Flexible cantilever retaining walls: design according to Eurocode 7 and classical methods

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ABSTRACT: The paper presents a comparative study on the design of flexible cantilever retaining walls using classical limit equilibrium methods and a finite element model. The conventional approaches of introducing safety in the calculations are referred as well as the Cases B+C approach established in Eurocode 7. The results of the design, in particular the wall embedded height and the maximum bending moment, provided by the distinct approaches are discussed and compared. A discussion is presented on the way to deal with Cases B+C approach when finite element analyses instead of limit equilibrium methods are used. Considering a simple numerical case study, the results from a set of finite element simulations, considering cantilever walls designed by means of a classical and Cases B+C approaches, are discussed. Some practical conclusions are drawn.

1 INTRODUCTION

The conventional design methods of flexible cantilever walls are limit equilibrium methods, which involve the consideration of active and passive earth pressures in close relation with an assumed movement of the wall as a rigid body. The design mainly consists of calculating the embedded wall height and the bending moment distribution.

Figure 1a shows the idealized distribution of earth pressures on the wall assumed in a well-known method of design (Padfield & Mair, 1984). A point close to the wall tip, P, is the centre of rotation of the wall. Above that point passive pressures in front of the wall are assumed and active pressures are considered at the opposite side; from P to the wall tip passive pressures occur behind the wall whereas active pressures are mobilized in front of it.

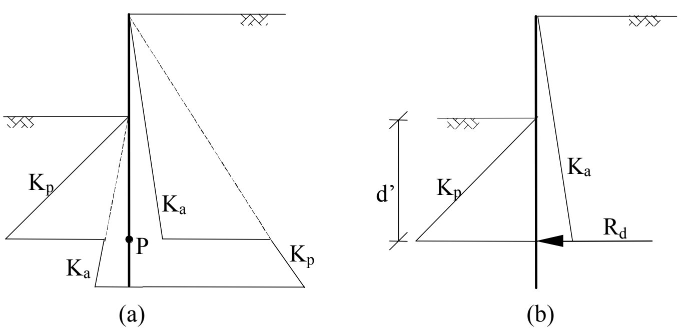


Figure 1. Limit equilibrium method of design: (a) - idealized distribution of earth pressures on the wall; (b) - system considered in the equilibrium equations.

As shown in Figure 1b, the pressures close to the wall tip are, for practical purposes, replaced by the respective resultant, R_d, applied at P. An equation of moment equilibrium in relation to P allows the determination of d'. With an equation involving horizontal forces, R_d is obtained. The distribution of the bending moments between the top of the wall and P can then be easily computed.

The actual embedded depth, d, is then taken as equal to 1.2d'. A check is recommended to ensure that R_d is lower or equal to the resultant of the earth pressures exerting between d' and 1.2d'.

Using this methodology, several approaches related to the introduction of safety in the calculations are known:

approach 1 - a classical method is to divide the passive pressures in front of the wall by a factor, F_p, ranging from 1.5 to 2.0 (Fig. 2a);

approach 2 - Burland et al. (1981) proposed an adjustment to the later approach, introducing a factor, F_r, by which the net available passive earth pressures (that is, the resultant of the passive and active pressures on the embedded length) should be divided (Fig. 2b); a value equal to 2.0 was proposed for that coefficient;

approach 3- an alternative method consists of increasing the embedded wall length by a factor F_d; a value of 1.3 is normally adopted;

approach 4- partial safety factors according EC 7 (see Table 1) can also be applied; if live loads at the surface are assumed as null, Case C corresponds to consider design values of permanent actions equal to

