

Multi-Resolution Rolling Stochastic Operation of a Hybrid PV-Battery-Pumped-Storage System in Electricity Markets

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Resumo

A integração de produção renovável variável, armazenamento eletroquímico e armazenamento hidroelétrico reversível cria problemas de operação em que decisões de mercado, estados físicos e incerteza futura estão fortemente interligados. É desenvolvida uma formulação de operação estocástica em horizonte rolante para um sistema híbrido composto por produção fotovoltaica, bateria e central hidroelétrica reversível, inspirado no contexto físico e de mercado do Alqueva. O problema é formulado como um programa linear inteiro misto que coordena despacho físico, compromissos comerciais, correções intradiárias, desvios residuais e evolução dos estados de armazenamento.

A metodologia combina cenários conjuntos para preços de eletricidade, disponibilidade fotovoltaica e afluências naturais com uma representação temporal multi-resolução. O horizonte próximo é descrito com resolução de 15 minutos e horária, enquanto o futuro mais distante é representado através de blocos semanais e mensais agregados. Esta estrutura preserva informação económica relevante para a gestão da água e da energia armazenada sem tornar o problema anual diretamente intratável. Uma função de valor terminal, calibrada a partir de experiências de continuação, é usada para aproximar o valor económico da água armazenada para além do horizonte explícito.

O estudo numérico compara controladores determinísticos, estocásticos e com função de valor terminal, bem como referências simplificadas sem a mesma estrutura multi-resolução. A análise avalia receita realizada, trajetórias de armazenamento, comportamento operacional e desempenho computacional em simulação fechada. Na trajetória realizada de 2025, os resultados mostram ganhos relevantes da representação estocástica face ao controlador determinístico, enquanto a função de valor terminal calibrada altera apenas marginalmente o desempenho anual quando o seu efeito é transferido para o controlador operacional através de metas diárias de armazenamento.

Palavras-chave: operação estocástica, horizonte rolante, armazenamento hidroelétrico reversível, bateria, energia fotovoltaica, mercados de eletricidade, programação linear inteira mista.

Abstract

The integration of variable renewable generation, electrochemical storage, and pumped-storage hydropower creates operational problems in which market decisions, physical states, and future uncertainty are strongly coupled. A rolling stochastic operation formulation is developed for a hybrid system composed of photovoltaic generation, a battery energy storage system, and pumped-storage hydropower, inspired by the physical and market context of Alqueva. The problem is formulated as a mixed-integer linear program that coordinates physical dispatch, commercial commitments, intraday corrections, residual deviations, and storage-state evolution.

The methodology combines joint scenarios for electricity prices, photovoltaic availability, and natural inflows with a multi-resolution time representation. The near horizon is described at 15-minute and hourly resolution, while the more distant future is represented through aggregated weekly and monthly blocks. This structure preserves economically relevant information for water and stored-energy management without making the annual rolling problem directly intractable. A calibrated terminal value function, estimated from continuation experiments, approximates the economic value of stored water beyond the explicit horizon.

The numerical study compares deterministic, stochastic, and terminal-value rolling controllers, together with simplified benchmark formulations without the same multi-resolution structure. The analysis evaluates realized revenue, storage trajectories, operational behaviour, and computational performance in closed-loop simulation. On the realized 2025 trajectory, the results show a material gain from stochastic representation relative to the deterministic controller, while the calibrated terminal value function changes annual performance only marginally when its effect is transferred to the operational controller through daily storage targets.

Keywords: stochastic operation, rolling horizon, pumped-storage hydropower, battery energy storage, photovoltaic generation, electricity markets, mixed-integer linear programming.

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*Não sou nada.
Nunca serei nada.
Não posso querer ser nada.
À parte isso, tenho em mim todos os sonhos do mundo.*

Álvaro de Campos

*Sê todo em cada coisa. Põe quanto és
No mínimo que fazes.*

Ricardo Reis

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Acronyms

BESS battery energy storage system.

CVaR conditional value-at-risk.

DA day-ahead.

ERSE Portuguese energy-services regulator.

ID continuous intraday.

IDA intraday auction.

MILP mixed-integer linear program.

OMIE Iberian electricity market operator.

PSH pumped-storage hydropower.

PV photovoltaic.

REN Portuguese transmission system operator.

SDG Sustainable Development Goal.

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Chapter 1

Introduction

1.1 Context, Motivation, and Problem Definition

Electrification of transport and heating, rising cooling demand, and digital infrastructure are increasing electricity consumption (International Energy Agency, 2025). Electricity systems must accommodate this growth while reducing greenhouse-gas emissions, preserving affordability, and maintaining security of supply (United Nations, 2015; United Nations Department of Economic and Social Affairs, 2025). In Europe, the Green Deal and Climate Law make climate neutrality by 2050 a binding policy objective and require a transformation of electricity generation, network operation, market design, and end-use consumption (European Commission, 2024b, 2025). The rapid expansion of variable solar and wind generation is central to that transformation, but it also increases the need for forecasting, balancing, reserve provision, network flexibility, and storage (Ergun et al., 2025; He et al., 2024).

Flexibility is the ability of the power system to adapt production, consumption, storage, and exchanges to changing conditions. It may be provided by dispatchable generation, demand response, interconnection, storage, or market mechanisms that permit positions to be revised closer to delivery. Storage is particularly relevant because it decouples the time of electricity production from the time of use. Pumped-storage hydropower (PSH) provides large-scale, long-duration energy shifting, whereas a battery energy storage system (BESS) provides rapid response for short-term arbitrage, balancing, and ancillary services (Yang et al., 2025; Garttan et al., 2025). Coordinating these technologies with photovoltaic (PV) generation turns the operational problem into one of portfolio management: the controller must allocate limited battery energy, reservoir water, and market opportunities over several time scales (Zisos et al., 2023).

Electricity markets provide the economic setting in which this flexibility is valued. In liberalized European systems, participants form positions in the day-ahead (DA) market, revise them through intraday markets as forecasts improve, and face balancing settlement when physical delivery differs from commercial schedules (European Commission, 2024a; Conejo et al., 2010). For a hybrid storage portfolio, a DA commitment made under uncertain prices or solar production may become suboptimal as new information arrives. Later corrections are constrained by market-gate

closures, inherited commitments, and the physical storage state; residual deviations are settled in balancing. The value of flexibility therefore depends on both physical capability and market timing (Schulze and McKinnon, 2016; Shinde et al., 2021; Silva et al., 2022).

Forecasts are valuable in this setting because they support decisions rather than merely describe the future. Stochastic optimization represents plausible trajectories of price, renewable production, and inflow, while rolling-horizon control updates the decision problem as information becomes available (Fleten et al., 2024; Gulotta et al., 2023; Gneiting et al., 2023; Silvente et al., 2018; Erdinç, 2023). In each rolling cycle, updated scenarios and the current physical and commercial states define a finite-horizon problem; the current physical actions and gate-binding market decisions are implemented, the state advances, and the problem is solved again. The controller must therefore balance immediate actions against the longer-term opportunity value of stored energy and water.

The case study draws on the Alqueva complex in southern Portugal as a physical and operational reference scale for reservoir-based storage and renewable integration (EDIA, S.A., 2025a). It does not reproduce a complete multipurpose-water operating policy; Chapter 3 defines the system boundary, infrastructure data, and excluded water-management obligations.

Existing studies establish the individual elements of this problem, including hybrid storage coordination, sequential market participation, stochastic scheduling, rolling updating, and non-myopic storage valuation. As synthesized in Chapter 2, the remaining gap concerns their conjunction rather than any individual component. The research problem is therefore to develop an energy-market-aware rolling optimization framework that coordinates a hybrid PV–BESS–PSH portfolio under uncertain electricity prices, photovoltaic availability, and natural inflows. The scope is sequential energy-market participation with residual imbalance settlement. Reserve and ancillary-service products remain relevant extensions, but they are outside the main numerical comparison.

1.2 Objectives, Contributions, and Research Questions

The general objective is to develop and evaluate a rolling stochastic market-operation framework for an Alqueva-inspired hybrid PV–BESS–PSH portfolio in sequential electricity markets. The work formulates a rolling mixed-integer linear program (MILP) that coordinates physical dispatch, storage evolution, day-ahead and intraday commitments, and residual imbalance settlement under joint scenarios for prices, photovoltaic availability, and natural inflow. Its planning–control architecture combines multi-resolution daily planning with short-horizon rolling control and, in one variant, a calibrated terminal water-value representation. The controller families are evaluated through closed-loop annual economic, operational, and computational measures.

The contribution lies in integrating these elements within one executable architecture rather than treating any of them as individually novel. The formulation keeps physical storage states and sequential commercial commitments in the same rolling MILP; the planning–control decomposition transfers long-horizon reservoir guidance through daily targets; and the closed-loop annual

environment compares the resulting controllers through realized economic, physical, and computational outcomes.

Three research questions operationalize these objectives:

- RQ1.** Does retaining a stochastic representation of electricity prices, photovoltaic availability, and natural inflow improve realized closed-loop energy-market performance relative to a deterministic representative-trajectory controller?
- RQ2.** What is the effect of adding calibrated terminal water valuation to the daily planning layer of the stochastic planning–control architecture, and does this additional continuation signal materially change reservoir management or annual value over a finite rolling horizon?
- RQ3.** Can the multi-resolution planning and rolling-control architecture, implemented through repeated MILP solves, represent near-term market and storage dynamics while remaining computationally tractable over a full annual closed-loop simulation?

RQ1 evaluates the uncertainty representation, **RQ2** isolates the terminal-valuation mechanism in the daily planning layer, and **RQ3** tests computational viability. The questions are assessed in closed loop through realized annual profit, reservoir-adjusted final value, revenue decomposition, reservoir trajectories, solve times, optimality gaps, and scenario-count scalability rather than through the objective value of a single open-loop schedule.

1.3 Related Projects and Publications

This dissertation was developed within the STOR-HY project at INESC TEC, which provides the research context for hybrid storage, market participation, and operational decision support.

The two submitted manuscripts cited in Chapter 2 also originate from STOR-HY. Kumar et al. (2025) examine the co-optimization of solar, battery, and pumped-storage resources in energy and reserve markets, whereas Taglioni et al. (2025) address short-term pumped-storage scheduling in day-ahead and intraday markets with hybrid renewable integration. They are cited for transparency and to distinguish their scopes from the present dissertation; their submitted status is indicated in the bibliography.

1.4 Dissertation Structure

Chapter 2 reviews hybrid storage operation, sequential electricity markets, stochastic scheduling, scenario construction, rolling control, and terminal storage valuation, and positions the methodological gap addressed in this dissertation. Chapter 3 defines the Alqueva-inspired system boundary, physical assets, market chronology, uncertainty sources, and modelling scope. Chapter 4 develops the forecasting and scenario-generation process, the multi-resolution rolling MILP, the

controller–simulator interface, and the terminal-value treatment. Chapter 5 presents the experimental design, calibration, annual closed-loop comparison, sensitivity analyses, and computational results. Chapter 6 answers the research questions, summarizes the contributions, and discusses limitations and future work.

Chapter 2

Literature Review

The operational problem spans hybrid-system operation, sequential market participation, stochastic optimization, scenario construction, and rolling-horizon control. These streams connect physical coordination, uncertain renewable production and inflows, commercial commitments, and the intertemporal value of storage.

The chapter first reviews hybrid-asset coordination and market-aware scheduling, then examines uncertainty, rolling control, scenario construction, and terminal storage value. The final synthesis compares the closest operational models and identifies the conjunction of components addressed by the methodology.

2.1 Integrated Operation of Hybrid PV–BESS–PSH Systems

Hybrid renewable-storage systems are commonly motivated by complementarity between resources. Photovoltaic generation provides low-marginal-cost energy but is variable and only partly controllable. Battery storage provides fast and granular flexibility but is limited by energy capacity, state of charge, power ratings, and degradation. Pumped-storage hydropower provides large-scale energy shifting and system flexibility, but its operation is constrained by pumping and generating modes, reservoir dynamics, hydraulic limits, and unit operating restrictions. The main modelling issue is not whether each technology has individual value, but how their constraints, time scales, and opportunity costs interact when they are operated as a single plant.

Durvasulu et al. (2024) review the rationale for adding batteries to hydropower plants and emphasize that hybridization changes both flexibility value and operating trade-offs. From a hydro–PV perspective, Yuan et al. (2021) show that the combined output of a renewable-hydro plant depends not only on expected renewable production, but also on the reliability level imposed on the schedule. More recently, Xiao and Zhu (2026) formulate a market-operation problem combining PV, hydropower, and PSH under uncertainty and risk treatment. These studies support an integrated treatment of renewable production and storage, especially when uncertainty affects the ability to meet a committed schedule.

The two submitted manuscripts introduced in Section 1.3 also originate from STOR-HY. They are included because of this common context and their technical proximity, but they do not define the literature gap. Kumar et al. (2025) jointly represent PV, BESS, and PSH resources in energy and reserve markets through deterministic full-period scheduling over historical data. Taglioni et al. (2025) address short-term pumped-storage scheduling with hybrid renewable integration across DA and intraday markets, but remain closer to deterministic rolling MILP scheduling than to joint price, renewable, and inflow scenarios. Neither combines the same physical detail, rolling information updates, stochastic market exposure, and long-horizon storage valuation.

The pumped-storage component requires more physical detail than a generic storage abstraction. Borghetti et al. (2008) show that variable head, unit commitment, and reservoir dynamics can be represented in MILP form, while Filipe et al. (2019) and Wang et al. (2025) demonstrate the importance of operating regions, pumping and generating feasibility, and storage dynamics in market scheduling. This literature supports treating PSH as a constrained hydroelectric asset rather than as a lossless storage block.

The battery introduces state-of-charge, power, efficiency, and cycling-cost constraints. Wang et al. (2016) show that coordinated wind–battery operation depends on battery-state decisions and renewable uncertainty, while Mirzaei Alavijeh et al. (2025) show that degradation assumptions affect profitability and preferred market schedules. Battery flexibility is therefore limited and economically costly rather than free balancing capacity.

Uncertainty also affects hybrid-system design. In a simulation-optimization study of renewable generation with pumped hydro storage, Zisos et al. (2023) show that it can alter storage sizing and economic valuation. It therefore influences capacity value, operating timing, and the trade-off between profit and reliability rather than acting only as a disturbance around a fixed deterministic design.

Taken together, the literature supports a differentiated view of flexibility. Both batteries and pumped storage can shift energy, but they do not provide the same service over the same time scale. The battery is better suited to rapid corrections, short-duration arbitrage, and near-term balancing of forecast errors. Pumped storage is better suited to larger energy shifts, sustained market positioning, and management of stored-water value. Coordinated operation is necessary because these resources can substitute for one another in some periods, complement one another in others, and compete for the same market opportunities while facing different physical and economic constraints.

2.2 Market Participation as Part of the Operating Problem

Electricity-market participation cannot be separated from physical scheduling. In liberalized electricity markets, a plant operator does not only choose a physical dispatch trajectory. The operator also creates commercial positions that are settled through a sequence of market layers. Day-ahead commitments, intraday adjustments, reserve commitments, and imbalance settlement all affect the value of the same physical flexibility. Models that collapse this sequence into a single revenue term

can obscure the chronology of decisions and misrepresent the incentives faced by a storage-rich plant.

The Iberian rules provide a concrete instance of this chronology: a DA position is established before delivery, intraday mechanisms allow it to be revised, and residual deviations are settled through the balancing layer (OMIE, 2024; REN, 2026; Entidade Reguladora dos Serviços Energéticos, 2025). Chapter 3 defines the operators, procedures, and simplified sequence retained for the control problem.

Multistage stochastic bidding models are particularly relevant because they represent market decisions as sequential commitments under gradually revealed uncertainty. Silva et al. (2022) model virtual-power-plant participation across DA, intraday, and balancing markets, showing that the full market sequence can increase profit and reduce imbalances relative to reduced market-participation cases. In the Spanish market, Wozabal and Rameseder (2020) formulate virtual-power-plant bidding across the DA market and six intraday auctions under uncertain prices and renewable production, reporting that stochastic policies outperform reduced sequential benchmarks. Schulze and McKinnon (2016) make a related point for day-ahead and intraday stochastic unit commitment: the value of stochastic programming depends on the information structure and on the timing of recourse decisions, not only on the presence of uncertainty. The central lesson is that intraday and balancing layers are not simply extra revenue streams; they are later decision and settlement stages that respond to residual uncertainty from earlier commitments.

These contributions are strongest on market chronology and recourse structure, but they are not direct substitutes for a hybrid PV–BESS–PSH dispatch formulation. Silva et al. (2022) use a virtual-power-plant abstraction, and Schulze and McKinnon (2016) focus on stochastic unit commitment rather than on the hydraulic and electrochemical coupling of a specific hybrid storage plant. They show why the sequence of markets matters, but they do not resolve the physical coordination problem between battery state of charge, pumped-storage reservoirs, photovoltaic availability, and natural inflows.

Storage-specific studies reinforce the same point. Löhdorf and Wozabal (2023) show that multimarket coordination can be valuable when volatility and liquidity make later markets operationally important. Finnah et al. (2022) integrate day-ahead and intraday self-scheduling for storage through approximate dynamic programming. Both support modelling market participation and storage evolution together rather than assigning market revenues to a separately optimized physical schedule.

Hydropower bidding studies add the plant-side constraints that link market participation with operational feasibility. Aasgård (2017) connects hydropower bidding with start-up representation, unit-commitment costs, and operating restrictions. More broadly, Aasgård et al. (2019) review mathematical and statistical methods for hydropower producers participating in multiple sequential short-term markets and emphasize that storing electricity as water creates a specific bidding problem. Fleten et al. (2024) emphasize that stochastic day-ahead hydropower bidding must produce feasible, interpretable market actions and must be benchmarked against operational practice. For storage-rich systems, the value of a commercial position depends on both the market price and

the future feasibility of the plant state left by that position.

Reserve participation further illustrates the need for market-aware scheduling. Heredia et al. (2018) combine day-ahead, intraday, reserve, and imbalance participation for a wind-BESS virtual power plant, while Filipe et al. (2019) study variable-speed pumped storage in energy and frequency-restoration reserve markets. Related stochastic formulations combine renewable-storage or virtual-power-plant scheduling with reserve redispatch and ancillary services (Fernández-Muñoz and Pérez-Díaz, 2022; Fusco et al., 2023). Vannoni and Sorce (2024) further show that acceptance probabilities and risk preferences affect ancillary-service value. Flexibility value therefore depends on both the settlement layer and the product's operating requirements.

Reserve-oriented studies often concentrate on the additional product definition rather than on the full interaction between sequential energy markets, imbalance settlement, and hybrid storage dynamics. This distinction matters because reserve revenue can make flexibility appear more valuable, but the feasibility and opportunity cost of that flexibility still depend on the same stored energy, stored water, and forecasted renewable production used for energy-market operation.

Balancing-market literature clarifies the residual layer after forward positions are established. Shinde et al. (2021) study stochastic dispatch under alternative imbalance-pricing structures and show that imbalance settlement has its own pricing logic. The relevant distinction is therefore between commercial positions fixed before delivery and realized physical operation, whose deviations may alter the relative performance of storage policies.

2.3 Stochastic Scheduling and Uncertainty Representation

Uncertainty enters hybrid market operation through several channels. Photovoltaic availability depends on weather conditions. Natural inflows affect water availability and future pumping and generation opportunities. Electricity prices reflect market fundamentals, renewable output, demand, congestion, and system conditions. Balancing and imbalance prices can also differ from forward prices. A deterministic schedule based on a single trajectory is a transparent benchmark, but it cannot represent the range of outcomes that may occur after commitments have been made.

Conejo et al. (2010) present stochastic programming as a central framework for electricity-market decision-making under uncertainty. In a stochastic scheduling model, the controller evaluates decisions across several possible future trajectories rather than against a single expected path. This is especially important for storage assets. A decision to discharge, charge, pump, or generate changes the future state of the plant, and the value of that future state depends on uncertain prices and uncertain renewable production. The value of stochastic scheduling is not merely better *ex post* accounting of forecast errors; it can change the decisions selected before those errors are realized.

Market-sequencing studies apply this logic directly. Silva et al. (2022) model sequential information revelation across DA, intraday, and balancing decisions, while Gulotta et al. (2023) combine day-ahead bidding with rolling short-term recourse. Their results show that stochastic-programming stages should match the decision sequence of the market and plant.

However, stochastic market models differ substantially in the uncertainty objects they use and in the plant physics they retain. Some formulations emphasize market stages but use simplified asset representations. Others describe renewable uncertainty in detail but do not carry that uncertainty through a market-settled storage dispatch model. For hybrid storage operation, the stochastic representation must be conditional on the information available at each decision time and must preserve the joint evolution of the exogenous variables that affect both revenue and feasibility.

Chance-constrained formulations instead impose a probability level for satisfying delivery, security, or feasibility requirements. Yuan et al. (2021) represent reliability in hydro–PV scheduling, while Wen et al. (2021) connect photovoltaic uncertainty with security-constrained day-ahead planning and reserve requirements. This approach balances expected profit against the risk of failing to meet a committed schedule.

Robust and hybrid robust–stochastic formulations provide stronger protection against adverse realizations. Baharvandi et al. (2020), Nasiri et al. (2021), Spiller et al. (2025), and Sun et al. (2025) apply related ideas to distribution networks, multi-energy systems, BESS operation, and wind-storage bidding. Scenario-based objectives may instead penalize tail exposure through conditional value-at-risk (CVaR) (Herding et al., 2024; García and Mínguez, 2014). These approaches show that uncertainty treatment should reflect risk attitude, data availability, and the consequences of constraint violations.

Point forecasts summarize uncertainty in one value or path, whereas probabilistic forecasts retain dispersion through densities, quantiles, ensembles, trajectories, or scenarios. Gneiting et al. (2023) emphasize calibration, scoring, benchmarking, and spatio-temporal structure in solar forecasting; Lago et al. (2021) show that electricity-price evaluation depends on the benchmark, market, test period, and metric; and Narajewski and Ziel (2020) model intraday price trajectories including no-trade effects. For storage operation, joint temporal paths are particularly important because decisions depend on how prices, renewable availability, and inflows evolve together.

Forecast-verification literature provides important criteria, but it does not define the optimization model by itself. A probabilistic forecast can be statistically well calibrated while still being inconvenient for a mixed-integer dispatch problem if it is not expressed as operationally usable trajectories, scenarios, or conditional inputs. This creates a modelling bridge between probabilistic forecasting and stochastic optimization: the uncertainty representation must be judged both statistically and operationally.

The literature suggests a hierarchy of uncertainty treatments. A deterministic formulation supports transparency and benchmarking. A scenario-based stochastic formulation is appropriate when credible samples, forecast-error models, or scenario-generation procedures are available. A robust or hybrid robust–stochastic formulation is appropriate when adverse outcomes must be controlled more explicitly. The main point is not simply to add uncertainty, but to embed uncertainty in the same decision structure that governs storage dynamics, market chronology, and recourse opportunities.

2.4 Rolling-Horizon and Multi-Timescale Control

Hybrid storage operation is naturally multi-timescale. Day-ahead positions are created before delivery. Intraday positions are updated closer to real time. Physical dispatch is constrained by hourly or sub-hourly plant dynamics. Storage value extends beyond the immediate optimization window. A single static optimization with perfect information cannot represent this structure. Rolling-horizon control is both a computational strategy and a modelling representation of sequential decision-making.

In a rolling-horizon controller, a finite look-ahead problem is solved repeatedly, only the earliest decisions are implemented, and the physical and commercial states are updated with new information. Silvente et al. (2018) and Gulotta et al. (2023) apply this principle in energy-management and post-day-ahead stochastic control. In storage markets, Finnah et al. (2022) and Schindler et al. (2024) similarly coordinate near-term trading with future storage value.

Rolling-horizon control is also suited to this setting because different parts of the horizon require different temporal detail. The near term may require detailed battery operation, pumping and generating decisions, and market delivery constraints. Later periods may only need coarser guidance through forecasts, scenarios, aggregated blocks, or terminal value approximations. This creates a trade-off between operational detail and computational tractability. Repeated optimization over a moving horizon allows detailed near-term scheduling while retaining enough future information to avoid myopic use of storage.

The value of stochastic information depends on how decisions are revised over time (Schulze and McKinnon, 2016). Erdinç (2023) shows the benefit of rolling re-optimization when renewable production differs from forecasts, while Lin et al. (2020) carry future storage value into wind-pumped-storage dispatch through approximate dynamic programming. The common requirement is to retain future value through either explicit look-ahead or an approximation.

The open issue is how much of the future each rolling problem should represent. Detailed long horizons are expensive when combined with integer decisions and scenarios, whereas short horizons can become myopic. Approximate dynamic-programming approaches such as Lin et al. (2020) use value functions, while rolling MILP approaches often retain explicit finite look-ahead. A hybrid controller requires a tractable compromise between near-term feasibility and longer-term opportunity cost.

Future storage value can be transmitted in two related ways. A separate planning or dynamic-programming model may provide marginal water or continuation values to a shorter-term controller; Schindler et al. (2024) illustrate this through a hydropower planner-trader decomposition. This keeps the operational problem compact, but its quality depends on how well the external value signal matches the market sequence, constraints, and forecast updates. Alternatively, one formulation may retain an explicit look-ahead region while representing distant periods more coarsely. The methodology combines these ideas through multi-resolution planning, reservoir targets, and terminal treatment.

The literature supports a rolling controller for hybrid storage operation. Such a controller is

not a single plan for the entire future, but a repeated decision process in which information is updated, previously fixed commercial commitments are respected, and only the near-term part of the optimized trajectory is implemented. This logic is also the natural bridge between forecast construction and market operation.

2.5 Scenario Generation, Reduction, and Forecast Updating

The quality of scenario-based optimization depends on the uncertainty representation that feeds it. Scenarios must be statistically credible, operationally relevant, and computationally manageable. Poorly designed scenarios can add stochastic detail without improving decision quality, producing schedules that are too conservative, too optimistic, or inconsistent with actual forecast-error behaviour.

Latorre et al. (2007) show that hydro-inflow scenario-tree construction affects both the uncertainty representation and the resulting optimization problem. Renewable studies likewise construct photovoltaic and wind trajectories that preserve spatial and temporal dependence (Golestaneh et al., 2016; Tang et al., 2018; Zhou et al., 2024). These results support joint trajectories rather than independent marginal samples.

Scenario reduction provides the complementary tractability mechanism. Heitsch and Roemisch (2003) approximate a large discrete distribution with a smaller representative set, which is essential when stochastic problems are solved repeatedly. Excessive reduction may nevertheless remove the uncertainty structure that drives storage and market decisions. García and Mínguez (2014) further show that reduction quality can depend on the downstream criterion, particularly for tail-sensitive objectives.

Scenario-reduction papers solve an important distributional approximation problem, but they do not remove the need to choose scenarios that are meaningful for the specific operating model. A reduced set for a hybrid plant should preserve the joint price, PV, and inflow patterns that drive charging, pumping, generation, imbalance exposure, and terminal storage value. The number of retained scenarios is not only a statistical parameter. It is also constrained by solver time, machine capacity, and the temporal granularity of the resulting MILP.

A second distinction is required between unconditional scenario generation and conditional forecast updating. Unconditional scenarios describe possible future trajectories from a general historical or probabilistic model. Conditional scenarios reflect the information available at the decision time, such as recent prices, updated photovoltaic forecasts, current reservoir levels, current battery state of charge, and updated system conditions.

Stochastic market studies represent sequential information revelation and update operation after day-ahead bidding as short-term information changes (Silva et al., 2022; Gulotta et al., 2023). Because scenario quality affects the measured value of stochastic scheduling (Schulze and McKinnon, 2016), scenario construction is part of controller design rather than detached preprocessing.

For hybrid PV–BESS–PSH operation, disciplined scenario design means focusing on uncertainty sources that materially affect decisions: photovoltaic availability, market prices, natural

inflows, and the residual settlement consequences of deviations. The scenario set should preserve dependence across these quantities when that dependence is relevant. For example, electricity prices are influenced by weather and renewable output, while photovoltaic availability and inflows are themselves weather-related. Treating these series as independent may remove precisely the relationships that determine storage value. A joint scenario set is more appropriate than separate marginal scenarios that are combined without regard to dependence.

2.6 Terminal Storage Value and Operating-Cost Realism

Finite-horizon scheduling creates an end-of-horizon problem. If a model maximizes only revenue inside the visible horizon, it may empty the battery or reservoir near the end of the optimization window even when stored energy would be valuable shortly afterwards. This behaviour is not an insight; it is an artefact of the truncated horizon. Storage scheduling requires terminal state constraints, terminal value functions, sufficiently long horizons, or some combination of these mechanisms.

In hydropower, this issue is expressed through water value. Löhndorf et al. (2013) connect short-term hydro-storage trading with longer-term storage value through approximate dynamic programming. Stored water depends on future opportunities and reservoir constraints as well as the current price, so rolling pumped-storage operation requires terminal treatment that avoids systematic boundary depletion.

Hydropower scheduling is often decomposed across time scales: a long- or medium-term planning model estimates the marginal value of stored water under future inflow, demand, and price conditions, and a short-term operational model then uses this marginal value as a continuation signal when deciding whether to turbine, pump, or preserve water. The strengths and limitations of this decomposition were discussed in Section 2.4. For terminal treatment, the relevant connection is to rolling-horizon truncation: terminal constraints, salvage values, approximate value functions, and coarse look-ahead representations are different ways of preventing the finite-horizon controller from treating the terminal stock as worthless.

Battery state of charge has an analogous future opportunity value, although the physical interpretation differs. It may support later price opportunities, forecast-error correction, reserve provision, or imbalance mitigation, while cycling creates degradation costs. Its terminal treatment must therefore recognize both future flexibility and the cost of using that flexibility.

Pumped-storage operation also has wear-related costs. Savin et al. (2020) review hydropower start-stop costs, and Eggen and Belsnes (2023) model operation-related maintenance. Aggressive cycling may therefore reduce net value once maintenance and residual-life effects are considered.

The modelling implication is not that a short-term scheduling study must solve the full long-term asset-health problem. Rather, the boundary of the model must be explicit. If detailed battery degradation and hydropower maintenance costs are excluded, the results are operational-market

outcomes before full asset-health accounting. If simplified cycling costs or terminal values are included, they are tractable approximations of longer-term effects rather than complete degradation or maintenance models.

2.7 Synthesis and Research Gap

The reviewed literature supports the individual ingredients of the problem: hybrid storage coordination, sequential market participation, uncertainty representation, rolling re-optimization, and non-myopic storage valuation. The review is problem-oriented rather than systematic. Table 2.1 is therefore limited to operational optimization studies that overlap with at least two dimensions central to this dissertation: hybrid-asset coordination, sequential short-term market participation, explicit uncertainty, repeated decision updating, and intertemporal storage valuation. Review articles, market-rule documents, forecasting and scenario-generation methods, physical linearization studies, and operating-cost studies are excluded from the table because they support individual modelling choices rather than provide comparable end-to-end operating models; they remain discussed in the thematic sections above.

The table describes how each component is represented instead of assigning binary scores. This avoids treating related but materially different formulations as equivalent. The two project manuscripts are retained because their asset and market settings are particularly close to the dissertation; their unpublished status and project relationship were identified in Sections 1.3 and 2.1.

Table 2.1: Methodological comparison of the operational models most closely related to the dissertation.

Reference	Asset representation	Market sequence	Uncertainty representation	Decision updating	Intertemporal storage treatment
Kumar et al. (2025)	PV, BESS, and variable-head PSH	DA energy and reserve	Deterministic historical trajectories	Full-period schedule	Reservoir and battery balances, without rolling continuation treatment
Taglioni et al. (2025)	PV, BESS, and PSH	DA and intraday	Deterministic forecasts	24-hour rolling horizon	Short finite horizon, without a separate long-horizon value mechanism
Xiao and Zhu (2026)	PV, hydropower, and PSH	Electricity-market sales after local load	Scenario-based stochastic optimization with CVaR for PV, load, and price uncertainty	Case-study scheduling problem	Reservoir dynamics, without a separate planning-control or continuation-value layer
Silva et al. (2022)	Wind, PV, and generic storage in a virtual power plant	DA, intraday, and balancing	Multistage stochastic market and renewable uncertainty	Sequential market stages with information revelation	Storage state within the market horizon, without hydropower water value
Wozabal and Rameseder (2020)	Wind virtual power plant	DA and six intraday auctions	Multistage stochastic prices and wind production	One-day Markov decision process	No physical storage or reservoir continuation value

Continued on next page

Table 2.1: Methodological comparison of closely related operational models (continued).

