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How to statistically handle firms with negative output: an analysis of the Portuguese economy

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Abstract

Roughly one-fifth of Portuguese firms report negative firm output (measured as gross value added) each year. Using a full-economy firm-level database for 601,908 Portuguese firms between 2018 and 2023, this dissertation documents how negative output is distributed across sectors, regions and firm-size classes, traces the five-year fate of the 2018-negative-output firms, and reviews treatment options for handling zero and negative values in applied work. Negative output is highly concentrated in real-estate activities and holding companies and is most prevalent in tourism-oriented southern districts and Madeira. Zero-employee entities, representing 21% of all firms, account for almost half of the negative output cases. Only 28% of 2018-loss-makers recover to zero or positive output by 2023, while 26% fail and 25% become temporarily inactive. The study concludes with practical guidelines and recommendations for researchers modelling firm-level data, combining economic treatments and statistical treatments, including the inverse hyperbolic sine (IHS) and log-modulus transformations.

JEL Codes: D22, D24, G33, L25.

Keywords: Negative Firm Output, Statistical Analysis, Portuguese Economy.

Resumo

Cerca de um quinto das empresas portuguesas apresentam, todos os anos, Valor Acrescentado Bruto (VAB) negativo. Utilizando uma base de dados de 601.908 empresas portuguesas entre 2018 e 2023, esta dissertação documenta como o VAB negativo é distribuído entre diferentes setores de atividade económica, regiões e dimensões de empresas, acompanhando o percurso, ao longo de cinco anos, das empresas com VAB negativo em 2018 e revê opções para o tratamento de valores nulos e negativos em estudos empíricos. O VAB negativo está fortemente concentrado em atividades imobiliárias e sociedades gestoras de participações sociais e é mais significativo nos distritos do sul de Portugal orientados para o turismo e na Madeira. As empresas com zero trabalhadores, que representam 21% de todas as empresas, correspondem a quase metade dos casos de VAB negativo. Apenas 28% das empresas com VAB negativo em 2018 recuperam para um valor nulo ou positivo em 2023, enquanto 26% cessam atividade e 25% tornam-se temporariamente inativas. O estudo conclui com orientações práticas e recomendações para investigadores que trabalham com dados empresariais longitudinais, combinando tratamentos de natureza económica e estatística, incluindo as transformações *inverse hyperbolic sine* (IHS) e *log-modulus*.

Classificação JEL: D22, D24, G33, L25.

Palavras-chave: Valor Acrescentado Bruto Negativo, Análise Estatística, Economia Portuguesa.

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1. Introduction

In the Portuguese economy, by 2023, 19.7% of firms reported a negative output, meaning their total turnover was smaller than their intermediate consumption – the consumption before any labor and financial costs were applied.

These are firms with clear financial problems or limited productive activity, which could be temporary if they result from sudden shocks such as natural disasters, which can completely stop sales, while still accruing some intermediate consumption. But negative output can also be persistent and an indication of a systemic problem with the industry or the firm which will result in its closing. Hence, it is important to historically characterize the negative output firms in the Portuguese market and understand how many reported temporary negative output and recovered in the following years, and for how many the negative output was a signal of future closing/bankruptcy.

Negative output poses problems for statistical analysis. In many models, natural logarithms of the variables are used, to deal with outliers and reduce heteroskedasticity. Since the logarithm cannot handle zeros or negatives, for firms with negative values, many times their respective logs are zero-floored, or the observations are dropped. These ad hoc choices systematically exclude or minimize firms in financial distress and distort the interpretation of results. In addition, the possible relationship between negative firm output and technical insolvency (negative equity) is explored.

This dissertation aims to characterize firms with negative output in the Portuguese economy between 2018 and 2023, analyze how it is distributed across sectors, regions and firm sizes, and examine the outcomes of firms that record negative output, including recovery, inactivity or legal exit. It also proposes treatment recommendations for researchers working with firm-level output values, by combining economic based sample refinements, focusing on genuinely productive firms, and statistical solutions that allow non-positive values to be retained in further modelling approaches. These recommendations are then applied to the 2018-2023 firm output Portuguese data.

The analysis uses a dataset of 601,908 firms from SABI, between 2018 and 2023. It combines variables such as identification and classification indicators, key financial values and employment data.

This study reveals that negative firm output is a persistent feature of the Portuguese economy. Across 2018-2023, around one in five firms records negative output in any given year, with cases concentrated in real-estate and holding companies and the tourism-oriented regions. Firms with zero employees represent 21% of all firms but close to half of all negative output cases. Following the firms that reported negative output in 2018, fewer than one third recovered to zero or positive output by 2023, while about one quarter exit through bankruptcy or dissolution and another quarter become temporarily inactive. This suggests that negative output often precedes a business's failure.

This dissertation shows that excluding sectors that do not represent productive activity and removing zero-employee firms significantly reduces the share of negative-output firms, yielding a clearer view of productive capacity. It also illustrates how log-like transformations such as the inverse hyperbolic sine and log-modulus can be used to handle zero and negative output values.

The following sections are organized as follows: Section 2 reviews the literature and formal definitions, Section 3 describes the data and methods used, Section 4 presents a characterization of firm output in Portugal, Section 5 proposes treatments to firm output samples and Section 6 concludes.

2. Literature Review

This section reviews the main concepts and empirical evidence regarding negative firm output. It defines firm output, discusses the causes of negative values, introduces the notion of technical insolvency, and shows statistical issues and proposed solutions to handle non-positive values.

2.1. Firm output

Firm output is used interchangeably with Gross Value Added as the value obtained from subtracting intermediate consumption from turnover. For this dissertation, firm output is the naming used, and is calculated at factor cost, defined by:

$$\text{Firm Output}_{fc} = (\mathbf{TR} + \mathbf{CI} + \mathbf{SUB} + \mathbf{CP}) - (\mathbf{CGSMC} + \mathbf{SES})$$

Where

TR – Turnover

CI – Change in inventories

SUB – Operating subsidies

CP – Capitalized production (work performed for own use)

CGSMC – Cost of goods sold and materials consumed

SES – Supplies and external services

All the following references to firm output use this factor cost version. This means that indirect taxes are excluded, and operating subsidies are included. The opposite happens when calculating output at market prices. Firm output at market prices would be more adequate for studies focused on fiscal or consumer impact, being less useful in this case.

2.2. Causes of negative firm output

Negative firm output may happen in three distinct circumstances: firm-specific pressures, sector wide crises or downturns, and macro-level crises.

2.2.1. Firm-specific pressures

At an individual firm level, firms may report negative output when they are being outcompeted or priced out of the market by rivals. Aggressive competition can directly undermine a firm's performance and output levels. Classic strategic theory notes that when competitive rivalry escalates, firms might engage in competitive moves and countermoves in an escalating fashion, and all competitors might end up worse off (Porter, 1980). Empirically, Derfus et al. (2008) prove this, describing a "Red Queen" effect where a firm's actions cause competitors to retaliate, eventually negating the initiator's gains, sometimes even reaching lower performance than its starting point. Specifically for smaller firms, recent evidence relates competitive pressure to lower output for Small and Medium Enterprises (SMEs) in developing economies (Javadi et al., 2025). This "dark side" of competition suggests that an increase of competitive pressure, through new entrants undercutting prices or rivals innovating, can push an individual firm's turnover below intermediate consumption, causing negative firm output in some years.

Besides competitive pressures, there are many other reasons for firm-specific negative output. There's the case of young firms, which often incur high costs to set up the business and production before any turnover arrives (Yang & Aldrich, 2017).

Also, firms with high customer concentration can experience negative output when losing a major customer. A major customer's bankruptcy leads the supplying firm to face lower operating margins (Kolay et al., 2016), which can tip firm output into negative territory.

Other firm-specific events can cause disruptions in production and nudge a firm into negative output, such as technical outages, cyber-attacks, labor strikes, accidents (especially for firms with manufacturing plants) and loss of human resource talent/skill (Katsaliaki et al., 2021).

2.2.2. Sector wide crises or downturns

A second type of shock stems from industry or sector-wide crises, where an entire sector experiences a downturn and shrinks the output of firms within it. These crises can emerge from structural changes, external shocks or demand slumps that target a particular industry.

The dot-com bubble collapse in 2000 was a sector specific crisis that affected technology/internet firms. After extreme speculation that soared stock prices, the subsequent recession caused bankruptcies and job losses across the sector (Ljungqvist & Wilhelm Jr., 2003). The lack of investment in tech firms during this crash (Khan et al., 2024) most likely brought many to negative firm output.

In a different sector, Woodall et al. (2011) showcase the downturn of the forest products sector in the northern United States from 2000 to 2011. This downfall was caused by a decline in demand for these products, whether by a lack of global competitiveness, or a decrease in print paper (with the popularization of electronics), or decrease in new housing projects (Woodall et al., 2011). The number of wood mill closures increased dramatically, and negative firm output in this sector likely also increased, although this is not explicitly shown in the literature.

Overall, firm output lacks representation in sector wide crises or economic downturns studies. However, the literature shows that a sector in crisis often suffers a sharp decline in sales and profit margins, and an increase in bankruptcies. As such, it is safe to assume that the number of firms recording negative output increased during these moments, due to demand collapse, technological obsolescence, and so forth.