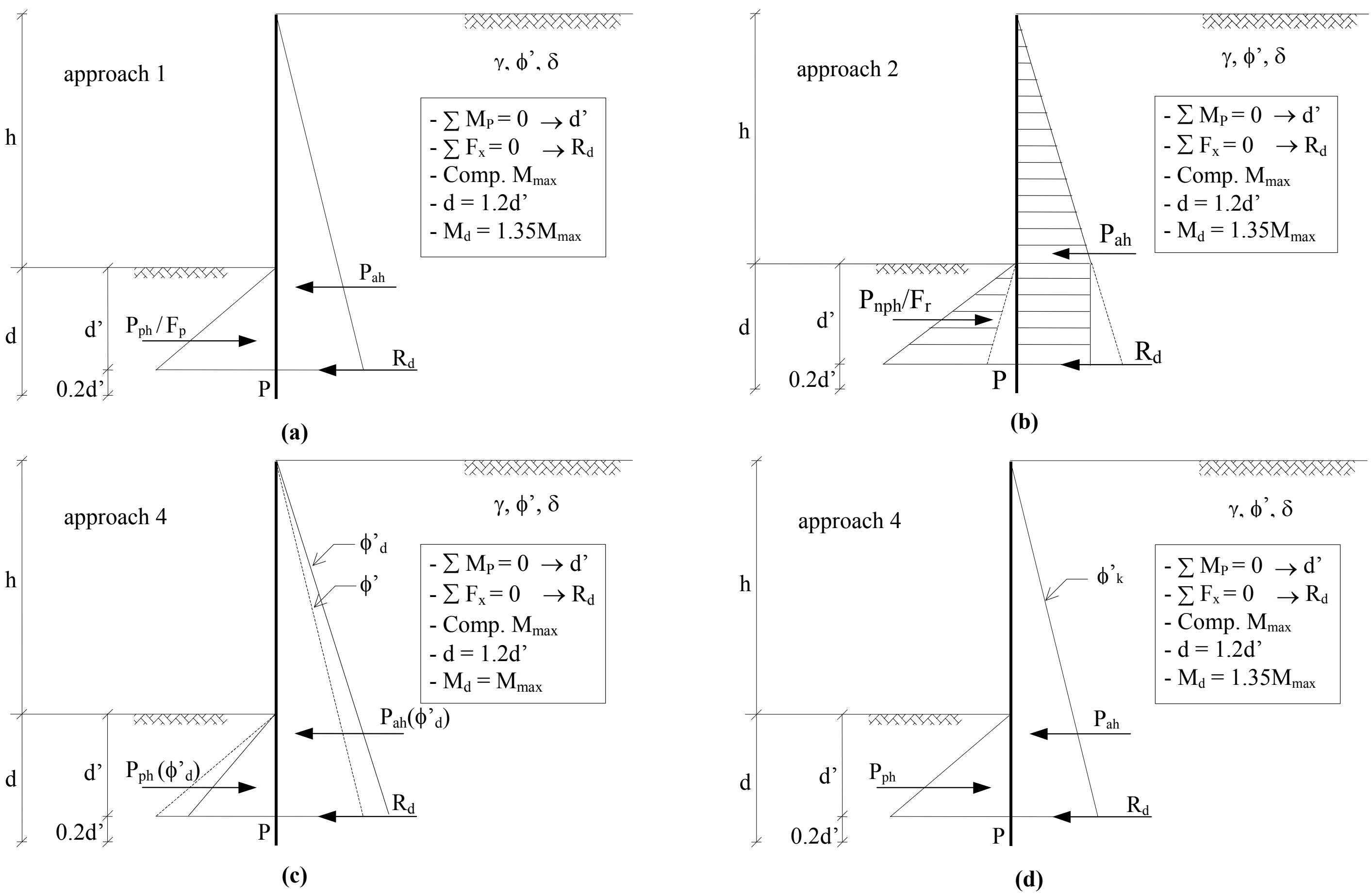


Figure 2. Approaches for the introduction of safety in the design of flexible cantilever retaining walls: (a) dividing the passive thrust by F_p; (b) dividing the net passive thrust by F_r; (c) EC 7 - Case C; (d) EC 7 - Case B

the characteristic ones ($\gamma_G = 1$), together with design values of the soil parameters obtained on the basis of partial safety factors γ_M greater than 1.0 (Fig. 2c); for Case B, characteristic strength parameters are used in combination with a factor of 1.35 to multiply permanent actions or their effect (Fig. 2d).

In order to compare results of the maximum wall bending moment, the values obtained from the approaches 1 to 3, which can be considered characteristic values, are multiplied by 1.35.

Table 1 (ENV 1997-1:1994). Partial factors – ultimate limit states in persistent and transient situations

Case	Actions (γ_F)			Ground Properties (γ_M)		
	Permanent (γ_G)		Variab. (γ_Q)	tan φ'	c'	c_u
	Unfav.	Fav.	Unfav.			
A	1.00	0.95	1.50	1.10	1.20	1.20
B	1.35	1.00	1.50	1.00	1.00	1.00
C	1.00	1.00	1.30	1.25	1.60	1.40

Note: Case A is only relevant to buoyancy problems, where hydrostatic forces comprise the main unfavourable action.

2 STUDY USING THE CLASSICAL LIMIT EQUILIBRIUM METHOD

2.1 *General*

In this section a brief comparative study on the application of the approaches just described is presented. An excavation in a homogeneous soil is considered with the angle of shearing resistance φ′ varying from 20° to 40°, in order to cover the most common range of the parameter from plastic clayey soils to dense sands; values of the soil unit weight have been adjusted following the variation of φ′(18kN/m^3 to 22 kN/m^3). Two conditions have been considered for the angle defining the strength of the soil-to-wall interface: $\delta = 0$ and $\delta = (2/3)\phi'$. The water table was assumed below the wall tip (W.L.0) or coincident with the bottom of the excavation (W.L.1). The active and passive earth pressure coefficients have been obtained on the basis of the theory of Caquot and Kérisel.

For the conditions assumed in the study – namely those concerning the water table and the homogeneity of the soil interested by the excavation – the embedded wall height is proportional to the excavation depth, h, whereas the maximum bending moment is proportional to γh^3, with the constants of proportionality being determined by the specific conditions of each problem.

2.2 *Comment on the results*

Figure 3 summarizes the results obtained. The graphs on the left concern the wall height whereas the design bending moments are presented on the right. The results from *approach 3* are not shown in

