

```
#include "ElementProperties.hpp"
```

```
#define SHAPE_FUNC(expression) [](Real r, Real s, Real t) -> Real { return expression; }
```

```
using namespace FEAPACK;
```

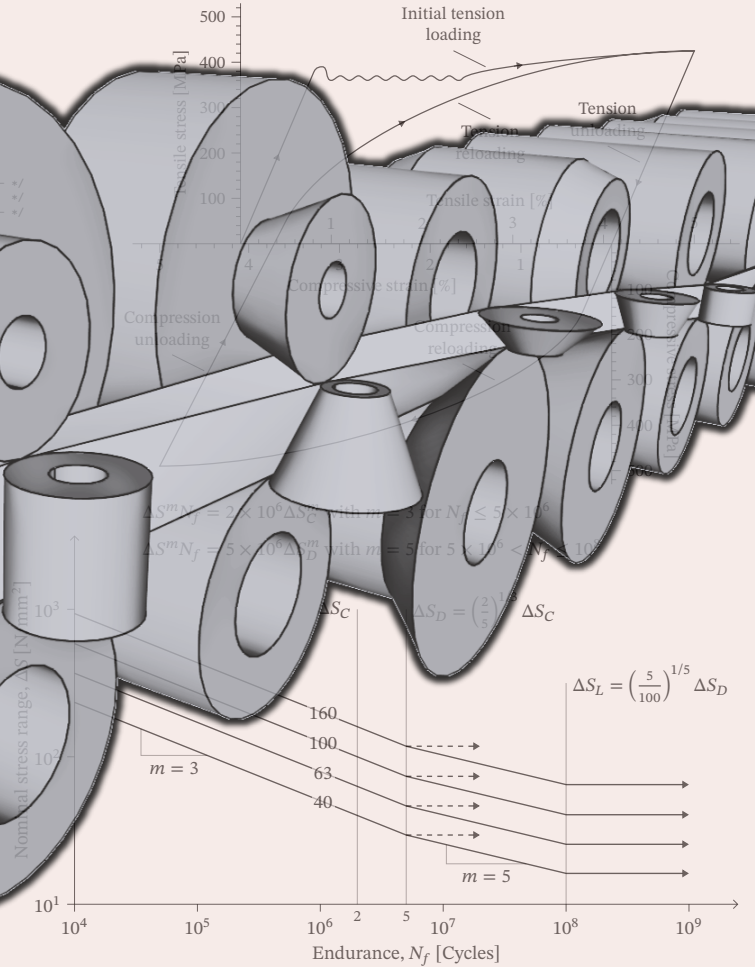
```
template<ElementType> static const std::vector<Point>& get_nodal_pts();
template<ElementType> static const std::vector<PointWeightPair>& get_int_pts(Bool clean);
template<ElementType> static const std::vector<ShapeFunction>& get_shape_funcs();
template<ElementType> static const std::vector<ShapeFunctionDerivatives>& get_derive_shape_funcs();
template<ElementType> static const std::vector<SurfaceDefinition>& get_surf_defs();
```

```
/* ----- */
/* ELEMENT TYPE PLANE_8 */
/* ----- */
```

```
template<
static const std::vector<Point>& get_nodal_pts<ElementType::PLANE_8>() {
    static const std::vector<Point> nodal_pts {
        { -1.0, -1.0, 0.0 },
        { 1.0, -1.0, 0.0 },
        { 1.0, 1.0, 0.0 },
        { -1.0, 1.0, 0.0 },
        { 0.0, -1.0, 0.0 },
        { 1.0, 0.0, 0.0 },
        { 0.0, 1.0, 0.0 },
        { -1.0, 0.0, 0.0 },
    };
    return nodal_pts;
}
```

```
template<
static const std::vector<PointWeightPair>& get_int_pts<ElementType::PLANE_8>(Bool clean_reduced) {
    static const std::vector<PointWeightPair> ips_full {
        { { -0.7745966692414834, -0.7745966692414834, 0.0 }, 0.3086419753086420 },
        { { 0.7745966692414834, -0.7745966692414834, 0.0 }, 0.3086419753086420 },
        { { 0.7745966692414834, 0.7745966692414834, 0.0 }, 0.3086419753086420 },
        { { -0.7745966692414834, 0.7745966692414834, 0.0 }, 0.3086419753086420 },
        { { 0.0, 0.0, 0.0000000000000000, -0.7745966692414834, 0.0 }, 0.4938271604938271 },
        { { 0.7745966692414834, 0.0000000000000000, 0.0 }, 0.4938271604938271 },
        { { 0.0000000000000000, 0.7745966692414834, 0.0 }, 0.4938271604938271 },
        { { -0.7745966692414834, 0.0000000000000000, 0.0 }, 0.4938271604938271 },
        { { 0.0, 0.0000000000000000, 0.0000000000000000, 0.0 }, 0.7901234567901234 },
    };
    static const std::vector<PointWeightPair> ips_reduced {
        { { -0.5773502691896258, -0.5773502691896258, 0.0 }, 1.0 },
        { { 0.5773502691896258, -0.5773502691896258, 0.0 }, 1.0 },
        { { 0.5773502691896258, 0.5773502691896258, 0.0 }, 1.0 },
        { { -0.5773502691896258, 0.5773502691896258, 0.0 }, 1.0 },
    };
    return reduced ? ips_reduced : ips_full;
}
```

```
template<
static const std::vector<ShapeFunction>& get_shape_funcs<ElementType::PLANE_8>() {
    static const std::vector<ShapeFunction> shape_funcs {
        SHAPE_FUNC( -0.25*(r - 1.0)*(s - 1.0)*(r + s + 1.0) ),
        SHAPE_FUNC( -0.25*(r + 1.0)*(s - 1.0)*(r - s - 1.0) ),
        SHAPE_FUNC( 0.25*(r + 1.0)*(s + 1.0)*(r + s - 1.0) ),
        SHAPE_FUNC( 0.25*(r - 1.0)*(s + 1.0)*(r - s - 1.0) ),
        SHAPE_FUNC( 0.5*(r*r - 1.0)*(s - 1.0) ),
        SHAPE_FUNC( -0.5*(s*s - 1.0)*(r + 1.0) ),
        SHAPE_FUNC( -0.5*(r*r - 1.0)*(r + 1.0) ),
        SHAPE_FUNC( 0.5*(s*s - 1.0)*(r - 1.0) ),
    };
    return shape_funcs;
}
```



Fatigue assessment procedures for cold-formed thin-walled steel structures

Carlos Daniel Santos Souto

Thesis submitted as part of the Doctoral Program in Mechanical Engineering of the Faculty of Engineering of the University of Porto, supervised by Abílio Manuel Pinho de Jesus, José António Fonseca de Oliveira Correia, and Marco Paulo Lages Parente

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‘Much of my work has come from being lazy. I didn’t like writing programs, so I started work on a system to make them easier to write.’

John Backus
The father of Fortran

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Abstract

Rack structures, commonly found in warehouses and distribution centres, are composed of thin-walled cold-formed mild steel profiles (S355MC steel, in this work) connected via non-preloaded bolted joints (made of S350GD steel). These structures enable efficient storage-and-retrieval (S&R) operations for palletized goods, often integrating automated systems such as pallet carrier shuttles to enhance logistics performance. However, the cyclic loading from heavy palletized goods, combined with dynamic actions from automated S&R systems, introduces fatigue damage, including fatigue cracking, which poses safety concerns and increases downtime and maintenance costs.

Manufacturing residual stresses from cold working and strain hardening, combined with the use of non-preloaded bolted connections, are key factors influencing fatigue behaviour. Residual stresses increase the complexity of the structural and fatigue analyses by altering mean stress levels during loading. In particular, tensile residual stresses can accelerate crack initiation and propagation when aligned with structural tensile stresses. However, residual stress determination is inherently complex. Additionally, the use of snug-tight bolted connections, where the applied clamping force is not predetermined, can lead to a limited shear slip resistance under load. This issue is exacerbated by low-friction zinc coatings in galvanized materials. Shear slip, if allowed or prevented by friction, directly affects the fatigue behaviour.

Despite the need to account for fatigue in the design of thin-walled structures, existing European regulations do not comprehensively cover the topic of cold-formed mild steel and snug-tight bolted connections, which require specific considerations. Within this context, the FASTCOLD European research project (project no. 745982) aimed to develop fatigue assessment procedures for constructional details used in rack structures. In parallel, the European Committee for Standardization is in the process of updating the EN 1993-1-9 standard (Eurocode 3). The revision, currently under approval as prEN 1993-1-9, introduces new considerations for non-preloaded bolted connections and rolled products. Ultimately, the timing and efforts made to update current fatigue standards highlight the importance of this investigation.

In addition to regulatory considerations, modern-day engineering projects often prioritize cost-efficiency, leading contractors to propose thinner structural members to remain competitive. However, structural integrity and long-term durability must still be ensured. To address this challenge, damage-tolerant approaches are beginning to gain traction, ensuring or extending the structure's service life even in the presence of visible cracks or defects.

The primary objective of this PhD thesis is the development of fatigue assessment procedures that can be applied to complex real-world cases. To this end, a comprehensive overview of the main fatigue data generated during the FASTCOLD project is made, in which the results of several full- and small-scale fatigue testing campaigns are presented in a unifying manner. This unification is made possible by applying regression-based stress-life ($S-N$) approaches in

which fatigue actions are evaluated numerically via finite element modelling. This methodology enables the assessment of general and complex multiaxial cases while facilitating the construction of master $S-N$ curves based on a more representative measure of stresses acting on a simplified pseudo-elastic material. In the case of the bolted joints, the numerical models can additionally account for bolt preload levels and frictional sliding effects. Ultimately, the results demonstrate that numerical models are capable of predicting the fatigue crack initiation sites, so that the fatigue analysis can be adapted accordingly. This represents an improvement over classical $S-N$ approaches, which rely on engineering measures and intricate detail categorization schemes that continue to grow in complexity as standards evolve. The aim is also to modernize current fatigue design approaches, as the use of numerical tools for stress determination is becoming more widely adopted for design purposes.

Still, regression-based approaches require extensive fatigue testing, which is often impractical or costly. Thus, alternative numerical procedures are explored, incorporating a more realistic elastoplastic material behaviour during the local evaluation of fatigue models at critical locations. This eliminates direct fatigue testing of components, relying instead on empirical properties of the plain material, which are more easily obtainable, and on accurate numerical modelling. For the investigated profiles, the critical regions coincide with the cold-formed corners. Hence, this study examines how manufacturing residual stresses affect local fatigue life prediction models, applying a state-of-the-art framework to a case study involving a specific thin-walled Z-profile used as guide rails for pallet carrier shuttles in automated S&R systems. The fatigue properties of the plain material are determined via low- and high-cycle fatigue testing, while the manufacturing straining history is assessed via finite element modelling of the press braking and roll forming processes. The resulting cold-formed geometries and residual stress fields serve as the initial state in structural analyses, integrating manufacturing effects into the subsequent fatigue analyses. Importantly, results show that accounting for residual stresses significantly improves life predictions, preventing unsafe overestimations.

Lastly, a novel damage-tolerant approach is developed, extrapolating from well-known engineering principles based on linear elastic fracture mechanics and stable fatigue crack growth. Preliminary studies are performed to identify a suitable approach for stress intensity factor determination and to establish a worst-case scenario for a cracked Z-profile case study. Then, a parameterized finite element model is created, based on which a simplified analytical model is derived, which is used for computing stress intensity factors, critical crack lengths, and corresponding critical loads during the static design of a cracked profile, delimiting the domain of stable fatigue crack growth based on the fracture toughness of the material. By including Paris' law, the residual fatigue life of the cracked Z-profile is determined, resulting in design curves for distinct profile thicknesses and initial crack lengths. Finally, exploratory research is performed to account for manufacturing residual stresses in the estimation of the residual fatigue life. Throughout this analysis, using the experimental data from previous full-scale fatigue testing, a qualitative comparison is made to verify the predicting capabilities of the derived approach. It is shown that predicted residual fatigue lives may be overestimated, highlighting the importance of including residual stresses for improved accuracy.

Keywords: Thin-walled structures; Cold-formed mild steel; Snug-tight bolted joints; Residual stresses; Stress-life approaches; Local fatigue approaches; Fatigue testing; Damage tolerance; Finite element modelling; Machine learning.

Resumo

As estruturas de estantes industriais (*racks*), utilizadas em armazéns e centros de distribuição, são compostas por perfis finos de aço macio conformado a frio (aço S355MC, neste trabalho) e ligados por juntas aparafusadas não pré-esforçadas (feitas de aço S350GD). Estas estruturas permitem operações eficientes de armazenamento e recolha (A&R) de produtos paletizados, e podem integrar sistemas automatizados, tais como *shuttles* porta-paletes, para melhorar o desempenho logístico. Contudo, o carregamento cíclico de mercadorias pesadas, combinado com as ações dinâmicas de sistemas automatizados de A&R, resulta em dano por fadiga, incluindo fissuração, o que levanta questões de segurança e aumenta os tempos de inatividade e os custos de manutenção.

As tensões residuais de fabrico provenientes do trabalho a frio e encruamento, bem como o uso de ligações aparafusadas não pré-esforçadas, são fatores-chave que influenciam o comportamento à fadiga. As tensões residuais aumentam a complexidade das análises estruturais e à fadiga ao alterar os níveis de tensão média durante o carregamento. Em particular, tensões residuais de tração podem acelerar a iniciação e propagação de fendas quando alinhadas com tensões estruturais de tração. Contudo, a determinação de tensões residuais é um processo complexo. Adicionalmente, a utilização de ligações aparafusadas “bem apertadas”, onde a força de aperto aplicada não é predeterminada, pode levar a uma resistência limitada ao deslizamento em corte durante o carregamento. Este problema é agravado por revestimentos de zinco de baixo atrito em materiais galvanizados. O deslizamento em corte, se permitido ou prevenido por atrito, afeta diretamente o comportamento à fadiga.

Apesar da necessidade de considerar a fadiga no dimensionamento de estruturas de parede fina, as normas europeias atuais não cobrem de forma abrangente o tema do aço macio conformado a frio e das ligações aparafusadas “bem apertadas”, que exigem considerações específicas. Neste contexto, o projeto de investigação europeu FASTCOLD (projeto n.º 745982) teve como objetivo desenvolver procedimentos de avaliação à fadiga para detalhes de construção utilizados em estruturas de estantes industriais. Em paralelo, o Comité Europeu de Normalização está em processo de atualização da norma EN 1993-1-9 (Eurocódigo 3). A revisão, atualmente em aprovação com a designação de prEN 1993-1-9, introduz novas considerações para ligações aparafusadas não pré-esforçadas e produtos laminados. Por fim, os recentes esforços feitos para atualizar as normas de fadiga atuais realçam a importância desta investigação.

Para além das exigências normativas, os projetos de engenharia modernos tendem a privilegiar a relação custo-eficácia, levando os projetistas a propor elementos estruturais mais finos para se manterem competitivos. Contudo, a integridade estrutural e a durabilidade a longo prazo devem ser asseguradas. Para abordar este desafio, as abordagens de tolerância ao dano

estão a começar a ganhar terreno, garantindo ou estendendo a vida útil da estrutura mesmo na presença de fendas ou defeitos visíveis.

O objetivo principal desta tese de doutoramento é o desenvolvimento de procedimentos de avaliação à fadiga que possam ser aplicados a casos complexos do mundo real. Para este efeito, é apresentada uma visão geral abrangente dos principais dados de fadiga gerados durante o projeto FASTCOLD, na qual os resultados de várias campanhas de ensaio de fadiga em escala real e em pequena escala são apresentados de forma unificadora. Esta unificação é possibilitada através da aplicação de abordagens de tensão-vida ($S-N$) baseadas em regressão, nas quais as ações de fadiga são avaliadas numericamente via modelação por elementos finitos. Esta metodologia permite a avaliação de casos multiaxiais gerais e complexos, ao mesmo tempo que facilita a construção de curvas $S-N$ mestres, sendo estas baseadas numa medida mais representativa das tensões que atuam num material pseudo-elástico simplificado. No caso das ligações aparafusadas, os modelos numéricos podem adicionalmente contabilizar os níveis de pré-esforço dos parafusos e os efeitos de deslizamento com atrito. Em última análise, os resultados demonstram que os modelos numéricos são capazes de prever os locais de iniciação de fenda por fadiga, para que a análise à fadiga possa ser adaptada em conformidade. Isto representa uma melhoria relativamente às abordagens $S-N$ clássicas, que dependem de medidas de engenharia e esquemas complexos de categorização de detalhes que continuam a aumentar em complexidade à medida que as normas evoluem. O objetivo é também modernizar as abordagens de dimensionamento à fadiga atuais, uma vez que o uso de ferramentas numéricas para a determinação de tensões está a tornar-se mais amplamente adotado para fins de projeto.

Ainda assim, as abordagens baseadas em regressão exigem ensaios de fadiga extensivos, algo que é frequentemente demorado e dispendioso. Assim sendo, são explorados procedimentos numéricos alternativos, incorporando um comportamento elastoplástico do material mais realista durante a avaliação local de modelos de fadiga em zonas críticas. Isto elimina o ensaio direto à fadiga de componentes, havendo uma base alternativa nas propriedades empíricas do material virgem, que são mais facilmente obtidas, e em modelação numérica rigorosa. Para os perfis investigados, as regiões críticas coincidem com os cantos conformados a frio. Assim sendo, este estudo examina como as tensões residuais de fabrico afetam os modelos locais de previsão da vida à fadiga, onde é aplicada uma abordagem inovadora a um caso de estudo que envolve um perfil em Z de parede fina específico, sendo este utilizado como guia para *shuttles* porta-paletes em sistemas automatizados de A&R. As propriedades de fadiga do material virgem são determinadas por ensaios de fadiga de baixo e alto ciclo, enquanto o histórico de encruamento é avaliado via modelação por elementos finitos dos processos de quinagem e perfilagem. As geometrias conformadas a frio resultantes e os campos de tensões residuais servem como estado inicial em análises estruturais, integrando os efeitos de fabrico nas análises à fadiga subsequentes. É importante salientar que os resultados mostram que a contabilização das tensões residuais melhora significativamente as previsões de vida, prevenindo sobrestimativas inseguras.

Por último, uma nova abordagem de tolerância ao dano é desenvolvida, extrapolando a partir de princípios de engenharia bem estabelecidos baseados na mecânica da fratura linear elástica e no crescimento estável de fenda por fadiga. São realizados estudos preliminares para identificar uma abordagem adequada para a determinação de fatores de intensidade de tensão e para estabelecer um caso mais desfavorável relativamente a um perfil em Z fissurado.

Em seguida, é criado um modelo de elementos finitos parametrizado, com base no qual é derivado um modelo analítico simplificado, utilizado para calcular fatores de intensidade de tensão, comprimentos críticos de fenda, e cargas críticas correspondentes durante o dimensionamento estático de um perfil fissurado, delimitando o domínio de crescimento estável de fenda por fadiga com base na tenacidade à fratura do material. Ao incluir a lei de Paris, a vida à fadiga residual do perfil em Z fissurado é determinada, resultando em curvas de projeto para diferentes espessuras de perfil e comprimentos de fenda iniciais. Finalmente, o efeito das tensões residuais de fabrico é explorado durante a geração de previsões de vida à fadiga residual. Ao longo desta análise, utilizando os dados experimentais de ensaios anteriores de fadiga em escala real, é feita uma comparação qualitativa para verificar as capacidades de previsão da abordagem construída. É demonstrado que as vidas à fadiga residuais previstas podem ser sobrestimadas, realçando a importância de incluir as tensões residuais para melhorar a precisão das previsões.

Palavras-chave: Estruturas de parede fina; Aço macio conformado a frio; Ligações aparafusadas “bem apertadas”; Tensões residuais; Abordagens tensão-vida; Abordagens de fadiga locais; Ensaios de fadiga; Tolerância ao dano; Modelação por elementos finitos; Aprendizagem automática.

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List of contents

Acknowledgements	i
Abstract	iii
Resumo	v
List of contents	viii
List of figures	xii
List of tables	xix
List of symbols	xxi
List of abbreviations	xxvii
1 Introduction	1
1.1 Background and motivation	1
1.2 Dissertation layout and objectives	4
2 Literature review	7
2.1 Mechanical behaviour of metals	7
2.1.1 Linear elasticity	7
2.1.2 Strain hardening	12
2.1.3 Cold forming	17
2.1.4 Manufacturing residual stresses	21
2.2 Metal fatigue	26
2.2.1 Fatigue damage stages	27
2.2.2 Fatigue damage factors	29
2.2.3 Total life approaches	31
2.2.3.1 Stress-life approaches	32
2.2.3.2 Strain-life approaches	33
2.2.3.3 Multiaxial fatigue	35
2.2.3.4 Variable amplitude loading	36
2.2.3.5 Statistical analysis	36
2.2.4 Damage-tolerant approach	39
	ix

List of contents

2.2.4.1	Stress intensity near the crack tip	40
2.2.4.2	Virtual crack closure technique	40
2.2.4.3	Virtual crack extension method	42
2.2.4.4	The J-integral	42
2.2.4.5	Stable fatigue crack growth	43
2.2.4.6	Theory of critical distances	44
2.2.5	Fatigue testing	45
2.2.6	Fatigue in cold-formed thin-walled profiles	46
2.3	Finite element modelling	49
2.3.1	Pure-displacement formulation of continuum elements	50
2.3.1.1	Isoparametric elements	50
2.3.1.2	Gaussian quadrature rule	52
2.3.1.3	Static equilibrium equations	53
2.3.1.4	Dynamic equilibrium equations	56
2.3.2	Finite element locking and alternative formulations	57
2.3.3	Nonlinear and nonsmooth finite element analysis	60
3	Mechanical testing of the plain materials	65
3.1	Static tensile strength testing	66
3.2	Low-cycle fatigue testing	68
3.3	High-cycle fatigue testing	73
3.4	Summary of the obtained material constants	76
4	Fatigue behaviour of bolted joints used in rack structures	79
4.1	Experimental details	80
4.2	Numerical details	82
4.3	Results and discussion	84
4.3.1	Failure modes and general behaviour	84
4.3.2	Fatigue life predictions using global approaches	89
4.3.3	Fatigue life predictions using a local approach	97
5	Numerical determination of cold forming residual stresses	103
5.1	Case study	104
5.2	Finite element modelling of the press braking process	104
5.3	Finite element modelling of the roll forming process	107
5.4	Results and discussion on cold forming residual stresses	110
5.5	The effect of pre-forming coiling operations	115
5.6	Comparison against results found in the literature	124
6	Fatigue behaviour of cold-formed thin-walled mild steel members	127
6.1	Fatigue testing campaigns	128
6.2	Numerical modelling of the experimental setups	133
6.3	Fatigue life predictions based on a global approach	135
6.3.1	Simplified stress computation	139
6.3.2	Overview of the global approach	142
6.4	Fatigue life predictions based on a local approach	143

6.4.1	Overview of the local approach	144
6.4.2	Application of the local approach	146
7	Development of a damage-tolerant approach	151
7.1	Numerical determination of stress intensity factors	152
7.1.1	Force-based extrapolation approach	153
7.1.2	Displacement-based extrapolation approach	154
7.1.3	Virtual crack extension method	155
7.1.4	Virtual crack closure technique	156
7.1.5	The J-integral	157
7.2	Parametric study on crack parameters	158
7.2.1	Base model results and sensitivity to mesh resolution	160
7.2.2	Effects of crack shape	163
7.2.3	Influence from modes II and III	166
7.2.4	Effects of load and boundary conditions	166
7.3	Simplified approach for design purposes	168
7.4	Estimating the residual fatigue life of a cracked profile	172
7.5	Accounting for residual stresses in the estimation of residual fatigue life	176
8	Conclusions	179
8.1	Concluding remarks	179
8.2	Future work	187
A	List of publications	189
B	Using machine learning for stress intensity factor determination	193
	References	204

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List of figures

1.1	Rack structures in a warehouse setting.	1
1.2	Pallet carrier shuttles as an automated storage-and-retrieval system.	2
1.3	Fatigue cracking observed in thin-walled cold-formed structural components.	3
2.1	Infinitesimal volume element describing a general stress state at a point.	9
2.2	Kinematics in two-dimensional continuum mechanics.	10
2.3	Illustration of typical engineering stress-strain curves.	13
2.4	Illustration of engineering stress-strain curves of high-strength and mild steels.	14
2.5	Illustration of true and engineering stress-strain curves.	14
2.6	Mathematical models for strain hardening.	15
2.7	Illustration of a typical hysteresis loop and the Bauschinger effect in metals.	17
2.8	Illustration of the roll forming process for producing a generic C-channel.	18
2.9	A flower diagram showing the incremental forming required to produce a generic C-channel.	19
2.10	Illustration of a typical roll forming line.	19
2.11	Illustration of the press braking process.	20
2.12	Storage of sheet metal coils.	20
2.13	Corner bending notation.	22
2.14	Through-thickness distribution of residual stresses in corner bending of thin metal sheets assuming an elastic-perfectly plastic material.	23
2.15	Results from distinct residual stress modelling approaches.	25
2.16	Fatigue damage stages and driving factors from fracture mechanics.	28
2.17	Fatigue crack initiation at leading edges of PSBs in a cycled copper crystal.	29
2.18	Illustration of mean stress effects.	30
2.19	Typical Wöhler curves, also known as stress-life or $S-N$ curves, used in fatigue design codes.	32
2.20	Total strain amplitude vs. endurance via the superposition of elastic and plastic strain components.	34
2.21	Constant and variable amplitude fatigue loading scenarios.	36
2.22	Example of a median $\epsilon-N$ curve and corresponding 95% confidence bands resulting from a statistical analysis following the ASTM E739 standard.	38
2.23	The three basic modes of fracture mechanics.	40
2.24	Two-step finite element analysis of a crack.	41
2.25	The J-integral: a path-independent line integral along any contour Γ that encloses the crack tip.	43

List of figures

2.26	Paris' law and the different stages of stable fatigue crack growth.	44
2.27	Illustration of distinct TCD methods.	45
2.28	Schematic representation of a load train in a servohydraulic testing machine.	45
2.29	Examples of loading fixtures used in axial fatigue testing machines.	46
2.30	Finite element discretization of a plate and corresponding sparse stiffness matrix (non-zero entries).	49
2.31	Finite element mesh comprised of isoparametric bilinear elements.	50
2.32	Shape functions of a bilinear isoparametric element.	52
2.33	Illustration of shear locking due to the inability of first-order finite elements in capturing the kinematics of bending.	58
2.34	Cantilever plate subjected to a transverse load (meshed).	58
2.35	Accounting for shear locking via correct element type selection.	59
2.36	The hourglass effect caused by zero-energy modes in reduced-integration elements.	60
2.37	Illustration of the incremental force method with Newton-Raphson iterations.	62
2.38	Example of a nonlinear and nonsmooth 1D contact problem.	62
2.39	Node-to-surface vs. surface-to-surface contact.	63
3.1	S350GD zinc coating under a scanning electron microscope.	66
3.2	Tensile yield curves of S350GD steel.	66
3.3	Nominal tensile yield curves of S355MC steel. Specimens taken from both the longitudinal and transverse directions.	67
3.4	Tensile yield curves of S355MC steel.	68
3.5	Experimental LCF testing setup.	69
3.6	Geometry of the smooth specimens used in the LCF experiments.	69
3.7	Evolution of maximum stress during load cycling.	71
3.8	Cyclic stress-strain hysteresis loops for the tested S355MC steel grade.	71
3.9	Cyclic stress-strain and median strain-life curves derived from LCF test results.	72
3.10	Comparison of the obtained LCF test data against results from the literature for similar S355 steel grades.	73
3.11	Experimental HCF testing setup.	74
3.12	Geometry of the smooth specimens used in the HCF experiments.	74
3.13	Extrapolation of LCF results into the HCF regime and comparison with actual HCF test data.	75
3.14	Final median stress-life curve and comparison of the obtained test data against results from the literature for similar S355 steel grades. Data is mean stress corrected.	76
4.1	Geometry of the tested bolted joint specimens.	81
4.2	Bolt holes used in the tested bolted joints.	82
4.3	Finite element models of the tested bolted joints in Abaqus/CAE.	83
4.4	Typical failure modes of snug-tight bolted joints in reference conditions.	84
4.5	Typical failure modes of snug-tight bolted joints with distinct load ratios for the same maximum applied load of 26 kN.	85
4.6	Typical failure modes of snug-tight bolted joints with a 4+4 bolt arrangement.	85
4.7	Typical failure modes of preloaded bolted joints.	86

4.8	Elastic stress amplitude considering the σ_{yy} component.	87
4.9	Typical fracture surfaces of the tested bolted joints.	89
4.10	Reference mean $S-N$ curve.	90
4.11	Reference mean $S-N$ curve with mean stress correction.	91
4.12	Direct comparison of the generated $S-N$ data using remote stresses.	92
4.13	Overview of the generated $S-N$ data.	93
4.14	Generated $S-N$ data using remote stresses vs. fatigue design curves from Eurocode 3.	94
4.15	Generated $S-N$ data using net stresses vs. fatigue design curves from Eurocode 3.	95
4.16	Generated $S-N$ data using peak pseudo-elastic maximum principal stresses vs. fatigue design curves from Eurocode 3.	96
4.17	Elastoplastic maximum principal stress amplitude.	98
4.18	Fatigue life predictions vs. experimental data based on distinct sources for the Basquin-Coffin-Manson fatigue constants.	99
4.19	Fatigue life predictions vs. experimental data based on optimized Basquin-Coffin-Manson fatigue constants for the S350GD material.	100
4.20	Computed nodal fatigue life prediction fields (examples).	101
5.1	Section geometry of the investigated Z-profiles.	104
5.2	Numerical simulation of the press braking process assuming a plane strain condition.	105
5.3	Numerical simulation of the full press braking process for the investigated Z-profile.	106
5.4	Roll forming model in Abaqus/CAE.	108
5.5	Initial meshed sheet used in the roll forming simulations and corresponding boundary conditions.	109
5.6	Roll forming flower diagram of the investigated Z-profile.	109
5.7	Simulation of the roll forming process in Abaqus/CAE.	110
5.8	Nominal vs. numerically obtained cold-formed geometry.	111
5.9	Equivalent plastic strain in press-braked and roll-formed Z-profiles.	112
5.10	Transverse residual stress in press-braked and roll-formed Z-profiles.	113
5.11	Longitudinal residual stress in press-braked and roll-formed Z-profiles.	114
5.12	Definition of corner paths used to analyse the distribution of cold forming residual stresses.	115
5.13	Equivalent plastic strain distribution along path b for the investigated cold-formed corner.	115
5.14	Transverse and longitudinal residual stress distributions along path b for the investigated cold-formed corner.	116
5.15	Transverse and longitudinal residual stress distributions along path a for the investigated cold-formed corner.	116
5.16	Simulation of the coiling process in Abaqus/CAE.	117
5.17	Longitudinal residual curvature after pre-forming coiling and uncoiling operations.	118
5.18	Equivalent plastic strain and longitudinal stress distributions along path b on a fully coiled sheet.	119

List of figures

5.19	Equivalent plastic strain and longitudinal stress distributions along path b on a flattened sheet after coiling and uncoiling.	119
5.20	Longitudinal residual stress distribution along path b on a sheet after coiling operations.	120
5.21	Effects of pre-forming coiling operations in the transverse residual stress field of press-braked Z-profiles.	121
5.22	Effects of pre-forming coiling operations in the longitudinal residual stress field of press-braked Z-profiles.	122
5.23	Effects of pre-forming coiling operations in the transverse and longitudinal residual stress distributions along path b for the investigated cold-formed corner.	123
5.24	Effects of pre-forming coiling operations in the transverse and longitudinal residual stress distributions along path a for the investigated cold-formed corner.	123
5.25	Comparison of cold forming residual stresses due to press braking along path b for the investigated cold-formed corners.	124
5.26	Comparison of cold forming residual stresses due to roll forming along path b for the investigated cold-formed corners.	125
6.1	Purpose-built setup used during the FASTCOLD project (University of Porto) for the fatigue testing of full-scale Z-profiles.	130
6.2	Downward and upward load cases considered during the fatigue testing of full-scale Z-profiles.	130
6.3	Strain gauge used for fatigue crack detection and monitoring.	131
6.4	Purpose-built setup used during the FASTCOLD project (University of Pisa) for the fatigue testing of full-scale Ω -uprights [14, 142].	131
6.5	Extraction of small-scale specimens from full-scale sections [14, 101].	132
6.6	Purpose-built testing device used during the FASTCOLD project (Aachen University) in the fatigue testing of small-scale specimens [14, 90].	132
6.7	Pseudo-elastic maximum principal stresses on full-scale Z-profiles.	133
6.8	Validation of the finite element models pertaining to the full-scale Z-profiles.	134
6.9	Pseudo-elastic maximum principal stresses on full-scale Ω -uprights.	134
6.10	Pseudo-elastic maximum principal stresses on small-scale specimens (plane strain models).	135
6.11	Overview of the generated $S-N$ data for the tested full-scale Z-profiles and corresponding small-scale specimens.	136
6.12	Overview of the generated $S-N$ data for the tested full-scale Z-profiles and all small-scale specimens.	137
6.13	Overview of the generated $S-N$ data for the tested full-scale Z-profiles and all small-scale specimens categorized by the r_i/t ratio.	137
6.14	Overview of the generated $S-N$ data for the tested full-scale Ω -uprights.	138
6.15	Generated $S-N$ data for the tested full-scale Ω -uprights grouped by r_i/t ratio.	138
6.16	Global overview of the generated $S-N$ data for Z-profiles and Ω -uprights (full- and small-scale) grouped by r_i/t ratio.	139
6.17	Example of 4 configurations generated by the parameterized finite element model.	140

6.18	Fit of the simplified model for stress computation to the numerically generated results.	141
6.19	Peak pseudo-elastic maximum principal stress determination: fit of the simplified analytical model (surfaces) to the available numerical results (points).	142
6.20	Overview of the framework for fatigue life predictions based on global approaches.	143
6.21	Overview of the framework for fatigue life predictions based on a local approach.	145
6.22	Residual stress effect on the structural response of a loaded Z-profile.	147
6.23	Residual stress effect on the fatigue life prediction field for the investigated Z-profile.	148
6.24	Fatigue life predictions vs. experimental data.	149
6.25	Endurance of cold roll-formed Z-profiles: experimental data vs. numerical predictions.	149
7.1	Uniaxially loaded rectangular plate with a central crack.	152
7.2	Node numbering used for the force-based extrapolation approach.	153
7.3	Stress intensity factors determined via the force-based extrapolation approach for distinct mesh refinement levels.	154
7.4	Stress intensity factors determined via the displacement-based extrapolation approach for distinct mesh refinement levels.	155
7.5	Stress intensity factors determined via the virtual crack extension method.	156
7.6	Stress intensity factors determined via the virtual crack closure techniques.	157
7.7	Stress intensity factors determined via the J-integral approach.	158
7.8	Finite element model for crack modelling: mesh and boundary conditions.	159
7.9	Loaded crack in base model.	161
7.10	Path-(in)dependence property of stress intensity factors (base model).	161
7.11	Distribution of stress intensity factors across the thickness (base model).	162
7.12	Stress intensity factor sensitivity to mesh resolution.	162
7.13	Influence of the finite element type choice in the computed values of stress intensity factors.	163
7.14	Crack length: 1 mm vs. 10 cm crack.	163
7.15	Effects of crack shape on the resulting stress intensity factors.	164
7.16	Through-thickness vs. half-thickness rectangular crack.	164
7.17	Example of a semi-elliptical crack.	165
7.18	Influence from modes II and III on stress intensity factor values.	166
7.19	Effect of mid-span support on the resulting stress field.	167
7.20	Effect of mid-span support on stress intensity factors.	167
7.21	Effect of load magnitude on stress intensity factors.	168
7.22	Fit of the simplified model for stress intensity factor determination based on the available numerical results.	170
7.23	Determination of stress intensity factors: fit of the simplified analytical model (surfaces) to the available numerical results (points); example for $F = 6$ kN.	171
7.24	Critical load F_c , for which the stress intensity factor $\bar{K}$ is equal to the fracture toughness K_c , as a function of the crack length a	172
7.25	Critical load as a function of the crack length for various material thicknesses.	172

List of figures

7.26	Fatigue design curves for estimating the residual endurance of a cracked profile.	174
7.27	Effect of material thickness in the estimation of residual endurance ($r = 8$ mm, $a_i = 1$ mm, $R_\sigma = 0$).	174
7.28	Effect of stress ratio in the fatigue crack propagation rate of S355 mild steel. . .	175
7.29	Residual fatigue life: prediction vs. experimental data.	175
7.30	Residual stress effect on the critical load F_c as a function of the crack length a . .	177
7.31	Predicted residual fatigue life with and without residual stress effect vs. experimental data.	178
B.1	Parameterized finite element model for stress intensity factor determination viewed as a “black box”.	194
B.2	Surrogate model: the fully connected feedforward neural network under development for stress intensity factor determination.	196
B.3	Neural network training and validation losses.	198
B.4	Surrogate neural network model vs. physics-based finite element model for stress intensity factor determination.	198
B.5	Trained neural network vs. fitted analytical model for stress intensity factor determination.	199

List of tables

3.1	Technical delivery conditions of the S350GD and S355MC steel grades.	65
3.2	Measurements of the test section and LCF test parameters for each specimen. .	70
3.3	Main LCF results obtained for the tested S355MC steel grade and derived from the pseudo-stabilized hysteresis loops.	70
3.4	HCF test parameters, main results, and measurements of the test section. . . .	75
3.5	Summary of the empirically determined material constants for the investigated S350GD and S355MC mild steel grades.	77
4.1	Fatigue testing matrix for the bolted joints.	80
4.2	Additional geometric properties of the bolted joint specimens.	81
4.3	Elastic stress concentration factors, K_t , and ratio of the force acting on the inner plate's net cross-section to the applied fatigue load, $F_{\text{net}}/F_{\text{applied}}$	88
4.4	Effects of the different tested conditions on the fatigue strength of the bolted joints.	93
4.5	Effects of (a) reducing the cross-sectional area and (b) increasing the stress ratio on the fatigue endurance of the bolted joints.	93
4.6	Derived Basquin and Coffin-Manson fatigue constants for the S355MC and S350GD mild steel grades.	97
5.1	Observed simulation times for the cold forming simulations.	110
5.2	Observed simulation times for the pre-forming coiling simulations.	118
6.1	Fatigue details investigated during the FASTCOLD project.	129
7.1	Cold forming residual stress and final maximum principal stress at fatigue crack initiation site.	176
8.1	Fatigue categories for structural details found in rack structures.	186

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List of symbols

Roman symbols

A	Area; crack surface area; polynomial coefficient.
A_0	Initial, undeformed area.
A_s	Nominal stress area.
a	Crack length; Stüssi model parameter.
a_c	Critical crack length.
a_i	Initial crack length.
B	Polynomial coefficient.
$\mathbf{B}$	Strain-displacement matrix.
b	Fatigue strength exponent; Stüssi model parameter; thickness.
C	Paris' law coefficient.
$\mathbf{C}$	Matrix of element nodal coordinates.
c	Elastic core length; fatigue ductility exponent.
D	Fatigue damage accumulation.
$\mathbf{D}$	Stress-strain matrix.
d_0	Nominal bolt hole diameter.
E	Young's modulus or modulus of elasticity in tension or compression.
e	Engineering or nominal total strain.
e_e	Engineering or nominal elastic strain.
e_p	Engineering or nominal plastic strain.
F	Force; cumulative distribution function.
F_c	Critical load leading to unstable crack growth.
F_p	Plastic force; Quantile function value for the F -distribution.
F_x	Internal force along x .
F_y	Internal force along y .

List of symbols

f	Frequency; function.
$\mathbf{f}$	External force vector.
$\mathbf{f}_p$	External point force vector.
$\mathbf{f}_s$	External surface-distributed force vector.
$\mathbf{f}_b$	External body-distributed force vector.
G	Shear modulus or modulus of rigidity; strain energy release rate.
g	Acceleration due to Earth's gravity.
I	Second area moment.
$\mathbf{I}$	Second-rank identity tensor; identity matrix.
J	J-integral.
$\mathbf{J}$	Jacobian matrix.
K	Bulk modulus; hardening coefficient; stress intensity factor.
K'	Ramberg-Osgood hardening coefficient.
$\bar{K}$	Equivalent or effective stress intensity factor.
K_c	Fracture toughness.
K_f	Fatigue notch factor.
K_t	Stress concentration factor.
$\mathbf{K}$	Stiffness matrix.
k	Stiffness; sample size.
L	Length; characteristic material length parameter.
L_0	Initial, undeformed length.
$\mathbf{L}$	Matrix of direction cosines.
l	Length.
M_x	Longitudinal plastic moment.
M_z	Transverse plastic moment.
$\mathbf{M}$	Mass matrix.
m	Line slope; Paris' law exponent.
N	Load cycle number; shape function.
N_f	Number of load cycles to failure (endurance).
$\mathbf{N}$	Matrix of shape functions.
n	Hardening exponent.
n'	Ramberg-Osgood hardening exponent.
$\mathbf{n}$	Eigenvector; principal axis.

P	Probability of failure.
q	Notch sensitivity index.
R	Load ratio.
R^2	Coefficient of determination.
$\bar{R}^2$	Adjusted coefficient of determination.
R_e	Engineering strain ratio.
R_{eH}	Upper yield strength.
R_{eL}	Lower yield strength.
R_m	Ultimate tensile strength.
R_p	Offset yield strength.
R_S	Engineering stress ratio.
R_ε	Strain ratio.
R_σ	Stress ratio.
r	Radius; first natural coordinate.
r_{th}	Yield threshold coiling radius.
r_x	Coiling radius.
r_z	Bend or corner radius.
$\mathbf{r}$	Natural coordinates vector.
S	Engineering or nominal stress.
S_m	Engineering or nominal mean stress.
S_u	Nominal ultimate tensile strength.
$\mathbf{S}$	Second-order Cartesian tensor.
s	Curvilinear coordinate; second natural coordinate.
t	Time; thickness; third natural coordinate.
t_p	Quantile function value for the t -distribution.
$\mathbf{t}$	Surface traction vector.
u	Displacement along the first Cartesian coordinate; unknown variable.
$\mathbf{u}$	Displacement vector.
v	Displacement along the second Cartesian coordinate.
W	Strain energy density.
W_i	Internal strain energy.
W_e	Work done by external loads.
w	Displacement along the third Cartesian coordinate; width.

List of symbols

X	Independent variable.
x	First Cartesian coordinate; transverse direction; random or independent variable.
$\mathbf{x}$	Cartesian coordinates vector.
Y	Dependent variable; geometric correction factor.
y	Second Cartesian coordinate; through-thickness direction.
z	Third Cartesian coordinate; longitudinal or rolling direction.

Greek symbols

α	Taras-Greiner fatigue strength factor; Weibull location parameter.
β	Weibull scale parameter.
Γ	Contour.
γ	Weibull shape parameter.
γ_j	Engineering shear strain.
Δ	Change in value or variation; range.
ΔK_{th}	Fatigue crack propagation limit or threshold.
ΔS_C	Detail category.
ΔS_D	Fatigue limit for constant amplitude loading.
ΔS_L	Fatigue limit for variable amplitude loading.
$\Delta \sigma_{th}$	Fatigue limit or threshold.
ε	Strain; true strain, natural strain, or logarithmic strain.
$\bar{\varepsilon}$	Equivalent or effective strain.
ε_0	Pre-strain.
ε_a	Strain amplitude.
$\bar{\varepsilon}_a$	Amplitude of equivalent strain.
ε_e	True elastic strain.
ε'_f	Fatigue ductility coefficient.
ε_i	Principal strains.
$\varepsilon_{i,a}$	Amplitude of principal strains within load cycle.
ε_{ij}	Strain tensor.
ε_m	Mean strain.
ε_{max}	Maximum value of strain within load cycle.
ε_{min}	Minimum value of strain within load cycle.

ε_p	True plastic strain.
$\bar{\varepsilon}_p$	Equivalent plastic strain.
ε	Strain tensor represented as a matrix.
$\underline{\varepsilon}$	Strain tensor represented as a vector.
κ	Material damping property parameter.
λ	Lamé's first parameter; eigenvalue.
μ	Lamé's second parameter; shear modulus; friction coefficient.
ν	Poisson's ratio.
Π	Potential energy.
ρ	Mass density.
σ	Stress; true stress; standard deviation.
$\bar{\sigma}$	Equivalent or effective stress.
σ_∞	Applied remote stress.
σ_a	Stress amplitude.
$\bar{\sigma}_a$	Amplitude of equivalent stress.
σ_f	Failure strength.
σ'_f	Fatigue strength coefficient.
σ_i	Principal stresses.
$\sigma_{i,a}$	Amplitude of principal stresses within load cycle.
$\sigma_{i,m}$	Mean value of principal stresses within load cycle.
σ_{ij}	Stress tensor.
σ_m	Mean stress.
σ_{major}	Major principal stress.
σ_{max}	Maximum value of stress within load cycle.
σ_{min}	Minimum value of stress within load cycle.
σ_u	Ultimate tensile strength.
σ_x	Transverse stress.
σ_y	Yield strength.
σ_z	Longitudinal stress.
σ	Stress tensor represented as a matrix.
$\underline{\sigma}$	Stress tensor represented as a vector.
Ω_e	Finite element domain.

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List of abbreviations

AISI	American Iron and Steel Institute
AM	Area method of the TCD
CAD	Computer-aided design
CAE	Computer-aided engineering
CEN	European Committee for Standardization
EU	European Union
FEA	Finite element analysis
FEM	Finite element method
FEUP	Faculty of Engineering of the University of Porto
HCF	High-cycle fatigue
ISO	International Organization for Standardization
LCF	Low-cycle fatigue
LEFM	Linear-elastic fracture mechanics
LM	Line method of the TCD
PB	Press-braked; press braking
PID	Proportional-integral-derivative
PM	Point method of the TCD
PRODEM	Doctoral Program in Mechanical Engineering
PSB	Persistent slip band
RF	Roll-formed; roll forming
RFCS	Research Fund for Coal and Steel
SD	Standard deviation
SEM	Scanning electron microscope
S&R	Storage-and-retrieval
TCD	Theory of critical distances

List of abbreviations

ULCF	Ultra-low-cycle fatigue
VCCT	Virtual crack closure technique
VCEM	Virtual crack extension method
VHCF	Very-high-cycle fatigue
VM	Volume method of the TCD

Chapter 1

Introduction

This thesis was conducted as part of the Doctoral Program in Mechanical Engineering (PRO-DEM) of the Faculty of Engineering of the University of Porto (FEUP). Its primary objective is to present fatigue assessment procedures for cold-formed thin-walled mild steel structures. While the current focus is on these specific structures and their structural members, the methodologies presented in this document, which are based on experimental fatigue testing and advanced finite element analysis (FEA), are versatile and can be adapted to other areas of mechanical and civil engineering in which fatigue phenomena must be considered. Finally, it is relevant to note that much of the work presented herein has already been peer-reviewed and published in the literature mentioned below (see [Section 1.2](#)).

1.1 Background and motivation

Rack structures ([Figure 1.1](#)) are characterized by the application of thin-walled cold-formed mild steel profiles joined using non-preloaded bolted connections associated with a limited shear slip resistance. The main purpose of these structures is to enable efficient storage-and-retrieval (S&R) operations for palletized goods and stocks. In modern warehouses and distribution centres, rack structures may also accommodate automated S&R systems, such as pallet carrier shuttles ([Figure 1.2](#)), in order to meet the demands of advanced logistics and maximize operational efficiency.



Figure 1.1: Rack structures in a warehouse setting; © 2024 NEDCON BV (reproduced with permission).

1: Introduction

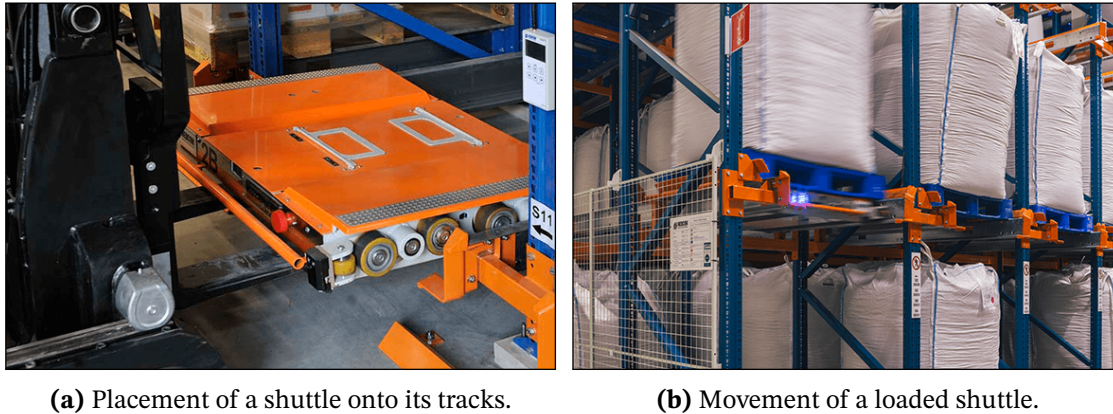
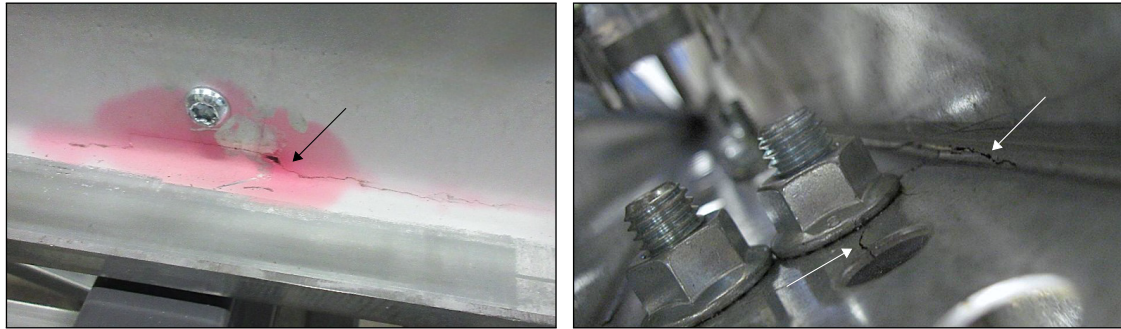


Figure 1.2: Pallet carrier shuttles as an automated storage-and-retrieval system; © 2024 NED-CON BV (reproduced with permission).

The cyclic storage of heavy palletized goods, combined with dynamic actions from S&R systems, leads to fatigue phenomena, such as fatigue cracking (Figure 1.3), which raises safety concerns and increases maintenance costs. Furthermore, the use of cold-formed mild steel members raises additional questions about the effect of residual stresses on fatigue performance. In fact, cold forming refers to a family of metal shaping processes performed below the recrystallization temperature of the material, resulting in a significant formation of residual stresses caused by cold working and strain hardening [1]. In short, residual stresses result from the complex straining history induced during manufacturing; however, their impact on fatigue performance is not necessarily negative. In general, residual stresses can be thought of in much the same way as static structural stresses when the material is subjected to dynamic fatigue loads, i.e., they tend to alter the mean stress level of the load cycle. Therefore, a simplified approach is to consider that residual stresses tend to be favourable when compressive, or detrimental when tensile [2]. Regardless, the determination of manufacturing residual stresses can be quite a complex task, as shown later. On a separate note, the use of non-preloaded bolted connections as a joining method, in which bolts are not tightened to a specific predetermined and controlled value (also known as snug-tight bolts), may result in a reduced shear slip resistance during load bearing. This issue can be further exacerbated when galvanized steel members are used, in which a layer of zinc is metallurgically bonded to the base metal as a corrosion protection technique, resulting in low-friction surfaces. Ultimately, a significant change in the joint's fatigue behaviour is expected based on whether shear slip occurs or is prevented by friction.

To summarize, fatigue is a significant matter in the design of rack structures and associated S&R systems. Specific concerns that require thorough understanding include the impact of residual stresses and the use of non-preloaded bolted connections. Nevertheless, at the time of writing, current EU regulations do not comprehensively cover the fatigue design of thin-walled structures and cold-formed steel members. Considering the Eurocode 3 standards, the EN 1993-1-1 standard [3] covers the static design of thick sections, while the EN 1993-1-3 standard [4] covers that of thin cold-formed members and sheeting. However, in regard to fatigue, the EN 1993-1-9 standard [5] only covers the fatigue design of thick hot-rolled sections. Moreover, the EN 15512 standard [6], dedicated to the structural design of adjustable



(a) Cracks in a cold-formed corner.

(b) Cracks near a bolted joint.

Figure 1.3: Fatigue cracking observed in thin-walled cold-formed structural components.

pallet racking systems, acknowledges fracture due to fatigue as a form of ultimate limit state; however, the standard makes no further reference to fatigue, later stating that ‘it is therefore implicit that normal rack structures are not subject to fatigue’. Meanwhile, there are North American standards, such as the AISI S100-16 standard [7], addressing structural members and connections made of cold-formed steel subject to cyclic loading within the elastic range of stresses of frequency and magnitude sufficient to initiate cracking and progressive failure, defining the limit state of fatigue.

Within this background, the FASTCOLD research project, funded by the Research Fund for Coal and Steel (RFCS) of the EU (project no. 745982), aimed the development of specific assessment procedures for the fatigue design of thin cold-formed members commonly used in the rack industry, filling the identified gap in current European fatigue design standards. As part of this project, in which the current author participated, preliminary research was performed. This, combined with a large dataset generated based on several fatigue testing campaigns conducted during the FASTCOLD project, spawned the current PhD research.

Besides the contributions arising from the FASTCOLD project, the European Committee for Standardization (CEN) via the CEN/TC 250 technical committee for structural Eurocodes, is currently in the process of updating the EN 1993-1-9 standard [5], where the prEN 1993-1-9 revision [8] is currently under approval. This revision includes new considerations for non-preloaded bolted connections, as well as for rolled products. Naturally, findings resulting from the current research will be compared against these newly proposed Eurocode 3 updates. Regardless, the timing and efforts made by the CEN to update current fatigue standards highlight the importance of the topic at hand.

Furthermore, in modern-day projects, a client may invite several contractors to submit proposals, often prioritizing affordability. Contractors compete by offering the lowest cost while balancing quality and innovation. To reduce costs and win bids, contractors are then motivated to propose thinner structural members, but structural integrity, regulatory compliance, and long-term durability must still be ensured. Ultimately, advanced engineering techniques for predicting the residual fatigue life of a profile in which a visible fatigue crack has already started to develop may be crucial in extending the structure’s service life. In this context, the present work also aims to develop a novel damage-tolerant approach, based on engineering principles of linear-elastic fracture mechanics (LEFM) and stable fatigue crack growth, that can be applied in real-world scenarios.

