation. Secondly, this ex-ante contract definition transfer to managers the preemption behavior, ensuring that they will dispute the role of market leader. Note that interests are maintained aligned even if a firm is preempted by the rival, ensuring that the follower manager will wait until the followers' optimal trigger is achieved. Thirdly, by defining this optimal contract, shareholders do not need to know the behavior of some variables, such as Y , σ or α , being free to pursuit other activities.

3.2. Ex-ante Sub-optimal Contract Definition and Agency Risk. In this subsection we analyze the ex-ante incentives of managers concerning the impact of a sub-optimal contract design on firms value losses in consequence of agency issues. In order to do so, we compare the aggregated value of manager and shareholders with the agency-free model of Dixit and Pindyck (1994).

Using the Pareto criterion, the multiplicity of pre-game contract mixes will be reduced to one optimal combination $\phi^*(w_a, w_i, Ks)$, since its resulting equilibria guarantees that the best possible result is undoubtedly achieved. In most of the circumstances (see Lemma. (3.2)) this labor contract structure Pareto-dominates all other combinations of the aggregate value of shareholders and manager of the leader firm, implying that no deviation will be profitable. In some specific circumstances, this contract is not the only possible contract combination that achieves the non-agency value result. Although, we may observe, in some sub-optimal cases, the absence of value deterioration comparing with the non-agency solution, these situations are not optimal combinations since the resulting triggers are not aligned, promoting value misappropriation between players.

Ex-ante, both firms will try to achieve the leader role so, the expected value of successful preemptive strategy will overcome at $Y \in [Y_s^L, Y_m^F)$ the follower strategy, which becomes a non credible strategy. Consequently, ex-ante, each firm's principal and agent will embody the leader's value function as the expected value when choosing the value-sharing factor ϕ which will be the same for both firms, due to their symmetric nature and perfect information.

Comparing the non-agency leader value function $L(Y)$ of Dixit and Pindyck (1994) with the aggregate value function $\Omega(Y, \phi)$ (stated on equation (3.38)), we want to study the incentives of both firms shareholders and managers when defining the labor contract under the expectation of gaining the leader role. Therefore:

LEMMA 3.2. *There exists a consistent tangible point at $\phi^*(w_a, w_i, Ks)$, such that:*

$$(3.41) \quad \Omega(Y, \phi^*) = L(Y), \text{ for } \phi = \phi^*$$

Other contract combinations may result in value loss, such that:

$$(3.42) \quad \Omega(Y, \phi) < L(Y), \text{ for } \phi \neq \phi^*$$

Proof 3.2: See appendix.

Outside the optimal contract mix ($\phi \neq \phi^*$) there are value misappropriation between firms' shareholders and respective managers in consequence of triggers misaligning, although, in some specific circumstances, we may not see aggregated value destruction compared with the non-agency. These results implies that the optimal labor contract is the only one that fully mitigates all agency costs guaranteeing that the aggregate value of shareholders and managers always equals the non-agency result in the leader firm, which corresponds to the expected role for both firms.

If, for some unexpected reason, a sub-optimal contract combination is chosen, then both firms' shareholders and managers face ex-ante agency risk resulting from the pre-emptive behavior distortion, and the consequent potential value misappropriation and value destruction that may result in sub-optimal contexts. This expectation distortion, produces an ex-ante incentive for both firms to chose this optimal contract design which always provides the best possible solution for the shareholders and manager, specifically, the Pareto optimal equilibrium. We prove that ϕ^* provides the first best solution for the aggregate leader firm - ex-ante expected result - corresponding to a Pareto equilibrium and also that this solution fully eliminates all agency costs.

4. Comparative Statics and Numerical Example

In this section we will study some analytical relations among the fundamental variables. The first subsection concerns the study of the sensitiveness of those variables, in both leader and follower firms. The second subsection will be responsible for the presentation of an empirical application, accompanied with a visual presentation and a respective characterization. Whenever necessary, the data presented on Table (1) will be used to illustrate the descriptions.

