

Faculdade de Economia da Universidade do Porto

Essays on Industry Dynamics and on Endogenous Economic Growth

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Aos meus Pais,
À minha Mulher,
Aos meus dois Filhos.

Nota Biográfica

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Sumário

A presente tese está organizada em seis ensaios independentes, ligados pelo tema do crescimento económico de longo prazo e dinâmica industrial.

O primeiro ensaio sistematiza a evidência empírica dispersa que relaciona a dinâmica das firmas com a dinâmica macroeconómica, tanto de curto como de longo prazo. Apresenta-se também uma breve revisão da literatura teórica mais recente sobre o tema, procurando relacionar os seus resultados com a referida evidência empírica.

O segundo ensaio estuda um modelo de *tournament* com I&D vertical e horizontal sob uma especificação *lab-equipment*. Deste quadro resultam efeitos fortes de composição inter-I&D que permitem explicar (qualitativamente) a conhecida correlação empírica negativa ou não significativa entre a intensidade agregada de I&D e a taxa de crescimento económico.

O ensaio seguinte propõe um mecanismo analítico capaz de explicar a distribuição empírica das firmas e da produção entre sectores *high* e *low-tech*. Aí se desenvolve um modelo de *directed technical change* endógeno que junta os mecanismos de expansão de variedades e de *quality ladders*, no quadro de uma especificação *lab-equipment* e sob diferentes regimes de efeitos de escala.

O quarto ensaio estuda o impacto dos *animal spirits* na estrutura industrial, nos níveis de consumo e no crescimento económico, no contexto de um modelo de *tournament* com I&D vertical e horizontal. O modelo prevê um efeito positivo dos *animal spirits* no nível do consumo per capita em *balanced growth path* (BGP), mas um impacto nulo no crescimento económico. Porém, uma economia sujeita a choques de expectativas apresentará uma velocidade de convergência para o BGP variável no tempo.

O quinto ensaio fornece uma justificação teórica para o Modelo de Inventário Permanente usado para estimar o *stock* de conhecimento tecnológico, ao mostrar que a acumulação deste *stock* pode ser representada como um processo dinâmico com taxa de depreciação endógena, constante em BGP.

O último ensaio estuda uma distribuição não degenerada dos preços de um bem homogéneo, no quadro de um modelo de *directed technical change* endógeno. Deriva-se analiticamente uma função densidade de probabilidade e mostra-se a sua relação com os parâmetros tecnológicos e de inovação do modelo.

Summary

This thesis is organised into six independent essays, linked through the theme of long-run aggregate growth and industry dynamics.

The first essay brings together, in a systematised fashion, the scattered empirical evidence relating firm dynamics with both short- and long-run macroeconomic dynamics. The essay also gives a brief account of the most recent theoretical literature on firm dynamics and macroeconomic dynamics, and tries to relate it to the empirical findings.

The second essay develops a single-technology tournament model of horizontal and vertical R&D under a lab-equipment specification. This setup gives rise to strong inter-R&D composition effects that offer a (qualitative) explanation for the well-known negative or insignificant empirical correlation between aggregate R&D intensity and both firm size and economic growth.

The next essay proposes an analytical mechanism to explain the data on the distribution of firms and production across high- and low-tech sectors. It builds a model of endogenous directed technical change that merges the expanding-variety with the quality-ladders mechanism, within a lab-equipment specification and under distinct regimes of scale effects.

The fourth essay studies the effect of animal spirits on the industrial structure, consumption level and economic growth in a tournament model of vertical and horizontal R&D. The model predicts a positive effect of animal spirits on the balanced-growth-path (BGP) level of per-capita consumption without impacting on economic growth. However, an economy that is subject to expectations shocks converges to BGP at a time-varying speed.

The fifth essay gives a theoretical background to the Perpetual Inventory Model used to estimate the technological-knowledge stock, by showing that the technological-knowledge accumulation can be represented as a dynamic process with an endogenous depreciation rate, which remains stable along the BGP.

The last essay studies a non-degenerate price distribution for the homogeneous good within a model of endogenous directed technical change. A probability density function is analytically derived and shown to be related to the technology and innovation parameters of the model.

Contents

1. Introduction	15
2. Stylised Facts and Other Empirical Evidence on Firm Dynamics, Business Cycle and Growth	20
2.1. Introduction	21
2.2. Stylised facts from firm-level data	22
2.3. Firm dynamics and the business cycle	23
2.4. Firm dynamics and economic growth	26
2.4.1. Industry life cycle	26
2.4.2. Time series evidence	27
2.4.3. Cross-country evidence	30
2.5. Theoretical literature on firm dynamics	33
2.5.1. Business cycle models	33
2.5.2. Economic growth models	35
2.6. Concluding remarks	38
3. Growth and Firm Dynamics with Horizontal and Vertical R&D	39
3.1. Introduction	40

Contents

3.2. The model	45
3.2.1. Production and price decisions	46
3.2.2. R&D	49
3.2.2.1. Vertical R&D free-entry and dynamic arbitrage conditions .	50
3.2.2.2. Horizontal R&D free-entry and dynamic arbitrage conditions	52
3.2.2.3. Consistency arbitrage condition	54
3.2.3. Households	56
3.2.4. Macroeconomic aggregation and equilibrium innovation rate	57
3.2.5. The dynamic general equilibrium	59
3.3. Equilibrium dynamics	60
3.3.1. The balanced-growth path	60
3.3.2. Aggregate transitional dynamics	63
3.4. Comparative-statics analysis	68
3.5. Conclusion	71
4. Directed Technical Change, Scale Effects and Industrial Structure	73
4.1. Introduction	74
4.2. The model	79
4.2.1. Production and price decisions	80
4.2.2. R&D	84
4.2.2.1. Vertical R&D free-entry and dynamic arbitrage conditions .	85
4.2.2.2. Horizontal R&D free-entry and dynamic arbitrage conditions	88
4.2.2.3. Consistency arbitrage conditions	90
4.2.3. Households	92
4.2.4. Macroeconomic aggregation and equilibrium innovation rates	93
4.2.5. The dynamic general equilibrium	95

4.2.6. The balanced-growth path	96
4.3. Labour endowment, scale effects and industrial structure	100
4.3.1. Number of firms, production and firm size	100
4.3.1.1. “Cross-country” perspective	100
4.3.1.2. “Intra-country” perspective	103
4.3.2. Concentration index	104
4.3.2.1. Labour endowment, scale effects and concentration	104
4.3.2.2. Dynamic entry costs, concentration and aggregate growth	108
4.3.3. Regime of scale effects implied by the data	110
4.4. Concluding remarks	111
5. Animal Spirits and the Composition of Innovation in a Lab-Equipment R&D	
Model with Transition	113
5.1. Introduction	114
5.2. Related literature on SSE	119
5.3. The benchmark model	121
5.3.1. Households	121
5.3.2. Production and price decisions	122
5.3.3. R&D	125
5.3.4. The symmetric general-equilibrium BGP	129
5.4. The model with asymmetric equilibria	131
5.4.1. R&D	131
5.4.2. The asymmetric general equilibrium BGP	135
5.5. Stationary sunspot equilibria	138
5.6. Transitional dynamics under animal spirits	143
5.7. Concluding remarks	148

6. Technological-Knowledge Dynamics in Lab-Equipment Models of Quality Ladders	151
6.1. Introduction	152
6.2. The quality-ladders model	154
6.2.1. Production	154
6.2.2. Vertical R&D	155
6.2.3. Households	156
6.3. Aggregate resource constraint and firms' market-value dynamics	157
6.4. Technological-knowledge dynamics	160
6.5. Conclusion	162
7. Equilibrium Price Distribution with Directed Technical Change	164
7.1. Introduction	165
7.2. Model	166
7.3. Balanced-growth path	169
7.4. Equilibrium price distribution	170
7.5. Comparative statics	173
7.6. Conclusion	174
Bibliography	175
A. Appendix to Chapter 3	188
A.1. Derivation of equation (3.31)	188
A.2. Necessary conditions for the existence of a BGP	189
A.3. Proof of Proposition 3.1	191
A.4. Proof of Proposition 3.2	192
A.5. Proof of Proposition 3.3	192

B. Appendix to Chapter 4	195
B.1. Data on high- and low-tech sectors	195
B.2. Derivation of equation (4.32)	197
B.3. Detrended dynamic system and interior steady state	200
B.4. Proxy for quality-adjusted production	204
B.5. Concentration index	206
B.6. The impact of the dynamic horizontal-entry costs in the industrial structure	207
B.7. Schumpeterian hypothesis	210
C. Appendix to Chapter 5	214
C.1. Model with static decreasing returns to horizontal R&D	214
C.2. Derivation of the aggregate resource constraint with “waves of enthusiasm” .	216
C.3. Existence and uniqueness of $A_p > 1$	218
C.4. Transitional dynamics	219

List of Figures

3.1.	Phase diagram in the detrended variables (x, z) for the piecewise-smooth system (3.40a)-(3.40b). Curves $\dot{x} = 0$ and $\dot{z} = 0$ are the isoclines and curve $\mathbf{W}^s$ is the stable manifold, which is the only equilibrium trajectory.	64
3.2.	Transitional dynamics of N , Q and average firm size in stationarised terms, when $x(0) > x^*$	67
4.1.	Relative number of firms and relative average firm size under the three regimes of scale effects.	102
4.2.	Concentration index in the case of $0 \leq \epsilon < \bar{\epsilon}_1$ or $\epsilon > 1$ (panel (a)), and in the case of $\bar{\epsilon}_1 < \epsilon < 1$ (panel (b)).	107
5.1.	Time-path for x under the two regimes: $\theta = 1$ (pointed line) and $\theta > 1$ (solid line).	146
5.2.	Transitional dynamics of x (left panel) and z (right panel) with an unanticipated expectations shock at t_1	147
7.1.	Probability density function for $P_L = P_H = P$ (left) and $P_L \neq P_H$ (right) .	172

List of Figures

B.1. Relative number of firms, relative production, relative average firm size, relative R&D intensity, concentration index, and selected correlations. The data concerns the 1995-2005 average for 23 European countries.	196
B.2. Concentration index and “market shares”	208
B.3. Correlation between relative R&D intensity and relative firm size. The data concerns the 1995-2005 average for nine European countries, and is available from the on-line Eurostat database (link at http://epp.eurostat.ec.europa.eu).	213
C.1. Phase diagram in the detrended variables (x, z) for the system (C.14a)-(C.14b), with $\theta = 1$ (lower intersection) and $\theta > 1$ (upper intersection). Curves $\dot{x} = 0$, $\dot{z} = 0$, $\dot{x}^s = 0$, and $\dot{z}^s = 0$ are the linearised isoclines.	222

1. Introduction

The endogenous growth literature has investigated extensively the idea that the technological progress is driven by market forces. Since innovation requires profit-seeking firms to undertake R&D, the factors determining the incentives to R&D activity in imperfect markets are central to the seminal work of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). These contributions, however, downplay the role of market structure.

In contrast, the identification of the determinants of market structure follows a long-standing tradition in Industrial Organisation. In recognising the endogenous character of those determinants, the literature has frequently emphasised the interrelation between market structure and innovative activity (e.g., Dasgupta and Stiglitz, 1980; Sutton, 1998), hence formally accommodating the early Schumpeterian view that the pace of innovation is related to market structure. Indeed, market structure is a key factor determining R&D activities because it captures features like the size and number of firms, barriers to entry and diversification; these factors determine market rivalry, which, in turn, determines the opportunities and constraints that firms face when planning R&D.

The interplay between long-run aggregate growth, innovative activity and market structure is analysed by a recent strand of the literature. Whereas some papers explore the interdependence between structure and growth by focusing on the strategic interaction of firms

1. Introduction

in an oligopolistic framework (e.g., van de Klundert and Smulders, 1997; Peretto, 1999; Aghion, Bloom, Blundell, Griffith, and Howitt, 2005), others emphasise the role of a specific factor within a monopolistic-competition setting – e.g., the number of firms (Peretto and Smulders, 2002), average firm size (e.g., Peretto, 1998) or firm size distribution (e.g., Thompson, 2001; Klette and Kortum, 2004).

This thesis proposes a theoretical contribution to the theme of long-run aggregate growth and industry dynamics, where the former is driven by R&D and the latter is measured by firm size and/or the number of firms in the context of monopolistic competition. In particular, the thesis comprises six independent essays, each of which aims at contributing to a particular strand of the literature.

The first essay aims at systematising the scattered empirical evidence relating firm dynamics with both short- and long-run macroeconomic dynamics. There are numerous studies that focus on firm-level data while controlling for macroeconomic conditions. From these studies a fairly robust set of empirical regularities pertaining to entry, exit, growth and the size distribution of firms has emerged. However, the literature that focuses explicitly on the interplay between firm dynamics and the business cycle is roughly confined to the US experience and to the cyclical properties of firm entry and exit, while the studies about the relationship between firm dynamics and economic growth are limited and unsystematic. We also give a brief account of the most recent theoretical literature on firm dynamics and macroeconomic dynamics, and try to relate it to the empirical findings.

The next three essays address a number of specific and, to some extent, independent empirical questions under a common analytical framework: a theoretical dynamic general-equilibrium model of endogenous growth, with simultaneous vertical and horizontal R&D under a full lab-equipment specification.

The second essay develops a single-technology tournament model of horizontal and vertical

1. Introduction

R&D, with the key feature that the overall growth rate is endogenous, as the splitting of the growth rate between the intensive and the extensive margin is itself endogenous. This setup gives rise to strong inter-R&D composition effects, while making economic growth and firm dynamics closely related, both along the balanced-growth path (BGP) and transition. The model hence offers a (qualitative) explanation for the negative or insignificant empirical correlation between aggregate R&D intensity and both firm size and economic growth, a well-known puzzle in the growth literature (e.g., Bassanini, Scarpetta, and Visco, 2000; Pagano and Schivardi, 2003). In the existing models, featuring a knowledge-driven specification (e.g., Howitt, 1999; Peretto and Smulders, 2002; Peretto and Connolly, 2007), re-compositions between vertical and horizontal R&D are not able to produce a negative correlation between total R&D intensity and economic growth, as the latter only changes in response to vertical innovation. Our results hence show the compatibility between innovation as the ultimate source of economic growth and the empirical evidence concerning the link between aggregate R&D intensity and both firm size and economic growth.

The third essay proposes an analytical mechanism to explain the data on the distribution of firms and production across high- and low-tech sectors. Empirical data suggests a significant variability of industrial structures across countries. A few regularities can be however pointed out: (i) the number of firms and total production are smaller in high- than in low-tech sectors (i.e., *relative* number of firms and relative production below unity); (ii) average firm size is larger in the high-tech sectors (i.e., *relative* average firm size above unity); (iii) relative production tends to be positively (cross-country) correlated to the relative number of firms; (iv) the relative number of firms tends to be (slightly) negatively (cross-country) correlated to relative average firm size. The essay builds a two-technology model of endogenous directed technical change (e.g, Kiley, 1999; Acemoglu and Zilibotti, 2001) that merges the expanding-variety with the quality-ladders mechanism, and explores

1. Introduction

the complementarity between them within a lab-equipment specification and under distinct regimes of scale effects. The model generates a specific set of BGP results with respect to the relationship between industrial structure, concentration and economic growth. Under small positive scale effects, the model is able to account for all the empirical regularities.

The fourth essay studies the effect of animal spirits on the composition of aggregate R&D, the industrial structure, consumption level and economic growth in a tournament model of vertical and horizontal R&D, both along the BGP and transition. The model is developed in the light of a number of empirical observations pertaining to R&D and growth: the existence of (i) “waves of enthusiasm”, according to which new industries are temporarily flooded by higher R&D expenditure (e.g., Klepper, 1996); (ii) insignificant or negative correlation between aggregate R&D intensity and the per-capita GDP growth rate (e.g., Bassanini, Scarpetta, and Visco, 2000); (iii) time-varying speeds of convergence, driven by technological factors (e.g., Bernard and Jones, 1996). By considering a lab-equipment specification, the model predicts a positive effect of animal spirits on the BGP level of per-capita consumption without impacting on economic growth and on aggregate vertical R&D. However, transition is slower under “waves of enthusiasm”, implying a longer period in which growth rates are higher than the BGP level. An economy that is subject to expectations shocks then converges at a time-varying speed.

The two last essays are shorter in length and explore particular results derived from the basic analytical model studied in the second and the third essays.

The fifth essay focuses on the long-run dynamics of the technological knowledge stock. Empirical estimates of the stock of ideas or technological knowledge usually build on the Perpetual Inventory Model (PIM), according to which the stock value suffers geometric depreciation at an arbitrated constant “obsolescence” rate (e.g., Coe and Helpman, 1995; Park, 2004). As pointed out by, e.g., Bitzer and Stephan (2007), this approach implies

1. Introduction

that, similarly to the process of physical-capital accumulation, a constant fraction of the technological-knowledge stock is lost with the passage of time and thus, if all R&D stops, that stock converges in the long run to zero. In this essay, we argue that the dynamics of the technological-knowledge stock can be represented by a mechanism similar to the one used for physical-capital accumulation, as long as we consider the endogeneity of the depreciation (obsolescence) rate and take a long-run (BGP) view of the process of technological-knowledge accumulation. This approach is in line with the notion of “endogenous obsolescence” explored by Caballero and Jaffe (1993), while it gives theoretical background to the assumption of geometric depreciation at a constant rate in the PIM.

The sixth essay focuses on the determinants of equilibrium price dispersion in homogeneous-good markets. Several models have been developed to identify these determinants. Work in this area includes models that assume *ex ante* heterogeneity in firms’ production costs and/or consumers’ search costs (e.g., Carlson and McAfee, 1983), or that assume information on prices is imperfect with otherwise identical agents (e.g., McAfee, 1995). This essay relates closely to the models that feature *ex ante* heterogeneous agents, while unveiling a non-degenerate price distribution within an endogenous-growth model. In particular, we show that the two-technology productive and innovative structure adopted in Acemoglu and Zilibotti (2001) allows for the analytical derivation of the equilibrium probability density function of homogeneous-good prices. This framework makes possible the study of a topic still untreated by the literature: the relationship between price distribution and innovation-related factors. In accord with casual empiricism, prices are distributed with positive probability over a closed interval and, under broad conditions, the mode of prices is smaller than their mean.

2. Stylised Facts and Other Empirical Evidence on Firm Dynamics, Business Cycle and Growth¹

Abstract: *This essay brings together, in a systematised fashion, the scattered empirical evidence relating firm dynamics with both short-run and long-run macroeconomic dynamics. There are numerous studies that focus on firm-level data while controlling for macroeconomic conditions. From these studies a fairly robust set of empirical regularities pertaining to entry, exit, growth and the size distribution of firms has emerged. However, the literature that focuses explicitly on the interplay between firm dynamics and the business cycle is roughly confined to the US experience and to the cyclical properties of firm entry and exit, while the studies about the relationship between firm dynamics and economic growth are limited and unsystematic. We also give a brief account of the most recent theoretical literature on firm dynamics and macroeconomic dynamics, and try to relate it to the empirical findings.*

Keywords: *empirical evidence; firm dynamics; business cycle; economic growth.*

JEL Classification: *L11*

¹The results in this chapter have been published as Gil (2009b) and also appeared as a working paper (Gil, 2008).

2.1. Introduction

The main goal of this essay is to bring together, in a systematised fashion, the scattered empirical evidence relating firm dynamics with both short- and long-run macroeconomic dynamics. There are numerous studies that focus on firm-level data while controlling for macroeconomic conditions, which cover a considerable range of variables, industries and countries. From these studies, what is already a rather robust set of empirical regularities, or stylised facts, pertaining to entry, exit, growth and the size distribution of firms has emerged. However, the literature that focuses explicitly on the interplay between firm dynamics and the business cycle is roughly confined to the US experience and to the cyclical properties of firm entry and exit, whereas systematic studies of the relationship between firm dynamics and economic growth are almost non-existent.

It is worth mentioning that the analysis of firms' behaviour has often been constrained by the lack of cross-country comparability of the underlying data, which may explain the lack of studies of this sort. On the other hand, the empirical evidence offered in the literature comes, in some cases, from simple tabulations of data on firm dynamics, or from case studies with hardly any statistical treatment, whereas in others it results from more structured econometric and time series analysis. This state-of-affairs becomes apparent in the lines that follow.

This essay also makes a brief digression into the more recent developments concerning the theoretical literature of firm dynamics. There is already a large number of papers that develop models of firm dynamics related to either business cycles or long-term growth. Yet the ability of the former set of models to address the empirical facts is far more compelling than in the latter case. This reflects both the nature of the aforementioned empirical evidence and the features of the theoretical models in each case.

The rest of the essay comprises five sections. The next section lists the main stylised facts that have emerged from a large number of studies using firm-level data with respect to entry, exit, growth and the size distribution of firms. Even though the facts are well-known, we choose to include them as they relate closely to some of the empirical evidence reported in the other sections of the paper. Section 2.3 presents the empirical regularities pertaining to firm dynamics over the business cycle, including not only the largely studied phenomena of firm entry and exit, but also some references to the dynamics of firm size and the number of firms. Section 2.4 is an attempt to summarise the empirical evidence about firm dynamics set in the context of economic growth, and it features three topics: the behaviour of firm dynamics over large time spans, throughout the industry life cycle and also across countries. Section 2.5 gives a brief account of the theoretical literature on the interrelation between firm dynamics and both short-run and long-run macroeconomic dynamics, and relates it to the empirical findings reported in the earlier sections. We conclude with Section 2.6.

2.2. Stylised facts from firm-level data

This section draws heavily from the list of stylised facts about firm dynamics summarised by Klette and Kortum (2004) and Cabral (2007); see references therein.²

1. In a given year, entry and exit occur simultaneously, with entry and exit rates being highly correlated across industries. Firms that enter and firms that exit are smaller than average incumbent size.
2. Smaller firms have a lower probability of survival, but those that survive tend to have higher growth rates than larger firms, especially for overall small size levels (for larger

²The first empirical studies about firm dynamics were strongly motivated by Gibrat (1931)'s seminal work, and the hypothesis known as Gibrat's law or the Law of Proportionate Effect that supported Gibrat's purely statistical model. However, as very well noted by Caves (1998), the modern list of stylised facts is essentially a list of exceptions to and extensions of Gibrat's law.

2. *Stylised Facts and Other Empirical Evidence on Firm Dynamics, Business Cycle and Growth*

firms growth rates tend to be unrelated to firm size).

3. Younger firms have a lower probability of survival, but those that survive tend to have higher growth rates than older firms.
4. The variance of growth rates is lower for larger firms.
5. The size distribution of firms is highly skewed, with the pattern of right-skewness suggesting that there are proportionally more small firms than large firms with respect to the lognormal distribution.
6. The productivity levels tend to be higher for larger firms.
7. Productivity levels and R&D across firms are positively related, whereas productivity growth is not strongly related to firm R&D.
8. R&D intensity is independent of firm size.
9. The distribution of R&D intensity is highly skewed, and a considerable fraction of firms report zero R&D.

2.3. Firm dynamics and the business cycle

As noted by Caves (1998, p. 1958), the “studies of intertemporal entry-exit linkages” – as those that support the empirical evidence described in Section 2.2 – usually “control for macroeconomic conditions in various ways and degrees”. However, one may be interested in studying explicitly the interplay between the business cycle and the dynamics of firm entry, exit and growth. Some recent papers explore this avenue, but the reported evidence concerns almost exclusively firm entry and exit in the US. In general, these papers use, in general, Hodrick-Prescott filtered data and mainly calculate cross-correlations between

variables of interest. The main point is that the number of firms varies (procyclically) over the business cycle.

1. Net entry (measured as net business formation, i.e., the difference between new incorporations and failures) and real profits comove, and both are strongly procyclical (Bilbiie, Ghironi, and Melitz, 2007). The correlation between net entry and output (measured by real GDP) ranges over the interval 0.70-0.73 (see Lewis, 2006; Bergin and Corsetti, 2005; and Devereux et al, 1996). Net entry tends to lead output and profit expansions by one quarter (see Devereux et al, 1996; and Bilbiie, Ghironi, and Melitz, 2007).
2. Entry (measured as new incorporations) is procyclical; its correlation with output ranges over the interval 0.35-0.55 (see Lewis, 2006; Bergin and Corsetti, 2005; and Devereux, Head, and Laphamb, 1996); its correlation with output growth has been calculated around 0.28 (see Campbell, 1998). Entry tends to lead output by one quarter (see Devereux, Head, and Laphamb, 1996); however, it tends to lag output growth by one quarter (see Campbell, 1998).
3. Similar to capital investment, entry and net entry are more volatile than output over the cycle (see Lewis, 2006).
4. Entry covaries positively with total factor productivity growth (see Campbell, 1998).
5. Exit is countercyclical; its correlation has been calculated around -0.42 with respect to output (see Devereux, Head, and Laphamb, 1996) and -0.51 with respect to output growth (see Campbell, 1998). Exit is coincident with output movements (see Devereux, Head, and Laphamb, 1996), but it tends to lead output growth by four quarters (see Campbell, 1998).

2. *Stylised Facts and Other Empirical Evidence on Firm Dynamics, Business Cycle and Growth*

6. Net entry responds positively and significantly to expansionary monetary policy shocks, as estimated by a 5-variable recursive VAR (see Lewis, 2006; and Bergin and Corsetti, 2005).
7. Firm size (measured as employment per firm) and product diversification (measured as the ratio of primary industry payrolls to total company payrolls or the number of four-digit manufacturing categories in which the companies operate) move together, in a procyclical fashion, leading to product space expansions during upturns (Jovanovic, 1993). The number of firms varies procyclically, but with less variation than firm size (Jovanovic, 1993).

Related to this evidence on the cyclical properties of net business formation are the findings from the research on labour market dynamics, in particular on job creation and job destruction due to the births and deaths of plants. A key stylised fact in this literature is that job destruction varies more over time than job creation, meaning that job turnover moves countercyclically. In a much cited study, Davis and Haltiwanger (1992) report that the correlation with net employment growth in the US is of -0.57. Konings (1995) cites several plant-level panel data studies that show that there exists a large amount of gross job creation and gross job destruction at all phases of the business cycle and even within narrowly defined sectors, with evidence for the US, UK, Germany, Italy and other European countries (see references in the paper). The author reports the yearly gross job flow rates for the UK and discusses its cyclical properties. He finds that gross job reallocation, the sum of job creation and destruction, is countercyclical, reflecting the fact that fluctuations in the job destruction rate are far more pronounced than fluctuations in the job creation rate. Interestingly, Campbell (1998) also reports that firm exit rate varies more (higher standard deviation over time) than firm entry rate, according to data for the US.

2.4. Firm dynamics and economic growth

This section is an attempt to summarise the empirical evidence about firm dynamics set in the context of economic growth, as the literature on this topic is sparse at best and lacks systematic nature. We organise the existing evidence around three topics. The more structured analysis comes from the literature on industry life cycle, which we present first. The second topic lists time-series evidence over long time horizons, while the the final topic reports some cross-country evidence.

2.4.1. Industry life cycle

An important line of research on firm dynamics tracks entrants to determine their subsequent growth and mortality rates along the industry life cycle. The stylised facts reported below are selected from Klepper (1996). This author lists the empirical regularities (see references therein) concerning how entry, exit, market structure and technological change vary from the birth of “technologically progressive” industries through maturity.

1. At the beginning of the industry, the number of *entrants* may rise over time or it may attain a peak at the start of the industry and then decline over time, but in both cases the number of entrants eventually becomes small.
2. The number of *producers* grows initially and then reaches a peak, after which it declines steadily despite continued growth in industry output.
3. Eventually the rate of change of the *market shares* of the largest firms declines and the leadership of the industry stabilises.
4. The diversity of *competing versions* of the product and the number of major product innovations tend to reach a peak during the growth in the number of producers and then fall over time.

2.4.2. Time series evidence

This second topic gathers scattered empirical evidence with respect to the behaviour of firm and industry-related variables over long time horizons, larger than the typical business cycle, in an attempt to devise secular trends.

1. Industry firm-size distributions appear to be stable over time (Klepper and Thompson, 2006). These distributions remain relatively unchanged even though there is substantial shifting of relative position within each distribution as firms grow and decline.³
2. Relative firm size is mean reverting over long time horizons, although not over short horizons (Klepper and Thompson, 2006), in the sense that there is a tendency for large firms to decline and small firms to grow relative to one another⁴ – the Galtonian regression-to-the-mean phenomenon (see Baldwin, 1995, ch. 5).
3. Differences in entry between industries do not persist for very long, although there is a very large cross-section (industry-level) variation in entry. Rates of entry are rarely high or persistently low over time in particular industries, but, rather, entry seems to come in bursts that are not highly synchronized across industries (Geroski, 1995).⁵
4. Historically, absolute firm size (employment per firm) and product diversification have moved together. They followed the secular increase in the capital-labour ratio over almost all of the past century, but from the 1980s onwards firm size and diversification have both declined (Jovanovic, 1993).⁶ However, this movement has not been

³As noticed by Caves (1998, p. 1976), the processes described by Facts 1, 2 and 4, in Section 2.2 above, are “typically consistent with the size distribution of firms (concentration) being stable over time”.

⁴These authors note that mean reverting in firm size is rightly what one would expect from a model combining a stationary firm-size distribution (see Fact 1 in this subsection) and persistent churning of firms (see Fact 1 in Section 2.2, above). However, this may be at odds with Fact 3 in Subsection 2.4.1.

⁵This piece of evidence reports us back to the industry life cycle approach. See Fact 1, in Subsection 2.4.1, above.

⁶A decline in establishment size is also reported by Carlsson (1989) for the period 1972-1982 in several

homogeneous across countries (see van Ark and Monnikhof, 1996).

5. The number of firms and establishments in manufacturing exhibited an upward trend over almost all of the past century (see Maddison, 1994).
6. Time series data shows a positive correlation between industry turnover and the process of productivity growth (Cabral, 2007).

As far as the evolution of the number of incumbent firms is concerned, the main focus in the literature has been on industry-level data. For example, according to Klepper and Thompson (2006), the laser industry experienced a steady growth in the number of producers over time, while Klepper and Simons (1997) report that certain industries, such as automobiles, tyres, and television receivers, experienced extremely sharp shakeouts in the number of producers as they aged despite robust growth in total production. This evidence leads us back to the industry life cycle approach (see Subsection 2.4.1, above). Carlsson (1989) finds an increase in the total number of establishments (US, Japan and 6 European countries) in manufacturing over a 10-year period (1973-1983). However, this is too short a period to identify a long-term trend, only concerns manufacturing and describes establishments instead of firms; moreover, the number of establishments decreased in 4 of those 8 countries. Maddison (1994) presents a far longer time-series data set, but one which is confined to the number of establishments in US manufacturing. According to this author, the number increased from 205 thousand to 355 thousand (a growth rate of 73%) between 1899 and 1986, whereas the average US manufacturing establishment rose in size

industries and countries. The author advances a couple of hypothesis to explain this finding: (i) "deglomeration" or specialization, that is, the selling-off or disinvestment of non-core businesses in order to free up scarce resources (particularly management time) to defend and nurture core business activities - the perception of a tougher and more uncertain business climate after the mid-1970s than during the 1960s and early 1970s is an important motive, but the "back-to-basics" movement can also be viewed as a result of the recognition that the conglomerate merger wave of the earlier decades had simply gone too far; (ii) the emergence of new computer-based technology which improves the quality and productivity of small or medium scale production relative to standardized mass-production techniques which dominated for the previous 150 years.

from 24 to 54 employees (a growth rate of 125%). Interestingly, the author also reports that GDP per person employed rose almost fourfold over that period, the proportion of people with higher education rose ninefold, and the stock of machinery and equipment per person employed rose about fifteenfold. These findings seem to be at odds with the reference to the capital-labour ratio in the aforementioned Fact 4. In contrast, Ehrlich (1985) finds a “relative stability of establishment sizes” in a long time-series data base (1900-1970) concerning manufacturing in the US, Japan and eight European countries. However, this stability is far more evident in the post-World War II period. Overall, the author concludes that “growth and contraction of establishments sizes cancel each other out over some longer period” (Ehrlich, 1985, p. 285). The increase in the number of firms and establishments and the stability of their average size over the long run are confirmed by Laincz and Peretto (2006), who analyse more recent data for the US. Finally, Jovanovic (1993) also reports an increase in sales per firm over time for the US.

With respect to econometric studies that explicitly analyse the link between firm size distribution and economic growth, the examples are scarce. Lucas (1978) attempts to link firm size distribution (more precisely, average firm size - employment per firm) to economic growth, measured by the growth in GNP per capita, by resorting to time-series data over the period 1900-1970. The data covers various sectors, and not only manufacturing. The results “show a clear and accurately measured effect of GNP per capita on average firm size. The estimated elasticity is in the range 0.8-1.0. The independent effect of trend on firm size [introduced as a simple means of correcting for labour improving quality over time] is negative, with firm size declining (GNP held fixed) at an annual rate of 0%-2.5%. Together, these two variables do an excellent job of accounting for the secular behaviour of firm size (however it is measured), and for large-scale cyclical movements (mainly the Great Depression)” (Lucas, 1978, p. 521). Following a different approach, Shaffer (2006) explores the

empirical association between economic growth (measured by the growth rates of median household income) and initial firm/establishment size at the country level, therefore explicitly testing for the theoretical prediction of a causal link running from firm/establishment size to economic growth. The findings indicate that smaller establishments in each of the four sectors studied are significantly and robustly associated with faster subsequent growth of median household income.

2.4.3. Cross-country evidence

The analysis of cross-country evidence has been hindered by several conceptual issues, as well as by measurement problems induced by cross-country differences in coverage, unit of observation, classification of activity and data quality (see, e.g., Bartelsman, Scarpetta, and Schivardi, 2003; Bartelsman, Haltiwanger, and Scarpetta, 2005). Nevertheless, it is possible to present some empirical evidence, as referred to in some literature on firm dynamics.

1. The pattern of right-skewness of the firm size distribution is common to several countries, despite the significant differences in size and level of economic development (Cabral, 2007).
2. Summary measures such as the k-firm concentration ratio take on similar values in the same industry in different countries (Klepper and Thompson, 2006). The relative weight of firms with fewer than 20 employees also shows a very low degree of dispersion between countries of different sizes and levels of GDP per capita (see Bartelsman et al., 2003).
3. The average size of incumbents varies widely across sectors and countries. It is generally smaller in most European countries than in the US due to differences in both the sectoral composition of the economy and within-industry peculiarities (Bartelsman et

al., 2003). Smaller countries tend to have a size distribution skewed towards smaller firms, but the average firm size does not map precisely with the overall dimension of the country, the underlying technological level of the industry or its degree of maturity (see Bartelsman et al., 2005).⁷ However, a positive relation between average firm size and the aggregate growth rate has been found (see Pagano and Schivardi, 2003).⁸

4. The evolution of the firm size distribution of a given cohort, very skewed at birth but gradually becoming more symmetric, is common to several countries (Cabral, 2007).
5. The turnover processes show very little qualitative difference among countries. According to Caves (1998), less developed countries appear to exhibit more turnover associated with their concentration on activities with smaller sunk costs. Yet, Bartelsman et al. (2005) report that the level of turnover varies across countries roughly independently from either the level or the growth rate of GDP per capita.
6. Large countries (measured by population size) tend to have a larger number of firms and lower concentration rates than small countries, whereas countries with larger rates of entry tend to exhibit a smaller number of firms (Sherer and Ross, 1990).
7. Barriers to entry tend to be smaller in more developed countries (Cabral, 2007).

As we can see, the reported evidence in the present subsection is generically limited to essentially qualitative facts, and is often unclear. At best, it gives us the idea that once a minimum threshold of development is surpassed, and provided we get sufficiently aggregated data to abstract from movements connected to industry life-cycle specificities, both the dimension and the level of development of an economy are weakly connected to the

⁷Using the data on firm size reported by Bartelsman et al. (2003), together with OECD data on national accounts, we find that the coefficient of correlation between the average firm size and GDP level is of 0.5 (0.38 if we exclude US from the sample), whereas between average firm size and GDP per capita is of 0.14.

⁸These authors conduct a cross-section study at the aggregate level and find that average firm size (employment per firm) matters for growth through its effects on R&D intensity.

characteristics of the observed firm size distribution and firm turnover.

As Bartelsman et al. (2005) points out, the evidence of significant cross-country differences in firm characteristics, their market dynamics and post-entry performance must be explained not only by differences in sectoral composition of the economy but also by “salient differences in market characteristics and in business environment” (Bartelsman et al., 2005, p. 38), which may differ from country to country, even when levels of development are identical. Nevertheless, one cannot overlook the conceptual and measurement problems that still plague this sort of analysis. In spite of recent developments, sponsored by international organisations such as the OECD and the World Bank (see, e.g., Bartelsman, Scarpetta, and Schivardi, 2003; Bartelsman, Haltiwanger, and Scarpetta, 2005), the lack of statistical comparability is still a serious constraint on cross-country comparisons.

The alternative approach, which exploits the time variation within countries, allows the country-specific measurement error to be removed. But this approach is only useful if sufficiently long time series are available, covering at least two complete business cycles. However, the longer the time series, the higher the probability of facing changes in the concepts and methodology adopted by the statistical system within each country, which implies that the measurement error in a country can no longer be taken as time invariant.⁹ On the other hand, one must take into account that development and growth processes at the country level are not homogeneous over time. Structural changes happen in a continuous time scale and industry and firm-level indicators are sensitive to them. The literature on industry life-cycle helps to shed some light on this. The use of aggregate data at the highest level possible should certainly help to mitigate this problem, but deep structural changes should be expected even at that level.

⁹The studies conducted by Lucas (1978) and Maddison (1994) are good examples of this. But, of course, one may argue that this is still a minor problem when compared to those issues afflicting cross-country comparisons.

2.5. Theoretical literature on firm dynamics

In this section, we summarise some of the recent developments on the theoretical side with respect to firm dynamics and macroeconomic dynamics, especially as they address (some of) the empirical facts presented in the previous two sections.

2.5.1. Business cycle models

There is recent macro literature that looks at the relationship between the business cycle and firms' entry-and-exit decisions as a specific transmission mechanism that enhances the ability of the models of short-term economic fluctuations to replicate the properties of fluctuations of actual economies.¹⁰ More specifically, several stochastic dynamic general-equilibrium models of monopolistic competition have been developed to study the effect of entry and exit on the dynamics of the business cycle, explicitly treating the number of firms as endogenous. However, although entry and exit are essentially dynamic phenomena, this literature has frequently formalised them as non-intertemporal.

One widely-used approach is to have instantaneous free entry, so that the number of active firms is such that it sets period-by-period profits to zero. Examples of models that assume a static zero-profit entry condition are Chatterjee and Cooper (1993), Devereux, Head, and Laphamb (1996) and, more recently, Bergin and Corsetti (2005), Jaimovich (2007) and Jaimovich and Floetotto (2008). In Dos Santos Ferreira and Dufourt (2006), besides the static profitability condition, the number of active firms must also satisfy a "sustainability condition" for free entry.¹¹ In contrast, some papers adopt a genuine dynamic entry model,

¹⁰These studies chiefly pertain to the US economy and the respective cross-correlations between output, consumption, investment, real wages and hours worked

¹¹The latter condition merely imposes that any potential entrant is deterred from becoming active because it correctly realises that no output would entail a positive profit, given its conjecture of the demand level and of the cumulative sectoral output supplied by its active competitors.

such as Lewis (2006) and Bilbiie, Ghironi, and Melitz (2007). Here, the free-entry condition equates the expected present discounted value of profits to the (sunk) entry cost. Thus, profits are allowed to vary over time and the number of firms is a true state variable. The models cited above are generally compatible with Facts 1 and 3, in Section 2.3. Lewis (2006) and Bergin and Corsetti (2005) also address Fact 6.

We also emphasise Campbell (1998), who presents a perfectly-competitive vintage capital model of endogenous capital obsolescence and replacement that explicitly considers aggregate uncertainty, as well as idiosyncratic uncertainty experienced by plants. Because the decision to exit is irreversible, this idiosyncratic uncertainty raises the value of the option to operate in the future. Each firm chooses the number and types of plants to purchase at the beginning of the period, which plants to close at the end of the period, and the plant's employment to maximise current profits. Given its particular structure, the model is able to address Facts 1-5.

Finally, in a very recent paper, Brito and Dixon (2009) consider the entry and exit of firms in a perfectly-competitive deterministic dynamic general-equilibrium model with capital. Entry is determined by a dynamic free-entry condition, so that the process of entry and the accumulation of capital interact in an explicitly dynamic setting, whose equilibrium is saddle-point stable with a two-dimensional stable manifold. Transitional dynamics towards the steady state can, under certain circumstances, be non-monotonic with respect to the number of firms and average firm size, measured as output per firm. Interestingly, this allows the model to address Facts 1 and 7.¹²

¹²To be concrete, the model is able to qualitatively replicate the facts: "Net entry tends to lead output and profit expansions" and "The number of firms varies procyclically, but with less variation than firm size".

2.5.2. Economic growth models

Several studies analyse firm dynamics in the context of non-stationary economies. In this short digression, we focus mainly on endogenous-growth models with horizontal entry (expansion of varieties). On the one hand, in order to explicitly consider the feedbacks between firm dynamics and long-term aggregate growth, a general-equilibrium endogenous-growth setup seems to be the most appropriate. On the other, a time-varying product space provides, in general, a richer setting than vertical entry (quality ladders) per se for studying firm entry, growth and evolution of the size distribution. Finally, the parallel with the business-cycle models cited above is also more obvious if we concentrate on growth models with horizontal entry in a monopolistic-competition setting.¹³

Examples of such models are Dinopoulos and Thompson (1998), Young (1998) and Howitt (1999), who merge the quality-ladders with the expanding-variety mechanism,¹⁴ so that each product line added to the market, as the outcome of horizontal R&D, observes an independent quality-ladders process, as the outcome of vertical R&D. Peretto (1998) presents a model where incumbents perform cost-reducing R&D, while entrants introduce new varieties.¹⁵ Luttmer (2007a) describes a model of monopolistic competition with aggregate and (multi-product) firm growth based on horizontal R&D. The implied firm size distri-

¹³We leave untreated the specific aspects of market structure related to the strategic interaction of firms, more akin to the industrial-organisation literature and that go beyond firm dynamics as analysed in this paper. In fact, there is a large set of models that studies the interplay between long-run aggregate growth and market structure by focusing on the strategic interaction of firms in an oligopolistic framework (e.g., Van de Klundert and Smulders, 1997; Peretto, 1999; Aghion et al., 2001; Aghion et al., 2005; Reis and Traca, 2008).

¹⁴The seminal paper with respect to endogenous growth based on an expanding-variety mechanism is Romer (1990). This is a model where monopolistic-competitive firms make decisions on horizontal R&D (i.e., R&D targeted at the creation of new goods) in order to maximise intertemporal profits. Other early papers, following a different approach, are Stokey (1988) and Young (1991). In these models, aggregate growth is sustained by learning-by-doing effects, which, in turn, induce a change over time of the subset of a given continuum of goods (varieties) that is actually produced.

¹⁵More recent papers in this line are, e.g., Peretto and Smulders (2002) and Peretto and Connolly (2007). Funk (2008) studies horizontal entry in the context of a vintage-knowledge model, where endogenous growth is induced by old and new knowledge produced by cost-reducing R&D.

bution, with the number of products per firm as support, follows from an (exogenous) entrepreneurial skill distribution. Thompson (2001) derives a stationary firm size distribution, with relative productivity per firm as support, based on a version of Dinopoulos and Thompson (1998).

All the models above predict an increase in the number of firms along the balanced-growth path, thus addressing Fact 5, in Subsection 2.4.2. They also predict firm size stability, in line with Ehrlich (1985)'s findings stated in Subsection 2.4.2 (see also Fact 4, in that subsection). Peretto (1998) and Howitt (1999) establish that economies with higher long-run growth rates tend to have a market structure characterised by larger firms (Fact 3, Subsection 2.4.3). Peretto (1998) predicts that large countries (measured by population size) tend to have a larger number of firms (Fact 6, Subsection 2.4.3).

Along different lines, Klette and Kortum (2004) build a quality-ladders model of monopolistic-competitive multi-product firms. There is no horizontal entry, in the sense that the overall product space is constant. However, each firm's product space is time-varying: while a firm adds new goods to its portfolio as a result of R&D, there is a probability that some other firm will innovate on a good it is currently producing, thus rendering it obsolete. The economy grows as innovations raise the quality of the overall measure of products. In turn, Luttmer (2007b) studies a dynamic monopolistic-competitive model of entry and exit. Incumbent firms become more productive at an idiosyncratic exogenous stochastic rate, whereas the economy-wide distribution of productivity improves over time as a result of selection and imitation by entrants. Klette and Kortum (2004) and Luttmer (2007b) derive a stationary firm size distribution, the former with support given by the number of products per firm and the latter by relative productivity per firm.

By predicting stationary firm size distributions along with long-run aggregate growth, Klette and Kortum (2004), Luttmer (2007b,a) and Thompson (2001) manage to address

Fact 1, in Subsection 2.4.2, and Facts 1 and 2, in Subsection 2.4.3. Except for Thompson (2001), the models are also compatible with Fact 2, in Subsection 2.4.2, and Fact 5, in Subsection 2.4.3.¹⁶

Finally, outside the endogenous-growth literature, an early paper by Lucas (1978) analyses a model of capital accumulation without entry and exit, but where the (exogenous) distribution of managerial talent determines firm size distribution. If the elasticity of substitution between capital and labour is less than one, the model predicts that average firm size is positively correlated with the level of capital per-capita of the economy. The author's own empirical findings corroborate the theoretical results - see Subsection 2.4.2.

The aforementioned models are not, in general, compatible with Facts 3 and 6, in Subsection 2.4.2, and Facts 4 and 7, in Subsection 2.4.3. These pieces of empirical evidence, which mainly concern industry-level barriers to entry and turnover rates, are closer to the industrial-organisation grounds, and as such are not easily addressed by what are still rather stylised aggregate growth models – namely constrained in their set up by the strong assumptions usually needed to generate balanced-growth-path equilibria.

The aforementioned models are not, in general, compatible with Facts 3 and 6, in Subsection 2.4.2, and Facts 4 and 7, in Subsection 2.4.3. These pieces of empirical evidence, mainly concerning industry-level barriers to entry and turnover rates, are closer to Industrial-Organisation grounds, and as such are not easily addressed by what are still rather stylised aggregate growth models – namely constrained in their set up by the strong assumptions usually needed to generate balanced-growth-path equilibria.

¹⁶Thompson (2001) does not incorporate firm exit.

2.6. Concluding remarks

In this essay, we try to systematise the scattered empirical evidence relating firm dynamics with both short-run and long-run macroeconomic dynamics. While it is rather straightforward to put in a nutshell the relevant empirical evidence concerning firm dynamics and the business cycle, matters get complicated when we search for empirical evidence set in the context of economic growth, whatever the chosen approach - cross-country or time-series analysis.

The empirical evidence with respect to firm dynamics and the business cycle is strongly quantitative in nature, although essentially limited to the US experience and to the study of firm entry and exit. The dynamics of firm size and number of firms has deserved less attention. On the other hand, the approach to theoretical modelling followed by this literature allows for precise numerical results, usually obtained through extensive model calibration. Thus, it is fairly straightforward to map the theoretical predictions with the empirical facts.

In contrast, the empirical evidence concerning firm dynamics over the long-run is sparse, commonly set in qualitative terms and often contradictory. In spite of recent developments, sponsored by international organisations such as the OECD and the World Bank, the lack of statistical comparability due to conceptual and measurement problems is still a serious constraint, especially on cross-country comparisons. The fact that growth models tend to produce mainly qualitative, and not always unequivocal, results adds to the overall difficulty of matching theoretical predictions and empirical findings.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D¹

Abstract: *This essay develops a tournament model of horizontal and vertical R&D under a lab-equipment specification. A key feature is that the overall growth rate is endogenous, as the splitting of the growth rate between the intensive and the extensive margin is itself endogenous. This setup gives rise to strong inter-R&D composition effects, while making economic growth and firm dynamics closely related, both along the balanced-growth path and transition. The model hence offers a (qualitative) explanation for the negative or insignificant empirical correlation between aggregate R&D intensity and both firm size and economic growth, a well-known puzzle in the growth literature.*

Keywords: *endogenous growth; vertical and horizontal R&D; firm dynamics; transitional dynamics.*

JEL Classification: *O41, D43, L16*

¹The results in this chapter appeared as two working papers (Gil, Brito, and Afonso, 2008; Gil, Brito, and Afonso, 2010).