Reference	Asset representation	Market sequence	Uncertainty representation	Decision updating	Intertemporal storage treatment
Gulotta et al. (2023)	PV, battery, electric-vehicle, and thermal storage	DA bidding followed by real-time and ancillary-service operation	Two-stage and rolling stochastic formulations	DA stage plus rolling real-time management	Short-term storage states, without hydropower or long-horizon water value
Schindler et al. (2024)	Interconnected hydropower reservoirs	Spot and reserve markets	Bilayer stochastic formulation	Annual planner and daily trader	End-of-day reservoir targets transmit long-horizon guidance
Lin et al. (2020)	Wind farms and PSH	DA economic dispatch	Stochastic sampling within approximate dynamic programming	Sequential forward solution of the dispatch problem	Trained piecewise-linear approximate value functions
Löhndorf et al. (2013)	Hydro storage	Interday and intraday trading	Stochastic Markov decision process	Multistage trading decisions	Long-term storage opportunity represented through approximate dual dynamic programming
This dissertation	PV, BESS, and variable-head PSH	DA, intraday auctions, continuous intraday correction, and imbalance settlement	Joint price, PV, and inflow scenarios conditional on each rolling issue	15-minute rolling control with a daily planning layer	Multi-resolution look-ahead, reservoir targets, and terminal water value

The table does not suggest that any individual component is absent from the literature. Hybrid physical models, sequential stochastic bidding, rolling control, and long-horizon water valuation are all established ideas. The remaining gap is their conjunction. The closest hybrid-asset studies retain deterministic or single-schedule treatments; the closest stochastic rolling-market studies use generic storage or omit pumped-storage water value; and the strongest long-horizon hydropower formulations do not represent the same PV–BESS–PSH portfolio and Iberian energy-market sequence. The dissertation is positioned at this intersection rather than claiming novelty for the individual components.

The methodology developed in the following chapters combines these dimensions in one rolling framework. It coordinates the three asset types, distinguishes commercial commitments from physical delivery, represents joint uncertainty conditionally on each decision time, and carries longer-term reservoir information through a multi-resolution planning layer and terminal treatment. The contribution lies in this integrated formulation and its closed-loop evaluation. Closed-loop evaluation repeatedly observes the current state, solves the decision problem, implements only the executable actions, and updates the state using realized data. Performance is measured from this executed sequence rather than from a single optimization objective; integration refers to the components represented within the decision framework.

Chapter 3

Problem Description

The previous chapter reviewed the literature that motivates coordinated operation of hybrid renewable-storage systems under uncertainty. This chapter defines the operational problem considered before the mathematical formulation is introduced. It describes the reference system, the physical assets, the market chronology, the uncertain exogenous quantities, the temporal structure, and the modelling assumptions that delimit the control problem.

3.1 Case Study and System Boundary

The reference context is the Alqueva region in southern Portugal. Alqueva is a multipurpose water infrastructure associated with irrigation, hydropower production, water supply, regional development, and renewable-energy integration. EDIA identifies Alqueva as Europe's largest strategic water reserve and presents the project as an infrastructure whose role extends beyond electricity production alone (EDIA, S.A., 2025a,b,c). The scale of the reservoir is central to this role: EDIA reports a total storage capacity of approximately $4,150 \text{ hm}^3$, a usable capacity of approximately $3,150 \text{ hm}^3$, and a water surface of about 250 km^2 at full storage level (EDIA, S.A., 2025b,c). These magnitudes explain why Alqueva is not only a local hydropower asset, but also a long-duration water and energy-storage infrastructure. The context combines water management, dispatchable hydro capability, renewable production, and the possibility of storage-based market participation.

The system considered is a stylized hybrid energy portfolio inspired by that setting. It combines three physical components: a PV plant, a BESS, and PSH. The model isolates the features required for short-term market-aware operation: variable solar production, limited battery energy, reservoir-based pumped-storage operation, natural inflows, and commercial commitments created in sequential electricity markets. The reservoir is represented through hydropower and storage constraints. Irrigation demand, urban water supply, environmental flow obligations, strategic drought reserves, and institutional water-allocation rules remain outside the operational boundary. The resulting controller represents electricity-market operation for an Alqueva-inspired storage asset within that boundary.

Figure 3.1 summarizes both the system boundary and the decision framework developed in this dissertation. Exogenous photovoltaic availability, natural inflow, and market data, together with market prices and cleared positions, provide the information used to construct forecasts and joint scenarios. These scenarios feed the daily planning layer and the rolling operational controller. The planning layer provides reservoir targets to the operational controller and may, depending on the controller variant, include a terminal-value signal. The rolling controller returns market-position decisions and physical-dispatch instructions.

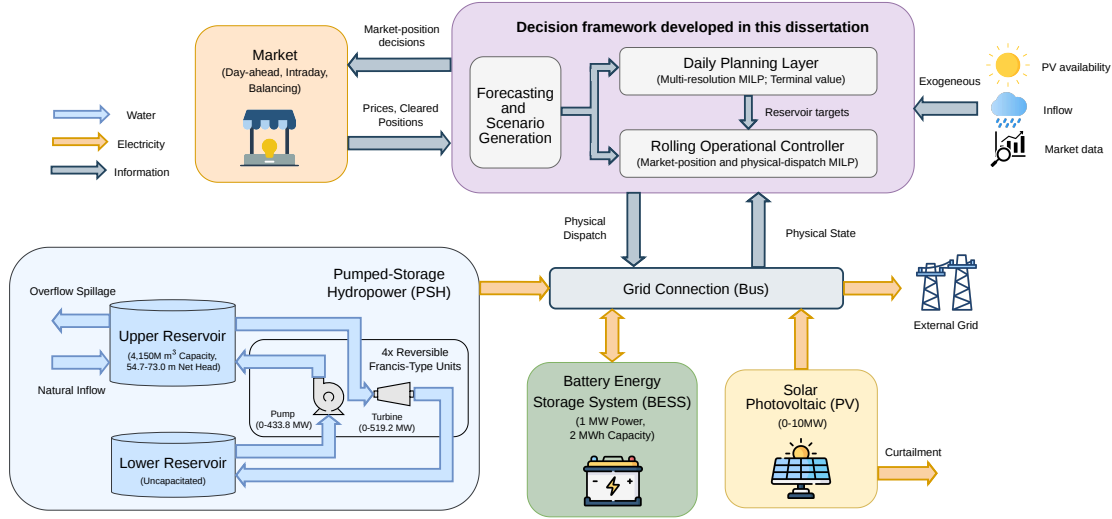


Figure 3.1: Physical system, decision framework, and information flows of the hybrid PV-BESS-PSH portfolio.

The control problem is formulated from the perspective of an operator that schedules the hybrid portfolio over a rolling horizon. At each decision time, the physical state includes the battery state of charge, reservoir storage, and relevant operating status, while the commercial state contains the market positions that have already cleared and are therefore fixed. Together with the history of observed exogenous quantities, these states define the information available to the decision framework. Future prices, photovoltaic availability, and natural inflows remain uncertain. The forecasting layer converts the available information into conditional trajectories, while the planning and operational layers use those trajectories to coordinate market commitments and asset operation. In the experiments, historical residual continuations are sampled using calendar and current-state information, anchored to the current observation, and assembled into joint scenarios. Fast-forward reduction selects the retained paths and redistributes their probability weights. Section 4.3 details this procedure.

3.2 Physical Components of the Hybrid System

The PV component represents variable renewable generation. Its output is treated as an exogenous availability profile: the controller can use available photovoltaic production directly, store

part of it when feasible, or allow curtailment when production exceeds profitable or feasible use. Photovoltaic generation has low marginal production cost, but it is neither fully dispatchable nor perfectly predictable. Its value depends on the ability of the storage assets and market decisions to absorb, shift, or hedge its variability.

The BESS provides short-term electrical storage. Its state is the stored electrical energy, commonly represented by state of charge. The relevant constraints are maximum charging and discharging power, energy-capacity limits, charging and discharging efficiencies, and restrictions that prevent simultaneous charging and discharging. The battery can respond rapidly and is well suited to near-term correction of forecast errors or price-driven repositioning. Its energy capacity is limited compared with the pumped-storage reservoir, and frequent cycling may have economic consequences even when detailed degradation is not explicitly modelled.

PSH provides larger-scale storage through the movement of water between reservoirs. In generating mode, stored water is released through turbines to produce electricity. In pumping mode, electricity is consumed to move water back to the upper reservoir. The relevant state is the stored water volume or equivalent energy content, while the relevant controls are pumping and generating flows or powers. The plant is constrained by reservoir bounds, pumping and generating capacities, efficiencies, hydraulic relationships, and operating-mode restrictions. The value of stored water is intertemporal: water used in the current period cannot be used in a later period unless it is replenished by pumping or natural inflow.

The three assets are coupled through the electrical balance and through market commitments. Photovoltaic production can supply market delivery, charge the battery, or reduce the need for pumped generation. The battery can smooth short-term deviations and support intraday corrections. The pumped-storage plant can shift larger energy volumes and preserve value across longer horizons. Coordinated scheduling is necessary because the same market opportunity may be served by different assets, but the future consequences of using each asset are different.

3.3 Market Environment and Decision Sequence

The market setting is based on the sequential structure of Iberian electricity-market participation. Iberian electricity market operator (OMIE) is the nominated electricity market operator for the Iberian Peninsula and manages day-ahead and intraday electricity markets for Spain and Portugal (Grupo OMI, 2026). The market rules define the framework for day-ahead and intraday trading, while Portuguese transmission system operator (REN) publishes Portuguese market information for system services and balancing (OMIE, 2024; REN, 2026). The procedures governing the global management of the Portuguese electricity system are specified in the *Manual de Procedimentos da Gestão Global do Sistema do setor elétrico*, approved by Portuguese energy-services regulator (ERSE) through Diretiva n.º 7/2025 (Entidade Reguladora dos Serviços Energéticos, 2025). The simplified chronology distinguishes the intraday auction (IDA) stages from continuous intraday (ID) corrections. Commercial positions are created before physical delivery, can be partially revised closer to real time, and are finally compared with realized delivery.

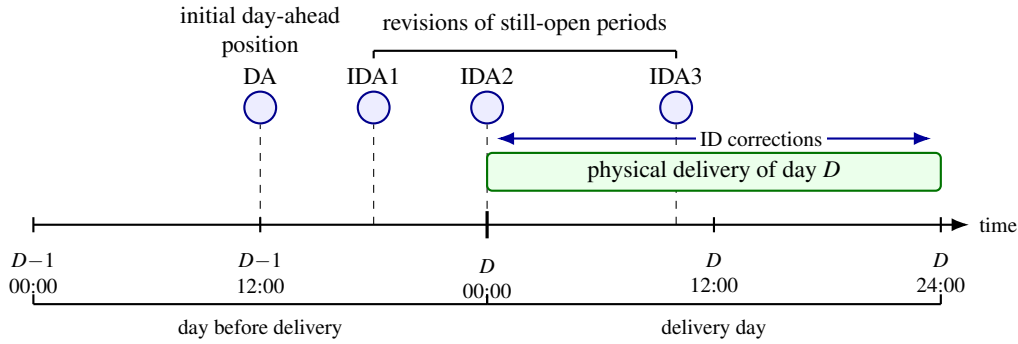


Figure 3.2: Reduced timing of day-ahead commitment, intraday revisions, and physical delivery. Gate locations are schematic and not drawn to scale.

Figure 3.2 summarizes the initial day-ahead commitment, subsequent intraday revisions, and physical delivery. Intraday refers to proximity to delivery rather than requiring every transaction to occur on delivery day D : revisions can begin on $D - 1$. The model distinguishes the three intraday auctions, IDA1–IDA3, from continuous intraday correction, ID. IDA1 and IDA2 cover the full next-day schedule, whereas IDA3 covers 12:00–24:00 of the delivery day. Delivery does not freeze every commercial position simultaneously. The overlap shown between delivery and the later correction opportunities is deliberate: once day D has begun, products whose relevant gates have passed are fixed, whereas still-open delivery periods may continue to be revised. The exact gate sequence used in the numerical experiments is part of the experimental setup, with detailed Iberian market-product timing abstracted away.

The first market layer is the DA market. A day-ahead position establishes a scheduled energy commitment before the delivery day. This position is valuable because it provides access to the main energy market, but it is also risky because it is chosen before all uncertainty has been resolved. At the time of the day-ahead decision, future photovoltaic availability, inflows, and later prices are not known exactly.

The second layer consists of intraday correction opportunities. In the real market environment, intraday auctions and continuous intraday trading allow market participants to revise their commercial positions as delivery approaches and forecasts improve, including while earlier periods of the delivery day are already being executed. In the simplified representation retained for the controller, the three intraday auctions provide sequential corrections to still-open delivery periods after the day-ahead schedule has been established. Continuous intraday trading is represented only as a bounded correction for the interval currently being executed, not as an order-book process or as future recourse over the remaining horizon. This preserves its operational role without attempting to reproduce every bid format, liquidity condition, or settlement detail.

Intraday correction may respond to updated forecasts, changes in storage state, or economic opportunities; it does not require a constraint violation. For example, lower photovoltaic availability than anticipated in a contracted sale may be accommodated by storage dispatch and, where trading remains possible, a reduction of the commercial position. Any remaining difference be-

tween delivery and the revised position is settled as a residual imbalance after execution, rather than corrected through a separate balancing decision stage.

The final layer is residual deviation settlement. During delivery, the physical operation of the hybrid system may differ from the commercial position previously established. Deviations can arise from forecast errors, physical constraints, storage limits, or strategic decisions. The model distinguishes between commercial schedules and realized physical operation. This distinction is essential: a storage asset may be physically able to produce or consume energy, but the economic consequence depends on the commercial position to which that physical delivery is compared.

Real market participation is based on bids and offers. A participant may submit price-quantity pairs, block orders, or other product-specific bid formats depending on the market layer. A more detailed model could choose how much energy to offer at different prices and represent bid acceptance at market clearing as an uncertain event determined by the realized clearing price. The reduced model does not take this strategic-bidding view. It treats prices as exogenous and optimizes cleared quantities or position corrections against those prices. Acceptance is therefore implicit: a quantity selected at an active gate is treated as cleared at the corresponding exogenous price and subsequently enters the inherited commercial position. This is equivalent to a price-taking assumption: the hybrid portfolio is not assumed to influence the market-clearing price, and it is not treated as a strategic market maker. Appendix D.2 outlines how this scope could be extended to explicit price-quantity offer blocks while preserving the price-taking assumption.

Reserve and ancillary-service mechanisms are also part of the real operating environment, especially for storage assets with fast response or pumping and generating flexibility. Including them explicitly would require additional product definitions, such as reserve-capacity reservation, availability constraints, activation uncertainty, activation-energy settlement, and compatibility between reserve provision and energy-market dispatch. These mechanisms are acknowledged as relevant, but they are not included in the market sequence retained for the core control problem. The represented market scope is energy-market scheduling with intraday correction and residual imbalance settlement, not full flexibility-market co-optimization. Appendix D.3 outlines how reserve-capacity commitments and scenario-dependent activation could be incorporated as an extension.

The retained market scope is therefore a forward commitment, later correction opportunities, gate-dependent fixation, and residual deviation settlement. Bid curves, network constraints, strategic price impact, reserve activation, and detailed regulatory settlement formulas remain outside the core problem.

3.4 Uncertainty Sources

Three exogenous uncertainty sources are central to the problem: electricity prices, PV availability, and natural inflows. Each affects the value of storage in a different way, and each evolves over time after initial market decisions have been made.

Electricity prices determine the revenue or cost associated with market positions and physical operation. Day-ahead and intraday prices influence whether the hybrid system should sell energy, buy energy for pumping or battery charging, preserve stored energy, or accept deviations. Price uncertainty is particularly important for storage because the value of charging or pumping in one period depends on future selling opportunities.

PV availability determines how much renewable energy can be used by the portfolio. Forecast errors in PV production affect both physical feasibility and commercial exposure. If realized PV output is lower than expected, previously scheduled sales may need to be supplied by storage or settled as deviations. If output is higher than expected, the system may need to store, sell, or curtail the surplus depending on market access and storage capacity.

Natural inflows affect the water balance of the pumped-storage system. Inflows can increase future generating capability and alter the opportunity cost of using stored water. Even when inflow uncertainty is smaller than price or photovoltaic uncertainty over the short term, it matters because reservoir state is a slow variable and because water availability links current dispatch to future flexibility.

These uncertainty sources are not independent in a practical sense. Weather conditions influence PV availability and can also affect system prices through aggregate renewable production and demand. Market prices across consecutive periods are temporally correlated. Inflows also have seasonal and weather-related structure. For this reason, the forecasting and scenario-construction layer should represent joint trajectories rather than independent marginal samples whenever possible. The methodology in Chapter 4 adopts this idea by formulating the rolling problem conditionally on the information available at each decision time.

3.5 Temporal Structure of the Problem

The problem spans several time scales. Physical dispatch and battery operation require short intervals, market positions become fixed at specific gates, and stored water and electricity retain value beyond the immediate delivery period.

A uniform high-resolution horizon would be unnecessarily large, whereas a uniformly coarse horizon would obscure near-term market and physical constraints. The model therefore represents the near horizon in detail and aggregates later periods into coarser planning blocks.

At each decision time, the controller optimizes over this look-ahead horizon, implements the current executable decisions, updates the physical and commercial states, and repeats the process when new information arrives.

This multi-resolution structure separates executable control from longer-horizon guidance: the detailed and hourly layers represent near-term delivery, while weekly and monthly layers indicate how water and energy should be preserved or repositioned. Chapter 4 formulates these layers and Chapter 5 reports their numerical settings.

3.6 Modelling Scope and Assumptions

The model is a control-oriented abstraction that retains photovoltaic variability, battery state of charge, pumped-storage water balance, sequential market commitments, residual deviation settlement, and uncertain exogenous quantities. It does not reproduce every engineering or institutional detail.

For tractability, the photovoltaic plant uses an availability profile, the battery uses aggregate power, energy, and efficiency constraints, and pumped storage uses aggregate pumping, generating, reservoir, and hydraulic relationships. Cell-level degradation, detailed maintenance, and full asset-health modelling are excluded.

The market assumptions follow Section 3.3: prices are exogenous, intraday trading is represented through correction layers, balancing through residual settlement, and reserves are excluded. Appendix D documents possible bidding and reserve extensions.

The information structure is nonanticipative: decisions may use the current physical state, fixed positions, observed history, and available forecasts, but not future realizations. The simulator evaluates decisions against the realized trajectory, whereas the controller receives only conditional forecasts and scenarios.

Within this scope, the problem can be stated as follows: operate a hybrid PV–BESS–PSH portfolio over a rolling horizon, under uncertain prices, photovoltaic availability, and natural inflows, while respecting physical constraints and sequential market commitments. The following chapter turns this problem description into a forecasting layer, a rolling simulation environment, and a mixed-integer optimization formulation.

Chapter 4

Methodology

This chapter develops a rolling stochastic market-operation framework for a hybrid PV–BESS–PSH system. At each decision time, the controller observes the physical state and fixed commitments, receives updated joint scenarios, and solves an operational MILP for the next market and dispatch actions. A separate daily problem uses the multi-resolution planning horizon and transfers longer-term reservoir guidance to the executable rolling problem through soft targets.

Figure 4.1 provides a visual index. The chapter proceeds from the controller interface and simulator to forecasting and scenario generation, followed by the rolling optimization framework and terminal-value extension.

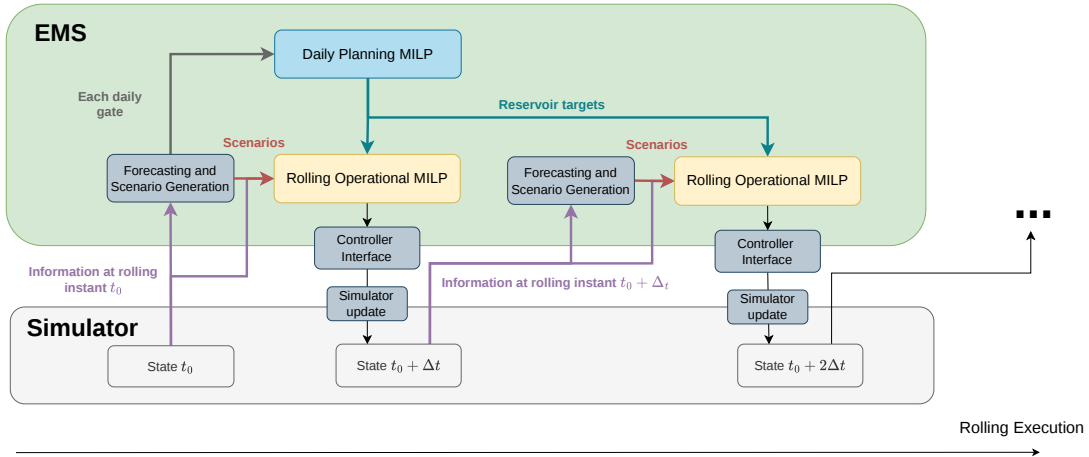


Figure 4.1: Rolling methodology and planning–control timing within the energy management system (EMS). The operational loop is repeated every 15 minutes. When planning targets are used, they are initialized at simulation start and refreshed at the daily day-ahead gate.

The notation used throughout this chapter is collected in Appendix A.

4.1 Controller Interface and Rolling Architecture

The controller interface links optimization to the rolling simulation. At each 15-minute decision time t_0 , it passes the available state and forecast representation to the optimizer and translates the solution into executable physical and commercial actions. Conditional plans for later intervals remain distinct from decisions that modify the simulated system.

The interface receives the physical state, including stored-energy and reservoir levels; the commercial state, containing cleared positions that remain binding; and the observed history of prices, PV availability, and inflow. Together these quantities define the information set $\mathcal{F}_{t_0}$ used to generate the conditional forecasts in Section 4.3.

The optimizer does not receive the future realized trajectory. It implements only the first physical action and the commercial decisions that become binding at the current gate; all later horizon decisions are provisional recourse that may be revised after new observations.

Forecast generation, operational optimization, execution, and simulator updating form the 15-minute loop shown in Figure 4.1. Controllers based on planning targets initialize the daily planning problem at simulation start and refresh its targets at the daily day-ahead gate. This solve creates no commitments and advances no physical state. At every operational step, newly binding market quantities enter the commitment book, the current continuous-intraday correction and first dispatch action are executed, and all later decisions remain provisional.

4.2 Rolling Simulation Environment

The rolling simulator carries the realized system evolution and exogenous path, whereas the forecasting layer maps currently available information into conditional future trajectories. Forecasts are constructed before implementation; realizations are revealed only as the simulator advances.

After implementing the current actions, the simulator advances one interval, propagates storage states, applies realized PV availability and inflow, and records deviations between commercial positions and physical delivery. It retains cleared positions and applies market revenues and *ex post* deviation settlement; balancing is therefore a settlement layer rather than an additional recourse stage.

The updated physical state, commercial state, and observations define the next information set. Repetition produces the closed-loop trajectory: the sequence of implemented actions and realized states generated by this feedback process. Performance is computed *ex post* from that trajectory; the objective of an individual MILP remains conditional on the information available when that problem was issued.

All controllers use the same rolling loop and realized path, differing only in the uncertainty representation and controller design being compared.

4.3 Forecast Representation and Scenario Generation

At each rolling decision time t_0 , the forecasting layer maps the information set $\mathcal{F}_{t_0}$ defined in Section 4.1 into a representation of uncertainty in electricity prices, PV availability, and natural inflow over the optimization horizon.

The forecasting layer is defined by its outputs rather than by a particular algorithm. It must be causal, temporally consistent with both the native and aggregate model intervals, and computationally suitable for repeated re-issuance. The required outputs are joint scenario paths, probability weights, detailed-horizon inputs, and compatible aggregate look-ahead inputs.

Historical residual sampling supplies these outputs in the experiments through a data-based forecast-generation procedure rather than a fitted parametric time-series model. The discussion of parametric models below concerns methodological alternatives, not algorithms implemented in the experiments. The optimization formulation does not depend on the specific forecasting method.

At issue time t_0 , the ideal probabilistic description is the conditional distribution $\mathbb{P}(X_{t_0+1:t_0+H} \mid \mathcal{F}_{t_0})$, where $X_{t_0+1:t_0+H}$ denotes the joint future trajectory of the exogenous variables over the forecast horizon. Since the MILP cannot operate directly on this distribution, it is approximated by the finite reduced scenario set $\mathcal{S}_{t_0}^{red} = \{(X_{t_0+1:t_0+H}^\omega, \hat{p}_\omega)\}_{\omega \in \Omega}$, with $\sum_{\omega \in \Omega} \hat{p}_\omega = 1$.

Here, Ω is the finite scenario-index set, ω identifies one joint future path, and $\hat{p}_\omega$ is its weight. The probability measure $\mathbb{P}$ is distinct from these discrete weights. Although the notation suppresses the dependence on t_0 , the scenario contents and weights are regenerated at every issue time. Each scenario contains the joint evolution of prices, PV availability, and inflow, with uncertainty beginning after the observed current interval.

4.3.1 Forecast Representation and Modelling Choice

A point forecast replaces the conditional distribution with one representative value or path, whereas a probabilistic forecast retains dispersion through densities, quantiles, ensembles, or scenarios. The representative point depends on the loss function: squared error selects the conditional mean, absolute error the median, and asymmetric losses the corresponding quantiles.

For stochastic optimization, the relevant requirement is a coherent uncertainty object that can enter the decision model. Collapsing the distribution to one path may remove the variation that creates storage option value, recourse, and asymmetric exposure. A scenario representation is therefore appropriate when decisions depend materially on the spread and cross-dependence of future quantities.

The relevant time series are neither simply stationary nor independent. Prices contain intraday, weekly, and seasonal structure together with spikes and regime changes; photovoltaic availability follows weather and daylight; and inflow has hydrological persistence and seasonality. Their joint evolution matters because the controller couples market exposure, renewable production, storage, and water availability.

Parametric time-series models provide interpretable representations of autocorrelation and seasonality and can be appropriate for individual point forecasts or marginal distributions. The present

problem instead requires coherent joint multi-step paths and consistency between detailed and aggregate horizons. Extending separate models to this setting requires an additional dependence, sampling, and aggregation specification, while electricity-price regimes may also be difficult to capture parametrically. Historical residual sampling offers a direct way to obtain temporally coherent continuations while retaining empirical shapes, persistence, and part of the observed dependence structure. It is therefore adopted as a practical trade-off rather than as a generally superior forecasting method; its representativeness and stationarity limitations are stated explicitly in the sampling procedure below.

4.3.2 Price Representation Within the Stochastic Forecast

The market layers use different price representations.

Day-ahead and discrete intraday-auction prices are stochastic inputs over future delivery intervals and support commercial recourse at their corresponding gates.

By contrast, the continuous intraday layer is represented only as a current-step correction. The current continuous-intraday reference price $\hat{\pi}_{t_0}^{ID}$ values the immediate sell correction, while the spread-adjusted buy price is $\hat{\pi}_{t_0}^{ID, buy} = \hat{\pi}_{t_0}^{ID} + \hat{s}^{ID}$. The annual experiments use a fixed purchase premium $\hat{s}^{ID} = 10$ EUR/MWh as a modelling assumption rather than an empirically calibrated bid–ask spread. Future intraday recourse is therefore represented through the discrete IDA layers, while continuous intraday trading is limited to the current step. Extending it over the horizon would require assumptions on liquidity, executable quantities, spreads, and trading deadlines that are outside the model scope.

Balancing exposure is not a forward recourse decision and has no separate scenario process. It appears *ex post* through settlement of deviations between the commercial position and realized delivery.

4.3.3 Historical Residual Sampling

Historical residual sampling reuses continuation patterns rather than perturbing each interval independently, thereby preserving empirical persistence, ramps, daily profiles, and sustained regimes.

Sampling from the full archive would mix seasonally and operationally dissimilar periods. Candidate continuations are therefore restricted using information available at the issue time, primarily calendar similarity and the current residual state. The objective is a conditionally relevant pool rather than one unique analogue day.

The method assumes that, after conditioning on calendar and current-state information, historical residual continuations remain representative of future uncertainty. Structural change in markets, renewable production, or hydrology may violate this stationarity assumption; filtering and anchoring mitigate but do not remove the limitation.

Let $\mathcal{J}$ index the modelled price, PV availability, and inflow series. For each uncertainty family $j \in \mathcal{J}$, the sampled continuation is anchored to the currently observed value through the forecast gap $\hat{g}_{t_0}^j = x_{t_0}^{j, real} - \hat{x}_{t_0}^{j, 50}$. Here, $x_{t_0}^{j, real}$ is the realized current value and $\hat{x}_{t_0}^{j, 50}$ is the conditional

central forecast at the issue time. Equation (4.1) then constructs the scenario path.

$$\hat{x}_{t_0+h}^{j,\omega} = \hat{x}_{t_0+h}^{j,50} + \hat{r}_h^{j,\omega} + \hat{\rho}_j^h \hat{g}_{t_0}^j \quad h \in \mathcal{H} \quad (4.1)$$

The term $\hat{r}_h^{j,\omega}$ represents the sampled historical residual continuation, while $\hat{\rho}_j^h \hat{g}_{t_0}^j$ propagates the current forecast gap forward. The persistence parameter $\hat{\rho}_j$ is raised to the lead time h so that the effect of the current mismatch decays geometrically over the forecast horizon. If $\hat{\rho}_j$ is small, the sampled path reverts quickly toward the central forecast. If it is close to one, the currently observed deviation remains influential for longer.

This anchoring step is methodologically important because it avoids an artificial jump between the observed present and the sampled future. The uncertainty representation remains independent of the optimization result itself. Scenario generation depends only on information available at time t_0 , which preserves causality and avoids look-ahead bias.

4.3.4 Joint Scenario Construction and Aggregate Intervals

Historical residual sampling generates joint scenario trajectories for electricity prices, PV availability, and natural inflows. The sampled family-specific paths are assembled on a common time support into the joint raw scenario bank $\mathcal{S}_{t_0}^{raw}$, which supplies both detailed-horizon and aggregate look-ahead inputs.

The detailed forecast uses the native simulation grid, while the later region uses aggregate intervals $\ell \in \mathcal{L}$ of increasing duration. Both are derived from the same scenarios and differ only in temporal resolution.

Energy and water aggregate naturally as interval totals, whereas price aggregation must retain enough internal profile information to remain meaningful for storage decisions. Section 4.4.1 defines the corresponding horizontal-block structure.

For each aggregate interval ℓ , let $\mathcal{T}(\ell)$ denote the native-grid intervals represented by ℓ . Equation (4.2) obtains the corresponding forecast quantity by applying the family-specific aggregation operator $A_\ell^j(\cdot)$ to that native-grid path.

$$\hat{x}_\ell^{j,\omega} = A_\ell^j \left(\left\{ \hat{x}_t^{j,\omega} \right\}_{t \in \mathcal{T}(\ell)} \right) \quad (4.2)$$

The operator $A_\ell^j(\cdot)$ averages prices over sufficiently homogeneous periods and converts PV availability and inflow into energy or volume totals. Aggregate inputs are therefore a coarser discretization of the same scenario paths, not a separate forecast.

4.3.5 Scenario Reduction

The raw scenario bank may be too large to be passed directly to a rolling MILP that is solved repeatedly over a long simulation. It is therefore replaced by a smaller representative subset,

$\mathcal{S}_{t_0}^{raw} \rightarrow \mathcal{S}_{t_0}^{red}$, with $N_{red} \ll N_{raw}$. Reduction should preserve price levels and spreads, renewable and inflow conditions, temporal shape, and operationally significant extremes.

