**FACULDADE DE ECONOMIA DO PORTO**

# ESSAYS ON NEW ECONOMIC GEOGRAPHY

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**TESE DE DOUTORAMENTO EM ECONOMIA**

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O que mais me fascina é ser o primeiro a descobrir o que os outros só irão saber depois.
Este sentimento é a mão invisível da minha investigação.

*To Maria and Armando, my parents*,

*Filipe and Marta, my brother and my sister.*

# Nota Biográfica

Vasco Leitão de Carvalho Gomes Leite nasceu a 17 de Abril de 1977, no Porto.

Em Novembro de 2003 licenciou-se em economia e finanças, pela Universidade Fernando Pessoa com média final de 16 valores, defendendo a monografia com o título "Os determinantes que definem a Vodafone um alvo apetecível para TMN- Teoria e Aplicação".

Foi consultor de gestão na Indeve - Investigação e Desenvolvimento Empresarial. Actualmente é economista na Comissão de Coordenação e Desenvolvimento Regional do Norte.

Em Setembro de 2004, iniciou o Doutoramento em Economia, na faculdade de Economia do Porto.

Após a conclusão da parte escolar do programa de doutoramento, apresentou os artigos da tese em encontros científicos tais como: "II Economic Theory Workshop in Vigo", "2nd Meeting of the Portuguese Economic Journal", "Simpósio de Analisis Economico 2008 em Zaragoza", "XXXIV Reunión de Estudios Regionales 2008 em Baeza".

# Acknowledgments

Being a man, I have had the feeling that a doctoral thesis is like conceiving a child. And contrary to what would happen in a normal pregnancy, there were numerous people who helped me in this quest.

I thank my friends, all of them, on all sides, and of all time. I have learned with their experiences, successes and failures, and this knowledge is an important contribute to my thesis. I have no intention to write their names in this short text. True friendship is much older than writing.

I thank the doctoral committee to have significantly improved the working conditions of doctoral candidates in the last years. I am referring in particular to the room PhD, where I spent thousands of hours doing this thesis. I am also grateful to FCT for the PhD scholarship.

This thesis also benefited considerably from the comments of Diego Puga, Jose Maria Chamorro Rivas, Pascal Mossay, Olga Alonso-Villar, Carlos Hervés Beloso, Alper Çenesiz and anonymous referees of the Portuguese Economic Journal.

I want to really thank my supervisors, Sofia Castro and João Correia da Silva for always supporting me, as well as encouraging me to do this work. I have only one word to describe them - brilliant.

*E agradeço à Mónica, a minha namorada, a ternura revelada durante esta etapa da nossa vida. Também desejo que realize os seus sonhos vida fora. E finalmente, aos meus pais e*

*irmãos, agradeço-lhes tudo com pequenas palavras num grande presente. Esta obra fica, é deles, e para sempre.*

# Sumário

Aplicando modelos de equilíbrio geral com concorrência imperfeita, o objectivo desta tese é explicar de que forma os custos de transporte assimétricos, os bens não transaccionáveis e as vantagens tecnológicas regionais afectam a distribuição espacial da actividade económica, na presença de forças de aglomeração.

No primeiro capítulo, generalizamos o modelo de Krugman (1991) incluindo custos de transporte assimétricos entre regiões e (assimétricos) custos de transporte que são internos às regiões.

No segundo capítulo, generalizamos o modelo de Krugman (1991) incluindo um parâmetro para a percentagem de trabalhadores industriais na economia. Nós relaxamos a restrição de que a percentagem de trabalhadores industriais é igual à percentagem da despesa em bens industriais.

No terceiro capítulo, nós generalizamos o modelo "core-periphery" resolvido analiticamente em Forslid e Ottaviano (2003), considerando um terceiro sector que produz bens não transaccionáveis (serviços).

No quarto capítulo, nós generalizamos o modelo "core-periphery" resolvido analiticamente em Forslid e Ottaviano (2003), incorporando uma vantagem tecnológica para todas as empresas que operam em uma das regiões. A vantagem tecnológica consiste em considerar uma região com um custo fixo mais baixo.

# Summary


Applying general equilibrium models with imperfect competition, the aim of this thesis is to explain how asymmetric trade costs, non-tradable goods and regional technological advantages affect the spatial distribution of the economic activity in the presence of agglomeration economies.

In the first chapter, we generalize the model of Krugman (1991) to allow for asymmetric trade costs between regions and for (asymmetric) trade costs that are internal to the regions.

In the second chapter, we generalize the model of Krugman (1991) including a parameter for the share of industrial workers in the economy. Therefore, we relax the constraint that the share of industrial workers in the economy and the share of spending in industrial goods are the same.

In the third chapter, we generalize the analytically solvable core-periphery model of Forslid and Ottaviano (2003) by considering a third sector, which produces non-tradable goods (services).

In the fourth chapter, we generalize the analytically solvable core-periphery model of Forslid and Ottaviano (2003) incorporating a technology advantage for all industrial firms operating in one of the regions. The technology advantage consists in modelling a region with a lower fixed cost.

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# Chapter 1

# Introduction

It is clear that at a macro level there appears a core-periphery structure where we observe high disparities within the same country. Some regions agglomerate the highest share of income and economic activity, while the other ones remain undeveloped and poor.

An identical spatial structure emerges at the global scale. For instance, in 2000, $83\%$ of the world GDP was concentrated in three regions (East Asia, EU, and NAFTA). In 1980, the same regions concentrated about $70\%$. Hence, concentration of the world GDP in these regions has been self-enforcing in the last years.

New economic geography explains this outcome using general equilibrium models with imperfect competition, where firms produce goods under increasing returns to scale. Considering perfect mobility of some factors of production, Krugman (1991), in a seminal paper, shows that agglomeration is an endogenous result motivated by the balance among three important effects, namely, “market access effect”, “cost-of-living effect” and “crowding effect”.

The market access effect is represented by the idea that firms get large sales by operating in a big market. Under increasing returns to scale, large sales make firms more profitable, leading to the entry of new ones. This increases the production of new varieties at a lower price causing a fall in the price index (cost-of-living effect), which in turn increases the real income of the population. However, an increase in the number of firms and workers in a region intensifies the competition (crowding effect). Hence, when the first two effects are stronger than the last one, agglomeration is an optimal economic result.

Some key economic parameters change the balance between these forces. Indeed, trade costs, the size of the industrial sector and the love for variety constitute the main explanatory factors of agglomeration. Krugman (1991) shows that an economy characterized by low trade costs, high love for variety and high spending in industrial products, will concentrate all economic activity in one region.

Afterwards, the recent literature has provided an understanding of a variety of matters in relation with the location of economic activity such as globalization and inequality of the nations (Krugman and Venables, 1995), regional inequalities (Puga, 1999), economic growth (Fujita and Thisse, 2003), economic development (Murata, 2008) and qualification (Mori and Turrini, 2005; Toulemonde, 2006).

The paper of Krugman and Venables (1995) studies how the globalization process effects the location of industry and gains from trade with immobility of labor. They show that the long term decline in transportation costs, leading to growing integration of world markets can produce first a division of the world into rich and poor regions, and then convergence in income and economic structure between these regions. Simple geography models like Krugman (1991) respond in a monotone way to declining transportation costs; when these costs fall below a critical level, industry concentrates in one region. In an opposite way, here, because labor is immobile (and thus wage differentials between regions emerge), continuing reductions in transportation costs eventually lead to a reindustrialization of the low-wage region.

A closely related result is founded in Puga (1999). The paper studies the relationship between the regional integration and regional differences in production structures and income levels. He shows that when workers do not move across regions, at low trade costs firms become increasingly sensitive to cost differentials leading industry to spread out again. In this case it will be firms that move and this can bring about convergence both in terms of industrial employment and of income.

Another contribution was made by Fujita and Thisse (2003). The authors study how the rate of growth generated by innovations is related to the degree of spatial concentrations of the activities. They show that when the economy moves from dispersion to agglomeration, innovation follows a much faster pace. In fact the model strongly supports the idea that agglomeration and growth reinforce each other.

Murata (2008) presents a model of structural change (without innovation) and agglomeration. Unlike Krugman (1991) the author endogenizes not only the degree of agglomeration, but also the expenditure and employment shares. He shows that a decline in transportation costs, by enhancing consumers' purchasing power shifts the demand from agricultural to non-agricultural good and induces the reallocation of labor from agriculture to non-agricultural activities. Thus, a substantial decline in transportation costs gives rise to agglomeration of non-agricultural activities.

The qualification of the workers and the spatial distribution of economic activity are mentioned in the models of Toulemonde (2006) and Mori and Turrini (2005). Toulemonde (2006) elaborated a model of new economic geography in which the decision to invest in skills acquisition depends on the demand for skills by firms from the industry sector. An increase in the number of firms in one region induces more workers from that region to become skilled which in turn raises the demand for the goods produced by firms from the industrial sector and reinforces the movement of firms to that region.

Mori and Turrini (2005) do not study the decision to invest in skills as Toulemonde (2006). They show that when skill levels of agents are not equal, spatial sorting according to skill levels arise as general phenomena. Agents with low skill may not be able to endure the tough competition at large agglomerations, and may rather avoid these locations. Conversely, since the highest skilled care less about competition, they will seek for the locations where they can fully exploit agglomeration.

Despite the numerous contributions to understanding the location of economic activities in space, some of limitations of the mainstream literature lies on its assumptions to capture the reality. Frequently, it is assumed that: (i) trade costs are identical between and within regions; (ii) the share of the industrial activity in the economy is equal to the share of spending in industrial goods; (iii) the economy only comprises two sectors (an agricultural and industrial sector) ignoring for instance, a non-tradable sector (service sector). It is also always assumed that all firms have the same technology in the economy (see these assumptions in Krugman, 1991; Fujita et al., 2001; Ottaviano and Thisse, 2004).

In reality, it is clear that trade costs are highly variable across regions, and the share of spending in non-tradable goods is too high in the economy to be simply neglected by the standard literature. Moreover, some regions are more competitive than others, where firms share a technology advantage increasing the welfare of the population.

In this context, the main motivation behind this thesis is to provide new explanations for the spatial distribution of the economic activity using new assumptions. Applying general equilibrium models with imperfect competition, this research aims at investigating mainly how asymmetric trade costs, non-tradable goods and regional technological advantages affect the spatial distribution of the economic activity in the presence of agglomeration economies. We study the influence of each these aspects one at a time.

The thesis has of four additional chapters. The chapters are independent in the sense that each chapter deals with a specific economic problem and, accordingly, its contribution can be considered independently of the remaining thesis. The chapters are linked through the theme of new economic geography.

In chapter 2, we generalize the model of Krugman (1991) to allow for asymmetric trade costs between regions and for (asymmetric) trade costs that are internal to the regions.

In chapter 3, we generalize the model of Krugman (1991) including a specific parameter for the share of industrial workers in the economy. Therefore, we relax the constraint that the percentage of industrial workers and the share of spending in industrial goods are the same.

In chapter 4, we generalize the core-periphery model of Forslid and Ottaviano (2003) to allow for a third sector, specifically a non-tradable sector (or service sector). We study how trade costs in the industrial sector and love for variety in the population affect simultaneously the spatial location of industrial and service activity.

In chapter 5, we generalize the core-periphery model of Forslid and Ottaviano (2003) to allow for a technology advantage in which all firms have lower fixed costs in one of the regions.

Our study will be based on analytical solution of models together with simulations. All numerical results in this thesis were obtained using MATLAB. Consistent with the mainstream methodology in new economic geography, we develop general equilibrium models capturing essential features of firms' interaction, and consumers' behavior in imperfectly competitive sectors.

Our method comprehends the following stages:

(i) Set up the model: In this stage we introduce the new assumption into standard core-periphery models so as to provide a suitable model for our economics concerns.

(ii) Equilibrium: In this stage, we rely on an analytical approach complemented by simulations to identify and to characterize the equilibrium. Depending on the nature of the model in the first stage, we might seek for different types of equilibrium, namely concentration, symmetric and asymmetric dispersion equilibrium;

(iii) Economic analysis: At this stage, we investigate the implications of the equilibrium characterized in the second stage. We point out the differences and similarities between our model and the standard one.

The technical proofs are all presented in appendices to each chapter.

# Chapter 2

# The core periphery model with asymmetric inter-regional and intra-regional trade costs

* The results in this chapter have been published as Leite at al.(2009a) and also appeared as a working paper (Leite et al., 2008).

## 2.1 Introduction

What is the impact of asymmetric internal and external trade costs on the spatial distribution of the industrial activity and on the welfare of the different interest groups in an economy?

Trade costs, broadly defined by Anderson and Wincoop (2004):

"*include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself: transportation costs (both freight costs and time costs), policy barriers (tariffs and nontariff barriers), information costs, contract enforcement costs, costs associated with the use of different currencies, legal and regulatory costs, and local distribution costs (wholesale and retail).*"

It is clear that trade costs are highly variable across countries. They are higher in landlocked countries than in coastal countries (Limão and Venables, 2001) and higher in developing countries than in industrialized countries (Anderson and Wincoop, 2004). Differences in trade costs, particularly those associated with the distance to the larger markets, explain some of the income inequality across countries (Redding and Venables, 2004).

The economic relevance of trade costs is beyond doubt, being equivalent (in industrialized countries) to a 170 % *ad valorem* tax equivalent, that can be decomposed into a 55% domestic trade cost, associated with local distribution, and a 74 % international trade cost (Anderson and Wincoop, 2004).

A monotonic relationship between trade costs and location of the economic activity is one of the main theoretical findings of the 'New Economic Geography' literature. If trade costs are high, economic activity is dispersed across regions, while if trade costs are low, then economic activity becomes concentrated in one region.[1]

[1]The concept of "region" may refer to locations ranging from small geographical regions like cities (Fujita

This recent literature has allowed an understanding of a variety of matters in its relation with the location of economic activity such as trade policy (Baldwin *et al.*, 2003), economic development (Murata, 2002), qualification (Mori and Turrini, 2005; Toulemonde, 2006), quality of the infrastructure (Martin and Rogers, 1995) and the structure of the transport network (Fujita and Mori, 1996; Mun, 2004). A welfare analysis of the agglomeration process was carried out by Ottaviano, Tabuchi and Thisse (2002).

In spite of the empirical evidence, most of the theoretical work has neglected the differences in trade costs across regions and the trade costs that are internal to a region, focusing on the case of symmetric trade costs associated with trade across regions.[2]

In this paper, we extend the model introduced by Krugman (1991) to allow for: (i) the existence of intra-regional (internal) trade costs, possibly different between regions; and (ii) the existence of asymmetric inter-regional (external) trade costs.

By asymmetric external trade costs we mean that the cost of trading from region 1 to region 2 is different from the cost of trading from region 2 to region 1. The assumption that trade costs from region 1 to region 2 are identical to those from region 2 to region 1 is pervasive in the existing literature. Nevertheless, it is clear that some trade barriers like tariffs and import quotas are unilateral (at least asymmetric) and that there may be different degrees of trade liberalization.[3] This asymmetry was illustrated by Krugman and Venables (1995, Section V) in the form of a unilateral import tariff.

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and Krugman, 1995; Mori, 1997) to larger areas such as countries or even continents (Krugman and Venables, 1995).

[2]See, for example, the works of Krugman (1991), Fujita and Krugman (1995), Mori (1997), Ottaviano and Puga (1998), Puga (1999), Fujita, Krugman and Venables (2001), Baldwin *et al.* (2003), Ottaviano and Thisse (2004) and the references therein.

[3]See Baldwin *et al.* (2003) for an overview on trade policy and economic geography.

There are papers that consider internal and external trade costs in new economic geography models, but none addressing the problem of extending the base model of Krugman (1991).

Martin and Rogers (1995) have considered asymmetric internal and external transportation costs in an economy with two countries, but in their model workers are immobile between regions. Therefore, it does not capture agglomeration as a self-reinforcing process generated by demand-linkage and cost-linkage circular causality.[4]

A kind of internal and external transportation costs also appears in Mansori (2003), who considers a country composed by two identical regions which trade with the rest of the world. Each region has a different cost of trading with the rest of the world, and there is also trade between the two regions.

Behrens et al. (2007) proposed a model in which there are two identical countries formed by two regions between which labor is mobile, while there is no international labor mobility. Goods can be traded both nationally and internationally at different costs. Particularly, they assume that countries have different internal trade costs, but still an identical external trade cost.

In sum, we study an economy in which there are agglomeration forces generated by the mobility of the industrial population, allowing for asymmetric internal and external trade costs. Technically, we extend Krugman's (1991) model to accommodate four different trade costs: external trade costs from region 1 to region 2 and from 2 to 1, and internal trade costs

[4]When workers migrate to a region, the size of the market increases, fostering economic activity in this region (demand-linkage). When economic activity is transfered to a region, trade costs in this region decrease, attracting workers (cost-linkage). See Baldwin *et al.* (2003) for a detailed explanation of the difference between the basic core-periphery model (Krugman, 1991) and the footloose captal model (Martin and Rogers, 1995).

within region 1 and within region 2.[5]

We show that in the case of symmetric trade costs, the model is equivalent to the original model of Krugman (1991) with trade cost equal to the ratio between the external and the internal trade cost. The trade cost considered in the existing literature can, thus, be interpreted as the ratio between external and internal trade costs. Recall that this ratio was measured as a $74\%$ tax by Anderson and Wincoop (2004).[6] The measures of the total trade cost as a $170\%$ tax and of the domestic component as a $55\%$ tax are irrelevant for the agglomeration process.

Not surprisingly, we find that industrial activity tends to shift to the region with lower internal trade costs, and to the region with higher cost of importing (lower cost of exporting).

Considering a decrease of the internal trade cost of a region, we find that, in this region, the real wages of the workers increase in the short-run, and economic activity increases in the long-run (workers migrate to the region). In terms of welfare, we observe a "win-lose" situation. In the short-run, the welfare of workers improves in this region but worsens in the other region, while the welfare of farmers improves in both regions (because prices go down).

In the case of a unilateral decrease in the cost of importing, there are different possible effects. If the size of the industrial sector is small, the real wages of the workers decrease in the short-run and economic activity decreases in the long-run. If the industrial sector is large, we arrive at the opposite conclusion. The real wages of the other regions' workers

[5] We do not address the issue of assigning trade costs to agricultural goods. See Fujita, Krugman and Venables (2001, chapter 7).

[6] A tax of $74\%$ corresponds to an iceberg cost parameter of $0.57 = 1/1.74$. To receive 1 unit, the customer pays $1.74$.

always increases. The welfare of farmers always improves in the region, and worsens in the other region.

In the next section we present an extension of the model of Krugman (1991) which accommodates different internal and external trade costs. Section 2.3 includes the main results, being divided into three subsections which corresponds to the cases of symmetric trade costs, asymmetric internal trade costs, and asymmetric external trade costs. Section 2.4 concludes the chapter with some remarks. Proofs of the analytical results can be found in the appendixes.

## 2.2 The model

The economy comprises two sectors (agriculture and industry) and two regions (1 and 2). Regions are symmetric in terms of technology and preferences, but may have different internal and external trade costs.

The agricultural sector is perfectly competitive and produces a homogeneous good under constant returns to scale, using only labor supplied by farmers, who are immobile between regions.

The manufacturing sector is monopolistic competitive and produces a continuum of varieties of a horizontally differentiated product using only labor supplied by workers, who are mobile between regions.

Both farmers and workers share a utility function of the form

$$U = C_M^{\mu} C_A^{1-\mu}, \tag{2.1}$$

where $C_A$ is consumption of the agricultural goods and $C_M$ is consumption of a manufactures aggregate. This functional form implies that $0 < \mu < 1$ is the share of spending on manufactured goods.

The manufacturing firms produce horizontally differentiated products, with the manufactures aggregate being defined as:

$$C_M = \left[ \int_0^N c_i^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)},$$

where $N$ is the quantity of horizontally differentiated products, $c_i$ is the consumption of the $i$ differentiated product and $\sigma > 1$ is the elasticity of substitution among the products. A low $\sigma$ means that the products have a high degree of differentiation (or that the consumers have a high preference for variety).

A fraction $0 < \mu < 1$ of the population works in the manufacturing sector, while the remaining, $1 - \mu$, works in the agricultural sector.[7] Farmers are evenly distributed between the regions, thus the agricultural population in each region is fixed and equal to $\frac{1-\mu}{2}$.

The industrial population in regions 1 and 2 is $L_1$ and $L_2$, with $L_1 + L_2 = \mu$. We denote the share of workers in region 1 by $f = \frac{L_1}{L_1+L_2}$ (the share of workers in region 2 is, obviously, $1 - f$).

Production of each variety requires a fixed input involving $\alpha > 0$ units of labor and a variable input involving $\beta > 0$ units of labor, supplied by the industrial workers. The cost function, in region $j$, is:

$$CT_j = W_j(\alpha + \beta x_i),$$

[7]The coincidence between the share of population in each sector and the share of spending on each sector only implies the equality between wages in both sectors, in equilibrium.

where $CT_j$ is the cost to produce one unit of some variety, $W_j$ is the nominal wage of the workers in region $j$, and $x_i$ is the output produced by the firm.

Given the profit-maximization pricing behavior of manufacturing firms that operate in a monopolistic competitive sector, the price of any manufactured product in region $j$ is:

$$p_j = \frac{\sigma}{\sigma - 1} \beta W_j.$$

Free entry of firms into the manufacturing sector drives profits to zero, which implies that all firms produce the same output, given by:

$$x_i = \frac{\alpha(\sigma - 1)}{\beta}.$$

Each firm employs the same number of workers, therefore:

$$\frac{L_1}{L_2} = \frac{n_1}{n_2},$$

where $n_j$ is the number of firms in region $j$.

The agricultural sector is perfectly competitive and has constant returns to scale. One unit of labor supplied by farmers is used to produce one unit of the agricultural good. Trade costs in this sector are neglected, therefore, the price of the agricultural good is the same in both regions, and chosen as the numeraire.

$$p_A = W_A = 1,$$

where $p_A$ is the price of the agricultural good and $W_A$ is the nominal wage of the farmers in both regions.

So far, we have presented the model of Krugman (1991). We now extend it by allowing for the existence of asymmetric trade costs between regions as well as different internal trade costs.

The trade of manufactures involves an *iceberg* trade cost. Of each unit of manufactures shipped from region $i$ to region $j$, only a fraction $0 < \tau_{ij} < 1$ arrives. Thus, a high $\tau_{ij}$ corresponds to a low trade cost. The trade of agricultural products is assumed to be costless.[8]

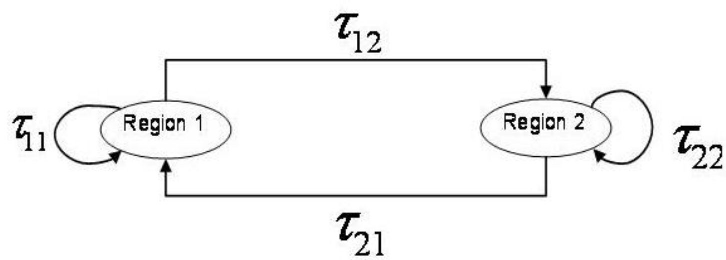


Figure 2.1: Asymmetric internal and external trade costs.

Figure 2.1 represents the configuration of the economy with two regions and four different trade costs. The parameter $\tau_{12}$ represents the cost of shipping the manufactured goods from region 1 to region 2, while $\tau_{21}$ represents the cost necessary to ship the manufactured goods from region 2 to region 1. We designate these as external trade costs. We also consider trade costs for goods that are produced and consumed in the same region, $\tau_{11}$ and $\tau_{22}$, and designate them as internal trade costs. We assume that the internal trade costs are lower than the external trade costs.

### 2.2.1 Short-run equilibrium

In the short-run, the spatial distribution of workers is taken as given (migrations do not occur). We start by computing output and nominal wages. After we compute prices and

[8]This assumption was relaxed by Adrian (1996). Agricultural transport costs were shown to render agglomeration of industry less likely.

check whether there are incentives for workers to migrate by comparing the real wages in each region.

We denote consumption in region $i$ of a representative region $j$ product by $Cji$.

In region $1$, the price of a local product is $p_1/\tau_{11}$, while the price of an imported product is $p_2/\tau_{21}$. Consumption is given by:

$$C_{11} = \left(\frac{p_1}{\tau_{11}}\right)^{-\sigma} \text{ and } C_{21} = \left(\frac{p_2}{\tau_{21}}\right)^{-\sigma}.$$

The expenditure on local manufactures, $E_{11}$, and on foreign manufactures, $E_{21}$, is:

$$E_{11} = \left(\frac{p_1}{\tau_{11}}\right)^{1-\sigma} n_1 \quad \text{and} \quad E_{21} = \left(\frac{p_2}{\tau_{21}}\right)^{1-\sigma} n_2.$$

Given $E_{11}$ and $E_{21}$, we define $Z_{11}$ as the ratio between region 1's expenditure on local manufactures and region 1's expenditure on manufactures imported from region 2:

$$Z_{11} = \frac{E_{11}}{E_{21}} = \left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma} \frac{n_1}{n_2} = \left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma} \frac{L_1}{L_2}. \tag{2.2}$$

With a similar procedure we obtain $Z_{12}$, the ratio between region 2's spending on region 1 products and local products:

$$Z_{12} = \frac{E_{12}}{E_{22}} = \left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma} \frac{L_1}{L_2}. \tag{2.3}$$

Let $Y_1$ and $Y_2$ denote the nominal regional income, which is equal to the sum of the incomes in the agricultural and the manufacturing sectors:

$$Y_1 = \frac{1-\mu}{2} + W_1L_1 \text{ and } Y_2 = \frac{1-\mu}{2} + W_2L_2. \tag{2.4}$$

The nominal wage of workers in region 1 is equal to the spending on region 1's manufactures:

$$L_1W_1 = \frac{Z_{11}}{1+Z_{11}}\mu Y_1 + \frac{Z_{12}}{1+Z_{12}}\mu Y_2 \Leftrightarrow W_1 = \frac{\mu}{L_1}\left[\frac{Z_{11}}{1+Z_{11}}Y_1 + \frac{Z_{12}}{1+Z_{12}}Y_2\right]. \tag{2.5}$$

Similarly, the nominal wage of workers in region 2 is:

$$W_2 = \frac{\mu}{L_2}\left[\frac{1}{1+Z_{11}}Y_1 + \frac{1}{1+Z_{12}}Y_2\right]. \tag{2.6}$$

Expressions (2.4)-(2.6) imply that the sum of the nominal wages across all agents is invariant:

$$L_1W_1 + L_2W_2 = \mu. \tag{2.7}$$

Equations (2.2)-(2.4) allow us to write (2.5) and (2.6) in the following way:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_1L_1)}{1+\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_2L_2)}{1+\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}}\right]; \tag{2.8}$$

$$W_2 = \frac{\mu}{L_2}\left[\frac{\frac{1-\mu}{2}+W_1L_1}{1+\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\frac{1-\mu}{2}+W_2L_2}{1+\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}}\right]. \tag{2.9}$$

Equations (2.8) and (2.9) constitute a system that determines $W_1$ and $W_2$ for a given distribution of workers between regions 1 and 2. Using (2.7), equation (2.8) can be written as a function of $W_1$ and equation (2.9) can be written as a function of $W_2$.

Workers are interested not in nominal wages but in real wages, and these depend on the cost of living in each region.

The price indices, $P_1$ and $P_2$, reflect the relationship between expenditure and utility for individuals in region 1 and region 2, respectively. These depend on the price of the agricultural products (normalized to 1) as well as on the price indices of manufactured goods, $P_{M1}$ and $P_{M2}$.

$$P_{M1} = \gamma\left[f\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau_{21}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}};$$

$$P_{M2} = \gamma \left[ f \left( \frac{W_1}{\tau_{12}} \right)^{1-\sigma} + (1-f) \left( \frac{W_2}{\tau_{22}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

where $f = \frac{L_1}{L_1+L_2}$ and $\gamma = \frac{\sigma\beta}{\sigma-1} \left[ \frac{\mu}{\frac{\alpha}{\beta}(\sigma-1)} \right]^{\frac{1}{1-\sigma}}$.

We have $P_1 = P_A^{1-\mu} P_{M1}^{\mu} = P_{M1}^{\mu}$ and $P_2 = P_A^{1-\mu} P_{M2}^{\mu} = P_{M2}^{\mu}$. Therefore:

$$P_1 = \gamma \left[ f \left( \frac{W_1}{\tau_{11}} \right)^{1-\sigma} + (1-f) \left( \frac{W_2}{\tau_{21}} \right)^{1-\sigma} \right]^{-\frac{\mu}{\sigma-1}}; \tag{2.10}$$

$$P_2 = \gamma \left[ f \left( \frac{W_1}{\tau_{12}} \right)^{1-\sigma} + (1-f) \left( \frac{W_2}{\tau_{22}} \right)^{1-\sigma} \right]^{-\frac{\mu}{\sigma-1}}. \tag{2.11}$$

The maximization of (2.1) yields consumption of each product (manufacture aggregate and agricultural good), and given the price index of each of them, we obtain the utility in region i for workers, $U_i^M$, and farmers, $U_i^A$:

$$U_i^M = \mu^{\mu}(1-\mu)^{1-\mu} \frac{W_i}{P_i} \quad \text{and} \quad U_i^A = \mu^{\mu}(1-\mu)^{1-\mu} \frac{1}{P_i}. \tag{2.12}$$

Workers seek the region with the highest utility or, equivalently, the highest real wage. From equations (2.8), (2.9), (2.10) and (2.11), we obtain the relative real wage, $\omega_1/\omega_2$:

$$\frac{\omega_1}{\omega_2} = \frac{\frac{W_1}{P_1}}{\frac{W_2}{P_2}} = \frac{W_1 \left[ f \left( \frac{W_1}{\tau_{12}} \right)^{1-\sigma} + (1-f) \left( \frac{W_2}{\tau_{22}} \right)^{1-\sigma} \right]^{\frac{\mu}{1-\sigma}}}{W_2 \left[ f \left( \frac{W_1}{\tau_{11}} \right)^{1-\sigma} + (1-f) \left( \frac{W_2}{\tau_{21}} \right)^{1-\sigma} \right]^{\frac{\mu}{1-\sigma}}}.$$

### 2.2.2 Long-run Equilibrium

The short-run equilibrium variables are determined taking as given the amount of industrial workers in each region, $f$. The long-run equilibrium is a situation where migration does not occur. We say that it is stable if it is robust to small perturbations of the distribution of workers across regions.

Dispersion is a long-run equilibrium configuration if regions have the same real wage. It is stable if a small migration to region $1$ decreases the real wage in region $1$, implying that the initial configuration is reestablished. Precisely:

$$\frac{\omega_1}{\omega_2}|_{f=f^*} = 1 \text{ and } \left[\partial\left(\frac{w_1}{w_2}\right)/\partial f\right]|_{f=f^*} < 0.$$

If the equilibrium share of population in region 1 is 0.5 ($f^* = 0.5$), we say that dispersion is symmetric (otherwise, it is asymmetric).

Concentration is a long-run equilibrium configuration if all workers are concentrated in the region that has the highest real wage. Unless real wages exactly coincide, it is stable.

$$f^* = 1 \text{ and } \frac{\omega_1}{\omega_2}|_{f=f^*} \geq 1 \quad \text{(concentration in region 1).}$$

$$f^* = 0 \text{ and } \frac{\omega_1}{\omega_2}|_{f=f^*} \leq 1 \quad \text{(concentration in region 2).}$$

## 2.3 Results

In this section, we consider three different cases.

1. Symmetric internal and external trade costs: ($\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i$).
2. Asymmetric internal trade costs: ($\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} \neq \tau_{22}$).
3. Asymmetric external trade costs: ($\tau_{12} \neq \tau_{21}$ and $\tau_{11} = \tau_{22} = \tau_i$).

We provide analytical results for short-run equilibria and simulations describing the long-run behavior.

### 2.3.1 Symmetric internal and external trade costs

Suppose that regions have equal internal trade costs, $\tau_{11} = \tau_{22} = \tau_i$, and symmetric external trade costs, $\tau_{12} = \tau_{21} = \tau_e$.

In a short-run equilibrium, the relative real wage, $\frac{\omega_1}{\omega_2}$, is the same as in the model of Krugman (1991), with $\tau = \frac{\tau_e}{\tau_i}$.[9] In this sense, we can reinterpret the trade cost in the model of Krugman (1991) as a ratio between external and internal trade costs.

**Proposition** 2.3.1. *When regions have equal internal trade costs, $\tau_{11} = \tau_{22} = \tau_i$, and equal external trade costs, $\tau_{21} = \tau_{12} = \tau_e$, the ratio between $\omega_1$ and $\omega_2$ is the same as in the case in which regions have only an external trade cost equal to $\tau = \frac{\tau_e}{\tau_i}$.*

The proof of this result is provided in the appendix.

Figure 2.2[10] illustrates the effect of a decrease of internal trade costs on the relative real wage (external trade costs are kept constant).

The bold line in figure 2.2 corresponds to short-run equilibria $(f, \frac{\omega_1}{\omega_2})$ in which there are no internal trade costs ($\tau_i = 1$) while the external trade costs are equal to 0.5 (the ratio between external and internal trade costs is $\tau = \frac{\tau_e}{\tau_i} = 0.5$). Any combination between internal and external trade costs such that $\frac{\tau_e}{\tau_i} = 0.5$ leads to the same relative real wage (Proposition 2.3.1), and therefore to the same spatial distribution of economic activity.

The dotted line corresponds to short-run equilibria in which the internal trade costs are 2/3 and the external trade costs are 0.5. In this case, the ratio between the external an internal trade costs is $\tau = \frac{\tau_e}{\tau_i} = 0.75$.

[9]Recall that the model of Krugman (1991) only considers the external trade cost.

[10]To plot this figure, we have set $\tau_e = 0.5$, $\mu = 0.3$ and $\sigma = 4$.

