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The role of fiscal and monetary policies on growth and wages

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Doctoral Thesis

PhD in Economics

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Biography

Fernando Brustolin's research primarily examines the impact of technological change on economic growth and wage differentials, with a particular emphasis on the role of innovation and automation in shaping labor market dynamics. His scholarly work seeks to advance the understanding of structural transformations driven by technological progress and their broader implications for income inequality, productivity, and long-term development.

Since 2009, he has held a position in information technology management at the Institute for Applied Economic Research (IPEA) in Brazil, where he has contributed to the integration of technological solutions in policy-oriented research. He earned an MSc in Economics from the University of Brasília (2016) and a BSc in Computer Science from the University of São Paulo (2008). His research aims to provide empirical insights and theoretical contributions that inform both academic debate and policy design in the context of technological change.

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Resumo

Esta tese analisa a forma como as políticas públicas e os enquadramentos institucionais influenciam o crescimento económico e a desigualdade salarial em economias caracterizadas por mudança tecnológica endógena e trabalho heterogéneo. É constituída por três ensaios teóricos que estudam mecanismos distintos, mas interligados, através dos quais a inovação, os instrumentos de política económica e as instituições moldam resultados económicos e distributivos de longo prazo.

O primeiro ensaio examina a interação entre as políticas fiscal e monetária num modelo de crescimento baseado no conhecimento, com investigação e desenvolvimento (I&D) horizontalmente diferenciada e trabalho qualificado e não qualificado. Os efeitos das políticas operam através de restrições de liquidez (cash-in-advance) e de subsídios à I&D. Embora estes instrumentos não afetem a taxa de crescimento de longo prazo, influenciam a intensidade da I&D, o prémio de qualificação e os níveis de bem-estar. Em condições economicamente plausíveis, os seus efeitos de longo prazo sobre a intensidade da I&D, a desigualdade salarial e a velocidade de circulação da moeda são complementares, sendo esta complementaridade mais forte durante a dinâmica de transição.

O segundo ensaio analisa o impacto dos sindicatos na desigualdade salarial e no crescimento económico no contexto de um modelo de mudança tecnológica dirigida. Os efeitos do aumento do poder negocial dos sindicatos dependem da sua orientação para o emprego ou para os salários, bem como da importância relativa dos canais de conhecimento tecnológico e de oferta de trabalho no prémio de qualificação.

O terceiro ensaio desenvolve um enquadramento para analisar o impacto da inteligência artificial no crescimento endógeno e na desigualdade salarial, modelando a IA como uma tecnologia de uso geral. A qualidade institucional é determinante para moderar os incentivos à inovação, os ganhos de produtividade e os resultados distributivos.

Abstract

This thesis analyzes how public policies and institutional arrangements influence growth and wage inequality in economies characterized by endogenous technological change and heterogeneous labor. It consists of three theoretical essays that examine distinct but interconnected mechanisms.

The first essay examines the interaction between fiscal and monetary policy in a knowledge-based, horizontally differentiated R&D growth model with skilled and unskilled labor. Policy effects operate through cash-in-advance constraints and R&D subsidies. Although fiscal and monetary policies do not affect the long-run growth rate, they influence R&D intensity, the skill premium, and welfare levels. Under economically plausible conditions, their long-run effects on R&D intensity, wage inequality, and the velocity of money are complementary. The analysis also highlights the importance of transition dynamics, showing that policy complementarity in welfare is stronger along the adjustment path.

The second essay examines how labor unions affect wage inequality and economic growth within a DTC framework. By introducing collective bargaining for unskilled workers, the model shows that the effects of increased union bargaining power depend on whether unions are oriented toward employment or wages, and on the relative strength of the technological-knowledge and labor-supply channels that affect the skill premium. This framework reconciles ambiguous empirical findings, demonstrating that stronger unions can compress or amplify wage inequality while affecting unskilled employment, the labor income share, and growth.

The third essay develops a framework to analyze the impact of artificial intelligence on endogenous growth and wage inequality, modeling AI as a general-purpose technology that can both complement and substitute human labor. Institutional quality plays a central role in moderating innovation incentives, productivity gains, and distributional outcomes. While AI adoption can accelerate long-run growth, weak institutions and insufficient redistribution may intensify economic polarization.

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1. Introduction

The study of long-run economic growth occupies a central place in macroeconomics, and technological progress has long been identified as a key driver of sustained increases in output, holding capital and labor constant (Romer 2019). This thesis argues that the long-run effects of technological change on growth and inequality depend critically on the interaction between fiscal and monetary policy instruments, labor market institutions, and broader institutional and governance arrangements. Although the three subsequent chapters focus on distinct mechanisms, they share a common analytical perspective on endogenous technological change in heterogeneous-labor economies.

Models of endogenous economic growth internalize the process of idea, or knowledge, creation within the economy, thereby linking technological progress to market incentives and other structural factors (Acemoglu 2009b). A prominent class of such models is that of expanding product varieties, or horizontal growth, developed in the spirit of Romer (1990) and Jones (1995). In this framework, research and development (R&D) activity generates an ever-growing number of intermediate input varieties used in final production. Aggregate productivity increases with the number of available varieties, which serves as a proxy for the economy's stock of technological knowledge and symmetrically enhances the productivity of all types of labor, irrespective of sectoral allocation.

By contrast, models of directed technical change (DTC), following Acemoglu (2002b), relax this neutrality assumption by allowing technological change to benefit certain factors more than others. In this approach, technological change exhibits both "direction" and "bias" (Acemoglu 2009a). Direction refers to which factor experiences an increase in relative demand as a result of innovation. At the same time, bias arises because an increase in the relative supply of a factor induces technological change that favors that same factor, reinforcing its productivity advantage.

The Romer (1990) model became the canonical framework for endogenous growth driven by R&D. Jones (1995), however, highlighted the presence of scale effects in this class of models that lacked empirical support. To address this issue, he proposed a modification that eliminates scale effects, yielding a framework in which long-run growth results from the allocation of resources to R&D but is ultimately determined by exogenous parameters governing population growth and technological opportunities. This insight gave rise to the class of semi-endogenous growth models associated with the Romer–Jones tradition.

Subsequent contributions have extended this framework in several directions, including the explicit incorporation of policy instruments to shape innovation incentives. In particular, fiscal interventions, such as taxes and R&D subsidies, can affect the investment in innovation and, in some cases, growth outcomes (e.g., Atkeson and Burstein 2019; Peretto 2007; Zeng and Zhang 2007). Money demand can be represented by placing real balances directly in household utility or by modeling transaction costs; approaches that are functionally equivalent under certain conditions outlined by Feenstra (1986). Currently, sectoral cash-in-advance (CIA) constraints is a common choice to integrate monetary mechanisms into endogenous growth settings (e.g., Chu and Cozzi 2014; Gil and Iglésias 2019).

Several extensions of horizontal endogenous growth models have incorporated labor heterogeneity. In particular, Afonso (2006) and Okada (2012)introduce skilled and unskilled labor into endogenous growth frameworks, enabling the analysis of wage differentials in innovation-driven economies. On the empirical side, a substantial body of evidence documents a sustained increase in the wage premium for skilled labor over recent decades (e.g., McAdam and Willman 2017; Song et al. 2018).

A leading explanation for the rise in wage inequality is skill-biased technological change (SBTC), which is typically formalized within models of directed technical change. From this perspective, innovation endogenously shifts toward skill-complementary technologies, raising the relative productivity and demand for skilled labor. The resulting bias in technological knowledge tends to dominate the countervailing effect of an exogenous expansion in the supply of skills, thus sustaining or increasing the skill premium (e.g., Li et al. 2013; McAdam and Willman 2017). The underlying mechanism operates through market-size effects: a larger relative supply of a factor expands the market for complementary technologies, thereby strengthening incentives for R&D directed toward it and reinforcing the associated wage differential.

While this extensive literature has substantially advanced the understanding of endogenous growth, monetary mechanisms, and wage inequality, these strands have primarily developed in parallel. Models that explicitly incorporate policy instruments typically abstract from labor heterogeneity and distributional consequences, whereas frameworks designed to explain the skill premium often emphasize the direction of technological change without explicitly modeling macroeconomic policy interactions. Moreover, the role of labor market institutions, as well as broader institutional and governance arrangements, in shaping innovation incentives and distributional outcomes remains only partially integrated into growth theory, despite growing interest in institutionalized wage-setting and automation-driven technological change.

This thesis is organized into three theoretical essays, each addressing a distinct mechanism through which policy or institutions shape innovation, economic growth, and wage inequality, with effects that are conditional on the underlying economic and institutional context. This work contributes to the endogenous growth literature by examining the interaction between policy instruments, institutional arrangements—including labor market institutions—and technological change within a set of growth models featuring heterogeneous labor.

Chapter 2 sets up a horizontal growth model with R&D subsidies and CIA constraints to analyze the complementarity of the fiscal and monetary policy effects on a set of real macroeconomic variables; Chapter 3 considers an unskilled labor trade union in a DTC model to explore how unions' bargaining power influence the technological bias, the income-labor share, wage inequality and growth; Chapter 4 models AI as a dual-factor technology in an overlapping-generations setting to examine its impact on growth and inequality, emphasizing the moderating role of institutions.

Taken together, these essays offer a coherent perspective on how technological change, policy instruments, and institutions interact to shape long-run growth and wage inequality. The final chapter concludes the thesis by synthesizing the main results, discussing their broader implications for growth theory and policy, and outlining avenues for future research.

2. Growth, wage inequality and welfare: complementarity between fiscal and monetary policies

This theoretical paper presents a comprehensive analysis of the effects of monetary and fiscal policy on economic growth in the short, medium, and long term within the context of a knowledge-based horizontal R&D growth model. The model incorporates both skilled and unskilled labor. The propagation of policy effects into the model is facilitated by cash-in-advance constraints and R&D subsidies. Under certain conditions, both policies exert effects in the same direction on a range of significant real macroeconomic variables in the long run. There is a notable complementarity in the long-run effects on R&D intensity, skill premium, and the velocity of money. Complementarity on long-run welfare is possible if the policy effects on output dominate the effects on intermediate goods production costs and consumption. Economic conditions that favor such a complementarity include the labor market conditions of high Frisch elasticity of labor supply in conjunction with low unemployment rates, a high rate of R&D discoveries, and an accelerated growth rate. Considering the transition dynamics when accounting for welfare tends to make policies more complementary, and such an effect is more pronounced at the beginning of the transition.

Keywords: endogenous growth; policy complementarity; fiscal policy; monetary policy; skill premium; welfare.

JEL Classification: O40; O31; J31; E52; E41; E62.

2.1. Introduction

The effects of policies on a set of relevant real macroeconomic variables —economic growth rate, real interest rate, R&D intensity, welfare, skill premium, and velocity of money— have long been debated among economists. However, some important questions remain. Is the impact on each variable conclusive? What conditions contribute to the complementarity between fiscal and monetary policy effects in the long run? For example, the theoretical and empirical literature suggests that, in the long run, inflation tends to have (i) an inconclusive impact on the skill premium (e.g., Amaral 2017), (ii) a negative effect on economic growth (e.g., Chu and Lai 2013), R&D intensity (e.g., Chu et al. 2015), and welfare (e.g., Camera and Chien 2014), and (iii) a positive influence on the velocity of money (e.g., Caballé and Hromcová 2010).

Several works on the endogenous growth literature focus on policy optimality. For example, Chu and Cozzi (2014) look for the optimal nominal interest (the monetary policy instrument) while considering different cash-in-advance (CIA) constraints. On the other hand, Gomez and Sequeira (2014) investigate the optimal subsidy structure under alternative tax schemes (the fiscal policy instruments). Unlike these analyses, we focus on the results of fiscal and monetary policy used concomitantly. Although it is not difficult to find literature analyzing the effects of the policy interaction (e.g., Dixit and Lambertini 2003; Fragetta and Kirsanova 2010; Muscatelli et al. 2004), there is a lack of studies regarding which conditions favor complementarity or substitutability between policies. This paper theoretically investigates the existence of substitutability or complementarity for the effects of fiscal policy (in the form of R&D subsidies) and monetary policy (the control of the nominal interest rate) while analyzing the macroeconomic relationships between the mentioned variables.

We propose a general equilibrium monetary (semi-)endogenous-growth model with horizontal knowledge-driven R&D that extends the Romer-Jones model (Jones 1995; Romer 1990). In particular, we incorporate two types of labor, skilled and unskilled, money demand through CIA constraints, and direct and indirect R&D subsidies to analyze whether both monetary and fiscal policies can be used in substitution for each other to induce similar effects on the aforementioned set of variables and, if their effects are complementary in the long-run. This analytical setup, in which horizontal R&D activity performed by skilled labor is the engine of economic growth, has not yet been used in the present context. For example, Lloyd-Ellis (1999) and Okada (2012) considered skill differentiation in horizontal growth models to study the rise of the skill premium without considering any policy. In contrast, Espinosa-Vega and Yip (1999), Chu and Cozzi (2014), and Arawatari et al. (2018) considered policies in endogenous growth setups but analyzed a smaller set of variables.

Both empirical (e.g., Almus and Czarnitzki 2003) and theoretical (e.g., Gomez and Sequeira 2014; Peretto 2007) studies point towards positive effects of direct and indirect subsidies to R&D on economic growth through an improvement in technological knowledge. Moreover, the dominating literature suggests a negative relationship between inflation (and demanding CIA constraints) and economic growth (Benhabib and Spiegel 2009; Chu et al. 2015). However, if the relations between these two policy types and economic growth are long debated among economists, the same cannot be said about the relations between them and a more extensive set of real macroeconomic variables like the skill premium, R&D intensity, welfare, and the velocity of money. By incorporating skilled and unskilled labor in the model, we can explore the relationship between the proposed policy instruments and the skill premium in knowledge-driven models like ours.

The final-goods sector produces a composite homogeneous good in a perfectly competitive market by using skilled and unskilled labor, inelastically supplied by the households, and a continuum of non-durable varieties of intermediate goods. Potential entrants into the intermediate-goods sector devote skilled labor to horizontal R&D, intending to discover new blueprints/designs, which are the aggregate outcome of R&D. Such effort is determined as the result of market forces, assuming profit-maximizing firms. Each intermediate-good variety, resulting from a design discovery, is produced using foregone output by its inventor in an imperfectly competitive market to be used by perfectly competitive firms in the homogeneous final good production. The monetary authority determines the money supply, and the model incorporates money demand via CIA constraints on consumption, production of intermediate goods, and R&D activities. Monetary policy, exercised through the nominal interest rate control, thus affects both production and households' decisions. With a change in this instrument, a force transmits different inflation costs, distorting the incentives and the use of economic resources across different sectors.

As a result of the close relationship between the production of intermediate goods and R&D, fiscal policy can stimulate the latter through a direct subsidy for investment in that activity or by subsidizing the production of intermediate goods. Monetary policy can encourage both activities without demanding sector-specific CIA restrictions. Our choice for monetary policy modeling considers that households use credit to pay for a portion of their consumption and that liquidity is a determinant of production, as evidence shows (Bates et al. 2009; McLean 2011). Thus, in addition to a CIA constraint on consumption, we also consider CIA constraints in intermediate goods production (e.g., Gil and Iglésias 2019) and the creation of new designs in R&D activities (e.g., Chu and Cozzi 2014).[1] Rational infinitely-lived households provide labor inelastically, maximize the utility obtained from the consumption of the aggregate final good, pay taxes, and earn income from labor and investments in financial assets and monetary balances. The way we organize R&D in the model allows us to discuss the policy effects on skill premium and welfare out of the skill-biased technological change (SBTC) context.

Complementarity and substitutability are defined in opposition to each other in consumer preferences between goods or goods' usage as inputs for production. For Jankowski and Wlezien (1993) there is substitutability between policies if different instruments can achieve the same objective. Such a condition is necessary, but more is needed to distinguish between complementarity and substitutability in policies. In our case, complementarity happens: "the effect of each policy is greater when conducted in conjunction with the other

[1]Generally, it is considered a CIA constraint on R&D investment because firms have strong R&D cash flow sensitivity (Brown and Petersen 2015) or to finance the blueprint production due to working capital requirements (Chu and Cozzi 2014). Here, the restriction on intermediate goods production indirectly affects R&D, the focus of this work. At the same time, the constraint on consumption is an apparatus with a limited effect that allows the model to work in case the other CIA parameters are zeroed.

than in isolation" (Eckardt and Neugart 2022).[2] We apply this idea to analyze the short- and medium-run transition quantitatively. However, it is possible to analyze the signs of cross-derivative steady-state expressions and improve the understanding of the policy effects.

In most empirical studies, the monetary-fiscal policy interaction is addressed in the context of strategic games, looking at the policymakers' actions, which can be more complementary or substitute (e.g., Muscatelli et al. 2004); others, however, look for optimal strategies to coordinate policy actions and are more distant from our topic (e.g., Dixit and Lambertini 2003; Fragetta and Kirsanova 2010).[3] A different strand of literature on these policy interactions examines whether the policy direction follows the economic cycle (e.g., Silva and Vieira 2017). We contribute to this body of knowledge by examining the interactions between fiscal and monetary policy and the conditions that foster long-run complementarity. In the theoretical context of the proposed endogenous growth model, conditions that are easy to satisfy permit both policies to induce the same effects on R&D intensity, skill premium, the velocity of money, output, and consumption, showing complementarity if jointly used. Complementarity in welfare, measured by the discounted lifetime utility, is only numerically detected when the model is extended to consider that labor is not perfectly inelastically supplied and households' leisure is valued (i.e., labor disutility). Lower inflation, a considerable level of subsidies in R&D, a higher growth rate, a higher rate of invention's arrival, higher rates of labor supply (or a lower unemployment rate), a less unequal distribution of the wage bill, and lower production costs in the intermediate goods sector are the economic conditions that we find favoring complementarity.

This work also adds to the literature on the skill premium. In this regard, the increase in wage inequality in favor of skilled labor (skill premium) has been significant in recent decades, according to empirical evidence (e.g., McAdam and Willman 2017). At first sight, this seems puzzling, given that an increase in skill supply accompanied such movement —e.g., workers with a bachelor's or advanced degree rose from 17% of the workforce in 1975 to 40% in 2019 (U.S. Census Bureau 2021, author's computation). The literature on SBTC is one of the most important explanations for this pattern. It implies a shift in endogenous production of technological knowledge that favors skilled over unskilled labor, increasing its relative productivity and, thus, its relative demand (e.g., Acemoglu

[2]The modern formalization of such a numerical concept goes back to Topkis's (1978) supermodular analysis framework, that does not "require any particular differentiability or convexity assumptions." It can be applied to "discrete or qualitative policy indicators," being more flexible than Edgeworth's idea of complementarity, that relies on the sign of cross-partial derivatives of an objective function, which can be restrictive (Macedo and Martins 2008).

[3]See Weber (2005) for a historical perspective on complementarity. See Vives (2008) for an overview of the "supermodular games" theory from which the "games of strategic complementarities" literature developed.

2002b). According to this literature, the increase in skill demand induced by technological knowledge bias dominates the (exogenous) increase in skill supply.[4] Complementing such an explanation, Lloyd-Ellis (1999) adds that the skill premium is also a result of the skewed distribution of the labor force capacity in absorbing new technologies. A more recent literature branch, connected with the SBTC, explains the decline in labor share observed in the last decades, mainly in developed economies and severely for unskilled labor (e.g., Karabarbounis and Neiman 2013), as a consequence of robotization, which fosters growth but raises the skill premium (e.g., Acemoglu and Restrepo 2018b). In the model proposed here, the skill premium is determined by the relative skill supply and the shares of unskilled and skilled labor in aggregate output. In our analysis of how policy instruments can affect the skill premium, both in the steady state and during transition dynamics, we disregard any bias towards directed technological knowledge. Such a mechanism, proposed by the SBTC literature, is absent in the current context. The consideration of an aggregate R&D process is merely instrumental in isolating the policy effects from the importance of the technological-knowledge bias mechanism. Our results suggest that monetary policy (pursuing lower inflation) may have contributed to the observed rise in the skill premium in recent decades.

We analytically explore the monetary policy's long-run (steady state) and short-medium-run (transition) effects. Looking at the steady-state effects, we start by emphasizing that a unique positive saddle-path-stable steady state exists and that the inflation rate is an increasing function of the exogenous monetary policy variable, the nominal interest rate. As in other semi-endogenous growth models, inflation or any policy does not affect the economy's long-run growth rate. However, the model does not feature super neutrality of money, as in Sidrauski (1967) and Stockman (1981). This is because, along with inflation and the velocity of money, real variables are also affected by changes in the growth rate of the money supply. The nominal interest rate, or inflation, negatively affects R&D intensity and the skill premium; it does not change the real interest rate and tends to affect the velocity of money circulation positively and welfare negatively. However, the theoretical conditions to reverse the direction of the policy effects on welfare are determined. During the transition, the economy's growth rate and R&D intensity are always positively correlated and a negative inflation-growth relationship can be observed for a plausible calibration.

The CIA approach we take to introduce money demand on consumption is a particular case of a more general money-in-utility (MIU) function specification, in which individuals like

[4]This literature considers that when the supply of a type of labor increases, the market for technologies that complement it broadens, and this creates additional incentives for R&D aimed at those technologies; hence, the SBTC explanation interprets the rise in the skill premium as a consequence of the (exogenous) increase in the relative supply of skilled labor.

to retain money. Such an approach simplifies the resulting expressions and the calibration process. In an extension of the model, we remove the CIA constraint on consumption to include money in the utility function instead (Feenstra 1986) and show that the main qualitative results for monetary and fiscal policy remain unchanged and, under certain conditions, the models are entirely equivalent.

The remainder of this study proceeds as follows. The theoretical model is spelled out in Section 2.2, considering the production and price decisions, R&D activity, labor shares, skill premium, preferences, monetary authority, and government. Section 2.3 depicts and solves the dynamic general equilibrium for the long run, presents the static results, shows that fiscal and monetary policy can induce effects on the same direction on the analyzed variables, the conditions in which policies' effects are complementary, and discusses the model's general behavior during the transition. Section 2.4 presents two model extensions: one considers elasticity in the amount of labor supplied by households; the other considers a MIU specification instead of the CIA on consumption approach. Section 2.5 describes the model calibration to match the United States (U.S.) economy (1975–2019) and numerically analyzes the steady state and the transition dynamics. Such analysis complements the Section 2.3 findings and investigates the different funding for the policy changes and the economic conditions that foster complementarity between policies' effects on welfare in the long run. Section 2.6 concludes.

2.2. Model

This section describes the model's setup, emphasizing the interactions between firms, households, and policy authorities. We extend Okada (2012), which introduced labor heterogeneity (skilled and unskilled labor) in the standard endogenous R&D growth model without scale effects (Jones 1995; Romer 1990) by incorporating fiscal instruments (taxes and subsidies) and money demand (CIA constraints). The homogeneous final good is produced using skilled and unskilled labor, inelastically supplied by the households, and a continuum of varieties of non-durable intermediate goods. Potential entrants into the intermediate-goods sector devote skilled labor to horizontal R&D to discover new blueprints/designs, thus the aggregate outcome of R&D. Once new inventions are discovered, imperfectly competitive firms produce intermediate goods used by perfectly competitive firms in the homogeneous final good production. Following Peretto (2007), Zeng and Zhang (2007), and Atkeson and Burstein (2019), among others, we consider that the government can influence R&D activity through its intervention in terms of fiscal policy. In line with Chu and Cozzi (2014) and Gil and Iglésias (2019), among others, we also incorporate the demand for money into the model, considering that a constant portion of total consumption is paid through CIA and via sector-specific CIA restrictions to R&D activities and intermediate-goods production, while the monetary authority determines the money supply. By doing so, fiscal and monetary policies are, *mutatis mutandis*, substitutes for the main effects. Moreover, infinitely-lived households inelastically supply skilled and unskilled labor and maximize utility obtained by consuming the homogeneous final good, earning income from labor and investments in financial assets and money balances. It is a dynamic general-equilibrium endogenous growth model in which agents are rational and solve their problems, free-entry R&D conditions are met, and markets clear.

2.2.1. Production and prices

The final good is homogeneous, and the the following function gives the sector's aggregated production Y at time t:

$$Y(t) = L_L(t)^{\alpha} L_{H_Y}(t)^{\beta} \int_0^{N(t)} X_j(t)^{1-\alpha-\beta} dj, \quad 0 < \alpha + \beta < 1, \tag{2.1}$$

where L_L and L_{H_Y} represent unskilled and skilled labor levels, respectively, and X_j is the amount of the j^{th} intermediate good demanded. Thus α, β, and $1-\alpha-\beta$ are the shares of each production factor in the final output. Variable N represents the existent and thus available varieties of non-durable intermediate goods, X_j, and can be seen as "a tractable proxy for the technological complexity of the typical firm's production process or, alternatively, for the average degree of specialization of the factors employed by the typical firm" (Barro and Sala-i-Martin 2004, p. 287). These sectors operate under perfect competition, and the price of the final good normalizes all prices (and wages); i.e., they are defined in real terms. Being $\mathscr{P}_Y$ the nominal price of the final good and $\mathscr{P}_j$ the nominal price of intermediate good j, $p_j = \frac{\mathscr{P}_j}{\mathscr{P}_Y}$ is the real price of the good, w_L and w_{H_Y} are the real wages paid for unskilled labor and skilled labor, respectively. The firms' profits are represented by

$$\Pi_Y(t) = Y(t) - \int_0^{N(t)} p_j(t) X_j(t) dj - w_L(t) L_L(t) - w_{H_Y}(t) L_{H_Y}(t). \tag{2.2}$$

Substituting (2.1) in (2.2), the first-order conditions for $\max_{X_j(t),L_L(t),L_{H_Y}(t)} \Pi_Y(t)$ give the inverse demand functions for each intermediate good X_j, unskilled labor, and skilled labor:

$$\frac{\partial \Pi_Y}{\partial X_j}(t) = 0 \Leftrightarrow p_j(t) = \big(1-\alpha-\beta\big)\big) L_L(t)^{\alpha} L_{H_Y}(t)^{\beta} X_j(t)^{-\alpha-\beta}, \tag{2.3}$$

$$\frac{\partial \Pi_Y}{\partial L_L}(t) = 0 \Leftrightarrow w_L(t) = \alpha L_L(t)^{\alpha-1} L_{H_Y}(t)^{\beta} \int_0^{N(t)} X_j(t)^{1-\alpha-\beta} dj, \tag{2.4}$$

$$\frac{\partial \Pi_Y}{\partial L_{H_Y}}(t) = 0 \Leftrightarrow w_{H_Y}(t) = \beta L_L(t)^{\alpha} L_{H_Y}(t)^{\beta-1} \int_0^{N(t)} X_j(t)^{1-\alpha-\beta} dj. \tag{2.5}$$

The model assumes that the same firm carries out the invention process through R&D activities and produces such intermediate good variety. Moreover, once a new variety is developed, its creator (the firm) will retain perpetual monopoly rights over it. Thus, each intermediate good j is only produced by the firm that invented it, which produces only this good variety and acts as a monopolist in that good market. The production of one unit of any of these goods requires η units of forgone final output.[5] Regarding fiscal policy, we assume the government can subsidize the production of a firm j by paying an *ad valorem* fraction $z_x \in [0,1]$ of its production costs. In terms of monetary policy, it is introduced a CIA on intermediate-goods production, $\Omega_x \geq 0$. Firms use the money borrowed from households, subject to the nominal interest rate ι, to pay upfront costs. Since firms can only repay the borrowed amount once they earn revenue from production, households effectively provide credit to them (Chu and Cozzi 2014). However, different from Chu and Cozzi (2014), we consider that the service for such financial debt amounts to a fraction $\Omega_x \iota \in [0,1]$ of the firm's production costs, and not that the CIA constraint itself is defined in this interval. Then, the monopolist j facing the demand curve given by (2.3) will solve the following profit-maximization problem:

$$\max_{X_j(t)} \Pi_j(t) = \max_{X_j(t)} p_j(t) X_j(t) - (1 - z_x + \Omega_x \iota)\, \eta X_j(t). \tag{2.6}$$

Solving this problem, we find that the amount of goods each firm j produces is independent of any firm-or-good-specific variable. That is, the demand for any intermediate good is the

[5]To the best of our knowledge, the consideration of a parameter for the constant unitary cost exogenously defined in the intermediate goods' production is due to Okada (2012). Therefore, such specification is not more arbitrary than the usual formulation without a parameter for such cost, which is equivalent to considering that one unit of the final good is required to produce one unit of intermediate good, i.e., to set $\eta = 1$.

same (see Appendix A) and given by:

$$X_j(t) = X(t) = \frac{(1-\alpha-\beta)^{\frac{2}{\alpha+\beta}} L_L(t)^{\frac{\alpha}{\alpha+\beta}} L_{H_Y}(t)^{\frac{\beta}{\alpha+\beta}}}{\eta^{\frac{1}{\alpha+\beta}} (1-z_x+\Omega_x \iota)^{\frac{1}{\alpha+\beta}}}, \forall j. \tag{2.7}$$

Consequently, the prices of the intermediate goods will be the same, because substituting (2.7) back in (2.3) gives

$$p_j(t) = p(t) = \frac{1-z_x+\Omega_x \iota}{1-\alpha-\beta}\eta, \forall j. \tag{2.8}$$

Following Jones (1995), we consider that new inventions happen at a rate $\delta N(t)^{\phi}$ (per allocated skilled labor unit during a time unit), as if in a Poisson process in which the arrival rate of new ideas increases with the stock of existing designs, N. The reasoning for this specification is that it becomes easier to find an invention because spillovers make all firms take advantage of the accumulated knowledge. The parameter ϕ, with $0 < \phi < 1$, represents these spillover's effects ("standing on shoulders") magnitude and controls the R&D process elasticity of scale, which has "diminishing technological opportunities" (Jones and Williams 2000). Consequently, $\frac{1}{\delta N(t)^{\phi}}$ units of skilled labor are necessary to invent a new design,[6] each labor unit receiving a wage w_{H_N} in retribution. The costs to find a new blueprint can be reduced by a governmental *ad valorem* subsidy z_r, with $z_r \in [0,1]$, but are increased in the proportion $\Omega_r \iota$. This latter denotes the debt service for the money borrowed from households, where $\Omega_r \geq 0$ represents the CIA provided to investment in R&D. Then, the invention cost is

$$Z(t) = (1-z_r+\Omega_r \iota)\frac{w_{H_N}(t)}{\delta N(t)^{\phi}}. \tag{2.9}$$

As the new designs market is competitive, whenever the present value of future profits that are possible to extract as a monopolist producing the new variety equals its initial invention cost, a firm will choose to invest in the new product development and enter the intermediate market. That is, when the free-entry condition is satisfied:

$$\int_t^{\infty} \Pi(\upsilon) e^{-\int_t^{\upsilon} r(\omega) d\omega} d\upsilon = Z(t), \tag{2.10}$$

where r is the real interest rate, and Π is the profit of any firm in this market, with $\Pi_j(t) = \Pi(t) = (\alpha+\beta)\, p(t)$.

Representing the aggregated level of skilled workers in R&D by L_{H_N}, and considering

[6] Here, we diverge from Okada (2012) that required $\frac{\eta}{N(t)^{\phi}}$ units of labor to invent a new design, tying up the productivity in R&D with the intermediate goods unitary production cost.

that inventions happen at rate $\delta N(t)^{\phi}$, the instantaneous aggregated production of new varieties, $\dot{N}$, is given by the following function:

$$\dot{N}(t) = \delta L_{H_N}(t) N(t)^{\phi}. \tag{2.11}$$

In this specification, skilled labor is the only input used to increment technological knowledge, and R&D firms hire it until the free-entry condition is satisfied. R&D intensity is the ratio between the expenditures in R&D and aggregate output, and, in the current context, it is given by $\frac{L_{H_N}(t) w_{H_N}(t)}{Y(t)}$.

The real interest rate demanded by households for investment in R&D of new varieties, $r(t) = \frac{\Pi(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)}$, is found by the differentiation of both sides of (2.10) with respect to (w.r.t.) time (see Appendix B). Defining the growth rate of N as $G_N(t) \equiv \frac{\dot{N}(t)}{N(t)} = \delta L_{H_N}(t) N(t)^{\phi-1}$, the ratio of the demand for skilled labor in the final-goods sector to the demand for skilled labor in R&D as $T(t) \equiv \frac{1-u(t)}{u(t)}$, and the growth of the demand for skilled labor in the final-goods sector as $G_T(t) \equiv \frac{(1-\dot{u}(t))}{(1-u(t))}$, r is found as a function of the variables u and G_N (see Appendix C), that is,

$$r(t) = \frac{(\alpha+\beta)(1-\alpha-\beta)}{\beta(1-z_r+\Omega_r\iota)} G_N(t) T(t) + (1-\phi) G_N(t) - \frac{\alpha}{\alpha+\beta} G_T(t). \tag{2.12}$$

Considering that the wages of skilled workers are the same in both sectors (e.g., Lloyd-Ellis 1999), we can represent the common wage rate for skilled labor by $w_H(t) \equiv w_{H_Y}(t) = w_{H_N}(t)$. We assume that no skilled worker wishes to act as unskilled because the wage of the first labor type is higher than that of the latter and, equivalently, that the skill premium (see Appendix D),

$$\frac{w_H(t)}{w_L(t)} = \frac{\beta}{\alpha} \frac{L_L(t)}{L_H(t)} = \frac{\beta}{\alpha} \frac{(1-s)}{s} \frac{1}{1-u(t)}, \tag{2.13}$$

is greater than one.[7] Thus, the ratio between the share of total output paid as wages to skilled and unskilled labor positively determines the skill premium.[8] In contrast, the ratio between the skilled and unskilled portion of the total labor supply negatively influences it.

The exogenous defined variable s is the share of skilled labor in the labor force, with $0 < s < \frac{\beta}{\alpha+\beta}$ (where $\alpha+\beta$ is the labor share in total output); u is an endogenous variable

[7]Such an assumption is easily satisfied: $0 < s < \frac{\beta}{\alpha+\beta}$ and $0 < u(t) < 1 \Rightarrow \lim_{s \to \frac{\beta}{\alpha+\beta}-} \frac{\beta}{\alpha}\frac{1-s}{s} = 1$, $\lim_{s\to 0+} \frac{\beta}{\alpha}\frac{1-s}{s} = +\infty$ and $\frac{1}{1-u(t)} > 1 \Rightarrow \frac{w_H(t)}{w_L(t)} > 1$.

[8]Notice that $w_L(t) = \frac{\alpha Y(t)}{L_L(t)}$, $w_H(t) = \frac{\beta Y(t) + \beta/T(t) Y(t)}{L_{H_Y}(t) + L_{H_N}(t)}$, and $\frac{\beta}{1-u(t)} = \beta + \frac{\beta}{T(t)}$, where $\frac{\beta}{T(t)} = \frac{L_{H_N}(t) w_{H_N}(t)}{Y(t)}$ represents R&D intensity.

that represents the share of skilled workers engaged in R&D, with $0 < u(t) < 1$;[9] and $1-u(t)$ is the portion of skilled people working in the final-goods sector. The total population level is $L(t)$, and its exogenous growth rate is $\frac{\dot{L}(t)}{L(t)} \equiv n$; the unskilled labor is $L_L(t) = (1-s)\,L(t)$, the skilled labor working in the final-goods sector is $L_{H_Y}(t) = \big(1-u(t)\big)\,sL(t)$, the skilled labor in the intermediate-goods sector is $L_{H_N}(t) = u(t)sL(t)$, and the total skilled labor is $L_H(t) = L_{H_Y}(t) + L_{H_N}(t) = sL(t)$. Output *per capita* is found by substituting (2.7), $L_L(t)$, and $L_{H_Y}(t)$ in (2.1) and dividing the result by $L(t)$:

$$y(t) = \frac{(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}\,(1-s)^{\frac{\alpha}{\alpha+\beta}}\,s^{\frac{\beta}{\alpha+\beta}}\,\big(1-u(t)\big)^{\frac{\beta}{\alpha+\beta}}\,N(t)}{\eta^{\frac{1-\alpha-\beta}{\alpha+\beta}}\,(1-z_x+\Omega_x\iota)^{\frac{1-\alpha-\beta}{\alpha+\beta}}}. \tag{2.14}$$

2.2.2. Wealth and consumption

Households consume and collect income from inelastically supplied labor and investments in financial assets and money balances. Incomes may be subject to taxes that will be used by the government for domestic fiscal policy purposes, in particular, as a means to finance the costs of the subsidies. We consider that all consumers have the same preferences and that all households are equal, such that there is an infinite-lived representative household. The instantaneous utility is given by $U(c(t)) = \frac{c(t)^{1-\theta}-1}{1-\theta}$, where c is *per capita* consumption and θ is the inverse of the intertemporal elasticity of substitution, with $\theta > 0$ and $\theta \neq 1$. Thus, at time $t = 0$, households maximize the discounted intertemporal lifetime utility, $\int_0^\infty U(c(t))e^{(n-\rho)t}dt$, which is assumed to be bounded away from infinite,[10] subject to the flow budget constraint (2.16) and the CIA constraint (2.17), having perfect foresight concerning the technological-knowledge progress over time. In this problem resolution (see Appendix E), households account for the population growth at rate n (since each household grows at the same rate as the population) and discount the future at rate ρ, with $\rho > 0$ and $n-\rho < 0$. Then, the representative household's optimization problem can be stated in terms of *per capita* variables as

$$\max_{c(t),b(t)} \int_0^\infty U(c(t))e^{(n-\rho)t}dt = \max_{c(t),b(t)} \int_0^\infty \left(\frac{c(t)^{1-\theta}-1}{1-\theta}\right) e^{(n-\rho)t}dt, \tag{2.15}$$

[9] Generally speaking, if the extremes of the interval are included, the model will not work. That is, if $u(t) = 0$, there will be no research and no *per capita* growth of the economy (2.11). Case $u(t) = 1$, there will be no output from the final goods production (2.1).

[10] In fact, we assume that, over time and in equilibrium, the utility growth rate plus the population growth rate are constant and less than the rate at which the future is discounted, i.e., that the infinite horizon integral in the optimization problem will converge.

$$
\begin{aligned}
\text{s.t. } \dot{a}(t)+\dot{m}(t) = &(1-\tau_H)\, s w_H(t) + (1-\tau_L)\,(1-s)\, w_L(t) \\
&+ (1-\tau_a)\, r(t) a(t) + \imath b(t) - c(t) - \tau_g(t) \\
&- n a(t) - n m(t) - \pi(t) m(t) + \tau_m(t), \text{ and}
\end{aligned} \tag{2.16}
$$

$$
\text{s.t. } b(t) + \xi c(t) \leq m(t). \tag{2.17}
$$

Per capita real money balances are represented by m. A portion $\xi c(t)$ of this amount is used to finance household consumption. The other part, b, is lent to incumbent firms to finance the intermediate-good manufacturing and R&D investment. The amount of money used to provide credit, $\xi c(t) + b(t)$, is subject to the nominal interest rate $\imath$, but the assumption that all households are equal makes the terms related to the CIA on consumption cancel each other in (2.16). The variable a represents *per capita* real financial assets/wealth holdings in the form of ownership of the firms that produce intermediate goods in monopolistic competition. So, the total assets in the economy will be given by $A(t) = a(t)L(t) = Z(t)N(t)$. The real interest rate is given by r, and the government imposes the following: τ_a, *ad valorem* tax rate on assets; τ_g, *lump sum* tax over individuals; τ_L and τ_H, the *ad valorem* rates of the taxes on wages of unskilled and skilled workers, respectively. The inflation rate is given by π, corresponding to the growth rate of the nominal price of the final good, $\pi(t) \equiv \frac{\dot{P}_Y(t)}{P_Y(t)}$, and determines the cost of holding money; the *seigniorage* revenues earned by the monetary authority will return to each household member in the form of a *lump sum* transfer, denoted by τ_m. The aggregate flow of income received by households as interest on the CIA provided to firms in the form of bonds is $\imath B(t)$, which amounts to the portion $\Omega_x \imath$ of all intermediates goods production costs and to the share $\Omega_r \imath$ of all the invention costs. Thus, considering equations (2.6), (2.7), (2.9), and (2.11), the aggregate stock of CIA provided to firms is

$$
B(t) = b(t)L(t) = \Omega_x \eta N(t) X(t) + \Omega_r L_{H_N}(t) w_{H_N}(t). \tag{2.18}
$$

Following standard optimal control theory, we can consider an auxiliary Hamiltonian function in the resolution of the intertemporal utility maximization. In doing so, the usual approach in the literature is to consider that the (static) CIA constraint is binding, that is, $b(t) + \xi c(t) = m(t)$. From the necessary conditions, it is possible to derive a no-arbitrage condition between real money balances and real financial assets that amounts to the following Fisherian equation:

$$
\imath = (1-\tau_a)\, r(t) + \pi(t). \tag{2.19}
$$

The optimal path of consumption (the households' Euler equation) is given by

$$\frac{\dot{c}(t)}{c(t)} = \frac{(1-\tau_a)\,r(t)-\rho}{\theta}. \tag{2.20}$$

The transversality conditions are $\lim_{t\to\infty}\mu(t)a(t)=0$ and $\lim_{t\to\infty}\mu(t)m(t)=0$, where $\mu(t) = \left(1+\iota\xi\right)^{-1}c(t)^{-\theta}e^{(n-\rho)t}$ is the co-state variable. The existence of these limits will require, in addition to the already presented assumptions over the model's parameters, that $1-z_x+\Omega_x\iota > (1-\alpha-\beta)^2$ and $1-z_r+\Omega_r\iota > 0$ (see Appendix F).[11] These two inequalities mean it is impossible to completely subsidize the costs of R&D activities and intermediate-sector production. The former also ensures that c does not exceed y, and the latter guarantees that $u(t) < 1$ in steady state. Considering the existing relations in the economy described in this entire section, it is possible to aggregate the intertemporal budget constraint (2.16) by the whole population (see Appendix G) to find that $C(t) = Y(t) - \eta N(t)X(t)$ and

$$c(t) = y(t)\left[1-\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\right]. \tag{2.21}$$

As c is a constant share of y and both variables grow at the same rate, i.e., $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)}$, the relation between output and consumption is stable along the time. The resolution of the representative household's problem put in evidence the following restrictions on some variables allocation path: $0 < u(t) < 1$; $G_N(t) > 0$; $y(t) > 0$; $a(t) > 0$. Generally speaking, they mean that, at any instant, some level of skilled labor, but not all, is engaged in R&D, that the invention's stock does not decrease, and that some level of production and assets exist in the economy.

Out of a balanced growth path, consumption does not grow at a constant rate, and there is no closed-form solution to represent the welfare of an individual that we consider as being the present value of utility accumulated over an infinite time horizon (Caputo 2005, pp. 414–415) by a member of the representative household at an arbitrary instant t, i.e., $W(t) \equiv \max_{c(\upsilon),b(\upsilon)} \int_t^\infty U(c(\upsilon))e^{-\rho(\upsilon-t)}d\upsilon$.[12] Nevertheless, as will be shown in Section 2.3, there is a closed-form expression for welfare along the steady state.

[11] Satisfying the transversality conditions also requires $(1-\theta)\,G_N^*+n-\rho<0$, which is the assumed convergence condition of the integral in the household optimization, where G_N^* is the growth rate that solves the problem. This inequality always hold case $\theta > 1$, but requires $\theta > 1-\left(\frac{\rho}{n}-1\right)(1-\phi)$ and $\phi > 1-\frac{1}{\frac{\rho}{n}-1}$ case $0<\theta<1$.

[12] As the population grows in our model, we are not measuring welfare by the social utility accumulated over time. With a growing population, a specification of the form $\int_t^\infty U(c(\upsilon))L(\upsilon)e^{-\rho(\upsilon-t)}d\upsilon$ would result in a continually increasing welfare along time, even if *per capita* utility remains the same (or reduces proportionately less than the population growth).

2.2.3. Government and monetary authority

In addition to firms and individuals, technology can also be influenced by government policies and by the central bank (or monetary authority). Hence, a description of the government's budget and the monetary authority is needed to finalize the characterization of the economy. As stated, the government may intervene by imposing taxes on wages and financial assets and subsidizing the production of intermediate goods and R&D activities. The government runs a balanced budget, i.e., with no surplus or public debt, which implies

$$\tau_a r(t)A(t)+\tau_L w_L(t)L_L(t)+\tau_H w_H L_H(t)+\tau_g(t)L(t)=z_x\eta N(t)X(t)+z_r w_{H_N}(t)L_{H_N}(t). \tag{2.22}$$

Respectively, the terms on the left-hand side (l.h.s.) of (2.22) represent revenues from taxation over assets-income, unskilled labor, and skilled labor, and a lump-sum tax paid by all individuals. In contrast, terms on the right-hand side (r.h.s.) represent the government's expenditures subsidizing intermediate goods production and R&D.[13]

Concerning the monetary authority, we consider that the nominal interest rate, ι, is the policy instrument used to handle inflation, affecting the model variables through the CIA constraints (e.g., Bernanke and Mishkin 1997; Chu and Cozzi 2014). Hence, with a change in the nominal interest rate, the CIA constraints generate forces that transmit different inflation costs, which distort the incentives and the use of economic resources in the different sectors. We follow the literature and assume that the monetary authority chooses the nominal interest rate exogenously.[14] Thus, the inflation rate, π, is endogenously determined according to the Fisher equation (2.19). Hence, in line with the empirical observation indicating that the inflation rate is endogenous and indirectly affected by the monetary policy instrument (e.g., Bernanke and Mishkin 1997), the Fisher equation reveals that it is endogenous, in the sense that although influenced by the exogenous choice of the nominal interest rate, it relies on the real economic macroeconomic conditions reflected in the endogenous real interest rate (2.12).

Denoting the nominal money supply by $\mathscr{M}$, its growth rate by $G_{\mathscr{M}}(t)\equiv\frac{\dot{\mathscr{M}}(t)}{\mathscr{M}(t)}$, and the real money supply/balances by $M(t)\equiv\frac{\mathscr{M}(t)}{P_Y(t)}$, we find $\frac{\dot{M}(t)}{M(t)}=G_{\mathscr{M}}(t)-\pi(t)$. Hence, considering (2.19) and that $\frac{\dot{M}(t)}{M(t)}=\frac{\dot{m}(t)}{m(t)}+n$, it is possible to show that the growth rate of the nominal

[13]Bearing in mind the definitions of $L_L(t)$, $L_H(t)$, and R&D intensity, equations (2.4), (2.5), (2.13), (2.14), (2.21), and (F.2) in Appendix F, the government balanced budget (2.22) can be written as $\tau_a r(t)\frac{\beta(1-z_r+\Omega_r\iota)}{G_N(t)T(t)}Y(t)+\tau_L\alpha Y(t)+\tau_H\frac{\beta}{1-u(t)}Y(t)+\tau_g(t)L(t)=z_x\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}Y(t)+z_r\frac{\beta}{T(t)}Y(t)$.

[14]In addition to Chu and Cozzi (2014), recent papers along these lines are Chu and Ji (2016), Chu et al. (2017), and Chu et al. (2019), among others.

money supply is endogenously determined:

$$G_{\mathscr{M}}(t) = \frac{\dot{m}(t)}{m(t)} + n + \iota - (1 - \tau_a)\, r(t). \tag{2.23}$$

The monetary authority will endogenously adjust the nominal money growth rate to whatever level is needed for the interest rate ι to prevail. *Seigniorage* revenues earned by the monetary authority will be $\mathscr{S}(t) = \frac{\dot{\mathscr{M}}(t)}{P_Y(t)} = \dot{M}(t) + \pi(t)M(t)$. As usual in the literature, the monetary authority returns the *seigniorage* revenues to the population as a *per capita lump sum* transfer

$$\tau_m(t) \equiv \frac{\mathscr{S}(t)}{L(t)} = \dot{m}(t) + nm(t) + \pi(t)m(t). \tag{2.24}$$

It will be presented later that the relation (2.19) between the inflation rate and the nominal interest rate implies that, in steady-state equilibrium, we can extend all the comparative-statics results of shifts in ι also to shifts in the steady-state inflation rate, π^*. Therefore, in the steady state, one could consider the inflation rate or even the money supply growth rate as the policy variables directly controlled by the monetary authority.[15] The velocity of money circulation is given by $\mathscr{M}(t)V(t) = P_y(t)Y(t) \Leftrightarrow V(t) = \frac{Y(t)}{\mathscr{M}(t)/P_y(t)} = \frac{Y(t)}{M(t)}$, which simplifies to (see Appendix H):

$$V(t) = \left\{ \Omega_x \frac{(1-\alpha-\beta)^2}{1 - z_x + \Omega_x \iota} + \Omega_r \frac{\beta}{T(t)} + \xi \left[1 - \frac{(1-\alpha-\beta)^2}{1 - z_x + \Omega_x \iota} \right] \right\}^{-1}. \tag{2.25}$$

Finally, aggregate money in the economy is simply $M(t) = V(t)^{-1}Y(t)$, and it is clear from (2.25) that the CIA parameters Ω_r, Ω_x, and ξ directly determine how money is distributed across the sectors in the economy. If R&D is a more liquidity-constrained activity than production (Brown and Petersen 2015), and the relative importance of the interest over CIA in the flow of expenditures for firms far exceeds that for households (Bank for International Settlements 2021b, authors' computation), we expect that $\Omega_r > \Omega_x > \xi$.[16]

2.3. Dynamics and equilibrium

We proceed by analyzing the dynamic general equilibrium resulting from optimal decentralized behavior, which is described by the labor allocation between sectors and the growth

[15] However, considering the nominal interest rate as the policy instrument simplifies the analytical derivation of the model's steady-state equilibrium without changing the comparative statics results.

[16] It becomes clear from (2.25) that if one sets a CIA parameter to zero, such as simplifying the expressions to conduct some specific analysis, the other CIA parameters must increase to keep money velocity constant. Although we do not expect significant side effects for qualitative long-run analysis, such increased parameters can become disproportionately high. Therefore, one should be aware of quantitative issues that could arise, e.g., in the direction or magnitude of some static long-run effects (see subsections 2.3.2 and 2.5.2) or in the dynamic response to monetary policy changes.

of technological knowledge. The dynamics of the model are given by the system made by the following differential equations:

$$
\begin{aligned}
\frac{\dot{\left(1-u(t)\right)}}{\left(1-u(t)\right)} &= \frac{\alpha+\beta}{\alpha\left(1-\tau_a\right)+\beta\theta}\left\{\left(1-\tau_a\right)\frac{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}{\beta\left(1-z_r+\Omega_r\iota\right)}G_N(t)T(t)\right. \\
&\qquad \left. -\left[\theta-\left(1-\tau_a\right)\left(1-\phi\right)\right]G_N(t)-\rho\right\}, \text{ and} \qquad (2.26) \\
\frac{\dot{G}_N(t)}{G_N(t)} &= n-\left(1-\phi\right)G_N(t)-\frac{\left(\alpha+\beta\right)}{\alpha\left(1-\tau_a\right)+\beta\theta}T(t)\times \\
&\qquad \left\{\left(1-\tau_a\right)\frac{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}{\beta\left(1-z_r+\Omega_r\iota\right)}G_N(t)T(t)\right. \\
&\qquad \left. -\left[\theta-\left(1-\tau_a\right)\left(1-\phi\right)\right]G_N(t)-\rho\right\}. \qquad (2.27)
\end{aligned}
$$

Equation (2.26) is found log-differentiating (2.14) and substituting (2.20) and (2.12) in the result, while (2.27) is found log-differentiating (2.11) and substituting (2.26) in the result (see Appendix I).

Proposition 2.1. *The dynamic general equilibrium of the model is determined by the point* $\left(1-u^*, G_N^*\right)$*, a constant, unique, positive, and saddle-point solution to the system made by (2.26) and (2.27). In such a point,*

$$
\begin{aligned}
G_N^* &= \frac{n}{1-\phi}, \text{ and} \qquad (2.28) \\
u^* &= \frac{1}{T^*+1}, \text{ with} \qquad (2.29) \\
T^* &= \frac{\beta\left(1-z_r+\Omega_r\iota\right)\left\{\left(1-\phi\right)\rho+n\left[\theta-\left(1-\phi\right)\left(1-\tau_a\right)\right]\right\}}{n\left(1-\tau_a\right)\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \qquad (2.30)
\end{aligned}
$$

Proof. See Appendix J. □

Remark. The cases where $G_N(t)=0 \Rightarrow u(t)=0$ or $1-u(t)=0$ violate the restrictions on the allocation path of u and G_N, i.e., $0<u(t)<1$ and $G_N(t)>0$, and are not solutions of the household's problem. In the equilibrium point, $\dot{G}_N(t)=0$ and $\dot{\left(1-u(t)\right)}=0$. This latter relation implies that $G_T(t)=G_T^*=0$ in steady state.

In the long run, the economy will achieve stationarity —the demand for labor in R&D, u, and the technological-knowledge growth rate, G_N, will become constant— and embark on a balanced growth path. Even though there is no closed-form solution for the integral representing welfare, as mentioned in Section 2.2, it is possible to find an expression for the welfare of an individual along the balanced growth path. Thus, considering that the economy is in the steady state at instant $t = 0$, long-run welfare is given by

$$W^* = \frac{1}{1-\theta}\left[-\frac{1}{\rho} - \frac{c^*(0)^{1-\theta}}{(1-\theta)\,G_N^* - \rho}\right], \tag{2.31}$$

with *per capita* consumption level at the instant stationarity is achieved, c^*, being endogenously determined considering $N(t) = \left(\frac{\delta L_{H_N}(t)}{G_N(t)}\right)^{\frac{1}{1-\phi}}$ from (2.11), (2.14), (2.21), (2.28), and (2.29), for a given population level $L(0)$ (see Appendix K).

2.3.1. Transition dynamics

The phase diagrams presented in Figure 2.1 depict the system's possible dynamic behaviors in the saddle-point equilibrium neighborhood. As shown in Appendix J, both phase lines, given by the *loci* for $\dot{\left(1-u(t)\right)} = 0$ in (2.26) and $\dot{G}_N(t) = 0$ in (2.27), have negative slopes in the plane $\left(1-u(t), G_N(t)\right)$, as well the saddle path. The horizontal arrows of motion are distancing from the *locus* $\dot{\left(1-u(t)\right)} = 0$, and the vertical ones are approaching the *locus* $\dot{G}_N(t) = 0$. Furthermore, the two possible patterns for the arrows of motion in the panels of the figure indicate that, independently of which phase line is the steepest, the saddle path is steeper than both.

(a) (b) (c)

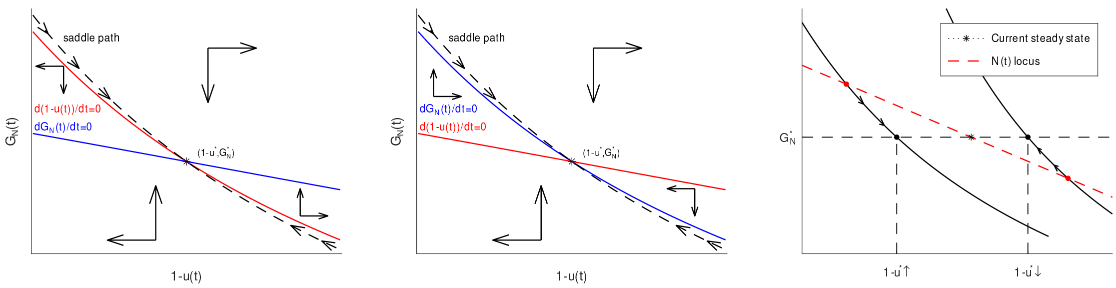


FIGURE 2.1: Phase diagram of the dynamic system with arrows of motion, phase lines, and saddle path in panels (a) and (b). The case in which the *locus* for $\dot{\left(1-u(t)\right)} = 0$ is steeper than the *locus* for $\dot{G}_N(t) = 0$ is in the left panel (a), while the right panel (b) presents the opposite situation. Panel (c) depicts the jump of the economy's state in case of change that does not affect G_N^* in the long run. A change that takes the economy from the current steady state $\left(1-u^*, G_N^*\right)$ increasing (decreasing) u in the long run and setting the new steady state to $\left(1-u^*\uparrow, G_N^*\right)$ $\left(\left(1-u^*\downarrow, G_N^*\right)\right)$ will make the state of the economy jump to the intersection of the current level of technological knowledge, given by the $N(t)$ *locus,* with the saddle path leading to the new equilibrium, given by the solid line on the left (right).

After an event that disturbs the economy and takes it from the steady state (or balanced-growth path), the agents' optimizing behavior moves it toward a long-run equilibrium reflecting the new economic conditions. In the presented setup, after a shock, the state of the economy should jump to a point over the saddle path that leads to the unique new steady state. The exogenous population growth rate and the internal forces of the model govern the dynamics during the transition to this long-run equilibrium, and the allocation of skilled labor in R&D, u, and the growth rate of technological knowledge, G_N, determine the economy's state. The values they assume along the saddle path determine the remaining variables.

