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Demand Response of Thermostatically Controlled Loads in Renewable Energy Communities

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Resumo

Com a crescente penetração das fontes de energia renovável e a criação das Comunidades de Energia Renovável (CER), a estrutura dos sistemas elétricos está a mudar radicalmente. Neste sentido, a flexibilidade do lado da procura é um elemento determinante na integração eficiente da produção renovável, na minimização dos custos energéticos, bem como na otimização do desempenho do sistema. Das alternativas flexíveis disponíveis no mercado, as Cargas Termostaticamente Controladas (TCL) destacam-se pela sua inércia térmica intrínseca, que permite o deslocamento do consumo no tempo sem afectar o conforto do consumidor.

Com a presente dissertação, um modelo de controlo ótimo para a gestão coordenada de TCLs num contexto de Comunidade de Energia Renovável com produção fotovoltaica é proposto e avaliado. O objectivo principal do presente modelo é a minimização do custo total de aquisição de energia elétrica, assegurando-se as restrições de conforto térmico e considerando a variabilidade dos preços horários proveniente do mercado diário.

A metodologia desenvolvida abrange:

(i) O modelo térmico de primeira ordem de acordo com a literatura científica de referência;

(ii) A modelização da carga base por meio de perfis normalizados de consumo de energia elétrica;

(iii) Perfis de produção fotovoltaica por meio de dados obtidos no PVGIS e subsequentemente adaptados para uma resolução quarto-horária; e

(iv) Dados de preços históricos do mercado OMIE.

O problema de controlo foi abordado como problema de otimização em horizonte finito e resolvido por Programação Linear Inteira Mista (MILP), ainda que com variáveis de decisão contínuas (convertidas para lógica binária posteriormente) e a modelização explícita da troca bidirecional com a rede elétrica. Adicionaram-se restrições para impedir a importação e exportação simultânea, assegurando-se assim a consistência económica e física no caso de preços muito baixos ou negativos.

No que diz respeito ao estudo de caso, foram considerados 22 residenciais com instalação fotovoltaica de potência nominal de 60 kW. Foi analisada a performance de três cenários: sem controlo (operação por histerese), otimização por Programação Linear com variáveis relaxadas e sem exportação, e formulação MILP, também com variáveis relaxadas, mas com exportação. Dos resultados alcançados, é possível afirmar que a coordenação ótima das TCLs leva a uma redução do custo energético face ao cenário sem controlo. Além disso, os limites de conforto térmico são respeitados. Foi também observado que existem retornos marginais decrescentes com o aumento da potência fotovoltaica instalada. Por outro lado, observou-se a existência de forte dependência do comportamento de ativação face à tarifa de injeção na rede.

Da análise da solução contínua face à solução discretizada, é possível afirmar que a abordagem é eficiente. Conclui-se que a combinação da flexibilidade térmica com a produção renovável distribuída é uma estratégia tecnicamente viável e economicamente interessante para a melhoria do desempenho energético das Comunidades de Energia Renovável.

Palavras chave: *demand response*, comunidades de energia renovável, cargas controláveis termostaticamente, programação linear, relaxação convexa

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Abstract

The increasing penetration of renewable energy sources and the emergence of Renewable Energy Communities (RECs) are reshaping the structure and operation of modern power systems. In this context, demand-side flexibility will play a decisive role in the integration of renewable generation, in reducing energy prices, and in enhancing overall system performance. Among the currently available flexible resources, Thermostatically Controlled Loads (TCLs) stand out due to their intrinsic thermal inertia, which allows load shifting over time without compromising consumer comfort.

This dissertation proposes and evaluates a control optimization model for the coordinated operation of TCLs within the framework of a Renewable Energy Community capable of photovoltaic generation. The main objective of this model is the minimization of total electricity cost, while satisfying thermal comfort constraints and accounting for time-varying day-ahead market prices.

The developed methodology:

(i) A first-order thermal model, consistent with scientific literature;

(ii) A base load model, using normalized residential consumption profiles;

(iii) Photovoltaic generation profiles obtained through PVGIS in a 15-minute resolution;

(iv) Historical price data from the OMIE market.

The control problem is formulated as a finite-horizon optimization problem and solved through Mixed-Integer Linear Programming (MILP), enabling the representation of the ON/OFF state of TCLs through binary variables and the explicit modeling of bidirectional exchange with the grid. Additional constraints were introduced to prevent simultaneous import and export of energy, ensuring the economic consistency and physical feasibility of the model, particularly in scenarios with very low or negative prices.

The case study analyses a community of 22 residential homes and a 60-kW photovoltaic installation under three different scenarios: (1) operation without coordinated control, (2) Linear Programming (LP) optimization with relaxed variables, with no exportation to the grid, and (3) MILP formulation with relaxed variables as well, but with the possibility of exporting energy to the grid. The results show that optimal coordination of TCLs leads to significant reductions in energy costs compared to the uncontrolled scenario, while always satisfying the thermal comfort constraints. Other findings include the strong dependence of the optimal coordinated control on the feed-in tariff value, and that as installed photovoltaic capacity increases the results show diminishing returns on costs savings. The comparison between the continuous and discrete solutions reveals only a slight deviation, validating the computational efficiency of the developed methodology. Overall, this methodology reveals that the integration of thermal flexibility with distributed renewable generation constitutes a technically feasible and economically advantageous strategy to enhance the energy efficiency of Renewable Energy Communities.

Keywords: Demand response, renewable energy communities, thermostatically controlled loads, linear programming, convex relaxation

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United Nations Sustainable Development Goals (SDGs)

This dissertation makes a direct contribution towards the achievement of the United Nations 2030 Agenda’s Sustainable Development Goals (SDGs), in particular **Goal 7, Affordable and Clean Energy,** and **Goal 11, Sustainable Cities and Communities.**

The development of an optimal control model for Thermostatically Controlled Loads in a Renewable Energy Community with photovoltaic production makes a direct contribution towards the achievement of a more sustainable future by encouraging the efficient use of renewable sources of energy and reducing the need for traditional energy from the conventional grid. It is shown how intelligent demand coordination makes it possible to minimize the costs of energy and maximize the use of renewable sources of energy, contributing towards a more sustainable future in terms of the efficient and affordable use of clean energy, in accordance with Goal 7.

The focus on a community of residential homes and the use of self-consumption of renewable energy also makes a direct contribution towards the achievement of Goal 11. The decentralized approach towards the use of renewable sources of energy is an important aspect in the development of more sustainable urban models.

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“Even if you are not ready for day
it cannot always be night.”
- Gwendolyn Brooks

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Acronyms and Symbols

Acronym list

ARIMA	AutoRegressive Integrated Moving Average
ARIMAX	AutoRegressive Integrated Moving Average with Exogenous Inputs
BTN	Baixa Tensão Normal
CEC	Citizens' Energy Community
DGEG	Direção-Geral de Energia e Geologia
DGO	Distribution Grid Operator
DLC	Direct Load Control
DR	Demand Response
DRP	Demand Response Program
DSO	Distribution System Operator
DSM	Demand Side Management
EGAC	Entity for the Collective Self-Consumption Management
ERSE	Entidade Reguladora dos Serviços Energéticos
EU	European Union
EV	Electric Vehicle
GARCH	Generalized AutoRegressive Conditional Heteroskedasticity
HOE	Horário Oficial de Espanha
I/CL	Interruptible/Curtailable Load
IEA	International Energy Agency
JRC	Joint Research Centre
LP	Linear Programming
LSE	Load Serving Entity
MIBEL	Mercado Ibérico de Eletricidade
MILP	Mixed-Integer Linear Programming
OMI	Operador do Mercado Ibérico de Energia
OMIE	Operador do Mercado Ibérico de Energia (Espanha)
OMIP	Operador do Mercado Ibérico de Energia (Portugal)
PICASSO	Platform for the International Coordination of Automated Frequency Restoration and Stable System Operation
PV	Photovoltaic

PVGIS	Photovoltaic Geographical Information System
RC	Resistance-Capacitance
REC	Renewable Energy Community
RED II	Renewable Energy Directive II
RES	Renewable Energy Sources
SDGs	Sustainable Development Goals
SEN	Sistema Elétrico Nacional
TOU	Time-of-Use
TSO	Transmission System Operator
V2G	Vehicle-to-Grid
VPP	Virtual Power Plant

Symbol list

α	Thermal coefficient
β	Thermal parameter

Chapter 1

Introduction

The current energetic transition, boosted by the international decarbonization commitments and growing renewable penetration, has been deeply changing the traditional structure of electrical systems. The decentralization of energy production, coupled with digitalization and active consumer participation, guides a paradigm in which demand stops being passive and is given a relevant operational role in its systems

In this context, Renewable Energy Communities (REC) appear as an organizational model that promotes decentralized production, collective auto consumption and local energy sharing. However, the massive integration of variable renewable energy, such as photovoltaic, comes with a few operational challenges related to availability, time variability and lag between production and consumption.

Demand side flexibility constitutes a fundamental tool to mitigate these challenges. Between flexible resources available in the residential sector, the thermostatically controlled loads (TCLs), like heating or AC systems, distinguish themselves by their ability to reallocate consumption through time due to the building's thermal inertia, without compromising user comfort.

This thesis frames itself in this problem, proposing an optimal control model for the TCLs coordination in a renewable energy community with photovoltaic production, based on the previous works of Fontes et al. and Halder et al., simultaneously considering technical restrictions, comfort limits and economical signals from the daily energy market.

1.1 - Motivation

The growing consumption electrification and the rise of renewable penetration demand new energy management strategies that allow for the assurance of economic efficiency and stability in the electric system. The simple photovoltaic system installation isn't enough to maximize economical and energetic benefits. Due to solar production presenting a strong concentration within a time frame, it frequently lags from the higher consumption periods

On the other hand, electricity markets, such as the daily OMIE market, present high price volatility and, in some cases, very small or even negative values. This reality creates opportunities, but it also demands decision models capable of coherent technical and economic information integration.

The scientific literature has demonstrated that TCLs represent a promising resource for the demand response programs, working as distributed thermal storage. works as of Halder et al. e Fontes et al. established strong foundations for the optimal management of these loads. However, most studies focus on either price response or aggregated consumption modulation without explicitly integrating community contextualized local renewable production.

As so the core motivation of this thesis consists of:

- Integrating thermal flexibility and photovoltaic production in the same control model;
- Evaluating the realistic market context load coordination economic impact;
- Contributing to the renewable energy community's role comprehension in energetic transition.

This dissertation aims to bring the theoretical control models of operational realities in modern energy communities closer.

1.2 - Objectives

The general goal of this thesis consists in the evaluation and development of an optimal control model for thermostatically controlled loads, based on the works of [1], [2] with the explicit integration in renewable energy communities with photovoltaic production, minimizing the total energy cost and assuring thermal comfort restrictions are followed.

To reach this general goal, the following specific objectives were defined:

- Modeling base residential load based on normalized profiles and annual consumption statistics data;
- Apply a first order thermal model to represent in-house dynamic temperature;
- Integration of photovoltaic production obtained through simulation and converted to 15-minute resolution;
- Assimilate energy market price signals in its decision models;
- Devise the control problem as a finite horizon optimization problem;

- Implement two distinct approaches: Linear Programming (LP) with control variable relaxation and Mixed-Integer Linear Programming formulation and explicit import/export mechanics modeling;
- Compare the different operational scenarios energetic and economic performance;
- Evaluate the influence of photovoltaic power and the injection tariff in the obtained results.
- Quantify decision variable discretization impact;

Through these objectives, it’s shown that optimal coordination of thermostatically controlled loads can significantly contribute to the economic and energetic efficiency of renewable energy communities.

In the development of this work, some simplifications were considered, in order to facilitate the development of the control problem formulation and the system’s analysis. More specifically, the following was considered:

- Energy price is known for the time horizon considered;
- Only energy price is taken into account (disregarding grid usage tariffs, taxes and other items considered in an electricity bill);
- The utilized PV energy generation is considered a perfectly known forecast;
- Each dwelling’s baseload is also known for the whole period;
- External temperature forecast is considered perfect.

Chapter 2

Background / State of the art

Energy is an extremely important determinant in shaping human and social development. This relationship between energy consumption and development was first described in the nineteenth century by Herbert Spencer [3] who described a relationship between a society's ability for production to its capacity for using energy. In the 1950s, this idea was extended by Leslie White [4],], who looked at development as a continuous process in which development, from less to more complex forms, is a result of improvements in energy consumption. This view was reinforced by Cottrell [5], who highlighted that energy is a key constraint to humanity's development. Without it, development, as well as growth, is restricted.

Energy access is an essential factor in development. The lack of it often results in poverty, as it leads to a higher dependency on human labor, as well as less efficient biomass-based energy transformations, which in turn, also emphasizes differences in energy equity.

The access to energy is an essential factor in the process of development, and the lack thereof has been identified to worsen poverty, increasing the need for human labor and inefficient energy transformations through biomass. This also leads to energy equity issues and their effects on the process of development. The use of fossil fuels has likewise led to severe effects on the global climate, characterized by the emission of greenhouse gases, thereby making it a crisis in Europe and around the world.

In 2019, the European Union adopted the Clean Energy for All Europeans package [6] setting targets that include a 40% reduction in emissions relative to 1990 levels, a 32% share of renewable energy in final consumption, and a 32.5% improvement in energy efficiency by 2030. Achieving these goals requires innovation in energy supply systems and significant strengthening of infrastructure. The package also focused on consumers as the center of the energy transition by fostering participation and mobilization of local resources. Additionally, to remain aligned with the 2030 Agenda and the Paris Agreement, renewable sources will need to achieve at least 63%

of the global energy mix by 2040, guiding a steady shift in power generation portfolios, distancing from fossil fuels and focusing on renewables.

Furthermore, centralized generation does not always satisfy energy demand efficiently everywhere. Therefore, investing in decentralized energy generation becomes more attractive. The use of distributed renewable energy increases renewable energy penetration. However, not all individuals can afford or have enough space for personal renewable energy installations. In such a scenario, Renewable Energy Communities (RECs) can play an important role.

2.1 - Renewable energy communities (RECs)

2.1.1 - Definition and European framework

RECs are groups of individuals or organizations that collaborate in the energy transition, producing renewable energy and meeting energy needs with sustainable sources. They comprise a local approach, empowering actors to participate in energy production and management.

The EU defined and regulated energy communities, forcing EU countries to integrate the definitions into their respective national frameworks. A distinction is made between RECs and Citizens' Energy Communities (CECs) [7]. Both are settled on voluntary collaboration, community ownership and participatory governance, consist of non-profit nature and have the main objective of generating environmental, social and economic benefits for members and the community, including alignment between local production and demand and promotion of energy efficiency.

Although RECs may be considered a specific subgroup of CECs, they are distinguished by a more clearly defined geographical focus and a more inherent connection to renewable sources. They seek to support the local production and consumption within a defined area, without the demand for total self-sufficiency. Their performance depends on optimized planning and operation that reconcile internal exchanges and interaction with the external network, ensuring economic viability and maximizing environmental and social benefits.

Energy sharing occurs via public infrastructure or market mechanisms, allowing for the share of surpluses and external transactions, therefore reinforcing efficiency and resilience. RED II establishes a framework set up to accelerate an equitable and sustainable transition, supporting the creation of RECs and local involvement in renewable energy production [8], addressing energy poverty and climate change, while promoting citizen ownership of the energy future.

RECs can promote more equal access to clean energy by vulnerable populations and can also support public policy goals: improved public health (through reduction of energy poverty), job creation associated with management/operation, and affordable energy for participants. They can also reduce dependence on fossil fuels.

2.1.1.1 - Renewable Energy Communities in Portugal

The concept was introduced on October 2019 by the Law-Decree No. 162/2019, partially transposing Directive 2018/2001/EU (in force from December 2018). Since then, several changes to the **National Electric System (SEN)** have impacted the approach to renewables and, specifically, the requirements for the creation and operation of communities, reflecting adaptation to decentralized production and collective self-consumption.