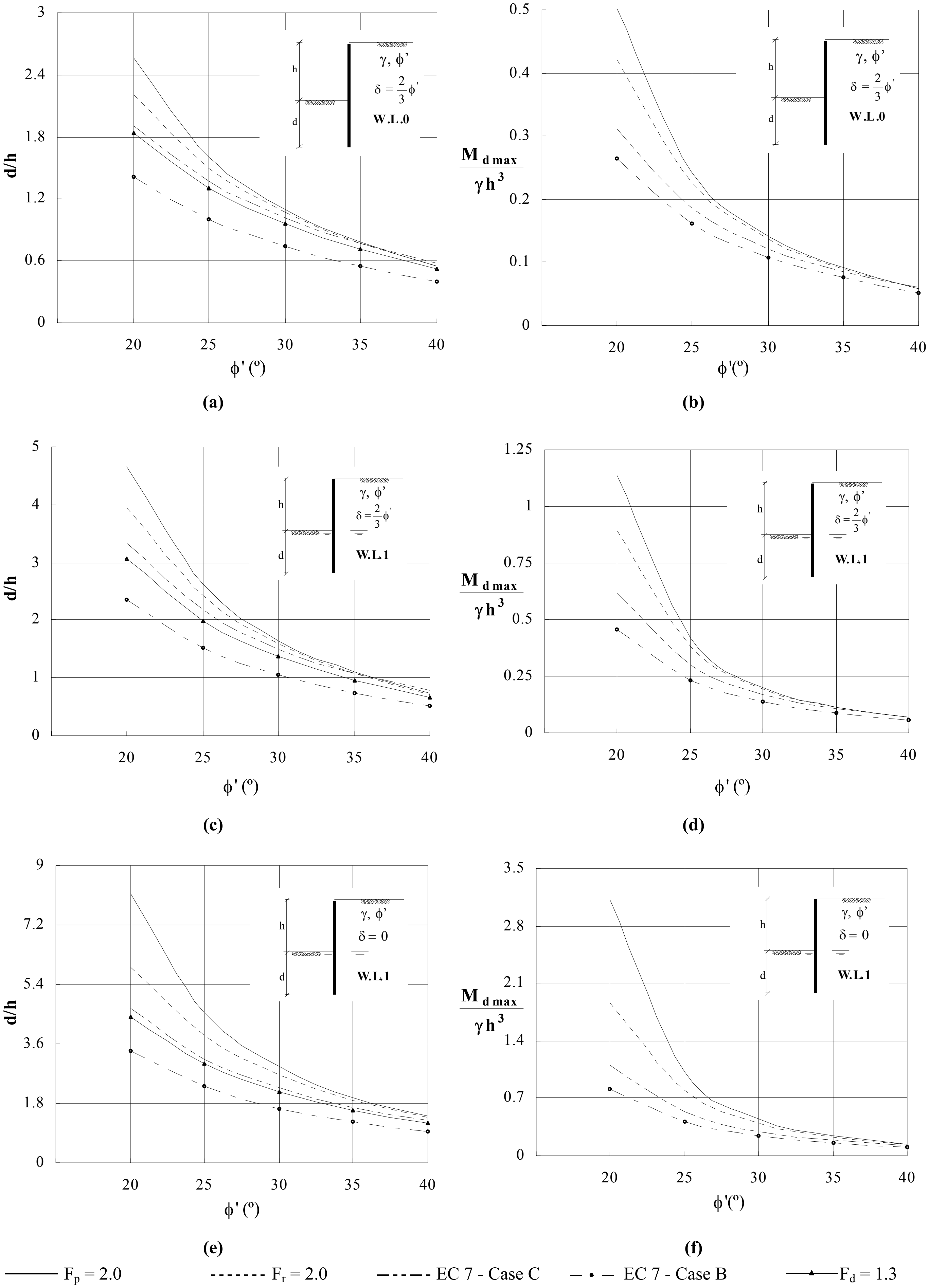


Figure 3. Embedded wall height and design wall bending moment: (a,b) water level below the wall tip and δ = (2/3) φ'; (c.d) water level at the base of the cut and δ = (2/3) φ'; (e,f) water level at the base of the cut and δ = 0

the graphs on the right because the design value of the wall bending moment coincides with the one given by the Case B.

The observation of the figure shows that: i) the quantitative differences among the results of the various approaches tend to evanesce with the increase of the angle of shearing resistance; ii) the use of $F_p = 2.0$ is the most conservative approach for values of φ’ less than 35º; iii) $F_r = 2.0$ leads to results very similar to the previous one for values of φ′ greater than 30°; iv) *approach 3* (F_d = 1.3) is more economical than the others; v) results from EC 7 - Case C lie between the ones from *approaches 2* and *3* for φ′ less than 35° and are the most conservative for φ′ over that value; vi) when EC 7 is applied to this type of retaining walls, Case C determines not only the sizing of the structure (note that the embedded height from Case B marginally exceeds the one corresponding to a limit equilibrium situation) but also the strength of the structural elements. These trends seem to be independent on the water table conditions and on the resistance of the soil-to-wall interface.

Comparing now the two first layers of graphs of Figure 3, the remarkable influence of the position of ground water surface can be easily evaluated. For example, when the angle of shearing resistance φ’ is equal to 30º, it can be observed that rising the water level from the wall tip up to the base of the excavation leads to an increase of around 50% on the embedded wall height and 40% on the maximum bending moment.

A comparison of the second and third layers of graphs of the same figure, which correspond to the same water conditions, permits an assessment to the influence of the soil-to-wall angle of friction. It can be observed that a smooth interface implies a very substantial increase on the embedded wall height and on the design bending moments.

3 STUDY USING A FINITE ELEMENT MODEL

3.1 *Numerical case study*

This part of the paper presents a study using a finite element model of an excavation 5 m deep in a homogeneous layer of sand, supported by a cantilever diaphragm wall 0.4 m thick. Surface surcharges are not considered.

Analyses have been performed in effective stresses and plane strain conditions. It is assumed that the wall installation does not affect the at-rest state of stress.