Variables L and N do not jump with economic shocks, as the population constantly grows at an exogenous rate, and the technological knowledge accumulates only through R&D activity (2.11). In the case of a shock at instant t, both variables keep their value unchanged. Variables u and G_N jump to the point in the space state that lies over the saddle path and satisfies (2.11), i.e., $G_N(t) = \delta su(t)L(t)N(t)^{\phi-1}$, computed with the value of the parameters that reflect the new economic conditions, and L and N at their historical values. The direction of the variables' jump depends on the economy's current state and technological level and the new long-run steady state resulting from the change. Notice that, in this model context, given the values of L and N, equation (2.11) establishes a linear and negative relation in the state space $\left(1-u(t), G_N(t)\right)$ that represents the *locus* for the current technological level, $N(t)$. As the saddle path also has a negative slope (see Appendix J), implying a positive relationship between u and G_N during the transition, any change that takes the economy from its balanced growth path and does not affect the long-run growth rate – as a policy change – and increases (decreases) u, in the long run, will set the new steady state to the left (right) of the current one and above (below) G_N^*. Consequently, u and G_N, and the skill premium (2.13), will jump to a point where they are above (below) the new steady-state values, while the velocity of money (2.25) jumps to a point where its value is below (above) the long-run level. To illustrate a jump in the economy, panel (c) of Figure 2.1 plots the *locus* for the predetermined variable $N(t)$ and its intersections with possible saddle paths for changes at instant t that shift u up or down without affecting G_N in the long run.[17]

Then, the transition starts from the value to which variables jumped, except for L and N, which stay at their historical levels. If u and G_N jump to values greater (lower) than their long-run levels, both variables continuously decrease (increase) in the direction of their new steady-state values. Consequently, the rate of change of $1-u(t)$, $G_T(t) \equiv$

[17]As we are interested in the policy interactions, we do not plot or discuss parameter changes that can shift G_N in the long run. However, the analysis is analogous even if the economy is not in a steady state. If the new long-run equilibrium lies to the left (right) of current $u(t)$ and above (below) the *locus* for $N(t)$, the economy will jump to a state where u and G_N are greater (smaller) than their current values but below (above) their long-run levels.

$\frac{\dot{(1-u(t))}}{1-u(t)} = \frac{\alpha+\beta}{\beta}\left(\frac{\dot{c}(t)}{c(t)} - G_N(t)\right)$ (see Appendix I), will assume a positive (negative) value during the transition, continually and asymptotically decreasing (increasing) to zero. Notice, in equations (2.26) and (2.27), that fiscal and monetary policy positively affect G_T in the short-medium-run path; that is, a greater (lower) z_r or a lower (greater) ι implies a faster transition if $G_T(t) > (<)\, 0$. Along the transition path, $G_T(t) > (<)\, 0$ implies that: output and consumption *per capita* jumped to a point below (above) their long-run level but are growing at a faster (slower) pace than the technological knowledge, i.e., $\frac{\dot{y}(t)}{y(t)} = \frac{\dot{c}(t)}{c(t)} > (<)\, G_N(t)$; skill premium is decreasing (increasing), i.e., $\frac{\dot{w}_L(t)}{w_L(t)} > (<)\, \frac{\dot{w}_H(t)}{w_H(t)}$ (see Appendix N); the velocity of money is increasing (decreasing); and, as consumption is growing above (below) its long-run growth rate but from a diminished (increased) level, welfare accounting transition is less (greater) than long-run welfare, i.e., $W(t) < (>)\, W^*$. This last result means that the extent to which the consumption level falls (rises), its growth rate during the transition, and the rate at which the future is discounted ultimately determine if a change will improve (or worsen) welfare when the short-medium-run is considered.

2.3.2. Effects of changes in the economic conditions

Recall that the main macroeconomic variables we are interested in are the growth rate of the economy in *per capita* terms, G_N^*, R&D intensity and the share of skilled labor demanded in R&D, u^*, the skill premium, $\left(w_H/w_L\right)^*$, the real interest rate, r^*, inflation, π^*, the velocity of money, V^*, and welfare, W^*. In addition to the restrictions on the parameters stated in Section 2.2, Assumption 2.1 will be considered in analyzing the long-run effects on these variables of a parameter's value permanent and unanticipated increase (see Appendix L). For most cases, the direction of the effect is unequivocal, as shown in Table 2.1, which outlines such findings. Proposition 2.2 states a general characteristic of semi-endogenous growth models. Proposition 2.3 allows the extension of the comparative static results regarding u^* to R&D intensity, while propositions 2.4 to 2.6 summarize the most important analytical results related to monetary and fiscal policies in the context of our model.

TABLE 2.1: Steady-state effects of a permanent and unanticipated parameter increase on the main variables

	Structural parameters on production, preferences and population									Fiscal policy: subsidies and taxes parameters					Monetary policy: CIA and nominal interest rate			
	α	β	η	δ	ϕ	θ	ρ	n	s	z_x	z_r	τ_a	τ_L	τ_H	Ω_x	Ω_r	ξ	ι
$\partial G_N^*/\partial(.)$	0	0	0	0	+	0	0	+	0	0	0	0	0	0	0	0	0	0
$\partial u^*/\partial(.)$	−	−	0	0	+	−	−	+	0	0	+	−	0	0	0	−	0	−
$\partial (w_H/w_L)^*/\partial(.)$	−	±	0	0	+	−	−	+	−	0	+	−	0	0	0	−	0	−
$\partial r^*/\partial(.)$	0	0	0	0	+	+	+	+	0	0	0	+	0	0	0	0	0	0
$\partial \pi^*/\partial(.)$	0	0	0	0	−	−	−	−	0	0	0	0	0	0	0	0	0	+
$\partial V^*/\partial(.)$	±	±	0	0	−	+	+	−	0	±	−	+	0	0	±	±	−	±
$\partial W^*/\partial(.)$	±	±	−	+	±	±	±	±	±	±	±	±	0	0	±	±	0	±

Notes: assuming $\alpha+\beta > 1/2$; "+" for positive effects, "−" for negative, "0" for none, and "±" for not analytically determined.

Proposition 2.2. *In the long run, the growth rate of N is independent of policy (fiscal and monetary) and only determined by n (population growth rate) and ϕ (R&D elasticity of scale).*

Proof. Direct from equation (2.28). □

In semi-endogenous growth models that follow Jones (1995), the long-run output *per capita* growth rate is a function of the exogenous population growth rate unaffected by policies. In the current context, through the nominal interest rate, monetary policy indirectly controls inflation (2.19) without affecting G_N^* in the long run. Case inflation is not seen as a uniquely monetary phenomenon, the common structural parameters determining both G_N^* and π^* produce effects that go in opposite directions and establish a negative relationship between these variables. Both theoretical situations are consistent with empirical literature that found the inflation-growth relationship negative or insignificant in many cases (e.g., Benhabib and Spiegel 2009).

Assumption 2.1. *In aggregate, most of the output is used to compensate for the labor allocated in final-goods production, i.e., $\alpha+\beta > 1/2$.*

The share of labor in final-goods production, i.e., $\alpha+\beta$, is crucial in determining many derivatives' signs, and the CIA constraints are necessary for the model to transmit the monetary policy effects. Thus, Assumption 2.1 allows us to unequivocally determine the direction of most effects on the analyzed variables in the case of a parameter's increase.

Proposition 2.3. *In the long run and under Assumption 2.1, the effects of a change in any parameter go in the same direction on the share of skilled labor demanded in R&D and on R&D intensity.*

Proof. R&D intensity is $\frac{L_{H_N}(t)w_{H_N}(t)}{Y(t)} = \frac{\beta}{T(t)}$ and $T \equiv \frac{1-u(t)}{u(t)}$. Considering (2.30), $\frac{\partial}{\partial\beta}\frac{\beta}{T^*} = \frac{-\beta[2(\alpha+\beta)-1]}{(1-\alpha-\beta)(\alpha+\beta)T^*} < 0$, and for the other parameters except β, $\frac{\partial}{\partial(\cdot)}\frac{\beta}{T^*} = \frac{\beta}{(1-u^*)^2}\frac{\partial u^*}{\partial(\cdot)}$. □

In fact, in an economy like ours, where all investment in R&D is due to the labor employed pursuing new inventions, this is an expected result. Notice that R&D intensity rises with the wages paid for skilled labor, which is expected to increase with the demand for this type of labor. This way, Proposition 2.3 formalizes the intuitive notion that all comparative static results related to the share of skilled labor engaged in R&D activities can be extended to R&D intensity, which was omitted in Table 2.1.[18]

Proposition 2.4. *In the long run and under Assumption 2.1, the real interest rate is not affected by a permanent and unanticipated increase in ι (the nominal interest rate); inflation is positively affected, R&D intensity and the skill premium are negatively affected, and the effects on the velocity of money and welfare are ambiguous.*

Proof. From (2.12) and 2.28, $r^* = \frac{\rho+\theta G_N^*}{1-\tau_a}$ and $\frac{\partial r^*}{\partial\iota} = 0$. From (2.19), $\frac{\partial\pi^*}{\partial\iota} > 0$. From (2.30), $\frac{\partial T^*}{\partial\iota} = \frac{\Omega_r T^*}{1-z_r+\Omega_r\iota} \geq 0$ and $\frac{\partial u^*}{\partial(\cdot)} = -(u^*)^2\frac{\partial T^*}{\partial(\cdot)} \Rightarrow \frac{\partial u^*}{\partial\iota} \leq 0$; from (2.13), $\frac{\partial\left(w_H^*/w_L^*\right)}{\partial\iota} = \frac{-\Omega_r u^*\left(w_H^*/w_L^*\right)}{1-z_r+\Omega_r\iota} < 0$. From (2.25), $\frac{\partial V^*}{\partial\iota} = \left[\frac{\Omega_x^2(1-\alpha-\beta)^2}{(1-z_x+\Omega_x\iota)^2} + \frac{\Omega_r^2\beta}{(1-z_x+\Omega_x\iota)T^*} - \frac{\xi\Omega_x(1-\alpha-\beta)^2}{(1-z_x+\Omega_x\iota)^2}\right](V^*)^2$. See Appendix K for $\frac{\partial W^*}{\partial\iota}$. □

Remark. $\frac{\partial V^*}{\partial\iota} > 0$ case the positive effects on V^* that come from the CIA in intermediate goods production and R&D dominate the negative effect that comes from the CIA in consumption; to this, it is sufficient (but not necessary) that the stock of CIA in intermediate goods production is not too small, i.e., $\Omega_x > \xi$. Welfare is negatively affected by ι, $\frac{\partial W^*}{\partial\iota} < 0$, case R&D is not dependent on CIA, i.e., case $\Omega_r = 0$, or case the portion of skilled labor engaged in R&D and the subsidies to intermediate goods production are not too high, i.e., $u^* < 1/2$ and $\alpha+\beta-z_x+\Omega_x\iota > 0$. A positive inflation-welfare relationship, $\frac{\partial W^*}{\partial\iota} > 0$, is possible and, case $\Omega_x = 0$, for this happen it is sufficient that $\alpha+\beta < u^*\left[\alpha+\beta+\beta(1-\phi)\right]$. In such a case, the *Friedman rule* is not verified.

As in Chu and Cozzi (2014), R&D decreases in the nominal interest rate if such activity is cash-constrained, $\Omega_r \neq 0$, but different to their results, R&D never increases with ι and the sign of $\frac{\partial u^*}{\partial\iota}$ is not related to the strength of the CIA in production, Ω_x. The negative effect

[18] Proposition 2.3 also exemplifies how Assumption 2.1 simplified the analysis to unequivocally determine the effects' direction on the interest variables. In the absence of Assumption 2.1, other restrictions could be required. For example, we could have assumed that the effects on R&D intensity and the demand for skilled labor in R&D should move in the same direction instead of deriving it. Such a choice would result in a constrained parameter space for the time preferences (θ and ρ) and the R&D elasticity of scale (ϕ) parameters. See equations (2.29) and (2.30).

of inflation —indirectly controlled through ι— on R&D intensity aligns with the empirical finding of "a statistically significant negative relationship between the inflation rate and the R&D share of GDP" (Chu et al. 2015). In our model, the nominal interest rate negatively affects the skill premium in the long run. The higher nominal interest rate increases the invention costs, represented by Z in (2.9), and the marginal cost of intermediate-goods production, which demands new inventions and is represented by $(1 - z_x + \Omega_x \iota)\, \eta$ in (2.6). Such costs increase repel investment, and R&D intensity falls. A lower level of investment in R&D also decreases the demand for skilled labor, a resource that becomes more abundant to be used in the final goods sector. In such conditions, the real wage paid for skilled labor falls proportionally more than that for unskilled labor. With less income, consumption falls, and output adjusts to the new demand, resources available, and costs. The lowered skilled-labor wage mitigates the increased price of intermediate goods (2.8) in the final goods' production. The discussed negative long-run skill premium relationship with ι results from the competing demand for skilled labor between final goods production and R&D.

In Table 2.1, the reduction of R&D intensity shows that investment falls proportionally more than output and consumption; money's velocity increases because, in real terms, the monetary base's shrinking due to inflation is proportionally more significant than the fall in output.

The positive effect of ι on V^* is in line with previously found empirical long-run relationship between the nominal interest rate and the "monetary base velocity" (e.g., McGrattan 1998) and with previous theoretical developments (e.g., Caballé and Hromcová 2010). Furthermore, the relationship between ι and W^* tends to be negative, meaning that inflation harms households' welfare in the aggregate in most cases. Similarly, in most theoretical situations analyzed in Camera and Chien (2014), persistent increased inflation leads to welfare losses in the long run. In their work, welfare variation is positively linked to inflation when capital is added to their model. Here, the theoretical possibility of a positive inflation-welfare relationship in the long run also exists and relates to production not being cash-constrained, and very high levels of R&D intensity and skill premium. As it is clear from the equation for the invention costs (2.9), investment in R&D will boost if the sector is less cash-constrained or subsidized, and, of course, a division of the wage bill that is more favorable to skilled labor raises the skill premium (2.13).

In such a situation, in a economy where most of the skilled labor is allocated to R&D, probably requiring that this activity be heavily subsidized, in the case of permanent inflation, the marginal gains of redirecting more of the scarce skilled labor to production would generate much higher marginal gains than in the most usual cases in which there is considerably more labor in production than in R&D. This way, in the absence of the adverse effects from the intermediate sector on welfare, the positive effects on production (and consumption) could prevail.

Despite such demanding conditions, the theoretical possibility of a positive inflation-welfare relationship in the long run, which contradicts the *Friedman rule*, was also found in previous works. For example, in the model of Espinosa-Vega and Yip (1999), the relationship direction is dependent on the degree of risk aversion of the agents, so inflation and growth can be positively related, allowing welfare to rise together. Our result also relates to Huang et al. (2021) who, in a long-run setup with horizontal and vertical growth, theoretically showed the possibility of increasing real consumption in response to inflation, provided that only horizontal R&D is cash-constrained. In their case, growth is positively related to inflation, and the Mundell-Tobin effect arises. Neither situation is possible in our setup.

Even though inflation does not affect the economy's long-run growth rate, the model has no money super neutrality, contrary to some empirical evidence (e.g., Deev and Hodula 2016). Changes in "the steady-state rate of growth of the money supply," given by $G^*_{\mathscr{M}} = \iota + n - \rho + (1-\theta)\, G^*_N$, considering (2.23) and (2.28), do "affect the value of real economic variables" (Danthine 2008), as R&D intensity, the skill premium, welfare, and the output level, and not only the monetary side of the economy, represented by inflation and the velocity of money.

Proposition 2.5. *In the long run and under Assumption 2.1, the real interest rate and inflation are not affected by a permanent and unanticipated increase in z_r (R&D subsidies); R&D intensity and the skill premium are positively affected, the velocity of money is negatively affected, and the effect on welfare is ambiguous.*

Proof. From (2.12) and (2.28), $\frac{\partial r^*}{\partial z_r} = 0$; from (2.19), $\frac{\partial \pi^*}{\partial z_r} = 0$. From (2.30), $\frac{\partial T^*}{\partial z_r} = \frac{-T^*}{1-z_r+\Omega_r \iota} < 0 \Rightarrow \frac{\partial u^*}{\partial z_r} > 0$, from (2.13), $\frac{\partial \left(w^*_H/w^*_L\right)}{\partial z_r} = \frac{\beta(1-s)\frac{\partial u^*}{\partial z_r}}{\alpha s(1-u^*)^2} > 0$. From (2.25), $\frac{\partial V^*}{\partial z_r} = \frac{-\Omega_r \beta (V^*)^2}{(1-z_r+\Omega_r \iota)T^*} < 0$. See Appendix K for $\frac{\partial W^*}{\partial z_r}$. □

Remark. Welfare is positively affected by z_r case the portion of skilled labor engaged in R&D is not too high, i.e., $u^* < 1/2$ imply $\frac{\partial W^*}{\partial z_r} > 0$. A negative welfare relationship with subsidies to R&D, $\frac{\partial W^*}{\partial z_r} < 0$, is possible and, case $\Omega_x = 0$, for this happen it is sufficient that $\beta \geq \alpha$ and $u^* > \frac{2}{3-\phi}$.

The real interest rate is unaffected by policies in the long run, and its pattern during the transition is ambiguous. From equation (2.12), it is not possible to determine if r jumps above or below its long-run level. Moreover, bearing in mind that $r(t) = \frac{\Pi(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)}$, we have for the dynamics that

$$\frac{d\,r(t)}{dt} = \frac{(\alpha+\beta)(1-\alpha-\beta)}{\beta\,(1-z_r+\Omega_r\iota)}\left(\frac{G_N(t)}{u(t)}\right)\left(\frac{\dot{G}_N(t)}{G_N(t)} - \frac{\dot{u}(t)}{u(t)}\right) + (1-\phi)\,\dot{G}_N(t) - \frac{\alpha}{\alpha+\beta}\dot{G}_T(t). \tag{2.32}$$

Notice in the r.h.s. of $\frac{d\,r(t)}{dt}$, in (2.32), that the first term will move in the same direction as G_N because $\left|\frac{\dot{G}_N(t)}{G_N(t)}\right| > \left|\frac{\dot{u}(t)}{u(t)}\right|$ (see Appendix N), as well as the second term. However, the last one will move in the opposite direction because G_T is continually going to 0, which, considering the preceding discussion in this subsection, means $\mathrm{sign}(\dot{G}_N/G_N) = \mathrm{sign}(\dot{G}_T)$. The inflation path in transition is also ambiguous, as π is determined by (2.19) and follows an opposite pattern than that displayed by the real interest rate. The model features a negative inflation-growth relationship if the last term in $\frac{d\,r(t)}{dt}$ is dominated by the first two. This negative relationship is expected for changes that increase (decrease) u^*. Once the velocity of money jumps to a smaller (greater) level, it indicates that the real monetary base is shrinking (expanding) more than the contraction (expansion) in output. In such a situation, the result for inflation would be in line with the empirical literature (e.g., Benhabib and Spiegel 2009), as discussed after Proposition 2.2, and the real interest rate pattern during the transition would qualitatively follow that presented by u and G_N. Notice that the first term in the r.h.s. of (2.32) is affected by the inverse of skilled labor share in the wage bill, $\left(\frac{\beta}{\alpha+\beta}\right)^{-1}$, and the third term is affected by the unskilled share $\frac{\beta}{\alpha+\beta}$, which influences the direction of the inflation-growth relationship during transitions. Such results relate to the semi-endogenous growth model in Arawatari et al. (2018), in which the nonlinearity in the inflation-growth relationship depends on the ability distribution in the population. In our case, however, it is related to the income distribution between groups with different skills.

In the model, changes in ι and z_r will require changing collected taxes to fund the variations the subsidies paid by the government. Because a fixed share of the intermediate goods production (2.6) and R&D activities (2.9) are subsidized, the portion of aggregate output directed to such expenditures rises more than the overall economy when policies stimulate such sectors. As all money collected from taxes is used to pay subsidies, ending up in the budget constraint (see Appendix G), and the seigniorage revenue returns to households, it is not necessary to adjust a tax to compensate for the subsidy level variation, simplifying the comparison of the policy instruments' effects.[19]. In this matter, z_r was preferred as a fiscal policy instrument to be compared with ι because it goes directly into R&D activity, the engine of the model's endogenous growth.[20] Moreover, as presented in Table 2.1, z_x affects only the long-run welfare, and τ_a is the only policy parameter affecting the long-run real interest rate.

When the government fosters R&D through direct subsidies, increasing z_r, the resources that should be paid upfront to achieve a new blueprint are reduced, i.e., Z falls, and R&D investment becomes relatively more attractive, which increases the demand for skilled labor in this activity, u^*, and pushes its relative wage up. With more income, households consume more, and output and welfare rise. The higher output pushes up wages for both labor types, but the increased demand in R&D makes skilled labor wages rise relatively more, which is reflected in the higher skill premium. Despite this increased economic activity, there is relatively less money in circulation because investment in R&D now attracts more resources; the constant portion of CIA in R&D expands the monetary base in a proportion greater than output, reducing V^*.

Assumption 2.2. *In the long run, most skilled labor is allocated in production and not in R&D activity, the stock of CIA to intermediate goods production is not too small, and the subsidies for intermediate goods production are not too high, i.e.,* $u^* < 1/2$, $\Omega_x > \xi$, *and* $\alpha + \beta - z_x + \Omega_x \iota > 0$.

As stated in the remarks of Propositions 2.4 and 2.5, Assumptions 2.1 and 2.2 are sufficient to unequivocally have $\frac{\partial V^*}{\partial \iota} > 0$, $\frac{\partial W^*}{\partial z_r} > 0$, and $\frac{\partial W^*}{\partial \iota} < 0$.

Proposition 2.6. *In the long run, and under Assumptions 2.1 and 2.2, there is complementarity between fiscal (an increase of direct R&D subsidies, z_r) and monetary policy (a decrease of the nominal interest rate, ι) effects on R&D intensity, the skill premium, and the velocity of money. Complementarity between the effects on the welfare is possible if R&D spillovers are not too small, and the complementary effects on the output level (at the time welfare is being computed) are strong enough to dominate the effects on the intermediate goods production costs and on the intertemporal utility from consumption, i.e., case* $\frac{\phi}{1-\phi} T^* \geq \frac{2\beta}{\alpha+\beta} + \frac{\alpha\beta(1-\phi)}{T^*(\alpha+\beta)^2}$ *and* $-\frac{\partial^2 W^*}{\partial z_r \partial \iota} = \frac{-c^*(0)^{-\theta}}{(1-\theta)G_N^* - \rho} \left(-\frac{\partial^2 c^*(0)}{\partial z_r \partial \iota} + \frac{\theta}{c^*(0)} \frac{\partial c^*(0)}{\partial z_r} \frac{\partial c^*(0)}{\partial \iota} \right) \geq 0$, *where* $-\frac{\partial^2 c^*(0)}{\partial z_r \partial \iota} = -\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota} + \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota} \left(\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota} - \frac{\partial^2 y^*(0)}{\partial z_r} \frac{\Omega_x}{1-z_x+\Omega_x \iota} \right)$.

[19]Notice that τ_L or τ_H also does not affect any variable in Table 2.1. However, case labor is not perfectly inelastically supplied, as in Subsection 2.4.1 extension, changes in τ_L or τ_H affect the supply of labor and, consequently, the long-run skill premium and welfare.

[20]In the Gomez and Sequeira (2014) model, subsidies to R&D lead to greater welfare gains than spending the same resources to subsidize intermediate goods production. Moreover, from 2010 to 2018, government support for private R&D activity in the U.S. "was largely skewed towards direct funding" (OECD 2021).

Proof. Recall that effects of both policies go in the same direction on welfare, R&D intensity, the skill premium, and the velocity of money, as stated in propositions 2.4 and 2.5; see also Appendix M. □

Remark. $\frac{\theta}{c^*(0)}\frac{\partial c^*(0)}{\partial z_r}\frac{\partial c^*(0)}{\partial \iota}$ represent the effects on intertemporal utility from consumption, $\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\left(\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota}-\frac{\partial^2 y^*(0)}{\partial z_r}\frac{\Omega_x}{1-z_x+\Omega_x\iota}\right)$ are the effects on the intermediate goods production costs, and $-\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota}$ are the effects on the output level. The relative magnitude of the effects from changes in ι and z_r on R&D intensity and the skill premium is given by the CIA constraint in R&D: $\left|\frac{\partial \beta/T^*}{\partial \iota}\right| = \Omega_r \left|\frac{\partial \beta/T^*}{\partial z_r}\right|$ and $\left|\frac{\partial\left(w_H^*/w_L^*\right)}{\partial \iota}\right| = \Omega_r \left|\frac{\partial\left(w_H^*/w_L^*\right)}{\partial z_r}\right|$ (see Appendix M).

Once the mild conditions stated in Assumptions 2.1 and 2.2 are satisfied, changes with opposite signs in z_r and ι will induce unambiguous long-run qualitative effects that go in the same direction on R&D intensity, skill premium, and the velocity of money. Moreover, these assumptions make it possible to determine the complementarity or substitutability between monetary and fiscal policy based on the sign of the cross derivatives of the analyzed variables w.r.t. z_r and ι. Under such conditions, it is possible to have complementarity or substitutability between fiscal and monetary policies effects on welfare. Empirical literature about the effects of fiscal and monetary policy interaction beyond the cycle is rare, but Zhang et al. (2022) is a recent exception. The authors concluded that loose monetary policy weakens the effects of R&D subsidies in 2010s China, which supports the possibility of policy substitutability in the long run. In the current context, where utility is taken exclusively from consumption, long-run welfare is a proxy to assess the policy effects on the overall economy's future performance in *per capita* terms once the consumption level is a fixed proportion of output (2.21). The condition established to have complementarity in the effects on the output level are easy to satisfy, but this is not sufficient to establish complementariy between the effects on welfare.

Regarding the skill premium, our setup suggests that fiscal and monetary policy are countervailing forces to the effects of an increase in the skill supply in the absence of the SBTC mechanism. The skill premium does not rise in response to an increase in s due to the model's lack of such a mechanism. Although theoretically in line with the Romer-Jones (Jones 1995; Romer 1990) tradition and supported by "some evidence," the reduction of the skill premium due to an increase in the supply of skilled labor predicted by our model contradicts most of the empirical evidence (Afonso and Magalhães 2020). As pointed out in the latter proposition remark, Ω_r gives the relative strength of monetary policy to fiscal policy in affecting the skill premium and R&D intensity. Moreover, the more dependent R&D is on the CIA, the greater the effects of monetary policy on the investment that generates long-run growth.

2.4. Model extensions

According to Proposition 2.6, the conditions in which there is substitutability or complementarity between fiscal and monetary policy effects on welfare, for the model presented in Section 2.2, depends on the strength of the policy effects on the steady state output level and the interaction of the isolated policy effects on future consumption. Thus, it is convenient to consider simple extensions to the model before conducting a numerical analysis to investigate the conditions that favor substitutability or complementarity. The first extension drops the assumption that labor is perfectly inelastically supplied. We also consider an alternative approach to modeling household money demand directly into the utility function, based on the premise that households are better off when holding money balances, as suggested by (Feenstra 1986).

2.4.1. Elastic labor supply

In this case, we assume that each household is endowed with amounts of skilled and unskilled labor and collectively chooses the portion of each labor type to supply, $0 < l_H(t) \leq 1$ and $0 \leq l_L(t) \leq 1$ (skilled and unskilled, respectively), which, in turns, reduce its utility, giving rise to voluntary unemployment. To ensure that the model aligns with the figures of the observed employed and unemployed population for each labor type, it is necessary to introduce a parameter $0 < l_F \leq 1$ that gives the relative size of the labor force in the entire population, or, in other words, to drop the assumption that the whole population, and all households' members, are able to work. This way, the aggregate level of skilled and unskilled labor employed becomes, respectively, $L_H(t) = sl_F L(t) l_H(t)$ and $L_L(t) = (1-s)\, l_F L(t) l_L(t)$, while $(1-l_F)\, L(t)$ gives the level of the population not in the labor force.

Production. Propagating such changes into the model, the expression for the real interest rate becomes

$$r_e(t) = \frac{(\alpha+\beta)\,(1-\alpha-\beta)}{\beta\,(1-z_r+\Omega_r\iota)} G_N(t) T(t) + (1-\phi)\, G_N(t) - \frac{\alpha}{\alpha+\beta}\left(G_T(t) + \frac{\dot{l}_H(t)}{l_H(t)} - \frac{\dot{l}_L(t)}{l_L(t)}\right), \tag{2.33}$$

which only differs from the baseline case (2.12) in its last term, i.e., during the transition. In this extension, the skill premium represents the relative labor demand and is given by

$$\frac{w_{H_e}(t)}{w_{L_e}(t)} = \frac{\beta}{\alpha}\frac{(1-s)}{s}\frac{1}{1-u(t)}\frac{l_L(t)}{l_H(t)}. \tag{2.34}$$

The only difference to Section 2.2 model (2.13) is the negative relation between the skill premium and the relative labor supplied by households, $\frac{l_H(t)}{l_L(t)}$. Output *per capita* becomes

$$y_e(t) = \left[\frac{(1-\alpha-\beta)^2}{\eta\left(1-z_x+\Omega_x \iota\right)}\right]^{\frac{1-\alpha-\beta}{\alpha+\beta}} \left[(1-s)\, l_L(t)\right]^{\frac{\alpha}{\alpha+\beta}} \left[s\left(1-u(t)\right) l_H(t)\right]^{\frac{\beta}{\alpha+\beta}} l_F N(t). \tag{2.35}$$

Notice that $l_L(t)^{\frac{\alpha}{\alpha+\beta}} l_H(t)^{\frac{\beta}{\alpha+\beta}} \leq 1$, and, for interior solutions of the household problem the inequality is strict, which implies that for a labor force of the same size, $l_F = 1$, and the same technological level, N, real output *per capita* is smaller than in the case in which labor is inelastically supplied (2.14).

Households. Households are composed of skilled and unskilled individuals able to supply labor (that can be employed or not) and (optionally) by individuals not in the labor force. Here, in addition to the consumption allocation, $c(t)$, they choose how much of each labor type, $l_H(t)$ and $l_L(t)$, to supply. As in Section 2.2, we assume that the distribution of the different individuals over the entire population is found in each household, allowing us to express their problem in *per capita* terms and solve the intertemporal optimization similarly as done in Appendix E, with instantaneous utility taking the form $\frac{c(t)^{1-\theta}-1}{1-\theta} - \frac{l_H(t)^{1+\frac{1}{\gamma_H}}}{1+\frac{1}{\gamma_H}} - \frac{l_L(t)^{1+\frac{1}{\gamma_L}}}{1+\frac{1}{\gamma_L}}$, where $\gamma_H > 0$ and $\gamma_L > 0$ are the Frisch-elasticity for each labor type supply, and, instead of (2.16), the budget constraint is

$$\begin{aligned}
\dot{a}(t) + \dot{m}(t) = {} & (1-\tau_H)\, s l_F l_H(t) w_{H_e}(t) + (1-\tau_L)(1-s)\, l_F l_L(t) w_{L_e}(t) \\
& + (1-\tau_a)\, r_e(t) a(t) + \iota b(t) - c_e(t) - \tau_{g_e}(t) \\
& - na(t) - nm(t) - \pi(t) m(t) + \tau_m(t).
\end{aligned}$$

The ratio of the additional first-order conditions in the household's problem, regarding $l_H(t)$ and $l_L(t)$, gives the relative labor supply, $\frac{w_H(t)}{w_L(t)} = \frac{(1-s)(1-\tau_L) l_H(t)^{\frac{1}{\gamma_H}}}{s(1-\tau_H) l_L(t)^{\frac{1}{\gamma_L}}}$. Substituting this in (2.34) results in

$$\frac{l_H(t)^{1+\frac{1}{\gamma_H}}}{l_L(t)^{1+\frac{1}{\gamma_L}}} = \frac{\beta}{\alpha} \cdot \frac{1-\tau_H}{1-\tau_L} \cdot \frac{1}{1-u(t)}, \tag{2.36}$$

representing the relative labor market equilibrium. With the imperfections introduced in the model by the preferences for labor supply, distinct taxes over wages distort the amount of each labor type employed in the economy, affecting the skill premium (2.34), output (2.35), consumption, and welfare (2.37).

Relative equilibrium. From the expansion of the invention's production function (2.11), it is straightforward to find that when u, l_H and l_L are constant, the steady state value of G_N is the same of that for the model as presented in Section 2.2 (2.28). Furthermore, when we equalize the consumption's growth rate, obtained from (2.21), (2.33), and (2.35), with G_N^*, we will find that the stationary expressions for T^* (2.30) and u^* (2.29) are still the same. The comparative statics results for such variables, summarized in Table 2.1, and also for r^*, π^*, and V^*, remain unchanged. With l_H^* and l_L^* being the steady state values for the household choice for labor supply, such that (2.36) is satisfied, it is possible to use (2.11), (2.21) and (2.35) to determine the initial consumption level and compute the steady-state welfare, similarly as done in Subsection 2.3.2:

$$W_e^* = \frac{1}{1-\theta}\left[-\frac{1}{\rho} - \frac{c^*(L(0))^{1-\theta}}{(1-\theta)\,G_N^* - \rho}\right] - \frac{1}{\rho}\left(\frac{{l_H^*}^{1+\frac{1}{\gamma_H}}}{1+\frac{1}{\gamma_H}} + \frac{{l_L^*}^{1+\frac{1}{\gamma_L}}}{1+\frac{1}{\gamma_L}}\right). \tag{2.37}$$

2.4.2. Money-in-utility function

In this case, we show that the CIA on consumption specification we adopted in the model presented in the previous sections is similar to a special case of the more general MIU function specification. Thus, we drop the CIA-on-consumption specified in the model detailed in Section 2.2 and solved in Section 2.3 to consider an MIU specification instead, with instantaneous utility given by $U(c(t), m_c(t)) = \frac{\left(c(t)^{\zeta} m_c(t)^{\sigma}\right)^{1-\theta} - 1}{1-\theta}$, representing a direct benefit (utility) for the households in holding money, measured by the marginal rate of substitution $\frac{\sigma c(t)}{\zeta m_c(t)}$, with $m_c(t) = \varepsilon(t) m(t)$, being m the total money balances in real terms, ε the portion of total money held for consumption purposes, and $\zeta, \sigma > 0$ the coefficients of the individual preferences for consumption and money, respectively. In this scenario, the representative household solves (2.38) instead of (2.15), subject to the same intertemporal budget restrictions (2.16) and CIA constraint (2.39) instead of (2.17):

$$\max_{c(t), b(t), \varepsilon(t)} \int_0^{\infty} \left[\frac{\left(c(t)^{\zeta} m_c(t)^{\sigma}\right)^{1-\theta} - 1}{1-\theta}\right] e^{(n-\rho)t} dt \tag{2.38}$$

$$\text{s.t. equation (2.16),}$$

$$\text{s.t. } b(t) \leq \left(1 - \varepsilon(t)\right) m(t). \tag{2.39}$$

The integral in the problem is assumed to converge, that is $\left(\zeta + \sigma\right)(1-\theta)\, G_N^* + n - \rho < 0$, and the transversality conditions are the same as those for problem (2.15), i.e.,

$\lim_{t\to\infty}\mu(t)a(t) = 0$ and $\lim_{t\to\infty}\mu(t)m(t) = 0$, but the co-state variable is now

$$\mu(t) = \frac{\zeta\left(c(t)^{\zeta} m_c(t)^{\sigma}\right)^{1-\theta} e^{(n-\rho)t}}{c(t)}.$$

From the optimality conditions for ε and b it is found that the portion of money to hold for consumption is a constant fraction of total consumption, i.e., $\varepsilon(t) = \frac{1}{\iota}\frac{\sigma c(t)}{\zeta m(t)} \Rightarrow m_c(t) = \frac{\sigma}{\iota\zeta}c(t) \Rightarrow \frac{\dot{m}_c(t)}{m_c(t)} = \frac{\dot{c}(t)}{c(t)}$, and the Euler equation for consumption from baseline specification (2.20) now turns to be

$$\frac{\dot{c}(t)}{c(t)} = \frac{(1-\tau_a)\,r(t)-\rho}{1-(1-\theta)\left(\zeta+\sigma\right)}. \tag{2.40}$$

The portion of money lent to firms is $B(t) = \left(1-\varepsilon(t)\right)M(t)$ that is equivalent to $M(t) = B(t) + \frac{\sigma}{\iota\zeta}C(t)$, and the velocity of money circulation (2.25) is replaced by

$$V(t) = \left\{\frac{\Omega_x(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota} + \frac{\Omega_r\beta}{T(t)} + \frac{\sigma}{\zeta\iota}\left[1-\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\right]\right\}^{-1}. \tag{2.41}$$

Equations (2.40) and (2.41), and the other Section 2.2 results, lead to a dynamic system made by equations (2.42) and (2.43) instead of (2.26) and (2.27):

$$\begin{aligned}
\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))} &= \frac{\alpha+\beta}{\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right]}\left\{(1-\tau_a)\frac{(\alpha+\beta)(1-\alpha-\beta)}{\beta(1-z_r+\Omega_r\iota)}G_N(t)T(t)\right.\\
&\quad\left.+\left[(1-\tau_a)(1-\phi)+(1-\theta)\left(\zeta+\sigma\right)-1\right]G_N(t)-\rho\right\}, \text{ and}
\end{aligned} \tag{2.42}$$

$$\begin{aligned}
\frac{\dot{G}_N(t)}{G_N(t)} &= n-(1-\phi)G_N(t) - \frac{(\alpha+\beta)}{\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right]}T(t)\times\\
&\quad\left\{(1-\tau_a)\frac{(\alpha+\beta)(1-\alpha-\beta)}{\beta(1-z_r+\Omega_r\iota)}G_N(t)T(t)\right.\\
&\quad\left.+\left[(1-\tau_a)(1-\phi)+(1-\theta)\left(\zeta+\sigma\right)-1\right]G_N(t)-\rho\right\}.
\end{aligned} \tag{2.43}$$

Following the same procedure as in the baseline case to solve the system (see Appendix J) yields a different result for T^* and, consequently, u^*. Results (2.28) and (2.29) continue the same. However, instead of (2.30), we find

$$T^* = \frac{\beta(1-z_r+\Omega_r\iota)\left\{(1-\phi)\rho - n\left[(1-\phi)(1-\tau_a)+(1-\theta)\left(\zeta+\sigma\right)-1\right]\right\}}{n(1-\tau_a)(\alpha+\beta)(1-\alpha-\beta)}. \tag{2.44}$$

Notice that $\left(\zeta+\sigma\right)(1-\theta)\,G_N^*+n-\rho<0 \Rightarrow (1-\theta)\left(\zeta+\sigma\right)G_N^*-\rho<0$, and steady-state welfare, which is not given by (2.31) anymore, is

$$W^* = \frac{1}{1-\theta}\left\{-\frac{1}{\rho}-\frac{\sigma}{\iota\zeta}\frac{c^*(L(0))^{(1-\theta)(\zeta+\sigma)}}{\left[(1-\theta)\left(\zeta+\sigma\right)G_N^*-\rho\right]}\right\}, \tag{2.45}$$

with *per capita* consumption level at the instant stationarity is achieved, c^*, being endogenously determined using (2.21), (2.14), (2.11), (2.28), (2.29), and (2.44), for a given population level. The equilibrium is saddle-path stable case $\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right]$ >0 (see Appendix J).

Comparative statics. Considering (2.41), (2.44), and (2.45), there is no difference in the sign of the partial derivatives presented in Table 2.1. Concerning the instruments of policy analyzed in Subsection 2.5.2, $\frac{\partial W^*}{\partial z_r}$ is still given by the expressions presented in Proposition 2.5, from (2.41),

$$\begin{aligned}\frac{\partial V^*}{\partial\iota} = \Bigg\{&\frac{\Omega_x^2(1-\alpha-\beta)^{\frac{1}{1-\alpha-\beta}}}{(1-z_x+\Omega_x\iota)^2}+\frac{\Omega_r^2\beta}{(1-z_x+\Omega_x\iota)\,T^*}\\ &+\frac{\sigma}{\zeta\iota^2}\left[1-\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\right]-\frac{\sigma\Omega_x(1-\alpha-\beta)^2}{\zeta\iota\,(1-z_x+\Omega_x\iota)^2}\Bigg\}\left(V^*\right)^2,\end{aligned}$$

and $\frac{\partial W^*}{\partial\iota}$ expression is in Appendix K. In this specification of the model, it is sufficient that $\Omega_x>\frac{\sigma}{\zeta\iota}$, $u^*<1/2$, and $\alpha+\beta-z_x+\Omega_x\iota>0$ to have $\frac{\partial V^*}{\partial\iota}>0$, $\frac{\partial W^*}{\partial\iota}<0$, and $\frac{\partial W^*}{\partial z_r}>0$, and monetary and fiscal policy induce the same long-run qualitative effects on the velocity of money and welfare. To summarize, the additional complexity of an MIU specification does not change the main qualitative results regarding fiscal and monetary policy.

Specialization. Despite being founded on different reasoning, in terms of modeling, the CIA in consumption adopted simplifies the particular case of the MIU specification discussed in this section, in which $\zeta+\sigma=1$. Case $\zeta+\sigma=1$, utility $U(c(t),m_c(t))$ becomes a monotonically increasing transformation of the utility function in the baseline model;[21] equations (2.40), (2.42), (2.43), and (2.44), reduce to their counterparts (2.20), (2.26), (2.43), and (2.30); the steady state is saddle-path stable, with the model presenting

[21]Case $\zeta+\sigma=1$, $U(c(t),m_c(t))=\left(\frac{\sigma}{\iota\zeta}\right)^{\sigma(1-\theta)}U(c(t))+\left[\left(\frac{\sigma}{\iota\zeta}\right)^{\sigma(1-\theta)}-1\right]\frac{1}{1-\theta}$, where $U(c(t))$ is the utility function of the Section (2.2) specification and $U(c(t),m_c(t))$ is the utility function of this subsection specification.

the same dynamics for the state variables u and G_N and differing only by the monetary base representation and the velocity of money (2.41).

2.5. Numerical analysis

Considering the steady-state relations determined in Section 2.3 and the available data, we now calibrate the model and its extensions to match some stylized facts of a real-world economy. Following the calibration, we complement the results of the previous qualitative analysis in the long-run static scenario and during the transition dynamics for the baseline model. Next, we compare the long-run effects on skill premium and welfare when different taxes are used to finance the policy changes considered and analyze the complementarity between fiscal and monetary policy, considering the cases in which labor supply is perfectly inelastic or elastic, and heterogeneous or homogeneous labor is considered.

2.5.1. Calibration and data

We used the U.S. economy between 1975 and 2019 as a reference for the model's calibration. In such a process, we considered that (i) the economy was in a balanced growth path in the period, (ii) steady-state reference values are the average from the data for the period, and (iii) the time unit in our model is one year. Table 2.2 summarizes the calibration for the baseline model, presented in Section 2.2, while Table 2.3 presents the calibrated model results for some variables and their reference values. Calibrations for the model's extensions are discussed at the end of this subsection. The temporal boundary adopted is due to the data set of mean earnings of workers (U.S. Census Bureau 2021) that we used to compute the reference for the skill premium.

Table 2.2: Baseline model calibration for the U.S. economy (1975–2019)

		Parameter	Value	Source
Structural	α	share of unskilled labor in the aggregate output	0.4	$\alpha+\beta=0.7$ (Afonso 2006); α and β match skilled and unskilled labor share in the wage bill (U.S. Census Bureau 2021, authors' computation).
	β	share of skilled labor (not engaged in R&D) in the aggregate output	0.3	
	η	real production cost of an intermediate-good unit	1	Equivalent to omitting it from the specification (e.g., Barro and Sala-i-Martin 2004).
	δ	rate of arrival of new ideas	0.1	Ratio of granted patents (USPTO 2024) to FTE researchers (OECD 2024).
	ϕ	R&D process elasticity of scale	0.44	Computed such that G_N^* (2.28) matches data (Feenstra et al. 2015).
	θ	inverse of the intertemporal elasticity of substitution	2.5	Vissing-Jørgensen (2002).
	ρ	households' discount rate	0.03	Jones (1995).
	n	population growth rate	0.01	Jones (1995).
	s	share of skilled workers in the labor force	0.29	U.S. Census Bureau (2021, authors' computation).
Fiscal	z_x	share of intermediate production costs subsidized	0.02	Computed to minimize the difference from $z_x \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota} + z_r \frac{\beta}{T^*}$ to data.
	z_r	share of R&D costs subsidized	0.13	Portion of GERD performed by the Government sector (OECD 2024).
	τ_a	*ad valorem* tax on assets returns	0.004	Computed to satisfy (2.22) such that $\tau_H = 2\tau_L$ and wages taxes pay approx. 4/5 of overall subsidies.
	τ_L	*ad valorem* tax on unskilled wages	0.004	
	τ_H	*ad valorem* tax on skilled wages	0.008	
Monetary	Ω_x	relative size of CIA stock in intermediate-goods production	3	Minimize the difference from β/T^* and V^* to the data, s.t. $0 < \Omega_x \iota/(1-z_x+\Omega_x \iota) <$ 0.41 and $0.41 \leq \Omega_r \iota/(1-z_r+\Omega_r \iota) < 1$.
	Ω_r	relative size of CIA stock in R&D activities	9.6	
	ξ	relative size of CIA stock in consumption	0.16	Computed such that $\xi\,(C/Y)^*$ matches data.
	ι	nominal interest rate	0.113	Computed to satisfy (2.19), such that r^* and π^* match data.

Notes: Subsection 2.2.2 and Appendix F discuss restrictions on the fiscal parameters; see Table 2.3 for the reference values guiding calibration.

The reference for G_N^* was computed discounting the population growth rate from the real GDP growth rate, as found in "Penn World Table version 10.0" (Feenstra et al. 2015). R&D intensity is the ratio of aggregate nominal investment in R&D to nominal GDP (U.S.

Bureau of Economic Analysis 2024d,e). From 1981 to 2019, we considered estimates for total researchers per thousand employees from the Organisation for Economic Co-operation and Development (OECD 2024) to compute the share of skilled labor in R&D,[22] and the percentage of gross domestic expenditure on R&D (GERD) performed by the Government sector as the amount of R&D subsidized. The skill premium, the share of skilled workers in the employed population, and the share of skilled labor in the aggregate wage bill were computed from the mean earnings of workers 18 years and over (U.S. Census Bureau 2021). We consider that skilled labor is provided by workers who have completed college or an advanced degree. The remaining labor force provides unskilled labor. The internal rate of return (Feenstra et al. 2015) is the target for r^*, the velocity of M2 money stock (Federal Reserve Bank of St. Louis 2021) for V^*, and the consumer price index (World Bank 2021) gives the reference for π^*. As the strength of CIA in R&D is more significant than in manufacturing, the "flow of debt service payments divided by the flow of income," or "debt service ratio" (DSR) (Drehmann et al. 2015), observed in private non-financial corporations (Bank for International Settlements 2021b) is assumed as the cap for the model long-run DSR in intermediate goods' production and the floor for long-run DSR in R&D.[23] Investigating the precautionary role of money in U.S. firms, Bates et al. (2009) observed that the average cash-to-assets ratio evolved from 10.6% in 1980 to 23.2% in 2006. The ratio of the nominal total consumer credit (Board of Governors of the Federal Reserve System (US) 2022) to nominal GDP gives the model's relative consumer credit to output, while the ratio of credit provided to non-financial corporations to GDP (Bank for International Settlements 2021a), by banks and by all sectors, are the reference values for the stock of bonds providing credit to firms in the model.

[22]This series starts in 1981, but its average value is consistent with that computed by Jones (2002) for 1979–1993.

[23]This series starts in 1999. Therefore, without further consideration, we assumed the average value for the available period as a reference.

TABLE 2.3: Results of the baseline model calibrated to the U.S. economy (1975–2019)

Variable or expression		Model's result	Reference value
G_N^*	growth rate of technological knowledge	1.8%	1.8%
u^*	share of skilled labor in R&D	8.5%	3.5%
$\frac{\beta}{T^*}$	R&D intensity	2.8%	2.8%
$\left(w_H/w_L\right)^*$	skill premium	2.0	2.0
r^*	real interest rate	7.5%	7.5%
π^*	inflation	3.8%	3.8%
V^*	velocity of money circulation	1.8	1.8
$\frac{\Omega_x \iota}{1-z_x+\Omega_x \iota}$	DSR in intermediate goods production	25%	$\leq 41\%$
$\frac{\Omega_r \iota}{1-z_r+\Omega_r \iota}$	DSR in R&D	52%	$> 41\%$
$\left(B/A\right)^*$	firm's cash-to-assets ratio	14%	11%–23%
$\left(B/Y\right)^*$	CIA to firms / GDP	29%	24%–62%
$\xi \left(C/Y\right)^*$	CIA to households / GDP	15%	15%
$z_x \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota} + z_r \frac{\beta}{T^*}$	Overall subsidies / GDP	0.52%	0.51%

Source: authors' computation.

Regarding the parameters' calibration, the aggregate labor share, given by $\alpha + \beta = 0.7$, is usual in the literature (e.g., Afonso 2006), and α and β, distinctively, follow the share of unskilled and skilled labor in the aggregate wage bill found in data, respectively 58% and 42%. The adopted n is typical in the literature (e.g., Jones 1995) and very close to the value observed. Equation (2.28) determines ϕ and equation (2.19) determines ι, with the involved variables assuming their reference values. While recent work suggests that the inverse of the elasticity of intertemporal substitution can assume very high values (e.g., Best et al. 2020; Yagihashi and Du 2015), we took for θ a value slightly higher than usual assumptions and consistent with previous empirical findings (Vissing-Jørgensen 2002). The discount rate is in line with theoretical literature, in which it is usual to find a value of 0.02 in the case of no population growth (e.g., Afonso 2006). Here, we consider 0.02 plus the population growth rate, matching Jones (1995). Parameters s and z_r are set directly to the value observed in data. The sum of federal, state, and local government expenditures with subsidies relative to GDP (U.S. Bureau of Economic Analysis 2024a,d,f,g) are used to find z_x. Assuming that skilled labor has a tax rate twice of unskilled, τ_a, τ_L, and τ_H are calibrated such that taxes collected from labor and capital invested in R&D in the model's economy, used only to pay subsidies, mimic the relative importance of personal and corporate U.S. government tax receipts (U.S. Bureau of Economic Analysis 2024b,c).

The value chosen for ξ approximates the model's amount of credit in consumption to the data and is insensitive to all Ω_x possible values. Ω_x and Ω_r were determined to minimize the differences from the resulting long-run R&D intensity and velocity of money to their reference values. Despite that, the values for these parameters diverge from those found in the CIA literature that do not consider the amount of money in the economy and its velocity in calibration; for example, in Chu and Cozzi (2014) the CIA in R&D, Ω_r, is considered in the interval $[0,1]$, whereas the benchmark value for it in Huang et al. (2021) is 0.45. In our model's context, for a nominal interest rate ranging from 6% to 11%, such values of CIA constraints would lead to a DSR 5 to 20 times smaller than the reference values found in data (see Table 2.3), and a velocity of money more than two times higher. Regarding such a puzzling situation, as our results are not qualitatively affected by these choices for the CIA constraints, we prefer to target the amount of money and credit observed. Although such higher values for Ω_x and Ω_r could be unreasonable in different setups or for other intents (e.g., one in which firms' default risks are considered), the steady state values for the monetary variables yielded by the model are in line with a set of measures obtained from different sources and far from the upper bounds we encountered.

To complete calibration, we considered the ratio of average granted patents by the U.S. Patent and Trademark Office (USPTO 2024) to the average of full-time equivalent (FTE) researchers employed (OECD 2024) in the period as the lower bound for the rate of arrival of new ideas, and set δ to such value. The unitary cost of an intermediate-good production was set to the unit, $\eta = 1$, which is equivalent to omitting it, as done in other R&D growth models without capital (e.g., Barro and Sala-i-Martin 2004).

With all parameters calibrated, we *ex-post* confirmed that the computed $\left(B/Y\right)^*$ was plausible and did not exceed the broader measure of credit provided to firms—non-financial corporations—by all sectors and did not fall short of that provided by banks, a more restrictive measure. Finally, the value u^* generated by the model calibrated to represent a real-world economy is much above the reference. In the context of our model, capital is not considered, and the only input in aggregate R&D is skilled labor. In reality, in addition to scientists, some capital is also necessary for these production processes. Therefore, as R&D intensity matches data, we interpret the portion of u^* that exceeds its reference value as the contribution of this very specialized capital used in reality but considered in the form of labor in the model (i.e., in equivalent labor units).

Model extensions. The calibration listed in Table 2.2 is followed for the extensions developed in Section 2.4. To calibrate the remaining parameters for the case of elastic labor supply (Subsection 2.4.1), we used monthly national-level data for the U.S. from the U.S. Bureau of Labor Statistics (BLS 2024) to compute $l_H(t)$ and $l_L(t)$ for the available observations and determine values for the Frisch elasticities of labor supply, $\gamma_H = 0.011$ and $\gamma_L = 0.031$, that minimize the average of the relative residual for $l_L(t)$, computed using the labor market equilibrium equation (2.36), with $\tau_H = \tau_L = 0$.[24] The relative size of the

[24]Because new values for $l_H(t)$ and $l_L(t)$ will only be computed in case the steady state $\left(1-u^*, G_N^*\right)$ changes, we considered a series with more observations than that used in the calibration of the baseline model to search for γ_H and γ_L values more appropriate to different combinations of the labor supply variables.

labor force was set to the value observed in data, $l_F \approx 0.653$. For the MIU function case (Subsection 2.4.2), we respectively set the relative preferences for consumption and money to $\zeta = 1 - \sigma$ and $\sigma \approx 0.018$, such that $\zeta + \sigma = 1$ and the results for the money velocity in this extension (2.41) and in the baseline model (2.25) are equal, i.e., $\frac{\sigma}{\zeta \iota} = \xi$. These choices make the MIU extension yield the same results as the baseline case and will be relaxed in the numerical analysis of complementarity between policies.

Population, labor supply, output, and consumption. Between 1975 and 2019, the U.S. population averaged approximately 270 million, which is the value we considered in the subsequent numerical simulations unless otherwise stated. To compute output and consumption for the Subsection 2.4.1 model, it is necessary that the household's choices of each labor type supply can be computed, which, considering (2.36), requires that the supply of one labor type is known. As this will not be the case, we overcome such limitation considering that in their internal process of choice, household members iteratively compute future long-run welfare (2.37) for the cases in which the supply of only one labor type changes, choosing the change that leads to greater welfare and repeating this until the better is to not change. Considering (2.37), such a recursive process can be loosely specified as $\Upsilon_{j+1}(l_H, l_L) = \max\left\{W_e^*(l_H, l_L), \Upsilon_i(\varphi_H(l_L), l_L), \Upsilon_j(l_H, \varphi_L(l_H))\right\}$, where j indexes the iteration step, and $\varphi_H(l_L)$ and $\varphi_L(l_H)$ are the functions found solving (2.36) for l_L and l_H, respectively. The considered initial rates for skilled and unskilled labor supply (or the labor type approximate average employment rate observed in the period) are, respectively, $l_H^* \approx 0.972$ and $l_L^* \approx 0.93$.

2.5.2. Comparative statics and transition dynamics

Comparative statics. The numerically computed long-run effects of a permanent and unanticipated increase in a parameter's value are summarized in Table 2.4 and complement the analytical results discussed in Section 2.3. Except for $\frac{\partial W^*}{\partial \alpha}$, all other results hold under Assumption 2.1 for the case of labor inelastically supplied (baseline model of Section 2.2). A Monte Carlo process, described in Appendix O, was used to run one million simulations in which all parameters can change to values lying in the interval of Table 2.2's calibration $\pm 10\%$ for the baseline model and the extension including elastic labor supply (Subsection 2.4.1).

TABLE 2.4: Long-run effects on the main variables for a permanent and unanticipated parameter increase in the neighborhood of the baseline calibration.

