MACROECONOMIC TRADE-OFFS AND MONETARY POLICY IN THE EURO AREA

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Tese de Doutoramento em Economia

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Biographic Note

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In spite of all these contributions, I remain the sole responsible for this thesis, so that its limitations, imperfections, and possible errors, are my exclusive liability.

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Abstract

This thesis studies the trade-offs between the levels (Phillips trade-off) and the variances (Taylor trade-off) of the unemployment gap and inflation in the aggregate Euro Area, throughout the last three decades of the XX\textsuperscript{th} Century, and their implications for the Area's monetary policy regime.

The Phillips trade-off is modelled within an unobserved components model, estimated by maximum likelihood with the kalman filter, featuring the Phillips curve and the Okun Law as main measurement equations, in which the NAIRU has a stochastic drift, whilst trend real output is modelled with a constant drift. Asymmetry tests do not reject the hypothesis that the Phillips curve has been linear, but clearly indicate a convex asymmetry in the Okun Law equation, during 1972-2000. It is shown that the forward-looking new keynesian Phillips curve works well for the Euro Area, once some deviation from rational expectations is allowed, and a model-consistent and time-varying NAIRU is used in computing the unemployment gap.

As for the Taylor trade-off, a marked improvement at around 1986 is documented, and inverse control is used to show that the emergence of a well-defined aggregate monetary policy regime in the Area, targeting a low rate of inflation, is part of the explanation for the greater macroeconomic stability. Two methods - optimal control with GMM estimation, and dynamic programming with FIML estimation - are employed to estimate the loss function of the notional central bank of the Area, and find a regime of strict inflation targeting with a significant interest rate smoothing and the target slightly above 2.5 percent. Both methods generate some signals that milder supply shocks and an improved ability of policymakers to maintain interest rates closer to their optimal path are also part of the explanation for the Taylor trade-off improvement. Finally, it is suggested a method for testing asymmetry in a central bank loss function, which is used to interpret the monetary policy regime results for the Area in an alternative way.
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Chapter 1

Unemployment-Inflation Trade-Offs and Monetary Policy in the Euro Area: an Introductory Overview

This chapter provides a brief theoretical and historical background for the studies in this thesis, and draws up the boundaries of the research. It is organised as follows.

In section 1.1, we present the main theoretical foundations of our work, and very briefly link up those backgrounds to the central tasks of each chapter. Section 1.2 discusses the meaning, motivations and objectives of studying the case of the Euro Area taken as a whole, for the specific period of the last three decades of the XXth Century. Section 1.3 summarises and delimits the scope of the thesis, briefly describing its central objectives, main methodological aspects, and some topics somehow related to the enquiry but not explicitly included in the study. Then, section 1.4 presents the structure of the thesis.

1.1. Unemployment-Inflation Trade-Offs: Levels and Variances

This thesis studies the trade-offs between unemployment and inflation, and their implications for monetary policy, in the aggregate Euro Area throughout the three last decades of the XXth century.

The key foundation of our analysis is the Phillips curve - the empirical regularities linking (wage) inflation to unemployment, detected by Phillips (1958, 1959), developed by Lipsey (1960) as a relation between price inflation and unemployment, and formalised by Samuelson and Solow (1960). We study the Phillips relation in the context of expectations augmented equations, as outlined by Friedman (1968) and Phelps (1967). Therefore, our work is developed consistently with the natural rate theory - the hypothesis that there is a transitory trade-off between the levels of unemployment and inflation but no permanent trade-off. In this context, the relevant concept and measure is the deviation of unemployment from the natural rate (NR), the
unemployment gap, and not merely the level of unemployment - a most relevant issue because the NR is not necessarily constant. More specifically, we address the Phillips trade-off in the context of the Calvo-Taylor-Rotemberg new keynesian formulation of the Phillips curve - described, *inter alia*, by Roberts (1995). In this new keynesian Phillips curve - explicitly derived from microeconomic foundations - expectations of inflation are forward-looking, and supposedly rational in Muth's (1961) sense.

The most notable recent contentious issues in the Phillips curve theory relate to the modelling of inflation expectations and to the curve's functional form, and both decisively shape the theoretical environment of this thesis. First, rational expectations are known to generate Phillips relations where inflation is a jump-variable, in contrast to its empirical persistence - Fuhrer and Moore (1995) -, and a serious ongoing research effort is trying to formalise sound alternatives to purely rational expectations. Second, an important tide of the Phillips curve literature of the 90s - see, for instance, Laxton *et al.* (1995) - has argued that the convex asymmetry present in the original Phillips' (1958, 1959) and Lipsey's (1960) curves, is actually significant in the recent data of the US and some developed countries. There are several empirical and theoretical motives for Phillips asymmetry, some additional reasons specific to the case of an aggregate of nations such as the Euro Area, and certainly important policy consequences, so the assessment of this matter is crucial in our research.

Within this theoretical context - of expectations-augmented Phillips equations compatible with the NR hypothesis -, Taylor (1979, 1994, 1998a) has shown that there is a permanent trade-off between the variances of unemployment and inflation. The Taylor trade-off means that - in a given monetary policy regime, structure of shocks, and magnitude of the Phillips trade-off - if the economy is disturbed by a shock that moves the gap and inflation in opposite directions, monetary policy can not bring both simultaneously on target. Rather, the best achievable combinations of variability of the gap and inflation around their targets draw a convex negatively sloped curve, known as efficiency policy frontier, or Taylor curve.

In summary, then, the current state of monetary theory, which frames our research, sees monetary policy as faced with two inter-linked trade-offs: the first-moments and the second-moments trade-off - respectively, the Phillips and the Taylor
trade-off. Transitorily, policy can choose between negatively related levels of unemployment gaps and inflation, but in the medium-to-long-run unemployment is attracted by the NR and monetary policy merely determines, ultimately, the rate of inflation. The short-run Phillips curve may be asymmetric, meaning that the marginal rate of substitution between the gap and inflation would vary with the cyclical state of the economy. The long-run Phillips curve is assumed to be a vertical line. In the medium-to-long-run, the best achievable combinations of variability of the gap and inflation around their targets, that monetary policy can obtain, are negatively related. This Taylor curve is convex for intermediate levels of volatility, but turns quickly into a steep curve, according to most estimates - Taylor (1998a).

Besides the functional form issue above referred, the analysis of the first-moments' trade-off is complicated by the fact that the position of the Phillips curve is affected not only by unobserved expectations of inflation, but, also, by the unobserved NR - which, in Friedman's (1968) definition, was not a static concept. In view of the apparent trends in the unemployment rate in recent episodes, and the implied instability in the trade-off - documented, for instance, by King et al. (1995) - a literature has emerged treating the NR explicitly as a time-varying parameter – Staiger et al. (1997a, b), Gordon (1997).

The analysis of the second-moments' trade-off is complicated by the fact that the position of the Taylor curve depends not only on somehow stable determinants - monetary policy regime and Phillips elasticity - but also upon the nature and variability of the shocks affecting the economy. Moreover, the actual position of the economy may be closer or further away from the efficiency frontier, as actual monetary policy is closer or afar from optimal policy - Fuhrer (1994, 1997a). However, the determinants of an estimated evolution of the volatility trade-off throughout a certain period can be traced out, assuming that the monetary authority behaves optimally for a given structural model and loss function. Specifically, inverse control theory can be used to back out the monetary policy regime’s structural coefficients, the central bank loss function coefficients - Salemi (1995). Joint estimation of the central bank's optimality condition and of the structural dynamic model of the economy, allows identification not
only of the policy regime, but also of the role played by the first-moments trade-off, supply shocks, and policy efficiency - Favero and Rovelli (2001) and Dennis (2001).

The central bank loss function, itself, has received attention from a parallel literature that has been debating its functional form, specifically arguing that in some cases it may not be quadratic, but rather asymmetric around the implicit targets for inflation and/or the gap - Goodhart (1998), Cukierman (2000, 2001).

The thesis connects to the outlined theoretical background as follows.

Chapter 2 deals with the estimation of the Phillips trade-off and of the model-consistent time-varying NR of the aggregate Euro Area with quarterly data from 1970:I to 2000:II., including tests of the hypothesis of asymmetry in the trade-off. This chapter also offers estimates of the Okun's (1970) Law, relating the unemployment and output gaps, tests of the possible asymmetry in that relation, and estimates of potential real output and the output gap in the Area. Results using standard adaptive expectations of inflation are compared to estimation of a new keynesian Phillips curve using Ball's (2000) concept of near-rational expectations - which is a promising alternative to purely rational expectations.

Chapter 3 uses the NR's and unemployment gap's recursive estimates of the previous chapter, updated by four quarters to cover the period 1972:I-2001:II, assesses the implicit evolution of the Taylor trade-off throughout that period, and draws implications for the monetary regime of the Area as a whole. The study puts a special emphasis in the period after 1986:I, in view of the apparent improvement in the volatility trade-off at the mid-80s. Specifically, we evaluate the role possibly played by the emergence of a well-defined aggregate monetary policy regime targeting a low rate of inflation, in that improvement. The framework employed also generates some indications about the possible contribution of the other determinants of the Taylor trade-off - Phillips trade-off, and supply shocks volatility - and of the position of the economy relative to it - policy optimality. Finally, this chapter brings together the literature on possible asymmetry of the central bank loss function with the inverse control literature that backs out the policymakers' loss function from the data.
1.2. The case of the Aggregate Euro Area

This thesis studies the case of the Euro Area considered as a whole. Specifically, we use quarterly macroeconomic time series of the aggregate of the Euro Area, as offered in the Area Wide Model Database (AWMD) published with Fagan et al. (2001) for 1970-1998, and, thereafter, as published in issues of the European Central Bank (ECB) Monthly Bulletin. The aggregate Euro Area data are weighted averages of nation-level macroeconomic data, computed as described by Fagan et al. (2001), meaning that the member-states are treated as if they were regions of a nation, whose data must be added to obtain the data of the relevant aggregate economic area.\(^1\)

Hence, we study the Euro Area’s macroeconomic trade-offs and their implications for monetary policy as if the Area was actually an economic and monetary entity throughout the last three decades of last century. However, the well-known facts are that the Single Market is a 1990s' process, and that the European Monetary Union (EMU) was launched only in January of 1999.

Simulating the study of the macroeconomics and monetary policy of the Euro Area beforehand its actual existence seems, nevertheless, both justifiable and necessary.

First, it seems justifiable to study the Area as a whole, having in mind its gradual process of economic and monetary integration since the late 70s. Monetary Integration began in 1979 with the creation of the European Monetary System (EMS) and its Exchange Rate Mechanism (ERM), and was deepened with the Basle-Nyborg agreement of 1987. Since the middle of the 80s controls to cross-the-border capital flows within the European Community were gradually relaxed, and exchange-rate realignments were infrequent - with the exception of the transitory 1992-1993 crisis. Economic integration was spurred by the European Single Act, in 1986, and further deepened with the Single Market in 1992, which has been completed during the 90s. By the mid 80s, a great part of the convergence of member-states' rates of inflation to German standards had already been achieved, and by the late 80s national short-term interest rates had also experienced a visible convergence.

\(^1\) It is beyond the scope of this work to assess, or even discuss, the methods used by the official sources to aggregate the nation-level data to sum-up to the Area data.
Second, it seems necessary to accumulate empirical evidence on the macroeconomics of the aggregate of the Euro Area, as the conduction of a single monetary policy by the ECB calls for the existence of as much knowledge as possible about the behaviour of the Area. In light of the well-known Lucas (1976) critique, knowledge about the Area's behaviour throughout our sample is not direct information regarding its behaviour after the possible structural break at 1999. We do not claim to be uncovering the trade-offs and the monetary policy regime of the EMU - there is simply not enough data yet for any researcher to aim at that task - but rather of the period covering roughly the last thirty years of the European Community's integration process. This is valuable information *per se*, for what it reveals about the integration process itself. Additionally, the estimates obtained are information prior to the EMU that may be important for a better understanding of the Area's reactions even within the EMU regime. Furthermore, historical data and estimates are required for having sound estimates of structural measures at the outset of the EMU, as for instance the level of the NR, and the position of the Area's trade-off between the volatility of the unemployment gap and inflation's volatility.

The Area has evidently experienced fundamental changes along the studied period, so we deal with structural instability in a number of ways throughout this thesis. First, the estimation methods used in chapter 2 are recursive techniques - namely, maximum likelihood with the Kalman filter. On one hand, this requires a large amount of initial data for its proper initialisation, but on the other hand, the time-varying parameters' estimates that it generates by the end of the sample are arguably not too dependent on the earlier data of the sample. Second, in chapter 3, we run adequate structural stability tests when we estimate, by full information maximum likelihood, a structural aggregate-supply / aggregate-demand model of the Area. Third, we end up, in that chapter, focusing on a period corresponding roughly to the second half of our sample, for which we identify a clear and new monetary structure in the Area.
1.3. Delimiting the Research: Objectives, Assumptions, Prior Research Options, and Methods

This thesis considers the unemployment-inflation trade-off in the vein of Mankiw's (2000) definition of it - inexorable and mysterious. Inexorable, in the sense that it is taken for granted that there exists a Phillips trade-off (and, hence, a Taylor trade-off), and that the challenge faced by the researcher is merely to model it successfully. Mysterious, in the sense that the efforts to model and test alternative specifications for the trade-off, in this research, do not incorporate any profound attempts at understanding the causes explaining its existence.

As implied by the brief review in section 1.1, there are plenty contentious issues in the present state of the analysis of the short-run (levels) trade-off - even within simple Phillips curve models, which merely describe, rather than explain, the trade-off and the course of the NR. They consist of, most notably, the implications of modelling inflation expectations as forward-looking, the choice of the functional form of the Phillips curve, and the method of modelling the path of the NR along time. As also suggested above, increased efforts have been devoted recently to the assessment of the long-run trade-off between the fluctuations of unemployment and inflation around their targets. Recent research has shown that this second-moments trade-off can furnish valuable information about the adopted monetary policy regime and about the optimality of actual monetary policy, given a certain dynamic structure of the aggregate-supply and aggregate-demand of the economy, and its shocks. In addition, once the analysis generates knowledge about the central bank loss function - the monetary policy regime - it is possible to enhance the enquiry with formal tests on the loss function functional form, namely in search of the kind of asymmetries that have been suggested recently by a parallel literature.

The objective of this thesis is to offer empirical evidence on these topics, trying to contribute to their ongoing literature. They are simultaneously of important theoretical and policy relevance per se - equally the Phillips and the Taylor trade-off are at the heart of macroeconomic theory and are crucial for the study of monetary policy. Furthermore, we will add to the literature on these topics with some empirical refinements - summarised in section 1.4 and described in detail within each chapter.
Moreover, our choice of the case of the aggregate Euro Area (during the last three decades of the XXth Century) as the specific historical episode to be studied, enhances the relevance of the research, as suggested by the arguments in section 1.2 above. With this regard, it is noteworthy that the thesis offers the first pieces of evidence on asymmetry tests of the Phillips trade-off and the Okun Law relation in the Area. In addition, it offers the first assessment of the Area's Taylor trade-off, and of the aggregate Euro Area monetary regime prior to the EMU, including formal tests of asymmetry of the notional central bank's loss function.

Before moving into the description of the structure of the thesis - section 1.4 below - some preliminary theoretical discussions are in order, for a better definition of the boundaries of the thesis. They essentially relate to assumptions and empirical options, made at the outset of the enquiry, which evidently influence it.

1.3.1 Natural Rate (NR) versus Non Accelerating Inflation Rate of Unemployment (NAIRU)

The modern, Friedman-Phelps, Phillips curve theory relates inflation to expectations of inflation and to the gap between actual unemployment and the natural rate (NR). This has been defined by Friedman (1968, page 8) as follows:

"At any moment of time, there is some level of unemployment which has the property that it is consistent with equilibrium in the structure of real wage rates. (...) The natural rate of unemployment (...) is the level that would be ground out by the walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the costs of mobility, and so on."\(^2\)

The original NR concept corresponds, then, to the rate of unemployment that balances the economy's competitive dynamic general equilibrium system, and its level - which is typically positive - depends upon the specific microeconomic structural characteristics of the economy. Even though the NR depends upon such microeconomic features, it has also a particular macroeconomic attribute - it is the level of the

unemployment rate that balances the aggregate-supply and aggregate-demand and, therefore, stabilises the rate of inflation. In the context of expectations-augmented Phillips relations, it follows that the actual unemployment rate can only diverge from the NR temporarily, following some shock that drives inflation away from expected inflation. In Friedman's words *(op cit, page 11),*

"(...) there is always a temporary tradeoff between inflation and unemployment; there is no permanent tradeoff. The temporary tradeoff comes not from inflation per se, but from unanticipated inflation, which generally means, from a rising rate of inflation."

According to the natural rate hypothesis, then, monetary policy actions can only affect real economic activity transitorily, as can be read from the following more recent words of the co-author of the modern Phillips curve theory *(Phelps, 1995, page15),*

"Monetary policy can make a permanent difference only to nominal variables: a policy to generate a finite increase or decrease in the inflation rate will generate only a transient dip of the actual unemployment rate relative to the path it would otherwise have taken. In particular, the actual unemployment rate, though occasionally hit by such shocks, is constantly homing in on the natural rate."

Modigliani and Papademos *(1975)* focused on the macroeconomic attribute of the NR and called it the non-accelerating inflation rate of unemployment (NAIRU) - a designation that turned out to be vastly more popular, thereafter, than the NR original name.

Some authors have argued that there are fundamental differences between the NR and the NAIRU concepts - for instance, Tobin *(1995, 1997), Rogerson *(1997), Galbraith *(1997), Phelps and Zoega *(1997)* - and some have tried to estimate the NR differently from the NAIRU - for instance, Staiger *et al.* *(2001)*, and Grant *(2002)*.

In contrast, Ball and Mankiw *(2002)* consider that the NAIRU is approximately a synonym for the NR. Solow *(1998a, page 5)* also considers them synonyms, even though rejecting both designations - the NR because the word natural suggests more than could be seriously argued for that particular state of the economy, and the NAIRU because it is terrible English. Solow chooses, instead, the term neutral rate of unemployment.
We adopt this view that the NAIRU and the NR are two sides of the same coin. The NAIRU is simply the equilibrium unemployment rate that can be estimated out of macroeconomic models including the Phillips curve. In turn, the NR is the estimate of the equilibrium unemployment rate that would be obtained from a structural model featuring all the microeconomic variables and forces that explain the flexible price equilibrium unemployment rate, according to Friedman-Phelps' theory.

The problem with the estimation of the NR is that it is virtually impossible to build a model that properly includes all its microeconomic determinants - demographic, institutional, informational, technological, etc… For instance, Dixon (1995, pages 57, 64, 70) notes that the NR concept is so vague and broad that no empirical implementation combining all its microeconomic elements in a coherent model has been achieved so far, and probably never will. More recently, Taylor (1998a, pages 35-36) also mentions that there are still great uncertainties about the microeconomic determinants of the NR.

Hence, the NAIRU seems to be the best measure of the NR that can be obtained, which explains that the recent literature has focused in estimation of the NAIRU - as an empirical estimate of the NR based on its macroeconomic attribute - from macroeconomic models including Phillips relations. But even researchers using macroeconomic models that include structural modelling of both the labour and goods markets, in the spirit of Layard et al. (1991), have been finding difficulties in finding plausible, significant and robust estimates of the NAIRU - see Cassino and Thornton (2002) for a recent example.

The most recent research has focused, then, on estimation of the NAIRU from very small-scale macroeconomic models featuring only one Phillips equation - in some cases a wage-equation, but, more frequently, a price equation. The most popular approach has been to model the NAIRU as a time-varying parameter in unobserved-components models with a Phillips equation in the measurement system, estimated by maximum-likelihood with the kalman filter - following Gordon (1997) and Staiger et al. (1997a, b). This approach amounts to give up explaining the NAIRU from its structural

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3 Even among the NR theory followers, there are divergences regarding the time that convergence to the NR takes. For instance, Phelps (1995, p. 21-22) says that he himself believes that the economy can take a non-equilibrium path for a long time, even indefinitely, because expectations can not be perfectly rational.
microeconomic determinants, but simply try to model its time-path subject to two constraints - that it varies smoothly, and that it fulfils its macroeconomic attribute of stabilising inflation. The second constraint is compatible with the natural rate theory, as reviewed above, and the first is also consistent with that theory, as can be read in Solow (1998a, pages 5-6):

"The neutral rate of unemployment defined by a standard accelerationist model does not have to be constant in time. Other sources of inflationary pressure help to determine the current neutral rate. The usual suspects include the demographic composition of the labour force, exogenously caused increases in food and energy prices, similar impulses from import prices, imposition or removal of formal or informal price controls, and still others. (...) The neutral rate might also respond to occasional well-defined changes in the environment of the labour market - like the scope, duration, generosity of unemployment insurance benefits; the strength and aggressiveness of trade unions; the presence or absence of restrictions on layoffs by employers - or to characteristics of product markets, like the intensity of international and domestic competition (...). What the model can not tolerate is the need to postulate fairly frequent, spontaneous and unexplained changes in the neutral rate itself."

In this thesis, we follow the Gordon approach in the estimation of the aggregate Euro Area NAIRU, and consider it the best estimate of the NR that could possibly be obtained.

The main features of this approach - that the NAIRU is a time-varying coefficient with a path modelled but not explained by the model, and that the best available estimate of the NR is the NAIRU - seem, moreover, particularly suited for the specific case under study. First, it seems unreasonable to try to model the Euro Area NAIRU as a constant, in view of the persistent rise in European unemployment during the 70s, 80s and part of the 90s, and its apparent decrease more recently. Blanchard and Wolfers (2000) argue that a combination of negative shocks with an unfavourable institutional framework in factors and goods markets explains the increase in structural unemployment in Europe from the 70s until the mid-90s, and that the shocks-

4 Staiger et al. (2001) tried to explain their estimated time-varying NAIRU for the U.S. and its states with structural determinants such as demographic, educational, industry characteristics, and indicators of labour market policy, with no success neither at the aggregate nor at the state level.

5 This approach is supported by recent evidence for the US in Staiger et al. (2001), who estimated the U.S. NR and NAIRU separately, using macroeconomic data. Specifically, they estimate the NR as the low frequency component of the unemployment rate, as suggested by Hall (1999), and the NAIRU as the time-varying equilibrium rate in a Phillips equation - and they obtain estimates of the NR and the NAIRU that are not significantly different.
institutions combination has improved ever-since. Second, any estimation of the NR from its structural microeconomic determinants would be arguably impossible because of the high diversity of structural determinants of the NR across the Area's countries, at least during most of the sample period.\footnote{Because of the difficulties in explaining the time-path of the NR from its structural determinants, even in the case of individual countries, a recent literature has developed models explaining the U.S. NAIRU decline since the mid-90s with macroeconomic phenomena. The most popular recent hypothesis explains the path of the NAIRU in the U.S. with a theory of wage aspirations lagging behind changes in the productivity growth rate - see Staiger et al. (2001), Ball and Moffit (2001), Mankiw (2001) and Ball and}

Furthermore, the NAIRU estimates of the NR seem also suitable because the aim of this thesis is to shed light on the macroeconomics of the Area with an eye on monetary policy. In fact, the NAIRU is precisely estimated from the link between aggregate economic activity and inflation that is relevant for monetary policy.

In this context, the correct specification of the Phillips curve is crucial for an adequate identification of the NAIRU as a consistent estimate of the NR. First, all the significant transitory price shocks must be added to the equation, as the movements they cause in inflation are exogenous, and therefore not related to deviations of unemployment from the equilibrium rate of unemployment. Second, the Phillips equation's functional form must be adequately identified, as in the case of a significant non-linearity there is a distinction between the stochastic NAIRU and the NAIRU that would prevail in the long-run, steady-state, deterministic equilibrium - which, in that case, would fit better the NR concept. Both these specification issues are considered in chapter 2, and we find no significant asymmetry in the Euro Area Phillips relation in our sample.

Accordingly, throughout this thesis we use the designations NR, NAIRU, neutral rate of unemployment, equilibrium rate of unemployment, and flexible prices rate of unemployment, as synonyms capturing the microeconomic and macroeconomic attributes reviewed above. Moreover, we use the term trend unemployment with a similar meaning, as we compute the NAIRU from a trend-cycle decomposition model that is not a mechanical detrending filter, as it includes the Phillips equation in its measurement system - see the details in chapter 2.
1.3.2 Real Unit Labour Costs (Labour Share of Income) versus Unemployment Gaps as Marginal Costs proxy in New Keynesian Phillips Curves

While the theoretical new keynesian Calvo-Rotemberg Phillips curve explains inflation with the current real marginal cost - besides expectations of inflation for next period -, empirical applications of the theory have typically used output or unemployment gaps as proxy for the real marginal cost. There is, however, a recent exception to this rule.

Gali and Gertler (1999) and Sbordone (2002) have argued that real unit labour costs - or, equivalently, the labour share of income - are the most theoretically sound and empirically successful proxy variables for the real marginal cost in optimising forward-looking Phillips curves. Gali et al. (2001) showed that detrended output enters such optimising Phillips curve with the wrong sign, both in the US and in the aggregate Euro Area data of the AWMD, while the labour share of income has well-behaved estimated coefficients. These authors have popularised their approach to the new keynesian Phillips relation as the New Keynesian Phillips Curve (NKPC).

The NKPC's replacement of the gap by the labour share has been, however, the object of criticism in some recent literature.

First, it has been noted that the NKPC splits the Phillips link between real activity and inflation into two pieces, the link between labour costs and inflation, and the link between real activity and the labour costs, focusing only on the first and failing to explain the second - Roberts (2001). It follows that the NKPC assumes that in each period labour supply adjusts completely and instantaneously to labour demand, therefore disregarding the labour market institutions and its imperfections and frictions that are at the heart of the Phillips theory. As has been quite effectively summarised in Nelson and Nikolov (2002, page 16),

"… the most these studies provide is a model of price-setting conditional on marginal cost - not a direct model relating inflation to a measure of excess demand."

Mankiw (2002). For the U.K. case, Hatton (2002) also assessed the ability of productivity growth to explain the NAIRU over several historical episodes, finding that it could only be a partial explanation.

This puzzle associated to the sign and significance of the Phillips elasticity in forward-looking new keynesian Phillips curves is documented as well by McCallum and Nelson (1999a), and Nelson and Nikolov (2002)
Second, Roberts (2001) has noted that the use of the labour share, or unit labour costs, as proxy for the marginal cost, means that the average labour productivity is used as measure of the marginal product of labour. Roberts has shown, for the US case, that it is the cyclical part of average labour productivity that correlates with inflation, and that that variable is dominated by a standard gap measure of economic activity, when both are included as explanatory variables in an optimising Phillips curve. Then, he concludes that, in the NKPC, the labour's share may simply be playing the role of a traditional economic activity variable.

Third, Rudd and Whelan (2002) have found evidence that the labour share of income does not appear to drive inflation, as the discounted sum of current and future labour shares explains very little of the observed variation in inflation. Thus, they argue that there is no compelling reason for replacing conventional output gap measures with the labour share in monetary policy analysis models, as the latter does not outperform the former as proxy for real marginal cost.

In view of these criticisms, we argue that the gains of swapping from the traditional gap-based Phillips equation to a labour share equation are still not satisfactorily clarified. Hence, in this thesis we do not follow the NKPC approach.

Our approach, instead, is to try to refine the specification and estimation of the standard gap-based new keynesian Phillips curve, addressing two of its features that are problematic and, as such, may be the cause of its empirical failure: rationality of expectations of inflation, and theoretical consistency of the gap.

As regards expectations, it is well known that the attempts to empirically validate the rational expectations hypothesis, with formal econometric tests, have systematically failed. Hence, the empirical problems with the new keynesian Phillips curve may be caused by the use of purely rational expectations of inflation, and not by the use of a gap as proxy for the real marginal cost.

With respect to the gap measure, the empirical problems with new keynesian Phillips curves could derive from the typical use of a mechanically detrended real activity indicator, instead of the theoretically relevant gap measuring deviations of real activity from the flexible price equilibrium.
Regarding expectations of inflation, since Fuhrer and Moore (1995) showed that some aggregate-level deviation from purely rational expectations is needed for new keynesian Phillips equations to fit some basic macroeconomic evidence, a vast literature has been seeking to build a theory of such deviations from rationality. In chapter 2, we develop one suggestion from this literature - Ball's (2000) hypothesis of near-rational, limited information, expectations of inflation - and show how it solves the inconsistency between forward-looking behaviour by optimising agents, and evidence of inflation inertia, preserving a reasonable and significant estimate of the Phillips trade-off.

The necessity of estimating real activity gaps that measure deviations of real activity from its flexible-price equilibrium - the ones that, in theory, can proxy for real marginal costs - has been receiving an increased interest in the literature - Woodford (2001a). Some recent studies have tried to design methods for estimating theory-consistent gaps, showing that they differ from mechanical detrending methods, and also that labour costs do not outperform such gaps in optimising new keynesian Phillips equations - see, for instance, Neiss and Nelson (2001, 2002) and Nelson and Nikolov (2002). In chapter 2, we estimate unemployment and output gaps using a structural time series method, which is considered a hybrid detrending technique combining statistical and theoretical criteria. Specifically, we estimate the NAIRU and potential output from an unobserved components model, that features in its measurement system the Okun Law and a new keynesian Phillips equation. The latter, guarantees that the estimated gaps are actually associated to demand and inflationary pressures, and, hence, that the estimated NAIRU and potential output have a flexible-price equilibrium meaning. All in all, our gap measure should prove superior to mechanically detrended unemployment or output, and compare well to the estimates of Gali et al. (2001) for the Euro Area business cycles.

Overall, the empirical success of chapter 2 seems to confirm that the standard gap-based new keynesian optimising (forward-looking) Phillips relation works well, once purely rational expectations are replaced by a more realistic hypothesis, and once a theoretically-driven and consistent gap measure is employed.
1.3.3 Taylor Rule versus Structural Approaches to Policymakers’ Preferences

Since Taylor’s (1993) seminal paper, the majority of monetary policy analysis has been conducted within frameworks featuring simple reaction functions linking short-term interest rates (the instrument) to deviations of inflation and the activity gap from their desired values (the targets), known as Taylor rules. Ensuing refinements of Taylor's original set-up include the estimation - rather than calibration - of the coefficients of the rule, the reaction of the policy instrument to expected - rather than contemporary - inflation and real gaps, and the explicit inclusion of an element of partial adjustment of the instrument - interest rate smoothing.\(^8\)


As regards the specific case of the Euro Area, the monetary policy of the European Central Bank (ECB) since the beginning of the European Monetary Union (EMU) in 1999 has also been studied in the context of Taylor rules. Mihov (2001) showed that the Euro Area interest rates during 1999-2000 have been closer to those predicted by a Taylor rule estimated with weighted average data of Germany, France and Italy, than to those predicted by a rule estimated with Germany data. Faust *et al.* (2001) used estimates of the Bundesbank rule and applied it to EMU-wide aggregates to simulate interest rates in the Area. Alesina *et al.* (2001) calibrated several alternative rule formulations and assessed which matched better actual ECB policy in 1999-2000 - an exercise updated by Gali (2002b). More recently, Begg *et al.* (2002) checked the coherence of the ECB actions throughout 2001 with the rule in Alesina *et al.* (2001), suggested a new rule, and compared the ECB policy with that of a Fed-in-Frankfurt, on the basis of a policy rule estimated with recent US data.

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\(^8\) See Mankiw (2001) for an exception - an analysis of recent US monetary policy with a rule without instrument inertia and with interest rates reacting to contemporaneous unemployment and core inflation.
Taylor rules have also been used in research trying to detect structural changes in monetary policy. Most particularly, in view of the apparent break in macroeconomic volatility observed at the beginning of the 80s in the U.S., studies using Taylor rules have detected a structural change in U.S. policy reaction function from the pre-Volcker period (until 1979) to the Volcker-Greenspan era.\(^9\) The typical strategy has been to estimate such rules over the different periods and then assess whether its coefficients are different - see, for instance, Clarida \textit{et al.} (2000). This identification of a change in U.S. monetary policy in the 80s seems to be robust, as it has also been detected by alternative literatures - for instance, the New Keynesian Phillips Curve literature, see Gali \textit{et al.} (2002).\(^10\)

However, the use of Taylor rules in monetary policy analysis has recently been the object of a number of criticisms.

First, Svensson (2001a, b, c) has argued that contemporary monetary policy - goal-directed, forward-looking, and conducted by rational policymakers - can only be described as a commitment to a targeting rule, not to a mechanical instrument rule. Minford \textit{et al.} (2001, 2002) have demonstrated analytically the lack of identification of the Taylor rule, showing that different policy rules such as money growth targeting and exchange-rate targeting have a Taylor rule representation that resembles that implied by a true Taylor rule. An empirical illustration of this argument has been offered in Razzak (2001) with US data, who showed that the money base-nominal GDP targeting rule of McCallum (1988) can be expressed as a Taylor rule, under rather trivial conditions.

Second, the estimation of Taylor rules has been shown to be somewhat fragile - Florens \textit{et al.} (2001) showed that the typical generalised method of moments (GMM) estimates of Taylor rules may exhibit large small-sample bias, high imprecision, and high variation across the specific type of GMM estimator.\(^11\) They suggested the use of full-information maximum likelihood (FIML) estimation, using additional equations

\(^9\) In many studies of the US monetary policy, the quarters 1979:III-1982:II are not used, as these correspond to the monetarist experiment, a period of unusual operating procedures - non-borrowed reserves targeting - which is significantly different from previous and subsequent regimes.

\(^10\) These authors identify technology shocks with the method suggested in Gali (1999). See the foundations of the NKPC in Gali and Gertler (1999), and recent developments in the survey by Gali (2002a).

\(^11\) See Favero and Marcellino (2001) for a recent discussion and empirical simulations concerning the information set used as instrument set in GMM estimation of Taylor rules.
describing the structure of the economy, arguing that it performs better in terms of precision and centricity in small samples. Recent empirical research along these lines, which appears to confirm the capability of the FIML approach, includes Jondeau and LeBihan (2000), and, for a sub-set of countries of the Euro Area, Clausen and Hayo (2002a, b). More decisively, Lindé (2001a), and Boivin and Giannoni (2002), have documented the U.S. monetary policy shift in the 80s showing that the coefficients of the policy rule function change, while those of aggregate-supply and aggregate-demand functions do not, within a FIML estimation of such three-equation macroeconomic models with US data.

Third, and most importantly, Favero (2001a), Favero and Rovelli (1999, 2001), and Dennis (2001) pointed out that policy reaction functions are reduced-form equations, whose coefficients are complex convolutions of two types of deep parameters - those describing the structure of the economy, and those defining the preferences of policymakers. Hence, Taylor rule coefficients do not directly identify a monetary policy regime - the central bank loss function -, and therefore its estimates do not allow direct study of changes in policymakers' preferences and efficiency. However, estimates of the policymakers' preferences - policy regime - can be backed out from estimated policy reaction functions - policy rule -, under certain identification conditions described by Dennis (2000).

In view of the criticisms above portrayed, we do not pursue a Taylor rule approach in chapter 3. As our central aim is to assess the contribution of a monetary policy regime change in the aggregate Euro Area to the improvement in its macroeconomic volatility around the mid 80s, we choose a framework that allows direct estimation of the structural coefficients describing the policymakers' preferences. Specifically, we use two alternative set-ups combining inverse control and a suitable estimation technique, to back out from the data the central bank loss function - optimal control and GMM, and dynamic programming and FIML.

An Appendix to that chapter shows results of estimation of simple forward-looking policy reaction functions with our aggregate Euro Area data, for our sample and the relevant sub-samples, illustrating its inability to offer estimates of the monetary policy regime.
1.4. Structure of the Thesis

The remainder of this thesis is organised as follows.

Chapters 2 and 3 describe our empirical analysis of the macroeconomic trade-offs and monetary policy of the Euro Area throughout the last three decades of the XX\textsuperscript{th} Century. Then, chapter 4 summarises the results of the research, and presents suggestions for future work.

Chapters 2 and 3 can be read independently, as they report self-sufficient research entities, with specific literature reviews, hypothesis, empirical work and conclusions. However, they are intimately related, so that their successive reading recounts a whole research story that is inter-linked and consistent. First, their broad object is the same, as they both focus on the macroeconomics of the Euro Area during the last three decades of the XX\textsuperscript{th} Century. Second, the variability trade-off that motivates chapter 3 has its theoretical foundations in the levels trade-off, studied in chapter 2. Hence, the third connection: the empirical work of chapter 3 uses the unemployment gap estimates obtained with the kalman filter in chapter 2.

In chapter 2 we test the linearity of the Phillips trade-off, as well as of the Okun Law, allowing for four alternative possible functional forms that nest the linear case and do not impose convexity nor concavity. We compare the results obtained with standard adaptive expectations in a backward-looking Phillips curve, with those obtained with near-rational expectations in a optimising, forward-looking, new keynesian curve. This chapter offers consistent estimates of the time-varying NAIRU and potential output, as the testing and estimation is performed in a small unobserved components model featuring the Phillips and Okun relation as main measurement equations - estimated by maximum likelihood with the kalman filter. The system successfully estimates all the hyper-parameters in the model, so that the trend-cycle decompositions of unemployment and output are obtained with no calibration of the signal-to-noise ratio. We report the smoothed estimates of the unobserved components, as well as adequate confidence bands, computed by Monte Carlo integration to comprise all the sources of uncertainty involved in kalman filter estimation. We also compare the cyclical turning points estimated by our model with some alternative estimates in the literature.
In chapter 3 we use the estimates of the unemployment gap obtained in chapter 2, to document an apparent improvement in the Taylor trade-off of the Area around the mid-80s. Based on the macroeconomic and policy record of the Area member-states, we argue that an important part of the explanation for the volatility trade-off improvement lies in the emergence of a well-defined monetary policy regime of low and stable inflation in the aggregate Area at 1986. Modelling the structure of the Area macroeconomy with a version of the Rudebusch-Svensson model, we use inverse control theory to back out from the data the policymakers' deep preference parameters - the coefficients of the loss function of the aggregate Area notional central bank. We employ two alternative methods recently used in research for the U.S. case by separate researchers, in conditions that render comparisons of results possible, therefore enhancing the robustness of our results. In addition to comparability, our strategy improves on the U.S. case literature in that we do not use an official measure of the NAIRU available at the final of the sample, but use a quasi-real-time estimate. We end the chapter suggesting a method of testing for asymmetry in policymakers' preferences across recessions and expansions, and applying it to the Euro Area regime post-86, thus presenting an alternative interpretation for the policy regime estimate.
Chapter 2


2.1. Introduction

This chapter presents estimates of the trade-off between the unemployment gap and inflation changes in the Euro Area as a whole. The trade-off is modelled in a simple macroeconomic model based on Phillips and Okun relations, and requires the estimation of the unobserved gaps in output and unemployment.

The unobserved components (UC) model has been first used by Watson (1986), in the context of the trend and cycle decomposition of the US real output. This statistical framework essentially postulates that the trend follows a random walk process and the cycle a stationary autoregressive process, with these two components mutually uncorrelated. In econometric terms, the unobserved cycle and trend are treated as time varying parameters, thus being estimated (by maximum likelihood) recursively through a Kalman (1960) filter. Clark (1989) augmented the model with a drift in the random walk driving the trend.

Kuttner (1994) extended the UC model in order to include a measurement equation derived from economic theory – the Phillips equation – showing that this additional information improves its performance in decomposing output. The resulting UC model successfully combines economic theory with econometrics and has been often considered to have important advantages over both pure statistical (mechanical) techniques and structural trend-cycle decompositions.¹

Gordon (1997) pioneered the use of this framework to decompose the unemployment rate into trend and cycle (unobserved) components. Specifically, he

adapted the UC set-up to the context of his triangle model of inflation – a Phillips equation in which inflation is explained by inertia, demand and supply shocks – in order to estimate a time varying NAIRU ("Non-Accelerating Inflation Rate of Unemployment") for the US.\textsuperscript{2} Gordon's motivation to admit that the equilibrium US unemployment rate might have been falling was the observation of a systematic fall in the US unemployment rate since 1994 without any upsurge in inflation. The most recent studies on this subject seem to confirm that the US macroeconomic performance during the 90s is well explained by the evolution of the trends of unemployment and productivity, that is, by a significant fall in the NAIRU - Staiger, Stock and Watson (2001). The empirical hypothesis of a time-varying NAIRU has satisfactory theoretical foundations, as Friedman’s concept of ‘natural rate’ was not one of a constant parameter (Friedman, 1968).

Following Gordon's seminal paper, many recent studies have estimated time-varying NAIRUs and the associated unemployment gaps from UC systems based on a Phillips equation. For example: Gerlach and Smets (1997) and Laubach (2001) with G7 countries data; Kichian (1999) with Canadian data; McAdam and McMorrow (1999) with US, Japan and EUR-15 data; the OECD area has been extensively treated in Richardson \textit{et al} (2000), with time-varying NAIRUs for 21 OECD countries; Peersman and Smets (1998) and Gerlach and Smets (1999) with EMU data; Orlandi and Pichelman (2000) with annual Euro Area data; and Irac (2000), Estrada \textit{et al} (2000) and Meyler (1999), focusing on individual member-States of the European Union, namely France, Spain and Ireland.

At a more institutional level, OECD research has recently adopted the Kalman filter estimates of quarterly time-varying NAIRUs, stating that the method should be used as a starting point for actual policy analysis and conception. Accordingly, the OECD NAIRU indicators will be based on this methodology - see Boone (2000), Richardson \textit{et al} (2000) and OECD (2000).\textsuperscript{3}

\textsuperscript{2} The NAIRU concept, first used by Modigliani and Papademos (1975), has been seen as an empirical 'natural rate', i.e. one that may be estimated from Phillips equations. The theoretical literature has discussed the concepts of 'natural rate', 'potential output rate', 'flexible prices rate', 'equilibrium rate' and 'NAIRU', but it is beyond the scope of this chapter to add to this discussion. Throughout this text, the terms \textit{NAIRU, equilibrium unemployment and trend unemployment rate} will be used as synonymous.

\textsuperscript{3} Differently, the researchers at the European Commission's D-G for Economic and Financial Affairs seem to prefer the production function approach (McMorrow and Roeger, 2001). This option may be
Moreover, the Gordon-style method has been used by the European Central Bank (ECB) research in the computation of the quarterly Euro-area "trend unemployment rate" included in the Area Wide Model Database (AWMD, hereafter) published with Fagan et al (2001). The ECB itself has recently summed up its interest in the concept and estimation of potential output - which applies as well to the equilibrium unemployment rate -, for the purpose of assessing the trade-off (European Central Bank, 2000a, page 37):

"The main interest of the ECB in the concept and estimation of potential output arises in the context of its stability-oriented monetary policy strategy (...) this strategy has two pillars. (...) Measuring potential output and its growth rate is an important issue under both pillars. Under the first pillar, (...), a measure of trend growth helps derive the reference value for growth in the broad monetary aggregate M3. Under the second pillar, (…), potential output growth and the output gap may be useful indicators for assessing the potential for inflationary pressures in the short to the medium term."

In addition to the arguments and results from these studies and to the institutional appraisal, the consideration of a time-varying NAIRU seems almost unavoidable in the Euro Area case, because of the persistent rise in European unemployment in the 1970s, 1980s and part of the 1990s. It does not seem realistic to try to explain such systematic increase in unemployment unless we admit an increase in the equilibrium unemployment rate itself, as has been extensively argued by Olivier Blanchard. Furthermore, there are reasons to believe that this process is now under inversion, i.e. the NAIRU may be now decreasing – see Blanchard (1999, 2000a, 2000b, 2000c, 2000d), Blanchard and Wolfers (2000), and Blanchard and Giavazzi (2001).
Having thus concluded that a time-varying NAIRU mechanism should be included in the estimation of the trade-off, we set out to address some of the shortcomings of the Gordon (1997) method as used in the research reported by Fabiani and Mestre (2000) and Fagan et al (2001). These shortcomings seem to affect the AWMD's trend unemployment series and may have important (technically as well as for policy making) consequences.

The problems include difficulties in the estimation of the signal-to-noise ratio in the decomposition of the unemployment rate, options concerning the specification of inflation expectations in the Phillips equation and the functional forms of the measurement equations, and also some statistical issues in computing confidence bands for the estimated NAIRU.

Based on recent literature about these problems, this chapter offers new tests and estimates of the Euro Area equilibrium unemployment rate, trend output and the associated real gaps. This is pursued by estimation of a small unobserved components macro model, that complements Gordon's measurement system with an Okun equation. This equation provides additional information necessary for the estimation of the standard deviation of the innovation to trend unemployment, thus achieving a proper estimation of the signal-to-noise ratio of unemployment. The estimation itself uses a conventional state-space framework and Kalman filter techniques. But some novelty is added furthermore by the consideration of a near-rational forward-looking model and, especially, of a battery of encompassing tests for asymmetries in the Phillips and Okun equations. Adequate confidence bands are also computed, for statistical evaluation purposes.

The rest of the chapter is planned as follows. In section 2.2, we explain in detail the problems implicit in current estimates for the Euro area, as well as my attempts to

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6 Some recent research projects at the ECB have also been estimating unobserved components systems of inflation, unemployment and real output of the aggregate Euro area, in work parallel to this chapter's - see the ECB's working papers by Camba-Mendez and Palenzuela (2001) and Fabiani and Mestre (2001). Some discussion of essential differences between these studies and this chapter's is offered below.

7 Fabiani and Mestre (2000, page 20) report having performed some non-nested tests of a linear Phillips equation against the alternative in which unemployment enters in logs rather than in levels; however, they do not show these results. They point that they are aware of the limitation of considering only the
overcome each of them. Section 2.3 presents the resulting model and its state-space format that allows the estimation with the Kalman filter. In section 2.4, the sources and main statistical characteristics of the data are described. Section 2.5 summarises the results of the research, while on section 2.6 we offer some concluding remarks.

2.2. Addressing Current Problems in the Estimation of the Euro Area NAIRU

In this section the problematic issues in the estimation of the Euro Area NAIRU are described in detail. Furthermore, the solutions used below in this chapter to overcome those problems are explained thoroughly.

2.2.1. Estimation of the NAIRU’s Innovation Standard Deviation

When estimating his model, Gordon (1997) initially feared that the time-varying NAIRU would pick-up all the variation in the Phillips' equation residual. In practice, however, the problem that many researchers faced was precisely the opposite - the 'pile-up' problem. In short, the maximum likelihood estimator of the variance of the innovation to an unobserved component that has low variation (low true standard deviation) is biased to zero because a large amount of probability in its distribution piles-up at zero. This difficulty in estimating the signal-to-noise ratio may exist even when the cyclical unemployment gap is modelled and, therefore, the system uses the unemployment rate information besides the inflation rate information - see, *inter alia*, Laubach (2001).

Stock and Watson (1998) defined median unbiased estimators for the coefficient variance in a time-varying parameter model, thus offering a statistical solution for the problem - which was used in Gordon (1998a) and Staiger, Stock and Watson (2001).

Apel and Jansson (1999a) (1999b) suggested a solution more derived from economic theory and used it successfully for some developed countries. Specifically, they included an Okun equation in the system, relating the unemployment and the logarithmic functional form, and they also seem to be aware of the limitation of not specifying nested tests.
output gap, thus adding more information to it and achieving a reasonable estimation of
the variance of the innovation to the NAIRU.\textsuperscript{8}

This solution is very interesting not only because it is driven by economic theory and
stylised facts, but also because it provides an estimate of the Okun relation, which
has important policy applications. Moreover, it allows for the discussion of many of the
relevant specification issues associated to this relation, such as the possible lags in the
association between the gaps and the specific functional form of the equation. The latter
topic has been receiving a large interest recently, with some researchers testing for
asymmetry in the cyclical relation between unemployment and output - see Mayes and

We follow Apel-Jansson's suggestion, extending the basic Gordon measurement
system by including the Okun equation. This will provide estimates of the (quarterly)
Euro Area Okun relation and allow testing for asymmetry in this relation, with a model-
consistent method that attempts to improve on the ones recently used in the literature.\textsuperscript{9}

Camba-Mendez and Palenzuela (2001) and Fabiani and Mestre (2001) also
estimated unobservable systems for the Euro area including the Okun relation, in
research parallel to this chapter's. However, they use different methods for identifying
the model that best fits the data, as discussed below. Ross and Ubide (2001) assessed
the results from multiple trend-cycle decomposition methods, for aggregate Euro Area

\textsuperscript{8} Okun (1970) suggested that the Law could be either estimated as a relation between the first-differences
of unemployment and output or, alternatively, as a relation between the deviations of unemployment and
output from their trends (i.e., their gaps).

\textsuperscript{9} Mayes and Viren (2000) and Viren (2001) tested asymmetry using U gaps and Y gaps estimated by
linear and purely mechanical techniques, while in this chapter the technique is mixed statistical-
thoretical and the gaps are estimated simultaneously with the asymmetry testing. Also, they used
threshold models (piecewise linear regressions) while in this chapter more complex non-linearities will be
investigated. Harris and Silverstone (2001) modelled and tested asymmetry in the (short-run) error-
correction mechanism implied by the (long-run) co-integration relation that they identified between the
levels of unemployment and output. Specifically, they tested a unique error-correction mechanism
(representing the adjustment of unemployment and output - i.e. \(\Delta u \) and \(\Delta y\) - to their long-run equilibrium)
versus the hypothesis of an asymmetric ECM, using the threshold auto-regressive model. Lee (2000)
tested asymmetry in the relation between the first differences of unemployment and output and also in the
relation between the unemployment and output gaps, using gaps pre-computed from three alternative
methods (Hodrick-Prescott filter, Beveridge-Nelson decomposition and a Gordon-style Unobserved-
components decomposition), within a threshold regression framework. Cuaresma (2000) also used a
piecewise-linear approach, to test for asymmetry in the contemporaneous relation between output and
unemployment gaps, comparing the outcomes of two alternative methods for trend-cycle decomposition -

26
data, including four UC models, among which is the one we identify in this chapter, although always in a linear framework.

2.2.2. Inflation Expectations in the Phillips Equation

The Phillips equation in Gordon's (1997, 1998a) triangle model of inflation - which is, as discussed above, the ECB's Research framework - is of the kind

\[
\pi_t = A(L)\pi_t + \gamma(u^n_t - u_t) + \omega S_t + \varepsilon_{phi}^t
\]

where the lag polynomial \(A(L)\) is arbitrarily chosen and is meant to proxy for inflation expectations and inertia.\(^{10}\) The underlying expectations formation hypothesis is clearly the backward-looking Cagan-Friedman's adaptive expectations rule.

The New Keynesian Phillips curve, derived from explicit optimization behaviour and microeconomic foundations explaining the existence of nominal stickiness, reached a quite different equation, sometimes known as the Calvo-Rotemberg Phillips equation:

\[
\pi_t = E_t\pi_{t+1} + \gamma(u^n_t - u_t) + \omega S_t + \varepsilon_{phi}^t
\]

This is forward-looking, because it derives from a model in which agents know that they can not change prices every periods and, so, condition their decision on the price they're setting for current period (t) on some expectation of the future prices (period t+1) prevailing in the Economy. Expectations are assumed to be model-consistent and rational – that is, agents are supposed to use efficiently all the information available by the time they form expectations, and they formulate the forecast out of the true model of the economy, which supposedly they know. As

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\(^{10}\) As is well known, since Friedman (1968) and Phelps (1967), the Phillips (1958) equation includes inflation expectations as the variable explaining the position of the curve in the \([\pi, U]\) space. Therefore, the curve complies with the Natural rate theory: in the long-run (when expectations can not be wrong) there is no relation between nominal and real macro-economic variables. In the equations in the text, S stands for some proxy for supply-shocks, which is crucial for a correct empirical specification of the equation - see the discussion below in section 2.3.
opposed to the Friedman-Phelps', this equation has reasonable microeconomic foundations.\footnote{For a simple deduction of the Calvo-Rotemberg Phillips equation see, \textit{inter alia}, Roberts (1995).}

In spite of its theoretical virtues, however, the New Keynesian Phillips equation is at odds with the data. As Ball (1994a, 1994b, 1997) stressed, it fails to explain why disinflations are costly in the real world. Fuhrer and Moore (1995) and Estrella and Fuhrer (1998) showed that the purely forward-looking New Keynesian model can not generate the high degree of persistence that is observed in actual inflation data, because the model has price level inertia and this precludes price changes (inflation) inertia.