The adopted method follows the fast-forward selection logic of Heitsch and Roemisch (2003). It iteratively selects representatives in a normalized feature space and reassigns discarded probability mass to the nearest retained path. The resulting subset remains interpretable and maps directly to the market and physical inputs.

Conceptually, reduction can be described in a feature space. Each raw scenario is mapped to a feature vector z_ω , and the retained subset is chosen so as to represent the geometry of the original scenario cloud. Equation (4.3) gives a generic representative-subset formulation.

$$\mathcal{S}_{t_0}^{red} \in \arg \min_{\substack{\mathcal{R} \subseteq \mathcal{S}_{t_0}^{raw} \\ |\mathcal{R}| = N_{red}}} \sum_{\omega \in \mathcal{S}_{t_0}^{raw}} \hat{p}_\omega^{raw} \min_{\omega' \in \mathcal{R}} d(z_\omega, z_{\omega'}) \quad (4.3)$$

The distance $d(\cdot, \cdot)$ is the mean absolute difference between range-normalized features from the detailed and aggregate trajectories. The fast-forward procedure selects scenarios successively rather than solving Equation (4.3) exactly. Each retained probability $\hat{p}_{\omega'}^{red}$ equals its original mass plus that of the discarded scenarios assigned to it, so the reduced probabilities sum to one.

The retained count N_{red} is an experimental parameter. Larger sets enrich the uncertainty representation but add scenario-indexed variables and constraints, so the choice depends on the horizon, reduction method, solver, and computational budget. Chapter 5 reports the selected count together with runtime and decision-sensitivity evidence.

At issue time t_0 , $\mathcal{S}_{t_0}^{red}$ enters the stochastic MILP; the deterministic benchmark instead uses one path with probability one. The following section defines how these inputs enter the constraints and nonanticipative decisions.

4.4 Rolling Optimization Framework

This section defines the stochastic market-dispatch formulation shared by the daily planning and rolling operational problems introduced in Section 4.1. It combines PV, BESS, and PSH operation with day-ahead, three discrete intraday-auction, and current-step continuous-intraday decisions. At decision time t_0 , inherited commitments are parameters, quantities becoming binding are node-common decisions, and later actions remain scenario-dependent recourse.

Figure 4.2 summarizes the chronological 15-minute and hourly region and the weekly and monthly continuation packs. The planning problem uses the complete structure, whereas the executable operational problem retains only the chronological region and receives reservoir guidance from the daily solve.

4.4.1 Decision Horizon and Time Architecture

The horizon contains an explicit chronological region and a coarser planning region. Delivery intervals $t \in \mathcal{T}^{near}$ comprise two 15-minute days followed by five hourly days in the operational

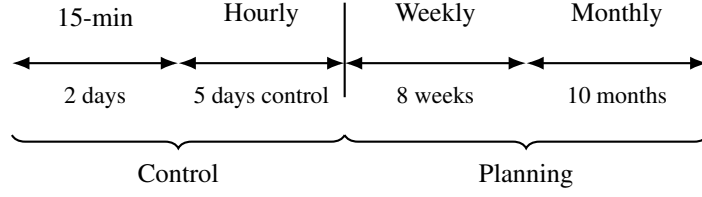


Figure 4.2: Multi-resolution control and planning horizon used in the methodology. The operational control solve uses two detailed days followed by five hourly days, while the daily planning layer extends the look-ahead through weekly and monthly packs. The numerical lengths shown are design choices of the adopted controller architecture, not structural limitations of the multi-resolution approach.

solve. Both segments use the same dispatch and commercial-position equations and differ only in temporal resolution. The daily planning layer extends the look-ahead through weekly and monthly packs and provides reservoir targets at the end of the seven-day operational horizons issued over the subsequent 24-hour control period.

Market status at t_0 partitions the detailed intervals. Let $\mathcal{A}^{IDA} = \{IDA1, IDA2, IDA3\}$ and $s \in \mathcal{A}^{IDA}$ index an auction. The sets $\mathcal{T}^{DA,fix}$ and $\mathcal{T}^{s,fix}$ contain cleared quantities, while $\mathcal{T}^{DA,open}$ and $\mathcal{T}^{s,open}$ contain quantities that remain decision variables. Their gate subsets identify quantities that become binding in the current solve. The aggregate set $\mathcal{T}^{fix}$ contains intervals whose complete commercial position is already fixed.

Fixed quantities are inherited parameters, gate quantities become binding after the current solve, and remaining open quantities are scenario-dependent provisional recourse reconsidered at later gates.

IDA1 and IDA2 cover the full next-day schedule, whereas IDA3 covers 12:00–24:00. The mapping follows the Iberian auction sequence (OMIE, 2024) but abstracts from bid formats, detailed order rules, and liquidity.

No balancing decision set is introduced. Consistent with the Portuguese framework, residual deviations are settled *ex post* rather than optimized at another gate (Entidade Reguladora dos Serviços Energéticos, 2025). The variable $\delta_{t,\omega}^{bal,+}$ denotes delivery above the commercial position and $\delta_{t,\omega}^{bal,-}$ delivery below it. Their coefficients may differ, although the numerical experiments use symmetric ordinary residual-deviation costs.

Eight weekly and ten monthly packs continue the horizon beyond the detailed region. They are planning approximations, not executable schedules, and retain longer-term price, water, and storage information without a year of native-grid scenario-indexed variables.

Sequential aggregation merges adjacent intervals while retaining their order, as shown in Figure 4.3. Coarser intervals replace several delivery prices with fewer averages; if taken too far, this loses the price signal needed for storage feasibility and market decisions.

Daily or weekly PV energy and inflow totals remain meaningful cumulative quantities, but one average price may not represent the hours in which energy is available or needed.

Weekly and monthly packs therefore use horizontal blocks that group recurring intra-pack

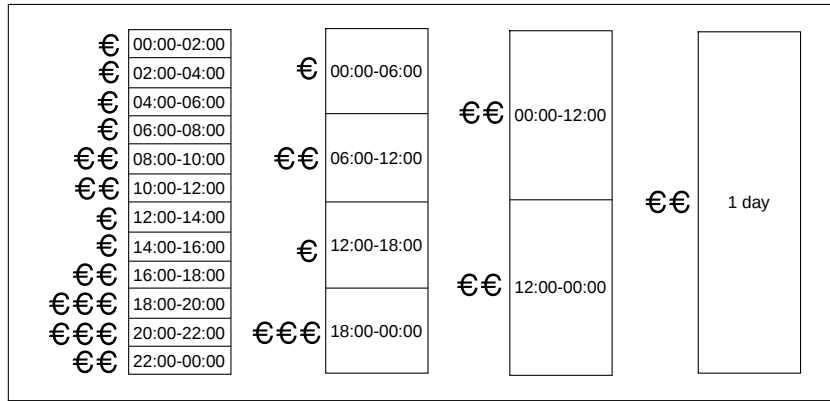


Figure 4.3: Sequential aggregation of delivery intervals. Consecutive periods are merged into longer chronological intervals, so the represented stock still moves through time in order. The price signal becomes less granular as several delivery-period prices are replaced by fewer averages.

periods, such as the same six-hour window across all days. Prices are averaged within these more homogeneous segments, while PV availability and inflow become energy and volume totals. A six-hour block pooled over a seven-day pack represents 42 hours, rather than one six-hour interval of a representative chronological day. Its price is an average over those periods, and its energy and water quantities cover their combined duration. Monthly blocks follow the same principle with the number of days represented by the pack; internal daily chronology is suppressed.

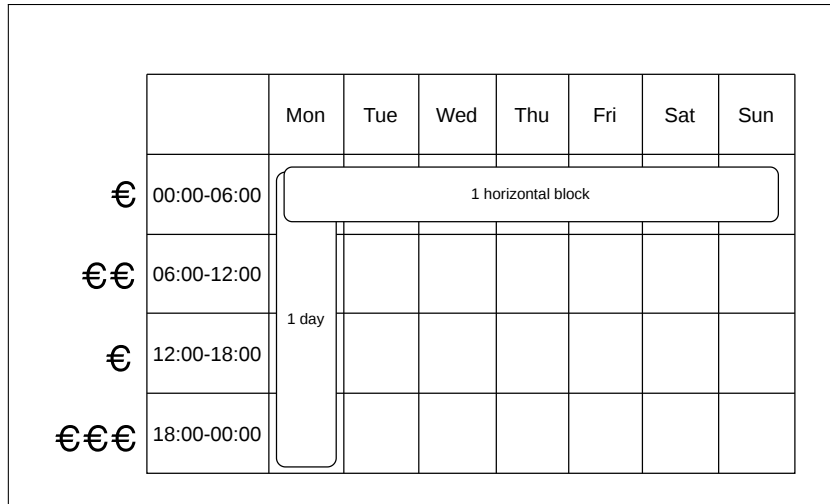


Figure 4.4: Horizontal aggregation inside a planning pack. Blocks pool recurring time-of-day segments across the pack, preserving part of the price profile without representing every internal interval chronologically.

These blocks pool similar periods rather than forming a chronological sub-sequence. They retain more price-profile information than one pack average but suppress internal chronology.

The blocks represent the visible coarse continuation region, while a terminal term values the omitted tail, following the terminal-value principle used in hydropower scheduling (Löhndorf et al., 2013; Lin et al., 2020).

4.4.2 Chronological and Pooled Stock Representations

The time-aggregation distinction has a direct stock-balance consequence. To express it generically, let S_τ denote the stock available at the opening of chronological interval τ , and let ΔS_τ denote its net change during that interval. Equation (4.4) propagates the closing stock of each interval into the next one.

$$S_{\tau+1} = S_\tau + \Delta S_\tau \quad \tau = 1, \dots, n \quad (4.4)$$

Every successor stock is therefore represented explicitly, allowing storage bounds and subsequent decisions to depend on the state inherited from the preceding interval.

For a horizontal pack p , let S_p^i and S_p^f denote its opening and closing stocks, and let ΔS_b denote the contribution of internal block b . Equation (4.5) retains only the aggregate pack balance.

$$S_p^f = S_p^i + \sum_{b \in \mathcal{B}_p^r} \Delta S_b \quad (4.5)$$

The internal contributions are chosen jointly, but no intermediate stock is introduced between them. In Figures 4.5 and 4.6, X and Y denote generic inputs and outputs, while ΔS denotes the associated contribution to the stock balance. The figures illustrate the two structures before Sections 4.4.4 and 4.4.5 specialize them to reservoir volume and BESS stored energy.

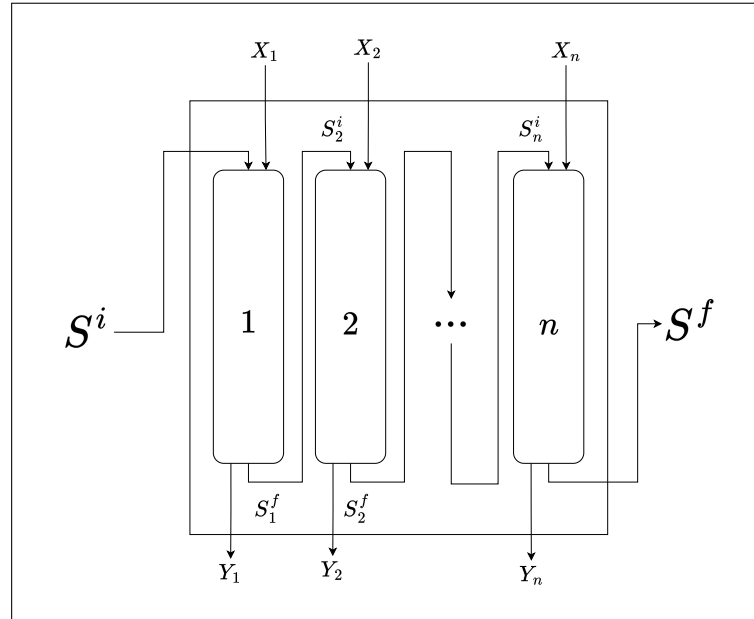


Figure 4.5: Chronological stock propagation. Each internal interval receives the stock left by the previous one, so every output is evaluated from the state available when that interval is reached.

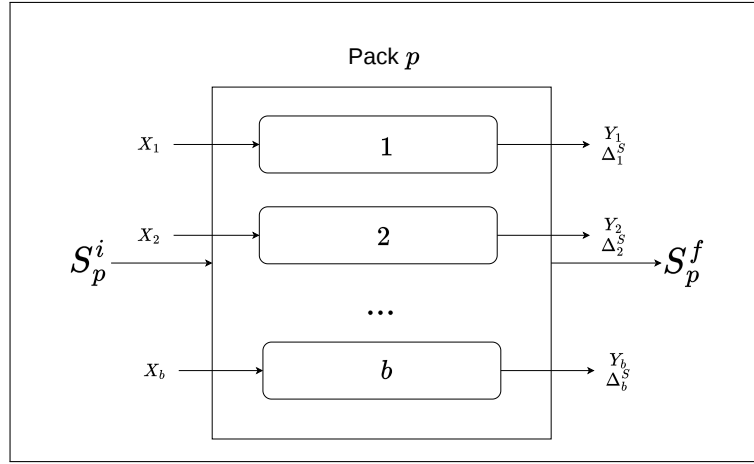


Figure 4.6: Pooled aggregate stock balance. A whole pack is represented by one opening stock, one closing stock, and internal aggregate contributions, without introducing intermediate stock states inside the pack.

Equation (4.4) underlies the 15-minute and hourly regions, whereas Equation (4.5) underlies the weekly and monthly planning packs. The pooled representation preserves each pack's net water and energy balance and the chronology between packs, but not intermediate within-pack feasibility. Consequently, an internal horizontal block is not an executable schedule and may not be feasible as an isolated chronological action. This limitation is accepted in exchange for carrying longer-horizon opportunity information in a repeatedly solved stochastic MILP.

4.4.3 Commercial Commitments and Market Recourse

Equation (4.6) defines the accumulated commercial position as the sum of the day-ahead position, the corrections from the intraday-auction stages $s \in \mathcal{A}^{IDA}$, and the current-step continuous intraday correction.

$$Q_{t,\omega}^{comm} = Q_{t,\omega}^{DA} + \sum_{s \in \mathcal{A}^{IDA}} \Delta_{t,\omega}^s + \Delta_{t,\omega}^{ID} \quad (4.6)$$

For each active auction gate, with $s \in \mathcal{A}^{IDA}$, $t \in \mathcal{T}^{s,gate}$, and $\omega \in \Omega$, the formulation introduces separate nonnegative quantities $Q_{t,\omega}^{buy,s} \geq 0$ and $Q_{t,\omega}^{sell,s} \geq 0$. Their signed correction is $\Delta_{t,\omega}^s = Q_{t,\omega}^{sell,s} - Q_{t,\omega}^{buy,s}$. A positive value of $\Delta_{t,\omega}^s$ corresponds to an additional sale in auction s , whereas a negative value corresponds to a net buyback. For future intervals in $\mathcal{T}^{s,open} \setminus \mathcal{T}^{s,gate}$, $\Delta_{t,\omega}^s$ is represented directly as a signed recourse variable; the buy-sell decomposition is required only when the current gate makes the correction binding.

At a market gate, however, the correction is not an independent speculative trading variable. It modifies a commercial position that may already contain quantities cleared in earlier markets. Let $\bar{Q}_t^{pre}$ denote the accumulated inherited position for delivery interval t immediately before the active gate is applied. The resulting position is $Q_{t,\omega}^{post,s} = \bar{Q}_t^{pre} + Q_{t,\omega}^{sell,s} - Q_{t,\omega}^{buy,s}$. The lower bound $\underline{Q}_t$ is the most negative net position supported by maximum pumping and battery charging, whereas $\bar{Q}_{t,\omega}$ is the maximum export supported by pumped-storage generation, battery discharge, and the

scenario-dependent available PV power. Equations (4.7a) and (4.7b) define these bounds.

$$\underline{Q}_t = - \left(\sum_{g \in \mathcal{G}} \hat{P}_g^{PSH,pump,max} + \hat{P}^{B,ch,max} \right) \quad (4.7a)$$

$$\overline{Q}_{t,\omega} = \sum_{g \in \mathcal{G}} \hat{P}_g^{PSH,turb,max} + \hat{P}^{B,dis,max} + \hat{P}_{t,\omega}^{PV,av} \quad (4.7b)$$

Equation (4.8) constrains the final post-gate position to this physical envelope.

$$\underline{Q}_t \leq Q_{t,\omega}^{post,s} \leq \overline{Q}_{t,\omega} \quad (4.8)$$

This envelope is imposed on the position after the new gate correction, not on inherited quantities that have already cleared in previous markets. Because gate corrections are node-common, the scenario-dependent upper bound enforces the selected commitment against every retained scenario.

To prevent simultaneous buying and selling in the same auction and delivery interval, the formulation imposes a binary direction variable $z_{t,\omega}^s$, with $z_{t,\omega}^s = 1$ selecting selling and $z_{t,\omega}^s = 0$ selecting buying. A second binary variable $a_{t,\omega}^s$ indicates whether a gate correction is active. Equations (4.9a)–(4.9f) define the complete gate-correction system relative to the distance between the inherited position and the physical envelope.

$$Q_{t,\omega}^{sell,s} \leq [\overline{Q}_{t,\omega} - \overline{Q}_t^{pre}]_+ z_{t,\omega}^s \quad (4.9a)$$

$$Q_{t,\omega}^{buy,s} \leq [\overline{Q}_t^{pre} - \underline{Q}_t]_+ (1 - z_{t,\omega}^s) \quad (4.9b)$$

$$Q_{t,\omega}^{sell,s} \leq [\overline{Q}_{t,\omega} - \overline{Q}_t^{pre}]_+ a_{t,\omega}^s \quad (4.9c)$$

$$Q_{t,\omega}^{buy,s} \leq [\overline{Q}_t^{pre} - \underline{Q}_t]_+ a_{t,\omega}^s \quad (4.9d)$$

$$z_{t,\omega}^s \in \{0, 1\} \quad (4.9e)$$

$$a_{t,\omega}^s \in \{0, 1\} \quad (4.9f)$$

where $[x]_+ = \max\{x, 0\}$. A position above the export envelope can be bought back, while one below the import envelope can be unwound through an additional sale. These are commercial corrections to the inherited book; physical capability is enforced on the post-gate position and dispatch.

An additional physicality condition prevents the gate from overshooting the planned physical delivery merely to create financial arbitrage between market layers. When an active gate correction is a sale, Equation (4.10) prevents the post-gate position from exceeding the planned physical net injection. When it is a buyback, Equation (4.11) prevents that position from remaining below the planned injection.

$$Q_{t,\omega}^{post,s} \leq P_{t,\omega}^{net} + M_{t,\omega} (1 - z_{t,\omega}^s + 1 - a_{t,\omega}^s) \quad (4.10)$$

$$Q_{t,\omega}^{post,s} \geq P_{t,\omega}^{net} - M_{t,\omega} (z_{t,\omega}^s + 1 - a_{t,\omega}^s) \quad (4.11)$$

where $M_{t,\omega} = \bar{Q}_{t,\omega} - \underline{Q}_t$ relaxes the inactive direction. If $a_{t,\omega}^s = 1$, the correction must move the commercial position toward the physical plan; otherwise, the directional restrictions are inactive. This excludes DA–IDA re-trading unsupported by planned operation.

The continuous intraday correction is different because it applies only to the interval currently being executed. Let $\hat{Q}_{t_0}^{ID,max}$ denote the currently executable quantity cap, set to zero when the interval is not tradable, and let $z_{t_0}^{ID}$ select selling rather than buying. Equations (4.12a)–(4.12c) impose the cap and prevent simultaneous buying and selling.

$$0 \leq Q_{t_0}^{sell,ID} \leq \hat{Q}_{t_0}^{ID,max} z_{t_0}^{ID} \quad (4.12a)$$

$$0 \leq Q_{t_0}^{buy,ID} \leq \hat{Q}_{t_0}^{ID,max} (1 - z_{t_0}^{ID}) \quad (4.12b)$$

$$z_{t_0}^{ID} \in \{0, 1\} \quad (4.12c)$$

Equation (4.13) then defines the signed correction over the complete detailed horizon.

$$\Delta_{t,\omega}^{ID} = \begin{cases} Q_{t_0}^{sell,ID} - Q_{t_0}^{buy,ID} & t = t_0 \\ 0 & t \neq t_0 \end{cases} \quad \forall t \in \mathcal{T}^{near} \quad \forall \omega \in \Omega \quad (4.13)$$

The quantities are node-common by construction because they carry no scenario index. The finite cap prevents the continuous layer from becoming an unlimited balancing substitute, while the price-taking representation remains a simplified approximation of intraday liquidity.

Market quantities that have already cleared are treated as fixed commitments in the current optimization. Equation (4.14) fixes day-ahead quantities after market closure.

$$Q_{t,\omega}^{DA} = \bar{Q}_t^{DA} \quad \forall t \in \mathcal{T}^{DA,fix} \quad \forall \omega \in \Omega \quad (4.14)$$

Equation (4.15) applies the same logic to every intraday-auction correction after its gate has passed.

$$\Delta_{t,\omega}^s = \bar{\Delta}_t^s \quad \forall s \in \mathcal{A}^{IDA} \quad \forall t \in \mathcal{T}^{s,fix} \quad \forall \omega \in \Omega \quad (4.15)$$

This structure encodes the sequential nature of the market. Earlier layers remain frozen, whereas later layers may still modify the accumulated commercial position on intervals that remain open to them.

Quantities committed at the current gate must be unique before the future scenario is known, as imposed in Section 4.4.8. The optimizer chooses these common quantities jointly with scenario-dependent physical plans and couples them through the imbalance relation. Open future intervals retain provisional recourse until a later solve reaches their gate.

Although the commercial position is not forced to equal the physical plan, arbitrary market quantities need not remain in the feasible region. For intervals whose commercial position is chosen by the current solve, Equation (4.16) applies the physical bounds defined in Equation (4.7) to $Q_{t,\omega}^{comm}$.

$$\underline{Q}_t \leq Q_{t,\omega}^{comm} \leq \bar{Q}_{t,\omega} \quad \forall t \in \mathcal{T}^{near} \setminus \mathcal{T}^{fix} \quad \forall \omega \in \Omega \quad (4.16)$$

The envelope removes unsupported new commitments from the feasible set rather than relying only on an imbalance penalty. Its upper bound reflects pumped-storage generation, battery discharge, and available photovoltaic production; its lower bound reflects pumping and battery charging. Already-cleared commitments remain binding and are not constrained retroactively. Later gates may reduce inherited exposure, while any residual inconsistency is settled through imbalance variables.

4.4.4 Detailed Physical Dispatch

The physical dispatch couples photovoltaic generation, battery operation, and pumped-storage operation. Unless stated otherwise, the equations in this subsection apply to every $t \in \mathcal{T}^{near}$ and $\omega \in \Omega$. Equation (4.17) defines the net power injected into the grid.

$$P_{t,\omega}^{net} = P_{t,\omega}^{PSH,turb} - P_{t,\omega}^{PSH,pump} + P_{t,\omega}^{B,dis} - P_{t,\omega}^{B,ch} + P_{t,\omega}^{PV,use} \quad \forall t \in \mathcal{T}^{near} \quad \forall \omega \in \Omega \quad (4.17)$$

Equation (4.18) splits the mismatch between physical injection and commercial position into positive and negative balancing imbalances.

$$P_{t,\omega}^{net} - Q_{t,\omega}^{comm} = \delta_{t,\omega}^{bal,+} - \delta_{t,\omega}^{bal,-} \quad \forall t \in \mathcal{T}^{near} \quad \forall \omega \in \Omega \quad (4.18)$$

Both imbalance components are nonnegative, with $\delta_{t,\omega}^{bal,+} \geq 0$ and $\delta_{t,\omega}^{bal,-} \geq 0$. A positive imbalance means that physical delivery exceeds the contracted commercial position, whereas a negative imbalance means that delivery falls short of it. These residual deviations are not decision variables of a separate balancing stage. In the objective they are priced or penalized through the coefficients $\hat{c}_{t,\omega}^{bal,+}$ and $\hat{c}_{t,\omega}^{bal,-}$, which may be asymmetric. Setting these coefficients equal gives the symmetric deviation-penalty case used for ordinary residual deviations in the numerical experiments.

Equation (4.18) defines the mismatch but does not itself impose its economic cost. In future optimization intervals, delivery and deviations are conditional on scenario ω ; Equation (4.42) subtracts the corresponding deviation costs from the maximization objective. After the current action is executed, the simulator calculates and settles the residual deviation using realized physical delivery and the cleared commercial position.

When the coefficients are equal and their common value $\hat{c}_{t,\omega}^{bal}$ is strictly positive, the positive–negative split is unnecessary and adds a redundant representation of the same symmetric deviation cost. A single absolute-deviation variable $\delta_{t,\omega}^{bal,abs}$ can then replace the pair through Equations (4.19a)–(4.19c).

$$\delta_{t,\omega}^{bal,abs} \geq P_{t,\omega}^{net} - Q_{t,\omega}^{comm} \quad (4.19a)$$

$$\delta_{t,\omega}^{bal,abs} \geq Q_{t,\omega}^{comm} - P_{t,\omega}^{net} \quad (4.19b)$$

$$\delta_{t,\omega}^{bal,abs} \geq 0 \quad (4.19c)$$

With the objective contribution $-\hat{c}_{t,\omega}^{bal} \delta_{t,\omega}^{bal,abs}$, the variable equals the absolute residual deviation at optimum. This alternative reduces the variable count in the symmetric case; the split formulation remains necessary when positive and negative deviations require distinct coefficients.

Equation (4.20) bounds photovoltaic dispatch by exogenous availability. The quantity $\hat{P}_{t,\omega}^{PV,av}$ is a forecast-scenario parameter, whereas $P_{t,\omega}^{PV,use}$ is the dispatch decision. The current interval uses observed availability; later dispatch decisions are conditional on their scenario inputs. The difference between available and used photovoltaic power is curtailed production.

$$0 \leq P_{t,\omega}^{PV,use} \leq \hat{P}_{t,\omega}^{PV,av} \quad \forall t \in \mathcal{T}^{near} \quad \forall \omega \in \Omega \quad (4.20)$$

Let Δ_t denote the duration, in hours, of explicit dispatch interval t . Equation (4.21) defines the BESS state transition.

$$E_{t+1,\omega}^B = E_{t,\omega}^B + \hat{\eta}^{B,ch} P_{t,\omega}^{B,ch} \Delta_t - \frac{1}{\hat{\eta}^{B,dis}} P_{t,\omega}^{B,dis} \Delta_t \quad (4.21)$$

Let $u_{t,\omega}^B = 1$ enable charging and $u_{t,\omega}^B = 0$ enable discharging. Equations (4.22a)–(4.22d) impose the power, mode, and energy bounds. The battery may remain idle under either value of $u_{t,\omega}^B$, but it cannot charge and discharge simultaneously. The energy bound is imposed on every successor state, including the terminal state generated by the final detailed transition.

$$0 \leq P_{t,\omega}^{B,ch} \leq \hat{P}^{B,ch,max} u_{t,\omega}^B \quad (4.22a)$$

$$0 \leq P_{t,\omega}^{B,dis} \leq \hat{P}^{B,dis,max} (1 - u_{t,\omega}^B) \quad (4.22b)$$

$$u_{t,\omega}^B \in \{0, 1\} \quad (4.22c)$$

$$0 \leq E_{t+1,\omega}^B \leq \hat{E}^{B,max} \quad (4.22d)$$

Equation (4.23) defines the upper-reservoir state transition.

$$V_{t+1,\omega}^{res} = V_{t,\omega}^{res} + q_{t,\omega}^{pump} \Delta_t - q_{t,\omega}^{turb} \Delta_t + \hat{q}_{t,\omega}^{nat} \Delta_t - q_{t,\omega}^{spill} \Delta_t \quad (4.23)$$

The reservoir volume and spillage domains are imposed by Equations (4.24a) and (4.24b). As with the battery, the volume bound applies to each successor state and therefore covers the terminal detailed state.

$$\underline{V}^{res} \leq V_{t+1,\omega}^{res} \leq \bar{V}^{res} \quad (4.24a)$$

$$q_{t,\omega}^{spill} \geq 0 \quad (4.24b)$$

Environmental, irrigation, and other multipurpose releases lie outside the system boundary defined in Chapter 3; consequently, the formulation does not retain a separate controlled-release decision. The variable $q_{t,\omega}^{spill}$ collects non-generating reservoir outflow. No independent spillway-capacity parameter is imposed: spillage is a nonnegative, penalized feasibility variable whose usable amount remains limited by the reservoir balance and lower storage bound. Let $\underline{h}$ and $\bar{h}$ denote the lower

and upper heads represented by the operating-point grid. Equation (4.25) defines the affine head approximation over the modelled reservoir range.

$$h_{t,\omega} = \underline{h} + \frac{\bar{h} - \underline{h}}{\bar{V}^{res} - \underline{V}^{res}} (V_{t,\omega}^{res} - \underline{V}^{res}) \quad (4.25)$$

The sets $\mathcal{K}^{turb}$ and $\mathcal{K}^{pump}$ index the sampled turbine and pumping operating points, including the zero-flow rows, while $\mathcal{K}^{turb,+}$ and $\mathcal{K}^{pump,+}$ contain only their positive-flow points. The variables $\lambda_{k,t,\omega}^{turb}$ and $\lambda_{k,t,\omega}^{pump}$ are the corresponding convex-combination weights. The surface shape follows the type of polynomial efficiency-curve representation described by Diniz et al. (2007), but the numerical samples are representative case-study values rather than manufacturer data. Following Borghetti et al. (2008) and Kumar et al. (2025), the nonlinear power–flow–head relations are replaced by convex combinations. Equations (4.26a)–(4.26f) give the turbine representation.

$$P_{t,\omega}^{PSH,turb} = \sum_{k \in \mathcal{K}^{turb}} \lambda_{k,t,\omega}^{turb} \bar{P}_k^{turb} \quad (4.26a)$$

$$q_{t,\omega}^{turb} = \sum_{k \in \mathcal{K}^{turb}} \lambda_{k,t,\omega}^{turb} \bar{q}_k^{turb} \quad (4.26b)$$

$$h_{t,\omega} = \sum_{k \in \mathcal{K}^{turb}} \lambda_{k,t,\omega}^{turb} \bar{h}_k^{turb} \quad (4.26c)$$

$$\sum_{k \in \mathcal{K}^{turb}} \lambda_{k,t,\omega}^{turb} = 1 \quad (4.26d)$$

$$\sum_{k \in \mathcal{K}^{turb,+}} \lambda_{k,t,\omega}^{turb} \leq u_{t,\omega}^{turb} \quad (4.26e)$$

$$\lambda_{k,t,\omega}^{turb} \geq 0 \quad \forall k \in \mathcal{K}^{turb} \quad (4.26f)$$

Equations (4.27a)–(4.27f) give the analogous pumping representation.

$$P_{t,\omega}^{PSH,pump} = \sum_{k \in \mathcal{K}^{pump}} \lambda_{k,t,\omega}^{pump} \bar{P}_k^{pump} \quad (4.27a)$$

$$q_{t,\omega}^{pump} = \sum_{k \in \mathcal{K}^{pump}} \lambda_{k,t,\omega}^{pump} \bar{q}_k^{pump} \quad (4.27b)$$

$$h_{t,\omega} = \sum_{k \in \mathcal{K}^{pump}} \lambda_{k,t,\omega}^{pump} \bar{h}_k^{pump} \quad (4.27c)$$

$$\sum_{k \in \mathcal{K}^{pump}} \lambda_{k,t,\omega}^{pump} = 1 \quad (4.27d)$$

$$\sum_{k \in \mathcal{K}^{pump,+}} \lambda_{k,t,\omega}^{pump} \leq u_{t,\omega}^{pump} \quad (4.27e)$$

$$\lambda_{k,t,\omega}^{pump} \geq 0 \quad \forall k \in \mathcal{K}^{pump} \quad (4.27f)$$

Here $\bar{P}_k$, $\bar{q}_k$, and $\bar{h}_k$ denote the coordinates of the discretized operating points. The numerical case study uses three positive flow points and three head points for each mode. A zero-flow row with zero power is included at every sampled head, so each representation contains $4 \times 3 = 12$ points. The weights always sum to one and therefore reconstruct the common head. When a mode binary

is zero, its positive-flow weights are disabled and only its zero-flow row can carry weight; when the binary is one, the positive operating surface becomes available.