2.2.3. Macro-level crises

The third category includes crises that affect nearly all sectors in a whole region, country, or globally. The global crises tend to create contractions in the economy that diminish output at the firm level, globally in the case of worldwide recessions or pandemics and more regionally regarding natural disasters.

The most recent example of a worldwide crisis is the COVID-19 pandemic, which started in 2020 and created an unprecedented impact on businesses worldwide. In the span of a few weeks, public health measures and the public's concern over the COVID-19 virus led to a suppression of consumer spending, disrupted supply chains and mandated closures of commercial establishments. In turn, firms across various sectors experienced a great drop in output, sales, and other such performance indicators. These negative effects came from both the demand and supply sides (Guerrieri et al., 2022) and the magnitude of this crisis is proven in empirical analyses. With international firm-level financial data, Hu and Zhang (2021) show that financial performance worsened significantly during the COVID-19 outbreak. More specifically, it is proven that Return on Assets (ROA) is negatively associated with the severity of the pandemic (measured by the number of COVID-19 cases). In the United States, in the second quarter of 2020, small business' sales dropped by an average of 29%, with a quarter of those reporting losses of more than 50% (Bloom et al., 2021). Large firms also exhibited financial distress (Ahmad, 2021). Similarly, Shen et al. (2020) found a significant negative impact of COVID-19 on firm performance in Chinese listed companies. Although its impact can be larger in different sectors and countries, large-scale crises like COVID-19 can bring down firm output and eventually increase the number of firms with negative output. The results shown in Section 4 support this, for Portugal, where there was an increase of over ten thousand firms with negative output, and the share of firms with negative output grew by 2 percentage points (19.9% to 21.9%) between 2019 and 2020.

Natural disasters, although a much more regional phenomenon, are still macro-level causes for negative firm output. Wildfires, floods, hurricanes or earthquakes can destroy manufacturing plants, warehouses, retail spaces, etc. These can impose extraordinary costs on firms through the loss of production capacity, raw materials and inventory, and, at the same time, reduce sales (Yamamoto & Naka, 2021). It constitutes a sharp drop in firm output for those affected that can bring many to negative territory. In Portugal, the most common example of this are the severe yearly wildfires, affecting firms mostly in the central and northern region of the country. Sousa (2021) predicts an economic activity decline of up to 3% in municipalities with extensive burnt area. 2017, a year where the central region of Portugal endured particularly extreme wildfires, the facilities of 483 firms were partially or entirely destroyed, and losses were estimated at 260 million euros in property damage (CTI, 2018). These are clearly disasters where firms are forced into negative output. However, it is important to note these are often one to two year occurrences of negative firm output, as firms can get back into a solid financial performance after receiving insurance payouts and special subsidies from the government (Jorge & Zhao, 2024).

2.3. Technical insolvency

This dissertation mentions the term technical insolvency when referring to firms with negative equity. This arises when the total liabilities surpass a fair valuation of the firm's assets (Altman, 1968). This condition is directly diagnosed from the balance sheet and is distinct from cash-flow insolvency (inability to pay debts as they are due).

As such, it is purely an accounting notion, distinguishable from the court-based insolvency proceedings (often called "bankruptcy"), which are legal processes initiated through the courts that determines how a firm's assets are distributed among creditors (DePamphilis, 2019). Although different, these two concepts are connected. In the case of Portuguese insolvency law (Código da Insolvência e da Recuperação de Empresas, Art. 3.º and 20.º), a "manifest excess of liabilities over assets in the last approved balance sheet" (a criterion that is close to technical insolvency) can be invoked by directors of the firm, creditors, or the public prosecutor to request a court-declared bankruptcy.

Technical insolvency is a strong warning signal for bankruptcy. Ohlson (1980) elaborates a probabilistic bankruptcy model, using a negative equity dummy variable which is a significant predictor of failure. Hjelseth et al. (2022) show that negative equity in two consecutive years significantly increased bankruptcy probability for Norwegian firms (2010-2021 data). Fachrudin (2021) finds that a technical insolvency dummy variable improves one and two years ahead insolvency prediction for Indonesian listed firms.

2.4. Statistical issues related to negative firm output

This subsection scans the scientific literature and non-academic sources for specific examples of the treatment of negative firm output values, while Subsection 2.5 looks at broader examples of dealing with non-positive values and showcases proposed solutions to these issues.

The prevalence of negative firm output in firms is significant, not just in Portugal, but also internationally. In Portugal, the share of firms that report negative output sits at around 20% (2018-2023), as shown in Subsection 4.1, while in the United States the share was around 15% for public firms between 2007-2017 (Hartman-Glaser et al., 2019), and in the Flanders region of Belgium the share was 14% in 2019 (Konings et al., 2023). Despite the lack of information in the literature, firms with negative output are prevalent in the global economy and create challenges for multiple statistical modeling and analysis circumstances.

First, growth rates may not be well-defined when firm output reaches non-positive values in standard growth rates and when reaching negative values in symmetric growth rates. To circumvent this issue, Jorge and Zhao (2024) assign zeros to observations with negative firm output, in order to avoid the denominator of the symmetric growth rate from ever being zero.

Second, logarithmic transformations are not defined for non-positive values. These transformations are commonly used to account for outliers. For example, when calculating productivity, both a World Bank policy research working paper (Francis et al., 2020) and a firm-level study in Portugal by the OECD (Mergulhão & Pereira, 2021) drop all non-positive output observations from their respective datasets. Francis et al. (2020) do not remove the zero values because the established logarithmic transformation is $ln(x + 1)$ over $ln(x)$. In a Portuguese policy evaluation report, the authors tested for replacing all negative output values with 1, in order to be able to apply a logarithmic transformation, but instead discarded this analysis to firm output due to probable bias (EY-Parthenon, 2023).

Third, many performance ratios place firm output in the denominator, and non-positive firm output can make these metrics undefined or economically meaningless. One solution is, as discussed previously, dropping all non-positive observations, as is done by Kehrig and Vincent (2017) when studying labor share in the United States. Differently, researchers may change the denominator, redefining the ratio. As an example, Hartman-Glaser et al. (2019) replace the ratio of capital income to firm output with the ratio of capital income to sales as a proxy for firm-level capital shares in their firm-level analysis, allowing for a direct comparison across firms. In a governmental environment, Spain's Ministry of Industry and Tourism instructs that for non-positive firm output values the energy intensity criterion is assessed using a specific fixed value rather than the computed ratio (Ministerio de Industria y Turismo, 2020).

These cases show that negative firm output produces ad hoc choices (zero-floors, exclusions, redefinitions and fixed ratios) that keep models computable but may distort their analysis and interpretation.

2.5. Proposed solutions for dealing with non-positive values

Besides the solutions present in the literature to deal with negative firm output shown in the previous subsection, prior studies also propose approaches for non-positive values in other variables, which could potentially be applied to firm output.

Two common solutions are the inverse hyperbolic sine (IHS) and the log-modulus transformations, which are especially useful when log-like transformations are needed while including non-positive values. This subsection reviews these two main "log-like" transformations designed for variables that may be non-positive, and then briefly summarizes additional approaches used in the literature.

One prominent solution is the inverse hyperbolic sine (IHS) transformation. The IHS is defined, using the natural logarithm, as:

$$\operatorname{arcsinh}(y) = \ln(y + \sqrt{y^2 + 1})$$

It is defined for all real values of y, including $y \leq 0$, since $y + \sqrt{y^2 + 1}$ is always positive. The IHS was originally proposed by Johnson (1949) and introduced to econometrics literature by Burbidge et al. (1988) as an alternative to the standard logarithmic transformations for variables that can bear negative values, like household net worth. It behaves similarly to the logarithm for large magnitudes and is approximately linear for values near zero. As such, and like the logarithmic transformation, it reduces the effect of outliers, while being well-defined at $y = 0$, yielding $arcsinh(0) = 0$ (Bellemare & Wichman, 2020). Due to its properties, the IHS transformation has been used to model outcomes such as earnings or wealth (Burbidge et al., 1988; Caroll & Dynan, 1999; Bellemare & Wichman, 2020).