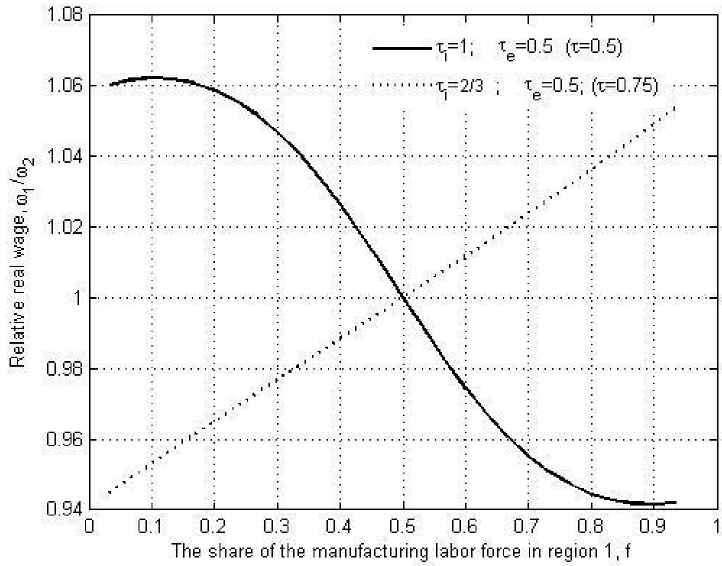


Figure 2.2: The spatial distribution of the economic activity with symmetric internal and external trade costs.

When the ratio between the internal and external trade costs changes, there is a change in the short-run equilibria. In this example, the decrease of internal trade costs (from $\tau_i = 2/3$ to $\tau_i = 1$) changes the equilibrium configuration from agglomeration to symmetric dispersion.

### 2.3.2 Asymmetric internal trade costs

What is the impact of a unilateral decrease in the internal trade costs:

- On the relative real wage, in the short-run?
- On the welfare of each interest group, in the short-run?
- On the distribution of industrial activity, in the long-run?

We provide analytical results for the first two questions, and use simulation to characterize the distribution of economic activity in the long-run. Proofs can be found in the appendix.

For the analytical results, we have chosen parameter values such that the initial long-run equilibrium is characterized by symmetric dispersion of economic activity. We start out with regions that have symmetric trade costs ($\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i$) and we consider a marginal decrease of the internal trade cost of region 2 (an increase in $\tau_{22}$).[11]

### Short-run effect on the relative real wage

Figure 2.3[12] compares short-run equilibria in the symmetric case ($\tau_{22} = \tau_{11}$) with short-run equilibria when region 2 has lower internal trade costs (i.e. $\tau_{22} > \tau_{11}$). Of course that, starting from a symmetric setting, analogous results are obtained for $\tau_{11} > \tau_{22}$, by interchanging the regions. The curve which depicts the short-run equilibria moves downwards, which means that workers in region 2 will have a higher real wage than workers in region 1, in the short-run. In particular, for $f = 0.5$, the relative real wage in region 1, $\frac{\omega_1}{\omega_2}$, is a decreasing function of $\tau_{22}$, and the economy moves from point A to point B.

According to proposition 2.3.2, and considering as a starting point a symmetric dispersion equilibrium in which $\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{22} = \tau_{11} = \tau_i$, this outcome occurs for any value of $\sigma$ and $\mu$.

**Proposition** 2.3.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $L_1 = L_2$. The relative real wage, $\frac{\omega_1}{\omega_2}$, is a decreasing function of $\tau_{22}$, for any $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

### Short-run effect on the welfare of the four interest groups

[11] Recall that, for each unit produced and sold in region 2, a fraction $\tau_{22}$ is consumed, while $1 - \tau_{22}$ is dissipated as trade costs.

[12] To plot this figure, we have set $\tau_e = 0.5$, $\mu = 0.3$ and $\sigma = 4$.

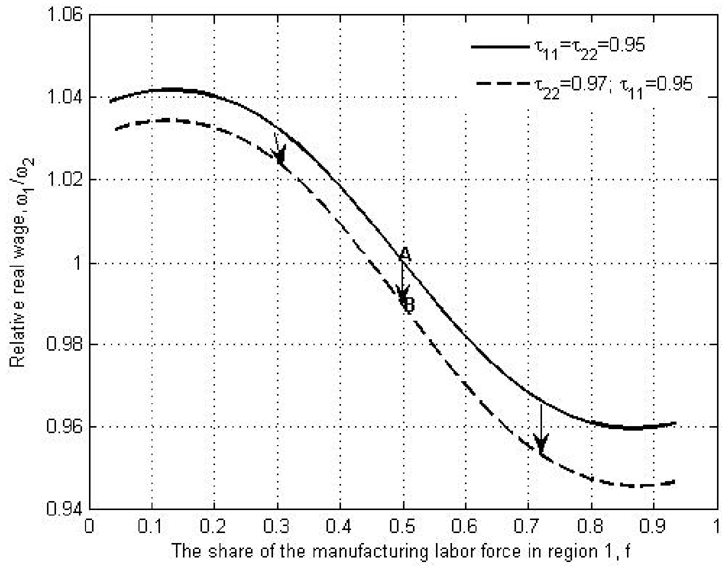


Figure 2.3: Short-run equilibria and asymmetric internal trade costs.

There are four interest groups in this economy, namely, the workers and the farmers in each region. Here we analyze the effect of variations of $\tau_{22}$ on the utility, (2.12), of each interest group.

Notice that the utility of the workers coincides with the real wage, except for the constant term, $\mu^{\mu}(1-\mu)^{1-\mu}$.

The decrease of the internal trade costs in region 2, (increase in $\tau_{22}$) influences the nominal wages and the price index in both regions, causing a "win-lose" outcome in which the workers in region 1 are the losers (Lemma 2.3.1), while the workers in region 2 are the winners (Lemma 2.3.2).

**Lemma** 2.3.1. *Let* $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ *and* $L_1 = L_2$. *The real wage in region 1,* $\omega_1$, *is a decreasing function of* $\tau_{22}$, *for any* $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ *and* $\sigma > 1$.

**Lemma** 2.3.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The real wage in region 2, $\omega_2$, is an increasing function of $\tau_{22}$, for $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

As the nominal wages of the farmers are always equal to 1, their real wages only depend on the price index. Therefore, all welfare effects stem from the cost-of-living effect. We show in Lemma 2.3.3 and Lemma 2.3.4 that the price indices in both regions are decreasing functions of $\tau_{22}$.

**Lemma** 2.3.3. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$. The price index in region 1, $P_1$, is a decreasing function of $\tau_{22}$ for $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

**Lemma** 2.3.4. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i$ and $L_1 = L_2$, the price index in region 2, $P_2$, is a decreasing function of $\tau_{22}$ for $0 < \tau_e < \tau_i < 1$, $\mu \in (0, 1)$ and $\sigma > 1$.*

In the short-run, a decrease in the internal trade costs of one region benefits the farmers in both regions.

#### Long-run effect on the distribution of industrial activity

Figures 2.4[13] and 2.5[14] illustrate the possible effects of a decrease in the internal trade costs of region 2. In the long-run, the economic activity can be asymmetrically dispersed between regions (figure 2.4) or fully concentrated in region 2 (figure 2.5).

Point A represents the initial long-run equilibrium (in which regions have symmetric internal and external trade costs). Industrial activity is equally divided between the regions ($L_1 = L_2 \Leftrightarrow f = 0.5$). The real wages are obviously identical in both regions (otherwise there would exist incentives for the workers to move to the region with higher real wages).

[13]To plot this figure, we have set $\tau_e = 0.5$, $\mu = 0.3$ and $\sigma = 4$.

[14]To plot this figure, we have set $\tau_e = 0.4$, $\mu = 0.3$ and $\sigma = 4$.

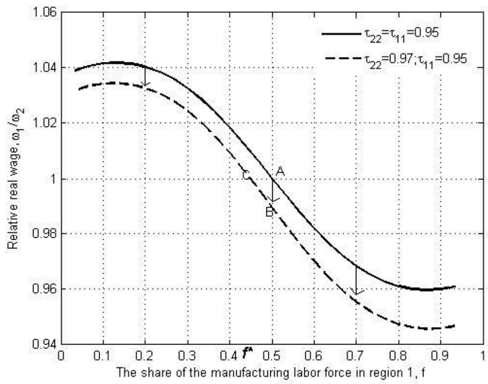


Figure 2.4: Asymmetric dispersion with asymmetric internal trade costs.

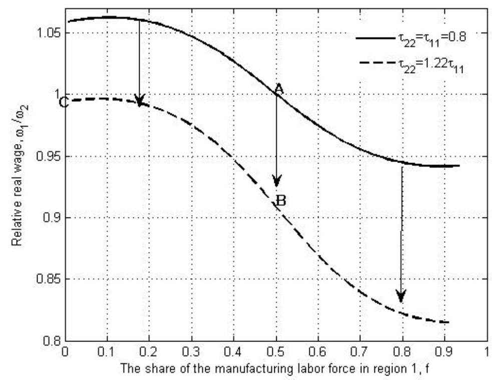


Figure 2.5: Concentration with asymmetric internal trade costs.

However, when $\tau_{22} > \tau_{11}$, the economy finds a new short-run equilibrium, point B, in which $\omega_2 > \omega_1$.

This attracts workers from region 1. There is migration to region 2 until the real wages coincide (figure 2.4) or until all workers have migrated to region 2 (figure 2.5).

Figure 2.4 shows an asymmetric dispersion equilibrium (point C) with $f^* < 0.5$ and $\omega_1 = \omega_2$, while figure 2.5 shows a concentration equilibrium (point C) with $f^* = 0$ and $\omega_2 > \omega_1$.

If economic activity is initially concentrated in a region, then a variation of internal trade costs may preserve this configuration (figure 2.6), or may imply that concentration can only occur in the region with the lower internal trade costs (figure 2.7)[15].

An increase in $\tau_{22}$ favors concentration in region 2, as the basin of attraction is enlarged. Indeed, for $\tau_{22} = 0.97$ and $\tau_{11} = 0.95$, we observe from figure 2.6, that any $f < 0.6$ (point C) is sufficient to induce concentration in region 2.

[15] To plot these figures, we have set $\tau_e = 0.75$, $\mu = 0.3$ and $\sigma = 4$.

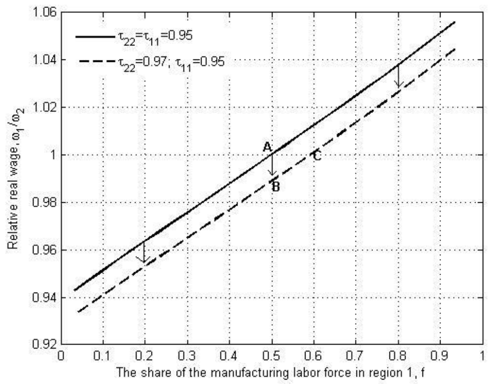


Figure 2.6: Agglomeration in any region with asymmetric internal trade costs.

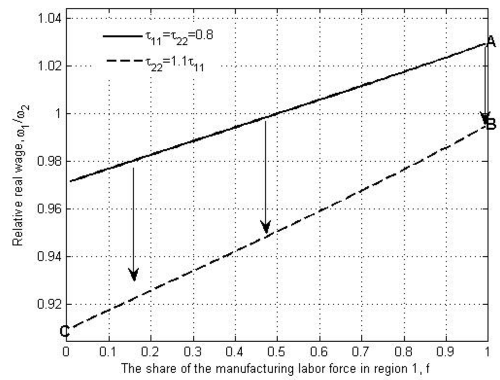


Figure 2.7: Agglomeration in only one region with asymmetric internal trade costs.

In figure 2.7, we illustrate a case in which all industrial activity is initially concentrated in region 1, with $f = 1$ and $\omega_1 > \omega_2$ (point $A$) and consider a decrease in the internal trade costs in region 2 (an increase in $\tau_{22}$). If the new relative real wage is above 1, the decrease in the internal trade costs in region 2 will have no effect on the spatial distribution of the industrial activity. But if the new relative real wage is below 1 (point B), then the impact on the location of the industry is drastic. In our illustration, for a $\tau_{22} = 1.1\tau_{11}$, all industrial activity relocates from region 1 to region 2 (point $C$ with $f = 0$ and $\omega_2 > \omega_1$).

### 2.3.3 Asymmetric external trade costs

What is the impact of an asymmetry between the cost of exporting and the cost of importing:

- On the relative real wage, in the short-run?
- On the welfare of each interest group, in the short-run?

- On the distribution of industrial activity, in the long-run?

We provide analytical results for the first two questions, and we use numerical methods to characterize the distribution of economic activity in the long-run. The proofs may be found on the appendix.

For the analytical results, we focus on the case in which the initial long-run equilibrium is characterized by symmetric dispersion of economic activity and we suppose that regions have initially symmetric trade costs, $\tau_{12} = \tau_{21} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i$. Then, we consider a marginal decrease of the cost of trading goods from region 1 to region 2 ($\tau_{12} > \tau_{21}$), keeping constant the cost of trading goods from region 2 to region 1 ($\tau_{21}$). We may interpret $\tau_{12} > \tau_{21}$ as the case in which region 2 has a lower cost of importing.[16]

We study the two types of long-run equilibrium: dispersion and concentration.

### Short-run effect on the relative real wage

Figures 2.8[17] and 2.9[18] show the impact of a decrease in cost of trade from region 1 to region 2 (increase in $\tau_{12}$) on the relative real wage, in the short-run (starting from a initial dispersion that is unstable and stable, respectively).

In the case illustrated in figure 2.8, the relative real wage of region 1 decreases (from point $A$ to $B$), while figure 2.9 shows the opposite effect. The direction of the effect depends on the weight of the industrial sector in the economy.

[16] The trade costs from region 1 to region 2, $\tau_{12}$, are supported by consumers in region 2. As $p_1$ is the price of the manufactured products produced in region 1 (determined in the market), then $\frac{p_1}{\tau_{12}}$ is the total price supported by consumers in region 2 when they purchase a manufacture produced in region 1. Then, if $\tau_{12}$ increases, the price paid by consumers in region 2 decreases.

[17] To plot this figure, we have set $\tau_i = 0.95$, $\mu = 0.96$ and $\sigma = 4$.

[18] To plot this figure, we have set $\tau_i = 0.95$, $\mu = 0.3$ and $\sigma = 4$.

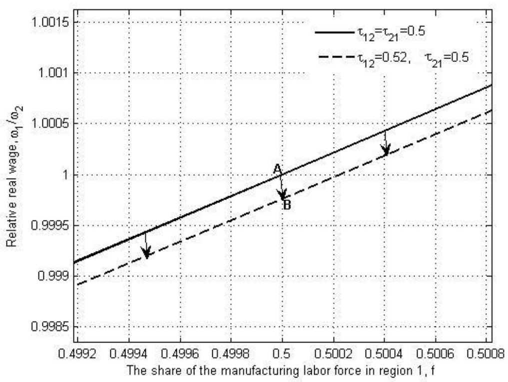


Figure 2.8: Liberalization is good.

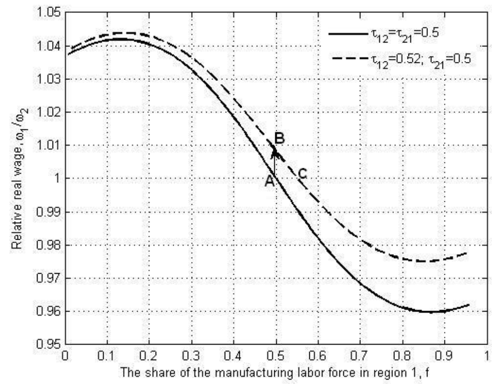


Figure 2.9: Liberalization is bad.

**Proposition** 2.3.3. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{22} = \tau_{11} = \tau_i$ *and* $L_1 = L_2$*. There is a* $\mu^*(\sigma, \tau) \in (0,1)$ *such that:* $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} > 0$ *for* $\mu \in (0, \mu^*)$ *and* $\frac{d(\omega_1/\omega_2)}{d\tau_{12}} < 0$ *for* $\mu \in (\mu^*, 1)$.

### Short-run effect on the welfare of each interest group

The four interest groups are the workers and the farmers in each of the regions. The utilities (2.12) coincide with the real wages, except for a constant. Therefore, we study the impact of $\tau_{12}$ on the real wages of each group.

We show that a decrease in cost of trade from region 1 to region 2 (an increase in $\tau_{12}$) increases the welfare of workers in region 1 (Proposition 2.3.4), whereas the effect on the welfare of workers in region 2 can be positive or negative (Proposition 2.3.5).

**Proposition** 2.3.4. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{11} = \tau_{22} = \tau_i$ *and* $L_1 = L_2$*. The real wage in region 1,* $\omega_1$*, is an increasing function of* $\tau_{12}$*, for any* $0 < \tau_e < \tau_i < 1$*,* $\mu \in (0,1)$ *and* $\sigma > 1$.

**Proposition** 2.3.5. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{22} = \tau_{11} = \tau_i$ *and* $L_1 = L_2$*. For any* $0 < \tau_e < \tau_i < 1$*, there is a* $\mu^*(\sigma, \tau) \in (0,1)$ *such that:* $\frac{d\omega_2}{d\tau_{12}} < 0$ *for* $\mu \in (0, \mu^*)$ *and*

$\frac{d\omega_2}{d\tau_{12}} > 0$ *for* $\mu \in (\mu^*, 1)$.

As the nominal wages of the farmers are always equal to 1, their real wages only depend on the price index. We show that the price index in region 1, $P_1$, is an increasing function of $\tau_{12}$ (Lemma 2.3.5) and that the price index in region 2, $P_2$, is a decreasing function of $\tau_{12}$ (Lemma 2.3.6). This means that a decrease in the cost of trading manufactured goods from region 1 to region 2 (an increase in $\tau_{12}$) benefits the farmers of region 2 and penalizes the farmers of region 1.

**Lemma** 2.3.5. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{11} = \tau_{22} = \tau_i$ *and* $L_1 = L_2$*. Then,* $P_1$ *is an increasing function of* $\tau_{12}$ *for any* $0 < \tau_e < \tau_i < 1$*,* $\mu \in (0, 1)$ *and* $\sigma > 1$.

**Lemma** 2.3.6. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{11} = \tau_{22} = \tau_i$ *and* $L_1 = L_2$*. Then,* $P_2$ *is a decreasing function of* $\tau_{12}$ *for any* $0 < \tau_e < \tau_i < 1$*,* $\mu \in (0, 1)$ *and* $\sigma > 1$.

### Long-run effect on the distribution of industrial activity

Figures 2.10 and 2.11[19] depict the long-run distribution of industrial activity. Depending on the extent of the asymmetry of external trade costs, the economic activity can be asymmetrically distributed between regions or fully concentrated in one region.

In figure 2.10, we present an asymmetric dispersion equilibrium. An increase in $\tau_{12}$ increases the relative real wage in region 1 attracting new workers to the region. The migration to region 1 leads to a decrease in $\omega_1/\omega_2$. This process continues until a new long-run equilibrium is reached, point C, with $\omega_1 = \omega_2$.

Figure 2.11 illustrates how a strong increase in $\tau_{12}$ may generate catastrophic agglomeration in region 1, producing a core-periphery structure (point C).

[19]To plot these figures, we have set $\tau_i = 0.95$, $\mu = 0.3$ and $\sigma = 4$.

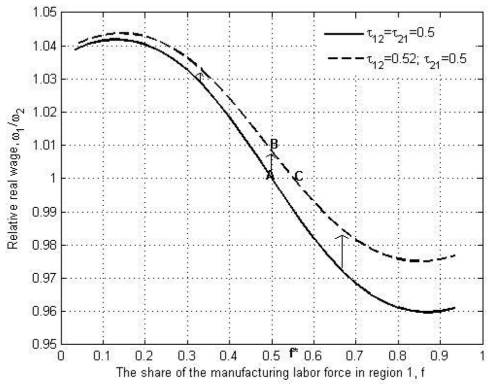


Figure 2.10: Asymmetric dispersion with asymmetric external trade costs.

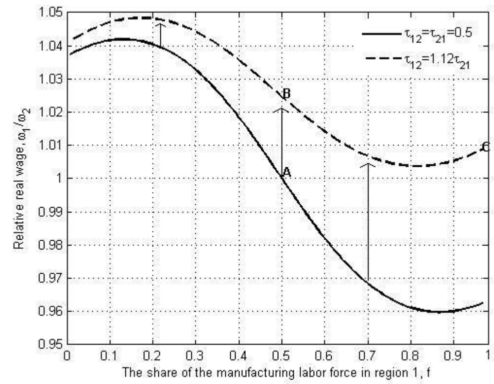


Figure 2.11: Concentration with asymmetric external trade costs.

Figures 2.12 and 2.13[20] describe the case in which there is an initial concentration of economic activity. Figure 2.12 deals with a case in which concentration may occur in any of the regions, while figure 2.13 shows a case in which all economic activity becomes concentrated in region 2.

Figure 2.12 shows that an increase in $\tau_{12}$ enlarges the basin of attraction of the equilibrium in which all economic activity is concentrated in region 1.

Figure 2.13 illustrates an environment in which region 2 initially concentrates all industrial activity (point $A$, with $f = 0$ and $\omega_2 > \omega_1$). An increase in $\tau_{12}$ raises the relative real wage in region 1, in the short-run (point B). As the real wage is higher in region 1, workers migrate from region 2 to region 1. In the long-run, there is full agglomeration in region 1 (point C).

There is a threshold degree of asymmetric trade liberalization between regions that generates the relocation of all industrial activity from one region to the other. However,

[20]To plot these figures, we have set $\tau_i = 0.95$, $\mu = 0.3$ and $\sigma = 4$.

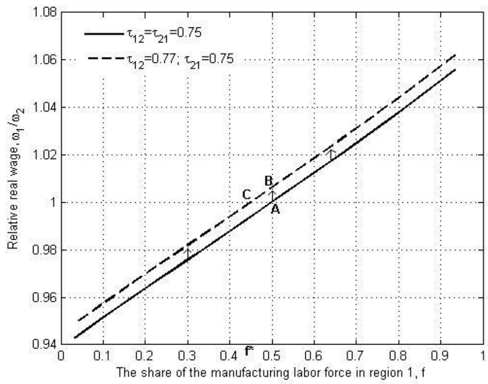


Figure 2.12: Agglomeration in any region with asymmetric external trade costs.

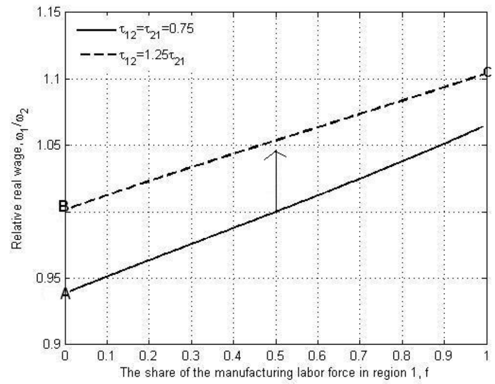


Figure 2.13: Agglomeration in only one region with asymmetric external trade costs.

there will be no migration of workers until the the asymmetry in trade liberalization reaches this threshold.

## 2.4 Concluding remarks

We have extended the model of Krugman (1991) in order to study the effects of internal and external trade costs on the location of industrial activity as well as on the welfare of the agents. The existence of an asymmetric dispersion equilibrium is not surprising, given that under distinct trade costs, the regions no longer are identical.

We find that industrial activity in a region is enhanced, *ceteris paribus*, by lower internal trade costs and by higher costs of importing (lower costs of exporting). The fact that asymmetries in the external trade costs lead to relocation of economic activity is a natural result. However, it was not present in the work of Martin and Rogers (1995). In their model, differentials in external trade costs only increase the sensitivity of industrial location to differentials in the internal trade costs.[21]

From the point of view of welfare, a decrease of the internal trade cost of a region benefits the workers of this region and the farmers of both regions, while the workers of the other region become worse off.

A decrease in the cost of trading from region 1 to region 2 benefits the workers of region 1 and the farmers of region 2, while the farmers of region 1 become worse off. The effect on the welfare of the workers of region 2 is positive if the weight of the industrial sector is large, and negative otherwise.

The analytical results obtained for short-run equilibria support the numerical evidence obtained for the long-run. In the long-run, numerical results indicate a new feature, namely that of sudden agglomeration in some instances.

[21] If internal trade costs are equal, then differentials in external trade costs have no effect.

# Appendices

# Appendix A

# Mathematical proofs

## A.1 Claims

**Claim** A.1.1. *If $L_1 = L_2$, then $W_1 + W_2 = 2$.*

*When $L_1 = L_2$, $\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{22} = \tau_{11} = \tau_i$, then $W_1 = W_2 = 1$.*

*Proof.* Substituting $L_1 = L_2 = \frac{mu}{2}$ in (2.7), we obtain $W_1 + W_2 = 2$.

If the trade costs are symmetric ($\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{22} = \tau_{11} = \tau_i$), then $W_1 = W_2 = 1$ is a short-run equilibrium, as we verify below.

The nominal wage in region 1 is:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_1L_1)}{1+\left(\frac{W_1\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_2L_2)}{1+\left(\frac{W_1\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}\frac{L_1}{L_2}}\right].$$

Substituting $W_1 = W_2 = 1$, $L_1 = L_2 = \frac{\mu}{2}$, $\tau_e = \tau_{12} = \tau_{21}$ and $\tau_i = \tau_{11} = \tau_{22}$, we obtain:

$$\begin{aligned}
1 &= 2\left[\frac{\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}\frac{1-\mu+\mu}{2}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}} + \frac{\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}\frac{1-\mu+\mu}{2}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}\right] \Leftrightarrow \\
1 &= \left[\frac{\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}} + \frac{\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}{1+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}\right] \Leftrightarrow \\
1 &= \frac{2+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}}{2+\left(\frac{\tau_e}{\tau_i}\right)^{1-\sigma}+\left(\frac{\tau_e}{\tau_i}\right)^{\sigma-1}} \Leftrightarrow \\
1 &= 1.
\end{aligned}$$

This short-run equilibrium is unique (Mossay, 2006).

□

**Claim** A.1.2. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$, and $L_1 = L_2$. Then, $P_1 = P_2 \geq 1$.*

*Proof.* Substituting $W_1 = W_2 = 1$, $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{22} = \tau_{11} = \tau_i$ and $f = \frac{1}{2}$ into (2.10) and (2.11), we obtain:

$$P_1 = \left(\frac{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}{2}\right)^{\mu/(1-\sigma)},$$

and

$$P_2 = \left(\frac{\tau_e^{\sigma-1} + \tau_i^{\sigma-1}}{2}\right)^{\mu/(1-\sigma)}.$$

We verify that $P_1 = P_2$, and since $\tau_i$ and $\tau_e$ are greater or equal than 1, $P_1 = P_2 \geq 1$.

□

**Claim** A.1.3. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{22} = \tau_{11} = \tau_i$ *and* $L_1 = L_2$*. Then,* $0 < \omega_1 = \omega_2 \leq 1$.

*Proof.* The real wage is simply given by the ratio between nominal wage ($W_1 = W_2 = 1$) and the price index ($P_1 = P_2 \geq 1$).

$$\omega_1 = \omega_2 = \frac{1}{P_1} = \frac{1}{P_2} = \left(\frac{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}{2}\right)^{\mu/(\sigma-1)}.$$

We clearly have $0 < \omega_1 = \omega_2 \leq 1$.

□

**Claim** A.1.4. *Let* $L_1 = L_2$*. In the short-run:*

$$\frac{dW_1}{d\tau_i} = -\frac{dW_2}{d\tau_i}, \quad \frac{dW_1}{d\tau_e} = -\frac{dW_2}{d\tau_e} \quad \textit{and} \quad \frac{dW_1}{d\tau_{22}} = -\frac{dW_2}{d\tau_{22}}.$$

*Proof.* This is a straightforward consequence of $W_1 + W_2 = 2$.

□

**Claim** A.1.5. *Let* $\tau_{21} = \tau_{12} = \tau_e$*,* $\tau_{11} = \tau_{22} = \tau_i = 1$ *and* $L_1 = L_2$*. An increase in* $\tau_{22}$ *decreases* $W_1$*, for any* $0 < \tau_e < \tau_i < 1$*,* $\mu \in (0,1)$ *and* $\sigma > 1$.

*Proof.* We want to prove that $\left.\frac{dW_2}{d\tau_{22}}\right|_{L_1=L_2} > 0$.

By Claim A.1.1, we can substitute $W_1 = 2 - W_2$ in equation (2.9).

$$\frac{W_2}{2} = \frac{\frac{1-\mu}{2} + (2 - W_2)\frac{\mu}{2}}{1 + \left(\frac{(2-W_2)\tau_{21}}{W_2\tau_{11}}\right)^{1-\sigma}} + \frac{\frac{1-\mu}{2} + \frac{W_2\mu}{2}}{1 + \left(\frac{(2-W_2)\tau_{22}}{W_2\tau_{12}}\right)^{1-\sigma}} = \frac{A}{B} + \frac{C}{D}. \tag{A.1}$$

We compute $\frac{dW_2}{d\tau_{22}}$ (denoted, below, as $W_2'$) by implicit differentiation, substituting $W_2 = 1$, $\tau_{21} = \tau_{12} = \tau_e$ and $\tau_{11} = \tau_{22} = \tau_i = 1$.

$$\frac{W_2'}{2} = \frac{A'}{B} - \frac{B'A}{B^2} + \frac{C'}{D} - \frac{D'C}{D^2},$$

where

$$A = C = \frac{1}{2}, \quad A' = -\frac{\mu}{2}W_2', \quad C' = \frac{\mu}{2}W_2', \quad B = 1 + \tau^{1-\sigma}, \quad D = 1 + \tau^{\sigma-1},$$

$$B' = 2(\sigma-1)\tau^{1-\sigma}W_2', \quad D' = 2(\sigma-1)\tau^{\sigma-1}\left(W_2' - \frac{1}{2\tau_i}\right).$$

With some manipulation:

$$\frac{W_2'}{2} = -\frac{\frac{\mu}{2}W_2'}{1+\tau^{1-\sigma}} + \frac{\frac{\mu}{2}W_2'}{1+\tau^{\sigma-1}} - \frac{(\sigma-1)\tau^{1-\sigma}W_2'}{(1+\tau^{1-\sigma})^2} - \frac{(\sigma-1)\tau^{\sigma-1}\left(W_2' - \frac{1}{2\tau_i}\right)}{(1+\tau^{\sigma-1})^2} \Leftrightarrow$$

$$\Leftrightarrow \quad W_2' \left[1 + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}} + \frac{(\sigma-1)\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2} + \frac{(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\right] = \frac{(\sigma-1)\tau^{\sigma-1}}{2\tau_i\,(1+\tau^{\sigma-1})^2}.$$

Using the fact that $\frac{\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2} = \frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}$:

$$W_2' = \frac{\frac{(\sigma-1)\tau^{\sigma-1}}{2\tau_i(1+\tau^{\sigma-1})^2}}{1 + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}} + \frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}}. \tag{A.2}$$

It should be clear that $W_2' = \frac{dW_2}{d\tau_{22}}$ is positive. Then, from Claim (A.1.4), we know that $\frac{dW_1}{d\tau_{22}}$ is negative.

□

**Claim** A.1.6. *Let $\tau_{21} = \tau_{12} = \tau_e$, $\tau_{11} = \tau_{22} = \tau_i =$ and $L_1 = L_2$. An increase in $\tau_{12}$ increases $W_1$, for any $\tau_i \in (0,1)$ $\tau_e \in (0,\tau_i)$, $\mu \in (0,1)$ and $\sigma > 1$.*

*Proof.* From Claim A.1.4, we know that $\frac{dW_2}{d\tau_{12}} = -\frac{dW_1}{d\tau_{12}}$. Denote $\frac{dW_2}{d\tau_{12}}$ by $W_2'$.

Recalling (A.1):

$$\frac{W_2}{2} = \frac{\frac{1-\mu}{2} + \frac{\mu}{2}(2 - W_2)}{1 + \left(\frac{2-W_2}{W_2}\frac{\tau_{21}}{\tau_{11}}\right)^{1-\sigma}} + \frac{\frac{1-\mu}{2} + \frac{\mu}{2}W_2}{1 + \left(\frac{2-W_2}{W_2}\frac{\tau_{22}}{\tau_{12}}\right)^{1-\sigma}} = \frac{A}{B} + \frac{C}{D}.$$

Then:

$$\begin{aligned}
\frac{W_2'}{2} &= \frac{-\frac{\mu}{2}W_2'B - (1-\sigma)\left(\frac{2-W_2}{W_2}\frac{\tau_{21}}{\tau_{11}}\right)^{-\sigma}\frac{\tau_{21}}{\tau_{11}}\frac{-W_2'W_2 - W_2'(2-W_2)}{W_2^2}A}{B^2} + \\
&+ \frac{\frac{\mu}{2}W_2'D - (1-\sigma)\left(\frac{2-W_2}{W_2}\frac{\tau_{22}}{\tau_{12}}\right)^{-\sigma}\left[\frac{-W_2'W_2 - W_2'(2-W_2)}{W_2^2}\frac{\tau_{22}}{\tau_{12}} + \frac{2-W_2}{W_2}\left(-\frac{\tau_{22}}{\tau_{12}^2}\right)\right]C}{D^2} = \\
&= \frac{-\frac{\mu}{2}W_2'(1+\tau^{1-\sigma}) + (1-\sigma)\tau^{-\sigma}\tau W_2'}{(1+\tau^{1-\sigma})^2} + \\
&+ \frac{\frac{\mu}{2}W_2'(1+\tau^{\sigma-1}) + (1-\sigma)\tau^{\sigma}\left[W_2'\tau^{-1} + \frac{\tau_i}{2\tau_e^2}\right]}{(1+\tau^{\sigma-1})^2}.
\end{aligned}$$

Equivalently (using $W_1' = -W_2'$):

$$\begin{aligned}
\frac{W_1'}{2} &= \frac{W_1'\left[(1-\sigma)\tau^{1-\sigma} - \frac{\mu}{2}(1+\tau^{1-\sigma})\right]}{(1+\tau^{1-\sigma})^2} + \\
&+ \frac{W_1'\left[(1-\sigma)\tau^{\sigma-1} + \frac{\mu}{2}(1+\tau^{\sigma-1})\right]}{(1+\tau^{\sigma-1})^2} - \frac{\frac{1-\sigma}{2\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}.
\end{aligned}$$

Solving for $W_1'$:

$$W_1' = \frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\frac{1}{1-A-B}, \tag{A.3}$$

in which:

$$A = \frac{2(1-\sigma)\tau^{1-\sigma} - \mu(1+\tau^{1-\sigma})}{(1+\tau^{1-\sigma})^2};$$

$$B = \frac{2(1-\sigma)\tau^{\sigma-1} + \mu(1+\tau^{\sigma-1})}{(1+\tau^{\sigma-1})^2}.$$

It is straightforward to see that $1 - A - B > 0$ (only the last term is negative):

$$1 - A - B = 1 + \frac{2(\sigma-1)\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2} + \frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}}.$$

□

## A.2 Proof of lemmas

### A.2.1 Lemma 2.3.1

*Proof.* We want to show that $\left.\frac{d\omega_1}{d\tau_{22}}\right|_{L_1=L_2} < 0$.