	Structural parameters on production, preferences and population									Fiscal policy: subsidies and taxes parameters					Monetary pol.: CIA and nom. interest rate			
	α	β	η	δ	ϕ	θ	ρ	n	s	z_x	z_r	τ_a	τ_L	τ_H	Ω_x	Ω_r	ξ	ι
$\partial G_N^*/\partial(.)$	0	0	0	0	+	0	0	+	0	0	0	0	0	0	0	0	0	0
$\partial u^*/\partial(.)$	−	−	0	0	+	−	−	+	0	0	+	−	0	0	0	−	0	−
$\frac{\partial\left(\frac{w_H}{w_L}\right)^*}{\partial(.)}$	−	[+]	0	0	+	−	−	+	−	0	+	−	0	0	0	−	0	−
$\frac{\partial\left(\frac{w_{He}}{w_{Le}}\right)^*}{\partial(.)}$	[−]	[+]	[0]	[0]	[+]	[−]	[−]	[+]	[−]	[0]	[+]	[−]	[0]	[0]	[0]	[−]	[0]	[−]
$\partial r^*/\partial(.)$	0	0	0	0	+	+	+	+	0	0	0	+	0	0	0	0	0	0
$\partial \pi^*/\partial(.)$	0	0	0	0	−	−	−	−	0	0	0	0	0	0	0	0	0	+
$\partial V^*/\partial(.)$	[+]	[+]	0	0	−	+	+	−	0	[−]	−	+	0	0	[−]	[−]	−	[+]
$\partial W^*/\partial(.)$	[±]	[−]	−	+	[+]	[−]	[−]	[−]	[+]	[+]	[+]	[−]	0	0	[−]	[−]	0	[−]
$\partial W_e^*/\partial(.)$	[±]	[±]	[−]	[+]	[+]	[−]	[−]	[±]	[−]	[+]	[+]	[±]	[+]	[+]	[−]	[±]	[0]	[±]

Notes: assuming $\alpha+\beta > 1/2$; "+" for positive effects, "−" for negative, and "0" for none; the numerically determined effects in [brackets] hold case parameters are set in the interval of Table 2.2 $\pm 10\%$; the direction of effects "[±]" vary for some simulated calibrations; at Table 2.2 calibration: $\partial W^*/\partial\alpha > 0$, $\partial W_e^*/\partial\alpha > 0$, $\partial W_e^*/\partial\beta > 0$, $\partial W_e^*/\partial n < 0$, $\partial W_e^*/\partial\tau_a < 0$, $\partial W_e^*/\partial\Omega_r < 0$, and $\partial W_e^*/\partial\iota < 0$; variables $\frac{w_{He}}{w_{Le}}$ and W_e^* defined in Subsection 2.4.1, the remaining in Section 2.2.

Computed results for the policy effects on the velocity of money and welfare on baseline model, $\frac{\partial V^*}{\partial \iota} > 0$, $\frac{\partial W^*}{\partial z_r} > 0$, and $\frac{\partial W^*}{\partial \iota} < 0$, also hold case the parameters calibrated from data (α, β, δ, ϕ, s, z_x, z_r, Ω_x, Ω_r, ξ, and ι) are allowed to vary in a range of $\pm 60\%$ from that on Table 2.2, for calibrations simulated with a similar Monte Carlos process. Such wide ranges of values for the parameters suggest that Propositions 2.4 and 2.5 are robust to calibrations targeting other economies. When elastic labor supply is considered and households suffer from labor disutility, there is the possibility of a welfare improvement from an increase in the output share accruing to skilled labor, population growth rate, asset tax rate, or nominal interest rate. In theses cases, the welfare improvement comes through the reduction of labor supply. Analogously, in this model, there is a welfare reduction due to an increase in the skilled portion of the labor force because its unemployment rate is lower than that of unskilled labor resulting that the negative effect households suffer from working more dominates the raise in utility caused by increased output and consumption.

Transition dynamics. As discussed in Subsection 2.3.1, there is only one stable trajectory through which the economy reaches the steady state, implying that the optimizing behavior of the agents makes the economy jump to a point over such a saddle path after some change in the economic conditions. Therefore, the system's internal forces determine the state of the economy during the transition. Knowing the values assumed by u and G_N over the saddle path and the population level, L, makes it possible to compute the technological knowledge level, N, and the remaining variables in the model. The calibrated model's saddle path was numerically determined by an adapted reverse shooting approach (see Appendix P).[25]

For the baseline model (Section 2.2), the paths of the analyzed variables, except for welfare, during the transition after the fiscal policy (increase of z_r), monetary policy (decrease of ι), and their combined changes discussed in Subsection 2.3.2 follow similar patterns along the time, as presented in Figure 2.2, in which shocks of approximately a standard deviation (s.d.) are considered, that is, 2.8 p.p. for R&D subsidies and 2.9 p.p. for the nominal interest rate. When comparing the effects of different policy instruments, Gomez and Sequeira (2014) normalized the sizes of the parameter changes by the amount of resources devoted to each subsidy. Here, we choose to let them at the computed s.d., because both changes increase the aggregate output spent with subsidies by approximately 0.1 p.p., i.e., from 0.52% to 0.61% for z_r and 0.59% for ι.

[25] Shooting's general approach "is to guess the unknown initial condition, solve the system as an initial value problem, and see if the terminal conditions to the initial value problem are close enough to the steady-state equilibrium" (Herbert and Stemp 2002). Reverse shooting consists of finding a point in a trajectory that converges to the steady state and integrating time backward to determine the path that led to it. In the model's steady-state neighborhood, the points to find lie over the linearized version of the system's stable manifold, which is tangent to the saddle path, according to the stable's manifold theorem.

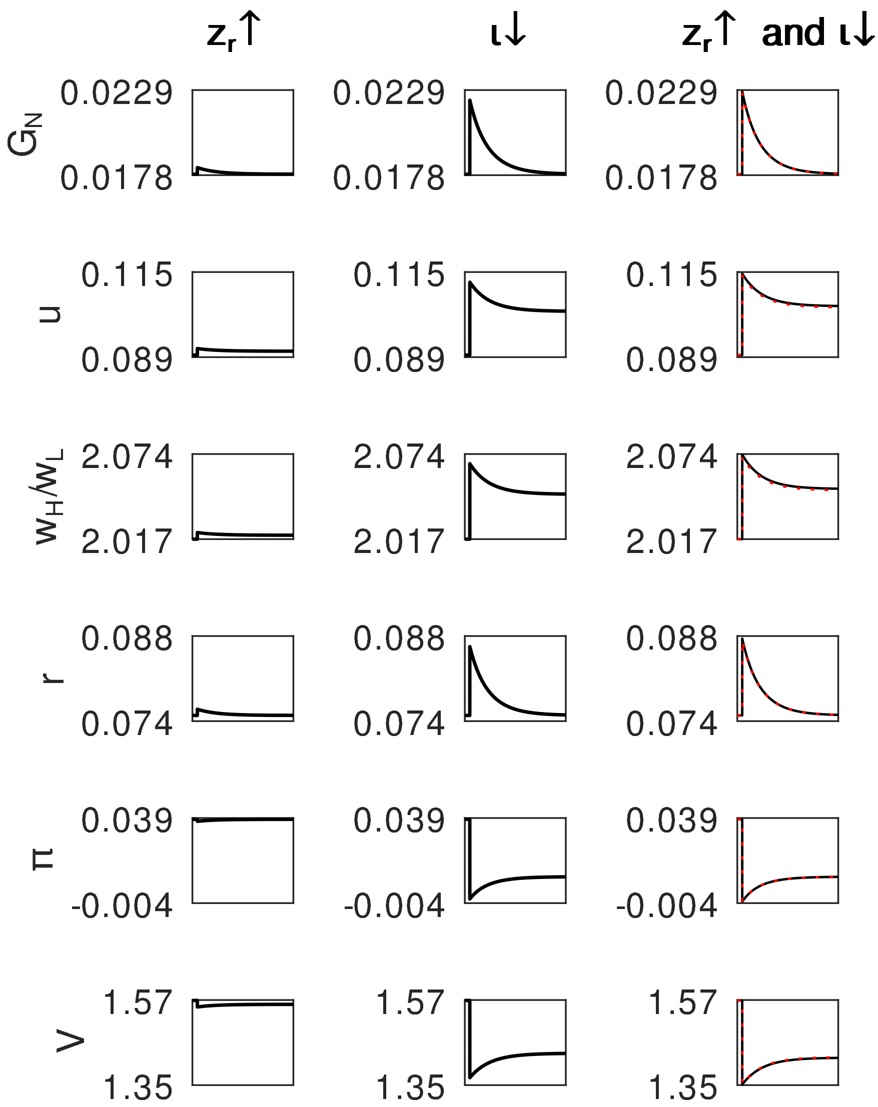


FIGURE 2.2: Dynamics of baseline model (Section 2.2) after policy changes, relative to Table 2.2 calibration; fiscal policy (z_r increases by 2.8 p.p.) effects are depicted in the left column, monetary policy (ι decreases by 2.9 p.p.) in the center, and the combined policies case in the right; the red dotted lines in the right column plot the sum of the isolated policy effects.

Figure 2.2 also illustrates the weaker sensitivity of the variables to fiscal policy stated in Proposition 2.6, compared to the effects of monetary policy. In all cases, the effects on a variable go in the same direction and, although not always distinguishable in the right column plots, the combined policies are stronger than the sum of the isolated policy effects, at the instant of the shocks and the beginning of the transition path. As already discussed, these policies only affect G_N and r during the transition, which will return to their previous steady-state levels in the long run. The negative inflation-growth relationship presented, qualitatively opposed to that of u and G_N and in the same direction as that of V, confirms the expected patterns for inflation and the real interest rate discussed in Subsection 2.3.1. Such a result regarding inflation complements Proposition 2.2 and is compatible with the empirical findings of a negative or insignificant relationship with the growth rate in the cases of a moderate inflation level (e.g., Benhabib and Spiegel 2009), as is the case for the calibration considered. The model extension with elastic labor supply (Subsection 2.4.1) variables' paths are very similar and are presented in Appendix Q (Figure Q.1).

2.5.3. Policy funding

In the model, fiscal and monetary policy will require an increase in collected taxes to fund the increase in the subsidies paid by the government. Because a fixed share of R&D and intermediate goods production activities is subsidized, the portion of aggregate output directed to such expenditures rises more than the overall economy when policies stimulate such sectors. The relative level of subsidies to GDP in the intermediate sector is given by $z_x \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota}$ and in R&D by $z_r \frac{\beta}{T^*}$, which means that the aggregate relative subsidy level is affected by most of the model's parameters (see Subsection 2.3.2, and Tables 2.1 and 2.4). After a change, seigniorage revenues transferred to individuals, τ_m, adjust according to (2.24), and the level of lump sum taxes, τ_g, adjusts conforming to (2.22), such that the government does not incur debt. The difference between these endogenous variables will determine the net amount to be transferred to or collected from each individual in the budget constraint (2.16). Tables 2.5 and 2.6 compare some long-run variables' response regarding policy changes financed entirely by only one kind of tax.

TABLE 2.5: Long-run fiscal policy funding.

#	Variable	Initial	$z_r\uparrow,\tau_g\uparrow$	$z_r\uparrow,\tau_a\uparrow$	$z_r\uparrow,\tau_H\uparrow$	$z_r\uparrow,\tau_L\uparrow$
1	β/T^*	0.02954	0.02997	0.02984	0.02997	0.02997
2	r^*	0.07494	0.07494	0.07521	0.07494	0.07494
3	π^*	0.03836	0.03836	0.03836	0.03836	0.03836
4	$\tau_m L/Y$	0.01360	0.01360	0.01360	0.01360	0.01360
5	$\tau_g L/Y$	0.00000	0.00089	0.00000	0.00000	0.00000
6	$\tau_a\frac{r^*\beta(1-z_r+\Omega_r\iota)}{G_N^*T^*}$	0.00097	0.00097	0.00184	0.00097	0.00097
7	$\tau_H\beta/(1-u^*)$	0.00264	0.00264	0.00264	0.00353	0.00264
8	$\tau_L\alpha$	0.00160	0.00160	0.00160	0.00160	0.00249
9	$z_x\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}$	0.00136	0.00136	0.00136	0.00136	0.00136
10	$z_r\beta/T^*$	0.00384	0.00474	0.00472	0.00474	0.00474
11	Tot. Subsidies/Y	0.00520	0.00610	0.00608	0.00610	0.00610
12	$(w_H/w_L)^*$	2.01701	2.01964	2.01887	2.01964	2.01964
13	W^*	22.20981	22.21023	22.21011	22.21023	22.21023
14	l_H^*	0.97199	0.97199	0.97199	0.97197	0.97199
15	l_L^*	0.93000	0.92996	0.92997	0.93000	0.92990
16	$(w_{H_e}/w_{L_e})^*$	1.92988	1.93232	1.93161	1.93242	1.93219
17	W_e^*	22.01976	22.02278	22.02191	22.02270	22.02297
18	— consump.	0.00000	0.00290	0.00207	0.00290	0.00290
19	— lab. supp.	0.00000	0.00012	0.00008	0.00004	0.00032

Notes: long-rung values for the model with inelastic heterogeneous labor is defined in Section 2.2 and extended with elastic labor supply in Subsection 2.4.1; models initially calibrated as in Subsection 2.5.1 and with changes in z_r alone (subsidies' variation funded with the endogenous lump-sum tax) and in z_r together with τ_a, τ_H, or τ_L for different sources of funding to the subsidies' variation; values in first 11 rows are the same in both models; the values in 4–11 rows are the aggregate value relative to output for seigniorage revenues (#4), lump-sum taxes (#5), taxes over assets (#6), skilled labor (#7), and unskilled labor (#8), subsidies to intermediate sector (#9) and R&D (#10), and the total subsidies (#11) match the total collected with taxes; values in rows 12-13 are exclusive of the Section 2.2 model and, in rows 14–19, of its Subsection 2.4.1 extension; rows 18–19 decompose the welfare variation due to the changes in consumption and labor supply, respectively.

If labor is inelastically supplied, the effects on most variables are the same, except if financed by the tax on assets' returns, which leads to smaller increments in R&D intensity, skill premium, welfare, and the aggregate subsidies relative to output.

TABLE 2.6: Long-run monetary policy funding.

#	Variable	Initial	$\iota\downarrow, \tau_g\uparrow$	$\iota\downarrow, \tau_a\uparrow$	$\iota\downarrow, \tau_H\uparrow$	$\iota\downarrow, \tau_L\uparrow$
1	β/T^*	0.02954	0.03444	0.03433	0.03444	0.03444
2	r^*	0.07494	0.07494	0.07515	0.07494	0.07494
3	π^*	0.03836	0.00936	0.00936	0.00936	0.00936
4	$\tau_m L/Y$	0.01360	0.00345	0.00345	0.00345	0.00345
5	$\tau_g L/Y$	0.00000	0.00069	0.00000	0.00000	0.00000
6	$\tau_a \frac{r^*\beta(1-z_r+\Omega_r\iota)}{G_N^* T^*}$	0.00097	0.00097	0.00165	0.00097	0.00097
7	$\tau_H\beta/(1-u^*)$	0.00264	0.00268	0.00267	0.00337	0.00268
8	$\tau_L\alpha$	0.00160	0.00160	0.00160	0.00160	0.00229
9	$z_x\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}$	0.00136	0.00146	0.00146	0.00146	0.00146
10	$z_r\beta/T^*$	0.00384	0.00448	0.00446	0.00448	0.00448
11	Tot. Subsidies/Y	0.00520	0.00594	0.00592	0.00594	0.00594
12	$(w_H/w_L)^*$	2.01701	2.04703	2.04635	2.04703	2.04703
13	W^*	22.20981	22.21389	22.21383	22.21389	22.21389
14	l_H^*	0.97199	0.97199	0.97199	0.97199	0.97199
15	l_L^*	0.93000	0.92959	0.92960	0.92965	0.92954
16	$(w_{H_e}/w_{L_e})^*$	1.92988	1.95774	1.95710	1.95786	1.95763
17	W_e^*	22.01976	22.04936	22.04889	22.04918	22.04952
18	— consump.	0.00000	0.02829	0.02785	0.02829	0.02829
19	— lab. supp.	0.00000	0.00132	0.00129	0.00113	0.00147

Notes: long-rung values for the model with inelastic heterogeneous labor is defined in Section 2.2 and extended with elastic labor supply in Subsection 2.4.1; models initially calibrated as in Subsection 2.5.1 and with changes in ι alone (subsidies' variation funded with the endogenous lump-sum tax) and in ι together with τ_a, τ_H, or τ_L for different sources of funding to the subsidies' variation; values in first 11 rows are the same in both models; the values in 4–11 rows are the aggregate value relative to output for seigniorage revenues (#4), lump-sum taxes (#5), taxes over assets (#6), skilled labor (#7), and unskilled labor (#8), subsidies to intermediate sector (#9) and R&D (#10), and the total subsidies (#11) match the total collected with taxes; values in rows 12-13 are exclusive of the Section 2.2 model and, in rows 14–19, of its Subsection 2.4.1 extension; rows 18–19 decompose the welfare variation due to the changes in consumption and labor supply, respectively.

When Subsection 2.4.1 model extension is considered, the adjustment in labor supply tends to be through the unskilled type, which is more elastic. Comparing the funding options, the supply of unskilled labor is lower when the increase in subsidies is paid with taxes over wages received by this labor type than when other taxes are used. The intuition is straightforward: when the tax on wages for the labor type rises, there is less incentive for households to supply such labor type, and firms will be unwilling to pay the wage level required to keep the previous employment level. Similarly, the supply of skilled labor is reduced when taxes on their wage are increased to finance an increase in subsidies in consequence of fiscal policy. However, only unskilled labor adjusts when the rise in the relative skilled wage is strong enough, as is the case in the monetary policy change presented in Table 2.6. In such a situation, contrasting with the fiscal policy change in Table 2.5, the higher demand for skilled labor in R&D puts its relative wage at a level where households prefer to adjust unskilled labor despite the increase in τ_H.

The pattern of the differences in welfare is the opposite of the unskilled labor supply; that is, in comparison with the case in which a lump sum tax pays the rise in subsidies, it is higher when paid by taxes on unskilled labor and lower in the other cases. Regarding the skill premium, workers require higher wages to compensate for increased taxes. Thus, compared with the value achieved with an increase in the lump sum tax, it is higher when taxes on skilled labor pay for the increase in subsidies and lower than or equal to the other funding options. Regarding the complementarity on welfare, in the cases analyzed in this subsection, the effects of both policies used in conjunction do not exceed the sum of those presented in Tables 2.5 and 2.6 (see Tables Q.1 and Q.2 in Appendix Q). The role of the different taxes for funding the subsidies on welfare complementarity will be explored in the following subsection.

2.5.4. Complementarity

To better understand the conditions that favor the complementarity between fiscal and monetary policy on welfare, we evaluate complementarity for the models with inelastic (Section 2.2) and elastic (Subsection 2.4.1) labor supply, resulting from permanent policy changes relative to Subsection 2.5.1 calibration. Following Macedo and Martins (2008), in our context, quantitatively discarding the existence of complementarity between monetary (a decrease in ι) and fiscal (an increase in z_r) policy regarding their effects on welfare is equivalent to evaluating that $W|_{z_r\uparrow,\iota\downarrow} - W|_{s0} < \left(W|_{\iota\downarrow} - W|_{ss}\right) + \left(W|_{z_r\uparrow} - W|_{ss}\right)$. Thus, there is complementarity case

$$W|_{z_r\uparrow,\iota\downarrow} + W|_{s0} - W|_{z_r\uparrow} - W|_{\iota\downarrow} \geq 0, \tag{2.46}$$

in which $W|_{s0}$ is the welfare evaluated at the economy's state before policy action, $W|_{\iota\downarrow}$ is welfare evaluated at the instant of a monetary policy intervention, $W|_{z_r\uparrow}$ for a fiscal policy change, and $W|_{z_r\uparrow,\iota\downarrow}$ case both policies are conducted in conjunction.

Table 2.7 summarizes the analysis of complementarity or substitutability on long-run welfare effects, considering (2.46), for the models with inelastic and elastic labor supply, heterogeneous and homogeneous labor types, and considering and not considering the transition path. For all cases in Table 2.7, complementarity on welfare is favored when the transition path is accounted for. In the dynamics of the cases of homogeneous labor supply in Table 2.7, the complementarity of the effects increases with the period of the transition considered. In contrast, in the heterogeneous labor cases, complementarity falls right after the policy intervention and assumes a humped shape when more time is considered (see Figure Q.2 in Appendix Q).

TABLE 2.7: Long-run fiscal and monetary policy complementarity or substitutability on welfare effects.

Labor modeling	**Initial welfare**	**Policy effect**			
		$z_\tau \uparrow$ (+2.8 p.p.)	**$\iota \downarrow$ (-2.9 p.p.)**	**$z_\tau \uparrow, \iota \downarrow$**	**l.h.s.(2.46)**
Inelastic heterogeneous	22.209812	+0.000419	+0.004079	+0.004400	-0.000098
" with transition		+0.000084	+0.001266	+0.001348	-0.000002
Inelastic homogeneous	22.220946	+0.000045	+0.000437	+0.000472	-0.000011
" with transition		+0.000009	+0.000282	+0.000326	+0.000035
Elastic heterogeneous	22.019759	+0.003020	+0.029605	+0.031987	-0.000638
" with transition		+0.000624	+0.009245	+0.009898	+0.000029
Elastic homogeneous	22.211507	+0.000364	+0.003513	+0.003788	-0.000089
" with transition		+0.000078	+0.002289	+0.002643	+0.000277

Notes: the model with inelastic heterogeneous labor is defined in Section 2.2 and extended with elastic labor supply in Subsection 2.4.1; models calibrated as in Subsection 2.5.1; homogeneous labor is achieved by setting $\alpha = 0$, $\beta = 0.7$ and $s = 1$; initial welfare and effects without considering the transition path computed from steady state expressions (2.31) and (2.37); welfare with transition obtained from the numerical integration of the discounted utility path towards its new steady state value.

By splitting the analysis of the welfare accumulated only at the beginning of the transition path from that of long-run welfare, while considering policy changes of different strengths, we find that complementarity on the long-run effects decreases with the strength of the policy. However, no relationship between policy strength and complementarity is evident. Such an analysis can be illustrated by plotting the difference in welfare variation in the long run and during the first years of the transition path. Figure 2.3 presents plots for the baseline model (inelastic heterogeneous labor supply). The results are similar for the other (three) non-plotted model variations and a broader range of monetary policy shocks. That is, Figure 2.3, panel (a), suggests that substitutability (complementarity) increases (decreases) with the strength of the policy change. While in panel (b), during the transition, the strength of the policies does not directly determine if the policy effects are more or less complementary. The latter result is reinforced by analyzing the complementarity sensitivity to the policy strength during the transition (Appendix Q, Figure Q.3). For the different policy strengths and models considered, the dynamics of welfare accumulation follow the ones described above, and complementarity is more pronounced at the beginning of the transition (before the transition half-life).

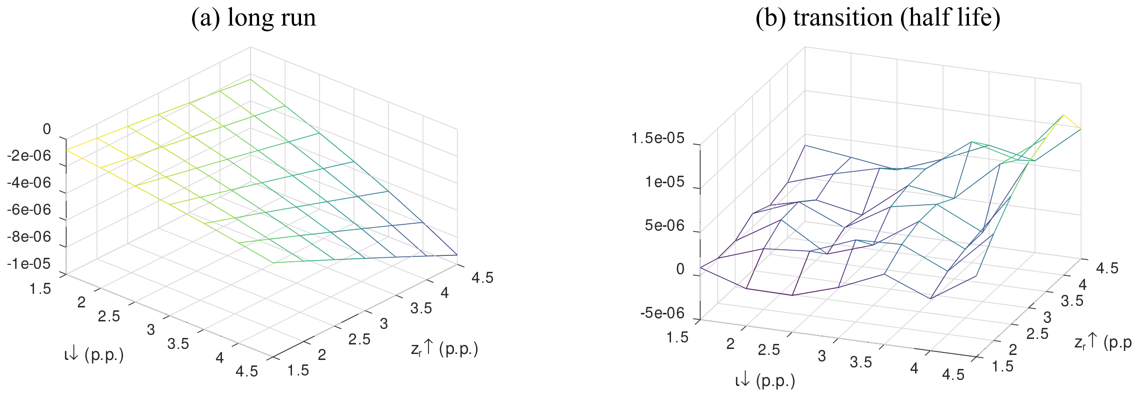


FIGURE 2.3: The difference from the effects on the baseline model's welfare of fiscal and monetary policy used together to the sum of the policies' effects isolated for different sizes (in p.p.) of change in policy parameters ι (monetary) and z_r (fiscal); (a) left panel shows the difference in long-run welfare not considering the utility transition path; (b) right panel shows the difference in utility accumulated over the transition half-life; in both panels, level zero equals the sum of the isolated effects; negative values indicate substitutability; only the results for the model considering inelastic heterogeneous labor supply (Section 2.2) are plotted due to their similarity with the other model variations.

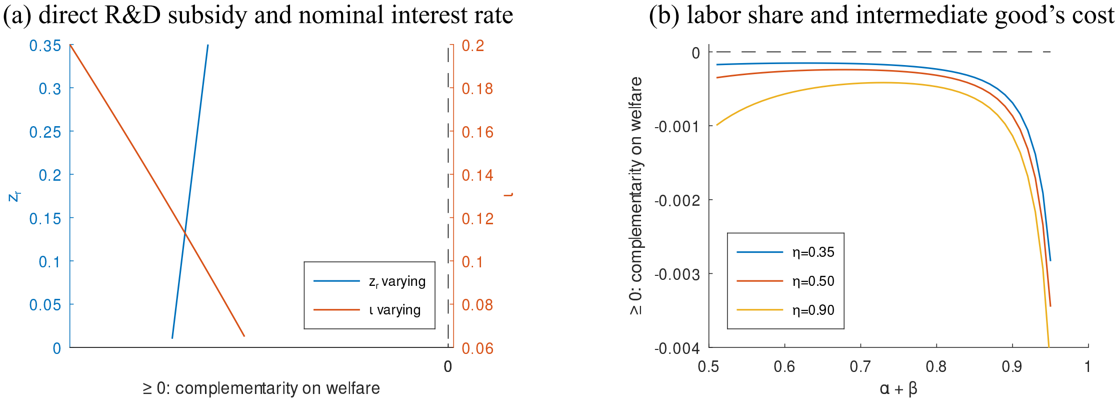


FIGURE 2.4: The difference from the effects on welfare of fiscal (z_r increases 2.8 p.p.) and monetary policy (ι decreases 2.9 p.p.) used together to the sum of the policies' effects isolated, case the economic environment changes, departing from Subsection 2.5.1 calibration; (a) in the left panel, direct R&D subsidy ($z_r \in [0.01, 0.35]$) and the nominal interest rate ($\iota \in [0.065, 0.2]$) vary; (b) in the right panel, total labor share ($\alpha + \beta \in [0.51, 0.95]$) varies under different intermediate good's production cost ($\eta \in \{0.35, 0.50, 0.90\}$); zero or positive values indicate complementarity.

In Figure 2.4, panel (a), different from the observed in Figure 2.3, in which stronger policy changes reduce the complementarity on long-run welfare, reacting to policy changes of constant strengths, complementarity is favored by an economic environment with greater levels of R&D subsidies and lower nominal interest rates, which translates as lower long-run inflation levels. In Figure 2.4, panel (b), we observe that the total labor share relationship with complementarity is nonlinear and that lower production costs of intermediate goods favor complementarity.

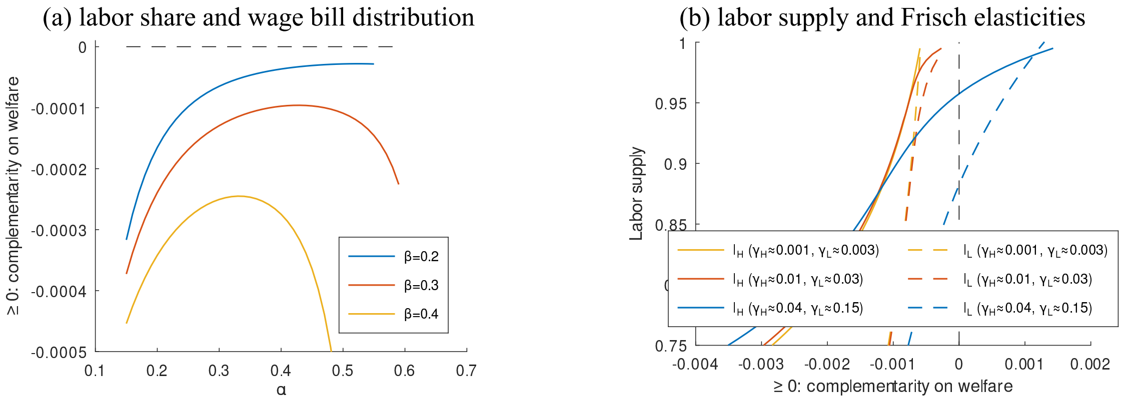
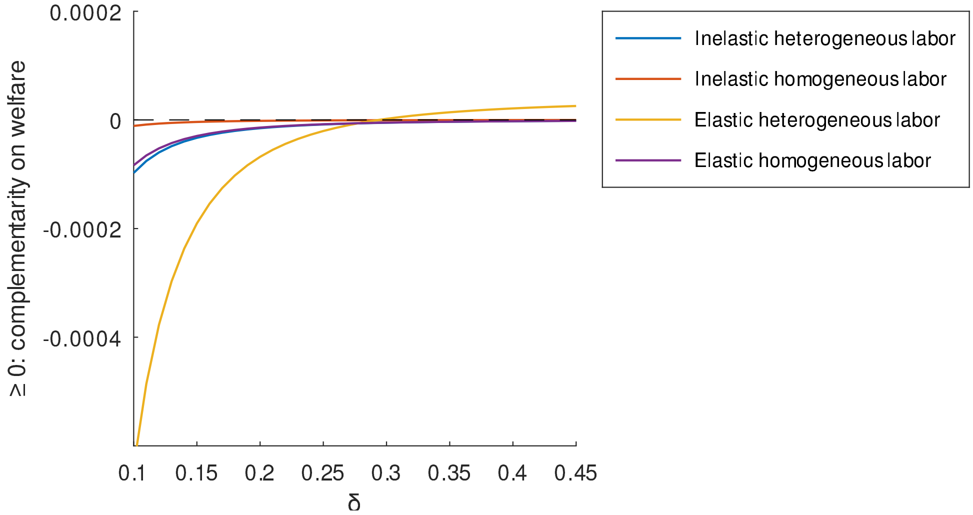


FIGURE 2.5: The difference from the effects on welfare of fiscal (z_r increases 2.8 p.p.) and monetary policy (ι decreases 2.9 p.p.) used together to the sum of the policies' effects isolated, case the economic environment changes, departing from Subsection 2.5.1 calibration; (a) in the left panel, different total labor share ($\alpha+\beta$) and distribution of the wage bill towards unskilled ($\alpha/[\alpha+\beta]$) or skilled ($\beta/[\alpha+\beta]$), for $\alpha \in [0.15, 0.59]$ and $\beta \in \{0.2, 0.3, 0.4\}$; (b) in the right panel, skilled and unskilled labor supply vary under different Frisch elasticities, $(\gamma_H, \gamma_L) \in \{(0.001, 0.003), (0.01, 0.03), (0.04, 0.15)\}$, satisfying (2.36) of Subsection 2.4.1 model; zero or positive values indicate complementarity.

In Figure 2.5, panel (a) depicts that a lower share of the wage bill paid to skilled labor reduces the skill premium and favors complementarity, which, in panel (b), is favored by higher labor supply elasticities. Figure 2.6 illustrates that, varying the determinants of the invention's production, the result of equation (2.46) increases as ϕ and δ grow. However, complementarity can only be detected for the Subsection 2.4.1 model (elastic heterogeneous labor supply) with the invention's arrival rate much above the calibrated value, $\delta \geq 0.30$ or with high spillovers in R&D, $\phi \geq 0.65$ (which substantially increases the long-run growth rate of the model by one p.p., since $\phi \approx 0.65 \Rightarrow G_N^* \approx 2.8\%$ with the other parameters unchanged).

(a) arrival rate of inventions



(b) long-run growth rate (or R&D spillovers, ϕ)

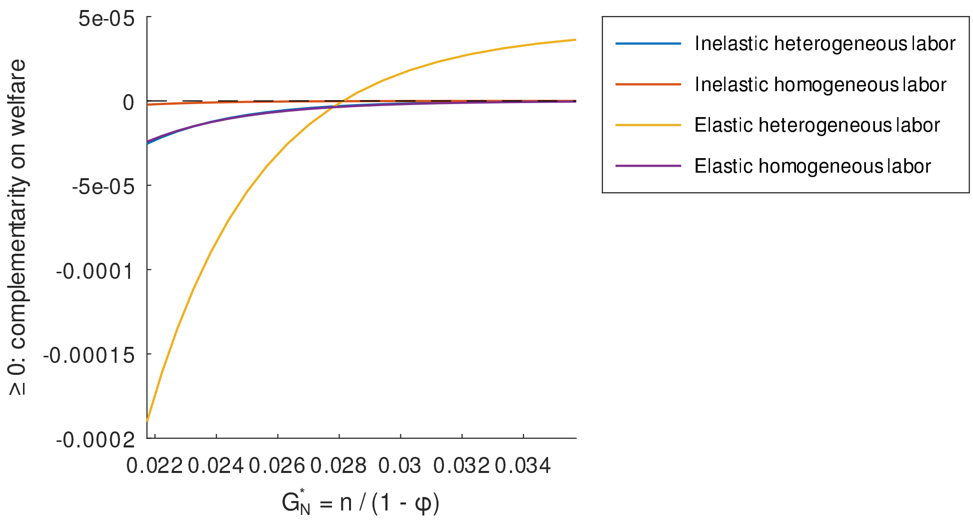
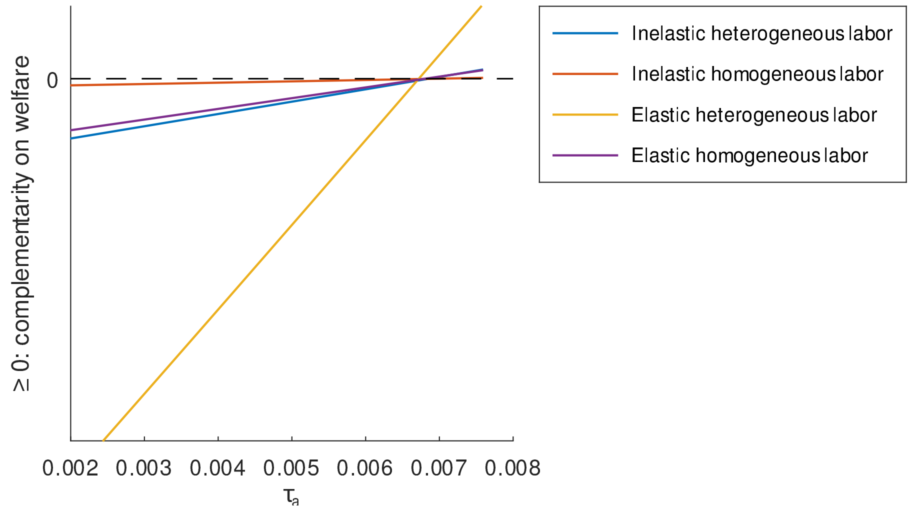


FIGURE 2.6: The difference from the effects on welfare of fiscal (z_r increases 2.8 p.p.) and monetary policy (ι decreases 2.9 p.p.) used together to the sum of the policies' effects isolated for the different model variations, case the determinants of the invention production function (2.11) vary; models calibrated as in Subsection 2.5.1; (a) in the left panel, the arrival rate of inventions vary ($\delta \in [0.01, 0.99]$); (b) in the right panel, R&D spillovers, which determines G_N^*, vary ($\phi \in [0.2, 0.8]$); zero or positive values indicate complementarity.

Fiscal and monetary policy require a rise in collected taxes to fund the increase in the subsidies paid by the government. Because a fixed share of R&D and intermediate goods production activities is subsidized, the portion of aggregate output directed to such expenditures rises more than the overall economy when policies stimulate such sectors. Figure 2.7 illustrates the policy complementarity sensitivity to taxes other than the lump sum progressively used to finance such an aggregate subsidy increase. The difference in the concomitant policy effects on welfare to the sum of the isolated effects increases with the taxes on R&D assets, τ_a, in all models analyzed. Taxes on wages generate a nonlinear response, but only in the model considering elastic heterogeneous labor supply (Subsection 2.5.1). Recall that, as parameter ϕ, τ_a positively affects the real interest rate in the long run (see Table 2.1). However, because household gains net of income are reduced, an increase in τ_a disincentivizes investment in R&D, partially offsetting the positive effects from policy.

(a) tax on R&D assets returns



(b) taxes on skilled and unskilled wages

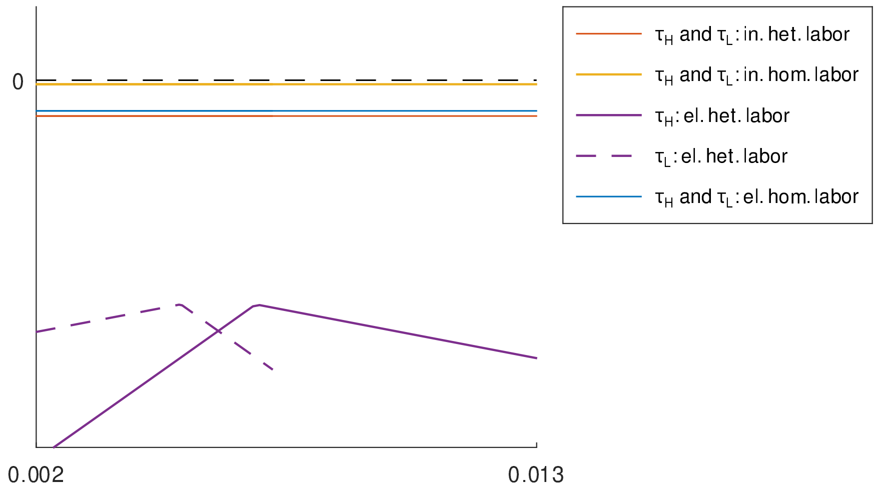


FIGURE 2.7: The difference from the effects on welfare of fiscal (z_r increases 2.8 p.p.) and monetary policy (ι decreases 2.9 p.p.) used together to the sum of the policies' effects isolated for the different model variations, case taxes other than the lump sum vary after a policy change; models calibrated as in Subsection 2.5.1; (a) in the left panel, the tax on R&D assets varies ($\tau_a \in [0.002, 0.008)$); (b) in the right panel, taxes on wages vary ($\tau_a \in [0.002, 0.013]$ and $\tau_L \in [0.002, 0.007]$); zero or positive values indicate complementarity.

In most of the analyzed scenarios, only the model with heterogeneous elastic labor supply (Subsection 2.4.1) showed complementarity for the fiscal and monetary policy effects in the long run. It is relevant that the combined policy effects tend to surpass the sum of isolated effects in the long run solely when households lose utility by supplying labor (Subsection 2.4.1). Several theoretical conditions were identified as favoring complementarity between fiscal and monetary policies: lower inflation, a considerable level of subsidies in R&D, a higher growth rate, a higher rate of invention's arrival, a higher labor supply (or a lower unemployment rate), a less unequal distribution of the wage bill, and lower production costs in the intermediate goods sector. Analyzing the U.S. economy from 1970 to 2001, Muscatelli et al. (2004) concluded that fiscal and monetary policy were used as substitutes until the 1980s and in a more complementary way in the 1990s. However, their work does not consider heterogeneity in labor nor an R&D or intermediate goods sector. Similar to their work, although using different model classes, our baseline structure also "suggests a degree of substitutability" between policies; in the case of a perfectly inelastic labor supply, complementarity between these policies is transitory if it exists. Nevertheless, different from that, our model allows for complementarity in the long-run policies' effects when imperfections are considered in the labor market.

Adjusting the model calibration to reflect some identified conditions makes the model show complementarity in the long-run policy effects on welfare. For example, setting $\alpha = 0.35$ and $\beta = 0.27$, with a total labor share of $\alpha + \beta = 0.62$, $\delta = 0.2$ for the invention's arrival rate, and $\phi = 0.5 \Rightarrow G_N^* \approx 2\%$ for the long-run growth rate. We also verified that, in such modified calibration, the complementarity in the policy effects is due to the contribution of the labor disutility variation in welfare. Considering only the second term in the r.h.s. of (2.37) to evaluate complementarity, similar to equation (2.46), the result is positive. However, the result is negative if we evaluate only the first term. As expected from the results discussed in the current subsection, the labor market conditions determine whether there is long-run complementarity or not in the Subsection 2.4.1 model.

Regarding other changes that push the model in the direction of complementarity, we consider it unlikely that the share of skilled labor in the wage bill decreases much, or that a significant deviation on the Frisch elasticities, γ_H and γ_L, is possible in our setup. Moreover, defining a proper value to η can be problematic, but to $y(t)$ approximate the output *per capita* (in thousands of US$, 1975-2019), given the population level (in millions), observed in the same year, η should be set to a value below 0.35, which would increase the complementarity of the effects. For the taxes used to fund an increase in aggregate subsidies paid by the government, changing the tax rate on wages has little practical importance once they are already calibrated near the points that deliver more complementarity for the welfare effects. In addition, an increase in the tax on R&D assets has long-run effects contrary to those of the analyzed policies and is also unlikely to be used in conjunction.

2.6. Conclusion

This work analyzed the complementarity between fiscal policy (in the form of R&D subsidies) and monetary policy (the control of inflation by the nominal interest rate) in the context of a two-sector knowledge-driven horizontal R&D growth model with skilled and unskilled labor that includes cash-in-advance (CIA) constraints in production and consumption, and subsidies in R&D and intermediate-goods production. The analysis is motivated by the open question of whether fiscal and monetary instruments act as substitutes or reinforce each other when innovation is at the core of long-run growth. It was shown that the model's dynamic system has a unique saddle-path-stable equilibrium, determining the economy's state through the growth rate of technology and the portion of skilled labor demanded in R&D. In the long run, or when the model becomes stationary, the economy embarks on a balanced-growth path and grows, in real *per capita* terms, at the same rate R&D discoveries are found, a result that follows from the decentralized optimizing behavior of the agents. This result is consistent with semi-endogenous growth theory, which allows for a focus on level, allocation, and welfare effects, thereby shifting the analysis from long-run growth rates.

In the context of this endogenous growth setup, the complementarity or substitutability between fiscal and monetary policy was studied from a theoretical perspective. It was verified that when labor supply is perfectly inelastic in the long run, and under mild conditions, fiscal and monetary policies can be used to induce the same effects on R&D intensity, skill premium, the velocity of money, output, and consumption. However, when used in conjunction, there is complementarity in their effects on these variables. That is, the two policy instruments are not just substitutes but can reinforce each other through their interaction with cash-in-advance constraints and the allocation of skilled labor between production and R&D. Complementarity for the effects on welfare is possible if the effects on the steady state output level (at the moment welfare is computed for) are strong enough to dominate the effects on intermediate goods production costs and intertemporal utility from consumption considered in long-run welfare (2.31).

The model admits the theoretical possibility of a positive long-run relationship between inflation and welfare when the financial constraints in the intermediate goods sector are sufficiently weaker than in R&D, illustrating departures from the Friedman rule in innovation-driven economies. However, a negative relationship between inflation and growth is expected during the transition path.

Moreover, our theoretical setup suggests that fiscal and monetary policy can contribute to explaining part of the observed rise in the skill premium, with monetary policy holding a prominent position, once its effects are magnified by the higher CIA constraint encountered in R&D compared to other sectors.

The numerical analysis of the model calibrated to the U.S. economy (1975–2019) also revealed that these policies can be complementary if the transition path is considered, especially during the initial phases of the adjustment path, when the difference between joint policy changes' effects and the sum of isolated policy effects is more noticeable. However, for reasonable values of the parameters, policy complementarity for long-run welfare effects was only confirmed in the case where labor is heterogeneous and elastically supplied, highlighting the role of labor market imperfections in shaping the policy implications for welfare.

The discussion on policy funding further showed that the method used to finance higher subsidies or monetary expansions is not neutral. The use of different tax instruments leads to varying labor supply responses, which in turn affect the skill premium and welfare, even when aggregate policy effects remain qualitatively unchanged. The simulations showed that households typically adjust by reducing the supply of unskilled labor, due to its relatively higher elasticity.

Simulating departures from the baseline calibration, we identified several conditions that foster policy complementarity for the effects on welfare in the long run, among which we highlight the importance of the labor market conditions with high Frisch elasticity of labor supply in conjunction with low unemployment rates, high rate of R&D discoveries, and relatively high growth rates. Additional aspects favoring complementarity include lower inflation, sizable R&D subsidies, a more equitable distribution of the wage bill, and lower production costs for intermediate goods.

Overall, the results of this work suggest that labor market frictions and innovation incentives fundamentally shape the effects of the interaction between fiscal and monetary policy. Moreover, it highlights the importance of considering the transitional path in welfare analysis, rather than focusing solely on steady-state growth effects.

It is worth mentioning that the theoretical conditions we identified still have to be empirically tested, especially if there is a connection between the decline of the labor share, which favors complementarity in our setup and was observed in recent decades (e.g., Karabarbounis and Neiman 2013), and the "more complementary way" fiscal and monetary policy have been used after the 1990s (Muscatelli et al. 2004). Establishing such links empirically would help assess whether the mechanisms emphasized by the model can account for observed changes in policy coordination.

Another point that deserves exploration is how the results regarding complementarity on long-run welfare are affected if the assumption of fixed labor supply is dropped. For example, assuming that labor supply decreases steadily in the long run (e.g., Boppart and Krusell 2020)could improve the robustness of our findings.

3. The role of labor unions in shaping wage inequality and economic growth

This paper introduces a directed technical change growth model with a trade union representing unskilled labor in the bargaining with firms for the wage and employment level, resulting in the direction of the effects of increased union bargaining power depending on the union's orientation towards employment or wage and whether the "technological knowledge" or the "labor-supply" channel dominates the skill premium, reconciling theory with the ambiguous results of previous empirical applications. More bargaining power can reduce the skill premium if unions are employment (wage) oriented and technological knowledge dominates (is dominated by) the labor supply.

Keywords: directed technical change; labor union; endogenous growth; skill premium; income-labor share.

JEL Classification: J31; J51; O33; O38; O41.

3.1. Introduction

Since the early 1980s, most Western economies, with the United States (U.S.) at the head, have experienced increased wage inequality between skilled and unskilled workers. Looking at the case of the U.S., Katz and Murphy (1992), Bound and Johnson (1992), Krueger (1993), Doms et al. (1997), Berman et al. (1998), and Allen (2001), among others, empirically determined that in the heart of such effect was the phenomenon of technological-knowledge bias towards skilled workers, which consequently leads to an increase in demand for these workers and a decrease in demand for unskilled workers. In theoretical terms, Acemoglu (1998, 2002a) begins by building a model of growth based on two R&D sectors, the so-called Directed Technical Change (DTC) model, which uses the market-size channel (relative supply of skills) to explain the mechanism causing technological-knowledge bias: an increase in the relative supply of skilled labor to firms makes R&D directed at those workers more profitable and, as a result, their productivity increases and thus their demand. That is, the labor endowment influences the direction of technological knowledge. In the long run, technologies that use the more abundant type of labor are favored, and the positive effect of the induced technical change on the skill premium (wage of skilled labor relative to unskilled labor) dominates the usually negative direct effect of the increase on the relative supply of skills (Afonso 2006; Loebbing 2022). Therefore, the technological knowledge bias and the resulting rise in inequality can be interpreted as a result of the exogenous increment in the relative supply of skills (or in the relative size of the market for technology complementary to skilled labor).

Relatedly, a considerable amount of literature links the rise in the skill premium in the last decades to the labor de-unionization observed in the period. Alongside other research, Katz and Autor (1999) empirically show that the decline in the union's bargaining power in the U.S. may also explain the increased skill premium between 1970 and 1999. Conversely, Card et al. (2004) and Visser and Checchi (2011) sought to examine whether an increase in the union's bargaining power contributes to compressing the skill premium. In a broader sense, the decline in unionization in the U.S. contributed to the observed growth in inequality after the 1980s (e.g., Card et al. 2004; DiNardo et al. 1996; Fortin et al. 2021). From a theoretical perspective, workers' unions have been represented in technological-driven growth models, but most work does not relate them to skill premium or inequality issues. For example, Palokangas (1996) and Chang et al. (2007) analyzed union's effects on growth, unemployment, and welfare; Lingens (2003, 2007) used different setups to examine how unionization affects growth.

Thus, although growth models are widely used to study the skill premium as a result of innovation-based technical change, only a bunch of works have incorporated unions in such theoretical frameworks to analyze their connection to the skill premium in the long run. To this matter, Acemoglu et al. (2001) linked technical change to de-unionization and inequality through a framework in which greater levels of skill-biased technical change (SBTC) erode the benefits provided by unions and, consequently, unionization and the wage compression promoted by them. More recently, Neto et al. (2019) explored that unions may affect the investment in technology complementary to the workers they represent. Their results suggest that case unions' preferences are oriented for employment (wage) are stronger, an increase in their bargaining power reduces (increases) the skill premium, skill to unskilled employment ratio, and the technological knowledge bias directed to skilled labor. Afonso et al. (2023) used a DTC model, in which union participation is depressed by technological knowledge, to study the union's relationship with the labor relocation of unskilled-intensive activities observed during globalization in previous decades. They concluded that unskilled labor relocation and de-unionization contributed to the rising inequality in advanced economies. Santos et al. (2025) proposed a DTC model in which higher societal individualism levels interfere with the capacity of unions to reduce the skill premium.

Connected to these studies, there is a theoretical discussion regarding the impact of labor unions on the path of the income-labor share and the economic growth rate. At these levels, there is also a vibrant empirical literature for developed countries, where R&D activity is more intense. Jayadev (2007) and Checchi and García-Peñalosa (2010), for example, note that an increase in the union's bargaining power positively affects income-labor share in the case of (developed) OECD countries; in this sense, the observed decline in the union's bargaining power "offers" an explanation for the observed reduction in the income-labor share. In terms of economic growth, however, the empirical studies are less conclusive in terms of results; indeed, some suggest that an increase in the union's bargaining power in the OECD favors economic growth (e.g., Asteriou and Monastiriotis 2004), while others conclude the opposite (e.g., Nickell and Layard 1999).

While previous studies have separately examined the impact of union bargaining power on wage inequality (Neto et al. 2019), income-labor share (Checchi and García-Peñalosa 2010), and economic growth (Asteriou and Monastiriotis 2004), we are unaware of a theoretical model integrating these three dimensions into a single framework. This paper addresses this gap by extending the DTC model with horizontal innovation by Acemoglu (2002a) to incorporate labor unions and analyze their impact on these interconnected variables. In this work, inequality and the union's bargaining power are connected through a mechanism in which the direction of the effects depends not only on the union's orientation toward employment but also on the strength of technological knowledge bias. The hypotheses that emerge from such a modeling guide the analysis: (i) Are the effects of the union's bargaining power on these variables dependent on its preferences toward employment or wage? (ii) Does the interaction between the technological knowledge bias and the relative supply of different labor types change such effects qualitatively?

Here, the skilled labor supply affects the skill premium directly and not only through the technological knowledge bias, as in Neto et al. (e.g., 2019). The proposed model reveals that the direction of the labor unions' bargaining power effects on wage inequality can be determined by the union orientation toward wage or employment and by the strength of the technological knowledge bias channel relative to the labor supply channel on the skill premium.[1] An increased bargaining power when labor unions are employment-oriented intensifies unskilled employment, stimulating economic growth and improving the income-labor share.[2] If the unions are wage-oriented, it generates technological-knowledge bias in favor of skilled workers.[3] Regarding economic growth, in contrast to the findings of Nickell and Layard (1999), who argue that unions hinder growth, our model predicts that

[1] Increasing union bargaining power has an ambiguous effect on income inequality in Chang and Hung (2016) and compresses wage inequality in Neto et al. (2019).

[2] Increasing union bargaining power depresses economic growth if the union is employment-oriented in the model with capital accumulation proposed by Chang et al. (2007), has no influence on the economic growth rate in the model with endogenous product market structure developed by Ji et al. (2016), and influences economic growth in the model with two-country R&D presented by Chu et al. (2016).

[3] Increasing union bargaining power raises the income-labor share when the union is wage-oriented in the model proposed by Chang and Hung (2016).

when unions are employment-oriented, they may enhance it. Essential empirical methods were used to illustrate that the theoretical findings in this paper regarding the direction of the bargaining power effects on the analyzed variables are plausible. To this matter, we combined data from the Organisation for Economic Co-operation and Development (OECD 2023) with the University of Amsterdam, the International Labour Organization (ILO 2024), and the Penn World Table 10.01 (Feenstra et al. 2015).

Our contribution to the literature is twofold. First, we call attention to the significance of the technological knowledge bias strength relative to the labor supply when unions' effects on the skill premium are analyzed. Our results that consider such forces in combination with the union's preferences complement those of Neto et al. (2019). Additionally, we add to the literature considering the relevance of the union's orientation when exploring the interesting topic of economic growth and labor markets in the presence of technological change. In this sense, the possibility of different qualitative responses on the skill premium, income-labor share, and growth may help to reconcile previous ambiguous results.

The remainder of this paper is organized as follows. Section 3.2 constructs an R&D-based growth model featuring directed technical change with labor unions. Section 3.3 discusses the economy's balanced growth equilibrium and then analyzes the effects of the union's bargaining power on employment, technological knowledge bias, skill premium, income-labor share, and economic growth. Data and the quantitative methods used to assess the theoretical effects on the skill premium are spelled out in Section 3.4. Concluding remarks are provided in Section 3.5.

3.2. Theoretical Setup

3.2.1. Overview

Rational infinitely-lived dynastic households inelastically supply labor, skilled and unskilled, maximize utility obtained with the consumption of the homogeneous final good of its members, pay taxes, and earn income from labor and investments in financial assets. From the production side, the aggregate final good comprises tasks from two sectors, unskilled (L-sector) and skilled (H-sector), which, in turn, use specific labor and a continuum of specific non-durable machines/intermediate goods: a DTC model. Each intermediate good is produced in monopolistic competition using a design that grants a perpetual protected patent and units of aggregate final good as inputs. This design is sold by the R&D sector, where there is perfect competition and, therefore, free entry, and every potential entrant dedicates aggregate final goods to produce or invent successful horizontal designs. Therefore, there is a technological knowledge bias favoring the H-sector if the number of varieties of machines produced in this sector is higher relative to the L-sector, which, in turn, causes each skilled worker to produce relatively more output and a wage differential between sectors, i.e., the skill premium.

The model that follows extends the DTC framework to consider a labor union bargaining with firms on behalf of unskilled workers. It is a dynamic general-equilibrium endogenous growth model, implying that firms and households are rational and solve their problems, free-entry R&D conditions are satisfied, and markets clear.

3.2.2. Consumers

The economy is populated by a fixed number of infinitely-lived households that consume and collect income from investments in financial assets and labor. There is a representative household, that inelastically supplies labor. We assume that consumers have perfect foresight concerning the technological change over time and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximize discounted lifetime utility. With a constant intertemporal elasticity of substitution (CIES) instantaneous utility function, the infinite horizon lifetime utility is

$$U = \int_0^\infty \left\{ \frac{C(t)^{1-\theta} - 1}{1-\theta} \right\} e^{-\rho t} dt, \tag{3.1}$$

where $C(t)$ is the consumption of the household at time t; $\theta \geq 1$ represents the inverse of the elasticity of intertemporal substitution, which we assume that is greater than or equal to 1, following the literature (e.g., Afonso and Magalhães 2020; Barro and Sala-i-Martin 2004); $\rho > 0$ is the time preference rate to ensure that U is bounded away from infinity if C is constant over time, or, more precisely, if $(1-\theta)g - \rho < 0$, where g is the consumption steady-state growth rate.

The utility maximization problem of the consumer is subject to the following intertemporal budget constraint:

$$\sum_{n=L,H} \dot{K}_n(t) = (1-\upsilon) \cdot \sum_{n=L,H} \left[r_n(t) \cdot K_n(t) + w_n(t) \cdot L_n(t) + \Pi_{Y_n}(t) \right] +$$
$$+ u\left(\overline{L}_L - L_L\right) - C(t) - S(t), \tag{3.2}$$

where $K_H = V_H A_H$ and $K_L = V_L A_L$ denote the household's real financial assets of skilled and unskilled monopolistic firms owned by the representative household;[4] r_H and r_L are the rate of return on skilled and unskilled designs, respectively; w_H and w_L are the wages paid

[4] As will become apparent below, A_H and A_L are the number of skilled and unskilled machines, respectively, and V_H and V_L correspond to the net present value of the profits of each skilled and unskilled producer of machines, respectively.

to the employed $n=H$ and $n=L$, respectively; L_H and $\overline{L}_L$ are the inelastic supply of skilled and unskilled labor, respectively, and thus $\overline{L}_L - L_L$ is the unemployment of unskilled labor; u is the unemployment benefit; $\Pi_{Y_H} + \Pi_{Y_L}$ are profits transferred from skilled-intensive and unskilled-intensive task firms; υ is the income tax rate; and, S is a lump-sum tax levied by the government. From standard dynamic optimization (see Appendix R), we derive the optimal path of consumption (the households' Euler equation),

$$g_C \equiv \frac{\dot{C}(t)}{C(t)} = \frac{(1-\upsilon)\, r(t) - \rho}{\theta}, \tag{3.3}$$

where r is the real interest rate and common rate of return of skilled and unskilled designs, and the transversality conditions are $\lim_{t\to+\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot K_n(t) = 0$, with $n \in \{L, H\}$.