1.2 Dissertation layout and objectives

Below, the layout of this dissertation is presented alongside the specific goals outlined for each chapter. This overview aims to clarify the structure of the current PhD research.

Chapter 1: Introduction – This chapter presents the background and motivation for the current PhD research, highlighting a clear gap in current European design standards for fatigue in thin-walled structures and cold-formed steel. Key concerns that require thorough understanding include the impact of manufacturing residual stresses and the use of non-preloaded bolted connections. Additionally, the research goals are presented.

Chapter 2: Literature review – This chapter presents a literature review designed to familiarize the reader with the theoretical background supporting this research. It offers a comprehensive overview, referencing both historically significant publications and recent studies that define the current state of the art. The chapter is organized into three main sections, each focusing on the mechanical behaviour of metals, metal fatigue, and numerical modelling.

Chapter 3: Mechanical testing of the plain materials – The goal of this chapter is to present empirical data related to the materials considered in this work, i.e., the S350GD and S355MC mild steels, along with the methods employed during testing and post-processing. Specifically, three types of experiments are performed: quasi-static tensile strength testing to assess the strain hardening behaviour of both materials, as well as dynamic strain-controlled low-frequency tests and load-controlled high-frequency tests to respectively evaluate the low- and high-cycle fatigue behaviours of the S355MC material. Naturally, the empirical models and constants derived in this chapter are referenced throughout this research. For the derived literature, see references [9, 10].

Chapter 4: Fatigue behaviour of bolted joints used in rack structures – This chapter presents specific fatigue assessment procedures for both preloaded and non-preloaded (snug-tight) bolted joints used in rack structures. Due to low-friction zinc coatings (galvanized plate material) and uncertainties regarding the preload level of the bolts, shear slip resistance may be limited, impacting failure modes. The main objective is to incorporate shear slip such that the fatigue behaviour of the joints is accurately modelled. During the FASTCOLD project, a fatigue testing campaign was conducted considering various parameters such as bolt arrangement, preload level, bolt hole manufacturing process, stress ratio, and plate cross-section size. From the resulting fatigue data, a classical global $S-N$ approach, which relies on engineering stress measures and detail categorization, yields multiple load vs. endurance relationships. The aim of this research is to simplify this complex detail categorization scheme by constructing a master mean $S-N$ curve that covers several joint parameters. This is achieved through a more refined measure of the stresses acting on the material, which are computed via finite element modelling while considering the bolt preload level, accurate friction coefficients, and a simplified pseudo-elastic material. Additionally, an alternative numerical procedure is explored that incorporates a more representative elastoplastic material behaviour, which is then followed by locally evaluating fatigue prediction models at critical locations. This method predicts fatigue life independently of bolted joint test results, relying only on the fatigue prop-

erties of the plain material and accurate numerical modelling. A standalone summary of the research presented within this chapter is given in reference [11].

Chapter 5: Numerical determination of cold forming residual stresses – The primary objective of this chapter is the numerical determination of residual stress and strain fields generated by cold forming manufacturing processes, such as press braking and roll forming. Additionally, effects due to auxiliary pre-forming processes, specifically coiling operations of the sheet material, are also investigated. The aim is to utilize the resulting residual fields and cold-formed geometries as the initial state in subsequent structural and fatigue analyses, which are dealt with in the next chapter. To this end, the manufacturing processes are numerically simulated via elastoplastic FEA. When considering a complex process such as roll forming, the resulting numerical work is both challenging and time-consuming. Nevertheless, an accurate determination of residual stresses is key to understand their potential effect on the fatigue performance of the produced structural component. For the derived literature, see references [9, 12, 13].

Chapter 6: Fatigue behaviour of cold-formed thin-walled mild steel members – This chapter presents specific fatigue assessment procedures for cold-formed thin-walled mild steel members. It includes a global summary of the main fatigue data generated during several full- and small-scale testing campaigns performed within the FASTCOLD project. A global approach is used to develop fatigue design rules via the categorization of structural details commonly found in the rack industry. This categorization is accomplished via regression-based *S-N* approaches; however, fatigue actions are evaluated numerically, allowing for the assessment of general and complex multiaxial cases. This numerical evaluation also aims to modernize current fatigue design approaches, as the use of numerical tools for stress determination is becoming more widely adopted for design purposes. Another objective is to investigate how manufacturing residual stresses affect fatigue life predictions made using alternative local approaches. To this end, a state-of-the-art framework is proposed and applied to a case study involving a specific thin-walled profile used as guide rails for pallet carrier shuttles in automated S&R systems, where the cold-formed corners are identified as the critical locations. The mechanical and fatigue properties of the plain material are first determined, followed by an evaluation of the material's straining history due to manufacturing, information that is then included in the subsequent structural analysis from which fatigue life predictions are made at critical locations. In regard to the relevant literature, reference [12] presents a preliminary and exploratory research onto these matters. Regarding the global approach, reference [14] presents a complete and standalone view into this topic, as well as a summary of the main FASTCOLD project results. Regarding the alternative local approach and corresponding residual stress effect, see reference [9] as a comprehensive and detailed paper dedicated to this subject.

Chapter 7: Development of a damage-tolerant approach – The focus of this chapter is to develop a damage-tolerant approach for the prediction of residual fatigue life of a profile in which a visible fatigue crack has already started to develop. The objective is for the derived approach to be applicable in real-world scenarios, extrapolating from well-known engineering principles based on LEFM and stable fatigue crack growth. To this end, finite element model-

1: Introduction

ling is used for the numerical determination of stress intensity factors, to perform a parametric study on crack parameters, and to derive a subsequent simplified analytical model which can then be used for the static design of a cracked profile based on the fracture toughness of the material. Additionally, by relying on Paris' law, the analysis can be further extended in view of estimating the residual fatigue life of a cracked profile. Finally, exploratory research is also presented to account for residual stresses in the estimation of the residual fatigue life.

Chapter 8: Conclusions – This chapter presents the major conclusions of this PhD thesis, along with suggestions for possible derived future works.

Appendix A: List of publications – This appendix lists several research papers published within the scope of this thesis.

Appendix B: Using machine learning for stress intensity factor determination – In Chapter 7, a damage-tolerant approach is developed based on a simplified analytical model that replaces a parameterized finite element model for stress intensity factor determination. As an alternative, in Appendix B, a surrogate neural network model is trained to emulate the physics-based numerical model. This data-driven approach represents a modern alternative capable of learning complex mappings from finite element modelling, while still producing results that are in good agreement with the closed-form expressions derived from fracture mechanics.

Chapter 2

Literature review

In this chapter, a literature review is made, aiming to introduce the reader to the underlying theoretical background that sustains this research. This review should provide a comprehensive overview, referencing historically relevant publications as well as recent works that establish the current state of the art. This chapter is divided into three main sections:

- In the first section, a review is made on the mechanical behaviour of metals, which starts by recalling basic concepts from linear elasticity and then delves into their strain hardening behaviour. Afterwards, a review on manufacturing processes based on cold forming and resulting residual stresses is made.
- In the second section, a global review on metal fatigue is made, presenting its history, general concepts, and currently adopted approaches. Afterwards, the current state of the art is reviewed in regard to fatigue in cold-formed steel and thin-walled structures with a focus on residual stress effects.
- In the third section, a brief introduction to numerical modelling via finite element procedures is given. Because it is challenging to cover such a vast topic in a few pages, the focus is limited to the fundamentals and on key advanced topics necessary for the modelling of cold forming processes, resulting in nonlinear and nonsmooth analyses.

2.1 Mechanical behaviour of metals

This section presents a literature review on the mechanical behaviour of metals, covering the topics of linear elasticity, strain hardening, cold forming, and residual stresses. The main objective is to present the basic concepts from mechanical engineering that are used throughout this work.

2.1.1 Linear elasticity

The first formal studies on the mechanical behaviour of metals began in the 17th century when Robert Hooke (1635-1703) first discovered the law of springs, which today is known as Hooke's law [15]. This law states that the force F required to extend or compress a spring by some length ΔL scales linearly with respect to that change in length, i.e., $F = k\Delta L$, where k is

2: Literature review

an empirically determined constant scale factor that characterizes the stiffness of the spring. However, Hooke's law is only valid for small displacements, i.e., when ΔL is small compared to the maximum possible deformation of the spring. Nevertheless, with his discoveries, Hooke paved the way to the theory of elasticity.

Today, the theory of elasticity is intrinsically related to the concepts of stress and strain. Stress, denoted σ , is defined as the intensity of force at a point [15], i.e.,

$$\sigma = \frac{\partial F}{\partial A} \quad \text{as} \quad \partial A \rightarrow 0. \quad (2.1)$$

In an uniaxial case – the simplest case – the stress state of a loaded body is uniform; hence, the normal stress σ (compressive or tensile) is given as the normal force F per unit of instantaneous cross-sectional area A , i.e.,

$$\sigma = \frac{F}{A}, \quad (2.2)$$

in which σ is called true stress. For convenience, engineers may also define stress as the normal force F per unit of initial, undeformed cross-sectional area A_0 , i.e.,

$$S = \frac{F}{A_0}, \quad (2.3)$$

where S is called engineering or nominal stress. An infinitesimal normal strain $d\varepsilon$ is defined as the infinitesimal change in length dl over the instantaneous length l [15], i.e.,

$$d\varepsilon = \frac{dl}{l}; \quad (2.4)$$

therefore, the normal strain ε is given by integrating from an initial, undeformed length L_0 to a current, deformed length L , i.e.,

$$\varepsilon = \int_{L_0}^L \frac{dl}{l} = \ln\left(\frac{L}{L_0}\right). \quad (2.5)$$

This finite form of strain is known as true strain, natural strain, or logarithmic strain. Again, for convenience, it is common to define the so-called engineering or nominal strain e as

$$e = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}. \quad (2.6)$$

For increasingly small strains, engineering and true strain measures become increasingly similar.

Using the definition of engineering stress, shown in (2.3), and the definition of engineering strain, shown in (2.6), Hooke's law may be restated as

$$S = Ee \quad \text{where} \quad E = \frac{kL_0}{A_0}, \quad (2.7)$$

in which the engineering stress is linearly related to the engineering strain via the modulus of elasticity in tension or compression, denoted E , which is also known as Young's modulus

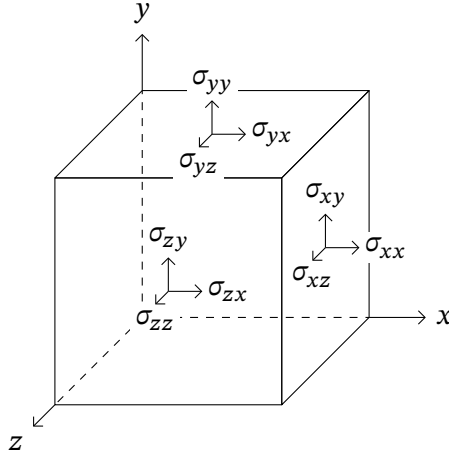


Figure 2.1: Infinitesimal volume element describing a general stress state at a point (without stress variations).

after Thomas Young (1773-1829) and is a measure of rigidity or stiffness of a solid material. Generally, this constitutive law, so-called since it relates kinematic to kinetic quantities, is only valid for small strains, and any material that obeys it is said to be linear-elastic or Hookean. Moreover, under small strain assumptions, the following is also valid:

$$\sigma = E\varepsilon. \quad (2.8)$$

Thus far, the uniaxial stress state was considered for introducing the concepts of stress and strain. However, in a three-dimensional case – the general case – the stress state of a loaded body is more complex; therefore, nine stress components are required to fully describe the stress state at any given point, as shown in [Figure 2.1](#). In fact, stress is a second-order tensor, denoted σ_{ij} , and it may be represented as a matrix:

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}; \quad (2.9)$$

or, since $\sigma_{ij} = \sigma_{ji}$, as the following vector:

$$\underline{\sigma} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} \equiv \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix}. \quad (2.10)$$

The first subscript of σ_{ij} denotes the normal to the plane on which the force acts, whereas the second denotes the direction of the force. Hence, repeated subscripts (e.g., σ_{xx}) denote normal stresses, while mixed subscripts (e.g., σ_{xy}) denote shear stresses.

As shown in [Figure 2.2](#), a change in the initial, undeformed configuration of a continuum body may be described via rigid-body displacements and deformations. Rigid-body motion consists in translation and rotation of the body without changing its shape or size. Deformation may be decomposed into two components, namely, distortion, which relates to changes in

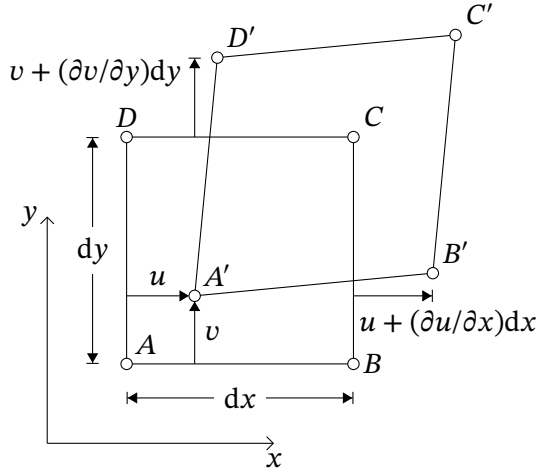


Figure 2.2: Kinematics in two-dimensional continuum mechanics.

shape, and dilatation, which relates to changes in size/volume. Strain measures must then be able to correctly describe the deformation of a body while excluding rigid-body motion. Similarly to stress, strain is also a second-order tensor, denoted ε_{ij} . Moreover, under small strain assumptions, normal strains may be defined as fractional extensions or contractions, $\Delta L/L_0$; hence, from Figure 2.2,

$$\varepsilon_{xx} = \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}} = \frac{(\partial u/\partial x)dx}{dx} = \frac{\partial u}{\partial x}, \quad (2.11)$$

and, similarly,

$$\varepsilon_{yy} = \frac{\overline{A'D'} - \overline{AD}}{\overline{AD}} = \frac{(\partial v/\partial y)dy}{dy} = \frac{\partial v}{\partial y}. \quad (2.12)$$

Shear strains are defined in terms of the angles between $\overline{AB}$ and $\overline{A'B'}$ and between $\overline{AD}$ and $\overline{A'D'}$; hence, under small strain assumptions, from Figure 2.2,

$$\varepsilon_{xy} = \frac{(\partial v/\partial x)dx}{dx} = \frac{\partial v}{\partial x} \quad \text{and} \quad \varepsilon_{yx} = \frac{(\partial u/\partial y)dy}{dy} = \frac{\partial u}{\partial y}. \quad (2.13)$$

Additionally, the engineering shear strain is defined as the sum of these angles, i.e., $\gamma_{xy} = \gamma_{yx} = \varepsilon_{xy} + \varepsilon_{yx}$. Thus, for the general three-dimensional case, the small strain tensor is given by

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (2.14)$$

and it may be represented as

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \equiv \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \quad \text{or} \quad \underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix} \equiv \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{Bmatrix}. \quad (2.15)$$

Moreover, the engineering shear strain can be defined as

$$\gamma_{ij} = 2\varepsilon_{ij} \quad \text{for } i \neq j. \quad (2.16)$$

It has been implied that stress and strain tensors are dependent on the chosen coordinate system; nevertheless, a stress state based on one set of orthogonal axes can be expressed using another. The components of a general second-order Cartesian tensor $\mathbf{S}$ may be transformed using the following expression [16]:

$$\mathbf{S}' = \mathbf{L}\mathbf{S}\mathbf{L}^T, \quad (2.17)$$

where $\mathbf{L}$ is the matrix of direction cosines between two orthogonal coordinate systems. Additionally, it is possible to determine a set of orthogonal axes ($\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$) in which $\mathbf{S}'$ becomes a diagonal matrix. This corresponds to a standard eigenproblem [16]:

$$\mathbf{S}\mathbf{n}_i = \lambda_i\mathbf{n}_i \Leftrightarrow (\mathbf{S} - \lambda_i\mathbf{I})\mathbf{n}_i = \mathbf{0}, \quad (2.18)$$

where $\mathbf{I}$ is the second-rank identity tensor, $\mathbf{n}_i$ denotes the eigenvectors, and λ_i denotes the corresponding eigenvalues. The non-trivial solution is obtained when $|\mathbf{S} - \lambda_i\mathbf{I}| = 0$. If $\mathbf{S}$ corresponds to $\boldsymbol{\sigma}$ or $\boldsymbol{\varepsilon}$, λ_i corresponds to the so-called principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) or principal strains ($\varepsilon_1 \geq \varepsilon_2 \geq \varepsilon_3$), respectively [16]. Principal stresses and strains are particularly interesting because they are invariant under orthogonal (rotational) coordinate transformations; thus, principal stresses are commonly used by engineers for describing equivalent or effective stresses (denoted $\bar{\sigma}$) following yield or failure criteria, the most used of which are [15]:

$$\text{Tresca yield criterion: } \bar{\sigma} = \sigma_1 - \sigma_3, \quad (2.19)$$

$$\text{von Mises yield criterion: } \bar{\sigma} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}. \quad (2.20)$$

If uniaxial tension is applied, from (2.8) one may conclude that $\varepsilon = \sigma/E$; however, in a three-dimensional continuum solid, extension in one direction results in contractions in the remaining Cartesian directions. This is the so-called Poisson effect [15], named after Siméon Poisson (1781-1840). Thus, taking into account these Poisson contractions, Hooke's law can be stated, for the general three-dimensional case, as [15]

$$\begin{aligned} \varepsilon_{xx} &= (1/E)[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})], & \varepsilon_{yz} &= \sigma_{yz}/(2G), \\ \varepsilon_{yy} &= (1/E)[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})], & \varepsilon_{zx} &= \sigma_{zx}/(2G), \\ \varepsilon_{zz} &= (1/E)[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})], & \varepsilon_{xy} &= \sigma_{xy}/(2G), \end{aligned} \quad (2.21)$$

where ν is the so-called Poisson's ratio, i.e., a measure of the Poisson effect, and G is known as the shear modulus or modulus of rigidity. After some algebraic manipulation, Hooke's law can be stated as the following, more convenient, matrix equation:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{pmatrix} = \begin{bmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \end{bmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{pmatrix}, \quad (2.22)$$

2: Literature review

or simply

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}, \quad (2.23)$$

where $\mathbf{D}$ is known as the stress-strain matrix and, in fact, is the matrix representation of a fourth-order tensor (i.e., it performs a linear map between stress and strain, two second-order tensors); λ and μ are, respectively, Lamé's first and second parameters, named after Gabriel Lamé (1795-1870), and these are related to E , ν , and G :

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu \equiv G = \frac{E}{2(1+\nu)}. \quad (2.24)$$

As previously mentioned, deformation may be decomposed into dilatation and distortion; thus, it is common to decompose the strain tensor into volumetric and deviatoric components, where the former is related to dilatation and the latter to distortion [16]:

$$\boldsymbol{\varepsilon} = \text{vol}(\boldsymbol{\varepsilon}) + \text{dev}(\boldsymbol{\varepsilon}), \quad \text{vol}(\boldsymbol{\varepsilon}) = \frac{1}{3} \text{tr}(\boldsymbol{\varepsilon})\mathbf{I}, \quad \text{dev}(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon} - \text{vol}(\boldsymbol{\varepsilon}), \quad (2.25)$$

where $\mathbf{I}$ is the second-rank identity tensor. The same idea can be applied to the stress tensor [16]:

$$\boldsymbol{\sigma} = \text{vol}(\boldsymbol{\sigma}) + \text{dev}(\boldsymbol{\sigma}), \quad \text{vol}(\boldsymbol{\sigma}) = \frac{1}{3} \text{tr}(\boldsymbol{\sigma})\mathbf{I}, \quad \text{dev}(\boldsymbol{\sigma}) = \boldsymbol{\sigma} - \text{vol}(\boldsymbol{\sigma}), \quad (2.26)$$

where $\text{vol}(\boldsymbol{\sigma})$ is known as hydrostatic stress or pressure. Considering this decomposition, Hooke's law for isotropic materials may be stated as [16]

$$\boldsymbol{\sigma} = 3K \text{vol}(\boldsymbol{\varepsilon}) + 2G \text{dev}(\boldsymbol{\varepsilon}), \quad (2.27)$$

where it becomes clear that K , the bulk modulus, given by

$$K = \frac{E}{3(1-2\nu)}, \quad (2.28)$$

is a measure of how resistant a material is to dilatation, while G is a measure of how resistant a material is to distortion.

2.1.2 Strain hardening

Most engineering materials cannot be considered linear-elastic, unless under small strain assumptions. In particular, metals subjected to large strains exhibit a strain hardening effect after an initial linear-elastic regime. Strain hardening, also known as work hardening, is related to permanent plastic deformations caused by microstructural changes in the material as a reaction to internal stresses. These microstructural changes are due to the movement of the so-called dislocations within the crystal lattice structure, a phenomenon known as slip [15]. Consequently, metals are known as elastoplastic materials.

The stress vs. strain relationship is determined experimentally via a quasi-static uniaxial tensile test, in which a smooth and slender specimen is stretched along its central axis. The stress-strain curve is then a representation of the performance of the specimen as the applied

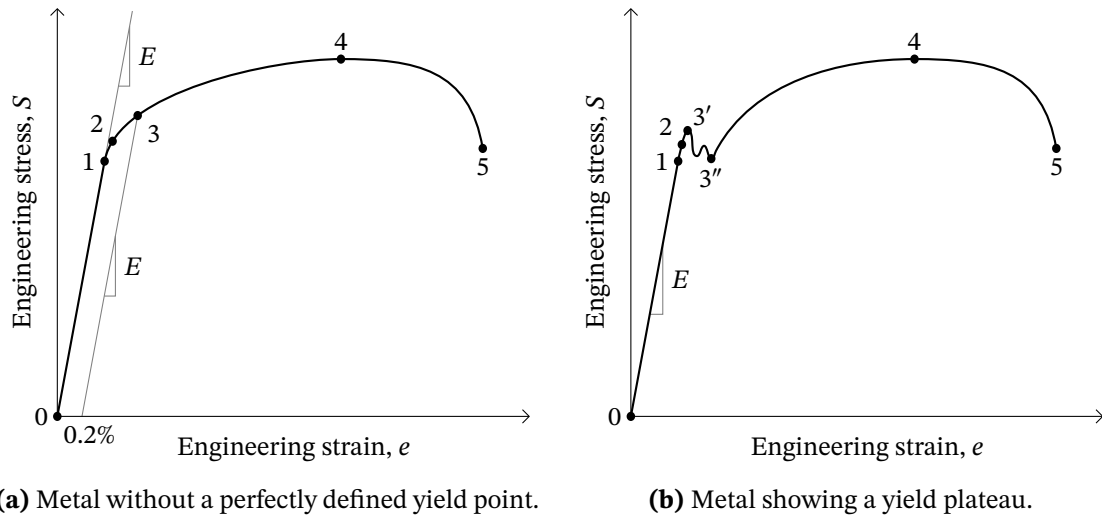


Figure 2.3: Illustration of typical engineering stress-strain curves. Point 0 is the origin, 1 is the proportional limit, 2 is the elastic limit, 3 is the offset yield strength or proof stress, 3' is the upper yield strength, 3'' is the lower yield strength, 4 is the ultimate tensile strength, and 5 denotes specimen fracture.

load increases monotonically until fracture is achieved. In fact, such a test generally results in a load vs. elongation curve, which is then converted into an engineering stress vs. engineering strain curve. Because engineering measures of stress and strain are obtained by dividing the load and elongation by constant factors, which pertain to the undeformed geometry of the specimen, the load-elongation curve has the same shape as the engineering stress-strain curve.

A typical engineering stress-strain curve is illustrated in Figure 2.3a. In this figure, from point 0, the origin, to point 1, the proportional limit, the stress-strain relationship is linear-elastic, i.e., stress is linearly proportional to strain. The slope of this straight line segment corresponds to the modulus of elasticity. Point 2, the elastic limit, which may coincide with point 1 depending on the material and precision of the measurement equipment, represents the stress value below which strains are elastic and completely reversible once the applied stress is removed. Beyond the elastic limit, strains are a combination of elastic and plastic portions, where the former are reversible, and the latter are permanent. Point 3, the offset yield strength, represents the stress value required to produce a small specified amount of permanent plastic deformation. In ISO and CEN standards, the offset yield strength is referred to as the proof stress for 0.1%, 0.2%, and 0.5% of permanent elongation, denoted $R_{p0.1}$, $R_{p0.2}$, and $R_{p0.5}$, respectively. The offset yield strength is commonly used for design and specification purposes since it avoids practical difficulties in the precise determination of the proportional and elastic limits. Point 4, the ultimate tensile strength, denoted R_m in ISO and CEN standards, represents the maximum load that a metal can withstand under the considered testing conditions. The line segment defined by points 2 and 4 represents the strain hardening region, in which the volume of the specimen remains constant during plastic deformation: as the specimen elongates, its cross-sectional area decreases uniformly. Finally, point 5 denotes the fracture of the specimen. The line segment defined by points 4 and 5 represents the so-called necking region, in which the cross-sectional area of the specimen disproportionately decreases locally, at

2: Literature review

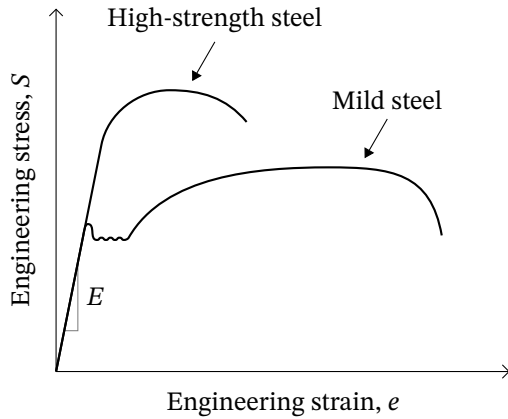


Figure 2.4: Illustration of engineering stress-strain curves of high-strength and mild steels.

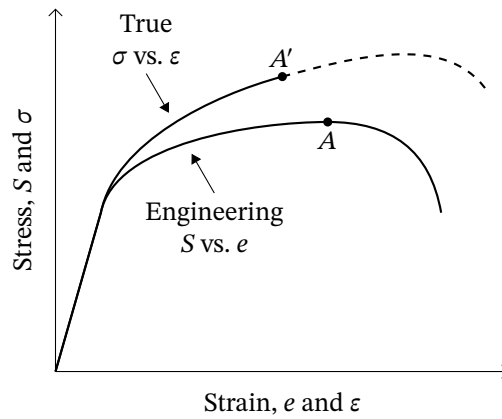


Figure 2.5: Illustration of true and engineering stress-strain curves. Point A is the ultimate tensile strength.

a point in the specimen that is slightly weaker than the rest. Further plastic deformations are concentrated in this region, and the ability of the specimen to resist deformation decreases, leading to failure.

While Figure 2.3a illustrates the stress-strain curve of a metal without a perfectly defined yield point, where the transition from elastic to plastic behaviour is gradual, Figure 2.3b shows that some metals exhibit a so-called yield plateau. The yield plateau is most common in annealed or hot-rolled low-carbon steels, while cold-rolled and high-carbon steels do not show a yield plateau [17–19]. Moreover, in ISO and CEN standards, the yield plateau is characterized via the upper and lower yield strengths, respectively denoted R_{eH} and R_{eL} (points 3' and 3'' in Figure 2.3b). R_{eH} corresponds to the sudden levelling or drop in stress as the material behaviour transitions from elastic to plastic, while R_{eL} corresponds to the lowest stress value within the yield plateau.

The shape of the stress-strain curve of a metal may depend on its composition, heat treatment, and pre-straining history, as well as on the adopted conditions during testing, namely, strain rate, temperature, orientation of the material, and size and shape of the specimen. Figure 2.4 illustrates the distinctiveness of stress-strain curves from high-strength and mild steels. In general, mild steels are preferable as a construction material in structural engineering since their ductility allows for a more forgiving design. High-strength alloys are generally used when high requirements of strength and durability are needed, e.g., they are commonly used in high-strength leaf springs. Even though the increase in yield strength provided by high-strength steels is a considerable advantage, some challenging downsides need to be accounted for. For example, the weldability of high-strength steels is lower than that of mild steels, and, importantly, it appears to decrease as the strength of the material increases [20]. Moreover, high-strength steels are more brittle, and they tend to fail in a sudden manner, leading to the unexpected failure of a loaded structural member or mechanical part.

Thus far, the stress-strain relationship was presented using engineering quantities. However, engineering measures of stress and strain can be converted to true measures. Assuming that deformation is uniform and volume is constant, i.e., $LA = L_0A_0$, and by recalling the

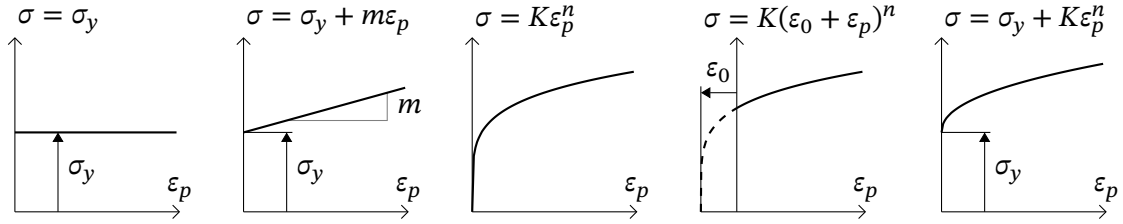


Figure 2.6: Mathematical models for strain hardening.

definition of engineering strain shown in (2.6), we get that $A_0/A = e + 1$. Combining this with the definitions of true and engineering stresses, shown in (2.2) and (2.3), we get that

$$\sigma = S(e + 1). \quad (2.29)$$

Moreover, by combining the definitions of true and engineering strains, shown in (2.5) and (2.6), respectively, one obtains that

$$\varepsilon = \ln(e + 1). \quad (2.30)$$

It should be noted that these equations are only valid when deformation is uniform; hence, they are not valid within the necking region. Figure 2.5 presents an illustration of true and engineering stress-strain curves.

It is often convenient to represent the stress-strain relationship using mathematical models. As illustrated in Figure 2.6, there are various equations capable of describing the strain hardening behaviour in true stress-strain curves [15, 17]. These strain hardening models relate true stress with true plastic strain (σ vs. ε_p). The simplest model is one where no strain hardening is considered after the initial yielding, i.e.,

$$\sigma = \sigma_y, \quad (2.31)$$

where σ_y is the true yield stress. This model represents an idealization of an elastic-perfectly plastic material, and it may be successfully used in cases where plastic strains are within the yield plateau, if one exists. Another simple model is obtained by considering a linear strain hardening behaviour:

$$\sigma = \sigma_y + m\varepsilon_p. \quad (2.32)$$

However, in metals, the strain hardening rate decreases with increasing strain; hence, the model shown above has limited applications. In most cases, a log-log plot of the true stress-strain curve will reveal an almost linear strain hardening behaviour; consequently, a power law is a good approximation:

$$\sigma = K\varepsilon_p^n. \quad (2.33)$$

This model is perhaps the most commonly used and is particularly attractive since it may be written as $\log \sigma = \log K + n \log \varepsilon_p$, in which the hardening coefficient K and hardening

2: Literature review

exponent n may be fitted to the available experimental data via linear regression. The power law shown above may be improved as follows:

$$\sigma = K(\varepsilon_0 + \varepsilon_p)^n, \quad (2.34)$$

where ε_0 pertains to the amount of material pre-strain prior to tensile testing. Another variation of this model is

$$\sigma = \sigma_y + K\varepsilon_p^n. \quad (2.35)$$

Both these models are more satisfying than the original shown in (2.33), since the latter results in zero stress when the plastic strain is zero.

There are also models capable of describing the full true stress-strain curve, namely, the well-known relationship due to Ramberg and Osgood (1943) [21], originally given as

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + K' \left(\frac{\sigma}{E} \right)^{n'}, \quad (2.36)$$

which essentially assumes that the elastic portion of strain, ε_e , can be modelled with a straight line, while the plastic portion, ε_p , can be modelled with a power law. The total strain is then obtained by adding the elastic and plastic portions. Nevertheless, the material behaviour described by this model is nonlinear at all stress levels; however, depending on the hardening exponent, the nonlinearity becomes significant only at stress levels approaching or exceeding σ_y . Additionally, one may realize that the plastic portion of the Ramberg-Osgood relationship is equivalent to the strain hardening model shown in (2.33), and that the Ramberg-Osgood constants are related to the aforementioned hardening coefficient and exponent:

$$K' = \left(\frac{E}{K} \right)^{\frac{1}{n}} \quad \text{and} \quad n' = \frac{1}{n}. \quad (2.37)$$

Thus, the Ramberg-Osgood relationship may also be stated as

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K} \right)^{\frac{1}{n}}. \quad (2.38)$$

Metals are also considered hysteretic materials in the sense that the stress-strain relationship is dependent on the straining history. Figure 2.7 illustrates the typical hysteretic behaviour of metals in tension-compression-tension loading. First, the virgin material is loaded in tension, yielding once the stress level is above the elastic limit, followed by a strain hardening behaviour. At 5% of tensile strain, the applied load is gradually removed, and elastic strains are relieved. The material is then reloaded in compression, and one may notice that the material now yields at a lower level of stress than when it first yielded. This is the so-called Bauschinger effect, named after Johann Bauschinger (1834-1893), in which the yield strength increases in the direction of plastic flow and decreases in other directions. After reloading the material in compression, the load is again reversed, and the stress-strain curve forms a so-called hysteresis loop. After a full load cycle, the second tensile stress-strain curve presents a distinct behaviour from the initial yielding behaviour. A stress-strain curve pertaining to a load-cycled material is usually called cyclic stress-strain curve or simply cyclic curve.

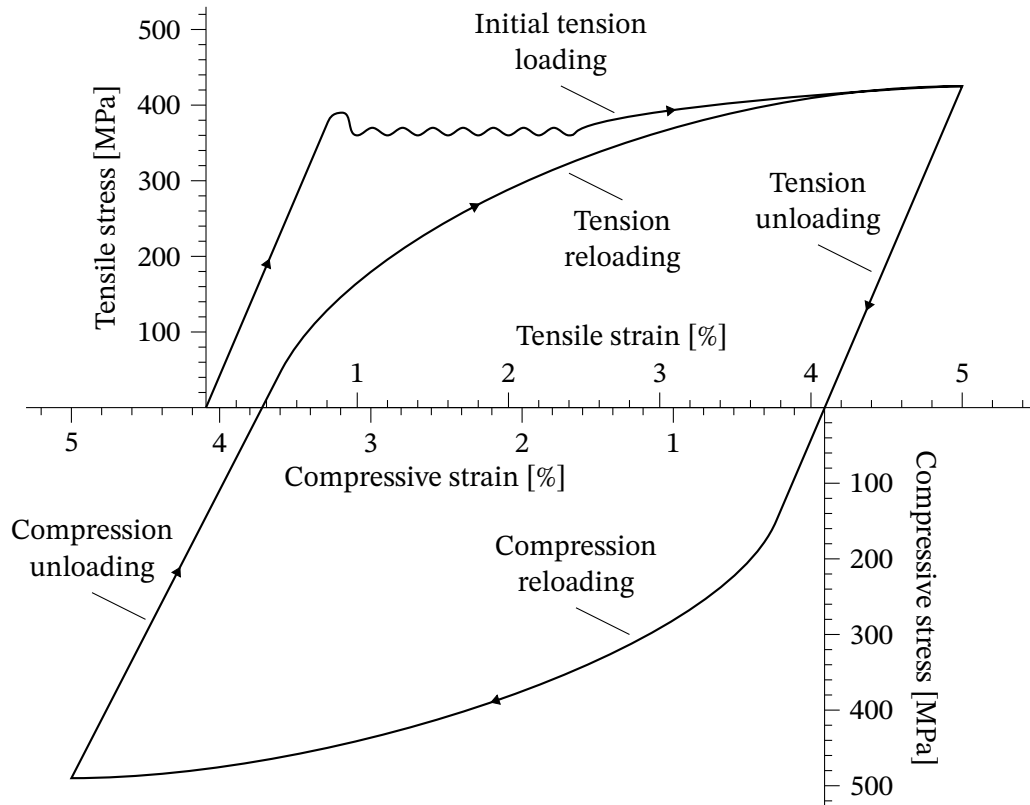


Figure 2.7: Illustration of a typical hysteresis loop and the Bauschinger effect in metals.

2.1.3 Cold forming

Since the stress-strain behaviour of metals is affected by their straining history, there is a relationship between manufacturing process and mechanical performance of the produced part. In metallurgy, manufacturing processes are generally categorized into six main groups: primary shaping, forming, dividing, joining, modifying, and coating [1]. Primary shaping pertains to the creation of initial shapes from molten metals, which are then used in subsequent operations. Forming refers to the technological process of deforming metal in a controlled manner beyond its elastic limit in order to obtain a desired shape via permanent plastic deformations, and it is considered a non-material removal process, i.e., the initial mass is retained after forming. Dividing is the local separation of material, e.g., slit cutting, whereas joining is the assembly of individual workpieces in order to create sub-assemblies, e.g., spot welding. Modifying pertains to the modification of material properties and characteristics, e.g., heat treating processes such as annealing. Finally, coating refers to the application of thin layers onto the material, usually to protect it from the environment, e.g., hot-dip galvanization and painting. Although all manufacturing sub-processes may impact the stress-strain history of the material via the formation or relief of internal residual stresses, in this work, the main focus is on the effect caused by forming processes.

In general, forming is classified as hot or cold forming. In hot forming, the forming process is performed above the recrystallization temperature of the metal and, as a result, manufac-

turing residual stresses are less relevant [22] and are mainly due to non-uniform cooling [23]. Meanwhile, cold forming is performed below the recrystallization temperature, and a significant formation of residual stresses is caused by cold working and strain hardening [1]. Although hot forming presents some advantages over cold forming, e.g., metals are more pliable at high temperatures and residual stresses are avoided, it also has considerable disadvantages, e.g., the high temperatures may impact negatively on geometric tolerances [24], and the process is generally more expensive as a consequence of the energy input required for heating [25]. Moreover, even though residual stresses increase the complexity of the stress-strain history of the material, it should be noted that they do not necessarily impact negatively on its mechanical performance.

In this work, two cold forming processes are investigated in view of manufacturing residual stresses, namely, press braking and roll forming. Both processes are used in sheet metal forming to produce thin-walled profiles. Large-scale productions tend to rely on roll forming, which can be defined, in general, as the process of forming sheet metal along straight, longitudinal, parallel bend lines with multiple pairs of contoured rolls without changing the thickness of the material at room temperature [26]. Essentially, in roll forming, a long and flat sheet of metal is fed into a roll forming mill, where rolls with specific contours incrementally shape the sheet's section via transverse bending until the desired profile is obtained, as illustrated in Figure 2.8. Additionally, this process is usually illustrated via the so-called flower diagram, as shown in Figure 2.9, where several forming increments are superimposed. In fact, each increment shown in the flower diagram represents the cross-section of the sheet as it passes through the corresponding forming station. Importantly, it is necessary to understand that roll forming is a continuous forming process, where the metal sheet is flat on one end of the production line and has the desired cross-section upon exiting the forming mill, as shown in

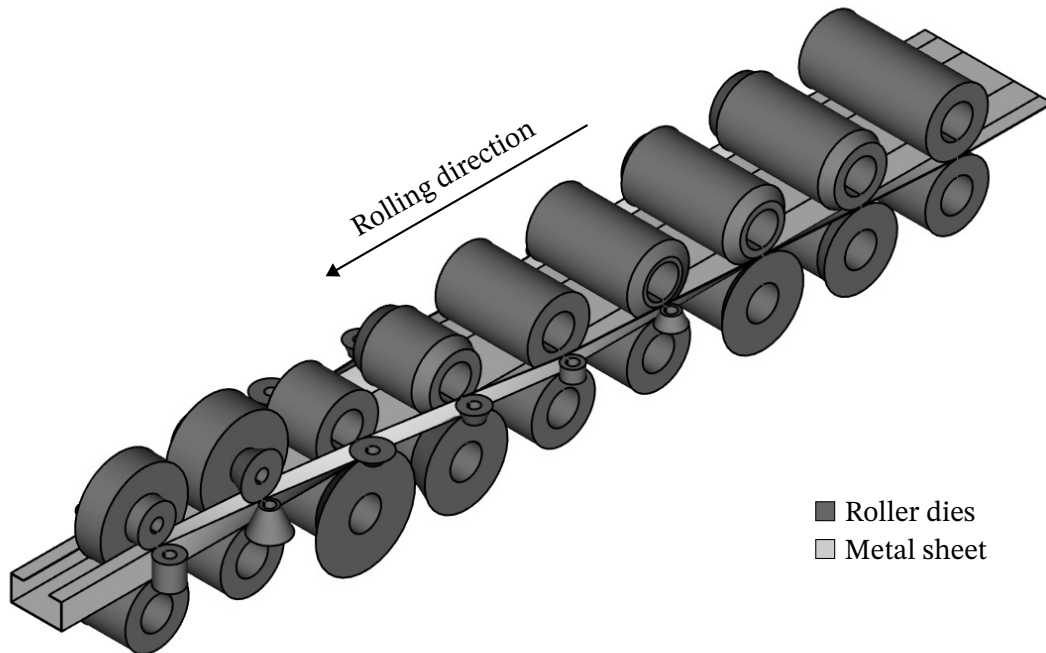


Figure 2.8: Illustration of the roll forming process for producing a generic C-channel.

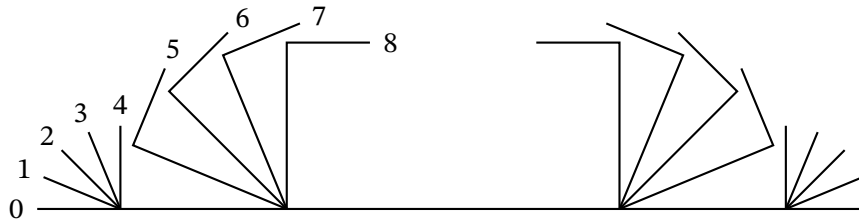


Figure 2.9: A flower diagram showing the incremental forming required to produce a generic C-channel.

Figure 2.8. Of course, due to the continuity of the metal sheet, redundant deformations are present between forming increments, i.e., certain parts of the sheet undergo elongation and/or bending which is later reversed via shrinkage and/or bending back. These redundant deformations, which are not shown in flower diagrams, may occur in the longitudinal (i.e., rolling) and transverse directions, and they are usually associated with manufacturing defects such as longitudinal bowing, twisting, and edge waviness [26]. Moreover, as shown in [Figure 2.10](#), the actual roll forming process is usually coupled to additional pre- and post-forming processes, e.g., flattening of the sheet, hole and cut-out punching, and final cutting to length.

Press braking is an alternative procedure for corner bending. In this work, a form of press braking known as air bending is considered. As shown in [Figure 2.11](#), in this process, a metal sheet is placed between a blunt punch and a die to be formed. The punch is then displaced against the die, pressing and forming the sheet while creating three contact points: one at the tip of the punching tool and one in each side of the V-shaped opening of the die. In contrast to bottom bending, in air bending, the sheet does not get fully pressed to the bottom of the die. Instead, the displacement of the punch is used to control the obtained bend angle or to accommodate different behaving materials.

While roll forming is generally faster, more cost-effective, and less labour-intensive in large-scale productions, press braking can be competitive for short runs, avoiding the design and investment costs of roll forming lines. Moreover, while roll forming is a more involved forming process entailing redundant deformations and a more complex stress-strain history, press braking can be regarded as a pure transverse bending process, where a plane strain condition is usually assumed [27].

In sheet metal forming, sheets are usually delivered and stored as coils, as pictured in [Figure 2.12](#). In roll forming, the coiled sheet is unwound directly into the production line and cut to length after forming, as shown in [Figure 2.10](#), whereas in press braking, the sheet is un-

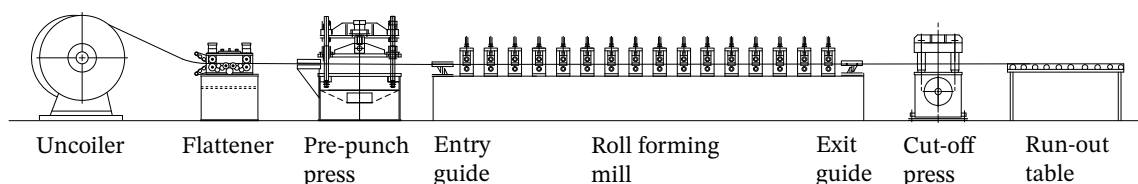


Figure 2.10: Illustration of a typical roll forming line. From Halmos [26]; © 2005 Taylor and Francis Group (reproduced with permission).

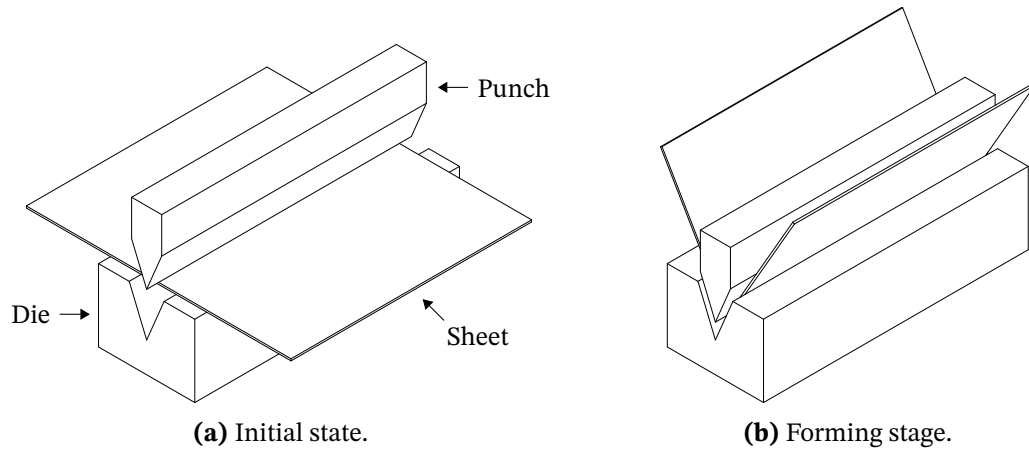


Figure 2.11: Illustration of the press braking process.

wound, possibly flattened, and cut to length prior to forming. Although residual stresses due to cold forming are expected to be the most significant, research efforts have been made into determining and characterizing pre-forming residual stresses due to coiling (i.e., winding of the material) and uncoiling (i.e., unwinding of the material) [13, 27–30]. Under certain conditions, which are discussed in [Section 2.1.4](#), a residual longitudinal curvature is left on the sheet after uncoiling. This curvature may be concave or convex, depending on the orientation of the coil during uncoiling. Moreover, if the residual curvature is large enough, a subsequent flattening operation may be required to guarantee geometric tolerances. Some authors consider flattening as an elastic process, i.e., the sheet is flattened temporarily while subjected to an external flattening load, whereas other authors view flattening as the permanent removal of the residual longitudinal curvature via controlled plastic deformations.



Figure 2.12: Storage of sheet metal coils. From Halmos [26]; © 2005 Taylor and Francis Group (reproduced with permission).

2.1.4 Manufacturing residual stresses

Approaches for the determination of residual stresses are reviewed by Díaz et al. (2021) [27]. In this regard, these approaches are based on experimental, numerical, or, in simple cases, analytical methods. Experimental techniques for the determination of residual stresses are usually categorized into destructive, semi-destructive, and non-destructive methods, based on the destructiveness of the measurement [27]. Sectioning denotes a family of widely used destructive methods based on transverse or longitudinal cutting followed by a qualitative [31] or quantitative [32, 33] analysis of the altered geometry due to the release of residual stresses. Alternatively, semi-destructive methods are based on the registration of relieved strains around a probing location. For example, hole drilling, which is the most common semi-destructive method [27], is based on incrementally drilling and registering relieved surface strains around the drilled hole using a strain gauge rosette. A residual stress distribution across the drilled thickness is then computed using a standardized algorithm, as detailed in ASTM E837 [34]. Meanwhile, in non-destructive methods, such as the X-ray diffraction method, the determination of residual stresses is usually limited at surface locations; however, this method is more versatile in terms of probing region, i.e., destructive and semi-destructive probing locations are sometimes limited to flat surface regions, while the X-ray diffraction method can be used to measure residual stresses in bend zones and corners [13, 27].

Computer-aided engineering (CAE) can also be used in the determination of residual stresses, where the finite element method (FEM) is usually employed. However, numerical simulations of cold forming processes are generally applied within the scope of optimizing the design of forming tools rather than computing residual stresses [27]. Nevertheless, such simulations are generally computationally expensive since they include complex tooling geometries and kinetics, as well as several nonlinearities, namely, kinematic and force nonlinearities due to contact-based forming, material nonlinearity due to the elastoplastic behaviour of metals, and geometric nonlinearity due to large deformations and rotations [13, 27, 35, 36]. Although several commercially available software applications based on FEM have been successfully used to model cold forming [13, 27], the investigation of manufacturing residual stress effects on mechanical performance is still very much an open research field.

Several distinct approaches have been proposed in the literature for the determination of manufacturing residual stresses without resorting to FEA. Perhaps the simplest and most straightforward is the analytical solution for the derivation of residual stresses due to corner bending and coiling operations proposed by Moen et al. (2008) [28], in which the following assumptions are made: (a) the derived prediction equations are based on beam mechanics, (b) the sheet thickness remains constant during forming, (c) the sheet neutral axis (where there are no internal stresses nor strains) is located at half the thickness and remains in this location during forming, (d) an elastic-perfectly plastic material is considered, (e) a plane strain state is assumed, and (f) membrane residual stresses are null. However, these assumptions may not be representative of real-world phenomena, e.g., regarding assumptions (b) and (f), it is possible for roll forming to be performed with an applied longitudinal tension just above the yield point of the material, a process known as tension roll forming [26], which is used to reduce excessive deformations due to initial roll contact. In this case, where bending is performed with an applied tension, a small reduction of the sheet thickness is expected [28, 29] and low-magnitude membrane residual stresses may be present [28]. Moreover, regarding assumption (c), a small shift in the neutral axis location towards the inside of the corner is

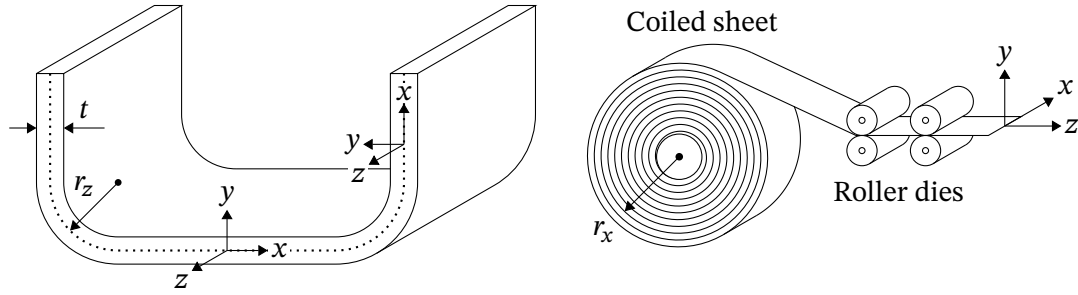


Figure 2.13: Corner bending notation: x is the transverse direction; y is the through-thickness direction; z is the longitudinal or rolling direction; t is the sheet thickness; r_z is the bend or corner radius; and r_x is the coiling radius.

expected during forming due to plasticity effects [28, 30]. Regarding assumption (d), strain hardening is expected to be significant during cold forming; however, considering an elastic-perfectly plastic material may be a reasonable approximation for coiling and uncoiling, where small plastic strains are expected, if any, within the yield plateau of the stress-strain curve. Finally, regarding assumption (e), a plane strain condition may be appropriate for press braking; however, this assumption may not be compatible with roll forming due to its incremental forming nature and the aforementioned redundant deformations.

Considering the presented assumptions and the notation shown in Figure 2.13, Moen et al. (2008) [28] proposed the following equations for the determination of transverse and longitudinal stresses in fully plastic corner bending:

$$\sigma_x^{\text{bending}} = \begin{cases} -\sigma_y & \text{if } 0 < y \leq +t/2, \\ +\sigma_y & \text{if } -t/2 \leq y < 0, \\ 0 & \text{if } y = 0, \end{cases} \quad \text{and} \quad \sigma_z^{\text{bending}} = \nu \sigma_x^{\text{bending}}, \quad (2.39)$$

in which the transverse bending stress magnitude outside the neutral axis corresponds to the yield stress since no strain hardening is considered. Of course, stresses are compressive and tensile above and below the neutral axis, respectively, and null within it. Moreover, the longitudinal stress is dictated by the Poisson effect, where the Poisson's ratio is $\nu = 0.5$ during plastic yielding.

Following the plastic corner bending step, the so-called spring-back effect takes place, where elastic strains are relieved and only permanent plastic deformations remain. Due to the spring-back effect, the distribution of transverse and longitudinal stresses (the latter due to the Poisson effect) across the thickness is expected to be nonlinear, following a zigzag pattern. To account for the change in stress caused by the spring-back effect, Moen et al. (2008) [28] first defined a plastic moment (force couple) given by

$$M_z^{\text{bending}} = F_p \frac{t}{2} = \frac{\sigma_y t^2}{4}, \quad (2.40)$$

which is then applied elastically through the thickness:

$$\sigma_x^{\text{spring-back}} = \frac{M_z^{\text{bending}} y}{I} = \frac{3\sigma_y}{t} y \quad \text{and} \quad \sigma_z^{\text{spring-back}} = \nu \sigma_x^{\text{spring-back}}. \quad (2.41)$$

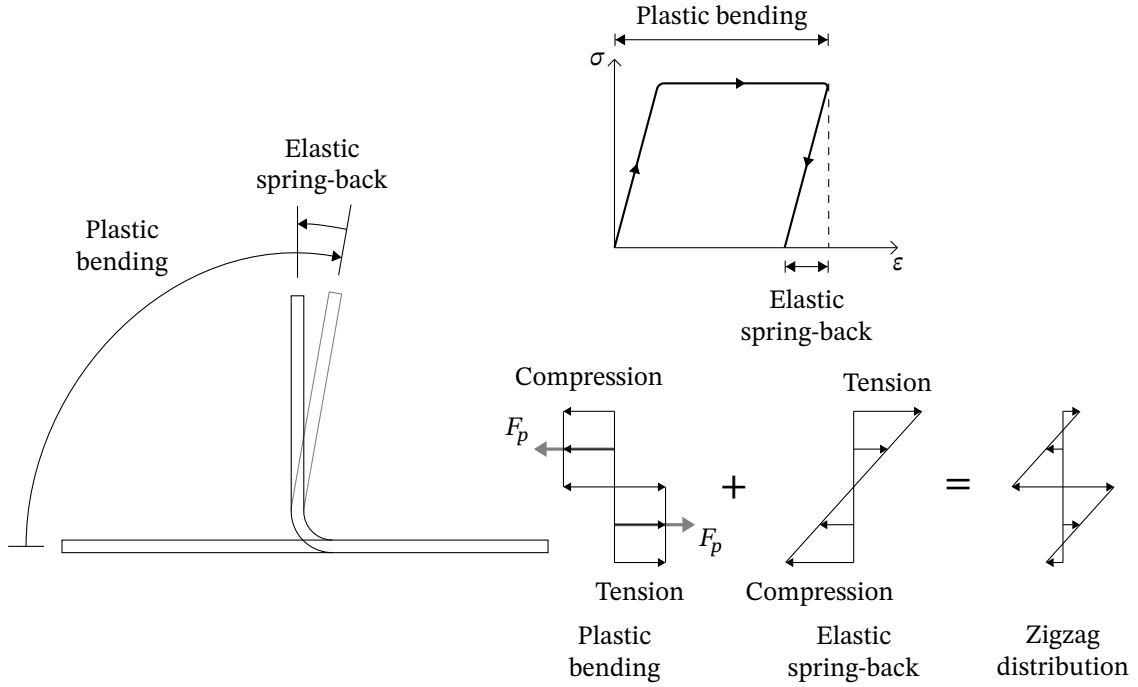


Figure 2.14: Through-thickness distribution of residual stresses in corner bending of thin metal sheets assuming an elastic-perfectly plastic material.