These models, while empirically successful, seem to lack any deep theoretical foundations for the assumed expectations' rule.\footnote{These are not identical models, as the reasons considered for the existence of backward-looking behaviour and the specific combination of backward and forward-looking expectations vary considerably. This does not preclude considering these studies as part of a family of models designed to deal with one problem: that, in order to fit the data, the New Keynesian rational expectations' sticky-price model needs to be augmented by additional lags of inflation that are not predicted by the model. Other studies in which similar hybrid Phillips equations are used but where the weights on past and future inflation are not estimated, but rather set \textit{a-priori}, include McCallum and Nelson (2000), Jensen (2001), Soderstrom (2001) and Walsh (2001a). The first two attach weights to lagged inflation of, respectively, 0.5 and 0.3. The third uses a baseline weight of 0.5 but also tests 0.75. The latter, also uses 0.3 as the benchmark value, but studies the performance of his model with weights ranging from 0 to 1.} Moreover, the estimated weight of the backward-looking part of expectations varies considerably across studies. For instance, Roberts (2001) estimates this weight to be around 50 percent. Fuhrer (1997b) found a point estimate of 0.8 for the US and could not statistically rule out complete non-rationality. Rudebusch (2002b), using survey data to proxy for expectations, estimated a coefficient of 0.71 for US quarterly data, but found the forward-looking component to be statistically different from zero. Similar estimates and inference have been obtained by Lindé (2001a), from a FIML estimation of a system composed of the new keynesian
Phillips equation, an optimising IS equation and a policy rule.\textsuperscript{13} Jondeau and Le Bihan (2001) document that the estimates of the weight of backward-looking expectations are highly sensitive to the estimation method - single-equation GMM and FIML estimation of a system similar to Lindé's (2001a) - and to the extension of leads and lags of inflation. They obtain estimates between 0.34 and 0.73 for the US and between 0.26 and 0.64 for the Euro Area. For the specific case of the Euro Area, Smets (2000) finds a weight of 0.48 in a GMM estimation with annual 1974-1998 data, while Doménech \textit{et al.} (2001b) obtain a weight of 0.46 in a GMM estimation with aggregate EMU quarterly data for 1986:I-2000:IV. More recently, based on simulations, Soderlind \textit{et al.} (2002) argue that one of the conditions for a New Keynesian model to mimic the behaviour of US main macro-economic time series is that the backward-looking behaviour in the Phillips equation has a weight of, at least, 0.9.

In contrast to this literature, Gali and Gertler (1999) suggest that, for the US case, the backward-looking component, although improving the statistical fit, should have only a minor weight, when a measure of the real marginal cost is used in place of the output gap. Within this so-called New Keynesian Phillips Curve set-up, Gali \textit{et al} (2001) estimated a weight of about 0.3 for inflation lags in a Phillips equation of the Euro Area, and about 0.4 for the US. However, Rudd and Whelan (2001) have shown that Gali and Gertler's estimation procedure, based in GMM single-equation estimation, is likely to suggest that forward-looking behaviour is very important even if the true model contains no such behaviour.\textsuperscript{14} And Ma (2002) showed that the parameters in the hybrid backward-forward-looking Phillips curve are weakly identified, so that conventional GMM statistics are inappropriate and, thus, Gali and Gertler's conclusions are problematic. Lindé (2001a) has documented and explained the bias and the difficulties, in general, of single-equation estimation methods to generate reliable estimates of the New Keynesian Phillips Curve - especially in the presence of (even mild) measurement errors. Jondeau and Le Bihan (2001) also find, in hybrid Phillips curves using the real marginal cost, that single-equation GMM estimation generates

\textsuperscript{13} He shows, with simulations, that this empirical approach pins down quite well the true parameters of a calibrated model - even with large measurement errors and some model mispecification.

\textsuperscript{14} Rudd and Whelan proposed a test of the NKPC based on its closed-form solution, in which current inflation equals a discounted stream of expected future output gaps (or unit labour costs). For the US case, they found that expectations play a very small role in explaining why high and significant
smaller estimates of the backward-looking part of inflation dynamics than does FIML estimation of a system enhanced with IS and policy rule equations - but that it is always significant. In their results, for the Euro Area, the difference is from 0.25 to 0.47, and the weight in the US can be as large as 0.48.

Many recent empirical studies of the Phillips curve have adopted several alternative pragmatic approaches to proxy for inflation expectations. Basically, four main different types of pragmatic proxy measures have been used. First, some studies have used time series data of inflation expectations' surveys, as was the case in Clark et al. (1995, 1996, 2001), Roberts (1995, 1997), and Laxton, Rose and Tambakis (1999). Second, others have estimated inflation expectations from some observed bond-market yields, as did Clark et al. (1996), Debelle and Laxton (1997), Yates (1998) and Debelle and Vickery (1998). Third, Pyythia (1999) and Mayes and Virén (2000) used the OECD's Secretariat official forecasts of price inflation published with the Economic Perspectives bi-annual bulletin. Fourth, another branch of the literature has proxied inflation expectations with the one-step-ahead forecasts of inflation obtained from some univariate statistical model fitted to inflation observed data. For example, Meyler (1999) used the predictions from an ARIMA model and Ash et al (2000) considered predictions from the Hodrick-Prescot filter. Dupasquier and Ricketts (1997, 1998) proxied canadian inflation expectations with the predictions of a three-state Markov switching regime model that endogenously picked-up periods of low, medium and high inflation.

The first two pragmatic approaches above are not feasible at this stage in the Euro area case. As to the first, many member-states do not have any inflation expectations' survey and, accordingly, there is no such survey for the Area. In what regards the second method, there is not enough homogeneity in the maturity and trading conditions of the indexed bonds of the Euro member-states. The third method could only be used in studies of annual or bi-annual data, whilst the focus in this chapter is on quarterly data. The fourth approach seems better suited for the problem at hand, but demands stronger theoretical foundations, so that its pragmatism can be, in any way,
legitimated by the theoretical debate on backward versus forward-looking expectations described above. This legitimisation can be found in a recent work by Ball (2000).

Addressing the forward-versus-backward-looking expectations debate, Mankiw (2000) stated that what the data are crying for is adaptive expectations. However, Ball (2000) noted that adaptive expectations are subject to the Lucas (1976) critique in that they are not necessarily adequate for all monetary regimes.

To solve the dilemma (between rational expectations Phillips equations failing to fit the facts and backward-looking equations being subject to the Lucas (1976) critique) Ball (2000) suggested the concept of near-rational inflation expectations. Ball's hypothesis is that real-world agents form inflation expectations considering only the past information on inflation, but use it optimally, identifying and estimating the best linear univariate forecasting model:

"The deviation from rationality is the fact that forecasts are univariate: agents ignore relevant variables such as output and interest rates. Aside from this limitation, agents' forecasts are optimal: they use inflation data as best they can. Metaphorically, one can imagine firms who use Box-Jenkins techniques to select an ARIMA model for inflation, but who do not go to the added trouble of using multivariate techniques."

This near-rational rule is then, in practice, a limited-information (or weak) rational expectations rule, which has some microeconomic foundations, in the spirit of Akerloff and Yellen (1985a, 1985b) model. It describes agents that, faced with the high costs of gathering and processing the whole information set needed for rational expectations, and knowing that these costs won't pay-off, limit their information set and form forward-looking expectations solely on the basis of past inflation.

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15 Even though he stressed that this pragmatic solution to the New Keynesian Phillips curve puzzle is not entirely satisfactory as adaptive expectations lack foundation in the microeconomic theories of price adjustment. In recent work, he suggested a model of slow dissemination of information (sticky information) instead of 'sticky prices' as a way of dealing with the New Keynesian Phillips Curve puzzle, - see Mankiw and Reis (2001a, 2001b). Although with different foundations, in practice, Mankiw and Reis' model is very similar to Fischer's (1977). Rudd and Whelan (2001) stress that their results against the New Keynesian Phillips equation do not necessarily imply that agents formulate expectations in a backward-looking manner, and that alternative avenues of research, like Mankiw and Reis', must be pursued.

16 Ball (2000), page 9. The classification 'optimal' seems a bit exaggerated in this context, as the near-rational rule proposed is merely that of a best linear unbiased univariate forecasting model.

17 Ball notes that it is assumed that (i) agents know the true time-series process that characterises inflation in each specific monetary regime, (ii) identify it ever-since its first period and (iii) estimate it with data from the whole sample period. Although not stronger than rational expectations', these hypotheses are strong and Ball admits that (ii) and (iii) could be alleviated with some learning modelling.
In this research we use Ball's (2000) concept of near-rational expectations. Specifically, a series of inflation (changes) expectations is computed exogenously to the model, as predictions from the best ARIMA model identified and estimated with Box-Jenkins methods for the adopted deflator series - and then it is used as data in the Phillips equation. Therefore, this equation is forward-looking - and, so, New Keynesian - but is also compatible with the inflation inertia observed in the present monetary regime.\footnote{Throughout the chapter, the results for the baseline near-rational expectations model are compared to the ones of standard adaptive expectations regressions - in which the coefficients on some lags of inflation changes are estimated within the Phillips equation. Note, however, that our adaptive expectations' models do not include \textit{arbitrary} lags of inflation, but have the lag polynomial truncated at the same extension of the optimal univariate ARIMA identified and estimated (3 quarters).}

\subsection*{2.2.3. The Phillips Equation Functional Form}

For the UC trend-cycle decomposition set-up to work well, it is crucial that the measurement equations are correctly specified. In this chapter's system this includes, notably, a correct choice of the Phillips and Okun's equations functional forms.

Recently, a great interest has been devoted to the possibility that the Phillips equation is in fact asymmetric and not linear.\footnote{The case for convex Okun equations has also been made - see Mayes and Viren (2000) and Viren (2001). But it has been made less out of theoretical foundations - which the Okun relation does not have at all - and more from stylised facts: specifically, the observation that in expansions the unemployment rate tends to decrease less than it tends to increase in recessions.} As Clark and Laxton (1997) recalled us, the original Phillips (1958) and Lipsey (1960) formulations related some price change measure to the inverse of unemployment.

There are several reasons why the Phillips relation may be asymmetric, both theoretical and empirical.

Empirically, the asymmetry in the inflation-output trade-off provides good explanations for many stylised facts of the post-war industrial economies, most of which have been reviewed, for the US case, by De Long and Summers (1988). More recently, Macklem (1995) noted that Phillips asymmetry could explain the apparent inflationary bias seen in recent macroeconomic behaviour and the need for deep and long recessions during the 80s and 90s to reverse the inflation build-up of the 70s.
The extensive literature of empirical testing for asymmetric effects of monetary policy has also been considered as evidence of asymmetry in the Phillips curve. This literature has addressed two distinct types of policy effects asymmetries: asymmetry in the effects of monetary stimulus versus contraction, on one hand, and asymmetry in the effects of policy actions in periods of positive output change versus periods of negative output growth. As Lo and Piger (2001) argue, convexity of the Phillips curve is theoretically associated to the second type of asymmetry, but because the state variables behind the two asymmetries are often correlated, it is difficult to determine which asymmetry is driving the empirical results. Notably, for our research, a convex aggregate supply could also show up empirically as a first type of asymmetry. Yet, most of the literature has been incapable to distinguish between both types of asymmetry, and to resolve some of their conflicting results, so we review both.

Within the first type of asymmetry, Cover (1992) studied US quarterly post-War data and found evidence that negative monetary shocks have significant negative effects on output, whilst the output effects of positive monetary shocks are not statistically significant. Analysing annual data of a panel of 18 European countries (1953-1990), and using a similar method, Karras (1996a) confirmed Cover's (1992) results, which suggested the existence of Phillips asymmetry also in the European case. Relaxing some constraints implicit in Cover's (1992) method, Karras (1996b), and Karras and Stoke (1999a, 1999b) offered additional evidence supporting his results. Moreover, they extended them by showing that the observed asymmetry in the effects of monetary shocks seems to arise from both aggregate-supply (Phillips) convexity and a credit ("pushing on a string") mechanism - both in quarterly seasonally adjusted US data for 1960:IV-1993:IV - (1999a) - and in quarterly data from 1963-1993 of a panel of 12 OECD countries - (1999b). Notably, Karras and Stokes (1999a), also offer evidence that asymmetries in the effects of monetary shocks on output are positively correlated with the level of inflation and that this asymmetry is due to an asymmetric response of investment and not of consumption.20 Karras and Stokes (1999b) showed that the asymmetry in the effects of positive versus negative shocks survives even when the cyclical situation of the economy is controlled for, thus suggesting that these may be
two co-existing phenomena. Thoma (1994) presents similar results, based on the significance of rolling sample Granger causality tests of money over output. On the contrary, Weise (1999) finds no evidence of a statistically different impact of M1 decreases versus that of M1 increases, within a trivariate VAR of output, prices and money. Also, Belongia (1996) finds that Cover's (1992) results are not robust to a change from the traditional monetary aggregate to a *divisia* index. Senda (2001) studies prewar and postwar data of OECD countries, and finds that the asymmetry between the effects of positive and negative monetary shocks depends upon the level of trend inflation, and suggests that a model with price-adjustment costs and positive trend inflation can explain the empirical results.

Within the second type of asymmetry, Macklem (1995) noted that the effects of the monetary tightening of the late 80s were slow to emerge when the economy was, initially, above the trend, but seemed to increase more than proportionately when it began operating below the trend.\(^21\) Weise (1999) reports evidence in favour of a convex aggregate supply curve by finding that significant aggregate demand shocks in either direction have stronger output effects when economic activity is weak and stronger price effects when economic activity is strong. Using a switching-regime model of real output to distinguish expansions from recessions, Garcia and Schaller (2002) also find larger effects of monetary actions during recessions than during expansions. With quarterly Austrian data from 1976:I-1998:IV, Kaufmann (2001) estimated an univariate model of real GDP including the Austrian three-month interest rate as a measure of monetary policy, within a Bayesian framework using Markov Chain Monte Carlo simulation methods, and found a significant negative effect of monetary policy during periods of below-average growth, and insignificant effects during periods of normal and

\(^{20}\) Shen (2000) also finds an association of the asymmetry to the level of inflation, in Taiwan data, as he reports results suggesting that positive monetary shocks have a positive effect on output when inflation is low, no effect for medium-range rates of inflation, and negative effects for high inflation rates.

\(^{21}\) Note that although asymmetric effects of monetary policy over the cycle - type II asymmetry - may be a symptom of a convex Phillips curve, they are not necessarily caused by such a convexity. In fact, there is at least one major alternative mechanism that could explain policy effects' asymmetries - the credit market effects discussed in Bernanke and Gertler (1989, 1995). These are associated to credit market imperfections, to the fact that firms are typically exposed to external financing but more heavily during recessions than during expansions, and to the fact that this external finance exposure influences firms' net value. These conditions create a 'financial accelerator' mechanism that acts more significantly during recessions than during expansions. In recessions, cash flow are lower, firms are more dependent from external finance and the values of the collateral of corporate debt are depressed, which makes firm's external finance premium more sensitive to interest rate changes.
above-average growth. At a more global European level, Peersman and Smets (2001a) studied the (a)symmetry in the effects of Area-wide monetary policy impulses on industrial production growth of seven EMU economies - Germany, France, Italy, Spain, the Netherlands, Belgium and Austria - during 1978-1998. Specifically, they computed the Area monetary impulses as the historical contribution of the monetary policy shocks to the Euro area short-term interest rate, as estimated from a structural VAR of the whole Euro area (i.e. as the cumulated effects of current and past monetary policy shocks on the interest rate). Then, they estimated a two states (recession and expansion) Markov Switching Regime Model, finding that the monetary impulses have significantly larger effects on industrial production growth during recessions than in booms, especially in Germany, France, Spain, Italy and Belgium.

Recently, Lo and Piger (2001) augment a standard univariate unobserved components trend-cycle decomposition of real US output with lagged monetary policy variables explaining the cyclical component, and allowing their coefficients to undergo Markov regime switching with no a-priori identification of the type of asymmetry involved. Their results show that when cyclical output is negative and falling, monetary policy is significantly more effective than otherwise, which is consistent with type-II asymmetry but not type-I asymmetry. In what regards the theoretical models of asymmetry, their results are only partially consistent with Phillips asymmetry - because monetary effects become different when the recession is deepening, but not when the economy is equally below trend but the transitory component is rising. They are, on the contrary, consistent with the Bernanke and Gertler (1989, 1995) credit-channel model.

In another strand of the literature, asymmetry is also suggested by the finding that some estimates of the benefice ratio of inflationary episodes seem significantly inferior to the estimated sacrifice ratio of disinflations. And the non-normality in the errors of Phillips line regressions, noted by some researchers, could also mean that the effect of the output gap on inflation is asymmetric and the errors could be picking up that wrong specification of those equations.

22 Portugal, Luxembourg, Ireland and Greece were excluded apriori of the analysis due to data limitations, while Finland was excluded because of statistical evidence that its macroeconomic cycle was not synchronised with the common cycle shared by the other States studied, at the outset of the EMU.
24 See, for instance, Fisher, Mahadeva and Whitley (1997), page 78.
Theoretically, Ball, Mankiw and Romer (1988) argued that a model with New-Keynesian characteristics implies a convex Phillips curve, and Ball and Mankiw (1994) provided possible explanations for why microeconomic rigidities generate asymmetries in the macroeconomic cycle and in the effects of shocks. These essays made asymmetry a part of the New Keynesian counter-revolution.\(^\text{25}\)

Recent surveys of theoretical New Keynesian models of pricing behaviour that imply asymmetry in the short-run adjustment of prices to aggregate demand shocks are given in Dupasquier and Ricketts (1998) and Macklem (1997). Most prescribe that the Phillips equation is convex - the capacity constraint model, the menu costs model, the contracts model, the downward nominal wage rigidity model and the efficiency wages model.

The rationale for the convexity in the Phillips relation varies considerably across these theories. The capacity constraint theory highlights the increasing difficulties of firms to increase production as they get closer to full capacity, and so justifies the increased elasticity of prices to demand at high levels of capacity utilisation. The menu-costs and the contracts theories stress that the frequency and size of price and wage revisions tend to increase with the inflation level. The downward nominal wage rigidity postulates that in contemporary economies the nominal wages have floors which imply that excess supply shocks have more real effects (and less nominal effects) than excess demand shocks have. The efficiency wages approach complements the downward nominal wages story by providing efficiency reasons for higher than equilibrium nominal wages, more prone to increase than to decrease.

On the other hand, there is one theoretical alternative - the monopolistically competitive model - that implies a concave functional form. In this model, oligopolistic competition causes firms to be more reluctant to raise prices than to lower them, because of the fear of losing market share.\(^\text{26}\)

\(^{25}\) This is not to say that these authors' arguments were undisputed - see the discussion in Akerlof, Rose and Yellen (1988).

\(^{26}\) Eisner (1997) and Stiglitz (1997) report evidence of a concavity in the US Phillips curve. Dupasquier and Ricketts (1997) perform a simple test (see pages 139-141) using data from Canada and the US, and reject the hypothesis of a concavity in the Phillips relation. More recently, Filardo (1998) studied US data between 1959 and 1997, and found evidence that the Phillips relation is convex when the economy is overheated (operating well above the trend) but is concave when the economy is weak (well below the trend). This evidence is seen as a possible explanation for the apparently contradictory results on the shape of the US Phillips curve.
Additionally to these global reasons, there are reasons specific to the EMU that strengthen the case in favour of a non-linear Phillips equation in this Area. Specifically, it has been argued that the aggregation of sectors of activity and regions with different cyclical positions and, possibly, different Phillips slopes, results in an asymmetric aggregate relation between inflation and the cyclical condition of the economy - see Mayes and Viren (2000) and Demertezis and Hallett (1995, 1998). This could be significant and tend to persist as the Euro Area still exhibits weak labour mobility across sectors and sub-regions.

Asymmetry in the Phillips equation would have important policy implications. With (convex) asymmetry, excess demand has more inflationary effects than excess supply has deflationary effects, which means that on average the unemployment gap will have to be negative for the inflation rate to be stable.

Figure 2.1 below illustrates this idea. In a deterministic world, the long-run Phillips curve would hold, as there are no surprises or shocks, and a null unemployment gap would be an equilibrium consistent with no inflation changes, even with Phillips curve asymmetry. This is the intersection of the Phillips curve to the [0,0] point in the plot. But if inflation and real activity vary stochastically - as they systematically do - and if the Phillips curve is convex, stationarity of the inflation rate implies that the unemployment gaps can not average to zero, but rather must be, on average, negative. This is termed Gap\textsuperscript{E} in the figure, which is simply the mean value of the unemployment gaps that occur if the inflation changes remain bounded symmetrically around zero, given an asymmetric Phillips curve. This is what might be called the stochastic equilibrium gap, which, as the figure clearly shows, is not identical to the deterministic equilibrium gap (zero), but rather is negative.\textsuperscript{27}

Most of the functional forms tested in this chapter allow for the data to choose between a convex or a concave configuration of the function, when estimating each form's crucial parameters - see below. However, it can not be denied that the author's aprioris favoured a convex specification in detriment of the concave - which could be inferred from the written text at some stages.

\textsuperscript{27} Laxton et al (1995), Clark et al (1995, 1996), Clark and Laxton (1997), Laxton et al (1999) and Isard et al (2001), \textit{inter alia}, state these arguments not in terms of deterministic and stochastic equilibrium gaps, but in terms of deterministic and stochastic NAIRUs. Their terminology is associated to the fact that they model the asymmetry in relation to the level of unemployment and not to the unemployment gap. In this chapter, the concepts, as well as the asymmetry tests below, are always developed in terms of...
Convex asymmetry would, therefore, call for a more activist stabilisation policy, taking pre-emptive action against any forecasted excess demand situation, because a subsequent policy to deliberately disinflate would be more costly than one to pre-emptively resist a similar rise in inflation. Clark et al. (2001), in their model for evaluation of alternative monetary policy rules in the US - including a convex Phillips curve - document this argument: an inflation forecast based rule dominates a rule that is not forward-looking but in which interest rates react more to positive than to negative gaps.

Moreover, a stabilisation policy that reduces the variability of real economic activity - and, as result, of inflation changes - gains particular importance in a convex economy, as it would place the (stochastic) equilibrium unemployment gap closer to zero. On this, see the arguments and the simulations in Laxton et al. (1995) (p.361-371), Clark et al. (1995, 1996), Clark and Laxton (1997) and Laxton et al (1999).

unemployment gaps, because that is the only meaningful concept and testable magnitude in a model of a time-varying NAIRU.

On the contrary, a concave Phillips curve would imply that a policy to pre-emptively resist inflationary pressures of a given size is more costly than a policy to deliberately disinflate.

These authors have also argued that policy errors arising from an incorrect assumption that the economy is linear would be more costly than the error of adopting a policy reaction function that assumes
The question of the possible asymmetry in the Phillips relation is, however, far from settled. The tests results have been mixed, depending on the countries, the specific time-series under study and the empirical methods and technical options adopted.\(^{30}\)

In the earlier stages of this literature, the NAIRU was assumed to be a constant or, at least, was estimated out of the model where asymmetry was to be tested - generally with the Hodrick-Prescott filter. This branch of the literature includes the studies by Chada \textit{et al} (1992), Turner (1995), Laxton \textit{et al} (1995), Clark \textit{et al} (1995, 1996, 2001), Fisher \textit{et al} (1997), Dupasquier and Ricketts (1997, 1998), Yates (1998), Filardo (1998), Pyythia (1999) and Mayes and Viren (2000). The asymmetric models considered in this literature include threshold linear models, quadratic, cubic and hyperbole functions.\(^{31}\)


However, these studies estimate asymmetric models without any formal pre-testing of the asymmetry. Furthermore, instead of relating the asymmetry to the unemployment gap, they specify it as related to the level of unemployment, which, in the modern framework of time-varying NAIRU, may not necessarily reflect the cyclical situation of the economy. A recent result that may corroborate this point, has been offered in Hamilton (2001): applying his parametric method of flexible nonlinear inference to US annual inflation and unemployment data from 1949 to 1997, he found no evidence of nonlinearity of the Phillips function on the unemployment level.\(^{32}\)

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30 Note that, just like in this text, at least three basic technical problems must be solved in these studies, which may affect the results. They are (i) the choice of the specific asymmetric functional form, (ii) the choice of the econometric technique for estimating the output/unemployment gap and (iii) the modelling of inflation expectations.

31 In a somehow different set-up, Eliasson (1999) modelled the non-linearity of constant-NAIRU Phillips curves as a logistic smooth transition autoregression.

32 Nor he found any evidence of nonlinearity as function of the inflation level. In his estimates, the time trend was the only regressor with a significant nonlinear effect on the inflation rate. This could mean that the nonlinearity that he captures in his model may be the result of the time-variation of the supply-shocks variables - that he does not proxy for in the regression - and/or of the time-variation of the NAIRU.
Recently, StAubyn (2000) suggested an empirical strategy that allows the formal testing of asymmetry encompassing the null hypothesis of linearity within the estimation of an unobserved components model designed to identify the NAIRU and the gap. He tested for asymmetry in the US Phillips curve specifying the Phillips' slope parameter as a possible function of the recent unemployment gap or inflation changes, a method that Martins and StAubyn (2001) also applied to Portuguese data.

In this chapter, this strategy is used to test for asymmetry in the Euro Area Phillips and Okun equations, thus offering the first formal joint test of these non-linearities. First, some preliminary evidence is detected using a linear spline (piece-wise linear or threshold) approach. Then, results from two functional forms quite common in the literature are supplied - quadratic and hyperbole - and, in addition, evidence using the exponential function is also offered. On the whole, these tests should provide reasonably robust statistical evidence regarding the problem at hand.

As mentioned, we consider and test for a non-linear Okun relation, jointly with the possibly non-linear Phillips curve. This is itself of interest - as stressed in recent literature by Mayes and Viren (2000), Lee (2000), Cuaresma (2000), Viren (2001), and Harris and Silverstone (2001) - and deepens the knowledge about the Euro Area cyclical behaviour. Moreover, it enables a more precise definition of the trade-off, by providing an indirect test of the hypothesis that its linearity/non-linearity depends upon the gap considered - unemployment or output. In our model, we have the opportunity to offer a first piece of evidence on this for the aggregate Euro Area, with the further advantage of using model-consistent output and unemployment gaps.

2.2.4. Confidence Bands for the Estimated NAIRU and Gaps

As is well known, the uncertainty associated to unobserved components' estimates surpasses the variance of the errors in their one-step ahead forecasts that is given by the Kalman filter. In fact, besides this uncertainty due to the estimation of the time-varying parameters, there is also the uncertainty due to the estimation of the hyper-parameters of the likelihood function. The Kalman filter does not generate a measure of this total uncertainty, and yet this is important for some assessment of the utility of the estimates for the practical conduct of monetary policy.
Hamilton (1986, 1994) suggested the use of Monte Carlo integration to compute an approximation to the total uncertainty around time varying parameters' estimates.\textsuperscript{33}

Using a large number (N) of randomly generated sets of hyper-parameters - drawn from their estimated distribution under the assumption of multivariate normality - his method computes measures of the two testable sources of asymmetry. These are filter uncertainty - the one that would occur even if the hyper-parameters were perfectly known - and parameter uncertainty - the one associated to the uncertainty about the hyper-parameters' estimates.

In this study, Hamilton's measure of uncertainty is computed, with N=10,000, for the estimated NAIRU and unemployment gap of the Euro area 1970:I-2000:II.

2.3. The Model

The model is a refined and extended version of the unobserved components framework recently popularised by Gordon (1997).

The basic extension consists of the inclusion of an Okun equation and a transition equation for trend real output, and was suggested by Apel and Jansson (1999a, 1999b) as a way of solving the 'pile-up' problem.

The most important refinements lie in the treatment of inflation expectations and in the testing for asymmetries in the measurement equations, which will allow a better specification of the system.

It consists of the following three measurement (2.1 to 2.3) and four state/transition (2.4 to 2.7) equations:\textsuperscript{34}

\begin{align*}
\Delta \pi_t &= E_t\Delta \pi_{t+1} + \gamma(u^n_t - u_t) + \alpha S_t + \epsilon_t^{phi} \quad (2.1) \\
y_t &= y^p_t + \theta(u^n_t - u_t) + \epsilon_t^{ok} \quad (2.2) \\
u_t &= u^n_t - (u^n_t - u_t) \quad (2.3)
\end{align*}

\textsuperscript{33} See Hamilton (1994), chapter 13, section 13.7. 'Statistical inference with the Kalman filter'.

\textsuperscript{34} Equation (1) and (3) are presented here in their baseline linear specification. The asymmetric functional forms considered in this study and the specific asymmetry are described below.
Equation (2.1) is a New Keynesian Phillips equation of the Calvo-Rotemberg type - specified in inflation changes -, which as is well known, explains inflation with inflation expectations and the demand pressure in the economy.\(^\text{35}\) We denote by \(E_t\), as usual, expectations formed in period \(t\), but, for the sake of realism, we consider that the information then available reports up to period \(t-1\) - which is a somehow standard assumption for quarterly data, see Rudebusch (2002a), but which we discuss further below in section 2.3.1.. The demand pressure is proxied by the difference between the NAIRU, \(u_t^n\) (which is allowed to vary along time), and observed unemployment, \(u_t\). Its action over wages and prices is the heart of the Phillips / Natural rate theory. In this empirical application, the function includes also some variable proxying for transitory supply-shocks (\(S_t\)), which influence inflation through a different channel than expectations and demand, having temporary effects that must be modelled in order to achieve the estimation of a NAIRU and not merely of a short-term NAIRU.\(^\text{36}\)

The second measurement equation is the Okun equation, relating the output gap to the current unemployment gap, which is a well-known empirical regularity with a long tradition in macroeconomics. In this model, it has the specific econometric role described above - of supplying more identifying information to the system and, thus, allowing for the estimation of all the hyper-parameters in the model, including the standard deviation of the NAIRU’s innovation, that otherwise would be affected by the 'pile-up problem'.

\(^{35}\) For a simple derivation of this New Keynesian Phillips relation see Roberts (1995).

\(^{36}\) The short-run NAIRU is the level of unemployment consistent with maintaining the current level of inflation in next period, while the NAIRU is the level that stabilises the inflation rate after all the temporary shocks that affect inflation have eroded their effects. For details on the distinction between the short-run NAIRU, the NAIRU and the long-term NAIRU (or Natural rate), see, *inter alia*, Estrella and Mishkin (1998) and Richardson *et al* (2000).
The third measurement equation, (2.3), stems from the seminal unobserved components model in Watson (1986) and decomposes the unemployment rate into the trend component, the NAIRU, and the cyclical unemployment gap. Technically, it assures that the NAIRU and the gap sum-up to the observed unemployment rate.

The state-system models the dynamics of trend and cycle unemployment and output - equations (2.4) to (2.7).

In equation (2.4), the NAIRU is assumed to follow a random walk, which is the standard assumption in the literature that, as in this chapter, aims at modelling the time-path of the natural rate without explaining it. The random walk process driving the NAIRU includes a drift which itself follows a random walk, rather than being a constant - which is modelled by equation (2.5). This drift specification is used to capture low frequency innovations to the stochastic trend such as long run structural breaks in trend growth rate - see Lo and Piger (2001). In other words, it is a standard procedure for modelling time series that exhibit for some period a specific systematic trend but may invert that trend and drift systematically in the opposite way - used, inter alia, by Gerlach and Smets (1997), Laubach (2001), and Kichian (1999). As discussed above, this is empirically and theoretically suited for the Euro Area case, in which equilibrium unemployment has been drifting up for some decades, seems to be decreasing currently and has the potential to decrease in the next years ahead.

Equation (2.6) models the unemployment gap as a stationary auto-regressive process of order 2. This is the assumption made in Watson (1986) and Kuttner (1994) about the US output gap, and has been adopted thereafter in several works that deal with unemployment gaps such as Apel and Jansson (1999a, 1999b), Rasi and Vilkari (1998) and Laubach (2001). Specifying the cyclical unemployment as a process that reverts to a zero mean captures the essence of Friedman's (1968) natural rate theory - that the unemployment rate can not drift away from the natural rate indefinitely.

Finally, equation (2.7) postulates that trend output follows a random walk with drift process. Here, however, the drift is a constant $g$ - as real output trends systematically up - and its estimated (positive) value represents the quarterly average growth rate of trend real output.
It should be noted that trend unemployment and trend output are specified as time-series processes with different dynamics - equations (2.4) and (2.7) - in accordance to the behaviour of the observed series. This is not the case in recent research parallel to this chapter's. For instance, the one reported in Camba-Mendez and Palenzuela (2001), where both trends are always modelled identically although allowing for three alternative processes: random walks with constant drift, local linear trends (random walk with stochastic drift) or smooth trends. Likewise, in a parallel piece of research on similar unobserved components systems estimated with aggregate Euro area data, Fabiani and Mestre (2001) also impose an identical process on trend unemployment and output - specifically, the local linear model. Ross and Ubide (2001), in turn, compared the performance of a number of alternative UC model formulations - including a linear version of our own - in terms of inflation forecasting and cyclical turning points identification, but did not actually performed any classic econometric identification of the model best fitting the data. We, instead, follow the behaviour of the observed series, and, as discussed below, the a-priori in this model's equation (2.7) was confirmed within our identification work, as the hypothesis of a stochastic drift versus that of a constant drift in trend real output was clearly rejected.

2.3.1. Inflation Expectations

A crucial issue in empirical studies of the Phillips equation is modelling expectations of inflation. As mentioned above in section 2.1, we follow the recent suggestion of Ball (2000), which reconciles the theoretical forward-looking New Keynesian Phillips equation with the inertia properties exhibited by recent inflation data. Specifically, it is supposed that agents are near-rational when forming expectations about future inflation, in the sense that they only use available inflation information. Following Ball's metaphor, this concept is used here assuming that agents are able to identify and estimate the best ARIMA process for inflation. In the present monetary regime, inflation has significant persistence and therefore is generally not stationary in its levels but only in its first differences. This means that agents identify and estimate the best ARMA process for inflation changes.
\[ \Delta \pi_t = \hat{\phi}_1 \Delta \pi_{t-1} + \hat{\phi}_2 \Delta \pi_{t-2} + \hat{\phi}_3 \Delta \pi_{t-3} + \ldots + \hat{\phi}_p \Delta \pi_{t-p} \]

This estimated model yields the rule that allows agents to compute expectations of inflation. To see how, note that the ARMA estimates can be written as

\[ \pi_t = \pi_{t-1} + \hat{\phi}_1 \Delta \pi_{t-1} + \hat{\phi}_2 \Delta \pi_{t-2} + \hat{\phi}_3 \Delta \pi_{t-3} + \ldots + \hat{\phi}_p \Delta \pi_{t-p} \]

Then, at period \( t \), agents form near-rational expectations of inflation for next period \( t+1 \) as follows:

\[ E_{t+1} \pi_{t+1} = \pi_t + \hat{\phi}_1 \Delta \pi_t + \hat{\phi}_2 \Delta \pi_{t-1} + \hat{\phi}_3 \Delta \pi_{t-2} + \ldots + \hat{\phi}_p \Delta \pi_{t-p+1} \]

Equation (1) in our model is a Calvo-Rotemberg New Keynesian Phillips equation transformed so that it is estimated in inflation changes and not levels. Disregarding the other elements in the Phillips relation (gap and supply-shocks), and assuming near-rational expectations of inflation, the transformation is simply

\[ \pi_t - \pi_{t-1} = E_t \pi_{t+1} - \pi_{t-1} + (\ldots) \]

\[ \Rightarrow \pi_t - \pi_{t-1} = \pi_t + \hat{\phi}_1 \Delta \pi_t + \hat{\phi}_2 \Delta \pi_{t-1} + \hat{\phi}_3 \Delta \pi_{t-2} + \ldots + \hat{\phi}_p \Delta \pi_{t-p+1} - \pi_{t-1} + (\ldots) \]

\[ \Rightarrow \Delta \pi_t = \Delta \pi_t + \hat{\phi}_1 \Delta \pi_t + \hat{\phi}_2 \Delta \pi_{t-1} + \hat{\phi}_3 \Delta \pi_{t-2} + \ldots + \hat{\phi}_p \Delta \pi_{t-p+1} + (\ldots) \]

This expression shows how Ball’s near-rational forward-looking Calvo-Rotemberg Phillips curves could be estimated, using the estimated coefficients from the best linear unbiased univariate time-series process for inflation (changes) and data until current period \( t \).

Besides forwardness, one other characteristic of standard formulations of New Keynesian theory is that they describe agents that set prices for period \( t \) after knowing the state of the Economy at that period \( t \). We depart from standard theory at this respect. In fact, we consider unrealistic for a model to assume that agents know the state of the Economy at \( t \) when making price decisions for that same period \( t \), because prices are one of the variables defining the state of the Economy at \( t \). In other words, if agents are deciding prices for period \( t \), that decision must surely be made at the very beginning of that period, when it is not realistic to assume that they could know anything about the state of the economy in a period that has not yet began. This argument would only be reinforced if one considers the lags involved in the publication of statistics describing the state of the Economy, and that we are using quarterly data.
Accordingly, we assume that agents decide prices for period (t), at the beginning of that period, taking into account expectations of inflation for next period (t+1), which they build using the available information on the state of the Economy, as of period (t-1) and the ARIMA coefficients estimates. Hence, in spite of being taken at period (t) for (t+1), expectations are in fact two-step-ahead forecasts, as only magnitudes relating to period (t-1) are assumed to be known when expectations at (t) are formed.

$$E_t \pi_{t+1} = E_t \left( \pi_t + \hat{\phi}_1 \Delta \pi_t + \hat{\phi}_2 \Delta \pi_{t-1} + \hat{\phi}_3 \Delta \pi_{t-2} + \ldots + \hat{\phi}_p \Delta \pi_{t-p+1} \right)$$

After some algebra, this can be expressed only in terms of known information at the beginning of period (t) - which relates to period (t-1) or previous - and of the ARMA coefficients. Assuming, for instance, p=3, we have:

$$E_t \pi_{t+1} = \pi_{t-1} + \left( \hat{\phi}_1 + \hat{\phi}_1^2 + \hat{\phi}_2 \right) \Delta \pi_{t-1} + \left( \hat{\phi}_2 + \hat{\phi}_1 \hat{\phi}_2 + \hat{\phi}_3 \right) \Delta \pi_{t-2} + \left( \hat{\phi}_3 + \hat{\phi}_1 \hat{\phi}_3 \right) \Delta \pi_{t-3}$$

Finally, as we estimate Phillips equations specified in inflation changes, the rule generating our series of near-rational forward-looking expectations of inflation changes is given by the following equation:

$$E_t \Delta \pi_{t+1} = \hat{\phi}_1^* \Delta \pi_{t-1} + \hat{\phi}_2^* \Delta \pi_{t-2} + \hat{\phi}_3^* \Delta \pi_{t-3}$$

(2.8)

where $$\hat{\phi}_1^* = \left( \hat{\phi}_1 + \hat{\phi}_1^2 + \hat{\phi}_2 \right)$$, $$\hat{\phi}_2^* = \left( \hat{\phi}_2 + \hat{\phi}_1 \hat{\phi}_2 + \hat{\phi}_3 \right)$$, and $$\hat{\phi}_3^* = \left( \hat{\phi}_3 + \hat{\phi}_1 \hat{\phi}_3 \right)$$.

Equation (2.8) closes our model, in the sense that the time series generated by (2.8) is taken as data in the Phillips equation (2.1), for each period (t).

We assume that this time-series of inflation expectations is orthogonal to the other regressors in the Phillips equation, namely the current unemployment gap and the lagged deviation of domestic inflation from imported inflation. In the latter, this means that it is assumed that the transmission of foreign inflation to domestic inflation changes takes longer than a quarter - which seems to be confirmed empirically with our data (see

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37 In our empirical analysis we have checked if results would differ significantly with standard Calvo and Taylor versions of New Keynesian Phillips equations – formulations assuming that agents use information contemporaneous to the formation of expectations. The difference is that the Taylor version includes explicitly a simple average of $$E_{t-1} \pi_{t-1}$$ and $$E_{t-1} \pi_{t-1}$$, while Calvo’s version only explicits $$E_{t-1} \pi_{t-1}$$ and considers $$E_{t-1} \pi_{t-1}$$ implicitly in the residual – see Ball (2000, page 18, equations (4) and (4’)). These formulations lead to smaller and not significant Phillips elasticity, even though retaining its correct sign. Also, the Phillips equation residuals behave worse than with our formulation. As for the rest, results are practically equivalent: the identification of the measurement equations is unchanged and the unobserved
below, section 5). Hence, the econometric framework used here does not suffer from the generated regressor problems, as discussed in Pagan (1984) and Pagan and Ullah (1988).\footnote{This model is a 'weak' or 'partly rational' expectations model of the class identified as \textit{model 2} in Pagan (1984) - pages 227-229. See below in section 5 the description of the model identification, where it is shown how the theoretical reasons for admitting the referred orthogonality have been confirmed empirically.}

In the empirical work below, the results from this 'near-rational expectations' model' are always compared to those from an adaptive expectations model'. The latter is the standard backward-looking Phillips model in which inflation (changes) expectations are proxied by lags of inflation (changes), with coefficients estimated within the Phillips equation simultaneously with the whole model. Note, however, that the extension of distributed lag of inflation is fixed at $p$, which is the extension identified with the Box-Jenkins method and adopted in the 'near-rational model'. Hence, the Gordon-style practice of including long and arbitrarily chosen lags of inflation in the Phillips equation will not be followed here, so that the results from both approaches remain comparable.

In the baseline model - the 'near-rational model' - expectations of inflation changes have an associated coefficient $\alpha$, to be estimated with the model. Given that the expectations' process is exogenously estimated with data for the entire period, the inclusion of this coefficient $\alpha$ is intended to allow for a correction, in each quarter, of the weight given to expectations in the Phillips relation.\footnote{The alternative option here could be having the ARIMA model for expectations estimated with coefficients up-dated with every new observation. However, because the number of observations available for this study is not that large, this would mean that, in practice, any robust estimation of the ARIMA process would leave only a small number of periods, at the end of the sample, in which coefficients updating could be done satisfactorily. Not restricting the $\alpha$ coefficient to be 1 across the whole sample is a simpler way of doing this with no practical disadvantages.}

It could be argued that the near-rational model may suffer from a problem of inconsistency of expectations. In fact, the Phillips equation (2.1) can be solved in order to the expectations term, generating a series of \textit{ex-post} inflation expectations that is not restricted to be identical to the one that is plugged into that equation from the (exogenous) estimation of equation (2.8). This issue is addressed below, with a brief discussion and an alternatively estimation, with simultaneous estimation of the whole set of equations (2.1)-(2.8).
2.3.2. State-space Format

Estimation by the Kalman filter requires that the system is written in state-space form. This form is as follows:

\[ O_t = \alpha U_t + \varepsilon_t^O \]  \hspace{1cm} (2.9)

\[ U_t = T U_{t-1} + g_t + \varepsilon_t^U \]  \hspace{1cm} (2.10)

where \( O \) stands for 'observables' and \( U \) for 'unobservables', and:

\[
\begin{bmatrix}
\gamma_t \\
u_t \\
\Delta \pi_t
\end{bmatrix}
\quad
\begin{bmatrix}
\alpha & \tau
\end{bmatrix}
\quad
\begin{bmatrix}
c_n \\
c_n \\
y_t \\
g
\end{bmatrix}
\quad
\begin{bmatrix}
0 \\
g_t
\end{bmatrix}
\]

\[
\begin{bmatrix}
\varepsilon_o \\
\varepsilon_{phi}
\end{bmatrix}
\quad
\begin{bmatrix}
\varepsilon_i \\
\varepsilon_{n}
\end{bmatrix}
\quad
\begin{bmatrix}
\varepsilon_j \\
\varepsilon_{gap}
\end{bmatrix}
\]

\[
T = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\quad
\begin{bmatrix}
\varepsilon_i^C \\
\varepsilon_i^C \\
\varepsilon_i^p \\
\varepsilon_i^p \\
\varepsilon_i^{gap} \\
\varepsilon_i^H
\end{bmatrix}
\]

with all innovations independent, serially uncorrelated and with normal distribution with zero expected values and constant variances.

In this model, \( \alpha_t \) is a vector composed of observed variables (expected inflation changes and the supply-shock proxy - the deviation of domestic inflation from imported...
inflation), zeros, ones and two parameters that are to be estimated - the Phillips relation slope parameter, $\gamma$, and the Okun law parameter, $\theta$.

$U_t$ is a vector of unobserved variables, composed of the coefficients associated to the expectations of inflation and the supply-shock proxy, $c_{1t}$ and $c_{2t}$, and of the unobservable variables that are to be estimated – trend output, equilibrium unemployment, unemployment gap and drift in the innovation to the equilibrium rate. It has transition equations defined by the matrix $T$. In the transition system, $\varepsilon_t^U$ is a vector of innovations, assumed to be normally distributed with a variance-covariance matrix $Q$:

$$\varepsilon_t^U \sim \mathcal{N}(0, Q)$$ (2.11)

This matrix $Q$ has, by assumption, all elements equal to zero except the diagonal ones associated with trend output, the NAIRU, the unemployment gap and the NAIRU’s drift - $\sigma_{\varepsilon^p}$, $\sigma_{\varepsilon^u}$, $\sigma_{\varepsilon^{\text{gap}}}$ and $\sigma_{\varepsilon^{\mu}}$ - so that these are the only parameters that really vary with time.

Estimation by the Kalman recursive equations requires the setting of initial values for the state vector. The starting values of trend output and the NAIRU were set to the corresponding observed values of output and unemployment, and those of the gap and drift were set accordingly to that assumption. Following the suggestion in the literature, relatively diffuse priors were adopted here by assuming large starting values for the unobserved' variance matrix. Specifically, the standard deviation of the one-step ahead predictions of the system's unobserved components were initialised at 5 percent for trend real output, 2 percentage points for the NAIRU and unemployment gap and at 0.75 percentage points for the NAIRU’s drift.

With these starting values for the state system and with initial conditions for the likelihood’s hyper-parameters, the Kalman iterations can proceed and maximisation of the maximum-likelihood function can be pursued. The log-likelihood function, $L$, is written on the system's one-step ahead prediction errors and their variance, as described
40 In the specific case of the baseline linear model of this study, \( L \) is a function of eleven unknown parameters, sometimes called hyper-parameters:

\[
L = L(\gamma, \theta, \rho_1, \rho_2, \sigma, \sigma_{\rho_1}, \sigma_{\rho_2}, \sigma_{\phi}, \sigma_{\theta}, \sigma_{\gamma}, \sigma_{\sigma_\rho}, \sigma_{\sigma_\theta})
\]  

(2.12)

which are, respectively, the slope parameters in the Phillips and Okun equations, the two auto-regressive parameters in the unemployment gap process, the average rate of growth of trend output, the standard deviations for the disturbances in the Okun and Phillips equations, and the standard deviations for the innovations in the NAIRU, the unemployment gap, trend output and the NAIRU drift.

Estimation was conducted with a GAUSS code, which uses the procedure Optmum to maximize the likelihood function. Standard deviations of the hyper-parameters' estimates were computed using the inverse of the Hessian (computed out of the maximisation process), while variances of the time-varying parameters were obtained from the Kalman filter recursive equations.

2.4. The Data: Sources and Basic Time Series Properties


The basic time-series properties of the data are reported in tables 2.1.A and 2.1.B. The inflation rate \( (\pi_t) \), the Unemployment rate \( (U_t) \) and the log of real output \( (Y_t) \) seem

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40 For details on the mechanics of the Kalman filter and estimation of UC systems by maximum likelihood, the reader should see Harvey (1989), Cuthbertson, Hall and Taylor (1992) or Hamilton’s (1994) chapter 13.

41 Estimation very often required some management of the maximization algorithm, the step-method and the algorithm for computing the gradient. The typical option was to initiate iterations with the ‘BFGS’ algorithm and the ‘stepbt’ step-method, and then switch to the ‘Newton’ algorithm and the ‘half’ step-method when the function had come closer to the maximum. Generally, the Richardson extrapolation of the forward difference method was used as the numerical gradient method, which can be very close to the analytical derivatives.

42 This data is a part of the AWMD - the Area Wide Model Database.

43 Results for augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are presented, as practice has revealed that PP tests very often help in clarifying the results of the (most common) ADF tests -
to be integrated of order 1. On the contrary, the series of inflation changes ($\Delta \pi_t$), the imports deflator's inflation rate ($\text{Imp} \pi_t$), the deviations of Euro Area inflation from imports inflation ($\text{Imp} \pi_t - \pi_t$), the changes of the unemployment rate ($\Delta u_t$) and the growth rates of real output ($\Delta Y_t$) seem to be I(0).