3.1. Introduction

A well-known puzzle in the growth literature is the insignificant or negative correlation between aggregate R&D intensity and the per-capita GDP growth rate found by several empirical studies (e.g., Bassanini, Scarpetta, and Visco, 2000); a related puzzle occurs with respect to the relation between R&D intensity and firm size (e.g., Pagano and Schivardi, 2003). The literature often underlines the complexity of the link between these variables (e.g., Audretsch and Keilbacha, 2008), and the several conceptual and measurement problems that afflict empirical analysis in this field (e.g., Bartelsman, Haltiwanger, and Scarpetta, 2005) to explain the lack of robust results. This essay highlights the role of inter-R&D composition effects in accounting for the above facts, by developing a model of simultaneous horizontal and vertical R&D by entrants within a lab-equipment specification. In the existing models, featuring a knowledge-driven specification (e.g., Peretto, 1998; Howitt, 1999; Peretto and Smulders, 2002; Peretto and Connolly, 2007), re-compositions between vertical and horizontal R&D are not able to produce a negative correlation between total R&D intensity and economic growth, as the latter only changes in response to vertical innovation. Our results hence show the compatibility between innovation as the ultimate source of economic growth and the empirical evidence concerning the link between aggregate R&D intensity and both firm size and economic growth.

The joint consideration of horizontal and vertical R&D in a framework that allows for the study of their dynamic interaction fits the vision that growth proceeds along an extensive (introduction of new goods) and an intensive margin (increase of good quality and reduction of marginal costs). This is in accord with the description of the process of industrial growth in both economic history and industry life-cycle literature (e.g., Freeman and Soete, 1997; Klepper, 1996). Furthermore, this approach allows us to consider economic growth and

3. *Growth and Firm Dynamics with Horizontal and Vertical R&D*

firm dynamics, measured by size and/or the number of firms, under the same framework.

This essay follows the “creative destruction”, or tournament, tradition in the simultaneous treatment of horizontal and vertical R&D: the entrant must choose between vertical and horizontal R&D, whereas the incumbent does not perform any R&D (e.g., Aghion and Howitt, 1998; Howitt, 1999; Segerstrom, 2000). Thus, we depart from the non-tournament approach, which postulates that vertical R&D is done by the incumbent and horizontal R&D is carried out by entrants (e.g., Young, 1998; Peretto, 1998, 1999; Dinopoulos and Thompson, 1998; Peretto and Smulders, 2002). Indeed, according to the stylised facts on entry (e.g., Geroski, 1995; Klepper, 1996), it (i) occurs mostly in already existing sectors, (ii) is driven mostly by new firms (instead of incumbents enduring diversification processes), and (iii) is closely related to R&D activities. In a model of R&D and firm dynamics under monopolistic competition, like ours, where R&D activity is the only mechanism by which entrants can make a competitive entry (by introducing either a new variety or a higher-quality version of an existing good), those facts imply that not only horizontal but also vertical R&D must be performed by entrants.²

The entry mechanism is a key component of the model. A potential entrant has to choose between introducing an innovation in an existing industry, specialised in the production of a particular variety, or introducing a new variety. In contrast to Howitt (1999), among others, we assume that there are no intersectoral spillovers in vertical R&D (e.g., Segerstrom, 2007; Etro, 2008). The entrant then faces a double uncertainty at each point in time: he/she does not know if a successful innovation will occur and in which industry it will take place. In particular, with respect to horizontal entry, the potential entrant has to compare the costs

²Vertical R&D by entrants is neither logically nor empirically incompatible with vertical R&D by incumbents, as attested by the extant papers on vertical R&D simultaneously by entrants and incumbent (e.g., Etro, 2004; Segerstrom, 2007). To simplify the structure of our model, we consider R&D by only one type of firm; in the light of the aforementioned empirical facts, we let that type be the entrants. A very recent paper with vertical R&D only by entrants is Francois and Lloyd-Ellis (2009).

3. *Growth and Firm Dynamics with Horizontal and Vertical R&D*

of creating a new variety with the expected benefits from introducing an innovation in any of the existing industries. We assume that the arrival of the innovation in any industry follows the Schumpeterian creative-destruction process with Poisson distribution, while its occurrence in a particular industry follows a uniform distribution.

Also crucial is the way R&D technology is characterised in the model. We consider a lab-equipment specification, whereas the literature predominantly assumes that R&D is knowledge-driven.³ In the latter case, the choice between vertical and horizontal R&D implies a division of labour between the two types of R&D. Since the total labour level is determined exogenously, there is a shortcoming: in the end, the rate of extensive growth is exogenous, i.e., the balanced-growth-path (BGP) flow of new goods occurs at the same rate as (or proportional to) population growth. The alternative assumption that R&D is of the lab-equipment type implies that the choice between vertical and horizontal innovation is related to the splitting of R&D expenditures, which are fully endogenous. Therefore, we endogenise the overall growth rate and thus the rate of extensive growth.

In particular, we develop a dynamic general equilibrium model where, from the point of view of households, wealth can be accumulated either by creating new firms or by accumulating capital (e.g., Brito and Dixon, 2009). A quality-ladders mechanism provides the non-physical capital accumulation (technological knowledge), whilst an expanding-variety mechanism offers the flow of new firms (new good lines). Average firm size is thus measured as technological-knowledge stock per firm, which, however, relates closely to production (sales) or financial assets per firm.

This creates a channel between vertical R&D (hence aggregate growth) and firm dynamics, both in the long run (BGP) and in the medium run (transition), due to the nature of

³Using Rivera-Batiz and Romer (1991)'s terminology, the assumption that homogeneous final good is the R&D input means that one adopts the “lab-equipment” version of R&D, instead of the “knowledge-driven” specification, in which labour is ultimately the only input.

3. *Growth and Firm Dynamics with Horizontal and Vertical R&D*

horizontal entry technology. We consider simultaneously dynamic and static decreasing returns to horizontal R&D: the former capture the negative spillovers associated with the number of varieties (e.g., due to the existence of proportional barriers to entry) and the latter implies new varieties are brought to the market gradually, instead of through a lumpy adjustment. This is in line with the stylised facts on entry (e.g., Geroski, 1995), according to which entry occurs mostly at small scale (adjustment costs penalise large-scale initial entry). So far, the literature has treated the two features separately (the former in, e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004; the latter in, e.g., Howitt, 1999; Jones and Williams, 2000). The first feature is crucial for the BGP properties of the model. The negative spillovers in horizontal entry per se determine a constant number of varieties/firms along the BGP; however, vertical R&D sustains variety expansion and aggregate growth at a positive finite rate. The second feature is key with respect to transitional dynamics. We show static decreasing returns to horizontal R&D introduce dynamic second-order effects in entry: the entry cost increases with the number of firms entering the market at a given instant. This friction in horizontal entry generates transitional dynamics, which operates through adjustments in average firm size. We also show that, given the interrelation between the two types of R&D, the dynamics is explicitly obtained from a piecewise-smooth dynamical system.

In the end, our framework gives rise to strong endogenous inter-R&D composition effects and makes economic growth and firm dynamics closely related: vertical R&D is the ultimate growth engine, whilst horizontal R&D builds an explicit link between aggregate and firm-dynamics variables.⁴

⁴This point has been made recently by Peretto and Connolly (2007), in a different analytical setup. Like Peretto (1998, 1999) and others, the authors build an endogenous growth model where incumbents do deterministic cost-reducing R&D, while entrants bring new products to the market, under a knowledge-driven specification. They consider fixed operating costs, which, unless the exogenous population growth rate is positive, determine a constant number of varieties along the BGP. Under this setting, only vertical R&D allows for growth “unconstrained by endowments”.

3. *Growth and Firm Dynamics with Horizontal and Vertical R&D*

One of the most interesting findings of the essay is the mixed result with respect to the BGP relation between R&D intensity and both economic growth and firm size, thereby lending theoretical support to the previously highlighted lack of clear-cut empirical relationship between those variables. Shifts in the BGP due to changes in preferences or in vertical-entry costs give rise to a positive correlation between R&D intensity and both growth and firm size, whilst changes in horizontal-entry costs yield a negative relationship. This is explained by an endogenous inter-R&D composition effect: an increase in horizontal-entry costs induces a shift of resources from horizontal to vertical R&D, but the effect on the growth rate of the increment in the vertical type is dominated by the decrease in the horizontal one. That is, there is a negative correlation between the growth rate and vertical R&D whose effect outweighs the positive correlation between the former and horizontal R&D.

Furthermore, in transitional dynamics the relationship between R&D intensity and both economic growth and firm size is unequivocally negative. Consider an emergent-market economy displaying a shallow market of differentiated goods, i.e., with a number of varieties too low relative to the technological-knowledge stock, hence firm size that is too large. Such an economy starts with a smaller vertical-innovation rate and higher economic growth than a mature (deep-) market economy. A lower vertical-innovation rate implies a higher real interest rate and lower vertical R&D, which secure the larger resources allocated to horizontal R&D. Along the transition path, the growth rate decreases, while the resources allocated to horizontal R&D are gradually re-targeted to vertical R&D, thus increasing the vertical-innovation rate. Thus, the latter and both the growth rate and firm size are negatively related. Due to a composition effect identical to that described for the BGP, R&D intensity moves in tandem with the vertical-innovation rate.⁵

⁵The negative relationship between economic growth and the vertical-innovation rate is also apparent in Aghion and Howitt (1998), but only for a specific set of parameter values.

Our model is close to Evans, Honkapohja, and Romer (1998), who build an expanding-variety mechanism with a lab-equipment specification exhibiting dynamic decreasing returns to scale. Here, variety expansion is ultimately sustained by physical-capital accumulation. Like us, Arnold (1998) and Funke and Strulik (2000) obtain an expanding-variety mechanism that is driven by non-physical capital accumulation, in their case in the form of human capital. However, there is no inter-R&D composition effect in these models, as they only feature horizontal R&D. In contrast, several recent models consider both horizontal and vertical innovation, focusing on the study of economic growth and firm dynamics under a common framework (e.g., van de Klundert and Smulders, 1997; Peretto, 1998, 1999, 2007; Thompson, 2001; Peretto and Smulders, 2002). As already stated, in as much as these models feature a knowledge-driven specification, shifts between vertical and horizontal R&D are unable to produce a negative correlation between total R&D intensity and growth, as the latter only changes in response to the intensive margin. The noteworthy exception is Peretto (2007), where R&D has a lab-equipment specification. However, this is a non-tournament model with deterministic vertical R&D.

The remainder of the essay is organised as follows. In Section 3.2, we present the model, giving a detailed account of the production, price and R&D decisions, and derive the dynamic general equilibrium. In Sections 3.3 and 3.4, we analyse the interior BGP and local-dynamics properties, and discuss their consistency with the empirical literature. Section 3.5 presents some concluding remarks.

3.2. The model

We build a dynamic general equilibrium model of a closed economy where a single competitively-produced final good can be used in consumption, production of intermediate goods and

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

R&D. The final good is produced by a (large) number of firms, each using labour and a continuum of intermediate goods indexed by $\omega \in [0, N]$. The economy is populated by L identical dynastic families, each endowed with one unit of labour that is inelastically supplied to final-good firms. Thus, the total labour level is L , which, by assumption, is constant over time. In turn, families make consumption decisions and invest in firms' equity.

Potential entrants can devote resources either to horizontal or to vertical R&D. Horizontal R&D increases the number of intermediate-good industries N , while vertical R&D increases the quality of the good of an existing industry, indexed by $j(\omega)$. Quality level $j(\omega)$ translates into productivity of the final producer from using the good produced by industry ω , $\lambda^{j(\omega)}$, where $\lambda > 1$ is a parameter measuring the size of each quality upgrade. By improving on the current best quality index j , a successful R&D firm will introduce the leading-edge quality $j(\omega) + 1$ and hence render inefficient the existing input supplied by the producer of ω . Therefore, the successful innovator will become a monopolist in ω . However, this monopoly, and the monopolist earnings that come with it, are temporary, because a new successful innovator will eventually substitute the incumbent.

3.2.1. Production and price decisions

The final-good firm has a constant-returns-to-scale technology using labour and a continuum of intermediate goods with measure $N(t)$, changing over time t , which is well-known from Barro and Sala-i-Martin (2004, ch. 6 and ch. 7)

$$Y(t) = A \cdot L^{1-\alpha} \cdot \int_0^{N(t)} \left[\lambda^{j(\omega,t)} \cdot X(\omega, t) \right]^\alpha d\omega, \quad 0 < \alpha < 1, \quad \lambda > 1, \quad (3.1)$$

where $A > 0$ is the total factor productivity, L is the labour input and $1 - \alpha$ is the labour share in production, and $\lambda^{j(\omega,t)} \cdot X(\omega, t)$ is the input of intermediate good ω measured

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

in efficiency units at time t .⁶ That is, we integrate the final-producer technology that is considered in variety-expansion (Barro and Sala-i-Martin, 2004, ch. 6) and quality-ladders (Barro and Sala-i-Martin, 2004, ch. 7) models.

Final producers are price-takers in all the markets they participate. They take wages, $w(t)$, and input prices $p(\omega, t)$ as given and sell their output at a price equal to unity. From the profit maximisation conditions, we determine the aggregate demand of intermediate good ω as

$$X(\omega, t) = L \cdot \left(\frac{A \cdot \lambda^{j(\omega, t)\alpha} \cdot \alpha}{p(\omega, t)} \right)^{\frac{1}{1-\alpha}}, \quad \omega \in [0, N(t)]. \quad (3.2)$$

The intermediate-good sector consists of a continuum $N(t)$ of industries. There is monopolistic competition if we consider the whole sector: the monopolist in industry ω fixes the price $p(\omega, t)$ but faces the isoelastic demand curve (3.2). We assume that the intermediate good is non-durable and entails a unit marginal cost of production, in terms of the final good, whose price is taken as given. Profit in industry ω is thus $\pi(\omega, t) = (p(\omega, t) - 1) \cdot X(\omega, t)$, and the profit maximising price is a constant markup over marginal cost⁷

$$p(\omega, t) \equiv p = \frac{1}{\alpha} > 1 \quad (3.3)$$

The quality of the intermediate good ω can be characterised by the quality index

$$q(\omega, t) \equiv \lambda^{j(\omega, t)\frac{\alpha}{1-\alpha}}. \quad (3.4)$$

⁶In equilibrium, only the top quality of each ω is produced and used; thus, $X(j, \omega, t) = X(\omega, t)$. Henceforth, we only use all arguments (j, ω, t) if they are useful for expositional convenience.

⁷We assume that $\frac{1}{\alpha} < \lambda$; i.e., if $\frac{1}{\alpha}$ is the price of the top quality, the price of the next lowest grade, $\frac{1}{\alpha\lambda}$, is less than the unit marginal cost. In this case, lower grades are unable to provide any effective competition, and the top-quality producer can charge the unconstrained monopoly price.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

Then, from (3.2) and (3.3), the aggregate quantity produced of ω is

$$X(\omega, t) = L \cdot (A \cdot \alpha^2)^{\frac{1}{1-\alpha}} q(\omega, t), \quad (3.5)$$

and the profit accrued by the monopolist in ω is

$$\pi(\omega, t) = \pi_0 \cdot L \cdot q(\omega, t), \quad (3.6)$$

which are both linear functions of $q(\omega, t)$. The constant $\pi_0 \equiv (1 - \alpha) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} / \alpha$ can be seen as the “basic” profit flow.⁸

As is well known, if there are no intersectoral spillovers, the aggregate quality index

$$Q(t) = \int_0^{N(t)} q(\omega, t) d\omega \quad (3.7)$$

measures the technological-knowledge level of the economy. This implies that aggregate output,

$$Y(t) = \left(A^{\frac{1}{\alpha}} \alpha^2 \right)^{\frac{\alpha}{1-\alpha}} \cdot L \cdot Q(t), \quad (3.8)$$

total resources devoted to intermediate-goods production,

$$X(t) = \int_0^{N(t)} X(\omega, t) d\omega = (A \alpha^2)^{\frac{1}{1-\alpha}} \cdot L \cdot Q(t), \quad (3.9)$$

and total profits,

$$\Pi(t) = \int_0^{N(t)} \pi(\omega, t) d\omega = \pi_0 \cdot L \cdot Q(t), \quad (3.10)$$

are all linear functions of $Q(t)$.

⁸This is the profit flow, constant over time, that accrues when $j = 0$ (i.e., $q = 1$).

3.2.2. R&D

We assume there is both vertical and horizontal R&D (e.g., Howitt, 1999; Segerstrom, 2000), that R&D is only performed by (potential) entrants, and that successful R&D leads to the set up of a new firm in either an existing or in a new industry. There is perfect competition among entrants and free entry in R&D business. As already stated, these assumptions are consistent with a number of stylised facts on entry (e.g., Geroski, 1995; Klepper, 1996).

The mechanism of entry is a key aspect of our model. A potential entrant has to choose between introducing an innovation in an existing industry, which is specialised in the production of a particular existing variety, or introducing a new variety. Differently from Howitt (1999) and others, we assume that there are no intersectoral spillovers in vertical R&D (e.g., Segerstrom, 2007; Etro, 2008). The entrant then faces a double uncertainty at each point in time: he/she does not know if a successful innovation will occur and in which industry it will take place. First, the arrival of an innovation in any extant industry follows the Schumpeterian creative-destruction process with Poisson distribution. This process may lead to entry, but it will also erode the monopoly power stemming from the innovation, regardless of which type, vertical or horizontal. Then, the dilemma for the potential entrant lies in introducing an innovation that will increase quality in an uncertain but existing industry or creating a new variety. In order to compare the benefits from the two types of entry, we assume that the quality level for the new industry will be the mean quality of the existing industries, while vertical entry will bring about an increase in quality by $\lambda^{\alpha/(1-\alpha)}$ for an existing but arbitrary industry.

3.2.2.1. Vertical R&D free-entry and dynamic arbitrage conditions

As is common in the literature, each new design is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. By improving on the current top quality level $j(\omega, t)$, a successful R&D firm earns monopoly profits from selling the leading-edge input of $j(\omega, t) + 1$ quality to final-good firms. A successful innovation will instantaneously increase the quality index in ω from $q(\omega, t) = q(j)$ to $q^+(\omega, t) = q(j + 1) = \lambda^{\alpha/(1-\alpha)} q(\omega, t)$. In equilibrium, lower qualities of ω are priced out of business.

Let $I_i(j)$ denote the Poisson arrival rate of vertical innovations (vertical-innovation rate) by potential entrant i in industry ω when the highest quality is j . The rate $I_i(j)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R_{vi}(j)$ committed by entrants at time t . As in, e.g., Barro and Sala-i-Martin (2004, ch. 7), $I_i(j)$ features constant returns in R&D expenditures, $I_i(j) = R_{vi}(j) \cdot \Phi(j)$, where $\Phi(j)$ is the R&D productivity factor, which is assumed to be homogeneous across i in ω . We assume

$$\Phi(j) = \frac{1}{\zeta \cdot L \cdot q(j+1)}, \quad (3.11)$$

where $\zeta > 0$ is a constant (flow) fixed vertical-R&D cost. Equation (3.11) incorporates a congestion effect (e.g., Segerstrom, 2007; Etro, 2008), implying vertical-R&D dynamic decreasing returns to scale (i.e., decreasing returns to cumulated R&D).⁹ To avoid the usual scale effect arising from labour level, (3.11) implies that an increase in market scale dilutes the effect of R&D outlays on innovation probability. Aggregating across i in ω , we get $R_v(j) = \sum_i R_{vi}(j)$ and $I(j) = \sum_i I_i(j)$, and thus

⁹The way Φ depends on j implies that the increasing difficulty of creating new qualities exactly offsets the increased rewards from marketing higher qualities - see (3.11) and (3.6). This allows for constant vertical-innovation rate over t and across ω along the BGP, i.e., a symmetric equilibrium (on asymmetric equilibria in quality-ladders models and its growth consequences, see Cozzi, 2007).

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

$$I(j) = R_v(j) \cdot \frac{1}{\zeta \cdot L \cdot q(j+1)}. \quad (3.12)$$

Observe that here $I(\omega, t) = I(j)$ is time-varying, and not constant as is usual in the literature.

As the terminal date of each monopoly arrives as a Poisson process with frequency $I(j)$ per (infinitesimal) increment of time, the present value of a monopolist's profits is a random variable. Let $V(j)$ denote the expected value of an incumbent firm with current quality level $j(\omega, t)$,¹⁰

$$V(j) = \pi(j) \int_t^\infty e^{-\int_t^s [r(\nu) + I(j)] d\nu} ds, \quad (3.13)$$

where r is the equilibrium market real interest rate and $\pi(j)$, given by (3.6), is constant in-between innovations. Free-entry prevails in vertical R&D such that the condition $I(j) \cdot V(j+1) = R_v(j)$ holds, which implies that

$$V(j+1) = \frac{1}{\Phi(j)} = \zeta \cdot L \cdot q(j+1). \quad (3.14)$$

Next, we determine $V(j+1)$ analogously to (3.13), then consider (3.14) and time-differentiate the resulting expression. Thus, if we also consider (3.6), we get the arbitrage condition facing a vertical innovator

$$\zeta = \frac{\pi_0}{r(t) + I(t)}. \quad (3.15)$$

It has several implications:¹¹ firstly, the present value of “basic” profits, using the effective

¹⁰We assume that entrants are risk-neutral and, thus, only care about the expected value.

¹¹Observe that, from (3.6) and (3.12), we have $\frac{\pi(\omega, t)}{\pi(\omega, t)} = I(\omega, t) \cdot \left[\dot{j}(\omega, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$ and $\frac{\dot{R}_v(\omega, t)}{R_v(\omega, t)} - \frac{\dot{I}(\omega, t)}{I(\omega, t)} = I(\omega, t) \cdot \left[\dot{j}(\omega, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$. Then, if we time-differentiate (3.14) by considering (3.13) and the equations above, we get $r(t) = \frac{\pi(j+1) \cdot I(j)}{R_v(j)} - I(j+1)$, which can then be re-written as (3.15).

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

rate of interest $r+I$ as a discount factor, should be equal to the fixed cost of entry; secondly, the rates of entry are symmetric across industries $I(\omega, t) = I(t)$; and, finally, the effective discount rate is constant over time. Then, the vertical-innovation rate is perfectly negatively correlated to the rate of return $I(t) = \pi_0/\zeta - r(t)$.

After solving equation (3.12) for $R_v(\omega, t) = R_v(j)$ and aggregating across industries ω , we determine total resources devoted to vertical R&D, $R_v(t)$, $R_v(t) = \int_0^{N(t)} R_v(\omega, t) d\omega = \int_0^{N(t)} \zeta \cdot L \cdot q^+(\omega, t) \cdot I(\omega, t) d\omega$. As the innovation rate is industry independent, then

$$R_v(t) = \zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot I(t) \cdot Q(t). \quad (3.16)$$

3.2.2.2. Horizontal R&D free-entry and dynamic arbitrage conditions

Variety expansion arises from R&D aimed at creating a new intermediate good. Again, innovation is performed by a potential entrant, which means that, because there is free entry, the new good is produced by new firms. Under perfect competition among R&D firms and constant returns to scale at the firm level, instantaneous entry is obtained as $\dot{N}_e(t) = \frac{1}{\eta} R_{ne}(t)$, where $\dot{N}_e(t)$ is the contribution to the instantaneous flow of the new good by R&D firm e at a cost of η units of the final good and $R_{ne}(t)$ is the flow of resources devoted to horizontal R&D by innovator e at time t . The cost η is assumed to be symmetric. Then, $R_n = \sum_e R_{ne}$ and $\dot{N}(t) = \sum_e \dot{N}_e(t)$, implying

$$R_n(t) = \eta \cdot \dot{N}(t). \quad (3.17)$$

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

We assume that the cost of setting up a new variety (cost of horizontal entry) is increasing in both the number of existing varieties, N , and the number of new entrants, $\dot{N}$,

$$\eta(t) = \phi \cdot N(t)^\sigma \cdot \dot{N}(t)^\gamma, \quad (3.18)$$

where $\phi > 0$ is a fixed (flow) cost, and $\sigma > 0$ and $\gamma > 0$. The dependence of η on N captures the assumption there are dynamic decreasing returns to scale. That is, the scale of the economy induces a negative externality in the form of a barrier to entry because it becomes costlier to introduce new varieties in large growing economies. The dependence of η on $\dot{N}$ means that the entry technology displays static decreasing returns to scale at the aggregate level, which we assume to be entirely external to the firm, but are compatible with the previous assumption of constant returns to scale at the firm level. The existing endogenous-growth literature deals with the two features separately: some models only display dynamic negative externalities (e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004, ch. 6) while others only assume static decreasing returns to scale (e.g., Arnold, 1998; Howitt, 1999; Jones and Williams, 2000).¹²

The dependence of the entry cost on the number of entrants introduces dynamic second-order effects in entry, implying that new varieties are brought to the market gradually, instead of through a lumpy adjustment. This is in line with the stylised facts on entry (e.g., Geroski, 1995): entry occurs mostly at small scale since adjustment costs penalise large-scale entry.

¹²We also depart from Howitt (1999) (see also Segerstrom, 2000) in that he hypothesises decreasing returns to scale to R&D at the firm level. Such an entry technology implies that (keeping our notation) $V = \eta = \frac{dR_n}{dN} > \frac{R_n}{N}$. In contrast, given our assumption of constant returns to scale, we have $V = \eta = \frac{R_n}{N} = \frac{dR_n}{dN}$ (see (3.21), below). Note that the price of entry, V , equals the marginal cost of entry, η , in both cases considered above; nevertheless, the assumption of constant returns eschews positive profits from entering, since $V = \frac{R_n}{N}$.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

We assume the innovator enters with the average quality level of the existing varieties,

$$\bar{q}(t) = \int_0^{N(t)} \frac{q(\omega, t)}{N(t)} d\omega = \frac{Q(t)}{N(t)}, \quad (3.19)$$

which is tantamount to having a uniform distribution with probability $1/N(t)$. As his/her monopoly power will be also terminated by the arrival of a successful vertical innovator in the future, the benefits from entry are given by

$$V(\bar{q}) = \bar{\pi}(t) \int_t^\infty e^{-\int_t^s [r(\nu) + I(\bar{q})] d\nu} ds, \quad (3.20)$$

where $\bar{\pi} = \pi_0 L \bar{q}$. The free-entry condition is now $\dot{N} \cdot V(\bar{q}) = R_n$, which simplifies to

$$V(\bar{q}) = \eta(t). \quad (3.21)$$

Substituting (3.20) into (3.21) and time-differentiating the resulting expression, yields the arbitrage equation facing a horizontal innovator

$$r(t) + I(t) = \frac{\dot{\bar{\pi}}(t)}{\dot{\eta}(t)} \quad (3.22)$$

3.2.2.3. Consistency arbitrage condition

Before deciding which type of R&D to perform, the potential entrant should evaluate the best type of entry. At the margin, she/he should be indifferent between the two types, implying there is a consistency condition between them. If we equate the effective rate of return $r + I$ for both types of entry by considering (3.15) and (3.22), we get the arbitrage

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

condition

$$\bar{q}(t) = \frac{Q(t)}{N(t)} = \frac{\eta(t)}{\zeta \cdot L}. \quad (3.23)$$

This condition is one of the key ingredients of the model. It equates the cost of the horizontal, η , to the average cost of vertical R&D, $\bar{q}\zeta L$. In response to a given exogenous shock, a lower (higher) N enhances (reduces) relative average quality, $1/N$, received by a newly-created variety, in order to compensate for a higher (lower) relative entry cost. The adjustment of the relative average quality $1/N$ guarantees that (3.23) has a finite and determined solution with respect to N without requiring (static) decreasing returns to horizontal R&D. This reflects our assumption of the absence of intersectoral spillovers in vertical innovation. In contrast, models that feature intersectoral spillovers in vertical innovation display a relative average quality which is independent of the number of varieties and, thus, the assumption of static decreasing returns to horizontal R&D (given the constant returns to vertical R&D) is necessary to ensure an interior equilibrium with non-zero vertical and horizontal innovative activity (e.g., Howitt, 1999).

Equation (3.23) can be equivalently recast as

$$\dot{N}(t) = x(Q(t), N(t)) \cdot N(t), \quad (3.24)$$

where

$$x(Q, N) = \left(\frac{\zeta \cdot L}{\phi} \right)^{\frac{1}{\gamma}} \cdot Q^{\frac{1}{\gamma}} \cdot N^{-\left(\frac{\sigma + \gamma + 1}{\gamma} \right)}, \quad (3.25)$$

which clarifies the mechanism of entry adopted in our model by explicitly incorporating a channel between vertical innovation and firm dynamics. It shows that the horizontal-entry

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

rate, $\dot{N}/N$, depends positively on Q and negatively on N : the first effect is an implication of the complementarity between the horizontal-entry rate and the technological-knowledge stock, and the second results from the barriers-to-entry effect incorporated in equation (3.18).

In a small time interval, the rate of growth of average quality is equal to the expected arrival rate of a successful innovation multiplied by the quality shift it introduces: $\dot{\bar{q}}/\bar{q} = I(t)(q^+ - q)/q$, where both the innovation rate and the quality shift are industry-independent. Time-differentiating (3.19), and using (3.24) yields

$$\dot{Q}(t) = (I(t) \cdot \Xi + x(Q(t), N(t))) \cdot Q(t). \quad (3.26)$$

where the quality shift is denoted by $\Xi \equiv (q^+ - q)/q = \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right)$. The innovation rate is endogenous and will be determined as an economy-wide function below. Equation (3.26) introduces a second dynamic interaction between the two types of entry, in this case between the number of varieties and the rate of growth of the quality index of the economy.

3.2.3. Households

The economy has L identical dynastic families who consume and collect income (dividends) from investments in financial assets (equity) and from labour. Each family is endowed with one unit of labour that is inelastically supplied. Thus, total labour supply, L , is exogenous and constant. We assume consumers have perfect foresight concerning the technological change over time and every household chooses the path of consumption $\{C(t), t \geq 0\}$ to maximise discounted lifetime utility

$$U = \int_0^\infty \left(\frac{C(t)^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt, \quad (3.27)$$

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + w(t) - C(t), \quad (3.28)$$

where a denotes household's real financial assets holdings. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $\lim_{t \rightarrow \infty} e^{-\int_0^t r(s) ds} a(t) \geq 0$ is also imposed. The optimal path of consumption Euler equation and the transversality condition are standard:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho) \quad (3.29)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0 \quad (3.30)$$

3.2.4. Macroeconomic aggregation and equilibrium innovation rate

The aggregate financial wealth held by all households is $L \cdot a(t) = \int_0^{N(t)} V(\omega, t) d\omega$, which, from the arbitrage condition between vertical and horizontal entry, yields $L \cdot a(t) = \eta(t) \cdot N(t)$. Taking time derivatives and comparing with (3.28), we get an expression for the aggregate flow budget constraint which is equivalent to the product market equilibrium condition (see Appendix A.1)

$$Y(t) = L \cdot C(t) + X(t) + R_v(t) + R_n(t). \quad (3.31)$$

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

If we substitute the expressions for the aggregate outputs (3.8) and (3.9) and for total R&D expenditures (3.16) and (3.17), we have

$$L \cdot H_Y \cdot Q(t) = L \cdot H_X \cdot Q(t) + L \cdot C(t) + \eta(t) \cdot \dot{N}(t) + \zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot I(t) \cdot Q(t) \quad (3.32)$$

where $H_Y \equiv \left(A^{\frac{1}{\alpha}} \cdot \alpha^2\right)^{\frac{\alpha}{1-\alpha}}$ and $H_X \equiv \left(A \cdot \alpha^2\right)^{\frac{1}{1-\alpha}}$. Solving for I , and using (3.23) and (3.24), we get the endogenous vertical-innovation rate at equilibrium,

$$I(Q, N, C) = \frac{1}{\zeta \cdot \lambda^{\frac{\alpha}{1-\alpha}}} \cdot \left(H_Y - H_X - \frac{C}{Q} - \zeta \cdot x(Q, N) \right), \quad (3.33)$$

as a function which is decreasing in consumption, increasing in the number of varieties and related in an ambiguous way with the aggregate quality level.¹³ However, if the quality level is high, the innovation rate will tend to be negatively related with Q . As function $I(Q, N, C)$ may be negative, the relevant innovation rate at the macroeconomic level is

$$I^+(Q, N, C) = \max \{ I(Q, N, C), 0 \}. \quad (3.34)$$

Again, we emphasise the complementarity between horizontal and vertical innovation, here made clear by the fact that, if N is too *low*, vertical R&D shuts down. This contrasts with, e.g., Peretto (2007) and Peretto and Connolly (2007). In these models, vertical R&D falls to zero when the number of varieties becomes too *high*, a result that basically reflects the assumption that horizontal R&D competes away scarce resources from vertical R&D.

From (3.15), we get the rate of return of capital as $r(Q, N, C) = r_0 - I^+(Q, N, C)$, where $r_0 \equiv \pi_0/\zeta$.

¹³The partial derivative with Q has the sign of $C(t)/Q(t) - \zeta/\gamma$, meaning that it may change over t .

3.2.5. The dynamic general equilibrium

The dynamic general equilibrium is defined by the allocation $\{X(\omega, t), \omega \in [0, N(t)], t \geq 0\}$, by the prices $\{p(\omega, t), \omega \in [0, N(t)], t \geq 0\}$ and by the aggregate paths $\{C(t), N(t), Q(t), I(t), r(t), t \geq 0\}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) vertical, horizontal and consistency free-entry conditions are met; and (iii) markets clear. The equilibrium paths can be obtained from the piecewise-smooth system

$$\dot{C} = \begin{cases} \frac{1}{\theta} \cdot (r_0 - \rho) \cdot C & \text{if } I(Q, N, C) \leq 0 \\ \frac{1}{\theta} \cdot (r_0 - I(Q, N, C) - \rho) \cdot C & \text{if } I(Q, N, C) > 0 \end{cases} \quad (3.35a)$$

$$\dot{Q} = \begin{cases} x(Q, N) \cdot Q & \text{if } I(Q, N, C) \leq 0 \\ (I(Q, N, C) \cdot \Xi + x(Q, N)) \cdot Q & \text{if } I(Q, N, C) > 0 \end{cases} \quad (3.35b)$$

$$\dot{N} = x(Q, N) \cdot N \quad (3.35c)$$

given $Q(0)$ and $N(0)$, and the transversality condition (3.30), which may be re-written as

$$\lim_{t \rightarrow \infty} e^{-\rho t} C(t)^{-\theta} \zeta \cdot L \cdot Q(t) = 0. \quad (3.36)$$

3.3. Equilibrium dynamics

3.3.1. The balanced-growth path

Let $g_y \equiv \dot{y}/y$ represent the growth rate of variable $y(t)$. As the functions in system (3.35a)-(3.35c) are homogeneous, a BGP may exist if the further necessary conditions are verified (see Appendix A.2): (i) the growth rates of consumption and of the quality index are equal, $g_C = g_Q = g$; (ii) the vertical-innovation rate is trendless, $g_I = 0$; and (iii) the growth rates of the quality index and the number of varieties are monotonously related, $g_Q = (\sigma + \gamma + 1) \cdot g_N$. Observe that $x = g_N$ is always positive if $N > 0$.

Proposition 3.1. Assume that $\mu \equiv (r_0 - \rho) / \theta > 0$ and that $\theta \geq 1$. Then there is a BGP only if the long-run level of the vertical-innovation rate is positive,

$$I^* = \frac{\mu(\sigma + \gamma)}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)} > 0, \quad (3.37)$$

and where the endogenous growth rates are also positive

$$g^* = \frac{\mu\Xi(\sigma + \gamma + 1)}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)} > 0, \quad (3.38)$$

$$g_N^* = \frac{\mu\Xi}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)} > 0. \quad (3.39)$$

We should emphasise the result that a BGP will only exist if $I > 0$ and there is no BGP in which $I = 0$. It is convenient to recast system (3.35a)-(3.35c), by using variable x as in (3.25), and variable $z \equiv C/Q$ into an equivalent system in detrended variables

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

$$\dot{x} = \begin{cases} -\left(\frac{\sigma}{\gamma} + 1\right) \cdot x^2 & \text{if } I(x, z) \leq 0 \\ \left[I(x, z) \cdot \Xi \cdot \frac{1}{\gamma} - \left(\frac{\sigma}{\gamma} + 1\right) \cdot x\right] \cdot x & \text{if } I(x, z) > 0 \end{cases} \quad (3.40a)$$

$$\dot{z} = \begin{cases} (\mu - x) z & \text{if } I(x, z) \leq 0 \\ \left[\mu - \left(\frac{1}{\theta} + \Xi\right) \cdot I(x, z) - x\right] \cdot z & \text{if } I(x, z) > 0 \end{cases} \quad (3.40b)$$

Proposition 3.2. Let $H_Y - H_X > \zeta \lambda^{\frac{\alpha}{1-\alpha}} (r_0 - \rho) / (1 + \theta \Xi)$. Then there is a unique BGP, such that $I(x^*, z^*) > 0$, $C^*(t) = z^* \cdot Q^*(t)$, $Q^*(t) = \phi / (\zeta L) \cdot (x^*)^\gamma \cdot N(0)^{\sigma+\gamma+1} \cdot e^{g^* \cdot t}$, $N^*(t) = N(0) \cdot e^{g_N^* \cdot t}$, where

$$x^* = g_N^*, z^* = H_Y - H_X - \zeta \cdot L \cdot \left(1 + \frac{\sigma + \gamma}{\Xi} \cdot \lambda^{\frac{\alpha}{1-\alpha}}\right) \frac{\mu \Xi}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)}. \quad (3.41)$$

Observe that $\lim_{\sigma \rightarrow \infty} g^* = \lim_{\gamma \rightarrow \infty} g^* = g_{no-entry}^*$ and that $g^* > 0$ requires $r^* - \rho > 0$. This condition also guarantees $g_N^* > 0$. Thus, under a sufficiently productive technology, our model predicts a BGP with constant positive g and g_N , where the former exceeds the latter by an amount corresponding to the growth of intermediate-good quality, driven by vertical innovation; to verify this, just replace (3.24) in (3.26) and solve to get $\dot{\bar{q}}/\bar{q} = \dot{Q}/Q - \dot{N}/N = I \cdot \Xi$, which is positive if $I > 0$. This implies that the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, which is consistent with the idea that industrial growth proceeds both along an intensive and an extensive margin. A similar result can be found in, e.g., Arnold (1998), Peretto (1998) and Howitt (1999).

But differently from the knowledge-driven literature, g_N^* is not a function of the (exogenous) population growth rate. The negative externality effect in (3.18) per se determines a constant N along the BGP (provided population growth is zero; see Barro and Sala-i-Martin,

2004, ch. 6);¹⁴ however, the variety expansion is sustained by technological-knowledge accumulation (independently of population growth), as the expected growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an (horizontal) entry cost, in spite of its increase with N . Hence, vertical R&D is the ultimate growth engine. In this sense, it is not necessarily the larger economy, measured by population size, that produces the greater number of varieties, but that with the larger technological-knowledge stock, which thus emerges as the relevant endogenous economic size measure.

Arnold (1998) and Funke and Strulik (2000) also obtain a positive g_N^* that is solely driven by knowledge accumulation. However, this occurs in the form of human-capital production, with the latter counterbalancing the increasing entry cost due to rising real wages caused by the positive impact of a growing N in labour marginal productivity. Evans, Honkapohja, and Romer (1998) build an expanding-variety mechanism with a lab-equipment specification exhibiting dynamic decreasing returns to scale, which is analogous to the one assumed here. In their model, the expansion of varieties is ultimately sustained by physical-capital accumulation. The monotonic positive relationship between g^* and g_N^* is qualitatively similar to the one observed in the models cited above.

We interpret the technological-knowledge stock per firm, $\bar{q}$, as a measure of average firm size (or $1/N$ as firm size relative to market size). Because $\dot{\bar{q}}/\bar{q} = I \cdot \Xi$, it expands at the growth rate of intermediate-good quality along the BGP. Alternative measures of firm size such as production (or sales) per firm, $X/N = H_X Q/N$ (see (3.9)), or financial assets per firm, $a/N = \eta = \zeta LQ/N$ (see (3.23)) produce the same result. Peretto (1998) takes employment per firm as an explicit measure of firm size, while physical capital per firm in efficiency

¹⁴In fact, the dependence of η on N is necessary to eschew the explosive growth that would occur if η were constant over t , or depended solely on $\dot{N}$, thus implying that a BGP would not exist. This is not the case in Barro and Sala-i-Martin (2004)'s basic model of pure expanding variety. It can be shown that the specification $\eta \equiv \eta(Q)$, $\eta' > 0$, $\eta'' < 0$, produces a similar result in our model.

units and human capital per firm can be interpreted as measures of firm size in Aghion and Howitt (1998, ch. 12), and Arnold (1998) and Funke and Strulik (2000), respectively.

These results are broadly supported by historical empirical evidence. The increase in sales per firm over time is referred to, e.g., by Jovanovic (1993) for the US, while Ehrlich (1985) finds a “relative stability of establishment sizes”, measured as employment per establishment, in a long time-series data base for the US, Japan and eight European countries. The increase in the number of firms and establishments over the long run is reported, e.g., by Maddison (1994). The last two empirical regularities are confirmed by Laincz and Peretto (2006), who analyse more recent data for the US.¹⁵

On the relationship between economic size and number of firms, Sherer and Ross (1990) state the robustness of the cross-sectional evidence that shows that large countries (measured by population size) tend to have a larger number of firms (and lower concentration rates); however, the relationship is far less clear when the comparison stands between intermediate and large-size countries. An explanation for this result may be that population size is not the best measure of economic size. Our model suggests that one should focus instead on endogenous measures of economic size, such as the technological-knowledge stock.

3.3.2. Aggregate transitional dynamics

Proposition 3.3. Under the previous assumptions, the BGP is determinate and is saddle-point stable. There is a piecewise-smooth continuous stable manifold which is positively sloped, implying there is a positive correlation between x and z along the transition to the BGP (see Figure 3.1). In addition, there is a magnitude $\tilde{x} \in \left(\mu, \frac{\Xi(\sigma+\gamma+1)+\sigma+\gamma}{\Xi(\sigma+\gamma+1)+\frac{1}{\theta}(\sigma+\gamma)}\mu\right)$, such that:

¹⁵See Gil (2009b) for a detailed discussion of the empirical literature relating firm dynamics with long-run economic growth.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

- (a) If $x(0) > \tilde{x}$, then there is a point $z(0)$ so that the pair (x, z) diminishes until the economy crosses the point $(\tilde{x}, \tilde{z})$ at t_0 . In $t \in [0, t_0]$, we have $I(t) = 0$; from t_0 onwards, $I(t)$ increases and the pair (x, z) will still diminish until it reaches the BGP levels (x^*, z^*) .
- (b) If $x^* < x(0) < \tilde{x}$, we have $0 < I(0) < I^*$ and $I(t)$ increases, while the pair (x, z) diminishes until it reaches the BGP levels.
- (c) If $0 < x(0) < x^*$, then $I(0) > I^* > 0$ and $I(t)$ decreases, while the pair (x, z) increases until it reaches the BGP levels.

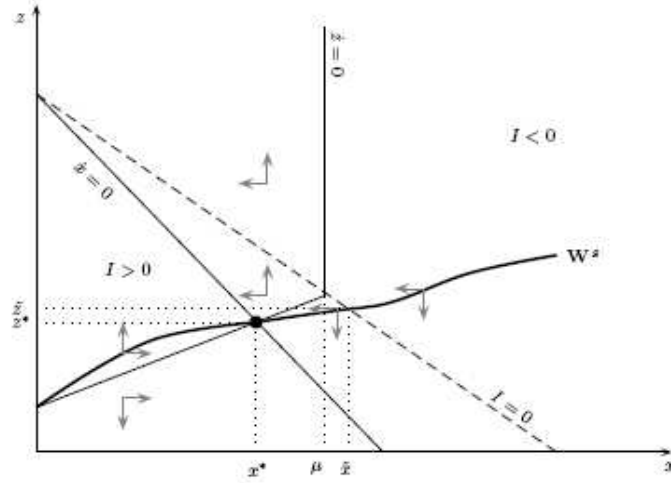


Figure 3.1.: Phase diagram in the detrended variables (x, z) for the piecewise-smooth system (3.40a)-(3.40b). Curves $\dot{x} = 0$ and $\dot{z} = 0$ are the isoclines and curve $\mathbf{W}^s$ is the stable manifold, which is the only equilibrium trajectory.

We focus on the empirically relevant case of $\theta > 1$ and consider deviations of x above its steady-state value, i.e., $x(0) > x^*$. This would be the case of an economy with a N too low

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

relative to Q and, thus, firm size too large: e.g., an emergent-market economy, displaying a shallow market of differentiated goods, in contrast to a mature-market economy, which is expected to boast a deep market of differentiated goods.¹⁶

Proposition 3.4. Consider an economy initially endowed with $N(0)$, such that $x^* < x(0) < \tilde{x}$, and $\theta > 1$. The transitional dynamics is characterised as follows. The economy experiences a decreasing z and $x (= g_N)$; this implies that more resources become available to R_v , boosting I and reducing r ; consequently, g falls due to the downward movement of g_N but less so due to the effect of accelerated vertical innovation, reflecting the increase in I . In stationarised terms, both N and Q grow along the transition path,¹⁷ but the former grows more than the latter, implying a falling firm size; also, C grows less than Q , whereas E decreases.

According to Proposition 3.4, our model exhibits the standard conditional convergence property (falling g and r towards the BGP), although through a specific mechanism. This consists of dynamic second-order effects in horizontal entry (see (3.18)), implying that new varieties are brought to the market gradually, as explained earlier. Since the aggregate production function exhibits constant returns in the accumulated factor, Q , this friction in horizontal entry is the only source of transitional dynamics. Its mechanism operates through imbalances between Q and N , i.e., adjustments in average firm size. An economy

¹⁶We use the following set of baseline parameter values to illustrate the transitional dynamics: $\gamma = 1.2$, $\sigma = 1.2$, $\phi = 1$, $\zeta = 0.9$, $\lambda = 2.5$, $\rho = 0.02$, $\theta = 1.5$, $\alpha = 0.4$, $A = 1$, $L = 1$. Given that along the BGP, $g_Q - g_N = (\sigma + \gamma)g_N$, the choice of values for σ and γ is such that $(\sigma + \gamma) = 2.4$, which is the ratio between the growth rate of the average firm size and the growth rate of the number of firms we have found in the empirical data (the data, which is available from the authors upon request, concerns 23 European countries in the period 1995-2005 and was taken from the Eurostat on-line database; link at <http://epp.eurostat.ec.europa.eu>). The values for λ , θ , ρ and α were set in line with previous work on growth and guided either by empirical findings or by theoretical specification, while the normalisation of A and L to unity at every t implies that all aggregate magnitudes can be interpreted as per capita magnitudes. The values of the remaining parameters were chosen in order to calibrate the BGP aggregate growth rate around 2.5 percent/year.

¹⁷In order to compute stationarised Q , Q_{stat} , let $Q(t) = Q_0 e^{g_Q(t)t}$ (with $Q_0 > 0$) and $Q_{stat}(t) = Q(t)e^{-g^*t}$; hence, $Q_{stat}(t) = Q_0 e^{(g_Q(t) - g^*)t}$. Stationarised N , C and E are computed from Q_{stat} , after numerical integration.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

with very few varieties relative to the technological-knowledge stock (i.e., with very high firm size) starts with a smaller I , a higher g_N , a higher g and a higher r than a mature market economy. A lower I implies a higher r (see (3.15)) and a lower R_v (see (3.12)), which secure the larger resources allocated to R_n . Note also that the higher g is solely justified by the higher g_N . Along the transition path, g and g_N decrease, whereas part of the resources allocated to R_n is gradually re-targeted to R_v . The consequent increase in I implies a falling r .

Thus, g and I are unequivocally negatively related in transitional dynamics. Due to the described inter-R&D composition effect, R&D intensity, $(R_v + R_n) / (Y - X)$, moves in tandem with I . Dinopoulos and Thompson (1998) report a similar re-balancing effect between vertical and horizontal R&D; however, their vertical-innovation rate falls in parallel with aggregate growth and the real interest rate along the transition path (as in Peretto, 1998). On the contrary, the medium-run negative relationship between aggregate growth and the vertical-innovation rate is also apparent in Aghion and Howitt (1998), but only for a specific set of parameter values.¹⁸

The latter results offer a theoretical explanation for the lack of positive correlation between innovation intensity (measured as R&D intensity) and economic growth found recently (e.g., Pessoa, 2010), particularly in countries situated below the technological frontier. In the context of our model, these would be countries approaching the BGP from above, and exhibiting a low speed of transition.¹⁹

¹⁸The mechanism in Aghion and Howitt (1998) is different from ours. They develop a quality-ladders model with physical capital as an input to R&D. Innovation, and hence aggregate growth, is stimulated by a rise in capital intensity towards its BGP level, whilst diminishing marginal returns to physical capital imply per se a fall in the aggregate-growth rate. For some parameter values, economic growth and the innovation rate move in opposite directions along the transition path.