The plant-wide mode binaries prevent simultaneous pumping and turbinng. Let $\mathcal{G}$ index the pumped-storage unit groups, let $\bar{n}_g$ be the number of available reversible units in group g , and let $n_{g,t,\omega}^{turb}$ and $n_{g,t,\omega}^{pump}$ denote the corresponding active-unit counts. Units within each group are represented with equal ratings, so dividing the group capacity by $\bar{n}_g$ gives the capacity assigned to each active unit. Equations (4.28a)–(4.28h) define these restrictions for every $g \in \mathcal{G}$.

$$u_{t,\omega}^{turb} + u_{t,\omega}^{pump} \leq 1 \quad (4.28a)$$

$$u_{t,\omega}^{turb} \in \{0, 1\} \quad u_{t,\omega}^{pump} \in \{0, 1\} \quad (4.28b)$$

$$0 \leq n_{g,t,\omega}^{turb} \leq \bar{n}_g u_{t,\omega}^{turb} \quad n_{g,t,\omega}^{turb} \in \mathbb{Z} \quad (4.28c)$$

$$0 \leq n_{g,t,\omega}^{pump} \leq \bar{n}_g u_{t,\omega}^{pump} \quad n_{g,t,\omega}^{pump} \in \mathbb{Z} \quad (4.28d)$$

$$u_{t,\omega}^{turb} \leq \sum_{g \in \mathcal{G}} n_{g,t,\omega}^{turb} \quad (4.28e)$$

$$u_{t,\omega}^{pump} \leq \sum_{g \in \mathcal{G}} n_{g,t,\omega}^{pump} \quad (4.28f)$$

$$P_{t,\omega}^{PSH,turb} \leq \sum_{g \in \mathcal{G}} \frac{\hat{P}_g^{PSH,turb,max}}{\bar{n}_g} n_{g,t,\omega}^{turb} \quad (4.28g)$$

$$P_{t,\omega}^{PSH,pump} \leq \sum_{g \in \mathcal{G}} \frac{\hat{P}_g^{PSH,pump,max}}{\bar{n}_g} n_{g,t,\omega}^{pump} \quad (4.28h)$$

Start counts are obtained from positive changes in the active-unit counts. Let $N_{g,t,\omega}^{turb,start}$ and $N_{g,t,\omega}^{pump,start}$ be the group-level start variables. Equations (4.29a)–(4.29e) define the transitions for every $g \in \mathcal{G}$ and the total starts used in the objective.

$$N_{g,t,\omega}^{turb,start} \geq n_{g,t,\omega}^{turb} - n_{g,t-1,\omega}^{turb} \quad (4.29a)$$

$$N_{g,t,\omega}^{pump,start} \geq n_{g,t,\omega}^{pump} - n_{g,t-1,\omega}^{pump} \quad (4.29b)$$

$$N_{g,t,\omega}^{turb,start} \geq 0 \quad N_{g,t,\omega}^{pump,start} \geq 0 \quad (4.29c)$$

$$N_{t,\omega}^{turb,start} = \sum_{g \in \mathcal{G}} N_{g,t,\omega}^{turb,start} \quad (4.29d)$$

$$N_{t,\omega}^{pump,start} = \sum_{g \in \mathcal{G}} N_{g,t,\omega}^{pump,start} \quad (4.29e)$$

At $t = t_0$, the predecessor counts are the observed operating status immediately before the horizon. Because the objective assigns a positive cost to every start, the inequalities attain the positive count changes without requiring the start variables themselves to be integer.

The implemented variable-head representation is compact. Because the weights are not restricted to a single local interpolation cell, the approximation is a convex-hull representation of the sampled operating surface rather than an exact local interpolation of the original nonlinear map. A finer grid, or an additional cell-selection structure, would represent the nonlinear power–

flow–head relation more accurately, especially in regions where efficiency changes rapidly, but it would also increase the number of continuous variables, binary variables, and constraint terms at every detailed time step and for every scenario. Since the rolling stochastic problem is solved repeatedly, this increase would propagate directly into model-build time, solver memory, and branch-and-bound complexity. The adopted grid reduces physical detail to limit computational cost while preserving the main dependence of pumped-storage power on both flow and head.

4.4.5 Coarse Planning Representation

This subsection specializes the pooled representation introduced above. The equations are written for a generic coarse layer $r \in \{W, M\}$, with chronological packs $p \in \mathcal{P}^r$ and horizontal blocks $b \in \mathcal{B}_p^r$. The scenario index is suppressed for readability; every balance and bound applies separately to each $\omega \in \Omega$.

Equation (4.30) defines the pooled reservoir balance of each coarse pack p .

$$V_p^{res,close} = V_p^{res,open} + \sum_{b \in \mathcal{B}_p^r} \left(\hat{V}_b^{nat} + V_b^{pump} - V_b^{turb} - V_b^{spill} \right) \quad (4.30)$$

Equation (4.31) defines the corresponding pooled BESS balance.

$$E_p^{B,close} = E_p^{B,open} + \sum_{b \in \mathcal{B}_p^r} \left(\hat{\eta}^{B,ch} E_b^{B,ch} - \frac{1}{\hat{\eta}^{B,dis}} E_b^{B,dis} \right) \quad (4.31)$$

Equations (4.32a) and (4.32b) impose the physical bounds at each pack boundary.

$$\underline{V}^{res} \leq V_p^{res,open} \leq \bar{V}^{res} \quad \underline{V}^{res} \leq V_p^{res,close} \leq \bar{V}^{res} \quad (4.32a)$$

$$0 \leq E_p^{B,open} \leq \hat{E}^{B,max} \quad 0 \leq E_p^{B,close} \leq \hat{E}^{B,max} \quad (4.32b)$$

Let κ^{turb} and κ^{pump} denote the aggregate water-to-energy conversion factors used in the coarse planning blocks. Equation (4.33) gives the aggregate export balance at the horizontal-block level.

$$E_b^{exp} = E_b^{PV} + \kappa^{turb} V_b^{turb} + E_b^{B,dis} - E_b^{B,ch} - \kappa^{pump} V_b^{pump} \quad (4.33)$$

The planned sale in the same horizontal block is allowed to differ from expected export, but the absolute mismatch is penalized. The block variables V_b^{turb} , V_b^{pump} , V_b^{spill} , E_b^{PV} , $E_b^{B,ch}$, $E_b^{B,dis}$, E_b^{sell} , and $\delta_b^{blk,\pm}$ are nonnegative, while E_b^{exp} is signed.

$$E_b^{exp} - E_b^{sell} = \delta_b^{blk,+} - \delta_b^{blk,-} \quad (4.34)$$

Thus, the planning layer does not force a perfect equality between aggregate physical export and planned sale, but it discourages large mismatches. This treatment is appropriate because the coarse blocks are planning abstractions rather than detailed executable schedules. In the objective, the

layer-specific mismatch variables are denoted $\delta_{w,\omega}^{W,\pm}$ and $\delta_{m,\omega}^{M,\pm}$ for weekly and monthly blocks, respectively. Equation (4.34) uses $\delta_b^{blk,\pm}$ as compact generic notation for either layer.

For a weekly horizontal block w , Equations (4.35a)–(4.35e) impose the duration-scaled operating limits.

$$V_w^{turb} \leq \hat{q}^{turb,max} \Delta_w \quad (4.35a)$$

$$V_w^{pump} \leq \hat{q}^{pump,max} \Delta_w \quad (4.35b)$$

$$E_w^{PV} \leq \hat{E}_w^{PV,av} \quad (4.35c)$$

$$E_w^{B,ch} \leq \hat{P}^{B,ch,max} \Delta_w \quad (4.35d)$$

$$E_w^{B,dis} \leq \hat{P}^{B,dis,max} \Delta_w \quad (4.35e)$$

The monthly layer uses the same inequalities with m and Δ_m replacing w and Δ_w .

4.4.6 Coupling and Terminal Conditions

The coarse planning layers are coupled to the detailed region through its terminal physical states. Let p_1^W denote the first weekly pack. Equations (4.36a) and (4.36b) initialize its reservoir and BESS stocks.

$$V_{p_1^W}^{res,open} = V_{T^{near}}^{res} \quad (4.36a)$$

$$E_{p_1^W}^{B,open} = E_{T^{near}}^B \quad (4.36b)$$

Equations (4.37a) and (4.37b) chain subsequent weekly packs chronologically.

$$V_{p+1}^{res,open} = V_p^{res,close} \quad p \in \mathcal{P}^W \quad p+1 \in \mathcal{P}^W \quad (4.37a)$$

$$E_{p+1}^{B,open} = E_p^{B,close} \quad p \in \mathcal{P}^W \quad p+1 \in \mathcal{P}^W \quad (4.37b)$$

The first monthly pack opens from the closing stock of the final weekly pack, and later monthly packs are chained analogously. This preserves chronology across packs even though chronology is pooled within each pack.

The fixed-terminal benchmark prevents end-of-horizon depletion relative to the storage states observed at t_0 . Equation (4.38) imposes this condition on the BESS.

$$E_T^B \geq E_{t_0}^B \quad (4.38)$$

Equation (4.39) imposes the corresponding upper-reservoir condition.

$$V_T^{res} \geq V_{t_0}^{res} \quad (4.39)$$

These floors are applied at every benchmark solve. The terminal index T denotes the final coarse closing stock in a multi-resolution problem and the end of the detailed-plus-hourly horizon in a

short operational problem. The daily planning-target controllers do not use the hydro floor as a hidden fallback: their planning reservoir terminal state is free unless a continuation-value term is selected, and their operational problem receives the soft target defined in Section 4.4.9.

4.4.7 Objective Function

Once the commercial layer, detailed physical layer, coarse planning representation, and terminal couplings have been defined, the decision-dependent objective can be separated into market, near-horizon operating, weekly, monthly, and terminal components. The rolling stochastic MILP maximizes the expected decision-dependent value Φ^{var} defined in Equation (4.40).

$$\Phi^{var} = \sum_{\omega \in \Omega} \hat{p}_{\omega} \left(\Phi_{\omega}^{mkt} + \Phi_{\omega}^{near} + \Phi_{\omega}^W + \Phi_{\omega}^M + \Phi_{\omega}^{term} \right) \quad (4.40)$$

Equation (4.41) defines the market component. It includes open day-ahead and intraday auction quantities. This covers future scenario-dependent auction corrections and the continuous intraday correction at the current step.

$$\begin{aligned} \Phi_{\omega}^{mkt} = & \sum_{t \in \mathcal{T}^{DA, open}} \hat{\pi}_{t, \omega}^{DA} Q_{t, \omega}^{DA} \Delta_t \\ & + \sum_{s \in \mathcal{A}^{IDA}} \sum_{t \in \mathcal{T}^{s, open}} \hat{\pi}_{t, \omega}^s \Delta_{t, \omega}^s \Delta_t \\ & + \left(\hat{\pi}_{t_0}^{ID, sell} Q_{t_0}^{sell, ID} - \hat{\pi}_{t_0}^{ID, buy} Q_{t_0}^{buy, ID} \right) \Delta_{t_0} \end{aligned} \quad (4.41)$$

Because $\Delta_{t, \omega}^s$ is signed, additional sales contribute positively and buybacks negatively. Corrections in $\mathcal{T}^{s, gate}$ become node-common and binding in the current solve, whereas the remaining intervals in $\mathcal{T}^{s, open}$ are priced provisional recourse decisions. The distinct continuous-intraday buy and sell prices retain the asymmetric same-step spread.

Equation (4.42) defines the near-horizon operating component. It includes imbalance, battery-degradation, spillage, and pumped-storage start-up costs.

$$\begin{aligned} \Phi_{\omega}^{near} = & - \sum_{t \in \mathcal{T}^{near}} \left[\hat{c}_{t, \omega}^{bal, +} \delta_{t, \omega}^{bal, +} + \hat{c}_{t, \omega}^{bal, -} \delta_{t, \omega}^{bal, -} \right. \\ & \left. + \hat{c}^{B, deg} \left(P_{t, \omega}^{B, ch} + P_{t, \omega}^{B, dis} \right) + \hat{c}^{spill} q_{t, \omega}^{spill} \right] \Delta_t \\ & - \sum_{t \in \mathcal{T}^{near}} \left(\hat{c}^{PSH, turb, start} N_{t, \omega}^{turb, start} + \hat{c}^{PSH, pump, start} N_{t, \omega}^{pump, start} \right) \end{aligned} \quad (4.42)$$

When the symmetric formulation in Equation (4.19) is used, the first two terms inside the bracket are replaced by $\hat{c}_{t, \omega}^{bal} \delta_{t, \omega}^{bal, abs}$. Duration Δ_t converts powers and flows into interval energy or volume, whereas start-up costs are event costs and are therefore not multiplied by duration.

Equations (4.43) and (4.44) define the weekly and monthly coarse-layer components.

$$\Phi_{\omega}^W = \sum_{w \in \mathcal{W}^{plan}} \left[\hat{\pi}_{w,\omega}^{DA,avg} E_{w,\omega}^{sell} - \hat{c}_{w,\omega}^{dev,W} \left(\delta_{w,\omega}^{W,+} + \delta_{w,\omega}^{W,-} \right) \right] \quad (4.43)$$

$$\Phi_{\omega}^M = \sum_{m \in \mathcal{M}^{plan}} \left[\hat{\pi}_{m,\omega}^{DA,avg} E_{m,\omega}^{sell} - \hat{c}_{m,\omega}^{dev,M} \left(\delta_{m,\omega}^{M,+} + \delta_{m,\omega}^{M,-} \right) \right] \quad (4.44)$$

The sale and mismatch variables in Equations (4.43) and (4.44) are already aggregate energy quantities, so no duration multiplier is required. The mismatch coefficients are obtained by applying the same imbalance-price rule to the corresponding scenario-dependent average day-ahead price.

The terminal component Φ_{ω}^{term} depends on the controller variant. It is zero when no terminal value is used or when terminal treatment is imposed only through hard constraints. For a value-function variant, Equation (4.45) adds the approximate continuation value of the closing state. Its positive contribution values water retained for use beyond the explicit horizon, so the maximization trades current revenue against estimated future opportunity. In the main Stochastic+Value Function controller, this term enters the daily planner and influences operational decisions through reservoir targets; it is not added directly to the rolling operational objective or counted as realized market revenue. Section 4.4.10.2 describes its estimation, and Section 5.3 reports the calibration.

$$\Phi_{\omega}^{term} = \hat{V}_{\theta}(x_{t_0+H,\omega}) \quad (4.45)$$

Section 4.4.10 defines this approximation and its calibration. The term affects planning decisions but is not counted as realized cashflow.

Cashflows associated with already-cleared quantities are fixed at t_0 . They matter for accounting but cannot affect the optimizer's choice. Equation (4.46) states this fixed component after the decision-dependent objective.

$$\begin{aligned} \Phi^{fix} = & \sum_{t \in \mathcal{T}^{DA,fix}} \hat{\pi}_t^{DA,fix} \bar{Q}_t^{DA} \Delta_t \\ & + \sum_{s \in \mathcal{Q}^{IDA}} \sum_{t \in \mathcal{T}^{s,fix}} \hat{\pi}_t^{s,fix} \bar{\Delta}_t^s \Delta_t \end{aligned} \quad (4.46)$$

Here $\hat{\pi}_t^{DA,fix}$ and $\hat{\pi}_t^{s,fix}$ are the clearing prices associated with the inherited day-ahead position $\bar{Q}_t^{DA}$ and intraday correction $\bar{\Delta}_t^s$, respectively. The complete accounting value is $\Phi^{tot} = \Phi^{fix} + \Phi^{var}$, but adding Φ^{fix} to Equation (4.40) would not change the selected policy.

The turbine and pumping start-up coefficients are simplified operating-cost proxies. Savin et al. (2020) review start-stop cost estimation for hydropower units, while Eggen and Belsnes (2023) show that detailed hydropower operating costs may depend on unit condition, start-stop history, ramping, part-load operation, and other asset-specific mechanisms. A symmetric turbine and pumping start-up coefficient can serve as a tractable proxy for discouraging unnecessary switching, but it is not a physical statement that turbine generation and pumping cause identical degradation. The numerical value used in the experiments is reported in Chapter 5.

4.4.8 Nonanticipativity

The formulation is stochastic, but only those decisions that are executable or become commercially binding at the current information node must be common across scenarios. Let n_0 denote the information node induced by $\mathcal{F}_{t_0}$, and let $\Omega(n_0) \subseteq \Omega$ be the set of scenarios branching from that node.

Let $\mathcal{X}^{exec}$ collect the variable families implemented at the current interval: $P^{B,ch}$, $P^{B,dis}$, $P^{PV,use}$, $P^{PSH,turb}$, $P^{PSH,pump}$, q^{turb} , q^{pump} , q^{spill} , u^B , u^{turb} , u^{pump} , n^{turb} , n^{pump} , $N^{turb,start}$, and $N^{pump,start}$. Equation (4.47) makes every current physical action node-common.

$$x_{t_0,\omega} = x_{t_0,\omega'} \quad \forall x \in \mathcal{X}^{exec} \quad \forall \omega \in \Omega(n_0) \quad \forall \omega' \in \Omega(n_0) \quad (4.47)$$

The interpolation weights are internal approximation variables rather than executable controls; equality of the current physical powers, flows, and mode variables is sufficient to make the implemented action common across scenarios.

If the current step coincides with a market gate, Equation (4.48) makes the day-ahead quantities committed at that gate node-common.

$$Q_{t,\omega}^{DA} = Q_{t,\omega'}^{DA} \quad \forall t \in \mathcal{T}^{DA,gate} \quad \forall \omega \in \Omega(n_0) \quad \forall \omega' \in \Omega(n_0) \quad (4.48)$$

Equation (4.49) imposes the corresponding generic condition for all three intraday auctions.

$$\Delta_{t,\omega}^s = \Delta_{t,\omega'}^s \quad \forall s \in \mathcal{A}^{IDA} \quad \forall t \in \mathcal{T}^{s,gate} \quad \forall \omega \in \Omega(n_0) \quad \forall \omega' \in \Omega(n_0) \quad (4.49)$$

Later recourse decisions within the horizon may differ across scenarios, because they are neither implemented immediately nor entered into the commitment book at time t_0 . They remain contingent plans that will be reconsidered after the next state observation and the next forecast update. The formulation does not impose nested nonanticipativity at every possible future information node. A full multistage scenario tree would represent gradual information revelation more explicitly, but it would also increase the number of variables and linking constraints in every rolling solve. The adopted approximation is retained for tractability, with the understanding that scenario-dependent future recourse may slightly overstate downstream flexibility and can influence the current decision through the look-ahead objective.

4.4.9 Daily Planning Target and Rolling Operational Solution

To avoid placing the complete multi-resolution horizon inside every 15-minute control solve, the adopted planning–control decomposition refreshes the full planning problem at a daily gate and transfers reservoir guidance to the short rolling problems. An initial planning solve supplies targets at simulation start.

At the daily day-ahead gate, the planner maximizes $\Phi^{plan} = \Phi^{var}$ over the complete multi-resolution horizon in Equation (4.40). Variants without terminal value set $\Phi_{\omega}^{term} = 0$ and impose

no reservoir terminal floor; the value-function variant uses Section 4.4.10. The solve returns one target R_v^{tar} for each rolling decision index v over the subsequent 24-hour control period, measured at the end of its seven-day operational horizon.

The operational solve at rolling decision index v then uses only the detailed two-day region and the five-day sequential hourly region. The weekly and monthly planning packs are removed from this executable control problem. Instead, Equation (4.50a) introduces their storage signal as a one-sided soft terminal floor, while Equation (4.50b) restricts the shortfall slack to be nonnegative.

$$V_{Top,\omega}^{res} + s_{v,\omega}^{res} \geq R_v^{tar} \quad \forall \omega \in \Omega \quad (4.50a)$$

$$s_{v,\omega}^{res} \geq 0 \quad \forall \omega \in \Omega \quad (4.50b)$$

The operational objective is $\Phi^{op} = \sum_{\omega \in \Omega} \hat{p}_{\omega} (\Phi_{\omega}^{mkt} + \Phi_{\omega}^{near} - c^{tar} s_{v,\omega}^{res})$, where c^{tar} is the penalty applied to terminal reservoir shortfall below the daily planning target. Ending above R_v^{tar} is allowed and is not penalized. This makes the target a guidance signal rather than a hard feasibility requirement.

The piecewise-linear terminal water value belongs only to the daily planner. The operational MILP receives the resulting soft floor R_v^{tar} , not the value function itself. This keeps the executable problem smaller while transmitting longer-term reservoir guidance. Chapter 5 reports the numerical and computational settings.

4.4.10 Terminal-Value Extension

In the value-function variant, the daily planner values its terminal reservoir state while retaining the BESS non-depletion condition. The planning trajectory is transferred through the soft targets above, and Figure 4.7 identifies the continuation omitted beyond the finite planning horizon.

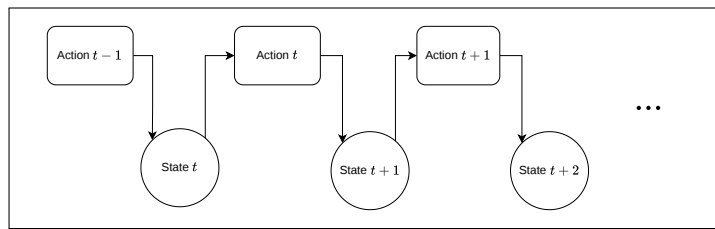


Figure 4.7: Schematic view of a finite explicit horizon embedded in an indefinitely continuing stochastic process. The rolling optimization represents only a finite visible segment and must assign an approximate continuation value to the terminal state left at the boundary of that segment.

The terminal approximation assigns the omitted downstream value to the closing reservoir state, preventing costless boundary depletion.

4.4.10.1 Continuation-Value Approximation

Let $x_{t_0+H,\omega}$ be the terminal state when the problem is solved at t_0 under scenario ω . Equation (4.51) augments the finite-horizon objective with its ideal continuation value V^* .

$$\max_{u_0} \sum_{\omega \in \Omega} \hat{p}_\omega [J_{t_0,\omega}^{\text{hor}}(u_0) + V^*(x_{t_0+H,\omega})] \quad (4.51)$$

where $J_{t_0,\omega}^{\text{hor}}$ is the explicit-horizon contribution, u_0 collects the rolling decisions, and $V^*(x)$ is the expected value of operating beyond the boundary from state x .

Because V^* depends on a high-dimensional physical, commercial, and contextual state, it is replaced by the low-dimensional parametric approximation $\hat{V}_\theta(x_{t_0+H,\omega})$, where θ denotes a finite set of parameters to be estimated. Equation (4.52) gives the resulting generic rolling objective.

$$\max_{u_0} \sum_{\omega \in \Omega} \hat{p}_\omega [J_{t_0,\omega}^{\text{hor}}(u_0) + \hat{V}_\theta(x_{t_0+H,\omega})] \quad (4.52)$$

The term $\hat{V}_\theta$ is an internal planning correction, not realized cashflow. It values the terminal state inside the daily planner, while reported profit remains the cash outcome of the simulated closed-loop trajectory.

The approximation remains MILP-compatible and permits marginal water values to vary by storage region rather than imposing a hard floor or constant salvage coefficient.

Terminal storage value may vary with season, hydrology, and price regime. Let $\sigma(t_0 + H)$ denote a coarse contextual index and write $\hat{V}_\theta(x_{t_0+H,\omega}) = \hat{V}_{\theta_{\sigma(t_0+H)}}(x_{t_0+H,\omega})$, so that different parameter sets may be associated with season, hydrological conditions, or price regime when supported by the calibration. The index does not impose a decomposition *a priori*; contexts are pooled when their estimated values do not differ materially.

The approximation family considered for $\hat{V}_\theta$ is piecewise linear. Piecewise-linear functions can be embedded naturally in a MILP, while still allowing the marginal value of storage to vary across different regions of the terminal state space. Rather than working directly on the full terminal state, the reduced representation is $z_{t_0+H,\omega} = g(x_{t_0+H,\omega})$, where $g(\cdot)$ selects or aggregates the terminal features judged most relevant for downstream value. The terminal term is then written as $\hat{V}_\theta(x_{t_0+H,\omega}) = \hat{V}_\theta(z_{t_0+H,\omega})$. Equation (4.53) gives a generic piecewise-linear representation.

$$\hat{V}_\theta(z) = \sum_{k \in \mathcal{K}^{\text{val}}} \lambda_k^{\text{val}}(z) v_k \quad (4.53)$$

where v_k are nodal values or segment parameters and $\lambda_k^{\text{val}}(z)$ are interpolation weights over the selected breakpoints. The weights are nonnegative, sum to one, reproduce z from the breakpoint coordinates, and are restricted to adjacent breakpoints. The reduced state may include several storage variables, but the numerical experiments retain only reservoir volume because the BESS is small relative to the hydro reservoir. Chapter 5 reports the selected context partition and final coefficients.

4.4.10.2 Estimation of the Approximation

The parameters θ are not known *a priori*. Let $\mathcal{K}^{train}$ be a set of training trajectories and $\Pi_k(\theta)$ the realized cash profit on trajectory k when the controller uses $\hat{V}_\theta$. Equation (4.54) gives a conceptual policy-level selection criterion.

$$\theta^* \in \arg \max_{\theta \in \Theta} \sum_{k \in \mathcal{K}^{train}} \Pi_k(\theta) \quad (4.54)$$

In Equation (4.54), $\Pi_k(\theta)$ contains realized receipts minus realized operating, degradation, start-up, and deviation costs. It excludes $\hat{V}_\theta$ from revenue: the candidate function affects profit only indirectly through the selected policy, avoiding circular credit during evaluation.

This criterion is conceptual rather than the adopted calibration because evaluating every candidate would require many rolling MILP simulations and produce an irregular integer-policy profit map.

The adopted continuation-value sweep treats the planning boundary as the start of a downstream problem. For each sampled date and forecast issue, that problem is solved from several initial reservoir volumes $V^{0,j}$ with a common terminal reference $\bar{V}^{ref}$, giving the optimized downstream objective $J_j = J(V^{0,j}; \bar{V}^{ref})$. With a common terminal reference, differences in J_j isolate the downstream value of additional initial water, estimated by Equation (4.55).

$$\hat{c}_j^{water} = \frac{J_j - J_{j-1}}{V^{0,j} - V^{0,j-1}} \quad V^{0,j} > V^{0,j-1} \quad (4.55)$$

The estimate measures downstream objective per additional cubic metre at the planning boundary, where the terminal state becomes the initial state of the omitted problem.

Slope samples are aggregated by reservoir band and any retained context to define the piecewise-linear marginal coefficients. Decreasing marginal values produce a concave integrated value in which water is most valuable at low storage. Chapter 5 reports the calibration.

A second sweep fixes initial storage and varies the terminal floor. It measures the in-horizon cost of stricter closing requirements rather than downstream continuation value, so it is used only as a feasibility and sensitivity diagnostic.

Chapter 5

Results

The numerical study evaluates the framework from Chapter 4 in closed loop. Each controller implements only its executable decisions before the physical and commercial states advance with realized data. Performance is therefore measured from annual realized operation rather than the objective of one optimization problem.

The comparisons provide the numerical evidence used in Chapter 6 to answer the three research questions. The Deterministic and Stochastic controllers isolate the effect of retaining a reduced uncertainty representation within the same planning-target architecture. The Stochastic and Stochastic+Value Function controllers isolate the incremental effect of terminal water valuation in the daily planning layer, while the benchmark cases provide additional comparative context. Computational viability is assessed through annual runtimes, achieved optimality gaps, and scenario-count diagnostics.

5.1 Experimental Design

All controllers are evaluated from 1 January to 31 December 2025 at 15-minute resolution, giving 35,040 rolling decisions on the same realized price, PV, and inflow path. At each step, the controller observes the current states, inherited commitments, and forecast issue before implementing the next action and newly binding market quantities. Each controller therefore produces one annual trajectory. The three stochastic scenarios are conditional inputs to each solve; annual metrics are not scenario averages, repeated simulated years, or Monte Carlo estimates.

The three main controllers use Section 4.4.9. Planning targets are initialized at the midnight simulation start and refreshed at the daily day-ahead gate at 12:00. Each planning MILP generates 96 reservoir targets for the subsequent 24-hour control period. The operational horizon contains two 15-minute days followed by five hourly days. The daily planning layer extends the look-ahead through eight weekly packs and ten monthly packs and supplies a seven-day terminal reservoir target for each operational decision in the control period.

The benchmarks instead solve two detailed and five hourly days without weekly, monthly, or target variables. The Operational Benchmark imposes terminal storage non-depletion, while

the Terminal-value Benchmark applies the calibrated water value directly to the short horizon. Planning and gate solves use a 0.1% relative gap target and 60 s limit; ordinary non-gate solves use 0.5% and 20 s. Any incumbent and achieved gap at the time limit are retained for computational analysis.

Table 5.1: Common simulation and optimization settings for the annual experiments.

Setting	Value used in the experiments
Simulated period	1 January–31 December 2025
Native rolling interval	15 minutes
Number of rolling decision times	35,040
Daily planning continuation, main controllers	8 weekly packs and 10 monthly packs
Benchmark horizon	2 detailed days and 5 hourly days; no weekly/monthly packs
Rolling operational horizon, main controllers	2 detailed days and 5 hourly days; no weekly/monthly packs
Terminal guidance in main operational solves	Soft reservoir floor derived from the daily planning target
Solver	Gurobi through Pyomo
MILP solve policy	0.1% relative gap target for daily planning and gate-time solves; 0.5% for ordinary non-gate operational solves; 20 s ordinary operational limit; 60 s planning and gate-time limit
Stochastic scenario count	3 retained joint scenarios
Scenario-reduction method	Fast-forward reduction on joint scenario features
Hardware	AMD Ryzen 5 2600 CPU, 6 cores / 12 logical processors, 16 GB RAM, Windows 10

Table 5.2 defines the five controller families to be compared. The names used consistently in the results are *Deterministic*, *Stochastic*, *Stochastic+Value Function*, *Operational Benchmark*, and *Terminal-value Benchmark*. The first three are the main controller variants. They isolate the effect of deterministic versus stochastic information and the incremental effect of adding calibrated water valuation to the daily planning layer under the same physical model, market sequence, operational horizon, and solver policy. The last two are reference-inspired benchmarks. They do not use the planning-target decomposition; instead, they solve the short control horizon directly with either a fixed terminal convention or a direct piecewise-linear terminal water value. Their exact benchmark formulations are given in Appendix B.

The clean within-architecture contrasts are Deterministic versus Stochastic and Stochastic versus Stochastic+Value Function. The two reference-inspired benchmarks provide context rather than pure one-factor ablations because they also differ in horizon, terminal treatment, and target-transfer logic. They test whether short-horizon joint dispatch or direct terminal valuation is sufficient without the planning-target architecture, but do not attribute the resulting gap to one component.

Table 5.2: Controller families defined for the annual rolling comparison.

Case	Forecast representation	Long-horizon or terminal treatment	Operational terminal treatment
Deterministic	Single representative trajectory	Full daily planning horizon without calibrated water value	Soft reservoir floor at the end of the operational horizon
Stochastic	Three-scenario joint representation	Full daily planning horizon without calibrated water value	Same soft reservoir floor
Stochastic+Value Function	Three-scenario joint representation	Full daily planning horizon with calibrated piecewise-linear reservoir value	Same soft reservoir floor; no direct value function in the operational solve
Operational Benchmark	Single representative trajectory	No daily planning target; no weekly/monthly packs; see Appendix B.1	Direct fixed storage non-depletion convention
Terminal-value Benchmark	Three-scenario joint representation	No daily planning target; no weekly/monthly packs; see Appendix B.2	Direct calibrated piecewise-linear terminal water value

5.2 Case-Study Parameters

The case study uses an Alqueva-inspired hybrid system composed of PV, BESS, and PSH. The physical and economic parameters are held fixed across all controller families so that differences in realized operation can be attributed to the information structure, terminal treatment, and benchmark design rather than to changes in asset capability.