Table 2.1 —Main legal alterations (since self-consumption on 2014)

Document	Alterations
Decreto-Lei n.º 153/2014	Legal framework of production for self-consumption
Decreto-Lei n.º 162/2019	Initiates collective self-consumption and CER
Despacho n.º 46/2019 (DGEG)	Rules for the operation and implementation of prior control
Regulamento n.º 266/2020 (ERSE)	Regulation of Self-Consumption of Electric Energy
Regulamento n.º 8/2021 (ERSE)	Regulation of Self-Consumption; revokes the former Regulation No. 266/2020
Decreto-Lei n.º 15/2022	Organization and operation of the SEM (production, storage, marketing and aggregation)
Decreto-Lei n.º 99/2024	Amends the regulatory framework applicable to renewable energies

2.1.1.2 - Power sharing methods

The sharing can be defined either by coefficients at the time of creation or by specific rules [9]:

- **Fixed coefficients:** percentage of energy allocated to each consumption point per period
- **Load-proportional coefficients:** DGO calculates the energy per point every 15 minutes based on the load
- **Hierarchical share:** priority groups; within the group it can be fixed or proportional
- **Dynamic share:** EGAC (“Entidade Gestora do Autoconsumo Coletivo”) defines periodically and recurringly the coefficients;
- **No definition:** DGO automatically assumes load-proportional sharing for members without defined coefficients

2.1.1.3 - EGAC (Entity for the Collective Self-Consumption Management)

EGAC manages the community. In a REC, the very own REC may act as EGAC and mediate interactions with the **DGO** and other legal entities (e.g., **DGEG**). Alternatively, it can be an entity dedicated to this purpose or an aggregator that manages the EGAC and the renewable projects that make up the community.

2.2 - Demand response

2.2.1 - Context and flexibility requirement

The growing prevalence of intermittent RES, dependent on the weather and uncertainty, increases the need for flexible electrical systems [10]. In the load and renewable generations RES penetration creates higher difference elevations (especially in peak hours), demanding more flexible conventional reservations; Otherwise, either the rise of real time market prices [11] or frequent curtailment can occur.

According to IEA [12], flexibility is the system's ability to alter consumption or generation patterns in the presence of unexpected or expected variations.

Currently, generation side flexibility is assured by manageable plants allowing for variations according to the speed their respective technologies allow, but its use demands caution due to additional operating costs and ecological consequences. Referred alternatives include smart grids, large scale storage and intergrid connections. However, they might be too expensive to implement. Therefore, DSM is considered the most promising solution [13].

2.2.2 - DSM: Evolution and strategies

From the operational point of view, demand has been historically considered not very flexible due to the necessary scale, but swaying demand isn't new. In [14], DSM appears with the 1970's energy crisis and it was formally defined in 1984 by its consumer side action and load curve shape. These strategies included [15]:

- Peak Clipping
- Valley Filling
- Load Shifting
- Strategic Conservation
- Strategic Load Growth
- Flexible Load Shape

Its definition evolved and now covers strategies that go from temporary to permanent changes through energy efficiency. DR is an integral feature of DSM and can be found under the temporary changes category.

2.2.3 - DR: Definition, benefits and consumer involvement

According to the IEA, DR consists of balancing grid demand by incentivizing consumers, usually with pricing or other monetary benefits to allocate their consumption to times of less demand or more availability. This way DR is the end-user's active response to stimuli (prices per period, incentives, penalizations for refusal when applied). The consumers became active grid management partakers, either getting monetary compensation or reduced costs. Additionally, they contribute for efficient management which can also have a beneficial environmental impact.

Aforementioned efficient management contributions include: faster contingency response, better existing grid capacity efficiency, less peak plant investment needs and higher reliability [16].

DR can, from DGO's point of view, also improve global production efficiency, not only by influencing tariffs, but also through technical aspects that mitigate costs and expansion/development system challenges.

The following additional information attributes DR with wholesale and consumer price proximity efficiency benefits, causing:

(i) Partakers financial benefits (savings and incentives) [17];
(ii) Market side financial benefits (avoids costly peak generation) [18];
(iii) Operation trust/security and lowering chance of interruption [17]
(iv) Market performance benefits (mitigates suppliers power when it comes to high pricing) [17]

They're also referred to as growth problematics (framework construction, renewable popularity, emissions) and they ascertain DR's potential for reliability, loss reduction, costs and economic efficiency [11]. Smart grids demand high active participation and allow bidirectional flow of information, broadening loads and promoting built in management (loads, storing and microgeneration, dedicated or EV), with both SEE and consumer benefits, including its use as a profit / loyalty tool by suppliers, reinforcing the need for a consumer centric approach without neglecting the impacts of decentralized generation on the grid [17].

2.2.4 - Demand response programs: Typologies and Mechanisms

The rapid advancements being made in the potential applications of distributed energy resources and flexible assets within local communities, together with the developments in power electronics and controlling equipment, has accelerated the transition from conventional networks into smart grids. This fundamentally alters the traditional power system operation, introducing new challenges for the DSO's and TSO's in maintaining not only a reliable and resilient operation of the system as well as an efficient one [19], [20].

Demand response can be described as changes in electricity usage by end-users in response to external signals, which implies the need for operational frameworks in order to define not only which external signal is used and how does the consumers respond to it, but also the eligibility, performance requirements and the compensation mechanism used [21].

From the point of view of TSO and DSO, these frameworks or programs bring clear advantages for the operation of the power system, however they must also be advantageous for the utilities, consumers, and aggregators in order to facilitate the provision of flexibility.

In the literature, these demand response programs are commonly classified into 2 main categories [22]:

- price-based programs;
- incentive-based programs.

Incentive-based programs typically allow the system operator to remotely adjust the demand of consumers participating in the respective DRP in exchange for financial incentives. In contrast, time-based programs rely on consumer reaction to dynamic price signals, which inherently introduces introducing behavioral uncertainty.

The literature further highlights that DR programs produce measurable impacts across different temporal scales, ranging from short-term operational balancing to long-term system planning.

2.2.4.1 - Demand Response Programs

Incentive-based DR programs are based on contractually defining the activation conditions under which the consumers, or aggregators acting on their behalf, must alter their consumptions in exchange for financial compensation. The activation of these flexible assets is typically requested by the system operator during periods of resource scarcity, system imbalance, or other contingencies [23].

Because the conditions of the load curtailment are previously defined and are subject to mechanisms that ensure its proper resolution (penalizing no-complying participants), incentive-based DR can be considered a dispatchable demand-side resource. For this reason, these resources can contribute to congestion management and participate in balancing markets. Some of the existing types are [24]:

- **Direct Load Control:** involves the remote control of end-use appliances by the system operator, utility or aggregator (e.g., washing machines, refrigerators, pumps). DLC has a long implementation history, so it’s considered a reliable mechanism for DR, Nevertheless, concerns related to flexibility and consumer privacy persist. These factors make this type of DR traditionally more suited for infrequent activations (ex: Peak shaving) rather than for continuous operation, which limits its renewable integration capabilities. Despite being used by utilities for some decades, DLC is only adopted by a small set of consumers, as a perceived intrusiveness and loss of user autonomy has hindered widespread adoption, a trend that may continue in the future

- **Interruptible/Curtailable Load (I/CL):** are contractual agreements in which the consumers agree to reduce their consumption for a scheduled timeframe in exchange for tariff discounts or financial credits. Penalties apply in the event of non-compliance. These programs are particularly suited to large industrial consumers that can tolerate temporary supply interruptions.
- **Demand Bidding:** allows large consumers or aggregators to submit bids specifying either the price at which they are willing to curtail load and the quantity of load reduction offered. If the bid is economically competitive relative to the alternative generation offers during market clearing, the system operator accepts and activates the demand reduction rather than dispatching additional generation. This mechanism enables higher remuneration during periods of elevated wholesale prices, analogous to supply side bidders, while still allowing consumers to maintain fixed retail tariffs.
- **Time-of-Use (TOU):** this type of tariff framework employs a predefined pricing schedule in which electricity prices vary according to time of day or season. They have been implemented in multiple countries as this structure reflects the real temporal variability of generation costs and demand patterns Typically, there are two approaches for pricing design:
    - Cost-based analysis, based on peak load pricing theory; and marginal cost principles;
    - Demand-response modelling using techniques such as fuzzy logic and price elasticity;.
    - A key limitation of TOU is that tariffs are typically determined months in advance, which is for contingency management or the integration of variable renewable generation. Standard implementations include tri-period tariffs (off-peak, shoulder, peak). Compared to more dynamic mechanisms, TOU offers lower price volatility for consumers but provides limited risk protection for suppliers.

2.3 - Demand response with TCL

The literature explores various types of loads that can participate in Demand Response [11]:

- **Lighting:** the frequency response via temporary reduction of light intensity increases the nadir frequency.
- **Electric vehicles (EVs) :** participation in the generation-demand balance via (i) passive response adjusting the charging rate after the measurement of the local frequency; or (ii) active response discharging part of the battery into the grid (V2G).
- **ESS in household appliances:** active studied response of the demand through the domestic ESS.

- **TCL (Thermostatically Controlled Load):** considered the most promising candidates, including refrigeration, heat pumps, and thermal storage tanks. Relevant due to their duty-cycle and easy on/off during frequency events; with continuous control methods, temporary reduction in consumption causes little discomfort. For participation, each piece of equipment must have a controller that receives an input signal (typically dependent on production) defining when to act.. As existing resources, they do not require installation investment; controllers have a low cost, making them more interesting in economic terms rather than ESS; they have high presence during peak hours, being an active resource available for control.

2.3.1 - DR with TCLs: evolution and thermal inertia

2.3.1.1 - Historical evolution and problem reemergence

The study of large groups of thermostatic loads began in the 1980's, with [25] as one example of such studies, focusing on modeling oscillatory effects on consumption after planned (DLC) or unplanned (*blackouts*) interruptions. After three decades, the topic resurfaced with the evolution of DR, motivated by new operational challenges, especially the integration of intermittent generation [26].

2.3.1.2 - Thermal inertia: modelling necessity and challenges

A pre-requisite to study the usage of thermal inertia in DR is a viable mathematical model to estimate the thermal behavior and predict the necessary energy to maintain the temperature within the limits. There are thermal models: based on historic data and an empirical model based on historic data and consumer behavior however, there are restrictions (failing to consider weather profiles and the absence of explicit thermal dynamics) [16].

A significant gap is identified: how to use the flexibility of the thermal inertia of TCLs to optimize energy at the household level. Optimal scheduling of thermal inertia and other DERs can minimize expenditure and shape demand; aggregation at the neighborhood level can reinforce *load shaping*. Thermal inertia allows for the adaptation of profiles to system needs, reducing consumer costs. The central difficulties are: (i) reliable thermal modelling that explicitly represents internal/ambient temperature, thermal capacity, thermal resistances, and stored energy, especially when integrated into optimization and aggregation tools; (ii) characterization of the randomness in the operation of TCLs and the absence of an aggregate behavior model, given the stochastic nature of residential demand; (iii) development of control patterns that optimize DR benefits for either consumers and operators [16].

2.4 - Energy markets

2.4.1 - Day ahead market and economic dispatch

Electricity is traded in markets: the market operator receives sell and buy bids, organizes them into merit curves (supply ascending; demand descending), and the intersection determines the outcome. This process is run for each hour of the next day (day-ahead). The solution corresponds to the economic dispatch (cost-optimal clearing). The system operator then checks technical feasibility (voltages, power flows, etc.); if there are no violations, the dispatch is validated and ancillary services are procured [27].

Ancillary services enable reliable and secure system operation, protecting against sudden variations, RES intermittency, and forecast errors. Traditionally, frequency and voltage control were supplied by conventional power plants; as these units are progressively decommissioned, new provision models are needed. The central topic here is the provision of ancillary services by demand-side resources. It is relevant to discuss how Demand-Side Management (DSM) integrates into markets to offer new ancillary services and support frequency stability [28].

2.4.2 - MIBEL: Structure and markets (day ahead, intraday, and forward)

MIBEL was created in 2001 and became operational on 1 January 2003 as a joint Portugal-Spain initiative enabling consumers in both systems to purchase electricity from any supplier active in Portugal or Spain [17]. There is a single Iberian market operator (OMI) with a Portuguese arm (OMIP) and a Spanish arm (OMIE). MIBEL's objectives include:

(i) Benefiting consumers,
(ii) Ensuring transparency and competition,
(iii) Providing a single reference price methodology,
(iv) Guaranteeing equal access,
(v) Improving sectoral economic efficiency.

Day ahead and intraday markets [29]: The day ahead market trades electricity for delivery on the following day; it forms prices for 24 hours/day and for every day of the year. In Portugal, the trading platform is operated by **OMIE** (historically OMEL), and trading follows the official market timetable (**HOE**) [26]. Hourly prices are determined by the intersection of supply and demand curves under a **single marginal price model**. When interconnection capacity cannot accommodate the scheduled cross border flows, market splitting occurs, separating Portugal and Spain into distinct price areas. The intraday market complements/adjusts day ahead positions and runs six daily sessions, each clearing prices for specific time windows.

2.5 - Aggregators: Role, specialization and processes

It is reiterated that most consumers don’t have a detailed conscience of their energy use, nor do they have the means to access markets directly; therefore, they resort to aggregators, who use DR programs to support participation and optimization. An aggregator is an intermediary entity between the final user and the system operator, aiming to optimize the usage of electricity with grid constraints and user interests, without compromising comfort or quality of service Aggregators can specialize by consumer type (residential, commercial/large industries, small industries), due to technological differences in monitoring/control and the legal differences.

Responsibilities are assigned to the aggregator: group customers and participate in wholesale to obtain lower prices; plan the usage of distributed power generation when it’s cheapest; manage and shift charges to off-peak hours; empower consumers to reduce costs without changing comfort/lifestyle [17]. Demand aggregators and production aggregators are established [17] The aggregator negotiates contracts to aggregate charge reduction/increase/shift and creates a set of controllable equipment; the obtained flexibility can be sold on the market at the best offer.

The demand aggregator collects searched information and offers and aggregated response (which may include customer production). The production aggregator collects information from dispersed generators and offers aggregated production; this aggregated production is designated “VPP”; there may be a combination of the two.

There’s mention of significant DR potential in small and medium sized clients yet unexplored; retail prices change slowly and are unattractive, reducing spontaneous response; the aggregators task is to enable response and use the result at wholesale. Listed procedures: service promotion; initial rentability study; installation of control and communication devices; payment of premium customers. Before operating, the aggregator must study user types and potential, as well as the cost/benefit of DR per equipment and impact on comfort, defining rewards to compensate for loss of comfort [17]. It is also indicated the aggregator requires specific competencies (knowledge of industries, ability to identify flexibility and limitations, technical capabilities for connection and charge integration).

Chapter 3

Method

In chapter 2, several of the topics adjacent to the main focus of this thesis were reviewed, in order to explain their interconnection and importance to the problem at hand, therefore allowing for its theme contextualization. In this chapter, the modelling of said problem will be elaborated and described. Further on this document, a case study will be presented which will use the methodology here presented as basis for obtaining the studied results.

What is intended in this section is to define a methodology which allows for the control of variable loads (in this case TCLs) in a manner that does not disrupt end user comfort (considering that TCLs alter the interior temperature of end users houses), but also brings the greatest financial benefits to all participants (end users, aggregators and load serving entities) and uses the maximum possible amount of renewable energy provided by the decentralized power plants that serve the energy community. This possibility, achieving controllability of electric loads for managing real-time energy imbalance, is very present in current literature, such as [30], [31]and [32]. The physical behavior of TCL can be accurately described by first-order thermal models derived from Newtonian heat transfer principles [25].

The methodology mentioned in the last paragraph, if implemented on real scenario, would use real and forecast data to apply the control methods and obtain the desired results. The data would be obtained by energy meters and meteorological stations. However, the work developed on this dissertation is purely theoretical, therefore, besides defining which data will be necessary, we must determine how the necessary data will be obtained.

The primary objective is to minimize electricity procurement costs while maintaining indoor thermal comfort and integrating photovoltaic generation under time-varying electricity prices. This approach is inspired by the works of Halder et al. [2] and Fontes et al. [1] but with the nuance of renewable energy production and the availability of that same energy to the consuming population. Therefore, this proposed methodology combines:

- A physics based thermal model of TCL;

- An energy production model based on PV forecasts and its integration;
- A price-driven economic optimization framework;
- A bidirectional grid interaction model;
- Mixed-integer linear programming (MILP) for Demand side management;.

The underlying philosophy considers that the aggregator (or LSE) plans beforehand the optimal consumption trajectory of all the controllable loads for a determined period [1]. This planning takes into account forecasts (or real data) of external temperature, energy price and energy production. Considering what was previously stated, the control problem is formulated as a finite-horizon optimization problem, supported by [33] and [34], which will determine the ON/OFF commands of each TCL and, consequently, the indoor temperature trajectory and the power flow at the grid connection point [35].

This method must incorporate economic signals (energy buy and sell price), grid interaction rules (no import and export at the same time) and technical constraints, such as thermal dynamics and user comfort bounds.