Table 2.1.A. Unit Root Tests - Augmented Dickey-Fuller (l=6)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>$\pi_t^+$</th>
<th>$\Delta \pi_t^{++}$</th>
<th>Imp $\pi_t$</th>
<th>Imp$\pi_t - \pi_t$</th>
<th>$U_t$</th>
<th>$\Delta U_t$</th>
<th>$Y_t^{+++}$</th>
<th>$\Delta Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(\rho_{-1})$</td>
<td>-25.24b</td>
<td>-163.23a</td>
<td>-55.16a</td>
<td>-69.16a</td>
<td>-1.43</td>
<td>-32.72a</td>
<td>-10.80</td>
<td>-90.02a</td>
</tr>
<tr>
<td>$\tau_{\rho_{-1}}$</td>
<td>-3.97b</td>
<td>-4.64a</td>
<td>-4.30a</td>
<td>-4.52a</td>
<td>-1.00</td>
<td>-2.73</td>
<td>-3.09</td>
<td>-4.30 a</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>8.68b</td>
<td>10.83a</td>
<td>9.26a</td>
<td>10.23a</td>
<td>2.13</td>
<td>4.00</td>
<td>5.20</td>
<td>9.32a</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>5.83b</td>
<td>7.22a</td>
<td>6.17b</td>
<td>6.83a</td>
<td>1.81</td>
<td>2.74</td>
<td>8.97a</td>
<td>6.22b</td>
</tr>
<tr>
<td>$T(\rho_{-1})$</td>
<td>-3.98</td>
<td>-155.30a</td>
<td>-33.65a</td>
<td>-63.67a</td>
<td>-0.99</td>
<td>-26.26a</td>
<td>-0.42</td>
<td>-84.17a</td>
</tr>
<tr>
<td>$\tau_{\rho_{-1}}$</td>
<td>-1.06</td>
<td>-4.56a</td>
<td>-3.18b</td>
<td>-4.31a</td>
<td>-2.04</td>
<td>-2.30</td>
<td>-1.18</td>
<td>-4.20a</td>
</tr>
<tr>
<td>$\Phi_1$</td>
<td>0.62</td>
<td>10.40a</td>
<td>5.06b</td>
<td>9.31a</td>
<td>2.66</td>
<td>2.76</td>
<td>8.36a</td>
<td>8.82a</td>
</tr>
</tbody>
</table>

Table 2.1.B. Unit Root Tests - Phillips-Perron (l=6)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\pi_t^+$</th>
<th>$\Delta \pi_t^{++}$</th>
<th>Imp $\pi_t$</th>
<th>Imp$\pi_t - \pi_t$</th>
<th>$U_t$</th>
<th>$\Delta U_t$</th>
<th>$Y_t^{+++}$</th>
<th>$\Delta Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z(\bar{\rho})$</td>
<td>-17.11</td>
<td>-149.55a</td>
<td>-34.41a</td>
<td>-40.76a</td>
<td>-1.16</td>
<td>-46.70a</td>
<td>-13.07</td>
<td>-99.37a</td>
</tr>
<tr>
<td>$Z(t^{-1}\bar{\rho})$</td>
<td>-3.51c</td>
<td>-14.65a</td>
<td>-4.53a</td>
<td>-5.05a</td>
<td>-0.38</td>
<td>-5.27a</td>
<td>-3.24</td>
<td>-8.79a</td>
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<tr>
<td>$Z(\Phi_3)$</td>
<td>6.58c</td>
<td>107.34a</td>
<td>11.06a</td>
<td>13.95a</td>
<td>1.60</td>
<td>14.02a</td>
<td>5.35</td>
<td>7.52b</td>
</tr>
<tr>
<td>$Z(\Phi_2)$</td>
<td>4.41c</td>
<td>71.56a</td>
<td>8.19a</td>
<td>10.02a</td>
<td>2.62</td>
<td>9.36a</td>
<td>29.57a</td>
<td>27.01a</td>
</tr>
<tr>
<td>$Z(\rho^*)$</td>
<td>-4.73</td>
<td>-151.01a</td>
<td>-30.79a</td>
<td>-41.15a</td>
<td>-2.00</td>
<td>-41.89a</td>
<td>-0.80</td>
<td>-97.32a</td>
</tr>
<tr>
<td>$Z(t^{-1}\rho^*)$</td>
<td>-1.46</td>
<td>-14.46a</td>
<td>-4.22a</td>
<td>-5.07a</td>
<td>-1.73</td>
<td>-4.85a</td>
<td>-1.77</td>
<td>-8.67a</td>
</tr>
<tr>
<td>$Z(\Phi_1)$</td>
<td>1.10</td>
<td>104.61a</td>
<td>10.71a</td>
<td>14.93a</td>
<td>3.62</td>
<td>11.76a</td>
<td>37.52a</td>
<td>39.68a</td>
</tr>
</tbody>
</table>

Table 2.1.A. Unit Root Tests - Augmented Dickey-Fuller (l=6)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\pi_t^+$</th>
<th>$\Delta \pi_t^{++}$</th>
<th>Imp $\pi_t$</th>
<th>Imp$\pi_t - \pi_t$</th>
<th>$U_t$</th>
<th>$\Delta U_t$</th>
<th>$Y_t^{+++}$</th>
<th>$\Delta Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z(\bar{\rho})$</td>
<td>-17.11</td>
<td>-149.55a</td>
<td>-34.41a</td>
<td>-40.76a</td>
<td>-1.16</td>
<td>-46.70a</td>
<td>-13.07</td>
<td>-99.37a</td>
</tr>
<tr>
<td>$Z(t^{-1}\bar{\rho})$</td>
<td>-3.51c</td>
<td>-14.65a</td>
<td>-4.53a</td>
<td>-5.05a</td>
<td>-0.38</td>
<td>-5.27a</td>
<td>-3.24</td>
<td>-8.79a</td>
</tr>
<tr>
<td>$Z(\Phi_3)$</td>
<td>6.58c</td>
<td>107.34a</td>
<td>11.06a</td>
<td>13.95a</td>
<td>1.60</td>
<td>14.02a</td>
<td>5.35</td>
<td>7.52b</td>
</tr>
<tr>
<td>$Z(\Phi_2)$</td>
<td>4.41c</td>
<td>71.56a</td>
<td>8.19a</td>
<td>10.02a</td>
<td>2.62</td>
<td>9.36a</td>
<td>29.57a</td>
<td>27.01a</td>
</tr>
<tr>
<td>$Z(\rho^*)$</td>
<td>-4.73</td>
<td>-151.01a</td>
<td>-30.79a</td>
<td>-41.15a</td>
<td>-2.00</td>
<td>-41.89a</td>
<td>-0.80</td>
<td>-97.32a</td>
</tr>
<tr>
<td>$Z(t^{-1}\rho^*)$</td>
<td>-1.46</td>
<td>-14.46a</td>
<td>-4.22a</td>
<td>-5.07a</td>
<td>-1.73</td>
<td>-4.85a</td>
<td>-1.77</td>
<td>-8.67a</td>
</tr>
<tr>
<td>$Z(\Phi_1)$</td>
<td>1.10</td>
<td>104.61a</td>
<td>10.71a</td>
<td>14.93a</td>
<td>3.62</td>
<td>11.76a</td>
<td>37.52a</td>
<td>39.68a</td>
</tr>
</tbody>
</table>

Notes: see table 2.1.A.

which is sometimes important, having in mind the lack of power of the unit root tests. We show results for all the alternative models, sequentially from the most complex to the simplest. The auto-regressive correction was truncated at the 6th lag, which, albeit arbitrary, is close to the Said and Dickey (1984) rule of thumb - $L = T (1/3)$ - that generally gives good results.
The properties are in line with general aprioristic analysis, and legitimate some of the specification options adopted here. They imply that the Phillips equation should be specified in inflation changes rather than in levels, as near-rational agents' expectations result from fitting a Box-Jenkins model to a stationary inflation series. In addition, they also suggest that the proxy for supply-shocks - imported inflation - can enter the Phillips equation alternatively in its levels or as the deviation from domestic inflation.

Table 2.2 reports the ARIMA model identified and estimated for the GDP deflator series. Specifically, Box-Jenkins methods suggested that a simple AR(3) process is the best univariate ARIMA model for output's inflation in the Euro area 1970-2000. As discussed above, each period’s one-step-ahead prediction of inflation changes given by this estimated model will be our series of expected inflation changes fed into the ‘near-rational expectations’ models.

Table 2.2 - Arima Model: GDP Deflator, Euro Area [1970:1-2000:II]

<table>
<thead>
<tr>
<th>Series = (1-B)^2 Log GDPdef_t</th>
<th>Estimate</th>
<th>T-Statistic</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ1</td>
<td>-0.4093</td>
<td>[-4.632]</td>
<td>(0.000)</td>
</tr>
<tr>
<td>φ2</td>
<td>-0.1425</td>
<td>[-1.511]</td>
<td>(0.134)</td>
</tr>
<tr>
<td>φ3</td>
<td>-0.2137</td>
<td>[-2.428]</td>
<td>(0.017)</td>
</tr>
<tr>
<td>φ(1):</td>
<td>-0.766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>0.9999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSRs</td>
<td>0.0011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>(29-3): 20.698</td>
<td></td>
<td>(0.757)</td>
</tr>
</tbody>
</table>

[ ]: t-statistics;
( ): Significance;

2.5. Results

2.5.1. Baseline Linear Model Results

The model was estimated with all reasonable specifications combining contemporary and lagged values of the explanatory variables in the Phillips and Okun equations. Individual significance statistics and global statistical quality indicators - especially normality and auto-correlation statistics of the measurement system's
residuals - were used to identify the most significant model. This was found to be the one in which the unemployment gap enters the Phillips and the Okun equations (only) contemporaneously to, respectively, the inflation changes and the real output gap.\textsuperscript{44}

The fact that additional lags of the unemployment gap are not statistically significant, in the Phillips equation, means that there is no evidence of the 'speed-limit effects' discussed since Turner (1995) - which would be a symptom of persistence, and possible hysteresis, in the Euro Area unemployment rate. This result is in line with Richardson \textit{et al's} (2000) findings that only in the UK and Spain, out of 21 OECD countries' quarterly data, were these effects significant, but not with the ones of McAdam and McMorrow (1999) obtained with annual aggregate data of the EU15.

The standard deviation of the residual of the Okun equation converged systematically to zero throughout the identification work, so it was restricted to that value during estimation.\textsuperscript{45}

We have alternatively estimated the model with a random-walk drift in the (random walk) process driving trend output, instead of the constant drift above defined. However, the variance of this random walk drift was not statistically significant and its estimated value showed no notable change along time, which confirmed the baseline apriori of a constant (positive) drift in Euro Area trend real GDP 1970-2000.

Similar identification work led to choosing the (one quarter) lagged deviation of the Euro area inflation from the imports' deflator inflation as the sole proxy for the supply shocks, in the Phillips equation. Economically, this variable is generally considered a good proxy for the inflationary pressures that originate in recent periods at the international level and will supposedly feed into domestic inflation with some lag. Historically, most supply shocks have indeed emerged outside the Euro Area and have

\textsuperscript{44}Differently, the experiments in Camba-Mendez and Palenzuela (2001) and Fabiani and Mestre (2001) rely on some specific aprioris and do not offer any identification work comparable to the one done here. Moreover, Fabiani and Mestre (2001) rely on OLS regressions (performed with proxies for the unobserved components obtained from Hodrick-Prescott filtering) to achieve some identification of the system. The fragility of this procedure might explain why they chose to report results from a number of slight variations of their baseline model, without being able to choose one preferred model. It may also explain why they included in the Phillips equation some supply-shocks proxy variables that were not statistically significant within this chapter's identification work - see comments below.

\textsuperscript{45}This means that, with this data, the model will generate unemployment and output gaps that are perfectly proportional. This is in sharp contrast to Astley and Yates' (1999) findings for the UK: using a structural VAR approach, they found that the unemployment and output gaps (as well as the capacity utilisation gap) exhibited very different dynamics.
subsequently affected the Area inflation - namely oil price shocks and commodities prices' shocks. In large and relatively closed economic areas like the US and the Euro one, this variable is generally seen as enough to model supply-side factors. It is assumed that the transmission of these shocks into domestic Euro area inflation takes longer than a quarter. This assumption was confirmed empirically, as the contemporary deviation of domestic Euro inflation from imported inflation was not statistically significant, contrary to what happens with the lagged deviation. This result also confirms that this supply-shocks proxy is orthogonal to the variables forming expectations, providing more content to the theoretical assumption that had previously discarded the possibility of a generated regressor problem.

One problem with this proxy in the specific case of the Euro Area is that the Euro Wide Area imports deflator may be an imperfect measure of actual imported inflation of the Euro Area. This is because this series was built by simple aggregation of the national imports deflators and includes changes in prices of trade between member-states - see Fagan et al (2001). As an alternative, two series of the International Monetary Fund's Financial Statistics were tried - 'world oil price' (average of the main 3 spot price indexes - Dubai, Brent and Texas) and 'non-fuel commodities', both in their levels, changes and as deviations from Euro Area inflation. Results were not significantly different and the model's statistical fit was actually worse.

In the attempt of modelling also domestic supply-shocks, experiments were conducted including several alternative measures of productivity in the Phillips equation. The variables considered here were, specifically, the changes of (i) the Solow residual from the Euro Area production function, (ii) the log of the ratio of real GDP to total employment, and (iii) the deviation of the series (ii) from its trend as given by, alternatively, the HP filter and a simple moving average. However these variables' estimated coefficients had the wrong sign or were not significant, and, in some cases, the model's residuals deteriorated significantly. It was also tried adding the deviation of (log) Unit Labour Costs from their trend - computed with the HP filter - but results were similarly not good.

Within the identification work, a model with hysteresis was also estimated. Specifically, hysteresis was modelled as in Jaeger and Parkinson (1994), specifying the
time process of the NAIRU as a random walk augmented with a *feed-back* effect from past estimated cycles. However, the coefficient associated to the lagged unemployment gap turned out to be not statistically significant.

Results are summed up in Table 2.3.

A first conclusion is that the results from the two alternative specifications of inflation (changes) expectations are very close. This means that Ball's (2000) concept of near-rational expectations, although technically different and more theoretically founded, leads to identical results - which are highly desirable - generated by adaptive expectations models, at least with this data-sample.\(^{46}\) The estimate of coefficient \(\alpha\), which is associated to inflation expectations in the Phillips equation, is quite close to its theoretical value of 1, and not statistically different from unity, which adds likelihood to the hypothesis of near-rational expectations of inflation.\(^{47}\)

All the coefficients have the expected signs and reasonable estimated values. Notably, the estimate of the Phillips elasticity is only slightly different across the expectations' hypothesis - 0.032 in the near-rational model, versus 0.0355 in the adaptive-expectations model - and is always correctly signed - a result that is at odds with the incorrect sign of the estimate obtained by Gali *et al.* (2001). All the coefficients are statistically significant, with one very important exception: the Phillips elasticity - one of the central coefficients of the model - is not statistically significant, having one-sided significance probability of about 17 percent. This result could be evidence of the *hysteresis* hypothesis first raised by Blanchard and Summers (1986), who argued that the natural rate model might be unable to describe the joint evolution of inflation and unemployment in European countries during the 70s and 80s.

\(^{46}\) Recall that in the adaptive expectations models in this chapter, the adopted extension for the lag polynomial (3 quarters) has been identified with the Box-Jenkins method, when specifying the optimal univariate model for inflation changes to be used in the near-rational models. While in our adaptive-expectations models the coefficients associated to lags of inflation changes are estimated within the Phillips equation - coefficients \(\phi\) in table 2.3 - in our near-rational expectations' models we use the roots estimated in the univariate Box-Jenkins model for inflation changes. These are then used to compute two-steps-ahead forecasts of inflation changes - \(E_{t+2}\Delta \pi_{t+1}\) - which are finally imputed in the Phillips equation as a series of expectations of changes of inflation.

\(^{47}\) We have also estimated the baseline model restricting the \(\alpha\) coefficient to 1. Although the Phillips elasticity estimate decreases slightly, its significance is virtually unchanged, like the rest of the results, especially the unobserved components' estimates.
Table 2.3  UC Model with Linear Phillips and Okun equations
GDP Deflator, Unemployment Rate, Real GDP ¹ [1970:1 - 2000:II]

<table>
<thead>
<tr>
<th>Specification of $\Delta\pi_t$ ²</th>
<th>Near-rational</th>
<th>Adaptive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phillips:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0320 [0.976] (0.329)</td>
<td>0.0355 [0.949] (0.342)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.990 [-0.051] (0.960) b</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>-</td>
<td>-0.389 (0.000)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-</td>
<td>-0.178 (0.050)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-</td>
<td>-0.277 (0.000)</td>
</tr>
<tr>
<td>$\phi(1)$</td>
<td>-</td>
<td>-0.843 [-0.952] (0.340) b</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.049 (0.000)</td>
<td>0.049 (0.000)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{\text{phi}}$</td>
<td>0.290 [14.931] (0.000)</td>
<td>0.283 [14.787] (0.000)</td>
</tr>
<tr>
<td><strong>Okun:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0235 [4.883] (0.000)</td>
<td>0.0236 [4.872] (0.000)</td>
</tr>
<tr>
<td><strong>Unobserved components:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>0.0060 [10.972] (0.000)</td>
<td>0.0060 [11.039] (0.000)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{\text{p}}$</td>
<td>0.0051 [13.226] (0.000)</td>
<td>0.0051 [13.198] (0.000)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{\text{n}}$</td>
<td>0.0872 [7.163] (0.000)</td>
<td>0.0872 [7.143] (0.000)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{\mu}$</td>
<td>0.0167 [1.953] (0.051)</td>
<td>0.0168 [1.952] (0.050)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.811 [23.279] (0.000)</td>
<td>1.811 [23.079] (0.000)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.833 [-10.774] (0.000)</td>
<td>-0.832 [-10.705] (0.000)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{c}$</td>
<td>0.0629 [4.639] (0.000)</td>
<td>0.0629 [4.626] (0.000)</td>
</tr>
<tr>
<td>Log L</td>
<td>-781.536</td>
<td>-780.880</td>
</tr>
<tr>
<td><strong>Residuals:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips equation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1.26 (0.53)</td>
<td>1.28 (0.53)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>6.140 (0.189)</td>
<td>1.987 (0.738)</td>
</tr>
<tr>
<td>Okun equation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>14.99 (0.00)</td>
<td>15.15 (0.00)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>4.529 (0.339)</td>
<td>4.593 (0.332)</td>
</tr>
<tr>
<td>Unemployment equation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>4.93 (0.09)</td>
<td>4.47 (0.11)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>8.743 (0.068)</td>
<td>8.890 (0.064)</td>
</tr>
</tbody>
</table>

a: Series transformed into: $\Delta\pi_t = (1-B)^2 \log GDP_{t/1}$; $U_t = U_t$; $Y_t = \log \text{real GDP}$

The coefficient $\omega$ is associated to $\pi_tIMP(i-1)-\pi_t(i-1)$. The coefficient $\alpha$ is associated to $E_t\pi_{t+1}$.

b: T-statistics and Significance for the test $H_0: \alpha = 1$; $H_0: \phi(1) = 1$.

c: Standardised residuals computed as defined in Harvey (1989), p. 256, 442.

[ ]: T-statistics; (): Significance; Significance probabilities relate to two-sided tests, except in 'Residuals'.

The standard deviation of the innovation of the first measurement equation (Okun equation) was restricted to zero as it systematically converged to that value.
However, this interpretation seems to be at odds with the lack of statistical significance of 'feed-backs' from cycle to trend and of 'speed-limit effects', related above.

The estimated Okun coefficient is statistically significant and says that each additional percentage point of gap in unemployment is associated with 2.35 additional percentage points of output away from its trend. This estimate is very close to 2, which is the current textbook benchmark for the US Okun's Law coefficient - Gordon (1998b) - and is also quite close to the mean of the estimates obtained by Lee (2000) with annual post-war data of 16 OECD countries.\textsuperscript{48}

Average trend real output growth is estimated at 0.6 percent per quarter, i.e. about 2.4 percentage points per year, which is in line with conventional wisdom on the subject. The fact that the hypothesis of a constant drift in trend real output could not be rejected means that there is no evidence of increases in the average growth rate of trend real output in the aggregate data for the Euro area up to 2000:II. Hence, at least with this method and until 2000:II, there is no statistical evidence of the 'New Economy' phenomena that has been under discussion in recent years, especially in the US case.\textsuperscript{49}

The standardised residuals of the two main measurement equations - Phillips and Okun - show no signs of significant auto-correlation, and the Phillips residuals seem normal. In contrast, the Okun equation residuals deviate significantly from normality.

In spite of the problems with the Okun equation, the model works well in the sense that it successfully decomposes the unemployment rate and the real output into reasonable trend and cycles that are in accordance with the conventional wisdom about the recent decades' macroeconomic cycles. Figure 2.2 shows the basic trend-cycle series, for the near-rational model (all the estimated unobserved components - NAIRU, unemployment gap, potential GDP and output gap - are identical in the 'adaptive expectations model'). The figure shows the typical negative skewness in the transitory

\textsuperscript{48} See especially the last row in Table 2, page 341. The unweighted mean of the estimates for the 6 Euro Area countries studied in that paper is 2.16, 1.86, 1.87 and 2.28 with, respectively, the first differences, Kalman filter, HP filter and BN filter gaps of unemployment and output. Note that Lee (2000) found considerable heterogeneity in Okun's coefficients across-countries and some evidence of structural change in the Law, with an overall reduction in its coefficients in recent decades.

\textsuperscript{49} This result seems to contradict those in McMorrow and Roeger (2001) for the EU-15 Area, obtained from a production function approach. Based on a growth accounting exercise, they estimate that the 'New Economy' sector - Information and Communication Technologies, ICT - may have contributed to an increase of between 0.25 and 0.33 of a percentage point to the EU-15 potential output's growth rate in the second half of the 1990s.
component - negative deviations from trend are deeper than positive deviations - that has been documented, for instance, by Sichel (1993). This evidence is somewhat suggestive of the Friedman (1993) plucking model of economic fluctuations - see Lo and Piger (2001) -, even though our unobserved components model is symmetric, and, thus, is not designed to capture asymmetries as the ones in Kim and Nelson's (1999) unobserved components model with Markov switching regimes.

**Figure 2.2 - Main Results from Baseline Linear Model**

---

**UNEMPLOYMENT AND SMOOTHED NAIRU**

**NEAR-RATIONAL EXPECTATIONS MODEL**

---

**UNEMPLOYMENT AND OUTPUT GAPS**

**NEAR-RATIONAL EXPECTATIONS MODEL**

---

U gap

Y gap

zero
Results point that the Euro Area was in a cyclical trough during the beginning of 1985. Then, a systematic expansion followed until a peak was reached in the beginning of 1991. Subsequently, the cyclical situation of the Euroland deteriorated continuously until a new trough was reached at the beginning of 1997 (with a slight recovery during 1994:III-1995:II that proved transitory). Finally, estimates show the Euro Area in a steady cyclical recovery since the beginning of 1998, and point to a positive situation since 2000:II.

All summed up, apart from the positive results, two main caveats remain. Firstly, the lack of statistical significance in the Phillips elasticity. Secondly, the possible poor quality of the estimation, shown by the non-normality in the Okun's equation residuals. Both these caveats seem to suggest that testing for asymmetries in the Phillips and - perhaps more importantly - in the Okun equation may be important for achieving an improvement in the specification of our empirical model.

2.5.2. Inflation Expectations: Model-consistency and Time-varying Coefficient

This section assesses to what extent the results obtained so far might be sensitive to the way inflation expectations are being treated. Specifically, we evaluate the impact of two extensions to the baseline model: first, forcing near-rational expectations to be model-consistent; and second, relaxing the restriction of a constant coefficient on inflation expectations in the Phillips equation.

Model-consistent Near-rational Expectations

The Ball-type univariate expectations given by equation (2.8) are, by definition, not model-consistent. In fact, this attribute of near-rational expectations is an essential part of the theoretical argument that they convey: real-world agents do not form expectations in a perfectly rational, model-consistent, way. The near-rational hypothesis assumes that agents are rational in the sense that they form forward-looking expectations - they try to forecast next quarter inflation - but are not perfectly rational in the sense that they use only information from the past of inflation to compute their
forecasts. A direct corollary to this argument is that agents do not know the true economic model, not even the trivariate unobserved components model that we use in this research - they only know the best linear unbiased univariate model for inflation.

Hence, it is only natural that the Ball-type expectations given by our equation (2.8) are inconsistent with the ex-post expectations given by the model when equation (2.1) is solved for $E_t \Delta \pi_{t+1}$. To illustrate this, the system is now re-estimated with joint estimation of the whole set of equations (2.1) to (2.8). Specifically, the Phillips equation (2.1) is now specified as

$$\Delta \pi_t = \phi_1^* \Delta \pi_{t-1} + \phi_2^* \Delta \pi_{t-2} + \phi_3^* \Delta \pi_{t-3} + \gamma (u^n_t - u_t) + \omega S_t + \epsilon^\phi_t$$

And the expectations' equation (2.8) is specified as

$$\Delta \pi_{t+1} = \phi_1^* \Delta \pi_{t-1} + \phi_2^* \Delta \pi_{t-2} + \phi_3^* \Delta \pi_{t-3} + \epsilon^{AR}_t \quad (2.8')$$

That is, this expectation's equation is estimated within the model with imposition of a cross-equation restriction of equality between the coefficients associated to the three lags of inflation (changes) included in this and in the Phillips' equation. Hence, this system corresponds to the adaptive expectations Phillips equations referred to before, extended with a fourth measurement equation - equation (2.8') above - which has three coefficients that by imposition equal those associated to lagged inflation in the Phillips' measurement equation. Note, however, that in equation (2.8') the measurement (observed) variable is the change in inflation in next quarter, while in the Phillips equation it is the current quarter change in inflation. The system has now one additional hyper-parameter to be estimated, the standard deviation of the innovation to the residual of the inflation changes equation, $\sigma \epsilon^{AR}_t$.

Results are summed-up in table 2.4.

The Phillips elasticity's estimate exhibits an important reduction, from 0.032 to 0.019, and its t-statistic decreases from 0.98 to 0.50, so that the significance level for the two-sided test that the true elasticity is zero is, now, 70 percent. However, the estimate of the Phillips trade-off coefficient retains the correct sign. The estimate of the Okun's elasticity increases only marginally from 0.0235 to 0.0238.
## Table 2.4 - Model With Model-Consistent Near-Rational Expectations Of Inflation GDP deflator, Unemployment rate, real GDP a [Euro area, 1970:I - 2000:II]

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>T-value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phillips:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0194</td>
<td>0.499</td>
<td>0.70</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>-0.199</td>
<td>-2.959</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.144</td>
<td>-2.047</td>
<td>0.04</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-0.085</td>
<td>-1.279</td>
<td>0.20</td>
</tr>
<tr>
<td>$\phi(1)$</td>
<td>-0.428</td>
<td>12.109</td>
<td>0.00</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.048</td>
<td>3.870</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma \varepsilon_{\text{phi}}$</td>
<td>0.281</td>
<td>14.786</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Okun:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0238</td>
<td>4.668</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Inflation changes:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma \varepsilon_{\text{AR}}$</td>
<td>0.333</td>
<td>14.251</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Unobserved components:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>0.0059</td>
<td>10.673</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma \varepsilon_{\text{p}}$</td>
<td>0.0051</td>
<td>12.900</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma \varepsilon_{\text{N}}$</td>
<td>0.0885</td>
<td>7.085</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma \varepsilon_{\text{mu}}$</td>
<td>0.0169</td>
<td>1.836</td>
<td>0.06</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.797</td>
<td>22.451</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.818</td>
<td>-10.231</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma \varepsilon_{\text{c}}$</td>
<td>0.0635</td>
<td>4.614</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Residuals:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>10.34</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>9.46</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Okun:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>17.67</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>4.19</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Unemployment:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>8.40</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>8.34</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Inflation changes:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>23.77</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>14.13</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

a: Series transformations - see table 2.3; The coefficient $\omega$ is associated to $\pi_{IMP(i-1)}-\pi_{(i-1)}$.
b: T-statistics and significance for the test $H_0: \phi(1) = -1$;  
c: Standardised residuals computed as defined in Harvey (1989), p. 256, 442.
Significance probabilities relate to two-sided tests, except in 'Residuals'.
All the other hyper-parameters' estimates show no visible change: notably, the average growth rate of trend real output is consistently estimated at 0.6 percent per quarter.

The estimates of $\phi_1^*$, $\phi_2^*$ and $\phi_3^*$ are quite different from their theoretical values that could be computed from the estimates of the AR (3) model for inflation changes shown in table 2.2. All the three coefficients are estimated at smaller values - especially the first two - and add up to -0.428, which is quite lower than the theoretical $\phi(1)$ that would be obtained from the univariate AR (3) estimates (-0.808).

The NAIRU estimates obtained from the model considered in this section are somehow above their counterparts from the near-rational expectations' model reported in table 2.3. This generates a level difference between the unemployment gap estimates of the two models, but their broad behaviour along the sample period is quite similar, so that there is no marked difference between the cyclical turning points they estimate.

The main indicators of quality of fit of the model in this section are evidently negative. In fact, the residuals of the measurement equations are much worse behaved than in the models reported in table 2.3: no residuals pass the Jarque-Bera test for the null of normality, at 5 percent of significance. The residuals of estimation of the equation modelling inflation changes, in turn, exhibit an high short-memory autocorrelation, clearly the consequence of the unit root in our quarterly time series of inflation - as that equation models two-step-ahead forecasts.

In the end, we choose to continue the empirical work within the framework defined in the previous section of this chapter, for two reasons. First, theoretically, Ball's concept of near-rational, limited-information expectations is not supposed to be model-consistent, as argued above. Second, empirically, the attempt in this section of forcing near-rational expectations of inflation - formulated two quarters ahead - to be model-consistent, has generated highly problematic empirical results.

**Time-varying Coefficient of Inflation Expectations**

Over the last two decades most developed countries have disinflated their economies and many changed their monetary policy regimes into explicit or implicit targeting of a low and stable rate of inflation. This context increased the attention to the
hypothesis that the weight that (at least partially) rational agents attribute to past inflation when forming expectations of inflation may vary as a function of the level of inflation, the monetary policy regime and its credibility. In Phillips equations, this would imply smaller coefficients associated to lags of inflation (as proxy for expectations) in times of low and stable inflation and policy institutions more strongly committed to low inflation. This had already been stated by Sargent (1971), when arguing that the practice of restricting the sum of the coefficients on lagged inflation to unity in Phillips regressions was not a necessary condition for the rational expectations' natural rate theory to be verified. Taylor (1998a, page 35), mentioned the hypothesis, and Poole (1998) speculated that errors in the quantification of the weight of expectations in Phillips equations might generate spurious shifts in the estimated NAIRU.

Brainard and Perry (2000) estimated price and wage (Phillips) equations for the US, with quarterly data from 1948 to 1998, allowing the intercept and the coefficients of all regressors - lagged inflation, the inverse of the unemployment rate and productivity - to be time-varying. They found that the only coefficient that showed significant time-variation was the one of lagged inflation - which means that the implicit estimated NAIRU showed no significant variation. More specifically, that coefficient's estimate was around 0.6 in 1960, increased systematically until it peaked at 0.8 by 1980 and then decreased systematically to an estimate of 0.4 in 1998, in line with the path of the inflation rate. Akerlof et al (2000) suggested some theoretical explanations for this phenomenon, within a simple model of wage and price setting. They estimated Phillips equations of the Brainard-Perry type, with US data, and found the estimates of the coefficients associated with the inflation expectations' proxies to be significantly larger in high inflation samples than in low inflation samples (lagged inflation and data from expectations surveys were used as alternative proxies). Kichian (2001) found similar evidence for Canada, when estimating Phillips equations relating inflation to lagged inflation, a supply-shock proxy and three alternative measures of the output gap estimated from three alternative time-series methods. More recently, Cogley and Sargent (2001) report results from bayesian estimation methods suggesting that the persistence of US inflation decreased in recent years, in line with Brainard-Perry-Akerlof. In contrast, Stock (2001), with frequency domain methods, and Pivetta and
Reis (2001), with bayesian time-varying parameter estimation and classical median unbiased estimation, find that the persistence of US inflation has been remarkably stable.

In order to test if there is any comparable phenomenon in the aggregate Euro area data, we estimated the baseline near-rational-expectations model specifying the $c_{1t}$ parameter (associated to the time-series of inflation changes expectations) as a random walk. Technically, this is achieved by relaxing the assumption that the innovation to this parameter, $\varepsilon_{1t}^c$, has null variance, hence allowing the $[1,1]$ element of the matrix Q to be estimated together with the other hyper-parameters of the model by maximisation of the likelihood function.

The estimates of the other hyper-parameters and the unobserved components of the model do not change significantly, and so are not reported. The estimate of parameter $c_{1t}$ at the end of the sample is unchanged at 0.99. The standard deviation of the innovation $\varepsilon_{1t}^c$ is estimated at 0.0000105, and is not statistically significant (one-sided significance probability 0.38). Accordingly, the first chart in figure 2.3 shows that while the coefficient's estimate changes significantly during the first half of the sample, then it stabilises, and its smoothed estimates are constant. Specifically, the smoothed estimate changes from 0.99006644 to 0.99006643, at period 8, and then stays unchanged throughout the rest of the sample.

It could be argued, however, that this experiment should be conducted within the adaptive expectations' model. In fact, performing this test within the 'near-rational expectations model' amounts to admit that what changes along time is how agents use the (exogenously computed) near-rational expectations. Akerlof's et al (2000) terminology seems to suggest, indeed, this type of interpretation, as they write that the Brainard-Perry hypothesis is one of 'near-rationality in the use of inflation expectations'. However, the New Keynesian Phillips theory states that agents attach always a weight of 1 to inflation expectations. What seems to be the issue in Brainard and Perry (2000) is, differently from Akerlof's et al (2000) suggestion, a change along time in the way near-rational agents compute their expectations based on inflation's past. Hence, in the 'near-rational expectations' model the experiment could be biased as it uses expectations generated as forecasts of an ARIMA model featuring constant parameters. If the
Brainard-Perry argument is true, these forecasts may be meaningless, at least in the final part of the sample, and what should be tested here is if lagged inflation has been receiving less weight, in recent times, when agents use it to proxy for expectations.

One interesting test would be modelling expectations as a weighted average of past inflation and the policy target, to assess if the relative weights of these regressors change as inflation decreases and the target gains credibility. Unfortunately, this specification can not be tested in this case, as there was no unique policy regime in the Euro area until 1999:I., with the EMU policy regime having too few data points for any robust empirical research. Moreover, the ECB's regime is not yet entirely clear, at the time we write this text, due to the inconsistency between the first and second pillars of its monetary strategy.50

In accordance with these arguments, we proceeded with a test closer to Brainard and Perry's (2000) procedures and quite in the spirit of our study. Specifically, we re-estimated the baseline adaptive expectations model admitting that the three coefficients associated to inflation changes at t-1, t-2 and t-3 in the Phillips equation may follow random walks processes. The estimates of the model's hyper-parameters and main unobserved components did not change significantly, and so are not reported. Most importantly, the main features of the estimates of trends and cycles of unemployment and output remained unchanged. The sum of the estimates of the parameters associated to inflation lags at the end of the sample is -0.8725, which compares to -0.844 in the original formulation of the model. The standard deviations of the innovations to the three time-varying parameters are estimated at 0.0000000051, 0.00065 and 0.000000083, respectively for lags 1, 2 and 3, all not statistically significant (one-sided significance probabilities of 0.49, 0.35 and 0.17, respectively). Accordingly, the second chart in figure 2.3 shows that the sum of the coefficient's estimates changes significantly during the first half of the sample and then stabilises. Its smoothed estimate decreases from -0.828 at the beginning of the sample to -0.8725 at the end, which is a larger movement than the one in the near-rational model but can not be considered significant.

Clearly, the order of magnitude of the change is not comparable to the ones found by Brainard-Perry, Akerlof et al. and Cogley-Sargent, and is in line with the results

50 See the chapter 3 in this thesis, for new results on this subject.
obtained by Stock, and Pivetta and Reis. Hence, there seems to be no statistical
evidence of the 'Brainard-Perry-Akerlof' phenomenon in aggregate Euro area data for
1970-2000, and so, at this stage, there seems to be no significant specification error in
modelling inflation expectations using constant parameters associated to past inflation.

Figure 2.3 - Coefficient of Inflation Expectations as Time-Varying Parameter
2.5.3.  Asymmetry Tests

Following StAubyn (2000) and Martins and StAubyn (2001), asymmetry is tested within estimation of the model. Specifically, the relevant elasticity – $\theta$ in the Okun relation and $\gamma$ in the Phillips' equation - is decomposed into a constant and a function of the cyclical state of the economy (measured by the last quarter estimated unemployment gap).\textsuperscript{51}

The rationale of the test is to have the data choosing which one of those components is statistically significant. The asymmetry hypothesis is tested encompassing the null of linearity, as linearity is only rejected – and asymmetry not rejected – if and only if the component of the elasticity associated to the cyclical state of the economy is statistically different from zero. In the opposite case, the equation collapses to the baseline linear specification.

It is worth noting that this test strategy is model-consistent, in the sense that asymmetry is tested simultaneously with the estimation of the real gap. It must also be stressed that asymmetry is specified as a relation between the relevant elasticity and the unemployment gap - not the level of the unemployment rate - because of the time-varying NAIRU framework adopted. In fact, any specific value of the unemployment rate could have different meanings concerning the cyclical situation of the economy, depending on the position of the NAIRU.

Our empirical strategy implies the previous definition of the asymmetric functional forms that are to be tested for, which obviously could affect the test results. One alternative interesting approach would be Hamilton's (2001) parametric method of flexible nonlinear inference, which has the advantage of not imposing \textit{ex-ante} the functional form but rather let the data choose the best function (and the explanatory variables that drive the nonlinearity). However, it is not clear how this method could be used in the context of an unobservable components model, where the arguments of the possible nonlinearity may be unobserved variables that are to be estimated as time-

\textsuperscript{51} Except in the linear spline function, where it is measured by current inflation changes net of expectations and supply shocks effects. This measure is used here simply to illustrate that it is theoretically and empirically equivalent to using the unemployment gap, as positive (negative) inflation changes (net of expectations and supply effects) are associated with positive (negative) gaps. Also, this formulation is simpler to estimate because inflation, expectations and supply-variables are observable and

67
varying parameters of the model, jointly with the asymmetry testing. Therefore, another
route is followed here in trying to minimise the risk that decisions on the functional
form may affect the test-results. Specifically, a reasonably large number of different
functional forms are tested in this study. Preliminary evidence is offered with the piece-
wise linear regression (spline equation), and then three possible functions are used - the
quadratic curve, the modified hyperbole used by Laxton et al. (1995) and the
exponential functional form (re-scaled so that it passes through the origin).

The spline function gives a good first approximation to the problem, but has three
main problems that, in practice, preclude its use as the final form adopted. First, its
derivative is discontinuous at the null unemployment gap. Second, it imposes the strong
assumption of allowing for only two different elasticities across the whole spectrum of
unemployment gaps. Third, it lacks any upper bound on the real gap.

These objections to the piece-wise linear regression would analogously apply to the
Markov regime-switching models, often used in literature testing for asymmetries, and
also, partly, to the (logistic) smooth transition auto-regression models also frequently
used elsewhere. In addition, models that allow for smooth and continuous change in the
relevant elasticity (letting it assume an infinity of values) seem to be more in line with
what may be the asymmetric behaviour of the real-world economy. The switching
regime models have a limited number of regimes and imply step changes of the
elasticity across regimes. The (logistic) smooth transition auto-regression is smooth in
the transition between regimes, but has also a limited number of regimes.52 In contrast,
the three functional forms described below imply smooth changes across a continuum
of possible parameters' values.

The quadratic form does not impose any upper bound on the gap, and, furthermore,
may include a phase of negative relation between the two variables of interest, that is
not economically of interest. But it is very simple to estimate, its derivative is
continuous - implying smooth and continuous asymmetry - and if the estimated gaps do
not fall into the undesired phase of the function, may be a good approximation to the
true non-linear function.

---

their coefficients are components of the unobserved components vector with true variances equal to zero
by imposition.
The exponential function (re-scaled) is very interesting in that, besides smooth and continuous, is always increasing and convex - although it does not impose any upper bound on the gap.

The Laxton's et al (1995) hyperbole seems also very convenient. It is always increasing and convex to the left of a 'wall' parameter - \( w \) in the expressions below - which has an important economic meaning. In fact, this parameter represents the value for the unemployment gap at which inflation changes (in the Phillips equation) and the output gap (in the Okun equation) increase without bound.

The analytic expressions of these functions (except for the spline) are as follows:

\[
\begin{align*}
\text{Quadratic: } & \quad \Delta \pi_t = A(L)\Delta \pi_t + k1(u^n_t - u_t) + k2(u^n_t - u_t)^2 + \omega S_t + \varepsilon_{t}^{\text{phi}} \\
\text{Hyperbole: } & \quad \Delta \pi_t = A(L)\Delta \pi_t + k2[w(u^n_t - u_t)/(w - (u^n_t - u_t))] + \omega S_t + \varepsilon_{t}^{\text{phi}} \\
\text{Exponential: } & \quad \Delta \pi_t = A(L)\Delta \pi_t + k2[\exp(u^n_t - u_t) - 1] + \omega S_t + \varepsilon_{t}^{\text{phi}}
\end{align*}
\]

These functional forms behave, for reasonable coefficients, as shown in figure 2.4.

52 Recent papers using the LSTAR models to test for asymmetry in the Phillips (or Aggregate-Supply) equation are Weise (1999) and Eliasson (1999).
Figure 2.4 - Non-Linear Functional Forms

PIECE-WISE LINEAR (SPLINE)

QUADRATIC
Their analytic expressions and specific tests when applied to the Phillips equation are summed up in table 2.5. It explains the transformation that is imposed on the elasticity of interest ($\gamma$) so that when it is multiplied by the gap, an analytical expression is generated which includes both a linear and a non-linear component.
In the table, the example is the $\gamma$ in the Phillips equation, but the same applies to the Okun's equation $\theta$.

Table 2.5 - Non-Linear Functional Forms and Asymmetry Tests

<table>
<thead>
<tr>
<th>Functional Form</th>
<th>Phillips equation: $\Delta \pi_t = A(L)\Delta \pi_t + \gamma(u^n_t - u_t) + \omega S_t + \epsilon^{phi}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear spline</td>
<td>$\Delta \pi_t = A(L)\Delta \pi_t + \gamma(u^n_t - u_t) + \omega S_t + \epsilon^{phi}_t$</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$\Delta \pi_t = A(L)\Delta \pi_t + k_1(u^n_t - u_t) + k_2(u^n_t - u_t)^2 + \omega S_t + \epsilon^{phi}_t$</td>
</tr>
<tr>
<td>Hyperbole</td>
<td>$\Delta \pi_t = A(L)\Delta \pi_t + k_1(u^n_t - u_t) + k_2[w/(w - (u^n_t - u_t)) + \omega S_t + \epsilon^{phi}_t]$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$\Delta \pi_t = A(L)\Delta \pi_t + k_1[u^n_t - u_t] + k_2[\exp(u^n_t - u_t) - 1] + \omega S_t + \epsilon^{phi}_t$</td>
</tr>
</tbody>
</table>

The hypothesis to test is if the coefficient $k_2$ in the models above is statistically different from zero, with the null being $H_0: k_2 = 0$. If that hypothesis can be rejected and the estimated coefficient is positive (negative), data tells that the equation of interest should be estimated with an asymmetric functional form - $k_1$ should be eliminated in the exponential and hyperbole forms - and will be convex (concave). If $k_2$ is not significantly different from zero, then the component associated to that parameter in the models above should be eliminated and the models collapse to the baseline linear model, ie $\gamma$ (or $\theta$) equals $k_1$. 
Table 2.6 - Asymmetry Tests - GDP deflator, Unemployment rate, real GDP a

<table>
<thead>
<tr>
<th>Specification of $\Delta \pi_t$</th>
<th>Near-rational</th>
<th>Adaptive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear spline</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Okun equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Phillips</td>
<td>Not rejected (0.13)</td>
<td>Rejected (0.06)</td>
</tr>
<tr>
<td>Given Asymmetric Phillips</td>
<td>Not rejected (0.11)</td>
<td>Rejected (0.04)</td>
</tr>
<tr>
<td>Phillips equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Okun</td>
<td>Not rejected (0.88)</td>
<td>Not rejected (0.56)</td>
</tr>
<tr>
<td>Given Asymmetric Okun</td>
<td>Not rejected (0.84)</td>
<td>Not rejected (0.58)</td>
</tr>
<tr>
<td><strong>Quadratic</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Okun equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Phillips</td>
<td>Rejected (0.01)</td>
<td>Rejected (0.00)</td>
</tr>
<tr>
<td>Given Asymmetric Phillips</td>
<td>Rejected (0.00)</td>
<td>Rejected (0.00)</td>
</tr>
<tr>
<td>Phillips equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Okun</td>
<td>Not rejected (0.71)</td>
<td>Not rejected (0.58)</td>
</tr>
<tr>
<td>Given Asymmetric Okun</td>
<td>Not rejected (0.14)</td>
<td>Not rejected (0.08)</td>
</tr>
<tr>
<td><strong>Hyperbole</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Okun equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Phillips</td>
<td>Rejected (0.00)</td>
<td>Rejected (0.00)</td>
</tr>
<tr>
<td>Given Asymmetric Phillips</td>
<td>Rejected (0.00)</td>
<td>Rejected (0.00)</td>
</tr>
<tr>
<td>Phillips equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Okun</td>
<td>Not rejected (0.86)</td>
<td>Not rejected (0.80)</td>
</tr>
<tr>
<td>Given Asymmetric Okun</td>
<td>Not rejected (0.62)</td>
<td>Not rejected (0.26)</td>
</tr>
<tr>
<td><strong>Exponential</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Okun equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Phillips</td>
<td>Rejected (0.00)</td>
<td>Rejected (0.00)</td>
</tr>
<tr>
<td>Given Asymmetric Phillips</td>
<td>Rejected (0.00)</td>
<td>Rejected (0.00)</td>
</tr>
<tr>
<td>Phillips equation is linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given Linear Okun</td>
<td>Not rejected (0.58)</td>
<td>Not rejected (0.88)</td>
</tr>
<tr>
<td>Given Asymmetric Okun</td>
<td>Not rejected (0.50)</td>
<td>Not rejected (0.25)</td>
</tr>
</tbody>
</table>

a: Series transformed into: $\Delta \pi_t = (1-B)^2 \log GDP \Delta t; U_t = U; Y_t = \log real GDP$

b: The null hypothesis of no asymmetry is not rejected, in spite of a significance probability that would point to rejection at 10 percent of confidence (although not at 5 percent), because the Phillips equation residuals are highly non-normal (Jarque-Bera statistics = 10.5, significance of 0.005 on the null of normality) and the estimated UCs have implausible values.

In the hyperbolic models above, the specific values used as 'wall parameters' (given unemployment gaps expressed in percentage points) were (i) in the near-rational models, $w^{OK}=2.4$ and $w^{PHI}=1.2$ and (ii) in the adaptive expectations models, $w^{OK}=2.1$ and $w^{PHI}=1.1$. For robustness analysis, it was also tried $w^{PHI}=2.0$ and $w^{PHI}=3.0$ in both models, leading to unchanged inference results.
Results are summarised in table 2.6.\textsuperscript{53}

Except for the linear spline function, the tests consistently reject the null hypothesis of linearity of the Okun relation. Due to the strong limitations of the spline form, mentioned above, we conclude that there is enough evidence to reject linearity of the Euro Area Okun relation, throughout 1970-2000. More specifically, since $k_2$ is consistently estimated with positive values, the evidence indicates that the Okun relation should be modelled with a convex functional form.

In what regards the Phillips relation, the evidence is even more clear: there is no evidence to reject the hypothesis that the Euro Area Phillips curve has been linear, between 1970 and 2000, at standard confidence levels.

\textbf{2.5.4. Detailed Results for the Preferred Model}

Given the evidence collected in last section, there is a decision to be made concerning which specific asymmetric functional form should be used in the estimation of the Okun equation in the model’s final version.

Table 2.6 summarises information from several statistical criteria considered determinant for that choice, for both near-rational and adaptive expectations models (Tables 2.7.A and 2.7.B, respectively).\textsuperscript{54} They include the significance of the Phillips and the Okun elasticities, the filter variance of the NAIRU and trend output, the mean square error of the model's one set-ahead forecast of inflation changes, the average and symmetry of the estimated unemployment gap and the normality of the measurement equations' residuals.

The scenario is fairly identical across both expectations' models, and, on the basis of that information, the quadratic functional form is chosen.

\textsuperscript{53} Note that in the models of (modified) hyperbole, the GAUSS procedure \emph{optimum} experienced problems in converging, when estimating the full model. Therefore, the following sequential testing method was used. Firstly, the 'wall' coefficients ($w$ in the table) were estimated in models that did not include any linear component in the Okun and/or Phillips coefficients (ie, $k_1=0$). Secondly, these estimates were calibrated into models including both the linear and the hyperbolic components of the coefficient of interest (ie $k_1$ and $k_2$ are free parameters), to perform the asymmetry test.

\textsuperscript{54} Results for the spline model are shown just for completeness, as that particular functional form is considered just a first approximation for testing asymmetry, not suitable for a final estimation of an asymmetric model.

<table>
<thead>
<tr>
<th>Functional Form</th>
<th>Spline</th>
<th>Quadratic</th>
<th>Hyperbole</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significance $\kappa^2 ,^a$</td>
<td>0.12726076</td>
<td>0.00984334</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Significance $\gamma ,^a$</td>
<td>0.43992148</td>
<td>0.037555</td>
<td>0.2142438</td>
<td>0.04253668</td>
</tr>
<tr>
<td>Variance NAIRU</td>
<td>0.45987524</td>
<td>0.34295078</td>
<td>0.57303305</td>
<td>0.31447141</td>
</tr>
<tr>
<td>Variance potential $Y$</td>
<td>0.0002721804</td>
<td>0.000191495</td>
<td>0.0003339774</td>
<td>0.00023053</td>
</tr>
<tr>
<td>Mean squared $\Delta \pi^\text{model} - \Delta \pi^\text{actual}$</td>
<td>0.078535338</td>
<td>0.077329</td>
<td>0.07776792</td>
<td>0.077181</td>
</tr>
<tr>
<td>U gap:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.31834714</td>
<td>-0.10856166</td>
<td>-1.0796429</td>
<td>-0.86141046</td>
</tr>
<tr>
<td>Min</td>
<td>-0.82107159</td>
<td>-1.2939133</td>
<td>-2.8863389</td>
<td>-2.3863680</td>
</tr>
<tr>
<td>Max</td>
<td>1.5308638</td>
<td>1.7867107</td>
<td>0.97220049</td>
<td>0.96661386</td>
</tr>
<tr>
<td>Normality of residuals Okun $^b$</td>
<td>0.00026782</td>
<td>0.042624258</td>
<td>0.0049417310</td>
<td>0.02412770</td>
</tr>
<tr>
<td>Normality of residuals $U. ,^b$</td>
<td>0.080302432</td>
<td>0.22057882</td>
<td>0.035479903</td>
<td>0.00387680</td>
</tr>
<tr>
<td>Normality of Residuals Phillips $^b$</td>
<td>0.46719619</td>
<td>0.78864932</td>
<td>0.28051516</td>
<td>0.27776090</td>
</tr>
</tbody>
</table>

a: Two-sided significance probability of the test of the null hypothesis that the coefficient is equal to zero.
b: One-sided significance probability of the Jarque-Bera statistics for testing the null hypothesis of normality.

Considering first the near-rational expectations' model, the quadratic form exhibits the best performance in 6 out of the 9 proposed criteria, including the most crucial: significance of Phillips' elasticity, variance of estimation of potential output, symmetry of the gaps, and normality of all measurement equations' residuals. Furthermore, it scores second in two more criteria – NAIRU’s variance and mean square error of inflation changes prediction - and, finally, shows a perfectly acceptable result in what
regards the significance of the asymmetric component of Okun Law's elasticity. In the adaptive expectations models, the quadratic form performs best in six criteria - significance of Phillips elasticity, filter variance of potential output, mean-square error of forecast of inflation changes, centricity of the gaps, and normality of the residuals of the Okun and unemployment equations.

Table 2.7.B - Criteria for Choice of Asymmetric Functional Form of Okun equation (GDP deflator, unemployment rate, real GDP) \(^a\) [Euro area, 1970:I - 2000:II]

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Spline</th>
<th>Quadratic</th>
<th>Hyperbole</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significance (\kappa_2^a)</td>
<td>0.058539</td>
<td>0.007137</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Significance (\gamma^a)</td>
<td>0.640296</td>
<td>0.054549</td>
<td>0.176881</td>
<td>0.171452</td>
</tr>
<tr>
<td>Variance NAIRU</td>
<td>0.68888263</td>
<td>0.34570911</td>
<td>0.5105046</td>
<td>0.2800522</td>
</tr>
<tr>
<td>Variance potential Y</td>
<td>0.00036059</td>
<td>0.0001924</td>
<td>0.0003073</td>
<td>0.000217738</td>
</tr>
<tr>
<td>Mean squared (\Delta \pi^*)</td>
<td>0.078177</td>
<td>0.076693</td>
<td>0.0772941</td>
<td>0.07680959</td>
</tr>
<tr>
<td>U gap:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.77028895</td>
<td>-0.10864352</td>
<td>-0.990894</td>
<td>-0.6945057</td>
</tr>
<tr>
<td>Min</td>
<td>-0.19677750</td>
<td>-1.2818523</td>
<td>-2.836103</td>
<td>-2.2426601</td>
</tr>
<tr>
<td>Max</td>
<td>2.1529710</td>
<td>1.8407730</td>
<td>0.9525220</td>
<td>1.0462479</td>
</tr>
<tr>
<td>Normality residuals Okun (^b)</td>
<td>0.00096677</td>
<td>0.03103709</td>
<td>0.0095715</td>
<td>0.010515</td>
</tr>
<tr>
<td>Normality residuals U. (^b)</td>
<td>0.10983604</td>
<td>0.21371748</td>
<td>0.04356165</td>
<td>0.0489565</td>
</tr>
<tr>
<td>Normality Residuals Phillips (^b)</td>
<td>0.66355932</td>
<td>0.70811918</td>
<td>0.80429955</td>
<td>0.818295</td>
</tr>
</tbody>
</table>

\(a\): Two-sided significance probability of the test of the null hypothesis that the coefficient is equal to zero.