Using (2.7), we write $W_2$ as function of $W_1$, and differentiate the expression using the chain rule and the Implicit Function Theorem.

$$\frac{d\omega_1}{d\tau_{22}} = \frac{d\left(\frac{W_1(W_1,W_2)}{P_1(W_1,W_2)}\right)}{d\tau_{22}} = \frac{\frac{\partial W_1}{\partial \tau_{22}} P_1(W_1) - \frac{dP_1}{d\tau_{22}} W_1}{P_1(W_1)^2}, \tag{A.4}$$

where

$$\frac{dP_1}{d\tau_{22}} = \left[\frac{\partial P_1(W_1)}{\partial \tau_{22}} + \frac{\partial P_1(W_1)}{\partial W_1}\frac{\partial W_1}{\partial \tau_{22}}\right].$$

From Claim A.1.1, when $L_1 = L_2$, $\tau_{11} = \tau_{22} = \tau_i$ and $\tau_{12} = \tau_{21} = \tau_e$, it is always the case that $W_1 = W_2 = 1$. Then (A.4) becomes:

$$\frac{d\omega_1}{d\tau_{22}} = \frac{\frac{\partial W_1}{\partial \tau_{22}} P_1(W_1) - \left[\frac{\partial P_1(W_1)}{\partial \tau_{22}} + \frac{\partial P_1(W_1)}{\partial W_1}\frac{\partial W_1}{\partial \tau_{22}}\right]}{P_1(W_1)^2}. \tag{A.5}$$

From Claims A.1.5 and A.1.2, we know that $\frac{\partial W_1}{\partial \tau_{22}} < 0$ and that $P_1(W_1) > 0$.

Since $P_1$ does not depend on the internal transportation costs of region 2, $\tau_{22}$, from (2.10) we have that $\frac{\partial P_1(W_1)}{\partial \tau_{22}} = 0$. Therefore, the second term in the numerator of (A.5) becomes:

$$\frac{\partial P_1(W_1)}{\partial W_1}\frac{\partial W_1}{\partial \tau_{22}} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$

$$\left\{\frac{1-\sigma}{2}\left[\left(\frac{1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]\right\}\frac{\partial W_1}{\partial \tau_{22}}.$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$:

$$\frac{\partial P_1(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}. \tag{A.6}$$

With $P_1$ at equilibrium values, $P_1 = \gamma\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}}$, and using (A.6), (A.5) becomes:

$$\begin{aligned}\frac{d\omega_1}{d\tau_{22}} &= \frac{\frac{dW_1}{d\tau_{22}}}{P_1} - \frac{\frac{\mu}{2}P_1\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}}{P_1^2} \\ &= \frac{\frac{dW_1}{d\tau_{22}}}{P_1} - \frac{\frac{\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}}{P_1}.\end{aligned}$$

To prove Lemma 2.3.1 we note that:

$$\frac{dW_1}{d\tau_{22}}\left[1 - \mu\frac{\tau_i^{\sigma-1} - \tau_e^{\sigma-1}}{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}}\right] < 0 \Rightarrow \frac{d\omega_1}{d\tau_{22}} < 0.$$

Given that $\tau_i > \tau_e$, it is clear that $0 < \frac{\mu\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)}{\tau_i^{\sigma-1} + \tau_e^{\sigma-1}} < 1$, finishing the proof.

□

### A.2.2 Lemma 2.3.2

*Proof.* We want to show that $\left.\frac{d\omega_2}{d\tau_{22}}\right|_{L_1=L_2} > 0.$

Using (2.7), we write $W_2$ as function of $W_1$, and differentiate the expression using the chain rule and the Implicit Function Theorem.

$$\frac{d\omega_2}{d\tau_{22}} = \frac{d\left(\frac{W_2(W_1,W_2)}{P_2(W_1,W_2)}\right)}{d\tau_{22}} = \frac{\frac{dW_2}{d\tau_{22}}P_2(W_1) - \frac{dP_2}{d\tau_{22}}W_2}{P_2(W_1)^2}, \tag{A.7}$$

where:

$$\frac{dP_2}{d\tau_{22}} = \left[\frac{\partial P_2(W_1)}{\partial \tau_{22}} + \frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}}\right]. \tag{A.8}$$

From (2.11), we compute the first term in (A.8):

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$

$$\frac{1-\sigma}{2}\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\left(-\frac{2-W_1}{\tau_{22}^2}\right).$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\tau_i^{\sigma-2}. \tag{A.9}$$

Secondly, we study the second term of (A.8):

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$

$$\left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1} - \left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}\frac{dW_1}{d\tau_{22}}.$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}. \tag{A.10}$$

With (A.9) and (A.10), equation (A.8) becomes:

$$\frac{dP_2}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left[\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}-\tau_i^{\sigma-2}\right].$$

With $P_2$ at the equilibrium value, $P_2 = \gamma\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}}$, equation (A.7) becomes:

$$\begin{aligned}\frac{d\omega_2}{d\tau_{22}} &= \frac{\frac{dW_2}{d\tau_{22}}}{P_2}-\frac{\frac{\mu}{2}P_2\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-1}\left[\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}-\tau_i^{\sigma-2}\right]}{P_2^2}\\
&= \frac{\frac{dW_2}{d\tau_{22}}}{P_2}-\frac{\mu\,\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)^{-1}\left[\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}-\tau_i^{\sigma-2}\right]}{P_2}.\end{aligned} \tag{A.11}$$

To prove that (A.11) is positive, we note that:

$$\frac{dW_2}{d\tau_{22}}-\mu\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)^{-1}\left(\tau_i^{\sigma-1}-\tau_e^{\sigma-1}\right)\frac{dW_2}{d\tau_{22}}+\mu\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)^{-1}\tau_i^{\sigma-2} =$$

$$= \frac{dW_2}{d\tau_{22}}\left(1-\mu\frac{\tau_i^{\sigma-1}-\tau_e^{\sigma-1}}{\tau_i^{\sigma-1}+\tau_e^{\sigma-1}}\right)+\frac{\mu\tau_i^{\sigma-2}}{\tau_e^{\sigma-1}+\tau_i^{\sigma-1}} > 0.$$

From Claims A.1.5 and A.1.4, we know that $\frac{dW_2}{d\tau_{22}} > 0$, thus the proof is finished.

□

### A.2.3 Lemma 2.3.3

*Proof.* By the chain rule:

$$\frac{dP_1}{d\tau_{22}} = \frac{\partial P_1(W_1)}{\partial \tau_{22}} + \frac{\partial P_1(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}}. \tag{A.12}$$

From (2.10), we know that $\frac{\partial P_1(W_1)}{\partial \tau_{22}} = 0$, and thus (A.12) becomes:

$$\begin{aligned}\frac{dP_1}{d\tau_{22}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\quad \frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{2-W_1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]\frac{dW_1}{d\tau_{22}}.\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{dP_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}.$$

As $\gamma > 0$, $\tau_i > \tau_e$ and (from Claim A.1.5) $\frac{dW_1}{d\tau_{22}} < 0$, the proof is finished.

□

### A.2.4 Lemma 2.3.4

*Proof.* By the chain rule:

$$\frac{dP_2}{d\tau_{22}} = \frac{\partial P_2(W_1)}{\partial \tau_{22}} + \frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}}. \tag{A.13}$$

From (2.11), we compute the first term of (A.13):

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$

$$\frac{1-\sigma}{2}\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\left(-\frac{2-W_1}{\tau_{22}^2}\right).$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$ above, we obtain:

$$\frac{\partial P_2(W_1)}{\partial \tau_{22}} = -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\tau_i^{\sigma-2}. \tag{A.14}$$

Secondly, we study the second term of (A.13):

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$

$$\left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1}-\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}\frac{dW_1}{d\tau_{22}}.$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2(W_1)}{\partial W_1}\frac{dW_1}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}. \tag{A.15}$$

With (A.14) and (A.15), we compute $\frac{dP_2}{d\tau_{22}}$:

$$\frac{dP_2}{d\tau_{22}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left[\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}}-\tau_i^{\sigma-2}\right]. \tag{A.16}$$

Given that, in (A.16), $\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}$ is positive, to finish the proof we need to show the following inequality:

$$\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{22}} < \tau_i^{\sigma-2} \Leftrightarrow \tau_i(1-\tau^{\sigma-1})\frac{dW_2}{d\tau_{22}} < 1.$$

Importing the expression (A.2):

$$\tau_i(1-\tau^{\sigma-1})\frac{\frac{(\sigma-1)\tau^{\sigma-1}}{2\tau_i(1+\tau^{\sigma-1})^2}}{1+\frac{\mu}{1+\tau^{1-\sigma}}-\frac{\mu}{1+\tau^{\sigma-1}}+\frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}} < 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{1-\tau^{\sigma-1}}{2}\,\frac{(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} < 1 + \frac{\mu}{1+\tau^{1-\sigma}} - \frac{\mu}{1+\tau^{\sigma-1}} + \frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}.$$

Since the expression on the left is lower than the last term of the expression on the right, the proof is finished.

□

### A.2.5 Lemma 2.3.5

*Proof.* Observe that:

$$\left.\frac{dP_1}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\partial P_1}{\partial \tau_{12}} + \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}. \tag{A.17}$$

As $\frac{\partial P_1}{\partial \tau_{12}} = 0$, (A.17) becomes:

$$\begin{aligned} \frac{dP_1}{d\tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1} - \left(\frac{2-W_1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]\right\}\frac{dW_1}{d\tau_{12}}. \end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{dP_1}{d\tau_{12}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1} + \tau_e^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}\right)\frac{dW_1}{d\tau_{12}}.$$

From Claim A.1.6, we know that $\frac{\partial W_1}{\partial \tau_{12}} > 0$, and the proof is finished.

□

### A.2.6 Lemma 2.3.6

*Proof.* Observe that:

$$\frac{dP_2}{\tau_{12}} = \frac{\partial P_2}{\partial \tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}. \tag{A.18}$$

Firstly, we study the first term in (A.18):

$$\begin{aligned} \frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left[\frac{1-\sigma}{2}\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\left(\frac{-W_1}{\tau_{12}^2}\right)\right]. \end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\begin{aligned} \frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{\frac{-\mu}{\sigma-1}-1}\left[\frac{\sigma-1}{2}\tau_e^{\sigma-2}\right] \\ &= -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\tau_e^{\sigma-2}. \end{aligned} \tag{A.19}$$

Secondly, we study the second term in (A.18), then:

$$\begin{aligned} \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1} - \left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}\frac{dW_1}{d\tau_{12}}. \end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right)\frac{dW_1}{d\tau_{12}}. \tag{A.20}$$

Adding (A.19) and (A.20), we obtain:

$$\frac{dP_2}{d\tau_{12}} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left[\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)\frac{\partial W_1}{\partial \tau_{12}}-\tau_e^{\sigma-2}\right].$$

From Claim A.1.6, $\frac{dW_1}{d\tau_{12}} > 0$, which implies that $\frac{dP_2}{d\tau_{12}} < 0$.

□

## A.3 Proof of propositions

### A.3.1 Proposition 2.3.1

*Proof.* Let $Z_{11}$ and $Z_{11}^K$ be the ratio between region 1's expenditure on local manufactures and that on manufactures from the other region in the model with equal internal and external trade costs and in the model without internal trade costs, respectively. Then, from (2.2), and for $\tau_{11} = \tau_{22} = \tau_i$, $\tau_{21} = \tau_{12} = \tau_e$ and $\tau = \dfrac{\tau_e}{\tau_i}$, we have:

$$Z_{11} = \frac{E_{11}}{E_{12}} = \left(\frac{W_1\tau_e}{W_2\tau_i}\right)^{1-\sigma}\frac{L_1}{L_2} = \left(\frac{W_1\tau}{W_2}\right)^{1-\sigma}\frac{L_1}{L_2} = {Z_{11}}^K.$$

Analogously, from (2.3):

$$Z_{12} = \frac{E_{21}}{E_{22}} = \left(\frac{W_1\tau_i}{W_2\tau_e}\right)^{1-\sigma}\frac{L_1}{L_2} = \left(\frac{W_1}{W_2\tau}\right)^{1-\sigma}\frac{L_1}{L_2} = {Z_{12}}^K$$

Next, we show that the nominal wages, $W_1$ and $W_2$, when regions have equal internal trade costs and equal external trades costs are the same as in the case in which regions have only an external trade costs, $\tau = \frac{\tau_e}{\tau_i}$.

From (2.8) and (2.9) with $\tau_i = \tau_{22} = \tau_{11}$ and $\tau_e = \tau_{12} = \tau_{21}$, the nominal wages satisfy:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1\tau_e}{W_2\tau_i}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_1L_1)}{1+\left(\frac{W_1\tau_e}{W_2\tau_i}\right)^{1-\sigma}\frac{L_1}{L_2}} + \frac{\left(\frac{W_1\tau_i}{W_2\tau_e}\right)^{1-\sigma}\frac{L_1}{L_2}(\frac{1-\mu}{2}+W_2L_2)}{1+\left(\frac{W_1\tau_i}{W_2\tau_e}\right)^{1-\sigma}\frac{L_1}{L_2}}\right]; \qquad \text{(A.21)}$$

$$W_2 = \frac{\mu}{L_2}\left[\frac{\frac{1-\mu}{2} + W_1 L_1}{1 + \left(\frac{W_1 \tau_e}{W_2 \tau_i}\right)^{1-\sigma} \frac{L_1}{L_2}} + \frac{\frac{1-\mu}{2} + W_2 L_2}{1 + \left(\frac{W_1 \tau_i}{W_2 \tau_e}\right)^{1-\sigma} \frac{L_1}{L_2}}\right]. \tag{A.22}$$

Since $\tau = \frac{\tau_e}{\tau_i}$, (A.21) and (A.22) become:

$$W_1 = \frac{\mu}{L_1}\left[\frac{\left(\frac{W_1 \tau}{W_2}\right)^{1-\sigma} \frac{L_1}{L_2}\left(\frac{1-\mu}{2} + W_1 L_1\right)}{1 + \left(\frac{W_1 \tau}{W_2}\right)^{1-\sigma} \frac{L_1}{L_2}} + \frac{\left(\frac{W_1}{W_2 \tau}\right)^{1-\sigma} \frac{L_1}{L_2}\left(\frac{1-\mu}{2} + W_2 L_2\right)}{1 + \left(\frac{W_1}{W_2 \tau}\right)^{1-\sigma} \frac{L_1}{L_2}}\right];$$

$$W_2 = \frac{\mu}{L_2}\left[\frac{\frac{1-\mu}{2} + W_1 L_1}{1 + \left(\frac{W_1 \tau}{W_2}\right)^{1-\sigma} \frac{L_1}{L_2}} + \frac{\frac{1-\mu}{2} + W_2 L_2}{1 + \left(\frac{W_1}{W_2 \tau}\right)^{1-\sigma} \frac{L_1}{L_2}}\right].$$

These expressions coincide with those of the classical model of Krugman. There is a single equilibrium, as shown by Mossay (2006). Therefore, the nominal wages coincide:

$$W_1 = W_1^K \qquad \text{and} \qquad W_2 = W_2^K.$$

The inclusion of internal trade costs changes the price index of manufactured goods. Let $P_{M1}$ be the price index of manufactured goods in the model with equal internal and external trade costs, and let $P_{M1}^K$ be the price index of manufactured goods in the model without internal trade costs. For $\tau_{11} = \tau_{22} = \tau_i$, $\tau_{12} = \tau_{21} = \tau_e$, and $\tau = \frac{\tau_e}{\tau_i}$, we have:

$$P_{M1} = \gamma\left[f\left(\frac{W_1}{\tau_i}\right)^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau_e}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \Leftrightarrow$$

$$\Leftrightarrow P_{M1}\tau_i = \gamma\left[f {W_1}^{1-\sigma} + (1-f)\left(\frac{W_2}{\tau}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = {P_{M1}}^K.$$

Doing the same for $P_{M2}$, we verify that the manufacturing price indexes increase to compensate for the internal "iceberg" trade costs:

$$P_{M1} = \frac{P_{M1}^K}{\tau_i} \qquad \text{and} \qquad P_{M2} = \frac{P_{M2}^K}{\tau_i}.$$

This implies that:

$$\omega_1 = \frac{W_1}{P_{M1}^{\mu}} = \frac{W_1^K}{(P_{M1}^K/\tau_i)^{\mu}} = \omega_1^K \tau_i^{\mu} \qquad \text{and} \qquad \omega_2 = \omega_2^K \tau_i^{\mu}.$$

The internal trade costs decrease the real wages in the same proportion, therefore, the relative real wage remains unaltered:

$$\frac{\omega_1}{\omega_2} = \frac{\omega_1^K \tau_i^{\mu}}{\omega_2^K \tau_i^{\mu}} = \frac{\omega_1^K}{\omega_2^K}.$$

□

### A.3.2 Proposition 2.3.2

*Proof.* We want to show that $\left.\dfrac{d\left(\omega_1/\omega_2\right)}{d\tau_{22}}\right|_{L_1=L_2} < 0.$

We have:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{22}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{22}}\omega_2 - \frac{d\omega_2}{d\tau_{22}}\omega_1}{\omega_2^2}. \tag{A.23}$$

From Claim A.1.3, we know that $\omega_1 = \omega_2 > 0$, therefore, (A.23) can be written as:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{22}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{22}} - \frac{d\omega_2}{d\tau_{22}}}{\omega_2}.$$

From Lemma 2.3.1 and Lemma 2.3.2, we know that $\frac{d\omega_1}{d\tau_{22}} < 0$ and $\frac{d\omega_2}{d\tau_{22}} > 0$ respectively, thus:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{22}}\right|_{L_1=L_2} < 0.$$

□

### A.3.3 Proposition 2.3.3

*Proof.* We want to know the sign of:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{12}}\omega_2 - \frac{d\omega_2}{d\tau_{12}}\omega_1}{\omega_2^2},$$

where

$$\frac{d\omega_1}{d\tau_{12}} = \frac{d\left(\frac{W_1}{P_1}\right)}{d\tau_{12}} = \frac{\frac{dW_1}{d\tau_{12}}P_1 - \left(\frac{\partial P_1}{\partial\tau_{12}} + \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right)W_1}{P_1^2}$$

and

$$\frac{d\omega_2}{d\tau_{12}} = \frac{d\left(\frac{W_2}{P_2}\right)}{d\tau_{12}} = \frac{\frac{dW_2}{d\tau_{12}}P_2 - \left(\frac{\partial P_2}{\partial\tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right)W_2}{P_2^2}.$$

When $L_1 = L_2$, $\tau_{22} = \tau_{11} = \tau_i$ and $\tau_{12} = \tau_{21} = \tau_e$, from Claims A.1.1, A.1.2 and A.1.3, we know that $W_1 = W_2 = 1$, $P_1 = P_2$ and $\omega_1 = \omega_2$. Simplifying:

$$\left.\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{d\omega_1}{d\tau_{12}} - \frac{d\omega_2}{d\tau_{12}}}{\omega_2},$$

with

$$\frac{d\omega_1}{d\tau_{12}} - \frac{d\omega_2}{d\tau_{12}} = \frac{\left(\frac{dW_1}{d\tau_{12}} - \frac{dW_2}{d\tau_{12}}\right)P_1 + \frac{\partial P_2}{\partial\tau_{12}} - \frac{\partial P_1}{\partial\tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}} - \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}}{P_1^2}.$$

From Claim A.1.4 and (2.10), we know that $\frac{dW_1}{d\tau_{12}} = -\frac{dW_2}{d\tau_{12}}$ and that $\frac{\partial P_1}{\partial\tau_{12}} = 0$. Then:

$$\frac{d\omega_1}{d\tau_{12}} - \frac{d\omega_2}{d\tau_{12}} = \frac{1}{P_1^2}\left[\frac{dW_1}{d\tau_{12}}\left(2P_1 + \frac{\partial P_2}{\partial W_1} - \frac{\partial P_1}{\partial W_1}\right) + \frac{\partial P_2}{\partial\tau_{12}}\right].$$

The next stage is to compute $\frac{\partial P_2}{\partial W_1}$, $\frac{\partial P_1}{\partial W_1}$ and $\frac{\partial P_2}{\partial\tau_{12}}$.

Using (2.11), we find:

$$\frac{\partial P_2}{\partial W_1} = -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}$$

$$\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1}-\left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right].$$

Replacing $W_1=1$, $\tau_{11}=\tau_{22}=\tau_i$, $\tau_{12}=\tau_{21}=\tau_e$ and $P_1=\gamma\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}}$:

$$\begin{aligned}\frac{\partial P_2}{\partial W_1} &= \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1}-\tau_i^{\sigma-1}\right)=\\ &= \mu P_1\frac{\tau_e^{\sigma-1}-\tau_i^{\sigma-1}}{\tau_e^{\sigma-1}+\tau_i^{\sigma-1}}=-\mu P_1\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}.\end{aligned}$$

Using (2.10) we find that $\frac{\partial P_1}{\partial W_1}=-\frac{\partial P_2}{\partial W_1}$:

$$\begin{aligned}\frac{\partial P_1}{\partial W_1} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{21}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}\\ &\quad \frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{11}}\right)^{-\sigma}\tau_{11}^{-1}-\left(\frac{2-W_1}{\tau_{21}}\right)^{-\sigma}\tau_{21}^{-1}\right]=\\ &= \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_i^{\sigma-1}+\tau_e^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_i^{\sigma-1}-\tau_e^{\sigma-1}\right).\end{aligned}$$

From (2.11), we compute $\frac{\partial P_2}{\partial \tau_{12}}$:

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\mu}{\sigma-1}\gamma\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma}+\left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1}\\ &\quad \frac{1-\sigma}{2}\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\left(-\frac{W_1}{\tau_{12}^2}\right)=-\frac{\gamma\mu}{2}\tau_e^{\sigma-2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1}+\tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\\ &= -\mu P_1\frac{\tau_e^{\sigma-2}}{\tau_e^{\sigma-1}+\tau_i^{\sigma-1}}=-\mu P_1\frac{\tau^{\sigma-1}\tau_e^{-1}}{1+\tau^{\sigma-1}}.\end{aligned}$$

Then, replacing:

$$P_1\left(\frac{d\omega_1}{d\tau_{12}}-\frac{d\omega_2}{d\tau_{12}}\right) \;=\; 2\frac{dW_1}{d\tau_{12}}\left(1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right)-\frac{\mu\tau_e^{-1}\tau^{\sigma-1}}{1+\tau^{\sigma-1}}.$$

The sign of $\frac{d(\omega_1/\omega_2)}{d\tau_{12}}$ is positive whenever:

$$\frac{dW_1}{d\tau_{12}}>\frac{\frac{\mu\tau_e^{-1}}{2}\frac{\tau^{\sigma}-1}{1+\tau^{\sigma-1}}}{1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}=\frac{\frac{\mu\tau_e^{-1}}{2}\tau^{\sigma-1}}{1+\tau^{\sigma-1}-\mu(1-\tau^{\sigma-1})}.$$

Importing the expression (A.3) for $W_1'>0$ obtained in Claim A.1.6:

$$W_1' \;=\; \frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\frac{1}{1-A-B},$$

where

$$A=-\frac{2(\sigma-1)\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2}-\frac{\mu}{1+\tau^{1-\sigma}},\quad \text{and} \quad B=-\frac{2(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}+\frac{\mu}{1+\tau^{\sigma-1}}.$$

Recalling that $\frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}=\frac{\tau^{1-\sigma}}{(1+\tau^{1-\sigma})^2}$, we know that $\frac{d(\omega_1/\omega_2)}{d\tau_{12}}>0$ if and only if:

$$\frac{\sigma-1}{1+4(\sigma-1)\frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}+\frac{\mu}{1+\tau^{1-\sigma}}-\frac{\mu}{1+\tau^{\sigma-1}}}>\frac{\frac{\mu}{2}(1+\tau^{\sigma-1})^2}{1+\tau^{\sigma-1}-\mu(1-\tau^{\sigma-1})}.$$

With some manipulation:

$$\frac{2(\sigma-1)}{(1+\tau^{\sigma-1})^2}>\frac{\mu+4\mu(\sigma-1)\frac{\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\mu^2\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}{1+\tau^{\sigma-1}-\mu(1-\tau^{\sigma-1})}.$$

It is clear that this is true when $\mu\to 0$. When $\mu\to 1$, the expression becomes:

$$\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}>1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}-\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}2\tau^{\sigma-1},$$

which is clearly false.

To show that the expression is true if and only if $\mu$ is lower or equal than some $\mu^*\in(0,1)$, notice that it is equivalent to a U-shaped parabola.

$$\frac{d\left(\omega_1/\omega_2\right)}{d\tau_{12}}>0\Leftrightarrow a\mu^2+b\mu+c>0,\quad \text{where} \quad a=\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}.$$

□

### A.3.4 Proposition 2.3.4

*Proof.* We want to show that $\left.\frac{d\omega_1}{d\tau_{12}}\right|_{L_1=L_2} > 0$.

$$\frac{d\omega_1}{d\tau_{12}} = \frac{d\left(\frac{W_1}{P_1}\right)}{d\tau_{12}} = \frac{\frac{dW_1}{d\tau_{12}}P_1 - \left[\frac{\partial P_1}{\partial \tau_{12}} + \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right]W_1}{P_1^2}. \tag{A.24}$$

When $L_1 = L_2$, $\tau_{11} = \tau_{22} = \tau_i$ and $\tau_{12} = \tau_{21} = \tau_e$, from Claim A.1.1 we know that $W_1 = 1$ and from (2.10) we find that $\frac{\partial P_1}{\partial \tau_{12}} = 0$. Then, simplifying (A.24):

$$\left.\frac{d\omega_1}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{dW_1}{d\tau_{12}}P_1 - \frac{\partial P_1}{\partial W_1}\frac{dW_1}{d\tau_{12}}}{P_1^2}. \tag{A.25}$$

From (2.10) we evaluate:

$$\frac{\partial P_1}{\partial W_1} = -\frac{\gamma\mu}{\sigma-1}\left[\frac{1}{2}(\tau_i^{\sigma-1} + \tau_e^{\sigma-1})\right]^{-\frac{\mu}{\sigma-1}-1}\frac{1-\sigma}{2}(\tau_i^{\sigma-1} - \tau_e^{\sigma-1}) = \mu P_1 \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} < P_1.$$

□

### A.3.5 Proposition 2.3.5

*Proof.* We want to show that $\left.\frac{d\omega_2}{d\tau_{12}}\right|_{L_1=L_2} < 0$.

$$\frac{d\omega_2}{d\tau_{12}} = \frac{d\left(\frac{W_2}{P_2}\right)}{d\tau_{12}} = \frac{\frac{dW_2}{d\tau_{12}}P_2 - \left(\frac{\partial P_2}{\partial \tau_{12}} + \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}\right)W_2}{P_2^2}. \tag{A.26}$$

From Claim A.1.1 we can rewrite (A.26) in the following way:

$$\left.\frac{d\omega_2}{d\tau_{12}}\right|_{L_1=L_2} = \frac{\frac{dW_2}{d\tau_{12}}P_2 - \frac{\partial P_2}{\partial \tau_{12}} - \frac{\partial P_2}{\partial W_1}\frac{dW_1}{d\tau_{12}}}{P_2^2}. \tag{A.27}$$

From Claim A.1.6 we know that $\frac{dW_1}{d\tau_{12}} > 0$ and from Claim A.1.4 we know that $\frac{dW_2}{d\tau_{12}} = -\frac{dW_1}{d\tau_{12}}$. Then, the sign of (A.27) is the same as the sign of:

$$-\frac{dW_1}{d\tau_{12}}\left(P_2 + \frac{\partial P_2}{\partial W_1}\right) - \frac{\partial P_2}{\partial \tau_{12}}.$$

From (2.11):

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left[\frac{1-\sigma}{2}\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\left(\frac{-W_1}{\tau_{12}^2}\right)\right].\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\begin{aligned}\frac{\partial P_2}{\partial \tau_{12}} &= -\frac{\gamma\mu}{\sigma-1}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left[\frac{\sigma-1}{2}\tau_e^{\sigma-2}\right] = \\ &= -\frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\tau_e^{\sigma-2} = -\mu P_2 \tau_e^{-1}\frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}}. \qquad \text{(A.28)}\end{aligned}$$

Again from (2.11):

$$\begin{aligned}\frac{\partial P_2}{\partial W_1} &= -\frac{\gamma\mu}{\sigma-1}\left\{\frac{1}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{1-\sigma} + \left(\frac{2-W_1}{\tau_{22}}\right)^{1-\sigma}\right]\right\}^{-\frac{\mu}{\sigma-1}-1} \\ &\times \left\{\frac{1-\sigma}{2}\left[\left(\frac{W_1}{\tau_{12}}\right)^{-\sigma}\tau_{12}^{-1} - \left(\frac{2-W_1}{\tau_{22}}\right)^{-\sigma}\tau_{22}^{-1}\right]\right\}.\end{aligned}$$

Replacing $W_1 = 1$, $\tau_i = \tau_{11} = \tau_{22}$ and $\tau_e = \tau_{12} = \tau_{21}$, we obtain:

$$\frac{\partial P_2}{\partial W_1} = \frac{\gamma\mu}{2}\left[\frac{1}{2}\left(\tau_e^{\sigma-1} + \tau_i^{\sigma-1}\right)\right]^{-\frac{\mu}{\sigma-1}-1}\left(\tau_e^{\sigma-1} - \tau_i^{\sigma-1}\right) = -\mu P_2 \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}. \qquad \text{(A.29)}$$

With (A.28) and (A.29), we observe that:

$$\frac{d\omega_2}{d\tau_{12}} < 0 \Leftrightarrow \frac{dW_1}{d\tau_{12}} \left(1 - \mu \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right) - \mu\tau_e^{-1} \frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}} > 0.$$

Importing the expression (A.3) obtained in Claim A.1.6:

$$\frac{dW_1}{d\tau_{12}} \quad = \quad \frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} \frac{1}{1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} - \mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}.$$

Thus $\frac{d\omega_2}{d\tau_{12}} < 0$ if and only if:

$$\frac{\frac{\sigma-1}{\tau_e}\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} \frac{1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}{1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} - \mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}} > \mu\tau_e^{-1}\frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}} \Leftrightarrow$$

$$\Leftrightarrow \frac{\sigma-1}{1+\tau^{\sigma-1}} \frac{1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}}{1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} - \mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}} > \mu \Leftrightarrow$$

$$\Leftrightarrow \frac{\sigma-1}{1+\tau^{\sigma-1}} \left(1-\mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right) - \mu\left[1+\frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} - \mu\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right] > 0.$$

The expression on the left is a U-shaped parabola, therefore, all we need to check is that the inequality is true when $\mu \to 0$ and false when $\mu \to 1$. It is easy to see that when $\mu \to 0$, the inequality is true. When $\mu \to 1$, the inequality becomes:

$$\frac{\sigma-1}{1+\tau^{\sigma-1}} - \frac{(\sigma-1)(1-\tau^{\sigma-1})}{(1+\tau^{\sigma-1})^2} - 1 - \frac{4(\sigma-1)\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2} + \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} \quad > \quad 0 \Leftrightarrow$$

$$\Leftrightarrow (\sigma-1)\left[\frac{-2\tau^{\sigma-1}}{(1+\tau^{\sigma-1})^2}\right] - 1 + \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} \quad > \quad 0.$$

Which is clearly false.

□

# Chapter 3

# The effect of population skills in the core-periphery model

## 3.1 Introduction

The new economic geography has been involved with well-defined phenomena, explaining mainly how economic integration affects the spatial distribution of the economic activity. These results are summarized in Baldwin et al.(2003).

Although its fundamental interest has been the effect of trade costs in the regional structure of the economic activity, the standard framework can be used to study other interesting issues, like the impact of the employment structure in the location of economic activity.

Krugman (1991), Krugman and Venables (1995), whose theory studies essentially agglomeration generated by demand, relate the expenditure share of industrial goods with spatial agglomeration.

By contrast, Ottaviano et al. (2002), whose theory is associated with supply, link the share of industrial employment with the degree of agglomeration. Their results do not depend on the expenditure share on the manufacturing sector because the absence of general equilibrium income effects - they use a quasi-linear utility function.

Applying a model reflecting effects both from demand and supply side, we generalize the model of Krugman (1991) including a precise parameter for the share of industrial employment in the economy.

Unlike Krugman (1991), we relax the constraint that the share of industrial employment is always equal to the share spending in industrial goods, and unlike Ottaviano, Tabuchi and Thisse (2002), we use a standard utility function, Cobb-Douglas/CES model, exhibitting general equilibrium income effects[1].

[1] In Pflüger and Südekum (2008a) there is a synthesis of different classes of utility functions applied in new economic geography models.

## 3.2 The model

This framework generalizes the core-periphery model of Krugman (1991) by including a specific parameter for the supply of industrial workers, $M$.

Krugman (1991) assumes that the total supply of industrial workers is:

$$M_1 + M_2 = \mu,$$

where, $M_1$ and $M_2$ are the supply of industrial workers in region 1 and 2 respectively, and $\mu$ is the share of spending in industrial goods. In our model, the share of industrial workers in the economy, $M$ can be different from the share of spending in industrial goods, $\mu$:

$$M_1 + M_2 = M, \text{ with } M \in (0, 1).$$

The economy comprises two regions and two sectors: an agricultural sector and an industrial sector. There are two factors of production: farmers and industrial workers ($M$). Farmers are immobile across regions, while industrial workers are mobile.

The supply of farmers is the same in each region[2], $\frac{1-M}{2}$, and the total population is normalized to unity.