¹⁹Available empirical evidence suggests slow transitions are the case in general. With a standard set of baseline parameters (see fn. 16), our model predicts a speed of convergence to the BGP of 2.2 percent/year, implying a half-life of roughly thirty-two years. This is within the range of estimates given by Barro and Sala-i-Martin (2004, ch. 11) for the US, Japan and several European countries.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

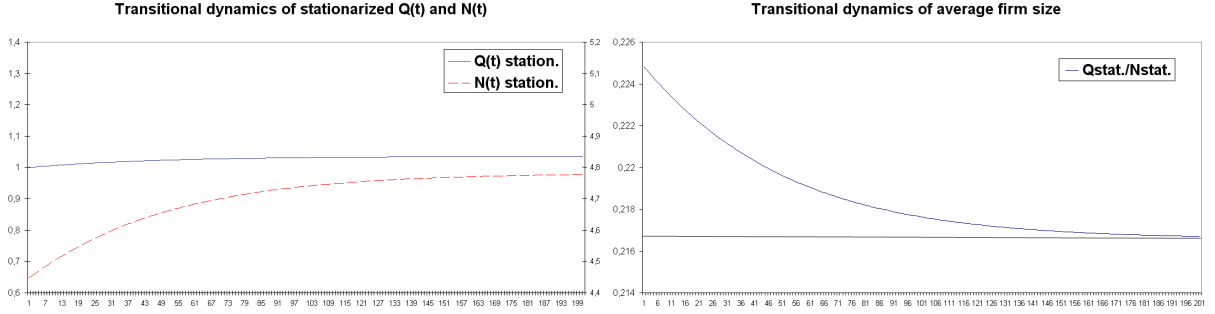


Figure 3.2.: Transitional dynamics of N , Q and average firm size in stationarised terms, when $x(0) > x^*$.

Along the transition path, Q/N and g are positively related (see Figure 3.2). In Aghion and Howitt (1998), firm size, measured as the physical-capital stock per firm in efficiency units, is commanded by the physical-capital stock along the transition path. As long as the model exhibits the convergence property, it produces a positive relationship between firm size and aggregate growth. The models by Peretto (1998), Arnold (1998) and Funke and Strulik (2000) also generate a positive relationship along the transition path between aggregate growth and firm size, measured as employment per firm in the former and the human-capital stock per firm in the other two. Our results then imply that R&D intensity and firm size display a negative relationship along the transition path, although their correlation may be either positive or negative with respect to shifts in the BGP, as explained below (see Section 3.4).

Empirical evidence of a positive correlation between aggregate growth and firm size (employment per firm) in the medium-run is provided by Jovanovic (1993) and Laincz and Peretto (2006), whilst Campbell (1998) reports evidence on the positive correlation between aggregate growth and the rate of entry (corresponding, in our model, to g_N). With respect to the latter relationship, it is noteworthy that the value of less than one predicted by our model is clearly matched by the empirical findings in Campbell (1998).

3.4. Comparative-statics analysis

From (3.23) and Proposition 3.2, we have

$$N^*(t) = \left(\frac{\zeta \cdot L}{\phi} \right)^{\frac{1}{\sigma+\gamma+1}} \cdot (x^*)^{\frac{-\gamma}{\sigma+\gamma+1}} \cdot (Q^*(t))^{\frac{1}{\sigma+\gamma+1}} \quad (3.42)$$

$$E^*(t) = x^* \cdot N^*(t) \quad (3.43)$$

Differentiate (3.38), (3.39), (3.42) and (3.43) with respect to the relevant structural parameters. The following proposition summarises the main results.

Proposition 3.5.

- (a) The aggregate growth rate, g^* , and the growth rate of the number of varieties, g_N^* , are: (i) decreasing in the fixed cost of vertical R&D, ζ , and in the elasticities of the horizontal entry cost function, σ and γ ; (ii) increasing in the size of quality upgrade, λ ; (iii) independent of the fixed cost of horizontal entry, ϕ .
- (b) For a given Q^* , the number of firms in the differentiated-good sector, N^* , (alternatively, firm size as measured by Q^*/N^*) is decreasing (increasing) in σ , ϕ and λ , and is increasing (decreasing) in γ and ζ .²⁰ Instantaneous entry, E^* , for a given Q^* , is decreasing in σ , ϕ and ζ , and is increasing in γ and λ .

The results in part (a) pertain to the growth effects and part (b) to the level effects of changes in the structural parameters. The lack of relationship between g^* and ϕ is noteworthy. Intuitively, it results from the dominant effect exerted by the vertical-innovation

²⁰In some particular cases, the alternative measures of firm size $\frac{X}{N} = H_X \frac{Q}{N}$ and $\frac{a}{N} = \zeta L \frac{Q}{N}$ yield different comparative static results from those in the text. For example, $\frac{a^*}{N^*}$ is increasing in ζ , whilst $\frac{Q^*}{N^*}$ and $\frac{X^*}{N^*}$ are both decreasing in that parameter.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

mechanism (the intensive margin) over the horizontal-entry dynamics (the extensive margin). Given the postulated horizontal entry technology, an equilibrium BGP with positive net entry occurs ultimately because entrants expect incumbency value to grow propelled by quality-enhancing R&D. However, ϕ influences N^* and E^* , reducing both - and thus increasing firm size - for a given value of Q^* , due to its impact on (3.23). Arnold (1998) and Funke and Strulik (2000) obtain a similar result with respect to both the aggregate growth rate and firm size, while Peretto (1998) predicts a similar relationship with firm size but the author establishes a positive relationship with the aggregate growth rate.

Changes in σ and in γ have a negative impact on g^* , but their effects contrast when it comes to the impact on N^* and E^* . Parameter σ exerts a first-order effect on N^* and E^* , qualitatively similar to the impact due to changes in ϕ , whereas γ exerts a second-order effect. These results suggest that industry policies aimed at reducing the variable costs faced by entrants may have opposing outcomes with respect to their impact on the market structure and industry dynamics, depending on whether they target σ or γ .

For a given Q^* , the correlation between N^* and g_N^* tends to be negative. This conforms with some of the empirical evidence on the rate of entry and the number of firms reported, e.g., by Sherer and Ross (1990). Since the same relationship applies to g^* , our model also predicts that economies with higher long-run growth rates tend to have a market structure characterised by larger firms, in line with Peretto (1998) and Aghion and Howitt (1998).²¹ A positive empirical relation between firm size (employment per firm) and the aggregate growth rate is found by Pagano and Schivardi (2003).

Our model offers mixed results with respect to the correlation between the vertical-innovation

²¹A positive correlation between firm size and growth is the general case in our model. However, that correlation is negative when a shift in the BGP is due to a variation in σ . Likewise, in Aghion and Howitt (1998), a change in the exogenous labour growth rate (implying a change in the rate of creation of differentiated goods, an effect qualitatively similar to a change in σ in our model) also implies a negative relationship between these two variables.

3. Growth and Firm Dynamics with Horizontal and Vertical R&D

rate, I^* (see (3.37)), and both g^* and firm size. Shifts in the BGP due to changes in the size of quality upgrade, λ , or in the elasticities of the entry cost function, σ and γ , yield a *negative* relationship between the two variables, whilst changes in the preferences parameters or in ζ give rise to a *positive* correlation. The same results apply to the relationship between g^* (or firm size) and R&D intensity, $(R_v^* + R_n^*) / (Y^* - X^*)$. The impact of changes in σ and γ (respectively, λ) is explained by an endogenous inter-R&D composition effect: an increase in these two parameters (decrease) induces a shift of resources from horizontal R&D to vertical R&D, but the effect on g^* of the increment in the vertical type is dominated by the decrease in the horizontal one. In the end, the observed negative correlation between g^* and R&D intensity reflects the negative correlation between g^* and R_v^* , whose effect outweighs the positive relation between g^* and R_n^* . A similar reasoning would apply to the relation between firm size and R&D intensity. Also in Peretto (1998), Dinopoulos and Thompson (1998) and others, a change in horizontal entry costs induces a change of the vertical-innovation rate in the same direction. However, this implies a positive correlation with aggregate growth, because the latter changes due solely to the intensive margin (the extensive margin is linked to the exogenous population growth rate).

According to, e.g., Bassanini, Scarpetta, and Visco (2000), empirical studies in general find a strong positive relationship between R&D intensity and growth at the sectoral and firm level, but a clear link is usually difficult to establish at the aggregate (cross-country) level. On the other hand, Pagano and Schivardi (2003) note that empirical firm-level studies on the relationship between firm size and innovation have failed to reach a clear conclusion. Nevertheless, those authors conduct a cross-section study at the aggregate level and find that average firm size matters for growth through its effects on R&D intensity, thus implicitly establishing a positive correlation between firm size and innovation intensity. The literature often places the emphasis on the complexity of the link between these variables

(Audretsch and Keilbacha, 2008), and the several conceptual and measurement problems that still afflict empirical analysis in this field (Bartelsman, Haltiwanger, and Scarpetta, 2005) to explain the lack of robust results. By producing mixed results in this respect, connected to the existence of an endogenous inter-R&D composition effect, our model lends theoretical support to the lack of clear-cut empirical findings. In other words, it shows that the shift of resources between vertical and horizontal R&D may hide the underlying positive relationship between R&D intensity and growth.

3.5. Conclusion

In this essay, we develop a non-scale tournament endogenous-growth model, where the expanding variety and the quality-ladders mechanism are merged under a lab-equipment specification. This assumption implies that the choice between vertical and horizontal innovation is related to the splitting of R&D expenditures, which are fully endogenous. Therefore, we endogenise the overall growth rate and thus the rate of extensive growth. Our framework gives rise to strong endogenous inter-R&D composition effects and makes economic growth and firm dynamics closely related: vertical R&D is the ultimate growth engine, whilst horizontal R&D builds an explicit link between aggregate and firm-dynamics variables.

The model predicts, under a sufficiently productive technology, a stationary BGP with constant positive growth rates, and where the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the general view that industrial growth proceeds both along an intensive and an extensive margin. The growth of the number of varieties is sustained by (endogenous) technological-knowledge accumulation, as the expected growth of intermediate-good quality makes it

3. *Growth and Firm Dynamics with Horizontal and Vertical R&D*

attractive for potential entrants to always put up an entry cost, in spite of its upward trend. In this setting, it is not necessarily the larger economies, measured by population size, that produce the greater number of distinct goods. Instead, the relevant distinction herein is based on an endogenous measure of economic size, such as the technological-knowledge stock.

We obtain specific results with respect to the impact of changes in the entry-cost parameters both in the aggregate growth rate and in the market structure along the BGP. We highlight (i) the lack of relationship between economic growth and the fixed horizontal entry cost, but the positive relation between the latter and firm size, (ii) the contrasting effect of changes in the two elasticity parameters of the entry cost function in firm size, (iii) the positive correlation between firm size and economic growth, and (iv) the mixed result with respect to the BGP relation between R&D intensity and both the aggregate growth rate and firm size.

We also conclude that the model exhibits the standard convergence property, though based on a rather distinct mechanism. The model produces results that differ from (or expand the results of) the early models of quality ladders with expanding variety. In particular, we obtain as a general result that medium-term economic growth and firm size are positively correlated, whereas R&D intensity and both medium-term economic growth and firm size move in opposite directions. The former result adds to the theoretical predictions already found in the literature, of positive correlation between economic growth and firm size measured either as employment per firm, human-capital stock per firm or physical-capital stock per firm in efficiency units, and which have had wide empirical support. Importantly, the latter result – together with (iv), above – offers a theoretical explanation for the insignificant or negative empirical correlation between aggregate R&D intensity and both economic growth and firm size, a well-known puzzle in the growth literature.

4. Directed Technical Change, Scale Effects and Industrial Structure

Abstract: *This essay represents a first attempt to devise an analytical mechanism to explain the data on the distribution of firms and production across high- and low-tech sectors. It builds a model of endogenous directed technical change that merges the expanding-variety with the quality-ladders mechanism, and explores the complementarity between them within a lab-equipment specification and under distinct regimes of scale effects. The model generates a specific set of balanced-growth-path results with respect to the relationship between industrial structure, concentration and economic growth. Under small positive scale effects, the model is able to account for all the empirical regularities.*

Keywords: *endogenous growth; industrial structure; high- and low-tech sectors; directed technical change; scale effects.*

JEL Classification: *O41, D43, L11, L16*

4.1. Introduction

The OECD classification of high- and low-tech sectors has been considered in a number of recent studies, namely in order to analyse empirically the impact of technology on industrial performance and globalisation (e.g., Pilat, Cimper, Olsen, and Webb, 2006), and on wage inequality (e.g., Cozzi and Impullitti, 2008). What our essay does is to look at the industrial structure *per se*, defined as the distribution of firms and production across high- and low-tech sectors. Empirical data, as illustrated by Figure B.1 (see Appendix B.1), suggests a significant variability of industrial structures across countries, considering the number of firms, total production and average firm size across high- and low-tech sectors. A few regularities can be however pointed out: (i) the number of firms and total production are smaller in high- than in low-tech sectors (i.e., *relative* number of firms and relative production below unity); (ii) average firm size is larger in the high-tech sectors (i.e., *relative* average firm size above unity); (iii) relative production tends to be positively (cross-country) correlated to the relative number of firms; (iv) the relative number of firms tends to be (slightly) negatively (cross-country) correlated to relative average firm size.

This essay proposes, for the first time in the literature, an analytical mechanism to explain those empirical facts. In particular, we develop a model of endogenous directed technical change (e.g., Acemoglu, 1998; Kiley, 1999; Acemoglu and Zilibotti, 2001) that combines vertical and horizontal R&D and scale effects related with factor endowment to analyse the industrial structure as characterised by the number of monopolistic firms, production and average firm size across sectors. Final-goods production uses either low- or high-skilled labour with labour-specific intermediate goods, while R&D can be directed to either the low- or the high-skilled labour complementary technology; hence, “sector” herein represents a group of firms producing the same type of labour-complementary intermediate goods.

As in, e.g., Cozzi and Impullitti (2008), we consider OECD high- and low-tech sectors as the empirical counterpart of the high- and low-skilled labour-complementary intermediate-good sectors.

This essay follows the “creative destruction”, or tournament, tradition in the simultaneous treatment of horizontal and vertical R&D: the entrant must choose between vertical and horizontal R&D, whereas the incumbent does not perform any R&D (e.g., Howitt, 1999; Segerstrom, 2000).¹ However, we consider a lab-equipment specification, whereas the literature predominantly assumes that R&D is knowledge-driven.² In the last case, the choice between vertical and horizontal innovation implies a division of labour between the two types of R&D. Since the total labour level is determined exogenously, in the end, the rate of extensive growth is exogenous, i.e., the balanced-growth-path (BGP) flow of new goods is a function of population growth. The alternative assumption that R&D is of the lab-equipment type implies the choice between vertical and horizontal innovation is related to the splitting of R&D expenditures, which are fully endogenous. Therefore, we endogenise the overall growth rate and thus the rate of extensive growth.

In particular, we develop a dynamic general equilibrium model where a quality-ladders mechanism provides the accumulation of non-physical capital (technological knowledge), whilst an expanding-variety mechanism offers the flow of new firms (new product lines).

Average firm size is measured by either production (sales) or financial assets per firm, both

¹Thus, we depart from the non-tournament approach, which postulates vertical R&D is made by the incumbent and horizontal R&D is carried out by entrants (e.g., Peretto, 1998, 1999; Dinopoulos and Thompson, 1998; Peretto and Smulders, 2002). Indeed, according to the stylised facts on entry (e.g., Geroski, 1995; Klepper, 1996), it (i) occurs mostly in already existing sectors, (ii) is driven mostly by new firms (instead of incumbents enduring diversification processes), and (iii) is closely related to R&D activities. In a model of R&D and firm dynamics under monopolistic competition, like ours, where R&D activity is the only mechanism by which entrants can make a competitive entry (by introducing either a new variety or a higher-quality version of an existing product), that implies that not only horizontal but also vertical R&D must be performed by entrants.

²Using Rivera-Batiz and Romer (1991)’s terminology, the assumption that homogeneous final good is the R&D input means that one adopts the “lab-equipment” version of R&D, instead of the “knowledge-driven” specification, in which labour is ultimately the only input.

commanded by technological-knowledge stock per firm. This framework makes economic growth and firm dynamics closely related: vertical R&D is the ultimate growth engine, while horizontal R&D permits an explicit link between aggregate and industry-level variables (number of firms, firm size and entry rate).³ By recognising the complementarity between R&D types within a lab-equipment specification, we find a specific set of results regarding the relation between industrial structure, concentration and economic growth.

In lab-equipment models, the scale effects are connected with the size of profits that accrue to the R&D successful firm; a larger market expands profits and, thus, the incentives to allocate resources to R&D, thereby increasing the aggregate growth rate.⁴ However, an increase in market scale also dilutes the impact of R&D outlays on innovation probability (e.g., Barro and Sala-i-Martin, 2004), since coordination, organisational, informational, marketing and transportation costs (e.g., Dinopoulos and Thompson, 1999) and rental protection actions by incumbents (e.g., Sener, 2008) (positively) related to market size make the introduction of new goods and the replacement of old ones increasingly difficult as the market grows. However, depending on the effectiveness of the referred costs and actions, these may partially, totally or over counterbalance the benefits of scale to innovative activity.

Accordingly, we consider a priori three distinct regimes of (net) scale effects: (i) large positive; (ii) small positive; and (iii) negative scale effects. By focusing on the BGP, we find that, depending on the regime of scale effects, the relationship between industrial structure and relative labour endowment (the ratio between high- and low-skilled labour

³This point has been made recently by Peretto and Connolly (2007), in a different analytical setup. Like Peretto (1998, 1999) and others, the authors build an endogenous growth model where incumbents do deterministic cost-reducing R&D, while entrants bring new products to the market, under a knowledge-driven specification. They consider fixed operating costs, which, unless the exogenous population growth rate is positive, determine a constant number of varieties along the BGP. Under this setting, only vertical R&D allows for growth “unconstrained by endowments”.

⁴The basic theory of industrial structure in IO literature suggests that concentration is determined by economies of scale relative to the market size. In our model, the role of market size is quite different.

measured in efficiency units) may differ significantly. Indeed, our results make clear the contrast between industrial structure under positive but small scale effects and under either negative or large positive scale effects. For instance, if there are small positive scale effects, the number of firms is greater in the sector with smaller average firm size, implying a right-skewed average-size distribution. In contrast, negative or large positive scale effects imply a left-skewed distribution, i.e., there are more firms in the sector with larger average firm size.

The model also provides a theoretical instrument to study the association between the relative labour endowment and concentration under each regime of scale effects. We consider a two-dimension aggregate concentration index by combining the share of the number of firms with the share of the output (sales) of each sector (high- and low-tech). The index compares sectors at the economy-wide level instead of individual firms within a sector, thus accommodating the view that traditional industry boundaries become decreasingly useful for economic analysis as new products and processes compete *across* industries (e.g., Kamien and Schwatz, 1975), which is a salient feature of our model. Under small positive scale effects, the index displays an inverted U-shaped behaviour with respect to the relative labour endowment, with the lower boundary corresponding to *total concentration of firms in one sector and of production in the other* and the upper boundary to a *uniform distribution of firms and production across sectors*. Under either negative or large positive scale effects, the index displays an U-shaped behaviour, with the lower boundary corresponding to the *uniform-distribution* case and the upper boundary to *total concentration of firms and production in a single sector*.

However, ultimately, only under a well-defined interval of small positive scale effects are the theoretical results able to account for the empirical data on industrial structure. The model is thus compatible with the well-known endogenous-growth debate over the counterfactual

character of large positive scale effects (e.g., Jones, 1995b).

In recognising the endogenous character of the determinants of industrial structure, the literature has frequently emphasised the interrelation between structure, and technology and innovative activity (e.g., Dasgupta and Stiglitz, 1980; Sutton, 1998). A recent strand of the literature studies the interplay between structure, innovative activity and long-term aggregate growth. Whereas some papers focus on the strategic interaction of firms in an oligopolistic framework (e.g., van de Klundert and Smulders, 1997; Peretto, 1999; Aghion, Bloom, Blundell, Griffith, and Howitt, 2005), others emphasise the role of a specific factor within a monopolistic-competition setting – e.g., the number of firms (Peretto and Smulders, 2002), average firm size (e.g., Peretto, 1998) or firm-size distribution (e.g., Thompson, 2001; Klette and Kortum, 2004). This essay relates closely to this literature, and in particular to the last set of papers, as it conducts the study of the interplay between growth, innovative activity and industrial structure in the context of monopolistic competition, using average firm size as the pivotal variable.

On the other hand, there are several studies analysing the link between the distribution of economic activity across industries and growth, either empirically (e.g., Fagerberg, 2000) or theoretically (e.g., Ngai and Pissarides, 2007; Bonatti and Felice, 2008; Acemoglu and Guerrieri, 2008). They derive the implications of different sectoral Total Factor Productivity (TFP) growth rates, suggesting that countries specialised in “technologically progressive” industries (high TFP growth) enjoy higher growth rates. By analysing the same particular dimension of industrial structure, our essay is related with this literature. However, by building on a mechanism of endogenous directed technical change, it is substantially different, namely by predicting constant TFP growth rates across sectors along the BGP (see Acemoglu and Zilibotti, 2001). Thus, concerning the link between growth and industrial structure, our results are set in *quantitative* terms, i.e., how many firms and how much

production are allocated to each sector vis-à-vis the others, and not in *qualitative* terms, i.e., in which specific sector is economic activity concentrated.

The remainder of the essay has the following structure. In Section 4.2, we give an overview of the model of directed technological change with vertical and horizontal R&D, present the general equilibrium and analyse the BGP properties of the model. In Section 4.3, we detail the comparative statics results, deriving predictions with respect to the relationship between BGP industrial structure, relative labour endowment and the regime of scale effects. Section 4.4 gives some concluding remarks.

4.2. The model

The model used herein is well-known from Acemoglu and Zilibotti (2001), augmented with vertical R&D, as introduced in Afonso (2006), and developed under varying regimes of scale effects. Thus, we study a directed technological change model with vertical and horizontal R&D, built into a dynamic general equilibrium setup of a closed economy where a single competitively-produced final good can be used in consumption, production of intermediate goods and R&D. The economy is populated by a fixed number of infinitely-lived households who inelastically supply one of two types of labour to final-good firms: low-skilled, L , and high-skilled labour, H . The final good is produced by a continuum of firms, indexed by $n \in [0, 1]$, to which two substitute technologies are available: the “Low” (respectively, “High”) technology uses a combination of L (H) and a continuum of L -specific (H -specific) intermediate goods indexed by $\omega_L \in [0, N_L]$ ($\omega_H \in [0, N_H]$).

Potential entrants can devote resources to either horizontal or vertical R&D, and directed to either the high- or the low-skilled labour-complementary technology. Horizontal R&D increases the number of industries, N_m , $m \in \{L, H\}$, in the m -complementary intermediate-

good sector,⁵ while vertical R&D increases the quality level of the good of an existing industry, indexed by $j_m(\omega_m)$. Then, the quality level $j_m(\omega_m)$ translates into productivity of the final producer from using the good produced by industry ω_m , $\lambda^{j_m(\omega_m)}$, where $\lambda > 1$ is a parameter measuring the size of each quality upgrade. By improving on the current best quality index j_m , a successful R&D firm will introduce the leading-edge quality $j_m(\omega_m) + 1$ and hence render inefficient the existing input supplied by the producer of ω_m . Therefore, the successful innovator will become a monopolist in ω_m . However, this monopoly, and the monopolist earnings that come with it, are temporary, because a new successful innovator will eventually substitute the incumbent.

4.2.1. Production and price decisions

Aggregate output at time t is defined as $Y(t) = \int_0^1 P(n, t) Y(n, t) dn$, which, since it is the numeraire, i.e., $\exp \left[\int_0^1 \ln P(n) dn \right] = 1$, may be recast as $Y(t) = \exp \left[\int_0^1 \ln Y(n, t) dn \right]$. Each final-good firm n has a constant-returns-to-scale technology using low- and high-skilled labour and a continuum of labour-specific intermediate goods with measure $N_m(t)$, $m \in \{L, H\}$, so that $N_{tot}(t) = N_L(t) + N_H(t)$ and

$$Y(n, t) = A \left[\int_0^{N_L(t)} (\lambda^{j_L(\omega_L, t)} \cdot X_L(n, \omega_L, t))^{1-\alpha} d\omega_L \right] [(1-n) \cdot l \cdot L(n)]^\alpha + A \left[\int_0^{N_H(t)} (\lambda^{j_H(\omega_H, t)} \cdot X_H(n, \omega_H, t))^{1-\alpha} d\omega_H \right] [n \cdot h \cdot H(n)]^\alpha, \quad 0 < \alpha < 1, \quad (4.1)$$

where $A > 0$ is the total factor productivity, $L(n)$ and $H(n)$ are the labour inputs used by firm n and α is the labour share in production, and $\lambda^{j_m(\omega_m, t)} \cdot X_m(n, \omega_m, t)$ is the input of m -complementary intermediate good ω_m measured in efficiency units at time t .⁶ An *absolute-*

⁵Henceforth, we will refer to the “ m -complementary intermediate-good sector” as “sector m ”.

⁶In equilibrium, only the top quality of each ω_m is produced and used; thus, $X_m(j, \omega_m, t) = X_m(\omega_m, t)$. Henceforth, we only use all arguments (j, ω_m, t) if they are useful for expositional convenience.

4. Directed Technical Change, Scale Effects and Industrial Structure

productivity advantage of H over L is captured by $h > l \geq 1$; a *relative*-productivity advantage of each labour type is determined by terms n and $(1 - n)$, implying that H is relatively more productive for larger n , and vice-versa. As explained below, at each t there is a competitive equilibrium threshold $\bar{n}$, which is endogenously determined, where the switch from one technology to the other becomes advantageous, so that each firm n produces exclusively with one technology, either L - or H -technology.

Following Acemoglu and Zilibotti (2001), we start by considering N_m as given. Final producers take the price of their final good, $P(n, t)$, wages, $w_m(t)$, and input prices $p_m(\omega_m, t)$ as given. From the profit maximisation conditions, we determine the demand of intermediate good ω_m by firm n

$$\begin{aligned} X_L(n, \omega_L, t) &= (1 - n) \cdot l \cdot L(n) \cdot \left[\frac{A \cdot P(n, t) \cdot (1 - \alpha)}{p_L(\omega, t)} \right]^{\frac{1}{\alpha}} \lambda^{j(\omega_L, t) \left(\frac{1 - \alpha}{\alpha} \right)} \\ X_H(n, \omega_H, t) &= n \cdot h \cdot H(n) \cdot \left[\frac{A \cdot P(n, t) \cdot (1 - \alpha)}{p_H(\omega, t)} \right]^{\frac{1}{\alpha}} \lambda^{j(\omega_H, t) \left(\frac{1 - \alpha}{\alpha} \right)} \end{aligned} \quad (4.2)$$

Intermediate-good sector m consists of a continuum $N_m(t)$ of industries. There is monopolistic competition if we consider the whole sector: the monopolist in industry $\omega_m \in [0, N_m(t)]$ fixes the price $p_m(\omega_m, t)$ but faces an isoelastic demand curve, $X_L(\omega_L, t) = \int_0^{\bar{n}} X_L(n, \omega_L, t) dn$ or $X_H(\omega_H, t) = \int_{\bar{n}}^1 X_H(n, \omega_H, t) dn$ (see (4.2)). We assume that intermediate goods are non-durable and entail a unit marginal cost of production, in terms of the final good, whose price is taken as given. Profit in ω_m is thus $\pi_m(\omega_m, t) = (p_m(\omega_m, t) - 1) \cdot X_m(\omega_m, t)$, and the profit maximising price is a constant markup over marginal cost

$$p_m(\omega_m, t) \equiv p = \frac{1}{1 - \alpha} > 1, \quad m \in \{L, H\}. \quad (4.3)$$

Given $\bar{n}$ and (4.3), we can then write the final-good output as

$$Y(n, t) = \begin{cases} A^{\frac{1}{\alpha}} P(n, t)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot (1-n) \cdot l \cdot L(n) \cdot Q_L(t) & , 0 \leq n \leq \bar{n} \\ A^{\frac{1}{\alpha}} P(n, t)^{\frac{1-\alpha}{\alpha}} \cdot (1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot n \cdot h \cdot H(n) \cdot Q_H(t) & , \bar{n} \leq n \leq 1 \end{cases}, \quad (4.4)$$

where the aggregate quality index

$$Q_m(t) = \int_0^{N_m(t)} q_m(\omega_m, t) d\omega, \quad q_m(\omega_m, t) \equiv \lambda^{j_m(\omega_m, t)(\frac{1-\alpha}{\alpha})}, \quad m \in \{L, H\}, \quad (4.5)$$

measures the technological-knowledge level in each sector m . With competitive final-good producers, economic viability of either L - or H -technology relies on the relative productivity and price of labour, as well as on the relative productivity and prices of intermediate goods, due to complementarity in production. Labour prices depend on quantities, H and L . In relative terms, the productivity-adjusted quantity of H is $\mathcal{H}/\mathcal{L}$, where $\mathcal{H} \equiv hH$ and $\mathcal{L} \equiv lL$. As for the productivity and prices of intermediate goods, they depend on complementarity with either H or L , on the technological knowledge embodied and on the mark-up. These determinants are summed up in Q_L and Q_H . The endogenous threshold $\bar{n}$ follows from equilibrium in the inputs markets, and relies on the determinants of economic viability of the two technologies, such that

$$\bar{n}(t) = \left[1 + \left(\frac{\mathcal{H} Q_H(t)}{\mathcal{L} Q_L(t)} \right)^{\frac{1}{2}} \right]^{-1}. \quad (4.6)$$

$\bar{n}$ implies that L - (H -)complementary technology is exclusively used by final-good firms indexed by $n \in [0, \bar{n}]$ ($n \in [\bar{n}, 1]$), and it can be related to the ratio of price indices of final goods produced with L - and H -technologies:

$$\frac{P_H(t)}{P_L(t)} = \left(\frac{\bar{n}(t)}{1 - \bar{n}(t)} \right)^\alpha, \text{ where } \begin{cases} P_L(t)^{\frac{1}{\alpha}} = P(n, t)^{\frac{1}{\alpha}} \cdot (1 - n) = e^{-\alpha} \bar{n}(t)^{-\alpha} \\ P_H(t)^{\frac{1}{\alpha}} = P(n, t)^{\frac{1}{\alpha}} \cdot n = e^{-\alpha} (1 - \bar{n}(t))^{-\alpha} \end{cases}. \quad (4.7)$$

In (4.7), we first define the price indices, P_L and P_H , by recognising that, in equilibrium, the marginal value product, $\frac{\partial}{\partial m(n)} (P(n, t)Y(n, t))$, must be constant over n , implying that $P(n, t)^{\frac{1}{\alpha}} \cdot (1 - n)$ and $P(n, t)^{\frac{1}{\alpha}} \cdot n$ must be constant over $n \in [0, \bar{n}]$ and $n \in [\bar{n}, 1]$, respectively. Then, by considering that at $\bar{n}$ the L - and the H - technology firms must break even, we relate P_L and P_H with $\bar{n}$. Equation (4.6) shows that if either the technology is highly H -biased or if there is a large relative supply of H , the share of final goods using the H -technology is large and $\bar{n}$ is small. By (4.7), small $\bar{n}$ implies a low P_H/P_L . In this case, the demand for $\omega_H \in [0, N_H]$ is low, which discourages R&D activities directed to H -technology.⁷

From (4.2), (4.3) and (4.7), we find the optimal intermediate-good production, $X_m(\omega_m)$, and, thus, the optimal profit accrued by the monopolist in ω_m

$$\pi_m(\omega_m, t) = \pi_{0m} \cdot P_m(t)^{\frac{1}{\alpha}} \cdot q_m(\omega_m, t), \quad m \in \{L, H\}, \quad (4.8)$$

where $\pi_{0L} \equiv \mathcal{L}A^{\frac{1}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) (1 - \alpha)^{\frac{2}{\alpha}}$ and $\pi_{0H} \equiv \mathcal{H}A^{\frac{1}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) (1 - \alpha)^{\frac{2}{\alpha}}$ are positive constants.

Total intermediate-good optimal production, $X_{tot} \equiv X_L + X_H \equiv \int_0^{N_L} X_L(\omega_L) d\omega_L + \int_0^{N_H} X_H(\omega_H) d\omega_H$, and total final-good optimal production, $Y_{tot} \equiv Y_L + Y_H \equiv \int_0^{\bar{n}} P(n)Y(n)dn + \int_{\bar{n}}^1 P(n)Y(n)dn$, are, respectively,

⁷Thus, labour structure affects the direction of R&D through the price channel, which appears in various papers, e.g., Acemoglu (2002), although always dominated by the market-size channel.

$$X_{tot}(t) = A^{\frac{1}{\alpha}} \cdot (1 - \alpha)^{\frac{2}{\alpha}} \cdot \left(P_L(t)^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot Q_H(t) \right) \quad (4.9)$$

and

$$Y_{tot}(t) = A^{\frac{1}{\alpha}} \cdot (1 - \alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot \left(P_L(t)^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot Q_H(t) \right). \quad (4.10)$$

X_{tot} and Y_{tot} are thus functions of the currently given Q_L and Q_H .

4.2.2. R&D

We assume there is both vertical and horizontal R&D (e.g., Howitt, 1999; Segerstrom, 2000), that R&D is only performed by (potential) entrants, and that successful R&D leads to the set up of a new firm in either an existing or in a new industry. There is perfect competition among entrants and free entry in R&D business. These assumptions are consistent with a number of stylised facts on entry (e.g., Geroski, 1995; Klepper, 1996).

The mechanism of entry is a key aspect of our model. A potential entrant has to choose between introducing an innovation in an existing industry, which is specialised in the production of a particular existing variety, or introducing a new variety. Differently from Howitt (1999) and others, we assume that there are no intersectoral spillovers in vertical R&D (e.g., Segerstrom, 2007; Etro, 2008). The entrant then faces a double uncertainty at each point in time: he/she does not know if a successful innovation will occur and in which industry it will take place. First, the arrival of an innovation in any extant industry follows the Schumpeterian creative-destruction process with Poisson distribution. This process may lead to entry, but it will also erode the monopoly power stemming from the innovation, regardless of which type, vertical or horizontal. Then, the dilemma for the potential entrant lies in introducing an innovation that will increase quality in an uncertain

but existing industry or creating a new variety. In order to compare the benefits from the two types of entry, we assume that the quality level for the new industry will be the mean quality of the existing industries, while vertical entry will bring about an increase in quality by $\lambda^{(1-\alpha)/\alpha}$ for an existing but arbitrary industry.

4.2.2.1. Vertical R&D free-entry and dynamic arbitrage conditions

As is common in the literature, each new design is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. By improving on the current top quality level $j_m(\omega_m, t)$, a successful R&D firm earns monopoly profits from selling the leading-edge input of $j_m(\omega_m, t) + 1$ quality to final-good firms. A successful innovation will instantaneously increase the quality index in ω_m from $q_m(\omega_m, t) = q_m(j_m)$ to $q_m^+(\omega_m, t) = q_m(j_m + 1) = \lambda^{(1-\alpha)/\alpha} q_m(\omega_m, t)$. In equilibrium, lower qualities of ω_m are priced out of business.

Let $I_m^i(j_m)$ denote the Poisson arrival rate of vertical innovations (vertical-innovation rate) by potential entrant i in industry ω_m when the highest quality is j_m . The rate $I_m^i(j_m)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R_{vm}^i(j_m)$ committed by entrants at time t . As in, e.g., Barro and Sala-i-Martin (2004, ch. 7), $I_m^i(j_m)$ features constant returns in R&D expenditures, $I_m^i(j_m) = R_{vm}^i(j_m) \cdot \Phi_m(j_m)$, where $\Phi_m(j_m)$ is the R&D productivity factor, which is assumed to be homogeneous across i in ω_m . We assume

$$\Phi_L(j_L) = \frac{1}{\zeta \cdot \mathcal{L}^\epsilon \cdot q_L(j_L + 1)} \text{ and } \Phi_H(j_H) = \frac{1}{\zeta \cdot \mathcal{H}^\epsilon \cdot q_H(j_H + 1)}, \quad (4.11)$$

where $\zeta > 0$ is a constant (flow) fixed vertical-R&D cost, such that $\zeta_L \equiv \zeta_H \equiv \zeta$, and $\epsilon \geq 0$ measures the degree of scale-effects removal. The latter captures the idea that

the difficulty of introducing new qualities and replacing old ones is proportional to the market size measured by employed labour in efficiency units (e.g., Barro and Sala-i-Martin, 2004), due to coordination, organisational and transportation costs (e.g., Dinopoulos and Thompson, 1999), as well as rental protection actions by incumbents (e.g., Sener, 2008); however, depending on the effectiveness of that costs and actions, they may partially ($0 < \epsilon < 1$), totally ($\epsilon = 1$) or over ($\epsilon > 1$) counterbalance the scale benefits on profits, which accrue to the R&D successful firm each t . Also, implicit in (4.11) is the assumption of the existence of dynamic decreasing returns to scale to vertical R&D (i.e., decreasing returns to cumulated R&D).^{8,9} Aggregating across i in ω_m , we get $R_{vm}(j_m) = \sum_i R_{vm}^i(j_m)$ and $I_m(j_m) = \sum_i I_m^i(j_m)$, and thus

$$I_L(j_L) = R_{vL}(j_L) \cdot \frac{1}{\zeta \cdot \mathcal{L}^\epsilon \cdot q_L(j_L + 1)} \text{ and } I_H(j_H) = R_{vH}(j_H) \cdot \frac{1}{\zeta \cdot \mathcal{H}^\epsilon \cdot q_H(j_H + 1)}. \quad (4.12)$$

As the terminal date of each monopoly arrives as a Poisson process with frequency $I_m(j_m)$ per (infinitesimal) increment of time, the present value of a monopolist's profits is a random variable. Let $V_m(j_m)$ denote the expected value of an incumbent firm with current quality

⁸The way Φ depends on j implies that the increasing difficulty of creating new product generations over t exactly offsets the increased rewards from marketing higher quality products; see (4.11) and (4.8). This allows for constant vertical-innovation rate over t and across ω in BGP (on *asymmetric* equilibrium in quality-ladders models and its growth consequences, see Cozzi, 2007).

⁹Sener (2008) contrasts the effects of rental protection actions with the expanding variety and the dynamic decreasing returns to R&D as scale-removal mechanisms within a quality-ladders model with knowledge-driven R&D specification. Observe, however, that the dynamic decreasing returns to R&D, as first introduced by Segerstrom (1998), and represented in our model by the term $1/q_m(j_m + 1)$ in (4.11), are neither necessary nor sufficient for the purpose of scale removal in a model with lab-equipment specification (though it plays a crucial role in guaranteeing a Poisson rate constant over ω_m and hence the existence of a symmetric equilibrium; see fn. 8). The same applies to the expanding variety mechanism, as it is clear if we let $\epsilon = 0$ in our results below.

level $j_m(\omega_m, t)$,¹⁰

$$V_m(j_m) = \pi_{0m} \cdot q_m(j_m) \int_t^\infty P_m(t)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s (r(v) + I_m(j_m)) dv} ds, \quad m \in \{L, H\}, \quad (4.13)$$

where r is the equilibrium market real interest rate, and $\pi_{0m} q_m(j_m) = \pi_m(j_m) P_m^{-\frac{1}{\alpha}}$, given by (4.8) and (4.7), is constant in-between innovations. Free-entry prevails in vertical R&D such that the condition $I_m(j_m) \cdot V_m(j_m + 1) = R_{vm}(j_m)$ holds, which implies that

$$V_L(j_L + 1) = \frac{1}{\Phi_L(j_L)} = \zeta \cdot \mathcal{L}^\epsilon \cdot q_L(j_L + 1) \text{ and } V_H(j_H + 1) = \frac{1}{\Phi_H(j_H)} = \zeta \cdot \mathcal{H}^\epsilon \cdot q_H(j_H + 1). \quad (4.14)$$

Next, we determine $V_m(j_m + 1)$ analogously to (4.13), then consider (4.14) and time-differentiate the resulting expression. Therefore, if we also consider (4.8), we get the arbitrage condition facing a vertical innovator

$$\zeta = \frac{\pi_0 \cdot \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}}}{r(t) + I_L(t)} = \frac{\pi_0 \cdot \mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}}}{r(t) + I_H(t)}, \quad (4.15)$$

where $\pi_0 \equiv \pi_{0L}/\mathcal{L} = \pi_{0H}/\mathcal{H}$.¹¹ It has two implications: the present value of “basic” profits $\pi_0 \cdot \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}}$ and $\pi_0 \cdot \mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}}$,¹² using the effective rate of interest $r + I_m$ as a discount factor, should be equal to the fixed cost of entry; and the rates of entry are symmetric across industries $I_m(\omega_m, t) = I_m(t)$.

After solving equation (4.12) for $R_{vm}(\omega_m, t) = R_{vm}(j_m)$ and aggregating across industries

¹⁰We assume that entrants are risk-neutral and, thus, only care about the expected value of the firm.

¹¹Observe that, from (4.8) and (4.12), we have $\frac{\dot{\pi}_m(\omega_m, t)}{\pi_m(\omega_m, t)} - \frac{1}{\alpha} \frac{\dot{P}_m(t)}{P_m(t)} = I_m(\omega_m, t) \cdot \left[\dot{j}_m(\omega_m, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$ and $\frac{\dot{R}_{vm}(\omega_m, t)}{R_{vm}(\omega_m, t)} - \frac{\dot{I}_m(\omega_m, t)}{I_m(\omega_m, t)} = I_m(\omega_m, t) \cdot \left[\dot{j}_m(\omega_m, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$. Thus, if we time-differentiate (4.14) by considering (4.13) and the equations above, we get $r(t) = \frac{\pi_m(j_m+1) \cdot I_m(j_m)}{R_{vm}(j_m)} - I_m(j_m + 1)$, which can then be rewritten as (4.15).

¹²These are the profit flows, constant over time, that accrue when $j_m = 0$ (i.e., $q_m = 1$).

ω_m , we determine total resources devoted to vertical R&D, $R_{vm}(t)$; e.g., with $m = L$, $R_{vL}(t) = \int_0^{N_L(t)} R_{vL}(\omega_L, t) d\omega_L = \int_0^{N_L(t)} \zeta \cdot \mathcal{L}^\epsilon \cdot q_L^+(\omega_L, t) \cdot I_L(\omega_L, t) d\omega_L$. As the innovation rate is industry independent, then

$$R_{vL}(t) = \zeta \cdot \mathcal{L}^\epsilon \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_L(t) \cdot Q_L(t) \text{ and } R_{vH}(t) = \zeta \cdot \mathcal{H}^\epsilon \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_H(t) \cdot Q_H(t). \quad (4.16)$$

4.2.2.2. Horizontal R&D free-entry and dynamic arbitrage conditions

Variety expansion arises from R&D aimed at creating a new intermediate good. Again, innovation is performed by a potential entrant, which means that, because there is free entry, the new good is produced by new firms. Under perfect competition among R&D firms and constant returns to scale at the firm level, instantaneous entry is obtained as $\dot{N}_m^e(t) = \frac{1}{\eta_m} \cdot R_{hm}^e(t)$, where $\dot{N}_m^e(t)$ is the contribution to the instantaneous flow of new m -complementary intermediate goods by R&D firm e at a cost of η_m units of the final good and $R_{hm}^e(t)$ is the flow of resources devoted to horizontal R&D by innovator e at time t . The cost η_m is assumed to be symmetric within sector m . Then, $R_{hm}(t) = \sum_e R_{hm}^e(t)$ and $\dot{N}_m(t) = \sum_e \dot{N}_m^e(t)$, implying

$$R_{hm}(t) = \eta_m \cdot \dot{N}_m(t), \quad m \in \{L, H\}. \quad (4.17)$$

We assume that the cost of setting up a new variety (cost of horizontal entry) is increasing in both the number of existing varieties, N_m , and the number of new entrants, $\dot{N}_m$,

$$\eta_m(t) = \phi \cdot N_m(t)^\sigma \cdot \dot{N}_m(t)^\gamma, \quad m \in \{L, H\}, \quad (4.18)$$

where $\phi > 0$ is a fixed (flow) cost, and $\sigma > 0$ and $\gamma > 0$. The dependence of η_m on N_m captures the assumption there are dynamic decreasing returns to scale. That is, the scale of the economy induces a negative externality in the form of a barrier to entry because it becomes costlier to introduce new varieties in large growing economies. The dependence of η_m on $\dot{N}_m$ means that the entry technology displays static decreasing returns to scale at the aggregate level, which we assume to be entirely external to the firm, but are compatible with the previous assumption of constant returns to scale at the firm level. The existing endogenous-growth literature deals with the two features separately: some models only display dynamic negative externalities (e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004, ch. 6) while others only assume static decreasing returns to scale (e.g., Arnold, 1998; Howitt, 1999; Jones and Williams, 2000).¹³

The dependence of the entry cost on the number of entrants introduces dynamic second-order effects in entry, implying that new varieties are brought to the market gradually, instead of through a lumpy adjustment. This is in line with the stylised facts on entry (e.g., Geroski, 1995): entry occurs mostly at small scale since adjustment costs penalise large-scale entry.

We assume the innovator enters with the average quality level of the existing varieties,

$$\bar{q}_m(t) = \int_0^{N_m(t)} \frac{q_m(\omega_m, t)}{N_m(t)} d\omega_m = \frac{Q_m(t)}{N_m(t)}, \quad m \in \{L, H\}, \quad (4.19)$$

¹³We also depart from Howitt (1999) (see also Segerstrom, 2000) in that he hypothesises decreasing returns to scale to R&D at the firm level. Such an entry technology implies that (keeping our notation) $V_m = \eta_m = \frac{dR_{hm}}{dN_m} > \frac{R_{hm}}{N_m}$. In contrast, given our assumption of constant returns to scale, we have $V_m = \eta_m = \frac{R_{hm}}{N_m} = \frac{dR_{hm}}{dN_m}$ (see (4.21), below). Note that the price of entry, V_m , equals the marginal cost of entry, η , in both cases considered above; nevertheless, the assumption of constant returns eschews positive profits from entering, since $V_m = \frac{R_{hm}}{N_m}$.

which is tantamount to having a uniform distribution with probability $1/N_m(t)$. As his/her monopoly power will be also terminated by the arrival of a successful vertical innovator in the future, the benefits from entry are given by

$$V_m(\bar{q}_m) = \pi_{0m} \cdot \bar{q}_m(t) \int_t^\infty P_m(t)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s [r(\nu) + I_m(\bar{q}_m)] d\nu} ds, \quad m \in \{L, H\}, \quad (4.20)$$

where $\pi_{0m} \bar{q}_m = \bar{\pi}_m P_m^{-\frac{1}{\alpha}}$. The free-entry condition is now $\dot{N}_m \cdot V(\bar{q}_m) = R_{hm}$, which simplifies to

$$V_m(\bar{q}_m) = \eta_m(t), \quad m \in \{L, H\}. \quad (4.21)$$

Substituting (4.20) into (4.21) and time-differentiating the resulting expression, yields the arbitrage equation facing a horizontal innovator

$$r(t) + I_m(t) = \frac{\dot{\pi}_m(t)}{\eta_m(t)}, \quad m \in \{L, H\}. \quad (4.22)$$

4.2.2.3. Consistency arbitrage conditions

Before deciding which type of R&D to perform, the potential entrant should evaluate the best type of entry in each sector m . At the margin, she/he should be indifferent between the two types, implying there is a consistency condition between them. If we equate the effective rate of return $r + I_m$ for both types of entry, from (4.15) and (4.22), we get the *intra-sector* arbitrage conditions

$$\bar{q}_L(t) = \frac{Q_L(t)}{N_L(t)} = \frac{\eta_L(t)}{\zeta \cdot \mathcal{L}^\epsilon} \text{ and } \bar{q}_H(t) = \frac{Q_H(t)}{N_H(t)} = \frac{\eta_H(t)}{\zeta \cdot \mathcal{H}^\epsilon}, \quad (4.23)$$

which are a key ingredient of the model. They equate the cost of the horizontal, η_L (respectively, η_H), to the average cost of vertical R&D, $\bar{q}_L \zeta \mathcal{L}^\epsilon$ ($\bar{q}_H \zeta \mathcal{H}^\epsilon$).

Following a similar reasoning, if we equate the effective rate of return for both sectors, e.g., by considering (4.15), an *inter-sector* arbitrage condition obtains

$$I_H(t) - I_L(t) = \frac{\pi_0}{\zeta} \left(\mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}} - \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}} \right). \quad (4.24)$$

On the other hand, (4.23) can be equivalently recast as

$$\dot{N}_m(t) = x_m(Q_m(t), N_m(t)) \cdot N_m(t), \quad m \in \{L, H\}, \quad (4.25)$$

where

$$x_L(Q_L, N_L) \equiv \left(\frac{\zeta}{\phi} \cdot \mathcal{L}^\epsilon \right)^{\frac{1}{\gamma}} \cdot Q_L^{\frac{1}{\gamma}} \cdot N_L^{-\left(\frac{\sigma+\gamma+1}{\gamma}\right)}, \quad (4.26a)$$

$$x_H(Q_H, N_H) \equiv \left(\frac{\zeta}{\phi} \cdot \mathcal{H}^\epsilon \right)^{\frac{1}{\gamma}} \cdot Q_H^{\frac{1}{\gamma}} \cdot N_H^{-\left(\frac{\sigma+\gamma+1}{\gamma}\right)}, \quad (4.26b)$$

which clarifies the adopted mechanism of entry by explicitly incorporating a channel between vertical innovation and firm dynamics. It shows that the horizontal-entry rate, $\dot{N}_m/N_m$, depends positively on Q_m and negatively on N_m : the first effect is an implication of the complementarity between the horizontal-entry rate and the technological-knowledge stock, and the second results from the barriers-to-entry effect incorporated in (4.18).