Parameter	Value	Description
$D(1)$	100	Monopolistic inverse demand function
$D(2)$	50	Duopolistic inverse demand function
K_s	\$1,000	Investment cost
r	0.05	Risk-free interest rate
σ	0.20	Instantaneous volatility
δ	0.03	Dividend-yield
w_i	\$5	Fixed wage for managing the idle project
w_a	\$4	Fixed wage for managing the active project
ϕ	—	Value-sharing rate

TABLE 1. The base case parameters.

4.1. Comparative Statics. Optimal trigger ϕ^* maintains the same properties stated on the previous chapter. Recalling equation (3.36), the higher the fixed idle wage w_i , the higher the necessary ϕ^* , representing a higher opportunity cost for the manager weighting on the investment timing decision. The inverse relation is valid for the remaining variables (w_a , r and K_s), meaning that the higher w_a and r , the lower the opportunity cost of the investment decision. Similarly, a superior K_s promotes a higher value that shareholders must abdicate in order to obtain their share on the firm's value.

Concerning the follower firm, we observe similar properties to the monopolistic case, namely the effect on optimal trigger for the follower shareholder and manager. Analyzing

the leader firm, in which we study each of the optimal trigger through an iterative process based on Table 1, we find a similar pattern to the followers result. Therefore:

$$(3.43) \quad \frac{\partial Y_s^F(\phi)}{\partial \phi} > 0$$

$$(3.44) \quad \frac{\partial Y_m^F(\phi)}{\partial \phi} < 0$$

$$(3.45) \quad \frac{\partial Y_s^L(\phi)}{\partial \phi} > 0$$

$$(3.46) \quad \frac{\partial Y_m^L(\phi)}{\partial \phi} < 0$$

In the shareholder's case, a higher (lower) value-sharing factor generates a higher (lower) shareholders optimal trigger, representing an increase on the value of the option to defer the project implementation, while the opposite stands for the manager. Since the manager chooses the real project implementation trigger then, any deviation from the optimal value-sharing factor will generate interests misaligning and agency issues.

Concretely, whenever ϕ changes, the $Z(Y)$ function represented on equation (0.64), will present a positively correlated change, affecting the optimal triggers Y_s^L and Y_s^F (Figure (1)). The opposite is valid for the $N(Y)$ function (stated on equation (0.70)) which presents an opposite relation with ϕ (Figure (2)), resulting in inverse movements of Y_m^L and Y_m^F .

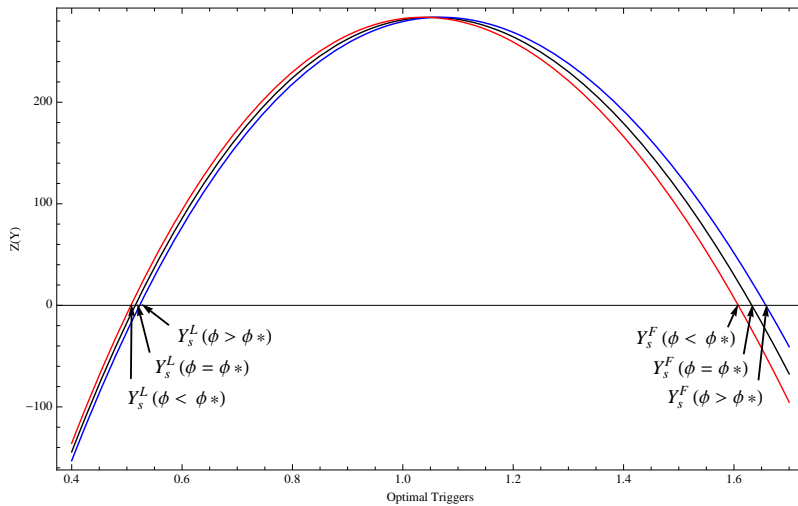


FIGURE 1. The triggers Y_s^L and Y_s^F for different levels of ϕ . Optimal ϕ^* and $Z(Y)$ calculated according to Table 1.

