

Faculdade de Engenharia da Universidade do Porto

Design of a Roll-Heave Decoupled Suspension for a Formula Student Prototype

Gabriel Peixoto



Master in Mechanical Engineering
Specialization Machine Design

Supervisor: Prof. Pedro Leal Ribeiro

September 28, 2026

Design of a Roll-Heave Decoupled Suspension for a Formula Student Prototype

Gabriel Peixoto

Master in Mechanical Engineering
Specialization Machine Design

September 28, 2026

Abstract

This dissertation presents the design, simulation, and structural verification of a roll-heave decoupled suspension for future Formula Student prototypes of the FSFEUP team. In a conventional suspension, a single elastic element at each corner reacts to the heave, pitch, and roll modes simultaneously, so its selection is always a compromise; a decoupled architecture routes the wheel motion into separate heave and roll elements, allowing the corresponding stiffnesses to be set independently.

The work follows a simulation-driven design process. Ride, pitch, and roll targets are derived from the aerodynamic operating envelope and translated into a suspension geometry, a set of motion ratios, and the corresponding elastic elements: heave springs of 83 and 100 N/mm at a motion ratio of 2.20, and roll springs of 50 N/mm at 2.08, meeting the roll gradient target of 0.94 deg/g. A seven-degree-of-freedom vehicle model in MATLAB returns heave, pitch, and roll modes at 3.57, 3.40, and 5.51 Hz, and shows that a four-corner layout at the same heave damping would leave the roll mode at a damping ratio of 0.17 against the 0.46 obtained here, the quantitative case for the decoupled architecture. A multibody model in Adams reproduces the derived rates within four per cent and measures a decoupling efficiency above 98 per cent on both elastic paths. The bellcrank mechanism is then designed in Siemens NX and verified in Abaqus. The geometry is driven by compliance and not by stress: the peak value of 313 MPa leaves a safety factor of 1.6 in 7075-T6 and a fatigue life three orders of magnitude longer than the duty requires, while the deflection of the roll path uses nearly its whole allowance. Integration into a surface model of the monocoque converts the remaining compliance into a requirement on the chassis, namely an attachment stiffness of 19 400 N/mm at the front and 25 900 N/mm at the rear, which holds the total loss in ride rate below two per cent. The architecture is finally compared with the conventional suspension of the FSFEUP03 prototype.

The elastic definition and the mechanism are carried to the level of detail the next design stage requires, while the kinematics should continue to be iterated as some characteristics, such as the lateral migration of the roll centre through roll, may warrant revision. Every elastic and geometric parameter is derived from a declared set of targets, so a change in those targets, be it a different aerodynamic map, a different roll gradient, or a different tyre, propagates through the same procedure, and the analytical and multibody models return the corresponding design without being rebuilt. The contribution is therefore both methodological and pedagogical: a documented and repeatable design procedure, a consistent set of vehicle dynamics parameters, and a simulation environment on which the team can iterate as the concept is developed into a prototype.

Keywords: Roll-heave decoupled suspension, Formula Student, Vehicle dynamics, Suspension kinematics, Ride and roll damping, Multibody simulation, Finite element analysis

Resumo

Esta dissertação apresenta o projeto, a simulação e a verificação estrutural de uma suspensão desacoplada em adornamento e oscilação vertical, destinada a futuros protótipos da equipa FSFEUP. Numa suspensão convencional, um único elemento elástico em cada canto reage em simultâneo aos três modos da massa suspensa, pelo que a sua seleção é sempre um compromisso; uma arquitetura desacoplada encaminha o movimento da roda para elementos separados, permitindo definir independentemente as rigidezes de oscilação vertical e de adornamento.

O trabalho segue um processo de projeto orientado por simulação. Os alvos de oscilação vertical, arfagem e adornamento derivam do envelope aerodinâmico e traduzem-se numa geometria, em relações de movimento e nos elementos elásticos: molas de oscilação vertical de 83 e 100 N/mm para uma relação de 2,20 e de adornamento de 50 N/mm para 2,08, cumprindo o gradiente de adornamento alvo de 0,94 °/g. Um modelo de sete graus de liberdade em MATLAB devolve estes três modos a 3,57, 3,40 e 5,51 Hz e mostra que uma arquitetura de quatro cantos, ao mesmo amortecimento vertical, deixaria o modo de adornamento a 0,17 contra os 0,46 aqui obtidos, o argumento quantitativo a favor do desacoplamento. Um modelo multicorpo em Adams reproduz as rigidezes derivadas a menos de quatro por cento e mede uma eficiência de desacoplamento superior a 98 por cento nos dois caminhos. O mecanismo de balancim, projetado em Siemens NX e verificado em Abaqus, é condicionado pela deformação e não pela tensão: o valor de pico de 313 MPa deixa um fator de segurança de 1,6 em 7075-T6 e uma vida à fadiga três ordens de grandeza acima do exigido, enquanto a deformação do caminho de adornamento consome quase toda a tolerância. A integração no modelo do monocoque converte a restante flexibilidade num requisito de rigidez das fixações ao chassis: 19 400 N/mm à frente e 25 900 N/mm atrás, mantendo a perda total de rigidez vertical abaixo de dois por cento. A arquitetura é por fim comparada com a suspensão convencional do protótipo FSFEUP03.

A definição elástica e o mecanismo estão ao nível de detalhe exigido pela fase seguinte, enquanto a cinemática deve continuar a ser iterada: a migração lateral do centro de adornamento é a primeira candidata a revisão. Todos os parâmetros elásticos e geométricos derivam de um conjunto declarado de alvos: alterá-los, seja o mapa aerodinâmico, o gradiente de adornamento ou o pneu, propaga-se pelo mesmo procedimento e os modelos devolvem o projeto correspondente. O contributo é, por isso, metodológico e pedagógico: um procedimento de projeto repetível, parâmetros de dinâmica de veículos coerentes e um ambiente de simulação sobre o qual a equipa pode iterar.

Palavras-chave: Suspensão desacoplada em adornamento e oscilação vertical, Formula Student, Dinâmica de veículos, Cinemática de suspensão, Amortecimento vertical e de adornamento, Simulação multicorpo, Elementos finitos

UN Sustainable Development Goals

The United Nations Sustainable Development Goals (SDGs) establish a shared global agenda for sustainability, innovation, and human development. This dissertation develops a suspension architecture for a student motorsport prototype, and its relation to that agenda is indirect but specific: it lies in how the design was arrived at, in the engineering capability it transfers to a student team, and in the material that a duty-derived verification avoids committing.

SDG 4.4 By 2030, substantially increase the number of youth and adults who have relevant skills, including technical and vocational skills, for employment, decent jobs and entrepreneurship

SDG 9.5 Enhance scientific research and upgrade the technological capabilities of industrial sectors, encouraging innovation

SDG 12.2 By 2030, achieve the sustainable management and efficient use of natural resources

Discussion. The contribution of this work to the SDG agenda is methodological rather than material. Its most defensible effect is educational (SDG 4.4): a student team is left with a design procedure, a set of verified parameters, and a simulation environment that can be re-run against new targets, which is a durable capability instead of a single artefact. The engineering contribution (SDG 9.5) lies in submitting an unconventional architecture to quantitative assessment instead of adopting it on reputation. The material contribution (SDG 12.2) is real but modest, and is stated as such: deriving the fatigue requirement from the service duty avoids the over-specification that a conventional life assumption would impose, and resolving the design in simulation reduces the hardware consumed in reaching it. The architecture itself is the heavier of the two compared here, by 1.2 kg on an assembly of 5.4 kg, so no claim of material saving is made for it. No claim is made regarding vehicle energy consumption, which this work neither targets nor measures.

SDG	Target	Contribution in this thesis	Performance indicators
4	4.4	The work is written as a transferable design procedure rather than as a single result: a documented chain from aerodynamic targets to geometry, stiffness, and damping, supported by a parametric multibody model and a MATLAB implementation reported in full. It equips successive generations of the FS-FEUP team with a method they can apply and question.	Documented design workflow; parametric models handed over; number of team members and subsequent prototypes using the procedure.
9	9.5	A suspension architecture uncommon in Formula Student is assessed against a conventional one on quantitative grounds, and the elastic and kinematic behaviour is verified in simulation before any hardware is committed. The decoupling is quantified through simulation.	Decoupling efficiency above 98% on both elastic paths over the full travel; agreement between analytical and multibody rates within 4%; requirements verified against independently computed values.
12	12.2	Component sizing is derived from the duty the vehicle actually sees: the fatigue life follows from a simulated lap and the design distance, and the alloy is selected against the stresses the component develops. Verification in simulation reduces the number of physical iterations needed to reach a sound design.	Fatigue life derived from a 585 km service history; alloy selected against a computed peak stress of 313 MPa; design iterations resolved in simulation before manufacture.

Agradecimentos

A elaboração desta dissertação não teria sido possível sem o contributo, incentivo e presença de várias pessoas, a quem gostaria de expressar a minha mais profunda gratidão.

Em primeiro lugar, deixo um agradecimento muito especial aos meus pais, à minha irmã e a toda a minha família pelo apoio constante que me deram, não apenas durante os momentos de maior exigência da realização desta dissertação, mas ao longo de todo o meu percurso académico.

Gostaria de expressar o meu sincero agradecimento ao meu orientador, Professor Pedro Leal Ribeiro, por toda a orientação, partilha de conhecimento e disponibilidade demonstrada ao longo deste trabalho.

Um agradecimento muito especial à equipa da Formula Student FEUP. Quero agradecer, em particular, aos líderes dos departamentos de Suspensão, Nuno Martins, de Chassis, Miguel Monteiro, de Aerodinâmica, Pedro Neutel e de Dinâmica Veicular, Luísa Carmo, pela contribuição para o desenvolvimento e enriquecimento deste projeto. Estendo também este agradecimento a todos os restantes membros destes departamentos e aos Mechanical Chief Engineers, Rui Soares e João Cardoso, pela ajuda ao longo desta jornada.

Gabriel Peixoto

“ You must always strive to be the best, but you must never believe that you are.”

Juan Manuel Fangio

Contents

1	Introduction	1
1.1	Context	1
1.1.1	Formula Student	1
1.1.2	Formula Student FEUP	2
1.1.3	Roll-Heave Decoupled Suspensions	3
1.2	Goal	4
1.3	Methodology	5
1.4	Structure	5
2	Literature Review	7
2.1	Types of Suspensions	7
2.2	Basic Vehicle Dynamics Concepts	8
2.2.1	Wheelbase and Track Width	8
2.2.2	Wheel Camber	8
2.2.3	Toe Angle	9
2.2.4	Kingpin Inclination, Scrub Radius and Spindle Length	10
2.2.5	Caster Angle and Mechanical Trail	12
2.2.6	Ackermann Steering Geometry	13
2.2.7	Slip Angle	14
2.2.8	Tyre Load Sensitivity	15
2.2.9	Ride and Roll Rate	16
2.2.10	Bellcrank Function, Motion Ratio and Wheel Rate	16
2.2.11	Load Transfer	18
2.2.12	Jacking	20
2.2.13	Bump Steer	20
2.3	Double-Wishbone Suspension	21
2.3.1	Parallel equal-length double wishbones	22
2.3.2	Converging equal-length double wishbones	22
2.3.3	Parallel unequal-length double wishbones	23
2.3.4	Converging unequal-length double wishbones	23
2.4	Suspension Modes	24
2.4.1	Heave	26
2.4.2	Roll	26
2.4.3	Pitch	26
2.4.4	Warp	26
2.5	Suspension Movement Control Mechanisms	27
2.5.1	Anti-Features	27
2.5.2	Conventional Suspension System	28

2.5.3	Roll-Heave Decoupling Mechanism	30
2.6	Roll-Heave Decoupled Suspensions in Formula Student	30
2.7	Formula Student Relevant Rules	33
2.8	Multibody Simulation in Suspension Design	35
2.8.1	Formulation	35
2.8.2	Numerical Solution	36
2.8.3	The ADAMS Formulation	36
2.8.4	Applications to Suspension Analysis	37
2.9	Chapter Conclusions	37
3	Suspension Geometry and Mode-Decoupling Architecture	39
3.1	Development Workflow	39
3.2	Design Requirements and Packaging Constraints	39
3.3	Vehicle Data and Dynamic Targets	42
3.3.1	KPI and Caster Influence on Camber Gain	44
3.3.2	Roll Centre Height Justification	44
3.4	Preliminary Kinematic Targets	48
3.5	Outer Hardpoint Definition	48
3.6	Suspension Geometry	49
3.6.1	First Suspension Geometry	49
3.6.2	Second Suspension Geometry	51
3.6.3	Final Suspension Geometry	52
3.7	Bellcrank Layout and Roll-Heave Decoupling Concept	53
3.8	Motion-Ratio Framework of the Decoupled Architecture	54
3.8.1	Design Implications of the Motion Ratios	56
3.9	Chapter Conclusions	57
4	Elastic and Damping Definition	59
4.1	Sizing Philosophy and Design Constraints	59
4.1.1	Displacement Limits	59
4.1.2	Load Cases	60
4.2	Ride Rates from the Aerodynamic Constraint	61
4.3	Heave Path Kinematics and Motion-Ratio Bounds	62
4.3.1	Kinematic Lower Bound	63
4.3.2	Static Position and the Adopted Motion Ratio	63
4.4	Anti-Features	64
4.4.1	Anti-Dive from the Nose-Dive Constraint	64
4.4.2	Anti-Lift and Anti-Squat from the Rebound Constraint	65
4.5	Spring Rates	66
4.5.1	Heave Springs	66
4.5.2	Roll Gradient Target from the Skidpad Aeromap	67
4.5.3	Required Total Roll Stiffness and its Distribution	68
4.5.4	Roll Spring and Motion-Ratio Selection	69
4.5.5	Sensitivity to the Roll Gradient Target	70
4.6	Vehicle Models	71
4.6.1	Quarter-Car Model	71
4.6.2	Seven-Degree-of-Freedom Model	73
4.6.3	Modal Behaviour and the Decoupling Argument	75
4.7	Damping Definition	76

4.7.1	Attainable Damping Ratios	76
4.7.2	Nominal Coefficients	77
4.7.3	The Roll Path and the Antisymmetric Modes	79
4.7.4	Verification Against the Selected Damper	79
4.7.5	Distribution of the Roll Damping	80
4.7.6	Limitations	82
4.8	Response at the Nominal Operating Point	82
4.9	Travel, Clearance and Requirement Verification	84
4.9.1	Combined Roll and Heave Travel	84
4.9.2	Verification of the Derived Requirements	85
4.9.3	Outputs for the Multibody Model	86
4.10	The Case for the Decoupled Architecture	87
4.10.1	The Two Architectures Share One Set of Equations	87
4.10.2	Stiffness: a Constraint that Depends on the Targets	88
4.10.3	Damping: a Constraint with no Corrective Device	89
4.10.4	Independence of Adjustment	90
4.11	Chapter Conclusions	91
5	Multibody Model and Kinematic Verification	93
5.1	Modelling Scope and Objectives	93
5.2	Model Description	95
5.3	Motion Ratio Verification	97
5.4	Rate Verification	98
5.5	Decoupling Efficiency	99
5.5.1	Single-Wheel Input and Left–Right Symmetry	101
5.6	Roll Centre Position and Migration	103
5.7	Camber Behaviour	104
5.8	Steering Kinematics	106
5.9	Steering Axis Geometry	109
5.9.1	Ackermann Geometry	110
5.10	Chapter Conclusions	112
6	Bellcrank Design, Structural Verification and Manufacturing Considerations	115
6.1	Design Requirements and Design Loads	115
6.1.1	Kinematic Requirements	115
6.1.2	Design Load Case	116
6.1.3	Stiffness Requirement	117
6.2	Pivot Bearing Selection	119
6.2.1	Duty and Design Load	120
6.2.2	Candidate Bearing Types	120
6.2.3	Selection Criterion	121
6.2.4	Selection and Revision of the Bore	121
6.3	Concept and Layout	122
6.3.1	Material Class and Manufacturing Route	122
6.3.2	Arrangement of the Attachment Points	123
6.3.3	Form of the Body	124
6.3.4	Assembly	125
6.4	Structural Verification by FEA	125

6.4.1	Strength Criterion	125
6.4.2	Load Case	126
6.4.3	Model and Boundary Conditions	126
6.4.4	Mesh	127
6.4.5	Results	128
6.4.6	Stiffness of the Two Load Paths	129
6.4.7	Fatigue Assessment	131
6.5	Selection of the Alloy	133
6.5.1	Strength at the Governing Location	133
6.5.2	Machining Behaviour and Surface Finish	134
6.5.3	Corrosion and Temper	134
6.6	Drawing Specification	135
6.7	Chapter Conclusions	135
7	Monocoque Integration and Compliance Requirements	137
7.1	Packaging	137
7.1.1	Installed Layout and Chassis Interfaces	138
7.1.2	Clearance over the Travel Range	139
7.1.3	Mass of the Installed Assembly	141
7.2	Compliance Budget	142
7.2.1	Axial Compliance of the Links	142
7.2.2	Allocation of the Budget	143
7.3	Sensitivity to Attachment Stiffness	144
7.3.1	Ride Rate: Closed-Form Sensitivity	144
7.3.2	Compliance Steer and Camber	146
7.4	Chapter Conclusions	149
8	Results, Comparative Analysis and Discussion	151
8.1	Results against the Objectives	151
8.2	Comparison with the Formula Student FEUP 03 (FSFEUP03) Suspension .	153
8.2.1	Kinematic Equivalence	153
8.2.2	Reachable Stiffness and Damping	154
8.2.3	Coupling of Adjustment	155
8.3	Discussion of Trade-offs	156
8.3.1	Mass, Parts and Volume	156
8.3.2	Ease of Adjustment	157
8.3.3	Complexity	157
8.4	Limitations	158
8.5	Chapter Conclusions	159
9	Conclusions and Future Work	161
9.1	Conclusions	161
9.2	Future Work	162
	References	165
A	Camber Change with Steering: Isolated Effects	169

B	Aerodynamic Maps	171
B.1	Straight-Line Aeromap	171
B.2	Skidpad Roll Sensitivity	172
C	Assembly of the 7-DOF Modal Model Matrices	175
D	MATLAB Implementation of the 7-DOF Modal Model	179
D.1	Implementation Choices	181
D.2	The Routines	181
D.3	Listings	184
E	Derivation of the Rocker Design Loads	193
E.1	Frame and Notation	193
E.2	Load Cases	193
E.3	Contact-Patch Forces	194
E.3.1	Vertical Load	194
E.3.2	Longitudinal and Lateral Forces	194
E.3.3	Aligning Torque	195
E.4	Equilibrium of the Upright	195
E.5	From the Pushrod to the Two Elements	196
E.6	Verification	196
E.7	Limitations	197
F	Modelling Entities in ADAMS/Car	199
F.1	Model Hierarchy	199
F.2	Geometry and Parameterisation	200
F.3	Parts	200
F.4	Constraints and Compliant Connections	200
F.5	Force Elements	201
F.6	Communicators	201
F.7	Test Rigs and Analyses	202
G	Front Left Bellcrank Technical Drawing	203

List of Figures

1.1	Formula Student competitions around the world - ©FSSwitzerland	1
1.2	FSFEUP02 competing at Formula Student Portugal 2025. ©Formula Student Portugal.	2
1.3	FSFEUP at Formula Student Switzerland 2026. ©Formula Student Switzerland.	3
1.4	Mode-decoupled damper arrangement, in which the heave and roll paths are actuated by separate units	4
2.1	Wheelbase representation	8
2.2	Front view of the suspension track	9
2.3	Camber angle	9
2.4	The effect of camber on wheel grip	10
2.5	Positive, Neutral, and Negative toe angle	11
2.6	Kingpin Inclination, Scrub Radius and Spindle Length Illustration	12
2.7	Caster Angle and Mechanical Trail	13
2.8	Ackermann Kinematic relationships during cornering	14
2.9	Slip angle representation at the tyre contact patch	15
2.10	Simplified representation of the suspension system	17
2.11	Bellcrank displacements from which the motion ratio is obtained	17
2.12	Paths of the lateral load transfer at an axle	19
2.13	Geometric condition for the absence of bump steer	21
2.14	Parallel equal-length double wishbones	22
2.15	Converging equal-length double wishbones	23
2.16	Parallel unequal-length double wishbones	24
2.17	Converging unequal-length double wishbones	25
2.18	The four suspension modes	25
2.19	T-bar mechanism for warp control	27
2.20	Anti-feature geometries at full compensation	29
2.21	Conventional Formula Student suspension	30
2.22	Mercedes-AMG One suspension	31
2.23	Mode-decoupled inboard layouts in Formula Student	32
3.1	Suspension development workflow	40
3.2	Control Volume of the wheel	41
3.3	Control Volume dimensions	41
3.4	Front suspension monocoque restrictions alongside the initial hardpoints.	42
3.5	Rear suspension monocoque restrictions alongside the initial hardpoints.	42
3.6	Front suspension monocoque restrictions alongside the final hardpoints.	43

3.7	Rear suspension monocoque restrictions alongside the final hardpoints. . . .	43
3.8	Isolated effect of the Kingpin Inclination (KPI) ($\text{caster} = 0^\circ$) on camber change across the steering range.	45
3.9	Isolated effect of the caster angle ($\text{KPI} = 0^\circ$) on camber change across the steering range.	45
3.10	Outer hardpoints within the rim envelope	49
3.11	Wireframe representation of the existing front FSFEUP03 suspension geometry.	50
3.12	Wireframe representation of the existing rear FSFEUP03 suspension geometry.	50
3.13	Wireframe representation of the first front suspension geometry.	51
3.14	Wireframe representation of the first rear suspension geometry.	51
3.15	Wireframe representation of the second front suspension geometry.	52
3.16	Wireframe representation of the second rear suspension geometry.	52
3.17	Wireframe representation of the final front suspension geometry.	53
3.18	Concept layout of the roll-heave decoupled mechanism.	54
3.19	Anti-tension device: the spring is compressed from the neutral position in both directions of relative bellcrank motion.	55
4.1	Two-Degree of Freedom (DOF) quarter-car model. The wheel-referred rate WR and damping coefficient c_w represent the suspension seen at the wheel, and the tyre is modelled as an undamped vertical spring K_t	72
4.2	Seven-DOF model of the roll-heave decoupled suspension. Each axle carries a heave element, acted on by both wheels jointly, and a roll element, acted on by their difference; the rear axle is identical in form to the front axle shown in (a).	74
4.3	Modal damping ratios of the seven-DOF model as a function of the front heave damping coefficient referred to the wheel, with $c_{h,R}$ held in proportion to the ride rates. The sprung ratios reach a ceiling well below the conventional target, while the wheel-hop modes reach the critically damped condition before that ceiling is attained.	77
4.4	Front share of the elastic lateral load transfer during a lateral acceleration ramp, for three roll damping distributions (above), and the peak departure from the steady-state value across the sweep (below). The total roll damping is constant.	81
4.5	Frequency response to a road displacement applied at all four wheels, at the damping of Table 4.11. Ride height above, dynamic tyre load below. . .	83
4.6	Transient responses at the nominal damping: lateral acceleration ramp, single-wheel kerb strike, and braking ramp.	84
4.7	Combinations of axle heave rate and mechanical axle roll stiffness reachable by each architecture. The four-corner layout is confined to the shaded region.	88
4.8	Combinations of heave and roll damping ratio reachable by each architecture. The four-corner layout is confined to a curve; the decoupled layout reaches the whole area.	90
5.1	Full-vehicle model of the decoupled suspension implemented in MacNeal-Schwendler Corporation (MSC) Adams/Car, in the static position.	95
5.2	Topology of the decoupled mechanism in the multibody model	96

5.3	Heave element displacement in parallel travel, from which the heave motion ratio is obtained.	98
5.4	Roll element displacement in opposite travel, from which the roll motion ratio is obtained.	99
5.5	Ride rate against wheel travel in parallel travel.	100
5.6	Wheel rate against wheel travel in parallel travel.	101
5.7	Roll element displacement in parallel travel	102
5.8	Heave element displacement in opposite travel	102
5.9	Element displacement at the front axle for a single-wheel input, with the left and right wheels driven in turn.	103
5.10	Element displacement at the rear axle for a single-wheel input.	104
5.11	Vertical migration of the roll centre through opposite travel.	105
5.12	Lateral migration of the roll centre through opposite travel, measured from the vehicle centre plane	105
5.13	Camber angle against wheel travel in parallel travel.	106
5.14	Camber angle against roll angle in opposite travel.	107
5.15	Toe angle against wheel travel at the front axle, before and after the instant centres were aligned with the tie-rod axis.	108
5.16	Toe angle against wheel travel in parallel travel	108
5.17	Toe angle of each wheel against roll angle in opposite travel.	109
5.18	Caster angle against wheel travel in parallel travel.	110
5.19	Kingpin inclination angle against wheel travel in parallel travel.	111
5.20	Steer angle of each front wheel against steering wheel input.	111
6.1	Ride rate of the front axle as a function of the stiffness of the bellcrank, with the spring, motion ratio and tyre rate of the adopted design. The dashed line marks the stiffness adopted as the requirement.	118
6.2	The front left bellcrank	124
6.3	Load and restraint arrangement of the structural model. The two applied forces are shown at the pushrod and roll element attachments, each along the axis of its link; the heave element attachment is restrained along its own axis and reacts the third force; the two bearing seats are coupled to reference points on the pivot axis and permit only rotation about it.	127
6.4	Von Mises stress in the rear right bellcrank under the design case, which is the most highly stressed of the four.	129
6.5	Displacement field of the front left bellcrank under the load used to characterise the heave path (left) and the roll path (right). The scales differ by an order of magnitude.	130
6.6	Logarithm of the life of the front left bellcrank, in repetitions of the lap, from the Brown–Miller assessment. The design requirement is 585 repetitions.	133
7.1	The roll–heave decoupling mechanism	138
7.2	The assembly installed on the vehicle, at both axles. The bodywork is shown as a surface and the monocoque structure itself is not represented	140
7.3	The two smallest clearances found over the travel range.	141
7.4	Front ride rate against wishbone pickup stiffness. The closed form is Equation (7.3) evaluated with $\sum t_j^2 = 0.320$ over the swept pickups; the marker is the requirement of Section 7.3.1. The departure of the lowest point from the curve is discussed in the text.	147

- 7.5 Camber compliance of the front wheel against pickup stiffness. The marker is the requirement of Section 7.3.1, at which the compliance is 0.019 deg/kN. 148
- 7.6 Toe compliance of the front wheel against pickup stiffness. The horizontal line is the kinematic bump steer over the full travel, referred to the same braking load so that the two are comparable on one axis. The marker is the requirement of Section 7.3.1, at which the compliance is 0.0136 deg/kN. 148
- D.1 Computational workflow. Blue denotes input, red the verification steps, and amber the stages that produce results reported in Chapter 4. 180

List of Tables

3.1	Baseline vehicle dimensions and mass properties used in the present work. .	44
3.2	Inertial properties of the sprung mass adopted as inputs to Chapter 4. The aerodynamic conditions on which the elastic sizing rests are established in Table 4.1.	44
3.3	Roll-centre height iterations and corresponding lateral load-transfer metrics.	47
3.4	Preliminary suspension design targets adopted as baseline references.	48
3.5	FSFEUP03 suspension hardpoint coordinates for the front-left quarter. . . .	50
3.6	FSFEUP03 suspension hardpoint coordinates for the rear-left quarter. . . .	50
3.7	First suspension hardpoint coordinates for the front-left quarter.	51
3.8	First suspension hardpoint coordinates for the rear-left quarter.	51
3.9	Second suspension hardpoint coordinates for the front-left quarter.	52
3.10	Second suspension hardpoint coordinates for the rear-left quarter.	52
3.11	Final suspension hardpoint coordinates for the front-left quarter.	53
4.1	Design constraints and load cases adopted for the elastic definition.	61
4.2	Aerodynamic and inertial loads at the two conditions used in this chapter. .	61
4.3	Anti-dive requirement as a function of the clearance limit and of the end-of-straight lift coefficient.	64
4.4	Designed heave springs.	67
4.5	Relative downforce variation with roll angle at the skidpad operating point, from the Computational Fluid Dynamics (CFD) aeromap.	67
4.6	Roll motion ratio and stroke limit for the available roll spring rates, 50/50 distribution.	69
4.7	Effect of the front/rear roll spring choice on the front lateral load transfer distribution.	70
4.8	Selected roll stiffness configuration.	70
4.9	Roll-side sizing re-evaluated for a range of roll gradient targets. The last column is the fraction of the mechanical roll stiffness that survives the series connection with the tyre.	71
4.10	Natural frequencies of the seven-DOF model at the adopted elastic configuration.	75
4.11	Nominal damping coefficients of the decoupled suspension.	78
4.12	Modal damping ratios at the nominal operating point.	78
4.13	Corner damping ratios of the same coefficients under the two conventions used in the literature.	78
4.14	Modal damping ratios with each damping path disabled in turn. The heave and roll paths partition the seven modes with no overlap.	79

4.15	Required coefficients referred to the damper, against the adjustment range of each valve setting. Ranges are equivalent linear coefficients at 0.25 m/s, with compression and rebound averaged. Each entry states where the adjustment range lies with respect to the requirement.	80
4.16	Front loaded-corner displacement around the friction ellipse, with the aerodynamic load scaled by the square of the speed compatible with each lateral acceleration. Element travels include the static compression and are referred to the suspension deflection, with the tyre deflection removed. The governing case is straight-line braking.	85
4.17	Requirements from which a design parameter was solved.	86
4.18	Requirements verified against independently computed values.	86
4.19	Elastic and damping definition outputs for the multibody model of Chapter 5.	87
4.20	Roll stiffness delivered by the corner springs alone, against the target of Section 4.5.3.	89
5.1	Verification matrix of the multibody model. Each quantity is compared against a value derived independently in an earlier chapter.	94
5.2	Data used to build the multibody model.	97
5.3	Analyses performed on the multibody model.	97
5.4	Motion ratios measured in the multibody model against the design values.	97
5.5	Rates at the static position, measured in the multibody model against the values derived in Chapter 4.	99
5.6	Residual coupling between the elastic paths	101
5.7	Toe variation with wheel travel and its consequences at the design condition.	107
5.8	Steering axis angles at the static position, against the preliminary targets of Section 3.4.	109
5.9	Samples adjacent to the reference steering position, used to interpolate the inner wheel angle. Magnitudes are shown.	112
6.1	Lever arms of the bellcrank about the pivot axis, and the pushrod displacement ratio of the outboard geometry, which together give the motion ratios through Equation 6.2.	116
6.2	Candidate design conditions for the bellcrank. Forces are axial, along the respective link, and negative values denote compression. The adopted condition is highlighted.	117
6.3	Effect of a loss of stiffness in the roll path on the roll gradient and on the roll angle at the design lateral acceleration of 1.6 g.	119
6.4	Stiffness required of the bellcrank at its allocated allowance for each path, referred to the wheel and to the pushrod attachment at which it is measured.	119
6.5	Comparison of the candidate pivot bearing types. The friction figure is the pivot friction torque referred back to the wheel through the bellcrank and the pushrod, evaluated at the rear design load.	121
6.6	Nominal properties of the candidate alloys and the derived specific quantities. Values are typical for wrought plate and are used for comparison between alloys, not as design allowables.	123
6.7	Definition of the structural model. All couplings connect the surface of the feature to a reference point at its centre; the two bearing seat reference points lie on the pivot axis.	126
6.8	Mesh refinement on the front right bellcrank under the design case.	128

6.9	Peak von Mises stress on each bellcrank under the design case of Section 6.1.2.	128
6.10	Measured stiffness of the four bellcranks, from the displacement of the pushrod attachment under 1000 N applied at that point, projected on the pushrod axis. The loss is evaluated against the allowance allocated to each path in Table 6.4: one per cent for heave and two per cent for roll.	130
6.11	Minimum life of each bellcrank from the Brown–Miller assessment, in repetitions of the lap. The requirement is 585 repetitions.	132
6.12	Factor of safety on the yield strength of 7075-T6, 503 MPa, at the peak stress of each bellcrank. The requirement is 1.5.	134
7.1	Inboard pickup points of the wishbones and the tie rod, with the direction along which a link in tension pulls on the chassis and the extreme link forces over the load cases of Appendix E. Positive values are tension. Coordinates are given for the left-hand corner; the right-hand corner is the mirror image in X	139
7.2	Mass of the assembly, by axle.	141
7.3	Structural stiffness required at the wheel, and the equivalent deflection at the static corner load, for three levels of admissible loss in ride rate. . . .	142
7.4	Link force per unit vertical wheel load, link length, and the resulting contribution to the summation of Equation (7.2).	143
7.5	Allocation of the admissible loss in ride rate along the heave load path. The bellcrank entry is the loss measured in Section 6.4.6; the remaining entries are requirements placed on parts of the path that have not yet been designed.	144
7.6	Load carried per unit vertical wheel load at each chassis attachment, and the share of the ride-rate compliance for which it is responsible.	145
7.7	Front ride rate against wishbone pickup stiffness, measured and predicted. .	146
8.1	Kinematic parameters of the two suspensions. The static values are read at ride height; the entries marked over the travel are the excursion of a quantity that varies along the stroke and are compressed to a single figure here.	154
8.2	Operating point of the FSFEUP03 suspension, derived from its spring and anti-roll bar rates. Both cars run a track of 1200 mm, so they share the boundary $K_\phi = \frac{1}{4}K_h t^2$ exactly.	154
8.3	Heave and roll damping of the FSFEUP03 over its design range of damping ratio. The heave coefficient is per wheel, referred to the wheel, as in Table 4.11.	155
8.4	Response of the two adjustment targets to a ten per cent change in each elastic element. The off-diagonal entries are what distinguish the two architectures.	156
8.5	Mass of the suspension assembly of each vehicle, by axle, including all components.	156
A.1	Camber change with steering produced by the kingpin inclination alone, in degrees, for caster = 0°.	169
A.2	Camber change with steering produced by the caster angle alone, in degrees, for KPI = 0°.	170
B.1	Straight-line aeromap: lift coefficient and aerodynamic balance as a function of front and rear ride height.	172

B.2	Skidpad roll sensitivity: absolute coefficients and centre of pressure as a function of body roll angle, with the relative change in lift referred to the zero-roll condition.	173
D.1	Routines of the modal analysis implementation.	179
E.1	Base load cases.	194

List of Symbols

Symbols are listed in the units in which they are quoted in the text. Where a quantity is given per axle, the subscripts F and R denote the front and rear axle; where it is given per corner, the subscripts FL , FR , RL and RR denote the four corners.

Latin symbols

Symbol	Description	Unit
a	Longitudinal distance from the sprung-mass centre of gravity to the front axle	m
A_j	Cross-sectional area of link j	mm ²
A_{ref}	Frontal reference area	m ²
a_x, a_y	Longitudinal and lateral acceleration	m/s ²
AD	Anti-dive	%
AL	Anti-lift	%
AS	Anti-squat	%
b	Longitudinal distance from the sprung-mass centre of gravity to the rear axle	m
C	Spring index, D_m/d	—
$\mathbf{C}$	Damping matrix	—
C_l	Lift coefficient	—
c_c	Critical damping coefficient of the corner	N s/m
c_{el}	Damping coefficient referred to the element	N s/m
c_h	Heave damping coefficient, per wheel, referred to the wheel	N s/m
c_r	Roll element damping coefficient	N s/m
c_w	Wheel-referred damping coefficient of the quarter-car model	N s/m
c_ϕ	Axle roll damping coefficient	N m s/rad
D_m	Mean coil diameter of the spring	mm
d	Spring wire diameter	mm
E_j	Young's modulus of link j	GPa
$E_j A_j$	Axial rigidity of link j	MN
F_F, F_R	Total heave force of the front and rear axle, both wheels included	N
F_L	Total aerodynamic downforce	N
$F_{L,F}, F_{L,R}$	Front and rear axle downforce	N
F_{pr}	Force in the pushrod	N

Symbol	Description	Unit
F_h	Force in the heave element	N
$F_{u,i}$	Force the suspension applies to the unsprung mass at corner i	N
F_w	Vertical force at the wheel	N
f_n	Natural frequency	Hz
G	Shear modulus of the spring steel	GPa
g	Gravitational acceleration	m/s ²
h_{CG}	Height of the centre of gravity	mm
h_{RC}	Roll-centre height	mm
h_{ref}	Reference height of the suspension	mm
h_U	Height of the centre of gravity of the unsprung mass	mm
I_p	Pitch inertia of the sprung mass	kg m ²
I_{yy}	Roll inertia of the sprung mass	kg m ²
K	Heave spring rate	N/mm
$\mathbf{K}$	Stiffness matrix	—
K_{att}	Stiffness of the chassis attachments referred to the wheel	N/mm
K_e	Stiffness of a structural path referred to the wheel	N/mm
K_h	Axle heave stiffness	N/mm
K_{links}	Axial stiffness of the suspension links referred to the wheel	N/mm
K_r	Roll spring rate	N/mm
K_t	Tyre vertical stiffness	N/mm
K_W	Wahl correction factor of the helical spring	—
K_ϕ	Axle roll stiffness	N m/deg
k_j	Stiffness of attachment j along its load direction	N/mm
k_x	Centroidal radius of gyration in pitch	mm
L	Wheelbase	mm
L_j	Length of link j	mm
LLTD	Lateral load transfer distribution	%
$\mathbf{M}$	Mass matrix	—
M_ϕ	Roll moment of the sprung mass per unit lateral acceleration	N m/g
MR	Heave motion ratio	—
MR _r	Roll motion ratio	—
MR _{h,L} , MR _{h,R}	Individual heave motion ratios, left and right	—
MR _{r,L} , MR _{r,R}	Individual roll motion ratios, left and right	—
m_s	Sprung mass	kg
$m_{s,c}$	Corner sprung mass	kg
$m_{u,c}$	Corner unsprung mass	kg
m_U	Unsprung mass of an axle	kg
N	Fatigue life	repetitions
n_a, n_t	Active and total number of spring coils	—
n_x, n_y	Longitudinal and lateral acceleration	g
$n_{x,a}, n_{x,b}$	Maximum longitudinal acceleration and deceleration	g
p	Admissible loss in ride rate	—
RR	Ride rate	N/mm

Symbol	Description	Unit
RR_{eff}	Effective ride rate including a compliant path in series	N/mm
r_h, r_r, r_{pr}	Bellcrank lever arms of the heave element, roll element and pushrod	mm
S	Available spring stroke	mm
t	Track width	mm
t_j	Force in link or attachment j per unit vertical wheel load	–
V	Reference vehicle speed	km/h
W	Vehicle weight	N
W_c	Sprung corner weight	N
W_s	Sprung weight	N
WR	Wheel rate	N/mm
WR_r	Wheel rate of the roll path, per wheel	N/mm
w	Total wheel travel	mm
z	Heave displacement of the sprung mass	mm
$z_{r,i}$	Road vertical displacement at corner i	mm
z_s, z_u	Sprung and unsprung displacement of the quarter-car model	mm
$z_{u,i}$	Vertical displacement of the unsprung mass at corner i	mm
$z_{w,F}, z_{w,R}$	Heave deformation of each axle, chassis relative to wheels	mm

Greek symbols

Symbol	Description	Unit
α	Tyre slip angle	°
β_R	Rear brake-force fraction	%
Γ_F, Γ_R	Roll moment of the front and rear axle	N m
γ	Camber angle	°
Δh	Roll moment arm, sprung-mass centre of gravity to the roll axis	mm
ΔW_x	Longitudinal load transfer	N
ΔW_y	Lateral load transfer	N
δ	Steer angle at the road wheel	°
δ_{aero}	Aerodynamic displacement target of the platform	mm
δ_{clr}	Clearance displacement limit of the platform	mm
δ_h, δ_r	Displacement of the heave and roll element	mm
δ_R	Chassis displacement at the rear axle	mm
δ_{pr}	Pushrod displacement	mm
δ_s	Element displacement	mm
δ_w	Wheel displacement	mm
η_F	Front share of the aerodynamic load	%
θ	Pitch angle of the sprung mass	°
θ_c	Caster angle	°
θ_k	Kingpin inclination	°

Symbol	Description	Unit
θ_1, θ_3	Slope of the line from the contact patch and from the wheel centre to the side-view instant centre	$^\circ$
λ	Vector of Lagrange multipliers of the multibody formulation	–
ξ	Modal damping ratio	–
ξ_h, ξ_p, ξ_ϕ	Damping ratio of the heave, pitch and roll mode	–
ρ	Air density; material density where noted	kg/m^3 ; g/cm^3
$\sigma_y, \sigma_u, \sigma_f$	Yield, ultimate and fatigue strength	MPa
σ_{ij}	Component of the stress tensor	MPa
τ	Corrected shear stress in the spring wire	MPa
Φ	Constraint equations of the multibody formulation	–
ϕ	Roll angle of the sprung mass	$^\circ$
$\phi_{\max}$	Maximum admissible roll angle	$^\circ$
$\phi_{w,F}, \phi_{w,R}$	Roll deformation of each axle	$^\circ$

Subscripts and superscripts

Subscript	Meaning
F, R	Front and rear axle
FL, FR, RL, RR	Front-left, front-right, rear-left and rear-right corner
L, R	Left and right side of an axle, in the individual motion ratios
h, r	Heave path and roll path
s, u	Sprung and unsprung
c	Referred to the corner
w	Referred to the wheel
pr	Referred to the pushrod
el	Referred to the elastic element
t	Tyre
j	Index of a link or of a chassis attachment
i	Index of a corner or of a vibration mode
eff	Effective value, including a compliant path in series
mec	Mechanical value, excluding the tyre in series
geom, elast	Geometric and elastic contribution to the lateral load transfer
nom	Design point of the aeromap
br	End-of-straight condition of the aeromap
max	Maximum admissible value
static	Value at the static position

Acronyms list

ARB	Anti-Roll Bar
CAD	Computer-Aided Design
CFD	Computational Fluid Dynamics
CG	Centre of Gravity
DIN	Deutsches Institut für Normung
DOF	Degree of Freedom
EN	European Standard
FDM	Fused Deposition Modeling
FEA	Finite Element Analysis
FEUP	Faculdade de Engenharia da Universidade do Porto
FS	Formula Student
FSAE	Formula Society of Automotive Engineers
FSFEUP	Formula Student FEUP
FSFEUP03	Formula Student FEUP 03
IC	Instant Centre
ISO	International Organization for Standardization
KPI	Kingpin Inclination
LMP	Le Mans Prototype
MR	Motion Ratio
MSC	MacNeal-Schwendler Corporation
NASCAR	National Association for Stock Car Auto Racing
OEM	Original Equipment Manufacturer
RC	Roll Centre
SAE	Society of Automotive Engineers

SLA	Short-Long Arm
TSAC	Tractive System Accumulator Container
VSA	Virtual Swing Arm

Chapter 1

Introduction

1.1 Context

1.1.1 Formula Student

Formula Student is an international engineering competition in which university teams conceive, design, manufacture, and race a small single-seat formula-style prototype. It originated as Formula SAE, first held in the United States in 1981 under the Society of Automotive Engineers (SAE), and has since grown into a network of events across Europe, Asia, Australia, and the Americas, as shown in Figure 1.1. Formula Student Germany, held annually at the Hockenheimring, is among the most competitive of them and gathers on the order of eighty teams in each of its classes.

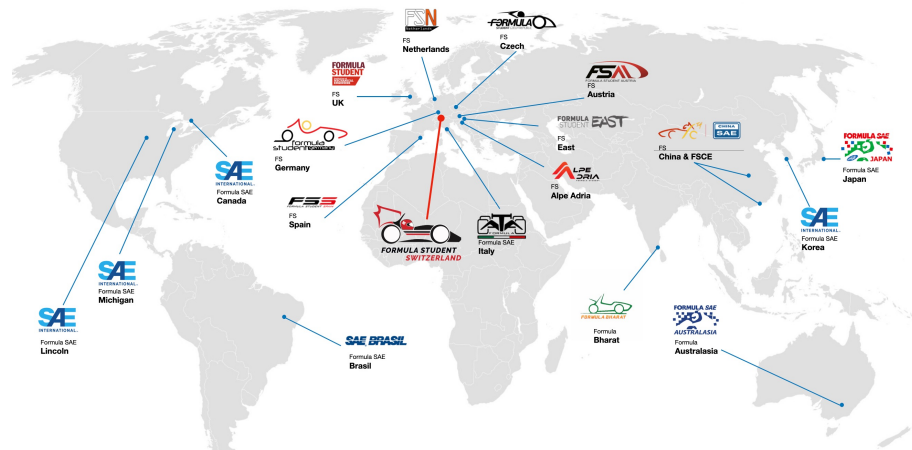


Figure 1.1: Formula Student competitions around the world - ©FSSwitzerland

Teams compete in one of the vehicle classes defined by the regulations, and the events are common to all of them. The static events assess the engineering design, the cost and manufacturing case, and the business case of the prototype. The dynamic events load the car in distinct and well-characterised ways: acceleration over a straight length, skidpad

on a figure-of-eight of constant radius, autocross over a single flying lap of a tight circuit, and endurance over a longer run that also rewards energy efficiency.

1.1.2 Formula Student FEUP

Formula Student FEUP (FSFEUP) is the team representing the Faculty of Engineering of the University of Porto. It was formed in the summer of 2021 by a group of engineering students and has since grown to around eighty members drawn from mechanical engineering, electrical and computer engineering, informatics, and industrial engineering and management.

The team's development has followed a two-year cycle for each prototype. FSFEUP01 was designed during 2022, a season in which the team competed in the static and concept events and collected its first international awards, and was manufactured in 2023. FSFEUP02 (Figure 1.2) followed the same path, designed in 2024 and entered in Concept Class that year, and manufactured in 2025 as an electric vehicle with an autonomous system, which allowed the team to enter the driverless category for the first time. That season produced the team's best results to date, including a third place in the Cost and Manufacturing categories in FSG.

In 2026, FSFEUP competed in the concept class in both FS Switzerland (FSCH) (Figure 1.3) and FS Portugal (FSPT), achieving an overall first place in both competitions.



Figure 1.2: FSFEUP02 competing at Formula Student Portugal 2025. ©Formula Student Portugal.



Figure 1.3: FSFEUP at Formula Student Switzerland 2026. ©Formula Student Switzerland.

The team is currently developing its third prototype, the FSFEUP 03, that will be manufactured in 2027. Alongside the development of FSFEUP 03, the team is also preparing the transition to one-year cycle, which includes a possible transition to a four wheel drive vehicle with a monocoque chassis, instead of a tubular chassis. Given this, the current work prepares a suspension geometry that has this into account.

1.1.3 Roll-Heave Decoupled Suspensions

In a conventional suspension, a single spring–damper unit at each corner reacts simultaneously to the symmetric (heave and pitch) and antisymmetric (roll) motions of the sprung mass. As a consequence, the choice of spring and damper is always a compromise between the response required in ride and the response required in roll, and the two cannot be tuned independently. A roll-heave decoupled suspension removes this compromise by routing the wheel motion into two separate elastic paths, one acting on the symmetric modes and one on roll. This separation allows the ride and roll stiffnesses, and their respective damping, to be set independently, giving finer control of the roll behaviour and of the aerodynamic platform.

The decoupled suspensions, as the one shown in Figure 1.4 are used in professional motorsport, and several Formula Student teams have successfully implemented them, particularly in combination with carbon monocoques [1–3]. The move of FSFEUP towards a monocoque platform in a near future therefore makes this an opportune moment to

establish the design framework and simulation tools for a decoupled suspension concept, so that future prototypes can adopt it on a validated basis.

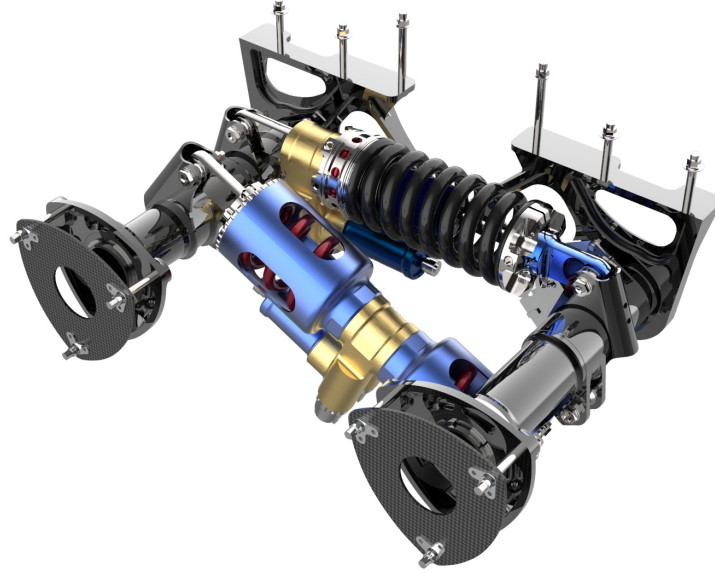


Figure 1.4: Mode-decoupled damper arrangement, in which the heave and roll paths are actuated by separate units [4]. ©Multimatic.

1.2 Goal

The main goal of this dissertation is to develop and verify the design of a roll-heave decoupled suspension intended for future FSFEUP monocoque prototypes. The contribution is primarily methodological: the work does not deliver a finished physical assembly, but a documented design process, a simulation environment, and a consistent set of parameters that will support the physical implementation of the concept in subsequent design cycles.

To this end, the following specific objectives are defined:

- Establish the mechanical principles of roll-heave decoupling and its kinematic implications for the suspension layout.
- Derive the vehicle dynamics targets from the aerodynamic operating requirements and translate them into a suspension geometry, subject to the constraints imposed by monocoque integration and by the Formula Student rules.
- Size the elastic and damping characteristics of the two paths, and examine the modal behaviour they produce, through analytical and multibody simulation.
- Design a refined bellcrank mechanism that delivers the decoupling, and verify its strength and stiffness by finite element analysis.

- Establish the packaging of the assembly within a monocoque and derive the attachment stiffness the chassis must provide.
- Compare the proposed architecture with the baseline FSFEUP03 suspension on the grounds of kinematics, reachable stiffness and damping, and adjustment.

1.3 Methodology

The work follows a simulation-driven development approach, structured as a sequence of stages in which the output of each stage feeds the next. The suspension targets are first derived from the vehicle's operating requirements, with particular emphasis on the aerodynamic platform: the admissible ride-height variation and body attitudes are obtained from the CFD aeromaps, and translated into ride, pitch, and roll targets. From these targets, the kinematic layout and the elastic system, comprising the heave and roll springs and their motion ratios, are defined.

Two simulation environments are used, with distinct purposes. The analytical models implemented in MATLAB, a quarter-car representation and a full-vehicle modal model, are sizing instruments: they set the modal frequency and damping targets and express the decoupling property in the governing equations. The multibody model built in MSC Adams is a verification instrument: it carries the real geometry and therefore measures what the assumptions of the analytical formulation cost, namely the variation of the motion ratios along the travel, the residual coupling between the two elastic paths, and the position and migration of the roll centre.

The bellcrank mechanism that realises the decoupling is then designed and verified by finite element analysis, for strength, for stiffness, and for fatigue life against a service duty derived from a simulated lap. Its integration into the monocoque establishes the packaging of the assembly and converts the compliance of the load path into a stiffness requirement on the chassis. Finally, the proposed architecture is compared against the FSFEUP03 suspension.

A comparison on lap time was envisaged at the outset but is not carried out here: the lap simulation tool in use by the team represents the suspension through corner springs and dampers only, and cannot express the modal elastic paths of a decoupled layout. The benefit of the architecture is therefore quantified analytically, through the combinations of stiffness and damping that each layout can reach.

1.4 Structure

The remainder of this dissertation is organised as follows.

- **Chapter 2** reviews the fundamentals of vehicle dynamics and suspension design relevant to the present work, including tyre behaviour, load transfer, suspension

modes and anti-features, and the state of the art of roll-heave decoupled suspensions in Formula Student.

- **Chapter 3** establishes the baseline geometry and the mode-decoupling architecture, defining the packaging constraints, the kinematic targets, the hardpoints, and the bellcrank concept and motion-ratio framework on which the decoupling relies.
- **Chapter 4** defines the elastic and damping characteristics of the two paths. The ride, pitch, and roll targets are derived from the aerodynamic constraints, the heave and roll springs and their motion ratios are sized, the anti-features are obtained from the displacement limits, and the damping is set on the seven-degree-of-freedom model, which also supplies the response at the resulting operating point and the case for the decoupled architecture against a conventional four-corner layout.
- **Chapter 5** presents the multibody model and the kinematic verification of the geometry: the motion ratios and the rates along the travel, the residual coupling between the two elastic paths, the roll-centre position and migration, and the camber and steering characteristics.
- **Chapter 6** designs the bellcrank that delivers the decoupling, selects its pivot bearings, verifies it for strength, stiffness and fatigue life by finite element analysis, and settles the alloy and the manufacturing route.
- **Chapter 7** establishes the packaging of the assembly within the monocoque and converts the compliance of the load path into the attachment stiffness the chassis has to provide.
- **Chapter 8** reports the results against the objectives, compares the proposed architecture with the FSFEUP03 suspension on kinematics, reachable stiffness and damping, coupling of adjustment, and mass, and discusses the trade-offs and the limitations of that comparison.
- **Chapter 9** draws the conclusions of the work and identifies the developments a subsequent design cycle should take up.

Chapter 2

Literature Review

2.1 Types of Suspensions

Three architectures span the range of automotive practice, and only one of them is a candidate for a Formula Student vehicle. The dependent beam axle connects the two wheels of an axle through a solid structural member, so that the displacement of one wheel is imposed on the other; the linkage that locates it governs the wheel path, the anti-dive, and the roll axis, but the coupling between the two sides cannot be removed by any choice of geometry. It survives in competition only where regulations mandate it, as in the North American National Association for Stock Car Auto Racing (NASCAR) championships [5, 6]. The MacPherson strut is independent, and its packaging efficiency has made it the dominant choice in passenger cars, but it behaves kinematically as a short-long arm layout in which the upper arm is infinitely long: the wheel loses negative camber in bump instead of gaining it, the strut axis cannot be inclined in side view without altering its front-view orientation, and the bending moment carried by the damper shaft introduces friction that has to be compensated by offsetting the spring line [5, 6].

Circuit racing requires the opposite of both: independent wheel control and the freedom to place the roll centre, the camber gain, and the steering axis independently of one another. The double-wishbone, or short-long arm (Short-Long Arm (SLA)), architecture provides it, and is accordingly the layout adopted throughout Formula Student and by the present work. This type of suspension is examined in detail in Section 2.3, after Section 2.2 introduce the concepts necessary to understand it [5, 6].

2.2 Basic Vehicle Dynamics Concepts

This section presents the vehicle dynamics concepts on which the design work of the following chapters is based.

2.2.1 Wheelbase and Track Width

The wheelbase and the track width are fixed at the beginning of the design process and determine much of the dynamic behaviour of the vehicle. The wheelbase is the horizontal distance between the centres of the front and rear contact patches, as shown in Figure 2.1; the distinction between this distance and the distance between the wheel centres only matters when the two axles use wheels of different diameter. The track width is the lateral distance between the contact patches of the two wheels of an axle (Figure 2.2) [5, 7].

The choice of wheelbase is a compromise between agility and stability. A short wheelbase reduces mass and allows the vehicle to change direction more quickly, which suits tight courses, while a long wheelbase gives more directional stability at high speed and, in categories with high downforce, more floor area for the aerodynamic package [5, 7].

The track width follows a simpler rule: a wider track reduces the lateral load transfer and the roll angle for a given lateral acceleration, so it is normally taken to the maximum that the regulations allow. Where the total width of the vehicle is limited and the rear tyres are wider than the front ones, the rear track has to be made slightly narrower than the front. In Formula Society of Automotive Engineers (SAE) and Formula Student (FS) the courses are tight and the vehicles are correspondingly small: track widths of approximately 1.2 m and wheelbases between 1.5 and 1.7 m are common, the lower end being set by the minimum of 1525 mm imposed by the regulations [5, 7].

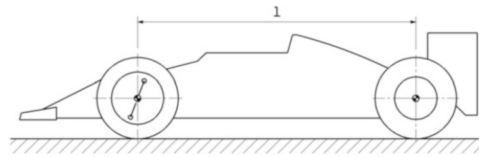


Figure 2.1: Wheelbase representation, where l denotes the distance between axles, written L throughout this work [5]. Reprinted with permission from M. Trzesniowski, Suspension System, 2023, Springer Nature. © 2023.

2.2.2 Wheel Camber

The camber is the angle, γ , between the wheel plane and a plane perpendicular to the road surface, measured in the front view. It is positive when the top of the wheel leans outwards and negative when it leans inwards, as shown in Figure 2.3 [5–7].

The camber varies with the movement of the suspension. In a geometry with negative camber gain, which is the usual choice in racing vehicles, compression takes the wheel

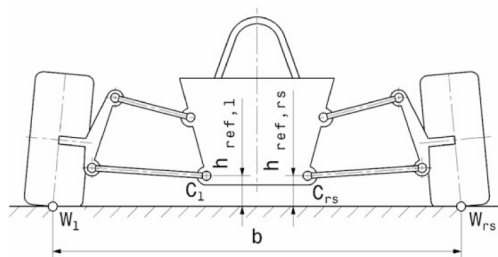


Figure 2.2: Front view of the suspension track, where b is the track width, written t throughout this work, h_{ref} the reference heights, and C the chassis pivot points [5]. Reprinted with permission from M. Trzesniowski, *Suspension System*, 2023, Springer Nature. © 2023.

towards negative camber and rebound takes it towards positive camber. In cornering the body rolls, so the loaded outer wheel is displaced towards positive camber and the unloaded inner wheel towards negative camber [5–7].

The relation between camber and the lateral force the tyre can produce is shown in Figure 2.4. The outer wheel reaches its highest lateral force at approximately -1° of camber, and the force falls quickly at positive values, while the inner wheel reaches its highest value at positive camber. With zero static camber the displacement produced in cornering therefore moves both tyres away from their most favourable condition, and racing vehicles are for this reason set with static negative camber, so that the positive displacement of the outer wheel ends at an angle close to the peak. The value cannot be increased without limit, since the inner wheel loses lateral force as the static camber becomes more negative, and since the highest longitudinal force, in acceleration and braking, is obtained at 0° [5–7].

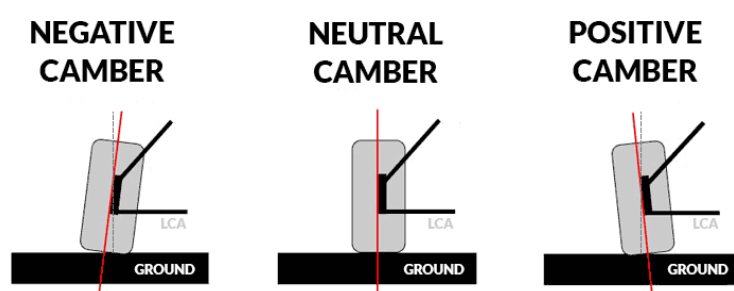


Figure 2.3: Camber angle [8]

2.2.3 Toe Angle

Seen from above, the static toe angle is the angle between the longitudinal axis of the vehicle and the longitudinal axis of the wheel, measured with the vehicle at rest. Toe is positive, or toe-in, when the leading edges of the two wheels of an axle converge towards

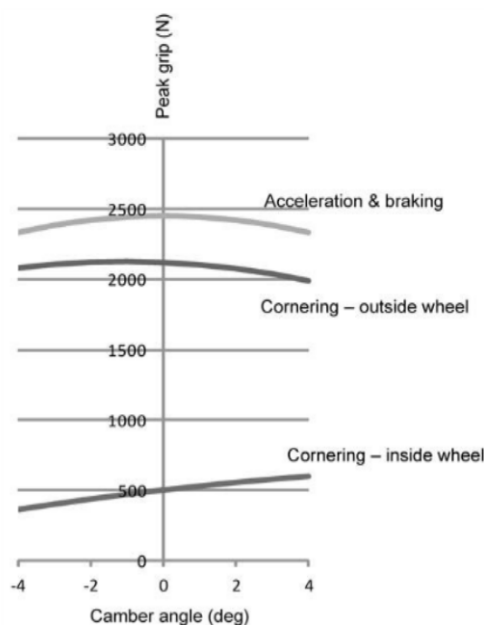


Figure 2.4: The effect of camber on wheel grip [7]. Reprinted with permission from D. Seward, *Race Car Design*, 2014, Palgrave Macmillan. © 2014.

the centreline of the vehicle, and negative, or toe-out, when they diverge. At zero toe the wheel planes are parallel to the longitudinal axis of the vehicle. The three cases are shown in Figure 2.5 [5, 9].

The static toe influences the response to steering inputs, the tyre wear, and the stability in a straight line. Toe-in increases directional stability and toe-out reduces it. The value chosen also has to compensate the deflection of the suspension and steering under load: the rolling resistance of a non-driven wheel pushes the contact patch rearwards, and a static toe-in is used so that the wheel runs parallel under load, which keeps the rolling resistance low [5, 9].

A driven wheel is loaded in the opposite direction, since the tractive force pulls the contact patch forwards, so driven front wheels are set with static toe-out for the same reason. Racing vehicles are usually driven by the rear axle, and that axle is set with toe-in. Besides increasing the directional stability, the toe-in keeps a small steady-state slip angle at the rear tyres, which pre-stresses the tyre and improves its capacity to transmit longitudinal tractive force to the track surface. Toe-out would produce a slip angle of the same magnitude, but it is not used at the rear because it reduces the yaw stability of the vehicle and increases the tendency to oversteer [5, 9].

2.2.4 Kingpin Inclination, Scrub Radius and Spindle Length

In the front view, the kingpin inclination (KPI) is the angle between the steering axis, which passes through the upper and lower ball joints, and the vertical. Two dimensions

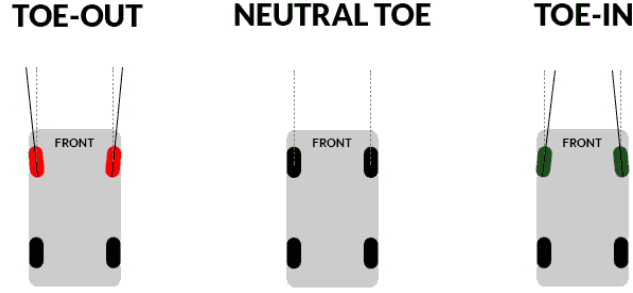


Figure 2.5: Positive, Neutral, and Negative toe angle [9]

are measured from the same axis: the scrub radius, which is the lateral distance between the steering axis and the centreline of the wheel at ground level, and the spindle length, or wheel centre offset, which is the horizontal distance between the steering axis and the wheel centre plane at axle height. The three are shown in Figure 2.6 [5–7].

The three parameters are set as a compromise between the space required for the brake caliper and disc inside the rim and the effects they produce in the behaviour of the vehicle. When the wheels are steered, the KPI inclines both of them towards positive camber, which reduces the lateral force the tyres can produce, so a low value is preferable. For a steering axis inclined by θ_k and no caster, the camber change at a steering angle δ is [7]

$$\Delta\gamma = \theta_k + \cos^{-1}(\sin \theta_k \cos \delta) - 90^\circ. \quad (2.1)$$

The KPI combined with a positive spindle length also raises the front of the vehicle when the wheels are steered away from the centre, so the weight of the vehicle helps to return the steering to the straight-ahead position. At higher speeds this effect is small compared with the one produced by the mechanical trail [5–7].

The scrub radius and the spindle length also act as lever arms that transmit disturbances to the steering. The spindle length applies the longitudinal forces at the wheel centre, which vary on an uneven surface as the rolling radius changes, and the resulting kickback in the steering is proportional to its length. The scrub radius applies the difference between the driving or braking forces at the two contact patches, and produces a steering torque that the driver feels. A zero scrub radius, known as centrepont steering, removes this lever arm, but the space available inside the rim rarely allows it. A negative scrub radius is instead used as a stabilising measure: if the longitudinal forces at the two wheels are unequal, the compliance of the steering system steers the wheels in the direction that opposes the resulting yaw moment, which helps to keep the vehicle in a straight line. The magnitude of the scrub radius is normally kept below 40 mm [5–7].

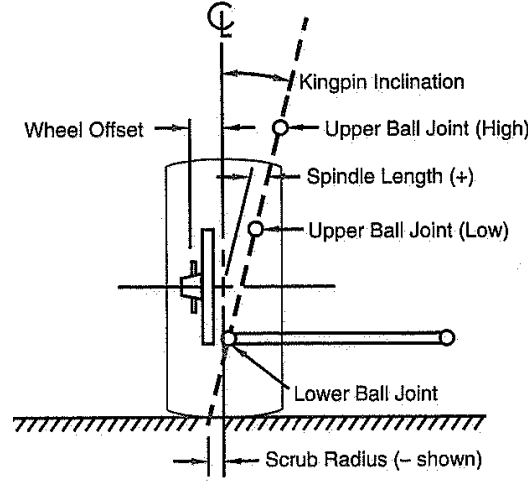


Figure 2.6: Kingpin Inclination, Scrub Radius and Spindle Length Illustration [6]. Reprinted with permission from W.F. Milliken and D.L. Milliken, *Race Car Vehicle Dynamics*, 1995, SAE International. © 1995 SAE International. Further distribution of this material is not permitted without prior permission from SAE.

2.2.5 Caster Angle and Mechanical Trail

The caster angle is the angle between the vertical and the steering axis, measured in the side view. Its two main consequences are a camber change with steering and the mechanical trail.