3.2.3. Government

To pay unemployment benefits, the government levies income taxes and household fixed taxes; i.e., the government chooses the income tax rate and unemployment benefits, balancing the budget through endogenous adjustment of household fixed taxes. The government's balanced budget constraint is then expressed in the form:

$$\upsilon \cdot \sum_{n=L,H} \left[r_n(t) \cdot K_n(t) + w_n(t) \cdot L_n(t) + \Pi_{Y_n}(t) \right] + S(t) = u\left(\overline{L}_L - L_L\right), \; u = s \cdot p_L \cdot Y_L; \tag{3.4}$$

i.e., to ensure that unemployment benefits exhibit sustained growth, following Chang et al. (2007), Chang and Hung (2016) and Chu et al. (2016), we specify $u = s \cdot p_L \cdot Y_L$, where: $p_L = \frac{P_L}{P_Y}$ is the real price of the output of the L-sector, Y_L is the unskilled-intensive final goods, and $s > 0$ is the proportionate rate between unemployment benefits and revenues for the unskilled-intensive good and can be treated as a policy parameter.[5]

3.2.4. Production and prices: final goods

Aggregate economy. Following Afonso and Gil (2022), the final good, Y,[6] which is unique, produced competitively in the H-sector and L-sector, respectively, Y_H and Y_L, has

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[5]Given that the government's tax revenues stated on the left-hand side of equation (3.4) will grow at the same rate as Y along the balanced-growth path, we specify that unemployment benefits u are proportional to $p_L Y_L$ so that the unemployment benefits (i.e., government expenditure) will grow at the same rate as Y, too.

[6]In this paper, when we mention "output," we are more precisely referring to the "value-added" of the economy. Since the output is produced with labor and capital and, in our case, the factors are labor and machines, we can interpret machines in the context of our model as replacing the role of capital in the production function.

the following Constant Elasticity of Substitution (CES) production function:

$$Y(t)=\left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},\,\gamma\in(0,1),\,\varepsilon>1, \tag{3.5}$$

i.e., (i) Y_H and Y_L are the output of (input from) the H-sector and the L-sector, respectively; (ii) γ is the distribution parameter that determines the relative importance of the H-sector; (iii) $\varepsilon\in(0,+\infty)$ is the elasticity of substitution between the two sectors: if $\varepsilon>(<)\,1$ then the outputs of the H and the L-sector are gross substitutes (complements)—from now on we consider that $\varepsilon>1$, as proposed by Acemoglu et al. (2012). The maximization problem of real profits is given by max $\Pi_Y=Y-p_HY_H-p_LY_L$, where $p_n=\frac{P_n}{P_Y}$ (i.e., $p_H=\frac{P_H}{P_Y}$ and $p_L=\frac{P_L}{P_Y}$), $n=H,\,L$, is the real price of the output of each sector.[7] After some calculations (see Appendix S), the relative price of the H-sector is

$$\frac{p_H}{p_L}=\frac{\gamma}{1-\gamma}\cdot\left(\frac{Y_H}{Y_L}\right)^{-\frac{1}{\varepsilon}}. \tag{3.6}$$

Given that the production function of final goods is homogeneous of degree one, we have $p_LY_L+p_HY_H=p_YY$ and

$$1=\left[\gamma^{\varepsilon}\cdot p_H^{1-\varepsilon}+(1-\gamma)^{\varepsilon}\cdot p_L^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{3.7}$$

Sectors of the economy. It is assumed that in each sector, skilled and unskilled, an infinite number of tasks are produced. Regarding the optimal decision of firms producing skilled-labor-intensive tasks and unskilled-labor-intensive tasks, similar to Chang et al. (2007), Chu et al. (2016), and Ji et al. (2016), the production functions of skilled and unskilled tasks are assumed to both exhibit decreasing returns to scale, which is justified by the existence of other fixed factors (e.g., land), thus ensuring that all firms earn a positive profit. As a result, firms producing unskilled tasks are willing to bargain with the existing union of unskilled workers in order to establish the respective wage, whereas the skilled wage is determined by the clearing condition for the skilled-labor market;[8] hence, only the unskilled wage is determined by collective bargaining.

[7] By real, we mean that is the ratio of the output price in each sector to the final output price. All prices and costs in this paper are to be regarded in this way.

[8] Indeed, the empirical evidence provided by Card (1998) reveals that the union's membership rate of unskilled workers is more intense but has been declining, being appropriate to focus on how the union's bargaining power affects unskilled workers. Moreover, Acemoglu et al. (2001) point out that skilled labor has greater bargaining power than unskilled labor, which makes skilled labor unwilling to join a labor union. Finally, as Checchi et al. (2010) pointed out, skilled labor treats wages as the reward for individual effort or talent (i.e., as reflecting their productivity) and, therefore, has less incentive to join a union.

We now discuss the optimizing decision of the task firm in each sector. Following Acemoglu (2002a), we assume that skilled- and unskilled-intensive tasks are produced by using, respectively, skilled and unskilled labor, L_H and L_L, and a continuum of specific complementary skilled and unskilled machine inputs, x_i^H and x_i^L, in the $i \in [0, A_H]$ and $i \in [0, A_L]$ intervals since A_n, $n = H, L$, is the number of varieties of skilled, $n = H$, and unskilled, $n = L$, machine inputs; the general production function is

$$Y_n(t) = L_n^{\alpha} \cdot \int_0^{A_n(t)} \left(x_i^n(t)\right)^{\beta} di, n = \{H, L\}, \tag{3.8}$$

where: $\frac{1}{1-\beta}$ is the elasticity of substitution between skilled machine inputs. To ensure that the skilled-intensive task firm earns a positive profit, the production function (3.8) exhibits decreasing returns to scale, i.e., $0 < \alpha + \beta < 1$. As stated, p_H is the real price of the H-sector, and $p_{x_i^H}$ is the real price of x_i^H; in turn, w_H is, as already stated, the skilled wage paid by skilled-intensive task firms. The real profits maximization problem in the n-sector is $\max\limits_{L_n, x_i^n} \Pi_{Y_n} = p_n \cdot Y_n - w_n \cdot L_n - \int_0^{A_n} p_{x_i^n} \cdot x_i^n \cdot di$. From its first order conditions (F.O.C.), $\frac{\partial \pi_{Y_n}}{\partial L_n} = 0$ and $\frac{\partial \pi_{Y_n}}{\partial x_i^n} = 0$, respectively, we have

$$w_n = \alpha \frac{p_n \cdot Y_n}{L_n} \Leftrightarrow w_n \cdot L_n = \alpha \cdot p_n \cdot Y_n \text{ and} \tag{3.9}$$

$$x_i^n = \left(\frac{\beta \cdot p_n}{p_{x_i^n}}\right)^{\frac{1}{1-\beta}} \cdot L_n^{\frac{\alpha}{1-\beta}} \Rightarrow \int_0^{A_n} p_{x_i^n} \cdot x_i^n \cdot di = \beta \cdot p_n \cdot Y_n; \tag{3.10}$$

i.e., in each sector, task firms pay the fraction α of its sales of output to hire labor, $w_n \cdot L_n = \alpha \cdot p_n \cdot Y_n$, and pay the fraction β to use all the respective machine inputs, i.e., $\int_0^{A_n} p_{x_i^n} \cdot x_i^n \cdot di = \beta \cdot p_n \cdot Y_n$. As a result, the final goods firm's profit is given by

$$\Pi_{Y_n} = (1 - \alpha - \beta) \cdot p_n \cdot Y_n = (1 - \beta) \cdot p_n \cdot Y_n - \alpha \cdot p_n \cdot Y_n = (1 - \beta) \cdot p_n \cdot Y_n - w_n \cdot L_n. \tag{3.11}$$

However, under a labor union, the previous analysis is not valid for the L-sector in which the unskilled wage is determined by collective bargaining. As in Pemberton (1988), Mezzetti and Dinopoulos (1991), Chang et al. (2007), and Chu et al. (2016), we consider a managerial labor union to capture internal conflicts between leadership and membership within a labor union. In particular, we assume that the objective function of a managerial labor union, U, contains two components: leadership, which cares about the number of members, and membership, which cares about the level of the wage rate. The union's objective can be written as the following Stone-Geary function, which is also supported by the empirical studies of Dertouzos and Pencavel (1981):

$$U = \left[(1-\upsilon)\, w_L - u\right]^{\delta} \cdot L_L^{\zeta},\ \delta > 0,\ \zeta > 0. \tag{3.12}$$

As already stated, U is the managerial labor union's objective function, and υ is the income tax rate. Moreover, u represents unemployment benefits, δ represents the importance of the wage surplus for unskilled labor, and ζ is the importance of the number of members.[9] Bearing in mind (3.12), we consider that both the managerial union and the unskilled-labor-intensive firm bargain over the unskilled wage rate, w_L, and the employment, L_L, through a generalized Nash bargaining solution—following Clark (1990), Goerke (1996, 2017), Chang et al. (2007), and García and Sorolla (2014). The larger the difference $\delta - \zeta$, the more the union approaches the extreme of a democratic (or populist) union; when $\delta = 1$, $\zeta = 1$, the union becomes risk neutral as in Maffezzoli (2001) and Zanetti (2007).

Let $\chi \in (0,1)$ denote the bargaining power of the union; $\max \Psi$ then states the bargaining problem but subject to the firm's demand for unskilled machine inputs:

$$\begin{gathered} \max_{w_L, L_L} \Psi = U^{\chi} \cdot \pi_{Y_L}^{1-\chi}, \\ s.t.\, x_i^L = \arg\max_{x_i^L} \pi_{Y_L},\ \chi \in (0,1). \end{gathered} \tag{3.13}$$

After some algebra, the bargaining solutions (see Appendix T) for the unskilled wage and employment are given by

$$w_L\colon \ \frac{\chi}{1-\chi} = \frac{L_L \left[(1-\upsilon)\cdot w_L - u\right]}{\delta \cdot (1-\upsilon) \cdot \pi_{Y_L}}; \text{ and} \tag{3.14}$$

$$L_L\colon \ \frac{\chi}{1-\chi} = \frac{w_L L_L - \alpha \cdot p_L \cdot Y_L}{\zeta \cdot \pi_{Y_L}}. \tag{3.15}$$

Considering (3.14) and (3.15) together, we have:

$$\underbrace{-\frac{\zeta \left[(1-\upsilon)\cdot w_L - u\right]}{\delta \cdot (1-\upsilon) \cdot L_L}}_{\text{union's indifference curve}} = \underbrace{\frac{\alpha \cdot p_L \cdot Y_L}{L_L^2} - \frac{w_L}{L_L}}_{\text{firm's iso-profit curve}}, \tag{3.16}$$

which resembles the contract curve in traditional union bargaining theory, representing all combinations of unskilled wage, w_L, and employment, L_L, in which the union's indifference

[9]According to the literature, there are two reasons to justify the union's focus on wages rather than on members' utility. First, Dertouzos and Pencavel (1981) provide empirical results to support that union utility is appropriate to model as a function of real wages. Second, McDonald and Solow (1981) and Oswald (1982) assume that workers tend to spend their total labor income on consumption and, therefore, the union utility function should be defined in the form of wages.

curve and the firm's iso-profit curve are tangential to each other. The contract curve is upward-sloping if the union is employment-oriented, $\zeta > \delta$. However, it is downward-sloping if the union is wage-oriented, $\zeta < \delta$.[10]

Bearing in mind the maximized L-sector real profits (3.11) and equation (3.15), we find that

$$w_L = \left[\frac{\zeta \cdot \chi \cdot (1-\alpha-\beta)}{1-\chi} + \alpha \right] \frac{p_L \cdot Y_L}{L_L}, \tag{3.17}$$

which represents all combinations of unskilled wage, w_L, and employment, L_L, that satisfies (3.16). In the case of $\zeta = 0$, the unskilled-intensive task firms pay the fraction α of its output to unskilled labor, thus replicating the situation without the labor union—see (3.10). In turn, a rise in the unions' bargaining power, χ, leads to an increase in the ratio between unskilled-labor income and unskilled-labor-intensive task firms revenues, $\frac{L_L \cdot w_L}{p_L \cdot Y_L}$. The relationship between χ and $\frac{L_L \cdot w_L}{p_L \cdot Y_L}$ is crucially vital for our analysis, as will be clear later on.

3.2.5. Production and prices: machines

As in Acemoglu (2002a), we assume that skilled and unskilled machine inputs are produced by using composite final good, and one unit of the composite final good produces $\frac{1}{\psi}$ units of machine inputs. Hence, the marginal cost of producing j is ψ; i.e., the effective cost of the machine good i is ψ in sectors L and H. Following Romer (1990), each $i \in [0, A_i]$ embodies a costly design created in the R&D sector, which is recovered if profits at each date are positive for a certain time in the future. This is assured by a patent law, which protects each leader firm's monopoly while, at the same time, spreading acquired technological knowledge to other firms almost without cost. The profit-maximization price of the monopolistic firms is determined by $\max_{p_{x_i^n}} \Pi_{x_i^n} = \left(p_{x_i^n} - \psi\right) \cdot x_i^n$, s.t. $x_i^n = \left(\frac{\beta \cdot p_n}{p_{x_i^n}}\right)^{\frac{1}{1-\beta}} \cdot L_n^{\frac{\alpha}{1-\beta}}$, which yields a price that is a constant over t and across i:

$$p_{x_i^n} = p_{x^n} = \frac{\psi}{\beta}. \tag{3.18}$$

Notice that $p_{x_i^n} = \frac{\psi}{\beta} \Leftrightarrow \psi = \beta \cdot p_{x_i^n} = \beta \cdot p_{x^n}$ and that $\beta < 1$ implies positive profits for each firm, $\Pi_{x_i^n} = (1-\beta) \cdot p_{x_i^n} \cdot x_i^n$, and the same positive markup across firms: $\frac{\Pi_{x_i^n}}{\psi \cdot x_i^n} = \frac{1-\beta}{\beta}$.

Based on the demand function of the type-i machine input in sector n in (3.10) and (3.18), we have that each machine input firm in sector n produces the same output. It faces the same

[10] From (3.16), the slope of the contract curve is $\frac{\partial w_L}{\partial L_L} = \frac{\delta \cdot \alpha}{\zeta - \delta} \frac{p_L \cdot Y_L}{L_L^2} \gtrless 0$ if $\zeta \gtrless \delta$, which means that when $\zeta > \delta$ ($\zeta < \delta$), the union is more (less) concerned about the employment level compared with the wage surplus. In traditional union bargaining theory, the slope of the contractual curve is determined by the risk preference of the union: risk-averse or risk-prone (Chang et al. 2007).

price and cost and, therefore, produces the same quantity, making our problem symmetric:

$$x_i^n = x^n = \left(\frac{\psi}{\beta}\right)^{\frac{-1}{1-\beta}} \cdot (\beta \cdot p_n)^{\frac{1}{1-\beta}} \cdot L_n^{\frac{\alpha}{1-\beta}}. \tag{3.19}$$

The monopolistic profit of each machine input firm is also symmetric: $\Pi_{x_i^n} = \Pi_{x^n} = (1-\beta) \cdot p_{x^n} \cdot x^n = (1-\beta) \cdot \left(\frac{\psi}{\beta}\right)^{\frac{-\beta}{1-\beta}} \cdot (\beta \cdot p_n)^{\frac{1}{1-\beta}} \cdot L_n^{\frac{\alpha}{1-\beta}} > 0$. We can also obtain the total monopolistic profit of all machine input firms in sector n as $\int_0^{A_n} \Pi_{x_i^n} \cdot di = A_n \cdot \Pi_{x^n} = A_n \cdot (p_{x^n} \cdot x^n - \psi \cdot x^n)$, which bearing in mind (3.18), (3.19), and that $\int_0^{A_n} p_{x_i^n} \cdot x_i^n \cdot di = A_n \cdot p_{x^n} \cdot x^n = \beta \cdot p_n \cdot Y_n$ (3.10), we have:

$$\int_0^{A_n} \Pi_{x_i^n} \cdot di = A_n \cdot \Pi_{x^n} = (1-\beta) \cdot \beta \cdot p_n \cdot Y_n, \tag{3.20}$$

i.e., tasks firms in sector n pays the fraction β of the respective revenues, $p_n \cdot Y_n$, to offset the expenses of all machine inputs in the sector. It follows from $\Pi_{x^n} = (1-\beta) \cdot p_{x^n} \cdot x^n$ that total revenues and total production costs for all machine input firms in sector n are $A_n \cdot p_{x^n} \cdot x^n = \beta \cdot p_n \cdot Y_n$ and $A_n \cdot \psi \cdot x^n = A_n \cdot \beta \cdot p_{x^n} \cdot x^n = \beta^2 \cdot p_n \cdot Y_n$, respectively.

3.2.6. Production and prices: R&D

As is usual in the R&D-growth literature (e.g., Barro and Sala-i-Martin 2004, Ch. 6), the production of each machine requires a design that grants a perpetual patent to its owner, which, in our case, is produced through horizontal R&D activities. Thus, a successful innovator retains exclusive rights over the use of his/her machine and, since there is perfect competition among entrants and free-entry in the R&D business, each successful R&D leads to the setup of a new firm in a new industry. Moreover, following Acemoglu (2002a), we have skilled and unskilled R&D activities, and that both types of R&D firms use composite final good to produce new varieties in the form of designs. We consider the following production function in the R&D sector:

$$\dot{A}_n(t) = \frac{1}{\eta_n} \cdot R_n(t), \tag{3.21}$$

where: $\eta_n > 0$ is a constant fixed (flow) cost, which can be interpreted as the amount of composite final good required for the invention of a unit of new designs in sector n; $A_n(t)$ is the number of available designs, $\dot{A}_n$ is the production of new designs, and R_n is the amount of composite final good used by in the n-sector R&D. As standard, the Hamilton-Bellman-Jacobi equation is $r_n(t) \cdot V_n(t) = \Pi_{x^n}(t) + \dot{V}_n(t)$,[11] where $V_n(t) \equiv \int_t^\infty \Pi_{x^n}(v) \cdot e^{-\int_t^v r_n(\omega) d\omega} dv$ is the net present discounted value of the design, and $r_n(t)$ is the interest rate at t; i.e., the

[11] This no-arbitrage condition reveals that the return on the R&D design, $r_n \cdot V_n$, is equal to the monopolistic profit from the machine input firm, Π_{x^n}, plus the capital gain, $\dot{V}_n$.

rate of return on the design. Thus, the R&D free-entry condition is

$$\eta_n = V_n(t), \tag{3.22}$$

where η_n and $V_n(t)$ represent, respectively, the flow cost and the return of investing one unit of final good in R&D sector.

3.2.7. Macroeconomic aggregation

Substituting (3.4) into (3.2) we have

$$\sum_{n=L,H}\left(\dot{V}_nA_n + V_n\dot{A}_n\right) = \sum_{n=L,H}\left[r_n \cdot K_n + w_n \cdot L_n + \Pi_{Y_n}\right] - C,$$

since $\dot{K}_n = \dot{V}_nA_n + V_n\dot{A}_n$. Then, by considering $r_n \cdot V_n = \Pi_{x^n} + \dot{V}_n$, $r_H = r_L = r$, $K_n = V_nA_n$, and $\Pi_{Y_n} = (1-\beta)\cdot p_n \cdot Y_n - w_n \cdot L_n$ in the previous expression, yields

$$\sum_{n=L,H} V_n\dot{A}_n = \sum_{n=L,H}\left[A_n \cdot \Pi_{x^n} + (1-\beta)\cdot p_n \cdot Y_n\right] - C. \tag{3.23}$$

Since the production function of the composite final goods (3.5) is homogeneous of degree one, from $\frac{\partial \Pi_Y}{\partial Y_H} = 0$ and $\frac{\partial \Pi_Y}{\partial Y_L} = 0$ we have $Y = \sum\limits_{n=L,H} p_nY_n$; moreover, considering also (3.20), (3.21), and (3.22) to substitute into (3.23), we can derive the economy's resource constraint:

$$Y = \sum_{n=L,H} A_n \cdot \psi \cdot x^n + \sum_{n=L,H} R_n + C. \tag{3.24}$$

Equation (3.24) indicates that the composite final goods can be used to produce machines, to produce designs in R&D, or to consume.

3.3. Balanced growth path

This section analyses the balanced growth path (BGP) equilibrium of the model such that consumers and firms solve their problems, there is free entry and absence of arbitrage opportunities in R&D, and markets clear. The BGP is characterized by a sequence of allocations $\{C,Y,Y_n,x^n,R_n,L_n,A_n\}_{t=0}^{\infty}$, prices $\{r,r_n,w_n,p_n,p_{x^n},V_n\}_{t=0}^{\infty}$ and policies $\{\upsilon,S_L,u\}_{t=0}^{\infty}$, $n=\{L,H\}$, such that at each time t: the representative household chooses C to maximize utility by knowing $\left\{r_n,w_n,\Pi_{Y_n},u,S\right\}_{t=0}^{\infty}$; the government budget constraint in (3.4) is balanced; competitive tasks firms produce Y by choosing $\{Y_H,Y_L\}$ to maximize profits, knowing prices $\{p_H,p_L\}$; skilled-intensive tasks firms produce Y_H by choosing $\left\{x^H,L_H\right\}$ to maximize profits, knowing $\left\{p_H,p_{x^H},w_H\right\}$; unskilled-intensive tasks firms produce Y_L by choosing $\left\{x^L\right\}$ to maximize profits, considering $\left\{p_L,p_{x^L}\right\}$ as given, and bargain with a managerial labor union to determine $\{w_L,L_L\}$; machine input firms produce x_n by setting

$\{p_{x^n}\}$ to maximize profits; R&D firms choose R_n to maximize profits taking $\{r, V_n\}$ as given; the market for tasks clears; i.e., equation (3.24) occurs; the skilled labor market clears and unskilled labor market is in equilibrium; i.e., $L_L + u = \overline{L}_L$.

We first deal with the determination of the level of unskilled employment, and then solve the equilibrium values of technological-knowledge bias, skill premium, the income-labor share, and economic growth rate.

3.3.1. Unskilled employment

Rearranging (3.17), we have

$$w_L \cdot L_L = \frac{\zeta \cdot \chi \cdot (1-\alpha-\beta) + \alpha (1-\chi)}{1-\chi} \cdot p_L \cdot Y_L. \tag{3.25}$$

In turn, by considering (3.25) in (3.11), we have:

$$\Pi_{Y_L} = (1-\beta) \cdot p_L \cdot Y_L - w_L \cdot L_L = \frac{\left(1-\chi-\zeta \cdot \chi\right)(1-\alpha-\beta)}{1-\chi} \cdot p_L \cdot Y_L. \tag{3.26}$$

Then, considering (3.14), (3.25), (3.26), and $u = s \cdot p_L \cdot Y_L$ (see Section 3.2.3), we can derive the level of unskilled employment:

$$L_L = \frac{(1-\upsilon)}{s} \cdot \left\{ (1-\alpha-\beta) \left[\frac{\zeta \cdot \chi}{1-\chi} - \frac{\delta \cdot \chi \cdot \left(1-\chi-\zeta \cdot \chi\right)}{(1-\chi)^2} \right] + \alpha \right\}. \tag{3.27}$$

From (3.27) the following Proposition should be stated:

Proposition 3.1. *Considering (3.27), an increase in the union's bargaining power, χ, increases the level of unskilled employment if the union is employment-oriented $\left(\zeta > \delta\right)$ and decreases it if, on the contrary, the union is strongly wage-oriented $\left(\zeta < \delta\right)$ and $\left|\zeta - \delta\right| > \frac{2 \cdot \delta \cdot \zeta \cdot \chi}{1-\chi}$.*

Proof. Differentiating (3.27) with respect to χ yields

$$\frac{\partial L_L}{\partial \chi} = \frac{(1-\alpha-\beta) \cdot (\upsilon - 1) \cdot \left[2 \cdot \delta \cdot \zeta \cdot \chi + \left(\zeta - \delta\right)(1-\chi)\right]}{s \cdot (\chi - 1)^3} > 0 \text{ if } \zeta > \delta. \tag{3.28}$$

In particular, we note that $\zeta < \delta$ and $\left|\zeta - \delta\right| > \frac{2 \cdot \delta \cdot \zeta \cdot \chi}{1-\chi} \Rightarrow \frac{\partial L_L}{\partial \chi} < 0$. □

When the union is employment-oriented $\left(\zeta > \delta\right)$ it is is willing to sacrifice wage surplus to increase the employment level and an upward-sloping contract curve is possible; i.e., there is a positive relationship between unskilled wage w_L and the level of unskilled employment

L_L. In face of a rise in the union's bargaining power χ, the union tends to bargain for a higher w_L and hence L_L rises in response. In turn, when the union is sufficiently wage-oriented $(\zeta < \delta)$ and $|\zeta - \delta| > \frac{2 \cdot \delta \cdot \zeta \cdot \chi}{1-\chi}$, the contract curve is downward-sloping; i.e., a negative relationship between w_L and L_L happens: a rise in the union's bargaining power χ leads the union to bargain for a higher w_L, and hence L_L falls in response. When $\zeta < \delta$, the union renounces some employment in favor of a higher wage surplus.

3.3.2. Technological-knowledge bias

Concerning the technological-knowledge bias, we start noting that regarding (3.6), from (3.8) and (3.19), we have

$$\frac{p_H}{p_L} = \left(\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon(1-\beta)}{\varepsilon(1-\beta)+\beta}} \left(\frac{A_H}{A_L}\right)^{-\frac{(1-\beta)}{\varepsilon(1-\beta)+\beta}} \left(\frac{L_H}{L_L}\right)^{-\frac{\alpha}{\varepsilon(1-\beta)+\beta}}. \tag{3.29}$$

In turn, by using (3.22), and, also, bearing in mind that

$$\text{(i) } \Pi_{x^n} = (1-\beta)\left(\frac{\psi}{\beta}\right)^{\frac{-\beta}{1-\beta}} (\beta \cdot p_n)^{\frac{1}{1-\beta}} L_n^{\frac{\alpha}{1-\beta}},$$

from (3.20) (Subsection 3.2.5), (ii) $r \cdot V_n = \Pi_{x^n} + \dot{V}_n$ (from Subsection 3.2.6), and (iii) $\dot{V}_n = 0$ from (3.22), we have:

$$\frac{p_H}{p_L} = \left(\frac{\eta_H}{\eta_L}\right)^{(1-\beta)\frac{\dot{A}_L}{A_L}} \left(\frac{L_H}{L_L}\right)^{-\alpha}. \tag{3.30}$$

Now, substituting (3.30) into (3.29), we can derive the steady-state technological-knowledge bias:[12]

$$\frac{A_H}{A_L} = \left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \left(\frac{L_H}{L_L}\right)^{\alpha(\varepsilon-1)} \left(\frac{\eta_H}{\eta_L}\right)^{-\beta-\varepsilon(1-\beta)}. \tag{3.31}$$

From (3.31) emerges the following Proposition.

Proposition 3.2. *Considering (3.31), an increase in the union's bargaining power, χ, increases the unskilled technological-knowledge bias when the union is employment-oriented $(\zeta > \delta)$, and can increase the skilled technological-knowledge bias when the union is wage-oriented $(\zeta < \delta)$.*

[12] For a detailed proof that $\frac{A_H}{A_L}$ converges to its steady-state value, see Acemoglu and Zilibotti (2001, Appendix 1). The intuition is the following: We start by proving that the number of varieties in each sector grows at the same rate, i.e., $\left(\frac{\dot{A}_H}{A_H}\right) = \left(\frac{\dot{A}_L}{A_L}\right)$. Let's suppose that $\left(\frac{\dot{A}_H}{A_H}\right) < \left(\frac{\dot{A}_L}{A_L}\right)$. This implies $\left(\frac{A_H}{A_L}\right) < \left(\frac{A_H}{A_L}\right)_{BGP}$, which implies that the relative profitability of H-sector is larger than its relative cost. This would drive entrants to increase relative R&D expenditures in the H-sector. This, in turn, would lead to an increase in the growth rate of the number of varieties in the H-sector, i.e., $\left(\frac{\dot{A}_H}{A_H}\right) > \left(\frac{\dot{A}_L}{A_L}\right)$, which would cause $\left(\frac{A_H}{A_L}\right)$ to increase, leading to the opposite effects described above until the no-arbitrage condition of investing in both sectors was met again, i.e., when $\frac{A_H}{A_L} = \left(\frac{A_H}{A_L}\right)_{BGP}$ and $\left(\frac{\dot{A}_H}{A_H}\right) = \left(\frac{\dot{A}_L}{A_L}\right)$. If $\left(\frac{\dot{A}_H}{A_H}\right) > \left(\frac{\dot{A}_L}{A_L}\right)$ the exact opposite dynamics would take place $\frac{A_H}{A_L}$ would decrease monotonically until it reached the BGP again.

Proof. Taking logarithms of (3.31) and differentiating the resulting equation with respect to χ, and, bearing in mind $\varepsilon - 1$ and (3.28), we obtain:

$$\frac{\partial \ln\left(\frac{A_H}{A_L}\right)}{\partial \chi} = -\frac{\alpha\,(\varepsilon-1)}{L_L}\frac{\partial L_L}{\partial \chi} < 0 \text{ if } \zeta > \delta. \tag{3.32}$$

Notice that $\zeta < \delta$ and $\left|\zeta - \delta\right| > \frac{2\cdot\delta\cdot\zeta\cdot\chi}{1-\chi} \Rightarrow \frac{\partial L_L}{\partial \chi} < 0 \Rightarrow \frac{\partial \ln\left(\frac{A_H}{A_L}\right)}{\partial \chi} > 0.$ □

As stated in Proposition 1, when the union is employment-oriented $\left(\zeta > \delta\right)$, there is a positive relationship between the unskilled wage w_L and the level of unskilled employment L_L; hence, a rise in the bargaining power of the union χ leads the union to bargain for a higher w_L, which, in turn, increases L_L. Higher level of L_L implies that the relative labor supply $\frac{L_H}{L_L}$ falls, which, due to the market-size channel emphasized by Acemoglu (2002a), leads the economy to promote unskilled technological-knowledge bias – i.e., a reduction in $\frac{A_H}{A_L}$. In turn, when the union is strongly wage-oriented, $\zeta < \delta$ and $\left|\zeta - \delta\right| > \frac{2\cdot\delta\cdot\zeta\cdot\chi}{1-\chi}$, in response to a rise in the bargaining power of the union χ, the relative labor supply $\frac{L_H}{L_L}$ increases and the economy generates skilled technological-knowledge bias – i.e., a rise in $\frac{A_H}{A_L}$ emerges.[13]

3.3.3. Skill premium

From (3.9), (3.17), (3.18), and (3.20), we reach the relationship among relative wages (see Appendix U), which depends on relative prices. Then, substituting (3.29) into the resulting expression, we have:

$$\frac{w_H}{w_L} = \underbrace{\alpha}_{\substack{\text{Skilled-labor-share}\\ \text{channel}}} \cdot \underbrace{\Omega}_{\substack{\text{Inverse-unskilled-labor-share}\\ \text{channel}}} \times \underbrace{\left(\frac{\eta_H}{\eta_L}\right)}_{\substack{\text{R\&D-technology}\\ \text{channel}}} \cdot \underbrace{\left(\frac{A_H}{A_L}\right)}_{\substack{\text{Technological-knowledge}\\ \text{channel}}} \cdot \underbrace{\left(\frac{L_H}{L_L}\right)^{-1}}_{\substack{\text{Labor-supply}\\ \text{channel}}}, \tag{3.33}$$

where: $\Omega \equiv \frac{1-\chi}{\zeta\cdot\chi\cdot(1-\alpha-\beta)+\alpha(1-\chi)}$; $\alpha = \frac{w_H\cdot L_H}{p_H\cdot Y_H}$ and $\frac{1}{\Omega}=\frac{w_L L_L}{p_L Y_L}$ – see (3.10) and (3.25), respectively. From (3.33), the skill premium is related to five channels. The first channel is the "skilled-labor share", which indicates that a rise in the skilled-labor share $\frac{w_H L_H}{p_H Y_H}$ increases the skill premium, due to the market size channel emphasized by Acemoglu (2002a). The second channel is "inverse of the unskilled-labor share", which indicates that a rise in the unskilled labor share $\frac{w_L L_L}{p_L Y_L}$ decreases the skill premium, also due to the market size

[13] Our analytical results are consistent with the numerical findings of the quality-ladder R&D model proposed by Neto et al. (2019).

channel emphasized by Acemoglu (2002a). The third channel is the "R&D-technology channel", which indicates that a rise in the relative fixed skilled-R&D flow cost, $\frac{\eta_H}{\eta_L}$, increases the skill premium. The fourth channel is the "technological-knowledge channel", which suggests that a rise in the technological-knowledge bias $\frac{A_H}{A_L}$ tends to stimulate the labor relative (skilled over unskilled) demand, thereby causing a rise in the skill premium. The fifth channel is the "labor supply", which indicates that a rise in the relative labor supply between skilled and unskilled labor leads to a decline in the skill premium.

Given the conflict between the last three channels—"R&D-technology", "technological-knowledge", and "labor-supply" in (3.33)—–, we can merge these three channels into two by solving (3.31) for $\left(\frac{L_H}{L_L}\right)^{-1}$ and substituting the result into (3.33), which gives rise to:

$$\frac{w_H}{w_L} = \left(\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\alpha(\varepsilon-1)}} \cdot \underbrace{\alpha}_{\substack{\text{Skilled-labor-share} \\ \text{channel}}} \cdot \underbrace{\Omega}_{\substack{\text{Inverse-unskilled-labor-share} \\ \text{channel}}} \times \underbrace{\left(\frac{\eta_H}{\eta_L}\right)^{\frac{\alpha(\varepsilon-1)-\beta-\varepsilon(1-\beta)}{\alpha(\varepsilon-1)}}}_{\substack{\text{R\&D-technology} \\ \text{channel}}} \cdot \underbrace{\left(\frac{A_H}{A_L}\right)^{\frac{\alpha(\varepsilon-1)-1}{\alpha(\varepsilon-1)}}}_{\substack{\text{Technological-knowledge} \\ \text{channel}}} . \tag{3.34}$$

From equation (3.34), the "technological-knowledge" channel dominates the "labor-supply" channel if $\alpha(\varepsilon-1) > 1$ is true, and under this situation $\frac{w_H}{w_L}$ is positively related to $\frac{A_H}{A_L}$. By contrast, the "technological-knowledge" channel falls short of the "labor-supply" channel if $\alpha(\varepsilon-1) < 1$ is true, and under this situation $\frac{w_H}{w_L}$ depends negatively upon $\frac{A_H}{A_L}$.

From (3.34), the union's bargaining power χ affects relative wages $\frac{w_H}{w_L}$ through the "inverse of the unskilled-labor share" channel, $\frac{1-\chi}{\zeta\cdot\chi\cdot(1-\alpha-\beta)+\alpha(1-\chi)}$, and the net effect between "technological-knowledge" channel and the "labor-supply" channel, $\left(\frac{A_H}{A_L}\right)^{\frac{\alpha(\varepsilon-1)-1}{\alpha(\varepsilon-1)}}$. Taking logarithms of (3.34) and differentiating the resulting equation with respect to χ, we obtain

$$\frac{\partial \ln \frac{w_H}{w_L}}{\partial \chi} = \frac{\zeta\cdot\chi\cdot(1-\alpha-\beta)+\alpha(1-\chi)}{1-\chi} \cdot \frac{\partial \Omega}{\partial \chi} + \frac{\alpha(\varepsilon-1)-1}{\alpha(\varepsilon-1)} \cdot \frac{\partial \ln\left(\frac{A_H}{A_L}\right)}{\partial \chi} \gtrless 0,$$

where: (i) $\frac{\partial \Omega}{\partial \chi} = -\frac{\zeta(1-\alpha-\beta)}{[\zeta\cdot\chi\cdot(1-\alpha-\beta)+\alpha(1-\chi)]^2} < 0$ reflects the effect of a rise in χ on $\frac{w_H}{w_L}$ via the "inverse of the unskilled-labor share" channel; i.e., a rise in the union's bargaining power χ increases the unskilled workers income $\frac{w_L L_L}{p_L Y_L}$, thereby resulting in a reduction in Ω, which, in turn, reduces $\frac{w_H}{w_L}$; (ii) from (3.32),

$$\frac{\partial \ln\left(\frac{A_H}{A_L}\right)}{\partial \chi} < (>) 0 \text{ if } \zeta > \delta \left(\zeta < \delta \text{ and } \left|\zeta - \delta\right| > \frac{2 \cdot \delta \cdot \zeta \cdot \chi}{1-\chi}\right),$$

showing that the effect of a rise in χ on the net effect between "technological-knowledge" channel and the "labor-supply" channel includes two components. The first component is the relationship between $\frac{A_H}{A_L}$ and χ. As stated in Proposition 3.2, when the union is employment-oriented $(\zeta > \delta)$, a rise in the union's bargaining power χ leads to a rise in w_L coupled with a rise in L_L – hence decreases $\frac{L_H}{L_L}$ –, which, in turn, decreases $\frac{A_H}{A_L}$. When the union is strongly wage-oriented, i.e.,

$$\zeta < \delta \text{ and } \left|\zeta - \delta\right| > \frac{2 \cdot \delta \cdot \zeta \cdot \chi}{1-\chi}, \tag{3.35}$$

a rise in χ leads to a rise in w_L accompanied by a reduction in L_L – hence a rise in $\frac{L_H}{L_L}$ –, which, in turn, rises $\frac{A_H}{A_L}$. The second component is the relationship between $\frac{w_H}{w_L}$ and $\frac{A_H}{A_L}$, which is positive when the "technological-knowledge" channel is relatively stronger (i.e., $\alpha(\varepsilon - 1) > 1$) and negative when the "technological-knowledge" channel is relatively weaker (i.e., $\alpha(\varepsilon - 1) < 1$). To summarize, the results are presented in Table 3.1.

TABLE 3.1: Effects of an increase in the union's bargaining power, χ, on the skill premium, $\frac{w_H}{w_L}$.

Effect direction		Conditions	Case
$\frac{\partial \ln \frac{w_H}{w_L}}{\partial \chi} < 0$	if	$\alpha(\varepsilon - 1) > 1$ and $\zeta > \delta$	1
		$\alpha(\varepsilon - 1) < 1$ and $\zeta < \delta$ and $\left|\zeta - \delta\right| > \frac{2\cdot\delta\cdot\zeta\cdot\chi}{1-\chi}$	2
$\frac{\partial \ln \frac{w_H}{w_L}}{\partial \chi} \gtrless 0$	if	$\alpha(\varepsilon - 1) < 1$ and $\zeta > \delta$	3
		$\alpha(\varepsilon - 1) > 1$ and $\zeta < \delta$ and $\left|\zeta - \delta\right| < \frac{2\cdot\delta\cdot\zeta\cdot\chi}{1-\chi}$	4
		otherwise	5

Hence, the effect of an increase in the union's bargaining power, χ, on the skill premium depends crucially on whether: (i) the union is employment-oriented or wage-oriented; and (ii) the "technological-knowledge" channel is relatively stronger or weaker than the "labor-supply" channel. Under both situations where $\zeta > \delta$ and $\alpha(\varepsilon - 1) > 1$ as well as where $\zeta < \delta$, $\left|\zeta - \delta\right| > \frac{2\cdot\delta\cdot\zeta\cdot\chi}{1-\chi}$ and $\alpha(\varepsilon - 1) < 1$, a rise in χ tends to lower $\frac{w_H}{w_L}$. In addition, a rise in χ decreases $\frac{w_H}{w_L}$ via a reduction in Ω. Accordingly, both negative effects reinforce each other, and hence a rise in χ gives rise to a negative impact on $\frac{w_H}{w_L}$. For the other two opposite situations, a rise in χ tends to stimulate $\frac{w_H}{w_L}$. Coupled with a rise in χ, which decreases $\frac{w_H}{w_L}$ via a reduction in Ω, both effects offset each other. Thus, a rise in χ has an ambiguous effect on $\frac{w_H}{w_L}$. For the remaining cases 3.1, the effect through the technological knowledge channel is also ambiguous. This discussion can be summarized in the following Proposition.

Proposition 3.3. *The effect of an increase in the union's bargaining power, χ, on the skill premium is negative (i) when the "technological-knowledge" channel dominates the "labor-supply" channel and the union is employment oriented or (ii) when "labor supply" dominates the "technological-knowledge" and the union is strongly wage-oriented.*

Remark. These conditions relate to Table 3.1, Cases 1 and 2, respectively.

This means that the direction of the effect (of a rise in χ) on the skill premium crucially depends on whether the union is employment-oriented or wage-oriented and whether the "technological-knowledge" channel is relatively stronger or weaker than the "labor-supply" channel.These results align with existing studies on the relationship between the union's bargaining power and income inequality. Neto et al. (2019) also indicate that the union's bargaining power can reduce the skill premium regardless of whether the union is employment-oriented or wage-oriented. Chang and Hung (2016) assume that the union is wage-oriented and point out that the union's bargaining power has an ambiguous effect on income inequality due to the endogeneity of labor's working hours. Chu et al. (2016) propose a two-country R&D-based growth model and find that an increase in the home country's employment-oriented union's bargaining power has a positive effect on the two countries' wage gap, while an increase in the home country's wage-oriented union's bargaining power has an ambiguous effect on the two countries' wage gap.

Here, counterintuitively, the increase of an employment-oriented union bargaining power depresses the skill premium when technological knowledge is relatively more potent than labor supply (Table 3.1, Case 1). Thus, the model reveals that the effect of the union's bargaining power on the skill premium depends not only on whether the union is employment-oriented or wage-oriented, but also on whether the "technological-knowledge" channel is relatively stronger or weaker than the "labor-supply" channel. Our proposal can be considered a contribution to the DTC literature (e.g., Acemoglu 1998, 2002a), as changes in the skill premium are also related to the technological-knowledge bias, although this relationship depends on whether the union is employment-oriented or wage-oriented.

In this sense, our analysis complements that of Neto et al. (2019), which examines how the union's preference for employment or wage affects the direction of the technological bias and the bargaining power effect on the skill premium. Their model production function is the special case in which $\varepsilon \to \infty$, i.e., the linear case of the CES. These distinctions imply that the technological bias influences the skill premium differently. In Proposition 3.3, the possibility of the union's bargaining power reducing the skill premium is linked to the relative strength of the "technological-knowledge" channel on the skill premium, independently of the bias direction. It is also worth noting that our theoretical results, which examine the relationship between union bargaining power and the skill premium, do not contradict the ambiguity found in existing empirical studies. For example, while Koeniger et al. (2007) find a negative relationship, Checchi and García-Peñalosa (2008) find a positive relationship. In the next Section, we empirically confirm that such ambiguous effects are possible.

3.3.4. Income-labor share

As previously shown, from (3.9) and (3.25) the skilled-labor income share and the unskilled-labor income share are, respectively,

$$\frac{w_H L_H}{Y} = \alpha \frac{p_H Y_H}{Y} \text{ and } \frac{w_L L_L}{Y} = \frac{1}{\Omega}\frac{p_L Y_L}{Y}. \tag{3.36}$$

Let $S_L \equiv \frac{w_H L_H + w_L L_L}{Y}$ denote the income-labor share. From (3.6), (3.30), and (3.36), we have that (see Appendix V)

$$S_L = \frac{\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + \frac{1}{\Omega}}{\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1}, \tag{3.37}$$

which differentiated with respect to χ yields

$$\begin{aligned}\frac{\partial S_L}{\partial \chi} &= \left(\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1\right)^{-2} \\ &\quad \times \left[\left(\alpha - \frac{1}{\Omega}\right) \cdot \frac{\eta_H}{\eta_L} \cdot \frac{\partial \frac{A_H}{A_L}}{\partial \chi} - \frac{1}{\Omega^2} \cdot \left(\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1\right) \cdot \frac{\partial \Omega}{\partial \chi}\right] \gtrless 0,\end{aligned} \tag{3.38}$$

where $\alpha < 1$ and $\Omega < 1 \Rightarrow \alpha - \frac{1}{\Omega} < 1$, $\frac{\partial \Omega}{\partial \chi} = \frac{-\zeta(1-\alpha-\beta)}{[\zeta \cdot \chi \cdot (1-\alpha-\beta) + \alpha(1-\chi)]^2} < 0$ (Subsection 3.3.3), and $\frac{\partial \ln\left(\frac{A_H}{A_L}\right)}{\partial \chi} < (>) 0$ if $\zeta > \delta$ ($\zeta < \delta$ and $\left|\zeta - \delta\right| > \frac{2 \cdot \delta \cdot \zeta \cdot \chi}{1-\chi}$) (3.32). The union's bargaining power χ affects the income-labor share S_L through the unskilled-labor-share channel and relative technologies $\frac{A_H}{A_L}$. A rise in the union's bargaining power χ leads unskilled workers to receive a higher value of $\frac{w_L L_L}{p_L Y_L} = \frac{1}{\Omega}$, thereby decreases Ω and increases S_L. Moreover, as stated in Proposition 3.2, when (i) the labor union is employment-oriented $(\zeta > \delta)$, a rise in the union's bargaining power χ decreases $\frac{A_H}{A_L}$, which, in turn, increases S_L, and when (ii) the union is strongly wage-oriented (3.35), a rise in χ increases $\frac{A_H}{A_L}$, which, in turn, decreases S_L. Hence, both positive effects on S_L reinforce each other when the labor union is employment-oriented $\left(\zeta > \delta\right)$ and thus a rise in χ increases S_L, whereas when the union is strongly wage-oriented (3.35) both a positive and a negative effects emerge wherefore we have an ambiguous effect of an increasing χ on S_L; i.e., $\zeta > \delta \Rightarrow \frac{\partial S_L}{\partial \chi} > 0$. Hence, these results lead us to establish the following proposition:

Proposition 3.4. *An increase in the union's bargaining power, χ, stimulates the income-labor share under an employment-oriented union, while it may either stimulate or depress the income-labor share under a strongly wage-oriented union.*

Proof. From (3.37), $\frac{\partial S_L}{\partial \chi} = \frac{\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{\partial \frac{A_H}{A_L}}{\partial \chi} - \frac{1}{\Omega^2} \cdot \frac{\partial \Omega}{\partial \chi}}{\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1} - \frac{\left(\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + \frac{1}{\Omega}\right) \cdot \frac{\eta_H}{\eta_L} \cdot \frac{\partial \frac{A_H}{A_L}}{\partial \chi}}{\left(\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1\right)^2}$, which leads to (3.38) (see Appendix V). □

Remark. Case the labor union is employment-oriented $\left(\zeta > \delta\right)$, an increase on χ generates an increase on both the skilled-labor income share and the unskilled-labor income share. On the former, $\frac{w_H L_H}{Y} = \alpha \cdot \gamma^{\varepsilon} \cdot p_H^{1-\varepsilon}$, the effect is due to the price channel only. On the latter, $\frac{w_L L_L}{Y} = \frac{1}{\Omega} \cdot (1-\gamma)^{\varepsilon} \cdot p_L^{1-\varepsilon}$, the positive effects on unskilled wage and employment level dominate the negative effect on the price.

Chang and Hung (2016) conclude that the union's bargaining power rises the income-labor share when the union is wage-oriented and we show that, in this case, the union's bargaining power may either stimulate or depress the income-labor share. Moreover, in the empirical studies performed by Jayadev (2007) and Checchi and García-Peñalosa (2010), the union's bargaining power is positively correlated with the income-labor share, as can happen here.

3.3.5. Economic growth

Now, we analyze the effect of the union's bargaining power on the balanced economic growth rate. To find this rate, we start by noting that from $\Pi_{x^n} = (1-\beta) \cdot p_{x^n} \cdot x^n = (1-\beta) \cdot \left(\frac{\psi}{\beta}\right)^{\frac{-\beta}{1-\beta}} \cdot (\beta \cdot p_n)^{\frac{1}{1-\beta}} \cdot L_n^{\frac{\alpha}{1-\beta}} > 0$ (Subsection 3.2.5), $\eta_n = V_n(t)$ (3.22) and $\frac{\dot{V}_n}{V_n} = 0$ in the steady state; from the Hamilton-Bellman-Jacobi equation (Subsection 3.2.6), the steady-state equilibrium real interest rate is

$$r = \frac{1-\beta}{\beta \cdot \eta_n \cdot \psi^{\frac{\beta}{1-\beta}}} \cdot L_n^{\frac{\alpha}{1-\beta}} \cdot p_n^{\frac{1}{1-\beta}}. \tag{3.39}$$

In turn, from (3.6), (3.7), and (3.30), we find p_L as a function of the technological-knowledge bias (see Appendix V). Substituting (3.31) in such a expression, we have

$$p_L = \left(\frac{1}{1-\gamma}\right)^{\frac{\varepsilon}{1-\varepsilon}} \cdot \left[\left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \cdot \left(\frac{\eta_H}{\eta_L}\right)^{(1-\varepsilon)(1-\beta)} \cdot \left(\frac{L_H}{L_L}\right)^{\alpha(\varepsilon-1)} + 1\right]^{\frac{1}{\varepsilon-1}}. \tag{3.40}$$

Then, substituting (3.40) into (3.39) yields

$$r = \frac{1-\beta}{\beta \cdot \eta_L \cdot \psi^{\frac{\beta}{1-\beta}}} \cdot \left(\frac{1}{1-\gamma}\right)^{\frac{\varepsilon}{(1-\beta)(1-\varepsilon)}} \times \left[\left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \cdot \left(\frac{\eta_H}{\eta_L}\right)^{(1-\beta)(1-\varepsilon)} \cdot (L_H)^{\alpha(\varepsilon-1)} + (L_L)^{\alpha(\varepsilon-1)}\right]^{\frac{1}{(1-\beta)(\varepsilon-1)}}. \quad (3.41)$$

Bearing in mind (3.3) and (3.39), the next proposition summarizes the dynamics of the economy both outside and in the BGP.

Claim. *There is a unique and stable BGP along which all the variables including output, consumption, number of machines, and wages in each sector grow at the following (steady-state) growth rate:*

$$g = \frac{(1-\upsilon)\cdot(1-\beta)}{\theta \cdot \beta \cdot \eta_L \cdot \psi^{\frac{\beta}{1-\beta}}} \cdot \left(\frac{1}{1-\gamma}\right)^{\frac{\varepsilon}{(1-\beta)(1-\varepsilon)}} \times \left[\left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \cdot \left(\frac{\eta_H}{\eta_L}\right)^{(1-\beta)(1-\varepsilon)} \cdot (L_H)^{\alpha(\varepsilon-1)} + (L_L)^{\alpha(\varepsilon-1)}\right]^{\frac{1}{(1-\beta)(\varepsilon-1)}} - \frac{\rho}{\theta}. \quad (3.42)$$

Proof. Bearing in mind the two conditions for free entry in R&D, $V_H = \eta_H$ and $V_L = \eta_L$, we have $\frac{\dot{V}_L}{V_L} = \frac{\dot{V}_H}{V_H} = 0$. Considering (3.27), a constant real interest rate (3.41) is only possible if $\frac{\dot{L}_H}{L_H} = \frac{\dot{L}_L}{L_L} = 0$, which implies that the technological-knowledge bias (3.31) is constant, i.e., $\frac{\dot{A}_H}{A_H} = \frac{\dot{A}_L}{A_L}$. From (3.7) and (3.30), we can infer $\frac{\dot{p}_L}{p_L} = \frac{\dot{p}_H}{p_H} = 0$. From $A_n \cdot \beta \cdot p_{x^n} \cdot x^n = \beta^2 \cdot p_n \cdot Y_n$ (Subsection 3.2.5) and equations (3.18) and (3.19), $\frac{\dot{A}_n}{A_n} = \frac{\dot{Y}_n}{Y_n}$. From (3.9), $\frac{\dot{w}_H}{w_H} = \frac{\dot{Y}_H}{Y_H}$; from (3.15), $\frac{\dot{w}_L}{w_L} = \frac{\dot{Y}_L}{Y_L}$. Then, we can derive:

$$\frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y}_L}{Y_L} = \frac{\dot{A}_H}{A_H} = \frac{\dot{A}_L}{A_L} = \frac{\dot{w}_H}{w_H} = \frac{\dot{w}_L}{w_L}. \quad (3.43)$$

Finally, considering the profit maximization of the final good firm, $p_H = \gamma Y_H^{\frac{-1}{\varepsilon}} Y^{\frac{1}{\varepsilon}}$ and $p_L = (1-\gamma) Y_L^{\frac{-1}{\varepsilon}} Y^{\frac{1}{\varepsilon}}$ (Appendix S), and equations (3.3), (3.24), and (3.43), we have:

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y}_L}{Y_L} = \frac{\dot{A}_H}{A_H} = \frac{\dot{A}_L}{A_L} = \frac{\dot{w}_H}{w_H} = \frac{\dot{w}_L}{w_L} = \frac{\dot{C}}{C},$$

which indicates that, along the balanced growth path, all of Y, Y_H,Y_L, A_H, A_L, w_H, w_L, and C grow at the common rate g. □

From Equation (3.42), the following Proposition can be stated concerning the impact of the union's bargaining power .

Proposition 3.5. *The union's bargaining power leads to a higher balanced economic growth rate under an employment-oriented union, while it leads to a lower balanced economic growth rate under a strongly wage-oriented union.*

Proof. Considering (3.28) and differentiating (3.42) with respect to χ, we can obtain the growth effect of χ:

$$\frac{\partial g}{\partial \chi} = \left[\left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \cdot \left(\frac{\eta_H}{\eta_L}\right)^{(1-\beta)(1-\varepsilon)} \cdot (L_H)^{\alpha(\varepsilon-1)} + (L_L)^{\alpha(\varepsilon-1)}\right]^{\frac{1-(1-\beta)(\varepsilon-1)}{(1-\beta)(\varepsilon-1)}} \times$$
$$\frac{\alpha \cdot (1-\upsilon)}{\theta \cdot \beta \cdot \eta_L \cdot \psi^{\frac{\beta}{1-\beta}}} \cdot \left(\frac{1}{1-\gamma}\right)^{\frac{\varepsilon}{(1-\beta)(1-\varepsilon)}} \cdot \frac{\partial L_L}{\partial \chi}.$$

□

Remark. Notice from (3.39) and (3.41) that, when the union is employment-oriented (strongly wage-oriented), i.e., $\zeta > \delta$ $\left(\zeta < \delta \text{ and } \left|\zeta - \delta\right| > \frac{2\cdot\delta\cdot\zeta\cdot\chi}{1-\chi}\right)$, the labor supply channel dominates (is dominated by) the price channel for the χ growth effects.

As already stated, under a strongly wage-oriented union, an increase in the union's bargaining power, χ, induces a higher unskilled wage rate, w_L, and a lower level of unskilled employment, L_L. A reduction in unskilled employment, L_L, and a constant level of skilled employment, L_H, provokes a fall in the balanced growth rate. In turn, under an employment-oriented union, an increase in the union's bargaining power, χ, leads to a higher unskilled wage rate, w_L, and a high level of unskilled employment, L_L, which results in a higher balanced growth rate.

Thus, the impact of an increase in the union's bargaining power, χ, on economic growth, g, depends on the orientation of the labor union, as also occurs in Chang et al. (2007) and Chu et al. (2016). This theoretical result of ours is compatible with the ambiguity found by the empirical literature; in effect, Nickell and Layard (1999) find evidence of an inverse effect of the union's bargaining power on economic growth for a panel of OECD countries, while Asteriou and Monastiriotis (2004) find that the union's bargaining power leads to a positive growth effect in 18 OECD countries.

Propositions 3.3 and 3.5 indicate that economies where unions focus on the employment level rather than being wage-oriented may achieve a more significant economic growth rate and lower wage inequality levels. However, policymakers willing to adjust trade union's bargaining power should sensibly consider each country's idiosyncrasies.

3.4. Exploring the data

We combine publicly available data from different sources to illustrate the theoretical findings presented in Section 3.3. While considering the relative labor supply and technological knowledge bias, we focus on the direction of the union's bargaining power effects on the skill premium. Such analysis begins by constructing measures to represent the primary variables involved: the relative supply of labor, the skill premium, and the relative technological knowledge bias. Then, the data is combined in an unbalanced panel, allowing us to find countries simultaneously satisfying the conditions of Proposition 3.3 and pointing in the expected direction in the other propositions. The results for these other propositions are promising. Although not statistically significant, they help to illustrate the concreteness of the predicted effects of the union's bargaining power on the analyzed variables.

