

Harnessing Quantum Optical States for Sensing and Imaging Applications

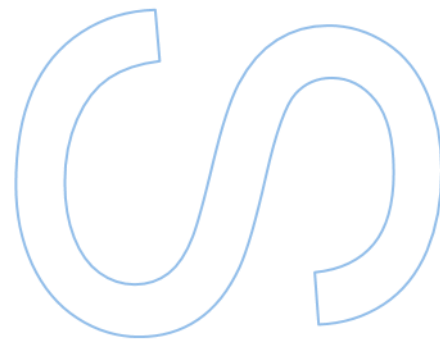
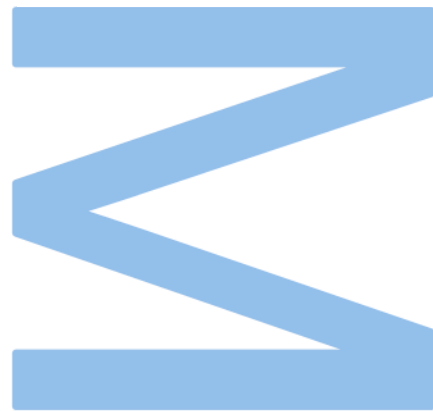
Carolina Ramos Gonçalves

Master in Physics

Department of Physics and Astronomy

Faculty of Sciences of the University of Porto

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Carolina Ramos Gonçalves

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Supervisor

Dr. Nuno Azevedo Silva, Area Manager, Faculdade de Ciências,
INESC TEC

Co-supervisor

Dr. Tiago D. Ferreira, Assistant Researcher, INESC TEC

Co-supervisor

Dr. Catarina Monteiro, Assistant Researcher, INESC TEC



“We have to remember that what we observe is not nature herself, but nature exposed to our method of questioning ”

Werner Heisenberg (1901-1976)

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Resumo

Nesta tese, exploramos os fundamentos teóricos e experimentais da ótica quântica. Partindo de uma compreensão aprofundada da estatística de fótons em diferentes estados de luz, avaliamos e medimos as respectivas assinaturas nas experiências seminais da primeira revolução quântica: os efeitos Hanbury-Brown-Twiss e Grangier-Roger-Aspect. Através da realização destas simples experiências, obtivemos os resultados esperados de $\langle g_{coh}^{(2)}(\tau) \rangle = 1.01283 \pm (5 \times 10^{-5})$ e $g_{Fock}^{(2)}(0) = 0.02 \pm 0.02$ (estes valores correspondem à média e ao ajuste Gaussiano dos respetivos dados, enquanto que os dados puros atribuem $g_{coh}^{(2)}(0) = 1.011 \pm 0.004$ e $g_{Fock}^{(2)}(0) = 0.0702183 \pm (4 \times 10^{-7})$, respetivamente), bem como uma melhor compreensão do equipamento experimental e das suas interfaces.

Após confirmar o comportamento não clássico dos estados de fóton único emitidos por *spontaneous parametric down-conversion* de tipo II, desenvolvemos e testamos um interferómetro de Hong-Ou-Mandel para aplicações de deteção. O êxito desta implementação levou-nos a aplicá-lo à imagiologia e, subsequentemente, à imagiologia birrefringente, um campo ainda pouco explorado mas promissor no domínio da imagiologia quântica baseada no efeito Hong-Ou-Mandel. Ambos os sistemas experimentais proporcionaram resultados bastante satisfatórios, alcançando uma precisão de $15 \mu m$ (a verdadeira sensibilidade está limitada pela banda espectral curta da fonte e espera-se que seja na ordem das centenas de μm) na diferença de caminho ótico e de 0.09° na diferença de polarização, com visibilidades até 73.5 %.

Embora se trate apenas de uma experiência preliminar, representa um passo significativo no avanço da investigação em ótica quântica experimental realizada internamente e que esta tecnologia poderá beneficiar diversas áreas da Ciência.

Palavras-chave: Ótica quântica, Sensorização quântica, Hanbury-Brown-Twiss, Grangier-Roger-Aspect, Hong-Ou-Mandel, Imagiologia quântica birrefringente

Abstract

In this thesis, we explore the theoretical and experimental fundamentals of quantum optics. Starting with a thorough understanding of the photon statistics of different light states, we evaluate and measure their signatures in the seminal experiments from the first quantum revolution: the Hanbury-Brown-and-Twiss and Grangier-Roger-Aspect effects. By performing these simple experiments, we could obtain the expected results of $\langle g_{coh}^{(2)}(\tau) \rangle = 1.01283 \pm (5 \times 10^{-5})$ and $g_{Fock}^{(2)}(0) = 0.02 \pm 0.02$ (these values correspond to the average and Gaussian fit os the respective data, while the raw data accuse $g_{coh}^{(2)}(0) = 1.011 \pm 0.004$ and $g_{Fock}^{(2)}(0) = 0.0702183 \pm (4 \times 10^{-7})$, respectively), as well as a better grasp at the experimental setup and interfaces.

After confirming the non-classical behaviour of single-photon states emitted through type-II spontaneous parametric down-conversion, we developed and tested a Hong-Ou-Mandel interferometer for sensing. The success of this implementation led us to apply it to imaging and then to birefringent imaging, a rather untapped but promising field in Hong-Ou-Mandel-based quantum imaging. Both setups allowed us very satisfactory results, achieving a precision of about $15 \mu m$ (true sensitivity is limited by the narrowband source and is expected to be in the order of hundreds of μm) in optical path difference and 0.09° in polarization difference and visibilities up to 73.5 %.

Although this is just a preliminary experiment, we believe it represents a huge step forward in in-house experimental quantum optical research and that this technology could benefit many areas of Science.

Keywords: Quantum optics, Quantum sensing, Hanbury-Brown-and-Twiss, Grangier-Roger-Aspect, Hong-Ou-Mandel, Birefringence quantum imaging

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List of Abbreviations

- BS** beam-splitter. viii, 18– 23, 26, 31, 32, 34, 35, 38– 40, 49
- CCD** charge-coupled device. 39, 40, 53, 56, 59, 60
- EM** electromagnetic. 3, 18, 19, 21, 42
- EMCCD** electron-multiplying charge-coupled device. 5
- FI** Fisher Information. v, viii, 48, 51, 52, 57, 62, 64
- FWHM** full width at half maximum. 29, 40, 44– 46
- GRA** Grangier-Roger-Aspect. v, viii, 13, 16, 22– 24, 26, 28– 30, 39, 63, 64
- HBT** Hanbury-Brown-and-Twiss. v, viii, 2, 13, 16– 18, 22, 24, 25, 28– 30, 63
- HOM** Hong-Ou-Mandel. v, viii, ix, 13, 26, 29– 35, 37– 51, 54– 60, 62– 64
- IR** infrared. 10, 12
- LED** light-emitting diode. 53, 59
- MLE** Maximum Likelihood Estimator. v, 48, 52, 62
- OCT** Optical Coherence Tomography. viii, 33, 34
- PBS** polarizing beam-splitter. 39, 40, 59
- QOCT** Quantum Optical Coherence Tomography. viii, 33, 34
- QPM** Quasi-Phase-Matching. viii, 7– 12
- SHG** Second Harmonic Generation. 5, 6
- SNR** Signal-to-Noise Ratio. 5, 47, 64
- SPAD** single-photon avalanche diode. 5, 16
- SPDC** Spontaneous Parametric Down-Conversion. v, vii, viii, 6– 13, 22, 24– 27, 40
- UV** ultraviolet. 10, 12

1. Introduction

For centuries, physicists have sought to understand the fundamental building blocks of nature, from the motion of celestial bodies to the behavior of atoms and light. Yet, time and again, reality has challenged their best ideas, and one of the most dramatic recent chapters in that ongoing reckoning was written in the 20th century with the advent of quantum mechanics, a theory that not only reshaped our understanding of nature but also laid the foundation for technologies such as semiconductors and lasers.

One hundred years later, what began as an exploration of puzzling atomic phenomena closer to fundamental physics has now evolved into the modern research field of quantum technologies, where quantum properties are harnessed to design new technology. Yet, bridging to real-world applications, the key question of quantum technologies is whether there is some advantage - the so called *quantum advantage* - that outshines a classical setup. More clearly speaking, its main research goal is to access the non-trivial, unique properties of quantum systems such as entanglement, superposition, and quantum coherence, in order to retrieve or process information in ways inaccessible when using classical methods (e.g. due to noise). These quantum advantages have been explored in multiple directions in the last decades, and can be organized into the four pillars of quantum technologies:

- **Quantum Computing:** harnesses quantum superposition and entanglement to solve problems exponentially faster than classical computers in specific tasks [1];
- **Quantum Communications:** ensures ultra-secure data transmission using quantum cryptographic protocols [2];
- **Quantum Simulation:** utilize systems following similar physical models to emulate complex quantum systems that are intractable with classical simulations [3, 4];
- **Quantum Sensing:** seeks to utilize quantum properties to enhance measurement precision beyond classical limits, enabling breakthroughs in metrology, navigation, and imaging [5].

Although these grand themes seem all very distinct, it is important to mention that they greatly contribute and influence each other, something we are used to witnessing in our

journey in learning Physics. But for the context of this thesis, we will focus on *quantum sensing* and, more specifically, on the field of *quantum imaging*, aiming to set a first stepping-stone for this new research line at the Center for Applied Photonics at INESC TEC.

At its core, quantum sensing is the art of preparing a well-defined quantum state, letting it interact with a target stimulus of the environment, and then reading out the tiny, state-dependent change with minimal added noise. It thus strongly hinges on creating, manipulating, and measuring a quantum state, which, in turn, is associated with a given quantum system. Although many different quantum systems can be utilized for various purposes and specifications, such as electrons [6], quantum dots [7, 8], nitrogen-vacancy centers [9], multilevel atomic systems [10], Bose-Einstein condensates [11], and topological states [12, 13], the non-classical states of light offer a versatile combination of room temperature operation, matured control techniques, and highly efficient detection. These qualities make photonics the natural key enabling technology for the work presented here. But before we can leverage those advantages, we start with a foundational question: what exactly makes a beam of light non-classical, and why are they so interesting in the context of quantum-enhanced sensing?

1.1. Quantum Description of Light and Non-classical States

Broadly speaking, we call a photonic state *non-classical* when its statistical properties cannot be reproduced by any classical mixture of coherent states. A widely used operational criterion is the presence of **sub-Poissonian photon statistics** (photon number distribution for which the variance is lower than the mean), which relates to the concept of photon antibunching, also referred to as negative Hanbury-Brown-and-Twiss (HBT) effect or anti-correlation effect.

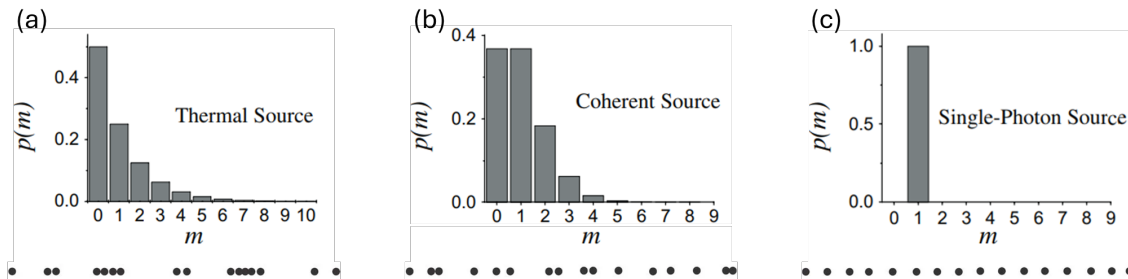


Figure 1.1: Types of Light: probability distribution of the number of photons for three sources with an average photon number $\langle n \rangle = 1$ and representation of their temporal distribution. (Adapted from [14])

This criterion is further illustrated in Fig. 1.1. Thermal or chaotic light exhibits strong photon number fluctuations due to the Bose–Einstein statistics of black-body radiation. This translates into a super-Poissonian photon distribution where photons tend to bunch. By contrast, coherent states of light follow a Poissonian photon number distribution, typically narrower than that of thermal light. For small average photon numbers, this distribution is discrete and asymmetric, but for large photon numbers it approaches a Gaussian distribution with the same mean and variance. Although the fluctuations are narrower than those of thermal light, they are still present and are known as photon noise. In this case, photons are emitted with random spacing. Then, we have single-photon sources that emit photons with a well-defined constant mode at approximately-regular time intervals, meaning the photons are separated by approximately regular time intervals (antibunching). This corresponds to a sub-Poissonian distribution with reduced fluctuations compared to the thermal and coherent states.

To make the above statements more precise, we can proceed by introducing a minimal quantum description for the electromagnetic field and how it connects to these statistics. Light is an electromagnetic (EM) wave described within classical electrodynamics by the vector potential $\mathbf{A}(\mathbf{r}, t)$ associated with the corresponding electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. Following a second-quantization methodology, we may promote the field to an operator by expanding the vector potential in plane-wave modes and introducing bosonic annihilation and creation operators $\hat{a}_{\mathbf{k},\lambda}$ and $\hat{a}_{\mathbf{k},\lambda}^\dagger$ over all propagation vectors ($\mathbf{k}$) and polarizations (λ) of light:

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k},\lambda} \sqrt{\frac{\hbar}{2\varepsilon_0\omega_{\mathbf{k}}V}} \left(\hat{a}_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} \mathbf{e}_{\mathbf{k},\lambda} + \hat{a}_{\mathbf{k},\lambda}^\dagger e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} \mathbf{e}_{\mathbf{k},\lambda}^* \right). \quad (1.1)$$

Working in the Coulomb gauge for simplicity ($\nabla \cdot \mathbf{A} = 0$ and assuming $\phi = 0$), we can proceed and write the equations for the electric and magnetic fields

$$\mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{A}}{\partial t} = i \sum_{\mathbf{k},\lambda} \sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2\varepsilon_0V}} \left(\hat{a}_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} - \hat{a}_{\mathbf{k},\lambda}^\dagger e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} \right) \mathbf{e}_{\mathbf{k},\lambda} \quad (1.2)$$

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A} = \sum_{\mathbf{k},\lambda} \sqrt{\frac{\hbar}{2\varepsilon_0\omega_{\mathbf{k}}V}} \left(\hat{a}_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} + \hat{a}_{\mathbf{k},\lambda}^\dagger e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)} \right) (\hat{\mathbf{k}} \times \mathbf{e}_{\mathbf{k},\lambda}). \quad (1.3)$$

and further compute the electromagnetic (EM) Hamiltonian

$$\hat{H} = \frac{1}{2} \int_V \left(\varepsilon_0 |\mathbf{E}|^2(\mathbf{r}, t) + \frac{1}{\mu_0} |\mathbf{B}|^2(\mathbf{r}, t) \right) d^3r = \sum_{\mathbf{k},\lambda} \hbar\omega_{\mathbf{k}} \left(\hat{a}_{\mathbf{k},\lambda}^\dagger \hat{a}_{\mathbf{k},\lambda} + \frac{1}{2} \right). \quad (1.4)$$

which results in a simple sum of harmonic oscillators over all propagation vectors and polarizations.

From this quantized description one can now construct the familiar classes of quantum states that exhibit the statistics shown in Fig. 1.1. Number (or Fock) states $|n\rangle$ are eigenstates of the number operator $\hat{n} = \hat{a}^\dagger \hat{a}$ (dropping the spectral and polarization index for simplicity), hence the single-mode Hamiltonian: $\hat{n} |n\rangle = n |n\rangle$. A single-photon state $|1\rangle$ is thus a pure state with a sharply defined photon number, i.e. a sub-Poissonian distribution. Coherent states $|\alpha\rangle$ are defined as eigenstates of the annihilation operator, $\hat{a} |\alpha\rangle = \alpha |\alpha\rangle$, and are the nearest quantum analogue to a classical monochromatic field (i.e. a laser). It is straightforward to demonstrate that the photon-number statistics are Poissonian,

$$P_{\text{coh}}(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}, \quad \bar{n} = |\alpha|^2, \quad (\Delta n)^2 = |\alpha|^2, \quad (1.5)$$

which for large mean photon number is approximated by a Gaussian with the same mean and variance, which is a direct consequence of the central limit theorem. Thermal or chaotic single-mode light is described by a Bose-Einstein probability distribution,

$$P_{\text{th}}(n) = \frac{\bar{n}^n}{(1 + \bar{n})^{n+1}}, \quad (1.6)$$

with $(\Delta n)^2 = \bar{n}(\bar{n} + 1) > \bar{n}$, exhibits super-Poissonian fluctuations and a tendency for detection events to cluster in time (bunching).

Still, for many quantum-technology applications, the raw photon-number statistics of non-classical states are secondary to a set of coherence and state-engineering properties that these allow. In particular, the ability to prepare and control coherent *superpositions* and to generate *entanglement* between modes or systems is far more important for tasks such as quantum communication, computing, and sensing.

In the quantum picture, superposition simply denotes a state that is composed of a linear combination of pure states, defining a given probability of the particle being in each particular state. Given the probabilistic nature of quantum states, particles described by a superposition of states have an uncertain physical quantity (such as position and momentum) until a measurement is made, which collapses the state.

In addition, when we have a multiple-variable system - for example, many particles or particle-field coupling - we can divide the class of superposition states into two categories: separable and entangled states. As the name suggests, separable states are used to describe de-coupled variables that allow us to write a given state and Hilbert space as a

tensor product of each individual variable:

$$\begin{aligned} |\Psi\rangle &= |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \dots \\ H &= H_1 \otimes H_2 \otimes \dots \end{aligned} \tag{1.7}$$

On the contrary, entangled states, are not separable, meaning that our system is deeply coupled. This is a unique quantum property, which enables obtaining information about one part of the system by measuring another. With this and other peculiar traits, entangled states violate Bell inequalities [15] and became very useful in areas such as quantum computing and cryptography [16].

These properties are crucial to quantum technologies and protocols, as they contribute to a higher correlation (a concept that shall be better explored in the next chapter) and thus may enhance resolution, precision and Signal-to-Noise Ratio (SNR) in imaging [17, 18], for example.

1.2. Production and Detection of Single-Photon States

So far, we have introduced single-photon states as a promising avenue for quantum technologies due to their non-classical properties. Yet, when bridging to actual quantum technologies, these theoretical entities present two experimental obstacles: producing and detecting them.

One might think that, by attenuating a laser to the point of having individual photons, we would have a light source inherently quantized. Yet, this is one of the most common misconceptions of experimental quantum optics, as the underlying statistics still make this a coherent state [19]. To circumvent coherent statistics, one could make use of fluorescent emission or solid-state emitters to produce single photons, as it has been proven to be able to produce non-classical light [20–22]*. This leads however to the second challenge: detection.

Technologically speaking, detecting a photon is not dramatically challenging: a single-photon avalanche diode (SPAD), a photomultiplier tube, or an electron-multiplying charge-coupled device (EMCCD) has long proven the capacity of detecting a lone photon efficiently. Yet, their high sensitivity also results in the detection of photons unrelated to our

*It has also been proved that we can turn a coherent or thermal source into a non-classical source by making the light go through any non-linear process whose rate is enhanced by the bunching of photons in one of the beams depleted by the phenomenon, such as two-photon absorption and Second Harmonic Generation (SHG) [23]. Furthermore, it was shown that two-level systems are able to produce single-photon states, provided that they have the right conditions (decoherence, filtering, strong coupling, resonant excitation, correlation, *etc* may surrender the emitter useless in terms of non-classical behavior) [24].

source, as well as "dark counts" (e.g. avalanche events without an external photon triggering the detector in a SPAD) [19]. Thus, the question shifts from "how can we detect a photon" to "how can we detect our photon and our photon only, distinguishing it from everything else?". In practice, it is impossible to filter out all noise and so, in order to overcome this issue, we need to work on the state production side, introducing the concept of photon-pair production via Spontaneous Parametric Down-Conversion (SPDC).

1.3. Spontaneous Parametric Down-Conversion (SPDC)

SPDC, illustrated in Fig. 1.2, is a non-linear optical effect in which a high-energy photon (pump) enters a transparent, non-centrosymmetric, non-linear crystal and splits into two other photons with lower energies (traditionally called signal and idler) [25, 26]. One can think of it as the reverse process to SHG. Unlike SHG, however, it cannot be explained by classical fields and we need to describe it with quantum mechanics [27]. This effect allows to create entanglement on several properties of the photons: polarization [28], angular orbital momentum [29], momentum [30] or energy [31].

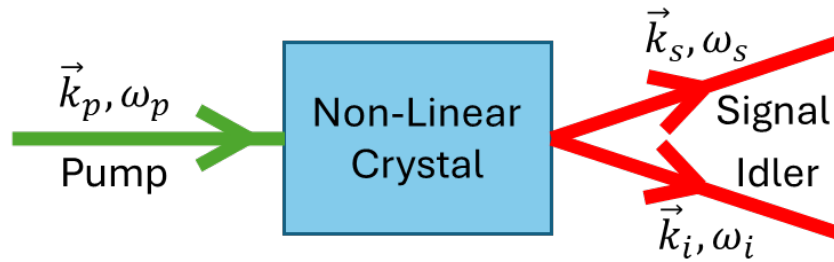


Figure 1.2: Simplified scheme of the SPDC process.

The name of this process is also very telling of its fundamentals [27, 32]:

- **spontaneous** (as opposed to stimulated): the photon pair is created without the presence of an external EM field at the target wavelength;
- **parametric** (related to the parametric oscillator model): can be described by a real second-order susceptibility tensor $\chi^{(2)}$; energy and momentum are conserved;
- **down-conversion**: the resulting photons have a lower energy than the incoming photon.

Since we are looking at a parametric process, it conserves both energy and momentum, meaning the generated photons can leave the crystal at different small angles (making it

easier to spatially separate the two beams of correlated photon pairs):

$$E_p = E_s + E_i \iff \omega_p = \omega_s + \omega_i \quad (1.8)$$

$$\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i + \mathbf{G}, \text{ where } |\mathbf{k}_j| = n_j \omega_j / c. \quad (1.9)$$

The vectors $\mathbf{k}_j$ and the scalars n_j are the wavevectors and the refractive indices (considering the wavelength and polarization), respectively, inside the crystal for each of the photons and $\mathbf{G}$ is the reciprocal lattice vector. Regarding the frequencies (ω_j) of the down-converted photons (Eq. 1.8), they are not necessarily equal (degenerate SPDC). In fact, this is typically more challenging to achieve due to the dispersion of the crystal.