The agricultural sector is perfectly competitive and produces a homogeneous good under constant returns to scale using only farmers. Transportation of agricultural output across regions is costless. The industrial sector produces a horizontally differentiated product using only industrial workers. Transportation of industrial goods is subject to iceberg transportation costs. For each unit of industrial good that is shipped to the other region, only a fraction $\tau$, with $0 < \tau < 1$, arrives.

[2]Krugman (1991) assumes that the supply of farmers in each region is $\frac{1-\mu}{2}$.

All the agents have the same preferences for consumption of industrial goods ($C_M$), and agricultural goods ($C_A$). The preferences are represented by the following utility function:

$$U = C_M^{\mu} C_A^{1-\mu},$$

$$C_M = \left[ \sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $\mu \in (0, 1)$, is the share of spending on industrial goods; $n$ is the number of varieties of industrial goods; $c_i$ is the consumption of the industrial good produced by firm $i$; and, finally, $\sigma > 1$ is the elasticities of substitution among industrial goods.

The agricultural sector produces a homogeneous good under constant returns to scale. The sector is perfectly competitive, therefore:

$$W_A = p_A,$$

where $W_A$ is the nominal wage of the farmers, and $p_A$ is the price of an agricultural good, taken as given by the firms and chosen to be the numeraire ($p_A = 1$).

All industrial firms in region $i$, for $i \in \{1, 2\}$ support a fixed cost of $\alpha$ units of industrial workers, and a variable cost of $\beta$ units of industrial workers per unit of good produced.

Firms choose a quantity, $q_i$, for $i \in \{1, 2\}$ to maximize profit. This implies that:

$$p_i = \frac{\sigma}{\sigma - 1} \beta W_i,$$

where $p_i$ is the equilibrium price of each firm.

Given the assumption of free entry, we obtain the quantity produced by each industrial firm in region $i \in \{1, 2\}$:

$$q_i = \frac{\alpha}{\beta}(\sigma - 1).$$

### 3.2.1 Short-run equilibrium

In the short-run, variables are determined by taking as given the amount of industrial workers in each region, $M_1$ and $M_2$.

We define $Z_{11}$ as the ratio between region 1's expenditure on local industrial goods and region 1's expenditure on industrial goods imported from region 2:

$$Z_{11} = \frac{M_1}{M_2}\left(\frac{W_1\tau}{W_2}\right)^{-(\sigma-1)}. \tag{3.1}$$

With a similar procedure we obtain $Z_{12}$, the ratio between region 2's spending on region 1 products and local products:

$$Z_{12} = \frac{M_1}{M_2}\left(\frac{W_1}{W_2\tau}\right)^{-(\sigma-1)}. \tag{3.2}$$

Let $Y_1$ and $Y_2$ denote the nominal regional income, which is equal to the sum of the incomes in the agricultural and the industrial sectors:

$$Y_1 = \frac{1-M}{2} + W_1 M_1, \text{ and } Y_2 = \frac{1-M}{2} + W_2 M_2. \tag{3.3}$$

The nominal wage of workers in region 1, $W_1$, is equal to the spending on region 1's industrial goods:

$$W_1 = \frac{\mu}{M_1}\left[\frac{Z_{11}}{1+Z_{11}}Y_1 + \frac{Z_{12}}{1+Z_{12}}Y_2\right], \tag{3.4}$$

Similarly, the nominal wage of workers in region 2 is:

$$W_2 = \frac{\mu}{M_2}\left[\frac{1}{1+Z_{11}}Y_1 + \frac{1}{1+Z_{12}}Y_2\right]. \tag{3.5}$$

For a given distribution of industrial workers in regions 1 and 2, equations (3.1) $-$ (3.5) are a system that determines $W_1$ and $W_2$.

However, workers are interested not in nominal wages but in real wages, and these depend on the cost of living in each region. Therefore the price index in region 1, $P_1$ is:

$$P_1 = \left[ M_1 W_1^{-(\sigma-1)} + M_2 \left( \frac{W_2}{\tau} \right)^{-(\sigma-1)} \right]^{-\mu/(\sigma-1)},$$

and the price index in region 2 is:

$$P_2 = \left[ M_1 \left( \frac{W_1}{\tau} \right)^{-(\sigma-1)} + M_2 W_2^{-(\sigma-1)} \right]^{-\mu/(\sigma-1)}.$$

Workers seek the region with the highest utility or, equivalently, the highest real wage. Then, the real wages of workers in region 1, $\omega_1$ is:

$$\omega_1 = \frac{W_1}{P_1},$$

and the real wage of workers in region 2, $\omega_2$ is:

$$\omega_2 = \frac{W_2}{P_2}.$$

### 3.2.2 Long-run equilibrium

The long-run equilibrium is a situation where migration does not occur. We say that it is stable if it is robust to small perturbations of the distribution of workers across regions.

Dispersion is a long-run equilibrium configuration if regions have the same real wage. It is stable if a small migration to region $1$ decreases the real wage in region $1$, implying that the initial configuration is reestablished. Precisely:

$$\frac{\omega_1}{\omega_2}|_{f=f^*=\frac{1}{2}} = 1 \text{ and } \left[ \partial \left( \frac{w_1}{w_2} \right) / \partial f \right] |_{f=f^*} < 0,$$

where $f = \frac{M_1}{M}$ is the share of industrial workers in region 1.

Concentration is a long-run equilibrium configuration where all industrial workers are concentrated in the region that has the highest real wage. Concentration in region 1 and in region 2 satisfies, respectively:

$$f^* = 1, \ \omega_1 \geq \omega_2 \ \text{and (for stability)} \ \exists \epsilon > 0 : 1 - \epsilon < f < 1 \Rightarrow \omega_1 \geq \omega_2;$$

$$f^* = 0, \ \omega_1 \leq \omega_2 \ \text{and (for stability)} \ \exists \epsilon > 0 : 0 < f < \epsilon \Rightarrow \omega_1 \leq \omega_2.$$

## 3.3 Results

### 3.3.1 The spatial distribution of the economic activity

In this section, we study how changes in the share of industrial workers in the economy, $M$, affect the spatial distribution of the economic activity. We use numerical examples to do this study.

For the case in which the share of the spending in industrial goods, $\mu$, is always equal to the share of industrial workers in the economy[3], $M$, we can see from figure[4] 3.1, that an increase in M (and $\mu$) make dispersion become unstable, and concentration will be an equilibrium configuration.

However, since $\mu$ is also a parameter representing preference for industrial goods, and considering intuitively, that $\mu$ varies slower than $M$, we should expect that an increase in the share of industrial activity in the economy does not imply, directly, an increase in the share of spending in industrial goods. Therefore, figure 3.2[5] illustrates how an increase in $M$

[3]This is the case in Krugman (1991).

[4]To plot this figure, we have set $\tau = 0.5$ and $\sigma = 4$.

[5]To plot this figure, we have set $\tau = 0.5$, $\mu = 0.3$ and $\sigma = 4$.

from 0.3 to 0.5, affects the spatial distribution of the economic activity, when $\mu$ is invariant. In this case, we can see that spatial distribution of economic activity is independent of $M$. Indeed, the curve that depicts all short-run equilibrium points $(\frac{\omega_1}{\omega_2}, f)$ remains unchanged for $M = 0.3$ and $M = 0.5$[6].

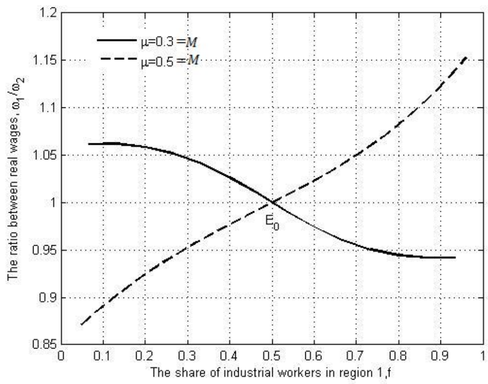


Figure 3.1: Dispersion becomes unstable.

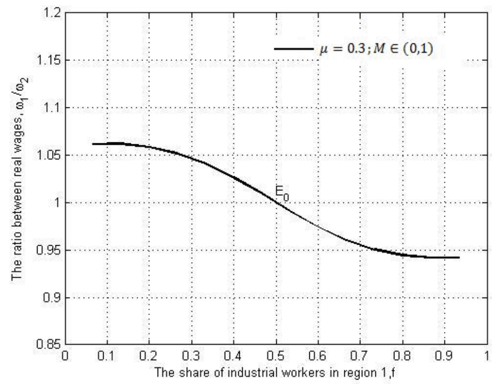


Figure 3.2: Industrial location remains unchanged.

### 3.3.2 Welfare analysis

It is clear in our numerical example, that for a given share of spending in industrial goods, a change in structure of labor supply does not affect the geographic location of the economic activity.

An interesting result arises when we study how a change in employment structure affects the welfare. Considering an equilibrium configuration in which regions have the same share of industrial activity (dispersion, $f = 0.5$) we examine how an increase in $M$ influences the

[6]Using other parameter values for M in our numerical example, we obtain the same result.

welfare (the real wage) of each industrial worker and the regional welfare (the real regional income).

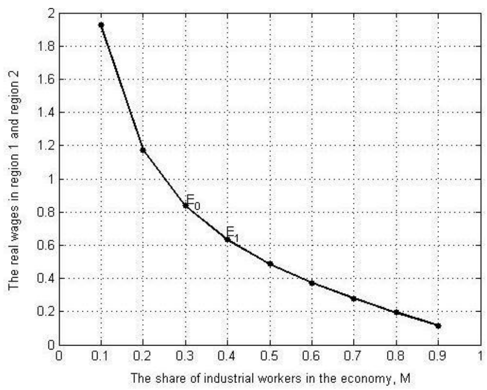


Figure 3.3: The real wage in both regions for different values of $M$.

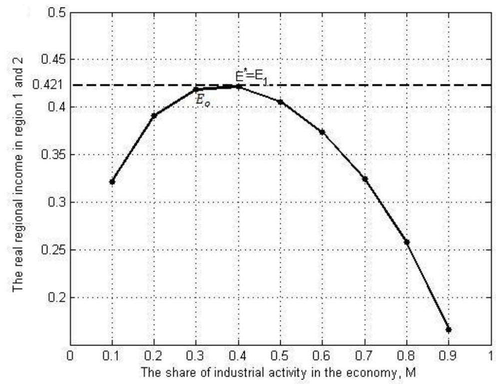


Figure 3.4: The optimal real regional income.

We can see from figure 3.3 that an increase in $M$ decreases the real wages of the industrial workers in each region[7]. Thus, for a given $\mu$, more industrial activity in the economy reduces the welfare of the industrial workers in each region.

An interesting result is the impact of $M$ on the regional welfare. From figure 3.4, we can see that there is a $M$ that maximizes the regional welfare[8]. Note that, for $\mu = 0.3$, the highest real regional income is obtained when $M = 0.4$, point $E^*$.

Since point $E^*$ represents a case in which the real wages of the industrial workers are lower than the real wages of the farmers, the maximum regional welfare would be attained by introducing a monetary incentive promoting industrial activity in the economy. Note that,

[7]Since initial equilibrium is dispersion, it is clear that real wages are equal in equilibrium. As an increase in M does not affects the spatial distribution of the economic activity, all changes in the employment structure produce the same impact on the welfare of regions, when f=0.5.

[8]To plot these figures, we have set $\tau = 0.5$, $\mu = 0.3$ and $\sigma = 4$.

without a monetary incentive, an increase in the regional welfare from point $E_0$ to $E^*$ is obtained with a decrease in the welfare of the industrial workers, from $E_0$ to $E_1$ (figure 3.3), and in this case, no farmer wants to become an industrial worker.

## 3.4 Concluding remarks

We extend the core-periphery model introducing a specific parameter for the share of industrial workers in the economy. We study how an increase in this parameter affects the geography of the economic activity and the welfare. A numerical example suggests that spatial distribution of the economic activity is independent of the employment structure, but there is a share of employment in the industrial sector that maximizes the regional welfare.

# Chapter 4

# A third sector in the core-periphery model:non-tradable goods

* The results in this chapter have been published in a working paper (Leite et al., 2009b)

## 4.1 Introduction

The literature on new economic geography has grown extensively over the two last decades. The idea of trade and geography in general equilibrium models was introduced for the first time by Krugman (1991), who developed a model illustrating how a country can endogenously become differentiated into an industrialized core and an agricultural periphery. In this model, trade costs are crucial to explain the spatial distribution of economic activity. If trade costs are high, industrial activity is dispersed across regions, while if trade costs are low, then industrial activity becomes concentrated in one region.

Despite it being a stylized fact that services (which are mainly non-tradable) have a very significant weight in the developed economies, representing more than two thirds of the total employment in the EU27, the standard literature in new economic geography assumes that regions have an "agricultural" sector (which produces perfectly tradable goods) and an "industrial" sector (which produces partially tradable goods).[1] A notable exception is the work of Helpman (1998) who substituted the agricultural sector by a perfectly competitive non-tradable goods sector (housing). Assuming that the location of this sector is exogenous, Helpman showed that housing acts as a dispersion force, by increasing the cost-of-living in a more populated region.

This result also appears in the economic geography model developed by Pflüger and Südekum (2008b), in which agents are assumed to have a logarithmic quasi-linear utility function and housing costs act in the spirit of Helpman (1998). They show that, starting from a situation of dispersion of industrial activity, falling trade costs lead to agglomeration. However, when trade costs become sufficiently low, the relative importance of housing

[1] See, for example, the works of Puga (1999), Fujita, Krugman and Venables (2001), Forslid and Ottaviano (2003) and Baldwin *et al.* (2003).

prices dominates the agglomeration forces, and dispersion occurs again. Contrasting with most new economic geography models, which feature 'bang-bang' phenomena (either symmetric dispersion or full agglomeration of the industrial activity in one of two regions), their model can generate partial agglomeration.

In the model of Behrens (2004), the absence of interregional trade is an endogenous outcome. Firms want to sell in the locations that allow them to make a positive profit. Depending on the level of trade costs and on the degree of competition, each good may be effectively traded in equilibrium or not. Behrens (2004) shows that when the trade costs are higher than a threshold value, all the industrial goods are non-tradable. In such an environment, the economy comprises only an agricultural (traditional) sector and a non-tradable goods sector. For this particular case, Behrens (2004) also shows that full and partial agglomeration in the non-tradable sector arises in a completely autarchic world, and the structure of the spatial economy is determined by the ratio of the mobile to immobile factor.

To the best of our knowledge, there is no model that explains the spatial distribution of the production of both tradable and non-tradable goods. In order to fill this gap, this paper generalizes the analytically solvable core-periphery model of Forslid and Ottaviano (2003) by considering a third sector, which produces non-tradable goods (services). Like the industrial sector, this service sector is assumed to be monopolistically competitive and mobile across regions.

Workers and firms operating in the industrial and service sectors move to the region with the highest utility level, until a spatial equilibrium is reached. We find that the resulting configuration may consist in full agglomeration, symmetric dispersion, or a combination of full agglomeration of industry with partial agglomeration of the service sector.

A strong preference for variety in the service sector is a very strong agglomeration force.

For any value of the trade costs, full agglomeration of industry and services in one region is an equilibrium whenever the elasticity of substitution among services is lower than a threshold value.

If the preference for variety of services is relatively weak, trade costs become crucial to explain the location of the economic activity. If trade costs are high, the industry and services become symmetrically dispersed across regions. If trade costs are low, then the industry becomes agglomerated in one region, while the services become only partially agglomerated. In this case, the region where all the industrial activity takes place will have more than one-half of the service sector activity.

## 4.2 The model

### 4.2.1 Basic setup

The model is an extension of the analytically solvable core-periphery model of Forslid and Ottaviano (2003) that incorporates a third sector of services (non-tradable goods).[2]

The economy comprises two regions and three sectors: an agricultural sector (perfectly tradable goods), an industrial sector (partially tradable goods) and a service sector (non-tradable goods). There are three factors of production: unskilled workers ($L$), industrial sector workers ($M$) and service sector workers ($S$). The unskilled workers are immobile across regions, while the industrial and service workers are mobile.

[2] It is straightforward to verify that by considering that the size of the service sector in null ($\mu_s = 0$), we obtain the model of Forslid and Ottaviano (2003).

We denote by $M_1$ and $M_2$, with $M_1 + M_2 = M$, the supply of industrial workers in regions 1 and 2, respectively, and by $S_1$ and $S_2$, with $S_1 + S_2 = S$, the supply of service workers in regions 1 and 2, respectively. The supply of unskilled workers is the same in each region, $L_1 = L_2 = L/2$, and the total population is normalized to unity, $L + M + S = 1$.

The agricultural sector is perfectly competitive and produces a homogeneous good under constant returns to scale using only unskilled labor. Transportation of agricultural output across regions is costless. The industrial sector and the service sector produce a horizontally differentiated product using sector-specific labor (fixed cost) and unskilled labor (variable cost).

Transportation of industrial goods and services is subject to iceberg transportation costs. For each unit of industrial good that is shipped to the other region, only a fraction $\tau_m \in (0, 1)$ arrives. The trade of services across regions is more costly: only a fraction $\tau_s \in [0, \tau_m]$ arrives. We will give particular attention to the case in which services are non-tradable across regions ($\tau_s = 0$).

All the agents have the same preferences for consumption of industrial goods ($C_M$), services ($C_S$) and agricultural goods ($C_A$). A natural extension of the utility function used by Krugman (1991) and by Forslid and Ottaviano (2003) to an economy with three sectors is the following:

$$\begin{aligned} U &= C_M^{\mu_m} C_S^{\mu_s} C_A^{1-\mu_m-\mu_s}, \\ C_M &= \left( \sum_{i=1}^{n_m} c_{m_i}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}}, \\ C_S &= \left( \sum_{i=1}^{n_s} c_{s_i}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}, \end{aligned} \tag{4.1}$$

where $\mu_m \in (0, 1)$ and $\mu_s \in (0, 1)$, with $\mu_m + \mu_s < 1$, are the shares of spending on industrial products and on services; $n_m$ and $n_s$ are the number of varieties of industrial goods and of services; $c_{m_i}$ and $c_{s_j}$ are the consumption of the industrial good produced by firm $i$ and of the service provided by the firm $j$; and, finally, $\sigma_m > 1$ and $\sigma_s > 1$ are the elasticities of substitution among industrial goods and among services.

### 4.2.2 Supply

#### Agricultural sector

In the agricultural sector, firms use unskilled labor to produce a homogeneous good under constant returns to scale. The production function is $q^A = L$, where $q^A$ is the amount of agricultural goods produced and $L$ is the quantity of unskilled labor employed. The cost function is $CT^A = W_a q^A$, where $W_a$ is the nominal wage of the unskilled workers employed. The profit function is:

$$\Pi^A = (p_a - W_a)q^A,$$

where $p_a$ is the price of an agricultural good, taken as given by the firms (perfect competition) and chosen to be the numeraire ($p_a = 1$).

The sector is perfectly competitive, therefore: $W_a = p_a = 1$.

#### Industrial sector

Firms in the industrial sector support a fixed cost of $\alpha_m$ units of industrial labor, and a variable cost of $\beta$ units of unskilled labor per unit of good produced. Since $W_a = 1$, the cost function is $CT^M = \alpha_m W_m + \beta q^M$, where $q^M$ is the quantity of industrial goods produced by an industrial firm and $W_m$ is the nominal wage of the industrial workers employed by the firm. The profit function is:

$$\Pi^M = p^M(q^M)q^M - \beta q^M - \alpha_m W_m. \tag{4.2}$$

Firms choose $q^M$ to maximize profit. This implies that:

$$p^M = \frac{\epsilon}{\epsilon - 1}\beta,$$

where $\epsilon$ is the price-elasticity of demand.

Since there is a large number of firms in the industrial sector, $\epsilon \approx \sigma_m$ (we have equality if there is a continuum of firms). Thus:[3]

$$p^M = \frac{\sigma_m}{\sigma_m - 1}\beta. \tag{4.3}$$

Given the assumption of free entry, the profit of each firm must be zero. Substituting (4.3) in (4.2), we obtain:

$$q^M = \frac{\alpha_m}{\beta}(\sigma_m - 1)W_m \tag{4.4}$$

Since an industrial firm employs $\alpha_m$ units of skilled labor, the total demand for skilled labor is $n_m\alpha_m$. Therefore, the number of firms must be:

$$n_m = \frac{M}{\alpha_m}. \tag{4.5}$$

**Service sector**

Firms in the service sector use $\alpha_s$ units of service workers as a fixed cost, and $\beta$ units of unskilled labor per unit of product.

As in the industrial sector, the price chosen by each firm is:

$$p^S = \frac{\sigma_s}{\sigma_s - 1}\beta.$$

[3] In the case of Cournot competition: $\frac{1}{\epsilon} = \frac{1}{\sigma_m} + s(1 - \frac{1}{\sigma_m})$, where $s$ is the market share of each firm. With many firms in the economy ($s \approx 0$), the price elasticity of demand, $\epsilon$, is approximately equal to the elasticity of substitution among the differentiated goods, $\sigma_m$.

The quantity produced by each firm is:

$$q^S = \frac{\alpha_s}{\beta}(\sigma_s - 1)W_s, \tag{4.6}$$

where $W_s$ is the nominal wage of the service workers employed by the firm.

And the number of firms is:

$$n_s = \frac{S}{\alpha_s}. \tag{4.7}$$

### 4.2.3 Demand

**Industrial sector**

Individual demand for each industrial variety is obtained from utility maximization (4.1) with respect to $c_{m_j}$. It can be shown that (Baldwin et al., 2003, pp. 38-39):

$$c_{m_j} = p_j^{-\sigma_m} \frac{\mu_m y}{\sum_{i=1}^{n_m} p_i^{1-\sigma_m}},$$

where $y$ is the income of the agent.

The industrial price index can be defined as:

$$P_m = \left( \sum_{i=1}^{n_m} {p_i}^{1-\sigma_m} \right)^{\frac{1}{1-\sigma_m}}. \tag{4.8}$$

Using (4.8), the individual demand for the industrial variety $j$ becomes:

$$c_{m_j} = \frac{p_j^{-\sigma_m}}{P_m^{1-\sigma_m}} \mu_m y.$$

Each firm sells its products in both regions. The price of a representative local industrial good is $p_{ii} = \frac{\sigma_m}{\sigma_m - 1}\beta$, and the price of a product that is exported from region $i$ to region $j$ is $p_{ij} = \tau_m^{-1} \frac{\sigma_m}{\sigma_m - 1}\beta$.

Since all manufacturing firms of a region set the same price, the industrial price index in region $i$ is:

$$\begin{aligned} P_{mi} &= \left(n_{mi}{p_{ii}}^{1-\sigma_m} + n_{mj}{p_{ij}}^{1-\sigma_m}\right)^{\frac{1}{1-\sigma_m}} = \\ &= \frac{\beta\sigma_m}{\sigma_m - 1}\left(n_{mi} + n_{mj}\tau_m^{\sigma_m - 1}\right)^{\frac{1}{1-\sigma_m}}, \end{aligned}$$

where $n_{mi}$ and $n_{mj}$ are the number of industrial firms in regions $i$ and $j$, respectively.

Defining $\phi_m = \tau_m^{\sigma_m - 1}$ as the degree of economic integration for the industrial sector (Baldwin et al., 2003), we obtain:

$$P_{mi} = \frac{\beta\sigma_m}{\sigma_m - 1}\left(n_{mi} + n_{mj}\phi_m\right)^{\frac{1}{1-\sigma_m}}. \tag{4.9}$$

Denoting the total demand of an industrial product that is produced in region $i$ and consumed in region $j$ by $C_{m_{ij}}$, we have:

$$C_{m_{ii}} = \frac{p_{ii}^{-\sigma_m}}{P_{mi}^{1-\sigma_m}}\mu_m Y_i \quad \text{and} \quad C_{m_{ij}} = \frac{p_{ij}^{-\sigma_m}}{P_{mj}^{1-\sigma_m}}\mu_m Y_j,$$

where $Y_i$ and $Y_j$ are the nominal incomes in regions $i$ and $j$, respectively.

Since $p_{ii} = \frac{\beta\sigma_m}{\sigma_m - 1}$ and $p_{ij} = \tau_m^{-1}\frac{\beta\sigma_m}{\sigma_m - 1}$, the above equations become:

$$C_{m_{ii}} = \frac{\left(\frac{\beta\sigma_m}{\sigma_m - 1}\right)^{-\sigma_m}}{P_{mi}^{1-\sigma_m}}\mu_m Y_i \quad \text{and} \quad C_{m_{ij}} = \frac{\tau_m^{\sigma_m}\left(\frac{\beta\sigma_m}{\sigma_m - 1}\right)^{-\sigma_m}}{P_{mj}^{1-\sigma_m}}\mu_m Y_j. \tag{4.10}$$

Denoting the output of an industrial firm in region $i$ by $q_{mi}$, we have:

$$q_{mi} = C_{m_{ii}} + \tau_m^{-1}C_{m_{ij}}. \tag{4.11}$$

Substituting (4.10) in (4.11), we obtain:

$$q_{mi} = \mu_m\left(\frac{\beta\sigma_m}{\sigma_m - 1}\right)^{-\sigma_m}\left(\frac{Y_i}{P_{mi}^{1-\sigma_m}} + \frac{\tau_m^{\sigma_m - 1}Y_j}{P_{mj}^{1-\sigma_m}}\right). \tag{4.12}$$

Replacing (4.9) in (4.12):

$$q_{m_i} = \mu_m \frac{\sigma_m - 1}{\beta\sigma_m}\left(\frac{Y_i}{n_{m_i} + \phi_m n_{m_j}} + \frac{\phi_m Y_j}{\phi_m n_{m_i} + n_{m_j}}\right).$$

Substituting (4.4) and (4.5) above, we obtain the nominal wage of the skilled workers in each region:

$$W_{mi} = \frac{\mu_m}{\sigma_m}\left(\frac{Y_i}{M_i + \phi_m M_j} + \frac{\phi_m Y_j}{\phi_m M_i + M_j}\right). \tag{4.13}$$

**Service sector**

All the expressions obtained in the previous subsection apply.

The individual demand for a service, $c_{s_j}$, is:

$$c_{s_j} = \frac{p_{s_j}^{-\sigma_s}}{P_s^{1-\sigma_s}}\mu_s y,$$

where $P_s = \left(\sum_{i=1}^{n_s} {p_i}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}$.

The internal and external demand for a service produced in region $i$ are:

$$C_{s_{ii}} = \frac{\left(\frac{\beta\sigma_s}{\sigma_s - 1}\right)^{-\sigma_s}}{P_{si}^{1-\sigma_s}}\mu_s Y_i \quad \text{and} \quad C_{s_{ij}} = \frac{\tau_s^{\sigma_s}\left(\frac{\beta\sigma_s}{\sigma_s - 1}\right)^{-\sigma_s}}{P_{sj}^{1-\sigma_s}}\mu_s Y_j.$$

The output of a service provider in region $i$ is:

$$q_{si} = \mu_s\left(\frac{\beta\sigma_s}{\sigma_s - 1}\right)^{-\sigma_s}\left(\frac{Y_i}{P_{si}^{1-\sigma_s}} + \frac{\phi_s Y_j}{P_{sj}^{1-\sigma_s}}\right),$$

where $\phi_s = \tau_s^{\sigma_s - 1}$ is the degree of economic integration in the service sector.

The price index of services in region $i$ is:

$$P_{si} = \frac{\beta\sigma_s}{\sigma_s - 1}\left(n_{si} + n_{sj}\phi_s\right)^{\frac{1}{1-\sigma_s}}, \tag{4.14}$$

where $n_{si}$ and $n_{sj}$ are the number of service providers in region $i$ and $j$, respectively.

The nominal wage of the skilled workers in the service sector in region $i$ is:

$$W_{si} = \frac{\mu_s}{\sigma_s}\left(\frac{Y_i}{S_i + \phi_s S_j} + \frac{\phi_s Y_j}{\phi_s S_i + S_j}\right). \tag{4.15}$$

**Regional income, perfect price index**

The nominal income in region $i$, is equal to the sum of the incomes in the agricultural, industrial and service sector:

$$Y_i = \frac{1 - M - S}{2} + W_{mi}M_i + W_{si}S_i, \quad \text{for } i = 1, 2. \tag{4.16}$$

The perfect price index of region, $P_i$, aggregates three price indices: the price index of the agricultural sector (normalized to 1), the price index of the industrial sector, $P_{mi}$, and the price index of the service sector, $P_{si}$.

We obtain the price index of industrial goods in region $i$, by substituting (4.5) into (4.9):

$$P_{mi} = \frac{\beta\sigma_m}{\sigma_m - 1}\left(\frac{M_i}{\alpha_m} + \frac{\phi_m M_j}{\alpha_m}\right)^{\frac{1}{1-\sigma_m}}, \quad \text{for } i, j = 1, 2.$$

Denote the share of industrial workers in region 1 by $f_m = \frac{M_1}{M}$. Substituting above:

$$P_{m1} = \frac{\beta\sigma_m}{\sigma_m - 1}\left(\frac{M}{\alpha_m}\right)^{\frac{1}{1-\sigma_m}}\left[f_m + (1 - f_m)\phi_m\right]^{\frac{1}{1-\sigma_m}},$$

and

$$P_{m2} = \frac{\beta\sigma_m}{\sigma_m - 1}\left(\frac{M}{\alpha_m}\right)^{\frac{1}{1-\sigma_m}}\left[\phi_m f_m + (1 - f_m)\right]^{\frac{1}{1-\sigma_m}}.$$

Substituting (4.7) into (4.14), and defining $f_s = \frac{S_1}{S}$ as the share of service sector workers in region 1, we obtain:

$$P_{s1} = \frac{\beta\sigma_s}{\sigma_s - 1}\left(\frac{S}{\alpha_s}\right)^{\frac{1}{1-\sigma_s}}\left[f_s + (1 - f_s)\phi_s\right]^{\frac{1}{1-\sigma_s}},$$

and

$$P_{s2} = \frac{\beta\sigma_s}{\sigma_s - 1}\left(\frac{S}{\alpha_s}\right)^{\frac{1}{1-\sigma_s}}\left[\phi_s f_s + (1 - f_s)\right]^{\frac{1}{1-\sigma_s}}.$$

Using the last four expressions, we obtain the perfect price indices for each region:

$$P_1 = \rho\left[f_m + (1 - f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}\left[f_s + (1 - f_s)\phi_s\right]^{\frac{\mu_s}{1-\sigma_s}}, \tag{4.17}$$

and

$$P_2 = \rho\left[\phi_m f_m + (1 - f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left[\phi_s f_s + (1 - f_s)\right]^{\frac{\mu_s}{1-\sigma_s}}, \tag{4.18}$$

where $\rho = \left(\frac{\beta\sigma_m}{\sigma_m - 1}\right)^{\mu_m}\left(\frac{\beta\sigma_s}{\sigma_s - 1}\right)^{\mu_s}\left(\frac{M}{\alpha_m}\right)^{\frac{\mu_m}{1-\sigma_m}}\left(\frac{S}{\alpha_s}\right)^{\frac{\mu_s}{1-\sigma_s}}$.

### 4.2.4 Short-run equilibrium

In the short-run, workers are immobile across regions. A short-run equilibrium consists in the equality of supply and demand. Aggregate prices, output and wages are endogenously determined.