The methodological approach has the following progression:

- Mathematical modelling of loads, generation and price signals;
- Thermal modelling, technical constraints;
- Formulation of the optimization problem;
- Implementation using MILP.

The remainder of this chapter details each modelling component, how data must be acquired and the complete optimization formulation.

3.1 - Control model overview

The control model framework developed intends to determine the optimal operation of multiple TCLs for demand response [36], considering renewable energy availability, the baseload of each household, electricity prices and thermal comfort boundaries. This problem can be formulated as an optimization problem that can follow a Linear Programming (LP) structure or a Mixed-Integer Linear Programming (MILP) structure. This choice is directly dependent on the chosen premises, but several previous works have shown how MILP can be utilized either in energy market optimization [19], for unit commitment and binary decisions [37] and more specifically related to the present work in the arbitrage of energy market prices with TCL aggregations [38].

3.1.1 - Linear programming formulation (without bidirectional exchange with the grid)

If the community is assumed to only import energy from the grid, then the grid interaction can be modelled using a single nonnegative variable,

$$p_k^{grid} \geq 0. \tag{3.1}$$

In this case, the objective is to minimize the energy import from the grid, therefore it can be described as:

$$min \sum_{k=1}^{K} \pi_k p_k^{grid}, \tag{3.2}$$

Subject to:

- Discrete thermal dynamics of TCL;
- Comfort boundaries of each TCL: $Li < x_{i,k} < U_i$;
- Energy balance considering only grid import:

$$p_k^{grid} \geq p^{base} + \sum_{i=1}^{N} P_i u_{i,k} - P_k^{PV}, \tag{3.3}$$

- Constraints $0 \leq u_{i,k} \leq 1$;
- $P_{imp,k} \geq 0$.

If the TCL control variables $u_{i,k}$ are allowed to vary continuously in the interval [0,1], the problem remains fully convex, [39][40], [41], and can be solved using standard LP solvers such as MATLAB's linprog.

The solution obtained is then converted to an ON/OFF logic, accounting for the devices minimum time of commutation, ensuring that the problem's solution is consistent with real equipment.

The continuous relaxation of binary variables (ON/OFF) is an approach that has been discussed in the literature and allows solving the LP with great computational efficiency. Halder et al. show that the discreet solution with binary variables leads to a MILP problem with a substantial dimension, whilst LP allows for a solution that requires less computational power.

Similarly, Fontes et al., demonstrate that control's relaxation allows the structural defining of a solution and usage of explicit strategies in certain situations.

In this regard, this convex relaxation has several advantages:

- Guaranteed global optimality;
- Fast computation even for large horizons;
- Scalability to large number of TCLs.

On the other hand, the LP formulation either assumes that no energy export is allowed or that export is implicitly neglected in the economic model.

3.1.2 - Limitation of the LP formulation

When an import/export dynamic is introduced, considering they are subject to different market values, two different variables must be used to represent each situation: $P_k^{imp} \geq 0$, $P_k^{exp} \geq 0$.

The power balance then becomes:

$$p^{base} + \sum_{i=1}^{N} P_i u_{i,k} - P_k^{PV} = P_k^{imp} - P_k^{exp} \tag{3.4}$$

Although this equality remains linear, it constrains only the difference between import and export. It does not explicitly prevent both variables from being simultaneously positive. Considering that, if electricity purchase is always greater than the feed-in tariff the LP solution will avoid situations where import and export are both true. However, in some scenarios, such as very low (or even negative) market prices the LP becomes economically inconsistent. The solver can artificially increase both import and export at the same time while preserving their difference, which will result in unbounded profits and consequently no real solutions to the problem. For that reason, LP formulation is no longer sufficient to ensure physical realism and boundedness.

3.1.3 - Mixed-integer linear programming formulation with bidirectional exchange

To ensure physical consistency and logical solutions, it must be enforced that the community either imports or exports energy, but never at the same time.

This condition is introduced using a binary variable, alongside linear constraints to enforce mutual exclusivity. Therefore, it is considered $y_k \in \{0,1\}$,

Where:

- $y_k = 1$ indicates import mode;
- $y_k = 0$ indicates export mode.

The following linear constraints enforce mutual exclusivity by use of a big M constraint logic [42].

$$P_k^{imp} \geq M y_k, \tag{3.5}$$

$$P_k^{exp} \geq M(1 - y_k), \tag{3.6}$$

Where M is a sufficiently large constant.

The above variables and constraints work to ensure that: if $y_k = 1$, then $P_k^{exp} = 0$ and, if $y_k = 0$, then $P_k^{imp} = 0$.

The introduction of binary variables transforms the problem into a MILP formulation.

In turn, this increases computation complexity. Nevertheless, the problem remains linear in all continuous variables, preserving interpretability of the model.

3.1.4 - Comparison between LP and MILP formulations

Despite increasing computational effort, MILP allows binary control and solutions, which would skip the step of converting the controls to an ON/OFF logic, while also allowing to explicitly include energy exportation in the control model. Below is a comparison table.

Table 3.1 — LP & MILP comparison.

Feature	**LP**	**MILP**
Continuous control – u	Yes	Optional
Binary control - u	No	Yes
Export allowed	No (only implicit)	Yes
Simultaneous import/export prevented	Price-dependent	Explicitly
Computational Complexity	Low	Moderate

Therefore, we can assume that LP formulation is preferred when less computational effort is desired and suitable for scenarios without export or when price structures guarantee no arbitrage. The MILP on the other hand is necessary for realistic modelling of bidirectional exchange under dynamic pricing conditions.

For that reason, MILP formulation was adopted in this work to represent realistic dynamic control and operation of the communities' TCL and accurately portray bidirectional grid exchange. However, it must be noted that, due to hardware constraints, a MILP formulation was used but applying a convex relaxation of the decision variables (similarly to the LP) and posterior conversion to binary variables.

3.1.5 - Contribution regarding state of the art

Considering the works of Halder et al. [2] and Fontes et al. [1], the biggest contribution this work presents is by integrating renewable generation on an energy community scenario in the control model. While in previous works the objective was only dependent on market prices (or total energy budget), in this work the availability of local energy is considered, which will help dictate the behavior and activation profile of aggregate TCLs.

3.2 - Load modelling

The load modeling is a fundamental element of the control model, since it determines the non-flexible power consumption and the contribution of the thermostatically controlled loads to the total load.

This section details the modeling of the community base load and the corresponding electrical power associated with the TCLs.

3.2.1 - Base load modelling

The community's total load has two main components [43]:

- Non-flexible base load, which is associated with the usual domestic power consumption (lighting, household appliances, electronic equipment) [44].
- Flexible load associated with TCLs, the operation of which is controlled by the proposed control model.

The base load is modelled using the typical consumption profiles, alongside data of average consumptions per household. These are derived from aggregated historical data, and its intention is to capture average residential consumption patterns in Portugal.

Usually, the typical consumption profiles describe a percentual temporal distribution of the total consumption for a year [45]. For that reason, the load profiles have to be multiplied by a flat value in kWh and only then converted to power, regarding the granularity of the data (hourly resolution, quarter-hourly resolution, etc.).

$$P_{i,k}^{base} = E_i^{year} p_k^{profile} \Delta t, \tag{3.7}$$

Where:

- E_i^{year} is the average annual consumption for unit i;
- $p_k^{profile}$ is the value of the chosen profile, at instant k,
- Δt is the time step in hours

Within the optimization model, this base load is treated as **non-controllable** and is introduced as an exogenous term in the power balance equation.

3.2.2 - TCL load modelling

The thermostatically controlled loads (TCLs) represent the flexible component of demand. Each TCL is characterized by nominal electrical load (P_i), corresponding to the power absorbed when the device is operating (ON state).

The total electrical load associated with the TCL of the community at instant (k) is given by:

$$P_k^{TCL} = \sum_{i=1}^{N} P_i u_{i,k}, \tag{3.8}$$

Where:

- N is the total number of TCLs;
- P_i is the nominal power of TCL (i);
- $u_{i,k}$ is the binary decision representing the ON/OFF state of TCL (i) at instant (k);

When $u_{i,k} = 1$ the TCL consumes its nominal power. When $u_{i,k} = 0$ its consumption is zero.

The electrical power of TLCs is directly related to the thermal parameter β_i in the dynamic model:

$$\dot{x}_i(t) = -\alpha_i(x_i - T_{out}) + \beta_i u_i(t)\,, \tag{3.9}$$

Parameter α is the coefficient that relates inside temperature evolution in regard to the difference between indoors temperature and outdoor temperature. The parameter β_i represents the variable rate of change of the indoor temperature induced by activating the device. These parameters depend on the thermal power of the heating system and the thermal capacity of the building.

Consequently, modeling electrical power is not only relevant to the energy balance but also directly influences the thermal dynamics and the feasibility of satisfying comfort constraints.

3.2.3 - Total community load

The total community load at instant (k) is given by:

$$P_k^{load} = P_k^{base} + P_k^{TCL}, \tag{3.10}$$

This total load is used in the balance equation with photovoltaic generation and the grid exchange:

$$P_k^{load} - P_k^{PV} = P_k^{imp} - P_k^{exp}, \tag{3.11}$$

The separation between base load and flexible load allows the control model to act exclusively on the TCLs component, while keeping non-flexible consumption unchanged.

3.3 - Generation modelling / forecast

In renewable energy communities, where energy management methods are used, energy production forecasts are essential for an efficient energy management, especially in Demand Response strategies with controllable loads. For that reason, its integration in the control model is essential, considering it affects the energy balance and, consequently, the optimal control decisions of the thermostatically controlled loads.

In real applications, the future renewable energy generation is not precisely known, so it must be estimated by means of forecasting models. There are three main categories of PV forecasting as stated by [46]:

- Physical models, based on meteorological data;
- Statistic models, based on historical data;
- Hybrid models, which use meteorological data and machine learning algorithms [47];

Therefore, in a real scenario, this control model would take as an input the result of one of these forecasting techniques.

3.3.1 - Approach used in this work

Considering the nonexistence of real data, it was necessary to use a reliable and accessible simulation tool to create synthetic PV generation profiles. The tool chosen was PVGIS (Photovoltaic Geographical Information System).

PVGIS is a widely use, free of charge, online, solar photovoltaic energy simulator, developed by the European Joint Research Center (JCR) that can calculates PV forecasts based on geographical location (available for the whole world) historical irradiation data and specific project parameters, such as tilt, orientation and PV power installed, and uses the methodologies described in [48], [49], [50], that solidifies its scientific validity.

For this work, hourly PV generation data was simulated for a specific place in Portugal, with tilt and orientation optimized automatically by the tool.

One of PVGIS limitations is that the minimum granularity it allows is hourly resolution. Therefore, the PV production profiles only account for 8760 hours (considering typical years), so it was necessary to convert the data to profiles with more instants.

3.3.2 - Conversion from hourly resolution to quarter-hourly resolution

Considering OMIE’s pricing structure, the time resolution used in this work is Δt = 15 minutes.

However, the PV Production profiles acquired from PVGIS are in an hourly resolution, so it is necessary to use temporal disaggregation methods to transform it into a profile with a quarter-hourly resolution.

Instead of assuming hourly constancy for each fifteen period inside the hours, a 2nd degree polynomial trendline based approach of extrapolation was used. In this manner, it was possible to represent as realistically as possible the soft variation of PV production in time). This is coherent with the expected curve of PV production in clear weather conditions.

3.3.2.1 - Method formulation

Consider a PV series $\{P_h^{PV}\}^H{}_{h=1}$, where H is the number of hourly observations in this chosen horizon. To convert this into a quarter hourly resolution, the steps taken were the following:

- Consider a time horizon t_h for which the extrapolation will be used;
    - In this case the time horizon for each extrapolation was a full day;
- Consider only the timeframe between the first solar hour (first instant with PV>0) and the last solar hour (last instant with PV>0);
- Adjust a 2nd degree polynomial trendline to that dataset, so the coefficients could be found:

$$\hat{P}^{PV}(t) = at^2 + bt + c, \tag{3.12}$$

- Evaluate the results of that equation for the rest of the time inside the considered time horizon with a quarter hour resolution;
- Enforce positive values of PV generation;

$$P_k^{PV} = \max\,(0, \hat{P}^{PV}(t_k)). \tag{3.13}$$

    - The max operator ensures physical consistency to the model.
- Preserve the original hourly values in the new quarter hourly array;

In this manner, it was possible to simultaneously preserve the variation of the trendline without defacing the original dataset utilized.

3.3.2.2 - Limitations of this approach

This approach to converting hourly resolution profiles to quarter hourly resolution profiles represents a compromise between computational simplicity and time horizon realism. Applying a trendline allows for a continuous curve with no abrupt, which is adequate to simulate general PV production trends without using more complex methodologies. However, synthetic and deterministic data does not consider forecasting errors or sudden changes due to clouds (for example) [46], [51].

Nonetheless, for aggregate simulation and behavioral analysis of TCL inside a REC, this approach is adequate and computationally efficient.

3.3.3 - Control model integration

The final PV series is used in the control model as an input that will determine, alongside the baseload, the amount of renewable energy available for TCL use, which will influence the optimal control decisions.

3.4 - Price forecast

Energy price forecast is a fundamental part of demand response models, considering that control decisions are heavily reliant on the price curve along the horizon. The inclusion of this forecast allows for an optimal control strategy that is not based only on technical criteria, such as thermal comfort, but also economic criteria, which in turn will minimize the community's total energy cost.

For thermostatically controlled loads, the availability for load shifting is relatively high, but its deployment is only economically efficient if the prices are known with certainty. In this context, having accurate pricing information allows the model to prepare for periods with high energy cost, by reducing or shifting power consumption, making use of the thermal dynamics in place, as well as identify low price periods, where it is beneficial to activate the majority of the loads.

In the literature, there are several methodologies for price forecasting, either through traditional statistic models (such as ARIMA [52], ARIMAX or GARCH [53]), machine learning models (such as Support Vector Machines, Random Forests or neural networks) [54].

However, although these models are very relevant in real world operations, the purpose of this work is not to evaluate energy price forecasting methods, but instead the impact of demand response in renewable energy communities. Therefore, for this work, historical data from an existing energy market was used for a predetermined period.

3.4.1 - Data used

The data considered was the historical data of day-ahead market prices. The granularity of this data has a hourly resolution and the unit is €/MWh.

To make it compatible with our control model, the prices must be converted to €/kWh and adjusted to the time resolution used (in this case, quarter hourly resolution).

Unit conversion is made in the following manner:

$$\pi_h^{€/kWh} = \frac{\pi_h^{M/kWh}}{1000}. \tag{3.14}$$

3.4.2 - Conversion from hourly to quarter hourly resolution

As happens with our PV generation forecast, the data is in hourly resolution and must be converted to quarter hourly resolution. However, in this case, the price is established as the price for each hour, so we can assume that the value for the next three periods is equal to the price of each hour.

3.4.3 - Feed-in tariff

As for the feed-in tariff, currently, most aggregators in Portugal have two plans for entities that intend on selling their excess renewable energy.

These plans can either be:

- Fixed price plan – where, at all times, the producers are paid a pre-determined price for all their energy, no matter the hour at which it is exported;
- Market indexed price plan – where producers are paid a tariff that is correspondent to the energy market price at the hour they energy was injected into the grid, minus a management fee the aggregator takes for representing the producing unit in the market.

The choice between each plan is ultimately customer dependent, but in this work, the plan considered was a fixed feed-in tariff plan, so that situations where market prices are negative, and therefore the community is penalized for selling energy, are avoided.

3.4.4 - Integration in the control model

The energy price (either for buying energy or selling energy) is incorporated in the objective function of the optimization problem:

$$min \sum_{k=1}^{K} \Delta t \, (\pi_k^{buy} p_k^{imp} - \pi^{sell} p_k^{exp}), \tag{3.15}$$

Where:

- π_k^{buy} is the OMIE price converted to €/kWh;
- π^{sell} is the feed-in tariff established on 3.4.3.

The possibility of negative energy prices introduces the scenario where the community can be paid to consume energy. This will have direct impact on the mathematical formulation (and method used to solve it), because it requires binary variables to stop import and export from happening simultaneously.

3.4.5 - Methodological considerations

Using OMIE prices allows for the simulation of a more real scenario, by making decisions based on real market prices that show volatility and extreme pricing events, as well as it allows the analysis of the impact of negative prices in the optimal control of TCL.