\(b\): One-sided significance probability of the Jarque-Bera statistics for testing the null hypothesis of normality.
With respect to two more criteria - significance of the convex component of the Okun equation and filter variance of the NAIRU - it scores second best performances, and finally, its results in terms of normality of the Phillips equation are unquestionably good.

The detailed estimation results of the model with linear Phillips equation and quadratic Okun equation are presented in Table 2.8.

Most coefficients' estimates do not change significantly, in comparison to the results shown in table 2.3, relative to the model with linear Phillips and Okun equations. However, two main improvements occur. First, the Okun equation residuals clearly improved, and are now very near to passing the Jarque-Bera test of the null of normality (significance of about 4 percent). Second, the Phillips elasticity is now significant at 5 percent.

These are not, obviously, minor changes - the Phillips relation is the theoretical basis of the model, and the empirical technique adopted presupposes normality of the stochastic processes modelled. The conclusion is that specifying the Okun relation as asymmetric is crucial for a correct specification of the proposed model.

The estimated Phillips coefficient tells us, in the near-rational expectations' model, that for each additional percentage point of unemployment gap, inflation changes by an additional 0.042 percentage points in the same quarter. The estimate of the Phillips trade-off elasticity is, as happened in the linear models, somehow smaller in the near-rational expectations' model than in the adaptive expectations' model - where it equals 0.053, instead of 0.042. However, the notable result here is that if rational expectations are replaced by forward-looking, but limited-information and near-rational expectations of inflation, then we obtain a correctly signed estimate of the Euro Area Phillips trade-off elasticity, which is, moreover, statistically significant at 5 percent of significance.

The Okun coefficients indicate that each additional percentage point of unemployment gap is associated with an additional deviation of output from its trend of 2.3 percentage points plus 2×0.008 of the unemployment gap.
<table>
<thead>
<tr>
<th>Specification of $\Delta \pi_t$</th>
<th>Near-rational</th>
<th>Adaptive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phillips:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.042 [1.91] (0.05)</td>
<td>0.053 [1.92] (0.05)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.022 [0.113] (0.91) $^b$</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>-0.402 (0.00)</td>
<td></td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.194 (0.03)</td>
<td></td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-0.287 (0.00)</td>
<td></td>
</tr>
<tr>
<td>$\phi(1)$</td>
<td>-0.884 [-0.46] (0.65) $^b$</td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.048 (0.00)</td>
<td>0.046 (0.00)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{\text{phi}}$</td>
<td>0.288 [15.08] (0.00)</td>
<td>0.280 [14.88] (0.00)</td>
</tr>
<tr>
<td><strong>Okun:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>0.023 [4.66] (0.00)</td>
<td>0.023 [4.74] (0.00)</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.008 [2.58] (0.01)</td>
<td>0.008 [2.69] (0.00)</td>
</tr>
<tr>
<td><strong>Unobserved components:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>0.0060 [12.82] (0.00)</td>
<td>0.0060 [12.35] (0.00)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{p}$</td>
<td>0.0047 [13.33] (0.00)</td>
<td>0.0047 [13.26] (0.00)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{N}$</td>
<td>0.093 [8.19] (0.00)</td>
<td>0.092 [8.19] (0.00)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{\mu}$</td>
<td>0.014 [1.87] (0.06)</td>
<td>0.015 [1.90] (0.05)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.821 [25.30] (0.00)</td>
<td>1.826 [25.83] (0.00)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.843 [-12.02] (0.00)</td>
<td>-0.847 [-12.26] (0.00)</td>
</tr>
<tr>
<td>$\sigma \varepsilon^{c}$</td>
<td>0.0581 [4.60] (0.00)</td>
<td>0.0586 [4.72] (0.00)</td>
</tr>
<tr>
<td>Log L</td>
<td>-788.315</td>
<td>-787.965</td>
</tr>
<tr>
<td><strong>Residuals:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips equation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>0.48 (0.79)</td>
<td>0.69 (0.71)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>6.09 (0.19)</td>
<td>1.96 (0.74)</td>
</tr>
<tr>
<td>Okun equation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>6.31 (0.04)</td>
<td>6.95 (0.03)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>1.72 (0.79)</td>
<td>1.61 (0.81)</td>
</tr>
<tr>
<td>Unemployment equation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>3.02 (0.22)</td>
<td>3.09 (0.21)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>6.64 (0.16)</td>
<td>7.74 (0.10)</td>
</tr>
</tbody>
</table>

*a: Series transformations - see table 2.3; The $\omega$ coefficient is associated to $\pi_{t-1}IMP(i-1)-\pi_{t-1}(i-1)$. The coefficient $\alpha$ is associated to $E_t \pi_{t+1}$.

$b: T$-statistics and Significance for the test $H_0: \alpha = 1$; idem for $H_0: \phi(1) = 1$;

c: Standardised residuals computed as defined in Harvey (1989), p. 256, 442.

[ ]: $T$-statistics; ( ): Significance; Significance probabilities relate to two-sided tests, except in 'Residuals'. The standard deviation of the innovation of the first measurement equation (Okun equation) was restricted to zero as it systematically converged to that value.
Figure 2.5 shows the main unobserved components estimated with this model.\textsuperscript{55} The central conclusion is that they do not change markedly in comparison to the ones of the baseline linear model and sketch the same story in what regards the dating of Euro area business cycles in the last decades.\textsuperscript{56} However, as the last chart in this figure shows, there are some differences in some specific estimates, such as the rate of decrease of the NAIRU's drift by the end of the sample.

\textbf{Figure 2.5 - Main Unobserved Components From Model With Quadratic Okun Curve}

\textsuperscript{55} Presented charts relate to the 'near-rational expectations' model'. As happened in the baseline linear model, the results with the 'adaptive expectations model' were identical and, as such, are not shown.

\textsuperscript{56} Note that the centre of the smoothed estimated gaps - and their amplitude in the particular case of the output gap - changes slightly from the linear to the quadratic models. Specifically, the minimum of the unemployment gap is now -1.97 (-1.66 in the linear model) and its maximum is 0.81 (1.08 formerly). The minimum of the output gap is now -3.22 (-3.90 in the linear model) and the maximum is 2.29 (2.55 formerly). This result is partly due to the change in the Okun equation functional form and partly caused by the fact that in the quadratic model the smoothed values are only approximations to the true smoothed values, because each period's output gap depends upon the last period's output gap.
The evolution of the NAIRU's estimated drift describes the path of the equilibrium unemployment in the Euro Area and is quite in line with the results of the literature of macro-labour that studied the subject. The path estimated with the quadratic model using near-rational expectations of inflation can be described as follows. At the beginning of the sample the NAIRU was drifting up at a rate of about 0.12 percentage points per quarter. During the 70s this positive drift persisted at values between 0.10
and 0.12 percentage points per quarter, therefore pushing the Euro Area equilibrium unemployment upwards at a more or less constant rate. Only after 1981:II did the drift begin decreasing. Between that quarter and 1994:I the NAIRU was drifting up but at decreasing rates (with some minor exceptions in 1990:IV and 1992:IV-1993:I). After 1994:II the estimated drift for the NAIRU is negative, which means that Euroland's equilibrium unemployment is decreasing, and at the end of the sample it was drifting down at a rate of about 0.075 percentage points per quarter.

Figure 2.6 displays scatter plots of the estimated unemployment and output gaps, showing the asymmetry in the Okun curve estimated with this model - which is, as seen above, statistically significant and important for the model's results. The smoothed estimates tell us that at the peak of booms, typically, the Euro Area economy has its real output growing at about 2 percent above the trend and unemployment 0.5 to 1 percentage point below the NAIRU. At the troughs real GDP grows between 2 and 3 percent below trend and unemployment is between 1.5 and 2 percentage points above the NAIRU.\(^{58}\)

Business cycle asymmetry is quite apparent in these charts: expansions generate smaller decreases in unemployment (increases in output), compared to the trend, than contractions generate increases in unemployment (decreases in output). As expected, asymmetry is more important the further away from trend the economy is, while at the trend's vicinity the Okun relation is approximately linear.

\(^{57}\) See Blanchard (1999), Blanchard and Wolfers (2000), Blanchard and Giavazzi (2001) and Blanchard (2000a, b, c, d).

\(^{58}\) As usual throughout this chapter, the reported results are from the 'near-rational expectations' model' and the charts from the adaptive expectations model' look similar.
2.5.5. Trend and Cycle in the Euro Area 1970-2000

In this section, results from the preferred model are further analysed, with the purpose of deepening their evaluation. First, confidence bands for the estimated NAIRU are computed. Then, the estimated NAIRU (and the corresponding gap) are compared to the trend unemployment rate (and the implied gaps) published in the AWMD. Moreover, the business cycle dating identified in this study are compared not only to the AWMD's but also to the ones implied by the recent analysis in Gali et al (2001). Then, the ex-post estimates of the unemployment gap resulting from the preferred model are
compared with their quasi-real-time estimates as of 1998:II, which is the last data-point in the AWMD and in Gali et al (2001). Finally, some computations illustrate the model's forecasting performance.

**Confidence Bands for the Euro Area NAIRU**

As mentioned above, Hamilton's (1986, 1994) method is now used to compute confidence bands for the estimated unobserved components of the preferred model. This Monte Carlo procedure is needed because the Kalman filter does not generate estimates of the whole uncertainty associated to the unobserved components' estimates. It estimates filter uncertainty only, therefore assuming perfect knowledge of the hyper-parameters of the likelihood function, which is clearly not the case.

In short, 10,000 sets of hyper-parameters were randomly generated, drawn from a multivariate normal distribution with averages equal to the parameter's point-estimates and variance-covariance matrix given by the inverse of the Hessian. The Kalman filter was then ran 10,000 times, with each of these hyper-parameters combinations, and the corresponding smoothed unobserved components, from T to 1, were computed.

Filter uncertainty is then given by the mean of the one-step smoothed forecast errors' variances for each observation t (from T to 1) across the N random samples. Parameter uncertainty is given by the mean square error of the smoothed UCs to the smoothed UCs of the estimated baseline model, for each observation t (from T to 1) across the N random sets of hyper-parameters. Total uncertainty, defined as a mean square error (MSE) for the UCs estimates around their true values, is given by the sum of these measures.  

The Monte Carlo experiment was conducted in GAUSS, using a code written by the author, which runs the Kalman filter, the Kalman smoother and computes all Hamilton's sources of uncertainty from observation T to 1. The random sets of hyper-parameters were generated with the instruction rgaussm, of the procedure mgauss.src of library distrib.lcg, written by Noack and Schlittgen (2000). For all Monte Carlo draws, the Kalman filter was initialised as follows. The Nairu and trend output's starting values

---

59 Filter uncertainty and parameter uncertainty are given by, respectively, equations 13.7.6. and 13.7.5. in Hamilton (1994).
were set at the actual values of unemployment and real output at time 1 - and the implied values for the gaps - just as was done in estimating the model. The parameters associated to inflation expectations and supply-shocks were initialised at the estimates obtained in the baseline model. The variance-covariance matrix of the unobserved components vector was initialised also at the estimates obtained in the preferred model above.

Results are summarised in table 2.9. The root of the mean square error (i.e. the standard error of estimate of the NAIRU and the gap around their true values) is estimated at 0.68 percentage points for the last available observation. Its average value across the sample is 0.59 percentage points and it ranges from a minimum of 0.42 to a maximum of 0.73 points. Hamilton's MSE is estimated at 0.46 percentage points, at observation T, from which about 0.29 are due to filter uncertainty and 0.17 to parameter uncertainty. As natural, filter uncertainty's maximum occurs at period T, as it increases systematically, from a minimum of 0.07 percentage points at period 1. Parameter uncertainty, on the other hand, evolves in a more complex and less systematic way. In our specific case, its maximum is observed at period 1 (0.42 percentage points) and its (0.05 percentage points) minimum occurs at 1980:I, while it averages to 0.22 points across the sample. At the end of the sample, there is no significant widening of the total uncertainty interval, because the increase in filter uncertainty is offset by a reduction of parameter uncertainty, which seems to be associated to the linearly decreasing path of unemployment since 1997.

The estimated uncertainty around the NAIRU is in line with the estimates reported in the recent literature. For instance, within the G7 countries, Richardson et al (2000) find average (full sample) standard errors (root of mean square errors) ranging from 0.2 in Japan to 1.1 percent point in France and the UK. OECD (2000) reports average and end-of-sample standard errors for 21 developed countries, the first ranging from 0.2 points in Japan and Austria to 1.4 points for Finland, and the second ranging from 0.3 percent and 1.8 percentage points (for the same countries). Irac (2000) places this uncertainty measure for France's NAIRU between 0.7 and 1.2 percentage points. Laubach (2001) finds standard errors between 0.54 and 1.98 percent points for the

---

60 Charts below show results for the near-rational expectations' models. The standard error for the adaptive expectations model at moment T is only slightly larger, at 0.70 percentage points.
NAIRUs of G7 countries, and for continental Europe countries his findings are 0.77 for Germany, 0.55 for Italy and 1.30 for France.61

Table 2.9 - NAIRU Estimates And Uncertainty [Euro area 1970:II-2000:II]

<table>
<thead>
<tr>
<th></th>
<th>2000:II</th>
<th>Average</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAIRU</td>
<td>9.15</td>
<td>7.40</td>
<td>10.15</td>
<td>1.72</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>0.05</td>
<td>-0.62</td>
<td>0.83</td>
<td>-1.95</td>
</tr>
<tr>
<td>MSE</td>
<td>0.46</td>
<td>0.36</td>
<td>0.53</td>
<td>0.18</td>
</tr>
<tr>
<td>Filter Parameter</td>
<td>0.29</td>
<td>0.15</td>
<td>0.29</td>
<td>0.07</td>
</tr>
<tr>
<td>Standard Error (RMSE)</td>
<td>0.68</td>
<td>0.59</td>
<td>0.73</td>
<td>0.42</td>
</tr>
<tr>
<td>95% Confidence Band</td>
<td>1.32</td>
<td>1.17</td>
<td>1.43</td>
<td>0.83</td>
</tr>
<tr>
<td>95% Conf. Interval (± 1.96*RMSE)</td>
<td>2.64</td>
<td>2.34</td>
<td>2.86</td>
<td>1.66</td>
</tr>
</tbody>
</table>


Figure 2.7 shows our preferred model estimates of the Euro area NAIRU and the unemployment gap with the corresponding 95 percent confidence bands (computed as 1.96*sqrt(MSE)). These amount to 1.32 percentage points by the end of the sample and are no smaller than 0.83 percentage points and no larger than 1.43 points, averaging to 1.17 points of percentage across the full sample.

As can be seen from the charts in that figure, actual unemployment is out of the 95 percent confidence bands only during the 4 to 5 years around the recessions' troughs. The Euro Area cyclical condition could be considered negative, with a 95 percent of confidence, since 1982:III, whereas the trough would be reached only two years and

61 Laubach (2001) page 228, table 4. Note that he stresses, across the paper, that Italy's results are hard to understand.
two quarters later, in 1985:I. In the most recent recession, the model would allow a confident classification of the Euro Area cyclical state as negative since 1993:IV, three years and one quarter before the 1997:I's trough.

**Figure 2.7. Estimated NAIRUs, Unemployment Gaps and Confidence Bands**
The model's performance is much worse at expansions, though, as one can never be 95 percent confident that the economy is *booming*, not even at the quarter of cyclical peak. In fact, in spite of a systematically increasing unemployment gap since the 1985 trough, until the peak at 1991:I., the lower bound of the 95 percent confidence interval around the gap never reached a positive value.

One could argue, tentatively, that the model does seem to generate worth information for the policymaker to react in an useful *timing* to the expansion\(^{62}\) (and its inflationary dangers), as the estimated gap is positive since 1989:III, which is almost two years before the peak.\(^{63}\) But the fact remains that unless we relax our confidence level\(^{64}\), the Euro Area economy could never be considered *booming* with this type of model.

These results concerning the imprecision in the (unobserved components) estimation of the NAIRU and the gap are not new. In fact, they have been around since the earlier days of the literature on time-varying NAIRUs, when Staiger, Stock and Watson (1997a, 1997b) estimated confidence bands for the US NAIRU so wide that one could never be confident that the actual unemployment was significantly different from the estimated NAIRU.

"The main finding is that the NAIRU is imprecisely estimated: a typical 95-percent confidence interval for the NAIRU in 1990 is 5.1 to 7.7 percent. (...) The imprecision suggests caution in using the NAIRU to guide monetary policy."\(^{65}\)

If anything, the confidence bands here compare slightly positively with the competing literature, as discussed above. But they do not solve the essential problem of NAIRU estimates lacking practical utility for the conduct of monetary policy.

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\(^{62}\) The plotted estimates of the NAIRU and the gap are smoothed estimates and are not real-time estimates, so this conclusion should be read with careful - see the discussions below.

\(^{63}\) Remember that the time lags between monetary policy actions and inflation reactions are typically placed around two years by many researchers, since the seminal *dating* in Friedman and Schwartz (1963).

\(^{64}\) Which seems to be what OECD (2000), Richardson et al (2000) and Irac (2000) have done, by graphing confidence bands computed with one only standard error in each side of the estimates.

\(^{65}\) Staiger, Stock and Watson (1997b), abstract. Note that they estimated uncertainty with a different method from this chapter's. In (1997a), they used the 'Delta method', while in (1997b) they used the 'Fieller method' - which they found superior to the latter on Monte Carlo simulations (for brief presentations of both methods see Staiger, Stock and Watson (1997b) page 37).
Euro Area NAIRU and Cycles: A Comparison to the AWMD

This chapter's estimates of the NAIRU and the gap are significantly different from those reported by the ECB in the Area Wide Model Database (series URT), as figure 2.8 shows.

First, the ECB's Research NAIRU is smoother than this chapter's and, accordingly, its unemployment gap is more volatile. Specifically, for the comparable period (1972:I-1998:II), the Area Wide Model Database (AWMD) NAIRU has a sample standard deviation of 1.98, while our series has a standard deviation of 2.62. The AWMD unemployment gap has a standard deviation of 1.3, while ours is around 0.86.

Second, the AWMD NAIRU average (6.96) is further away from the actual unemployment series average (7.87) than ours (7.24 percentage points). This implies that the average of this chapter's estimates of the unemployment gap is closer to zero than the ECBs': -0.63 against -0.92. As can be seen from the chart with estimates of the unemployment gap, in the 1990-91 cyclical peak the AWMD unemployment gap was positive for only four quarters and reached a maximum of 0.22 percentage points. This study's gap was positive for thirteen quarters and peaked at 0.83 points.

Third, between the beginning of the sample and 1984 the ECB's research NAIRU behaves almost as a deterministic time trend, while this chapter's series fluctuates considerably and drifts up much more markedly, in line with the path of actual unemployment.

Finally, the point estimates suggest that the model behind the AWMD systematically underestimated the NAIRU since 1978:III, in comparison to this chapter's model. In the recent periods, the AWMD NAIRU is much more sensitive to the evolution of actual unemployment than this chapter's estimates, which suggests more economic foundation of the latter in comparison to the former.

As the last chart in figure 2.8 shows, the deviation of this chapter's estimates from the ECB's research is not random and is very significant, peaking at around 1.5 percentage points at the end of 1993, which suggests that the AWMD trend unemployment rate may not be correct.
Figure 2.8. Estimated NAIRUs, Unemployment Gaps and Confidence Bands versus ECB's Estimates in AWMD
As table 2.10 shows, in three out of the six cyclical turning points, identified in the sample period, the AWMD unemployment gap changed its evolution simultaneously with our estimated unemployment gap. This happened whether *strictu sensu* - as in 1980:I - or with no more than a quarter of delay, as in 1974:I and 1991:I. The interesting regularity here is that the divergences occur always at troughs. Specifically, they occur at the recession of the mid-70s (7 quarters of delay), the 1985:I trough (4
quarters of leading) and the recession of the second half of the 90s (11 quarters of leading).

Table 2.10 – Cyclical Turning Points [Euro area, 1970:I - 2000:II]

<table>
<thead>
<tr>
<th>Cyclical Turning Point</th>
<th>Unemployment Gap [trivariate UC]</th>
<th>Inefficiency Wedge [GGS (2001)]</th>
<th>Unemployment Gap [ECB's AWMD]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>1974:I</td>
<td>-----</td>
<td>1974:II</td>
</tr>
<tr>
<td>Trough</td>
<td>1975:IV</td>
<td>-----</td>
<td>1977:III *</td>
</tr>
</tbody>
</table>

* Between the beginning of the ECB's AWMD trend unemployment rate estimates (1971:IV) and quarter 1980:III, this is the only period in which the unemployment gap implicit in the ECB's AWMD is negative. Moreover, it is only slightly below zero (-0.0222). Furthermore, between 1977:II and 1978:IV the AWMD unemployment gap evolved very closely to 0, although at positive values (except 1977:III, obviously). All this implies that it is very hard to consider this period as a standard cyclical downturn using the AWMD series.

The AWMD NAIRU is almost always inside the 95 percent confidence bands surrounding our estimates of the NAIRU. In the studied sample, only in the five quarters between 1972:I and 1973:I and in the five quarters from 1993:II to 1994:II can we be 95 percent confident that the AWMD estimates are different from ours.

This 1993-94 episode could be used to illustrate the practical relevance of the divergence between this chapter's model and the AWMD trend unemployment rate. In fact, if the Monetary Union and the European Central Bank were already in force by then, the AWMD series would suggest that the European Central Bank should lower interest rates as soon as 1994:II. On the contrary, this chapter's model indicates that the cyclical trough has been reached only in 1997:I, so only about then would it strongly suggest lowering interest rates. Euro Area inflation did reach a local minimum by 1994:II (quarterly rate of 0.45 percentage points) but it increased again until 1995:II (0.95 percentage points) and only from 1986:III on was it below 0.45 points. This means that an expansionary monetary action by 1994 would have been inappropriate. Obviously, it remains to be seen how different would these trend and cycle estimates be.
if the same data set ending by 1998:II was to be used with both models - a question that
will be addressed below.


Another interesting analysis of these results is to have them compared to Gali,
Gertler and Salido's (2001) (hereafter, GGS) indicator of capacity utilisation in the Euro

The GGS's theoretical and empirical framework is significantly different from
ours: they estimate a New Keynesian Phillips curve using not a real gap but a proxy for
the real marginal cost (specifically real unit labour costs) and their estimation technique
is GMM.  

Most importantly, they argue that the real marginal cost can be decomposed
into the *wage mark-up* - which indicates the level of frictions in the labour market and,
therefore, accounts for inflation inertia - and the ratio of the household's marginal cost
of labour supply to the marginal product of labour.  

They refer to the latter as the *inefficiency wedge*, and argue that it is a proportionate measure of output relative to the
efficient level of output, i.e. the output corresponding to the frictionless competitive
equilibrium. They further argue that the standard formulation of the New Phillips
relation based on the output (or unemployment) gap is correct only under the
assumption of constant wage mark-ups.

Figure 2.9 shows GGS's measure of the *inefficiency wedge* for the Euro area,
1970:I-1998:II, together with this chapter's estimate of the unemployment gap and an
published by the ECB (values to the right of the vertical line).  

The most striking

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66 Ross and Ubide (2001) offer a comparison of the Euro Area business cycles estimated from a number of alternative methods, also including Unobservable Components Models and GGS's inefficiency wedge.

67 For a recent critique of GGS's use of the labour share (or unit labour costs) as proxy for the real marginal cost in their New Keynesian Phillips relation, see Roberts (2001).


69 GGS's measure was replicated using the proxies and formulas they define for the practical decomposition of the log of real marginal cost into the sum of the logs of the wage mark-up and the log of the inefficiency wedge (see p. 1262-1263). Specifically, the (log of) inefficiency wedge is measured as the log of private real consumption, plus the log difference of total employment to the labour force, minus the log of real output deducted of the log difference of the employment to the labour force. For 1970:I-1998:II the source was the AWMD, while for 1998:III-2000:II data was taken from recent issues of the ECB's Monthly Bulletin. The inefficiency wedge for 1970:I-1998:II in the graph mimics perfectly the dotted line in the upper part of GGS's figure 5, page 1264.
conclusion is that between 1980 and 1998:II the cyclical turning points identified by both models are highly similar. This can be more clearly seen in Table 2.10. In fact, these models' estimated turning points match perfectly at the 1991:I's peak, they are just three quarters away from each other at the early 80s peak and at the mid 80s trough, and are separated by only two quarters at the 1997's trough.\footnote{At the beginning of the sample period, on the contrary, the two models generate different results. GGS's cyclical measure indicates that the Euro area was producing at a high capacity utilisation rate. They further indicate that the capacity utilisation decreased slowly but steadily until the end of 1980 - therefore, suggesting that there was not a typical business cycle in the Euro Area during the 70s. My model's estimates, on the contrary, are closer to a standard business cycle, albeit shorter and less ample than usual.\footnote{Gali, Gertler and Salido (2001), page 1262, equation (22).}}

**Figure 2.9. Unemployment Gap versus Gali, Gertler and Salido's (2001) Inefficiency Wedge**

These results are interesting at least on two grounds. First, at a general level, it is remarkable that two such different empirical frameworks, estimation methods and statistical information generate so similar business cycle turning points.

Second, these results seem to defy GGS's theoretical arguments stating that the formulation of the New Phillips curve based on the output gap is correct only under the assumption of a constant wage mark-up.\footnote{Gali, Gertler and Salido (2001), page 1262, equation (22).} In fact, as the bottom half of GGS's figure 5...
(p. 1264) shows, the wage mark-up seems to have changed markedly during this period. Yet, this chapter's model, based on a Phillips equation specified on the unemployment gap, generates a trend-cycle decomposition that is very close, in its turning points, to GGS's inefficiency wedge's turning points across almost 20 years.

One advantage of the unobserved components framework here is that the uncertainty associated to the estimation of the NAIRU can be measured. The computations of the Inefficiency Wedge, in turn, depend upon a set of simplifying assumptions and of decisions about proxies for unavailable statistical information which uncertainty can not be formally assessed.

**Ex-Post and Quasi-Real-Time Estimates of the Unemployment Gap**

It could be argued that the AWMD URT series and GGS's inefficiency wedge should be compared to the NAIRU that would be estimated with this chapter's model using data only up to 1998:II (last period of the AWMD).

The question here is that of the comparison of real-time or quasi-real-time estimates of the real gap to their ex-post estimates, on which there is a growing recent literature. In short, real-time estimates of the gaps may subsequently be subject to changes both due to data revisions and statistical revisions - the latter being changes in coefficient's estimates and, therefore, in estimates of the unobserved components, in recent periods, due to the availability of new data. Quasi-real-time estimates are not subject to data revisions, but are subject to statistical revisions.

In an important essay, Orphanides and Van Norden (1999) found that real-time estimates of the output gap are typically subject to revisions of the same order of magnitude of the estimated gap itself. They further found that the main cause for those revisions is not data revision, but statistical revision instead, as the end-of-sample estimates of trend output tend to be highly unreliable with most available methods, especially around turning points. In a more detailed framework, Tchaidze (2001) confirmed that it is the inclusion of leads of data in the estimation of real gaps that contributes the most to its ex-post revision. However, the results in Camba-Mendez

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72 The implication of real gap uncertainty for the design of monetary policy is potentially enormous and has received attention from a large recent literature. This analysis is, however, beyond the scope of this
and Palenzuela (2001) seem to indicate that unobserved components models similar to the one in this text, when applied to aggregate Euro area data, may suffer less from statistical uncertainty than the time-series studied in Orphanides and Van Norden (1999).

Figure 2.10 shows estimates of the unemployment gap of the Euro area, obtained with the model identified in this study when estimated with data up to only 1998:II. The first chart compares these quasi-real-time estimates at 1998:II to the ex-post estimates computed at 2000:II, showing that the additional 8 quarters of data generate substantial statistical revisions in the estimated unemployment gaps from 1995 onwards. These revisions increase steadily between 1995 and 1998, amounting from 0.18 percentage points at 1995:I to 0.87 percentage points at 1998:II, reaching 72 percent of the ex-post estimated gap at this quarter.

As the second and third charts in figure 2.10 show, the AWMD's trend unemployment and unemployment gaps are significantly different from our quasi-real-time estimates, at 95 percent of confidence, in three episodes. These are the periods from 1972:I to 1973:I, from 1993:I to 1994:IV and from 1997:I to 1998:II (the last six quarters of this sub-sample). Hence, even when quasi-real-time estimates as of 1998:II are used, there are still very large differences between this model's and the AWMD's gaps, which track back to a period of about 20 quarters before the end-of-sample.

The charts also show that the quasi-real-time estimates as of 1998:II would wrongly suggest the existence of a trough at 1994:II, therefore failing - similarly to the AWMD - to detect the 1997:I trough. One qualification of this result, is that while the AWMD's estimated gap at 1994:II was -2.89 percentage points - minus 0.3 points than at the previous trough - this model's quasi-real-time estimated gap amounted to -1.41 percentage points, about 0.5 points above its value at the previous trough. In spite of this qualification, the results suggest, essentially, that the policy use of real gaps estimated in real-time (from unobserved components systems) should be pursued with great caution.
Figure 2.10. Quasi-Real-Time versus Ex-Post Estimates of the Unemployment Gap

UNEMPLOYMENT GAP 1970-1998:II

NAIRU
The last chart in figure 2.10 illustrates the fact that the estimates as of 1998:II mimic the turning points identified with GGS's inefficiency wedge from 1980 to 1991, but fail to identify the 1997 trough. Obviously, the GGS method has the advantage, here, of not involving any statistical procedure, therefore not being subject to statistical uncertainty and revisions. This is the positive corollary of the disadvantage of that
method that we previously discussed, namely the impossibility of computing measures of uncertainty for this cyclical indicator.

**Inflation Forecasts**

Finally, figure 2.11 illustrates this chapter's model's performance in forecasting short-run inflation.

The first chart shows actual quarterly inflation rate and in-sample one-step-ahead forecasts of inflation for 1972:I-2000:II. Each period's quarterly inflation rate is predicted using the smoothed estimates of the Phillips equation's parameters, but only last quarter information on inflation expectations, domestic and imported inflation and the unemployment gap. As can be seen in the chart, the model tracks the short-run changes of inflation fairly well. The root of the mean square error (RMSE) is 0.28, which means that the model predicts next quarter's inflation, within the sample, with a 95 percent confidence band with around 0.55 percentage points.

The second chart in figure 2.11 looks with more detail into the model's predictive performance from 1996:I onwards. It shows that in spite of the somehow erratic movements of quarterly GDP inflation in 1999 and the first half of 2000, the model still performs well. Furthermore, the models' out of sample two-step-ahead predictions for 2000:III and 2000:IV are shown, to the right of the dashed vertical line. These forecasts use the smoothed estimates of the Phillips equation and information on inflation expectations, domestic and imported inflation and the estimated gap as of 2000:II. As the chart shows, the model's performance happens to be quite good in these two quarters. The root of the mean square error is 0.09 and the corresponding 95 percent confidence band would be around 0.18 percentage points.

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73 The inflation rates until 2000:II used in the charts and computations here are the initial ECB's estimates, which were used in this chapter's estimation - hence, they do not include subsequent revisions in this data. Figures for 2000:III, 2000:IV were drawn from the ECB's Monthly Bulletin of June 2001.
Figure 2.11. Actual Inflation versus Model Predictions of Inflation


2.6. Concluding Remarks

Using quarterly aggregate data for the Euro Area from 1970:I to 2000:II, this chapter estimates a simple macroeconomic unobserved components' model by maximum-likelihood, with the Kalman filter, whose main measurement equations are the Okun and Phillips relations.

The estimation method and the tests here provided address some of the shortcomings existing in the literature and thus the new estimates of the Euroland's NAIRU and real gaps improve on the ECB's estimates published in the AWMD on some grounds.

The system successfully estimates all the hyper-parameters in the model, including the standard deviation of the innovation in the NAIRU's random walk, thus circumventing the need to impose any arbitrary value for this or any other parameter.

This model's Phillips equation is not a backward-looking ad-hoc equation with arbitrary lags of inflation, but rather a forward-looking New Keynesian relation. This New-Keynesian Phillips equation has good empirical properties, due to the use of a recently suggested concept of near-rational expectations of inflation.

Tests of asymmetry in the Okun and the Phillips equation are provided, with a method that encompasses the linear null hypothesis and covering four possible non-linear functional forms. On the whole, the evidence does not allow rejection of the hypothesis that the Euro Area Phillips relation has been linear in the last three decades, whilst clearly allows rejection of that hypothesis for the Okun relation, in favour of convexity. We draw two immediate consequences from these findings. First, the choice of unemployment vs output gap to characterise the trade-off may not be irrelevant in terms of its linearity/non-linearity. Second, neither non-linearity nor linearity should be taken for granted a priori across samples. Not rejection of linearity in the Euro Area Phillips relation (at least until 2000:II) has important policy implications, which can be easily drawn from the discussion on asymmetries reviewed in this work.

The system is estimated fitting a quadratic function to the Okun equation, which seems to be the most suitable assumption for the data at hand, within the functional forms here considered. The results show that when near-rational expectations of
inflation are used in the forward-looking new keynesian Phillips equation, in place of purely rational expectations, the Phillips trade-off is estimated with the correct sign and is statistically significant, provided that the Okun equation is adequately modelled as a convex relation. Hence, although the point estimate of the Phillips elasticity turns out to be somehow smaller than that obtained from similar models with adaptive expectations, Ball's (2000) limited-information forward-looking expectations seems to work well in fitting both the data and the theoretical properties of the new keynesian Phillips curve.

The estimated model seems to capture reasonably well features of the macroeconomic data of the Euro Area during the 1970-2000 period. The peaks and troughs of the estimated unemployment gap seem in accordance with the conventional wisdom about recent European cycles and are very close to turning points identified by alternative sources with different frameworks, data and method. The model's performance in forecasting short-run movements of inflation is also satisfactory. However, comparison of ex-post estimates to quasi-real-time estimates of the main unobserved components shows that the model is subject to a significant amount of statistical uncertainty and revisions.

Monte Carlo integration shows that this chapter's model estimates the Euro Area equilibrium unemployment rate with a degree of confidence that compares well with the competing literature. The 95 percent confidence bands are estimated at \( \pm 1.32 \) percentage points at the end of the sample and range from 0.83 to 1.43 points, averaging to 1.17 percentage points for the full sample. During the 4 to 5 years around the cyclical troughs the model estimates gaps that are different from zero at a 95 percent of confidence. The behaviour of the model's confidence interval estimates is, however, not that good during expansions.

Hence, in the Euro Area, like in the other developed countries and Areas, the evidence so far calls (at the best) for a cautious use of the unemployment (output) gap in the practical conduct of monetary policy.

There are at least two avenues for future research along the lines of this chapter's.

First, the unobserved components model can be enhanced with additional equations, which may improve estimation of the unobserved variables by adding new information and restrictions from economic theory, as well as generate estimates of other
unobserved variables relevant for monetary policy analysis. An interesting extension would be to include in our model an aggregate demand (IS) equation, modelling the unemployment (output) gap as function of the expected gap and the real short-term interest rate, as derived from optimising new keynesian models. One alternative formulation - although less theory-driven - would write the IS as function of the gap between actual interest rate and its natural level, and then explicitly relate this natural rate of interest to the natural unemployment and potential output, as in Laubach and Williams (2001). Another extension of the model could consider a production function in the measurement system, and use information from production factors and technology. A recent work along these lines is Proietti et al.'s (2002) combination of a Phillips equation with a production function, enabling them to estimate the contribution of the production factors and productivity to trends and cycles.

A second path for future research relates to the modelling of inflation expectations - a crucial issue for the analysis of the Phillips trade-off, which, in spite of the good results in this chapter, remains open, and, as such, has been receiving enormous attention in the recent literature. On purely empirical grounds, data eventually available on aggregate Euro Area surveys of inflation expectations would be a natural candidate to proxy for inflation expectations, thus giving a pragmatic solution to the problem of expectations being unobservable. On theoretical - and not merely empirical - grounds, there is a growing recent literature of theories postulating deviations from full rationality, similarly to Ball's (2000) near-rationality hypothesis used in this chapter. Some of those theories seem to deserve a closer look, in the ongoing search for theoretical models of expectations simultaneously consistent with evidence from microeconomic and macroeconomic data. Examples of recent works along these lines are: Mankiw and Reis' (2001a, 2001b) hypothesis of slow dissemination of information; Ball and Croushore's (2001) hypothesis of agents forming expectations with data not completely up to date; Carroll's (2001) epidemiological model explaining deviations of household expectations from rational forecasts; Sims' (2001) model of agents with limited information-processing capacity and rational inattention - considered in Woodford's (2001b) model of monopolistically-competitive pricing with suppliers' perception of economic conditions contaminated by noise resulting from infinite-processing capacity; Orphanides and Williams' (2002) model of agents that rely on an
adaptive learning technology to form expectations - finite memory least squares learning -, resulting in perpetual learning because of the constant change of the economy structure - an approach building on the learning literature recently developed by Sargent (1999a) and Evans and Konkapohja (2001).
Chapter 3  

3.1. Introduction

Along the last thirty years, the macroeconomic performance of the Euro Area, in terms of the main policy objective variables, has changed markedly. Figure 3.1.A illustrates this idea, showing five years averages of (quarterly) inflation and unemployment gap variability around their desired targets (that is, square deviations from the desired levels), between 1972:I and 2001:II.

The unemployment gap is an updated version of the one computed in chapter 2, from an unobserved components model of stochastic NAIRU and trend GDP, featuring the Phillips and the Okun relations of the Area as measurement equations, which has been estimated by maximum likelihood using the Kalman filter. Inflation corresponds to the first differences of the log of the GDP deflator, the price index used in the gap estimation. The desired levels are assumed to be a null unemployment gap and a value of 0.5 percentage points of quarterly inflation, compatible with the usual assumption of 2 percent of annual inflation - typically used since Taylor (1993).

Figure 3.1.A shows the macroeconomic performance of the Euro Area clearly improving since the second half of the 80s, with both inflation and gap variances smaller than before, recording quite low levels and systematically decreasing further throughout 1996-2001.

1 The Phillips equation relates quarterly inflation changes to three lags of quarterly inflation changes, the current unemployment gap and the deviation of inflation in the Area from imported inflation in the previous quarter. The Okun equation relates the output gap to current period unemployment gap, according to a quadratic function. The NAIRU follows a random walk with a stochastic drift and trend output is modelled as a random walk with a constant drift. The presented charts employ the unemployment gap series given by the Kalman smoother, which uses the information of the full sample - see chapter 2 for details.
Figure 3.1 A - Macroeconomic Volatility, Euro Area 1972-2001

Figure 3.1 B - Ratios of Variability to the Absolute Value of the Average of Inflation And of the Unemployment Gap, Euro Area 1972-2001

Figure 3.1.B presents adjusted variability values - computed as the ratio of the variability measures of figure 3.1.A to the absolute value of the averages of the
corresponding variable - which allows a more robust reading. The picture is slightly different in what regards the gap variability, but fully confirms that the 1986-2001 period has been outstanding in terms of inflation performance. Adjusted inflation variability fell systematically during the whole period, but its fall has been especially large from the first to the second half of the 80s. It has recorded impressively small values since then, and has been further lowered until 2001:II.

We interpret these pictures within the framework set by Taylor (1979), who showed that macroeconomic models featuring a transitory trade-off between the levels of inflation and unemployment imply a permanent trade-off between the variability of inflation and unemployment around their desired levels. This second moments' trade-off means that when the economy suffers a supply shock, that changes inflation and the gap in opposite directions, the policymaker cannot bring both variables on track simultaneously with equal vigour. As further developed by Taylor (1993, 1994) and Fuhrer (1994, 1997a), any given monetary policy regime faces an efficiency policy frontier that represents the most favourable combinations of inflation and gap variability.\(^2\) The optimal policy frontier - sometimes called Taylor curve - is convex to the origin, its position is a function of the policy regime and of the variability of the shocks hitting the economy, and its curvature is a function of the structural behaviour of the economy, namely the Phillips elasticity. An efficient policy explores the best achievable combinations of macro stabilisation, and thus places the economy precisely on the Taylor curve, so the distance between actual macro performance and the frontier may be interpreted as a measure of the policy inefficiency. The specific point where the policymaker places the economy is a function of the relative weight attached to inflation and activity gap volatility in his loss function. This analysis is further complicated by the fact that the optimal policy frontier may change over time.\(^3\)

The history of the Euro Area economic policy since the mid-80s features, most notably, the German monetary policy leadership of the exchange rate mechanism of the

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\(^2\) See Taylor (1998 a, b, c) and Solow (1998b) for further discussion on the EPF. Studies of the empirical evidence on the variability trade-off include Debelle and Fischer (1994), Owyong (1996), Iscan and Osberg (1998), and Lee (1999).

\(^3\) Efficiency policy frontiers have been used to compare the best combinations of macro volatility achievable by alternative policy reaction functions - see, among many others, Defina et al. (1996), Black et al. (1997), and Ditmar et al. (1999). They've been also used to assess the robustness of some specific class of rules - see, for instance, Amano et al. (1999).
European monetary system, with rare exchange-rate realignments, and, since the mid-90s, a very high degree of macro and policy convergence ahead of 1999's EMU. These facts strongly suggest that an important part of the change in macroeconomic performance above described may be attributed to a policy regime change in the Area.

This chapter empirically investigates what lies behind the documented favourable shift in the volatility trade-off of the Euro Area, taking on the central hypothesis that a crucial part of the explanation lies in the emergence of a new and well-defined monetary policy regime in the aggregate Euro Area after 1986. In order to test this hypothesis, we estimate the Area policymakers' preferences, with a framework that simultaneously identifies the Area macroeconomic structure - which is also a determinant of the Taylor trade-off - focusing on the comparison of the period 1972-1985 with that of 1986-2001. Our framework, moreover, generates consistent estimates of the variation in inflation not explained by the model, and of the deviation of actual interest rates from their optimal path, which provide some information about two other factors affecting the volatility trade-off - supply shocks volatility, and monetary policy efficiency.4

**Policymakers' Preferences**

A recent literature has been attempting to estimate policymakers' preferences structural parameters. Three main streams stand out, the studies by Cecchetti *et al* (Cecchetti, McConnel and Quiros, 1999, Cecchetti and Erhmann, 1999, and Cecchetti, Flores-Lagunes and Krause, 2001), those of Favero and Rovelli (1999, 2001), and the one by Dennis (2001). While the former studies a broad cross-section of 23 OECD countries, the second and the third deal with the US case, focusing on the estimation of

4 We use official aggregate data of the Euro Area, from the Area Wide Model Database (AWMD) or computed from the AWMD, as described below. We study the Area as a whole, as our aim is to see if the aggregate data reveal any well-identified global economic structure, and policy regime, throughout a period in which (except for 10 observations at the end of the sample) nations, not the Area, were the formal economic units. We do not analyse nation-level data, so our analysis is set off from the opposite perspective of the literature that studies data from EMU members and searches for heterogeneity of monetary transmission mechanisms - a literature that has been reporting mixed results and where no clear consensus seems to exist. Recent references of this literature are, among many others, Clausen and Hayo (2002a, b), Ciccarelli and Rebucci (2002), Mihov (2001), Clements *et al.* (2001), Sala (2001), Leichter and Walsh (1999), Aksoy *et al.* (2002), and Dornbusch *et al.* (1998).
the role that a monetary policy regime change may have had in the improvement in U.S. macro-economic performance around 1980.\(^5\)

A common characteristic of all these studies is their use of the small macroeconomic model suggested by Rudebusch and Svensson (1999) to represent the dynamic structural behaviour of the macro-economy. One of the key features of this model is that it is compatible with the natural rate hypothesis.\(^6\) Moreover, its purely backward-looking aggregate-demand and aggregate-supply equations, though at odds with the functions derived from explicit micro optimisation - such as the models in Walsh (1998) - have proved to be quite successful in capturing the main properties of recent US macroeconomic data, especially the high persistence in inflation.

The distinctive feature of Cecchetti \textit{et al.} (1999), Cecchetti and Erhmann (1999), and Cecchetti \textit{et al.} (2001) is that their empirical strategies are based on fitting the second moments of the data. Their framework seems to be, however, affected by two main caveats. First, it does not estimate the policymakers' preference coefficients through formal econometric methods. Second, it does not include interest rate smoothing. Dennis (2001) showed that allowing for interest rate smoothing in a procedure that estimates the policy objective function by matching the model second moments with those of the data does not seem to work well. Specifically, he found too low estimates of the interest rate smoothing parameter, implying that the estimated policy rule fails to fit well the first moments of the nominal short-term interest rate.\(^7\)

Favero and Rovelli (1999, 2001) and Dennis (2001) frameworks are based on the estimation of the aggregate-supply/aggregate-demand system together with an

\(^5\) Rowe and Yetman (2000) have also empirically estimated the level of inflation targeted by the Bank of Canada and whether it alternatively targeted output. However, their empirical strategy seems more limited in scope, as it studies one only parameter of policymakers' preferences.

\(^6\) See Beyer and Farmer (2002) for an explanation of the macroeconomic change observed in the U.S. at 1980 - inflation, unemployment and nominal interest rate changing from upward drifting to downward drifting - developed in an alternative set-up featuring a long-run Phillips relation positively sloped, instead of the natural rate hypothesis. Their analysis is based on the co-integration properties of the data-set \{\(\pi\), \(u\), \(i\)\} and in its representation and estimation as a vector equilibrium correction model - featuring optimising IS and Phillips equations and a policy reaction function. It seems, however, to be subject to the criticism that the system equation describing monetary policy (the interest rate rule) is a reduced-form equation, in contrast to the other two equations in the system.

\(^7\) See Dennis' (2001) Appendix 2. We do not mean that second moments are not important, but only that estimation should focus on another approach. The main criteria to evaluate the quality of an estimation of policymakers' preferences is whether the optimal policy rule derived from the estimated preferences' coefficients does a good job in matching both the first and the second moments of the data.
interest rate equation describing conditions for optimality of policymaker's actions. To write those conditions, Favero and Rovelli use optimal control, while Dennis uses dynamic programming results, building on the use of inverse-control theory to interpret the policy equation of a VAR in Salemi (1995).

Favero and Rovelli (2001) augment the Rudebusch-Svensson structure with the policymaker Euler equation, that is, the first order conditions that solve the inter-temporal optimisation problem faced by the policy authority. They explicitly consider the cross-equation restrictions in the system, and jointly estimate the system with quarterly US data by GMM truncating the structural equations at four lags and the Euler equation at four leads. Their framework generates estimates of the policymaker preferences parameters - inflation target, relative weight on inflation-output stability, weight on interest rate smoothing - and of the equilibrium real interest rate. They measure the supply shocks affecting the Economy by the estimated variance of the residual of the aggregate supply equation, and measure the efficiency of policy with the volatility of actual interest rates around their estimated optimal path, given by the standard deviation of the Euler equation residual. Favero and Rovelli confirm the change in US monetary regime at the beginning of the 80s and successfully estimate the policy regime parameters, as well as - according to their assumptions - the change in policy efficiency, and in supply shocks.

Dennis (2001) uses Chow’s (1981) lagrangean method of dynamic programming to solve the infinite-horizon optimisation problem faced by the policymaker. He then takes the resulting optimal state-contingent policy reaction rule as an interest rate equation joining the dynamic constraints - aggregate-supply/aggregate-demand - in a three-equation system, which is estimated by FIML. The main advantage of Dennis' approach is the elimination of the need to truncate the policy horizon, thus letting the policymakers consider the entire forecast horizon when setting policy rates. Dennis framework builds on the use of inverse-control theory by Salemi (1995) to uncover the revealed preferences of the Federal Reserve across a number of sub-periods of a sample

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8 In Favero and Rovelli (1999) the structure of the economy is firstly estimated and then the policymaker's preferences are estimated given the aggregate-supply/demand coefficients estimates.  
9 Ozlale (2000) reports results from a similar exercise, but instead of using full information maximum likelihood, he uses the kalman filter to compute the likelihood function.
beginning in 1947 and ending in 1992. Inverse-control means, here, finding the coefficients of the central bank’s loss function, that produce the optimal feedback equation that best fits the data. Dennis confirms the essential results in the literature on policy regime changes in US recent history, and estimates the main policy regime parameters. There is, however, a discrepancy between Dennis’ and Favero and Rovelli’s estimated weights of interest rate smoothing, which deserves further discussion in section 3.4.3 below.

The Euro Area 1972-2001

In order to study the causes of the apparent improvement in the volatility trade-off of the Euro Area at the mid-80s, we use a refined and extended version of the Favero and Rovelli (2001) framework, and compare the results with the alternative one by Dennis (2001). These frameworks seem suitable, given the role that the emergence of a new monetary regime by 1986 seems to have played in the process.

Even though its observed shift in volatility parallels that of the US, the Euro Area case has important particularities that render our work potentially more complex. First, there was no formal unique European monetary policy regime until the EMU, in 1999, and, until then, the national monetary authorities operated within heterogeneous monetary policy institutional frameworks – see Cukierman (1992, chapter 19). Second, the Area time series are weighted averages of the 11 member-states forming the EMU, as published in the Area Wide Model Database (AWMD) by the ECB, and not original raw data. They may mask significant heterogeneity across the Area economies, in what regards cyclical positions, structure of the economy, shocks, efficiency of policies and policymakers preferences - see the references in footnote 4 above. Third, the output and unemployment gap series in the AWMD are subject to estimation contention - see chapter 2.

In spite of these difficulties, both the data and the history of events in the European monetary integration throughout 1979-1999 motivate our hypothesis that a new and well-identified policy regime exists de facto since the mid-80s. This is a

---

10 Dennis sets the temporal discount factor equal to 0.99, while Favero and Rovelli use 0.975. Both report the well-known difficulty in estimating this coefficient - see, inter alia, Ireland (1997).
regime associated to the leadership of the EMS Exchange Rate Mechanism by German monetary policy - which, itself, had initiated a new regime, stabilising the inflation objective at 2 percent per year, after gradually decreasing the target since the beginning of the 80s.

First, contrary to what had happened since the creation of the EMS in 1979, after April 1986 there were almost no realignments of exchange-rate parities within the system. Notably, no realignments at all existed in the 5 years between January of 1987 and the crisis of the summer of 1992, with the Basle-Nyborg agreement of September 1987, and the way it has been implemented, effectively strengthening the EMS. This exchange rate stability explains the finding that monetary policy autonomy has decreased, within the system, after 1986-87 - see, for instance, Loureiro (1996). Truly, some EMS countries had tightened their controls to international flows of capital during the turbulent initial period of the System, but these controls were effective only in periods immediately preceding realignments. As such, they were practically ineffective after 1986 - see, for instance, the differential between onshore and offshore interest rates for the franc and the lira in Gros and Thygesen (1992, page 121). Moreover, the 1987 European Single Market programme has been a crucial additional force pushing member-countries for a structural easing of foreign capital controls (on top of the monetary integration and the increasing ineffectiveness of controls due to the mounting sophistication of financial operations). In fact, since the middle of the 80s, all member-states began gradually liberalising their international capital movements with the other EEC nations. As a result, the actual level of capital controls imposed by the Community largest countries was already relatively unimportant well before the 1990 deadline for liberalisation (to which Spain, Portugal, Ireland and Greece had exceptions) - see European Commission (1997, pages 25-46 and Appendix C).