The PSH representation contains two Alqueva I units and two Alqueva II units operating under a shared hydraulic head approximation. Total turbine capacity is 519.2 MW and total pumping capacity is 433.8 MW. The upper reservoir usable range is 830–4150 hm³, with an initial volume of 3818 hm³, corresponding to 90% of usable capacity. The BESS has 1 MW charge power, 1 MW discharge power, 2 MWh energy capacity, and an initial state of charge of 1 MWh.

The variable-head PSH approximation uses the flow and head grids in Table 5.3. Each mode uses three positive flow points and three head points. A zero-flow row is included internally, so each mode has 12 operating points in the MILP. This approximation preserves the main dependence of power conversion on flow and hydraulic head while keeping the resulting optimization problem linear and repeatedly solvable. The reduced grid is used as a tractability choice for the repeated stochastic annual simulations.

Table 5.3: Grid values and efficiency ranges used in the variable-head pumped-storage approximation.

Grid	Values	Unit
Turbine flow	0.172, 4.586, 9.000	10 ⁶ m ³ /h
Pumping flow	0.530, 3.865, 7.200	10 ⁶ m ³ /h
Head	54.700, 63.850, 73.000	m
Turbine efficiency samples	0.8533–0.8707	scalar
Pumping efficiency samples	0.8540–0.8676	scalar

Table 5.4 records the remaining fixed inputs and identifies their basis. The physical representation follows the Alqueva hybrid-system context described by EDP and EDIA (EDP, 2020,

2022; EDIA, S.A., 2025b), while the detailed hydraulic limits follow the STOR-HY case-study parameterization (Kumar et al., 2025; Taglioni et al., 2025). The remaining economic coefficients are modelling assumptions. A symmetric 1000 EUR proxy is applied per turbine or pumping start, compared with the 722 EUR average start/stop estimate reported by Eggen and Belsnes (2023); it discourages switching but is not a calibrated degradation model. Positive and negative residual deviations also use the same unit penalty, retaining balancing as settlement rather than another strategic market.

The annual runs use an imbalance-price multiplier of 2.0. This coefficient determines the economic cost of residual deviations between commercial positions and realized physical delivery. The bid-curve diagnostic in Section 5.6 examines a related but distinct question: whether price-contingent DA submission can represent operating intentions that a single unconditional quantity cannot express.

Table 5.4: Fixed numerical inputs used in the annual case study and their basis.

Quantity	Value	Source or basis
PV peak availability	10 MW	Alqueva hybrid-system reference
BESS charge/discharge power	1 MW / 1 MW	Official Alqueva data
BESS energy range	0–2 MWh; initial energy 1 MWh	Alqueva data and model initialization
BESS charge/discharge efficiency	0.95 / 0.95	Model assumption
BESS throughput cost	2 EUR/MWh	Model assumption
PSH turbine power capacity	519.2 MW	Official Alqueva data
PSH pumping power capacity	433.8 MW	Official Alqueva data
Turbine and pump start cost	1000 EUR/start	Literature-based proxy
Upper reservoir range	830–4150 hm ³ ; initial state 3818 hm ³	Alqueva and STOR-HY case data
Safe head range	52.1–74.0 m	STOR-HY case data
Maximum turbine/pump flow	9.0 / 7.2 hm ³ /h	STOR-HY case data
Spillage cost	0.0001 EUR/m ³	Model assumption
Continuous-intraday purchase premium	10 EUR/MWh above the sell reference price	Model assumption
Imbalance penalty coefficient	Larger of balancing reference price and 2.0 times commercial reference price	Model assumption

The parameterization is PSH-dominated: PV and BESS capacities are small relative to the four reversible units, reflecting the represented Alqueva scale rather than a balanced capacity-expansion design. Reservoir operation, market timing, and stochastic price treatment therefore dominate the annual ranking, although the smaller assets preserve the hybrid electrical coupling. Alternative capacities can be represented without changing the formulation.

The reported revenues assume exogenous day-ahead and intraday-auction prices, a strong abstraction for a 500 MW-class asset. This common price-taking environment isolates controller differences; continuous intraday trading is additionally bounded by tradability and volume. The annual comparison is therefore relative rather than a prediction of strategic trading profit.

Table 5.5 summarizes the realized exogenous series shared by all controllers. Natural inflow enters as a 15-minute volume converted to flow, with water quantities rescaled internally for numerical conditioning. The continuous-intraday row summarizes the carried price series, while executable recourse remains limited by the tradability and liquidity assumptions in Chapter 4.

Table 5.5: Summary statistics of the annual exogenous input series.

Series	Min.	Mean	Median	Max.	Unit
Day-ahead price	-5.00	66.14	70.00	240.00	EUR/MWh
IDA1 price	-15.00	65.52	70.75	240.10	EUR/MWh
IDA2 price	-100.00	65.20	70.01	260.00	EUR/MWh
IDA3 price	-25.63	53.17	52.37	263.47	EUR/MWh
Continuous-intraday price	-174.59	62.66	66.07	337.83	EUR/MWh
PV availability	0.00	2.02	0.02	10.00	MW
Natural inflow q^{nat}	0.000	0.699	0.452	3.159	10^6 m ³ /h

Table 5.6 compares realized 15-minute trajectories on sampled delivery days with the reduced three-scenario fan. Each row is an exogenous quantity evaluated across the fan, not an individual scenario. Envelope coverage is the fraction of realizations between the retained scenario minimum and maximum; weighted-path mean absolute error (MAE) compares realizations with the probability-weighted scenario path; and mean width is the average difference between the retained maximum and minimum. For example, the 76.5% day-ahead coverage means that 76.5% of sampled observations lie inside this envelope. It is neither a scenario probability nor a nominal confidence level.

Table 5.6: Forecast-envelope diagnostics by exogenous quantity for the three retained scenarios.

Quantity	Envelope coverage	Weighted-path MAE	Mean envelope width	Unit
Day-ahead price	76.5%	23.12	80.19	EUR/MWh
PV availability	53.6%	1.02	0.35	MW
Natural inflow q^{nat}	51.1%	662,884	1,254,189	m ³ /h

With three retained scenarios, the fan is a tractable decision input rather than a calibrated predictive interval. Its day-ahead price envelope covers about three quarters of sampled observations, while PV and inflow coverage is roughly one half. The fan therefore exposes the optimizer to directional uncertainty and joint movement without reproducing the full raw distribution. Scenario weights vary by issue, and the annual stochastic gain is conditional on this reduced fan rather than a fully calibrated probabilistic forecast.

5.3 Terminal Water-Value Calibration

Terminal water valuation follows Section 4.4.10.2. For sampled dates, the downstream problem is solved from several initial reservoir levels under the same forecast issue and terminal reference.

Differences in optimized objective estimate the marginal continuation value of additional initial water.

The calibration used 100 dates, split evenly between spring–summer and autumn–winter, and swept initial storage over usable-capacity deciles. A common terminal level within each sweep isolates the effect of additional initial water. Alternative closing references produced the same qualitative pattern, so the midpoint convention supplies the reported coefficients. Each solve used the planning layer’s three scenarios, multi-resolution horizon, and solver, after which marginal estimates were pooled by storage band.

Figure 5.1 shows the seasonal marginal continuation-value estimates. Seasonal differences exist, but they are small relative to the confidence intervals and do not create a clearly distinct seasonal ordering across all bands. The annual experiments use one pooled curve formed from the aggregated decile-band estimates. The controller interpretation remains simple: water has a decreasing marginal continuation value as the reservoir fills, but the value function does not switch discontinuously by season.

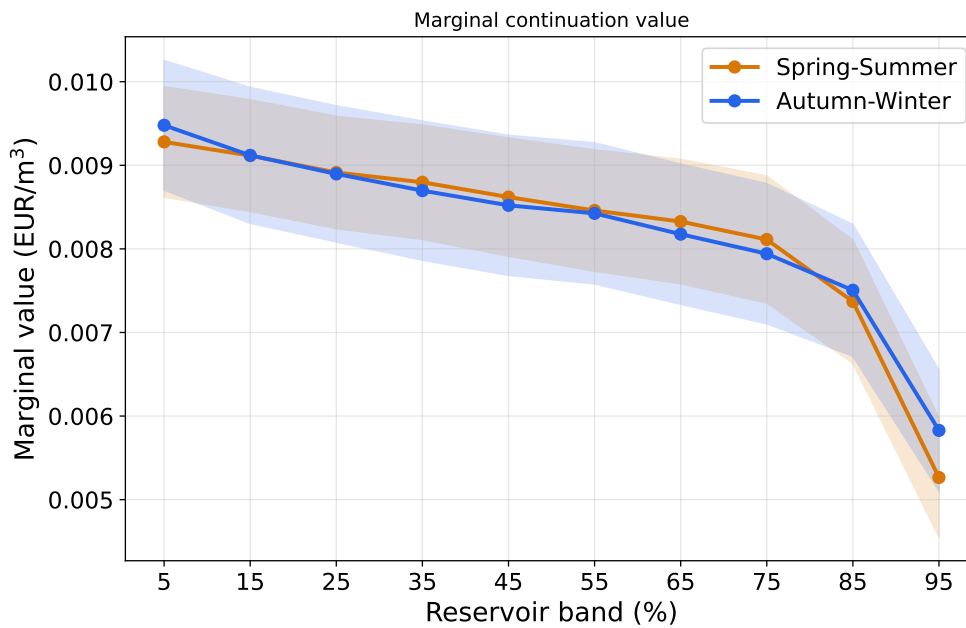


Figure 5.1: Seasonal marginal continuation-value estimates.

Table 5.7 reports the pooled coefficients. Stochastic+Value Function uses them in daily planning and transfers their effect through targets; Terminal-value Benchmark applies the same function directly to the short horizon. Marginal values are shown in EUR/m³ and, using a 63.05 m representative head and 0.90 turbine efficiency, in EUR/kWh. Integrated values are measured from the lower usable-storage bound. Their decreasing marginal pattern produces a concave value function.

Figure 5.2 shows the marginal coefficients and their integrated piecewise-linear value. Appendix D.4 compares smoother alternatives. The adopted form preserves the decreasing marginal

Table 5.7: Pooled terminal water-value coefficients used by the terminal-value controllers.

Reservoir band	Marginal [EUR/m ³]	value	Marginal R/kWh]	value [EU-	Integrated upper breakpoint	value at [M EUR]
0–10%	0.00942		0.06093		3.13	
10–20%	0.00917		0.05932		6.17	
20–30%	0.00896		0.05797		9.15	
30–40%	0.00881		0.05698		12.07	
40–50%	0.00864		0.05584		14.94	
50–60%	0.00850		0.05499		17.76	
60–70%	0.00832		0.05380		20.53	
70–80%	0.00810		0.05237		23.22	
80–90%	0.00746		0.04825		25.69	
90–100%	0.00552		0.03569		27.52	

pattern while remaining directly MILP-compatible.

The fixed-initial-state terminal-floor sweep is only a target-sensitivity diagnostic, as distinguished from continuation calibration in Section 4.4.10.2.

5.4 Comparison of Rolling Controllers

The annual comparison combines economic outcomes with reservoir, battery, dispatch, spillage, and PV utilization indicators so that cash gains are not assessed independently of the terminal physical state. Annual statistics use the full year; detailed operational diagnostics exclude the first day to avoid the empty initial commitment book.

Economic entries are calculated from realized closed-loop actions and settlements. Market revenue accumulates DA and intraday cashflows; net profit subtracts realized imbalance and operating costs. Final reservoir storage is measured over the usable range, and its estimated value applies the calibrated continuation curve relative to the lower usable bound. “Net + reservoir” adds this estimate to cash profit as an *ex post* residual-state diagnostic, not as realized revenue or a forecast-scenario objective.

Table 5.8 reports the five annual runs. Deterministic versus Stochastic isolates the retained uncertainty representation, Stochastic versus Stochastic+Value Function tests terminal valuation in daily planning, and the benchmarks test shorter reference-style architectures. Because final storage differs, the table reports both cash profit and the residual value obtained from Table 5.7.

The reservoir-adjusted value does not define a counterfactual policy: a materially different curve would require re-optimization rather than retrospective repricing.

Figure 5.3 shows annual storage and cumulative profit. The Stochastic and Stochastic+Value Function controllers earn 158.81 and 158.84 M EUR, respectively, while Deterministic earns 151.03 M EUR. They finish at 6.2%, 6.2%, and 6.7% usable storage. Approximately 1036 hm³

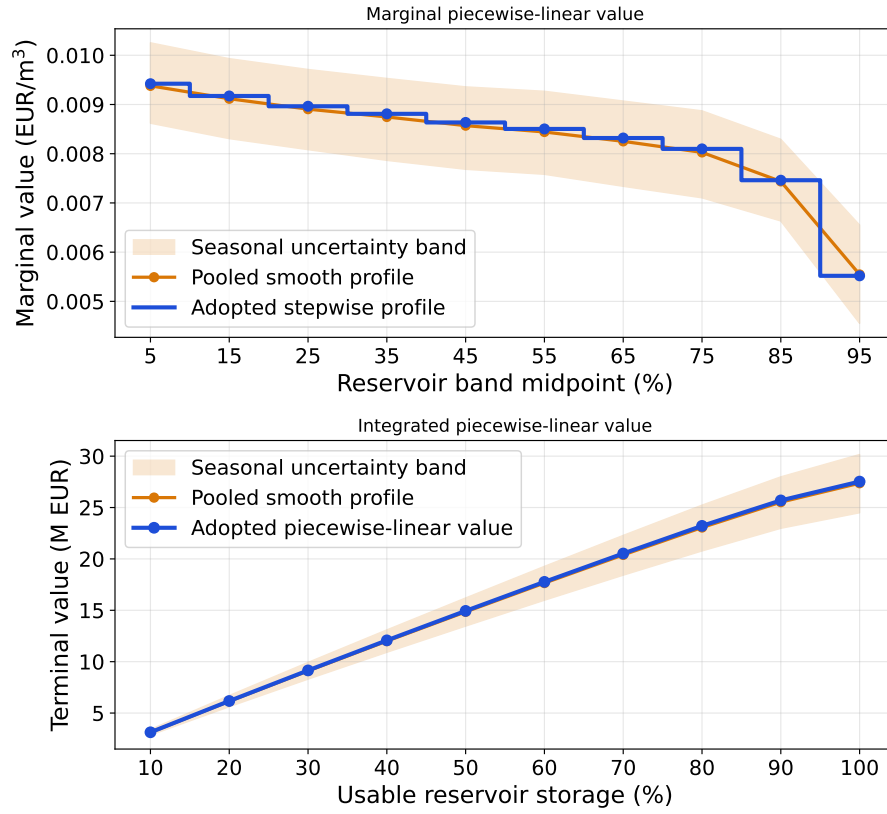


Figure 5.2: Pooled piecewise-linear terminal water-value representation.

Table 5.8: Primary annual comparison of rolling controllers.

Controller	Net profit [M EUR]	Market revenue [M EUR]	Imbalance cost [M EUR]	Operating cost [M EUR]	Final reservoir [% usable]	Reservoir value [M EUR]	Net + reservoir [M EUR]
Deterministic	151.03	153.87	0.52	2.32	6.7	2.11	153.14
Stochastic	158.81	161.80	0.43	2.56	6.2	1.94	160.75
Stochastic+Value Function	158.84	161.80	0.41	2.56	6.2	1.94	160.78
Operational Benchmark	127.44	130.41	0.54	2.43	85.0	24.47	151.90
Terminal-value Benchmark	127.98	148.00	15.80	4.21	23.2	7.12	135.10

remains in the stochastic cases in absolute terms, but these states lie near the lower modelled usable bound. This electricity-only drawdown is not a recommendation for the real multipurpose reservoir.

Because final storage is similar, the stochastic gain is not explained by preserving more water. Adjusted totals are 153.14, 160.75, and 160.78 M EUR for Deterministic, Stochastic, and Stochastic+Value Function, respectively, giving an observed 2025 gain of about 5% for the stochastic formulation.

This is a case-study outcome, not a long-run expected value. Section 5.6 examines how fixed-quantity commitments may compress stochastic price information.

Stochastic and Stochastic+Value Function differ by only 0.03 M EUR, have equal reported revenue and costs, and both finish at 6.2% usable storage. Their annual storage and profit trajectories are correspondingly almost indistinguishable. Section 6.1 discusses this small incremental effect under the adopted planning-target transfer mechanism.

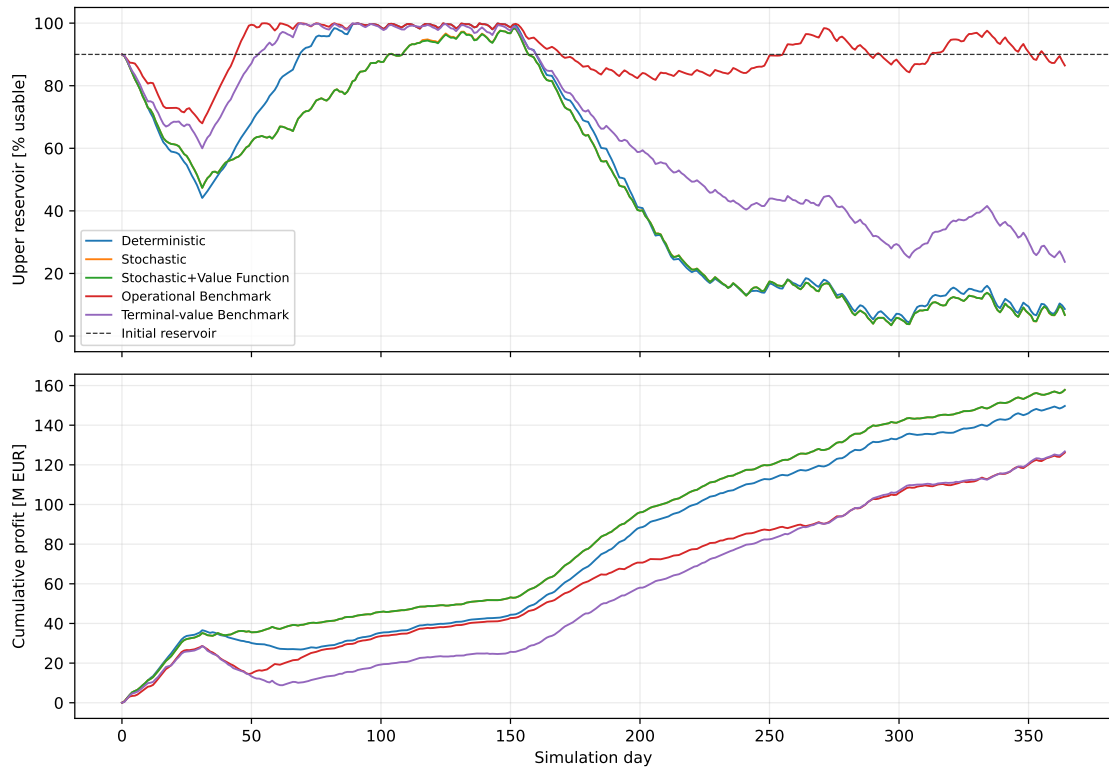


Figure 5.3: Annual reservoir and cumulative-profit trajectories for the five controller families.

Table 5.9 separates revenue, penalties, and operating costs. Relative to Deterministic, Stochastic reduces DA revenue from 198.64 to 185.98 M EUR, a decrease of 12.66 M EUR, while the ID/IDA component improves from -44.77 to -24.17 M EUR, an increase of 20.60 M EUR. The reported market revenue consequently rises from 153.87 to 161.80 M EUR, approximately 7.93 M EUR, and net profit increases from 151.03 to 158.81 M EUR, approximately 7.78 M EUR. Small differences in the component sums reflect rounding. Lower DA revenue is therefore compatible

with higher total revenue because intraday cashflows are substantially less negative; differences in imbalance and operating costs account for the remaining net-profit adjustment. This is an accounting decomposition of the realized annual outcomes, not an isolated causal test of why the stochastic decisions differ.

Table 5.9: Annual revenue decomposition.

Controller	DA revenue [M EUR]	ID/IDA revenue [M EUR]	Imbalance cost [M EUR]	Start cost [M EUR]	BESS cost [M EUR]	Net profit [M EUR]	Net + reservoir [M EUR]
Deterministic	198.64	-44.77	0.52	2.31	0.01	151.03	153.14
Stochastic	185.98	-24.17	0.43	2.55	0.01	158.81	160.75
Stochastic+Value Function	185.94	-24.15	0.41	2.55	0.01	158.84	160.78
Operational Benchmark	179.86	-49.45	0.54	2.42	0.01	127.44	151.90
Terminal-value Benchmark	171.84	-23.84	15.80	4.20	0.01	127.98	135.10

Terminal-value Benchmark incurs 15.80 M EUR in imbalance cost, compared with 0.41–0.54 M EUR for the other controllers, and also has larger start costs. These two components account for most of its lower net profit.

The relationship between the daily planning targets and the realized reservoir trajectories is reported as a supporting diagnostic in Appendix C.1 rather than as a primary controller-performance result.

5.5 Scenario-Count Sensitivity

The annual stochastic simulations retain three scenarios to limit the cost of 35,040 repeated solves. A spring checkpoint tests $N \in \{1, 3, 5, 7, 9\}$ under the same physical state, market context, forecast issue, solver policy, and raw scenario bank. The single-scenario case is the deterministic limit, while $N \geq 3$ retains a stochastic fan.

Figure 5.4 shows that $N = 1$ follows a distinct pumping and turbinning schedule and closes the 48-hour horizon at lower storage. The $N = 3, 5, 7, 9$ trajectories form a tighter cluster. Across all counts, closing storage spans about 1.3 percentage points of usable capacity.

This local result supports the three-scenario design but does not establish annual robustness. It distinguishes one path from a small stochastic fan more clearly than three from the larger retained counts. Appendix C.2 reports the computational effect; validating the annual ranking would require repeated full-year runs or broader checkpoint coverage.

The figure reports conditional plans and closing states, not realized profit: only the first action would be implemented and the remaining trajectory is provisional. A profit comparison would require executing every policy over a common subsequent path.

5.6 Day-Ahead Bid-Curve Diagnostic

The annual controllers commit one unconditional net DA quantity before the clearing price is known. A storage portfolio may instead prefer to buy below the opportunity cost of water, remain

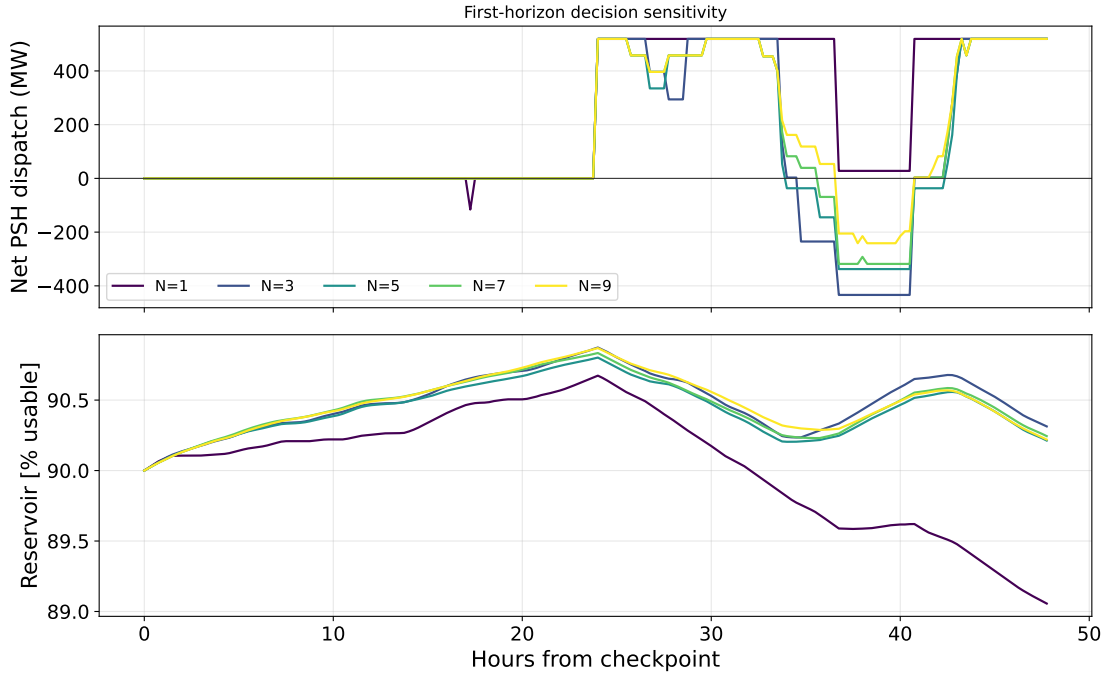


Figure 5.4: Checkpoint sensitivity of the first-horizon policy to the number of retained scenarios.

neutral near it, and sell above it. The reduced-form diagnostic isolates this distinction for one 15-minute product using a price fan and local marginal water values.

This is not a sixth annual controller or an estimate of annual bidding value. It tests only whether a nonanticipative price-contingent submission can retain information compressed by an unconditional quantity.

The price fan contains five equally weighted DA clearing prices, $\{30, 40, 60, 70, 100\}$ EUR/MWh. The separate opportunity-cost model uses $\lambda = 60$ EUR/MWh as the representative stored-energy value, within the range of Table 5.7. With round-trip efficiency 0.754, this corresponds to a pumping breakeven near 45 EUR/MWh. Five pumping slices span 45–37 EUR/MWh and five generating slices span 60–70 EUR/MWh. These slices are local marginal-value inputs, not additional scenarios or an approximation of the complete annual value curve.

Equation (5.1) defines the simpler water-value heuristic using a deadband around the representative value.

$$q^{heur}(p) = \begin{cases} -\sum_{g \in \mathcal{G}} \hat{P}_g^{PSH, pump, max} & p < \lambda - \Delta \\ 0 & \lambda - \Delta \leq p \leq \lambda + \Delta \\ \sum_{g \in \mathcal{G}} \hat{P}_g^{PSH, turb, max} & p > \lambda + \Delta \end{cases} \quad (5.1)$$

Here negative quantities denote pumping purchases and positive quantities denote turbine sales; $\Delta = 10$ EUR/MWh is the deadband half-width and $\mathcal{G}$ indexes the PSH unit groups. With $\lambda = 60$ EUR/MWh, the heuristic buys below 50 EUR/MWh, remains neutral from 50 to 70 EUR/MWh,

and sells above 70 EUR/MWh, but cannot express partial quantities within the marginal-value stack.

The fixed-quantity controller submits one quantity for all five prices, whereas the bid curve submits a monotone function $q(p)$. Low prices can therefore accept pumping, central prices little or no quantity, and high prices generation. Although the fan's expected price provides no directional signal to an unconditional risk-neutral quantity, its dispersion remains usable by the curve.

Table 5.10: Reduced-form single-product day-ahead bid-curve diagnostic.

Policy	Expected value EUR/product	Q(30) MW	Q(40) MW	Q(60) MW	Q(70) MW	Q(100) MW
Fixed quantity	0	0.0	0.0	0.0	0.0	0.0
Water-value heuristic curve	1,158	-433.8	-433.8	0.0	0.0	519.2
Optimized stochastic curve	1,295	-433.8	-260.3	0.0	415.4	519.2

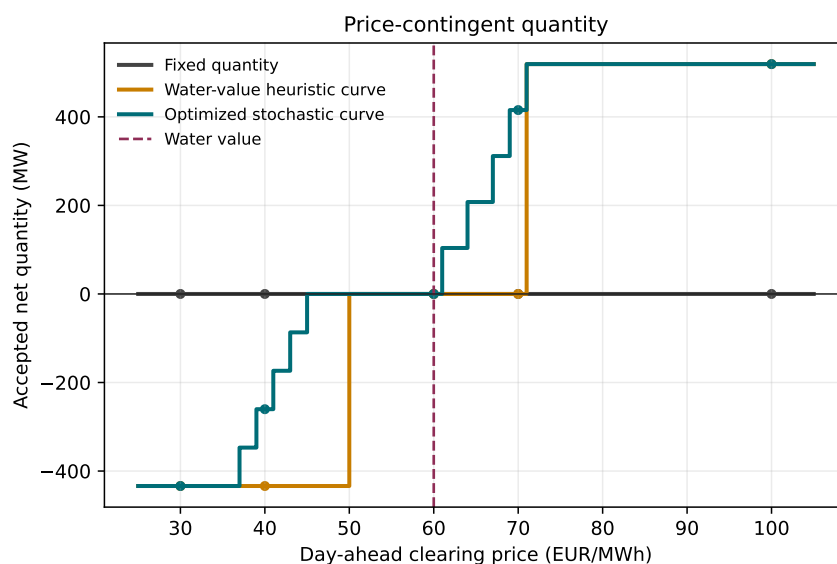


Figure 5.5: Reduced-form single-product DA bid-curve diagnostic. The vertical line marks the representative water value; negative quantities denote net purchase for pumping and positive quantities denote net sale.

Table 5.10 and Figure 5.5 show that the fixed formulation collapses to zero, while the optimized curve buys 260 MW at 40 EUR/MWh, remains neutral at 60 EUR/MWh, and sells 415 MW at 70 EUR/MWh. Expected storage-adjusted value rises from zero to 1,295 EUR for the 15-minute product. The diagnostic therefore demonstrates the local value of price-contingent quantities when the unconditional expected-price decision is neutral.

The fixed formulation may observe several price and physical scenarios but must compress them into one inherited DA commitment. Later imbalance or intraday correction cannot make that accepted quantity price-contingent. The bid curve instead conditions the cleared quantity on

the market price, aligning low-price states with pumping or neutrality and high-price states with generation.

5.7 Computational Performance

Computational performance is assessed through annual runtime, per-step runtime, MILP time, and achieved relative gap across the repeated rolling solves.

The common solver policy is defined in Section 5.1. Stored per-step runtime separates the complete rolling step, including forecast preparation and simulator updating, from MILP time. In Table 5.11, total time is the sum of recorded runtimes for all 35,040 steps of the annual 15-minute simulation, representing 8,760 hours of simulated operation. It excludes pauses between execution sessions and is distinct from simulated duration. For example, Stochastic requires approximately 61.93 hours of recorded execution, or 6.36 seconds per rolling step. The table also reports the mean final gap across solves.

Table 5.11: Computational-performance summary for the annual rolling simulations.

Controller	Total time [h]	Mean step time [s]	Mean MILP time [s]	Max MILP time [s]	Mean gap [%]
Deterministic	12.00	1.23	1.12	3.02	0.020
Stochastic	61.93	6.36	3.03	16.59	0.024
Stochastic+Value Function	61.96	6.37	3.04	15.00	0.024
Operational Benchmark	12.07	1.24	1.15	3.41	0.029
Terminal-value Benchmark	55.65	5.72	3.08	22.32	0.039

Deterministic and Operational Benchmark are faster because they use one representative trajectory; the other cases retain scenario-dependent structure. Mean gaps remain below the configured tolerances, while Terminal-value Benchmark has the largest maximum solve time and mean gap. Appendix C.2 reports the separate scenario-count experiment.

Chapter 6

Conclusion

This chapter answers the research questions, states the contributions, and identifies the assumptions that limit the results.

6.1 Main Findings

The answers use realized closed-loop trajectories rather than isolated optimization objectives. Numerical differences are outcomes of the simulated 2025 path, not expectations over internal forecast scenarios.