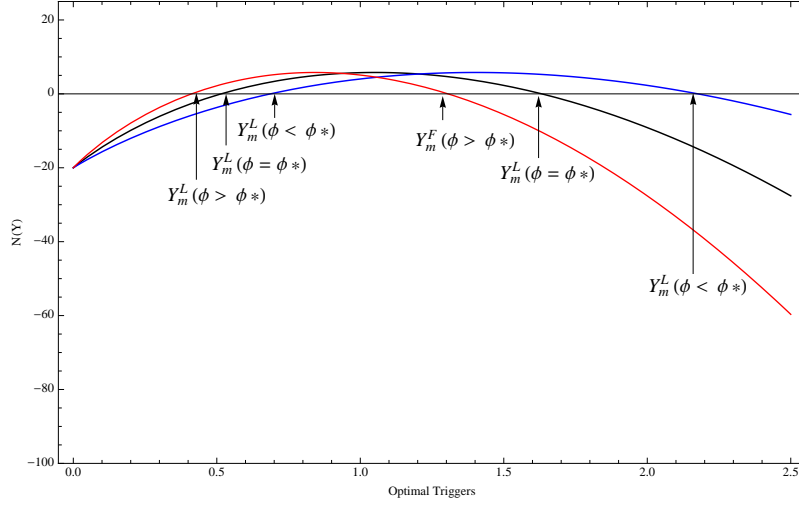


FIGURE 2. The triggers Y_m^L and Y_m^F for different levels of ϕ . Optimal ϕ^* and $N(Y)$ calculated according to Table 1.

Considering Y^{L*} and Y^{F*} as the leader firm and follower firm optimal investment triggers in the non-agency model and considering a fixed w_i and w_a ($w_i > w_a$), we observe that:

$$(3.47) \quad Y_s^F \rightarrow \frac{\beta}{\beta-1} \frac{\delta}{D(2)} \left(K_s + \frac{w_a - w_i}{r} \right) < Y^{F*} \quad \text{as } \phi \rightarrow 0$$

$$(3.48) \quad Y_m^F \rightarrow +\infty \quad \text{as } \phi \rightarrow 0$$

$$(3.49) \quad Y_s^F \rightarrow +\infty \quad \text{as } \phi \rightarrow 1$$

$$(3.50) \quad Y_m^F \rightarrow \frac{\beta}{\beta-1} \frac{\delta}{D(2)} \frac{w_i - w_a}{r} \quad \text{as } \phi \rightarrow 1$$

$$(3.51)$$

$$(3.52) \quad Y_s^L < Y^{L*} \quad \text{as } \phi \rightarrow 0$$

$$(3.53) \quad Y_m^L \rightarrow +\infty \quad \text{as } \phi \rightarrow 0$$

$$(3.54) \quad Y_s^L \rightarrow +\infty \quad \text{as } \phi \rightarrow 1$$

$$(3.55) \quad Y_m^L < Y^{L*} \quad \text{as } \phi \rightarrow 1$$

$$(3.56)$$

and by fixing ϕ , and remember that $\phi \in (0, 1)$, we see that:

$$(3.57) \quad Y_s^F \rightarrow \frac{\beta}{\beta - 1} \frac{\delta}{D(2)} \frac{1}{1 - \phi} K_s > Y^{F*} \quad \text{as } w_a \rightarrow w_i$$

$$(3.58) \quad Y_m^F \rightarrow 0 \quad \text{as } w_a \rightarrow w_i$$

$$(3.59) \quad Y_s^L > Y^{L*} \quad \text{as } w_a \rightarrow w_i$$

$$(3.60) \quad Y_m^F \rightarrow 0 \quad \text{as } w_a \rightarrow w_i$$

These results (figures (1) and (2)) mean that in a suboptimal contract $\phi \neq \phi^*$, both firm's managers will present sub-optimal behavior, choosing triggers that do not fulfill the expectations of the shareholders. This leads to sooner investments, if a higher-than-optimal share of the value is given or later investments if the value-sharing factor is not enough.

The results of equations (3.47) and (3.52) show that, in the absence of a value-sharing compensation, the entry moment of both firms' shareholders will be lower than the non-agency model but, since the ex-post investment fixed wage does not surpass the idle project fixed wage ($w_a < w_i$), the managers will choose not to invest, maintaining a higher fixed salary (equations (3.48) and (3.53)). In opposition, as represented in equations (3.49) and (3.54), $\phi = 1$ translates the hypothetical scenario where shareholders give the total project value to their managers. In this case, shareholders will incur in a cost K_s but, since they do not get any cash-flows, they prefer not to invest. Nevertheless, in this case, the firms' real entry points (Y_m^L and Y_m^F) will only depend on the managers investment cost, $\frac{w_i - w_a}{r}$, tending to its minimal values.