Although the vector $\mathbf{G}$ is usually assumed to be zero in theoretical analysis, it plays a significant role in practice by contributing to phase-matching. Experimentally, the phase-matching is achieved by modifying the refractive indices that affect the photons, which can be controlled by crystal orientation, cutting, or temperature tuning, as well as by engineering the reciprocal lattice vector through Quasi-Phase-Matching (QPM).

Each phase-matching technique has its pros and cons so it is hard to say one is better than the other; that is decided case-by-case, depending on the crystal, target and pump wavelength and goal of the experiment:

- **Angular tuning** is very accurate and stable but it is not always physically possible to achieve, since it is limited by the Snell-Descartes law and the birefringence of the crystal, and creates some walk-off. This phenomenon occurs when the angle between the propagation direction and the optical axis of the crystal has a value other than 0° or 90° . This causes the Poynting vector and the propagation vector for extraordinary rays not to be parallel and, as a result, ordinary and extraordinary rays with parallel propagation vectors quickly diverge from one another as they travel through the crystal. This walk-off effect limits the spatial overlap of the waves and decreases the efficiency of any non-linear mixing process, including SPDC [32].
- In the case of **temperature tuning**, a very precise temperature control is required and is limited to crystals where the amount of birefringence is strongly temperature-dependent [33].
- In **QPM** [34], the crystal is replaced by a periodically poled material in which the positive c axis alternates in orientation with a period $\Lambda \propto L_{coh}$ [32]. For imperfect phase-matching (wavevector mismatch), the amplitude of the product of the non-linear

mixing process follows a sinusoidal function along its propagation in the crystal so, alternating the orientation of its axis at a specific distance, will force the amplitude to increase when it would decrease (vertical dotted lines in Fig. 1.3). This process has very striking advantages, namely its flexibility and tunability, high-efficiency (when using longer crystals), the possibility to use isotropic or weakly birefringent materials and the ability to achieve collinear and degenerate SPDC if needed, among others [32].

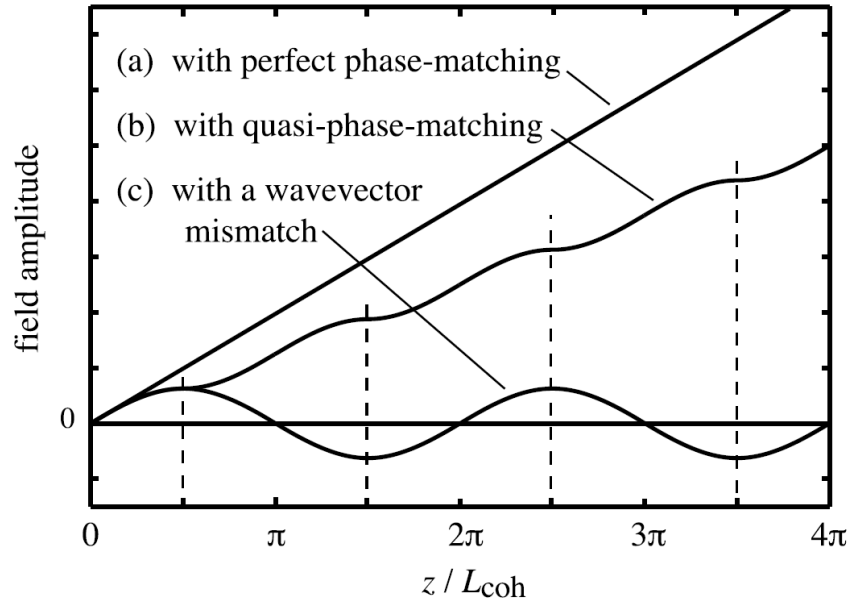


Figure 1.3: Comparison of the spatial variation of the field amplitude of the generated wave in a non-linear optical interaction for three different phase matching conditions (perfect phase-matching, QPM and wavevector mismatch) [32].

It is worth noting that, in some cases, it might be advantageous to combine different types of tuning in order to achieve a more precise process [35].

Besides the types of phase-matching, there are three types of SPDC processes, which mainly differ on the polarization differences between the three photons involved, and the crystals used but, consequently, each create down-converted photons with some interesting properties.

1.3.1 Type-I

In type-I crystals, the polarizations of the signal and idler photons are parallel (both orthogonal to the pump's) and determined by crystal orientation [36]. In terms of phase-matching, we can achieve it without resorting to quasi-phase-matching, meaning that we may take $G = 0$ and calculate (usually resorting to numerical approaches since it is a

transcendental equation) the angular separation of the down-converted photons by combining the previous equations. Assuming that the pump photons are polarized along the extraordinary axis and the idler and signal's along the ordinary:

$$n_p^e \omega_p = n_s^o \omega_s \cos \theta_s + n_i^o (\omega_p - \omega_s) \sqrt{1 - \left(\frac{n_s^o \omega_s}{n_i^o (\omega_p - \omega_s)} \sin \theta_s \right)^2} \quad (1.10)$$

$$\text{degenerate case } (\omega_s = \omega_i = \omega_p/2 \text{ and } \theta_i = \theta_s^*) : \cos \theta_s = \frac{n_p^e}{n_s^o} \quad (1.11)$$

This (typically) non-zero angle and rotational symmetry of the phase-matching condition around the pump beam result in a conical emission pattern, as seen in the last inset of Fig. 1.4. From the previous equations, we can see that, for some crystals, it is possible to achieve collinear type-I SPDC [37] but, due to its easier tunability, it is usually required to resort to QPM (specially if the photons are not degenerate in frequency) [38].

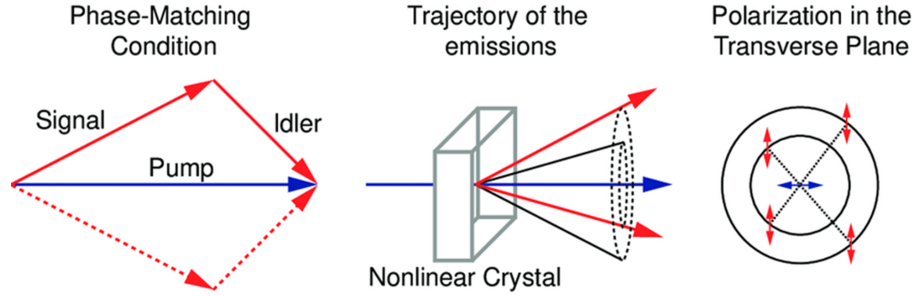


Figure 1.4: Fundamentals of Type-I SPDC [39].

The non-collinear type-I SPDC geometry offers a key experimental advantage: since the down-converted photons emerge at angles relative to the pump direction, they can be spatially separated from the intense pump beam, reducing background noise and simplifying optical filtering. It also allows the use of spatially correlated but well-separated beam paths, which is beneficial for coincidence detection and coupling into separate optical fibers or detectors [40].

By tuning the phase-matching and achieving degenerate emission we significantly enhance the indistinguishability of the photon pair, which is a crucial requirement for quantum interference, entanglement swapping, and linear optical quantum computing. When photons are indistinguishable in all degrees of freedom except the one being used for encoding (such as path or time-bin), they are more robustly entangled and their interference is stronger [41].

*This is only possible because they have the same polarization and thus the refractive indices are the same. This relation can be easily obtained from the conservation of momentum along the axis perpendicular to the pump's propagation vector.

In terms of materials, type-I SPDC commonly uses a wide variety of non-linear crystals (polled, in bulk and in waveguides) such as β -barium borate (BaB_2O_4 - BBO), lithium triborate (LiB_3O_5 - LBO), potassium dideuterium phosphate (KH_2PO_4 - KDP) and periodically-polled lithium niobate ($LiNbO_3$ - PPLN), each offering distinct advantages. BBO is favored for its high nonlinearity and broad transparency range (190 – 3300 nm), making it ideal for ultraviolet (UV) and visible SPDC, though it suffers from spatial walk-off due to birefringence [42]. LBO provides excellent thermal stability, low walk-off, and a high damage threshold, suitable for high-power applications [43]. KDP is inexpensive and available in large sizes with deep UV transparency, but has lower nonlinearity and is highly hygroscopic [44]. PPLN stands out for its extremely high effective nonlinearity via QPM, broad tunability from visible to mid- infrared (IR), and compatibility with integrated photonics, making it ideal for narrowband and telecom-wavelength entangled photon sources [45].

1.3.2 Type-II

In contrast with the previous process, type-II SPDC splits the pump photon into orthogonal polarizations signal and idler photons. Similarly, we can take $G = 0$ and deduce the angular separation to the pump propagation which takes the same form as Eq. 1.10, except we must change the polarization axis of only one of the down-converted photons due to their orthogonal polarizations.

However, even for the degenerate case, we can no longer easily obtain $\theta_i = \theta_s$ [25, 46] and the emission pattern forms two non-collinear, intersecting cones, usually with different apertures [47] (last inset of Fig. 1.5). The most interesting part of this process lies in the intersections of these cones, as these points play a crucial role in quantum optics since they correspond to a balanced superposition of two orthogonal polarizations, i.e. a maximally entangled, Bell state (most commonly, $|\Psi^-\rangle = 1/\sqrt{2}(|H, V\rangle - |V, H\rangle)$, which is especially useful for quantum teleportation, Bell inequality tests and quantum key distribution) [31, 36]. The fact that the down-converted photons are emitted entangled in their polarization states makes type-II SPDC very useful, for example, for representing qubits in quantum information [48, 49]; polarization entanglement is also possible with type-I SPDC but requires two crystals or interferometric processes [50]. However, if we generate the down-converted photon through QPM, we are able to observe $\theta_i = \theta_s$ and even collinear emission with the pump [38].

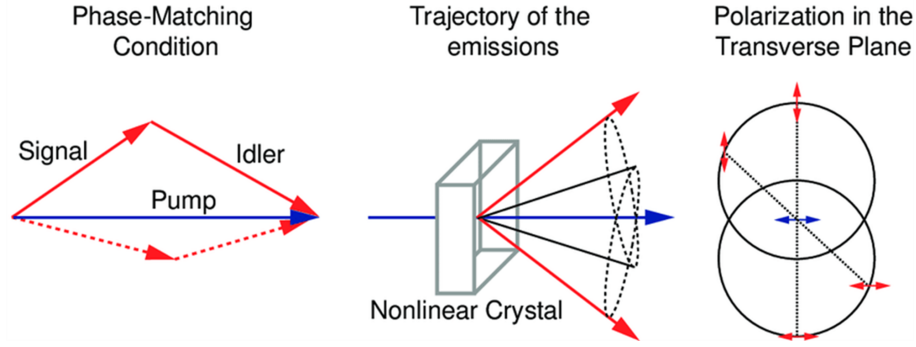


Figure 1.5: Fundamentals of Type-II SPDC [39].

The main disadvantage of this process is the need for careful compensation for timing and spatial overlap due to birefringence, which complicates significantly the alignment of the source [25, 51]. Usually experiments include compensation crystals or delay lines to restore indistinguishability.

Type-II SPDC commonly uses non-linear crystals such as BBO, periodic-poled potassium titanyl phosphate ($KTiOPO_4$ - PPKTP) and lithium niobate, both in bulk and periodical-poled (LN and PPLN). BBO presents the same qualities and demerits as it does for type-I, being widely favored for its broad transparency range, relatively high nonlinear coefficient, and ease of phase matching in the visible range [52]. However, it still exhibits significant spatial and temporal walk-off due to birefringence, requiring compensation [25, 51]. PPKTP, on the other hand, enables QPM with low walk-off, making it suitable for fiber coupling and narrowband entanglement, especially at longer wavelengths (e.g. for telecommunications) [53]. LN is useful in waveguide configurations for integrated photonic platforms, offering high non-linear efficiency and compatibility with electro-optic modulation [54]. The choice of material depends on experimental requirements, such as wavelength, entanglement quality and integration.

1.3.3 Type-0

In type-0 SPDC, all photons involved have the same polarization. If we aim for collinear emission, we obtain that either $n_p = n_i = n_s$ (which is very experimentally challenging, since traditional birefringence-based phase-matching is not possible) or $G \neq 0$ (which is achievable through periodic poling as a QPM mechanism [32]), as depicted in Fig. 1.6.

Although we are not going to focus on this process, QPM in type-0 SPDC is more flexible to tune, allowing for more efficient photon conversion, access to a vaster range of output

wavelengths (both deriving from a larger d_{33} non-linear coefficient), non-critical phase-matching (avoiding spatial walk-off) and collinear, room-temperature emission [55].

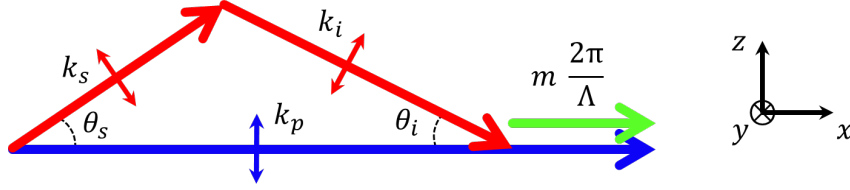


Figure 1.6: Fundamentals of (non-collinear) Type-0 SPDC [55].

Regarding materials, QPM devices based on PPLN guarantee a high nonlinearity but are not suitable for the generation of photons in the visible and near- IR spectrum, due to the low absorption edge and optical damages for blue to UV laser pumping [34]. To tackle this problem, periodic-poled $KTiOPO_4$ (PPKTP) crystals are used, which have a higher absorption edge and damage threshold to UV light, even if its nonlinearity is lower [56].

The following table summarizes the three SPDC types discussed throughout this section.

Table 1.1: Summary of SPDC Types described in the main text.

Feature	Type-0	Type-I	Type-II
Polarization	All photons have the same polarization (either o or e)	Pump has different polarization than signal/ idler	Signal and idler photons have orthogonal polarizations
Phase-Matching Conditions	n_p^e, n_s^e, n_i^e or n_p^o, n_s^o, n_i^o	n_p^e, n_s^o, n_i^o or n_p^o, n_s^e, n_i^e	n_p^e, n_s^o, n_i^e or n_p^o, n_s^e, n_i^o
Crystals used	PPLN, PPKTP, PPLT	BBO, LBO, KDP, PPLN	BBO, PPKTP, PPLN
Photon Output	Signal and idler have the same polarization, can be collinear	Signal and idler have the same polarization, orthogonal to the pump, form collinear light cones	Signal and idler have orthogonal polarizations, entangled where the non-collinear cones overlap
Advantages	High conversion efficiency, flexible tuning	Simplifies separation of pump from signal/ idler	Generates polarization-entangled photon pairs
Disadvantages	Requires periodically poled crystals, limited materials	Less suitable for polarization-entangled photons	Complex alignment, lower efficiency than Type-0

1.4. Objectives and Thesis Structure

Having introduced the fundamentals of quantum technologies and motivation for this work, as well as a brief overview of non-classical states of light and SPDC, the remainder of this thesis is structured to translate those concepts into concrete experimental advances in quantum sensing and imaging. The work is organized around three objectives, each developed in a dedicated section and supported by both pioneering and state-of-the-art references:

- **Establish a solid basis on single photon experiments:** as previously referred in the motivation, this dissertation aims to be a stepping-stone for the research line of quantum sensing at INESC TEC. Therefore, without previous experimental work, substantial effort was put into implementing, understanding, and documenting major experiments on the field of quantum sensing with single-photon states. Chapter 2 describes this endeavor, revisiting the seminal HBT and Grangier-Roger-Aspect (GRA) experiments with a quick historical overview of the theory of second-order correlation along with measurements for our SPDC and He-Ne sources. We then extend the discussion to heralded single-photon states, detailing the apparatus modifications required for anti-bunching measurements and benchmarking our detectors. Together, these experiments provide the experimental standards (timing resolution, source brightness, and computational interface) needed for the more demanding interference studies that follow.
- **Experimental observation of Hong-Ou-Mandel (HOM) effect:** the main goal of this thesis focuses on the use of HOM effect to develop a quantum sensing platform. In chapter 3, we introduce the HOM interference in a Mach–Zehnder geometry, beginning with the ideal case of equal-length paths and progressing to unequal-arm configurations used for sensing. Our measurements reproduce the canonical HOM dip and extract the spectral bandwidth of the SPDC photons along with other properties of the source and samples, thereby closing the gap with current state-of-the-art demonstrations [57]. This chapter validates both the single-photon source and the two-photon interference scheme.
- **Deploy a HOM platform for quantum imaging:** the final objective encloses the scientific novelty of this thesis, which concerns the extension of HOM interferometry to quantum microscopy setups, and in particular, to image birefringent materials. In Chapter 4 we describe the implementation and characterization of a HOM quantum microscopy platform by adding a motorized 2D stage to enable point-by-point scanning of a sample placed in one arm of the interferometer. In particular, we demonstrate quantum imaging of samples that vary in thickness and birefringence and discuss optimization strategies.

The class of birefringent and thickness-varying samples investigated here represents a significant step toward a practical and unexplored field in quantum imaging technology:

they both test the sensitivity of two-photon indistinguishability to physically relevant perturbations and serve as a realistic target for a quantum microscopy implementation. Besides, and looking forward beyond birefringence and thickness, the platform can be adapted to reveal other mechanisms that degrade photon indistinguishability - for example, changes in spatial mode, group-velocity dispersion, or spectral structure - which can also enable further proof-of-principle experiments to applied sensing.

1.5. Major Outputs and Contributions

In addition to the current document, this MSc dissertation has led to various communications and outputs in close collaboration with the QUANTOS team at the Center for Applied Photonics at INESC TEC. These include:

- Manuscripts:
 - [Preprint] C. Gonçalves, T. D. Ferreira, C. S. Monteiro, and N. A. Silva, “Quantum imaging of birefringent samples using Hong-Ou-Mandel interference,” arXiv preprint arXiv:2512.19637, 2025 [58]
- Communications
 - C. R. Gonçalves, T. D. Ferreira and N. A. Silva, “Exploring Hong-Ou-Mandel Microscopy for Birefringent Media”. Poster presentation on the Quantum Day @PT conference
 - C. R. Gonçalves, T. D. Ferreira and N. A. Silva, “Towards Hong-Ou-Mandel Microscopy for Birefringent Materials”. Oral presentation on the University of Porto’s Young Research Meeting (IJUP)
 - C. R. Gonçalves, T. D. Ferreira and N. A. Silva, “Towards Hong-Ou-Mandel Microscopy for Birefringent Materials”. Pitch presentation on the UP International Day of Light 2025 conference
 - C. R. Gonçalves, T. D. Ferreira and N. A. Silva, “Polarization-Sensitive Hong-Ou-Mandel Microscopy for Quantum Imaging Applications”. Oral presentation on the 9th Spanish & Portuguese Advanced Optical Microscopy Meeting (SPAOM)
- Conference Proceedings
 - T. D. Ferreira, C. Monteiro, C. Gonçalves, O. Frazão, and N. A. Silva, “Enhancing a polarimetric fiber sensor using Fisher information,” in 29th International

Conference on Optical Fiber Sensors, vol. 13639. SPIE, 2025, pp. 90–93.
DOI: 10.1117/12.3062938 [59]

- Distinctions
 - Best poster award on the Quantum Day @PT conference
 - Best pitch presentation on the UP International Day of Light 2025 conference

2. Probing the Nature of Light

The first objective of this work was to become thoroughly acquainted with the experimental equipment and methodologies necessary for quantum imaging. Ensuring proficiency with single-photon detection techniques, coincidence measurements, and time-resolved data acquisition was a prerequisite for the successful realization of more advanced quantum optical experiments, particularly those involving photon interference and entanglement. For this purpose, we chose to reproduce two cornerstone experiments in quantum optics: the HBT and GRA experiments.

Although neither of these experiments was originally designed as a tool for the characterization of light sources, they provide a powerful and widely accepted method for evaluating the statistical nature of optical fields through the second-order correlation function, $g^{(2)}(\tau)$. These measurements enabled us to distinguish between classical and non-classical states of light by observing photon bunching and antibunching phenomena. As will be discussed in Sections 2.1 and 2.2, these behaviors are indicative of thermal, coherent or single-photon states - related with the statistical signatures of classical/non-classical light states introduced in Section 1.1 - and serve as critical benchmarks in assessing the quantum properties of a light source.

Besides, reproducing the HBT and GRA experiments also served additional technical purposes that will be further utilized later for the development and deployment of our quantum imaging solution. On one hand, they offered direct and accessible means to validate the performance of our detection apparatus, particularly the SPAD, coincidence logic, and time-tagging module. On the other hand, these experiments provided a controlled framework to develop and test our own software for measurement automation and data analysis. Through this process, we established a robust experimental pipeline, ensuring that future measurements, especially those requiring precise timing, statistical fidelity, and high stability, could be carried out with confidence.

2.1. Hanbury-Brown-and-Twiss (HBT) Effect

Shortly after the first clear demonstration of electron interference by Clinton Davisson and Lester Germer, in 1927, via electron diffraction (providing experimental confirmation of the wave-particle duality proposed by de Broglie [27]), it was observed that both quantum mechanics and classical wave theories, such as classical electromagnetism (as previously

observed in Young's double slit experiment in 1801), could predict similar interference patterns [60]. However, it was soon recognized that such *first-order* interference experiments, which involve only the amplitudes of the wavefunctions and their interference, are not necessarily linked with underlying quantum features. To probe the intrinsic quantum nature of light and matter, it became necessary to explore additional photon statistical signatures related to intensity correlations and fluctuations in detection events, including the shot noise or quantum noise in photon arrival statistics [61].