When the wheels are steered, the inclination of the steering axis in the side view rotates the wheel about that axis, and the outer wheel gains negative camber while the inner wheel gains positive camber. As described in Section 2.2.2, this is the direction that keeps the loaded tyre closer to the vertical and increases the lateral force it can produce [5–7]. For a caster angle θ_c and a steering angle δ , the camber change is [7]

$$\Delta\gamma = \cos^{-1}(\sin\theta_c \sin\delta) - 90^\circ. \quad (2.2)$$

Together with the KPI, the caster angle also displaces the chassis vertically when the wheels are steered, an effect known as jacking, which is treated in Section 2.2.12 [5–7].

The mechanical trail is the longitudinal distance between the centre of the contact patch and the point at which the steering axis meets the ground, and it follows from the caster angle. It governs the self-centring torque, together with the pneumatic trail introduced in Section 2.2.7, and therefore the effort required at the steering wheel. A large caster angle is often combined with a steering axis offset forward of the wheel centre in the side view, so that the total mechanical trail, and with it the steering effort, does not become excessive. Both quantities are shown in Figure 2.7 [5–7].

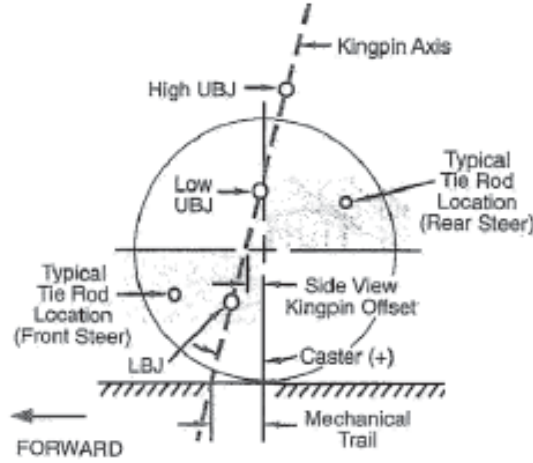


Figure 2.7: Caster Angle and Mechanical Trail [6]. Reprinted with permission from W.F. Milliken and D.L. Milliken, *Race Car Vehicle Dynamics*, 1995, SAE International. © 1995 SAE International. Further distribution of this material is not permitted without prior permission from SAE.

2.2.6 Ackermann Steering Geometry

When the front wheels are steered away from the straight-ahead position, the geometry of the steering linkage determines whether the two wheels turn through the same angle or whether one turns more than the other. At very low speed, where the tyres generate no appreciable lateral force, every wheel has to be tangent to a circular path about a common centre. The inner wheel follows a smaller radius than the outer one, so it has to be turned through a larger angle if the tyres are not to work against each other and scrub [5, 7].

This is the Ackermann condition, and it relates the ideal angles at the inner and outer wheels, $\delta_{A,i}$ and $\delta_{A,o}$, as [5]

$$\cot(\delta_{A,o}) = \cot(\delta_{A,i}) + \frac{j}{l}, \quad (2.3)$$

where l is the wheelbase and j is the distance between the steering axes at ground level (Figure 2.8). The geometry actually implemented is expressed as a percentage of this ideal,

$$\text{Percent Ackermann} = \frac{\Delta\delta}{\Delta\delta_A} \cdot 100\%, \quad (2.4)$$

in which $\Delta\delta$ is the difference between the inner and outer steer angles produced by the linkage and $\Delta\delta_A$ is the difference required by Equation 2.3 at the same steering position. A value of 100% corresponds to true Ackermann, 0% to parallel steering, and negative values to reverse Ackermann, in which the inner wheel turns through the smaller angle [5, 7].

At high lateral acceleration the picture changes. The tyres have to develop a slip angle to produce lateral force, and the load transfer leaves the inner wheels lightly loaded. A

lightly loaded tyre reaches its maximum lateral force at a smaller slip angle, so a linkage built for the low-speed condition steers the inner wheel beyond the angle at which it produces its greatest force. The tyre is then dragged at an excessive slip angle, which raises its temperature and adds drag. Parallel steering, or a small amount of reverse Ackermann, brings both tyres closer to their peak at the same time, and is therefore the usual choice in racing vehicles. In a rear-steer layout it is obtained by moving the steering rack forward with respect to the line joining the outer tie-rod ball joints. The penalty is at low speed, where the front wheels work against each other and the vehicle becomes difficult to manoeuvre by hand. Since fast corners require small steering angles, and the difference between the two geometries is correspondingly small there, this penalty is generally accepted [5, 7].

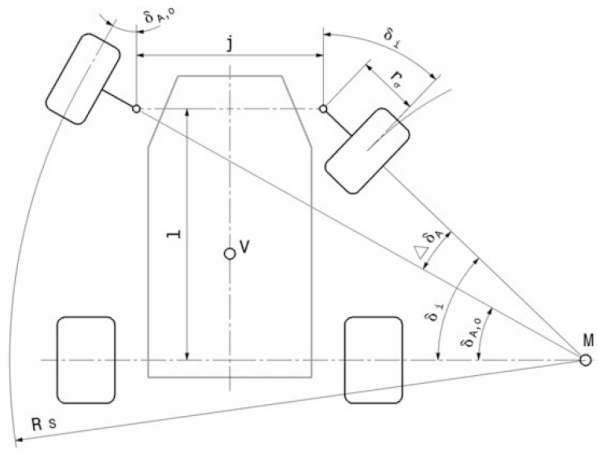


Figure 2.8: Ackermann Kinematic relationships during cornering [5]. Reprinted with permission from M. Trzesniowski, *Suspension System*, 2023, Springer Nature. © 2023.

2.2.7 Slip Angle

The slip angle, α , is the angle between the direction in which a tyre is travelling and its longitudinal centre plane, as shown in Figure 2.9. In spite of the name, its presence does not mean that the tyre is sliding on the road: it describes the elastic distortion of the tread within the contact patch [5, 7].

When a lateral force is applied, the tread deforms progressively as it enters the contact area, and the resulting angular deviation is the slip angle. The relation works in both directions: a lateral force applied to the tyre produces a slip angle, and a tyre made to run at a slip angle produces a lateral force. Along the contact patch this force builds from the leading edge, reaches a maximum, and falls as the tread leaves the road, so the resultant acts behind the centre of the contact patch. This offset is the pneumatic trail, and it adds to the mechanical trail of Section 2.2.5 to produce the self-aligning moment that returns the steering towards the centre [5, 7].

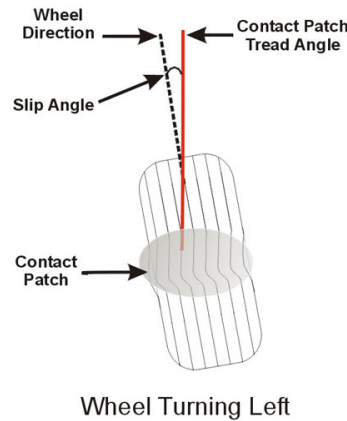


Figure 2.9: Slip angle representation at the tyre contact patch [10]

The lateral force increases with the slip angle up to a maximum, which for racing tyres occurs between 7° and 10° ; beyond that the tyre begins to slide and the force falls. The slip angles developed at the front and rear axles, α_f and α_r , in steady-state cornering determine the behaviour of the vehicle. When the two are equal the vehicle is neutral and holds its trajectory without correction at the steering wheel. When the front slip angle is the larger one, $\alpha_f > \alpha_r$, the vehicle understeers: the front axle does not produce the lateral force the trajectory requires, and the steering angle has to be increased to prevent the vehicle from running wide. When the rear slip angle is the larger one, $\alpha_f < \alpha_r$, the vehicle oversteers, and the steering has to be turned in the opposite direction, which is known as counter-steering [5, 7].

2.2.8 Tyre Load Sensitivity

The lateral force that a tyre can generate does not grow in direct proportion to the vertical load applied to it. When the lateral force developed at a given slip angle is divided by the vertical load, a lateral force coefficient, also called the lateral friction coefficient, is obtained. The peak value of this coefficient is higher at light loads and decreases as the vertical load increases; this behaviour is known as tyre load sensitivity [6]. The same effect can be described by expressing the friction force as a function of the wheel load rather than through a constant coefficient [5].

The consequence becomes evident when the grip of an axle is considered as a whole. Since the lateral force increases less than in proportion to the vertical load, transferring load across an axle, so that one wheel is pushed down while the opposite wheel is unloaded by the same amount, reduces the combined grip of the pair below the value available when both wheels carry equal loads [7]. Every increase in load transfer at one end of the vehicle therefore reduces the total lateral force that the corresponding axle can produce.

Tyre load sensitivity is the mechanism that links load transfer to handling. It explains why load transfer is kept as small as the rest of the design allows, why the front-to-rear

distribution of the lateral load transfer governs the balance of the vehicle (Section 2.2.11), and why the suspension geometry is arranged to keep the loaded outer tyre working as close as possible to its most favourable condition.

2.2.9 Ride and Roll Rate

The ride and roll rates relate a change in wheel loading to a change in the position of the body. The ride rate relates the change in the tyre normal force at one corner to a vertical displacement of the body, and is expressed in N/mm. The roll rate is the moment that opposes the roll of the body per degree of roll, produced by the difference between the tyre normal forces at the two sides of an axle. It is provided by the combination of the ride rates, the track width, and the anti-roll bar [6].

These rates determine the distribution of the tyre normal forces and therefore the lateral forces available at each corner. Their selection is iterative: the rates depend on the wheel loads that are expected, including aerodynamic downforce, while the wheel loads themselves depend on the distribution of those same rates. Racing vehicles use stiff springs, since the motion of the body in roll and pitch has to be controlled under high lateral, longitudinal, and aerodynamic loads [6].

The overall rate is composed of two contributions in series. The vertical force per unit displacement measured at the wheel centre with respect to the chassis is the wheel rate, WR, which is lower than the rate of the spring itself because of the motion ratio of the linkage. The tyre adds its own vertical stiffness, K_t , between the wheel centre and the ground, as represented in Figure 2.10. Combining the two as springs in series and solving for the wheel rate required to obtain a given ride rate, RR, gives [6]

$$\text{WR} = \frac{\text{RR} K_t}{K_t - \text{RR}}. \quad (2.5)$$

Since the vertical stiffness of the tyre is finite, the ride rate of the vehicle is always lower than the wheel rate [6].

2.2.10 Bellcrank Function, Motion Ratio and Wheel Rate

In pushrod- and pullrod-actuated suspensions the spring and damper are not mounted at the wheel, but are driven through a bellcrank. The bellcrank is the kinematic interface between the wheel-side linkage and the inboard elastic elements: it redirects the motion of the wheel towards a convenient position inside the chassis and, at the same time, sets the mechanical advantage between the travel of the wheel and the travel of the spring and damper [5–7].

That mechanical advantage is expressed by the motion ratio, MR, defined as the travel of the wheel per unit of travel of the spring, as illustrated in Figure 2.11. Neglecting compliance and friction, the stiffness seen at the wheel is not the stiffness of the spring, but

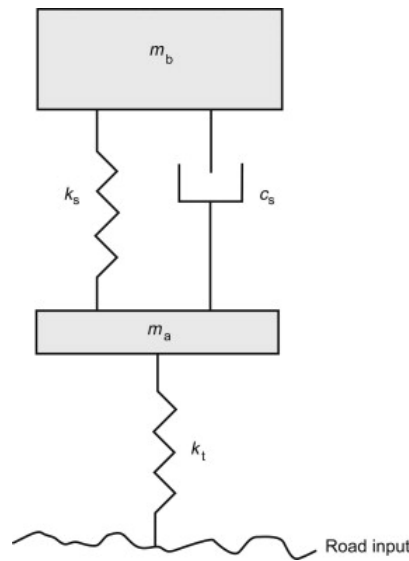


Figure 2.10: Simplified representation of the suspension system [11]

$$WR = \frac{K}{MR^2}, \quad (2.6)$$

where K is the spring rate. Since the relation is quadratic, small changes in the shape of the bellcrank, in the position at which the pushrod is picked up, or in the inclination of the damper produce large changes in the stiffness seen at the contact patch [5, 6].

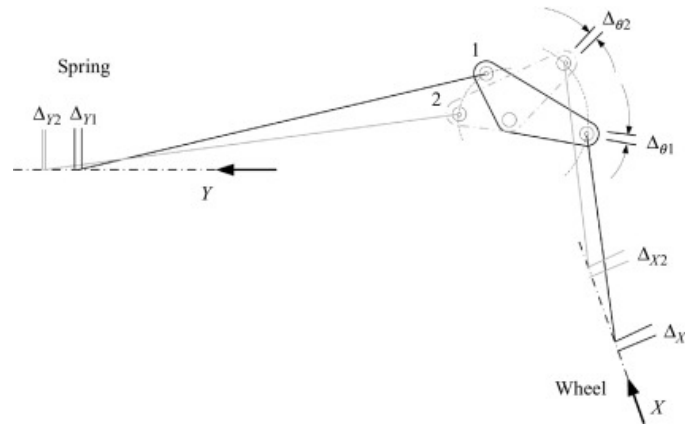


Figure 2.11: Bellcrank displacements from which the motion ratio is obtained. Δ_X denotes the travel at the wheel and Δ_Y the corresponding travel at the spring, shown at two positions of the mechanism; the motion ratio at each position is $MR = \Delta_X / \Delta_Y$ [12].

The motion ratio is not selected only to obtain a target wheel rate. It also determines the travel required from the spring, the velocity at which the damper operates, the volume occupied by the assembly, and the forces carried by the bellcrank pivots and by their attachments to the chassis. Since the two positions of Figure 2.11 give different values, the motion ratio is not constant along the travel. A nonlinear bellcrank can be used when

a progressive wheel rate is wanted, but it makes the correspondence between the analytical models and the response of the vehicle less direct, so a smooth and nearly linear response is normally preferred in the early stages of the design [5–7].

2.2.11 Load Transfer

Because the centre of gravity of the vehicle lies above the ground, any longitudinal or lateral acceleration produces an inertial reaction that, together with the tyre forces at road level, forms an out-of-balance couple. This couple redistributes the vertical load between the wheels, a phenomenon known as load transfer. In combination with the tyre load sensitivity described in Section 2.2.8, load transfer is one of the dominant factors governing the grip available at each axle and, ultimately, the balance of the car [6].

2.2.11.1 Longitudinal Load Transfer

Under acceleration or braking, the inertial reaction acts horizontally through the centre of gravity while the traction or braking force acts at the contact patch. Taking moments about a wheel contact line, the resulting transfer of vertical load between the front and rear axles is

$$\Delta W_x = \frac{W n_x h_{CG}}{L}, \quad (2.7)$$

where W is the vehicle weight, n_x the longitudinal acceleration normalised by gravity, $n_x = a_x/g$, and therefore dimensionless, h_{CG} the centre-of-gravity height, and L the wheel-base [6, 7]. The load is removed from the front axle and added to the rear under acceleration (squat), and the sign reverses under braking (dive). This is the load that the side-view anti-features react geometrically rather than elastically, as discussed in Section 2.5.1.

2.2.11.2 Lateral Load Transfer

During steady cornering, load is transferred from the inner to the outer wheels. Taking moments about the line joining the outer contact patches, the total lateral load transfer of the vehicle is

$$\Delta W_y = \frac{W n_y h_{CG}}{t}, \quad (2.8)$$

with $n_y = a_y/g$ the lateral acceleration in the same normalised form and t the track width [6, 7]. This total, however, does not describe the paths by which the load reaches the ground. As shown in Figure 2.12, it can be separated into an unsprung-mass contribution, a geometric contribution transmitted by the suspension links, and an elastic contribution transmitted by the springs, dampers, and anti-roll bar [5]:

$$\Delta W_{y,U} = m_U a_y \frac{h_U}{t}, \quad \Delta W_{y,\text{geom}} = m_s a_y \frac{h_{\text{RC}}}{t}, \quad \Delta W_{y,\text{elast}} = m_s a_y \frac{\Delta h}{t}, \quad (2.9)$$

$$\Delta W_y = \Delta W_{y,U} + \Delta W_{y,\text{geom}} + \Delta W_{y,\text{elast}}, \quad (2.10)$$

where m_U and m_s are the unsprung and sprung masses, h_U the unsprung-mass centre-of-gravity height, h_{RC} the roll-centre height, and $\Delta h = h_{\text{CG},s} - h_{\text{RC}}$ the roll moment arm, that is, the distance between the sprung-mass centre of gravity and the roll centre [5].

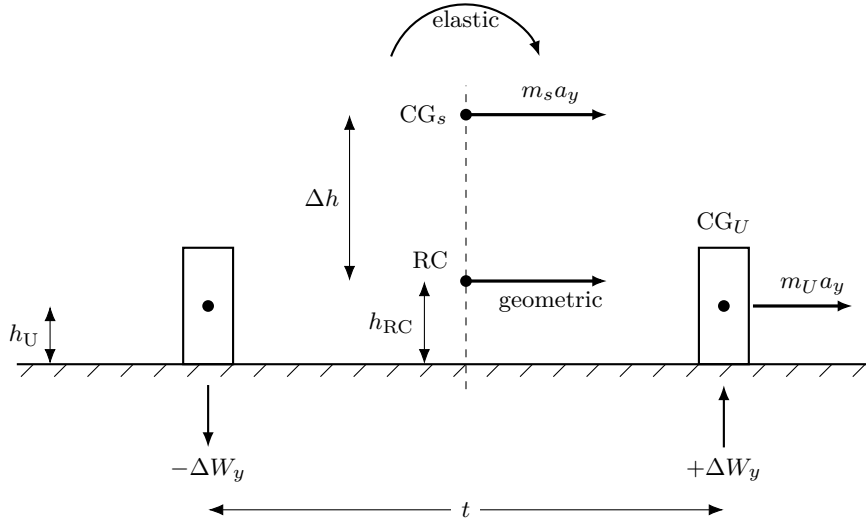


Figure 2.12: Paths of the lateral load transfer at an axle. The unsprung mass acts through h_U ; the geometric contribution is transmitted by the links and acts through the roll centre at h_{RC} ; the elastic contribution results from the roll moment of the sprung mass about the roll centre, with arm Δh .

The unsprung and geometric contributions are determined at each axle by the masses and by the roll-centre height of that axle. The elastic contribution, in contrast, arises from a single roll moment applied to the sprung mass, which is reacted by the two axles in proportion to their roll stiffness,

$$\Delta W_{y,\text{elast},i} = \frac{K_{\phi,i}}{K_{\phi,F} + K_{\phi,R}} \cdot \frac{m_s a_y \Delta h}{t_i}, \quad i \in \{F, R\}, \quad (2.11)$$

where $K_{\phi,i}$ is the roll stiffness of axle i [6]. The geometric contribution is governed directly by the roll-centre height and acts almost instantaneously during a steering input, whereas the elastic contribution is transmitted through the body-supporting springs and stabilisers and therefore develops over the roll transient. In the quasi-steady mid-corner state the sum of the three contributions equals the total of Equation 2.8 [5].

2.2.11.3 Lateral Load Transfer Distribution and Balance

At a given lateral acceleration, the total lateral load transfer of Equation 2.8 is fixed by the weight of the vehicle, the centre-of-gravity height and the track, none of which the suspension controls; its distribution between the front and rear axles, however, it does. The front lateral load transfer distribution is defined as

$$\text{LLTD}_F = \frac{\Delta W_{y,F}}{\Delta W_{y,F} + \Delta W_{y,R}}, \quad (2.12)$$

and is controlled by two mechanisms whose effects add: the elastic part is set by the front-to-rear distribution of roll stiffness, adjusted through spring rates and anti-roll bars, while the geometric part is set by the relative heights of the front and rear roll centres [6]. Because of tyre load sensitivity (Section 2.2.8), the axle that receives the larger share of the lateral load transfer loses proportionally more grip; shifting the distribution towards the front therefore promotes understeer, and shifting it rearward promotes oversteer. Adjusting this distribution is consequently the principal means by which the steady-state balance of the car is tuned [7].

2.2.12 Jacking

Jacking is the vertical displacement of the sprung mass produced by the lateral forces at the contact patches. The lateral force is transmitted to the chassis along the geometry of the suspension links, whose line of action in the front view passes through the roll centre; when the roll centre lies above the ground, that transmission has a vertical component that lifts the sprung mass. The effect is therefore inseparable from the roll-centre height and from the geometric part of the lateral load transfer of Section 2.2.11: a high roll centre shortens the roll moment arm and reduces the roll angle, but it also increases the share of the lateral force that is transmitted directly by the links, and with it the vertical displacement of the body and the variation of the tyre loads [6, 7].

A second source of jacking acts in the side view. As described in Sections 2.2.4 and 2.2.5, steering the wheels about an axis that is inclined both in the front view and in the side view raises the front of the vehicle, and the weight of the vehicle then contributes to returning the steering to the centre position [5, 6].

The roll-centre height is consequently a compromise. It is not chosen only to limit the roll angle, but also to keep the vertical displacement of the body and the resulting variation of the tyre loads within acceptable limits, and to obtain the camber behaviour described in Section 2.2.2 [5, 6].

2.2.13 Bump Steer

Bump steer is the variation of the toe angle produced by vertical wheel travel. It follows from the relative motion between the steering linkage and the suspension links, and is

therefore determined by the positions of the wishbone pivots, the position of the steering rack, the geometry of the upright, and the length of the tie rod [5, 7].

The variation vanishes when the tie rod describes the same arc as the point of the upright to which it is attached, which requires its inner and outer ends to lie on the axis defined by the instant centre of the wishbones, as shown in Figure 2.13. Any departure from that condition produces a toe change with travel. Small values are sometimes introduced deliberately to shape the response of the vehicle, but an uncontrolled toe variation reduces the predictability of the response and increases tyre scrub. In a Formula Student vehicle the suspension travel is small, so the toe change per millimetre of travel has to be correspondingly small if the behaviour of the contact patch is to remain consistent under braking, at turn-in, and over kerbs [5, 7].

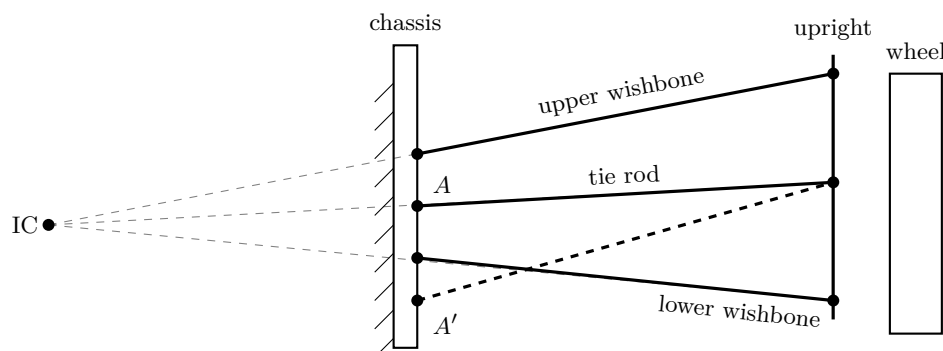


Figure 2.13: Geometric condition for the absence of bump steer, in the front view. With the inner end of the tie rod at A , on the line joining the instant centre IC to its outer end, the tie rod and the upright describe the same arc and the toe does not change with travel. With the inner end at A' , off that line, the two arcs differ and the toe varies through the travel.

2.3 Double-Wishbone Suspension

The double-wishbone suspension locates the wheel carrier with an upper and a lower A-arm. Together with the steering link and the spring and damper assembly, these two arms define the path that the wheel follows through its travel. The camber gain, the roll centre, the steering axis, and the anti-features can be defined almost independently of one another, which is the reason why this layout is used whenever the control of the contact patch is more important than the space that the suspension occupies [5, 6].

The behaviour of the linkage is defined by the geometry of the two arms, and in particular by whether they have equal or unequal length, and whether they are parallel or convergent in the front view. The four resulting combinations are described in the following sections.

2.3.1 Parallel equal-length double wishbones

As seen in Figure 2.14, in this configuration the upper and lower arms have the same length and are parallel to each other, so that the arms, the chassis, and the wheel carrier form a parallelogram. It is the simplest case, and it is useful mainly as a reference against which the other three are compared [5, 7].

In parallel travel the parallelogram keeps the wheel plane at a constant angle, so the camber does not change. The wheel does, however, move laterally, and the resulting displacement of the contact patch, known as scrub, drags the tyre across the road surface and increases wear and rolling resistance. Longer arms reduce the effect, but the width available in the vehicle limits how much they can be lengthened.

In roll the same geometry transmits the body angle directly to the wheel: a roll angle of 4.0° produces 4.0° of camber at both wheels. The loaded outer wheel is therefore pushed towards positive camber (Section 2.2.2), which lifts the inner edge of the tyre and reduces the lateral force it can carry. The loss can only be compensated by limiting the roll angle or by adding static negative camber, and both compensations have costs of their own. For this reason the parallel equal-length arrangement is not used in racing vehicles [5, 7].

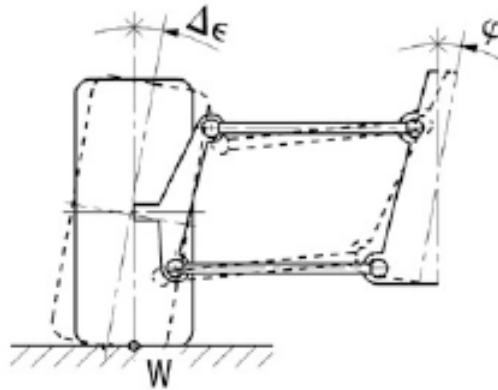


Figure 2.14: Parallel equal-length double wishbones [5]. Reprinted with permission from M. Trzesniowski, *Suspension System*, 2023, Springer Nature. © 2023.

2.3.2 Converging equal-length double wishbones

In this configuration the two arms still have the same length, but they are no longer parallel: they are inclined so that they converge towards the longitudinal centre of the vehicle, as illustrated in Figure 2.15. The instant centre in the front view is therefore at a finite distance, and the kinematic behaviour changes accordingly [7].

Because the arms swing along different arcs, the wheel no longer keeps a constant angle in parallel travel. As the wheel moves upward, the geometry inclines the top of the wheel towards the vehicle, which produces negative camber gain. This is the property that corrects the main limitation of the parallel arrangement. In roll, the loaded outer wheel is

compressed, and the negative camber it gains opposes the positive camber imposed by the body roll. The wheel plane is therefore kept closer to the vertical, and the lateral force available at the outer tyre is higher than in the parallel case [7].

The track width variation, however, remains. The arcs described by the equal-length arms still displace the wheel laterally in vertical travel, so the scrub of the parallel arrangement is not removed. Correcting the camber behaviour and the track variation at the same time requires arms of unequal length, which are treated in the following sections [7].

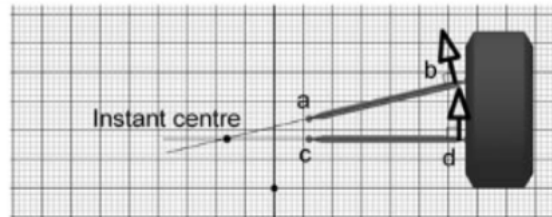


Figure 2.15: Converging equal-length double wishbones [7]. Reprinted with permission from D. Seward, *Race Car Design*, 2014, Palgrave Macmillan. © 2014.

2.3.3 Parallel unequal-length double wishbones

In this configuration the arms remain parallel at the static position, but the upper arm is shorter than the lower one (Figure 2.16). Since the two arms are parallel, their axes do not intersect and the instant centre lies at infinity, so the camber and the track do not vary at the design position. As soon as the wheel moves, however, the arms of different length rotate through different angles and cease to be parallel, and the camber therefore changes progressively with travel [5].

The shorter upper arm inclines the top of the wheel carrier towards the vehicle as the wheel rises, so the camber gained in bump is negative. In roll, the loaded outer wheel is compressed and gains negative camber, which opposes the inclination imposed by the body and keeps the tyre closer to the vertical. The unloaded inner wheel is displaced in the opposite direction and is taken to positive camber, but it carries little load and the effect on the total lateral force is small. The track variation is also smaller than in the equal-length arrangements, and the lengths of the arms and the positions of their pivots can be chosen to reduce it further [5].

The length of the upper arm cannot be reduced without limit. If it is too short, large compression can bring it close to the horizontal position in which the linkage reaches a dead point, and this constraint is particularly restrictive in vehicles that require long suspension travel [5].

2.3.4 Converging unequal-length double wishbones

Figure 2.17 shows that in this configuration the upper arm is shorter than the lower one, often about 80% of its length, and the two arms converge towards the centre of the vehicle.

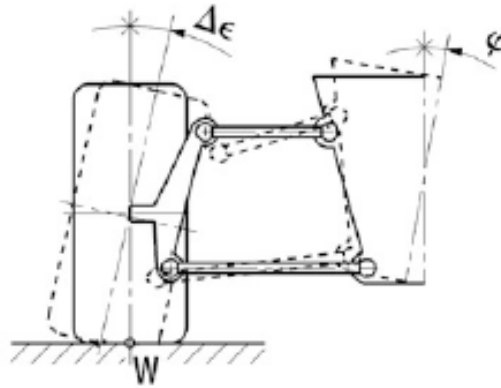


Figure 2.16: Parallel unequal-length double wishbones [5]. Reprinted with permission from M. Trzesniowski, *Suspension System*, 2023, Springer Nature. © 2023.

It is the arrangement normally understood by the term SLA, and it is the one used in racing vehicles [5, 7].

Since the lengths of the arms and the positions of their pivots can be chosen almost freely, this configuration offers the widest range of kinematic behaviour of the four. The camber variation in roll can be made small, or the loaded outer wheel can be taken towards negative camber, so that the wheel plane stays close to the vertical during cornering. A small amount of static negative camber, of the order of -2.0° , is normally added so that the outer wheel does not reach positive values. The position of the roll centre and its migration through the travel can also be controlled, which is not possible in the equal-length arrangements. In the side view, inclining the axes of the arms produces the anti-dive and anti-squat behaviour that limits the attitude change of the body under braking and acceleration. The track variation can be kept small in bump, but it becomes larger in rebound. The parameters are also strongly coupled: lowering the upper inner pivot, for example, raises the roll centre, increases the camber variation in parallel travel and reduces it in roll, so a single change cannot be assessed in isolation [5, 7].

2.4 Suspension Modes

The suspension governs the motion of the sprung mass relative to the wheels. That motion is decomposed into four modes: heave, roll, pitch, and warp, shown in Figure 2.18. The four are independent and together span every motion the four wheels can describe, so each is defined below by the pattern of equal displacements that isolates it, and an arbitrary set of wheel displacements resolves into a contribution from each. Controlling them is what reduces the variation of the tyre loads, and therefore preserves the mechanical grip, while keeping the attitude of the body within the range for which the aerodynamic package was designed [6].

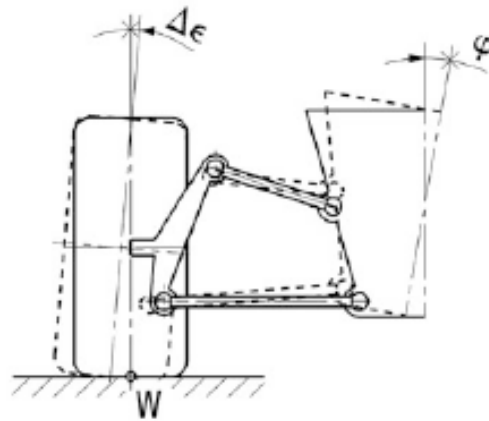


Figure 2.17: Converging unequal-length double wishbones [5]. Reprinted with permission from M. Trzesniowski, *Suspension System*, 2023, Springer Nature. © 2023.

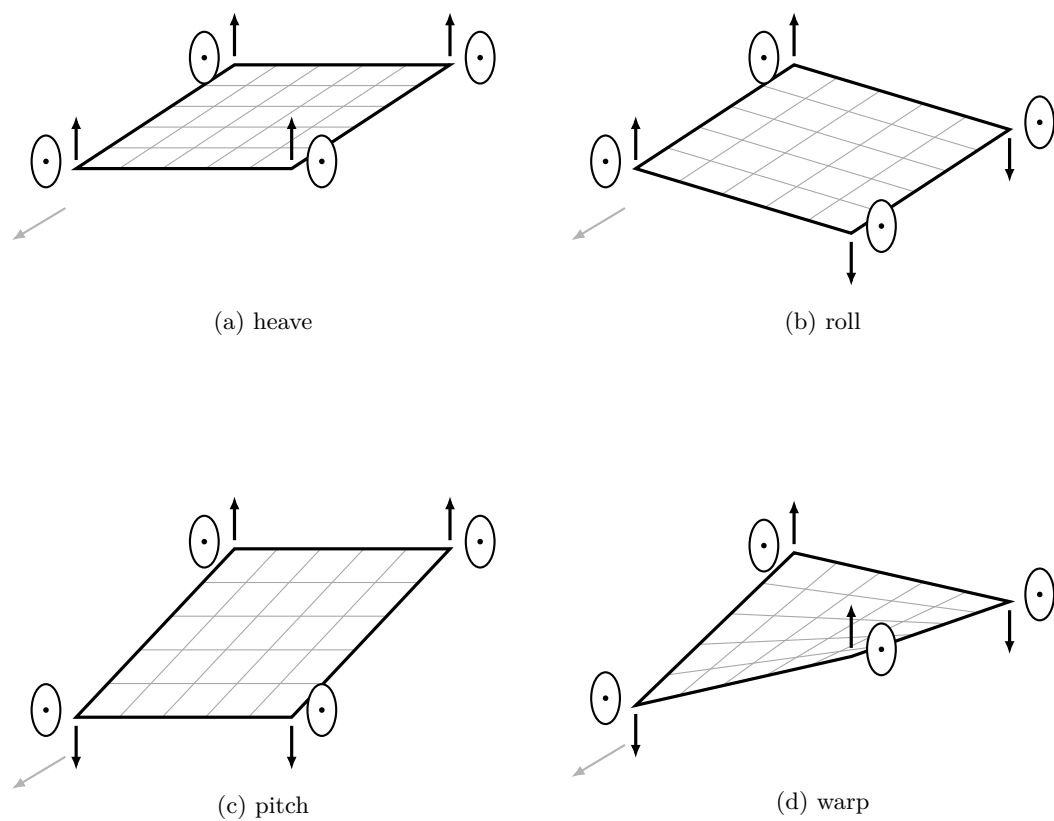


Figure 2.18: The four suspension modes. The arrows indicate the direction of the displacement of each wheel with respect to the chassis, and the grey arrow the direction of travel.

2.4.1 Heave

Heave is the equal and simultaneous vertical displacement of both wheels of an axle with respect to the chassis, as shown in Figure 2.18(a). It is produced by the variation of the aerodynamic downforce with speed and by symmetrical irregularities of the road surface [6].

In a conventional layout the heave motion is reacted by the corner springs and dampers. In vehicles that rely on aerodynamic load, such as Formula 1 and Le Mans Prototype (LMP) prototypes, small changes in ride height produce large changes in downforce, so the control of this mode is treated separately: a dedicated element at each axle, known as a heave spring, together with its damper, reacts the symmetric motion, and the corner elements are then free to be selected for the remaining modes [7].

2.4.2 Roll

Roll is the rotation of the sprung mass about its longitudinal axis, shown in Figure 2.18(b). It occurs when the two wheels of an axle are displaced vertically by equal amounts in opposite directions, and is produced by the lateral load transfer of Section 2.2.11 during cornering [6].

The roll gradient of the vehicle follows from the roll-centre heights and from the combined stiffness of the springs and of the anti-roll bars (Anti-Roll Bar (ARB)). The distribution of the roll stiffness between the front and the rear axle is the principal means of setting the steady-state balance of the vehicle, as described in Section 2.2.11 [6, 7].

2.4.3 Pitch

Pitch is the rotation of the sprung mass about its transverse axis, shown in Figure 2.18(c). It occurs when the front and rear axles are displaced vertically in opposite directions, and is produced by the longitudinal load transfer during acceleration, known as squat, and during braking, known as dive [6].

The mode is reacted by the corner springs and dampers, and the fraction of the longitudinal load transfer that reaches the chassis through the links rather than through the springs is set by the anti-dive and anti-squat geometries of Section 2.5.1. Its control matters because the attitude of the body displaces the aerodynamic centre of pressure along the vehicle during transient manoeuvres [7].

2.4.4 Warp

Warp, also called cross-articulation, is the displacement of diagonally opposite wheels by equal amounts in opposite directions, shown in Figure 2.18(d). It is the mode that determines whether the four tyres remain in contact with an uneven surface [6].

A high warp stiffness means that an uneven surface lifts one wheel of a diagonal, which removes its share of the grip and transfers load to the remaining three. The mode can be softened by connecting the front and rear anti-roll bars through a mechanism such as the T-bar of Figure 2.19, but such a connection also alters the heave and pitch stiffness, and the space it requires and the mass it adds mean that it is seldom used [7].

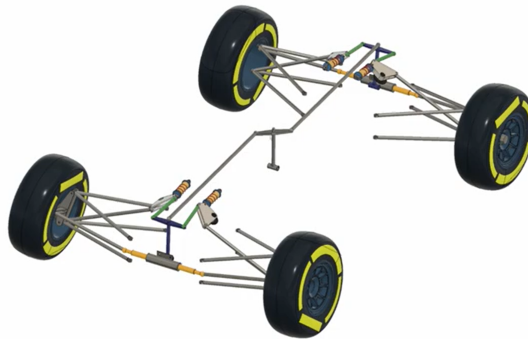


Figure 2.19: T-bar mechanism for warp control [13]

The complete separation of the four modes is obtained with interconnected hydraulic systems, which allow each mode to be set on its own, for example a high roll stiffness together with zero warp stiffness. Their complexity, cost, and mass restrict them to the categories of motorsport in which those penalties can be accepted [6, 7]. The mechanisms used to control these modes in the absence of such systems are the subject of the following section.

2.5 Suspension Movement Control Mechanisms

2.5.1 Anti-Features

Braking and acceleration transfer load between the axles, as shown in Section 2.2.11. Because that load is reacted by the springs, the ones at one end of the vehicle compress while the ones at the other extend, and the body pitches: the nose dives under braking and the rear squats under acceleration. Part of the transfer can be carried by the suspension links instead, and this is obtained by inclining their pivots on the chassis in the side view. The geometries that result are called anti-dive, anti-lift and anti-squat, or collectively anti-features [6, 7].

Which of them is available at each axle depends on where a longitudinal force exists. Under braking a force acts at both contact patches, so both axles can be given a compensating geometry: the front one is called anti-dive and the rear one anti-lift. Under acceleration the driving force reaches the rear wheels, and the corresponding geometry at that axle is called anti-squat, while the front axle of a rear-wheel-drive vehicle carries no longitudinal force and its lift cannot be reduced geometrically [6].

The geometry is constructed in the side view. Lines drawn through the upper and lower bearings of the upright, parallel to the inclined pivot axes on the chassis, intersect at the side-view instant centre, which defines a side-view swing axle. Under braking with outboard brakes the braking force acts at the contact patch and produces a moment about that instant centre which opposes the moment that causes the dive, so the degree of compensation follows from the position of the instant centre,

$$\% \text{ anti-dive} = \frac{F_{F\%} L \tan \theta_1}{h_{CG}} \times 100, \quad (2.13)$$

where $F_{F\%}$ is the fraction of the total brake force at the front axle, L the wheelbase, h_{CG} the height of the centre of mass, and θ_1 the slope of the line from the contact patch to the side-view instant centre [6]. When the resultant force passes through the instant centre the two moments cancel, which corresponds to 100% compensation: the front axle does not compress and the rear axle does not extend under braking [6, 7].

Under acceleration the driving torque is reacted by the chassis rather than by the wheel carrier, so the force enters the linkage at the wheel centre and not at the contact patch. The compensation is then

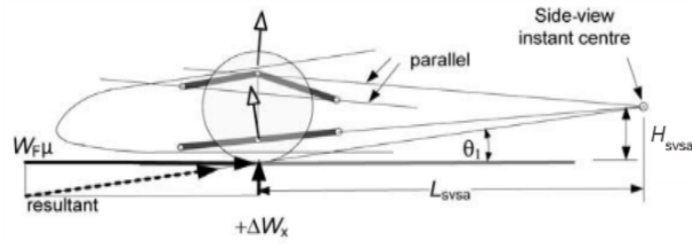
$$\% \text{ anti-squat} = \frac{L \tan \theta_3}{h_{CG}} \times 100, \quad (2.14)$$

where θ_3 is the slope of the line from the wheel centre to the side-view instant centre [6]. Figure 2.20 shows the three geometries at full compensation.

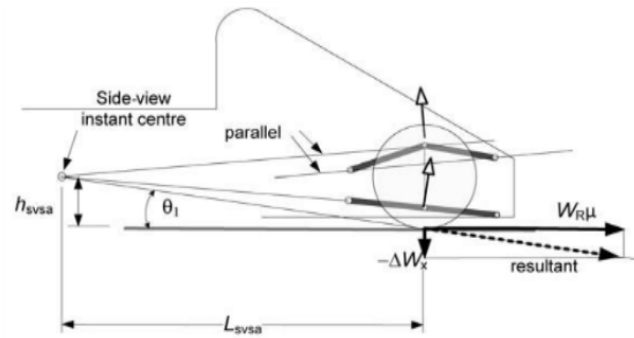
These geometries keep the attitude of the body stable, but they do so by routing the inertial reaction through the suspension links instead of the springs, and that has consequences. A high degree of compensation requires an instant centre that is high and close to the wheel, which produces large caster variation through the travel. The forces carried by the links increase the friction at the joints and reduce the ability of the suspension to follow the road, and under the braking forces of a vehicle with high downforce the suspension can become effectively locked. Compensation above about 50% increases the fluctuation of the wheel loads on uneven braking zones, which promotes locking of the front tyres and understeer when braking into a corner, and an excessive anti-squat produces the equivalent effect at the rear, with loss of traction. For these reasons, and because drivers use the pitch motion of the body to judge the deceleration of the vehicle, race cars use up to about 30% of brake pitch compensation, formula cars and sports prototypes use practically none, and the anti-squat is limited by the power-to-weight ratio of the vehicle and seldom exceeds 20% in circuit racing [5–7].

2.5.2 Conventional Suspension System

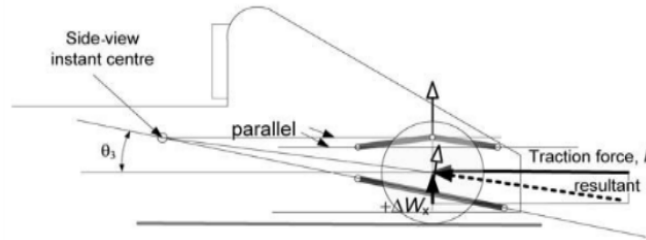
In a conventional racing suspension the heave, pitch, and roll modes are reacted by independent corner spring and damper assemblies, with an anti-roll bar (ARB) added at



(a) 100% anti-dive, front axle in braking



(b) 100% anti-lift, rear axle in braking



(c) 100% anti-squat, rear axle in acceleration

Figure 2.20: Side-view geometries corresponding to full compensation [7]. Reprinted with permission from D. Seward, *Race Car Design*, 2014, Palgrave Macmillan. © 2014.

each axle, as in the example of Figure 2.21. In roll the corner springs and the ARB act in parallel, so their stiffnesses add to give the roll stiffness of the axle [6, 7].

The consequence is that heave and roll are coupled. The corner springs have to support the sprung mass and control the heave displacement, and the rate chosen for that purpose already fixes a lower bound for the roll stiffness of the axle. The ARB can raise the roll stiffness without changing the heave rate, but the converse does not hold: the heave stiffness cannot be changed without also changing the roll stiffness and, with it, the lateral load transfer distribution [6, 7].

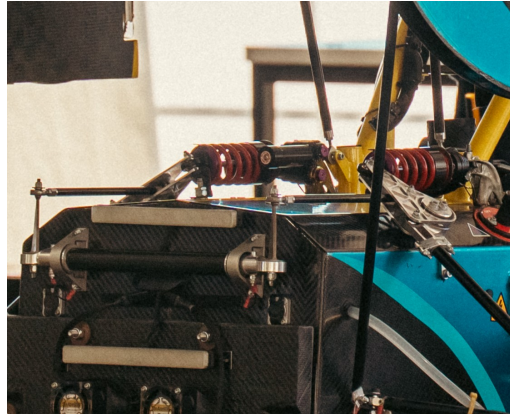


Figure 2.21: Conventional suspension of a Formula Student vehicle. ©Formula Student Germany.

2.5.3 Roll-Heave Decoupling Mechanism

The coupling described above is removed by routing the wheel motions through the bell-cranks and linkage so that the symmetric travel of the two wheels of an axle acts on one elastic element and the antisymmetric travel acts on another. The first is the heave element, which also reacts pitch, and the second is the roll element, which takes the place of the anti-roll bar. The heave stiffness can then be set from the ride-height and platform requirements and the roll stiffness from the cornering balance, without either constraining the other [1, 2].

The architecture does not remove the compromises of the design, it moves them. The motion ratios of the two paths, the packaging of the assembly, the friction of the additional joints, the stiffness of the mechanism, and the number of parts to be maintained all have to be defined, and they interact. Connecting the two wheels of an axle through the same mechanism makes the assembly compact in plan view but more demanding in the vertical direction, particularly when the dampers can no longer be placed longitudinally, so the choice of architecture reaches the packaging of the chassis and the volume available inside it [3, 14].

The principle is not new to motorsport. The heave elements described in Section 2.4.1 are the same idea applied to the symmetric mode alone, and a well-known road application is the suspension of the Mercedes-AMG One, shown in Figure 2.22.

2.6 Roll-Heave Decoupled Suspensions in Formula Student

The concept has been taken up by several Formula Student teams, and three student works document it in enough detail to be compared. They differ in scope: one reports a system that was built and raced [14], one designs and verifies a front axle without building it [2], and one develops the kinematics and the mechanical design with the support of numerical optimisation [1].

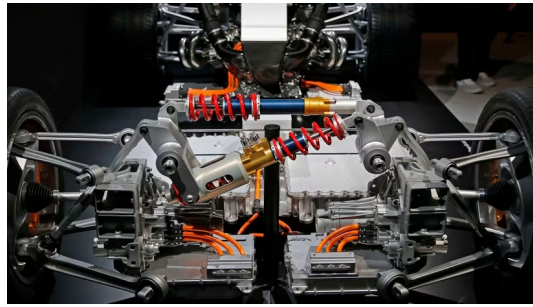


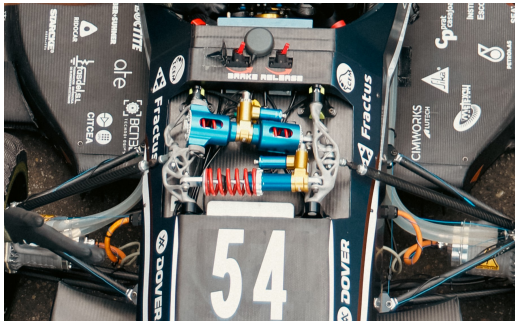
Figure 2.22: Mercedes-AMG One suspension [15]

Urban [14] documents the transition of TU Brno Racing from the layout of Dragon 8, in which each axle carried the two corner units, a separate heave spring and damper, and an anti-roll bar, to the fully decoupled layout of Dragon 9, in which each axle carries one unit acting only in roll and one acting in heave and pitch. Each axle therefore loses one spring and damper unit, one bellcrank, and the anti-roll bar, whose function is taken over by the roll element, and the mass of the two suspension systems together fell from 9.70 kg to 6.20 kg. The penalty is equally concrete and lies in the packaging. Connecting the two wheels of an axle through the same mechanism makes the assembly taller, because the dampers can no longer be placed longitudinally, and at the front axle this forced the lower wall of the monocoque upwards and, through the cross-section template rule, the upper wall with it. Reducing the size of the assembly, in order to recover that monocoque volume, became one of the objectives of the following car.

Díaz de Rábago [2] addresses the front axle alone and works from the required response backwards. A one-degree-of-freedom model is used for roll, since the quantity of interest is the roll angle at a given steady lateral acceleration, and a two-degree-of-freedom model for heave, so that the wheel-hop mode is represented. The targets adopted are a sprung mass frequency of 3.91 Hz and a roll gradient of 1 deg/g, which lead to a heave spring of 43.8 N/mm and a roll spring of 10.0 N/mm. The mechanism uses two asymmetric bellcranks, with the heave element between them and parallel to the ground and the roll element crossing diagonally, and the motion ratios obtained are 1.2 in heave and 0.8 in roll. Because the roll element has to be loaded in both directions of roll, an anti-tension device was designed so that it is compressed in both strokes. The parts were verified by finite element analysis, with a minimum safety factor of 1.3. The design was not manufactured.

Ventura Lecha [1] covers the widest scope of the three. The wishbone and bellcrank kinematics are defined with a derivative-free optimisation algorithm, DEFT-Funnel, chosen because the objective and constraint functions are evaluated by solving sets of equations numerically and by consulting a tyre model, so no analytical derivatives are available. The wishbone points are optimised to maximise the mean lateral force the tyres can produce over an autocross lap, evaluated on logged damper, steering and accelerometer data from a previous car, with the force averaged over a range of slip angle about its peak so that

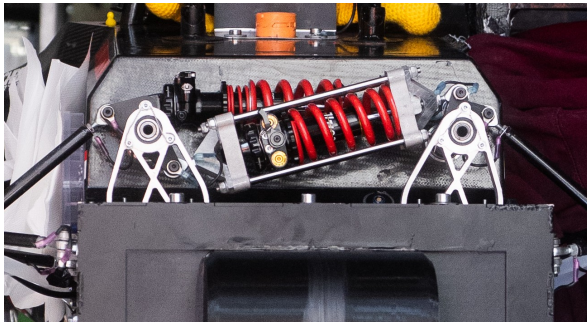
geometries with a sharp maximum are penalised; at the rear, bump steer is minimised in a separate run. The bellcrank points are then optimised for decoupling itself, the objective being the sum of the parasitic displacements: that of the heave spring under a roll of $\pm 1.5^\circ$ and that of the roll spring under $+15$ and -20 mm of heave. The dynamic requirements are established with analytical calculations, a seven-degree-of-freedom full-vehicle model, and vehicle data. Damper settings are examined for their effect on the load distribution during cornering, the compliance of the wishbones is used to size the tube diameters, and the loads at the bellcrank are derived as the basis for a topology optimisation left to future work.



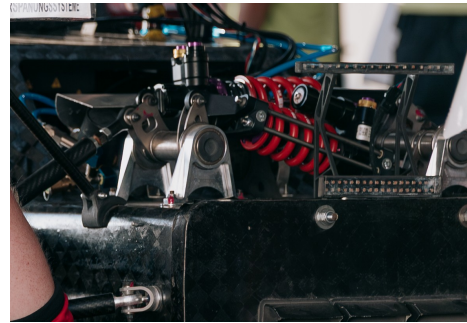
(a) BCN eMotorsport



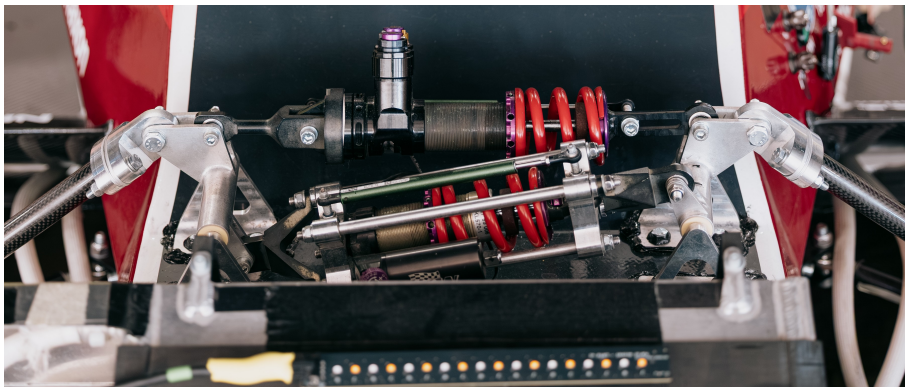
(b) Running Snail Racing Team



(c) Lions Racing Team



(d) municHMotorsport e.V.



(e) Formula Student Team Lausanne

Figure 2.23: Mode-decoupled inboard suspension layouts in Formula Student vehicles. © Formula Student Germany.

Read together, the three works agree on what the architecture gives and on what it costs. All three treat the motion ratio as a separate quantity for each mode rather than as a single installation ratio, and two of them arrive at asymmetric bellcranks in order to obtain different ratios in heave and in roll. All three identify the vertical dimension of the assembly as the binding packaging constraint. They differ in how far the design was taken: only the Brno system was manufactured and raced, and it is also the only one that reports the consequences of the architecture on the chassis itself. None of the three reports the damping behaviour of the two paths as a design constraint, and none derives the stiffness that the chassis attachments must provide for the intended rates to be reached at the wheel.

Beyond these academic works, several Formula Student teams have adopted mode-decoupled inboard layouts, as shown in Figures 2.23.

2.7 Formula Student Relevant Rules

In this section, the Formula Student Rules that most impact the suspension design are presented. These rules define hard boundaries for the admissible geometry, minimum wheel travel, rollover stability, and joint design philosophy. As a result, they directly influence the feasible track width, ride height, suspension travel budget, and attachment strategy of the proposed concept. The clauses reproduced below are therefore a selected subset of the official Formula Student Rules 2026, Version: 1.1 [16]; the original rule numbering is preserved for traceability, and the omitted intermediate rules are not considered directly relevant to the present suspension design.

T2.2 Ground Clearance

T2.2.1 The minimum static ground clearance of any portion of the vehicle, other than the tyres, including a driver, must be 30 mm. If an active suspension system is installed, the static ground clearance is measured in the lowest adjustable position.

T2.5 Suspension

T2.5.1 The vehicle must be equipped with fully operational front and rear suspension systems including shock absorbers and a usable wheel travel of at least 50 mm and a minimum jounce of 25 mm with driver seated.

T2.9 Wheelbase, Track and Rollover Stability

T2.9.1 The vehicle must have a wheelbase of at least 1525 mm.

T2.9.2 The smaller track of the vehicle (front or rear) must be no less than 75% of the larger track.

T2.9.3 The track and centre of gravity of the vehicle must combine to provide adequate rollover stability. This is tested by the Tilt Test IN7.

T3.4 Alternative Materials and Manufacturing

T3.4.4 Parts of the steering, brake and suspension system must not be manufactured using additive processes that produce anisotropic strength properties (e.g. consumer-grade Fused Deposition Modeling (FDM)). Additively manufactured parts are permitted only if proven to have equivalent strength in all load directions.

T 10 FASTENERS

T10.1 Critical Fasteners

T10.1.1 Critical fasteners are defined as bolts, nuts, and other fasteners utilised in the primary structure, the (autonomous) steering, (autonomous) brake, driver's harness, suspension systems and those specifically designated as critical fasteners in the respective rule.

T10.1.2 All threaded critical fasteners must be steel, at least 4 mm metric grade 8.8 (unmodified Original Equipment Manufacturer (OEM) parts 3 mm metric grade 8.8), equivalent size or of that specified in the referencing rule, whichever is larger. The strength classification of critical fasteners must be identifiable.

T10.1.3 All threaded critical fasteners must be of the type hexagon bolts (International Organization for Standardization (ISO) 4017, ISO 4014 or an equivalent standard.) or socket head cap screws (ISO 4762, Deutsches Institut für Normung (DIN) 7984, ISO 7379 or an equivalent standard.) including their fine-pitch thread versions.

T10.1.4 Any bolted joint in the primary structure and mounting of the Tractive System Accumulator Container (TSAC) to the chassis using either tabs or brackets, must have an edge distance ratio "e/D" of 1.5 or greater. "D" equals the hole diameter and "e" equals the distance from the hole centreline to the nearest free edge of the tab or bracket. Any tabs attaching suspension members to the primary structure are not required to meet this rule.

T10.1.5 For steering and suspension systems, alternative fasteners are allowed if equivalency to T10.1.2 and T10.1.3 can be shown.

T10.2 Securing Fasteners

T10.2.1 All critical fasteners must be secured from unintentional loosening by the use of positive locking mechanisms.

T10.2.2 The following methods are accepted as positive locking mechanisms:

- Correctly installed safety wiring.
- Cotter pins.
- Nylon lock nuts (ISO 7040, ISO 10512, European Standard (EN) 1663 or an equivalent standard) for locations where no temperature rating above 80°C is required.
- Prevailing torque lock nuts (DIN 980, ISO 7042 or an equivalent standard, and jet nuts or K-nuts).
- Locking plates.
- Tab washers.

Any locking mechanism based on pre-tensioning or an adhesive is not considered a positive locking mechanism.

T10.2.3 Snap or retaining rings according to DIN 471, DIN 472, or equivalent standard are allowed in unmodified OEM applications or for securing bearings or springs or brake disc floaters given that they do not bear any loads under normal driving conditions. The groove must be in pristine condition and manufactured according to the standard of the snap or retaining ring.

T10.2.4 A minimum of two full threads must project from any lock nut.

T10.2.5 All spherical rod ends and spherical bearings on the steering or suspension system must be in double shear or captured by having a screw/bolt head or washer with an outer diameter that is larger than the spherical bearing housing inner diameter.

T10.2.6 Adjustable tie-rod ends must be constrained with a jam nut to prevent loosening.

IN7.1.3 The vehicle will be placed upon the tilt table and to an angle of 60°. There must be no fluid leaks and all wheels must remain in contact with the tilt table surface.

2.8 Multibody Simulation in Suspension Design

The geometric quantities introduced in Section 2.2 — camber gain, roll centre migration, motion ratio, bump steer — are all consequences of the same spatial linkage, and none of them is constant along the suspension travel. Closed-form expressions exist only for planar idealisations, so the evaluation of a three-dimensional suspension over its working range is carried out numerically. The established tool for that purpose is multibody systems analysis.

2.8.1 Formulation

A multibody model represents the mechanism as a set of rigid bodies whose relative motion is restricted by kinematic constraints. Each body has six degrees of freedom and each joint

removes a number of them: a spherical joint removes three and a revolute joint removes five. Writing $\mathbf{q}$ for the vector of generalised coordinates of all bodies and $\Phi(\mathbf{q}, t) = \mathbf{0}$ for the set of constraint equations, the equations of motion follow from the Lagrange formulation with multipliers [17],

$$\mathbf{M}\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^T \boldsymbol{\lambda} = \mathbf{Q}, \quad \Phi(\mathbf{q}, t) = \mathbf{0}, \quad (2.15)$$

where $\mathbf{M}$ is the mass matrix, $\Phi_{\mathbf{q}}$ the Jacobian of the constraints, $\boldsymbol{\lambda}$ the vector of Lagrange multipliers and $\mathbf{Q}$ the vector of applied forces. The multipliers are the reaction forces transmitted by the joints, so a multibody solution gives the internal loads of the mechanism as well as its motion [17].

Equation 2.15 is not an ordinary differential equation but a system of differential-algebraic equations, since the constraints contain no derivatives of the unknowns. The formulations in use differ in how those constraints are enforced, and Bauchau and Laulusa [18] review the alternatives and the numerical difficulties that each of them brings.

2.8.2 Numerical Solution

Two properties of Equation 2.15 govern how it is solved. The first is that the system is of differential-algebraic type, for which the numerical treatment was established by Gear [19] for circuit simulation and later transferred to mechanical systems. The second is that the equations are numerically stiff, because the eigenvalues of a suspension model are widely separated: the sprung-mass modes lie well below the wheel-hop modes, and the constraints themselves behave as infinitely stiff connections. Explicit integration would need a time step set by the fastest mode, so implicit and stiffly stable schemes are used instead. Negrut et al. [20] discuss the low-order formulas used for this purpose and the compromise each of them makes between stability, order of accuracy and the drift of the constraint equations.

2.8.3 The ADAMS Formulation

ADAMS, an acronym for Automatic Dynamic Analysis of Mechanical Systems, is the implementation used in this work, and it originates in the combination of the formulation of Section 2.8.1 with the numerical treatment of Section 2.8.2. Orlandea et al. [21, 22] set out the formulation in two companion papers: sparse matrix algorithms and stiff integration methods, both developed for electrical circuit simulation, are applied to the equations of motion and constraints of a three-dimensional mechanism. The sparsity comes from the choice of an absolute coordinate formulation, in which every body carries its full set of coordinates and the joints appear as explicit constraints, so that each joint couples only the two bodies it connects and the Jacobian $\Phi_{\mathbf{q}}$ has few non-zero entries. Eliminating the constraints would give a smaller system, but a dense one.

The same papers report the first applications, one of which is the front suspension of a passenger car, and a companion paper by Orlandea and Chace [23] deals only with vehicle

suspension simulation. Suspension analysis has therefore been a driving application of the code since its origin.

The program offers three classes of analysis, and they serve different purposes. A kinematic analysis solves only $\Phi(\mathbf{q}, t) = \mathbf{0}$, which requires the mechanism to have no remaining degrees of freedom once the prescribed motion is imposed; this is the analysis used to extract the geometric characteristics of a suspension as a function of wheel travel. A static analysis solves for the equilibrium configuration under applied loads. A dynamic analysis integrates the full system in time. The distinction matters here: the kinematic quantities of a suspension are properties of the linkage alone and are obtained without reference to mass or to time, whereas the modal behaviour requires the dynamic solution.

2.8.4 Applications to Suspension Analysis

The literature applies these tools to suspension design in two ways. The first is kinematic and elastokinematic characterisation, in which the model is exercised over the travel range to obtain the geometric quantities that classical analysis can only approximate, and to quantify the additional wheel displacement produced by the compliance of the bushings and links. Blundell and Harty [24] set out the approach in full, and Rocca and Russo [25] illustrate its elastokinematic side, modelling a multilink suspension with compliant joints and identifying the joint stiffnesses from measured response. This is the use made of the model in Chapter 5, and of the bushing sweep of Chapter 7.

The second is the comparison of design alternatives. Prastiyo and Fiebig [26] modelled in ADAMS two double-wishbone suspensions that differed only in the rate characteristic of the spring, one linear and one progressive, and compared their road holding and ride over modelled road profiles. Holding every other parameter constant is what makes the difference attributable to the spring alone, and it is practical only in simulation. Their motivation applies to the present context as well: small teams usually validate a suspension late, with limited apparatus or by driver assessment, so a change found at that stage is expensive.

The same approach has been applied within Formula Student. Harsh [27] developed a full-vehicle multibody model of a Formula Student car, with a tyre model extended to include temperature effects, and validated it against skidpad and endurance data recorded by the team, showing that a model of this kind can be brought to a level of agreement that supports decisions at component level.

2.9 Chapter Conclusions

This chapter established the theoretical foundation on which the remainder of the dissertation builds. The main suspension layouts were reviewed, and the vehicle dynamics concepts that govern their behaviour were introduced, from the tyre-level mechanisms of

slip angle and load sensitivity, through the geometric parameters that shape the wheel kinematics, to the load transfer that links them to the handling balance of the vehicle.

Three ideas carry into the chapters that follow. Tyre load sensitivity establishes why load transfer, and in particular its front-to-rear distribution, governs the grip and the balance of the vehicle. In a conventional suspension a single elastic element at each corner reacts to the heave, pitch, and roll modes at the same time, so its selection is always a compromise, and this is the limitation that the roll-heave decoupled architecture removes. And the bellcrank, through the motion ratio and the wheel rate it produces, is what translates the design targets into physical spring rates, which in a decoupled layout cannot be described by a single motion ratio.

The review of decoupled suspensions in Formula Student confirmed the relevance of the concept and identified what the existing work leaves open. The three studies examined treat the motion ratio separately for each mode and identify the vertical dimension of the assembly as the binding packaging constraint, but none of them treats the damping of the two paths as a design constraint, and none derives the stiffness that the chassis attachments must provide for the intended rates to be reached at the wheel. Neither is documented within the team.

Four requirements for the present work follow from the chapter. The suspension geometry has to preserve the camber control, the bump steer and the roll centre behaviour described in Section 2.2. The bellcrank and linkage have to be defined through motion ratios and wheel rates and not through nominal spring values alone. The decoupling has to produce a separation between the heave and roll paths that survives the real geometry, without introducing excessive friction, compliance or mass. And all of this has to remain compatible with the rules of Section 2.7 and with the space available inside the chassis. Chapter 3 takes these requirements as its starting point and establishes the baseline geometry and the mode-decoupling architecture adopted in this work.

Chapter 3

Suspension Geometry and Mode-Decoupling Architecture

This chapter translates the conclusions of the literature review and the design constraints into the final geometric definition of the proposed suspension. The objective is to define a technically justified suspension layout and a feasible roll-heave decoupling architecture. To that end, the main packaging constraints, geometric hardpoints, and bellcrank requirements are established and combined into the final concept. The geometry presented here reflects the iterations and refinements driven by the anti-feature derivation of Chapter 4 and by the bump steer correction.

3.1 Development Workflow

Figure 3.1 shows the workflow adopted in the present work and the chapter in which each stage is presented. The present chapter covers the first of those stages: the packaging constraints, the hardpoints, the baseline geometry, and the bellcrank architecture.

3.2 Design Requirements and Packaging Constraints

The geometry is bounded by the Formula Student rules of Chapter 2 and by the architecture of the prototype: the in-wheel motor, the planetary gearbox, the available chassis hardpoints, and the minimum wheel travel.