Equations (3.57) and (3.59) shows that if w_a tend to w_i , shareholders lose wage savings, so that the option to delay the investment is still valuable at the optimal entry points Y^{L*} and Y^{F*} . Although the investment cost is higher for the shareholder, if $w_i > w_a$, managers will have a lower opportunity cost to invest so take the decision sooner (equations (3.58) and (3.60)).

4.2. Numerical Example. We will now present a numerical example, considering data on Table (1). We consider a scenario where two firms intend to invest in a new market, being both symmetric under perfect information. Both firms shareholders contract a manager entity to monitor the investment opportunity and its key value-drivers, choosing the investment entrance when adequate. Players will design an optimal contract, using a fixed component and a contingent element based on projects cash-flows, in order to maximize the expected aggregate value of shareholders and manager. This contract framework

transfers to managers the incentive to preempt the rival firm, implying that managers' optimal trigger will equal the desired shareholders investment trigger. The resulting leader, determined exogenously

We will analyze the analytical result, considering equation (3.36) for the optimal ϕ , equations (3.9) and (3.18) to determine the optimal trigger point for the follower firms shareholders and manager respectively, and equation (0.64) and (0.70) from the Appendix to iteratively determine the leader firms shareholder and manager optimal entry triggers. We find the following output results:

Output	Value
ϕ^*	0.02
$Y_s^F = Y_m^F = Y^{F*}$	\$1.63
$Y_s^L = Y_m^L = Y^{L*}$	\$0.52

TABLE 2. The output values for the parameters presented in Table 1.

In both firms, negotiation between shareholders and respective managers will lead to the value-sharing rate of 0.02, the price that ensures ex-ante preemptive optimal behavior of both managers. This optimal contract will guarantee the alignment of managers and shareholders, leading to the leader entry trigger when Y hits \$0.52 and to the subsequent follower entry moment, when a price of \$1.63 is achieved.

Figure (4) shows the ex-ante incentive of both firms' shareholders and managers have to negotiate the optimal contract. In this specific numerical example, the optimal contract $\phi^*(w_a, w_i, K_s)$ is the only combination that ensures value preservation, since all other possible value sharing components ϕ result in a lower expected aggregated value. Nonetheless, we reiterate that this may not always happen, so that other sub-optimal solutions may preserve the aggregate firm value, although it fails in aligning triggers.

Figures 3(a) and 3(b) concern, respectively, the value functions of both firms shareholders and the value function of both firms managers. Since both firms shareholders and respective managers sign the optimal contract mix $\phi^*(w_a, w_i, K_s)$, then the entry moment chosen by managers will fulfill shareholders requirements, equaling the optimal non-agency equilibria (Y^{F*} and Y^{L*}). This is graphically visible on figure 3(c) which also shows that, in the presence of optimal contractual relations, the aggregated shareholders and managers value equals the non-agency value solution, confirming figure (4) result.

5. Conclusion

This chapter analyzes the impact of agency relations in a non-exclusive investment opportunity, overcoming the usual assumption that managers are fully aligned with owners or that the investment opportunity is managed by the shareholders. In this model the