Second, there was a clear convergence of the EMU member-states inflation rates toward the German rate, between 1980 and 1986, and thereafter rates have moved broadly in tandem, with a very gradual final completion of the nominal convergence process throughout 1987-1999. To see this, figure 3.2 depicts the EMU member-states inflation rates during 1974-1999 (except for Portuguese inflation, which has been an outlier during the 70s and the 80s), with German inflation in bold - clearly showing a nominal convergence with Germany ahead of 1986. Furthermore, there has also been a
visible nominal convergence of EMU countries to Germany's equilibrium in the path of short-term interest rates. Specifically, the dispersion of the EMU members' short-term interest rates relative to the German one trended down between 1983:III (when it recorded a root mean square error of 3.5) until 1999:I (when it reached zero). This dispersion was already below 2.5 in 1986:I, and moved below the average of the 1974-1998 period (1.7) ever since 1988:II.

**Figure 3.2 - Consumer Price Index Inflation, Founding-States of EMU (Excluding Portugal) 1974:I-1999:IV**

While authors writing before the 1992-93 exchange rate crisis had already written about a New EMS - see Giavazzi and Spaventa (1990) and Gros and Thygesen (1992) - others writing in the aftermath of the crisis were less optimistic - see Loureiro (1996). Our argument is that the subsequent history of events and macroeconomic performance allows the restatement of the hypothesis, as exchange markets crisis turned out to have no significant structural consequences.

Our hypothesis of a new monetary regime, of a somewhat informal nature, after 1986, and its role of gradual anticipation of 1999's EMU, is compatible with arguments put forth previously, albeit in a different context, by McCallum (1997). The crucial point is that, in many episodes of monetary history, institutional changes lag behind
actual policy changes, which in turn reflect, with some lag, evolving social beliefs and attitudes. Muscatelli and Trecroci (2000) surveyed evidence favouring this hypothesis and Muscatelli et al. (2000) offered empirical evidence in favour of this class of arguments, related to the recent upsurge of inflation targeting regimes.

Our hypothesis is also compatible with evidence from forward-looking policy rules, and their interpretation, in Doménech et al. (2001a). First, they show that such rules can explain the behaviour of the quarterly aggregate EMU short-term interest rate after the mid-80s, but not before. Second, they show that the fall in the volatility of inflation in the Area, between 1986 and 1994, is associated to the convergence of the Area coefficient of response of interest rates to inflation, to the Bundesbank rule's coefficient. This led Doménech et al. (2001b) to study their small macroeconomic model for the Area - composed of a forward-looking policy rule and hybrid backward-forward-forward-looking IS and Phillips equations - with data beginning in 1986:I.

In a framework closer to ours, the recent evidence in De Grauwe and Piskorski (2001) seems also to reinforce our hypothesis. They simulate policy and macroeconomic outcomes for the Euro Area member-states, in two alternative systems - one of targeting weighted national data, and another of targeting aggregate Area data. Their simulations, based on a model estimated with 1984-1998 data and policymakers preferences calibrated to some alternative sets of preferences, suggest that no significant changes in loss and volatility arise from aggregate data targeting - even though their estimation period begins eight quarters before ours.11

We do not claim, however, that the policy regime that we may identify for the aggregate Euro Area between 1986 and 2001 is a good approximation to the current and future policy regime of the Area. In search for the reasons behind the volatility trade-off improvement by the mid-80s, we intend to document the Area policy regime existing before the creation of the EMU, rather than document the EMU policy regime, which will most probably be structurally different from the past.

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11 Peersman and Smets (1999) estimate optimal Taylor rules for the aggregate of Germany, France, Austria, Belgium and the Netherlands, using data ending in 1997:IV but tracking back to 1975:I, which seems less supported by the data and history of the Euro Area.
Research - both theoretical and empirical - of the object of policymakers' preferences is, undoubtedly, far from exhausted. The value we try to add, in this chapter, to ongoing research, can be summarised in four features.

First, we seek to identify the role played by the emergence of a new monetary regime of the aggregate Euro Area in the improvement of the Area volatility trade-off after 1985. In order to do so, we use frameworks recently applied to the US case, which generate formal estimates of the deep preference parameters of monetary policymakers - the central bank loss function - together with the dynamic structure of the macro-economy - which also determines the second-moments trade-off. Moreover, the methods offer some indication about the change in the volatility of supply shocks and in the degree of optimality of monetary policy - which influence, respectively, the slope of the Taylor trade-off curve, and the position of the economy relative to that efficiency frontier. By using frameworks recently applied to the US case, we render comparisons of results possible.

Second, we try to improve on the empirical strategy of Favero and Rovelli (2001), and Dennis (2001), in what regards the gap series used in the estimation. Following a standard practice in the literature, they use measures of the gap available from official sources at the time of their research - the typical alternative being some *ad-hoc* filtering of the raw latest available data. Recently, it has been shown that this practice inhibits a correct estimation of policy reaction functions - see Croushore and Stark (1999, section VI), Orphanides (2000, 2001a, 2001b) and Nelson and Nikolov (2001, 2002).\(^{12}\) The first have reconstructed US real-time data from vintages, or snapshots, of data available at quarterly intervals in real-time - an approach also used by Egginton *et al.* (2002) to build a real-time macro data set for the UK. The second and third have read the policymakers' real-time perceptions about the state of the economy out of Staff forecasts presented to the policy committee, the discussions reported in the minutes of the committee meetings, and other policymakers documented statements. Here, we can not develop any of these approaches, as we study an Area with no aggregate real-time statistical data and a notional central bank, for almost the whole 

\(^{12}\) The published version of Croushore and Stark (1999) - Croushore and Stark (2001) - does not include Section VI, relative to Taylor rules. More recently, Croushore and Stark (2002) present evidence of other type of macroeconomic research that is also not robust to different data-sets, and Stark and Croushore (2001) show how the data vintage may matter for forecasting exercises.
sample period. Instead, we use a *quasi-real-time* estimate of the gap: a series computed at each quarter with final revised data, but using information of up until that quarter only, as described in next section. We argue that this seems to be the best that can be achieved to approximate data available to policymakers in real-time, in the case of the Euro Area before 1999.

Third, we offer comparable evidence obtained from the two alternative empirical frameworks previously used to study the US case by different researchers. Comparison of the results obtained by Favero and Rovelli (2001) to Dennis' (2001), on the US case, is not straightforward, because of three main reasons. First, they calibrate the discount factor to slightly different values (0.975 and 0.99, respectively). Second, they perform estimation with somehow different samples - 1961:I-1979:II versus 1980:III-1998:III in Favero and Rovelli, and 1966:I-1979:III versus 1982:I-2000:II in Dennis. Third, they use potential output series from different sources - the Bureau of Economic Analysis of the U.S Department of Commerce, and the Congregational Budget Office, respectively. Differences of this type are not present in this chapter, so that comparing the two approaches' results is now possible, for our case. This has two advantages. On one hand, it allows for a more robust answer to the questions motivating our research, while, on the other, allows for drawing some results of a more methodological nature that can prove useful in interpreting the results - including those for the US.

Fourth, we extend the basic framework used for the US case, by formally testing the hypothesis that the policymakers' preferences in the Euro Area may have been asymmetric across recessions and expansions. The testing framework brings together two strands of the literature that have been dissociated - the estimation of policymakers' preferences and the study of asymmetric policy preferences. Specifically, we show that the Favero and Rovelli (2001) type of approach can be extended to estimate possibly asymmetric policymaker loss functions, for a given specific target of inflation. Moreover, in addition to the standard hypothesis of asymmetric reaction of policy to inflation and gaps across expansions and recessions, we test the hypothesis of asymmetry in interest rate smoothing as a function of the cyclical state of the economy.
The rest of the chapter is outlined as follows. Section 3.2 presents our structural model, describes the data and offers structural stability tests and other useful preliminary results. Section 3.3 explains the estimation framework based on optimal control, and describes the results. Section 3.4 explains the alternative framework based on dynamic programming, summarises the estimation results, and compares them to those of the previous section. Section 3.5 presents and applies our method for testing for asymmetry in the central bank loss function across recessions and expansions. Section 3.6 offers some concluding remarks.

3.2. A Model for Monetary Policy Analysis of the Euro Area

3.2.1. Aggregate Supply and Aggregate Demand

We model the structure of the Euro Area macro-economy with a simple backward-looking aggregate-supply/aggregate-demand system similar to the one applied by Rudebusch and Svensson (1999), and Rudebusch (2001a) to US data.

The model, in its original form for quarterly data, is given by:  \[ (3.1) \]

\[
\begin{align*}
x_t &= c_{x1} x_{t-1} + c_{x2} x_{t-2} - c_r \sum_{i=1}^{4} (i - \pi_{t-i}) + e^d_t \\
\pi_t &= c_{\pi1} \pi_{t-1} + c_{\pi2} \pi_{t-2} + c_{\pi3} \pi_{t-3} + c_{\pi4} \pi_{t-4} + c_x x_{t-1} + e^s_t
\end{align*}
\]

where \( x \) represents the gap, \( \pi \) is inflation, and \( i \) stands for nominal short-term interest rate. The first equation is an IS relation - representing aggregate demand -, linking the output gap to its lags, to the average of the real interest rate during the previous four quarters, and to a stochastic demand shock. The second equation is a Phillips relation - representing aggregate supply -, linking the inflation rate to its lags, to the lagged output gap and to a stochastic supply shock.  \[ (3.1) \]

Their motivation for using this model was threefold. First, tractability and transparency of results; Second, good fit to recent US data; Third, proximity to many

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13 Rudebusch and Svensson (1999), page 207.
14 See Fair (2001c) for one of the most extensive criticisms of this model, on the grounds that it is not enough empirically based.
policymakers' views about the dynamics of the economy, and to the spirit of many policy-oriented macro-econometric models, including some models used by central banks. The essence of this view is that the short-term interest rate is the policy instrument, output gaps are the main real indicator, monetary policy acts with a transmission lag, and aggregate supply is represented by a short-run Phillips curve with adaptive expectations and conforming to the natural rate hypothesis.

Our own reasons for using this model are essentially three, besides their first and third motivations.

First, it has been widely used in recent empirical studies of monetary policy rules or regimes. This includes, for the US case (besides the above cited), Orphanides (1998), Ball (1999), Favero and Rovelli (1999, 2001), Ozlale (2000), Dennis (2001), and Meyer et al (2001), and, for European countries, Peersman and Smets (1998, 1999), Taylor (1999c), Clausen and Hayo (2002 a, b), and Aksoy et al. (2002). The intensive use of the model does not mean that the problem of finding a simple macro-econometric model that effectively represents actual developed economies is solved - see the discussions in Cukierman (2001) and Fair (2001c). Rather, it is due to its reasonable theoretical and empirical properties - from which Goodhart (2000) stresses the realistic inclusion of monetary transmission lags. In addition, using it may facilitate useful comparison to results obtained elsewhere in the literature.

Second, even though most of the studies using the model relate to the US, it could be argued that the structural behaviour of the Euro Area may be broadly similar to that of the US economy, as both are large and relatively closed economies - see Rudebusch and Svensson (2002). Particularly supportive of this argument is Agresti and Mojon's (2001) finding that the business cycle of the aggregate Euro Area is highly comparable to the US business cycle in a number of respects. And, similarly, Angeloni et al.'s (2002) findings that the Euro-wide responses of output and inflation to monetary policy actions are quite close to those generally reported for the US. In fact, both Peersman and Smets (1998, 1999) and Taylor (1999c) had estimated this small model with aggregate data from a core of EMU countries, obtaining statistical fits comparable to the ones for the US. Aksoy et al. (2002) estimate the model with data of the 11 EMU states, augmenting the baseline model with an effect from the trade weighted average of
the output gap of the rest of the EMU members, to each member-state gap in its IS equation.

Third, this model is particularly appropriate in our case, because the unemployment gap series that we use has been computed within a system that features a Phillips equation similar to the one in this model - see chapter 2.

Data and model identification

Model (3.1) has been identified for US data. We now identify the specific formulation of this model that best fits our Euro Area aggregate data. The data are quarterly aggregate time-series for the Euro Area beginning in 1972:II and ending in 2001:II.

The inflation rate is the annualised GDP inflation, computed as 4 times the first differences of the log of quarterly GDP deflator. Our proxy for exogenous supply shocks is the deviation of imported inflation from domestic inflation, computed as 4 times the first difference of the log of the quarterly imports deflator of the Area minus the annualised domestic inflation rate. The nominal short-term interest rate is the quarterly average of the 3-month interest rate EURIBOR. For 1970-1998, the source is the Area Wide Model Database published in Fagan et al (2001), whilst for subsequent periods compatible updates are taken from several issues of the ECB's Monthly Bulletin.

The unemployment gap is measured in percentage points and computed as the NAIRU minus actual unemployment rate at each quarter, so that positive values correspond to expansions. It is an update of the series computed in chapter 2, from an unobserved components model featuring the Phillips (with adaptive expectations) and Okun relations as main measurement equations, estimated by maximum likelihood using the Kalman filter. The update, due to the 4 additional observations for 2000:III-2001:II, did not change the model identification nor the estimates of its main
parameters, so that the behaviour of the unsmoothed unobserved components remains fairly unchanged.\(^{16}\)

Our gap series has thus been obtained in a system with a Phillips equation similar to the one in this chapter, and is not a full-sample estimate.\(^{17}\) Rather, it is the optimal estimate of the gap at each time period, given the identified model and the available information until that period - as it is given by the kalman filter updating equations, not by the end of sample smoother.\(^{18}\) Hence, we depart from the standard practice in the literature - Favero and Rovelli (2001), Dennis (2001) - of using some estimate of the gap obtained from an official source and available at the time of research. These departures may significantly enhance the consistency of the empirical exercise. First, the consistency in the supply-side modelling may reduce the probability of spurious regression problems. Second, the use of a quasi-real-time estimate of the gap approximates the policymakers' real-time perceptions about the state of the economy that are behind their actual policy decisions and, thus, allows for a better estimation of their preferences.\(^{19}\)

Orphanides (2000, 2001 a, 2001 b, 2002) has shown that using real-time data is crucial for the \textit{ex-post} evaluation of US monetary policy. As Orphanides (2001 b, page 7) writes,

\(^{16}\) Differently, the smoothed unemployment gap would change significantly at the sample's last periods even though the parameters' estimates do not change, because of the statistical revision effect - see the discussions in chapter 2.

\(^{17}\) Ball and Tchaidze (2002) analyse the Federal Reserve policy performance in 1987-1995, and 1996-2000, estimating Taylor rules with an unemployment gap computed as actual unemployment minus an average of real-time time-varying NAIRUs, estimated by several researchers using unobserved components models with the Phillips equations as main measurement equation.

\(^{18}\) Muscatelli \emph{et al} (2000) adopt a similar approach in their estimation of forward-looking Taylor rules, concerning the gap and inflation expectation series they use. Actually, both are estimated using a structural time-series approach and the kalman filter, implying that each period observations use data relative to the past and present only, and not full sample information, which is closer to the information available to policymakers when deciding interest rates.

\(^{19}\) Assuming that the policymaker uses our trend-cycle decomposition model and limits its information to the series in the model, there are essentially two differences between our quasi-real time estimates and strict real-time estimates. First, real-time estimates are published with a lag and are provisional, that is, they are subject to subsequent revisions - which Dynan and Elmendorf (2001), for instance, have shown to generate quite significant changes in the US output data, especially around troughs. Second, real-time estimates may be affected by changes in the model identification and/or parameter estimates. Hence, the estimate for any quarter may be revised whenever new data is available and the model is re-estimated. As discussed in the text, the only way of unravelling the true policymakers' real-time perceptions would be to analyse the information in data actually used in policy committee meetings, which is not possible in our research.
"(...) this practice [of relying on ex-post constructed data as proxies for the information available to policymakers] can lead to misleading descriptions of historical policy and obscure the behaviour suggested by information available to policymakers in real time. (...) the main difficulty arises from the fact that monetary policy decisions are based and reflect policymaker perceptions of the state of the economy at the time policy is made."

The estimated policy reaction functions in Orphanides (2001 b, 2002) suggest that US monetary policy during the 70s great inflation was not significantly different from the 80-90s long boom policy, in what regards the response of interest rates to inflation forecasts. Rather, the 70s Great Inflation seems to have been the result of a too activist response of policy to real-time potential output measures that were known to be over optimist only much later.\textsuperscript{20}

For the UK case, Nelson and Nikolov (2001) develop an approach similar to Orphanides' to infer real-time policymakers' perceptions of the state of the economy. They show that the UK's Great Inflation has been caused, in part, by an overvaluation of potential output in real-time during the 70s - like in the US case, because policymakers failed to correctly evaluate in real-time the negative trend productivity shocks of the late 60s and early 70s.\textsuperscript{21} Then, in Nelson and Nikolov (2002), they show that the full explanation combines the real-time gap measurement error with policymakers behaving with monetary policy neglect - the latter explaining a larger part of the rise in inflation during the 70s.\textsuperscript{22}

Orphanides uses as real-time data the forecasts included in the Greenbook by the Fed Board Staff, for each Fed Open Market Committee meeting, prepared during the middle month of each quarter. He complements this information with the estimates of potential output prepared by the President's Council of Economic Advisors, which he

\textsuperscript{20} Orphanides (2001 b) pages 7-8, 12-13, 17-22. Orphanides (2001 b, footnote 13, page 14) suggests that this misperception of the change in the path of trend productivity may have been a widespread phenomenon and thus explain the generalised increase in inflation in most developed countries in the 70s and its decline during the 80s.

\textsuperscript{21} Note that the UK's Great Inflation is longer than the US, having been identified as early as in 1965 (the year of the first documented reference to "stagflation") and having a last episode still as recently as in 1990 - see Nelson and Nikolov (2002).

\textsuperscript{22} Their concept of monetary policy neglect means, in short, that because of a widespread believe that inflation was a cost-push phenomenon and that monetary policy would be ineffective in controlling it, policy featured a too low response of interest rates to inflation and a too large response to (misperceived) gaps. Moreover, monetary policy tended to target an average real interest rate below the natural interest rate - which Nelson and Nikolov modelled as an intercept misalignment of the UK's policy rule during most of the 70s.
argues were treated as data by the Fed Board Staff until 1980. In Orphanides (2002), he shows that a one-sided moving average of real-time unemployment data would generate compatible unemployment gap real-time estimates. Perez (2001) confirmed Orphanides' result, estimating rules with data from the Greenbook combined with the real-time data set constructed by Croushore and Stark (1999). Remarkably, this finding contradicts those in Clarida et al (2000) and Taylor (1999 a), who, differently, use ex-post data. However, Mehra (2001) finds that during the 70s the US Fed does seem to have violated the Taylor principle, with estimation using solely the real-time data set of Croushore and Stark (1999). In this chapter, we can not adopt approaches similar to those of Orphanides, Perez, Croushore-Stark, Mehra, Nelson and Nikolov, and Egginton et al, so we adopt a time-series approach, motivated by our use of the kalman filter to estimate the gap.

Figure 3.3 shows the significant differences between the ex-post (smoothed) gap and our quasi-real time (unsmoothed) series. That difference is the quasi-real time gap measurement error, or uncertainty, faced by policymakers. The volatility and persistence properties of our quasi-real time uncertainty series are in line with the range of values in Orphanides et al. (2000) and Rudebusch (2001a, 2002b) for their real-time measurement errors. Specifically, a standard error of 0.41 percentage points (versus 0.5 to 1 percentage point in Orphanides and Rudebusch), and a first-order auto-regressive root estimated at 0.94 (versus 0.75 to 0.95). Our quasi-real time measurement error is also compatible with the uncertainty around the NAIRU estimates obtained by Monte Carlo integration in chapter 2 - average root of mean square error of 0.6 percentage points. Hence, we believe that our measure approaches fairly well the real-time measure that, unfortunately, we can not know.

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23 See Taylor (2000) for a criticism of this procedure. Note that no reconstruction of real-time data can be immune to criticism.

24 Coenen et al. (2001) study the profile of revision of the main macroeconomic variables in the Euro Area during 1999 and 2000, using the numbers published in the ECB Monthly Bulletin. As each bulletin edition uses information available up to some days before a Governing Council meeting, they consider the first publication of a variable number for a specific month/quarter as its first provisional estimate and, thus, its real-time estimate available to policymakers. Their method is feasible, and should be highly helpful in future research on EMU policymaking with real-time data.
Preliminary individual least squares regressions and full information maximum likelihood estimation of the system suggest - on the basis of individual significance statistics - that the specification that best adjusts the Rudebusch-Svensson model to our quarterly Euro Area data is the following:\textsuperscript{25}

\[
\begin{align*}
    x_t &= c_0 + c_1 x_{t-1} + c_2 x_{t-2} + c_3 x_{t-3} + c_4 \left( i_{t-3} - \pi_{t-3} \right) + e_t^d \\
    \pi_t &= c_5 \pi_{t-1} + c_6 \pi_{t-2} + c_7 \pi_{t-3} + c_8 \pi_{t-4} + c_9 x_t + c_{10} \left( \text{Im} \, \pi_{t-1} - \pi_{t-1} \right) + e_t^s
\end{align*}
\]

\[(3.2)\]

Our model differs from the original Rudebusch-Svensson formulation in several details. First, in the Phillips equation, the unemployment gap explains inflation contemporaneously, not lagged. Second, in that equation, we consider a supply shock measured by the deviation of imported inflation from domestic inflation, in the previous period. This exogenous shock is included for compatibility with the Phillips equation used in chapter 2 to estimate the time-varying NAIRU, and means that our version of the Rudebusch-Svensson model is not one of a completely closed economy, differently

\textsuperscript{25} Differently, Taylor (1999 c) and Peersman and Smets (1999) estimate the specification identified by Rudebusch and Svensson (1999) for the US case, using weighted data of selected European countries.
from the versions typically used for US studies. Third, in the IS equation, we find that it takes 3 quarters for the real interest rate changes to begin impacting on the gap, and that no statistical gains are obtained by considering a moving average of real interest rates. Fourth, in that equation, we find that the gap, lagged by three periods, is statistically significant, meaning that our gap measure has more persistence than the measures of the US gap used by Rudebusch and Svensson.

The real interest rate effect, in the IS equation, is clearly statistically significant, so we have no need to extend it with financial variables, contrary to what Goodhart and Hofmann (2000) find in their developed countries sample.26

Interestingly, our identification implies a pattern of transmission of monetary actions to the gap that is similar to that in Peersman and Smets (2001b), in spite of the marked differences in empirical method and data.27 It is also compatible with Angeloni et al.'s (2002) extensive reading of the evidence on the Euro Area transmission of monetary policy. Specifically, interest rate changes affect output temporarily, with effects peaking at more or less one year, while inflation hardly moves during the first year and, then, gradually falls over the subsequent few years.28

**Structural Stability Tests**

One potential problem with this model is that, in theory, it is subject to the Lucas (1976) critique. In fact, if the true dynamic behaviour of inflation and the gap includes forward-looking elements - as dynamic general equilibrium analysis and the New

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26 More recently, Goodhart and Hofmann (2002) estimate backward-looking IS equations for the G7 countries including several detrended financial variables, in addition to the real interest rate, namely real exchange rates, real house prices and real equity prices. They find all these statistically significant and argue that the optimal monetary policy rule given by a model without property and equity prices in its IS function yields significantly higher loss than would be obtained from an IS equation including those financial variables, if the true model does include them.

27 Peersman and Smets (2001b) estimate identified VARs with quarterly series of the AWMD, from 1980 to 1998. Their endogenous variables are the levels of real GDP, consumer price index, 3 month nominal interest rates, M3 and of the real effective exchange rate (all in logs except for interest rates). The exogenous variables they consider are a world commodity price index, US real GDP and the US short-term interest rate. Note that actual real output is used instead of an output gap measure, and that it is the log of the price level that is used rather than its changes.

28 See McAdam and Morgan (2001) and van Els et al. (2001), *inter alia*, for overviews and experiments with the transmission of monetary policy actions in the Euro Area within large-scale macro-econometric models.
Keynesian theory prescribes -, the reduced form coefficients of this backward-looking model are not stable when the monetary policy rule changes.

Simulations in Lindé (2000) suggest that the Lucas critique may be quantitatively important for the Rudebusch-Svensson model, both statistically and economically. He generated artificial data from a version of Cooley and Hansen (1995) real business cycle model, augmented by four alternative nominal money growth policy rules, and found that changing the rule at the middle of the data-set caused the model coefficients to change with higher probability than usual significance levels.29

In complete contrast, Rudebusch (2002a) simulations suggest that the empirical relevance of the Lucas critique is not significant. He generated data from several New Keynesian models, with varying degrees of forward-looking behaviour and discrete shifts between alternative policy rules calibrated to mimic estimates typical of the US post-war literature, finding that the model reduced form coefficients would only appear unstable in rather extreme and unrealistic parameterisations. Estrella and Fuhrer (1999) had previously offered estimation results pointing to the same conclusion. With US quarterly data for 1966-1997, they estimated the Rudebusch-Svensson model, a forward-looking New Keynesian model with Roberts' (1995) Phillips equation and McCallum and Nelson's (1999b) IS equation, and, finally, a similar forward-looking model with persistent errors - closing all the models with a similar Taylor rule equation. Then, they computed likelihood ratio tests for structural instability of unknown timing of the system, failing to reject stability of the backward-looking model, while rejecting it in both versions of the forward-looking models.

As Estrella and Fuhrer (1999) note, since the Lucas critique is an empirical matter, every model should be tested for stability before being used for policy analysis. Following this advice, we put our model to test for structural stability over the entire

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29 This does not imply that purely forward-looking, optimising, IS-Phillips models should be preferred in actual empirical studies. In fact, Lindé (2001b) sheds some light on this issue, by estimating the three possible alternative versions of the IS-Phillips model - entirely backward-looking, entirely forward-looking and hybrid backward-forward-looking - using data simulated from a calibrated dynamic general equilibrium model with flexible prices and completely forward-looking properties. His main findings are twofold. First, GMM estimation of the system can actually detect that forward-looking behaviour is more important than backward-looking behaviour (weighting 0.7 in both IS and Phillips equations), in contrast to the empirical results from US data, which attribute more weight to backward-looking behaviour. Second, structural instability due to policy regime changes is not easier to detect in the forward-looking
sample 1972:1-2001:II. We skip stability tests for each equation individually and analyse the structural stability of our model when estimated by full information maximum likelihood. One reason is that a system approach to stability analysis improves the econometric ability of detecting breaks. A second reason is that we'll be using the system and not individual equations in our estimation of policymakers' preferences.\textsuperscript{30}

We analyse structural stability with the Andrews and Fair (1988, equation 3.6, page 623) Wald statistic for testing pure structural change - that is, a significant change in all the coefficients in the model.\textsuperscript{31} Because we do not have any clear \textit{a-priori} about the timing of possible structural breaks, the analysis is placed within the Andrews (1993) framework for testing parameter instability with unknown change point. Specifically, the relevant statistic is Andrews' (1993, equations 4.1 and 4.2, page 835)

\[
\sup_{\pi \in \Pi} W_T (\pi)
\]

where $\Pi$ is a set with closure in (0,1). Following Andrews' suggestion, which is well suited for our sample size and number of coefficients, we set the trimming at 15 percent, thus defining $\Pi = (0.15,0.85)$, which amounts to give up 16 observations at each end of the sample. Actually, the 16 observations trim is the minimum possible, because we are estimating 12 parameters and the model is estimated conditional on 4 lagged observations of inflation (and 3 of the gap). Accordingly, we compute the sequence of the Wald statistic as function of all possible break-dates within this interval and compare the sup W statistic with the critical value given in Andrews (1993, table I, page 840). We have checked that the regressions are globally statistically significant for both sub-samples near the trimming points, in order to make sure that relevant regressions were being compared.

\textsuperscript{30} Stability analysis in US applications of the model have typically been conducted with single equation least squares estimation - see Rudebusch's (2001a) and Rudebusch and Svensson's (1999, 2002).

\textsuperscript{31} On this statistic, see also Hamilton (1994, pages 424-426) and Greene (2001, pages 292-293). As is well-known, in spite of the asymptotic equivalence between this statistic and the alternatives (likelihood-ratio and lagrange multiplier), in finite samples the Wald statistic is typically larger, which means that it is the most severe test statistic of this class, that is, tends to reject the null more often.
Figure 3.4.A displays (in log scale) the Andrews and Fair (1988) Wald statistic sequence for testing a pure break in the model, together with the adequate Andrews (1993) critical value (for a model with 12 coefficients, 15 percent trimming, and 5 percent of significance). The figure shows that the supW statistic has a value around 898 at 1997:II, which is remarkably above the 30.16 Andrews (1993) critical value. Thus, there is strong empirical evidence to reject the null hypothesis of no structural change in our IS/Phillips model for the period 1972-2001.

This result is in sharp contrast with the failure to reject stability typically obtained in the US case - see Rudebusch (2001a), Rudebusch and Svensson (2002) and, with mixed forward-backward-looking equations, Rudebusch (2002b). However, these studies perform tests covering breakpoints only until the end of 1992, at most, so they could be missing more recent structural breaks. For instance, with our data, the test statistic only begins exceeding the critical value in 1993:III.

In order to assess whether the structural break harms our hypothesis of a new monetary regime from 1986 on, we proceed to the estimation of the break-date. Following Hansen (2001), we estimate the timing of the structural break by least squares, because of the limitations of the Wald statistic for that estimation - see Hansen (2001, section 3). Specifically, we take as an estimate of the break-date the sample split that minimises the sum of square residuals (over the middle 70 percent of the sample), that is, the split that minimises the sum of the variance of sub-sample 1 and 2 residuals. Figure 3.4.B depicts such variances for the Phillips and the IS equation, when they are jointly estimated by FIML. The residual variances are minimised at the same quarter for both equations - 1995:II - which is clearly due to the interactions between aggregate demand and supply in the model. Figures 3.4.A and 3.4.B indicate that, if we restrict our analysis to the 1977-1995 period in search of another significant structural break in the model, the best candidate seems to be 1983. Hence, the stability tests results are not incompatible with the hypothesis of a stable monetary regime beginning in 1986, but suggest that the structural behaviour of the Euro Area economy changed significantly in 1995.32

32 We have checked whether the results are quantitatively different if we specify the IS function considering the average of the real interest rate during the four previous quarters, instead of the real rate lagged by three quarters. This formulation - the original Rudebusch-Svensson IS model devised for US data - would probably be more robust to some changes in the lag structure of the effects of the real
interest rate over spending throughout the sample. The results were, however, fairly similar to those above reported in the text and figures: there is evidence of a structural break, which is estimated at 1995:1 for the IS and 1996:III for the Phillips equation.
The structural break in 1995 is perhaps associated to changes in the Area economy ahead of the 1999 EMU. It is hardly surprising that these changes are statistically significant around three and a half years in advance, in view of the well-known nominal convergence process and, also, as it seems sensible that such deep changes are anticipated and gradual.

It could be argued that in view of the stability tests results, the estimates should be restricted to the period 1986-1995. However, we estimate the new monetary regime with 1986-2001 data. We do so for two main reasons. First, because of data scarcity: 34 quarters of data (the 38 quarters of 1986:I-1995:II, minus 4 quarters needed for the model initialisation), covering only one business cycle, would hardly offer comfortable degrees of freedom in estimation. Second, because we believe that every regime switch is gradual. At times, in our empirical analysis, we check these arguments, comparing the results that would come out of estimation with data truncated at 1995:II, with those of our baseline period 1986:I-2001:II. As discussed below, estimation is very problematic most of those times, apparently because of insufficiency of data, but when possible, generates results that are not dramatically different. What our results suggest, instead, is that the estimation of the EMU policy regime may combine post 1999 data with data beginning already in 1995:III - which may be useful for future research, in view of the data scarcity problem. In this regard, the use of monthly data may be especially useful in future research focusing on the EMU policy regime, to enhance the degrees of freedom in its estimation. Monthly data would also be suitable in view of the twice-a-month periodicity of the ECB’s Governing Council, as it could lead to several interest rate changes within some quarters.

Preliminary Monetary Regime Evidence: Taylor Rules

As a first approximation to our problem, we report in Appendix 1 the results of estimation of a forward-looking version of the Taylor rule of the kind suggested by Clarida et al. (1998), with the Euro Area data for 1972:I-2001:II.
The results in table A3.2 clearly show that there is a structural break in the Area forward-looking Taylor rule at 1985:IV. They also show that actual monetary policy can be well described by such a Taylor rule during 1986:I-2001:II, but not before. The results are illustrated in figures A3.1-A3.3.

The estimated rule for the period since 1986:I has a coefficient of 2.37 on one-year-ahead expected inflation and a partial adjustment coefficient of 0.77. This rule suggests that the 1986:I-2001:II policy regime in the Euro Area has been one of strict inflation targeting, as the coefficient on the gap is not statistically significant. However, if past unemployment gaps are excluded from the instrument set for GMM estimation, a significant coefficient on the current gap would be estimated in the Taylor rule, meaning that the unemployment gap, while not being a final goal of policy, does bring useful information for policymaking. The estimated rule suggests, also, that the notional monetary policy of the Area, during 1986:I-2001:II had an element of interest rate smoothing, as the estimated partial adjustment coefficient is high and statistically significant. Whether the (typically observed) partial adjustment in interest rates has any economic content, or, instead, merely reflects econometric problems such as error-in-variables or omitted variable problems, is still an open issue - which will also be of relevance below in this chapter, when discussing the specification of the central bank loss function.

Figure A.3.4 shows that, if the 1986-2001 policy rule had been followed during 1972-1985, actual short-term interest rates would have been much higher than they actually were, especially in the last seven years of the 70s.

The Appendix shows that, for the case of the period 1986-2001 in the Euro Area, the Taylor rule framework does not estimate precisely the inflation target, and illustrates the inability of the framework to estimate the policymakers' structural

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33 We include a Andrews and Fair (1988) Wald test for a structural break in the rule. Fair (2000, 2001a) criticises the Clarida et al. (1998, 2000) type of empirical strategy for not offering formal tests of the rule structural stability, arguing that the changes in coefficients found in the US case after 1982, though economically relevant, are not statistically significant.

34 In addition to the political and macroeconomic arguments, operational factors too imply that it would be highly implausible that a clear monetary regime is apparent in the data before the 80s. In fact, by then German monetary policy was based in three tools other than short-term interest rates - reserve requirements, discount-window quotas and the Lombard rate - see Bernanke and Mihov (1997). Hence, empirical studies that assume short-term interest rates as the policy instrument and indicator are not at all likely to detect any significant policy regime prior to the 1980s.
preference parameters - see also the discussion in chapter 1, section 1.3.3.. In order to accomplish that task, we must include the central bank loss function in our model and write it in an empirically suited form, which is what we now proceed to do.

### 3.2.2. Central Bank Loss Function

Following the standard assumption in the literature, we model central bank (policymakers) preferences with an inter-temporal loss function that is quadratic on the variability of inflation and the gap around their desired levels ($\pi^*$ and 0), as well as of the change in interest rates, with future values discounted at rate $\delta$.\[ L = E_t \sum_{i=0}^{\infty} \delta^i \left[ (\pi_{t+i} - \pi^*)^2 + \lambda (\pi_{t+i} - \pi^*)^2 + \mu (\pi_{t+i} - \pi^*)^2 \right] \] (3.3)

**Flexible Inflation Targeting**

The inclusion of the gap variability in $L$ is a broad formulation that is generally considered compatible with the statutes of most modern central banks, such as the US Fed, which have a priority commitment to price stability but also some second order objective about growth and employment.\

Svensson (2001c) argues that even formal Inflation Targeting regimes are dual, because once the inflation target is chosen, policy minimises the deviations of inflation and output from their targets.

However, the ECB statutes, similarly to the Bundesbank's, are not entirely clear on the significance that real activity stabilisation has in its legal mandate, as they merely state that (Chapter II, article 2º)

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\[35\] We assume that the relevant arguments in the Euro Area policymaker loss function are variables of the aggregate Area, deliberately disregarding the possibility that nation-level economic performance might affect the decisions of some or all members of the Governing Council (GC). Meade and Sheets (2002) study the decisions of the ECB GC between March 1999 and August 2001 and find evidence not incompatible with a regional bias hypothesis. Aksoy et al. (2002) study the effects of different policy decisions procedures by the ECB's GC. They simulate four procedures, combining Area-wide versus national-level concerns by ECB Board members and Governors of national central banks: a nationalistic rule, a consensual procedure, an ECB-rule, and an EMS-rule. Investigating this topic is, however, beyond the scope of this chapter.

\[36\] We do not explore Walsh (2001b) suggestion that what monetary authorities - at least the US Fed - monitor is the growth in demand relative to growth in potential, which corresponds not to the gap but to its changes.
“[...] the primary objective of the ESCB shall be to maintain price stability. Without prejudice to the objective of price stability it shall support the general economic policies in the Community [...]”

Moreover, the ECB Governing Council, when announcing its stability-oriented monetary policy strategy, clarified that (ECB, 1998, article 2º)

"As mandated by the Treaty establishing the European Community, the maintenance of price stability will be the primary objective of the ESCB. Therefore, the ESCB's monetary policy strategy will focus strictly on this objective."

This led some authors – for instance, Goodhart (1998) – to argue that output is not supposed to enter the true ECB objective function.

However, Goodhart recognises that there are two good reasons for the gap to enter any central bank's revealed preferences. First, current inflation and gap are the critical variables for forecasting future inflation – which is the actually targeted variable in contemporary policy regimes, due to the lags in policy implementation and transmission. Second, because of the variability trade-off, central banks must react to supply shocks with an eye on output, avoiding too fast a reversion of inflation to its target, as the lagged effects of a sharp monetary policy can lead to excessive output and instrument instability - see Goodhart (2000). A third reason for the unemployment (or output) gap to enter specifically the ECB's loss function derives from the finding by Begg et al. (2002) that cyclical output seems to lead cyclical inflation, in the Euro Area, by about 2 to 3 quarters.

On the contrary, there are relevant reasons that may justify that policymakers, in general, concentrate their forecasting and monitoring efforts in inflation. First, there certainly are difficulties in the estimation of output or unemployment gaps, implying very large uncertainty surrounding contemporary real gap estimates - see chapter 2 and the references therein. This is the reason behind McCallum's (2001a) argument that monetary policy should not respond strongly to output gaps. Second, empirical evidence suggests that official output growth forecasts are significantly less accurate than inflation forecasts produced by the same sources - see the recent evidence for 13 European countries in Oller and Barot (2000).

37 In the policy reaction function's literature there are arguments contrasting with Goodhart's views. For instance, Batini and Haldane (1999) offer evidence suggesting that feed-back policy rules based on inflation forecasts are output-encompassing.
All summed-up, we choose to specify our baseline loss, L, according to a flexible inflation targeting regime (Svensson, 1997), because it nests the strict inflation targeting case - King's (1997) inflation nutter, minimising inflation volatility and not the gap variance, considered unrealistic by Svensson (2001b, c). Hence, we let our empirical evidence freely discriminate which of these systems fits better the revealed preferences of the Euro Area notional policymaker during 1986-2001.38

**Interest Rate Smoothing**

Theoretical central bank loss functions assume that inflation and some activity gap are the only final goals of policy - see Cukierman (1992) and Walsh (1998). However, optimal interest rates simulated from models with such loss functions are substantially more volatile than actual short-term interest rates. Hence, following the standard practice in the literature, we consider a loss function in which the policymaker also dislikes variations in the policy instrument, the nominal short-term interest rates. This accounts for the well documented fact that central banks change interest rates in small discrete steps in the same direction over extensive periods, and reverse the path of rates only infrequently - see, for instance, Rudebusch (1995), Goodhart (1996, 1998), Lowe and Ellis (1998) and Sack and Wieland (2000). And accounts for the evidence that, moreover, the persistence in rates surpasses the serial correlation in the typical policy goal variables (inflation and gap) - see Sack (2000). Similarly, the policy rules literature shows that allowing for partial adjustment in the interest rate is a necessary condition for a good fit of forward-looking estimated policy reaction rules - see Clarida and Gertler (1997) and Clarida et al. (1998).39

A large research effort has been devoted to devising the foundations of interest rate smoothing, and the literature offers five explanations for partial adjustment of interest rates by optimising central banks - see the reviews by Lowe and Ellis (1998), Sack and Wieland (2000), Srour (2001), and Cobham (2001).38

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38 Throughout this chapter we use the expression inflation targeting in a broad sense, meaning that monetary policy is clearly committed to a goal of price stability, from which it does not deviate because of other possibly existing goals. Neither the Bundesbank nor the ECB, at the time we write, could be considered inflation targeters in the strict institutional sense defined, for instance, in Svensson (2001b), most especially because of failure to meet the transparency and accountability criteria.
First, central banks smooth rates in order to promote financial stability. According to this explanation, central banks are concerned not only with macroeconomic management (inflation and activity stabilisation), but also with a third final policy goal - the stability of the financial system - and must trade-off both - see Cukierman (1992), Rudebusch (1995), Goodhart (1996), Lowe and Ellis (1998) and Mishkin (1999). Frequent, abrupt or erratic changes in interest rates would harm banks profits and generate additional volatility in securities markets, both of which are very sensitive to interest rates changes. Hence, the probability of a financial crisis is minimised, cæteris paribus, by interest rates smoothing.

Second, smoothing short-term rates is the best way to achieve final goals of macro policy, because of the environment of forward-looking agents, as firstly suggested by Goodfriend (1991) and developed by Woodford (1999) - see also Levin et al (1999), Goodhart (1996, 1998) and Sack and Wieland (2000). If agents are forward-looking in a manner that output and prices do not react to changes in day-to-day rates, but only to changes in longer term rates (short and medium term rates), then the monetary authority should not change short-term rates frequently. A policy moving interest rates gradually induces, at each rate change, expectations of additional future interest rate movements, and that allows agents to anticipate the path of longer-term rates, thus affecting their demand decisions. Then, the authority ends up performing better in terms of stabilising output and prices, while maintaining a low level of volatility of the short-term interest rate. This is one of the reasons behind Goodhart (1998) argument that central bankers should try to minimise the number of reversals of short-term rates. Lowe and Ellis (1998) note also that too frequent directional changes in rates would render ineffective the announcement impact of monetary policy actions.

A third reason for instrument changes to be smooth is that policymakers face three important uncertainties when conducting policy - data, parameter and model uncertainty. From these, data and model specification uncertainties can not be

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39 Taylor (1999a) does not include partial adjustment in his rules because a moving average of recent past inflation is part of the explanatory variables.

40 Chapter 7, pages 117-135. Cukierman also offers an alternative explanation (the optimal seigniorage hypothesis), where interest rates smoothing are seen as a by-product of a policy attempt to optimise seigniorage and other taxes in a way that minimises the present value of the social costs of financing public expenditures.
summarised by a single probability distribution, thus being more complex than parameter uncertainty - see Cagliarini and Heath (2000).

Gradualism may be a reaction of policymakers to data uncertainty, i.e., in the perception that policymakers have about the state of the economy in real-time. This uncertainty may be due to publication lags, data revisions and statistical uncertainty in estimating potential output (natural rate), which is unobservable - see, among many others, Orphanides (1998, 2001a), Orphanides et al (2000), Smets (1999), and Dynan and Elmendorf (2001). However, the extent to which data uncertainty explains instrument smoothing is still unclear, as it has not been possible to disentangle the real-time measurement error of the policymaker - the conscience of which would generate intentional interest rate smoothing - from that of the econometrician. Uncertainty about the state of the economy is, perhaps, more clearly an explanation for intentional policy gradualism if one has in mind that policy should react to the forecasts of output and inflation and not merely to the actual cyclical situation of the economy - Goodhart (1996, 1998). These forecasts, formulated some quarters ahead, are inevitably smoother than actual data turns out to be, as ex-post data includes the effect of shocks that can not be forecasted. Hence, as Goodhart (2000) argues, interest rate data is more persistent than actual inflation and gap data, because forecasters underestimate the extent of upward/downward pressure on inflation at the start of upswings/downswings.42

In what regards the structure of the economy, many have argued that gradualism is an optimal reaction to uncertainty about parameters and the monetary transmission mechanism - see, for instance, Estrella and Mishkin (1998) and Batini et al. (1999). In fact, Goodhart (1998) considers parameter uncertainty - known also as Brainard, or

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41 One alternative to interest-rate smoothing as a response to uncertainty about the state of the Economy, based on non-linear revisions of the NAIRU estimate, has been recently suggested by Meyer et al (2001). Coenen et al. (2001) suggest a different approach, based on the possible informational role of money. They apply their approach within the Coenen and Wieland (2000) model of the Euro Area, augmented with a Coenen and Vega (2001) demand for money and an empirically calibrated model of the revision process of aggregate Euro Area output.

42 A tentative corollary to this reasoning is that we could use a loss function with no imposed rates inertia, in empirical analysis, if we had access to the real-time series of inflation and gap expectations that policymakers used when deciding policy. However, the controversy on the method of reconstruction of real-time gaps for the US - see Taylor (2000), Mehra (2001), and Perez (2001) - suggests that we may be a long way from achieving any efficient reconstruction of policymakers' true real-time data. Actually, English et al. (2002) estimate backward and forward-looking Taylor-type rules with US data available in real-time to policymakers (Croushore and Stark, as well as Orphanides' data) and find that the partial adjustment coefficient estimate keeps being positive and significant.
multiplicative uncertainty - the central explanation for why interest rates are changed in small steps, especially when coupled with long and variable lags of policy transmission. Empirical studies typically do find parameter uncertainty to significantly increase policy gradualism. Most studies, however, suggest that this source of uncertainty may have only modest empirical effects, when considered by itself - Debelle and Cagliarini (2000), Peersman and Smets (1999) and Rudebusch (2001a) - or even when combined with data uncertainty - Sack (2000).

Blinder (1998) emphasises that the policymaker is typically uncertain about the general specification of the model describing the economy and the transmission of policy, and suggests that simulations of several alternative models should be averaged out to - at least roughly - account for that uncertainty. Cagliarini and Heath (2000) formalise an approach to optimal control in which the policymaker wishes to compare alternatives under all probability distributions in order to make their decisions, and then use a sensible rule to choose the specific interest rate value within the selected interval. They report simulations with a model close to Rudebusch-Svensson's showing that their approach achieves a good replication of the smoothness in actual interest rates. Recent empirical work in a similar spirit, by Favero and Milani (2001) and Castelnuovo and Surico (2001), is suggestive that model uncertainty could indeed be an important reason for the actual interest rate inertia observed in the US case.

A fourth reason for the existence of interest rate smoothing, is Goodhart's (1996, 1998) argument that central bankers avoid interest rate changes that they might have to

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43 Brainard, multiplicative or coefficient (parameter) uncertainty is the uncertainty beyond that inherent to the additive errors (shocks) in model equations, which is the sample uncertainty. With shocks uncertainty only, certainty-equivalence holds, that is, policy should be conducted based on the best forecast for the shocks, thus using the model coefficients point estimates. In contrast, with parameter uncertainty - uncertainty about the specific parameter values, assuming, however, that they are draw from a known probability distribution - it is known since Brainard (1967) that certainty-equivalence does not hold and gradualism becomes optimal. In short, in a context of parameter uncertainty, large reactions of interest rates to inflation shocks increase the variance of inflation, so that policymakers must trade-off between immediate correction of the inflation deviation from target (bias) and a good performance in controlling its (long-run) variability - recall that \(E(\pi^2) = [E(\pi)]^2 + \text{var}(\pi)\). Hence, policymakers do not immediately offset entirely the shocks, but leave some residual consequences of shocks to be dealt with during next periods.

44 The results of a literature based on the minimax approach to robust control have been the opposite: policies that are robust (in that sense) to general specification model uncertainty tend to be more aggressive - see, for instance, Sargent (1999b), and Onatski and Stock (2000). However, Blinder's (1998) approach seems to be closer to how actual policymakers in the real world deal with model uncertainty, than the minimax approach - see Cagliarini and Heath (2000). Tetlow and Muehlen (2002) recently
reverse in some near future because that would be perceived by the public as evidence of inconsistency, error or irresolution. Mishkin (1999), Caplin and Leahy (1997), and Lowe and Ellis (1998) also join this argument. In a world of uncertain forecasts, central banks tend not to change interest rates - especially to raise them - until evidence of actual, rather than predicted, shifts in the inflationary pressure are in the public domain. This explains the evidence in Goodhart (1996) that short-term rates do not lead inflation in recent data of a group of developed countries.

A fifth reason pointed out in the literature is Rudebusch (2001b) argument that, in an era of low inflation, small interest rate changes make it less likely that the zero bound of nominal interest rates is reached.\textsuperscript{45}

In spite of the several arguments above, there seems to be no clear consensus in the literature, so far, about the meaning of interest rate smoothing. Specifically, it remains an open issue to know if the evidence of partial adjustment of interest rates has a structural economic meaning or whether it is merely a sign of econometric problems - see Goodhart (1996, 1998), Ball (1999), Rudebusch (2001a, 2001b) and Svensson (2001b). Some have argued that the serial correlation of short-term rates could be due to the omission of a persistent variable that influences policy - Gerlach and Schnabel (2000). Others, like Rudebusch (2001a), have argued that it could be the result of persistent measurement errors in the state variables - in which case smoothing may be an illusion created, for instance, by the use of final data in empirical research, instead of the real-time data used by policymakers. For instance, Mehra (2001) finds no significant interest rate smoothing in Taylor-type rules estimated with US 1979:III-1987:IV real-time data.\textsuperscript{46} And simulations by Lansing (2002), from a small model calibrated with US data properties, suggest that failure to account for the measurement error in the Fed's real-time perceptions about potential output can explain as much as half of the apparent degree of inertia in the US federal funds rate. However, other researchers as Orphanides review certainty-equivalence, versus robust and bayesian control, to conclude that these sophisticated techniques do not necessarily improve the policymaker's performance.

\textsuperscript{45} Rudebusch (2001a) finds evidence that each of the analysed uncertainties - model structure, coefficients, and data - can not, by itself, fully explain the reduced and inertial response of interest rates to the economy state, but some combinations of those uncertainties could replicate the stodginess observed in historical rules. Data uncertainty seems to be a common element to all those combinations.