In a small time interval, the growth rate of average quality is equal to the expected arrival rate of a successful innovation multiplied by the quality shift it introduces: $\dot{\bar{q}}_m/\bar{q}_m = I_m(t)(q_m^+ - q_m)/q_m$, where both the innovation rate and the quality shift are industry-independent. Time-differentiating (4.19), and using (4.25) yields

$$\dot{Q}_m(t) = (\Xi \cdot I_m(t) + x_m(Q_m(t), N_m(t))) \cdot Q_m(t), \quad m \in \{L, H\}, \quad (4.27)$$

where the quality shift is denoted by $\Xi \equiv (q_m^+ - q_m)/q_m = \left(\lambda^{\frac{1-\alpha}{\alpha}} - 1\right)$. The innovation rate is endogenous and will be determined as an economy-wide function below. Equation (4.27) introduces a second dynamic interaction between the two types of entry, in this case between the number of varieties and the growth rate of the aggregate quality index.

4.2.3. Households

The economy is populated by a fixed number of infinitely-lived households who consume and collect income from investments in financial assets (equity) and from labour. Households inelastically supply one of two types of labour: low-skilled, L , and high-skilled labour, H . Thus, total labour supply, $L + H$, is exogenous and constant. We assume consumers have perfect foresight concerning the technological change over time and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximise discounted lifetime utility

$$U = \int_0^\infty \left(\frac{C(t)^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt, \quad (4.28)$$

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + w_L(t) \cdot L + w_H(t) \cdot H - C(t), \quad (4.29)$$

where a denotes households' real financial assets holdings. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $\lim_{t \rightarrow \infty} e^{-\int_0^t r(s) ds} a(t) \geq 0$ is also imposed. The optimal path of consumption Euler equation and the transversality condition are standard,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho) \quad (4.30)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0. \quad (4.31)$$

4.2.4. Macroeconomic aggregation and equilibrium innovation rates

The aggregate financial wealth held by all households is

$$a(t) = \sum_{m=L,H} a_m(t) = \sum_{m=L,H} \int_0^{N_m(t)} V_m(\omega_m, t) d\omega_m,$$

which, from the arbitrage condition between vertical and horizontal entry, yields $a(t) = \sum_{m=L,H} \eta_m(t) \cdot N_m(t)$. Taking time derivatives and comparing with (4.29), we get an expression for the aggregate flow budget constraint which is equivalent to the product market equilibrium condition (see Appendix B.2)

$$Y_{tot}(t) = X_{tot}(t) + C(t) + R_h(t) + R_v(t) \quad (4.32)$$

where $R_h = \sum_{m=L,H} R_{hm}$ and $R_v = \sum_{m=L,H} R_{vm}$. Substituting the expressions for the aggregate outputs (4.10) and (4.9), and for total R&D expenditures (4.16) and (4.17), we

have

$$\begin{aligned} \chi_Y \cdot \Gamma(t) &= \chi_X \cdot \Gamma(t) + C(t) + \eta_L(t) \cdot \dot{N}_L(t) + \eta_H(t) \cdot \dot{N}_H(t) + \\ &+ \zeta \cdot \mathcal{L}^\epsilon \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_L(t) \cdot Q_L(t) + \zeta \cdot \mathcal{H}^\epsilon \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_H(t) \cdot Q_H(t), \end{aligned} \quad (4.33)$$

where $\chi_Y \equiv A^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{2(1-\alpha)}{\alpha}}$, $\chi_X \equiv A^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{2}{\alpha}}$ and $\Gamma(t) \equiv P_L(t)^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot Q_L(t) + P_H(t)^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot Q_H(t)$. Solving for, e.g., I_L , and using (4.23) and (4.25), we get the endogenous vertical-innovation rate at equilibrium in sector L

$$\begin{aligned} I_L(Q_L, Q_H, N_L, N_H, C, I_H) &= \frac{1}{\zeta \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot \mathcal{L}^\epsilon} \left\{ (\chi_Y - \chi_X) \left(P_H^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot \frac{Q_H}{Q_L} + P_L^{\frac{1}{\alpha}} \cdot \mathcal{L} \right) - \frac{C}{Q_L} \right\} - \\ &- \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^\epsilon \cdot \frac{Q_H}{Q_L} \cdot I_H - \frac{1}{\lambda^{\frac{1-\alpha}{\alpha}}} \cdot \left[\left(\frac{\mathcal{H}}{\mathcal{L}} \right)^\epsilon \cdot \frac{Q_H}{Q_L} \cdot x_H(Q_H, N_H) + x_L(Q_L, N_L) \right], \end{aligned} \quad (4.34)$$

as a function which is decreasing in consumption, increasing in the number of varieties and related in an ambiguous way with the aggregate quality level in each sector $m \in \{L, H\}$. Observe, in particular, that P_L and P_H are (non-linear) functions of Q_H/Q_L alone (see (4.6) and (4.7)). If we further use (4.24) to eliminate I_H from (4.34), we get $I_L \equiv I_L(Q_L, Q_H, N_L, N_H, C)$. As functions $I_m(Q_L, Q_H, N_L, N_H, C)$ can be negative, the relevant innovation rates at the macroeconomic level are

$$I_m^+(Q_L, Q_H, N_L, N_H, C) = \max \{I_m(Q_L, Q_H, N_L, N_H, C), 0\}, \quad m \in \{L, H\}. \quad (4.35)$$

From (4.15), we get the rate of return of capital as $r(Q_L, Q_H, N_L, N_H, C) = r_{0m} - I_m^+(Q_L, Q_H, N_L, N_H, C)$,

where $r_{0L} \equiv \pi_0 \mathcal{L}^{1-\epsilon} P_L^{\frac{1}{\alpha}} / \zeta$ and $r_{0H} \equiv \pi_0 \mathcal{H}^{1-\epsilon} P_H^{\frac{1}{\alpha}} / \zeta$.

4.2.5. The dynamic general equilibrium

The dynamic general equilibrium is defined by the allocation $\{X_m(\omega, t), \omega \in [0, N_m(t)], t \geq 0, m \in \{L, H\}\}$, by the prices $\{p_m(\omega, t), \omega \in [0, N_m(t)], t \geq 0, m \in \{L, H\}\}$ and by the aggregate paths $\{C(t), N_m(t), Q_m(t), I_m(t), r(t), t \geq 0, m \in \{L, H\}\}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) vertical, horizontal and consistency free-entry conditions are met; and (iii) markets clear. The equilibrium paths can be obtained from the piecewise-smooth system

$$\dot{C} = \begin{cases} \frac{1}{\theta} \cdot (r_{0L} - \rho) \cdot C & \text{if } I_L(\cdot) \leq 0 \\ \frac{1}{\theta} \cdot (r_{0L} - I_L(Q_L, Q_H, N_L, N_H, C) - \rho) \cdot C & \text{if } I_L(\cdot) > 0 \end{cases} \quad (4.36a)$$

$$\dot{Q}_m = \begin{cases} x_m(Q_m, N_m) \cdot Q_m & \text{if } I_m(\cdot) \leq 0 \\ (I_m(Q_L, Q_H, N_L, N_H, C) \cdot \Xi + x_m(Q_m, N_m)) \cdot Q_m & \text{if } I_m(\cdot) > 0 \end{cases} \quad (4.36b)$$

$$\dot{N}_m = x_m(Q_m, N_m) \cdot N_m \quad (4.36c)$$

given $Q_m(0)$ and $N_m(0)$, $m \in \{L, H\}$, and the transversality condition (4.31), which may be re-written as

$$\lim_{t \rightarrow \infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot \zeta \cdot (\mathcal{L}^\epsilon \cdot Q_L(t) + \mathcal{H}^\epsilon \cdot Q_H(t)) = 0. \quad (4.37)$$

4.2.6. The balanced-growth path

Let $g_y \equiv \dot{y}/y$ represent the growth rate of variable $y(t)$. As the functions in system (4.36a)-(4.36c) are homogeneous, a BGP may exist if the further necessary conditions are verified: (i) the growth rates of consumption and of the quality indices are equal, $g_C = g_{Q_L} = g_{Q_H} = g$; (ii) the growth rates of the number of varieties are equal, $g_{N_L} = g_{N_H}$; (iii) the vertical-innovation rates and the final-good price indices are trendless, $g_{I_L} = g_{I_H} = g_{P_L} = g_{P_H} = 0$; and (iv) the growth rates of the quality indices and the number of varieties are monotonously related, $g_{Q_L}/g_{N_L} = g_{Q_H}/g_{N_H} = (\sigma + \gamma + 1)$, $g_{N_m} \neq 0$. Observe that $x_m = g_{N_m}$ is always positive if $N_m > 0$, $m \in \{L, H\}$.

It is convenient to recast system (4.36a)-(4.36c), by using variables x_m as in (4.26a)-(4.26b) and variables $z_L \equiv C/Q_L$ and $Q \equiv Q_H/Q_L$, into an equivalent system in detrended variables. If we focus on the region of the state space where $I_m(\cdot) > 0$, $m \in \{L, H\}$, then

$$\dot{x}_L = \left[\frac{\Xi}{\gamma} \cdot I_L - \left(\frac{\sigma + \gamma}{\gamma} \right) \cdot x_L \right] \cdot x_L, \quad (4.38a)$$

$$\dot{z}_L = \left[\frac{1}{\theta} \cdot (r_{0L} - \rho) - \left(\frac{1}{\theta} + \Xi \right) \cdot I_L - x_L \right] \cdot z_L, \quad (4.38b)$$

$$\dot{x}_H = \left[\frac{\Xi}{\gamma} \cdot I_L - \left(\frac{\sigma + \gamma}{\gamma} \right) \cdot x_H + \frac{\Xi}{\gamma} \cdot \frac{\pi_0}{\zeta} \cdot \left(\mathcal{H}^{1-\epsilon} \cdot P_H^{\frac{1}{\alpha}} - \mathcal{L}^{1-\epsilon} \cdot P_L^{\frac{1}{\alpha}} \right) \right] \cdot x_H, \quad (4.38c)$$

$$\dot{Q} = \left[\Xi \cdot \frac{\pi_0}{\zeta} \cdot \left(\mathcal{H}^{1-\epsilon} \cdot P_H^{\frac{1}{\alpha}} - \mathcal{L}^{1-\epsilon} \cdot P_L^{\frac{1}{\alpha}} \right) + x_H - x_L \right] \cdot Q. \quad (4.38d)$$

where $I_L \equiv I_L(Q, x_L, x_H, z_L) = I_L(Q_L, Q_H, N_L, N_H, C)$. By equating $\dot{x}_m = \dot{z}_L = \dot{Q} = 0$, it is straightforward to show that there is just one interior steady state (see Appendix B.3)

$$\tilde{Q} = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{1-2\epsilon}, \quad (4.39)$$

$$\tilde{x}_L = \tilde{x}_H = \frac{\frac{\Xi}{\theta} (\tilde{r}_{0L} - \rho)}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)}, \quad (4.40)$$

$$\tilde{z}_L = (\chi_Y - \chi_X) \left(\tilde{P}_H^{\frac{1}{\alpha}} \mathcal{H} \tilde{Q} + \tilde{P}_L^{\frac{1}{\alpha}} \mathcal{L} \right) - \left(\zeta \lambda^{\frac{1-\alpha}{\alpha}} \tilde{I}_L + \zeta \tilde{x}_L \right) \left(\mathcal{H}^\epsilon \tilde{Q} + \mathcal{L}^\epsilon \right), \quad (4.41)$$

where $\tilde{I}_L = \tilde{I}_H = (\sigma + \gamma) \tilde{x}_L / \Xi = (\sigma + \gamma) \tilde{x}_H / \Xi$; $\tilde{r}_{0L} \equiv \pi_0 \mathcal{L}^{1-\epsilon} \tilde{P}_L^{\frac{1}{\alpha}} / \zeta = \tilde{r}_{0H} \equiv \pi_0 \mathcal{H}^{1-\epsilon} \tilde{P}_H^{\frac{1}{\alpha}} / \zeta$; $\tilde{P}_L = e^{-\alpha} \left[1 + (\mathcal{H}/\mathcal{L})^{1-\epsilon} \right]^\alpha$ and $\tilde{P}_H = e^{-\alpha} \left[1 + (\mathcal{H}/\mathcal{L})^{\epsilon-1} \right]^\alpha$. Assume that $\tilde{r}_{0m} - \rho > 0$; then, (4.39)-(4.41) is a steady-state equilibrium with balanced growth in the usual sense,¹⁴ such that the endogenous growth rates are positive

$$\tilde{g}_{N_L} = \tilde{g}_{N_H} = \tilde{x}_L = \tilde{x}_H > 0, \quad (4.42)$$

$$\tilde{g}_{Q_L} = \tilde{g}_{Q_H} = \tilde{g} = \frac{\frac{\Xi}{\theta} (\tilde{r}_{0L} - \rho) (\sigma + \gamma + 1)}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)} > 0. \quad (4.43)$$

Moreover, the relative intermediate-good output and the relative number of firms (i.e., sector H vis-à-vis sector L) are

$$\tilde{X} \equiv \left(\frac{\tilde{X}_H}{\tilde{X}_L} \right) = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{1-\epsilon}, \quad (4.44)$$

$$\tilde{N} \equiv \left(\frac{\tilde{N}_H}{\tilde{N}_L} \right) = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{\frac{1-\epsilon}{\sigma+\gamma+1}}. \quad (4.45)$$

¹⁴It can also be shown that the interior equilibrium $(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q})$ is saddle-point stable, since the linearised system around the steady state has three eigenvalues with negative real part and one with positive real part. Also, since the dimension of the stable manifold equals the number of predetermined variables, the equilibrium is determinate.

Since firm size may be measured as production (or sales) per firm, e.g., $X_L/N_L = \chi_X \cdot P_L^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot Q_L/N_L$ (see (4.9) with $m = L$), then, by considering (4.44) and (4.45), we get relative firm size, $(X/\tilde{N}) = (\mathcal{H}/\mathcal{L})^{\frac{(1-\epsilon)(\sigma+\gamma)}{\sigma+\gamma+1}}$.¹⁵

However, since we wish to confront our theoretical results with the data on production for a number of countries (see Appendix B.1), and since data is presented in a quality-adjusted base by the national statistics offices (e.g., Eurostat, 2001), we find it convenient to compute production also in quality-adjusted terms. Reiterating the steps as in Subsection 4.2.1, we find total intermediate-good quality-adjusted production to be (e.g., with $m = L$) $\mathfrak{X}_L = \int_0^{N_L} \int_0^{\bar{n}} \lambda^{j_L(\omega_L)} \cdot X_L(n, \omega_L) dn d\omega_L = A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{2}{\alpha}} P_L^{\frac{1}{\alpha}} l L \check{Q}_L$, where $\check{Q}_L = \int_0^{N_L} \lambda^{j_L(\omega_L)^{\frac{1}{\alpha}}} d\omega_L$, and $\mathfrak{X}_{tot} = \mathfrak{X}_L + \mathfrak{X}_H$. We cannot find an explicit algebraic expression for the BGP value of $\check{Q}_m$. However, as shown in Appendix B.4, we can build an adequate proxy for $\check{Q}_m$ as a function of Q_m and N_m , which is $\widehat{\check{Q}_m} = Q_m^{\frac{1}{1-\alpha}} \cdot N_m^{-\left(\frac{\alpha}{1-\alpha}\right)}$. Accordingly, we define the proxy $\widehat{\mathfrak{X}_m} = X_m \cdot (Q_m/N_m)^{\frac{\alpha}{1-\alpha}}$ for $\mathfrak{X}_m$. Thus, having in mind (4.39) and (4.44), we consider the following quality-adjusted measures of relative production and relative firm size along the BGP

$$\left(\tilde{\mathfrak{X}}\right) = \tilde{X} \cdot \left(\frac{\tilde{Q}}{\tilde{N}}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\mathcal{D}_1(\epsilon)} \quad (4.46)$$

$$\frac{\left(\tilde{\mathfrak{X}}\right)}{\tilde{N}} = \left(\frac{\tilde{X}}{\tilde{N}}\right) \cdot \left(\frac{\tilde{Q}}{\tilde{N}}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\mathcal{D}_2(\epsilon)} \quad (4.47)$$

where $\mathcal{D}_1(\epsilon) \equiv \{1 - \alpha + \sigma + \gamma - \epsilon[1 + (1 + \alpha)(\sigma + \gamma)]\} / [(\sigma + \gamma + 1)(1 - \alpha)]$ and $\mathcal{D}_2(\epsilon) \equiv \{\sigma + \gamma - \epsilon[\alpha + (1 + \alpha)(\sigma + \gamma)]\} / [(\sigma + \gamma + 1)(1 - \alpha)]$. It will be important to define the

¹⁵An alternative measure of firm size is financial assets per firm, $\frac{a_L}{N_L} = \eta_L = \zeta \mathcal{L}^\epsilon \frac{Q_L}{N_L}$. Then, from our assumptions, namely $\zeta_L = \zeta_H \equiv \zeta$, we have $\left(\frac{\tilde{a}}{\tilde{N}}\right) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^\epsilon \left(\frac{\tilde{Q}}{\tilde{N}}\right) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\frac{(1-\epsilon)(\sigma+\gamma)}{\sigma+\gamma+1}}$, where $a \equiv \frac{a_L}{a_H}$. That is, the alternative measures of relative firm size $\frac{\tilde{X}}{\tilde{N}}$ and $\frac{a}{N}$ coincide along the BGP, whatever ϵ .

critical values for ϵ , $\bar{\epsilon} \in \{\bar{\epsilon}_1, \bar{\epsilon}_2\}$, such that $\mathcal{D}_1(\bar{\epsilon}_1) = 0$ and $\mathcal{D}_2(\bar{\epsilon}_2) = 0$. That is,

$$\bar{\epsilon}_1 = \frac{1 - \alpha + \sigma + \gamma}{1 + (1 + \alpha)(\sigma + \gamma)},$$

$$\bar{\epsilon}_2 = \frac{\sigma + \gamma}{\alpha + (1 + \alpha)(\sigma + \gamma)}.$$

Since $0 < \alpha < 1$, then $0 < 1 - \alpha \leq \bar{\epsilon}_1 < 1/(1 + \alpha) < 1$ and $0 \leq \bar{\epsilon}_2 < 1/(1 + \alpha) < 1$, with $\bar{\epsilon}_2 < \bar{\epsilon}_1$ for σ, γ finite.

Observe that $\tilde{g} > 0$ requires $\pi_0 \mathcal{L}^{1-\epsilon} \tilde{P}_L^{\frac{1}{\alpha}} / \zeta - \rho = \pi_0 \mathcal{H}^{1-\epsilon} \tilde{P}_H^{\frac{1}{\alpha}} / \zeta - \rho > 0$. Since, $\tilde{g} = \tilde{g}_C = (r - \rho) / \theta$, then $\tilde{r} > \rho$ must occur. This condition also guarantees $\tilde{g}_{N_m} > 0$, $m \in \{L, H\}$. Thus, our model predicts, under a sufficiently productive technology, a BGP with constant positive g and g_{N_m} , where the former exceeds the latter by the growth of intermediate-good quality; to verify this, just replace (4.25) in (4.27), and solve to get $\dot{Q}_m / Q_m - \dot{N}_m / N_m = I_m \cdot \Xi$, which is positive if $I_m > 0$. This implies that the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the view that industrial growth proceeds both along an intensive and an extensive margin (e.g., Howitt, 1999; Peretto and Connolly, 2007). Variety expansion is sustained by endogenous technological-knowledge accumulation (independently of population growth), as the expected growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an entry cost, in spite of its increase with N_m .

4.3. Labour endowment, scale effects and industrial structure

Now, we discuss the comparative statics of the BGP,¹⁶ namely concerning the impact of changes in the relative endowment of high-skilled labour, $\mathcal{H}/\mathcal{L}$, on the relative number of firms, N , production (sales), $\hat{\mathfrak{X}}$, and average firm size, $\hat{\mathfrak{X}}/N$.¹⁷ This exercise is carried out by considering four critical values of the degree of scale-effects removal, ϵ : $\epsilon = 0$, which corresponds to no scale-effects removal (e.g., Acemoglu and Zilibotti, 2001); $\epsilon = 1$, which corresponds to exact scale-effects removal (e.g., Barro and Sala-i-Martin, 2004), under which $\mathcal{H}/\mathcal{L}$ has no impact on N ; $\epsilon = \bar{\epsilon}_2$ (respectively, $\epsilon = \bar{\epsilon}_1$), which corresponds to incomplete scale-effects removal, under which $\mathcal{H}/\mathcal{L}$ has no impact on $\hat{\mathfrak{X}}/N$ (no impact on $\hat{\mathfrak{X}}$). Accordingly, we define three regimes of (net) scale effects: (i) “large” positive scale effects (defined by either $0 \leq \epsilon < \bar{\epsilon}_1$ or $0 \leq \epsilon < \bar{\epsilon}_2$); (ii) “small” positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$ or $\bar{\epsilon}_2 < \epsilon < 1$); and (iii) negative scale effects ($\epsilon > 1$). We find that, as the regime of scale effects changes, the relationship between relative labour endowment and industrial structure may differ significantly.

4.3.1. Number of firms, production and firm size

4.3.1.1. “Cross-country” perspective

Below we summarise the main results with respect to the impact of changes in $\mathcal{H}/\mathcal{L}$ on the industrial structure, as characterised by the number of firms, output and average firm size in sector H vis-à-vis sector L . One can interpret this exercise as a *cross-section* comparison of industrial structures between countries with different levels of $\mathcal{H}/\mathcal{L}$.

¹⁶Henceforth, the $\sim$ is omitted for the sake of simplicity.

¹⁷The comparative statics concerning the remaining structural parameters and endogenous variables can be found in Gil, Brito, and Afonso (2010).

Proposition 4.1. Let a country have a *larger* $\mathcal{H}/\mathcal{L}$. Then it will have:

- (i) A *larger* N but a *smaller* $\hat{\mathfrak{X}}/N$, if there are “small” positive scale effects ($\bar{\epsilon}_2 < \epsilon < 1$);
- (ii) *Larger* N , $\hat{\mathfrak{X}}$ and $\hat{\mathfrak{X}}/N$, if there are “large” positive scale effects ($0 \leq \epsilon < \bar{\epsilon}_2$);
- (iii) *Smaller* N , $\hat{\mathfrak{X}}$ and $\hat{\mathfrak{X}}/N$, if there are negative scale effects ($\epsilon > 1$).

In (i) the response of $\hat{\mathfrak{X}}$ to a higher $\mathcal{H}/\mathcal{L}$ is ambiguous: if $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$, $\hat{\mathfrak{X}}$ increases though less than N , and if $\bar{\epsilon}_1 < \epsilon < 1$, $\hat{\mathfrak{X}}$ falls and N increases. To better understand the mechanism that links $\mathcal{H}/\mathcal{L}$ to the industrial structure, observe first that the results above stem from the different response of $\hat{\mathfrak{X}}$ and N to $\mathcal{H}/\mathcal{L}$ (see (4.39) and (4.45)). This mirrors the dominant impact of scale effects on $\hat{\mathfrak{X}}$ through the vertical-innovation mechanism, imposing only through the latter on the horizontal-innovation mechanism, and thereby affecting N .¹⁸ Furthermore, the effect on N is dampened by the friction in horizontal entry dynamics introduced by the entry-cost function (4.18). The higher sensitivity of $\hat{\mathfrak{X}}$ to scale effects is clear from the following: $\hat{\mathfrak{X}}$ is constant with respect to $\mathcal{H}/\mathcal{L}$ when $\epsilon = \bar{\epsilon}_1 < 1$ (i.e., incomplete removal of scale effects), while N is constant with respect to $\mathcal{H}/\mathcal{L}$ when $\epsilon = 1$ (i.e., exact removal of scale effects).

This also explains the effect of the elasticities of the horizontal entry cost function, σ and γ , on the value of the threshold $\bar{\epsilon} \in \{\bar{\epsilon}_1, \bar{\epsilon}_2\}$: the lower σ and γ , the lower $\bar{\epsilon}$, i.e., the higher the degree of scale effects below which there is falling relative production and firm size with $\mathcal{H}/\mathcal{L}$. These results make clear the contrast between industrial structure under positive but “small” scale effects and under either negative or “large” positive scale effects. Figure 4.1 depicts the relationship between $\mathcal{H}/\mathcal{L}$ and both $\hat{\mathfrak{X}}/N$ and N under the three regimes of (net) scale effects.

¹⁸Notice that, given the postulated horizontal entry technology and our lab-equipment specification, the vertical-innovation mechanism ultimately commands the horizontal entry dynamics along the BGP, meaning that a BGP with increasingly costly entry only occurs because entrants expect incumbency value to grow propelled by quality-enhancing R&D.

4. Directed Technical Change, Scale Effects and Industrial Structure

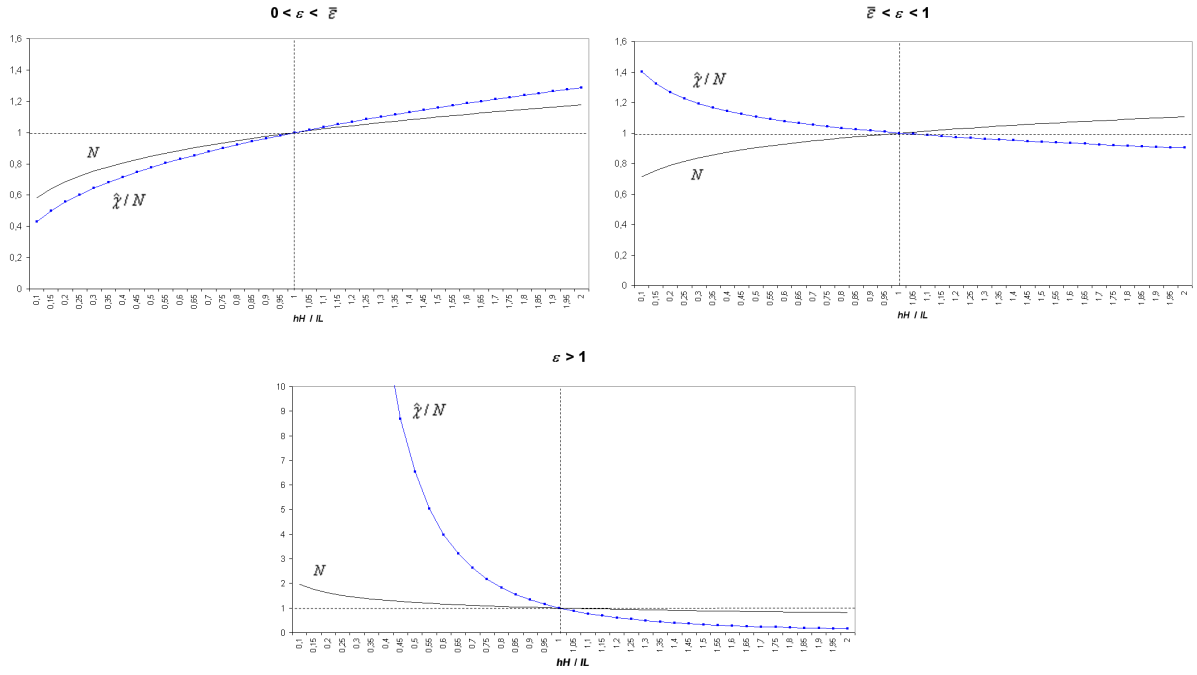


Figure 4.1.: Relative number of firms and relative average firm size under the three regimes of scale effects.

Cross-country evidence in Figure B.1, Appendix B.1, suggests that the relative number of firms tends to be (a) positively correlated with relative production, but (b) (slightly) negatively correlated with relative average firm size. Relationship (a) is compatible with our results, if scale effects are either negative or positive and “large” ($0 \leq \epsilon < \bar{\epsilon}_1$); however, relationship (b) is in line with our results in the case of “small” positive scale effects ($\bar{\epsilon}_2 < \epsilon < 1$). If $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$, both (a) and (b) are compatible with the predictions of our model.

4.3.1.2. “Intra-country” perspective

Now, we focus on the comparison between the relative number of firms, N , relative output, $\hat{\mathfrak{X}}$, and relative average firm size, $\hat{\mathfrak{X}}/N$, for a given level of relative labour endowment, $\mathcal{H}/\mathcal{L}$, i.e., we look at the industrial structure in an *intra-country* perspective.

Proposition 4.2. A given $\mathcal{H}/\mathcal{L}$ above unity corresponds to (vice versa for $\mathcal{H}/\mathcal{L}$ below unity):

- (i) N above and $\hat{\mathfrak{X}}/N$ below unity, if there are “small” positive scale effects ($\bar{\epsilon}_2 < \epsilon < 1$);
- (ii) N , $\hat{\mathfrak{X}}$ and $\hat{\mathfrak{X}}/N$ above unity, if there are “large” positive scale effects ($0 \leq \epsilon < \bar{\epsilon}_2$);
- (iii) N , $\hat{\mathfrak{X}}$ and $\hat{\mathfrak{X}}/N$ below unity, if there are negative scale effects ($\epsilon > 1$).

In (i), see that if $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$, $\hat{\mathfrak{X}}$ is above unity with $\hat{\mathfrak{X}} < N$, and, if $\bar{\epsilon}_1 < \epsilon < 1$, $\hat{\mathfrak{X}}$ is below unity. From Proposition 4.2, we conclude that, provided $\mathcal{H}/\mathcal{L} \neq 1$ and $\epsilon \neq 1$, the industrial structure is characterised by a skewed BGP distribution of average firm size between sectors L and H as follows. Under “small” positive scale effects, the number of firms is greater in the sector with smaller average firm size ($N > 1$ and $\hat{\mathfrak{X}}/N < 1$, or vice versa), implying a right-skewed average-size distribution. In contrast, negative or “large” positive scale effects imply a left-skewed average-size distribution, i.e., there are more large-average firms than small-average firms ($N > 1$ and $\hat{\mathfrak{X}}/N > 1$, or vice versa).¹⁹ Thus, only in the case of positive

¹⁹If $\epsilon = \bar{\epsilon}_2$ and $\frac{\mathcal{H}}{\mathcal{L}} \neq 1$, the two sectors have a different number of firms, but their average size is the same. If $\epsilon = 1$ and $\frac{\mathcal{H}}{\mathcal{L}} \neq 1$, the opposite is true: the two sectors exhibit the same number of firms, but with

but “small” scale effects does average-size distribution obtain a skewness which is in line with the IO stylised facts on the size distribution of *individual* firms (see, e.g., Sutton, 1997; Cabral and Mata, 2003). The mechanism behind this result is the same as the one described in Proposition 4.1, above (see again Figure 4.1).

According to the data in Figure B.1, in Appendix B.1, the relative number of firms and relative production display values below unity. This is in line with our results with respect to N and $\hat{\mathfrak{X}}$ given: $\mathcal{H}/\mathcal{L} < 1$, if scale effects are “large” positive ($0 \leq \epsilon < \bar{\epsilon}_1$); $\mathcal{H}/\mathcal{L} > 1$, if scale effects are negative. On the other hand, data shows relative firm size systematically above unity. This is compatible, in theory, with the observed relative production and relative number of firms below unity - implying a right-skewed firm size distribution - if we have $\mathcal{H}/\mathcal{L} < 1$ combined with positive scale effects in the interval $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$.

4.3.2. Concentration index

4.3.2.1. Labour endowment, scale effects and concentration

We are interested in the relationship between the level of relative labour endowment and a synthetic measure of industry concentration. Concentration measures widely used in the IO literature are, e.g., the k -firm concentration ratio (the sum of the market share of the k^{th} biggest firms in a given industry) and the Herfindahl index (the quadratic sum of the shares of all firms in a given industry).

However, to choose the concentration measure, one must take into account two particular features of our model. First, the relevant “market” to measure the degree of concentration is established at the economy-wide level, since the economy comprises a continuum of

different average size. If $\frac{\mathcal{H}}{\mathcal{L}} = 1$, the average-firm-size distribution is uniform (same number of firms and same average firm size), whatever $\epsilon \geq 0$.

monopolistic industries producing imperfectly substitutable goods. Second, the equilibrium values of the relevant variables (namely, output and the number of firms) are non-stationary, since they are defined along a BGP characterised by positive growth rates. Due to the first feature, the k -firm concentration ratio is meaningless and, owing to the second feature, the Herfindahl index is not appropriate (it tends to zero as N_m grows at the rate $g_N > 0$ along the BGP).²⁰

Since a similar reasoning applies to other commonly used concentration measures, we propose an own measure. We consider a two-dimension aggregate concentration index, by combining the “market share” of the number of firms, N_m/N_{tot} , with the “market share” of the quality-adjusted output (sales), $\widehat{\mathfrak{X}}_m/\widehat{\mathfrak{X}}_{tot}$, of each sector $m \in \{L, H\}$, as follows

$$\mathfrak{I} \equiv \frac{\widehat{\mathfrak{X}}_L}{\widehat{\mathfrak{X}}_{tot}} \cdot \frac{N_L}{N_{tot}} + \frac{\widehat{\mathfrak{X}}_H}{\widehat{\mathfrak{X}}_{tot}} \cdot \frac{N_H}{N_{tot}} = \frac{\widehat{\mathfrak{X}} \cdot N + 1}{\widehat{\mathfrak{X}} \cdot N + \widehat{\mathfrak{X}} + N + 1} \quad (4.48)$$

The proposed concentration index compares sectors at the economy-wide level instead of individual firms within a sector and is constant along the BGP (see (4.45) and (4.46)). The aggregate character accommodates the view, noted by Kamien and Schwatz (1975), that traditional industry boundaries seem to become decreasingly useful for the purpose of economic analysis as new products and processes compete *across* industry lines, which is a salient feature of our model.

The proposition below characterise the behaviour of the concentration index with respect to $\mathcal{H}/\mathcal{L}$.

Proposition 4.3.

(i) $\mathfrak{I}$ displays a U-shaped behaviour with respect to $\mathcal{H}/\mathcal{L}$, with a minimum $\mathfrak{I} = 1/2$ at

²⁰Observe that the version of the concentration ratio usually applied to symmetric equilibria, $1/N_m$, also tends to zero along the BGP.

4. Directed Technical Change, Scale Effects and Industrial Structure

$\mathcal{H}/\mathcal{L} = 1$ such that $1/2 \leq \mathfrak{J} \leq 1$, if there are “large” positive ($0 \leq \epsilon < \bar{\epsilon}_1$) or negative scale effects ($\epsilon > 1$);

- (ii) $\mathfrak{J}$ displays an inverted U-shaped behaviour with respect to $\mathcal{H}/\mathcal{L}$, with a maximum $\mathfrak{J} = 1/2$ at $\mathcal{H}/\mathcal{L} = 1$ such that $0 \leq \mathfrak{J} \leq 1/2$, if there are “small” positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$).

As shown in detail in Appendix B.5, the boundaries to the concentration index are to be interpreted as follows: $\mathfrak{J} = 1$ refers to *total concentration of firms and production in a single sector* and $\mathfrak{J} = 0$ refers to *total concentration of firms in one sector and of production in the other*. We identify the extreme case $\mathfrak{J} = 1$ as industry concentration of type I and $\mathfrak{J} = 0$ as industry concentration of type II. In contrast, $\mathfrak{J} = 1/2$ implies a *uniform distribution of firms and production across sectors*.²¹

By interpreting this exercise again as a cross-country comparison of industrial structures, we conclude that, depending on the regime of scale effects, the industrial structure of a given country is characterised by different concentration patterns. In the case of either negative or “large” positive scale effects, a country with a higher $\mathcal{H}/\mathcal{L} < 1$ is expected to have a *lower* concentration index, with $\mathfrak{J} = 1/2$ as the lower boundary, i.e., a *uniform distribution*; a country with a higher $\mathcal{H}/\mathcal{L} > 1$ is expected to have a *higher* concentration index, with $\mathfrak{J} = 1$ as the upper boundary, i.e., *concentration of type I*. In the case of “small” positive scale effects, a country with a higher $\mathcal{H}/\mathcal{L} < 1$ is expected to have a *higher* concentration index, with $\mathfrak{J} = 1/2$ as the upper boundary; a country with a higher $\mathcal{H}/\mathcal{L} > 1$ is expected to have a *lower* concentration index, with $\mathfrak{J} = 0$ as the lower boundary, i.e., *concentration of type II*.

Figure 4.2 illustrates the non-monotonic relationship between the concentration index and

²¹To be rigorous, when $\epsilon = \bar{\epsilon}_1$ or $\epsilon = 1$, then $\mathfrak{J} \rightarrow \frac{1}{2}$ for $\frac{\mathcal{H}}{\mathcal{L}} > 1 \wedge \frac{\mathcal{H}}{\mathcal{L}} \rightarrow \infty$ or $\frac{\mathcal{H}}{\mathcal{L}} < 1 \wedge \frac{\mathcal{H}}{\mathcal{L}} \rightarrow 0$. We interpret these as “degenerate” cases associated with specific levels of scale effects, $\epsilon = \bar{\epsilon}_1$ and $\epsilon = 1$ (see Appendix B.5).

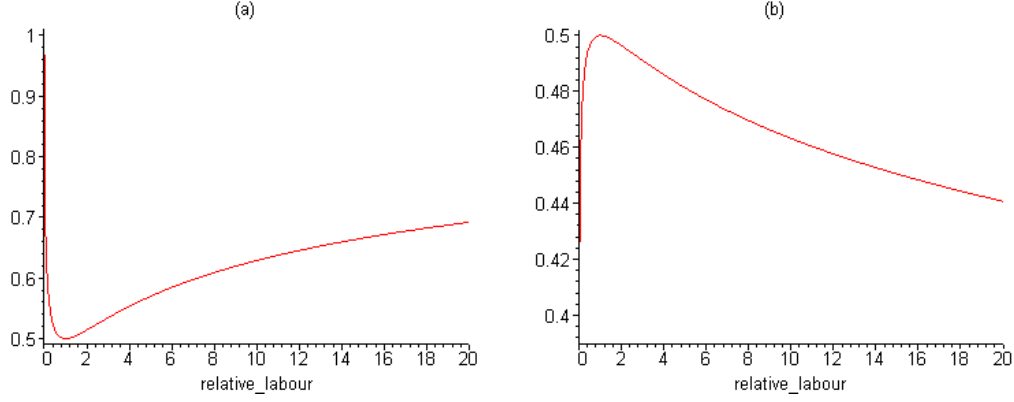


Figure 4.2.: Concentration index in the case of $0 \leq \epsilon < \bar{\epsilon}_1$ or $\epsilon > 1$ (panel (a)), and in the case of $\bar{\epsilon}_1 < \epsilon < 1$ (panel (b)).

the level of relative labour endowment. Panel (a) depicts the case of $0 \leq \epsilon < \bar{\epsilon}_1$ and $\epsilon > 1$. Panel (b) depicts the case of $\bar{\epsilon}_1 < \epsilon < 1$. Observe that the concentration index exhibits a symmetric behaviour with respect to $\mathcal{H}/\mathcal{L}$ in geometric terms, i.e., if we define the function $\mathfrak{J} \equiv \mathfrak{J}(\mathcal{H}/\mathcal{L})$, for a given ϵ , then it can be shown that $\mathfrak{J}(\mathcal{H}/\mathcal{L}) = \mathfrak{J}\left((\mathcal{H}/\mathcal{L})^{-1}\right)$.²² On the other hand, it can also be shown that the *higher* σ and γ , the *less pronounced* the rate of change of $\mathfrak{J}$ with respect to $\mathcal{H}/\mathcal{L}$.

Computation of the concentration index by using the data in Figure B.1, Appendix B.1, shows that the index displays values above $1/2$, hence implying industry concentration of type I. This is compatible with our results in the case of negative or “large” positive scale effects. On the other hand, the concentration index and both relative production and the relative number of firms display a negative correlation across countries, which is also compatible with our model. Indeed, let $N < 1$ and $\hat{\mathfrak{X}} < 1$, which are in accord with the data on the relative number of firms and relative production (see Subsection 4.3.1.2,

²²First note, from (4.46) and (4.45), that $((\mathcal{H}/\mathcal{L})^{-1})^{\mathcal{D}_1} = 1/\hat{\mathfrak{X}}$ and $((\mathcal{H}/\mathcal{L})^{-1})^{\frac{1-\epsilon}{\sigma+\gamma+1}} = 1/N$. Second, let $\mathfrak{J}_{1/\hat{\mathfrak{X}}} = \frac{\frac{1}{\hat{\mathfrak{X}} \cdot N} + 1}{\frac{1}{\hat{\mathfrak{X}} \cdot N} + \frac{1}{\hat{\mathfrak{X}}} + \frac{1}{N} + 1}$. Finally, multiply both numerator and denominator by $\hat{\mathfrak{X}} \cdot N$, to get $\mathfrak{J}_{1/\hat{\mathfrak{X}}} = \frac{1 + \hat{\mathfrak{X}} \cdot N}{1 + \hat{\mathfrak{X}} \cdot N + \hat{\mathfrak{X}} + N} = \mathfrak{J}$ (see (4.48)).

above); if those values reflect $\mathcal{H}/\mathcal{L} < 1$ with positive scale effects (Proposition 4.2), then higher N and $\hat{\mathfrak{X}}$ due to higher $\mathcal{H}/\mathcal{L}$ (Proposition 4.1) must imply a lower concentration index, approaching $1/2$ from above; if those values reflect $\mathcal{H}/\mathcal{L} > 1$ with negative scale effects, then lower N and $\hat{\mathfrak{X}}$ due to higher $\mathcal{H}/\mathcal{L}$ must imply a higher concentration index, approaching 1 from below.

Notably, given the intervals for ϵ in Proposition 4.3, the results above are compatible with those in Subsection 4.3.1, if we have $\mathcal{H}/\mathcal{L} < 1$ combined with positive scale effects in the interval $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$.

4.3.2.2. Dynamic entry costs, concentration and aggregate growth

Now, we explore the association between the concentration index and aggregate long-term growth, g , by considering the simultaneous impact of changes in the elasticities of the horizontal entry cost function, σ and γ , on both variables, for a given relative labour endowment, $\mathcal{H}/\mathcal{L}$.²³ The following propositions summarise the effect of σ and γ on g and $\mathfrak{J}$.

Proposition 4.4. The aggregate growth rate, g , is *decreasing* in the elasticities σ and γ .

Proposition 4.5.

- (i) $\mathfrak{J}$ is decreasing in σ and γ (approaching $\mathfrak{J} = 1/2$ from above), if there are “large” positive ($0 \leq \epsilon < \bar{\epsilon}_1$) or negative scale effects ($\epsilon > 1$), and $\mathcal{H}/\mathcal{L} \neq 1$;
- (ii) $\mathfrak{J}$ is increasing in σ and γ (approaching $\mathfrak{J} = 1/2$ from below), if there are “small” positive scale effects ($\bar{\epsilon}_1 < \epsilon < 1$), and $\mathcal{H}/\mathcal{L} \neq 1$;
- (iii) $\mathfrak{J}$ is independent of σ and γ , if $\epsilon = 1$ or $\mathcal{H}/\mathcal{L} = 1$.

²³Notice that σ and γ are the only primitive parameters that influence simultaneously g and $\mathfrak{J}$, for a given set $(\epsilon, \mathcal{H}/\mathcal{L})$ (see (4.43), (4.39), (4.45) and (4.44)).

4. Directed Technical Change, Scale Effects and Industrial Structure

From Propositions 4.4 and 4.5, we conclude that there is a *positive relationship between aggregate growth and concentration of type I*, when scale effects are negative or positive and “large”, and a *positive relationship between aggregate growth and concentration of type II*, when scale effects are positive but “small”.

According to the data in Figure B.1, Appendix B.1, the concentration index and aggregate per capita growth display a positive correlation, which is in line with our model whatever the level of (non-null) scale effects.

Notably, given the intervals for ϵ in Proposition 4.5, the results above are compatible with those in Subsections 4.3.1 and 4.3.2.1, if we have positive scale effects in the interval $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$.

It is clear from the above how the general equilibrium nature of our model allows for simultaneous determination of aggregate growth and industrial structure, as a subset of the technology parameters (σ, γ) that determine the aggregate growth rate also influence concentration. Finally, it should also be clear that our results with respect to the association between aggregate growth and industrial structure concern its *quantitative* dimension (i.e., how many firms and how much production are allocated to each sector vis-à-vis the others) and not its *qualitative* dimension (i.e., concentration of economic activity in a specific type of sector), as pursued by the literature of structural change (e.g., Fagerberg, 2000 and Bonatti and Felice, 2008).²⁴

²⁴Appendix B.6 analyses the impact of changes in the dynamic horizontal-entry costs, represented by the elasticities σ and γ , in the asymmetry of the industrial structure. In turn, Appendix B.7 explores a version of the Schumpeterian hypothesis by studying the interrelation between R&D intensity, measured as the ratio between R&D outlays and output, and average firm size.

4.3.3. Regime of scale effects implied by the data

Before concluding, it is useful to make a final assessment of the model by establishing which regime of scale effects is implied by the data. As shown throughout Section 4.3, the model generates a number of results, which depend on the assumed regime of scale effects, ϵ , and the level of relative labour endowment, $\mathcal{H}/\mathcal{L}$ (above or below unity). However, the confrontation of the theoretical results with the data, plus the requirement that the those results are consistent with each other, allows us to select among them and the associated interval for ϵ and $\mathcal{H}/\mathcal{L}$. We then conclude that the regime of scale effects that is able to account for all the empirical regularities corresponds to positive scale effects in the interval $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_1$, combined with $\mathcal{H}/\mathcal{L} < 1$.

While the selected interval for $\mathcal{H}/\mathcal{L}$ is in accordance with the empirical data (e.g., Acemoglu, 2003), the case for ϵ may depend on the size of its interval. It varies with the elasticities of the horizontal entry cost function, σ and γ , such that $\lim_{\gamma, \sigma \rightarrow \infty} \bar{\epsilon}_1 = \lim_{\gamma, \sigma \rightarrow \infty} \bar{\epsilon}_2 = 1/(1 + \alpha)$, and $\bar{\epsilon}_1|_{\gamma, \sigma=0} = 1 - \alpha$ and $\bar{\epsilon}_2|_{\gamma, \sigma=0} = 0$. With $\alpha = 0.6$, which is a standard value for the labour share in production in the endogenous-growth literature, then $1/(1 + \alpha) = 0.625$. Thus, with σ and γ large enough but finite, one can denote as “small” positive the scale effects that make the theoretical results compatible with the data. This should then be in line with the well-known endogenous-growth debate over the counterfactual character of *large* positive scale effects (e.g., Jones, 1995b).

On the other hand, the described dependence of $\bar{\epsilon} \in \{\bar{\epsilon}_1, \bar{\epsilon}_2\}$ on σ and γ makes clear the role of the horizontal-entry mechanism in explaining the data on industrial structure with an empirically reasonable regime of scale effects: without entry (corresponding to $\sigma, \gamma \rightarrow \infty$, since $\lim_{\sigma \rightarrow \infty} \tilde{g} = \lim_{\gamma \rightarrow \infty} \tilde{g} = \tilde{g}_{no-entry}$), the selected interval for ϵ collapses into $\epsilon = 1/(1 + \alpha)$; hence only two regimes of scale effects exist, corresponding to the early defined regimes of

“large” positive and negative scale effects.

4.4. Concluding remarks

This essay studies a specific dimension of the industrial structure, that of the distribution of firms and production across sectors, by building an endogenous general equilibrium growth model of directed technical change that merges the expanding-variety with the quality-ladders mechanism. The model allows us to accommodate the view that the relationship between industrial structure, innovative activity and aggregate growth is not causal: they are simultaneously (endogenously) determined.

The model represents a first attempt to devise an analytical mechanism to explain the data on the number of firms, production and average firm size in high-tech vis-à-vis low-tech sectors in a number of countries. It also provides a theoretical instrument to study the association between concentration, measured at the aggregate level, and long-run aggregate growth.

Results hinge on the assumption that scale effects connected to the size of profits that, in each period, accrue to the R&D successful firm may be negative, positive or null. By focusing on the BGP, we find that, as the regime of scale effects changes, the relation between industrial structure and relative labour endowment measured in efficiency units may differ significantly. If anything, the confrontation with the data suggests that the empirical relevance of our results depends on the existence of small positive scale effects. On the other hand, it is well known the endogenous-growth debate over the counterfactual character of large positive scale effects. Given this, we underline the practical importance of distinguishing small from large positive scale effects, since, in that case, the predictions of the model suffer a change of sign due to the “regime switch”.