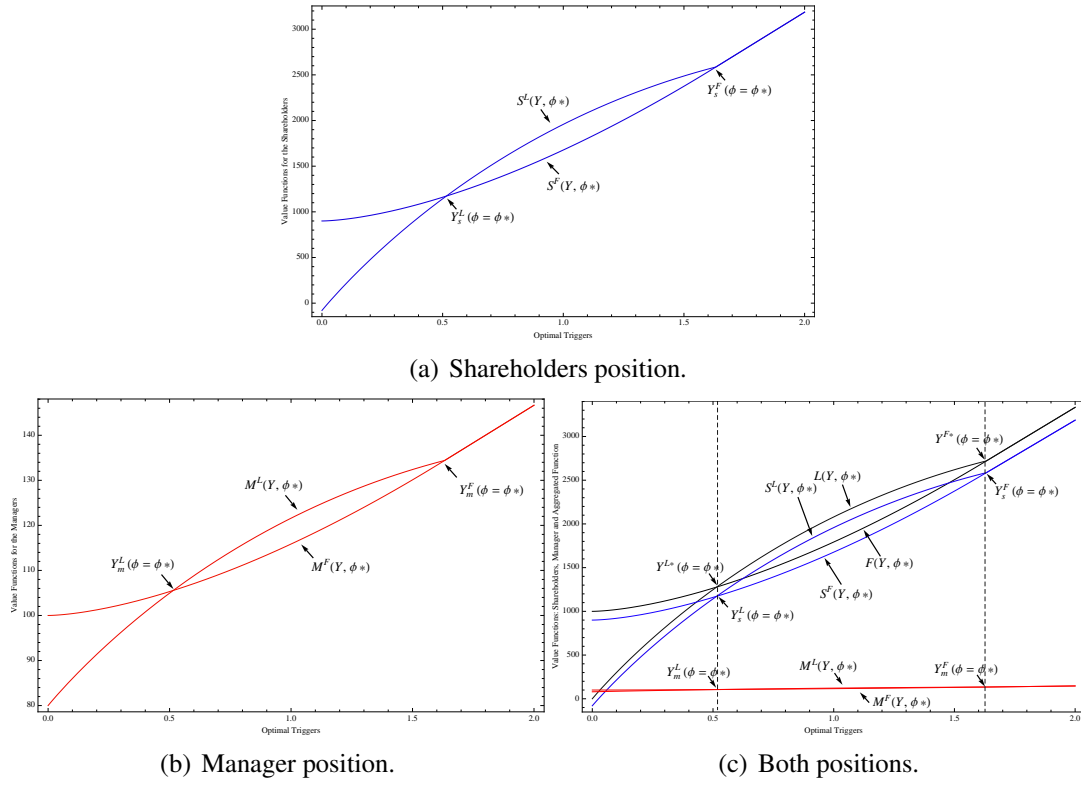


FIGURE 3. The value functions for the shareholders and for manager of both leader and follower firms. The parameters are according to Table 1, and $\phi = \phi^* = 0.02$.

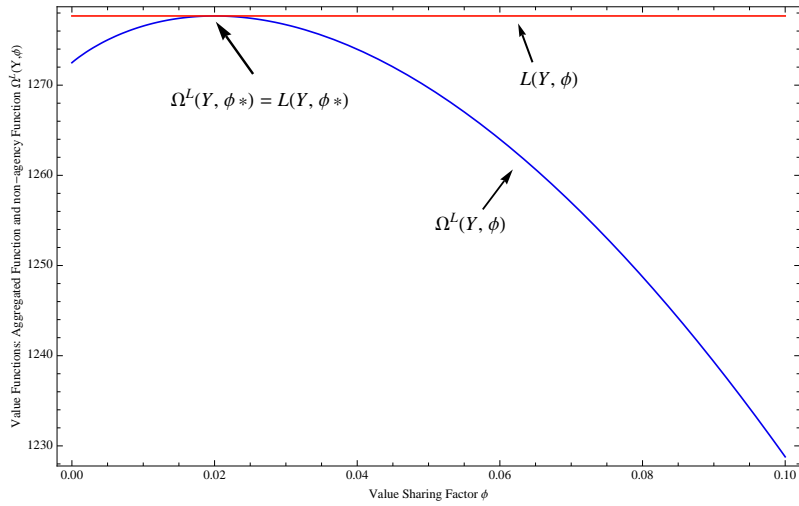


FIGURE 4. The aggregated value function $\Omega(Y, \phi)$ compared with the non-agency leader function $L(Y)$ for different levels of ϕ calculated according to Table 1.

interaction between two firms, that aim to achieve a leader role in this market, is transferred to their respective managers which will maximize their own utility when taking the investment decision. Consequently, shareholders delegate the decision role to managers, which may lead to interest misaligning under sub-optimal contract design due to the information asymmetry that arises between shareholders and managers when the game begins. Considering that each firms' shareholders and respective managers have risk bearing asymmetry, the agency contract design plays a crucial role in interest aligning while taking the investment timing decision.