Equations (4.13), (4.15), (4.16), (4.17) and (4.18) determine the short-run equilibrium of the model. We recall these equations:

$$\begin{aligned}
W_{m1} &= \frac{\mu_m}{\sigma_m}\left(\frac{Y_1}{M_1 + \phi_m M_2} + \frac{\phi_m Y_2}{\phi_m M_1 + M_2}\right), \\
W_{m2} &= \frac{\mu_m}{\sigma_m}\left(\frac{Y_2}{M_2 + \phi_m M_1} + \frac{\phi_m Y_1}{\phi_m M_2 + M_1}\right), \\
W_{s1} &= \frac{\mu_s}{\sigma_s}\left(\frac{Y_1}{S_1 + \phi_s S_2} + \frac{\phi_s Y_2}{\phi_s S_1 + S_2}\right),
\end{aligned}$$

$$W_{s2} = \frac{\mu_s}{\sigma_s}\left(\frac{Y_2}{S_2+\phi_s S_1}+\frac{\phi_s Y_1}{\phi_s S_2+S_1}\right),$$

$$Y_1 = \frac{1-M-S}{2}+W_{m1}M_1+W_{s1}S_1,$$

$$Y_2 = \frac{1-M-S}{2}+W_{m2}M_2+W_{s2}S_2,$$

$$P_1 = \rho\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}\left[f_s+(1-f_s)\phi_s\right]^{\frac{\mu_s}{1-\sigma_s}},$$

$$P_2 = \rho\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left[\phi_s f_s+(1-f_s)\right]^{\frac{\mu_s}{1-\sigma_s}}.$$

Solving these equations, we find the nominal wages of the workers in each region:[4]

$$\begin{aligned} W_{m1} &= \overline{C}\sigma_m 2\phi_m M_1\left[\sigma_s-\mu_s\frac{S_1S_2\left(1-\phi_s^2\right)}{S_1S_2\left(1+\phi_s^2\right)+\phi_s\left(S_1^2+S_2^2\right)}\right]+ \\ &+ \overline{C}\sigma_m M_2\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(\phi_m^2\frac{S_1-\phi_s S_2}{S_1+\phi_s S_2}+\frac{S_2-\phi_s S_1}{\phi_s S_1+S_2}\right)\right\}, \\ W_{m2} &= \overline{C}\sigma_m 2\phi_m M_2\left[\sigma_s-\mu_s\frac{S_1S_2\left(1-\phi_s^2\right)}{S_1S_2\left(1+\phi_s^2\right)+\phi_s\left(S_1^2+S_2^2\right)}\right]+ \\ &+ \overline{C}\sigma_m M_1\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(\phi_m^2\frac{S_2-\phi_s S_1}{\phi_s S_1+S_2}+\frac{S_1-\phi_s S_2}{S_1+\phi_s S_2}\right)\right\}, \\ W_{s1} &= \overline{D}\sigma_s 2\phi_s S_1\left[\sigma_m-\mu_m\frac{M_1M_2\left(1-\phi_m^2\right)}{M_1M_2\left(1+\phi_m^2\right)+\phi_m\left(M_1^2+M_2^2\right)}\right]+ \\ &+ \overline{D}\sigma_s S_2\left\{\sigma_m\left[\phi_s^2+1+\frac{\mu_s}{\sigma_s}(\phi_s^2-1)\right]-\mu_m\left(\phi_s^2\frac{M_1-\phi_m M_2}{M_1+\phi_m M_2}+\frac{M_2-\phi_m M_1}{\phi_m M_1+M_2}\right)\right\}, \end{aligned}$$

[4]See Appendix B.1 for detailed calculations.

$$
\begin{aligned}
W_{s2} &= \overline{D}\sigma_s 2\phi_s S_2\left[\sigma_m - \mu_m \frac{M_1 M_2\left(1-\phi_m^2\right)}{M_1 M_2\left(1+\phi_m^2\right)+\phi_m\left(M_1^2+M_2^2\right)}\right]+ \\
&+ \overline{D}\sigma_s S_1\left\{\sigma_m\left[\phi_s^2+1+\frac{\mu_s}{\sigma_s}(\phi_s^2-1)\right]-\mu_m\left(\phi_s^2\frac{M_2-M_1\phi_m}{\phi_m M_1+M_2}+\frac{M_1-\phi_m M_2}{M_1+\phi_m M_2}\right)\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{C} &= \frac{\mu_m\sigma_s\frac{1-M-S}{2}(S_1+\phi_s S_2)(\phi_s S_1+S_2)}{R}, \\
\overline{D} &= \frac{\mu_s\sigma_m\frac{1-M-S}{2}(M_1+\phi_m M_2)(\phi_m M_1+M_2)}{R},
\end{aligned}
$$

and

$$
\begin{aligned}
R &= \left\{\sigma_m\left(M_1+\phi_m M_2\right)\left[\sigma_s\left(S_1+\phi_s S_2\right)-S_1\mu_s\right]-M_1\mu_m\sigma_s\left(S_1+\phi_s S_2\right)\right\}\times \\
&\times \left\{\sigma_m\left(\phi_m M_1+M_2\right)\left[\sigma_s\left(\phi_s S_1+S_2\right)-S_2\mu_s\right]-M_2\mu_m\sigma_s\left(\phi_s S_1+S_2\right)\right\}- \\
&- \left[M_1\mu_m\phi_m\sigma_s\left(\phi_s S_1+S_2\right)+S_1\mu_s\phi_s\sigma_m\left(\phi_m M_1+M_2\right)\right]\times \\
&\times \left[M_2\mu_m\phi_m\sigma_s\left(S_1+\phi_s S_2\right)+S_2\mu_s\phi_s\sigma_m\left(M_1+\phi_m M_2\right)\right].
\end{aligned}
$$

The real wages of the industrial sector workers in regions 1 and 2 are $\omega_{m1} = \dfrac{W_{m1}}{P_1}$ and $\omega_{m2} = \dfrac{W_{m2}}{P_2}$, and the real wages of the service sector workers in regions 1 and 2 are $\omega_{s1} = \dfrac{W_{s1}}{P_1}$ and $\omega_{s2} = \dfrac{W_{s2}}{P_2}$.

### 4.2.5 Long-run equilibrium

In the long-run, the skilled workers of the industrial and service sectors choose their location with the objective of maximizing their utility (equivalently, they move to the region with the highest real wage).

Migration is assumed to be determined by the following processes:

$$\dot{f}_m = \frac{df_m}{dt} = \begin{cases} \omega_{m1}(f_s, f_m) - \omega_{m2}(f_s, f_m), & \text{if } 0 < f_m < 1 \\ \min\{0, \omega_{m1}(f_s, f_m) - \omega_{m2}(f_s, f_m)\}, & \text{if } f_m = 1 \\ \max\{0, \omega_{m1}(f_s, f_m) - \omega_{m2}(f_s, f_m)\}, & \text{if } f_m = 0 \end{cases}$$

and

$$\dot{f}_s = \frac{df_s}{dt} = \begin{cases} \omega_{s1}(f_s, f_m) - \omega_{s2}(f_s, f_m), & \text{if } 0 < f_s < 1 \\ \min\{0, \omega_{s1}(f_s, f_m) - \omega_{s2}(f_s, f_m)\}, & \text{if } f_s = 1 \\ \max\{0, \omega_{s1}(f_s, f_m) - \omega_{s2}(f_s, f_m)\}, & \text{if } f_s = 0, \end{cases}$$

where $t$ is time, which is left implicit to simplify notation.

A distribution of economic activity, $(f_s^*, f_m^*)$, is a steady-state if and only if $\dot{f}_m = \dot{f}_s = 0$ at $(f_s^*, f_m^*)$. A long-run equilibrium is a stable steady-state.

The sufficient conditions for stability are the following:

$(i)$ $\quad f_x^* = 0 \;\Rightarrow\; (\omega_{x1} - \omega_{x2})\,|_{(f_s^*, f_m^*)} < 0$, for $x \in \{s, m\}$;

$(ii)$ $\quad f_x^* = 1 \;\Rightarrow\; (\omega_{x1} - \omega_{x2})\,|_{(f_s^*, f_m^*)} > 0$, for $x \in \{s, m\}$;

$(iii)$ $\quad f_x^* \in (0,1) \wedge f_y^* \in \{0,1\} \;\Rightarrow\; \frac{\partial(\omega_{x1} - \omega_{x2})}{\partial f_x}\,|_{(f_s^*, f_m^*)} < 0$, for $(x, y) \in \{(s, m), (m, s)\}$;

$(iv)$ $\quad (f_s^*, f_m^*) \in (0,1)^2 \;\Rightarrow\; det(J)|_{(f_s^*, f_m^*)} > 0$ and $tr(J)|_{(f_s^*, f_m^*)} < 0$, where:

$$J = \begin{bmatrix} \frac{\partial(\omega_{m1} - \omega_{m2})}{\partial f_m} & \frac{\partial(\omega_{m1} - \omega_{m2})}{\partial f_s} \\ \frac{\partial(\omega_{s1} - \omega_{s2})}{\partial f_m} & \frac{\partial(\omega_{s1} - \omega_{s2})}{\partial f_s} \end{bmatrix}.$$

## 4.3 The case in which services are non-tradable

In this section, we study the case in which services are asymptotically non-tradable ($\tau_s \to 0$ and, thus, $\phi_s \to 0$).[5] This is the case for most services related to arts, entertainment, real estate, rental, wholesale trade, education and health services.

### 4.3.1 Short-run equilibrium

We start by computing, for each sector, the difference between the real wages of the skilled workers in region 1 and region 2 (see Appendix B.2). We obtain:

$$\omega_{m1} - \omega_{m2} = \frac{\mu_m K}{\phi_m \left[f_m^2 + (1-f_m)^2\right] K_1 + (1-f_m) f_m K_2} \times \left(\bar{\omega}_{m1} - \bar{\omega}_{m2}\right) \quad (4.19)$$

and

$$\omega_{s1} - \omega_{s2} = \frac{\mu_s K \left\{\phi_m \left[f_m^2 + (1-f_m)^2\right] + f_m (1-f_m)\left(1+\phi_m^2\right)\right\}}{\left\{\phi_m \left[f_m^2 + (1-f_m)^2\right] K_1 + (1-f_m) f_m K_2\right\}} \left(\bar{\omega}_{s1} - \bar{\omega}_{s2}\right), \quad (4.20)$$

where:

$$K_1 = \sigma_m (\sigma_s - \mu_s) \left[\sigma_m (\sigma_s - \mu_s) - \sigma_s \mu_m\right],$$

$$K_2 = \sigma_m (\sigma_s - \mu_s) \left[\sigma_m (\sigma_s - \mu_s)\left(1+\phi_m^2\right) - 2\mu_m \sigma_s\right] + \mu_m^2 \sigma_s^2 \left(1-\phi_m^2\right),$$

[5]We do not consider the limit case because, with $\tau_s = 0$, the demand of the agricultural workers is indeterminate when services are concentrated in the other region. When restricted to $C_S = 0$, they are indifferent between any attainable consumption vector.

$$K = \frac{\sigma_m^{1-\mu_m}\sigma_s^{1-\mu_s}(1-M-S)(\sigma_m-1)^{\mu_m}(\sigma_s-1)^{\mu_s}}{2\alpha_m^{\frac{\mu_m}{\sigma_m-1}}\alpha_s^{\frac{\mu_s}{\sigma_s-1}}\beta^{\mu_m+\mu_s}M^{1-\frac{\mu_m}{\sigma_m-1}}S^{-\frac{\mu_s}{\sigma_s-1}}},$$

$$\bar{\omega}_{m1} = \frac{2\phi_m f_m(\sigma_s-\mu_s)+(1-f_m)\left[\sigma_s(\phi_m^2+1)+\frac{\sigma_s\mu_m}{\sigma_m}(\phi_m^2-1)-\mu_s(\phi_m^2+1)\right]}{\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{\frac{\mu_s}{1-\sigma_s}}},$$

$$\bar{\omega}_{m2} = \frac{2\phi_m(1-f_m)(\sigma_s-\mu_s)+f_m\left[\sigma_s(\phi_m^2+1)+\frac{\sigma_s\mu_m}{\sigma_m}(\phi_m^2-1)-\mu_s(\phi_m^2+1)\right]}{\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{\frac{\mu_s}{1-\sigma_s}}},$$

$$\bar{\omega}_{s1} = \frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{1-f_m(1+\phi_m)}{1-f_m(1-\phi_m)}}{\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{1-\frac{\mu_s}{\sigma_s-1}}},$$

$$\bar{\omega}_{s2} = \frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{f_m(1+\phi_m)-\phi_m}{f_m(1-\phi_m)+\phi_m}}{\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{1-\frac{\mu_s}{\sigma_s-1}}}.$$

### 4.3.2 Long-run equilibrium

In this section, we study the long-run equilibrium of the model. The following are possible spatial equilibrium configurations:

(i) concentration of industry and services in the same region;

(ii) concentration of industry in one region with asymmetric dispersion of services;

(iii) symmetric dispersion of industry and services.

Figure 4.1 illustrates the possible equilibrium solutions. In the vertical axis we have the share of industrial workers in region 1 and in the horizontal axis we have the share of

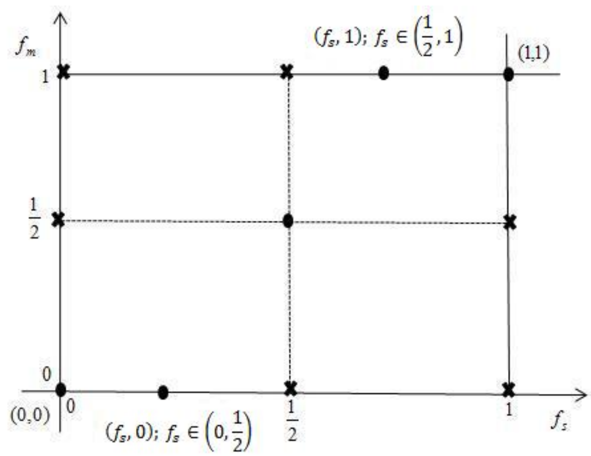


Figure 4.1: Possible equilibrium configurations

service workers in region 1. The possible equilibrium solutions are signaled by black dots. The black crosses signal configurations which are never an equilibrium.

### Concentration of industry and services in the same region

Concentration of the industrial and service activity in region 1, $(f_s^*, f_m^*) = (1,1)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(1,1)} \geq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(1,1)} \geq 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall (f_s, f_m) \in (1-\epsilon, 1]^2$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \geq 0, \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)} \geq 0. \end{cases}$$

Similarly, concentration of the industrial and service activity in region 2, $(f_s^*, f_m^*) = (0,0)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(0,0)} \leq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(0,0)} \leq 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall (f_s, f_m) \in [0, \epsilon)^2$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \leq 0, \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)} \leq 0. \end{cases}$$

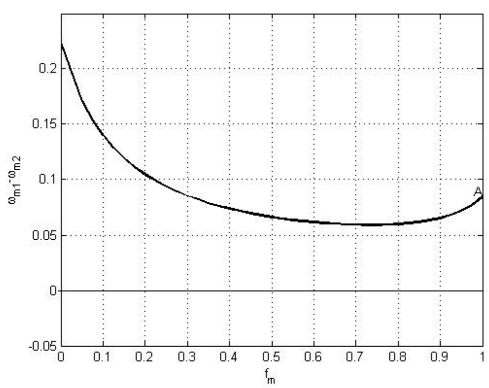


Figure 4.2: Concentration of industry.

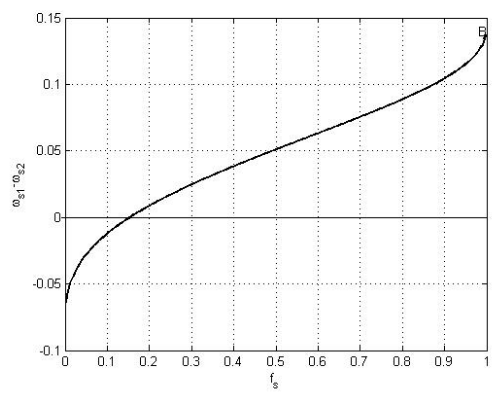


Figure 4.3: Concentration of services.

Figures 4.2 and 4.3 illustrate the existence of full agglomeration of the industrial and service activity in one region, when $\sigma_s \leq \mu_s + 1$.[6]

In figure 4.2, we assume that all the services are concentrated in region 1, $f_s = 1$, and we study the relationship between the spatial distribution of industry and the difference between the real wages of the industrial workers across regions. We find that the real wage is always higher in region 1. Thus, all industrial workers migrate to region 1.

[6]To plot these figures, we have set $\tau_m = 0.5$, $\mu_m = 0.4$, $\sigma_m = 4$, $\mu_s = 0.4$ and $\sigma_s = 1.3$.

In figure 4.3, we assume that all the industry is concentrated in region 1, $f_m = 1$, and we study how the spatial distribution of services affects the difference between the real wages of the service sector workers across regions. We find that if the service sector activity in region 1 is high enough, the service sector workers obtain a higher real wage in region 1. We conclude that full agglomeration of industry and services is an equilibrium.

**Lemma** 4.3.1. *Concentration of both sectors in a single region is an equilibrium if and only if* $\sigma_s \leq \mu_s + 1$.

This result (which is proved in appendix B.4) shows that trade costs in the industrial sector are irrelevant to explain concentration of industry and services activity in a single region, when the degree of differentiation in the service sector is high and services are non-tradable. For any value of the trade costs in the industrial sector, a high preference for variety of services is a sufficient condition to induce full concentration of industry and services. Under this condition, even if industrial goods are perfectly tradable, all the industrial firms locate their production in the same region. This contrasts with the results obtained in the classical model.

From now on, we will frequently assume that $\sigma_s > \mu_s + 1$ (no black-hole condition).

### Concentration of industry and asymmetric dispersion of services

Concentration of industrial activity in region 1 with asymmetric dispersion of the service activity, $(f_s^*, f_m^*) = (s, 1)$, with $s \in (0.5, 1)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(s,1)} \geq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(s,1)} = 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall(f_s, f_m) \in (s-\epsilon, s+\epsilon) \times (1-\epsilon, 1]$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \geq 0 \\ \left.\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s}\right|_{(f_s,f_m)} < 0 \end{cases}$$

Similarly, concentration of industrial activity in region 2 with asymmetric dispersion of the service activity, $(f_s^*, f_m^*) = (s, 0)$, with $s \in (0, 0.5)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(s,0)} \leq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(s,0)} = 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall(f_s, f_m) \in (s-\epsilon, s+\epsilon) \times [0, \epsilon)$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \leq 0 \\ \left.\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s}\right|_{(f_s,f_m)} < 0 \end{cases}$$

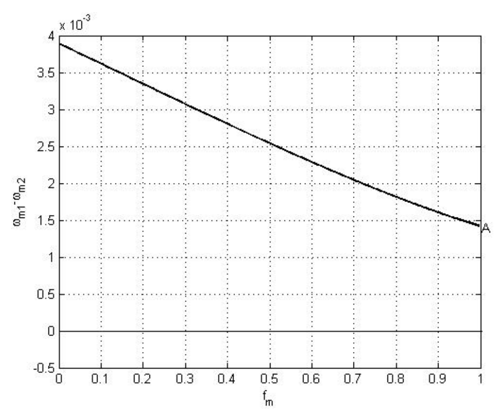


Figure 4.4: Concentration of industry.

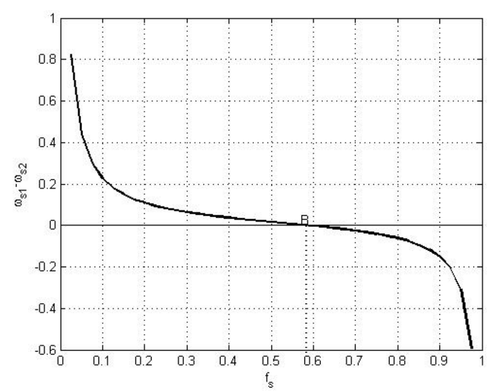


Figure 4.5: Asymmetric dispersion of services.

Figures 4.4 and 4.5 illustrate an equilibrium with full concentration of industry and asymmetric dispersion of services.[7]

With $f_s = 0.586$, we can see from figure 4.4 that the real wage of the industrial workers is always higher in region 1. Thus, industrial workers locate in region 1.

In figure 4.5, we assume that all the industrial activity is concentrated in region 1, $f_m = 1$, and study how the spatial distribution of the service sector activity affects the difference between the real wages of the service sector workers across regions. The migration of the service sector workers leads to an equilibrium with $f_s = 0.586$, as the real wages of the service sector workers coincide.

We conclude that the concentration of industry in region 1 and the asymmetric dispersion of services (58.6% in region 1) constitutes an equilibrium.

The following lemma describes the conditions for the existence of an equilibrium in which the industry is concentrated while the service sector is asymmetrically dispersed.

**Lemma** 4.3.2. *Concentration of industrial activity in region 1 or 2 and asymmetric dispersion of the service activity is an equilibrium if* $\sigma_s > \mu_s + 1$ *and:*

$$\left[\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m}\right]^{\frac{\mu_s}{\mu_s+1-\sigma_s}} \phi_m^{1-\frac{\mu_m(\sigma_s-1)}{(\sigma_m-1)(\sigma_s-1-\mu_s)}} - \frac{\phi_m^2\left[\sigma_s(1+\frac{\mu_m}{\sigma_m})-\mu_s\right]+\sigma_s(1-\frac{\mu_s}{\sigma_m})-\mu_s}{2(\sigma_s-\mu_s)} > 0.$$

To prove lemma 4.3.2, we use the following result. It states that when the elasticity of substitution of services is high enough to satisfy what we may call the black-hole condition, then a movement of service workers to a region decreases the attractiveness of this region to the service workers.

[7] To plot these figures, we have set $\tau_m = 0.825$, $\mu_m = 0.4$, $\sigma_m = 4$, $\mu_s = 0.4$ and $\sigma_s = 4$. We have also set $f_s = 0.586$ in figure 4.4 and $f_m = 1$ in figure 4.5.

**Lemma** 4.3.3. *When $\sigma_s > \mu_s + 1$, an increase in the share of services in region 1, $f_s$, decreases the difference between the real wages in the service sector ($\omega_{s1} - \omega_{s2}$).*

### Symmetric dispersion of industry and services

Symmetric dispersion of the industrial activity and service activity, $(f_s^*, f_m^*) = (0.5, 0.5)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(0.5,0.5)} = 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(0.5,0.5)} = 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if $det(J)|_{(f_s,f_m)=(0.5,0.5)} > 0$ and $tr(J)|_{(f_s,f_m)=(0.5,0.5)} < 0$, where $J$ is the Jacobian matrix of the model described by expressions (4.19) and (4.20):

$$J = \begin{bmatrix} \frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_m} & \frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_s} \\ \frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} & \frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s} \end{bmatrix}.$$

The following result implies that all these derivatives are well defined.

**Claim** 4.3.1. *The differences $\omega_{m1} - \omega_{m2}$ and $\omega_{s1} - \omega_{s2}$ are continuous and differentiable functions of $f_m$ and $f_s$, for $(f_s, f_m) \in (0, 1)$.*

Symmetric dispersion is always a steady-state.

**Lemma** 4.3.4. *When $(f_s, f_m) = (0.5, 0.5)$, the real wages in the industrial sector and in the service sector are equal across regions ($\omega_{m1} = \omega_{m2}$ and $\omega_{s1} = \omega_{s2}$).*

We already know that $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s} < 0$. Calculating the remaining elements of the Jacobian matrix, we obtain the following results.

**Lemma** 4.3.5. *When* $(f_s, f_m) = (0.5, 0.5)$*, the migration of service sector workers to region 1 increases the difference between the real wages of the industrial sector workers in region 1 and region 2, that is,* $\frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_s} > 0$*.*

**Lemma** 4.3.6. *When* $(f_s, f_m) = (0.5, 0.5)$*, the migration of industrial sector workers to region 1 increases the difference between the real wages of the service sector workers in region 1 and region 2, that is,* $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} > 0$*.*

These lemmas are important to explain the stability of the dispersion configuration and the following figures are useful for an intuitive understanding of the dynamics.[8]

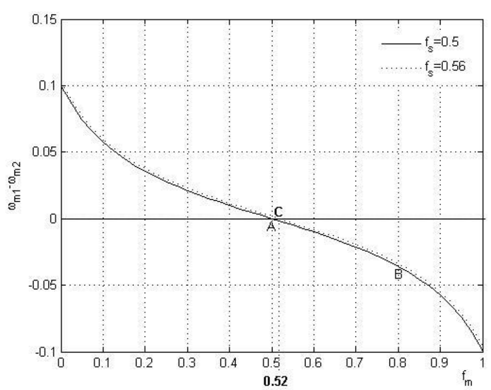


Figure 4.6: Dispersion of industry.

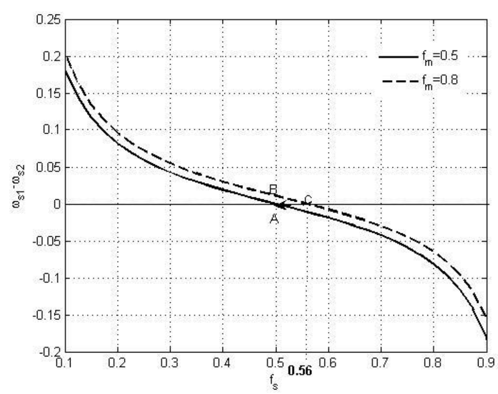


Figure 4.7: Dispersion of services.

Assume that the economy is initially located in point A, with $(f_s, f_m) = (0.5, 0.5)$. We have $\omega_{m1} = \omega_{m2}$ (figure 4.6) and $\omega_{s1} = \omega_{s2}$ (figure 4.7). Consider an increase in the number of industrial workers in region 1, to $f_m = 0.8$ (point B). From Lemma 4.3.6, an increase in $f_m$ increases $\omega_{s1} - \omega_{s2}$, and thus the curve in figure 4.7 moves up. The service sector workers would tend to move to region 1, until $f_s = 0,56$. On the other hand, with

[8]To plot figures 4.6 and 4.7, we have set $\mu_m = 0.4$ $\sigma_m = 4$, $\mu_s = 0.4$, $\sigma_s = 4$ and $\tau_m = 0.5$.

$f_s = 0.56$, the curve in figure 4.6 moves up, and the industrial workers would also migrate, until $f_m = 0.52$. With $f_m = 0.52$, the resulting $f_s$ would be lower than 0.56, giving rise to a new $f_m$, lower than 0.52. It seems that this process continues until $(f_s, f_m) = (0.5, 0.5)$, suggesting that symmetric dispersion is an equilibrium.

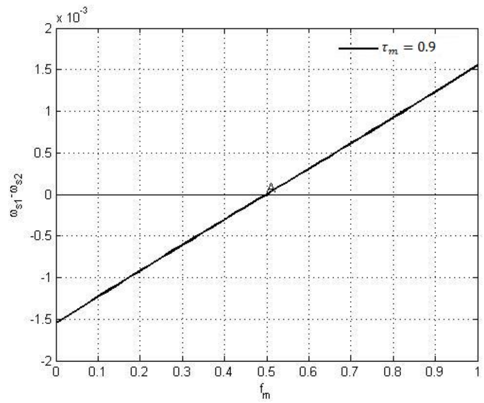


Figure 4.8: Dispersion becomes unstable.

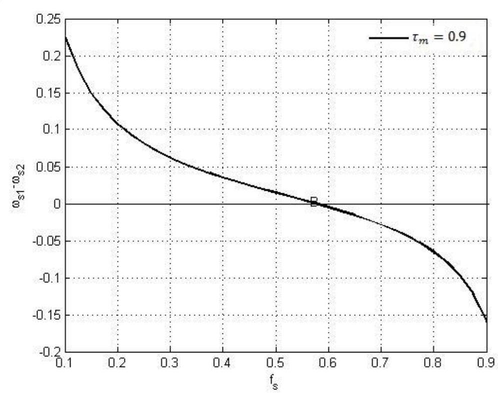


Figure 4.9: Asymmetric dispersion.

We can see from figure 4.8 that dispersion is unstable when $\tau_m = 0.9$[9]. An increase in $f_m$ increases the difference between the real wages of the industrial workers in region 1 and region 2, attracting workers from region 2 to region 1. From lemma 4.3.6, an increase in $f_m$ also increases the difference between the real wages in the service sector Therefore, in the long-run, we will have an asymmetric dispersion of the service sector activity and full concentration of the industrial activity.

In Appendix B.4 we calculate the Jacobian matrix. Here, we compute $det(J)$ and $tr(J)$ using a numerical example. Figure 4.10 illustrates a case in which symmetric dispersion of the industrial and service activity is an equilibrium for low values of $\phi_m$.[10]

[9]To plot figures 4.8 and 4.9, we have set $\mu_m = 0.4$ $\sigma_m = 4$, $\mu_s = 0.4$, $\sigma_s = 4$ and $\tau_m = 0.9$. Additionally, we have set $f_s = 0.5$ to plot figure 4.8 and $f_m = 1$ to plot figure 4.9.

[10]In figures 4.10 and 4.11, it is also assumed that $\sigma_m = 4$, $\sigma_s = 4$, $\mu_m = 0.4$ and $\mu_s = 4$.

For the parameter values in our numerical example, $tr(J)$ is negative for any $\phi_m \in (0, 1)$. Therefore, the sign of the determinant is crucial for the stability. From figure 4.10, we can see that $det(J)$ is positive for low values of $\phi_m$. This means that the eigenvalues of $J$ have negative real parts and symmetric dispersion of industry and services is an equilibrium.

For high values of $\phi_m$, the symmetric dispersion becomes unstable. In this case, asymmetric dispersion of the service activity with full concentration of the industrial activity in one region becomes the equilibrium.

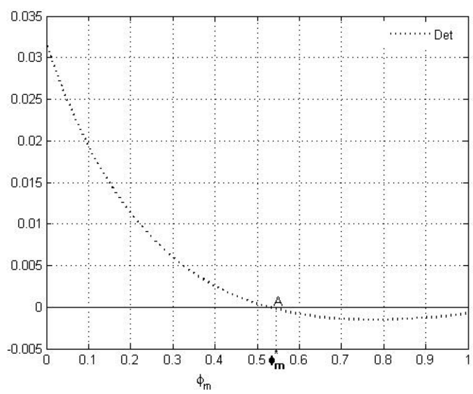


Figure 4.10: $Det(J)$ as a function of $\phi_m$.

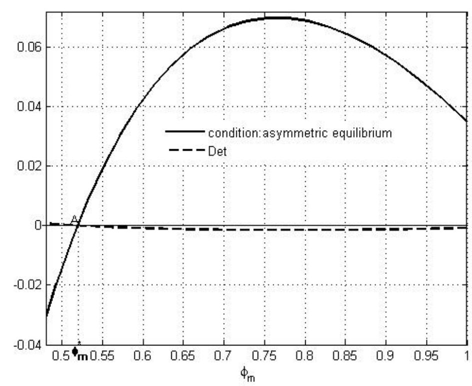


Figure 4.11: Threshold value for $\phi_m$.

This result can be viewed in figure 4.11, where we also plot the "asymmetric condition" in lemma 4.3.2, as a function of $\phi_m$. When $\phi_m$ is higher than $\phi_m^*$, concentration of all industrial activity in one region with asymmetric dispersion of the service activity becomes an equilibrium. Therefore, point A is a threshold value for $\phi_m$.

In particular, when $\sigma_s > \mu_s + 1$, the economy can have two distinct equilibrium configurations. For high trade costs (low $\phi_m$), we find that symmetric dispersion of services and industry is an equilibrium, while for low trade costs (high $\phi_m$), we find that concentration of industry with asymmetric dispersion of services is an equilibrium.

**Configurations which are never an equilibrium**

We also show that the following configurations are never an equilibrium:

(i) concentration of services and concentration of industry in different regions;

(ii) symmetric dispersion of services and concentration of industry;

(iii) concentration of services together with dispersion of industry.

**Lemma** 4.3.7. *Concentration of each sector in different regions is never an equilibrium.*

**Lemma** 4.3.8. *Symmetric dispersion of services and concentration of industry is never an equilibrium.*

**Lemma** 4.3.9. *Concentration of services together with dispersion of industry is never an equilibrium.*

## 4.4 The case in which services are tradable

In this section we show, numerically, that a different spatial configuration of economic activity, concentration of services with dispersion of industry, can appear when services are tradable ($0 < \phi_s \leq \phi_m < 1$).

Consider an initial equilibrium, in which both industry and services are concentrated in region 1. Figure 4.12 illustrates how a decrease in the trade cost of services (an increase in $\tau_s$) affects the spatial distribution of industry, when all services are concentrated in region 1 (point A). The main result is that a fall in the trade cost of services leads to an equilibrium in which industry becomes asymmetrically dispersed (point C), while the service sector remains concentrated in region 1 (see figure 4.13).[11]

[11] To plot figures 4.12 and 4.13, we have set $\tau_m = 0.5$, $\mu_m = 0.4$ $\sigma_m = 4$, $\mu_s = 0.4$, $\sigma_s = 1.3$. We also set

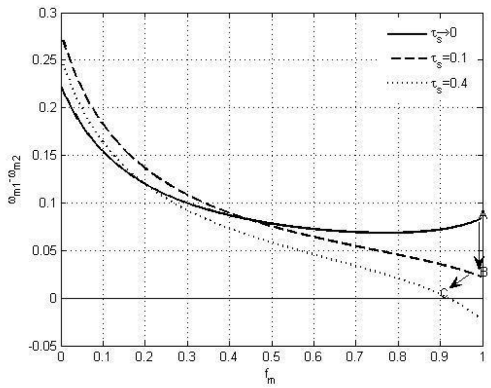


Figure 4.12: Industry becomes dispersed.

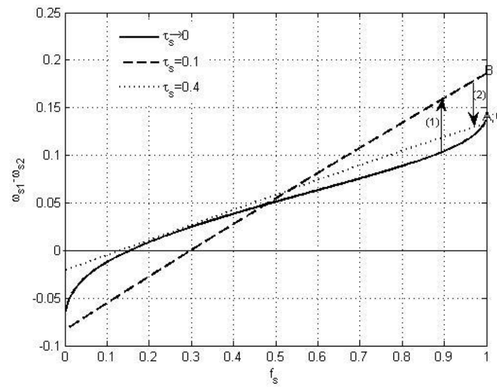


Figure 4.13: Concentration of services.

The same decrease in the trade cost of services (from $\tau_s \to 0$ to $\tau_s = 0.4$) may not change the initial equilibrium configuration, being compatible with symmetric dispersion of both sectors (set $\tau_m = 0.5$, keeping fixed the remaining parameters) or asymmetric dispersion of services with full concentration of industry (set $\tau_m = 0.825$).

## 4.5 Concluding remarks

We have extended the footloose entrepreneur model (Forslid and Ottaviano, 2003) to allow for a third sector: a monopolistic competitive sector of services, assumed to be non-tradable across regions.

We find that the strength of the preference for variety of services is crucial to explain the spatial distribution of the industrial and service activity. When the elasticity of substitution among services is below a certain threshold, full concentration of industry and services

$f_s = 1$ in figure 4.12 and $f_m = 1$ in figure 4.13.

in a single region is always an equilibrium. But this threshold value for the elasticity of substitution among services seems a bit too low to be attainable in modern economies, as it corresponds to a very high price markup over marginal cost.[12] Based on this model, we should not expect, therefore, full concentration of industry and services in a single region.

With a higher elasticity of substitution among services, which is more likely, the spatial distribution of economic activity depends on the trade costs in the industrial sector. If these trade costs are high, symmetric dispersion of the services and industrial activity is an equilibrium. If they are low, concentration of industry with asymmetric dispersion of services is an equilibrium (in this case, the industrialized region has more than 50% of the service sector activity).

Taking into account the existence of non-tradable goods, with specialized workers which are mobile across regions, should provide new insights about the determinants of the spatial organization of economic activity. We hope that this model may be seen as a step in this direction.

[12] With services representing 50% of the economic activity ($\mu_s = 0.5$), at this threshold ($\sigma_s = 1 + \mu_s$), we obtain $p_s = \frac{\sigma_s}{\sigma_s - 1}\beta = 3\beta$ (the price of a representative service is equal to 3 times the marginal cost). A lower price markup is obtained if we consider a higher elasticity of substitution among services.

# Appendices

# Appendix B

# Mathematical proofs

## B.1 Short-run equilibrium

The nominal wages of the industrial and service sector workers are:

$$W_{mi} = \frac{\mu_m}{\sigma_m}\left(\frac{Y_i}{M_i + \phi_m M_j} + \frac{Y_j \phi_m}{\phi_m M_i + M_j}\right), \tag{B.1}$$

$$W_{si} = \frac{\mu_s}{\sigma_s}\left(\frac{Y_i}{S_i + \phi_s S_j} + \frac{Y_j \phi_s}{\phi_s S_i + S_j}\right). \tag{B.2}$$

The regional nominal incomes are:

$$Y_i = \frac{1 - H - S}{2} + W_{mi} M_i + W_{si} S_i. \tag{B.3}$$

Our goal in this section is to find $W_{mi}$ and $W_{si}$ as functions of the parameters of the model. First, we determine $Y_i$ and $Y_j$. Then, we substitute these into $W_{mi}$ and $W_{si}$.