On the empirical side, further research should be devoted to fill the data gap on the magnitude of scale effects and relative labour endowment in efficiency units, in order to assess the quantitative relevance of our mechanism. In particular, observe that our cross-country analysis hinges mostly on the assumption that countries with different industrial structures also display different relative labour ratios; however, one may as well expect a priori that distinct industrial structures occur with similar relative labour ratios. Some effort might also be devoted to the collection of data on the composition of the technological-knowledge stock between high- and low-tech sectors. On the other hand, a larger set of countries than the one used here, in particular including the US and Japan for all variables, is desirable in order to guarantee robustness of empirical results.

Finally, in this essay, the whole of our theoretical analysis was conducted in terms of the BGP. However, the study of the transitional dynamics should be an objective for future work. The qualitative characterisation of the local dynamics properties might allow one to find to what extent variations in a country's initial conditions (namely the inherited number of firms and stock of technological knowledge) lead to non-monotonic time paths of both industrial structure and skill premium towards the BGP. On the other hand, given the role played by factor endowment and scale effects, it should be only natural to extend the model to an open-economy framework, in particular focusing on intermediate-good international trade and its impact on the cross-country industrial structure.

5. Animal Spirits and the Composition of Innovation in a Lab-Equipment R&D Model with Transition¹

Abstract: *This essay revisits the issue of self-fulfilling “waves of enthusiasm” as stationary rational expectations equilibria in tournament models of horizontal and vertical R&D. By considering a lab-equipment specification, the model predicts a positive effect of animal spirits on the balanced-growth-path (BGP) level of per-capita consumption without impacting on economic growth and on aggregate vertical R&D. However, transition is slower under “waves of enthusiasm”, implying a longer period in which growth rates are higher than the BGP level. An economy that is subject to expectations shocks then converges at a time-varying speed. On average over time, transition is longer but less “painful” - i.e., with higher per-capita consumption levels - than otherwise.*

Keywords: *animal spirits; endogenous growth; horizontal and vertical R&D; time-varying speed of transition.*

JEL Classification: *O41, E32, D43, L16*

¹The results in this chapter appeared as a working paper (Gil, 2009a).

5.1. Introduction

This essay studies the effect of animal spirits, or sunspots, on the composition of aggregate R&D, the industrial structure (number of firms and average firm size), consumption level and economic growth in an endogenous-growth model of vertical and horizontal R&D, both along the balanced-growth path (BGP) and transition. The model is developed in the light of a number of empirical observations pertaining to R&D and growth: the existence of (i) “waves of enthusiasm”, according to which new industries are temporarily flooded by higher R&D expenditure (e.g., Klepper, 1996); (ii) insignificant or negative correlation between aggregate R&D intensity and the per-capita GDP growth rate (e.g., Bassanini, Scarpetta, and Visco, 2000); (iii) time-varying speeds of convergence, driven by technological factors (e.g., Bernard and Jones, 1996).

Formally, this essay develops a non-scale tournament model of quality ladders (vertical R&D) and expanding variety (horizontal R&D),² where, different from the literature, a lab-equipment specification is considered. The existence of indeterminate deterministic BGPs, each of which is saddle-point stable, is then proved: the economy is free to select any of these equilibria depending on how animal spirits coordinate (rational) entrepreneurial expectations. This extends Cozzi (2005)’s analysis of Howitt (1999) to a different framework, which allows one to show that Cozzi’s “waves of enthusiasm” do not have any effect on BGP growth rates. Indeed, in our essay the higher vertical R&D in the new sectors

²In the tournament, or “creative destruction”, models of horizontal and vertical R&D, the entrant must choose between vertical and horizontal R&D, whereas the incumbent does not perform any R&D (e.g., Howitt, 1999; Segerstrom, 2000). The non-tournament models, in turn, postulate vertical R&D is made by the incumbent and horizontal R&D is carried out by entrants (e.g., Dinopoulos and Thompson, 1998; Peretto and Smulders, 2002). Indeed, according to the stylised facts on entry (e.g., Geroski, 1995; Klepper, 1996), it (i) occurs mostly in already existing sectors, (ii) is driven mostly by new firms (instead of incumbents enduring diversification processes), and (iii) is closely related to R&D activities. In a model of R&D and firm dynamics under monopolistic competition, like ours, where R&D activity is the only mechanism by which entrants can make a competitive entry (by introducing either a new variety or a higher-quality version of an existing product), that implies that not only horizontal but also vertical R&D must be performed by entrants.

tends to crowd out the R&D from the more mature sectors. Therefore, balanced growth is more immune to animal spirits than expected. In contrast, the speed of transition is reduced under “waves of enthusiasm”, which implies the economy experiences a longer period in which growth rates are higher than the BGP level.

Since Azariadis (1981), Cass and Shell (1983) and others, it is well-known that we can construct economic environments in which changes in economic agents’ beliefs per se, if shared by everyone, influence current choices in such a way that the variations in beliefs are self-fulfilling, i.e., there are sunspot equilibria. More specifically, these are rational expectations equilibria that are perfectly correlated with stochastic factors that do not affect the preferences, endowment and production set of any individual; hence, only purely extrinsic uncertainty affects prices and allocations.

This essay is concerned with the existence of sunspot equilibria in a continuous-time endogenous-growth setting with multiple stationary equilibria, as highlighted by Cozzi (2005). We focus on the concept of Stationary Sunspot Equilibria (SSE),³ built as a randomisation over multiple deterministic stationary equilibria, in the tradition of Cass and Shell (1983). Due to the existence of multiple equilibria, there exists a coordination problem between firms, which cannot be solved by referring solely to the fundamentals of the economic system. Following a common practice in the literature, we assume that firms tackle this coordination problem by referring to some extrinsic stochastic process, which “selects” a specific equilibrium over the others.

We adopt Cozzi (2005)’s approach in order to show that our lab-equipment model admits multiple deterministic stationary equilibria. Cozzi highlights the asymmetric BGPs ad-

³If the extrinsic factors are subject to a stationary stochastic process, then the sunspot equilibria are called “stationary”, that is, the effect of beliefs agents hold about their environment does not vanish asymptotically. As argued by, e.g., Azariadis and Guesnerie (1986), this property is relevant namely because stationary beliefs are likely to be the asymptotic outcome of many stable learning processes (e.g., Woodford, 1990; and, more recently, Evans, Honkapohja, and Marimon, 2007).

mitted by tournament models that simultaneously consider quality ladders and expanding variety, addressing the often observed waves of innovations characterised by a flood of quality improving R&D into newly introduced sectors (e.g., Gort and Klepper, 1980; Klepper, 1996). Cozzi assumes that, as soon as a new good is introduced, there will be a “wave of enthusiasm” for that sector, in the sense that the new product line attracts more vertical R&D than the older ones, thus implying a “supernormal” process of creative destruction in the new sector. He also assumes that “enthusiasm” disappears after the first quality improvement, which implies that, along the BGP, horizontal innovation will be discouraged owing to an expected increase in the rate of creative destruction until the first quality jump. Therefore, the model predicts an asymmetric equilibrium (indeed a continuum of asymmetric equilibria) with a larger vertical R&D intensity engaged in improving the quality of already improved products, and thus of aggregate vertical R&D intensity, at the expense of a lower horizontal R&D intensity.

Cozzi explores a version of Howitt (1999), who – similarly to, e.g., Segerstrom (2000), Peretto and Smulders (2002) and Peretto and Connolly (2007) – presents a growth model that features the two types of R&D within a knowledge-driven specification. Under this setup, the choice between vertical and horizontal R&D implies a division of labour between the two types of R&D. Since the total labour level is determined exogenously, there is a shortcoming: in the end, the rate of extensive growth is exogenous, i.e., the BGP flow of new goods occurs at the same rate as (or proportional to) population growth.

In contrast, we consider a (non-scale) lab-equipment specification.⁴ This assumption implies that the choice between vertical and horizontal innovation is related to the splitting of R&D expenditures, which are fully endogenous. Therefore, we endogeneise the overall growth rate

⁴Using Rivera-Batiz and Romer (1991)’s terminology, the assumption that the final good is the R&D input means that one adopts the “lab-equipment” version of R&D, instead of the “knowledge-driven” specification, in which labour is the only input.

and thus the rate of extensive growth. Also differently from Howitt (1999) and others, we assume that there are no intersectoral spillovers in vertical R&D (e.g., Segerstrom, 2007; Etro, 2008). As shown in detail elsewhere,⁵ this framework gives rise to strong endogenous inter-R&D composition effects, this way offering a (qualitative) explanation for the negative or insignificant empirical correlation between aggregate R&D intensity and both firm size and economic growth, a well-known puzzle in the growth literature (e.g., Bassanini, Scarpetta, and Visco, 2000; Pessoa, 2010).⁶

One of the most interesting findings of the essay is that animal spirits have an effect on the composition of R&D without impacting on aggregate vertical R&D: in our general-equilibrium setting, reduced outlays in “mature” industries compensate for the increased R&D intensity in newly-born industries. Thus, we have an inter-R&D composition effect combined with an intra-(vertical-)R&D composition effect. Since vertical R&D is the ultimate growth engine,⁷ this implies that animal spirits have no impact on the aggregate growth rate; however, there is a positive welfare effect, given the positive impact on the level of per-capita consumption. These results contrast with the, respectively, positive and negative effects obtained by Cozzi. A positive relationship between animal spirits (optimistic expectations of firms) and the BGP aggregate growth is also found in an endogenous-growth setup by, e.g., Drugeon and Wigniolle (1996) and Nishimura and Shigoka (2006). In contrast, our positive welfare effect is in line with the business-cycle models by, e.g., Chatterjee, Cooper, and Ravikumar (1993) and Dos Santos Ferreira and Dufourt (2006).

⁵See Gil, Brito, and Afonso (2010).

⁶In the models with a knowledge-driven specification, re-compositions between vertical and horizontal R&D are not able to produce a negative correlation between total R&D intensity and economic growth, as the latter changes in response to the intensive margin only.

⁷As explained later, the negative spillovers in horizontal entry per se determine a constant number of varieties/firms along the BGP (e.g., Romer, 1990); however, vertical R&D sustains variety expansion and aggregate growth at a positive finite rate, because the expected growth of goods quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an entry cost, in spite of its increase over time.

However, an also important result is that transition is slower under “waves of enthusiasm”. This implies that the economy spends more time in transition vis-à-vis the symmetric case, hence experiencing a longer period in which aggregate growth rates are higher and aggregate vertical innovation rates are lower than in the BGP. A catching-up economy that is subject to expectations shocks oscillates between an “expansive phase”, corresponding to a higher *level* and a slower *rate of decline* of per-capita consumption in effective terms, and a “recessive phase”, where the opposite is true. Therefore, on average over time, the economy endures a longer but less “painful” transition – i.e., with higher per-capita consumption levels – than otherwise.

The model exhibits, under sunspots, a time-varying speed of convergence, an empirically relevant feature emphasised by, e.g., Bernard and Jones (1996) and Castellacci (2008). In the theoretical models by Eicher and Turnovsky (2001) and Brito and Dixon (2009), among others, such a result occurs because of the interactions between two state-variables under a two-dimensional stable manifold. The distinctive feature of our essay is that the time-varying speed of adjustment entirely results from self-fulfilling expectations held by the entrepreneurs, in the context of a single-dimensional stable manifold.

These results suggest that animal spirits retain their relevance for the purpose of economic growth, aggregate vertical R&D and welfare, especially for those economies experiencing a transition process.

The rest of the essay has the following structure. The next section briefly discusses the related literature on SSE. Section 5.3 outlines the symmetric equilibrium produced by the model of vertical and horizontal R&D within a lab-equipment specification. Section 5.4 focus on the class of asymmetric general equilibria characterised by supernormal waves of creative destruction in the newly introduced intermediate goods. Section 5.5 is concerned with the existence of SSE obtained as a randomisation over the multiple deterministic

equilibria. Section 5.6 studies the local-dynamics properties of the asymmetric equilibria and the sunspot transition. Finally, section 5.7 gives some concluding remarks.

5.2. Related literature on SSE

Distinct forms of SSE have been explored by the literature, in the context of different classes of general-equilibrium rational-expectations models.

The seminal work by Cass and Shell (1983) describes SSE obtained as a lottery, or randomisation, over *multiple* deterministic equilibria within a discrete-time overlapping-generation model. Using the stationary transition probabilities (from sunspot to no-sunspot activity and vice versa) as weighting factors, SSE are derived as the convex combination of the multiple solutions to the deterministic optimisation problem. Azariadis and Guesnerie (1986) derive a sufficient condition for the existence of SSE around the *single* deterministic stationary equilibrium (or deterministic cycle) also within a discrete-time overlapping-generation model. The authors identify a subset of all stationary transition probability matrices for which SSE exist as the solution of a “stochastic deformation” of the deterministic optimisation problem. Woodford (1986) presents a method of constructing SSE generated by a stationary transition probability matrix in a discrete-time model that exhibits the *indeterminacy* of equilibrium near a steady state. Our essay is in line with the first approach, by studying a model with multiple stationary equilibria and constructing SSE as a randomisation over these equilibria. In this respect, the SSE we study have a simpler origin than the other forms of SSE extant in the literature.

Like us, more recently some authors have studied SSE in continuous-time infinitely-lived agents models of endogenous growth. Drugeon and Wigniolle (1996) develop an extension of Grossman and Helpman (1991), and establish sufficient conditions for the existence of

a SSE around the single deterministic stationary equilibrium, which include restrictions from above on the stationary transition probabilities of a continuous-time Markov process. Nishimura and Shigoka (2006) show how to construct a SSE in multisector endogenous-growth models with local indeterminacy,⁸ based on, e.g., Lucas (1988) and Romer (1990).⁹ Those papers exhibit a positive relationship between sunspots (optimistic expectations of firms) and the long-run aggregate growth (just like Cozzi, 2005), while in our model sunspots have no effect on long-run aggregate growth.

By focusing on the endogenous number of firms, we also relate to some papers in the business-cycle literature with endogenous firm entry and exit. Dos Santos Ferreira and Dufourt (2006) study a discrete-time real-business-cycle (RBC) model, while Chatterjee, Cooper, and Ravikumar (1993) present a discrete-time two-sector overlapping-generation model. In both models, there is Cournot competition and variations in the number of active firms are associated with the aggregate fluctuations due to sunspots, constructed as a randomisation between multiple deterministic steady states. Jaimovich (2007) studies an RBC model with firm entry and exit where the SSE are derived upon local indeterminacy of the deterministic steady state. Notably, in all these models, sunspots have a positive effect on the number of active firms, whereas in our growth model (similarly to Cozzi, 2005), sunspots have a negative effect on the number of firms.¹⁰

⁸This paper extends the continuous-time analysis in Shigoka (1994).

⁹Also within the endogenous-growth literature, Francois and Lloyd-Ellis (2003) study the effects of animal spirits on long-run growth by developing an extension of Grossman and Helpman (1991) where the realisation of innovations is separated in time from their implementation. However, in their model, expectations are deterministic.

¹⁰Note also that we study the particular class of SSE of order two, i.e. with two possible events or states of nature, as in Cass and Shell (1983), Azariadis and Guesnerie (1986), Drugeon and Wigniolle (1996), Nishimura and Shigoka (2006), among others, but in contrast to, e.g., Shigoka (1994) and Dos Santos Ferreira and Dufourt (2006).

5.3. The benchmark model

In this section, we first present the setup of the full-endogenous growth model of quality ladders and expanding variety and then derive its symmetric equilibrium.

We explore a dynamic general equilibrium model of a closed economy where a single competitively-produced final good can be used in consumption, production of intermediate goods, and R&D. The final good is produced by a (large) number of firms each using labour and a continuum of intermediate inputs indexed by $\omega \in [0, N]$. The economy is populated by fixed infinitely-lived households who inelastically supply labour to final-good firms. In turn, families make consumption decisions and invest in firms' equity.

Potential entrants can devote resources either to horizontal or to vertical R&D. Horizontal R&D increases the number of intermediate-good industries N , while vertical R&D increases the quality of the good of an existing industry, indexed by $j(\omega)$. Quality level $j(\omega)$ translates into productivity of the final producer from using the good produced by industry ω , $\lambda^{j(\omega)}$, where $\lambda > 1$ is a parameter measuring the size of each quality upgrade. By improving on the current best quality index j , a successful R&D firm will introduce the leading-edge quality $j(\omega) + 1$ and hence render inefficient the existing input supplied by the producer of ω . Therefore, the successful innovator will become a monopolist in ω . However, this monopoly is temporary, because a new successful innovator will eventually substitute the incumbent.

5.3.1. Households

The economy is populated by a fixed number of infinitely-lived households who consume and collect income from investments in financial assets (equity) and from labour. Households inelastically supply labour to final-good firms; thus, total labour supply, L , is exogenous and

constant. We assume consumers have perfect foresight concerning the aggregate rate of technological change over time,¹¹ and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximise the discounted lifetime utility

$$U = \int_0^\infty \left(\frac{C(t)^{1-\Theta} - 1}{1-\Theta} \right) e^{-\rho t} dt, \quad (5.1)$$

where $\rho > 0$ is the subjective discount rate and $\Theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + w(t) \cdot L - C(t), \quad (5.2)$$

where a denotes households' real financial assets holdings, r the equilibrium market real interest rate and w the real labour wage. The initial level of wealth $a(0)$ is given, whereas the non-Ponzi games condition $\lim_{t \rightarrow \infty} e^{-\int_0^t r(s) ds} a(t) \geq 0$ holds. The optimal path of consumption satisfies the well-known differential Euler equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\Theta} (r(t) - \rho), \quad (5.3)$$

as well as the transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} C(t)^{-\Theta} a(t) = 0$.

5.3.2. Production and price decisions

The final-good firm has a constant-returns-to-scale technology using labour and a continuum of intermediate goods with measure $N(t)$, changing over time t , which is well-known from Barro and Sala-i-Martin (2004, ch. 6 and ch. 7)

¹¹As we will see below, the uncertainty in R&D at the industry level creates jumpiness in microeconomic outcomes. However, as the probabilities of successful R&D across industries are independent and there is a continuum of industries, this jumpiness is not transmitted to macroeconomic variables.

$$Y(t) = L^{1-\alpha} \cdot \int_0^{N(t)} \left[\lambda^{j(\omega,t)} \cdot X(\omega, t) \right]^\alpha d\omega, \quad 0 < \alpha < 1, \quad \lambda > 1, \quad (5.4)$$

where L is the labour input and $1 - \alpha$ is the labour share in production, and $\lambda^{j(\omega,t)} \cdot X(\omega, t)$ is the input of intermediate good ω measured in efficiency units at time t .¹² That is, we integrate the final-producer technology that is considered in variety-expansion and quality-ladders models (Barro and Sala-i-Martin, 2004, ch.6 and ch. 7, respectively).

Final producers are price-takers in all the markets they participate. They take wages, $w(t)$, and input prices $P(\omega, t)$ as given and sell their output at a price equal to unity. From the profit maximisation conditions, we determine the aggregate demand of intermediate good ω as

$$X(\omega, t) = L \cdot \left(\frac{\lambda^{j(\omega,t)\alpha} \cdot \alpha}{P(\omega, t)} \right)^{\frac{1}{1-\alpha}}, \quad \omega \in [0, N(t)]. \quad (5.5)$$

The intermediate-good sector consists of a continuum $N(t)$ of industries. There is monopolistic competition if we consider the whole sector: the monopolist in industry ω fixes the price $P(\omega, t)$ but faces the isoelastic demand curve (5.5). We assume that the intermediate good is non-durable and entails a unit marginal cost of production, in terms of the final good, whose price is taken as given. The profit maximising price in industry ω is a constant markup over marginal cost $P(\omega, t) \equiv P = 1/\alpha > 1$,¹³ which implies the aggregate quantity produced of ω

¹²In equilibrium, only the top quality of each ω is produced and used; thus, $X(j, \omega, t) = X(\omega, t)$. Henceforth, we only use all arguments (j, ω, t) if they are useful for expositional convenience.

¹³We assume that $\frac{1}{\alpha} < \lambda$; i.e., if $\frac{1}{\alpha}$ is the price of the top quality, the price of the next lowest grade, $\frac{1}{\alpha\lambda}$, is less than the unit marginal cost. In this case, lower grades are unable to provide any effective competition, and the top-quality producer can charge the unconstrained monopoly price.

$$X(\omega, t) = L \cdot \left(\lambda^{j(\omega, t)\alpha} \cdot \alpha^2 \right)^{\frac{1}{1-\alpha}}. \quad (5.6)$$

Using the results above we get the profit accrued by the monopolist in ω

$$\pi(\omega, t) = \tilde{\pi} \cdot L \cdot \lambda^{j(\omega, t)\left(\frac{\alpha}{1-\alpha}\right)}, \quad (5.7)$$

where $\tilde{\pi} \equiv (1 - \alpha) \cdot \alpha^{\frac{2}{1-\alpha}} / \alpha$.

Substituting (5.6) in (5.4) yields the aggregate output

$$Y(t) = \alpha^{\frac{2\alpha}{1-\alpha}} \cdot L \cdot Q(t), \quad (5.8)$$

where

$$Q(t) = \int_0^{N(t)} \lambda^{j(\omega, t)\left(\frac{\alpha}{1-\alpha}\right)} d\omega \quad (5.9)$$

is the intermediate-input aggregate quality index, which measures the technological-knowledge level of the economy, since, by assumption, there are no intersectoral spillovers. Total resources devoted to intermediate input production are also a linear function of $Q(t)$

$$X(t) = \int_0^{N(t)} x(\omega, t) d\omega = \alpha^{\frac{2}{1-\alpha}} \cdot L \cdot Q(t), \quad (5.10)$$

as are total profits

$$\Pi(t) = \int_0^{N(t)} \pi(\omega, t) d\omega = \tilde{\pi} \cdot L \cdot Q(t). \quad (5.11)$$

5.3.3. R&D

We assume there is both vertical and horizontal R&D (e.g., Howitt, 1999; Segerstrom, 2000), that R&D is only performed by (potential) entrants, and that successful R&D leads to the set up of a new firm in either an existing or in a new industry. There is perfect competition among entrants and free entry in R&D business. These assumptions are consistent with a number of stylised facts on entry (e.g., Geroski, 1995; Klepper, 1996).

As in the standard model of quality ladders, vertical R&D constitutes the search for new designs that lead to a higher quality of existing intermediate goods. Each new design is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. By improving on the current top quality level $j(\omega, t)$, a successful R&D firm earns monopoly profits from selling the leading-edge input of $j(\omega, t) + 1$ quality to final-good firms. A successful innovation will instantaneously increase the quality index in ω from $q(\omega, t) = q(j) \equiv \lambda^{j(\omega, t)\alpha/(1-\alpha)}$ to $q^+(\omega, t) = q(j+1) = \lambda^{\alpha/(1-\alpha)}q(\omega, t)$. In equilibrium, lower qualities of ω are priced out of business.

Let $I_i(j)$ denote the Poisson arrival rate of vertical innovations (vertical-innovation rate) by potential entrant i in industry ω when the highest quality is j . Rate $I_i(j)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R_{vi}(j)$ committed by entrants at time t . As in, e.g., Barro and Sala-i-Martin (2004, ch. 7), $I_i(j)$ features constant returns in R&D expenditures, $I_i(j) = R_{vi}(j) \cdot \Phi(j)$, where $\Phi(j)$ is the R&D productivity factor, which is assumed to be homogeneous across i in ω . We assume

$$\Phi(j) = \frac{1}{\zeta \cdot L \cdot q(j+1)} \quad (5.12)$$

where $\zeta > 0$ is a constant flow fixed vertical-R&D cost. With (5.12), we wish to capture the idea that the larger is the scale of expected production of a firm, $L \cdot q(j+1)$ (see

(5.6)), the larger are the costs necessary to discover and develop the associated technology: e.g., construction of prototypes and samples, new assembly lines and training of workers. These assumptions guarantee that spending in R&D increases at the same rate as output, delivering a BGP without scale effects, in line with the last generation of quality-ladders models (e.g., Jones, 1995a; Barro and Sala-i-Martin, 2004).¹⁴ Also, differently from Howitt (1999) and others, we assume that there are no intersectoral spillovers in vertical R&D (e.g., Segerstrom, 2007; Etro, 2008). Aggregating across i in ω , we get $R_v(j) = \sum_i R_{vi}(j)$ and $I(j) = \sum_i I_i(j)$, and thus

$$I(j) = R_v(j) \cdot \frac{1}{\zeta \cdot L \cdot q(j+1)} \quad (5.13)$$

As the terminal date of each monopoly arrives as a Poisson process with frequency $I(j)$ per (infinitesimal) increment of time, the present value of a monopolist's profits is a random variable. Let $V(j)$ denote the expected value of an incumbent firm with current quality level $j(\omega, t)$,¹⁵ such that $V(j) = \pi(j) \int_t^\infty e^{-\int_t^s (r(v) + I(j)) dv} ds$, where $\pi(j)$, given by (5.7), is constant in-between innovations. Along the BGP, r and I are constants; hence, we can further write

$$V(j) = \frac{\pi(j)}{r + I(j)}. \quad (5.14)$$

On the other hand, free-entry prevails in vertical R&D such that the condition $I(j) \cdot V(j+1) = R_v(j)$ holds, which implies that

¹⁴Moreover, the increasing difficulty of creating new qualities over t due to the growth in j exactly offsets the increasing rewards from marketing higher qualities - see (5.12) and (5.7). As shown below, this allows for a constant Poisson rate along the BGP.

¹⁵We assume that entrants are risk-neutral and, thus, only care about the expected value of the firm.

$$V(j+1) = \frac{1}{\Phi(j)} = \zeta \cdot L \cdot q(j+1). \quad (5.15)$$

By substituting (5.14) into (5.15) and using (5.7) to simplify, we get the arbitrage equation facing a vertical innovator

$$r = \frac{\tilde{\pi}}{\zeta} - I \quad (5.16)$$

According to (5.16), the rates of entry are symmetric across industries, $I(j) = I$.

Variety expansion arises from R&D aimed at creating a new intermediate good. Again, innovation is performed by a potential entrant, which means that, because there is free entry, the new good is produced by a new firm. Under perfect competition among R&D firms and constant returns to scale at the firm level, instantaneous entry is obtained as $\dot{N}_e(t) = \frac{1}{\eta} R_{ne}(t)$, where $\dot{N}_e(t)$ is the contribution to the instantaneous flow of the new good by R&D firm e at a cost of η units of the final good and $R_{ne}(t)$ is the flow of resources devoted to horizontal R&D by innovator e at time t . The cost η is assumed to be symmetric, with $\eta \equiv \eta(t) = N(t)^\sigma$, $\sigma > 0$ (e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004, ch. 6).¹⁶ Then, $R_n = \sum_e R_{ne}$ and $\dot{N}(t) = \sum_e \dot{N}_e(t)$, implying

$$R_n(t) = \eta(t) \cdot \dot{N}(t) \quad (5.17)$$

We assume the innovator enters with the average quality level of the existing varieties (e.g., Dinopoulos and Thompson, 1998; Howitt, 1999),

¹⁶That is, the cost of setting up a new variety (cost of horizontal entry) is increasing in the number of existing varieties, N . The scale of the economy induces a negative externality in the form of a barrier to entry because it becomes costlier to introduce new varieties in large growing economies. Analytically, the positive dependence of η on N is necessary to eschew the explosive growth that would occur in our model, e.g., if η were constant over time. This is not the case in Barro and Sala-i-Martin (2004, ch. 6)'s basic model of pure expanding variety.

$$\bar{q}(t) = \int_0^{N(t)} \frac{q(\omega, t)}{N(t)} d\omega = \frac{Q(t)}{N(t)}, \quad (5.18)$$

which is tantamount to having a uniform distribution with probability $1/N$. As his/her monopoly power will be also terminated by the arrival of a successful vertical innovator in the future, the benefits from entry are given by $V(\bar{q}) = \bar{\pi}(t) \int_t^\infty e^{-\int_t^s [r(\nu) + I(\bar{q})] d\nu} ds$, where $\bar{\pi} = \tilde{\pi} L \bar{q}$. Analogously to $V(j)$ in (5.14), we then have

$$V(\bar{q}) = \frac{\bar{\pi}(t)}{r + I(\bar{q})}. \quad (5.19)$$

The free-entry condition is $\dot{N} \cdot V(\bar{q}) = R_n$, which simplifies to

$$V(\bar{q}) = \eta(t). \quad (5.20)$$

Substituting (5.19) into (5.20), yields the arbitrage equation facing a horizontal innovator

$$r + I = \frac{\bar{\pi}(t)}{\eta(t)}. \quad (5.21)$$

Finally, before deciding which type of R&D to perform, the potential entrant should evaluate the best type of entry. At the margin, she/he should be indifferent between the two types, implying there is a consistency condition between them. If we equate the effective rate of return $r + I$ for both types of entry by considering (5.16) and (5.21), we get the arbitrage condition

$$\bar{q}(t) = \frac{Q(t)}{N(t)} = \frac{\eta(t)}{\zeta \cdot L}. \quad (5.22)$$

This condition is one of the key ingredients of the model. It equates the cost of the horizontal, η , to the average cost of vertical R&D, $\bar{q}\zeta L$.

5.3.4. The symmetric general-equilibrium BGP

The aggregate financial wealth held by all households is $a(t) = \int_0^{N(t)} V(\omega, t) d\omega$, which, from the arbitrage condition between vertical and horizontal entry, yields $a(t) = \eta(t) \cdot N(t)$. Taking time derivatives and comparing with (5.2), we get an expression for the aggregate flow budget constraint which is equivalent to the product market equilibrium condition (see Gil, Brito, and Afonso, 2010)

$$Y(t) = C(t) + X(t) + R_v(t) + R_n(t). \quad (5.23)$$

The dynamic general equilibrium is defined by the allocation $\{X(\omega, t), \omega \in [0, N(t)], t \geq 0\}$, by the prices $\{p(\omega, t), \omega \in [0, N(t)], t \geq 0\}$ and by the aggregate paths $\{C(t), N(t), Q(t), I(t), r(t), t \geq 0\}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) vertical, horizontal and consistency free-entry conditions are met; and (iii) markets clear.

We now derive and characterise the BGP. Let $g_y \equiv \dot{y}/y$ represent the growth rate of variable $y(t)$. Along the BGP, the aggregate resource constraint (5.23) is satisfied with Y , X , C , R_v and R_n growing at the same constant rate. By considering (5.8) and by time-differentiating (5.22) with $\eta(t) = N(t)^\sigma$, the following necessary conditions for the existence of a BGP are derived: (i) $g_C = g_Q = g$; (ii) $g_I = 0$; and (iii) $g_Q/g_N = (\sigma + 1)$, $g_N \neq 0$. Observe that g is the long-run aggregate growth rate and that g_Q and g_N are monotonically related.

If we assume that the number of industries, N , is large enough to treat Q as time-differentiable and non-stochastic, then we can time-differentiate (5.18) to get $\dot{Q}(t) =$

$\int_0^{N(t)} \dot{q}(j, \omega, t) d\omega + q(N, t) \dot{N}(t)$. After some algebraic manipulation of the latter, we can write, for $I > 0$,

$$g_Q = I \cdot \Xi + g_N, \quad \Xi \equiv \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right). \quad (5.24)$$

Next, solve (5.3) with respect to r and note that, along the BGP, $g_C = g$, to get $r = \rho + \Theta g$. The latter, combined with $g = (1 + \sigma) \cdot g_N$, (5.24) and (5.16), and solved with respect to g , yields

$$g = \frac{\mu \cdot \Xi \cdot (\sigma + 1)}{\Xi \cdot (\sigma + 1) + \frac{1}{\Theta} \sigma}, \quad (5.25)$$

where $\mu \equiv (\tilde{\pi}/\zeta - \rho)/\Theta$. Observe that $\lim_{\sigma \rightarrow \infty} g = g_{no-entry}$ and that $g > 0$ requires $\mu > 0$. Since, from (5.3), $g = g_C = (r - \rho)/\Theta$, then $r > \rho$ must occur; this condition also guarantees $g_N > 0$.¹⁷ Thus, under a sufficiently productive technology, our model predicts a BGP with constant positive g and g_N , where the former exceeds the latter by an amount corresponding to the growth of intermediate-good quality, driven by vertical innovation; to verify this, just check (5.24) and solve to get $\dot{Q}/Q - \dot{N}/N = \Xi \cdot I$, which is positive if $I > 0$. This implies that the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the view that industrial growth proceeds both along an intensive and an extensive margin (e.g., Howitt, 1999; Peretto and Connolly, 2007).

But differently from the knowledge-driven literature, g_N is not linked to the (exogenous)

¹⁷Also, considering $a(t) = \eta(t) \cdot N(t)$ and (5.22), we rewrite the transversality condition as

$$\lim_{t \rightarrow \infty} e^{-\rho t} C(t)^{-\theta} \zeta \cdot L \cdot Q(t) = \lim_{t \rightarrow \infty} e^{-\rho t} \left(\frac{C(t)}{Q(t)} \right)^{-\theta} \zeta \cdot L \cdot \left(\hat{Q} e^{gt} \right)^{1-\theta} = 0 \quad (5.26)$$

where $Q = \hat{Q} e^{gt}$ and $\hat{Q}$ denotes detrended Q . Thus, the transversality condition implies $\rho > (1 - \theta)g$; i.e., $r > g$, since $g = \frac{1}{\Theta} (r - \rho)$. This condition also guarantees that attainable utility is bounded, i.e., the integral (5.1) converges to infinity.

population growth rate. The dynamic decreasing returns due to $\eta(t) = N(t)^\sigma$, per se, determine a constant N along BGP (see Barro and Sala-i-Martin, 2004, ch. 6); however, the N expansion is sustained by technological-knowledge accumulation, as the expected growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an entry cost, in spite of its increase with N .

5.4. The model with asymmetric equilibria

We now adopt Cozzi (2005)’s approach in order to show that our model admits (at least) a continuum of asymmetric stationary deterministic equilibria, besides the symmetric equilibrium derived in the previous section.

5.4.1. R&D

We focus on the class of asymmetric general equilibria characterised by supernormal waves of creative destruction (“waves of enthusiasm”) in the newly introduced intermediate goods, and which satisfy the following assumption:

Assumption 5.1. Consider $\theta \in (1, \infty)$, constant across industries and over time. When a new intermediate good ω is introduced, vertical R&D will be θ times higher in industry ω than in the product lines that have experienced at least one quality jump (“mature” industries). After the next quality jump occurs in ω , its vertical R&D level will become equal to the R&D carried out in the other “mature” industries.

This assumption is in line with some stylised facts on industry life cycle (e.g., Gort and Klepper, 1980; Klepper, 1996), according to which frequently new industries are initially

and quickly developed by new entrants. Now, let N^E denote the number of industries that have not experienced the first quality jump and I the average Poisson rate in the “mature” industries. Since all new industries start without having innovated in the vertical direction, N^E increases as far as N increases; however, at any t , there is also a number $\theta \cdot I \cdot N^E$ that innovates and leaves the group of N^E . Thus, $\dot{N}^E = -\theta \cdot I \cdot N^E + \dot{N}$, from which, with N^E/N constant in BGP, we have

$$\frac{N^E}{N} = \frac{g_N}{g_N + \theta I}. \quad (5.27)$$

This then implies that the average Poisson rate of vertical innovation at the aggregate level is given by AI , where $A \equiv [1 + (\theta - 1)g_N / (g_N + \theta I)] \in (1, \theta), \forall g_N, I > 0, \theta \in (1, \infty)$; i.e., A is a weighted average of θ and 1, such that AI exceeds I . Provided that I and g_N are constant along the BGP, A is also constant.

As the monopoly power of a vertical innovator will eventually be terminated by the arrival of a successful innovation with average frequency AI per (infinitesimal) increment of time, the average expected value of the monopolist’s profits with current quality level j is

$$V^a(j) = \frac{\pi(j)}{r + A \cdot I(j)}. \quad (5.28)$$

In other words, under Assumption 5.1, financial markets recognise the “enthusiasm” about the newly-introduced industries and thus are willing to channel savings to aggregate vertical innovation according to the arbitrage condition $r = \pi/V^a - AI$: investors require that the real interest rate, r , equals the average dividend rate, π/V^a , plus the average rate of capital gain $-AI$. The latter term incorporates the fact that all industries are undertaking vertical R&D every period but some are still waiting for their first innovation in the vertical direction. By substituting (5.28) into (5.15) and using (5.7) to simplify, we get the arbitrage

equation facing a vertical innovator

$$r + A \cdot I = \frac{\tilde{\pi}}{\zeta}. \quad (5.29)$$

Now we turn to horizontal innovation under Assumption 5.1. The latter implies that the terminal date of the first monopolist in a newly introduced product line arrives with frequency θI ; hence, the benefits from entry are given by

$$V^E(\bar{q}) = \frac{\bar{\pi}(t)}{r + \theta \cdot I(\bar{q})} = \Omega \cdot V^a(\bar{q}), \quad (5.30)$$

where $\Omega \equiv [1 - (\theta - A)I / (r + \theta I)] \in (0, 1), \forall r, I > 0, \theta \in (1, \infty)$ captures the negative effect of increased creative destruction on the value of the monopolist firm (i.e., $V^E < V^a$, due to $\theta > 1$). Provided that I , A and r are constant along the BGP, Ω is also constant. This implies that the horizontal R&D free-entry condition can be rewritten as

$$\eta(t) = \Omega \cdot V^a(\bar{q}). \quad (5.31)$$

Substituting $V^a(\bar{q})$, determined analogously to (5.28), into (5.31), yields the arbitrage equation facing a horizontal innovator

$$r + A \cdot I = \frac{\bar{\pi}(t)}{\eta(t)} \Omega. \quad (5.32)$$

Finally, if we equate the effective average rate of return $r + AI$ for both types of entry by considering (5.29) and (5.32), we get the consistency arbitrage condition

$$\frac{Q(t)}{N(t)} = \frac{\eta(t)}{\zeta \cdot L} \cdot \Omega^{-1}. \quad (5.33)$$

This is our model's version of arbitrage equation (H') in Cozzi (2005) (itself a generalisation of equation (H) in Howitt, 1999).¹⁸ Having in mind that $0 < \Omega < 1$, it is clear from (5.33) that “waves of enthusiasm” imply a smaller N vis-à-vis the symmetric equilibrium implicit in (5.22): a lower N enhances average quality, Q/N , for a given Q (i.e., relative average quality $1/N$) received by a newly-born industry, in order to compensate for the higher subsequent creative destruction rate ($\Omega^{-1} > 1$) due to $\theta > 1$. On the right-hand side of (5.33), we get $\Omega^{-1} \rightarrow 1$ with $\theta \rightarrow 1$, and $\Omega^{-1} \rightarrow +\infty$ with $\theta \rightarrow +\infty$. In the latter case, N must approach zero in order to elevate relative average quality to infinity.

Notice that the described mechanism: (i) stems from our assumption of no intersectoral spillovers in vertical innovation, in as much as the latter implies that the BGP relative average quality is given by $1/N$ (see (5.18)); (ii) is only weakened by $\eta \equiv \eta(N)$, $\eta' > 0$, in (5.33); (iii) does not depend qualitatively on the presence of *static* decreasing returns to horizontal R&D.

In contrast, Howitt (1999)'s model features both intersectoral spillovers in vertical innovation and *static* decreasing returns to horizontal R&D; the former assumption implies that relative average quality is independent of the number of industries in BGP (see Howitt's equation (13)), and thus the latter assumption is necessary to ensure that the consistency condition between vertical and horizontal innovation has a finite and determined solution (with respect to the amount of R&D; see Howitt's equation (H)). “Waves of enthusiasm”, as analysed by Cozzi (2005), imply a lower number of industries than in the symmetric equilibrium in Howitt (1999)'s model, too, but the mechanism runs from horizontal R&D - which decreases as a direct effect of decreasing marginal returns to horizontal R&D in Cozzi's equation (H') - to the number of industries.¹⁹

¹⁸To see this, consider Cozzi's arbitrage equation with “waves of enthusiasm” as a certain event, i.e., $p_E = 1$.

¹⁹We can easily add static decreasing returns to horizontal R&D to our model, as in Howitt (1999) and Cozzi (2005), such that $\dot{N} = \frac{1}{\eta}\phi(R_n)$, $\phi' > 0$, $\phi'' < 0$, and $\frac{Q}{N} = \frac{\eta}{\zeta \cdot \phi'(R_n)} \cdot \Omega^{-1}$. In this case, the impact

5.4.2. The asymmetric general equilibrium BGP

In order to derive the asymmetric equilibrium characterised by “waves of enthusiasm” in an explicit general-equilibrium setting, we must take into account that the aggregate financial wealth held by all households, equity being taken at its market value, is now

$$a(t) = \eta(t) \cdot \Omega^{-1} \cdot N(t), \quad (5.34)$$

while the households’ flow budget constraint is

$$\dot{a}(t) = r(t)a(t) + w(t)L - C(t) + \Lambda(t), \quad (5.35)$$

where Λ is a real pure profit. By substituting (5.34) in (5.35), we then get the product market equilibrium condition (see Appendix C.2).

$$Y(t) = X(t) + C(t) + R_n(t) + A \cdot R_v(t), \quad (5.36)$$

where $A \cdot R_v$ is the aggregate vertical R&D and R_v is the vertical R&D conducted by the “mature” industries.

The necessary conditions for the existence of a BGP continue to be those presented in Subsection 5.3.4.

If we time-differentiate (5.18), and take into account the average rate of vertical innovation at the aggregate level, $AI > 0$, we have

$$g_Q = A \cdot I \cdot \Xi + g_N, \quad (5.37)$$

of $\Omega^{-1} > 1$ on the right-hand side of (5.33) will be matched by a decrease in both N and R_n . See Appendix C.1 for details.

which is our version of vertical innovation equation (14) in Cozzi (2005).²⁰ Then, solve (5.3) with respect to r , to get $r = \rho + \Theta g$. The latter, combined with (5.29) and (5.37), yields

$$g = \left(\frac{\sigma + 1}{\sigma} \right) \cdot A \cdot I \cdot \Xi = \left(\frac{\sigma + 1}{\sigma} \right) \cdot \left(\frac{\bar{\pi}}{\zeta} - \rho - \Theta g \right) \cdot \Xi. \quad (5.38)$$

By solving (5.38) with respect to g , we find

$$g = \frac{\mu \cdot \Xi \cdot (\sigma + 1)}{\Xi \cdot (\sigma + 1) + \frac{1}{\Theta} \sigma}, \quad (5.39)$$

which is the BGP aggregate growth rate under asymmetric equilibrium satisfying Assumption 1. However, (5.39) equals the expression for the BGP aggregate growth rate under symmetric equilibrium (i.e., $\theta = 1 \Rightarrow A = \Omega = 1$), as found in (5.25). Given $g = (1 + \sigma) \cdot g_N$, the same applies to g_N .

Now, let s denote a given variable value along the symmetric BGP, as derived for the model in Section 5.3. Then, from (5.39) and (5.29), we have, respectively

$$g = g^s \Leftrightarrow g_N = g_N^s, \quad (5.40)$$

$$A \cdot I = \frac{\bar{\pi}}{\zeta} - \rho - \Theta g = I^s \Leftrightarrow I = \frac{I^s}{A} < I^s. \quad (5.41)$$

Given (5.33) and our assumption of $\eta(t) = N(t)^\sigma$, we find, for a given Q ,

$$N = (\Omega \cdot \zeta \cdot L \cdot Q)^{\frac{1}{\sigma+1}} < N^s. \quad (5.42)$$

²⁰To see this, use $p_E = 1$ in Cozzi's equation.

By solving (5.33) with respect to η and substituting in (5.17), we get, for a given Q ,

$$R_n = \eta \cdot \dot{N} = \Omega \cdot g_N \cdot \zeta \cdot L \cdot Q < R_n^s, \quad (5.43)$$

and, by solving (5.13) with respect to R_v and aggregating across ω ,

$$A \cdot R_v = A \cdot I \cdot \zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot Q = R_v^s \Leftrightarrow R_v = \frac{R_v^s}{A} < R_v^s. \quad (5.44)$$

Finally, if we substitute (5.8), (5.10), (5.43) and (5.44) in (5.36), we get, for a given Q ,

$$C = \left(\mathcal{B} - \Omega \cdot g_N \cdot \zeta - A \cdot I \cdot \zeta \cdot \lambda^{\frac{\alpha}{1-\alpha}} \right) \cdot L \cdot Q > C^s, \quad (5.45)$$

where $\mathcal{B} \equiv \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}$. Observe that, given Q and L , (5.45) is the productivity-adjusted level of per-capita consumption.

In Appendix C.3, we prove the existence of a finite and unique $A > 1$, given the set $(\theta; g_N^s, I^s)$, that solves for the asymmetric BGP (5.39)-(5.45). From A , one computes I and Ω . Moreover, because the number $\theta \in (1, \infty)$ is arbitrary, it can then be shown that there is a continuum of asymmetric BGPs (see Section 5.5, below, for more detail).

Thus, in our model, the “waves of enthusiasm” have no impact on aggregate vertical R&D intensity (that is, for a given Q , or a given Y – see (5.8)), as reduced outlays in “mature” industries compensate for the increased R&D intensity in newly-born industries. This intra-R&D composition effect is induced by the fact that financial markets link the *effective* return to vertical R&D (the real interest rate plus the average rate of creative destruction) to the fundamentals, which must be the same whether we consider the symmetric or the asymmetric equilibrium.²¹ The latter effect, together with the fact that, as a consequence

²¹Notice term $\frac{\tilde{\pi}}{\zeta}$ in (5.16) and (5.29).

of our lab-equipment specification, vertical innovation is the ultimate growth engine, in the sense that it sustains both variety expansion and aggregate growth, implies that the “waves of enthusiasm” have no impact on either the growth rate of the number of varieties or the aggregate growth rate. This result contrasts with the positive effect obtained by Cozzi (2005).

Our results coincide with Cozzi’s in what concerns the negative impact of “waves of enthusiasm” on horizontal R&D intensity. However, as already alluded above with respect to the effect of “waves of enthusiasm” on N , the mechanism at work is subtly different in our case: it predicts a lower horizontal R&D in response to a lower N vis-à-vis the symmetric equilibrium, since the latter requires a smaller $\dot{N}$ to sustain a given BGP growth rate, g_N , independently of the type of returns to horizontal R&D we postulate.²² Given the fact that, in our model, “waves of enthusiasm” have no impact on the productivity-adjusted level of per-capita final-good production, $Y/(QL)$ (see (5.8)), then the lower horizontal R&D intensity implies that more resources become available for per-capita consumption, also in productivity-adjusted terms, $C/(QL)$. This positive welfare effect contrasts with the negative effect found by Cozzi.

5.5. Stationary sunspot equilibria

In the previous sections, we described a set of multiple stationary deterministic equilibria (BGPs): one symmetric equilibrium and a continuum of asymmetric equilibria characterised by “waves of enthusiasm”. This section is concerned with the existence of stationary equilibria in which all firms expect a “wave of enthusiasm” for the new intermediate good, such that “waves of enthusiasm” constitute self-fulfilling stationary rational expectations

²²Observe that the assumption of $\eta \equiv \eta(N)$, $\eta' > 0$, has no qualitative effects on the described mechanism.

equilibrium outcomes.

Due to the existence of multiple equilibria, there exists a coordination problem between firms,²³ which cannot be solved by referring solely to the fundamentals of the economic system. As is standard in the literature, we explicitly assume that firms tackle this coordination problem by referring to some extrinsic stochastic process, as described below:

Assumption 5.2. At each t , all R&D firms observe an exogenous variable $u(t) \in \{0, 1\}$ and establish a link between this observation and the existence of a “wave of enthusiasm” towards a newly created intermediate good ω at t : “no wave of enthusiasm” ($u(t) = 0$) or “wave of enthusiasm” ($u(t) = 1$). Consider $p \in (0, 1)$, constant across industries and over t . With probability p , $u(t) = 1$, i.e., vertical R&D will be $\theta > 1$ times higher in the new industry than in the “mature” industries. With probability $1 - p$, $u(t) = 0$, i.e., the new industry is immediately as R&D intensive as the “mature” industries.

Observe that, like in Cozzi (2005), the nature of extrinsic uncertainty is industry specific. As the probabilities of “waves of enthusiasm” across industries are independent and there is a continuum of industries, uncertainty is not transmitted to macroeconomic variables. Therefore, the SSE studied herein do not generate aggregate uncertainty.²⁴

Having Assumptions 5.1 and 5.2 in mind, SSE are then built as a randomisation over the multiple deterministic stationary equilibria (e.g., Cass and Shell, 1983; Dos Santos Ferreira and Dufourt, 2006). Firstly, consider the vertical-innovation arbitrage condition and the aggregate growth rate, which are now given by

²³As noted by Cozzi (2005), since after the first quality jump all sectors experience the same degree of vertical innovation, before that jump R&D firms are indifferent among sectors, which opens the door namely to an asymmetric allocation of R&D.