Robert Hanbury Brown and Richard Twiss showed that light from a thermal (chaotic) extended source exhibits enhanced intensity correlations within the temporal coherence time set by the bandwidth of observation. This idea came to its full potential as a second-order interferometer, originally proposed as an astronomical tool for measuring the size of a star. Indeed, considering the star as a sum of thermal emitters, this creates a random intensity pattern that flickers within the coherence time. By using two telescopes separated by a distance that can be varied, we can probe the intensity correlations of this random pattern. In particular, if the telescopes are spatially close enough, we are effectively looking at the same part of the random pattern, meaning that both detectors will measure the same variations in time and so the correlation will increase. By increasing the distance between the two telescopes and compensating for the introduced time delay, we start to measure different parts of the random pattern, and the extra intensity correlations will naturally decrease. Thus, by fitting how the excess correlation falls with baseline, we are able to measure the angular size of the star under observation.

While the HBT experiment provided an improved technique to measure stellar angular sizes, it also showed that intensity correlations can probe phenomena beyond a simple wave-or-particle interpretation. Despite initial controversy over the quantum interpretation of photon “bunching”, the HBT experiment laid the foundation for modern quantum optics by revealing the inadequacy of classical wave theories in describing intensity correlations. It demonstrated the need for a statistical quantum description of light, highlighting the subtle interplay between its wave-like and particle-like properties [62, 63].

The HBT effect arises from the tendency of bosons, such as photons, to bunch together due to Bose-Einstein statistics [64]*. In the quantum regime, this bunching can be interpreted as a consequence of two-photon interference between indistinguishable detection paths [65–67]. The method's key advantages include its experimental simplicity, as it does

*This particular paper showed how to understand the HBT effect in terms of electric field fluctuations, contributing to the complete acceptance of this phenomenon and the creation of the field of quantum optics.

not require phase stability, and its robustness to atmospheric or ionospheric fluctuations, which do not affect intensity correlations [61]. As such, the original HBT setup (Fig. 2.1) was both minimal and highly resilient, making it ideal for astronomical applications.

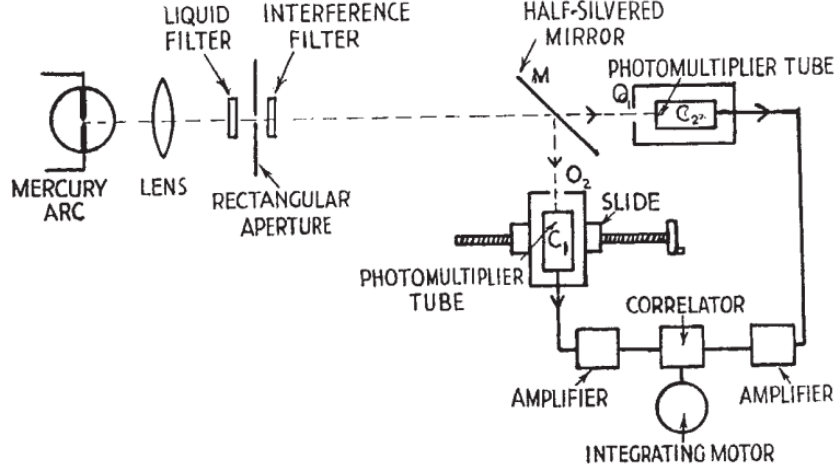


Figure 2.1: Simplified schematic of the original HBT experiment's setup [67]. Later, for astronomical measurements (its original purpose), the setup changed and upscaled.

This experiment can be further exploited as a means of characterizing light. The most important aspect is the correlation of the detector signals simultaneously, i.e., at $\tau = 0$, being τ the time delay between the detectors. We define the second-order* correlation function, which is directly related to the coincidences between detectors A and B on each arm of the interferometer, as follows:

$$g^{(2)}(\tau) = \frac{\langle I_A(t + \tau) \cdot I_B(t) \rangle}{\langle I_A(t + \tau) \rangle \cdot \langle I_B(t) \rangle} \quad (2.1)$$

$$= \frac{\langle : \hat{n}_A(\tau) \hat{n}_B : \rangle}{\langle \hat{n}_A \rangle \cdot \langle \hat{n}_B \rangle} = \frac{\langle : \hat{a}_A^\dagger(\tau) \hat{a}_A(\tau) \hat{a}_B^\dagger \hat{a}_B : \rangle}{\langle \hat{a}_A^\dagger \hat{a}_A \rangle \cdot \langle \hat{a}_B^\dagger \hat{a}_B \rangle} = \frac{\langle \hat{a}_B^\dagger \hat{a}_A^\dagger(\tau) \hat{a}_A(\tau) \hat{a}_B \rangle}{\langle \hat{a}_A^\dagger \hat{a}_A \rangle \cdot \langle \hat{a}_B^\dagger \hat{a}_B \rangle}, \quad (2.2)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the creation and annihilation operators, respectively, $\hat{n} = \hat{a}^\dagger \hat{a}$ is the number operator, $: \dots :$ indicates normal ordering of operators and $\langle \dots \rangle$ denotes a temporal average, where the delay does not matter unless it is between quantities being integrated (regarding the absence of the delay term on the denominator in Eq. 2.2).

In order to perform this experiment, the beam-splitter (BS) (half-silvered mirror in Fig. 2.1) is essential, as it allows us to examine correlations between two independent detection channels that receive light from the same source. This arrangement makes it possible

*In Eq. 2.1, we can see that $g^{(2)}(\tau)$ is a second-order quantity in intensity. However, it is usually described as a fourth-order quantity in EM field. Since we are working in the quantum domain and defining $g^{(2)}(\tau)$ with four creation/ annihilation operators (Eq. 2.2), it is actually more intuitive to consider it a fourth-order quantity, further distinguishing it from linear phenomena.

to measure $g^{(2)}(\tau)$ accurately, free from single-detector saturation and conserving the temporal fluctuations of the source. Consequently, before analyzing the second-order correlation function for different light states, we need to know how the BS works. The BS is composed of four ports, as seen in Fig. 2.2, which affect the EM field and their respective operators according to the following transference matrix:

$$\begin{bmatrix} \mathbf{E}_3 \\ \mathbf{E}_4 \end{bmatrix} = \begin{bmatrix} Re^{i\phi_{13}} & Te^{i\phi_{23}} \\ Te^{i\phi_{14}} & Re^{i\phi_{24}} \end{bmatrix} \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \hat{a}_3 \\ \hat{a}_4 \end{bmatrix} = \begin{bmatrix} Re^{i\phi_{13}} & Te^{i\phi_{23}} \\ Te^{i\phi_{14}} & Re^{i\phi_{24}} \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix}. \quad (2.3)$$

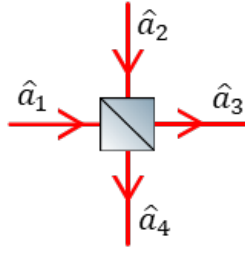


Figure 2.2: Simplified schematic of a BS.

If we take a 50:50 BS, both R and T are equal to $1/\sqrt{2}$ but the phase is trickier to define. Usually they are set to $\phi_{13} = \phi_{14} = \phi_{23} = 0$ and $\phi_{24} = \pi$ [68] or $\phi_{13} = \phi_{24} = 0$ and $\phi_{14} = \phi_{23} = \pi/2$ [69], though it depends on the fabrication process even if there are some fixed relations. For the sake of simplicity, we shall take the former conditions to illustrate our examples. Since we will be writing our states in the Fock basis, it is in our best interest to define the creation operators in each port:

$$\begin{bmatrix} \hat{a}_3 \\ \hat{a}_4 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} \quad (2.4)$$

$$\Leftrightarrow \begin{cases} \hat{a}_3 = \frac{1}{\sqrt{2}} (\hat{a}_1 + \hat{a}_2) \\ \hat{a}_4 = \frac{1}{\sqrt{2}} (\hat{a}_1 - \hat{a}_2) \end{cases} \quad (2.5)$$

$$\Leftrightarrow \begin{cases} \hat{a}_3^\dagger = \frac{1}{\sqrt{2}} (\hat{a}_1^\dagger + \hat{a}_2^\dagger) \\ \hat{a}_4^\dagger = \frac{1}{\sqrt{2}} (\hat{a}_1^\dagger - \hat{a}_2^\dagger) \end{cases} \quad (2.6)$$

$$\Leftrightarrow \begin{cases} \hat{a}_1^\dagger = \frac{1}{\sqrt{2}} (\hat{a}_3^\dagger + \hat{a}_4^\dagger) \\ \hat{a}_2^\dagger = \frac{1}{\sqrt{2}} (\hat{a}_3^\dagger - \hat{a}_4^\dagger) \end{cases}. \quad (2.7)$$

Let us assume our input state $|\psi\rangle$ goes into port 1 and, in port 2, there is a vacuum state $|0\rangle$, i.e. $|\Psi\rangle = |\psi\rangle_1 \otimes |0\rangle_2$. By inputting this state on Eq. 2.2 and using the relations derived

in Eqs. 2.5 and 2.6, we get:

$$\begin{aligned}
 g^{(2)}(0) &= \frac{\langle \hat{a}_4^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_4 \rangle}{\langle \hat{a}_3^\dagger \hat{a}_3 \rangle \cdot \langle \hat{a}_4^\dagger \hat{a}_4 \rangle} \\
 &= \frac{\langle (\hat{a}_1^\dagger - \hat{a}_2^\dagger) (\hat{a}_1^\dagger + \hat{a}_2^\dagger) (\hat{a}_1 + \hat{a}_2) (\hat{a}_1 - \hat{a}_2) \rangle}{\langle (\hat{a}_1^\dagger + \hat{a}_2^\dagger) (\hat{a}_1 + \hat{a}_2) \rangle \cdot \langle (\hat{a}_1^\dagger - \hat{a}_2^\dagger) (\hat{a}_1 - \hat{a}_2) \rangle} \\
 &= \frac{\langle \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 \rangle}{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle^2} \quad (\text{all } \hat{a}_2 \text{ and } \hat{a}_2^\dagger \text{ trace out}) \quad (2.8) \\
 &= \frac{\langle \hat{a}_1^\dagger (\hat{a}_1 \hat{a}_1^\dagger - 1) \hat{a}_1 \rangle}{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle^2} \quad (\text{bosonic commutation relations}) \\
 &= \frac{\langle \hat{n}_1^2 \rangle - \langle \hat{n}_1 \rangle}{\langle \hat{n}_1 \rangle^2}.
 \end{aligned}$$

For time delays much larger than the coherence time ($\tau \gg \tau_c$), the photons detected become statistically independent, because the light field has lost its phase coherence. This means that the second-order correlation will tend to the unity, as the photons originate random arrivals, regardless of the statistics of the source [70].

Let us now look at concrete examples, by applying to port 1 of the BS a Fock state, a coherent state and thermal light, and observing their signature $g^{(2)}(0)$.

Fock States: A Fock state is a quantum state with a well-defined number of particles in a given mode of the field, represented as $|\psi\rangle = |n\rangle$ or as a superposition $|\psi\rangle = \sum_n c_n |n\rangle$; $\sum_n c_n^* c_n = 1$. We can then calculate their statistical properties and second-order correlation as follows:

$$\langle \hat{n} \rangle = \langle n | \hat{n} | n \rangle = n \quad (2.9)$$

$$\langle \hat{n}^2 \rangle = \langle n | \hat{n}^2 | n \rangle = n \langle n | \hat{n} | n \rangle = n^2 \quad (2.10)$$

$$\implies g_{Fock}^{(2)}(0) = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2} = \frac{n^2 - n}{n^2} = 1 - \frac{1}{n} \quad (2.11)$$

From this expression, it is clear that, as we increase the number of photons, the correlation will converge to 1, never actually reaching it, so, we can say that $g_{Fock}^{(2)}(0) < 1$. Particularly, we can also see that, for a single photon ($n = 1$), $g_{Fock}^{(2)}(0) = 0$. This is due to the fact that a lone photon will either be reflected or transmitted, being impossible to detect it simultaneously on both detectors, i.e. they are completely anti-correlated. As we distance ourselves from the perfect overlap ($\tau \neq 0$), the correlation increases to 1, obtaining a correlation function with a single negative-amplitude peak at $\tau = 0$, offset of 1 and width

of the order of several nanoseconds (the same as the coherence time of photons emitted by a single-photon sources). Overall, we can state that $g_{Fock}^{(2)}(\tau) \leq 1$.

Coherent States (Laser): The quantum states of the EM field that most closely resemble a classical monochromatic wave can be expressed in the Fock basis as a special superposition of number states: $|\psi\rangle = |\alpha\rangle = \sum_{n=0} e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$. These states have interesting statistics, being eigenstates of the annihilation operator and having Poissonian statistics, as mentioned previously:

$$\begin{aligned} \langle \hat{n} \rangle &= \sum_{n,n'=0} e^{-|\alpha|^2} \frac{\alpha^{*n'} \alpha^n}{\sqrt{n'!} \sqrt{n!}} n \underbrace{\langle n' | n \rangle}_{\delta_{n,n'}} = e^{-|\alpha|^2} \sum_{n=0} \frac{(|\alpha|^2)^n}{n!} n \\ &= e^{-|\alpha|^2} |\alpha|^2 \sum_{n=0} \frac{(|\alpha|^2)^{n-1}}{(n-1)!} = e^{-|\alpha|^2} |\alpha|^2 \underbrace{\sum_{n=0} \frac{(|\alpha|^2)^n}{n!}}_{e^{|\alpha|^2}} \end{aligned} \quad (2.12)$$

$$\begin{aligned} \langle \hat{n}^2 \rangle &= \sum_{n,n'=0} e^{-|\alpha|^2} \frac{\alpha^{*n'} \alpha^n}{\sqrt{n'!} \sqrt{n!}} n^2 \langle n' | n \rangle = e^{-|\alpha|^2} \sum_{n=0} \frac{(|\alpha|^2)^n}{n!} n^2 \\ &= e^{-|\alpha|^2} |\alpha|^2 \sum_{n=0} \frac{(|\alpha|^2)^{n-1}}{(n-1)!} n = e^{-|\alpha|^2} |\alpha|^2 \sum_{n=0} \frac{(|\alpha|^2)^n}{n!} (n+1) \\ &= e^{-|\alpha|^2} |\alpha|^2 \left(|\alpha|^2 \sum_{n=0} \frac{(|\alpha|^2)^n}{n!} + \sum_{n=0} \frac{(|\alpha|^2)^n}{n!} \right) \\ &= |\alpha|^4 + |\alpha|^2 \end{aligned} \quad (2.13)$$

$$\Rightarrow g_{coh}^{(2)}(0) = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2}{\langle \hat{n} \rangle^2} = \frac{|\alpha|^4 + |\alpha|^2 - |\alpha|^2}{(|\alpha|^2)^2} = 1 \quad (2.14)$$

Independently of the intensity/ mean photon number ($I \propto \langle \hat{n} \rangle = |\alpha|^2$), $g_{coh}^{(2)}(0) = 1$. This also happens for other delays, as stated before, so we would witness a constant $g_{coh}^{(2)}(\tau) = 1$ for any value of τ , confirming the random, independent temporal distribution of the photons described in Section 1.1. We can picture this phenomenon as the state dividing itself in numerous Fock states at the BS, resulting in their simultaneous detection regardless of the delay.

Thermal Light: quantum states that describe light in thermal equilibrium (e.g., black-body radiation, incandescent lamps) are actually mixed states, represented by a density matrix, with no fixed phase and a random number of photons in each mode. Still, as seen previously, they are described by Bose-Einstein statistics (super-Poissonian, Eq. 1.6),

and a second-order correlation can be obtained with ease:

$$(\Delta n)^2 = \langle n^2 \rangle - \langle n \rangle^2 \quad (\text{definition of variance}) \quad (2.15)$$

$$= \langle n \rangle^2 + \langle n \rangle^* \quad (\text{chaotic light for a fixed mean photon number}) \quad (2.16)$$

$$\begin{aligned} \Rightarrow g_{th}^{(2)}(0) &= \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2} \\ &\stackrel{(\text{Eq. 2.15})}{=} \frac{(\Delta n)^2 + \langle n \rangle^2 - \langle n \rangle^2}{\langle n \rangle^2} \\ &\stackrel{(\text{Eq. 2.16})}{=} \frac{\langle n \rangle^2 + \langle n \rangle + \langle n \rangle^2 - \langle n \rangle^2}{\langle n \rangle^2} \\ &= 2 \end{aligned} \quad (2.17)$$

We can conclude that thermal light is highly correlated, as opposed to the other light states we analyzed. However, this value of correlation is only observed at $\tau = 0$. Since τ_c is usually in the order of femtoseconds and the typical time resolution of the detectors is in the order of nanoseconds, the second-order correlation will drop very rapidly, and this peak is very hard to detect, as shall be pointed out in Section 2.3. Overall, we can say that, for thermal/ chaotic light, $g_{th}^{(2)}(\tau) \geq 1$.

2.2. Grangier-Roger-Aspect (GRA) Experiment

In a standard HBT setup with a single-photon source, where only the outputs of a BS are monitored without any heralding detector, the measured second-order correlation function often exhibits a sharp positive peak at $\tau = 0$ (indicative of photon bunching) even though SPDC ideally produces single photons when conditioned. This occurs because the HBT setup integrates over all detection events, including multi-pair emissions, background counts, and the dominant vacuum component of SPDC, none of which are excluded from the measurement. Consequently, the measured $g^{(2)}(0)$ becomes greater than 1, reflecting the super-Poissonian statistics of the unconditioned SPDC output [71, 72]. In order to obtain the expected results, we must introduce a third detector to herald the presence of a signal photon by detecting its twin, thereby conditioning the measurement on a known single-photon input. This post-selection suppresses multi-photon and background contributions, resulting in a clear demonstration of photon antibunching with $g^{(2)}(0) \rightarrow 0$,

*Retrieving the definition of non-classical light states introduced in Section 1.1, we can see that thermal light could never be non-classical as the variance is always higher than the mean (super-Poissonian statistics) and, in the case of coherent states, equal (Poissonian statistics).

consistent with the quantum prediction for a single-photon state [73]. These contrasting outcomes highlight the importance of conditional measurements in revealing the quantum nature of light and underscore the limitations of unheralded second-order correlation measurements for characterizing non-classical sources, consequently demonstrating the relevance of the GRA experiment. Effectively, for high-photon-count sources, the two experiments are equivalent.

This experiment was first proposed and carried out in 1986, by Alain Aspect, Jean Dalibard, and Gérard Roger, with the aim of investigating the behavior of single photons at a beam splitter and verifying the quantum mechanical prediction that a single photon can interfere with itself [73]. The motivation stemmed from extending the interpretation of interference - traditionally applied to ensembles of photons - to individual quantum particles, in line with the superposition principle of quantum mechanics. In their setup (see Fig. 2.3), a heralded single-photon source was used to ensure that only one photon was present in the interferometer at a time. The core of the experiment involved monitoring coincidence events at two detectors placed at the output ports of a BS, while a third detector was used to herald the presence of a single photon. The observation that the probability of simultaneous detection at both output ports was nearly zero provided compelling evidence against any classical wave-like splitting of the photon and was interpreted as a manifestation of the indivisibility of single photons.

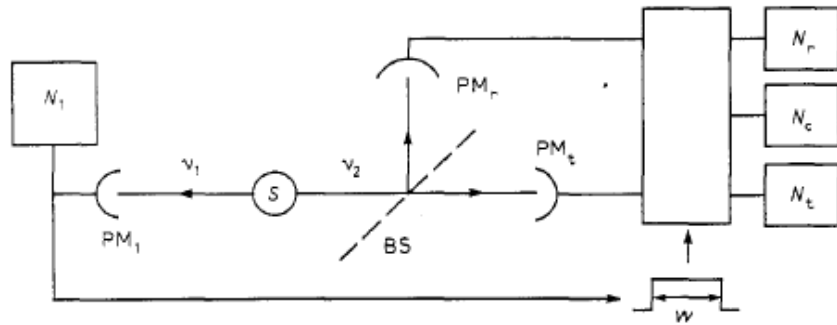


Figure 2.3: Simplified schematic of the original GRA experiment's setup [73]. N_i denotes the number of counts on photomultiplier PM_i (except for N_c , which represents the number of coincidences between the three detectors within the coincidence window w) while ν_1 and ν_2 are the frequencies of the photons emitted by the photon-pair source S .

More significantly, the experiment provided one of the earliest clear demonstrations of photon antibunching - a hallmark of single-photon emission and a distinctly non-classical statistical signature. This effect cannot be explained by classical wave theories of light and therefore served as strong support for the quantum nature of electromagnetic radiation.

The results of this experiment not only reinforced the necessity of a quantum field theory description of light, but also contributed substantially to the development of experimental quantum optics and to foundational tests of quantum mechanics [27].

While the configuration resembles that of the HBT interferometer, the GRA experiment is fundamentally motivated by the high sensitivity of single-photon detection and the advantageous property that certain quantum light sources, such as SPDC, emit photons in correlated pairs. By introducing a third detector (Fig. 2.3) to monitor the idler photon, a heralding mechanism is established that effectively tags the emission of its twin, the signal photon. This trigger allows for the identification of genuine single-photon events and significantly improves the signal-to-noise ratio by discriminating against background counts in the output detectors of the interferometer [74].

In the context of the GRA experiment, the measured second-order correlation function $g_{GRA}^{(2)}(\tau)$ is expected to exhibit a gaussian-shaped temporal profile. This characteristic shape arises not from the underlying photon statistics alone, but from a combination of experimental factors such as timing jitter in the detectors, electronic resolution of the time-tagging system, and statistical fluctuations due to finite counting rates. These sources of uncertainty effectively convolve the ideal correlation signal with a Gaussian response function, resulting in a broadened peak centered around zero delay. The width of this Gaussian profile therefore provides insight into the temporal resolution of the system rather than the coherence properties of the photon source itself [75].

2.3. Experimental Setup

Overall, the HBT and GRA experiments described in previous sections are very similar, with the only major setup distinction being the addition of the idler path for post-selection in the GRA. Thus, we implemented a single setup for both experiments, allowing to quickly switch from one configuration to the other and allowing the immediate comparison of results under the same conditions. Schematics and photos of the implemented setup are presented in Fig. 2.4.