The outer hardpoints are constrained by the internal volume of the rim and by the powertrain components housed inside the wheel assembly (Figures 3.2 and 3.3). The rim has an inner diameter of 231 mm, and an effective design diameter of 215 mm was adopted, so that no component may come within 8 mm of the rim. This margin covers the manufacturing and assembly tolerances of the wheel assembly and the deflection of the hub, the bearing and the rim under peak cornering and braking loads. The clearance has to hold over the whole operating envelope and not only at the static position: the

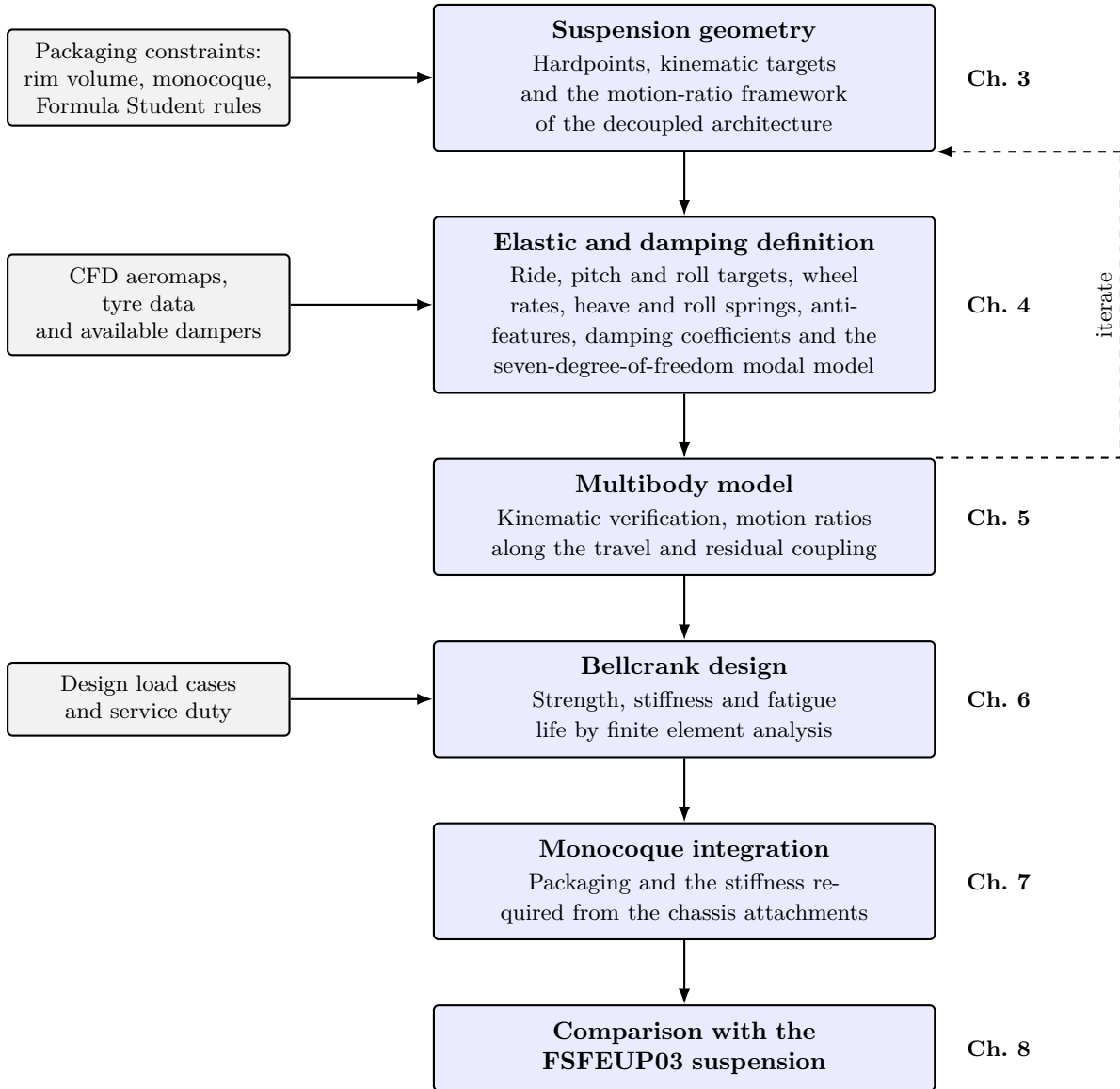


Figure 3.1: Development workflow adopted in the present work, with the chapter in which each stage is presented.

wishbones rotate about their chassis axes by up to 6.5° over 30 mm of jounce, and in steering the rim rotates about the kingpin axis relative to the links by up to the maximum steer angle of about 11.5° , so the position of closest approach is not known in advance. The same 215 mm diameter had already been adopted in the FSFEUP03 project and confirmed in service, and is retained here as a conservative first-order value; the clearance of the final geometry is verified in Chapter 7.

The chassis interface constrains the geometry from the other side. The available space has to accommodate the suspension links, the angular motion of the bellcrank, and the travel envelopes of the spring and damper elements.

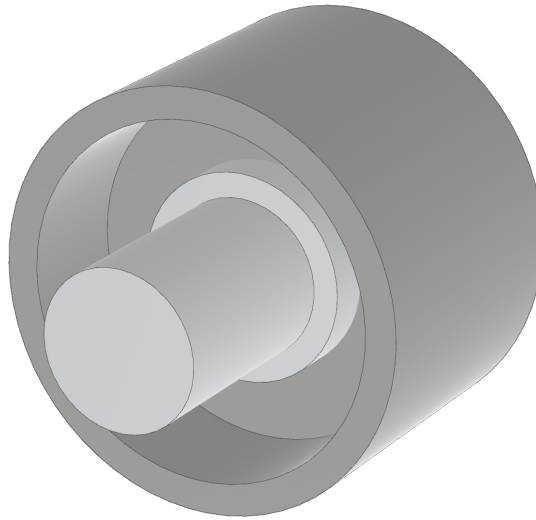


Figure 3.2: Control Volume of the wheel

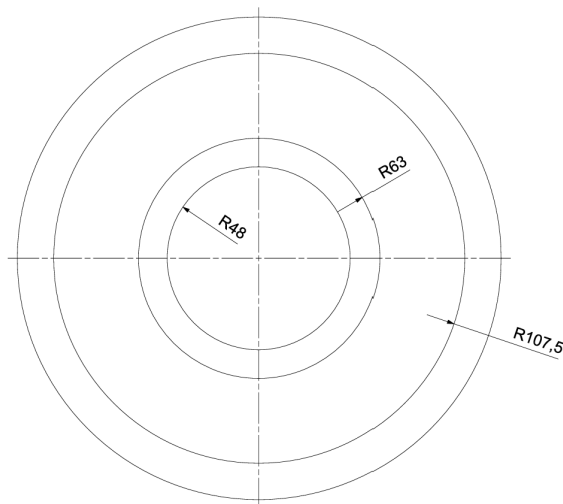


Figure 3.3: Control Volume dimensions

The geometric restrictions imposed by the monocoque on the front suspension are the following:

- The side wall, highlighted in yellow in Figure 3.4, must maintain a near-vertical profile.
- The monocoque floor, also indicated in yellow in Figure 3.4, must be positioned below the pedal-box template, allowing for the required clearance margin and panel thickness, which are represented in red.
- The steering rack, and with it the outer tie-rod pickup points, must be located below the monocoque floor.

On the rear suspension, the monocoque imposes that all side-wall planes, highlighted in yellow in Figure 3.5, remain parallel and near-vertical.

In Figures 3.4 and 3.5 the planes are shown together with the initial hardpoints, defined for the tubular chassis, and therefore they do not respect the restrictions listed above. After the iterations presented in Section 3.6 all of them are respected, as shown in Figures 3.6 and 3.7.

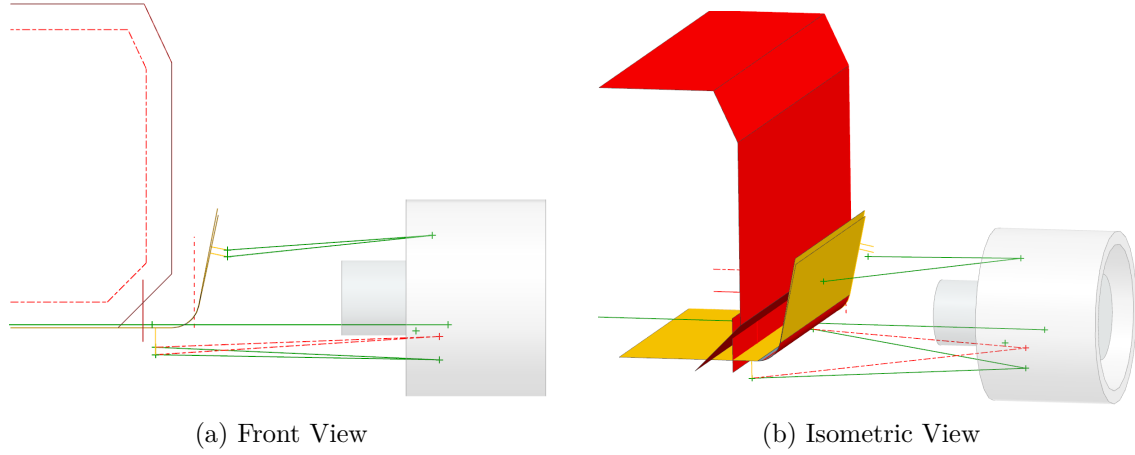


Figure 3.4: Front suspension monocoque restrictions alongside the initial hardpoints.

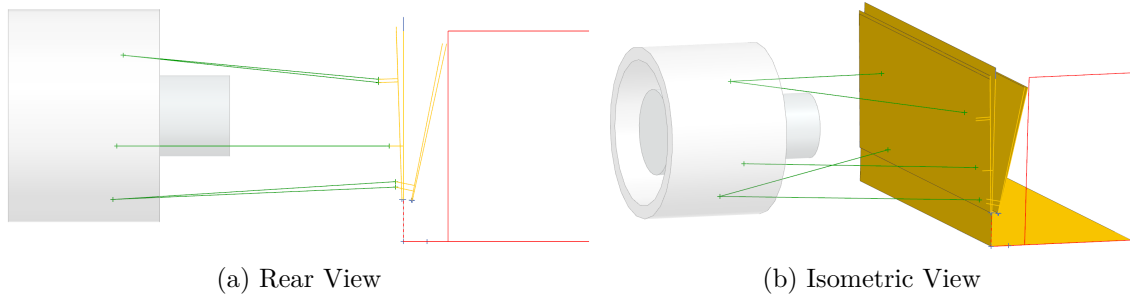


Figure 3.5: Rear suspension monocoque restrictions alongside the initial hardpoints.

3.3 Vehicle Data and Dynamic Targets

As established in Chapter 2, the tyre is the only interface through which the vehicle exchanges load with the track, so the purpose of the suspension geometry is to keep it working close to its most favourable condition under combined lateral, longitudinal and aerodynamic loading. Three requirements follow for the baseline kinematics:

- **Camber gain:** the loaded outer wheel gains negative camber in cornering, opposing the inclination imposed by the body roll and keeping the wheel plane close to the vertical.

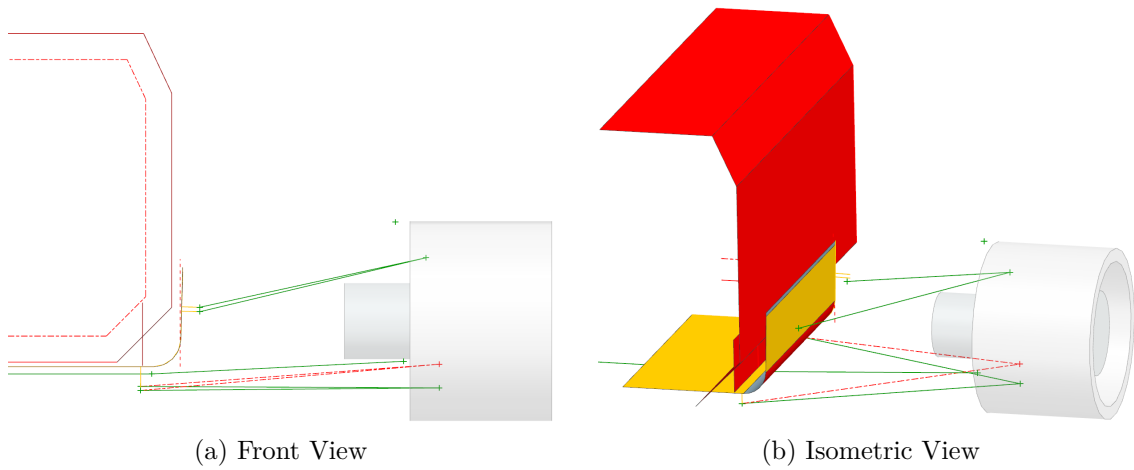


Figure 3.6: Front suspension monocoque restrictions alongside the final hardpoints.

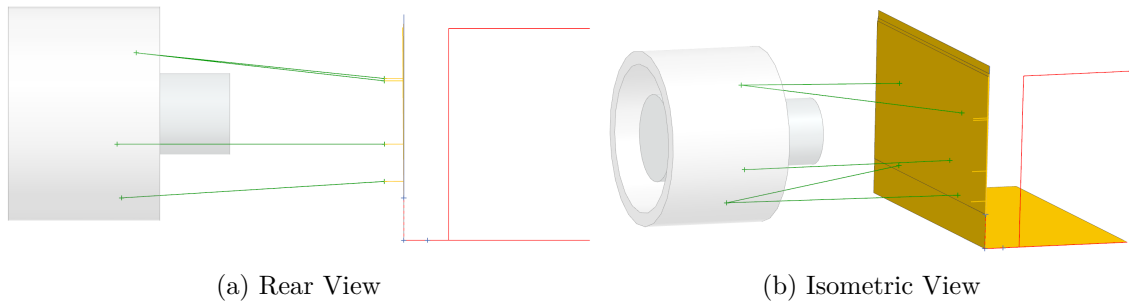


Figure 3.7: Rear suspension monocoque restrictions alongside the final hardpoints.

- **Bump steer:** the toe variation over the whole wheel travel is kept small, so that the response to a steering input does not change with the position of the suspension.
- **Roll centre migration:** the movement of the roll centre through the travel is kept controlled, so that the geometric share of the lateral load transfer does not shift abruptly.

At this stage the tyre is represented by a set of acceleration limits rather than by a force and moment model. The vehicle is expected to sustain $1.6g$ in traction, $1.8g$ in braking and $1.6g$ laterally. These values are not the mechanical grip limits of the tyre alone: they already include the load the aerodynamic package is expected to produce, and they are adopted here as the load cases that drive the preliminary sizing of the suspension and the first structural estimates. They set the magnitude of the longitudinal and lateral load transfers, and through them the suspension travel and the roll stiffness required to hold the aerodynamic platform.

The vehicle dimensions and mass properties on which these targets rest, and which serve as inputs to the analyses of Chapter 4, are given in Tables 3.1 and 3.2.

Table 3.1: Baseline vehicle dimensions and mass properties used in the present work.

Parameter	Value
Wheelbase [mm]	1530
Front track [mm]	1200
Rear track [mm]	1200
Front ride height [mm]	37.5
Rear ride height [mm]	37.5
Sprung mass (including 68 kg driver) [kg]	227
Unsprung mass [kg]	48.6
Total mass [kg]	276
Sprung mass Centre of Gravity (CG), x [mm]	0
Sprung mass CG, y [mm]	821.64
Sprung mass CG, z [mm]	296.04
Unsprung mass CG, x [mm]	0
Unsprung mass CG, y [mm]	765.00
Unsprung mass CG, z [mm]	205.05
Total mass CG, x [mm]	0
Total mass CG, y [mm]	811.66
Total mass CG, z [mm]	280.00

The total mass centre of gravity height is the measured quantity; the height of the sprung mass follows from it and from the unsprung mass distribution.

Table 3.2: Inertial properties of the sprung mass adopted as inputs to Chapter 4. The aerodynamic conditions on which the elastic sizing rests are established in Table 4.1.

Parameter	Value
Sprung mass roll inertia, I_{yy} [kg m ²]	22.9
Sprung mass pitch inertia, I_p [kg m ²]	146.6
Pitch centre, x [mm]	0
Pitch centre, y [mm]	507.79
Pitch centre, z [mm]	34.97

3.3.1 KPI and Caster Influence on Camber Gain

The effects described in Sections 2.2.5 and 2.2.4 can be separated and quantified. Figures 3.8 and 3.9 show each one in isolation, with the other angle set to zero, across the steering range. The two behave differently with the direction of steer: the camber produced by the KPI is an even function of the steering angle, so both wheels are taken towards positive camber, while the camber produced by the caster is an odd function, negative at the outer wheel and positive at the inner one. The two surfaces were produced by the Formula Student FEUP (FSFEUP) vehicle dynamics department, and the data behind them are reproduced in Appendix A.

3.3.2 Roll Centre Height Justification

The roll centre (Roll Centre (RC)) heights set the roll moment arm and the division of the lateral load transfer between its geometric and elastic parts, so they are established

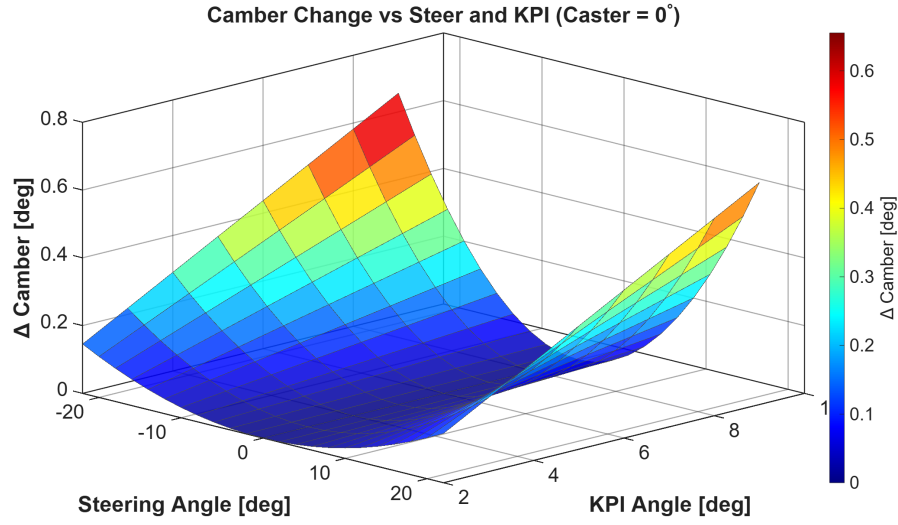


Figure 3.8: Isolated effect of the KPI (caster = 0°) on camber change across the steering range.

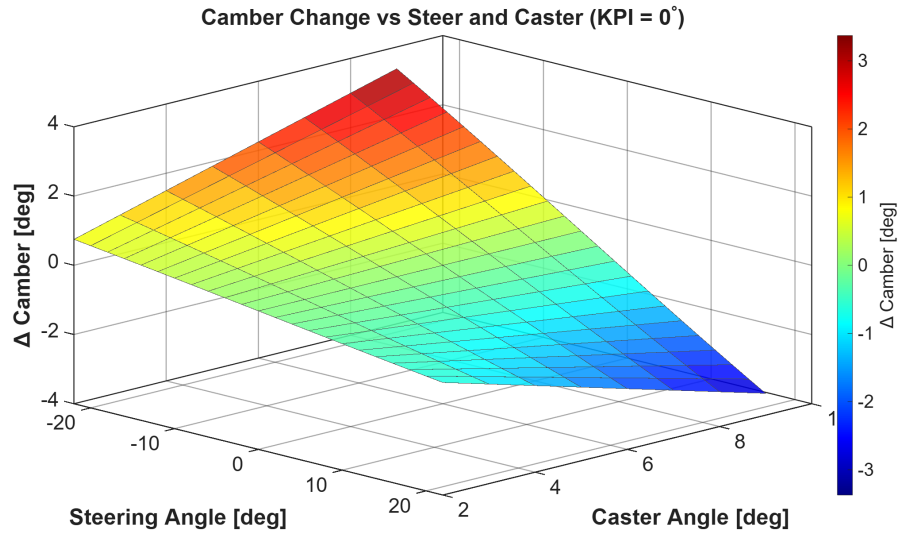


Figure 3.9: Isolated effect of the caster angle (KPI = 0°) on camber change across the steering range.

before the inner hardpoints are placed. A study was carried out in which the front and rear heights were varied and the three contributions of Equation 2.9 were evaluated for each combination, at the design lateral acceleration of 1.6 g.

Two criteria guided the selection. The first is a front load transfer distribution close to 50%, which corresponds to a neutral steady-state balance; it is adopted as the baseline because the balance is afterwards tuned through the front-to-rear distribution of roll stiffness in Chapter 4, and a neutral geometric starting point leaves that adjustment its full range in both directions. The second is a geometric share of the total load transfer of approximately 30%, which is a declared allocation rather than a derived value. It follows from the trade-off the roll-centre height carries: raising the roll centres shortens the roll

moment arm and reduces the roll angle, but it also increases the share of the lateral force transmitted directly by the links, and with it the jacking of the sprung mass and the migration of the roll centre through the travel, as described in Section 2.2.12 [5, 6]. A third of the transfer carried geometrically keeps the roll angle within reach of the springs without placing the roll centres high enough for jacking to become a design problem.

Table 3.3 shows that the total load transfer of the vehicle is essentially unchanged across the combinations, so the grip available is not affected by the choice; what changes is how that transfer is divided. The two criteria are met over a broad region rather than at a single point: every combination in the two sweeps returns a front load transfer distribution between 47.5 and 48.7% and a geometric share between 31.0 and 32.6%, while the front geometric share varies by 3.5 percentage points over the whole study. That is the useful result of the study, and it is what settles the selection: over the range examined the division of the load transfer is insensitive to the roll-centre heights, so no combination within the region is preferable to another on these grounds, and an error of a few millimetres in either height is of no consequence to the balance. The combination of 90 mm at the front and 96 mm at the rear was adopted from within that region, returning a front load transfer distribution of 48.5%, a geometric share of 31.9% and a front geometric share of 29.5%. These heights are the values used in the analyses of Chapter 4, and they are the starting point for the two-dimensional construction of the virtual swing arm (Virtual Swing Arm (VSA)) and of the instant centre (Instant Centre (IC)) that locate the inner pickup points.

Table 3.3: Roll-centre height iterations and corresponding lateral load-transfer metrics, evaluated at a steady lateral acceleration of 1.6 g. Ten of the twenty-seven combinations evaluated are shown, arranged as two sweeps in which a single height is varied at a time. The adopted configuration is highlighted.

Metric	Baseline	Rear sweep, $h_{RC,F} = 90$ mm				Front sweep, $h_{RC,R} = 97$ mm				
Front RC height, $h_{RC,F}$ [mm]	80	90	90	90	90	89	90	91	92	93
Rear RC height, $h_{RC,R}$ [mm]	100	100	98	96	94	97	97	97	97	97
Lateral load-transfer components [N]										
Total load transfer – front	474.4	481.2	482.7	484.3	485.9	482.8	483.5	484.2	484.9	485.6
Total load transfer – rear	523.9	516.9	515.3	513.7	512.0	515.1	514.5	513.8	513.1	512.4
Elastic load transfer – per axle	299.9	292.9	294.4	296.0	297.6	295.9	295.2	294.5	293.8	293.1
Geometric load transfer – front	110.0	123.7	123.7	123.7	123.7	122.4	123.7	125.1	126.5	127.8
Geometric load transfer – rear	159.5	159.5	156.3	153.1	149.9	154.7	154.7	154.7	154.7	154.7
Geometric load-transfer fraction	31.0%	32.6%	32.2%	31.9%	31.5%	31.9%	32.0%	32.2%	32.4%	32.5%
Front load-transfer distribution	47.5%	48.2%	48.4%	48.5%	48.7%	48.4%	48.5%	48.5%	48.6%	48.7%
Front geometric load-transfer fraction	26.8%	29.7%	29.6%	29.5%	29.4%	29.3%	29.5%	29.8%	30.1%	30.4%
Rear geometric load-transfer fraction	34.7%	35.3%	34.7%	34.1%	33.5%	34.3%	34.4%	34.4%	34.5%	34.5%

3.4 Preliminary Kinematic Targets

Before the first hardpoint iteration, a set of target ranges is fixed to guide the geometry and the later simulation work. They are not final values; they are the references from which the first layout is built, and they are listed in Table 3.4.

Table 3.4: Preliminary suspension design targets adopted as baseline references.

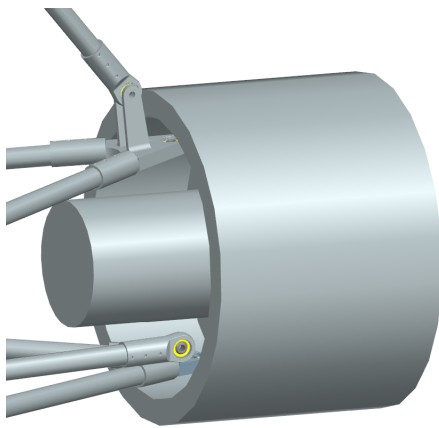
Parameter	Design objective	Initial target / reference
Bump steer	Limit toe variation over the operational wheel-travel range to preserve predictable steering response	As close to zero as possible in straight-line heave, with smooth monotonic variation
Scrub radius	Limit steering kickback and excessive steering effort at the contact patch	< 40 mm
Caster angle	Provide self-aligning behaviour and favourable camber gain during steering	6°
Kingpin inclination (KPI)	Balance steering-axis packaging, scrub radius, and camber variation	3°–7°
Ackermann	Define the front steering relation required for corner-entry response	Outer wheel: 11.54°; inner wheel: 10.15° at the reference steering position
Anti-dive	Reduce front pitch motion under braking through side-view geometry	30% initial reference
Anti-squat	Reduce rear pitch motion under traction through side-view geometry	30% initial reference
Anti-lift	Control rear-axle pitch behaviour under braking, if required by the final pitch balance	Derived from the anti-squat geometry

The initial anti-dive and anti-squat references are taken from the values reported for the mode-decoupled suspension of BCN eMotorsport [1].

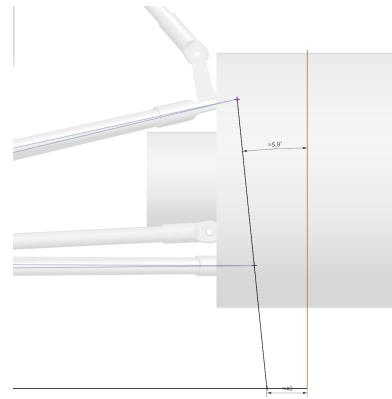
3.5 Outer Hardpoint Definition

The admissible region for the outer hardpoints follows from the constraints of Section 3.2 and from the preliminary targets of Section 3.4. Within this region the wheel-side points were placed first, since they set the envelope inside which the chassis-side points are later searched.

These points were obtained by adapting the existing FSFEUP03 geometry to the new architecture, which includes the front in-wheel motors, a different damper packaging, and an Ackermann target that the FSFEUP03 design could not accommodate. The main modification follows from the wheel-side packaging: to keep the scrub radius below 40 mm, a higher KPI had to be accepted, rising from the original 3.5° to 5.9° (Figure 3.10). The wishbone outer points were kept unchanged from this definition onwards; apart from a



(a) Components within the rim envelope.



(b) Steering axis, kingpin inclination and scrub radius.

Figure 3.10: Outer hardpoints within the rim envelope. In (b) the steering axis is drawn through the two ball joints; the kingpin inclination is measured from the wheel centre plane and the scrub radius at ground level.

minor adjustment of the front tie-rod outer point, the iterations of Section 3.6 varied only the chassis-side coordinates.

3.6 Suspension Geometry

The suspension geometry is the result of the iterative process outlined in Section 3.1, in which the initial FSFEUP03 hardpoints were progressively refined to meet the new packaging constraints and dynamic targets.

3.6.1 First Suspension Geometry

With the outer points and the preliminary targets defined, the first inner-point iteration uses the anti-features and the roll and pitch centre targets as its main references. The purpose of this baseline is not to represent the final hardpoint set, but to provide a coherent starting point that satisfies the principal constraints identified above, with the inner hardpoints placed within the admissible chassis-side regions defined by the monocoque interface.

The existing FSFEUP03 geometry is represented in Figures 3.11 and 3.12, where:

- A** – Upper wishbone chassis forward;
- B** – Upper wishbone chassis rearward;
- C** – Upper wishbone upright;
- D** – Lower wishbone chassis forward;

E – Lower wishbone chassis rearward;

F – Lower wishbone upright;

G – Tie rod chassis;

H – Tie rod upright.

The corresponding coordinates are given in Tables 3.5 and 3.6, and are the geometric reference from which the new layout is adapted.

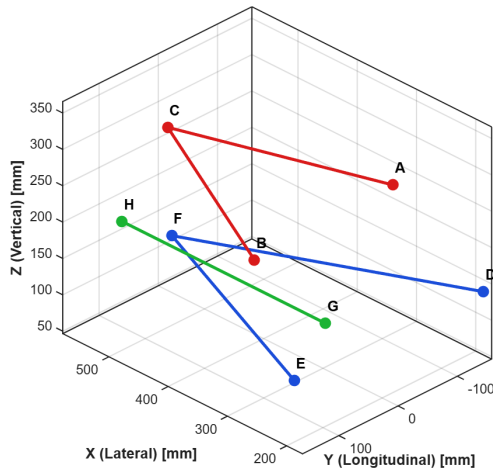


Figure 3.11: Wireframe representation of the existing front FSFEUP03 suspension geometry.

Table 3.5: FSFEUP03 suspension hardpoint coordinates for the front-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	280.00	−98.25	266.74
B	280.00	135.69	259.22
C	543.50	18.54	286.14
D	187.00	−157.92	133.63
E	187.00	161.28	141.78
F	553.27	1.75	126.33
G	220.98	75.00	171.37
H	564.12	75.00	171.37

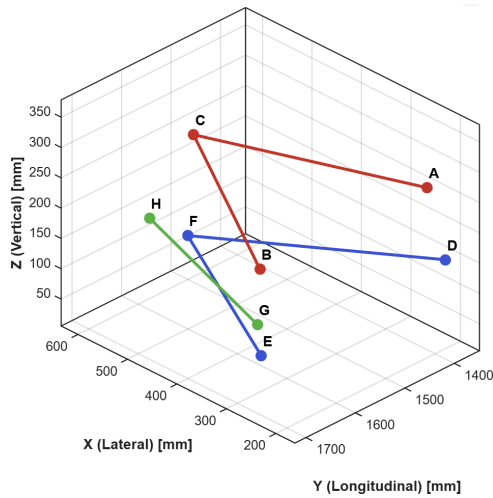


Figure 3.12: Wireframe representation of the existing rear FSFEUP03 suspension geometry.

Table 3.6: FSFEUP03 suspension hardpoint coordinates for the rear-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	250.00	1366.29	245.33
B	250.00	1704.41	249.20
C	553.48	1535.00	277.74
D	230.00	1349.13	127.61
E	230.00	1721.49	121.16
F	565.45	1535.00	106.50
G	237.37	1722.05	170.00
H	560.91	1616.25	170.00

After applying the packaging changes described in the previous sections, the resulting geometry is defined by an updated set of hardpoints (Tables 3.7 and 3.8), represented in Figures 3.13 and 3.14.

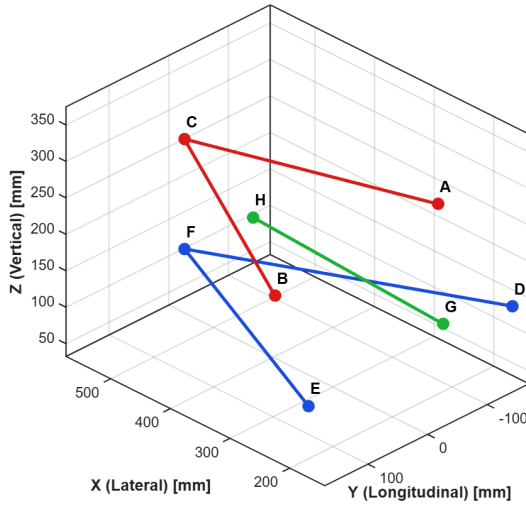


Figure 3.13: Wireframe representation of the first front suspension geometry.

Table 3.7: First suspension hardpoint coordinates for the front-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	243.57	-120.00	257.53
B	243.57	153.35	243.25
C	530.48	18.54	286.08
D	168.64	-170.00	127.65
E	168.64	172.33	130.59
F	547.43	1.23	121.37
G	182.88	-68.00	139.00
H	502.00	-68.00	155.00

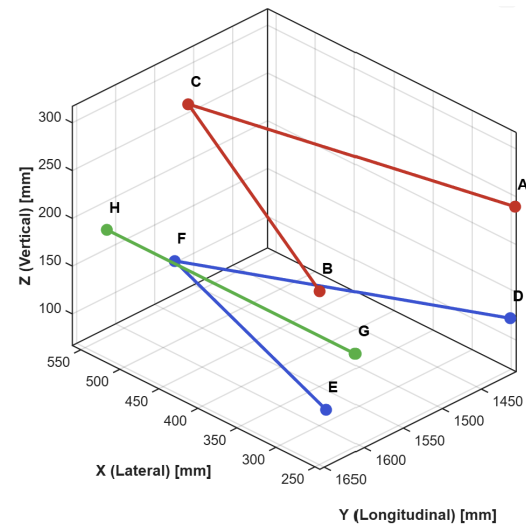


Figure 3.14: Wireframe representation of the first rear suspension geometry.

Table 3.8: First suspension hardpoint coordinates for the rear-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	245.21	1405.30	238.97
B	244.00	1655.50	253.36
C	538.00	1530.00	277.74
D	243.98	1412.75	125.75
E	244.00	1647.50	126.11
F	555.45	1530.00	106.50
G	243.60	1611.25	170.00
H	560.91	1611.25	170.00

3.6.2 Second Suspension Geometry

The geometry is not defined in a single step. In this iteration the side-view geometry was adjusted to incorporate the pitch-control targets derived in Chapter 4 and the Ackermann relation verified in Chapter 5: the wishbone pivots were inclined to obtain the anti-dive

of 32.8% and anti-lift of 4.6%. The resultant geometries are represented in Figures 3.15 and 3.16, with the hardpoints of Tables 3.9 and 3.10.

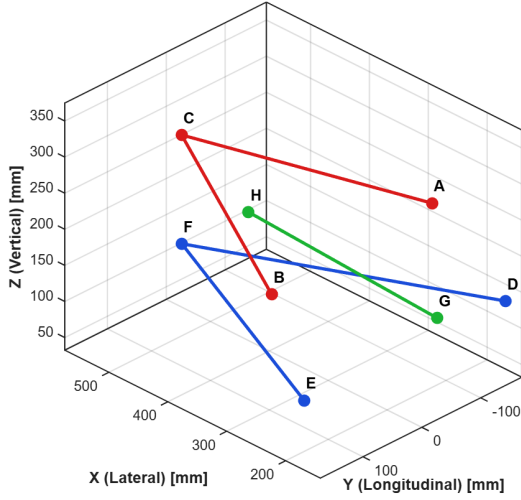


Figure 3.15: Wireframe representation of the second front suspension geometry.

Table 3.9: Second suspension hardpoint coordinates for the front-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	243.57	-120.00	252.10
B	243.57	152.44	236.92
C	530.48	18.54	286.08
D	168.64	-170.00	126.70
E	168.64	172.34	128.94
F	547.43	1.23	121.37
G	182.88	-68.00	139.00
H	503.30	-68.00	155.00

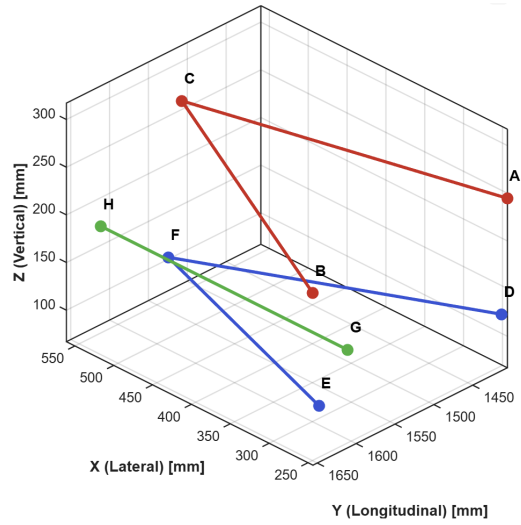


Figure 3.16: Wireframe representation of the second rear suspension geometry.

Table 3.10: Second suspension hardpoint coordinates for the rear-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	243.96	1405.31	244.70
B	244.00	1655.50	247.47
C	538.00	1530.00	277.74
D	244.00	1412.49	125.89
E	244.00	1647.50	125.96
F	555.45	1530.00	106.50
G	243.60	1611.25	170.00
H	560.91	1611.25	170.00

3.6.3 Final Suspension Geometry

Inclining the wishbone pivots displaced the front suspension instant centres and introduced excessive bump steer. To correct it, the wishbone chassis pivots were repositioned so that the instant centres return to the line through the tie-rod ends, keeping the tie-rod coordinates unchanged; this correction is detailed in Chapter 5. The rear geometry was not affected and remains as given in Table 3.10. The resulting front hardpoints are presented in Figure 3.17 and Table 3.11.

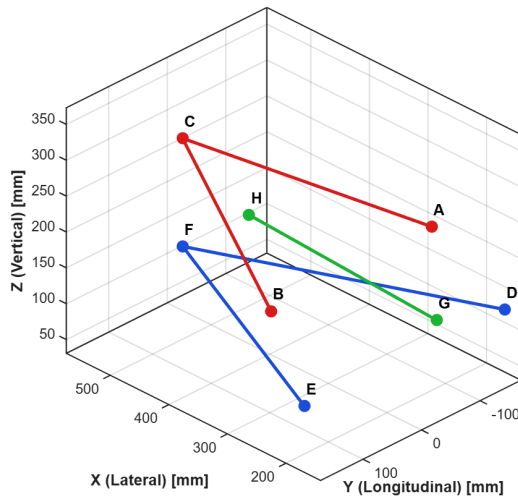


Figure 3.17: Wireframe representation of the final front suspension geometry.

Table 3.11: Final suspension hardpoint coordinates for the front-left quarter.

Point	X [mm]	Y [mm]	Z [mm]
A	243.57	−120.00	223.41
B	243.57	154.37	217.75
C	530.48	18.54	286.08
D	168.64	−170.00	118.13
E	168.64	172.30	123.15
F	547.43	1.23	121.37
G	182.88	−68.00	139.00
H	503.30	−68.00	155.00

3.7 Bellcrank Layout and Roll–Heave Decoupling Concept

Once a feasible baseline geometry is defined, the suspension concept must be connected to the inboard elastic system. This integration is achieved through a bellcrank layout compatible with the roll–heave decoupled architecture introduced in Chapter 2, as the one shown in Figure 3.18.

Each axle carries two bellcranks, one per corner, driven by the respective pushrod. The two bellcranks are connected to each other by two elastic paths instead of the single spring–damper unit per corner of a conventional layout. The first path is the *heave element*, mounted so that it is displaced only when both wheels move in the same direction; the second is the *roll element*, mounted so that it is displaced only when the wheels move in opposite directions. Symmetric wheel travel therefore loads the heave element alone, and antisymmetric travel loads the roll element alone, which is what makes the anti-roll bar of a conventional suspension unnecessary.

The roll element requires an additional provision. The relative motion of the two bellcranks reverses sign between left-hand and right-hand roll, but a coil spring can only be loaded in compression. An anti-tension device is therefore used. The damper is held between two collars, and a pair of parallel rods connects the two ends of the unit along the outside of the dampers. This arrangement inverts the load path in the extension stroke: whichever way the unit moves, one collar advances towards the other and the spring is compressed. The spring is thus compressed in both the compression and the extension stroke.

The reference state is the neutral position, in which both collars rest against their seats and the damper sits at its installed length, in the static position (Figure 3.19a). Wheel travel in either roll direction compresses the spring from this state, and the collar spacing is reduced by the same amount whether the unit shortens (Figure 3.19b) or lengthens

(Figure 3.19c). When the roll direction reverses, the spring first returns to the neutral position, where its force passes through zero, and is then compressed again by the opposite collar. The roll rate is therefore symmetric about the neutral position, and the spring never sees tension. The damper is used in the same way in both roll directions: its compression curve acts as the vehicle rolls away from the neutral position, and its rebound curve acts as it returns to it.

Two consequences follow. No preload is required to keep the spring in compression, and the full compression stroke is available in each roll direction, which doubles the relative bellcrank travel that a given spring length can accommodate. This is what allows the high roll motion ratio adopted in Section 3.8.1.

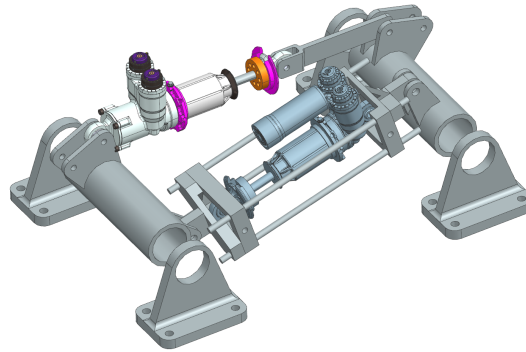


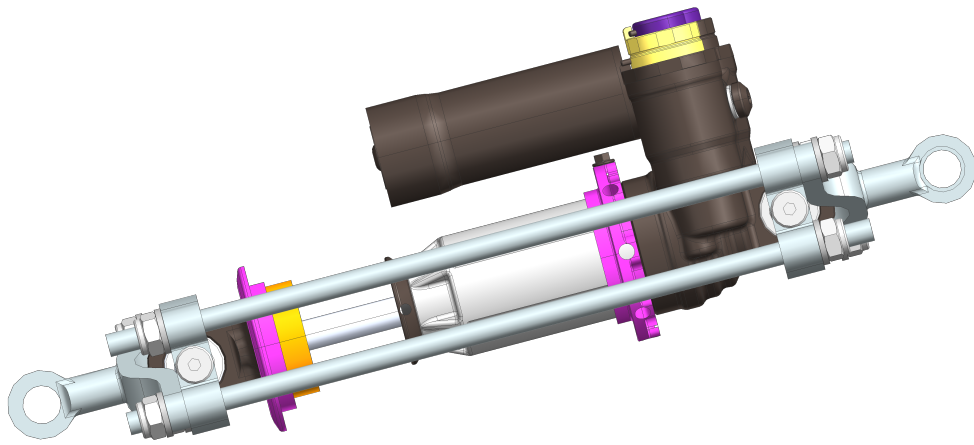
Figure 3.18: Concept layout of the roll–heave decoupled mechanism.

The layout presented here is the functional concept, with the bellcranks represented as rigid bodies of arbitrary shape. Their detailed geometry follows from the load cases of Chapter 6, and the final assembly, once the elastic elements are selected and the mechanism is packaged against the monocoque, is presented in Chapter 7.

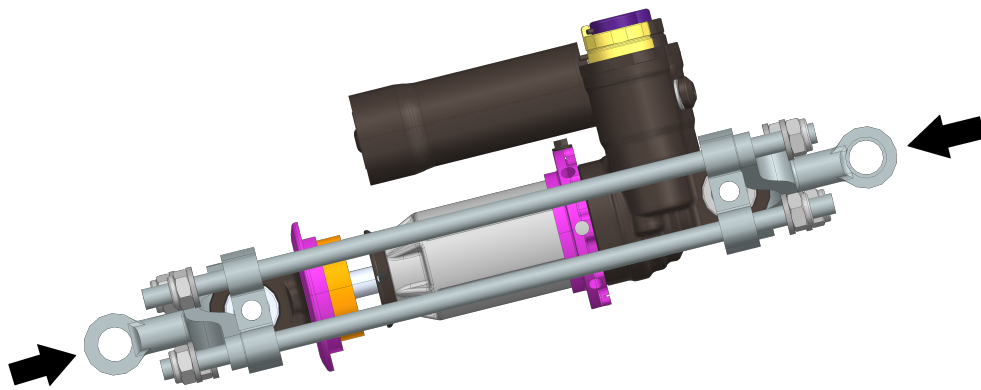
3.8 Motion-Ratio Framework of the Decoupled Architecture

The bellcrank layout of Section 3.7 routes the wheel motion into two distinct elastic paths, each mounted between the two bellcranks of an axle. This has a direct consequence on the kinematic description of the suspension. In a conventional four-corner arrangement, a single Motion Ratio (MR) per corner, defined as the wheel displacement divided by the spring displacement, is sufficient to characterise the elastic behaviour. In the decoupled architecture the relationship between wheel motion and element displacement is mode-dependent and asymmetric, and six motion ratios are used to describe it, of which four are independent. The anti-tension device does not participate in this description: it only enforces that the roll element is loaded in compression for either roll direction.

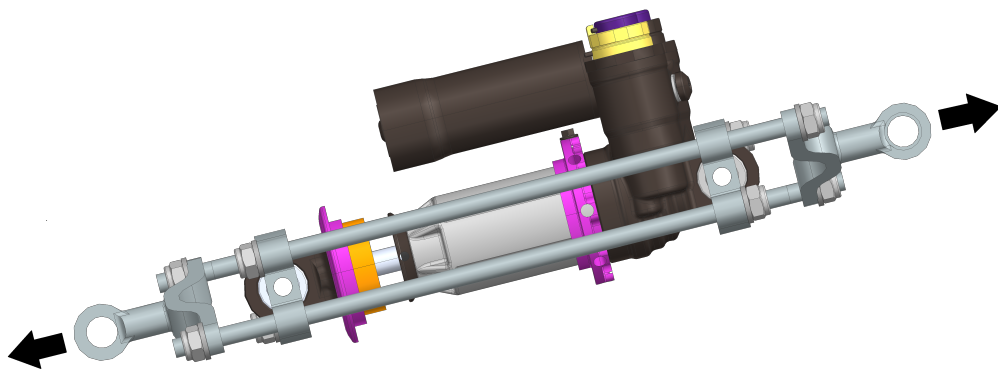
The four *individual* motion ratios relate each element to the jounce of a single wheel, the opposite wheel being held fixed. The bellcrank geometry does not guarantee that the



(a) Neutral position



(b) Compression stroke



(c) Extension stroke

Figure 3.19: Anti-tension device: the spring is compressed from the neutral position in both directions of relative bellcrank motion.

two sides are equivalent, so these ratios may take distinct values on the left and right; whether they do on the present design is established in Section 5.5:

- **Individual heave motion ratios** $\text{MR}_{h,L}$ and $\text{MR}_{h,R}$: jounce of the left and right wheel, respectively, per unit displacement of the heave element.
- **Individual roll motion ratios** $\text{MR}_{r,L}$ and $\text{MR}_{r,R}$: jounce of the left and right wheel, respectively, per unit displacement of the roll element.

The heave and roll motion ratios relate each element to the chassis motion it controls, and follow from the individual ones. Superposing the contributions of the two wheels gives the element displacement in each mode,

$$\delta_h = \frac{z_L}{\text{MR}_{h,L}} + \frac{z_R}{\text{MR}_{h,R}}, \quad \delta_r = \frac{z_L}{\text{MR}_{r,L}} - \frac{z_R}{\text{MR}_{r,R}}, \quad (3.1)$$

from which the heave motion ratio MR , defined for equal wheel displacements, and the roll motion ratio MR_r , defined for equal and opposite wheel displacements, are the harmonic means of the corresponding pair:

$$\text{MR} = \frac{2}{\frac{1}{\text{MR}_{h,L}} + \frac{1}{\text{MR}_{h,R}}}, \quad \text{MR}_r = \frac{2}{\frac{1}{\text{MR}_{r,L}} + \frac{1}{\text{MR}_{r,R}}}. \quad (3.2)$$

For a symmetric axle the two members of each pair are equal and the heave or roll ratio reduces to their common value.

The heave and roll motion ratios govern the corresponding wheel rates and, through them, the natural frequencies of the sprung mass. The individual motion ratios capture the left–right asymmetry of the bellcrank geometry and are required whenever a single wheel is excited in isolation, as in the load cases of the bellcrank mechanism. The heave motion ratio MR is that of the shared heave element, on which both wheels act jointly, so that the element displacement under symmetric travel z is $2z/\text{MR}$, and it is in this sense that it is carried through Chapter 4. The motion ratios are evaluated numerically, by imposing a finite vertical displacement $\pm\Delta z$ on the wheels in the heave or roll pattern and measuring the resulting element displacement. Because they vary along the suspension travel, any wheel rate derived from them is position-dependent rather than constant.

3.8.1 Design Implications of the Motion Ratios

Two consequences of this framework govern the bellcrank geometry.

The first is that the individual roll motion ratio is independent of the heave motion ratio. In a conventional suspension the motion ratio is designed to rise with upward wheel travel, stiffening the corner towards the end of its stroke, but in roll this couples the two wheels unfavourably, since the progression cannot be applied to the loaded side alone without disturbing the heave response. The decoupled architecture removes this

constraint, allowing the individual roll motion ratio to be shaped so that the loaded outer wheel becomes softer in roll, which reduces the jacking tendency discussed in Section 2.5.1 and improves mechanical grip on the wheel carrying the larger share of the lateral load.

The second is that the two elements impose opposite preferences on the pushrod position. The roll element benefits from a high motion ratio, since the anti-tension device doubles the usable stroke and a high roll motion ratio keeps the roll damper within its favourable velocity and displacement window. The heave element benefits from a lower motion ratio, since it sees the combined displacement of both wheels, and a high motion ratio would demand a disproportionately stiff and long-stroke heave spring. The pushrod position is therefore one of the central packaging trade-offs of the bellcrank design, and it is resolved with the stiffness targets of Chapter 4.

3.9 Chapter Conclusions

This chapter established the geometric and kinematic basis of the decoupled suspension. Starting from the design requirements and packaging constraints, the tyre data and the vehicle dynamic targets were defined, including the roll-centre heights justified through the load-transfer decomposition and the preliminary kinematic targets for camber, caster, and bump steer. From these, the outer hardpoints and the baseline suspension geometry were established.

The core of the chapter is the mode-decoupling architecture: the bellcrank layout that routes the wheel motion into separate heave and roll elements, the anti-tension device that keeps the roll element in compression, and the motion-ratio framework that describes the mechanism. Because the bellcranks are not necessarily symmetric, four independent motion ratios are needed, from which the heave and roll ratios follow, and together they set the pushrod geometry through two opposing requirements. The virtual-work relations linking the motion ratios to the wheel rates are derived in Section 4.3, where they are first used.

The baseline geometry and the motion-ratio framework provide the inputs for the elastic and damping definition of Chapter 4 and for the simulation environment of Chapter 5.

Chapter 4

Elastic and Damping Definition

This chapter translates the vehicle-level dynamic targets into ride rates, wheel rates, spring rates, motion-ratio bounds, anti-feature percentages and damping coefficients. It builds on the baseline geometry and the roll-heave architecture of Chapter 3, and two quantities introduced there as preliminary references are refined here: the anti-dive value adopted in Section 3.4 and the bellcrank architecture of Section 3.7. The elastic and damping sides are treated together because the damping cannot be defined without the vehicle models that the stiffness sizing also relies on, and because the case for the decoupled architecture rests on both at once.

4.1 Sizing Philosophy and Design Constraints

Conventional ride-rate sizing in race-car design starts from a target natural frequency, from which the wheel rate and the spring rate are derived [6, 7], an approach well established for vehicles with limited aerodynamic load but which does not by itself guarantee that the aerodynamic platform stays within its admissible operating window.

The present work therefore adopts an aerodynamic-driven sizing philosophy. The most restrictive physical constraint, the maximum chassis displacement before the aerodynamic package contacts the ground, is used as the primary design driver. The front and rear ride rates follow directly from it, and the remaining quantities are obtained either from the same constraint set or from other physical limits. The natural frequencies of the sprung mass are not imposed; they are computed afterwards, in Section 4.6.

4.1.1 Displacement Limits

Two distinct displacement limits govern the derivation, and the distinction between them runs through the whole chapter.

The first is a clearance limit. It follows from the static ride height of 37.5 mm defined in Chapter 3, reduced by a 7.5 mm margin for kerbs, surface irregularities and dynamic

overshoot, giving an admissible chassis travel of $\delta_{\text{clr}} = 30$ mm. This is an absolute bound, and it is the limit against which the braking event is verified in Section 4.4.1.

The second is an aerodynamic limit, and it is the one the elastic sizing is built on. Spending the whole clearance on the steady aerodynamic load would leave nothing for transient events, so half of it is allocated to the load carried in steady-state running and half is held in reserve for braking, kerbs and the superposition of roll,

$$\delta_{\text{aero}} = \frac{1}{2} \delta_{\text{clr}} = 15 \text{ mm.} \quad (4.1)$$

The split is a declared allocation and not a derived quantity. Deriving it would require the aerodynamic load and balance to be characterised continuously along the ride-height excursion rather than at the discrete attitudes available here, which is identified as future work.

4.1.2 Load Cases

Two aerodynamic conditions are carried through the chapter and must not be confused. The first is the design point, the attitude and speed at which the CFD evaluation was run and at which the ride rates of Section 4.2 are sized. The second is the end of a straight, the instant at which the vehicle is at its maximum speed with the platform fully descended and the brakes about to be applied, and it is the condition against which the clearance is verified in Section 4.4.1. Every quantity carrying the superscript *nom* belongs to the first and every quantity carrying *br* to the second.

With that distinction established, the constraints and load cases are collected in Table 4.1.

The reference velocity of 100 km/h is the expected speed at the end of the acceleration event, and is therefore the most demanding combination of aerodynamic load and braking deceleration. The total wheel travel of 55 mm is the regulatory 50 mm requirement (Section 2.7) with a 5 mm safety margin. The geometric anti-dive ceiling of 50 % is the maximum compatible with practical link inclinations on the prototype and avoids the documented drawbacks of excessive anti-dive discussed in Section 2.5.1.

The total aerodynamic downforce at a given speed is [6, 7]

$$F_L = \frac{1}{2} \rho V^2 A_{\text{ref}} |C_l|, \quad (4.2)$$

split between the axles as $F_{L,F} = \eta_F F_L$ and $F_{L,R} = (1 - \eta_F) F_L$. The longitudinal load transfer reacted by the springs is Equation 2.7 written for the sprung mass alone, which acts through the height of its own centre of gravity [6, 7],

$$\Delta W_x = \frac{m_s n_x g h_{s,\text{CG}}}{L}, \quad (4.3)$$

Table 4.1: Design constraints and load cases adopted for the elastic definition. The accelerations are the longitudinal and lateral components of the acceleration of the centre of mass, resolved in the vehicle axes and normalised by gravity, and V is the speed of the same point; nom denotes the design point and br the end-of-straight condition.

Parameter	Symbol	Value
Clearance displacement limit	δ_{clr}	30 mm
Aerodynamic displacement target	δ_{aero}	15 mm
Heave spring stroke	S	50 mm
Total wheel travel	w	55 mm
Maximum longitudinal deceleration	$n_{x,b}$	1.8 g
Maximum longitudinal acceleration	$n_{x,a}$	1.6 g
Design lateral acceleration	n_y	1.6 g
Reference vehicle speed	V	100 km/h
Lift coefficient at design point	C_l^{nom}	-3.34
Lift coefficient, end of straight	C_l^{br}	-3.72
Frontal reference area	A_{ref}	1.1 m ²
Air density	ρ	1.225 kg/m ³
Aero load fraction at design point	η_F^{nom}	46.3 %
Aero load fraction, end of straight	η_F^{br}	57.7 %
Rear brake-force fraction	β_R	45 %
Geometric anti-dive ceiling	AD_{max}	50 %

with $m_s = 227$ kg, $h_{s,\text{CG}} = 0.296$ m and $L = 1.530$ m from Table 3.1. The two conditions are both read from the straight-line aeromap and are listed in Table 4.2, and the full grid from which they are taken is reproduced in Appendix B. The roll sensitivity used in Section 4.5.2 comes from a separate skidpad evaluation at the static ride height, so its coefficients are not directly comparable with those of Table 4.2.

Table 4.2: Aerodynamic and inertial loads at the two conditions used in this chapter.

Quantity	Design point	End of straight
Lift coefficient $ C_l $	3.34	3.72
Front aero load fraction η_F	46.3 %	57.7 %
Total downforce at 100 km/h	1737.9 N	1935.0 N
Front-axle downforce $F_{L,F}$	804.8 N	1116.9 N
Rear-axle downforce $F_{L,R}$	933.1 N	818.1 N
<i>Longitudinal transfer through the springs</i>		
Braking, $n_{x,b} = 1.8$ g, $\Delta W_{x,b}$		775.6 N
Acceleration, $n_{x,a} = 1.6$ g, $\Delta W_{x,a}$		689.4 N

4.2 Ride Rates from the Aerodynamic Constraint

The ride rate of an axle relates the change in tyre normal force at the ground, per corner, to the vertical displacement of the sprung mass (Section 2.2.9). Requiring the chassis displacement under the design-point aerodynamic load to equal δ_{aero} gives

$$\text{RR}_F = \frac{F_{L,F}^{\text{nom}}}{2\delta_{\text{aero}}} = 26.8 \text{ N/mm}, \quad \text{RR}_R = \frac{F_{L,R}^{\text{nom}}}{2\delta_{\text{aero}}} = 31.1 \text{ N/mm}. \quad (4.4)$$

Both the magnitude and the distribution of the load come from a single CFD evaluation, so they correspond to the same operating condition. That evaluation was run at 81.5 km/h, and the loads of Table 4.2 follow from scaling its coefficients to the reference velocity through Equation 4.2.

The clearance verification uses a different condition. Its critical instant is the end of a straight, immediately before the brakes are applied: the vehicle is at its maximum speed, the platform has descended under the full aerodynamic load, and the braking weight transfer arrives before the speed has fallen appreciably. The coefficients are read from the straight-line aeromap at the lowest front and rear ride heights of the grid.

Beyond that instant the attitude pitches, which loads the front axle further, while the speed falls, which unloads it; the two effects oppose one another and their net evolution would require a coupled aerodynamic and vehicle simulation, so the end-of-straight condition is adopted as the representative instant.

Because the tyre vertical stiffness is finite, the wheel rates must compensate for the elastic series effect of the tyre. From Equation 2.5, with $K_t = 125.314 \text{ N/mm}$,

$$\text{WR} = \frac{\text{RR} K_t}{K_t - \text{RR}}, \quad (4.5)$$

which gives $\text{WR}_F = 34.1 \text{ N/mm}$ and $\text{WR}_R = 41.4 \text{ N/mm}$.

4.3 Heave Path Kinematics and Motion-Ratio Bounds

The bellcrank layout of Section 3.7 routes the symmetric component of the wheel motion through a single heave element mounted between the two bellcranks of each axle. Both wheels act on that element, so its displacement is proportional to the sum of the two wheel displacements, and the relation between wheel rate and spring rate differs from the conventional arrangement of one spring per corner.

The relation follows from virtual work applied to the motion ratios of Section 3.8. Equating the work of the wheel forces to that stored in the heave spring, with both wheels loading it simultaneously,

$$2F_w \delta_w = F_h \delta_h, \quad F_w = \text{WR} \delta_w, \quad F_h = K \delta_h, \quad (4.6)$$

which together with the kinematic relation $\delta_h = 2\delta_w/\text{MR}$ gives

$$\text{WR} = \frac{2K}{\text{MR}^2}, \quad \delta_h = \frac{2\delta_w}{\text{MR}}. \quad (4.7)$$

The factor of two relative to the conventional $\text{WR} = K/\text{MR}^2$ reflects the two wheels loading one element.

The roll path follows from the same argument applied to an antisymmetric input. With the two wheels displaced by $+z$ and $-z$, the roll element carries the difference of the two contributions and is displaced by $\delta_r = 2z/\text{MR}_r$, while the wheel forces form a couple and do work $2F_w z$, so that

$$2F_w z = F_r \delta_r, \quad F_w = \text{WR}_r z, \quad F_r = K_r \delta_r, \quad (4.8)$$

which gives the per-wheel roll-path wheel rate. The axle roll stiffness follows from it geometrically: the same pair of displacements corresponds to a body roll angle $\phi = 2z/t$ and the couple they produce is $F_w t$, so $K_\phi = F_w t/\phi$, and

$$\text{WR}_r = \frac{2K_r}{\text{MR}_r^2}, \quad K_\phi = \frac{1}{2} \text{WR}_r t^2, \quad (4.9)$$

with K_r the roll spring stiffness, MR_r the individual roll motion ratio and t the track width. These are the elastic input for the roll sizing of Section 4.5; the remainder of the present section concerns the heave path.

4.3.1 Kinematic Lower Bound

The total wheel travel w must be accommodated within the available spring stroke $S = 50$ mm, which gives

$$\text{MR} \geq \frac{2w}{S} = \frac{2 \times 55}{50} = 2.2. \quad (4.10)$$

The travel adopted is $w = 55$ mm: the regulations require 50 mm with at least 25 mm of jounce, and the additional 5 mm is carried as margin.

4.3.2 Static Position and the Adopted Motion Ratio

The travel available in each direction follows from where the element sits under the static load. At equilibrium the element carries $F_h = \text{MR} W_c$, with W_c the sprung corner weight, and with $K = \text{WRMR}^2/2$ the static compression is

$$\delta_h^{\text{static}} = \frac{F_h}{K} = \frac{2}{\text{MR}} \frac{W_c}{\text{WR}}. \quad (4.11)$$

The spring is installed at its free length with the damper fully extended, so this is also the extension travel available before it unseats. The design values give 13.7 mm at the front and 13.1 mm at the rear, leaving 36.3 mm and 36.9 mm of element travel in compression.

Referred to the wheel through $\delta_w = \delta_h \text{MR}/2$, the same figures are 15.1 mm of rebound and 39.9 mm of jounce at the front. The total wheel travel is therefore $S\text{MR}/2 = 55$ mm, so selecting the motion ratio is equivalent to selecting the total travel, and the rebound

share is the static wheel deflection W_c/WR , which does not depend on the motion ratio. The jounce share exceeds the minimum 25 mm the regulations demand.

There is no upper bound imposed, but raising the motion ratio is unfavourable on two counts: the spring rate required for a given wheel rate grows as MR^2 , and the element displacement, and with it the damper velocity, falls as $1/MR$. The lower bound is therefore adopted,

$$MR = 2.2, \quad (4.12)$$

this being the smallest motion ratio that accommodates the full wheel travel within the available stroke, and therefore the one that requires the lowest spring rate. The lower rate is favourable beyond the springs: it reduces the forces transmitted to the bellcrank pivots and to the chassis attachments, which eases the structural verification of Chapter 6.

4.4 Anti-Features

4.4.1 Anti-Dive from the Nose-Dive Constraint

With the front ride rate fixed by the aerodynamic constraint, the anti-dive percentage is no longer a free parameter: it follows from requiring that the front chassis displacement, under maximum braking with the end-of-straight aerodynamic load, does not exceed δ_{clr} .

The vertical force per front wheel is the irreducible aerodynamic contribution $F_{L,F}/2$ plus the residual mechanical contribution $(1 - AD)\Delta W_{x,b}/2$, the anti-dive geometry carrying a fraction AD of the longitudinal transfer through the links rather than through the springs. Imposing the constraint,

$$\frac{F_{L,F} + (1 - AD)\Delta W_{x,b}}{2} = \delta_{\text{clr}} RR_F, \quad AD = 1 - \frac{2\delta_{\text{clr}} RR_F - F_{L,F}}{\Delta W_{x,b}}, \quad (4.13)$$

which with the values of Tables 4.1 and 4.2 gives a requirement of 36.4 %.

That figure carries more uncertainty than its two decimal places suggest, and the sensitivity is worth stating before the geometry is compared against it. Differentiating Equation 4.13 with respect to the clearance limit gives $2RR_F/\Delta W_{x,b}$, or 6.9 percentage points of anti-dive per millimetre, and with respect to the aerodynamic load gives $1/\Delta W_{x,b}$, or 1.4 points per per cent of downforce. Table 4.3 evaluates both.

Table 4.3: Anti-dive requirement as a function of the clearance limit and of the end-of-straight lift coefficient.

δ_{clr} [mm]	28	29	30	31	32
AD required [%]	50.3	43.4	36.4	29.5	22.6
C_l^{br} relative to nominal	−5 %		nominal		+5 %
AD required [%]	29.3		36.4		43.6

The second row is the decisive one, because it is not a margin that was chosen. Five per cent is a modest allowance for the uncertainty of a single CFD evaluation, yet it moves the requirement by seven points in either direction. The requirement is therefore of the order of 36 %, known to something like ± 7 points.

The geometry implemented in Chapter 3 delivers

$$\boxed{\text{AD} = 32.8 \%}, \quad (4.14)$$

which leaves 28.5 N of the longitudinal transfer to be carried by the springs beyond what the requirement assumes, and therefore 0.5 mm of additional front displacement. The front axle descends 30.5 mm rather than 30.0 mm at the critical instant, against a static ride height of 37.5 mm: the margin against contact falls from 7.5 mm to 7.0 mm. The shortfall is a fraction of the uncertainty in the requirement itself, and the geometry is retained.

This value supersedes the preliminary 30 % adopted in Section 3.4 from literature benchmarks, and sits well within the geometric ceiling of 50 %, so the pitch-control authority is obtained without forcing the links towards the drawbacks of high anti-dive discussed in Section 2.5.1.

At the rear, under maximum acceleration with the design-point aerodynamic load and with no anti-squat geometry, the chassis displacement is

$$\delta_R = \frac{F_{L,R} + \Delta W_{x,a}}{2RR_R} = 26.1 \text{ mm}, \quad (4.15)$$

leaving 3.9 mm of margin. The superposition is deliberately conservative, since maximum acceleration occurs at low speed where the aerodynamic load is a fraction of its design-point value, and the two do not in fact coincide. The bump-side constraint at the rear is therefore not binding, in contrast to the front nose-dive constraint. The rebound side requires a separate treatment.