However, it is also important to note that:

- This method only works as an applicable real-life example if the time horizon used in solving the control problem is 24 hours or less (because day-ahead market prices are published on the day before);
- This method only considers energy price and no additional cost that is usually part of the billing (such as grid access tariffs, social tariffs, etc.).

This simplification is consistent with a deterministic approach, usually adopted in academic studies of energy control [19].

3.5 - Thermal modelling

Considering Demand Response in this work will be attained by control of thermostatically controlled loads, the thermal modelling is a central topic of the control strategy developed, because it will be a deciding factor for coordination of the activation of the flexible loads, in a manner that allows them to make the best use of the renewable energy available.

Thermal modelling is a way to express mathematically the evolution of interior temperature considering outside temperature conditions and heat changing actions (such as turning on the air conditioning). These models are used in energy efficiency simulation tools, such as EnergyPlus, that determine the thermal balance for different zones [55].

However, for optimization and control analysis of electrical devices, it is sometimes preferred to use simplified models that maintain the most important aspects of thermal dynamics without sacrificing computational processing power. It has been shown that models of superior order are able to capture more complex thermal dynamics, even for multiple coupled thermic zones. Nevertheless, studies also show that more simple RC models are able to provide representations that are precise enough to study demand response and load management systems, if correctly parametrized [56]. The additional complexity of higher order models marginally increases the precision, while heavily increasing computational effort. Not only that but, in demand response studies, it is important that the methods applied are computationally efficient, fast and physically interpretable, all of which can be acquired with low-order RC models.

For all the above reasons, the inside temperature of each dwelling is calculated, and consequently controlled, by a low order model which translates the dominant behaviour of heat transfer between the outside and the inside. The thermal model applied in this work is also consistent with those applied in the literature by Fontes et al. and Halder et al., whose articles serve as the basis for this dissertation.

In Halder et al. [2], the thermal dynamics of each TCL is modelled as a discreet linear system:

$$x_{k+1} = ax_k + bu_k + d, \tag{3.16}$$

Where:

- x_k is the internal temperature;
- u_k is the control variable (ON/OFF).

The underlying continuous formula that served as basis is:

$$\dot{x}(t) = -\frac{1}{RC}\left(x(t) - T_{out}(t)\right) + \frac{P}{C}u(t)\,, \tag{3.17}$$

This corresponds to a first order RC model. If $\alpha = \frac{1}{RC}$ and $\beta = \frac{P}{C}$, it is possible to obtain the structure used in this work:

$$\dot{x}(t) = -\alpha(x - T_{out}) + \beta_u\,. \tag{3.18}$$

Therefore, the thermal model is equivalent to the model utilized in Halder et. al.

3.6 - TCL behavior

Thermostatically Controlled Loads (TLCs) exhibit a particular dynamic behavior resulting from the interaction between three fundamental elements:

- the thermal inertia of the building,
- the user-imposed comfort constraints and
- the ON/OFF control logic

Understanding this behavior is essential not only for the correct formulation of the optimization problem but also to make sure that the implemented model is physically coherent [57].

3.6.1 - Thermal comfort and operating band

Traditionally, the operation of a TCL is governed by a thermostat that maintains the indoor temperature within a comfort band delimited by a lower limit L and an upper limit U: $L \leq T_{i,k} \leq U$.

This interval represents the acceptable comfort zone for the user and constitutes a fundamental constraint in the optimization problem.

In the TCL control literature, this comfort band is often modeled as an admissible operating region within which thermal dynamics can evolve freely, with device activation being determined by the need to prevent violation of the bounds. As the work of Halder et al. claims, this admissible region plays a central role in the optimal control formulation, enabling the exploitation of thermal flexibility without compromising comfort.

In this work, the L and U limits are treated as linear constraints within the MILP optimization problem, ensuring that the calculated thermal trajectory permanently obliges to the comfort interval.

3.6.2 - ON/OFF dynamics and hysteresis

In a real scenario, most TCLs follow an ON/OFF control logic with hysteresis. The equipment is activated when the temperature reaches the lower limit and is deactivated when it reaches the upper limit (or vice versa in cooling mode). This behavior can be described as a binary system, alternating between two operating modes:

- ON Mode: Positive thermal load is supplied.
- OFF Mode: Passive evolution according to the natural thermal dynamic.

Nevertheless, for optimization purposes, this logic is reformulated through a convex relaxation of a binary variable:

$$u_{i,k} \in [0,1], \tag{3.29}$$

Where:

- $u_{i,k} = 1$ represents the ON state at full power;
- $u_{i,k} = 0$ represents the OFF state.
- Any other value is an intermediate state

This reformulation is consistent with the approach of Fontes et al., who adopts a binary representation of TCL activation in order to enable the integration of it in a mathematical programming framework. Like so, these continuous variables will then be converted to binary representation.

3.6.3 - Thermal flexibility and inertia

The main characteristic that makes TCLs suitable for demand response programs is their thermal inertia. The thermal capacity of a building allows consumption to be shifted over time without immediately violating comfort constraints.

From a physical standpoint, when the heating system is switched off, the indoor temperature does not change instantaneously but gradually evolves. Thus, enabling temporary periods of zero consumption.

This phenomenon is explored in Halder et al., where the thermal flexibility is used to shape the aggregated consumption of TCL populations. The ability to anticipate operation (pre-heating) or postpone the power consumption constitutes the basis of the optimal control strategy applied in this work.

3.6.4 - Switching constraints and operation realism

In real systems, excessive and frequent switching between ON and OFF states may lead to:

- Premature equipment degradation,
- Reduced efficiency,
- Perceived thermal discomfort.

With this being said, literature acknowledges the importance of imposing additional restrictions, such as minimum up time and minimum down time, particularly in models that rely on binary variables [58].

Even though the present work focuses primarily on energy and economic modeling, the use of binary variables within the MILP allows such additional constraints to be added straightforwardly if required, while maintaining consistency with the hybrid nature of real TCL operation.

3.7 - Control model application

In this section we present the complete mathematical formula used in the Control Model developed throughout this chapter. This problem is treated as a Mixed-Integer Linear Programming (MILP) problem which integrates:

- TCLs Thermal Dynamics
- Comfort Restrictions
- Non-Manageable Base Load
- Photovoltaic Production
- Daily OMIE market prices
- Bidirectional exchanges with the grid
- Arbitrage prevention in the instance of negative prices.

The use of MILP in thermal load management and renewable integration is widely documented in the literature of energy control and demand response. Being particularly adequate when there are discrete decisions related to the ON/OFF state of the devices [38].

3.7.1 - Temporal horizon and discretization

The problem is solved in a finite horizon of (K) periods. With temporal discretization of $\Delta t = 15\ minutes$

All external parameters, such as External temperature, photovoltaic production and market pricing, are considered known throughout the horizon, with the assumption of perfect prediction.

This approach is consistent with the day-ahead market framing, where the prices are known one day in advance. This way, the daily resolution of the optimization problem constitutes a coherent approximation with the operational reality.

Although in theory the formulation allows for the solving of complete annual horizons, such approach will entail a high number of binary variables and therefore higher computational effort.

Not only that, but also given that the used market pricing comes from day-ahead, price values are published with daily advance and the Optimal decision is reviewed daily in operational reality, it was concluded that a 24-hour time horizon establishes a methodologically coherent and computationally efficient approach.

This strategy is consistent with regular practices in optimal clearing models and great short-term planning of electric systems being sufficient for capturing the interaction between pricing, PV generation and thermal flexibility.

3.7.2 - Decision variables

For each TCL $i = 1, \dots, N$ and instant $k = 1, \dots, K$, defines:

- $u_{i,k} \in [0,1]$ - ON/OFF state of TCL;
- $T_{i,k}$ - Internal temperature;
- $P_k^{imp} \geq 0$ - Imported power from the grid;
- $P_k^{exp} \geq 0$ -power exported to the grid;
- $y_k \in \{0,1\}$ - Binary variable that defines the import/export

The global decision vector can be represented as:

$$\mathrm{z} = \begin{bmatrix} u \\ T \\ p^{imp} \\ p^{exp} \\ y \end{bmatrix}, \tag{3.20}$$

The presence of binary variables follows the common approach in the optimal thermal load management as in Halder et al. e Fontes et al., that use discrete variables to modulate TCLs operational state.

3.7.3 - Objective

The objective consists of minimizing the liquid energy cost throughout the horizon:

$$min \sum_{k=1}^{K} \Delta t \, (\pi_k^{buy} p_k^{imp} - \pi^{sell} p_k^{exp}), \tag{3.15}$$

Where:

- π_k^{buy} is the OMIE price converted to €/kWh;
- π^{sell} is the feed-in tariff established on 3.5.3.

3.7.4 - Thermal Dynamics

This structure is consistent with discrete linear first order models used in Halder et al. e Fontes et al.

$$\dot{x}(t) = -\alpha(x - T_{out}) + \beta_u \ . \tag{3.18}$$

3.7.5 - Comfort restrictions

$$L \leq T_{i,k} \leq U, \tag{3.21}$$

These restrictions directly correspond to the operational limits of the TCL and are formulated as linear restrictions assuring MILP structure compatibility.

3.7.6 - Power balance

The community total power is:

$$P_k^{load} = P_k^{base} + \textstyle\sum_{i=1}^{N} p_i u_{i,k}, \tag{3.22}$$

The energy balance at the end of grid connection is:

$$P_k^{load} - P_k^{PV} = P_k^{imp} - P_k^{exp}, \tag{3.11}$$

3.7.7 - Simultaneous import and export prevention

In the presence of negative prices, a documented occurrence in the European markets with high renewable penetration, stopping artificial arbitrage is necessary.

So, the following is implemented:

$$P_k^{imp} \geq M y_k, \tag{3.5}$$

$$P_k^{exp} \geq M(1 - y_k), \tag{3.6}$$

The above variables and constraints work to ensure that: if $y_k = 1$, then $P_k^{exp} = 0$ and, if $y_k = 0$, then $P_k^{imp} = 0$.

The introduction of binary variables transforms the problem into a MILP formulation.

The use of this technique (Big-M formulation) is common in optimization problems with mutually excluding decisions.

3.7.8 - Final structure

The problem is formally defined as:

$$\min_{z} f^T z \ , \tag{3.23}$$

Subject to:

- $Az \leq b$;
- $A_{eq}z \leq b_{eq}$;
- $l \leq z \leq u$;
- $z_j \in \{0,1\}$ para $j \in I$

3.8 - Conclusion

In this chapter, the developed method was presented and explained its application for solving optimization control problems with TCLs in a Renewable Energy Community context.

The approach is based on a short-term deterministic formulation, solved with a 15-minute timestep for a horizon of 24 hours. These timeframes are coherent with the structure of OMIE's day ahead market, which is the market considered for the case study to be presented in section 4. The chosen timeframes guarantee some realistic operation of the model, considering that all prices for the day-ahead known with enough time to run the solver.

This formulation explicitly models the following:

- Non flexible baseload, built based on typical load profiles and data of average annual consumptions for a regular household;
- Photovoltaic energy production, obtained from PVGIS and extrapolated to a quarter-hourly resolution by means of a 2nd degree polynomial equation;
- Energy price signal, taken directly from OMIE;
- TCL thermal dynamics, modelled through a first order discreet linear system (RC model);
- The operational behavior of TCLs, including thermal comfort boundaries and ON/OFF decisions;
- Bidirectional exchange with the grid (power import/export);

The final formulation of this problem as a Mixed-Integer Linear Programming (MILP) was shown as necessary due to the existence of very low, or even negative, energy prices, which require additional constraints of the problem, to prevent artificial arbitrage and simultaneous energy import and export.

Overall, the structure of the model is aligned with the work developed by Halder et al. [2] and Fontes at al. [1], adopting the same base thermal dynamic and overall control problem philosophy whilst adapting it to a renewable energy community scenario.

In the following chapter, the case study will be described, as well as the proposed methods for analysis and results obtained.

Chapter 4

Case Study

With the objective of evaluating the performance and applicability of the developed control model, a case study was considered for a small dimension renewable energy community on which all participants had a TCL installed with the same characteristics.

This particular configuration intends to represent a realistic scenario of collective self-consumption in low voltage, which is figured in the Portuguese regulatory framework.

4.1 - Renewable energy community structure

The renewable energy community is composed by 22 houses in a building, situated in Porto, Portugal, and it is considered that the renewable energy production comes from a power plant installed in the electrical riser panel of the building.

In this manner, this community makes no use of the national electrical grid, therefore no grid access tariffs have to be paid, which allows for a simpler economic analysis of the model.

Each dwelling is assumed to have:

- A non-flexible base load, modelled by taking typical residential load profiles;
- A flexible load, which is a thermostatically controlled load, in this case an air conditioning unit.

Therefore, the population of TCLs is given by N, where: $N = 22$

4.2 - Renewable energy system – PV power plant

This community is served by a PV power plant which generates energy to be shared with each household. The considered PV power plant for this community has the following nominal power: $P_{inst}^{PV} = 60kW$.

The energy production profile was obtained from PVGIS, considering the irradiation in Porto, Portugal with optimal tilt and slope (given by PVGIS) and converted to a quarter-hourly resolution, as described in 3.3.1 -

4.3 - Load base modelling

In a real context, baseload profiles would be obtained from the historical data of each consumption point, which would allow for an individual and detailed analysis of the energy consumption behavior of each user. However, that kind of data is unavailable for this study, therefore, an alternative approach was taken, based on data published by certified entities regarding energy consumption.

E-Redes, the Distribution Grid Operator in Portugal, publishes every year the normalized consumption profiles for low voltage clients [59]. For 2025, these profiles are organized by categories that illustrate different consumption patterns:

- Profile BTN A
- Profile BTN B
- Profile BTN C

From these, profile BTN C is the one that more accurately portrays the behavior of domestic clients, according to E-Redes. For that reason, that was the chosen profile.

Now, this profile is presented as a permillage of the whole annual consumption for each quarter hour. So, it was necessary to use statistical data about the average annual consumption for a typical household.

As stated by the annual report “Energia em Números”, the average household annual consumption stands between 2500 and 5000 kWh/year [60]. Therefore, 3500kWh/year was assumed as the basis to create the parameters of each household.

From there, Gaussian noise was applied to the 22 households, creating different annual consumptions for each member of the population.

Table 4.1 — Population Annual Consumption.

TCL ID	Annual Baseload
1	3823
2	4600
3	2145
4	4017
5	3691
6	2715
7	3240
8	3706
9	5647
10	5162
11	2690
12	5321
13	3935
14	3462
15	3929
16	3377
17	3426
18	4394
19	4345
20	4350
21	3903
22	2776

The annual consumption of each unit was then multiplied by the load profile from E-Redes and processed as described in Method, which resulted in an annual quarter-hourly consumption profile for each dwelling.

With this approach, we can guarantee a credible representation of residential consumption for different users, assuming a realistic profile, even though synthetically generated. This allows us to process the data in the optimization control model, without compromising validity.

4.4 - Thermostatically controlled load

It was considered that all dwelling were architecturally equivalent and were equipped with the same air conditioning units, which allowed to normalize the population of TCL and consider that all devices used the same power when heating internal thermal mass.

PTCL = 2kW;

Nevertheless, each TCL is modelled individually, which allows:

- Detailed analysis of temperature evolution in each household;
- Exploring distributed thermal flexibility;
- Evaluating the aggregate consumption of the community;

So, for instant k, the power used by any TCL is described as: $P_i = 2 \times u_{i,k}$

This calculated power will be summed with the baseload power of each dwelling, which will result in the total load of the community

$$P_k^{total} = P_k^{base} + P_k^{TCL}, \quad (3.24)$$

4.5 - Thermal comfort and temperature bounds

The temperature bounds and agreed temperature band with each user will be one of the constraints and inputs that will dictate the behavior of each TCL.

So, as was done for the annual total consumption, in this case, a base lower and upper bound were selected $L = 16$ and $U = 24$

And then a Gaussian noise was applied to generate different comfort bands for each TCL, which resulted in the following dataset.

Table 4.2 — TCL Temperature Boundaries.

TCL ID	L	U
1	17	23
2	18	24
3	17	24
4	17	25
5	17	25
6	16	25
7	16	23
8	15	24
9	17	23
10	15	23
11	15	24
12	15	26
13	12	23
14	18	24
15	16	24
16	15	25
17	18	23
18	14	24
19	16	25
20	16	25
21	16	26
22	16	24

4.6 - Economic parameters

As stated in the Method chapter, the energy prices considered were extracted from historical data of a real energy market. In this case, considering the case study describes a REC in Portugal, OMIE was chosen and data of 2024 was selected.