An elastic-perfectly plastic constitutive law, with a Mohr-Coulomb failure envelop and non-associated plastic flow, has been adopted for the soil. A similar behaviour was considered for the soil-to-wall interface. Reinforced concrete was assumed as linear elastic (E = 20 GPa, ν = 0.2).

The finite element mesh is represented in Figure 4 together with the parameters assumed for the sand and the two conditions concerning ground water: water level below the wall tip (W.L.0) or coinciding with the base of the excavation (W.L.1). The mesh includes 3260 nodal points, 1008 eight-node quadrangular isoparametric elements to represent the soil and the wall and 96 six-node joint elements to represent the soil-to-wall interface. The excavation is simulated in 19 stages.

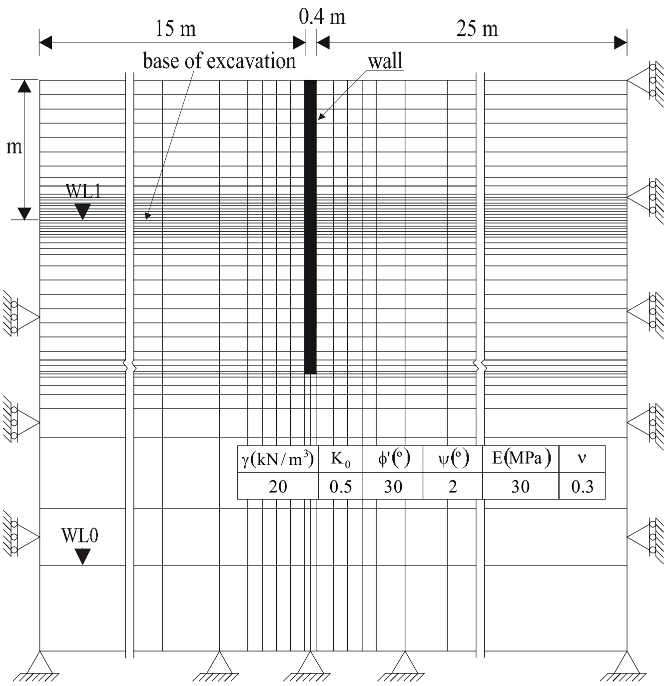


Figure 4. Finite element mesh and main assumptions of the numerical case study

3.2 *Wall designed according approach 2 (F_r = 2.0)*

A few finite element analyses have been carried out with the wall embedded height obtained from the limit equilibrium method combined with the above called *approach 2*, for the two conditions concerning the water level and the two values of the soil-to-wall angle of shearing resistance. Figures 5 to 7 summarize the results of the analysis combining the water level coinciding with the bottom of the excavation (W.L. 1) and δ = (2/3) φ′. They show, respectively, the evolution of the lateral wall displacements, the distribution of the final wall bending moments and the effective normal earth pressures on both sides of the wall at the end of the excavation (5 m).

The comparison of the maximum bending moments with the values provided by the limit equilibrium method will be done later.

In Figure 7 the dashed lines correspond to the horizontal components of the theoretical active (behind the wall) and passive (in front of the wall) earth pressures, or to their algebraic addition, computed for the values of δ effectively mobilized in the analyses. These values, which are shown on the left

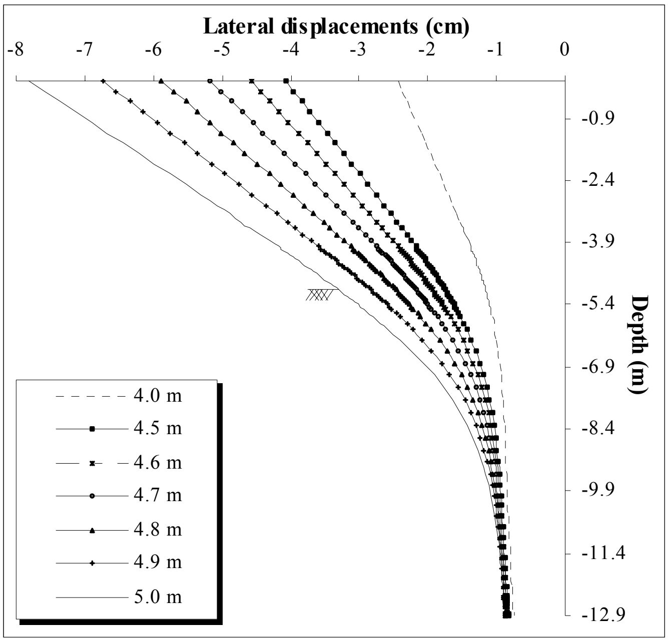


Figure 5. Wall designed according *approach 2* (W.L.1, δ = (2/3) ϕ'): evolution of the lateral wall displacements