3.4.1. Main variables

Skill premium and relative labor supply. Using data from the International Labour Organization (ILO 2024) on average monthly earnings of employees and employment levels, yearly values for the countries' skill premiums can be computed. Skilled labor was considered the labor type supplied by workers with tertiary education or an advanced degree, more precisely, those classified by the ILO as having an "advanced" level of education, or, equivalently, in ISCED-11 levels 5–8 (UNESCO 2012). The remaining labor force was aggregated as unskilled labor. The skill premium was computed using the "mean weekly hours actually worked per employee" and "hourly earnings" for countries with no available monthly data. The inverse of relative labor type supply, (L_L/L_H), as it enters equation (3.33), was computed directly from the working population levels.[14]

Relative technological knowledge level and trade union bargaining power. The empirical literature on wage differentials is vast. Still, more work needs to be done addressing the technological bias between different skill levels. Related to our problem, Bound and Johnson (1992) estimated intra-industry rates of technical change directly from microdata for the U.S. from 1973 to 1988, without attempting to compute the skilled-to-unskilled technological bias. Using wage and employment data for the U.S., Acemoglu (2002a) estimated such bias of technological knowledge in a straightforward manner. Unfortunately, the procedure used in the latter work is tied to assumptions regarding the

[14]ILO (2024) datasets utilized: average hourly earnings of employees by sex and education - annual (EAR_4HRL_SEX_EDU_CUR_NB_A); average monthly earnings of employees by sex and education - annual (EAR_4MTH_SEX_EDU_CUR_NB_A); mean weekly hours actually worked per employee by sex and education - annual (HOW_XEES_SEX_EDU_NB_A); employment by sex, age and education (thousands) (EMP_TEMP_SEX_AGE_EDU_NB_A); and, working-age population by sex, age and education (thousands) - annual (POP_XWAP_SEX_AGE_EDU_NB_A).

elasticity of substitution between sectors, and it does not account for the aggregate variation in the economy's total factor productivity (TFP). As Allen (2001) pointed out, the "purest" conceptual measure of technical change is TFP, "but it turns out to be completely unrelated to wage patterns empirically." Thus, we attempt to compute an index for the technological knowledge bias evolution in different countries directly from macroeconomic aggregate observables by decomposing the product of labor $\left[\Lambda(L_H+L_L)\right]^{\alpha_H+\alpha_L}$ as $(\Lambda_H L_H)^{\alpha_H}+(\Lambda_L L_L)^{\alpha_L}$, where Λ is TFP; α_H and α_L are the output share for the respective labor type; Λ_H and Λ_L are the specific factor productivity of each labor type (computed from such decomposition and not from our theoretical model with which it is related but not the same). To estimate Λ_H and Λ_L, we consider $\frac{w_H}{w_L}=\frac{\alpha_H}{\alpha_L}\frac{\Lambda_H}{\Lambda_L}\frac{L_L}{L_H}$ as an auxiliary equation to solve the linear system obtained by taking the logs of both equations. Notice that the latter resembles equation (3.33) without R&D costs, which do not change over time, i.e., $\frac{\eta_H}{\eta_L}$ is constant in our model. Then, taking values for the aggregate labor share and total factor productivity from the Penn World Table 10.01 (Feenstra et al. 2015) and the labor share and employment level from the ILO (2024) data already mentioned, our technological knowledge bias index can be defined as $\mathrm{TK}\equiv\frac{\Lambda_H}{\Lambda_L}$ and computed for the countries.

The bargaining power of trade unions is not directly observable. However, one commonly considered measure is the "union membership rate among active workers" (Koeniger et al. 2007), also known as union density. Here, the trend in union density, UD, obtained from the OECD/AIAS ICTWSS database version 1.1 (OECD 2023), is considered a proxy for union bargaining power. Table 3.2 brings the number of countries and periods with data available for the (inverse of) relative label supply, skill premium, technological knowledge bias, and union density trend.

TABLE 3.2: Basic description of computed variables

	Countries	Period	Mean	Min.	Max.
Inverse of rel. labor supply	154	1987–2023	10.063	0.425	542.050
Skill premium	123	1991–2023	1.926	0.339	8.232
Technological knowledge bias	89	1991–2019	0.738	0.343	3.917
Union density trend	55	1960–2021	0.345	0.045	0.939

3.4.2. Union's bargaining power effects

As discussed in Section 3.3, an increase in the bargaining power of trade unions may affect the skill premium through the "technological-knowledge" bias and the "labor supply" channels. When the former dominates the latter, the skill premium is positively related to technological knowledge bias, which, in turn, is negatively related to the bargaining power if trade unions are oriented toward employment. Conversely, if "labor supply" dominates the "technological knowledge" and unions are wage-oriented, the prediction is that the technological knowledge bias has a negative relationship with the skill premium and is positively related to the union's bargaining power. The theoretically expected directions of the union's bargaining power effects on the skill premium and the other analyzed variables guide the identification of the countries that better fit the cases analyzed in Proposition 3.3 (the resulting country grouping is listed in Appendix W, Table W.1). Running separate pooled ordinary least squares (pooled OLS) regressions in the subsets of our panel defined by the countries the cases identified in Table 3.1, it is possible to confirm the adverse effect of UD on the skill premium, as predicted in Proposition 3.3. The base specification tested is derived from (3.33), assuming $\frac{\eta_H}{\eta_L} = 1$ and including UD in the model. The results for such regressions are presented in Table 3.3.

TABLE 3.3: Pooled OLS regressions for Table 3.1's Cases 1 and 2

	Dependent variable:			
	$\log(w_H/w_L)$			
	Case 1		Case 2	
	(1)	(2)	(3)	(4)
log(UD)	0.138	-0.287^{**}	-0.152	-0.146^{*}
	(0.379)	(0.080)	(0.092)	(0.056)
log(TK)	0.822	-0.094	0.060	0.012
	(0.924)	(0.526)	(0.255)	(0.131)
$\log(\alpha)$	-0.022		-0.046	
	(0.523)		(0.167)	
$\log(\Omega)$	-0.170		-0.192	
	(0.831)		(0.246)	
$\log(L_L/L_H)$		0.407^{**}		0.031
		(0.108)		(0.061)
Constant	0.980	-0.527	0.054	0.288^{*}
	(2.179)	(0.439)	(0.487)	(0.091)
Countries	9	9	7	7
Median(obs)	14	14	13	13
Max(obs)	18	18	16	16
Observations	121	121	93	93
R^2	0.216	0.497	0.710	0.683

Notes: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Cluster-robust variance estimators (by country, CV_3 with Satterthwaite d.f.).

Data for the different countries are pooled together to estimate the coefficients. However, the observations are clustered to compute the standard errors, which means that the inference unit becomes the country, rather the observations themselves (e.g., MacKinnon et al. 2023). Conditional on the country groups for cases 1 and 2, significant results are observed for the negative effect of the unions' bargaining power on the skill premium. Despite the close cluster sizes observed, due to the limited amount of information available, it remains challenging to determine whether the asymptotic inference is reliable. Table 3.4 compares the inference for the hypothesis of a zero regressor for the union density coefficient in the regressions presented in Table 3.3, computed using cluster-robust variance estimators (CRVEs) and wild bootstrap simulation methods.[15] Notice, Table 3.3, that none of the regressors are significant for the specification that excludes the inverse of relative labor supply used for regressions (1) and (3). A specification that considered the inverse of relative labor supply and the coefficients for the labor share was tested and omitted due to multicollinearity issues, as corroborated by the computation (see Appendix W, Table W.2) of variance inflation factors (VIF; Fox and Monette 1992).

Thus, the regression specification that better captures the effects of the union's bargaining power on the skill premium is the one used in Table 3.3, regressions (2) and (4), given by

$$\log({}^{w_H}/_{w_L}) = \lambda_0 + \lambda_1 \log(\mathrm{UD}) + \lambda_2 \log(\mathrm{TK}) + \lambda_3 \log({}^{L_L}/_{L_H}) + \mu. \tag{3.44}$$

Table W.3, in Appendix W, presents the results of OLS regressions based on (3.44) for the remaining cases in Table 3.1. Because the mechanism described in Proposition 3.3 is indirect and mediated by the technological knowledge bias, we consider a two-stage least squares (2SLS) specification, yielding results that are not statistically significant but go in the directions stated in Propositions 3.2 and 3.3 (see Appendix W, Tables W.4, W.5, and W.6). In addition, for the groups of countries in the cases considered in Proposition 3.3, OLS regressions with UD as the single regressor also point to effects in the directions expected in Propositions 3.1, 3.4, and 3.5 (see Appendix W, Table W.7). Although not all are statistically significant, these other results are promising. Moreover, despite the limited data available, the overall empirical results support the theoretical predictions in Section 3.3, particularly the mechanism by which the union's bargaining power affects wage differentiation, as determined by the union's orientation and technological knowledge

[15] CV_1 and CV_3 (Bell and McCaffrey 2002) are the cluster generalized versions of the popular HC_1 (Hinkley 1977) and HC_3 (MacKinnon and White 1985) robust estimators, of which CV_3 is the more "conservative" (e.g., MacKinnon et al. 2023), i.e., less frequently rejects the null hypothesis of a zero regressor. The degrees-of-freedom (d.f.) correction is based on a standard Satterthwaite-type approximation, which yields different d.f. for each coefficient in the model. The simulation method "wild cluster restricted" (WCR) account for heteroskedasticity of unknown form and employ a frequentist approach to compute the confidence intervals and p-values, without relying on the determination of asymptotic standard errors (MacKinnon et al. 2023).

TABLE 3.4: CIs and p-values for Table 3.3's regressions

	Estimate	CI Lower	CI Upper	p-value
Reg. (1): CV_1, Satt $t(0.646, 5.51)$	0.138	-0.396	0.673	0.544
Reg. (1): WCR bs., Rademacher	0.138	-0.498	1.110	0.898
Reg. (1): WCR bootstrap, Webb	0.138	-0.483	1.070	0.880
Reg. (1): CV_3, Satt $t(0.365, 4.76)$	0.138	-0.850	1.127	0.731
Reg. (2): CV_1, Satt $t(-5.38, 4.58)$	-0.287	-0.428	-0.146	0.004
Reg. (2): WCR bs., Rademacher	-0.287	-0.376	-0.119	0.012
Reg. (2): WCR bootstrap, Webb	-0.287	-0.376	-0.120	0.014
Reg. (2): CV_3, Satt $t(-3.61, 3.43)$	-0.287	-0.523	-0.051	0.029
Reg. (3): CV_1, Satt $t(-6.21, 3.58)$	-0.152	-0.223	-0.080	0.005
Reg. (3): WCR bs., Rademacher	-0.152	-0.274	-0.005	0.047
Reg. (3): WCR bootstrap, Webb	-0.152	-0.274	-0.023	0.040
Reg. (3): CV_3, Satt $t(-1.65, 2.42)$	-0.152	-0.486	0.183	0.218
Reg. (4): CV_1, Satt $t(-3.68, 4.78)$	-0.146	-0.250	-0.043	0.016
Reg. (4): WCR bs., Rademacher	-0.146	-0.303	0.054	0.094
Reg. (4): WCR bootstrap, Webb	-0.146	-0.317	0.047	0.078
Reg. (4): CV_3, Satt $t(-2.62, 2.60)$	-0.146	-0.340	0.048	0.092

Note: Clustered by country, p-values for coefficient of $\log(\mathrm{UD}) = 0$ hypothesis.

bias.

3.5. Concluding remarks

This paper proposes an endogenous growth model with directed technical change, incorporating labor unions, to investigate the impact of union bargaining power on economic growth, wage inequality, and the labor income share. The theoretical model revealed that the effects of increased union bargaining power heavily depend on whether the union is oriented towards employment or wages, as well as the relative strength of the technological knowledge channel versus the labor supply channel in determining the skill premium. This approach sheds light on the ambiguities observed in empirical literature regarding the relationship between union power and wage inequality, suggesting that different union orientations and technological contexts can lead to contrasting outcomes.

The theoretical results suggest that increasing union bargaining power can stimulate growth in unskilled employment and raise the labor income share when the union is employment-oriented. However, if the union is wage-oriented, increased bargaining power tends to favor skilled workers, thereby widening the skill premium and promoting a technological bias towards innovations that benefit skilled workers. This, in turn, can reduce the labor income share and lower the long-term economic growth rate. Thus, the effects of unions on economic growth and wage inequality are highly contextual, shaped by the complex interplay between union preferences and the economy's structural characteristics.

Moreover, the empirical tests conducted provide support for the theoretical predictions regarding the effects of union bargaining power, particularly on the skill premium or, more broadly, wage inequality. In countries where unions predominantly focus on employment, increased union bargaining power tends to reduce wage inequality. Conversely, in cases where unions prioritize wage increases, such a negative effect of union power on wage inequality is only observed when the labor supply channel is relatively stronger than the technological bias channel. These findings help explain why prior studies, such as Koeniger et al. (2007) and Checchi and García-Peñalosa (2008), have identified ambiguous outcomes regarding the effects of unions on wage inequality across different countries.

This study also makes important contributions to the literature on endogenous growth and directed technical change. By incorporating labor unions into a model of technological change, we offer a more comprehensive view of the mechanisms that govern income distribution between skilled and unskilled workers, as well as how union bargaining power influences the direction of technological progress. By linking these elements to long-term economic growth, the study advances the understanding of how labor market institutions and technological innovations interact to shape development trajectories.

Regarding policy implications, the results suggest that policies encouraging employment-oriented union negotiations, rather than wage-focused negotiations, may be more effective in promoting sustainable economic growth and reducing wage inequality. However, the effectiveness of such policies depends on the specific context of each country, including labor market structures and the strength of the technological knowledge orientation of the economy. Therefore, policymakers must carefully consider the underlying conditions before implementing labor reforms aimed at adjusting union bargaining power. Countries in which trade unions are strongly oriented towards wages face more demanding conditions to succeed in such policies and, perhaps, could benefit more from policies pointed to determinants of the skill premium and growth not analyzed in detail in the present work, as the relative costs of R&D technologies that complement different types of labor.

Other policies outside the scope of the current work may also be relevant. For example, fiscal interventions, such as progressive taxation, could counteract the inequality-enhancing effects of wage-oriented unions. Additionally, investment in skill formation and technology adoption policies could balance the negative consequences of wage-oriented unions on unskilled employment.

Ultimately, the limitations of this study highlight promising areas for future research. The lack of detailed data on union orientation and the challenge of quantifying the technological knowledge bias torward skilled or unskilled labor present obstacles to directly applying the theoretical results without further empirical work. Additionally, it would be valuable to explore how other institutional factors, such as the tax system and innovation policies, interact with union power to influence long-term economic outcomes.

Data availability and software usage

All the datasets analyzed in this paper are publicly available. They were specified and cited in Section 3.4 and included among the entries in the References section at the end of the main text. All analyses were performed using R Statistical Software (v4.4.1; R Core Team 2024); ILO (2024) data sets were obtained using Rilostat (v2.2.0; Bescond 2024); Penn World Table 10.01 (Feenstra et al. 2015) was obtained from DataverseNL using dataverse (v0.3.14; Kuriwaki et al. 2024); IV regressions were fitted with ivreg (Fox et al. 2025); cluster-robust variance-covariance matrix were computed with clubSandwich (Pustejovsky 2024) and wild cluster bootstrap inference with fwildclusterboot (Fischer and Roodman 2021).

4. Growth, artificial intelligence, and inequality: the moderating role of institutions

This chapter develops a comprehensive theoretical framework to analyze the impact of artificial intelligence (AI) on long-run economic growth and income inequality, emphasizing the moderating role of institutional quality. Drawing on the foundations of endogenous growth theory and institutional economics, AI is modeled as a form of automation that can substitute for human capital and, more broadly, as a general-purpose technology (GPT) capable of transforming production processes, labor markets, and innovation dynamics across the economy. In line with recent advances in the literature, AI is conceptualized as a dual-factor technology that can both complement and substitute for human labor, thereby generating complex and potentially conflicting effects on the supply of skills, productivity, and wage distribution.

By extending a canonical R&D-based endogenous growth model, this essay incorporates AI alongside traditional drivers of technological progress and explicitly embeds institutional quality as a key channel defining economic outcomes. Institutions influence not only innovation incentives and R&D productivity, but also the extent to which new technologies diffuse across firms and workers through education, reskilling, and redistributive mechanisms. Within this framework, the interaction among automation technologies, skill supply, and institutional arrangements generates feedback mechanisms that jointly determine growth trajectories and inequality dynamics.

The analysis shows that AI adoption can accelerate long-term growth by enhancing productivity and stimulating innovation. However, these gains are not automatic. When institutional quality is weak, substituting AI for skilled labor may depress incentives for education, reduce the supply of skills, and exacerbate wage polarization, ultimately leading to suboptimal growth outcomes and lower social welfare. Conversely, inclusive institutions, i.e., characterized by effective governance, broad access to education, and redistributive policies, can mitigate these adverse effects by sustaining the supply of skills, supporting innovation, and promoting a more equitable distribution of income.

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Keywords: endogenous growth; artificial intelligence; automation; skill premium; institutions; skills supply.

JEL Classification: E130; E240; J220; O310; O410.

4.1. Introduction

In the 21st century, technological development has accelerated at a remarkable pace, largely through the use of robots and artificial intelligence (AI) in tasks previously performed humans (Acemoglu and Restrepo 2018a; Ribeiro and Prettner 2025). In this context, AI emerges as a general-purpose technology (GPT) with the potential to transform entire business processes and sectors, from goods-producing technologies to the dynamics of research and development activities that support growth and innovation. That is, GPTs can be seen as pervasive enabling technologies with the potential to bring "generalized productivity gains" throughout the economy (Bresnahan and Trajtenberg 1995).

Historically, GPTs have played a dual role in economic development, driving productivity and innovation while also widening the gaps between economies based on institutional quality and resource availability. Similar concerns arise as the adoption of AI in economic activities broadens, and it is established as a GPT. That is, AI diffusion may become central in driving the aggregate growth trajectory, notwithstanding the importance of adequate institutions and investments in human capital (Helpman 1998). As a result, economies with better governance structures and higher investments in research and development (R&D) and education are likely to experience accelerated growth and reduced inequality through the adoption of AI. On the other hand, stagnation and rising inequality may be the outcome in economies with limited capacity to adapt to technological change.

AI can operate to complement skilled labor in knowledge-intensive tasks, but also to substitute for human capital by automating cognitive tasks (Bloom et al. 2025). These dual, opposing dynamics introduced by AI are distinct from those of previous technological revolutions, which have been biased toward the skilled portion of the labor force (e.g., Acemoglu 2002a). In addition to automating tasks, reversing the skill premium trajectory observed over the last few decades can reduce the costs of skilled-labor-intensive processes, such as R&D. This dynamic fosters a feedback loop in which advancements in AI amplify innovation, further enhancing aggregate output.

Economies with higher levels of human capital and stronger institutions are likely better equipped to reap the unevenly distributed potential benefits of AI adoption. According to North (1990), institutions are the "rules of the game" in defining economic, political, and social interactions, shaping incentives for innovation and influencing the distribution of the benefits of technological adoption. Inclusive institutions promote broad-based development, whereas "extractive institutions" tend to concentrate resources and perpetuate inequality (Acemoglu and Robinson 2012). In the context of AI, for example, inclusive institutions can accelerate innovation by sustaining the supply of skills even as economic incentives decline, mitigate adverse labor-market effects by fostering reskilling initiatives, and provide better social protection. In contrast, extractive institutions may exacerbate labor market fragmentation and limit broader societal benefits.

Endogenous growth theory, following the ideas of Romer (1990) and Lucas (1988), for example, places the internal mechanisms of innovation, human capital accumulation, and technological progress at the heart of economic development, providing the core framework for analyzing the impacts of new technology adoption on economic development. This paper contributes to the growth literature in three main ways. First, we propose a model that explicitly considers an institutional quality channel as affecting R&D productivity and the share of the population able to complete tertiary education and acquire skills. In this model, production and innovation follow the lines established by Romer (1990) and Jones (1995) in their R&D-based growth model, which has been further extended to include two labor types (Okada 2012). Second, institutional quality is related to the endogenous outcomes of progressive and redistributive policy decisions, as well as to the investments in education and reskilling. Third, robots and artificial intelligence are two sources of automation in the production technology (Bloom et al., 2025) of an overlapping-generations general equilibrium framework in which individuals choose whether to acquire skills or not (Prettner and Strulik 2020).

Within this setup, the present work aims to address two key questions related to the adoption of AI in production. (i) How are endogenous growth trajectories affected? (ii) In what ways do institutional frameworks moderate their economic impacts? The introduction of AI into production increases economic growth and investment in R&D; however, because skilled wages are compressed, it may reduce the supply of skilled labor. In the analyzed scenarios, better institutions, resulting from more redistributive transfers or reskilling initiatives for workers affected by AI, may increase the supply of skills or, at the very least, moderate the reduction in individuals' decisions to acquire skills. Additionally, better institutions also reduce inequality, as measured by the skill premium. Without strong institutions, the adoption of AI can exacerbate wage polarization and reduce incentives to acquire skills.

The remainder of the paper is structured as follows. Section 4.2 reviews the relevant literature, discussing the foundations of endogenous growth theory, the economics of GPTs, and the relationship between technological adoption and institutions. Section 4.3 presents the theoretical model, which considers individual decisions regarding skill acquisition in an economy where AI and robots can substitute or complement skilled and unskilled labor, respectively. Section 4.4 discusses simulation results focusing on the substitutability between AI and skilled labor, as well as the role of institutions in shaping economic trajectories, moderating undesirable effects, and promoting the redistribution of gains from technological advances. Finally, Section 4.5 concludes with a brief discussion of the broader implications of AI adoption, outlining avenues for future research.

4.2. Related literature

To analyze the implications of artificial intelligence (AI), we connected three different strands of macroeconomic literature: (i) endogenous growth and human capital, (ii) automation and directed technical change, and (iii) institutions and redistribution.

Lucas (1988) discusses the importance of human capital accumulation in growth, distinguishing between its "internal effects", i.e., the benefits that accrue directly to the individuals, and the "external effects" related to how people influence others' productivity. Romer (1990) pointed out that growth driven by technological knowledge improvements should be scaled by the amount of human capital allocated to the research process. Moreover, it shows the importance of knowledge as a non-rivalrous good, allowing (at least a portion of) it to be used to generate more knowledge. As a result of deliberate investments in innovation, the economy grows alongside the number of inventions available. Jones (1995) introduces the notions of congestion ("stepping on toes") and intertemporal spillover ("standing on shoulders") effects into the innovation process to modulate the scale effects present in such models. In our model, human capital is represented by skilled labor, which does not accumulate but is the only input into a Romer-Jones R&D process that generates cumulative technological knowledge, thereby affecting aggregate growth. Finite-lived individuals compare their lifetime payoffs to decide whether to remain unskilled or invest in skill acquisition.

More recently, Prettner and Strulik (2020) proposed mechanisms similar to those developed by Romer (1990) and Jones (1995) within an overlapping-generations (OLG) framework, in which the automation of unskilled labor affects returns to education, thereby influencing technological development through R&D and the skill premium. We follow the endogenous acquisition decision they proposed, also allowing investment in R&D to be affected by institutional quality.

Directed technical change (DTC), following Acemoglu (2002a), holds that different technologies can complement one another and that the relative prices and market sizes of these technologies determine the direction of innovation. Such a "direction" indicates which factor relative demand will increase with a technology change. In the context of AI, the central insight from this theory is that technological progress can complement or substitute for specific types of labor, as in task-based models in which the automation of existing tasks displaces labor previously used to perform them. An opposite effect happens from the creation of new tasks in which labor is preferred or has some comparative advantage. In such models, the net effect on employment and wages depends on the elasticities of substitution across the different labor types and on the rates at which new tasks are created and existing tasks are automated. Instead of explicitly modeling a task-based mechanism, Bloom et al. (2025) used nested constant elasticity of substitution (CES) structures to theoretically represent substitutability between unskilled labor and robots, between skilled labor and AI, and between the output of these two groups. The production side of our model follows the specification of Bloom et al. (2025).

There is a vast empirical literature documenting that the adoption of automation increased output at the cost of reduced demand for unskilled labor performing routine tasks. For example, using a "panel data on robot adoption within industries in seventeen countries" (1993–2007), Graetz and Michaels (2018) find that robots increased manufacturing productivity and depressed routine manual employment. Autor and Dorn (2013) document employment polarization in the U.S. (1980–2005) consistent with task displacement and reallocation. Martinez (2018) documents the link between industry-level automation and the decline in the U.S. labor share (1972–2010). Brynjolfsson et al. (2017) consider AI a general-purpose technology (GPT) according to the criteria set forth by Bresnahan and Trajtenberg (1995), highlighting its importance in enabling "complementary innovations that could multiply their impact." However, at the firm level, effects are heterogeneous, gains are unevenly distributed, and complementary investments, including human capital development, may be necessary to realize the benefits of AI adoption. This way, we distinguish AI from robots in our model and allow them to have different elasticities of substitution for skilled and unskilled labor, respectively.

Regarding the GPTs' absorption into an economy, North (1990) defines institutions as the formal and informal rules that determine how economic incentives operate. Lucas (1988) sees human capital accumulation as a "social activity" that may be affected by policies. Acemoglu and Robinson (2012) argue that inclusive institutions promote broad-based participation and innovation, whereas extractive institutions concentrate rents and exacerbate inequality. Policy-oriented work emphasizes that investment in education and reskilling, as well as progressive taxation and redistributive policies, are essential to enable workers to complement new technologies and to reduce their exposure to adverse shocks (Aghion et al. 2020; Autor et al. 2021; Piketty 2014; Stephany and Teutloff 2024). In our model, institutional quality amplifies R&D productivity and the portion of the population able to acquire skills, i.e., to complete tertiary education, generating feedback loops: better institutions stimulate innovation and broaden skills demand; broader skills supply and higher R&D intensity, in turn, increase the returns to sustaining more inclusive institutions.

Summarizing the main ingredients of the model presented in the following section, it builds on the framework of horizontal growth, from Romer (1990) and Jones (1995), and on OLG models that explicitly endogenize the skill-acquisition decision (e.g., Prettner and Strulik 2020). It considers the ideas of automation substituting and complementing labor explored in Hémous and Olsen (2014, 2022) and Bloom et al. (2025), which are also aligned with the effects found in task-based approaches, as in Acemoglu and Restrepo (2018a, 2019), and with empirical observations regarding the skill premium in the lines of Autor and Dorn (2013) and Graetz and Michaels (2018). Our choice to implement automation in a nested constant elasticity of substitution (CES) function with two factors, skilled labor that can be substituted by AI, and unskilled labor, which can be substituted by robots, with expanding varieties innovation, allows us to capture the idea of complementarity and substitutability present in task-based models without explicitly modeling it. Such choices lead to a tractable general-equilibrium structure. The introduction of an institutional quality element that affects R&D productivity and the share of the population able to attain tertiary education creates a feedback loop that also influences wage dispersion and growth.

The proposed setup allows us to study how institutional quality, resulting from investments in education and redistributive and progressive policies, can moderate the effects of AI on long-run growth and on trajectories of innovation and inequality. A possibility that is underexplored in existing automation-growth models.

4.3. Theoretical setup

This paper develops a comprehensive theoretical framework that examines the impact of Artificial Intelligence (AI) and institutional quality on economic growth. To analyze how institutional arrangements—such as public transfers and education expenditures—shape an economy's ability to adapt to technological change, we adapted the Prettner and Strulik (2020) overlapping-generations framework version of the foundational endogenous growth model by Jones (1995) and Romer (1990). The chosen approach is particularly well-suited for studying the interaction between automation, skill-biased technological progress, and education because it endogenizes the decision to acquire skills in response to changing incentives (Lucas 1988; Prettner and Strulik 2020). The model captures the dynamic feedback between policy instruments, the skill premium, and individual choices regarding education and (re)skilling. As new technologies substitute for labor, skilled and unskilled workers exposed to automation, in the form of robots or artificial intelligence, are pressed to continue studying to stay employed (Acemoglu and Restrepo 2018b; Autor et al. 2021). The modeling choices we make will enable us to assess how institutional quality measures, such as subsidized education or redistribution through transfers, affect labor allocation, innovation, and overall inequality in a general equilibrium framework (Acemoglu and Robinson 2012; North 1990).

The model considers three main agents interacting in an open economy: individuals, firms, and government. Individuals live for two periods and enter the economy after high school, when they must decide whether to remain unskilled and enter the labor market directly or to pursue higher education to become a skilled worker, incurring the disutility of studying for a fraction of their lives during this period (Prettner and Strulik 2020). Decisions regarding skill acquisition and savings are made during the first period of life. In the following period, individuals are retired, living off their previous savings and some eventual transfers from the government, and die with certainty by the end of this time period. Unskilled individuals provide labor unconditionally for the production of final goods, while skilled individuals may also engage in research and development (R&D) activities.

Firms are profit-maximizing entities that produce final and intermediate goods and coordinate the R&D activities that drive innovation and are the engine of growth. As in Jones (1995) and Romer (1990), horizontal innovation expands the existing variety of intermediate goods, empowering all production factors in the final goods sector. Institutional quality is an exogenous aspect that influences economic outcomes, representing the government's policy choices regarding income redistribution and educational support (Acemoglu and Robinson 2012; North 1990). The government collects taxes, which can be used to implement redistribution policies and enhance labor adaptability by fostering education or reskilling initiatives. Together, these agents create a dynamic environment where the adoption and diffusion of AI impact production, innovation, and income distribution.

4.3.1. Individuals

Utility. There are two types of individuals, unskilled and skilled, denoted by $j = L, H$, respectively, who take utility from consumption when they are at the working age, period represented by the subscript t, and when they are retired, represented by $t+1$. Their utility can be represented by

$$u_{j,t} = \log\left(c_{j,t}\right) + \zeta \log\left(\bar{R} s_{j,t}\right) - \mathbb{1}_{[j=H]} v\left(a\right), \tag{4.1}$$

where $c_{j,t}$ is the consumption of young adults, ζ is the discount factor, and $\bar{R}$ is the exogenous gross interest rate on savings $s_{j,t}$such that $c_{j,t+1} = \bar{R} s_{j,t}$ refers to consumption in the second period of life. $\mathbb{1}_{[j=H]}$ is a function that assumes the value of one when utility is evaluated to a skilled individual and zero otherwise. With a representing the individual ability level, $v\left(a\right)$ represents the disutility experienced from exerting effort in tertiary education, and is defined as

$$v(a) = \xi \log\left(\frac{\psi}{a - a_{\min}}\right), \text{ case } a \geq a_{\min}\text{; and,} \tag{4.2}$$
$$v(a) = \infty, \text{ otherwise;}$$

with $a_{\min}$ being the minimum ability level required to obtain a college degree and become skilled, and ξ and ψ the auxiliary parameters to calibrate such a function. Notice that the infinite disutility individuals with ability below such level suffer represent the idea that "not all members of society are willing or capable to obtain a college degree" (Prettner and Strulik 2020), and that $a > \psi + a_{\min}$ implies, i.e., there are some individuals who "experience pleasure" in exerting effort to achieve a higher degree.

Budget and decisions. Both skilled and unskilled individuals are endowed with one unit of labor to supply at time t, for which they receive a wage $w_{j,t}$ over which they pay a tax rate $\tau_{w,j}$. Individuals give up a fraction η_j of their time, which is devoted to studying. At this point, we diverge from Prettner and Strulik (2020) and to discuss the effects of increasing educational requirements for unskilled individuals, we do not assume η_L equals zero (and costs zero).[1] When young, or, at their first period of life, individuals receive government transfers $T_{j,t}$, consume the amount $c_{j,t}$, and save $s_{j,t}$. In the second period of life, when retired, they use the savings invested in the previous period to consume $c_{j,t+1}$. Then, the budget constraint is

$$\left(1 - \tau_j\right)\left(1 - \eta_j\right) w_{j,t} + T_{j,t} = c_{j,t} + s_{j,t}, \tag{4.3}$$

and each individual j-type individual chooses $c_{j,t}$ and $s_{j,t}$ when maximizing (4.1) s.t. (4.3). Substituting $c_{j,t+1} = \bar{R} s_{j,t}$ in (4.3), the Lagrangian of this problem can be written as

$$\begin{aligned}\mathscr{L} = {} & \log\left(c_{j,t}\right) + \zeta \log\left(c_{j,t+1}\right) - \mathbb{1}_{[j=H]} v(a) + \\ & \mu \left\{\left(1 - \tau_j\right)\left(1 - \eta_j\right) w_{j,t} + T_{j,t} - c_{j,t} - \frac{c_{j,t+1}}{\bar{R}}\right\},\end{aligned} \tag{4.4}$$

[1]The prevalence of secondary education is a recent phenomenon. For example, in the United States, it was only in the mid 1990s that high school graduates achieved 85% of the population up to 24 years, and it took another three decades for this figure to rise to around 90% (CPS Historical Time Series Tables on School Enrollment, Table 5-a, U.S. Census Bureau 2023).

The first-order conditions of the problem will yield the optimal consumption and saving decisions for both individual types, which are (see Appendix X), respectively,

$$c_{j,t} = \frac{1}{1+\zeta}\left[\left(1-\tau_j\right)\left(1-\eta_j\right)w_{j,t}+T_{j,t}\right] \text{ and} \tag{4.5}$$

$$s_{j,t} = \frac{\zeta}{1+\zeta}\left[\left(1-\tau_j\right)\left(1-\eta_j\right)w_{j,t}+T_{j,t}\right]. \tag{4.6}$$

That is, a fraction $\frac{1}{1+\zeta}$ of the received income is consumed, and a fraction $\frac{\zeta}{1+\zeta}$ is saved. As usual in the overlapping generations setup, the older generation owns the capital used in production along with the labor supplied by the younger generation. To simplify the problem, it is assumed that the capital stock entirely depreciates when individuals die, such that there are no bequests or inheritances.

Substituting (4.2), and results (4.5) and (4.6) back into (4.1), we find the indirect utility, conditioned on education, over which individuals decide whether to become skilled or to stay unskilled. Rearranging the resulting equation reveals to the ability threshold above which individuals will choose to engage in tertiary education,

$$a_t^* = \psi\left[\frac{(1-\tau_H)(1-\eta_H)w_{H,t}+T_{H,t}}{(1-\tau_L)(1-\eta_L)w_{L,t}+T_{L,t}}\right]^{-\frac{1+\zeta}{\xi}} + a_{\min}. \tag{4.7}$$

The threshold depends on after-tax, after-study-time net wages and transfers, linking policy and wages to skill supply.[2]

4.3.2. Research and development

The competitive R&D sector, based in Jones (1995), considers that new machines are invented at a rate $\bar{\delta}_t$ per effective allocated skilled labor unit during a period of time, that is,

$$A_t - A_{t-1} = \bar{\delta}_t L_{H,A,t}, \tag{4.8}$$

where A_{t-1} is the stock of existing designs that comes from the previous period. On your turn, the arrival rate is defined as

$$\bar{\delta}_t = \frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}}, \tag{4.9}$$

which is equivalent to assuming a Poisson process in which new ideas arrive at the rate δA_{t-1}^{ϕ}, which increases with A_{t-1}. The reasoning behind this specification is that it becomes easier to find an invention because spillovers enable all firms to benefit from the accumulated knowledge. The parameter ϕ, with $0 < \phi \leq 1$, controls the magnitude of

[2]See also Lucas (1988), on the role of human capital incentives.

these intertemporal knowledge spillovers' effect, or "standing on shoulders" effect (Jones 2002). The parameter λ, with $0 \leq \lambda \leq 1$, in turn, controls the strength of the congestion externality, or the "stepping on toes" (Jones 2002) effect that happens due to duplication and overlapping efforts in research. Given that $w_{H,A,t}$ represents the wage received by each unit of skilled labor engaged in R&D, and $p_{A,t}$ represents the price for which R&D firms sell a new blueprint. Thus, these firms optimize $p_{A,t}\left(A_t - A_{t-1}\right) - w_{H,A,t}L_{H,A,t}$ to determine the optimal amount of skilled labor to employ, given by

$$w_{H,A,t} = \bar{\delta} p_{A,t} \Leftrightarrow w_{H,A,t} = \frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} p_{A,t}. \tag{4.10}$$

Inventions are assumed to be protected by a patent length of one period of time[3] and investment in R&D for a new machine will occur whenever the profits that are possible to extract by the monopolist intermediate good firm producing the new variety, i, equals the price paid to the R&D firm. This way, in the current specification, skilled labor is the only input used to increment technological knowledge, and R&D firms hire it until the free-entry condition is satisfied. The growth rate of A can be encountered by merging (4.8) and (4.9):

$$g_{A,t} \equiv \frac{A_t - A_{t-1}}{A_{t-1}} = \delta A_{t-1}^{\phi-1} L_{H,A,t}^{\lambda}. \tag{4.11}$$

In turn, R&D intensity is the ratio between total expenditures in R&D and aggregate output, and, in the current context, it is given by the ratio of wages paid to researchers in R&D sector (4.10) and aggregate output, that is, by

$$\frac{w_{H,A,t}L_{H,A,t}}{Y_t} = \frac{\delta p_{A,t} L_{H,A,t}^{\lambda} A_{t-1}^{\phi}}{Y_t}. \tag{4.12}$$

4.3.3. Baseline production

To illustrate the core mechanisms, we first solve (derivations in Appendix Y) the model using the more straightforward production function developed by Okada (2012), which builds upon the Romer-Jones (1995; 1990) semi-endogenous growth model framework, distinguishing between skilled and unskilled labor types and considering that technological knowledge accumulates and plays the role of the total-factor productivity (TFP). Subsequently, we relax this assumption to incorporate automation, in the form of artificial intelligence and robots, as production factors that substitute for skilled and unskilled labor, respectively, embedded into nested constant elasticity of substitution (CES) functions,

[3]"This simplification helps to avoid intertemporal (dynastic) problems of patent holding and patent pricing while keeping the basic incentive to create new knowledge intact" (Strulik et al. 2013).

similar to the production technology used in the partial equilibrium model of (Bloom et al. 2025).

Final goods. Consider that final goods production is carried out in a competitive sector, according to the following technology:

$$Y_{O,t} = L_{L,t}^{\alpha_O} L_{H,Y,t}^{\beta_O} \int_0^{A_t} X_{O,i,t}^{1-\alpha-\beta} \, di, \quad 0 < \alpha + \beta < 1,$$

where the subscript O indicates symbols used only in the current subsection model, and $L_{L,t}$ and $L_{H,Y,t}$ represent the amount of unskilled and skilled labor employed in production, respectively. $X_{O,i,t}$ is the amount of the i^{th} intermediate good demanded, with α_O, β_O, and $1 - \alpha_O - \beta_O$ being the shares of each factor in the output. These factors have their contribution remunerated by $w_{O,L,t}$, $w_{O,H,Y,t}$, and $(1 + \tau_X)\, p_{O,i,t}$, respectively, with τ_X being an *ad valorem* tax imposed over the intermediate goods price. All prices are expressed in real terms, i.e., normalized by the unitary price of the final good. Variable A_t represents the existing varieties of non-durable intermediate goods, i, and can be seen as the available technological knowledge in an economy. When solving its problem, the representative final goods firm finds

$$p_{O,i,t} = \frac{1 - \alpha_O - \beta_O}{1 + \tau_X} L_{L,t}^{\alpha_O} L_{H,Y,t}^{\beta_O} X_{O,i,t}^{-\alpha_O - \beta_O}, \tag{4.13}$$

$$w_{O,L,t} = \alpha_O L_{L,t}^{\alpha-1} L_{H,Y,t}^{\beta} \int_0^{A_t} X_{O,i,t}^{1-\alpha-\beta} \, di = \alpha_O \frac{Y_t}{L_{L,t}}, \text{ and} \tag{4.14}$$

$$w_{O,H,Y,t} = \beta_O L_{L,t}^{\alpha_O} L_{H,Y,t}^{\beta_O - 1} \int_0^{A_t} X_{O,i,t}^{1-\alpha_O-\beta_O} \, di = \beta_O \frac{Y_t}{L_{H,Y,t}}. \tag{4.15}$$

Intermediate goods. A firm in the intermediate-goods sector purchases a patent for a variety i produced in R&D sector. It then monopolizes the production of this item for the entire period of time, which is assumed to be the duration of the patent rights. After that, in future periods, this firm, or any other producer, continues producing that variety, but without any margin. It is assumed that one unit of the intermediate variety is produced from one unit of physical capital, which is remunerated by the exogenous interest rate $\bar{R}$. Considering (4.13) to solve the profit-maximization problem of the new vintage producers,

$$\Pi_{O,i,t} = \max_{X_{i,t}} p_{O,i,t} X_{O,i,t} - \bar{R} X_{O,i,t},$$

we will find that the amount of goods produced by any patent-protected intermediate good firm i is the same:

$$X_{O,t} = X_{O,i,t} = \frac{(1-\alpha_O-\beta_O)^{\frac{2}{\alpha+\beta}} L_{L,t}^{\frac{\alpha_O}{\alpha_O+\beta_O}} L_{H,Y,t}^{\frac{\beta_O}{\alpha_O+\beta_O}}}{(1+\tau_X)^{\frac{1}{\alpha_O+\beta_O}} \bar{R}^{\frac{1}{\alpha_O+\beta_O}}}, \forall i. \tag{4.16}$$

Producers of older varieties, non-protected by patent laws and identified by the subscript j, set their prices at the marginal cost, i.e., $p_{O,j,t} = \bar{R}$. Substituting this for $p_{O,i,t}$ in (4.13) gives the amount of $X_{O,j,i,t}$ to be produced, which, combined to (4.16), can be expressed by

$$X_{O,j,t} = (1-\alpha_O-\beta_O)^{\frac{1}{-\alpha_O-\beta_O}} X_{O,i,t}.$$

The aggregate demand for new varieties is $(A_t - A_{t-1}) X_{O,i,t}$, and, for the already existing ones, is $A_{t-1}X_{O,j,t} = A_{t-1}(1-\alpha_O-\beta_O)^{\frac{1}{-\alpha_O-\beta_O}} X_{O,i,t}$. Thus, considering all varieties, the aggregate demand for intermediate goods is

$$\begin{aligned} \tilde{A}_{O,t}X_{O,t} &\equiv (A_t - A_{t-1}) X_{O,i,t} + A_{t-1}X_{O,j,t}, \text{ with} \\ \tilde{A}_{O,t} &= \left[(1-\alpha_O-\beta_O)^{\frac{1}{-\alpha_O-\beta_O}} - 1\right] A_{t-1} + A_t. \end{aligned} \tag{4.17}$$

Labor allocation and wages between sectors. Considering that the population is constant and total labor is $L = L_{L,t} + L_{H,t}$, where $L_{L,t} = F\left(a_t^*\right)L$ and $F(a)$ is the cumulative distribution function (c.d.f.) of ability in the population, the system made by the following equations determine the equilibrium conditions of the model, where *locus* M_O comes from (4.17), and *locus* N_O from (4.7):

$$\begin{aligned} M_O :& w_{O,H,A,t} - w_{O,H,Y,t} = 0, \\ N_O :& = \left(a_t^* - a_{\min}\right)^{\frac{\xi}{1+\zeta}} \left[(1-\tau_H)(1-\eta_H) w_{O,H,t} + T_{O,H,t}\right] \\ & - \psi\left[(1-\tau_L)(1-\eta_L) w_{O,L,t} + T_{O,L,t}\right] = 0. \end{aligned} \tag{4.18}$$

That is, equilibrium arises when the wages received by scientists (or skilled workers in R&D) equalizes with the wage level paid to skilled workers in the productive side of the economy and the supply of skills equals its demand in production and R&D. Then, assuming that the population ability is uniformly distributed in the interval $(0,1)$, such that $F(a) = a$, and, considering that $L = L_{L,t} + L_{H,t}$ and $L_{H,t} = L_{H,Y,t} - L_{H,A,t}$, and equations (4.7), (4.8), (4.9), (4.14), (4.15), and (4.16), the previous equations give origin to the following system of implicit equations in the space $\left(L_{H,t}, L_{H,A,t}\right)$:

$$\begin{cases} M_O: & \delta\Lambda_6\left(L_{H,t}-L_{H,A,t}\right)-\beta\Lambda_4\Lambda_5 A_{t-1}^{1-\phi}L_{H,A,t}^{1-\lambda}-\beta\delta\Lambda_5 A_{t-1}L_{H,A,t}=0, \\ N_O: & \beta\Lambda_1\Lambda_2\Lambda_4\left(\frac{L-L_{H,t}}{L}-a_{\min}\right)^{\frac{\xi}{1+\zeta}}\left(L-L_{H,t}\right)^{\frac{\alpha}{\alpha+\beta}}\left(L_{H,t}-L_{H,A,t}\right)^{\frac{-\alpha}{\alpha+\beta}}A_{t-1} \\ & +\beta\delta\Lambda_1\Lambda_2\left(\frac{L-L_{H,t}}{L}-a_{\min}\right)^{\frac{\xi}{1+\zeta}}\left(L-L_{H,t}\right)^{\frac{\alpha}{\alpha+\beta}}\left(L_{H,t}-L_{H,A,t}\right)^{\frac{-\alpha}{\alpha+\beta}}L_{H,A,t}^{\lambda}A_{t-1}^{\phi} \\ & +\left(\frac{L-L_{H,t}}{L}-a_{\min}\right)^{\frac{\xi}{1+\zeta}}T_{,H,t}-\psi T_{L,t} \\ & -\alpha\psi\Lambda_1\Lambda_3\Lambda_4\left(L-L_{H,t}\right)^{\frac{-\beta}{\alpha+\beta}}\left(L_{H,t}-L_{H,A,t}\right)^{\frac{\beta}{\alpha+\beta}}A_{t-1} \\ & -\alpha\delta\psi\Lambda_1\Lambda_3\left(L-L_{H,t}\right)^{\frac{-\beta}{\alpha+\beta}}\left(L_{H,t}-L_{H,A,t}\right)^{\frac{\beta}{\alpha+\beta}}L_{H,A,t}^{\lambda}A_{t-1}^{\phi}=0, \end{cases} \tag{4.19}$$

$$\begin{aligned} \text{with } \Lambda_1 &= \left\{\frac{\left(1-\alpha_O-\beta_O\right)^2}{\left(1+\tau_X\right)\bar{R}}\right\}^{\frac{1-\alpha_O-\beta_O}{\alpha_O+\beta_O}}, \\ \Lambda_2 &= \left(1-\tau_H\right)\left(1-\eta_H\right), \\ \Lambda_3 &= \left(1-\tau_L\right)\left(1-\eta_L\right), \\ \Lambda_4 &= \left(1-\alpha_O-\beta_O\right)^{\frac{1}{-\alpha_O-\beta_O}}, \\ \Lambda_5 &= \left(1+\tau_X\right), \text{ and} \\ \Lambda_6 &= \left(\alpha_O+\beta_O\right)\left(1-\alpha_O-\beta_O\right). \end{aligned}$$

When solving their problems, individuals and firms take the transfers $T_{H,t}$ and $T_{L,t}$ as given, because these agents, individually, are unable to internalize the effects of their decisions on these variables. This way, it is possible to show that $\frac{dL_{H,A,t}}{dL_{H,t}} > 0$ in both *loci*, that the M curve starts at the origin and the N curve is to the right of the origin; so, in the region of the space where a solution exists, the solution is unique, and the N slope is greater than the M slope.

Static results. Regarding the ability level threshold, a^*, some mechanics of the model are straightforward from $L = L_{L,t} + L_{H,t}$ and (4.18), that is, $\frac{\partial L_{L,t}}{\partial a^*} = L > 0$ and $\frac{\partial L_{H,t}}{\partial a^*} = -L < 0$. Additionally, it is possible to find that the endogenous variables of (4.19) are affected by changes in a^*.

Proposition 4.1. *If technological knowledge equally increases the productivity of unskilled and skilled labor, an increase in the population share able to achieve tertiary education unequivocally raises the overall supply of skills, $L_{H,Y}$, and the skilled labor allocated to R&D, $L_{H,A,t}$.*

Proof. In Appendix Y: from the system (4.19), we find $\frac{\partial L_{H,A,t}}{\partial a_t^*} < 0$ (Y.22); and, from (4.14) and (4.15), we find that $\frac{\partial w_{L,t}}{\partial a_t^*} < 0$ (Y.23). □

Different from the case analyzed in Prettner and Strulik (2020), where technological evolution competes with unskilled labor, while the complementary skilled labor is favored by A, in the case where both labor types benefit from the technological knowledge, both the skills supply and skilled labor allocation in R&D increase (decrease) with the decrease (increase) in a^*. That is, the adjustment in one endogenous variable is not at the cost of the other in absolute terms, and both variables move in the same direction. In addition, it is also possible to show from the system (4.19) that $\frac{\partial L_{H,A,t}}{\partial a_t^*} < 0$ and $\frac{\partial L_{H,Y,t}}{\partial a_t^*} < 0$; from (4.11) that $\frac{\partial g_{A,t}}{\partial a_t^*} < 0$; and, from (4.14) and (4.15), that $\frac{\partial w_{L,t}}{\partial a_t^*} < 0$ and $\frac{\partial}{\partial a_t^*}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right) > 0$. With less skilled labor being supplied, firms in the final goods sector will be better off substituting workers who will demand a higher relative wage for unskilled labor that now receives a lower wage. Analogously, the higher relative wages paid in exchange for scientists' labor, make the investment in R&D less attractive, i.e., the free-entry condition will be satisfied at a lower level of labor. Such changes in the labor market and in the production and R&D sectors lead to a permanent reduction in the growth rate. Finally, notice that it is only when the government transfers, $T_{L,t}$ and $T_{H,t}$, are treated as exogenous or in the absence of them, that it is possible to show, from 4.7, that the intuitive results that an increase in the utility costs the individuals incur to stay unskilled leads to an increase in the skills supply, i.e., $\frac{\partial a^*}{\partial \eta_L} < 0$, and, conversely, that the increase in the utility costs to achieve tertiary education increase the skill supply, i.e., $\frac{\partial a^*}{\partial \eta_H} > 0$.

4.3.4. Production with robots and artificial intelligence

We proceed by incorporating, as production factors, robots that substitute for unskilled labor and artificial intelligence (AI) that substitutes for skilled labor. In this sense, we adapt the nested constant elasticity of substitution (CES) production function from Bloom et al. (2025). We consider that technical knowledge accumulates and plays the role of TFP, while capital is represented by the intermediate goods, as in the previous subsection. Despite the model differences, the steps required to achieve equilibrium are essentially the same; therefore, the exposition will focus only on the points where it differs from the baseline model presented above.

Final goods. Consider the following production function:

$$Y_t = \int_0^{A_t} x_{i,t}^{\alpha}\, di \left\{ \beta_3 \left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1) P_t^{\theta} \right]^{\frac{\gamma}{\theta}} + (1-\beta_3) \left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2) G_t^{\sigma} \right]^{\frac{\gamma}{\sigma}} \right\}^{\frac{1-\alpha}{\gamma}}, \tag{4.20}$$

where, as in the baseline model, Y_t is aggregate output, $x_{i,t}$ represents the amount of i-variety intermediate goods (machines) used in final-goods production, which also represents the capital owned by the older generation, and A_t is the amount of existing intermediate varieties that also represents the economy's technological knowledge level. Additionally, P_t refers to the stock of industrial robots that can replace unskilled labor $L_{L,t}$, and G_t is the stock of AI that can substitute for skilled labor in production. $L_{H,Y,t}$, and β_1, β_2, and β_3 refer to the output shares of the respective factor in the CES functions; α is the elasticity of substitution between the intermediate goods and the outermost aggregative CES, representing the aggregate contribution of both labor types and their automation counterpart (robots and AI). The degree of substitutability between unskilled labor and robots is controlled by θ, and between skilled labor and AI is controlled by σ. Additionally, γ determines the substitution between the CES for unskilled labor and robots, and the CES for skilled labor and AI. For these parameters, it is considered that $\alpha, \beta_1, \beta_2, \beta_3 \in (0,1)$ and $\gamma, \theta, \sigma \in (0,1]$.

In this setting, labor and automation are unconditionally supplied and paid according to their marginal products. The problem solved by the final good representative firm is

$$\begin{aligned}\Pi_{Y,t} = \max Y_t - \int_0^{A_t} (1+\tau_X)\, p_{x,i,t} x_{i,t}\, di - w_{L,t} L_{L,t} - w_{H,Y,t} L_{H,Y,t} \\ - (1+\tau_P)\, p_{P,t} P_t - (1+\tau_G)\, p_{G,t} G_t,\end{aligned}$$

where the prices $p_{x,i,t}$, $p_{P,t}$, $p_{G,t}$, $w_{L,t}$, and $w_{H,Y,t}$ are given. Then, from the first-order conditions (FOCs), we have

$$p_{x,i,t} = (1+\tau_X)^{-1}\frac{\partial Y_t}{\partial x_{i,t}} = (1+\tau_X)^{-1}\alpha x_{i,t}^{\alpha-1}L_{\text{CES},t}^{1-\alpha}, \tag{4.21}$$

$$w_{L,t} = (1-\alpha)\beta_1\beta_3 L_{L,t}^{\theta-1}L_{\text{L,CES},t}^{\gamma-\theta}L_{\text{CES},t}^{1-\alpha-\gamma}\int_0^{A_t} x_{i,t}^{\alpha}\,di, \text{ and} \tag{4.22}$$

$$w_{H,Y,t} = (1-\alpha)\beta_2(1-\beta_3)L_{H,Y,t}^{\sigma-1}L_{H,\text{CES},t}^{\gamma-\sigma}L_{\text{CES},t}^{1-\alpha-\gamma}\int_0^{A_t} x_{i,t}^{\alpha}\,di, \text{ where} \tag{4.23}$$

$$L_{L,\text{CES},t} = \left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1)P_t^{\theta}\right]^{\frac{1}{\theta}},$$

$$L_{H,\text{CES},t} = \left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2)G_t^{\sigma}\right]^{\frac{1}{\sigma}}, \text{ and}$$

$$L_{\text{CES},t} = \left[\beta_3 L_{L,\text{CES},t}^{\gamma} + (1-\beta_3)L_{H,\text{CES},Y,t}^{\gamma}\right]^{\frac{1}{\gamma}}.$$

Results (4.22) and (4.23) lead to an expression for the skill premium,

$$\frac{w_{H,Y,t}}{w_{L,t}} = \frac{\beta_2(1-\beta_3)}{\beta_1\beta_3}L_{L,t}^{1-\theta}L_{H,Y,t}^{\sigma-1}\left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1)P_t^{\theta}\right]^{\frac{\theta-\gamma}{\theta}}\left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2)G_t^{\sigma}\right]^{\frac{\gamma-\sigma}{\sigma}}, \tag{4.24}$$

from which it is possible to confirm, unequivocally, that such an inequality measure declines with higher allocations of $L_{H,Y,t}$ and raises case $L_{L,t}$ increases (see Appendix Z) for $\frac{\partial}{\partial L_{L,t}}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)$ (Z.1) and $\frac{\partial}{\partial L_{H,Y,t}}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)$ (Z.2). Moreover, when substitution between AI and skilled labor is stronger than between output from the skilled and unskilled labor CES, an increase in the AI usage reduces the skill premium, i.e., $\text{sign}\left[\frac{\partial}{\partial G_t}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)\right] = \text{sign}(\gamma-\sigma)$ (Prettner and Strulik 2020). Analogously, $\text{sign}\left[\frac{\partial}{\partial P_t}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)\right] = \text{sign}(\theta-\gamma)$ (Z.3).

Intermediate goods. As in the baseline specification, the interest rate is assumed to be exogenous, and the profit-maximization problem of a new vintage producer is also similar, $\Pi_{x,i,t} = \max_{X_{i,t}} p_{x,i,t}x_{i,t} - \bar{R}x_{i,t}$, and, again, we find that the amount of goods produced by the patent-protected intermediate good firms is constant across all i varieties:

$$x_{i,t} = X_t = \left[\frac{\alpha^2}{(1+\tau_X)\bar{R}}\right]^{\frac{1}{1-\alpha}} L_{\text{CES},t}, \forall i. \tag{4.25}$$

Producers of non-patent-protected varieties, identified by the subscript j, set their prices at the marginal cost, i.e., $p_{x,j,t} = \bar{R}$, and face the (adjusted to be expressed in terms of x_i)

demand $x_{j,t} = \alpha^{\frac{1}{\alpha-1}} x_{i,t}$. The aggregate demand for intermediate goods is

$$\tilde{A}_t = \left(\alpha^{\frac{1}{\alpha-1}} - 1\right) A_{t-1} + A_t = \left(\alpha^{\frac{1}{\alpha-1}} + g_A\right) A_{t-1}, \tag{4.26}$$

with g_A given by (4.11). Then, substituting (4.25) and (4.26) in (4.20), the production function can be rewritten as

$$\begin{aligned} Y_t = & \left[\frac{\alpha^2}{(1+\tau_X)\bar{R}}\right]^{\frac{\alpha}{1-\alpha}} \left(\alpha^{\frac{1}{\alpha-1}} + g_A\right) A_{t-1} \\ & \times \left\{\beta_3 \left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1) P_t^{\theta}\right]^{\frac{\gamma}{\theta}} + (1-\beta_3) \left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2) G_t^{\sigma}\right]^{\frac{\gamma}{\sigma}}\right\}^{\frac{1}{\gamma}}. \end{aligned} \tag{4.27}$$

4.3.5. Institutions

Government. Although considered exogenous to the agents with respect to their choice variables, the results of their decisions are influenced by the policy parameters and variables. In addition to defining transfers to the individuals, $T_{j,t}$, and taxes on wages, τ_j, the government may also intervene in the economy by imposing taxes on capital usage in production, τ_X, and on robots, τ_P, and artificial intelligence, τ_G. Thus, the aggregate amount of collected taxes is given by

$$\mathscr{T}_t = \tau_L w_{L,t} L_{L,t} + \tau_H w_{H,t} L_{H,t} + \tau_X p_X \tilde{A}_t X_t + \tau_P p_P P_t + \tau_G p_G G_t.$$

Regarding the government expenditures, we consider that a fixed portion, $\kappa_E > 0$, of the aggregate output, Y_t, is used to pay for the years of education individuals receive before the age that they enter the labor market. Additionally, a portion $\kappa_L \geq 0$ ($\kappa_H \geq 0$) of Y_t is used on reskilling initiatives for unskilled (skilled) workers exposed to automation. To let the government budget balanced, the amount of the collected taxes not used on these initiatives is transferred back to workers according the following rule $\chi = \frac{T_{L,t}}{T_{H,t}}$, assuming that all individuals of the younger cohort receive a transfer, i.e., $T_{L,t} > 0$ and $T_{H,t} > 0$.