²⁴Thus, the deterministic nature of the households’ optimisation program is not affected by the consideration of sunspots (see Subsection 5.3.1, above).

$$r + A_p \cdot I = \frac{\tilde{\pi}}{\zeta} \quad (5.46)$$

$$g_Q = A_p \cdot I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) + g_N \quad (5.47)$$

where $A_p \equiv [1 + (\theta - 1)g_N p / (g_N + \theta I)] \in (1, \infty), \forall g_N, I > 0, p \in (0, 1), \theta \in (1, \infty)$ and $p = 1 \Rightarrow A_p = A$.

Secondly, consider the expected value of the first monopolist in a newly introduced product line, which is now

$$V(\bar{q}) = \frac{\bar{\pi}(t)}{r + I} \quad \text{with probability } 1 - p, \quad (5.48a)$$

$$V^E(\bar{q}) = \frac{\bar{\pi}(t)}{r + \theta I} \quad \text{with probability } p, \quad (5.48b)$$

Recall from Subsection 5.3.3 that the contribution to the instantaneous flow of new goods by R&D firm e in the horizontal-R&D sector is $\dot{N}_e = \frac{1}{\eta} R_{ne}$. Given perfect competition among innovator firms, each of them takes as given the marginal value of entry, V or V^E . Under Assumption 5.2, R&D firms solve the lottery²⁵

$$R_{ne} = \operatorname{argmax} \left[p \cdot \left(\dot{N}_e \cdot V^E - R_{ne} \right) + (1 - p) \cdot \left(\dot{N}_e \cdot V - R_{ne} \right) \right]$$

The single-valuedness and continuity of the latter, for a given p , is guaranteed by the strict concavity and continuity of the maximand with respect to R_{ne} . After aggregation, the

²⁵Without sunspots, R&D firms solve $R_{ne} = \operatorname{argmax} \left(\dot{N}_e \cdot V - R_{ne} \right)$, with the associated first-order condition implying (5.20). Given constant returns to scale, this is equivalent to consider directly the free-entry condition $\dot{N}_e \cdot V = R_{ne}$, as in Subsection 5.3.3.

associated first-order condition implies the horizontal R&D free-entry condition

$$\eta(t) = p \cdot V^E(\bar{q}) + (1 - p) \cdot V(\bar{q}) = \Omega_p \cdot V^a(\bar{q}) \quad (5.49)$$

where $\Omega_p \equiv [1 - (\theta - A_p)I / (r + \theta I)] [1 + (1 - p)(\theta - 1)I / (r + I)] \in (0, 1), \forall r, I > 0, p \in (0, 1), \theta \in (1, \infty)$ and $p = 1 \Rightarrow \Omega_p = \Omega$.

Then, the consistency arbitrage condition (5.33) holds now with Ω replaced by Ω_p . On the right-hand side of (5.33), we have $\Omega_p^{-1} \rightarrow 1$ with $p \rightarrow 0 \vee \theta \rightarrow 1$, and $\Omega_p^{-1} \rightarrow (1 - p)^{-1}$ with $\theta \rightarrow +\infty$. Thus, in the latter case, with $p \in (0, 1)$, N need not approach zero and hence elevate relative average quality to infinity. The combination of (5.3), (5.46) and (5.47) yields (5.39), while the BGP relationships (5.40)-(5.45) apply now with Ω and A replaced by Ω_p and A_p , respectively. In Appendix C.3, we prove the existence of a finite and unique $A_p > 1$, given the set $(\theta, p; g_N^s, I^s)$, that solves for the asymmetric BGP under Assumption 5.2.

Having the above in mind, let the variables in the state without “waves of enthusiasm” be indexed by 0, whereas the ones in the state with “waves of enthusiasm” be indexed by 1. We define formally a SSE as follows:

Definition. For a given $\theta \in (1, \infty)$, a SSE is a quintuple $(\Omega_u, A_u, p)_{u \in \{0, 1\}}$, with $\Omega_0 = A_0 = 1$, $\Omega_1 = \Omega_p \in (0, 1)$, $A_1 = A_p \in (1, \infty)$ and $p \in (0, 1)$, that yields the equality parts in (5.40)-(5.45), with Ω and A replaced by Ω_u and A_u , $u \in \{0, 1\}$, respectively.

A continuous monotonic relationship between the BGP values of the endogenous variables under Assumption 5.1 and 5.2 and, respectively, θ and p is needed, such that a continuum of SSE exists. By using the implicit function theorem to compute $dA_p/d\theta$ and $d\Omega_p/d\theta$, it is easy to verify that A_p and Ω_p relate monotonically with θ . To be more specific, A_p *increases* monotonically with θ at a decreasing rate ($\lim_{\theta \rightarrow \infty} dA_p/d\theta = 0$), whereas Ω_p

decreases monotonically with θ also at a decreasing rate. With respect to the latter, we add that, on one hand, the direct negative effect of θ on Ω_p overweighs the positive effect of θ through A_p on Ω_p and, on the other hand, the decreasing marginal direct effect of θ on Ω_p overweighs the impact of $\lim_{\theta \rightarrow \infty} dA_p/d\theta = 0$.

Re-iterating the same steps as above, one can give a proof of the monotonic relationship between p and, respectively, A_p and Ω_p . Because sunspots are the random factors that, with probability p , choose one specific realisation from the underlying multiple deterministic equilibria, and because the numbers θ and p are arbitrary, then there is a continuum of SSE.

With these ingredients, we are able to rewrite Cozzi (2005)’s Proposition 1 as

Proposition 5.1. A continuum of BGPs exist parametrised by “waves of enthusiasm” probabilities $p \in (0, 1)$ and amplitudes $\theta \in (1, \infty)$. Comparing such BGPs, the *larger* p and θ , the *smaller* the number of firms, aggregate horizontal R&D and vertical R&D in “mature” industries, and the *larger* vertical R&D in newly-born industries and the level of effective per-capita consumption. Aggregate vertical R&D, the growth rate of the number of varieties and of intermediate-good quality, and thus the aggregate growth rate, do not depend on either p or θ .

Thus, with respect to the impact of sunspots, or animal spirits, (optimistic expectations of firms) on the number of firms, our growth model predicts a negative relationship similarly to Cozzi (2005). The opposite result obtains in the business-cycle models with endogenous firm entry by, e.g., Chatterjee, Cooper, and Ravikumar (1993), Dos Santos Ferreira and Dufourt (2006) and Jaimovich (2007). In our study, animal spirits have no effect on long-run aggregate growth, in contrast to Cozzi (2005), but also to, e.g., the endogenous-growth models analysed in Drugeon and Wigniolle (1996) and Nishimura and Shigoka (2006). These papers all exhibit a positive relation between animal spirits and the aggregate growth rate.

Finally, with respect to welfare effects, the positive impact we find of animal spirits on per-capita consumption is in line with, e.g., Chatterjee, Cooper, and Ravikumar (1993) and Dos Santos Ferreira and Dufourt (2006), but opposes to the negative effect described by Cozzi (2005).

5.6. Transitional dynamics under animal spirits

The previous sections showed that the aggregate growth rate and the aggregate vertical R&D are invariant with respect to animal spirits along the BGP. Now, we investigate whether the same is true along the transition path towards the BGP (all proofs appear in Appendix C.4).

In order to exhibit transitional dynamics, the model is extended by considering

$$\eta(t) = N(t)^\sigma \dot{N}(t)^\gamma, \quad (5.50)$$

where σ and γ are positive constants. This specification means that the horizontal entry cost is not only increasing in the number of existing varieties, N , but also in the number of new entrants, $\dot{N}$. This mechanism introduces dynamic second-order effects in entry, implying that new varieties are brought to the market gradually, instead of through a lumpy adjustment. This is in line with the stylised facts on entry (e.g., Geroski, 1995): entry occurs mostly at small scale since adjustment costs penalise large-scale entry.

The arbitrage condition between vertical and horizontal R&D continues to be (5.33). Then, by substituting (5.50) in (5.33) and solving in order to $\dot{N}$, we obtain

$$\dot{N}(t) = x(Q(t), N(t)) \cdot N(t), \quad (5.51)$$

where

$$x(Q(t), N(t)) = (\zeta \cdot L)^{\frac{1}{\gamma}} \cdot (\Omega(t) \cdot Q(t))^{\frac{1}{\gamma}} \cdot N(t)^{-\left(\frac{\sigma+\gamma+1}{\gamma}\right)}. \quad (5.52)$$

Observe that Ω now depends on t , since I and r vary along the transition path. It follows that the necessary conditions for the existence of a BGP are now: (i) $g_C = g_Q = g$; (ii) $g_I = 0$; and (iii) $g_Q/g_N = (\sigma + \gamma + 1)$, $x \neq 0$.

It is then convenient to recast system (5.3), (5.37) and (5.51), by using variable x as in (5.52) and variable $z \equiv C/Q$, into an equivalent system in detrended variables. Since the linearised system around the steady state (x^*, z^*) has two distinct real eigenvalues with opposite signs, the dynamics are saddle-path stable.

We focus on the empirically relevant case of $\Theta > 1$. Consider deviations of x above its BGP value, i.e., $x(0) = x_0 > x^*$. This would be the case of an economy with a N too low relatively to Q - e.g., an emergent-market economy, displaying a shallow market of differentiated goods, in contrast to a mature-market economy, which is expected to boast a deep market of differentiated goods. Thus, we have

Proposition 5.2. Assume that $\Theta > 1$ and $AI > 0$, and consider an economy initially endowed with $N(0)$ varieties, such that $x(0) > x^*$. The transitional dynamics is characterised as follows. The economy experiences a decreasing z and $x (= g_N)$; this implies that more resources become available to AR_v , boosting AI and reducing r ; consequently, g falls due to the downward movement of g_N but less so due to the effect of accelerated vertical innovation, reflecting the increase in AI .

According to Proposition 5.2, the model exhibits the standard conditional convergence property (falling g and r towards the interior BGP), although through a specific mechanism. This consists of dynamic second-order effects in horizontal entry, implying that new varieties

are brought to the market gradually, as stated above. An economy with very few varieties relative to the technological-knowledge stock starts with a smaller AI , a higher g_N , and a higher g than a mature market economy. A lower AI implies a higher r (see (5.29)) and a lower AR_v (see (5.13)), which secure the larger resources allocated to R_n . Along the transition path, g and g_N decrease, whereas part of the resources allocated to R_n is gradually re-targeted to AR_v . The consequent increase in AI implies a falling r . Moreover, with A also falling towards the BGP, I increases more than AI along transition, implying an intra-(vertical-)R&D composition effect favouring the “mature” industries. The unequivocal negative correlation between growth and the aggregate vertical innovation rate - which also applies to R&D intensity, $(AR_v + R_n) / (Y - X)$, - is a new result in the theoretical growth literature, although it has been reported in empirical studies (Bassanini, Scarpetta, and Visco, 2000; Pessoa, 2010).

We notice that the qualitative characterisation in Proposition 5.2 also applies to the symmetric case ($\theta = 1 \Rightarrow A = \Omega = 1$).²⁶ However, it can be shown that the speed of transition is smaller under “waves of enthusiasm”, as illustrated in Figure 5.1.²⁷ This implies that the economy spends more time in transition vis-à-vis the symmetric case, as emphasised by Proposition 5.3.

²⁶Gil, Brito, and Afonso (2010) study the transitional dynamics for the symmetric case in detail.

²⁷We use the following set of baseline parameter values: $\gamma = 1.2$, $\sigma = 1.2$, $\phi = 1$, $\zeta = 0.9$, $\lambda = 2.5$, $\rho = 0.02$, $\Theta = 1.5$, $\alpha = 0.4$, $L = 1$, $\theta = 3$. Given that along the BGP, $g_Q - g_N = (\sigma + \gamma)g_N$, the choice of values for σ and γ is such that $(\sigma + \gamma) = 2.4$, which is the ratio between the growth rate of the average firm size and the growth rate of the number of firms we have found in the empirical data (the data concerns 23 European countries in the period 1995-2005 and is available from the Eurostat on-line database; link at <http://epp.eurostat.ec.europa.eu>). The values for λ , θ , ρ and α were set in line with previous work on growth and guided either by empirical findings or by theoretical specification, while the normalization of A and L to unity at every t implies that all aggregate magnitudes can be interpreted as per capita magnitudes. The values of the remaining parameters were chosen in order to calibrate the BGP aggregate growth rate around 2.5 percent/year.

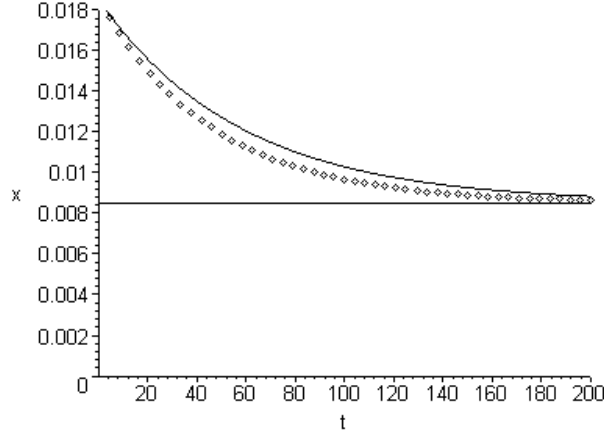


Figure 5.1.: Time-path for x under the two regimes: $\theta = 1$ (pointed line) and $\theta > 1$ (solid line).

Proposition 5.3. Consider that, from time $t = t_0$, a given economy is on a transition path characterised by $\theta = \theta_0 = 1$ (no “waves of enthusiasm”). At $t = t_1 > t_0$, the economy suffers an (unanticipated) expectations shock, such that henceforth $\theta = \theta_1 > 1$ (“waves of enthusiasm”). The economy instantaneously adjusts through an upward jump in z , towards a new transition path characterised by a slower speed of convergence to the BGP. The economy will henceforth spend more time in transition vis-à-vis the symmetric case, hence experiencing a longer period in which growth rates, x and g , are higher and the vertical innovation rate, AI , is lower than in the BGP.

Proposition 5.3 makes clear that there is a medium-term positive impact of animal spirits on the growth rate along with a negative impact on vertical innovation rate. Figure 5.2 depicts the time-path for x and z as described in Proposition 5.3. Observe that from t_1 the time-path lines are less steep, reflecting the fact that the speed of transition is reduced when the economy shifts to a regime with “waves of enthusiasm”.

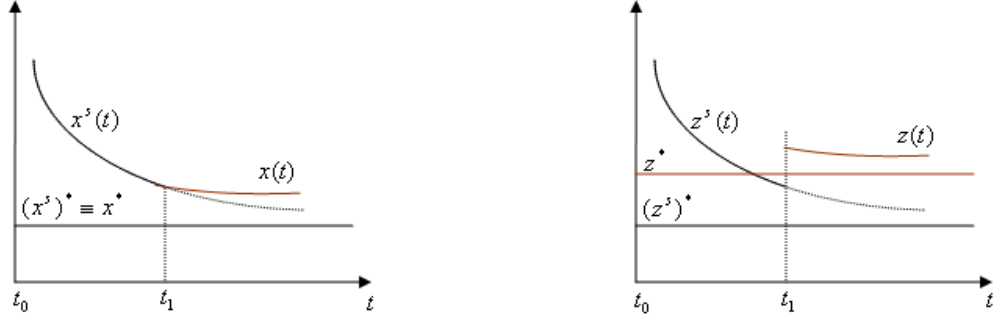


Figure 5.2.: Transitional dynamics of x (left panel) and z (right panel) with an unanticipated expectations shock at t_1 .

Thus, the model develops a mechanism, complementary to the friction in horizontal entry, that slows down the transition. Available empirical evidence suggests slow transitions are the case in general. With the chosen set of baseline parameters (see fn. 27), the speed of convergence to the BGP is 2.2 percent/year, implying a half-life of roughly thirty-two years, which is within the range of estimates given by Barro and Sala-i-Martin (2004, ch. 11) for the US, Japan and several European countries. This result contrasts with the rather higher values obtained with the standard Ramsey model calibrated with reasonable parameter values (see Ortigueira and Santos, 1997).

In a sunspot transition, the economy oscillates between an “expansive phase” (i.e., with “waves of enthusiasm”) corresponding to a higher *level* and a slower *rate of decline* of per-capita consumption in effective terms, and a “recessive phase” (i.e., with no “waves of enthusiasm”), where the opposite is true. Therefore, a catching-up economy that is subject to expectations shocks then converges at a time-varying speed. On average over time, transition is longer but less “painful”- i.e., with higher per-capita consumption levels - than otherwise.

The time-varying speed of convergence (of the linearised system) exhibited by our model is in line with the empirical results emphasised by, e.g., Bernard and Jones (1996) and, more recently, Castellacci (2008). In the theoretical models by Eicher and Turnovsky (2001) and Brito and Dixon (2009), among others, such a result occurs because of the interactions between two state-variables. Indeed, two-dimensional stable manifolds imply that the convergence speeds will vary through time, thereby introducing important flexibility to the transitional dynamics. The distinctive feature of our model is that time-varying speed of adjustment entirely results from self-fulfilling expectations held by the entrepreneurs, in the context of a single-dimensional stable manifold.

To sum up, these results suggest that animal spirits retain their relevance for the purpose of economic growth, aggregate vertical R&D and welfare, especially for those economies enduring a transition process.

5.7. Concluding remarks

In this essay, we revisit the issue of self-fulfilling “waves of enthusiasm” as stationary rational expectations equilibria in endogenous-growth models that feature both the quality-ladders and the expanding-variety mechanism. For that purpose, we develop a model that merges the two mechanisms under a non-scale lab-equipment specification without intersectoral spillovers.

The model predicts, under a sufficiently productive technology, a stationary BGP with constant positive growth rates, and where the consumption growth rate equals the growth rate of the number of varieties plus the growth rate of intermediate-good quality, in line with the general view that industrial growth proceeds both along an intensive and an extensive margin. Nevertheless, different from the knowledge-driven models, the growth

of the number of varieties is not linked to the exogenous population growth rate. It is sustained by endogenous technological-knowledge accumulation, as the expected growth of intermediate-good quality makes it attractive for potential entrants to always put up an entry cost, in spite of its upward trend.

In line with Cozzi (2005), this essay focuses on asymmetric equilibria derived from self-fulfilling prophecies à la Cass and Shell (1983), instead of any asymmetry in market fundamentals, such as cost structures or technologies. It shows that the BGP composition of aggregate R&D, the industrial structure (number and average firm size) and the per-capita consumption level can be affected in a relevant manner by animal spirits.

However, by considering a vertical and horizontal R&D model in some aspects distinct from Howitt (1999), our results coincide only partially with those presented by Cozzi (2005). Both models predict a smaller number of firms in the asymmetric equilibrium with sunspots, in order to enhance the returns to horizontal entry, given subsequent higher obsolescence (creative destruction) rate. In our model, returns are increased because relative average quality is higher when the number of varieties is lower, whereas in Cozzi (2005), returns are increased because marginal returns to horizontal R&D are higher when R&D outlays are lower, which in turn imply a smaller number of industries in equilibrium.

More importantly, balanced growth dynamics are distinct. In our model, long-run aggregate growth is not affected by animal spirits, while Cozzi's model predicts a higher aggregate growth rate, due to a higher aggregate rate of vertical innovation. The mechanism behind our result is twofold. On the one hand, we consider an explicit general BGP equilibrium where financial markets link the *effective* return to vertical R&D (the real interest rate plus the aggregate rate of creative destruction) to the fundamentals, which must be the same whether we consider the symmetric equilibrium or the asymmetric equilibrium with “waves of enthusiasm”. This induces an intra-R&D composition effect between “mature” and

newly-born industries, thus dampening the impact of animal spirits on aggregate vertical R&D intensity. The latter, together with the fact that in our model vertical innovation is the ultimate growth engine, in the sense that it sustains both variety expansion and aggregate growth, implies that animal spirits have no impact on the growth rate of the number of varieties and on the aggregate growth rate along the BGP. Nevertheless, our model predicts a positive impact on the BGP level of per-capita consumption, in contrast to Cozzi's negative welfare effect.

Furthermore, there is an important impact of animal spirits in transitional dynamics. As the speed of transition towards the BGP is smaller when there are "waves of enthusiasm", this implies the economy experiences a longer period with growth rates above and vertical aggregate R&D below the BGP level, before completing the convergence process. Besides, there is also a positive welfare effect under sunspot transition.

This set of results suggests that the risk that policy intervention, by acting itself as a potential source of extrinsic uncertainty, sees its effectiveness reduced in a long-term perspective – as explained by Cozzi (2005) – may ultimately be more relevant to the impact of public policy on the industrial structure and the level of per-capita consumption than on the long-run aggregate growth rate. Instead, one should expect a medium-term impact on both economic growth rates and per-capita consumption levels. With long transitions, as seems to be the case empirically, this suggests then that animal spirits retain their relevance for the purpose of policy intervention as far as economic growth and welfare are concerned.

6. Technological-Knowledge Dynamics in Lab-Equipment Models of Quality Ladders¹

Abstract: *The Perpetual Inventory Model (PIM) assumes that, in each period, an arbitrary constant fraction of technological-knowledge stock is lost. This essay gives a theoretical background to the PIM by showing that the technological-knowledge accumulation follows a dynamic process with an endogenous depreciation rate, which remains stable along the balanced-growth path.*

Keywords: *technological-knowledge dynamics; endogenous depreciation rate; Perpetual Inventory Model.*

JEL Classification: *O30, O41*

¹The results in this chapter have been published as Gil and Afonso (2010) and also appeared as a working paper (Gil and Afonso, 2008).

6.1. Introduction

Empirical estimates of the stock of ideas or technological knowledge usually build on the Perpetual Inventory Model (PIM), according to which the stock value suffers geometric depreciation at an arbitrated constant “obsolescence” rate (e.g., Coe and Helpman, 1995; Frantzen, 1998; and Park, 2004). This approach is not free of criticism, as “a constant depreciation rate implies that depreciation takes place in a mechanical way”, independently of whether R&D is carried out or not (Bitzer and Stephan, 2007, p. 181). Similarly to the process of physical capital accumulation, this mechanism implies that a constant fraction of the technological-knowledge stock is lost with the passage of time and thus, if all R&D stops, that stock converges in the long run to zero.

Bitzer and Stephan (2007) study an econometric model that takes into account the Schumpeterian link between present R&D and the depreciation of past R&D capital stock, in order to estimate the technological-knowledge stock. The authors build a stock-flow equation according to which every R&D investment first induces an increase in the R&D capital stock, but thereafter renders the existing stock obsolete. The depreciation rate depends on past R&D investments and is therefore not constant as in the PIM. This dependency yields the desirable result that the R&D capital stock converges to a positive constant if R&D ceases.

In this essay, we argue that the dynamics of the technological-knowledge stock can be represented by a mechanism similar to the one used for physical-capital accumulation, as long as we take into account the endogeneity of the depreciation (obsolescence) rate and take a long-run (balanced-growth-path, BGP) view of the process of technological-knowledge accumulation. This approach is in line with the notion of “endogenous obsolescence” explored by Caballero and Jaffe (1993), while it gives theoretical background to the assumption of

geometric depreciation at a constant rate in the PIM. Also, we make explicit the link between our concept of technological knowledge and the measure of knowledge stock proposed by Griliches (1979).

Our contribution is defined within a lab-equipment framework (Rivera-Batiz and Romer, 1991) nested in an endogenous growth model of quality ladders in the intermediate-good sector (e.g., Aghion and Howitt, 1998, ch. 3, and Barro and Sala-i-Martin, 2004, ch. 7). As in the standard model, the final-good production function is Cobb-Douglas, each quality-adjusted intermediate good is produced by a single-product firm with a constant-return technology, only (potential) entrants do R&D and innovation arrival follows a Poisson process. The dynamic equation for the technological-knowledge stock is obtained through the explicit derivation of the aggregate resource constraint from households' balance sheet and flow budget constraint. This contrasts with the standard procedure in quality-ladders literature, which assumes directly the aggregate resource constraint.

Similarly to the standard growth model with physical capital accumulation, the stock of technological knowledge increases with the flow of gross investment (vertical R&D) and decreases due to depreciation (obsolescence) over time. Contrary to physical-capital accumulation models, the depreciation rate is endogenous (it is the Poisson arrival rate of vertical innovation), since it depends on R&D activity itself, thus reflecting the Schumpeterian process of creative destruction. Moreover, the depreciation rate is constant along the BGP. Hence, according to our model-based approach, one should not view the assumption of a constant depreciation rate as “a serious drawback of the PIM” (Bitzer and Stephan, 2007, p. 181), as it conforms with the theoretical prediction of a wide class of endogenous growth models of vertical innovation. Also, the endogeneity of the depreciation rate in the growth models of quality ladders allows for its explicit computation through proper calibration of the model, after the determination of its BGP equilibrium, in contrast to the

arbitrary choice of values in the standard PIM applications.

The essay is organised as follows. Section 6.2 briefly sketches the standard model of quality ladders in a lab-equipment framework. In section 6.3, the aggregate resource constraint and the accumulation equation for the firms' total market value are derived. In section 6.4, the technological-knowledge dynamics is made explicit. Section 6.5 concludes.

6.2. The quality-ladders model

We follow the standard multi-sector model of quality ladders (Aghion and Howitt, 1998, ch. 3; Barro and Sala-i-Martin, 2004, ch. 7). This is a dynamic general equilibrium model of a closed economy where there is a single competitively-produced final good, Y , that can be used in consumption, C , production of intermediate goods, X , and vertical R&D activities, R . Final output is the numeraire (that is, we set its price equal to unity). Labour, L , is inelastically supplied by households to final-good firms and, by assumption, does not vary over time. In turn, families invest in firms' equity.

6.2.1. Production

The final-good production function is

$$Y(t) = L(t)^{1-\alpha} \int_0^N \lambda(\omega, t) x(\omega, t)^\alpha d\omega \quad (6.1)$$

where $L(t)$ is labour input; $(1 - \alpha)$, $0 < \alpha < 1$, is the labour share in production; $x(\omega, t)$ is the amount used of the latest generation of the intermediate good ω , weighted by its quality level $\lambda(\omega, t)$, and $N > 0$ is the measure of how many different intermediate goods ω exist, which we assume to be constant.

Each firm in the final-good sector seeks to maximise profit, taking the price of ω relative to the final-good price, $p(\omega, t)$, and the labour wage, $w(t)$, also relative to the final-good price, as given. The intermediate good is nondurable and entails a unit marginal cost of production, measured in terms of the final good. Since there is a continuum of intermediate goods, one can assume that firms are atomistic and take as given the price of final output (numeraire); monopolistic competition, therefore, prevails and firms face isoelastic demand curves. The optimal intermediate good price is the usual monopoly price markup, $p(\omega, t) \equiv p = 1/\alpha > 1$, constant over time and across industries. The quantity produced of intermediate good ω is $x(\omega, t) = L [\lambda(\omega, t)\alpha^2]^{\frac{1}{1-\alpha}}$ and the profit accrued by the monopolist in ω is $\pi(\omega, t) = L \cdot \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} \lambda(\omega, t)^{\frac{1}{1-\alpha}}$. Substituting in (6.1) and aggregating across the economy yields $Y(t) = L \cdot (\alpha^2)^{\frac{\alpha}{1-\alpha}} \int_0^N \lambda(\omega, t)^{\frac{1}{1-\alpha}} d\omega$, $X(t) = \int_0^N x(\omega, t) d\omega = L \cdot (\alpha^2)^{\frac{1}{1-\alpha}} \int_0^N \lambda(\omega, t)^{\frac{1}{1-\alpha}} d\omega$, and $\Pi(t) = \int_0^N \pi(\omega, t) d\omega = L \cdot \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} \int_0^N \lambda(\omega, t)^{\frac{1}{1-\alpha}} d\omega$.

Having in mind that, in equilibrium, labour wage, w , and intermediate-good price, p , are equated to the marginal product of labour and the marginal product of intermediate goods, respectively, the following aggregate relations are derived: $wL = (1 - \alpha)Y$; $X = \alpha^2 Y$, $pX = \alpha Y$ and $\Pi = X \cdot (p - 1) = \alpha Y - \alpha^2 Y$.

Now, let $q(\omega, t) \equiv \lambda(\omega, t)^{\frac{1}{1-\alpha}}$ and define the representative intermediate good as the average of all intermediate goods, such that its quality is $\bar{q} \equiv E_\omega(q)$, the average of q over industries, i.e., $\int_0^N q(\omega, t) d\omega = \bar{q}(t)N$. Hence, $X(t) = x(\bar{q})N$ and $\Pi(t) = \pi(\bar{q})N$.

6.2.2. Vertical R&D

Firms decide over their optimal vertical-R&D level, which constitutes the search for new designs that lead to a higher quality of existing intermediate goods. Each new design is granted a patent, meaning that a successful researcher retains exclusive rights over the use of his/her improved intermediate good. Only (potential) entrants can do R&D and

innovation arrival follows a Poisson process, with instantaneous rate of R&D success, I .² With free-entry into each vertical R&D race and perfect competition among entrants, the R&D expenditures of individual entrants will be negligible. Thus, we have the free-entry condition

$$I(\omega, t) \cdot V(\omega, t) = R(\omega, t) \quad (6.2)$$

where V is the *expected* discounted value of profits associated to the next innovation, $V(\omega, t) = \int_t^\infty \pi(\omega, s) e^{-\int_t^s (r(v) + I(\omega, v)) dv} ds$, where r is the equilibrium market real interest rate. This equation reflects the fact that, if a profit flow can stop when a Poisson event with arrival rate I occurs, then we can calculate the *expected* present value of the stream of profit as if it never stops, but adding I to the discount rate. Thus, we can interpret $r + I$ as an *effective* discount rate. V can be interpreted as the market value of the patent or the value of the monopolist firm owned by households. From (6.2), we can aggregate across ω to get $R(t) = \int_0^N R(\omega, t) d\omega$.

6.2.3. Households

Households consume and collect income (dividends) from investments in financial assets (equity) and labour income. They choose the trajectory of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximise a standard discounted lifetime utility. Intertemporal utility is maximised subject to the flow budget constraint

²The specific way each innovation arrival impacts on intermediate-good quality level λ depends on whether we consider a model with intersectoral spillovers (Aghion and Howitt, 1998, ch. 3) or not (Barro and Sala-i-Martin, 2004, ch. 7). In any case, we take the usual assumption that each entrant possesses the same R&D technology, specified such that I is constant across industries at every t , i.e., $I(\omega, t) = I(t)$, and also constant over time in balanced-growth path.

$$\dot{a}(t) = r(t)a(t) + w(t)L - C(t) \quad (6.3)$$

where a stands for households' financial assets (equity) holdings, measured in terms of final-good output. The maximisation of discounted lifetime utility is also subject to the initial level of wealth, and to the usual no-Ponzi-schemes and transversality conditions. Households take the real labour wage, w , and the real rate of return on financial assets, r , as given. The latter equals dividend payments in units of asset price corrected by the Poisson death rate

$$r(t) = \frac{\pi(t)}{V(t)} - I(t) \quad (6.4)$$

This equation can be read as an arbitrage condition for investors, which requires that the real interest rate equals the dividend rate, π/V , plus the rate of capital gain, $-I$.

6.3. Aggregate resource constraint and firms' market-value dynamics

Consider the representative intermediate good, with quality $\bar{q}$. The balance sheet of households equates the value of equity holdings to the market value of firms, that is

$$a(t) = V(\bar{q})N \quad (6.5)$$

Hence, we can characterise the change in the value of equity as

$$\dot{a}(t) = \dot{V}(\bar{q})N \quad (6.6)$$

By substituting (6.3) in the left-hand side of (6.6) and letting $K \equiv VN$, we get

$$\dot{K}(t) = r(t)K(t) + w(t)L - C(t) = [r(t) + I(t)]K(t) + w(t)L - C(t) - I(t)K(t) \quad (6.7)$$

Also, from (6.4) and (6.5), we observe that

$$r(t) + I(t) = \frac{\pi(\bar{q})}{V(\bar{q})} \Leftrightarrow [r(t) + I(t)]a(t) = \pi(\bar{q})N \quad (6.8)$$

The equation above implies that $\Pi = \pi N = (r + I)K$ for the representative intermediate good. Using the latter, together with $wL = (1 - \alpha)Y$ and $\Pi = \alpha Y - \alpha^2 Y$ in (6.7), and simplifying with $X = \alpha^2 Y$, yields

$$\dot{K}(t) = Y(t) - X(t) - C(t) - I(t)K(t) \quad (6.9)$$

which is the accumulation equation for K , i.e., firms' total market value. If we solve (6.9) in order to Y , we have the aggregate resource constraint

$$Y(t) = X(t) + C(t) + R(t) \quad (6.10)$$

where

$$R(t) = \dot{K}(t) + I(t)K(t) \Leftrightarrow \dot{K}(t) = R(t) - I(t)K(t) \quad (6.11)$$

Equation (6.10) is a *product market equilibrium condition*, according to which total final-good output is allocated among consumption, production of intermediate goods and vertical R&D expenditures.

As for (6.11), the first term on the right-hand side, R , represents the *gross investment* in

technological knowledge through vertical R&D at time t , whereas the second term, IK , represents the *depreciation* of the total market value of firms (i.e., lived patents) due to the impact (obsolescence) of the stochastic arrival of vertical innovations on the existing technological-knowledge stock, i.e., the Schumpeterian effect of creative destruction. This means that $R = 0 \Rightarrow I = 0$ and, thus, $\dot{K} = 0$. Equation (6.11) has obvious similarities to the accumulation equation of physical capital in the standard Ramsey model. However, the depreciation rate displayed by our model is not exogenous, but rather an endogenous function of vertical R&D activity, in line with the notion of “endogenous obsolescence” explored by Caballero and Jaffe (1993).

According to our assumptions, in a BGP equilibrium with $R > 0$, the Poisson rate $I > 0$ is constant, meaning that (6.11) can be rewritten as

$$\dot{K}(t) = R(t) - IK(t) \tag{6.12}$$

The latter is identical to the dynamic equation postulated under the PIM, and where technological-knowledge stock is measured as “R&D capital stock” (e.g., Coe and Helpman, 1995).³ Within our general-equilibrium setting, the “R&D capital stock” equals firms’ total market value, K , and thus households’ total financial assets, a .

Our operationalisation of the concept of technological-knowledge stock can be linked to the measure of knowledge stock proposed by Griliches (1979) and analysed recently by Klette and Kortum (2004). For Griliches, the “technical knowledge” stock is some lag function of *past* R&D. Klette and Kortum (2004) propose a measure of knowledge stock conditional on past R&D expenditures, R , considering that the appropriate discount rate on past R&D is the intensity of creative destruction. In our model, this is the Poisson arrival rate I . Hence,

³An alternative approach is to apply PIM to patent-count data in order to compute the “stock of ideas” (see, e.g., Porter and Stern, 2000, who, however, do not consider obsolescence, and Pessoa, 2005).

if we let K denote the R&D capital stock and take I as time-variable, we can write

$$K(t) = \int_{t_0}^t e^{-\int_s^t I(\tau) d\tau} R(s) ds \quad (6.13)$$

where t_0 is the time on which the first intermediate-good line was born.⁴ If we time-differentiate (6.13), we get $\dot{K}(t) = R(t) - I(t)K(t)$, which is (6.11).

6.4. Technological-knowledge dynamics

In section 6.3 we studied the dynamics of technological-knowledge stock through the dynamics of firms' total market value, K . In order to perform a direct study of the former, we need a measure of R&D effectiveness. Let the instantaneous probability of R&D success be given by a relation exhibiting constant returns in R&D expenditures⁵

$$I(\omega, t) = R(\omega, t)\Phi(\omega, t) \quad (6.14)$$

where the function Φ , to be defined below, is the same for every ω and captures the effect of the current technological knowledge in ω on R&D effectiveness. By substituting (6.14) in (6.2), we get

$$V(\omega, t) = \frac{1}{\Phi(\omega, t)} \quad (6.15)$$

We must also define a measure of technological-knowledge stock. In a quality-ladders model *without* intersectoral spillovers, such as Barro and Sala-i-Martin (2004, ch. 7), the relevant

⁴For an alternative measure of knowledge stock, set within a model that takes into account obsolescence and diffusion effects, see Caballero and Jaffe (1993).

⁵The assumption of constant returns in R&D activities, instead of decreasing returns as in, e.g., Segerstrom and Zolnieriek (1999), simplifies the analysis but does not change the results in any fundamental way.

measure is the *aggregate quality index* $A(t) \equiv \int_0^N \lambda(\omega, t)^{\frac{1}{1-\alpha}} d\omega$, with $\Phi(\omega, t) \equiv \frac{1}{\zeta \lambda(\omega, t)^{\frac{1}{1-\alpha}}}$, where $\zeta > 0$ is a fixed cost of doing R&D. Substitute the latter in (6.15), to get $V(\bar{q}) = \zeta \bar{q}(t)$, with $\bar{q}(t) = \frac{A(t)}{N}$. From here, we have

$$a(t) = V(\bar{q})N = \zeta A(t) \quad (6.16)$$

and, thus

$$\dot{A}(t) = \frac{1}{\zeta} R(t) - I(t)A(t) \quad (6.17)$$

In a quality-ladders model *with* intersectoral spillovers, such as Aghion and Howitt (1998, ch. 3), the relevant measure is the *leading-edge quality index* $\lambda^{max} \equiv \max [\lambda(\omega), \omega \in [0, N]]$, for each t , with $\Phi(\omega, t) \equiv \frac{1}{\zeta \lambda^{max}(t)}$. Following the same steps as before, we get

$$a(t) = V(\bar{q})N = \zeta N \lambda^{max}(t) \quad (6.18)$$

and, thus

$$\dot{\lambda}^{max}(t) = \frac{1}{\zeta N} R(t) - I(t)\lambda^{max}(t) \quad (6.19)$$

Equations (6.17) and (6.19) make clear the role of I as the endogenous depreciation rate of technological knowledge.⁶ Hence, according to our model-based approach, one should not view the assumption of a constant depreciation rate as “a serious drawback of the PIM” (Bitzer and Stephan, 2007, p. 181), as it conforms with the theoretical prediction of a wide class of endogenous growth models of vertical innovation. The constancy of the depreciation

⁶Notably, it can be shown equations (6.16) and (6.18) guarantee that, given $R(t) = \int_0^N R(\omega, t) d\omega$ and (6.11), the consistency condition $\int_0^N R(\omega, t) d\omega = \dot{K}(t) + I(t)K(t)$ is satisfied.

rate along the BGP implies that the view of a constant fraction of technological-knowledge stock being lost with the passage of time is a good approximation in the long run, whereas the endogeneity of the depreciation rate ensures that, if all R&D stops, that stock does not converge to zero in the long run.⁷

In this setting, the depreciation rate can be explicitly computed through proper calibration of the model, after the determination of its BGP equilibrium. We present a simple illustration, based on the analytical expression for the BGP value of I derived in Barro and Sala-i-Martin (2004, ch. 7) and the following set of baseline parameter values: $\zeta = 0.8$, $\lambda = 2.5$, $\rho = 0.02$, $\theta = 1.5$, $\alpha = 0.4$, $L = 1$.⁸ The obtained Poisson arrival rate is of 3 percent along the BGP.⁹ This result contrasts with the arbitrary values chosen in the standard PIM applications, typically between 5 and 20 percent (e.g., Coe and Helpman, 1995).

6.5. Conclusion

Empirical estimates of technological-knowledge stock usually build on the PIM, according to which the stock value suffers geometric depreciation at an arbitrated constant “obsolescence” rate. This approach has been contested recently in the literature, since, similarly to the process of physical-capital accumulation, this mechanism implies that a constant fraction of technological-knowledge stock is lost with the passage of time. Thus, if all R&D stops,

⁷Indeed, the model used by Bitzer and Stephan (2007) yields a constant depreciation rate when (lagged) R&D investment grows at the same rate as technological-knowledge stock, which is exactly what happens in BGP equilibrium in the quality-ladders models.

⁸The values for λ , θ , ρ and α were set in line with previous work on growth and guided either by empirical findings or by theoretical specification. The values of the remaining parameters were chosen in order to calibrate the BGP aggregate growth rate around 2.5 percent/year. The normalization of L to unity at every t implies that the results of the model do not depend on the value of the growth rate of that variable (either zero or not), and also that all aggregate magnitudes can be interpreted as per capita magnitudes.

⁹Interestingly, this value corresponds to the average of the estimates provided by Caballero and Jaffe (1993) for the creative-destruction rate in the US.

that stock converges in the long run to zero.

In this essay, we argue that the dynamics of technological-knowledge stock can indeed be represented by such a mechanism, as long as we consider the endogeneity of the depreciation rate and take a long-run (BGP) view of the process of technological-knowledge accumulation. In this setting, we present the explicit computation of the depreciation rate through proper calibration of the model as an alternative route to the arbitrary choice of values in the standard PIM applications.

7. Equilibrium Price Distribution with Directed Technical Change¹

Abstract: *This essay studies a non-degenerate price distribution for the homogeneous good within a model of endogenous directed technical change. A probability density function is analytically derived and shown to be related to the technology and innovation parameters of the model.*

Keywords: *price distribution; directed technical change; scale effects; labour endowment.*

JEL Classification: *D41, D43, O41.*

¹The results in this chapter have been published as Gil, Figueiredo, and Afonso (2010) and also appeared as a working paper (Gil, Figueiredo, and Afonso, 2009).

7.1. Introduction

Several models have been developed to identify the determinants of equilibrium price dispersion in homogeneous-good markets. Work in this area includes models that assume ex ante heterogeneity in firms' production costs and/or consumers' search costs (e.g., Carlson and McAfee, 1983), or that assume information on prices is imperfect with otherwise identical agents (e.g., McAfee, 1995). More recently, Kultti and Virrankoski (2003, 2004) explore a model with ex ante symmetric agents and publicly and costlessly known prices, but which considers sellers' capacity constraint and the possibility of more than one seller in a location.

This essay relates closely to the models that feature ex ante heterogeneous agents, while unveiling a non-degenerate price distribution within an endogenous-growth model. In our setup, technical change takes the form of expanding variety, and can be low- or high-skilled labour complementary (directed technical change). This framework makes possible the study of a topic still untreated by the literature: the relationship between price distribution and innovation-related factors.

In particular, we show that the two-technology productive and innovative structure adopted in Acemoglu and Zilibotti (2001) allows for the analytical derivation of the equilibrium probability density function (pdf) of homogeneous-good prices. This result relies on ex ante heterogeneity among producers, but no direct assumption is made with respect to the pdf of firms' production costs. Instead, the posited production function, which combines either low- or high-skilled labour with labour-specific intermediate goods, implies a uniform distribution of firms' relative competitive advantage in adopting high- versus low-skilled labour-complementary technology. Moreover, consumers are homogeneous and only care about the aggregate "consumer price index", i.e., the price of a continuously divisible basket

of homogeneous goods (the composite final good).²

The derived pdf of prices is an equilibrium distribution in the sense that it emanates from equilibrium conditions at the micro level: (i) the value product of low- and of high-skilled labour complementary technology must break even at the margin, and (ii) the marginal value product of each type of labour must be equalised across homogeneous-good firms. The stationarity of the equilibrium pdf is then obtained by considering the balanced-growth path (BGP), at the macro level.

Under an R&D technology with a varying degree of net scale effects benefiting innovative activity, we show that price mean and dispersion along the BGP depend on the scale effects and on the relative labour endowment measured in efficiency units. In accord with casual empiricism, prices are distributed with positive probability over a closed interval and, under broad conditions, the mode of prices is smaller than their mean.

7.2. Model

The model used herein is the one by Acemoglu and Zilibotti (2001), augmented with a varying degree of scale effects in R&D technology.

The economy is populated by fixed infinitely-lived households who inelastically supply one of two types of labour: low-skilled, L , and high-skilled labour, H . Households choose an aggregate consumption plan, $\{C(t), t \geq 0\}$, to maximize $U = \int_0^\infty \left(\frac{C(t)^{1-\theta}-1}{1-\theta} \right) e^{-\rho t} dt$, subject to a standard flow budget constraint and a No-Ponzi game condition; $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the coefficient of relative risk aversion. The plan satisfies the Euler equation $\dot{C}(t)/C(t) = (r(t) - \rho)/\theta$, where $r(t)$ is the real interest rate at

²These simplifying assumptions help to show clearly the role of the innovation-related factors in explaining the price distribution.

time t .

The composite final good, Y , is produced by a continuum of competitive firms, indexed by $n \in [0, 1]$, and can be used in consumption, production of intermediate goods, X , and R&D, R . At t , $Y(t) = \int_0^1 P(n, t) Y(n, t) dn = \exp \left[\int_0^1 \ln Y(n, t) dn \right]$, since the price of Y is normalised to one, $P_Y = \exp \left[\int_0^1 \ln P(n) dn \right] = 1$; P_Y can be interpreted as an aggregate “consumer price index”.³

To produce the final good, two substitute technologies are available to firm n : the Low (High) technology uses a combination of L (H) and a continuum of L - (H -)specific intermediate goods indexed by $\omega_L \in [0, N_L(t)]$ ($\omega_H \in [0, N_H(t)]$), such that

$$Y(n, t) = \left[\int_0^{N_L(t)} x_L(n, \omega_L, t)^{1-\alpha} d\omega_L \right] [(1-n) \cdot L(n)]^\alpha + \left[\int_0^{N_H(t)} x_H(n, \omega_H, t)^{1-\alpha} d\omega_H \right] [n \cdot h \cdot H(n)]^\alpha, \quad 0 < \alpha < 1, \quad (7.1)$$

where $x_m(n, \omega_m, t)$, $m \in \{L, H\}$, is the input of m -complementary intermediate good ω_m , used by n ; $N_m(t)$ is the measure of intermediate goods in sector m ,⁴ which grows over t due to successful R&D (technical change); $L(n)$ and $H(n)$ are the labour inputs and α is the labour share in production. An *absolute*-productivity advantage of H over L is captured by $h > 1$; a *relative*-productivity advantage of each labour type is determined by terms n and $(1-n)$, implying that H is relatively more productive for larger n .

Economic viability of either L - or H -technology relies on the relative productivity and price of labour, as well as on the relative productivity and prices of intermediate goods, due to complementarity in production. At t there is an endogenous equilibrium threshold final

³The expressions for Y and P_Y are the symmetric Cobb-Douglas functions $Y = Y(1)^{\frac{1}{n}} \cdot Y(2)^{\frac{1}{n}} \dots Y(n)^{\frac{1}{n}}$ and $P(1)^{\frac{1}{n}} \cdot P(2)^{\frac{1}{n}} \dots P(n)^{\frac{1}{n}}$ for $n \rightarrow \infty$. Thus, Y and P_Y are constructed following a geometric-aggregation procedure.

⁴Henceforth, we refer to the “ m -complementary intermediate-good sector” as “sector m ”.

good,

$$\bar{n}(t) = \left[1 + \left(\frac{hH}{L} \frac{N_H(t)}{N_L(t)} \right)^{\frac{1}{2}} \right]^{-1},$$

where the switch from one technology to the other becomes advantageous, and which depends on the degree of directed technical change, N_H/N_L . $\bar{n}$ implies that L - (H -) technology is exclusively used by final-good firms indexed by $n \in [0, \bar{n}]$ ($n \in [\bar{n}, 1]$), such that $Y(t) \equiv Y_L(t) + Y_H(t) \equiv \int_0^{\bar{n}(t)} P(n, t) Y(n, t) dn + \int_{\bar{n}(t)}^1 P(n, t) Y(n, t) dn$. The threshold can be related to the ratio of price indices of final goods produced with L - and H -technologies:

$$\frac{P_H(t)}{P_L(t)} = \left(\frac{\bar{n}(t)}{1 - \bar{n}(t)} \right)^\alpha, \text{ where } \begin{cases} P_L(t)^{\frac{1}{\alpha}} \equiv P(n, t)^{\frac{1}{\alpha}} \cdot (1 - n) = e^{-\alpha} \bar{n}(t)^{-\alpha} \\ P_H(t)^{\frac{1}{\alpha}} \equiv P(n, t)^{\frac{1}{\alpha}} \cdot n = e^{-\alpha} (1 - \bar{n}(t))^{-\alpha} \end{cases}. \quad (7.2)$$

In (7.2), we first define the price indices, P_L and P_H , by recognising that, in equilibrium, the marginal value product, $\frac{\partial}{\partial m(n)} (P(n, t) Y(n, t))$, $m \in \{L, H\}$, must be constant over n , implying that $P(n, t)^{\frac{1}{\alpha}} \cdot (1 - n)$ and $P(n, t)^{\frac{1}{\alpha}} \cdot n$ must be constant over $n \in [0, \bar{n}]$ and $n \in [\bar{n}, 1]$, respectively. Then, considering that at $\bar{n}$ the L - and the H - technology firms must break even, we relate P_L and P_H with $\bar{n}$.