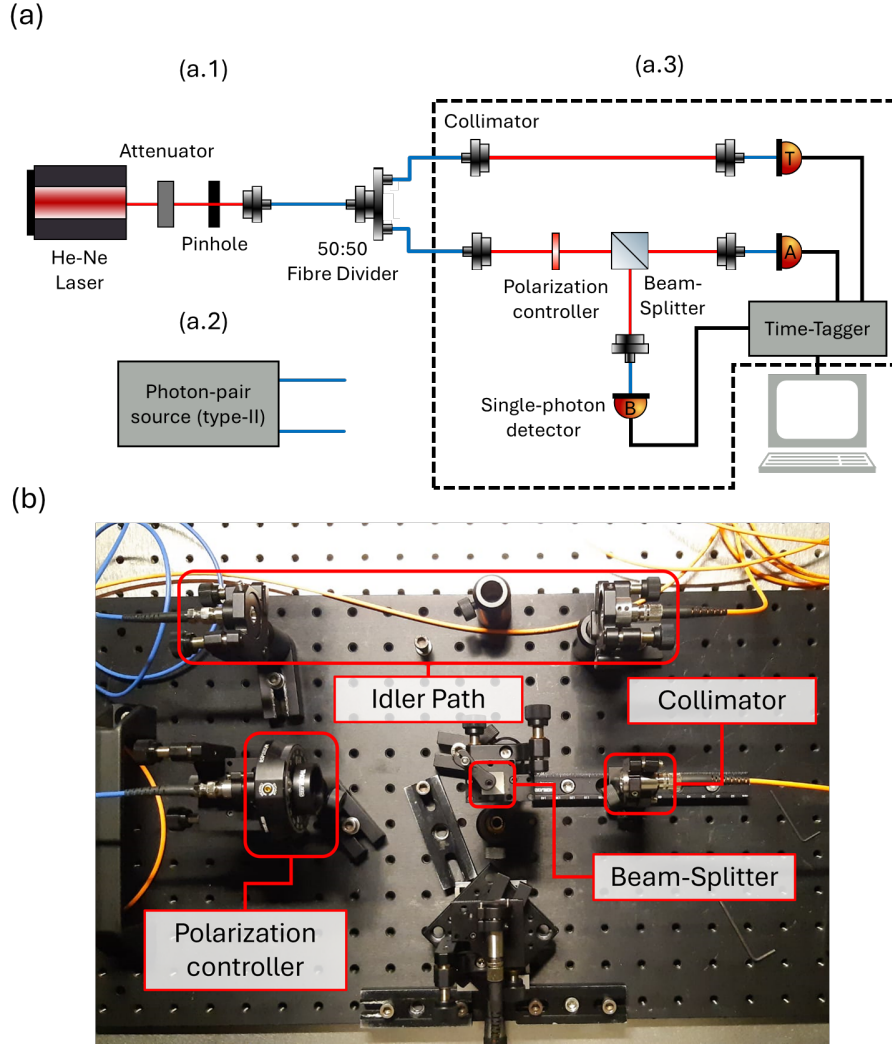


Figure 2.4: (a) Schematic diagram depicting 1. SPDC source and 2. Laser source that can be connected to 3. the experimental setup, where the dotted line represents the black box we used to reduce the noise from background light; (b) image detailing the main setup inside the box (excluding the detectors and the time tagger). The blue lines represent optical fiber; the red ones, light propagating in free space; and the black ones are electrical signal paths. Note that the upper arm corresponds to the idler and does not play any role in the HBT experiment; and that the filters are not represented but would be added right before the output collimators.

Starting from the source side, we utilized two sources: an SPDC source corresponding to SPDC810N type-II photon-pair source from Thorlabs (producing photons with (810 ± 1) nm of wavelength, > 100 kHz emission rate and a maximum pump power of 40 mW) [76]; and a laser source, corresponding to a 632.8 nm Novette He-Ne laser from Uniphase (now Lumentum) [77]. To switch between sources during the experiments, we added/removed the filters and connected the input optical fibers to each source. To work with the SPDC source, a band-pass filter (centered around 810 nm and with a bandwidth of 10 nm) was placed right before each collimator connected to the detectors, to reduce the noise of background light. The polarization controller (half-wave plate) on the path of the

signal photon was used to ensure a balanced division in the BS. This would normally be unnecessary, but we noticed that the measurements in those detectors were not of the same amplitude and, thus, verified that the BS we were using was slightly polarization-dependent for this wavelength. Since this mishap was corrected with the polarization controller, there was no need to repeat the experiment with a different, non-polarizing BS (though this was later corrected for the HOM setup). In the case of the He-Ne laser, we simply removed the filters. Note that in the He-Ne source we have also introduced a pin-hole to filter the spatial modes, and two additional attenuators - a fixed and a variable one, in order to achieve similar detection rates to the GRA measurements. This ensured that the single-photon detectors did not saturate or were damaged during the measurements, but also that the statistics for both experiments were similar.

Advancing to the detection side, we used three SPDMH2F single photon detector modules from Thorlabs (suitable for measurements from 400 to 1000 *nm*, with a 60 % and 70 % detection efficiency for the wavelengths of the SPDC and coherent source, respectively, and a time resolution of 1000 *ps*) [78], a Time-Tagger 20 from Swabian Instruments (with a precision of 1 *ps* and dead time of 6 *ps*, being able to register up to 9×10^6 tags per second) [79].

In terms of the experimental alignment procedure, we employed an auxiliary laser beam, co-propagating along the same optical axis as the photon emission by plugging it to the collimators, to facilitate the first alignment of optical components such as the collimators and the BS. This technique, widely used in optics experiments involving very low-intensity light, allows for the efficient and accurate positioning of optical elements before switching to the actual photon-pair source for fine-tuning [48, 71]. Furthermore, the low intensity of the down-converted photons, a consequence of both the probabilistic nature of the SPDC process and limitations in pump power, underscores the importance of optimizing spatial mode overlap and minimizing losses throughout the system [32, 73].

2.3.1 Determination of the Delay between Idler and Signal Arms

With the experiment set up, the next step was to adjust the delay between the detectors, by synchronizing them with detector T (trigger; the one in the upper arm in Fig. 2.4). Since the SPDC photons are definitely correlated, the best way to determine the time misalignment between the channels was to perform correlation measurements. We took an array of data for each detector and represented the correlation between them with different delays

over the trigger detector. The peaks of those curves are centered at the relative delays for each detector. These values were then employed digitally to compensate the time misalignment between the two channels [79]. During this first task, we tried replacing the single-mode fibers connected to the SPDC source with multi-mode fibers, in order to improve the photon acceptance into the fiber, which resulted in the observation of twice the detection rates, achieving around 5×10^5 photons per second in the trigger detector with 70 % pump power (which we maintained throughout this experiment).

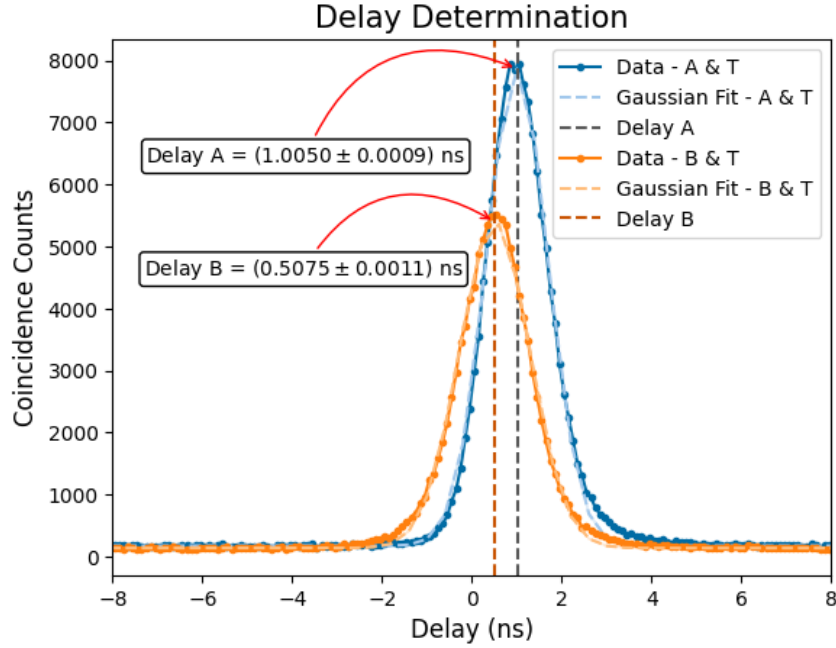


Figure 2.5: Delay determination for synchronization of the detectors.

One might be able to notice a slight skewness on the right side of our data before it converges to the minimum counts (Fig. 2.5). This was most likely due to reflections in the cables connecting the detectors to the time-tagger. We tried minimizing them as much as possible by switching the cables and adapters, spatially separating the cables (in an attempt to reduce crosstalk), and decreasing the pump power, but we could not eliminate the skewness completely.

2.4. Experimental Results

In terms of data analysis for the HBT and the GRA experiment, we can define the second-order correlation function in terms of the detected signals as:

$$g_{HBT}^{(2)} = \frac{R_{AB}}{R_A R_B \Delta t} = \frac{N_{AB}}{N_A N_B} \quad (2.18)$$

$$g_{GRA}^{(2)} = \frac{R_{TAB} R_T}{R_{TA} R_{TB}} = \frac{N_{TAB} N_T}{N_{TA} N_{TB}}, \quad (2.19)$$

where R_i denotes the detection rate in detector i (if there is more than one letter in subscript, the quantity represents the coincidence rate between these detectors), Δt is the coincidence window and N_i is the total number of detections of that detector within that window ($R_i = N_i / \Delta t$). For the experiments, we utilized a coincidence window of 2 ns , a delay range of -60 to 60 ns and 10000 measurements that were averaged (taking about one hour and a half to complete the GRA experiment and two hours for the HBT). The results obtained for both experiments are depicted in Fig. 2.6.

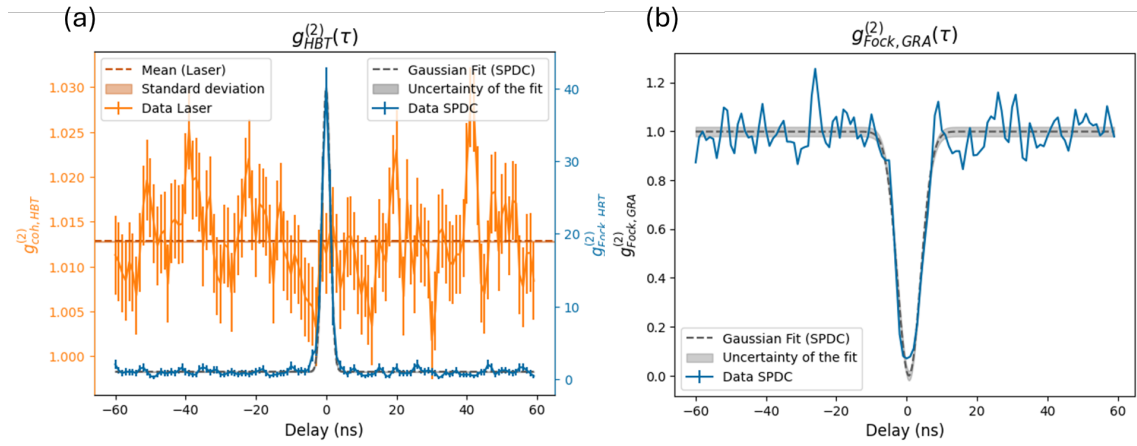


Figure 2.6: Results of the (a) HBT and (b) GRA experiments. The error bars are all represented but some are too small in comparison to the scale of the graphic*.

Starting with the analysis of the HBT experiment results (Fig. 2.6a), we noticed that the coherent source follows the expected behavior of the overall shape of the $g_{coh}^{(2)}(\tau)$ function. Indeed, and quantitatively, we obtained $g_{coh}^{(2)}(0) = 1.011 \pm 0.004$ and, averaging of all measurements and delays, the mean correlation value $\langle g_{coh}^{(2)}(\tau) \rangle = 1.01283 \pm (5 \times 10^{-5})^\dagger$ with $\chi^2 = 0.0027$ and standard deviation $\sigma = 0.004775$. These results align with the expected $g_{coh}^{(2)}(\tau) = 1$ and confirm the theory previously discussed in this chapter.

*To reinforce our choice of building a black box to shield the setup from background light, we share that our previous measurements presented very visible error bars in the same graphic scale due to background noise, even with the bandpass filters.

† the uncertainty of the mean value was obtained from the standard deviation of the raw $g_{coh}^{(2)}$ (which were averaged over all measurements) and divided but the squared root of the number of delay values used.

Looking now at the results for the Fock state in the HBT configuration, we observe the behavior described previously, which departs from the expected second-order correlation for single-photons. Indeed, in the absence of a herald photon for post-selection, we obtain a strong correlation $g_{Fock,HBT}^{(2)}(0) > 1$ that corresponds to the thermal-related bunching of those photons (not a true single-photon state).

As previously stated, the single-photon second-order correlation $g_{Fock}^{(2)}(0) = 0$ may still be observed if we consider the GRA experiment with post-selection of the single-photon states. Indeed, and looking at the results for this case (Fig. 2.6b), we observe that the second-order correlation at $\tau = 0$ was $g_{Fock}^{(2)}(0) = 0.0702183 \pm (4 \times 10^{-7})^*$ which deviates slightly from our predictions of achieving $g_{Fock}^{(2)}(0) = 0$. Nonetheless, since there is always some noise we cannot remove, the result is very satisfactory: if we apply Eq. 2.11, we obtain that our average mode is $\langle n \rangle = 1.0755213 \pm (4 \times 10^{-7})$ instead of $n = 1$, confirming a slight deviation from a perfect single-photon source.

Regarding the shape of the curve, we also obtained the expected behavior of an inverted gaussian peak as a function of the delay with an offset around 1 and $g_{Fock}^{(2)} \leq 1$. The actual parameters (with $\chi^2 = 0.61$) indicated an offset of 0.999 ± 0.007 with a gaussian centered at $\tau_0 = (0.64 \pm 0.14) \text{ ns}$ and full width at half maximum (FWHM) of $(7.7 \pm 0.3) \text{ ns}$. Furthermore, the gaussian fit accused $g_{Fock}^{(2)}(0) = 0.02 \pm 0.02$, which translates to $\langle n \rangle = 1.02 \pm 0.02$. The result associated to the width of this dip may be related to the spectral bandwidth and coherence (or wavepacket) length of the photons and influence the precision we can achieve in sensing and imaging using this source [80]. In this case, we are looking at a rather large FWHM, which translates into a long coherence length and narrow spectral width (the latter was confirmed by the datasheet of the photon-pair source [76] with a photon bandwidth of $< 0.25 \text{ nm}$).

2.5. Concluding Remarks

Through the execution of the HBT and GRA experiments, we could understand the physical meaning of second-order correlations, as well as the different signatures of single-photon and coherent states. The results attained allowed us to confirm and visualize the theory behind non-classical light and compare it to coherent light.

Furthermore, we acquired several critical competencies that directly support the successful implementation of HOM interferometer, our main and more adventurous task for this

*the uncertainties of the $g_{Fock}^{(2)}$ values were obtained from the standard deviation over all measurements divided by the squared root of the number of measurements.

thesis. First, both experiments required precise alignment of optical components, an essential skill for ensuring the spatial and temporal overlap of photons, which shall be most useful in our next endeavors.

Additionally, the process of configuring the time-tagger and synchronizing the detectors provided the foundation for managing timing resolution and coincidence detection. By allowing the measurement of second-order correlations, these experiments allowed us to quantify photon statistics and assess the quality of photon sources, especially in terms of temporal coherence and single-photon purity.

On the computational side, the development of an interface to the equipment and subsequent data analysis ensured that we could freely tune the parameters of the experiment and quickly analyse and save the results. Furthermore, the GRA experiment introduced the concept of heralded detection, enabling better discrimination between signal and noise, which, consequently, exposed us to three-detector schemes and the synchronization challenges that arise in multi-channel detection, improving our readiness to handle the dual-arm architecture of the HOM interferometer.

In summary, the practical and theoretical insights gained from the HBT and GRA experiments, ranging from photon statistics and detector calibration to coincidence timing and signal discrimination, laid the necessary groundwork for successfully executing and interpreting HOM interference, which is a more complex but fundamentally-related quantum optical phenomenon.

3. Hong-Ou-Mandel (HOM) Microscopy

With the groundwork of photon correlations established in the previous chapter, we now turn to a phenomenon, Hong-Ou-Mandel (HOM) effect, that builds upon these concepts, introducing quantum interference that we will later use to implement a quantum sensing strategy.

The HOM effect is perhaps one of the most striking manifestations of quantum features of light and interference: when two indistinguishable photons meet at a balanced beam splitter, quantum interference suppresses the possibility of them leaving by different ports, producing the characteristic HOM dip in coincidence counts. The position, shape, and visibility of this dip is sensitive to path delays, polarization changes, and other sample-induced distinguishability on the photons. Thus, in the context of sensing, it enables depth-resolved measurements with axial resolution, high robustness to phase noise and dispersion, and operation at ultra-low optical powers, which are important attributes for quantum imaging.

In this chapter, we will introduce the two-photon interference mechanism and its theoretical description, outline practical considerations for achieving indistinguishability, and show how HOM visibility can be used for sensing for probing sample thickness, refractive index, and polarization/birefringence. We then present our experimental interferometer design, source characterization, and representative measurements that illustrate the capabilities and limitations of HOM-based imaging, setting the stage for birefringent microscopy in the following chapter [81, 82].

3.1. Overview of the HOM Effect and Applications in Quantum Technology

When we have a photon entering a 50:50 BS (see Fig. 2.2), we know that it has equal probability of being transmitted or reflected.

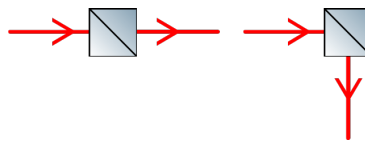


Figure 3.1: Possible outcomes when one photon enters a 50:50 BS.

In terms of a wavefunction, we may represent the state in terms of the pair of input ports (1,2) and output ports (3,4) has (according to the relations defined in Eq. 2.7):

$$|1,0\rangle_{1,2} = \frac{1}{\sqrt{2}} \left(|1,0\rangle_{3,4} - |0,1\rangle_{3,4} \right). \quad (3.1)$$

In the case of having two photons entering a 50:50 BS through different ports, we would expect a superposition of the following exiting states:

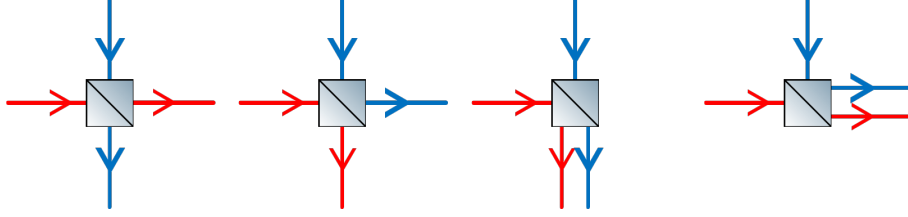


Figure 3.2: Possible outcomes when two photons enter a 50:50 BS through different ports.

which translates into the following wavefunction (according to the relations defined in Eq. 2.7):

$$|1,1'\rangle_{1,2} = \frac{1}{2} \left(|1,1'\rangle_{3,4} - |1',1\rangle_{3,4} - |0,2\rangle_{3,4} + |2,0\rangle_{3,4} \right). \quad (3.2)$$

However, if the initial photons are completely indistinguishable, the first two terms in Eq. 3.2 cancel out (destructive quantum interference). As a result, the probability amplitude corresponding to the scenario in which the two photons exit through different output ports of the BS vanishes and, consequently, both photons are detected exiting through the same output port. This phenomenon is referred to as photon bunching and is a typically bosonic behaviour, as well as a hallmark of the HOM quantum interference [57], which arises from the symmetrization of the two-photon wavefunction.

Bridging to the experimental side, we must first ensure a fundamental indistinguishability (frequency, polarization, spatial mode, etc). Then, we may vary a path delay of one of the photons (Fig. 3.3a) in order to change between a perfect/non-perfect temporal overlap at the BS. When enough indistinguishability is obtained, we start observing a dip in the coincidence counts between the two detectors, since only one of the detectors is receiving the two photons at a time, achieving, ideally, no coincidences for perfect indistinguishability, as seen in Fig. 3.3b.

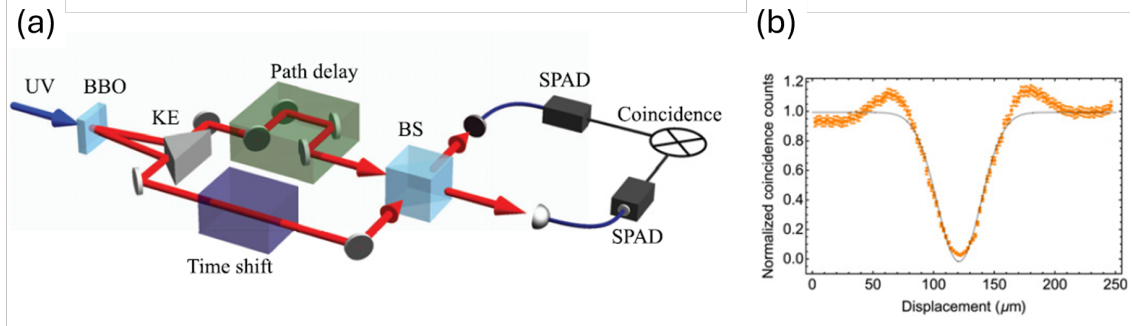


Figure 3.3: General setup for HOM microscopy (a) and typical HOM dip (b) [57].

Depending on the electronics and sources utilized, tracking the center of this dip is typically done with a time resolution varying from femtoseconds [83] to attoseconds [81]. This corresponds to a spatial resolution of μm to $\text{\AA}$, which combined with the fact that it can be done at ultra-low levels of light (down to single photon), offers a unique quantum advantage for photo-sensitive samples that can be capitalized in biology [81] or 2D materials [84], for example.