Therefore, the energy buy price at each instant is correspondent to the hourly values of the OMIE market for the year 2024.

As for the feed-in tariff, it was considered a fixed feed-in tariff based on prices that aggregators are practicing. The typical price range is between 25€/MWh and 40€/MWh, there the chosen value was: $\pi^{sell} = 30€/MWh$

4.7 - Simulation horizon

Considering the available power consumption profiles from E-Redes, the temporal resolution of this simulation is 15 minutes.

And considering the structure of the day-ahead OMIE market, the horizon for the simulation is 24 hours.

These choices are consistent with the real data available, but also with the necessity to limit the computational dimension on the MILP.

Chapter 5

Simulation Scenarios

5.1 - Scenario 1 – No control operations (reference)

In the first scenario, TCLs operate autonomously, following a traditional hysteresis thermostat logic, without any economical optimization.

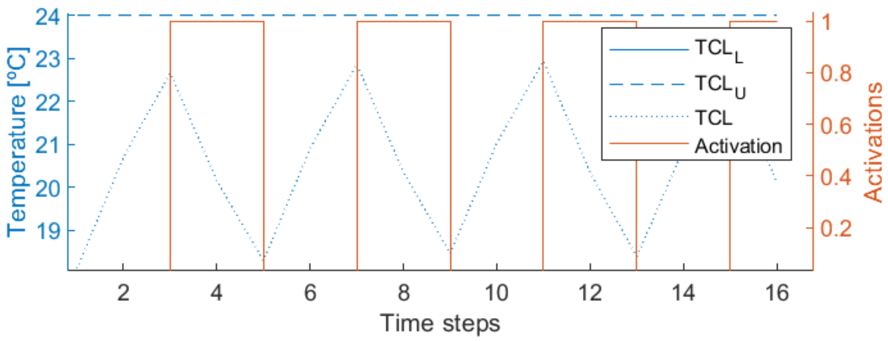

Figure 5.1.1 – Hysteresis mode of functioning of a TCL with no control.

In this case:

- Each TCL turns on when the temperature reaches the inferior limit L
- Turns of when it reaches the superior Limit U
- There is no coordination between dwellings
- The photovoltaic production simply passively diminishes the grid import.
- It isn't considered structured output

This Scenario represents the conventional operation of residential dwellings, being the baseline for the comparison.

Studies about demand response problems frequently use this as a reference for quantifying energy and efficiency gains [22].

5.2 - Scenario 2 – LP (No export)

In the second scenario, a Linear Programming based optimization model is applied with the following assumptions:

- Continuous control variables
- Absence of grid export
- Only importation allowed
- Schedule pricing from OMIE

The problem remains computationally and convexily efficient, granting:

- Thermal flexibility's maximum theoretical potential evaluation
- Realist Binary Behavior comparison
- Relaxed and discrete solution difference analysis

The use of continuous relaxations for the thermal flexibility analysis is standard in literature, serving as a lower bound.

5.3 - Scenario 3 – MILP

In the third scenario, the complete model developed in this thesis is applied, including:

- ON/OFF binary variables
- Bidirectional grid exchanges
- Arbitrage prevention through Big-M restriction
- Negative price consideration
- Feed in tariff;

This is the most believable scenario from the technical and economical point of view, representing an optimized operation of the integrated energy community in the wholesale market.

Chapter 6

Results

6.1 - Energy analysis

6.1.1 - Total power profile

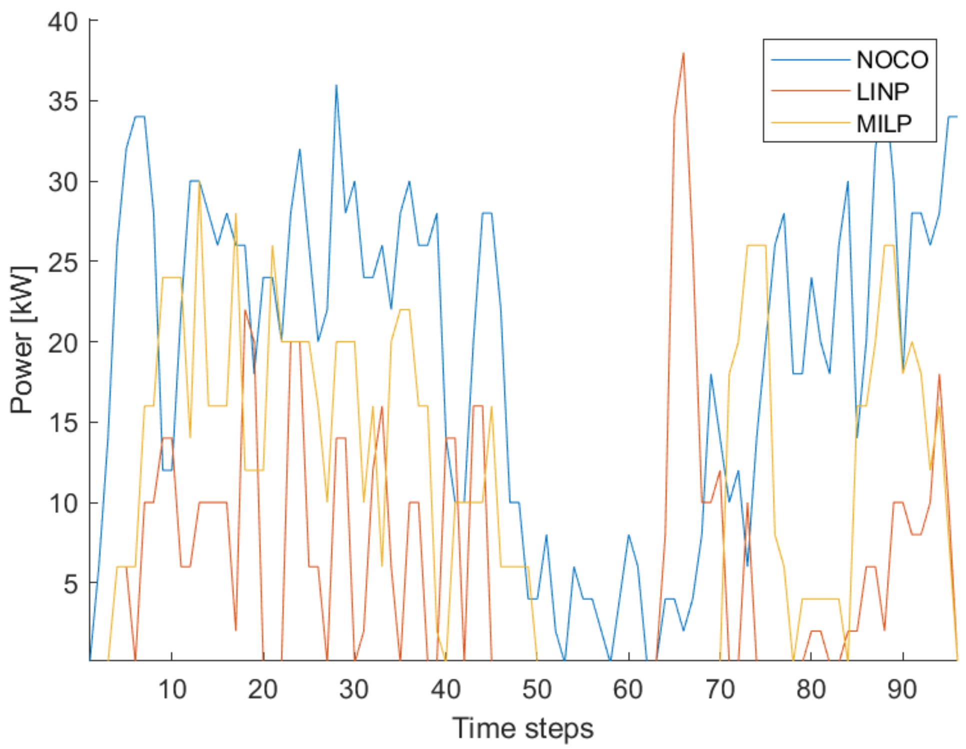

Figure 6.1.1.1 – Total Power of Aggregate TCLs, on the 59th day tested, considering No Control, Linear Programming control model and MILP control model.

The differences observed between the LP and MILP results stem from structural differences in the formulations, which are:

- The explicit inclusion of grid export in the MILP;
- Logical constraints preventing simultaneous import and export;
- The logic behind the treatment of negative electricity prices.

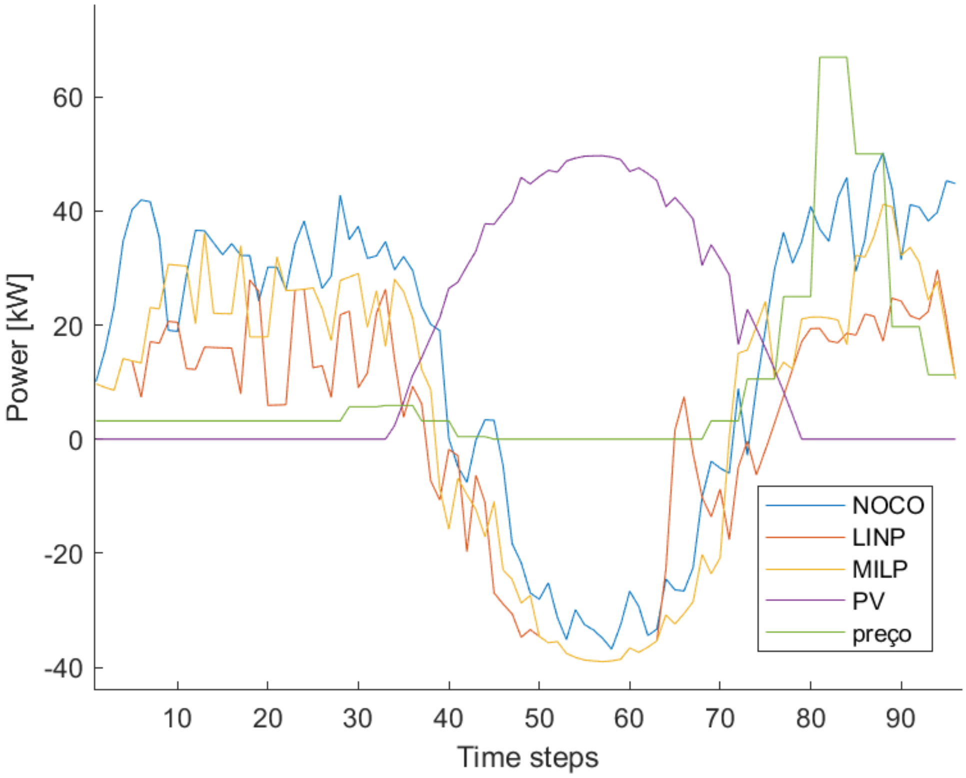

Figure 6.2.1.2 – Total Net Power of the REC, for each control method, juxtaposed with PV Power and Price Signal

For identical PV capacities, the MILP model show higher average grid imports compared to the LP formulation, but very similar final total costs. This can be explained by the fact that the LP scenario does not consider energy exportation, which forces the model to prioritize self-consumption of the PV generation. On the other hand, the MILP framework introduces an additional degree of freedom by allowing energy export and using it to generate revenue. This leads to a different allocation of energy flows, which can represent a more realistic operation of the aggregate loads of the community.

6.1.2 - Selfconsumption and selfsufficiency

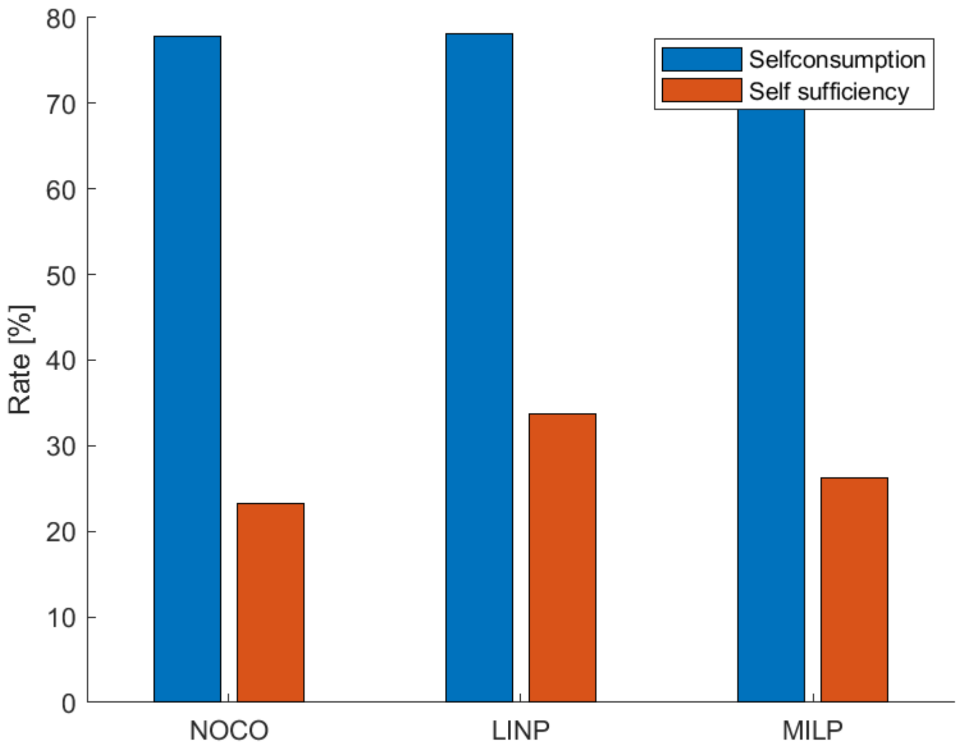

Figure 6.3.2.1 – Average selfconsumption and selfsufficiency rate for each control method (No Control, LP and MILP).

This trend highlights an important structural limitation: the available thermal flexibility within the community is finite and does not scale proportionally with installed PV capacity.

Beyond a certain penetration level, additional PV generation exceeds the system's ability to internally absorb energy through thermal flexibility. As a result, surplus energy is exported rather than consumed locally.

The LP formulation consistently yields higher self-consumption values than the MILP. This is largely attributable to the absence of export in the LP model, which implicitly enforces internal utilization of PV generation. By contrast, the MILP model allows explicit export, resulting in lower self-consumption ratios in high-PV scenarios.

A similar pattern is observed in self-sufficiency levels. While increasing PV capacity improves the community's degree of energy independence, the improvement becomes progressively smaller as penetration rises. Again, the LP formulation tends to slightly overestimate self-sufficiency relative to the MILP.

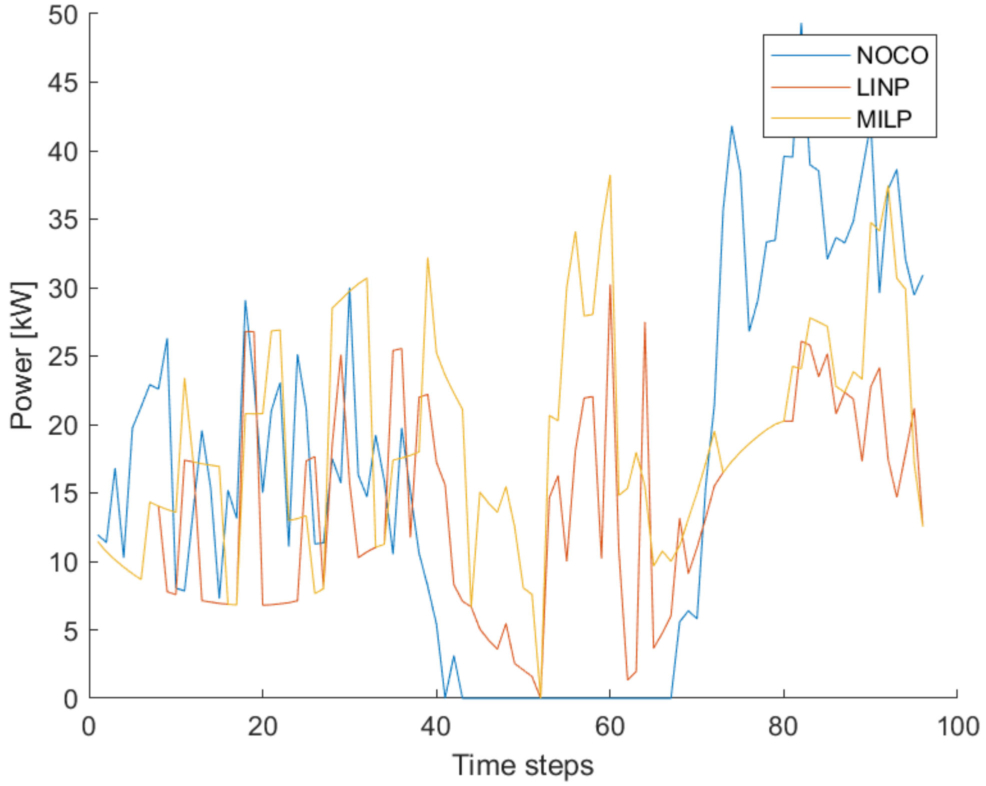

Figure 6.4.2.2 – Comparison of the grid import diagrams of each control method for 59th day tested

6.1.3 - Peak shaving

The optimized control strategies demonstrate a measurable reduction in grid import peaks compared to uncontrolled operation. Thermal loads effectively act as distributed energy buffers, shifting consumption toward periods of higher PV production or lower market prices.

This behavior aligns with the established literature on demand response, where thermostatically controlled loads are recognized as suitable resources for peak reduction and load shifting.

However, the peak reduction effect stabilizes as PV capacity increases. This reinforces the notion of structural saturation: once the available thermal flexibility has been fully exploited, further increases in PV capacity no longer yield proportional improvements in peak shaving performance.