Intermediate-good sector m consists of monopolistically competitive firms that produce $\omega_m \in [0, N_m(t)]$ to meet isoelastic demand curves, derived from the final-good firms' profit maximisation conditions. Since one unit of ω_m is produced with one unit of Y , the profit-maximisation price fixed by the monopolist in ω_m yields the constant mark-up $p(\omega_m, t) \equiv p = 1/(1 - \alpha) > 1$. Bearing in mind (7.2) and the mark-up p , we then derive the optimal intermediate-good production, $X_L(\omega_L, t) = \int_0^{\bar{n}(t)} x_L(n, \omega_L, t) dn = (1 - \alpha)^{\frac{2}{\alpha}} L P_L(t)^{\frac{1}{\alpha}}$ and $X_H(\omega_H, t) = \int_{\bar{n}(t)}^1 x_H(n, \omega_H, t) dn = (1 - \alpha)^{\frac{2}{\alpha}} h H P_H(t)^{\frac{1}{\alpha}}$. It follows that the optimal profits

accrued by the monopolist in ω_m are

$$\pi_L(\omega_L, t) = \bar{\pi} \cdot L \cdot P_L(t)^{\frac{1}{\alpha}} \text{ and } \pi_H(\omega_H, t) = \bar{\pi} \cdot h \cdot H \cdot P_H(t)^{\frac{1}{\alpha}}, \quad \bar{\pi} \equiv \left(\frac{\alpha}{1 - \alpha} \right) (1 - \alpha)^{\frac{2}{\alpha}}. \quad (7.3)$$

Technical change takes the form of increases over t in N_L and N_H . R&D firms face a sunk cost of η units of Y to design a new variety, which is protected by a patent. The laws of motion of N_m , $m \in \{L, H\}$, are

$$\dot{N}_L(t) = \frac{1}{\eta L^\epsilon} \cdot R_L(t) \text{ and } \dot{N}_H(t) = \frac{1}{\eta (hH)^\epsilon} \cdot R_H(t), \quad (7.4)$$

where R_m , $m \in \{L, H\}$, denotes the flow of resources to improve N_m , with $R_L + R_H = R$. Unlike Acemoglu and Zilibotti (2001), we consider $\epsilon \geq 0$, where $1 - \epsilon$ measures the degree of net scale effects. The latter capture the idea that the difficulty of introducing new varieties is proportional to the market size measured by employed labour in efficiency units, e.g., due to coordination, organisational and transportation costs (Dinopoulos and Thompson, 1999); however, depending on the effectiveness of those costs, they may partially ($0 < \epsilon < 1$), totally ($\epsilon = 1$) or over ($\epsilon > 1$) counterbalance the benefits of scale to innovation, which are connected to the size of profits that accrue to the R&D successful firm (see (7.3)).

7.3. Balanced-growth path

Along the BGP, N_L and N_H grow at the same rate (the economic growth rate), which implies that $\bar{n}$, P_L and P_H are constants (see (7.2)). It follows that $\pi_L(\omega_L)$ and $\pi_H(\omega_H)$ in (7.3) are also constants. Thus, the present value of profits can be written as $V_m = \pi_m \int_t^\infty e^{[-\int_t^s r(\nu) d\nu]} ds$, $m \in \{L, H\}$.

Moreover, with free entry into R&D and positive R , V_m must equal the cost of invention (see (7.4)). This implies that r is constant and given by $r = \pi_L / (\eta L^\epsilon) = \pi_H / [\eta (hH)^\epsilon]$, which then yields

$$\frac{\tilde{P}_H}{\tilde{P}_L} = \left(\frac{L}{hH} \right)^{\alpha(1-\epsilon)}, \text{ where } \begin{cases} \tilde{P}_L = e^{-\alpha} \tilde{n}^{-\alpha} \\ \tilde{P}_H = e^{-\alpha} (1 - \tilde{n})^{-\alpha} \end{cases}, \tilde{n} = \left[1 + \left(\frac{hH}{L} \right)^{1-\epsilon} \right]^{-1}. \quad (7.5)$$

With $\epsilon = 0$, equation (7.5) becomes Acemoglu and Zilibotti's (2001) equation (15).

Proposition 7.1

(i) If $\epsilon = 1$ or $\frac{hH}{L} = 1$, then $\tilde{P}_L = \tilde{P}_H = \tilde{P} = \left(\frac{2}{e} \right)^\alpha \Leftrightarrow \tilde{n} = \frac{1}{2}$;

(ii) if $0 \leq \epsilon < 1$ and

a. $\frac{hH}{L} < 1$, then $\tilde{P}_L < \tilde{P}_H = \left(\frac{\tilde{n}}{1-\tilde{n}} \right)^\alpha \tilde{P}_L \Leftrightarrow \tilde{n} > \frac{1}{2}$;

b. $\frac{hH}{L} > 1$, then $\tilde{P}_L > \tilde{P}_H = \left(\frac{\tilde{n}}{1-\tilde{n}} \right)^\alpha \tilde{P}_L \Leftrightarrow \tilde{n} < \frac{1}{2}$;

(iii) if $\epsilon > 1$ and

a. $\frac{hH}{L} < 1$, then $\tilde{P}_L > \tilde{P}_H = \left(\frac{\tilde{n}}{1-\tilde{n}} \right)^\alpha \tilde{P}_L \Leftrightarrow \tilde{n} < \frac{1}{2}$;

b. $\frac{hH}{L} > 1$, then $\tilde{P}_L < \tilde{P}_H = \left(\frac{\tilde{n}}{1-\tilde{n}} \right)^\alpha \tilde{P}_L \Leftrightarrow \tilde{n} > \frac{1}{2}$.

7.4. Equilibrium price distribution

We now derive the equilibrium price distribution for the homogeneous good along the BGP.^{5,6}

⁵Henceforth, the $\sim$ is omitted.

⁶For technical statistical details see, e.g., Rohatgi (1976).

Theorem 7.1 Let n be a random variable with pdf $f(n)$ and $y = \varphi(n)$ a random variable with pdf $g(y)$. If φ is a piecewise monotonic function, then $g(y) = \sum_{i=1}^k f(\varphi_i^{-1}(y)) \left| \frac{d\varphi_i^{-1}(y)}{dy} \right|$, where k is the number of sub-domains in which φ is monotonic and φ_i^{-1} denotes the inverse function of φ in the sub-domain i , $i = 1, \dots, k$.

From Section 7.2,

$$n \sim U(0, 1) \Rightarrow f(n) = \begin{cases} 1, & 0 \leq n \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

and, given (7.2),

$$y = P(n) = \begin{cases} (1 - n)^{-\alpha} P_L, & 0 \leq n < \bar{n} \\ n^{-\alpha} P_H, & \bar{n} \leq n \leq 1 \end{cases}.$$

Importantly, P_L and P_H are constants over n and, along the BGP, also over t (see (7.5)).

Thus, from Proposition 7.1 and Theorem 7.1, we get

Proposition 7.2 The random variable $y = P(n)$ has the pdf:

$$\begin{aligned} \text{(i) If } P_L = P_H = P, g(y) &= \begin{cases} \frac{2}{\alpha P} \left(\frac{y}{P} \right)^{-\frac{1}{\alpha}-1}, & P \leq y \leq 2^\alpha P \\ 0, & \text{otherwise} \end{cases} \\ \text{(ii) if } P_L < P_H, g(y) &= \begin{cases} \frac{1}{\alpha P_L} \left(\frac{y}{P_L} \right)^{-\frac{1}{\alpha}-1}, & P_L \leq y < P_H \\ \frac{1}{\alpha P_L} \left(\frac{y}{P_L} \right)^{-\frac{1}{\alpha}-1} + \frac{1}{\alpha P_H} \left(\frac{y}{P_H} \right)^{-\frac{1}{\alpha}-1}, & P_H \leq y \leq \bar{n}^{-\alpha} P_H \\ 0, & \text{otherwise} \end{cases} \\ \text{(iii) if } P_L > P_H, g(y) &= \begin{cases} \frac{1}{\alpha P_H} \left(\frac{y}{P_H} \right)^{-\frac{1}{\alpha}-1}, & P_H \leq y < P_L \\ \frac{1}{\alpha P_L} \left(\frac{y}{P_L} \right)^{-\frac{1}{\alpha}-1} + \frac{1}{\alpha P_H} \left(\frac{y}{P_H} \right)^{-\frac{1}{\alpha}-1}, & P_L \leq y \leq \bar{n}^{-\alpha} P_H \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

7. Equilibrium Price Distribution with Directed Technical Change

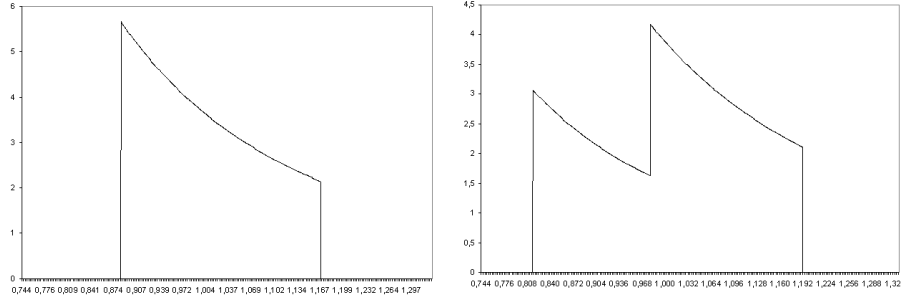


Figure 7.1.: Probability density function for $P_L = P_H = P$ (left) and $P_L \neq P_H$ (right)

The pdf of $P(n)$ is truncated from above and below, i.e., prices are distributed with positive probability over a closed interval, in line with casual empiricism. Figure 7.1 depicts the pdf for $P_L = P_H = P$ ($\epsilon = 1 \vee hH/L = 1$) and $P_L \neq P_H$ ($\epsilon \neq 1 \wedge hH/L \neq 1$).

From Proposition 7.2, we have

Proposition 7.3 The mean and variance of $y = P(n)$ are $E(P(n)) = \frac{P_H}{\alpha-1} \left[\bar{n}^{-\alpha} - \left(\frac{\bar{n}}{1-\bar{n}} \right)^{-\alpha} - 1 \right]$ and $Var(P(n)) = P_H^2 \left\{ \frac{1}{2\alpha-1} \left[\bar{n}^{-2\alpha} - \left(\frac{\bar{n}}{1-\bar{n}} \right)^{-2\alpha} - 1 \right] - \frac{1}{(\alpha-1)^2} \left[\bar{n}^{-\alpha} - \left(\frac{\bar{n}}{1-\bar{n}} \right)^{-\alpha} - 1 \right]^2 \right\}$.

Proposition 7.4 The mode of $y = P(n)$, $\mu_o(P(n))$, is:

(i) $\mu_o(P(n)) = P$, if $P_L = P_H = P \Leftrightarrow \bar{n} = \frac{1}{2}$;

(ii) $\mu_o(P(n)) = P_L$, if

a. $\bar{n} \in (\frac{1}{2} - \frac{1}{2+8\alpha}, \frac{1}{2})$, when $P_L > P_H$;

b. $\bar{n} \in (\frac{1}{2} + \frac{1}{2+8\alpha}, 1]$, when $P_L < P_H$.

(iii) $\mu_o(P(n)) = P_H$, if

a. $\bar{n} \in [0, \frac{1}{2} - \frac{1}{2+8\alpha})$, when $P_L > P_H$;

b. $\bar{n} \in (\frac{1}{2}, \frac{1}{2} + \frac{1}{2+8\alpha})$, when $P_L < P_H$.

7.5. Comparative statics

We discuss the impact of changes in hH/L and ϵ on price mean and dispersion. Given equation (7.5) and Proposition 7.3, we have

Proposition 7.5 For given α and $hH/L \neq 1$, $E(P(n))$ and $Var(P(n))$ are decreasing functions of $\epsilon \in [0, 1)$ and increasing functions of $\epsilon \in (1, \infty)$.

Proposition 7.6 For given α and $\epsilon \neq 1$, $E(P(n))$ and $Var(P(n))$ are decreasing functions of $hH/L \in [0, 1)$ and increasing functions of $hH/L \in (1, \infty)$.

Thus, countries with (i) larger scale effects (in modulus), given α and $hH/L \neq 1$ or (ii) larger imbalances between high- and low-skilled labour endowments in efficiency units, given α and $\epsilon \neq 1$, are expected to have larger price mean and variance. This result also corresponds to a larger variation coefficient $\sqrt{\frac{Var(P(n))}{E(P(n))^2}} = \sqrt{\frac{(\alpha-1)^2}{2\alpha-1} \frac{\bar{n}^{-2\alpha} - (\frac{\bar{n}}{1-\bar{n}})^{-2\alpha} - 1}{[\bar{n}^{-\alpha} - (\frac{\bar{n}}{1-\bar{n}})^{-\alpha} - 1]^2}} - 1$.

We also investigate under which conditions the mode of $P(n)$ is smaller than its mean, as we intuitively expect the empirical case. From Propositions 7.3 and 7.4, we find

Proposition 7.7 $E(P(n)) > \mu_o(P(n))$ requires:

(i) when $\mu_o(P(n)) = P_L = P_H = P$, $\alpha > 2^\alpha - 1$;

(ii) when $\mu_o(P(n)) = P_H$,

a. $\bar{n} \in \left[0, \frac{1}{2} - \frac{1}{2+8\alpha}\right)$, for $P_L > P_H$;

b. $\bar{n} \in \left(\frac{1}{2}, \frac{1}{2} + \max\left\{\frac{1}{2+8\alpha}, \frac{1+\alpha-2^\alpha}{2\alpha(1-\alpha)}\right\}\right)$, for $P_L < P_H$.

(iii) when $\mu_o(P(n)) = P_L$,

a. $\bar{n} \in \left(\frac{1}{2} - \max\left\{\frac{1}{2+8\alpha}, \frac{1+\alpha-2^\alpha}{2\alpha(1-\alpha)}\right\}, \frac{1}{2}\right)$, for $P_L > P_H$;

b. $\bar{n} \in \left(\frac{1}{2} + \frac{1}{2+8\alpha}, 1\right]$, for $P_L < P_H$.

Whatever $0 < \alpha < 1$, the inequality in (i) is universally verified, while (ii)a and (iii)b are always verified under Proposition 7.4. The restrictiveness of the conditions in (ii)b and (iii)a depends on the specific value of α .

We focus on the latter two cases, as they preclude extreme values for $\bar{n}$, and hence for hH/L . Let $\alpha = 0.4$, a standard value in the endogenous-growth literature. Then, (iii)a $\bar{n} \in (0.31, \frac{1}{2})$ and (ii)b $\bar{n} \in (\frac{1}{2}, 0.69)$. Consequently, if $hH/L = 0.2$ (or $hH/L = 5$), then equation (7.5) implies $\frac{1}{2} \leq \epsilon \leq 1.5$; if $hH/L = 0.65$ (or $hH/L = 1.5$), equation (7.5) implies $0 \leq \epsilon \leq 2$. Thus, with a relatively large imbalance in hH/L , small scale effects (in modulus) are required to $E(P(n)) > \mu_o(P(n))$. However, if hH/L is sufficiently close to one, no constraint is imposed on scale effects from above ($\epsilon \geq 0$), although they still cannot be too negative (ϵ cannot exceed unity by too much). Only in case (i) with $hH/L = 1$ (see Proposition 7.1) are there no constraints on scale effects.

7.6. Conclusion

This essay analytically derives a non-degenerate price distribution for the homogeneous good within a model of endogenous directed technical change.

By obtaining an explicit result for the equilibrium price distribution along the BGP, we are able to make testable predictions with respect to the price mean and dispersion of countries with different levels of relative labour endowment and scale effects. In particular, an empirically compatible result with respect to the mode of prices may require small scale effects, in modulus. This conforms with the well-known endogenous-growth literature debate over the counterfactual character of large (positive) scale effects (Jones, 1995b).

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A. Appendix to Chapter 3

A.1. Derivation of equation (3.31)

The aggregate financial wealth held by all households is $L \cdot a(t) = \int_0^{N(t)} V(\omega, t) d\omega$. If we consider the arbitrage condition between vertical and horizontal entry, we get

$$L \cdot a(t) = \eta(t) \cdot N(t) \quad (\text{A.1})$$

Hence, we can characterise the change in the value of equity as

$$L \cdot \dot{a}(t) = \eta(t) \cdot \dot{N}(t) + \dot{\eta}(t) \cdot N(t) \quad (\text{A.2})$$

Solve (3.22) in order to $\dot{\eta}$ and, together with (A.1) and (3.28), substitute in (A.2) to get

$$\begin{aligned} r(t) \cdot L \cdot a(t) + L \cdot w(t) - L \cdot C(t) &= \eta(t) \cdot \dot{N}(t) - \bar{\pi} \cdot N(t) + \\ &+ (r(t) + I(t)) \cdot \eta(t) \cdot N(t) + \frac{\dot{\bar{\pi}}(t)}{\bar{\pi}(t)} \cdot \eta(t) \cdot N(t) = 0 \Leftrightarrow \end{aligned}$$

$$\Leftrightarrow (L \cdot w(t) + \bar{\pi}(t) \cdot N(t)) - L \cdot C(t) - \eta(t) \cdot \dot{N}(t) - \left(I(t) \cdot \eta(t) \cdot N(t) + \frac{\dot{\bar{\pi}}(t)}{\bar{\pi}(t)} \cdot \eta(t) \cdot N(t) \right) = 0 \Leftrightarrow$$

$$\Leftrightarrow Y(t) - X(t) - L \cdot C(t) - \eta(t) \cdot \dot{N}(t) - \left(I(t) \cdot \eta(t) \cdot N(t) + \frac{\dot{\bar{\pi}}(t)}{\bar{\pi}(t)} \cdot \eta(t) \cdot N(t) \right) = 0 \quad (\text{A.3})$$

where we used $Y = w \cdot L + p \cdot X \Leftrightarrow Y - X = w \cdot L + \bar{\pi} \cdot N$.¹ Next, recall that $R_n = \eta \cdot \dot{N}$ and $R_v = I \cdot \eta \cdot N + \frac{\dot{\bar{\pi}}}{\bar{\pi}} \cdot \eta \cdot N$,² such that (A.3) reads

$$Y(t) = X(t) + L \cdot C(t) + R_n(t) + R_v(t)$$

which is (3.31).

A.2. Necessary conditions for the existence of a BGP

First, we prove (i) and (ii). Substituting $R_v(t) = I(t) \cdot \zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot Q(t)$, (3.8), (3.9) and (3.17) in (3.31), and simplifying the resultant considering (3.23) and the fact that, along the BGP, $\dot{N} = g_N N$, we have

$$H_Y Q(t) = H_X Q(t) + C(t) + \zeta g_N + I(t) \zeta \lambda^{\frac{\alpha}{1-\alpha}} Q(t) \quad (\text{A.4})$$

¹Having in mind (3.1), (3.3), (3.8), (3.9) and (3.10), and that, in equilibrium, w and p are equated to the marginal product of labour and the marginal product of intermediate goods, respectively, it is easily shown that $w \cdot L = (1 - \alpha) \cdot Y$, $X = \alpha^2 \cdot Y$, $p \cdot X = \alpha \cdot Y$ and total profits $\Pi = X \cdot (p - 1) = \alpha \cdot Y - \alpha^2 \cdot Y$. Also, have in mind that given (3.10) and (3.19), total profits can be represented as $\Pi = \bar{\pi} \cdot N$.

²To see this, recall that $\frac{\dot{\bar{\pi}}}{\bar{\pi}} = I \cdot \left[\dot{j} \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right] = I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right)$ for $\dot{j} = 1$ and small λ , and use it together with the consistency condition (3.23), solved in order to ηN , in (3.16), to get $R_v = I \cdot \zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot Q = I \cdot \eta \cdot N + \frac{\dot{\bar{\pi}}}{\bar{\pi}} \cdot \eta \cdot N$.

where $H_Y \equiv \left(A^{\frac{1}{\alpha}} \alpha^2\right)^{\frac{\alpha}{1-\alpha}}$ and $H_X \equiv (A\alpha^2)^{\frac{1}{1-\alpha}}$. Time-differentiating (A.4) and solving in order to $\dot{I}$, yields

$$\begin{aligned} \dot{I}(t) &= \frac{1}{\zeta \lambda^{\frac{1}{1-\alpha}} Q(t)} \left\{ \dot{Q}(t) \left[H_Y - H_X - \zeta g_N - I(t) \zeta \lambda^{\frac{\alpha}{1-\alpha}} \right] - \dot{C}(t) \right\} \Leftrightarrow \\ &\Leftrightarrow g_I = \frac{1}{\zeta \lambda^{\frac{\alpha}{1-\alpha}} I(t)} \left\{ g_Q \left[H_Y - H_X - \zeta g_N - I(t) \zeta \lambda^{\frac{\alpha}{1-\alpha}} \right] - g_C \frac{C(t)}{Q(t)} \right\} \end{aligned} \quad (\text{A.5})$$

Along the BGP, g_I , g_Q and g_C must be constant. Consider three scenarios:

(a) Suppose $g_I, g_Q, g_C \neq 0$ and $g_I \neq g_Q \neq g_C$. From (A.5), this implies that g_I changes as I and $\frac{C}{Q}$ change over time, which is a contradiction.

(b) Suppose $g_I, g_Q, g_C \neq 0$ and $g_Q = g_C = g$, then we have, from (A.5),

$$g_I = \frac{1}{\zeta \lambda^{\frac{\alpha}{1-\alpha}} I(t)} \left\{ g \left[H_Y - H_X - \zeta g_N - I(t) \zeta \lambda^{\frac{\alpha}{1-\alpha}} - \frac{C(t)}{Q(t)} \right] \right\} \Leftrightarrow g_I = 0$$

which is a contradiction.

(c) Suppose $g_I = 0$ and $g_Q \neq g_C$, then we have, from (A.5),

$$g_C = \frac{Q}{C} g_Q \left[H_Y - H_X - \zeta g_N - I(t) \zeta \lambda^{\frac{\alpha}{1-\alpha}} \right]$$

This implies that g_C changes as $\frac{Q}{C}$ changes over time, which is also a contradiction.

Then, it must be true that, along the BGP, $g_Q = g_C = g$ and $g_I = 0$. □

The proof of (iii) is as follows. From (3.24), using (3.23), we have

$$\frac{\dot{N}(t)}{N(t)} = g_N(t) = \left(\frac{\zeta L}{\phi} \right)^{\frac{1}{\gamma}} Q^{\frac{1}{\gamma}} N^{-\left(\frac{\sigma+\gamma+1}{\gamma} \right)} \equiv x(Q, N) \quad (\text{A.6})$$

Since g_N must be constant along the BGP, then, by time-differentiating the equation above and equating to zero, we find

$$x \left[\frac{1}{\gamma} \frac{\dot{Q}(t)}{Q(t)} - \left(\frac{\sigma + \gamma + 1}{\gamma} \right) \frac{\dot{N}(t)}{N(t)} \right] = 0 \Leftrightarrow \frac{g_Q}{g_N} = (\sigma + \gamma + 1)$$

provided $x \neq 0$. □

A.3. Proof of Proposition 3.1

Proof. Using the Caballé and Santos (1993) decomposition, we write $C(t) = c(t)e^{g_C t}$, $Q(t) = \tilde{q}(t)e^{g_Q t}$ and $N(t) = n(t)e^{g_N t}$. We can build a detrended system in $(c, \tilde{q}, n)$ only if $g_C = g_Q = g$ and $g_Q = g_N(\sigma + \gamma + 1)$. If those conditions hold, then $x(\tilde{q}, n) = x(Q, N)$ and $I^+(C, Q, N) = I^+(c, \tilde{q}, n)$, and an equivalent system for the detrended variables is obtained: $\dot{n} = (x(\tilde{q}, n) - g_N) \cdot n$, $\dot{c} = (\mu - I^\pm(c, \tilde{q}, n) - \theta \cdot g) \cdot (c/\theta)$, $\dot{\tilde{q}} = (I^\pm(c, \tilde{q}, n) \cdot \Xi + x(\tilde{q}, n) - g_Q) \cdot \tilde{q}$. Now, consider the steady state of this system $(c^*, \tilde{q}^*, n^*)$. Firstly, observe that if we have $H_y - H_x - c^*/\tilde{q}^* - \zeta n^*/\tilde{q}^* < 0$, then $I = I^- = 0$. In this case, there is no BGP because we should have $g_Q = g_N = 0$, which contradicts the fact that $g_N = x(\tilde{q}, n) > 0$ if $\tilde{q} > 0$ and $n > 0$. Secondly, consider $H_y - H_x - c^*/\tilde{q}^* - \zeta n^*/\tilde{q}^* > 0$. In this case, we get a steady state for $I = I^+ = I^*$, as in (3.37), which is positive if $\mu > 0$. Then, there is a unique BGP, $C^*(t) = c^*e^{g^* t}$, $Q^*(t) = \tilde{q}^*e^{g^* t}$, $N^*(t) = n^*e^{g_N^* t}$, in which the long run growth rates g^* and g_N^* , as in (3.38) and (3.39), are positive if $\mu > 0$. At last, the transversality condition $\lim_{t \rightarrow \infty} \zeta L(c^*)^{-\theta} \tilde{q}(t) e^{-(\rho + (\theta - 1)g^*)t} = 0$ holds if $\theta \geq 1$. □

A.4. Proof of Proposition 3.2

Proof. Consider the change in variables, $z = C/Q$ and $x = [(L\zeta)/\phi]^{1/\gamma} Q^{1/\gamma} N^{-(\sigma+\gamma+1)/\gamma}$, and the equivalent system (3.40a)-(3.40b) in detrended variables and Proposition 1. If the BGP is unique in the initial system (3.35a)-(3.35c), it is also unique in this new system. We can also prove that, in this new system, an admissible steady state $(x^*, z^*) \in \mathbb{R}_{++}^2$, can also exist if $I^* = I^+(x^*, z^*) > 0$. However, those steady-state level variables should be positive. Sufficient conditions for this are that $\mu > 0$ and that the isocline $\dot{z} = 0$ crosses the axis z , for $x = 0$, at non-negative values, that is $H_y - H_x > \zeta \lambda^{\alpha/(1-\alpha)} (r_0 - \rho)/(1 + \theta\Xi)$. At last, as is usual in endogenous growth models, if we consider the three original variables, C , Q , and N , the dynamic system is under-determinate. As a natural choice for a determinate variable is the number of varieties, we can determine explicit values along the BGP by choosing $N(0)$ as given. Therefore, the economy will be placed along a BGP if $Q(0) = \phi/(L\zeta)(x^*)^\gamma N(0)^{\sigma+\gamma+1}$. $\square$

A.5. Proof of Proposition 3.3

Proof. Consider the system (3.40a)-(3.40b) and its admissible steady state (x^*, z^*) , where $I^* = I^+(x^*, z^*) > 0$. The Jacobian evaluated at that steady state is

$$J(x^*, z^*) = \begin{pmatrix} a_{11}z^* & a_{12}x^* \\ a_{21}z^* & a_{22}x^* \end{pmatrix} = \frac{1}{\lambda^{\frac{\alpha}{(1-\alpha)}}} \begin{pmatrix} (1 + \theta\Xi)z^*/(\theta\zeta) & (1 - \theta)x^*/\theta \\ -\Xi z^*/(\gamma\zeta) & -[\Xi(\sigma + \gamma + 1) + \sigma + \gamma]x^*/\gamma \end{pmatrix}$$

As $\det(J(x^*, z^*)) = -[\Xi(\sigma + \gamma + 1) + \sigma + \gamma]/(\gamma\zeta\lambda^{\frac{\alpha}{(1-\alpha)}}) < 0$, then $J(x^*, z^*)$ has one negative, $\lambda_s < 0$, and one positive eigenvalue. The slope of the eigenspace associated to the negative eigenvalue is $-(a_{22}x^* - \lambda_s)/(a_{21}z^*)$. As $a_{21} < 0$ then the slope has the same sign as

$a_{22}x^* - \lambda_s$. After some algebra, we get

$$a_{22}x^* - \lambda_s = -\frac{1}{2\lambda^{\frac{\alpha}{1-\alpha}}} \left[\left(\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma) \right) \frac{x^*}{\gamma} + \frac{(1 + \theta\Xi)z^*}{\theta\zeta} \right] + \Delta(J(x^*, z^*))^{1/2}$$

where the discriminant of the Jacobian is

$$\begin{aligned} \Delta(J(x^*, z^*)) &= \left\{ \frac{1}{2\lambda^{\frac{\alpha}{1-\alpha}}} \left[\left(\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma) \right) \frac{x^*}{\gamma} + \frac{(1 + \theta\Xi)z^*}{\theta\zeta} \right] \right\}^2 + \\ &\quad + \frac{4z^*x^*}{\zeta\gamma} \Xi \left(\sigma + \gamma + 1 - \frac{1}{\theta} \right). \end{aligned}$$

Then $a_{22}x^* - \lambda_s > 0$ if $\theta \geq 1$ and the local stable manifold is positively sloped in the neighbourhood of (x^*, z^*) .

The phase diagram in Figure 3.1 allows for a geometric characterisation of the dynamics of transition. Observe that curve $I^+(x, z) = 0$, that we will call switching curve, divides the state space into two zones: in the northeast area, where, as $I^+(x, z) < 0$, we set $I = 0$ and the dynamics is given by the first branch, and in the southwest area, where we have $I^+(x, z) > 0$ and the dynamics is given by the second branch. The isocline $\dot{x} = 0$ is negatively sloped and lies entirely in the second branch. The isocline $\dot{z} = 0$ passes through the two branches: it has a positive intercept in the second branch if $H_y - H_x > \zeta\lambda^{\alpha/(1-\alpha)}(r_0 - \rho)/(1 + \theta\Xi)$, has a positive slope and cuts the switching curve at point $x = \mu$, where it is continuous but piecewise-smooth, and is vertical in the first branch. As x is a predetermined variable, this means that if $x(0) < \mu$ then the transition path lies entirely in the second branch, and approaches the BGP values (x^*, z^*) as shown in the figure because the slope of the stable manifold is flatter than the isocline $\dot{z} = 0$. This can be easily proved

if it is observed that

$$\left. \frac{dz}{dx} \right|_{\dot{z}=0} - \left. \frac{dz}{dx} \right|_{W^s} = -\frac{a_{12}x^*}{a_{11}z^*} + \frac{a_{22}x^* - \lambda_s}{a_{21}z^*} = \frac{\lambda_u + a_{11}z^*}{a_{11}a_{21}(z^*)^2} \lambda_s > 0.$$

If $x(0) > \tilde{x}$, the transition path is a concatenation of a transition path lying in the first branch with the transition path in the second branch. This can be proved if we observe that: first, the paths in the first branch to the right of the isocline $\dot{z} = 0$ cross the switching curve in a point $\tilde{x} > \mu$ but smaller than $\frac{\Xi(\sigma+\gamma+1)+\sigma+\gamma}{\Xi(\sigma+\gamma+1)+\frac{1}{\theta}(\sigma+\gamma)}\mu$, which is the projection of the steady state z^* into the switching curve, because of the slope of the stable manifold; second, the paths starting in the first branch to the right of the isocline $\dot{z} = 0$ approach the switching curve and cross it. We can prove that there is no other form of collision, by computing the projections of the vector fields in both sides of the switching curve

$$\frac{\partial I^+}{\partial z} \dot{z}^- + \frac{\partial I^+}{\partial x} \dot{x}^- = \frac{\partial I^+}{\partial z} \dot{z}^+ + \frac{\partial I^+}{\partial x} \dot{x}^+ = \frac{1}{\zeta \lambda^{\frac{\alpha}{1-\alpha}}} \left[\left(\frac{\sigma + \gamma}{\gamma} x^2 + \zeta(x - \mu)z \right) \right] > 0$$

because the collision should take place for $x > \mu$. This means that the path coming from the first branch approaches the switching curve with the same direction as the one defined by the vector field for $I^+ < 0$, that is, both x and z decrease, which means that it will cross the switching boundary and continue with the same direction inside the second branch. We cannot determine the crossing point exactly, but we know that it should be in the intersection of the stable manifold in the second branch with the switching boundary. This means that the stable manifold is piecewise-smooth and lies in the two branches as depicted in Figure 3.1. □

B. Appendix to Chapter 4

B.1. Data on high- and low-tech sectors

In this appendix we illustrate cross-country data with respect to the number of firms, production, average firm size (production/number of firms) and R&D intensity (R&D/production) in high-tech vis-à-vis low-tech manufacturing sectors, and also aggregate per capita growth rates (see Figure B.1). The source is the Eurostat on-line database (link at <http://epp.eurostat.ec.europa.eu>), where the OECD classification of high-tech and low-tech sectors is considered.¹ We also compute the aggregate concentration index for each country by applying the data to (4.48).

¹Data with respect to output (Gross Value Added) and R&D intensity (R&D/production) is also available from the on-line OECD STAN Indicators Database (link at <http://stats.oecd.org>). According to the latter, the pattern depicted by Figure B.1, with $X^* < 1$ and $R^*/Q^* > 1 > 1$ for all countries, is also verified for the non-European countries belonging to the OECD, namely the US, Canada, Mexico, Australia, Korea and Japan.

B. Appendix to Chapter 4

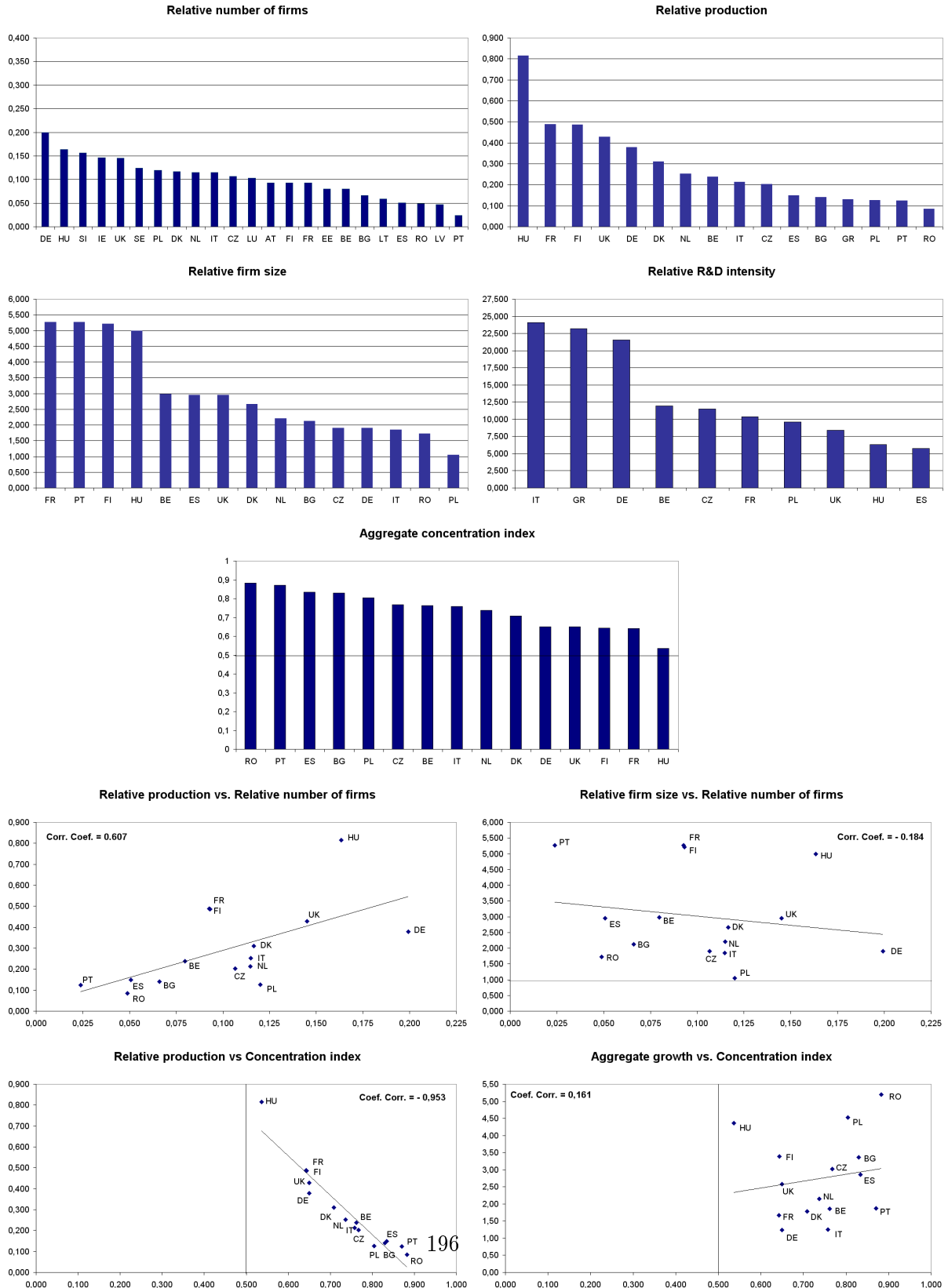


Figure B.1.: Relative number of firms, relative production, relative average firm size, relative R&D intensity, concentration index, and selected correlations. The data concerns the 1995-2005 average for 23 European countries.

B.2. Derivation of equation (4.32)

We derive the aggregate resource constraint from the households' balance sheet and flow budget constraint. Firstly, consider the production function (4.1), such that, given threshold (4.6), we have firm n value product (time indexes are omitted)

$$P(n) \cdot Y(n) = \begin{cases} P(n) \cdot A \cdot \left[\int_0^{N_L} (\lambda^{j_L(\omega_L)} \cdot X_L(n, \omega_L))^{1-\alpha} d\omega_L \right] [(1-n) \cdot l \cdot L(n)]^\alpha & , 0 \leq n \leq \bar{n} \\ P(n) \cdot A \cdot \left[\int_0^{N_H} (\lambda^{j_H(\omega_H)} \cdot X_H(n, \omega_H))^{1-\alpha} d\omega_H \right] [n \cdot h \cdot H(n)]^\alpha & , \bar{n} \leq n \leq 1 \end{cases} \quad (\text{B.1})$$

Since, in equilibrium, the wage paid to each unit of human capital $m \in \{L, H\}$ is equal to its marginal value product, we use (B.1) to get

$$\begin{aligned} w_L &= \frac{\partial(P(n)Y(n))|_{0 \leq n \leq \bar{n}}}{\partial L(n)} \Leftrightarrow w_L \cdot L(n) = \alpha \cdot P(n) \cdot Y(n) |_{0 \leq n \leq \bar{n}} \\ w_H &= \frac{\partial(P(n)Y(n))|_{\bar{n} \leq n \leq 1}}{\partial H(n)} \Leftrightarrow w_H \cdot H(n) = \alpha \cdot P(n) \cdot Y(n) |_{\bar{n} \leq n \leq 1} \end{aligned} \quad (\text{B.2})$$

Aggregating (B.2) across n and simplifying with the following labour-market clearing conditions

$$\begin{aligned} \int_0^{\bar{n}} L(n) dn &= L \Leftrightarrow L(n) = \frac{L}{\bar{n}} \\ \int_{\bar{n}}^1 H(n) dn &= H \Leftrightarrow H(n) = \frac{H}{1-\bar{n}} \end{aligned} \quad (\text{B.3})$$

yields²

$$\begin{aligned} \int_0^{\bar{n}} w_L \cdot L(n) dn &= \alpha \int_0^{\bar{n}} P(n) \cdot Y(n) dn \Leftrightarrow w_L L = \alpha Y_L \\ \int_{\bar{n}}^1 w_H \cdot H(n) dn &= \alpha \int_{\bar{n}}^1 P(n) \cdot Y(n) dn \Leftrightarrow w_H H = \alpha Y_H \end{aligned} \quad (\text{B.4})$$

²Given Cobb-Douglas technology, expenditures across final goods are equalised (i.e., $P(n)Y(n)$ is constant over n), which implies $L(n)$ and $H(n)$ constant over $n \in [0, \bar{n}]$ and $n \in [\bar{n}, 1]$, respectively.

where $Y_L = \int_0^{\bar{n}} P(n)Y(n)dn$, $Y_H = \int_{\bar{n}}^1 P(n)Y(n)dn$, such that $Y_{tot} \equiv Y_L + Y_H = \int_0^1 P(n)Y(n)dn$.

On the other hand, from the derivation of (4.10), we know that $Y_L = A^{\frac{1}{\alpha}} \cdot (1 - \alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot P_L^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot Q_L$ and $Y_H = A^{\frac{1}{\alpha}} \cdot (1 - \alpha)^{\frac{2(1-\alpha)}{\alpha}} \cdot P_H^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot Q_H$. Also, from the derivation of (4.9), we learn that $X_L = \int_0^{N_L} X_L(\omega_L)d\omega_L = A^{\frac{1}{\alpha}} \cdot (1 - \alpha)^{\frac{2}{\alpha}} \cdot P_L^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot Q_L$ and $X_H = \int_0^{N_H} X_H(\omega_H)d\omega_H = A^{\frac{1}{\alpha}} \cdot (1 - \alpha)^{\frac{2}{\alpha}} \cdot P_H^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot Q_H$. Therefore, it is easy to show that, given (4.3),

$$X_m = (1 - \alpha)^2 Y_m \Leftrightarrow p_m X_m = (1 - \alpha) Y_m, \quad m \in \{L, H\} \quad (\text{B.5})$$

We put (B.4) and (B.5) together to get aggregate gross income

$$Y_m = w_m m + p X_m, \quad m \in \{L, H\} \quad (\text{B.6})$$

or, since total profits in each technology group are $\pi_m N_m = \int_0^{N_m} (p_m(\omega_m) - 1) \cdot X_m(\omega_m) d\omega_m = (p - 1) \cdot X_m$, to get aggregate value added

$$Y_m - X_m = w_m m + \pi_m N_m, \quad m \in \{L, H\} \quad (\text{B.7})$$

Secondly, consider aggregate financial wealth held by all households in sector m , $a_m = \int_0^{N_m} V_m(\omega_m) d\omega_m$. If we also consider the arbitrage condition between vertical and horizontal entry, we get

$$a = a_L + a_H = \eta_L N_L + \eta_H N_H \quad (\text{B.8})$$

which, by time-differentiation, becomes

$$\dot{a} = \eta_L \dot{N}_L + \dot{\eta}_L N_L + \eta_H \dot{N}_H + \dot{\eta}_H N_H \quad (\text{B.9})$$

Next, we solve (4.22) with respect to $\dot{\eta}$ and, together with (B.8) and (4.29), substitute in (B.9) to get

$$\begin{aligned}
 r(a_L + a_H) + w_L L + w_H H - C &= \eta_L (r + I_L) N_L - \pi_L N_L + \eta_L \left(\frac{\dot{\pi}_L}{\pi_L} - \frac{1}{\alpha} \frac{\dot{P}_L}{P_L} \right) N_L + \\
 &+ \eta_H (r + I_H) N_H - \pi_H N_H + \eta_H \left(\frac{\dot{\pi}_H}{\pi_H} - \frac{1}{\alpha} \frac{\dot{P}_H}{P_H} \right) N_H + \eta_L \dot{N}_L + \eta_H \dot{N}_H \Leftrightarrow \\
 &\Leftrightarrow w_L L + w_H H + \pi_L N_L + \pi_H N_H = C + I_L \eta_L N_L + I_H \eta_H N_H + \eta_L \dot{N}_L + \eta_H \dot{N}_H + \\
 &+ \left(\frac{\dot{\pi}_L}{\pi_L} - \frac{1}{\alpha} \frac{\dot{P}_L}{P_L} \right) \eta_L N_L + \left(\frac{\dot{\pi}_H}{\pi_H} - \frac{1}{\alpha} \frac{\dot{P}_H}{P_H} \right) \eta_H N_H \tag{B.10}
 \end{aligned}$$

By using (B.7) in (B.10) and recalling $R_h = R_{hH} + R_{hL}$; $R_v = R_{vH} + R_{vL}$; $R_{hm} = \dot{\eta}_m N_m$ and $R_{vm} = I_m a_m + \left(\frac{\dot{\pi}_m}{\pi_m} - \frac{1}{\alpha} \frac{\dot{P}_m}{P_m} \right) \eta_m N_m$,³ we find

$$Y_L - X_L + Y_H - X_H = C + R_{hH} + R_{hL} + R_{vH} + R_{vL} \Leftrightarrow$$

$$\Leftrightarrow Y_{tot} = X_{tot} + C + R_h + R_v$$

which is (4.32).

³To see this, recall that $\frac{\dot{\pi}_m}{\pi_m} - \frac{1}{\alpha} \frac{\dot{P}_m}{P_m} = I_m \cdot \left[\dot{j}_m \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right] = I_m \cdot \left(\lambda^{\frac{1-\alpha}{\alpha}} - 1 \right)$ for $\dot{j}_m = 1$ and small λ , and use it together with the consistency condition (4.23) solved with respect to $\eta_m N_m$, in (4.16), to get, again with $m = L$, $R_{vL} = I_L \zeta \mathcal{L}^\epsilon \lambda^{\frac{1-\alpha}{\alpha}} Q_L = I_L a_L + \left(\frac{\dot{\pi}_L}{\pi_L} - \frac{1}{\alpha} \frac{\dot{P}_L}{P_L} \right) a_L$.

Finally, observe that since the real interest rate r consists of dividend payments in units of asset price minus the Poisson death rate, i.e., $r = \frac{\pi_m}{V_m} - I_m$, for each t (see, e.g., (4.21) and (4.22)), then $a_m = V_m N_m \Rightarrow \pi_m N_m = (r + I_m) a_m$. From here and (B.7), we rewrite (4.29) as

$$\begin{aligned} \dot{a} &= r(a_L + a_H) + w_L L + w_H H - C = \\ &= \left(\frac{\pi_L}{V_L} - I_L \right) a_L + \left(\frac{\pi_H}{V_H} - I_H \right) a_H + w_L L + w_H H - C = \\ &= Y_L - X_L + Y_H - X_H - I_L a_L - I_H a_H - C \end{aligned} \tag{B.11}$$

If we replace (4.32), solved in order to $R_v + R_h$, in (B.11), we obtain

$$\dot{a} = R_v + R_h - (I_L a_L + I_H a_H) \tag{B.12}$$

which is the accumulation equation for a . The first two terms on the right-hand side of (B.12) represent the *gross investment* in technological knowledge at time t , whereas the third term represents the *depreciation* (obsolescence) of the existing technological-knowledge stock due to the stochastic arrival of vertical innovations (i.e., as j jumps to $j + 1$) in each sector $m \in \{L, H\}$.

B.3. Detrended dynamic system and interior steady state

The fixed points of the system (4.38a)-(4.38d) can be found by equating $\dot{x}_L = 0$, $\dot{z}_L = 0$, $\dot{x}_H = 0$ and $\dot{Q} = 0$. However, given the structure of our model, we can solve first for the

latter equation and use the result to solve jointly $\dot{z}_L = 0$ and $\dot{x}_L = 0$. The solution for $\dot{x}_H = 0$ follows directly from the latter.