Note that the resolution is connected with the shape of the dip itself, which allows us to retrieve some properties of our photon source. Indeed, its overall shape reflects the temporal/spectral profile of the photon wave packets [85], with a width inversely proportional to the length of the wave packet or the coherence length of the down-shifted photons [82]. Furthermore, asymmetries can indicate dispersion, chirp, or additional imperfections [86, 87], being thus also useful to probe characteristics of the source itself. With this knowledge about the HOM dip, we have a powerful tool for testing the indistinguishability of photons. If the dip reaches zero (visibility $\alpha = 1$), it indicates that the photons are perfectly indistinguishable. Any deviation from zero at perfect alignment suggests some degree of distinguishability, and thus, along with the previous information that we can extract from the data, we can use the HOM effect to characterize photon pair sources (for example, [88–91]).

Another significant advantage of HOM imaging lies in its ability to provide intrinsic axial resolution, making it particularly valuable for depth-resolved microscopy (similar to Optical Coherence Tomography (OCT), though it can be complementary, especially in the context of quantum-enhanced precision and sensitivity) [92, 93]. Such capabilities can be realized using an interferometric configuration similar to that shown in Fig. 3.3a (transmission) or in Fig. 3.4 (reflection). The second configuration is commonly used for Quantum Optical Coherence Tomography (QOCT) and makes use of a refined version of the HOM

setup designed explicitly for depth profiling, achieving enhanced performance in scattering media and improved robustness to dispersion [94–96].

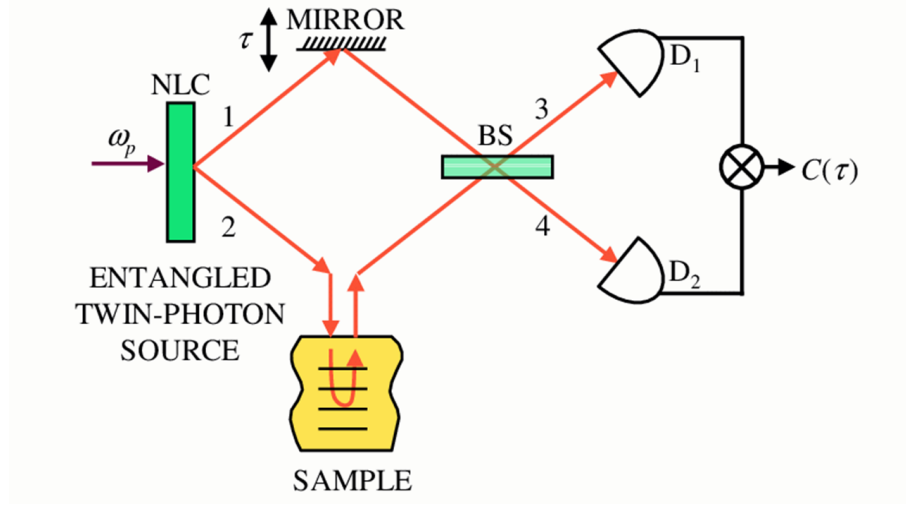


Figure 3.4: Schematic of QOCT, a technique created by implementing the HOM effect to OCT [94].

Although we focused on indistinguishable photons entering the inputs that are associated with a symmetric state with bosonic behaviour, one might also ask what would happen if we have anti-symmetric input state. With photon sources, this can be achieved for example by tuning the polarizations on each arm, realizing a Bell state $|\Psi^-\rangle$:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|H, V\rangle_{1,2} - |V, H\rangle_{1,2}) = -\frac{1}{\sqrt{2}} (|H, V\rangle_{3,4} - |V, H\rangle_{3,4}) \quad (3.3)$$

In this case, the anti-symmetric state maintains its anti-symmetry after going through the BS (which applies a symmetric unitary transformation), witnessing anti-bunching (a typically fermionic behaviour). In terms of data analysis, it would give rise to an inverted dip (the correlation/ coincidences would be maximum for perfect alignment). This is important for Bell state analysis, which is also a test we can perform with this technique [57]. Since all of the other three Bell states are symmetric, they bunch and are harder to distinguish through this simple measurement (though it is still possible with the addition of half-wave plates or other polarization-related components [97]).

Overall, the HOM effect originated a plethora contributions to many fields in Science, some of which include improving resolution in imaging and sensing [81–83, 98], QOCT [92–94], quantum metrology and quantum state engineering (for example, by creating $N00N$ states, similarly to Eq. 3.2 for indistinguishable photons, where we created one with $N = 2$) [99–107], quantum state analysis (especially Bell states) [97], characterization of sources of indistinguishable photon pairs [88–91], quantum communication and

computation [108–114] and even non-photonic systems [115–125].

3.2. Theoretical Framework

Although we already provided a quick preview of the statistical and phenomenological behaviour of a HOM interferometer, we can tackle the situation more formally, as described in Ref. [126]. In this analysis, we neglected absorption and other fluctuations in the photon number, as well as changes in spatial mode and other degrees of freedom, unless stated.

Starting with the case where we do not have a sample in the beam path, the overall state is written as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\phi_1\rangle_1 |\phi_2\rangle_2 + |\phi_2\rangle_1 |\phi_1\rangle_2) = \frac{1}{\sqrt{2}}(|\phi_1\phi_2\rangle_{1,2} + |\phi_2\phi_1\rangle_{1,2}), \quad (3.4)$$

where $|\cdot\rangle_i$ denotes the photon ϕ_i on arm i , meaning that we have two photons in the system but in an unknown arm. The corresponding density matrix representation is given by

$$\rho = |\psi\rangle \langle \psi|. \quad (3.5)$$

Then, we need to define the total action of the BS on both states, which is no longer described by Eq. 2.4 but is written as a direct product of both its individual actions, as follows

$$BS_{total} = BS \otimes BS. \quad (3.6)$$

and the output density matrix, after the beam splitter is given by

$$\rho_{out} = BS_{total} \rho BS_{total}^\dagger \quad (3.7)$$

Note that the density matrix changed basis to one where the order now represents the detectors after ports 3 and 4 ($|\phi_1\phi_2\rangle_{1,2} \rightarrow |\phi_3\phi_4\rangle_{3,4}$). Thus, we can define the probability of measuring double coincidences using projective operators. We cannot, however, forget that we are dealing with a superposition:

$$P_c = P_{34} + P_{43} = |\phi_3\phi_4\rangle_{3,4} \langle \phi_3\phi_4| + |\phi_4\phi_3\rangle_{3,4} \langle \phi_4\phi_3|. \quad (3.8)$$

The probability of measuring double coincidences is given by the estimation of the projector in density matrix fashion, which applied to ρ_{out} gives

$$p_c = \text{Tr}[\rho_{out} P_c] = 0 \quad (3.9)$$

However, this model only give us the dip value, where the two photons are indistinguishable. To better model the experiment, we need to consider the distinguishable and the indistinguishable situations, as well as the transition between the two. For that, we will consider the two distinguishable states to construct the density matrix describing that mixed state

$$|\psi_{dis}^1\rangle = |\phi_1\phi_2\rangle_{1,2} \rightarrow \rho_{12} = |\phi_1\phi_2\rangle_{1,2} \langle\phi_1\phi_2| \quad (3.10)$$

$$|\psi_{dis}^2\rangle = |\phi_2\phi_1\rangle_{1,2} \rightarrow \rho_{21} = |\phi_2\phi_1\rangle_{1,2} \langle\phi_2\phi_1|$$

$$\rho_{dist} = \frac{1}{2}\rho_{12} + \frac{1}{2}\rho_{21} \quad (3.11)$$

Thus, the total density matrix that takes into account the indistinguishable and distinguishable states is given by

$$\rho_{total} = p\rho_{indist} + (1-p)\rho_{dist} \quad (3.12)$$

where $p \in [0, 1]$ represents the amount of indistinguishable photons in the systems and requires a model. For example, a common model for p accounts for the temporal overlap between the two photons wavepackets. If each photon can be described by a gaussian distribution [127] with a coherence length l_c (associated with the coherence time), the wavepacket can be modeled as

$$\psi(x, x_0) = \frac{\sqrt{2}e^{-\frac{2(x-x_0)^2}{l_c^2}}}{\sqrt[4]{\pi} \sqrt{l_c}}, \quad \text{with } l_c = \frac{\lambda^2}{\Delta\lambda}, \quad (3.13)$$

and so, we can define p by looking at their overlap:

$$p(x_0, x_1) = \frac{2}{\sqrt{\pi} l_c} \int_{-\infty}^{\infty} e^{-\frac{2(x-x_0)^2}{l_c^2}} e^{-\frac{2(x-x_1)^2}{l_c^2}} dx = e^{-\frac{(x_0-x_1)^2}{l_c^2}}. \quad (3.14)$$

We are now ready to write the new ρ_{out} in the same fashion as Eq. 3.7 and obtain the coincidence probability as

$$p_c(x_0, x_1) = \text{Tr}[\rho_{out} P_c] = \frac{1}{2} \left(1 - e^{-\frac{(x_0-x_1)^2}{l_c^2}} \right), \quad (3.15)$$

which produces the usual dip shape that we foreshadowed. However, it is important to mention that this result only applies to Gaussian wavepackets. Although the outcome is

somewhat similar, the shape of the dip may differ; for example, a top hat wavepacket will generate a triangular distribution of probability, a triangular wavepacket will overlap in a cubic polynomial function, and a sinc-shaped one will result in another sinc.

To describe the experimental data more accurately, we can further improve the previous equation by introducing two phenomenological parameters. First, we may introduce the α parameter that measures the maximum visibility obtained experimentally, accounting for errors, fluctuations, and other perturbations; Secondly, we introduce a β parameter that takes into account absorption/photon losses introduced in a path (e.g., by a sample), making this parameter axial dependent. With these modifications, the expression becomes:

$$g_{HOM}^{(2)} \propto P_c = \frac{\beta}{2} \left(1 - \alpha e^{-\frac{\Delta x^2}{l_c^2}} \right), \quad (3.16)$$

where $\Delta x = x_0 - x_1$, from which we can define a visibility V parameter as

$$V = \frac{C_{\text{base}} - C_{\text{dip}}}{C_{\text{base}}} = \frac{NP_c(\infty) - NP_c(0)}{NP_c(\infty)} = \alpha \quad (3.17)$$

Looking at the visibility parameter, it is straightforward to conclude that it does not depend on β , meaning that it can be utilized to probe a sample in multiple points (e.g. scanning microscope) even if its absorption is spatially inhomogeneous. Also, as it is independent of N , the interferometer also works at lower light intensities, even if it decreases the temporal statistics of the dip (which can be corrected by longer acquisition times).

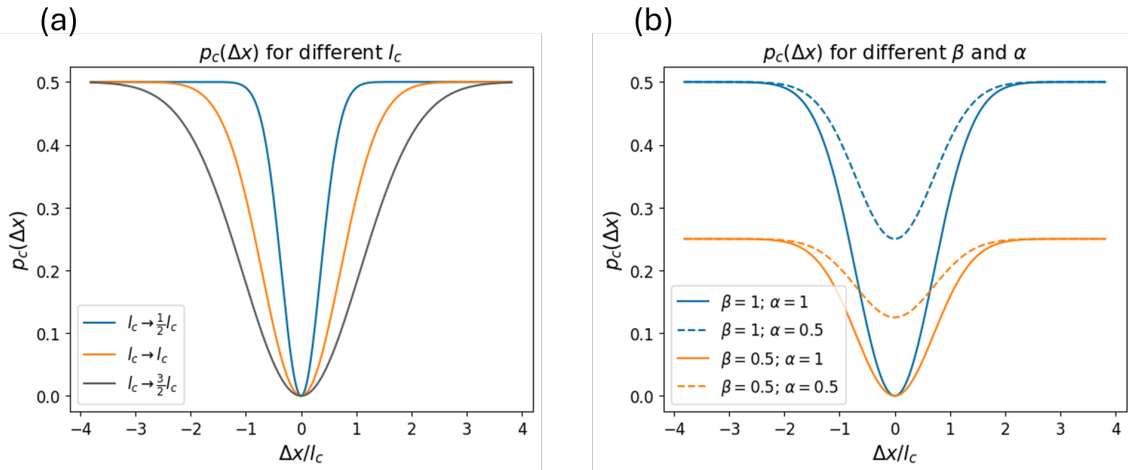


Figure 3.5: Theoretical predictions of the behaviour of the HOM dip. Lower coherence lengths will narrow our curve (a) while external influences will decrease the offset and low visibilities will reduce the depth of the dip (b).

At this point, we already have all the theory needed to understand path distance measurement, the easiest application of HOM in quantum sensing. The insertion of a sample in the interferometer will alter the optical path of the signal photon (if not other properties on top of it) and displace the dip. Considering that the sample does change the other degrees of freedom of photons, we can model this displacement with the exponent term in Eq. 3.16 suffering a small transformation $x_0 - x_1 \rightarrow x_0 - x_1 - \delta x$, where δx is the optical path added by the sample and takes into account its refractive index.

3.3. Experimental Implementation of the HOM Interferometer

The experimental setup implemented (Fig. 3.6) is composed of two arms, one for each down-converted photon, that converge on a 50:50 BS, which is followed by a band-pass filter and collimator connected to a single-photon detector at each exit port.

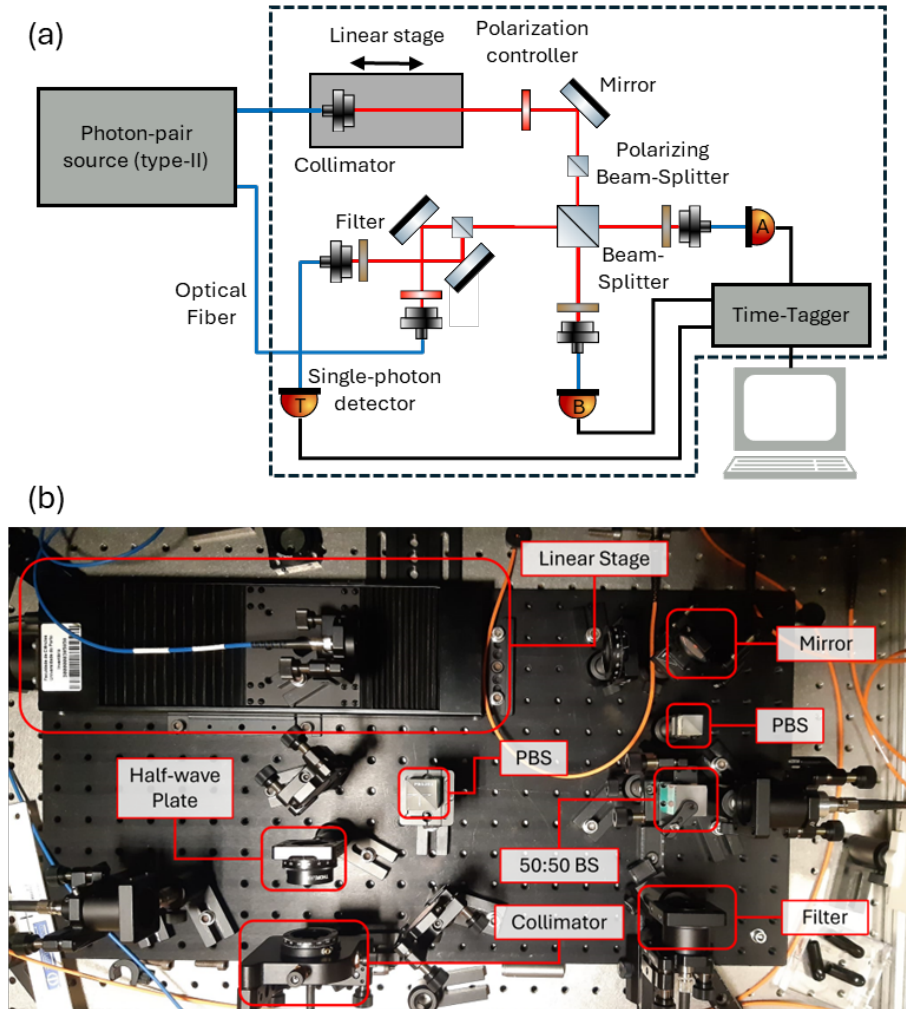


Figure 3.6: Setup for the HOM interferometer: (a) schematic diagram of the experimental setup, where the dotted line represents the black box we used to reduce the noise from background light; (b) image detailing the main setup inside the box (excluding the detectors and the time tagger). The blue lines represent optical fiber; the red ones, light propagating in free space; and the black ones are electrical signal paths.

The upper arm, through which travels the idler photon, starts with a collimator placed on top of a computer-controlled NRT150 motorized linear translation stage [128] in order to, as mentioned before, achieve perfect overlap at the BS, similar to the path delay illustrated in Fig. 3.3a. Then, there is a half-wave plate to maximize counts and a polarizing beam-splitter (PBS) to ensure polarization indistinguishability: the PBS projects photon's polarization state to its orientation, and by varying the rotation of the half-wave plate, we can make sure that the vast majority of photons will be transmitted into the BS where the HOM effect takes place. By placing the same type of PBS in the same orientation in the other arm, we successfully ensure that they have the same polarization, while maximizing the detection rate.

The lower arm, through which travels the signal photon, follows mostly the same setup (except for the linear stage). However, we make use of the PBS to direct some photons into a third detector and, similarly to the synchronization performed in the GRA experiment, use it for a quick delay determination to make sure we are operating in the zero delay (after the BS) regime, where the correlation between the down-converted photons is maximum.

3.3.1 Alignment Considerations and Polarization Control

To observe the HOM effect it is required for the photons to be sufficiently indistinguishable. For an imperfect indistinguishability, the visibility of the dip decreases, as the effect is diminished. In this section of the work, we describe the alignment of the system, following methods previously described in Refs. [129–131].

The very first step, similarly to any other free-space optical setup, was to level and align the components with an alignment laser. Since this particular setup has a lot of similarities with a Mach-Zehnder interferometer, the assembly would be fairly simple if it were not for the very meticulous approach that we took in order to provide the best alignment possible. This thorough first alignment included special attention to the back-reflections in every BS and PBS, and a sweep along the possible positions of the collimator of the idler photon in the linear stage (to ensure the alignment would not be ruined during the measurement).

Secondly, we placed two charge-coupled device (CCD) cameras in our setup: one in the near-field right after the 50:50 BS and other in the far-field (after a temporary mirror system to elongate the distance). With these cameras, we were able to guarantee perfect

overlap of the beams through a finer tuning of the alignment. We would also add 30 *mm* convergent lenses before both CCD to verify the alignment in the Fourier plane.

Finally, we took advantage of the small coherence length of a broadband optical source by Multiwave Photonics (which uses amplified spontaneous emission to emit polarized light around $\lambda_0 = 1050 \text{ nm}$ with a FWHM of 50 *nm* and output power of 20 *mW* [132]) to observe interference in its spectrum. We connected the fiber that would go into detector B to a spectrometer and analyzed the spectrum after the interference while changing the position in the linear stage (still observing the near-field CCD). This allowed us to perfect the alignment (camera) and estimate the position in the linear stage where the HOM effect would happen (as the interference fringes would be more visible).

At this point, we had ensured that the photons would interfere in the BS. However, since we knew that our photons resulted from a type-II SPDC, their polarizations would be orthogonal, and so it was required that that degree of freedom was adjusted in order to obtain indistinguishability. That is the function of the PBS in each arm, as stated previously. By tuning the rotation of the half-wave plate and monitoring the photon count rate in real time (thanks to the time-tagger software), we could maximize the detection rate, ensuring the best statistics possible.

3.3.2 Source Analysis and Indistinguishability

Even after all this very meticulous alignment, it is common to fail to observe the HOM effect. Indeed, this was one of the major challenges encountered during the experimental work: it was first hypothesized that the dip could be out of range for the stage, which we tested by moving the whole stage, re-aligning everything, and measuring it again. After many attempts and even directly measuring the paths with a ruler, it was ruled out that this was the origin of the problem, concluding that it was related with the source.

In Section 1.3, it was mentioned that the phase-matching conditions needed to be met for the SPDC process to occur. In this case, we knew that they were met, as we could measure photons on both fibers; however, we did not know if those were degenerate or non-degenerate. This realization propelled us to characterize the photon-pair source in the relevant ways that could be controlled: crystal temperature and pump power.

The first test performed was to see how the wavelengths of the photons changed with the temperature of the non-linear crystal, taking inspiration from a similar analysis in Ref. [53]. To put this to test, a spectrometer was connected directly to each output of the pair

source individually. We measured each spectrum at different temperatures, taking both the signal and idler spectra before changing the temperature. The pump power was set to 95 % in order to obtain the best statistics possible in a short time.

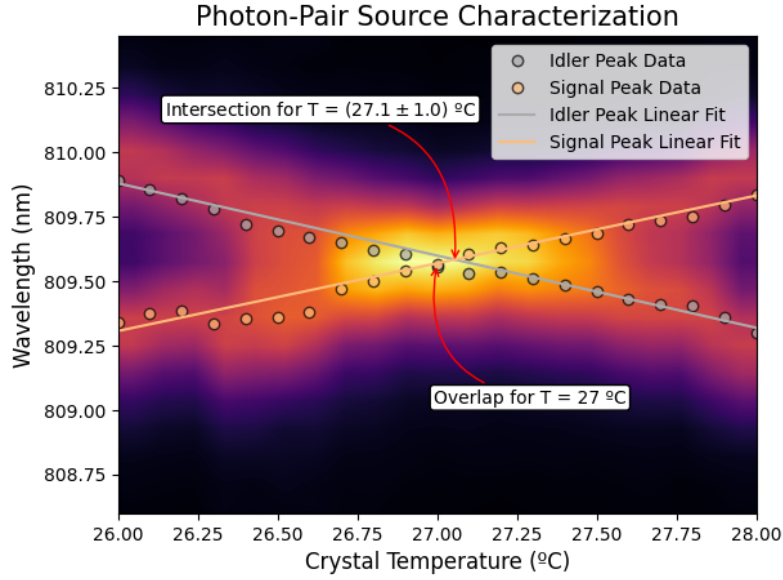


Figure 3.7: Characterization of the photon-pair source as a function of the temperature (performed with 95 % pump power).