Another log-like approach is the log-modulus transformation, introduced by John and Draper (1980). This solution maintains the sign of the original observation while applying a logarithmic scale to its magnitude (the absolute value). The transformation can be written as:

$$L(y) = \operatorname{sign}(y) \cdot \ln(|y| + 1)$$

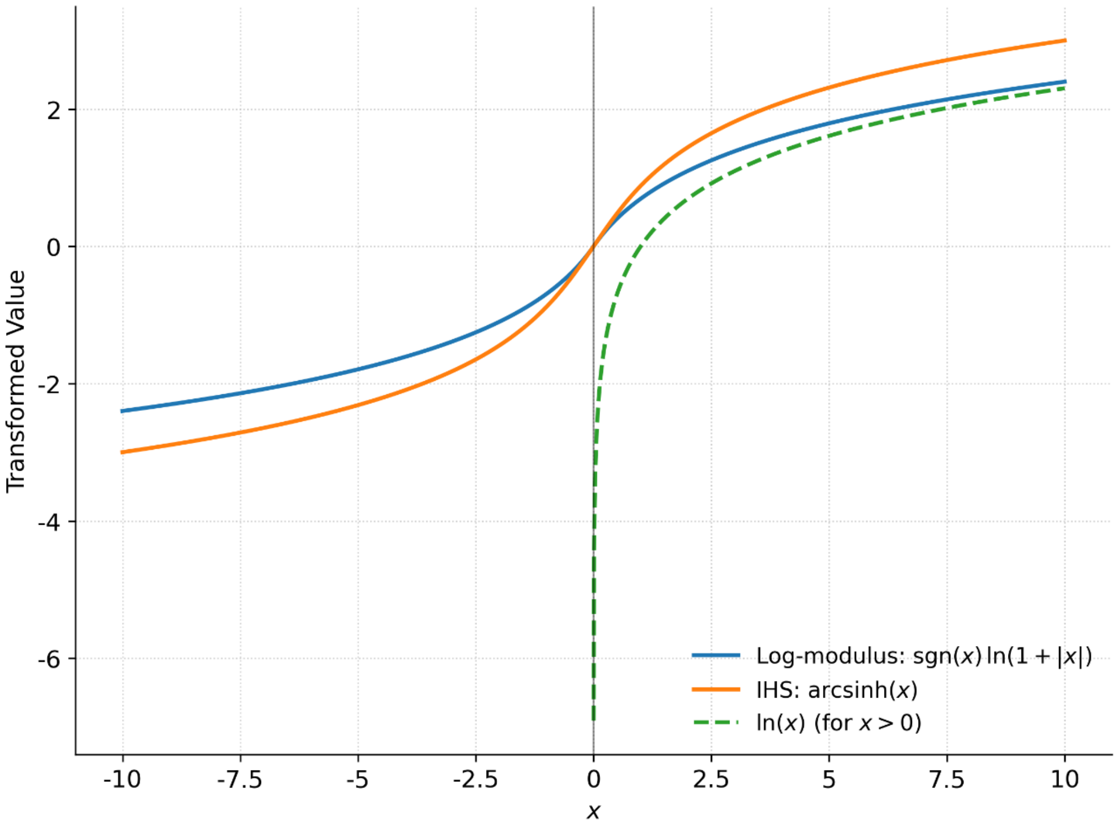

Figure 2-1. Comparison of Log, IHS and Log-Modulus Transformations

It takes the natural logarithm of the absolute value of y plus 1 and then multiplies it by the sign of y so it retains the observation's original sign. Like IHS, log-modulus yields 0 when $y = 0$, is defined for negative values and reduces skewness and scale range (John & Draper, 1980). Although more rarely used in econometrics than IHS, it has been used in financial analysis to perform Knowledge Discovery in Databases (KDD) of financial reports, overcoming problems faced by the use of ratios in financial predictive modelling (Trigueiros & Sam, 2016).

Overall, these two solutions are straightforward to implement and useful options for transforming firm output data to a log-like scale, while including non-positive values. Figure 2-1 illustrates the behavior of both transformations, as well as the standard logarithmic transformation (which is only defined for positive values), over a continuous x.

In addition to these approaches, several others have been proposed in the literature which handle negative values. Yeo and Johnson (2000) proposed a transformation which extends the Box-Cox transformation (Box & Cox, 1964) to accommodate zero and negative values. This method requires estimating a parameter λ through maximum likelihood estimate. A different approach is to add a sufficiently large constant c to the variable so that all values become positive and then applying the logarithm, like $ln(y + c)$. This is a common fix, as Bellégo et al. (2022) notes, 48% of articles published in the American Economic Review (2016-2020) used it when faced with the problem of logs of zeros. However, the choice of c is essentially arbitrary and challenging to justify (Karakaplan et al., 2020). An unconventional solution is to extend the logarithm into the complex number domain. Proposed by Karakaplan et al. (2020), it replaces negative values with their absolute values and acknowledges the logarithm's sign component as an imaginary term. Although an interesting methodological exploration, its economic interpretation becomes difficult due to its extension to complex number domain.

In summary, the literature offers a wide range of solutions for dealing with zero or negative values. Table 2-1 describes the solutions presented, focusing on approaches already applied to negative firm output (from Subsection 2.4) and the two main "log-like" transformations intended for variables that may take non-positive values.

Table 2-1. Potential solutions to handle negative firm output (where Y = firm output)

Group	Solution	Formula/method	Source
A. Applied solutions to negative output	Exclude non-positive Y	Drop Y≤0 from analysis	Francis et al. (2020); Mergulhão & Pereira (2021), etc.
	Zero-flooring	Set Y=0 for Y≤0 (or Y=1 if you intend to log Y)	Jorge & Zhao (2024)
	Ratio redefinitions	Redefine or drop invalid ratios	Hartman-Glaser et al. (2019)
	Fixed ratios	Set ratio to a constant for Y≤0	Ministerio de Industria y Turismo (2020)
B. Negative-capable transformations	Inverse hyperbolic sine (IHS)	$\mathrm{arcsinh}(\mathrm{Y}) = \ln\left(\mathrm{Y} + \sqrt{\mathrm{Y}^2 + 1}\right)$	Burbidge et al. (1988); Bellemare & Wichman (2020)
	Log-modulus	$\mathrm{L}(\mathrm{Y}) = \mathrm{sign}(\mathrm{Y}) \cdot \ln(\lvert\mathrm{Y}\rvert + 1)$	John & Draper (1980); Trigueiros & Sam (2016)

3. Methodology

This section presents the methodological approach used in this dissertation. It outlines the data extraction process, as well as its preliminary procedures. It also shows the treatment of inactive firms and the calculations that were made. Lastly, some limitations are presented for this methodology.

3.1. Data extraction – database description

This dissertation uses firm-level data from the SABI database, provided by Bureau Van Dijk and Informa D&B. The SABI database includes financial and legal indicators of Portuguese and Spanish firms.

The dataset constructed for this dissertation includes all Portuguese firms available in the SABI database with a known value for at least one of the years between 2018 and 2023. This means that each firm can contribute between one and six yearly observations. In total, it consists of 601,908 Portuguese unique firms. The dataset includes firms of three different legal forms, such as *Sociedades Anónimas* (S.A.), *Sociedades por Quotas* (Lda.) and *Sociedades Unipessoais* (Unipessoal, Lda.). It does not cover self-employed individuals who operate under independent contractor regimes (e.g. *recibos verdes*), as these entities fall outside the scope of corporate financial filings.

The variables extracted contain both financial and non-financial firm-level indicators, as listed below:

- Identification and classification variables:
    - Firm name
    - Tax identification number
    - District
    - Municipality
    - Main economic activity (CAE Rev.3 code)
    - Operational status

- Key financial indicators (2018-2023):
  - Firm output
  - Equity
  - Liquidity ratio
  - Cash and cash equivalents
- Employment data:
  - Number of employees (2018-2023)

All the raw values of the dataset were extracted from the SABI database in 2025. 2024 was not considered in this analysis due to the large number of values still missing. All monetary values are reported in thousands of euros.

3.2. Preliminary data procedures

Firms were classified by economic activity using three levels of aggregation derived from the five-digit CAE code. Following the *Instituto Nacional de Estatística* (INE) guidelines on the Portuguese classification of economic activities (CAE Rev. 3), the "Group" column consists of the first three digits of CAE while the "Division" column represents the first two digits. The last column groups every "Division" into 21 different "Sections", from A to U, according to the same guidelines. The bulk of the economic activity-based analysis was done with "Sections", while "Divisions" and "Groups" were used to detail the inner workings of specific values obtained for some sections, as deemed necessary.

The dataset was checked for duplicate firms by confirming that every row contained a unique tax identification number (*NIF*).

3.3. Treatment of inactive firms

It is important to clarify the "Operational Status" variable, as it is used to classify firms as inactive, which is then used in the analysis on the recovery of firms after recording negative output.

This variable indicates each firm's current legal situation. Although it is undated, it reflects the most recent information filed through the annual IES return (*Informação Empresarial Simplificada*). It is composed of the following categories:

- Active
- Temporarily inactive
- Dissolution
- Liquidation
- Insolvency
- Merger
- Legal closure
- Extinction

For this dissertation, two groups are created from these categories, as shown in Table 3-1, named inactive and failed firms.

Although Legal closure and Extinction are not necessarily indicative of a failed-like state, they are aggregated with the Failed group solely for succinct reporting, given their negligible share (together representing 0.07% of observations).

These groupings allow for a clear separation between firms that are currently operating and those that have left the market, whether temporarily or permanently as a result of a merger or a bankruptcy-like situation. This ensures consistency of the recovery analysis in Section 4.

Table 3-1. Grouping of firms by operational status

Group	Operational statuses included	Rationale
Inactive	Temporarily inactive, Dissolution, Liquidation, Insolvency, Merger, Legal closure, Extinction	These firms no longer conduct normal business operations or have been absorbed by another entity.
Failed	Dissolution, Liquidation, Insolvency, Legal closure, Extinction	These legal states typically arise when a company can no longer viably continue.

3.4. Handling of n.a. (missing) values and zero employee firms

Most statistics reported in Section 4 are percentage weights or derived from the raw SABI counts. Two practical issues arise: cells with *n.a.* (missing) values and firms that report zero employees.