We propose an optimal contract mix, through fixed wages and a value-sharing component, that enforces optimal aligned behavior of both firms' managers. This contract, determined ex-ante, immediately ensures optimal preemptive behavior when determining the leader role, but also ensures that the exogenously determined follower firm manager will take the optimal investment decision. Since interest aligning is ensured ex-ante, shareholders of both firms need not to follow the evolution of the investment opportunity value drivers in order to guarantee optimal behavior.

Deviations from the optimal contract lead to sub-optimal investment triggers, which may lead in most of the circumstances to shareholders and managers aggregated value destruction. This situation incentivizes the contractual negotiation to tend to the optimal contract, reflecting the relative opportunity cost of manager and shareholders.

Appendix

Lemma 2.1: There exists a unique point $Y_s^L \in (0, Y_s^F)$ such that:

$$(0.61) \quad S^L(Y_s^L) - K_s = S^F(Y_s^L)$$

$$(0.62) \quad S^L(Y) - K_s < S^F(Y), \text{ for } Y < Y_s^L$$

$$(0.63) \quad S^L(Y) - K_s \geq S^F(Y), \text{ for } Y > Y_s^L$$

PROOF. 2.1 We define the function $Z(Y) = S^L(Y) - K_s - S^F(Y)$ which represents the gain of preempting the rival paying the investment cost against being preempted. Rearranging using equations (3.22) and (3.8) we have:

$$(0.64) \quad Z(Y) = (1 - \phi) \frac{YD(1)}{r - \alpha} + \left(\frac{Y}{Y_s^F} \right)^{\beta_1} \frac{1}{\beta_1 - 1} \left[\beta_1 \left(1 - \frac{D(1)}{D(2)} \right) - 1 \right] \left(K_s - \frac{w_i - w_a}{r} \right) - K_s + \frac{w_i - w_a}{r}$$

Calculating $Z(Y)$ at $Y = 0$ we get $Z(0) = -K_s + \frac{w_i - w_a}{r}$, and $Z(Y)$ at $Y = Y_s^F$ we have $Z(Y_s^F) = 0$. Deriving $Z(Y)$ at Y_s^F results in:

$$(0.65) \quad Z'(Y)|_{Y=Y_s^F} = -(1 - \phi)(\beta_1 - 1) \frac{D(1) - D(2)}{r - \alpha} < 0$$

since $\beta_1 > 1$, $\phi < 1$, $r > \alpha$ and $D(1) > D(2)$.

We prove uniqueness of Y_s^L demonstrating strict concavity of $Z(Y)$ over the interval $(0, Y_s^F)$. The second derivative of $Z(Y)$ is:

$$(0.66) \quad Z''(Y) = \beta_1 \left(\frac{Y}{Y_s^F} \right)^{\beta_1} \frac{(K_s - \frac{w_i - w_a}{r})}{Y^2} \left[\beta_1 \left(1 - \frac{D(1)}{D(2)} \right) - 1 \right] < 0$$

These results guarantee that Y_s^L is unique over the interval $(0, Y_s^F)$. □

Lemma 2.2: There exists a unique point $Y_m^L \in (0, Y_m^F)$ such that:

$$(0.67) \quad M^L(Y_m^L) - K_s = S^F(Y_m^L)$$

$$(0.68) \quad M^L(Y) < M^F(Y), \text{ for } Y < Y_m^L$$

$$(0.69) \quad M^L(Y) \geq M^F(Y), \text{ for } Y > Y_m^L$$

PROOF. 2.2 Defining the function $N(Y) = M^L(Y) - M^F(Y)$ we intend to comprehend the managerial benefit of preempting its rival . Rearranging using equations (3.30) and (3.17) we have:

$$(0.70) \quad N(Y) = \phi \frac{YD(1)}{r - \alpha} + \left(\frac{Y}{Y_s^F} \right)^{\beta_1} \frac{1}{\beta_1 - 1} \left[\beta_1 \left(1 - \frac{D(1)}{D(2)} \right) - 1 \right] \left(\frac{w_i - w_a}{r} \right) - \frac{w_i - w_a}{r}$$

Calculating $N(Y)$ at $Y = 0$ and at $Y = Y_m^F$ we get $N(0) = -\frac{w_i - w_a}{r}$ and $N(Y_m^F) = 0$. Deriving $N(Y)$ at Y_s^F results in:

$$(0.71) \quad N'(Y)|_{Y=Y_m^F} = -\phi(\beta_1 - 1) \frac{D(1) - D(2)}{r - \alpha} < 0$$