For RQ1, retaining the reduced stochastic representation improved the observed annual outcome relative to the deterministic representative-trajectory controller. Under the same planning-target architecture, the Stochastic controller earned 7.78 M EUR more realized net profit and 7.61 M EUR more after the final reservoir adjustment, while both controllers ended with similarly low reservoir storage. The revenue decomposition associates this difference primarily with less negative intraday revenue rather than with lower operating costs. This result supports the use of stochastic information in the case study, but does not establish a general expected gain across years or operating trajectories. The bid-curve diagnostic further shows that uncertainty representation and market representation are connected: unconditional DA quantities compress scenario-dependent price information into one commitment, whereas a price-contingent submission can preserve part of that information.

For RQ2, adding the calibrated terminal water-value function to the stochastic daily planning layer did not materially change reservoir management or annual value. The Stochastic+Value Function controller differed from the Stochastic controller by only 0.03 M EUR in net profit, and both finished at 6.2% usable reservoir storage. The value function is therefore supported as an interpretable modelling and diagnostic component, but not as a demonstrated source of additional closed-loop value under the adopted target-transfer mechanism. This conclusion concerns the implementation tested here: once converted into daily reservoir targets, the marginal continuation signal acted mainly as a tunable storage-preservation mechanism.

For RQ3, the multi-resolution planning and rolling-control architecture was computationally tractable for the annual case study. All five controller runs completed the 35,040-step simulation. The stochastic planning-target variants required approximately 3.0 s of mean MILP time per rolling step, well below the 15-minute control interval, and their mean reported gaps were 0.024%. This demonstrates practical solvability for the retained three-scenario representation on the reported hardware, not unrestricted scalability. The checkpoint experiment shows that solve time rises materially as scenarios are added, so richer uncertainty representations remain constrained by the repeated-solution budget.

6.2 Contributions

The methodological contribution is the integration of hybrid-asset operation, sequential market commitments, joint uncertainty, and intertemporal storage guidance within an executable rolling framework. The formulation in Section 4.4 couples photovoltaic dispatch, battery energy, and variable-head pumped storage with day-ahead commitments, intraday corrections, and residual imbalance costs. It distinguishes inherited commitments, currently binding decisions, and provisional future recourse, while joint price, photovoltaic, and inflow paths provide multivariate uncertainty inputs on a common time grid. The contribution concerns the conjunction of these established components in the operating problem identified by the literature review.

The planning–control decomposition in Section 4.4.9 provides a mechanism for transmitting longer-term reservoir guidance to repeated short-horizon decisions. Daily multi-resolution planning supplies soft reservoir targets, while the operational problems retain chronological dispatch and market constraints and are reissued as information changes. This architecture connects coarse long-horizon storage opportunity with executable near-term operation without solving the complete planning problem at every 15-minute interval.

The terminal-water-value procedure provides an estimated continuation signal tied to reservoir storage. The calibration in Section 5.3 derives marginal values from downstream optimization sweeps and represents them through a piecewise-linear function. Its comparison with the otherwise equivalent stochastic controller also establishes an empirical limitation: adding this signal to the daily planner produces little additional annual value under the tested target-transfer mechanism. The contribution is therefore the estimation procedure and its assessed role in the architecture, rather than a demonstrated profit improvement from terminal valuation.

The closed-loop evaluation framework supplies complementary empirical evidence. The controller comparison in Section 5.4 uses a common realized annual path and assesses cash profit alongside revenue components, final storage, and computational performance. Section 5.5 examines local decision sensitivity to the retained scenario count, and Section 5.6 isolates how fixed-quantity submissions compress price-dependent operating intentions. Together, these analyses identify both the observed benefit of stochastic information and the conditions that limit its practical expression in market decisions.

6.3 Relation to the Sustainable Development Goals

The contribution to the United Nations Sustainable Development Goals (SDGs) lies in the operational layer of the energy transition. The analysis does not address access policy or infrastructure investment directly, but it studies how renewable generation and storage can be operated more efficiently once such assets are available. This connection matters because the transition to sustainable electricity systems depends not only on installing low-carbon technologies, but also on using them in a way that preserves reliability, reduces avoidable waste, and makes flexibility available when the system needs it (United Nations, 2015).

The strongest connection is with SDG 7, which concerns access to affordable, reliable, sustainable, and modern energy (United Nations Department of Economic and Social Affairs, 2025). Coordinated operation of PV, BESS, and PSH assets can help convert variable renewable production into a more controllable energy resource. By scheduling generation, pumping, battery use, and market commitments jointly, the proposed framework supports a more efficient use of existing clean-energy infrastructure. In practical terms, this type of decision support can help shift renewable energy from low-value or surplus periods to periods of higher demand, reduce unnecessary curtailment, and improve the economic value of storage-backed renewable production.

The analysis also relates to SDG 13 on climate action. Flexible low-carbon resources are important for power systems with high shares of variable renewable generation, because they can reduce the need for fossil-fuelled balancing and peaking resources when renewable output and demand do not coincide. A rolling stochastic controller does not by itself guarantee emissions reductions, since those depend on the wider generation mix and market design. By improving the way renewable-storage portfolios respond to uncertainty, the methodology supports the broader technical conditions required for deeper renewable integration and lower-carbon electricity operation.

6.4 Limitations

The participant is modelled as a price taker, which may be restrictive for a large storage asset in low-liquidity hours. The annual baseline also uses unconditional quantities rather than complete price-quantity bid curves. The diagnostic in Chapter 5 isolates this interface effect, and Appendix D.2 outlines an explicit price-taking formulation, but neither treatment represents price impact or strategic market clearing.

The intraday representation is also simplified. Auctions are signed correction layers, while continuous trading is a bounded current-step correction. This preserves market chronology but omits order-book depth, bid–ask dynamics, execution risk, and price impact.

Historical residual sampling preserves empirical temporal structure, while joint scenario assembly approximates cross-variable dependence. The representation depends on the historical record and the scenario-selection rules. The three retained scenarios are a tractability choice, not a calibrated predictive distribution, and the forecast diagnostic indicates only central-band coverage

for PV and inflow. The physical representation is likewise a compact variable-head approximation, while start and degradation costs are proxies rather than plant-specific maintenance models.

The stochastic formulation is a rolling two-stage approximation rather than a complete multistage scenario tree. Current executable actions and gate-binding quantities are nonanticipative, whereas later plans may differ by scenario because they are recomputed before execution. This can overestimate future flexibility; nested nonanticipativity would be stricter but materially larger.

The numerical comparison covers one realized 2025 trajectory and is therefore a closed-loop case study, not a statistical estimate of long-run performance. The approximately 5% reservoir-adjusted stochastic gain may depend on that year's volatility, hydrology, forecast quality, and price regime. Generalization requires multiple historical or coherent synthetic out-of-sample years that preserve dependence among prices, renewable availability, and inflow.

The Alqueva-inspired reservoir is represented as an electricity-storage state, not as a multipurpose water reserve. Irrigation, urban supply, environmental flows, drought security, and institutional allocation rules are excluded. The low final storage levels therefore express an electricity-only valuation and are not an operational policy for the real infrastructure.

The piecewise-linear terminal value depends on sampled dates, scenarios, storage bands, and the pooling rule. It approximates average continuation value rather than a fully state-contingent marginal value and may smooth scarcity effects at low storage.

The horizontal weekly and monthly packs suppress intermediate storage states and are not executable chronological schedules. Their usefulness is supported indirectly by closed-loop performance, not by equivalence to a full chronological long-horizon solution. Formal validation would compare their targets with smaller high-resolution planning solves or add conservative intra-pack restrictions.

The imbalance-price multiplier and continuous-intraday liquidity cap simplify balancing exposure, which in practice depends on portfolio and system imbalance direction, settlement rules, and market conditions. All controllers face the same approximation, but absolute profits and possibly part of their ranking remain conditional on it.

Reserve and ancillary services are excluded. Because reserve capacity competes with arbitrage for power, energy, and water headroom, the reported energy-market-plus-imbalance profits do not represent the full market value of the hybrid portfolio.

Computational limits constrain scenario count and temporal detail. Network constraints, detailed outages, environmental restrictions, and reserve obligations are also excluded. Appendix C reports computational diagnostics, while Appendix D.3 sketches a reserve extension.

6.5 Future Work

The most direct extension is a complete stochastic bidding controller with market-compliant bid formats, acceptance rules, settlement conventions, and annual closed-loop evaluation. Appendix D.2 gives the price-taking formulation direction.

If the portfolio can affect prices, volume-dependent price impact, residual-demand approximations, order-book models, or strategic market-clearing formulations would be required instead of the price-taking MILP.

Reserve co-optimization would add capacity, activation, and deliverability constraints, as outlined in Appendix D.3, and test whether remunerated flexibility changes the low terminal-storage outcomes.

A production-oriented reservoir model should add seasonal storage limits, strategic reserves, environmental flows, irrigation schedules, drought penalties, or explicit trade-offs between electricity revenue and water security. These constraints would quantify the cost of preserving water for non-electricity uses.

Forecasting could use trained probabilistic models with stronger treatment of regimes and cross-variable dependence, while risk-averse objectives such as CVaR could address downside exposure. Statistical validation should cover multiple historical or coherent synthetic years. Decomposition, stronger scenario reduction, and improved warm starts could support richer scenario sets and bidding formulations; Appendices D.1, C.2, and C.3 describe these directions.

Terminal value could include battery state, smoother approximations such as Appendix D.4, and a more direct transfer of marginal continuation information into the operational objective. Recalibration using duals from linear-programming relaxations, fixed-integer re-solves, or decomposition cuts could better represent local water scarcity, but should be validated through closed-loop out-of-sample performance across terminal conventions, scenario counts, and hydrological regimes.

6.6 Final Remarks

Hybrid renewable-storage operation is not only a dispatch problem and not only a forecasting problem. It is a sequential decision problem in which physical storage states, uncertain future conditions, market commitments, and settlement consequences interact over several time scales. The proposed framework shows how these elements can be represented in a rolling stochastic MILP with an explicit multi-resolution horizon and calibrated water-value signal. The analysis also highlights that market-interface assumptions matter: stochastic information is most valuable when the commercial decision can express how accepted quantities should change with the realized clearing price. Bridging this gap through explicit bidding, richer intraday liquidity modelling, and stronger probabilistic forecasts is the main step between the research model and an operational decision-support tool.

References

- Aasgård, E. K., Fleten, S.-E., Kaut, M., Midthun, K., and Perez-Valdes, G. A. (2019). Hydropower bidding in a multi-market setting. *Energy Systems*, 10(3):543–565.
- Aasgård, E. K. (2017). Hydropower Bidding Using Linearized Start-Ups. *Energies*, 10(12):1975.
- Baharvandi, A., Aghaei, J., Nikoobakht, A., Niknam, T., Vahidinasab, V., Giaouris, D., and Taylor, P. (2020). Linearized Hybrid Stochastic/Robust Scheduling of Active Distribution Networks Encompassing PVs. *IEEE Transactions on Smart Grid*, 11(1):357–367.
- Borghetti, A., D’Ambrosio, C., Lodi, A., and Martello, S. (2008). An MILP Approach for Short-Term Hydro Scheduling and Unit Commitment With Head-Dependent Reservoir. *IEEE Transactions on Power Systems*, 23(3):1115–1124.
- Conejo, A. J., Carrión, M., and Morales, J. M. (2010). *Decision Making Under Uncertainty in Electricity Markets*, volume 153 of *International Series in Operations Research & Management Science*. Springer US, Boston, MA.
- Diniz, A. L., Esteves, P. P. I., and Sagastizabal, C. A. (2007). A mathematical model for the efficiency curves of hydroelectric units. In *2007 IEEE Power Engineering Society General Meeting*, pages 1–7, Tampa, FL, USA.
- Durvasulu, V., Balliet, W. H., Lopez, C. J., Lin, Y., Li, B., Alam, S. M. S., Mahalik, M. R., Kwon, J., and Mosier, T. M. R. (2024). Rationale for adding batteries to hydropower plants and tradeoffs in hybrid system operation: A review. *Renewable and Sustainable Energy Reviews*, 202:114673.
- EDIA, S.A. (2025a). Alqueva. EDIA Alqueva project page. Available at: <https://www.edia.pt/en/alqueva/> (accessed 2026-06-13).
- EDIA, S.A. (2025b). Alqueva Multipurpose Project. EDIA Alqueva Multipurpose Project page. Available at: <https://www.edia.pt/en/alqueva/multipurpose-alqueva-project/> (accessed 2026-06-13).
- EDIA, S.A. (2025c). The territory. EDIA territorial description of the Alqueva reservoir. Available at: <https://www.edia.pt/en/alqueva/the-territory/> (accessed 2026-06-13).
- EDP (2020). XFLEX HYDRO: Increasing the Flexibility of Hydropower. EDP technical description of the Alqueva I and II reversible units. Available at: <https://www.edp.com/en/innovation/innovation-projects/xflex-hydro-increasing-flexibility-hydropower> (accessed 2026-07-17).

- EDP (2022). Alqueva's Floating Solar Panels. EDP description of the floating photovoltaic and battery installation at Alqueva. Available at: <https://www.edp.com/en/media/edp-stories/alquevas-floating-solar-panels> (accessed 2026-07-17).
- Eggen, A. O. and Belsnes, M. (2023). Operation related maintenance and reinvestment costs for hydropower scheduling. *Energy Systems*.
- Entidade Reguladora dos Serviços Energéticos (2025). Diretiva n.º 7/2025: Aprova o manual de procedimentos da gestão global do sistema do setor elétrico. Manual de Procedimentos da Gestão Global do Sistema do setor elétrico (MPGGS). Available at: https://www.erse.pt/media/jeddlfqg/cpl27_mpggs_vs-pext.pdf (accessed 2026-06-13).
- Erdinç, F. G. (2023). Rolling horizon optimization based real-time energy management of a residential neighborhood considering PV and ESS usage fairness. *Applied Energy*, 344:121275.
- Ergun, S., Dik, A., Boukhanouf, R., and Omer, S. (2025). Large-Scale Renewable Energy Integration: Tackling Technical Obstacles and Exploring Energy Storage Innovations. *Sustainability*, 17(3):1311.
- European Commission (2024a). Electricity Market Design. European Commission energy-market policy page. Available at: https://energy.ec.europa.eu/topics/markets-and-consumers/electricity-market-design_en (accessed 2026-06-13).
- European Commission (2024b). The European Green Deal. European Commission policy page. Available at: https://commission.europa.eu/strategy-and-policy/priorities-2019-2024/european-green-deal_en (accessed 2026-06-13).
- European Commission (2025). European Climate Law. European Commission climate-action page. Available at: https://climate.ec.europa.eu/eu-action/european-climate-law_en (accessed 2026-06-13).
- Fernández-Muñoz, D. and Pérez-Díaz, J. I. (2022). Optimisation models for the day-ahead energy and reserve scheduling of a hybrid wind-battery virtual power plant. arXiv:2206.13784 [eess].
- Filipe, J., Bessa, R. J., Moreira, C., and Silva, B. (2019). Optimal bidding strategy for variable-speed pump storage in day-ahead and frequency restoration reserve markets. *Energy Systems*, 10(2):273–297.
- Finnah, B., Gönsch, J., and Ziel, F. (2022). Integrated day-ahead and intraday self-schedule bidding for energy storage systems using approximate dynamic programming. *European Journal of Operational Research*, 301(2):726–746.
- Fleten, K. K., Aasgård, E. K., Xing, L., Grøttum, H. H., Fleten, S.-E., and Gundersen, O. E. (2024). Applying and benchmarking a stochastic programming-based bidding strategy for day-ahead hydropower scheduling. *Computational Management Science*, 21(2):44.
- Fusco, A., Giofrè, D., Francesco Castelli, A., Bovo, C., and Martelli, E. (2023). A multi-stage stochastic programming model for the unit commitment of conventional and virtual power plants bidding in the day-ahead and ancillary services markets. *Applied Energy*, 336:120739.
- García, R. and Mínguez, R. (2014). Iterative scenario based reduction technique for stochastic optimization using conditional value-at-risk. *Optimization and Engineering*, 15(2):355–380.

- Garttan, G., Alahakoon, S., Emami, K., and Jayasinghe, S. G. (2025). Battery Energy Storage Systems: Energy Market Review, Challenges, and Opportunities in Frequency Control Ancillary Services. *Energies*, 18(15):4174.
- Gneiting, T., Lerch, S., and Schulz, B. (2023). Probabilistic solar forecasting: Benchmarks, post-processing, verification. *Solar Energy*, 252:72–80.
- Golestaneh, F., Gooi, H. B., and Pinson, P. (2016). Generation and evaluation of space–time trajectories of photovoltaic power. *Applied Energy*, 176:80–91.
- Grupo OMI (2026). Spot Market OMIE. Grupo OMI market description. Available at: <https://www.grupoomi.eu/en/spot-market-omie> (accessed 2026-06-13).
- Gulotta, F., Crespo del Granado, P., Piscicella, P., Siface, D., and Falabretti, D. (2023). Short-term uncertainty in the dispatch of energy resources for VPP: A novel rolling horizon model based on stochastic programming. *International Journal of Electrical Power & Energy Systems*, 153:109355.
- He, C., Geng, H., Rajashekara, K., and Chandra, A. (2024). Analysis and Control of Frequency Stability in Low-Inertia Power Systems: A Review. *IEEE/CAA Journal of Automatica Sinica*, 11(12):2363–2383.
- Heitsch, H. and Roemisch, W. (2003). Scenario Reduction Algorithms in Stochastic Programming. *Computational Optimization and Applications*, 24:187–206.
- Herding, R., Ross, E., Jones, W. R., Endler, E., Charitopoulos, V. M., and Papageorgiou, L. G. (2024). Risk-aware microgrid operation and participation in the day-ahead electricity market. *Advances in Applied Energy*, 15:100180.
- Heredia, F.-J., Cuadrado, M. D., and Corchero, C. (2018). On optimal participation in the electricity markets of wind power plants with battery energy storage systems. *Computers & Operations Research*, 96:316–329.
- International Energy Agency (2025). Electricity 2025. Analysis and forecast to 2027. Available at: <https://www.iea.org/reports/electricity-2025> (accessed 2026-06-13).
- Kumar, P. P., Reis, D. M., Soares, T. A., Silva, B., and Santiago, J. (2025). Co-optimization of hybrid solar, battery and pumped hydropower systems in energy and reserve markets. Manuscript submitted for publication.
- Lago, J., Marcjasz, G., De Schutter, B., and Weron, R. (2021). Forecasting day-ahead electricity prices: A review of state-of-the-art algorithms, best practices and an open-access benchmark. *Applied Energy*, 293:116983.
- Latorre, J. M., Cerisola, S., and Ramos, A. (2007). Clustering algorithms for scenario tree generation: Application to natural hydro inflows. *European Journal of Operational Research*, 181(3):1339–1353.
- Lin, S., Fan, G., Jian, G., and Liu, M. (2020). Stochastic economic dispatch of power system with multiple wind farms and pumped-storage hydro stations using approximate dynamic programming. *IET Renewable Power Generation*, 14(13):2507–2516.
- Löhndorf, N. and Wozabal, D. (2023). The Value of Coordination in Multimarket Bidding of Grid Energy Storage. *Operations Research*, 71(1):1–22.

- Löhndorf, N., Wozabal, D., and Minner, S. (2013). Optimizing Trading Decisions for Hydro Storage Systems Using Approximate Dual Dynamic Programming. *Operations Research*, 61(4):810–823.
- Mirzaei Alavijeh, N., Khezri, R., Mazidi, M., Steen, D., and Le, A. T. (2025). Profit benchmarking and degradation analysis for revenue stacking of batteries in Sweden’s day-ahead electricity and frequency containment reserve markets. *Applied Energy*, 381:125151.
- Narajewski, M. and Ziel, F. (2020). Ensemble forecasting for intraday electricity prices: Simulating trajectories. *Applied Energy*, 279:115801.
- Nasiri, N., Zeynali, S., Ravadanegh, S. N., and Marzband, M. (2021). A hybrid robust-stochastic approach for strategic scheduling of a multi-energy system as a price-maker player in day-ahead wholesale market. *Energy*, 235:121398.
- OMIE (2024). Day-ahead and intraday electricity market rules. Non-binding English translation of the market operating rules. Available at: https://www.omie.es/sites/default/files/2024-06/R_23052024_EN_0.pdf (accessed 2026-06-13).
- REN (2026). Energy Market Information System. REN market information portal. Available at: <https://mercado.ren.pt/EN/Electr/MarketInfo> (accessed 2026-06-13).
- Savin, O., Badina, C., Baroth, J., Charbonnier, S., and Bérenguer, C. (2020). Start and Stop Costs for Hydro Power Plants: A Critical Literature Review. In *Proceedings of the 30th European Safety and Reliability Conference and 15th Probabilistic Safety Assessment and Management Conference*, pages 342–349. Research Publishing Services.
- Schindler, K., Rujeerapaiboon, N., Kuhn, D., and Wiesemann, W. (2024). A Planner-Trader Decomposition for Multimarket Hydro Scheduling. *Operations Research*, 72(1):185–202.
- Schulze, T. and McKinnon, K. (2016). The value of stochastic programming in day-ahead and intra-day generation unit commitment. *Energy*, 101:592–605.
- Shinde, P., Hesamzadeh, M. R., Date, P., and Bunn, D. W. (2021). Optimal Dispatch in a Balancing Market With Intermittent Renewable Generation. *IEEE Transactions on Power Systems*, 36(2):865–878.
- Silva, A. R., Pousinho, H. M. I., and Estanqueiro, A. (2022). A multistage stochastic approach for the optimal bidding of variable renewable energy in the day-ahead, intraday and balancing markets. *Energy*, 258:124856.
- Silvente, J., Kopanos, G. M., Dua, V., and Papageorgiou, L. G. (2018). A rolling horizon approach for optimal management of microgrids under stochastic uncertainty. *Chemical Engineering Research and Design*, 131:293–317.
- Spiller, M., Scrocca, A., Andreotti, D., Rancilio, G., Bovera, F., and Merlo, M. (2025). A Hybrid Robust-Stochastic Operation Model for BESS in the Day-Ahead and Automatic Frequency Restoration Markets. In *2025 21st International Conference on the European Energy Market (EEM)*, pages 1–6.
- Sun, S., Liu, Y., Tang, Z., Tu, M., Du, X., and Liu, J. (2025). A hybrid stochastic-robust bidding model for wind-storage system in day-ahead market considering risk preference. *Electric Power Systems Research*, 238:111148.

- Taglioni, F., Reis, D., Kumar, P., and Soares, T. (2025). Short-term optimal scheduling of pumped storage plants in day-ahead and intraday markets with hybrid renewable integration. Manuscript submitted for publication.
- Tang, C., Wang, Y., Xu, J., Sun, Y., and Zhang, B. (2018). Efficient scenario generation of multiple renewable power plants considering spatial and temporal correlations. *Applied Energy*, 221:348–357.
- United Nations (2015). The 17 Goals. United Nations Sustainable Development Goals portal. Available at: <https://sdgs.un.org/goals> (accessed 2026-06-13).
- United Nations Department of Economic and Social Affairs (2025). Goal 7: Ensure access to affordable, reliable, sustainable and modern energy for all. United Nations Sustainable Development Goals portal. Available at: <https://sdgs.un.org/goals/goal7> (accessed 2026-06-13).
- Vannoni, A. and Sorce, A. (2024). Integrated energy and ancillary services optimized management and risk analysis within a pay-as-bid market. *Applied Energy*, 371:123628.
- Wang, H., Liao, S., Cheng, C., Liu, B., Fang, Z., and Wu, H. (2025). Short-term scheduling strategies for hydro-wind-solar-storage considering variable-speed unit of pumped storage. *Applied Energy*, 377:124336.
- Wang, Y., Zhou, Z., Botterud, A., Zhang, K., and Ding, Q. (2016). Stochastic coordinated operation of wind and battery energy storage system considering battery degradation. *Journal of Modern Power Systems and Clean Energy*, 4(4):581–592.
- Wen, X., Abbes, D., and Francois, B. (2021). Stochastic Optimization for Security-Constrained Day-Ahead Operational Planning Under PV Production Uncertainties: Reduction Analysis of Operating Economic Costs and Carbon Emissions. *IEEE Access*, 9:97039–97052.
- Wozabal, D. and Rameseder, G. (2020). Optimal bidding of a virtual power plant on the Spanish day-ahead and intraday market for electricity. *European Journal of Operational Research*, 280(2):639–655.
- Xiao, P. and Zhu, J. (2026). Optimal operation of a photovoltaic-hydropower-pumped storage system in the electricity market. *Electric Power Systems Research*, 253:112495.
- Yang, W., Zhao, Z., Pérez-Díaz, J. I., Hunt, J. D., Vagnoni, E., Nøland, J. K., Quaranta, E., Wang, R., Li, X., and Cheng, Y. (2025). Pumped storage hydropower operation for supporting clean energy systems. *Nature Reviews Clean Technology*, 1(7):454–473.
- Yuan, W., Wang, X., Su, C., Cheng, C., Liu, Z., and Wu, Z. (2021). Stochastic optimization model for the short-term joint operation of photovoltaic power and hydropower plants based on chance-constrained programming. *Energy*, 222:119996.
- Zhou, R., Chen, S., Han, Y., Liu, Q., Chen, Z., and Hu, W. (2024). Probabilistic spatiotemporal scenario generation method for dynamic optimal power flow in distribution networks. *International Journal of Electrical Power & Energy Systems*, 155:109667.
- Zisos, A., Sakki, G.-K., and Efstratiadis, A. (2023). Mixing Renewable Energy with Pumped Hydropower Storage: Design Optimization under Uncertainty and Other Challenges. *Sustainability*, 15(18):13313.

Appendix A

Methodology Nomenclature

This appendix collects the recurring notation used in Chapter 4. Symbols introduced only inside the formulation-extension appendices are defined locally in those sections rather than repeated here. The table is separated from the main methodology chapter so that the methodological development can be read without an extended notation block at the beginning.

Table A.1: Methodology nomenclature used in Chapter 4.

Symbol	Description	Unit / domain
Sets and horizon indices		
t_0	Rolling decision time at which the controller is solved.	time
$t \in \mathcal{T}^{near}$	Delivery interval in the explicit detailed and hourly dispatch region.	index
$h \in \mathcal{H}$	Forecast lead time measured from t_0 .	index
$\ell \in \mathcal{L}$	Aggregate forecast interval used when native-grid paths are coarsened.	index
$\omega, \omega' \in \Omega$	Scenario indices in the reduced scenario set.	index
$j \in \mathcal{J}$	Uncertainty-family index for price, PV availability, and inflow quantities.	index
$s \in \mathcal{A}^{IDA}$	Intraday-auction stage, with $\mathcal{A}^{IDA} = \{IDA1, IDA2, IDA3\}$.	index
$r \in \{W, M\}$	Coarse planning-layer type, weekly or monthly.	index
$p \in \mathcal{P}^r$	Coarse planning pack in layer r .	index
$b \in \mathcal{B}_p^r$	Horizontal block inside coarse pack p of layer r .	index
$\mathbf{v}$	Rolling decision index associated with a daily reservoir target.	index
$w \in \mathcal{W}^{plan}$	Weekly horizontal planning block used in the objective.	index
$m \in \mathcal{M}^{plan}$	Monthly horizontal planning block used in the objective.	index
$k \in \mathcal{K}^{turb}$	Turbine operating-point index in the variable-head approximation.	index
$k \in \mathcal{K}^{pump}$	Pumping operating-point index in the variable-head approximation.	index
$\mathcal{K}^{turb,+}$	Positive-flow turbine operating points.	set
$\mathcal{K}^{pump,+}$	Positive-flow pumping operating points.	set
$g \in \mathcal{G}$	Pumped-storage unit group.	index
$\mathcal{T}(\ell)$	Native-grid intervals represented by aggregate forecast interval ℓ .	set
$\mathcal{T}^{DA,fix}$	Delivery intervals with fixed day-ahead positions.	set
$\mathcal{T}^{DA,gate}$	Delivery intervals committed at the current day-ahead gate.	set

Symbol	Description	Unit / domain
$\mathcal{T}^{s,fix}$	Delivery intervals with fixed corrections from intraday auction s .	set
$\mathcal{T}^{s,gate}$	Delivery intervals committed at the current gate of intraday auction s .	set
$\mathcal{T}^{s,open}$	Delivery intervals whose corrections in intraday auction s remain decision-relevant.	set
$\mathcal{T}^{fix}$	Delivery intervals whose complete accumulated commercial positions are fixed across all represented market layers.	set
$\mathcal{T}^{DA,open}$	Day-ahead intervals whose positions remain decision-relevant.	set
T^{near}	Terminal index of the explicit detailed and hourly dispatch region.	index
T	Terminal index of the represented optimization horizon.	index
T^{op}	Terminal index of the short rolling operational horizon.	index
n_0	Information node induced by the information set at t_0 .	node
$\Omega(n_0)$	Scenarios branching from information node n_0 .	set
$\mathcal{X}^{exec}$	Variable families corresponding to physical actions implemented in the current interval.	set
Forecast and scenario quantities		
$\mathcal{F}_{t_0}$	Information available when the forecast is issued at time t_0 .	information set
$X_{t_0+1:t_0+H}$	Joint future trajectory of exogenous quantities over the forecast horizon.	mixed
$\mathcal{S}_{t_0}^{raw}$	Raw scenario bank generated at issue time t_0 .	set
$\mathcal{S}_{t_0}^{red}$	Reduced scenario set passed to the MILP.	set
N_{raw}	Number of raw candidate scenarios before reduction.	count
N_{red}	Number of reduced scenarios passed to the MILP.	count
$\hat{p}_\omega$	Probability weight of scenario ω .	probability
$x_{t_0}^{j,real}$	Realized current value of uncertainty family j at issue time t_0 .	family-dependent
$\hat{x}_{t_0+h}^{j,50}$	Conditional central forecast for uncertainty family j at lead time h .	family-dependent
$\hat{x}_{t_0+h}^{j,\omega}$	Scenario-path value of uncertainty family j at lead time h in scenario ω .	family-dependent
$\hat{x}_\ell^{j,\omega}$	Aggregate forecast quantity for family j and interval ℓ .	family-dependent
$\hat{r}_h^{j,\omega}$	Sampled historical residual continuation for uncertainty family j in scenario ω .	family-dependent
$\hat{g}_{t_0}^j$	Current forecast gap used to anchor sampled paths to the observed value.	family-dependent
$\hat{p}_j$	Persistence factor applied to the current forecast gap of uncertainty family j .	scalar
$A_\ell^j(\cdot)$	Aggregation operator mapping native-grid values of family j into aggregate interval ℓ .	operator
z_ω	Feature vector used to compare scenario ω during scenario reduction.	vector
$d(z_\omega, z_{\omega'})$	Distance between scenarios ω and ω' in feature space.	scalar
$\hat{P}_{t,\omega}^{PV,av}$	Available PV power forecast in interval t and scenario ω .	MW
$\hat{q}_{t,\omega}^{nat}$	Natural inflow forecast in interval t and scenario ω .	m ³ /h
$\hat{E}_w^{PV,av}$	Available PV energy over weekly horizontal block w .	MWh
$\hat{V}_b^{nat}$	Natural inflow volume over coarse horizontal block b .	m ³
Market inputs and commitments		