\textsuperscript{46} However, three features of his Taylor-type rule are unorthodox: i) interest rates respond to lagged (not expected future) inflation and gaps; ii) rates also react to a real bond rate; iii) all interest rates data are averages of the first month of each quarter.
(2001a) and Perez (2001), estimate Taylor rules with US data available in real-time at Federal Open Market Committee meetings, and still find large and significant partial adjustment coefficients. Rudebusch (2001b) noted the possible confusion between partial adjustment and non-inertial rules with serially correlated shocks caused by error-in-variables problems. In order to support his argument, he argued that the empirical evidence from the term structure of interest rates is at odds with the assumption that central banks smooth interest rates, because financial markets do not seem to have information about interest rates in future quarters, under rational expectations. He then suggests that, in policy analysis, rules with partial adjustment should be replaced by non-inertial rules with serially correlated shocks. These shocks, reflecting missing variable or measurement errors, are observationally equivalent to partial adjustment, but do imply the lack of predictability of future quarterly rates that is observed in term structure regressions.\footnote{Rudebusch admits that there may be other hypothesis of reconciliation of the policy rule and term structure empirical results. For instance, if expectations of interest rates are not predominantly rational, then the term structure estimates can not be interpreted to have the implications they have in Rudebusch's}

Empirical evidence from recent research has, however, challenged Rudebusch's (2001b) arguments. First, English \textit{et al.} (2002) derive testable implications of Rudebusch's serially correlated errors hypothesis, and the standard partial adjustment hypothesis, for an equation explaining the change in actual interest rates with the change in the interest rate suggested by the policy rule and lagged actual and rule's rates. Their estimates and bootstraps simulations, with US data for 1987:I-2000:IV, show that while there are omitted serially correlated errors in estimated policy rules, the partial adjustment term in those estimated rules is significant and remains so even once serially correlated errors are permitted. Second, Favero (2001b) shows that US interest rates for a wide range of maturities co-move with rates for similar maturities obtained by simulating forward a small macro model composed of Rudebusch-Svensson IS and Phillips relations, and a Taylor rule with a partial adjustment coefficient as large as 0.92. In short, when rational expectations are replaced by model-consistent, limited information, expectations, data in a model with interest rate smoothing does not reject the term structure hypothesis. Third, Gourinchas and Tornell's (2001) results on the forward-premium puzzle (positive predictable excess returns, another paradox of
modern finance similar in nature to the term-structure puzzle) also defy Rudebusch's arguments. In survey data from the G7, 1986-1995, they find that agents systematically under-react to innovations in the interest rates, misperceiving shocks as more transitory than they are. Their results document significant market anomalies in the short-run, and imply that financial markets data should not be used to derive results, as Rudebusch's (2001b), that depend upon the strict validity of rational expectations and perfect market clearing.

Before ending this section describing our model for monetary policy analysis in the Euro Area, some notes on money and exchange rates are in order.

Following the currently consensual monetary policy analysis framework - see, for instance, McCallum (2001b) - , no monetary aggregate is included in our model. That stems from the assumption that money is not relevant for policy, neither as an instrument nor as an intermediate target. As a result, though, the model estimated coefficients should be interpreted with some caution, since no distinction between money supply and money demand surfaces in the model and, thus, some coefficients may be reflecting mixed effects from demand and policy changes.

Some observers could find the assumption of excluding money from policy incompatible with the fact that the Bundesbank pursued explicit monetary targets since 1974, and with the fact that the ECB bases its policy on two pillars, the first being actually a monetary aggregate growth targeting - see ECB (1998). However, it has been demonstrated that the Bundesbank should be considered much more an inflation targeter than a money targeter - see Von Hagen (1995) and Bernanke and Mihov (1997). Specifically, its money targets have always been defined as function of some inflation target and a projection of potential output growth, and since 1986 its inflation target has always been defined at 2 percent per year, the level considered compatible with price stability. This is compatible with evidence in Muscatelli et al. (2000), who show that monetary aggregates are not statistically significant in forward-looking policy reaction functions of the Bundesbank, estimated with inflation expectations and output gaps paper. Other instance would be an intermediate degree of partial adjustment together with some serially correlated shocks.
given by structural time-series models estimated with the kalman filter. Von Hagen (1999) has argued that the role of money targets in the Bundesbank framework seems to have been merely political, in the sense that they were adopted in order to solve coordination problems at the launching and consolidation of a new monetary regime of irrevocable commitment to price stability. The ECB, in turn, has presented the rationale for its two-pillar strategy as a way to cope with the uncertainty faced by policymakers, with each pillar offering information from two alternative theoretical paradigms of inflation, both being useful for monetary policy - ECB (2000). Still, it has recognised the difficulties of integrating an active role for money into conventional real economic models. Mihov (2001) has found no evidence of relevance of the ECB's first pillar in its monetary policy and argued that, like the Bundesbank's, its action can be described as forecast inflation targeting. Rudebusch and Svensson (2002) classify the ECB's policy first pillar a weak monetary targeting, as money growth is not considered an intermediate target variable that should systematically be brought in line with the reference value, but one between many reference indicators of the risks to price stability.48 Begg et al. (2002) show that the ECB interest rate decisions throughout 1999-2001 are not significantly correlated to the money growth indicator and the association between these variables has the wrong sign. Gali (2002b) reviews the arguments in favour and against the relevance of the ECB's first pillar, and argues that the case for the money growth pillar is weak. Most notably, he notes that the quantity theory identity, behind it, is only one among many relations between nominal and real variables holding in the long run, and that the empirical evidence on it is not stronger than for several other relations. Moreover, he reviews empirical evidence on the role that money might have as second pillar indicator for future inflation, concluding that it is far from clear. On one hand, Trecroci and Vega (2000) and Gerlach and Svensson (2002) find that, in the Euro Area, nominal money growth does not seem to have any marginal forecasting power for future inflation, over the output gap and the real money

48 Recent research has shown that in realistic macroeconomic models monetary targeting performs worse than inflation targeting. See Svensson (1997, 1999a, and 1999b) for theoretical examples of such research, and Rudebusch and Svensson (2002) for an empirical example. One theoretical framework that could be closer to the ECB monetary framework has been recently suggested in Christiano and Rostagno (2001). Their monetary authority manages short-term interest rates according to the state of the macroeconomy - that is, following a Taylor rule - as long as money growth falls within a specified target range, but abandons the interest rate rule for a constant money growth rule if the monetary target range is violated.
gap. On the other hand, Altimari (2001) suggests that money might be useful to anticipate medium term and low frequency trends in Euro Area inflation. Recently, Svensson (2002) argues that the long run correlation between money and inflation is largely irrelevant for the conduct of monetary policy, and recalls non-Euro-Area evidence that money growth is a poor predictor of inflation at horizons relevant for monetary policy - Estrella and Mishkin (1997) and Stock and Watson (1999). He calls for an urgent reform of the ECB monetary policy strategy, including the demolition of the first pillar.

Another important feature of our model is the absence of an exchange-rate variable, and, thus, the preclusion of the exchange rate as an intermediate or final target. Exchange-rates may be a significant argument in policy functions of open economies - as shown by Black et al. (1997) for Canada, Batini and Haldane (1999) for the UK and Conway et al. (1998) for New Zealand. However, they are less likely to matter in a policy rule of a large and relatively closed Area like the Euro Area - see Peersman and Smets (1999). The evidence in Clarida and Gertler (1997) indicates that even the Bundesbank concerns with the DMark exchange-rate, when conducting monetary policy, were essentially related to its importance as a determinant of domestic inflationary pressures and not as a final target per se. This is supported by evidence in Muscatelli et al. (2000), who show that the exchange-rate is not statistically significant in forward-looking policy reaction functions of the Bundesbank. The role of the exchange-rate seems to be similar for the ECB, in view of its two-pillar monetary strategy definition - see ECB (2000b). In fact, these arguments find validation in a recent text by ECB officials, Gaspar and Issing (2002). They state that, being the Area a large and closed economy, the Euro exchange rate could neither be an intermediate target nor a final objective, but merely one of the variables to be considered when assessing the economic situation of the Area and looking at the transmission of policy. In our model, we consider these roles of the exchange-rate implicitly through the exogenous shock variable in the Phillips curve, which is the lagged deviation of imported from domestic inflation. Anyhow, as Angeloni et al. (2002) have argued, the transmission of monetary policy in such an Area operates mainly through domestic channels. Actually, Clausen and Hayo (2002 a) have recently found that the exchange-
rate channel does not seem to play a critical role in monetary policy transmission in the aggregate of the main Euro Area countries.

3.3. Estimation of Policymakers' Preferences using Optimal Control and GMM

3.3.1. Framework

The central bank inter-temporal optimisation problem is:

\[
\max_{\{i_t\}_{t=0}^\infty} \sum_{t=0}^{\infty} \delta^t \left[ (\pi_{t+1} - \pi^*)^2 + \lambda x_{t+1}^2 + \mu (i_{t+1} - i_{t+1})^2 \right]
\]

subject to:

\[
x_t = c_0 + c_1 x_{t-1} + c_2 x_{t-2} + c_3 x_{t-3} + c_4 (i_{t-3} - \pi_{t-3}) + e^d_t
\]

\[
\pi_t = c_5 \pi_{t-1} + c_6 \pi_{t-2} + c_7 \pi_{t-3} + c_8 \pi_{t-4} + c_9 x_t + c_{10} (\text{Im} \pi_{t-4} - \pi_{t-4}) + E^s_t
\]

We circumscribe our analysis to a discretionary monetary policy regime, implying that the monetary authority solves the optimisation problem in each period, sequentially choosing the policy decision that minimises loss given the state of the economy in that period and its structural dynamic behaviour. In fact, commitment would not be a realistic assumption, in spite of its theoretical superiority, not only in view of the historical facts of the Area, but also because the absence of a commitment technology is still, nowadays, a practical unsolved problem.

Adopting the method of Optimal Control to solve this problem - see, inter alia, Chiang (1992) - we calculate the first-order conditions for minimisation of loss, which lead to the following Euler equation:

\[
E_t \sum_{i=0}^\infty \delta^i \left[ (\pi_{i+1} - \pi^*) \frac{\partial \pi_{i+1}}{\partial i_t} \right] + E_t \sum_{i=0}^\infty \delta^i \lambda \left[ x_{i+1} \frac{\partial x_{i+1}}{\partial i_t} \right] + \left[ \mu (i_{i+1} - i_{i+1}) - \mu \delta E_t (i_{i+1} - i_{i+1}) \right] = 0 \tag{3.4}
\]

In order to estimate this equation it is necessary to truncate its lead polynomials at some reasonable temporal horizon. Favero and Rovelli (2001) use a 4 quarters horizon. Dennis (2001) criticises this choice - which implies \( \delta = 0 \) for all \( i \geq 5 \) -
arguing that most of the impact of monetary policy on inflation happens beyond the one-year-ahead horizon. However, three arguments stand in favour of the lead truncation of the Euler equation at 4 quarters.

First, the loss function seems to be more related to Goodhart’s (2000) concept of forecast horizon - the horizon of the economic prospects motivating current policy decisions -, than to his concept of policy horizon - the date at which the objectives of policy are to be obtained, considering the policy transmission lags. Simulations in Batini and Haldane (1999) suggest that the optimal forecasting horizon is between 3 and 6 quarters, when annualised quarterly inflation is considered. Muscatelli et al. (2000) show that estimated forward-looking policy reaction functions for the US, Japan, Germany, UK, Canada, Sweden and New Zealand, strongly indicate that actual policy decisions involve forecast horizons of inflation not beyond 4 quarters ahead. Boivin and Giannoni (2002) find that forward-looking policy reaction functions for the US seem to be associated with a forecast horizon of no more than 1 quarter for the gap and 3 quarters for inflation. Doménech et al. (2001a) find that forecast horizons of 1 quarter for inflation and the gap, in the US case, and of 2 quarters for inflation and 1 for the gap, in the case of EMU aggregate data, generates the best statistical fit for Clarida et al. (1998, 2000) rules. Moreover, it is well known that, as a practical matter, forecasting the state of the macro-economy over a medium term is highly difficult, so that many macroeconomic projections are formulated for only a one-year-ahead horizon. For instance, the projections in the IMF World Economic Outlook of October 2001 include forecasts for 2001 and 2002. The OECD reports macroeconomic projections for the 3 semesters subsequent to the information cut-off date, but its Secretariat analyses the range of risks involved in the short-term and provides some information on alternative

50 The optimal forecast horizon is a function of the relative degree of backward and forward looking behaviour of the economy model - if agents were purely forward-looking, policy could react merely to current inflation, whilst if they are essentially backward-looking policy should have a longer forecasting horizon. For instance, Batini and Nelson (2000) did find smaller optimal horizons in a partly forward-looking model than in a VAR. The results in Batini and Haldane relate to a model in which backward-looking behaviour in the Phillips relation dominated forward-looking behaviour, but the latter did exist and had a weight of 20 percent. In the Rudebusch-Svensson model used here, agents are purely backward-looking, so it could be argued that policy should perhaps be slightly more forward-looking than Batini and Haldane’s prescription.
scenarios. In the US, the Greenbook of statistical data available to each Federal Open Market Committee meeting only reports consistently forecasts of inflation not more than four quarters ahead - see Perez (2001). The Bank of England Inflation Report, while including forecasts for GDP growth and inflation that extend into a horizon of 8 quarters, seems to be closer to a policy horizon exercise, because it typically forecasts inflation on target at the end of the 2-year horizon. Actually, the optimal feedback horizon must be shorter than the policy horizon: if authorities want to achieve the target 8 quarters ahead, they must surely react to forecasted deviations of the variable from the target at some earlier period. In addition, the Report includes an explicit measure of the dispersion of the forecasts made by the Policy Committee, showing how it increases with the lead. Hence, it seems that the truncation used by Favero and Rovelli is in line with the forecasting needs and abilities of real-world monetary policy authorities and therefore can not be significantly at odds with their actual behaviour, even though their theoretical loss function may have an infinite horizon.

Second, as Favero and Rovelli (2001) have argued, a natural cutting point for the future horizon of the Euler equation emerges anyway, even if we consider a theoretical infinite horizon loss function. In fact, the weight attached to expectations of future gaps and inflation decreases as the time-lead increases, meaning that expectations of the state of the economy carry less relevant information for the present conduct of policy as they relate to periods further away in the future.

Finally, expanding the horizon in the Euler equation would complicate it and bring collinearities to the system, causing great difficulties to estimation. It is worth noting that our option is consistent with the standard practice in the estimation of

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51 See, for instance, the Summary of projections, page viii, in OECD (2001), presenting forecasts up until the second semester of 2002 with information dating up to May 2001. Recently, Oller and Barot (2000) find that, contrary to the first year's projections, the second-year ahead forecasts made by OCDE for 13 European countries during 1971-1997 appear to contain a (positive) bias. The European Commission seems to go slightly further into the future than OECD - see, for instance, European Commission (2001), the Supplement A "Economic Trends" of European Economy, nº 10-11, October-November 2001, where macroeconomic projections for 2002 and 2003 are published. 52 Here we are using the forecasting and feed-back horizon as equivalent concepts, even though the latter has been developed in the slightly different context of simple policy reaction rules. Specifically, for Batini and Nelson (2000), the feed-back horizon is the future period for which authorities should form the inflation forecast entering a simple inflation forecast-based policy rule. 53 See for example the so-called "probability fans" of the GDP and RPIX Inflation projections in the Inflation Report of August, Bank of England (2001), pages 47 and 49. The dispersion of the forecasts is
forward-looking policy reaction functions. Boivin and Giannoni (2002) truncate the forecast horizon at 1 quarter for output and 2 quarters for inflation, while Muscatelli et al. (2000) and Orphanides (2001b) truncate the inflation forecast horizon at 4 quarters.

Once the Euler equation is truncated at 4 quarters ahead, its partial derivatives components can be expressed as functions of the IS and Phillips parameters, thus building into the Euler equation the cross-equation restrictions. This ensures that the loss function is being properly minimised subject to the constraints given by the economy structure.

\[
E_t \sum_{i=0}^{4} \delta^i \left[ (\pi_{t+i} - \pi^*) \frac{\partial \pi_{t+i}}{\partial \pi_t} + E_t \sum_{i=0}^{4} \delta^i \lambda \frac{\partial x_{t+i}}{\partial \pi_t} + \left[ \mu(i_t - i_{t-1}) - \mu \delta E_t(i_{t+1} - i_t) \right] \right] = 0
\]

\[<=> \delta^3 E_t \left[ (\pi_{t+3} - \pi^*) \frac{\partial \pi_{t+3}}{\partial \pi_t} \right] + \delta^4 E_t \left[ (\pi_{t+4} - \pi^*) \frac{\partial \pi_{t+4}}{\partial \pi_t} \right] + \lambda \delta^3 E_t \left[ x_{t+3} \frac{\partial x_{t+3}}{\partial \pi_t} \right] + \left[ \mu(i_t - i_{t-1}) - \mu \delta E_t(i_{t+1} - i_t) \right] = 0
\]

Expanding the partial derivatives, (3.5) turns into

\[
\delta^3 E_t (\pi_{t+3} - \pi^*) \left[ \frac{\partial \pi_{t+3}}{\partial x_{t+3}} \frac{\partial x_{t+3}}{\partial \pi_t} \right] + \\
\delta^4 E_t (\pi_{t+4} - \pi^*) \left[ \frac{\partial \pi_{t+4}}{\partial x_{t+4}} \frac{\partial x_{t+4}}{\partial \pi_t} + \frac{\partial \pi_{t+4}}{\partial \pi_{t+3}} \frac{\partial \pi_{t+3}}{\partial x_{t+3}} \frac{\partial x_{t+3}}{\partial \pi_t} \right] + \\
\lambda \delta^3 E_t x_{t+3} \left[ \frac{\partial x_{t+3}}{\partial \pi_t} \right] + \\
\lambda \delta^4 E_t x_{t+4} \left[ \frac{\partial x_{t+4}}{\partial x_{t+3}} \frac{\partial x_{t+3}}{\partial \pi_t} \right] + \left[ \mu(i_t - i_{t-1}) - \mu \delta E_t(i_{t+1} - i_t) \right] = 0
\]

Then, the IS, Phillips and Euler equations - respectively, (3.7), (3.8), and (3.9) - can be jointly estimated as a system, generating estimates of the structural parameters \(c_0\) through \(c_{10}\), as well as of the policymakers structural preferences parameters \(\mu, \lambda\) and \(\pi^*\):

\[
x_t = c_0 + c_1 x_{t-1} + c_2 x_{t-2} + c_3 x_{t-3} + c_4 (i_{t-3} - \pi_{t-3}) + \epsilon_t^d
\]

based on the time series recent historical variance, which it is then skewed to reflect the balance of risks attached to the forecasts by the Committee members' judgement.
\[ \pi_t = c_5 \pi_{t-1} + c_6 \pi_{t-2} + c_7 \pi_{t-3} + c_8 x_{t-4} + c_9 x_t + c_{10} (\ln \pi_{t-1} - \pi_{t-1}) + e_t^x \]  
\( (3.8) \)

\[ \delta \delta E_t (\pi_{t-3} - \pi^*) \left[ c_9 c_4 + c_9 c_{10} c_4 + c_9 c_5 c_4 + c_9 c_5 c_6 c_4 \right] + 
\lambda \delta E_t x_{t-3} \left[ c_4 + \delta E_t x_{t-4} \right] + \left[ \mu (i_t - i_{t-1}) - \mu \delta E_t x_{t-3} \right] + e_t^\pi = 0 \]  
\( (3.9) \)

Following Favero and Rovelli, the ratio \(-c_0/c_4\) is the estimate of the real equilibrium interest rate. The estimates of \(\mu, \lambda,\) and \(\pi^*\), describe the monetary policy regime, and allow for answering the question motivating this chapter, namely, whether the emergence of a well-defined monetary policy regime had a role in the improvement in the volatility trade-off in the Euro Area since 1986. The standard error of the Phillips equation residual offers some information about the volatility of supply shocks, and the standard deviation of the Euler equation residual is some indication about the optimality of monetary policy - i.e. the success of policymakers in setting interest rates close to their optimal path. Our interpretation of these two standard errors of estimation is more cautious than the interpretations in Favero and Rovelli (2001). Firstly, because, as discussed in section 3.2.2, the absence of variables such as money from the model may change the meaning of some coefficients and residuals. Secondly, and more specifically in respect of the Euler equation standard error, because the estimated deviation of actual interest rates from their optimal path may reflect specification problems, which can not be disentangled from their possible economic content.

We replace expectations of inflation and the gap by their actual observations and, because the expectations errors pile-up in the Euler equation residuals, we proceed to estimate the system with GMM, using lagged values of all the variables in the model as instruments.\(^{54}\) This is a suitable way of estimating the system, as it is reasonable to assume that policymakers are rational - within the model - and, when optimising, use all the available information efficiently to forecast inflation and the gap.\(^{55}\)

\(^{54}\) Our estimation procedure mimics Favero and Rovelli's (2001), in order to compare results. Specifically, we use the (one-step weighting matrix) GMM procedure of Eviews, and we use as instruments all the variables in the model lagged up until 4 quarters. The instruments choice also safeguards our analysis from the criticism described in Wooldridge (2001, page 90), as it is clear that we did not search over different sets of moment conditions until some desired result was achieved. Inflation enters the instrument set in changes instead of levels, because of its high persistence throughout most of the sample.

\(^{55}\) Note that GMM implies assuming that Central Banker’s expectations are rational, but not the public’s. Actually, using the Rudebusch-Svensson model to characterise the structure of the economy amounts to assume that agent’s expectations are not rational, as it is a backward-looking model.
In order to obtain an heteroskedasticity and autocorrelation consistent covariance matrix we use Andrews and Mohanan's (1992) pre-whitening and a Bartlett Kernel to weight the auto-covariances, with a bandwidth estimated with Andrews' (1991) estimator.\textsuperscript{56} Covariance estimation is especially difficult in cases, like ours, of serially correlated moment conditions and small samples (while total observations amount to 118, we have only 62 observations for the post-1986 regime). This explains our choice of the two-step estimator, that is, a one-step weighting matrix version of GMM.\textsuperscript{57}

3.3.2. Results

Table 3.1 reports results of estimation with the whole sample, 1972:I-2001:II. The empirical framework seems to be well designed, as the J test of the over-identifying restrictions does not reject the orthogonality conditions (significance of 0.7), indicating that the instruments are suited for the system. However, the model does not adjust well to the whole sample, as the main structural coefficients are not precisely estimated.\textsuperscript{58} This is true for the Phillips elasticity - the sensitivity of inflation to the real gap - and for the IS elasticity - sensitivity of the gap to real interest rates -, in spite of their theory-compatible signs. Moreover, it is also true for the policymakers' preferences. The point estimate of the inflation level supposedly targeted by monetary authorities is 4.6 percent, but its estimate is highly imprecise - a Wald test does not allow rejection of the hypothesis that the true target is any value between 0 and 9.2 percent, at 5 percent of significance. Both the weights attached to the variability of the gap and to the variability of interest rates are not significantly different from zero. The equilibrium real interest rate is estimated at 2.3 percent, but this point estimate is the result of the ratio of two parameters ($c_0$ and $c_4$) both very imprecisely estimated.

\textsuperscript{56} Pre-whitening is indicated in cases of very strong first-order auto-correlation (unit-root or close to unit-root processes) plaguing the moments, and is meant to flatten the moments spectrum at the neighbourhood of the zero frequency, so that the kernel estimator becomes unbiased - Canova (1999b).

\textsuperscript{57} See the discussions in Hansen et al (1996), Anderson and Sorensen (1996), Burnside and Eichenbaum (1996), Christiano and Den Haan (1996), Canova (1999b) and Wooldridge (2001), on the finite-sample problems of GMM. Hansen et al (1996), argued that in small samples the continuous up-dating estimator performs better, but Florens \textit{et al}. (2001) show Monte Carlo results indicating that the two-step estimator generates results closer to maximum likelihood, and parameters estimators that are not strongly biased, in the estimation of forward-looking Taylor rules.

\textsuperscript{58} As above in section 3.2 addressing the structural stability of the system IS-Phillips we have tried to robustify the aggregate demand equation by considering the average of the real interest rate during the
<table>
<thead>
<tr>
<th>Table 3.1 – Model with Baseline Loss Function</th>
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<table>
<thead>
<tr>
<th></th>
<th>Estimates</th>
<th>T-statistics</th>
<th>Significance Prob</th>
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<tbody>
<tr>
<td><strong>IS equation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C0</td>
<td>0.0399</td>
<td>1.46</td>
<td>0.14</td>
</tr>
<tr>
<td>C1</td>
<td>1.255</td>
<td>13.16</td>
<td>0.00</td>
</tr>
<tr>
<td>C2</td>
<td>-0.186</td>
<td>-1.15</td>
<td>0.25</td>
</tr>
<tr>
<td>C3</td>
<td>-0.159</td>
<td>-1.69</td>
<td>0.09</td>
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<tr>
<td>C4</td>
<td>-0.017</td>
<td>-1.90</td>
<td>0.06</td>
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<tr>
<td><strong>Phillips equation:</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>C5</td>
<td>0.562</td>
<td>10.63</td>
<td>0.00</td>
</tr>
<tr>
<td>C6</td>
<td>0.297</td>
<td>5.16</td>
<td>0.00</td>
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<tr>
<td>C7</td>
<td>-0.049</td>
<td>-0.74</td>
<td>0.46</td>
</tr>
<tr>
<td>C8</td>
<td>0.175</td>
<td>2.77</td>
<td>0.01</td>
</tr>
<tr>
<td>C9</td>
<td>0.083</td>
<td>1.13</td>
<td>0.26</td>
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<tr>
<td>C10</td>
<td>0.052</td>
<td>8.58</td>
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<tr>
<td><strong>Euler equation:</strong></td>
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<tr>
<td>(\pi^*)</td>
<td>4.62</td>
<td>1.96</td>
<td>0.05</td>
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<tr>
<td>(\lambda)</td>
<td>0.117</td>
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<td>0.71</td>
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<tr>
<td>(\mu)</td>
<td>0.0103</td>
<td>1.03</td>
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<tr>
<td>(\sigma(\varepsilon_{IS}))</td>
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<tr>
<td>(\sigma(\varepsilon_{PH}))</td>
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<tr>
<td>(\sigma(\varepsilon_{EU}))</td>
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<td><strong>J-test</strong></td>
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<td>32.08 *</td>
<td>0.70</td>
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<tr>
<th>Fitted series' (\sigma)</th>
<th>Data's (\sigma)</th>
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<tr>
<td>U. Gap</td>
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</tr>
<tr>
<td>Inflation</td>
<td>3.533</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>2.982</td>
</tr>
</tbody>
</table>

| Residuals IS :             |                   |
| Sample mean                | -0.001             | 0.97 ** |
| Jarque-Bera                | 26.751             | 0.00   |
| Durbin-Watson              | 1.95               |        |
| Q(4)                       | 10.658             | 0.03   |
| Q(28)                      | 41.438             | 0.05   |

| Residuals Phillips :       |                   |
| Sample mean                | 0.051              | 0.62 ** |
| Jarque-Bera                | 3.11               | 0.21   |
| Durbin-Watson              | 1.96               |        |
| Q(4)                       | 1.057              | 0.90   |
| Q(28)                      | 29.20              | 0.40   |

| Residuals Euler :          |                   |
| Sample mean                | -0.004             | 0.01 ** |
| Jarque-Bera                | 10.31              | 0.01   |
| Durbin-Watson              | 0.55               |        |
| Q(4)                       | 218.55             | 0.00   |
| Q(28)                      | 1109.16            | 0.00   |

Estimation: two-step GMM. Instruments: constant, \(\Delta \pi_{it}\), \(U_{gap_{it}}\), \(stir_{it}\), \((I_{\pi_{it}}-\pi_{it})\), \(i=1, \ldots, 4\); Discount factor: \(\delta=0.975\); Variance-Covariance matrix HAC consistent: Andrews and Mohanan (1992); pre-whitening: Bartlett kernel, bandwidth Andrews (1991); Signific. probabilities are for one-sided tests. \(\dagger\) Imprecisely estimated because based on at least one coefficient with too large standard error; \(*\): \(J\times\text{nobs}\); \(**\): \(H_0\): Mean=0; Fitted series: observed series minus residuals of estimation of each series' equation.

previous year - as in the original Rudebusch-Svensson model. The results were extremely similar to those reported in table 3.1.

The J test for the over-identifying restrictions does not allow rejection of the moments used in estimation for any sub-sample, but its results are much more comfortable for the period 1986-2001, suggesting that this period is more suitable for estimation, which is compatible with our policy regime hypothesis.

The main elasticity-coefficients of the IS and Phillips equations have correct signs and are well estimated in both samples, but their estimates change markedly. The persistence of inflation does not change and remains compatible with a unit root process. The persistence of the gap is in line with the expected behaviour for the series in the second sub-sample, but not in the first sub-sample, where the coefficients associated to its lags sum-up to an unreasonable value of -0.22.

The estimates of the inflation target and the real equilibrium interest rate document a sharp change in macroeconomic conditions and monetary regime in 1985. The inflation target is estimated with good precision, at 9.4 percent in 1972-1985 and 2.9 percent in 1986-2001, and the equilibrium real interest rate is also precisely estimated, as 0.9 percent in the first sub-sample and 4.4 percent in the second. However, the results for the two other main coefficients describing policymakers' preferences are not so good. The estimate of $\lambda$ has an unreasonable negative sign, and is only significant at 9 percent, in the second sub-sample. The estimate of $\mu$, although changing from negative to positive across the sub-samples, remains significant only at 9 percent in the 1986:1-2001:II period.
Table 3.2 – Model with Baseline Loss Function

<table>
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<td><strong>IS equation:</strong></td>
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<tr>
<td>C0</td>
<td>0.025</td>
<td>2.04</td>
<td>0.04</td>
<td>0.093</td>
<td>3.36</td>
<td>0.00</td>
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<td>C1</td>
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<td>0.00</td>
<td>1.254</td>
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<tr>
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<td>0.00</td>
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<td><strong>Phillips eq:</strong></td>
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<td>0.167</td>
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<td>0.217</td>
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<td>0.135</td>
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<td><strong>Euler eq:</strong></td>
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<tr>
<td>π*</td>
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<td>33.40</td>
<td>0.00</td>
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<td>-3.07</td>
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<td>-0.18</td>
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<td>0.09</td>
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<td>µ</td>
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<td>-1.11</td>
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<td>0.008</td>
<td>1.69</td>
<td>0.09</td>
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<tr>
<td>r*</td>
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<tr>
<td>σ(εEU)</td>
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<td>0.0093</td>
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<td><strong>J-test</strong></td>
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<td>0.398</td>
<td>24.66</td>
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Fitted series σ

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<tr>
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<th>U. Gap</th>
<th>Inflation</th>
<th>Interest Rate</th>
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<td>U. Gap</td>
<td>0.824</td>
<td>0.847</td>
<td>0.512</td>
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<td>Inflation</td>
<td>2.057</td>
<td>2.148</td>
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<tr>
<td>Interest Rate</td>
<td>2.332</td>
<td>2.329</td>
<td>2.659</td>
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Residuals IS:

<table>
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<tr>
<th></th>
<th>Mean</th>
<th>Jarque-Bera</th>
<th>D-W</th>
<th>Q(4)</th>
<th>Q(28)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.006</td>
<td>24.704</td>
<td>1.97</td>
<td>2.154</td>
<td>46.751</td>
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<tr>
<td>Jarque-Bera</td>
<td>0.80</td>
<td>0.00</td>
<td>1.13</td>
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<tr>
<td>D-W</td>
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<td>Q(4)</td>
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<td>0.031</td>
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<td>Q(28)</td>
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<td>24.195</td>
<td>0.67</td>
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Resid. Phillips

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<th>Mean</th>
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<th>D-W</th>
<th>Q(4)</th>
<th>Q(28)</th>
</tr>
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<tbody>
<tr>
<td>Mean</td>
<td>0.068</td>
<td>0.965</td>
<td>1.91</td>
<td>0.499</td>
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<td>Jarque-Bera</td>
<td>0.72</td>
<td>0.62</td>
<td>1.22</td>
<td>0.97</td>
<td>0.01</td>
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<td>D-W</td>
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<td>Q(4)</td>
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<td>0.031</td>
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<td>24.195</td>
<td>0.67</td>
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<tr>
<td>Q(28)</td>
<td>0.35</td>
<td>0.54</td>
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<td>264.55</td>
<td>0.67</td>
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Resid. Euler:

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<th>Mean</th>
<th>Jarque-Bera</th>
<th>D-W</th>
<th>Q(4)</th>
<th>Q(28)</th>
</tr>
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<tbody>
<tr>
<td>Mean</td>
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<td>1.180</td>
<td>0.41</td>
<td>81.029</td>
<td>208.71</td>
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<tr>
<td>Jarque-Bera</td>
<td>0.77</td>
<td>0.54</td>
<td>0.73</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>D-W</td>
<td>0.00</td>
<td>1.702</td>
<td></td>
<td>84.129</td>
<td>264.55</td>
</tr>
<tr>
<td>Q(4)</td>
<td>0.35</td>
<td>0.43</td>
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<td>84.129</td>
<td>264.55</td>
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<tr>
<td>Q(28)</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td>264.55</td>
<td>0.00</td>
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Estimation: two-step GMM. Instruments: constant, Δπt-i, Ugap_t-i, stir_t-i, (I_t_π−π)_t-i, i=1,...,4;
Discount factor: δ=0.975; Variance-Covariance matrix HAC: Andrews and Mohanan (1992) pre-whitening; Bartlett kernel, bandwith estimated with Andrews (1991) method;
Prob.: one-sided significance probability; *: J×NOBS; **: H0: Mean=0;
Fitted series: observed series minus residuals of estimation of each series' equation.
The problems in table 3.2 with the estimates of $\mu$ and $\lambda$, clearly require further inquiry in order to characterise the Euro Area policy regime since 1986. We now explore the hypothesis that the regime has been one of strict inflation targeting with interest rate smoothing (SITIRS). This step implies estimating a system composed of the above defined IS and Phillips equations - (3.7) and (3.8) -, together with a new formulation for the Euler equation - replacing (3.9) - that does not include any term associated to the unemployment gap:

$$
\delta^3 E_t (\pi_{t+3} - \pi^*) [c9.c4] + \delta^4 E_t (\pi_{t+4} - \pi^*) [c9.c1.c4 + c5.c9.c4] + [\mu(i_t - i_{t-1}) - \mu \delta E_t (i_{t+1} - i_t)] + e_i^p = 0
$$  

(3.10)

Table 3.3 reports the estimation of this model for the whole sample. They are not qualitatively different - nor quantitatively, in most cases - from those of the baseline loss function of flexible inflation targeting with interest rate smoothing. 59

Table 3.4 reports estimates of the strict inflation targeting with interest rate smoothing regime for 1972:I-1985:IV, versus 1986:I-2001:II. The J test for the over-identifying restrictions suggests, even more clearly than in the baseline loss, that the moments are well suited for estimation of the system in 1986:I-2001:II, but somewhat less in 1972:I-1985:IV. The estimates of the Phillips and IS coefficients are reasonable and quite precise, and the dynamics of the gap does not show the anomaly apparent in table 3.2 for the first sub-sample.

The results are compatible with the hypothesis that the Euro Area monetary regime changed significantly in 1985, and imply that, during 1986-2001, the Euro Area has had a policy regime of strict inflation targeting with interest rate smoothing. In contrast, it is difficult to characterise the regime of 1972-1985. Our estimates indicate that during 1986-2001 the notional Euro Area monetary authority targeted inflation at around 2.73 percent and managed interest rates with a positive and very significant degree of smoothing. The change in monetary regime has been drastic, from an implicit inflation target of 9.2 percent, and an interest rate smoothing component of L that is not well identified, in the previous period.

59 As in the model with the baseline loss, we have tried to robustify the aggregate demand equation by considering the average of the real interest rate during the previous year. Again, the results were very similar to those reported in table 3.3.
Table 3.3 – Model with Strict Inflation Targeting with Interest Rate Smoothing (SITIRS) Loss Function

<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimates</th>
<th>T-statistics</th>
<th>Significance Prob</th>
</tr>
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<tr>
<td><strong>IS equation:</strong></td>
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<tr>
<td>C0</td>
<td>0.0445</td>
<td>1.73</td>
<td>0.08</td>
</tr>
<tr>
<td>C1</td>
<td>1.219</td>
<td>13.99</td>
<td>0.00</td>
</tr>
<tr>
<td>C2</td>
<td>-0.125</td>
<td>-0.81</td>
<td>0.42</td>
</tr>
<tr>
<td>C3</td>
<td>-0.187</td>
<td>-2.01</td>
<td>0.05</td>
</tr>
<tr>
<td>C4</td>
<td>-0.019</td>
<td>-2.21</td>
<td>0.03</td>
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<tr>
<td><strong>Phillips equation:</strong></td>
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<td></td>
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</tr>
<tr>
<td>C5</td>
<td>0.553</td>
<td>10.61</td>
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</tr>
<tr>
<td>C6</td>
<td>0.325</td>
<td>6.04</td>
<td>0.00</td>
</tr>
<tr>
<td>C7</td>
<td>-0.030</td>
<td>-0.47</td>
<td>0.64</td>
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<tr>
<td>C8</td>
<td>0.138</td>
<td>2.35</td>
<td>0.02</td>
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<tr>
<td>C9</td>
<td>0.102</td>
<td>1.41</td>
<td>0.16</td>
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<tr>
<td>C10</td>
<td>0.049</td>
<td>8.20</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Euler equation:</strong></td>
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<tr>
<td>( \pi^* )</td>
<td>5.69</td>
<td>4.83</td>
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<tr>
<td>( \mu )</td>
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<td>0.34</td>
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<tr>
<td>( r^* )</td>
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<tr>
<td>( \sigma(\varepsilon_{IS}) )</td>
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<tr>
<td>( \sigma(\varepsilon_{PH}) )</td>
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<tr>
<td>( \sigma(\varepsilon_{EU}) )</td>
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<td><strong>J-test</strong></td>
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<tr>
<td><strong>Fitted series' ( \sigma )</strong></td>
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<tr>
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<tr>
<td>Interest Rate</td>
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<td><strong>Residuals IS :</strong></td>
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<tr>
<td>Sample mean</td>
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<tr>
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<td>Durbin-Watson</td>
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</tr>
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<td>Sample mean</td>
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<td>0.65 **</td>
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</tr>
<tr>
<td>Jarque-Bera</td>
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Estimation: two-step GMM. Instruments: constant, \( \Delta \pi_{i-1}, \Delta \pi_{i-1} \), \( \pi_{i-1}, (\pi_{i-1}-\pi_{i-1}), i=1,\ldots,4 \); Discount factor: \( \delta=0.975 \); Variance-Covariance matrix HAC: Andrews and Mohanan (1992) pre-whitening; Bartlett kernel, bandwidth estimated with Andrews (1991) method; Significance probabilities relate to one-sided tests; \( \dagger \) Im precisely estimated because based on coefficients from which at least one has too large standard error; *: \( J \times \text{nobs} \); **: H0: Mean=0. Fitted series: observed series minus residuals of estimation of each series’ equation.
Table 3.4 – Model with Strict Inflation Targeting with Interest Rate Smoothing (SITIRS) Loss Function


<table>
<thead>
<tr>
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<td><strong>Estimates</strong></td>
<td><strong>T-stats</strong></td>
<td><strong>Prob.</strong></td>
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<tr>
<td><strong>IS equation:</strong></td>
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<td></td>
</tr>
<tr>
<td>C0</td>
<td>0.040</td>
<td>3.57</td>
</tr>
<tr>
<td>C1</td>
<td>1.168</td>
<td>26.10</td>
</tr>
<tr>
<td>C2</td>
<td>0.050</td>
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<td>C4</td>
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<tr>
<td><strong>Phillips eq:</strong></td>
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</tr>
<tr>
<td>C5</td>
<td>0.566</td>
<td>13.40</td>
</tr>
<tr>
<td>C6</td>
<td>0.352</td>
<td>6.95</td>
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<tr>
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<td>-0.038</td>
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<tr>
<td>C8</td>
<td>0.104</td>
<td>2.02</td>
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<tr>
<td>C9</td>
<td>0.122</td>
<td>3.42</td>
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<tr>
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<td>5.57</td>
</tr>
<tr>
<td><strong>Euler eq:</strong></td>
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<tr>
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<tr>
<td>( \sigma(e_{PH}^I) )</td>
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<tr>
<td>( \sigma(e_{EU}^I) )</td>
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<td>J-test</td>
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<td><strong>Data ( \sigma )</strong></td>
<td><strong>Data ( \sigma )</strong></td>
</tr>
<tr>
<td>U. Gap</td>
<td>0.821</td>
<td>0.847</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.063</td>
<td>2.148</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>2.324</td>
<td>2.329</td>
</tr>
<tr>
<td><strong>Residuals IS:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.004</td>
<td>0.32 **</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>33.164</td>
<td></td>
</tr>
<tr>
<td>D-W</td>
<td>1.92</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>2.333</td>
<td>0.67</td>
</tr>
<tr>
<td>Q(28)</td>
<td>48.85</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Resid.Phillips</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.043</td>
<td>0.19 **</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>0.668</td>
<td>0.72</td>
</tr>
<tr>
<td>D-W</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>0.912</td>
<td>0.92</td>
</tr>
<tr>
<td>Q(28)</td>
<td>53.69</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Resid. Euler:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.003</td>
<td>0.31 **</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>2.055</td>
<td>0.36</td>
</tr>
<tr>
<td>D-W</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>Q(4)</td>
<td>135.35</td>
<td>0.00</td>
</tr>
<tr>
<td>Q(28)</td>
<td>306.48</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Estimation: two-step GMM. Instruments: constant, \( \Delta \pi_{it}, \text{Ugap}_{it}, \text{stir}_{it}, (I_{it} - \pi_{it})_{it}, \) \( i=1, \ldots, 4; \)
Discount factor: \( \delta=0.975; \) Variance-Covariance matrix HAC: Andrews and Mohanan (1992) pre-whitening; Bartlett kernel, bandwidth estimated with Andrews (1991) method;
Prob.: One-sided significance probability. *: J×NOBS; **: H0: Mean=0;
Fitted series: observed series minus residuals of estimation of each series' equation.
The change in the Area macroeconomic conditions is also clear in our estimates, most notably, in the increase of the equilibrium real interest rate from 1.34 to 4.51 percent.

Table 3.4 includes some indication that the improvement in the volatility trade-off of the Area since 1986 may not have been entirely due to the policy regime change, and that milder supply shocks and an increase in the optimality of interest rates may have also contributed to the volatility reduction. In fact, the standard deviation of the residual of the Phillips equation falls from 1.38 in the first period to 0.81 in the second, i.e. about 41 percent, and the standard deviation of the residual of the Euler equation falls from 0.021 to 0.011, about 47 percent. The interpretation of the latter is even more tentative than that of the former, though. Actually, in addition to the specification issues above mentioned, the failure to identify a well-defined regime in the first sub-period defies the interpretation of the Euler equation as an optimising relation during that period, implying that standard errors of estimation of the Euler equations may not be comparable across sub-samples.

Table 3.5 offers some sensitivity analysis concerning the date of emergence of the monetary policy regime of strict inflation targeting with interest rate smoothing. It suggests that the regime switch has been somehow abrupt, and that the critical years have been 1985 and 1986 - in agreement with our hypothesis built above in section 1 on the basis of the history of events and macroeconomic performance of the Area. In fact, while the estimate of the inflation target is relatively stable somewhat below 3 percent since early 1985, during that year the point estimate and the statistical significance of the weight of interest rate smoothing in the loss function increase gradually. In addition, the suitability of the moment conditions, as evaluated by the J statistic, also improves markedly since the second quarter of 1985. The results obtained for 1986:I-2001:II would be confirmed if the sample-period was assumed to begin at 1986:III, and would be broadly matched if samples beginning until 1987:I were used. After that, the table may be reflecting the small sample numerical problems to which GMM estimation of such a system is highly sensitive. Actually, the aptness of the sample moment conditions falls sharply, as indicated by the J test significance probability, and the ability of estimation to identify the interest rate smoothing weight deteriorates.
considerably - even though the estimates of the inflation target and of the equilibrium real interest rate remain broadly stable.

Table 3.5 – Sensitivity Analysis: Date of Emergence of Policy Regime of Strict Inflation Targeting with Interest Rate Smoothing (SITIRS)

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>π*</th>
<th>μ</th>
<th>r*</th>
<th>J-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984:1 - 2001:II</td>
<td>3.066</td>
<td>0.00</td>
<td>0.008</td>
<td>0.52</td>
</tr>
<tr>
<td>1984:II - 2001:II</td>
<td>3.067</td>
<td>0.00</td>
<td>0.006</td>
<td>0.68</td>
</tr>
<tr>
<td>1984:III - 2001:II</td>
<td>3.218</td>
<td>0.00</td>
<td>0.005</td>
<td>0.57</td>
</tr>
<tr>
<td>1984:IV - 2001:II</td>
<td>3.145</td>
<td>0.00</td>
<td>0.003</td>
<td>0.66</td>
</tr>
<tr>
<td>1985:I - 2001:II</td>
<td>2.961</td>
<td>0.00</td>
<td>0.005</td>
<td>0.40</td>
</tr>
<tr>
<td>1985:II - 2001:II</td>
<td>2.857</td>
<td>0.00</td>
<td>0.004</td>
<td>0.37</td>
</tr>
<tr>
<td>1985:III - 2001:II</td>
<td>2.792</td>
<td>0.00</td>
<td>0.003</td>
<td>0.20</td>
</tr>
<tr>
<td>1985:IV - 2001:II</td>
<td>2.816</td>
<td>0.00</td>
<td>0.008</td>
<td>0.10</td>
</tr>
<tr>
<td>1986:1 - 2001:II</td>
<td>2.726</td>
<td>0.00</td>
<td>0.014</td>
<td>0.01</td>
</tr>
<tr>
<td>1986:II - 2001:II</td>
<td>2.528</td>
<td>0.00</td>
<td>0.010</td>
<td>0.05</td>
</tr>
<tr>
<td>1986:III - 2001:II</td>
<td>2.719</td>
<td>0.00</td>
<td>0.011</td>
<td>0.01</td>
</tr>
<tr>
<td>1986:IV - 2001:II</td>
<td>2.390</td>
<td>0.00</td>
<td>0.012</td>
<td>0.09</td>
</tr>
<tr>
<td>1987:I - 2001:II</td>
<td>2.597</td>
<td>0.00</td>
<td>0.011</td>
<td>0.07</td>
</tr>
<tr>
<td>1987:II - 2001:II</td>
<td>2.666</td>
<td>0.00</td>
<td>0.009</td>
<td>0.11</td>
</tr>
<tr>
<td>1987:III - 2001:II</td>
<td>2.648</td>
<td>0.00</td>
<td>0.007</td>
<td>0.19</td>
</tr>
<tr>
<td>1987:IV - 2001:II</td>
<td>2.721</td>
<td>0.00</td>
<td>0.005</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Estimation: two-step GMM. Instruments: constant, Δπt-i, Ugapt-i, stirt-i, (I π−π)it, i=1,…,4; Discount factor: δ=0.975; Variance-Covariance matrix HAC: Andrews and Mohanan (1992) pre-whitening; Bartlett kernel, bandwidth estimated with Andrews (1991) method; Significance probabilities relate to one-sided tests.

Figure 3.5 displays the fit of the Euler interest rate equation to the short-term interest rate between 1986:I and 2001:II, both for the flexible inflation targeting with interest rate smoothing regime and for the regime of strict inflation targeting with interest rate smoothing. It shows that the model fits satisfactorily the interest rates first moments, and that the fit improves significantly when the gap is dropped from the loss function - the mean square error falls from 1.01 to 0.42. Accordingly, the regime of strict inflation targeting matches better the data second moments - standard deviation of 3.11, versus 2.66 in the data and 3.41 in the flexible inflation targeting regime.
Figure 3.5 - Actual versus Fitted Interest Rate
Optimal Control and GMM, 1986:I-2001:II,
Flexible Inflation Targeting with Interest Rate Smoothing (FITIRS), Strict Inflation Targeting with Interest Rate Smoothing (SITIRS)

Note: Fitted interest rates obtained by dynamically solving the IS-Phillips-Euler system, using the coefficient estimates obtained for the sample period 1986:1-2001:II.

In summary, the answers we obtain with this method for the questions motivating this research are as follows. First, the emergence of a well-defined aggregate monetary policy regime targeting a low rate of inflation has been part of the causes for the apparent reduction of macroeconomic volatility in the Euro Area after 1986. Specifically, the regime is estimated as one of strict inflation targeting with interest rate smoothing, with the target level for inflation at around 2.7 percent. Second, there are some indications that supply shocks may have been milder since 1986 - as the standard deviation of the Phillips equation residual decreased by 41 percent - which could add to the explanation of the Taylor trade-off improvement. Third, there are some signals that the ability of the policymaker to run interest rates closer to their optimal path may have improved. In fact, the standard deviation of the Euler equation residual fell by 47 percent, which could mean that part of the reduction in macroeconomic volatility could have resulted from a better exploitation of the volatility possibilities frontier.
The results of the structural stability tests in the previous section had suggested that there is evidence of a structural break in our small macroeconomic model for the Area in 1995:II. We have tried to check whether this structural break could be affecting the results, attempting to estimate the model for 1986:I-1995:II. However, in order to estimate the system by GMM with such a limited amount of degrees of freedom, we have had to restrict significantly the number of instruments and, as a result, the estimation results (not reported) turned out to be far less precise than those for 1986:I-2001:II, inhibiting clear conclusions. Hence, it is not possible to estimate a policy regime for the 1986:I--1995:II period, with this framework.

Comparing our Euro Area results to those reported by Favero and Rovelli (2001) for the US, we see that both economies experienced significant monetary regime changes during the 80s, in both cases toward regimes targeting low inflation rates (estimated at 2.63 for the US and 2.73 for the Euro Area). According to our hypothesis and estimates, the Euro Area regime switch happened some years after the US switch, but seems more drastic than that of the US: the inflation target fell more and the real equilibrium interest rate increased more than in the US. The estimates suggest that the Euro Area policymaker has put more weight on interest rate smoothing in his objective function than the US Fed has did ($\mu_{US}^{=0.0085}$, versus $\mu_{EUR}^{=0.014}$). Finally, while the US policy regime seems to be one of flexible inflation targeting with interest rate smoothing - as $\lambda$ is statistically significant in Favero and Rovelli (2001) - our evidence suggests that the Euro Area notional monetary authority has been much more an inflation nutter.60

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60 Although statistically significant, the weight of the gap in the estimated Fed preferences in Favero and Rovelli is numerically very low. This may be considered as compatible with the prescription offered by Soderlind et al. (2002), that one of the conditions that a new keynesian model should fulfil in order to mimic the broad behaviour of US macroeconomic series is a small (virtually zero) preference for output stabilisation. Our own estimates for the Euro Area clearly comply with this condition, which is certainly important for its good performance in matching the low inflation volatility and the high persistence and volatility of our gap measure.
3.4. Estimation of Policymakers' Preferences using Dynamic Programming and FIML

3.4.1. Framework

There is currently a sizeable literature of monetary policy analysis using dynamic programming theory to solve the infinite-horizon policymaker optimization problem, given a small model describing the structural dynamics of the aggregate economy.\textsuperscript{61} In this literature, the loss function coefficients are typically calibrated, rather than estimated, and the economy structure is very often described with the Rudebusch and Svensson (1999) model.\textsuperscript{62}

Some authors have calibrated alternative loss functions and studied the stabilizing performance of the implied optimal linear policy rules, as is the case of Rudebusch and Svensson (1999, 2002) for the US. Others have studied the outcome of alternative policy rules under uncertainty, such being the case of Peersman and Smets (1999) with weighted-average data of a core of five EMU member-states. Others - Aksoy \textit{et al.} (2002) - have used this framework to study how different decision procedures by the ECB would affect the economic outcomes and welfare in the EMU member-states. More recently, some authors have identified ranges of specific parameter values of theoretically derived models that could best mimic the broad stylized facts in the US data - Soderlind \textit{et al.} (2002), with a new keynesian model.

Dennis (2001) has also used dynamic programming, but, in contrast to the referred literature, he has simultaneously estimated - by full information maximum likelihood (FIML) - the economy structure and the loss function coefficients describing the monetary policy regime, for several US historical samples. His framework builds on a prior exercise of estimation of the Federal Reserve revealed preferences, using inverse-control theory, by Salemi (1995). The main differences are in the specified loss function, and in Salemi’s use of an unrestricted VAR to model the macro-economy.