4.4.2 Anti-Lift and Anti-Squat from the Rebound Constraint

The anti-dive percentage was driven by a ground-clearance limit. The rear requirements are of a different nature: they concern the integrity of the spring-damper assembly.

The governing condition is that the heave spring remain in compression throughout the operating envelope. If dynamic extension reduces the compression to zero the spring leaves its seat and drops out of the load path, and an impulsive reaction is transmitted to the chassis when contact is recovered. A minimum residual compression of 5 mm is imposed at the worst rebound event of each axle, which is ten per cent of the spring stroke.

The worst rebound event at the rear is the mechanical-only component of maximum braking at low speed, where the aerodynamic load no longer compensates the longitudinal transfer. The element extension without anti-lift is $\Delta W_{x,b}/(WR_R MR) = 8.5$ mm, which against the static compression of 13.1 mm leaves only 4.6 mm. Imposing the constraint,

$$\delta_{s,R}^{\text{static}} - \frac{(1 - \text{AL}) \Delta W_{x,b}}{\text{WR}_R \text{MR}} \geq 5 \text{ mm}, \quad \boxed{\text{AL} \geq 4.6 \%}. \quad (4.16)$$

This is a small percentage, well below typical Formula Student values, and is achievable through a modest longitudinal inclination of the rear links.

In acceleration the rear axle works on the bump side. The chassis displacement of Equation 4.15 corresponds to 19.6 mm of suspension travel once the tyre deflection is removed, hence an element compression of $13.1 + 17.8 = 30.9$ mm, leaving 19.1 mm against the 50 mm stroke. Both margins are positive throughout the event, so no minimum anti-squat follows from the bump side,

$$\boxed{\text{AS}_{\min} = 0 \%}. \quad (4.17)$$

and the anti-squat is left to follow from the geometry that the anti-lift requirement fixes.

The two rear percentages are derived from independent constraints but are not independent design parameters, because a single side-view instant centre produces both: the braking force acts at the contact patch and the traction force at the wheel centre, so the same point gives one lever arm for anti-lift and a shorter one for anti-squat (Section 2.5.1). Of the two requirements it is the rebound constraint that binds, and the anti-squat is whatever the instant centre that satisfies it happens to produce. On the geometry implemented in Chapter 3 that instant centre lies marginally below the wheel-centre height, so the longer lever arm measured from the contact patch is positive while the shorter one measured from the wheel centre is not, and the axle returns $\text{AS} = -0.64 \%$. The consequence for the constraint above is 0.05 mm of additional element compression against the 19.1 mm of margin, so a value of that size is of no practical significance.

4.5 Spring Rates

4.5.1 Heave Springs

With the wheel rates fixed and the motion ratio adopted, the heave spring stiffness at each axle follows from Equation 4.7 as

$$K = \frac{\text{WRMR}^2}{2}, \quad (4.18)$$

giving $K_F = 83 \text{ N/mm}$ and $K_R = 100 \text{ N/mm}$.

4.5.1.1 Heave Spring Design

Because no catalogue spring meets the required rates, the two heave springs were designed as helical compression springs of round wire. For a spring of wire diameter d , mean coil diameter D_m and n_a active coils, the rate and the corrected shear stress are [28]

$$K = \frac{G d^4}{8 D_m^3 n_a}, \quad \tau = K_W \frac{8 F D_m}{\pi d^3}, \quad K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}, \quad (4.19)$$

with $C = D_m/d$ the spring index, K_W the Wahl correction and $G = 79.3$ GPa the shear modulus of the spring steel. The mean coil diameter is fixed by the damper body the spring surrounds, so the rate is set through the wire diameter and the number of active coils, the latter in half-coil increments.

The resulting designs are given in Table 4.4. The rates obtained are within 1.5 % of the values required by Equation 4.18, which is the resolution the half-coil increment allows, and the elastic quantities of this chapter are quoted at the required values.

Table 4.4: Designed heave springs.

	Front	Rear
Wire diameter d [mm]	8.0	8.0
Mean coil diameter D_m [mm]	43.5	43.5
Spring index C [-]	5.4	5.4
Active coils n_a [-]	6.0	5.0
Total coils n_t [-]	7.5	6.5
Rate K [N/mm]	82.2	98.7
Rate required [N/mm]	82.6	100.1
Solid length [mm]	60.0	52.0

4.5.2 Roll Gradient Target from the Skidpad Aeromap

The roll target combines two inputs taken from different conditions. The sensitivity of downforce to body roll comes from the CFD aeromap, which was evaluated at the steady-state skidpad condition,

$$a_y = \frac{v^2}{R} = \frac{11.61^2}{9.125} = 14.77 \text{ m/s}^2 = 1.51g. \quad (4.20)$$

The acceleration at which the resulting roll limit is applied is the design value of 1.6 g from Table 4.1, which is the highest lateral acceleration the vehicle is designed to sustain. The two differ, and the roll angle must be bounded at the design limit rather than at the condition the map happens to have been computed for.

The variation of downforce with body roll is listed in Table 4.5.

Table 4.5: Relative downforce variation with roll angle at the skidpad operating point, from the CFD aeromap.

Roll angle [°]	0	0.5	1.0	1.5	2.0	2.5
Downforce change [%]	0	+1.5	-7.1	-7.7	-11.1	-10.4

The limit is set from the shape of the table rather than from a declared percentage. The degradation is not uniform: it falls by 8.6 points between 0.5° and 1.0°, by only 0.6 further to 1.5°, and then by 3.4 more to 2.0°. The map therefore presents a plateau between 1.0° and 1.5° bounded by steep degradation on either side, and there is no reason

to place the limit inside a plateau: the roll angle is taken at its upper end, $\phi_{\max} = 1.5^\circ$, where the degradation is 7.7 %.

In the linear model of Section 4.5.3 the roll angle is proportional to the lateral acceleration, as Equation 4.23 shows, so the roll gradient is a constant and not a local slope. The limit adopted above therefore fixes it directly, by dividing the admissible roll angle by the acceleration at which it is not to be exceeded,

$$\boxed{\frac{d\phi}{dn_y} = \frac{\phi_{\max}}{n_y} = \frac{1.5^\circ}{1.6g} = 0.94 \text{ deg/g}}. \quad (4.21)$$

4.5.3 Required Total Roll Stiffness and its Distribution

The roll moment per unit of lateral acceleration is produced by the sprung weight acting through the roll moment arm Δh of Section 2.2.11, the distance from the sprung-mass centre of gravity to the roll axis. With $W_s = 2227 \text{ N}$ and $\Delta h = 202.8 \text{ mm}$,

$$M_\phi = W_s \Delta h = 451.6 \text{ Nm/g}. \quad (4.22)$$

The roll moment does not stay constant as the body rolls. Once the sprung mass has rotated by ϕ , its own weight acts through the horizontal offset of the centre of gravity from the roll axis and adds a moment in the direction of the roll, so that for small angles the equilibrium of the sprung mass reads

$$K_\phi \phi = M_\phi n_y + M_\phi \phi, \quad \frac{d\phi}{dn_y} = \frac{M_\phi}{K_\phi - M_\phi}, \quad (4.23)$$

the second term on the left being the destabilising jacking contribution, which is retained here rather than neglected [6]. Solving for the stiffness required to meet the target, with the gradient expressed in radians per g,

$$K_{\phi, \text{total}} = M_\phi \left(1 + \frac{1}{d\phi/dn_y} \right) = 28050 \text{ Nm/rad} = \boxed{489.6 \text{ Nm/deg}}, \quad (4.24)$$

measured at the ground, that is with the tyre in series with the springs. The tyre contributes a vertical stiffness at each contact patch and not a torsional one, but the two wheels of an axle are separated by the track, so that stiffness reaches the roll mode through the same geometric conversion as Equation 4.9: a roll angle ϕ displaces the two contact patches by $\pm t\phi/2$ and the resulting pair of vertical forces forms a couple, giving an axle roll stiffness of the tyres

$$K_{\phi, \text{tyre}} = \frac{1}{2} K_t t^2 = 1575 \text{ Nm/deg} \quad (4.25)$$

per axle, which is what acts in series with the mechanical roll stiffness of the springs.

The front-to-rear distribution governs the lateral load transfer distribution and, through it, the steady-state balance. As established in Section 2.2.11, the total lateral load transfer of each axle is the sum of an unsprung, a geometric and an elastic contribution; the first two are fixed by the masses and the roll-centre heights, and the elastic part is distributed between the axles in proportion to their roll stiffness.

A balanced 50/50 distribution is adopted as the baseline. Splitting Equation 4.24 equally gives 244.8 Nm/deg per axle at the ground, and removing the tyre roll stiffness of Equation 4.25 from the series combination yields the mechanical stiffness each pair of springs must provide,

$$K_{\phi,F}^{\text{mec}} = K_{\phi,R}^{\text{mec}} = 289.9 \text{ Nm/deg.} \quad (4.26)$$

Evaluating the load-transfer decomposition for this distribution gives $\text{LLTD}_F = 48 \%$, a marginally rear-biased balance, retained as the starting point with the deliberate possibility of biasing it forward through the front-to-rear ratio of the roll spring rates.

4.5.4 Roll Spring and Motion-Ratio Selection

The mechanical axle roll stiffness is delivered by the roll spring rate acting through the roll motion ratio. Combining Equation 4.9 gives $K_\phi = K_r t^2 / \text{MR}_r^2$, which solved for the motion ratio maps each available spring to a roll motion ratio,

$$\text{MR}_r = t \sqrt{\frac{K_r}{K_\phi^{\text{mec}}}}. \quad (4.27)$$

admissible roll motion ratio is bounded on two sides. Both stiffnesses enter in consistent units: K_r in N/mm and K_ϕ^{mec} converted to N mm/rad, so that the ratio is dimensionless and MR_r follows from the track in millimetres. The upper side is set by the range of spring rates the supplier offers; the lower side by the ± 25 mm the element has in each direction, since it sits at mid-stroke — the anti-tension device of Section 3.7 compresses the spring for either direction of roll — and the element travel is $2\delta_w / \text{MR}_r$, so a lower motion ratio demands a longer stroke for the same roll angle. The resulting window is given in Table 4.6.

Table 4.6: Roll motion ratio and stroke limit for the available roll spring rates, 50/50 distribution.

K_r [N/mm]	20	25	30	35	40	45	50
MR_r [-]	1.32	1.47	1.61	1.74	1.86	1.97	2.08
Element travel @ 1.5° [mm]	23.8	21.3	19.5	18.0	16.9	15.9	15.1
Roll angle at ± 25 mm [°]	1.6	1.8	1.9	2.1	2.2	2.4	2.5

A roll spring rate of 50 N/mm is selected for both axles, placing the roll motion ratio at $\text{MR}_r = 2.08$. This is the highest motion ratio attainable with the available spring range while meeting the roll gradient target, and it is preferred on packaging grounds: it eases

the routing of the roll element within the bellcrank assembly, keeps the roll damper in a favourable velocity range, and brings the roll motion ratio closer to that of the heave path. The roll element then uses 15.1 mm of its travel at the aerodynamically limited roll angle.

The balance can be biased forward within the same bellcrank geometry by softening the rear spring, since the front already uses the stiffest available. Table 4.7 gives the effect; the baseline 50/50 is retained, with the other two options available should testing call for a more understeer-biased balance. The selected roll stiffness configuration is presented in Table 4.8.

Table 4.7: Effect of the front/rear roll spring choice on the front lateral load transfer distribution.

Roll springs F/R [N/mm]	Front roll stiffness share [%]	LLTD _F [%]
50 / 50	50.0	48.0
50 / 45	52.6	49.7
50 / 40	55.6	51.7

Table 4.8: Selected roll stiffness configuration.

Parameter	Front	Rear
Roll spring rate, K_r [N/mm]	50	50
Roll motion ratio, MR_r [-]	2.08	2.08
Mechanical axle roll stiffness, K_ϕ^{mec} [Nm/deg]	289.9	289.9
Roll stiffness distribution [%]	50	50
Resulting roll gradient [deg/g]	0.94	
Front lateral load transfer distribution [%]	48	

4.5.5 Sensitivity to the Roll Gradient Target

The roll gradient is the one elastic target in this chapter whose supporting criterion carries a discretionary margin, and the whole roll-side sizing rests on it. Table 4.9 re-evaluates the derivation chain for a range of gradients, every column recomputed from the same equations.

The table reads from left to right and its columns are of two kinds. The gradient in the first column fixes the roll angle at the design acceleration in the second, and from that angle the table branches. One branch is aerodynamic and empirical: the downforce change in the third column is read from the skidpad aeromap of Table 4.5 at that angle, by linear interpolation between its tabulated points. The other branch is mechanical and analytical: the roll stiffness in the fourth column follows from the same angle through Equation 4.24, the mechanical stiffness of the springs from Equation 4.25 once the tyre is removed from the series combination, and the roll motion ratio from Equation 4.27; together they are what the target costs. The downforce and the stiffness are therefore two independent consequences of one roll angle and not of one another: no force in the third column acts on the stiffness in the fourth, and the table places them side by side only so that the exchange between them can be read on a single line.

Table 4.9: Roll-side sizing re-evaluated for a range of roll gradient targets. The last column is the fraction of the mechanical roll stiffness that survives the series connection with the tyre.

$d\phi/dn_y$ [deg/g]	ϕ at 1.6 g	Δ down- force [%]	K_ϕ [Nm/deg]	K_ϕ^{mec} [Nm/deg]	MR_r [-]	f_ϕ [Hz]	Series yield
0.4	0.64°	−0.9	1137.0	889.7	1.19	8.03	64 %
0.5	0.80°	−3.7	911.2	641.1	1.40	7.33	71 %
0.6	0.96°	−6.4	760.6	501.4	1.58	6.77	76 %
0.8	1.28°	−7.4	572.4	349.8	1.90	5.94	82 %
0.938	1.50°	−7.7	489.6	289.9	2.08	5.51	84 %
1.0	1.60°	−8.4	459.5	269.0	2.16	5.35	85 %
1.2	1.92°	−10.6	384.3	218.8	2.40	4.90	88 %

Moving from 0.938 to 0.6 deg/g recovers 1.3 percentage points of downforce for a 73 % increase in the mechanical stiffness the springs must deliver, and moving to 0.5 deg/g recovers 4.0 points for a 121 % increase. The tyre is the reason: because the mechanical roll stiffness acts in series with the tyre roll stiffness of 1575 Nm/deg per axle, the fraction that reaches the vehicle falls as the target is tightened, from 84 % at the adopted gradient to 71 % at 0.5 deg/g and 64 % at 0.4 deg/g. Beyond that point a growing share of any additional spring stiffness is absorbed by tyre compliance and never contributes to controlling the platform, and the attainable roll damping ratio falls with it.

The tabulated downforce change of Table 4.5 is not monotonic across the sampled range: it rises at 0.5° before falling, and recovers slightly between 2.0° and 2.5°. Whether these features are physical or reflect the resolution of the map has not been established, and the choice of 1.5° rests on the flat interval between 1.0° and 1.5° being real. That interval is worth 0.6 percentage points of downforce and 73 % of the mechanical roll stiffness the springs must deliver, so if the plateau proved to be an artefact the target would have to be revisited at 1.0°. Refining the aeromap over that interval is therefore the single most useful addition to the data supporting this section. The target of Equation 4.21 is retained.

4.6 Vehicle Models

The elastic definition is complete, but the damping cannot be established from it directly: the dynamics of the unsprung mass and the modal behaviour of the sprung mass have to be represented first. Two models are used, the simplest that supports each question.

4.6.1 Quarter-Car Model

The quarter-car model of Figure 4.1 is the simplest that represents the heave behaviour of a single corner with the tyre as a separate degree of freedom, and is the standard two-degree-of-freedom idealisation of ride analysis [6, 29]. It comprises a corner sprung mass $m_{s,c}$ supported on the wheel-referred rate WR and damping c_w , and a corner unsprung

mass $m_{u,c}$ connected to the ground through the tyre vertical stiffness K_t , the tyre damping being neglected.

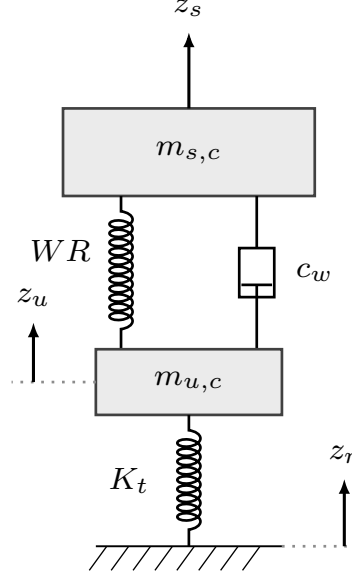


Figure 4.1: Two-DOF quarter-car model. The wheel-referred rate WR and damping coefficient c_w represent the suspension seen at the wheel, and the tyre is modelled as an undamped vertical spring K_t .

With z_s and z_u the sprung and unsprung displacements, the free-vibration equations are

$$\begin{aligned} m_{s,c} \ddot{z}_s &= -WR(z_s - z_u) - c_w(\dot{z}_s - \dot{z}_u), \\ m_{u,c} \ddot{z}_u &= WR(z_s - z_u) + c_w(\dot{z}_s - \dot{z}_u) - K_t z_u, \end{aligned} \quad (4.28)$$

which in matrix form are defined by

$$\mathbf{M} = \begin{bmatrix} m_{s,c} & 0 \\ 0 & m_{u,c} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_w & -c_w \\ -c_w & c_w \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} WR & -WR \\ -WR & WR + K_t \end{bmatrix}. \quad (4.29)$$

With the front-axle corner masses from Chapter 3, $m_{s,c} = 52.55$ kg and $m_{u,c} = 12.15$ kg, the front wheel rate and the tyre stiffness, the model returns a ride frequency of 3.58 Hz and a wheel-hop frequency of 18.33 Hz. The ride frequency recovers the value obtained from the ride rate directly, since the series combination of WR_F and K_t is RR_F ; the wheel-hop frequency is a new output.

For a multi-degree-of-freedom system the damping ratio of each mode is not a single scalar, because the damping matrix couples the modes. An estimate is obtained by projecting the system onto its modal coordinates and neglecting the off-diagonal terms of the modal damping matrix [30],

$$\xi_i = \frac{C_{ii}^*}{2\sqrt{M_{ii}^*K_{ii}^*}}, \quad \mathbf{M}^* = \mathbf{U}^\top \mathbf{M} \mathbf{U}, \quad \mathbf{C}^* = \mathbf{U}^\top \mathbf{C} \mathbf{U}, \quad \mathbf{K}^* = \mathbf{U}^\top \mathbf{K} \mathbf{U}, \quad (4.30)$$

with $\mathbf{U}$ the mass-normalised modal matrix. Coarser still is the single-degree-of-freedom idealisation, which discards the unsprung mass and places the sprung mass directly on the ride rate, the rate that supports it at the ground, giving $\xi = c_w/c_c$ with

$$c_c = 2\sqrt{RRm_{s,c}}. \quad (4.31)$$

This is the form in which damping ratios are quoted in the literature, and at the conventional compromise $\xi = 0.7$ [6] it returns $c_w \approx 1.66$ kNs/m for the front corner.

That figure must be treated with care. Discarding the unsprung mass is a poor approximation here: with $K_t/WR_F = 3.7$ a significant share of the suspension deflection is taken up by the tyre, and the damper sees correspondingly less relative velocity than the idealisation assumes. Solving the two-DOF system exactly with $c_w = 1.66$ kNs/m returns a ride damping ratio of 0.42 and a wheel-hop ratio of 0.65, not the 0.70 requested. The single-degree-of-freedom estimate is therefore retained only as an order of magnitude.

4.6.2 Seven-Degree-of-Freedom Model

The benefit of the decoupled architecture acts on the roll and pitch modes of the sprung mass, so a model of the complete vehicle is required; the seven-degree-of-freedom form adopted here, and shown in Figure 4.2, is the usual one for this purpose [27, 29]. It comprises a sprung mass with three degrees of freedom, heave z , pitch θ and roll ϕ , together with four unsprung masses with one vertical degree of freedom each,

$$\mathbf{x} = [z, \theta, \phi, z_{u,FL}, z_{u,FR}, z_{u,RL}, z_{u,RR}]^\top. \quad (4.32)$$

Heave is positive upwards, pitch positive nose-down and roll positive left-side-up. The inertial properties are the sprung mass, the pitch inertia I_p and the roll inertia I_{yy} of the sprung mass, listed in Table 3.2. In the vehicle coordinate system adopted here the X axis is transverse and the Y axis longitudinal, so pitch is a rotation about X and roll a rotation about Y . The pitch inertia was extracted from the Computer-Aided Design (CAD) assembly in Siemens NX from the centroidal radius of gyration $k_x = 803.5$ mm,

$$I_p = m_s k_x^2 = 146.6 \text{ kg} \cdot \text{m}^2. \quad (4.33)$$

Writing Newton's equations for the sprung mass and for each unsprung mass,

$$\begin{aligned} m_s \ddot{z} &= -(F_F + F_R), \\ I_p \ddot{\theta} &= a F_F - b F_R, & m_{u,c} \ddot{z}_{u,i} &= K_t (z_{r,i} - z_{u,i}) + F_{u,i}, \\ I_{yy} \ddot{\phi} &= -(\Gamma_F + \Gamma_R), \end{aligned} \quad (4.34)$$

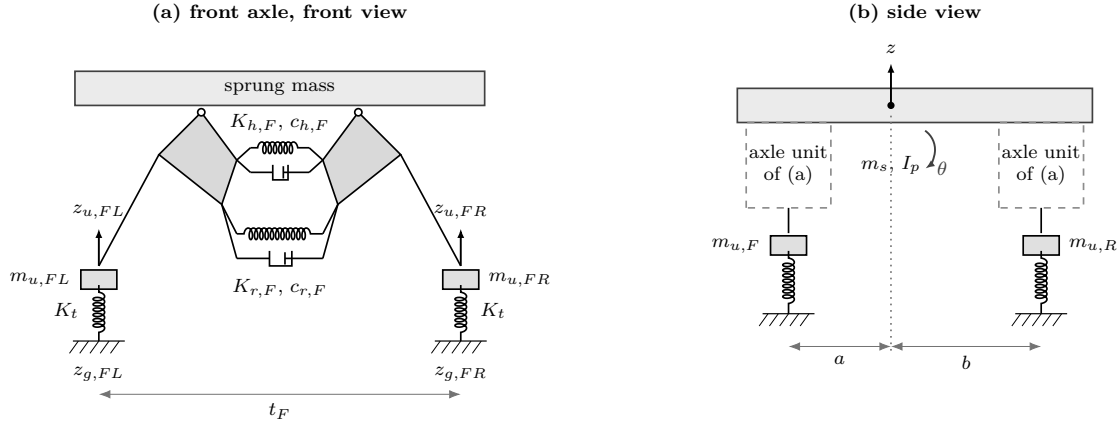


Figure 4.2: Seven-DOF model of the roll-heave decoupled suspension. Each axle carries a heave element, acted on by both wheels jointly, and a roll element, acted on by their difference; the rear axle is identical in form to the front axle shown in (a).

for $i \in \{FL, FR, RL, RR\}$, where

F_F, F_R	total heave force of the front and rear axle, both wheels included
Γ_F, Γ_R	roll moment of the front and rear axle
$F_{u,i}$	force the suspension applies to the unsprung mass at corner i
$z_{r,i}$	road vertical displacement at corner i
$z_{w,F}, z_{w,R}$	heave deformation of each axle, chassis relative to wheels
$\phi_{w,F}, \phi_{w,R}$	roll deformation of each axle
a, b	longitudinal distances from the sprung-mass centre of gravity to the front and rear axles, 0.822 m and 0.708 m
t_F, t_R	front and rear track widths.

The axle loads follow from the deformations and from the elastic and damping properties of each path,

$$\begin{aligned} F_F &= 2(WR_F z_{w,F} + c_{h,F} \dot{z}_{w,F}), & \Gamma_F &= K_{\phi,F} \phi_{w,F} + c_{\phi,F} \dot{\phi}_{w,F}, \\ F_R &= 2(WR_R z_{w,R} + c_{h,R} \dot{z}_{w,R}), & \Gamma_R &= K_{\phi,R} \phi_{w,R} + c_{\phi,R} \dot{\phi}_{w,R}, \end{aligned} \quad (4.35)$$

the factor of two in the heave terms accounting for both wheels of an axle deflecting by z_w , while the roll stiffness is already an axle-level quantity and needs none. Each corner receives half of the axle heave force and the roll moment divided by the track,

$$F_{u,FL} = \frac{1}{2}F_F + \frac{\Gamma_F}{t_F}, \quad F_{u,FR} = \frac{1}{2}F_F - \frac{\Gamma_F}{t_F}, \quad (4.36)$$

and analogously at the rear. The axle deformations, after linearisation under the small-angle assumption, are

$$\begin{aligned}
z_{w,F} &= z - a\theta - \frac{1}{2}(z_{u,FL} + z_{u,FR}), & \phi_{w,F} &= \phi - \frac{1}{t_F}(z_{u,FL} - z_{u,FR}), \\
z_{w,R} &= z + b\theta - \frac{1}{2}(z_{u,RL} + z_{u,RR}), & \phi_{w,R} &= \phi - \frac{1}{t_R}(z_{u,RL} - z_{u,RR}).
\end{aligned} \tag{4.37}$$

Collecting the coefficients yields the mass, damping and stiffness matrices of the system; their assembly is reported in Appendix C.

4.6.3 Modal Behaviour and the Decoupling Argument

The modal analysis returns three low-frequency modes of the sprung mass and four wheel-hop modes, collected in Table 4.10. The modes are identified by shape and not by their order in the spectrum: the mass-normalised mode shapes are compared after scaling the rotations by a characteristic length, a for pitch and $t/2$ for roll, so that all participations are expressed in metres, and the largest participation names the mode. Where that participation belongs to an unsprung coordinate, the mode is a wheel-hop mode and the phasing of the four corners is matched against the in-phase, roll-like, pitch-like and warp-like patterns to complete the name.

Table 4.10: Natural frequencies of the seven-DOF model at the adopted elastic configuration.

Mode	Character	f_n [Hz]
Pitch	sprung, symmetric, pitch-dominated	3.40
Heave	sprung, symmetric, heave-dominated	3.57
Roll	sprung, antisymmetric	5.51
Wheel hop, warp-like	unsprung, antisymmetric front-to-rear	17.59
Wheel hop, roll-like	unsprung, antisymmetric left-to-right	17.77
Wheel hop, pitch-like	unsprung, symmetric front-to-rear	18.33
Wheel hop, in-phase	unsprung, symmetric	18.75

Both sprung frequencies fall within the typical Formula Student band [1, 6], and they lie close to one another because the ride rates happen to be distributed in almost exactly the ratio b/a that decouples heave from pitch: the design gives $RR_F/RR_R = 0.863$ against $b/a = 0.862$, and a coupling term $RR_Rb - RR_Fa$ of -0.005 N/mm. That is a property of the design point, since the aerodynamic load distribution that sets the ride rates migrates as the platform descends, and it is reported as an outcome and was not adopted as an objective.

The roll mode sits 1.9 Hz above the other two because the roll inertia of the sprung mass is small relative to the roll stiffness the aerodynamic platform requires. The four wheel-hop modes lie between 17.6 and 18.8 Hz, a separation from the sprung modes of about five, which is the quantity that governs the attainable damping ratios in Section 4.7.

The decisive observation is structural rather than numerical, and follows from the form of $\mathbf{K}$ and $\mathbf{C}$. The heave elements populate the z and θ coordinates and the roll elements populate ϕ , with no entry common to both: the two elastic paths occupy disjoint blocks. A change to one path therefore leaves the frequencies and the damping ratios of

the other unaltered, which is the decoupling property stated qualitatively in Chapter 3 now expressed in the governing equations.

The model is linear: it assumes small angles, constant motion ratios, a fixed roll centre and a linear tyre, and does not represent the coupling between vertical, longitudinal and lateral dynamics. These assumptions are appropriate for setting the modal frequency and damping targets and for demonstrating the decoupling property. The first two of them are lifted by the kinematic verification of Chapter 5, where the motion ratios of Section 3.8 are measured along the travel and the roll centre is allowed to migrate; the small-angle and linear-tyre assumptions, and the absence of cross-axis coupling, remain limitations of the present treatment.

4.7 Damping Definition

The decoupled architecture makes four damping coefficients available, $c_{h,F}$, $c_{h,R}$, $c_{\phi,F}$ and $c_{\phi,R}$, against the two of a conventional four-corner layout. Two of them set the heave and pitch damping ratios, and the remaining two set the level of roll damping together with its front-to-rear distribution. The latter is the transient counterpart of the roll stiffness distribution: where the stiffness split governs the steady-state balance, the damping split governs the balance during the build-up of lateral acceleration.

4.7.1 Attainable Damping Ratios

The single-degree-of-freedom relation of Section 4.6.1 is monotonic in the damping coefficient and therefore implies that any damping ratio can be reached by making the damper stiff enough. That implication does not survive the introduction of the tyre as a separate degree of freedom. As the coefficient grows the damper progressively locks the unsprung mass to the sprung mass, and in the limit the vehicle oscillates as a single body on the undamped tyre springs, so the damping ratio of the sprung modes rises, reaches a ceiling, and falls again, as Figure 4.3 shows.

Across the sweep the modes are followed by continuation from the undamped system, each eigenvalue being matched to its nearest neighbour as the damping is introduced gradually, so that a mode is tracked by its shape and not by its position in the spectrum. This is what keeps the curves of Figure 4.3 continuous once the wheel modes become overdamped and the eigenvalue ordering changes.

The ceilings are $\xi_h = 0.52$ at $c_{h,F} \approx 2.16$ kNs/m for the heave mode and $\xi_p = 0.490$ at $c_{h,F} \approx 2.20$ kNs/m for the pitch mode, so the conventional target of 0.7 [6] is not attainable on this vehicle for any choice of damper. The reason is the limited separation between the sprung and unsprung frequencies. The ride frequencies imposed by the aerodynamic constraint are high, at 3.57 and 3.40 Hz, while the wheel-hop frequency sits near 18 Hz, giving a ratio close to 5.1 against the 8 to 12 typical of road vehicles [6]. A single damper

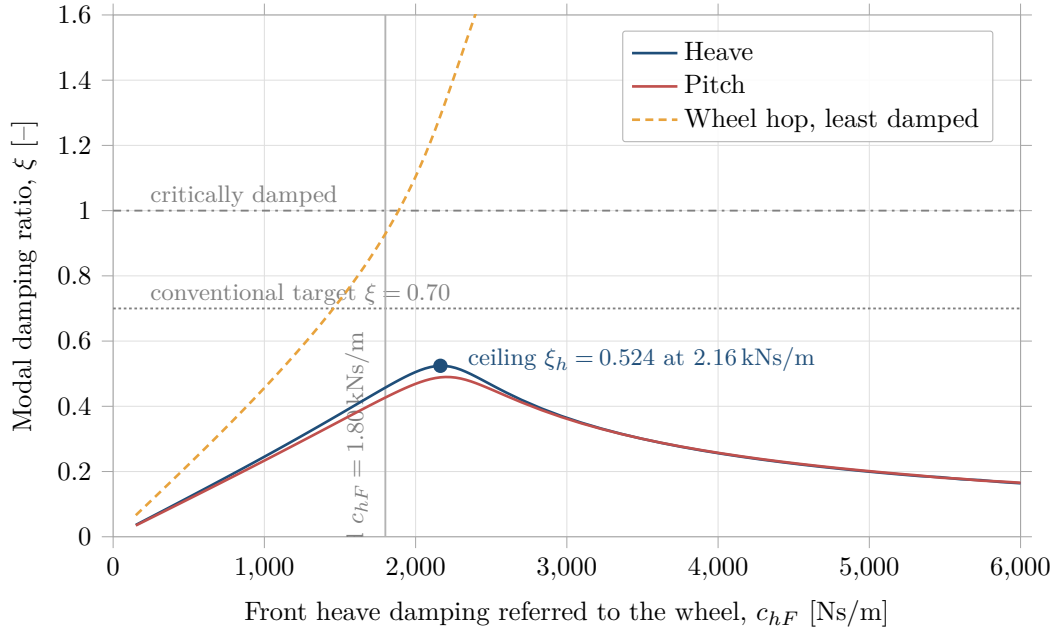


Figure 4.3: Modal damping ratios of the seven-DOF model as a function of the front heave damping coefficient referred to the wheel, with $c_{h,R}$ held in proportion to the ride rates. The sprung ratios reach a ceiling well below the conventional target, while the wheel-hop modes reach the critically damped condition before that ceiling is attained.

cannot serve the two frequency ranges independently, and the compromise between sprung-mass control and wheel control becomes binding at a much lower damping level than the quarter-car estimate suggests. This follows directly from the aerodynamic-driven sizing philosophy, and is a design constraint of this vehicle and not a deficiency of the concept.

4.7.2 Nominal Coefficients

The wheel modes provide the bound that the sprung modes cannot. Damping is raised until the least damped of the four wheel-hop modes reaches the critically damped condition, on the grounds that beyond it the wheel no longer oscillates but also no longer follows the road, so the contact-patch load varies more while the damper contributes nothing further to body control [6, 7]. That bound falls at $c_{h,F} = 1.89$ kNs/m referred to the wheel, where the heave damping ratio would be 0.48.

The design point must sit clear of it by at least the resolution of the hardware. The selected damper is adjusted in 19 discrete positions per circuit, and across that range the force at 0.25 m/s varies from 1600 to 4960 Ns/m at the element, so one position is worth some 187 Ns/m there and 77 Ns/m referred to the wheel. A round value of 1.80 kNs/m is therefore adopted, 1.2 positions below the bound, at the cost of a heave damping ratio of 0.46 instead of 0.48.

The rear coefficient is set in the same ratio as the ride rates, $c_{h,R}/c_{h,F} = \text{RR}_R/\text{RR}_F$, which is the condition for both axles to carry the same damping ratio. The roll coefficients

are then chosen to place the roll damping ratio at the level of the heave ratio, so that no mode of the sprung mass is markedly better damped than another, and split evenly between the axles in line with the 50/50 roll stiffness distribution.

Table 4.11: Nominal damping coefficients of the decoupled suspension.

Parameter	Symbol	Value
Front heave damping, per wheel, at the wheel	$c_{h,F}$	1.80 kNs/m
Rear heave damping, per wheel, at the wheel	$c_{h,R}$	2.09 kNs/m
Front axle roll damping	$c_{\phi,F}$	499 Nms/rad
Rear axle roll damping	$c_{\phi,R}$	499 Nms/rad
Front heave element coefficient	–	4.36 kNs/m
Rear heave element coefficient	–	5.05 kNs/m
Roll element coefficient	–	1.50 kNs/m

The damping ratios these coefficients produce on the complete model are listed in Table 4.12, alongside the projected values of Equation 4.30. The projection overestimates the sprung ratios by between three and eight per cent and the wheel-hop ratios by up to fifteen, which measures the modal damping coupling it neglects and is the reason the exact formulation is used throughout.

Table 4.12: Modal damping ratios at the nominal operating point.

Mode	f_n [Hz]	ξ exact	ξ projected
Pitch	3.40	0.43	0.42
Heave	3.57	0.46	0.43
Roll	5.51	0.46	0.42
Wheel hop, warp-like	17.59	0.29	0.26
Wheel hop, roll-like	17.77	0.33	0.32
Wheel hop, pitch-like	18.33	0.81	0.70
Wheel hop, in-phase	18.75	0.93	0.79

These ratios are lower than the figures usually quoted for this class of vehicle, and the difference is one of definition and not of hardware. Published values are almost invariably the single-degree-of-freedom ratio c_w/c_c of a single corner, computed with the tyre treated as rigid or absorbed into a ride rate. Evaluated on that basis the same coefficients give the corner figures of Table 4.13, which sit inside the conventional band, while the modal ratios of Table 4.12 describe the vehicle as a whole and are necessarily smaller, because part of the suspension deflection is taken up by the tyre and never reaches the damper.

Table 4.13: Corner damping ratios of the same coefficients under the two conventions used in the literature.

Definition	Front	Rear
$c_w/(2\sqrt{RRm_{s,c}})$, on the ride rate	0.76	0.76
$c_w/(2\sqrt{WRm_{s,c}})$, on the wheel rate	0.67	0.67

Both are reported because they answer different questions. The corner ratio is the one to quote when comparing damper hardware with other vehicles; the modal ratio governs how the vehicle actually settles, and it is the only one that can be compared against the ceiling of Section 4.7.1. The response sweep of Section 4.10.3 later provides an independent check on the choice: of all the combinations examined there, the one that minimises the ride-height variation lies within one per cent of the operating point adopted here.

4.7.3 The Roll Path and the Antisymmetric Modes

The block structure of $\mathbf{C}$ partitions the seven modes between the two paths, and the partition is exact. Table 4.14 evaluates the model with each path in turn disabled. Removing the roll dampers leaves the roll mode and the two antisymmetric wheel modes at zero damping and every other ratio unchanged; removing the heave dampers does the same to the remaining four. No mode draws damping from both paths, and no coefficient of one path appears in the ratios of the other.

Table 4.14: Modal damping ratios with each damping path disabled in turn. The heave and roll paths partition the seven modes with no overlap.

Mode	Nominal	$c_\phi = 0$	$c_h = 0$
Pitch	0.43	0.43	0
Heave	0.46	0.46	0
Roll	0.46	0	0.46
Wheel hop, warp-like	0.26	0	0.26
Wheel hop, roll-like	0.33	0	0.33
Wheel hop, pitch-like	0.81	0.81	0
Wheel hop, in-phase	0.93	0.93	0

This is the decoupling property of Section 4.6.3 in numerical form, and it is what allows either path to be tuned without disturbing the other. It also carries the one cost the architecture imposes. In a conventional four-corner layout the corner dampers act on every mode, so the antisymmetric wheel modes are damped whether or not anyone intended it; here they are reached only through c_ϕ . The warp-like mode is the least damped of the four at the nominal point, at $\xi = 0.26$, and it is the mode a single-wheel kerb strike excites, so its damping has to be rechecked whenever the roll coefficients are changed at the track.

4.7.4 Verification Against the Selected Damper

Converting the wheel-referred coefficients to the damper requires the relation of Equation 4.7: because both wheels of an axle act jointly on the central heave element,

$$c_{el} = \frac{c_h MR^2}{2}, \quad c_r = \frac{c_\phi MR_r^2}{t^2}, \quad (4.38)$$

the second following from the roll wheel-rate relation in the same way. The factor of two halves the required coefficient relative to the conventional single-pushrod result.

The selected hardware is the KW V5 Racing Formula Student damper, available in three valve settings, each adjustable in 19 positions per circuit. The supplier quotes the force at 0.25 m/s at the extremes of the adjustment range of each setting; converting those forces to equivalent linear coefficients at that velocity, and averaging compression and rebound to match the symmetric model, gives Table 4.15. The reference velocity is representative, since the heave element reaches 0.20 m/s for a 10 mm wheel amplitude at the heave natural frequency.

Table 4.15: Required coefficients referred to the damper, against the adjustment range of each valve setting. Ranges are equivalent linear coefficients at 0.25 m/s, with compression and rebound averaged. Each entry states where the adjustment range lies with respect to the requirement.

		Heave, front	Heave, rear	Roll
Required c at the element [Ns/m]		4356	5051	1499
Force implied at 0.25 m/s [N]		1089	1263	375
<i>Adjustment range of each valve setting [Ns/m]</i>				
#S2FS04 standard	1600–4960	covered	below	above
#S2FS05 soft	670–2360	below	below	covered
#S2FS06 hard	2480–7120	covered	covered	above

No single valve setting serves all three elements. The standard setting covers the front heave element but falls short of the rear, and is too stiff for the roll element even fully open; the soft setting covers only the roll element, and the hard setting covers both heave elements but not the roll one. The operating point is therefore realised with the two heave elements on the hard setting and the two roll elements on the soft one, which keeps the car to two damper specifications. Across the hard range each adjuster position is worth some 258 Ns/m at the element, so the front and rear heave dampers are the same unit about three positions apart, both well inside their range.

The magnitude of the required heave coefficients is itself a consequence of the architecture. The heave element moves at $2/MR = 0.91$ times the wheel velocity, so its effective velocity ratio is $MR/2 = 1.1$ rather than MR , but it carries the duty of both wheels. Had the same wheel rates been realised with one damper per corner at that same velocity ratio, each would require 2178 Ns/m at the front and 2525 Ns/m at the rear, or 545 N and 631 N at 0.25 m/s, comfortably within the standard setting. It is the concentration of two wheels' damping duty into a single central element that doubles the force demand, and this hardware cost grows with the motion ratio.

4.7.5 Distribution of the Roll Damping

The architecture offers four damping coefficients but only three of them appear in the modal results: changing the distribution of the roll damping between the axles, at constant total, leaves every damping ratio unaltered. It is nonetheless a design parameter, and the one adopted in Section 4.7.2 by analogy with the stiffness distribution has so far not been justified.

What it governs is the balance during the build-up of lateral acceleration. In steady state the lateral load transfer distribution is fixed by the roll stiffness distribution and the dampers carry nothing. While the roll angle is still changing, the roll dampers carry a moment proportional to the roll rate, apportioned between the axles by the damping split and not by the stiffness split. The transient distribution therefore departs from the steady one by an amount proportional to the difference between the two splits, and the departure vanishes when they are equal. This is a freedom the four-corner layout does not possess: there the roll damping of each axle is tied to its heave damping by Equation 4.43, so the split is fixed by the ride sizing and is unavailable for tuning.

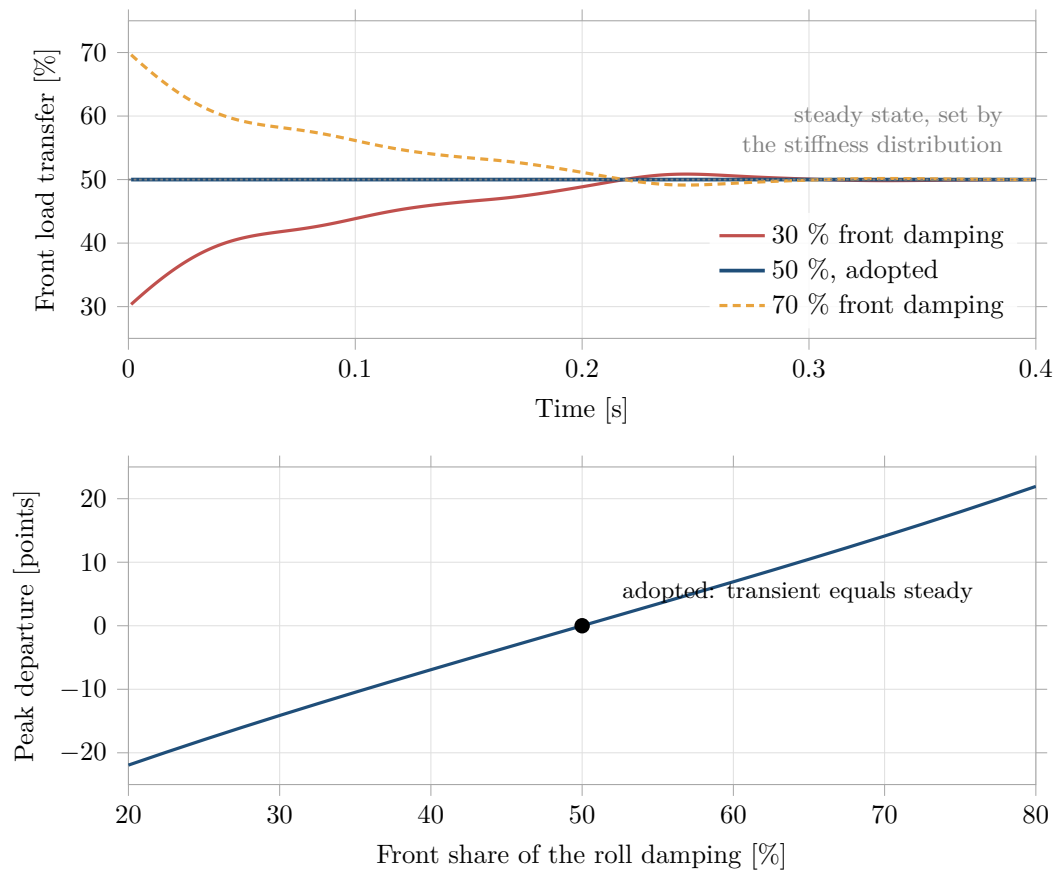


Figure 4.4: Front share of the elastic lateral load transfer during a lateral acceleration ramp, for three roll damping distributions (above), and the peak departure from the steady-state value across the sweep (below). The total roll damping is constant.

The sweep, presented in Figure ??, confirms the relation and gives its magnitude: the peak departure is linear in the split, at approximately 0.70 percentage points of load transfer per percentage point of damping distribution. At 30 % front damping the front axle carries only 35.9 % of the load transfer at the peak of the transient against 50 % in steady state, a departure of 14.1 points that decays over roughly 0.2 s; at 70 % the departure is of equal size and opposite sign.

The 50/50 distribution is therefore the unique setting for which the vehicle presents

the same balance during corner entry as in the steady state, and it is adopted on that basis. Whether a departure from it is desirable is a handling question and is not settled here: a deliberate rearward bias would produce transient oversteer on turn-in, which some drivers prefer, and the freedom to introduce it without touching the steady balance or any damping ratio is itself a product of the architecture. What this analysis establishes is the magnitude of the authority available and the neutral setting from which any such tuning should start.

4.7.6 Limitations

The treatment is linear and symmetric in compression and rebound, and the selected damper is neither. Its characteristic is digressive: at the mid-range setting the force at 0.026 m/s is about 2.4 times what a linear coefficient matched at 0.263 m/s would give [31]. The coefficients derived here therefore describe the damper correctly only near the reference velocity, and the full force–velocity characterisation belongs to the tuning stage. The asymmetry is milder, the two directions differing by some ten per cent at the reference velocity, so the average used throughout is a fair representation of the unit.

The supplier curves are dynamometer measurements of each valve setting, but the units fitted to the vehicle were not characterised individually and no on-car measurement was made, so the coefficients have not been verified on the assembled suspension. The motion ratios are taken as constant, whereas Section 3.8 shows that they vary along the travel, so the damper-referred coefficients are representative of the static position only. Finally, the tyre is represented as an undamped spring, which understates the wheel-hop damping ratios and is therefore conservative for the criterion of Section 4.7.2.

4.8 Response at the Nominal Operating Point

The state-space form of the seven-DOF model, extended with the road displacement at each wheel and with the lateral and longitudinal accelerations as inputs, supplies both the frequency and the transient responses from a single model.

Figure 4.5 shows the response to a road input applied at all four wheels. The ride-height gain, the quantity the aerodynamic sizing acts on, peaks at 1.81 at 3.97 Hz and falls below unity above 6 Hz, so the platform follows the road at low frequency, amplifies it around the sprung modes, and rejects it thereafter. The dynamic tyre load instead peaks at 17.6 Hz, in the wheel-hop region: the two metrics are governed by different parts of the spectrum, which is why they prefer different damping.

Figure 4.6 collects three transients. The lateral acceleration ramp confirms the roll-side sizing from the response side and not from the stiffness: the steady roll angle of 1.393° at 1.51 g corresponds to 0.922 deg/g against the 0.938 deg/g target, with 4.5 % overshoot and settling to within 5 % in 0.22 s.

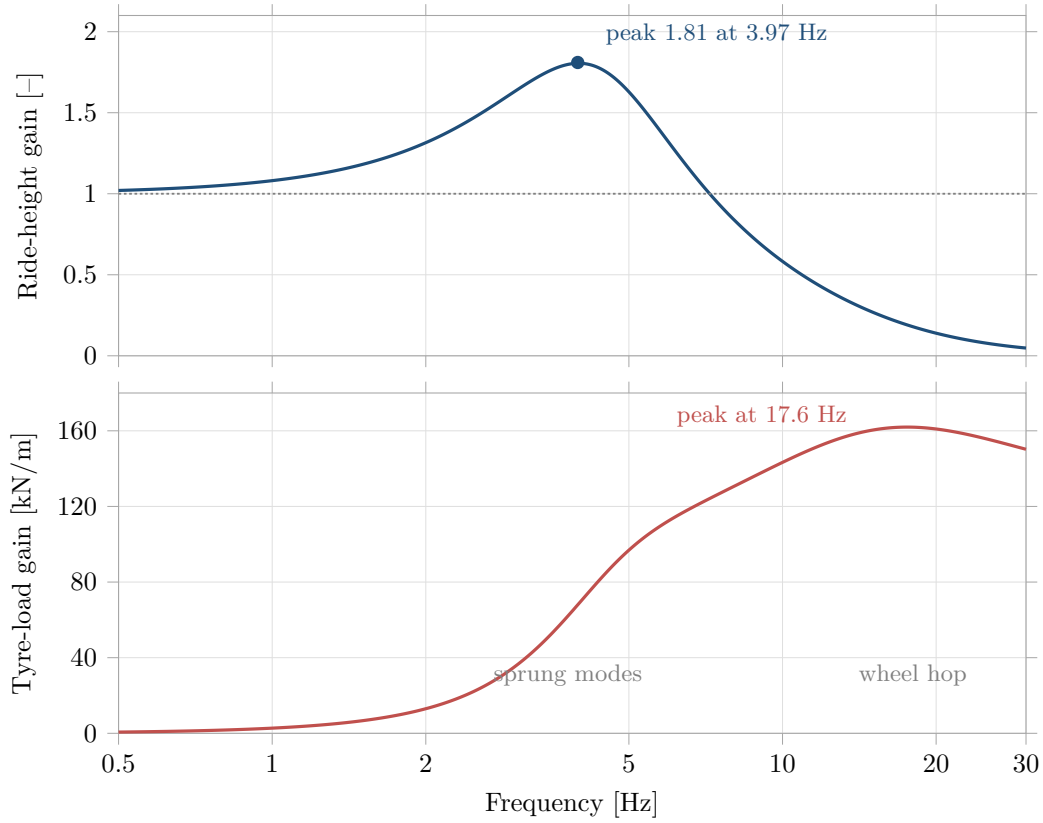


Figure 4.5: Frequency response to a road displacement applied at all four wheels, at the damping of Table 4.11. Ride height above, dynamic tyre load below.

The single-wheel kerb is the case that excites the warp mode. The struck wheel sees a peak dynamic load of 2409 N against 744 N at the opposite wheel, and the roll disturbance decays in 0.34 s. Since the warp mode is reachable only through the roll damper in this architecture, this recovery time is controlled by c_ϕ alone.

The braking ramp gives a front ride-height loss of 13.2 mm against a rear gain of 11.4 mm at 1.8 g. The asymmetry follows from the front axle being the softer of the two, so the load it gains under braking produces the larger excursion. The longitudinal input enters as a moment about the pitch centre, so the anti-features are represented and these are elastic excursions on top of the geometric reaction, not the free-suspension values.

Those two excursions amount to 0.92° of pitch, a gradient of 0.51 deg/g, and they carry an aerodynamic consequence. A dive of θ lowers the front axle by $a\theta$ and raises the rear by $b\theta$, and both displacements move the balance forward,

$$\frac{\partial \eta_F}{\partial \theta} = \left(-a \frac{\partial \eta_F}{\partial h_F} + b \frac{\partial \eta_F}{\partial h_R} \right) = 10.2 \text{ percentage points per degree.} \quad (4.39)$$

Both the gradient above and the attitude it is applied to are read from the straight-line aeromap of Appendix B, and not from the separate evaluation that supplies the design-point loads; the two are not interchangeable, and the migration is quoted here on the map

so that the baseline and its derivative come from one dataset. On that map the balance at the static attitude is 56.5 %, so at maximum braking the 9.4 percentage points this gives carry it forward to some 66 %. No pitch target was imposed by the sizing method, so this is reported as an outcome; it is accepted because the migration is transient and acts in the favourable direction, loading the front axle at the moment the front tyres carry the braking effort. Bounding transient balance migration independently would require lap simulation, and is identified as future work.

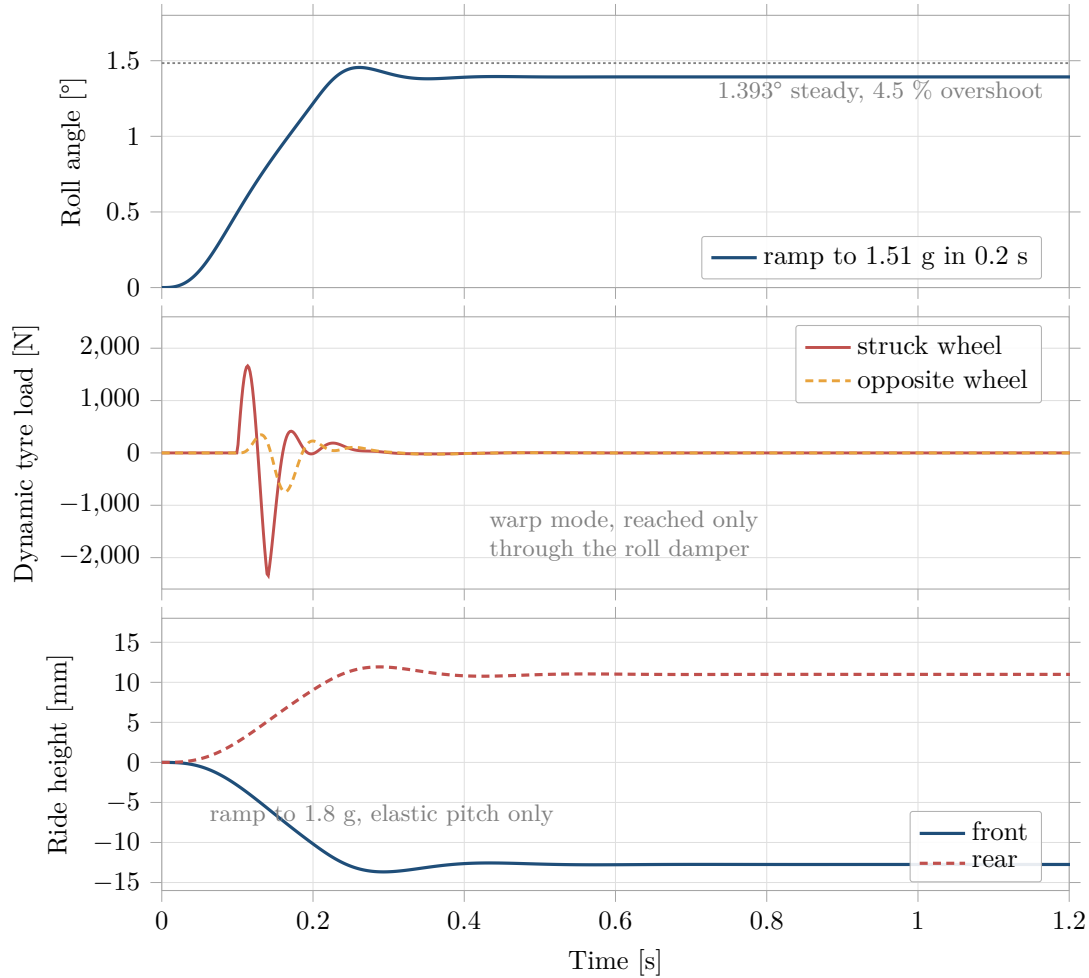


Figure 4.6: Transient responses at the nominal damping: lateral acceleration ramp, single-wheel kerb strike, and braking ramp.

4.9 Travel, Clearance and Requirement Verification

4.9.1 Combined Roll and Heave Travel

The stroke figures established above concern the heave path alone. In a four-corner layout that would be sufficient, since a single damper per corner sees every component of the wheel motion and its stroke bounds the wheel travel directly. The decoupled architecture

removes that guarantee: the heave element responds only to the symmetric component and the roll element only to the antisymmetric one, so neither constrains the other and a wheel can continue to travel through roll after the heave element has reached its limit.

Adding the two isolated maxima is unnecessarily severe, since maximum braking and maximum lateral acceleration cannot occur together. The tyres are limited to a combined envelope, taken here as the friction ellipse [6]

$$\left(\frac{n_x}{n_{x,\max}}\right)^2 + \left(\frac{n_y}{n_{y,\max}}\right)^2 = 1, \quad (4.40)$$

and the aerodynamic load must be scaled with the square of the speed as the envelope is traversed, since the two extremes occur at very different speeds: the skidpad lateral acceleration corresponds to approximately 42 km/h, where the front downforce is a sixth of its value at 100 km/h.

Table 4.16: Front loaded-corner displacement around the friction ellipse, with the aerodynamic load scaled by the square of the speed compatible with each lateral acceleration. Element travels include the static compression and are referred to the suspension deflection, with the tyre deflection removed. The governing case is straight-line braking.

n_x [g]	n_y [g]	Dive [mm]	Roll [mm]	Corner [mm]	Heave / roll element [mm]
1.80	0.00	30.5	0.0	30.5	35.5 / 0.0
1.56	0.75	9.8	7.4	17.2	20.7 / 7.1
1.16	1.16	8.7	11.4	20.1	19.8 / 10.9
0.90	1.31	7.8	12.8	20.6	19.0 / 12.4
0.31	1.49	5.2	14.6	19.8	16.9 / 14.0
0.00	1.51	3.6	14.8	18.5	15.6 / 14.3

Table 4.16 shows that the governing condition is straight-line braking and not any combined point, because the loss of aerodynamic load in the slower combined cases more than offsets the roll contribution. No element approaches its stroke limit: the heave element reaches 35.5 mm including static compression against the 50 mm of heave stroke, and the roll element 14.3 mm against the ± 25 mm it has in each direction. The verification also confirms that superimposing roll on the braking case does not worsen the clearance, because the two contributions peak at opposite ends of the speed range.

The result depends on the speed scaling and would differ if the vehicle were to sustain high lateral acceleration at high speed. On the Formula Student events considered here the highest sustained lateral accelerations occur in slow corners, so the assumption holds; on a circuit with fast corners it would have to be revisited. It also identifies straight-line braking, and not any combined manoeuvre, as the governing case for the kinematic clearance study of wishbone travel, driveshaft plunge and tyre clearance.

4.9.2 Verification of the Derived Requirements

The requirements derived through this chapter are collected here against the values the design achieves (Table 4.17). Three of them are satisfied by construction, because a design

parameter was solved from them, and the rest are verified, because the value follows from a calculation that could have failed.

Table 4.17: Requirements from which a design parameter was solved.

Requirement	Limit	Achieved	Parameter solved
Platform excursion under steady aerodynamic load	15 mm	15.0 mm	RR_F, RR_R
Residual spring compression in rebound	≥ 5 mm	5.0 mm	$AL \geq 4.57\%$
Roll angle at design lateral acceleration	1.5°	1.5°	K_ϕ

As seen in Table 4.18, only the first line does not close exactly, by the 0.53 mm that Section 4.4.1 attributes to the difference between the anti-dive requirement and the geometry implemented, and it is accepted there.

The roll gradient line carries most of the positive weight. The target was derived from the aeromap and used to size the roll springs; the achieved value was obtained by integrating a lateral acceleration ramp through the seven-DOF model, a route that shares no intermediate quantity with the sizing. The two agree to within 1.7 %, which is the strongest single piece of evidence that the elastic chain of this chapter is internally consistent.

The last two lines have least margin. The damper requirement is met only with a deliberate valve selection, and the in-phase wheel-hop mode reaches $\xi = 0.93$, close enough to critical that any further increase in heave damping during testing would push it past.

Table 4.18: Requirements verified against independently computed values.

Requirement	Limit / target	Achieved	Verdict
Chassis clearance in braking	30 mm	30.5 mm	see above
Roll gradient, transient response	0.938 deg/g	0.922 deg/g	satisfied
Front heave spring compression	≤ 50 mm	35.5 mm	satisfied
Rear heave spring compression	≤ 50 mm	31.0 mm	satisfied
Roll element travel at $\phi_{\max}$	≤ 25 mm	15.1 mm	satisfied
Rear chassis displacement in acceleration	≤ 30 mm	26.1 mm	satisfied
Loaded-corner clearance, combined event	< 37.5 mm	30.5 mm	satisfied
Total wheel travel	≥ 50 mm	55 mm	satisfied
Jounce travel	≥ 25 mm	39.9 mm	satisfied
Rear anti-features against practical ceiling	$\leq 20\%$	4.6 %	satisfied
Damper coefficients against adjustment range	KW V5 range	covered	satisfied, with valve selection
Wheel-hop damping below critical	< 1	0.93	satisfied

4.9.3 Outputs for the Multibody Model

Table 4.19 collects the quantities carried forward, at the values used in the multibody model.

Table 4.19: Elastic and damping definition outputs for the multibody model of Chapter 5.

Parameter	Symbol	Value
Front ride rate (target)	RR_F	26.83 N/mm
Rear ride rate (target)	RR_R	31.11 N/mm
Front wheel rate	WR_F	34.14 N/mm
Rear wheel rate	WR_R	41.38 N/mm
Heave motion ratio	MR	2.2
Roll motion ratio	MR_r	2.08
Front heave spring stiffness	K_F	83 N/mm
Rear heave spring stiffness	K_R	100 N/mm
Roll spring stiffness, both axles	K_r	50 N/mm
Front anti-dive (implemented)	AD	32.78 %
Rear anti-lift (derived, lower bound)	AL	4.57 %
Rear anti-squat (geometric by-product)	AS	-0.64 %
Heave natural frequency	$f_{n,h}$	3.57 Hz
Pitch natural frequency	$f_{n,p}$	3.40 Hz
Roll natural frequency	$f_{n,\phi}$	5.51 Hz
Front heave damping (per wheel)	$c_{h,F}$	1.80 kNs/m
Rear heave damping (per wheel)	$c_{h,R}$	2.09 kNs/m
Front axle roll damping	$c_{\phi,F}$	499 Nms/rad
Rear axle roll damping	$c_{\phi,R}$	499 Nms/rad
Heave damping ratio	ξ_h	0.46
Pitch damping ratio	ξ_p	0.43
Roll damping ratio	ξ_ϕ	0.46

4.10 The Case for the Decoupled Architecture

The stiffness and damping values above were each obtained from independent requirements, and nothing so far establishes that a conventional four-corner layout could not have satisfied all of them. This section examines that question and separates the parts of the answer that depend on the particular targets adopted here from the parts that do not.

4.10.1 The Two Architectures Share One Set of Equations

Writing the corner deflections of a four-corner layout in the coordinates of Equation 4.32 gives $d_{FL} = z_{w,F} + \frac{t}{2}\phi_{w,F}$ and $d_{FR} = z_{w,F} - \frac{t}{2}\phi_{w,F}$, so the strain energy of the two corner springs is

$$\mathcal{V} = \frac{1}{2} \left[2k z_{w,F}^2 + \frac{1}{2} k t^2 \phi_{w,F}^2 \right]. \quad (4.41)$$

This is the same quadratic form the decoupled architecture produces, with $WR \rightarrow k$ and $K_\phi \rightarrow \frac{1}{2} k t^2$. The two layouts are therefore not different models but the same model under different constraints, and the question reduces to which combinations of stiffness and damping each one can reach.

4.10.2 Stiffness: a Constraint that Depends on the Targets

Eliminating the corner rate between the heave and roll expressions,

$$K_\phi = \frac{1}{4}K_h t^2 + K_{\text{ARB}}, \quad K_{\text{ARB}} \geq 0, \quad (4.42)$$

with $K_h = 2k$ the axle heave rate. Once the corner springs are sized for the heave requirement the roll stiffness they produce is already determined, and an anti-roll bar can only add to it. The four-corner architecture is thus confined to the region above a straight line through the origin whose slope, $t^2/4 = 0.36 \text{ m}^2$, depends only on the track, whereas the decoupled architecture reaches the whole quadrant.

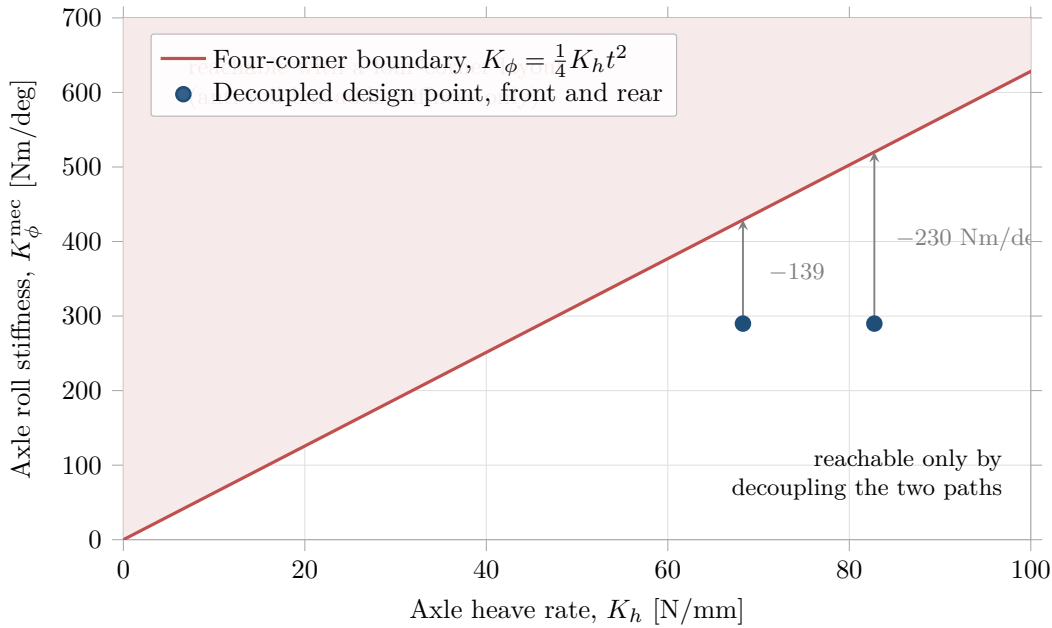


Figure 4.7: Combinations of axle heave rate and mechanical axle roll stiffness reachable by each architecture. The four-corner layout is confined to the shaded region.