Institutional quality. Institutional quality, Ω, can be seen as an exogenous factor that amplifies or dampens the effectiveness of R&D efforts, shaping the environment in which innovation occurs. Strong institutions enhance R&D productivity by providing robust intellectual property protection, funding public research, and fostering collaboration between the private and public sectors. Conversely, weak institutions hinder innovation by creating uncertainty or reducing incentives for investment. By linking A_t to Ω, the framework captures how governance and policy decisions can significantly influence the trajectory of technological progress and, consequently, economic growth.

The redistributive policies, expressed in terms τ_L and τ_H from the revenues side, and by $T_{L,t}$ and $T_{H,t}$ on the government expenditures, influence how the economic benefits of the technological progress and automation are shared across society; reducing inequality by reallocating the economic gains from automation. Higher redistributive efforts can reduce income inequality and provide resources for public goods like education, which in turn strengthen Ω.

In turn, education and reskilling investments, κ_E, κ_L, and κ_H, enable workers to increase the skill supply and adapt to technological disruptions caused by technology adoption. For example, taxation of AI-driven profits can fund reskilling initiatives, creating a virtuous cycle where institutions simultaneously enhance human capital and foster inclusive growth. By increasing the share of high-skilled workers $(L_{H,t})$, education and reskilling efforts promote inclusive economic growth. The effectiveness of institutions is modeled as the result of redistributive policies and investments on education:

$$\Omega = \left(\frac{1+\tau_H}{1+\tau_L}\right)^{\rho_\tau} \left(\frac{T_{L,t}}{T_{H,t}}\right)^{\rho_T} \left(\frac{\kappa_E}{E_{\min}}\right)^{\rho_E} (1+\kappa_L)^{\rho_L} (1+\kappa_H)^{\rho_H}, \tag{4.28}$$

where ρ_1, ρ_2, ρ_3, ρ_4, and ρ_5 modulate the relative importance of progressive taxation, $\frac{1+\tau_H}{1+\tau_L}$, redistributive transfers, $\frac{T_{L,t}}{T_{H,t}}$, education individuals receive before entering the labor market, $\frac{\kappa_E}{E_{\min}}$, and investments in reskilling and education for unskilled, κ_L, and skilled, κ_H, workers in determining institutional quality; i.e., the higher the Ω more inclusive are the institutions and greater are the innovation incentives and distributional outcomes (North 1990; Acemoglu and Robinson 2012).

The multiplicative structure of (4.28) reflects the view that strong institutions require progress in several dimensions, such that the different policies are complementary.[4] Aggregating these effects on a unique measure of institutional quality makes it easy to introduce feedback loops between policy decisions, which reflect the institutional environment, and growth. On the model, the link with R&D can be established by considering that

$$\delta = \delta_Q \Omega^{\rho_r},$$

where the rate of ideas arrival is now given by δ_Q and $\rho_r \geq 0$ controls the intensity of the institutional quality effects on the R&D (4.8) productivity $\bar{\delta}_t$. On the supply side, innovation can be fostered by institutions setting

$$a_{\min} = \frac{a_Q}{\Omega^{\rho_a}},$$

[4]The idea is that investment in education and reskilling allows labor to better complement new technologies, while redistributive transfers and progressive taxation improves the transition path (e.g., Autor et al. 2021; Piketty 2014; Stephany and Teutloff 2024).

affecting directly the skills supply decision (4.7), capturing the idea that individuals that grow and are educated on more inclusive and better socially-protected environments will likely be able to achieve greater educational attainment. Thus, the ability threshold to achieve tertiary education is now represented by a_Q, and the effect of the institutional quality on such parameter is moderated by ρ_a. With these minimal changes, which do not require altering any of the previously presented expressions, the model reinforces the role of institutional quality through feedback loops between the supply and demand for skills and innovation.

4.3.6. Dynamics and equilibrium

General equilibrium. As a result of the optimal decisions of individuals and firms, the general equilibrium of the model will be given by the set of time trajectories for real quantities, prices, wages, technologies, and policies,

$$\begin{aligned}\{&Y_t, X_t, L_{L,t}, L_{H,Y,t}, L_{H,A,t}, c_{L,t}, c_{H,t}, s_{L,t}, s_{H,t},\\ &p_{A,t}, p_{X,t}, p_{P,t}, p_{G,t}, w_{L,t}, w_{H,Y,t}, w_{H,A,t},\\ &A_t, P_t, G_t, T_{L,t}, T_{H,t}\}_{t=0}^{\infty}.\end{aligned}$$

Taking P and G as exogenous variables constant in the long run, the general equilibrium determination is similar to the case with the baseline production function and determined in the labor market, once the skilled wages in R&D and final goods equalizes, $w_{H,A,t} = w_{H,Y,t}$ and the optimal rate of skill supply and its allocation between these sectors satisfy (4.7). Considering $L = L_{L,t} + L_{H,t}$ and $L_{H,t} = L_{H,Y,t} - L_{H,A,t}$, where $L_{L,t} = F\left(a_t^*\right) L$, with the population ability uniformly distributed in the interval $(0,1)$, such that $F(a) = a$, the system that determines the model equilibrium is given by

$$M : w_{H,A,t} - w_{H,Y,t} = 0, \tag{4.29}$$

$$\begin{aligned}N :=& \left(a_t^* - a_{\min}\right)^{\frac{\xi}{1+\zeta}} \left[(1-\tau_H)(1-\eta_H) w_{H,t} + T_{H,t}\right]\\ &- \psi\left[(1-\tau_L)(1-\eta_L) w_{L,t} + T_{L,t}\right] = 0.\end{aligned} \tag{4.30}$$

Bearing in mind (4.7), (4.8), (4.9), (4.22), (4.23), and (4.25) it is possible to find expressions for $w_{L,t}$, $w_{H,Y,t}$, and $w_{H,A,t}$, as functions of $L_{H,t}$ and $L_{H,A,t}$ (see Appendix (Z)). Then, following the steps described in Appendix Y, equations (4.29) and (4.30) lead to a system of implicit equations in the space $\left(L_{H,t}, L_{H,A,t}\right)$, analogous to (4.19).

The equilibrium characterization is also similar to the baseline model in Subsection 4.3.3, with the M curve starting at the left and showing a smaller slope than N curve, as depicted in Figure 4.1, which plots these curves under different calibrations. Letting $\bar{L}_{H,t}$ be the value of $L_{H,t}$ that solves equation(4.30) as $L_{H,A,t}$ gets closer to zero from the right, i.e., $\bar{L}_{H,t}$ solves $\lim_{L_{H,A,t}\to 0^+} N(L_{H,t})$, an equilibrium can exist if $L_{H,t} > \bar{L}_{H,t}$.

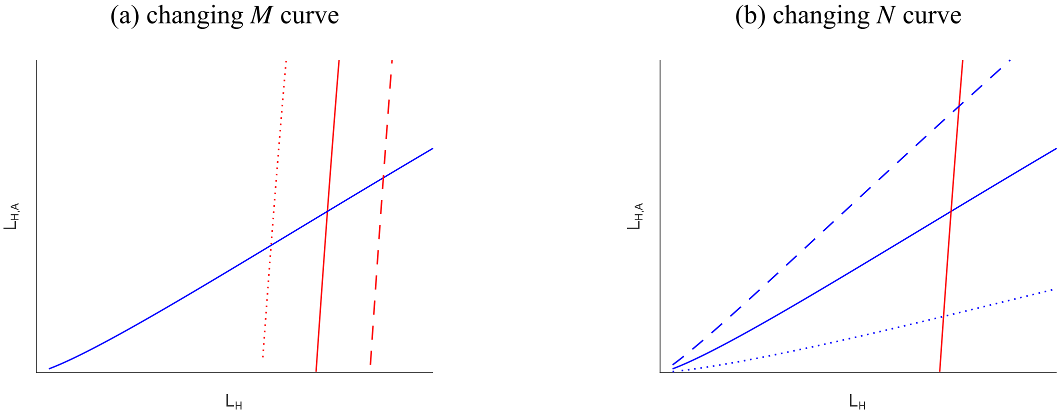

FIGURE 4.1: Curves starting near the origin, in blue, represent the M *locus*, while the steeper curves, in red, represent the N *locus*. The left panel (a) depicts changes that move the N curve left (right), dotted (dashed) red line, due to an increase (decrease) in a_{min}. On the right panel (b), the slope of the M curve increases (decreases), dashed (dotted) blue line, in response to an increase (decrease) in α.

Balanced growth path and transition dynamics. Considering (4.11) and that the P_t grows at the rate $g_{P,t}$, and that G_t grows at the rate $g_{G,t}$, the growth rate of aggregate output during can be derived from (4.27).

Proposition 4.2. *Case the demand for automation is increasing and bounded, the economy's growth rate during the transition is always higher than in the long run.*

Proof. From (4.27), we have:

$$g_{Y,t} = g_{A,t} + \frac{\beta_3(1-\beta_1)\left[\beta_1 L_L^\theta + (1-\beta_1)P_t^\theta\right]^{\frac{\gamma-\theta}{\theta}} P_t^\theta\, g_{P,t}}{\gamma\left\{\beta_3\left[\beta_1 L_{L,t}^\theta + (1-\beta_1)P_t^\theta\right]^{\frac{\gamma}{\theta}} + (1-\beta_3)\left[\beta_2 L_{H,Y,t}^\sigma + (1-\beta_2)G_t^\sigma\right]^{\frac{\gamma}{\sigma}}\right\}} + \frac{(1-\beta_2)(1-\beta_3)\left[\beta_2 L_{H,Y,t}^\sigma + (1-\beta_2)G_t^\sigma\right]^{\frac{\gamma-\sigma}{\sigma}} G_t^\sigma\, g_{G,t}}{\gamma\left\{\beta_3\left[\beta_1 L_{L,t}^\theta + (1-\beta_1)P_t^\theta\right]^{\frac{\gamma}{\theta}} + (1-\beta_3)\left[\beta_2 L_{H,Y,t}^\sigma + (1-\beta_2)G_t^\sigma\right]^{\frac{\gamma}{\sigma}}\right\}}. \tag{4.31}$$

In the long run, $g_{A,t}$ is constant A_{t-1} grows at the same rate of A_t, i.e., $g_{A,t}$; and $g_{P,t} = g_{G,t} = 0$; thus the dynamics becomes governed by the growth rate of technological knowledge, i.e., $\lim_{t\to\infty} g_{Y,t} = g_{A,t}$. □

Remark. Notice that the second and third terms in (4.31) resembles a weighted average of the growth rates of P_t and G_t.

As in Jones (1995), without a growing population, the model only presents a balanced growth path case $\phi = 1$. Case $\phi < 1$, the growth rate is decreasing and $\lim_{t\to\infty} g_{A,t} = 0$ in the long run, which will cease R&D investment and the demand for scientists, i.e., $\lim_{t\to\infty} L_{H,A,t} = 0$. In this case, the long-run equilibrium will be determined only by the value of $L_{H,t}$ which implicitly solves equation (4.30). Despite that, Prettner and Strulik (2020) argue that “the case of zero asymptotic growth is compatible with meaningful adjustment dynamics”.

Figure 4.2 exemplifies the long-run dynamics of the endogenous variables $L_{H,t}$ and $L_{H,A,t}$. Case $\phi = 1$, finding equilibrium considering constant P_t and G_t in the long run is a static problem. In such a situation, the transition between two different equilibria will be given by the exogenous path of P_t and G_t.

(a) states transition (b) variables’ path

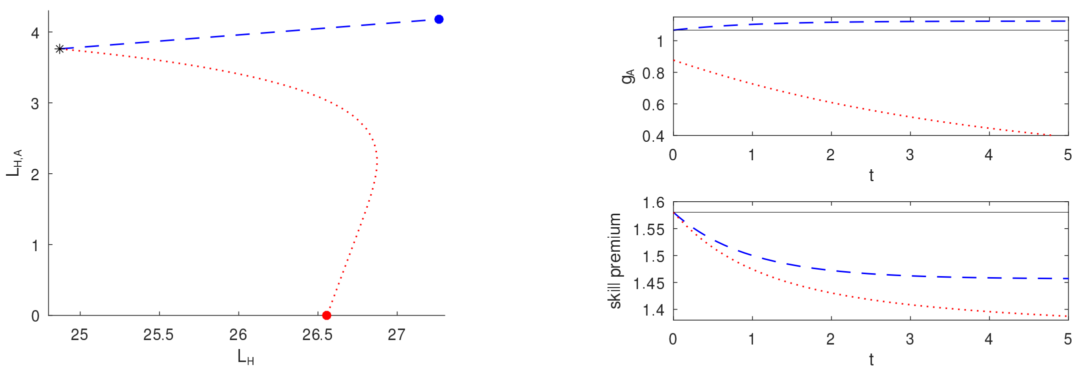

FIGURE 4.2: Examples of the model dynamics for the transitions from an initial state ($*$ marker), to two different steady-states ($\bullet$ markers) after a decrease in $a_{\min}$, which leads to an increase in the skill supply. The blue dotted lines show the transitions for the case where $\phi = 1$ and the red dotted lines for the case where $\phi < 1$. The left panel (a) plots the trajectory of the solutions to (4.29) and (4.30). The right panel (b) plots the paths for two variables of the model: the growth rate of A_t (upper plot) and the skill premium (bottom plot). Case $\phi = 1$, we observe in the upper plot that g_A increases together with the more abundant skilled labor; case $\phi < 1$, g_A goes down until it achieves zero on the distant steady state. The thin black lines on both panel (b) plots mark the variables’ level at the initial state ($*$).

To let the steady state continue to be tractable in a two-variable space, we assume that these two kinds of automation, P and G, are upper-bounded functions that converge to a constant value in the long run. Different dynamics for P and G automation can be assumed, but, here, we give forms considering that these variables converge to the constant values $\bar{P}$ and $\bar{G}$, respectively, and that more automation is available in the economy as its technological level evolves. Then, during transitions, the path of these variables would be given by

$$P_t = \bar{P}\left(1-e^{-\phi_P A_t}\right) \text{ and } G_t = \bar{G}\left(1-e^{-\phi_G A_t}\right),$$

respectively, where ϕ_P and ϕ_G control the rate of convergence. Notice, however, that such an assumption of an exogenous upper-bounded path for P and G is not innocuous, especially when carrying out long-run steady state analysis. It should be seen as a technical construct that, as already mentioned, allows us to keep the model two-dimensional and pin down real roots for the system made by equations (4.29) and (4.30).

4.4. Analysis and simulation

In this section, we present numerical simulations to illustrate the model's predictions. These simulations aim to examine the dynamic impacts of key parameters, including the substitutability between AI and skilled labor, σ, and institutional quality, Ω. While detailed numerical implementation may be deferred to future research, the proposed analysis outlines the mechanisms and insights that can be obtained.

4.4.1. Calibration

For the individuals and R&D characterization, our baseline calibration follows that found in Prettner and Strulik (2020), such that $\delta = 0.55$, $\eta_H = 0.11$, $\eta_L = 0$, $\psi = 0.1$, $\delta_Q = 0.55$, $\lambda = 0.5$, and $\xi = 1+\zeta$. Because our model is more sensitive to intertemporal knowledge spillovers, we set $\phi = 0.995$, which ensures a very slow transition to the corner solution. As we assume that there is always some transfer level to all workers, we have $\tau_L = \tau_H = 0.13$. To calibrate the production side of the economy, as described in Subsection 4.3.4, we follow Ribeiro and Prettner (2025), with $\alpha = 0.33$, $\gamma = 0.33$, $\theta = 0.75$, $\sigma = 0.5$, $\beta_1 = 0.9$, $\beta_2 = 0.95$, and $\beta_3 = 0.66$, while letting $\tau_X = \tau_P = \tau_G = 0$. For government and institutions, we make $\chi = 2$ (transfers will be neutral the closer their ratio is to the skill premium), $E_{\min} = 0.03$, $\kappa_E = 0.03$, $\kappa_L = \kappa_H = 0$, $\rho_\tau = \rho_T = \rho_E = \rho_L = \rho_r = \rho_a = 1$, $\phi_P = 0.2$, and $\bar{P} = 20L$. Both Prettner and Strulik (2020) and Ribeiro and Prettner (2025) target the United States economy after the second half of the 20th century.

4.4.2. Role of institutional quality

Institutional quality (4.28) amplifies the benefits of AI adoption by enhancing R&D efficiency and ensuring equitable distribution of economic gains. The equation governing technological growth is (4.8), where $\bar{\delta}$ (4.9) is the skilled productivity in the process of new inventions discovery. In the current model, skilled labor is the only input used to increment technological knowledge, and the decision of an individual to remain unskilled or to become skilled and increase the supply of skills in the economy is determined by

(4.7). Figure 4.3 illustrates the effects of a raise in Ω, as a consequence of an increase in the ratio of unskilled to skilled workers' transfers from the government, $\frac{T_{L,t}}{T_{H,t}}$, on the aggregate output growth rate (4.31), the relative skilled population, $\frac{L_{H,t}}{L}$, R&D intensity (4.12), skill premium (4.5).

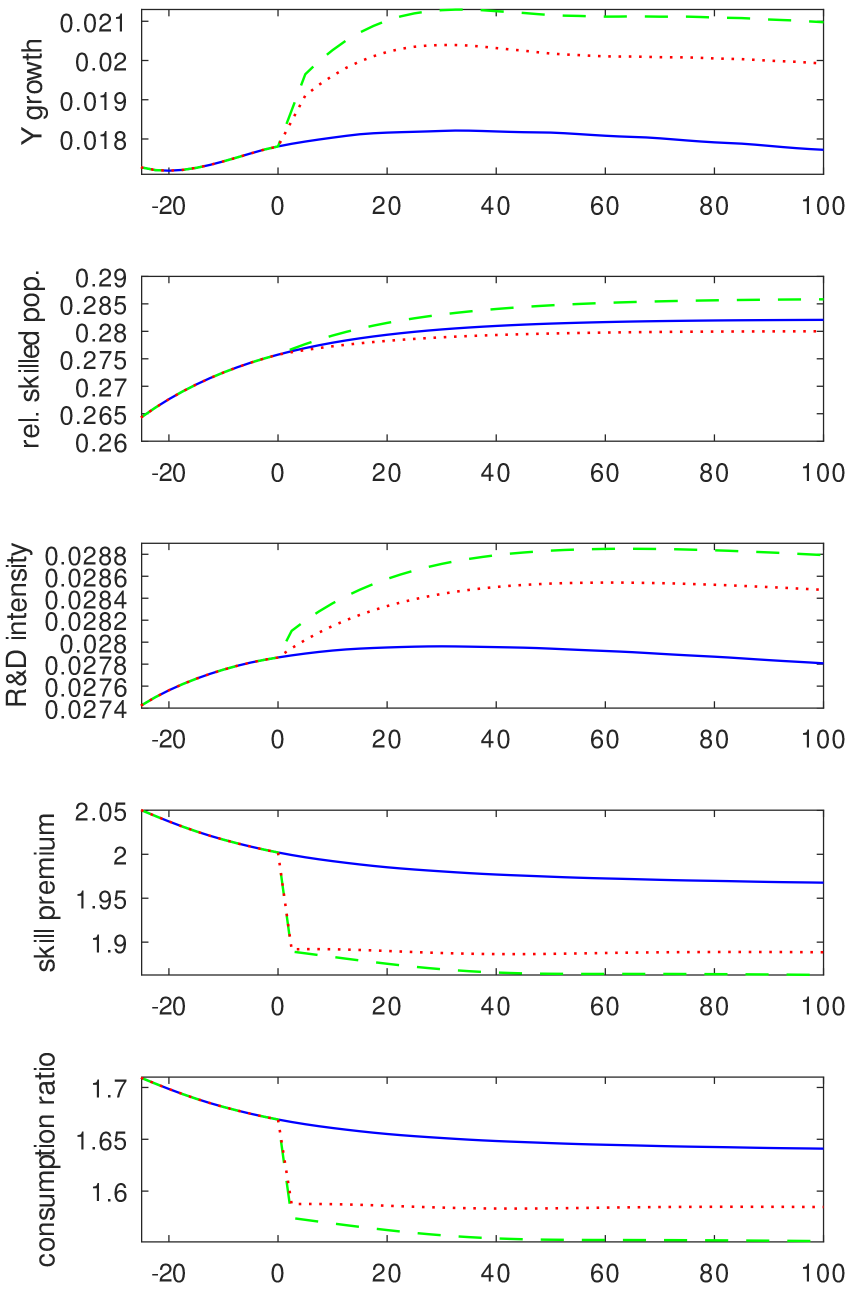

FIGURE 4.3: Changes in the transition path case AI is introduced in the economy at period 0. The solid blue line represents the path if this type of skilled labor automation is not introduced, while the green dashed line highlights the path if government transfers become more redistributive.

The introduction of automation for skilled labor raises output and R&D intensity, indicating a higher rise in skilled wages than in the overall economy. However, the gains for skilled labor substituted by AI are smaller than the gains for unskilled labor that complements such activities. The pressure on relative wages can be verified by analyzing the skill premium. These movements ultimately lead to a reduction in the incentives for individuals to attend college, thereby decreasing the proportion of the skilled population.

An increase in institutional quality, caused by redistributive policies, despite reducing the skill premium and the relative consumption of skilled workers, leads to a less unequal society and promotes more opportunities, allowing more individuals to access tertiary education and acquire skills. Better institutions facilitate R&D, thereby increasing the demand for skilled workers.

4.4.3. Substitutability between AI and labor

The parameter σ, which governs the substitutability between AI and skilled labor, $L_{H,Y}$, plays a central role in determining the distributional impacts of AI adoption. Figure 4.4 plots the variables' paths for the cases in which AI adoption happens, and skilled labor and AI substitutes more or less for each other in the $L_{H,\text{CES}}$ output than unskilled ($L_{L,\text{CES}}$) and skilled ($L_{H,\text{CES}}$) sector outputs substitute in the aggregate (L_{CES}), i.e., case $\sigma > \gamma$ or $\sigma < \gamma$.

High σ implies that AI replaces skilled labor more effectively, reducing the marginal product for this labor type. This results in a relatively lower w_H, skill premium, and less inequality when measured in terms of relative consumption. However, the gains in output are counterbalanced by the reduction in the supply of skills. As depicted in Figure 4.3, wages grow more rapidly than aggregate output, leading to an increase in R&D intensity. In turn, a σ below the elasticity of substitution between sectors, γ, implies that AI acts as a complement to $L_{H,Y}$, enhancing productivity without significant pressure on skilled labor. The increase in skilled labor income provides incentives for more individuals to acquire skills.

Mitigating losses. A variation of the scenario analyzed in Figure 4.4 occurs in the case of greater substitutability of AI for skilled labor, where individuals must give up wages to spend time reskilling for activities not yet automated; that is, η_H rises when AI is introduced into the economy. In such a situation, the government could redirect expenditures paid in the form of transfers to reskilling activities that mitigate the losses of skilled workers and reduce the decline in skilled labor supply. Such a situation is presented in Figure 4.5.

To mitigate the adverse effects of an additional investment in reskilling activities by workers as a consequence of labor substitution by AI, we simulate in the model the mechanism that compensates workers for the time spent reskilling. When the government chooses to use collected taxes to invest in reskilling workers affected by automation, the opportunities to acquire skills and conduct R&D rise in response to the improved institutional environment. This process highlights the crucial role of institutions in the economy's capacity to adapt to technological change.

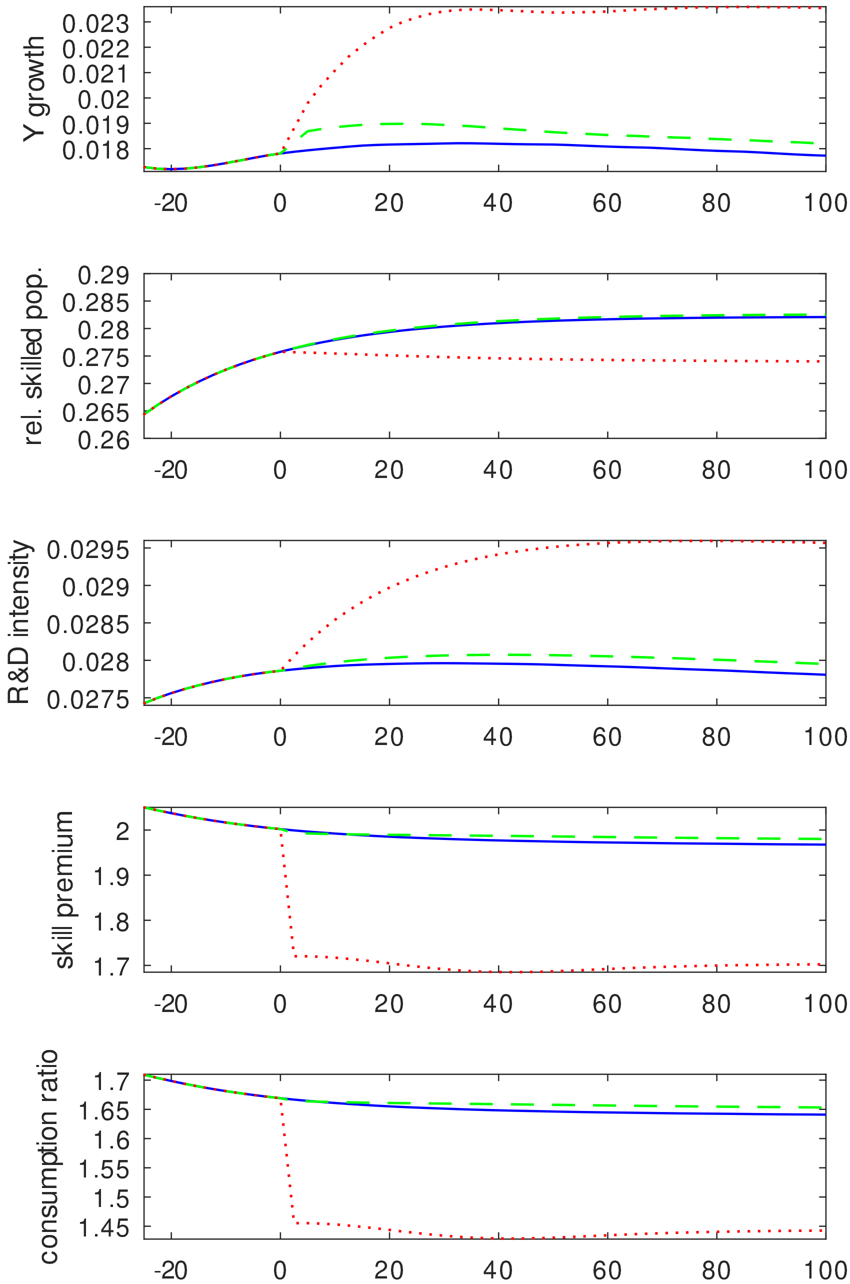

FIGURE 4.4: Changes in the transition path case AI is introduced in the economy at period 0 under different skilled labor-AI elasticity of substitution. The solid blue line represents the path if this type of skilled labor automation is not introduced, while the green dashed (red dotted) line highlights the path if skilled labor and AI are less (more) substitutable than the output of the skilled and unskilled sectors.

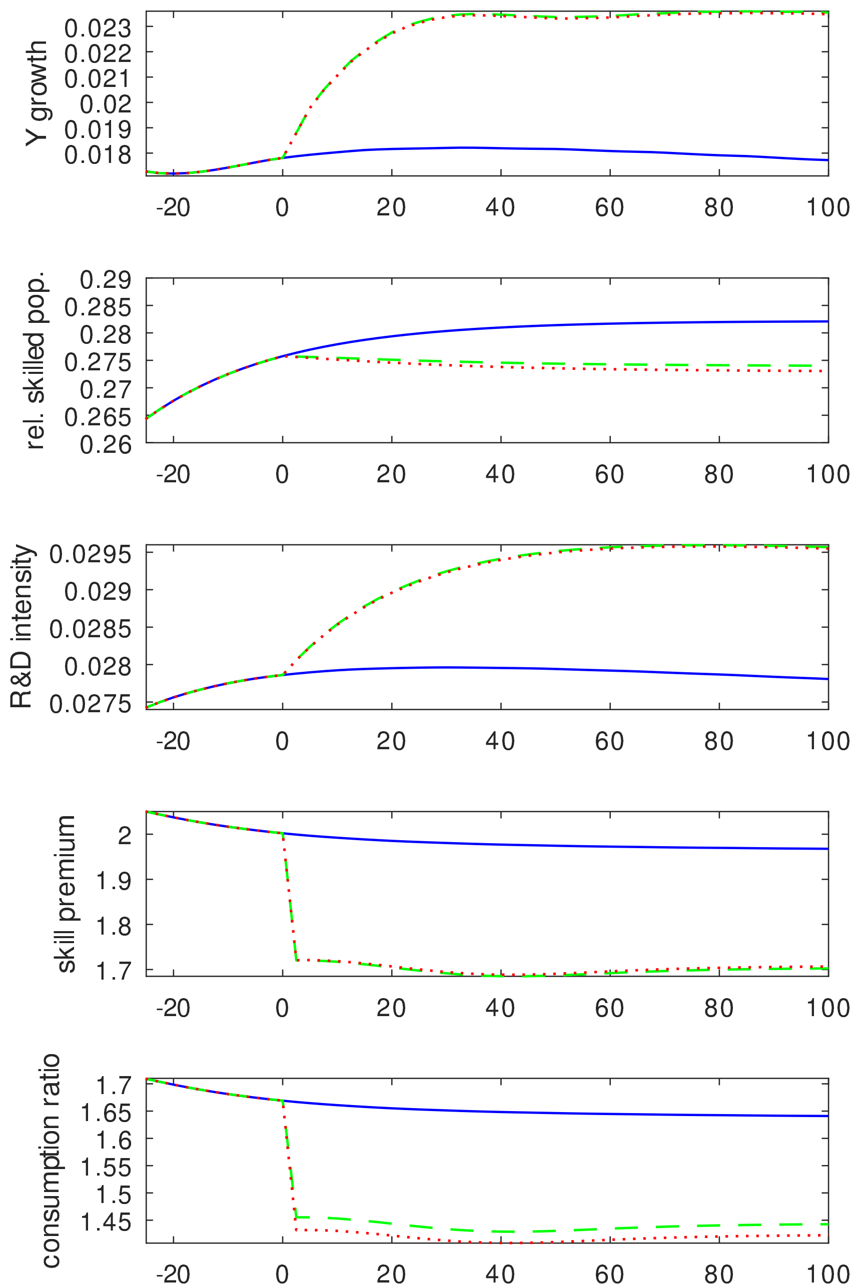

FIGURE 4.5: Changes in the transition path case AI is introduced in the economy at period 0 under different investment in workers' reskilling. The solid blue line represents the path if this type of skilled labor automation is not introduced, while the green dashed (red dotted) line highlights the path if there is (there is not) investment in reskilling for the affected labor type.

4.5. Concluding remarks

This paper develops a comprehensive endogenous growth framework to study the macroeconomic and distributional implications of Artificial Intelligence (AI). By modeling AI as a technology that can both complement and substitute for human labor, the analysis highlights how AI influences production, innovation, and the supply of skills. A central contribution of the model is to incorporate the idea that these effects are not entirely predetermined by technology: instead, institutions play a significant role in mediating them.

The framework captures AI's dual impact on labor markets. When AI complements human tasks, productivity, wages, and innovation incentives rise; when it substitutes for skilled labor, wage pressures emerge, incentives to acquire skills weaken, and inequality can intensify. The net effect depends mainly on the elasticity of substitution between AI and labor, as well as on how institutions affect workers' ability to adapt. In the absence of strong institutions, technological progress may follow a trajectory of faster growth but declining skill supply and rising polarization. By contrast, in economies with inclusive institutions, AI adoption can reinforce innovation while sustaining human capital accumulation.

A key feature of the model is that institutions operate through multiple, mutually reinforcing channels. Redistributive policies and progressive taxation moderate wage dispersion and smooth transition costs; better social protection, combined with investments in education and reskilling policies, expand the supply of skills; and stronger institutional environments enhance R&D productivity. These mechanisms generate feedback loops in which inclusive institutions support innovation and growth, which in turn strengthen the political and economic foundations for further institutional investment. As a result, institutional quality becomes a central determinant of whether AI leads to inclusive and sustainable growth or to persistent inequality.

By explicitly incorporating both AI and robots into an endogenous growth model with skill choice and institutional feedback, this paper advances the literature on automation and growth. The framework highlights tradeoffs individuals face when deciding whether to acquire skills in an economy where wage structures depend on automation. It clarifies the policy role in shaping these incentives. In doing so, the analysis bridges strands of research on skill acquisition, automation, and institutions within a unified general-equilibrium setting.

Several routes for future research emerge from this work. Alternative specifications for the diffusion and growth of automation technologies could be explored, as could more parsimonious or empirically grounded measures of institutional quality. Empirical validation using cross-country or sector-level data would be particularly valuable for testing the model's predictions and quantifying the strength of the institutional channel. Extending the framework to incorporate environmental considerations or endogenous and different returns for investments in AI and robots could further enhance its relevance.

Similar to previous GPTs, AI represents a profound technological shift whose economic consequences will unfold in the years to come. This paper underscores that the distribution of its gains is not an inevitable outcome of technology alone, but a policy-contingent result modulated by institutions, education systems, and redistribution. By strengthening institutions and investing in human capital, societies can harness AI's transformative potential while promoting inclusive, innovation-driven long-run development.

5. Concluding remarks

This thesis examined the roles of fiscal policy, monetary policy, and institutions in shaping long-run economic growth and wage dynamics within endogenous-growth frameworks featuring technological change and labor heterogeneity. Across three interconnected theoretical essays, the analysis emphasized that the effects of technological progress on growth, inequality, and welfare are mediated and moderated by policy design, institutional quality, and labor-market structures.

The first essay used a knowledge-driven, semi-endogenous growth model in which innovation relies on skilled labor and is subject to liquidity constraints to investigate the interaction between fiscal and monetary policy. By introducing cash-in-advance requirements for production and R&D alongside R&D subsidies, the model incorporated frictions through which policy can affect real economic variables even in the absence of long-run growth effects. The analysis showed that, under mild and empirically plausible conditions, fiscal policy (through direct R&D subsidies) and monetary policy (through inflation control) generate qualitatively similar long-run effects on R&D intensity, the skill premium, the velocity of money, output, and consumption.

Importantly, when jointly implemented, these policies can become complementary rather than merely substitutable, with the most substantial welfare-relevant effects arising along the transition path following policy changes. Welfare complementarity depends critically on labor-market characteristics and on how policy-induced gains in aggregate output compare with associated changes in consumption and intermediate-goods production costs. When labor supply is elastic, and households experience disutility from work, coordinated fiscal and monetary interventions are more likely to yield welfare gains even in the long run. These results underscore the importance of evaluating policy within a dynamic welfare framework, rather than relying exclusively on steady-state comparisons.

The second essay extended the analysis of innovation-driven growth by introducing labor unions into a framework of directed technical change. Addressing a gap in the literature, it jointly analyzed the effects of union bargaining power on wage inequality, the income–labor share, and economic growth. A key contribution is to show that the impact of unions is inherently context-dependent, hinging on whether unions are employment-oriented or wage-oriented and on whether technological-knowledge or labor-supply channels dominate the relative wage determination. Employment-oriented unions can foster unskilled employment, raise labor's income share, and support growth by mitigating excessive skill-biased innovation, whereas wage-oriented unions may reinforce inequality. Empirical evidence from a multi-country panel supported these theoretical predictions, helping reconcile divergent findings in the existing literature.

The third essay examined artificial intelligence as a general-purpose technology with far-reaching implications for growth and inequality. Modeling AI as both a complement to and a substitute for human labor, while explicitly accounting for institutional quality, the analysis captured key features of contemporary automation. While AI adoption can enhance productivity and accelerate long-run growth, its distributional consequences depend critically on the quality of economic and social institutions. Effective governance, inclusive education systems, and redistributive mechanisms enable societies to harness AI for inclusive growth, whereas weak institutions amplify polarization and inequality.

From a broader theoretical perspective, the three essays underscore the importance of transitional dynamics in understanding welfare-relevant outcomes. Across different mechanisms and institutional settings, the most consequential effects of policy changes, institutional arrangements, and technological breakthroughs often arise during periods of economic adjustment rather than solely in long-run equilibria. During these transitions, fiscal–monetary coordination, labor-market structures, and institutional quality play especially prominent roles.

Several avenues for future research naturally follow. Empirically testing the conditions for policy complementarity and institutional mediation highlighted here would be particularly valuable, given recent changes in labor shares, the diffusion of automation, and policy coordination. Extensions incorporating declining long-run labor supply, demographic change, or endogenous labor-force participation could further enrich the analysis of labor-market frictions and welfare. Finally, allowing for heterogeneous firms, international technology diffusion, or environmental constraints would broaden the framework's applicability to contemporary macroeconomic challenges.

Overall, this thesis demonstrates that innovation-driven growth is inseparable from institutional context and policy design. Fiscal and monetary policies, labor-market institutions, and governance structures jointly shape not only the pace of technological progress but also its distributional consequences. Understanding these interactions is therefore essential to achieving sustained and more inclusive economic development in an era of rapid technological change.

Appendices

Appendix A
Intermediate-goods profit maximization

Substituting (2.3) in (2.6), the problem an intermediate sector firm has to solve is

$$\Pi_j(t) = \max_{X_j(t)} \quad (1-\alpha-\beta)L_L(t)^{\alpha}L_{H_Y}(t)^{\beta}X_j(t)^{1-\alpha-\beta} - (1 - z_x + \Omega_x \iota)\,\eta X_j(t),$$

and its first-order condition is $\frac{\partial \Pi_j(t)}{\partial X_j(t)} = 0 \Leftrightarrow X_j(t)^{\alpha+\beta} = \frac{(1-\alpha-\beta)^2 L_L(t)^{\alpha} L_{H_Y}(t)^{\beta}}{(1-z_x+\Omega_x\iota)\eta}$. The number of goods each firm j will produce does not rely on any variable specific to this firm or good, which means that it will be the same for all firms and given by (2.7). Consequently, substituting (2.3) in (2.7), we find that the real price of any intermediate good will be the same and given by (2.8).

Appendix B
Investment decision

It is possible to solve the equation (2.10) differentiating both sides w.r.t. t. First, let us assume it exists a primitive function $R(t)$, for which the derivative is $r(t)$, and deal with the discount factor:

$$\begin{aligned}\frac{\partial}{\partial t}\left[e^{-\int_t^{\upsilon} r(\omega)d\omega}\right] &= e^{-\int_t^{\upsilon} r(\omega)d\omega}\frac{\partial}{\partial t}\left[-\int_t^{\upsilon} r(\omega)d\omega\right] = e^{-\int_t^{\upsilon} r(\omega)d\omega}\frac{\partial}{\partial t}\left[R(t)-R(\upsilon)\right]\\ &= e^{-\int_t^{\upsilon} r(\omega)d\omega}\left[1\,r(t)-0\,r(\upsilon)\right] = e^{-\int_t^{\upsilon} r(\omega)d\omega}r(t).\end{aligned}$$

Then, apply the product's rule to differentiate the expression under the integral sign:

$$\begin{aligned}\frac{\partial}{\partial t}\left[\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}\right] &= \frac{\partial}{\partial t}\left[\Pi(\upsilon)\right]e^{-\int_t^{\upsilon} r(\omega)d\omega} + \Pi(\upsilon)\frac{\partial}{\partial t}\left[e^{-\int_t^{\upsilon} r(\omega)d\omega}\right]\\ &= 0+\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}r(t) = r(t)\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}.\end{aligned}$$

Furthermore, use Leibniz´s formula, see Sydsæter et al. (2008, pp. 154–155), to differentiate the l.h.s. of equation w.r.t. t:

$$\begin{aligned}\frac{d}{dt}\left[\int_t^{\infty}\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}d\upsilon\right] &= -\Pi(t)e^{-\int_t^{t} r(\omega)d\omega}\frac{d}{dt}[t] + \int_t^{\infty}\frac{\partial}{\partial t}\left[\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}\right]d\upsilon\\ &= -\Pi(t)e^{0} + \int_t^{\infty} r(t)\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}d\upsilon\\ &= -\Pi(t)+r(t)Z(t).\end{aligned}$$

Solving the time-differentiation of the intermediate firm entry condition equation:

$$\begin{aligned}\frac{d}{dt}\left[\int_t^{\infty}\Pi(\upsilon)e^{-\int_t^{\upsilon} r(\omega)d\omega}d\upsilon\right]r(t) &= \frac{d}{dt}\left[Z(t)\right] \Leftrightarrow\\ -\Pi(t)+r(t)Z(t) &= \dot{Z}(t) \Leftrightarrow\\ r(t) &= \frac{\Pi(t)}{Z(t)}+\frac{\dot{Z}(t)}{Z(t)}.\end{aligned}$$

Appendix C
Real interest rate

Finding $\frac{\Pi}{Z}$. The optimal profit of any intermediate sector firm is found by substituting (2.8) and (2.7) in the expression for the firm profit:

$$\begin{aligned}\Pi(t) &= p(t)X(t) - \eta\,(1 - z_x + \Omega_x \iota)\,X(t) \\ &= \eta^{\frac{\alpha+\beta-1}{\alpha+\beta}}\,(\alpha+\beta)\,(1-\alpha-\beta)^{\frac{2-\alpha-\beta}{\alpha+\beta}}\,(1 - z_x + \Omega_x \iota)^{\frac{\alpha+\beta-1}{\alpha+\beta}} && \text{(C.1)} \\ &\times (1-s)^{\frac{\alpha}{\alpha+\beta}}\, s^{\frac{\beta}{\alpha+\beta}}\,\big(1-u(t)\big)^{\frac{\beta}{\alpha+\beta}}\,L(t). && \text{(C.2)}\end{aligned}$$

Considering $w_{H_Y}(t) = w_{H_N}(t)$ and substituting (2.5) in (2.9) it is possible to find the invention cost:

$$\begin{aligned}Z(t) &= (1 - z_r + \Omega_r \iota)\,\frac{w_{H_N}(t)}{\delta N(t)^{\phi}} \\ &= \beta\delta^{-1}\eta^{\frac{\alpha+\beta-1}{\alpha+\beta}}\,(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}\,(1 - z_x + \Omega_x \iota)^{\frac{\alpha+\beta-1}{\alpha+\beta}}\,(1 - z_r + \Omega_r \iota)\times \\ &\quad (1-s)^{\frac{\alpha}{\alpha+\beta}}\, s^{\frac{-\alpha}{\alpha+\beta}}\,\big(1-u(t)\big)^{\frac{-\alpha}{\alpha+\beta}}\,N(t)^{1-\phi}. && \text{(C.3)}\end{aligned}$$

Bearing in mind that $G_N(t) = \delta u(t) s L(t) N(t)^{\phi-1}$ and $T(t) \equiv \frac{1-u(t)}{u(t)}$, divide (C.2) by (C.3), multiply the result by $\frac{u(t)}{u(t)}$, and rearrange the terms to find

$$\frac{\Pi(t)}{Z(t)} = \frac{(\alpha+\beta)\,(1-\alpha-\beta)\,s\,\big(1-u(t)\big)\,L(t)}{\beta\delta^{-1}\,(1 - z_r + \Omega_r \iota)\,N(t)^{1-\phi}}\,\frac{u(t)}{u(t)} = \frac{(\alpha+\beta)\,(1-\alpha-\beta)}{\beta\,(1 - z_r + \Omega_r \iota)}\,G_N(t)T(t). \tag{C.4}$$

Finding $\frac{\dot{Z}(t)}{Z(t)}$. Recall $G_T(t) \equiv \frac{(1-\dot{u}(t))}{(1-u(t))}$ and log-differentiate (C.3):

$$\frac{\dot{Z}(t)}{Z(t)} = (1-\phi)\,\frac{\dot{N}(t)}{N(t)} - \frac{\alpha}{\alpha+\beta}\,\frac{\big(1-\dot{u}(t)\big)}{\big(1-u(t)\big)} = (1-\phi)\,G_N(t) - \frac{\alpha}{\alpha+\beta}\,G_T(t). \tag{C.5}$$

Real interest rate. Substituting (C.4) and (C.5) in $r(t) = \frac{\Pi(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)}$ gives (2.12).

Appendix D
Skill premium

Using (2.7), (2.4) can be rewritten as

$$
\begin{aligned}
w_L(t) &= \alpha L_L(t)^{\alpha-1} L_{H_Y}(t)^{\beta} \int_0^{N(t)} X(t)^{1-\alpha-\beta} dj = \alpha L_L(t)^{\alpha-1} L_{H_Y}(t)^{\beta} X(t)^{1-\alpha-\beta} N(t) \\
&= \alpha \eta^{\frac{\alpha+\beta-1}{\alpha+\beta}} (1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}} (1-z_x+\Omega_x \iota)^{\frac{\alpha+\beta-1}{\alpha+\beta}} \\
&\quad \times (1-s)^{\frac{-\beta}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}} \big(1-u(t)\big)^{\frac{\beta}{\alpha+\beta}} N(t),
\end{aligned} \tag{D.1}
$$

and (2.5) as

$$
\begin{aligned}
w_{H_Y}(t) &= \beta L_L(t)^{\alpha} L_{H_Y}(t)^{\beta-1} \int_0^{N(t)} X(t)^{1-\alpha-\beta} dj = \beta L_L(t)^{\alpha} L_{H_Y}(t)^{\beta-1} X(t)^{1-\alpha-\beta} N(t) \\
&= \beta \eta^{\frac{\alpha+\beta-1}{\alpha+\beta}} (1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}} (1-z_x+\Omega_x \iota)^{\frac{\alpha+\beta-1}{\alpha+\beta}} \\
&\quad \times (1-s)^{\frac{\alpha}{\alpha+\beta}} s^{\frac{-\alpha}{\alpha+\beta}} \big(1-u(t)\big)^{\frac{-\alpha}{\alpha+\beta}} N(t).
\end{aligned} \tag{D.2}
$$

Then, as $w_{H_Y}(t) = w_{H_N}(t) = w_H(t)$, dividing (D.2) by (D.1), equation (2.13) is found.

Appendix E
Household's problem

The unique representative household will grow at the same rate n of population, and, say that $\mathscr{H}(t)$ is the size of each household, with $\dot{\mathscr{H}}(t)/\mathscr{S}(t) = n \Leftrightarrow \mathscr{H}(t) = \mathscr{H}(0)e^{nt}$, then solving the household's infinite horizon optimization is the same as solving the problem of a representative individual time-discounted by $n-\rho$, that is, $\max \int_0^\infty \mathscr{H}(t)U(t)e^{-\rho t}dt \Leftrightarrow \max \int_0^\infty U(t)e^{(n-\rho)t}dt$. Though, the intertemporal representative household's problem can be stated by (2.15), subject to (2.16) and (2.17), with the transversality conditions

$$\lim_{t\to\infty}\mu(t)a(t) = 0 \text{ and } \lim_{t\to\infty}\mu(\mathrm{t})\mathrm{m}(\mathrm{t})\text{=}0.$$

Hamiltonian. For this optimal control problem, the following Hamiltonian can be defined:

$$\begin{aligned} H = {} & \left(\frac{c(t)^{1-\theta}-1}{1-\theta}\right)e^{(n-\rho)t} + \lambda(t)\left[b(t)+\xi c(t)-m(t)\right] \\ & + \mu(t)\left[(1-\tau_H)sw_H(t)+(1-\tau_L)(1-s)w_L(t)\right. \\ & \qquad \left. + (1-\tau_a)r(t)a(t)+\iota b(t)-c(t)-na(t)-nm(t)-\pi(t)m(t)+\tau(t)\right]. \end{aligned}$$

First-order conditions. The necessary conditions under the Maximum principle are

$$\frac{\partial H}{\partial c(t)} = 0 \Leftrightarrow \mu(t) = c(t)^{-\theta}e^{(n-\rho)t}+\lambda(t)\xi, \tag{E.1}$$

$$\frac{\partial H}{\partial b(t)} = 0 \Leftrightarrow \mu(t)\iota = -\lambda(t), \tag{E.2}$$

$$\frac{\partial H}{\partial a(t)} = -\dot{\mu}(t) \Leftrightarrow \frac{\dot{\mu}(t)}{\mu(t)} = n-(1-\tau_a)r(t), \tag{E.3}$$

$$\frac{\partial H}{\partial m(t)} = -\dot{\mu}(t) = -\mu(t)n-\mu(t)\pi(t)-\lambda(t), \text{ and} \tag{E.4}$$

$$\frac{\partial H}{\partial \mu(t)} = \dot{a}(t)+\dot{m}(t),\ \frac{\partial H}{\partial \lambda(t)} = 0.$$

Euler equation for consumption. Log-differentiating (E.1) w.r.t. t gives $\frac{\dot{\mu}(t)}{\mu(t)} = -\theta\frac{\dot{c}(t)}{c(t)} + n-\rho$, and substituting this in (E.3) gives (2.20).

Fisher equation. Substituting (E.2) in (E.4) gives $\frac{\dot{\mu}(t)}{\mu(t)} = \pi(t) + n - \iota$, which plugged in (E.3) leads to (2.19).

Appendix F
Transversality conditions

The transversality conditions are $\lim_{t\to\infty}\mu(t)a(t)=0$ and $\lim_{t\to\infty}\mu(t)m(t)=0$, and several restrictions on the values that parameters and variables can assume to satisfy them will be shown. First, we will find the equilibrium expressions that define μ, a, and m as a function of L (that is exogenously defined and will always be positive), and the variables u and G_N. Then, we will analyze which relations in these expressions are necessary to avoid a division by zero, i.e., that could make those limits indeterminate or go to infinity, while considering the model assumptions about the parameters and variables and their economic meaning. After that, the relations that ensure the limits' convergence to zero are analyzed.

Equilibrium expressions. From (E.1) and (E.2), we find μ; then, considering (2.14), (2.21), that (2.17) holds, $w_{H_Y}(t)=w_{H_N}(t)$, $a(t)=\frac{A(t)}{L(t)}=\frac{Z(t)N(t)}{L(t)}$, and that rearranging (2.11) we find $N(t)=\left(\frac{\delta L_{H_N}(t)}{G_N(t)}\right)^{\frac{1}{1-\phi}}$, we have

$$\begin{aligned}\mu(t)&=e^{(n-\rho)t}\left(1+\iota\xi\right)^{-1}c(t)^{-\theta}\\&=e^{(n-\rho)t}\left(1+\iota\xi\right)^{-1}\left[1-(1-\alpha-\beta)^2\left(1-z_x+\Omega_x\iota\right)^{-1}\right]^{-\theta}y(t)^{-\theta},\end{aligned}\tag{F.1}$$

$$\begin{aligned}a(t)&=\delta^{-1}\beta\left(1-z_r+\Omega_r\iota\right)s^{-1}\left(1-u(t)\right)^{-1}N(t)^{1-\phi}L(t)^{-1}\times\\&\qquad\frac{(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}}s^{\frac{\beta}{\alpha+\beta}}\left(1-u(t)\right)^{\frac{-\alpha}{\alpha+\beta}}N(t)}{\eta^{\frac{\alpha+\beta-1}{\alpha+\beta}}\left(1-z_x+\Omega_x\iota\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}}\\&=\beta\left(1-z_r+\Omega_r\iota\right)\left(1-u(t)\right)^{-1}u(t)G_N(t)^{-1}y(t),\text{ and}\end{aligned}\tag{F.2}$$

$$\begin{aligned}m(t)&=b(t)+\xi c(t)\\&=\left\{\frac{\Omega_x(1-\alpha-\beta)^{\frac{1}{1-\alpha-\beta}}}{1-z_x+\Omega_x\iota}+\frac{\Omega_r\beta u(t)}{1-u(t)}+\xi\left[1-\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\right]\right\}y(t),\text{ with}\end{aligned}\tag{F.3}$$

$$y(t)=\frac{(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}}s^{\frac{\beta}{\alpha+\beta}}\left(1-u(t)\right)^{\frac{\beta}{\alpha+\beta}}}{\eta^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(1-z_x+\Omega_x\iota\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}}\left(\frac{\delta L_{H_N}(t)}{G_N(t)}\right)^{\frac{1}{1-\phi}}.\tag{F.4}$$

From (2.29) and (2.30), we have

$$u^*=\frac{n(1-\tau_a)(\alpha+\beta)(1-\alpha-\beta)}{\beta\left(1-z_r+\Omega_r\iota\right)\left\{(1-\phi)\rho+n\left[\theta-(1-\phi)(1-\tau_a)\right]\right\}+n(1-\tau_a)(\alpha+\beta)(1-\alpha-\beta)}.\tag{F.5}$$

Avoiding divisions by zero Recall that u is a population share and notice that in (F.2) and (F.3) its exponents will never be negative, but $1-u(t)$ has negative exponents, then it is necessary that $0 \leq u(t) < 1$, which implies $T(t) = \frac{1-u(t)}{u(t)} > 0$. Notice that $n(1-\tau_a)(\alpha+\beta)(1-\alpha-\beta) > 0$ and

$$(1-\phi)\rho + n\left[\theta - (1-\phi)(1-\tau_a)\right] = (1-\phi)\left[\rho - n(1-\tau_a)\right] + n\theta > 0.$$

Considering (2.30) in satisfying $T(t) > 0$, it is necessary that

$$1 - z_r + \Omega_r \iota > 0.$$

Consequently, the denominator of (F.5) will be strictly positive and $u^* > 0$. Relation (F.1) requires $y(t) > 0$ (there is no negative production), and, from (F.4), an economy with any production level requires $u(t) > 0$ and $G_N(t) > 0$ at any instant. Notice that the restriction on the sign of G_N avoids a negative y and is also necessary to avoid a transition path free of discontinuities, i.e., $G_N(t) \neq 0$, to its positive steady-state value. We also have that $a(t) \geq 0$ because it would be meaningless a negative level of assets, and to satisfy this and avoid a division by zero in (F.2) and (F.3), it is necessary that $1 - z_x + \Omega_x \iota > 0$, a condition already satisfied case

$$1 - z_x + \Omega_x \iota > (1-\alpha-\beta)^2,$$

which is necessary to satisfy $u(t) > 0$ and $c(t) \leq y(t)$ (see Appendix G).

Convergence to zero Recall that $\frac{\dot{\mu}(t)}{\mu(t)} = -\theta\frac{\dot{c}(t)}{c(t)} + n - \rho$ (see Appendix E), and, in the long run, $\frac{\dot{m}(t)}{m(t)} = \frac{\dot{a}(t)}{a(t)} = \frac{\dot{y}(t)}{y(t)} = \frac{\dot{c}(t)}{c(t)} = G_N^*$ (see Appendix N). Then, to simplify the notation, consider $t = 0$ the instant at which the economy achieves the steady state, such that $\mu(t) = \mu(0)e^{(-\theta G_N^* + n - \rho)t}$, $a(t) = a(0)e^{G_N^* t}$, and $m(t) = m(0)e^{G_N^* t}$, with $\mu(0)$, $a(0)$, and $m(0)$ computed from (F.1), (F.2), and (F.3), respectively, with (F.4) considered and $L(t)$, $u(t)$, and $G_N(t)$ known. With these relations, in the steady state $\mu(t)a(t) = \mu(0)a(0)e^{[(1-\theta)G_N^* + n - \rho]t}$ and $\mu(t)m(t) = \mu(0)m(0)e^{[G_N^*(1-\theta) + n - \rho]t}$, which implies that even if stationarity is achieved at a different instant than zero, to satisfy the transversality conditions, it is necessary that

$$(1-\theta)G_N^* + n - \rho < 0, \tag{F.6}$$

which is the assumed convergence condition for the integral in the household's problem. Notice that, considering (2.28), (F.6) implies $(1-\theta)n + (n-\rho)(1-\phi) < 0$, which is always true for $\theta > 1$, but requires $\theta > 1 - \left(\frac{\rho}{n} - 1\right)(1-\phi)$ and $\phi > 1 - \frac{1}{\frac{\rho}{n} - 1}$, case $0 < \theta < 1$.