This time, $\nu < 0.5$ is assumed during elastic rebound.

According to Moen et al. (2008) [28], cold forming residual stresses are given by adding stresses due to plastic corner bending with those due to elastic spring-back:

$$\sigma_x^{\text{forming}} = \sigma_x^{\text{bending}} + \sigma_x^{\text{spring-back}} \quad \text{and} \quad \sigma_z^{\text{forming}} = \sigma_z^{\text{bending}} + \sigma_z^{\text{spring-back}}. \quad (2.42)$$

This residual stress modelling approach is illustrated in Figure 2.14. Additionally, the authors also derived an equation for equivalent plastic strain prediction based on the von Mises yield criterion, which is given by

$$\bar{\epsilon}_p^{\text{bending}} = \frac{2}{\sqrt{3}} \ln \left(1 + \left| \frac{y}{r_z} \right| \right). \quad (2.43)$$

This distribution exists only at the cold-formed corner locations, and it assumes an infinitesimally small elastic core located at $y = 0$.

Moen et al. (2008) [28] also proposed equations for modelling residual stresses due to coiling, uncoiling, and flattening, following the aforementioned assumptions and notation. In this regard, the r_x/t ratio, where r_x is the coiling radius and t is the sheet thickness, dictates the significance of residual stresses due to coiling operations. In particular, for high values of r_x/t , the material behaviour may remain within the elastic regime; consequently, no coiling effects are expected. When low values of r_x/t are considered, the material is expected to yield during coiling, and a residual longitudinal curvature is left on the sheet after uncoiling. Of course, r_x varies with the location of the material in the spiral-shaped coil, i.e., material closer

2: Literature review

to the centre of the coil presents higher curvature and coiling effects should be more relevant at these locations. Nevertheless, as the material is cut from the coil before or after forming to relatively small lengths, a constant mean r_x value is usually considered for any particular cold-formed section. Stresses on a fully coiled sheet are given by

$$\sigma_z^{\text{coiling}} = \begin{cases} +\sigma_y & \text{if } +c/2 \leq y \leq +t/2, \\ (Ey)/r_x & \text{if } -c/2 < y < +c/2, \\ -\sigma_y & \text{if } -t/2 \leq y \leq -c/2, \end{cases} \quad \text{and} \quad \sigma_x^{\text{coiling}} = \nu \sigma_z^{\text{coiling}}, \quad (2.44)$$

where $\nu < 0.5$ and

$$c = \frac{2r_x \sigma_y}{E} \leq t \quad (2.45)$$

indicates the size of the elastic core, and this expression is only valid if $r_x \leq r_{\text{th}}$, where

$$r_{\text{th}} = \frac{Et}{2\sigma_y} \quad (2.46)$$

denotes the threshold radius which initiates yielding from coiling. Otherwise, if $r_x > r_{\text{th}}$, the material behaviour remains in the elastic regime and the residual stress history is not affected.

If the material yielded during coiling, elastic spring-back will occur once the sheet is uncoiled. In this case, stresses are modelled via the following equations:

$$\sigma_z^{\text{uncoiling}} = -\frac{M_x^{\text{coiling}}}{I} y = -\frac{12M_x^{\text{coiling}}}{t^3} y \quad \text{and} \quad \sigma_x^{\text{uncoiling}} = \nu \sigma_z^{\text{uncoiling}}, \quad (2.47)$$

where $\nu < 0.5$ and

$$M_x^{\text{coiling}} = \sigma_y \left[\left(\frac{t}{2} \right)^2 - \frac{1}{3} \left(r_x \frac{\sigma_y}{E} \right)^2 \right] \quad (2.48)$$

is the plastic coiling moment. After uncoiling, a sheet that has yielded during coiling will present a residual curvature. The sheet may then be subsequently flattened, and in this case stresses are modelled by

$$\sigma_z^{\text{flattening}} = -\frac{Ey}{r_x^{\text{uncoiling}}} \quad \text{and} \quad \sigma_x^{\text{flattening}} = \nu \sigma_z^{\text{flattening}}, \quad (2.49)$$

where $\nu < 0.5$ and

$$r_x^{\text{uncoiling}} = \frac{1}{\frac{1}{r_x} - \frac{M_x^{\text{coiling}}}{EI}} = \frac{1}{\frac{1}{r_x} - \frac{12M_x^{\text{coiling}}}{Et^3}} \quad (2.50)$$

is the radius pertaining to the residual longitudinal curvature. Finally, an equation for equivalent plastic strain prediction in a fully coiled sheet is also given as

$$\bar{\epsilon}_p^{\text{coiling}} = \begin{cases} 0 & \text{if } |y| < c/2, \\ \frac{2}{\sqrt{3}} \ln \left(1 + \left| \frac{y}{r_x} \right| - \frac{\sigma_y}{E} \right) & \text{otherwise.} \end{cases} \quad (2.51)$$

Ultimately, the presented residual stress modelling approach due to Moen et al. (2008) [28] may take into account the effects of coiling, uncoiling, and flattening, as well as the effects due to corner bending. Nevertheless, other residual stress modelling approaches are available in the literature. For example, Rondal (1987) [37] proposed an incremental numerical method for the determination of residual stresses due to corner bending. This approach differentiates itself from the one due to Moen et al. by considering an elastoplastic material behaviour with linear isotropic hardening after a perfectly plastic regime (yield plateau). Additionally, Quach et al. (2004) [29] proposed a closed-form analytical solution for the determination of residual stresses due to coiling, uncoiling, and flattening. While the approach due to Moen et al. and the one due to Quach et al. both assume small plastic strains and strain hardening is neglected due to the presence of a yield plateau in the stress-strain curve (i.e., an elastic-perfectly plastic

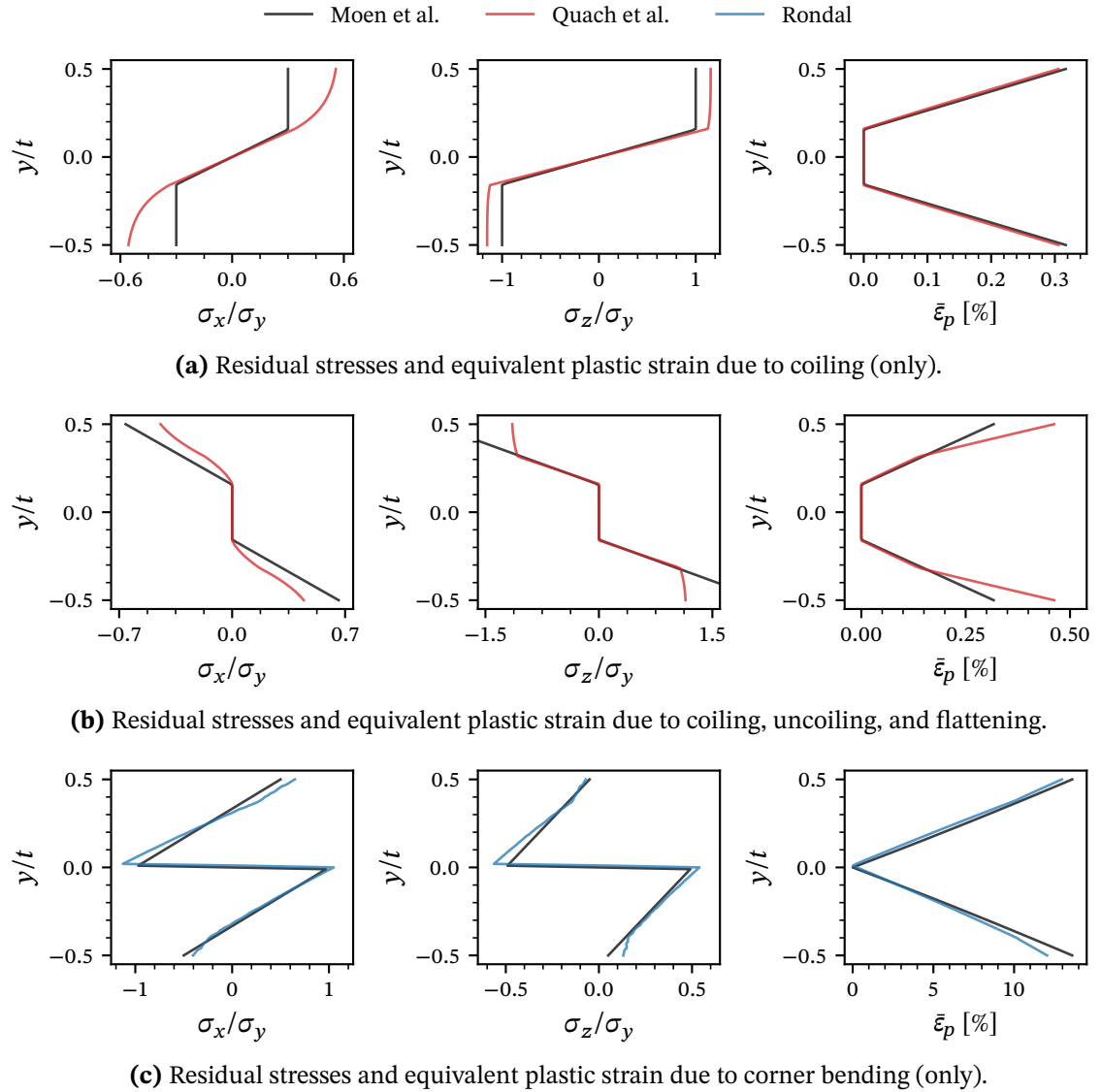


Figure 2.15: Results from distinct residual stress modelling approaches.

material is considered) and pure longitudinal bending in a plane strain condition is assumed, the approach due to Quach et al. is based on the Prandtl-Reuss flow rule and the von Mises yield criterion.

The approaches due to Rondal (1987) [37] and Quach et al. (2004) [29] are more intricate than the one due to Moen et al. (2008) [28]; consequently, these will not be detailed here. Nevertheless, Figure 2.15 showcases these three approaches for residual stress modelling. In this case, an arbitrary material is considered, defined by a Young's modulus of 200 GPa, a Poisson's ratio of 0.3, and a yield strength of 250 MPa, while the linear hardening model is defined by a slope of 1250 MPa; moreover, a sheet thickness of 2 mm is considered, as well as a coiling radius of 250 mm and a bend radius of 8 mm.

2.2 Metal fatigue

Nowadays, mechanical and structural engineers understand that materials behave differently when subjected to static or cyclic loads. In particular, metals are able to resist, without yielding, internal stresses caused by static loads if the stress levels are lower than the yield strength of the material. After the yield point, deformation in metals is decomposable into elastic and plastic portions, where the former is reversible during unloading and the latter is permanent. Permanent deformations are then associated with damage since mechanical and structural components would ideally operate within the elastic regime. Interestingly, however, when metals are subjected to cyclic or repeating loads, fatigue damage accumulation can lead to failure, even if the load level is low enough as not to reach the static yield point of the material.

The word *fatigue* originates from the Latin word *fatigare*, meaning 'tire out', and, today, it is used as a medical term to describe the overall feeling of physical and mental tiredness. Similarly, the term is used in engineering to describe material damage and failure that occurs when it is subjected to repeating loads. More precisely, fatigue can be defined as 'the progressive, localized, permanent structural change that occurs in materials subjected to fluctuating stresses and strains that may result in cracks or fracture after a sufficient number of fluctuations' [38]. Historically, it is believed that the first study on metal fatigue was conducted ca. 1829 by Wilhelm Albert [39], a German mining engineer who tested iron chains used in mine hoists, subjecting them to repeated load proof tests [2]. Fatigue research gained attention on a global scale with the increasing use of ferrous metals in railway bridges and train components. In 1842, the historic Versailles rail accident, in France, which consisted in a train derailment and subsequent fire, is believed to have caused the death of around 50 to 200 people (estimates vary) and hundreds of injuries. Later, it was determined that the derailment was caused by the failure of one of the axles of the leading locomotive, a consequence of fatigue damage accumulation [2, 40]. This incident led to a systematic research into the topic of metal fatigue, which at the time was poorly understood. Notably, ca. 1843, the renowned Scottish mechanical engineer, William Rankine, studied the 'unexpected breakage' of railway axles and the means of preventing it [41]. In this time period, the so-called crystallization theory gained traction, postulating that fatigue was caused by a change from a fibrous to a crystalline and brittle microstructure of the material, but this was later disproven in the beginning of the 20th century with the advances on microscopy [2, 42]. The term *fatigue* is believed to have been coined ca. 1854 by Frederick Braithwaite [2, 43], who employed the term exclusively to denote

the cracking of metals under repeated loading. Since then, several notable contributions to fatigue research have been made, which are discussed in the appropriate following sections.

Several failure mechanisms can be considered in the study of fatigue [2]. Variations of stresses and strains caused by externally applied mechanical loads result in mechanical fatigue. Creep denotes the permanent deformation observed in materials subjected to persistent mechanical stresses over long periods of time, whose effect may occur even when stress levels are below the yield strength of the material, especially when it is subjected to heat. The conjunction of cyclic mechanical and thermal loads leads to thermomechanical fatigue. Environmental factors are also at play: corrosion fatigue relates to the change in the chemical composition of a material subjected to harsh environments during load cycling, resulting in a more brittle, oxidized material that is prone to cracking. In tribology, sliding and rolling fatigue relate to the damaging kinetics acting on materials involved in sliding and rolling contact, respectively, while fretting fatigue describes the progressive deterioration of the material due to oscillatory relative motion and frictional sliding between contacting surfaces. These failure mechanisms are responsible for the majority of failures in structural and machinery components [2], which usually occur due to the presence of cyclic loads whose peak values are well below the safe loads estimated via a static design. Consequently, engineers are required to account for fatigue phenomena using dedicated and standardized fatigue design procedures in order to meet expected safety requirements, e.g., EN 1993-1-9 (Eurocode 3) [5].

Despite the significant advances made in fatigue research, today, several specialized scientific journals focused on this topic reveal that fatigue is still an active research field and research efforts are still being made to better understand the associated phenomena. The study of fatigue can be categorized into low-cycle fatigue (LCF) and high-cycle fatigue (HCF) studies [2]. The major difference is that, under LCF, stresses are high enough to cause appreciable plastic deformation prior to failure, i.e., stresses are above the static yield strength of the material and the life of the component is short, whereas in HCF, stresses are low enough so that the material remains primarily under the elastic regime, resulting in longer fatigue lives. More recently, the term ultra-low-cycle fatigue (ULCF) has been used to describe fatigue studies where fatigue lives are under 100 load cycles [44]. ULCF is distinguished from LCF since damage mechanisms in the former are distinct from those found in the latter. In particular, materials subjected to ULCF exhibit a damage behaviour resulting from a combination of ductile damage and typical LCF damage [44]. On the other end of the spectrum, the terms very-high-cycle fatigue (VHCF), where fatigue lives are over 10^7 load cycles [45], and gigacycle fatigue (over 10^9 load cycles) [46], have been used to refute the idea of a fatigue limit, which is a convenient practical concept stating that, if cyclic stresses and strains are low enough, the material will endure indefinitely.

2.2.1 Fatigue damage stages

Fatigue damage is a complex process where defects may nucleate in initially undamaged zones of the material, propagating in a stable manner while creating microscopic cracks. If cyclic loading continues, the growth and coalescence of microscopic flaws leads to a dominant macroscopic crack, which in turn may propagate until a critical crack length is reached, after which the component is expected to fail catastrophically. Generally, fatigue damage can be divided into the following stages [2], which are also shown in [Figure 2.16](#):

2: Literature review

1. Microstructural changes in the form of nucleation of micrometre-sized flaws along slip bands and grain boundaries.
2. Initial development of microscopic cracks, where a crack is long enough to be modelled by bulk properties of the material.
3. Formation of a dominant macroscopic crack due to the growth and coalescence of microscopic cracks. This stage generally constitutes the demarcation between crack initiation and propagation.
4. Stable propagation of the dominant macroscopic crack, where concepts from LEFM are valid.
5. The crack reaches a critical length and structural failure or complete fracture is achieved in one additional load cycle.

The total fatigue life of a component is viewed as the sum of the crack initiation and propagation periods before failure. Fatigue cracks can initiate even at low stress levels, well below the static yield strength of the material, due to micro-plasticity, where the formation and clustering of dislocations form a wall-like microstructure known as a persistent slip band (PSB) [2]. PSBs are named as such because they leave persistent surface markings on metals, which, even after being removed by polishing, reappear on the same location with continued cycling [2]. Since the beginning of the 20th century, as a result of the pioneering work by Ewing and Rosenhain (1900) [47] and Ewing and Humfrey (1903) [48], it is known that PSBs in polished surfaces are responsible for the so-called extrusions and intrusions, i.e., small localized surface elevations and depressions, that lead the initiation of fatigue cracks [2], as shown in Figure 2.17. Consequently, free surfaces offer more favourable conditions for potential fatigue crack initiation sites. Additional surface factors also promote the initiation of cracks, e.g., surface roughness and stress concentrations caused by inhomogeneous stress distributions due to notched geometries. However, internal cracks are also possible, mainly due to the presence of non-metallic inclusions, pores, and other types of microstructural defects, but they are less common [2, 38]. Once a macroscopic crack has formed, its propagating behaviour is generally modelled by fracture mechanics, which is discussed further.

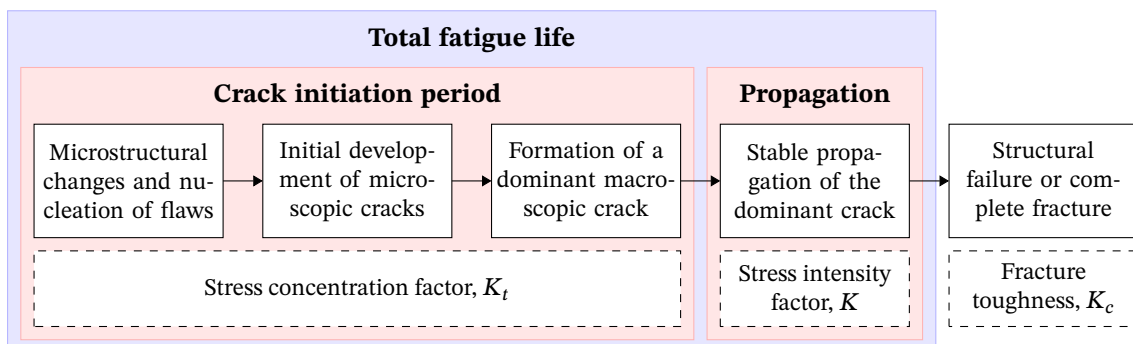


Figure 2.16: Fatigue damage stages and driving factors from fracture mechanics.

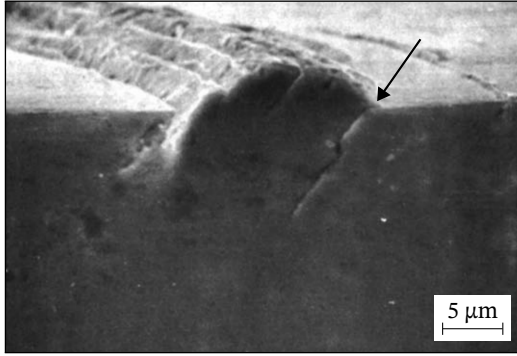


Figure 2.17: Fatigue crack initiation at leading edges of PSBs in a cycled copper crystal. From Bao-Tong and Laird [49]; © 1989 Pergamon Press (reproduced with permission).

2.2.2 Fatigue damage factors

Several factors play a role in the nucleation of micro-defects leading to the initiation of a fatigue crack. However, factors that contribute to the initiation of a crack do not necessarily have an effect on its propagation. The main factor of fatigue damage are internal stresses acting on the material. Cyclic stresses are usually described via three parameters: the stress range $\Delta\sigma$ as the algebraic difference between the maximum and minimum stresses in the load cycle, respectively, $\sigma_{\max}$ and $\sigma_{\min}$; the stress amplitude σ_a as half the stress range; and the mean stress σ_m as the arithmetic mean of the maximum and minimum stresses; additionally, the stress ratio, denoted R_σ , can also be defined as the ratio of the minimum to the maximum stress. Hence, these parameters are given by

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}, \quad \sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}, \quad \sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}, \quad R_\sigma = \frac{\sigma_{\min}}{\sigma_{\max}}. \quad (2.52)$$

Alternatively, if cyclic strains are considered, the parameters become the strain range $\Delta\varepsilon$, strain amplitude ε_a , mean strain ε_m , and strain ratio R_ε , which are analogous to the aforementioned stress parameters and are given by

$$\Delta\varepsilon = \varepsilon_{\max} - \varepsilon_{\min}, \quad \varepsilon_a = \frac{\varepsilon_{\max} - \varepsilon_{\min}}{2}, \quad \varepsilon_m = \frac{\varepsilon_{\max} + \varepsilon_{\min}}{2}, \quad R_\varepsilon = \frac{\varepsilon_{\min}}{\varepsilon_{\max}}. \quad (2.53)$$

As shown further, the fact that stresses and strains are the main factor of fatigue damage is highlighted in approaches for fatigue life prediction, which generally rely on stress vs. endurance or strain vs. endurance relationships, where endurance pertains to the number of load cycles or load reversals to failure, denoted N_f or $2N_f$, respectively.

The mean stress can be seen as the second most important factor of fatigue damage since it is well known that it has a great impact in the observed fatigue life [2]. The mean stress effect in metals is shown clearly in typical stress amplitude vs. endurance plots, as illustrated in Figure 2.18a, where, for the same stress amplitude, endurance decreases with increasing mean stress values. An alternative way to visualize the mean stress effect is via the so-called constant life diagrams, as illustrated in Figure 2.18b, where stress amplitude vs. mean stress relationships are plotted for the same constant endurance. In this regard, the classical relationships are the ones due to Gerber (1874) [50], Goodman (1899) [51], and Soderberg (1930)

2: Literature review

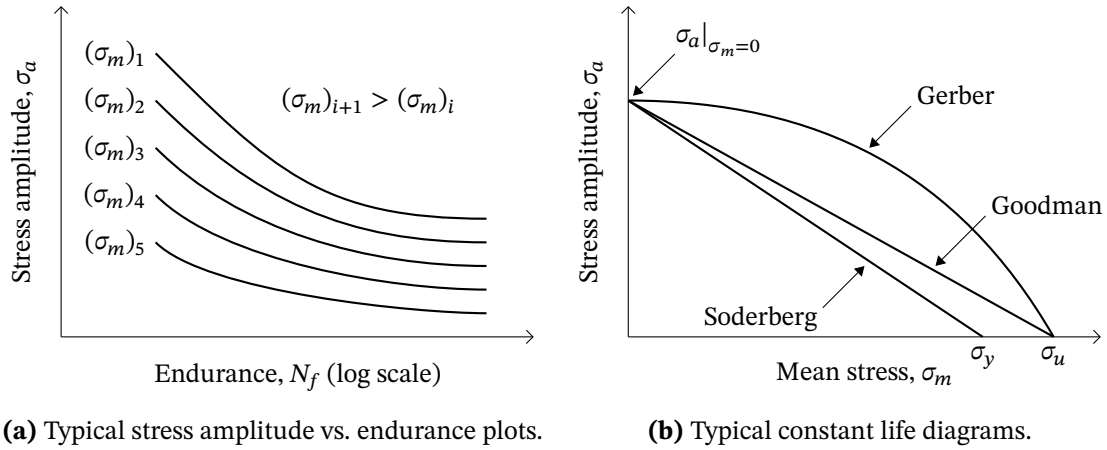


Figure 2.18: Illustration of mean stress effects.

[52], which are given by the following expressions [2, 38]:

$$\text{Gerber model: } \sigma_a = \sigma_a|_{\sigma_m=0} \left[1 - \left(\frac{\sigma_m}{\sigma_u} \right)^2 \right], \quad (2.54)$$

$$\text{Goodman model: } \sigma_a = \sigma_a|_{\sigma_m=0} \left(1 - \frac{\sigma_m}{\sigma_u} \right), \quad (2.55)$$

$$\text{Soderberg model: } \sigma_a = \sigma_a|_{\sigma_m=0} \left(1 - \frac{\sigma_m}{\sigma_y} \right), \quad (2.56)$$

where, in order to maintain a fixed fatigue life, the stress amplitude σ_a is related to the stress amplitude for fully reversed loading, denoted $\sigma_a|_{\sigma_m=0}$, the mean stress σ_m , and the yield strength or ultimate tensile strength of the material, respectively, σ_y and σ_u . As shown in Figure 2.18b, when the mean stress is zero, i.e., during fully reversed loading ($R_\sigma = -1$), the allowable stress amplitude for a component corresponds to its fatigue strength for the considered number of load cycles. However, as the mean stress increases, the allowable stress amplitude decreases and, in the limit, when the mean stress is equal to the yield strength or ultimate tensile strength of the material, the allowable stress amplitude is zero. The figure also reveals that the Soderberg model (2.56) is the most conservative, followed by the Goodman (2.55) and Gerber (2.54) models. Nevertheless, alternative models for mean stress correction are available in the literature; for example, Taras and Greiner (2010) [53], while investigating riveted bridge components, developed the following model:

$$\Delta\sigma = \frac{1 - R_\sigma}{1 - \alpha R_\sigma} \Delta\sigma|_{R_\sigma=0}, \quad (2.57)$$

which states that if the value of $\Delta\sigma|_{R_\sigma=0}$, which is the fatigue strength for a stress ratio of $R_\sigma = 0$ (zero-tension loading), is known, then the value of the fatigue strength $\Delta\sigma$ for any stress ratio can be computed, and vice versa. The parameter α is introduced as a fatigue strength reduction or increase factor, and the value of $\alpha = 0.60$ is provided for mild steels [53]. Later, Souto et al. (2021) [11] showed that $\alpha = 0.768$ worked well for non-preloaded bolted joints.

In actual structural and machinery components, where more intricate geometries and loadings are considered, the distribution of stresses is inhomogeneous, leading to stress concentrations at certain locations. The stress concentration factor K_t is defined as the ratio of a local peak value of stress to a remote nominal value of stress, and it is another factor of fatigue damage that generally contributes to a reduction in endurance; however, in general, a proportional relationship is not verified between K_t and N_f [2]. Consequently, in fatigue, K_t is usually replaced by the so-called fatigue notch factor, denoted K_f , which corresponds to the ratio of the fatigue strength of a smooth specimen to the fatigue strength of a notched specimen, where the fatigue strengths are evaluated at the same number of endured load cycles. The fatigue notch factor is empirically determined, being related to the strength and ductility of the material, as well as to the notch geometry. Moreover, the fatigue notch factor tends to the stress concentration factor for large notch root radii and for higher strength materials [2]. Additionally, the so-called notch sensitivity index can be defined as

$$q = \frac{K_f - 1}{K_t - 1}, \quad (2.58)$$

which varies from zero to the unity, i.e., from no notch effect to full notch effect.

Other fatigue damage factors are known, such as: the material, i.e., its ductility, hardenability, presence of microstructural defects and impurities, etc.; surface roughness, which is generally linked to have a deteriorating effect on fatigue resistance; size of the component or specimen, i.e., the fatigue strength of large components is lower than that of smaller ones; thermal effects, namely due to hysteretic heating during cycling with high strain rates, resulting in thermal softening; environment, e.g., corrosion fatigue; and residual stresses [2, 38].

From the mentioned fatigue damage factors, the effects of residual stresses on fatigue are perhaps the least understood. In this regard, numerical studies were performed by Souto et al. (2021) [12] to evaluate the fatigue behaviour of thin-walled, cold-formed, mild steel sections while accounting for residual stresses due to roll forming that were validated by correlation with experimental data. Meanwhile, Gothivarekar et al. (2021) [54] studied the fatigue behaviour of double-bent thick plates made of high-strength steel using elastoplastic numerical analysis to simulate fatigue loading while accounting for residual strains from press braking that were validated by a digital image correlation technique [55]. While Souto et al. concluded on a minimal residual stress effect, Gothivarekar et al. concluded on a clear fatigue life decrease. Moreover, Gerin et al. (2016) [56] studied the effects of cold extrusion on HCF of highly hardenable steel, while Chiou and Yang (2012) [57] studied the effects of pre-deformation on the fatigue behaviour of non-hardenable steel. While Gerin et al. concluded on a positive residual stress effect linked to a hardness value, Chiou and Yang showed the existence of a cyclic softening behaviour that increases with pre-strain level, resulting in a reduction of endurance. Based on the mentioned investigations, one may conclude that the effects of residual stresses are controversial, and they vary based on the considered cold forming process, material, and operating conditions of the produced part. Hence, further research supported by strong experimental campaigns is required to characterize the effects of residual stresses on fatigue.

2.2.3 Total life approaches

While fatigue is, in fact, a micromechanical process, continuum approaches are widely used to characterize the total fatigue life of a component based on bulk material properties. In

this regard, these approaches account for both the crack initiation and propagation periods using empirical formulations, where the total fatigue life, defined as the total number of load cycles to failure (i.e., endurance), is related to stresses and/or strains and possibly other fatigue damage factors. Traditionally, total life approaches are split into stress- and strain-based methodologies. Stress-based approaches, where stresses are considered as the main fatigue damage factor, are usually applied in HCF since, in this case, the stress-strain relationship is mainly linear-elastic. On the other hand, strain-based approaches, where strains are considered as the main fatigue damage factor, are usually employed in LCF, where, due to high strain amplitudes, the stress-strain relationship becomes nonlinear and the determination of true stresses is not so straightforward.

2.2.3.1 Stress-life approaches

Stress-life approaches were pioneered by August Wöhler during the second half of the 19th century [2], who is now known for the Wöhler curves, also known as stress-life or $S-N$ curves. As shown in Figure 2.19, modern $S-N$ curves used in fatigue design codes, e.g., EN 1993-1-9 (Eurocode 3) [5], are generally presented using base 10 logarithmic scales. Moreover, these curves are usually associated with the concepts of detail category and fatigue limit. The detail category, denoted ΔS_C , is a reference value, usually in N/mm^2 , pertaining to the fatigue strength of a component at a fixed number of load cycles (2×10^6 cycles in Eurocode 3). The fatigue limit of a component corresponds to the stress level below which it will endure in-

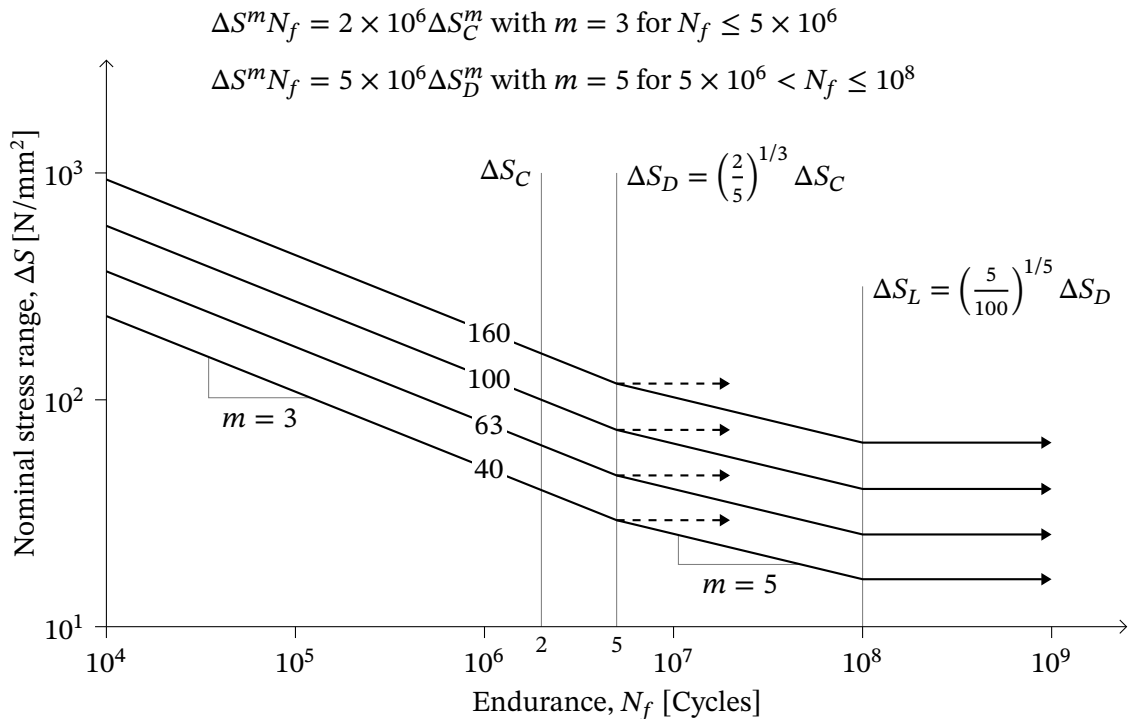


Figure 2.19: Typical Wöhler curves, also known as stress-life or $S-N$ curves, used in fatigue design codes.

definitely. This limit is based on empirical observations [2]; however, more recent studies in VHCF question the idea of a fatigue limit [45, 46]; nevertheless, the fatigue limit is a practical concept adopted by many. Eurocode 3 defines two fatigue limits, namely, a fatigue limit for constant amplitude loading, denoted ΔS_D , defined as the fatigue strength at 5×10^6 load cycles, and a subsequent limit for when stress spectra with stress levels above and below the constant amplitude fatigue limit are considered, denoted ΔS_L , defined as the fatigue strength at 10^8 load cycles.

Traditional approaches based on S - N curves are denoted as global approaches since they do not take into account the localized nature of fatigue damage and, instead, rely on the categorization of details and components. This may result in complex detail categorization systems for real-world applications. Generally, S - N approaches rely on HCF testing using smooth specimens or small-scale components, and nominal stresses are usually considered; nevertheless, recently, Souto et al. (2021) [11, 12] have shown S - N curves based on numerically determined peak stresses. After fatigue testing, a regression analysis is usually carried out (as discussed in Section 2.2.3.5), generating an S - N curve for the tested component.

Another stress-life approach is the one due to Basquin (1910) [58], where the true stress amplitude in fully reversed, constant amplitude fatigue testing is related to the number of load reversals to failure ($2N_f$) via the following equation [2]:

$$\frac{\Delta\sigma}{2} = \sigma'_f (2N_f)^b, \quad (2.59)$$

where σ'_f and b are the fatigue strength coefficient and exponent, respectively. The Basquin model can be applied as a local approach since the corresponding empirical constants can be obtained considering true stresses acting on the plain material, where smooth dog-bone or hourglass type specimens are tested; once calibrated, the model can be evaluated locally at critical locations within a component. This approach contrasts with the aforementioned global S - N approaches based on nominal stresses and detail categorization. Additionally, the mean stress effect can be accounted for via the following expression due to Morrow (1968) [59]:

$$\frac{\Delta\sigma}{2} = (\sigma'_f - \sigma_m)(2N_f)^b, \quad (2.60)$$

which is a modification of the original Basquin model assuming that the tensile mean stress $\sigma_m > 0$ reduces the fatigue strength σ'_f .

2.2.3.2 Strain-life approaches

Strain-life approaches are generally used in LCF and practical applications where engineering components undergo localized plastic flow. In this case, the stress-strain relationship is nonlinear and the computation of true stresses is no longer straightforward. The classical strain-life relationship is the one due to Coffin (1954) [60] and Manson (1954) [61], who independently proposed the following relationship between plastic strain amplitude and number of load reversals to failure [2]:

$$\frac{\Delta\epsilon_p}{2} = \epsilon'_f (2N_f)^c, \quad (2.61)$$

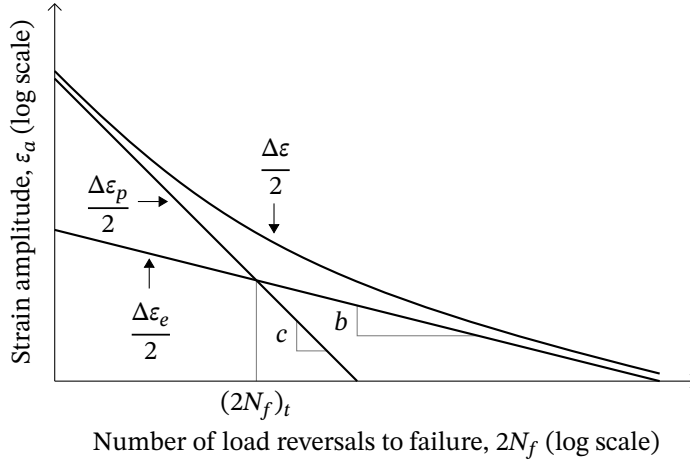


Figure 2.20: Total strain amplitude vs. endurance via the superposition of elastic and plastic strain components.

where ϵ'_f and c are the fatigue ductility coefficient and exponent, respectively. Similarly to the Basquin model, the Coffin-Manson model can also be applied as a local approach, where the empirical constants are obtained from the testing of smooth dog-bone or hourglass type specimens and true strains can be considered. Additionally, it is possible to combine the Basquin and Coffin-Manson models via the decomposition of strain into elastic and plastic components:

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2}, \quad (2.62)$$

where $\Delta\epsilon$, $\Delta\epsilon_e$, and $\Delta\epsilon_p$ are the total, elastic, and plastic strain ranges. Noting that

$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\sigma}{2E}, \quad (2.63)$$

the Basquin-Coffin-Manson model can be written as follows [2]:

$$\frac{\Delta\epsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f(2N_f)^c. \quad (2.64)$$

Moreover, the aforementioned Morrow's mean stress correction can also be applied to the Basquin-Coffin-Manson model:

$$\frac{\Delta\epsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + \epsilon'_f(2N_f)^c. \quad (2.65)$$

The Basquin-Coffin-Manson model is illustrated in [Figure 2.20](#), where the total strain amplitude vs. endurance relationship is obtained via the superposition of elastic and plastic strain components. The figure also shows a transition point from short to long fatigue lives, denoted $(2N_f)_t$, defined as the number of load reversals to failure where elastic and plastic strain amplitudes are equal. This point is given by

$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\epsilon_p}{2} \Rightarrow (2N_f)_t = \left(\frac{E\epsilon'_f}{\sigma'_f} \right)^{1/(b-c)}. \quad (2.66)$$

Consequently, if $2N_f \ll (2N_f)_t$ (short fatigue lives), then plastic strain amplitude is more prominent and the fatigue life of the material is dictated by ductility; otherwise, if $2N_f \gg (2N_f)_t$ (long fatigue lives), then elastic strain amplitude is more prominent and the fatigue life is dictated by the rupture strength [2].

2.2.3.3 Multiaxial fatigue

Thus far, the mentioned total life approaches are deemed valid for cyclic stresses within uniaxial loading. However, in practical applications, most engineering components undergo multiaxial cyclic loading. In this case, it is common to use equivalent stresses and strains [2]. Using the von Mises criterion, shown in (2.20), in multiaxial fatigue, the amplitude of equivalent stress, denoted $\bar{\sigma}_a$, is given as

$$\bar{\sigma}_a = \sqrt{\frac{1}{2}[(\sigma_{1,a} - \sigma_{2,a})^2 + (\sigma_{2,a} - \sigma_{3,a})^2 + (\sigma_{3,a} - \sigma_{1,a})^2]}, \quad (2.67)$$

where $\sigma_{i,a}$ are the amplitudes of principal stresses within the constant amplitude load cycle. Similarly, the equivalent mean stress, denoted $\bar{\sigma}_m$, is given as

$$\bar{\sigma}_m = \sqrt{\frac{1}{2}[(\sigma_{1,m} - \sigma_{2,m})^2 + (\sigma_{2,m} - \sigma_{3,m})^2 + (\sigma_{3,m} - \sigma_{1,m})^2]}, \quad (2.68)$$

where $\sigma_{i,m}$ are the mean values of the principal stresses within the constant amplitude load cycle. Consequently, the Basquin model can be rewritten in terms of equivalent stress as

$$\bar{\sigma}_a = \sigma'_f (2N_f)^b, \quad (2.69)$$

or, if considering the Morrow's mean stress correction, as

$$\bar{\sigma}_a = (\sigma'_f - \bar{\sigma}_m)(2N_f)^b. \quad (2.70)$$

Still within the von Mises maximum distortion energy theory, the amplitude of equivalent strain, denoted $\bar{\varepsilon}_a$, is given as

$$\bar{\varepsilon}_a = \frac{1}{1+\nu} \sqrt{\frac{1}{2}[(\varepsilon_{1,a} - \varepsilon_{2,a})^2 + (\varepsilon_{2,a} - \varepsilon_{3,a})^2 + (\varepsilon_{3,a} - \varepsilon_{1,a})^2]}, \quad (2.71)$$

where $\varepsilon_{i,a}$ are the amplitudes of principal strains within the constant amplitude load cycle. Consequently, the Basquin-Coffin-Manson model can be rewritten as

$$\bar{\varepsilon}_a = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c, \quad (2.72)$$

or, if considering the Morrow's mean stress correction, as

$$\bar{\varepsilon}_a = \frac{\sigma'_f - \bar{\sigma}_m}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c. \quad (2.73)$$

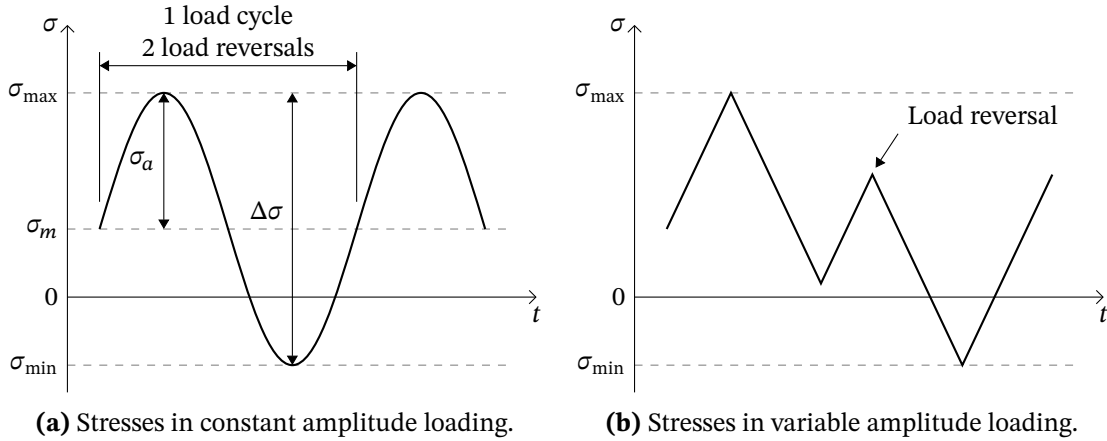


Figure 2.21: Constant and variable amplitude fatigue loading scenarios.

2.2.3.4 Variable amplitude loading

Thus far, the presented fatigue approaches pertain to constant amplitude cyclic loading scenarios, as shown in Figure 2.21a. However, in real-world applications, engineering components are most likely subjected to variable amplitude loading, as shown in Figure 2.21b, where the stress amplitude varies significantly under realistic loads. Hence, in variable amplitude loading, it is necessary to predict fatigue lives under distinct stress (or strain) amplitudes. For this purpose, the so-called cumulative damage rules are employed. In this regard, the most used rule is the one due to Palmgren (1924) [62], later restated and popularized by Miner (1945) [63], which is a linear rule given by

$$D = \sum_{i=1}^m \frac{n_i}{(N_f)_i}, \quad (2.74)$$

where n_i denotes the number of constant amplitude load cycles for m distinct constant amplitude blocks, and $(N_f)_i$ denotes the number of load cycles to failure at that stress or strain amplitude. If $D = 1$, where D denotes the fatigue damage accumulation, then failure is expected to occur.

In the Palmgren-Miner rule, $(N_f)_i$ is obtained via one of the aforementioned total life approaches, while n_i is obtained by performing cycle counts in a given stress or strain spectrum, also called stress-time or strain-time history. In this regard, Souto et al. (2019) [64] have developed a software application where, given a stress-time history, the fatigue damage accumulation D is computed based on a global S - N approach combined with the rainflow cycle counting algorithm, which is arguably the most popular approach for cycle counting since the algorithm has been standardized in the ASTM E1049 [65] standard.

2.2.3.5 Statistical analysis

Fatigue approaches are intrinsically related to testing. After a successful testing campaign, a regression analysis is usually carried out in order to fit some mathematical model to the obtained experimental data. For this purpose, ASTM E739 [66] purposes a standardized proced-

ure for the statistical analysis of linear stress-life ($S-N$) and strain-life ($e-N$) fatigue data. The term linear pertains to the linear relationship between stress or strain vs. endurance when using appropriate coordinates (i.e., logarithmic scales). The standard suggests using a linear model of the type $Y = A + BX$, where X denotes the independent variable, i.e., the selected and controlled variable (namely, stress, strain, or their logarithms), Y denotes the dependent variable, i.e., fatigue life or its logarithm, and A and B denote the model parameters to be fitted. Assuming that the fatigue data pertains to a random sample, no run-outs nor suspended tests are considered, and that the $S-N$ or $e-N$ relationship can indeed be described by the aforementioned linear model, then the maximum likelihood estimators of A and B are

$$\hat{A} = \bar{Y} - \hat{B}\bar{X}, \quad \hat{B} = \frac{\sum_{i=1}^k (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^k (X_i - \bar{X})^2}, \quad (2.75)$$

where the caret (^) and bar (̄) symbols denote estimate and average, respectively, and k is the total number of tested specimens (i.e., the sample size). The recommended expression for estimating the variance of the normal distribution for Y is

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^k (Y_i - \hat{Y}_i)^2}{k - 2}, \quad (2.76)$$

in which $\hat{Y}_i = \hat{A} + \hat{B}X_i$. The standard also provides formulas for the computation of the confidence intervals for parameters A and B :

$$\hat{A} \pm t_p \hat{\sigma} \left[\frac{1}{k} + \frac{\bar{X}^2}{\sum_{i=1}^k (X_i - \bar{X})^2} \right]^{1/2}, \quad \hat{B} \pm t_p \hat{\sigma} \left[\sum_{i=1}^k (X_i - \bar{X})^2 \right]^{-1/2}, \quad (2.77)$$

where $\hat{\sigma}$ is the estimated standard deviation and t_p (usually a tabulated value as a function of k) pertains to the t -distribution for the desired confidence level (usually 95%). Moreover, confidence bands for the median $S-N$ or $e-N$ curve as a whole are defined as

$$\hat{A} + \hat{B}X \pm \sqrt{2F_p} \hat{\sigma} \left[\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum_{i=1}^k (X_i - \bar{X})^2} \right]^{1/2}, \quad (2.78)$$

in which F_p (usually a tabulated value as a function of k) pertains to the F -distribution for the desired confidence level (again, usually 95%). Note that, if considering $X = \log \sigma_a$ and $Y = \log N_f$, then, after some algebraic manipulation, the considered linear model can be rewritten as

$$\sigma_a = (2 \times 10^A)^{-1/B} (2N_f)^{1/B}, \quad (2.79)$$

which corresponds to the Basquin model with

$$\sigma'_f \equiv (2 \times 10^A)^{-1/B}, \quad b \equiv \frac{1}{B}. \quad (2.80)$$

Instead, if considering $X = \log(\Delta \varepsilon_p/2)$ and $Y = \log N_f$, the linear model can be rewritten as

$$\frac{\Delta \varepsilon_p}{2} = (2 \times 10^A)^{-1/B} (2N_f)^{1/B}, \quad (2.81)$$

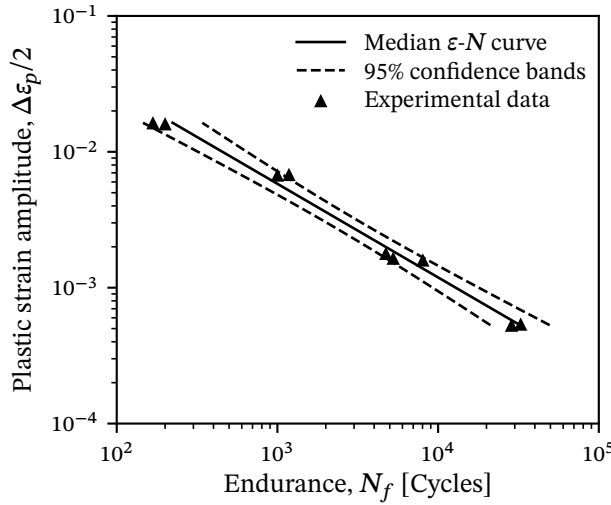


Figure 2.22: Example of a median ε - N curve and corresponding 95% confidence bands resulting from a statistical analysis following the ASTM E739 standard.

which corresponds to the Coffin-Manson model with

$$\varepsilon'_f \equiv (2 \times 10^4)^{-1/B}, \quad c \equiv \frac{1}{B}. \quad (2.82)$$

Hence, the presented standardized linear regression procedure can be used to fit the Basquin and Coffin-Manson models. An example is shown in Figure 2.22. Alternatively, the fit can be carried out via nonlinear programming techniques or, somewhat equivalently, by treating the fit process as a nonlinear least squares problem.

In metals with considerable microstructural irregularities and variable flaw size distribution across batches, usually arising from manufacturing, experimental scatter in fatigue data may be too large [2]. On effect, similar fatigue testing campaigns on the same material may yield distinct results. In the presented linear regression analysis, the stochastic nature of fatigue is accounted for in the confidence intervals for the fitted parameters and in the confidence bands for the median S - N or e - N curve as a whole. Another well-accepted approach is to establish safety bands using the so-called 68-95-99.7 rule (also known as the empirical rule), whose name stands for the percentage of values that lie within an interval estimate in a normal distribution, i.e., 68%, 95%, and 99.7% of the values lie within one, two, and three standard deviations of the mean, respectively; in other words, the standard deviation, multiplied by a factor of one, two, or three, can be used to draw safety bands for the desired confidence level.

Other statistical models and distributions can be applied in order to generate the so-called probabilistic S - N or e - N fields (known as P - S - N or P - e - N fields). In this regard, the state of the art is reviewed by Barbosa et al. (2019) [67]. Besides the presented ASTM approach, these authors also reviewed the probabilistic models due to Castillo and Fernández-Canteli (2009) [68], Sendekyj (1981) [69], and Stüssi (1955) [70], in which, only the Stüssi model was shown to be very effective in both LCF and HCF regions. The Stüssi model is based on a nonlinear function to represent S - N curves, which is given by

$$\Delta\sigma = \frac{\sigma_u + aN_f^b \Delta\sigma_{th}}{1 + aN_f^b}, \quad (2.83)$$

where $\Delta\sigma_{th}$ is the fatigue limit or threshold, and a and b are the model parameters. The corresponding probabilistic model is then built using the three-parameter Weibull distribution, whose cumulative distribution function is given by

$$F(x, \alpha, \beta, \gamma) = 1 - \exp\left[-\left(\frac{x - \alpha}{\beta}\right)^\gamma\right] \quad \text{with } x \geq \alpha, \quad (2.84)$$

where F corresponds to the probability of failure P ; α , β , and γ denote the Weibull location, scale, and shape parameters, respectively; and, considering the Stüssi model, the random variable x is

$$x = \Delta\sigma - \frac{\sigma_u + aN_f^b \Delta\sigma_{th}}{1 + aN_f^b}. \quad (2.85)$$

The fit of the various parameters pertaining to the probabilistic model is accomplished via nonlinear optimization. Nevertheless, a simpler approach is the one originally proposed by Weibull (1939) [71], who first introduced the concept of probability of failure at a given failure strength σ_f as

$$P = 1 - \exp\left[-\left(\frac{\sigma_f}{\beta}\right)^\gamma\right]. \quad (2.86)$$

The original Weibull cumulative distribution function can be rearranged into

$$\ln\left(\ln \frac{1}{1 - P}\right) = \gamma \ln \frac{\sigma_f}{\beta}, \quad (2.87)$$

and, in practice, a linear relationship of the double-logarithm of $1/(1 - P)$ (ordinate) is plotted against the logarithm of σ_f (abscissa), from which the β (related to the y -intercept) and γ (related to the slope) parameters are determined. Such a plot is known as a Weibull plot or diagram.

2.2.4 Damage-tolerant approach

The damage-tolerant approach aims the determination of the residual fatigue life of a fractured component based on the characterization of fatigue crack propagation made by applying the theory of fracture mechanics. Historically, fracture mechanics began in the 1920s with the pioneering work of Griffith (1921) [72] who formulated analytical criteria for the unstable extension of a crack in brittle solids based on the conservation of energy. Griffith postulated that an increase in surface energy due to crack extension must equal a decrease in potential energy of the system [2]. However, Griffith's idealized model for brittle fracture did not account for plastic energy dissipation near the crack tip; hence, it could not be directly applied to most engineering materials. Later, Orowan (1952) [73] extended the concepts of brittle fracture and their applications to metals by considering their elastoplastic nature. Afterwards, Irwin (1957) [74] further consolidated the theory of fracture mechanics by introducing the concept of strain energy release rate, denoted G , as the loss of total potential energy Π per the incremental increase of crack surface area A :

$$G = -\frac{d\Pi}{dA}. \quad (2.88)$$

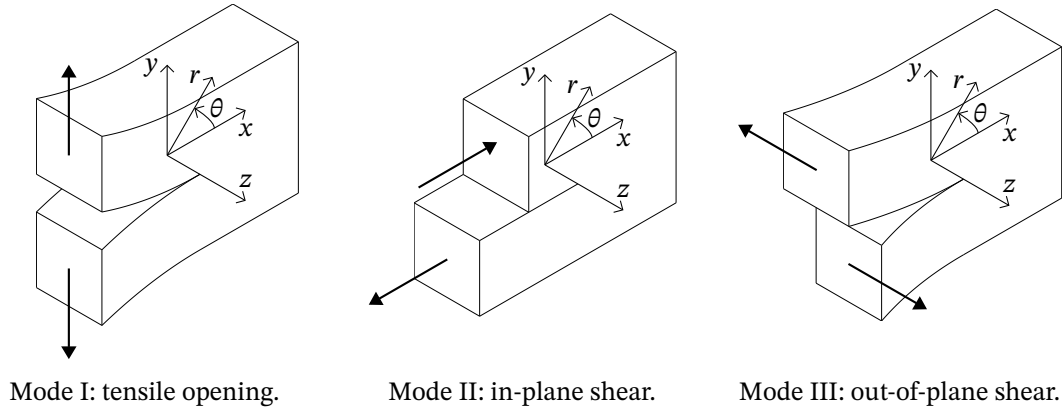


Figure 2.23: The three basic modes of fracture mechanics.