It was found that, around $T = 27^\circ\text{C}$, the photons had the same wavelengths and thus were indistinguishable in frequency (Fig. 3.7). We then set the crystal temperature accordingly and tried to measure the HOM dip with a pump power of 70 %, an empirical choice of a compromise between good statistics and not risking saturating the detectors. To our surprise, we still could not see the expected signature for this effect: we quickly concluded that the phase-matching should also depend on the pump power.

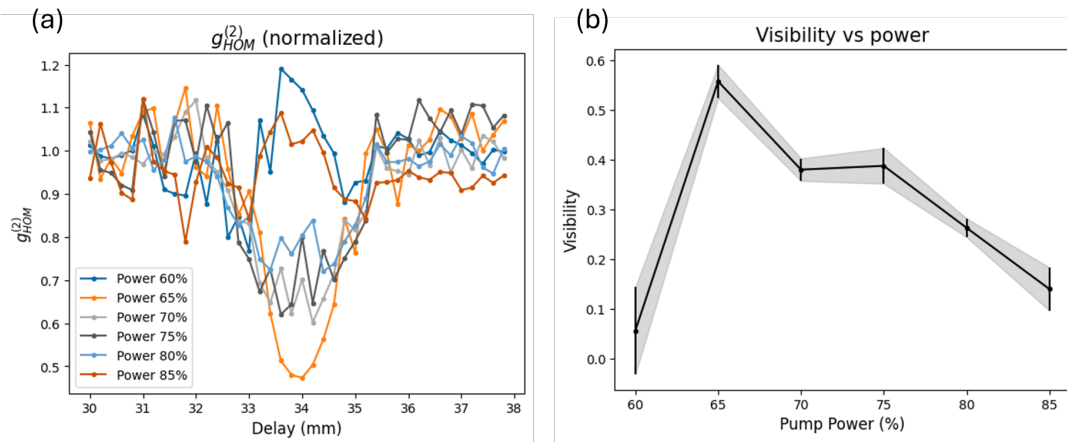


Figure 3.8: Source characterization for different pump powers at a fixed temperature: (a) normalized HOM dip for different pump powers; (b) visibility of the HOM dip as a function of the pump power.

For a set temperature, we measured the HOM dip with different pump powers (Fig. 3.8a) and saw that, for a pump power of 65 %, we could achieve the best visibility (Fig. 3.8b). Then, to double check the spectrometer results, the pump power was fixed to that value and the crystal temperature was changed, registering the value of the correlation near the bottom of the dip from which we calculated the visibility and looked for the optimal value, knowing that the offset of the curve would not change since it was still a free space measurement*. Following the expected behaviour (Fig. 3.9), we concluded that the photons were the most similar for $T = (27.60 \pm 0.04) ^\circ\text{C}$, at 65 % pump power.

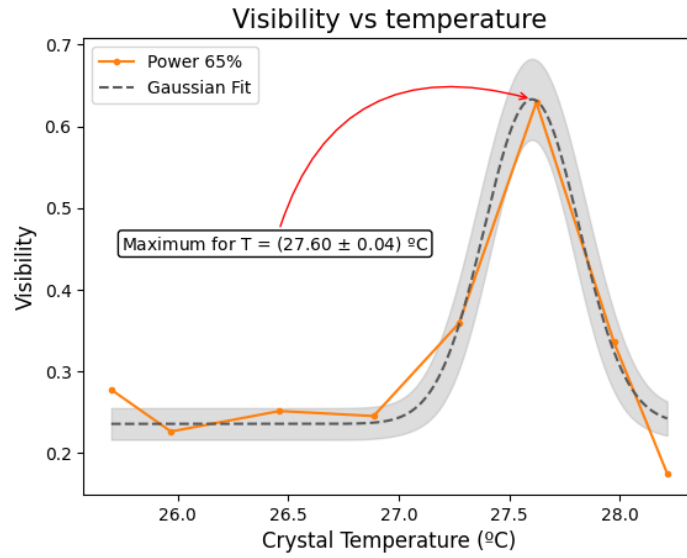


Figure 3.9: HOM dip's visibility as a function of crystal temperature for 65 % pump power.

We could just take our conclusions from these graphics, however we eventually figured that the analysis was incomplete: the pump power may induce non-linear effects in the crystal (such as the optical Kerr effect[†] [32, 133]) and changes the refractive indices, consequently modifying the phase-matching conditions; so, there should be an optimal temperature for each pump power for degenerate emission, which is not necessarily equal for all powers, supported by the different optimal temperatures for 95 % (Fig. 3.7) and 65 % (Fig. 3.9) powers. To verify our hypothesis and complete our characterization of the source, we evaluated the depth of the HOM dip (and thus the visibility, although to obtain the exact visibility we would need to double the already long measurement time) for different pump powers and crystal temperatures.

*The power and frequency we are using are too low to create any non-linear phenomena in the air that would interfere with this assumption.

[†]This effect describes the change of refractive index with a term proportional to the intensity of the EM field, i.e. $n = n_0 + n_2 I$, leading to self-phase modulation and Kerr lensing. As a second-order non-linear effect, it induces changes in the second-order electric susceptibility term, χ_2 .

The results in Fig. 3.10 give strength to our previous argument regarding non-linearities within the crystal, as for the same temperature, there are different power values that allow a good visibility. However, for each power there is only one optimal temperature for degenerate frequency emission (optimal visibility of the dip), corresponding to the darker spots on Fig. 3.10 (lowest dip).

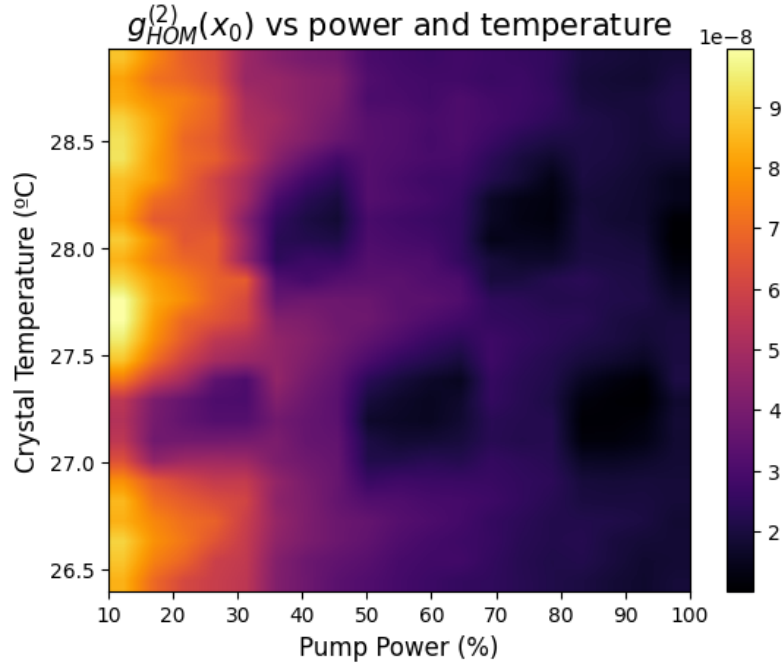


Figure 3.10: Complete characterization of the photon-pair source as a function of pump power and temperature of the crystal, evaluating the depth of the HOM dip.

We could also observe that, for a lower photon rate, i.e., a lower pump power, the dip was worse, whereas for higher intensities, it seems to become much better. One might feel like this is a fallacious conclusion, given that the offset and individual counts are most likely changing with different powers. Although this is a valid concern, our goal here was to determine the best temperature for each pump power to perform HOM interference and, regardless of that fine detail, higher pump powers would have taken preference in order to achieve the best statistics at a faster pace. With this information, we can safely and most efficiently* perform our experiments.

*The datasheet [76] states that the room temperature should not exceed 25 °C, which meant that, before the air conditioning cooled the room down during very hot summer days, the optimal temperature for degenerate emission was very different from expected, varying up to 0.7 °C.

3.3.3 Experimental Results

After ensuring the optimal conditions for indistinguishability of the photons, alignment, and collimator efficiency (i.e., optimizing the reception of photons for better statistics, by tuning the position and inclination of the collimators), we could measure a HOM dip, as observed in Fig. 3.11.

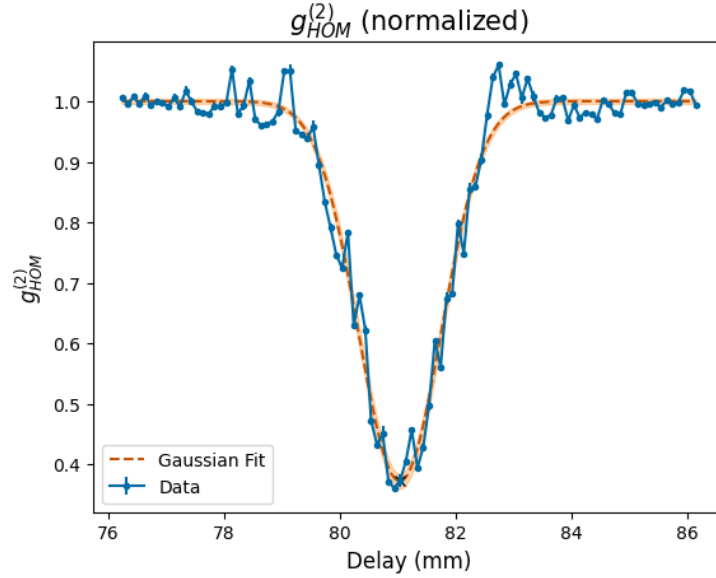


Figure 3.11: HOM dip (normalized).

With this preliminary result (Fig. 3.11), we achieved a precision of 0.015 mm (uncertainty of the center of the dip, according to the Gaussian fit [83]) and a visibility of $(62.9 \pm 1.2) \%$. The fit also accused a FWHM of $(1.68 \pm 0.04) \text{ mm}$, from which we can derive $l_c = (1.01 \pm 0.02) \text{ mm}$ and represents a spectral bandwidth of the order of $(0.649 \pm 0.015) \text{ nm}^*$ that reflects the narrowband nature of our source (matching the value in the datasheet of $< 0.25 \text{ nm}$ [76]), all with $\chi^2 = 0.10$.

The next test was to place a sample and perform optical path sensing. We found two acrylic pieces, about 1.5 mm thick each[†]. We measured the HOM dip in free space, with

*The gaussian fit applied to the data was based on Eq. 3.16, that states that $g_{HOM}^{(2)} \propto 1 - e^{-\Delta x^2/2\sigma^2}$ [83, 134], where $\text{FWHM} = 2\sigma\sqrt{2\log 2}$. We can write $l_c = \sqrt{2}\sigma$ from the comparison between the two expressions for $g_{HOM}^{(2)}$. Then, with Eq. 3.13, we can calculate the spectral bandwidth as $\Delta\lambda = \lambda_0^2/l_c$. Then, we can obtain the value for the coherence time, τ_c , by dividing l_c by the speed of light. There could be some factors along the way (related to the shape of the pulse [127]) that were neglected, however, this serves as an estimate of the order of magnitude of the quantities. The uncertainties were propagated according to the previous steps.

[†]Strictly speaking, it was hard to accurately measure the thickness of the samples, so the error associated with this quantity is of same order as the measurement.

one and then two samples. As expected, we can see the center of the dip moving back and maintaining a rather stable width.

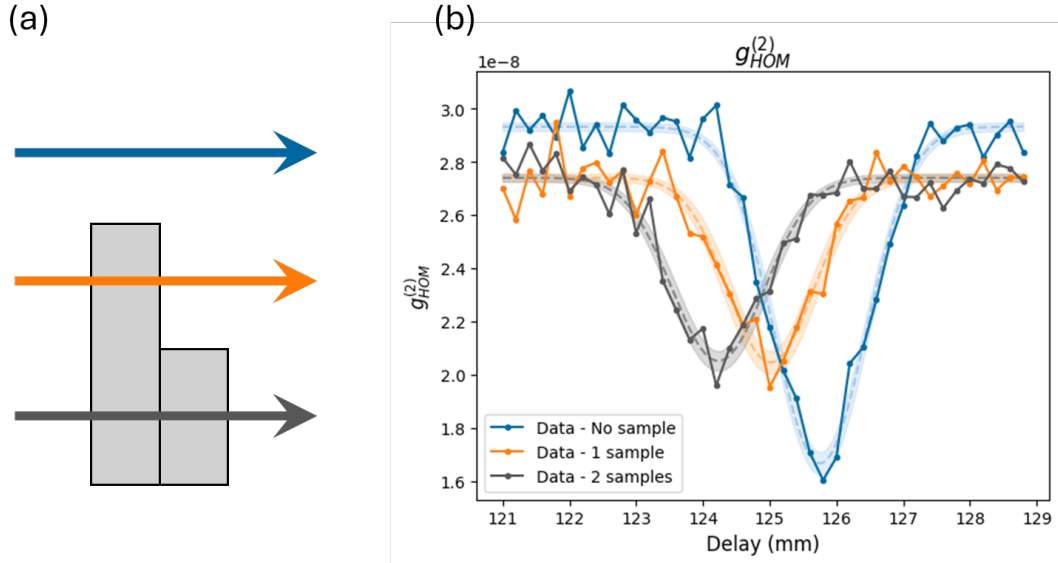


Figure 3.12: Comparison between the HOM dip for free space, one and two acrylic samples. (a) Visual representation of the samples. (b) HOM dip for the three cases. The dashed lines correspond to the Gaussian fit to each set of experimental data.

Table 3.1: Summary of the relevant parameters regarding the data on Fig. 3.12b.

Data	Center (mm)	FWHM (mm)	Visibility (%)	Offset ($\times 10^{-8}$)
No sample	125.76 ± 0.02	1.63 ± 0.06	43.1 ± 1.3	2.933 ± 0.016
One sample	125.01 ± 0.04	1.52 ± 0.11	25.3 ± 1.5	2.742 ± 0.016
Two samples	124.23 ± 0.04	1.63 ± 0.10	25.1 ± 1.2	2.741 ± 0.014

First, let us look at the main goal of this measurement: the determination of the thickness of the samples. By analyzing the drift in the center of the dip, we get the difference in optical path, $\Delta x'$. Since this value is related to the phase shift that the sample inputs on the photon's wavefunction, we need to take into consideration the variation in refractive index (Δn). Having said this, we can conclude that the thickness (Δx) is given by:

$$\Delta x = \frac{\Delta x'}{\Delta n}. \quad (3.18)$$

Taking this into account and considering that the refractive index of acrylic is 1.495 [135], we obtain:

$$\begin{aligned} \Delta x_{0,1} &= (1.497 \pm 0.096) \text{ mm}, \\ \Delta x_{0,2} &= (3.08 \pm 0.09) \text{ mm}, \\ \Delta x_{1,2} &= (1.58 \pm 0.11) \text{ mm}, \end{aligned} \quad (3.19)$$

where the subscripts represent the dips we are analyzing, for example, $\Delta x'_{0,1}$ is the difference in optical path between the data with one sample and none.

As expected, the thickness of the samples is around 1.5 mm . Conversely, we could take our *a priori* estimate of the thickness and calculate the refractive index, achieving the following results:

$$\begin{aligned} n_{0,1} &= 1 + (0.49 \pm 0.03) , \\ n_{0,2} &= 1 + (0.508 \pm 0.015) , \\ n_{1,2} &= \underbrace{1}_{n_{air}} + \underbrace{(0.52 \pm 0.04)}_{\Delta n_{1,2}} . \end{aligned} \tag{3.20}$$

Secondly, we would like to direct the attention to the FWHM of the dips. We can see that the values are very similar, with a standard deviation of 0.023. As mentioned before, the width of the dip is related to the coherence length of the photons (or the length of the wave packet) and thus remains rather unaltered by these simple samples.

Finally, we take a closer look at the visibilities and offsets. It is clear that, upon inserting the samples, the visibility and the offset decrease significantly. The dips related to the samples (both one and two) present similar values for these quantities, indicating that whatever is causing this worsening in their quality does not depend on the thickness of the sample (eliminating the possibility of absorption). Our hypothesis is that there might be some reflection in the samples and maybe some diffusion; there is also the chance that the samples were not placed perfectly perpendicular to the trajectory of the photons and thus slightly deviated the beam, consequently decreasing the detection rates and introducing some path distinguishability. Regardless of the reason, this change in offset could represent a hurdle for the time optimization of the sensing and imaging using this technique, although, for simpler cases, we can perform quick measurements of the full HOM dip and evaluate the best approach. For more complex samples, we may be doomed to either sacrifice the precision of our measurement or take a very long time to acquire the data.

3.4. Concluding Remarks

In this chapter, we explored the HOM effect as both a conceptual and practical tool for quantum imaging. We showed how the position, shape and visibility of the dip encode path delay and sample-induced distinguishability, and we built a simple statistical model

that allowed us to link those observables to the coherence length of the photons and their overlap.

On the experimental side, we detailed how to assemble the interferometer from scratch and all the necessary steps to experimentally observe the HOM dip. A full characterization of the single photon source was performed to study in detail how the temperature and pump power affect the indistinguishability of the photons and, consequently, the visibility of the HOM dip. The setup was further validated for HOM-based quantum sensing by measuring the thickness and refractive index of a known sample, having obtained an optical path precision of $15\ \mu\text{m}$ regarding the determination of the center of the Gaussian fit*, replicating successfully recent work reported on the literature for this research field [81, 98] even though we still have some room for improvement when we look at hyper-precision procedures [136, 137].

Furthermore, this initial work with a HOM interferometer allowed us to clarify some practical limits of our configuration, such as the narrowband source reducing the sensitivity to small path delays and the sample interfaces introducing reflections and deviating the single photons which reduces the dip depth. With this groundwork in place, the next chapter turns HOM contrast into a targeted imaging modality for birefringent samples, exploiting the clean mapping between polarization-induced distinguishability and visibility to produce quantitative, depth-robust contrast.

More generally speaking, HOM-based sensing has some advantages over classical methods that were highlighted in our tests, namely the absence of an absolute reference or phase stability, the resilience and high SNR (54.0 or 17.3 dB on average in our system) and the possibility to use low power light and non-damaging wavelengths [98].

*Since we are using a narrowband source, it would be more realistic to accuse a higher value for precision, in the order of hundreds of μm perhaps. The value we presented was, as mentioned previously, merely the uncertainty of the center of the Gaussian fit.

4. HOM-based Birefringent Imaging

The HOM interferometer explores controlled degrees of distinguishability between single photons to probe properties of a sample placed in one arm. In the previous chapter, we explored the theory of the effect and used it in a simple sensing application to measure path delays and infer refractive-index changes introduced by a known sample. However, because our single-photon source is narrowband, the setup proved relatively insensitive to small path delays, motivating us to explore another contrast mechanism. In this chapter, we extend HOM microscopy from axial path sensing to polarization contrast, further exploring Fisher Information (FI) and Maximum Likelihood Estimator (MLE) tools to identify optimal operating points (near the dip minimum) and estimators. Birefringent media imprint different phase delays on orthogonal polarization components, which in a two-photon interferometer translates into tunable distinguishability between the signal and idler photons. Since HOM coincidence visibility is suppressed in proportion to the polarization overlap, birefringence appears as a change in dip depth, while the dip position remains primarily set by path delay. In short, path and polarization perturbations separate into complementary channels: the dip center encodes optical thickness, and the visibility encodes polarization rotation/retardance.

4.1. Theoretical Framework

We start this chapter by extending the previous model for birefringent samples. A birefringent material is characterized by having different refractive indices depending on the polarization of the propagating light. For an arbitrary polarization state, this means that one component may propagate with a different speed and thus rotate the polarization of the beam. This variation in polarization will make the photon pair deviate from indistinguishability, which can in principle be observed with HOM microscopy. We can picture it by considering that only the projection along the idler's polarization contributes to the interference, while the orthogonal term obstructs the phenomenon:

$$\begin{aligned}
 |\psi\rangle_{1,2} &= \hat{a}_{1,H}^\dagger \left(\cos \theta \hat{a}_{2,H}^\dagger + \sin \theta e^{i\phi} \hat{a}_{2,V}^\dagger \right) |0\rangle \\
 &\stackrel{(\text{Eq. 2.7})}{=} \frac{1}{2} \left(\hat{a}_{3,H}^\dagger + \hat{a}_{4,H}^\dagger \right) \left[\cos \theta \left(\hat{a}_{3,H}^\dagger - \hat{a}_{4,H}^\dagger \right) + \sin \theta e^{i\phi} \left(\hat{a}_{3,V}^\dagger - \hat{a}_{4,V}^\dagger \right) \right] |0\rangle
 \end{aligned} \tag{4.1}$$

$$\stackrel{(\text{Eq. 2.2})^*}{\implies} g^{(2)}(0) \propto \sin^2 \theta. \tag{4.2}$$

resulting in a version of the Malus's law for the second-order correlation function. The coefficients θ and ϕ are related with the action of the half-wave plate, where θ is the rotation angle of the linear polarization and ϕ is a trivial phase. We can see that the depth of the HOM dip will decrease (higher $g^{(2)}(0)$) for larger rotations from the original axis (higher θ). We can also clearly retrieve the original result of $g^{(2)}(0) = 0$ if $\theta = 0^\circ$, i.e., no rotation of polarization and indistinguishable photons. Furthermore, for the maximum value of $\theta = 90^\circ$, the correlation becomes maximum, as well as the number of coincidences since the HOM effect is mostly lost.

Although the simple model obtained with Eq. 4.2 is valid, we can perform a more in-depth analysis of our system, starting with the action of the birefringent sample on our signal photon, which we can write as the following unitary Jones matrix:

$$J(\theta') = \begin{pmatrix} \cos \theta' & -\sin \theta' \\ \sin \theta' & \cos \theta' \end{pmatrix} \begin{pmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2} \end{pmatrix} \begin{pmatrix} \cos \theta' & \sin \theta' \\ -\sin \theta' & \cos \theta' \end{pmatrix}, \quad (4.3)$$

where δ is the retardance (phase delay between its orthogonal principal axes) and θ' is the fast-axis orientation measured from the laboratory horizontal, that will apply a rotation of $\theta = 2\theta'$ on the signal photon's polarization [138].