Appendix G
Macroeconomic aggregation and consumption

Budget constraint aggregation. Substituting the *lump sum* from the *seigniorage* revenues (2.24) in the household's budget constraint (2.16), expressed in *per capita* terms; multiplying all terms by $L(t)$; substituting $B(t)$ for the r.h.s. of (2.18); substituting the terms for the collected taxes for the r.h.s. of (2.22); substituting $\dot{a}(t)L(t) = \dot{A}(t) - nA(t)$; substituting $sw_H(t)L(t) = w_{H_Y}(t)L_{H_Y}(t) + w_{H_N}(t)L_{H_N}(t)$; and, rearranging the resulting expression, gives

$$\begin{aligned}\dot{A}(t) &= w_{H_Y}(t)L_{H_Y}(t) + w_L(t)L_L(t) + \iota\Omega_x\eta N(t)X(t) - z_x\eta N(t)X(t)\\ &\quad + w_{H_N}(t)L_{H_N}(t) + \iota\Omega_r w_{H_N}(t)L_{H_N}(t) - z_r w_{H_N}(t)L_{H_N}(t) - C(t) + r(t)A(t).\end{aligned} \tag{G.1}$$

Using $r(t) = \frac{\Pi(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)}$, $A(t) = Z(t)N(t)$, and $\Pi(t) = p(t)X(t) - (1 - z_x + \Omega_x\iota)\,\eta X(t)$ we have that

$$r(t)A(t) = p(t)N(t)X(t) - \eta N(t)X(t) + z_x\eta N(t)X(t) - \iota\Omega_x\eta N(t)X(t) + \dot{Z}(t)N(t). \tag{G.2}$$

As the final goods sector operates with zero profit, we have that

$$w_L(t)L_L(t) + w_{H_Y}(t)L_{H_Y}(t) = Y(t) - p(t)N(t)X(t). \tag{G.3}$$

Once Z (2.9) is the cost to increase N by one unit, the amount necessary to pay for R&D activities, at any instant, can be represented as $\dot{N}(t)Z(t) = (1 - z_r + \Omega_r\iota)\,L_{H_N}(t)w_{H_N}(t)$, and we have that

$$A(t) = Z(t)N(t) \Rightarrow \dot{A}(t) - \dot{Z}(t)N(t) = \dot{N}(t)Z(t) = (1 - z_r + \Omega_r\iota)\,L_{H_N}(t)w_{H_N}(t). \tag{G.4}$$

Then, substituting (G.2), (G.3), and (G.4) in (G.1) results in

$$C(t) = Y(t) - \eta N(t)X(t). \tag{G.5}$$

Consumption as a share of output. Notice that, using (2.7) and (2.14),

$$\frac{\eta N(t)X(t)}{L(t)} = \eta N(t)\frac{(1-\alpha-\beta)^{\frac{2}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}}s^{\frac{\beta}{\alpha+\beta}}\left(1-u(t)\right)^{\frac{\beta}{\alpha+\beta}}}{\eta^{\frac{1}{\alpha+\beta}}(1-z_x+\Omega_x\iota)^{\frac{1}{\alpha+\beta}}} = y(t)\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}. \tag{G.6}$$

Then, dividing (G.5) by $L(t)$ and substituting (G.6) in it gives (2.21), which implies $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)}$ and $u(t) > 0 \wedge c(t) \leq y(t) \Rightarrow (1-\alpha-\beta)^2 < 1 - z_x + \Omega_x \iota$.

Appendix H
Velocity of money circulation

Recall $L_{H_N}(t) = su(t)L(t)$ and $w_{H_Y}(t) = w_{H_N}(t)$, then, substituting (2.5) and (2.7) in (2.18), while considering (2.14) and $Y(t) = y(t)L(t)$, we find

$$\begin{aligned}
B(t) &= \Omega_x \eta^{\frac{\alpha+\beta-1}{\alpha+\beta}} (1-\alpha-\beta)^{\frac{2}{\alpha+\beta}} (1-z_x+\Omega_x \iota)^{-\frac{1}{\alpha+\beta}} \times \\
&\quad (1-s)^{\frac{\alpha}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}} \big(1-u(t)\big)^{\frac{\beta}{\alpha+\beta}} L(t)N(t) \\
&\quad + \Omega_r \eta^{\frac{\alpha+\beta-1}{\alpha+\beta}} \beta (1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}} (1-z_x+\Omega_x \iota)^{\frac{\alpha+\beta-1}{\alpha+\beta}} \times \\
&\quad (1-s)^{\frac{\alpha}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}} \big(1-u(t)\big)^{\frac{-\alpha}{\alpha+\beta}} u(t)L(t)N(t) \\
&= \left[\frac{\Omega_x (1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota} + \frac{\Omega_r \beta}{T(t)} \right] Y(t).
\end{aligned}$$

And, considering (2.21) and that the CIA constraint (2.17) binds, we have

$$M(t) = B(t) + \xi C(t) = \left\{ \frac{\Omega_x (1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota} + \frac{\Omega_r \beta}{T(t)} + \xi \left[1 - \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota} \right] \right\} Y(t). \tag{H.1}$$

Substituting (H.1) in $V(t) = \frac{Y(t)}{M(t)}$ yields (2.25).

Appendix I
Dynamic system

Finding G_T. Log-differentiating (2.21) w.r.t. time, while considering (2.14), we find

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)} = \frac{\beta}{\alpha+\beta}\frac{\dot{\overline{(1-u(t))}}}{1-u(t)} + \frac{\dot{N}(t)}{N(t)} = \frac{\beta}{\alpha+\beta}G_T(t) + G_N(t) \Leftrightarrow$$

$$G_T(t) \equiv \frac{\dot{\overline{(1-u(t))}}}{1-u(t)} = \frac{\alpha+\beta}{\beta}\left(\frac{\dot{c}(t)}{c(t)} - G_N(t)\right). \tag{I.1}$$

Variation of G_N. Log-differentiating $G_N(t) = \delta u(t) s L(t) N(t)^{\phi-1}$ w.r.t. time, bearing in mind $L_{H_N}(t) = su(t)L(t)$, $\frac{\dot{L}(t)}{L(t)} = n$, and $\dot{u}(t) = -\dot{\overline{(1-u(t))}} \Rightarrow \frac{\dot{u}(t)}{u(t)} = -\left[\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))}\frac{(1-u(t))}{u(t)}\right]$ $= -G_T(t)T(t)$, results in

$$\frac{\dot{G}_N(t)}{G_N(t)} = n - G_T(t)T(t) - (1-\phi)\,G_N(t). \tag{I.2}$$

Consumption flow. Substituting (2.12) in (2.20) yields

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[(1-\tau_a)\frac{(\alpha+\beta)(1-\alpha-\beta)}{\beta(1-z_r+\Omega_r\iota)}G_N(t)T(t)\right.$$
$$\left. + (1-\tau_a)(1-\phi)\,G_N(t) - (1-\tau_a)\frac{\alpha}{\alpha+\beta}G_T(t) - \rho\right]. \tag{I.3}$$

Equations (I.1) and (I.2), with $\frac{\dot{c}(t)}{c(t)}$ given by (I.3), describe the dynamics of the model. Substituting (I.3) in (I.1) and solving the result for G_T yields (2.26); substituting (2.26) in (I.2) we find (2.27).

Appendix J
Steady state and dynamics

Recall that $T(t) = \frac{1-u(t)}{u(t)} = \frac{1-u(t)}{1-(1-u(t))}$, and define

$$\Lambda_1 \equiv \frac{\alpha+\beta}{\alpha(1-\tau_a)+\beta\theta},$$
$$\Lambda_2 \equiv \frac{(1-\tau_a)(\alpha+\beta)(1-\alpha-\beta)}{\beta(1-z_r+\Omega_r\iota)}, \text{ and}$$
$$\Lambda_3 \equiv \theta-(1-\tau_a)(1-\phi),$$

such that it is possible to rewrite (2.26) and (2.27), respectively, as

$$\dot{\left(1-u(t)\right)} = \left(1-u(t)\right)\Lambda_1\left[\Lambda_2 G_N(t)\frac{1-u(t)}{1-\left(1-u(t)\right)} - \Lambda_3 G_N(t) - \rho\right], \text{ and} \tag{J.1}$$

$$\dot{G}_N(t) = G_N(t)\left\{n-(1-\phi)G_N(t) - \Lambda_1\frac{1-u(t)}{1-\left(1-u(t)\right)}\left[\Lambda_2 G_N(t)\frac{1-u(t)}{1-\left(1-u(t)\right)} - \Lambda_3 G_N(t) - \rho\right]\right\}. \tag{J.2}$$

Phase lines and steady state. Solve (J.1) with $\dot{\left(1-u(t)\right)} = 0$ and (J.2) with $\dot{G}_N(t) = 0$ for G_N to, respectively, obtain the G_N coordinate of the phase lines:

$$G_N(t) = \frac{\rho}{\Lambda_2 T(t) - \Lambda_3}, \text{ and} \tag{J.3}$$

$$G_N(t) = \frac{\rho T(t) + {}^{n}/_{\Lambda_1}}{\Lambda_2 T(t)^2 + \Lambda_3 T(t) + {}^{(1-\phi)}/_{\Lambda_1}}. \tag{J.4}$$

Equalize (J.3) and (J.4) and solve the result for T to find (2.30). Finally, substitute (2.29) and (2.30) in (J.3) to find (2.28).

Unique solution. The point $\left(1-u^*, G_N^*\right)$ is a constant solution (time-independent) of the differential equation system made by (2.26) and (2.27) but there are other two fixed points that solve such system: (i) $\left(0, \frac{n}{1-\phi}\right)$ and $(1,0)$. The economic problem under consideration requires that $0 < u(t) < 1$ and $G_N(t) > 0$ (see Subsection 2.2.2 and Appendix F), what rules out the latter two points as solutions for the optimal control problem. This way, the model has the unique steady-state solution $\left(1-u^*, G_N^*\right)$.

Local stability. The Jacobian for the system made by (J.1) and (J.2), w.r.t. $1-u(t)$ and $G_N(t)$, is

$$J = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix}, \text{ with}$$

$$j_{11} = \Lambda_1 \left\{ \Lambda_2 G_N(t) \left[\left(\frac{1-u(t)}{1-\left(1-u(t)\right)} \right)^2 + 2 \left(\frac{1-u(t)}{1-\left(1-u(t)\right)} \right) \right] - \Lambda_3 G_N(t) - \rho \right\},$$

$$j_{12} = \Lambda_1 \left(1-u(t)\right) \left(\Lambda_2 \frac{1-u(t)}{1-\left(1-u(t)\right)} - \Lambda_3 \right),$$

$$j_{21} = \Lambda_1 \frac{G_N(t)}{\left[1-\left(1-u(t)\right)\right]^2} \left[\rho - G_N(t) \left(2\Lambda_2 \frac{1-u(t)}{1-\left(1-u(t)\right)} - \Lambda_3 \right) \right], \text{ and}$$

$$j_{22} = n + \rho \Lambda_1 \frac{1-u(t)}{1-\left(1-u(t)\right)}$$
$$-2G_N(t) \left[1-\phi + \Lambda_1 \frac{1-u(t)}{1-\left(1-u(t)\right)} \left(\Lambda_2 \frac{1-u(t)}{1-\left(1-u(t)\right)} - \Lambda_3 \right) \right].$$

Notice that $\Lambda_1 > 0$, $\Lambda_2 > 0$, $G_N^* > 0$ (2.28), and $T^* > 0$ (2.30), which imply, at steady state, that $\Lambda_2 G_N(t) \frac{1-u(t)}{1-(1-u(t))} = \Lambda_2 G_N^* T^* = \rho + \Lambda_3 G_N^* > 0$. At steady state, the elements of J are $j_{11}^* = \frac{\Lambda_1}{u^*}\left(\rho + \Lambda_3 G_N^*\right)$, $j_{12}^* = \frac{\rho \Lambda_1 (1-u^*)}{G_N^*}$, $j_{21}^* = \frac{-\Lambda_1 G_N^*}{(u^*)^2}\left(\rho + \Lambda_3 G_N^*\right)$, and $j_{22}^* = -n - \rho \Lambda_1 T^*$, and we have $\det(J^*) = \frac{-n\Lambda_1}{u^*}\left(\rho + G_N^* \Lambda_3\right)$ and $\mathrm{tr}(J^*) = \Lambda_1\left(\rho + \Lambda_3 G_N^*\right) + \Lambda_1 \Lambda_3 G_N^* T^* - n$. As $\Lambda_1 > 0$ and $\rho + \Lambda_3 G_N^* > 0 \Rightarrow \det(J^*) < 0$, the eigenvalues of J^* are both real with opposite signs, which implies that the solution $\left(1-u^*, G_N^*\right)$ is an hyperbolic equilibrium and a saddle point. Under such conditions, at the steady state, the dynamic of such linear system is qualitatively, or topologically, equivalent to the non-linear dynamic system made by (2.26) and (2.27), according to the Hartman-Grobman Theorem.

Phase lines' slope. At steady state, there are horizontal arrows of motion distancing from the loci $\left(1-\dot{u(t)}\right) = 0$ given by (J.3) as $j_{11}^* > 0$, and there are vertical arrows of motion approaching the loci $\dot{G}_N(t) = 0$ given by (J.4) as $j_{22}^* < 0$. The slope of both phase lines are negative in the plane $\left(1-u(t), G_N(t)\right)$: $\frac{\partial T(t)}{\partial\left(1-u(t)\right)} > 0$; considering (J.3),

$\frac{\partial G_N(t)}{\partial(1-u(t))} = \frac{-G_N(t)}{(\Lambda_2 T(t)-\Lambda_3)} \frac{\partial T(t)}{\partial(1-u(t))} < 0$, as $G_N(t) > 0 \Rightarrow \Lambda_2 T(t) - \Lambda_3 > 0$; considering (J.4),

$$\begin{aligned}
\frac{\partial G_N(t)}{\partial\left(1-u(t)\right)} &= \left[\frac{\rho}{\rho T(t) + {}^{n}/_{\Lambda_1}} - \frac{2\Lambda_2 T(t) + \Lambda_3}{\Lambda_2 T(t)^2 + \Lambda_3 T(t) + {}^{(1-\phi)}/_{\Lambda_1}}\right] G_N(t) \frac{\partial T(t)}{\partial\left(1-u(t)\right)} \\
&= \left\{\frac{\rho\left(1-\phi\right) - n\Lambda_3 - \Lambda_2 T(t)\left(\rho\Lambda_1 T(t) + 2n\right)}{\Lambda_1\left(\rho T(t) + {}^{n}/_{\Lambda_1}\right)\left[\Lambda_2 T(t)^2 + \Lambda_3 T(t) + {}^{(1-\phi)}/_{\Lambda_1}\right]}\right\} G_N(t) \frac{\partial T(t)}{\partial\left(1-u(t)\right)} \\
&< 0,
\end{aligned}$$

because $\rho\left(1-\phi\right) - n\Lambda_3 = \rho\left(1-\phi\right) + n\left(1+\phi\tau_a - \tau_a\right) - n\left(\theta+\phi\right)$, from the assumption of convergence in the household optimization problem $\left(1-\theta\right)G_N^* + n - \rho < 0 \Leftrightarrow \rho\left(1-\phi\right) + n\left(\theta - 2 + \phi\right) > 0 \Leftrightarrow n\left(\theta+\phi\right) > 2n - \rho\left(1-\phi\right)$, thus $0 < 1 + \phi\tau_a - \tau_a < 2 \Rightarrow \rho\left(1-\phi\right) - n\Lambda_3 = \rho\left(1-\phi\right) + n\left(1+\phi\tau_a-\tau_a\right) - n\left(\theta+\phi\right) < 0$, and $G_N(t) > 0 \Rightarrow \Lambda_2 T(t)^2 + \Lambda_3 T(t) + {}^{(1-\phi)}/_{\Lambda_1} > 0$.

Saddle path slope. Considering that the negative eigenvalue of the linear system is

$$\lambda_2 = \frac{\mathrm{tr}(J^*) - \sqrt{\mathrm{tr}(J^*)^2 - 4\det(J^*)}}{2} < 0$$

and, as an eigenvector is defined up to a multiplicative constant that normalizes its other component to 1, we can consider that the eigenvector associated with λ_2 that gives the direction of the saddle path is $e_2 = (e_{21}, e_{22} = 1)^{\mathrm{T}}$. Then, as $J^* e_2 = \lambda_2 e_2 \Rightarrow j_{11}^* e_{21} + j_{12}^* = \lambda_2 e_{21} \Leftrightarrow e_{21} = \frac{j_{12}^*}{\lambda_2 - j_{11}^*} < 0$, with $j_{11}^* > 0$ and $j_{12}^* > 0$, due to the topological equivalence with the linear system dynamics at the steady state neighborhood, the saddle path has a negative slope.

Linearized system stable and unstable manifolds. In space $\left(1-u(t), G_N(t)\right)$, the $G_N(t)$ components of the stable and unstable manifolds as functions of the $1-u(t)$ components are respectively given by $G_N^{LS}(1-u(t)) = G_N^* + \frac{e_{22}}{e_{21}}\left[\left(1-u(t)\right) - (1-u^*)\right]$ and $G_N^{LU}(1-u(t)) = G_N^* + \frac{e_{12}}{e_{11}}\left[\left(1-u(t)\right) - (1-u^*)\right]$, where e_{11} and e_{12} are the components of the eigenvector e_1 associated with the positive eigenvalue λ_1 (Fuente 2000, pp. 473–477).

Money-in-utility specification. Considering the changes in the model, as specified in Section 2.4.2, and following the same steps we find

$$
\begin{aligned}
G_N(t) = {} & (1 - z_r + \Omega_r \iota)\,\beta\rho / \\
& \Big\{(1-\tau_a)(\alpha+\beta)(1-\alpha-\beta)\,T(t) \\
& -(1 - z_r + \Omega_r \iota)\,\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)-(1-\tau_a)(1-\phi)\right]\Big\} \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
G_N(t) = {} & \left((\alpha+\beta)\,\rho T(t) + n\left\{\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right]\right\}\right) / \\
& \Big\{(1-\tau_a)(\alpha+\beta)^2(1-\alpha-\beta)\,T(t)^2/\left[\beta(1 - z_r + \Omega_r \iota)\right] \\
& +(\alpha+\beta)\left[1-(1-\theta)\left(\zeta+\sigma\right)-(1-\tau_a)(1-\phi)\right]T(t) \\
& +(1-\phi)\left\{\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right]\right\} \quad \Big\}
\end{aligned}
$$

instead of (J.3) and (J.4), which leads to the unique solution given by (2.29), (2.28) and (2.44). Case $\zeta+\sigma=1$, the expressions involved in the dynamics simplify to the baseline specification. Case $\zeta+\sigma\neq 1$, considering $\Lambda_1 \equiv \frac{\alpha+\beta}{\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right]}$, $\Lambda_2 \equiv (1-\tau_a)\frac{(\alpha+\beta)(1-\alpha-\beta)}{\beta(1-z_r+\Omega_r\iota)}$, and $\Lambda_3 \equiv 1-(1-\theta)\left(\zeta+\sigma\right)-(1-\tau_a)(1-\phi)$, it is still possible to rewrite (2.42) and (2.43) as (J.1) and (J.2), and then, case $\Lambda_1 > 0 \Rightarrow \det(J^*) < 0$, i.e., case $\alpha(1-\tau_a)+\beta\left[1-(1-\theta)\left(\zeta+\sigma\right)\right] > 0$, the solution is a saddle point by the Hartman-Grobman Theorem.

Appendix K
Welfare

Considering that the economy is stationary at instant $t = 0$ and $L(0)$ is given, from (2.11), we find $N(t) = \left(\frac{\delta L_{H_N}(t)}{G_N(t)}\right)^{\frac{1}{1-\phi}}$. Substituting this, (2.28), and (2.29), in (2.14) yields that

$$y^*(0) = \frac{(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}}s^{\frac{\beta}{\alpha+\beta}}(1-u^*)^{\frac{\beta}{\alpha+\beta}}}{\eta^{\frac{1-\alpha-\beta}{\alpha+\beta}}(1-z_x+\Omega_x\iota)^{\frac{1-\alpha-\beta}{\alpha+\beta}}}\left(\frac{\delta s u^* L(0)}{G_N^*}\right)^{\frac{1}{1-\phi}},$$

and, considering (2.21), $c^*(0) = \left[1 - \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\right] y^*(0)$. From that instant $t = 0$ onward along the balanced growth path, the consumption level of an individual will be given by $c(t) = c^*(0)e^{G_N^* t}$. Then, case $(1-\theta)G_N^* - \rho < 0$, we have that

$$W^* = \int_0^\infty \left[\frac{c^*(L(0))^{1-\theta} - 1}{1-\theta}\right] e^{-\rho t} dt$$

results in (2.31); case $(1-\theta)G_N^* - \rho > 0$, the integral would be divergent; and, case $(1-\theta)G_N^* - \rho = 0$, it would simplify to $-\frac{1}{\rho(1-\theta)}$, independent of the consumption level at $t = 0$. Recall that in the household problem, it was assumed that $(1-\theta)G_N^* + n - \rho < 0$, which implies $(1-\theta)G_N^* - \rho < 0$.

Comparative statics.

$$\frac{\partial y^*(0)}{\partial z_r} = y^*(L(0))\left\{\frac{\alpha+\beta-u^*\left[\alpha+\beta+\beta(1-\phi)\right]}{(1-\phi)(\alpha+\beta)(1-u^*)u^*}\right\}\frac{\partial u^*}{\partial z_r};$$

$$\frac{\partial c^*(0)}{\partial z_r} = \frac{\partial y^*(L(0))}{\partial z_r}\left[1 - \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}\right];$$

$$\frac{\partial W^*}{\partial z_r} = \frac{-c^*(0)^{-\theta}}{\left[(1-\theta)G_N^* - \rho\right]}\frac{\partial c^*(0)}{\partial z_r}.$$

$$
\begin{aligned}
\frac{\partial y^{*}(0)}{\partial \iota} &= y^{*}(L(0))\left\{\frac{\alpha+\beta-u^{*}\left[\alpha+\beta+\beta\,(1-\phi)\right]}{(1-\phi)\,(\alpha+\beta)\,(1-u^{*})\,u^{*}}\right\}\frac{\partial u^{*}}{\partial \iota};\\
\frac{\partial c^{*}(0)}{\partial \iota} &= \frac{\partial y^{*}(L(0))}{\partial \iota}\left[1-\frac{(1-\alpha-\beta)^{2}}{1-z_{x}+\Omega_{x}\iota}\right]\\
&\quad -\frac{(\alpha+\beta-z_{x}+\Omega_{x}\iota)\,\Omega_{x}\,(1-\alpha-\beta)\,y^{*}(0)}{(1-z_{x}+\Omega_{x}\iota)^{2}\,(\alpha+\beta)};\\
\frac{\partial W^{*}}{\partial \iota} &= \frac{-c^{*}(0)^{-\theta}}{\left[(1-\theta)\,G_{N}^{*}-\rho\right]}\frac{\partial c^{*}(0)}{\partial \iota}.
\end{aligned}
$$

Notice that $\frac{\partial W^{*}}{\partial \iota}\neq 0$ iff $\Omega_{x}\neq 0$ or $\Omega_{r}\neq 0$. Case $\Omega_{x}\neq 0$ and $\Omega_{r}=0$, $\frac{\partial y^{*}(L(0))}{\partial \iota}=\frac{\partial u^{*}}{\partial \iota}=0\Rightarrow \frac{\partial c^{*}(L(0))}{\partial \iota}<0$ and $\frac{\partial W^{*}}{\partial \iota}<0$. Case $\Omega_{r}\neq 0$ and $\Omega_{x}=0$, $\alpha+\beta<u^{*}\left[\alpha+\beta+\beta\,(1-\phi)\right]$ implies $\frac{\partial W^{*}}{\partial z_{r}}<0$ and $\frac{\partial W^{*}}{\partial \iota}>0$.

MIU specification. Analogously to the baseline model, as the integral in the household problem (2.38) is assumed to converge, we have $(1-\theta)\left(\zeta+\sigma\right)G_{N}^{*}-\rho<0$ and W^{*} given by (2.45), with $\frac{\partial W^{*}}{\partial z_{r}}$, $\frac{\partial c^{*}(0)}{\partial z_{r}}$, $\frac{\partial y^{*}(0)}{\partial z_{r}}$ $\frac{\partial c^{*}(0)}{\partial \iota}$, and $\frac{\partial y^{*}(0)}{\partial \iota}$, as in the baseline model, and

$$
\frac{\partial W^{*}}{\partial \iota}=\frac{1}{(1-\theta)}\frac{\sigma}{\zeta\iota^{2}}\frac{c^{*}(0)^{(1-\theta)(\zeta+\sigma)}}{\left[(1-\theta)\left(\zeta+\sigma\right)G_{N}^{*}-\rho\right]}\left[1-\frac{\iota\,(1-\theta)\left(\zeta+\sigma\right)}{c^{*}(0)}\frac{\partial c^{*}(0)}{\partial \iota}\right].
$$

Appendix L
Long-run comparative statics

The differentiation of the steady-state value of the variables growth rate G_N^* (2.28); skilled labor allocation in R&D u^* (2.29); the skill premium $\left(w_H/w_L\right)^*$ (2.13); the real interest rate r^* (2.12); inflation π^*, determined from (2.19); the velocity of money circulation V^* (2.25); and steady-state welfare W^* (2.31); w.r.t. all the parameters of the model were carried out with the aid of the computer algebra system (CAS) Maxima —wxMaxima for Windows 23.05.1 (wxWidgetss 3.2.2, Maxima 5.47.0 with SBCL 2.3.2). The sign of the resulting expressions was manually analyzed and presented in Subsection 2.3.2.

Appendix M
Policies complementarity

Fiscal policy is considered an increase in z_r, while monetary policy is considered a decrease in ι. Thus, there are complementary between these policies on a variable if $\frac{\partial}{\partial z_r}\left[-\frac{\partial(\cdot)}{\partial \iota}\right] = -\frac{\partial^2(\cdot)}{\partial z_r \partial \iota} \geq 0$. For T^* (2.30) we have $\frac{\partial T^*}{\partial \iota} = \frac{\Omega_r T^*}{1-z_r+\Omega_r \iota} \geq 0$, $\frac{\partial T^*}{\partial z_r} = \frac{-T^*}{1-z_r+\Omega_r \iota} < 0$, and $-\frac{\partial^2 T^*}{\partial z_r \partial \iota} = -\frac{\Omega_r T^*}{(1-z_r+\Omega_r \iota)^2} - \frac{\Omega_r}{1-z_r+\Omega_r \iota}\frac{\partial T^*}{\partial z_r} = 0$. For u^* (2.29), $\frac{\partial u^*}{\partial \iota} = -(u^*)^2 \frac{\partial T^*}{\partial \iota} \leq 0$, $\frac{\partial u^*}{\partial z_r} = -(u^*)^2 \frac{\partial T^*}{\partial z_r} > 0$, and $-\frac{\partial^2 u^*}{\partial z_r \partial \iota} = 2u^* \frac{\partial u^*}{\partial z_r}\frac{\partial T^*}{\partial \iota} > 0$. For R&D intensity, $\frac{\partial \beta/T^*}{\partial \iota} = \frac{-\beta}{(T^*)^2}\frac{\partial T^*}{\partial \iota} < 0$, $\frac{\partial \beta/T^*}{\partial z_r} = \frac{-\beta}{(T^*)^2}\frac{\partial T^*}{\partial z_r} > 0$, and $-\frac{\partial^2 \beta/T^*}{\partial z_r \partial \iota} = -\frac{2\beta}{(T^*)^3}\frac{\partial T^*}{\partial z_r}\frac{\partial T^*}{\partial \iota} + \frac{\beta}{(T^*)^2}\frac{\partial^2 T^*}{\partial \iota \partial z_r} > 0$. For the skill premium (2.13), $\frac{\partial \left(w_H/w_L\right)^*}{\partial \iota} = \frac{-\Omega_r}{1-z_r+\Omega_r \iota} u^* \left(w_H/w_L\right)^* < 0$, $\frac{\partial \left(w_H/w_L\right)^*}{\partial z_r} = \frac{\beta(1-s)}{\alpha s (1-u^*)^2}\frac{\partial u^*}{\partial z_r} > 0$,

$$
\begin{aligned}
-\frac{\partial^2 \left(w_H/w_L\right)^*}{\partial z_r \partial \iota} = &\frac{\Omega_r}{(1-z_r+\Omega_r \iota)^2} u^* \left(w_H/w_L\right)^* \\
&+ \frac{\Omega_r}{1-z_r+\Omega_r \iota}\left[\frac{\partial u^*}{\partial z_r}\left(w_H/w_L\right)^* + u^* \frac{\partial \left(w_H/w_L\right)^*}{\partial z_r}\right] > 0.
\end{aligned}
$$

For the velocity of money (2.25), $\frac{\partial V^*}{\partial z_r} = \frac{-\Omega_r \beta (V^*)^2}{(1-z_r+\Omega_r \iota) T^*} < 0$, $\Omega_x > \xi$ implies

$$
\begin{aligned}
\frac{\partial V^*}{\partial \iota} = &\left[\frac{\Omega_x^2 (1-\alpha-\beta)^{\frac{1}{1-\alpha-\beta}}}{(1-z_x+\Omega_x \iota)^2} + \frac{\Omega_r^2 \beta}{(1-z_x+\Omega_x \iota) T^*} - \frac{\xi \Omega_x (1-\alpha-\beta)^2}{(1-z_x+\Omega_x \iota)^2}\right] \\
&\times \left(V^*\right)^2 > 0, \text{ and} \\
-\frac{\partial^2 V^*}{\partial z_r \partial \iota} = &\left[\frac{\Omega_x^2 (1-\alpha-\beta)^{\frac{1}{1-\alpha-\beta}}}{(1-z_x+\Omega_x \iota)^2} + \frac{\Omega_r^2 \beta}{(1-z_x+\Omega_x \iota) T^*} - \frac{\xi \Omega_x (1-\alpha-\beta)^2}{(1-z_x+\Omega_x \iota)^2}\right] \\
&\times \left(-2V^*\right)\frac{\partial V^*}{\partial z_r} > 0.
\end{aligned}
$$

For long-run welfare (2.31), recall from propositions 2.4 and 2.5 that $u^* < 1/2$ and $\alpha + \beta - z_x + \Omega_x \iota > 0$ imply $\frac{\partial W^*}{\partial \iota} < 0$ and $\frac{\partial W^*}{\partial z_r} > 0$. The additional condition that $\phi \left(T^*\right)^2 \geq \frac{2\beta T^*(1-\phi)}{\alpha+\beta} + \frac{\alpha\beta(1-\phi)^2}{(\alpha+\beta)^2}$ implies $\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota} \leq 0$; to see this, we used the same computer algebra system as in Appendix (L), to compute the cross derivative of (2.14), manipulate and combine the resulting terms, and take some limits to find such a condition, that is satisfied if $(T^*)^2 \geq \frac{\sqrt{\phi+4}+2}{2\phi}$, which, in your turn, is easy to be satisfied if ϕ (or, G_N^*) is not too low. Then, $\frac{\partial c^*(0)}{\partial z_r}\frac{\partial c^*(0)}{\partial \iota} \leq 0$, $-\frac{\partial^2 c^*(0)}{\partial z_r \partial \iota} = -\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota} + \frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x \iota}\left(\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota} - \frac{\partial^2 y^*(0)}{\partial z_r}\frac{\Omega_x}{1-z_x+\Omega_x \iota}\right)$ and

$$\phi \left(T^*\right)^2 \geq \frac{2\beta T^*(1-\phi)}{\alpha+\beta} + \frac{\alpha\beta(1-\phi)^2}{(\alpha+\beta)^2} \Rightarrow -\frac{\partial^2 y^*(0)}{\partial z_r \partial \iota} \geq 0.$$

Appendix N
Growth rates

During transitions. As $w_{H_Y}(t) = w_H(t)$, substituting (2.7) in (2.4) and in (2.5) gives expressions for $w_L(t)$ and $w_H(t)$ (see Appendix D), that can be log-differentiated to find $\frac{\dot{w}_L(t)}{w_L(t)} = G_N(t) + \frac{\beta}{\alpha+\beta} G_T(t)$ and $\frac{\dot{w}_H(t)}{w_H(t)} = G_N(t) - \frac{\alpha}{\alpha+\beta} G_T(t)$. Log-differentiating $a(t) = \frac{Z(t)N(t)}{L(t)}$, we find $\frac{\dot{a}(t)}{a(t)} = \frac{\dot{w}_H(t)}{w_H(t)} + (1-\phi) G_N(t) - n$ (see Appendix C). As $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)}$, it is possible to log-differentiate the expression for $y(t)$ and find that $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{w}_L(t)}{w_L(t)}$. It is possible to verify that $b(t)$ (2.18) is composed of two terms that grow at different rates, $\frac{\dot{\left(\Omega_x \eta N(t) X(t)/L(t)\right)}}{\Omega_x \eta N(t) X(t)/L(t)} = \frac{\dot{w}_L(t)}{w_L(t)}$ and $\frac{\dot{\left(\Omega_r L_{H_N}(t) w_{H_N}(t)/L(t)\right)}}{\Omega_r L_{H_N}(t) w_{H_N}(t)/L(t)} = \frac{\dot{w}_H(t)}{w_H(t)}$. Recall that u and G_N changes in the same direction during transitions and, rearranging (2.11) as $\frac{G_N(t)}{u(t)} = \delta s \frac{L(t)}{N(t)^{1-\phi}}$, notice that the ratio $\frac{L(t)}{N(t)^{1-\phi}}$ increases (decreases) case $N(t)^{1-\phi}$ grows faster (slower) than $L(t)$, i.e., case $G_N(t) > (<)\, G_N^*$, which implies, in both cases, a greater rate of converge for G_N than for u, that is, $\left|\frac{\dot{G}_N(t)}{G_N(t)}\right| > \left|\frac{\dot{u}(t)}{u(t)}\right|$. Moreover, notice that, as $1-u(t)$ and $u(t)$ are always adjusting in opposite directions but by the same absolute amount, $\frac{\dot{u}(t)}{u(t)} = \frac{-(1-u(t))G_T(t)}{u(t)} \Rightarrow \left|\frac{\dot{u}(t)}{u(t)}\right| = T(t)\left|G_T(t)\right|$, which means that $u(t) < (>)\, 1/2 \Rightarrow \left|\frac{\dot{u}(t)}{u(t)}\right| > (<) \left|G_T(t)\right|$.

During the balanced-growth path. $u(t) = u^*$, $G_N(t) = G_N^*$, and $G_T(t) = G_T^* = 0$, which implies that the real interest rate (2.12) and the velocity of money (2.25) will be constant, and that $\frac{\dot{w}_L(t)}{w_L(t)} = \frac{\dot{w}_H(t)}{w_H(t)} = \frac{\dot{a}(t)}{a(t)} = \frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)} = \frac{\dot{b}(t)}{b(t)} = \frac{\dot{m}(t)}{m(t)} = G_N^*$. $\frac{\dot{w}_L(t)}{w_L(t)} = \frac{\dot{w}_H(t)}{w_H(t)}$ and u^* imply that the skill premium (2.13) will be constant. $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{m}(t)}{m(t)}$ and a constant real interest rate imply that inflation (2.19) will be constant.

Appendix O
Robustness of long-run-effects direction

The local robustness of the derivatives was verified using a Monte Carlo approach. In this simulated process, values for all the parameters were randomized (pseudo-)aleatorially, and uniformly distributed over given intervals. Case the resulting calibration satisfies the model restrictions of Section 2.2 and Assumption 2.1, the previously loaded expressions are evaluated at the point made by the simulated parameters. It should have the same sign as the result previously computed at the baseline calibration to confirm a differentiation result. The process runs until it achieves the desired number of valid calibrations. Derivatives were computed with Maxima CAS (see Appendix L); simulations were carried out in R 4.4.2 for Windows (platform x86_64-w64-mingw32/x64, 64-bit).

Appendix P
Saddle path

The stable and unstable spaces of the linearization of the system made by equations (2.26) and (2.27) at the steady state $\left(1-u^*, G_N^*\right)$ are

$$S^L \equiv \left\{\left(1-u^*+\varepsilon, G_N^{LS}(1-u^*+\varepsilon)\right)\right\} \text{ and}$$
$$U^L \equiv \left\{\left(1-u^*+\varepsilon, G_N^{LU}(1-u^*+\varepsilon)\right)\right\},$$

with the functions G_N^{LS} and G_N^{LU} defined as in Appendix J and $0 < 1-u^*+\varepsilon < 1$. By the stable manifold theorem, S^L is tangent to the model's saddle path. Following Herbert and Stemp (2002), the saddle path trajectory is found by choosing Δ_u, such that the points $\left(1-u^*-\Delta_u, G_N^{LS}(1-u^*-\Delta_u)\right)$ and $\left(1-u^*+\Delta_u, G_N^{LS}(1-u^*+\Delta_u)\right)$ are initial conditions that make the dynamic system converge to the steady state as times go forward. Considering the Table 2.2 calibration, we tried different values until finding an adequate Δ_u that numerically solved the dynamic model with LSODE, the Livermore Solver for Ordinary Differential Equations (Hindmarsh 1980) —the only solver we were able to use to integrate backward in the GNU Octave 9.1.0 for Windows's standard distribution. The points for the saddle path plotted in Figure P.1 were computed using this solver but integrating the time backward. The figure also shows the stable and unstable manifolds of the model's dynamic system made by equations (2.26) and (2.27) linearized at the steady state, as in Appendix J. Once the saddle path is determined, the path of the other variables can be computed, as shown in Figures 2.2 and Q.1.

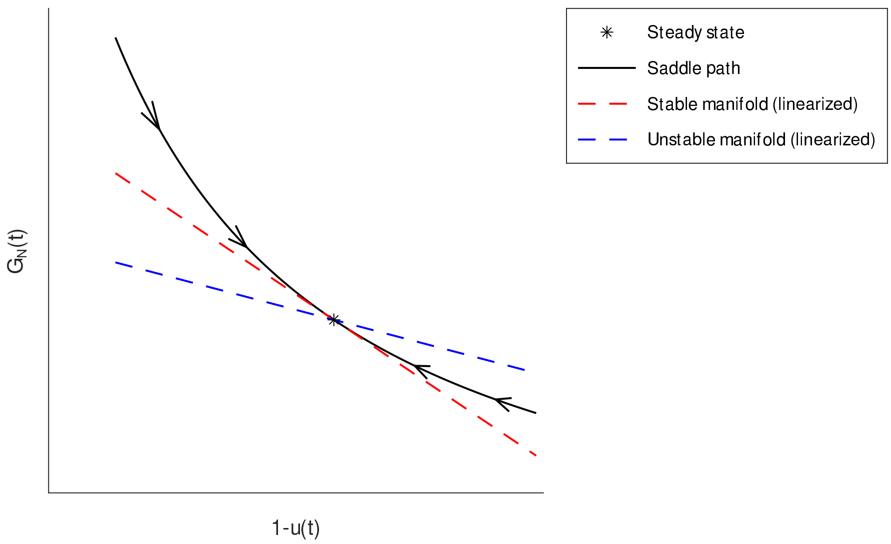

FIGURE P.1: Plots of the calibrated model's saddle path and the stable and unstable manifolds of the dynamic system linearization at the steady state.

Appendix Q
Other figures and tables

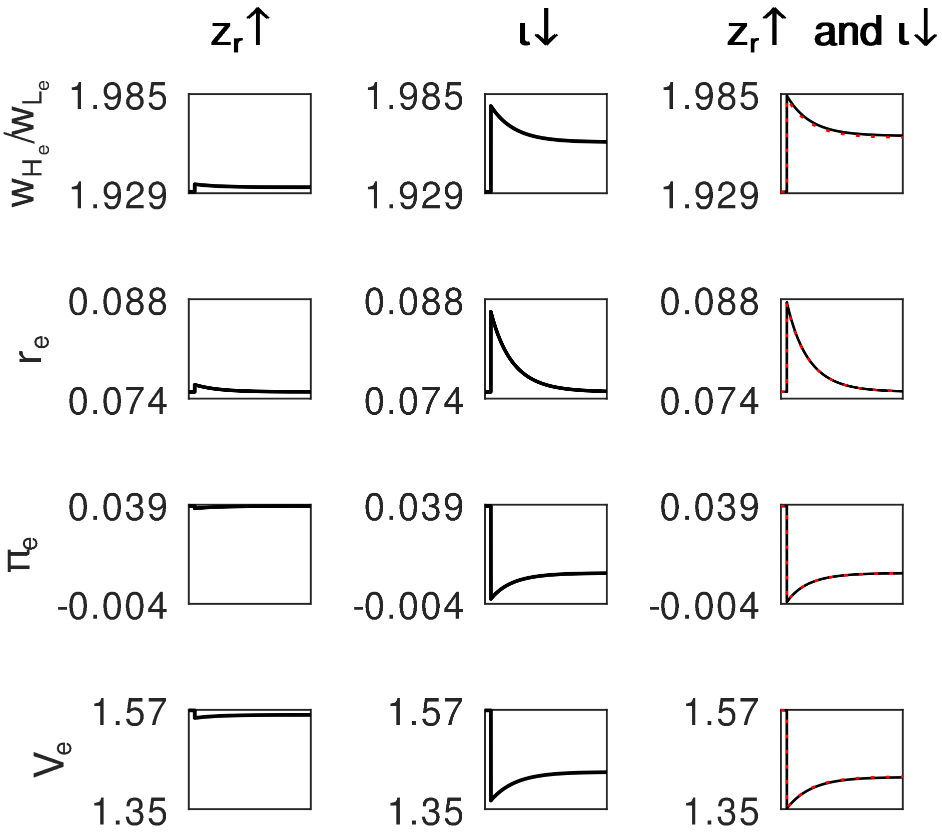

FIGURE Q.1: Dynamics of the model extended with elastic labor supply (Subsection 2.4.1) after policy changes, relative to Table 2.2 calibration; fiscal policy (z_r increases by 2.8 p.p.) effects are depicted in the left column, monetary policy (ι decreases by 2.9 p.p.) in the center, and the combined policies case in the right; the red dotted lines in the right column plot the sum of the isolated policy effects.

TABLE Q.1: Long-run fiscal and monetary policy funding.

#	Variable	Initial	$z_r\uparrow,\iota\downarrow,\tau_g\uparrow$	$z_r\uparrow,\iota\downarrow,\tau_a\uparrow$	$z_r\uparrow,\iota\downarrow,\tau_H\uparrow$	$z_r\uparrow,\iota\downarrow,\tau_L\uparrow$
1	β/T^*	0.02954	0.03503	0.03474	0.03503	0.03503
2	r^*	0.07494	0.07494	0.07548	0.07494	0.07494
3	π^*	0.03836	0.00936	0.00936	0.00936	0.00936
4	$\tau_m L/Y$	0.01360	0.00345	0.00345	0.00345	0.00345
5	$\tau_g L/Y$	0.00000	0.00175	0.00000	0.00000	0.00000
6	$\tau_a \frac{r^*\beta(1-z_r+\Omega_r\iota)}{G_N^* T^*}$	0.00097	0.00097	0.00267	0.00097	0.00097
7	$\tau_H\beta/(1-u^*)$	0.00264	0.00268	0.00268	0.00443	0.00268
8	$\tau_L\alpha$	0.00160	0.00160	0.00160	0.00160	0.00335
9	$z_x\frac{(1-\alpha-\beta)^2}{1-z_x+\Omega_x\iota}$	0.00136	0.00146	0.00146	0.00146	0.00146
10	$z_r\beta/T^*$	0.00384	0.00553	0.00549	0.00553	0.00553
11	Tot. Subsidies/Y	0.00520	0.00700	0.00695	0.00700	0.00700
12	$(w_H/w_L)^*$	2.01701	2.05061	2.04887	2.05061	2.05061
13	W^*	22.20981	22.21421	22.21406	22.21421	22.21421
14	l_H^*	0.97199	0.97199	0.97199	0.97199	0.97199
15	l_L^*	0.93000	0.92954	0.92956	0.92969	0.92942
16	$(w_{H_e}/w_{L_e})^*$	1.92988	1.96106	1.95944	1.96137	1.96080
17	W_e^*	22.01976	22.05175	22.05060	22.05129	22.05213
18	— consump.	0.00000	0.03052	0.02945	0.03052	0.03051
19	— lab. supp.	0.00000	0.00147	0.00139	0.00100	0.00186

Notes: long-rung values for the model with inelastic heterogeneous labor is defined in Section 2.2 and extended with elastic labor supply in Subsection 2.4.1; models initially calibrated as in Subsection 2.5.1 and with concomitant changes in z_r and ι without changing any tax parameter (subsidies' variation funded with the endogenous lump-sum tax) and in z_r and ι together with τ_a, τ_H, or τ_L for different sources of funding to the subsidies' variation; values in first 11 rows are the same in both models; the values in 4–11 rows are the aggregate value relative to output for seigniorage revenues (#4), lump-sum taxes (#5), taxes over assets (#6), skilled labor (#7), and unskilled labor (#8), subsidies to intermediate sector (#9) and R&D (#10), and the total subsidies (#11) match the total collected with taxes; values in rows 12-13 are exclusive of the Section 2.2 model and, in rows 14–19, of its Subsection 2.4.1 extension; rows 18–19 decompose the welfare variation due to the changes in consumption and labor supply, respectively.

TABLE Q.2: Long-run complementarity for different policy funding.

#	l.h.s.(2.46)	Initial	$z_r\uparrow, \iota\downarrow, \tau_g\uparrow$	$z_r\uparrow, \iota\downarrow, \tau_a\uparrow$	$z_r\uparrow, \iota\downarrow, \tau_H\uparrow$	$z_r\uparrow, \iota\downarrow, \tau_L\uparrow$
1	β/T^*	0.00000	0.00016	0.00011	0.00016	0.00016
2	$(w_H/w_L)^*$	0.00000	0.00095	0.00066	0.00095	0.00095
3	W^*	0.00000	-0.00010	-0.00007	-0.00010	-0.00010
4	$(w_{H_e}/w_{L_e})^*$	0.00000	0.00088	0.00061	0.00097	0.00086
5	W_e^*	0.00000	-0.00064	-0.00045	-0.00083	-0.00060
6	— consump.	0.00000	-0.00068	-0.00047	-0.00067	-0.00067
7	— lab. supp.	0.00000	0.00004	0.00003	-0.00016	0.00007

Notes: difference from the effects on the baseline model's welfare of fiscal and monetary policy used together (Table Q.1) to the sum of the policies' effects isolated for different sizes (in p.p.) of change in policy parameters z_r (Table 2.5) ι (Table 2.6) for the model with inelastic heterogeneous labor is defined in Section 2.2 and extended with elastic labor supply in Subsection 2.4.1; values for R&D intensity in the first line is the same in both models; values in rows 2-3 are exclusive of the Section 2.2 model and, in rows 4–7, of its Subsection 2.4.1 extension; rows 6–7 decompose the welfare variation due to the changes in consumption and labor supply, respectively.

(a) inelastic labor supply and elastic heterogeneous labor supply cases

(b) elastic homogeneous labor supply case

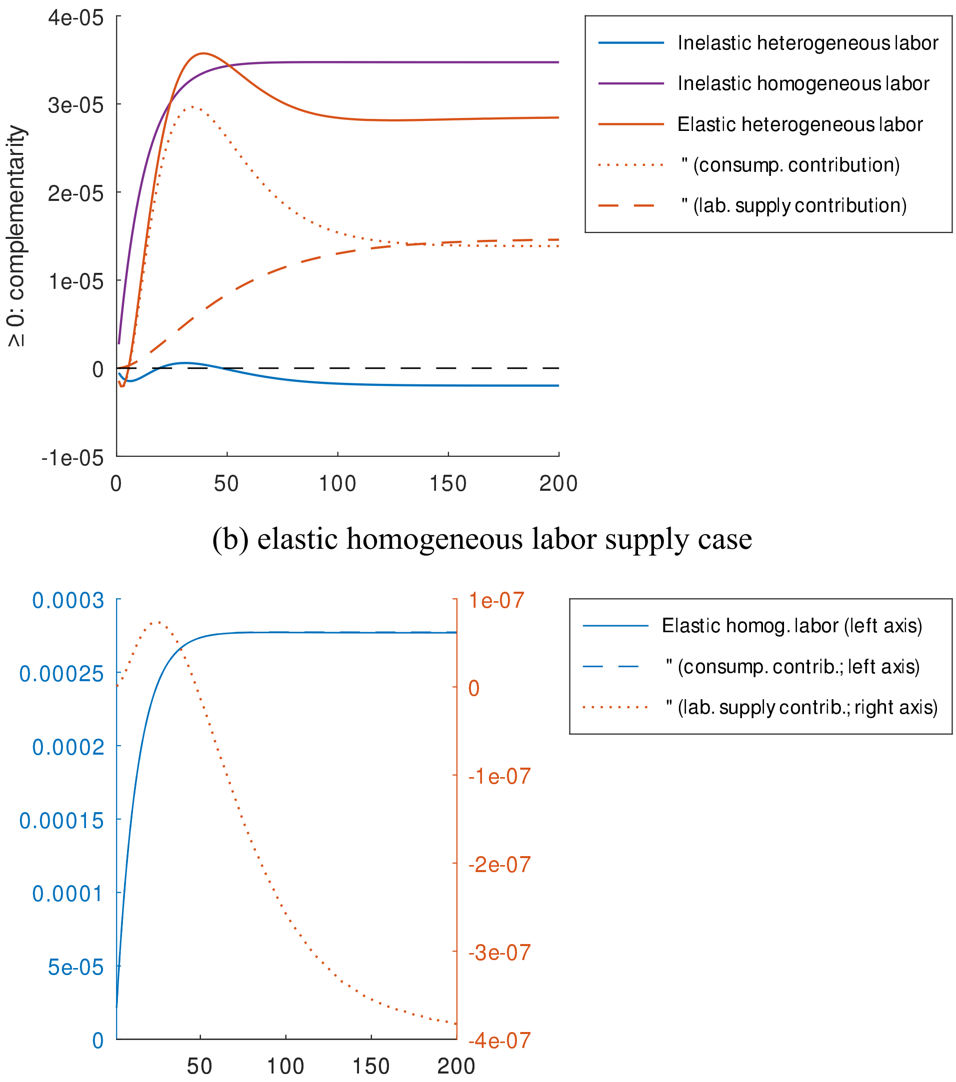

FIGURE Q.2: Evolution of policy complementarity/substitutability on welfare as more years of the transition path are considered; for the elastic labor supply cases, the differences of the effects on the first (due to consumption) and the second (due to labor supply) terms in the r.h.s. of (2.37) are also plotted.

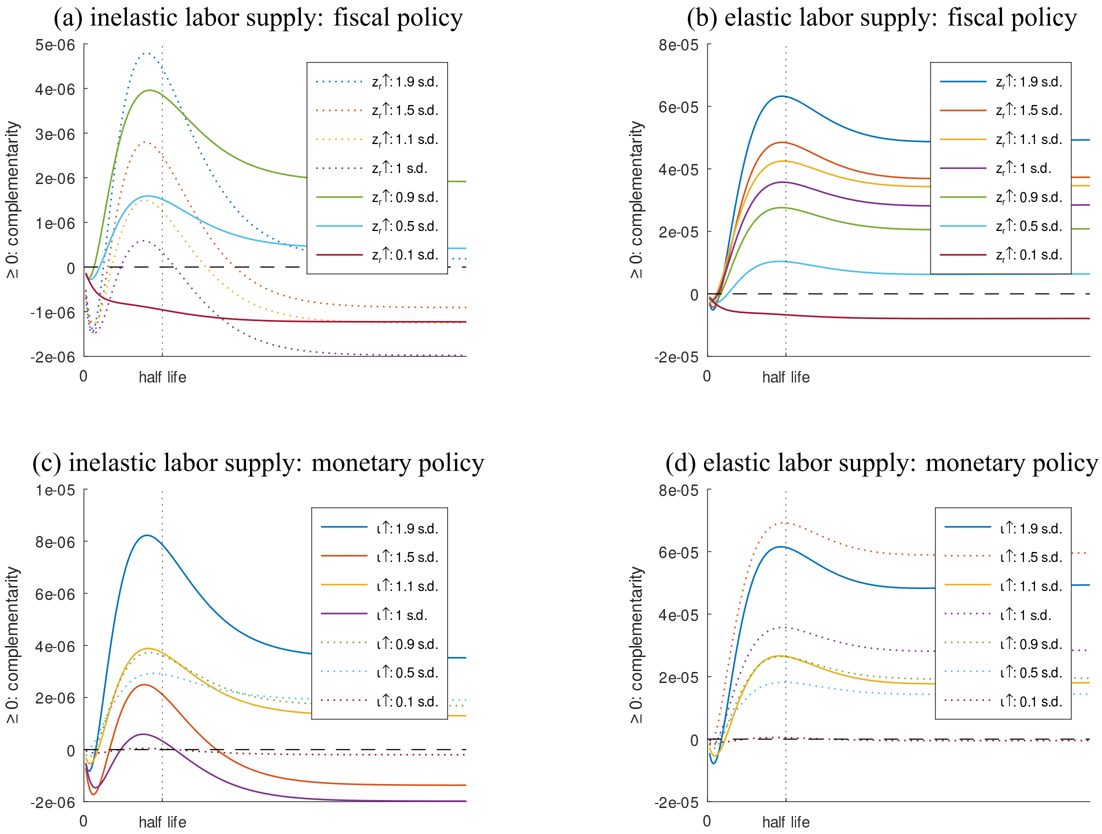

FIGURE Q.3: Evolution of policy complementarity/substitutability on welfare under different policy strength, as more years of the transition path are considered; baseline model with inelastic labor supply (Section 2.2) on the left panels and the model extension with elastic labor supply (Subsection 2.4.1) on the right panels; on the upper panels, monetary policy changes are hold in one s.d. (ι decreases 2.9 p.p.) while fiscal policy changes vary; on the bottom panels, fiscal policy changes are hold in one s.d. (z_r increases 2.8 p.p.) while monetary policy changes vary.

Appendix R
Consumer problem

Bearing in mind (3.1) and (3.2), from standard optimal control theory, we consider the auxiliary Hamiltonian function

$$
\begin{aligned}
H = & \left(\frac{C(t)^{1-\theta}-1}{1-\theta}\right)e^{-\rho t} \\
& + \lambda(t)\left[(1-\upsilon)\cdot\sum_{n=L,H}\left(r_n(t)\cdot K_n(t)+w_n(t)\cdot L_n(t)+\Pi_{Y_n}(t)\right)\right. \\
& \qquad \left. + u\left(\overline{L}_L - L_L\right) - C(t) - S(t)\right],
\end{aligned}
$$

where K_n, with $n \in \{L,H\}$, are the state variables; λ is the co-state variable; and C is the control variable. Then, the necessary conditions under the Maximum Principle are:

$$\frac{\partial H}{\partial C} = 0 \Leftrightarrow C^{-\theta}e^{-\rho t} - \lambda = 0 \Rightarrow \tag{R.1}$$

$$\frac{\dot{\lambda}}{\lambda} = -\theta\frac{\dot{C}}{C} - \rho, \tag{R.2}$$

$$\frac{\partial H}{\partial K_n} = -\dot{\lambda} \Leftrightarrow (1-\upsilon)\,r_n + \frac{\dot{\lambda}}{\lambda} = 0, \text{ and} \tag{R.3}$$

$$\frac{\partial H}{\partial \lambda} = \sum_{n=L,H}\dot{K}_n.$$

Rearranging (R.1) we have $C^{-\theta}e^{-\rho t} = \lambda$, and log-differentiating it w.r.t. time yields (R.2); substituting (R.2) into (R.3) results in the Euler equation for consumption (3.3).