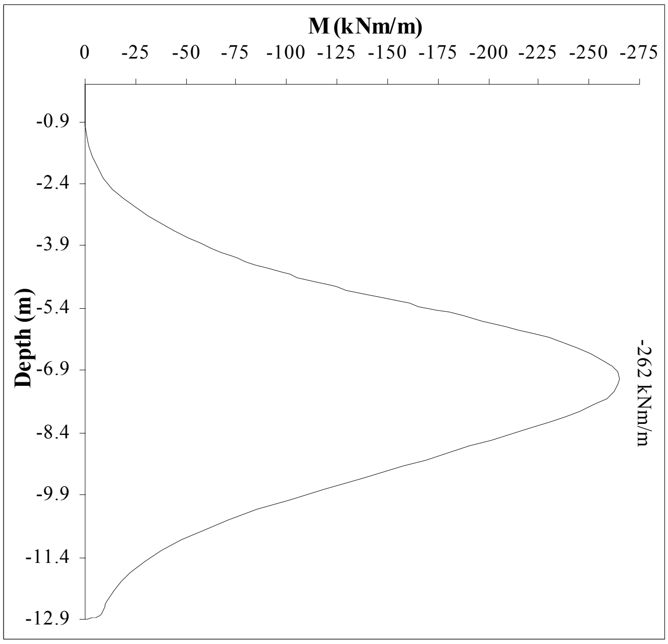
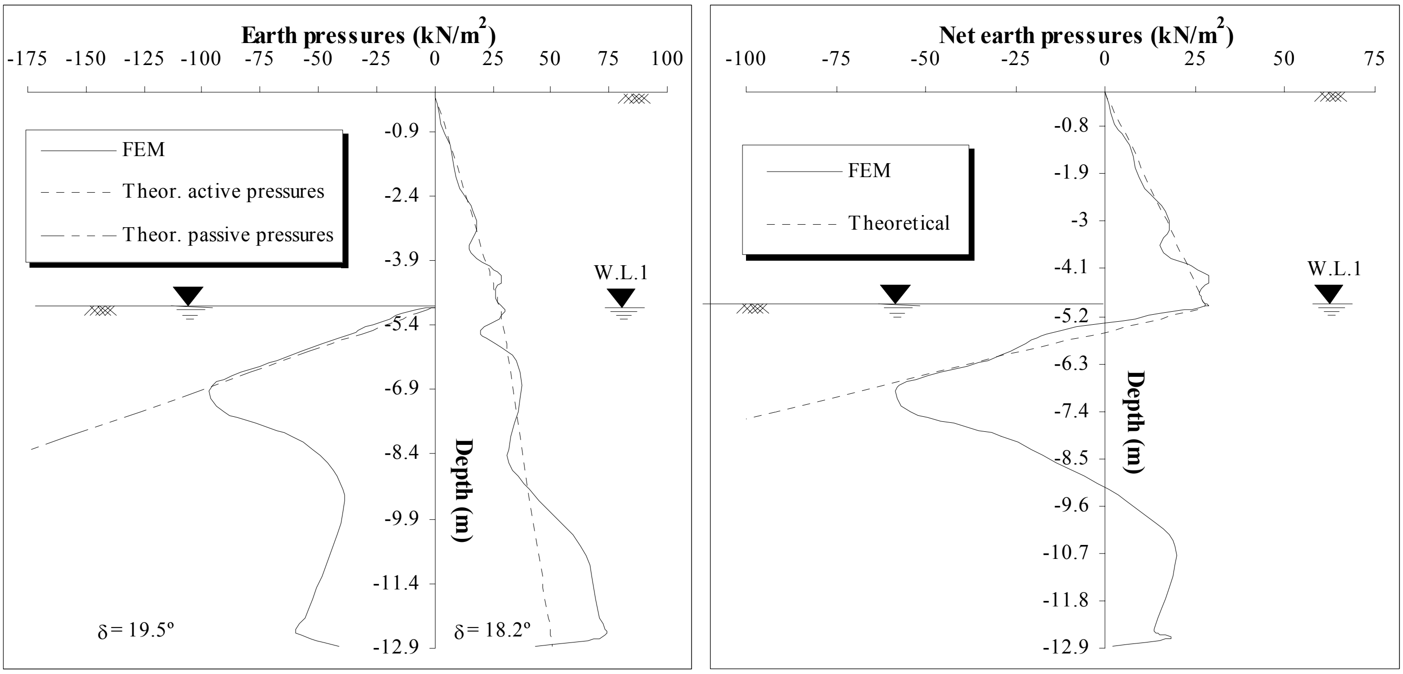


Figure 6. Wall designed according *approach 2* (W.L.1, δ = (2/3) ϕ'): distribution of the final wall bending moments



(a) (b)

Figure 7. Wall designed according *approach 2* (W.L.1, δ = (2/3) ϕ'): effective final normal earth pressures (a) on both sides of the wall and (b) showing the net pressures on the embedded height

graphs, are close to the one assumed in the limit equilibrium method (20°). Typically, the mobilized values of δ in front of the wall are greater than the ones at the opposite side.

3.3 *Wall designed according to Eurocode 7*

3.3.1 *General*

As was noted before, Case B provides an embedded wall height that corresponds to a safety factor marginally greater than 1.0 in relation to a limit state of rotation around the wall tip. Since the wall height will never be conditioned by this case, the finite element analyses presented in this paragraph considered the wall geometry provided by the limit equilibrium method explained in Figures 1 and 2 combined with the safety factors of Case C.

Two types of analyses have been carried out:

- *analyses CC*, ie analyses in which the wall height is calculated as referred above and characteristic permanent loads and design values of the strength parameters of the ground according to Case C are considered;

- *analyses CB*, ie analyses with wall height obtained in the same way as before, taking characteristic permanent loads and characteristic values of soil properties, and whose results in terms of internal structural forces are multiplied by 1.35 (this corresponds to Case B).