Substituting (B.1) and (B.2) in (B.3), and using $a = \frac{1-H-S}{2}$, we obtain:

$$Y_i = a + M_i \frac{\mu_m}{\sigma_m} \left( \frac{Y_i}{M_i + \phi_m M_j} + \frac{Y_j \phi_m}{\phi_m M_i + M_j} \right) + S_i \frac{\mu_s}{\sigma_s} \left( \frac{Y_i}{S_i + \phi_s S_j} + \frac{Y_j \phi_s}{\phi_s S_i + S_j} \right).$$

Rearranging, we obtain $Y_i$ as a function of $Y_j$:

$$\begin{aligned} Y_i \quad & \left[ 1 - \frac{\mu_m M_i}{\sigma_m \left( M_i + \phi_m M_j \right)} - \frac{\mu_s S_i}{\sigma_s \left( S_i + \phi_s S_j \right)} \right] = \\ = \quad & a + \left[ \frac{\mu_m M_i \phi_m}{\sigma_m \left( \phi_m M_i + M_j \right)} + \frac{\mu_s S_i \phi_s}{\sigma_s \left( \phi_s S_i + S_j \right)} \right] Y_j. \end{aligned}$$

For convenience, define:

$$b_m = \phi_m M_i + M_j,$$

$$c_m = M_i + \phi_m M_j,$$

$$b_s = \phi_s S_i + S_j,$$

$$c_s = S_i + \phi_s S_j.$$

With some manipulation, we obtain:

$$Y_i \frac{\sigma_m c_m \sigma_s c_s - \mu_m M_i \sigma_s c_s - \mu_s S_i \sigma_m c_m}{\sigma_m c_m \sigma_s c_s} = a + \frac{\mu_m M_i \phi_m \sigma_s b_s + \mu_s S_i \phi_s \sigma_m b_m}{\sigma_s b_s \sigma_m b_m} Y_j.$$

This is equivalent to:

$$Y_i = \frac{a \sigma_m \sigma_s c_m c_s + \left( \mu_m M_i \phi_m \sigma_s b_s + S_i \mu_s \phi_s \sigma_m b_m \right) c_m c_s b_m^{-1} b_s^{-1} Y_j}{\sigma_m c_m \left( \sigma_s c_s - S_i \mu_s \right) - M_i \mu_m \sigma_s c_s}. \tag{B.4}$$

The above equation yields $Y_i$ as a function of $Y_j$. By symmetry, we can write $Y_j$ as function of $Y_i$ as follows:

$$Y_j = \frac{a \sigma_m \sigma_s b_m b_s + \left( \mu_m M_j \phi_m \sigma_s c_s + S_j \mu_s \phi_s \sigma_m c_m \right) b_m b_s c_m^{-1} c_s^{-1} Y_i}{\sigma_m b_m \left( \sigma_s b_s - S_j \mu_s \right) - M_j \mu_m \sigma_s b_s}. \tag{B.5}$$

Substituting (B.5) in (B.4) and simplifying, we obtain:

$$Y_i = \frac{c_m c_s \left[\sigma_m b_m \left(\sigma_s b_s - S_j \mu_s\right) - M_j \mu_m \sigma_s b_s + M_i \mu_m \phi_m \sigma_s b_s + S_i \mu_s \phi_s \sigma_m b_m\right]}{a^{-1} \sigma_m^{-1} \sigma_s^{-1} R}, \tag{B.6}$$

and, by symmetry:

$$Y_j = \frac{b_m b_s \left[\sigma_m c_m \left(\sigma_s c_s - S_i \mu_s\right) - M_i \mu_m \sigma_s c_s + M_j \mu_m \phi_m \sigma_s c_s + S_j \mu_s \phi_s \sigma_m c_m\right]}{a^{-1} \sigma_m^{-1} \sigma_s^{-1} R}, \tag{B.7}$$

where:

$$\begin{aligned} R &= \left[\sigma_m c_m \left(\sigma_s c_s - S_i \mu_s\right) - M_i \mu_m \sigma_s c_s\right] \left[\sigma_m b_m \left(\sigma_s b_s - S_j \mu_s\right) - M_j \mu_m \sigma_s b_s\right] - \\ &\quad - \left(M_i \mu_m \phi_m \sigma_s b_s + S_i \mu_s \phi_s \sigma_m b_m\right) \left(M_j \mu_m \phi_m \sigma_s c_s + S_j \mu_s \phi_s \sigma_m c_m\right). \end{aligned}$$

Denoting by $Y_i^N$ and $Y_j^N$ the numerators of $Y_i$ and $Y_j$ in equations (B.6) and (B.7), we can rewrite (B.1) in the following way:

$$W_{mi} = \frac{\mu_m \sigma_s a}{c_m b_m R} \left(Y_i^N b_m + \phi_m Y_j^N c_m\right).$$

Replacing the expressions for $Y_i$ and $Y_j$, and setting $i = 1$ and $j = 2$, we obtain:

$$\begin{aligned} W_{m1} &= \frac{\mu_m a \sigma_m \sigma_s}{R} \left[\, c_s b_m \left(\sigma_s b_s - S_2 \mu_s\right) - M_2 c_s \sigma_m^{-1} \mu_m \sigma_s b_s + M_1 c_s \sigma_m^{-1} \mu_m \phi_m \sigma_s b_s + \right. \\ &+ S_1 \mu_s \phi_s c_s b_m + \phi_m b_s c_m \left(\sigma_s c_s - S_1 \mu_s\right) - M_1 \phi_m b_s \mu_m \sigma_s \sigma_m^{-1} c_s + \\ &+ \left. M_2 \phi_m^2 b_s \mu_m \sigma_s \sigma_m^{-1} c_s + S_2 \phi_m b_s \mu_s \phi_s c_m \,\right]. \end{aligned}$$

Denoting $\overline{C} = \frac{\mu_m c_s b_s \sigma_s a}{R}$, and replacing $b_m$, $c_m$, $b_s$ and $c_s$ by the corresponding expressions:

$$\begin{aligned} \frac{W_{m1}}{\overline{C}} &= \sigma_m \left(\phi_m M_1 + M_2\right) \left(\sigma_s - \mu_s \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1}\right) + \mu_m \sigma_s \left(\phi_m M_1 - M_2\right) + \\ &+ \phi_m \left[\sigma_m \left(M_1 + \phi_m M_2\right) \left(\sigma_s - \mu_s \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2}\right) + \mu_m \sigma_s (\phi_m M_2 - M_1)\right]. \end{aligned}$$

Manipulating:

$$\frac{W_{m1}}{\overline{C}} = \phi_m \sigma_m M_1 \left[2\sigma_s - \mu_s \left(\frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1} + \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2}\right)\right] +$$

$$+M_2 \left[\sigma_m \sigma_s \left(\phi_m^2 + 1\right) + \sigma_s \mu_m \left(\phi_m^2 - 1\right) - \sigma_m \mu_s \left(\phi_m^2 \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} + \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1}\right)\right].$$

It is easy to show that:

$$\frac{S_2 - \phi_s S_1}{\phi_s S_1 + S_2} + \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} = \frac{2 S_1 S_2 \left(1 - \phi_s^2\right)}{S_1 S_2 \left(1 + \phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}.$$

Substituting this expression, we determine $W_{m1}$ and (by symmetry) $W_{m2}$:

$$\frac{W_{m1}}{\overline{C}\sigma_m} = 2\phi_m M_1 \left[\sigma_s - \mu_s \frac{S_1 S_2 \left(1 - \phi_s^2\right)}{S_1 S_2 \left(1 + \phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}\right] +$$

$$+ M_2 \left[\sigma_s \left(\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}\phi_m^2 - \frac{\mu_m}{\sigma_m}\right) - \mu_s \left(\phi_m^2 \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} + \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1}\right)\right] \tag{B.8}$$

and

$$\frac{W_{m2}}{\overline{C}\sigma_m} = 2\phi_m M_2 \left[\sigma_s - \mu_s \frac{S_1 S_2 \left(1 - \phi_s^2\right)}{S_1 S_2 \left(1 + \phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}\right] +$$

$$+ M_1 \left[\sigma_s \left(\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}\phi_m^2 - \frac{\mu_m}{\sigma_m}\right) - \mu_s \left(\phi_m^2 \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1} + \frac{S_1 - \phi_s S_2)}{S_1 + \phi_s S_2}\right)\right], \tag{B.9}$$

where:

$$\overline{C} = \frac{\mu_m c_s b_s \sigma_s a}{R} = \frac{\mu_m \sigma_s a}{R}(S_1 + \phi_s S_2)(S_2 + \phi_s S_1),$$

and:

$$\begin{aligned} R &= \left\{\sigma_m \left(M_1 + \phi_m M_2\right)\left[\sigma_s \left(S_1 + \phi_s S_2\right) - S_1 \mu_s\right] - M_1 \mu_m \sigma_s \left(S_1 + \phi_s S_2\right)\right\} \times \\ &\times \left\{\sigma_m \left(\phi_m M_1 + M_2\right)\left[\sigma_s \left(\phi_s S_1 + S_2\right) - S_2 \mu_s\right] - M_2 \mu_m \sigma_s \left(\phi_s S_1 + S_2\right)\right\} - \end{aligned}$$

$$
\begin{aligned}
- & \quad \left[M_1\mu_m\phi_m\sigma_s\left(\phi_s S_1 + S_2\right) + S_1\mu_s\phi_s\sigma_m\left(\phi_m M_1 + M_2\right)\right] \times \\
\times & \quad \left[M_2\mu_m\phi_m\sigma_s\left(S_1 + \phi_s S_2\right) + S_2\mu_s\phi_s\sigma_m\left(M_1 + \phi_m M_2\right)\right].
\end{aligned}
$$

Equations (B.8) and (B.9) are explicit functions of the parameters of the model.

By analogy, we find the nominal wages in the service sector:

$$
\frac{W_{s1}}{\overline{D}\sigma_s} = 2\phi_s S_1\left[\sigma_m - \mu_m\frac{M_1M_2\left(1-\phi_m^2\right)}{M_1M_2\left(1+\phi_m^2\right)+\phi_m\left(M_1^2+M_2^2\right)}\right] +
$$

$$
+ S_2\left[\sigma_m\left(\phi_s^2 + 1 + \frac{\mu_s}{\sigma_s}\phi_s^2 - \frac{\mu_s}{\sigma_s}\right) - \mu_m\left(\phi_s^2\frac{M_1-\phi_m M_2}{M_1+\phi_m M_2} + \frac{M_2-\phi_m M_1}{M_2+\phi_m M_1}\right)\right] \tag{B.10}
$$

and

$$
\frac{W_{s2}}{\overline{D}\sigma_s} = 2\phi_s S_2\left[\sigma_m - \mu_m\frac{M_1M_2\left(1-\phi_m^2\right)}{M_1M_2\left(1+\phi_m^2\right)+\phi_m\left(M_1^2+M_2^2\right)}\right] +
$$

$$
+ S_1\left[\sigma_m\left(\phi_s^2 + 1 + \frac{\mu_s}{\sigma_s}\phi_s^2 - \frac{\mu_s}{\sigma_s}\right) - \mu_m\left(\phi_s^2\frac{M_2-\phi_m M_1}{M_2+\phi_m M_1} + \frac{M_1-\phi_m M_2)}{M_1+\phi_m M_2}\right)\right], \tag{B.11}
$$

where:

$$
\overline{D} = \frac{\mu_s c_m b_m \sigma_m a}{R} = \frac{\mu_s\sigma_m a}{R}(M_1+\phi_m M_2)(M_2+\phi_m M_1),
$$

and:

$$
\begin{aligned}
R & = \left\{\sigma_m\left(M_1+\phi_m M_2\right)\left[\sigma_s\left(S_1+\phi_s S_2\right) - S_1\mu_s\right] - M_1\mu_m\sigma_s\left(S_1+\phi_s S_2\right)\right\} \times \\
& \times \quad \left\{\sigma_m\left(\phi_m M_1+M_2\right)\left[\sigma_s\left(\phi_s S_1+S_2\right) - S_2\mu_s\right] - M_2\mu_m\sigma_s\left(\phi_s S_1+S_2\right)\right\} - \\
& - \quad \left[M_1\mu_m\phi_m\sigma_s\left(\phi_s S_1 + S_2\right) + S_1\mu_s\phi_s\sigma_m\left(\phi_m M_1 + M_2\right)\right] \times \\
& \times \quad \left[M_2\mu_m\phi_m\sigma_s\left(S_1 + \phi_s S_2\right) + S_2\mu_s\phi_s\sigma_m\left(M_1 + \phi_m M_2\right)\right].
\end{aligned}
$$

## B.2 Short-run equilibrium with $\tau_s \to 0$

With $\tau_s \to 0$, equations (4.13), (4.15), (4.16), (4.17) and (4.18) become:

$$\begin{aligned}
W_{m1} &= \frac{\mu_m}{\sigma_m M}\left[\frac{Y_1}{f_m+\phi_m(1-f_m)}+\frac{\phi_m Y_2}{\phi_m f_m+1-f_m}\right]\\
W_{m2} &= \frac{\mu_m}{\sigma_m M}\left[\frac{Y_2}{1-f_m+\phi_m f_m}+\frac{\phi_m Y_1}{\phi_m(1-f_m)+f_m}\right]\\
W_{s1} &= \frac{\mu_s Y_1}{\sigma_s S f_s}\\
W_{s2} &= \frac{\mu_s Y_2}{\sigma_s S(1-f_s)}\\
Y_1 &= \frac{1-M-S}{2}+W_{m1}Mf_m+W_{s1}Sf_s\\
Y_2 &= \frac{1-M-S}{2}+W_{m2}M(1-f_m)+W_{s2}S(1-f_s)\\
P_1 &= \rho\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{\frac{\mu_s}{1-\sigma_s}}\\
P_2 &= \rho\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}} (1-f_s)^{\frac{\mu_s}{1-\sigma_s}}.
\end{aligned}$$

From (B.8)-(B.11), considering $\phi^S \to 0$, we compute the nominal wages of the workers in each region:

$$\frac{W_{m1}}{\overline{C}_0\sigma_m} = 2\phi_m M_1\left(\sigma_s-\mu_s\right)+M_2\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)\right\}, \quad \text{(B.12)}$$

$$\frac{W_{m2}}{\overline{C}_0\sigma_m} = 2\phi_m M_2\left(\sigma_s-\mu_s\right)+M_1\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)\right\}, \quad \text{(B.13)}$$

$$\frac{W_{s1}}{\overline{D}_0 \sigma_s} = S_2 \left[ \sigma_m \left( 1 - \frac{\mu_s}{\sigma_s} \right) - \mu_m \frac{M_2 - \phi_m M_1}{\phi_m M_1 + M_2} \right], \tag{B.14}$$

$$\frac{W_{s2}}{\overline{D}_0 \sigma_s} = S_1 \left[ \sigma_m \left( 1 - \frac{\mu_s}{\sigma_s} \right) - \mu_m \frac{M_1 - \phi_m M_2}{M_1 + \phi_m M_2} \right], \tag{B.15}$$

where

$$\overline{C}_0 = \frac{\mu_m S_1 S_2}{R_0 \sigma_m},$$

$$\overline{D}_0 = \frac{\mu_s (M_1 + \phi_m M_2)(\phi_m M_1 + M_2)}{R_0 \sigma_s},$$

and

$$\begin{aligned} R_0 &= \frac{S_1 S_2}{a \sigma_m \sigma_s} \{ \left[ \sigma_m \left( M_1 + \phi_m M_2 \right) \left( \sigma_s - \mu_s \right) - M_1 \mu_m \sigma_s \right] \times \\ &\times \left[ \sigma_m \left( \phi_m M_1 + M_2 \right) \left( \sigma_s - \mu_s \right) - M_2 \mu_m \sigma_s \right] - M_1 M_2 \mu_m^2 \phi_m^2 \sigma_s^2 \} . \end{aligned}$$

Dividing the nominal wages in the industrial sector, (B.12) and (B.13), by the regional price level, we obtain the real wages of the industrial workers:

$$\omega_{m1} = \frac{2\phi_m f_m \left( \sigma_s - \mu_s \right) + (1 - f_m) \left\{ \sigma_s \left[ \phi_m^2 + 1 + \frac{\mu_m}{\sigma_m} (\phi_m^2 - 1) \right] - \mu_s \left( 1 + \phi_m^2 \right) \right\}}{R_m \left[ f_m + (1 - f_m) \phi_m \right]^{\frac{\mu_m}{1 - \sigma_m}} f_s^{\frac{\mu_s}{1 - \sigma_s}}} \tag{B.16}$$

and

$$\omega_{m2} = \frac{2\phi_m (1 - f_m) \left( \sigma_s - \mu_s \right) + f_m \left\{ \sigma_s \left[ \phi_m^2 + 1 + \frac{\mu_m}{\sigma_m} (\phi_m^2 - 1) \right] - \mu_s \left( 1 + \phi_m^2 \right) \right\}}{R_m \left[ \phi_m f_m + (1 - f_m) \right]^{\frac{\mu_m}{1 - \sigma_m}} \left( 1 - f_s \right)^{\frac{\mu_s}{1 - \sigma_s}}}, \tag{B.17}$$

where

$$\begin{aligned} R_m &= \frac{\rho M}{a \mu_m \sigma_m \sigma_s} \{ \left\{ \sigma_m \left[ f_m + \phi_m (1 - f_m) \right] \left( \sigma_s - \mu_s \right) - f_m \mu_m \sigma_s \right\} \times \\ &\times \left\{ \sigma_m \left[ \phi_m f_m + (1 - f_m) \right] \left( \sigma_s - \mu_s \right) - (1 - f_m) \mu_m \sigma_s \right\} - f_m (1 - f_m) \mu_m^2 \phi_m^2 \sigma_s^2 \} . \end{aligned}$$

With some manipulation, we find that the difference between the real wages in the industrial sector can be written as:

$$\omega_{m1} - \omega_{m2} = \frac{\mu_m K}{\phi_m \left[f_m^2 + (1 - f_m)^2\right] K_1 + (1 - f_m) f_m K_2} \left(\bar{\omega}_{m1} - \bar{\omega}_{m2}\right), \tag{B.18}$$

where:

$$K = \frac{\sigma_m^{1-\mu_m} \sigma_s^{1-\mu_s} (1 - M - S) (\sigma_m - 1)^{\mu_m} (\sigma_s - 1)^{\mu_s}}{2\alpha_m^{\frac{\mu_m}{\sigma_m - 1}} \alpha_s^{\frac{\mu_s}{\sigma_s - 1}} \beta^{\mu_m + \mu_s} M^{1 - \frac{\mu_m}{\sigma_m - 1}} S^{-\frac{\mu_s}{\sigma_s - 1}}},$$

$$K_1 = \sigma_m (\sigma_s - \mu_s) \left[\sigma_m (\sigma_s - \mu_s) - \sigma_s \mu_m\right],$$

$$K_2 = \sigma_m (\sigma_s - \mu_s) \left[\sigma_m (\sigma_s - \mu_s) \left(1 + \phi_m^2\right) - 2\mu_m \sigma_s\right] + \mu_m^2 \sigma_s^2 \left(1 - \phi_m^2\right),$$

$$\bar{\omega}_{m1} = \frac{2\phi_m f_m (\sigma_s - \mu_s) + (1 - f_m) \left\{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s \left(\phi_m^2 + 1\right)\right\}}{\left[f_m + (1 - f_m)\phi_m\right]^{\frac{\mu_m}{1 - \sigma_m}} f_s^{\frac{\mu_s}{1 - \sigma_s}}},$$

$$\bar{\omega}_{m2} = \frac{2\phi_m (1 - f_m) (\sigma_s - \mu_s) + f_m \left\{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s \left(\phi_m^2 + 1\right)\right\}}{\left[\phi_m f_m + (1 - f_m)\right]^{\frac{\mu_m}{1 - \sigma_m}} (1 - f_s)^{\frac{\mu_s}{1 - \sigma_s}}}.$$

Similarly, for the service sector, dividing the nominal wages in the service sector, (B.14) and (B.15), by the regional price level, we obtain the real wages of the service workers:

$$\omega_{s1} = \frac{\left[f_m + \phi_m (1 - f_m)\right] \left[\phi_m f_m + 1 - f_m\right] \left[\sigma_m \left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{1 - f_m - \phi_m f_m}{1 - f_m + \phi_m f_m}\right]}{R_s \left[f_m + (1 - f_m)\phi_m\right]^{\frac{\mu_m}{1 - \sigma_m}} f_s^{1 - \frac{\mu_s}{\sigma_s - 1}}}$$

and:

$$\omega_{s2} = \frac{\left[f_m + \phi_m (1 - f_m)\right] \left[\phi_m f_m + 1 - f_m\right] \left[\sigma_m \left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{f_m - \phi_m (1 - f_m)}{f_m + \phi_m (1 - f_m)}\right]}{R_s \left[\phi_m f_m + (1 - f_m)\right]^{\frac{\mu_m}{1 - \sigma_m}} (1 - f_s)^{1 - \frac{\mu_s}{\sigma_s - 1}}},$$

where

$$R_s = \frac{\rho S}{a \sigma_m \sigma_s} \left\{ \left[\sigma_m \left[f_m + \phi_m (1 - f_m)\right] (\sigma_s - \mu_s) - f_m \mu_m \sigma_s\right] \times \right.$$

$$\times \quad \left[\sigma_m\left(\phi_m f_m + 1 - f_m\right)\left(\sigma_s - \mu_s\right) - (1-f_m)\mu_m\sigma_s\right] - f_m(1-f_m)\mu_m^2\phi_m^2\sigma_s^2 \; \} \, .$$

Again, after some manipulation, we write the real wage differential in the service sector as:

$$\omega_{s1} - \omega_{s2} = \frac{\mu_s K\left\{\phi_m\left[f_m^2 + (1-f_m)^2\right] + f_m\left(1-f_m\right)\left(1+\phi_m^2\right)\right\}}{\phi_m\left[f_m^2 + (1-f_m)^2\right]K_1 + \left(1-f_m\right)f_m K_2}\left(\bar{\omega}_{s1} - \bar{\omega}_{s2}\right) \tag{B.19}$$

where:

$$\bar{\omega}_{s1} = \frac{\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m\frac{1-f_m(1+\phi_m)}{1-f_m(1-\phi_m)}}{\left[f_m + (1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{1-\frac{\mu_s}{\sigma_s - 1}}};$$

$$\bar{\omega}_{s2} = \frac{\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m\frac{f_m(1+\phi_m)-\phi_m}{f_m(1-\phi_m)+\phi_m}}{\left[\phi_m f_m + (1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left(1-f_s\right)^{1-\frac{\mu_s}{\sigma_s-1}}}.$$

# B.3 Proof of claim 4.3.1

Inspection of the expressions for $\omega_{m1} - \omega_{m2}$ and $\omega_{s1} - \omega_{s2}$ shows that continuity of these differences and their derivatives depends on the denominator in (B.18) not being zero. This is the case since $K_1$ and $K_2$ are positive. □

# B.4 Proof of lemmas

## B.4.1 Lemma 4.3.1

Since regions are symmetric, we only study concentration in region 1.

When the workers become concentrated in region 1, $R_m$ in (B.16) and (B.17), converges to:

$$\lim_{f_m \to 1^-} R_m = \frac{\rho M \phi_m}{a\mu_m\sigma_s}\left(\sigma_m\sigma_s - \mu_s\sigma_m - \mu_m\sigma_s\right)\left(\sigma_s - \mu_s\right) > 0.$$

Therefore, from (B.16) and (B.17), we have:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m1} = \frac{2\phi_m\left(\sigma_s-\mu_s\right)}{\lim_{(f_m,f_s)\to(1^-,1^-)} R_m} = \frac{2a\mu_m\sigma_s}{\rho M\left(\sigma_m\sigma_s-\sigma_m\mu_s-\mu_m\sigma_s\right)},$$

while:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m2} = \frac{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)}{\lim_{(f_m,f_s)\to(1^-,1^-)} R_m\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left(1-f_s\right)^{\frac{\mu_s}{1-\sigma_s}}}.$$

The numerator is positive, while the denominator goes to infinity. Thus:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m2} = 0.$$

We conclude that the industrial workers remain concentrated, as:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m1} > \lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m2}.$$

In the service sector, real wages in region 1 tend to:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{s1} \quad = \quad \lim_{(f_m,f_s)\to(1^-,1^-)} \frac{\phi_m(1-f_s)\left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)+\mu_m\right]}{R_s}.$$

Notice that:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \frac{R_s}{\phi_m(1-f_s)} \quad = \quad \frac{\rho S}{a\sigma_s}\left(\sigma_m\sigma_s-\sigma_m\mu_s-\mu_m\sigma_s\right)\left(\sigma_s-\mu_s\right).$$

Therefore:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{s1} \quad = \quad \lim_{(f_m,f_s)\to(1^-,1^-)} \frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)+\mu_m}{\frac{\rho S}{a\sigma_s}\left(\sigma_m\sigma_s-\sigma_m\mu_s-\mu_m\sigma_s\right)\left(\sigma_s-\mu_s\right)} > 0,$$

while:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{s2} = \frac{\phi_m^{\frac{1-\sigma_m-\mu_m}{1-\sigma_m}}\left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\right]}{\lim_{(f_m,f_s)\to(1^-,1^-)} R_s(1-f_s)^{\frac{\mu_s}{1-\sigma_s}}} =$$

$$= \frac{\tau_m^{\mu_m}\left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\right]}{\frac{\rho S}{a\sigma_s}\left(\sigma_m\sigma_s-\sigma_m\mu_s-\mu_m\sigma_s\right)\left(\sigma_s-\mu_s\right)} \lim_{(f_m,f_s)\to(1^-,1^-)} (1-f_s)^{\frac{\sigma_s-1-\mu_s}{1-\sigma_s}}.$$

The real wage of the service workers in the empty region tends to zero if $\sigma_s < 1+\mu_s$ and to plus infinity if $\sigma_s > 1+\mu_s$. Notice also that with $\sigma_s = 1+\mu_s$, we have $\omega_{s1} > \omega_{s2}$ (in the limit).

We conclude that concentration of both sectors in a single region is an equilibrium if and only if $\sigma_s \leq 1+\mu_s$. □

### B.4.2 Lemma 4.3.2

Since regions are symmetric, we only study the case in which all the industrial activity is concentrated in region 1.

When $f_m = 1$, the difference between the real wages of the service sector workers in (B.19) is:

$$\omega_{s1}-\omega_{s2} = \frac{\mu_s K}{K_1}\left[\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m}{f_s^{1-\frac{\mu_s}{\sigma_s-1}}} - \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{1-\frac{\mu_s}{\sigma_s-1}}}\right].$$

We begin our proof by calculating the values of $f_s \in (0,1)$ for which $\omega_{s1}-\omega_{s2}=0$. Since $K$ and $K_1$ are strictly positive and finite:

$$0 \;=\; \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m}{f_s^{1-\frac{\mu_s}{\sigma_s-1}}} - \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{1-\frac{\mu_s}{\sigma_s-1}}} \Leftrightarrow$$

$$\Leftrightarrow \quad \frac{1-f_s}{f_s} = \left\{\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m\right]}\right\}^{\frac{\sigma_s-1}{\sigma_s-1-\mu_s}}. \tag{B.20}$$

Simplifying we obtain:

$$f_s = \frac{1}{\left\{\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m\right]}\right\}^{\frac{\sigma_s-1}{\sigma_s-1-\mu_s}}+1}.$$

Notice that since $\phi_m^{\frac{\mu_m}{1-\sigma_m}} > 1$, we have:

$$0 < \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m\right]} < 1.$$

Therefore, since $\sigma_s > \mu_s + 1$, we have:

$$0 < \left\{\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m\right]}\right\}^{\frac{\sigma_s-1}{\sigma_s-1-\mu_s}} < 1$$

and

$$\frac{1}{2} < f_s < 1.$$

It remains to verify the equilibrium condition for the industrial sector.

When the skilled workers in the industrial sector are concentrated in region 1, the difference between the real wages of the industrial workers in (B.18) is:

$$\omega_{m1} - \omega_{m2} =$$

$$= \frac{\mu_m K}{\phi_m K_1}\left\{\frac{2\phi_m(\sigma_s-\mu_s)}{f_s^{\frac{\mu_s}{1-\sigma_s}}} - \frac{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right]-\mu_s(\phi_m^2+1)}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{\frac{\mu_s}{1-\sigma_s}}}\right\}.$$

Then, $\omega_{m1} - \omega_{m2} > 0$ if and only if:

$$\frac{2\phi_m(\sigma_s-\mu_s)}{f_s^{\frac{\mu_s}{1-\sigma_s}}} > \frac{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right]-\mu_s(\phi_m^2+1)}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{\frac{\mu_s}{1-\sigma_s}}} \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{1-f_s}{f_s}\right)^{\frac{\mu_s}{1-\sigma_s}} > \frac{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right]-\mu_s(\phi_m^2+1)}{2\phi_m(\sigma_s-\mu_s)\phi_m^{\frac{\mu_m}{1-\sigma_m}}}$$

Replacing equation (B.20), we obtain:

$$\left\{\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m\right]}\right\}^{\frac{-\mu_s}{\sigma_s-1-\mu_s}} > \frac{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right]-\mu_s(\phi_m^2+1)}{2(\sigma_s-\mu_s)\phi_m^{\frac{\mu_m}{1-\sigma_m}+1}}.$$

Rearranging we have:

$$\left[\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m}\right]^{\frac{-\mu_s}{\sigma_s-1-\mu_s}}\phi_m^{1-\frac{\mu_m(\sigma_s-1)}{(\sigma_m-1)(\sigma_s-1-\mu_s)}}$$

$$-\quad\frac{\phi_m^2\left[\sigma_s(1+\frac{\mu_m}{\sigma_m})-\mu_s\right]+\sigma_s(1-\frac{\mu_s}{\sigma_m})-\mu_s}{2(\sigma_s-\mu_s)} > 0.$$

Concentration of all the industrial activity in a region and asymmetric dispersion of the service activity is a steady-state when the above condition is satisfied. Lemma 4.3.3 provides the stability condition, guaranteeing that it is an equilibrium. □

### B.4.3 Lemma 4.3.3

From equation (B.19), the sign of $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s}$ is equal to the sign of $\frac{\partial(\bar{\omega}_{s1}-\bar{\omega}_{s2})}{\partial f_s}$.

Calculating the partial derivatives:

$$\frac{\partial\bar{\omega}_{s1}}{\partial f_s} = \left(-1+\frac{\mu_s}{\sigma_s-1}\right)\frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{1-f_m(1+\phi_m)}{1-f_m(1-\phi_m)}}{[f_m+(1-f_m)\phi_m]^{\frac{\mu_m}{1-\sigma_m}}}f_s^{-2+\frac{\mu_s}{\sigma_s-1}},$$

$$\frac{\partial\bar{\omega}_{s2}}{\partial f_s} = \left(1-\frac{\mu_s}{\sigma_s-1}\right)\frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{f_m(1+\phi_m)-\phi_m}{f_m(1-\phi_m)+\phi_m}}{[\phi_m f_m+(1-f_m)]^{\frac{\mu_m}{1-\sigma_m}}}(1-f_s)^{-2+\frac{\mu_s}{\sigma_s-1}}.$$

With $\sigma_s > \mu_s + 1$, we find that, for $f_s \in (0,1)$:

$$\frac{\partial\bar{\omega}_{s1}}{\partial f_s} < 0,$$

$$\frac{\partial \bar{\omega}_{s2}}{\partial f_s} > 0.$$

When $f_s \in \{0, 1\}$, one of these partial derivatives is null, but we still have $\dfrac{\partial \bar{\omega}_{s1}}{\partial f_s} < \dfrac{\partial \bar{\omega}_{s2}}{\partial f_s}$. $\square$

### B.4.4 Lemma 4.3.4

When $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$, we have $\bar{\omega}_{m1} = \bar{\omega}_{m2}$ and $\bar{\omega}_{s1} = \bar{\omega}_{s2}$.

Therefore, we also have $\omega_{m1} = \omega_{m2}$ and $\omega_{s1} = \omega_{s2}$. $\square$

### B.4.5 Lemma 4.3.5

We want to prove that $\frac{\partial(\omega_{m1} - \omega_{m2})}{\partial f_s} > 0$.

From (B.18), we know that the sign of $\frac{\partial(\bar{\omega}_{m1} - \bar{\omega}_{m2})}{\partial f_s}$ is the same.