Since equation (0.71) is negative, we prove that condition $N(Y)$ have at least on root in the interval $(0, Y_m^F)$. Additionally, it is necessary to prove that Y_m^L is unique by demonstrating strict concavity of $N(Y)$ over this interval:

$$(0.72) \quad N''(Y) = \beta_1 \left(\frac{Y}{Y_s^F} \right)^{\beta_1} \frac{w_i - w_a}{r Y^2} \left[\beta_1 \left(1 - \frac{D(1)}{D(2)} \right) - 1 \right] < 0$$

The second derivative of $N(Y)$ is represented on equation and guarantees that Y_m^L is the only root of $N(Y)$ in interval $(0, Y_m^F)$.

□

Lemma 3.1: The optimal contract mix $\phi^*(w_a, w_i, K_s)$, presented on equation (3.36) guarantees that manager chooses the leader shareholders' optimal trigger:

$$(0.73) \quad Y_s^L|_{\phi=\phi^*} = Y_m^L|_{\phi=\phi^*}$$

Implied from:

$$(0.74) \quad \arg \max_{\phi \in (0,1)} \Omega(Y, \phi) = \frac{K_m}{K_s}$$

PROOF. 3.1 The optimal contract mix ($\phi^*(w_a, w_i, K_s)$) of the leader firm is obtained by finding the maximum value possible for the aggregated leader firm value $\Omega(Y, \phi)$, as stated on equation (3.38). Therefore, we have:

$$(0.75) \quad \frac{\partial \Omega(Y, \phi)}{\partial \phi} = 0 \Rightarrow \phi = \phi^* = \frac{K_m}{K_s}$$

$$(0.76) \quad \frac{\partial^2 \Omega(Y, \phi)}{\partial \phi^2} < 0$$

□

Lemma 3.2: There exists a consistent tangible point at $\phi^*(w_a, w_i, K_s)$, such that:

$$(0.77) \quad \Omega(Y, \phi^*) = L(Y), \text{ for } \phi = \phi^*$$

Other contract combinations may result in value loss, such that:

$$(0.78) \quad \Omega(Y, \phi) < L(Y), \text{ for } \phi \neq \phi^*$$

PROOF. 3.2 Considering:

$$(0.79) \quad \Theta(Y, \phi) = L(Y) - \Omega(Y, \phi)$$

Our aim is to demonstrate that the optimal contract mix $\phi^*(w_a, w_i, K_s)$ totally mitigates all agency costs. Substituting $\phi = \phi^*$ in equation (0.79), we verify that $\Theta(Y, \phi^*) = 0$, implying that the aggregate value of shareholders and manager equals the non-agency solution.

We also want to understand if there are agency costs if $\phi \neq \phi^*$ and in what circumstances, comparatively to the non-agency solution. Nonetheless, equation will have different breaking points, meaning that when $\phi \neq \phi^*$ there will be a mismatch between the triggers Y_m^F and Y^F , breaking the aggregate function $\Theta(Y, \phi)$ in three branches. Consequently, we must study three different situations (where Y^F is the optimal investment trigger of follower firm in the agency-free model). So, rearranging equation (0.79):

$$(0.80) \quad \Theta(Y, \phi) = \begin{cases} \frac{\beta_1}{\beta_1 - 1} \left(1 - \frac{D(1)}{D(2)}\right) \left[K_s \left(\left(\frac{Y}{Y^F}\right)^{\beta_1} - \left(\frac{Y}{Y_s^F}\right)^{\beta_1} \right) + \frac{w_i - w_a}{r} \left(\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \right) \right] & \text{for } Y < Y_m^F \wedge \phi > \phi^* \\ \frac{\beta_1}{\beta_1 - 1} \left(1 - \frac{D(1)}{D(2)}\right) \left[K_s \left(\left(\frac{Y}{Y^F}\right)^{\beta_1} - \left(\frac{Y}{Y_s^F}\right)^{\beta_1} \right) + \frac{w_i - w_a}{r} \left(\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \right) \right] & \text{for } Y < Y^F \wedge \phi < \phi^* \\ \frac{Y(D(2) - D(1))}{\delta} - \frac{\beta_1}{\beta_1 - 1} \left(1 - \frac{D(1)}{D(2)}\right) \left[\left(\frac{Y}{Y_s^F}\right)^{\beta_1} K_s - \left(\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \right) \frac{w_i - w_a}{r} \right] & \text{for } Y^F < Y < Y_m^F \wedge \phi < \phi^* \end{cases}$$