3.4.1. Handling of n.a. (missing) values

SABI marks a field as *n.a.* when there is no number registered for that variable for that firm. This can be because the firm was not active in that year, e.g. a company created in 2019 will show as *n.a.* in the 2018 Firm output variable. But it can also be that the firm did not file the relevant item in the corresponding legal return for that fiscal year.

To deal with these missing values, for every weight-based calculation, all *n.a.* values were excluded from the counting of firms, in the numerator as well as the denominator. Concretely, when a share or percentage is reported that satisfies a condition in a given year, the denominator includes only firms that have a recorded value for that variable in that year, and the numerator counts how many of those firms satisfy the condition. Firms for which the relevant variable is *n.a.* in that year are ignored in both counts.

3.4.2. Handling zero employee firms

Roughly 21% of the firms observed (2018-2023 average) report zero employees. Such firms can be holding companies, firms that outsource all labor or newly created firms that have not yet hired staff.

As a baseline rule, firms with zero employees are included in all main indicators, graphs and tables. However, Subsection 4.2 showcases key indicators after excluding these firms, allowing readers to gauge whether their inclusion skews headline results.

3.5. Methodological limitations

Despite the range of the dataset, four structural limitations should be considered when interpreting the results in Section 4.

Firstly, as pointed out in Subsection 3.1, the dataset does not cover self-employed individuals who operate under independent contractor regimes (e.g. *recibos verdes*). Their output is reported in their personal income tax filings, thus falling outside the scope of SABI's database records.

Secondly, the variable "District", which is used in the grouping of firms for regional analysis, identifies the registered headquarters (fiscal address), not necessarily the location of the production. As such, firms with multiple production sites or off-district headquarters introduce some distortion into the regional maps, which must be kept in mind by the reader.

Finally, the sector analysis relies on the main economic activity (CAE Rev. 3 code) reported by the firm. Since firms can register multiple CAE codes, it's important to consider this limitation. This is especially relevant for firms with CAE codes associated with different sections (A to U), which could cause certain industries to appear smaller than they really are. However, since there is no data on the share of output originating from each firm's CAE code, taking the main CAE code only seems to be the best approach to group firms according to economic sector.

These limitations do not invalidate the results in Section 4, but they caution against over-interpreting some of them, especially the regional and sector breakdowns.

4. Firm output in Portugal

This section presents a characterization of firm output in Portugal between 2018 and 2023. It documents the prevalence of negative output, and its distribution across sectors, regions, zero and non-zero employee firms, and examines how negative output relates to firm recovery or exit and technical insolvency.

4.1. General characterization of firms with negative output in Portugal

Table 4-1 summarizes the prevalence of negative firm output across Portuguese firms during 2018-2023. The panel is held constant at the same 601,908 firms per year, with roughly 400-450 thousand of these having a known value for firm output in any given year. About one in five firms reported negative firm output, in every year except 2020, where the share rose to 21.9%, highlighting the first-year impact of the COVID-19 pandemic, as some firms halted or decreased production. Then, gradually, the incidence of negative output reverted to pre-pandemic levels.

The count of firms with missing values for output falls from 200 thousand in 2018 to around 150 thousand in 2023. This pattern reflects the panel design combined with firm demography: every year more companies are created than dissolved in Portugal (INE, 2025). New firms enter the panel with a valid output figure for their first year but naturally show *n.a.* for all preceding years, while dissolving firms report missing values for all years after the year of dissolution.

Table 4-1. Prevalence of negative firm output

	2018	2019	2020	2021	2022	2023
Firms in dataset	601,908	601,908	601,908	601,908	601,908	601,908
Firms with output n.a.	200,424	181,669	171,532	161,242	151,264	154,146
Firms with known output value	401,484	420,239	430,376	440,666	450,644	447,762
Firms with negative output	79,484	83,414	94,237	89,014	89,357	88,056
% of firms with negative output	**19.8%**	**19.9%**	**21.9%**	**20.2%**	**19.8%**	**19.7%**

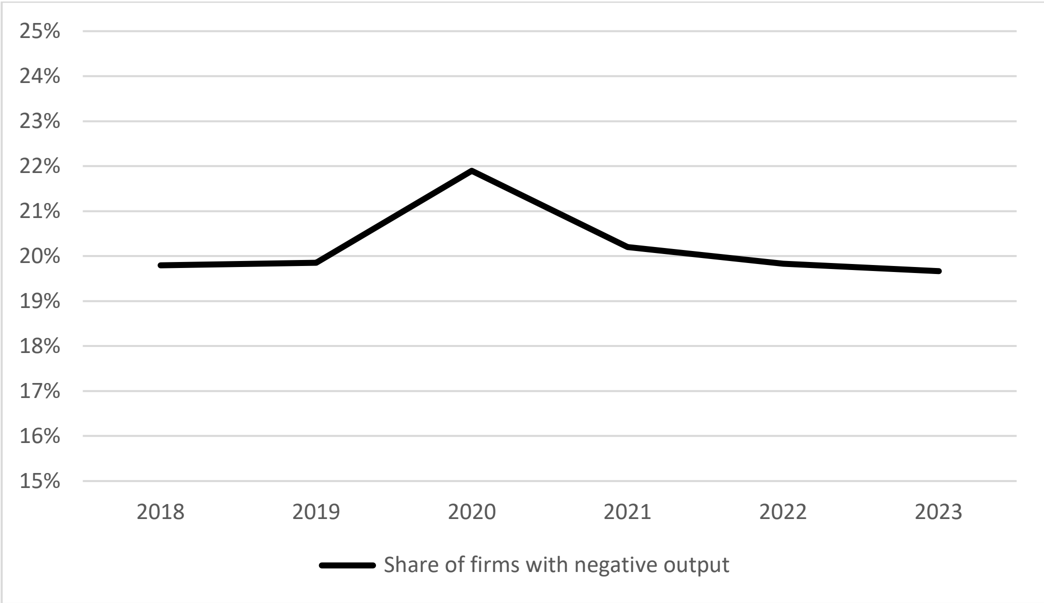

Figure 4-1. Share of firms with negative output

Figure 4-1 plots the share of negative output firms, showcasing the peak in 2020 and the following return to trend.

These figures confirm that negative firm output is not a marginal phenomenon in the Portuguese economy, being a persistent trait affecting about one-fifth of the sampled firms.

4.1.1. Sectors

Figure 4-2 ranks the 19 CAE Rev. 3 sections (A–S) by the share of firms that reported negative gross value added in 2018. Percentages span from barely 8% in Human-health & social-work activities (Q) to almost 45% in Real-estate activities (L).

The three sections reporting the highest share of firms with negative output are Real-estate activities (L), Electricity, gas, steam & air-conditioning supply (D) and Financial & insurance activities (K). A deeper look at these sections through 3-digit CAE codes shows what drives these outliers.

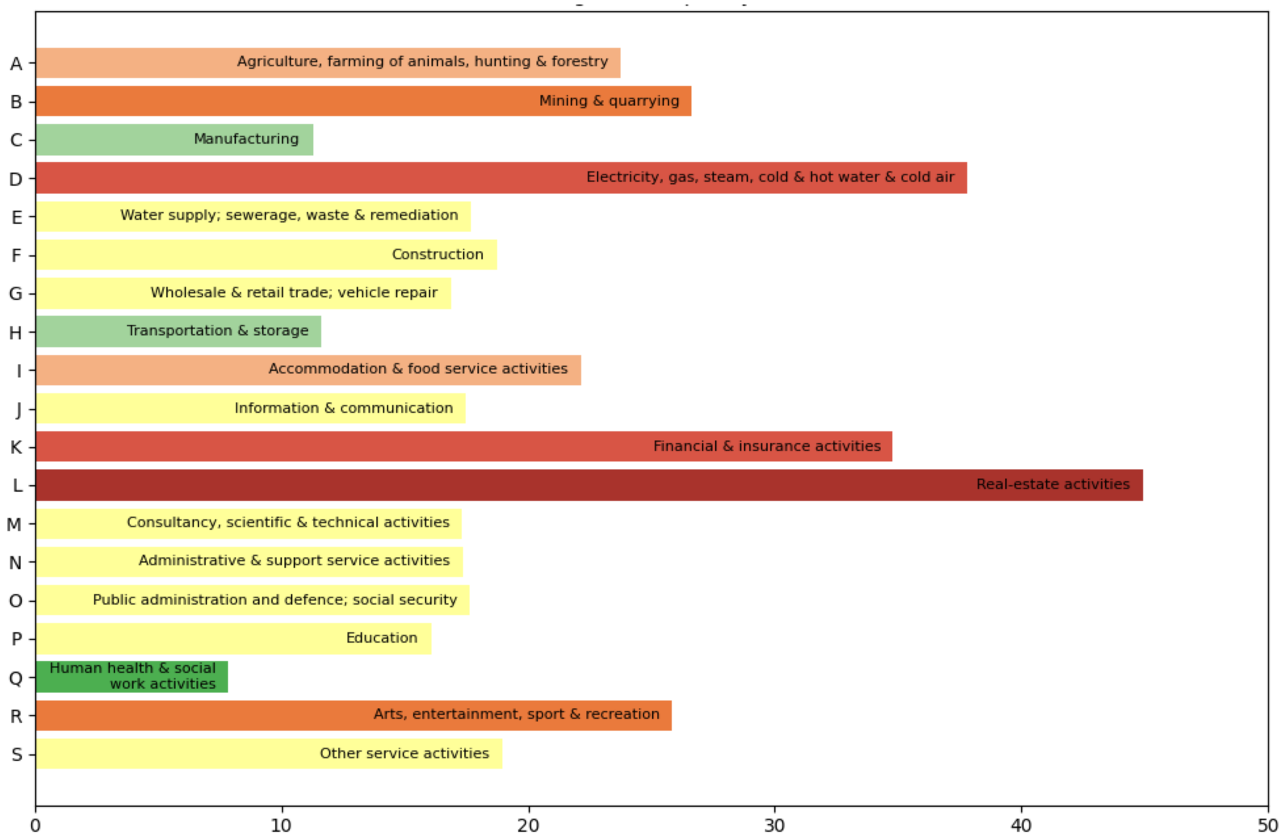

Figure 4-2. Share of firms with negative output by CAE section, 2018

(sections T and U omitted: together represent 3 firms, none negative)