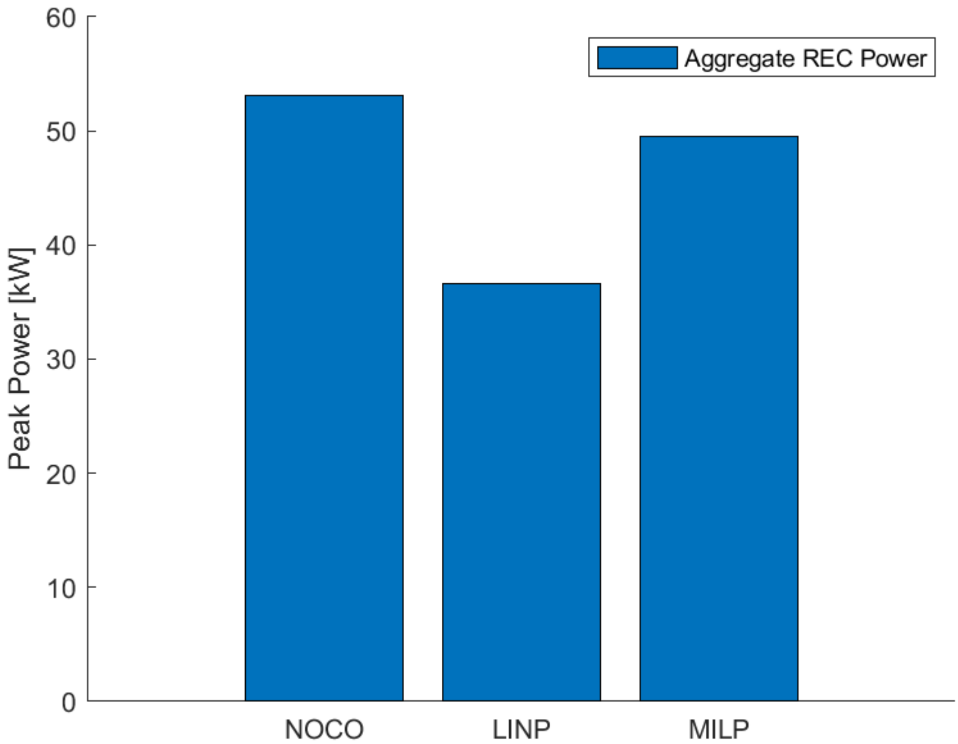

Figure 6.5.3.1 – Maximum Peak Power of the REC for No Control, LP and MILP

6.2 - Thermal analysis

The thermal model used in this work defines lower and upper bounds of thermal comfort for each user, which will help control each TCL. These bounds are defined by the parameters L and U of the TCL.

In the formulation of the optimization problems, with a convex relaxation of the decision variables, the solution has to, by definition, result in thermal dynamics that do not violate the thermal comfort constraints. However, this could happen when converting the decision variables to binary logic. Nonetheless, as stated in the method, a robust approach to guarantee that the conversion respects the boundaries was taken, therefore it can also be observed that in no moment the bounds were passed.

Not only that, but it also noted that even though the continuous solution allows for intermediate values of control variables, the resulting temperature curve stays inside the admissible region imposed by the problem. After converting to binary, it is also observed that the admissible region is respected, which allows the conclusion that there are no physical inconsistencies in the model.

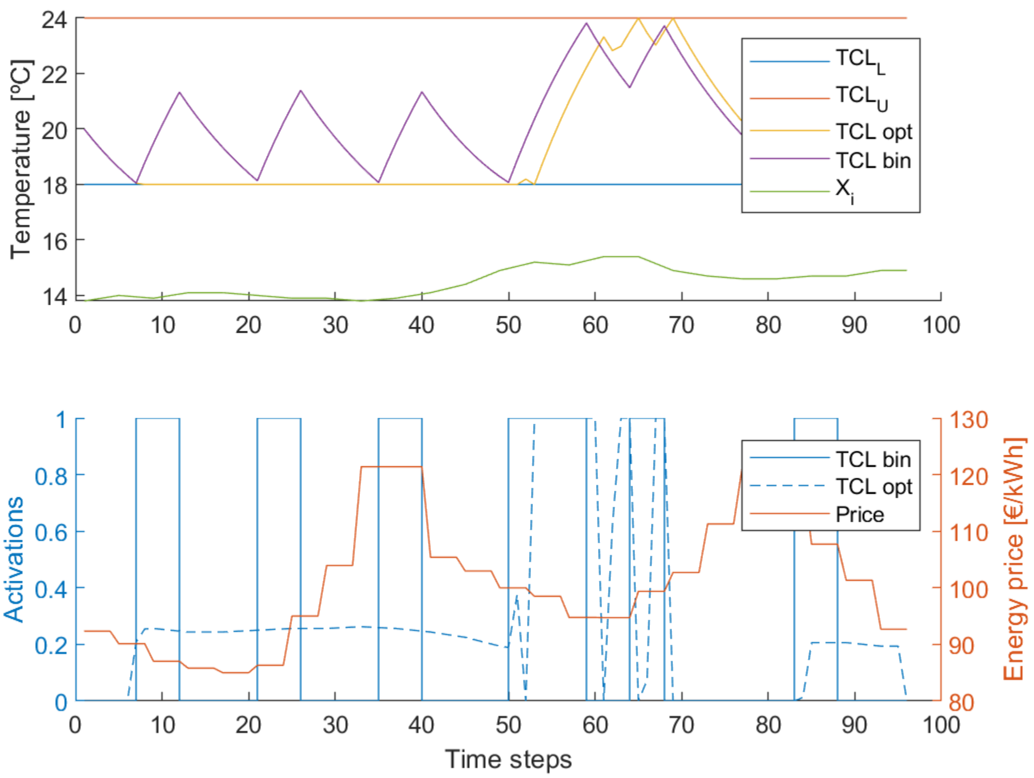

Figure 6.6.3.1 – Example of TCL no.2 behaviour on the 10th day tested; comparison between optimal decisions and binary decisions aligned with the inside temperature it causes, juxtaposed with the price signal and outside temperature.

From this, the following can be concluded:

- The mathematical formulation, regarding thermal dynamics and constraints, is consistent and was correctly implemented;
- The discretization of the control variables lead to thermal deviations that are not significant enough to steer the internal temperature outside of the defined bounds;
- Thermal flexibility is imposed and explored inside the thermal comfort band.

These results show that optimal control allows to the maximization of economic benefits, while maintaining the thermal requirements of each user.

6.3 - Performance and method analysis

6.3.1 - No control vs LP vs MILP

The comparison between the three considered methods allows the evaluation of the impact of coordinating demand and discretization of the control variables in the energetic and economic performance of the community.

It should be noted that, in both optimization cases, the problem was initially solved considering a convex relaxation of the decision variables in the interval [0,1]. Therefore, for the MILP formulation considered, this model does not directly solve a classical mixed-integer problem, but a relaxed formulation, followed by discretization, which is served the mixed-integer logic in the definition of import and export mechanics.

6.3.2 - Energy comparison

In the No Control scenario, the behavior of the TCL units is exclusively determined by local thermostat logic, where, whenever the indoor temperature is about to hit one of the boundaries, it changes state to prevent that. There is no coordination with renewable energy production, nor with the price signal.

For these reasons:

- Excess PV energy consumption is not optimized;
- The exportation is a result of only the mismatch between production and consumption;
- Peak consumption patterns are usually not altered by the increase of PV power installed;

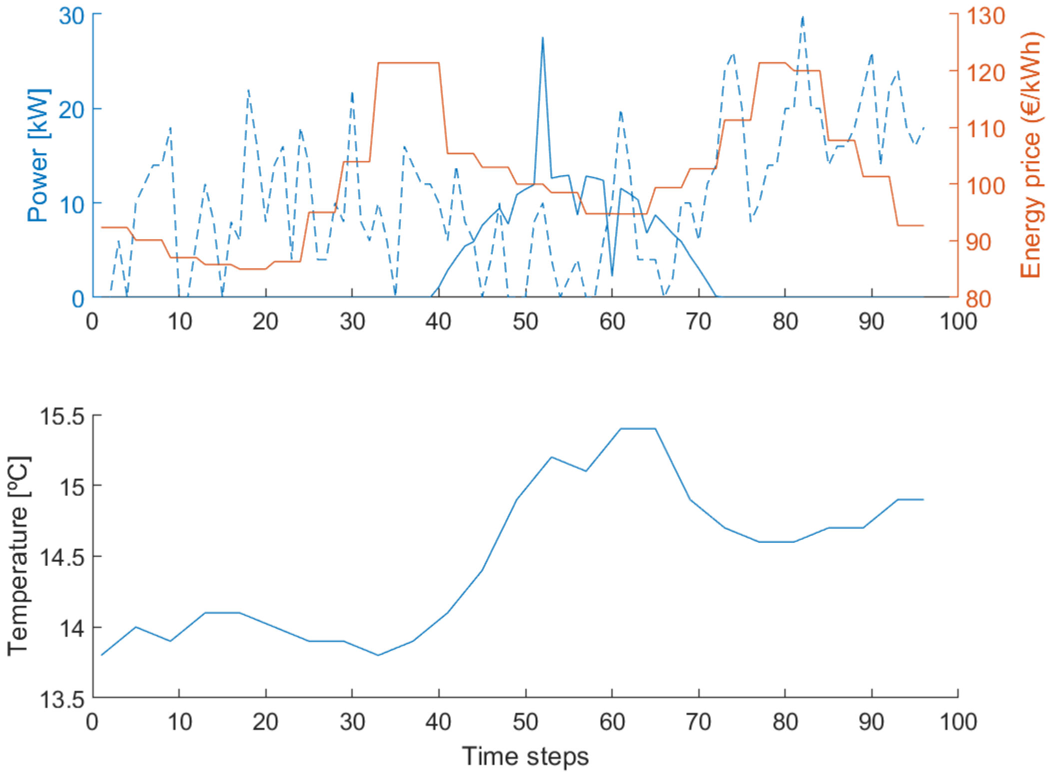

Figure 6.7.2.1 – No Control Scenario, day 10: Aggregate TCL Power vs PV Power, Price signal and outside temperature

In the LP scenario, there is an optimal maneuvering of flexible loads to maximize self-consumption. By having continuous decision variables at first, the model is able to move consumption to instants with the most energy production, reduce energy import from the grid when the price signal is high and optimize self-consumption in the community.

However, the continuous decision variables do not represent the real operation of TCL, considering these devices operate with binary logic (either ON or OFF). For that reason, the variables were then converted, which might cause a less optimal absorption of solar energy available.

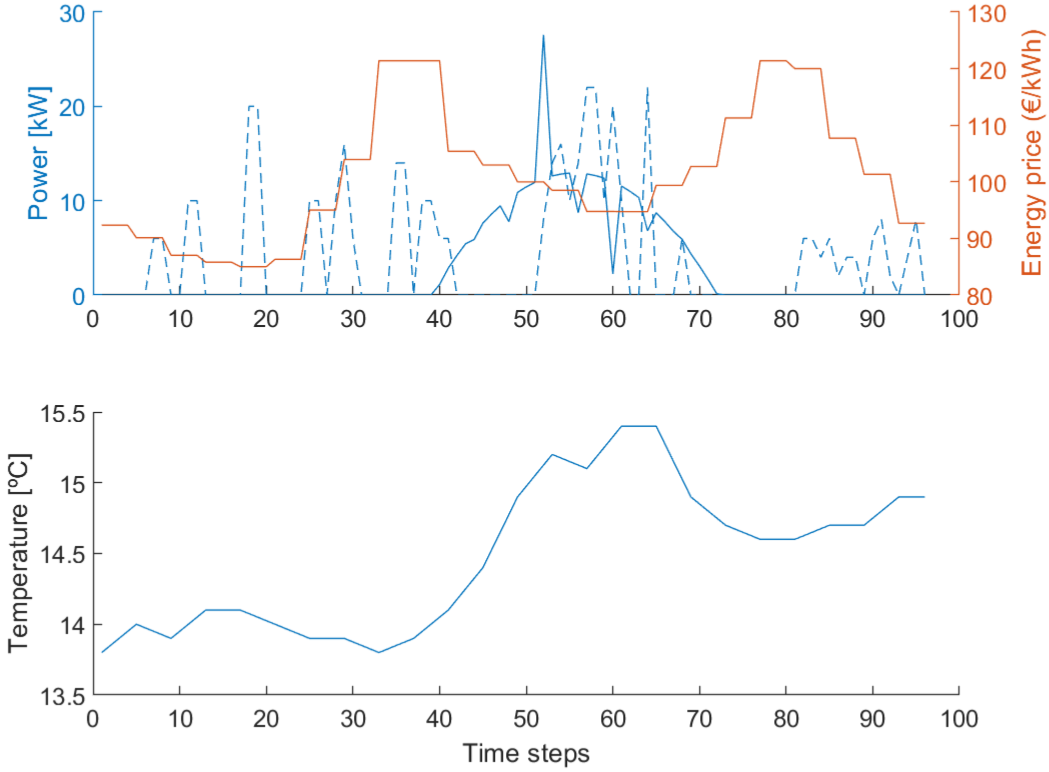

Figure 6.8.2.2 – Linear Programming Control Method, day 10: Aggregate TCL Power vs PV Power, Price signal and outside temperature

In the MILP formulation, a logic similar to the LP scenario was followed, however, the problem is able to enforce the prohibition of simultaneous import and export at the grid connection point, which in turn allows the valorization of injecting (and selling) excess energy. Consequently, the self-consumption profile is different from LP scenario, because the model will now prioritize selling energy in moments where the consumption price signal is too low.

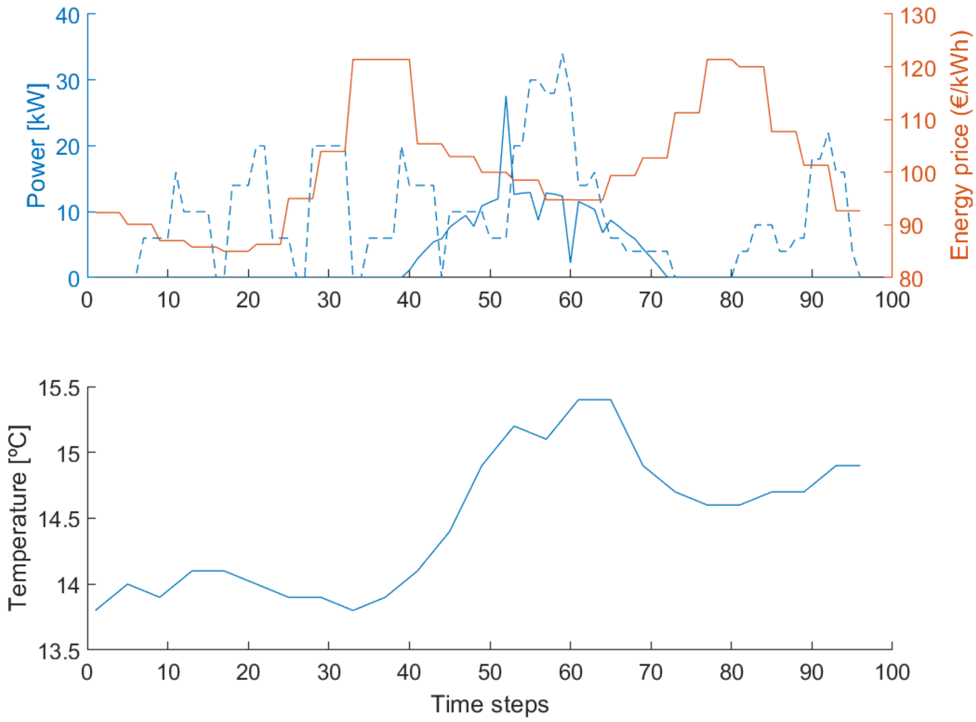

Figure 6.9.2.3 – Mixed-Integer Linear Programming Control Method, day 10: Aggregate TCL Power vs PV Power, Price signal and outside temperature

6.3.3 - Economic comparison

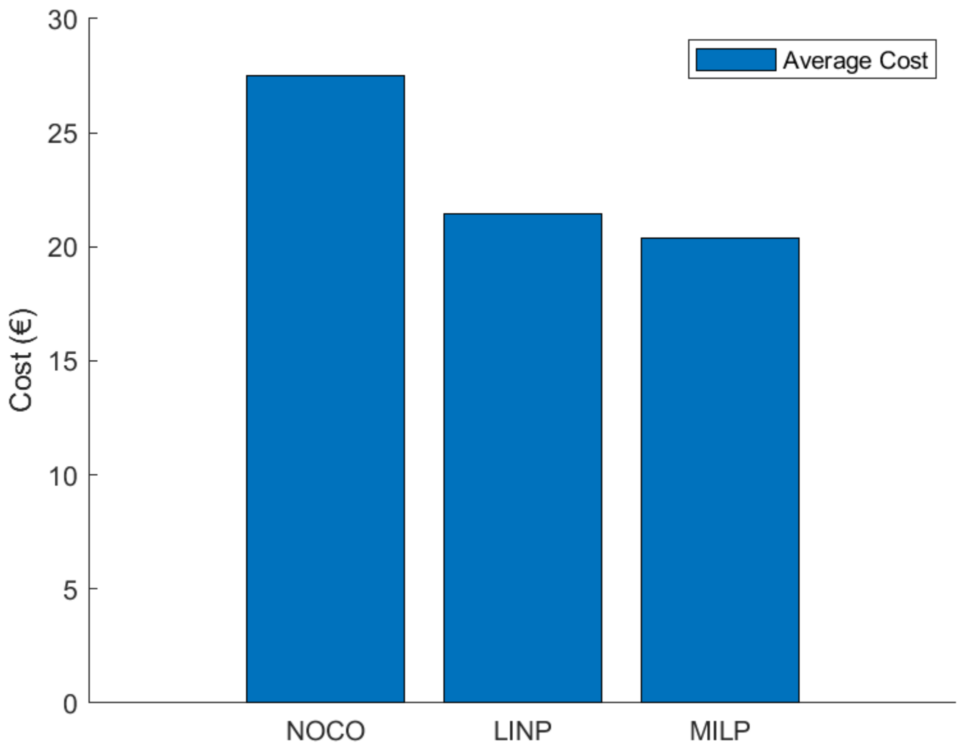

Figure 6.3.3.1 – Economic comparison of the average daily cost between the three control methods (post discretization for LP and MILP) for a period of 59 days between the 1st of January and 28th of February.