Calculating the partial derivatives:

$$\frac{\partial \bar{\omega}_{m1}}{\partial f_s} = \frac{2\phi_m f_m \left(\sigma_s - \mu_s\right) + (1 - f_m) \left\{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s \left(\phi_m^2 + 1\right)\right\}}{\frac{\sigma_s - 1}{\mu_s} \left[f_m + (1 - f_m)\phi_m\right]^{\frac{\mu_m}{1 - \sigma_m}} f_s^{1 - \frac{\mu_s}{\sigma_s - 1}}},$$

$$\frac{\partial \bar{\omega}_{m2}}{\partial f_s} = -\frac{2\phi_m (1 - f_m) \left(\sigma_s - \mu_s\right) + f_m \left\{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s \left(\phi_m^2 + 1\right)\right\}}{\frac{\sigma_s - 1}{\mu_s} \left[\phi_m f_m + (1 - f_m)\right]^{\frac{\mu_m}{1 - \sigma_m}} (1 - f_s)^{1 - \frac{\mu_s}{\sigma_s - 1}}}.$$

Substituting $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$, we find that:

$$\frac{\partial \left(\bar{\omega}_{m1} - \bar{\omega}_{m2}\right)}{\partial f_s} = \frac{2\phi_m \left(\sigma_s - \mu_s\right)}{\frac{\sigma_s - 1}{\mu_s} \left[\frac{1}{2}(1 + \phi_m)\right]^{\frac{\mu_m}{1 - \sigma_m}} 2^{-1 + \frac{\mu_s}{\sigma_s - 1}}} > 0.$$

$\square$

### B.4.6 Lemma 4.3.6

We want to prove that $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} > 0$. Using equation (B.19), we determine the partial derivative:

$$\begin{aligned}
\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} &= \frac{\partial}{\partial f_m}\left[\frac{\mu_s K\left\{\phi_m\left[f_m^2+(1-f_m)^2\right]+f_m\,(1-f_m)\left(1+\phi_m^2\right)\right\}}{\phi_m\left[f_m^2+(1-f_m)^2\right]K_1+(1-f_m)\,f_m K_2}\right](\bar{\omega}_{s1}-\bar{\omega}_{s2}) \\
&+ \frac{\partial\left(\bar{\omega}_{s1}-\bar{\omega}_{s2}\right)}{\partial f_m}\left[\frac{\mu_s K\left\{\phi_m\left[f_m^2+(1-f_m)^2\right]+f_m\,(1-f_m)\left(1+\phi_m^2\right)\right\}}{\phi_m\left[f_m^2+(1-f_m)^2\right]K_1+(1-f_m)\,f_m K_2}\right].
\end{aligned}$$

Since we are evaluating the derivative at $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$, the first term disappears:

$$\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} = \frac{\partial\left(\bar{\omega}_{s1}-\bar{\omega}_{s2}\right)}{\partial f_m}\left[\frac{\mu_s K\left(2\phi_m+1+\phi_m^2\right)}{2\phi_m K_1+K_2}\right].$$

The partial derivative of $\bar{\omega}_{s1}$ with respect to $f_m$ is:

$$\begin{aligned}
\frac{\partial\bar{\omega}_{s1}}{\partial f_m} &= \frac{1}{\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}\,f_s^{1-\frac{\mu_s}{\sigma_s-1}}}\times \\
&\times\ \mu_m\frac{(1+\phi_m)\left[1-f_m(1-\phi_m)\right]+(\phi_m-1)\left[1-f_m(1+\phi_m)\right]}{\left[1-f_m(1-\phi_m)\right]^2}+ \\
&+\ \frac{1}{\left\{\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}\,f_s^{1-\frac{\mu_s}{\sigma_s-1}}\right\}^2}\times \\
&\times\ \frac{\mu_m}{\sigma_m-1}\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}-1}f_s^{1-\frac{\mu_s}{\sigma_s-1}}(1-\phi_m)\times \\
&\times\ \left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{1-f_m\,(1+\phi_m)}{1-f_m\,(1-\phi_m)}\right].
\end{aligned}$$

Substituting $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$:

$$\left.\frac{\partial \bar{\omega}_{s1}}{\partial f_m}\right|_{(f_s,f_m)=\left(\frac{1}{2},\frac{1}{2}\right)} = \frac{\mu_m \left[(1+\phi_m)\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)+(\phi_m-1)\left(\frac{1}{2}-\frac{1}{2}\phi_m\right)\right]}{\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^{\frac{\mu_m}{1-\sigma_m}} 2^{-1+\frac{\mu_s}{\sigma_s-1}} \left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^2} +$$

$$+ \quad \frac{\frac{\mu_m}{\sigma_m-1}\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^{\frac{\mu_m}{1-\sigma_m}-1} 2^{-1+\frac{\mu_s}{\sigma_s-1}}(1-\phi_m)}{\left[\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^{\frac{\mu_m}{1-\sigma_m}} 2^{-1+\frac{\mu_s}{\sigma_s-1}}\right]^2} \left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{\frac{1}{2}-\frac{1}{2}\phi_m}{\frac{1}{2}+\frac{1}{2}\phi_m}\right].$$

Both terms are positive, therefore, $\frac{\partial \omega_{s1}}{\partial f_m} > 0$.

By symmetry, $\frac{\omega_{s2}}{\partial f_m} = -\frac{\partial \omega_{s1}}{\partial f_m}$. Hence, we conclude that $\left.\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m}\right|_{f_m=f_s=\frac{1}{2}}$ is positive. $\square$

### B.4.7 Lemma 4.3.7

Since regions are symmetric, we only need to study the case in which industry is concentrated in region 1 and services are concentrated in region 2. We look at points near $(f_m, f_s) = (1, 0)$ and at what happens when first $f_m \to 1$ and then $f_s \to 0$.

The difference between the real wages in the industrial sector when all the industry is located in region 1 and all the services are located in region 2 is:

$$\omega_{m1} - \omega_{m2} = \frac{K}{K_1}\left(\bar{\omega}_{m1} - \bar{\omega}_{m2}\right).$$

Notice that the constants $K$ and $K_1$ are strictly positive and finite.

Observe also that when $(f_m, f_s) = (1, 0)$, we have $\bar{\omega}_{m1} = 0$ and $\bar{\omega}_{m2} > 0$. Therefore, $\omega_{m1} < \omega_{m2}$.

Concentration of each sector in a different region is never an equilibrium. $\square$

### B.4.8 Lemma 4.3.8

Since regions are symmetric, it is enough to study the case in which services are symmetrically dispersed while the industry is concentrated in region 1.

The difference between the real wages in the service sector when $f_s = 0.5$ and $f_m = 1$ is:

$$\omega_{s1} - \omega_{s2} = \frac{4\mu_s K}{K_1} \left(\bar{\omega}_{s1} - \bar{\omega}_{s2}\right).$$

The constants $K$ and $K_1$ are strictly positive and finite.

With $(f_s, f_m) = \left(\frac{1}{2}, 1\right)$, we have $\bar{\omega}_{s2} = \bar{\omega}_{s1}\phi_m^{\frac{\mu_m}{\sigma_m - 1}} < \bar{\omega}_{s1}$. Therefore, $\omega_{s2} < \omega_{s1}$.

Symmetric dispersion of services with concentration of industry cannot be an equilibrium. □

### B.4.9 Lemma 4.3.9

Since regions are symmetric, it is enough to study the case in which industry is dispersed while services are concentrated in region 2 ($f_s = 0$).

From expression (B.18), the difference between the real wages in the industrial sector is:

$$\omega_{m1} - \omega_{m2} = \frac{\mu_m K}{\phi_m \left[f_m^2 + (1 - f_m)^2\right] K_1 + (1 - f_m) f_m K_2} \left(\bar{\omega}_{m1} - \bar{\omega}_{m2}\right).$$

With $f_s = 0$, we have $\bar{\omega}_{m1} = 0$ and $\bar{\omega}_{m2} > 0$.

This implies that $\omega_{m1} < \omega_{m2}$, because $K$, $K_1$ and $K_2$ are strictly positive and finite.

Dispersion of industry with concentration of services cannot be an equilibrium. □

# Chapter 5

# The core-periphery model with technology advantage

## 5.1 Introduction

In general, core-periphery models assume that regions have the same preferences and technology. In such framework, all consumers share the same utility function and firms produce using the same technology (see for example, Krugman, 1991 and Puga, 1999).

The same technology is represented by the idea that each firm in each region has an identical cost function, which includes a fixed cost and a variable cost. More precisely, all industrial firms in each region support a fixed cost of $\alpha$ units of a particular factor of production and a variable cost.

In this paper, we assume that a region has *a priori* a technological advantage, in which all firms have lower fixed costs. This means that they use less $\alpha$ units of skilled labor than all firms in the other region, to produce one unit of good.

A related paper was developed by Behrens and Thisse (2006) in which an economy is formed by two countries with different set-up costs and entry conditions. In this case, contrary to our model, countries have also different endowment sizes (as labor and capital), and consumers are immobile and supply labor locally, whereas capital is perfectly mobile.

Our model generalizes the analytically solvable core-periphery model of Forslid and Ottaviano (2003) incorporating a technology advantage for all industrial firms operating in one of the regions. We study how this technology advantage affects the spatial distribution of the economic activity and the welfare of the interest groups in the economy.

We find evidence that industrial activity in a region is enhanced, *ceteris paribus*, by lower home fixed costs and by higher foreign fixed costs. From the point of view of welfare, we conclude that all skilled workers in a region are better off if the technology advantage arise from a decrease (increase) in the home (foreign) fixed costs. Relatively to the welfare of the unskilled workers, we conclude that a technology advantage in a region increases the welfare of the unskilled workers in both regions if there is a fall in one of the fixed costs.

## 5.2 The model

### 5.2.1 Basic setup

The model is an extension of the analytically solvable core-periphery model of Forslid and Ottaviano (2003) that incorporates a technology advantage for all industrial firms operating in one of the regions. The technology advantage consists in modelling a region with a lower fixed cost.

The economy comprises two regions and two sectors: an agricultural sector, and an industrial sector. There are two factors of production: unskilled workers ($L$) and skilled workers ($M$). The unskilled workers are immobile across regions, while the skilled workers are mobile.

We denote by $M_1$ and $M_2$, with $M_1 + M_2 = M$, the supply of skilled workers in regions 1 and 2, respectively. The supply of unskilled workers is the same in each region, $L_1 = L_2 = L/2$. The total population is normalized to unity, $L + M = 1$.

The agricultural sector is perfectly competitive and produces a homogeneous good under constant returns to scale using only unskilled labor. Transportation of agricultural output across regions is costless. The industrial sector produces a horizontally differentiated product using skilled labor (fixed cost) and unskilled labor (variable cost).

Transportation of industrial goods is subject to iceberg transportation costs. For each unit of industrial good that is shipped to the other region, only a fraction $\tau$, with $0 < \tau < 1$

arrives.

All the agents have the same preferences for consumption of industrial goods ($C_M$), and agricultural goods ($C_A$). The preferences are represented by the following utility function:

$$U = C_M^{\mu} C_A^{1-\mu},$$

$$C_M = \left[ \sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $\mu \in (0, 1)$, is the share of spending on industrial products; $n$ is the number of varieties of industrial goods; $c_i$ is the consumption of the industrial good produced by firm $i$; and, finally, $\sigma > 1$ is the elasticities of substitution among industrial goods.

### 5.2.2 Supply

**Agricultural sector**

In the agricultural sector, firms use unskilled labor to produce a homogeneous good under constant returns to scale. The production function is $q^A = L$, where $q^A$ is the amount of agricultural goods produced and $L$ is the quantity of unskilled labor employed. The cost function is $CT^A = W_L q^A$, where $W_L$ is the nominal wage of the unskilled workers employed. The profit function is:

$$\Pi^A = (p_A - W_L) q^A,$$

where $p_A$ is the price of an agricultural good, taken as given by the firms (perfect competition) and chosen to be the numeraire ($p_A = 1$).

The sector is perfectly competitive, therefore:

$$W_L = p_A = 1.$$

**Industrial sector**

All industrial firms in region $i$, for $i \in \{1, 2\}$ support a fixed cost of $\alpha_i$ units of skilled labor, with $\alpha_1 \leq \alpha_2$, and a variable cost of $\beta$ units of unskilled labor per unit of good produced.

Since $W_L = 1$, the cost function for all firms in region $i$ is $CT_i = \alpha_i W_i + \beta q_i$, where $q_i$ is the quantity of industrial goods produced by an industrial firm in region $i$ and $W_i$ is the nominal wage of the industrial workers in region $i$.

The profit function for all manufacturing firms in region $i$ is:

$$\Pi_i = p_i(q_i)q_i - \beta q_i - \alpha_i W_i. \tag{5.1}$$

Firms choose $q_i$ to maximize profit. This implies that:

$$p_i = \frac{\epsilon}{\epsilon - 1}\beta,$$

where $\epsilon$ is the price-elasticity of demand.

Since there is a large number of firms in the industrial sector, $\epsilon \approx \sigma_m$ (we have equality if there is an infinite number of firms). Thus:[1]

$$p_i = \frac{\sigma}{\sigma - 1}\beta. \tag{5.2}$$

Given the assumption of free entry, the profit of each firm must be zero. Substituting (5.2) in (5.1), we obtain:

$$q_i = \frac{\alpha_i}{\beta}(\sigma - 1)W_i. \tag{5.3}$$

[1] In the case of Cournot competition: $\frac{1}{\epsilon} = \frac{1}{\sigma} + s(1 - \frac{1}{\sigma})$, where $s$ is the market share of each firm. With many firms in the economy ($s \approx 0$), the price elasticity of demand, $\epsilon$, is approximately equal to the elasticity of substitution among the differentiated goods, $\sigma$.

Since an industrial firm in region $i$ employs $\alpha_i$ units of skilled labor, the total demand for skilled labor in region $i$ is $n_i\alpha_i$. Therefore, the number of firms in region $i$ must be:

$$n_i = \frac{M_i}{\alpha_i}. \tag{5.4}$$

### 5.2.3 Demand

#### Industrial sector

Each industrial firm sells its products in both regions. The price of a representative local industrial good is $p_{ii} = \frac{\beta\sigma}{\sigma-1}$, and the price of a product that is exported from region $i$ to region $j$ is $p_{ij} = \tau^{-1}\frac{\beta\sigma}{\sigma-1}$, where $0 < \tau < 1$ is the transportation cost of industrial goods[2].

Since all industrial firms of a region set the same price, the industrial price index in region $i$ is:

$$\begin{aligned} P_i &= \left[n_i {p_{ii}}^{1-\sigma} + n_j {p_{ij}}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \\ &= \frac{\beta\sigma}{\sigma-1}\left(n_i + n_j\tau^{\sigma_m-1}\right)^{\frac{1}{1-\sigma}}, \end{aligned}$$

where $n_i$ and $n_j$ are the number of industrial firms in regions $i$ and $j$, respectively.

Defining $\phi = \tau^{\sigma-1}$ as the degree of economic integration for the industrial sector (see Baldwin et al., 2003), we obtain:

$$P_i = \frac{\beta\sigma}{\sigma-1}\left(n_i + n_j\phi\right)^{\frac{1}{1-\sigma}}. \tag{5.5}$$

[2]Transportation cost of industrial goods is subject to iceberg transportation costs. For each unit of industrial good that is shipped to the other region, only a fraction $0 < \tau < 1$ arrives.

Denoting by $C_{ij}$ the total demand of an industrial product that is produced in region $i$ and consumed in region $j$, we have:

$$C_{ii} = \frac{p_{ii}^{-\sigma}}{P_i^{1-\sigma}}\mu Y_i \quad \text{and} \quad C_{ij} = \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}}\mu Y_j,$$

where $Y_i$ and $Y_j$ are the nominal incomes in regions $i$ and $j$, respectively.

Since $p_{ii} = \frac{\beta\sigma}{\sigma-1}$ and $p_{ij} = \tau^{-1}\frac{\beta\sigma}{\sigma-1}$, the above equations become:

$$C_{ii} = \frac{\left(\frac{\beta\sigma}{\sigma-1}\right)^{-\sigma}}{P_i^{1-\sigma}}\mu Y_i \quad \text{and} \quad C_{ij} = \frac{\tau^{\sigma}\left(\frac{\beta\sigma}{\sigma-1}\right)^{-\sigma}}{P_j^{1-\sigma}}\mu Y_j. \tag{5.6}$$

Denoting by $q_i$ the output of an industrial firm in region $i$, we have:

$$q_i = C_{ii} + \tau^{-1}C_{ij}. \tag{5.7}$$

Substituting (5.6) in (5.7), we obtain:

$$q_i = \mu\left(\frac{\beta\sigma}{\sigma-1}\right)^{-\sigma}\left(\frac{Y_i}{P_{Mi}^{1-\sigma}} + \frac{\tau^{\sigma-1}Y_j}{P_j^{1-\sigma}}\right). \tag{5.8}$$

Replacing (5.5) in (5.8):

$$q_i = \mu\frac{\sigma-1}{\beta\sigma}\left(\frac{Y_i}{n_i+\phi n_j} + \frac{\phi Y_j}{\phi n_i + n_j}\right).$$

Substituting (5.3) and (5.4) above, we obtain the nominal wage of the skilled workers in each region:

$$W_1 = \frac{\mu}{\alpha_1\sigma}\left(\frac{Y_1}{\frac{1}{\alpha_1}M_1 + \frac{\phi}{\alpha_2}M_2} + \frac{\phi Y_2}{\frac{\phi}{\alpha_1}M_1 + \frac{1}{\alpha_2}M_2}\right),$$

and

$$W_2 = \frac{\mu}{\alpha_1\sigma}\left(\frac{Y_2}{\frac{1}{\alpha_2}M_2 + \frac{\phi}{\alpha_1}M_1} + \frac{\phi Y_1}{\frac{\phi}{\alpha_2}M_2 + \frac{1}{\alpha_1}M_1}\right).$$

Considering $\alpha = \frac{\alpha_1}{\alpha_2}$ as the ratio between the fixed costs, above equations become:

$$W_1 = \frac{\mu}{\sigma}\left(\frac{Y_1}{M_1 + \phi\alpha M_2} + \frac{\phi Y_2}{\phi M_1 + \alpha M_2}\right), \tag{5.9}$$

and

$$W_2 = \frac{\mu}{\sigma}\left(\frac{Y_2}{M_2 + \frac{\phi}{\alpha} M_1} + \frac{\phi Y_1}{\phi M_2 + \frac{1}{\alpha} M_1}\right). \tag{5.10}$$

**Regional income, perfect price index**

The nominal income in region $i$, $Y_i$, is equal to the sum of the incomes in the agricultural and industrial sector:

$$Y_i = \frac{L}{2} + W_i M_i, \text{ for } i = 1, 2. \tag{5.11}$$

The perfect price index of region, $P_i$, aggregates two price indices: the price index of the agricultural sector (normalized to 1) and the price index of the industrial sector, $P_{M_i}$.

We obtain the perfect price index in region $i$, substituting (5.4) into (5.5):

$$P_1 = \frac{\beta\sigma}{\sigma - 1}\left(\frac{1}{\alpha_1}\right)^{\frac{\mu}{1-\sigma}} (M_1 + \phi\alpha M_2)^{\frac{\mu}{1-\sigma}}, \tag{5.12}$$

and

$$P_2 = \frac{\beta\sigma}{\sigma - 1}\left(\frac{1}{\alpha_2}\right)^{\frac{\mu}{1-\sigma}} \left(M_2 + \frac{\phi}{\alpha} M_1\right)^{\frac{\mu}{1-\sigma}}. \tag{5.13}$$

### 5.2.4 Short-run equilibrium

In the short-run, workers are immobile across regions. A short-run equilibrium consists in the equality of supply and demand. Aggregate prices, output and wages are endogenously determined.

Equations (5.9), (5.10), (5.11), (5.12) and (5.13) determine the short-run equilibrium of the model. Here we recall these equations:

$$\begin{aligned}
W_1 &= \frac{\mu}{\sigma}\left(\frac{Y_1}{M_1+\phi\alpha M_2}+\frac{\phi Y_2}{\phi M_1+\alpha M_2}\right),\\
W_2 &= \frac{\mu}{\sigma}\left(\frac{Y_2}{M_2+\frac{\phi}{\alpha}M_1}+\frac{\phi Y_1}{\phi M_2+\frac{1}{\alpha}M_1}\right),\\
Y_1 &= \frac{L}{2}+W_1M_1,\\
Y_2 &= \frac{L}{2}+W_2M_2,\\
P_1 &= \frac{\beta\sigma}{\sigma-1}\left(\frac{1}{\alpha_1}\right)^{\frac{\mu}{1-\sigma}}\left(M_1+\phi\alpha M_2\right)^{\frac{\mu}{1-\sigma}},\\
P_2 &= \frac{\beta\sigma}{\sigma-1}\left(\frac{1}{\alpha_2}\right)^{\frac{\mu}{1-\sigma}}\left(M_2+\frac{\phi}{\alpha}M_1\right)^{\frac{\mu}{1-\sigma}}.
\end{aligned}$$

Simplifying this system of equations[3] we find the ratio between the real wages:

$$\frac{\omega_1}{\omega_2}=\frac{\frac{W_1}{P_1}}{\frac{W_2}{P_2}}=\alpha^{\frac{\mu}{1-\sigma}}\frac{W_1}{W_2}\frac{\left(M_2+\frac{\phi}{\alpha}M_1\right)^{\frac{\mu}{1-\sigma}}}{\left(M_1+\phi\alpha M_2\right)^{\frac{\mu}{1-\sigma}}}, \tag{5.14}$$

[3] see appendix C.1 for detailed calculations.

where:

$$\frac{W_1}{W_2} = \frac{\left(\phi M_2 + \frac{1}{\alpha} M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_2}{M_2 + \frac{\phi}{\alpha} M_1}\right) + \phi M_2 \frac{\left[2\phi M_2 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right](M_1 + \alpha\phi M_2)}{[2\phi M_1 + \alpha(1+\phi^2) M_2]\left(M_2 + \frac{\phi}{\alpha} M_1\right)}}{\frac{\left[2\phi M_2 + \frac{1}{\alpha}(1+\phi^2) M_1\right](M_1 + \phi\alpha M_2)}{[2\phi M_1 + \alpha(1+\phi^2) M_2]\left(M_2 + \frac{\phi}{\alpha} M_1\right)}\left(\phi M_1 + \alpha M_2\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \phi\alpha M_2}\right) + \phi M_1}$$

Defining the share of the skilled workers in region 1 by $f = \frac{M_1}{M_1 + M_2}$, (5.14) becomes:

$$\frac{\omega_1}{\omega_2} = \alpha^{\frac{\mu}{1-\sigma}} \frac{\left[(1-f) + \frac{\phi}{\alpha} f\right]^{\frac{\mu}{1-\sigma}}}{[f + \phi\alpha(1-f)]^{\frac{\mu}{1-\sigma}}} \times$$

$$\times \frac{\left[\phi(1-f) + \frac{1}{\alpha} f\right]\left[\frac{\sigma}{\mu} - \frac{(1-f)}{(1-f) + \frac{\phi}{\alpha} f}\right] + \phi(1-f)\frac{\left[2\phi(1-f) + \frac{1}{\alpha}\left(1+\phi^2\right) f\right][f + \alpha\phi(1-f)]}{[2\phi f + \alpha(1+\phi^2)(1-f)]\left[(1-f) + \frac{\phi}{\alpha} f\right]}}{\frac{\left[2\phi(1-f) + \frac{1}{\alpha}(1+\phi^2) f\right][f + \phi\alpha(1-f)]}{[2\phi f + \alpha(1+\phi^2)(1-f)]\left[(1-f) + \frac{\phi}{\alpha} f\right]}\left[\phi f + \alpha(1-f)\right]\left[\frac{\sigma}{\mu} - \frac{f}{f + \phi\alpha(1-f)}\right] + \phi f}$$

### 5.2.5 Long-run equilibrium

In the short-run equilibrium, real wages are determined by taking as given the amount of skilled workers in each region. The long-run equilibrium is a situation in which, in addition, workers do not wish to migrate. The long-run equilibrium is stable if it is robust to small perturbations of the distribution of workers across regions.

Dispersion is a long-run equilibrium configuration where $0 < f^* < 1$ and regions offer the same real wage. It is stable if a small migration to region $i$ decreases the real wage in region $i$, implying that the initial configuration is reestablished. Formally:

$$f^* \in (0,1), \frac{\omega_1}{\omega_2}|_{f=f^*} \text{ and (for stability) } \left.\frac{\partial(\omega_1/\omega_2)}{\partial f}\right|_{f=f^*} < 0$$

If the equilibrium share of population in region 1 is $f^* = 0.5$, we say that dispersion is symmetric. Otherwise, it is asymmetric.

Concentration is a long-run equilibrium configuration where all skilled workers are concentrated in the region that offers the highest real wage. Concentration in region 1 and in region 2 satisfies, respectively:

$$f^* = 1,\ \omega_1 \geq \omega_2 \text{ and (for stability) } \exists \epsilon > 0 : 1 - \epsilon < f < 1 \Rightarrow \omega_1 \geq \omega_2,$$

$$f^* = 0,\ \omega_1 \leq \omega_2 \text{ and (for stability) } \exists \epsilon > 0 : 0 < f < \epsilon \Rightarrow \omega_1 \leq \omega_2.$$

## 5.3 Results

We consider two distinct situations: (1) symmetric fixed costs, and (2) asymmetric fixed costs.

### 5.3.1 Symmetric fixed costs

**Proposition** 5.3.1. *When the number of skilled workers is the same in each region ($M_1 = M_2$), and regions have equal fixed costs, that is $\alpha = 1$, the real wages are the same in each region.*

*Proof.* When $\alpha = 1$ and $M_1 = M_2$, expression (5.14) becomes:

$$\begin{aligned} \frac{\omega_1}{\omega_2} &= \frac{(\phi M_1 + M_1)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1+\phi M_1}\right) + \phi M_1 \frac{\left[2\phi M_1+\left(1+\phi^2\right)M_1\right](M_1+\phi M_1)}{[2\phi M_1+(1+\phi^2)M_1](M_1+\phi M_1)}}{\frac{[2\phi M_1+(1+\phi^2)M_1](M_1+\phi M_1)}{[2\phi M_1+(1+\phi^2)M_1](M_1+\phi M_1)}(\phi M_1 + M_1)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1+\phi M_2}\right) + \phi M_1} \\ &\times \frac{(M_1 + \phi M_1)^{\frac{\mu}{1-\sigma}}}{(M_1 + \phi M_1)^{\frac{\mu}{1-\sigma}}} \Leftrightarrow \end{aligned}$$

$$\frac{\omega_1}{\omega_2} = \frac{(\phi M_1 + M_1)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1+\phi M_1}\right) + \phi M_1}{(\phi M_1 + M_1)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1+\phi M_2}\right) + \phi M_1} = 1$$

$\square$

This means that symmetric dispersion is an equilibrium for identical fixed costs. This is the case in Forslid and Ottaviano (2003).

### 5.3.2 Asymmetric fixed costs

In this section we study how a technology advantage affects the location of industrial activity. The technology advantage is modelled by having a region in which all industrial firms have lower fixed costs.

Figure 5.1[4] illustrates how a decrease in the fixed costs in region 1, relatively to those in region 2, affects the location of industrial activity. The bold line describes the case in which $\alpha = 1$, while the dotted line illustrates a situation in which $\alpha = 0.8$.

We can see from figure 5.1 that a decrease in $\alpha$, increases the relative real wage of the skilled workers in region 1 (in the short-run, the economy moves from point $E_o$ to $E_1$). In the long-run, there will be asymmetric dispersion, as region 1 will have more workers than region 2 (point $E_2$).

According to proposition 5.3.2, and considering as a starting point a symmetric dispersion equilibrium in which $\alpha = 1$ and $f_m = 0.5$, this outcome occurs for any value of $\phi \in (0, 1)$, $\mu \in (0, 1)$ and $\sigma > 1$.

[4]To plot this figure, we have set $\tau = 0.5$, $\mu = 0.3$ and $\sigma = 3$.

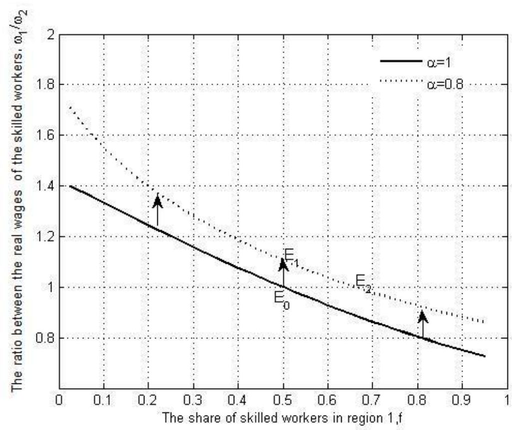


Figure 5.1: Asymmetric dispersion.

**Proposition** 5.3.2. *When the number of skilled workers is the same in each region ($M_1 = M_2$), and the fixed costs in region 1 are lower than in region 2, that is $\alpha < 1$, then the real wages in region 1 are higher than in region 2, for any value of $\phi \in (0,1)$, $\mu \in (0,1)$ and $\sigma > 1$.*

Figure 5.2[5] reveals an interesting result. When a technology advantage is sufficiently high, all industrial activity is concentrated in a single region. For instance, suppose that region 1 obtains a technology advantage ($\alpha = 0.6$). We can see from figure 5.2 an equilibrium configuration in which all industrial activity is concentrated in region 1 (point $E_2$).

According to proposition 5.3.3, concentration is an equilibrium in region 1 when $\frac{\alpha_1}{\alpha_2}$ is lower than a critical value, that is $\frac{\alpha_1}{\alpha_2} \leq \alpha^*$. And according to proposition 5.3.4, concentration is an equilibrium in region 2 when $\frac{\alpha_2}{\alpha_1} \leq \alpha^*$.

**Proposition** 5.3.3. *Concentration of all industrial activity in region 1 is an equilibrium when $\frac{\alpha_1}{\alpha_2} \leq \alpha^*$, with $\alpha^* = \frac{2\phi^{\frac{\mu}{1-\sigma}+1}}{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}}$.*

[5]To plot this figure, we have set $\tau = 0.5$, $\mu = 0.3$ and $\sigma = 3$.

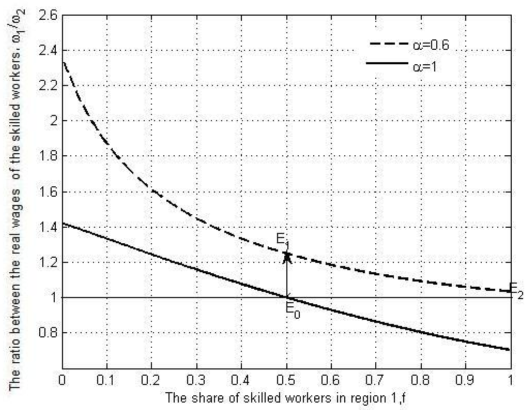


Figure 5.2: Concentration.

**Proposition** 5.3.4. *Concentration of all industrial activity in region 2 is an equilibrium when* $\frac{\alpha_2}{\alpha_1} \leq \alpha^*$ *with* $\alpha^* = \frac{2\phi^{\frac{\mu}{1-\sigma}+1}}{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}}$.

It is straightforward that when $\alpha^* < 1$, concentration only occurs in the region with the lowest fixed cost. But in the case in which, $\alpha^* > 1$, if there is concentration in the region with the highest fixed cost, then concentration may also occur in the other region.

Another interesting issue is the effectiveness of the technology advantage in attracting industrial activity when $\phi$ has different values.

Figure 5.3[6] illustrates a case in which for a given technology advantage in region 1, $\alpha = 0.75$, an increase in $\tau$ from $\tau = 0.5$ to $\tau = 0.75$ changes the spatial distribution of the economic activity. The economy moves from point $E_0$ (asymmetric dispersion) to point $E_1$, where all industrial activity is concentrated in region 1. Thus, a technology advantage is more effective in attracting industrial activity when economic integration is high.

According to proposition 5.3.5, an increase in $\phi$ increases the critical value $\alpha^*$. Therefore,

[6]To plot this figure, we have set $\alpha = 0.75$, $\mu = 0.4$, $\sigma = 3$.

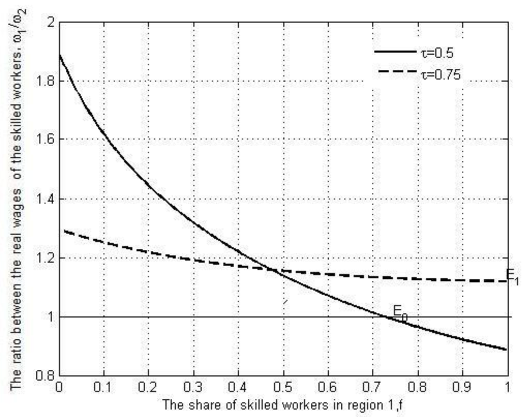


Figure 5.3: Asymmetric dispersion.

for low economic integration (or low $\phi$), only a high technology advantage generates an equilibrium configuration in which all industrial activity is concentrated in a region. But, for high economic integration, a small technology advantage is sufficient to produce the same outcome (concentration).

**Proposition** 5.3.5. *The critical point* $\alpha^* = \frac{2\phi^{\frac{\mu}{1-\sigma}+1}}{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}}$ *is an increasing function of* $\phi$.

We also study how a technology advantage affects the welfare of the population in region 1 and region 2. For this, we assume $f = 0.5$, so that there are as many workers in region 1 as in region 2. The technology advantage is obtained in two ways: (i) a decrease in the home fixed costs; (ii) an increase in the foreign fixed costs.

We suppose that region 1 obtains an technology advantage. Figure 5.4[7] illustrates a case in which region 1 decreases the fixed costs from 1 to 0.8. We can see that a fall in $\alpha_1$, *ceteris paribus*, raises the welfare of the skilled workers in region 1. Figure 5.5 illustrates a situation in which technology advantage in region 1 is obtained by an increase in the

[7]To plot figures 5.4 and 5.5 we have used $\tau = 0.5$, $\mu = 0.4$ and $\sigma = 3$.

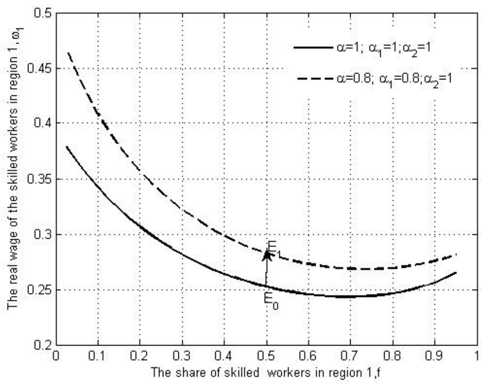


Figure 5.4: Welfare in region 1 with a decrease in $\alpha_1$.

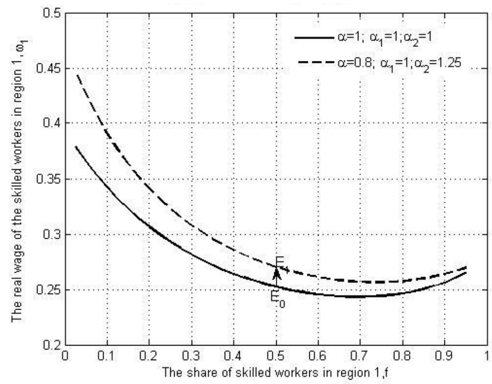


Figure 5.5: Welfare in region 1 with an increase in $\alpha_2$.

fixed costs of region 2. Even in this case, the welfare of the skilled workers in region 1 is improved.

Figure 5.6 and 5.7[8] illustrate how a technology advantage in region 1 affects the welfare of the skilled workers in region 2. We can see from figure 5.6, that a fall in $\alpha_1$, raises the welfare of the skilled workers in region 2. On the other hand, figure 5.7 illustrates a situation in which a technology advantage in region 1 is obtained by an increase in the fixed costs of region 2. In this case, skilled workers in region 2 are worse off.