\textsuperscript{61} See Litterman (1983) for an early example of a use of dynamic programming, together with a time-series representation of the dynamic behaviour of the economy. His model did not include, however, any final goal variables - inflation and output -, as it was meant to study the trade-off between interest rate and M1 volatility, in order to assess whether an optimal interest rate rule could be superior to the operational framework of the US 1979-82 monetarist experiment.

\textsuperscript{62} Examples of studies not using the Rudebusch-Svensson structural model are Fair's (2000, 2001a, 2001b) evaluations of alternative rules, with simulation methods, in his US and multi-country models.
dynamics - which implies that he conducted estimation with a two-step procedure because of the large number of parameters to estimate.\textsuperscript{63}

Dennis' approach is an effective alternative to the Favero and Rovelli (2001) framework, with possible advantages at two levels. First, the use of dynamic programming instead of optimal control eliminates the need for truncating the optimality conditions, rather considering the infinite horizon optimisation problem solved by the policymaker. Second, the use of FIML overcomes some weaknesses of GMM - namely related to serial correlation and non-stationarity of the moment conditions, and to the small dimension of the sample - which can interact to difficult the estimation of the variance-covariance matrix and, thus, the ability of GMM to generate efficient estimates. Certainly, FIML has its own weaknesses, most especially the need to assume normality of the system's innovations, small sample departure from asymptotic results, and higher sensitivity to mis specification problems.

Aware of both its advantages and disadvantages, we now proceed to estimate the Euro Area monetary policy regime of our sample following Dennis' method, for comparison with last section. The method can be described as follows.

Recall the dynamic optimization problem faced by the Monetary Authority:

\[
\text{Min}(L) = \text{Min} \ E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{2} \left( \pi_{t+i} - \pi^* \right)^2 + \lambda x_{t+i}^2 + \mu (i_{t+i} - i_{t+i-1})^2 \right] \quad (3.11)
\]

\textsuperscript{63} There are at least five additional important differences. First, Salemi uses monthly data, not based in national accounting statistics. Second, besides output, prices and interest rates, he also includes a measure of the money stock and a stock market index in his VAR. Third, he estimated the VAR with all series differenced, deseasonalized and linearly detrended. Fourth, he ran the inverse-control procedures and estimation for two alternative policy control variables - interest rate changes and, alternatively, growth rate of the money stock variable. Finally, Salemi's central bank loss function possibly includes all variables in the VAR, some of which specified in unusual ways - growth rates of output, prices, money, and stock index, and first differences of a short-term interest rate.
subject to\(^6^4\)

\[
x_t = c_0 + c_1 x_{t-1} + c_2 x_{t-2} + c_3 x_{t-3} + c_4 (i_{t-3} - \pi_{t-3}) + e_t^d
\]

\[
\pi_t = c_5 + c_6 \pi_{t-1} + c_7 \pi_{t-2} + c_8 \pi_{t-3} + (1 - c_6 - c_7 - c_8) \pi_{t-4} + c_9 x_t + c_{10} (\text{Im} \pi_{t-1} - \pi_{t-1}) + e_t^S
\]

The dynamic constraints can be written in state-space form as follows:

\[
A_i X_{t+1} = AX_t + Bu_t + C + E_{t+1} \tag{3.12}
\]

where \(u\) stands for the control variable - the interest rate, \(i\) - and \(X\) is the vector of state variables. Denoting \(\text{Im} \pi_{t-1} - \pi_{t-1}\) by \(I\pi_{t-1}\), the state-space form (3.12) can be further detailed as:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & -c_6 & 0 & 0 & 0 & 0 & 0 & -c_{10} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
\pi_{t+1} \\
\pi_t \\
\pi_{t-1} \\
x_{t+1} \\
x_t \\
x_{t-1} \\
i_t \\
i_{t-1} \\
\Delta_t \\
I_t
\end{bmatrix}
\]

\(^6^4\) Two minor variations from the model used in our Euler equation-GMM estimation are the inclusion of a constant in the Phillips equation \((c_5)\) and the imposition of dynamic homogeneity over that equation - i.e. restricting the sum of the coefficients on lagged inflation to 1. Both variations are made for ensuring comparability to Dennis' results for the US, and do not change qualitatively any results.
As the objective function is quadratic and the constraints are linear and stochastic, the problem fits into the stochastic optimal linear regulator framework - see, *inter alia*, Ljungqvist and Sargent (2000), and Hansen and Sargent (2001). We use the solution suggested by Chow (1981, 1983, 1997), which consists of introducing a vector of Lagrange multipliers, \( \lambda \), and setting to zero its derivatives in order to the control and states variables, thus obtaining a set of first-order conditions. In these, expectations at time 0 are replaced by expectations at time \( t \), highlighting that we are solving for the optimal closed-loop system, as we assume discretionary policy.

In order to solve these conditions for the control variable \( u \) and the multiplier \( \lambda \), Chow suggests approximating \( \lambda(X) \) by a linear function

\[
\lambda(X) = HX + h
\]  

\((3.13)\)

---

65 In this brief description of the solution method we follow closely Chow (1997), pages 22-24.

66 Notice the timing assumption of this model. At each period \( t \), the monetary authority observes the current state of the economy, that is, inflation and the gap up until period \( t \), together with the interest rates that it has set in the past. Then, he decides the interest rate for period \( t \). Then, period \( t \) demand
and the derivatives of the objective function with respect to the control and state
variables by linear functions as well

\[
\frac{\partial}{\partial X} L(X, u) = K_{11} X + K_{12} u + k_1
\]

(3.14)

\[
\frac{\partial}{\partial u} L(X, u) = K_{21} X + K_{22} u + k_2
\]

(3.15)

These three linear approximations, together with the linear constraint (3.12) and
the first-order conditions with respect to the state and the control, are the basis for the
solution.

From the first-order conditions relative to the control variable, and the linear
approximations (3.13) and (3.15), we obtain the optimal state-contingent linear policy
rule

\[
u_t = GX + g
\]

(3.16)

where

\[
G = -(K_{22} + \delta B' HB)^{-1} (K_{21} + \delta B' HA)
\]

(3.17)

\[
g = -(K_{22} + \delta B' HB)^{-1} [k_2 + \delta B'(HC + h)]
\]

(3.18)

Analogously, using the linear approximations (3.13) and (3.14) for adequate
substitution in the first-order conditions relative to the state-vector, we obtain

\[
H = K_{11} + K_{12} G + \delta A' H (A + BG)
\]

(3.19)

\[
h = (K_{12} + \delta A' HB) g + k_1 + \delta A' (HC + h)
\]

(3.20)

Combining equation (3.17) and (3.19), gives the matrix Riccati equation

\[
H = K_{11} + \delta A' HA - (K_{12} + \delta A' HB)(K_{22} + \delta B' HB)^{-1} (K_{21} + \delta B' HA)
\]

(3.21)

The Riccati equation can be solved iteratively for H. Given H, equation (3.17)
can be used to compute G, equation (3.18) to calculate g, and equation (3.20) to obtain
h.

and supply shocks occur, generating, together with the interest rate decided for \(t\), the new outcome of
the state-vector, at \(t+1\).
In our specific problem, k1 and k2 are equal to zero. We define the matrix $K_{11}$ as

$$
K_{11} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

Which implies that $K_{12}$ is set to zero.\(^{67}\) We further define $K_{21}$ and $K_{22}$ as follows:

$$
K_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
K_{22} = \mu
$$

The linear approximation to the dynamic system constraining the maximization of $L$ is simply the system (3.12) above, solved for the state vector at $t+1$ and excluding the innovations' vector, as follows:

$$
X_{t+1} = A_0^{-1}AX_t + A_0^{-1}B_i + A_0^{-1}C
$$

The estimation proceeds as follows. In each iteration, for certain given values of the model parameters – $c_0$, $c_1$, $c_10$, $\lambda$, $\mu$ - we solve the Riccati equation (3.21) iteratively for $H$. Then, $H$ is used to compute the coefficients of the optimal state-contingent policy rule, $G$, the constant of the optimal linear policy rule ($g$), and the constant of the solution to the Lagrangean ($h$).

The resulting optimal state-contingent policy rule is of the form

$$
i_t = g_0 + g_1x_t + g_2x_{t-1} + g_3x_{t-2} + g_4x_{t-3} + g_5x_t + g_6x_{t-1} + g_7x_{t-2} + g_8i_{t-1} + g_9i_{t-2} + g_{10}l_{t-1}
$$

\(^{67}\) Generally, there are alternative ways to write the linear approximation to the dynamic optimisation problem solution. We have tried a formulation in which $K_{12}$ is not set to zero. Specifically, we have written $K_{11}[10,10]=\mu$, $K_{11}[10,8]=\mu$, and $K_{12}[10]=-\mu$ (with $K_{22}$ and $K_{21}$ unchanged). Estimation results were not significantly different, showing that numerical equivalence also exists.
and joins the IS and Phillips in a system of three equations. We include an innovation in the interest rate equation, which, technically, solves the singularity in the variance-covariance matrix of the system that otherwise would exist.

The three equations system can be written as:

\[
F_0 Z_t = F_1 Z_{t-1} + F_2 Z_{t-2} + F_3 Z_{t-3} + F_4 Z_{t-4} + F_5 I\pi_{t-1} + K + \Xi_t
\]

(3.23)

where \(F_0, F_1, F_2, F_3, F_4\) and \(F_5\) are \((3 \times 3)\) matrices of coefficients adequately defined, and

\[
Z_t = \begin{bmatrix} \pi_t \\ x_t \\ h_t \end{bmatrix}, \quad I\pi_{t-1} = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}, \quad K = \begin{bmatrix} c_5 \\ c_0 \\ g_0 \end{bmatrix}, \quad \Xi_t = \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^d \\ \varepsilon_t^d \end{bmatrix}
\]

The vector of residuals of the system, \(\Xi_t\), is assumed to follow a multivariate normal distribution with mean zero and variance-covariance matrix \(\Omega\), with no additional restrictions. Under this assumption, we compute the log-likelihood function of the data from 1 to \(T\), conditional on the four first observations, using the square of the regression residuals as estimate of the variance-covariance matrix. The log-likelihood function is maximized with respect to the model parameters - IS, Phillips and loss function coefficients - by standard numerical methods.

As in last section, we estimate the real equilibrium interest rate as the ratio of the estimates \([c_0/(-c_4)]\). In what regards the monetary policy regime, this set-up estimates \(\lambda\) and \(\mu\), but does not offer a direct estimate of the inflation target, so we compute it as the sample average of the nominal interest rate minus the estimate of the real equilibrium interest rate. As in section 3.3, we adopt the standard error of the Phillips equation residual as some indication of the volatility of supply shocks. Analogously to the standard deviation of the Euler equation residuals in section 3.3, we interpret the

\[68\] In addition to the current information used in the Taylor (1993) rule, the optimal rule includes past deviations of inflation and the gap from their targets, as well as past values of the control variable. This additional past information allows policy to react more completely to economic conditions - see Bullard and Schaling (2001) for an example, showing how lagged state variables are crucial to account for changes in trend productivity.

\[69\] See, for instance, Chow (1983), page 170.

\[70\] We are indebted to Richard Dennis for sharing his GAUSS code, which is the basis for our own codes of this section. Estimation uses the procedure Optmum to minimise the symmetric of LogLik, with the
standard error of the optimal linear policy rule residual as some indication about the optimality of monetary policy - i.e., the ability of policymakers to run actual interest rates close to their optimal path. As before, this interpretation is exploratory, as the control theory attributes this error to specification imperfections.\(^71\)

### 3.4.2. Results

Table 3.6 reports the results of estimation of the system for the whole sample.

That table confirms that it is not possible to estimate reasonably and precisely the central bank loss function coefficients for the whole 1972-2001 period. Moreover, as the IS intercept is not statistically different from zero, the real equilibrium interest rate is not well estimated, which, in turn, precludes the estimation of a precise inflation target.

The optimal policy rule is close to a simple auto-regression of the interest rate, with a root of 0.97, showing almost no reaction of interest rates to inflation and a very moderate reaction to the gap. This rule achieves a good fit to the data second moments, especially those of inflation and interest rate, but not that good to the level of the interest rate.

\(^{71}\) BFGS algorithm and the Stepbt step method. Standard errors of the estimates are computed using the inverse of the Hessian matrix as estimator for the parameters variance-covariance matrix.

\(^{71}\) Building on inverse-control theory, Salemi (1995) and Dennis (2001) interpret this innovation as resulting from the fact that the econometrician does not observe the whole information used by the central bank to conduct policy.
Table 3.6 – Model with Baseline Loss Function

<table>
<thead>
<tr>
<th></th>
<th>Estimates</th>
<th>T-statistics</th>
<th>Significance Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IS equation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C0</td>
<td>0.006</td>
<td>0.33</td>
<td>0.37</td>
</tr>
<tr>
<td>C1</td>
<td>1.156</td>
<td>14.34</td>
<td>0.00</td>
</tr>
<tr>
<td>C2</td>
<td>-0.043</td>
<td>-0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>C3</td>
<td>-0.181</td>
<td>-2.35</td>
<td>0.01</td>
</tr>
<tr>
<td>C4</td>
<td>-0.0062</td>
<td>-1.54</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Phillips equation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>-0.052</td>
<td>-0.51</td>
<td>0.31</td>
</tr>
<tr>
<td>C6</td>
<td>0.524</td>
<td>6.76</td>
<td>0.00</td>
</tr>
<tr>
<td>C7</td>
<td>0.272</td>
<td>3.10</td>
<td>0.00</td>
</tr>
<tr>
<td>C8</td>
<td>-0.096</td>
<td>-1.10</td>
<td>0.14</td>
</tr>
<tr>
<td>C9</td>
<td>0.250</td>
<td>1.53</td>
<td>0.06</td>
</tr>
<tr>
<td>C10</td>
<td>0.038</td>
<td>3.62</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Policy regime:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ</td>
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<td>0.24</td>
</tr>
<tr>
<td>µ</td>
<td>31.503</td>
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<tr>
<td>π*</td>
<td>7.76†</td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ*</td>
<td>0.90†</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ (ε</td>
<td>IS)</td>
<td>0.149</td>
<td></td>
</tr>
<tr>
<td>σ (ε</td>
<td>PH)</td>
<td>1.090</td>
<td></td>
</tr>
<tr>
<td>σ(ε</td>
<td>OPR)</td>
<td>0.656</td>
<td></td>
</tr>
<tr>
<td><strong>Optimal Policy Rule</strong></td>
<td></td>
<td>Sums ††</td>
<td>Long-run</td>
</tr>
<tr>
<td>Gπt</td>
<td>0.019</td>
<td>0.039</td>
<td>1.300</td>
</tr>
<tr>
<td>Gπt-1</td>
<td>0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gπt-2</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gπt-3</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gx1</td>
<td>0.305</td>
<td>0.165</td>
<td>5.500</td>
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<tr>
<td>Gx1-1</td>
<td>-0.079</td>
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</tr>
<tr>
<td>Gx1-2</td>
<td>-0.061</td>
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<tr>
<td>Gi1</td>
<td>0.972</td>
<td>0.970</td>
<td>-</td>
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<td>Gi1-2</td>
<td>-0.002</td>
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<tr>
<td>GIπt-1</td>
<td>0.001</td>
<td>0.001</td>
<td>0.033</td>
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<tr>
<td><strong>Fitted series' σ</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U. Gap</td>
<td>0.674</td>
<td>0.691</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>3.578</td>
<td>3.573</td>
<td></td>
</tr>
<tr>
<td>Interest Rate</td>
<td>2.976</td>
<td>2.978</td>
<td></td>
</tr>
</tbody>
</table>

Estimation: FIML
Discount factor: δ=0.975; Standard-errors: square root of diagonal elements of inverse of the Information Matrix (Hessian); Significance probabilities relate to one-sided tests; † Imprecisely estimated because based on at least one coefficient with too large standard error; †† Sum of optimal policy rule coefficients of inflation, gap, interest rate and imp. inflation, respectively; π* = Inflation target, estimated as ( ̅i - r*), where ̅i is the sample average of nominal interest rate; Fitted series: observed series minus residuals of estimation of each series' equation.
The results of estimation of the system in the two sub-samples, 1972-1985 and 1986-2001, are reported in table 3.7.

There are some signs of a change of monetary policy regime from the first to the second sub-sample. First, for 1986-2001, the system estimates with precision a real equilibrium interest rate (4.78 percentage points) and, thus, the inflation target (2.41 percentage points). Second, the optimal linear feed-back rule changes markedly across the two sub-samples. The coefficient of reaction of current interest rates to its recent past decreases from 0.94 to 0.85. The cumulative short-term reaction of interest rates to gaps increases from 0.09 to 0.17 and, notably, the reaction of interest rates to current and recent inflation increases from 0.07 to 0.34. The change in the long-run responses of policy to inflation and the gap highlights more clearly the change in the conduct of policy across the sub-samples. While in 1972-1985 the long-run reaction of policy to the gap is higher than the reaction to inflation (1.56 versus 1.18), in the 1986-2001 period optimal policy is far more oriented towards price stability, as interest rates react to inflation more than they react to gaps - 2.28 versus 1.17.

As expected, our results show that the optimal linear policy reaction function would be activist with respect to inflation in both sub-samples, as the estimate of the long-run response of interest rates to inflation is larger than 1 in 1972-1985 as well as in 1986-2001.

The system mimics the data second moments better in the second period (especially for the gap and interest rates), and also fits the data first moments far better after 1985 - notably, the standard error of the interest rate equation residuals falls from 0.82 to 0.46. In spite of all these signs of a regime change at 1985, in table 3.7 we fail to estimate with reasonable precision the policymaker loss function coefficients, not only for 1972-1985 but also for 1986-2001.

---

72 Formal tests of the significance of such changes are not offered, as the coefficients of the optimal policy rule are not directly estimated, but instead are computed with dynamic programming results from the estimates of the model structural coefficients (IS, Phillips, and central bank's loss function).

73 Recall that in our policy reaction function the long-run response of interest rates to, for instance, inflation, is given by \((g_1\pi_t + g_2\pi_{t-1} + g_3\pi_{t-2} + g_4\pi_{t-3})/(1 - g_8\pi_t - 1 - g_9\pi_t - 2)\).

74 A long-run response of nominal interest rate to inflation below 1 is destabilising in modern small models such as ours, Clarida et al. (2000), and the FRB/US model described in Reifschneider et al. (1999), but not necessarily in larger and richer models - see Fair (2000, 2001a, 2001b). See also Woodford (2001a, page 233) for a more complete description and discussion of the determinacy condition, known as 'Taylor principle'.
### Table 3.7 – Model with Baseline Loss Function

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IS equation:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_0$</td>
<td>0.004</td>
<td>0.100</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1.053</td>
<td>1.244</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.138</td>
<td>-0.321</td>
</tr>
<tr>
<td>$C_3$</td>
<td>-0.300</td>
<td>0.077</td>
</tr>
<tr>
<td>$C_4$</td>
<td>-0.018</td>
<td>-0.021</td>
</tr>
<tr>
<td><strong>Phillips equation:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_5$</td>
<td>-0.126</td>
<td>0.084</td>
</tr>
<tr>
<td>$C_6$</td>
<td>0.440</td>
<td>0.617</td>
</tr>
<tr>
<td>$C_7$</td>
<td>0.334</td>
<td>0.104</td>
</tr>
<tr>
<td>$C_8$</td>
<td>-0.047</td>
<td>0.001</td>
</tr>
<tr>
<td>$C_9$</td>
<td>0.348</td>
<td>0.237</td>
</tr>
<tr>
<td>$C_{10}$</td>
<td>0.031</td>
<td>0.060</td>
</tr>
<tr>
<td><strong>Policy regime:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>138.043</td>
<td>-4.297</td>
</tr>
<tr>
<td>$\mu$</td>
<td>68.250</td>
<td>0.423</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>10.22†</td>
<td>2.41</td>
</tr>
<tr>
<td>$r^*$</td>
<td>0.20†</td>
<td>4.78</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{IS})$</td>
<td>0.163</td>
<td>0.128</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{PH})$</td>
<td>1.368</td>
<td>0.808</td>
</tr>
<tr>
<td>$\sigma(\varepsilon_{OPR})$</td>
<td>0.818</td>
<td>0.463</td>
</tr>
<tr>
<td><strong>Optimal Policy Rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{\pi_t}$</td>
<td>0.030</td>
<td>0.067</td>
</tr>
<tr>
<td>$G_{\pi_{t-1}}$</td>
<td>0.019</td>
<td>0.068</td>
</tr>
<tr>
<td>$G_{\pi_{t-2}}$</td>
<td>0.010</td>
<td>0.052</td>
</tr>
<tr>
<td>$G_{\pi_{t-3}}$</td>
<td>0.008</td>
<td>0.048</td>
</tr>
<tr>
<td>$G_{x_t}$</td>
<td>0.200</td>
<td>1.561</td>
</tr>
<tr>
<td>$G_{x_{t-1}}$</td>
<td>-0.042</td>
<td>-0.053</td>
</tr>
<tr>
<td>$G_{x_{t-2}}$</td>
<td>-0.069</td>
<td>0.015</td>
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<tr>
<td>$G_{i_{t-1}}$</td>
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<td>0.855</td>
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<tr>
<td>$G_{i_{t-2}}$</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>$G_{\pi_{t-1}}$</td>
<td>0.001</td>
<td>0.018</td>
</tr>
<tr>
<td><strong>Fitted series' $\sigma$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U. Gap</td>
<td>0.526</td>
<td>0.503</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.997</td>
<td>1.246</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>3.097</td>
<td>2.681</td>
</tr>
</tbody>
</table>

Estimation: FIML; Discount factor: $\delta=0.975$; Standard-errors: square root of diagonal elements of inverse of the Information Matrix (Hessian); Significance probabilities relate to one-sided tests; † Imprecisely estimated because based on at least one coefficient with too large standard error; †† Sum of optimal policy rule coefficients of inflation, gap, interest rate and imp. inflation, respectively; $\pi^*$ = Inflation target, estimated as $(\bar{i} - r^*)$, where $\bar{i}$ is the sample average of nominal interest rate; Fitted series: observed series minus residuals of estimation of each series' equation.
Motivated by the results in table 3.7, in the appendix, and in section 3.3, we now estimate regimes of strict inflation targeting with interest rate smoothing for the periods considered. Estimates for the whole sample are given in table 3.8. The main results are similar to those obtained with the baseline loss function. We fail to estimate precisely the real equilibrium interest rate, the inflation target, and the loss function coefficients.

Table 3.9 reports the estimates of strict inflation targeting regimes with interest rate smoothing for the two relevant sub-samples. As with the flexible inflation targeting loss function, the monetary regime break at 1985 is highly apparent: for 1972:I-1985:IV, most estimates of regime coefficients are not statistically significant, while for 1986:I-2001:II the opposite happens. The system estimates, with high precision, a real equilibrium rate of 4.6 percentage points and an inflation target of 2.6 percentage points during 1986:I-2001:II.

The optimal policy reaction function changes markedly across the sub-samples, with the reaction to past interest rates decreasing from 0.94 to 0.90 and the short-run reaction to inflation and output increasing to about 0.22 cumulative points for each.\textsuperscript{75} Compared to the homologous rule estimated with the baseline loss function, the 1986:I-2001:II optimal rule now features larger responses of interest rates to the gap both in the short-run - 0.23 versus 0.17 - and in the long-run - 2.34 versus 1.17. This long-run reaction surpasses the long-run response of interest rates to inflation - which keeps being estimated at 2.21. Hence, although the gap has not been an independent target in the loss function, it still has been important for policy making - most probably as a leading indicator of inflation.

The interest rate fitted by this loss model is observationally equivalent to that fitted by the baseline loss model, rendering a graphical comparison useless - the root of the mean square deviation between them is 0.07 percentage points. In contrast, given a loss function of strict inflation targeting, the regimes estimated for the two sub-samples differ significantly. In order to illustrate this, figure 3.6 shows that the optimal state-contingent policy rule implied by the estimates from the second sub-sample, performs worse in adjusting interest rates of the first sub-sample period. The standard error is 1.34 in 1972-1985, quite above the 0.464 computed for 1986-2001.
## Table 3.8 – Model with Strict Inflation Targeting Loss Function

<table>
<thead>
<tr>
<th></th>
<th>Estimates</th>
<th>T-statistics</th>
<th>Significance Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IS equation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C0</td>
<td>0.014</td>
<td>0.82</td>
<td>0.20</td>
</tr>
<tr>
<td>C1</td>
<td>1.158</td>
<td>14.45</td>
<td>0.00</td>
</tr>
<tr>
<td>C2</td>
<td>-0.053</td>
<td>-0.42</td>
<td>0.34</td>
</tr>
<tr>
<td>C3</td>
<td>-0.158</td>
<td>-2.10</td>
<td>0.02</td>
</tr>
<tr>
<td>C4</td>
<td>-0.009</td>
<td>-2.16</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Phillips equation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>-0.034</td>
<td>-0.34</td>
<td>0.37</td>
</tr>
<tr>
<td>C6</td>
<td>0.511</td>
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</tr>
<tr>
<td>C7</td>
<td>0.266</td>
<td>3.04</td>
<td>0.00</td>
</tr>
<tr>
<td>C8</td>
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<td>-0.96</td>
<td>0.17</td>
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<tr>
<td>C9</td>
<td>0.395</td>
<td>3.41</td>
<td>0.00</td>
</tr>
<tr>
<td>C10</td>
<td>0.037</td>
<td>3.54</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Policy regime:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu)</td>
<td>32.119</td>
<td>1.40</td>
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</tr>
<tr>
<td>(\pi^*)</td>
<td>8.79 †</td>
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</tr>
<tr>
<td>(r^*)</td>
<td>1.63 †</td>
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<td>(\sigma(\varepsilon_{IS}))</td>
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<tr>
<td>(\sigma(\varepsilon_{PH}))</td>
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<tr>
<td>(\sigma(\varepsilon_{OPR}))</td>
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<tr>
<td><strong>Optimal Policy Rule</strong></td>
<td></td>
<td>Sums ††</td>
<td>Long-run</td>
</tr>
<tr>
<td>(G_\pi)</td>
<td>0.025</td>
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<td>1.472</td>
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<tr>
<td>(G\pi_{t-1})</td>
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<td>(G\pi_{t-2})</td>
<td>0.007</td>
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<tr>
<td>(G\pi_{t-3})</td>
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<tr>
<td>(G_x)</td>
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</tr>
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<td>-</td>
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<td>(G\pi_{t-4})</td>
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<td>(G\pi_{t-4})</td>
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<td>0.001</td>
<td>0.028</td>
</tr>
<tr>
<td><strong>Fitted series' (\sigma)</strong></td>
<td></td>
<td>Data (\sigma)</td>
<td></td>
</tr>
<tr>
<td>U. Gap</td>
<td>0.686</td>
<td>0.691</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>3.613</td>
<td>3.573</td>
<td></td>
</tr>
<tr>
<td>Interest Rate</td>
<td>2.982</td>
<td>2.978</td>
<td></td>
</tr>
</tbody>
</table>

Estimation: FIML; Discount factor: \(\delta=0.975\); Standard-errors: square root of diagonal elements of inverse of the Information Matrix (Hessian); Significance probabilities relate to one-sided tests; † Imprecisely estimated because based on at least one coefficient with too large standard error; †† Sum of optimal policy rule coefficients of inflation, gap, interest rate and imp. inflation, respectively; \(\pi^*\) = Inflation target, estimated as \((\bar{i} - r^*)\), where \(\bar{i}\) is the sample average of nominal interest rate; Fitted series: observed series minus residuals of estimation of each series’ equation.

---

75 In both sub-samples, the in-quarter response of the instrument to the gap is about three times the size of its response to inflation, which is in line with Peersman and Smets (1999) and Aksoy et al. (2002).
Table 3.9 – Model with Strict Inflation Targeting Loss Function


<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimates</strong></td>
<td><strong>T-Stats</strong></td>
</tr>
<tr>
<td><strong>Estimates</strong></td>
<td><strong>T-Stats</strong></td>
</tr>
<tr>
<td>IS equation:</td>
<td></td>
</tr>
<tr>
<td>C0</td>
<td>0.006</td>
</tr>
<tr>
<td>C1</td>
<td>1.053</td>
</tr>
<tr>
<td>C2</td>
<td>0.133</td>
</tr>
<tr>
<td>C3</td>
<td>-0.289</td>
</tr>
<tr>
<td>C4</td>
<td>-0.020</td>
</tr>
<tr>
<td>Phillips equation:</td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>-0.116</td>
</tr>
<tr>
<td>C6</td>
<td>0.425</td>
</tr>
<tr>
<td>C7</td>
<td>0.328</td>
</tr>
<tr>
<td>C8</td>
<td>-0.034</td>
</tr>
<tr>
<td>C9</td>
<td>0.462</td>
</tr>
<tr>
<td>C10</td>
<td>0.028</td>
</tr>
<tr>
<td>Policy regime:</td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>56.98</td>
</tr>
<tr>
<td>( \pi^* )</td>
<td>10.14 †</td>
</tr>
<tr>
<td>( r^* )</td>
<td>0.28 †</td>
</tr>
<tr>
<td>( \sigma (\varepsilon_{\text{IS}}) )</td>
<td>0.164</td>
</tr>
<tr>
<td>( \sigma (\varepsilon_{\text{PH}}) )</td>
<td>1.367</td>
</tr>
<tr>
<td>( \sigma (\varepsilon_{\text{OPR}}) )</td>
<td>0.827</td>
</tr>
<tr>
<td>Optimal Policy Rule</td>
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</tr>
<tr>
<td>G( \pi_t )</td>
<td>0.031</td>
</tr>
<tr>
<td>G( \pi_{t-1} )</td>
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</tr>
<tr>
<td>G( \pi_{t-2} )</td>
<td>0.009</td>
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<tr>
<td>G( \pi_{t-3} )</td>
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<td>G( x_t )</td>
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<td>G( x_{t-1} )</td>
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<td>G( x_{t-2} )</td>
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<td>G( i_{t-1} )</td>
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</tr>
<tr>
<td>G( i_{t-2} )</td>
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<tr>
<td>G( \pi_{t-1} )</td>
<td>0.001</td>
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<td>Fitted series' ( \sigma )</td>
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<tr>
<td>U. Gap</td>
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<tr>
<td>Inflation</td>
<td>2.031</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>3.083</td>
</tr>
</tbody>
</table>

Estimation: FIML; Discount factor: \( \delta = 0.975 \);
Standard-errors: square root of diagonal elements of inverse of the Information Matrix (Hessian);
Significance probabilities relate to one-sided tests;
† Imprecisely estimated because based on at least one coefficient with too large standard error;
†† Sum of optimal policy rule coefficients of inflation, gap, interest rate and imp. inflation, respectively;
\( \pi^* = \text{Inflation target, estimated as } (\bar{i} - r^*) \), where \( \bar{i} \) is the sample average of nominal interest rate;
Fitted series: observed series minus residuals of estimation of each series' equation.
Most importantly, the panel relative to 1986:I-2001:II in table 3.9 shows the most precise estimation of the loss function weights that we achieve within the framework of this section. Specifically, the coefficient associated to interest rate smoothing during 1986:I-2001:II is estimated at 2, and is significant at 5 percent, which can be considered a reasonably precise estimate. Also, the point estimate of 2 is not as unreasonable as many of the others in tables 3.6 through 3.9, and seems more plausible than the estimates for the US case described in Dennis (2001). In addition, and interestingly enough, our estimate is compatible with the interval of values for the interest rate smoothing weight that Soderlind et al. (2002) have identified as necessary for their small New-Keynesian model to mimic the persistence and volatility in US inflation, gap and short-term interest rates ($1 \leq \mu \leq 2$).\footnote{Our estimate is smaller than the value that Fair (2000, 2001a) uses for stochastic simulations with his US model (2.0, which corresponds to 4.0, because inflation and the gap weight 0.5 each), as well as than the calibration in Fair's (2001b) simulations (9.0, which is actually 18.0). However, Fair's weight is attached to the deviations of interest rate changes to their baseline path and not to interest rate changes themselves, so that their values are not actually comparable to our weight $\mu$.}
It is often considered hard to accept, by conventional wisdom, that the instrument smoothing is so important, compared to inflation stabilization - see, for instance, Rudebusch (2001a). However, as Soderlind et al. (2002) note, there seems to be a paradox between the generalized acceptance of high coefficients on lagged interest rate in Taylor-type rules, and the notion that high weights of interest rate smoothing in L are hard to accept. Moreover, if, as argued in section 3.2.2, policy inertia results merely from the excessive smoothness of inflation forecasts, compared to its *ex-post* volatility, then it reflects serially correlated forecast (and possibly measurement) errors of inflation (and possibly gaps). In that case, the weight attached to interest rate smoothing is directly related to the inflation (and possibly gap) goals, making its large estimated magnitude less hard to accept.

In table 3.9 there is a marked decrease of the estimated Phillips elasticity across the sub-samples - from 0.462 to 0.141 - which, taken at face value, would mean that this method identifies a much more horizontal Taylor curve in the second period. This result is, however, at odds with the fact that the time-varying NAIRU used to compute the unemployment gap employed in this chapter's estimation has been estimated, in chapter 2, from an unobserved components model with a constant Phillips elasticity. Moreover, it contradicts the results obtained with optimal control and GMM, described in table 3.4, which exhibit a constant estimate of the Phillips elasticity, in harmony with chapter 2, of around 0.12. Furthermore, the result is unreasonable, as the estimate for the first sub-period is too large, and there is no reason to expect a decrease in the Phillips elasticity, after 1985, to one third of its hypothetical value of the first sample period. We suspect that the anomaly in the estimate relative to 1972:I-1985:IV is associated to the fact that FIML may generate very wrong results whenever a model is not well specified, as it tries to adjust the coefficients of the equation with the largest residuals in order to minimize them. Hence, we choose to disregard the 1972:I-1985:IV sub-sample results.

In short, the answers we obtain, with this method, for the relevant questions in this research, are as follows. First, the emergence of a well-defined aggregate monetary policy regime targeting a low rate of inflation, is part of the explanation of the reduction in macroeconomic volatility of the Euro Area since 1986. The regime is estimated to be one of strict inflation targeting, with a target rate of inflation around 2.6 percent, with a significant degree of interest rate smoothing. Second, there are signals that the supply
shocks impacting on the Area have been milder after 1986, as the standard error of the Phillips residual decreased by 41 percent, which could have caused, also, an inward shift of the Taylor efficiency frontier. Finally, there are some signals that the ability of policymakers to maintain interest rates closer to their optimal path has improved after 1986, as the standard error of the residual of the optimal linear policy rule decreased by an impressive 45 percent across the sub-samples. As in section 3.3, these signals can only be considered tentative, however: first, control theory ascribes an econometric meaning to the deviation of the control variable from its optimal path; second, the residuals have different meanings across sub-samples, as there is no clear policy regime in the first one.

The results obtained by Dennis (2001), for the US case, confirm the policy regime switch in that country at (the beginning of) the 80s, likewise we identify for the Euro Area at 1986. Moreover, Dennis' estimate of the output gap weight in the Fed's loss function after 1982 is highly imprecise and, hence, not statistically significant, suggesting a regime of strict inflation targeting with interest rate smoothing. This is similar to the regime that we estimate for the Euro Area after 1986, and is compatible with Soderlind et al. (2002) condition that a very small preference for output stabilization is needed for a new keynesian model to match inflation's low volatility and output's high volatility and persistence. Interestingly, according to Dennis' estimates, in the Volcker-Greenspan policy regime the level of inflation targeted by the US Fed is estimated at 1.4 percent, which is fairly below our estimate for the notional Euro Area central bank.

We end this section examining the sensitivity of the results to the structural break detected above for the Area IS-Phillips model around 1995:II. Specifically, we have estimated the strict inflation targeting regime (with interest rate smoothing) with data throughout 1986:1-1995:II. The weight of interest rate smoothing is estimated at 1.74, instead of 1.99, and with higher imprecision - significance level of 12 percent. The overall fit of the model is slightly worse than in table 3.9, which is especially true for the model ability to mimic the interest rates second moments - with fitted rates exhibiting somewhat an excessive volatile. Hence, no significant differences in policy
regime seem to exist, whilst the data scarcity problem seems to affect more severely the estimation of the system.\textsuperscript{77}

### 3.4.3. Optimal Control and GMM \textit{versus} Dynamic Programming and FIML

The FIML estimation based on a dynamic programming solution of the policymaker problem, and the GMM estimation with an optimal control solution, yield the same answers to the questions motivating this research.

Both set-ups suggest that the sample period should be partitioned, and react well to the partition at 1985:IV. Both suggest that it is only possible to identify an equilibrium real interest rate in the second sub-sample, and broadly agree in their estimate (4.5 - 4.6). Both frameworks precisely estimate very similar inflation targets for the 1986:1-2001:II period (2.7 - 2.6), and fail in doing so for 1972:1-1985:IV. Both clearly indicate that the Euro Area policy regime of 1986:1-2001:II, has been one of strict inflation targeting with interest rate smoothing. Both calculate a decrease in the standard error of the residuals of their optimizing equations slightly above 45 percent, after 1986 - which is remarkable, given the differences between the Euler equation and the optimal linear policy rule. And both agree in their estimation of the fall in the standard error of the Phillips curve residuals, after 1986 - precisely 41 percent for both.

As reviewed above, there is less agreement between the estimates obtained from these two approaches for the US case. First, the inflation targets estimated for the most recent regime are 2.63 percent in Favero and Rovelli (2001) and 1.38 percent in Dennis (2001). Second, and most particularly, while Favero and Rovelli find a statistically significant weight of the output gap in the Fed's loss function, Dennis finds a point estimate of that weight that is much larger but he can not rule out the hypothesis that the true coefficient is zero. In view of the similarity of our broad results across both methods - except for the divergence discussed below - we conclude that the differences

\textsuperscript{77} We have also checked the sensitivity of our results to the timing assumption in the model. Specifically, we have estimated the model assuming that the central bank watches inflation and the gap up until period (t-1), when deciding his policy for period (t). This gives an optimal state contingent linear policy rule with inflation and the gap lagged one period in comparison to the specification in the text - resembling the Taylor rule used in Cogley and Sargent (2001). Our estimation results show an increase in the lag of optimal interest rates to actual rates, but the essence of the econometric results is unchanged.
between Favero and Rovelli results and Dennis' can originate from the disparities in their empirical procedures discussed in section 3.1 above.

Coming back to the Euro Area case, the Euler equation estimated by GMM fits the first and second moments of interest rate somewhat worse than the optimal state-contingent linear policy rule estimated with FIML. The root of the mean square error (RMSE) of fitted rates is 0.62 for the Euler equation, and is 0.46 in the optimal-policy-rule estimated by FIML. The standard deviation of the series of fitted interest rates is 3.11 in GMM and 2.63 in FIML, against a sample standard deviation of 2.65.

Figure 3.7 – Actual versus Fitted Interest Rate, 1986:I-2001:II, Loss Function: SITIRS, Dynamic Programming and FIML, and Optimal Control and GMM

Notes: GMM fitted interest rates obtained by dynamically solving the IS-Phillips-Euler system, using the coefficient estimates obtained for the sample period 1986:I-2001:II; FIML fitted interest rates computed as observed interest rates minus residuals of estimated optimal policy rule for the sample period 1986:I-2001:II.

Figure 3.7 shows that the interest rates fitted by the Euler equation (GMM) and by the optimal policy reaction rule (FIML) are not dramatically different. By construction, they are not, however, strictly comparable: the series of GMM fitted
interest rates is obtained by dynamically solving the estimated IS-Phillips-Euler system, assuming perfect knowledge of the inflation rate at 3 and 4 quarters ahead.\textsuperscript{78}

One important difference that is apparent in the figure is that the rates fitted with the FIML and dynamic programming approach tend to lag actual and GMM fitted rates. The contemporaneous correlation between the interest rates fitted by the two methods is 0.966, whereas it increases to 0.978 when the GMM interest rates are lagged once. This reflects the dominance of the auto-regression element in the optimal policy rule, which is associated to the high estimate of the interest rate smoothing weight obtained with the dynamic programming approach: 1.999, versus 0.014 in Euler-GMM.

We come now to the heart of the difference between the results in sections 3.3 and 3.4: the estimate of \( \mu \). The difference already exists - and is even quantitatively more important - between results for the US in Favero and Rovelli (2001) and Dennis (2001). When comparing his results to those of Favero and Rovelli, Dennis suggested that the difference appears to stem from two facts. First, GMM models the interest rate changes - in the Euler equation - while the interest rate equation in FIML is estimated in levels. Second, GMM implies a truncation of the policy horizon - in practice, it assumes that \( \delta = 0 \), for all quarters \( i \geq 5 \) - while FIML considers the infinite horizon optimization problem when estimating the policymakers' optimal reaction function. This could be important, for Dennis, as it takes long and variable lags for monetary policy to impact on the real economy. Soderlind et al. (2002) also points out this second reason as the main explanation for the divergences in results.

There are also econometric differences possibly affecting the results. From certain points of view, the FIML approach is more restrictive and sensitive than GMM, as it requires the assumption of normality of the residuals of the structural system, while GMM depends only on a set of orthogonality conditions and not on probabilistic assumptions - see Wooldridge (2001). Also, FIML may be more sensitive in that it adjusts the coefficient estimates to improve the fit of the equation with worse mean square errors of the system. On the other hand, GMM is more sensitive to non-

\textsuperscript{78} If the fitted interest rate from the GMM approach was computed as that of the FIML framework - observed series minus interest rate equation estimation residuals - we would obtain a fitted series observationally equivalent to the actual series. This is caused by the small magnitude of the Euler equation residuals, when estimated - as in this research - using actual future values of inflation in place of its expected values.
stationarity of the moment conditions - Hamilton (1994, page 424) -, than FIML is to non-stationary time-series in the system. Also, GMM could suffer more from small-sample problems, which especially difficult the estimation of the variance-covariance matrix when the moments are serially correlated as in our case. It is not clear, at this stage, the net effect of all these econometric particularities.

Favero and Milani (2001), and Castelnuovo and Surico (2001), have offered evidence somewhat suggestive that the interest-rate-smoothing puzzle could be solved by some consideration of the model uncertainty faced by policymakers, using Granger's (2000) thick modeling approach.

Another hypothesis is that the puzzle could be caused by the fact that the Euler/GMM framework uses actual future values of inflation, in place of expectations, while the dynamic-programming/FIML approach uses only actual current and lagged state variable values. If policy inertia is caused by expectations errors - the excessive smoothness of inflation forecasts being passed on to the policy rates path - it could happen that the Euler/GMM framework generates lower interest rates smoothing weights estimates. If this hypothesis is true, then the estimates of the degree of optimal policy inertia from both methods would only converge if inflation expectations were replaced, in the Euler equation, by expectations available to policymakers in real-time. We have no chance, however, of testing such an hypothesis, at least for the time being.

Finally, we explore another possible explanation. The dynamic programming/FIML approach is based on the lagrangean method of solution to the optimisation problem, which, as Chow (1997, page 25) notes, always finds an optimal control function, even when the system does not reach a steady state. Now, the state vector converges to an equilibrium if and only if the matrix governing the dynamics of $X_t$ under optimal control has all its characteristic roots smaller than unity in absolute value - see Ljungqvist and Sargent (2000, chapter 4). From equations (3.22) and (3.16) above, we have.

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80 In subsequent research, Castelnuovo and Surico (2002) fix $\mu = 0.2$ - the standard value assumed by Rudebusch and Svensson (1999, 2002) - and estimate the inflation and gap variability weights minimising the distance of the interest rate fitted by the optimal state-contingent linear rule to that fitted by the unconstrained estimate of the rule.
\[ X_{t+1} = A_0^{-1}AX_t + A_0^{-1}B_i + A_0^{-1}C \quad \Leftrightarrow \quad X_{t+1} = \tilde{A}X_t + \tilde{B}i_t + \tilde{C} \]

\[ \Leftrightarrow X_{t+1} = \tilde{A}X_t + \tilde{B}(GX_t + g) + \tilde{C} \quad \Leftrightarrow \quad X_{t+1} = (\tilde{A} + \tilde{B}G)X_t + (\tilde{B}g + \tilde{C}) \]

Hence, the optimal dynamic system for \( X_t \) will only be stable - \( X_t \) will converge to a unique stationary distribution - if the maximum absolute value of the eigenvalues of matrix \( (\tilde{A} + \tilde{B}G) \) is strictly smaller than unity. Some of the studies elsewhere in the literature check for this condition. For instance, studying data of the 11 EMU member-states within a dynamic programming framework close to ours, Aksoy et al. (2002) report maximum eigenvalues of 0.99 for all countries.

In studies that use dynamic programming to compute the optimal linear policy rule, given estimates of the structural parameters of the model, roots so close to unity do not create problems. However, when dynamic programming is used together with non-linear estimation, such proximity to non-stationarity may create numerical problems.

Table 3.10 shows that the maximum value in modulus of the characteristic roots of the optimal dynamic matrix for the state vector is almost always numerically indistinguishable from one, in this study. The eigenvalue that is further away from unity is the one of the strict inflation targeting regime estimated for 1986:I-2001:II, in which case the maximum root of the system is 0.983. Interestingly, this is, among our estimates, the case where the loss weights are most precisely estimated and, indeed, have more reasonable point estimates.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>( L(\pi_t, x_t, \Delta i_t) )</td>
<td>0.996</td>
<td>0.992</td>
<td>1.007</td>
</tr>
<tr>
<td>( L(\pi_t, \Delta i_t) )</td>
<td>0.995</td>
<td>0.991</td>
<td>0.983</td>
</tr>
</tbody>
</table>

Values reported are the maximum of the characteristic roots, in modulus, of matrix \( (\tilde{A} + \tilde{B}G) \), which is the optimal dynamics of the state vector given by the solution to the dynamic optimization problem.

As before, we finalize this section by checking whether the results would be different if the estimation period is restricted to 1986:I-1995:II, to assess if the well
identified monetary regime beginning in 1986 has significantly changed in 1995. We find that the greater absolute value of the eigenvalues of the optimal state-vector dynamics is 0.985, for that period, which is very much close to the maximum absolute value of the characteristic roots for 1986:I-2001:II.\footnote{Moreover, we have checked (i) whether results would change when data is limited to more recent observations, and (ii) whether they would change if the interest rate smoothing parameter was the only to be estimated by maximum likelihood. As to (i), we have estimated this model for periods 1988:I-2001:II, 1989:I-2001:II and 1990:I-2001:II. The smallest estimate we obtained for the interest-rate smoothing weight has been 1.31, with 1 percent significance, for 1988-2001. The maximum absolute value of the eigenvalues of the state optimal dynamics is 0.967, the lowest characteristic root we obtained across all the sub-samples. In what regards (ii), we have estimated the model inputting the estimates for the IS and Phillips parameters obtained by estimation of that system, leaving only the interest rate smoothing parameter to be estimated by maximum likelihood. Its estimate changed slightly to 1.873, but its t-statistic increased to 3.03 (significance $\cong 0.01$). The optimal feed-back rule did not change significantly. This model records}

In the end, we consider the interest rate smoothing question an unsolved puzzle. We have offered arguments that seem to suggest that neither GMM results with optimal control, nor FIML results with dynamic programming, should be considered superior, with our present knowledge of the problem. Yet, we have shown that there are reasons to cast some doubts on the numerical results of FIML estimation based on the lagrangean method of solution to the dynamic programming problem. Fortunately, the results from both methods are qualitatively identical, so our conclusions in this research seem to be reasonably robust.


3.5.1. Definition of Asymmetric Policy Preferences

So far, we have modelled policymakers' preferences symmetrically with respect to the cyclical and inflationary state of the economy. Formally, and following the literature, we have considered quadratic loss functions, which attach equal weight to positive and negative deviations of the goal variables from their targets. In such functions, the loss increases linearly with the distance of the goal variables from the target, meaning that it is more important to return to the target the further away from it the variable is. Both these characteristics of quadratic loss functions are considered
attractive and intuitive - see, for instance, Svensson (2001c). In addition, quadratic functions are tractable, while large analytical complexities could arise from polynomial loss functions of higher order.

However, there is a growing recent literature focusing on departures of central bank loss functions from the quadratic form.

Cukierman (2000, 2001) challenges the assumption that, given an inflation level, positive real gaps are as disliked as negative gaps of the same dimension, and that, given a real gap level, positive and negative deviations of inflation from target cause the same loss.\(^8\) He focuses on inflationary bias asymmetry, designing a loss function in which central bankers strictly target inflation in expansions, but react to the output gap at recessions. That specification is meant to analytically describe his observation that the political establishment is sensitive to the social costs of recessions, and that in democratic societies, even independent, but accountable, central banks, are not totally insensitive to the wishes of the political establishment. This argument had also been suggested by Goodhart (1998), who further argued that this asymmetry could derive from the fact that policymakers are typically uncertain about the current and future state of the economy, thus having a natural tendency to delay restrictive policy actions for longer than expansionary measures. Blinder (1998, p.19-20) has been profusely cited as an insider confirmation of his hypothesis

“In most situations the CB will take far more political heat when it tightens pre-emptively to avoid higher inflation than when it eases pre-emptively to avoid higher unemployment”.

This asymmetry hypothesis offers a new explanation for the inflation bias observed throughout much of the XX\(^{th}\) Century: in the presence of uncertainty, a central bank with such a loss function makes the probability of erring on the side of tightness smaller than that of erring on the side of ease. This explanation, moreover, has the appealing feature of not implying the assumption that policymakers target output above its natural level - central to the Kydland and Prescott (1977), and Barro and Gordon (1983), dynamic inconsistency paradigm. In fact, recent statements by central bank

\(^8\) Also a similar ability to mimic the first and second moments of data. The maximum absolute value of eigenvalues of the matrix of optimal dynamics of the state-vector is, in this case, 0.9814.
insiders - Vickers (1998) and Blinder (1998) - as well as remarks from academics - McCallum (1995, 1997) - had been noting, that it is unlikely that contemporaneous central banks systematically target output levels above expected natural output.

Inflationary bias asymmetry was not the exclusive centre of Goodhart's (1998) attention, though, as he argued that there are reasons to believe that central bankers have an inner-conflict that may result in two opposing types of asymmetry. On one hand, they are not immune to social pressures and feel great dissatisfaction when the economy is in a recession, so they would tend to strengthen their actions when output is below potential than when it is above. But, on the other hand, they want to credibly pursue their main objective of price stability and thus tend to have a stronger attitude against inflation pulling above the target than against inflation below the target - which would generate a deflationary bias. These two aims are conflicting, as inflation is typically a pro-cyclical variable, and Goodhart (1998, page 18) raised the hypothesis that they could perhaps balance out. Nobay and Peel (1998) studied theoretically a model of a central bank with asymmetric preferences, allowing, a-priori, for both inflationary and deflationary asymmetry.

Deflationary bias asymmetry, like inflationary asymmetry, is caused by the combination of asymmetric preferences and the uncertainty typically faced by policymakers. If a central bank needs to build a new credibility as inflation fighter, in order to consolidate a regime of low and stable inflation, his reaction to uncertainty is to prefer to fail on the low-inflation than on the high-inflation side of the target. This deflationary bias has actually been the subject of some references during the 1990s, in light of the context of disinflation and commitment to low inflation in developed countries since the mid-80s - see Fischer (1994). Hence, this asymmetry hypothesis may be more relevant to explain recent episodes than the inflationary bias asymmetry - for instance, the nominal convergence process ahead of the EMU, and the definition of the

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82 This reasoning of Cukierman (2000, 2001) seems to be hard to observe in practice, because inflation and output are significantly correlated - which could be an additional reason to use quadratic loss functions.