2.2.4.1 Stress intensity near the crack tip

Eventually, energy-based fracture mechanics lead the way to LEFM where the critical conditions for the growth of a crack are obtained via linear-elastic stress analyses. The first basic concept of LEFM is that the crack surface propagation in an arbitrarily loaded solid can always be characterized as a combination of three basic modes of separation, which are illustrated in Figure 2.23. Moreover, extending the work of Westergaard (1939) [75], Irwin (1957) [74] quantified the stress state near the tip of a linear-elastic crack in terms of the so-called stress intensity factor, denoted K . In fact, the stress intensity factor is defined for each mode as follows [2] (see Figure 2.23 for the commonly used coordinates at the crack tip):

$$K_I = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_{yy}(r, 0), \quad K_{II} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_{xy}(r, 0), \quad K_{III} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_{yz}(r, 0). \quad (2.89)$$

The stress intensity factor is extremely useful since the stress field is asymptotic and a singularity is expected at the crack tip; hence, local peak stresses cannot be used directly to characterize the crack front. In practice, the formulas shown in (2.89) are not necessarily used by engineers. Instead, the expression

$$K = Y \sigma_\infty \sqrt{\pi a} \quad (2.90)$$

is widely used, where a pertains to the crack length, σ_∞ pertains to an applied remote stress, and Y is an empirically determined geometric correction factor used to account for the type and geometry of the crack. Additionally, in LEFM, under monotonic, quasi-static loading conditions, the unstable propagation of a crack is expected once K reaches a critical value, namely, the so-called fracture toughness, denoted K_c , which is empirically determined and sensitive to various fatigue damage factors.

2.2.4.2 Virtual crack closure technique

The virtual crack closure technique (VCCT) [76], which is usually applied in conjunction with FEA, is an approach to determine stress intensity factors for arbitrary geometries and loadings without the need for experimental testing. In VCCT, a two-step FEA is conducted, as shown

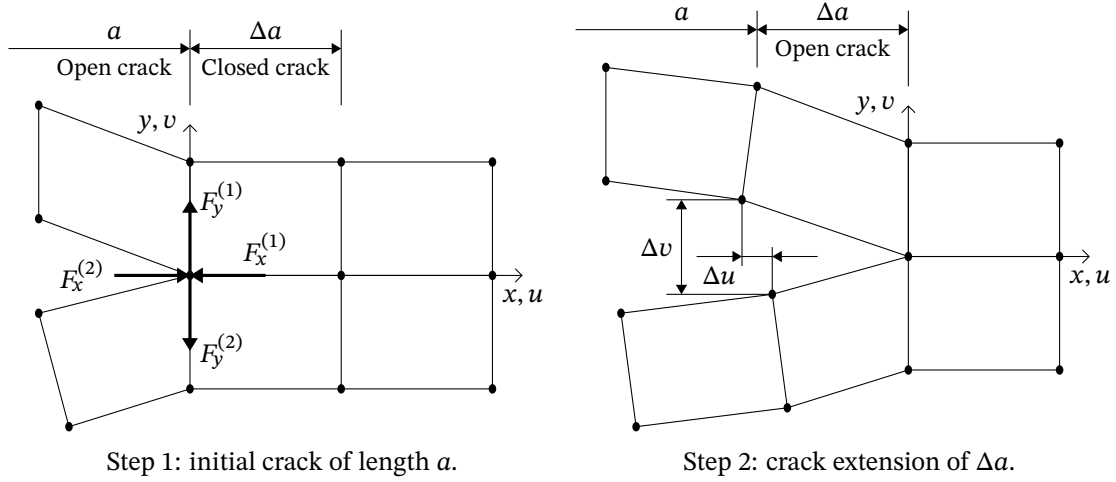


Figure 2.24: Two-step finite element analysis of a crack.

in Figure 2.24, where, in the first step, an analysis is performed considering a crack of length a , which is modelled as a geometric discontinuity by decoupling finite elements through their nodes. In the second step, a new analysis is performed considering the exact same parameters as in the previous step, except that a crack of length $a + \Delta a$ is now modelled, where the crack length increment Δa is dictated by the length of the finite element edges. From the first step, the internal nodal forces F_x and F_y are extracted from the crack tip node, and from the second step, the relative nodal displacements Δu and Δv , pertaining to the crack tip of the previous step, are obtained. It should be noted that a local coordinate system is considered, aligned with the crack plane. For the case shown in Figure 2.24, where bilinear elements are considered, the strain energy release rate is given as follows [76]:

$$G_I = -\frac{1}{2t\Delta a}F_y\Delta v, \quad G_{II} = -\frac{1}{2t\Delta a}F_x\Delta u. \quad (2.91)$$

Assuming a plane stress condition, the strain energy release rate can be related to the stress intensity factor by the following expressions [76]:

$$K_I = \sqrt{G_I E}, \quad K_{II} = \sqrt{G_{II} E}, \quad (2.92)$$

where E is the Young's modulus. If trilinear elements are considered in a general three-dimensional case, the strain energy release rate is given as follows [76]:

$$G_I = -\frac{1}{2\Delta A}F_y\Delta v, \quad G_{II} = -\frac{1}{2\Delta A}F_x\Delta u, \quad G_{III} = -\frac{1}{2\Delta A}F_z\Delta w, \quad (2.93)$$

where ΔA denotes the change in crack surface area. The stress intensity factor is then obtained by the following expressions, assuming a plane strain or anti-plane shear condition [2, 76]:

$$K_I = \sqrt{\frac{G_I E}{1 - \nu^2}}, \quad K_{II} = \sqrt{\frac{G_{II} E}{1 - \nu^2}}, \quad K_{III} = \sqrt{\frac{G_{III} E}{1 + \nu}}. \quad (2.94)$$

Formulas for higher order elements are also available, e.g., see reference [76].

2.2.4.3 Virtual crack extension method

The virtual crack extension method (VCEM) [77] is another approach for the determination of stress intensity factors considering arbitrary geometries and loadings. The VCEM is very similar to the aforementioned VCCT and, in fact, both approaches share the same two-step FEA procedure. However, in the VCEM, the strain energy release rate is determined via the following expression for computing the total potential energy of a discrete (or discretized) system:

$$\Pi = \frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u} - \mathbf{u}^T \mathbf{f}, \quad (2.95)$$

where $\mathbf{u}$ is the nodal displacements vector, $\mathbf{f}$ is the applied nodal forces vector, and $\mathbf{K}$ is the stiffness matrix; hence, by recalling the definition of G , shown in (2.88), one may assume the following finite difference approximation:

$$G \approx -\frac{\Delta \Pi}{\Delta A} = -\frac{{}^a \Pi - {}^{a+\Delta a} \Pi}{\Delta A} \quad \text{with} \quad \Delta A \sim 0, \quad (2.96)$$

here ${}^a \Pi$ is obtained from the first-step of the FEA, assuming a crack length of a , and ${}^{a+\Delta a} \Pi$ is obtained from the second-step, where a crack of length $a + \Delta a$ is modelled. It is also possible to decompose G into the three basic modes of LEFM by using a local coordinate system aligned with the crack plane, as shown in Figure 2.24. The stress intensity factor is then obtained via the expressions shown in (2.92) and (2.94). The advantage of the VCEM is that it is a simple and straightforward procedure, while the obvious disadvantage is that the accuracy of this method is dependent on an arbitrary choice for the ΔA parameter, which is also inherently related to the finite element mesh resolution.

2.2.4.4 The J-integral

As shown in the VCCT and VCEM approaches, the determination of stress intensity factors mainly consists in computing the strain energy release rate. For this purpose, Rice (1968) [78] introduced the so-called J-integral, which is a path-independent line integral along any contour Γ that encloses the crack tip, as shown in Figure 2.25. The J-integral is defined as

$$J = \int_{\Gamma} \left(W dy - \mathbf{t} \frac{\partial \mathbf{u}}{\partial x} ds \right), \quad (2.97)$$

where W is the strain energy density, $\mathbf{t}$ is the surface traction vector, and $\mathbf{u}$ is the displacement vector. Rice also showed that, for a linear-elastic material, J is equivalent to G .

Some commercially available FEA packages are capable of evaluating the J-integral. This approach is particularly attractive because, in the user's perspective, it is simple to use, requires low computational efforts, and provides accurate results, even with coarser meshes [79]. Moreover, it is common in FEA to use the divergence theorem in order to convert the J-integral from a contour integral into a so-called equivalent domain integral, and the integration domain is then defined using rings of elements surrounding the crack tip [79]. Even though the J-integral is path-independent, in practice, several distinct paths are defined to analyse the convergence of numerical results. Usually, the first contour is defined by the elements

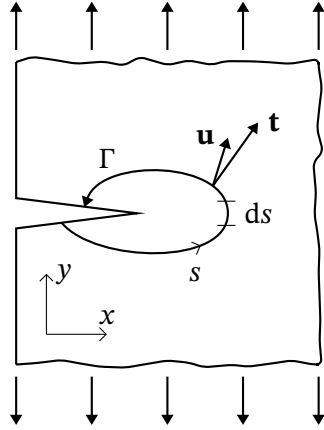


Figure 2.25: The J-integral: a path-independent line integral along any contour Γ that encloses the crack tip.

directly connected to the crack tip node, and the following contours are defined by surrounding rings of elements, increasingly farther away from the crack tip. Nevertheless, alternative approaches for computing the J-integral are shown in the literature, e.g., see reference [80].

2.2.4.5 Stable fatigue crack growth

As previously mentioned, the aim of the damage-tolerant approach is to determine the residual fatigue life of a fractured component. This is usually carried out by applying the principles of LEFM, which are applicable during low cyclic stresses, where the zone of plastic deformation ahead of the crack tip is a minor perturbation in an otherwise elastic field [2]. In this regard, Paris and Erdogan (1963) [81] established what is now known as Paris' law (also known as the Paris-Erdogan equation), which is given as

$$\frac{da}{dN} = C(\Delta K)^m, \quad (2.98)$$

in which the fatigue crack growth rate da/dN is related to the stress intensity factor range ΔK by a power relationship, where C and m are empirical constants. The predicted number of load cycles to failure can be computed by integrating the crack propagation law from an initial crack length a_i to a critical crack length a_c , i.e.,

$$N_f = \int_{a_i}^{a_c} \frac{da}{C(\Delta K)^m}. \quad (2.99)$$

Despite being a popular approach, the Paris power law is only applicable over a portion of the total stable crack propagation curve. Figure 2.26 shows the three stages of stable crack growth: in stage A, ΔK is close to the fatigue crack propagation limit or threshold, denoted ΔK_{th} , below which cracks do not propagate at detectable rates; stage B is where Paris' law is applicable and a linear relationship between $\log(da/dN)$ vs. $\log(\Delta K)$ is verified; finally, stage C pertains to high values of ΔK , where the fatigue crack growth rate increases rapidly until the crack propagation becomes unstable.

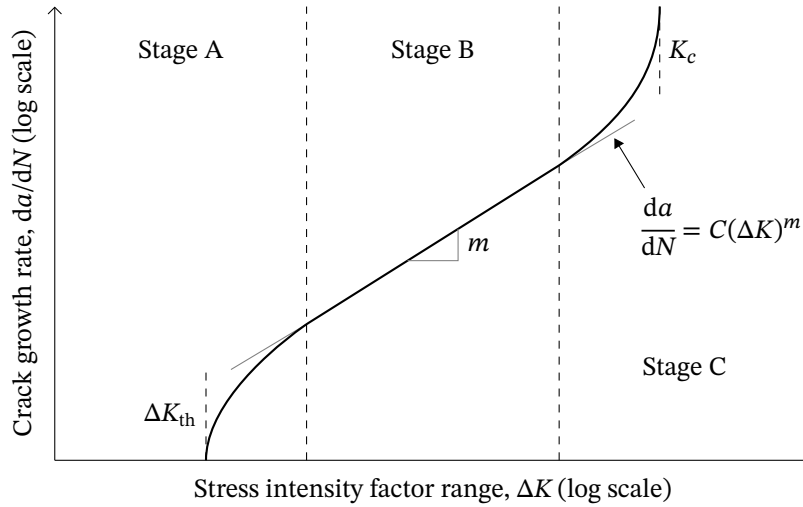


Figure 2.26: Paris' law and the different stages of stable fatigue crack growth.

2.2.4.6 Theory of critical distances

The theory of critical distances (TCD) is a group of methods that employ the concept of a characteristic material length parameter, denoted L [82]. Essentially, in TCD methods, effective stresses are considered for fatigue, rather than using peak values computed directly at the hot-spot region, which are generally less reliable due to large stress gradients found near the notch root. The simplest TCD method is known as the point method (PM), in which failure is assumed to occur once the cyclic stress range value at a distance $L/2$ from the notch root is equal to the fatigue strength of the component [82]. Another approach is the so-called line method (LM), which is similar to the PM, except that stress values are averaged over the distance $2L$ from the notch root [82]. Ultimately, effective stresses for these approaches are given as follows [82]:

$$\text{TCD-PM: } \bar{\sigma} = \sigma(L/2), \quad (2.100)$$

$$\text{TCD-LM: } \bar{\sigma} = \frac{1}{2L} \int_0^{2L} \sigma(l) dl. \quad (2.101)$$

The LM is an extension of the simpler PM; moreover, the LM can also be extended into the area method (AM) and then into the volume method (VM) [82]. These distinct approaches are illustrated in Figure 2.27.

Regarding the characteristic material length parameter, it is understood as a material property, and it can be derived from LEFM [82]:

$$L = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{\Delta \sigma_{th}} \right)^2, \quad (2.102)$$

where $\Delta \sigma_{th}$ is the fatigue limit of a plain specimen and ΔK_{th} is the fatigue crack propagation threshold.

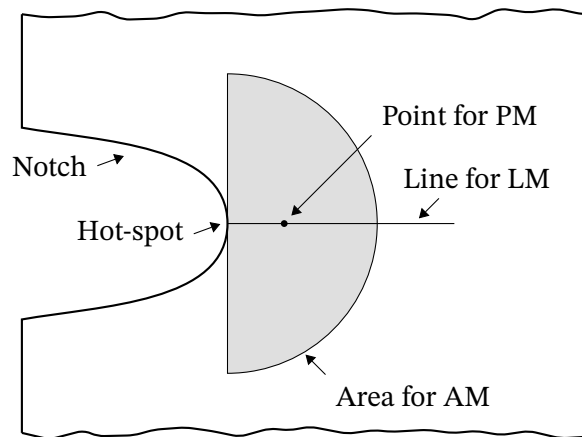


Figure 2.27: Illustration of distinct TCD methods.

2.2.5 Fatigue testing

Fatigue research is intrinsically related to fatigue testing. Generally, fatigue testing machines consist of the same basic components, namely, a load train, a control system, and monitors. The load train, illustrated in Figure 2.28, consists of the load frame, grips, fixtures, drive system, and test specimen. The load frame of the machine is what reacts to the forces applied to the specimen, while the drive system is responsible for generating those loads. Fully-electric systems use electric motors and act on the specimen via load actuators (e.g., cams or rotating grips), whereas in servohydraulic machines, electric motors drive hydraulic pumps who provide controllable pressure for a hydraulic piston actuator, and, in electromagnetic machines, the specimen is loaded using resonance and high-frequency excitations. In regard to the control system, it is used to initiate testing, adjust and maintain the controlled testing parameters, and terminate the test at predefined events, such as a load drop due to spe-

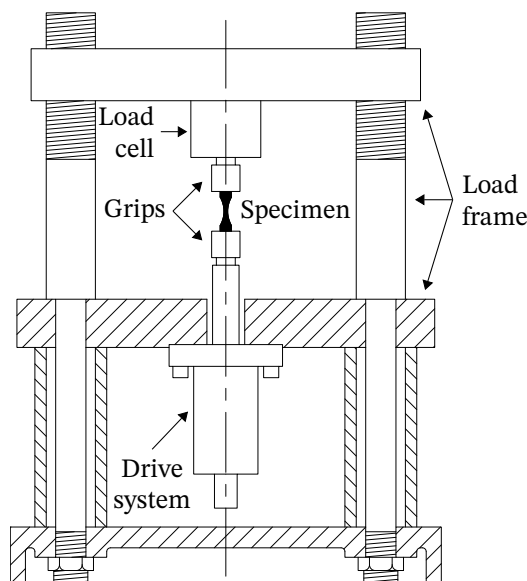


Figure 2.28: Schematic representation of a load train in a servohydraulic testing machine.

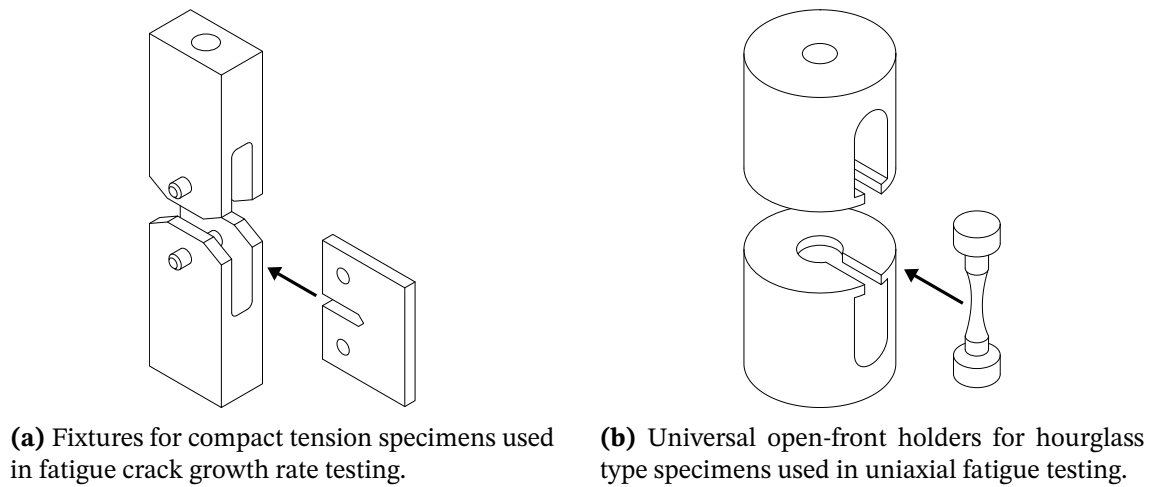


Figure 2.29: Examples of loading fixtures used in axial fatigue testing machines.

cimen failure. During testing, a continuously modulated control of time-varying loads and displacements is required, which is generally performed via a feedback control loop mechanism known as a proportional-integral-derivative (PID) controller. Finally, monitoring of loads, strains, displacements, and cycle counts is accomplished via sensors, such as load cells inserted into the load train, pressure transducers, and dynamic strain gauge extensometers. These sensors may provide an output signal to the PID controller or simply to a readout and storage device for subsequent data analyses.

Generally, fatigue testing machines are somewhat universal and different loading fixtures are attached to the grips to alter the mode of loading and to accommodate different types of specimens, providing versatile testing conditions. Nevertheless, several types of fatigue testing machines exist, such as axial, bending, and torsional testing machines. For example, in metal fatigue, an axial testing machine can accommodate compact tension specimens used in fatigue crack growth rate testing, as well as smooth dog-bone and hourglass type specimens used in uniaxial fatigue (see [Figure 2.28](#) and [Figure 2.29](#)).

Finally, it can be noted that fatigue testing is usually conducted by following a standardized procedure, e.g., the ASTM E466 [83] proposes a standard practice for conducting force-controlled constant amplitude axial fatigue tests, the ASTM E606 [84] proposes a standardized test method for strain-controlled fatigue, and the ASTM E647 [85] proposes a standard method for the measurement of fatigue crack growth rates. Additionally, the ASTM E468 [86] proposes a standard practice for the presentation of constant amplitude fatigue test results.

2.2.6 Fatigue in cold-formed thin-walled profiles

As previously mentioned, cold-formed steel members are shaped from mild steel sheets, strips, plates, or bars through processes like press braking or roll forming conducted at room temperature [87]. These manufacturing methods allow for the creation of diverse shapes and profiles, imparting these members with a high strength-to-weight ratio, making them advantageous in various applications [88, 89]. However, cold-formed steel members contain internal residual stresses, introduced during the manufacturing process due to cold working and strain

hardening, that remain in the produced part even after the external loads or thermal gradients that caused them have been removed [27]. In cold forming, the magnitude of internal residual stresses is significantly influenced by the cold-formed corner's bending radius and sheet thickness [90]. Moreover, the impact of residual stresses on fatigue life can be either detrimental or beneficial, depending on whether they are tensile or compressive in nature, affecting the fatigue behaviour of the produced part while under cyclic loading [91, 92].

In Section 2.1.4, we discussed approaches for the determination of residual stresses, which is the first step to analyse any possible residual stress effect. These approaches can be based on experimental [13, 31, 93], numerical [13, 30, 54, 55], or, in simple cases, analytical methodologies [28, 29, 37]. In this regard, the current state of the art is reviewed by Díaz et al. (2021) [27]. Regardless, the determination of residual stresses is difficult to accomplish when considering thin-walled profiles, particularly when nonlinear distributions across the thickness are expected [27], as in the case of cold bending [27, 28]. Therefore, when experimental results are presented in the literature, they are usually limited to the flat regions [13, 93, 94] or, when considering results at the cold formed corners, these are usually limited to the surfaces [13, 31, 93–97]. Weng and White (1990) [33, 98] were among the first researchers to provide a through-thickness distribution of residual stresses in press-braked steel corners. To this end, experimental hole drilling and sectioning techniques were used [27]. However, while the current work is focused on thin-walled mild steel profiles, Weng and White considered thick plates made of high-strength steel. Later, Quach et al. (2006) [30] numerically simulated the press braking process used to produce the specimens considered by Weng and White, and a good match was verified between experimental measurements of residual stresses and numerical predictions. As for roll forming, Tong et al. (2012) [99] used experimental hole drilling and X-ray diffraction methods to measure the longitudinal residual stress distribution across the thickness of cold roll-formed thick-walled sections made of mild steel. Later, Souto et al. (2022) [13] used similar techniques for the determination of residual stresses in cold roll-formed thin-walled mild steel profiles. Besides, Alegre et al. (2019) [100] proposed advances in experimental techniques, developing a new application of the hole drilling method for the evaluation of residual stresses concentrated on the rounded corners generated by rolling operations.

In regard to fatigue, the residual stress effect is currently an active research field. Independently and nearly concurrently, Gothivarekar et al. (2021) [54] and Souto et al. (2021) [12] purposed similar frameworks for fatigue life prediction based on elastoplastic finite element analysis followed by a local approach applied via a post-processing procedure. In both studies, the manufacturing process of the specimens is simulated numerically via finite element modelling in order to obtain a residual stress field due to cold forming. These results are then used as the initial state during a subsequent structural loading analysis, from which fatigue life predictions are performed at critical locations using local models based on empirically determined material constants that are previously obtained via low- and high-cycle fatigue testing campaigns of the plain material. In both studies, adequate fatigue life predictions were achieved when compared against experimental findings.

Castiglioni and Menghini (2022) [101] presented fatigue results pertaining to structural details made of cold-formed mild steel. This study focused on a more practical small-scale fatigue testing campaign, where specimens were extracted from full-scale profiles; nevertheless, an agreement was found between full- and small-scale test data, showcasing the wide

applicability and integrity of the considered approaches. The authors also presented a parametric study on the effect of geometric properties on fatigue endurance, in which numerical simulations were used to evaluate the effective stress level inside the cold-formed corner as a function of the real geometry of the specimen that often is distinct from the nominal specification. Parametric studies were carried out to estimate the influence of certain geometric parameters on the stress geometric variation factor. The authors also generated fatigue design curves compatible with Eurocode 3.

Geers et al. (2021) [90] tested specimens sampled from sheet material used in cold roll-forming. A purpose-built testing device was used based on the principles of harmonic motion and resonance of single-mass oscillators, resulting in a large number of fatigue tests. The profiled sheets manufactured from the raw material were also subjected to cyclical loads and, despite a conservative assessment, detail categories were formulated based on the cold-formed corner radius to material thickness ratio. The authors also managed to generate fatigue design curves compatible with EN 1993-1-9 [5].

Mashiri et al. (2013) [102] studied the fatigue behaviour of hollow sections with thicknesses lower than 4 mm. The effects of galvanizing, adopted steel grade, stress ratio and tube wall thickness on the long-term behaviour were investigated. They concluded that *S-N* curves lie above the ones available in the Australian, European, and Canadian standards.

Heinilä et al. (2008) [103] performed numerical and experimental studies on the fatigue crack paths developed in the corner of square hollow sections at both small- and full-scale. During testing, mixed mode (I, II, and III) crack growths with distinct paths were observed due to a multiaxial stress state. It was found that fatigue crack paths depend on the type and range of cyclic loading, material properties, and residual stress conditions at the critical locations.

In regard to regulations, cold-formed steel and thin-walled structures are completely missing from current versions of European standards for fatigue design. In fact, Eurocode 3 covers the static design of thick sections in EN 1993-1-1 [3] and that of cold-formed members and sheeting in EN 1993-1-3 [4]; however, in regard to fatigue design, EN 1993-1-9 [5] only covers thick, hot-rolled sections. Furthermore, the EN 15512 standard [6], dedicated to the structural design of adjustable pallet racking systems, adopts a design approach assisted by testing aimed at assessing the ultimate strength and stiffness of members and connections to be used for static verifications. Nevertheless, no rules or recommendations are given to investigate the long-term fatigue behaviour. While the standard acknowledges fracture due to fatigue as a form of ultimate limit state, it later states that ‘it is therefore implicit that normal rack structures are not subject to fatigue...’ [6]. Nevertheless, there are North American standards, such as the AISI S100-16 standard [7], addressing structural members and connections made of cold-formed steel subject to cyclic loading within the elastic range of stresses of frequency and magnitude sufficient to initiate cracking and progressive failure, defining the limit state of fatigue. In this regard, important research was carried out by Klippstein et al. (1980-1985) [104–106] and by Yu and LaBoube (2010) [107]. In Australia, the AS/NZS 4600 standard [108], based on the work of Hancock (2007) [109], defines minimum requirements for the design of cold-formed structural members from carbon or low-alloy steel sheet, strip, plate or bar used for load-carrying purposes in buildings. It is also applicable for structures other than buildings, provided appropriate allowances are made for dynamic effects, since fatigue and fracture are not covered.

Mashiri et al. (2013) [102] reviewed several fatigue design specifications, including the Eurocode 3 [5], the UK Department of Energy's Guidance [110], the Australian Standard on steel structures [111], and the IIW recommendations [112], noting a lack of fatigue design rules for tubular profiles with thicknesses lower than or equal to 4 mm. On the other hand, while the identified gap is still present in current versions of Eurocode 3, the CEN is in the process of updating the EN 1993-1-9 standard [5]. The revision, currently under approval as prEN 1993-1-9 [8], introduces new considerations for rolled products in the form of new detail categories and $S-N$ curves.

2.3 Finite element modelling

The FEM is a widely used numerical technique for converting governing differential equations into systems of algebraic equations, in which a continuous system is discretized by subdividing the original domain into small and simpler parts called finite elements, forming a mesh where these elements are connected by their nodes. Each node of the finite element mesh is then associated with degrees of freedom, which, in structural analyses, typically represent generalized displacements that describe the deformed configuration of the discrete system relative to a reference state. In fact, the main objective of the FEM is to compute these nodal displacement fields, from which strains and stresses are subsequently derived.

Within the domain of each element, unknown functions are approximated using element shape functions. While the FEM approximates an unknown function over the whole domain, the global approximating functions are never explicitly defined; instead, they are implicitly generated by the shape functions of the individual elements. Equilibrium equations are then determined for each element and later assembled into a global system via the so-called direct stiffness method. In general, this results in large sparse systems of algebraic equations (see Figure 2.30).

In the FEM applied to solids and structures, elements are usually categorized as solid (continuum) elements or structural elements. Solid elements, derived directly from solid mechanics, include plane stress, plane strain, axisymmetric, and general 3D elements, where only translational degrees of freedom are considered. Structural elements, originating from struc-

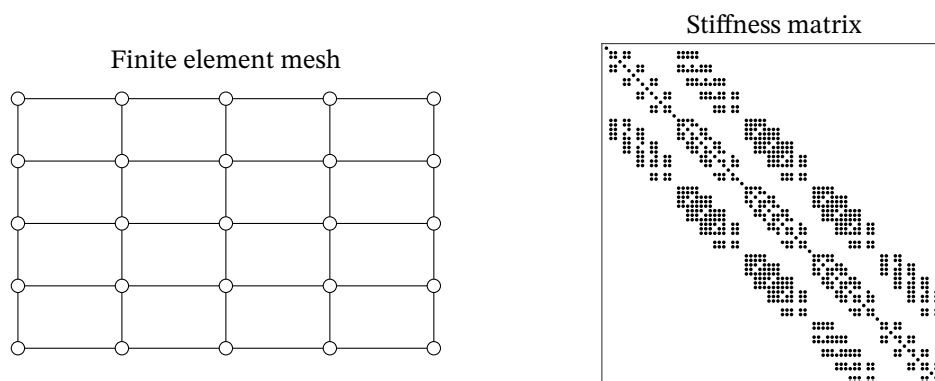


Figure 2.30: Finite element discretization of a plate and corresponding sparse stiffness matrix (non-zero entries).

tural mechanics, include trusses, beams, plates, shells, and membranes, and may involve both translational and rotational degrees of freedom.

2.3.1 Pure-displacement formulation of continuum elements

To understand the basic principles of the FEM, let us examine the pure-displacement formulation of continuum elements in 2D. Note that the concepts presented here can be easily extended to 3D. First, consider the following finite element approximation for some unknown function u :

$$u(\mathbf{x}) \approx \sum_{i=1}^n N_i(\mathbf{x}) u_i, \quad (2.103)$$

where $\mathbf{x} = [x \ y]^T$ are the physical coordinates within the domain of a given element where u is to be evaluated, while u_i is the value of the unknown function at the element node $i = 1, \dots, n$ and N_i is the corresponding shape function, which is evaluated to null at all nodes of the element except at node i , where it is evaluated to the unity. In other words, the value of u at any given point within the element's domain is obtained by interpolating nodal values using a linear combination of the corresponding shape functions. Note that a vector notation is preferred, as in $u(\mathbf{x})$ instead of $u(x, y)$, so that equations such as the one shown in (2.103) are valid for both 2D and 3D problems.

2.3.1.1 Isoparametric elements

We now introduce the concept of isoparametric elements, allowing for the analysis of complex geometries by mapping the physical coordinates of each element to a natural coordinate system through a transformation, i.e., a change of variables (see Figure 2.31). Therefore, we may write that

$$\mathbf{x}(\mathbf{r}) \approx \sum_{i=1}^n N_i(\mathbf{r}) \mathbf{x}_i, \quad (2.104)$$

where $\mathbf{r} = [r \ s]^T$ are the natural coordinates within the domain of a given element where the physical coordinates $\mathbf{x}$ are to be evaluated, and $\mathbf{x}_i$ are the physical coordinates of the element

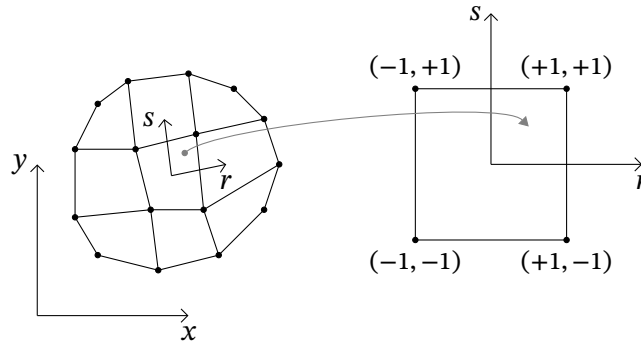


Figure 2.31: Finite element mesh comprised of isoparametric bilinear elements.

nodes. This is useful because shape functions are now the same for every element of the same type since they are defined as functions of the natural coordinates. Using the element's natural representation also makes integration much simpler (as shown later).

As an example, consider the bilinear isoparametric solid element (a 2D quadrilateral element with 4 nodes). In this case, we have the following bilinear approximation:

$$\bar{u}(r, s) = a_{00} + a_{10}r + a_{01}s + a_{11}rs = \begin{bmatrix} 1 & r & s & rs \end{bmatrix} \begin{Bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{Bmatrix}, \quad (2.105)$$

and, by considering each element node, we have that

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} 1 & r_1 & s_1 & r_1s_1 \\ 1 & r_2 & s_2 & r_2s_2 \\ 1 & r_3 & s_3 & r_3s_3 \\ 1 & r_4 & s_4 & r_4s_4 \end{bmatrix} \begin{Bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{Bmatrix} = \begin{bmatrix} +1 & -1 & -1 & +1 \\ +1 & +1 & -1 & -1 \\ +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \end{bmatrix} \begin{Bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{Bmatrix}. \quad (2.106)$$

By inverting the matrix, the polynomial coefficients are obtained:

$$\begin{Bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{Bmatrix} = \frac{1}{4} \begin{bmatrix} +1 & +1 & +1 & +1 \\ -1 & +1 & +1 & -1 \\ -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}, \quad (2.107)$$

and, by substituting (2.107) into (2.105):

$$\bar{u}(r, s) = \frac{1}{4} \begin{bmatrix} 1 & r & s & rs \end{bmatrix} \begin{bmatrix} +1 & +1 & +1 & +1 \\ -1 & +1 & +1 & -1 \\ -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}. \quad (2.108)$$

Finally, by performing the above matrix multiplication:

$$\bar{u}(r, s) = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \equiv \sum_{i=1}^4 N_i(r, s)u_i, \quad (2.109)$$

in which

$$\begin{aligned} N_1 &= \frac{1}{4}(1-r)(1-s), \\ N_2 &= \frac{1}{4}(1+r)(1-s), \\ N_3 &= \frac{1}{4}(1+r)(1+s), \\ N_4 &= \frac{1}{4}(1-r)(1+s), \end{aligned} \quad (2.110)$$

are the element shape functions. As previously mentioned, N_i is evaluated to null at all nodes of the element except at node i , where it is evaluated to the unity, as plotted in Figure 2.32.

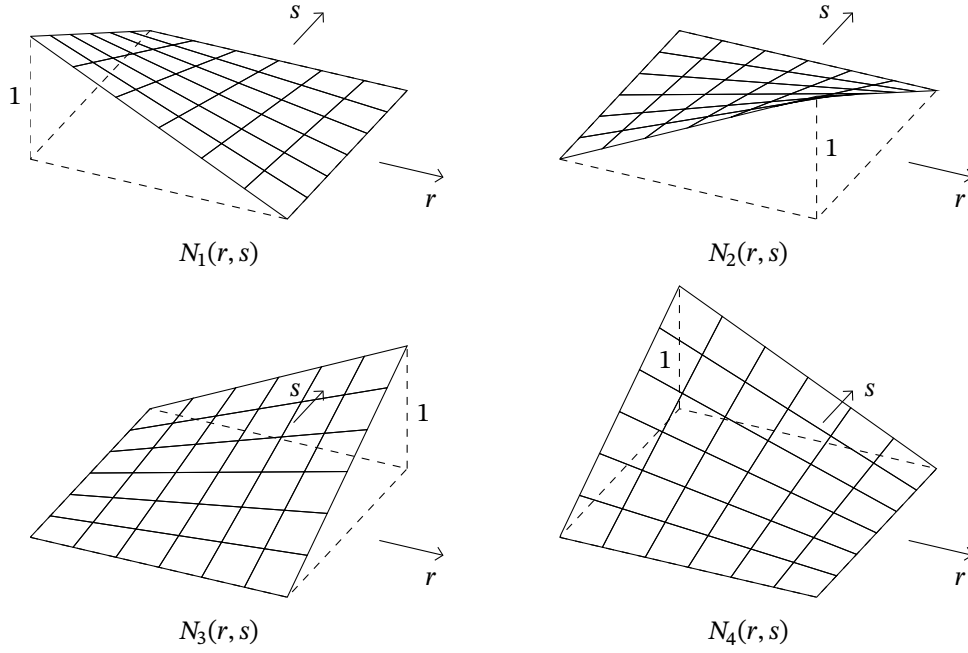


Figure 2.32: Shape functions of a bilinear isoparametric element.

2.3.1.2 Gaussian quadrature rule

The Gaussian quadrature rule is commonly used in the FEM to evaluate definite integrals. This quadrature rule yields exact results for polynomials of degree $2n - 1$ or lower by choosing n suitable quadrature points (also called Gauss points or integration points) and corresponding weights. This is referred to as a full-integration scheme. Alternatively, if fewer than n points are chosen, a reduced-integration scheme is being used. For the bilinear element, we get the following:

$$\iiint_{\Omega_e} f(x, y) d\Omega_e = b \int_{-1}^{+1} \int_{-1}^{+1} f(r, s) |\mathbf{J}(r, s)| dr ds \approx b \sum_{i=1}^2 \sum_{j=1}^2 f_{ij} |\mathbf{J}_{ij}| \omega_i \omega_j, \quad (2.111)$$

where b is the out-of-plane thickness of the 2D element; f_{ij} denotes the value of f at the integration point (r_i, s_j) ; ω_i and ω_j are the corresponding weights; and $|\mathbf{J}_{ij}|$ is the determinant of the Jacobian matrix, also evaluated at the integration point, which accounts for the change of coordinates (from $\mathbf{x}$ to $\mathbf{r}$) and represents the change in volume arising from that mapping. The Jacobian matrix is given, in 2D, by

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix} = \frac{\partial \mathbf{x}}{\partial \mathbf{r}} \approx \sum_{i=1}^n \frac{\partial N_i}{\partial \mathbf{r}} \mathbf{x}_i, \quad (2.112)$$

in which n is the number of element nodes. In matrix form, we get:

$$\mathbf{J} = \mathbf{N}_r \mathbf{C} \Leftrightarrow \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix} = \begin{bmatrix} N_{1,r} & \cdots & N_{n,r} \\ N_{1,s} & \cdots & N_{n,s} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix}. \quad (2.113)$$

When relying on the Gaussian quadrature rule, secondary variables, such as strains and stresses, are computed at the integration points. However, for visualization of the results in the finite element mesh, results are usually required at the nodes. Consequently, it is necessary to perform an extrapolation from the integration points to the element nodes. If the element only contains one integration point (known as constant deformation elements), then the element nodal values are equal to the value found at the integration point. If the number of integration points differs from the number of element nodes, the extrapolation can be achieved by using the least squares method to fit some polynomial model to the known data at the integration points. Alternatively, if the number of integration points is the same as the number of element nodes, then the nodal values can be directly computed by first defining the following matrix equation:

$$\begin{bmatrix} N_1(\mathbf{r}_1) & \cdots & N_n(\mathbf{r}_1) \\ \vdots & \ddots & \vdots \\ N_1(\mathbf{r}_n) & \cdots & N_n(\mathbf{r}_n) \end{bmatrix} \begin{Bmatrix} u_1 \\ \vdots \\ u_n \end{Bmatrix}_{\text{Nodes}} = \begin{Bmatrix} u_1 \\ \vdots \\ u_n \end{Bmatrix}_{\text{IPs}}, \quad (2.114)$$

where the right-hand side is a vector of the known values of a function evaluated at the integration points (IPs) of the element; the left-hand side contains a known matrix of shape functions evaluated at the integration points, as well as a vector of the unknown nodal values; hence, the nodal values can be computed by solving the linear system of equations shown above.

This extrapolation process returns, for the same global mesh node, as many values as there are elements connected to that node. If a sufficiently refined mesh is used, these values should be similar; nevertheless, a straightforward approach is to average the nodal values (using their arithmetic mean) obtaining, in this way, a single value per mesh node. This final process is called averaging or smoothing. Note that the final smoothing could obscure severe discontinuities between element domains, which indicates that the finite element mesh is too coarse.

2.3.1.3 Static equilibrium equations

To perform a linear-elastic static analysis, equilibrium equations need to be formulated. To this end, one may start by recalling Green's small strain tensor:

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad \text{and} \quad \gamma_{ij} = 2\varepsilon_{ij} \quad \text{for} \quad i \neq j, \quad (2.115)$$

where u_i is the displacement along the x_i coordinate. By applying a finite element approximation, we get that

$$\frac{\partial u_i}{\partial x_j} \approx \sum_{k=1}^n \frac{\partial N_k}{\partial x_j} u_{ik}. \quad (2.116)$$

2: Literature review

Therefore, the strain tensor (represented as a vector) can be computed by the following strain-displacement relationship, which is valid for the plane stress case:

$$\underline{\varepsilon} = \mathbf{B}\mathbf{u} \Leftrightarrow \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} N_{1,x} & 0 & \cdots & N_{n,x} & 0 \\ 0 & N_{1,y} & \cdots & 0 & N_{n,y} \\ N_{1,y} & N_{1,x} & \cdots & N_{n,y} & N_{n,x} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{Bmatrix}, \quad (2.117)$$

where $\mathbf{B}$ is the so-called strain-displacement matrix. Notice that the strain-displacement relationship requires the physical derivatives of the shape functions; however, in isoparametric elements, the shape functions are defined as functions of the natural coordinates. To overcome this, the chain rule is first applied:

$$\frac{\partial N_i}{\partial r} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial r} \quad \text{and} \quad \frac{\partial N_i}{\partial s} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial s}, \quad (2.118)$$

or, in matrix form:

$$\mathbf{N}_r = \mathbf{J}\mathbf{N}_s \Leftrightarrow \begin{bmatrix} N_{1,r} & \cdots & N_{n,r} \\ N_{1,s} & \cdots & N_{n,s} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix} \begin{bmatrix} N_{1,x} & \cdots & N_{n,x} \\ N_{1,y} & \cdots & N_{n,y} \end{bmatrix}, \quad (2.119)$$

where, for any given point within the isoparametric element's domain, $\mathbf{N}_r$ and $\mathbf{J}$ are known. Hence, the physical derivatives of the shape functions are given by:

$$\mathbf{N}_s = \mathbf{J}^{-1}\mathbf{N}_r. \quad (2.120)$$

Once the strain-displacement relationship is known, the next step is to define the stress-strain relationship. In a linear analysis, this is obtained via the linear elasticity theory; specifically, the generalized Hooke's law for isotropic materials is used, which can be expressed for the plane stress case as

$$\underline{\sigma} = \mathbf{D}\underline{\varepsilon} \Leftrightarrow \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}, \quad (2.121)$$

where $\mathbf{D}$ is the so-called stress-strain matrix.

Once the strain-displacement and stress-strain relationships are established, the next step to formulate the equilibrium equations is to recall the principle of virtual work, which states that the variation of internal strain energy equals the work done by external nodal loads:

$$\delta W_i = \delta W_e \Leftrightarrow \iiint_{\Omega_e} \hat{\underline{\varepsilon}}^T \underline{\sigma} d\Omega_e = \hat{\mathbf{u}}^T \mathbf{f}, \quad (2.122)$$

where the caret (^) denotes virtual quantities and $\mathbf{f}$ is introduced as the external equivalent nodal forces vector (the computation of which is discussed below). By substituting in the strain-displacement relationship shown in (2.117):

$$\iiint_{\Omega_e} \mathbf{B}^T \hat{\mathbf{u}}^T \underline{\sigma} d\Omega_e = \hat{\mathbf{u}}^T \mathbf{f} \Leftrightarrow \iiint_{\Omega_e} \mathbf{B}^T \underline{\sigma} d\Omega_e = \mathbf{f}, \quad (2.123)$$

where an expression for the internal nodal loads is obtained (left-hand side), which, of course, must be equal to the applied external nodal loads. By substituting in the stress-strain relationship shown in (2.121), the following is obtained:

$$\iiint_{\Omega_e} \mathbf{B}^T \mathbf{D} \boldsymbol{\varepsilon} d\Omega_e = \mathbf{f}. \quad (2.124)$$

We may then reapply the strain-displacement relationship, as done previously, resulting in:

$$\iiint_{\Omega_e} \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega_e \mathbf{u} = \mathbf{f}, \quad (2.125)$$

which is the standard matrix equation for a linear static analysis. In a more compact notation, we get

$$\mathbf{K} \mathbf{u} = \mathbf{f}, \quad (2.126)$$

where $\mathbf{u}$ is the nodal displacements vector to be computed, $\mathbf{f}$ is the applied nodal forces vector, and $\mathbf{K}$ is the so-called element stiffness matrix that is usually computed using the Gaussian quadrature rule, as previously discussed for the bilinear element:

$$\mathbf{K} = \iiint_{\Omega_e} \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega_e \approx b \sum_{i=1}^2 \sum_{j=1}^2 \mathbf{B}_{ij}^T \mathbf{D}_{ij} \mathbf{B}_{ij} |\mathbf{J}_{ij}| \omega_i \omega_j. \quad (2.127)$$

Regarding the equivalent nodal forces vector $\mathbf{f}$, it is usually computed from three distinct components:

$$\mathbf{f} = \mathbf{f}_p + \mathbf{f}_s + \mathbf{f}_b, \quad (2.128)$$

where $\mathbf{f}_p$ denotes the applied point loads, $\mathbf{f}_s$ the distributed surface loads (such as pressures), and $\mathbf{f}_b$ the distributed volume or body loads (related to inertia loads such as gravity). While $\mathbf{f}_p$ is usually known directly, $\mathbf{f}_s$ and $\mathbf{f}_b$ are computed via integration as they pertain to distributed loads. For each element, the equivalent nodal loads due to body forces are computed as follows [113]:

$$\mathbf{f}_b = \iiint_{\Omega_e} \mathbf{H}^T \mathbf{f}^b d\Omega_e, \quad (2.129)$$

where $\mathbf{f}^b$ is a vector containing the individual components of the load along the global axes and $\mathbf{H}$ is known as the displacement interpolation matrix given by [113]

$$\mathbf{H} = [\mathbf{N}_1 \mathbf{I} \quad \cdots \quad \mathbf{N}_n \mathbf{I}], \quad (2.130)$$

where $\mathbf{I}$ is the identity matrix. The equivalent nodal loads due to surface forces are computed analogously; however, the integration is performed over element surfaces. Conveniently, an element surface may itself be treated as a lower dimension element.

Once the equilibrium equations for each element in the finite element mesh are known, or, in other words, once $\mathbf{K}$ and $\mathbf{f}$ are determined for each element, these are assembled into a

2: Literature review

global system of equations, where each nodal degree of freedom receives contributions from all elements connected to that node. Symbolically, we have

$$\mathbf{K}^{\text{global}} = \sum_i \mathbf{K}_i^{\text{element}} \quad \text{and} \quad \mathbf{f}^{\text{global}} = \sum_i \mathbf{f}_i^{\text{element}}. \quad (2.131)$$

The process of assembling the element matrices by this direct addition is called the direct stiffness method [113]. In practice, a local-to-global degree of freedom mapping is required, accounting for deactivated degrees of freedom due to prescribed nodal values (boundary conditions). The global system is then solved via static condensation (also known as Guyan reduction [114]). In this approach, we have

$$\begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{mn} \\ \mathbf{K}_{nm} & \mathbf{K}_{nn} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_m \\ \mathbf{u}_n \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_m \\ \mathbf{f}_n \end{Bmatrix}, \quad (2.132)$$

in which $\mathbf{u}_m$ and $\mathbf{u}_n$ are the unknown and prescribed nodal displacements, respectively, and $\mathbf{f}_m$ denotes the applied nodal loads while $\mathbf{f}_n$ denotes the unknown reaction forces. The unknowns are computed, symbolically, as shown:

$$\begin{aligned} \mathbf{u}_m &= \mathbf{K}_{mm}^{-1}(\mathbf{f}_m - \mathbf{K}_{mn}\mathbf{u}_n), \\ \mathbf{f}_n &= \mathbf{K}_{nm}\mathbf{u}_m + \mathbf{K}_{nn}\mathbf{u}_n. \end{aligned} \quad (2.133)$$

2.3.1.4 Dynamic equilibrium equations

In dynamic problems, where inertia effects cannot be discarded, the equilibrium equations are defined as follows [113]:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}, \quad (2.134)$$

where $\mathbf{u}$, $\dot{\mathbf{u}}$, and $\ddot{\mathbf{u}}$ are, respectively, the nodal displacements, velocities, and accelerations vectors. We introduce $\mathbf{M}$ as the mass matrix [113]:

$$\mathbf{M} = \iiint_{\Omega_e} \rho \mathbf{H}^T \mathbf{H} d\Omega_e, \quad (2.135)$$

in which ρ is the mass density of the element's material and $\mathbf{H}$ is the aforementioned displacement interpolation matrix. $\mathbf{C}$ is also introduced as the damping matrix [113]:

$$\mathbf{C} = \iiint_{\Omega_e} \kappa \mathbf{H}^T \mathbf{H} d\Omega_e, \quad (2.136)$$

where κ is a material damping property parameter. Finally, note that $\mathbf{f}$ is the equivalent nodal forces vector as shown in (2.128); however, the component $\mathbf{f}_b$ now includes velocity-dependent damping forces and acceleration-dependent inertia forces [113]:

$$\mathbf{f}_b = \iiint_{\Omega_e} \mathbf{H}^T (\mathbf{f}^b - \rho \mathbf{H}\ddot{\mathbf{u}} - \kappa \mathbf{H}\dot{\mathbf{u}}) d\Omega_e. \quad (2.137)$$

The global versions of the mentioned matrices and vectors can be obtained via the direct stiffness assembly method, in the same way as previously discussed for the stiffness matrix.

Dynamic equilibrium equations can be solved using several approaches, e.g., via direct integration or modal superposition methods, where the latter introduces a transformation to the equilibrium equations so that the solution phase is computationally more efficient [113]. Regardless, several formulations can be made to solve the equilibrium equations, which are usually based on replacing the time derivatives by finite differences, leading to either explicit or implicit integration schemes. Implicit schemes are inherently stable and allow for larger time increments, while explicit schemes become unstable when large enough time increments are chosen. However, explicit schemes can be more efficient since their formulation may allow for simpler algebraic computations. In practice, explicit schemes are typically used in short time-duration simulations, such as impact simulations, whereas implicit schemes are preferred in general for their increased stability and accuracy. For example, in the Newmark method [113] we assume

$$\begin{aligned} {}^{t+\Delta t}\dot{\mathbf{u}} &= {}^t\dot{\mathbf{u}} + ((1 - \delta){}^t\ddot{\mathbf{u}} + \delta{}^{t+\Delta t}\ddot{\mathbf{u}})\Delta t, \\ {}^{t+\Delta t}\mathbf{u} &= {}^t\mathbf{u} + {}^t\dot{\mathbf{u}}\Delta t + \left(\left(\frac{1}{2} - \alpha\right){}^t\ddot{\mathbf{u}} + \alpha{}^{t+\Delta t}\ddot{\mathbf{u}}\right)\Delta t^2, \end{aligned} \quad (2.138)$$

where the superscripts indicate the current or next time frames, while α and δ are parameters that can be determined to adjust integration accuracy and stability. In the original method, also called constant average acceleration method or trapezoidal rule, $\alpha = 1/4$ and $\delta = 1/2$ are considered. In this case, the equilibrium equations become

$$\left(\frac{4}{\Delta t^2}\mathbf{M} + \frac{2}{\Delta t}\mathbf{C} + \mathbf{K}\right){}^{t+\Delta t}\mathbf{u} = {}^{t+\Delta t}\mathbf{f} + \mathbf{M}\left(\frac{4}{\Delta t^2}{}^t\mathbf{u} + \frac{4}{\Delta t}{}^t\dot{\mathbf{u}} + {}^t\ddot{\mathbf{u}}\right) + \mathbf{C}\left(\frac{2}{\Delta t}{}^t\mathbf{u} + {}^t\dot{\mathbf{u}}\right), \quad (2.139)$$

which are solved for ${}^{t+\Delta t}\mathbf{u}$; after that, the computation of ${}^{t+\Delta t}\ddot{\mathbf{u}}$ and ${}^{t+\Delta t}\dot{\mathbf{u}}$ is performed using (2.138).

2.3.2 Finite element locking and alternative formulations

It is well-known that fully integrated first-order finite elements following a pure-displacement formulation, i.e., elements whose primary variables are displacements, velocities, and accelerations (and other quantities such as strains and stresses are derived from those variables), suffer from the so-called locking effect [113]. The locking effect simply means that the element formulation presents higher-than-expected stiffness when comparing numerical results with theoretical and experimental ones. Consequently, finite element locking must be addressed.

Shear locking is related to the inability of first-order finite elements in capturing the kinematics of bending. As an example, consider a beam subjected to pure bending, as shown in Figure 2.33. Due to the linear nature of the element's geometry, the element cannot correctly model the kinematics associated with bending in the sense that the curvature of the deformed shape is not captured. On effect, a spurious shear stress is introduced, causing the element to reach equilibrium with smaller displacements, essentially making it stiffer than it should be.

Shear locking can be reduced by refining the finite element mesh, by using second-order finite elements, or by resorting to more sophisticated element formulations. For example, in Abaqus, the so-called incompatible mode formulation can be used to account for shear locking, where additional degrees of freedom are introduced into the first-order element, enhancing the element's deformation gradient [79, 115]. These enhancements allow for a linear

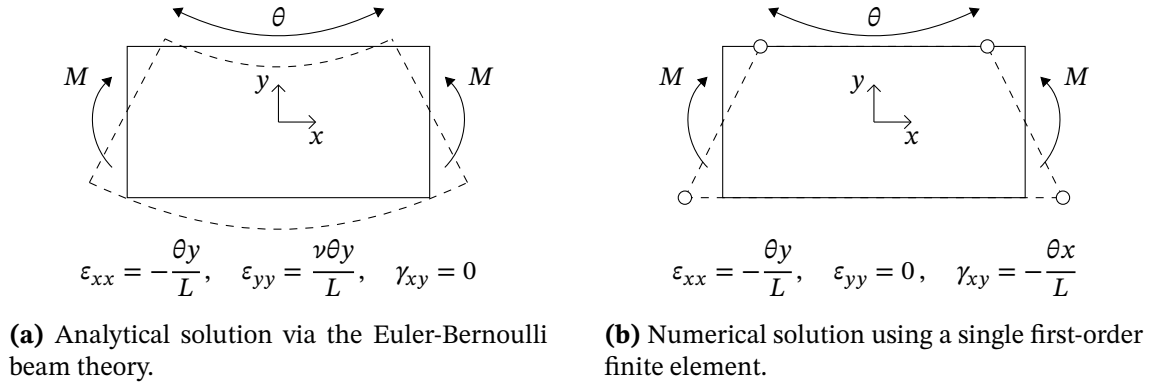


Figure 2.33: Illustration of shear locking due to the inability of first-order finite elements in capturing the kinematics of bending. Note: L is the considered length.

variation of the deformation gradient across the element's domain, eliminating the parasitic shear stress [79, 115]. Moreover, incompatible mode elements can produce results in bending problems that are comparable to quadratic elements but at a significantly lower computational cost [115].