Following the approach we took for simpler cases, we now describe the total operator that will act on the input state, which is now sensitive to polarization. Assuming that it starts with a linear horizontal polarization on both photons, where only the photons in the signal arm ($|\cdot\rangle_1$) are affected by the sample, we have:

$$\begin{aligned} J_{op}(\theta' = \theta/2) = & |\phi_1 H\rangle_{1\ 1} \langle \phi_1 H| \otimes J(\theta/2) \otimes |\phi_2 H\rangle_{2\ 2} \langle \phi_2 H| \otimes I + \\ & + |\phi_1 H\rangle_{2\ 2} \langle \phi_1 H| \otimes I \otimes |\phi_2 H\rangle_{1\ 1} \langle \phi_2 H| \otimes J(\theta/2), \end{aligned} \quad (4.4)$$

where I is the identity. After rewriting our state as $|\psi\rangle \rightarrow J_{op}(\theta/2) |\psi\rangle$ and repeating the procedure detailed on the previous chapter, we apply the BS and update our projector

* $\hat{n}_i = \hat{n}_{i,H} + \hat{n}_{i,V} = \hat{a}_{i,H}^\dagger \hat{a}_{i,H} + \hat{a}_{i,V}^\dagger \hat{a}_{i,V}$

operators as:

$$\begin{aligned}
 P_{34}^{pol} &= |\phi_3 H \phi_4 H\rangle_{3,4} \langle \phi_3 H \phi_4 H| + |\phi_3 V \phi_4 H\rangle_{3,4} \langle \phi_3 V \phi_4 H| + \\
 &\quad + |\phi_3 H \phi_4 V\rangle_{3,4} \langle \phi_3 H \phi_4 V| + |\phi_3 V \phi_4 V\rangle_{3,4} \langle \phi_3 V \phi_4 V| \\
 P_{43}^{pol} &= |\phi_4 H \phi_3 H\rangle_{3,4} \langle \phi_4 H \phi_3 H| + |\phi_4 V \phi_3 H\rangle_{3,4} \langle \phi_4 V \phi_3 H| + \\
 &\quad + |\phi_4 H \phi_3 V\rangle_{3,4} \langle \phi_4 H \phi_3 V| + |\phi_4 V \phi_3 V\rangle_{3,4} \langle \phi_4 V \phi_3 V| \\
 P_c^{pol} &= P_{34}^{pol} + P_{43}^{pol}.
 \end{aligned} \tag{4.5}$$

Then, the coincidence probability is given by the usual Malus' law:

$$p_c^{pol} = \frac{1}{2} \sin^2 \left(\frac{\delta}{2} \right) \sin^2 \theta, \tag{4.6}$$

which supports the validity of Eq. 4.2, given that half-wave plates have $\delta = \pi$.

Analogous to the previous chapter's steps, it is straightforward to show that the coincidence probability will be given by

$$P_c(\Delta x, \theta) = \frac{\beta}{2} \left(1 - \alpha e^{-\frac{\Delta x^2}{l_c^2}} \cos^2 \theta \right), \tag{4.7}$$

where α measures the maximum visibility obtained experimentally and β is a parameter that takes into account every source of absorption/ photon loss by the sample, as explained in the previous chapter. This equation shows that the HOM dip's visibility follows a $\cos^2 \theta$. This theoretical prediction is depicted in Fig. 4.1 with $\beta = \alpha = 1$.

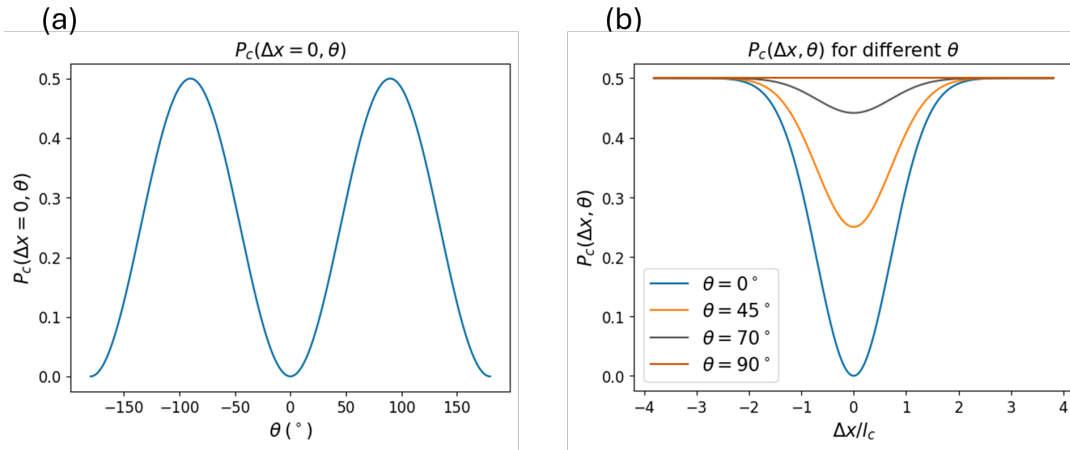


Figure 4.1: Effects of the polarization difference in the HOM dip. (a) Variation of the coincidence probability (akin to visibility) with the difference in angle between the photons' polarizations. (b) Evolution of the coincidence probability upon changing the optical path for different fixed angles between the photons' polarizations.

4.1.1 Fisher Information (FI)

To explore the best way of extracting the birefringence parameters from the raw photon-count records, we followed standard multinomial-estimation theory, as explored in Ref. [81], and considered that the probability of one of the photons being lost is giving by γ (taking into account loss terms makes the predictions more accurate, bridging the gap between theory and experiment). Furthermore, we also assume that the HOM interferometer has three mutually exclusive outcomes: no clicks on either detector ($i = 0$), one of the detectors clicks ($i = 1$) and both detectors clicks ($i = 2$). The probabilities of these outcomes can be written as function of the coincidence probability $P_c(\Delta x, \theta)$ as

$$\begin{pmatrix} P(0|\theta) \\ P(1|\theta) \\ P(2|\theta) \end{pmatrix} = \begin{pmatrix} \gamma^2 & \gamma^2 \\ 2\gamma(1-\gamma) & 1-\gamma^2 \\ 1-\gamma^2-2\gamma(1-\gamma) & 0 \end{pmatrix} \begin{pmatrix} P_c(\Delta x, \theta) \\ 1-P_c(\Delta x, \theta) \end{pmatrix}, \quad (4.8)$$

where $1 - P_c(\Delta x, \theta)$ is interpreted as the probability of measuring a bunching event and $P(i|\theta)$ corresponds to the probability of measuring i events giving that the sample introduces a rotation of θ on the signal photons' polarization.

The FI quantifies how much information the experimental data has about the parameter that we want to measure, in this case θ [139]. For the present case and considering the statistical model presented above, the FI is calculated as

$$\begin{aligned} F_\theta &= \sum_i \frac{(\partial_\theta P(i|\theta))^2}{P(i|\theta)} \\ &= \frac{\alpha^2 (\gamma - 1)^2 (\gamma + 1) (1 - \cos 4\theta)}{2 \left(V(\theta) - e^{\frac{\Delta x^2}{l_c^2}} \right) \left(\gamma V(\theta) - V(\theta) - 3\gamma e^{\frac{\Delta x^2}{l_c^2}} - e^{\frac{\Delta x^2}{l_c^2}} \right)}. \end{aligned} \quad (4.9)$$

The FI is directly linked to the Cramér-Rao inequality that establishes a lower bound on the variance σ_θ^2 of any unbiased estimator as $\sigma_\theta^2 \geq F_\theta^{-1}$ [140, 141]. This kind of study has been conducted for many quantum and classical technologies, in order to extract the maximum information from the systems and increase precision [59, 81, 142].

For this system, the FI is maximized for $\Delta x = 0$. This is to be expected, as the bottom of the dip corresponds to the position that enables the tracking of the visibility, associated with θ , with the maximum dynamic range that we can achieve experimentally.

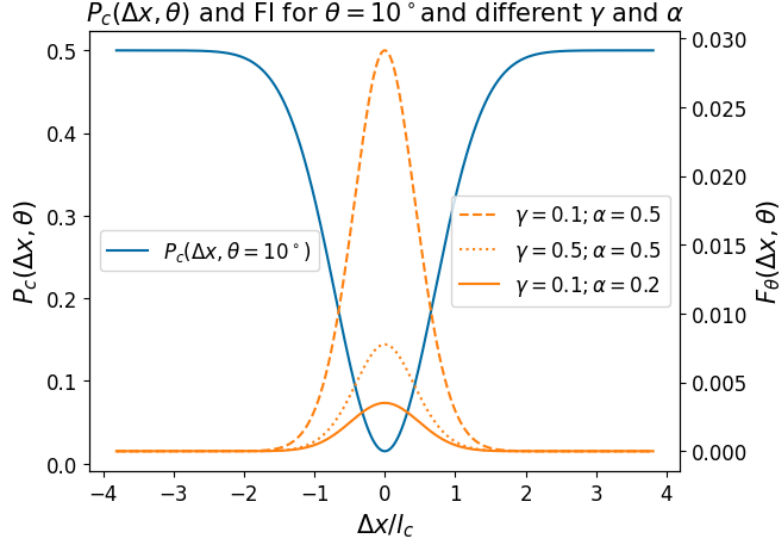


Figure 4.2: Coincidence probability and FI upon changing the optical path for a fixed $\theta = 10^\circ$ and different γ and α .

Additionally, we can easily observe in Fig. 4.2 some other intuitive conclusions about our system, such as the fact that a lossy system (higher γ) will provide less information, as well as one with less visibility (lower α). However, the former is related to the overall loss in statistics, which would undoubtedly reduce the amount of information we can extract; whereas the latter reduces greatly the performance of our system, diminishing its dynamic range and precision.

4.1.2 Maximum Likelihood Estimator (MLE)

After $N = N_0 + N_1 + N_2$ trials, the data is fully described by the counts $\mathbf{N} = (N_0, N_1, N_2)$, whose probability distribution is multinomial,

$$L(\theta) = \Pr(\mathbf{N}|\theta) = \frac{N!}{N_0! N_1! N_2!} \prod_{i=0}^2 P_i(\theta)^{N_i}, \quad (4.10)$$

because the events are independent. The MLE, in this case denoted by θ_{MLE} , maximizes $L(\theta)$ or, equivalently, its logarithm $\ell(\theta) = \sum_i N_i \ln P_i(\theta)$. For a single parameter, the MLE satisfies the stationary condition $\partial \ell(\theta) / \partial \theta = 0$, and solving that equation in order to θ gives us the following expression

$$\alpha \cos^2 \theta_{MLE} = \underbrace{\frac{\gamma N_1 - N_1 + 3\gamma N_2 + N_2}{(\gamma - 1)(N_1 + N_2)}}_V e^{\frac{\Delta_x^2}{2l_c^2}}, \quad (4.11)$$

for $\Delta x = 0$, point of maximum information, V is the visibility that varies from point to point on the sample. Thus, for imaging the birefringence of a sample, the sample can be raster-scanned in the transverse plane, and, by measuring the respective N_i counts, the value of θ can be estimated. However, as detailed in the previous chapter, in order to remove the local absorption of the sample, we followed a different approach for measuring the local visibility. In particular, for every pixel (m, n) , two images were acquired: a dip frame at $\Delta x \approx 0$ and a baseline frame at $|\Delta x| \gg l_c$, where both were integrated for equal durations. The normalized visibility is then obtained as

$$\begin{aligned} V(\theta)_{(m,n)} &= \frac{C_{\text{base}}(m, n) - C_{\text{dip}}(m, n)}{C_{\text{base}}(m, n)} \\ &= \frac{NP_c(\infty, \theta_{m,n}) - NP_c(0, \theta_{m,n})}{NP_c(\infty, \theta_{m,n})} \\ &= \alpha \cos^2 \theta_{m,n} , \end{aligned} \quad (4.12)$$

which ideally ranges from 0 (no birefringence) to 1 (orthogonal polarizations) and confirms our previous inference. Once again, the visibility does not depend on β , reiterating that this system can be used with lower-intensity light. We can also go a step further and infer that we can retrieve the angle difference between the photons' polarizations as

$$\theta = \arccos \left(\sqrt{\frac{V}{\alpha}} \right) . \quad (4.13)$$

Furthermore, the sample also introduces a path delay on the signal photon; however, for samples with small profile variations, and since our single-photon source is narrowband, this delay is negligible. More precisely, for the Gaussian temporal model considered here, the indistinguishability factor due to the delay is $e^{-\Delta x^2 / l_c^2}$, which for a change of 1% would require a path delay of $\delta x \sim 100 \mu\text{m}$, assuming $l_c = 1 \text{ mm}$.

4.2. Implementation of Imaging Setup

For the new setup, there were no changes regarding the polarization, but adaptations on the imaging part. We placed a 2D moving stage between a $4f$ lenses system (composed of two 50 mm plane-convex lenses), to better focus the beam on the samples, and included a white light-emitting diode (LED) and CCD with a polarizer for reference. One might notice that, in Fig. 4.3a, the labels of those last components are written in red; that is because these are not used during the imaging, only to take a reference picture. The camera is also used in the alignment phase and, using the laser and the LED, we adjust the camera to verify where the beam will hit the sample.

Regarding the $4f$ system, it is important to mention that, although the spot reaches its minimum size at the focal point of the system, that is also where it creates a larger speckle pattern with the smallest grain size and the highest contrast, corresponding to the strongest modulation of spatial frequencies [143–146]. A simpler way to visualize the problem, that is more evident experimentally, is that the spot size of our beam becomes of the order of the structures we are sampling, which deforms and diffracts the beam, making it more sensitive to small defects. As a compromise, we picked thinner samples and placed them just a bit after the focal point of the first lens of the system. Consequently, the pictures taken with the camera were slightly blurry but the statistics for the HOM dip, as we shall show in the next section, allowed visibilities up to 73.5 %.

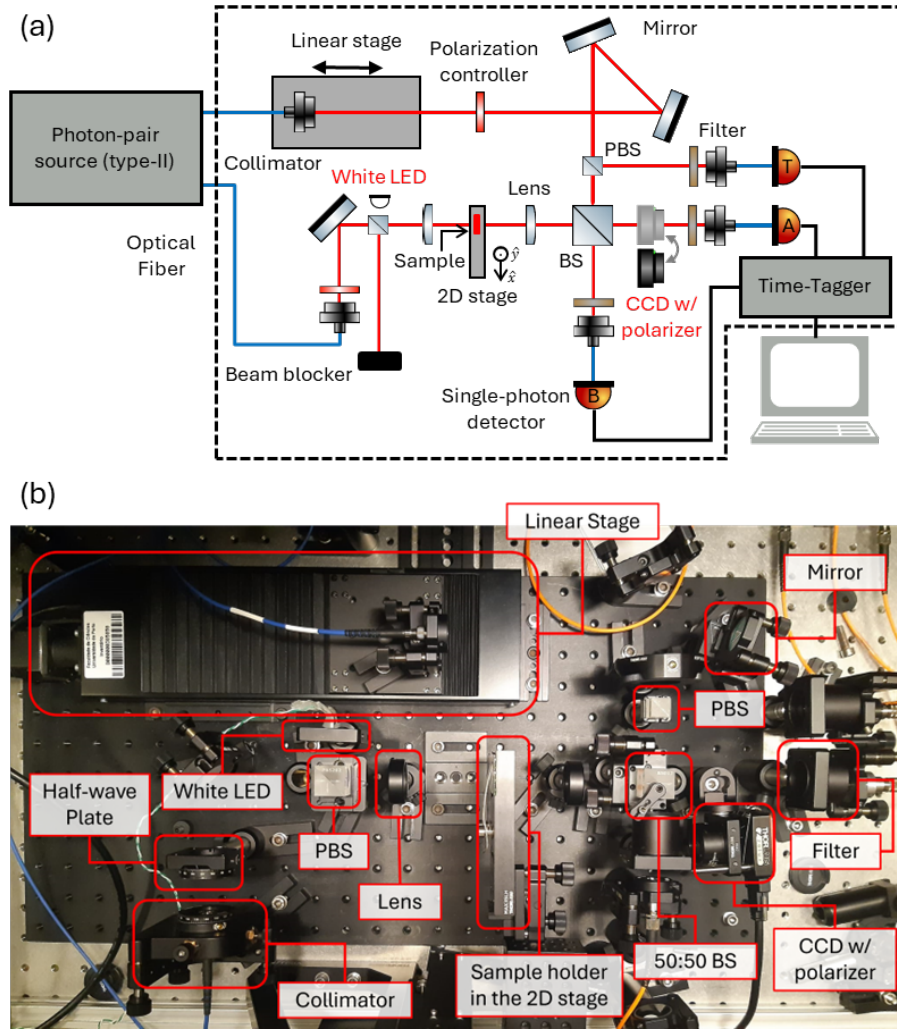


Figure 4.3: Setup for the HOM microscope for birefringent imaging. (a) schematic diagram of the experimental setup, where the dotted line represents the black box we used to reduce the noise from background light; (b) image detailing the main setup inside the box (excluding the detectors and the time tagger). The blue lines represent optical fiber; the red ones, light propagating in free space; and the black ones are electrical signal paths.

We need to keep in mind that this methodology assumes a constant optical path length, i.e., the thickness and effective refractive index do not change throughout the sample. This is facilitated by the fact that our source is narrowband, making it less sensitive to variations in optical path. It would require a much more time-consuming, full measurement of the dip per pixel in order to probe both quantities. This could constitute future works with this setup or with even more degrees of freedom, such as spatial mode and perhaps non-linear phenomena.

4.3. Experimental Results

This section includes all the steps taken towards the realization of our goal: birefringent imaging. We can divide into three main categories where we:

1. Applied the imaging system to replicate a sample with varying thickness,
2. Tested the viability of using the visibility of the HOM dip to probe the polarization rotation, following the conclusions from Section 4.1,
3. Evaluated the efficiency of the birefringent imaging, decoupling that quantity from the optical path and conducting some tests to show the accuracy of this technique to determine the absolute angle of rotation applied to the signal photon.

4.3.1 Optical Path Imaging

Before jumping straight into birefringent imaging, we performed a quick measurement of a sample made up of two microscope slides glued to each other with some displacement, as to test the imaging system on three regions (similar to the test in Fig. 3.12). For this reconstruction, we measured the coincidences in ten positions of the linear stage (with almost 4 s of acquisition time and a coincidence window of 1 ns), in order to more accurately detect the position of the dip, which is represented in Fig. 4.4b. Although the reconstructed image does not have much spatial resolution (as it is just a 10×10 image), we can still easily differentiate the zones where we have free space, one and two slides. It is especially interesting to observe the part of the slide that is not completely in overlap with the other and still correctly identify its thickness. We can also observe in the colorbar of the second image that the position of the dip (values in mm) did not vary significantly, similarly to what we obtained in Fig. 3.12; this points once more to the narrowband nature of our source, which decreases our precision.

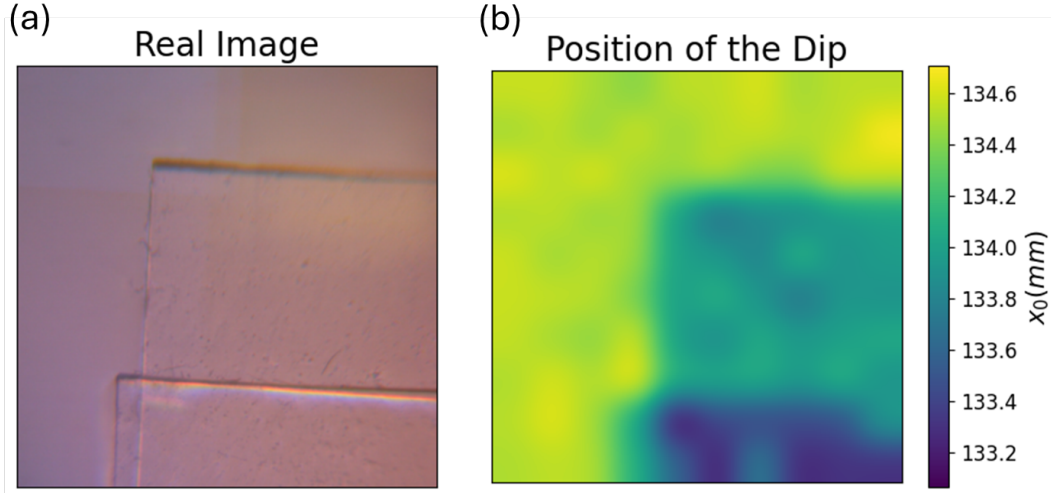


Figure 4.4: Thickness profile of two microscope slides glued with displacement. Figure a) is the image given by the CCD with a polarizer and b) is the reconstructed image given by the center position of the HOM dip.

4.3.2 Validation of the Polarization Sensing Mechanism

In order to verify our calculations and theoretical predictions, we performed a quick test by placing a rotating half-wave plate on the signal arm of the interferometer (Fig. 3.6) and measuring the HOM dip for different rotations of the half-wave plate (a θ' rotation of the plate induces a $\theta = 2\theta'$ rotation of the polarization axis and $\theta = 0$ does not necessarily mean that the angle difference between the polarizations of the two photons is zero, since we did not calibrate as such). This experiment allowed us to verify that only the visibility is changing (see Fig. 4.5), as predicted by Eqs. 4.7 and 4.12 and Fig. 4.1b.

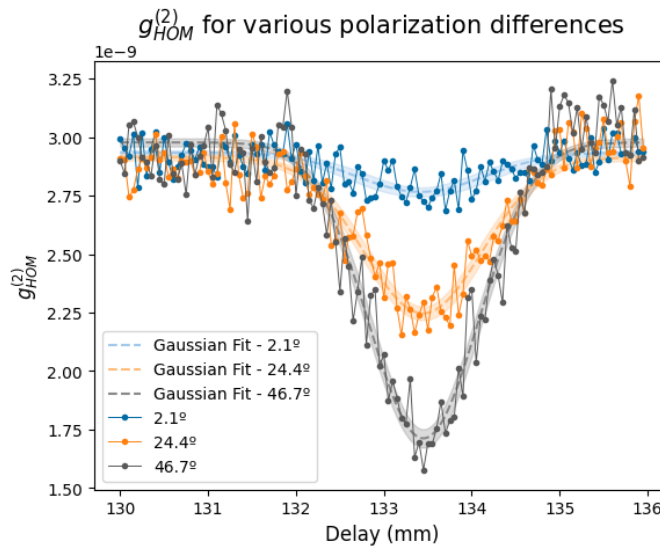


Figure 4.5: HOM dip for various polarization differences between the idler and signal photons. The angles detailed are those of the half-wave plate, θ' . They are the most similar for a rotation of 46.7° (best visibility/ indistinguishability).