For the present vehicle the requirements lie outside that region, as Table 4.20 shows: the corner springs demanded by the heave sizing over-deliver roll stiffness by 48 % at the front and 79 % at the rear, and the anti-roll bar stiffness needed to reach the target is negative on both axes. No passive anti-roll element can remove stiffness that the main springs have already introduced, so within a four-corner layout the only remedies would be to soften the springs, abandoning the ride-height tolerance the aeromap imposes, or to accept a roll gradient below the target. Dual-rate corner springs are a partial exception, and they escape the bound of Equation 4.42 only because that bound assumes a linear corner rate; the two rates still act on both paths at once, so the coupling itself remains.

This outcome is specific to the targets adopted and should be read as a verification, not as a property of the architecture. Solving the same relations of Section 4.5.5 for the roll gradient at which the required mechanical axle stiffness equals the 429.0 and 520.0 Nm/deg

Table 4.20: Roll stiffness delivered by the corner springs alone, against the target of Section 4.5.3.

Axle	K_h [N/mm]	Corner springs alone [Nm/deg]	Anti-roll bar required [Nm/deg]
Front	68.3	429.0	−139.1
Rear	82.8	520.0	−230.1

the corner springs deliver, the boundary is crossed at approximately 0.67 deg/g at the front and 0.58 deg/g at the rear: had the roll target been tightened past those values the corner springs would fall short instead, and an anti-roll bar would close the difference. The reference decoupled designs surveyed in Chapter 2 sit on that side of the boundary, so the stiffness result reported here is a feature of this vehicle’s combination of a stiff aerodynamic platform requirement with a moderate roll gradient, not a general demonstration.

4.10.3 Damping: a Constraint with no Corrective Device

The damping side admits no such escape. Applying the same coordinate transformation to the corner dampers gives

$$c_\phi = \frac{1}{2} c_h t^2, \quad (4.43)$$

an equality and not an inequality, with c_h the per-wheel coefficient. The anti-roll bar has no counterpart in damping: no passive element in common use adds roll damping without adding heave damping. A four-corner layout therefore has a single free parameter per axle and traces a curve in the plane of the two damping ratios, whereas the decoupled architecture reaches an area. No choice of target, no aerodynamic package and no tyre can relax this, because it follows from the kinematics of the layout alone.

The consequence for this vehicle is severe. Rebuilding the seven-DOF model with the corner-spring roll stiffness of Table 4.20 and the corner-damper roll damping of Equation 4.43, at the same heave coefficients, gives a roll mode at 6.64 Hz with a damping ratio of 0.17, against the 0.46 the decoupled layout provides at identical heave and pitch ratios. In the same configuration one wheel-hop mode reaches $\xi = 1.74$ and is overdamped, the condition the selection criterion of Section 4.7.2 exists to avoid. The conventional layout therefore fails on both counts at once: the roll mode is substantially underdamped while a wheel mode is overdamped, and neither can be corrected without disturbing the heave damping the platform control depends on.

To place the comparison on a response metric instead of on the coefficients, in Figure 4.8 the two damping ratios were swept independently and two figures of merit evaluated at each point: the root-mean-square front ride-height variation, which is the quantity the aerodynamic platform requirement acts on, and the root-mean-square dynamic tyre load, which stands for grip. Both follow from integrating the squared transfer function against a road displacement spectrum falling as the inverse square of frequency, with the left and right tracks taken as statistically independent. That last assumption matters: a road

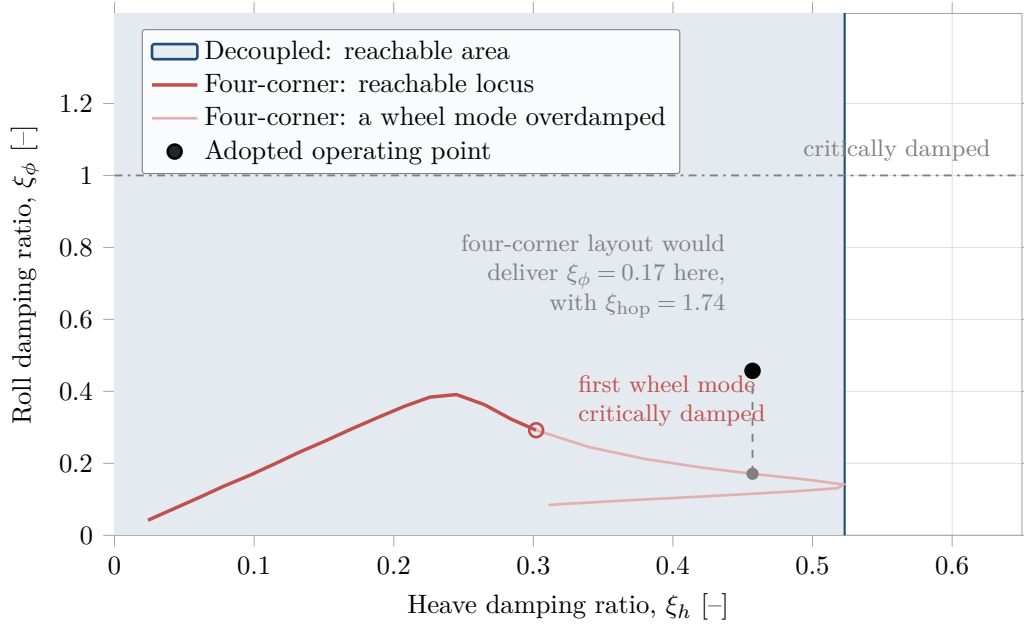


Figure 4.8: Combinations of heave and roll damping ratio reachable by each architecture. The four-corner layout is confined to a curve; the decoupled layout reaches the whole area.

input applied equally to all four wheels excites only the symmetric modes, and a sweep built on it would be insensitive to the roll damping altogether.

The two metrics prefer different points. The lowest ride-height variation occurs at $\xi_h = 0.41$ with $\xi_\phi = 0.92$, and the lowest tyre-load variation at $\xi_h = 0.24$ with $\xi_\phi = 0.37$; relative to the adopted operating point the improvements are 0.6 % and 9.2 %. Neither is reachable with a four-corner layout: at the heave damping the ride-height metric prefers, Equation 4.43 fixes $\xi_\phi = 0.16$ against the 0.92 the metric calls for, and it cannot be made up, because raising it would require stiffening the corner dampers and with them the heave damping the platform control depends on. The improvements in the response metrics are modest, at around one and nine per cent, but they are unreachable, and the roll damping of the conventional layout would be deficient whether or not those points were sought.

4.10.4 Independence of Adjustment

A third consequence follows from the block structure of $\mathbf{K}$ and $\mathbf{C}$, and is independent of whether either set of targets happens to be reachable. In the decoupled architecture the heave coefficients populate the z and θ coordinates and the roll coefficients populate ϕ , with no overlap, so a change to one path leaves the other untouched.

In a four-corner layout every coefficient enters every coordinate. Changing a corner spring moves the heave and the roll stiffness together, and the anti-roll bar recovers only the roll component afterwards; changing a corner damper moves all three damping ratios at once. Two of the design freedoms exploited in this chapter have no equivalent there: the roll damping level of Section 4.7.2, and its front-to-rear distribution of Section 4.7.5,

which governs the balance during corner entry without disturbing the steady balance or any damping ratio.

The case for the architecture therefore rests on the independence of the two paths and not on the numerical values of any one target: the stiffness requirements of this vehicle could have been met conventionally, the damping requirements could not, and the freedom to tune either path in isolation exists only in the decoupled layout.

4.11 Chapter Conclusions

This chapter defined the elastic and damping characteristics of the decoupled suspension from an aerodynamic-driven sizing approach. The ride rates follow directly from the chassis displacement the aerodynamic package tolerates, and every other elastic quantity follows from them or from another physical limit: the wheel rates through the tyre series relation, the motion-ratio lower bound from the wheel travel that must fit within the spring stroke, the heave spring rates from the summing relation, and the roll springs from a roll gradient target read off the skidpad aeromap. The anti-features were derived from binding displacement constraints instead of assumed, the front from the clearance limit in braking and the rear from the requirement that the heave spring stay in compression in rebound.

One result of the elastic sizing is worth isolating. The anti-dive requirement is sensitive to the aerodynamic load at the sizing instant to the extent of seven percentage points for five per cent of downforce, so the implemented 32.8 % sits comfortably inside the uncertainty of the requirement it is checked against, at a cost of half a millimetre of clearance.

On the damping side the conventional target of $\xi = 0.70$ was shown to be unattainable for any choice of damper. With the tyre carried as a separate degree of freedom the sprung ratios reach a ceiling of 0.52 in heave and 0.49 in pitch, a consequence of the limited separation between the sprung and wheel-hop frequencies that the aerodynamic-driven ride rates impose. An explicit criterion was therefore adopted, the highest sprung damping for which no wheel mode reaches the critically damped condition; the wheel modes supply the bound at 1.89 kNs/m referred to the wheel, and the operating point was placed one adjuster position inside it.

Three results follow from the architecture itself and not from the numerical values. The roll path is the sole source of damping for the roll mode and for both antisymmetric wheel modes, so the warp-like mode is governed by c_ϕ alone and must be rechecked whenever the roll coefficients change. The concentration of two wheels' damping duty into a single central element doubles the force demand on the hardware and moves the heave requirement into the hard valve setting of the selected damper. And the front-to-rear distribution of the roll damping, which leaves every damping ratio unchanged, governs

the balance during the build-up of lateral acceleration, the 50/50 split being the unique setting for which the transient distribution equals the steady one.

The comparison against a four-corner layout closes the case for the architecture. On the stiffness side the conventional layout fails on this vehicle but would not on a vehicle with a tighter roll target, so that result is a verification and not a demonstration. On the damping side the relation between roll and heave damping is an equality with no corrective device available, and at the heave damping adopted here a conventional layout leaves the roll mode at a damping ratio of 0.17 while overdamping a wheel mode. No aerodynamic package, target or tyre can relax that constraint, and it is the substance of the case for the decoupled architecture on this vehicle.

The values collected in Table 4.19 are the elastic and damping inputs for the multibody model of Chapter 5.

Chapter 5

Multibody Model and Kinematic Verification

The dissertation uses two simulation environments with distinct purposes. The analytical models of Chapter 4 — the quarter-car representation and the seven-DOF modal model — are sizing instruments: they set the frequency and damping targets and demonstrate the decoupling property on constant motion ratios and a fixed roll centre. The multibody model described here is a verification instrument: it carries the real geometry and therefore measures what those assumptions cost. Each model is presented in the chapter in which it is used, and this chapter is concerned only with the second.

5.1 Modelling Scope and Objectives

The multibody model is used for the quantities that depend on the real geometry and that the analytical formulation of Chapter 4 cannot produce: the motion ratios and their variation along the travel, the residual coupling between the two elastic paths, the roll-centre position and its migration, and the steering and camber characteristics. The modal behaviour and the damping ratios are not evaluated here, because the seven-DOF model of Section 4.6.2 obtains them from the same elastic data without numerical differentiation, and a multibody extraction would add scatter without adding physics. The division between the two environments is therefore methodological, and each quantity is produced once, by the model best suited to it.

Four idealisations bound what the results of this chapter can be taken to mean. The links and the bellcranks are rigid and the joints are ideal, so the model returns the kinematics of the mechanism and not the deflection of its members; the compliance of the load path is treated separately in Chapter 7, where the chassis attachments are replaced by bushings and their stiffness is swept. The tyre is represented by its free radius and its vertical rate alone, with no force and moment model, which is sufficient for kinematic quantities but excludes any statement about grip or balance. The analyses are quasi-static: the

suspension is moved through prescribed displacement patterns and the resulting geometry is measured, so no inertial or damping effect participates in the results. The anti-tension device of Section 3.7 is not represented, because it enforces the sign of the load carried by the roll element without altering the motion of the mechanism.

The quantities the model is required to return, the values they are checked against, and the section in which each is reported are collected in Table 5.1. Every reference value was derived in an earlier chapter and none was obtained from the multibody model itself, so each line of the table is a comparison between two independent routes to the same quantity.

Table 5.1: Verification matrix of the multibody model. Each quantity is compared against a value derived independently in an earlier chapter.

Quantity	Reference value	Derived in	Reported in
Heave motion ratio, MR	2.200	Sec. 4.3.2	Sec. 5.3
Roll motion ratio, MR_r	2.080	Sec. 4.5.4	Sec. 5.3
Ride rates, front / rear	26.8 / 31.1 N/mm	Sec. 4.2	Sec. 5.4
Wheel rates, front / rear	34.1 / 41.4 N/mm	Sec. 4.2	Sec. 5.4
Coupling between elastic paths	zero by construction	Sec. 3.8	Sec. 5.5
Left–right symmetry of the individual motion ratios	not guaranteed by the layout	Sec. 3.8	Sec. 5.5.1
Roll-centre heights, front / rear	90 / 96 mm	Sec. 3.3.2	Sec. 5.6
Roll-centre migration	no reference value	—	Sec. 5.6
Camber gain	negative camber on the loaded wheel, monotonic	Sec. 3.4	Sec. 5.7
Bump steer	as close to zero as the packaging allows	Sec. 3.4	Sec. 5.8
Caster angle	6°	Tab. 3.4	Sec. 5.9
Kingpin inclination	3–7°	Tab. 3.4	Sec. 5.9
Ackermann, outer / inner	11.54 / 10.15°	Tab. 3.4	Sec. 5.9.1

Two kinds of entry appear in that table and they are not read in the same way. A quantity such as the motion ratio is a design parameter that the geometry was built to deliver, so agreement confirms that the bellcrank arms were placed as intended and a departure would be a construction error rather than a finding. A quantity such as the roll-centre migration has no reference value at all: it is a property of the layout that the front-view construction of Chapter 3 cannot produce, and the model is the only instrument in this work that reports it. The first kind verifies the design and the second characterises it, and each section states which of the two its result belongs to.

5.2 Model Description

The suspension was built in MSC Adams/Car from the final hardpoints of Section 3.6, and Figure 5.1 shows the resulting full-vehicle assembly in the static position. Each corner comprises the upright, the upper and lower wishbones, the tie rod and the pushrod, all as rigid bodies connected by ideal joints, with the wheel carried on a vertical spring representing the tyre.

The analyses reported in this chapter were run on the front and rear suspension assemblies rather than on the full vehicle, each assembled with its steering subsystem and a suspension test rig. The rig imposes the prescribed displacement patterns of Table 5.3 directly at the wheels, which is what makes them available as inputs, and it takes the tyre free radius and vertical rate from its own parameters rather than from the tyre property file. The tyre in these analyses is therefore a vertical spring of the stated rate acting at the stated radius, and the force and moment coefficients contained in the property file play no part in any result reported here. That is consistent with the scope of Section 5.1, since no quantity in this chapter requires a tyre force.

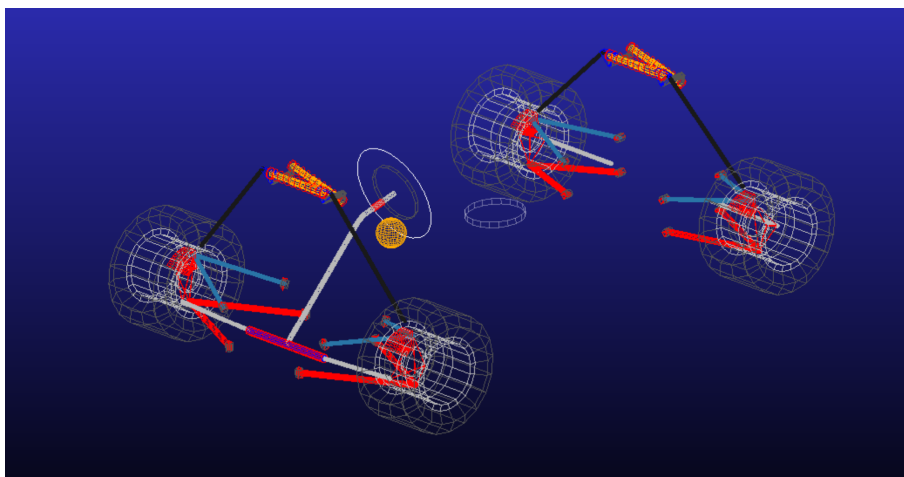
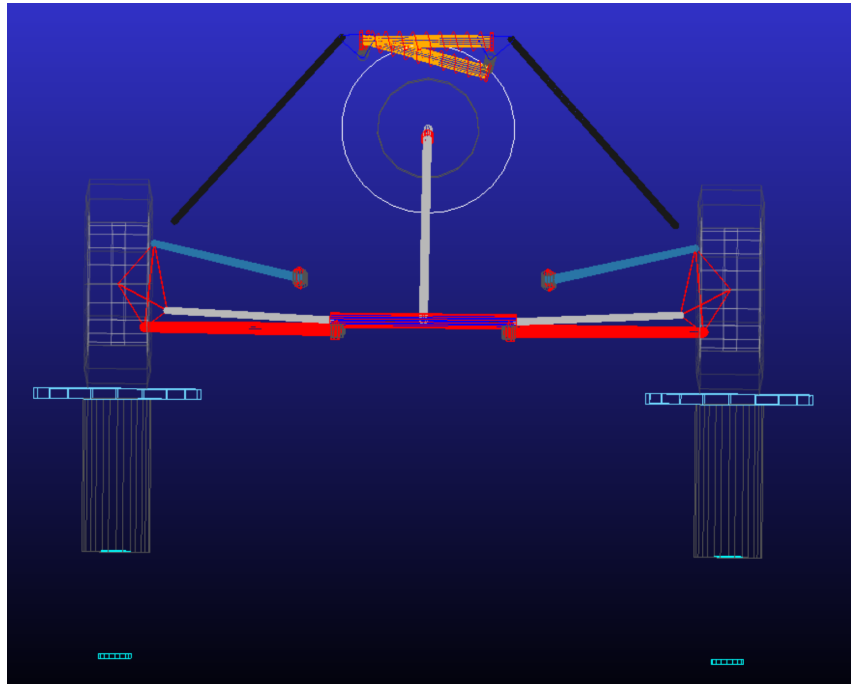


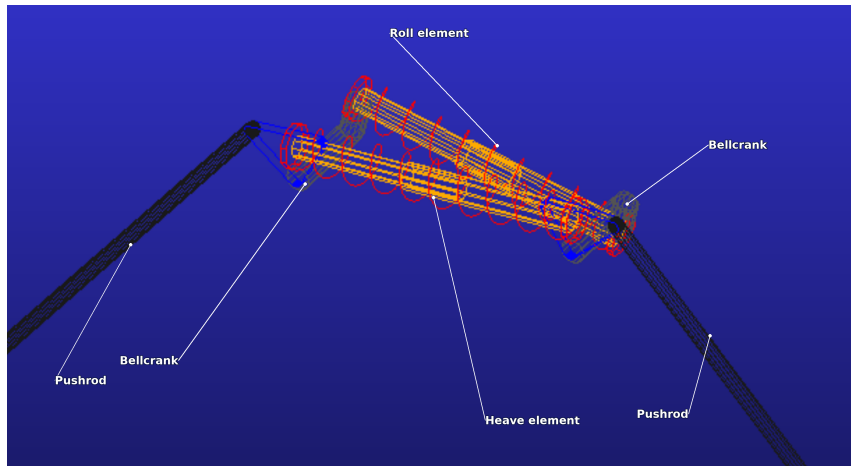
Figure 5.1: Full-vehicle model of the decoupled suspension implemented in MSC Adams/Car, in the static position.

The departure from a conventional model is inboard of the pushrods. Each axle carries two bellcranks, one per corner, each pivoting on the chassis about its own axis, and the two elastic elements are mounted between them rather than between a bellcrank and the chassis: the heave element connects the two bellcranks so that it is displaced by the sum of the two wheel displacements, and the roll element connects them so that it is displaced by their difference. The topology is shown in Figure 5.2. Because both elements react against a moving body at each end, no element in the model has a fixed chassis reaction point, and this is the feature that every result of this chapter ultimately measures.

The model was adapted from an existing Formula Society of Automotive Engineers (FSAE) decoupled-suspension model supplied by MSC, into which the geometry and the



(a) Front view of the axle



(b) Detail of the bellcrank pair and the two elastic elements

Figure 5.2: Topology of the decoupled mechanism in the multibody model, at the front axle. Each pushrod drives its own bellcrank, and the two elastic elements are mounted between the two bellcranks rather than between a bellcrank and the chassis.

elastic data of the present design were substituted. The data it was built from are collected in Table 5.2, with the source of each value.

The analyses run on the model are listed in Table 5.3, with the travel imposed in each and the quantities extracted from it. The travel imposed in parallel motion is the design travel of each axle established in Section 4.3.2, which is not symmetric about the static position: the element is installed at its free length, so the rebound available is the static wheel deflection and the remainder of the 55 mm falls on the jounce side. In opposite

Table 5.2: Data used to build the multibody model.

Input	Value	Source
Front hardpoints	Tab. 3.11	Sec. 3.6
Rear hardpoints	Tab. 3.10	Sec. 3.6
Heave spring rate, front / rear	83 / 100 N/mm	Tab. 4.19
Roll spring rate, both axles	50 N/mm	Tab. 4.19
Heave element damping, front / rear	4.36 / 5.05 kNs/m	Tab. 4.11
Roll element damping, both axles	1.50 kNs/m	Tab. 4.11
Heave element installed length, front / rear	279.0 / 280.0 mm	Sec. 4.3.2
Roll element installed length, front / rear	262.3 / 272.7 mm	Sec. 4.3.2
Tyre free radius	205.74	Data
Tyre vertical rate	125.314 N/mm	Tab. 4.1
Wheelbase / track	1530 / 1200 mm	Tab. 3.1
Centre of gravity height	280 mm	Tab. 3.1
Drive / brake distribution, front	50 / 55 %	Vehicle configuration

motion the travel is symmetric by nature, and ± 27.5 mm corresponds to 2.6° of body roll on the 1200 mm track, which covers the design roll angle of 1.5° with margin.

Table 5.3: Analyses performed on the multibody model.

Analysis	Input	Quantities extracted
Parallel wheel travel	-15.1 to $+39.9$ mm front, -14.4 to $+40.6$ mm rear	Heave motion ratio, ride and wheel rates, camber gain, bump steer, caster, kingpin inclination
Opposite wheel travel	± 27.5 mm, equivalent to 2.6° of roll	Roll motion ratio, coupling of the heave path to roll, roll-centre migration, camber in roll, roll steer
Single-wheel travel	± 27.5 mm, one wheel at a time	Left-right symmetry of the individual motion ratios
Steering sweep	To $\pm 11.5^\circ$ at the road wheel	Ackermann relation

Every analysis is run about the static position, which the installed element lengths of Table 5.2 establish.

5.3 Motion Ratio Verification

Before any result is extracted, the model is checked against the quantities it should reproduce by construction. The motion ratios are the natural choice, since every stiffness in Chapter 4 depends on them and the wheel rate scales with their square.

Table 5.4: Motion ratios measured in the multibody model against the design values.

Path	Design	Measured
Heave, front	2.20	2.20
Heave, rear	2.20	2.20
Roll, front	2.08	2.08
Roll, rear	2.08	2.08

As seen in Table 5.4, all four paths agree within one per cent, which establishes that the hardpoints implemented in the model reproduce the geometry from which the elastic sizing was derived. The motion ratio is not exactly constant along the travel, as verified in Figure 5.3 for heave motion and Figure 5.4 for roll motion, which is the one assumption of Chapter 4 the model can test; its variation is quantified through its effect on the rate in Section 5.4, and it is also the physical origin of the residual coupling examined next.

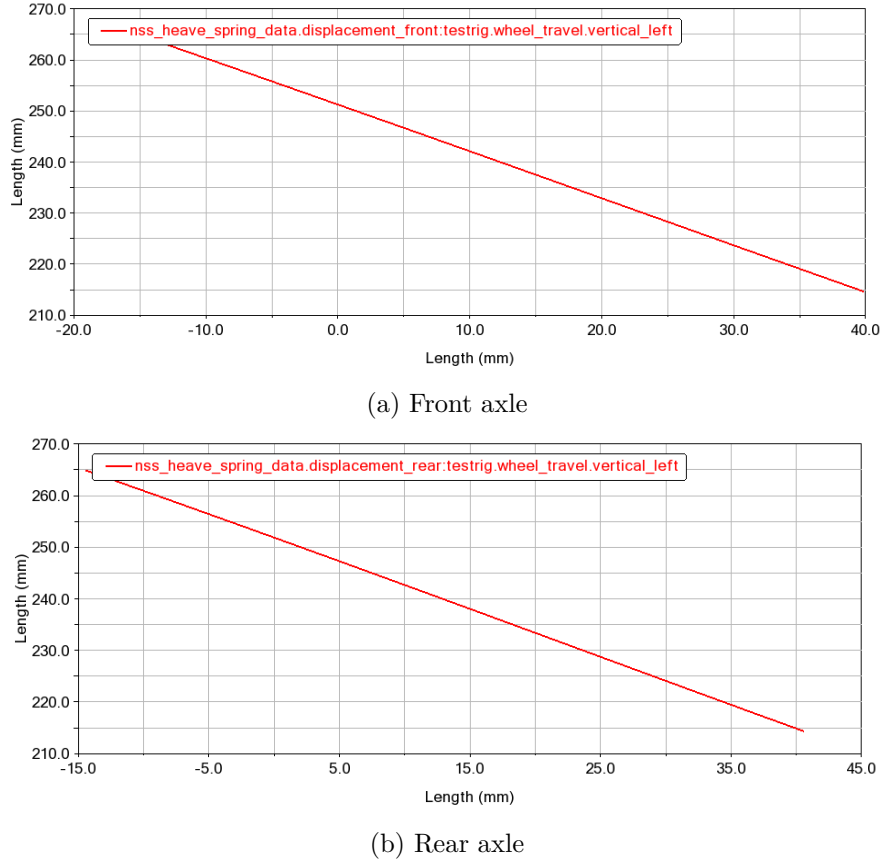


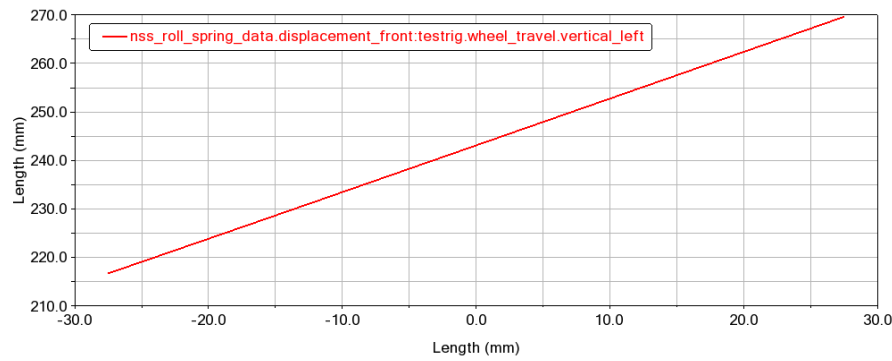
Figure 5.3: Heave element displacement in parallel travel, from which the heave motion ratio is obtained.

5.4 Rate Verification

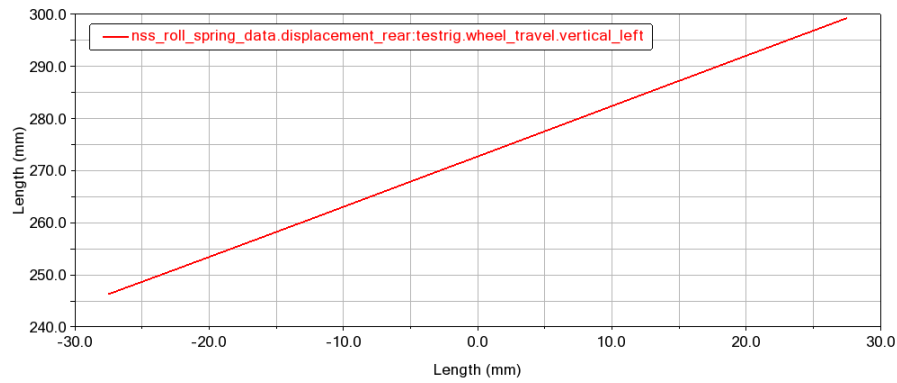
The motion ratios establish that the kinematics are correct, but the quantity the elastic sizing actually delivers is the rate at the wheel, and it depends on the square of the motion ratio. It is therefore checked directly.

The rear traces are shown to +33 mm (Figures 5.5 and 5.6), beyond which the differentiated rate becomes unreliable close to the end of the travel.

As verified in Table 5.5, the front axle reproduces the derived values almost exactly, to within a few tenths of a per cent on both rates. The rear exceeds them by two to four per cent, in the same direction on both rates.



(a) Front axle



(b) Rear axle

Figure 5.4: Roll element displacement in opposite travel, from which the roll motion ratio is obtained.

Table 5.5: Rates at the static position, measured in the multibody model against the values derived in Chapter 4.

Quantity	Derived	Measured	Deviation
Front ride rate [N/mm]	26.8	26.9	+0.3 %
Rear ride rate [N/mm]	31.1	31.8	+2.1 %
Front wheel rate [N/mm]	34.1	34.2	+0.2 %
Rear wheel rate [N/mm]	41.4	42.9	+3.7 %

The rate is not constant along the travel, and the figures give the variation directly. At the front it spans 25.4 to 27.1 N/mm and at the rear 30.4 to 32.4 N/mm, in both cases rising to a maximum near the middle of the jounce travel and falling thereafter. Referred to the static value that is a variation of some three per cent, so the treatment of the rate as constant in Chapter 4 is supported over the whole travel and not only over the small excursions the aerodynamic platform requirement allows.

5.5 Decoupling Efficiency

Two tests isolate the effect. In parallel travel the roll element should not move at all, and its displacement measures the coupling of the roll path to heave; in opposite travel

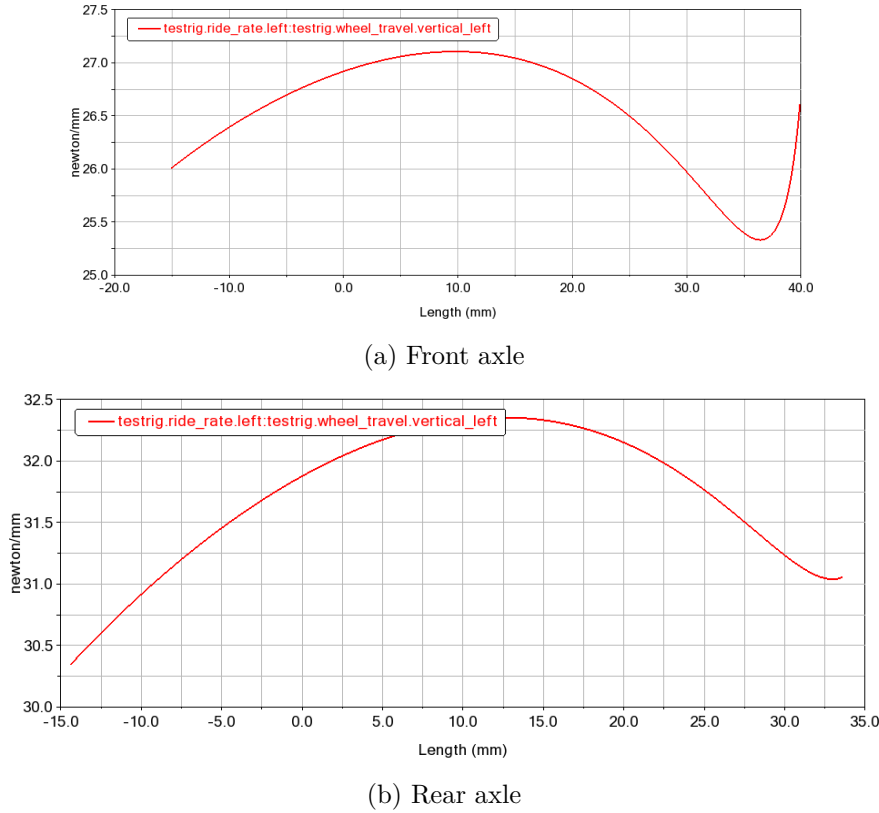


Figure 5.5: Ride rate against wheel travel in parallel travel.

the heave element should not move, and its displacement measures the converse. Each parasitic displacement is normalised by the travel of the element that the same test drives.

The architecture therefore achieves about 99% decoupling on the heave path and 98.5 % on the roll path, as represented in Figures 5.7 and ??, and quantified in Table 5.6 with the two axles agreeing closely enough to confirm that the geometry is consistent from one end of the vehicle to the other. Although, it is important to highlight that most of the coupling occurs near the end of the travel, so in the major of the events, the coupling is below 0.5 % in the both paths. Both parasitic displacements originate in the same second-order geometric effect identified below, corresponding to even functions of the wheel travel, symmetric parabolas with their extremum at the static position. A left-right asymmetry in the rocker geometry would produce odd functions, linear through the origin, and none is observed.

The coupling is therefore a second-order geometric effect arising from the variation of the motion ratio measured in Section 5.3, and it decays quadratically as the travel is reduced: at the design roll angle of 1.5° the parasitic force in the heave element is 13.4 N at the front and 16.2 N at the rear, against the 1134 N and 1315 N those elements carry statically.

The practical consequence is a small elastic jacking effect: rolling the vehicle shortens the heave element slightly and therefore raises the chassis. At the design roll angle the

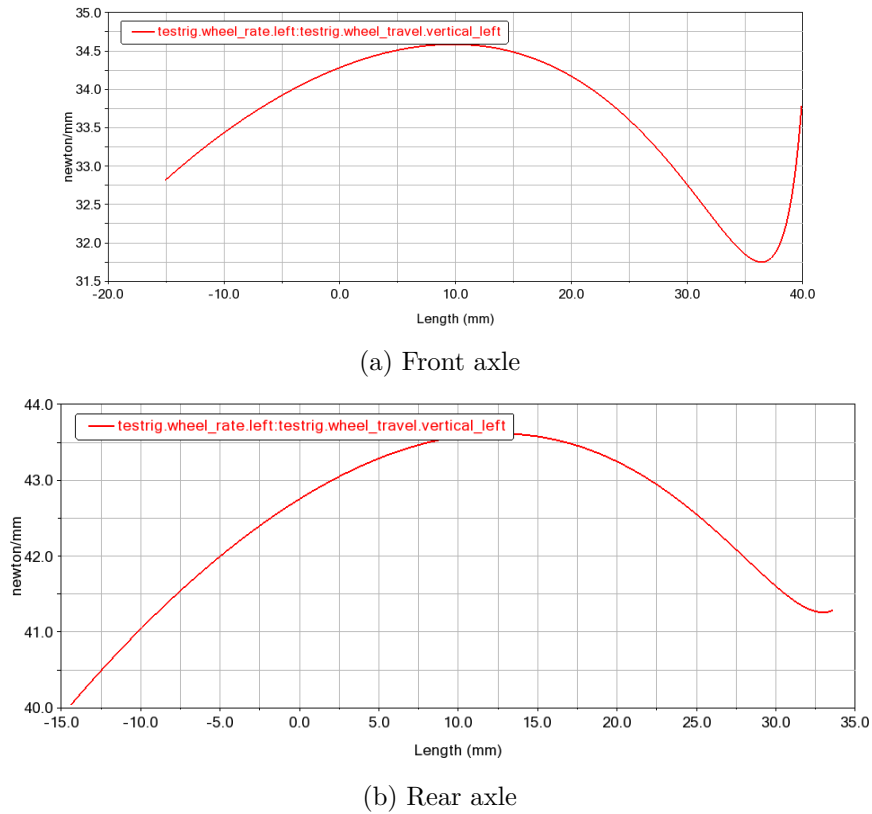


Figure 5.6: Wheel rate against wheel travel in parallel travel.

Table 5.6: Residual coupling between the elastic paths

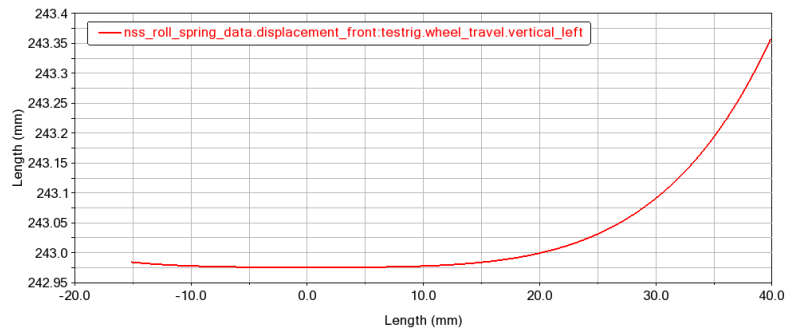
Test	Axle	Parasitic travel	Reference travel	Coupling
Parallel travel, roll element	Front	0.383 mm	39.9 mm	0.96 %
Parallel travel, roll element	Rear	0.400 mm	40.4 mm	0.99 %
Opposite travel, heave element	Front	0.282 mm	28.75 mm	0.99 %
Opposite travel, heave element	Rear	0.446 mm	29.00 mm	1.54 %

rise is 0.2 mm, against the 15 mm of platform excursion the aerodynamic sizing allows. It is negligible for platform control, but it is a real effect that the analytical model cannot see, and it is reported here for that reason.

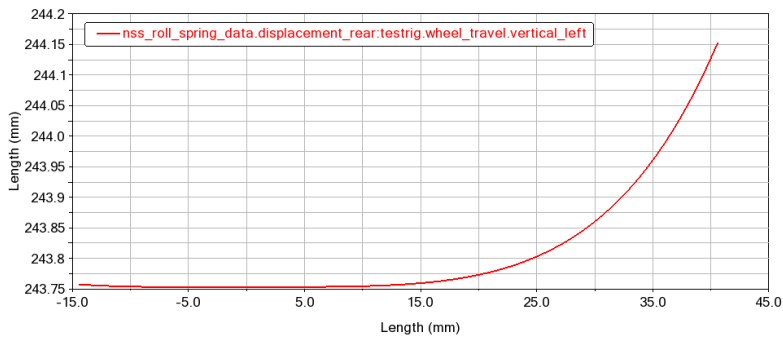
5.5.1 Single-Wheel Input and Left–Right Symmetry

Parallel and opposite travel are the symmetric and antisymmetric combinations of the wheel inputs, and a suspension that behaved differently on the two sides would still satisfy both tests if the asymmetry happened to cancel. Displacing one wheel at a time removes that possibility, since each side is then exercised in isolation.

The result shown in Figures 5.9 and 5.10 is unambiguous. The heave element traces for a left-wheel and a right-wheel input coincide exactly, which is expected of a element that responds to the sum of the two wheel displacements irrespective of which wheel supplies

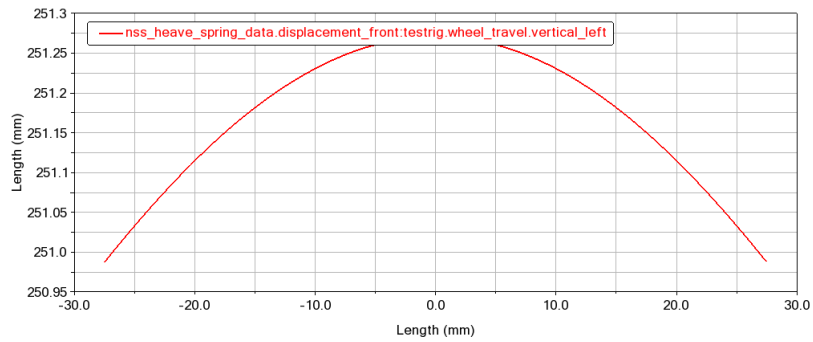


(a) Front axle

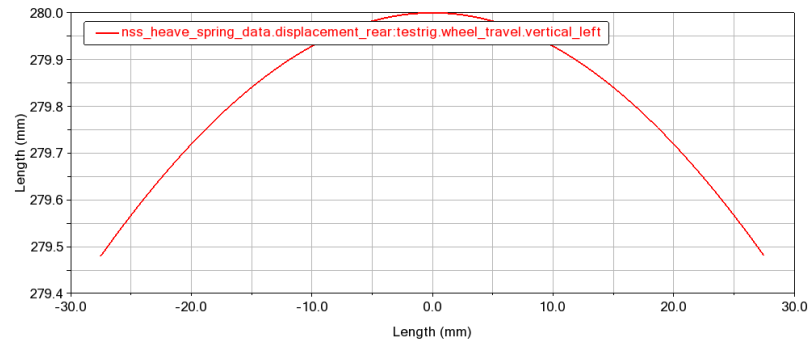


(b) Rear axle

Figure 5.7: Roll element displacement in parallel travel



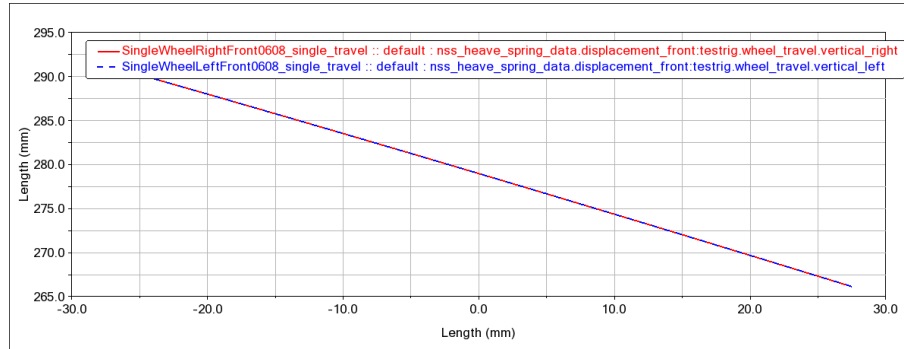
(a) Front axle



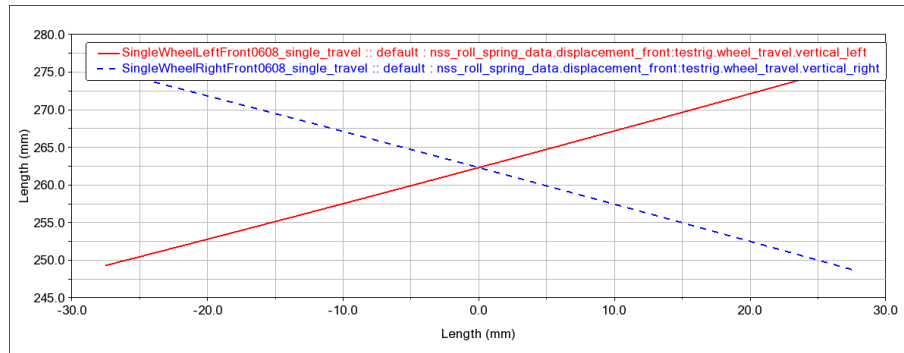
(b) Rear axle

Figure 5.8: Heave element displacement in opposite travel

it. The roll element traces are exact mirror images about the static position, which is likewise expected of a differencing element. Both hold at each axle.



(a) Heave element



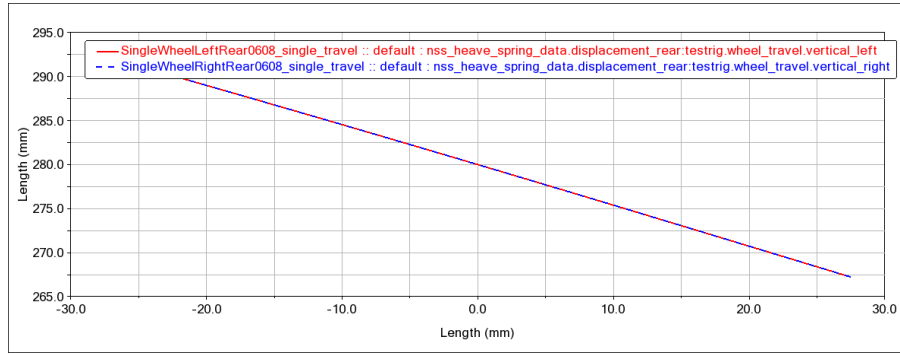
(b) Roll element

Figure 5.9: Element displacement at the front axle for a single-wheel input, with the left and right wheels driven in turn.

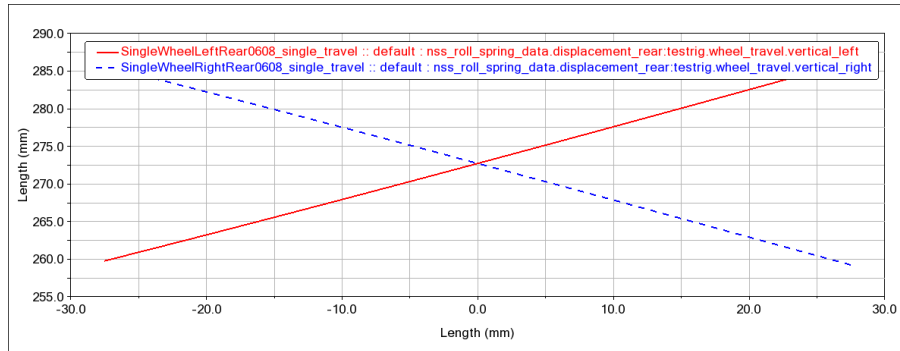
Section 3.8 admits distinct individual motion ratios on the left and right sides, since the bellcrank geometry does not guarantee that the two are equivalent; establishing whether they differ on the present design is one of the questions the model was built to answer. The measurements resolve it: the parasitic curves reported above are even functions, which excludes an asymmetric contribution, and the single-wheel traces coincide or mirror to within the resolution of the analysis. The individual motion ratios therefore take the same value on both sides of each axle, and the residual coupling is a second-order geometric effect rather than a left-right imbalance.

5.6 Roll Centre Position and Migration

The roll-centre heights selected in Section 3.3.2 were obtained from a front-view construction. The multibody model returns 88 and 94.3 mm, against the design values of 90 and 96 mm, as shown in Figure 5.11. The residual difference of approximately 1.7 mm may corresponds to the tyre deflection under the static corner load, which the analytical construction does not include.



(a) Heave element



(b) Roll element

Figure 5.10: Element displacement at the rear axle for a single-wheel input.

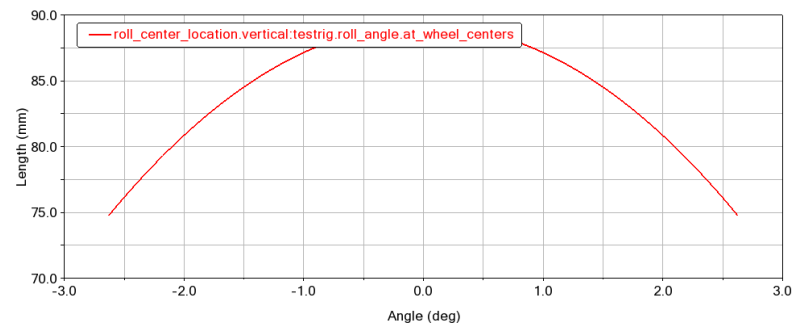
The migration of the roll centre through roll is reported in both components, because the lateral component (Figure 5.12) is the larger of the two and is not visible in the vertical trace alone. Over ± 27.5 mm of opposite travel, equivalent to 2.6° of roll, the rear roll centre descends 28 mm and displaces laterally by 320 mm. Scaled to the design roll angle this is a lateral migration of approximately 180 mm, or thirty per cent of the half track, against a vertical migration of some 16 mm.

A lateral migration of that magnitude moves the roll axis out of the vehicle centre plane and introduces an asymmetric jacking component that the symmetric roll analysis of Chapter 4 does not represent. Both lateral traces are linear in wheel travel, which is the favourable aspect of the result: the migration is proportional to roll angle and does not change character through the travel, so its effect on the balance is at least predictable.

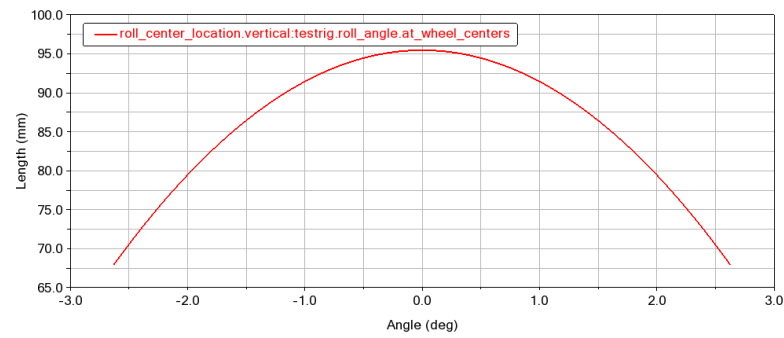
The qualitative conclusion, that the lateral migration dominates the vertical one and is large enough to warrant attention, is common to both front and rear axles.

5.7 Camber Behaviour

The camber characteristics are reported for both excitation modes, since the decoupled architecture separates them and they are governed by the same front-view geometry that sets the roll centre.

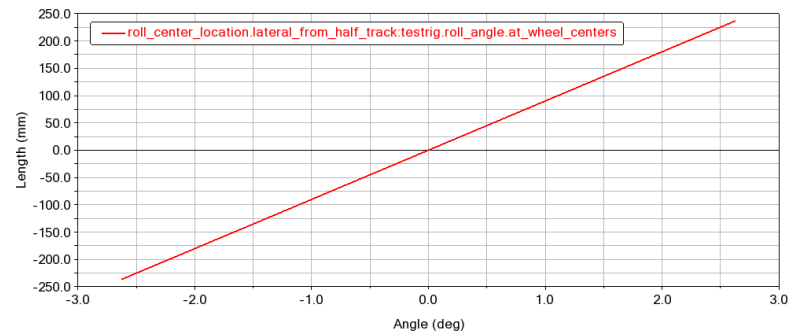


(a) Front axle



(b) Rear axle

Figure 5.11: Vertical migration of the roll centre through opposite travel.

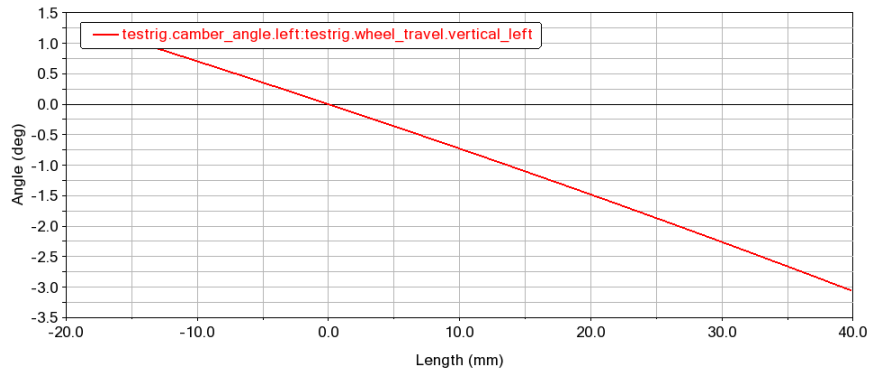


(a) Front axle

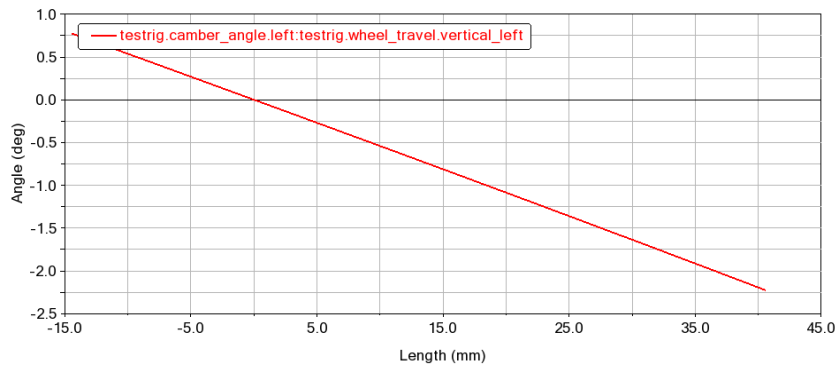


(b) Rear axle

Figure 5.12: Lateral migration of the roll centre through opposite travel, measured from the vehicle centre plane



(a) Front axle



(b) Rear axle

Figure 5.13: Camber angle against wheel travel in parallel travel.

Both traces are linear across the whole travel, as seen in Figure 5.13, which satisfies the requirement of monotonic variation stated in Section 3.4. The front axle gains camber at approximately -0.08 deg per millimetre of wheel travel and -0.78 deg per degree of roll (Figure 5.14).

The second figure is the one with direct handling significance. A gradient of -1.0 deg per degree of roll would hold the wheel plane vertical relative to the road, so the measured -0.78 represents a camber compensation of 78 %: at the design roll angle of 1.5° the loaded wheel leans outwards by some 0.33° relative to the road, having gained 1.17° of negative camber relative to the chassis. That is a conventional compromise, since full compensation demands a low roll centre and a long swing arm and conflicts with the camber gain in heave.

5.8 Steering Kinematics

The toe variation with wheel travel was refined by aligning the suspension instant centres with the tie-rod axis at the front and with the toe-link axis at the rear, and the result is represented in Figure 5.16. The front axle, which initially exhibited a gradient of 0.023 deg/mm, was brought to 0.00135 deg/mm on the jounce side; the rear axle stands at

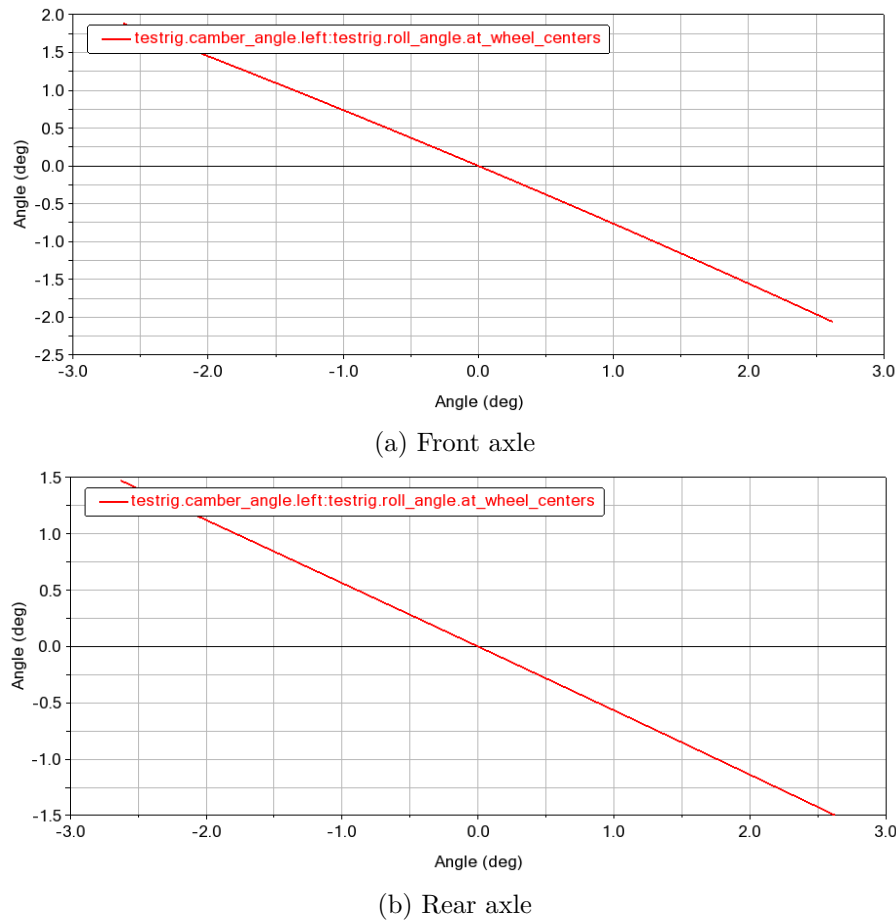


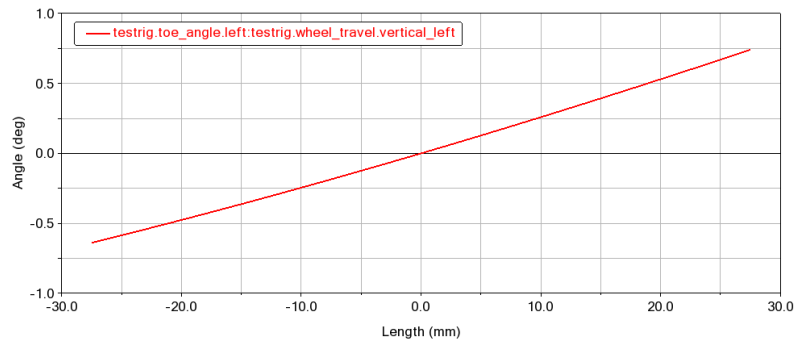
Figure 5.14: Camber angle against roll angle in opposite travel.

0.00042 deg/mm, as reported in Table 5.7. Both are well inside the qualitative requirement of Section 3.4, and the resulting toe excursions across the whole travel are 0.055° at the front and 0.022° at the rear. Figure 5.15 shows the front axle before and after the correction, on the same scale.

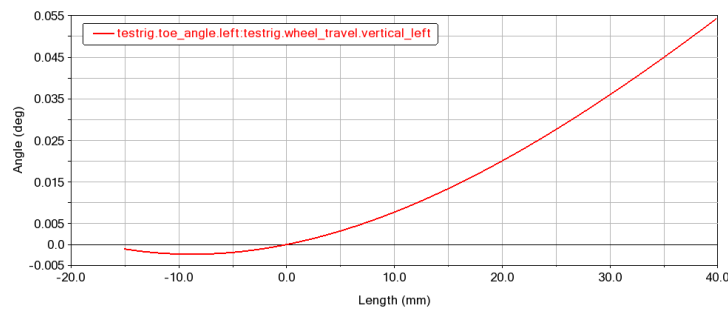
Table 5.7: Toe variation with wheel travel and its consequences at the design condition.

Quantity	Front	Rear
Gradient, jounce side [deg/mm]	0.00135	0.00042
Gradient, rebound side [deg/mm]	-0.00063	0.00072
Toe excursion over the travel [°]	0.057	0.022
Total toe change over δ_{aero} [°]	0.047	0.0040
Roll steer coefficient [deg/deg]	0.0060	-0.0016

The parity of the curve is what governs the roll steer (Figure 5.17). A curve that is exactly even in wheel travel, with its minimum at the static position, produces no axle steer at all: in roll the rising and falling wheels toe in the same rotational sense and their contributions cancel. Any departure from that condition introduces an odd component about the static position, and it is that component alone that produces roll steer.

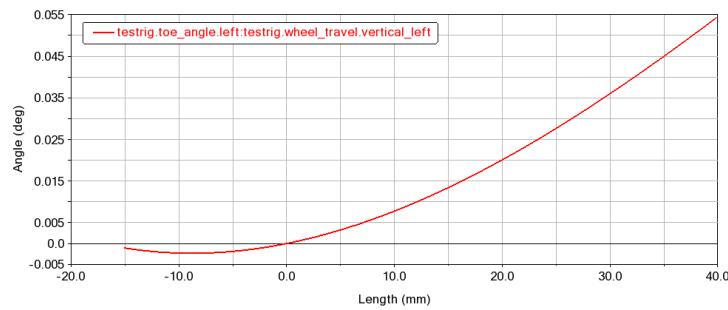


(a) Before the correction

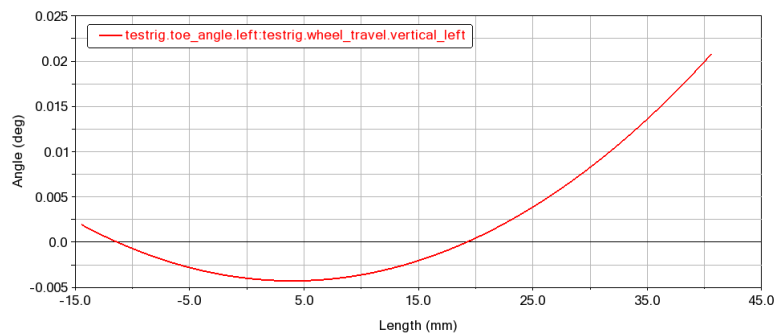


(b) After the correction

Figure 5.15: Toe angle against wheel travel at the front axle, before and after the instant centres were aligned with the tie-rod axis.



(a) Front axle



(b) Rear axle

Figure 5.16: Toe angle against wheel travel in parallel travel

Both values are more than an order of magnitude below what would constitute a deliberate handling contribution, and they are of opposite sign, so the residual effects of the two axles partly offset. For reference, the front roll steer coefficient before the geometry was revised was 0.105 deg per degree, which is comparable to the roll understeer that a designer would introduce intentionally.

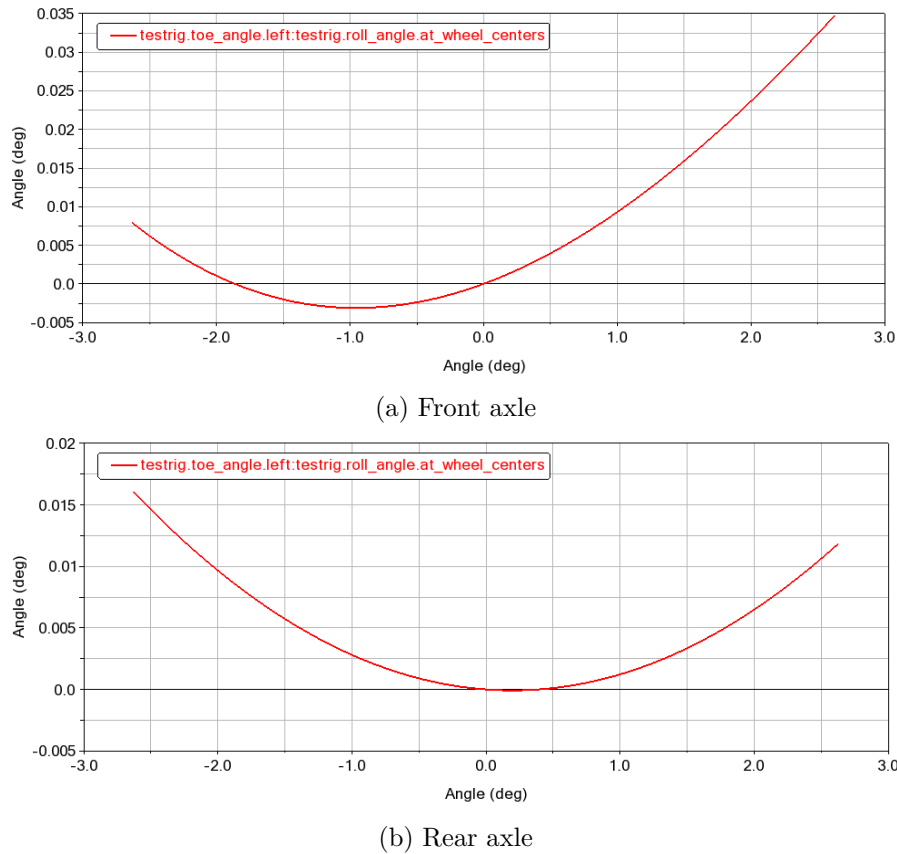


Figure 5.17: Toe angle of each wheel against roll angle in opposite travel.

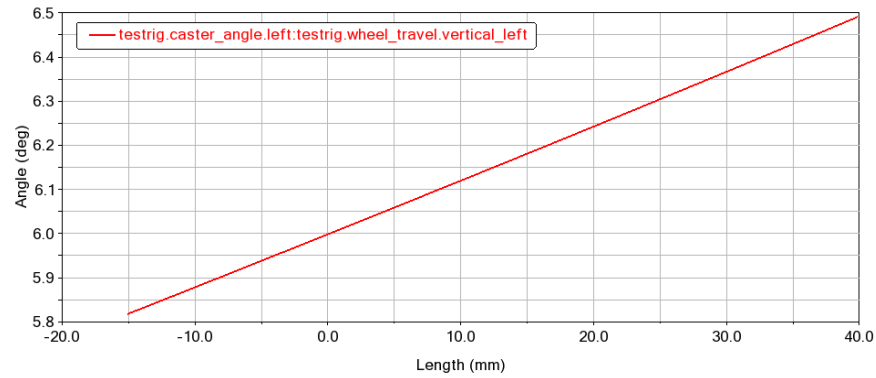
5.9 Steering Axis Geometry

The inclination of the steering axis is reported last in Table 5.8.

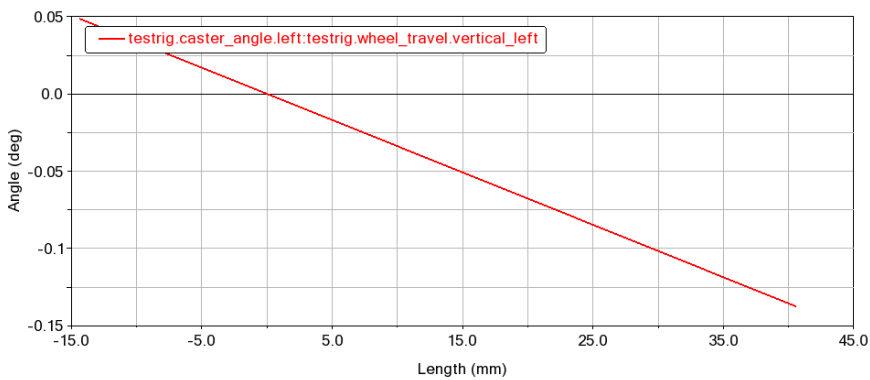
Table 5.8: Steering axis angles at the static position, against the preliminary targets of Section 3.4.