For Real-estate activities, it is "Buying and selling of own real-estate" and "Renting of own or leased real-estate", reporting negative output in 53% and 42% of firms, respectively. These firms often hold assets that generate capital gains rather than operating revenue, so intermediate consumption can exceed output regularly.

In the case of Electricity, gas, steam and AC supply, the worst performing group is "Production, transmission and trade of electricity" at 39%. This may be caused by price swings in the energy market, as well as high fixed costs.

Finally, for Financial & insurance activities, negative output concentrates on "Management activities of holding companies", reporting a significant 76% share. Holding companies regularly record negative output since they focus on management and asset holding, with the subsidiaries bearing almost all production and often remaining profitable.

In the period between 2018 and 2023, the COVID-19 spike in 2020 and 2021 is different in intensity across sectors. While almost all sectors registered an increase in the share of negative output firms, it was more significant for Transportation and storage (H) and Arts, entertainment, sport & recreation (R), where the shares increased by around 37%, and also substantial for Accommodation & food service activities (I), Administrative & support service activities (N), and Other service activities (S), with the share of negative output firms growing by around 30%, 28% and 24% respectively. This is consistent with the impact of COVID-19 restrictions on tourism and contact-intensive services.

4.1.2. Regions

Figure 4-3 displays the regional incidence of negative output in 2018, at the district level. The share of firms reporting negative output vary from 14% in Bragança to 27% in Madeira.

A clear difference emerges between south and north of the country, especially the interior north districts, with Bragança, Vila Real, Guarda and Viseu all below 16%, compared to the southern districts of Faro, Beja and Évora all above 19%. The tourism centered districts of the south contrast with the manufacturing heavy northern districts, showing higher shares of firms with negative output.

Portugal's capital Lisbon records the highest share on the mainland (24%), while Madeira leads country wide at 27%. As mentioned in Subsection 3.5, the district variable reflects headquarters' location. Many firms register their headquarters in Lisbon for administrative convenience or in Madeira to benefit from that region's special tax regime, even when production occurs elsewhere. This headquarters bias may therefore overstate the true concentration of negative output production in those two districts.

Over 2018-2023 (see Appendix, Table 8-1 for full district 2018-2023 data), the ranking is essentially unchanged: Lisbon, Madeira, Beja and Faro remain the highest incidence areas, indicating structural rather than cyclical differences. There's a clear increase across every district in 2020, due to the COVID-19 pandemic, and a recovery in 2021, with 2022-2023 returning to 2018-2019 values.

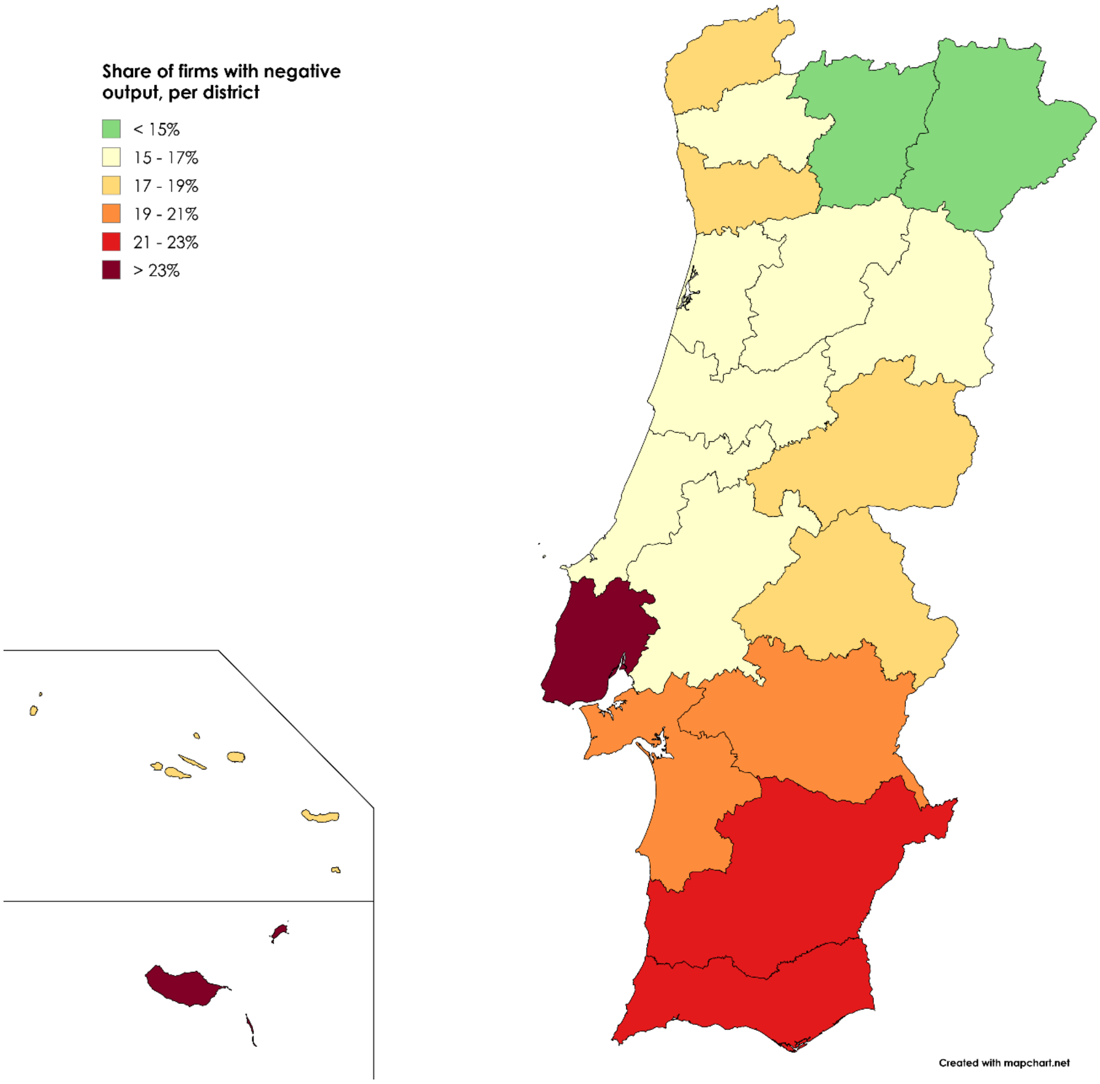

Figure 4-3. Share of firms with negative output, by district (created with mapchart.net)

Overall, the sector results reinforce the geographic findings: activities linked to tourism and speculative real-estate markets, concentrated in the south and in Madeira, exhibit the highest prevalence of negative firm output, whereas manufacturing dominated northern regions sit at the lower end of the spectrum.

4.2. Zero or more employees – differences in firm output

Table 4-2 compares firms that report at least one employee to those that report zero employees in 2018 and 2023 (observations with n.a. employee counts are excluded from the calculations).

Zero-employee firms make up about 21% of all firms in both years. Furthermore, almost half of these zero-employee firms register negative output. These shares (47% in 2018 and 49% in 2023) are roughly four times the rate observed among firms with employees. This means that zero-employee firms make up for almost half of every year's negative firm output cases.

The incidence of negative output in zero-employee firms is based on the inherent characteristics of these types of firms. Many of these firms are meant for asset-holding purposes, such as holding companies or real-estate firms, which tend to record negative output (see Subsection 4.1.1). For example, real-estate firms represent about 22% of all zero-employee firms.

Another driver is firm age: newly registered companies often report zero employees in their first year while incurring start-up purchases and service fees. With sales still ramping up, many of these young firms record negative output.