As an application example, consider a cantilever steel plate subjected to a transverse load, as shown in Figure 2.34. A linear-elastic static analysis is conducted using Abaqus. The material is defined by a Young's modulus of $E = 210$ GPa and a Poisson's ratio of $\nu = 0.3$. The plate has a length of $a = 100$ mm, a width of $b = 10$ mm, and an out-of-plane thickness of $t = 1$ mm. The transverse load has magnitude of $P = 200$ N. A plane stress condition is assumed and the CPS4 (first-order pure-displacement formulation, susceptible to shear locking), CPS4I (first-order incompatible mode formulation), and CPS8 (second-order pure-displacement formulation) elements are tested. As shown in Figure 2.35, shear locking can be eliminated by using the CPS4I or CPS8 elements.

Volumetric locking is another case where finite elements present higher stiffness than expected. This type of locking is related to (near-)incompressible materials (i.e., $\nu \approx 0.5$). Generally, the deformation of an infinitesimal element can be interpreted as a combination of two components: distortion and dilatation. This leads to the decomposition of stresses into deviatoric and hydrostatic pressure components. Consequently, the strain energy can also be decomposed into two parts. Dilatation, which relates to changes in volume, is usually constrained via the penalty or Lagrange multipliers method so that the incompressible material remains with its original volume. Pure-displacement formulation elements rely on the penalty method, where the penalty parameter is actually the bulk modulus of the material

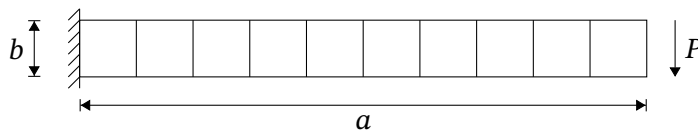


Figure 2.34: Cantilever plate subjected to a transverse load (meshed).

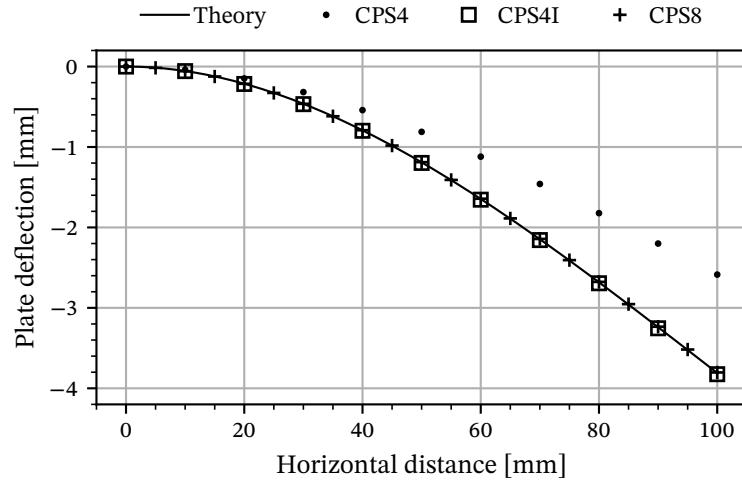


Figure 2.35: Accounting for shear locking via correct element type selection: theoretical (Euler-Bernoulli beam theory) vs. numerical solution of the problem shown in Figure 2.34.

(which approaches infinity as the Poisson's ratio tends to 0.5). Hence, if the element is subjected to a large dilatation, its strain energy tends to infinity, leading to numerical convergence issues and parasitic stiffening. To account for volumetric locking, mixed formulations are usually employed, where the incompressibility constraint is enforced via the Lagrange multipliers method, making the multiplier a new and independent solution variable. In this case, the multiplier is directly related to the hydrostatic pressure in the element. In Abaqus, the so-called hybrid incompressible solid element formulation is available, which is a mixed formulation combined with a selective-integration scheme, where full-integration is used for the evaluation of deviatoric stresses and reduced-integration is used for the hydrostatic pressure evaluation [79], which is known in the literature as the B-bar method or as constant dilatation elements [13].

An alternative way to account for both forms of locking is to use a reduced-integration scheme in regular pure-displacement formulation elements. Since the pure-displacement formulation yields an overestimation of the stiffness, then, by not accurately evaluating the displacement-based element stiffness during numerical integration, one may expect improved results [113]. As K. J. Bathe puts it best, 'this should be the case if the error in the numerical integration compensates appropriately for the overestimation of structural stiffness due to the finite element discretization' [113]. In other words, approximation errors during numerical integration can be used to balance out finite element locking effects. This is very convenient since the computational cost is also reduced by using a reduced-integration scheme, and one may keep the simpler pure-displacement formulation without going into more complex formulations. However, reduced-integration brings a significant downside: the hourglass effect.

The hourglass effect is related to the so-called zero-energy deformation modes. For example, consider the case shown in Figure 2.33: if a reduced-integration scheme were to be used here, a single integration point would be considered at the centroid of the element, lying exactly at the neutral axis of the beam where all components of stress and strain are zero. In that case, this bending mode of deformation would be, in fact, a zero-energy mode because no

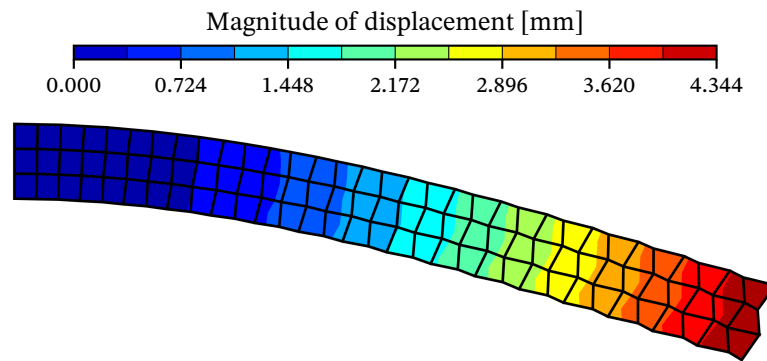


Figure 2.36: The hourglass effect caused by zero-energy modes in reduced-integration elements. Deformation scale factor: 5.

strain energy would be generated. Thus, the element would present no stiffness in this mode, allowing for spurious deformations due to indeterminate nodal displacements. Moreover, this zero-energy mode can propagate throughout the mesh, producing meaningless results [79]. For example, consider the previously presented problem shown in Figure 2.34, however, this time with a more refined mesh of reduced-integration pure-displacement formulation elements (CPS4R in Abaqus); for this case, results are shown in Figure 2.36, where one can see the typical hourglass-shaped elements due to the zero-energy modes that the middle layer elements are subjected to, since their integration points lie exactly along the neutral axis of the plate. The hourglass effect can be reduced by refining the finite element mesh, by using full-integration elements, or by introducing artificial hourglass stiffness in reduced-integration elements (the so-called hourglass control, which is also available in Abaqus and enabled by default).

2.3.3 Nonlinear and nonsmooth finite element analysis

When analysing real-world systems, analyses tend to be nonlinear. There are several types of nonlinearities in mechanical systems, for example [116]:

- **Geometric nonlinearity** represents the general cases where the relationship between kinematic quantities (i.e., strain and displacement) is nonlinear. This type of nonlinearity occurs when large deformations and/or large rotations are considered.
- **Kinematic nonlinearity**, also called **boundary nonlinearity**, represents the cases where boundary conditions depend on the deformation of the structure. For example, when two bodies are squished together, the displacements on the contact area are constrained such that they cannot penetrate each other; however, as deformation increases, so does the contact area, making the analysis nonlinear.
- **Force nonlinearity** represents the cases where load magnitudes and/or directions depend on the deformation of the structure. For example, when a cantilever beam in bending is subjected to a pressure, the load direction depends on the beam deformation since, by definition, a pressure is applied perpendicularly to the loaded surface. Moreover, if

the magnitude of a surface or body load is defined per unit of deformed area or volume, the analysis also becomes nonlinear.

- **Material nonlinearity** represents the general cases where the stress-strain relationship (i.e., the constitutive model) is nonlinear. Nonlinear constitutive models include, for example, elastoplastic and hyperelastic materials.

In general, the incremental force method is used to solve nonlinear problems [116]. In a static analysis, this method starts by considering the base state of the system as the initial candidate solution. The solution is then updated by incrementally increasing the applied loads. To this end, a pseudo-time variable is used, which can be understood as a simple load multiplier that usually ranges from 0 (initial state) to 1 (fully deformed state). Moreover, since the analysis is nonlinear, for each load/time increment, an iterative process based on the Newton-Raphson method [113, 116] is used to guarantee that the candidate solution is converging. Additionally, in more sophisticated solvers, the incrementation process is usually adaptive, i.e., the load/time increment updates depending on if the solution is converging quickly or not converging at all.

Assuming we know the solution at time t , the solution at time $t + \Delta t$ must ensure that the externally applied loads are in equilibrium with the internal forces:

$${}^{t+\Delta t}\mathbf{f}_{\text{ext}} - {}^{t+\Delta t}\mathbf{f}_{\text{int}} = \mathbf{0}. \quad (2.140)$$

Additionally, the iterative procedure based on the Newton-Raphson method is [113]

$$\begin{aligned} \Delta \mathbf{f}^{(k-1)} &= {}^{t+\Delta t}\mathbf{f}_{\text{ext}} - {}^{t+\Delta t}\mathbf{f}_{\text{int}}^{(k-1)}, \\ {}^{t+\Delta t}\mathbf{K}^{(k-1)} \Delta \mathbf{u}^{(k)} &= \Delta \mathbf{f}^{(k-1)}, \\ {}^{t+\Delta t}\mathbf{u}^{(k)} &= {}^{t+\Delta t}\mathbf{u}^{(k-1)} + \Delta \mathbf{u}^{(k)}, \end{aligned} \quad (2.141)$$

with the initial conditions

$${}^{t+\Delta t}\mathbf{u}^{(0)} = {}^t\mathbf{u} \quad \text{and} \quad {}^{t+\Delta t}\mathbf{f}_{\text{int}}^{(0)} = {}^t\mathbf{f}_{\text{int}}. \quad (2.142)$$

The iterative procedure is performed for $k = 1, 2, 3, \dots$ until the out-of-balance load vector $\Delta \mathbf{f}^{(k-1)}$ or the displacement increments $\Delta \mathbf{u}^{(k)}$ are sufficiently small. Moreover, ${}^{t+\Delta t}\mathbf{K}^{(k-1)}$ denotes the current tangent stiffness matrix. In this case, where the matrix is updated every iteration, the full Newton-Raphson iteration scheme is considered, as shown in Figure 2.37a. While this scheme requires fewer iterations until convergence is achieved, each iteration is expensive due to the computation of the tangent stiffness matrix. Alternatively, one may use the same tangent stiffness matrix during the iterative procedure, denoted ${}^t\mathbf{K}$. In this case, the modified Newton-Raphson iteration scheme is considered, as shown in Figure 2.37b, where the tangent stiffness matrix is computed per increment (instead of per iteration), which requires more iterations, but each iteration is also cheaper.

In structural analyses, particularly when modelling contact problems, the solution may become nonsmooth. This happens because the system's behaviour can change abruptly when a previously non-contacting surface starts to interact with another, or when sliding occurs during frictional contact. The contact introduces discontinuities which require more advanced

2: Literature review

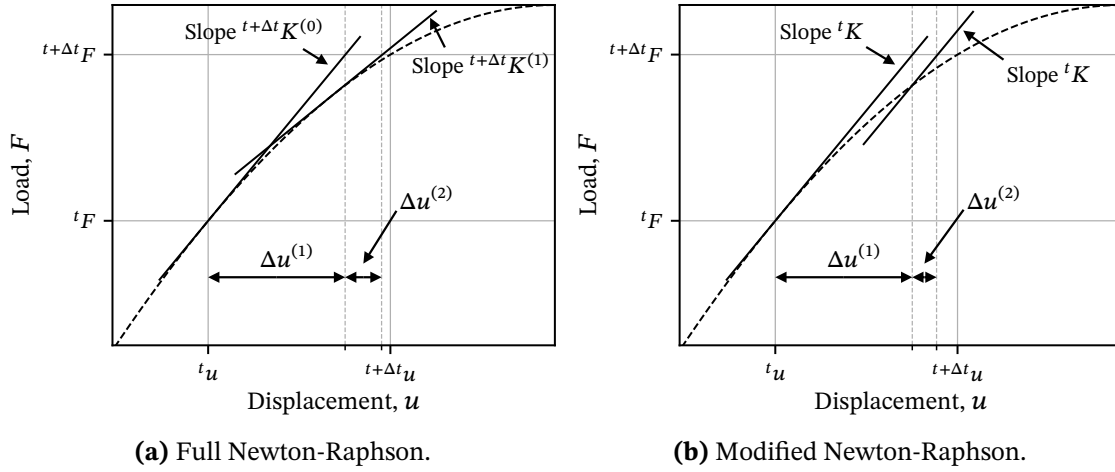


Figure 2.37: Illustration of the incremental force method with Newton-Raphson iterations.

numerical techniques to handle the transitions. A basic example is illustrated in Figure 2.38, in which both normal and tangential contact are considered.

In commercially available finite element software, contact constraints are enforced via the penalty method or other approaches based on Lagrange multipliers, where the latter are computationally more expensive, but also more accurate. Besides the enforcement of contact constraints, a contact discretization algorithm is also required, where the concepts of main/master surface and corresponding secondary/slave surface are introduced. In traditional node-to-surface contact, slave nodes are constrained from penetrating into the master surface, while the master surface is allowed to penetrate into the slave surface, as shown in Figure 2.39. Because of this, it is important to discretize the slave surface with a finer finite element mesh in comparison to the master surface; for illustrative purposes, this is not done in Figure 2.39. When considering surface-to-surface contact, contact conditions are enforced in an average sense over the slave surface. In this case, small nodal penetrations may still occur, but large

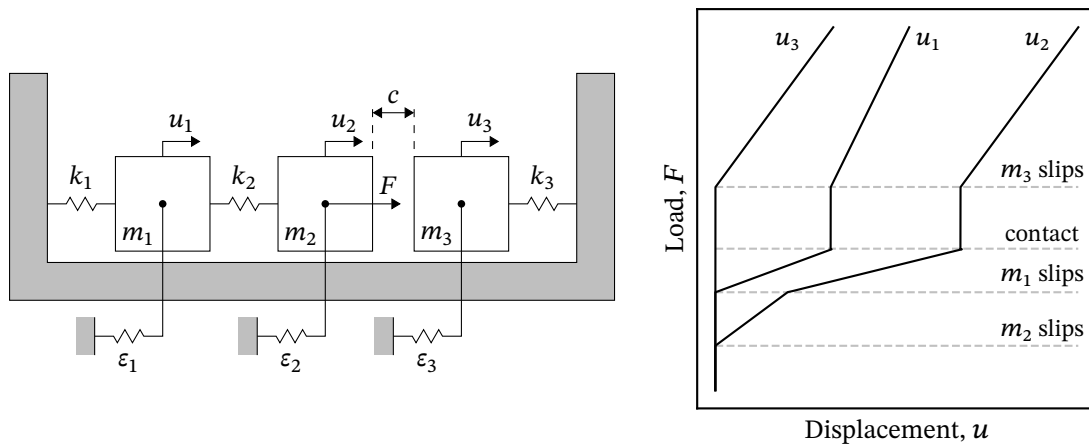


Figure 2.38: Example of a nonlinear and nonsmooth 1D contact problem.

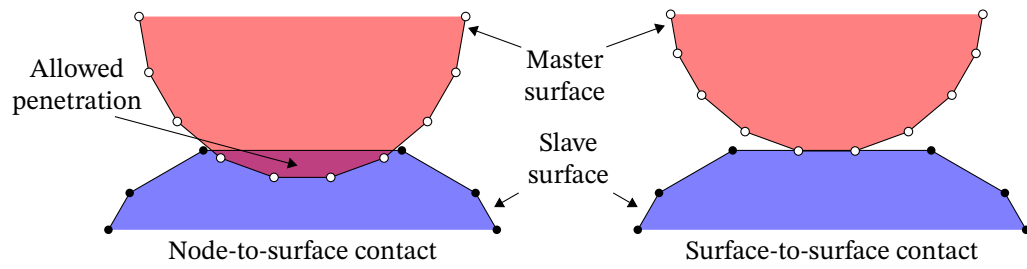


Figure 2.39: Node-to-surface vs. surface-to-surface contact.

surface penetrations are prevented. Ultimately, the surface-to-surface contact discretization is generally the preferred approach; however, it comes with an increased computational cost. Finally, a contact tracking algorithm is also required when sliding occurs between contact pairs.

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Chapter 3

Mechanical testing of the plain materials

This chapter presents the empirical information regarding the materials considered throughout this work, namely, the S350GD and the S355MC mild steel grades. The S350GD is a zinc-coated cold-rolled steel, whereas the S355MC is an uncoated thermomechanically rolled, i.e., hot-rolled, steel. Additionally, according to their respective technical delivery conditions [117, 118], Table 3.1a presents the main mechanical properties of both steel grades, whereas Table 3.1b presents their chemical composition.

The S350GD zinc coating is a result of hot-dip galvanization. The coating thickness was measured from a single plate sample. The plate was cut and the resulting exposed surface was polished in preparation for measuring using a scanning electron microscope (SEM), as shown in Figure 3.1. According to 45 measurements, the average zinc coating thickness is 13.65 μm with a standard deviation of 4.07 μm .

Table 3.1: Technical delivery conditions of the S350GD and S355MC steel grades [117, 118].

(a) Main mechanical properties.

Steel grade	$R_{p0.2}$ [MPa] min.	R_{eH} [MPa] min.	R_m [MPa] min.	R_m [MPa] max.
S350GD	350	–	420	–
S355MC	–	355	430	550

(b) Chemical composition in percentage by mass.

Steel grade	C % max.	Mn % max.	Si % max.	P % max.	S % max.	Al % min.	Nb % max.	V % max.	Ti % max.
S350GD	0.20	1.70	0.60	0.10	0.045	–	–	–	–
S355MC	0.12	1.50	0.50	0.025	0.020	0.015	0.09	0.20	0.15

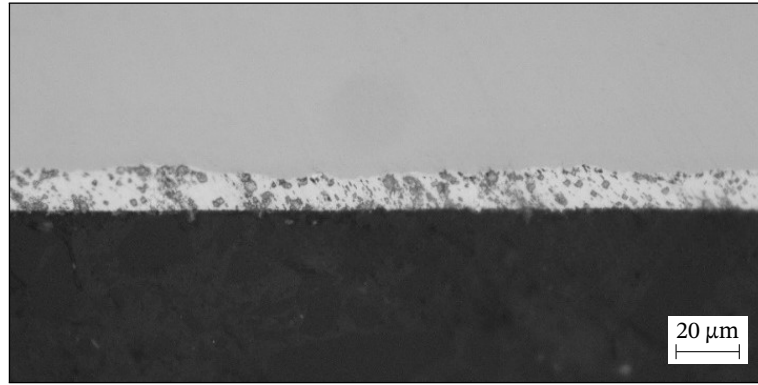


Figure 3.1: S350GD zinc coating under a scanning electron microscope.

3.1 Static tensile strength testing

Quasi-static tensile strength testing was conducted during the FASTCOLD project for both the S350GD and S355MC steel grades. Figure 3.2 presents the tensile yield curves of two dog-bone type specimens of S350GD. As discussed in Section 2.1.2, Figure 3.2 shows that the S350GD steel does not present a yield plateau since it is a cold-rolled steel, i.e., it has been pre-strained during manufacturing.

To allow for the material definition in the forthcoming numerical models, a strain hardening model was fitted to the available experimental data. The equation shown in (2.34) is chosen since it includes a parameter, ϵ_0 , pertaining to the amount of material pre-strain. The fit was carried out using the SciPy library for scientific computing [119]. The fit process is treated as a nonlinear least squares problem, which is solved via unconstrained minimization; more specifically, the Levenberg-Marquardt algorithm is employed [120, 121]. The data used for the fitting process is shown in Figure 3.2b, where the necking region is discarded. The obtained

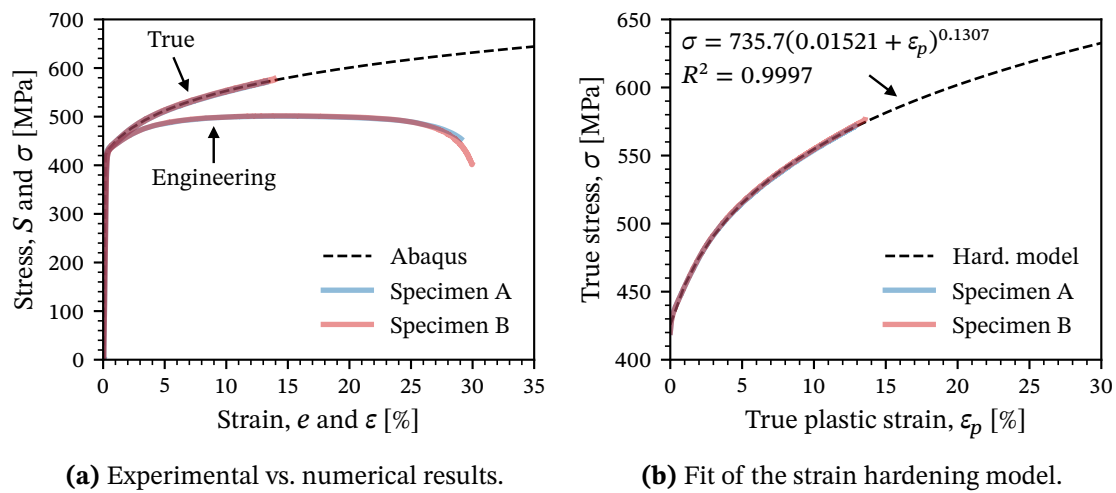


Figure 3.2: Tensile yield curves of S350GD steel.

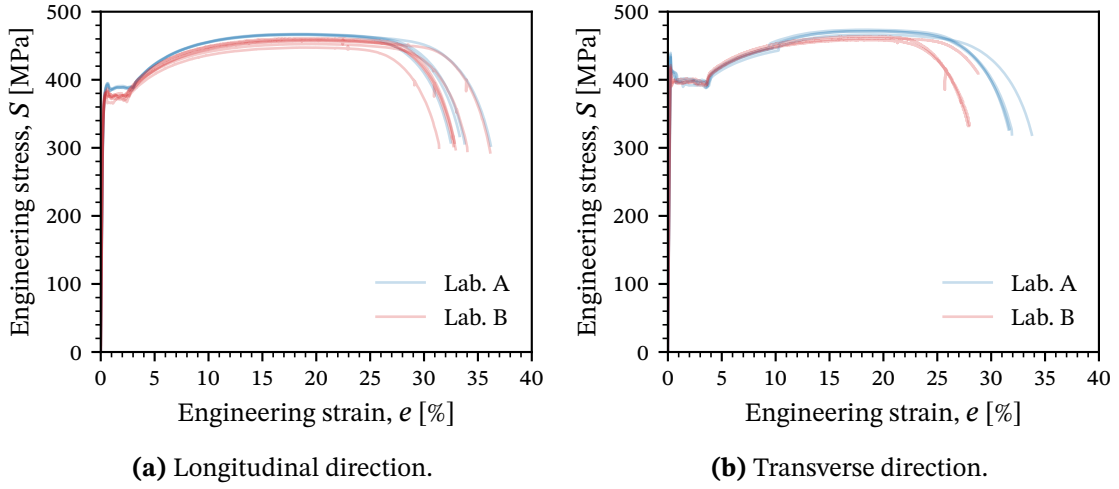


Figure 3.3: Nominal tensile yield curves of S355MC steel. Specimens taken from both the longitudinal and transverse directions.

model parameters are $K = 735.7$ MPa, $n = 0.1307$, and $\varepsilon_0 = 0.01521$, resulting in a coefficient of determination of $R^2 = 0.9997$. Regarding the elastic portion of the stress-strain curves, a Young's modulus of approximately 210 GPa was found by fitting a linear stress vs. strain model to the available data.

Regarding the S355MC steel, it was tested along the longitudinal (i.e., rolling) and transverse directions. The tests were performed at two laboratories, both partners within the FAST-COLD project: RWTH Aachen University (laboratory A) tested specimens taken directly from the sheet metal coil (virgin material), while University of Porto (laboratory B) tested specimens taken from the flat region of a cold roll-formed Z-profile. Laboratory A tested 4 specimens for each direction, whereas laboratory B tested 6 specimens for the longitudinal direction and 3 specimens for the transverse direction. A total of 17 specimens were tested. The results are shown in Figure 3.3, which reveals that the S355MC steel presents a yield plateau, as expected from a hot-rolled mild steel (as discussed in Section 2.1.2); moreover, the material presents a minor anisotropic behaviour, especially within the yield plateau region.

For the material definition, a strain hardening model was fitted to the available experimental data from laboratory A (virgin material), considering the results from the longitudinal (i.e., rolling) direction. A modified version of the hardening model shown in (2.33) is considered in order to account for the yield plateau of the material:

$$\sigma = \begin{cases} \sigma_y & \text{if } \varepsilon_p \leq \varepsilon_p^0, \\ K\varepsilon_p^n & \text{otherwise,} \end{cases} \quad (3.1)$$

where ε_p^0 corresponds to the plastic strain value at the end of the yield plateau. The fit was again carried out using the SciPy library. The data used for the fitting process is shown in Figure 3.4b, where the necking region is discarded. The obtained model parameters are $K = 745.4$ MPa, $n = 0.1648$, $\sigma_y = 394.5$ MPa, and $\varepsilon_p^0 = 0.0225$, resulting in a coefficient of determination of $R^2 = 0.9992$. Regarding the elastic portion of the stress-strain curves, a Young's modulus

3: Mechanical testing of the plain materials

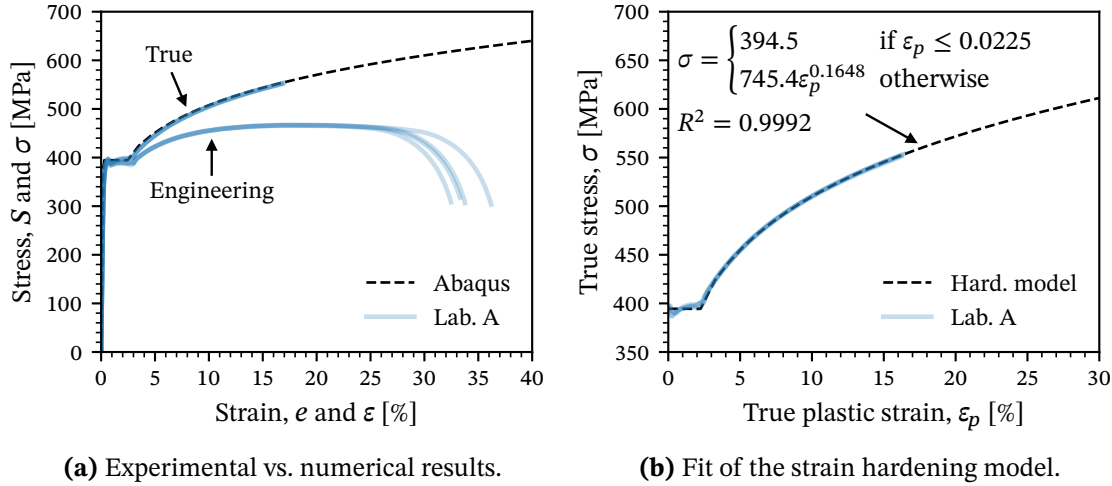


Figure 3.4: Tensile yield curves of S355MC steel.

of approximately 200 GPa was found by fitting a linear stress vs. strain model to all available data.

In order to validate the material definition, an uniaxial test was simulated in Abaqus for both the S350GD and S355MC steel grades, where a single C3D8R element was used (a trilinear hexahedral solid element with 8 nodes and 1 integration point). As shown in Figure 3.2a and Figure 3.4a, a very good match is visible between the numerical and experimental data.

3.2 Low-cycle fatigue testing

Strain-controlled fatigue tests were performed on smooth and polished specimens of S355MC steel in order to characterize the LCF behaviour of the nominally homogeneous material via the experimental determination of fatigue properties. The cyclic stress-strain curve is also derived from experiments, allowing for an investigation on the cyclic stability of the material, i.e., whether the material will harden, soften, or remain stable due to cyclic plastic straining.

The adopted testing methodology is the one proposed in the ASTM E606 standard [84]. The ASTM E739 standard procedure [66] is used for the subsequent statistical analysis. An INSTRON 8801 servohydraulic testing system (rated up to 100 kN) is used with an INSTRON 2620-602 dynamic extensometer (12.5 mm gauge length with ± 2.5 mm travel) for direct strain measurement and closed loop strain control. The knife-edge probes of the extensometer are attached to the specimen via o-rings acting as elastic retainers. The specimens were uniaxially loaded at room temperature and in air. Special care was taken when clamping the specimens to ensure that their major axis closely coincided with the load axis, minimizing bending and shear strains. For this purpose, lines were drawn in the specimens to assist in the alignment. The testing setup is shown in Figure 3.5.

The geometry of the specimens, which is within the specifications of the ASTM E606 standard [84], is shown in Figure 3.6. The width (w) and thickness (t) of the reduced test section were measured prior to testing using callipers. The measurements are shown in Table 3.2.

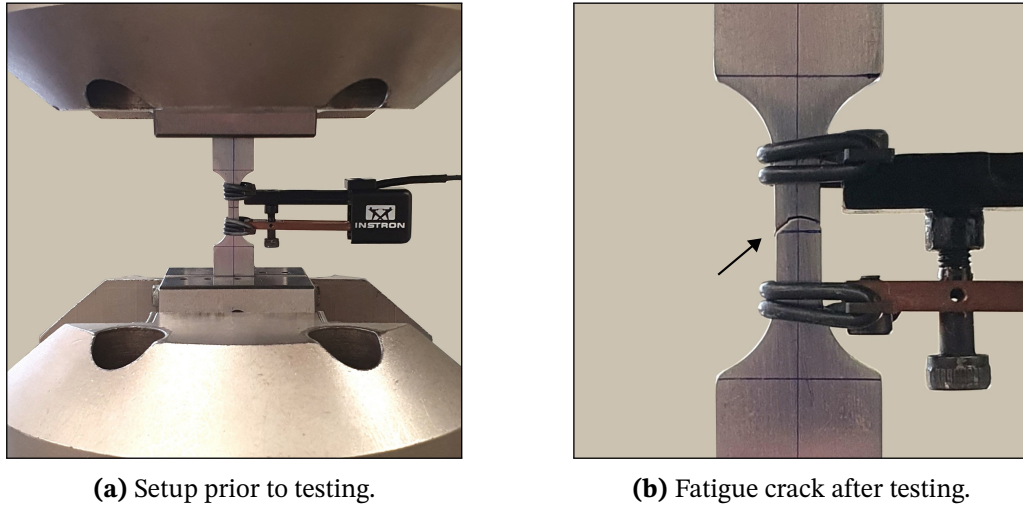


Figure 3.5: Experimental LCF testing setup.

To minimize possible surface effects, all surfaces of the specimens were polished using an appropriate sequence of sanding paper grits.

The specimens were extracted from the flat region of a cold roll-formed thin-walled Z-profile (3.5 mm of material thickness), avoiding the cold-formed corners due to the presence of residual stresses. A total of 10 specimens were tested, where half were extracted in alignment with the longitudinal direction of the profile, while the remaining specimens were extracted from the transverse direction.

LCF tests were performed for a strain ratio equal to $R_e = 0.1$, following a sinusoidal waveform whose frequency was adjusted to maintain a constant average strain rate (which is twice the product of frequency and strain range, $de/dt = 2f\Delta e$) of 0.8 %/s [20]. In accordance with the ASTM E606 standard [84], constant average strain rate testing was chosen since it is often preferred to constant frequency testing. Table 3.2 summarizes the parameters adopted for each test. According to the sampling recommendations found in the ASTM E739 standard [66], the presented testing campaign falls within the ‘research and development testing of components and specimens’ category, as a replication of 40 % is verified, defined as

$$\% \text{ replication} = 100 \times \left(1 - \frac{\text{total number of different stress or strain levels used in testing}}{\text{total number of specimens tested}} \right).$$

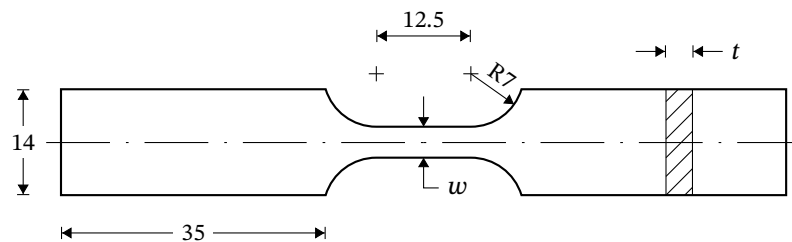


Figure 3.6: Geometry of the smooth specimens used in the LCF experiments.

3: Mechanical testing of the plain materials

Table 3.2: Measurements of the test section and LCF test parameters for each specimen. Note: the letters L and T identify longitudinal and transverse specimens, respectively.

Specimen	w [mm]	t [mm]	A [mm ²]	R_e [–]	de/dt [%/s]	Δe [%]	f [Hz]
LCF-L01	3.98	3.50	13.93	0.10	0.80	0.50	0.80
LCF-L02	3.95	3.50	13.83	0.10	0.80	1.00	0.40
LCF-L03	4.00	3.50	14.00	0.10	0.80	2.00	0.20
LCF-L04	3.97	3.47	13.78	0.10	0.80	0.80	0.50
LCF-L05	3.95	3.55	14.02	0.10	0.80	0.40	1.00
LCF-T01	3.99	3.52	14.04	0.10	0.80	0.50	0.80
LCF-T02	3.98	3.49	13.89	0.10	0.80	1.00	0.40
LCF-T03	3.95	3.50	13.83	0.10	0.80	2.00	0.20
LCF-T04	3.97	3.50	13.90	0.10	0.80	0.65	0.62
LCF-T05	4.00	3.55	14.20	0.10	0.80	0.40	1.00

In all instances, testing was conducted until full separation of the specimen was observed. Moreover, fatigue cracks initiated and propagated within the test region, as shown in [Figure 3.5b](#). All tests were deemed valid and no run-outs were assumed.

Since testing was conducted under strain-controlled conditions, the exact fatigue life was determined by analysing the evolution of maximum stress during load cycling. As shown in [Figure 3.7](#), a decrease in the maximum stress value is verified as the number of load cycles increases. Eventually, a sudden drop of this value indicates that a complete through-crack has formed. Based on this criterion, [Table 3.3](#) shows the obtained fatigue lives.

[Figure 3.8a](#) presents the cyclic stress-strain hysteresis loops registered during testing. Some cycles are omitted for a better visualization of the cyclic behaviour, namely, the first cycle corresponding to the initial yield behaviour, as well as the last few cycles corresponding to an imminent specimen separation.

[Figure 3.8b](#) presents the stabilized hysteresis loops, which are artificially centred at the origin for visualization purposes. The half-life criterion is considered, which may coincide

Table 3.3: Main LCF results obtained for the tested S355MC steel grade and derived from the pseudo-stabilized hysteresis loops.

Specimen	Δe [%]	Δe_e [%]	Δe_p [%]	ΔS [MPa]	S_m [MPa]	N_f [Cycles]
LCF-L01	0.50	0.283	0.217	540.8	5.29	2.31×10^4
LCF-L02	1.00	0.372	0.628	691.1	8.51	1.53×10^3
LCF-L03	2.00	0.444	1.556	750.1	3.15	5.90×10^2
LCF-L04	0.80	0.375	0.425	604.0	18.6	5.68×10^3
LCF-L05	0.40	0.271	0.129	483.6	11.5	3.84×10^4
LCF-T01	0.50	0.308	0.192	558.3	18.2	1.42×10^4
LCF-T02	1.00	0.380	0.620	684.1	9.19	2.78×10^3
LCF-T03	2.00	0.435	1.565	769.1	1.79	4.90×10^2
LCF-T04	0.65	0.326	0.324	603.6	9.30	7.92×10^3
LCF-T05	0.40	0.264	0.136	515.5	8.01	3.71×10^4

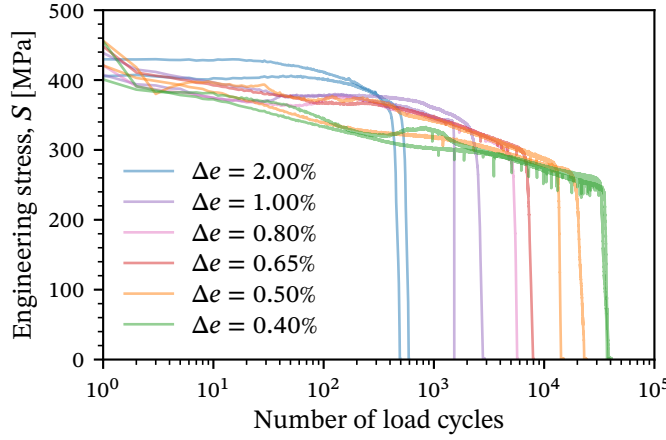


Figure 3.7: Evolution of maximum stress during load cycling.

with a stabilized cyclic behaviour criterion for the cases that showed stabilization; otherwise, the half-life criterion corresponds to a pseudo-stabilized behaviour [20]. Figure 3.7 reveals stabilization for the two largest strain range values considered and controlled during testing, whereas no stabilization of the cyclic behaviour is observed for the remaining cases, as the maximum stress value tends to decrease during cycling.

Using the pseudo-stabilized hysteresis loops, while disregarding the aforementioned artificial translation made for visualization purposes, the elastic and plastic strain ranges were computed. The plastic strain range was determined as the width of the corresponding hysteresis loop for when the load magnitude crosses the horizontal axis, i.e., for $S = 0$. The elastic strain range was then computed from the total strain range decomposition into elastic and plastic components. Moreover, the pseudo-stabilized hysteresis loops were also used to determine the stress range and mean stress values. These results are shown in Table 3.3.

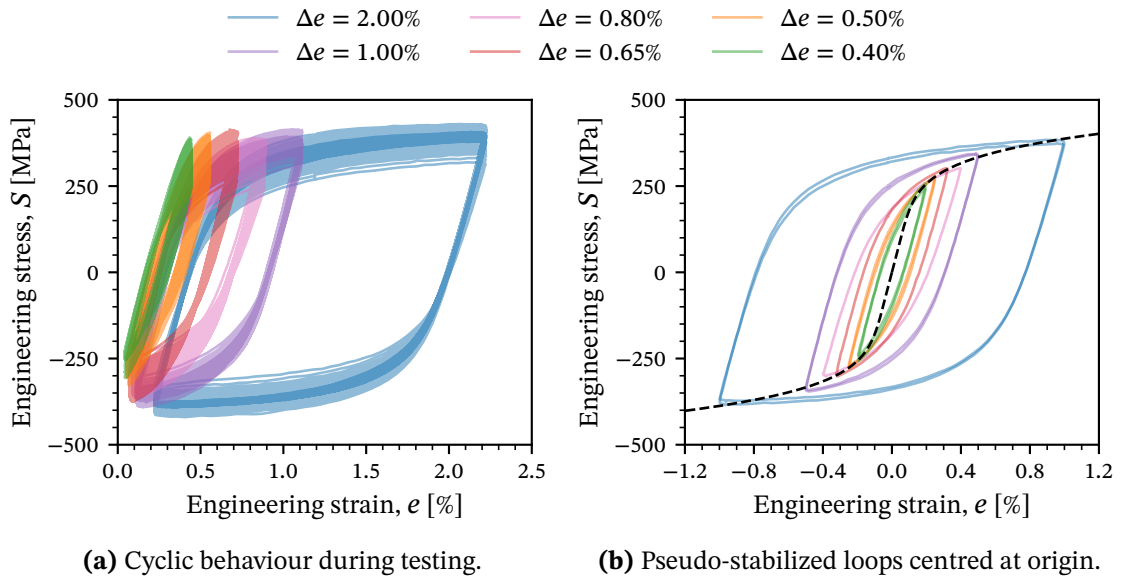


Figure 3.8: Cyclic stress-strain hysteresis loops for the tested S355MC steel grade.

3: Mechanical testing of the plain materials

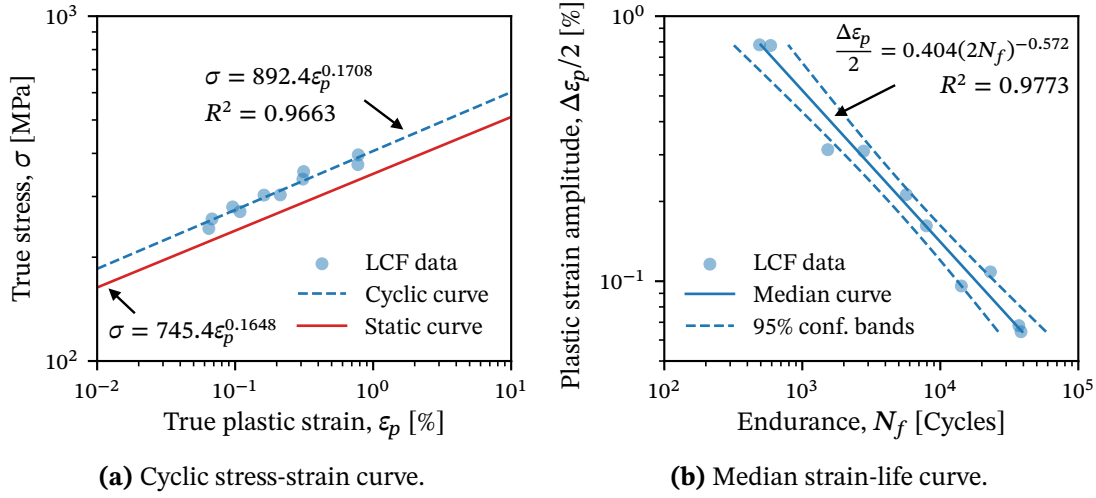


Figure 3.9: Cyclic stress-strain and median strain-life curves derived from LCF test results.

Using Table 3.3, a cyclic stress-strain curve can be derived from the obtained LCF test data. The fitting process is identical to the one described in Section 3.1. The classical power law shown in (2.33) is chosen as the hardening model. Results are shown in Figure 3.9a, where the previously fitted static stress-strain curve is also plotted for comparison. From fitting, the parameters $K = 892.4$ MPa and $n = 0.1708$ are obtained ($R^2 = 0.9663$). By comparing the static and cyclic curves, one may realize that they are close to parallel, i.e., they share a similar hardening exponent: $n = 0.1648$ (static) vs. $n = 0.1708$ (cyclic). However, the cyclic curve presents a larger hardening coefficient: $K = 745.4$ MPa (static) vs. $K = 892.4$ MPa (cyclic); and this difference mainly arises from the fact that the virgin material shows a yield plateau which is then absent in the load-cycled material. In other words, the material appears to harden after load cycling, but hardening only occurs within the initial load cycle, after which the material softens until fracture. Additionally, Figure 3.8b presents another representation of the cyclic curve, where it crosses the tips of the pseudo-stabilized hysteresis loops. However, this second cyclic curve differs slightly from the one presented in Figure 3.9a, as the former is based on engineering measures of stress and strain, while the latter is based on true measures.

As detailed in Section 2.2.3.5, the ASTM E739 standard practice [66] is used to generate a median strain-life curve ($\Delta\epsilon_p/2$ vs. N_f). Results are shown in Figure 3.9b, where a good coefficient of determination is verified ($R^2 = 0.9773$). Moreover, as presented, the median strain-life curve corresponds to the Coffin-Manson model, from which the fatigue ductility coefficient and fatigue ductility exponent can be obtained: $\epsilon'_f = 0.4042$ and $c = -0.5715$.

Figure 3.10 presents a direct comparison of the obtained experimental data against results extracted from the literature, in which similar strain-controlled low-frequency fatigue testing campaigns are performed for similar S355 steel grades but distinct tested strain ratios: De Jesus et al. (2012) [20] considered $R_e = -1$, Carvalho et al. (2015) [122] considered $R_e = 0$, and $R_e = 0.1$ is considered in this work. In all cases, a constant average strain rate of $d\epsilon/dt = 0.8$ %/s is considered during testing. The figure shows that results are similar for the distinct testing campaigns, revealing that a mean strain effect is not captured for the considered material, at least not within the considered LCF regime. Furthermore, a median strain-life curve corres-

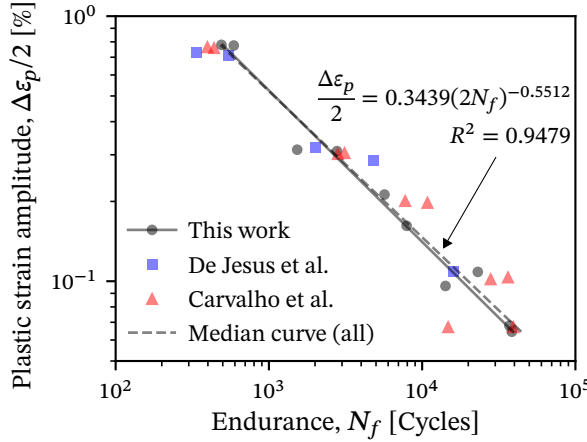


Figure 3.10: Comparison of the obtained LCF test data against results from the literature for similar S355 steel grades.

ponding to the Coffin-Manson model is generated considering all the available data, resulting in a fatigue ductility coefficient of $\epsilon'_f = 0.3439$, a fatigue ductility exponent of $c = -0.5512$, and a coefficient of determination of $R^2 = 0.9479$.

3.3 High-cycle fatigue testing

In the previous section, the LCF behaviour of the considered S355MC mild steel was successfully characterized via strain-controlled low-frequency fatigue testing. However, it is now necessary to investigate the material's HCF behaviour. While the LCF behaviour is generally described via a plastic strain vs. endurance relationship, the HCF behaviour is usually described via a stress vs. endurance relationship. From the LCF test results shown in Table 3.3, it is already possible to construct a stress vs. endurance relationship, in which a linear-elastic constitutive model can be used to compute the stress given the obtained elastic strain range derived from the pseudo-stabilized stress-strain hysteresis loops. Nevertheless, such results are still within the LCF range, where the observed fatigue lifetimes are within 490 and 38,450 load cycles. Hence, some HCF test data is required to at least confirm that the tendencies observed within the LCF region are still valid for HCF.

To assist in the characterization of the HCF behaviour of the considered S355MC material, load-controlled high-frequency fatigue tests were performed on smooth and polished specimens. The adopted testing methodology is the one proposed in the ASTM E466 standard [83]. As in the previous LCF testing campaign, the ASTM E739 standard procedure [66] is used for the subsequent statistical analysis. A RUMUL TESTRONIC high-frequency resonant testing machine (rated up to 100 kN) is used. The specimens were uniaxially loaded at room temperature and in air. The testing setup is shown in Figure 3.11, whereas the geometry of the specimens, which is within the specifications of the ASTM E466 standard, is shown in Figure 3.12. The holes at the ends of the specimens are used for alignment purposes only and the alignment pin is removed before loading. This ensures that the major axis of the specimen coincides with the load axis, minimizing bending and shear strains. Furthermore, the width (w) and thickness (t) of the reduced test cross-section were measured prior to testing using callipers. The measurements are shown in Table 3.4. Finally, to minimize any possible surface effects, all surfaces of the specimens were polished using an appropriate sequence of

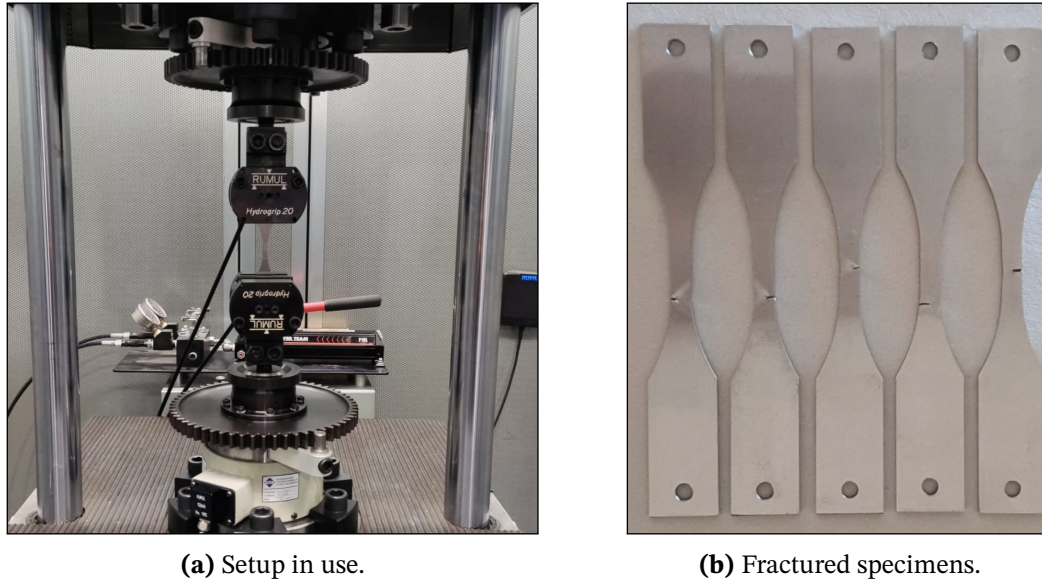


Figure 3.11: Experimental HCF testing setup.

sanding paper grits, which was also done for the LCF specimens.

As in the previous LCF testing campaign, the HCF test specimens were extracted from the flat region of a cold roll-formed thin-walled Z-profile, avoiding the cold-formed corners due to the presence of residual stresses. A total of 9 specimens were tested, where all were extracted in alignment with the longitudinal direction of the profile.

HCF tests were performed considering a stress ratio of $R_S = 0.1$, which is a natural extension to the linear regime of the strain ratio value of $R_e = 0.1$ considered during the previous LCF testing campaign. Loading was applied following a constant amplitude sinusoidal waveform whose frequency is related to a resonant frequency of the system (~ 100 Hz), within the typical range where frequency effects are negligible [83]. Table 3.4 summarizes the parameters adopted for each test. Testing was conducted until a visible crack formed within the test region (see Figure 3.11b); however, some run-outs were verified. Results are shown in Table 3.4.

As previously discussed, from the LCF test results shown in Table 3.3, a stress vs. endurance relationship can be constructed, in which the obtained elastic strain range derived from the pseudo-stabilized stress-strain hysteresis loops is converted to stress via Hooke's law. Results are shown in Figure 3.13, where a good coefficient of determination is obtained for the

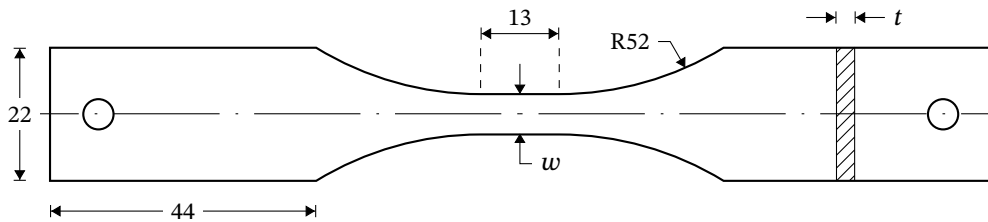


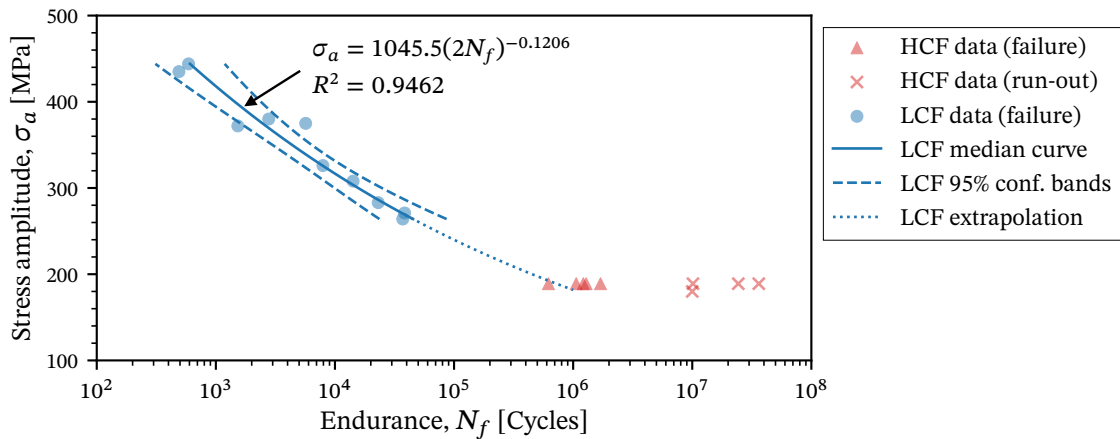
Figure 3.12: Geometry of the smooth specimens used in the HCF experiments.

Table 3.4: HCF test parameters, main results, and measurements of the test section.

Spec.	w [mm]	t [mm]	A [mm ²]	R_S [-]	ΔS [MPa]	S_m [MPa]	N_f [Cycles]	Run-out
HCF-1	6.63	2.95	19.56	0.1	360	220	1.0×10^7	✓
HCF-2	6.69	3.00	20.07	0.1	378	231	3.6×10^7	✓
HCF-3	6.64	2.98	19.79	0.1	378	231	1.1×10^6	
HCF-4	6.61	3.02	19.96	0.1	378	231	6.2×10^5	
HCF-5	6.56	3.00	19.68	0.1	378	231	1.2×10^6	
HCF-6	6.62	3.00	19.86	0.1	378	231	1.7×10^6	
HCF-7	6.62	2.99	19.79	0.1	378	231	2.4×10^7	✓
HCF-8	6.60	2.96	19.54	0.1	378	231	1.0×10^7	✓
HCF-9	6.67	3.00	20.01	0.1	378	231	1.3×10^6	

generated median stress-life curve (σ_a vs. N_f), which was obtained via the ASTM E739 standard practice [66]. Nevertheless, these results are still based on the LCF testing campaign. However, if the median stress-life curve, which corresponds to the Basquin model, is extrapolated to the HCF region, it coincides nicely with the obtained HCF data, as shown in Figure 3.13, indicating that results based on LCF testing are still valid within the HCF regime. Moreover, a fatigue strength coefficient of $\sigma'_f = 1045.5$ MPa and a fatigue strength exponent of $b = -0.1206$ are verified ($R^2 = 0.9462$).