Symbol	Description	Unit / domain
$\hat{\pi}_{t,\omega}^{DA}$	Day-ahead price for interval t in scenario ω .	EUR/MWh
$\hat{\pi}_t^{DA,fix}$	Clearing price associated with an already-fixed day-ahead position.	EUR/MWh
$\hat{\pi}_{t,\omega}^s$	Price of intraday auction stage s for interval t in scenario ω .	EUR/MWh
$\hat{\pi}_t^{s,fix}$	Clearing price associated with an already-fixed correction from intraday auction s .	EUR/MWh
$\hat{\pi}_{t_0}^{ID}$	Current continuous-intraday reference price used for the immediately executed interval.	EUR/MWh
$\hat{\pi}_{t_0}^{ID,sell}$	Current continuous-intraday sell price used for the immediately executed interval.	EUR/MWh
$\hat{s}^{ID}$	Buy-side premium or spread applied to the current continuous-intraday correction.	EUR/MWh
$\hat{\pi}_{t_0}^{ID,buy}$	Current continuous-intraday buy price, equal to $\hat{\pi}_{t_0}^{ID} + \hat{s}^{ID}$.	EUR/MWh
$\hat{Q}_{t_0}^{ID,max}$	Tradability-dependent cap on the current continuous-intraday correction.	MW
$\hat{c}_{t,\omega}^{bal,+}$	Unit coefficient applied to positive residual imbalance in interval t and scenario ω .	EUR/MWh
$\hat{c}_{t,\omega}^{bal,-}$	Unit coefficient applied to negative residual imbalance in interval t and scenario ω .	EUR/MWh
$\hat{c}_{t,\omega}^{bal}$	Common unit coefficient used when positive and negative residual imbalances are penalized symmetrically.	EUR/MWh
$\hat{\pi}_{w,\omega}^{DA,avg}$	Scenario-dependent average day-ahead price used in weekly and monthly planning sale terms.	EUR/MWh
$\hat{\pi}_{m,\omega}^{DA,avg}$		
$\bar{Q}_t^{DA}$	Already-cleared day-ahead commercial position.	MW
$\bar{\Delta}_t^s$	Already-cleared correction from intraday auction stage s .	MW
$\bar{Q}_t^{pre}$	Accumulated inherited commercial position immediately before the active gate.	MW
Physical and economic parameters		
Δ_t	Duration of explicit dispatch interval t .	h
Δ_w	Duration of weekly planning block w .	h
Δ_m	Duration of monthly planning block m .	h
$\hat{P}^{B,ch,max}$	Maximum BESS charging power.	MW
$\hat{P}^{B,dis,max}$	Maximum BESS discharging power.	MW
$\hat{E}^{B,max}$	Maximum BESS stored energy.	MWh
$\hat{\eta}^{B,ch}$	BESS charging efficiency.	scalar
$\hat{\eta}^{B,dis}$	BESS discharging efficiency.	scalar
$\hat{P}_g^{PSH,turb,max}$	Maximum turbine power of pumped-storage group g .	MW
$\hat{P}_g^{PSH,pump,max}$	Maximum pumping power of pumped-storage group g .	MW
$\bar{n}_g$	Number of available reversible units in pumped-storage group g .	count
$\hat{q}^{turb,max}$	Maximum plant-level turbine flow.	m ³ /h
$\hat{q}^{pump,max}$	Maximum plant-level pumping flow.	m ³ /h
$\underline{V}^{res}, \bar{V}^{res}$	Lower and upper bounds of modelled upper-reservoir volume.	m ³
$\underline{h}, \bar{h}$	Lower and upper hydraulic heads represented by the operating-point grid.	m
$\hat{c}^{B,deg}$	BESS cycling or degradation proxy cost.	EUR/MWh
$\hat{c}^{spill}$	Spillage cost coefficient.	EUR/m ³
$\hat{c}^{PSH,turb,start}$	Pumped-storage turbine start-up cost coefficient.	EUR/start
$\hat{c}^{PSH,pump,start}$	Pumped-storage pumping start-up cost coefficient.	EUR/start
$\hat{c}_{w,\omega}^{dev,W}$	Price-derived unit penalty for weekly export-sales mismatch.	EUR/MWh

Symbol	Description	Unit / domain
$\hat{c}_{m,\omega}^{dev,M}$	Price-derived unit penalty for monthly export-sales mismatch.	EUR/MWh
$\underline{Q}_t$	Physical lower bound on the post-gate commercial position in interval t .	MW
$\overline{Q}_{t,\omega}$	Scenario-dependent physical upper bound on the post-gate commercial position in interval t .	MW
$M_{t,\omega}$	Big-M relaxation coefficient used in the gate-correction physicality constraints.	MW
$\bar{P}_k^{turb}, \bar{q}_k^{turb}$ $\bar{h}_k^{turb}$	Power, flow, and head coordinates of turbine operating point k .	mixed
$\bar{P}_k^{pump}, \bar{q}_k^{pump}$ $\bar{h}_k^{pump}$	Power, flow, and head coordinates of pumping operating point k .	mixed
Market decision variables		
$Q_{t,\omega}^{DA}$	Day-ahead commercial position selected by the MILP.	MW
$Q_{t,\omega}^{sell,s}$	Sell quantity committed at the active gate of intraday auction s .	MW
$Q_{t,\omega}^{buy,s}$	Buy quantity committed at the active gate of intraday auction s .	MW
$\Delta_{t,\omega}^s$	Signed intraday-auction correction; decomposed into sell minus buy quantities at an active gate.	MW
$Q_{t_0}^{sell,ID}$	Current-step continuous-intraday sell quantity.	MW
$Q_{t_0}^{buy,ID}$	Current-step continuous-intraday buy quantity.	MW
$z_{t_0}^{ID}$	Binary direction variable for the current continuous-intraday correction.	binary
$\Delta_{t,\omega}^{ID}$	Signed continuous-intraday correction, nonzero only at the current interval.	MW
$Q_{t,\omega}^{comm}$	Accumulated commercial position after all modelled market layers.	MW
$Q_{t,\omega}^{post,s}$	Commercial position after applying the correction of active gate s .	MW
$z_{t,\omega}^s$	Binary direction variable for gate correction s , equal to one for selling and zero for buying.	binary
$a_{t,\omega}^s$	Binary activation variable indicating whether gate correction s is active.	binary
Detailed physical decision variables		
$P_{t,\omega}^{net}$	Net physical power injected into the grid.	MW
$P_{t,\omega}^{PV,use}$	PV power used by the controller.	MW
$P_{t,\omega}^{B,ch}$	BESS charging power.	MW
$P_{t,\omega}^{B,dis}$	BESS discharging power.	MW
$E_{t,\omega}^B$	BESS stored energy.	MWh
$u_{t,\omega}^B$	Binary BESS direction variable that enables charging when one and discharging when zero.	binary
$P_{t,\omega}^{PSH,turb}$	Pumped-storage turbine power.	MW
$P_{t,\omega}^{PSH,pump}$	Pumped-storage pumping power.	MW
$q_{t,\omega}^{turb}$	Turbined water flow.	m ³ /h
$q_{t,\omega}^{pump}$	Pumped water flow.	m ³ /h
$q_{t,\omega}^{spill}$	Non-generating reservoir outflow represented as spillage.	m ³ /h
$V_{t,\omega}^{res}$	Reservoir volume.	m ³
$h_{t,\omega}$	Hydraulic head implied by reservoir states.	m
$N_{t,\omega}^{turb,start}$	Total number of turbine starts across pumped-storage groups.	starts
$N_{t,\omega}^{pump,start}$	Total number of pumping starts across pumped-storage groups.	starts
$N_{g,t,\omega}^{turb,start}$	Turbine starts in pumped-storage group g .	starts

Symbol	Description	Unit / domain
$N_{g,t,\omega}^{pump,start}$ $n_{g,t,\omega}^{turb}$ $n_{g,t,\omega}^{pump}$ $u_{t,\omega}^{turb}$ $u_{t,\omega}^{pump}$ $\lambda_{k,t,\omega}^{turb}$ $\lambda_{k,t,\omega}^{pump}$ $\delta_{t,\omega}^{bal,+}$ $\delta_{t,\omega}^{bal,-}$ $\delta_{t,\omega}^{bal,abs}$	Pumping starts in pumped-storage group g. Number of active turbine units in pumped-storage group g. Number of active pumping units in pumped-storage group g. Binary activation variable for turbine operating mode in the variable-head approximation. Binary activation variable for pumping mode in the variable-head approximation. Convex-combination weight of turbine operating point k. Convex-combination weight of pumping operating point k. Positive imbalance, meaning physical delivery above the commercial position. Negative imbalance, meaning physical delivery below the commercial position. Absolute residual imbalance used when positive and negative deviations have the same unit coefficient.	starts integer integer binary binary scalar scalar MW MW MW
Coarse planning variables		
$V_p^{res,open}$ $V_p^{res,close}$ $E_p^{B,open}$ $E_p^{B,close}$ E_b^{sell} E_b^{exp} E_b^{PV} $E_b^{B,ch}$ $E_b^{B,dis}$ V_b^{turb} V_b^{pump} V_b^{spill} $\delta_b^{blk,+}$ $\delta_b^{blk,-}$ $\delta_{w,\omega}^{W,+}$ $\delta_{w,\omega}^{W,-}$ $\delta_{m,\omega}^{M,+}$ $\delta_{m,\omega}^{M,-}$ κ^{turb} κ^{pump}	Opening and closing reservoir volumes of coarse pack p. Opening and closing BESS energy states of coarse pack p. Planned energy sale in coarse horizontal block b; scenario-indexed in the stochastic objective. Physical export energy in coarse horizontal block b; scenario-indexed in the stochastic objective. PV energy represented in coarse horizontal block b. BESS charging energy in coarse horizontal block b. BESS discharging energy in coarse horizontal block b. Turbined water volume in coarse horizontal block b. Pumped water volume in coarse horizontal block b. Spilled water volume in coarse horizontal block b. Positive and negative export-sales mismatch in generic coarse block b. Weekly positive and negative export-sales mismatch variables. Monthly positive and negative export-sales mismatch variables. Aggregate water-to-energy conversion factors used in coarse planning blocks.	m^3 MWh MWh MWh MWh MWh m^3 m^3 m^3 MWh MWh MWh MWh MWh/m³
Objective, planning-target, and terminal-value symbols		
Φ^{fix} Φ^{var} Φ_{ω}^{mkt} Φ_{ω}^{near}	Cashflow contribution of already-cleared market quantities. Expected decision-dependent value maximized by the rolling MILP. Scenario market-cashflow and near-horizon operating-cost components.	EUR EUR EUR EUR

Symbol	Description	Unit / domain
Φ_{ω}^W	Scenario cashflow components of the weekly and monthly planning layers.	EUR
Φ_{ω}^M	Scenario terminal-objective contribution; zero when no terminal value is used.	EUR
Φ_{ω}^{term}	Complete accounting value, equal to $\Phi^{fix} + \Phi^{var}$.	EUR
Φ^{tot}	Objective of the full daily planning problem.	EUR
Φ^{plan}	Objective of the short rolling operational problem.	EUR
Φ^{op}	Reservoir target extracted for rolling decision index v .	m ³
R_v^{tar}	Slack variable for terminal reservoir shortfall below R_v^{tar} .	m ³
$s_{v,\omega}^{res}$	Penalty applied to terminal reservoir-target shortfall.	EUR/m ³
c^{tar}	Terminal state reached at the end of the explicit horizon.	state vector
$x_{t_0+H,\omega}$	Collection of rolling decision variables selected at decision time t_0 .	vector
u_{t_0}	Scenario contribution of the explicit represented horizon.	EUR
$J_{t_0,\omega}^{hor}$	Exact, ideal downstream continuation-value function.	EUR
V^*	Parametric approximate terminal value function.	EUR
$\hat{V}_{\theta}$	Parameter vector of the approximate terminal value function.	vector
θ	Context index associated with the terminal date of the represented horizon.	index
$\sigma(t_0 + H)$	Reduced terminal-state representation used by the terminal value function.	vector
$z_{t_0+H,\omega}$	Mapping from full terminal state to reduced terminal-state features.	function
$g(\cdot)$	Breakpoint or node index set for the piecewise-linear terminal value.	set
$\mathcal{K}^{val}$	Interpolation weight of value-function node k at reduced state z .	scalar
$\lambda_k^{val}(z)$	Nodal value or segment parameter of the terminal value approximation.	EUR
v_k	Set of training trajectories used in the conceptual parameter-selection problem.	set
$\mathcal{X}^{train}$	Realized cash profit on training trajectory k under terminal-value parameters θ , excluding the terminal-value term itself.	EUR
$\Pi_k(\theta)$	Feasible parameter set for terminal-value calibration.	set
Θ	Initial reservoir volume imposed at sweep point j .	m ³
$V^{0,j}$	Common downstream terminal reservoir reference in the continuation sweep.	m ³
$\bar{V}^{ref}$	Optimized downstream objective obtained at sweep point j .	EUR
J_j	Finite-difference estimate of marginal downstream water value.	EUR/m ³
$\hat{c}_j^{water}$		

Appendix B

Benchmark Controller Definitions

This appendix collects the two benchmark controller definitions used in the numerical comparison. Both benchmarks retain the same physical plant boundary but do not use the daily planning-target decomposition. Instead, they solve the short rolling control horizon directly. The first benchmark adapts a deterministic hybrid co-optimization perspective with a fixed terminal storage convention. The second benchmark keeps stochastic information and applies the calibrated terminal water-value function directly at the end of the same short horizon.

B.1 Operational Hybrid Co-Optimization Benchmark

This section defines the deterministic benchmark inspired by the hybrid co-optimization structure proposed by Kumar et al. (2025). That study is used as a contextual asset-stack comparison, because it models the coordinated operation of PV, BESS, and PSH at Alqueva under one deterministic optimization problem. Its original setting, however, is not a rolling operational controller: the optimization is performed as a deterministic full-period scheduling problem over historical data and includes reserve-market participation. The benchmark defined here retains the deterministic hybrid co-optimization logic of Kumar et al. (2025), but translates it into a rolling market-operation setting that is compatible with the simulator, the annual data, and the market scope adopted in the main experiments.

At each rolling decision time t_0 , the benchmark solves a deterministic version of the market-dispatch problem with the same plant model used in Chapter 4. The uncertain exogenous quantities are replaced by one representative forecast trajectory, with $\Omega^{det} = \{\bar{\omega}\}$ and $\hat{p}_{\bar{\omega}} = 1$, so that electricity prices, PV availability, and natural inflow enter the optimization only through $\hat{\pi}_t$, $\hat{P}_t^{PV}$, and $\hat{Q}_t^{nat}$. No scenario probabilities, scenario-reduction step, or stochastic nonanticipativity constraints are required.

The benchmark keeps the detailed 15-minute control layer and the sequential hourly continuation layer, but does not include the explicit weekly and monthly horizontal planning packs. Equation (B.1) states its short-horizon objective.

$$\max \Phi_{\bar{\omega}}^{mkt} + \Phi_{\bar{\omega}}^{near} \quad (\text{B.1})$$

All detailed physical and market constraints remain unchanged. The benchmark keeps the same battery stock balance, upper-reservoir balance, variable-head pumped-storage representation, unit-commitment logic, gate-dependent market commitments, and imbalance treatment as the main formulation. The comparison isolates the consequences of deterministic information and the absence of explicit coarse future packs rather than simplifying the plant model.

Because the benchmark omits the daily planning target and the weekly and monthly planning layers, it requires an explicit terminal rule to avoid end-of-horizon depletion. Let $V_{t_0}^{res}$ and $E_{t_0}^B$ denote the upper-reservoir volume and battery energy observed at the start of the solve. Equations (B.2) and (B.3) impose the adopted rolling non-depletion convention.

$$V_T^{res} \geq V_{t_0}^{res} \quad (\text{B.2})$$

$$E_T^B \geq E_{t_0}^B \quad (\text{B.3})$$

These constraints do not estimate continuation value. They only prevent the controller from creating artificial profit by shifting storage depletion beyond the represented horizon. In that sense, the benchmark remains close to the deterministic scheduling logic of Kumar et al. (2025): storage is coordinated jointly with market participation, but no separate long-horizon stochastic valuation layer is introduced.

The benchmark is executed in the same closed-loop simulation as the other controllers. At each time step, the current physical state and inherited commitments are observed, the representative deterministic forecast is issued over the visible horizon, problem (B.1)–(B.3) is solved, and only the next executable dispatch action and any quantities fixed by the current market gate are implemented. The simulator then advances with realized prices, realized PV availability, and realized inflow.

Under this interpretation, the benchmark may be read as an operational rolling counterpart of the deterministic hybrid co-optimization philosophy of Kumar et al. (2025). It preserves the integrated PV–BESS–PSH market-dispatch structure, but does not use stochastic recourse, explicit coarse future market blocks, a daily planning target, or a calibrated terminal water-value function. Relative to the proposed controller, the comparison indicates the combined effect of those omitted elements rather than isolating a single modelling feature.

B.2 Terminal-Value Benchmark Without Coarse Planning Packs

This section defines a benchmark based on the long-term/short-term decomposition commonly used in hydropower scheduling. It is motivated by the approximate dynamic programming framework of Löhndorf et al. (2013) and by the approximate value-function treatment adopted by Lin et al. (2020). In both cases, detailed short-term operating decisions are coordinated with an external representation of downstream storage value rather than with an explicit long-horizon market chronology.

At each rolling decision time t_0 , the benchmark solves the same short market-dispatch problem as the operational controller: two days at 15-minute resolution followed by five sequential hourly days, without explicit weekly or monthly planning packs. The benchmark keeps the reduced joint scenario set Ω with probabilities $\hat{p}_\omega$, so prices, PV availability, and natural inflow remain stochastic inside the short horizon.

The omitted future is represented directly at the terminal boundary of this short horizon. Let $V_{T,\omega}^{res}$ denote the upper-reservoir volume at the terminal boundary under scenario ω . The objective contains the terminal term $\Phi^{TV} = \sum_{\omega \in \Omega} \hat{p}_\omega \hat{V}(V_{T,\omega}^{res})$, where $\hat{V}(\cdot)$ is the calibrated concave piecewise-linear terminal water-value function reported in Chapter 5. This is the same function used by the stochastic value-function planning controller, but here it is applied directly in the rolling short-horizon benchmark rather than first being converted into a daily reservoir target.

The battery does not receive a separate terminal value term. Equation (B.4) retains its terminal non-depletion condition.

$$E_{T,\omega}^B \geq E_{t_0}^B \quad \forall \omega \in \Omega \quad (\text{B.4})$$

whereas the reservoir is valued through Φ^{TV} . This choice reflects the much larger long-horizon economic leverage of the reservoir in the case study.

Equation (B.5) summarizes the complete benchmark problem.

$$\begin{aligned}
 & \max \quad \sum_{\omega \in \Omega} \hat{p}_{\omega} \left[\Phi_{\omega}^{mkt} + \Phi_{\omega}^{near} + \hat{V}(V_{T,\omega}^{res}) \right] \\
 & \text{subject to} \quad \text{all detailed-region and hourly-layer constraints of Chapter 4} \\
 & \quad \quad \quad E_{T,\omega}^B \geq E_{t_0}^B \quad \forall \omega \in \Omega \\
 & \quad \quad \quad \text{no explicit weekly planning-pack variables or constraints} \\
 & \quad \quad \quad \text{no explicit monthly planning-pack variables or constraints}
 \end{aligned} \tag{B.5}$$

This comparison is informative because the calibrated terminal valuation is held fixed while the explicit coarse market chronology and the daily planning-target mechanism are removed. If the benchmark performs similarly to the proposed value-function controller, then the daily multi-resolution planning layer may be adding little beyond the terminal value itself. If a material performance gap remains, that gap can be attributed to the information retained by the explicit weekly and monthly planning blocks and to the way that information is translated into daily reservoir targets.

The benchmark is stochastic and value-informed, but not multi-resolution in the same sense as the proposed value-function controller. It relies on the calibrated terminal water-value function to represent the omitted future, while removing the explicit weekly and monthly market structure and the separate daily planning step.

Appendix C

Supporting Numerical Diagnostics

This appendix collects numerical diagnostics that support, but do not determine, the primary controller comparison. It first compares the daily planning targets with the realized reservoir trajectories, then reports the scenario-count scalability experiment and the solver-reuse strategies considered for repeated rolling MILP solution.

C.1 Planning-Target Tracking

Figure C.1 compares the realized reservoir trajectories of the three planning-target controllers with the shifted seven-day reservoir targets produced by their daily planning layers. The target associated with each rolling decision is plotted at the corresponding operational-horizon boundary. This alignment makes the two series comparable in time, but it does not turn the target into a directly executable reservoir schedule.

The broad seasonal movements are similar, while the high-frequency target variation is followed only approximately. The figure is therefore useful for checking that the daily planning signal reaches the operational layer, but it is not a quantitative tracking test and does not establish that the realized trajectory is optimal relative to the planning targets. For this reason, it is treated as a supporting diagnostic rather than as a primary performance result.

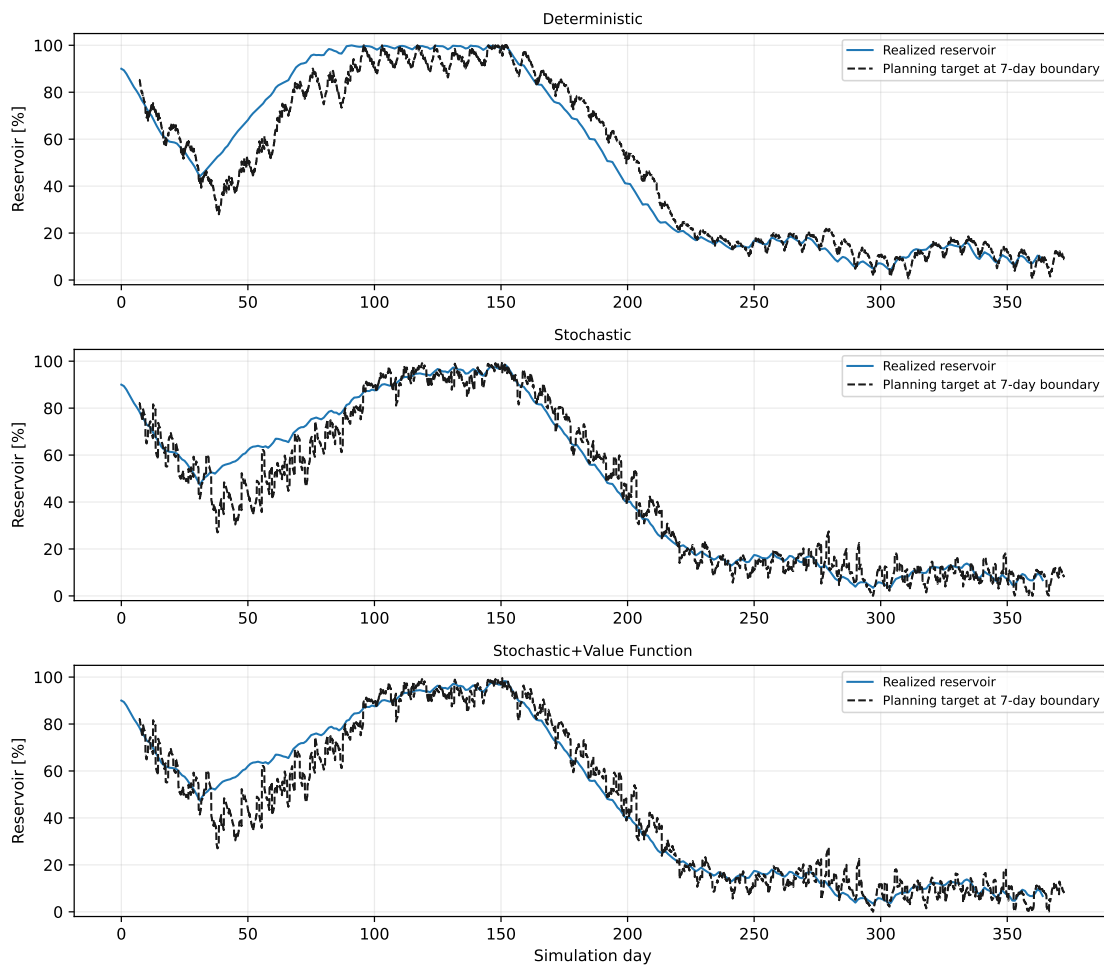


Figure C.1: Realized reservoir trajectories and shifted seven-day planning targets for the three daily-planning controllers.

C.2 Scenario-Count Scalability

This section reports a fixed-checkpoint timing diagnostic for the stochastic rolling MILP. It isolates the computational effect of the number of retained scenarios in the same fixed-quantity controller used by the annual stochastic runs. The diagnostic does not use the bid-curve formulation.

The starting point is the stored state of the stochastic annual run at simulation step 7584. This preserves the reservoir state, battery state, active market books, and inherited commitments from a previously executed annual run rather than constructing a synthetic operating state. At that decision time, a historical forecast issue is built from raw joint price, photovoltaic, and inflow scenarios and reduced to $N \in \{1, 2, 3, 4, 5, 7, 9, 12, 16, 24, 32, 40, 64\}$ retained scenarios. The main sweep up to 32 retained scenarios uses 40 raw paths. The two additional stress points, $N = 40$ and $N = 64$, use 64 raw paths because the retained count cannot exceed the raw candidate set. Each retained scenario count has 20 checkpoint readings, so the figure shows the full sample band rather than a single representative timing. The solver and market settings match the current annual configuration: Gurobi, a relative gap target of 10^{-3} , a 20 s solver limit, OMIE intraday prices, a 50 MW continuous-intraday trade cap, and an imbalance-price factor of 2.0.

Table C.1 reports the scenario-reduction time, median relative gap, and the median wall time for the complete MILP build-and-solve call. Figure C.2 shows the same measurements as box plots with every sample point retained and the mean values connected. All 260 solves were feasible. The median build-and-solve time increased from 0.810 s with one retained scenario to 3.109 s with the three-scenario setting used in the annual stochastic runs, 8.622 s with nine scenarios, 28.934 s with 32 scenarios, 44.879 s with 40 scenarios, and 64.842 s with 64 scenarios. The high-count wall times can exceed the nominal 20 s solver limit because the measured interval includes model construction and result extraction in addition to the solver's internal optimization time. Some individual $N = 40$ and $N = 64$ stress samples also terminate with larger reported optimality gaps under the time limit; these samples are retained in the plotted band and do not affect the conclusion that such large retained sets are computationally costly for repeated rolling control.

Table C.1: Checkpoint solve time as a function of retained scenario count, with 20 readings per count.

Scenarios	Repeats	Reduction (s)	Median (s)	Max (s)	Median gap
1	20	1.194	0.810	1.060	0.0000
2	20	1.409	1.662	1.728	0.0000
3	20	1.700	3.109	4.290	0.0000
4	20	1.717	3.883	5.025	0.0000
5	20	1.556	4.860	6.139	0.0000
7	20	2.318	7.006	8.331	0.0000
9	20	2.710	8.622	12.102	0.0005
12	20	3.227	11.869	13.834	0.0004
16	20	2.954	15.404	19.950	0.0004
24	20	4.321	24.349	29.007	0.0005
32	20	5.923	28.934	39.789	0.0004
40	20	7.074	44.879	56.090	0.0004
64	20	8.090	64.842	74.241	0.0010

These timings are implementation-level evidence for the selected rolling-horizon setup, not an

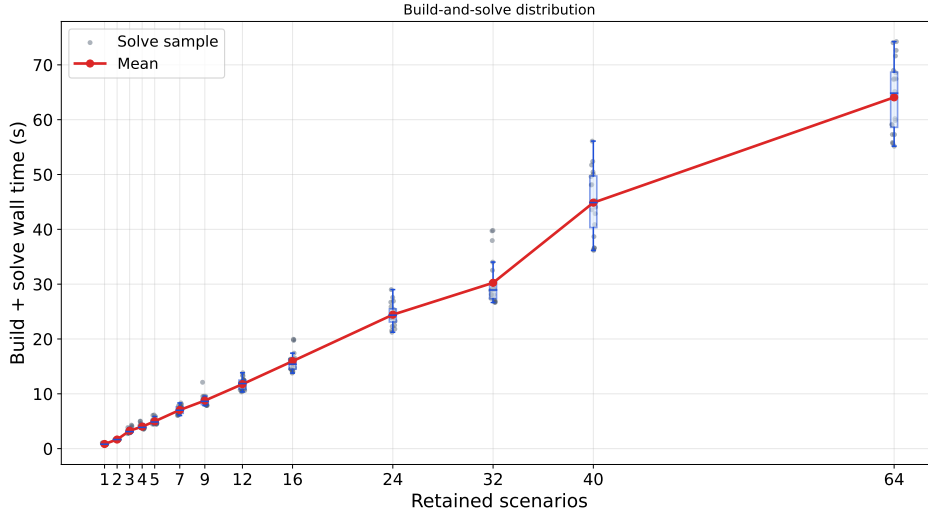


Figure C.2: Measured checkpoint MILP build-and-solve wall time as the retained stochastic scenario count increases. Each retained count has 20 readings.

asymptotic complexity proof. The experiment uses a short fixed-checkpoint design: a representative real checkpoint is solved repeatedly rather than running full multi-day simulations for every scenario count. The result supports the use of a small reduced scenario set in the annual experiments. Retaining three scenarios keeps the stochastic controller in the low-second wall-time range on this checkpoint, while materially larger retained sets increase the repeated rolling-control cost without changing the physical model or market interface.

C.3 Warm-Start and Solver-Reuse Strategies

Rolling-horizon optimization repeatedly solves closely related MILP instances: after each executed interval, the horizon is shifted, the physical state is updated, new forecasts are issued, and a new problem is solved. Warm-start and solver-reuse strategies are natural candidates for reducing the computational burden of repeated stochastic dispatch. The difficulty is that similarity is not identity. Forecasts, storage states, fixed market commitments, and gate-closure windows change at every decision time, and the weekly and monthly planning blocks change their relation to the detailed horizon as time advances.

Transferring the full previous solution is the least robust strategy. Copied continuous trajectories start from an outdated physical state and may violate stock-balance equations, power limits, or terminal constraints, so the solver may reject the start, spend time repairing it, or discard it during presolve. When scenario sets are regenerated at each decision time, a scenario index in one solve also need not correspond to the same uncertainty path in the next. Full-solution transfer is only safe when the mapping between old and new variables is explicit and infeasible values are filtered before being passed to the solver.

A more defensible strategy transfers only the discrete operating pattern over the overlapping part of the horizon: pumping versus turbinning modes, BESS charging versus discharging, unit activation, start-up indicators, and buy/sell directions. These variables encode combinatorial structure, are less sensitive than continuous quantities to small changes in prices or inflows, and can help the branch-and-bound search find a feasible incumbent early. If the transferred pattern is provided only as a solver start, optimality is not restricted; if the binaries are instead fixed and only the

continuous variables are re-optimized, the problem becomes an evaluation of a chosen discrete policy and should not replace the full MILP unless a heuristic policy is explicitly accepted. The benefit is problem-dependent: it is most likely when the integer search dominates and consecutive forecasts are similar, and least likely when presolve dominates or the previous pattern quickly becomes economically implausible.

Solver reuse is a different idea: a persistent solver interface keeps the model attached to the solver and updates only the data that changed, avoiding repeated model translation and solver start-up overhead. This is only safe while the model structure is unchanged, so a conservative implementation requires a structural fingerprint covering scenario counts, time steps, planning blocks, fixed-commitment masks, gate windows, terminal-condition mode, and operating-point grids, with a full rebuild whenever the fingerprint changes.

Exploratory profiling did not justify either route for the validated runs. The tested persistent Pyomo/Gurobi interface reproduced the one-shot results on a short equivalence check but was slower in wall-clock terms: a two-hour stochastic benchmark required approximately 82.1 s with the one-shot route and 103.2 s with the persistent route, because the model structure changed too often for true mutable-parameter reuse and the repeated model-translation step exceeded the overhead it was meant to remove. The numerical experiments use the ordinary one-shot MILP solve. This profiling result is implementation-specific: discrete-pattern warm starts may become valuable as the number of integer decisions grows, and persistent reuse may be beneficial under a formulation with fixed dimensions and fully mutable data. Any future use of these strategies should identify the transferred variables, distinguish a solver start from a fixed heuristic policy, and compare both runtime and closed-loop economic performance against the one-shot reference.

Appendix D

Formulation Extensions and Terminal-Value Alternatives

This appendix collects formulation extensions and supporting alternatives that follow directly from the modelling structure. The first three extensions address risk aversion, explicit price-quantity bidding, and reserve-market participation. The bidding formulation supports the diagnostic reported in Chapter 5; the other extensions are included as formulation references rather than elements of the main numerical comparison. The final section discusses smoother quadratic alternatives to the piecewise-linear terminal water-value approximation used in the implemented MILP.

D.1 Risk-Aversion Extensions

The stochastic formulation in Chapter 4 is risk-neutral: it maximizes probability-weighted expected profit over the reduced scenario set. Expected profit does not distinguish between policies with the same mean but different downside exposure.