Table 4-2. Prevalence of negative firm output in staffed versus zero-employee firms

	2018	2023
Firms with ≥ 1 employee	315,730	352,239
… of which report negative output	39,488	41,109
Share of negative output (≥ 1 employee)	12.5%	11.7%
Firms with 0 employees	84,829	94,441
… of which report negative output	39,807	46,679
Share of negative output (0 employees)	46.9%	49.4%

4.3. Recovery of firms after negative output

Negative firm output can lead to a failed business. As such, it is important to measure how firms recover after recording negative output. Table 4-3 tracks the 79,484 firms that recorded negative output in 2018 and shows their situation five years later. “Recovery” is defined as having non-negative output in 2023, while the legal outcomes are measured with the Operational status variable, grouped as shown previously in Table 3-1. The following Table 4-3 also provides a version of this analysis that excludes firms with zero employees.

These results show that recovery remains uncommon. Only 28% of the original negative output firms (32% among firms with employees) achieved non-negative output by 2023. Zero-employee firms reduce recovery rates, due to their asset-holding features mentioned previously. Around 26% of firms “fail”, meaning that they reach a permanent legal ending (excluding mergers, which are rare at < 1%). Roughly two out of three firms still record losses (negative output) or have exited the market.

Table 4-3. Outcome in 2023 of firms that registered negative output in 2018

2023 condition	All firms (79,484)	Share	Excluding 0 employee firms (39,488)	Share
Recovered (output ≥ 0)	**22,590**	**28.4%**	**12,793**	**32.4%**
Output still negative	25,352	31.9%	10,476	26.5%
Output *n.a.*	31,542	39.7%	16,219	41.1%
Active	38,615	48.6%	22,140	56.1%
Temporarily inactive	19,730	24.8%	6,768	17.1%
Merger	787	1.0%	260	0.7%
Failed	**20,352**	**25.6%**	**10,320**	**26.1%**

Note: observations with n.a. employee counts are excluded from the right-hand panel; firms with n.a. 2023 output are counted in the “Output n.a.” row.

Even after excluding firms with zero employees, these findings show that negative output is a strong warning sign. A firm that reports losses in 2018 faces:

- A one in three chance of recovery;
- A one in four chance of legally exiting the market;
- A one in six chance of becoming temporarily inactive.

4.4. Technical insolvency and its relationship with negative firm output

As mentioned in Subsection 2.3, a firm is considered technically insolvent when it reports negative equity. In Portugal, the share of firms in technical insolvency sits at around a quarter of all firms. This share is very stable during 2018 to 2023, fluctuating between 24.12% and 25.00%., as shown in Table 4-4. Noticeably, there is not the same COVID-19 spike that is observed in firm output, with a less than 1 percentage point increase between 2019 and 2020, with this 2020 value being equal to 2018's. This suggests that demand shocks (such as COVID-19) impact firm output deeper than equity.

Geographically, technical insolvency is distributed similarly to negative output, higher in Lisbon, Madeira, Setúbal and Faro. However, these variables behave differently when dividing firms by economic activity. Figure 4-4 showcases these differences, with the high negative output share CAE sections mentioned in Subsection 4.1.1 being around the average for technical insolvency. On the other hand, the ones that show a higher share of technical insolvency are Accommodation & food service activities (I), Education (P) and Arts, entertainment, sport & recreation (R) at about 40% and Other service activities (S) at 50%.

Table 4-4. Prevalence of technical insolvency

	2018	2019	2020	2021	2022	2023
Firms with known equity value	401,486	420,241	430,378	440,667	450,645	447,762
Firms in technical insolvency	100,227	101,346	107,609	109,435	110,711	109,993
% of firms in technical insolvency	**24.96%**	**24.12%**	**25.00%**	**24.83%**	**24.57%**	**24.57%**

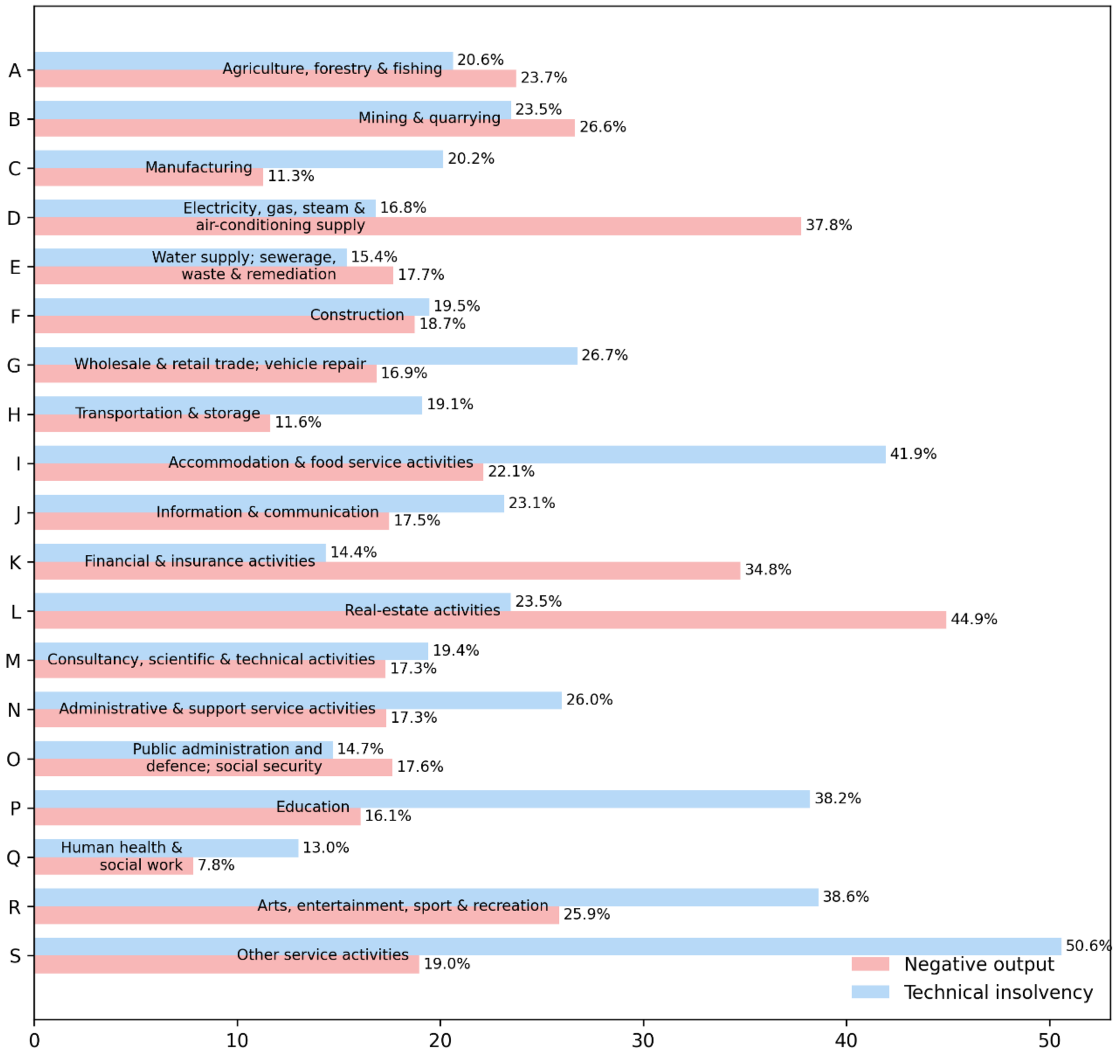

Figure 4-4. Share of firms with negative output & technically insolvent by CAE section, 2018

Overall, technical insolvency does not point to a direct relationship with negative output, either in the overall economy or by economic activity.

5. Treatment Recommendations

This section introduces treatment recommendations when dealing with firm output data. The treatments are divided into two different outlooks: economic treatments, where certain types of firms are excluded, and statistical treatments, that tackle the problems that arise with negative firm output values.

To exemplify the treatment recommendations when dealing with a sample of an economy's firm output, this section takes the values for the Portuguese economy for 2023. As shown in Subsection 4.1, in 2023 around 19.7% of the 447,762 firms reported negative output. Figure 5-1 displays the distribution of firm output with a histogram, where negative output firms are shown in orange and non-negative firms in green. The y-axis uses a logarithmic scale to accommodate the large differences in frequency across the range of output values. Roughly 99.5% of firms are placed in the two bins nearest to zero on the histogram, which corresponds to output values between approximately -6.6 and +6.6 million euros. There are also large outliers, with 38 firms being over 200 million euros of output.