83 The implementation of inflation targeting regimes in some countries during the 90s was meant to deal with this inner conflict of central bankers, especially important in systems where governments have had fully discretionary power over monetary policy. For a recent evaluating review of the first decade of inflation targeting see Mishkin and Schmidt-Hebbel (2001).
ECB inflation target have been considered by some observers as asymmetric in the
deflationary sense.\footnote{As Mishkin and Schmidt-Hebbel (2001, pages 26-30) recently reviewed, there is not yet any consensus on what may be the optimal long-run inflation target level, but all Inflation Targeters have been choosing long-run inflation goals slightly above zero (typically some interval between 1 and 2, or 3, percent). This has arguably been their pragmatic solution to the conflict between the need for credibility and the desire of minimising the probability of deflation.}

There are also some references in the literature to a somehow different type of asymmetry - the hypothesis that monetary authorities increase rates in different pattern from that of decreases in interest rates. Goodhart (1996, 1998) noted that because interest rate increases are treated as bad news, monetary authorities could be led to increase rates less regularly, and in larger jumps, than they decrease them. Cukierman (1992, page 121) wrote

“(...) it is pretty clear that banks dislike large unexpected swings in interest rates, particularly if they are upward.”

However, most direct comparisons between the pattern of interest rate increases and decreases in the literature have not found significant evidence of differences - see Rudebusch (1995) and Goodhart (1996).

The empirical literature of monetary policy asymmetry has, so far, tested for policymakers' asymmetry within the framework of monetary policy reaction functions of the Taylor (1993) rule type. This is the case of, \textit{inter alia}, Blinder (1997), Clarida and Gertler (1997), Clarida \textit{et al} (1998, 1999), Dolado \textit{et al}. (2000, 2002), Bec \textit{et al}. (2001), Orphanides and Wieland (2000), and Martin and Milas (2001). This approach is not satisfactory, not only because of the empirical difficulties in identification and estimation of forward-looking Taylor rules, but also because these rules are not direct evidence on the deep policymakers' preference parameters, as reviewed in chapter 1 of this dissertation. In turn, the studies developed within frameworks alternative to that of Taylor-type rules, either have not offered formal evidence on the policymakers' loss function coefficients - Mishkin and Posen (1997), Gerlach (2000) - or have done so in frameworks restricted by somewhat stringent assumptions - Ruge-Murcia (2001 a, b).

Next, we present a new framework to test for asymmetries in a central bank loss function, and use it to enhance our knowledge about the monetary policy regime of the Euro Area since 1986:I – the period when a well-defined monetary policy regime seems
to have emerged in the Area. Our testing framework brings together two strands of the literature that have been dissociated so far - the estimation of policymakers' preferences and the study of asymmetric policy preferences. Specifically, we show that the approach based on optimal control and GMM estimation used in section 3.3 of this chapter can be extended to allow for formally testing the hypothesis that policymakers' loss function coefficients may be asymmetric across recessions and expansions, for a given specific target of inflation. Our framework nests the standard case of a quadratic loss function - that is, symmetry of preferences - and does not impose *ex-ante* inflationary or deflationary asymmetry. Moreover, in addition to the standard hypothesis of asymmetric reaction of policy to inflation and gaps across expansions and recessions, we test the hypothesis of asymmetry in interest rate smoothing as a function of the cyclical state of the economy.

### 3.5.2. Tests of Asymmetric Policy Preferences

**Preliminary remarks**

In section 3.3, with the central bank loss function constrained to a quadratic functional form, we have estimated the inflation target at 2.73 percentage points (with optimal control and GMM) and have found that the unemployment gap is not statistically significant in the Euro Area policymakers' objective function during 1986:I-2001:II. We now offer an alternative interpretation of those findings, by considering the official inflation target implicitly assumed throughout the period and allowing for asymmetry in policymakers' preferences across recessions and expansions.

The implicit official inflation target of the German central bank has been 2 percent per year, since 1986. This has been the level of inflation that the Bundesbank has considered compatible with price stability, since that year, and the basic statutory objective of the Bundesbank is price stability. The ECB inflation target interval is also not incompatible with a point target of 2 percent, even though some authors - as Gali (2002b) - note that the reference value for money growth implies a target range between 1 and 2 percent, and others - Svensson (2002) - criticise the asymmetry in the target. Here, we find reasonable to adopt 2 percent as the official inflation target in the Euro Area during 1986:I-2001:II.
The inflation target estimated with a quadratic loss function, 2.73 percentage points, and the official target of 2 percent, can be reconciled by allowing for asymmetry in the loss function across recessions and expansions. Econometrically, there is a problem of under-identification in models trying to estimate simultaneously the inflation target and an unconstrained loss function functional form. If a quadratic loss is assumed, we can estimate the inflation target, whereas if an inflation target is imposed, estimation calculates a functional form for the loss function. These two exercises yield results that are observationally equivalent in the limit case of a completely unconstrained loss functional form. We now assess whether the marked difference between the official inflation target and 2.73 percent can be addressed with a loss function that is asymmetric with respect to business cycles, thus offering a new view of section 3.3 results.

A simple inspection of the data allows some preliminary informal conclusions. The sample average of the unemployment gap is -0.16 (percentage points) and that of the inflation rate is 3.05 (percent per year). Our estimate of the inflation target is not statistically different from the sample average (significance probability of 31 percent in a Wald test), but is significantly above the official 2 percent target (Wald test significance probability of 0.007). Hence, real activity has been close to potential, on average, while inflation has been, on average, 1 percentage point per year above the official target – i.e., there has been somewhat an inflationary bias in the Area during 1986:I-2001:II.

One alternative interpretation would be considering that the 2 percent target might not have been a binding target, but merely a reference number with a political role, perhaps similar to the one played by money target values in Germany according to Von Hagen (1999). If so, our best guess for the true inflation target would be the estimate of 2.73 percent associated to a symmetric loss function (with the associated interval of statistical uncertainty). Because of the under-identification problem described above, we can not econometrically address this question.

Another alternative interpretation would be pointing out that the excess of average inflation over the official target might have been the result of a gradual disinflationary process beginning in 1986, with the definition of the new 2 percent
target, and only ending later in the sample. This interpretation is, however, at odds with the precise estimation of an inflation target for the 1986:I-2001:II period, which should not have been possible with such a disinflationary scenario. Moreover, modelling loss asymmetry across expansions and recessions should control for such a scenario of systematic fall in inflation.

Our choice of a regime of strict inflation targeting is also further scrutinized in this section. Specifically, our framework allows testing whether the rejection of the statistical significance of the unemployment gap term in the loss function is robust to the possibility of an asymmetric response of policy to the cyclical state of the economy. The relevant hypothesis here seems to be whether the coefficient associated to the gap element is statistically significant (and correctly signed) in recessions.

**Framework and Results**

We assess asymmetry of the loss function extending the optimal control framework and GMM framework of Favero and Rovelli (2001) using dummy variables to distinguish between quarters in which the economy is in a recession - negative unemployment gap - from quarters in which it is in an expansion - positive unemployment gap. Specifically, we specify a threshold quadratic loss function: a quadratic function with weights, attached to each objective variable, that can assume two different values, one at expansions and other at recessions.  

---

85 We do not use the linex form used by Nobay and Peel (1998) and Ruge-Murcia (2001 a, 2001 b), as its behaviour does not diverge significantly from that of the threshold quadratic, for reasonable loss weights and the values of gaps and inflation that exist in the data. In the presence of a significant asymmetry, the linex functional form behaves exponentially to one side of 0 and linearly to the other side of 0, while the threshold quadratic behaves always non-linearly, with different increasing rates to the left and right-hand-side of 0. For reasonable coefficients, these functions cross each other at some point to the left and some point to the right of 0. Further away from those intersection points, the linex increases at increasingly higher rates than the quadratic threshold, in its exponential branch, and at increasingly lower rates than the asymmetric quadratic, in its linear branch. However, these unbounded divergences are only significant (for reasonable coefficients) at points that are mostly out of the range of unemployment gaps and inflation deviations (from target) present in our sample.
The loss function of the strict inflation-targeting regime is now defined as

\[
L = E_t \sum_{i=0}^{\infty} \delta^i \left[ \frac{1}{2} \phi^{REC} (\pi_{t+i} - \pi^*)^2 + \mu^{REC} (i_{t+i} - i_{t+i-1})^2 \right], \quad \pi_{t+i} < 0
\]

\[
L = E_t \sum_{i=0}^{\infty} \delta^i \left[ \frac{1}{2} ((1 - \phi^{REC})(\pi_{t+i} - \pi^*)^2 + \mu^{EXP} (i_{t+i} - i_{t+i-1})^2 \right], \quad \pi_{t+i} \geq 0
\]

The dummies are defined with respect to the cyclical state of the economy, but they relate very similarly to the deviation of inflation from target, because of the contemporaneous positive and significant association between the gap and inflation – the statistically significant Phillips elasticity.\(^{86}\) We chose to normalise the weight associated by policymakers to deviations of inflation from the target during expansions to one minus the weight associated during recessions, in order to restrict the number of parameters to estimate and, thus, facilitate the convergence of the estimation criteria.

As in section 3.3 above, we truncate the optimal control problem 4 quarters ahead, and, considering the cross-equation restrictions, obtain the following first order conditions for either state of the economy:

\[
\delta^3 E_t \phi^{REC} (\pi_{t+3} - \pi^*)[c9,c4] +
\delta^3 E_t \phi^{REC} (\pi_{t+4} - \pi^*)[c9,c1,c4 + c5,c9,c4] +
\left[ \mu^{REC} (i_t - i_{t-1}) - \mu^{REC} \delta E_t (i_{t+i} - i_t) \right] + e^p_t = 0 \quad \pi_{t+i} < 0
\]

\[
\delta^3 E_t (1 - \phi^{REC}) (\pi_{t+3} - \pi^*)[c9,c4] +
\delta^3 E_t (1 - \phi^{REC}) (\pi_{t+4} - \pi^*)[c9,c1,c4 + c5,c9,c4] +
\left[ \mu^{EXP} (i_t - i_{t-1}) - \mu^{EXP} \delta E_t (i_{t+i} - i_t) \right] + e^p_t = 0 \quad \pi_{t+i} \geq 0
\]

These are then merged into one single Euler equation, using the dummy variables referred to above, generating a Euler equation that allows for asymmetries in both the inflation gap and the interest rate smoothing elements, while nesting the symmetry case.\(^{87}\) Symmetry exists when \(\mu^{EXP}\) and \(\mu^{REC}\) are not significantly different from each other and when \(\phi^{REC}\) is not statistically different from \((1-\phi^{REC})\), and, thus,

\(^{86}\) We have run the tests with dummies distinguishing between inflation above and below the target, and the results were fairly similar.

\(^{87}\) Note that this strategy changes the normalisation adopted for the inflation-gap term in the loss function from 1 to 0.5. This changes the numerical estimates of the other coefficients but does not change the statistical behaviour of the model - normalisation at 1 is as arbitrary as at 0.5.
these are the two null hypothesis of interest to test. Rejection of any of the null hypothesis is statistically incompatible with symmetry and is compatible with asymmetry of the loss function in that particular argument.

The resulting Euler equation is estimated jointly with the Phillips and IS equations, using GMM as in section 3.3, setting the inflation target at the official 2 percent level. We test our hypothesis using Wald tests on the relevant coefficients.

Table 3.11 summarises the results of these tests, for 1986:1-2001:II, within the strict inflation targeting regime.

Table 3.11 – Tests of Asymmetry of Loss Function - Strict Inflation Targeting with Interest Rate Smoothing, $\pi=2\%$


<table>
<thead>
<tr>
<th>MODEL: Strict Inflation targeting with interest rate smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$ target</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>$\mu^\text{REC}$</td>
</tr>
<tr>
<td>$\mu^\text{EXP}$</td>
</tr>
</tbody>
</table>

**H0:** $\mu^\text{EXP}=\mu^\text{REC}$

The interest rate smoothing part of $L$ is not asymmetric with respect to the cyclical state of the Economy, given that the inflation-gap element of $L$ is symmetric

<table>
<thead>
<tr>
<th>Official: 2%</th>
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</thead>
<tbody>
<tr>
<td>$\mu^\text{REC}$</td>
</tr>
<tr>
<td>$\mu^\text{EXP}$</td>
</tr>
</tbody>
</table>

**H0:** $\phi^\text{EXP}=\phi^\text{REC}$

The inflation-gap part of $L$ is not asymmetric with respect to the cyclical state of the Economy, given that the interest rate smoothing element of $L$ is symmetric

<table>
<thead>
<tr>
<th>Official: 2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^\text{REC}$</td>
</tr>
<tr>
<td>$\phi^\text{EXP}$</td>
</tr>
</tbody>
</table>

**H0:** $\phi^\text{EXP}=\phi^\text{REC}$ \land $\mu^\text{EXP}=\mu^\text{REC}$

The inflation-gap part of $L$ and the Interest rate smoothing element of $L$ are symmetric with respect to the cyclical state of the Economy

<table>
<thead>
<tr>
<th>Official: 2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^\text{REC}$</td>
</tr>
<tr>
<td>$\phi^\text{EXP}$</td>
</tr>
<tr>
<td>$\mu^\text{REC}$</td>
</tr>
<tr>
<td>$\mu^\text{EXP}$</td>
</tr>
</tbody>
</table>

Estimation: two-step GMM. Instruments: constant, $\Delta\pi_{t-i}, \text{Ugap}_{t-i}, \text{stir}_{t-i}, (I\pi-\pi)_{t-i}, i=1,\ldots,4$;
Discount factor: $\delta=0.975$; Variance-Covariance matrix HAC: Andrews and Mohanan (1992) pre-whitening; Bartlett kernel, bandwidth estimated with Andrews (1991) method;
In models where the inflation-gap part of $L$ is not allowed to be asymmetric, its weight in $L$, $\phi$, is 0.5.
The table shows that, given an inflation target of 2 percent, there is no statistical
evidence of threshold asymmetry in the coefficient associated to interest rate smoothing
in the loss function, that is, the policymaker did not change policy rates differently in
recessions and expansions. In contrast, there is significant evidence of asymmetry in the
inflation-gap element of the loss function when the official 2-percent target is used: the
Wald test for the null of equality of the inflation-gap coefficients between recessions
and expansions has significance of 0.0015. Those coefficients estimates are 0.78 for
recessions and 0.22 for expansions, meaning that the monetary authority disliked
inflation deviations from the 2 percent target during recessions more than three times it
disliked the deviations that existed during expansions, in average.

We now inspect whether, given the 2 percent official inflation target and loss
coefficients possibly changing across cyclical states, the unemployment gap may weight
significantly in policymaker's preferences. Specifically, in this new formulation of the
problem, the unemployment gap may be significant in only one of the two cyclical
states of the economy, or may have significantly different coefficients in recessions and
expansions.

The flexible inflation targeting with interest rate smoothing policy regime with
possible threshold asymmetries has a loss function defined by

\[
L_i = E_i \sum_{t=0}^{\infty} \delta^t \left\{ \sum_{s=REC, EXP} \left[ \phi_s^{REC} (\pi_{i,t} - \pi^s)^2 + \lambda_s^{REC} (x_{i,t})^2 + \mu_s^{REC} (i_{i,t} - i_{i,t-1})^2 \right] x_{t+i} < 0 \right\}
\]

\[
L_i = E_i \sum_{t=0}^{\infty} \delta^t \left\{ \sum_{s=REC, EXP} \left[ \phi_s^{EXP} (\pi_{i,t} - \pi^s)^2 + \lambda_s^{EXP} (x_{i,t})^2 + \mu_s^{EXP} (i_{i,t} - i_{i,t-1})^2 \right] x_{t+i} \geq 0 \right\}
\]

The Euler equations, one for each cyclical state of the economy, obtained by
truncating the optimal control problem 4 quarters ahead and considering the cross-
equation restrictions arising from the structure of the economy, are as follows:
As before, these are then merged into one single equation, using the dummy variables referred to above. We thus have an Euler equation that allows for asymmetries in the inflation gap, the unemployment gap, and the interest rate smoothing elements, and nests the symmetry case. Symmetry exists when $\mu^{\text{EXP}}$ and $\mu^{\text{REC}}$, and $\lambda^{\text{EXP}}$ and $\lambda^{\text{REC}}$ are not significantly different from each other and when $\phi^{\text{REC}}$ is not statistically different from $(1-\phi^{\text{REC}})$, so that these are the three null hypotheses of interest to test. We estimate this Euler equation jointly with the Phillips and IS equations, using GMM as above, setting the inflation target at the official 2 percent level, and then test the hypothesis using Wald tests on the relevant coefficients.

Table 3.12 summarises the results. There is no statistical evidence of asymmetry in the policymakers' preferences with respect to interest rate smoothing, as in the strict inflation targeting regime. In contrast, there is evidence of asymmetry both in the $(\pi-\pi^*)$ and in the unemployment gap element of the loss function, when they are considered independently. The results indicate that, if the inflation target has actually been the official 2 percent target, the way the notional monetary authority of the Euro Area managed interest rates, during 1986:I-2001:II, reveals that it disliked recessions but actually liked positive gaps. For instance, taking the coefficients in the panel testing only $\lambda^{\text{EXP}}=\lambda^{\text{REC}}$, it placed a weight of 0.111 on each percentage point of negative gap, and a weight of –0.361 on each percentage point of positive gap. In what regards deviations of inflation from 2 percent, policymakers disliked the deviations during recessions more than 9 times they disliked inflation deviations from target during expansions (weights of, respectively, 0.905 and 0.095, in the panel testing only for $\phi^{\text{EXP}}=\phi^{\text{REC}}$).
Table 3.12 – Tests of Asymmetry of Loss Function - Flexible Inflation Targeting with Interest Rate Smoothing, $\pi=2\%$


MODEL: Flexible Inflation targeting with interest rate smoothing

<table>
<thead>
<tr>
<th>$\pi$ target</th>
<th>Coeff.</th>
<th>Estimate</th>
<th>T-stat.</th>
<th>Signific. Probability</th>
<th>Wald Test</th>
<th>Statistical Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu^\text{REC}$</td>
<td>0.014</td>
<td>2.90</td>
<td>0.00</td>
<td>1.74</td>
<td>Not</td>
</tr>
<tr>
<td></td>
<td>$\mu^\text{EXP}$</td>
<td>0.003</td>
<td>0.51</td>
<td>0.61</td>
<td>(0.19)</td>
<td>Rejected</td>
</tr>
</tbody>
</table>

$H_0: \mu^\text{EXP}=\mu^\text{REC}$

The interest rate smoothing part of $L$ is not asymmetric with respect to the cyclical state of the Economy, given that the inflation-gap and unemployment-gap elements are symmetric

Official: 2%  

| $\lambda^\text{REC}$ | 0.111 | 1.90 | 0.06 | 4.82 | Rejected |
| $\lambda^\text{EXP}$ | -0.361 | -2.20 | 0.03 | (0.03) | |

$H_0: \lambda^\text{EXP}=\lambda^\text{REC}$

The unemployment-gap part of $L$ is not asymmetric with respect to the cyclical state of the Economy, given that the inflation-gap and the interest rate smoothing elements are symmetric

Official: 2%  

| $\phi^\text{REC}$ | 0.905 | 8.78 | 0.00 | 15.44 | Rejected |
| $\phi^\text{EXP}$ | 0.095 | 0.92 | 0.36 | (0.00) | |

$H_0: \phi^\text{EXP}=\phi^\text{REC}$

The inflation-gap part of $L$ is not asymmetric with respect to the cyclical state of the Economy, given that the interest rate smoothing and the unemployment gap elements of $L$ are symmetric

Official: 2%  

| $\lambda^\text{REC}$ | 0.085 | 1.41 | 0.16 | 3.88 | Rejected |
| $\lambda^\text{EXP}$ | -0.314 | -2.07 | 0.04 | (0.05) | |
| $\mu^\text{REC}$ | 0.007 | 1.79 | 0.08 | 0.08 | Not |
| $\mu^\text{EXP}$ | 0.006 | 2.11 | 0.04 | (0.78) | Rejected |

$H_0: \lambda^\text{EXP}=\lambda^\text{REC} \land \mu^\text{EXP}=\mu^\text{REC}$

The unemployment-gap and the interest rate smoothing element of $L$ are not asymmetric with respect to the cyclical state of the Economy, given that the inflation-gap part of $L$ is symmetric

Official: 2%  

| $\phi^\text{REC}$ | 0.918 | 9.38 | 0.00 | 18.22 | Rejected |
| $\phi^\text{EXP}$ | 0.822 | 0.84 | 0.40 | (0.00) | |
| $\mu^\text{REC}$ | 0.016 | 2.45 | 0.02 | 1.70 | Not |
| $\mu^\text{EXP}$ | 0.006 | 1.40 | 0.16 | (0.19) | Rejected |
The inflation-gap and the unemployment-gap elements of $L$ are not asymmetric with respect to the cyclical state of the Economy, given that the interest rate smoothing part $L$ is symmetric.*

<table>
<thead>
<tr>
<th>Official: 2%</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^{REC}$</td>
<td>0.333</td>
<td>1.33</td>
<td>0.19</td>
<td>0.45</td>
<td>Not</td>
</tr>
<tr>
<td>$\phi^{EXP}$</td>
<td>0.666</td>
<td>2.67</td>
<td>0.01</td>
<td>(0.51)</td>
<td>Rejected</td>
</tr>
<tr>
<td>$\lambda^{REC}$</td>
<td>0.108</td>
<td>1.27</td>
<td>0.21</td>
<td>2.60</td>
<td>Not</td>
</tr>
<tr>
<td>$\lambda^{EXP}$</td>
<td>-0.660</td>
<td>-1.37</td>
<td>0.17</td>
<td>(0.11)</td>
<td>Rejected</td>
</tr>
</tbody>
</table>

* Sample: 1986:2-2001:2; GMM estimator highly volatile to small changes in estimated sample.

When we test for asymmetry simultaneously in the inflation-gap and unemployment-gap elements of $L$, there is no evidence of asymmetry. This result is perhaps associated to complex interactions between the inflation and unemployment gap asymmetries. GMM estimation of these models is, actually, very problematic: reasonable convergence fails for the entire 1986:I-2001:II period, and the estimates reported are for 1986:II-2001:II. Furthermore, the estimates are highly volatile to small changes in the sample. Taken together, these facts mean that the results from this model are not reliable. The explanation for this may be associated to the contemporaneous Phillips relation, which is creating an econometric problem of identification of the source of asymmetry.

The individual significance statistics of the loss function coefficients in the second and third panels of table 3.12 (allowing for, in turn, $\lambda^{EXP} \neq \lambda^{REC}$ and $\phi^{EXP} \neq \phi^{REC}$), suggest that the model with asymmetry in the unemployment gap weight seems to be preferable. In fact, it does not seem sensible to have inflation eliminated from the central bank loss in expansions. Moreover, the flexible inflation targeting model with
different coefficients on the unemployment gap weight, across recessions and expansions, fits actual interest rates far better than a model of flexible inflation targeting and asymmetry in the component of inflation deviations from 2 percent. Specifically, the mean square error is 0.68 in the former and 1.28 in the latter.

In summary, we draw four main conclusions from the analysis in this section. First, if we assume 2 percent to be the official inflation target, and allow for different policymakers' preferences between recessions and expansions, there is statistical evidence of inflationary asymmetry in the loss function of the notional monetary authority of the Euro Area during 1986:I-2001:II. Second, under those assumptions, the regime best characterising the policy regime in the Area, during 1986:I-2001:II, is one of flexible inflation targeting with interest rate smoothing and asymmetry in the unemployment gap weight in the loss function, in which authorities only disliked recessions and actually liked expansions. Third, we can not determine whether the notional monetary authority of the Euro Area, in 1986-2001, had symmetric preferences and followed a monetary policy of targeting inflation at 2.7 percent, or whether it disliked recessions but not expansions, and targeted inflation at the official 2 percent level. The only way to solve this observational equivalence would be to exogenously obtain precise and credible information on the true inflation target, or on its statistical distribution, pursued by the monetary authority. Fourth, at a more methodological level, we suggest an extension to the optimal control framework with GMM estimation, and show that it is useful to assess the possibility of asymmetries in central bank loss functions. This avenue of research should be fruitful, in the future, once credible information about the true inflation target is available.

Further explorations

In view of the structural break detected at 1995:II in section 3.2 above, we now look at the period 1995:II-2001:II, to examine whether the well-defined monetary policy regime beginning in 1986:I experienced any marked shift by mid-90s.

We have computed the mean square error (MSE) of all possible models of symmetric and asymmetric loss functions, for the reasonable inflation targets, and have come out with the result that is shown in figure 3.8. The figure indicates that during
1995:II-2001:II, the symmetric loss model with a 2 percent inflation target adjusts better to the interest rates data than the model with a 2.73 percent target (MSEs of, respectively, 0.20 and 0.34).

**Figure 3.8 - Actual versus Fitted Interest Rate, 1995:I-2001:II, Loss Function: Quadratic, SITIRS, Optimal Control and GMM Model Coefficients Estimated Throughout 1986:1-2001:II**

\[ \pi^* = 2.73, \text{ and } \pi^* = 2 \]

![Figure 3.8](image)

Note: Fitted interest rates obtained by dynamically solving the IS-Phillips-Euler system, using the coefficient estimates obtained for the sample period 1986:1-2001:II with, \( \pi^* = 2 \) and, alternatively, \( \pi^* = 2.73 \).

Hence, this evidence indicates that the policy regime that emerged after 1986 may have experienced a significant change by 1995:II, ahead of the EMU in 1999. Specifically, the inflation target may have been reduced from about 2.7 to 2 percent or, put alternatively, the EMU policy regime (after 1995) may not be suffering from the inflationary bias recorded for the whole 1986:I-2001:II period. This evidence is compatible with our structural stability tests in section 3.2, and is also compatible with the inflation target defined in the ECB statutes - which, as reviewed above, may more likely induce a deflationary bias, than an inflationary one.
These conclusions can only be considered tentative, however, as they are subject to numerous qualifications. Most importantly, the 1995:II-2001:II is too short a period for us to be able to reach conclusive and robust conclusions: the adjusted rates of figure 3.8 have been simulated with coefficient estimates obtained with the data for the whole 1986:I-2001:II period, as the post 1995:I quarterly observations do not allow for any robust GMM estimation of a new policy regime. Investigating this post-1995:II regime is clearly a path for future research.

3.6. Concluding Remarks

The empirical research in this chapter shows that the emergence of a well-defined monetary policy regime in the aggregate Euro Area is part of the explanation for the apparent improvement in the volatility trade-off between inflation and the gap of the Area since 1986. This result is noteworthy, bearing in mind the institutional prevalence of national monetary policies until 1999, and confirms with formal evidence the well-known stylised fact that monetary policy autonomy of member-states of the ERM fell markedly since the mid-80s, with the advent of the German leadership of the System. Notably, the two alternative methods employed - optimal control with GMM estimation, and dynamic programming with FIML estimation - both indicate that the Area monetary policy regime post-86 has been one of strict inflation targeting with a significant interest rate smoothing, with the inflation target estimate located somewhat above 2.5 percentage points.

We find signs that the improvement in the Taylor trade-off of the Area since 1986 may have been caused, in addition, by somewhat milder supply shocks - which move the efficiency policy frontier - and by some increase in the ability of policymakers to run interest rates closer to their optimal path. These signs are remarkably robust across the two optimisation-estimation methods used in the research. Most extraordinarily, the two employed methods generate similar estimates for the decrease in the standard error of the residuals of their optimizing interest rate equations, from 1972-1985 to 1986-2001: 47 and 45 percent, respectively for the Euler and the optimal linear policy rule equation.
Estimated forward-looking Taylor rules are also compatible with the unemployment gap not showing up as a significant argument in the loss function, in the post-1986 Euro Area regime. The gap is valuable information, however, for monetary policymaking, as apparent in the need to include past unemployment gaps in the instrument sets for GMM estimation of the Taylor rule.

The finding that a well-identified monetary policy regime seems to have existed in the Area after 1986, implies that Rudebusch and Svensson's (2002) use of (1961-1996) US data to draw lessons for Euro-system monetary versus inflation targeting may have been unwarranted, as post-86 aggregate Area data could have been used.

With our new optimal control and GMM based approach to modelling loss function asymmetries across expansions and recessions, we present our policymakers' preferences estimates in an alternative form. Specifically, we show that the data alone can not discriminate between the Euro Area notional policymaker having targeted inflation at 2.7 percent, with a quadratic loss excluding the unemployment gap, and, alternatively, having flexibly targeted inflation at 2 percent and disliking negative but liking positive unemployment gaps. We discuss informational conditions necessary for solving this observational equivalence.

We confirm that interest rate smoothing is an open problematic issue, not only with regard to theoretical explanations but also concerning empirical estimation. The two methods used in this chapter yield quite different estimates for the instrument inertia in the Euro Area loss function since 1986:I, as happens in previous studies for the US case. Our assessment of this issue suggests that the dynamic programming with FIML approach may suffer from numerical problems, and that the optimal control with GMM estimation method seems to yield a loss function that is closer to its theoretical foundations.

Finally, there are some indications that the well-defined monetary policy regime that emerged in the Euro Area after 1986:I may have changed by 1995:II, ahead of the EMU. Specifically, the actual inflation target may have switched from about 2.7 to the official 2 percent target, or, put alternatively, the policy regime since 1995 may not be suffering from any inflationary bias. However, the data available so far does not allow a precise scrutiny of this question.
This chapter has suggested several avenues for future empirical research on the Euro Area policymakers' preferences, from which we emphasize four. First, when a sufficient amount of additional future data is collected, estimation of policymakers preferences of the EMU monetary policy regime should be pursued, focusing on post-1995:II data. Second, when possible, monthly data, instead of quarterly data, should be used, not only to enhance the degrees of freedom of estimation, but also in view of the periodicity of the ECB's Governing Council meetings. Third, when precise and credible information on the official inflation target is available, our framework may be applied to investigate possible asymmetries in the ECB loss function. Fourth, when real-time data available to ECB policymakers is available, their preferences may be estimated with greater precision, and perhaps the interest rate smoothing puzzle may be clarified.
Appendix


A.3.1. The Model

Taylor rule: \[ r_t^* = \bar{r} + \beta \left( \pi^e_{t+4} / \Omega_t \right) - \pi^* + \gamma \left( x_t^e / \Omega_t \right) \] (A3.1)

Partial adjustment constraint: \[ r_t = (1 - \rho) r_t^* + \rho r_{t-1} + \nu_t \] (A3.2)

Where \( r_t^* \) is the level of the short-term interest rate that policymakers would like to set at quarter t, \( \bar{r} \) is the equilibrium nominal short-term interest rate - that is, the level that would prevail if inflation and the gap were to equal their target levels, respectively \( \pi^* \) and \( 0 \). \( \pi^e_{t+4} / \Omega_t \) stands for the expectation that policymakers make, with information available at period t, for the rate of inflation four quarters ahead and, similarly, \( x_t^e / \Omega_t \) is the policymakers expectation of the current period gap, made with information available at each period. In the partial adjustment equation, \( \rho \) represents the degree of interest rate smoothing, and the residual \( \nu_t \) is meant to model irregular components and inefficiency in the conduction of policy.

Defining \( \alpha = \bar{r} - \beta \pi^* \), and merging equations (A3.1) and (A3.2), we obtain

\[ r_t = (1 - \rho) \alpha + (1 - \rho) \beta \left( \pi^e_{t+4} / \Omega_t \right) + \gamma \left( x_t^e / \Omega_t \right) + \rho r_{t-1} + \nu_t \] (A3.3)

Now, the expectation of period t gap with information available at period t corresponds precisely to our gap series, which has been computed with the kalman filter updating equations. Hence, we replace that expectation by \( x_t \), that is, the current period observation of our gap. The expectation of inflation four quarters ahead - compatible with the 12-month-ahead expectation in Clarida et al. (1998) and with our discussion of
the forecast horizon of policymakers in the text - is replaced by the actual observation of inflation at $t+4$. Then, the equation residual, $\epsilon_t$, is a linear combination of the error $\nu_t$ and the inflation expectation error. The model can, then, be estimated by GMM, using the orthogonality conditions implied by the fact that, if policymakers are rational, the equation residual is uncorrelated with information available at period $t$, which includes information relating up until period $t-1$.

Hence, the equation to estimate is:

$$r_t = (1 - \rho)\alpha + (1 - \rho)\beta(\pi_{t+4}) + (1 - \rho)\gamma(x_t) + \rho r_{t-1} + \epsilon_t$$  \hspace{1cm} (A3.4)

To obtain the inflation target implicit in the estimated coefficients, we take as estimator of the equilibrium nominal short-term interest rate, $\bar{r}$, the sample average of the short-term interest rate. Given the estimates of $\alpha$ and $\beta$, it is straightforward to obtain an estimate of $\pi^*$.

### A.3.2. Results

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<tr>
<td><strong>Coefficients:</strong></td>
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<tr>
<td>$R^2$</td>
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<tr>
<td>DW</td>
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<tr>
<td>S.E. regression</td>
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<td>J-test</td>
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**RMSE interest rate** 3.21  
**Fitted series $\sigma$** Data $\sigma$:  
**Interest Rate** 2.96 3.87

Estimation: equation (A3.4), by GMM. 
Instruments: $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}, i_{t-1}, i_{t-2}, i_{t-3}, i_{t-4}, I\pi_{t-1}, I\pi_{t-2}, I\pi_{t-3}, I\pi_{t-4}$, where $I\pi$ is the imports inflation rate minus the domestic inflation rate.  
No prewhitening; HAC variance-covariance - Bartlett kernel, Bandwidth = 4.  
Significance probabilities relate to one-sided tests.

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Estimation: equation (A3.4), by GMM.
Instruments: πt-1, πt-2, πt-3, πt-4, x1t-1, x1t-2, x1t-3, x1t-4, 1t-1, 1t-2, 1t-3, 1t-4, Iπt-1, Iπt-2, Iπt-3, Iπt-4, where Iπ is the imports inflation rate minus the domestic inflation rate.
No prewhitening; HAC variance-covariance - Bartlett kernel, Bandwidth = 4.
Significance probabilities relate to one-sided tests.
† Imprecisely estimated because based on coefficients from which at least one has too large standard error.
Figure A.3.1 - Short-Term Interest Rate 1972:I-2001:II: Actual and Fitted with Forward-Looking Taylor Rule

Figure A.3.2 - Short-Term Interest Rate 1972:I-1985:IV: Actual and Fitted with Forward-Looking Taylor Rule
Figure A.3.3 - Short-Term Interest Rate 1986:I-2001:II: Actual and Fitted with Forward-Looking Taylor Rule

Chapter 4

Conclusions and Reflections for Future Research

This chapter finishes the thesis, summarising its most central results, and suggesting some scopes for further work.

4.1. Summary of Results

This study contributes to the monetary policy analysis literature offering an empirical enquiry of the macroeconomic trade-offs and monetary policy in the Euro Area, focusing on aggregate Area data of the last three decades of the XX\textsuperscript{th} Century. Chapter 2 has analysed the Phillips trade-off, with a special emphasis on testing for possible asymmetry in the Phillips relation, on modelling expectations as near-rational, and in the consistent estimation of the Area's time-varying NAIRU. The analysis is conducted within a small unobserved components model featuring the Phillips and Okun relations as main measurement equations, estimated by maximum likelihood with the kalman filter. Chapter 3 has focused on the Taylor trade-off and in its implications for the monetary policy regime and the economic structure of the Area throughout the sample period. The analysis compares results of two alternative inverse control procedures and estimation methods - optimal control with GMM estimation, and dynamic programming with FIML estimation. The former method is, then, used to discuss possible asymmetries in the central bank loss function.

As regards the Phillips relation, the analysis shows that once purely rational expectations are replaced by a more realistic hypothesis - specifically Ball's (2000) near-rational, limited-information expectations -, and once time-variation of the NAIRU is allowed, then the new keynesian Phillips curve - optimising, forward-looking - works well for the Euro Area. In fact, the Phillips elasticity is estimated with the correct sign, and is statistically significant - when the theoretical equations in the measurement system are modelled with proper functional forms. Moreover, the estimated model-
consistent NAIRU implies a path for the unemployment gap that mimics the business cycle dating generated by the alternative New Keynesian Phillips Curve (NKPC) approach, based on unit labour costs instead of gaps - as reported in Gali et al. (2001). Hence, our analysis suggests that the empirical problems with the new Keynesian Phillips curve, documented in some literature, are more likely related to the use of rational expectations of inflation, and/or of mechanically detrended activity time-series, than to the choice of the proxy variable for real marginal costs. Moreover, it shows that the limited-information rational expectations are a viable alternative to the pragmatic but somewhat unfounded approach of specifying mixed forward and backward-looking expectations of inflation, often used in the literature.

Our asymmetry tests clearly indicate that there is not enough evidence to reject the hypothesis that the Phillips relation has been linear in the aggregate Euro Area throughout 1970-2000. In contrast, the evidence gathered strongly suggests that there is a statistically significant and economically important asymmetry in the Okun Law. Notably, our asymmetry tests nest the linear specification, cover four alternative functional forms, do not impose convexity nor concavity, and use model-consistent NAIRU and potential output series - all desirable features that allow for a good degree of confidence in the obtained results. The evidence of convexity in the Area's Okun equation is consistent with results for most developed countries elsewhere in the literature. In contrast, our evidence of linearity of the Phillips curve challenges several recent strands of literature, which have been stressing that there are supposedly important theoretical and empirical reasons for the Phillips curve to be asymmetric, especially when aggregated data of regions or nations is scrutinised. Remarkably, our evidence seems to defy a literature that has been specifying convex Phillips curves, for some developed countries, without offering convincing preliminary evidence of the allegedly important convexities in the trade-off. Taken together, our Phillips and Okun equations' asymmetry tests results indicate that the choice of unemployment versus output gap to characterise the trade-off may not be irrelevant for its functional form, and that neither linearity nor non-linearity should be taken for granted a-priori across samples.
In chapter 2, the choice of the specific unobserved components' model to be estimated - based on wide-ranging preliminary identification tests - allows some conclusions, which are also of interest per se:

First, in both the (linear) Phillips equation and in the (quadratic) Okun relation, the unemployment gap relates only contemporaneously to the explained variables. Hence, no speed-limits effects are present in the Phillips trade-off, in contrast to some earlier literature on the subject, but in accordance with more recent results elsewhere in the literature.

Second, the trade-off equation is augmented with the deviation of domestic inflation from imported inflation in the previous quarter, which seems to summarise well the exogenous shocks affecting the inflation of the Area during the sample period. Notably, variables such as productivity and exchange-rates were not found to be statistically relevant in the Phillips equation. This result contrasts with some literature that has been including a large number of such variables possibly affecting inflation exogenously, without testing for their individual significance.

Third, the NAIRU is successfully modelled as a random walk with a drift which itself follows a random walk, while the random walk process driving potential real output has, in turn, a constant drift. This identification result challenges some recent literature that had been imposing similar stochastic processes for the NAIRU and potential real output of the Euro Area. With our model for the NAIRU, we find no evidence of hysteresis, when it is modelled as a feed-back from lagged gaps to the neutral rate of unemployment.

One exercise in chapter 2 that also allows some worthy of note conclusions, is a systematic comparison between the results of estimation of the identified model and those of a standard backward-looking Phillips curve. It turns out that most hyperparameters' estimates are quite close, as well as the estimates for the unobserved components, and the results of the asymmetry tests. The most detectable difference happens in the estimate of the Phillips' trade-off elasticity - which decreases from 0.053 in the adaptive expectations model, to 0.042 in the forward-looking equation with near-rational expectations of inflation, even though its statistical significance remains stable at 5 percent. These point estimates are equivalent to about 0.21 and 0.17, respectively, if
an annualised rate of inflation had been used in the estimation, which are not inferior to estimates obtained in recent research for the US case. Hence, our results clearly challenge the studies that had been claiming that there was no Phillips relation in Europe, and has identified conditions for a proper detection of that significant trade-off. These include most especially, a proper choice of functional forms for the Phillips and Okun equations, a model-consistent time-varying NAIRU, and some deviation of expectations from full rationality.

Finally, the confidence bands for our estimates of the time-varying NAIRU, including all the sources of uncertainty - filter and parameter -, are in line with the related research on different economic areas in different places in the literature. Essentially, they are very wide and advise caution when using such NAIRU estimates for the conduct of monetary policy.

In chapter 3, we take on the unemployment gap estimates of chapter 2, and document a striking improvement in the Taylor trade-off of the Euro Area, around 1986. The simultaneous fall in the volatility of the gap and inflation continues during the 1990s, although less impressively than at the turn from the first to the second half of the 80s. This phenomenon mimics the evolution of the macroeconomic volatility in other developed areas, such as the US - although, in comparison to that case, with some lag.

This evaluation of the Taylor trade-off, taken together with the stability of the coefficients associated to lagged inflation in the Phillips equation of chapter 2 - when estimated as time-varying coefficients, in section 2.5.2 - conveys a clear story regarding the time-series properties of the Euro Area inflation rate throughout the period. In short, the Euro Area inflation has become less volatile, but has maintained its persistence - a story that seems quite close to that of the US, according to some recent research on this topic.

Bearing in mind that the gap series used to assess the Taylor trade-off has been estimated, in chapter 2, with a constant Phillips elasticity, we come across a transparent interpretation about the joint evolution of both unemployment-inflation trade-offs of the Area throughout 1970-2001, from the very beginning of chapter 3. Under the assumptions of our estimation of chapter 2 - most especially a time-varying NAIRU -
we find out that the Phillips trade-off - between the levels of the gap and inflation - has been constant, while the Taylor trade-off - between the variances of the gap and inflation - has decreased markedly since around 1986. This finding has been confirmed, in chapter 3, as a change in the Phillips elasticity is rejected, in favour of other structural changes driving the simultaneous improvement in the gap and inflation volatility.

In search for the causes behind the improvement in the second-moments trade-off, we reach a crucial conclusion about the monetary history of the Area, in chapter 3. We find that one important cause for that improvement is that, after 1986, a well-identified monetary policy regime targeting a low rate of inflation has emerged in the aggregate data of the Area. Specifically, assuming a standard quadratic functional form, we successfully estimate a loss function of the Area's notional central bank, with aggregate data since 1986:I, and identify the monetary regime as one of strict inflation targeting - at slightly above 2.5 percent - with a significant degree of interest rate smoothing. Hence, our research offers formal evidence that clarifies the extent of a well-known stylised fact of the Area's monetary integration in the years before the creation of EMU - that sovereignty of national monetary policy had been given up by EMS countries well before formal loss of monetary autonomy in 1999. Moreover, our finding implies that the use of US data to draw lessons for Euro-system monetary policy, in some recent research, may have been unwarranted, as aggregate Euro Area data after 1986 might have been used instead.

In addition, chapter 3 offers signals that milder supply shocks and higher monetary policy efficiency - the ability of policymakers to maintain actual interest rates close to their optimal path - also seem to have contributed the fall in the Area macroeconomic volatility since 1986, in addition to the policy regime change. In fact, the standard error of the residuals of both our inflation equation, and the equation describing the optimality conditions for the policy instrument, decreased markedly from 1972-1985 to 1986-2001.

In chapter 3, we make use of two alternative inverse control and estimation procedures recently used for the US case by autonomous researchers - optimal control with GMM, and dynamic programming with FIML estimation - to study the role of the
monetary regime in the Taylor trade-off improvement of the Euro Area. This enhances the robustness of our conclusions regarding the Euro Area and, in addition, sheds some light into the causes of some discrepancies in the results about the US case.

With respect to robustness of our results, both methods identify a regime of strict inflation targeting with interest rate smoothing, and estimate the inflation target slightly above 2.5 percent. Also, both estimate identical percent changes in the standard errors of their Phillips and optimising interest rate equations, from 1972-1985 to 1986-2001, which may be considered, somewhat, signals about the changes in volatility of supply shocks and optimality of monetary policy.

The striking broad accordance that we reach with these methods, on the Euro Area case, is not seen in their use by independent researchers on the US case. Hence, we suspect that the divergence of results in the U.S. case is due to differences in details of the empirical frameworks - namely data sources, discount factor calibration, and sample period delimitation. One particular divergence that we do observe in our results with the two methods, which also appears in the results of the US literature, is the estimate of the weight of interest rate smoothing in the central bank's loss function. Our literature review and econometric work confirm that interest rate smoothing remains an open problematic issue, both theoretically and empirically. Our assessment of the issue suggests that the dynamic programming with FIML approach may suffer from numerical problems, and that the optimal control with GMM estimation method seems to estimate a loss function that is closer to its theoretical foundations.

Overall, our results show that there is an outstanding resemblance between the causes broadly identified as explaining the fall in macroeconomic volatility during the 80s in the case of the Euro Area and in the case of the US. Essentially, the change of monetary policy regime, towards a targeting of a low rate of inflation, is now known to have been crucial for that favourable outcome in both areas. The notable singularity of the Euro Area, here, is that this development has occurred in the context of a de facto loss of national monetary autonomy from most its member-states, in favour of the leadership of the EMS policy by Germany - the country with the best record as inflation fighter.
Having in mind the prevalence of formal monetary and economic autonomy by the Euro Area member-states until 1999, it is also noteworthy that our work shows how the well-known Rudebusch-Svensson model describes the dynamic structure of the Area macro-economy, at least since 1986, once a proper model identification is done. The similarity with the modelling of the US economy is remarkable, as is the contrast with difficulties in fitting this model to some smaller and more open individual economies. A test of stability of this model suggests that the structure of the Area macroeconomy may have changed somewhat ahead of the EMU creation, probably at 1995, which may be valuable information for future research on the EMU macro and monetary regime.

Some exploratory analysis at the end of chapter 3, based on the structural stability tests results, offers some indications that the monetary policy regime that emerged in the Euro Area after 1986:1 may have changed by 1995:II, probably ahead of the EMU regime. Specifically, the actual inflation target may have moved closer to the official 2 percent target. However, we note that the data available so far does not allow a precise scrutiny of this question - we argue that only when more data is available can any study of the EMU monetary policy regime be accomplished.

The issues pertaining to the specific vintage of statistical data used in monetary policy analysis receive a large attention, both in chapter 2 and chapter 3 of our work.

In chapter 2, we show how the end-of-sample kalman filter estimates of the unemployment gap are sensitive to statistical uncertainty, computing the NAIRU and the gap with quasi-real-time data as of 1998:II, for a more fair comparison with the estimates in the ECB's AWMD. This statistical uncertainty, associated to the somewhat large confidence bands around unobserved components estimates and their typical widening by the end of samples, advises caution in the use of output or unemployment gap estimates in the conduction of monetary policy.

A natural corollary of the previous reasoning is that the statistical uncertainty also advises caution in the choice of the estimates of the gaps that are to be used in monetary policy analysis. Hence, our decisions regarding the unemployment gap data to be used in chapter 3. In fact, motivated by chapter 2 results, and by results in the
expanding literature on the importance of real-time data for monetary policy analysis, we use the unemployment gap obtained in chapter 2 with the kalman filter, and not with the smoother. We describe this recursive estimate as a quasi-real-time unemployment gap, and argue that it is, for the object of our study, the closest a researcher can get to the notional true data available to notional Euro Area policymakers in real-time, before 1999. Furthermore, as the specification of the Philips equation in both chapters is quite similar, this choice of data adds consistency to our empirical exercise. Here, we diverge from the approach taken in studies of the US policymakers' preferences - where estimates of gaps available from official sources at the timing of the research have been used - and the success of our empirical analysis seems to encourage our approach.

The interest devoted in chapter 2 to the possible asymmetry of the Phillips and Okun equations is paralleled, in chapter 3, by the study of possible asymmetry in the central bank loss function across expansions and recessions. Here, we integrate the literature of estimation of policymakers' preferences - specifically, the optimal control with GMM framework - with the literature assessing deviations of the loss function functional form from the standard quadratic approach. We employ the developed method in our case, assuming an official inflation target of 2 percent, and present our policymakers' preference estimates in an alternative form. Specifically, we show that the data alone can not discriminate between a strict inflation targeting regime with a 2.7 percent target, with a quadratic loss, and, alternatively, a flexible inflation targeting regime with the official 2 percent target, in which policymakers disliked negative, but liked positive, unemployment gaps. Finally, we discuss the information requirements needed for overcoming this observational equivalence.

4.2. Reflections for Future Research

Monetary policy analysis is, at the moment, an extremely vivid area of research, with a huge ongoing effort, and plenty scope for new enquiries. Knowledge about the specific case of the Euro Area is, by itself, far from satisfactory, and clearly calls for additional research - especially because of the structural break caused by the EMU. This section mentions some possible extensions of the studies conducted in chapters 2 and 3,
motivated by some of the obtained results, or by recent developments of the author's perception about the topics in those chapters.

First, the unobserved components models in chapter 2 could be enriched with additional measurement equations, suggested by economic theory, maybe improving the estimation of the NAIRU and potential output - not only their precision but also their flexible-price equilibrium attribute. A natural candidate would be a production function, and the inclusion of data from production factors - which could enable the estimation of the contribution of factors and their productivity to potential activity and cycles. Another natural candidate would be an aggregate demand, or IS, function, which would bring interest rate data into the model. Including an IS function seems particularly important, as it could solve one inconsistency between the results of chapter 2 and chapter 3. While in chapter 2 we do not reject the stability of the average growth rate of potential real output, in chapter 3 we observe a marked increase in the estimate of the equilibrium real interest rate, at the mid-80s - and these variables should be intimately associated, according to economic theory.

In chapter 3, in order to retain comparability with previous studies on the US case, we adopted the backward-looking standard version of the Rudebusch-Svensson model. One second possible extension of this research would be to consider a structural model with optimising, forward-looking, IS and Phillips equations, which have more theoretical foundations. One natural way - along the lines of this thesis - of holding the empirical success of the model, would be to use near-rational expectations of inflation in both functions. This would obviously apply to chapter 2, also, if the system were extended with an IS function, as suggested above.

A third extension of the study relates to the crucial topic of expectations of inflation. Our use of Laurence Ball's near-rational expectations of inflation has proved quite successful in our empirical work, and that is a crucial development in this thesis. However, expectations of inflation remain at the heart of modern monetary theory, and there are plenty ongoing developments of models of learning and deviation from full rationality. These ongoing contributions deserve a closer look in future research, as they may strengthen the theoretical roots of imperfect rationality.
This thesis offers an explanation for the reduction in the volatility of the gap and inflation, which includes the emergence of a well-defined monetary policy regime targeting a low rate of inflation. It follows that this explanation also helps in understanding the fall in the level of inflation. The thesis does not, however, explain the level of the unobserved component implicit in the estimated unemployment gap - the NAIRU. Although the explanation of the path of the Euro Area NAIRU has been intentionally excluded from this thesis, as mentioned in chapter I, it could be a fourth extension of our work. In fact, there now seems to be room for such a research programme, in view of recent explanations of the evolution of the US NAIRU based on determinants that can be measured at the macroeconomic level - such as the relation between the change in trend productivity and workers' wage aspirations.

Finally, the fifth extension relates to the study of the EMU's monetary policy regime, and to the possibility of using the empirical frameworks employed in chapter 3, for that purpose, in future research. Several conditions are necessary to make that study possible, and others are advisable to strengthen its results. First, when a sufficient amount of additional data is collected, estimation of policymakers' preferences in the EMU policy regime could be achieved, with the structural stability tests in chapter 3 suggesting that data posterior to 1995:II can be used for that purpose. Second, when possible, monthly data, instead of quarterly data, should be used, not only to enhance the degrees of freedom of estimation, but also in view of the periodicity of the ECB's Governing Council meetings. Third, when precise and credible information on the official inflation target is available, our optimal control with GMM estimation framework for testing for asymmetries in loss functions may be applied to investigate possible asymmetries in the ECB loss function. Fourth, when real-time data available to ECB policymakers is available, their preferences may be estimated with greater precision, and perhaps the interest rate smoothing puzzle can be clarified.
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