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CORRECTION

Correction to: A classification of nilpotent compatible Lie algebras

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Explanation of the mistake

We have made a computational mistake when calculating the orbits of some cocycles in Section 4.4 of the original paper. This makes it so that three families of orbits (and thus three compatible Lie algebra isomorphism families), which previously depended on the field structure, now do not, and now depend on one less parameter. Thus, the classification is independent of the base field (as long as the characteristic is not 2).

Errata

Section 4.4 : In the classification of the extensions of $\mathsf{NCL}_{3,2}$

The computation of the last orbit (page 23) should read

> If $\beta' \neq 0$, consider the automorphism
>
> $$\phi = \begin{pmatrix} 1/\beta' & -\alpha'/\beta' & 0 \\ 0 & 1 & 0 \\ 0 & -\delta'/\beta' & 1/\beta' \end{pmatrix}, \text{ which sends } \gamma_1 + \alpha'\gamma_4 + \beta'\gamma_5 + \delta'\gamma_6 \text{ to } \frac{1}{\beta'}(\gamma_1 + \gamma_5).$$

The original article can be found online at https://doi.org/10.1007/s12215-024-01126-z.

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and be followed by

> We thus have all cases covered and we have the following orbits (already taking cohomology into account):
>
> $$\mathcal{O}([\gamma_1]),\quad \mathcal{O}([\gamma_4]),\quad \mathcal{O}([\gamma_1+\beta\gamma_4]),\quad \mathcal{O}([\gamma_1+\gamma_5]),\quad \mathcal{O}([\gamma_1+\gamma_6]),\quad \beta\in\mathbb{K}^\times \tag{11}$$
>
> where the third orbit is a one-parameter family. We will now show that all of these orbits are pairwise distinct.

At the end of the same page, the discussion regarding the parameter β in $\mathcal{O}([\gamma_1+\beta\gamma_5])$ should be disregarded, and the discussion regarding $\mathcal{O}([\gamma_1+\beta\gamma_4])$ and $\mathcal{O}([\gamma_1+\beta'\gamma_5])$ should have the parameter β' replaced by 1.

Finally, the list of algebras that are obtained from $\mathsf{NCL}_{3,2}$ should read

> Thus, we have obtained all the distinct $\mathrm{Aut}(\mathsf{NCL}_{3,2})$-orbits. Using the same representatives as in (11) we obtain the following extensions.
>
> - $\mathsf{NCL}_{4,6}$, with relations $[e_1,e_2]=e_3$, $[e_2,e_3]=e_4$;
> - $\mathsf{NCL}_{4,7}$, with relations $[e_1,e_2]=e_3$, $\{e_2,e_3\}=e_4$;
> - $\mathsf{NCL}^{\beta}_{4,8}$, with relations $[e_1,e_2]=e_3$, $[e_2,e_3]=e_4$, $\{e_2,e_3\}=\beta e_4$ $(\beta'\neq 0)$;
> - $\mathsf{NCL}_{4,9}$, with relations $[e_1,e_2]=e_3$, $[e_2,e_3]=e_4$, $\{e_1,e_3\}=e_4$;
> - $\mathsf{NCL}_{4,10}$, with relations $[e_1,e_2]=e_3$, $[e_2,e_3]=e_4$, $\{e_1,e_2\}=e_4$.

Section 4.4 : In the classification of the extensions of $\mathsf{NCL}_{3,3}$

This whole subsection should now read

> Since $\mathsf{NCL}_{3,3}=(\mathsf{NCL}_{3,2})^s$, we can use Propositions 4.1 – 4.3 to obtain the list of one-dimensional extensions of $\mathsf{NCL}_{3,3}$ by just considering the switched copies of the algebras in the corresponding list for $\mathsf{NCL}_{3,2}$.
> They are the following:
>
> - $\mathsf{NCL}_{4,11}$, with relations $\{e_1,e_2\}=e_3$, $\{e_2,e_3\}=e_4$;
> - $\mathsf{NCL}_{4,12}$, with relations $[e_2,e_3]=e_4$, $\{e_1,e_2\}=e_3$;
> - $\mathsf{NCL}^{\beta}_{4,13}$, with relations $[e_2,e_3]=\beta e_4$, $\{e_1,e_2\}=e_3$, $\{e_2,e_3\}=e_4$ $(\beta'\neq 0)$;
> - $\mathsf{NCL}_{4,14}$, with relations $[e_1,e_3]=e_4$, $\{e_1,e_2\}=e_3$, $\{e_2,e_3\}=e_4$;
> - $\mathsf{NCL}_{4,15}$, with relations $[e_1,e_2]=e_4$, $\{e_1,e_2\}=e_3$, $\{e_2,e_3\}=e_4$.

Section 4.4 : In the classification of the extensions of $\mathsf{NCL}^{\alpha}_{3,4}$

Equation (12) on page 25 should now read

> We consider the same orbits as before, namely
>
> $$\mathcal{O}([\gamma_1]),\quad \mathcal{O}([\gamma_4]),\quad \mathcal{O}([\gamma_1+\beta\gamma_4]),\quad \mathcal{O}([\gamma_1+\gamma_5]),\quad \mathcal{O}([\gamma_1+\gamma_6]),\quad \beta\neq 0. \tag{12}$$

At the bottom of the same page, it should read

Thus, we have the following *distinct* orbits

$$\mathcal{O}([\gamma_1]), \quad \mathcal{O}([\gamma_4]), \quad \mathcal{O}([\gamma_1 + \beta\gamma_4]), \quad \mathcal{O}([\gamma_1 + \gamma_5]), \quad \beta \neq 0, \tag{13}$$

and using these representatives, we obtain the following extensions

- $\mathsf{NCL}^{\alpha}_{4,16}$, with relations $[e_1, e_2] = e_3$, $[e_2, e_3] = e_4$, $\{e_1, e_2\} = \alpha e_3$;
- $\mathsf{NCL}^{\alpha}_{4,17}$, with relations $[e_1, e_2] = e_3$, $\{e_1, e_2\} = \alpha e_3$, $\{e_2, e_3\} = e_4$;
- $\mathsf{NCL}^{\alpha,\beta}_{4,18}$, with relations $[e_1, e_2] = e_3$, $[e_2, e_3] = e_4$, $\{e_1, e_2\} = \alpha e_3$, $\{e_2, e_3\} = \beta e_4 \quad (\beta \neq 0)$;
- $\mathsf{NCL}^{\alpha}_{4,19}$, with relations $[e_1, e_2] = e_3$, $[e_2, e_3] = e_4$, $\{e_1, e_2\} = \alpha e_3$, $\{e_1, e_3\} = e_4$.

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