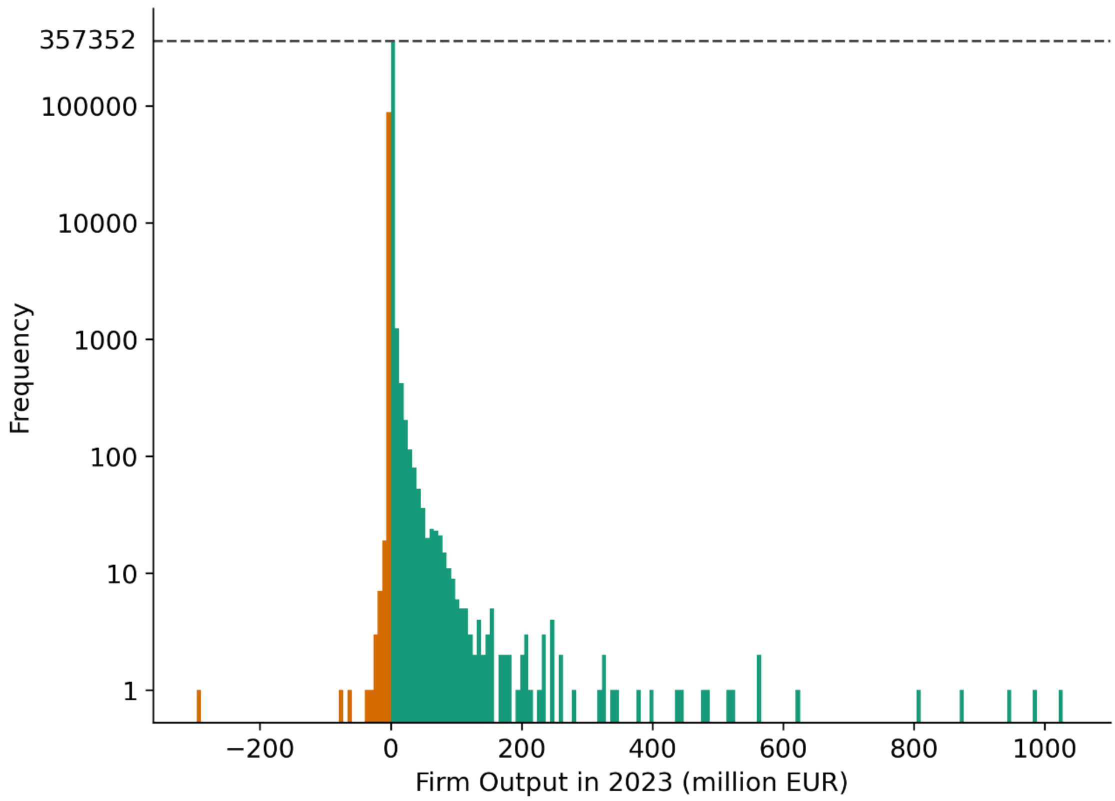

Figure 5-1. Histogram of firm output for Portuguese firms in 2023

5.1. Economic treatments

When researchers analyze firm output, their objective is typically to evaluate the productive capacity of a specific group of firms (sector, region, country, etc.). If the focus were instead on overall performance, they could work with sales or profits, which are more commonly studied, and, in the case of sales, do not face the same statistical issues that arise with negative values. From this production perspective, it is therefore reasonable to exclude certain types of firms from the analysis, according to their economic activity or other characteristics.

The first practical refinement is removing firms operating in sectors that do not represent production activity. This includes, according to the economic activity sectors named in Subsection 4.1.1, Real-estate activities (L), where firms often function as asset-holding instruments, Financial & insurance activities (K), which contain holding companies and financial firms that behave much differently from the rest of the economy, and Public administration and defence; social security (O), a sector that includes firms that do not behave as typical market producers such as municipal entities and volunteer firefighters associations.

The second refinement consists of excluding all firms with zero employees. These entities generally do not have production capacity, many exist merely for asset holding, while others are newly established firms that have not reached the production stage. Given the focus on productive output, this exclusion offers a more realistic perspective of the economy.

When applying these refinements to the 2023 data, the number of firms is reduced from 447,762 to 321,653 and the share of firms with negative output drops from 19.7% to 10.2%. Figure 5-2 represents the updated histogram, after these two treatments to the 2023 output data. It shows that including firms that do not correspond to productive activity inflates the incidence of negative firm output values.

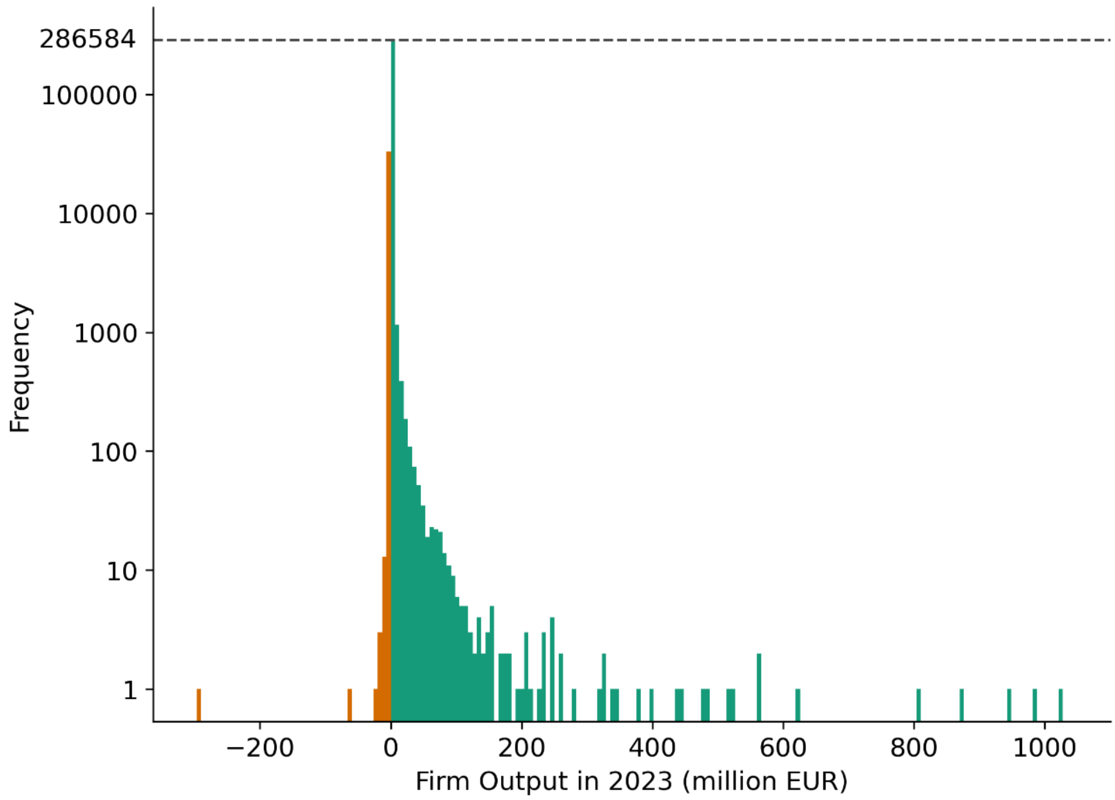

Figure 5-2. Histogram of firm output for Portuguese firms in 2023 after refinement

While these treatment examples use Portuguese data for 2023, the principles are applied to other economies. Similar empirical analyses of firm output should consider excluding from their sample financial firms and others that do not possess production capacity.

5.2. Statistical treatments

After the treatments introduced in Subsection 5.1, Figure 5-2 still shows some outliers and some negative values for firm output. As such, if there's a need to apply a log-like transformation for statistical procedures (to deal with the large outliers), two treatments introduced in Subsection 2.5 are easily appliable. These are the Inverse Hyperbolic Sine (IHS) and log-modulus transformations.

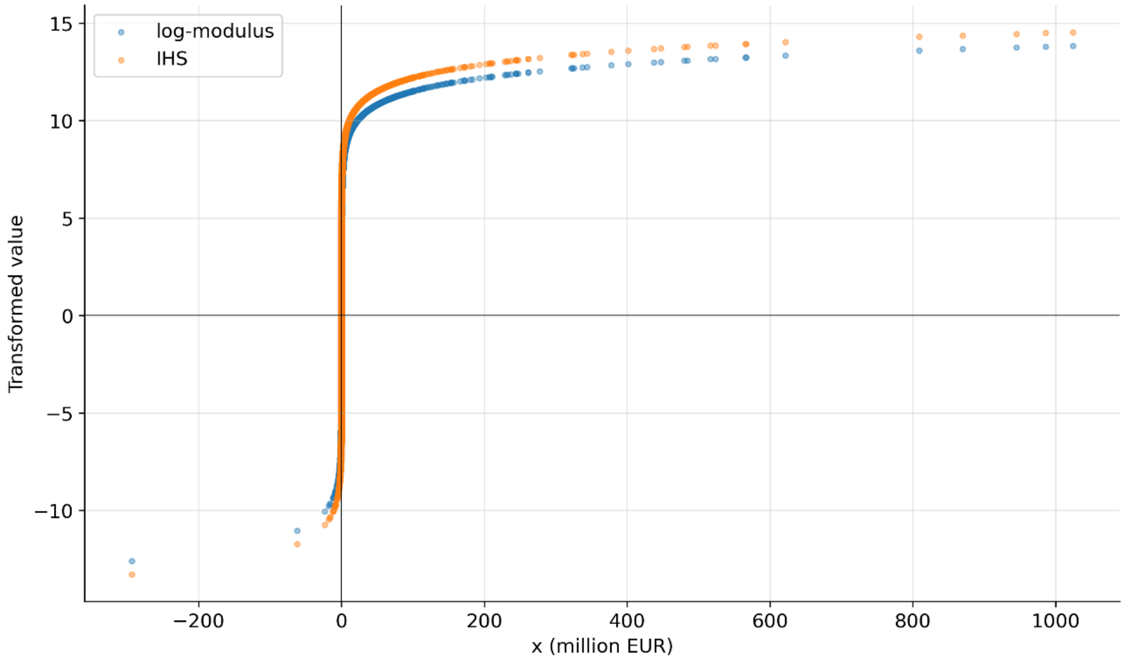

Figure 5-3. IHS and log-modulus transformations applied to Firm Output for Portuguese Firms in 2023 after refinement

Figure 5-3 plots these transformations and shows their compressing properties. It is important to note that the transformations were applied to the units of measure of the raw data, in thousands of Euros, although the Figure displays output in millions of Euros for ease of interpretation.