It should be noted that results shown in Figure 3.13 should be corrected in terms of mean stress. For the LCF test data, the mean stress values are extracted from the corresponding pseudo-stabilized stress-strain hysteresis loops; results are shown in Table 3.3. For the HCF test data, the mean stress values are easily computed based on the considered stress ratio; results are shown in Table 3.4. The Gerber model for mean stress correction is considered, as detailed in Section 2.2.2 and shown in (2.54). The Gerber model is chosen since it is generally the most adequate for ductile alloys subjected to tensile mean stresses [2], as is the case of the

**Figure 3.13:** Extrapolation of LCF results into the HCF regime and comparison with actual HCF test data.

3: Mechanical testing of the plain materials

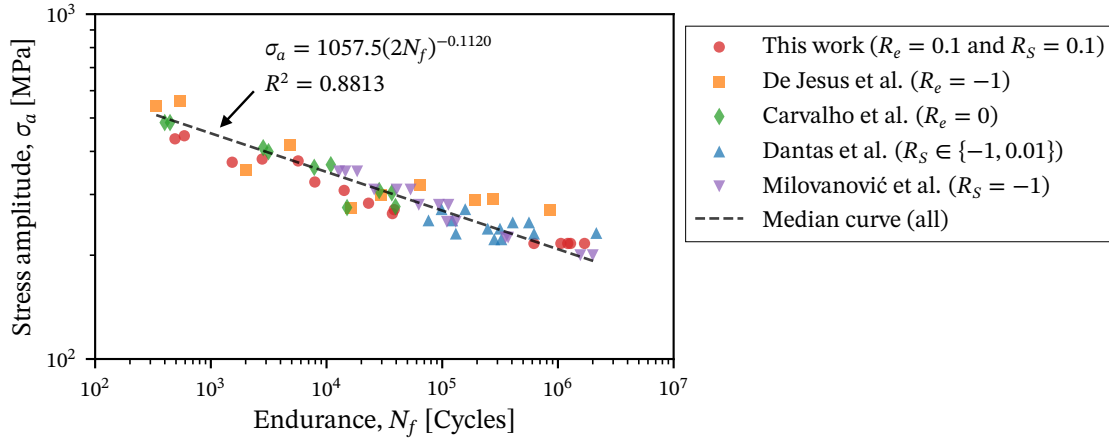


Figure 3.14: Final median stress-life curve and comparison of the obtained test data against results from the literature for similar S355 steel grades. Data is mean stress corrected.

investigated S355MC material. Final results are shown in Figure 3.14, where the mean stress correction is applied. In this figure, LCF and HCF results are joined together and run-outs are discarded.

To complement the obtained experimental data, results extracted from the literature are also included in Figure 3.14. All data pertains to similar S355 steel grades tested via constant amplitude uniaxial loading of smooth and polished specimens. De Jesus et al. (2012) [20] performed strain-controlled tests while considering a strain ratio of -1 . Carvalho et al. (2015) [122] also performed strain-controlled tests, but this time considering a strain ratio of 0 . Dantas et al. (2021) [123] performed load-controlled tests while considering a stress ratio of -1 and 0.01 . Milovanović et al. (2022) [124] also performed load-controlled tests while considering a stress ratio of -1 . In all cases, run-outs are discarded, and a mean stress correction is applied whenever necessary. The ASTM E739 standard practice [66] is used to generate the final median stress-life curve while considering all the available data. A very good fit is verified, resulting in the following Basquin constants: $\sigma'_f = 1057.5$ MPa and $b = -0.1120$ ($R^2 = 0.8813$).

3.4 Summary of the obtained material constants

A final synopsis of the empirically determined material constants for the investigated S350GD and S355MC mild steel grades is presented in Table 3.5. Further details are of course presented above.

3.4: Summary of the obtained material constants

Table 3.5: Summary of the empirically determined material constants for the investigated S350GD and S355MC mild steel grades.

(a) Quasi-static hardening properties (stress vs. plastic strain models).

Case	Model	K [MPa]	n [-]	ε_0 [-]	σ_y [MPa]	ε_p^0 [-]	R^2 [%]
S350GD, static (Figure 3.2b)	$\sigma = K(\varepsilon_0 + \varepsilon_p)^n$	735.7	0.1307	0.01521	-	-	99.97
S355MC, static (Figure 3.4b)	$\sigma = \begin{cases} \sigma_y & \text{if } \varepsilon_p \leq \varepsilon_p^0 \\ K\varepsilon_p^n & \text{otherwise} \end{cases}$	745.4	0.1648	-	394.5	0.02250	99.92
S355MC, cyclic (Figure 3.9a)	$\sigma = K\varepsilon_p^n$	892.4	0.1708	-	-	-	96.63

(b) Low- and high-cycle fatigue properties (Basquin and Coffin-Manson models).

Case	Model	ε'_f [-]	c [-]	σ'_f [MPa]	b [-]	R^2 [%]
S355MC, LCF (Figure 3.10)	$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f(2N_f)^c$	0.3439	-0.5512	-	-	94.79
S355MC, HCF (Figure 3.14)	$\sigma_a = \sigma'_f(2N_f)^b$	-	-	1057	-0.1120	88.13

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Chapter 4

Fatigue behaviour of bolted joints used in rack structures

As mentioned in [Section 2.2](#), corrosion fatigue is related to the change in the chemical composition of a material subjected to harsh environments during load cycling, resulting in a more brittle, oxidized material that is prone to cracking. To mitigate the effects of corrosion fatigue, corrosion protection techniques are required, which constitute an important aspect of structural and fatigue design. In this regard, cold-formed, thin-walled, mild steel profiles are often supplied with zinc coatings for corrosion protection, where a layer of zinc is metallurgically bonded to the base metal by using a hot-dip galvanization process. This technique is relatively recent but widely used, presenting several economic and environmental advantages [\[125\]](#). In regard to metal fatigue, it has been shown that the metallurgical bond can affect the crack initiation site and the fatigue behaviour of the zinc-coated steel [\[125, 126\]](#). Berto et al. (2016) [\[127, 128\]](#) have shown a very limited reduction in the fatigue life of thick bolted joints made of galvanized mild steel when compared to uncoated joints. Kashyzadeh et al. (2013-2017) [\[129, 130\]](#) studied the effect of coating thickness on the fatigue life of galvanized medium carbon steel, and a significant reduction rate of the fatigue endurance for an increasing coating thickness was found; moreover, the galvanized coating was found to be the most appropriate, among others investigated, for small coating thicknesses, which then becomes too brittle for higher thicknesses.

Rack structures are characterized by the application of cold-formed, thin-walled, possibly galvanized, mild steel members and snug-tight bolted connections using medium strength bolts. These connections act mainly as shear splices in bearing, whose behaviour is influenced by the low-friction zinc coatings and snug-tight bolts. Even when using preloaded bolts, the reduced plate thicknesses and low friction coefficients may result in a limited slip resistance, which may be further reduced in time due to bolt preload relaxation [\[131, 132\]](#). Gomes et al. (2019) [\[133\]](#) researched the monotonic and failure behaviours of bolted joints made of thin plates of S350GD and S355MC mild steel grades applied for rack structures, where the former is a zinc-coated steel obtained by hot-dip galvanization. By performing monotonic slip tests and monotonic static tests, while considering distinct preload levels, bolt configurations, and coatings (no coating, zinc, and paint), the zinc-coated S350GD presented consistently the lowest friction coefficients. The researchers also showed that, for each tested configuration, the monotonic behaviour is distinct, and FEA can be successful in simulating the load-

4: Fatigue behaviour of bolted joints used in rack structures

displacement relationship and in identifying the monotonic failure modes.

In this chapter, the fatigue behaviour of double shear bolted joints made of thin zinc-coated S350GD steel plates is investigated. This investigation is made possible by the publicly available experimental results from the FASTCOLD EU research project, which are used as a starting point. Several parameters were considered during the FASTCOLD's testing campaign: (a) number of bolts used and their arrangement; (b) preload level of the bolts; (c) manufacturing process of the bolt holes; (d) applied stress ratio; and (e) plate cross-section size (width).

The main objective of this chapter is to use global approaches based on $S-N$ curves to compare and discuss the results obtained for the different tested cases, using either gross or net nominal stresses when adequate. The obtained $S-N$ data is also plotted against fatigue design curves from the current version of Eurocode 3 and possible upcoming versions. Additionally, two new state-of-the-art approaches are also investigated [11, 12]: the first is similar to the classical global $S-N$ approach, but it considers peak pseudo-elastic maximum principal stresses, which include all sources of stress concentration and are obtained via elastic numerical analysis; the second is a local approach based on the Basquin-Coffin-Manson model with mean stress correction used in conjunction with elastoplastic numerical analysis, creating a predictive fatigue model as an alternative to the aforementioned regression $S-N$ approaches. The obtained failure modes and fracture surfaces are also presented and discussed.

4.1 Experimental details

All tested specimens were made of thin plates of zinc-coated S350GD cold-rolled mild steel (plate thickness of 2 mm). The available empirical information regarding the considered material is presented in [Chapter 3](#).

[Table 4.1](#) presents the fatigue testing matrix used in the FASTCOLD project, where two

Table 4.1: Fatigue testing matrix for the bolted joints.

Bolt type	Bolt array	Preload level	Hole production	Load ratio	Number of tests	Max. load [kN]	Notes
M12	1+1	Snug-tight	Punched	0.1	13	18, 22, 26	A, B
M12	1+1	Snug-tight	Punched	0.3	4	26	B
M12	1+1	Snug-tight	Punched	0.5	4	26	B
M12	1+1	Snug-tight	Punched	0.1	4	26	C
M12	1+1	Preloaded	Punched	0.1	12	19.5, 22, 26	–
M12	4+4	Snug-tight	Punched	0.1	8	50, 60	–
M12	4+4	Snug-tight	Drilled	0.1	8	50, 60	–
M16	1+1	Snug-tight	Punched	0.1	2	22, 28	D
M16	1+1	Preloaded	Punched	0.1	2	28, 34	D
M16	4+4	Snug-tight	Punched	0.1	3	35, 45, 70	D
M16	4+4	Preloaded	Punched	0.1	1	77	D

^A Reference test conditions.

^B Included in the reference mean $S-N$ curve.

^C Reduced cross-sectional area.

^D Preliminary tests (same dimensions, except larger hole diameter).

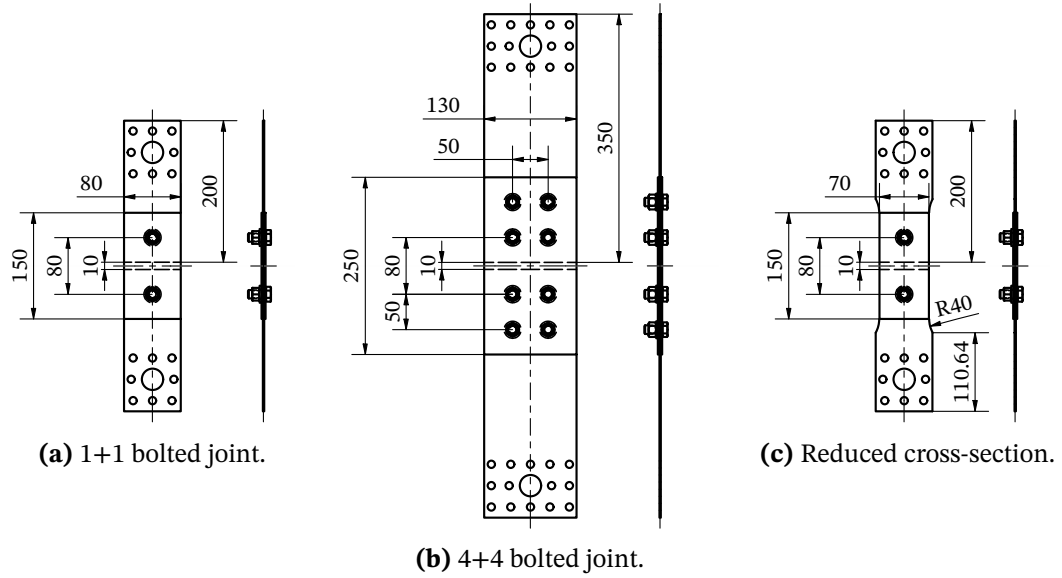


Figure 4.1: Geometry of the tested bolted joint specimens (dimensions in mm); plate thickness of 2 mm. Also see Table 4.2.

MTS servohydraulic axial fatigue testing machines were used. Additionally, Figure 4.1 shows the geometry of the tested bolted joint specimens, where the hole pattern at the ends were used for gripping the specimens using thick steel plates. The main fatigue tests were performed using M12 bolts (grade 8.8) with corresponding nuts and washers; however, preliminary tests used M16 bolts (grade 8.8), an uncommon size for the rack industry. Regarding the bolt arrangement, two options were considered, namely, 1+1 and 4+4 bolt arrays, as shown in Figure 4.1. Regarding the bolt preload force level, two options were considered: the ‘preloaded’ level, defined in EN 1090-2 [134] as 70% of the bolt’s ultimate tensile resistance, i.e.,

$$F_{\text{preloaded}} = 0.7S_uA_s, \quad (4.1)$$

where S_u is the bolt’s nominal ultimate tensile strength and A_s is the bolt’s nominal stress area; the second preload force level is denoted as the ‘snug-tight’ level, defined in this work as 25% of the preloaded level, i.e.,

$$F_{\text{snug-tight}} = 0.25F_{\text{preloaded}}. \quad (4.2)$$

Table 4.2: Additional geometric properties of the bolted joint specimens.

Bolt type	Bolt array	Cross-section	Gross area, A_{gross} [mm ²]	Net area, A_{net} [mm ²]	Hole diameter, d_0 [mm]
M12	1+1	Normal	160	132	14
M12	1+1	Reduced	140	112	14
M12	4+4	Normal	260	204	14
M16	1+1	Normal	160	124	18
M16	4+4	Normal	260	188	18

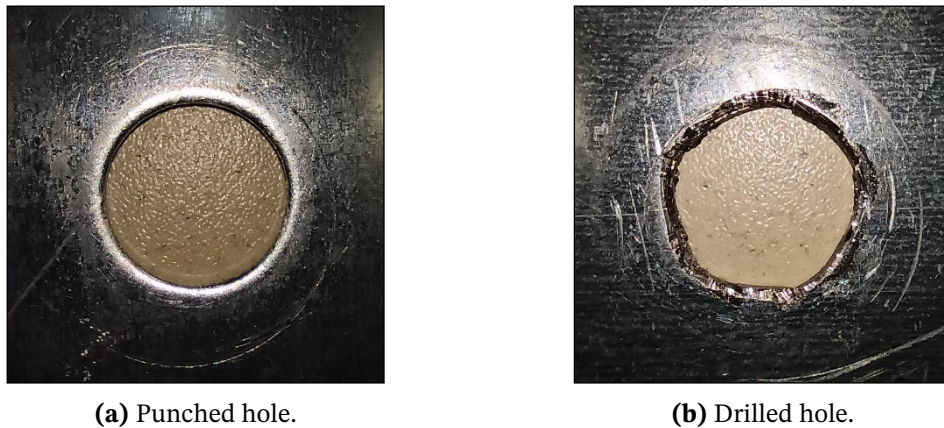


Figure 4.2: Bolt holes used in the tested bolted joints.

The snug-tight level was introduced to represent the usual operating conditions found in the rack industry. Moreover, following the EN 1090-2 [134] recommendations, the torque method was used to apply the bolt's preload force level. Regarding the hole production process, two scenarios were considered: holes produced by punching, which should represent holes produced during manufacturing, and holes produced by drilling with a twist/helical drill, which should represent on-site drilled holes made during assembly. As shown in Figure 4.2, the hole production by drilling resulted in poor quality holes. Regarding the load ratio, the values of 0.1, 0.3, and 0.5 were considered, which were combined with distinct maximum loads (see Table 4.1). Finally, regarding the size of the cross-sectional area, Figure 4.1c shows the geometry of specimens with a reduced cross-section.

4.2 Numerical details

Finite element models of the bolted joints were built within the Abaqus/CAE environment and all tested conditions were individually simulated. Moreover, two types of analyses were performed: elastic analyses, whose results were used in global approaches, and elastoplastic analyses, whose results were used in local approaches. The S350GD material definition is presented in Chapter 3. In the elastic analyses, only the elastic properties of the material are defined, whereas in the elastoplastic analyses the strain hardening behaviour is also modelled via the fitted mathematical model (see Chapter 3) in conjunction with multilinear kinematic hardening and the von Mises yield surface criterion. Unlike isotropic hardening, in the kinematic hardening model, which is based on the work of Lemaitre and Chaboche (1990) [135], the yield surface shifts in stress space so that straining in one direction reduces the yield strength in the other Cartesian directions, simulating in this way the Bauschinger effect as discussed in Section 2.1.2.

In all instances, 1/4 models were considered, as shown in Figure 4.3, taking advantage of the symmetries along the joints' length and width axes. In fact, the geometry of the joints allows for 1/8 models; however, this would unfortunately prevent using the bolt load option available in Abaqus, which is used to apply the bolt preload.

Regarding interactions, contact surface pairs were defined as follows: bolt shank surfaces

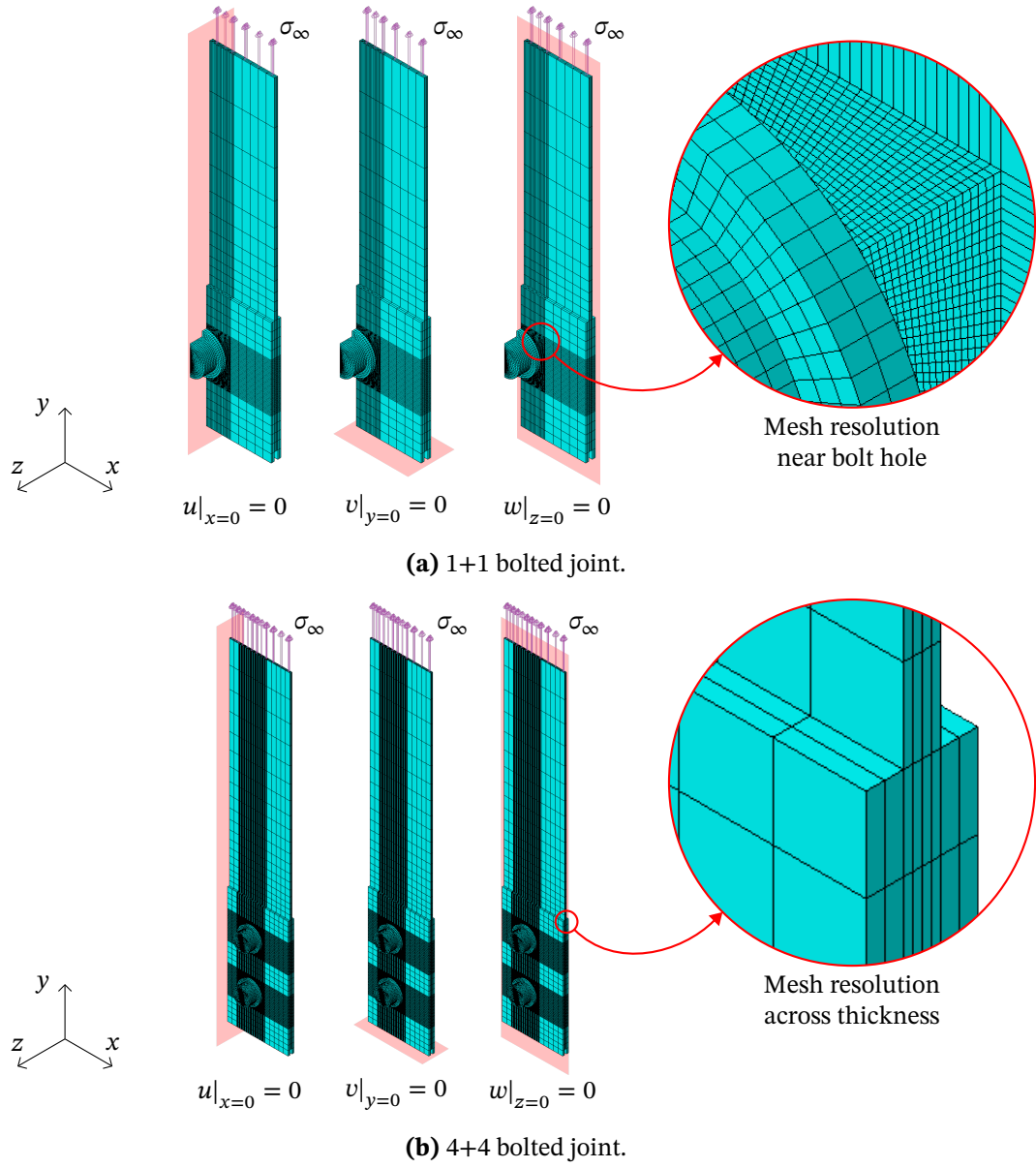


Figure 4.3: Finite element models of the tested bolted joints in Abaqus/CAE.

and hole surfaces; bolt heads and outside surface of the cover plates; inside surface of the cover plates and middle plate surfaces. Contact is modelled using a surface-to-surface discretization and a finite sliding formulation, while the penalty method is used to enforce the constraints arising from hard normal contact and tangential contact with a friction coefficient of $\mu = 0.12$. The friction coefficient is the average value from monotonic static tests performed in reference [133] for the same type of bolted joints. Moreover, unlike the method of Lagrange multipliers, the penalty method does not require additional system variables to enforce the contact constraints, at the expense of negligible nodal penetrations – this is the preferred approach for avoiding unnecessary computational costs.

4: Fatigue behaviour of bolted joints used in rack structures

The quasi-static analyses consist of four load steps: the first step is used to apply the bolt preload level using a bolt load available in Abaqus; the second step is used to perform a transition from a force-controlled bolt load to a fixed-length bolt load, allowing for preload relaxation during stretching; the third and fourth steps are used, respectively, to apply the maximum and minimum fatigue loads within the constant amplitude load cycle.

Regarding the finite element mesh, first-order elements are used with a reduced integration scheme. The mesh is comprised mostly by C3D8R brick elements, but some C3D6 wedge elements are used in the core of the bolts due to their cylindrical geometry. The element edge size is approximately 0.2 mm in the hole and bolt shank regions, while a 5 mm global edge size is defined; outside the contact zones, element edge size varies linearly from 5 to 25 mm along the length axis of the middle plate; across the thickness, 4 element layers are used in the middle plate (the most critical), while 2 layers are used for the cover plates. The reduced integration scheme is used to both reduce the computational cost and to help avoid volumetric locking during fully plastic yielding ($\nu = 0.5$).

4.3 Results and discussion

Experimental and numerical results are shown in the following sections. The failure modes and general behaviour of the investigated bolted joints, as well as the resulting fracture surfaces, are discussed in the first section. The following two sections present the fatigue life predictions via global and local approaches, respectively.

4.3.1 Failure modes and general behaviour

Figure 4.4 shows the typical failure modes of the tested snug-tight bolted joints in reference conditions. As seen in this figure, in snug-tight bolted joints, fatigue cracks consistently initiated within the bolt hole region, approximately aligned with the theoretical net cross-section, and they propagated perpendicularly to the load direction, as expected. However, the fig-

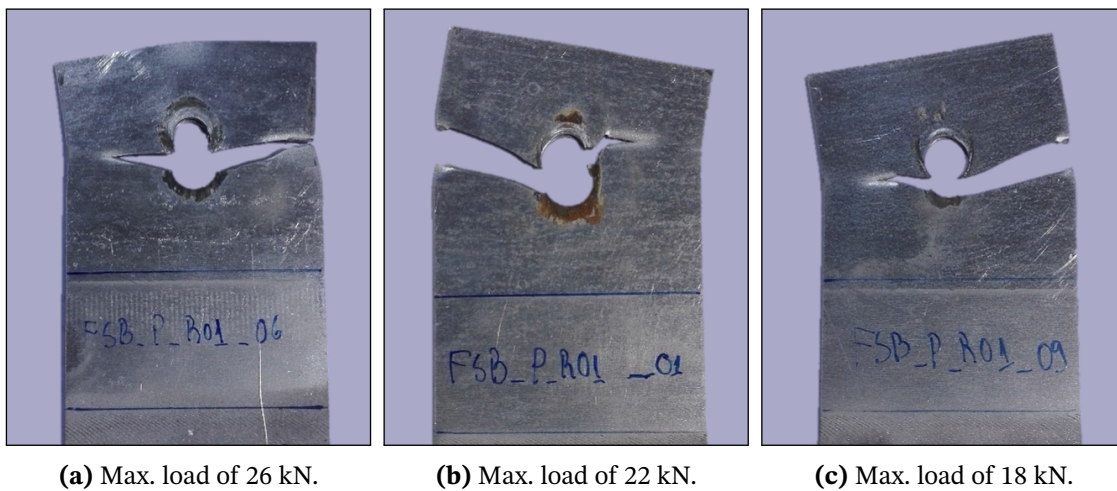


Figure 4.4: Typical failure modes of snug-tight bolted joints in reference conditions.

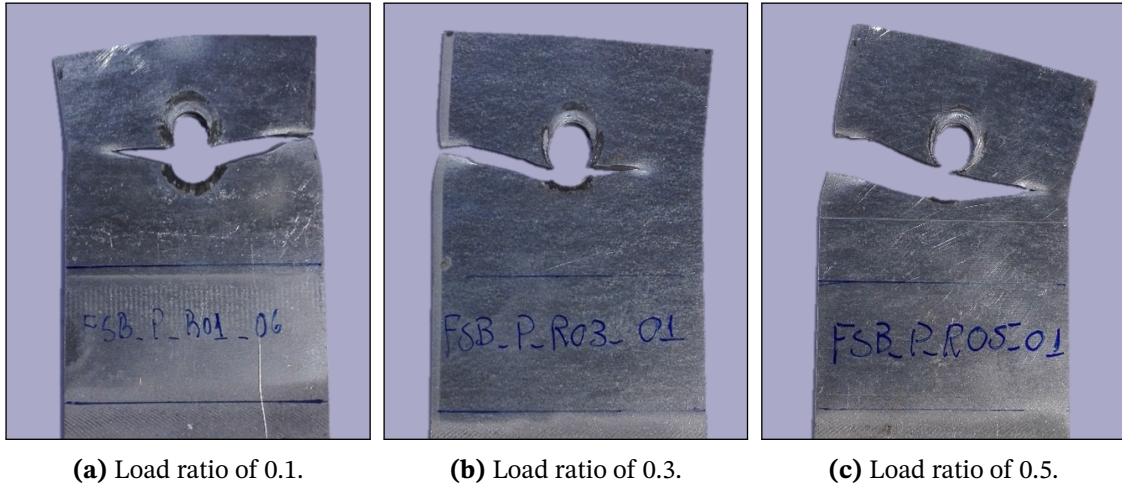


Figure 4.5: Typical failure modes of snug-tight bolted joints with distinct load ratios for the same maximum applied load of 26 kN.

ure also reveals that the initiation site tends to move away from the centre of the hole as the maximum applied load decreases (i.e., as the stress range decreases). Additionally, as shown in Figure 4.5, the same tendency is found for increasing load ratios (i.e., for decreasing stress ranges). Consequently, one may conclude that, under low stress ranges, the load is transmitted more prominently by friction between the joint's plates and the effects of stress concentration due to the pin-loaded bolt holes are less relevant.

Figure 4.6 shows the typical failure modes of the tested snug-tight bolted joints using a 4+4 bolt arrangement. As seen in this figure, fatigue cracks consistently initiated in alignment with the bolt holes (i.e., within the net cross-section), in the bolt row farthest away from the joint lap. The cracks also propagated perpendicularly to the load direction, as expected.

Figure 4.7 shows the typical failure modes of the tested preloaded bolted joints. In speci-

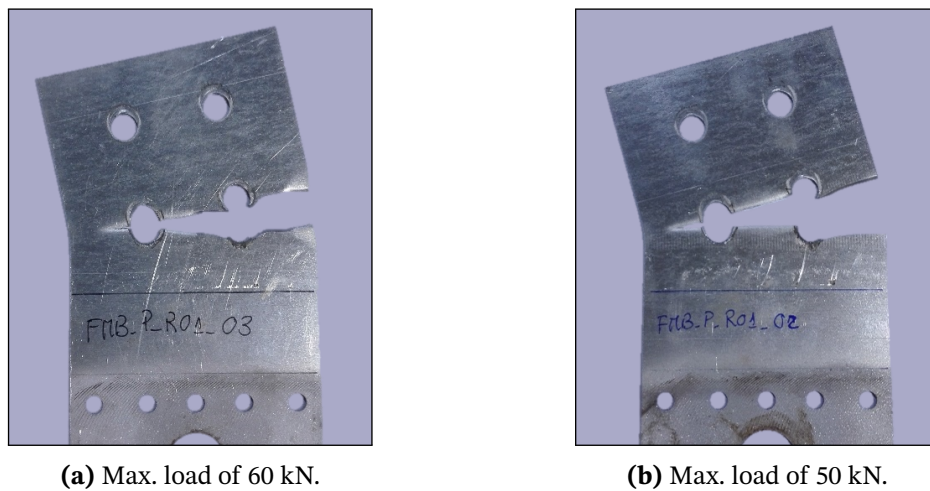


Figure 4.6: Typical failure modes of snug-tight bolted joints with a 4+4 bolt arrangement.

4: Fatigue behaviour of bolted joints used in rack structures

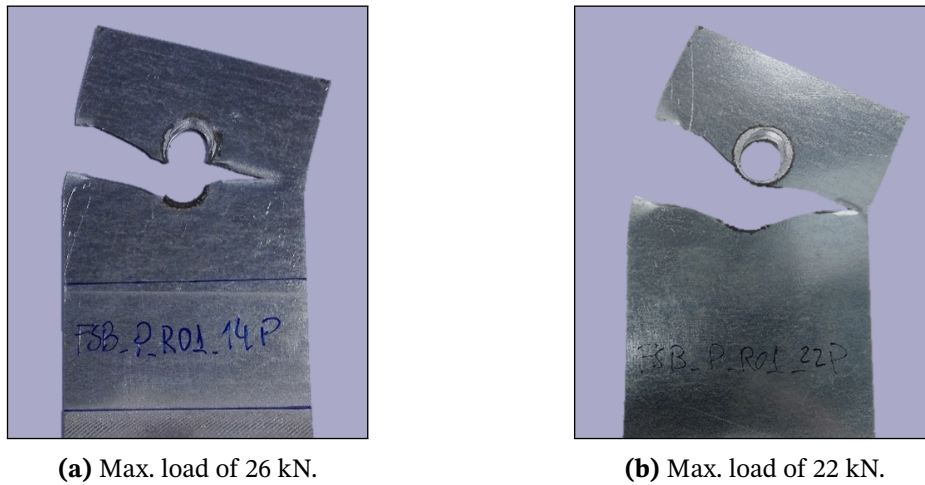


Figure 4.7: Typical failure modes of preloaded bolted joints.

mens subjected to lower stress ranges, fatigue cracks initiated and propagated outside the bolt holes, in the gross cross-section, as shown in [Figure 4.7b](#). In specimens subjected to higher stress ranges ([Figure 4.7a](#)), fatigue cracks initiated and propagated in alignment with the bolt holes, i.e., in the net cross-section, in a very similar way to what is observed in snug-tight bolted joints.

Based on the presented failure modes, one may conclude that the crack initiation site is influenced by the bolt preload level and the applied stress range. For higher bolt preload levels and/or lower applied stress ranges, fatigue loads are significantly transmitted by friction between the plates and the stress concentration due to the hole geometry is not so significant. In this case, the crack initiation site tends to move away from the hole region and into the gross cross-sectional area of the plate. In contrast, for lower bolt preload levels and/or higher applied stress ranges, friction effects are less prominent and the fatigue loads are mainly transmitted via the pin-loaded bolt holes. Consequently, the initiation site is found in the net cross-section, inside the bolt hole region.

In conclusion, significant shear slip may occur between the joint's plates if the bolt preload level is insufficient or the fatigue loading is too high. In these cases, failure is characterized by the pin-loaded hole(s) (see, for example, [Figure 4.4a](#)). In contrast, if the bolt preload level is enough to prevent shear slip, then failure can occur in the gross cross-sectional area (see [Figure 4.7b](#)). In this case, fretting fatigue should be a relevant fatigue damage factor, which, as discussed in [Section 2.2](#), arises from *small* relative displacements between two contacting surfaces, leading to their progressive deterioration due to frictional sliding. This phenomenon can significantly reduce the endurance of a component, and failure may occur at low stress ranges, well below the fatigue limit of the non-fretted material [136]. Sun et al. (2016) [136] state that 'there are two competitive modes in the failure of bolted joints: fatigue failure initiate at the hole edge and fretting fatigue failure initiate at the plate surface', and with the present research one may arrive at that same conclusion.

These findings are corroborated by numerical analysis. Using the finite element models presented in [Section 4.2](#), [Figure 4.8](#) presents, for chosen examples, the elastic stress amplitude within the fatigue load cycle considering the σ_{yy} component (the most relevant). Additionally,

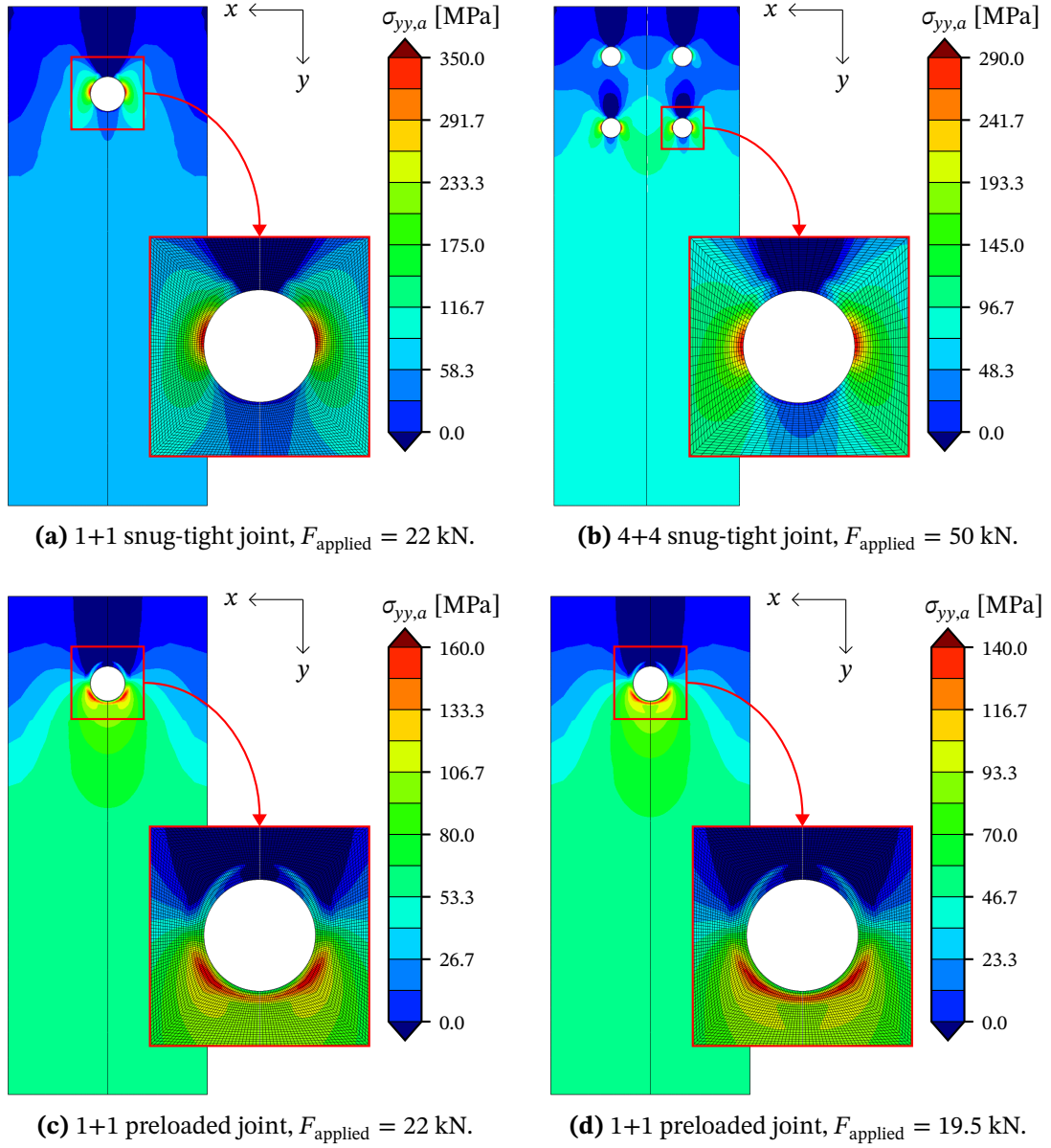


Figure 4.8: Elastic stress amplitude considering the σ_{yy} component.

Table 4.3 presents the elastic stress concentration factors, $K_t = \sigma_{yy}/\sigma_{\infty}$, and the $F_{\text{net}}/F_{\text{applied}}$ ratio, where F_{applied} is the applied fatigue load (maximum value) and F_{net} is the corresponding force acting on the inner plate's net cross-section obtained via integration of the numerical results in that area:

$$F_{\text{net}} = \iint_{A_{\text{net}}} \sigma_{yy} dA = 2 \int_0^{w/2} \int_{-t/2}^{t/2} \sigma_{yy} dz dx, \quad (4.3)$$

where σ_{yy} is null within the hole regions of the net cross-section.

4: Fatigue behaviour of bolted joints used in rack structures

Table 4.3: Elastic stress concentration factors, K_t , and ratio of the force acting on the inner plate's net cross-section to the applied fatigue load, $F_{\text{net}}/F_{\text{applied}}$. Notes: cross-section A pertains to the bolt row farthest away from the joint lap and B is the closest.

Conditions	F_{applied} [kN]	$F_{\text{net}}/F_{\text{applied}}$		$K_t = \sigma_{yy}/\sigma_{\infty}$ (peak values)	
		Cross-sec. A	Cross-sec. B	Cross-sec. A	Cross-sec. B
M12, 1+1, snug-tight	26	96.9%	–	5.78	–
	22	95.9%	–	5.63	–
	18	94.6%	–	5.40	–
M12, 1+1, preloaded	26	82.5%	–	3.31	–
	22	78.9%	–	2.72	–
	19.5	76.0%	–	2.24	–
M12, 4+4, snug-tight	60	96.8%	43.4%	3.55	1.70
	50	95.9%	42.5%	3.43	1.59

As shown in Table 4.3, the $F_{\text{net}}/F_{\text{applied}}$ ratio tends to decrease for decreasing applied loads, indicating that the load is more prominently transmitted by friction as the applied load decreases in magnitude; moreover, this effect is again more prominent for preloaded joints, as expected. Regarding 4+4 bolted joints, these results undoubtedly verify that the net cross-section relative to the bolt row farthest away from the joint lap is the most critical.

Table 4.3 also shows that stress concentration factors are significantly lower for preloaded bolted joints and tend to decrease for decreasing applied loads, again corroborating that friction effects are more prominent in these cases. Regarding 4+4 bolted joints, K_t values of 3.55 and 3.43 were found for the bolt row farthest away from the joint's centre, while the values of 1.70 and 1.59 were found for the closest bolt row. According to the literature [137], K_t values of 3.85 and 2.60 can be approximated for the farthest and closest bolt rows, respectively, but an open hole (without a bolt) is assumed for the less critical net cross-section. Moreover, deviations from the literature could also be attributed to the fact that reference [137] assumes a ratio of hole diameter to hole pitch of 0.2 (where pitch is the spacing between bolt holes in the same row), while a value of 0.28 is considered in this study for 4+4 joints with M12 bolts. Regarding 1+1 snug-tight bolted joints, an average K_t of 5.6 was obtained, and according to reference [137] one may expect a value of 5.2 for a pin-loaded hole, indicating once more that, for these cases, friction effects are less relevant and snug-tight joints are similar in principle to a pin-loaded hole in a plate.

It can also be noted that the location of the maximum value of the nodal elastic stress amplitudes correlates well with the observed crack initiation sites: directly compare Figure 4.8a vs. Figure 4.4b, Figure 4.8b vs. Figure 4.6b, and Figure 4.8c vs. Figure 4.7b. Hence, numerical results are capable of predicting the crack initiation site for both cases where (a) the site is in the net cross-section due to high amounts of shear slip (e.g., see Figure 4.8a) and (b) the site is close to the gross cross-section due to low amounts of shear slip (e.g., see Figure 4.8d).

A SEM was used to evaluate the fracture surfaces of the tested specimens using a FEI Quanta 400 FEG equipment. Figure 4.9 compares the obtained fracture surfaces of four bolted joint specimens. The figure shows typical fatigue striations, indicating an incremental and stable crack growth. Fatigue dimples are also visible, which are typical of final ductile fail-

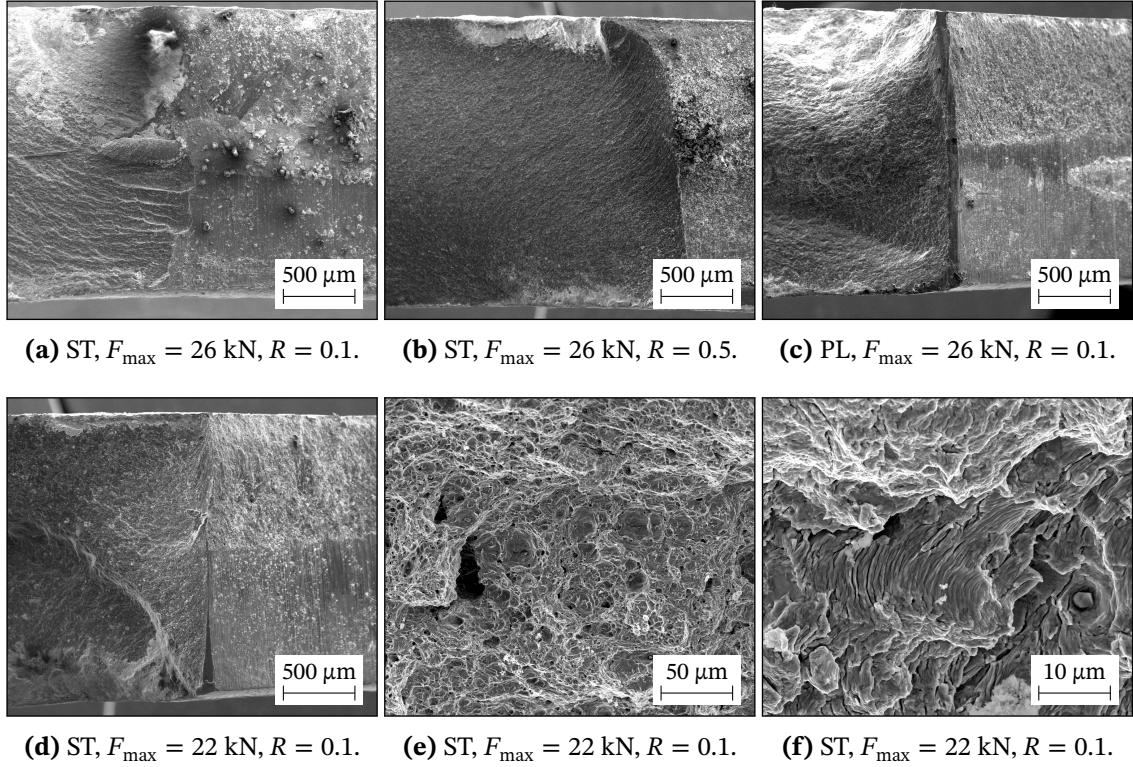


Figure 4.9: Typical fracture surfaces of the tested bolted joints (1+1 bolt configuration with punched holes). Notes: ST stands for snug-tight bolts; PL stands for preloaded bolts; $F_{\max}$ is the maximum load; R is the load ratio.

ure. Additionally, fretting effects may be identified due to the contamination of the fracture surfaces with zinc coating material, shown by the brighter (white) regions.

4.3.2 Fatigue life predictions using global approaches

In order to generate S - N curves for the tested bolted joints, the total fatigue life is considered, which includes the crack initiation and propagation periods; moreover, an infinite fatigue life (run-out) is assumed at 5×10^6 load cycles, which is consistent with the fatigue limit for constant amplitude loading defined in EN 1993-1-9 (Eurocode 3) [5]. The mean S - N curves are obtained via the ASTM E739 standard procedure [66], as detailed in Section 2.2.3.5. Additionally, as a measure of the goodness of fit, the coefficient of determination (R^2) is determined, while the standard deviation (SD) is used to establish confidence bands.

Initially, in order to directly compare the effects of the different tested conditions, remote nominal stresses are used to generate the S - N data. In this case, the gross cross-sectional area of the loaded plate is considered, i.e.,

$$S_{\text{remote}} = \frac{F_{\text{applied}}}{A_{\text{gross}}} \quad \text{with} \quad A_{\text{gross}} = wt. \quad (4.4)$$

Figure 4.10 presents the reference mean S - N curve including the 1+1 bolted joints with snug-tight M12 bolts, punched holes, normal cross-sectional area, and three tested load ratios.

4: Fatigue behaviour of bolted joints used in rack structures

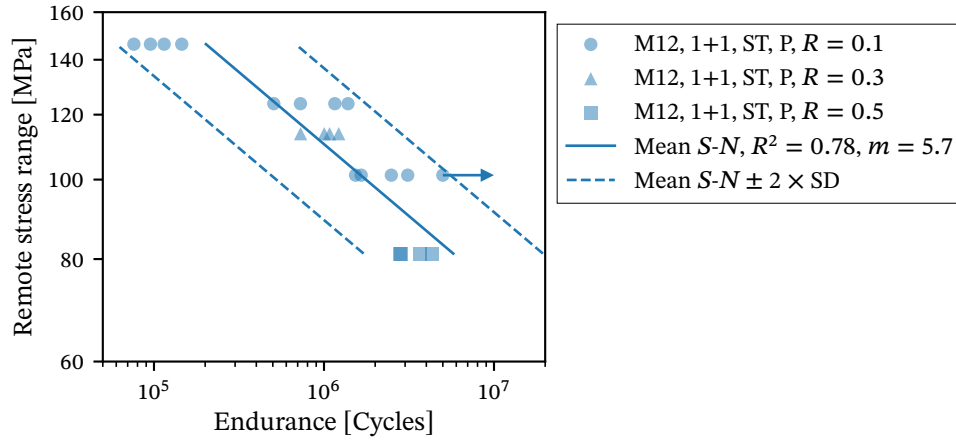


Figure 4.10: Reference mean $S-N$ curve. Notes: ST stands for snug-tight bolts; P stands for punched holes; SD stands for standard deviation.

A coefficient of determination of $R^2 = 0.78$ is verified, as well as a slope of $m = 5.7$. The R^2 value can be increased by performing a mean stress correction to the obtained $S-N$ data. As detailed in Section 2.2.2, the Gerber (2.54), Goodman (2.55), Soderberg (2.56), and Taras-Greiner (2.57) models can be used for the mean stress correction. For the Gerber, Goodman, and Soderberg models, the required material parameters, namely the yield and ultimate tensile strengths of the material, were extracted from its technical delivery conditions (as presented in Chapter 3). Regarding the Taras-Greiner model, the fatigue strength factor α was optimized to maximize the R^2 value, resulting in $\alpha = 0.768$. As shown in Figure 4.11, all mean stress correction models were successful in improving the R^2 value; however, this also increases the slope of the mean $S-N$ curve. Since the original value of $m = 5.7$ is the most consistent with the values found in EN 1993-1-9 (Eurocode 3) [5], the $S-N$ data shown hereinafter is presented without a mean stress correction.

Figure 4.12 presents a direct comparison between the effects of the several tested conditions regarding the tested bolted joint specimens. Table 4.4 and Table 4.5 present a summary of Figure 4.12. Figure 4.12a shows a comparison between 1+1 and 4+4 bolted joints, where 4+4 bolted joints show a 75% increase in fatigue strength when comparing this value from the mean $S-N$ curve at 2×10^6 load cycles. This value is important as it corresponds to the detail category in EN 1993-1-9 (Eurocode 3) [5]. Figure 4.12b shows a comparison between punched and drilled bolt holes, where specimens with drilled holes show a 20% decrease in the fatigue strength value at 2×10^6 load cycles. This can be explained by the poor quality of the drilled holes, as shown in Figure 4.2. Figure 4.12c shows a comparison between results for specimens with normal and reduced cross-sections, where, unexpectedly, specimens with a reduced cross-sectional area show a higher fatigue resistance. This could potentially be explained by a more intense plastic action (hardening) that lead to a more favourable residual stress distribution. Figure 4.12d shows a comparison between bolted joints with snug-tight and preloaded bolts, where specimens with preloaded bolts show a 20% higher fatigue strength at 2×10^6 load cycles. This is expected due to the beneficial effect of the load being transferred more prominently by friction, as previously discussed in Section 4.3.1.

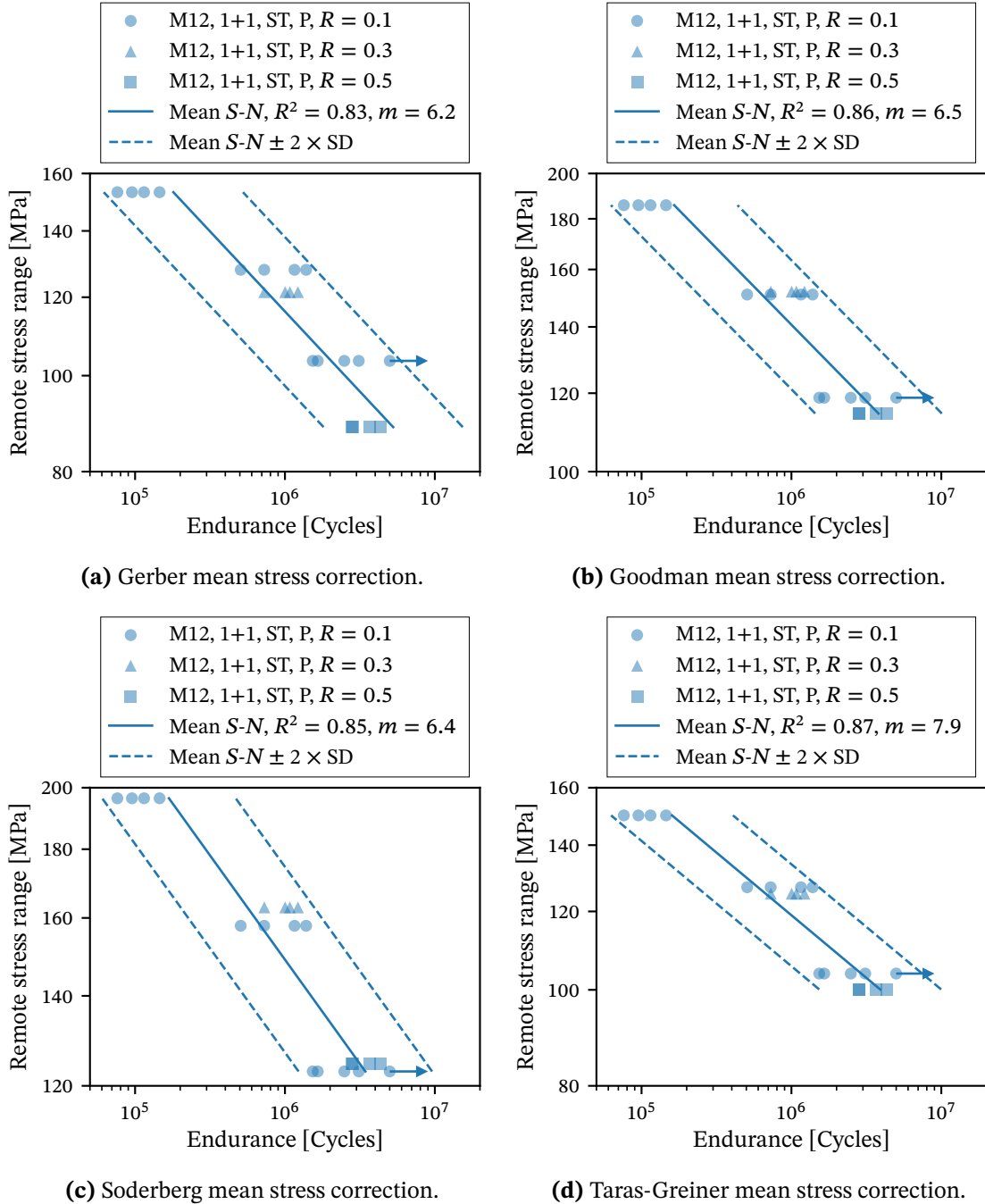


Figure 4.11: Reference mean $S-N$ curve with mean stress correction. Notes: ST stands for snug-tight bolts; P stands for punched holes; SD stands for standard deviation.