This distinction is relevant for market operation under uncertainty. Extreme but plausible combinations of low prices, adverse PV availability, inflow deviations, and imbalance prices can lead to poor realized outcomes. A purely risk-neutral objective may accept such exposure if it is compensated by higher expected profit in more frequent scenarios. A risk-averse objective exchanges part of this expected profit for improved performance in the lower tail of the profit distribution. In electricity-market optimization, CVaR has often been used for this purpose because it can be represented linearly in scenario-based stochastic programs and can be combined with MILP formulations; Herding et al. (2024) use it for risk-aware market participation, while García and Mínguez (2014) discuss scenario-based stochastic optimization with CVaR and its computational implications.

Let x denote the complete set of decision variables of a rolling solve, and let $\Pi_\omega(x)$ be the profit contribution associated with scenario $\omega \in \Omega$. Equation (D.1) states the abstract risk-neutral objective.

$$\max_x \sum_{\omega \in \Omega} \hat{p}_\omega \Pi_\omega(x) \quad (\text{D.1})$$

where $\hat{p}_\omega$ is the probability of scenario ω . Scenario loss is defined as $L_\omega(x) = -\Pi_\omega(x)$, so low profit becomes high loss. For a confidence level $\alpha \in (0, 1)$, conditional value-at-risk is the expected

loss in the upper tail beyond the value-at-risk threshold. Equation (D.2) gives its standard finite-scenario epigraph representation.

$$\text{CVaR}_\alpha(L) = \min_{\eta, \xi} \left[\eta + \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \hat{p}_\omega \xi_\omega \right] \quad (\text{D.2})$$

subject to

$$\xi_\omega \geq L_\omega(x) - \eta \quad \forall \omega \in \Omega \quad (\text{D.3})$$

$$\xi_\omega \geq 0 \quad \forall \omega \in \Omega \quad (\text{D.4})$$

Here η is an auxiliary variable representing the value-at-risk threshold and ξ_ω measures the excess loss of scenario ω above that threshold. The confidence level α determines how deep into the tail the risk measure looks. For example, $\alpha = 0.95$ focuses on the worst five percent of the loss distribution, as represented by the available scenario probabilities.

A simple risk-averse extension combines expected profit with a penalty on tail losses:

$$\max_{x, \eta, \xi} \left[\sum_{\omega \in \Omega} \hat{p}_\omega \Pi_\omega(x) - \lambda \left(\eta + \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \hat{p}_\omega \xi_\omega \right) \right] \quad (\text{D.5})$$

subject to the original operational constraints and to the additional constraints (D.3)–(D.4). The parameter $\lambda \geq 0$ controls the strength of risk aversion. When $\lambda = 0$, the formulation collapses to the risk-neutral expected-profit objective. As λ increases, the optimizer gives more importance to the adverse tail and becomes willing to sacrifice expected profit in exchange for better protection against severe low-profit scenarios.

The sign convention in (D.5) follows from the profit-maximization objective. Conditional value-at-risk is usually defined for losses, so the term $L_\omega(x) = -\Pi_\omega(x)$ converts low-profit scenarios into high-loss scenarios, and the objective subtracts the resulting tail-risk measure. Equivalently, one may define a lower-tail conditional value-at-risk directly on profit and add it to the objective with an appropriate sign. The loss-based convention is often clearer because it matches the standard minimization interpretation of conditional value-at-risk.

In the rolling market-dispatch problem, the risk-averse term would not change the physical meaning of the constraints. It would change the way the optimizer ranks feasible policies across scenarios. A risk-neutral policy may schedule aggressive sales or deeper storage depletion if those decisions increase average profit. A risk-averse policy may instead reserve more water or BESS energy, reduce exposure to uncertain imbalance settlements, or avoid commercial positions that are profitable on average but harmful in adverse price realizations. The resulting controller can be less profitable in expectation while being more robust against rare but damaging outcomes.

This extension also interacts with scenario generation and reduction. If the scenario set underrepresents extreme outcomes, the conditional value-at-risk term cannot protect against them. Conversely, if scenario reduction removes too many adverse scenarios, the risk term may become artificially weak. This is one reason why García and Mínguez (2014) emphasize that scenario reduction for conditional value-at-risk problems is decision-dependent: the relevant information is concentrated in the tail scenarios that determine the risk measure. A risk-averse version of the rolling controller would require checking not only the number of retained scenarios, but also whether the retained set preserves the economically relevant adverse tail.

The formulation above is included as a direct extension of the stochastic MILP; Chapter 5 does not evaluate it numerically. Its implementation would require selecting α , calibrating λ , and comparing policies using both mean realized profit and downside metrics over out-of-sample

rolling simulations. The expected qualitative trade-off is clear: larger λ should reduce exposure to worst-case or tail scenarios, but it can also reduce average annual revenue by making the controller more conservative.

D.2 Explicit Bidding Extensions

The market model used in the main annual comparison represents market participation through quantities evaluated at exogenous price scenarios. This is a price-taking scheduling abstraction: market-clearing prices are uncertain inputs, and the optimizer decides how much energy should be committed or corrected at each market stage. Real electricity markets are richer than this abstraction because participants submit price-quantity bids or offers rather than one unconditional quantity. Explicit bidding models are a natural extension when the objective is not only to schedule a quantity against a forecast price, but to construct a curve whose accepted quantity depends on the realized clearing price.

This appendix describes the price-taking day-ahead bid-curve extension that follows from the same modelling structure. Chapter 5 uses a reduced-form one-product diagnostic to isolate the mechanism, rather than an annual bid-curve controller. The market price remains exogenous, and the hybrid portfolio is not assumed to influence market clearing. The additional decision is a monotone signed net-quantity curve over a finite price grid. Negative quantities denote net purchase for pumping or charging at low prices, and positive quantities denote net sale. The price grid is fixed within a solve so that clearing can be represented linearly, but it may be rebuilt at each rolling decision time from the newest probabilistic price forecast. In a full rolling implementation, the levels can be centred on a forecast statistic such as the weighted median of the day-ahead price scenarios for each delivery interval and shifted by fixed offsets. In the reduced-form diagnostic, the grid is specified directly by the five illustrative clearing prices used in the price fan. Other choices, such as forecast quantiles or market tick-aligned levels, would fit the same formulation.

The key distinction from the annual fixed-quantity formulation is that the optimizer does not choose one day-ahead quantity for each scenario. It chooses one submitted curve before the clearing price is known. The scenarios are then used to evaluate that common curve: each scenario has a possible clearing price, and that price selects the quantity that would be accepted in that scenario. The same scenario also carries its own PV availability, inflow, and future price path, so physical feasibility and imbalance exposure are evaluated jointly with the accepted quantity. The scenario probabilities remain probabilities of complete future paths. The probability of accepting a given price bucket is only an aggregation induced by those joint scenarios, not a separate independent input.

This structure is consistent with stochastic bidding models in electricity markets. Fleten et al. (2024) discuss stochastic-programming-based hydropower bidding and the need to translate optimization results into feasible bids, while Herding et al. (2024) formulate feasible market bidding curves within a stochastic MILP. Vannoni and Sorce (2024) also show that market-product modelling may require explicit treatment of offer acceptance and settlement rules.

Consider a market interval $t \in \mathcal{T}^{bid}$, an ordered price grid $\mathcal{R}_t = \{0, \dots, R_t - 1\}$, and ordered price levels $\rho_{t,0} < \rho_{t,1} < \dots < \rho_{t,R_t-1}$. Let $B_{t,r}$ denote the signed net quantity submitted for the price bucket associated with level r . The curve is nonanticipative: the quantities $B_{t,r}$ are chosen before the day-ahead clearing price is known. Equation (D.6) imposes monotonicity.

$$B_{t,r} \leq B_{t,r+1} \quad 0 \leq r \leq R_t - 2 \quad (\text{D.6})$$

The submitted curve should remain inside broad product and plant limits. Let $\overline{Q}_t^{buy,firm}$ and $\overline{Q}_t^{sell,firm}$ denote admissible buy and sell magnitudes available before the clearing price is known. Equation (D.7) imposes a simple curve-level bound.

$$-\overline{Q}_t^{buy,firm} \leq B_{t,r} \leq \overline{Q}_t^{sell,firm} \quad r \in \mathcal{R}_t \quad (D.7)$$

These bounds do not need to force every price bucket to be physically deliverable in every scenario. Only the bucket selected by a scenario's clearing price becomes an accepted market position in that scenario. The feasibility-critical constraints should be applied to the accepted quantity after price clearing, through the same physical dispatch, storage-balance, and imbalance equations used by the main controller. The reduced-form diagnostic in Chapter 5 suppresses this physical detail and uses the pumped-storage import and export limits directly, because its purpose is to isolate the market-interface effect rather than to reproduce the full rolling plant model.

For each scenario ω , Equation (D.8) maps the day-ahead clearing price $\lambda_{t,\omega}$ to one bucket of the submitted curve.

$$\kappa(t, \omega) = \max \{r \in \mathcal{R}_t : \rho_{t,r} \leq \lambda_{t,\omega}\} \quad (D.8)$$

with the convention that prices below the first level select bucket 0, while prices above the final level select bucket $R_t - 1$. The accepted day-ahead quantity in scenario ω is then

$$Q_{t,\omega}^{DA} = Q_t^{inh} + B_{t,\kappa(t,\omega)} \quad (D.9)$$

The index $\kappa(t, \omega)$ is computed from the scenario price and the fixed price grid; it is not a binary decision variable. Equation (D.9) is linear. The scenario probabilities remain the joint probabilities of the full price, PV, and inflow paths. Equation (D.10) aggregates them by selected price bucket for reporting; no independent acceptance-probability variable is introduced.

$$\mathbb{P}\{\kappa(t, \omega) = r\} = \sum_{\omega \in \Omega: \kappa(t,\omega)=r} \hat{p}_\omega \quad (D.10)$$

This is only a diagnostic aggregation of the same joint scenarios; it does not separate price uncertainty from the physical uncertainty carried by each path.

After (D.9), $Q_{t,\omega}^{DA}$ is treated like any other accepted day-ahead position in the market-dispatch model. It enters the commercial-position balance, the physical tracking relation, and the objective under the same scenario ω . Thus, a high-price scenario with low PV availability does not merely accept a high sale; it also tests whether that sale can be served by the PSH, BESS, available PV, and reservoir state in that same future path. Conversely, a low-price scenario can accept a buy or neutral position without imposing the high-price sale quantity as a commitment in that scenario.

When no retained scenario clears a particular bucket, that bucket has zero probability in the expected-revenue objective. It is not economically identified by the stochastic program. In the implementation, such zero-mass buckets are tied to the nearest bucket that is selected by at least one scenario, with the lower bucket chosen in an exact tie. This prevents *ex post* realized-price evaluation from reading arbitrary monotone values in price regions that were not represented by the retained scenario set.

Equation (D.11) gives the day-ahead revenue contribution under uniform-price settlement.

$$R_{t,\omega}^{DA,bid} = \lambda_{t,\omega} Q_{t,\omega}^{DA} \quad (D.11)$$

This term is linear because both $\lambda_{t,\omega}$ and $\kappa(t, \omega)$ are scenario parameters. If a pay-as-bid settlement rule were considered instead, the revenue expression would have to be changed to value

the accepted blocks at their submitted prices. The diagnostic in Chapter 5 uses the uniform-price interpretation because the day-ahead Iberian market clears at a market price rather than through pay-as-bid settlement.

The accepted bid quantity enters the same commercial-position logic as the scheduled quantities in the main formulation. The physical dispatch variables must still satisfy PV, BESS, PSH, reservoir, network, and imbalance relations. The bid curve changes how the day-ahead commercial position is generated; it does not remove the need to serve the accepted position physically or settle deviations. A curve that is profitable in expectation may still perform poorly if it creates commitments that require excessive pumping, battery cycling, or imbalance exposure in some scenarios.

The rolling interpretation is essential. The curve is optimized before the day-ahead gate closes. Once the market clears, Equation (D.12) determines the inherited quantity from the submitted curve $B_{t,r}^*$ and realized price λ_t^{real} .

$$Q_t^{fix} = Q_t^{inh} + B_{t,\kappa(t,real)}^* \quad (D.12)$$

Only this accepted quantity becomes a fixed commercial position. The unaccepted parts of the curve are not inherited commitments. This distinction is the main modelling advantage relative to a single unconditional quantity: the optimizer can expose different quantities to different clearing prices, and the realized market fixes only the quantity associated with the cleared price.

This also clarifies the deterministic comparison. A controller with one physical forecast trajectory may still be paired with a distribution over possible day-ahead clearing prices, but then it is no longer an unconditional deterministic schedule. It is a deterministic-physics, probabilistic-clearing benchmark. If the curve is optimized against only one price atom, then only the bucket selected by that atom is economically identified; the other curve segments are tie-filled or shaped mainly by monotonicity and capacity bounds. A deterministic bid-curve benchmark is better interpreted either as deterministic physical operation combined with an exogenous price distribution, or as a fixed heuristic rule based on marginal water value. Such a comparator is informative, but it is not the same object as the unconditional deterministic controller in the annual baseline.

The signed cumulative representation can be related to a more traditional block-offer representation. A pure sell-side curve could instead define nonnegative incremental blocks $b_{t,r}$ accepted whenever $\rho_{t,r} \leq \lambda_{t,\omega}$, giving $Q_{t,\omega}^{bid} = \sum_r \mathbf{1}\{\rho_{t,r} \leq \lambda_{t,\omega}\} b_{t,r}$. A pure purchase curve would reverse the acceptance inequality. The implemented diagnostic uses the signed net curve because the same hybrid plant may rationally buy at low prices and sell at high prices in the same delivery interval. It is closer to a combined demand-and-supply schedule than to a single sell-only offer stack.

With exogenous price scenarios and fixed price levels inside each solve, the bid-curve extension remains a linear stochastic MILP. In this context, fixed price levels are input parameters during the solve; the grid itself may still vary across intervals, market gates, or rolling decision times. A finer grid gives a more flexible curve but increases model size and can leave some buckets weakly identified when few scenarios fall near them. A coarser grid is less flexible but easier to interpret and more robust for small retained scenario sets.

This price-taking formulation is not strategic bidding. If the submitted curve materially affects the market-clearing price, then $\lambda_{t,\omega}$ is no longer an exogenous scenario parameter and the clearing rule becomes endogenous. That setting usually leads to equilibrium, complementarity, or bilevel formulations rather than the direct stochastic MILP extension described here. The formulation preserves the same market-power assumption as the main model: prices are uncertain and externally determined, while the portfolio chooses feasible quantities conditional on those prices.

The annual baseline experiments in Chapter 5 use unconditional market quantities, and the reduced-form diagnostic reported there isolates the price-contingent market-interface mechanism

without quantifying its annual value. The additional requirements of a full bid-curve implementation are discussed with the future-work directions in Chapter 6.

D.3 Reserve-Market Extensions

Energy-market dispatch uses flexibility by converting stored energy, stored water, and renewable availability into scheduled electricity positions. Reserve-market participation uses the same flexibility differently: part of the available operating range is withheld from the energy schedule and sold as capacity that can be activated by the system operator. For BESS and PSH assets, the reserve commitment competes with arbitrage, imbalance correction, and storage preservation even though both technologies can provide fast upward or downward response. Reserve revenue is not an independent premium added after the energy schedule; it changes the feasible set of the dispatch problem.

The literature reflects this coupling. Filipe et al. (2019) study variable-speed pumped-storage participation in day-ahead energy and frequency-restoration reserve markets. Fernández-Muñoz and Pérez-Díaz (2022) and Fusco et al. (2023) show that energy and reserve products can be co-optimized under uncertainty in renewable-storage and virtual-power-plant settings. Vannoni and Sorce (2024) further emphasize that ancillary-service value depends on acceptance, activation, settlement, and risk preferences. A reserve extension should represent both the commercial commitment of capacity and the uncertain physical activation of that capacity.

Let $t \in \mathcal{T}^{res}$ denote a market interval in which reserve products may be offered. Upward and downward reserve capacities are represented by the nonnegative variables $R_t^{up} \geq 0$ and $R_t^{down} \geq 0$. These variables represent committed capacity rather than activated energy. Let $\alpha_{t,\omega}^{up}$ and $\alpha_{t,\omega}^{down}$ be activation fractions in scenario ω , with values between zero and one. Equations (D.13) and (D.14) bound the corresponding activated quantities.

$$0 \leq A_{t,\omega}^{up} \leq \alpha_{t,\omega}^{up} R_t^{up} \quad t \in \mathcal{T}^{res} \quad \omega \in \Omega \quad (\text{D.13})$$

$$0 \leq A_{t,\omega}^{down} \leq \alpha_{t,\omega}^{down} R_t^{down} \quad t \in \mathcal{T}^{res} \quad \omega \in \Omega \quad (\text{D.14})$$

If the activation fraction is treated as an exogenous scenario parameter, these constraints are linear. A deterministic activation-duration requirement can be represented by replacing the scenario activation fractions with product-specific reserve-duration parameters.

The nonanticipativity structure separates capacity commitment from realized activation. Reserve capacity is a commercial commitment made before activation is known, so R_t^{up} and R_t^{down} should be common across scenarios that share the same information at the reserve gate. Activation variables $A_{t,\omega}^{up}$ and $A_{t,\omega}^{down}$, by contrast, may differ by scenario because activation is part of the uncertain future. In this sense, reserve capacity is a nonanticipative commitment of flexibility, whereas reserve activation is a scenario-dependent use of that committed flexibility.

Reserve commitments must be compatible with ordinary dispatch. For a BESS, upward reserve can be delivered by increasing discharge or reducing charge, while downward reserve can be delivered by increasing charge or reducing discharge. Equations (D.15) and (D.16) impose simplified power headroom.

$$P_{t,\omega}^{B,dis} + R_t^{B,up} \leq \bar{P}^{B,dis} \quad t \in \mathcal{T}^{res} \quad \omega \in \Omega \quad (\text{D.15})$$

$$P_{t,\omega}^{B,ch} + R_t^{B,down} \leq \bar{P}^{B,ch} \quad t \in \mathcal{T}^{res} \quad \omega \in \Omega \quad (\text{D.16})$$

Power headroom alone is not enough. The storage state must also remain compatible with a possible activation over the required delivery duration. If τ^{up} and τ^{down} denote product-specific

activation durations, then a conservative BESS energy-feasibility approximation is

$$E_{t,\omega}^B \geq \underline{E}^B + \tau^{up} R_t^{B,up} \quad t \in \mathcal{T}^{res} \quad \omega \in \Omega \quad (\text{D.17})$$

$$E_{t,\omega}^B \leq \bar{E}^B - \tau^{down} R_t^{B,down} \quad t \in \mathcal{T}^{res} \quad \omega \in \Omega \quad (\text{D.18})$$

Efficiency factors can be included by scaling the reserve-energy terms in (D.17)–(D.18). More detailed formulations may also include degradation costs caused by reserve activation.

For PSH, the same principle applies, but the feasible reserve directions depend on the operating mode. Equations (D.19) and (D.20) give generic headroom conditions.

$$P_{t,\omega}^{H,gen} + R_t^{H,up,gen} \leq \bar{P}_{t,\omega}^{H,gen} \quad (\text{D.19})$$

$$P_{t,\omega}^{H,pump} + R_t^{H,down,pump} \leq \bar{P}_{t,\omega}^{H,pump} \quad (\text{D.20})$$

with additional constraints linking reserve provision to turbine, pumping, or switching modes. If reserve can only be provided while already synchronized in a compatible mode, the reserve variables must be bounded by the corresponding commitment binaries. If fast switching is allowed by the product and by the unit, additional start-up, synchronization, or minimum-availability constraints are required. The reservoir balance must also include the water effect of reserve activation, since activated upward generation consumes additional water and activated downward pumping may store additional water.

Reserve products add capacity revenue and, depending on the market design, activation revenue or cost. Equation (D.21) gives a generic expected-profit contribution.

$$\sum_{t \in \mathcal{T}^{res}} \left(\pi_t^{cap,up} R_t^{up} + \pi_t^{cap,down} R_t^{down} \right) + \sum_{\omega \in \Omega} \hat{P}_\omega \sum_{t \in \mathcal{T}^{res}} \left(\pi_{t,\omega}^{act,up} A_{t,\omega}^{up} + \pi_{t,\omega}^{act,down} A_{t,\omega}^{down} - C_{t,\omega}^{act} \right) \quad (\text{D.21})$$

where $\pi_t^{cap,\cdot}$ are reserve-capacity prices, $\pi_{t,\omega}^{act,\cdot}$ are activation settlement prices, and $C_{t,\omega}^{act}$ collects activation-related operating costs. If reserve acceptance itself is uncertain, the capacity variables can be combined with the bidding logic of Section D.2: offered reserve capacity is selected before clearing, accepted capacity becomes a fixed commitment, and activation is then represented conditionally on the accepted capacity.

The objective extension creates an opportunity-cost trade-off. Increasing reserve capacity can increase capacity revenue, but it reduces the feasible energy-market dispatch region by reserving power and energy headroom. A reserve-aware optimizer may choose lower energy arbitrage in some intervals to preserve flexibility for reserve products, or reject reserve capacity when the expected activation consequences are too costly. This is precisely why reserve co-optimization should be embedded in the dispatch problem rather than evaluated as an external revenue adder.

In a rolling formulation, reserve commitments follow the same chronology as other market decisions. At a decision time, reserve capacity for market gates that have not closed can be optimized using the information set then available. Once a reserve product clears, the accepted reserve capacity becomes part of the inherited commercial and physical state for the relevant delivery intervals. Later rolling solves must respect that capacity commitment even if the updated energy-price forecast would otherwise prefer a different dispatch.

The stochastic distinction between commitment and activation is also important for nonanticipativity. If reserve capacity is sold before the activation signal is known, the same reserve-capacity decision must apply across all scenarios that are indistinguishable at the reserve gate. Scenario-dependent activation can then change dispatch, storage states, imbalance exposure, and future opportunity costs. If activation is severe in only a small subset of scenarios, a risk-neutral model

may still sell reserve capacity when the capacity payment is large enough. A risk-averse extension, such as the conditional-value-at-risk formulation in Section D.1, could penalize policies that perform poorly under rare but intense activation events.

Chapter 5 does not evaluate this extension numerically. Implementing it would require specifying the reserve products, gate chronology, acceptance rules, activation-duration requirements, settlement prices, and non-delivery penalties of the target market. It would also increase the size of the stochastic MILP, because reserve headroom constraints must hold across scenarios and because activation affects storage trajectories. The formulation above shows that reserve-market participation can be added consistently: reserve capacity is modelled as a nonanticipative commitment, activation is modelled as a scenario-dependent realization, and both are coupled to the same physical storage constraints used for energy-market dispatch.

D.4 Quadratic Terminal Water-Value Alternatives

The terminal water-value approximation used in the rolling controller is piecewise linear. This choice is deliberate: it preserves compatibility with the MILP formulation while allowing the marginal value of stored water to vary across reservoir-storage bands. A smoother alternative is possible if the estimated marginal water values are first approximated by linear functions and then integrated. The resulting terminal value is piecewise quadratic rather than piecewise linear.

Let $z \in [0, 1]$ denote normalized usable reservoir storage and let $\mu(z)$ denote the marginal value of stored water. The calibrated continuation runs produce estimates over storage bands. Equation (D.22) treats them as constant within each band.

$$\mu(z) = \mu_i \quad z \in [z_i, z_{i+1}] \quad (\text{D.22})$$

Equation (D.23) gives the corresponding piecewise-linear value function.

$$V(z) = V(z_i) + \mu_i(z - z_i) \quad z \in [z_i, z_{i+1}] \quad (\text{D.23})$$

After conversion from normalized storage to physical volume, this is the structure embedded in the rolling controller. The slope μ_i is the economic reward assigned to an additional cubic metre of terminal water in band i .

A quadratic alternative joins the marginal values by straight lines. Equation (D.24) gives one such segment.

$$\mu(z) = a_i z + b_i \quad z \in [z_i, z_{i+1}] \quad (\text{D.24})$$

Equation (D.25) integrates that marginal value.

$$V(z) = V(z_i) + \int_{z_i}^z (a_i u + b_i) du = V(z_i) + \frac{a_i}{2} (z^2 - z_i^2) + b_i (z - z_i) \quad (\text{D.25})$$

The value function is quadratic inside the segment. If the marginal value is decreasing, then $a_i < 0$, so the value function is concave. This does not imply that the value of water is negative. As long as $\mu(z) > 0$, the terminal value $V(z)$ still increases with storage, but at a decreasing rate.

A coarser variant fits a small number of straight lines to the marginal-value estimates. In the diagnostic shown below, one line is fitted to the first eight decile points, covering the lower and middle reservoir range, and a second line is fitted through the last two decile points, covering the high-storage range where the estimated marginal value falls more sharply. Integrating this marginal approximation gives a two-region quadratic value function. Such a representation

is smoother and lower-dimensional than a decile-by-decile step function, but it changes the optimization structure. A quadratic terminal reward moves the problem away from a pure MILP unless it is re-linearized or handled through a quadratic-capable solver interface. For that reason, the adopted controller keeps the piecewise-linear representation as the operational formulation.

Figure D.1 compares the marginal-value approximations. Figure D.2 then compares the integrated terminal values implied by those marginal curves. Because the areas under the marginal curves are similar, the integrated values remain close. Figure D.3 reports the difference relative to the adopted piecewise-linear value function in thousands of euros.

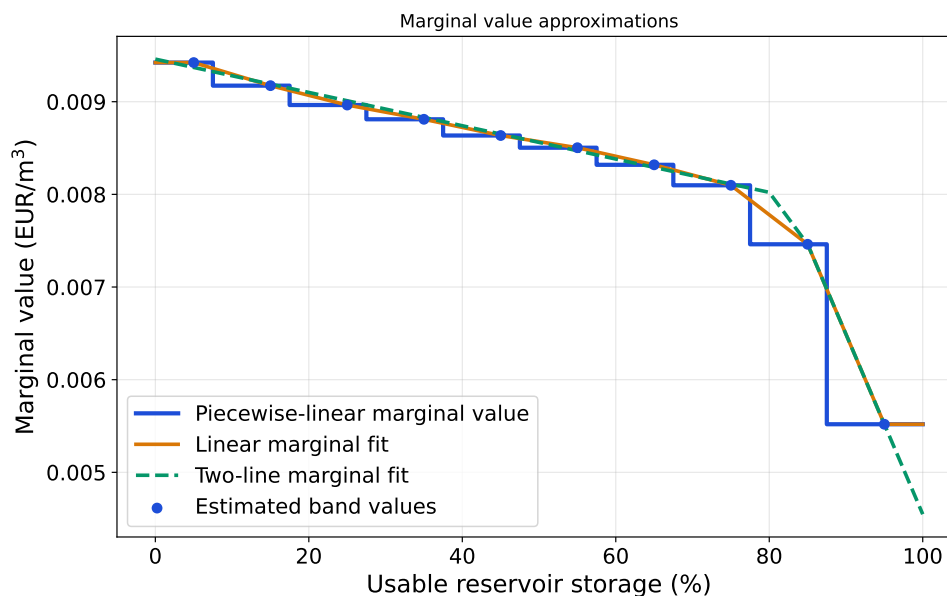


Figure D.1: Marginal-value approximations derived from the calibrated continuation-value coefficients.

The numerical comparison confirms that the smoother alternatives do not materially alter the total calibrated terminal value. The adopted piecewise-linear curve reaches approximately 27.52 million EUR at full usable storage, while the integrated linear-marginal and two-line alternatives reach nearly identical values. The maximum internal difference between the integrated linear-marginal alternative and the piecewise-linear value function is below 0.1 million EUR. This explains why the integrated curves nearly overlap.

This appendix presents the quadratic form as a modelling extension rather than as a replacement used in the numerical experiments. A quadratic terminal value could be appropriate if a smoother continuation-value signal were required, or if the optimizer were allowed to solve a quadratic formulation directly. However, for repeated rolling stochastic optimization, the piecewise-linear form gives the main economic effect, preserves interpretability of the marginal water values, and remains naturally compatible with the MILP structure.

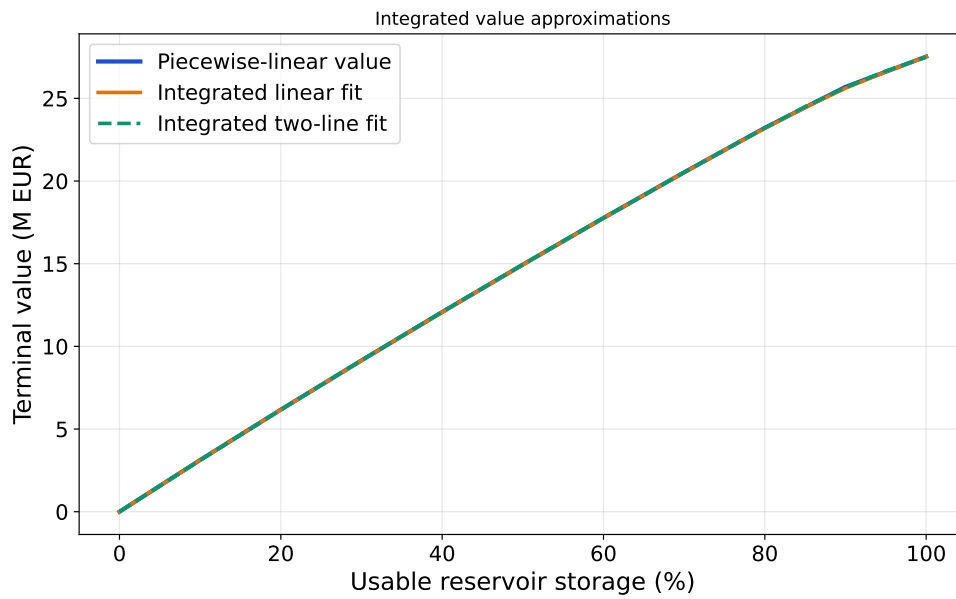


Figure D.2: Integrated terminal-value approximations implied by the alternative marginal-value fits.

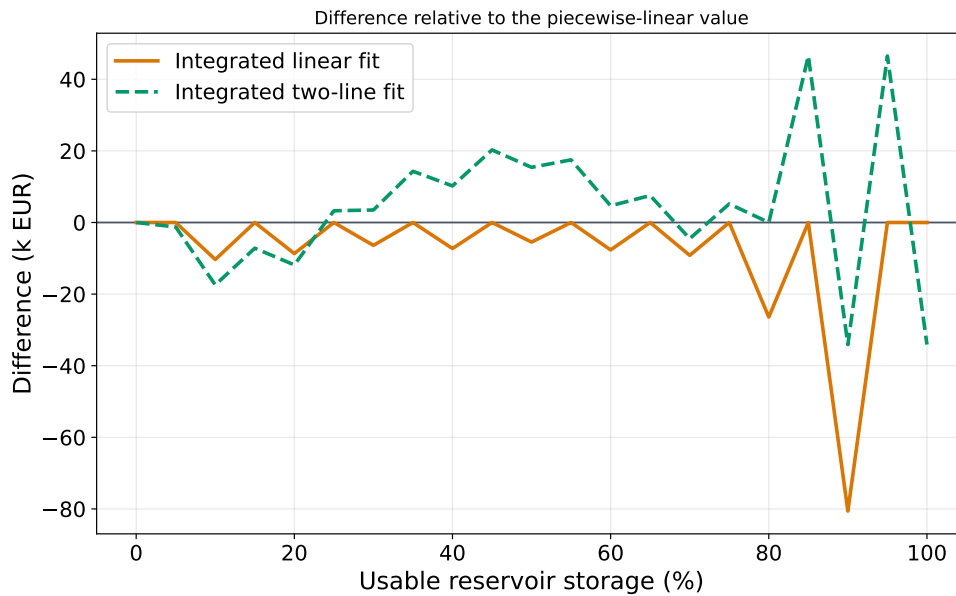


Figure D.3: Difference between the alternative integrated value functions and the adopted piecewise-linear value function.