Appendix S
Relative price of H sector

The maximization problem is $\max_{Y_H,Y_L} \Pi_Y = Y - p_H Y_H - p_L Y_L$, subject to (3.5), that is equivalent to

$$\max_{Y_H,Y_L} \Pi_Y = \left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - p_H Y_H - p_L Y_L;$$

the F.O.C. are

$$\frac{\partial(\cdot)}{\partial Y_H} = 0 \Leftrightarrow p_H = \gamma Y_H^{\frac{-1}{\varepsilon}} \left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}, \text{ and} \tag{S.1}$$

$$\frac{\partial(\cdot)}{\partial Y_L} = 0 \Leftrightarrow p_L = (1-\gamma) Y_L^{\frac{-1}{\varepsilon}} \left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}. \tag{S.2}$$

Notice, from (3.5), that $Y^{\frac{1}{\varepsilon}} = \left[\gamma Y_H^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}}$; substituting it in (S.1) and (S.2) we find

$$p_H = \gamma Y_H^{\frac{-1}{\varepsilon}} Y^{\frac{1}{\varepsilon}} \text{ and } p_L = (1-\gamma) Y_L^{\frac{-1}{\varepsilon}} Y^{\frac{1}{\varepsilon}}, \tag{S.3}$$

which allows to determine (3.6) and each sector's output: $Y_H = \left(\frac{\gamma}{p_H}\right)^{\varepsilon} Y$ and

$$Y_L = \left(\frac{1-\gamma}{p_L}\right)^{\varepsilon} Y.$$

After some manipulation, substituting the last in $P_Y Y = P_H Y_H + P_L Y_L$ results in (3.7).

Appendix T
Contract curve

Substituting (3.8) and (3.10) in (3.11) we find

$$\pi_{Y_L}^{1-\chi} = \left\{ (1-\beta)\cdot p_n \cdot \left[L_L^{\alpha} \cdot \int_0^{A_L(t)} \left(x_i^L(t) \right)^{\beta} di \right] - w_L \cdot L_L \right\}^{1-\chi}. \tag{T.1}$$

Considering (3.12) and (T.1), the bargaining problem (3.13) can be represented by

$$\max_{w_L, L_L} \Psi = \left\{ \left[(1-\upsilon) w_L - u \right]^{\delta} \cdot L_L^{\zeta} \right\}^{\chi} = \left[(1-\upsilon) w_L - u \right]^{\delta\chi} \cdot L_L^{\zeta\chi} \cdot$$
$$\left\{ (1-\beta)\cdot p_L \cdot \left[L_L^{\alpha} \cdot \int_0^{A_L(t)} \left(x_i^L(t) \right)^{\beta} di \right] - w_L \cdot L_L \right\}^{1-\chi},$$

with $\int_0^{A_L(t)} \left(x_i^L(t) \right)^{\beta} di = \left(\frac{\beta \cdot p_L}{1} \right)^{\frac{\beta}{1-\beta}} \cdot L_L^{\frac{\alpha\beta}{1-\beta}} \cdot \int_0^{A_L(t)} \left(\frac{1}{p_{x_i^L}} \right)^{\frac{\beta}{1-\beta}} di$. Then, the F.O.C. are $\frac{\partial \Psi}{\partial w_L} = 0 = \frac{\delta\chi(1-\upsilon)U^{\chi} \cdot \pi_{Y_L}^{1-\chi}}{(1-\upsilon)w_L - u} - \frac{U^{\chi} \cdot \pi_{Y_L}^{1-\chi} L_n}{\pi_{Y_L}}$ and

$$\begin{aligned} \frac{\partial \Psi}{\partial L_L} &= 0 \\ &= \frac{\zeta\chi U^{\chi} \cdot \pi_{Y_L}^{1-\chi}}{L_L} + \frac{(1-\chi) U^{\chi} \cdot \pi_{Y_L}^{1-\chi}}{\pi_{Y_L}} \\ &\times \left\{ (1-\beta)\cdot p_L \cdot \left[L_L^{\alpha} \cdot \int_0^{A_L(t)} \left(x_i^n(t) \right)^{\beta} di \right] \left\{ \frac{\alpha}{L_L} + \frac{\alpha\beta}{(1-\beta) L_L} \right\} - w_L \right\} \Leftrightarrow \end{aligned} \tag{T.2}$$

$$\frac{\zeta\chi}{1-\chi} = -\frac{L_L \left\{ (1-\beta)\cdot p_L \cdot Y_L \cdot \left\{ \frac{\alpha}{L_L} + \frac{\alpha\beta}{(1-\beta)L_L} \right\} - w_L \right\}}{\pi_{Y_L}}. \tag{T.3}$$

Equations (T.1) and (T.3), lead, respectively, to (3.14) and (3.15). Equalizing (3.14) and (3.15), leads to (3.16). Solving (3.16) for w_L, leads to $w_L = -\frac{Y_L \cdot p_L \cdot \alpha \cdot \delta \cdot \upsilon + L_L \cdot u \cdot \zeta - Y_L \cdot p_L \cdot \alpha \cdot \delta}{(L_L \cdot \zeta - L_L \cdot \delta)\cdot \upsilon - L_L \cdot \zeta + L_L \cdot \delta}$ and $\frac{\partial w_L}{\partial L_L} = \frac{Y_L \cdot p_L \cdot \alpha \cdot \delta}{L_L^2 \cdot \zeta - L_L^2 \cdot \delta}$.

Appendix U
Skill premium

Rearranging (3.20) and substituting $\Pi_{x^n} = (1-\beta)\cdot\left(\frac{\psi}{\beta}\right)^{\frac{-\beta}{1-\beta}}\cdot(\beta\cdot p_n)^{\frac{1}{1-\beta}}\cdot L_n^{\frac{\alpha}{1-\beta}}$ and (3.18) in it, we find the n-sector output,

$$A_n\cdot\Pi_{x^n} = (1-\beta)\cdot\beta\cdot p_n\cdot Y_n \Leftrightarrow$$
$$Y_n = \frac{A_n\cdot\Pi_{x^n}}{(1-\beta)\cdot\beta\cdot p_n} = \frac{A_n\cdot(1-\beta)\cdot p_{x_i^n}\cdot x_i^n}{(1-\beta)\cdot\beta\cdot p_n}$$
$$= \beta^{-1}\cdot p_n^{-1}\cdot A_n\cdot\frac{\psi}{\beta}\cdot\left(\frac{\psi}{\beta}\right)^{\frac{-1}{1-\beta}}\cdot(\beta\cdot p_n)^{\frac{1}{1-\beta}}\cdot L_n^{\frac{\alpha}{1-\beta}},$$

and, from this, the relative output:

$$\frac{Y_H}{Y_L} = \frac{p_H^{\frac{1}{1-\beta}-1}\cdot A_H\cdot L_H^{\frac{\alpha}{1-\beta}}}{p_L^{\frac{1}{1-\beta}-1}\cdot A_L\cdot L_L^{\frac{\alpha}{1-\beta}}}. \tag{U.1}$$

Using (3.9) for w_H, and equations (3.17) and (U.1), we find

$$\frac{w_H}{w_L} = \frac{\frac{\alpha\cdot p_H\cdot Y_H}{L_H}}{\left[\frac{\zeta\cdot\chi\cdot(1-\alpha-\beta)}{1-\chi}+\alpha\right]\frac{p_L\cdot Y_L}{L_L}}$$
$$= \frac{\alpha}{\left[\frac{\zeta\cdot\chi\cdot(1-\alpha-\beta)+\alpha(1-\chi)}{1-\chi}\right]}\frac{p_H^{\frac{1}{1-\beta}}}{p_L^{\frac{1}{1-\beta}}}\frac{A_H}{A_L}\frac{L_L^{1+\frac{\alpha}{1-\beta}}}{L_H^{1+\frac{\alpha}{1-\beta}}}. \tag{U.2}$$

Substituting $\Omega \equiv \frac{1-\chi}{\zeta\cdot\chi\cdot(1-\alpha-\beta)+\alpha(1-\chi)}$ and (3.29) in (U.2) results in (3.33). Solving (3.31) for $\left(\frac{L_H}{L_L}\right)^{-1}$, we find

$$\left(\frac{L_H}{L_L}\right)^{-\alpha(\varepsilon-1)} = \left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon}\left(\frac{\eta_H}{\eta_L}\right)^{-\beta-\varepsilon(1-\beta)}\left(\frac{A_H}{A_L}\right)^{-1} \Leftrightarrow$$
$$\left(\frac{L_H}{L_L}\right)^{-1} = \left(\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\alpha(\varepsilon-1)}}\left(\frac{\eta_H}{\eta_L}\right)^{\frac{-\beta-\varepsilon(1-\beta)}{\alpha(\varepsilon-1)}}\left(\frac{A_H}{A_L}\right)^{\frac{-1}{\alpha(\varepsilon-1)}}. \tag{U.3}$$

Substituting (U.3) into (3.33), gives rise to (3.34).

Appendix V
Income labor share

Substituting (S.3) into (3.36) we find

$$\frac{w_H L_H}{Y} = \alpha \frac{p_H Y_H}{Y} = \alpha \cdot \gamma^{\varepsilon} \cdot p_H^{1-\varepsilon}, \text{ and } \frac{w_L L_L}{Y} = \frac{1}{\Omega} \frac{p_L Y_L}{Y} = \frac{1}{\Omega} \cdot (1-\gamma)^{\varepsilon} \cdot p_L^{1-\varepsilon}. \tag{V.1}$$

Solving (3.6) for p_H and substituting the result in (3.7) we find

$$\begin{aligned} 1 &= \gamma^{\varepsilon} \cdot \left[p_L \cdot \frac{\gamma}{1-\gamma} \cdot \left(\frac{Y_H}{Y_L} \right)^{-\frac{1}{\varepsilon}} \right]^{1-\varepsilon} + (1-\gamma)^{\varepsilon} \cdot p_L^{1-\varepsilon} \\ &= p_L^{1-\varepsilon} \cdot \left(\frac{1}{1-\gamma} \right)^{-\varepsilon} \cdot \left[\frac{\gamma}{1-\gamma} \cdot \left(\frac{Y_H}{Y_L} \right)^{\frac{\varepsilon-1}{\varepsilon}} + 1 \right] \Leftrightarrow \\ p_L^{1-\varepsilon} &= \left(\frac{1}{1-\gamma} \right)^{\varepsilon} \cdot \left[\frac{\gamma}{1-\gamma} \cdot \left(\frac{Y_H}{Y_L} \right)^{\frac{\varepsilon-1}{\varepsilon}} + 1 \right]^{-1}. \end{aligned} \tag{V.2}$$

Solving (3.6) for $\frac{Y_H}{Y_L}$ and substituting the result in (V.2), substituting (3.30) in (V.3), and substituting (U.3) in (V.4) leads to

$$p_L^{1-\varepsilon} = \left(\frac{1}{1-\gamma} \right)^{\varepsilon} \cdot \left[\left(\frac{\gamma}{1-\gamma} \right)^{\varepsilon} \cdot \left(\frac{p_H}{p_L} \right)^{1-\varepsilon} + 1 \right]^{-1} \tag{V.3}$$

$$= \left(\frac{1}{1-\gamma} \right)^{\varepsilon} \cdot \left[\left(\frac{\gamma}{1-\gamma} \right)^{\varepsilon} \cdot \left(\frac{\eta_H}{\eta_L} \right)^{(1-\beta)(1-\varepsilon)} \left(\frac{L_H}{L_L} \right)^{-\alpha(1-\varepsilon)} + 1 \right]^{-1} \tag{V.4}$$

$$= \left(\frac{1}{1-\gamma} \right)^{\varepsilon} \cdot \left(\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1 \right)^{-1}. \tag{V.5}$$

Substituting (V.1), (3.30), and (U.3) into $S_L = \frac{w_H L_H + w_L L_L}{Y}$, we find (V.6); substituting (V.5) in (V.6) yields:

$$\begin{aligned}
S_L &= \alpha \cdot \gamma^{\varepsilon} \cdot p_H^{1-\varepsilon} + \frac{1}{\Omega} \cdot (1-\gamma)^{\varepsilon} \cdot p_L^{1-\varepsilon} \\
&= \alpha \gamma^{\varepsilon} p_L^{1-\varepsilon} \left[\left(\frac{\eta_H}{\eta_L} \right)^{(1-\beta)} \left(\frac{L_H}{L_L} \right)^{-\alpha} \right]^{1-\varepsilon} + \frac{1}{\Omega} (1-\gamma)^{\varepsilon} p_L^{1-\varepsilon} \\
&= p_L^{1-\varepsilon} \cdot (1-\gamma)^{\varepsilon} \cdot \left(\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + \frac{1}{\Omega} \right) \\
&= \frac{\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + \frac{1}{\Omega}}{\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1}.
\end{aligned} \tag{V.6}$$

Then, we have $\frac{\partial S_L}{\partial \chi} = \frac{\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{\partial \frac{A_H}{A_L}}{\partial \chi} - \frac{1}{\Omega^2} \cdot \frac{\partial \Omega}{\partial \chi}}{\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1} - \frac{\left(\alpha \cdot \frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + \frac{1}{\Omega} \right) \cdot \frac{\eta_H}{\eta_L} \cdot \frac{\partial \frac{A_H}{A_L}}{\partial \chi}}{\left(\frac{\eta_H}{\eta_L} \cdot \frac{A_H}{A_L} + 1 \right)^2}$, which can be rearranged as (3.38).

Appendix W
Regressions

Grouping countries. Our main interest lies in finding groups of countries that simultaneously satisfy the theoretical predictions stated in Section 3.3's propositions. The chain of events established in Propositions 3.2 and 3.3 can be estimated through two-stage regressions. We fit the model made of equations (W.1) and (W.2) with two-stage linear square (2SLS) and classify the countries into four groups (matching proposition's cases), according to the expected sign of the coefficients for $\log(\mathrm{UD})$ and $\widehat{\log(\mathrm{TK})}$ in the first and second stages of the regression, respectively. The search procedure is iterative, and at each iteration, we add the non-classified country that improves the group regression in terms of p-values for the $\widehat{\log(\mathrm{TK})}$ coefficient, weak instruments test, or Wu-Hausman test, until no country that improves the results is left. It is a simple minimization algorithm that performs a sequential search in a discrete space. Only improvements for a country inclusion that lets the UD effect direction for the group of countries (resulting from the ordinary least square regressions presented in Table W.7) as theoretically expected in propositions 3.1, 3.4, and 3.5, are considered. The resulting country classification is presented in Table W.1.

Multicollinearity issues. Table W.2 presents the variance inflation factors (VIFs) of the pooled OLS regressions in Table 3.3, along with two omitted specifications, due to multicollinearity issues, for Table 3.1, Cases 1 (–) and 2 (*).

Table 3.1, Cases 3, 4, and 5. Table W.3 presents the results of OLS regressions fitting equation (3.44) for the groups of countries not included in Cases 1 and 2 of Table 3.1.

Chained effect. If $\log(\mathrm{TK})$ is endogenously determined with $\log(\frac{w_H}{w_L})$, in equation (3.44), an instrumental variables (IV) estimation specified by

$$\log(\frac{w_H}{w_L}) = \lambda_{2,0} \qquad + \lambda_{2,2}\widehat{\log(\mathrm{TK})} + \lambda_{2,3}\log({}^{L_L}/_{L_H}) + \mu_2 \tag{W.1}$$

$$\log(\mathrm{TK}) = \lambda_{1,0} + \lambda_{1,1}\log(\mathrm{UD}) \qquad + \lambda_{1,3}\log({}^{L_L}/_{L_H}) + \mu_1 \text{ and} \tag{W.2}$$

can be carried out, where $\log(\mathrm{UD})$ is considered the instrument. $\widehat{\log(\mathrm{TK})}$ indicates that, instead of the values observed in data, the fitted values resulting from the first stage regression (W.2) are considered a regressor for the second stage (W.1). Although not justifiable

TABLE W.1: Country classification

Country (code)	**Classification**	**Country (code) – cont.**	**Classification – cont.**
Argentina (ARG)	(other case)	Indonesia (IDN)	Case 2
Australia (AUS)	(other case)	Ireland (IRL)	Case 3
Austria (AUT)	(other case)	Italy (ITA)	(other case)
Belgium (BEL)	(other case)	Latvia (LVA)	(other case)
Brazil (BRA)	(other case)	Lithuania (LTU)	(other case)
Bulgaria (BGR)	Case 1	Luxembourg (LUX)	(other case)
Canada (CAN)	(other case)	Malta (MLT)	Case 2
Chile (CHL)	(other case)	Mexico (MEX)	Case 1
Colombia (COL)	Case 1	Netherlands (NLD)	(other case)
Costa Rica (CRI)	(other case)	Norway (NOR)	(other case)
Croatia (HRV)	Case 2	Poland (POL)	Case 1
Cyprus (CYP)	Case 2	Portugal (PRT)	Case 1
Czech Republic (CZE)	Case 3	Romania (ROU)	Case 1
Denmark (DNK)	Case 2	Serbia (SRB)	Case 1
Estonia (EST)	(other case)	Slovakia (SVK)	Case 1
Finland (FIN)	Case 2	Slovenia (SVN)	Case 3
France (FRA)	(other case)	South Africa (ZAF)	Case 4
Germany (DEU)	(other case)	Spain (ESP)	(other case)
Greece (GRC)	(other case)	Sweden (SWE)	(other case)
Hungary (HUN)	Case 1	Switzerland (CHE)	(other case)
Iceland (ISL)	Case 2	Turkey (TUR)	(other case)
India (IND)	(other case)	United Kingdom (GBR)	(other case)
...	...	United States (USA)	(other case)

TABLE W.2: VIF for Table 3.1's Cases 1 and 2 regressions

	(–)	(1)	(2)	(*)	(3)	(4)
$\log(\text{UD})$	3.770	3.703	3.205	4.891	3.195	3.229
$\log(\text{TK})$	2.929	2.625	2.710	2.430	2.389	2.292
$\log(L_L/L_H)$	5.083		2.135	98.830		5.010
$\log(\alpha)$	4.875	2.189		67.436	5.711	
$\log(\Omega)$	2.221	1.431		4.239	1.448	

in the face of Table 3.3 results, an IV specification is plausible due to the measure of technological knowledge bias defined in Subsection 3.4.1 simultaneous determination with the skill premium (e.g., Wooldridge 2010). Also, it allows the investigation of Proposition 3.3 indirect mechanism. The regressions results for the groups of countries (for which more than one country was found) are presented in Table W.4. The inference comparison for the first and second-stage regressors of interest are in Tables W.5 and W.6, respectively, where "WRE" refers to the nonasymptotic method Wild Restricted Efficient bootstrap (MacKinnon et al. 2023).

TABLE W.3: Pooled OLS regressions for Table 3.1's Cases 3, 4, and 5

	Dependent variable:		
	$\log(w_H/w_L)$		
	Case 3	Case 4	Other cases
log(UD)	0.196	−0.587	−0.110
	(0.130)	(1.336)	(0.066)
log(TK)	0.279	1.175***	0.220
	(0.735)	(0.173)	(0.267)
$\log(L_L/L_H)$	−0.024	1.493***	0.211
	(0.178)	(0.397)	(0.112)
Constant	0.937	−1.912	0.279*
	(0.595)	(2.178)	(0.152)
Countries	3	1	25
Median(obs)	14	18	15
Max(obs)	16	18	27
Observations	41	18	403
R^2	0.572	0.608	0.435

Notes: *p<0.1; **p<0.05; ***p<0.01
Cluster-robust variance estimators (by country, CV_3 with Satterthwaite d.f.).
Robust variance estimaton for cases with one country (HC_3).

Other effects. The regressions presented in Table W.7 show that, despite not being significant for the countries in Cases 1 and 2 of Proposition 3.3, the direction of the union's bargaining power effects on the other analyzed variables are in line with Propositions 3.1, 3.4, and 3.5.

Table W.4: Instrumental variable regressions for Table 3.1's Cases 1, 2, and 3

	Dependent variable:					
	$\log(\text{TK})$	$\log(w_H/w_L)$	$\log(\text{TK})$	$\log(w_H/w_L)$	$\log(\text{TK})$	$\log(w_H/w_L)$
	Case 1		Case 2		Case 3	
$\log(\text{UD})$	-0.294		0.208		-0.081	
	(0.116)		(0.098)		(0.032)	
$\widehat{\log(\text{TK})}$		0.880^{*}		-0.691		-2.137
		(0.314)		(0.437)		(1.781)
$\log(L_L/L_H)$	0.176^{***}	0.235^{*}	0.304	0.245	0.091	0.197
	(0.031)	(0.090)	(0.280)	(0.178)	(0.046)	(0.208)
Constant	-1.208^{***}	0.649^{**}	-0.493	-0.058	-0.622^{*}	-0.566
	(0.220)	(0.164)	(0.217)	(0.242)	(0.069)	(0.849)
Weak instr.		6.434^{**}		4.505^{**}		6.588^{**}
Wu-Hausman		13.007^{***}		6.853^{**}		2.284
Countries	9	9	7	7	3	3
Median(obs)	14	14	13	13	14	14
Max(obs)	18	18	16	16	16	16
Observations	121	121	93	93	41	41
R^2	0.631	0.369	0.564	0.202	0.914	-0.447

Notes: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
The left column, in each case, refers to the 1st stage regression.
Cluster-robust variance estimaton (by country, CV_3 with Satterthwaite d.f.).

Table W.5: Confidence intervals and p-values for Table W.4 regressions (1st stage)

	Estimate	CI Lower	CI Upper	p-value
Case 1: CV_1, Satt $t(-5.15, 3.96)$	-0.294	-0.454	-0.135	0.007
Case 1: WCR bs., Rademacher	-0.294	-0.396	-0.106	0.012
Case 1: WCR bootstrap, Webb	-0.294	-0.397	-0.106	0.015
Case 1: CV_3, Satt $t(-2.54, 2.49)$	-0.294	-0.711	0.122	0.102
Case 2: CV_1, Satt $t(2.60, 3.82)$	0.208	-0.018	0.435	0.063
Case 2: WCR bs., Rademacher	0.208	-0.291	0.342	0.141
Case 2: WCR bootstrap, Webb	0.208	-0.235	0.345	0.151
Case 2: CV_3, Satt $t(2.12, 2.78)$	0.208	-0.119	0.535	0.131
Case 3: CV_1, Satt $t(-2.91, 1.84)$	-0.081	-0.212	0.050	0.111
Case 3: WRE bs., Rademacher	-0.081	-0.115	0.009	0.250
Case 3: WRE bs., Webb	-0.081	-0.699	0.370	0.332
Case 3: CV_3, Satt $t(-2.57, 1.10)$	-0.081	-0.404	0.241	0.217

Note: Clustered by country, p-values for coefficient of $\log(\text{UD}) = 0$ hypothesis.

TABLE W.6: Confidence intervals and p-values for Table W.4 regressions (2nd stage)

	Estimate	CI Lower	CI Upper	p-value
Case 1: CV_1, Satt $t(4.02, 3.96)$	0.880	0.271	1.489	0.016
Case 1: WRE bs., Rademacher	0.880	-0.122	1.639	0.068
Case 1: WRE bootstrap, Webb	0.880	-0.095	1.606	0.063
Case 1: CV_3, Satt $t(2.80, 2.49)$	0.880	-0.246	2.006	0.084
Case 2: CV_1, Satt $t(-1.65, 3.82)$	-0.691	-1.877	0.496	0.178
Case 2: WRE bs., Rademacher	-0.691	-Inf	Inf	0.133
Case 2: WRE bootstrap, Webb	-0.691	-Inf	Inf	0.133
Case 2: CV_3, Satt $t(-1.58, 2.78)$	-0.691	-2.145	0.764	0.219
Case 3: CV_1, Satt $t(-1.34, 1.84)$	-2.137	-9.595	5.320	0.321
Case 3: WRE bs., Rademacher	-2.137	-Inf	Inf	0.250
Case 3: WRE bs., Webb	-2.137	-Inf	Inf	0.346
Case 3: CV_3, Satt $t(-1.20, 1.10)$	-2.137	-20.269	15.994	0.427

Note: Clustered by country, p-values for coefficient of $\widehat{\log(\mathrm{TK})} = 0$ hypothesis.

TABLE W.7: POLS regressions for Propositions 3.1, 3.4, and 3.5

	Dependent variable:					
	L_L	S_L	g	L_L	S_L	g
	Proposition 3.3 - Case 1			Proposition 3.3 - Case 2		
UD	0.067	0.541***	0.217	−0.014	−0.027	−0.593
	(0.094)	(0.100)	(0.446)	(0.043)	(0.023)	(0.360)
Constant	0.410***	0.259***	0.031***	0.432***	0.367***	0.027***
	(0.016)	(0.017)	(0.004)	(0.026)	(0.014)	(0.008)
R^2	0.004	0.198	0.002	0.001	0.015	0.031

Note: *p<0.1; **p<0.05; ***p<0.01

Appendix X
Individuals' decisions

At period t, individuals choose $c_{j,t}$ and $s_{j,t}$ when maximizing (4.1) and (4.3). Considering $c_{j,t+1} = \bar{R}s_{j,t}$, the problem Lagrangian is given by (4.4) and the first-order conditions are:

$$\frac{\partial \mathscr{L}}{\partial c_{j,t}} = 0 \Leftrightarrow \mu = \frac{1}{c_{j,t}}, \tag{X.1}$$

$$\frac{\partial \mathscr{L}}{\partial c_{j,t+1}} = 0 \Leftrightarrow \zeta \frac{1}{c_{j,t+1}} = \mu \frac{1}{\bar{R}}. \tag{X.2}$$

Rearranging (4.3), we find $\frac{c_{j,t+1}}{\bar{R}} = \left(1-\tau_j\right)\left(1-\eta_j\right) w_{j,t} + T_{j,t} - c_{j,t}$, and substituting (X.1) in (X.2) we find $\frac{c_{j,t+1}}{\bar{R}} = \zeta c_{j,t}$. Then, equalizing these results yields (4.5), and substituting (4.5) back into (4.3) leads to (4.6).

Substituting the results (4.5) and (4.6) back into (4.1), we find

$$\begin{aligned}
u_{j,t} = {} & \log\left\{\frac{1}{1+\zeta}\left[\left(1-\tau_j\right)\left(1-\eta_j\right) w_{j,t} + T_{j,t}\right]\right\} \\
& + \zeta \log\left\{\bar{R}\frac{\zeta}{1+\zeta}\left[\left(1-\tau_j\right)\left(1-\eta_j\right) w_{j,t} + T_{j,t}\right]\right\} \\
& - 1_{[j=H]} v(a) \\
= {} & \left(1+\zeta\right)\log\left[\left(1-\tau_j\right)\left(1-\eta_j\right) w_{j,t} + T_{j,t}\right] \\
& + \zeta \log\left(\bar{R}\zeta\right) - \left(1+\zeta\right)\log\left(1+\zeta\right) - \mathbb{1}_{[j=H]} v(a).
\end{aligned}$$

Then,

$$\begin{aligned}
u_{L,t} &= \left(1+\zeta\right)\log\left[(1-\tau_L)(1-\eta_L) w_{L,t} + T_{L,t}\right], \text{ and} \\
u_{H,t} &= \left(1+\zeta\right)\log\left[(1-\tau_H)(1-\eta_H) w_{H,t} + T_{H,t}\right] - v(a).
\end{aligned}$$

An individual will engage in tertiary education if $u_{H,t} \geq u_{L,t}$, i.e., considering the previous results, case

$$v(a) \leq \left(1+\zeta\right)\log\left[\frac{(1-\tau_H)(1-\eta_H) w_{H,t} + T_{H,t}}{(1-\tau_L)(1-\eta_L) w_{L,t} + T_{L,t}}\right].$$

Substituting (4.2) in the previous result and solving it for equality leads to the threshold ability level (4.7).

Appendix Y
Baseline production

For the sake of simplicity and easier comparison with the model that considers robots and artificial intelligence in the productive side of the economy, we drop the subscript O from the notation in this section.

Final goods. The production function to be considered is

$$Y_t = L_{L,t}^{\alpha} L_{H,Y,t}^{\beta} \int_0^{A_t} X_{i,t}^{1-\alpha-\beta}\, di, \quad 0 < \alpha + \beta < 1. \tag{Y.1}$$

The representative firm solves the following optimization each period:

$$\Pi_{Y,t} = \max_{X_{i,t}, L_{L,t}, L_{H,Y,t}} Y_t - \int_0^{A_t} (1+\tau_X)\, p_{i,t} X_{i,t}\, di - w_{L,t} L_{L,t} - w_{H,Y,t} L_{H,Y,t}. \tag{Y.2}$$

Substituting (Y.1) in (Y.2), the first-order conditions (FOCs) give the inverse demand functions for each intermediate good X_i, unskilled labor, and skilled labor; respectively:

$$\frac{\partial \Pi_{Y,t}}{\partial X_{i,t}} = 0 \Leftrightarrow p_{i,t} = \frac{1-\alpha-\beta}{1+\tau_X} L_{L,t}^{\alpha} L_{H,Y,t}^{\beta} X_{i,t}^{-\alpha-\beta}, \tag{Y.3}$$

$$\frac{\partial \Pi_{Y,t}}{\partial L_{L,t}} = 0 \Leftrightarrow w_{L,t} = \alpha L_{L,t}^{\alpha-1} L_{H,Y,t}^{\beta} \int_0^{A_t} X_{i,t}^{1-\alpha-\beta}\, di = \alpha \frac{Y_t}{L_{H,Y,t}}, \text{ and} \tag{Y.4}$$

$$\frac{\partial \Pi_{Y}}{\partial L_{H,Y,t}} = 0 \Leftrightarrow w_{H,Y,t} = \beta L_{L,t}^{\alpha} L_{H,Y,t}^{\beta-1} \int_0^{A_t} X_{i,t}^{1-\alpha-\beta}\, di = \beta \frac{Y_t}{L_{H,Y,t}}. \tag{Y.5}$$

Intermediate goods. Considering (Y.3), a monopolistic firm i solves

$$\Pi_{i,t} = \max_{X_{i,t}} p_{i,t} X_{i,t} - \bar{R} X_{i,t} = \max_{X_{i,t}} \frac{1-\alpha-\beta}{1+\tau_X} L_{L,t}^{\alpha} L_{H,Y,t}^{\beta} X_{i,t}^{-\alpha-\beta} - \bar{R} X_{i,t}, \tag{Y.6}$$

which has the following FOC:

$$\frac{\partial \Pi_{i,t}}{\partial X_{i,t}} = 0 \Leftrightarrow X_{i,t}^{\alpha+\beta} = \frac{(1-\alpha-\beta)^2 L_{L,t}^{\alpha} L_{H,Y,t}^{\beta}}{(1+\tau_X)\bar{R}}. \tag{Y.7}$$

Substituting (Y.7) back in (Y.3), we find that all varieties will be charged the same price:

$$p_{i,t} = p_{x,t} = \frac{\bar{R}}{1-\alpha-\beta}, \forall i. \tag{Y.8}$$

Substituting (Y.8) in (Y.3) and solving for $\bar{R}$ gives

$$\bar{R} = \frac{(1-\alpha-\beta)^2}{1+\tau_X} L_{L,t}^{\alpha} L_{H,Y,t}^{\beta} X_{i,t}^{-\alpha-\beta}. \tag{Y.9}$$

Intermediate firms of non-patent-protected older vintages j solve $\Pi_{j,t} = \max_{X_{ijt}} p_{j,t}X_{j,t} - \bar{R}X_{j,t} = 0$ and set their price at the marginal cost, i.e., $\frac{\partial \Pi_{j,t}}{\partial X_{j,t}} = 0 \Leftrightarrow p_{j,t} = \bar{R}$. Substituting this last result for $X_{i,t}$ in (Y.3) gives

$$\bar{R} = \frac{1-\alpha-\beta}{1+\tau_X} L_{L,t}^{\alpha} L_{H,Y,t}^{\beta} X_{j,t}^{-\alpha-\beta}. \tag{Y.10}$$

Equalizing (Y.9) and (Y.10), we find X_j demand as a function of X_i:

$$X_{j,t} = (1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}} X_{i,t}.$$

Because the demand for each new variety is independent of i, from (Y.9), we can write

$$X_{i,t} = X_t = \frac{(1-\alpha-\beta)^{\frac{2}{\alpha+\beta}} L_{L,t}^{\frac{\alpha}{\alpha+\beta}} L_{H,Y,t}^{\frac{\beta}{\alpha+\beta}}}{(1+\tau_X)^{\frac{1}{\alpha+\beta}} \bar{R}^{\frac{1}{\alpha+\beta}}}. \tag{Y.11}$$

Substituting (Y.8) and (Y.11) in (Y.6) gives

$$\Pi_{i,t} = \frac{(\alpha+\beta)\bar{R}X_{i,t}}{1-\alpha-\beta} = \frac{(\alpha+\beta)(1-\alpha-\beta)^{\frac{2-\alpha-\beta}{\alpha+\beta}} L_{L,t}^{\frac{\alpha}{\alpha+\beta}} L_{H,Y,t}^{\frac{\beta}{\alpha+\beta}}}{(1+\tau_X)^{\frac{1}{\alpha+\beta}} \bar{R}^{\frac{1-\alpha-\beta}{\alpha+\beta}}}. \tag{Y.12}$$

The demand for newer varieties is $(A_t - A_{t-1})X_{i,t}$, and for older varieties, it is $A_{t-1}X_{j,t} = A_{t-1}(1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}} X_{i,t}$. Thus, letting $\tilde{A}_t$ denote the adjusted aggregate demand for $X_{j,t}$ and considering (4.11) and (Y.11), the adjusted aggregate amount intermediate goods used in the final goods' sector is

$$\int_0^{A_t} X_{i,t}\, di \equiv \tilde{A}_t X_t = (A_t - A_{t-1})X_t + A_{t-1}(1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}} X_t, \text{ with} \tag{Y.13}$$

$$\tilde{A}_t = \left[(1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}} + g_A\right] A_{t-1}. \tag{Y.14}$$

General equilibrium. Using (Y.13) allows to rewrite (Y.1) as

$$Y_t = L_{L,t}^{\alpha} L_{H,Y,t}^{\beta} \tilde{A}_t X_t^{1-\alpha-\beta}. \tag{Y.15}$$

Substituting (Y.11), (Y.14), and (Y.15) in Y.4 gives

$$w_{L,t} = \alpha L_{L,t}^{\frac{-\beta}{\alpha+\beta}} L_{H,Y,t}^{\frac{\beta}{\alpha+\beta}} \left[(1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}} + g_A \right] A_{t-1} \left\{ \frac{(1-\alpha-\beta)^{\frac{2}{\alpha+\beta}}}{(1+\tau_A)^{\frac{1}{\alpha+\beta}} \bar{R}^{\frac{1}{\alpha+\beta}}} \right\}^{1-\alpha-\beta}. \tag{Y.16}$$

Substituting (Y.11), (Y.14), and (Y.15) in (Y.5) gives

$$w_{H,Y,t} = \beta L_{L,t}^{\frac{\alpha}{\alpha+\beta}} L_{H,Y,t}^{\frac{-\alpha}{\alpha+\beta}} \left[(1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}} + g_A \right] A_{t-1} \left\{ \frac{(1-\alpha-\beta)^{\frac{2}{\alpha+\beta}}}{(1+\tau_X)^{\frac{1}{\alpha+\beta}} \bar{R}^{\frac{1}{\alpha+\beta}}} \right\}^{1-\alpha-\beta}. \tag{Y.17}$$

Substituting (Y.12) in the free-entry condition, $p_{A,t} = \Pi_{i,t}$, and the result in (4.10) gives

$$w_{H,A,t} = \frac{\delta(\alpha+\beta)(1-\alpha-\beta)^{\frac{2-\alpha-\beta}{\alpha+\beta}} L_{L,t}^{\frac{\alpha}{\alpha+\beta}} L_{H,Y,t}^{\frac{\beta}{\alpha+\beta}} A_{t-1}^{\phi}}{(1+\tau_X)^{\frac{1}{\alpha+\beta}} \bar{R}^{\frac{1-\alpha-\beta}{\alpha+\beta}} L_{H,A,t}^{1-\lambda}}. \tag{Y.18}$$

Skilled wages in R&D and production sectors converge in equilibrium, i.e., $w_{H,A,t} = w_{H,Y,t}$, thus, equalizing (Y.17) and (Y.18) we find

$$\delta \Lambda_6 L_{H,Y,t} = \beta (\Lambda_4 + g_A) \Lambda_5 A_{t-1}^{1-\phi} L_{H,A,t}^{1-\lambda}, \tag{Y.19}$$

$$\begin{aligned} \text{where } \Lambda_4 &= (1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}}, \\ \Lambda_5 &= (1+\tau_X), \text{ and} \\ \Lambda_6 &= (\alpha+\beta)(1-\alpha-\beta). \end{aligned} \tag{Y.20}$$

Substituting (Y.14) in (4.7) and rearranging we find

$$\begin{aligned} (a^* - a_{\min})^{\frac{\theta}{1+\zeta}} \left[\beta \Lambda_1 \Lambda_2 (\Lambda_4 + g_A) A_{t-1} L_{L,t}^{\frac{\alpha}{\alpha+\beta}} L_{H,Y,t}^{\frac{-\alpha}{\alpha+\beta}} + T_{H,t} \right] = \\ \psi \left[\alpha \Lambda_1 \Lambda_3 (\Lambda_4 + g_A) A_{t-1} L_{L,t}^{\frac{-\beta}{\alpha+\beta}} L_{H,Y,t}^{\frac{\beta}{\alpha+\beta}} + T_{L,t} \right], \end{aligned} \tag{Y.21}$$

$$\text{with } \Lambda_1 = \left\{ \frac{(1-\alpha-\beta)^2}{(1+\tau_A)\bar{R}} \right\}^{\frac{1-\alpha-\beta}{\alpha+\beta}},$$
$$\Lambda_2 = (1-\tau_H)(1-\eta_H), \text{ and}$$
$$\Lambda_3 = (1-\tau_L)(1-\eta_L).$$

Then, substituting $L_{L,t} = a_t^* L$, $L_{H,t} = L - L_{L,t}$, and $L_{H,Y,t} = L_{H,t} - L_{H,A,t}$ in (Y.19) and (Y.21), results in the *loci* M and N, respectively, in system (4.19).

Slopes. Because $L_{H,t}$ and $L_{H,A,t}$ are implicitly defined by the equations in (4.19), and bearing in mind that $L_{H,A,t}$ is also a function of $L_{H,t}$, we have that

$$\frac{dM}{dL_{H,t}} \equiv \frac{\partial M}{\partial L_{H,t}} + \frac{\partial M}{\partial L_{H,A,t}} \frac{dL_{H,A,t}}{dL_{H,t}} = 0 \Leftrightarrow \frac{dL_{H,A,t}}{dL_{H,t}} = -\frac{\partial M / \partial L_{H,t}}{\partial M / \partial L_{H,A,t}} > 0,$$

where

$$\frac{\partial M}{\partial L_{H,A,t}} = -\delta \Lambda_6 A_{t-1}^{\phi} - \beta \delta \Lambda_5 A_{t-1}^{\phi} - \beta (1-\lambda) \Lambda_4 \Lambda_6 A_{t-1} L_{H,A,t}^{-\lambda} < 0$$

and

$$\frac{\partial M}{\partial L_{H,t}} = \delta \Lambda_6 A_{t-1}^{\phi} > 0.$$

Similarly, for N curve we have $\frac{dL_{H,A,t}}{dL_{H,t}} = -\frac{\partial N / \partial L_{H,t}}{\partial N / \partial L_{H,A,t}} > 0$; it is easy to verify that $\frac{\partial N}{\partial L_{H,t}} < 0$, because all terms go in the same direction; regarding $\frac{\partial N}{\partial L_{H,A,t}}$ we can show that, with proper initial conditions, i.e., A_{t-1} high enough, $\frac{\partial N}{\partial L_{H,A,t}}$ is positive if $\beta > \delta\lambda$ or $\frac{\beta \Lambda_4}{\alpha+\beta} > \delta\lambda$, which tends to be true, because $\Lambda_4 > 1$, $\frac{1}{\alpha+\beta} > 1$, $\delta < 1$, and $\lambda < 1$. In

$$\begin{aligned}
\frac{\partial N}{\partial L_{H,A,t}} = & \frac{\alpha\beta\Lambda_1\Lambda_2\Lambda_4}{\alpha+\beta} \left(\frac{L-L_{H,t}}{L} - a_{\min} \right)^{\frac{\theta}{1+\zeta}} \left(L_{H,t} - L_{H,A,t} \right)^{-1-\frac{\alpha}{\alpha+\beta}} A_{t-1} \\
& + \frac{\alpha\beta\delta\Lambda_1\Lambda_2}{\alpha+\beta} \left(\frac{L-L_{H,t}}{L} - a_{\min} \right)^{\frac{\theta}{1+\zeta}} \left(L_{H,t} - L_{H,A,t} \right)^{-1-\frac{\alpha}{\alpha+\beta}} L_{H,A,t}^{\lambda} A_{t-1}^{\phi} \\
& + \beta\delta\lambda\Lambda_1\Lambda_2 \left(\frac{L-L_{H,t}}{L} - a_{\min} \right)^{\frac{\theta}{1+\zeta}} \left(L_{H,t} - L_{H,A,t} \right)^{\frac{-\alpha}{\alpha+\beta}} L_{H,A,t}^{\lambda-1} A_{t-1}^{\phi} \\
& + \frac{\alpha\beta\psi\Lambda_1\Lambda_3\Lambda_4}{\alpha+\beta} \left(L - L_{H,t} \right)^{\frac{-\beta}{\alpha}} \left(L_{H,t} - L_{H,A,t} \right)^{\frac{-\alpha}{\alpha+\beta}} A_{t-1} \\
& + \frac{\alpha\beta\delta\psi\Lambda_1\Lambda_3}{\alpha+\beta} \left(L - L_{H,t} \right)^{\frac{-\beta}{\alpha}} \left(L_{H,t} - L_{H,A,t} \right)^{\frac{-\alpha}{\alpha+\beta}} L_{H,A,t}^{\lambda} A_{t-1}^{\phi} \\
& - \alpha\delta\lambda\psi\Lambda_1\Lambda_3 \left(L - L_{H,t} \right)^{\frac{-\beta}{\alpha}} \left(L_{H,t} - L_{H,A,t} \right)^{\frac{\beta}{\alpha+\beta}} L_{H,A,t}^{\lambda-1} A_{t-1}^{\phi},
\end{aligned}$$

notice the sign of the combination of the fourth line with the last one will be defined by the expression $\beta\frac{\Lambda_4}{\alpha+\beta}L_{H,A,t}^{1-\lambda}A_{t-1}^{1-\phi}-\delta\lambda\left(L_{H,t}-L_{H,A,t}\right)$, and that, because population is fixed, an appropriate value for A_{t-1} ensures $L_{H,A,t}^{1-\lambda}A_{t-1}^{1-\phi}>L_{H,t}-L_{H,A,t}$.

Also, notice in (4.19) that $(0,0)$ is a solution of M, which starts at the origin of $\left(L_{H,t},L_{H,A,t}\right)$ space. In turn, in N curve, $L_{H,A,t}=0\rightarrow L_{H,t}>0$, that is, N curve starts right of the origin in the space. Thus, in the region of the space where a solution exists, the solution is unique and N slope is greater than M slope.

Comparative statics. The derivative of $L_{H,A}$ is implicitly defined from (Y.19), which can be expressed as

$$\delta\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)L_{H,Y,t}A_{t-1}^{\phi}=\beta\left(1+\tau_X\right)\tilde{A}_tL_{H,A,t}^{1-\lambda}\Leftrightarrow$$

$$\begin{aligned}
\frac{\partial\log}{\partial a^*}\left[\delta\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)L_{H,Y,t}A_{t-1}^{\phi}\right]&=\frac{\partial\log}{\partial a^*}\left[\beta\left(1+\tau_X\right)\tilde{A}_tL_{H,A,t}^{1-\lambda}\right]\Leftrightarrow\\
\frac{\partial\log}{\partial a^*}\left[L_{H,Y,t}\right]&=\frac{\partial}{\partial a^*}\log\tilde{A}_t+(1-\lambda)\frac{\partial}{\partial a^*}\log\left[L_{H,A,t}\right]\Leftrightarrow\\
\frac{-L-\frac{\partial L_{H,A,t}}{\partial a^*}}{L_{H,Y,t}}&=\frac{\delta\lambda\frac{\partial L_{H,A,t}}{\partial a^*}L_{H,A,t}^{\lambda-1}A_{t-1}}{\tilde{A}_t}+(1-\lambda)\frac{\frac{\partial L_{H,A,t}}{\partial a^*}}{L_{H,A,t}}\Leftrightarrow\\
\frac{-L-\frac{\partial L_{H,A,t}}{\partial a^*}}{L_{H,Y,t}}&=\frac{\partial L_{H,A,t}}{\partial a^*}\left[\frac{\delta\lambda L_{H,A,t}^{\lambda}A_{t-1}+(1-\lambda)\tilde{A}_t}{\tilde{A}_tL_{H,A,t}}\right]\Leftrightarrow\\
\frac{-L}{L_{H,Y,t}}&=\frac{\partial L_{H,A,t}}{\partial a^*}\left[\frac{\delta\lambda L_{H,A,t}^{\lambda}A_{t-1}+(1-\lambda)\tilde{A}_t}{\tilde{A}_tL_{H,A,t}}+\frac{1}{L_{H,Y,t}}\right].
\end{aligned}$$

Because $\frac{\delta\lambda L_{H,A,t}^{\lambda}+(1-\lambda)\tilde{A}_t}{\tilde{A}_tL_{H,A,t}}+\frac{1}{L_{H,Y,t}}>0$, we have

$$\frac{\partial L_{H,A,t}}{\partial a_t^*}<0. \tag{Y.22}$$

From $L_{H,Y,t}=L_{H,t}-L_{H,A,t}$ we have $\frac{\partial L_{H,Y}}{\partial a^*}<0$:

$$\begin{aligned}
\frac{\partial L_{H,Y,t}}{\partial a_t^*}&=-\frac{\partial L_{L,t}}{\partial a_t^*}-\frac{\partial L_{H,A,t}}{\partial a_t^*}=-L-\frac{\partial L_{H,A,t}}{\partial a_t^*}\\
&=\frac{L}{L_{H,Y,t}}\left\{\left[\frac{\delta\lambda L_{H,A,t}^{\lambda}A_{t-1}+(1-\lambda)\tilde{A}_t}{\tilde{A}_tL_{H,A,t}}+\frac{1}{L_{H,Y,t}}\right]^{-1}-L_{H,Y,t}\right\}.
\end{aligned}$$

Notice that $\left[\frac{\delta\lambda L_{H,A,t}^{\lambda}A_{t-1}+(1-\lambda)\tilde{A}_t}{\tilde{A}_t L_{H,A,t}}+\frac{1}{L_{H,Y,t}}\right]^{-1}-L_{H,Y,t}<0$ because

$$\left[\frac{\delta\lambda L_{H,A,t}^{\lambda}A_{t-1}+(1-\lambda)\tilde{A}_t}{\tilde{A}_t L_{H,A,t}}+\frac{1}{L_{H,Y,t}}\right]^{-1}<L_{H,Y,t}\Leftrightarrow$$

$$\frac{\delta\lambda L_{H,A,t}^{\lambda}A_{t-1}+(1-\lambda)\tilde{A}_t}{\tilde{A}_t L_{H,A,t}}>0\Rightarrow\frac{\partial L_{H,Y}}{\partial a^*}<0. \tag{Y.23}$$

From (4.11) we wave $\frac{\partial g_{A,t}}{\partial a_t^*}=\delta\lambda L_{H,A,t}^{\lambda-1}\frac{\partial L_{H,A,t}}{\partial a_t^*}<0.$

From (Y.16) we have

$$\frac{\partial w_{L,t}}{\partial a_t^*}=\frac{-\beta}{\alpha+\beta}\frac{w_{L,t}}{L_{L,t}}\frac{\partial L_{L,t}}{\partial a_t^*}+\frac{\beta}{\alpha+\beta}\frac{w_L}{L_{H,Y,t}}\frac{\partial L_{H,Y,t}}{\partial a_t^*}+\frac{w_{L,t}}{\left[(1-\alpha-\beta)^{\frac{1}{-\alpha-\beta}}+g_{A,t}\right]}\frac{\partial g_{A,t}}{\partial a_t^*}<0.$$

From (Y.16) and (Y.17) we have

$$\frac{w_{H,Y,t}}{w_{L,t}}=\frac{\beta}{\alpha}\frac{L_{L,t}}{L_{H,Y,t}}\Leftrightarrow$$

$$\frac{\partial}{\partial a_t^*}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)=\frac{1}{L_{L,t}}\frac{w_{H,Y}}{w_{L,t}}\frac{\partial L_{L,t}}{\partial a_t^*}-\frac{1}{L_{H,Y,t}}\frac{w_{H,Y,t}}{w_{L,t}}\frac{\partial L_{H,Y,t}}{\partial a_t^*}>0.$$

From (Y.17) we will find $\frac{\partial w_{H,Y,t}}{\partial a_t^*}\gtrless 0$, and from (Y.18) we will find $\frac{\partial w_{H,A,t}}{\partial a_t^*}\gtrless 0$.

Appendix Z
Production with robots and artificial intelligence

Considering (4.24), we have

$$\frac{\partial}{\partial L_{L,t}}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)=\left[\frac{1-\theta}{L_{L,t}}+\frac{\beta_1(\theta-\gamma)L_{L,t}^{\theta}}{\left[\beta_1 L_{L,t}^{\theta}+(1-\beta_1)P_t^{\theta}\right]L_{L,t}}\right]\frac{w_{H,Y,t}}{w_{L,t}}$$
$$=\left\{\frac{(1-\gamma)\beta_1 L_{L,t}^{\theta}+(1-\theta)(1-\beta_1)P_t^{\theta}}{\left[\beta_1 L_{L,t}^{\theta}+(1-\beta_1)P_t^{\theta}\right]L_{L,t}}\right\}\frac{w_{H,Y,t}}{w_{L,t}}>0, \tag{Z.1}$$

$$\frac{\partial}{\partial L_{H,Y,t}}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)=\left[\frac{\sigma-1}{L_{H,Y,t}}+\frac{(\gamma-\sigma)\beta_2 L_{H,Y,t}^{\sigma}}{\left[\beta_2 L_{H,Y,t}^{\sigma}+(1-\beta_2)G_t^{\sigma}\right]L_{H,Y,t}}\right]\frac{w_{H,Y,t}}{w_{L,t}}$$
$$=\left\{\frac{(\gamma-1)\beta_2 L_{H,Y,t}^{\sigma}+(\sigma-1)(1-\beta_2)G_t^{\sigma}}{\left[\beta_2 L_{H,Y,t}^{\sigma}+(1-\beta_2)G_t^{\sigma}\right]L_{H,Y,t}}\right\}\frac{w_{H,Y,t}}{w_{L,t}}<0, \tag{Z.2}$$
$$\frac{\partial}{\partial P_t}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)=\left[\frac{(\theta-\gamma)(1-\beta_1)P_t^{\theta-1}}{\beta_1 L_{L,t}^{\theta}+(1-\beta_1)P_t^{\theta}}\right]\frac{w_{H,Y,t}}{w_{L,t}},\text{ and} \tag{Z.3}$$
$$\frac{\partial}{\partial G_t}\left(\frac{w_{H,Y,t}}{w_{L,t}}\right)=\left[\frac{(\gamma-\sigma)(1-\beta_2)G_t^{\sigma-1}}{\beta_2 L_{H,Y,t}^{\sigma}+(1-\beta_2)G_t^{\sigma}}\right]\frac{w_{H,Y,t}}{w_{L,t}}.$$

The problem an intermediate sector firm has to solve in the current production technology context (4.20) is

$$\Pi_{x,i,t}=\max_{X_{i,t}}\quad(1+\tau_A)^{-1}\alpha x_{i,t}^{\alpha}L_{\text{CES},t}^{1-\alpha}-\bar{R}x_{i,t},$$

from which the first-order condition (FOC) is

$$\frac{\partial\Pi_{x,i,t}}{\partial x_{i,t}}=0\Leftrightarrow\bar{R}=(1+\tau_X)^{-1}\alpha^2 x_{i,t}^{\alpha-1}L_{\text{CES},t}^{1-\alpha}.$$

The number of goods each firm i will produce does not rely on any variable specific to this firm or good, which means that it will be the same for all firms and given by (4.25). Consequently, solving (4.21) for $x_{i,t}$ and substituting the result in (4.25), we find that the real price of any intermediate good will be the same for all i, and given by

$$\bar{R}=\alpha p_{x,i,t}\Leftrightarrow p_{x,i,t}=\frac{\bar{R}}{\alpha},\forall i. \tag{Z.4}$$

Substituting (Z.4) back in $\Pi_{x,i,t} = p_{x,i,t}x_{i,t} - \bar{R}x_{i,t}$ yields

$$\Pi_{x,i,t} = \frac{1-\alpha}{\alpha}\bar{R}x_{i,t}. \tag{Z.5}$$

From (4.9), (4.10), the free-entry condition, $p_{A,t} = \Pi_{i,t}$, (4.25), and (Z.5) we find the wage for skilled labor in R&D sector:

$$\begin{aligned}
w_{H,A,t} &= p_{A,t}\bar{\delta} = \Pi_{x,i,t}\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} = \frac{1-\alpha}{\alpha}\bar{R}x_t\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} \\
&= \left[\frac{\alpha^{1+\alpha}}{(1+\tau_X)\bar{R}^{\alpha}}\right]^{\frac{1}{1-\alpha}}\frac{\delta(1-\alpha)A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}}L_{\mathrm{CES},t} \qquad \text{(Z.6)}\\
&= \left[\frac{\alpha^{1+\alpha}}{(1+\tau_X)\bar{R}^{\alpha}}\right]^{\frac{1}{1-\alpha}}\frac{\delta(1-\alpha)A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} \\
&\times\left\{\beta_3\left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1)P_t^{\theta}\right]^{\frac{\gamma}{\theta}} + (1-\beta_3)\left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2)G_t^{\sigma}\right]^{\frac{\gamma}{\sigma}}\right\}^{\frac{1}{\gamma}}.
\end{aligned}$$

Substituting (4.25) and (4.26) in (4.22) gives

$$\begin{aligned}
w_{L,t} &= (1-\alpha)\beta_1\beta_3 L_{L,t}^{\theta-1}L_{L,\mathrm{CES},t}^{\gamma-\theta}L_{\mathrm{CES},t}^{1-\alpha-\gamma}\tilde{A}_t X_t^{\alpha} \\
&= (1-\alpha)\beta_1\beta_3 L_{L,t}^{\theta-1}\left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1)P_t^{\theta}\right]^{\frac{\gamma-\theta}{\theta}} \\
&\times\left\{\beta_3\left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1)P_t^{\theta}\right]^{\frac{\gamma}{\theta}} + (1-\beta_3)\left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2)G_t^{\sigma}\right]^{\frac{\gamma}{\sigma}}\right\}^{\frac{1-\gamma}{\gamma}} \\
&\times\left(\alpha^{\frac{1}{\alpha-1}} + g_A\right)A_{t-1}\left[\frac{\alpha^2}{(1+\tau_X)\bar{R}}\right]^{\frac{\alpha}{1-\alpha}}.
\end{aligned}$$

Substituting (4.25) and (4.26) in (4.23) gives

$$\begin{aligned}
w_{H,Y,t} &= (1-\alpha)\beta_2(1-\beta_3)L_{H,Y,t}^{\sigma-1}L_{H,\mathrm{CES},t}^{\gamma-\sigma}L_{\mathrm{CES},t}^{1-\alpha-\gamma}\tilde{A}_t X_t^{\alpha} \qquad \text{(Z.7)}\\
&= (1-\alpha)\beta_2(1-\beta_3)L_{H,Y,t}^{\sigma-1}\left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2)G_t^{\sigma}\right]^{\frac{\gamma-\sigma}{\sigma}} \\
&\times\left\{\beta_3\left[\beta_1 L_{L,t}^{\theta} + (1-\beta_1)P_t^{\theta}\right]^{\frac{\gamma}{\theta}} + (1-\beta_3)\left[\beta_2 L_{H,Y,t}^{\sigma} + (1-\beta_2)G_t^{\sigma}\right]^{\frac{\gamma}{\sigma}}\right\}^{\frac{1-\gamma}{\gamma}} \\
&\times\left(\alpha^{\frac{1}{\alpha-1}} + g_A\right)A_{t-1}\left[\frac{\alpha^2}{(1+\tau_X)\bar{R}}\right]^{\frac{\alpha}{1-\alpha}}.
\end{aligned}$$

Equalizing the skilled wages in both sectors, considering (4.25), (4.26), (Z.6) and (Z.7), we find

$$\alpha\delta L_{\mathrm{CES},t}^{\alpha+\gamma}-(1+\tau_X)\,\beta_2\,(1-\beta_3)\,L_{H,Y,t}^{\sigma-1}L_{H,A,t}^{1-\lambda}L_{H,\mathrm{CES},t}^{\gamma-\sigma}\left(\alpha^{\frac{1}{\alpha-1}}+g_A\right)A_{t-1}^{1-\phi}=0.$$

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