$$(0.81) \quad \text{sgn}\{\Theta(Y, \phi)\} = \begin{cases} \Theta(Y, \phi) > 0 & \text{if } \frac{\left(\frac{1}{Y^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}}{\left(\frac{1}{Y_m^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}} > \phi^*, \text{ for } Y < Y_m^F \wedge \phi > \phi^* \\ \Theta(Y, \phi) > 0 & \text{if } \frac{\left(\frac{1}{Y^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}}{\left(\frac{1}{Y_m^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}} > \phi^*, \text{ for } Y < Y^F \wedge \phi < \phi^* \\ \Theta(Y, \phi) > 0 & \text{if } K_s \left(\frac{Y}{Y_s^F}\right)^{\beta_1} > \frac{YD(2)}{\delta} \frac{\beta_1 - 1}{\beta_1} + \frac{w_i - w_a}{r} \left[\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \right], \text{ for } Y^F < Y < Y_m^F \wedge \phi < \phi^* \end{cases}$$

This result proves that, in most of the circumstances that $\phi \neq \phi^*$, there are agency costs and, consequently, the non-agency value $L(Y)$ is higher than the aggregated result $\Omega(Y, \phi)$. Nonetheless, the most important observation is that the optimal labor contract $\phi^*(w_a, w_i, Ks)$ always mitigates all agency costs, providing the first best solution.

□

CHAPTER 4

Conclusion

Real options theory frequently assumes that investment decision is directly taken by the owners or that, if there is a manager, he is fully aligned with them. This thesis provides a contribute to the existing literature by dropping this assumption and considering the existence of a managerial entity which manages the investment opportunity and takes the investment decision when and whether appropriate.

Therefore, we consider that shareholders of a firm, which intends to enter in a new market, need to hire a manager to supervise the investment option and take the implementation decision, which may be justified by several possible reasons for instance, lack of knowledge or opportunity cost matters. Consequently, shareholders will offer a contract mix composed by a continuous fixed wage while the project is idle and a combination of a continuous fixed wage and a variable value sharing component when the project is active.

Since manager is a utility maximizer agent, his acts may differ from the shareholders desired choices, when a sub-optimal contract is agreed, generating deviated investment decisions. So, we find that under some conditions, and since there are information asymmetry shareholders ignore the options value drivers evolution - an agency issue arises. Nonetheless, providing an optimal contract mix that properly represents the relative opportunity costs of manager and shareholders, the latter can ensure optimal agent behavior without the need of further supervision. So, through this simple but adequate contract structure, we can simultaneously embody the agency issue and provide an optimal solution that mitigates the risks and the consequent agency costs.

This development was firstly extended to an exclusive investment solution in Chapter 2, where we provide the analytical formulation of the problem in an exclusive option scenario, further solving it and providing three additional extensions concerning the existence of impatient managers, the existence of a hidden rival and the inclusion of management effort costs. In chapter 3 we develop a duopolistic leader-follower framework where the same agency problem is embodied and solved by a similar optimal contract design. Each firms shareholders and respective manager have perfect information about the rivals contract definition, so that preemptive behavior leads the firms to negotiate the same contract. Sub-optimal contract design will motivate undesired investment decisions that generate

value misappropriation and, in some circumstances, may lead to aggregate value destruction. Consequently, each firm's principal and agent negotiate ex-ante the optimal contract guaranteeing interest alignment and optimal preemptive behavior.

Concluding, these achievements give a simple but meaningful contribution to Real Options Agency Theory, so that the foundations to further developments may flourish. As a result, future possible paths may be the extension to an oligopolistic framework and a perfect competition environment, the development of a duopolistic model where rival firm contract negotiation is unknown so that, both firms' shareholders and managers work with ex-ante uncertainty.

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