This two types of analyses seem to be the proper way of applying finite element models to this problems of soil-structure interaction in the context of the methodology "Case B + Case C" of Eurocode 7. This can be explained with the help of Figure 8, which refers, for instance, to the structural failure of the cantilever wall by bending. The curve corresponds to the infinite number of combinations of values of "actions" and "resistance" that lead to the limit state. It then divides two regions: collapse and stability. Point P represents the position of the "safe" structure according to EC 7. The vertical distance from P to the curve corresponds to the margin of safety provided by Case B – to be checked with the help of f. e. *analyses CB* - whereas the horizontal distance represents the margin of safety from Case C – to be verified by using *CC analyses*.

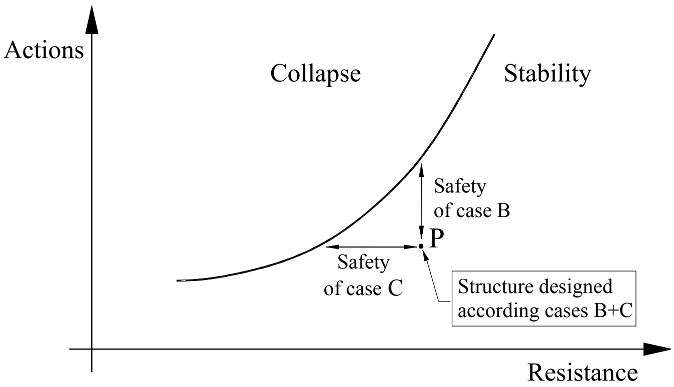


Figure 8. Scheme of safety checks for the Cases B+C approach of EC 7

3.3.2 *Discussion of the results*

Finite element analyses of the two types CC and CB were undertaken for the two ground water conditions and the two values of δ. The wall embedded height calculated by the limit equilibrium method (combined with Case C safety factors) was adopted in the numerical analyses presented next.

The results for ground water coinciding with the base of the excavation (W.L.1) and $\delta = (2/3)\phi'$ are shown in Figures 9 to 11, concerning the lateral wall displacements (final results, Figure 9a, and the evolution with the progress of the excavation, Figure 9b), final effective normal earth pressures and wall bending moments, respectively. The bending moments from analysis CB, represented in Figure 11, correspond to the finite element results multiplied by the factor 1.35.

It should be noted that in the analysis CC the wall is in a situation of imminent collapse when the excavation depth attains 5 m, which is obvious from the observation of Figure 9b, because the embedded height is the one derived from the limit equilibrium calculations using the design (reduced) values of ground resistant properties, which have also been introduced in the finite element analysis. Naturally, the displacements from analysis CB are much lower because this corresponds to the same wall height combined with the characteristic values of the soil strength parameters.

The pattern of the distribution of final earth pressures on the embedded wall height is quite distinct in the two analyses, particularly with regard to the very high pressures on the back of the wall close to the tip in analysis CC. This is in agreement with the idealized distribution of earth pressures shown in Figure 1 for a wall in limit equilibrium conditions, which is, in a large extent, the case of this analysis.

3.4 *Overall comparison of the results*

In this paragraph a comparison is made among the results of all the finite element analyses, as well as the results of the limit equilibrium calculations previously performed to obtain the wall embedded height which is an input in the numerical simulations.

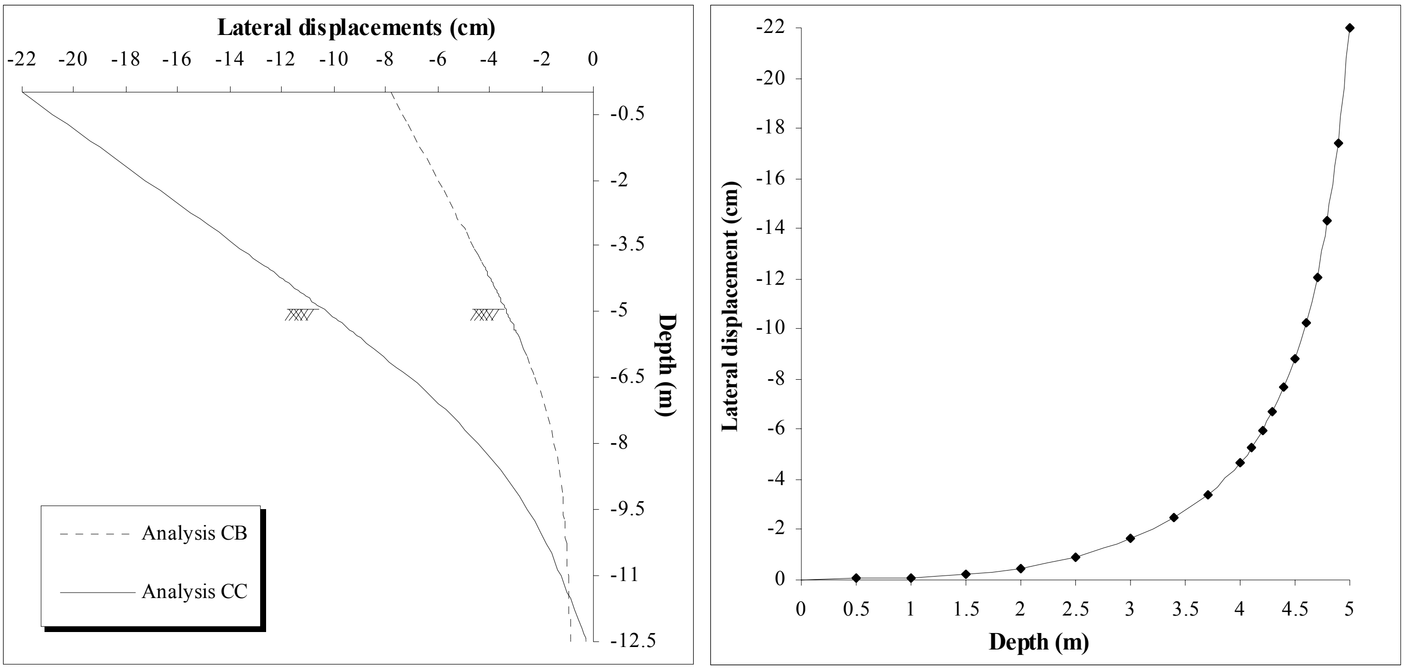
Table 2 summarizes the results in terms of wall embedded height and maximum wall bending moments. The two assumed ground water conditions and values of the soil-to-wall interface resistance are included, as well as the three distinct types of finite element analyses: the so-called CB and CC, as well as the ones taking the wall height from the classical *approach 2* ($F_r = 2.0$). For each case two distinct values of the wall bending moment are shown: the one given by the limit equilibrium calculation (LEC) and the finite element model result (FEM). All the bending moments are multiplied by the factor 1.35 except the finite element results from analyses CC.