Quantity	Target	Front	Rear
Caster angle [°]	6	6.0	0.0
Kingpin inclination [°]	3–7	5.8	5.8

The front caster angle of 6.0° meets its target exactly, and varies by $\pm 0.34^\circ$ across the travel (Figure 5.18). The rear caster is zero at the static position by construction, since that axle does not steer, and its variation of $\pm 0.09^\circ$ is an order of magnitude smaller.



(a) Front axle



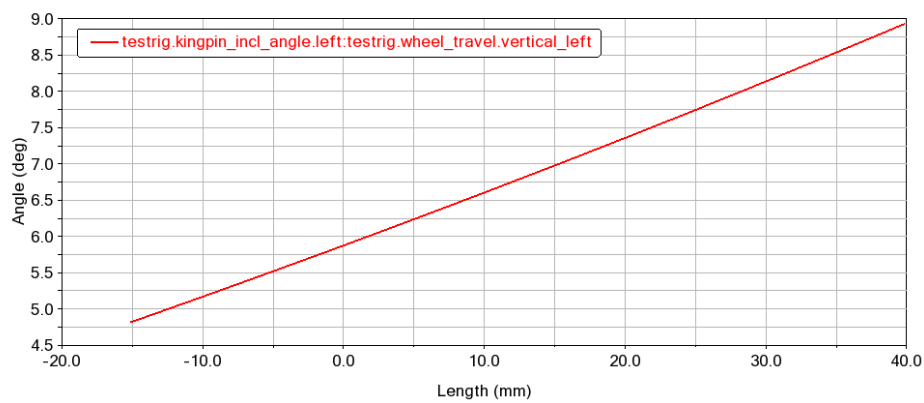
(b) Rear axle

Figure 5.18: Caster angle against wheel travel in parallel travel.

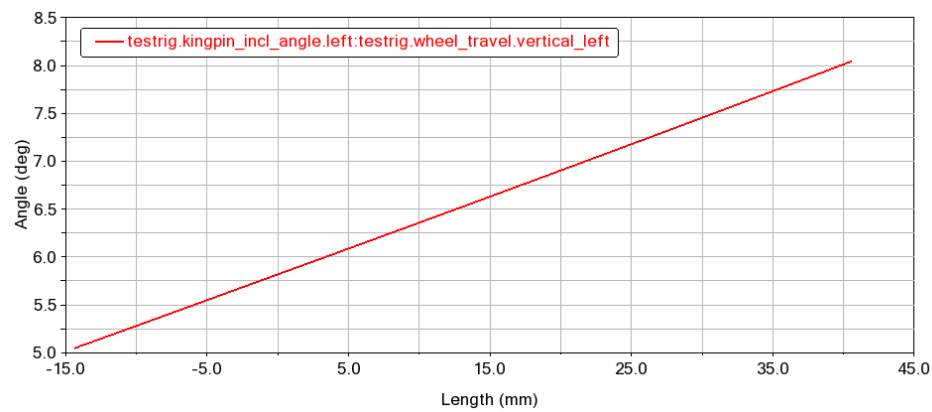
The kingpin inclination is 5.8° at both axles, inside the three to seven degree band adopted as a target (Figure 5.19). Its variation with travel is considerably larger than that of the caster and is linear, from 4.8° in full rebound to 8.9° in full jounce at the front, a gradient of 0.075 deg/mm , so the upper bound of the target band is exceeded beyond approximately 15 mm of jounce. Over the aerodynamic platform excursion of Section 4.1.1 the angle reaches 7.0° , at the edge of the band but within it. The consequence of a rising kingpin inclination is an increase in the steering torque required and in the jacking of the steered wheel, both of which grow only in the compressed part of the travel where the vehicle is already loaded.

5.9.1 Ackermann Geometry

The steering relation adopted for this vehicle is the one the team intended for the FSFEUP03 but could not implement there, the packaging of the chassis and suspension having prevented it. It is therefore verified here as an implementation requirement rather than examined as a design variable: the choice of the target itself lies outside the scope of the present work.



(a) Front axle



(b) Rear axle

Figure 5.19: Kingpin inclination angle against wheel travel in parallel travel.

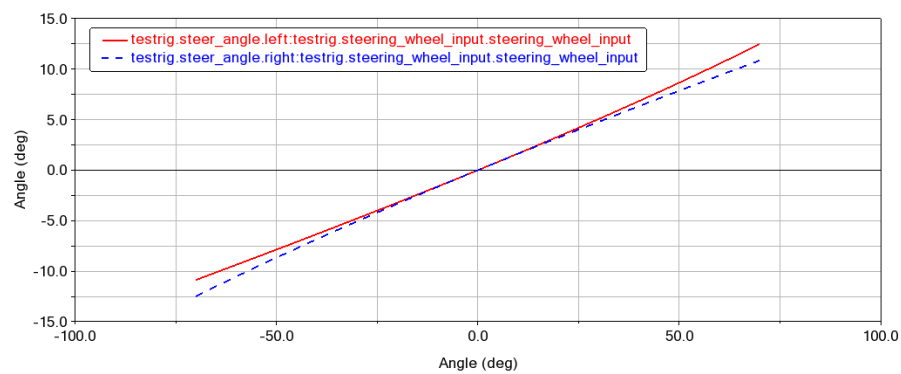


Figure 5.20: Steer angle of each front wheel against steering wheel input.

The target is specified in Section 3.4 as 11.54° at the outer wheel with 10.15° at the inner one, at the reference steering position. Since that position is not a sampled point of the analysis, as seen in Figure 5.20, the inner angle is obtained by linear interpolation between the two adjacent samples, listed in Table 5.9.

Table 5.9: Samples adjacent to the reference steering position, used to interpolate the inner wheel angle. Magnitudes are shown.

Inner wheel [°]	Outer wheel [°]
10.2407	11.6620
10.1988	11.6074
10.1570	11.5527
10.1151	11.4982
10.0733	11.4437

Interpolating the two highlighted rows at an outer angle of exactly 11.540° gives an inner angle of 10.147° , against the specified 10.15° . The target is therefore met to within 0.003° .

The relation is an anti-Ackermann one, the outer wheel steering more than the inner. For an outer angle of 11.54° on this wheelbase and track the implemented geometry delivers 10.15° , which corresponds to approximately -66% Ackermann. The intent is to exploit the fact that at high lateral acceleration the outer tyre carries almost all of the lateral force and reaches its peak at a larger slip angle than the lightly loaded inner one. Whether that reasoning holds for the events considered is not assessed here; the requirement was to reproduce the geometry, and it is reproduced.

5.10 Chapter Conclusions

This chapter carried the design of the preceding chapters into a multibody model and measured the quantities that depend on the real geometry.

The design parameters are reproduced. The four motion ratios agree with their design values within one per cent, the front rates to within a few tenths of a per cent, and the rear rates within four per cent. The assumption of a constant rate, on which the elastic sizing of Chapter 4 rests, is supported over the whole travel and not only near the static position: the rate spans 25.4 to 27.1 N/mm at the front and 30.4 to 32.4 N/mm at the rear, some three per cent about the static value, rising to a maximum near the middle of the jounce travel and falling thereafter.

The decoupling property survives the real geometry. Over the full travel the roll path is decoupled from heave to 99.0 % and the heave path from roll to between 98.5 and 99.0 %, and the residual is concentrated at the ends of the travel: over the excursions the vehicle actually uses it stays below half a per cent on both paths. The residual is a second-order geometric effect and not a left–right imbalance, since all four parasitic curves are even

functions of the travel and the single-wheel analyses show the individual motion ratios to be equal on the two sides of each axle.

The kinematic characteristics meet the targets of Chapter 3. The camber compensation in roll is 78 %, the front bump steer gradient was reduced by a factor of seventeen from its value before the geometry was revised, and the caster, kingpin inclination and Ackermann relation all fall inside their target bands, the last to within 0.003° . The roll-centre heights come out at 88 and 94.3 mm against the design values of 90 and 96 mm, a difference of the order of the tyre deflection at static load, which the front-view construction that produced the design values does not include.

Two results are characterisations rather than verifications, and both concern quantities no earlier chapter could produce. The kingpin inclination rises linearly through the travel and leaves the upper bound of its target band beyond approximately 15 mm of jounce, reaching 8.9° at full travel; over the aerodynamic platform excursion it remains within the band. And the roll centre migrates laterally far more than vertically, by some 180 mm at the design roll angle against 16 mm vertically, which displaces the roll axis from the vehicle centre plane and introduces an asymmetric jacking component that the symmetric analysis of Chapter 4 does not represent. Both lateral traces are linear in wheel travel, so the effect grows in proportion to roll angle and does not change character through the travel.

Chapter 6

Bellcrank Design, Structural Verification and Manufacturing Considerations

The bellcrank, also called the rocker, is the component through which the movement of the pushrod is transmitted to the heave and roll elements, and it is the component about which the least freedom remains by the time it is designed. The lever arms it must provide follow from the motion ratios adopted in Chapter 4, the loads it must carry follow from the load cases of Appendix E, and the stiffness it must have follows from the rate targets that the Chapter 4 derives. This chapter establishes those requirements, selects the pivot bearings and the layout that satisfy them, verifies the resulting component by finite element analysis, and settles the material and the manufacturing route.

6.1 Design Requirements and Design Loads

Three groups of requirements govern the component, and they are collected here so that every choice made later in the chapter can be traced to one of them. The envelope available to the bellcrank, the clearance it has to keep to the chassis and the stiffness required of its attachments are not treated here: they belong to the integration of the suspension into the monocoque and are established in Chapter 7.

6.1.1 Kinematic Requirements

The bellcrank has three attachment points besides its pivot: the pushrod, the heave element and the roll element. Their distances from the pivot axis are not free, since the ratio of the lever arms is one of the two factors that produce the motion ratios adopted earlier.

The other factor lies outside the component. The outboard geometry sets the ratio between the displacement of the wheel and the displacement of the pushrod, which by virtual work is the inverse of the ratio of the forces they carry,

$$\frac{\delta_w}{\delta_{pr}} = \frac{F_{pr}}{F_w}, \quad (6.1)$$

while the bellcrank sets the displacement of each element per unit displacement of the pushrod through the ratio of the corresponding arms. The individual motion ratios of Section 3.8 are the product of the two,

$$\text{MR}_h = \frac{F_{pr}}{F_w} \frac{r_{pr}}{r_h}, \quad \text{MR}_r = \frac{F_{pr}}{F_w} \frac{r_{pr}}{r_r}. \quad (6.2)$$

A ratio of lever arms is therefore not a motion ratio, and Table 6.1 keeps the two contributions separate. The motion ratios the arms below produce are those adopted in Chapter 4 and verified against the multibody model in Section 5.3.

Table 6.1: Lever arms of the bellcrank about the pivot axis, and the pushrod displacement ratio of the outboard geometry, which together give the motion ratios through Equation 6.2.

	Front	Rear
Pushrod arm r_{pr} [mm]	53.8	46.3
Heave element arm r_h [mm]	30.0	30.0
Roll element arm r_r [mm]	31.8	31.7
r_{pr}/r_h [-]	1.79	1.54
r_{pr}/r_r [-]	1.69	1.46
Pushrod ratio $\delta_w/\delta_{pr} = F_{pr}/F_w$ [-]	1.22	1.42

Two features of the arrangement are requirements and not consequences. The heave and roll arms must differ, since equal arms would give equal motion ratios and remove the freedom to size the two paths independently. And the attachment of the roll element is not mirrored between the two bellcranks of an axle: they pick it up at different heights, which is what makes an antisymmetric wheel input compress that element while a symmetric input leaves it undisturbed.

6.1.2 Design Load Case

The twenty-five load cases of Appendix E produce three candidate conditions for the bellcrank: the case that develops the largest force in the heave element, the case that develops the largest force in the roll element, and the end-of-travel condition in which the heave element reaches the usable damper stroke. Table 6.2 collects them.

Only one of the three needs to be analysed, and the reason lies in how the component is restrained. The bellcrank is a free body acted on by three forces in equilibrium, so the structural model applies two of them and reacts the third: the attachment of the heave element is restrained along the axis of that element, and the heave force appears

Table 6.2: Candidate design conditions for the bellcrank. Forces are axial, along the respective link, and negative values denote compression. The adopted condition is highlighted.

Bellcrank	Condition	Pushrod [N]	Heave el. [N]	Roll el. [N]
Front	Braking 1.8 g, bump on FL	-2406	-3304	-953
Front	Cornering 1.6 g, bump on FR	-2994	-2815	+2408
Front	Heave element at end of travel	-2292	-4110	≈ 0
Rear	Acceleration 1.6 g, bump on RL	-3084	-3590	-1107
Rear	Cornering 1.6 g, bump on RR	-3882	-3172	+2668
Rear	Heave element at end of travel	-3196	-4932	≈ 0

as a reaction instead of an input. The load applied to the model is therefore the pair of pushrod and roll element forces alone.

Read in those terms, the cornering case is the largest of the three in both applied components at once. At the front it develops 2994 N in the pushrod against 2406 and 2292 N for the other two, and 2408 N in the roll element against 953 and zero; at the rear the same ordering holds, with 3882 N against 3084 and 3196 N, and 2668 N against 1107 and zero. Neither of the remaining conditions applies a larger force in either direction, so neither can produce a more severe stress state, and the cornering case is adopted as the single design condition for the structural verification of Section 6.4.

6.1.3 Stiffness Requirement

The bellcrank is not only a load-carrying member. It sits in the load path between the wheel and the elastic elements, so any deflection of its own adds to the deflection of the spring and of the tyre. The three act in series, and the quantity the vehicle actually experiences is their combination: the ride rate. Writing K_e for the stiffness of the bellcrank referred to the wheel, and defining the ride rate as $\frac{1}{RR} = \frac{1}{WR} + \frac{1}{K_t}$, the effective ride rate considering the bellcrank compliance is given by:

$$\frac{1}{RR_{\text{eff}}} = \frac{1}{WR} + \frac{1}{K_t} + \frac{1}{K_e} = \frac{1}{RR} + \frac{1}{K_e} \quad (6.3)$$

Defining the absolute loss in ride rate as $\Delta RR = RR - RR_{\text{eff}}$, the exact relative loss is $\frac{\Delta RR}{RR} = \frac{RR}{RR + K_e}$. Assuming that the bellcrank is significantly stiffer than the suspension ($K_e \gg RR$), this relationship can be approximated as:

$$\frac{\Delta RR}{RR} \approx \frac{RR}{K_e} \quad (6.4)$$

To ensure that this relative loss does not exceed an admissible fraction p (i.e., $\frac{\Delta RR}{RR} \leq p$), evaluating the exact mathematical condition yields the minimum stiffness requirement:

$$K_e \geq RR \frac{1-p}{p}. \quad (6.5)$$

The reference for the criterion is the ride rate because the bellcrank is in series with the tyre as well as with the spring, so the loss it produces is a fraction of the combined rate, and it is the ride rate that governs the natural frequencies and the control of the aerodynamic platform established in Chapter 4.

Figure 6.1 shows why a requirement of this kind has to be stated as a threshold. Below roughly 1000 N/mm the ride rate falls away steeply, and a small change in the stiffness of the component produces a large change in the behaviour of the vehicle; above about 2500 N/mm the curve has flattened, and the stiffness gained for each unit of mass added to the component no longer buys any change in the rate the vehicle sees. The requirement is placed at the knee of that curve, where the component ceases to matter, and not at a value chosen for its own sake.

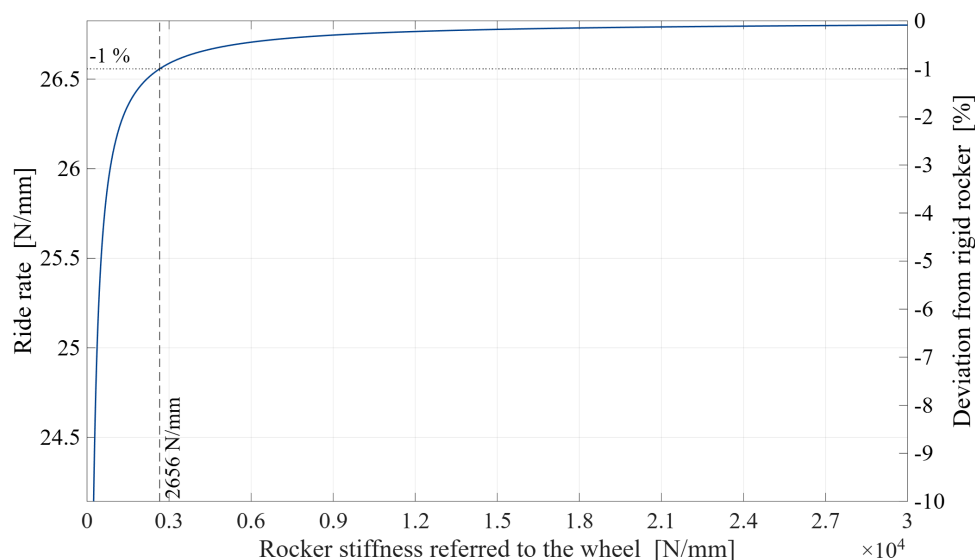


Figure 6.1: Ride rate of the front axle as a function of the stiffness of the bellcrank, with the spring, motion ratio and tyre rate of the adopted design. The dashed line marks the stiffness adopted as the requirement.

The admissible loss is not the same for both paths, because the quantities they feed are constrained very differently. The heave path determines the ride rate and through it the ride-height excursion under aerodynamic load, which is the binding constraint from which the entire elastic design of Chapter 4 is derived. The roll path determines the roll gradient, whose target was itself established in Section 4.5.2 as deliberately more permissive, on the grounds that roll occurs at low speed and under a fraction of the aerodynamic load.

The consequence of the difference can be quantified. Table 6.3 evaluates what a loss in the roll path costs, and the figures are small: two per cent moves the roll angle at the design lateral acceleration by three hundredths of a degree, which on the sensitivity of Table 4.9 corresponds to about two tenths of a percentage point of downforce in the cornering condition. Taken at its worst, with the whole of the allowance appearing at one axle and none at the other, the same two per cent displaces the front elastic share of the

lateral load transfer by half a percentage point; in practice the effect is smaller, since the four bellcranks are of the same design and their compliances are similar.

Table 6.3: Effect of a loss of stiffness in the roll path on the roll gradient and on the roll angle at the design lateral acceleration of 1.6 g.

Loss [%]	Roll gradient [deg/g]	Roll at 1.6 g [°]
0	0.938	1.500
1	0.948	1.516
2	0.957	1.531

The admissible loss is therefore allocated as one per cent to the heave path and two per cent to the roll path.

The threshold has to be transformed to the bellcrank, since the stiffness above is referred to the wheel whereas the finite element model reports a displacement at an attachment point of the component. The two are related by the same transformation that connects the wheel rate to the spring rate, so that at the pushrod attachment $K_{pr} = K_e (F_{pr}/F_w)^2$, with the ratio of Table 6.1. Table 6.4 collects the thresholds in the form in which they are checked.

Table 6.4: Stiffness required of the bellcrank at its allocated allowance for each path, referred to the wheel and to the pushrod attachment at which it is measured.

Axle	Path	Allowance [%]	At the wheel [N/mm]	At the pushrod [N/mm]
Front	Heave	1	2656	3979
Front	Roll	2	965	1446
Rear	Heave	1	3079	6199
Rear	Roll	2	965	1943

This requirement accounts for the bellcrank alone. The chassis attachments and the suspension links are further compliances in the same series chain, and their allocation within the overall budget is set out in Chapter 7.

6.2 Pivot Bearing Selection

The pivot bearings are selected before the bellcrank body is shaped, because the bore fixes the diameter of the journals at the ends of the shaft and the outside diameter fixes the supports that carry them, and both bound the geometry that follows. Each bellcrank is carried on two bearings placed at -35 and 160 mm along the pivot axis, measured from the heave pivot point, so that all three applied loads fall inside the bearing span.

Two decisions are taken here and they are not closed at the same point. The type of bearing follows from the duty and is settled in this section. The size within the series does not: every candidate carries the design load with a wide margin, so the rating does

not discriminate between sizes, and what the bore actually governs is the section of the journal that carries the load into the supports. That is a structural question, answered in Section 6.4.5, and Section 6.2.4 records the iteration it produced.

6.2.1 Duty and Design Load

The design load follows from the twenty-five load cases of Appendix E: 4061 N at the front and 4780 N at the rear, on the bearing nearer the pushrod, which carries three to four times the load of the far bearing. The governing condition is always cornering or braking with a single-wheel bump superimposed, that being the only condition in which the heave and the roll element are loaded at the same time. The two bellcranks of an axle are not identical, so the envelope of the two is applied to both.

The axial load is negligible. Both pushrod hardpoints lie in the plane normal to the pivot axis, so the pushrod force has no axial component in the nominal geometry. The bearings therefore have to locate the bellcrank axially and not to react a load.

The motion is oscillating and slow. Full wheel travel corresponds to some 48° of bellcrank rotation on either axle, at the frequency of the sprung-mass modes, and the bellcrank never completes a revolution. This characteristic separates the candidate bearing types more than the load does.

6.2.2 Candidate Bearing Types

To evaluate the best trade-off for the bellcrank pivot, four bearing families were selected based on their distinct performance characteristics relevant to suspension kinematics. Spherical plain bearings are traditionally suited for high-load oscillating duty and misalignment tolerance, although with higher friction. Conversely, rolling-element bearings (cylindrical roller, needle, and double-row ball) were considered to explore low-friction alternatives: roller and needle bearings offer exceptional radial load capacity in a compact package, while double-row ball bearings provide a compromise capable of supporting combined loads.

Four candidates were considered, one representative from each of these families available at the required 15 mm bore size. This specific diameter was determined through the iterative design of the bellcrank, rather than being an initial constraint. Table 6.5 collects the properties that distinguish them.

The candidates separate into two groups. The rolling-element bearings share a friction coefficient two orders of magnitude below that of the plain spherical bearing, and all of them carry the design load with a wide static margin. The spherical plain bearing is the only one intended for oscillating duty.

Table 6.5: Comparison of the candidate pivot bearing types. The friction figure is the pivot friction torque referred back to the wheel through the bellcrank and the pushrod, evaluated at the rear design load [32][33].

	Cylindrical roller NJ 202 ECP	Ball, double row 4202 ATN9	Needle, inner ring NA 4902.2RS	Spherical plain GE 15 C
Bore [mm]	15	15	15	15
Outside diameter [mm]	35	35	28	26
Width [mm]	11	14	13	12
Friction coefficient [-]	0.0011	0.0015	0.0015	0.05
Friction at the wheel [N]	0.6	0.8	0.9	41
Contact	line	point	line	sliding
Intended for oscillation	no	no	no	yes
Misalignment tolerance	very low	low	low	high

6.2.3 Selection Criterion

Static capacity does not discriminate: every candidate exceeds the design load by at least a factor of two. Packaging separates only the two 35 mm options from the rest, since the outside diameter sets the size of the chassis support in which the bearing is housed.

The criterion that decides is friction, and it decides because of what this suspension is for. The decoupled architecture exists to control the platform through separate heave and roll paths, and the damper forces it relies on are smallest at small suspension amplitudes. Pivot friction acts against exactly those forces. Referred to the wheel, the spherical plain bearing contributes 41 N against 0.9 N for the needle bearing, at a wheel velocity of 0.1 m/s where the rear damper produces 209 N. Twenty per cent of the damping force lost to friction, against half of one per cent, is not a marginal difference.

The disadvantage of a rolling-element bearing in this duty is false brinelling, in which small-amplitude oscillation never moves the contact points far enough to redistribute the lubricant, so that the raceways are marked without the bearing ever completing a revolution. The concern is bounded here: the bellcrank rotation over the operating range is of the order of tens of degrees, well beyond the pitch angle between adjacent rolling elements, so the contact points do move.

6.2.4 Selection and Revision of the Bore

A needle roller bearing with an inner ring is adopted at each of the eight pivots. It provides the friction of a rolling-element bearing in a smaller outside diameter than either rolling-element alternative, and the inner ring removes the hardened and ground shaft that a drawn-cup needle bearing would require. The integral seals retain the grease and exclude the contamination that governs the practical life of a bearing operating far below its load limit.

The size within the series was not settled at this stage, for the reason given at the head of the section, and the first iteration of the geometry used the NA 4901.2RS, of 12 mm

bore. Under the design case of Section 6.1.2 the journal at that diameter proved to be the most highly stressed region of the component, and the bore was carried to the next size in the series, the NA 4902.2RS of 15 mm. Since both the bending and the torsional stiffness of a shaft section vary with the fourth power of its diameter, that step relieves the journal without carrying the chassis supports, and with them the envelope of the whole assembly, any further than the packaging allows.

6.3 Concept and Layout

The requirements of Section 6.1 fix the ratios between the three lever arms, and the bearing selection of Section 6.2 fixes the diameter and the width of the journals. Neither fixes the absolute length of the arms, nor the arrangement of the three attachments about the pivot, nor the form of the body that connects them.

The absolute size is settled first, and the argument is short. Since the motion ratios and the forces in the three links follow from the ratios of the arms alone, scaling all three arms together leaves both unchanged; what it does change is the moment the body carries, which grows in proportion to their length. The arms are therefore made as small as the journals and the material required around each attachment bore allow, and the values of Table 6.1 are the result.

What remains are the arrangement of the attachments about the pivot and the form of the body, and they are taken in that order, since the arrangement of the points defines the load path whose shape the body then has to follow.

6.3.1 Material Class and Manufacturing Route

The component is machined from solid, a laser-cut and welded fabrication having been set aside, so the material is restricted to those suited to substantial removal from plate or bar stock. The route is settled before the body is shaped, since it imposes a minimum internal radius on every feature and makes distortion the practical risk of manufacture.

The class is settled here and the alloy is not. The class enters the deflections through the elastic modulus and the geometry through the manufacturing route, both of which the sections that follow depend on; the alloy enters only the allowable against which the stresses are read, and is therefore chosen in Section 6.5, once they are known. Five materials are compared, spanning the three classes of structural metal.

The specific stiffness is the same for all five, to within three per cent, even though the densities differ by a factor of almost three and the moduli by a factor of three. No available structural metal therefore reduces the compliance of the bellcrank at equal mass, so the deflection requirement of Section 6.1.3 is met through geometry alone and plays no part in the material choice.

The two non-aluminium candidates are set aside on the same criterion for opposite reasons. The steel is the weakest of the five on specific strength, at 55 kNm/kg against

Table 6.6: Nominal properties of the candidate alloys and the derived specific quantities. Values are typical for wrought plate and are used for comparison between alloys, not as design allowables.[34]

Alloy	Class	σ_y [MPa]	σ_u [MPa]	E [GPa]	ρ [g/cm ³]	σ_y/ρ [kNm/kg]	E/ρ [MNm/kg]
6061-T6	Aluminium	276	310	68.9	2.70	102	25.5
2024-T351	Aluminium	325	470	73.1	2.78	117	26.3
7075-T6	Aluminium	503	572	71.7	2.81	179	25.5
25CrMo4	Steel	435	670	205.0	7.85	55	26.1
Ti-6Al-4V	Titanium	880	950	113.8	4.43	199	25.7

179 kNm/kg for 7075-T6, so a steel bellcrank of equal static margin would weigh more than three times as much; for a component whose mass acts through the motion ratio on the very quantities this dissertation sizes, that penalty is not recoverable elsewhere. Titanium is the opposite case, with the highest specific strength of the five, and it is set aside on manufacturability and cost: its low thermal conductivity concentrates heat at the cutting edge and its reactivity with most tool materials promotes adhesion and diffusion wear, so that cutting speeds and tool life fall an order of magnitude below those achievable in aluminium [35].

6.3.2 Arrangement of the Attachment Points

The motion ratios fix the ratios of the arms but say nothing about the angles between them, and those are what the layout decides. Each of the three links is placed perpendicular to its own arm at the static position.

The criterion follows from how the moment about the pivot is produced. A link at an angle θ to its arm develops a moment $Frs\sin\theta$, so the effective arm is $r\sin\theta$, and the motion ratio that the arms are meant to deliver is obtained exactly only where $\sin\theta = 1$. Away from that position the effective arm falls, and it falls slowly at first: the derivative of $\sin\theta$ vanishes at 90° , so the perpendicular arrangement is the one in which the effective arm, and with it the motion ratio, is least sensitive to the rotation of the bellcrank. That is also the condition that keeps the mechanism furthest from the alignment of a link with its own arm, where the moment about the pivot would collapse and the ratio would run away.

The perpendicular condition can be met at one position only, and the static position was adopted for it. That choice is not the one that gives the least variation over the travel. The interval swept is not symmetric about the static position, so setting the perpendicular condition at the middle of that interval would place the stationary point of $\sin\theta$ at the centre of the range instead of near one end, and the motion ratio would vary less across the travel as a result. Establishing that position and the variation it would recover is identified as future work; the consequence of the choice made here is measured in Section 5.4, where the rate is found to vary by some three per cent about its static value.

The three attachments are placed within the bearing span. With the two bearings at -35 and 160 mm along the pivot axis, every applied load falls between them, so the shaft is loaded in bending between two supports and not as a cantilever, and the moment it carries follows from the offset between each journal and the face of its support. The pushrod, which applies the largest of the three forces, is placed nearer the bearing that Section 6.2.1 identifies as carrying three to four times the load of the other.

6.3.3 Form of the Body

The two elements are not picked up in the same way. The heave element is attached immediately above the pivot axis, in the plane in which the pushrod acts, while the roll element is attached out on the body, offset along that axis, and the two bellcranks of an axle pick it up at different heights. That offset is what the architecture requires, and it is also what makes the two load paths through the component so different in length, as Section 6.4.6 measures.

The body adopted is a shaft carrying the three attachments as lugs projecting from it, and is shown in Figure 6.2. Its two ends are turned down to the bore of the selected bearing and serve as the journals on which the inner rings are mounted, the bearings themselves being housed in the chassis supports.

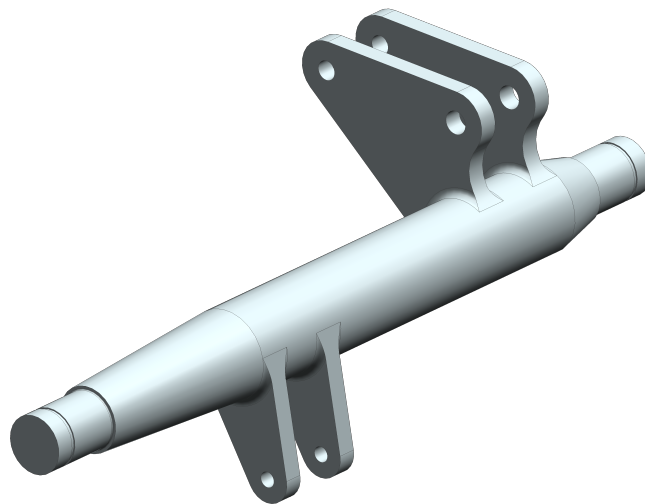


Figure 6.2: The front left bellcrank. The pushrod and heave element attachments are carried by a single clevis block at one end of the shaft, the roll element attachment by a pair of ears offset along the pivot axis, and the two journals are turned at the ends of the shaft itself.

The form follows from where the three attachments are. The pushrod and the heave element are picked up close together at one end and the roll element out along the axis, so the material that connects them is long in one direction and compact in the other two. Machining from solid imposes on every feature a minimum internal radius set by the diameter of the smallest tool that can reach it.

6.3.4 Assembly

The bellcrank is carried in two supports on the chassis, one at each end of its shaft. Each inner ring is mounted on its journal, seated against a shoulder of the bellcrank on one side and retained by a circlip on the other, while the outer ring seats against a face of the chassis support on one side and is retained by a circlip on the other. The two ends are assembled in the same way. Each of the three links is attached by a bolt in double shear, the lugs being machined as clevises.

6.4 Structural Verification by FEA

The bellcrank is verified by finite element analysis in Abaqus. Four models are required, one per corner. The two axles differ in their lever arms, the pushrod acting at 53.80 mm from the pivot axis at the front and at 46.30 mm at the rear against 30.00 mm for the heave element on both, and the two bellcranks of an axle are not a mirrored pair either, since the roll element is picked up at a different height on each.

6.4.1 Strength Criterion

The criterion is that the bellcrank sustain the design case of Section 6.1.2 without yielding, with a factor of 1.5 on the yield strength of the material, and that it reach the design life of Section 6.4.7 without initiating a fatigue crack.

The limit state is yielding. For this component that is not merely a conservative choice: permanent deformation at the journals or at either element attachment would alter the lever arms of Table 6.1, and with them the motion ratios on which the rates and the damping of Chapter 4 depend. A bellcrank that has yielded is therefore no longer the component that was designed, even if it remains intact.

The magnitude of the factor follows the convention of aeronautical practice, in which the ultimate load is taken as one and a half times the limit load. No factor is prescribed for suspension components by the Formula Student regulations, so a value has to be adopted and justified, not cited. Applying it to the yield strength rather than to the ultimate strength is the more demanding of the two for the aluminium alloys of Table 6.6, whose ratio of yield to ultimate strength is high and whose ductility beyond yield is limited; the allowable that results is evaluated in Section 6.5, once the alloy is fixed. What the factor absorbs is the uncertainty the analysis does not represent — load cases derived from a vehicle model and not measured on the car, a quasi-static and linear elastic analysis with idealised connections at the bores, and nominal material properties. It does not absorb the dynamic content of a road input, which is already carried on the load side by the vertical factor of 1.5 g applied at one wheel in the design case, so that the two allowances are not counted twice.

6.4.2 Load Case

The single design case of Section 6.1.2 is applied: cornering at 1.6 g with a bump on the outer wheel, at the forces highlighted in Table 6.2. In cornering the loaded pushrod is the one on the outer wheel, so applying the case to both sides of an axle is an envelope; since the two bellcranks are not mirror images of one another, neither result can be inferred from the other. No symmetry is exploited within the model, and the case is itself asymmetric.

The direction of each applied force is that of the end-of-travel position and not that of the static geometry. The reason is that the governing case does not occur at the static position: the bellcrank can rotate only about 34.6° at the front and 35.3° at the rear from static before the heave element reaches the end of its stroke, whereas the loads of the design case would demand between 42 and 66° on the loaded bellcrank. The mechanism therefore reaches its stop under this condition, and the end-of-travel orientation is the correct one for the loaded bellcrank and not an approximation to it.

6.4.3 Model and Boundary Conditions

The bellcrank is a free body acted on by three forces in equilibrium, and the model exploits that: two of the three are applied and the third is replaced by the restraint that reacts it. The attachment of the heave element is held along the axis of that element, and the force in it is a reaction and not an input. Since the free body is statically determinate, any one of the three could have been substituted in this way without altering either the reactions or the stresses; the heave element is chosen because it is where the rotation is reacted in the condition being analysed. Section 6.4.2 established that the mechanism reaches the end of the heave element's stroke under the design case, so at that position the element bears against its stop and the rotation about the pivot is carried by it, which is what the restraint represents.

Some restraint there is necessary in any event: the applied forces being in equilibrium makes the resultant zero, not the rotation restrained, and leaving it free produces a rigid-body mode and a singular stiffness matrix.

Table 6.7 collects the definition of the model, and Figure 6.3 shows it on the geometry.

Table 6.7: Definition of the structural model. All couplings connect the surface of the feature to a reference point at its centre; the two bearing seat reference points lie on the pivot axis.

Feature	Prescribed	Value
<i>Applied</i>		
Pushrod attachment	Force along the pushrod axis	Table 6.2
Roll element attachment	Force along the element axis	Table 6.2
<i>Restrained</i>		
Heave element attachment	Translation along the element axis	0
Both bearing seats	All degrees of freedom except rotation about the pivot axis	0

The bearing seats reproduce what the bearings and their supports permit, which is rotation about the pivot axis and nothing else. That leaves exactly one freedom in the model, and it is the one the heave element restraint removes.

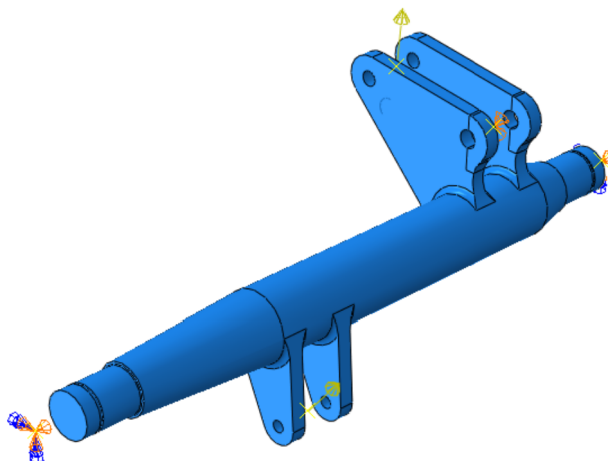


Figure 6.3: Load and restraint arrangement of the structural model. The two applied forces are shown at the pushrod and roll element attachments, each along the axis of its link; the heave element attachment is restrained along its own axis and reacts the third force; the two bearing seats are coupled to reference points on the pivot axis and permit only rotation about it.

6.4.4 Mesh

The bellcrank is meshed with quadratic tetrahedral elements, the ten-node tetrahedron C3D10 of the Abaqus element library, which carries a quadratic interpolation of the displacement field and therefore represents a linear variation of strain within each element [36]. Linear tetrahedra are avoided for the reason that follows from that same property: the four-node element is a constant-strain element, so it cannot reproduce the strain gradient that bending imposes across the section and behaves too stiffly under it, and the stresses it returns are least reliable where the gradient is steepest, which is precisely at the fillets that govern this component [37, 38].

Stresses in the immediate neighbourhood of the coupling reference points are not evaluated, since the coupling constraint is a modelling device and not a physical connection: it concentrates the applied load on the boundary of the bore, where the stress it produces has no dimension of its own and grows without bound as the mesh is refined. Results are read at a distance of several elements from those regions.

The element size was established by refinement, on the front right bellcrank, where the global maximum falls at a fillet and no region has to be excluded from the evaluation, so that the curve refers unambiguously to the quantity reported. Table 6.8 collects the peak von Mises stress and the displacement of the loaded point at each level.

Table 6.8: Mesh refinement on the front right bellcrank under the design case.

Element size [mm]	Elements [-]	Nodes [-]	Peak stress [MPa]	Displacement [mm]
10	6 123	11 538	143.7	0.40
8	8 781	15 900	191.9	0.40
4	24 551	41 368	190.6	0.41
3	39 650	64 862	186.3	0.41
2	87 485	138 147	184.6	0.41
1.5	204 164	313 628	186.6	0.41

The peak occurs at a fillet of finite radius, where the elastic solution is bounded and the discretised one converges to it. Between the two finest levels it changes by 1.1 per cent and the displacement by 0.2 per cent, which establishes that the value reported is a stress concentration resolved by the mesh and not an artefact of it. The coarsest level is the exception and is retained to show why: at 10 mm the fillet is not spanned by a single element, and the peak returned is a quarter below the converged value because the concentration is not represented at all.

The 2 mm mesh, highlighted in the table, is the one used for the four models of Section 6.4.5; it lies within 1.1 per cent of the finest level at a tenth of its size.

6.4.5 Results

Table 6.9 collects the peak von Mises stress on each of the four bellcranks under the design case, read in element-nodal values without averaging. The stresses do not depend on the alloy, since the analysis is linear elastic and the component is of a single material; the factors of safety they permit are established in Section 6.5, where the alloy is selected against them.

Table 6.9: Peak von Mises stress on each bellcrank under the design case of Section 6.1.2.

Bellcrank	Peak stress [MPa]
Front left	155.5
Front right	184.6
Rear left	175.8
Rear right	312.6

Three of the four fall in a narrow band between 155 and 185 MPa, and the fourth does not. The rear right bellcrank reaches 313 MPa, as verified in Figure 6.4, sixty-nine per cent above the highest of the other three, and it is the governing component of the set.

The difference is not an axle effect. Were it one, the two rear bellcranks would both stand above the two front ones, and they do not: the rear left is the second lowest of the four. What separates the two sides of an axle is the attachment of the roll element, which is picked up at a different height on each bellcrank and is therefore the one feature of the four that is not mirrored. The angle between the roll element load and the pushrod

load consequently differs from one side to the other, and with it the extent to which the two superpose at the pushrod lug fillet. On the rear right the two act in directions close enough for their effects to add at that fillet, whereas on the rear left they act more nearly at right angles and the peak is correspondingly lower.

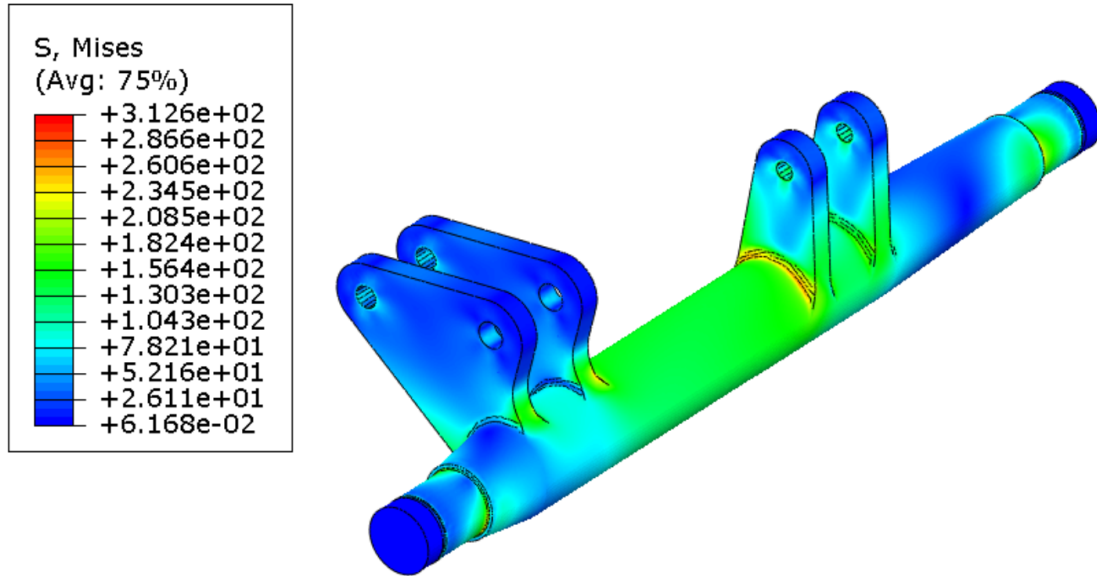


Figure 6.4: Von Mises stress in the rear right bellcrank under the design case, which is the most highly stressed of the four.

The peak on the governing bellcrank sets the requirement on the alloy. Sustaining 313 MPa with the factor of 1.5 on yield of Section 6.4.1 calls for a yield strength of at least 469 MPa, which is the figure the selection of Section 6.5 has to meet.

6.4.6 Stiffness of the Two Load Paths

The results above establish the stress. What remains is the requirement of Section 6.1.3, which is stated on the deflection of the bellcrank and is measured in a separate pair of load steps.

Each path is characterised by its own load step in the same model, so that the compliance of one is not contaminated by the load applied through the other. In both steps a force of 1000 N is applied at the pushrod attachment, along the pushrod axis, and nothing else is applied; the two steps differ only in which element attachment is restrained. Restraining the heave attachment along its own axis, together with the two bearing seats, measures the deformation of the bellcrank between the pushrod and the heave element, which is the compliance the heave path sees; restraining the roll attachment instead measures the corresponding quantity for the roll path. The stiffness follows as $K = F/\delta$ from the displacement of the loaded point, taken relative to the pivot axis so that rigid-body motion permitted by the restraints is excluded, and projected onto the pushrod axis.

Table 6.10 collects the measured values for the four bellcranks. All four satisfy their allowance on both paths, but by very unequal margins: the heave path is stiffer than required by a factor of between four and seven on every bellcrank, while the roll path uses most of the two per cent it has been given.

Table 6.10: Measured stiffness of the four bellcranks, from the displacement of the pushrod attachment under 1000 N applied at that point, projected on the pushrod axis. The loss is evaluated against the allowance allocated to each path in Table 6.4: one per cent for heave and two per cent for roll.

Bellcrank	Heave path			Roll path		
	δ	K	Loss	δ	K	Loss
	[mm]	[N/mm]	[%]	[mm]	[N/mm]	[%]
Front left	0.039	25419	0.16	0.54	1854	1.6
Front right	0.037	26846	0.15	0.51	1977	1.5
Rear left	0.029	34263	0.18	0.50	2010	2.0
Rear right	0.038	26445	0.24	0.46	2169	1.8
Allowance		1.00			2.00	

These are the values of the geometry as finally drawn. The sections were arrived at iteratively, and it was the roll allowance that drove the iterations throughout: no intermediate form failed the strength criterion, and every change made to the component was made to recover compliance on that path.

The load is applied along the axis of the pushrod in the end-of-travel position.

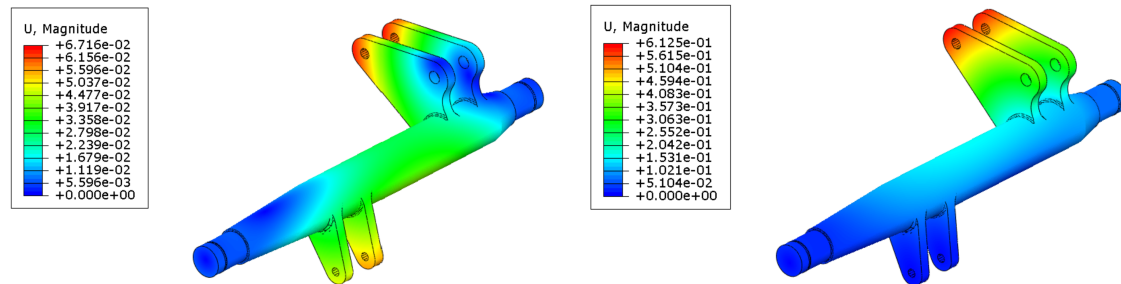


Figure 6.5: Displacement field of the front left bellcrank under the load used to characterise the heave path (left) and the roll path (right). The scales differ by an order of magnitude.

The difference between the two figures is geometric. The paths share everything from the pushrod to the pivot and separate only at the end: the heave element is picked up in the same block as the pushrod, whereas the roll element is picked up out along the body, offset along the pivot axis. The roll path is therefore carried in bending and torsion of the body over most of its length, and the displacement field of Figure 6.5 confirms it: the gradient is distributed along the body instead of concentrating at the element attachment.

Two things follow. The margin on the heave path is not material available to be removed, since almost none of the structure serves that path alone. And compliance on

the roll path is recovered only in the section of the body, whose bending and torsional stiffness both vary with the fourth power of its diameter.

That is what makes stiffness, and not strength, the requirement that sizes this component. The peak stress is a local concentration at a fillet, relieved by opening its radius or adding material at the root of the lug, neither of which costs appreciable mass; the compliance of the roll path is distributed, and is bought only in material where the mass of the component is. The rear right bellcrank, tightest of the four on strength at a factor of 1.6, is no exception: the geometry that would relieve it costs nothing against the roll allowance.

The stiffness reported here is an upper bound on that of the assembled component, which includes clearance in the bearings and compliance at the bolted attachments that the model does not represent.

6.4.7 Fatigue Assessment

The bellcrank is not loaded between a working state and an unloaded one: the roll element reverses between cornering to one side and to the other, and the force in the pushrod follows the vertical load on the wheel throughout a lap. The cycle that damages the component is therefore not defined by the design case, and the assessment is made against a service history instead.

A design life has to be stated, and it follows from the duty the component is expected to see. The distance adopted is 585 km, the target the team sets for the running of the vehicle, which enters the analysis as 585 repetitions of a representative autocross lap of about 93 s.

The component is required to survive some 10^4 significant cycles, which places it in the finite-life region of the curve and well short of the 5×10^8 at which the fatigue strength of aluminium alloys is conventionally quoted.

The history is taken from a lap simulation of the FSFEUP03, for which a model of an autocross lap is available. What is taken from it is the description of the manoeuvre — the lateral and longitudinal accelerations and the tyre forces at each wheel — and not the loads in its suspension, which belong to a different architecture with different motion ratios and cannot be transferred by scaling. Those channels are passed through the load model of Appendix E, the same model that produced the twenty-five load cases, so that the force histories obtained are those of the present vehicle throughout. The lap is not perfectly symmetric between left-hand and right-hand corners, as no real circuit is, so each bellcrank sees its own history and the four are analysed separately.

The stress history at every node is assembled by superposition, and not by solving the structure at each instant. Because the attachment of the heave element is restrained, the free body carries two independent applied forces and not three, and each of the two is resolved into two components in the reference frame of the bellcrank itself, giving four unit stress fields. The decomposition is what accommodates the rotation of the component,

which sweeps close to 48° on either axle over the travel: a single unit case applied in a fixed direction would represent only one position of it. With the forces expressed in components fixed to the body, the superposition

$$\sigma_{ij}(t) = \sum_{k=1}^4 \sigma_{ij}^{(k)} F_k(t) \quad (6.6)$$

is exact within the linear elastic model, at every node and for every component of the stress tensor.

The assessment is carried out in fe-safe on the superposed history, using the Brown–Miller criterion with the Morrow correction for mean stress. A critical-plane criterion is appropriate here because the two force channels do not vary in proportion to one another, so the stress state does not remain proportional through the lap and the orientation of the principal directions moves; a criterion based on an equivalent scalar stress would not represent it. The highest peak stress of Table 6.9 is under two thirds of the yield strength of the alloy, so the notch correction the solver applies returns the elastic values and the assessment is one of high-cycle fatigue conducted with a strain-life model. That curve is corrected for a machined finish.

Table 6.11 collects the minimum life returned on each bellcrank. Every one of them clears the 585 repetitions required by the design distance by at least three orders of magnitude.

Table 6.11: Minimum life of each bellcrank from the Brown–Miller assessment, in repetitions of the lap. The requirement is 585 repetitions.

Bellcrank	$\log_{10} N$	N [repetitions]	Location of the minimum
Front left	7.36	2.3×10^7	Change of section and pushrod lug fillet
Front right	6.13	1.3×10^6	Attachment bores
Rear left	8.60	4.0×10^8	Change of section next to the bearing seat
Rear right	9.52	3.3×10^9	Body between the lugs and changes of section

The lowest of the four is read at the attachment bores of the front right bellcrank (Figure 6.6), where the coupling constraints are applied and the stresses are a product of the idealisation, which is why they are excluded from the static results of Section 6.4.5. The fatigue calculation does not admit the same treatment: the life is computed at every node from the stress history at that node, and the minimum over the model is reported wherever it falls. Where it coincides with a bore it is therefore a lower bound and not an estimate, and the physical minimum on that component lies above the figure tabulated. On the remaining three the minimum falls at a change of section or at a lug fillet, which are physical locations, and there it is read as it stands.

The order of the four is not that of the static analysis, and the reversal is informative. The rear right bellcrank carries the highest peak stress of the set under the design case, at 312.6 MPa, and returns the longest fatigue life of the four. The two are governed by

different things: the design case is a single extreme event carrying a vertical factor of 1.5 g at one wheel that no part of the lap reproduces, whereas the damage accumulated over the lap follows the amplitude of the cycles the lap actually contains.

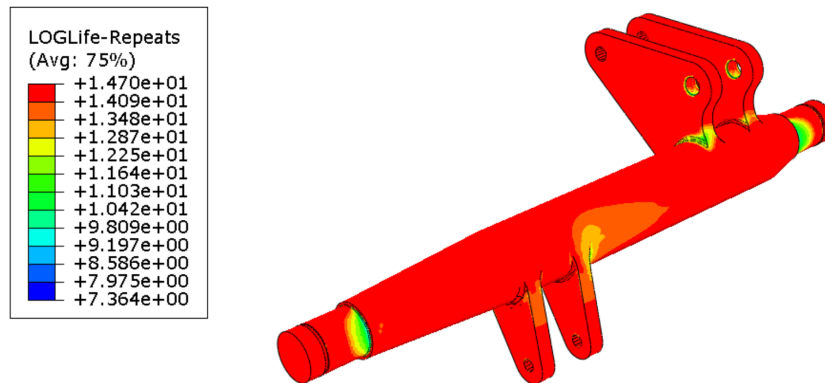


Figure 6.6: Logarithm of the life of the front left bellcrank, in repetitions of the lap, from the Brown–Miller assessment. The design requirement is 585 repetitions.

The history is quasi-static: a lap simulation reproduces the manoeuvre content of the duty — braking, cornering, traction — but not the content of the road surface, so kerbs and the high-frequency vertical input do not appear in it, and a single circuit is represented. The allowance for that input is made on the side of the static verification instead, through the vertical factor of 1.5 g carried by the design case. The result therefore establishes that the duty the vehicle is designed for does not consume the fatigue capacity of these components; the size of the margin should not be read as a measure of how much capacity remains.

6.5 Selection of the Alloy

The class was settled in Section 6.3.1 and what remains is the choice between the three aluminium alloys of Table 6.6, made here because this is the first point at which the stresses the component develops are known.

6.5.1 Strength at the Governing Location

The criterion that decides is strength. Against the peak stress of 313 MPa on the rear right bellcrank, 6061-T6 would yield outright, its yield strength of 276 MPa lying below the stress developed, and 2024-T351 reaches a factor of 1.04, which leaves it at yield in all but name. Neither meets the 1.5 required by Section 6.4.1. 7075-T6 gives 1.61 and is the only one of the three that satisfies the criterion.

With a yield strength of 503 MPa the allowable is 335 MPa. Table 6.12 gives what each bellcrank achieves in that alloy.

Table 6.12: Factor of safety on the yield strength of 7075-T6, 503 MPa, at the peak stress of each bellcrank. The requirement is 1.5.

Bellcrank	Peak stress [MPa]	Factor of safety [-]
Front left	155.5	3.23
Front right	184.6	2.72
Rear left	175.8	2.86
Rear right	312.6	1.61

The margin on the rear right bellcrank is the one that decides the alloy, at seven per cent above the requirement against factors above 2.7 on the other three. That margin is a property of the fillet as drawn and not of the size of the component, for the reason given in Section 6.4.6, and it would be recovered by local geometry at negligible mass. The alloy is nonetheless selected against the geometry that was analysed.

6.5.2 Machining Behaviour and Surface Finish

The higher strength of 7075-T6 is often assumed to make it the more difficult alloy to machine, and in terms of cutting forces and tool load it is. The comparison is not one-sided, however. In dry ball-end milling of aircraft 7000-series plate, Yeganefar et al. [39] report less burr formation and better surface quality on 7075-T6 than on the softer 7050-T7451, while the latter was the more machinable alloy on the remaining attributes. The distinction matters for this component, whose bearing seats and attachment bores carry the tightest tolerances on the part and where the number of finishing operations is governed by burr formation and surface finish, not by cutting force.

Surface finish is not a matter of appearance here. Yang et al. [40] establish a quantitative relation between surface roughness and fatigue life in machined aluminium, treating the surface topography as a series of micro-notches with an associated stress concentration factor. Since the bellcrank is a fatigue-loaded component with a reversing load path, and since the strain-life curve used in Section 6.4.7 is corrected for a machined finish, the roughness specified on the drawing is part of the structural specification of the component.

6.5.3 Corrosion and Temper

The peak-aged temper of 7075 is the most susceptible of the alloy's tempers to intergranular stress corrosion cracking. Oñoro [41] compared several precipitation-ageing treatments of 7075 in salt spray fog and in sodium chloride solution and found every temper to be susceptible, but with T6 consistently the worst and T73 consistently the best; the over-aged temper achieves that resistance at the cost of strength. For a vehicle stored and run in the open the exposure is real but mild compared with the marine and aerospace service in which those results were obtained, and the strength of the peak-aged temper is required by the stress on the governing bellcrank. The peak-aged condition is therefore retained

and the component is anodised, with the over-aged temper recorded as the alternative should service experience indicate that it is needed.

Aluminium 7075-T6 is adopted.

6.6 Drawing Specification

The bellcrank is machined from solid, and three of its features are specified for reasons established earlier in this chapter.

The features that carry the tightest tolerances are the two bearing seats and the three attachment bores. In Appendix G seats are toleranced for the fit required by the inner ring of the bearing selected in Section 6.2, and for coaxiality with one another, which the needle bearing requires. The bores are toleranced in position relative to the pivot axis, since it is their distance from that axis that produces the motion ratios of Table 6.1, and a departure there propagates into the rates of Chapter 4.

The third is the surface finish. A general roughness of $R_a 3.2$ is called out, reduced to $R_a 0.8$ at the attachment bores and $R_a 0.4$ at the bearing seats, where the tolerances are tightest and the material is most highly stressed.

6.7 Chapter Conclusions

The chapter delivers four bellcranks, one per corner, machined from 7075-T6 and pivoting on needle roller bearings.

None of the requirements they meet originated in the component itself. The lever arms follow from the motion ratios through the pushrod force ratio of the outboard geometry, and the implemented geometry reproduces the adopted ratios to within a quarter of a per cent. The design load is a single condition, cornering at 1.6 g with a bump on the outer wheel, which was shown to envelope the other candidates in both of the forces the structural model applies.

The pivot bearing was selected on friction and not on capacity, every candidate carrying the design load with a wide margin. Referred to the wheel, a spherical plain bearing would consume a fifth of the damper force at low suspension velocity against half of one per cent for a needle bearing, and in a suspension whose purpose is to control the platform through its damping that difference decides.

Stiffness, and not strength, is what sizes the component. The roll path consumes most of the two per cent of compliance allocated to it on all four bellcranks, while the heave path is stiffer than required by a factor of between four and seven and the peak stresses give factors between 1.6 and 3.2 against the 1.5 required.

Fatigue was assessed against a service history and not against the design case, and every bellcrank clears the design distance by at least three orders of magnitude. That margin measures the severity of the lap analysed, which contains the manoeuvre and not

the road surface; the allowance for the road input sits on the side of the static verification instead.

The alloy was selected after the analysis and not before it, since the stress field does not depend on it, and of the three only 7075-T6 meets the criterion at the governing location.

Chapter 7

Monocoque Integration and Compliance Requirements

The bellcrank and its elements do not act on the wheel directly: every force they develop reaches the contact patch through the suspension links and through the points at which those links attach to the chassis. Any compliance in that path acts in series with the heave and roll elements and therefore reduces the rates that Chapter 4 established, and displaces the wheel in ways that a purely kinematic model cannot show. This chapter first establishes how the assembly is arranged within the vehicle and which interfaces it requires, and then derives how stiff those interfaces have to be for the effects above to remain within a stated tolerance.

The monocoque exists at this stage as an outer geometry, which is enough to place the assembly and to verify that it clears the structure over the travel, and it is used for both in Section 7.1. What it does not yet have is a structural design, so there is nothing whose deflection could be measured. The analysis is therefore posed in the opposite direction to the usual one: it does not measure what a given structure deflects, it states the stiffness the structure must provide. The outcome is a set of requirements that can be handed to the design of the monocoque, which is the form in which this part of the work is useful.

7.1 Packaging

The bellcrank, the bearings selected in Section 6.2 and the two elements form an assembly whose position within the vehicle is fixed by the hardpoints of Chapter 3. What remains to be established is how that assembly sits relative to the chassis structure: the volume it occupies, the clearance it retains over the full range of travel, and the points at which it has to be supported.

7.1.1 Installed Layout and Chassis Interfaces

The pivot axis runs longitudinally and the two elements span laterally between the two bellcranks of an axle. The heave element is shared by both bellcranks and the roll element connects one to the other, so neither is grounded to the chassis. The anti-tension mechanism of Section 3.7 acts on the damper and is likewise internal to the assembly. The chassis therefore carries the suspension at two distinct groups of points: the supports of the bellcrank pivots, which are particular to this architecture, and the inboard pickups of the wishbones and the tie rod, which follow from the geometry of Chapter 3 and are common to any pushrod layout. Figure 7.1 shows the assembly with the four pivot supports in place and no other connection to the chassis.

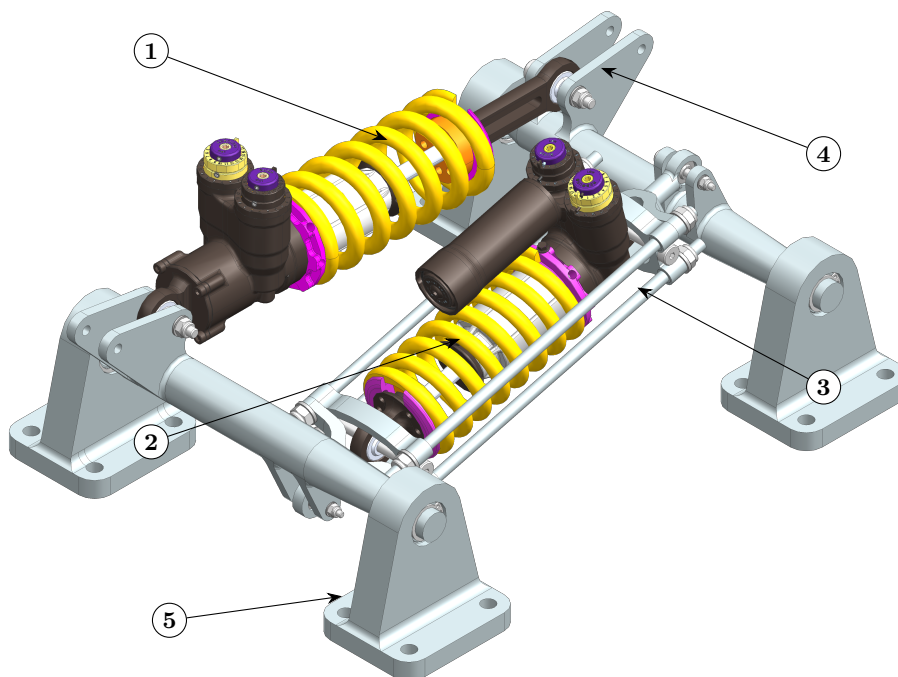


Figure 7.1: The roll-heave decoupling mechanism of one axle. **1** heave element; **2** roll element; **3** parallel rods of the anti-tension device; **4** bellcrank; **5** chassis support of the pivot. Neither element is grounded to the chassis: the only connections are the four pivot supports.

Bellcrank pivot supports

Each bellcrank is carried on two chassis supports, one at each end of its body, placed at -35 and 160 mm along the pivot axis measured from the heave pivot point. The needle bearings of Section 6.2 are housed in those supports and run on the turned-down ends of the body, so each support carries a radial load and the span between them is 195 mm.

The three attachment points all lie within that span: the pushrod and the heave element act at the heave pivot station and the roll element 87.5 mm from it. The body

is therefore loaded in bending between its two supports with no overhang, which is what allows the reaction at each support to be obtained from the lever rule and used in Section 7.3.

Wishbone and tie-rod pickups

Each link is a two-force member, so the load it applies to the chassis acts along the link axis and its direction is fixed by the geometry. Only the magnitude changes between load cases. Table 7.1 lists, for each inboard pickup, the direction in which a link in tension pulls on the chassis and the largest tensile and compressive forces found over the twenty-five load cases of Appendix E. The values are the envelope of the two sides of each axle.

Table 7.1: Inboard pickup points of the wishbones and the tie rod, with the direction along which a link in tension pulls on the chassis and the extreme link forces over the load cases of Appendix E. Positive values are tension. Coordinates are given for the left-hand corner; the right-hand corner is the mirror image in X .

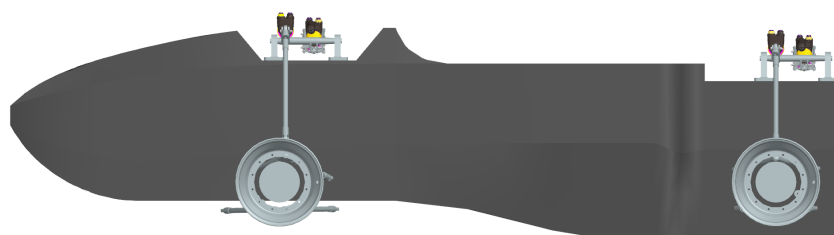
Axle	Pickup	Position (X, Y, Z) [mm]	Direction	Force range [N]
Front	Upper wishbone, fore	(243.6, -120.0, 223.4)	(0.884, 0.427, 0.193)	-1166 to +1720
Front	Upper wishbone, aft	(243.6, 154.4, 217.8)	(0.884, -0.418, 0.210)	-1112 to +1475
Front	Lower wishbone, fore	(168.6, -170.0, 118.1)	(0.911, 0.412, 0.008)	-2497 to +4927
Front	Lower wishbone, aft	(168.7, 172.4, 123.1)	(0.911, -0.412, -0.004)	-3286 to +3835
Front	Tie rod	(182.9, -68.0, 139.0)	(0.999, 0.000, 0.050)	-1351 to +877
Rear	Upper wishbone, fore	(244.0, 1405.3, 244.7)	(0.916, 0.388, 0.103)	-849 to +2566
Rear	Upper wishbone, aft	(244.0, 1655.5, 247.5)	(0.916, -0.391, 0.094)	-1437 to +2091
Rear	Lower wishbone, fore	(244.0, 1412.5, 125.9)	(0.934, 0.352, -0.058)	-5690 to +4745
Rear	Lower wishbone, aft	(244.0, 1647.5, 126.0)	(0.934, -0.352, -0.058)	-3675 to +5840
Rear	Tie rod	(243.6, 1611.2, 170.0)	(1.000, 0.000, 0.000)	-756 to +1039

The lower wishbone carries by far the largest loads at both axles, reaching 5.8 kN in tension at the rear aft pickup and 5.7 kN in compression at the rear fore pickup, and the two lower pickups reverse sign between traction and braking. The upper wishbone reaches under half of that level and the tie rod under a fifth of it. The governing cases are, in every instance, one of the base manoeuvres combined with a single-wheel bump at the corner in question, which is consistent with the bellcrank loads of Appendix E.