From an economic standpoint, both optimization methods result in substantial reductions of the energy cost when compared to the No Control scenario. This confirms that optimizing demand is the leading factor in reducing the final cost of energy for the community.

Considering the continuous decisions, the MILP formulation presents usually better results than the LP scenario. However, after binary conversion, there is a worsening of the results, more substantial in the MILP. This reflects the following:

- Granularity loss of the continuous solution;
- The impossibility of operating in intermediate states;
- Having different loads profiles, as a consequence of binary operation, which leads to different import and export profiles.

Nonetheless, the increase caused by binary logic is small enough to consider that this translates to a good approximation of the implementable real control of the TCL.

6.3.4 - Discretization gap

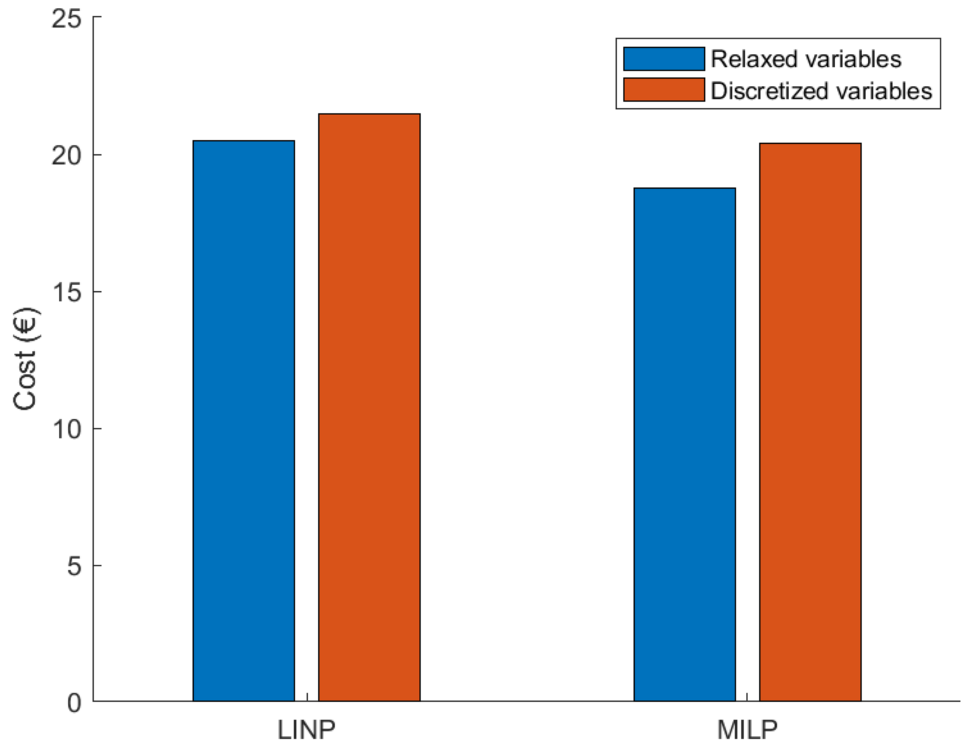

Figure 6.10.4.1 – Comparison of the average daily cost between the three control methods before and after discretization of the decision variables.

Considering the problem was solved in a continuous regime and discretized *a posteriori* it was necessary to analyze the gap in the result of the minimized function as a consequence of the discretization.

In this context, it is calculated as

$$Gap(\%) = \frac{C_{binary} - C_{continuous}}{C_{continuous}} \times 100, \tag{6.1}$$

which translates perceptually the cost difference between the continuous and the binary solutions.

This indicator measures the economic impact of discretization ex-post.

The results show that this gap is limited, usually to a value under 10%, depending on the level of renewable energy production. This means that:

- The solution is a good approximation of the implementable, real solution;
- The efficiency loss due to discretization is relatively low;
- The adopted approach is a valid and computationally efficient compromise between physical realism and modelling simplification.

It is also important to note that the gap tends to be bigger the more PV production there is. This reflects the higher sensitivity of the solution to TCL activation decisions when there is more excess energy.

6.3.5 - Global interpretation

This analysis allows three conclusions:

- Optimization is the most impactful factor;
    - The difference between LP and MILP is negligible, however, when compared to No Control scenario, they present substantially better results;
- The convex relaxation of decision variables results in the greatest minimization of the objective function;
    - The LP scenario shows the maximum theoretical potential of thermal flexibility, acting as a methodological reference;
- Discreet modelling is necessary for realism;
    - The MILP formulation captures the possibility of import and exportation based on price signals and excess energy available, representing the real-world strategies that are implementable in consumption optimization.

6.4 - Sensibility analysis

6.4.1 - PV power

This analysis examines how increasing local generation affects the energy cost, the degree of self-consumption, the level of self-sufficiency, and the community's import/export profile. In doing so, it assesses the structural impact of increasing renewable penetration within the community, from both economic and energy perspectives. It further evaluates the relevance of the MILP formulation compared with the LP approximation.

6.4.1.1 - Impact of PV on energy cost

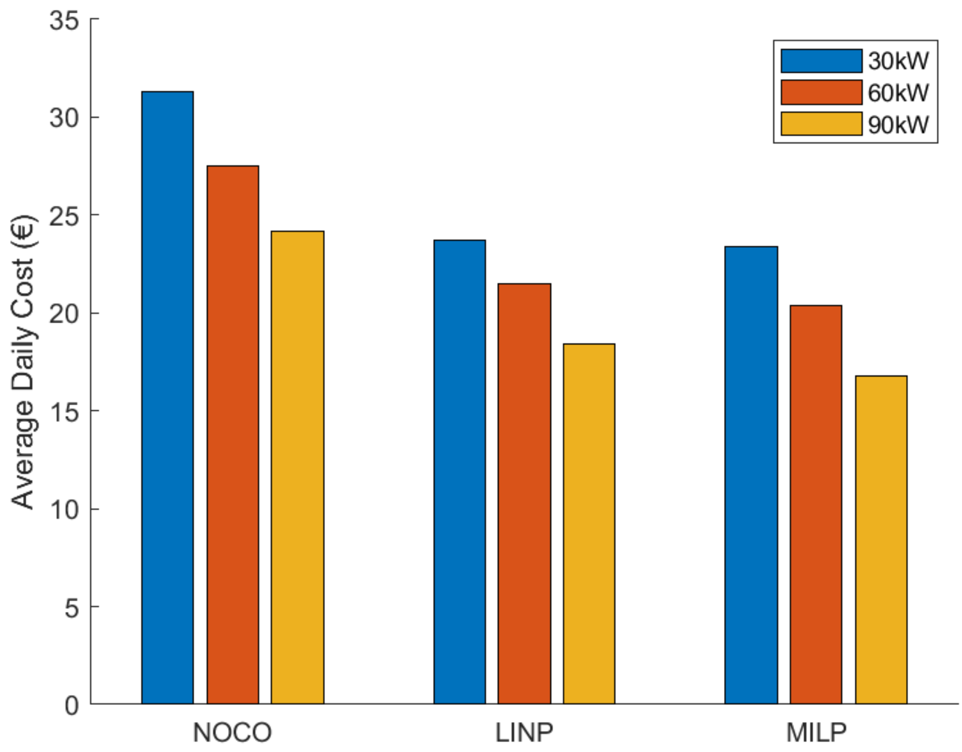

Figure 6.11.1.1.1 – Comparison of average daily cost for the tested period between the three methods for three different levels of PV penetration in the REC.

A systematic reduction in the average daily energy cost is observed as the installed PV capacity increases, across all scenarios considered (no control, LP, and MILP). This behavior is consistent with the community's decreasing dependence on the grid as local production grows.

However, the cost reduction is not linear. Increasing PV capacity from 30 kW to 60 kW yields a substantial decrease in the average cost, whereas the further increase to 90 kW results in a proportionally smaller reduction. This reveals the presence of diminishing marginal returns associated with increasing installed capacity, driven by temporal mismatch between production and demand.

In the optimized scenarios (LP and MILP), the additional economic benefit provided by thermal flexibility becomes more pronounced as installed capacity rises. Specifically, at 90 kW, the cost difference between the uncontrolled scenario and the optimized scenarios is greater than at 30 kW. This confirms that demand flexibility becomes progressively more valuable under high renewable penetration, where there is a larger volume of energy to manage.

Moreover, the difference between LP and MILP grows slightly with capacity, reflecting the increasing importance of binary constraints and explicit export modelling in regimes with structural surplus.

6.4.1.2 - Impact of PV on self-consumption

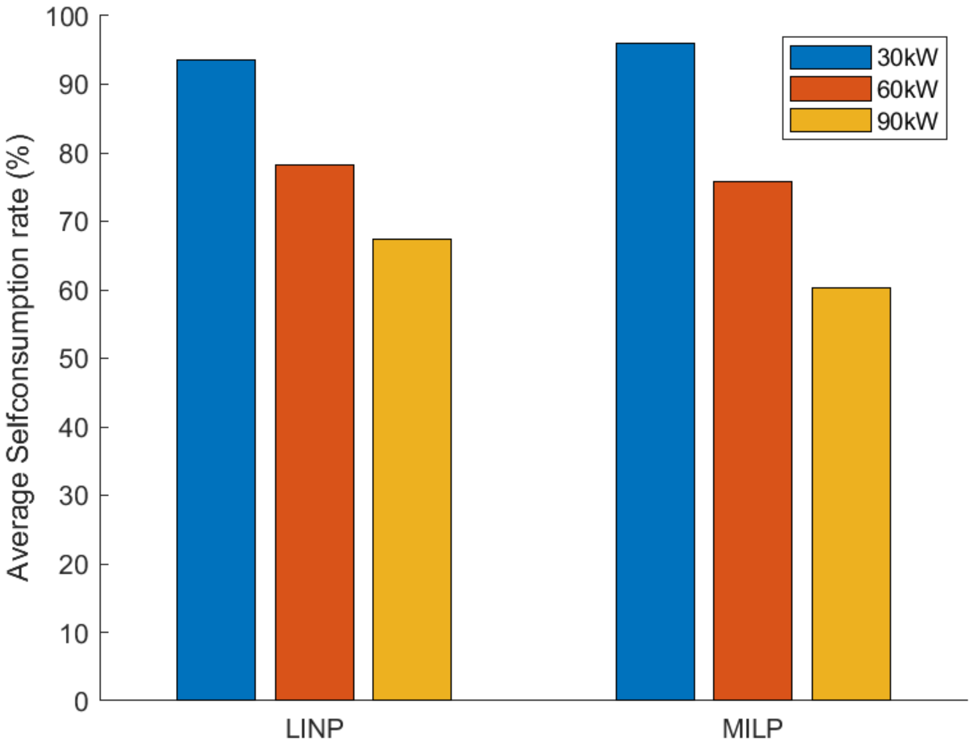

Figure 6.12.1.2.1 – Comparison of average selfconsumption rate for the tested period between LP and MILP methods for three different levels of PV (30kW, 60kW and 90kW).

The self-consumption rate decreases as PV capacity increases. At 30 kW, production is close to the community’s aggregated demand, resulting in very high self-consumption. As capacity increases to 60 kW and then to 90 kW, significant surpluses arise during high-irradiance hours, inevitably leading to larger export volumes.

This trend aligns with studies on collective self-consumption, which show that oversizing PV without additional storage or flexibility tends to reduce the share of local consumption.

Comparing LP and MILP, the LP model predicts higher self-consumption, particularly at high PV levels. This results from the LP model's inability to export surplus, forcing TCL activation to absorb excess generation. While this behavior artificially boosts the selfconsumption metric, it does not necessarily represent the economically optimal solution when export is allowed.

This finding highlights a key conceptual distinction: **maximizing self-consumption is not equivalent to minimizing total cost**.

6.4.1.3 - Impact of PV on self-sufficiency

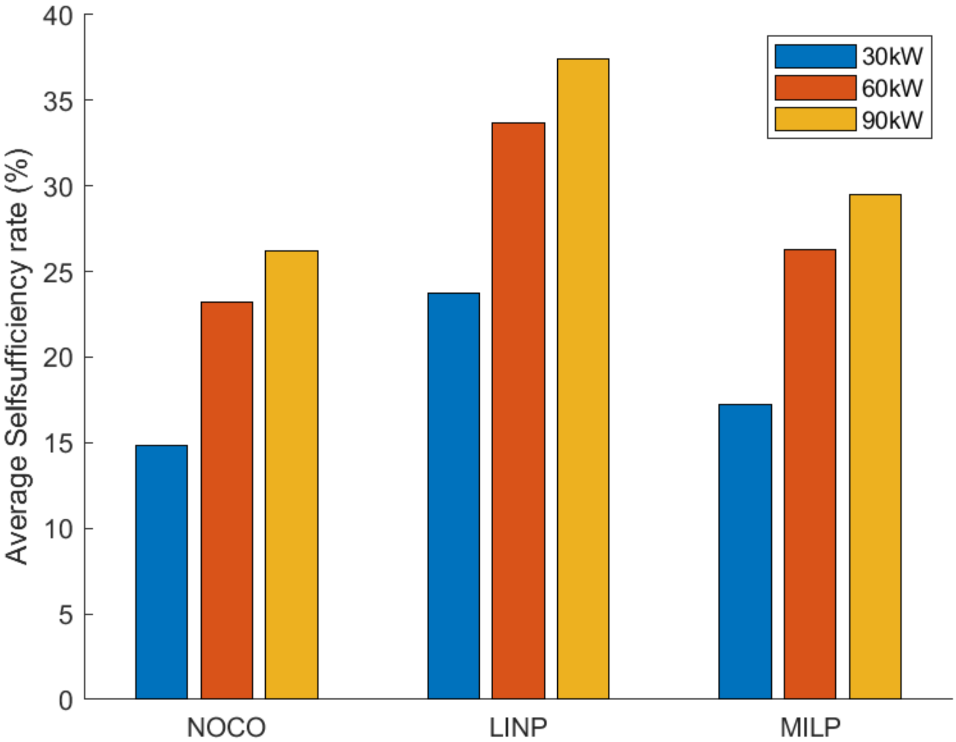

Figure 6.13.1.3.1 – Comparison of average selfconsumption rate for the tested period between LP and MILP methods for three different levels of PV (30kW, 60kW and 90kW).

The community's energy self-sufficiency increases with installed PV capacity, reflecting the higher contribution of local generation to meeting total demand. However, as with cost, the increase exhibits diminishing returns at higher capacity levels.

Moving from 30 kW to 60 kW yields a substantial increase in the fraction of demand served locally, whereas the increase from 60 kW to 90 kW is more modest because much of the additional production occurs when demand is already largely met.

Optimization (LP and MILP) amplifies the self-sufficiency gains relative to the uncontrolled scenario, demonstrating that thermal flexibility can reduce imports even without additional PV capacity.

6.4.1.4 - Impact of PV on grid export

Energy export becomes progressively more relevant as PV capacity increases. At 30 kW, export is negligible. At 60 kW, export occurs regularly during central daylight hours. At 90 kW, the community clearly becomes an exporting entity, with structural surpluses during many periods.

This behavior reveals three distinct operating regimes:

- Absorption regime (30 kW) - most generation is consumed locally.
- Intermediate regime (60 kW) - significant local consumption coexists with regular export.
- Export regime (90 kW) - production frequently exceeds local absorption capacity.

The MILP formulation becomes particularly important in the latter two regimes, as the choice between exporting and absorbing surplus gains economic significance.

6.4.1.5 - Impact of PV on peak shaving

Despite the substantial increase in PV generation, the peak import only decreases moderately as capacity rises. This occurs because the peak demand typically falls outside solar production hours, especially in the evening.