In what concerns the wall embedded height, the differences between the classical *approach 2* and EC 7-Case C results could be anticipated bearing in mind the parametric study presented in section 3.

About the design wall bending moments, some comments may be done;

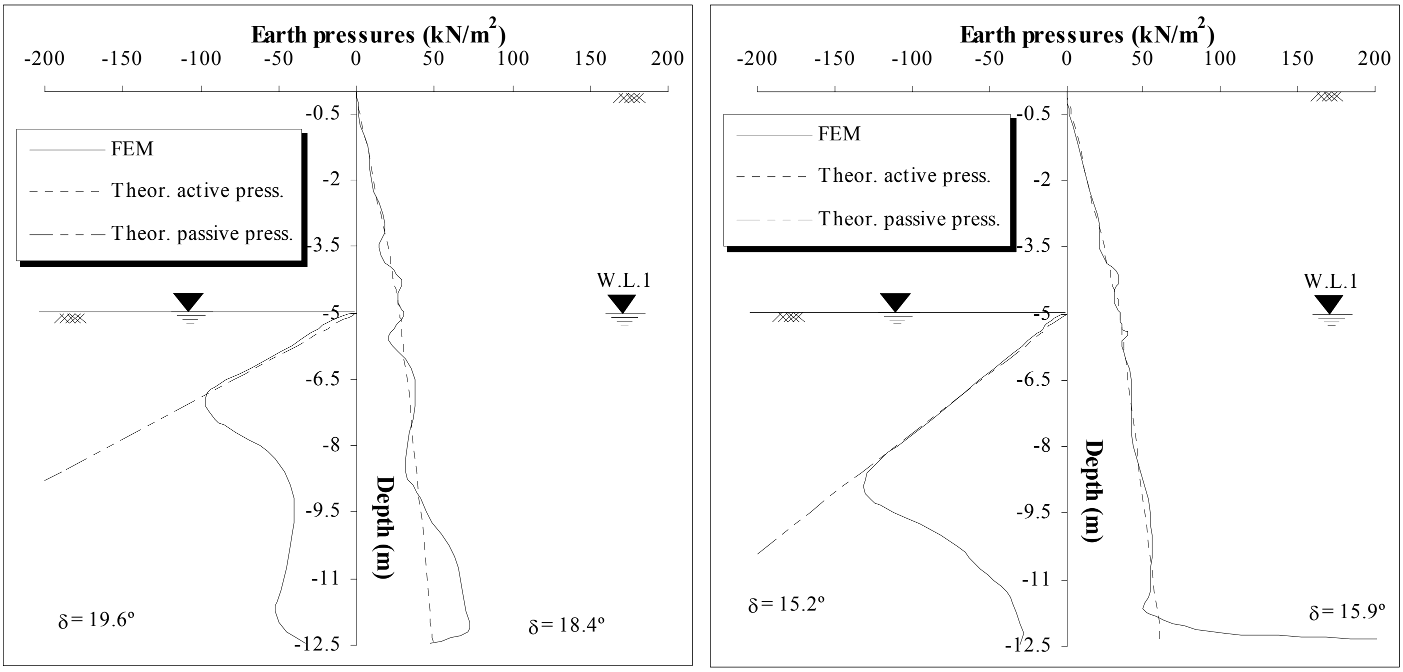
i) the moments from LEC are always greater than the ones from FEM; however, the discrepancies are substantial from *approach 2* (63 to 94%) and quite small for the analyses CC (8 to 16%); this is due to the fact that in this type of analyses the wall is very close to a limit equilibrium condition at the end of the excavation;

ii) it is rather curious to observe that maximum moments are provided by *approach 2* when LEC are used and by analyses CC using FEM;



(a) (b)

Figure 9. Lateral wall displacements (W.L1, δ = (2/3) ϕ’): (a) final distribution from analyses CB and CC; (b) - evolution of the displacement of the top of the wall with the current excavation depth in analysis CC