4: Fatigue behaviour of bolted joints used in rack structures

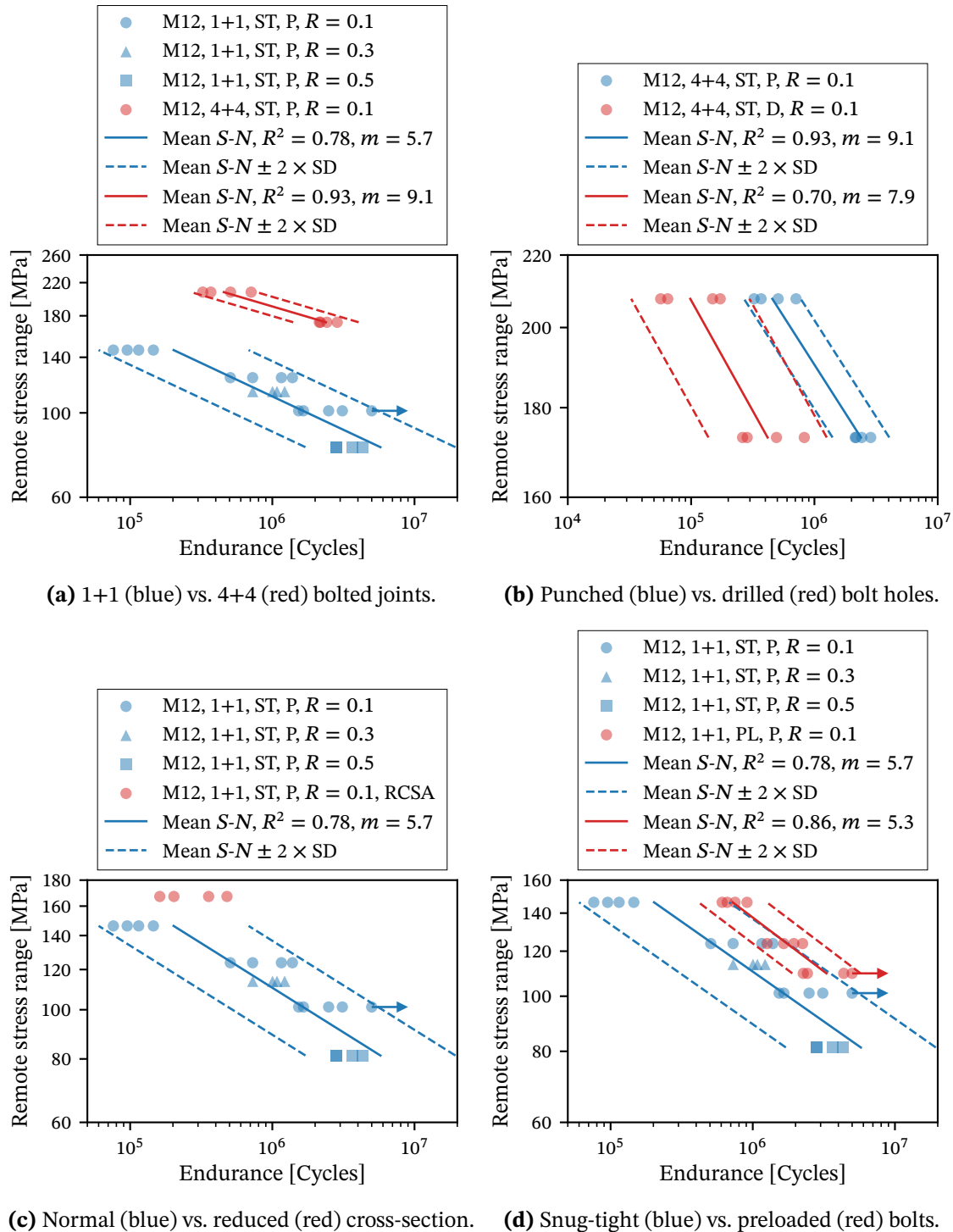


Figure 4.12: Direct comparison of the generated $S-N$ data using remote stresses. Notes: ST stands for snug-tight bolts; PL stands for preloaded bolts; P stands for punched holes; D stands for drilled holes; RCSA stands for reduced cross-sectional area; SD stands for standard deviation.

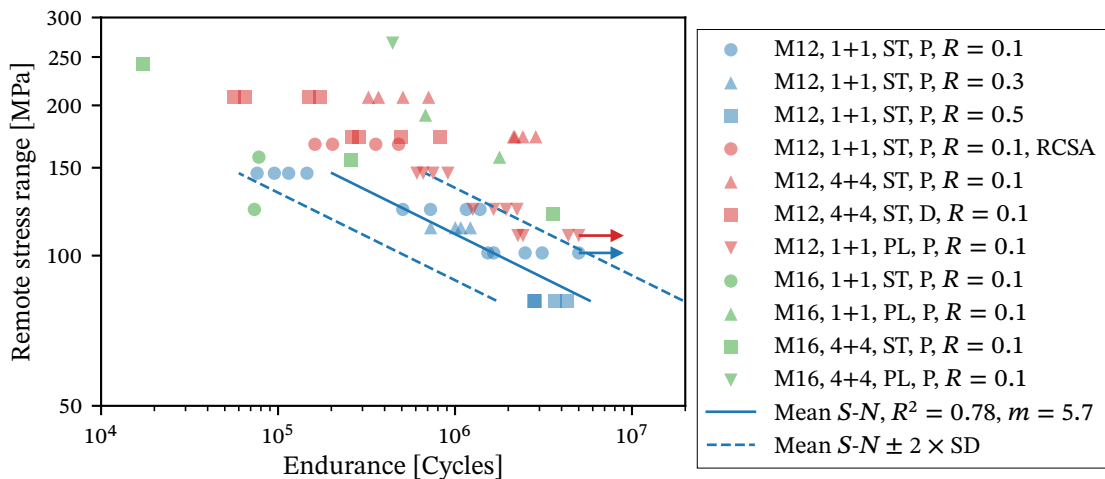
Table 4.4: Effects of the different tested conditions on the fatigue strength of the bolted joints.

Fatigue strength at 2×10^6 load cycles (from the mean $S-N$ curve) [MPa]					
Bolt array (snug-tight)		Hole production (4+4)		Preload level (1+1)	
1+1	4+4	Punched	Drilled	Snug-tight	Preloaded
100	175 (+75%)	175	140 (−20%)	100	120 (+20%)

Table 4.5: Effects of (a) reducing the cross-sectional area and (b) increasing the stress ratio on the fatigue endurance of the bolted joints.

Fatigue endurance (for an applied maximum load of 26 kN) [Cycles]			
Reference conditions $R = 0.1$	Reduced cross-section $R = 0.1$	Normal cross-section $R = 0.3$	Normal cross-section $R = 0.5$
7.6×10^4	2.0×10^5	1.0×10^6	3.7×10^6
9.5×10^4	3.6×10^5	1.1×10^6	2.8×10^6
1.5×10^5	4.8×10^5	1.2×10^6	2.8×10^6
1.1×10^5	1.6×10^5	7.3×10^5	4.3×10^6
Average: 1.1×10^5	Average: 3.0×10^5	Average: 1.0×10^6	Average: 3.4×10^6

Figure 4.13 shows an overview of the generated $S-N$ data for all tested conditions, including preliminary tests with bolted joints using M16 bolts, which represent an uncommon size in the racking industry. The reference mean $S-N$ curve, built using the results from 1+1 snug-tight connections in reference conditions while including different load ratios (see Table 4.1), can be seen as the lower band for the obtained data.

**Figure 4.13:** Overview of the generated $S-N$ data. Notes: ST stands for snug-tight bolts; PL stands for preloaded bolts; P stands for punched holes; D stands for drilled holes; RCSA stands for reduced cross-sectional area; SD stands for standard deviation.

4: Fatigue behaviour of bolted joints used in rack structures

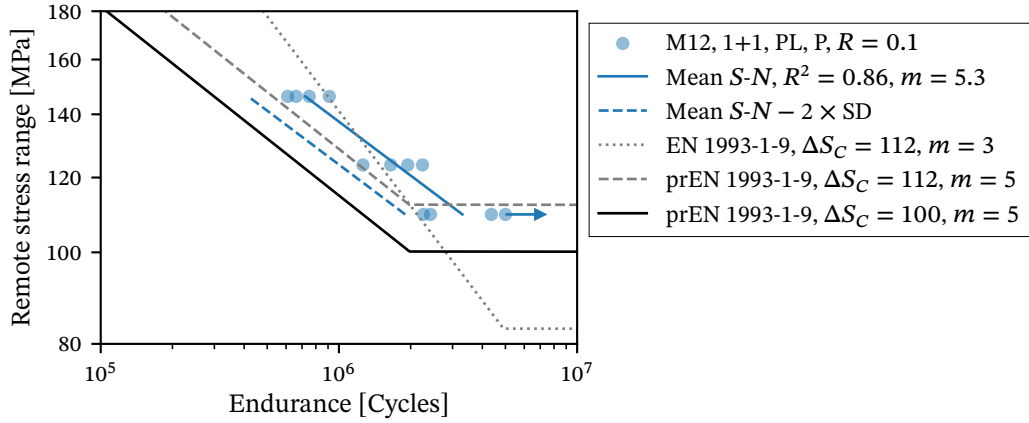


Figure 4.14: Generated S - N data using remote stresses vs. fatigue design curves from Eurocode 3. Notes: PL stands for preloaded bolts; P stands for punched holes; SD stands for standard deviation.

The obtained S - N data is compared against fatigue design curves from EN 1993-1-9 (Eurocode 3) [5]. First, regarding preloaded bolted joints, the current version of Eurocode 3 proposes a category for double covered symmetrical joints with preloaded high-strength bolts, where $\Delta S_C = 112$ MPa (detail category) and $m = 3$ (slope) for an endurance of $N_f \leq 5 \times 10^6$ load cycles (fatigue limit). Additionally, the CEN/TC 250 is currently developing a new version of EN 1993-1-9. The new version is designated as prEN 1993-1-9 [8]. One of the changes proposed in this revision is to update the slope of the aforementioned detail category from $m = 3$ to $m = 5$ and truncate the curve for $N_f \leq 2 \times 10^6$ load cycles. In both versions of EN 1993-1-9, nominal stress ranges are expected to be calculated considering the gross cross-sectional area of the plate, as shown in (4.4). In accordance, Figure 4.14 shows the obtained results for preloaded bolted joints matched against the corresponding fatigue design curves. As seen in this figure, the results of this research agree very well with the prEN 1993-1-9 revision, i.e., the updated detail category is much more consistent with the available data; however, a low safety margin is verified. Therefore, the upcoming fatigue detail category of $\Delta S_C = 112$ MPa with $m = 5$ may be recommended; however, since the beneficial preload effect on thin zinc-coated plates may be less relevant due to low friction coefficients, it would be advisable to use a more conservative fatigue design curve, e.g., the upcoming detail category of $\Delta S_C = 100$ MPa with $m = 5$, as shown in Figure 4.14.

In regard to the snug-tight bolted joints, the current version of EN 1993-1-9 (Eurocode 3) [5] has no considerations for non-preloaded bolts. This changes with the prEN 1993-1-9 revision [8], which introduces two new fatigue design curves for double covered symmetrical joints with non-preloaded normal bolts in holes with normal clearances. A distinction is made for drilled or reamed holes ($\Delta S_C = 90$ MPa with $m = 5$) vs. punched or thermally cut holes ($\Delta S_C = 71$ MPa with $m = 3$). For these applications, prEN 1993-1-9 recommends using the net cross-sectional area of the plate for the computation of nominal stress ranges, i.e.,

$$S_{\text{net}} = \frac{F_{\text{applied}}}{A_{\text{net}}} \quad \text{with} \quad A_{\text{net}} = t(w - nd_0), \quad (4.5)$$

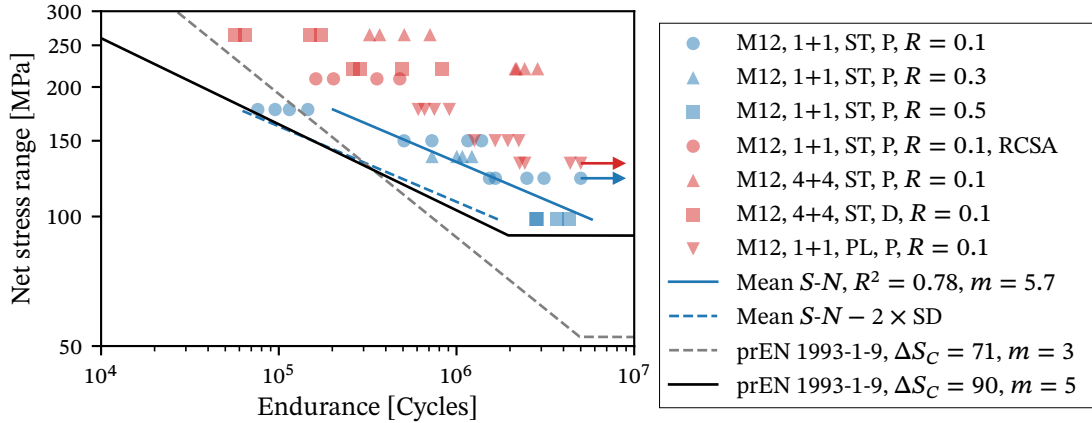


Figure 4.15: Generated $S-N$ data using net stresses vs. fatigue design curves from Eurocode 3. Notes: ST stands for snug-tight bolts; PL stands for preloaded bolts; P stands for punched holes; D stands for drilled holes; RCSA stands for reduced cross-sectional area; SD stands for standard deviation.

where t is the plate thickness, w is the plate width, n is the number of bolt holes in the cross-section, and d_0 is the nominal bolt hole diameter. Using this measure of stress, Figure 4.15 shows the obtained results compared against the newly proposed detail categories. As seen in this figure, the new detail category of $\Delta S_C = 90$ MPa with $m = 5$ may be recommended, as it is almost coincident with the reference mean $S-N$ curve translated by minus twice the standard deviation (delimiting the 97.72% survival proportion). This category is then a conservative but adequate approach for various types of bolted joints, including the preloaded connections, while also being safe for positive load ratios between 0.1 and 0.5, i.e., mean stress correction measures are not required. It may also be noted that the detail category of $\Delta S_C = 71$ MPa with $m = 3$ is not a good match, as shown in Figure 4.15; hence, it is not recommended.

Thus far, $S-N$ data was generated using engineering measures of stress considering the gross or net cross-sectional area of the critical plates depending on if the joints rely on preloaded or snug-tight bolts, respectively. In fact, in this classical approach, because the stress state of the critical plate is non-homogeneous, this results not in a true stress acting on the material vs. fatigue endurance relationship, but rather on a load represented by a nominal stress vs. fatigue endurance relationship. This classical approach is widely used as it is a very practical approach, since the determination of true stresses acting on the material is not straightforward. Nevertheless, with this approach, each loading scenario needs to be catalogued into detail tables. Hence, as design standards grow to include more real-world loading scenarios, detail categorization systems become increasingly more convoluted. Still, it is impossible to catalogue every possible structural detail and loading combination, leaving more complex cases outside the standards' considerations. For example, it has been shown in Section 4.3.1 that the crack initiation site may be found in the net or gross cross-section of the critical plate depending on both the applied stress range and preload level of the bolts; hence, there is an uncertainty regarding the crack initiation site, as preloaded bolted joints may behave similarly to snug-tight joints. Even the prEN 1993-1-9 revision [8], which will include considerations for non-preloaded bolts, will not consider these aspects.

4: Fatigue behaviour of bolted joints used in rack structures

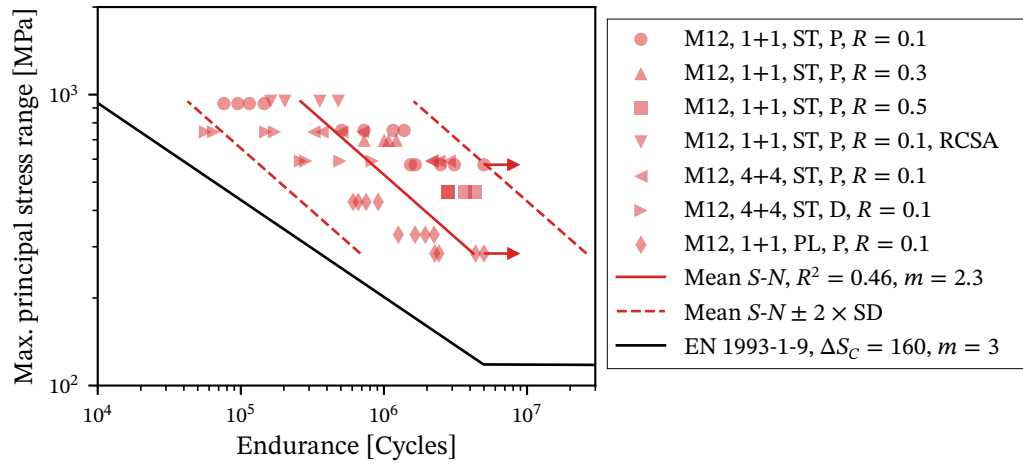


Figure 4.16: Generated S - N data using peak pseudo-elastic maximum principal stresses vs. fatigue design curves from Eurocode 3. Notes: ST stands for snug-tight bolts; PL stands for preloaded bolts; P stands for punched holes; D stands for drilled holes; RCSA stands for reduced cross-sectional area; SD stands for standard deviation.

In order to avoid convoluted detail categorization systems, an alternative procedure is tested. This approach relies on numerically determined peak pseudo-elastic maximum principal stresses, extracted from the most critical mesh node present in the finite element results. As maximum principal stresses are invariant under orthogonal coordinate transformations, no particular coordinate system is required, capturing in this way all sources of stress concentration and simplifying the analysis of complex multiaxial cases. Moreover, it is understood from fracture mechanics that cracks tend to initiate and propagate perpendicularly to the maximum principal stress direction since it usually represents the direction of tensile stresses. The term *pseudo-elastic* is used simply to denote that, in this approach, stresses are based on a linear-elastic material model, whereas plasticity effects are present during testing. This approach presents some major advantages:

- It captures a more representative nonlinear quasi-static response of the loaded bolted joint arising from (a) the normal hard contact between bolt shanks and bolt holes and (b) the frictional tangential contact between the plates. The nonlinear response results in a local stress ratio that may be distinct from the applied remote stress ratio. This has direct implications on the stress range value that should be considered for the fatigue analysis.
- The bolt preload can be included in the numerical modelling, and it was shown in [Section 4.3.1](#) that finite element models are capable of predicting if significant shear slip will occur based on the bolt preload and fatigue load levels. Consequently, this approach results in no ambiguity regarding the crack initiation site, removing the necessity for distinct bolted joint detail categories that may be based on distinct stress measures.

The new results are shown in [Figure 4.16](#), revealing a single master S - N curve that can be used for all tested conditions. Moreover, the generated S - N data is consistent with the

highest fatigue resistance available in the current version of EN 1993-1-9 [5] ($\Delta S_C = 160$ MPa with $m = 3$), which is also introduced in the prEN 1993-1-9 revision [8] for sharp notch effects. Regardless, the stress determination approach used to generate the S - N data shown in Figure 4.16 is distinct from what is used in Eurocode 3.

4.3.3 Fatigue life predictions using a local approach

Fatigue life predictions based on a local approach are performed using the Basquin-Coffin-Manson model with mean stress correction, as discussed in Section 2.2.3. To account for the multiaxial stress state of the critical plate, especially around the bolt hole regions, maximum principal stresses and strains are considered, which are obtained from elastoplastic FEA. As explained previously, maximum principal stresses and strains are independent of the chosen coordinate system, and they capture all sources of stress concentration. Moreover, it is understood from fracture mechanics that cracks tend to initiate and propagate perpendicularly to the maximum principal stress direction since it usually represents the direction of tensile stresses. Figure 4.17 presents the results from elastoplastic FEA for selected examples, where the elastoplastic maximum principal stress amplitude within the fatigue load cycle is shown. The critical regions correlate well with the observed fatigue crack initiation sites.

A Matlab script that loads the FEA results from Abaqus/CAE was used to compute the fatigue life predictions, solving the Basquin-Coffin-Manson equation for each node of the finite element mesh using a derivative-free search method (`fminsearch` function). The critical spot of the plate is defined as the node with the lowest fatigue life prediction. Computing fatigue lives at the nodes, instead of at the integration points, also allows this computation to be performed directly at the surfaces, which are seen as the critical regions.

However, the Basquin-Coffin-Manson fatigue constants were not available for the investigated S350GD mild steel grade, as these were only determined in this work for the S355MC steel due to material availability constraints. Consequently, the S350GD fatigue constants must be estimated. To this end, one may assume that the fatigue constants for both the S350GD and S355MC mild steel grades are similar. In this regard, the corresponding fatigue constants are shown in Table 4.6, considering the LCF and HCF experiments described in Chapter 3 and the LCF experiments from De Jesus et al. (2012) [20]. Alternatively, Table 4.6 also shows the Basquin-Coffin-Manson fatigue constants estimated directly for the S350GD material consid-

Table 4.6: Derived Basquin and Coffin-Manson fatigue constants for the S355MC and S350GD mild steel grades.

Fatigue constants ^A	De Jesus et al. (2012) S355MC	This work S355MC	Osgood (1982) AISI 1015 (S235JR)	Bäumel and Seeger (1990) S350GD (estimated)
σ'_f	952 MPa	1057 MPa	827 MPa	870 MPa
ϵ'_f	+0.7371	+0.3439	+0.95	+0.590
b	-0.0890	-0.1120	-0.11	-0.087
c	-0.6640	-0.5512	-0.64	-0.580

^A Basquin model: $\sigma_a = \sigma'_f(2N_f)^b$; Coffin-Manson model: $\Delta\epsilon_p/2 = \epsilon'_f(2N_f)^c$.

4: Fatigue behaviour of bolted joints used in rack structures

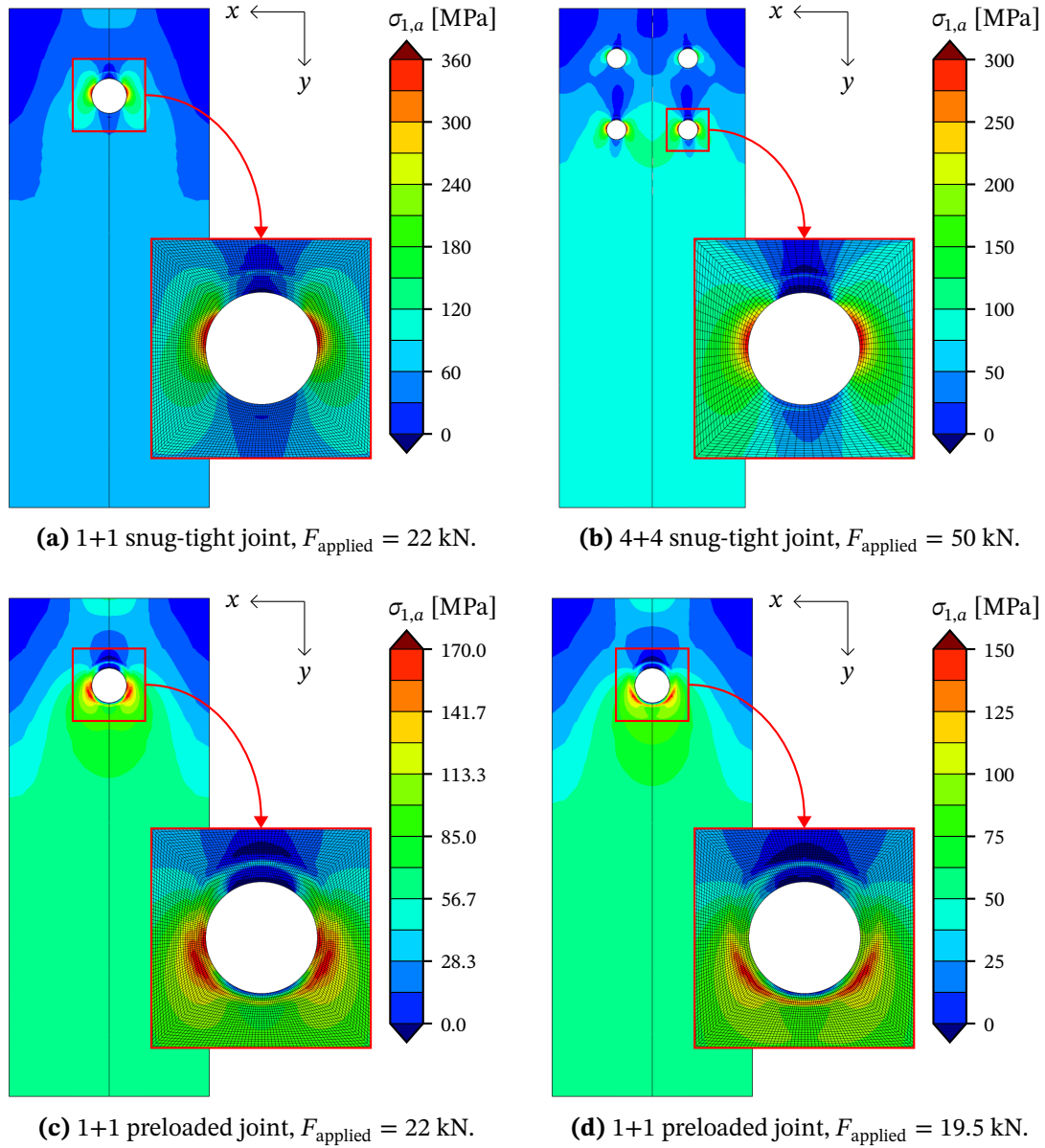


Figure 4.17: Elastoplastic maximum principal stress amplitude.

ering the uniform material law proposed by Bäumel and Seeger (1990) [138], which has been derived from a large amount of collected fatigue data. This universal slopes method estimates that: $\sigma'_f = 1.5\sigma_u$, $\epsilon'_f = 0.59$ if $\sigma_u/E \leq 0.003$, $b = -0.087$, and $c = -0.58$. For further comparisons, Table 4.6 also shows the corresponding fatigue constants for the S235JR material, as presented by Osgood (1982) [139].

Considering the distinct sources for the Basquin-Coffin-Manson fatigue constants shown in Table 4.6, Figure 4.18 presents the corresponding fatigue life predictions vs. experimental observations. The figure shows that, for most cases, fatigue life predictions are underestimated (safe), but for the preloaded bolted joints subjected to the lowest two levels of load magnitude,

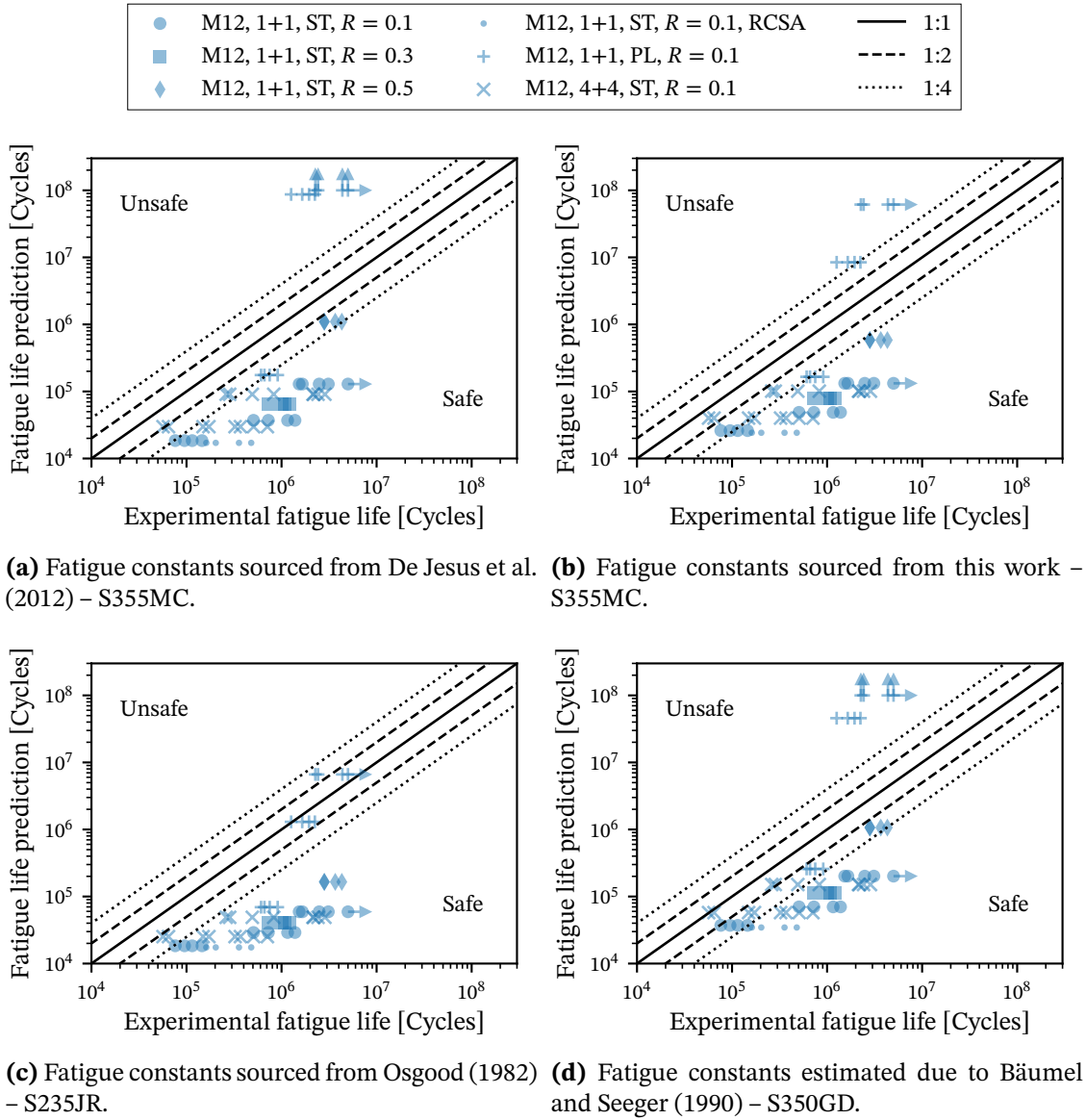


Figure 4.18: Fatigue life predictions vs. experimental data based on distinct sources for the Basquin-Coffin-Manson fatigue constants.

fatigue life predictions are overestimated (unsafe). This could be attributed to fretting fatigue, since this damage factor is not taken into account by the current FEA-based local approach and should be the most relevant in these cases where large shear slip is prevented by the preload level of the bolts. Regardless, for all cases, the predictive capabilities of this approach could be improved, since there are still uncertainties regarding the Basquin-Coffin-Manson fatigue constants for the considered S350GD material.

Consequently, a subsequent study is performed in view of optimizing the Basquin-Coffin-Manson fatigue constants so that fatigue life predictions become better aligned with experimental observations. To this end, the mean squared error (MSE) is minimized using Mat-

4: Fatigue behaviour of bolted joints used in rack structures

lab's constrained nonlinear programming solver (fmincon function). For the starting point of the optimization process, the fatigue constants estimated for the S350GD material using the approach due to Bäuml and Seeger (1990) are considered. Additionally, the σ'_f and ε'_f parameters were constrained to not change more than $\pm 10\%$ in regard to the initial estimated values, and the b and c parameters were constrained within the following intervals: $b \in [-0.12, -0.05]$; $c \in [-0.7, -0.5]$. This ensures that the optimized fatigue constants are within expected, literature-based values [2]. Finally, because it has been shown that fretting fatigue can still be an issue for the predictive model, the optimization process has been performed twice: once including and once excluding the two aforementioned cases susceptible to fretting (i.e., in the second approach, these points are not considered for the MSE minimization). The first approach resulted in $\sigma'_f = 954$ MPa, $\varepsilon'_f = 0.649$, $b = -0.12$, and $c = -0.5$. The second approach resulted in $\sigma'_f = 956$ MPa, $\varepsilon'_f = 0.6487$, $b = -0.0933$, and $c = -0.5$. Ultimately, the major difference between the two approaches is the resulting b parameter (i.e., the fatigue strength exponent), in which both values are consistent with the literature discussed above. However, the value of $b = -0.0933$ (derived using the second approach in which cases susceptible to fretting were removed from the MSE minimization) more closely aligns with the value estimated from the approach due to Bäuml and Seeger (1990). The final results are shown in Figure 4.19, in which an improved predictive capability of the FEA-based local approach is verified, while still acknowledging that fretting fatigue is an issue for the predictive model. Nevertheless, while these final results are based on an inverse analysis, Chapter 6 shows that, for a different case study based on the S355MC material, this approach still performs well when the required fatigue constants have been duly determined beforehand.

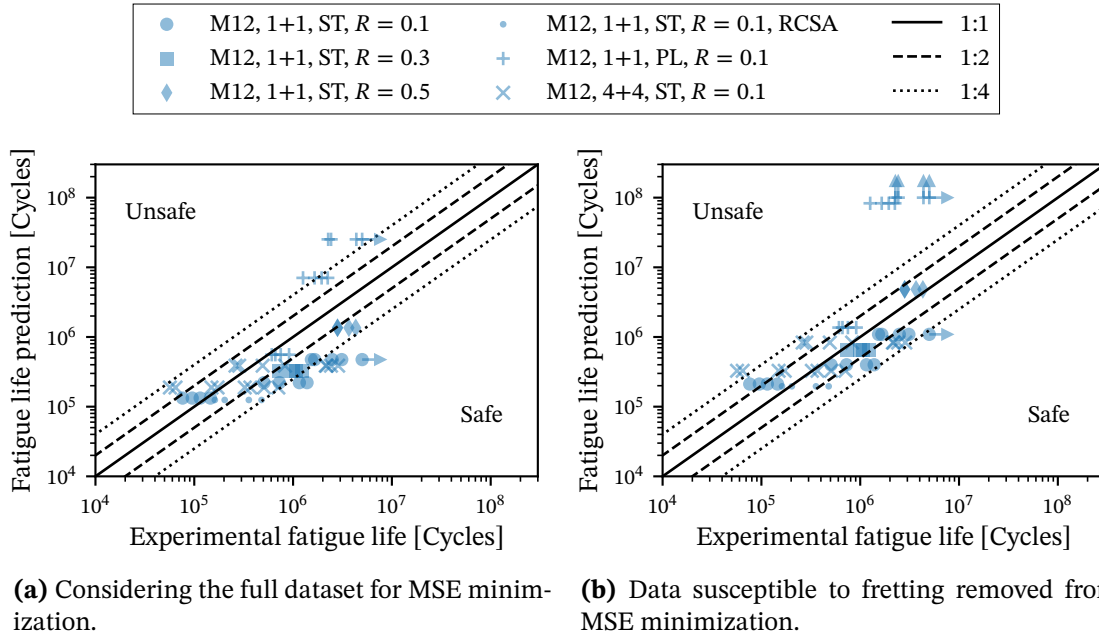


Figure 4.19: Fatigue life predictions vs. experimental data based on optimized Basquin-Coffin-Manson fatigue constants for the S350GD material.

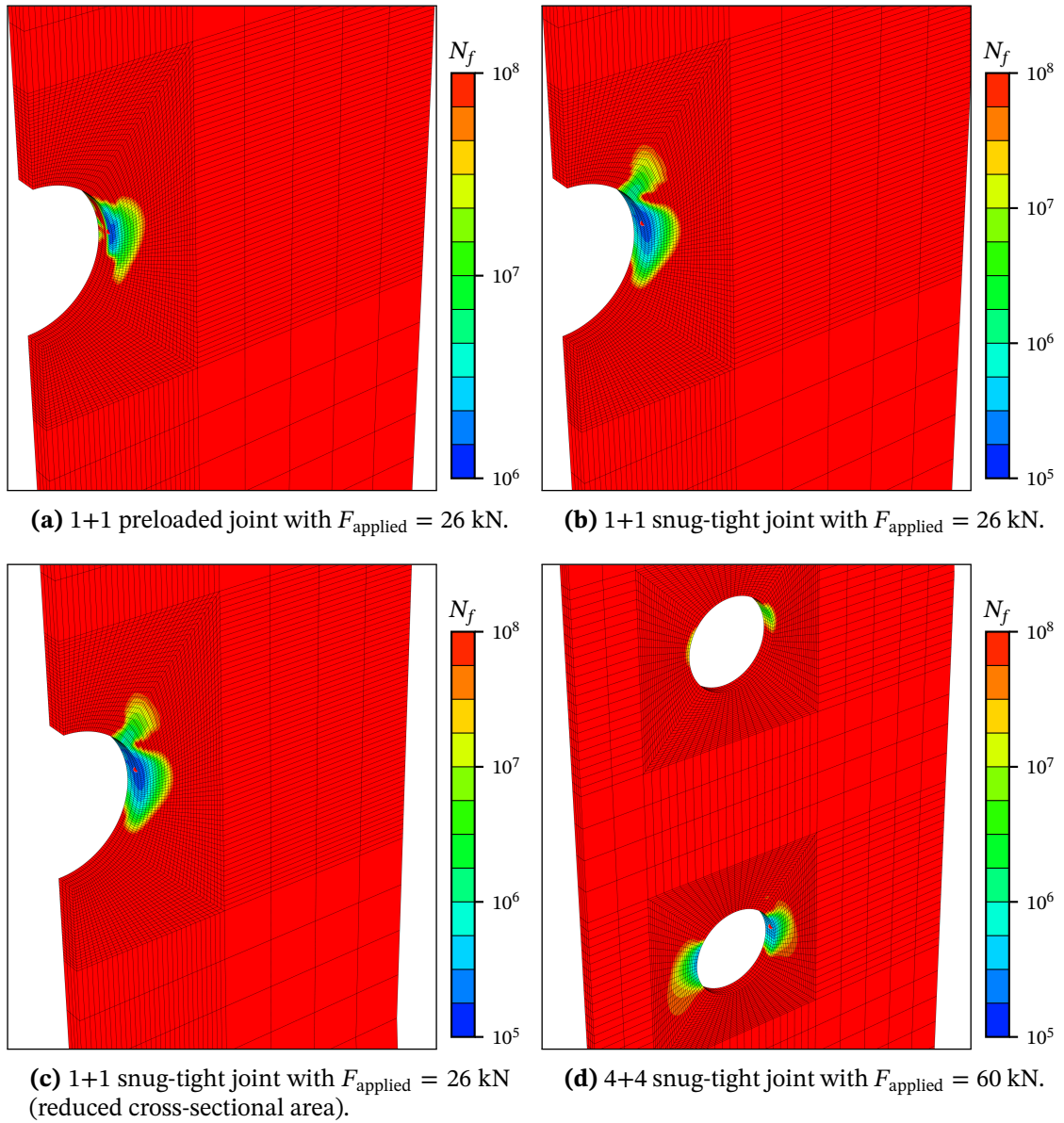


Figure 4.20: Computed nodal fatigue life prediction fields (examples).

Considering the approach whose final results are shown in [Figure 4.19b](#), [Figure 4.20](#) shows the corresponding computed nodal fatigue life prediction fields for some selected examples. As shown in this figure, the critical region is located within the bolt hole contour, which is consistent with the experimental observations.

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Chapter 5

Numerical determination of cold forming residual stresses

As discussed in more detail in [Section 2.1.3](#), cold forming of metallic profiles is usually carried out via roll forming or press braking, where the former is a more complex manufacturing process involving redundant deformations and the latter can be regarded as a simpler pure bending process. Nevertheless, both processes are used to produce thin-walled profiles via controlled permanent plastic deformations and, consequently, manufacturing residual stresses are left in the final product. Moreover, sheet metal coils are generally used in cold forming as the base material, and the required operations of coiling, uncoiling, and flattening can introduce additional residual stresses even before the forming process, further increasing the complexity of the final product's straining history.

Today, the determination of manufacturing residual stresses is an active research topic since it is known that such stresses can have a positive, negative, or neutral impact on structural performance and on fatigue, as previously discussed in [Section 2.2.2](#). However, the determination of residual stresses can be quite the complex task. As discussed in [Section 2.1.4](#), residual stresses may be determined experimentally, numerically, or, in simple cases, analytically. A review on the current state of the art in residual stress determination is given by Díaz et al. (2021) [27].

This chapter focuses on the numerical determination of manufacturing residual stresses acting on cold-formed thin-walled profiles. The roll forming and press braking manufacturing processes are considered and the resulting residual stress fields are compared. Furthermore, the effect of pre-forming operations, i.e., the effects of coiling, uncoiling, and flattening of the base material, are also investigated. This study relies on FEA that is generally computationally expensive due to the inclusion of complex tooling geometries and kinetics, as well as several nonlinearities, namely, kinematic and force nonlinearities due to contact-based forming, material nonlinearity due to the elastoplastic behaviour of metals, and geometric nonlinearity due to large deformations and rotations. Additionally, special considerations must be taken into account in order to avoid shear locking that possibly arises during bending and volumetric locking found during fully plastic yielding.

In this chapter, the objective is to numerically simulate the manufacturing process of a thin-walled profile in order to obtain representative residual stress and strain fields. The end goal is to use those results as the initial state in subsequent structural simulations with the

aim of enriching local fatigue damage models with a more complete straining history. This subsequent and novel study is presented in the next chapter.

5.1 Case study

A cold-formed, thin-walled, mild steel Z-profile is considered as a case study. In particular, these profiles are used as guide rails for pallet carrier shuttles in automated solutions for S&R (see Figure 1.2). The R8 web-to-flange corner (see Figure 5.1) is regarded as the critical spot during normal operating conditions, as shown in Figure 1.3a, since the lower flange deflects while supporting the weight of a passing shuttle, creating a stress concentration in the corner region. Consequently, a special focus is given to this location during the investigation of manufacturing residual stresses.

Figure 5.1 shows the relevant section geometry parameters of the investigated Z-profiles. Additional parameters are not shown due to confidentiality agreements between the manufacturer and the partners of the FASTCOLD project. Regarding the material, the investigated Z-profiles are made of S355MC, an uncoated hot-rolled mild steel. The plain material and its properties are presented in Chapter 3.

5.2 Finite element modelling of the press braking process

Air bending, a form of press braking, is considered. As shown in Section 2.1.3, in air bending, a metal sheet is placed between a blunt punch and a die to be formed. The process consists of two stages. In the first, the forming stage, the punch is displaced against the fixed die, pressing and forming the sheet while creating three contact points: one at the tip of the punching tool and one at each side of the V-shaped opening of the die. In the final stage, the rebound stage, the punch is retreated, and the spring-back effect takes place, where elastic strains are relieved and only permanent plastic deformations remain. In general, this corner bending process is considered constant along the longitudinal axis and a plane strain condition may be assumed.

The numerical simulations are carried out within the Abaqus/CAE environment. The models include three physical parts: the punch, die, and sheet. While the sheet is of course

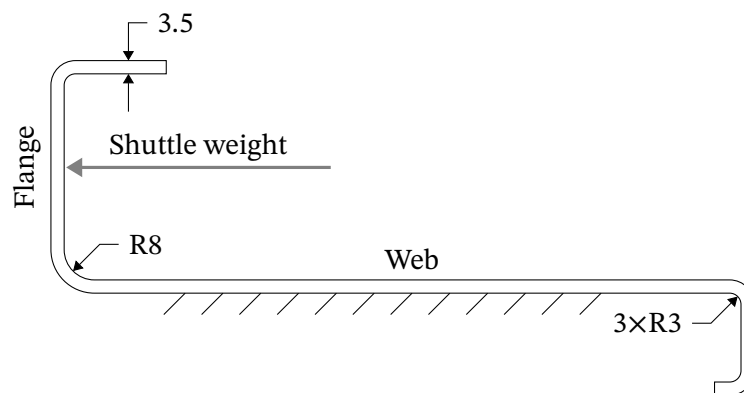


Figure 5.1: Section geometry of the investigated Z-profiles (dimensions in mm).

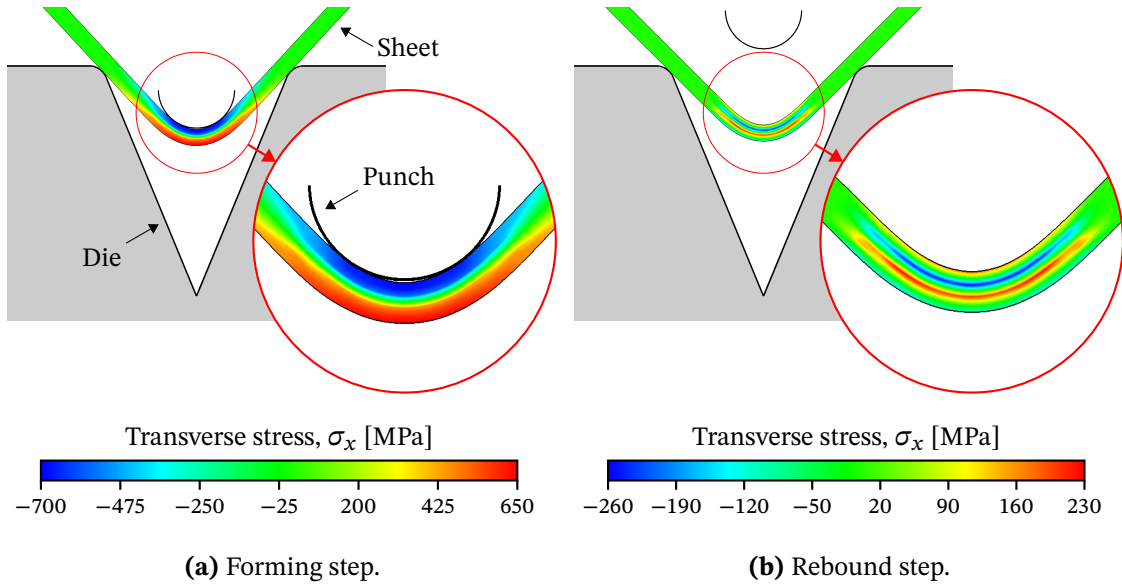


Figure 5.2: Numerical simulation of the press braking process assuming a plane strain condition.

deformable, the punch and die are modelled as analytical rigid parts. Moreover, the geometry of the parts should represent real-world values found in the industry. Regarding the S355MC steel grade, it is considered elastoplastic. The elastic properties consist of the Young's modulus and Poisson's ratio, whereas plasticity is modelled via isotropic strain hardening combined with the von Mises yield surface criterion. The strain hardening behaviour is defined via the fitted strain hardening model presented in Section 3.1. Section 3.1 also shows a validation of the material definition by correlation of experimental and numerical results pertaining to uniaxial tensile tests.

As shown in Figure 5.2, two main steps are considered per bend: the first step is to apply the forming loads by advancing the punch against the fixed die, and the second step is to allow for material spring-back by retreating the punch. A dynamic analysis is considered following an implicit scheme for the time integration, and a quasi-static application is used. A dynamic analysis is chosen because initial rigid body motion along the vertical axis is allowed for the sheet (rigid body motion along the horizontal axis is constrained via boundary conditions arising from corner bending symmetry). However, once contact is established between the sheet, punch, and die during the simulation, the sheet can no longer have rigid body motion since it is constrained by the contact kinetics. Moreover, the quasi-static application is used because the objective is to determine the final static response and, in this type of application, inertia effects are introduced primarily to regularize unstable behaviour. Additionally, a step prior to the forming step is introduced in order to apply gravity effects, which propagate across the following steps. Gravity is introduced to stabilize the sheet's position during the final rebound step since contact between the punch and sheet is broken, which would allow for unstable rigid body motion of the sheet in the vertical axis.

As shown in Figure 5.3, the procedure mentioned above is repeated for each corner, meaning that a simulation is performed per bend. Each subsequent simulation uses the results of

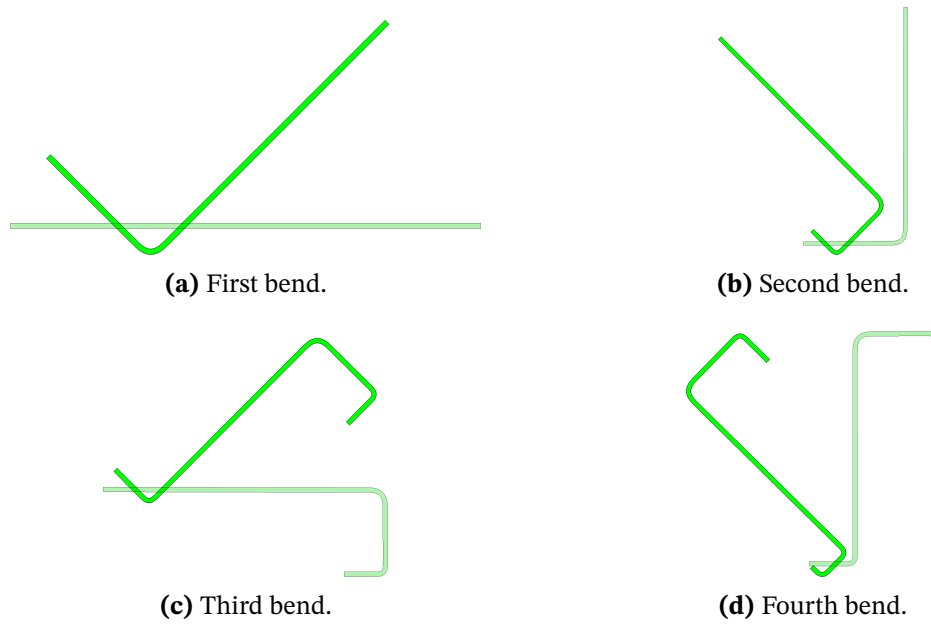


Figure 5.3: Numerical simulation of the full press braking process for the investigated Z-profile.

the previous as the initial state. Additionally, supplementary coordinate systems are created in order to correctly locate the punch and die tools. Regarding the displacement of the punch, a displacement is applied so that a consistent bend angle of 90 ± 0.5 deg is achieved after spring-back.

Regarding the contact interactions between the punch-sheet and die-sheet surface pairs, a hard pressure-overclosure is considered for the normal contact, whereas tangential contact is considered frictionless. A node-to-surface contact discretization is used, allowing for some penetrations of the master surfaces (i.e., the tool surfaces) into the slave surfaces (i.e., the sheet surfaces). Still, this approximation was found to be acceptable while significantly decreasing the computational cost when compared against a more sophisticated surface-to-surface discretization. However, as discussed below, solid and shell elements will be considered, and a surface-to-surface discretization is used in the latter case to account for the shell thickness. Regardless, a finite sliding formulation is considered, allowing for sliding kinematics between contact surfaces, and contact constraints are enforced via the penalty method, which is the less computationally expensive approach available in Abaqus since it does not add additional unknown variables, unlike other methods based on Lagrange multipliers, nor does it add additional contact iterations; however, the penalty-based contact allows for nodal penetrations between contact surfaces, but these were found to be insignificant.

A parametric study on the finite element mesh is performed aiming to identify the finite element type that strikes the best balance between quality of results and computational efficiency. To this end, only finite elements that are not susceptible to locking are considered. Three approaches are investigated: in the first approach, a 2D plane strain condition is assumed, where the CPE4R and CPE4IH bilinear solid elements are used; in the second approach, a general 3D case is assumed, where the C3D8R and C3D8IH trilinear solid elements

are used; in the third approach, the sheet is modelled as a shell, and the S4R bilinear shell element is used. The CPE4R and C3D8R solid elements follow a pure-displacement formulation with reduced integration and hourglass control, while the CPE4IH and C3D8IH solid elements follow an incompatible mode and hybrid incompressible solid formulations with selective integration, and the S4R element is a general purpose thin and thick conventional shell element with reduced integration and hourglass control.

As discussed in [Section 2.3.2](#), reduced integration in pure-displacement formulation elements may be used as a numerical ‘trick’ so that the error in the numerical integration may compensate appropriately for the stiffness overestimation, avoiding both shear and volumetric locking. The hourglass control associated to these elements is introduced to minimize the so-called zero-energy deformation modes arising from reduced integration schemes. As an alternative to this numerical ‘trick’, the incompatible mode and hybrid incompressible solid formulations are used to account for shear and volumetric locking, respectively. Moreover, regarding shell elements, the Abaqus shell element formulation allows for thickness changes and nonlinear stress/strain distributions across the thickness. Generally, no more than nine integration points across the thickness are required if using Simpson’s rule, or five points if using Gaussian quadrature, for severe nonlinear applications [115]; on effect, Simpson’s rule with nine integration points is chosen for the integration across the shell thickness, allowing for results at the surfaces without extrapolation.

During this parametric study, equivalently sized meshes are used. In all cases, transverse element edge size is exactly 1 mm within the bend regions and approximately 2 mm within the straight regions. For solid models, element edge size is exactly 0.5 mm across the thickness, resulting in seven element layers, whereas in the shell-based model, Simpson’s rule with nine integration points is chosen for the integration across the thickness. In 3D models, element edge size is approximately 25 mm in the longitudinal direction. Ultimately, this results in 1,376 nodes and 1,197 elements for the 2D plane strain models, 50,912 nodes and 43,092 elements for the general 3D cases, and 6,364 nodes and 6,156 elements for the shell-based model.

5.3 Finite element modelling of the roll forming process

The roll forming manufacturing process is presented in [Section 2.1.3](#). In short, in roll forming, a long flat sheet of metal is fed into a roll forming machine, where rolls with specific contours incrementally shape the sheet’s section via transverse bending until the desired profile is obtained. It is important to point out that roll forming is a continuous forming process, where the metal sheet is flat on one end of the production line and has the desired cross-section upon exiting the forming mill. Moreover, due to the continuity of the material, the sheet is inherently subjected to redundant deformations between forming increments, increasing the complexity of the final product’s straining history.

The numerical work is again carried out in the Abaqus/CAE environment. Within the FASTCOLD framework, the manufacturer of the Z-profile made available the CAD drawings of each individual forming station present in the real roll forming machine. These drawings were then used during the modelling process, guaranteeing in this way that representative geometries and kinetics are considered during the forming simulation. A total of 14 stations and 45 rolls are present in the roll forming line, as shown in [Figure 5.4](#). Additionally, the rolls are modelled as analytical rigid parts, while the deformable metal sheet shares the same

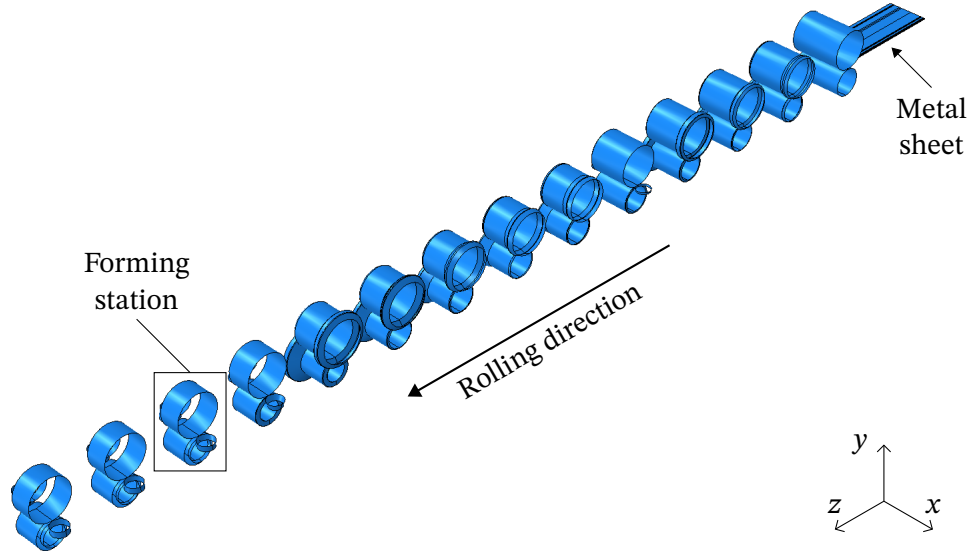


Figure 5.4: Roll forming model in Abaqus/CAE.

material definition used in the press braking simulations.

For consistency with the press braking simulations, a quasi-static dynamic analysis is considered with implicit time integration. A dynamic analysis is chosen even though a static analysis could be carried out since the sheet has no rigid body motion, as discussed below. Still, a dynamic analysis was found to be more stable, allowing for larger time increments. The quasi-static application is used again because the objective is to determine the final static response, where inertia effects are mainly used for stabilization purposes. Moreover, the contact modelling approach is also kept consistent with the press braking simulations.

During the modelling procedure, a common simplification of the roll forming process is taken into account. In this approach, contact is considered frictionless; on effect, the rolls are not required to actually roll. Additionally, the sheet is fixed in space with minimal boundary conditions that prevent rigid body motion without preventing the natural transverse and longitudinal deformations that occur during the forming process (see Figure 5.5). The forming process advances via prescribed displacements in the rolls, which move towards the sheet parallel to the longitudinal direction. Additionally, the sheet's length is roughly double the distance between forming stations, guaranteeing that the sheet is always in contact with two stations, which is necessary to properly model longitudinal deformations.