This confirms that PV alone is not an effective peak reduction mechanism. Peak mitigation primarily arises from thermal flexibility, which shifts consumption from the peak period to earlier hours. Hence, optimal TCL control plays a more decisive role than simply increasing PV capacity.

6.4.2 - Feed-in tariff sensitivity

To evaluate how the economic framework for energy export affects the community's optimal operation, a sensitivity analysis was performed on the feed in tariff, while keeping PV capacity fixed at 60 kW and maintaining all other system parameters constant (outdoor temperature profile, TCL thermal parameters, and OMIE day ahead market prices).

Three feed-in tariff values were considered,

- 0 €/MWh
- 30 €/MWh
- 100 €/MWh

with the goal being to understand how the export incentive influences the optimal balance between self-consumption, export, and import, as well as the community's total energy cost.

6.4.2.1 - Impact of feed-in tariff on total cost

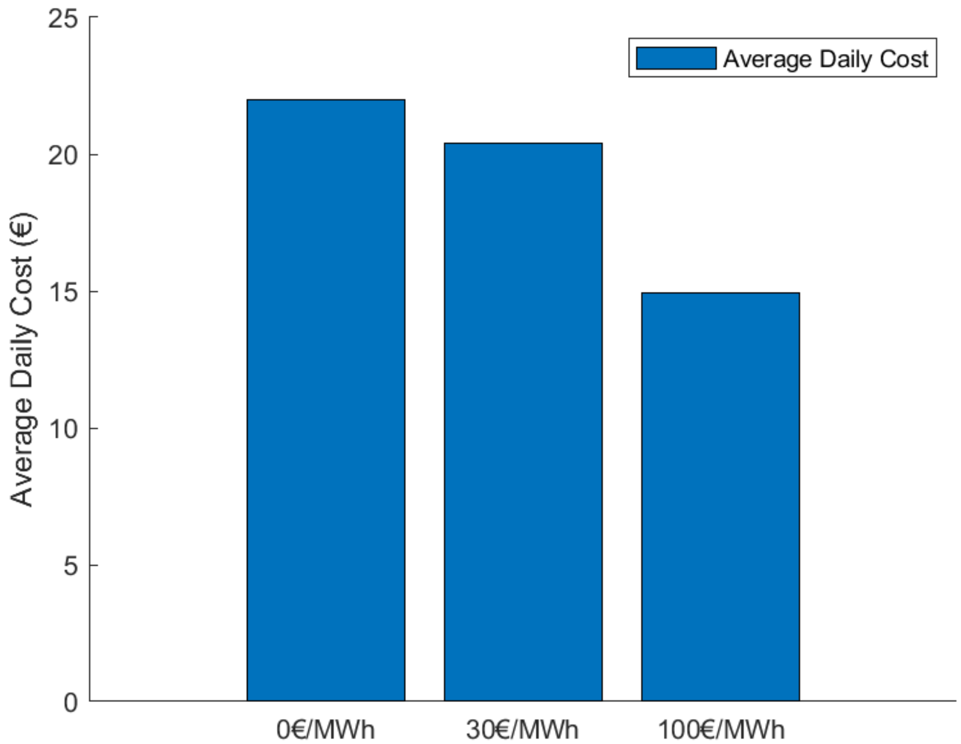

Figure 6.14.2.1.1 – Comparison of average daily cost for the tested period, for MILP method, considering three different feed-in tariffs.

A systematic reduction in the average daily energy cost is observed as the feed in tariff increases. For 60 kW of PV, the cost evolves approximately from:

- 21.98 €/day at 0 €/MWh,
- 20.37 €/day at 30 €/MWh,
- 14.91 €/day at 100 €/MWh.

The increase from 0 to 30 €/MWh produces a moderate cost reduction, while the jump to 100 €/MWh yields a much more pronounced decrease. This shows that remunerating export fundamentally shifts the economic balance of the system.

This behavior is consistent with optimal dispatch theory in electricity markets, where the decision to consume locally or export depends on the comparison between the marginal cost of import and the marginal revenue from selling [61]. As the tariff approaches or exceeds the marginal acquisition cost, exporting becomes economically competitive with forced self-consumption.

Higher tariffs also increase the daily variability of the cost, reflecting greater sensitivity to fluctuations in PV production and market prices.

6.4.2.2 - Impact of feed-in tariff on export

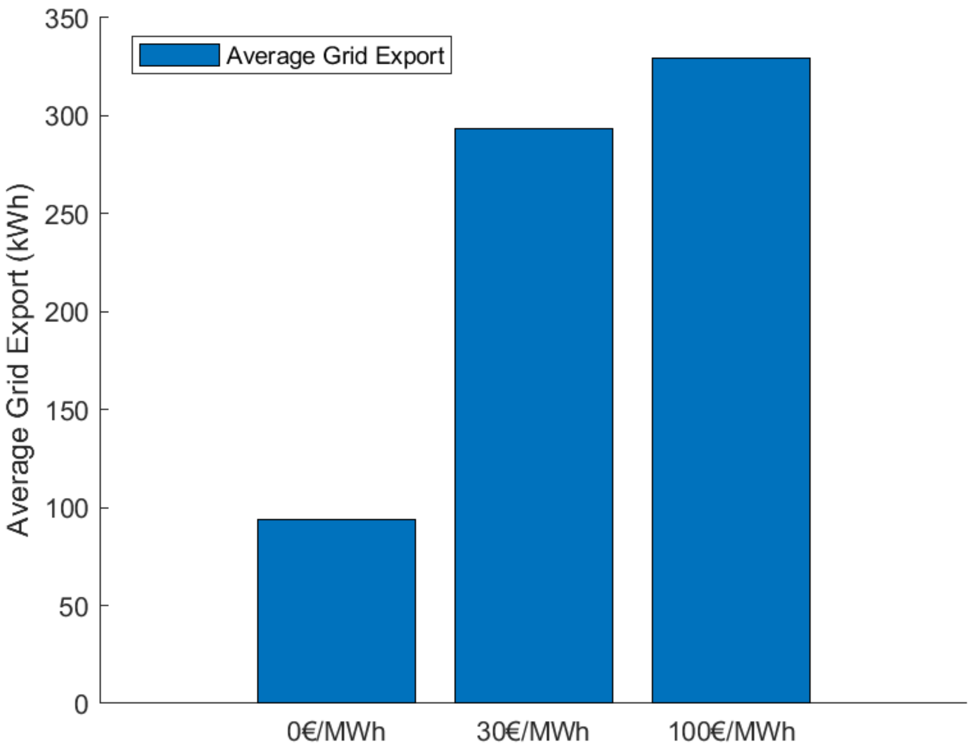

Figure 6.15.2.2.1 – Comparison of average daily energy exportation for the tested period, for MILP method, considering three different feed-in tariffs.

Average daily export increases sharply when the tariff rises from 0 to 30 €/MWh, from approximately 94 kWh/day to around 295 kWh/day. This >200% increase shows that the MILP model responds strongly to the economic incentive, exporting surplus whenever marginal revenue justifies it.

When the tariff increases to 100 €/MWh, export rises further to roughly 330 kWh/day. However, this increment is smaller than the previous one, suggesting a physical constraint: export becomes limited not by price but by the maximum available surplus.

This confirms that explicit modelling of export is essential for correctly capturing the economic impact of high-renewable communities.

6.4.2.3 - Impact of feed-in tariff on self-consumption

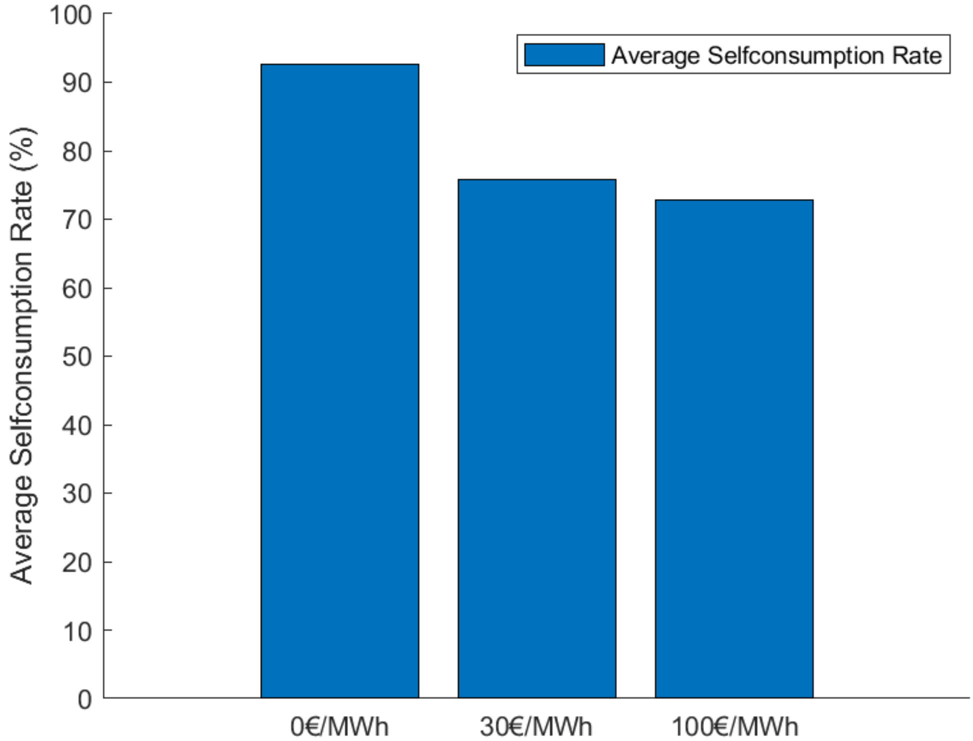

Figure 6.16.2.3.1 – Comparison of average selfconsumption rates for the tested period, for MILP method, considering three different feed-in tariffs.

Conversely, the average self-consumption rate decreases as the tariff increases:

- **~92.5%** at 0 €/MWh,
- **75.7%** at 30 €/MWh,
- **72.7%** at 100 €/MWh.

When export is not remunerated (or poorly remunerated), the model maximizes local consumption by strategically activating TCLs. However, when export becomes profitable, the model no longer prioritizes maximum self-consumption, shifting instead to global economic optimization.

6.5 - Discussion of results

The analysis carried out in this chapter provides a systematic evaluation of the impact of optimal control of thermostatically controlled loads (TCLs) on the energetic and economic performance of a renewable energy community with distributed PV generation.

The results consistently demonstrate that introducing an optimized control model yields significant improvements relative to the uncontrolled scenario. Although PV generation naturally reduces grid dependence, it is the coordination of demand that unlocks additional value, by reducing imports during high price periods or strategically managing surplus energy.

From an energy performance standpoint:

- Optimal control reduces peak importation, albeit moderately.
- Thermal flexibility acts as an effective form of short-term distributed storage.
- Thermal inertia is exploited without violating comfort constraints, validating the modelling framework.

These findings align with demand response literature, where thermal inertia is frequently interpreted as a form of virtual storage.

From an economic standpoint, both formulations, LP and MILP, generate considerable savings. However, the difference between the continuous and discretized solutions is limited, suggesting that the convex relaxation offers a close approximation to physically implementable behavior. The relaxation gap remained moderate, confirming the methodological suitability of the approach.

The PV capacity sensitivity analysis identified three distinct operating regimes (dominant absorption, intermediate operation, and structural export) and showed that the marginal value of thermal flexibility increases with renewable penetration. This aligns with studies on distributed generation integration and collective self-consumption.

The feed in tariff sensitivity analysis demonstrated that regulatory context plays a decisive role. With low tariffs, the system tends to maximize self-consumption; with high tariffs, export becomes economically dominant.

Throughout the entire time horizon analyzed, no violations of thermal constraints were observed in either the continuous or discrete solutions. This confirms the consistency of the dynamic model and its suitability for representing TCL physical behavior.

In practical terms, the developed models show potential for application in:

- Renewable energy communities with high PV penetration,
- Aggregated residential systems dominated by thermal loads,
- Local energy planning studies evaluating the economic impact of distributed generation.

The linear programming based approach with subsequent discretization also offers computational efficiency, enabling simulations over extended time horizons (daily or seasonal) at low computational cost.

In summary, the results confirm that integrating thermal flexibility with local renewable generation is an effective strategy to reduce energy costs, improve utilization efficiency, and enhance grid interaction. The coherence between empirical results and theoretical foundations in the literature supports the scientific validity and practical relevance of the model in the context of the energy transition and the decentralization of production.

Chapter 7

Conclusion

The objective of the present dissertation was to develop and evaluate an optimal control model for TCL in a REC scenario with photovoltaic production and economic signals from an energy market, following the philosophy of Fontes et al [1].

Based on the theoretical foundations' existent in the literature, a methodological extension was proposed that explicitly incorporates local renewable production and the possibility of bidirectional exchange with the grid, while simultaneously maintaining the thermal comfort of users.

The developed methodology is based on a deterministic finite horizon formulation, solved with a 15-minute discretization and daily horizon, in line with the structure of the day-ahead market. The problem was modelled as a Mixed-Integer Liner Programming (MILP) with convex relaxation of decision variables, allowing:

- Representation of realistic binary behavior of TLCs;
- Prevent simultaneous import and export;
- Correctly incorporate situations of very low or negative prices;
- Maximize the economic benefit for the community.

The structural analysis of the thermal model demonstrated that the feasibility of the problem critically depends on the relation between thermal parameters, reinforcing the importance of physical-mathematical consistency before applying optimization techniques.

7.1 - Main Results

The results obtained allowed several relevant conclusions to be drawn.

Firstly, it was verified that the optimized coordination of TCLs leads to significant reductions in total energy costs when compared to the uncontrolled scenario. Demand optimization was found to be more decisive than simply increasing installed photovoltaic capacity.

Secondly, it was confirmed that thermal flexibility acts as a form of virtual energy storage, allowing consumption to be displaced to periods of higher renewable energy production or lower market prices, without violating comfort limits.

The comparison between the LP formulation and the MILP formulation has shown that:

- Convex relaxation provides a theoretical lower bound on the optimal cost;
- Subsequent discretization introduces a moderate deviation (discretization gap);
- The economic impact of discretization is relatively reduced, validating the adopted approach.

Additionally, the sensitivity analysis to photovoltaic potency highlighted the existence of three distinct operating regimes: absorption regime, intermediate regime and structural export regime. The presence of decreasing marginal returns was observed as renewable penetration increases.

Finally, no violations of thermal restrictions were found in any of the analyzed scenarios, confirming the consistency of the thermal modelling and its computational implementation.

7.2 - Contributions of the Dissertation

The main contributions of this work can be summarized as follows:

- Explicit integration of local photovoltaic production, in a REC scenario, in the optimal control of TCLs;
- Robust MILP formulation facing negative prices and arbitrage;
- Structural analysis of the feasibility of the thermal model;
- Systematic comparison between without control operations, LP and MILP;
- Sensitivity study to renewable penetration and injection tariff.

By extending the TCL control methodology to the specific context of Renewable Energy Communities, this dissertation contributes to the comprehension of the role of thermal flexibility in the decentralized energy transition.

7.3 - Work limitations

Despite the results obtained, there are limitations that must be acknowledged:

- Perfect price forecast, production and outdoor temperature was assumed;
- The thermal model adopted is of first order;
- Additional grid costs or regulatory charges were not considered;
- Uncertainties and stochastic events were not modelled.

These simplifications are coherent with a deterministic academic approach but limit direct applicability in a real environment without additional adaptations.

7.4 - Future work

Based on the conclusions obtained, several lines of future research are identified:

- Integration of stochastic price and production forecasting models;
- Implementation of rolling horizon control;
- Inclusion of minimum up/down time restrictions;
- Integration with system service markets (aFRR, mFRR);
- Multi-objective analysis (cost vs emissions);
- Experimental testing with real measurement data.
- Optimal control with other loads (EV, by V2G methods, for example);

7.5 - Final considerations

The energy transition requires not only an increase in renewable production but also intelligent mechanisms for coordinating demand. This work demonstrates that exploiting residential thermal inertia, when combined with market signals and distributed production, can generate significant economic benefits without compromising user comfort.

The integration of distributed flexibility in Renewable Energy Communities represents a technically feasible economically relevant solution that is aligned with European decarbonization objectives.

Thus, it is concluded that the optimal coordination of thermostatically controlled loads represents an effective tool for increasing the efficiency of collective self-consumption and reinforcing the sustainability of the decentralized electricity system.

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