(a) (b)

Figure 10. Effective final normal earth pressures on both sides of the wall (W.L.1, δ = (2/3) ϕ’): (a) - *analysis CB*; (b) - *analysis CC*

Table 2. Summary of results in terms of wall embedded height and design bending moment

Approach	*Approach 2* (F_r = 2.0)	*Approach EC 7 analyses CB*	*Approach EC 7 analyses CC*	*Approach 2* (F_r = 2.0)	*Approach EC 7 analyses CB*	*Approach EC 7 analyses CC*
Water level below wall tip (W.L.0) δ = (2/3) ϕ’				δ = 0		
d (m)	5.3	5.1	5.1	8.3	7.3	7.3
M_d (kNm/m) LEC	-344	-	-304	-597	-	-470
M_d (kNm/m) FEM	-211	-211	-261	-353	-353	-424
Water level 5 m deep (W.L.1) δ = (2/3) ϕ’				δ = 0		
d (m)	7.9	7.5	7.5	13.2	11.3	11.3
M_d (kNm/m) LEC	-487	-	-423	-998	-	-745
M_d (kNm/m)FEM	-262	-262	-367	-514	-512	-688

iii) the values from the finite element analyses with the wall designed according to *approach 2* and the CB type analyses are very close or even coincident; this arises from the fact that the wall embedded height is very similar in the two types of analyses and both consider characteristic values of the shear strength parameters;

iv) the moments from CC analyses considerably exceed (20 to 40%) the ones from the approaches just referred; it seems, then, that the factor of 1.25 applied to the shear strength parameters is more severe than the use of a factor of 1.35 multiplying the effect of the actions derived from the finite element analyses performed with the characteristic values of the soil and the soil-to-wall interface resistant parameters.

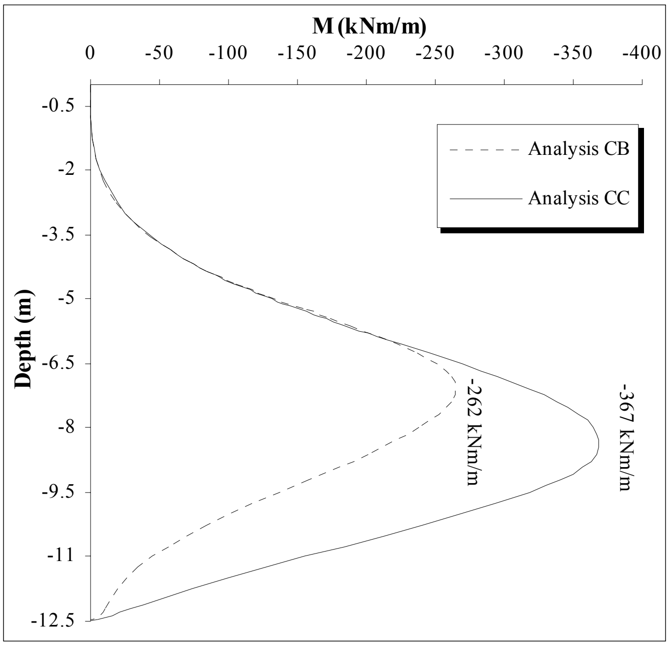


Figure 11. Distribution of the design wall bending moments (W.L.1, $\delta = (2/3)\ \phi'$) from analyses CB and CC

4 CONCLUSIONS

The purpose of the paper has been to present a comparative study on the design of flexible cantilever retaining walls. Conventional limit equilibrium calculations were carried out with the safety introduced by means of distinct ways, including the Cases B+C approach, established in Eurocode 7. A brief parametric study, lead to the following main conclusions:

- classical *approach 1*, as well as *approach 2*, involving a safety factor of 2.0 in the passive or net passive resistance, seem to be very conservative for soils with angle of shearing resistance lower than 25º to 30°;
- when the *approach Cases B+C* is used, the later case appears to be critical to both wall height and structural strength design; it gives results that in most cases lie between the approaches referred above and the one (*approach 3*) which considers the use of a safety factor on the embedded wall length.

Since this type of retaining wall involves a complex soil-structure interaction, finite element models have an enormous potential as design tools. However, limit equilibrium calculations are still very useful as they provide the wall height, which is a basic input for the finite element calculations.

The application of finite element analyses in the context of the *Cases B+C approach* of EC 7 is still in a phase of clarification. The paper presents a tentative procedure that can be adopted for flexible retaining walls. Basically, it consists of considering in the analyses the (actual) geometry of the structure obtained from Case C limit equilibrium calculations. Two finite element analyses – the so-called analyses CB and CC - are then carried out with the introduction of the safety factors for ground properties and for actions of the two cases.

The overall comparison of the results in terms of wall bending moments from the finite element analyses, permitted to draw the following main conclusions:

- the values from the limit equilibrium calculations are always greater than the ones from the respective finite element analyses;
- the design bending moments from analyses CC are greater than the results provided by analyses CB, which corroborates the conclusions of the parametric study using limit equilibrium calculations;
- for the conditions of the numerical case study, the use of Eurocode 7 together with the finite element analyses as proposed in the paper, would provide a reduction of 4 to 14% in the embedded wall height and of 24 to 31% in the design bending moment, in comparison with the use of the classical *approach 2* and limit equilibrium calculations.

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