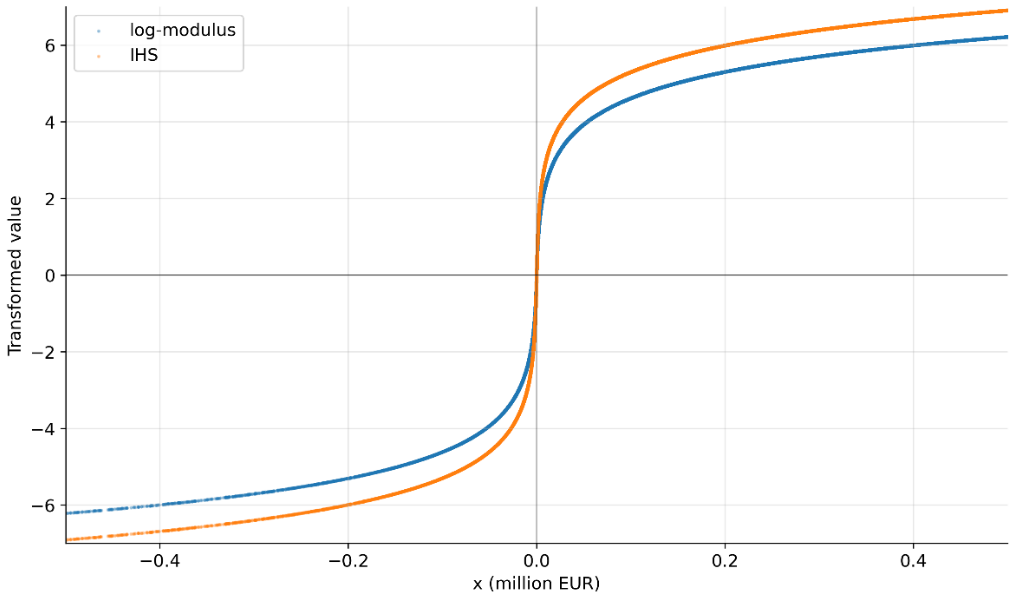

Figure 5-4. Zoomed-in view of the IHS and log-modulus transformations near zero output

Since readability of the transformations near zero is difficult, Figure 5-4 provides a close-up view of the firms with output between -500 and 500 thousand euros.

These statistical treatments seem to be fit for a sample of firms and their output. However, researchers must weigh the impact of the transformation when interpreting results. The literature suggests some problems with logarithmic and log-like transformations (Chen & Roth, 2024; Ciani & Fisher, 2019) that should be considered. Nonetheless, these transformations appear reasonable to apply and be used in further research of firm output, such as regression analysis.

6. Conclusions

This section summarizes the main findings and contributions of the dissertation. It highlights the key results, their implications, acknowledges limitations and outlines directions for future research.

Contributions and scope. This research contributes to the literature by covering firm output, especially the prevalence, characteristics and outcomes of firms with negative output, using full-economy firm-level data for Portugal, with 601,908 unique firms, between 2018 and 2023. It points out how negative output can originate from firm-specific pressures, sector wide downturns or macro-level crises, while describing statistical issues that can arise from handling negative output values and discussing potential solutions. Also, it offers treatment recommendations for firm-level data when dealing with firm output. The dissertation bridges the gap between a descriptive firm-level description of firm output and the technical discussion on how to handle non-positive values.

Main findings. This study finds that, in Portugal, nearly one fifth of firms reported negative output in each year from 2018 to 2023. This high prevalence illustrates how simply dropping all firms with negative output, as is often done in the literature, may severely bias subsequent analysis. Negative output cases are substantially concentrated in specific sectors of economic activity, such as real-estate and financial activities. Firms in these sectors do not resemble typical productive firms and are often asset-holding instruments, with intermediate costs consistently exceeding their turnover. Geographically, the highest occurrences of negative firm output occur in the tourism-oriented southern districts and in Madeira.

Firm size also proves to be a significant factor in the occurrence of negative output. Firms with zero employees represent 21% of all firms and account for nearly half of all negative output cases. This is likely because many zero-employee firms are used for asset holding or they could be newly created firms with minimal productive capacity.

Negative output tends to endure or culminate in the firm exiting the economy. The analysis shows how, of the firms reporting negative output in 2018, only 28% had recovered to non-negative output in 2023, while 26% "failed" and entered bankruptcy or dissolution and 25% became temporarily inactive. Therefore, a majority of negative output firms failed or remained unproductive over five years, which suggests that negative output can be a precursor to business failure.

To improve the economic analysis of firm output, this dissertation proposes treatment recommendations, through the exclusion of certain types of firms from the sample, with the goal of focusing on genuinely productive firms. Excluding firms in real-estate activities, financial firms, and public administration entities and removing all zero employee firms creates a more realistic view of the production capacity of the economy. In the Portuguese 2023 data, applying these exclusions reduced total firm count by 28% and diminished the share of negative output firms from 19.7% to 10.2%, representing how "non-productive" firms expand the prevalence of negative output.

From a statistical viewpoint, the presence of zero or negative output values poses a challenge to modelling, as the widely used logarithmic transformation cannot be applied to non-positive values. This dissertation suggests that, if researchers need to apply a log-like transformation (in data with multiple levels of magnitude, to compress extreme outliers), the inverse hyperbolic sine (IHS) and the log-modulus transformations are viable and effectively handle negative output firms. The application of these treatments allows for a more representative sample of firms with production capacity, without ad hoc droppings of negative observations.

While the results are specific to Portugal in the period between 2018 and 2023, the issues raised by negative output and the recommended treatments are appropriate to any country or region's firm-level dataset where non-positive values are common and the production of that economy is the intended focus of analysis.

Limitations. The limitations of the methodology and findings in this dissertation must be considered. The analysis is restricted to firms included in the SABI database, which does not cover self-employed individuals who operate under independent contractor regimes. Also, regional and economic activity classifications are based on the registered headquarters and main CAE code of the firm, which does not fully capture the multiple geographic or activity nature of a firm's production. These factors probably introduce some distortion into the regional and sectoral assessments.

Further research. Future research could apply a similar firm output approach to other countries to evaluate whether, and which, characteristics documented for Portugal hold in different settings. In addition, an analysis that weighs firm size could also be useful. In this dissertation, the only grouping done by size was by dividing zero and non-zero employee firms, further work can use total sales or groupings by number of employees to evaluate firms at different sizes.

Also, a natural next step is to use one of the suggested alternative log-like transformations into econometric models that include firm output or even utilize both transformations and compare their results. Finally, further studies could combine firm output with other firm-level data, infer correlations between output and other variables, and grant a richer analysis of the mechanisms behind a firm's persistent negative output and recovery.

Final remarks. Overall, the evidence suggests that negative firm output is significant and economically meaningful. It is an early warning signal for many firms, and it forces researchers to make thorough methodological decisions when using firm-level data in assessments of the productive characteristics of an economy.

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8. Appendix

Table 8-1. Share of firms with negative output by district, 2018-2023 (%)

District	2018	2019	2020	2021	2022	2023	Avg 18–23
Portugal	19.80	19.85	21.90	20.20	19.83	19.67	20.21
Viana do Castelo	17.59	17.75	19.56	18.33	17.70	17.71	18.11
Braga	15.98	16.00	17.17	16.08	15.78	15.91	16.15
Vila Real	14.72	14.54	16.53	14.85	15.14	14.73	15.09
Bragança	14.44	15.82	17.04	17.42	18.39	18.94	17.01
Porto	18.80	18.89	20.72	19.08	18.77	18.62	19.15
Aveiro	16.23	16.70	17.40	16.22	16.48	16.16	16.53
Viseu	15.82	15.15	16.48	15.02	14.97	14.82	15.38
Guarda	15.14	15.15	16.20	15.10	15.13	15.21	15.32
Coimbra	16.45	16.09	17.70	16.31	15.80	16.03	16.40
Castelo Branco	17.28	16.99	19.07	17.27	17.16	17.66	17.57
Leiria	15.73	16.20	17.56	16.81	16.26	16.26	16.47
Santarém	16.92	16.89	18.03	17.39	17.21	16.74	17.20
Portalegre	18.93	19.11	19.51	18.01	18.63	18.55	18.79
Lisboa	23.99	24.12	27.13	24.92	24.39	23.94	**24.75**
Setúbal	19.05	18.83	20.43	19.31	18.92	19.14	19.28
Évora	19.35	18.93	20.33	19.44	18.66	19.02	19.29
Beja	22.62	21.26	23.33	20.11	19.73	21.32	21.39
Faro	21.81	21.82	24.80	22.29	21.04	21.15	22.15
Açores	17.27	17.64	21.49	17.78	18.80	17.96	18.49
Madeira	26.94	26.47	29.26	26.54	25.49	24.85	**26.59**

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