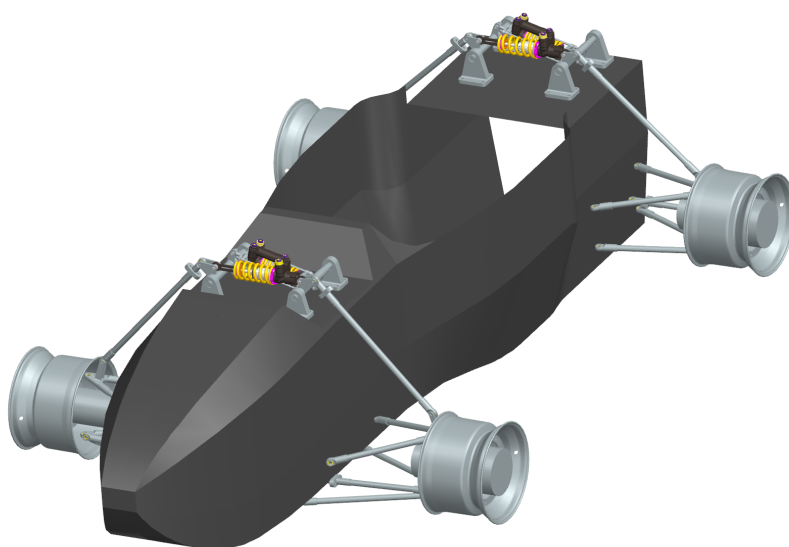
In Figure 7.2, it's possible to see an isometric and a lateral view of the decoupling mechanism integrated in a monocoque model, made only with 2D surfaces, that was adapted to this suspension from the existing model within the team.

7.1.2 Clearance over the Travel Range

The clearance was verified in the Motion application of Siemens NX, on the model of Figure 7.2 with the suspension integrated into the monocoque geometry. The mechanism was driven over the full travel range and no interference was found between any two components.



(a) Side view.



(b) Isometric view.

Figure 7.2: The assembly installed on the vehicle, at both axles. The bodywork is shown as a surface and the monocoque structure itself is not represented

Two cases are close enough to be worth recording. The first is between the heave element and the left bellcrank at full rebound, where the clearance falls to 2.5 mm. The second is between the front upper wishbone and the motor housing, at 3 mm, in full bump. Figure 7.3 shows both.

Neither is an interference, but neither leaves room for the geometry to move. A change to any of the parameters that set the travel or the bellcrank rotation would consume a clearance of this size, so both are recorded as constraints on any subsequent revision and not as findings closed by this check.

A remedy exists for each, and in both cases it costs something established earlier. The heave clearance is opened by moving the heave element attachment further from the pivot, which raises the lever arm of Table 6.1 and so the other lever arms also have to be corrected. The wishbone clearance is opened by widening the spread of the upper A-arm, which moves the inboard pickups of Table 7.1 and with them the kinematics of Chapter 3.

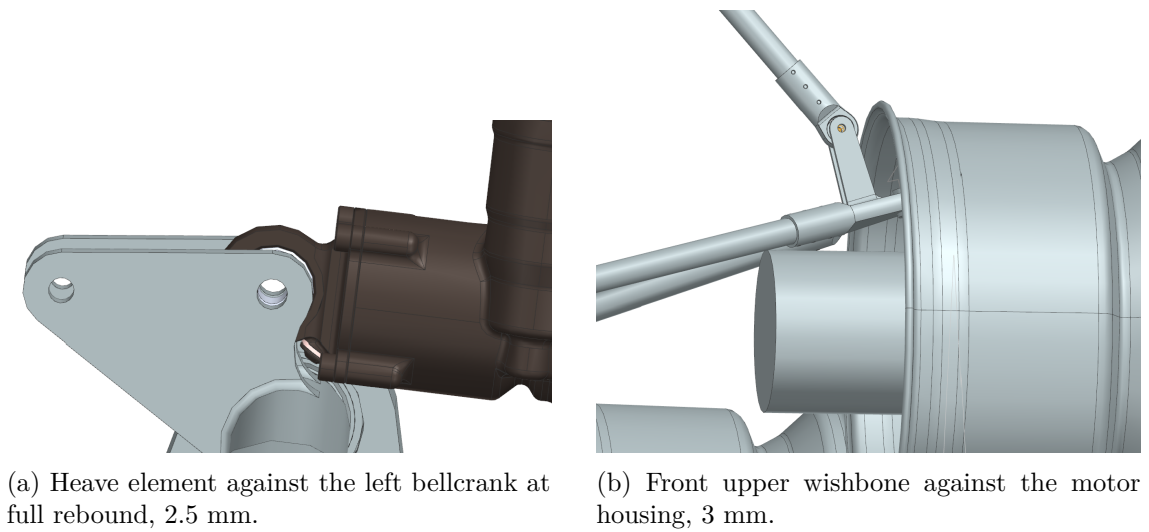


Figure 7.3: The two smallest clearances found over the travel range.

Neither is a local fix, and that is the reason both are left as they stand and identified here for future work.

7.1.3 Mass of the Installed Assembly

Table 7.2 gives the mass of the components that make up the assembly, weighed per axle.

Table 7.2: Mass of the assembly, by axle.

Component	Front [g]	Rear [g]	Total [g]
Bellcrank, left	256.1	223.5	479.6
Bellcrank, right	256.4	224.8	481.2
Heave element	783	729	1512
Roll element	1158	1155	2313
Total with the rest of the components	2746	2627	5373

The two bellcranks of an axle agree to within 1.3 g, which is the check that the two sides, although not mirror images of one another, were drawn to the same section. The front pair is fourteen per cent heavier than the rear, which follows from the longer pushrod arm of Table 6.1 and the larger body it requires.

Every component in the table is sprung, and none of it is carried by the wheel. The four bellcranks account for less than a fifth of the total, so the mass of the assembly is dominated by the elements and by the damper-spring elements. That is worth stating plainly: reducing the mass of the bellcranks further would have almost no effect on the mass of the assembly, and the stiffness requirement of Section 6.4.6 is what should govern their section, as it does.

The figures exclude the chassis supports since they were not sized, but includes the bearings and the fasteners.

7.2 Compliance Budget

The criterion is stated on the ride rate RR and not on the wheel rate, so that it matches the bellcrank stiffness requirement of Section 6.1.3 and, through it, the ride frequencies from which the whole elastic sizing of Chapter 4 follows. The tyre is therefore already part of the series path, and the requirement placed on the structure is correspondingly less severe than it would be if referred to the wheel rate alone.

For a structural path of stiffness K_e in series with a suspension of ride rate RR , the effective rate is

$$\frac{1}{RR_{\text{eff}}} = \frac{1}{RR} + \frac{1}{K_e}, \quad \frac{\Delta RR}{RR} \approx \frac{RR}{K_e} \quad \text{for } K_e \gg RR. \quad (7.1)$$

Fixing the admissible loss p therefore fixes the required stiffness, $K_e \geq RR(1-p)/p$. Table 7.3 evaluates this for the ride rates of Chapter 4, and expresses the same requirement as the deflection the suspension is allowed to lose at the static corner load. That deflection is referred to the wheel; the form in which the requirement is checked on the structure itself, attachment by attachment, is derived in Section 7.3.

Table 7.3: Structural stiffness required at the wheel, and the equivalent deflection at the static corner load, for three levels of admissible loss in ride rate.

Axle	Required K_e [N/mm]			Deflection at static load [mm]		
	1 %	2 %	5 %	1 %	2 %	5 %
Front	2656	1315	510	0.194	0.392	1.011
Rear	3080	1524	591	0.194	0.392	1.012

The admissible deflection is the same on both axles, to three decimal places. This is not a coincidence: the deflection allowed at a given loss is the static deflection of the suspension multiplied by $p/(1-p)$, and the static deflection is 19.21 mm at the front against 19.22 mm at the rear, the two axles having been sized to nearly the same ride frequency. The requirement in deflection is therefore a property of the ride frequency and not of the axle, which is convenient for the chassis design: a single displacement target applies at every attachment.

7.2.1 Axial Compliance of the Links

One contribution can be evaluated in closed form, without recourse to the multibody model. Each link is a two-force member, so under a unit vertical load at the contact patch it carries a force t_j fixed by the geometry, and the wheel displacement that its axial extension produces follows from Castigliano's theorem,

$$\frac{1}{K_{\text{links}}} = \sum_j \frac{t_j^2 L_j}{E_j A_j}, \quad (7.2)$$

where L_j is the length of link j and $E_j A_j$ its axial rigidity. Table 7.4 lists t_j and L_j for both axles.

Table 7.4: Link force per unit vertical wheel load, link length, and the resulting contribution to the summation of Equation (7.2).

Link	Front			Rear		
	t_j [-]	L_j [mm]	$t_j^2 L_j$ [mm]	t_j [-]	L_j [mm]	$t_j^2 L_j$ [mm]
Upper wishbone, fore	+0.364	324.7	43.1	+0.379	321.1	46.0
Upper wishbone, aft	+0.098	324.7	3.1	+0.376	321.1	45.4
Lower wishbone, fore	+0.124	415.7	6.4	+0.195	333.4	12.7
Lower wishbone, aft	+0.403	415.6	67.3	+0.195	333.4	12.7
Tie rod	-0.074	320.8	1.7	0.000	317.3	0.0
Pushrod	-1.224	487.4	730.5	-1.419	451.4	908.9
Total			852.3			1025.8

The summation is dominated by the pushrod, which contributes 86 % of the total at the front and 89 % at the rear: it is the longest member in the vertical load path and the only one carrying more than the applied load, at 1.22 and 1.42 times the wheel force respectively. The tie rod contributes nothing at the rear and almost nothing at the front, which is expected of a member that reacts steering and not vertical load. Any effort spent stiffening the links should therefore go to the pushrod.

The links are pullwound carbon-fibre tubes of 14 mm bore and 16 mm outside diameter, giving a section of 47.12 mm². The stack alternates high modulus and high strength fibres and is laid up predominantly at ± 10 degrees, so the axial modulus of the laminate is 203.7 GPa and the axial rigidity is $EA = 9.60$ MN. Substituting in Equation (7.2) gives $K_{\text{links}} = 11261$ N/mm at the front and 9356 N/mm at the rear, that is a loss in ride rate of 0.24 % and 0.33 % respectively, of which the pushrod alone accounts for 0.20 and 0.30 points.

These figures cover the tubes only. The bonded inserts and the spherical rod ends at each end of a link add compliance in series that has not been quantified here, so the values above are a lower bound on the compliance of the assembled links. The terminations are therefore given an allocation of their own in Table 7.5, which converts them from an unknown into a requirement in the same manner as the rest of the chapter.

7.2.2 Allocation of the Budget

Table 7.5 distributes the admissible loss among the contributions that make up the path. Each allocation is a requirement that can be verified independently of the others. The bellcrank is the one entry already settled: Section 6.1.3 allowed it one per cent of ride rate on the heave path, and the four components measured in Section 6.4.6 consume between 0.15 and 0.24 of that. It is the consumed value, and not the allowance, that is carried into the budget below; the remainder of the bellcrank allowance is not reassigned.

Table 7.5: Allocation of the admissible loss in ride rate along the heave load path. The bellcrank entry is the loss measured in Section 6.4.6; the remaining entries are requirements placed on parts of the path that have not yet been designed.

Contribution	Front [%]	Rear [%]	Basis
Bellcrank	0.25	0.25	measured, 0.15 to 0.24
Link tubes	0.24	0.33	calculated from EA
Link terminations	0.25	0.25	requirement
Chassis attachments	1.00	1.00	requirement
Total	1.74	1.83	

The total is held below 2 %. At that level the front ride rate falls from 26.83 to 26.37 N/mm and the ride frequency from 3.57 to 3.54 Hz, and the ride-height excursion on which the aerodynamic constraint of Section 4.2 acts grows by the same proportion, from 15 to 15.3 mm. That increase is absorbed by the travel reserve declared there, so the aerodynamic requirement is not disturbed.

The allocation given to the terminations corresponds to a stiffness of 10 732 N/mm at the front and 12 444 N/mm at the rear, referred to the wheel, or 0.048 mm of additional deflection at the static corner load on both axles. Expressed in a form that can be checked against a bonded joint, it is the compliance that a further 597 mm of pushrod tube would introduce at the front and 383 mm at the rear, so the four terminations of a corner have to be stiffer than that length of the tube they join. Should a coupon test of the terminations show otherwise, the deficit is taken from the chassis allocation, which is the only entry in the table with room to spare.

The chassis allocation of 1 % is the entry carried into Table 7.3 and into the sensitivity study of Section 7.3. It covers the attachment points of Table 7.1 together with the bellcrank pivot supports, and how it divides among them is established there and not assumed here.

7.3 Sensitivity to Attachment Stiffness

The requirement of Section 7.2 is expressed at the wheel, whereas the quantity a chassis designer controls is the stiffness of an individual attachment acting in a particular direction. This section relates the two. The part of the relation that concerns the ride rate is obtained in closed form; the part that concerns compliance steer and camber is obtained from the multibody model, since it has no equivalent closed form.

7.3.1 Ride Rate: Closed-Form Sensitivity

An attachment that is compliant along the direction in which it carries load behaves, for the vertical load path, exactly as a link of finite axial rigidity does. Applying Equation (7.2)

with the attachment stiffness k_j in place of the link rigidity gives

$$\frac{1}{K_{\text{att}}} = \sum_j \frac{t_j^2}{k_j}, \quad (7.3)$$

where t_j is again the load carried by attachment j per unit vertical load at the contact patch. The sensitivity of the ride rate to each attachment is therefore governed by t_j^2 , and not by the magnitude of the load the attachment carries in the design load cases. The two orderings are different: Table 7.1 shows the lower wishbone carrying the largest forces, yet those forces arise under longitudinal and lateral manoeuvres and correspond to a small t_j .

The bellcrank pivot supports enter the same summation. In heave the pushrod force and the heave element force act on the bellcrank at the same station along the pivot axis and in directions that add instead of opposing, so their resultant is 3.15 times the wheel load at the front and 3.38 at the rear. Because both act at the heave pivot station, and the two supports lie at -35 and 160 mm from it, the lever rule places 82 % of that resultant on the near support and the remaining 18 % on the far one.

Table 7.6: Load carried per unit vertical wheel load at each chassis attachment, and the share of the ride-rate compliance for which it is responsible.

Attachment	Front		Rear	
	t_j [-]	Share [%]	t_j [-]	Share [%]
Upper wishbone, fore	+0.364	1.8	+0.379	1.7
Upper wishbone, aft	+0.098	0.1	+0.376	1.7
Lower wishbone, fore	+0.124	0.2	+0.195	0.5
Lower wishbone, aft	+0.403	2.2	+0.195	0.5
Tie rod	-0.074	0.1	0.000	0.0
Pivot support, -35 mm	+2.581	91.2	+2.774	91.3
Pivot support, 160 mm	+0.565	4.4	+0.607	4.4
Sum $\sum t_j^2$	7.306		8.424	

Over nine tenths of the ride-rate compliance attributable to the chassis is therefore concentrated in a single attachment per bellcrank, the pivot support nearest the heave element. The five wishbone and tie-rod pickups together account for less than five per cent. This is a consequence of the architecture and not of the particular geometry: the pivot support is the one attachment that lies within the vertical load path, and it carries the sum of the two forces the bellcrank develops.

If every attachment is given the same stiffness, Equation (7.3) reduces to $k \geq K_e \sum_j t_j^2$, that is 19400 N/mm at the front and 25900 N/mm at the rear for the 1 % allocation of Table 7.5. A single figure per axle, verifiable point by point, is the most convenient form in which to hand the requirement to the design of the monocoque, and it is the form adopted here. A designer who finds one attachment difficult to stiffen may redistribute using the shares in Table 7.6, at the cost of stiffening another.

7.3.2 Compliance Steer and Camber

The closed form above accounts for the ride rate, which is a scalar displacement along the load path. It says nothing about the change in wheel orientation that the same compliances produce, because that depends on how the attachment displacements combine through the linkage geometry. Those effects are the ones a purely kinematic model cannot show, and they are evaluated in the multibody model of Chapter 5 by replacing the ideal joints at the chassis attachments with bushings of finite stiffness, as described in Appendix F.

Model preparation

The ideal joints at the wishbone and tie-rod pickups were replaced by bushings with a straight-line force-deflection characteristic, so that the stiffness is a value set directly and not the slope of a curve. The rotational stiffnesses were set to zero: the links are carried on spherical bearings at both ends and react no moment.

With every pickup set to 10^8 N/mm the model reproduces the kinematic results of Chapter 5, which establishes the reference against which every subsequent run is measured. Each pickup is oriented on the vehicle axes and not along its link, so the sweep applies equal stiffness in the plane in which the wishbone carries load and holds the remaining direction rigid. Because the wishbone is modelled as one rigid body on two bushings, whereas the assembled suspension uses two tubes with spherical ends, the model transmits a moment that the hardware does not. The stiffness it demands is therefore the more severe of the two, and it is that value which is adopted.

Ride rate

All four wishbone pickups of a front corner were given the same stiffness and swept over four decades (Figure 7.4), with the ride rate read at the static position in each run. Table 7.7 lists the results against Equation (7.3) written in its exact form, $1/RR_{\text{eff}} = 1/RR + \sum t_j^2/k$. The summation here runs over the swept pickups alone and amounts to 0.320, and not over the whole of Table 7.6: the pivot supports are represented in this model as ideal joints and are not part of the sweep.

Table 7.7: Front ride rate against wishbone pickup stiffness, measured and predicted.

k [N/mm]	Measured [N/mm]	Predicted [N/mm]	Loss [%]
Rigid	26.918	—	—
10^5	26.916	26.916	0.007
10^4	26.896	26.895	0.082
10^3	26.691	26.688	0.843
10^2	23.924	24.783	11.12

Agreement is to three decimal places over the three highest stiffnesses, and the loss falls with a log-log slope of -1.01 against the -1 the equation requires. The lowest stiffness departs from the prediction, which is expected: at a loss of 11 % the suspension

is well outside the range in which a structural path an order of magnitude stiffer than the suspension was assumed, and the rate curve loses the progression seen over the travel in the other runs. Within the range the requirement occupies, the closed form describes the behaviour, and it is on that basis that it is applied in Section 7.3.1 to the pivot supports, which the present model cannot sweep without altering its topology.

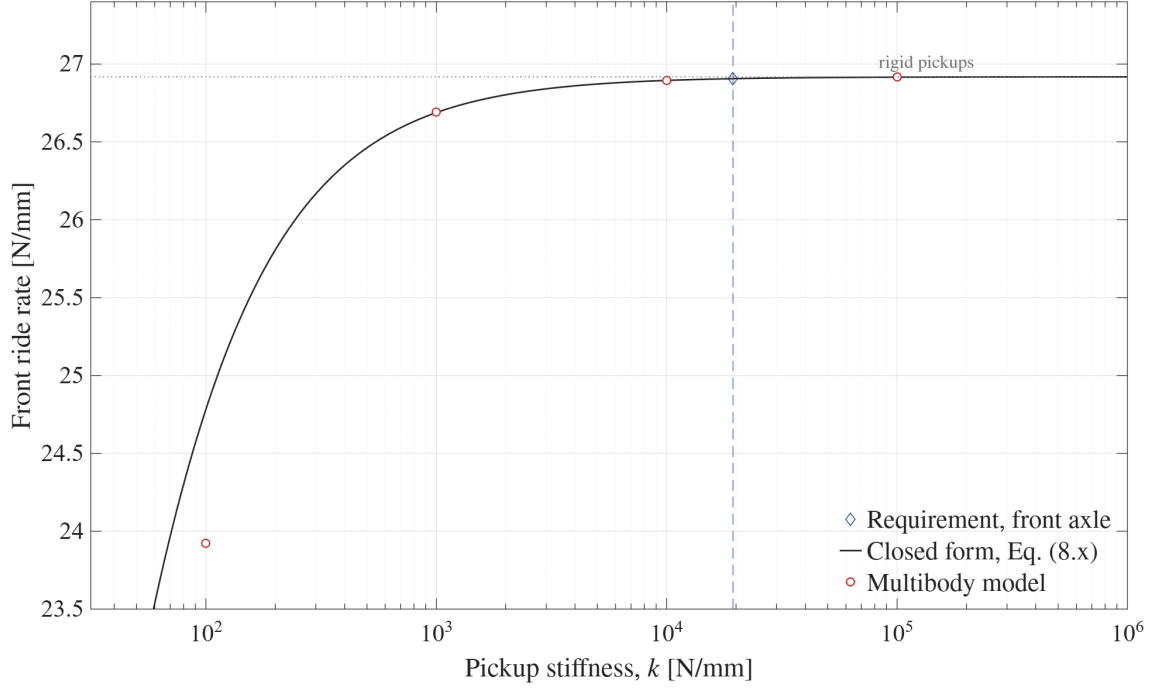


Figure 7.4: Front ride rate against wishbone pickup stiffness. The closed form is Equation (7.3) evaluated with $\sum t_j^2 = 0.320$ over the swept pickups; the marker is the requirement of Section 7.3.1. The departure of the lowest point from the curve is discussed in the text.

Camber and compliance steer

Camber and toe were obtained from static load analyses in which the wheel centre height is held fixed and the contact-patch force is ramped, so that the change in wheel orientation is compliance alone with no contribution from vertical travel. Camber was taken from the outer wheel under the lateral load of the 1.6 g case and toe from the braking load of the 1.8 g case. With the pickups rigid both quantities are of the order of 10^{-6} degrees, so the values are read directly and not differenced against a reference. Figures 7.5 and 7.6 give both against the pickup stiffness.

Both quantities go as the inverse of the pickup stiffness, as the ride rate does, but they compare very differently with the kinematic behaviour they perturb. At the required stiffness the camber compliance is 0.019 deg/kN, which under the lateral load of the case amounts to 0.035 degrees against roughly four degrees of kinematic camber gain over the travel: two orders of magnitude smaller, and of no consequence.

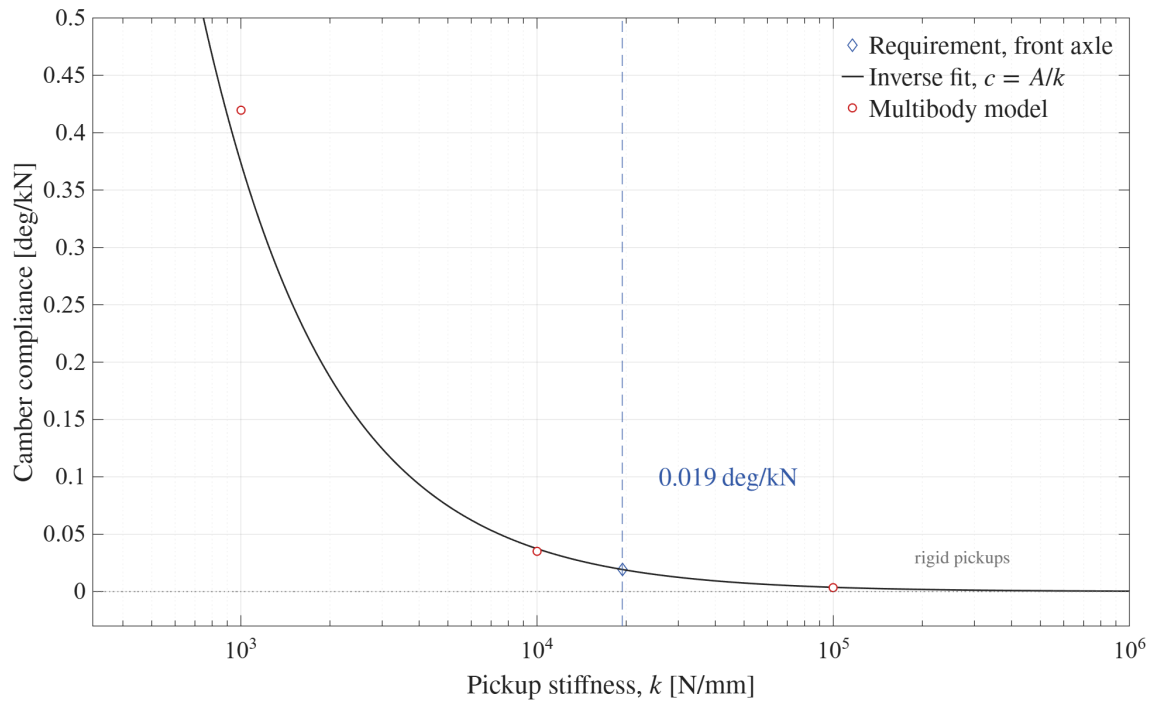


Figure 7.5: Camber compliance of the front wheel against pickup stiffness. The marker is the requirement of Section 7.3.1, at which the compliance is 0.019 deg/kN.

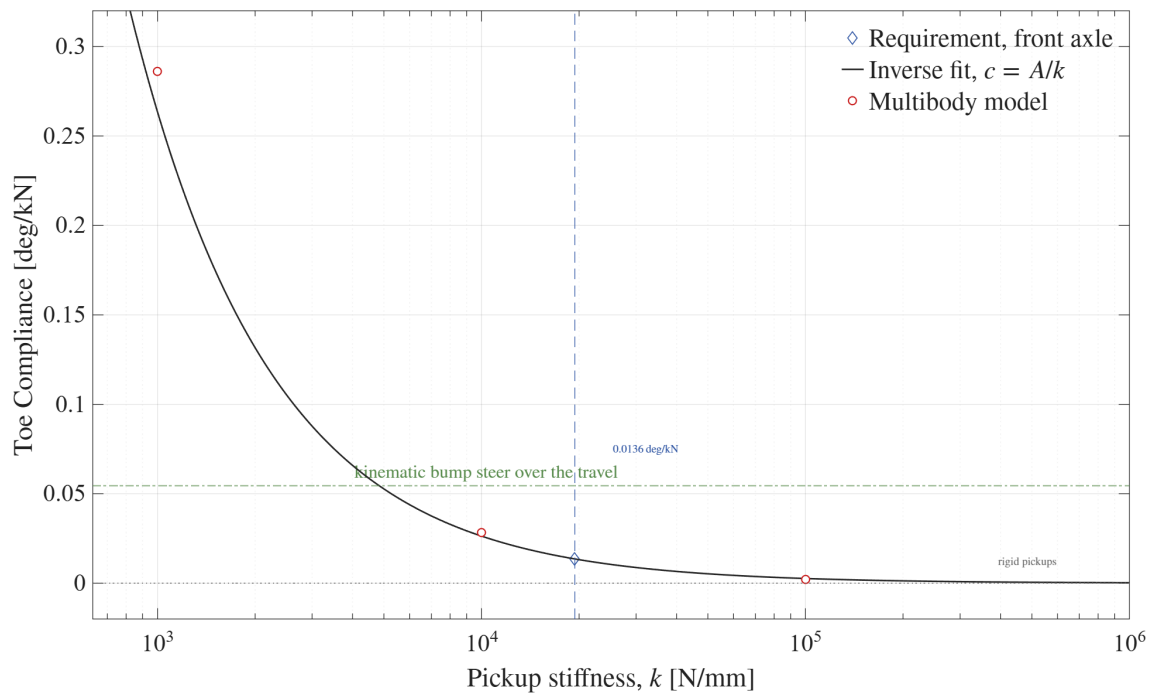


Figure 7.6: Toe compliance of the front wheel against pickup stiffness. The horizontal line is the kinematic bump steer over the full travel, referred to the same braking load so that the two are comparable on one axis. The marker is the requirement of Section 7.3.1, at which the compliance is 0.0136 deg/kN.

The compliance steer does not enjoy that margin. At the same stiffness it is 0.0136 deg/kN, against a kinematic bump steer that is only four times larger over the whole of the travel. Figure 7.6 shows the two lines crossing at a little under 5000 N/mm, which is to say that a chassis a quarter as stiff as required would produce as much toe change through compliance as the geometry produces through travel. The reason is not that the compliance is large but that the bump steer is small: the tie-rod position was tailored to make it so, and a quantity deliberately reduced to near zero is easily matched by a secondary effect. Holding the compliance steer an order of magnitude below the kinematic variation would call for attachments near 10^5 N/mm, five times the requirement the ride rate sets.

That is not adopted here. The requirement remains the one derived from the ride rate, and the compliance steer is recorded as the quantity with the least margin, to be revisited if the chassis design finds the requirement easy to exceed. It is also the one result of this chapter that the closed form could not have produced, which is what justifies the multibody study.

Two points are worth stating explicitly. The requirement derived here is the *local* stiffness of the attachments; the global torsional stiffness of the chassis is a separate question and is outside the scope of this work. And the attachments are not required to be equally stiff in every direction: what matters at each point is the direction in which it carries load, which for the links is the axis given in Table 7.1 and for the pivot supports is the plane normal to the pivot axis.

7.4 Chapter Conclusions

The chapter delivers one number per axle to the design of the monocoque: every attachment carrying the suspension is to be stiffer than 19400 N/mm at the front and 25900 N/mm at the rear, along the direction in which it carries load.

That requirement comes from a budget and not from a convention. The admissible loss in ride rate is held below two per cent in total, of which the bellcrank consumes between 0.15 and 0.24, the link tubes 0.24 at the front and 0.33 at the rear, and one per cent is allocated to the chassis; the terminations of the links, which were not quantified, are given an allocation of their own so that they enter the budget as a requirement and not as an omission. At that total the front ride frequency falls from 3.57 to 3.54 Hz and the ride-height excursion grows from 17.5 to 17.8 mm, which the travel reserve absorbs.

Where that stiffness has to be found is not evenly distributed. Over nine tenths of the compliance the chassis is responsible for sits at a single attachment per bellcrank, the pivot support nearest the heave element, because it is the one attachment lying inside the vertical load path and it carries the sum of the two forces the bellcrank develops. The five wishbone and tie-rod pickups together account for less than five per cent, and the ordering has nothing to do with the magnitude of the loads those pickups carry in the design cases.

Two results are recorded as the quantities with the least margin, and not as findings closed here. The compliance steer at the required stiffness is within a factor of four of the bump steer the geometry produces over the full travel, and that proximity is a consequence of the bump steer having been made small, not of the compliance being large; holding it an order of magnitude below would cost five times the stiffness the ride rate demands, and is not adopted. And the assembly clears the structure over the full travel with two clearances of 2.5 and 3 mm, neither of which leaves room for the geometry to move.

The requirement derived here is the local stiffness of the attachments. The global torsional stiffness of the chassis is a separate question and is outside the scope of this work.

Chapter 8

Results, Comparative Analysis and Discussion

The preceding chapters established the geometry, sized the elastic and damping elements, verified the rocker structurally, and set out what the architecture requires of the mono-coque. This chapter draws those outcomes together and places them against the FS-FEUP03 suspension, the car the team is designing and the practical reference for anything proposed here.

The comparison is arranged so that what the two suspensions share is settled before what separates them is claimed. Their kinematics are common ground, and establishing that is what licenses attributing the remaining differences to the elastic system rather than to the linkage. The FSFEUP03 is not a control case: it was designed for a different mass and a different set of targets, so a difference in outright performance between the two cars would report those choices rather than the architecture. It enters as a reference point, a real operating point to place on the reachable sets already derived, and a real set of masses and part counts against which the cost of the decoupled layout can be stated.

8.1 Results against the Objectives

Six objectives were set at the outset. The first five are settled by the chapters that precede this one and are taken in turn below, each with the result that settles it and a reference to where it was obtained; none of the figures already presented is reproduced here. The sixth, the comparison with the FSFEUP03 suspension, is the subject of the remainder of this chapter.

Separation of the heave and roll modes

The architecture was adopted to make the two elastic paths independent, and the multi-body model confirms that it does. The decoupling efficiency is greater than 98 % on both paths, with a maximum parasitic travel of 0.45 mm.

The corresponding statement in the elastic model is the block structure of $\mathbf{K}$ established in Appendix C: the heave wheel rates populate the heave and pitch coordinates and leave the roll coordinate untouched, while the axle roll stiffnesses populate the roll coordinate alone.

Targets derived from the aerodynamic envelope

The second objective was to derive the dynamic targets from the aerodynamic operating requirements rather than from a convention, and to carry them into a geometry the monocoque and the regulations admit. Chapter 4 does the first: the ride rates follow from the chassis displacement the aerodynamic package tolerates, the roll gradient from the skidpad aeromap, and the anti-features from binding displacement limits, so that no elastic quantity rests on an assumed natural frequency. Chapter 3 does the second: the hardpoints are placed within the envelope the monocoque interface and the rim volume leave free, and the resulting travel and track satisfy the rules of Section 2.7.

Freedom to set roll and heave independently

The same block structure is what the objective was really about. A conventional four-corner layout is bound by $K_\phi \geq \frac{1}{4}K_h t^2$, an anti-roll bar being able to add roll stiffness but not to remove it, so heave and roll cannot be assigned in any order. With the two paths carried by separate elements that constraint disappears, and the ride frequencies and the roll gradient were set against their own targets rather than against each other, as Chapter 4 sets out. The same separation holds in C, which is what allowed the heave and roll damping ratios to be assigned independently and, beyond that, the front-to-rear distribution of the roll damping to be used as a parameter that leaves every damping ratio unchanged and acts only on the transient.

Structural feasibility of the mechanism

The architecture is only useful if the component it requires can be built, and the bellcrank is the component that carries the whole of both paths. Four were verified, one per corner, and all four meet the strength criterion with the governing one at a factor of 1.6 on the yield strength of the selected alloy against the 1.5 required. All four meet their compliance allowance on both paths, and it is the compliance and not the stress that sizes them, for the reason established in Section 6.4.6. The fatigue assessment against a service history clears the design distance by at least three orders of magnitude on every one. The component that results weighs between 224 and 256 g, which is under a fifth of the mass of the assembly it belongs to.

Integration into the vehicle

The assembly was placed in the vehicle, cleared over the full travel, and reduced to a requirement that can be handed on. Every attachment carrying the suspension is to be stiffer than 19400 N/mm at the front and 25900 N/mm at the rear, a requirement derived from a compliance budget that holds the total loss in ride rate below two per cent, and over nine tenths of the chassis share of that budget sits at one attachment per bellcrank. Two clearances of 2.5 and 3 mm were found and are recorded as constraints on any subsequent revision. The complete statement is in Section 7.4.

What was not achieved

A lap time simulation comparison between the FSFEUP03 and an equivalent prototype carrying the suspension designed here would have been the final point of the project, but it was not possible: the templates the team has access to in IPG CarMaker, the lap time simulation tool it uses, cannot represent a suspension of this type.

8.2 Comparison with the FSFEUP03 Suspension

8.2.1 Kinematic Equivalence

The geometry of Chapter 3 derives from the FSFEUP03 hardpoints, so the two suspensions are expected to agree on everything the linkage governs. Establishing that they do is what licenses attributing the differences reported later to the elastic system and not to the geometry. Table 8.1 places the two side by side.

A difference is taken to matter when it would produce a change in wheel load or wheel orientation comparable to the effects the elastic comparison claims. For the roll centre height that threshold is of the order of ten millimetres, since the geometric share of lateral load transfer varies with it directly; for camber and toe over the travel it is a tenth of a degree, which is the resolution at which either is set on the car. Three entries pass those thresholds, and each is a declared decision of an earlier chapter rather than a consequence of the architecture: the roll centre heights were selected in Section 3.3.2, the anti-dive was derived from the clearance limit in Section 4.4.1, and the front bump steer was reduced by realigning the tie rod with the instant centres of the front linkage in Chapter 3. The remaining entries agree to within the thresholds above, which is what licenses attributing the elastic differences reported later to the elastic system alone.

Of the three, the bump steer is the one that could be misread later. The front bump steer of this suspension is smaller than that of the FSFEUP03 because the tie rod was realigned with the instant centres of the front linkage. That is a hardpoint correction and has no connection to the decoupling of the two elastic paths: the same correction could have been applied to a four-corner suspension and would have produced the same result there. It is stated here so that the reduced bump steer is not read later as an outcome

Table 8.1: Kinematic parameters of the two suspensions. The static values are read at ride height; the entries marked over the travel are the excursion of a quantity that varies along the stroke and are compressed to a single figure here.

Parameter	FSFEUP03	This work	Difference
<i>Steering axis</i>			
Kingpin inclination [°]	3.5 / 4.0	5.8 / 5.8	+2.3 / +1.8
Caster [°]	6.0 / 0.0	6.0 / 0.0	-
Mechanical trail [mm]	11.3 / 0.0	11.3 / 0.0	-
Scrub radius [mm]	39.0 / 27.2	40.0 / 33.4	+1.0 / +6.2
<i>Load transfer geometry</i>			
Roll centre height [mm]	72.0 / 86.3	88.0 / 94.3	+16.0 / +8.0
Anti-dive [%]	20.0	32.78	+12.78
<i>Wheel orientation over the travel</i>			
Camber gain [°/mm]	-0.041 / -0.051	-0.075 / -0.056	-0.034 / -0.005
Bump steer [°/mm]	0.0018 / 0.0038	0.0014 / 0.0004	-0.0004 / -0.0034
<i>Steering</i>			
Ackermann [%]	-42.0	-66.0	-24.0

of the architecture, and so that the small margin between it and the compliance steer of Section 7.3.2 is understood as the consequence of having reduced it.

8.2.2 Reachable Stiffness and Damping

The FSFEUP03 carries corner springs of 40 and 35 N/mm at the front and rear, on motion ratios of 1.30 and 1.23, giving wheel rates of 23.67 and 23.13 N/mm. In series with a tyre of about 125 N/mm those wheel rates return ride rates of 19.908 and 19.555 N/mm, which are the values the vehicle dynamics report of that car states.

Roll stiffness comes from two sources. The springs contribute $\frac{1}{2}WRt^2$ at each axle, and the anti-roll bar adds the rest: 72 N/mm on a motion ratio of 2.82 at the front, and a bar adjustable between 1.87 and 3.60 N m/deg on a ratio of 0.122 at the rear. Table 8.2 collects the result.

Table 8.2: Operating point of the FSFEUP03 suspension, derived from its spring and anti-roll bar rates. Both cars run a track of 1200 mm, so they share the boundary $K_\phi = \frac{1}{4}K_h t^2$ exactly.

	Front	Rear
Wheel rate [N/mm]	23.7	23.1
Axle heave stiffness K_h [N/mm]	47.3	46.3
K_ϕ from springs [N m/deg]	297	291
K_ϕ from the bar [N m/deg]	114	126 to 242
K_ϕ total [N m/deg]	411	416 to 533
Share carried by the bar [%]	28	30 to 45

Placed on the reachable set of Figure 4.7, the point necessarily lies inside it, since the four-corner layout was what produced the set. What the position reports is the distance from the boundary, and that distance is the work the anti-roll bar had to do: the bar raises the roll stiffness 38 % above what the springs alone provide at the front, and between 43

and 83 % at the rear. Reaching the same roll stiffness without a bar would have required springs stiffer by that proportion, and with them a ride rate the targets did not allow. The design point of this work lies on the other side of the same boundary, and reaches it with no additional device at all.

The damping comparison is of a different kind, because the FSFEUP03 report does not fix a damping coefficient: it establishes the critical damping per corner, 2038.8 N s/m at the front and 2234.0 N s/m at the rear, and selects a working range of damping ratio between 0.6 and 0.9 (Table 8.3), leaving the setting to be found in testing. The car therefore occupies a segment rather than a point.

Table 8.3: Heave and roll damping of the FSFEUP03 over its design range of damping ratio. The heave coefficient is per wheel, referred to the wheel, as in Table 4.11.

	Front		Rear	
	$\xi = 0.6$	$\xi = 0.9$	$\xi = 0.6$	$\xi = 0.9$
c_h [N s/mm]	1.22	1.83	1.34	2.01
c_ϕ [N m s/rad]	881	1321	965	1448

That the car occupies a segment and not a region is the whole of the argument. In Chapter 4 it was shown that a four-corner layout satisfies $c_\phi = \frac{1}{2}c_h t^2$ as an equality, with no device standing outside it in the way an anti-roll bar stands outside the stiffness relation. Whatever damping ratio the team settles on in testing, the car moves along that curve and cannot leave it.

There is a further consequence, and it is one the stiffness comparison conceals. The anti-roll bar contributes roll stiffness but no roll damping. Since the damping ratio in roll varies as $c_\phi/\sqrt{K_\phi}$, adding a bar that carries 29 % of the roll stiffness lowers that ratio by a factor of 0.843 relative to the heave ratio the dampers were set to. A heave ratio of 0.6 to 0.9 therefore leaves the roll mode at 0.51 to 0.76, and no adjustment of the bar corrects it: stiffening the bar to recover roll stiffness lowers the roll damping ratio further. The decoupled architecture is not subject to this, because the element that provides roll stiffness carries its own damper.

8.2.3 Coupling of Adjustment

Section 4.6.3 argued for the independence of the two adjustments from the block structure of the stiffness and damping matrices, which is a structural argument rather than a numerical one. With the rates of Table 8.2 the same statement can be made in numbers. Table 8.4 gives, for each architecture, how the two targets of adjustment respond to a ten per cent change in each elastic element.

On the conventional layout the corner spring sets both quantities, so no change to it leaves the other alone, and the bar recovers only part of what the spring displaced. On the decoupled layout the off-diagonal entries are zero by construction rather than by tuning.

Table 8.4: Response of the two adjustment targets to a ten per cent change in each elastic element. The off-diagonal entries are what distinguish the two architectures.

Architecture	Change of +10 % in	Roll gradient	Ride frequency
FSFEUP03	corner springs	−6.6 %	+4.9 %
	anti-roll bar	−2.8 %	0
Decoupled	heave element	0	+4.9 %
	roll element	−9.1 %	0

The table also reports something the block structure alone does not. The roll element returns 9.1 % of roll gradient for a 10 % change, against 2.8 % from the anti-roll bar, because the bar carries only 29 % of the roll stiffness while the springs carry the rest. The conventional adjustment is not merely coupled: it is the less effective of the two per unit of change, and its authority is bounded by how much of the roll stiffness the bar was given in the first place.

8.3 Discussion of Trade-offs

The decoupled layout reaches a region of the stiffness and damping space that a four-corner suspension cannot, and it separates two adjustments that the conventional layout binds together. Neither of those comes free. This section states what they cost, and then whether the exchange is worth making.

8.3.1 Mass, Parts and Volume

Table 8.5 places the two assemblies side by side.

Table 8.5: Mass of the suspension assembly of each vehicle, by axle, including all components.

	FSFEUP03 [kg]	This work [kg]	Difference
Front	1.88	2.75	+0.87, +46 %
Rear	2.25	2.63	+0.38, +17 %
Total	4.13	5.37	+1.24, +30 %

The decoupled assembly is heavier by 1.24 kg, or thirty per cent, and almost three quarters of that penalty falls on the front axle. Part of it is not architectural: the FSFEUP03 runs a wheel rate of 23.7 N/mm at the front against 34.1 N/mm here, so its springs are sized for about seven tenths of the rate and are correspondingly lighter, and that difference reports the targets each car was given and not the layout. The remainder is the cost of carrying the roll path in an element of its own, and it is paid mostly at the front, where the two axles are otherwise closest in mass.

The count of parts moves in the opposite direction, and against the expectation the word “decoupled” invites. Both vehicles carry four dampers, four springs and four bell-cranks. The FSFEUP03 carries in addition a Z-bar at the front and a U-bar at the rear,

each with its pair of drop links, which the decoupled layout does not have: the roll stiffness is carried by an element that is already a damper and a spring, so no separate device is needed to produce it. Against that, the decoupled layout adds the anti-tension mechanism shown in Chapter 3 and the structure that carries the roll element between the two bellcranks. The two are therefore comparable in the number of parts, and the mass penalty above is paid in the size of the components and not in their number.

The cost that is unambiguous is volume. The two elements of each axle span the full width between the bellcranks, across the centre of the vehicle, which a four-corner layout leaves free. That volume is not recoverable: it is where the architecture puts the elements, and any structure the monocoque needs in that region has to be routed around them.

8.3.2 Ease of Adjustment

One cost is not a matter of mass or space but of how the suspension is handled at an event. On the FSFEUP03 the roll stiffness is changed by rotating the blade of the anti-roll bar, which needs no tools and is done between runs. On the decoupled layout the equivalent change means fitting a different spring to the roll element, which means dismantling. For a team that tunes between sessions this is a real loss, and it should be weighed against what the same comparison gives back.

What it gives back is authority and independence. The bar on the FSFEUP03 carries 29 % of the roll stiffness, so the range over which the blade can move the roll gradient is bounded by that share, and moving it drags the roll damping ratio with it, since the bar adds stiffness without adding damping. The roll element of the decoupled layout carries all of the roll stiffness and carries its own damper, so the change is larger in effect and leaves the heave mode and the roll damping untouched. The conventional adjustment is quicker, within a narrower window and with side effects; the decoupled one is slower, over a wider window and with none.

Two qualifications belong here. The first is that neither architecture decouples the roll gradient from its distribution between the axles: raising the roll stiffness at one axle changes both the total and the balance, and the decoupling this work is concerned with is between roll and heave, not between roll and its distribution. The second is that the difficulty of adjustment is a property of how the roll element was built here and not of the architecture. An adjustable preload, or a variable-rate element, would recover the convenience without touching the principle. That is outside the scope of this work, and is recorded in Chapter 9 among the developments a later car could take up.

8.3.3 Complexity

The answer depends on what the car is asked to do, and the entries above say for which kind of car it falls each way.

Three of the gains are real and none of them is a matter of degree. The roll and heave stiffnesses are set against their own targets instead of against each other. The roll damping is not bound to the heave damping, which the conventional layout cannot avoid: an anti-roll bar adds roll stiffness without adding roll damping, so the roll mode of the FSFEUP03 sits at a damping ratio of 0.51 to 0.76 while its heave mode is at 0.6 to 0.9, and no setting of the bar corrects it. And the two adjustments are independent by construction and not by tuning, with the roll element returning three times the authority per unit of change that the bar returns.

The costs are equally concrete. The assembly weighs 5.37 kg for the two axles, it occupies the centre of the car where a four-corner layout occupies less volume, it places a stiffness requirement of 19400 and 25900 N/mm on attachments the monocoque design has yet to accommodate, and changing the roll stiffness means dismantling where the conventional car needs no tools.

The exchange is worth making for a car whose performance depends on controlling the platform: one carrying enough downforce for ride height to matter, running on a surface smooth enough that the damping is set for body control and not for compliance, and tuned between events rather than between runs. Those are the conditions under which the roll damping ratio and the ride-height excursion are the quantities that decide, and both are what this architecture buys.

It is not worth making for a car without appreciable downforce, or one run on a rough surface, or one whose setup is found by trying settings between sessions. There the mass and the volume are paid in full, the independence of adjustment is worth less than the speed of it, and a well-chosen anti-roll bar reaches a roll gradient that is adequate at a fraction of the cost. The present vehicle falls in the first category, which is why the architecture was adopted, and the honest statement of the result is that the decision follows from the aerodynamic target and not from the merit of decoupling in the abstract.

8.4 Limitations

Four limitations bound what this comparison establishes.

No lap simulation was run, so no claim is made about outright performance. What is compared are architecture-level quantities: the reachable combinations of stiffness and damping, the coupling between the two adjustments, and the cost in mass, parts and volume.

The two cars differ in mass and in the targets each was given, and the FSFEUP03 runs 23.7 N/mm at the wheel against 34.1 N/mm here. A difference in any quantity that depends on those choices reports them and not the architecture, which is why the comparison is confined to quantities that do not.

Neither suspension is compared against test data, because neither has been tested in the form described here. The FSFEUP03 rates are as designed and not as measured;

the decoupled system has not been built, so its masses come from the CAD model, its compliance is a requirement and not a measurement, and the elastic and damping design rests on a model whose parameters were verified against independent calculations and not against a vehicle.

And the multibody model represents each wishbone as one rigid body on two bushings, whereas the assembled suspension uses two tubes with spherical ends. The model therefore transmits a moment the hardware does not and demands more of the chassis than the assembled suspension will, as noted in Section 7.3.2.

One result of the design, and not of the comparison, is carried forward without being closed. The compliance steer at the required attachment stiffness is within a factor of four of the kinematic bump steer, which is the smallest margin anywhere in this work. It arises because the bump steer was deliberately made small, so the two are close by construction and not through any weakness in the structure, but it is the quantity a later revision should check first.

8.5 Chapter Conclusions

This chapter placed the design against the suspension of the FSFEUP03 and stated what the decoupled architecture costs.

The two suspensions share their linkage topology and derive from the same hardpoint set, but they are not identical: the geometry was revised in Chapter 3, and the revisions are named where they occur so that the elastic comparison can be read against them and not through them.

The comparison itself is confined to what the architecture governs. On the stiffness plane the FSFEUP03 reaches its roll target only because the anti-roll bar carries close to a third of the roll stiffness, and reaching the same target without one would have called for springs the ride targets did not allow. On the damping plane it has no equivalent device, so it occupies a curve and not a region, and the bar it does carry adds roll stiffness without adding roll damping: its roll mode is damped less than its heave mode, and no setting of the bar corrects that. That consequence is the one the stiffness comparison conceals and is the clearest thing the decoupled layout buys.

The independence of the two adjustments, argued from the block structure of the matrices, was restated in numbers. On the conventional layout no change to the corner spring leaves the other target alone and the bar recovers only part of what the spring displaces; on the decoupled layout the off-diagonal terms are zero by construction, and the roll element carries more authority per unit of change because it carries all of the roll stiffness instead of a share of it.

Four costs follow from the architecture. The assembly is thirty per cent heavier, it occupies the centre of the vehicle, it concentrates its compliance requirement on a chassis that has yet to be designed, and its roll stiffness is changed by dismantling where the

conventional car needs no tools. Part of the mass difference reports the targets each car was given and not the layout, and the count of parts is comparable, but neither of those makes the penalty disappear.

Where the exchange falls was answered and not deferred: it is worth making for a car whose performance depends on controlling the platform, and it is not worth making for one without appreciable downforce or one tuned between runs. The present vehicle is the first kind, which is why the architecture was adopted, and the decision therefore follows from the aerodynamic target and not from any general merit of decoupling.

Chapter 9

Conclusions and Future Work

The preceding chapters derived the geometry, sized the two elastic paths, verified them against the real kinematics, designed the mechanism that delivers them, and placed the whole against the suspension the team is currently designing. This chapter draws those outcomes together. The conclusions state what the design and verification of a roll-heave decoupled suspension for FSFEUP establishes and what it does not, while the future work section outlines concrete directions for continuation, both within the current team cycle and in subsequent iterations of the project.

9.1 Conclusions

The primary objective of this dissertation was to develop and verify the design of a roll-heave decoupled suspension for the monocoque prototypes the team intends to build, and to deliver it as a procedure that can be re-run rather than as a single result. It was met through a chain in which each stage supplies the next, and Chapter 8 settles each objective against the result that closes it.

The decoupling was measured, and it survives the real geometry: the two elastic paths remain separate to better than 98.4 % over the whole travel and to better than half a per cent over the excursions the vehicle uses, the residual being a second-order geometric effect and not a left-right imbalance. In the elastic model the same property is the block structure of the stiffness and damping matrices, which is what allows the ride frequencies and the roll gradient to be set against their own targets instead of against each other.

The case for the architecture rests on the damping and not on the stiffness. The corner springs of a four-corner layout bind the roll stiffness to the heave stiffness through a relation to which an anti-roll bar can add but from which it cannot subtract, and that constraint is binding on this vehicle but would not be on one with a tighter roll target, so it is a verification of this design and not a general demonstration. The corner dampers instead bind the roll damping to the heave damping through an equality that no aerodynamic package, tyre or target can relax: at the heave damping this vehicle requires,

a conventional layout would leave the roll mode at a damping ratio of 0.17 against the 0.46 obtained here, while overdamping a wheel mode.

The elastic definition was driven by the aerodynamic platform and not by a target natural frequency, so every quantity in it follows from a declared physical limit, the anti-features included. The chain closes on itself in one place worth recording: the roll gradient target was derived from the aeromap and used to size the roll springs, while the value achieved came from a lateral acceleration ramp integrated through the seven-degree-of-freedom model, and the two agree to within 1.7 %.

The component the architecture requires can be built, and it is compliance and not strength that sizes it: the four bellcranks meet the strength criterion with safety factors greater than 1.5 and clear the design distance in fatigue by three orders of magnitude, yet the roll path consumes most of the compliance allowance it was given while the heave path is stiffer than required several times over. The integration then converts the remaining compliance into a requirement that can be handed on, an attachment stiffness above 19400 N/mm at the front and 25900 N/mm at the rear, over nine tenths of which is demanded at one attachment per bellcrank.

This architecture also has drawbacks, the assembly is thirty per cent heavier than the one it is compared against, it occupies the centre of the vehicle, and its roll stiffness is changed by dismantling where the conventional car needs no tools. The exchange is worth making for a car whose performance depends on controlling the aerodynamic platform and is not worth making for one without appreciable downforce or one tuned between runs, so the decision follows from the aerodynamic target of this vehicle and not from any general merit of decoupling.

What the work does not establish is bounded, and is stated plainly. No lap simulation was run, so no claim of outright performance is made and the comparison is confined to architecture-level quantities; and nothing was built or tested, so the masses come from the CAD model and the compliance is a requirement and not a measurement. Three aspects were addressed at a preliminary level and would benefit from further investigation: the compliance of the link terminations, the design of the chassis supports that carry the bellcrank pivots, and the non-linear characteristic of the selected damper.

9.2 Future Work

The present study opens several avenues for future developments, both in terms of immediate next steps and longer-term research directions. The following points are suggested:

- **Quantification of the benefit on lap time:** The foremost gap is that the comparison of Chapter 8 reports what each architecture can reach and not what either is worth over a lap. Closing it is blocked by a tool and not by a method, since the lap simulation environment available to the team represents the suspension through corner springs and dampers alone.

- **Further development of the kinematics:** Two quantities are recorded as characterisations rather than as closed results. The roll centre migrates laterally by some 180 mm at the design roll angle against 16 mm vertically, which displaces the roll axis from the vehicle centre plane and introduces an asymmetric jacking component that the symmetric analysis does not represent; a front-view study in which the migration enters as a design quantity alongside the static height would address it. The perpendicular condition of the bellcrank links was set at the static position, whereas placing it at the middle of the interval swept would reduce the variation of the motion ratio, presently some three per cent, and with it the residual coupling that variation produces.
- **Revision of the hardpoints:** As it was reported, there is critical clearance between one of the wishbones and the motor housing control volume, so this requires a new iteration of the hardpoints to solve this issue.
- **Refinement of the aerodynamic data and of the targets derived from it:** Two of the declared allocations on which the elastic sizing rests are limited by the resolution of the aerodynamic data rather than by the method. The split of the clearance limit between steady aerodynamic load and transient reserve would only become a derived quantity if the aerodynamic load and balance were characterised continuously along the ride-height excursion instead of at discrete attitudes. The roll gradient target rests on the flat interval of the skidpad aeromap between 1.0° and 1.5° of roll being real, and that interval is worth 0.6 percentage points of downforce and 73 % of the mechanical roll stiffness the springs must deliver, so refining the map over it is the single most useful addition to the data supporting the elastic definition.
- **Design of the chassis supports and closure of the compliance items:** The supports that carry the bellcrank pivots were not sized and are excluded from the mass of the assembly, yet they are where over nine tenths of the chassis compliance sits, so their design is the first structural task the monocoque work should take up. The bonded inserts and spherical rod ends of the links enter the budget as an allocation and would be converted into a measurement by a coupon test of a terminated link. The two clearances of 2.5 and 3 mm found over the travel are not interferences, but the remedy for each costs something established earlier, so both remain constraints on any subsequent revision.
- **Exploration of alternative bellcrank designs:** The bellcrank was designed assuming a geometry that can be made through machining. Although, as seen in Chapter 6 the stiffness is the dominating factor in the design, so other geometries and manufacturing methods must be explored. A bellcrank made with a hollow section with higher outer diameter, maintaining the stiffness and reducing the mass,

may be considered, considering welding as the manufacturing method. The development of a topologically refined bellcrank is also an option, as seen in some teams mentioned in Section 2.6. A version of this last one was started, but it lacked the level of detail necessary to be included in this work.

- **Improvement of the model fidelity:** The wishbones are represented in the multi-body model as one rigid body on two bushings, whereas the assembled suspension uses two tubes with spherical ends, so the model transmits a moment the hardware does not and demands more of the chassis than the assembled suspension will; modelling each link as two two-force members would remove that conservatism. The fatigue assessment rests on a lap simulation that reproduces the manoeuvre content of the duty but not the content of the road surface, and a measured wheel-load history from the running vehicle would replace both that history and the vertical factor carried in the design case.
- **Manufacture and physical validation:** Every result reported in this dissertation is a computed one, and the natural sequence is therefore to manufacture one axle of the mechanism, measure the two motion ratios and the residual coupling on a rig, and measure the compliance of the assembled load path against the requirement derived in Chapter 7. Such validation would also allow the assumptions carried through the elastic definition to be replaced by measurements, and would establish whether the decoupling efficiency obtained in simulation is reproduced by the hardware.
- **Implementation in future prototypes:** Finally, the design procedure presented in this dissertation should be applied to subsequent FSFEUP prototypes. Whether by adopting the geometry and the elastic definition developed here or by following the same procedure against new targets, the analytical and multibody models return the corresponding design without being rebuilt, and this work is intended to serve as a transferable foundation for continuous improvement across future team cycles.

References

- [1] Marc Ventura Lecha. Design and optimization of a mode-decoupled suspension system for a Formula Student single-seater. Bachelor's thesis, Escola Tècnica Superior d'Enginyeria Industrial de Barcelona, Universitat Politècnica de Catalunya, Barcelona, January 2025. URL <https://upcommons.upc.edu/entities/publicati/b59a10c9-8567-479a-9ed6-9552aabdd5fc>.
- [2] José Díaz de Rábago Areilza. Analysis and design of a roll-heave decoupled front suspension for a Formula Student prototype. Bachelor's thesis, Universidad Pontificia Comillas, Escuela Técnica Superior de Ingeniería (ICAI), Madrid, 2024. URL <http://hdl.handle.net/11531/84167>.
- [3] Josep Villalba Nicolau. Diseño, fabricación y optimización de una suspensión desacoplada en heave y roll para un prototipo de Formula Student. Bachelor's thesis, Universitat Politècnica de València, 2022. URL <https://riunet.upv.es/handle/10251/184111>.
- [4] Lawrence Butcher. Multimatic makes DSSV damper tech available to all Formula SAE and Formula Student teams. Professional Motorsport World, June 2021. URL <https://www.pmw-magazine.com/news/race-series-news/multimatic-makes-dssv-damper-tech-available-to-all-formula-sae-and-formula-student-teams.html>.
- [5] Michael Trzesniowski. *Suspension System*. Springer Fachmedien Wiesbaden, Wiesbaden, 2023. ISBN 978-3-658-39846-0. doi: 10.1007/978-3-658-39847-7.
- [6] William F. Milliken and Douglas L. Milliken. *Race Car Vehicle Dynamics*. SAE International, Warrendale, PA, 1995. ISBN 978-1-56091-526-3.
- [7] Derek Seward. *Race Car Design*. Palgrave Macmillan, London, 2014. ISBN 978-1-137-03014-6.
- [8] Learn camber, caster, and toe | suspension alignments. Come and Drive It, n.d.. URL <https://www.comeanddriveit.com/suspension/camber-caster-toe>.
- [9] Toe — geometry explained. Suspension Secrets, May 2017. URL <https://suspensionsecrets.co.uk/toe/>.
- [10] Tyre slip angle — geometry explained. Suspension Secrets, n.d.. URL <https://suspensionsecrets.co.uk/tyre-slip-angle/>.
- [11] Passive suspension system — an overview. ScienceDirect Topics, n.d.. URL <https://www.sciencedirect.com/topics/engineering/passive-suspension-system>.

- [12] Bell crank. ScienceDirect Topics, n.d. URL <https://www.sciencedirect.com/topics/engineering/bell-crank>.
- [13] KYLE.ENGINEERS. The ultimate in mechanical grip — how does fully mode decoupled suspension work? YouTube, AAAA. URL <https://www.youtube.com/watch?v=5eh5lZu2JqM>. Video.
- [14] Marek Urban. Design of Formula Student wheel suspensions. Diploma thesis, Brno University of Technology, Faculty of Mechanical Engineering, Brno, 2020.
- [15] HotCars. How the Mercedes-AMG One’s Formula 1-derived suspension works. HotCars, 2023. URL <https://www.hotcars.com/mercedes-amg-one-may-be-most-complicated-road-car-ever/>.
- [16] *Formula Student Rules 2026*. Formula Student Germany, 2025. URL <https://www.formulastudent.de/fsg/rules/>. Version 1.1.
- [17] Edward J. Haug. *Computer Aided Kinematics and Dynamics of Mechanical Systems, Volume I: Basic Methods*. Allyn and Bacon, Boston, 1989. ISBN 978-0-205-11669-8.
- [18] Olivier A. Bauchau and André Laulusa. Review of contemporary approaches for constraint enforcement in multibody systems. *Journal of Computational and Nonlinear Dynamics*, 3(1):011005, 2008. doi: 10.1115/1.2803258.
- [19] C. W. Gear. Simultaneous numerical solution of differential-algebraic equations. *IEEE Transactions on Circuit Theory*, 18(1):89–95, 1971. doi: 10.1109/TCT.1971.1083221.
- [20] Dan Negrut, Laurent O. Jay, and Naresh Khude. A discussion of low-order numerical integration formulas for rigid and flexible multibody dynamics. *Journal of Computational and Nonlinear Dynamics*, 4(2):021008, 2009. doi: 10.1115/1.3079784.
- [21] N. Orlandea, M. A. Chace, and D. A. Calahan. A sparsity-oriented approach to the dynamic analysis and design of mechanical systems—part 1. *Journal of Engineering for Industry*, 99(3):773–779, 1977. doi: 10.1115/1.3439312.
- [22] N. Orlandea, D. A. Calahan, and M. A. Chace. A sparsity-oriented approach to the dynamic analysis and design of mechanical systems—part 2. *Journal of Engineering for Industry*, 99(3):780–784, 1977. doi: 10.1115/1.3439313.
- [23] N. Orlandea and M. A. Chace. Simulation of a vehicle suspension with the ADAMS computer program. In *SAE International Automotive Engineering Congress and Exposition*, 1977. doi: 10.4271/770053. SAE Technical Paper 770053.
- [24] Michael Blundell and Damian Harty. *The Multibody Systems Approach to Vehicle Dynamics*. Butterworth-Heinemann, Oxford, 2 edition, 2015. ISBN 978-0-08-099425-3.
- [25] E. Rocca and R. Russo. A feasibility study on elastokinematic parameter identification for a multilink suspension. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 216(2):153–160, 2002. doi: 10.1243/0954407021528995.

- [26] Wisnu Prastiyo and Wieslaw Fiebig. Multibody simulation and statistical comparison of the linear and progressive rate double wishbone suspension dynamical behavior. *Simulation Modelling Practice and Theory*, 108:102273, 2021. doi: 10.1016/j.simpat.2021.102273.
- [27] Diwakar Harsh. Full vehicle model of a Formula Student car. Master’s thesis, Delft University of Technology, Delft, 2018.
- [28] Richard G. Budynas and J. Keith Nisbett. *Shigley’s Mechanical Engineering Design*. McGraw-Hill Education, New York, 11 edition, 2020. ISBN 978-0-07-339821-1.
- [29] Thomas D. Gillespie. *Fundamentals of Vehicle Dynamics*. SAE International, Warrendale, PA, 1992. ISBN 978-1-56091-199-9.
- [30] Singiresu S. Rao. *Mechanical Vibrations*. Pearson, Hoboken, NJ, 6 edition, 2017. ISBN 978-0-13-436130-7.
- [31] KW automotive. KW V5 racing Formula Student: Technology and testing. Technical documentation, KW automotive GmbH, 2026.
- [32] *Rolling bearings catalogue*. SKF, 2023.
- [33] *Spherical plain bearings and rod ends*. SKF, 2019.
- [34] MatWeb. Material property data. Online material property database. URL <https://www.matweb.com/>.
- [35] E. O. Ezugwu and Z. M. Wang. Titanium alloys and their machinability—a review. *Journal of Materials Processing Technology*, 68(3):262–274, 1997. doi: 10.1016/S0924-0136(96)00030-1.
- [36] Dassault Systèmes. *Abaqus Analysis User’s Guide*. Dassault Systèmes Simulia Corp., Providence, RI, 2022. Section: Solid (continuum) elements.
- [37] Robert D. Cook, David S. Malkus, Michael E. Plesha, and Robert J. Witt. *Concepts and Applications of Finite Element Analysis*. John Wiley & Sons, New York, 4 edition, 2002. ISBN 978-0-471-35605-9.
- [38] A. O. Cifuentes and A. Kalbag. A performance study of tetrahedral and hexahedral elements in 3-D finite element structural analysis. *Finite Elements in Analysis and Design*, 12(3–4):313–318, 1992.
- [39] Ali Yeganefar, Seyed Ali Niknam, and Victor Songmene. Machinability study of aircraft series aluminium alloys 7075-t6 and 7050-t7451. *Transactions of the Canadian Society for Mechanical Engineering*, 44(3):427–439, 2020. doi: 10.1139/tcsme-2019-0215.
- [40] Yali Yang, Hao Chen, Wang Feng, Sha Xu, Yongfang Li, and Ruoping Zhang. Fatigue life analysis for 6061-t6 aluminum alloy based on surface roughness. *PLoS ONE*, 16(6):e0252772, 2021. doi: 10.1371/journal.pone.0252772.
- [41] J. Oñoro. The stress corrosion cracking behaviour of heat-treated Al–Zn–Mg–Cu alloy in modified salt spray fog testing. *Materials and Corrosion*, 61(2):117–123, 2010. doi: 10.1002/maco.200905255.
- [42] *Adams/Car User’s Guide*. Hexagon (MSC Software), 2023.