Then, fixing the path delay, we measured the depth of the HOM dip (if we had the offset value to normalize the values, it would be equivalent to one minus the visibility). This measurement would help us determine whether the system is dependent and sufficiently sensitive to changes in polarization.

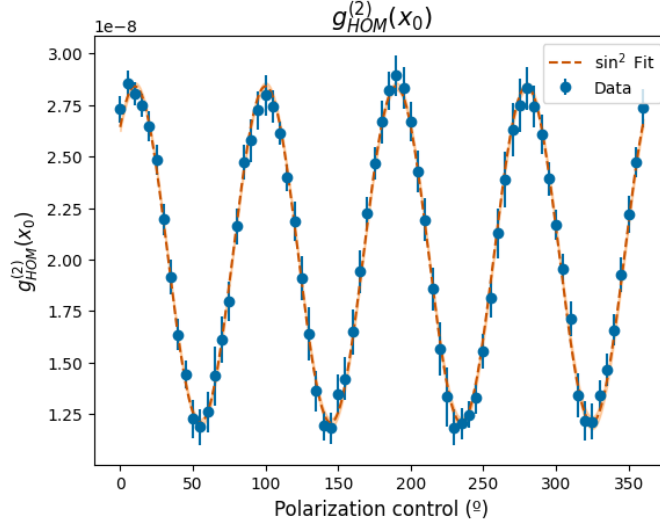


Figure 4.6: Depth of the dip as a function of the angle of the half-wave plate in the signal photon's arm. The polarization control angle induces double the rotation on the photon's polarization.

As we can see in Fig. 4.6, the HOM dip is sensitive to polarization, describing a very well behaved sinusoidal function, similar to the Malus' Law and predicted by Eqs. 4.2 and 4.12 and Fig. 4.1a and confirmed in other works such as Ref. [126]. This indicates that it is feasible to use this technique to image birefringent samples.

In the imaging application, we could find the angle for which the half-wave plate would allow the best visibility of the HOM dip, however, this would be very inefficient and time-consuming. Instead, we settled for smooth samples with fairly constant thickness in order to measure without changing the position in the linear stage and only look at the changes in visibility, given that we just proved that the bottom of the dip would provide the most FI. Although this method leaves room for degeneracies (since the behaviour is periodic and the dip is symmetric), we found this to be the most time-efficient method.

This is rather uncharted territory, although there are already recent developments proving the potential of this technique [75, 147–149]. Nonetheless, it constitutes an important step towards more complex sensing regarding the degree of freedom that is varying and even quantum state analysis.

4.3.3 Birefringent Imaging

Our new sample was a single microscope slide with shards of a half-wave plate rotated by different angles. With similar statistics, we measured the HOM dip for different pieces and found that, akin to the results in Fig. 4.3, the center of the dip did not change significantly; however, the visibility clearly varied (Fig. 4.7). We selected some points on the sample to measure the full HOM dip and better illustrate their behaviour, where we can see their center x_0 barely shifting (standard deviation $\sigma = 0.033$), even between the absence (upper right inset) and superposition (upper left inset) of shards, while the visibility difference is commensurable. Furthermore, we would expect that the center of the dip measured in the slide glass would present the largest x_0 but, the average measurements of the upper center inset was the one to actually exhibit that tendency, which once again brings our attention to the fact that the narrowband source we used is not extremely sensitive to changes in optical path.

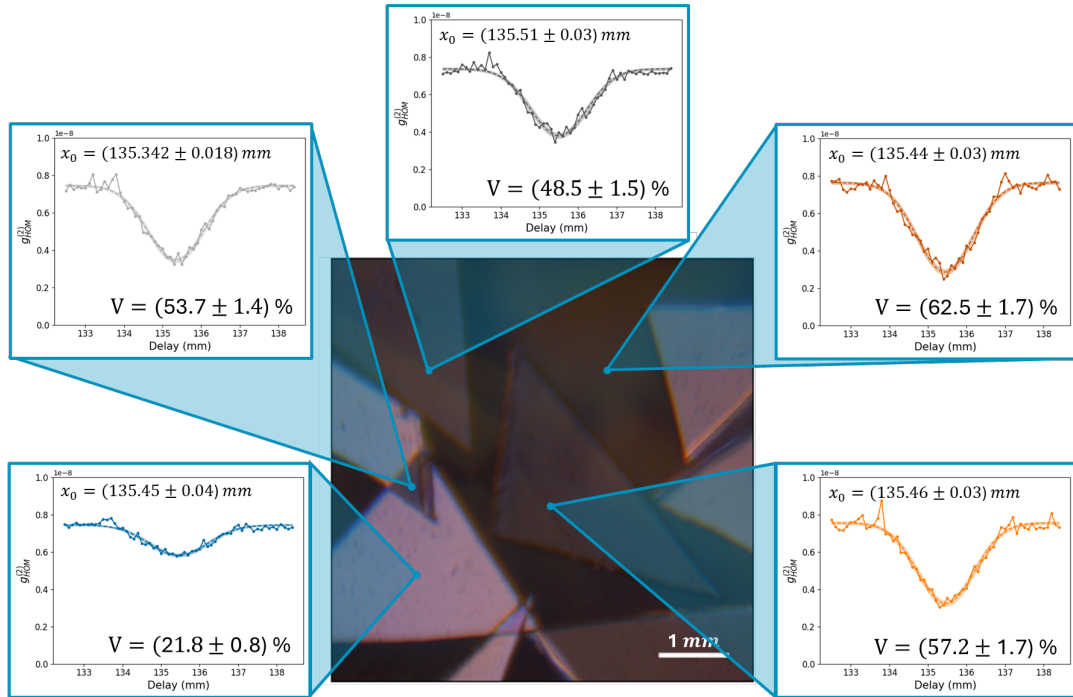


Figure 4.7: Examples of the varying behaviour of the HOM dip throughout our birefringent sample.

We then measured, for the whole sample, the coincidences with the linear stage on the bottom of the dip and far from it (for the offset or reference measurement) and reconstructed the image through the visibility values. Since it included an interesting part where two shards overlapped, we proceeded to take a close-up image of the sample to better illustrate the precision of the procedure. Furthermore, it is worth noting that the edges of

the shards appear with a very low visibility, most likely due to reflection or diffraction. However, it actually works in our favor, as it allows for a good identification of the boundaries of the pieces, even when the difference in the photon's polarization is hardly measurable. Additionally, we must mention that, given the position of the white LED next to the signal arm's PBS, the polarization images we see in Figs. 4.7 and 4.8 are negatives. In order to correct that in the grey-scale polarization image, we represented, from here on, the negative of the original data.

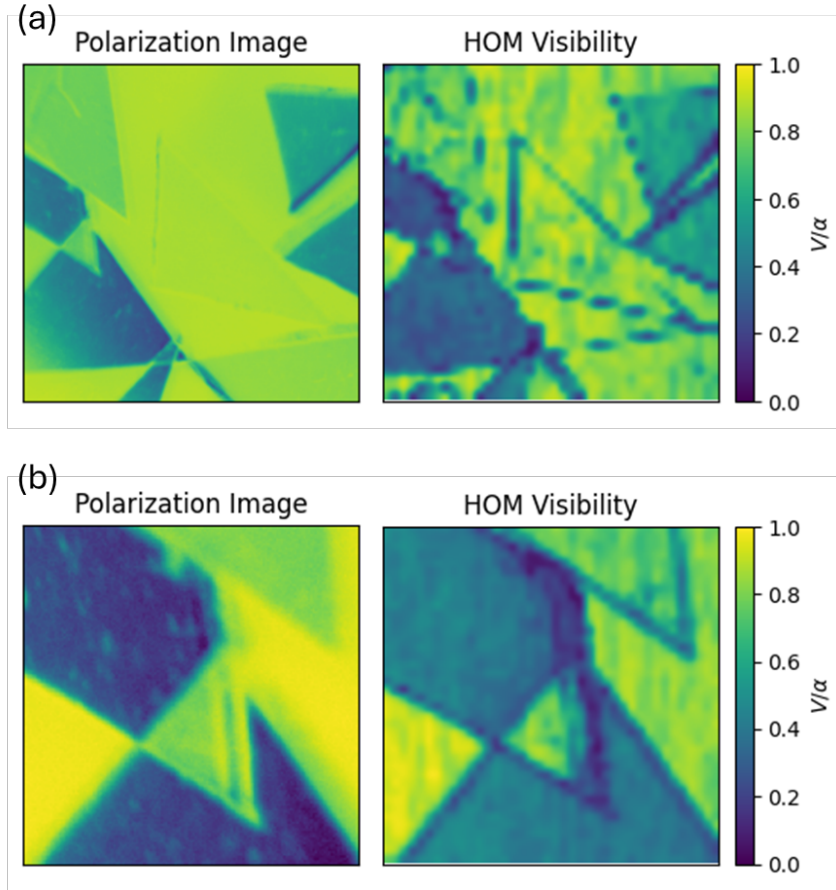


Figure 4.8: Comparison between the images from the camera with the polarizer and the visibility of the HOM dip. (a) Full sample. (b) Close-up around the superposition of shards.

From the previous results, we can see there is a correspondence between the polarization image and the visibility. This observation led us to perform a calibration, finding the relation between these two quantities. In order to take this test, we placed a rotating half-wave plate in the signal arm and measured the HOM dip and took a grey-scale picture on our CCD camera with a polarizer for different angles. This data is represented in the following figure, along with the sinusoidal fit that we found described the data the best. We also input vertical lines with the actual color measured by the camera to better visualize the relation.

By finding which angle of the half-wave plate achieved the best visibility, we figured that must be the angle for which the photons have equal polarization, and so we applied a transformation to represent the results as a function of the angle difference between the polarizations of the photons. This approach could, however, have its associated errors, exposed by the fact that the minimum visibility does not reach zero nor correspond to a 90° difference. Actually, the whole test has a systematic error that we must keep in mind: the half-wave plate is not the most suitable for this wavelength. Even with a polarization camera, which we later got access to, this error is still noticeable. Furthermore, using just a regular CCD with a polarizer becomes a problem concerning the specifications of the camera, such as exposition and gain, which means we would need another calibration for every time we changed them because of a new sample's picture. Having said that, we recognize that these results only constitute a proof-of-concept for this calibration and it requires further testing with the appropriate equipment.

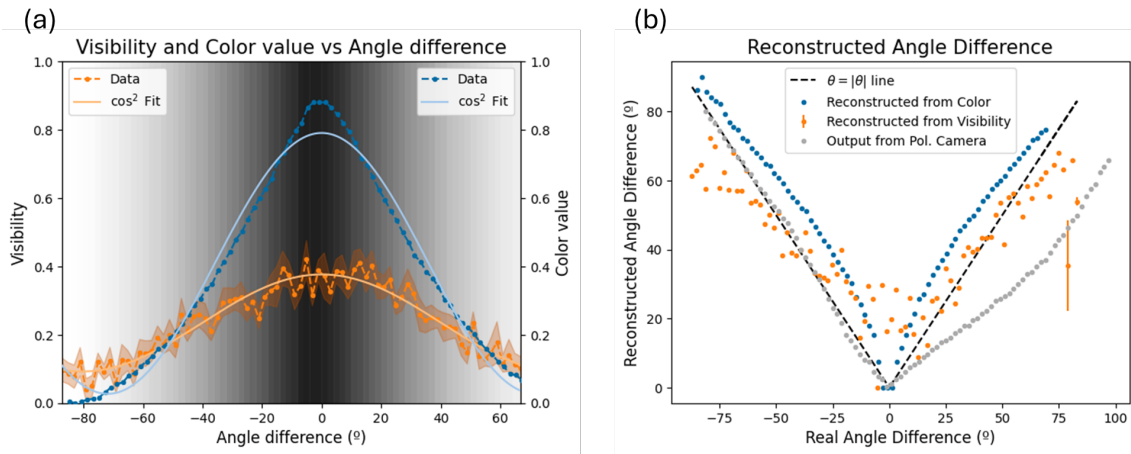


Figure 4.9: Relation between the visibility of the HOM, the color value of the polarization image and the polarization difference.

Using the polarization camera, we took a more accurate polarization image and, along with the conclusions from Eq. 4.13, we represented the angle difference applied by the sample. With our data from the HOM visibility, as seen in Fig. 4.9, we were limited by the periodical behaviour of the visibility, which is symmetrical in relation to the zero angle rotation and originates a degeneracy in visibility, as expected by Eq. 4.12. Even then, this approach with a more suitable equipment, namely the polarization camera, is more useful than to work with the CCD with the polarizer, although more expensive.

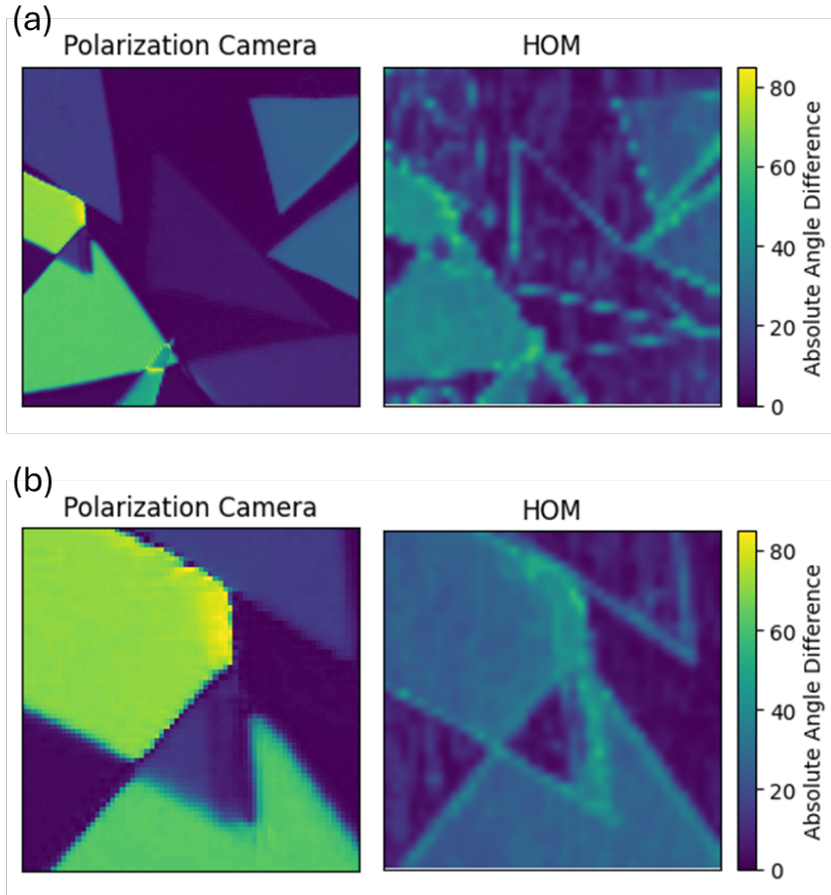


Figure 4.10: Reconstruction of the angle difference (absolute rotation angle) applied by the samples*. (a) Full sample. (b) Close-up around the superposition of shards.

Besides the general advantages of this technique (that were mentioned in the previous section and still hold), we were able to now demonstrate other perks of this setup. The sensitivity to phase and sample-induced distinguishability makes this a very versatile technique with potential to be integrated in numerous fields for various different measurements, and could be subject to future tests for more complex samples are degrees of freedom.

Additionally, the previous calibration attempt allowed us to attain a resolution of 0.09° for the absolute polarization difference between the two photons (uncertainty of the center of the visibility in Fig. 4.9, according to the $\cos^2(\theta - \theta_0)$ fit). High-end commercial polarimeters (e.g., Rudolph Autopol [150], Bellingham + Stanley ADP series [151, 152]) specify angular resolutions up to 0.001° . However, there are still little to no published works quantifying the angular sensitivity of quantum polarization microscopy.

*The polarization camera pictures were taken posterior to the removal of the sample from its original position so the positioning is not perfectly equal to the original data. We did our best to get as close as possible though.

4.4. Concluding Remarks

This chapter elevated the possibility of using HOM interferometry for polarization-contrast imaging. We formalized how birefringence and path delay map onto two separable observables that modify the dip: the dip center for optical thickness and the dip visibility for polarization rotation/ retardance. This allowed us to conclude that both properties can be measured independently, which is possible here because we employ a narrowband single-photon source, less sensitive to differences in optical path.

Extending the physical framework introduced in the previous chapter, we developed a simple statistical model. With the FI and MLE analysis, we identified the dip minimum as the most informative operating point and provided a model-consistent estimator for the experimental data. Experimentally, a raster-scan protocol that acquires paired dip/ baseline frames produced normalized visibility maps on test samples, and targeted measurements confirmed that visibility tracks polarization while the center remains largely unchanged. Using this framework, we demonstrated quantitative birefringence contrast and achieved an angular resolution on the order of 0.09° under realistic laboratory conditions.

At the same time, the data highlighted practical limits and clear upgrade paths. The methodology implemented in this chapter leaves a $\cos^2 \theta$ degeneracy that complicates the retrieval of the rotation angle without auxiliary polarimetry. To lift this degeneracy, additional measurements are required, for example, changing the polarization reference and tracking how the visibility varies between measurements. This idea is currently being explored in the laboratory.

5. Conclusions and Outlook

This thesis aimed to explore and experimentally demonstrate fundamental quantum interference phenomena, in particular the HBT, GRA, and HOM effects, using photon-pair sources. The ultimate goal was to establish the foundations for quantum sensing and imaging protocols using non-classical states of light, which would then allow us to setup a HOM-based birefringence quantum imaging system.

The HBT and GRA experiments confirmed the non-classical nature of our photon source, with $g_{coh}^{(2)}(\tau) = 1.01283 \pm (5 \times 10^{-5})$ and $g_{Fock}^{(2)}(0) = 0.02 \pm 0.02$ respectively, consistent with expectations for coherent and single-photon states (these values correspond to the average and Gaussian fit os the respective data, while the raw data accuse $g_{coh}^{(2)}(0) = 1.011 \pm 0.004$ and $g_{Fock}^{(2)}(0) = 0.0702183 \pm (4 \times 10^{-7})$). These experiments were instrumental in consolidating our knowledge on the theoretical and experimental fundaments of the topic, calibrating our detectors, understanding the code, automating the interface and validating our timing electronics.

Regarding the HOM interferometer, we could observe a clear interference dip with a visibility of $(62.9 \pm 1.2) \%$ and precision of $15 \mu m$ (which we suspect is too little), allowing depth-resolved measurements and the estimation of the spectral bandwidth of the source. The success of the implementation of the HOM-based system was made possible through methodic alignment, troubleshooting and thorough characterization of the source to achieve indistinguishability of the photons. With this setup, we could determine the difference in optical path (which in turn can be used to attain the thickness or refractive index of a sample) through the drift of the center of the dip, and perform birefringent imaging through the variation in visibility of the dip, which went up to 73.5% . We are particularly optimist about the latter, as even the boundaries between samples with similar birefringence and small defects are very noticeable and we were able to provide a proof-of-concept of a calibration between the visibility and the color value. Having said that and noting the we could achieve a precision of 0.09° in polarization difference, this constitutes a very good first step towards developing a quantum polarization microscope.

Although these are hardly recent techniques, we still faced our fair share of setbacks. The implementation of the HOM interferometer required precise temporal and spatial

overlap of the photon paths, so aligning optical components and achieving sufficient indistinguishability was definitely our first challenge. Understanding the functioning of the equipment and processing of its acquired data also took some time but was perfectly expected from a brand-new setup, although we were taken by surprise by the source (as was made clear in Section 3.3.2). Finally, through very compelling discussions and sometimes frustrating testing, the most challenging part was the implementation of the birefringence imaging, as we struggled with the procedure, optimization of the measurements and choice of samples.

It is vital to reinforce the importance of improving quantum sensing and technology, since it has some crucial advantages over classical methods, as highlighted in our tests. From the absence of absolute reference or phase stability, resilience and high SNR to the possibility to use low-power light and non-damaging wavelengths, this sensitivity to phase and sample-induced distinguishability makes HOM-based systems very flexible and could become standard in a very near future, as soon as the price and time-efficiency problems are tackled. Consequently, by building and characterizing our quantum interference setup, this work contributes to the growing experimental toolbox of quantum optics and paves the way for future applications in quantum imaging, metrology, and photonic quantum information processing. Additionally, it constitutes an important step within INESC TEC, as it represents the first experimental work with quantum optical equipment in the research facility.

Since this was the work developed in just a year (cut short by belated arrival of the equipment), we already have some ideas that could be explored in the future:

- Enhance the spectral purity of the photon source to increase HOM visibility;
- Integrate adaptive optics and waveform shaping or use a spatial light modulator for spatial mode tweaking in order to maximize the FI of the input state [59];
- Apply the imaging system to more complex samples (for example, mineral thin sections or biological tissue);
- Experiment with other degrees of freedom that induce distinguishability, such as spatial mode, dispersion and non-linearities;
- Develop a hybrid system that leverages the qualities from HOM and GRA for sensing and imaging;

- Perform a more careful calibration to connect the values of visibility to the difference in polarization angle;
- Devise a new approach that bypasses the degeneracy in polarization angle.

In conclusion, this thesis has laid a solid experimental foundation for the development of quantum imaging systems based on fundamental photon interference effects. Through the design, alignment, and characterization of quantum optical setups, we have taken a meaningful step toward making practical quantum sensing technologies more available and common-ground. We have also started a new chapter in in-house experimental quantum optics that has a lot of room to grow.

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