We conclude that if a technology advantage does not imply an increase in the foreign fixed costs, skilled workers in both regions are better off when $f_m = 0.5$.

Unskilled workers are another group in the economy. They are immobile between regions and their welfare only depends on the price index of the region where they live. In the next figures[9], we study how a technology advantage in region 1 affects the welfare of the unskilled workers in both regions.

[8]To plot these figures we have chosen $\tau = 0.5$, $\mu = 0.4$ and $\sigma = 3$

[9]To plot the following figures we have chosen $\tau = 0.5$, $\mu = 0.4$ and $\sigma = 3$

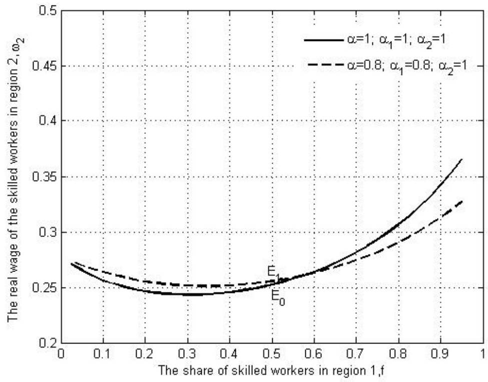


Figure 5.6: Welfare in region 2 with a decrease in $\alpha_1$.

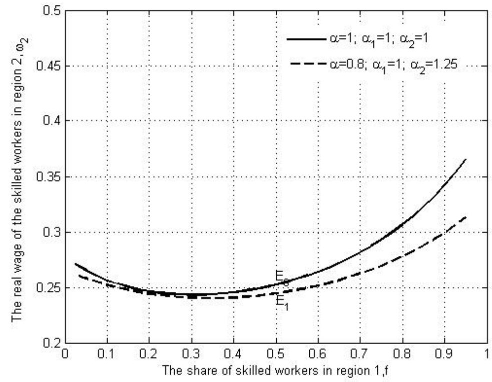


Figure 5.7: Welfare in region 2 with an increase in $\alpha_2$.

We can see from figure 5.8 that a fall, *ceteris paribus*, in the fixed cost of the region 1, $\alpha_1$, increases the real wages of the unskilled workers in that region, whereas, a rise in the foreign fixed cost ($\alpha_2$) decreases the real wages of the unskilled workers in region 1 (figure 5.9).

Doing the same analysis for region 2, figure 5.10 illustrates that a decrease in the fixed cost in region 1, ($\alpha_1$) increases the real wages of the unskilled workers in region 2, but an increase in the fixed cost in region 2 decreases the real wages in that region.

We conclude that a technology advantage in a region increases the welfare of the unskilled workers in both regions if there is a fall in one of the fixed costs. We also conclude, that a technology advantage obtained by a rise in the foreign fixed costs worsens the welfare of the unskilled workers in both regions.

We formulate the following proposition expressing the relation between the fixed costs and the welfare (or the real wages) of the unskilled workers.

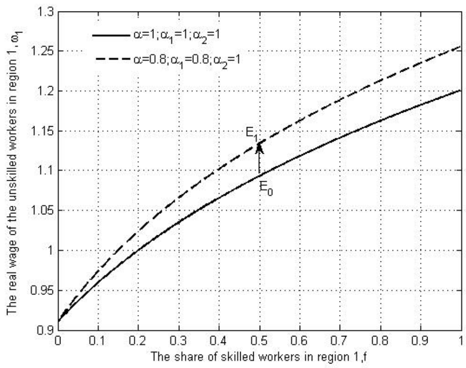


Figure 5.8: The welfare of the unskilled workers in region 1 when $\alpha_1$ decreases.

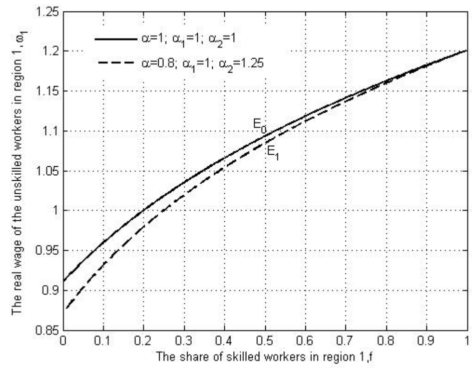


Figure 5.9: The welfare of the unskilled workers in region 1 when $\alpha_2$ increases.

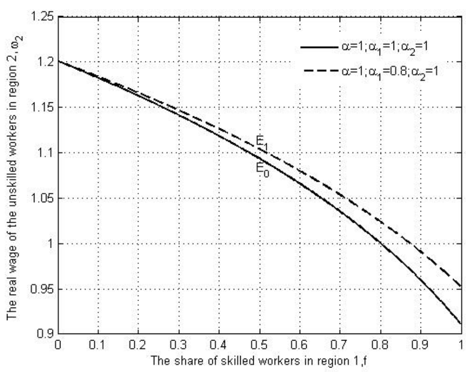


Figure 5.10: The welfare of the unskilled workers in region 2 when $\alpha_1$ decreases.

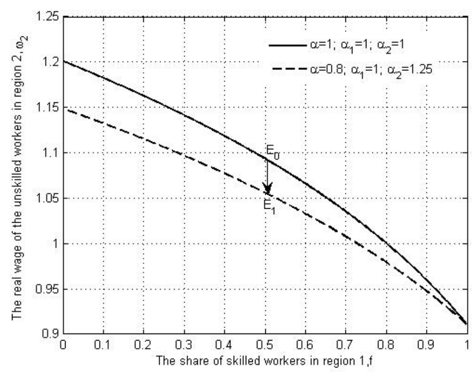


Figure 5.11: The welfare of the unskilled workers in region 2 when $\alpha_2$ increases.

**Proposition** 5.3.6. *The real wage of the unskilled workers in region $i$ is a decreasing function of $\alpha_i$ and $\alpha_j$ for any $\phi \in (0,1)$, $\mu \in (0,1)$ and $\sigma > 1$.*

## 5.4 Concluding remarks

We have extended the footloose entrepreneur model (Forslid and Ottaviano, 2003) including a technology advantage for all industrial firms operating in one of the regions. The technology advantage is modelled by having a region in which all industrial firms have lower fixed costs.

We find that industrial activity in a region is enhanced, *ceteris paribus*, by lower home fixed costs and by higher foreign fixed costs. We also find that a technology advantage is more effective in attracting industrial activity when economic integration is high.

From the point of view of welfare, we conclude that all skilled workers in a region are better off if the home (foreign) fixed costs decreases (increases). But, if a technology advantage does not imply an increase in the foreign fixed costs, skilled workers in both regions are better off.

Relatively to the welfare of the unskilled workers, we conclude that a technology advantage in a region increases the welfare of the unskilled workers in both regions if there is a fall in one of the fixed costs. But, a technology advantage obtained by a rise in the foreign fixed costs worsens the welfare in both regions.

# Appendices

# Appendix C

# Mathematical proofs

## C.1 Short-run equilibrium

Equations (5.9), (5.10), (5.11), (5.12) and (5.13) determine the short-run equilibrium of the model. We recall these equations here.

$$
\begin{aligned}
W_1 &= \frac{\mu}{\sigma}\left(\frac{Y_1}{M_1+\phi\alpha M_2}+\frac{\phi Y_2}{\phi M_1+\alpha M_2}\right), \\
W_2 &= \frac{\mu}{\sigma}\left(\frac{Y_2}{M_2+\frac{\phi}{\alpha}M_1}+\frac{\phi Y_1}{\phi M_2+\frac{1}{\alpha}M_1}\right), \\
Y_1 &= \frac{L}{2}+W_1M_1, \\
Y_2 &= \frac{L}{2}+W_2M_2,
\end{aligned}
$$

$$P_1 = \frac{\beta\sigma}{\sigma-1}\left(\frac{1}{\alpha_1}\right)^{\frac{\mu}{1-\sigma}}(M_1+\phi\alpha M_2)^{\frac{\mu}{1-\sigma}},$$

$$P_2 = \frac{\beta\sigma}{\sigma-1}\left(\frac{1}{\alpha_2}\right)^{\frac{\mu}{1-\sigma}}\left(M_2+\frac{\phi}{\alpha}M_1\right)^{\frac{\mu}{1-\sigma}}.$$

Substituting $Y_1$ and $Y_2$ int $W_1$ and $W_2$, we obtain:

$$W_1 = \frac{\mu}{\sigma}\left(\frac{\frac{L}{2}+W_1M_1}{M_1+\phi\alpha M_2}+\frac{\phi\left(\frac{L}{2}+W_2M_2\right)}{\phi M_1+\alpha M_2}\right),$$

and

$$W_{M_2} = \frac{\mu}{\sigma}\left(\frac{\frac{L}{2}+W_2M_2}{M_2+\frac{\phi}{\alpha}M_1}+\frac{\phi\left(\frac{L}{2}+W_1M_1\right)}{\phi M_2+\frac{1}{\alpha}M_1}\right).$$

By convenience, we denote:

$$a = \frac{L}{2},$$

$$b_1 = \phi M_1+\alpha M_2,$$

$$c_1 = M_1+\phi\alpha M_2,$$

$$b_2 = \phi M_2+\frac{1}{\alpha}M_1,$$

$$c_2 = M_2+\frac{\phi}{\alpha}M_1.$$

Using this notation, the above expressions become:

$$W_1 = \frac{\mu}{\sigma}\left(\frac{a+W_1M_1}{c_1}+\frac{\phi\left(a+W_2M_2\right)}{b_1}\right),$$

and

$$W_2 = \frac{\mu}{\sigma}\left(\frac{a + W_2 M_2}{c_2} + \frac{\phi\,(a + W_1 M_1)}{b_2}\right).$$

After some algebra, we obtain:

$$W_1\left(\frac{\sigma}{\mu} - \frac{M_1}{c_1}\right) = \frac{a}{c_1} + \frac{\phi a}{b_1} + \frac{\phi}{b_1} M_2 W_2 \tag{C.1}$$

and

$$W_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right) = \frac{a}{c_2} + \frac{\phi a}{b_2} + \frac{\phi}{b_2} M_1 W_1. \tag{C.2}$$

Substituting (C.2) into (C.1), we obtain:

$$W_1\left[\frac{\sigma}{\mu} - \frac{M_1}{c_1} - \frac{\phi^2 M_2 M_1}{b_1 b_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)}\right] = \frac{a}{c_1} + \frac{\phi a}{b_1} + \frac{\phi a M_2}{b_1 c_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)} + \frac{\phi^2 a M_2}{b_1 b_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)}.$$

Substituting (C.1) into (C.2), we obtain:

$$W_2\left[\frac{\sigma}{\mu} - \frac{M_2}{c_2} - \frac{\phi^2 M_1 M_2}{b_2 b_1\left(\frac{\sigma}{\mu} - \frac{M_1}{c_1}\right)}\right] = \frac{a}{c_2} + \frac{\phi a}{b_2} + \frac{\phi a M_1}{b_2 c_1\left(\frac{\sigma}{\mu} - \frac{M_1}{c_1}\right)} + \frac{\phi^2 a M_1}{b_2 b_1\left(\frac{\sigma}{\mu} - \frac{M_1}{c_1}\right)}.$$

After some manipulation, we have that:

$$\begin{aligned} W_1 \quad &= \quad \left[\frac{b_1 b_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)}{\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{c_1}\right) b_1 b_2 - \phi M_2 M_1}\right] \times \\ &\times \quad \left[\frac{a}{c_1} + \frac{\phi a}{b_1} + \frac{\phi a M_2}{b_1 c_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)} + \frac{\phi^2 a M_2}{b_1 b_2\left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)}\right], \end{aligned}$$

and

$$W_2 = \left[\frac{b_2 b_1\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)}{\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)\left(\frac{\sigma}{\mu}-\frac{M_2}{c_2}\right)b_2 b_1-\phi M_1 M_2}\right]\times$$

$$\times \left[\frac{a}{c_2}+\frac{\phi a}{b_2}+\frac{\phi a M_1}{b_2 c_1\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)}+\frac{\phi^2 a M_1}{b_2 b_1\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)}\right].$$

Since there is a common term, the above equations become:

$$W_1 = \frac{H}{b_1}\left[\frac{ab_1\left(\frac{\sigma}{\mu}-\frac{M_2}{c_2}\right)}{c_1}+\left(\frac{\sigma}{\mu}-\frac{M_2}{c_2}\right)\phi a+\frac{\phi a M_2}{c_2}+\frac{\phi^2 a M_2}{b_2}\right],$$

and

$$W_2 = \frac{H}{b_2}\left[\frac{ab_2\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)}{c_2}+\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)\phi a+\frac{\phi a M_1}{c_1}+\frac{\phi^2 a M_1}{b_1}\right],$$

with

$$H = \frac{b_2 b_1}{\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)\left(\frac{\sigma}{\mu}-\frac{M_2}{c_2}\right)b_2 b_1-\phi M_1 M_2}.$$

After some algebra, the nominal wages of the skilled workers in region 1 and 2 are:

$$W_1 = \frac{H}{b_1}\left[\frac{a\left(\frac{\sigma}{\mu}-\frac{M_2}{c_2}\right)(b_1+\phi c_1)}{c_1}+\frac{\phi M_2 a\,(b_2+\phi c_2)}{c_2 b_2}\right]$$

and

$$W_2 = \frac{H}{b_2}\left[\frac{a\left(\frac{\sigma}{\mu}-\frac{M_1}{c_1}\right)(b_2+\phi c_2)}{c_2}+\frac{\phi M_1 a\,(b_1+\phi c_1)}{c_1 b_1}\right].$$

The ratio of the real wages is:

$$\frac{\omega_1}{\omega_2} = \frac{\frac{W_1}{P_1}}{\frac{W_2}{P_2}} = \alpha^{\frac{\mu}{1-\sigma}} \frac{W_1}{W_2} \frac{\left(M_2 + \frac{\phi}{\alpha} M_1\right)^{\frac{\mu}{1-\sigma}}}{\left(M_1 + \phi\alpha M_2\right)^{\frac{\mu}{1-\sigma}}}, \tag{C.3}$$

where:

$$\frac{W_1}{W_2} = \frac{c_2 b_2 \left(\frac{\sigma}{\mu} - \frac{M_2}{c_2}\right)(b_1 + \phi c_1) + \phi\,(b_2 + \phi c_2)\, c_1 M_2}{c_1 b_1 \left(\frac{\sigma}{\mu} - \frac{M_1}{c_1}\right)(b_2 + \phi c_2) + \phi\,(b_1 + \phi c_1)\, c_2 M_1} \Leftrightarrow$$

$$\frac{W_1}{W_2} = \frac{\left(\phi M_2 + \frac{1}{\alpha} M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_2}{M_2 + \frac{\phi}{\alpha} M_1}\right) + \phi M_2 \frac{\left[2\phi M_2 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right](M_1 + \alpha\phi M_2)}{\left[2\phi M_1 + \alpha(1+\phi^2) M_2\right]\left(M_2 + \frac{\phi}{\alpha} M_1\right)}}{\frac{\left[2\phi M_2 + \frac{1}{\alpha}(1+\phi^2) M_1\right](M_1 + \phi\alpha M_2)}{\left[2\phi M_1 + \alpha(1+\phi^2) M_2\right]\left(M_2 + \frac{\phi}{\alpha} M_1\right)}\left(\phi M_1 + \alpha M_2\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \phi\alpha M_2}\right) + \phi M_1}.$$

Defining the share of the skilled workers in region 1 by $f = \frac{M_1}{M_1+M_2}$, the above equation becomes:

$$\begin{aligned}
\frac{\omega_1}{\omega_2} &= \alpha^{\frac{\mu}{1-\sigma}} \frac{\left[(1-f) + \frac{\phi}{\alpha} f\right]^{\frac{\mu}{1-\sigma}}}{\left[f + \phi\alpha(1-f)\right]^{\frac{\mu}{1-\sigma}}} \times \\
&\times \frac{\left[\phi(1-f) + \frac{1}{\alpha} f\right]\left[\frac{\sigma}{\mu} - \frac{(1-f)}{(1-f) + \frac{\phi}{\alpha} f}\right] + \phi(1-f) \frac{\left[2\phi(1-f) + \frac{1}{\alpha}\left(1+\phi^2\right) f\right]\left[f + \alpha\phi(1-f)\right]}{\left[2\phi f + \alpha(1+\phi^2)(1-f)\right]\left[(1-f) + \frac{\phi}{\alpha} f\right]}}{\frac{\left[2\phi(1-f) + \frac{1}{\alpha}(1+\phi^2) f\right]\left[f + \phi\alpha(1-f)\right]}{\left[2\phi f + \alpha(1+\phi^2)(1-f)\right]\left[(1-f) + \frac{\phi}{\alpha} f\right]}\left[\phi f + \alpha(1-f)\right]\left[\frac{\sigma}{\mu} - \frac{f}{f + \phi\alpha(1-f)}\right] + \phi f}.
\end{aligned}$$

## C.2 Proof of propositions

### C.2.1 Proposition 5.4.2

*Proof.* We want to prove that $\frac{\omega_1}{\omega_2} > 1$, when $M_1 = M_2$ and $\alpha < 1$. From (C.3), we have that:

$$\frac{\omega_1}{\omega_2} = \frac{W_1}{W_2} \frac{P_2}{P_1}.$$

It is sufficient to show that both $\frac{W_1}{W_2}$ and $\frac{P_2}{P_1}$ are bigger than 1.

First, we show that $\frac{P_2}{P_1} > 1$. Notice that for $M_1 = M_2$:

$$\frac{P_2}{P_1} = \alpha^{\frac{\mu}{1-\sigma}} \frac{\left(M_1 + \frac{\phi}{\alpha} M_1\right)^{\frac{\mu}{1-\sigma}}}{\left(M_1 + \phi\alpha M_1\right)^{\frac{\mu}{1-\sigma}}} \Leftrightarrow$$

$$\frac{P_2}{P_1} = \left(\frac{\alpha M_1 + \phi M_1}{M_1 + \phi\alpha M_1}\right)^{\frac{\mu}{1-\sigma}} = \left(\frac{\alpha + \phi}{1 + \alpha\phi}\right)^{\frac{\mu}{1-\sigma}}.$$

For $\frac{P_2}{P_1}$ to be bigger than 1 we must have $\frac{\alpha+\phi}{1+\alpha\phi} < 1$. This occurs for:

$$\alpha + \phi < 1 + \alpha\phi \Leftrightarrow (1-\phi)\,\alpha < 1 - \phi \Leftrightarrow \alpha < 1 \text{ and } \phi < 1.$$

Second, we show that $\frac{W_1}{W_2} > 1$ when $\alpha < 1$.

Notice that for $M_1 = M_2$:

$$\frac{W_1}{W_2} = \frac{\left(\phi M_1 + \frac{1}{\alpha} M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \frac{\phi}{\alpha} M_1}\right) + \phi M_1 \frac{\left[2\phi M_1 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right]\left(M_1 + \alpha\phi M_1\right)}{\left[2\phi M_1 + \alpha\left(1+\phi^2\right) M_1\right]\left(M_1 + \frac{\phi}{\alpha} M_1\right)}}{\frac{\left[2\phi M_1 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right]\left(M_1 + \phi\alpha M_1\right)}{\left[2\phi M_1 + \alpha\left(1+\phi^2\right) M_1\right]\left(M_1 + \frac{\phi}{\alpha} M_1\right)}\left(\phi M_1 + \alpha M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \phi\alpha M_1}\right) + \phi M_1} \Leftrightarrow$$

$$\frac{W_1}{W_2} = \frac{\left(\phi M_1 + \frac{1}{\alpha} M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \frac{\phi}{\alpha} M_1}\right) + \phi M_1 A}{A\left(\phi M_1 + \alpha M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \phi\alpha M_1}\right) + \phi M_1}$$

where:

$$A = \frac{\left[2\phi M_1 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right]\left(M_1 + \alpha\phi M_1\right)}{\left[2\phi M_1 + \alpha\left(1+\phi^2\right) M_1\right]\left(M_1 + \frac{\phi}{\alpha} M_1\right)}.$$

Observe that $\frac{W_1}{W_2} > 1$ since:

$$\left(\phi M_1 + \frac{1}{\alpha} M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \frac{\phi}{\alpha} M_1}\right) + \phi M_1 A >$$

$$A\left(\phi M_1 + \alpha M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \phi\alpha M_1}\right) + \phi M_1 \Leftrightarrow$$

$$\Leftrightarrow \quad \left(\phi M_1 + \frac{1}{\alpha} M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \frac{\phi}{\alpha} M_1}\right)$$

$$- \quad A\left(\phi M_1 + \alpha M_1\right)\left(\frac{\sigma}{\mu} - \frac{M_1}{M_1 + \phi\alpha M_1}\right) + \phi M_1 (A-1) > 0.$$

It is clear that above condition is satisfied if:

- $A = \dfrac{\left[2\phi M_1 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right]\left(M_1 + \alpha\phi M_1\right)}{\left[2\phi M_1 + \alpha\left(1+\phi^2\right) M_1\right]\left(M_1 + \frac{\phi}{\alpha} M_1\right)} > 1.$

- $\dfrac{\phi M_1 + \frac{1}{\alpha} M_1}{\phi M_1 + \alpha M_1} > A.$

Concerning the first item, we have that:

$$\frac{\left[2\phi M_1 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right]\left(M_1 + \alpha\phi M_1\right)}{\left[2\phi M_1 + \alpha\left(1+\phi^2\right) M_1\right]\left(M_1 + \frac{\phi}{\alpha} M_1\right)} > 1 \Leftrightarrow$$

$$\Leftrightarrow \quad 2\phi^2 M_1^2 + 2\phi^2 \alpha M_1^2 + \frac{1}{\alpha}(1+\phi^2) M_1^2 + (1+\phi^2) M_1^2 \phi >$$

$$2\phi^2 M_1^2 + \frac{1}{\alpha} 2\phi^2 M_1^2 + \alpha(1+\phi^2) M_1^2 + (1+\phi^2) M_1^2 \phi \Leftrightarrow$$

$$\Leftrightarrow \quad 2\phi^2 \alpha M_1^2 - \frac{1}{\alpha} 2\phi^2 M_1^2 > \alpha(1+\phi^2) M_1^2 - \frac{1}{\alpha}(1+\phi^2) M_1^2 \Leftrightarrow$$

$$\Leftrightarrow \quad 2\phi^2 M_1^2 \left(\alpha - \frac{1}{\alpha}\right) > (1+\phi^2) M_1^2 \left(\alpha - \frac{1}{\alpha}\right)$$

$$\Leftrightarrow \quad M_1^2 \left(\alpha - \frac{1}{\alpha}\right)\left(\phi^2 - 1\right) > 0.$$

We conclude that this condition is verified if $\alpha < 1$.

Concerning the second condition, we have that:

$$\frac{\phi M_1 + \frac{1}{\alpha} M_1}{\phi M_1 + \alpha M_1} > \frac{\left[2\phi M_1 + \frac{1}{\alpha}\left(1+\phi^2\right) M_1\right]\left(M_1 + \alpha\phi M_1\right)}{\left[2\phi M_1 + \alpha\left(1+\phi^2\right) M_1\right]\left(M_1 + \frac{\phi}{\alpha} M_1\right)} \Leftrightarrow$$

$$\Leftrightarrow \quad \frac{\phi + \frac{1}{\alpha}}{\phi + \alpha} > \frac{\left[2\phi + \frac{1}{\alpha}\left(1+\phi^2\right)\right]\left(1 + \alpha\phi\right)}{\left[2\phi + \alpha\left(1+\phi^2\right)\right]\left(1 + \frac{\phi}{\alpha}\right)} \Leftrightarrow$$

$$\Leftrightarrow \quad \frac{\left(\phi + \frac{1}{\alpha}\right)\left(1 + \frac{\phi}{\alpha}\right)}{\left(\phi + \alpha\right)\left(1 + \alpha\phi\right)} > \frac{\left[2\phi + \frac{1}{\alpha}\left(1+\phi^2\right)\right]}{\left[2\phi + \alpha\left(1+\phi^2\right)\right]}.$$

By convenience above condition is rewritten as:

$$\frac{A}{B} > \frac{C}{D}.$$

It is satisfied if:

$$A > C \Leftrightarrow \left(\phi + \frac{1}{\alpha}\right)\left(1 + \frac{\phi}{\alpha}\right) > \left[2\phi + \frac{1}{\alpha}\left(1+\phi^2\right)\right] \text{ and}$$

$$B < D \Leftrightarrow \left(\phi + \alpha\right)\left(1 + \alpha\phi\right) < \left[2\phi + \alpha\left(1+\phi^2\right)\right].$$

Notice that:

$$A > C \Leftrightarrow$$

$$\Leftrightarrow \phi + \frac{\phi^2}{\alpha} + \frac{1}{\alpha} + \frac{\phi}{\alpha^2} > \left[2\phi + \frac{1}{\alpha}\left(1+\phi^2\right)\right] \Leftrightarrow$$

$$\Leftrightarrow \phi\left(1 + \frac{1}{\alpha^2}\right) + \frac{1}{\alpha}\left(1+\phi^2\right) > 2\phi + \frac{1}{\alpha}\left(1+\phi^2\right) \Leftrightarrow$$

$$\Leftrightarrow 1 + \frac{1}{\alpha^2} > 2 \Leftrightarrow \alpha^2 < 1.$$

Observe that:

$$B < D \Leftrightarrow$$

$$\phi + \alpha\phi^2 + \alpha + \alpha^2\phi < 2\phi + \alpha(1+\phi^2) \Leftrightarrow$$

$$\Leftrightarrow \phi + \alpha^2\phi + \alpha\left(1+\phi^2\right) < 2\phi + \alpha(1+\phi^2) \Leftrightarrow$$

$$\Leftrightarrow \phi\left(1+\alpha^2\right) < 2\phi \Leftrightarrow \alpha^2 < 1.$$

We conclude that $\frac{W_1}{W_2} > 1$ since $\alpha < 1$, and we finish the proof.

□

### C.2.2 Proposition 5.4.3

*Proof.* Recall that $\alpha = \frac{\alpha_1}{\alpha_2}$.

When $f = 1$, the ratio between the real wages becomes:

$$\frac{\omega_1}{\omega_2} = \alpha^{\frac{\mu}{1-\sigma}} \left(\frac{\phi}{\alpha}\right)^{\frac{\mu}{1-\sigma}} \frac{\frac{1}{\alpha}\frac{\sigma}{\mu}}{\frac{\frac{1}{\alpha}(1+\phi^2)\phi\left(\frac{\sigma}{\mu}-1\right)}{\frac{2\phi^2}{\alpha}} + \phi} = \frac{2\phi^{\frac{\mu}{1-\sigma}+2}\frac{\sigma}{\alpha\mu}}{(1+\phi^2)\,\phi\left(\frac{\sigma}{\mu}-1\right) + 2\phi^3} =$$

$$= \frac{2\phi^{\frac{\mu}{1-\sigma}+1}\frac{\sigma}{\alpha\mu}}{(1+\phi^2)\left(\frac{\sigma}{\mu}-1\right) + 2\phi^2}.$$

Concentration is an equilibrium in region 1 if $\omega_1 \geq \omega_2$, that is:

$$2\phi^{\frac{\mu}{1-\sigma}+1}\frac{\sigma}{\alpha\mu} \geq \left(1+\phi^2\right)\left(\frac{\sigma}{\mu}-1\right) + 2\phi^2 \Leftrightarrow$$

$$\Leftrightarrow \quad \frac{\alpha_1}{\alpha_2} \leq \frac{2\phi^{\frac{\mu}{1-\sigma}+1}}{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}}.$$

□

### C.2.3 Proposition 5.4.4

*Proof.* Recall that $\alpha = \frac{\alpha_1}{\alpha_2}$.

When $f = 0$, the ratio between the real wages becomes:

$$\frac{\omega_1}{\omega_2} \quad = \quad \left(\frac{1}{\phi}\right)^{\frac{\mu}{1-\sigma}} \frac{\phi\left(\frac{\sigma}{\mu}-1\right)+\phi\frac{2\phi^2}{(1+\phi^2)}}{\frac{2\phi^2}{(1+\phi^2)}\frac{\alpha\sigma}{\mu}} = \frac{\left(1+\phi^2\right)\left(\frac{\sigma}{\mu}-1\right)+2\phi^2}{2\phi^{\frac{\mu}{1-\sigma}+1}\frac{\alpha\sigma}{\mu}}.$$

Concentration of all industrial activity in region 2 is an equilibrium if $\omega_1 \leq \omega_2$:

$$\left(1+\phi^2\right)\left(\frac{\sigma}{\mu}-1\right)+2\phi^2 \leq 2\phi^{\frac{\mu}{1-\sigma}+1}\frac{\alpha\sigma}{\mu} \Leftrightarrow \alpha \geq \frac{\left(1+\phi^2\right)\left(\frac{\sigma}{\mu}-1\right)+2\phi^2}{2\phi^{\frac{\mu}{1-\sigma}+1}\frac{\sigma}{\mu}} \Leftrightarrow$$

$$\Leftrightarrow \alpha \geq \frac{\phi^2\left(\frac{\sigma}{\mu}+1\right)+\frac{\sigma}{\mu}-1}{2\phi^{\frac{\mu}{1-\sigma}+1}\frac{\sigma}{\mu}} \Leftrightarrow \alpha \geq \frac{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}}{2\phi^{\frac{\mu}{1-\sigma}+1}} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\frac{\alpha_1}{\alpha_2}} \leq \frac{2\phi^{\frac{\mu}{1-\sigma}+1}}{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}} \Leftrightarrow \frac{\alpha_2}{\alpha_1} \leq \frac{2\phi^{\frac{\mu}{1-\sigma}+1}}{\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}}.$$

□

### C.2.4 Proposition 5.4.5

*Proof.* We want to prove that $\frac{\partial\alpha}{\partial\phi} > 0$. □

Notice that:

$$\frac{\partial\alpha}{\partial\phi} > 0 \Leftrightarrow \frac{\frac{\partial}{\partial\phi}\left(2\phi^{\frac{\mu}{1-\sigma}+1}\right)\left[\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}\right]-\frac{\partial}{\partial\phi}\left[\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}\right]2\phi^{\frac{\mu}{1-\sigma}+1}}{\left[\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}\right]^2} > 0.$$

Since the numerator is positive, the above condition is satisfied if:

$$2\left(\frac{\mu}{1-\sigma}+1\right)\phi^{\frac{\mu}{1-\sigma}}\left[\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}\right]-2\phi\left(1+\frac{\mu}{\sigma}\right)2\phi^{\frac{\mu}{1-\sigma}+1} > 0 \Leftrightarrow$$

$$\Leftrightarrow 2\phi^{\frac{\mu}{1-\sigma}}\left\{\left(\frac{\mu}{1-\sigma}+1\right)\left[\phi^2\left(1+\frac{\mu}{\sigma}\right)+1-\frac{\mu}{\sigma}\right]-2\phi^2\left(1+\frac{\mu}{\sigma}\right)\right\} > 0.$$

Since $2\phi^{\frac{\mu}{1-\sigma}}$ is positive, the above condition is satisfied if:

$$\phi^2\left(1+\frac{\mu}{\sigma}\right)\left(\frac{\mu}{1-\sigma}-1\right)+\left(\frac{\mu}{1-\sigma}+1\right)\left(1-\frac{\mu}{\sigma}\right) > 0 \Leftrightarrow$$

$$\Leftrightarrow \quad \phi < \sqrt{\frac{\left(\frac{\mu}{1-\sigma}+1\right)\left(\frac{\mu}{\sigma}-1\right)}{\left(1+\frac{\mu}{\sigma}\right)\left(\frac{\mu}{1-\sigma}-1\right)}}.$$

Observe that condition is satisfied if the square root is higher than 1, that is:

$$\left(\frac{\mu}{1-\sigma}+1\right)\left(\frac{\mu}{\sigma}-1\right) > \left(1+\frac{\mu}{\sigma}\right)\left(\frac{\mu}{1-\sigma}-1\right) \Leftrightarrow \frac{\mu}{\sigma} > \frac{\mu}{1-\sigma}.$$

Since the right-hand side is negative, we finish the proof.

### C.2.5 Proposition 5.4.6

*Proof.* The real wage of an unskilled worker in region 1 is:

$$\omega_1^L = \frac{W_1^L}{P_1}.$$

We want to prove that $\frac{d\omega_1^L}{d\alpha_1} < 0$ and $\frac{d\omega_1^L}{d\alpha_2} < 0$:

$$\frac{\partial \omega_1^L}{\partial \alpha_1} = \frac{1}{P_1^2}\left(\frac{\partial W_1^L}{\partial \alpha_1}P_1 - \frac{\partial P_1}{\partial \alpha_1}W_1^L\right).$$

and

$$\frac{\partial \omega_1^L}{\partial \alpha_2} = \frac{1}{P_1^2}\left(\frac{\partial W_1^L}{\partial \alpha_2}P_1 - \frac{\partial P_1}{\partial \alpha_2}W_1^L\right).$$

Since $W_1^L = 1$, above expressions become:

$$\frac{\partial \omega_1^L}{\partial \alpha_1} = -\frac{1}{P_1^2}\frac{\partial P_1}{\partial \alpha_1}$$

and

$$\frac{\partial \omega_1^L}{\partial \alpha_2} = -\frac{1}{P_1^2}\frac{\partial P_1}{d\alpha_2}.$$

We have:

$$\frac{\partial P_1}{\partial \alpha_1} = \frac{\mu}{1-\sigma}\frac{\beta\sigma}{\sigma-1}\left(\frac{M_1}{\alpha_1} + \frac{\phi}{\alpha_2}M_2\right)^{\frac{\mu}{1-\sigma}-1}\left(-\frac{M_1}{\alpha_1^2}\right) > 0$$

and

$$\frac{\partial P_1}{\partial \alpha_2} = \frac{\mu}{1-\sigma}\frac{\beta\sigma}{\sigma-1}\left(\frac{M_1}{\alpha_1} + \frac{\phi}{\alpha_2}M_2\right)^{\frac{\mu}{1-\sigma}-1}\left(-\frac{\phi M_2}{\alpha_2^2}\right) > 0$$

□

and the results follows.

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