Appendix A

Camber Change with Steering: Isolated Effects

The surfaces of Figures 3.8 and 3.9 were produced by the FSFEUP suspension department. Tables A.1 and A.2 give the underlying values, in degrees of camber change at the outer wheel, for steering angles between -22° and $+22^\circ$ and for each angle between 2° and 9° . Negative steering angles correspond to the wheel on the inside of the corner. In each table the other angle is set to zero.

Table A.1: Camber change with steering produced by the kingpin inclination alone, in degrees, for caster = 0° .

Steer [°]	KPI [°]							
	2	3	4	5	6	7	8	9
-22	0.146	0.218	0.291	0.364	0.437	0.510	0.583	0.655
-20	0.121	0.181	0.241	0.302	0.362	0.422	0.482	0.543
-18	0.098	0.147	0.196	0.245	0.294	0.343	0.392	0.440
-16	0.077	0.116	0.155	0.194	0.232	0.271	0.310	0.349
-14	0.059	0.089	0.119	0.149	0.178	0.208	0.238	0.267
-12	0.044	0.066	0.087	0.109	0.131	0.153	0.175	0.197
-10	0.030	0.046	0.061	0.076	0.091	0.106	0.122	0.137
-8	0.019	0.029	0.039	0.049	0.058	0.068	0.078	0.088
-6	0.011	0.016	0.022	0.027	0.033	0.038	0.044	0.049
-4	0.005	0.007	0.010	0.012	0.015	0.017	0.019	0.022
-2	0.001	0.002	0.002	0.003	0.004	0.004	0.005	0.005
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
+2	0.001	0.002	0.002	0.003	0.004	0.004	0.005	0.005
+4	0.005	0.007	0.010	0.012	0.015	0.017	0.019	0.022
+6	0.011	0.016	0.022	0.027	0.033	0.038	0.044	0.049
+8	0.019	0.029	0.039	0.049	0.058	0.068	0.078	0.088
+10	0.030	0.046	0.061	0.076	0.091	0.106	0.122	0.137
+12	0.044	0.066	0.087	0.109	0.131	0.153	0.175	0.197
+14	0.059	0.089	0.119	0.149	0.178	0.208	0.238	0.267
+16	0.077	0.116	0.155	0.194	0.232	0.271	0.310	0.349
+18	0.098	0.147	0.196	0.245	0.294	0.343	0.392	0.440
+20	0.121	0.181	0.241	0.302	0.362	0.422	0.482	0.543
+22	0.146	0.218	0.291	0.364	0.437	0.510	0.583	0.655

Table A.2: Camber change with steering produced by the caster angle alone, in degrees, for KPI = 0°.

Steer [°]	Caster [°]							
	2	3	4	5	6	7	8	9
-22	0.749	1.124	1.498	1.873	2.248	2.622	2.997	3.371
-20	0.684	1.026	1.368	1.710	2.052	2.394	2.736	3.078
-18	0.618	0.927	1.236	1.545	1.854	2.163	2.472	2.781
-16	0.551	0.827	1.103	1.378	1.654	1.929	2.205	2.481
-14	0.484	0.726	0.968	1.210	1.452	1.693	1.935	2.177
-12	0.416	0.624	0.832	1.040	1.247	1.455	1.663	1.871
-10	0.347	0.521	0.695	0.868	1.042	1.216	1.389	1.563
-8	0.278	0.418	0.557	0.696	0.835	0.974	1.113	1.253
-6	0.209	0.314	0.418	0.523	0.627	0.732	0.836	0.941
-4	0.140	0.209	0.279	0.349	0.419	0.488	0.558	0.628
-2	0.070	0.105	0.140	0.174	0.209	0.244	0.279	0.314
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
+2	-0.070	-0.105	-0.140	-0.174	-0.209	-0.244	-0.279	-0.314
+4	-0.140	-0.209	-0.279	-0.349	-0.419	-0.488	-0.558	-0.628
+6	-0.209	-0.314	-0.418	-0.523	-0.627	-0.732	-0.836	-0.941
+8	-0.278	-0.418	-0.557	-0.696	-0.835	-0.974	-1.113	-1.253
+10	-0.347	-0.521	-0.695	-0.868	-1.042	-1.216	-1.389	-1.563
+12	-0.416	-0.624	-0.832	-1.040	-1.247	-1.455	-1.663	-1.871
+14	-0.484	-0.726	-0.968	-1.210	-1.452	-1.693	-1.935	-2.177
+16	-0.551	-0.827	-1.103	-1.378	-1.654	-1.929	-2.205	-2.481
+18	-0.618	-0.927	-1.236	-1.545	-1.854	-2.163	-2.472	-2.781
+20	-0.684	-1.026	-1.368	-1.710	-2.052	-2.394	-2.736	-3.078
+22	-0.749	-1.124	-1.498	-1.873	-2.248	-2.622	-2.997	-3.371

Appendix B

Aerodynamic Maps

B.1 Straight-Line Aeromap

The aerodynamic coefficients used in Chapter 4 are read from the straight-line aeromap computed by the aerodynamics department of the team. The map is a 5×5 grid in front and rear ride height, evaluated at constant speed with zero roll and zero yaw. Table B.1 reproduces it in full, so that the two operating conditions of Table 4.2 can be traced back to the points they were taken from.

The lift coefficient is referenced to a frontal area of 1.1 m^2 and the aerodynamic balance is the front share of the total vertical aerodynamic load, obtained from the pitching moment about the rear axle and the wheelbase,

$$\eta_F = \frac{M_{\text{rear}}}{L |F_L|}. \quad (\text{B.1})$$

The design point adopted for the ride-rate sizing, $|C_l| = 3.343$ with $\eta_F = 46.31 \%$, comes from a separate evaluation of the baseline configuration at the static attitude and is therefore not one of the grid points below. The end-of-straight condition used for the clearance verification, $|C_l| = 3.722$ with $\eta_F = 57.72 \%$, is the first row of the table, at the lowest front and rear ride heights of the grid.

Two features of the map bear on how the values were selected. The lift coefficient rises monotonically as the front ride height falls, so the most descended attitude is also the one carrying the largest aerodynamic load, which is why the corner of the grid and not an interior point is the critical condition for the clearance verification. The aerodynamic balance moves forward with the same excursion, from 45.3% at the highest front ride height to 57.7% at the lowest, and it is this migration that makes the front axle, rather than the rear, the axle on which the clearance constraint binds.

Table B.1: Straight-line aeromap: lift coefficient and aerodynamic balance as a function of front and rear ride height.

h_F [mm]	h_R [mm]	C_l [-]	η_F [%]
103.75	20.0	-3.722	57.72
103.75	27.5	-3.852	61.06
103.75	35.0	-3.846	63.50
103.75	42.5	-3.906	64.35
103.75	50.0	-3.909	63.57
111.25	20.0	-3.417	53.91
111.25	27.5	-3.504	54.97
111.25	35.0	-3.519	56.85
111.25	42.5	-3.525	59.57
111.25	50.0	-3.594	61.40
118.75	20.0	-3.324	50.51
118.75	27.5	-3.319	52.13
118.75	35.0	-3.276	53.35
118.75	42.5	-3.379	55.04
118.75	50.0	-3.412	56.65
126.25	20.0	-3.162	47.45
126.25	27.5	-3.103	49.11
126.25	35.0	-3.253	49.40
126.25	42.5	-3.220	52.03
126.25	50.0	-3.283	53.55
133.75	20.0	-2.959	45.27
133.75	27.5	-3.009	46.29
133.75	35.0	-3.141	46.29
133.75	42.5	-3.066	47.99
133.75	50.0	-3.164	50.35

B.2 Skidpad Roll Sensitivity

The roll gradient target of Section 4.5.2 is derived from a separate evaluation at the steady-state skidpad condition, in which the body is rolled about the roll axis at the static ride height. The conditions are a turning radius of 9.125 m at 11.61 m/s, a front and rear ride height of 37.5 mm, roll centre heights of 72 mm at the front and 88 mm at the rear, zero toe and zero yaw, with the vehicle turning to the right. The steered wheels carry -0.7° of camber and 9.4° of steer on the inside, and 0.7° of camber and 12.4° of steer on the outside.

The absolute lift coefficient at zero roll, 2.754, is lower than the straight-line design point of 3.343 because the skidpad condition is evaluated at the static attitude and at a much lower speed; only the relative change with roll is carried into Chapter 4. The centre of pressure migrates with roll as well, from 57.4 % at zero roll to 53.9 % at the adopted limit, and it shows the same non-monotonic feature at 0.5° that the lift coefficient does. This balance migration is not accounted for in the roll gradient target, which is based on total downforce alone, and refining the map in that region would allow both effects to be

Table B.2: Skidpad roll sensitivity: absolute coefficients and centre of pressure as a function of body roll angle, with the relative change in lift referred to the zero-roll condition.

Roll [°]	C_l [-]	ΔC_l [%]	C_d [-]	η_F [%]
0.0	-2.754	—	1.114	57.36
0.5	-2.794	+1.5	1.104	58.55
1.0	-2.559	-7.1	1.108	54.57
1.5	-2.542	-7.7	1.095	53.89
2.0	-2.448	-11.1	1.082	54.24
2.5	-2.467	-10.4	1.089	54.42

treated together.

Appendix C

Assembly of the 7-DOF Modal Model Matrices

This appendix details the assembly of the mass, damping, and stiffness matrices of the seven-degree-of-freedom (DOF) full-vehicle model of Section 4.6.2. The generalised coordinate vector is the one defined in Equation 4.32,

$$\mathbf{x} = [z, \theta, \phi, z_{u,FL}, z_{u,FR}, z_{u,RL}, z_{u,RR}]^T. \quad (\text{C.1})$$

The matrices follow from the kinetic, potential, and dissipation functions of the system. The potential and dissipation functions are written directly in terms of the axle heave and roll deflections of Equation 4.37,

$$\begin{aligned} \mathcal{V} &= \frac{1}{2} \left[2WR_F z_{w,F}^2 + 2WR_R z_{w,R}^2 + K_{\phi F} \phi_{w,F}^2 + K_{\phi R} \phi_{w,R}^2 + K_t \sum_i z_{u,i}^2 \right], \\ \mathcal{D} &= \frac{1}{2} \left[2c_{hF} \dot{z}_{w,F}^2 + 2c_{hR} \dot{z}_{w,R}^2 + c_{\phi F} \dot{\phi}_{w,F}^2 + c_{\phi R} \dot{\phi}_{w,R}^2 \right], \end{aligned} \quad (\text{C.2})$$

the road excitation being set to zero for the free-vibration analysis. The factor of two multiplying the heave terms accounts for the two wheels of each axle deflecting equally under an axle heave displacement: WR_F , WR_R , c_{hF} and c_{hR} are defined per wheel, so each axle stores $2 \cdot \frac{1}{2} WR z_w^2$ and dissipates $2 \cdot \frac{1}{2} c_h \dot{z}_w^2$. No such factor appears on the roll terms, because $K_{\phi F}$, $K_{\phi R}$, $c_{\phi F}$ and $c_{\phi R}$ are already axle-level quantities.

The stiffness and damping matrices are then obtained as the Hessians of $\mathcal{V}$ and $\mathcal{D}$ with respect to the generalised coordinates and velocities, respectively, $K_{ij} = \partial^2 \mathcal{V} / \partial x_i \partial x_j$ and $C_{ij} = \partial^2 \mathcal{D} / \partial \dot{x}_i \partial \dot{x}_j$, which guarantees that both are symmetric by construction. The mass matrix is diagonal,

$$\mathbf{M} = \text{diag}(m_s, I_p, I_{yy}, m_{u,c}, m_{u,c}, m_{u,c}, m_{u,c}), \quad (\text{C.3})$$

with m_s the sprung mass, I_p and I_{yy} the pitch and roll inertias of the sprung mass about its centre of gravity, and $m_{u,c}$ the corner unsprung mass.

An equivalent and less error-prone route to the same result is to collect the four axle deflections of Equation 4.37 into a linear map $\mathbf{d} = \mathbf{T}\mathbf{x}$, with $\mathbf{d} = [z_{w,F}, z_{w,R}, \phi_{w,F}, \phi_{w,R}]^\top$ and

$$\mathbf{T} = \begin{bmatrix} 1 & -a & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 1 & b & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{t_F} & \frac{1}{t_F} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{t_R} & \frac{1}{t_R} \end{bmatrix}, \quad (\text{C.4})$$

so that the energies become quadratic forms in $\mathbf{d}$ and

$$\mathbf{K} = \mathbf{T}^\top \text{diag}(2WR_F, 2WR_R, K_{\phi F}, K_{\phi R}) \mathbf{T} + \text{diag}(0, 0, 0, K_t, K_t, K_t, K_t), \quad (\text{C.5})$$

with $\mathbf{C}$ following from the same expression with the damping coefficients in place of the stiffnesses and without the tyre term. This form is symmetric by construction and was the one used to generate the numerical results of Chapter 4.

The block structure of $\mathbf{K}$ in Equation C.8 makes the decoupling property explicit. The heave wheel rates WR_F and WR_R populate the heave–pitch block of the sprung-mass partition (rows and columns z and θ) but leave the roll coordinate ϕ untouched, whereas the axle roll stiffnesses $K_{\phi F}$ and $K_{\phi R}$ populate the roll coordinate exclusively. The same separation holds in $\mathbf{C}$: the heave damping coefficients c_{hF}, c_{hR} act on the symmetric (heave and pitch) modes and the roll damping coefficients $c_{\phi F}, c_{\phi R}$ act on the antisymmetric (roll) mode. This is the matrix-level statement of the independence between the two elastic paths that the decoupled architecture is designed to provide, and it is what allows the heave/pitch and roll damping ratios to be assigned separately, as discussed in Section 4.7.3.

The separation is not confined to the sprung-mass partition. Because the roll coefficients are the only ones entering the ϕ coordinate, they are also the only ones acting on the antisymmetric wheel modes, a consequence examined quantitatively in Chapter 4.

The natural frequencies and mode shapes are obtained from the eigenvalue problem $(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{u} = \mathbf{0}$. The modal damping ratios follow from Equation 4.30 once the modal matrix $\mathbf{U}$ is known, or exactly from the eigenvalues of the first-order form

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q}, \quad \mathbf{q} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad (\text{C.6})$$

whose fourteen eigenvalues form seven pairs, one per mode. Denoting by λ_a and λ_b the pair belonging to a given mode,

$$\omega_n = \sqrt{\lambda_a \lambda_b}, \quad \xi = -\frac{\lambda_a + \lambda_b}{2\omega_n}, \quad (\text{C.7})$$

which remains valid when a mode becomes overdamped and its two eigenvalues are real,

a situation that does occur at realistic damping levels for this vehicle and which Equation 4.30 cannot represent.

The stiffness matrix is

$$\mathbf{K} = \begin{bmatrix} 2WR_F + 2WR_R & -2WR_Fa + 2WR_Rb & 0 & -WR_F & -WR_F & -WR_R & -WR_R \\ -2WR_Fa + 2WR_Rb & 2WR_Fa^2 + 2WR_Rb^2 & 0 & WR_Fa & WR_Fa & -WR_Rb & -WR_Rb \\ 0 & 0 & K_{\phi F} + K_{\phi R} & -\frac{K_{\phi F}}{t_F} & \frac{K_{\phi F}}{t_F} & -\frac{K_{\phi R}}{t_R} & \frac{K_{\phi R}}{t_R} \\ -WR_F & WR_Fa & -\frac{K_{\phi F}}{t_F} & \frac{K_{\phi F}}{t_F^2} + K_t + \frac{WR_F}{2} & -\frac{K_{\phi F}}{t_F^2} + \frac{WR_F}{2} & 0 & 0 \\ -WR_F & WR_Fa & \frac{K_{\phi F}}{t_F} & -\frac{K_{\phi F}}{t_F^2} + \frac{WR_F}{2} & \frac{K_{\phi F}}{t_F^2} + K_t + \frac{WR_F}{2} & 0 & 0 \\ -WR_R & -WR_Rb & -\frac{K_{\phi R}}{t_R} & 0 & 0 & \frac{K_{\phi R}}{t_R^2} + K_t + \frac{WR_R}{2} & -\frac{K_{\phi R}}{t_R^2} + \frac{WR_R}{2} \\ -WR_R & -WR_Rb & \frac{K_{\phi R}}{t_R} & 0 & 0 & -\frac{K_{\phi R}}{t_R^2} + \frac{WR_R}{2} & \frac{K_{\phi R}}{t_R^2} + K_t + \frac{WR_R}{2} \end{bmatrix} \quad (\text{C.8})$$

and the damping matrix has the identical structure, with each stiffness term replaced by its damping counterpart and the tyre term omitted,

$$\mathbf{C} = \begin{bmatrix} 2c_{hF} + 2c_{hR} & -2ac_{hF} + 2bc_{hR} & 0 & -c_{hF} & -c_{hF} & -c_{hR} & -c_{hR} \\ -2ac_{hF} + 2bc_{hR} & 2a^2c_{hF} + 2b^2c_{hR} & 0 & ac_{hF} & ac_{hF} & -bc_{hR} & -bc_{hR} \\ 0 & 0 & c_{\phi F} + c_{\phi R} & -\frac{c_{\phi F}}{t_F} & \frac{c_{\phi F}}{t_F} & -\frac{c_{\phi R}}{t_R} & \frac{c_{\phi R}}{t_R} \\ -c_{hF} & ac_{hF} & -\frac{c_{\phi F}}{t_F} & \frac{c_{hF}}{2} + \frac{c_{\phi F}}{t_F^2} & \frac{c_{hF}}{2} - \frac{c_{\phi F}}{t_F^2} & 0 & 0 \\ -c_{hF} & ac_{hF} & \frac{c_{\phi F}}{t_F} & \frac{c_{hF}}{2} - \frac{c_{\phi F}}{t_F^2} & \frac{c_{hF}}{2} + \frac{c_{\phi F}}{t_F^2} & 0 & 0 \\ -c_{hR} & -bc_{hR} & -\frac{c_{\phi R}}{t_R} & 0 & 0 & \frac{c_{hR}}{2} + \frac{c_{\phi R}}{t_R^2} & \frac{c_{hR}}{2} - \frac{c_{\phi R}}{t_R^2} \\ -c_{hR} & -bc_{hR} & \frac{c_{\phi R}}{t_R} & 0 & 0 & \frac{c_{hR}}{2} - \frac{c_{\phi R}}{t_R^2} & \frac{c_{hR}}{2} + \frac{c_{\phi R}}{t_R^2} \end{bmatrix} \quad (\text{C.9})$$

Appendix D

MATLAB Implementation of the 7-DOF Modal Model

This appendix describes the MATLAB routines used to produce the modal and damping results of Chapter 4. The code depends only on base MATLAB.

Three decisions shape the implementation. The vehicle parameters are defined in one file and read from there by everything else. The matrices are assembled from a single linear map, described in Section D.1. And every result passes through a verification stage that compares the assembled model against values obtained by hand in the body of the dissertation. Table D.1 summarises the roles of the routines.

Table D.1: Routines of the modal analysis implementation.

File	Role
fsfeup_params.m	Vehicle parameters in SI units, gathered in one place, together with the reference values used for verification
fs_build_MCK.m	Assembles $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ from the deformation map of Equation C.4
fs_verify.m	Verification against Table 4.19 and Section 4.6.1
fs_modal.m	Natural frequencies, mode identification and exact damping ratios
fs_attainable.m	Damping ratio of every mode against the damping level, and the ceiling
fs_nominal.m	Operating point from the criterion of Section 4.7.2
fs_target_damping.m	Damping coefficients for prescribed damping ratios, with attainability check
fs_damper_check.m	Coefficients referred to the damper, against the adjustment range of the hardware
fs_state_space.m	Fourteen-state model with road, lateral and longitudinal inputs
fs_response.m	Frequency responses and the three transient cases
fs_conventional.m	The four-corner architecture as a constraint on the same equations
fs_design_space.m	Reachable stiffness and damping combinations
fs_roll_split.m	Roll damping distribution and the transient load-transfer balance
fs_sweep.m	Response metrics over the damping design space
fs_export_ceiling.m	Writes the data plotted in Figure 4.3
run_damping_study.m	Driver reproducing the numerical results reported in Chapter 4

Figure D.1 shows how the routines fit together. The chain is linear: the parameters are read once, the matrices are assembled from them, the assembly is checked, and only then are results produced. The single loop in the diagram is the parameter sweep, which re-assembles the damping matrix at every point.

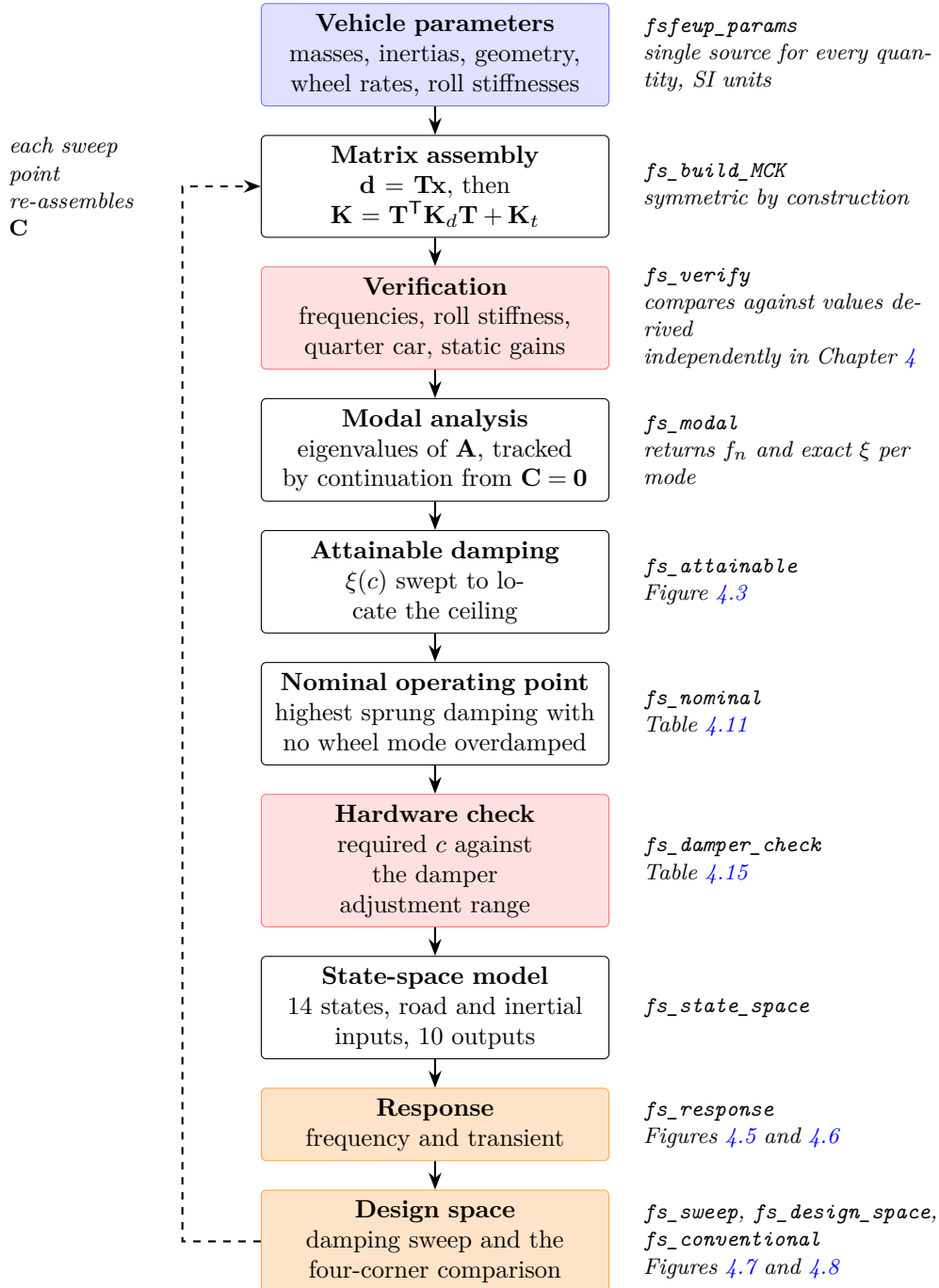


Figure D.1: Computational workflow. Blue denotes input, red the verification steps, and amber the stages that produce results reported in Chapter 4.

The verification stage sits between the assembly and every result. It compares the

assembled model against quantities obtained by independent routes: the heave and pitch frequencies of Section 4.6.3, the total roll stiffness of Equation 4.24, the quarter-car frequencies of Section 4.6.1, and three static gains of the state-space model evaluated against closed-form expressions. Agreement is required to within two per cent, except for the wheel-hop frequencies, where the reference value is itself approximate and the tolerance is ten per cent. Without these checks an assembly error would only surface at the interpretation stage, where it can be mistaken for a physical effect.

D.1 Implementation Choices

Two aspects of the implementation are worth describing: how the matrices are assembled, and how the modes are identified once damping is present.

The stiffness and damping matrices are assembled from a linear map. The four axle deflections are collected into $\mathbf{d} = \mathbf{T}\mathbf{x}$ of Equation C.4, and the matrices follow as $\mathbf{T}^T \mathbf{K}_d \mathbf{T}$. The result is symmetric by construction, which matters because a mistyped off-diagonal sign in a hand-written 7×7 matrix produces a model that still runs and still returns plausible frequencies.

The modes are tracked by continuation. Matching the complex eigenvectors of the state matrix against the undamped mode shapes works at low damping and scrambles once any mode becomes heavily damped, because the eigenvectors then cease to resemble the undamped shapes. The routine therefore introduces the damping gradually, $\mathbf{C} \rightarrow \alpha \mathbf{C}$ with α advancing from zero to one in sixty steps, and matches each eigenvalue to its nearest neighbour at the previous step. At $\alpha = 0$ the assignment is exact, so the chain is anchored and the mode identity survives the parameter sweeps.

D.2 The Routines

The routines are described below in the order of Figure D.1.

fsfeup_params

Returns a structure holding every vehicle quantity used anywhere in the implementation: masses and inertias, wheelbase and track, centre-of-gravity and roll-centre heights, wheel rates, tyre stiffness, axle roll stiffnesses and motion ratios. All values are stored in SI units, and the conversions from the millimetres and degrees of the chapter tables are performed here, once, so that no unit conversion appears inside any analysis routine. The structure also carries the reference values against which the assembly is later verified, and a switch that reproduces the assembly with and without the per-axle factor of Appendix C.

fs_build_MCK

Assembles the mass, damping and stiffness matrices for a given set of damping coefficients, by the route of Section [D.1](#).

fs_verify

Runs the checks that stand between the assembly and every result, and prints a pass or fail line for each. It compares the undamped frequencies against Section [4.6.3](#), the total roll stiffness against Equation [4.24](#), the roll-to-heave frequency ratio, the quarter-car frequencies and critical damping against Section [4.6.1](#), and three static gains of the state-space model against closed-form expressions: the roll angle under lateral acceleration, the ride-height response to a uniform road rise, and the dynamic tyre load in that same condition, which must vanish.

fs_modal

Returns the natural frequency, the damping ratio and the shape of each of the seven modes. Two damping ratios are returned: the projection of Equation [4.30](#), which neglects the off-diagonal modal damping terms, and the exact value obtained from the eigenvalues of the state matrix. Each mode is named from the dominant participation in its shape, with rotations scaled by a characteristic length so that participations are compared in metres.

fs_attainable

Sweeps the damping coefficient and records the damping ratio of every mode, exposing the ceiling on the attainable sprung damping. The relation is not monotonic: beyond a certain coefficient the damper begins to lock the unsprung mass to the sprung mass and the sprung damping falls again. The routine locates the maximum and reports it together with the wheel-hop ratios at that point, and its output is Figure [4.3](#).

fs_nominal

Selects the operating point. It bisects on the heave damping coefficient to find the largest value for which no wheel-hop mode reaches the critically damped condition, then bisects on the roll coefficient to place the roll damping ratio at the heave value, split evenly between the axles. It returns the four coefficients together with their values referred to the damper, and produces Table [4.11](#).

fs_target_damping

Solves the inverse problem: given target damping ratios, it returns the coefficients that produce them. Since the relation is not monotonic, a target above the ceiling has no

solution and a target below it has two. The routine checks attainability before solving and returns the lower-coefficient solution, which delivers the same sprung damping with less wheel-hop damping and less heat. It is used for imposing specific ratios; the operating point reported in the chapter comes from `fs_nominal`.

fs_damper_check

Converts the coefficients from the wheel to the damper, using the summing relation of Equation 4.38, and compares them with the adjustment range of each valve setting of the selected hardware. The supplier quotes forces at a single reference velocity, so each quoted force is converted to an equivalent linear coefficient at that velocity and compression and rebound are averaged. The routine also reports what a single damper per corner would require, which isolates the duty that the summing architecture concentrates into one element. Its output is Table 4.15.

fs_state_space

Rewrites the second-order system in first-order form with fourteen states, and attaches the inputs and outputs needed for the response analyses. The six inputs are the road displacement at each of the four wheels plus the lateral and longitudinal accelerations; the ten outputs are the ride height at each corner, the dynamic tyre load at each wheel, the chassis roll angle and the sprung heave acceleration. All six input columns are built at once, so the same object serves the frequency responses, the lateral step and the braking case without reassembly.

fs_response

Produces the results at the nominal operating point. In the frequency domain it evaluates the transfer functions from a road input to ride height, dynamic tyre load and sprung acceleration. In the time domain it integrates three cases with a fixed-step matrix exponential, so that no solver toolbox is required: a lateral acceleration ramp, a single-wheel kerb strike, and a braking ramp. Its outputs are Figures 4.5 and 4.6.

fs_conventional

Returns the parameter set of the equivalent four-corner architecture. The corner springs produce the same quadratic form as the decoupled layout, so no separate assembly is needed; the two architectures differ in that the conventional one is bound by $K_\phi \geq \frac{1}{4}K_h t^2$, an anti-roll bar being able to add roll stiffness but not to remove it. The routine reports the anti-roll bar stiffness that would be required to reach the target, a negative value indicating that the target is unreachable.

fs_design_space

Evaluates the region of the stiffness plane accessible to each architecture and the corresponding region of the damping plane. It writes the data plotted in Figures 4.7 and 4.8.

fs_roll_split

Sweeps the front-to-rear distribution of the roll damping with the total held constant, and records the front share of the elastic lateral load transfer during a lateral acceleration ramp. This parameter leaves every damping ratio unchanged, so its effect is visible only in the transient, and the routine reports the peak departure from the steady-state distribution and the time taken to recover it. Its output is Figure 4.4.

fs_sweep

Sweeps the two damping ratios independently and evaluates two figures of merit at each point: the root-mean-square ride height variation and the root-mean-square dynamic tyre load, both obtained by integrating the squared transfer function against a road displacement spectrum. The left and right tracks are treated as statistically independent. This matters for the roll damping: a road input applied equally to all four wheels excites only the symmetric modes, and the roll path would then have nothing to act on. The conventional architecture is added as a single locus rather than an area.

fs_export_ceiling

Writes the attainability curve to a text file, so that the figure is drawn in L^AT_EX from the data and remains vectorial and typeset in the document font.

run_damping_study

The driver. It runs the verification, reports the effect of the per-axle factor, locates the ceiling, establishes the nominal point, prints the resulting damping ratios against the projected values, repeats the analysis with the roll dampers removed to isolate their contribution to the antisymmetric modes, and checks the coefficients against the hardware. Running it reproduces the numerical results reported in Chapter 4.

D.3 Listings

The two routines that carry the implementation choices of Section D.1, and the one that verifies the assembly, are listed below.

```

1 function [M,K,C,T] = fs_build_MCK(p,c)
2 %FS_BUILD_MCK Assemble the 7-DOF mass, stiffness and damping matrices.
3 %
4 % [M,K,C,T] = FS_BUILD_MCK(p,c)
```

```

5 %
6 % p : parameter struct from FSFEUP_PARAMS
7 % c : damping struct with fields
8 %     c.hF    front heave damping, PER WHEEL, at the wheel    [Ns/m]
9 %     c.hR    rear  heave damping, PER WHEEL, at the wheel    [Ns/m]
10 %     c.phiF  front axle roll damping                        [Nms/rad]
11 %     c.phiR  rear  axle roll damping                        [Nms/rad]
12 %     Pass c = [] for the undamped system (C = 0).
13 %
14 % Generalised coordinates (Eq. 4.32):
15 %     x = [ z ; theta ; phi ; zuFL ; zuFR ; zuRL ; zuRR ]
16 %
17 % Assembly route. The four axle deflections of Eq. 4.37 are collected
18 % into the linear map
19 %
20 %     d = T*x ,    d = [ z_wF ; z_wR ; phi_wF ; phi_wR ]
21 %
22 % The energies of Eq. C.2 are then quadratic forms in d,
23 %
24 %     V = 1/2 * d' * Kd * d + 1/2 * Kt * sum(zu_i^2)
25 %     D = 1/2 * d' * Cd * d
26 %
27 % so the Hessians are K = T'*Kd*T + Ktyre and C = T'*Cd*T. This is the
28 % assembly of Appendix C written compactly, and it is symmetric by
29 % construction.
30 %
31 % See also FSFEUP_PARAMS, FS_MODAL, FS_STATE_SPACE.
32
33 if nargin < 2 || isempty(c)
34     c = struct('hF',0,'hR',0,'phiF',0,'phiR',0);
35 end
36
37 a = p.a; b = p.b; tF = p.tF; tR = p.tR;
38
39 %—— deformation map, Eq. 4.37 ——
40 %
41 %     z     theta   phi    zuFL    zuFR    zuRL    zuRR
42 % T = [   1      -a      0    -1/2    -1/2      0      0 ; % z_wF
43 %        1      b      0      0      0    -1/2    -1/2 ; % z_wR
44 %        0      0      1    -1/tF    1/tF      0      0 ; % phi_wF
45 %        0      0      1      0      0    -1/tR    1/tR ]; % phi_wR
46
47 %—— mass matrix, Eq. C.3 ——
48 % p.Ixx is the roll inertia of the sprung mass, I_yy in the text
49 M = diag([p.ms, p.Ip, p.Ixx, p.mu, p.mu, p.mu, p.mu]);
50
51 %—— stiffness and damping in deformation space ——
52 % n = 2 for the corrected assembly (see note in FSFEUP_PARAMS)
53 n = p.axleFactor;
54 Kd = diag([n*p.WRF, n*p.WRR, p.KphiF, p.KphiR]);
55 Cd = diag([n*c.hF, n*c.hR, c.phiF, c.phiR]);
56
57 Ktyre = diag([0 0 0 p.Kt p.Kt p.Kt p.Kt]);
58
59 K = T.'*Kd*T + Ktyre;
60 C = T.'*Cd*T;
61
62 %—— enforce exact symmetry against round-off ——

```

```

62 K = (K+K.)/2;
63 C = (C+C.)/2;
64 end

```

Listing D.1: `fs_build_MCK.m` — assembly of the mass, damping and stiffness matrices.

```

1 function R = fs_modal(M,K,C,p)
2 %FS_MODAL Complex modal analysis of the 7-DOF model in state space.
3 %
4 % R = FS_MODAL(M,K,C,p) returns a struct with
5 %   R.f      [7x1] undamped natural frequency of each mode      [Hz]
6 %   R.fd     [7x1] damped natural frequency (NaN if overdamped) [Hz]
7 %   R.zeta   [7x1] EXACT damping ratio of each mode              [-]
8 %   R.zetaP  [7x1] projected damping ratio, Eq. 4.30             [-]
9 %   R.label  {7x1} 'heave','pitch','roll','hop-inphase', ...
10 %   R.shape  [7x7] undamped mode shapes, mass-normalised
11 %   R.over   [7x1] logical, true where the mode is overdamped
12 %   R.wn     [7x1] undamped circular frequency                  [rad/s]
13 %
14 % Method
15 % -----
16 % Two damping ratios are returned:
17 %
18 % zetaP projection of Eq. 4.30 onto the undamped modes, neglecting the
19 %        off-diagonal terms of the modal damping matrix. This is the
20 %        expression derived in the text.
21 %
22 % zeta exact. The second-order system is written in first order form
23 %        with  $q = [x ; \dot{x}]$ ,
24 %         $\dot{q} = A*q$ ,  $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ 
25 %        and eig(A) gives 14 eigenvalues. Each mode owns two of them,
26 %        and from that pair
27 %         $\omega_n = \sqrt{\lambda_a * \lambda_b}$ 
28 %         $\zeta = -(\lambda_a + \lambda_b) / (2 * \omega_n)$ 
29 %        which holds for an underdamped pair (conjugates) and equally for
30 %        an overdamped mode (two real eigenvalues,  $\zeta \geq 1$ ). The
31 %        shorter form  $\zeta = -\text{real}(\lambda) / \text{abs}(\lambda)$  is avoided because it
32 %        loses meaning once a mode goes overdamped, which occurs in this
33 %        vehicle at realistic damping levels.
34 %
35 % Pairing and identification
36 % -----
37 % The 14 eigenvalues are assigned to the 7 modes by continuation from the
38 % undamped system, where the assignment is exact, in nStep increments of
39 % alpha. Each mode therefore keeps its identity across a parameter sweep,
40 % which the order returned by eig does not guarantee. Each mode is then
41 % named from the dominant participation in its undamped shape, with
42 % rotations scaled by a characteristic length (a for pitch, t/2 for roll)
43 % so that participations are compared in metres.
44 %
45 % See also FS_BUILD_MCK, FS_TARGET_DAMPING, FS_ATTAINABLE.
46
47 n = size(M,1);
48
49 %----- undamped reference basis -----
50 [U,W2] = eig(K,M);

```



```

51 wn = sqrt(abs(diag(W2)));
52 [wn,ord] = sort(wn);
53 U = real(U(:,ord));
54 U = U./sqrt(diag(U.'*M*U)).';           % mass-normalise: U'*M*U = I
55
56 %—— projected ratios, Eq. 4.30 ——
57 zetaP = diag(U.'*C*U)./(2*wn);
58
59 %—— exact eigenvalues, tracked by continuation from C = 0 ——
60 % Matching complex eigenvectors to undamped mode shapes fails once any
61 % mode becomes heavily damped: the eigenvectors stop resembling the
62 % undamped shapes and the assignment scrambles. The damping is therefore
63 % introduced gradually, C → alpha*C for alpha = 0..1, and at each step
64 % every eigenvalue is matched to its NEAREST neighbour from the previous
65 % step. At alpha = 0 the assignment is exact and unambiguous
66 % (eigenvalues are +/- i*wn), so the chain is anchored.
67 nStep = 60;
68 alpha = linspace(0,1,nStep);
69
70 % anchor: two eigenvalue slots per mode
71 trk = zeros(2*n,1);
72 for i = 1:n
73     trk(2*i-1) = 1i*wn(i);
74     trk(2*i)   = -1i*wn(i);
75 end
76 owner = reshape(repmat(1:n,2,1),[],1); % which mode owns each slot
77
78 MiK = M\K;  MiC = M\C;
79 for s = 2:nStep
80     As = [zeros(n) eye(n) ; -MiK -alpha(s)*MiC];
81     new = eig(As);
82     trk = matchNearest(trk,new);
83 end
84
85 %—— damping ratio from each tracked pair ——
86 zeta = zeros(n,1); wnEx = zeros(n,1); fd = nan(n,1); over = false(n,1);
87 for i = 1:n
88     la = trk(owner==i);
89     w = sqrt(la(1)*la(2));
90     if real(w) < 0, w = -w; end
91     w = abs(real(w));
92     wnEx(i) = w;
93     zeta(i) = -real(la(1)+la(2))/(2*w);
94     if zeta(i) >= 1
95         over(i) = true;
96     else
97         fd(i) = w*sqrt(1-zeta(i)^2)/(2*pi);
98     end
99 end
100
101 %—— identify each mode by dominant participation ——
102 label = cell(n,1);
103 scale = [1, p.a, p.tF/2];
104 for k = 1:n
105     v = abs(U(:,k)); v = v/max(v);
106     part = [v(1)*scale(1), v(2)*scale(2), v(3)*scale(3), max(v(4:7))];
107     [~,i] = max(part);

```

```

108     switch i
109         case 1, label{k} = 'heave';
110         case 2, label{k} = 'pitch';
111         case 3, label{k} = 'roll';
112         case 4, label{k} = ['hop-' hopPattern(U(4:7,k))];
113     end
114 end
115
116 R.f = wn/(2*pi); R.wn = wn; R.fd = fd;
117 R.zeta = zeta; R.zetaP = zetaP; R.label = label;
118 R.shape = U; R.over = over; R.wnExact = wnEx;
119
120 if exist('table','file') || exist('table','builtin')
121     R.tbl = table(label, R.f, zeta, zetaP, over, ...
122         'VariableNames',{'mode','f_Hz','zeta_exact','zeta_proj','overdamped'});
123 end
124 end
125
126 % =====
127 function s = hopPattern(v)
128 %HOPPATTERN Classify a wheel-hop mode by the phasing of the four corners.
129 % Order of v: [FL FR RL RR]
130 pat = [ 1 1 1 1 ; % all four in phase
131         1 -1 1 -1 ; % roll-like (left vs right)
132         1 1 -1 -1 ; % pitch-like (front vs rear)
133         1 -1 -1 1 ]; % warp-like (diagonal)
134 nm = {'inphase','roll','pitch','warp'};
135 v = real(v(:));
136 if norm(v) == 0, s = 'inphase'; return; end
137 v = v/norm(v);
138 [~,i] = max(abs(pat*v));
139 s = nm{i};
140 end
141
142 % =====
143 function out = matchNearest(prev,new)
144 %MATCHNEAREST Reorder NEW so that out(i) is the eigenvalue closest to
145 % prev(i). Greedy on the distance matrix, one-to-one.
146 m = numel(prev);
147 Dm = abs(prev(:) - new(:).'); % m x m distances
148 out = zeros(m,1);
149 usedNew = false(m,1);
150 usedOld = false(m,1);
151 [~,srt] = sort(Dm(:),'ascend');
152 for s = srt.'
153     [i,k] = ind2sub([m m],s);
154     if ~usedOld(i) && ~usedNew(k)
155         out(i) = new(k);
156         usedOld(i) = true;
157         usedNew(k) = true;
158     end
159 end
160 end

```

Listing D.2: fs_modal.m — frequencies, mode identification and exact damping ratios.

```

1 function ok = fs_verify(p,tol)
2 %FS_VERIFY Check the assembled model against independently derived values.
3 %
4 %   ok = FS_VERIFY(p)
5 %   ok = FS_VERIFY(p,tol)      tol as a fraction, default 0.02 (2 %)
6 %
7 %   Runs five checks and prints a PASS/FAIL table:
8 %
9 %       1 Undamped eigenvalues are purely imaginary.
10 %        A non-zero real part with C = 0 means a sign error in the assembly.
11 %
12 %       2 7-DOF undamped frequencies reproduce the modal results of
13 %        Section 4.6.3 (heave 3.57 Hz, pitch 3.40 Hz) and the wheel hop
14 %        frequency of Section 4.6.1 (~18 Hz).
15 %
16 %       3 3-DOF sprung reduction on ride rates reproduces the same two
17 %        frequencies and the total roll stiffness of Eq. 4.24
18 %        (489.6 Nm/deg), and the roll/heave frequency ratio exceeds 1.5.
19 %
20 %       4 Quarter-car of Section 4.6.1: ride 3.6 Hz, hop ~18 Hz, and
21 %        cw = 1.6 kNs/m from the single-DOF estimate on the front corner.
22 %
23 %       5 Static gains of the state-space model: roll angle under lateral
24 %        acceleration against Equations 4.22 and 4.24, and the rigid-body
25 %        response to a uniform road rise.
26 %
27 %   Called at the start of RUN_DAMPING_STUDY and before every sweep.
28
29 if nargin < 2 || isempty(tol), tol = 0.02; end
30 ok = true;
31
32 fprintf('\n=== fs_verify =====\n');
33 fprintf('axleFactor = %d (2 = corrected Eq. C.2)\n\n',p.axleFactor);
34
35 %——— check 1 and 2: full 7-DOF, undamped —————
36 [M,K] = fs_build_MCK(p,[]);
37 R = fs_modal(M,K,zeros(7),p);
38
39 maxZeta = max(abs(R.zeta));
40 ok = report('undamped: all zeta = 0', maxZeta, 0, 1e-8, ok, '%.2e');
41
42 fh = pick(R,'heave'); fp = pick(R,'pitch');
43 ihop = strcmp(R.label,'hop',3);
44 fhop = mean(R.f(ihop));
45
46 ok = report('7-DOF heave frequency [Hz]', fh, p.ref.fHeave, tol, ok);
47 ok = report('7-DOF pitch frequency [Hz]', fp, p.ref.fPitch, tol, ok);
48 ok = report('7-DOF mean hop frequency [Hz]', fhop, p.ref.fHop, 0.10, ok);
49
50 %——— check 3: 3-DOF sprung reduction on ride rates —————
51 T3 = [1 -p.a 0 ; 1 p.b 0 ; 0 0 1 ; 0 0 1];
52 n3 = p.axleFactor;
53 Ms = diag([p.ms, p.Ip, p.Ixx]);
54 Ks = T3.'*diag([n3*p.RRF, n3*p.RRR, p.KphiEffF, p.KphiEffR])*T3;
55 Ks = (Ks+Ks.)/2;
56 [U3,W3] = eig(Ks,Ms);

```

```

57 w3 = sqrt(diag(W3)); [w3,o3] = sort(w3); U3 = U3(:,o3);
58 sc3 = [1, p.a, p.tF/2];
59 lbl3 = cell(3,1); nm3 = {'heave','pitch','roll'};
60 for k = 1:3
61     [~,i] = max(abs(U3(:,k)).*sc3.'');
62     lbl3{k} = nm3{i};
63 end
64 f3 = w3/(2*pi);
65 ok = report('3-DOF heave frequency [Hz]', f3(strcmp(lbl3,'heave')), p.ref.fHeave, tol, ok);
66 ok = report('3-DOF pitch frequency [Hz]', f3(strcmp(lbl3,'pitch')), p.ref.fPitch, tol, ok);
67 ok = report('total roll stiffness [Nm/deg]', ...
68     (p.KphiEffF+p.KphiEffR)*pi/180, p.ref.KphiTot, tol, ok);
69
70 % roll/heave separation is reported for information: no numerical
71 % threshold is imposed anywhere in the sizing, so this is not a PASS/FAIL
72 ratio = f3(strcmp(lbl3,'roll'))/f3(strcmp(lbl3,'heave'));
73 fprintf(' %42s %10.3f (reported, not a criterion)\n', ...
74     'roll / heave frequency ratio', ratio);
75
76 %——— check 4: quarter car, Section 4.6.1 —————
77 msc = p.ms*(p.b/p.L)/2; % front corner sprung mass
78 Mq = diag([msc, p.mu]);
79 Kq = [ p.WRF, -p.WRF; -p.WRF, p.WRF + p.Kt ];
80 fq = sort(sqrt(eig(Kq,Mq)))/(2*pi);
81
82 ok = report('quarter-car ride frequency [Hz]', fq(1), p.ref.fQCride, 0.03, ok);
83 ok = report('quarter-car hop frequency [Hz]', fq(2), p.ref.fHop, 0.10, ok);
84
85 cc = 2*sqrt(p.RRF*msc);
86 ok = report('front cw at zeta = 0.7 [Ns/m]', 0.7*cc, p.ref.cwFront, 0.05, ok);
87
88 %——— check 5: static gains of the state-space model —————
89 [cN,~] = fs_nominal(p);
90 [Md,Kd,Cd] = fs_build_MCK(p,cN);
91 [~,io] = fs_state_space(Md,Kd,Cd,p);
92 Kdc = io.D - io.Cout*(io.A\io.B);
93
94 u = zeros(6,1); u(5) = 9.81; % 1 g lateral
95 phi = Kdc(strcmp(io.outputNames,'phi'),:)*u;
96 phiRef = p.ms*p.hRoll*9.81/(p.KphiEffF+p.KphiEffR);
97 ok = report('static roll at 1 g [deg]', phi*180/pi, phiRef*180/pi, tol, ok);
98
99 u = zeros(6,1); u(1:4) = 0.010; % 10 mm road rise
100 y = Kdc*u;
101 ok = report('ride height for 10 mm road rise [mm]', 1e3*y(1), 10, 1e-3, ok);
102 ok = report('dynamic tyre load for a rigid rise [N]', y(5), 0, 1e-6, ok);
103
104 fprintf('\n%s\n', repmat('-',1,56));
105 if ok
106     fprintf('ALL CHECKS PASSED — the assembly reproduces the reference values.\n');
107 else
108     fprintf('SOME CHECKS FAILED — debug the assembly before continuing.\n');
109 end
110 fprintf('=====\n\n');
111 end
112
113 % =====

```

```
114 function ok = report(name,val,ref,tol,ok,fmt)
115 if nargin < 6, fmt = '%.3f'; end
116 if ref == 0
117     good = abs(val) <= tol;
118 else
119     good = abs(val-ref)/abs(ref) <= tol;
120 end
121 fprintf([' %42s ' fmt ' (ref ' fmt ') %s\n'], name, val, ref, passStr(good));
122 ok = ok && good;
123 end
124
125 function s = passStr(good)
126 if good, s = 'PASS'; else, s = 'FAIL'; end
127 end
128
129 function f = pick(R,name)
130 i = find(strcmp(R.Label,name),1);
131 if isempty(i), f = NaN; else, f = R.f(i); end
132 end
```

Listing D.3: `fs_verify.m` — verification against the published values.

Appendix E

Derivation of the Rocker Design Loads

This appendix sets out the method used to obtain the loads applied to the bellcrank mechanism in Chapter 6. The chain runs from a set of vehicle-level load cases, through the contact-patch forces they produce, to the axial force in each suspension link, and finally to the force carried by the heave and the roll elements. The verification steps that support each stage are reported at the end.

E.1 Frame and Notation

The frame is the one used for the hardpoint tables of Chapter 3: X lateral and positive towards the left of the car, Y longitudinal and positive towards the rear, and Z vertical and positive upwards with the origin at ground level. Coordinates are in millimetres, forces in newtons and moments in newton-metres.

The four corners are indexed $i \in \{FL, FR, RL, RR\}$ throughout. A positive lateral acceleration a_y corresponds to a left-hand turn and therefore loads the right-hand wheels; a positive longitudinal acceleration a_x is a tractive condition and a negative one is braking.

E.2 Load Cases

Five base cases represent the extremes of the operating envelope. They are listed in Table E.1 and follow the case set used by the team for the FSFEUP03 prototype, so that the design loads of the two vehicles are directly comparable.

Cases 4 and 5 are the combined conditions of that inherited set and are retained for comparability rather than re-derived here; both lie inside the friction ellipse of Section 4.9.1, and neither governs any of the loads reported in this appendix.

Each base case is then repeated with an additional vertical load applied at one wheel at a time, representing a kerb strike or a bump encountered in the middle of the manoeuvre.

Table E.1: Base load cases.

Case	Condition	a_x [g]	a_y [g]
1	Maximum acceleration	+1.60	0.00
2	Maximum braking	−1.80	0.00
3	Maximum lateral	0.00	1.60
4	Combined traction and cornering	+0.72	0.72
5	Combined braking and cornering	−1.12	1.12

The additional load is 1.5 times the static load of that corner. With four corners, this produces twenty derived cases and a total of twenty-five.

The single-wheel cases are not a formality. In the decoupled architecture they are the condition that loads the heave and the roll element simultaneously, as Section E.5 shows, and they turn out to govern the design of the bellcrank.

E.3 Contact-Patch Forces

E.3.1 Vertical Load

The static vertical load at each wheel follows from the total mass and the longitudinal position of the centre of gravity. Longitudinal load transfer is distributed equally between the two wheels of an axle,

$$\Delta F_{z,x} = \frac{m a_x h_{CG}}{L}, \quad (\text{E.1})$$

and lateral load transfer is distributed between the axles using the lateral load-transfer distribution of Equation 2.12,

$$\Delta F_{z,y}^F = \text{LLTD} \frac{m a_y h_{CG}}{t_F}, \quad \Delta F_{z,y}^R = (1 - \text{LLTD}) \frac{m a_y h_{CG}}{t_R}. \quad (\text{E.2})$$

The bump contribution, where present, is added to the corner concerned. The resulting vertical load is floored at zero, since a wheel cannot pull on the road; when this occurs the corresponding wheel is treated as unloaded and its tyre forces vanish with it.

E.3.2 Longitudinal and Lateral Forces

The prototype is four-wheel driven, so both axles deliver tractive effort. The total tractive force is distributed between the axles in proportion to the vertical load each axle carries after load transfer, which is the traction-limited distribution and the one a functioning traction control approaches. Under braking the distribution is fixed by the brake balance, with the rear axle taking β_R of the total. Within an axle, the longitudinal force is distributed between the two wheels in proportion to their vertical loads.

The lateral force at each wheel is taken in proportion to its vertical load, so that the sum over the four wheels equals $m a_y$.

E.3.3 Aligning Torque

The tyre aligning torque is written as the lateral force acting through a total trail,

$$M_{z,i} = F_{y,i} t_i. \quad (\text{E.3})$$

The trail is not treated as a constant. Evaluating Equation E.3 against the aligning torques recorded in the FSFEUP03 load-case data shows that the implied trail grows in approximate proportion to the vertical load, with $t_i \approx k_t F_{z,i}$ and k_t between 0.0100 and 0.0133 mm/N across the entries with appreciable load. The mean value, $k_t = 0.01144$ mm/N, is adopted. The consequence that matters is qualitative: a lightly loaded inside wheel produces very little aligning torque even when the tyre is at its lateral limit, so the steering link is not sized by the unloaded side.

E.4 Equilibrium of the Upright

Each corner is a set of six two-force members: the two arms of the upper wishbone, the two arms of the lower wishbone, the tie rod, and the pushrod. Each carries an axial force only, so the corner has six unknowns and the static equilibrium of the upright supplies exactly six equations. The problem is therefore determinate and can be solved without reference to anything inboard of the pushrod.

This is the property that makes the method independent of the rocker design: the pushrod force follows from the wheel loads and the outboard geometry alone, and the same derivation applies to a conventional and to a decoupled layout.

Let $\mathbf{p}_j$ be the outboard end of link j , on the upright, and $\mathbf{q}_j$ its inboard end, on the chassis. The unit vector along the link, taken positive in tension, is

$$\mathbf{u}_j = \frac{\mathbf{q}_j - \mathbf{p}_j}{\|\mathbf{q}_j - \mathbf{p}_j\|}. \quad (\text{E.4})$$

With $\mathbf{F}_c$ and $\mathbf{M}_c$ the force and moment applied at the contact patch $\mathbf{r}_c$, equilibrium of forces and of moments about the origin gives

$$\underbrace{\begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_6 \\ \mathbf{p}_1 \times \mathbf{u}_1 & \cdots & \mathbf{p}_6 \times \mathbf{u}_6 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} T_1 \\ \vdots \\ T_6 \end{bmatrix} = - \begin{bmatrix} \mathbf{F}_c \\ \mathbf{r}_c \times \mathbf{F}_c + \mathbf{M}_c \end{bmatrix}, \quad (\text{E.5})$$

which is solved for the six axial forces T_j , positive in tension. The system is assembled and solved once per corner and per load case, that is one hundred times over the twenty-five cases.

Conditioning of $\mathbf{A}$ is monitored as a check on the geometry. The raw condition number is not meaningful, because the force rows are dimensionless while the moment rows carry

units of length; after non-dimensionalising the moment rows by a characteristic length of 600 mm the condition number is 32 at the front and 171 at the rear. Both are comfortable. The higher rear value reflects the rear tie rod lying close to the plane of the lower wishbone, which makes that axle the more sensitive of the two to errors in the hardpoint coordinates.

E.5 From the Pushrod to the Two Elements

The pushrod forces of the two wheels of an axle are decomposed into a symmetric and an antisymmetric part,

$$T_s = \frac{T_L + T_R}{2}, \quad T_a = \frac{T_L - T_R}{2}. \quad (\text{E.6})$$

By the block structure established in Section 4.6.2, the symmetric part is reacted entirely by the heave element and the antisymmetric part entirely by the roll element. What carries each part to its element is the bellcrank itself. Each bellcrank is a free body in equilibrium about its pivot, so the moment the pushrod applies at its arm r_{pr} is reacted by the two elements acting at their own arms r_h and r_r , listed in Table 6.1. Under a symmetric input both bellcranks of the axle act on the shared heave element in the same sense, and under an antisymmetric input both act on the roll element, so in each mode the force the element carries is the one a single bellcrank applies,

$$F_h = T_s \frac{r_{pr}}{r_h}, \quad F_r = T_a \frac{r_{pr}}{r_r}. \quad (\text{E.7})$$

The ratio of arms is not a motion ratio: by Equation 6.2 the two differ by the pushrod displacement ratio of the outboard geometry, so that $r_{pr}/r_h = MR_h F_w / F_{pr}$. Equation E.7 is written in the arms because they enter the moment balance directly and are exact, whereas the motion ratios carry the rounding of the values adopted in Chapter 4.

A load case that is symmetric across the axle, such as straight-line braking, produces $T_a = 0$ and loads the heave element alone. A single-wheel input produces $T_s = T_a = T/2$ and loads both elements at once, which is why the bump cases govern the bellcrank: it is the only condition in which both attachment points carry force simultaneously and the moment about the rocker pivot is largest.

E.6 Verification

Four independent checks are applied before the results are used.

Equilibrium residual. The residual $\|\mathbf{AT} - \mathbf{b}\|$ is of the order of 10^{-10} N in every corner and every case, confirming that the linear system is solved consistently rather than in a least-squares sense.

Static closure. The vertical wheel loads of the unaccelerated vehicle sum to the vehicle weight, 2703.6 N, confirming the load-transfer bookkeeping.

Virtual work. For a pure vertical load, the ratio between the pushrod force and the wheel force must equal the inverse of the ratio of their displacements, since $F_{pr} \delta_{pr} = F_w \delta_w$. The equilibrium solution gives $F_{pr}/F_w = 1.224$ at the front and 1.419 at the rear, obtained from statics alone, and these reproduce the displacement ratios of the kinematic model reported in Table 6.1. Force and displacement are two independent routes to the same quantity, so their agreement validates the geometry and the solver together.

E.7 Limitations

The derivation is quasi-static: the load cases represent sustained accelerations with a bump superimposed as an equivalent static load, and no dynamic amplification beyond that factor is applied. The suspension links are treated as rigid two-force members, so compliance in the bushings, in the rod ends and in the chassis attachment points is not represented, and neither is the friction in the spherical joints.

The geometry is evaluated in the static position. The motion ratios and the link directions both change along the suspension travel, so the forces reported here correspond to ride height and not to the extremes of the stroke. The pushrod force ratio varies by a few per cent over the design travel, which is small relative to the margin carried in the load factors, but it means that the results are design loads rather than an envelope.

Finally, the lateral force distribution assumes that every tyre operates at the same fraction of its vertical load, which neglects the load sensitivity of the tyre. This is conservative for the lightly loaded inside wheels and slightly unconservative for the heavily loaded outside ones.

Appendix F

Modelling Entities in ADAMS/Car

This appendix defines the modelling entities of ADAMS/Car that the description of the vehicle model in Chapter 5 relies on. The purpose is to fix the vocabulary rather than to reproduce the software documentation, to which the reader is referred for the associated procedures [42]. Each entity is described by the function it performs in the model and by the consequence its use has for the results.

F.1 Model Hierarchy

ADAMS/Car separates a vehicle model into three levels, and the separation is what makes a suspension model reusable across design iterations.

Template. The template holds the topology of a subsystem: which parts exist, how they are connected, and which quantities are parametric. It does not hold numerical values. Two suspensions of the same architecture but different geometry share one template.

Subsystem. A subsystem is an instance of a template with values assigned to its parameters — coordinates, masses, stiffnesses. Changing a hardpoint or a spring rate is a change to the subsystem, and requires no modification of the connectivity.

Assembly. An assembly gathers the subsystems that are to be analysed together, resolves the connections between them, and adds the test rig. A suspension assembly contains one axle and its steering; a full-vehicle assembly contains both axles, the body and the powertrain.

The consequence for the present work is that the geometric studies of Chapter 5 are carried out by editing subsystem values within a fixed template, so that the successive iterations of the hardpoint set differ only in the quantity under examination.

F.2 Geometry and Parameterisation

Hardpoints. Hardpoints are the named coordinate triples from which the rest of the template is constructed: the locations of the joints, of the link attachments and of the wheel centre. Every part, joint and force element in a template is defined relative to hardpoints rather than to absolute coordinates, so that moving a hardpoint moves everything attached to it consistently. This is the mechanism by which a change of suspension geometry propagates through the model.

Construction frames. Construction frames define orientations that hardpoints alone cannot express, such as the axis of a revolute joint or the direction along which a bushing is characterised. They are used where a direction is required in addition to a position.

Parameter variables. Parameter variables hold scalar quantities that drive the template, such as a track width or a toe setting, allowing a single value to control several dependent entities.

F.3 Parts

General parts. A general part is a rigid body with mass, centre of mass and inertia tensor. Wishbones, uprights and rockers are general parts. In a kinematic analysis their mass properties do not affect the result, since the solution depends only on the constraint equations; in a dynamic analysis they do.

Mount parts. A mount part is a massless placeholder that marks where a subsystem connects to something outside itself. When the assembly is built, each mount part is replaced by the part it is connected to — the chassis, or a part in an adjacent subsystem. If no such part is found, the mount is attached to ground. Mount parts are the reason a suspension template can be analysed on a test rig and, unchanged, be assembled into a full vehicle.

Flexible bodies. A part may be replaced by a modally reduced finite element representation where its deformation affects the result. This is not used in the present model, in which all parts are treated as rigid.

F.4 Constraints and Compliant Connections

Joints. A joint is an ideal constraint that removes degrees of freedom without permitting relative displacement in the constrained directions. A spherical joint removes three, a

revolute joint five, a translational joint five. Joints are the entities that appear in the constraint vector of Equation 2.15, and the reaction they carry is the corresponding Lagrange multiplier.

Bushings. A bushing replaces a joint by a compliant connection, transmitting force and moment through prescribed stiffness and damping in six directions instead of enforcing an exact constraint. The choice between a joint and a bushing determines whether the model is purely kinematic or elastokinematic: with joints, the wheel path is a property of the linkage alone; with bushings, it also depends on the loads applied.

Couplers and motions. Couplers impose a fixed ratio between the motions of two joints. Motions prescribe a displacement as a function of time, and are the means by which a test rig imposes wheel travel; a motion removes a degree of freedom, which is what allows a kinematic analysis to have a unique solution.

F.5 Force Elements

Springs and dampers. Spring and damper elements act along the line between two parts and are defined either by a constant rate or by a force-displacement or force-velocity table. A non-linear rate is therefore represented directly, without changing the topology of the model.

Bump stops and rebound stops. These are one-sided force elements that engage beyond a specified displacement, and represent the end of the available travel.

Tyre models. A tyre model generates the forces and moments at the contact patch as functions of slip, load and camber. It is required for handling analyses and is not involved in the kinematic studies, in which the wheel motion is imposed by the test rig.

F.6 Communicators

Communicators are the mechanism by which subsystems connect without any subsystem containing a reference to another. They have no counterpart in general-purpose multibody software and are the entity most specific to ADAMS/Car.

An output communicator publishes information from a subsystem: a part, a location, an orientation or a scalar value. An input communicator declares that a subsystem requires such information. When the assembly is built, each input is matched to an output of the same class and matching name, and the connection is resolved. A suspension template thus publishes the parts to which the steering rack attaches, and the steering template requests them, without either template naming the other.

Two consequences follow in practice. A subsystem whose input communicators are not matched will still assemble, with the unmatched connections defaulting to ground, which produces a model that runs and gives incorrect results; the assembly report is therefore checked for unmatched communicators before any analysis is accepted. And a template can be exchanged for another of the same architecture provided the communicator set is preserved.

F.7 Test Rigs and Analyses

A test rig is a subsystem that applies prescribed motions or loads to an assembly in place of the missing parts of the vehicle. The suspension test rig grounds the chassis and drives the wheel centres, so that the suspension is exercised over its travel independently of the rest of the vehicle.

The analyses used in this work fall into two of the three classes described in Section 2.8.3. A kinematic analysis requires the mechanism to have no remaining degrees of freedom once the rig motions are imposed, and returns the configuration of the linkage as a function of the imposed travel; it is the analysis behind the geometric characteristics reported in Chapter 5. A static analysis solves for equilibrium under applied loads and is used where the deflected position is required rather than a prescribed one.

The parallel wheel travel analysis displaces both wheels of an axle in phase and characterises the heave behaviour; the opposite wheel travel analysis displaces them in antiphase and characterises the roll behaviour. In the decoupled architecture these two analyses excite the two elastic paths separately, which is what allows the motion ratios of each path to be measured independently.

Appendix G

Front Left Bellcrank Technical Drawing