We are interested in the interior steady state, i.e., $\tilde{x}_L \neq 0 \wedge \tilde{z}_L \neq 0 \wedge \tilde{x}_H \neq 0 \wedge \tilde{Q} \neq 0$. Given (4.25), we know that $\tilde{x}_L = \tilde{x}_H$. Together with (4.38d), we find that $\dot{Q} = 0$ implies

$$\tilde{P} \equiv \left(\frac{\tilde{P}_H}{\tilde{P}_L} \right) = \left(\frac{\mathcal{L}}{\mathcal{H}} \right)^{\alpha(1-\epsilon)}, \quad (\text{B.13})$$

which also guarantees $\tilde{I}_L = \tilde{I}_H$ (see (4.24)). Next, substitute (B.13) in (4.7) and solve in order to $\frac{Q_H}{Q_L}$ to get

$$\tilde{Q} \equiv \left(\frac{\tilde{Q}_H}{\tilde{Q}_L} \right) = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{1-2\epsilon} \quad (\text{B.14})$$

From here, together with (4.6) and (4.7), we find

$$\tilde{P}_L = e^{-\alpha \tilde{n}^{-\alpha}} \text{ and } \tilde{P}_H = e^{-\alpha (1 - \tilde{n})^{-\alpha}} \quad (\text{B.15})$$

$$\tilde{n} = \left[1 + \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{1-\epsilon} \right]^{-1} \quad (\text{B.16})$$

From (4.9), (B.13) and (4.39), we then have

$$\tilde{X} \equiv \left(\frac{\tilde{X}_H}{\tilde{X}_L} \right) = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{1-\epsilon}$$

Now, we turn to the solution of $\dot{x}_L = 0$ and $\dot{z}_L = 0$. By replacing (B.13) and (B.14) in (4.34), we get the linear function $I_L \equiv I_L(x_L, x_H, z_L) = I_0 + I_1 x_H + I_2 x_L + I_3 z_L$, where $I_0 \equiv \frac{\Theta}{\zeta \lambda^{\frac{1-\alpha}{\alpha}} \mathcal{L}^\epsilon} (\chi_Y - \chi_X) \left[\tilde{P}_H^{\frac{1}{\alpha}} \mathcal{H} \tilde{Q} + \tilde{P}_L^{\frac{1}{\alpha}} \mathcal{L} \right]$, $I_1 \equiv -\frac{\Theta}{\lambda^{\frac{1-\alpha}{\alpha}}} \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{1-\epsilon}$, $I_2 \equiv -\frac{\Theta}{\lambda^{\frac{1-\alpha}{\alpha}}}$ and

B. Appendix to Chapter 4

$I_3 \equiv -\frac{\Theta}{\zeta \lambda^{\frac{1-\alpha}{\alpha}} \mathcal{L}^\epsilon}$, and $\Theta \equiv \left[1 + \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-\epsilon}\right]^{-1}$. Substituting in (4.38a) and (4.38b), equating $\dot{x}_L = 0$ and $\dot{z}_L = 0$ and solving for the interior steady state, yields

$$\tilde{z}_L = \left(-I_0 - I_1 \tilde{x}_H - I_2 \tilde{x}_L + \frac{\sigma + \gamma}{\Xi} \tilde{x}_L\right) \frac{1}{I_3} \quad (\text{B.17})$$

$$\tilde{x}_L = \frac{\frac{\Xi}{\theta} \left[\frac{\pi_0}{\zeta} \mathcal{L}^{1-\epsilon} \tilde{P}_L^{\frac{1}{\alpha}} - \rho \right]}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)} \quad (\text{B.18})$$

Given that $\tilde{x}_L = \tilde{x}_H$ and (B.15), we can write

$$\tilde{x}_H = \frac{\frac{\Xi}{\theta} \left[\frac{\pi_0}{\zeta} \mathcal{L}^{1-\epsilon} \tilde{P}_L^{\frac{1}{\alpha}} - \rho \right]}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)} = \frac{\frac{\Xi}{\theta} \left[\frac{\pi_0}{\zeta} \mathcal{H}^{1-\epsilon} \tilde{P}_H^{\frac{1}{\alpha}} - \rho \right]}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)} \quad (\text{B.19})$$

Also, from (4.25) and (4.26a)-(4.26b), we find

$$\tilde{g}_{N_L} = \tilde{g}_{N_H} = \tilde{x}_L = \tilde{x}_H \quad (\text{B.20})$$

and

$$\tilde{g}_{Q_L} = \tilde{g}_{Q_H} = \tilde{g} = \frac{\frac{\Xi}{\theta} \left[\frac{\pi_0}{\zeta} \mathcal{L}^{1-\epsilon} \tilde{P}_L^{\frac{1}{\alpha}} - \rho \right] (\sigma + \gamma + 1)}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)}$$

Finally, using (B.17) and the definition of I_0 , I_1 , I_2 and I_3 , we have

$$\tilde{I}_L = \tilde{I}_H = \frac{\sigma + \gamma}{\Xi} \tilde{x}_L = \frac{\frac{1}{\theta} \left[\frac{\pi_0}{\zeta} \mathcal{L}^{1-\epsilon} \tilde{P}_L^{\frac{1}{\alpha}} - \rho \right] (\sigma + \gamma)}{\Xi (\sigma + \gamma + 1) + \frac{1}{\theta} (\sigma + \gamma)} \quad (\text{B.21})$$

and

$$\begin{aligned}\tilde{z}_L &= (\chi_Y - \chi_X) \left[\tilde{P}_H^{\frac{1}{\alpha}} \mathcal{H} \tilde{Q} + \tilde{P}_L^{\frac{1}{\alpha}} \mathcal{L} \right] - \\ &\quad - \left(\zeta \lambda^{\frac{1-\alpha}{\alpha}} \tilde{I}_L + \zeta \tilde{x}_L \right) \left[\mathcal{H}^\epsilon \tilde{Q} + \mathcal{L}^\epsilon \right]\end{aligned}\tag{B.22}$$

The BGP values of N_L , N_H and C are derived from (4.26a)-(4.26b) and $z_L \equiv C/Q_L$, given Q_H (alternatively, Q_L). Thus,

$$\tilde{N}_L = \left(\frac{\zeta}{\phi} \mathcal{L}^\epsilon \right)^{\frac{1}{\sigma+\gamma+1}} (\tilde{x}_L)^{\frac{-\gamma}{\sigma+\gamma+1}} (\tilde{Q}_L)^{\frac{1}{\sigma+\gamma+1}}\tag{B.23a}$$

$$\tilde{N}_H = \left(\frac{\zeta}{\phi} \mathcal{H}^\epsilon \right)^{\frac{1}{\sigma+\gamma+1}} (\tilde{x}_H)^{\frac{-\gamma}{\sigma+\gamma+1}} (\tilde{Q}_H)^{\frac{1}{\sigma+\gamma+1}}\tag{B.23b}$$

$$\tilde{C} = \tilde{z}_L \tilde{Q}_L\tag{B.24}$$

Then, we use (B.23a) and (B.23b) to derive

$$\tilde{N} \equiv \left(\frac{\tilde{N}_H}{\tilde{N}_L} \right) = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{\frac{1-\epsilon}{\sigma+\gamma+1}}\tag{B.25}$$

Finally, from (4.23), and the households' balance sheet, we rewrite the transversality condition (4.31) as

$$\begin{aligned}&\lim_{t \rightarrow \infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot \zeta \cdot (\mathcal{L}^\epsilon \cdot Q_L(t) + \mathcal{H}^\epsilon \cdot Q_H(t)) = \\ &= \lim_{t \rightarrow \infty} e^{-\rho t} \cdot z_L(t)^{-\theta} \cdot \zeta \cdot (\mathcal{L}^\epsilon + \mathcal{H}^\epsilon \cdot Q(t)) \cdot Q_L(t)^{1-\theta} = 0,\end{aligned}\tag{B.26}$$

where $z_L \equiv C/Q_L$ and $Q \equiv Q_H/Q_L$ are stationary along the BGP, as shown above.

Let $Q_L = \hat{q}_L e^{gt}$, where $\hat{q}_L$ denotes detrended Q_L (thus stationary along the BGP), and substitute in (B.26), to see that the transversality condition implies $\rho \geq (1 - \theta)g$; using again $g = (r - \rho) / \theta$, the latter condition can be written alternatively as $r > g$. As it happens, this condition also guarantees that attainable utility is bounded, i.e., the integral (4.28) converges to infinity.

B.4. Proxy for quality-adjusted production

Given that j follows a Poisson distribution with parameter $I \cdot t$, that is, we have $j \sim Po(I \cdot t)$ over $[0, t]$, then

$$\begin{aligned} E\left(\lambda^{j\frac{1}{\alpha}}\right) &= E\left(\left(\lambda^{\frac{1}{\alpha}}\right)^j\right) = \sum_{j=0}^{\infty} \left(\lambda^{\frac{1}{\alpha}}\right)^j \frac{e^{-It} (It)^j}{j!} = \\ &= e^{It\lambda^{\frac{1}{\alpha}}} e^{-It} \sum_{j=0}^{\infty} \frac{e^{-It\lambda^{\frac{1}{\alpha}}} \left(It\lambda^{\frac{1}{\alpha}}\right)^j}{j!} = \\ &= e^{It\lambda^{\frac{1}{\alpha}}} e^{-It} = e^{-It(1-\lambda^{\frac{1}{\alpha}})} \end{aligned}$$

$$\begin{aligned} E\left(\lambda^{j\frac{1-\alpha}{\alpha}}\right) &= E\left(\left(\lambda^{\frac{1-\alpha}{\alpha}}\right)^j\right) = \sum_{j=0}^{\infty} \left(\lambda^{\frac{1-\alpha}{\alpha}}\right)^j \frac{e^{-It} (It)^j}{j!} = \\ &= e^{It\lambda^{\frac{1-\alpha}{\alpha}}} e^{-It} \sum_{j=0}^{\infty} \frac{e^{-It\lambda^{\frac{1-\alpha}{\alpha}}} \left(It\lambda^{\frac{1-\alpha}{\alpha}}\right)^j}{j!} = \\ &= e^{It\lambda^{\frac{1-\alpha}{\alpha}}} e^{-It} = e^{-It(1-\lambda^{\frac{1-\alpha}{\alpha}})} = \end{aligned}$$

B. Appendix to Chapter 4

Next, consider the random variables $\mathcal{Z} \equiv \lambda^{j\frac{1-\alpha}{\alpha}}$ and $\mathcal{K} \equiv \lambda^{j\frac{1}{\alpha}}$, as well as the sum of the random variables $\mathcal{Z}_i$, i.i.d. of $\mathcal{Z}$, in Q_m , and $\mathcal{K}_i$, i.i.d. of $\mathcal{K}$, in $\check{Q}_m$, $m \in \{L, H\}$. Then, we get

$$E(Q_m) = N e^{-I_m t (1 - \lambda^{\frac{1-\alpha}{\alpha}})} \quad (\text{B.27})$$

$$E(\check{Q}_m) = N e^{-I_m t (1 - \lambda^{\frac{1}{\alpha}})} \quad (\text{B.28})$$

Using $\ln(v+1) \approx v$ for v small enough, (B.27) and (B.28) can be rewritten as follows

$$E(Q_m) = N e^{I_m t (\frac{1-\alpha}{\alpha}) \ln \lambda} = N_m \lambda^{I_m t (\frac{1-\alpha}{\alpha})} \quad (\text{B.29})$$

$$E(\check{Q}_m) = N e^{I_m t (\frac{1}{\alpha}) \ln \lambda} = N_m \lambda^{I_m t (\frac{1}{\alpha})}. \quad (\text{B.30})$$

Thus, $E(\check{Q}_m)/E(Q_m) = \lambda^{I_m t (\frac{1}{\alpha} - \frac{1-\alpha}{\alpha})} = \lambda^{I_m t}$, which goes to ∞ as $t \rightarrow \infty$. However, given (B.29) and (B.30), we also have

$$\{E(Q_m)\}^{(\frac{1}{1-\alpha})} N_m^{-\left(\frac{\alpha}{1-\alpha}\right)} = N_m \lambda^{I_m t (\frac{1}{\alpha})} = E(\check{Q}_m) \quad (\text{B.31})$$

Since, in our model, Q_m is treated as a continuous deterministic variable, we consider the following proxy, $\hat{\check{Q}}_m$, as a deterministic version of (B.31)

$$\hat{\check{Q}}_m = Q_m^{\frac{1}{1-\alpha}} \cdot N_m^{-\left(\frac{\alpha}{1-\alpha}\right)}$$

It can then be shown that $\check{Q}_m/\hat{\check{Q}}_m = \text{constant}$.

B.5. Concentration index

In this appendix, we present in some detail the two-dimensional aggregate concentration index analysed in the text. Define the “market shares” $u_m \equiv \hat{\mathfrak{X}}_m / \hat{\mathfrak{X}}_{tot}$, $o_m \equiv N_m / N_{tot}$, $m \in \{L, H\}$, such that

$$\mathfrak{J} \equiv u_L \cdot o_L + u_H \cdot o_H = (1 - u_H) \cdot (1 - o_H) + u_H \cdot o_H \quad (\text{B.32})$$

According to the generic algebraic properties of $\mathfrak{J}$ as defined in (B.32), the index has as an upper boundary

$$\mathfrak{J} = 1 \Leftarrow (u_m, o_m = 0 \Leftrightarrow u_{m'}, o_{m'} = 1),$$

i.e., *total concentration of firms and production in a single sector*; and as a lower boundary

$$\mathfrak{J} = 0 \Leftarrow (u_m = 1, o_m = 0 \Leftrightarrow u_{m'} = 0, o_{m'} = 1),$$

i.e., *total concentration of firms in one sector and of production in the other*. Also of interest is

$$\mathfrak{J} = 1/2 \Leftarrow (u_m, o_m = 1/2 \Leftrightarrow u_{m'}, o_{m'} = 1/2),$$

i.e., *uniform distribution of firms and production across sectors*.

One also obtains $\mathfrak{J} = 1/2$ if $u_m = 1/2$, $o_m = 0 \Leftrightarrow u_{m'} = 1/2$, $o_{m'} = 1$, corresponding to a uniform distribution of production but total concentration of firms in one sector (or vice-versa). We treat this as a degenerate case in terms of our model, associated with specific levels of scale effects ($\epsilon = \bar{\epsilon}_1$ and $\epsilon = 1$), as shown next.⁴

⁴The generalisation of the concentration index to M technological sectors is straightforward: $\mathfrak{J}_M \equiv \sum_i^M u_i o_i$, defined in the interval $0 \leq \mathfrak{J}_M \leq 1$, with the lower and upper boundaries corresponding to the extreme concentration cases similarly to the case of $M = 2$, and $\mathfrak{J}_M = \frac{1}{M}$ corresponding to the uniform distri-

Proof of Proposition 4.3. ($0 \leq \mathfrak{J} \leq 1/2$ or $1/2 \leq \mathfrak{J} \leq 1$): Take (4.45) and (4.46) and see that

(i) if $0 \leq \epsilon < \bar{\epsilon}_1$ [respectively, $\epsilon > 1$], then $\mathcal{H}/\mathcal{L} > 1 \wedge \mathcal{H}/\mathcal{L} \rightarrow \infty \Rightarrow N, \hat{\mathfrak{X}} \rightarrow \infty$ [$\Rightarrow N, \hat{\mathfrak{X}} \rightarrow 0$] and $\mathcal{H}/\mathcal{L} < 1 \wedge \mathcal{H}/\mathcal{L} \rightarrow 0 \Rightarrow N, \hat{\mathfrak{X}} \rightarrow 0$ [$\Rightarrow N, \hat{\mathfrak{X}} \rightarrow \infty$], which, substituting in (4.48), yields $\mathfrak{J} \rightarrow 1$ in either case;

(ii) if $\bar{\epsilon}_1 < \epsilon < 1$, then $\mathcal{H}/\mathcal{L} > 1 \wedge \mathcal{H}/\mathcal{L} \rightarrow \infty \Rightarrow N \rightarrow \infty, \hat{\mathfrak{X}} \rightarrow 0$ and $\mathcal{H}/\mathcal{L} < 1 \wedge \mathcal{H}/\mathcal{L} \rightarrow 0 \Rightarrow N \rightarrow 0, \hat{\mathfrak{X}} \rightarrow \infty$, which yields $\mathfrak{J} \rightarrow 0$ in either case;

(iii)

(a) if $\mathcal{H}/\mathcal{L} = 1, \forall_{\epsilon > 0} \Rightarrow N, \hat{\mathfrak{X}} = 1$, which now yields $\mathfrak{J} = 1/2$;

(b) if $\epsilon = \bar{\epsilon}_1$ [$\epsilon = 1$], then $\mathcal{H}/\mathcal{L} > 1 \wedge \mathcal{H}/\mathcal{L} \rightarrow \infty \Rightarrow N \rightarrow \infty, \hat{\mathfrak{X}} = 1$ [$\Rightarrow N = 1, \hat{\mathfrak{X}} \rightarrow 0$] and $\mathcal{H}/\mathcal{L} < 1 \wedge \mathcal{H}/\mathcal{L} \rightarrow 0 \Rightarrow N \rightarrow 0, \hat{\mathfrak{X}} = 1$ [$\Rightarrow N = 1, \hat{\mathfrak{X}} \rightarrow \infty$], which yields $\mathfrak{J} \rightarrow 1/2$. $\square$

Figure B.2 depicts the concentration index, defined in terms of the “market shares” u_H and o_H . Panel (b) represents a correspondence between $\mathfrak{J}$ and u_H , where: ($u_H = 1; \mathfrak{J} = 1$), if $o_H = 1$; ($u_H = 1; \mathfrak{J} = 0$), if $o_H = 0$; ($u_H = 0; \mathfrak{J} = 1$), if $o_H = 1$; and ($u_H = 0; \mathfrak{J} = 0$), if $o_H = 0$. A similar characterisation applies to the correspondence between $\mathfrak{J}$ and o_H in panel (c).

B.6. The impact of the dynamic horizontal-entry costs in the industrial structure

In this appendix, we analyse the impact of changes in the dynamic horizontal-entry costs, represented by the elasticities σ and γ , in the asymmetry of the industrial structure, by

bution across sectors.

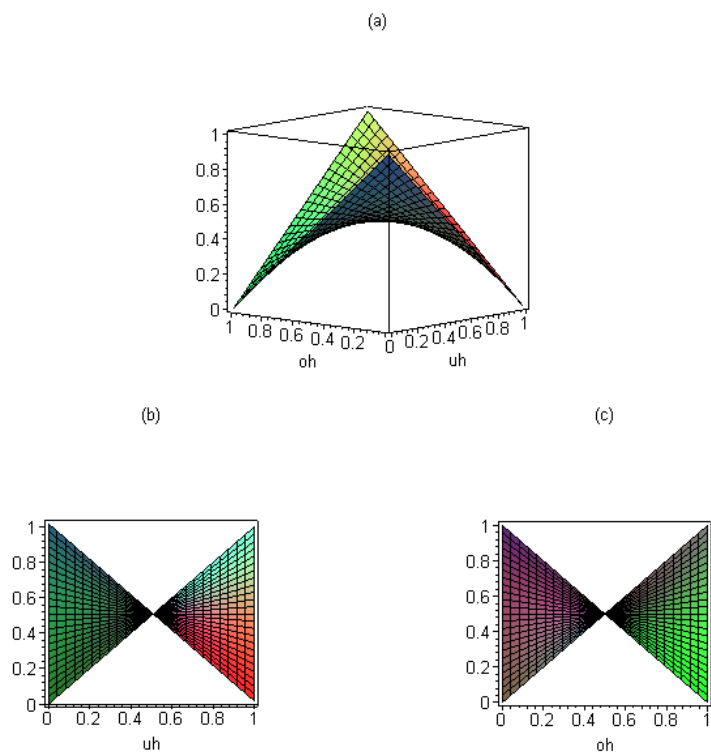


Figure B.2.: Concentration index and “market shares”

keeping $\mathcal{H}/\mathcal{L}$ as given.⁵

Proposition B.1. With $\mathcal{H}/\mathcal{L}$ above unity, an increase in σ and γ implies (vice versa with $\mathcal{H}/\mathcal{L}$ below unity):

- (i) With positive scale effects ($0 \leq \epsilon < 1$) – a decrease in N and increase in $\hat{\mathfrak{X}}/N$;
- (ii) With negative scale effects ($\epsilon > 1$) – an increase in N and a decrease in $\hat{\mathfrak{X}}/N$.⁶

In order to interpret the last proposition in terms of the influence of σ and γ in the asymmetry of the industrial structure, one must recall Proposition 4.2 and consider whether the initial BGP values of N and $\hat{\mathfrak{X}}/N$ are above or below unity. Thus, an increase in the dynamic entry costs leads to a decrease, in relative terms, in the number of firms in the sector that has more of them (in sector H vis-à-vis sector L , if $N > 1$, and vice versa, if $N < 1$), whatever $\epsilon \neq 1$, i.e., dynamic entry costs have always a *stabilising effect* with respect to the number of firms, provided there are some scale effects (whether positive or negative).

However, dynamic entry costs have a *stabilising effect* with respect to average firm size only when scale effects are positive but “small” ($\bar{\epsilon}_2 < \epsilon < 1$): an increase in σ and γ implies an increase, in relative terms, in the average firm size in the sector that has smaller firms. When scale effects are either negative or “large” positive (i.e., $\epsilon > 1$ or $0 \leq \epsilon < \bar{\epsilon}_2$), an increase in σ and γ leads to an increase, in relative terms, in the average firm size in the sector that has larger firms, i.e., dynamic entry costs have a *destabilising effect*. Given the described effect on N , we then find that *dynamic entry costs have a global stabilising effect on industrial structure only when scale effects are positive but “small”. Otherwise, the impact on average firm size countervails the effect on the number of firms*.

⁵Notice that this exercise may be interpreted as either an intra-country or a cross-country comparison.

⁶With $\frac{\mathcal{H}}{\mathcal{L}} = 1$ or $\epsilon = 1$, N is independent of σ and γ . With $\frac{\mathcal{H}}{\mathcal{L}} = 1$, $\epsilon = 1$ or $\epsilon = \bar{\epsilon}_2$, $\frac{\hat{\mathfrak{X}}}{N}$ is independent of σ and γ .

Finally, by joining Proposition B.1 with Propositions 4.4 and 4.5, we learn that a higher g is expected to come hand-in-hand with a higher concentration of type I (respectively, type II), but also with a less (more) *asymmetric* distribution of firm size across sectors.⁷

B.7. Schumpeterian hypothesis

A wide range of empirical interpretations of the Schumpeterian hypothesis has led to a diversity of tests, in particular, involving the relationship between R&D activity and both market power and firm size, each variously measured (e.g., Kamien and Schwatz, 1975). Less frequently recognised are the feedbacks between the latter variables and R&D activity. On the other hand, data availability has allowed more extensive investigation of the relation of R&D activity with firm size than with market power. Therefore, there is a large literature studying whether R&D activity increases more than proportionally with firms' output or sales, i.e., whether large firms are more R&D-intensive than small firms. At least among R&D-reporting firms, the evidence suggests that R&D increases in proportion to sales, pointing to R&D intensity independent of firm size (e.g., Klette and Kortum, 2004).

Having the above in mind, we now explore a version of the Schumpeterian hypothesis by studying the interrelation between R&D intensity, measured as the ratio between R&D outlays and output, and average firm size. As referred before, in our model there is no causality to be imputed to these relationships, since average firm size and R&D activity are simultaneously determined.

Firstly, use (4.16), (4.17), (4.23) and (4.39) to get $R_v \equiv R_{vH}/R_{vL} = (\mathcal{H}/\mathcal{L})^{1-\epsilon}$ and $R_n \equiv R_{nH}/R_{nL} = (\mathcal{H}/\mathcal{L})^{1-\epsilon}$. Then, combine the latter with (4.46) to find the R&D-intensity

⁷To be rigorous, since $\bar{\epsilon}_2 < \bar{\epsilon}_1$ for σ and γ positive and finite, one must also consider the interval $\epsilon \in (\bar{\epsilon}_2, \bar{\epsilon}_1)$, for which a higher concentration of type I relates to a more asymmetric distribution of firm size.

ratio

$$\frac{R}{\widehat{\mathfrak{X}}} = \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{\mathcal{D}_R(\epsilon)} \quad (\text{B.33})$$

where $R \equiv R_n = R_v$ and $\mathcal{D}_R(\epsilon) \equiv - \left(\frac{\alpha}{1-\alpha} \right) \left\{ \frac{-\epsilon[1+2(\sigma+\gamma)]+\sigma+\gamma}{\sigma+\gamma+1} \right\}$. Define the critical value for ϵ , $\bar{\epsilon}_R$, such that $\mathcal{D}_R(\bar{\epsilon}_R) = 0$,

$$\bar{\epsilon}_R = \frac{\sigma + \gamma}{1 + 2(\sigma + \gamma)},$$

and recall $\bar{\epsilon}_2 = \frac{\sigma+\gamma}{\alpha+(1+\alpha)(\sigma+\gamma)}$, such that $\mathcal{D}_2(\bar{\epsilon}_2) = 0$, where

$$\mathcal{D}_2(\epsilon) \equiv \frac{1}{(\sigma + \gamma + 1)(1 - \alpha)} \cdot \{\sigma + \gamma - \epsilon[\alpha + (1 + \alpha)(\sigma + \gamma)]\}$$

is the elasticity of $\mathcal{H}/\mathcal{L}$ in the BGP expression for $\widehat{\mathfrak{X}}/N$ (see (4.47)). Notice that $0 \leq \bar{\epsilon}_R < 1/2$ and that, since $0 < \alpha < 1$, then $0 \leq \bar{\epsilon}_R < \bar{\epsilon}_2 < 1/(1 + \alpha) < 1$, for σ, γ finite.

The relation between R&D intensity and average firm size, for a given $\mathcal{H}/\mathcal{L}$, is characterised as follows

Proposition B.2. A given $\mathcal{H}/\mathcal{L}$ above unity corresponds to (vice versa for $\mathcal{H}/\mathcal{L}$ below unity):

- (i) With “small” positive scale effects ($\bar{\epsilon}_2 < \epsilon < 1$) – $R/\widehat{\mathfrak{X}}$ above and $\widehat{\mathfrak{X}}/N$ below unity;
- (ii) With “large” positive scale effects ($0 \leq \epsilon < \bar{\epsilon}_2$) – $R/\widehat{\mathfrak{X}}$ below and $\widehat{\mathfrak{X}}/N$ above unity,
when $0 \leq \epsilon < \bar{\epsilon}_R$; $R/\widehat{\mathfrak{X}}$ and $\widehat{\mathfrak{X}}/N$ above unity, when $\bar{\epsilon}_R < \epsilon < \bar{\epsilon}_2$;
- (iii) With negative scale effects ($\epsilon > 1$) – $R/\widehat{\mathfrak{X}}$ above and $\widehat{\mathfrak{X}}/N$ below unity.

According to Proposition B.2, the version of the Schumpeterian hypothesis that relates positively R&D intensity and firm size is only predicted by our model when scale effects are positive and “large” in the interval $\bar{\epsilon}_R < \epsilon < \bar{\epsilon}_2$. Otherwise, the sector with smaller average

firm size exhibits higher levels of R&D intensity. One must also consider the following cases: if $\mathcal{H}/\mathcal{L} = 1$, whatever $\epsilon > 0$, R&D intensity and average firm size are both constant across sectors; if $\epsilon = \bar{\epsilon}_R$, whatever $\mathcal{H}/\mathcal{L} > 0$, R&D intensity is unchanged across sectors, irrespective of the behaviour of firm size; and if $\epsilon = \bar{\epsilon}_2$, firm size is constant across sectors, irrespective of the behaviour of R&D intensity. Thus, in the latter two particular cases, the model implies that R&D intensity is independent of firm size, which is one of the stylised facts that have emerged from firm-level empirical studies (Klette and Kortum, 2004).

Parallel results could be appreciated by conducting a “cross-country” analysis. In particular, if $\epsilon > \bar{\epsilon}_2$ (respectively, $0 \leq \epsilon < \bar{\epsilon}_R$), then the higher $\mathcal{H}/\mathcal{L}$, the higher (lower) $R/\hat{\mathfrak{X}}$ and the lower (higher) $\hat{\mathfrak{X}}/N$, thus implying a negative relationship between firm size and R&D intensity.

The data in Figure B.1 shows relative R&D intensity above unity, which is compatible, in theoretical terms, with the observed relative firm size also above unity if we have $\mathcal{H}/\mathcal{L} > 1$ combined with “large” positive scale effects in the interval $\bar{\epsilon}_R < \epsilon < \bar{\epsilon}_2$ and. In contrast, observe that, according to the cross-country evidence in Figure B.3, relative R&D intensity and relative firm size tend to be negatively correlated, a fact that is compatible with our theoretical results whatever the level of scale effects, except for the interval $\bar{\epsilon}_R < \epsilon < \bar{\epsilon}_2$. In any case, the data we present seems to corroborate the Schumpeterian hypothesis as defined at the technological-sector level, in contrast to the conclusions from firm-level empirical evidence, referred to above.

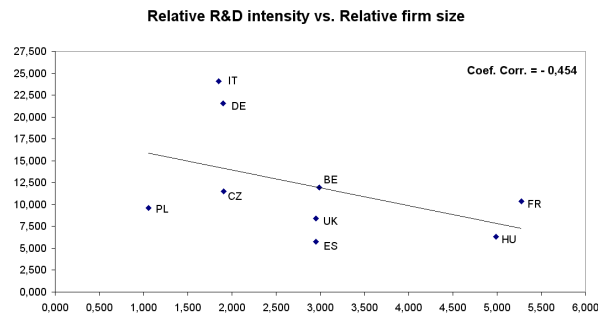


Figure B.3.: Correlation between relative R&D intensity and relative firm size. The data concerns the 1995-2005 average for nine European countries, and is available from the on-line Eurostat database (link at <http://epp.eurostat.ec.europa.eu>).

C. Appendix to Chapter 5

C.1. Model with static decreasing returns to horizontal R&D

We consider first the symmetric equilibrium. Let horizontal R&D technology (5.17), in the text, be replaced by (in this appendix we omit time subscripts for sake of simplicity)

$$\dot{N} = \frac{1}{\eta(N)}\phi(R_n) \tag{C.1}$$

where $\phi' > 0$, $\phi'' < 0$ and function η as defined in the text. Term $\phi(R_n)$ introduces *static* decreasing returns to horizontal R&D in the model. We keep term $1/\eta(N)$ in (C.1), as the *dynamic* decreasing returns to horizontal R&D implied by $\eta \equiv \eta(N)$, $\eta' > 0$, are necessary to eschew the explosive growth that would otherwise occur.¹ More specifically, let $\dot{N}_e(t)$ be the contribution to the instantaneous flow of new goods by R&D firm e in the horizontal-R&D sector and $R_{ne}(t)$ the flow of resources devoted to horizontal R&D by e at t (measured in units of final-good output Y), such that $\dot{N}_e = \frac{1}{\eta(N)}\phi_e(R_{ne})$. Given perfect competition among R&D firms, each of them takes as given the marginal value of entry, V . R&D firms solve $\max_{R_{ne}} \dot{N}_e \cdot V - R_{ne}$, with the associated first-order condition implying,

¹More generally, we need some type of dynamic friction in horizontal entry in order to eschew the explosive growth that would otherwise occur given the feedback between horizontal and vertical innovation in a lab-equipment setup. It can be shown that the specification $\eta \equiv \eta(Q)$, $\eta' > 0$, $\eta'' < 0$, produces a similar result.

under a convenient aggregation procedure, $V = \eta(N)/\phi'(R_n)$. We then get the consistency condition

$$\frac{Q}{N} = \frac{\eta(N)}{\zeta \cdot L \cdot \phi'(R_n)}. \quad (\text{C.2})$$

For concreteness, assume the specification $\phi(R_n) = R_n^{\sigma_1}$ and $\eta(N) = N^{\sigma_2}$, with $0 < \sigma_1 < 1$ and $\sigma_2 > 0$.² By considering the latter and that, along the BGP, $\dot{R}_n/R_n = \dot{Q} - Q = g$, time-differentiation of (C.6) yields the BGP necessary relationship

$$g = \left(\frac{1 + \sigma_2}{\sigma_1} \right) \cdot g_N, \quad (\text{C.3})$$

where $(1 + \sigma_2)/\sigma_1 > 1$. Solving (C.1) in order to R_n , together with (C.2), leads to

$$R_n = g_N \cdot \sigma_1 \cdot \zeta \cdot L \cdot Q, \quad (\text{C.4})$$

$$N = [\zeta \cdot L \cdot \phi'(R_n) \cdot Q]^{\frac{1}{\sigma_2+1}} = \left[(\zeta \cdot L \cdot \sigma_1 \cdot Q)^{\sigma_1} g_N^{1-\sigma_1} \right]^{\frac{1}{\sigma_2+1}}. \quad (\text{C.5})$$

Now, we focus on the asymmetric equilibrium under Assumption 1. Re-iterating the same steps as in Subsection 5.4.1, we find that the consistency condition (C.2) is replaced by

$$\frac{Q}{N} = \frac{\eta(N)}{\zeta \cdot L \cdot \phi'(R_n)} \cdot \Omega^{-1}, \quad (\text{C.6})$$

²Given this specification for $\phi(R_n)$, the aggregation procedure that allows one to derive (C.1), and thus (C.2), can be achieved by postulating $\dot{N}_e = \frac{1}{\eta(N)} R_{ne}^{\sigma_1} M^{\sigma_1-1}$, where M is the total number of firms in the horizontal-R&D sector (e.g., Segerstrom and Zolnierrek, 1999). With all R&D firms choosing the same amount of R&D, this technology allows for convenient aggregation, such that $\dot{N} = \frac{1}{\eta(N)} R_n^{\sigma_1}$, where $\dot{N} = M \cdot \dot{N}_e$ and $R_n = M \cdot R_{ne}$. Observe that, as $M \rightarrow \infty$, the aggregate flow of new goods does not change, but the individual contribution of any firm e becomes negligible.

where $\Omega \equiv [1 - (\theta - A)I / (r + \theta I)] \in (0, 1), \forall r, I > 0, \theta \in (1, \infty)$. Given (C.6), we get

$$R_n = g_N \cdot \sigma_1 \cdot \zeta \cdot L \cdot \Omega \cdot Q, \quad (\text{C.7})$$

$$N = [\zeta \cdot L \cdot \Omega \cdot \phi'(R_n) \cdot Q]^{\frac{1}{\sigma_2+1}} = [(\zeta \cdot L \cdot \Omega \cdot \sigma_1 \cdot Q)^{\sigma_1} g_N^{\sigma_1-1}]^{\frac{1}{\sigma_2+1}}. \quad (\text{C.8})$$

It is straightforward to show that the presence of $\phi(R_n)$ in (C.1) reduces the (negative) impact of “waves of enthusiasm” on R_n vis-à-vis the symmetric equilibrium (in absolute terms), because now Ω multiplies by $\sigma_1 < 1$ in (C.7). The effect of $\phi(R_n)$ on the (negative) impact of “waves of enthusiasm” on N is ambiguous: since R_n feeds back on N through $\phi'(R_n)$ - Ω multiplies by $\phi'(R_n)$ in (C.8) - the exact magnitude depends also on the level of $\phi'(R_n)$, in particular whether it is below or above unity.

C.2. Derivation of the aggregate resource constraint with “waves of enthusiasm”

Consider the households’ balance sheet (5.34), in the text. Hence, we can characterise the change in the value of equity as

$$\dot{a}(t) = \eta(t) \cdot \Omega^{-1} \cdot \dot{N}(t) + \dot{\eta}(t) \cdot \Omega^{-1} \cdot N(t). \quad (\text{C.9})$$

Substitute (5.35) in the left-hand side of (C.9) and $\dot{\eta}/\eta = \dot{Q}/Q - \dot{N}/N = AI \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right)$ - derived from (5.33) and (5.37) - in the right-hand side, to get

$$(r + A \cdot I) \cdot a(t) - A \cdot I \cdot a(t) + w(t) \cdot L - C(t) + \Lambda(t) =$$

$$= A \cdot I \cdot \left(\lambda^{\frac{\alpha}{1-\alpha}} - 1 \right) \cdot \eta(t) \cdot \Omega^{-1} \cdot N(t) + \eta(t) \cdot \Omega^{-1} \cdot \dot{N}(t). \quad (\text{C.10})$$

Then, using (5.28) solved in order to π , together with $Y - X = wL + \pi N$,³ in (C.10), we find

$$Y(t) - X(t) - C(t) + \Lambda(t) = A \cdot I \cdot \lambda^{\frac{\alpha}{1-\alpha}} \cdot a(t) + \eta(t) \cdot \Omega^{-1} \cdot \dot{N}(t). \quad (\text{C.11})$$

Finally, recall that $R_n = \eta \dot{N}$ and $R_v = I \lambda^{\frac{1-\alpha}{\alpha}} a$,⁴ and let

$$\Lambda(t) = (\Omega^{-1} - 1) \eta(t) \dot{N}(t), \quad (\text{C.12})$$

such that (C.11) reads

$$Y(t) = X(t) + C(t) + R_n(t) + A \cdot R_v(t),$$

which is (5.23), in the text. Observe that (C.12) means that the “real pure profit” term, Λ , in (5.35) must capture the net increase in the households’ balance sheet induced by horizontal entry, in turn explained by the fact that the average value from entry exceeds the average cost, $V > \eta$, when $\theta > 1$ (thus implying $\Omega^{-1} > 1$). Notice that Λ equals zero in the symmetric equilibrium, $\theta = 1$ (in that case, $\Omega^{-1} = 1$), which confirms that (5.35) generalises (5.2).

³Having in mind the price markup $p = \frac{1}{\alpha}$, equations (5.4), (5.8), (5.10) and (5.11), and that, in equilibrium, w and p are equated to the marginal product of labour and the marginal product of intermediate goods, respectively, it is easily shown that $wL = (1 - \alpha)Y$, $X = \alpha^2 Y$, $pX = \alpha Y$ and total profits $\Pi = X \cdot (p - 1) = \alpha Y - \alpha^2 Y$. Also, have in mind that, given (5.11) and (5.18), total profits can be represented as $\Pi = \bar{\pi} \cdot N$.

⁴By solving (5.13) with respect to R_v and aggregating across ω , we obtain $R_v = \int_0^N \Phi(\omega)^{-1} I(\omega) d\omega = I \zeta L \lambda^{\frac{\alpha}{1-\alpha}} Q$ (see (5.44)). From (5.33) and (5.34), we then get $R_v = I \zeta L \lambda^{\frac{\alpha}{1-\alpha}} Q = I \lambda^{\frac{\alpha}{1-\alpha}} a$.

C.3. Existence and uniqueness of $A_p > 1$

We give a sketch of the formal proof for the existence and uniqueness of the value of $A_p > 1$, given the set $(\theta, p; g_N^s, I^s)$, that solves for the asymmetric BGP both under Assumption 5.1 ($p = 1 \Rightarrow A_p = A$) and Assumption 5.2 ($p \in (0, 1)$). Recall from (5.41) that $I \equiv I(A_p) = I^s/A_p$, where $I^s = \bar{\pi}/\zeta - \rho - \Theta g$. Thus, we wish to show that the equation

$$A_p = 1 + (\theta - 1) \cdot \frac{g_N \cdot A_p}{g_N \cdot A_p + \theta \cdot I^s} \cdot p \equiv \chi(A_p) \quad (\text{C.13})$$

has a unique and finite solution, i.e., that the locus of A_p (i.e., the 45° line) and $\chi(A_p)$ coincide for a unique and finite value of $A_p > 1$. Firstly, see that $\chi(A_p)$ is a continuous increasing concave function, that is, for A_p finite,

$$\frac{d\chi(A_p)}{dA_p} = (\theta - 1) \frac{g_N \cdot p}{g_N \cdot A_p + \theta \cdot I^s} \left(1 - \frac{g_N \cdot A_p}{g_N \cdot A_p + \theta \cdot I^s} \right) > 0$$

and

$$\frac{d^2\chi(A_p)}{dA_p^2} = 2(\theta - 1) \frac{g_N^2 \cdot p}{(g_N \cdot A_p + \theta \cdot I^s)^2} \left(-1 + \frac{g_N \cdot A_p}{g_N \cdot A_p + \theta \cdot I^s} \right) < 0,$$

since $1 - (g_N A_p) / (g_N A_p + \theta I^s) > 0$. Secondly, note that $\lim_{A_p \rightarrow \infty} d\chi/dA_p = 0$. Thus, $\chi(A_p)$ is monotonically increasing such that $\exists A_p > 1, d\chi/dA_p < 1$. The latter, combined with $\chi(0) = 1$, implies that the curve $\chi(A_p)$ and the 45° line cross only once, at a given $A_p > 1$ finite.

C.4. Transitional dynamics

This appendix addresses transitional dynamics under Assumption 5.1. Similar results would follow under Assumption 5.2. It is easily shown that, with $AI > 0$, the system of ODE's (5.3), (5.37) and (5.51) is equivalent to the system in detrended variables

$$\dot{x} = \left[\frac{1}{\gamma} \cdot \frac{\dot{Q}}{Q} - \left(\frac{\sigma + \gamma + 1}{\gamma} \right) \cdot \frac{\dot{N}}{N} \right] \cdot x = A(x, z) \cdot I(x, z) \cdot \frac{\Xi}{\gamma} \cdot x - \left(\frac{\sigma}{\gamma} + 1 \right) \cdot x^2 \quad (\text{C.14a})$$

$$\dot{z} = \left(\frac{\dot{C}}{C} - \frac{\dot{Q}}{Q} \right) \cdot z = \mu \cdot z - \left(\frac{1}{\theta} + \Xi \right) \cdot A(x, z) \cdot I(x, z) \cdot z - x \cdot z \quad (\text{C.14b})$$

where $\mu \equiv (\tilde{\pi}/\zeta - \rho)/\Theta$ and $\Xi \equiv (\lambda^{\frac{\alpha}{1-\alpha}} - 1)$. Also, from (5.36) (see Appendix C.2), we have

$$A(x, z) \cdot I(x, z) = \frac{1}{\zeta \cdot L \cdot \lambda^{\frac{\alpha}{1-\alpha}}} \cdot (\mathcal{B} - z - \zeta \cdot L \cdot \Omega(x, z) \cdot x), \quad \mathcal{B} \equiv \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} \quad (\text{C.15a})$$

$$A(x, z) = \left[1 + (\theta - 1) \frac{x}{x + \theta \cdot I(x, z)} \right] \quad (\text{C.15b})$$

$$\Omega(x, z) = \left[1 - \frac{(\theta - A(x, z)) \cdot I(x, z)}{\frac{\pi}{\zeta} + (\theta - A(x, z)) \cdot I(x, z)} \right] \quad (\text{C.15c})$$

In (C.15c), we have replaced r with its expression in (5.29). Observe that the system (C.15a)-(C.15c) defines I , A and Ω as functions of x and z implicitly.⁵ Equations (C.14a)-(C.15c), plus the transversality condition and the initial condition $x(0)$, describe the tran-

⁵We continue to assume that N^E/N is constant (see (5.27)), such that (5.27) and (C.15b) hold along the transition path.

sitional dynamics and the BGP, by jointly determining the variables $(x(t), z(t))$. Then, we can determine the original variables $N(t)$ and $C(t)$, for a given $Q(t)$.

By equating $\dot{x} = 0$ and $\dot{z} = 0$, and considering $\mu > 0$, it is straightforward to show that there is just one interior steady-state equilibrium, with balanced growth in the usual sense, such that

$$x^* = g_N^* = \frac{\mu \Xi}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)} > 0 \quad (\text{C.16a})$$

$$z^* = \left(\frac{C}{Q}\right)^* = \mathcal{B} - \zeta \cdot L \cdot \left(\Omega^* + \frac{\sigma + \gamma}{\Xi} \cdot \lambda^{\frac{\alpha}{1-\alpha}}\right) \frac{\mu \Xi}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)} > 0 \quad (\text{C.16b})$$

Notice that, as shown in Section 5.4, $x^* = (x^*)^s$ and $z^* > (z^*)^s$, where s denotes the symmetric BGP. From x^* (and $(I^*)^s$) one computes A^* implicitly, and then I^* , Ω^* and z^* (see (C.15b), (5.41), (C.15c) and (C.16b)).

We qualitatively characterise the local dynamics properties in a neighbourhood of the asymmetric BGP, by studying the solution of the linearised system obtained from (C.14a) and (C.14b)

$$\begin{pmatrix} \dot{x}(t) \\ \dot{z}(t) \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} x(t) - x^* \\ z(t) - z^* \end{pmatrix} \quad (\text{C.17})$$

The elements of the Jacobian matrix, $J(x, z)$, in (C.17), evaluated at that steady state are

$$J_{11} = - \left[\left(\frac{\sigma + \gamma}{\gamma} \right) + \frac{\Xi}{\gamma(\Xi + 1)} \Omega^* \right] x^* - \frac{\Xi}{\gamma(\Xi + 1)} (x^*)^2 \left(\frac{\partial \Omega}{\partial x} \right)^* \quad (\text{C.18a})$$

$$J_{12} = -\frac{\Xi}{\gamma\zeta L(\Xi+1)}x^* - \frac{\Xi}{\gamma(\Xi+1)}(x^*)^2\left(\frac{\partial\Omega}{\partial z}\right)^* \quad (\text{C.18b})$$

$$J_{21} = -\left[-\left(\frac{1}{\Theta} + \Xi\right)\frac{\Omega^*}{(\Xi+1)} + 1\right]z^* + \frac{(\frac{1}{\Theta} + \Xi)}{(\Xi+1)}x^*z^*\left(\frac{\partial\Omega}{\partial x}\right)^* \quad (\text{C.18c})$$

$$J_{22} = \frac{(\frac{1}{\Theta} + \Xi)}{\zeta L(\Xi+1)}z^* + \frac{(\frac{1}{\Theta} + \Xi)}{(\Xi+1)}x^*z^*\left(\frac{\partial\Omega}{\partial z}\right)^* \quad (\text{C.18d})$$

Having in mind the implicit functions of x and z (C.15a)-(C.15c), it can be shown that $(\frac{\partial\Omega}{\partial x})^*, (\frac{\partial\Omega}{\partial z})^* > 0$, such that $\det(J^*) < 0$, where $J^* \equiv J(x^*, z^*)$. Thus, J^* has two distinct real eigenvalues with opposite signs

$$\delta_1 = \frac{1}{2}\left(\text{tr}(J^*) - \Delta(J^*)^{\frac{1}{2}}\right) < 0 \quad (\text{C.19})$$

$$\delta_2 = \frac{1}{2}\left(\text{tr}(J^*) + \Delta(J^*)^{\frac{1}{2}}\right) > 0 \quad (\text{C.20})$$

where $\Delta(J^*) \equiv \text{tr}(J^*)^2 - 4 \cdot \det(J^*)$. Therefore, the dynamics are saddle-path stable, where δ_1 determines the dynamics for the transversality condition to hold (see fn. 17, in the text). Having solved (C.17), the dynamics of I , A , Ω and g are derived simultaneously from (C.15a), (C.15b), (C.15c) and (5.37).

The slope of the eigenspace associated to the negative eigenvalue is $-(J_{22} - \delta_1)/J_{21}$. As $J_{21} < 0$ then the slope has the same sign as $J_{22} - \delta_1$. It can be shown that if $\Theta > \bar{\Theta}$, $\bar{\Theta} \equiv [(\Xi+1)(1-\Omega)/\Omega+1]^{-1}$,⁶ the local stable manifold is positively sloped

⁶Notice that $\bar{\Theta} < 1$, if $\Omega \in (0, 1)$, and $\bar{\Theta} = 1$, if $\Omega = 1$. The latter case is analysed in detail in Gil, Brito, and Afonso (2010).

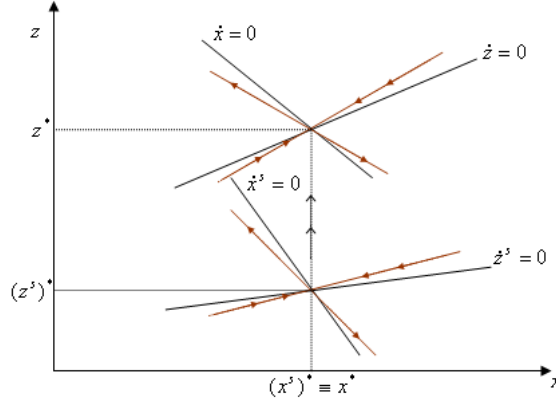


Figure C.1.: Phase diagram in the detrended variables (x, z) for the system (C.14a)-(C.14b), with $\theta = 1$ (lower intersection) and $\theta > 1$ (upper intersection). Curves $\dot{x} = 0$, $\dot{z} = 0$, $\dot{x}^s = 0$, and $\dot{z}^s = 0$ are the linearised isoclines.

in the neighbourhood of (x^*, z^*) . The phase diagram in Figure C.1 allows for a geometric characterisation of the dynamics of transition by considering, in particular, the empirically relevant case of $\Theta > 1$. The figure depicts both the symmetric and the asymmetric scenario with $AI > 0$, taking into account that $\frac{dz}{dx}|_{\dot{x}=0} = \frac{|J_{11}|}{|J_{12}|} < \frac{|J_{11}^s|}{|J_{12}^s|}$ (the unstable manifold is less steep in the asymmetric case) and $\frac{dz}{dx}|_{\dot{z}=0} = \frac{|J_{21}|}{|J_{22}|} > \frac{|J_{21}^s|}{|J_{22}^s|}$ (the stable manifold is steeper in the asymmetric case).

If $x(0) > x^*$, the transitional dynamics is characterised by decreasing pairs (x, z) and (x^s, z^s) towards the BGP. Then, g and g^s also decrease (see (5.37)), while AI and I^s increase along the transition (see (C.15a)). Provided A falls towards the BGP,⁷ it follows that I increases more than AI (see (C.15a)). Observe also that if the economy is initially at $((x^s)^*, (z^s)^*)$ and there is an expectations shock that moves the BGP to (x^*, z^*) , and since $x^* \equiv (x^s)^s$, the manipulation of z allows the economy to jump directly to the new long-run equilibrium

⁷Since we are focusing on local dynamics in a neighbourhood of the BGP, we make use of the fact that $(\frac{\partial A}{\partial x}), (\frac{\partial A}{\partial z}) > 0$ evaluated at the BGP.

without experiencing a transition (this movement is represented by the vertical arrows in Figure C.1).

Moreover, given (C.18a)-(C.18d) and (C.19)-(C.20), it can be shown that $|\delta_1| < |\delta_1^s|$ and $\delta_2 > \delta_2^s$. Thus, the model predicts a slower transition under “waves of enthusiasm”, as illustrated by Figure 5.1, in the text, implying that the economy spends more time in transition vis-à-vis the symmetric case.