Regarding the sheet's finite element mesh, shown in Figure 5.5, due to the high computational cost of the roll forming simulations, only the C3D8R and S4R elements are considered, where both have also been used in the press braking simulations. Regarding the mesh resolution, transverse element edge size is exactly 1 mm within the bend regions (compatible with the press braking simulations), and it increases linearly within the straight regions. For the solid model, element edge size is exactly 0.5 mm across the thickness, resulting in seven element layers, whereas in the shell-based model, Simpson's rule with nine integration points is chosen for the integration across the thickness (both cases being compatible with the press braking simulations). In the longitudinal direction, element edge size is exactly 8 mm. Ulti-

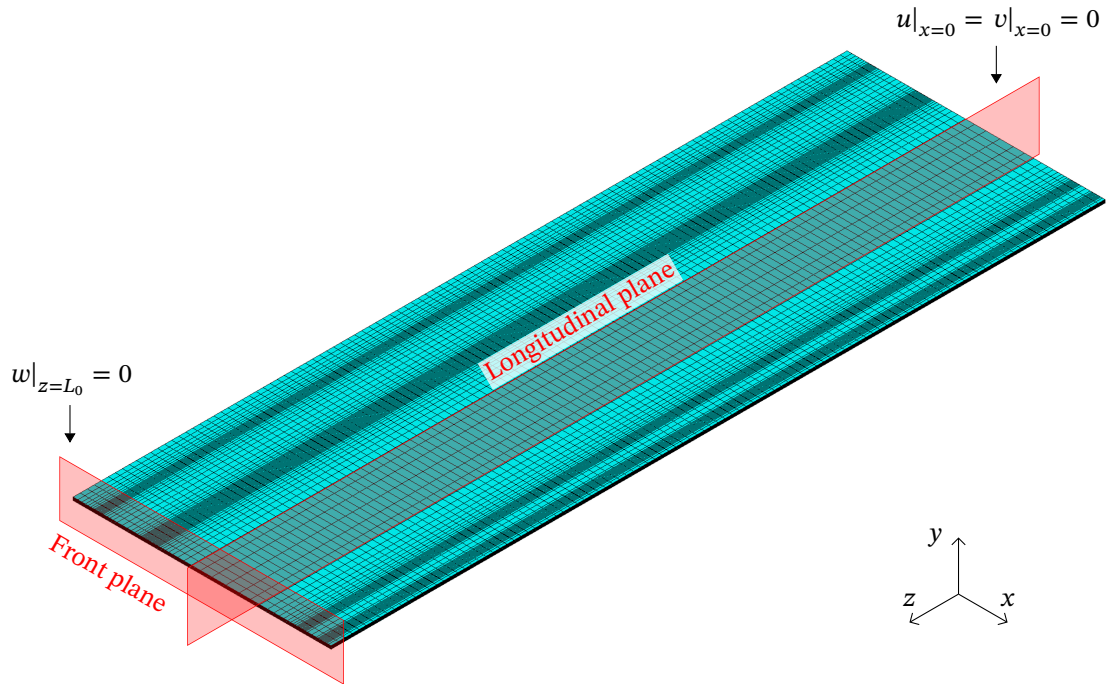


Figure 5.5: Initial meshed sheet used in the roll forming simulations and corresponding boundary conditions.

mately, this results in 87,912 nodes and 75,460 elements for the general 3D case, and 10,989 nodes and 10,780 elements for the shell-based model.

The Z-profile's roll forming flower diagram, which shows the incremental transverse bending of the sheet, where each increment corresponds to a forming station, is shown in Figure 5.6. This particular flower diagram was not obtained via some preprocessing tool; instead, it shows actual results from the numerical simulation of the roll forming process (i.e., it shows the mid-span cross-section of the deformed sheet aligned with each forming station). Additionally, Figure 5.7 shows part of the simulation of the roll forming process in Abaqus.

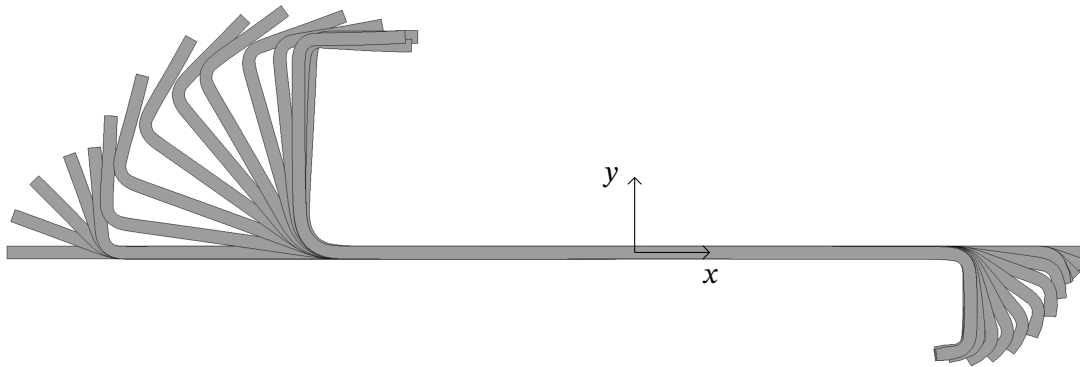


Figure 5.6: Roll forming flower diagram of the investigated Z-profile.

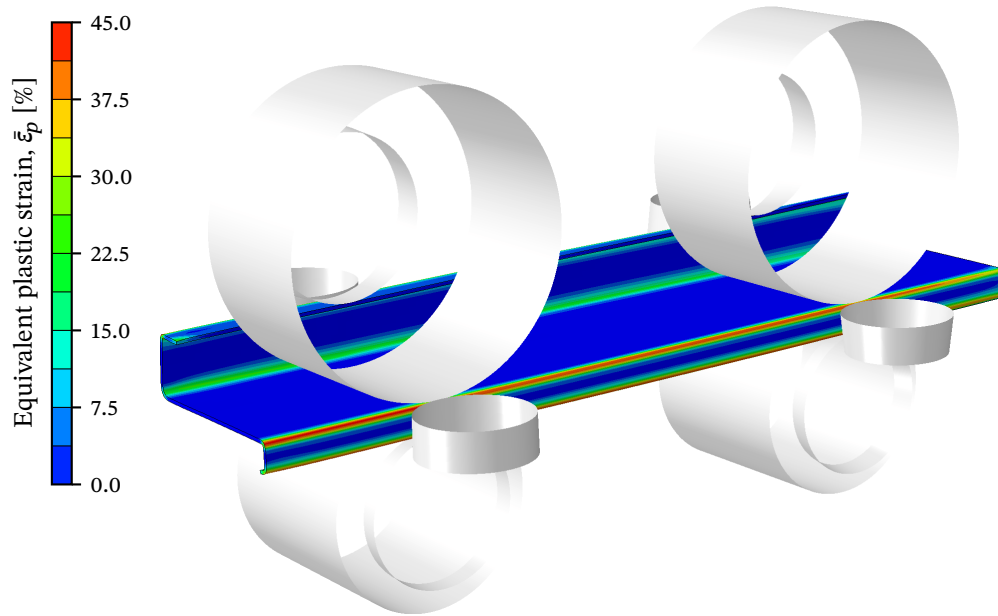


Figure 5.7: Simulation of the roll forming process in Abaqus/CAE (stations 11 and 12). The shown equivalent plastic strain scalar field reveals that the bends are the most plasticized zones, as expected.

5.4 Results and discussion on cold forming residual stresses

Table 5.1 presents the observed simulation times for the cold forming simulations. As shown in this table, the roll forming simulations are much more demanding than the press braking simulations. For press braking, the computational cost can be drastically reduced if a plane strain state is assumed, as expected, due to the large reduction in the required number of finite elements (including contact elements) and integration points. Moreover, the usage of incompatible mode and hybrid incompressible solid formulations may significantly increase

Table 5.1: Observed simulation times for the cold forming simulations.

Manufacturing process	Sheet modelling approach	Element type	Wall-clock time ^A
Press braking (PB)	2D plane strain solid	CPE4IH	00:01:56
		CPE4R	00:01:40
	General 3D solid	C3D8IH C3D8R	08:55:15 00:50:03
	Shell	S4R	00:55:09
Roll forming (RF)	General 3D solid	C3D8R	12 days
	Shell	S4R	24 days

^A On a consumer-grade 7th-generation quad-core processor running at 3.60 GHz with 32 GB of memory.

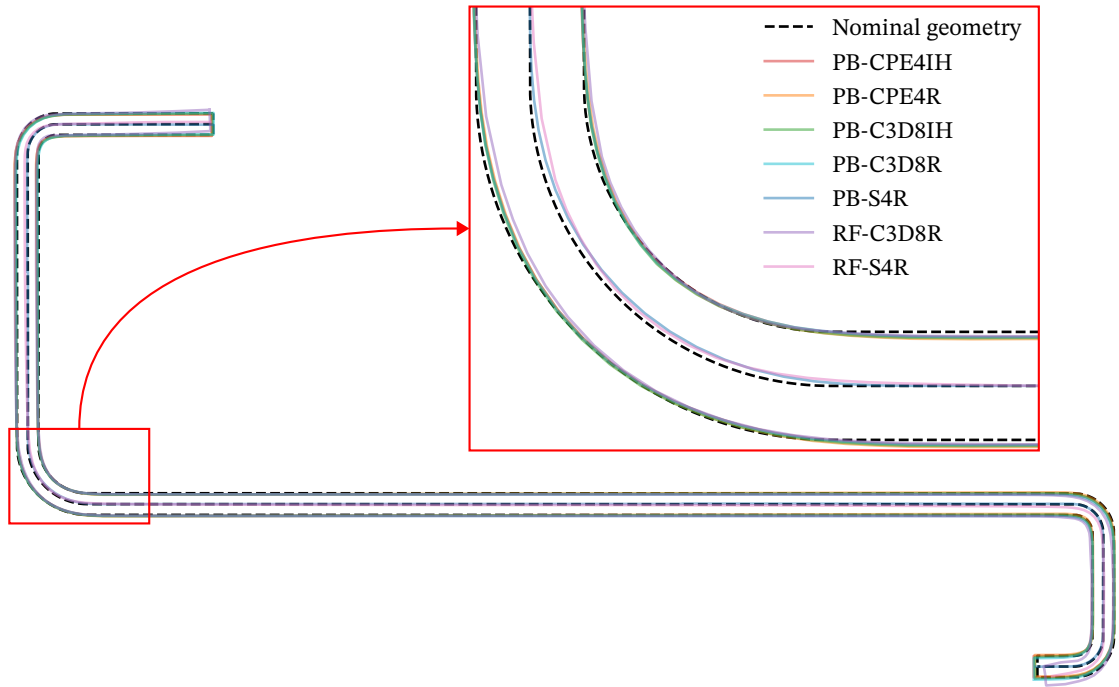


Figure 5.8: Nominal vs. numerically obtained cold-formed geometry.

the computational cost, arising from the increase in the number of degrees of freedom and integration points vs. pure-displacement formulation with reduced integration. Finally, regarding the usage of shell elements, one may expect, in general, a performance increase vs. using 3D solid elements due to a reduction in the number of finite elements, as the material thickness is not geometrically discretized; however, the expected performance gain of shell elements is outweighed by the loss of performance due to the increase in the number of contact elements, as shell elements rely on the surface-to-surface contact discretization to account for the shell thickness (whereas solid elements are used with node-to-surface contact) and a significant number of automatically generated contact elements is required. Therefore, it is not advisable to use shell elements in large contact problems, especially in ones where both surfaces of the shell are considered for contact.

Figure 5.8 shows the nominal section geometry for the investigated Z-profile vs. the shape obtained numerically from the cold forming simulations. As shown in this figure, the expected R8 corner geometry is obtained with very minor deviations. Moreover, minor thickness changes are verified at the middle of the cold-formed corner: for press braking, a reduction of 0.05 mm to 0.07 mm is verified (depending on the considered element type), whereas for roll forming, the thickness reduction is up to 0.09 mm. On average, a thickness reduction of 1.6% and 2.6% is verified for the press-braked and roll-formed corner, respectively. Additionally, in solid elements, normal stresses perpendicular to the middle surface of the Z-profile present a low magnitude of less than 25 MPa (less than 7.0% of the yield stress), validating the applicability of shell elements, since in these cases the mentioned stress component is neglected.

A global press braking vs. roll forming comparison is made by comparing the results obtained via finite element models based on the C3D8R element type. First, the equivalent

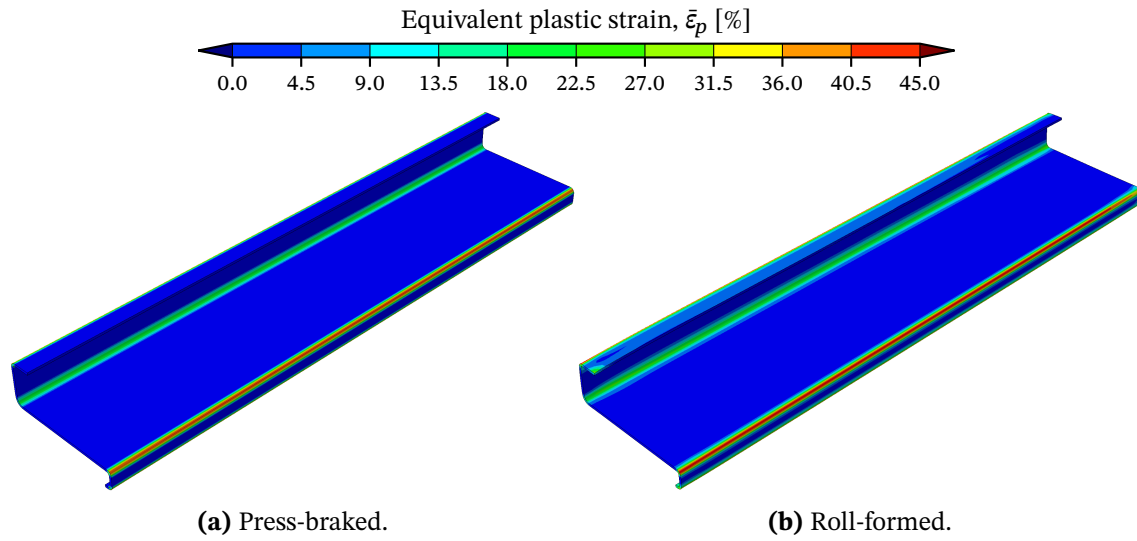


Figure 5.9: Equivalent plastic strain in press-braked and roll-formed Z-profiles.

plastic strain scalar fields are shown in Figure 5.9, revealing that both press-braked and roll-formed profiles present similar plastic zones concentrated in the cold-formed corners, as expected.

Figure 5.10 shows the transverse residual stress fields after press braking and roll forming, revealing that residual stresses in the press-braked profile are limited to the corner regions, which is expected since this manufacturing process can be regarded as a pure bending process. Meanwhile, the transverse residual stress field is more complex in the roll-formed profile. However, in the latter case, the high residual stress values found at the profile's ends are not particularly relevant since they are related to the sheet's entry and exit from forming stations and ideally the roll forming process should be modelled as a continuous process, where the sheet would not be subjected to the repetitive and harsh process of entering and exiting forming stations, but of course such modelling approach is not feasible due to constraints regarding computational costs. Additionally, the tensile residual stresses found inside the R8 corner are larger in magnitude for the roll-formed profile, whereas the compressive residual stresses found outside the R8 corner are larger in magnitude for the press-braked profile.

Figure 5.11 shows the longitudinal residual stress fields after press braking and roll forming, revealing that these stresses are less relevant in press-braked profiles, while high magnitude stress values are found on the web surfaces (near the R8 corner) of roll-formed profiles, as well as in the flanges (near the R3 corners). It should be noted that this distinction between press-braked and roll-formed profiles is found mainly outside the cold-formed corners, i.e., within the flat regions.

As previously mentioned, the web-to-flange corner (i.e., the R8 corner) is the critical spot of the Z-profile during normal operating conditions. Hence, it is in this location that residual stresses are more closely investigated. For this purpose, Figure 5.12 presents the definition of corner paths used to analyse the distribution of cold forming residual stresses. First, Figure 5.13 presents the equivalent plastic strain distributions along path b (middle of the corner, across the thickness). The analytical solutions due to Moen et al. (2008) [28] and Rondal (1987)

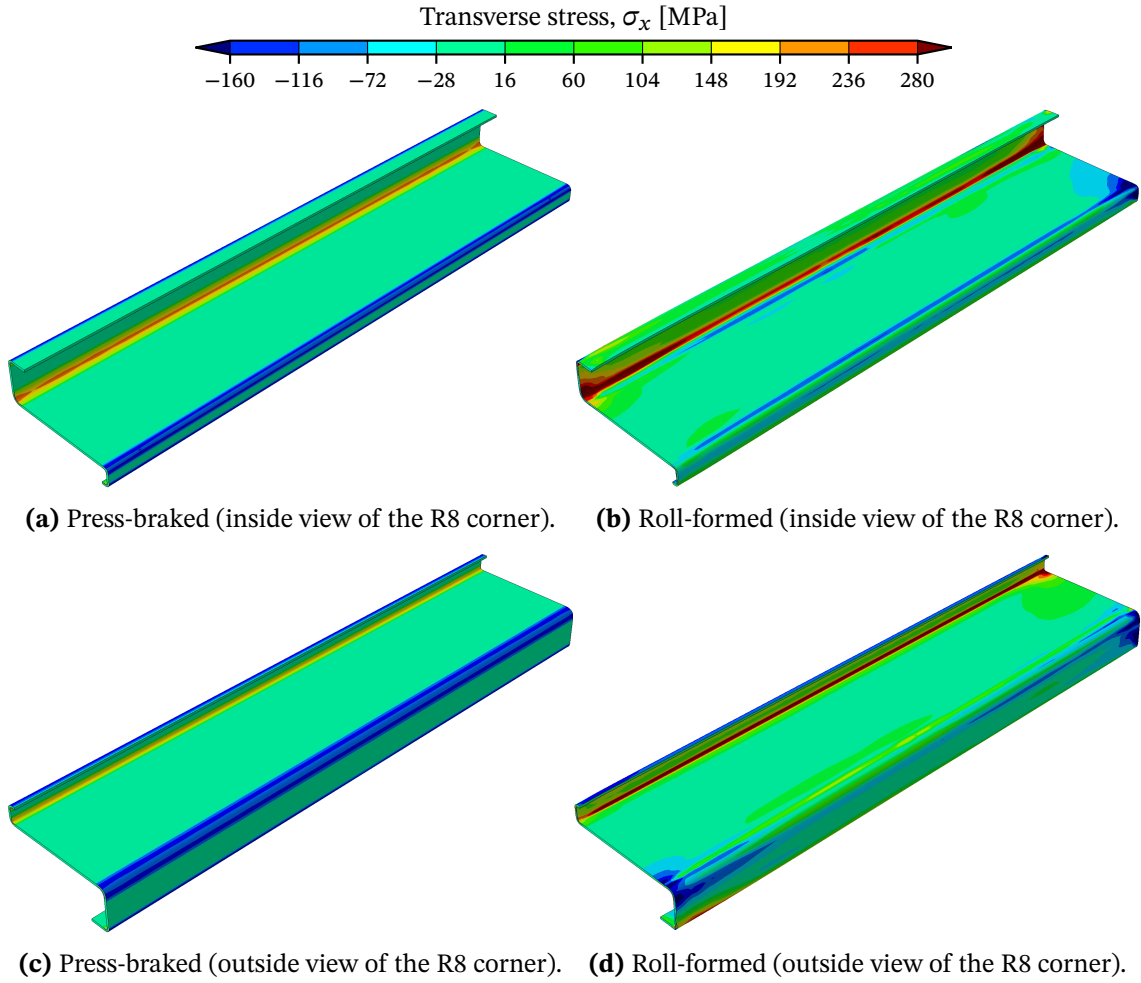


Figure 5.10: Transverse residual stress in press-braked and roll-formed Z-profiles.

[37], which were presented in [Section 2.1.4](#), are used for comparisons. Regarding these results, the following remarks can be made:

- For press braking, results from 2D plane strain solid models are almost identical to the corresponding results from general 3D solid models, validating the plane strain state.
- A shift in the neutral axis location towards the inside of the corner is expected during forming due to plasticity effects. This is indeed captured numerically, except for the S4R shell element. Moreover, the analytical solution due to Rondal also captures a shift in the neutral axis location, but a less significant one. Additionally, for the numerical results where the shift is captured, the neutral axis is located consistently between 40% and 45% of the nominal thickness.
- In press braking, solid elements following a pure-displacement formulation do not show the existence of an elastic centre (where null equivalent plastic strains are expected), which differs from the analytical solutions and the remaining numerical results. The

5: Numerical determination of cold forming residual stresses

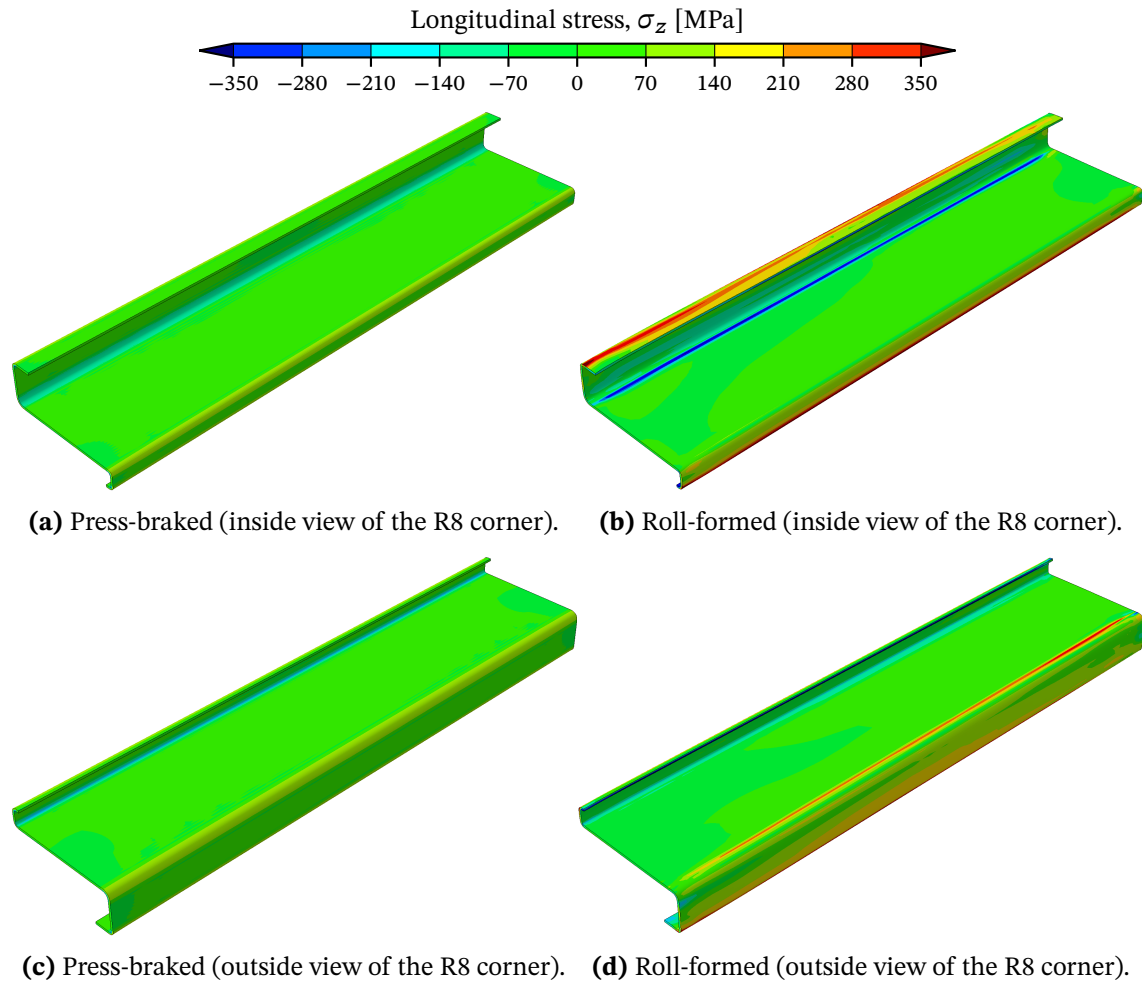


Figure 5.11: Longitudinal residual stress in press-braked and roll-formed Z-profiles.

same can be said for the roll forming results.

- Numerically and depending on the considered case, the equivalent plastic strain has a value of 20% to 30% at the considered crack initiation site, whereas analytically, a value of 17% to 19% is verified, depending on the considered solution.

Figure 5.14 presents the transverse and longitudinal residual stress distributions along path b (middle of the corner, across the thickness), which are characterized by a zigzag pattern due to elastic spring-back after elastoplastic bending. Moreover, along the considered path, transverse residual stresses represent the most relevant stress component, as expected. At the considered crack initiation site, most numerical results show a transverse residual stress value between 50% and 80% of the yield strength of the material, which is consistent with the analytical solutions. However, results arising from an incompatible mode and hybrid incompressible solid formulations show a higher value of around 115%. Regarding longitudinal residual stresses, values between 0% and –30% of the yield strength of the material are verified at the considered crack initiation site. Finally, these figures also show that the FEM is not cap-

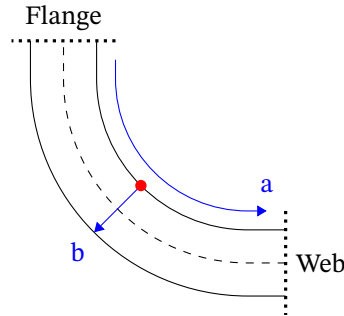


Figure 5.12: Definition of corner paths used to analyse the distribution of cold forming residual stresses. The dot represents the considered fatigue crack initiation site (point of interest).

able of perfectly capturing the abrupt discontinuity expected at around 50% of the normalized path distance; this is expected due to the interpolating nature of the FEM, where a continuous variation of stress and strain is assumed.

Figure 5.15 presents the transverse and longitudinal residual stress distributions along path a (inside the corner, along the surface). The considered crack initiation site corresponds to 50% of the normalized path distance. These figures showcase the difference between residual stress fields pertaining to press-braked and roll-formed profiles, which are found mainly outside the corner regions, i.e., within the flat web and flange regions.

5.5 The effect of pre-forming coiling operations

Sheet metal coils are generally used in cold forming and the required operations of coiling, uncoiling, and flattening can introduce additional residual stresses even before the forming process. As discussed in Section 2.1.4, the effects of pre-forming coiling operations are dictated by the r_x/t ratio, where r_x is the mean coiling radius and t is the sheet thickness, and more significant coiling effects are expected for lower values of r_x/t due to a more intense plastic action. During the FASTCOLD project, a ratio of around $r_x/t = 100$ was determined as a worst-case scenario for the rack industry, which is consistent with the literature [28–30, 140]. Consequently, this r_x/t value is considered hereinafter.

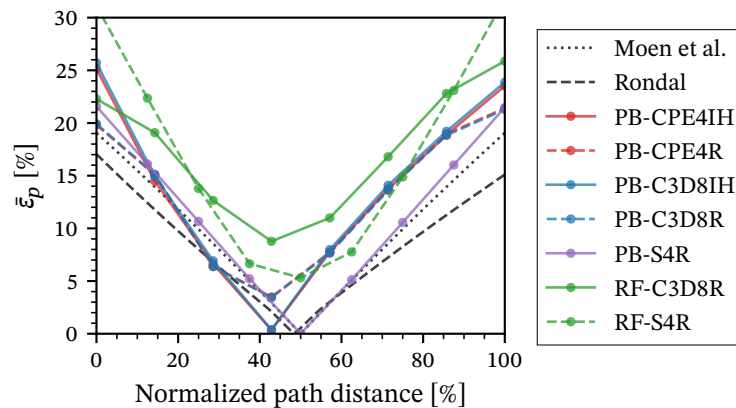


Figure 5.13: Equivalent plastic strain distribution along path b (middle of the corner, across the thickness) for the investigated cold-formed corner.

5: Numerical determination of cold forming residual stresses

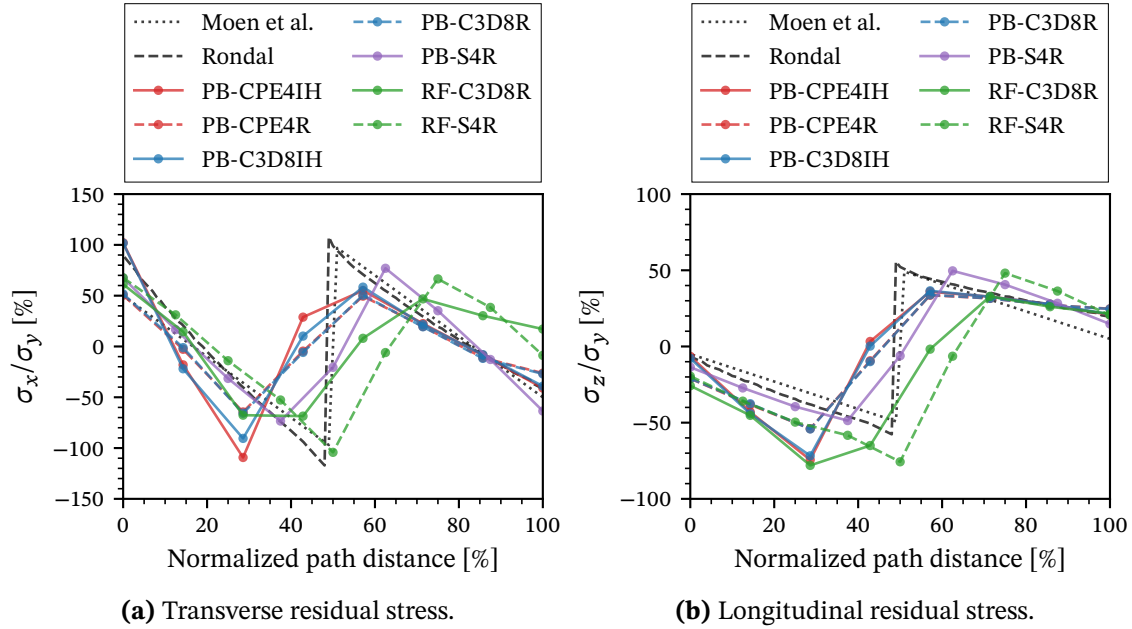


Figure 5.14: Transverse and longitudinal residual stress distributions along path b (middle of the corner, across the thickness) for the investigated cold-formed corner.

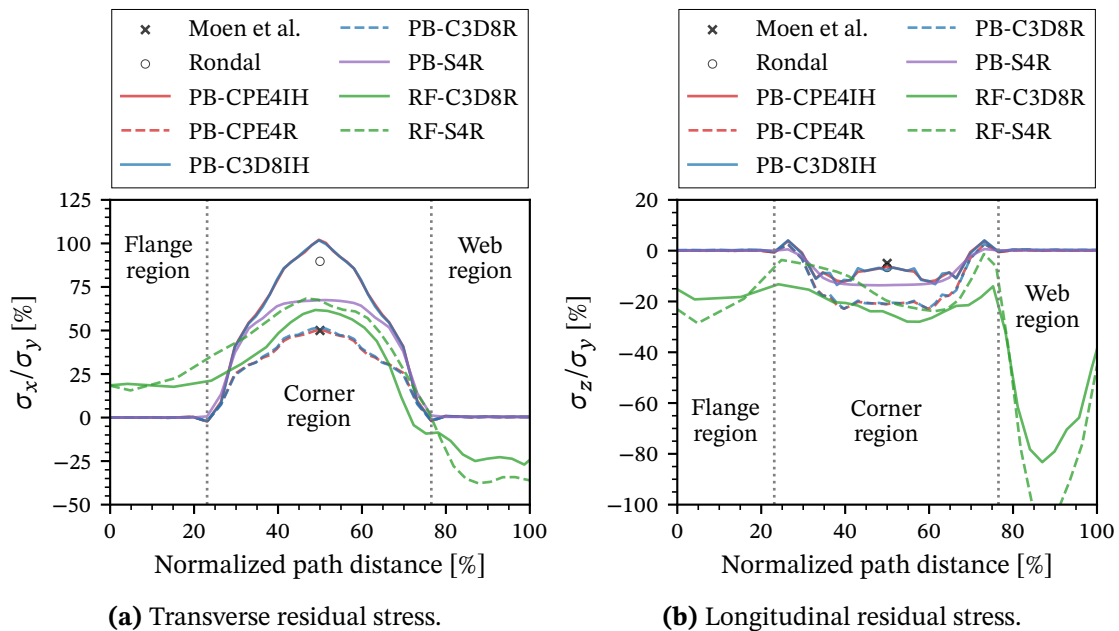


Figure 5.15: Transverse and longitudinal residual stress distributions along path a (inside the corner, along the surface) for the investigated cold-formed corner.

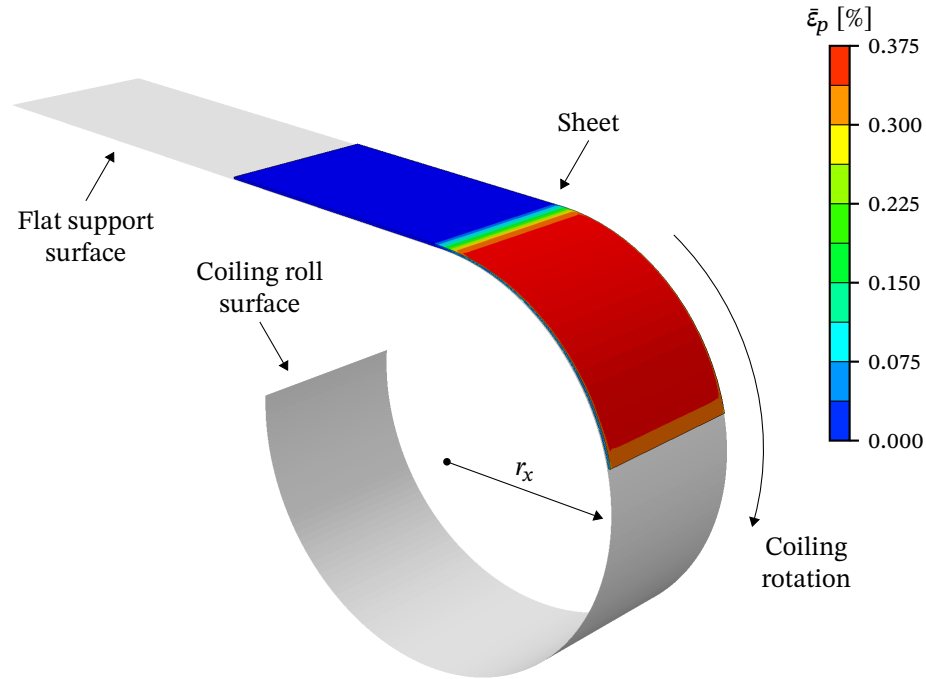


Figure 5.16: Simulation of the coiling process in Abaqus/CAE.

In this section, the objective is to consider the effects arising from pre-forming coiling operations in subsequent cold forming simulations. To this end, numerical simulations of the coiling processes are first conducted via the Abaqus/CAE environment. The modelling approach used for the coiling simulations is kept consistent with the one presented for the cold forming simulations, i.e., tools are considered rigid parts, the same material definition is used, the same contact modelling approach is considered, a quasi-static dynamic implicit analysis is conducted, the same finite element types that are not susceptible to locking are employed, as well as the same mesh resolution.

As shown in [Figure 5.16](#), the model includes the sheet and an analytical rigid surface that represents both a flat support and the coiling roll. While the surface is fixed, a rotation around the centre of the coiling roll is applied to the front of the sheet via kinematic coupling constraints, pulling and coiling the sheet. Moreover, separation between contacting surfaces is constrained, ensuring that the correct curvature is applied to the sheet. After the sheet has been fully coiled, the rotation is reversed, pushing the sheet and forcing it into a flat state due to no-separation contact, simulating the flattening operation. After the sheet has been coiled and subsequently flattened, the uncoiled state is obtained by gradually removing the no-separation contact constraints, releasing the sheet. Due to longitudinal residual stresses, the sheet curls, and a residual longitudinal curvature is left on the sheet. The sheet's state after these operations is shown in [Figure 5.17](#).

The simulation times for the pre-forming coiling simulations are shown in [Table 5.2](#). Of course, the computational cost is greatly reduced when considering a plane strain case. The usage of solid elements based on a pure-displacement formulation with reduced integration also decreases the computational cost since the incompatible mode and hybrid incompressible

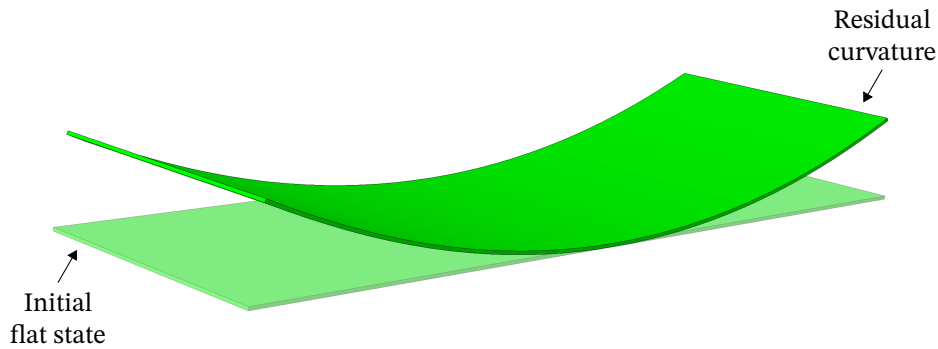


Figure 5.17: Longitudinal residual curvature after pre-forming coiling and uncoiling operations.

solid formulations rely on more integration points and increase the number of unknown variables. Moreover, as previously discussed, the expected performance gain of shell elements is outweighed by the loss of performance due to the increase in the number of contact elements, as shell elements require a surface-to-surface contact discretization, whereas solid elements are used with node-to-surface discretization. Ultimately, this study has showcased how shell elements can be inefficient when they are used in large contact problems, especially in ones where both surfaces of the shell are considered for contact.

Figure 5.18 shows the equivalent plastic strain and longitudinal stress distributions at the future corner location along path b (middle of the corner, across the thickness) on a fully coiled sheet. In general, a good agreement is found between numerical and analytical results, and any differences may be attributed to the fact that numerical results are based on more representative models while analytical results are based on additional simplifications. The figure also reveals an elastic core, as expected, but its location and size change based on the considered approach. Moreover, comparing Figure 5.13 vs. Figure 5.18a, it is clear that plastic strains in coiling have a much lower magnitude than those found after cold forming.

Figure 5.19 shows the equivalent plastic strain and longitudinal stress distributions at the future corner location along path b (middle of the corner, across the thickness) on a coiled, uncoiled, and flattened sheet. Some differences are found between numerical and analytical solutions, naturally arising from the distinct approaches employed in each method. Notably,

Table 5.2: Observed simulation times for the pre-forming coiling simulations.

Manufacturing process	Sheet modelling approach	Element type	Wall-clock time ^A
Coiling, uncoiling, and flattening	2D plane strain solid	CPE4IH	00:04:08
		CPE4R	00:03:31
	General 3D solid	C3D8IH	04:11:03
		C3D8R	01:49:28
	Shell	S4R	03:12:15

^A On a consumer-grade 7th-generation quad-core processor running at 3.60 GHz with 32 GB of memory.

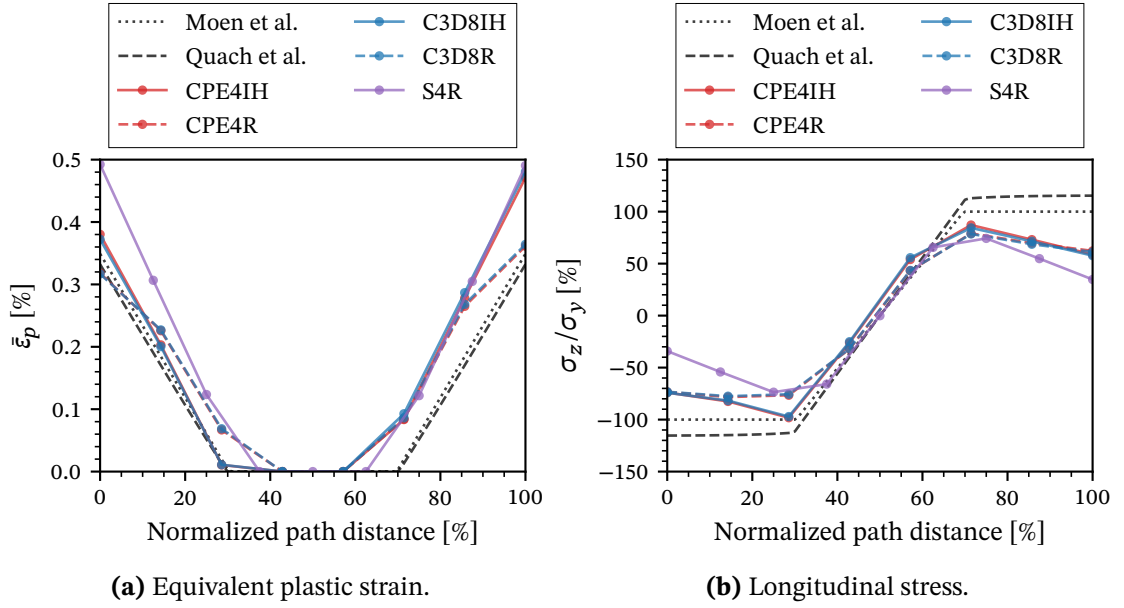


Figure 5.18: Equivalent plastic strain and longitudinal stress distributions at the future corner location along path b (middle of the corner, across the thickness) on a fully coiled sheet.

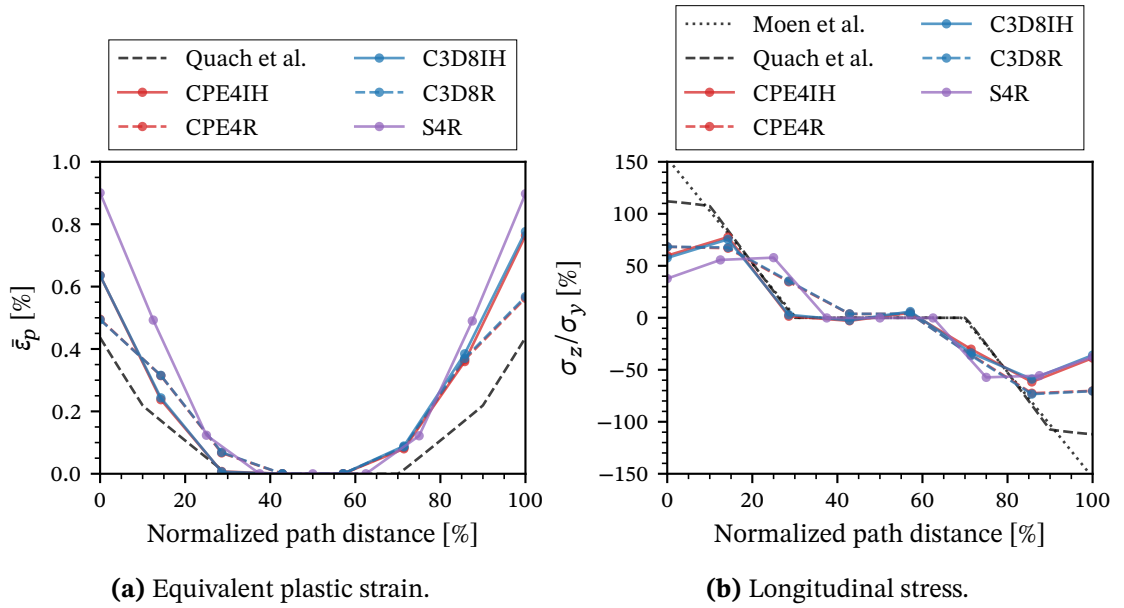


Figure 5.19: Equivalent plastic strain and longitudinal stress distributions at the future corner location along path b (middle of the corner, across the thickness) on a flattened sheet after coiling and uncoiling.

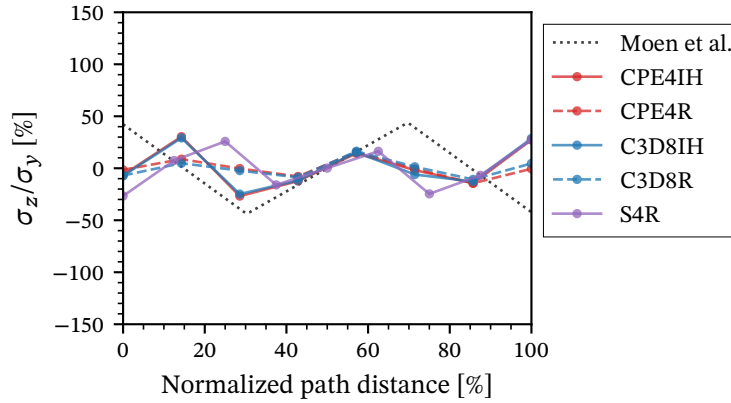


Figure 5.20: Longitudinal residual stress distribution along path b (middle of the corner, across the thickness) on a sheet after coiling operations.

it should be clarified that, in order to be consistent with the presented literature on the topic of coiling operations, and as discussed in [Section 2.1.4](#), the sheet's flattened state is enforced via external loads (in this case, arising from no-separation contact constraints). Hence, [Figure 5.19b](#) does not represent the final longitudinal residual stress field since the flattening loads are latter removed. The final longitudinal residual stress distribution across the thickness is shown in [Figure 5.20](#), where only the longitudinal component is shown because it is the most relevant one since transverse stresses are mainly due to the Poisson effect. As shown in this figure, the maximum magnitude of longitudinal residual stresses after coiling and uncoiling operations is around 50% of the yield strength of the material, which is significant.

[Figure 5.21](#) and [Figure 5.22](#) respectively compare the transverse and longitudinal residual stress fields of a press-braked Z-profile obtained with and without pre-forming coiling effects. As shown in these figures, coiling effects on the transverse residual stress field are minor, but they become significant when analysing the longitudinal residual stress field. Pre-forming coiling operations have a clear effect on the final residual stress field due to cold forming; however, these effects are found mainly in the straight regions, i.e., outside the cold-formed corners. Cold forming is then seen as the dominant process, contributing the most to the residual stresses found at those locations. It can also be noted that the corner bending process removes, almost completely, the longitudinal residual curvature due to pre-forming coiling operations (shown in [Figure 5.17](#)).

[Figure 5.23](#) shows the transverse and longitudinal residual stress distributions along path b (middle of the corner, across the thickness) for the press-braked Z-profile obtained with and without pre-forming coiling effects. The figure shows how, in this instance, coiling effects are negligible.

[Figure 5.24](#) shows the transverse and longitudinal residual stress distributions along path a (inside the corner, along the surface) for the press-braked Z-profile obtained with and without pre-forming coiling effects. The figure shows how coiling effects are minor in the transverse component but become relevant in the longitudinal one, particularly in the straight regions, i.e., outside the cold-formed corner.

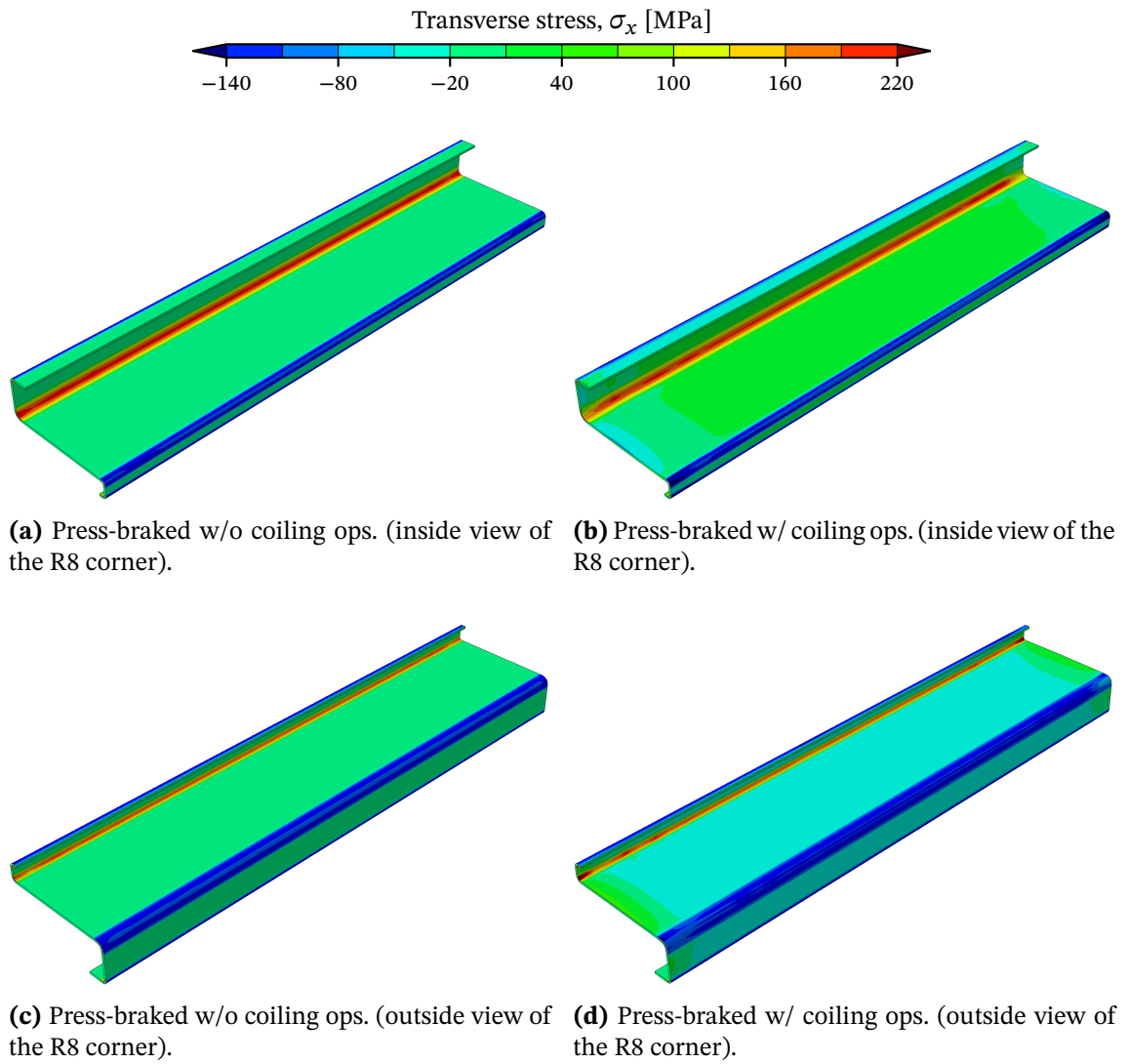


Figure 5.21: Effects of pre-forming coiling operations in the transverse residual stress field of press-braked Z-profiles.

5: Numerical determination of cold forming residual stresses

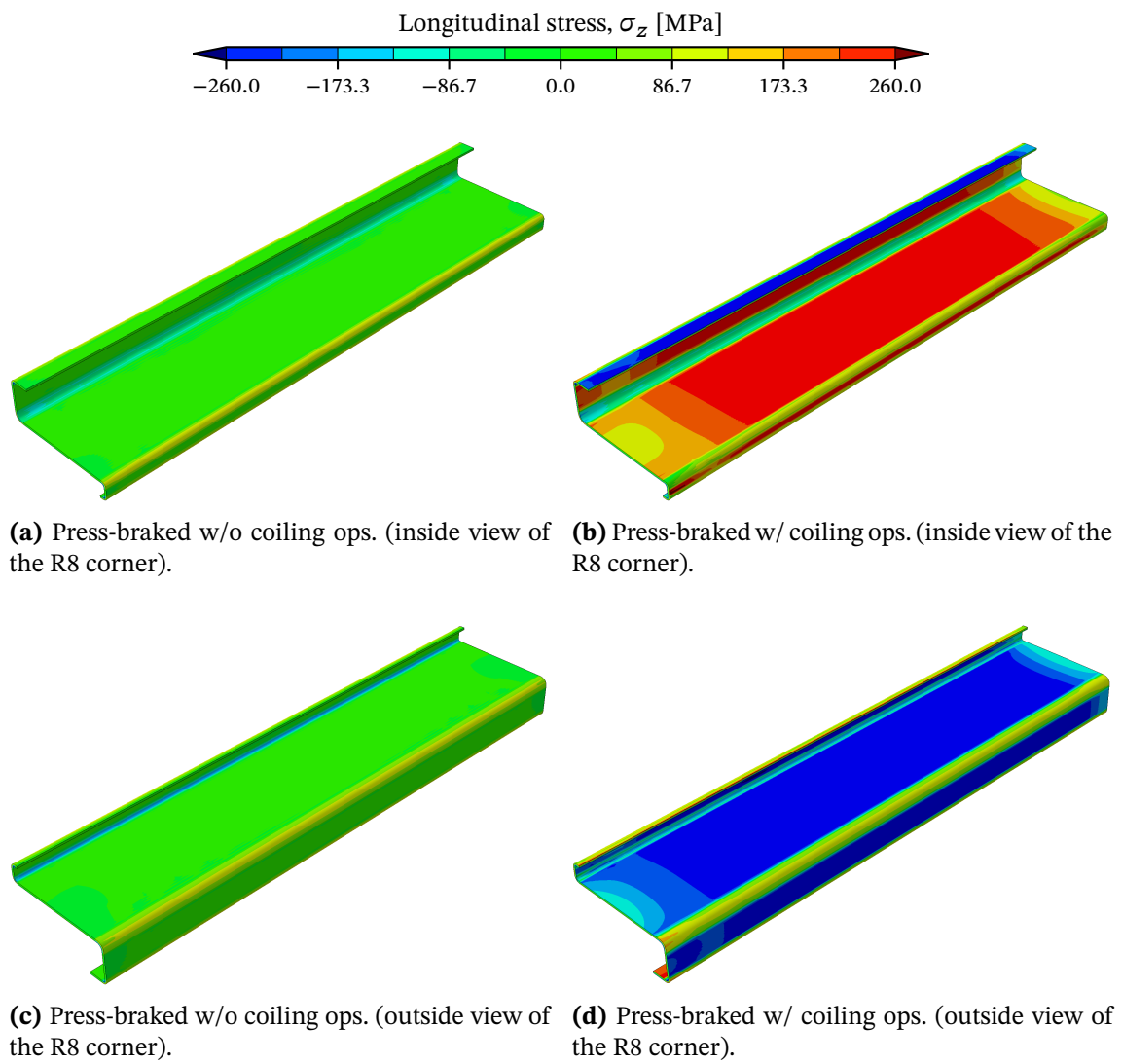


Figure 5.22: Effects of pre-forming coiling operations in the longitudinal residual stress field of press-braked Z-profiles.

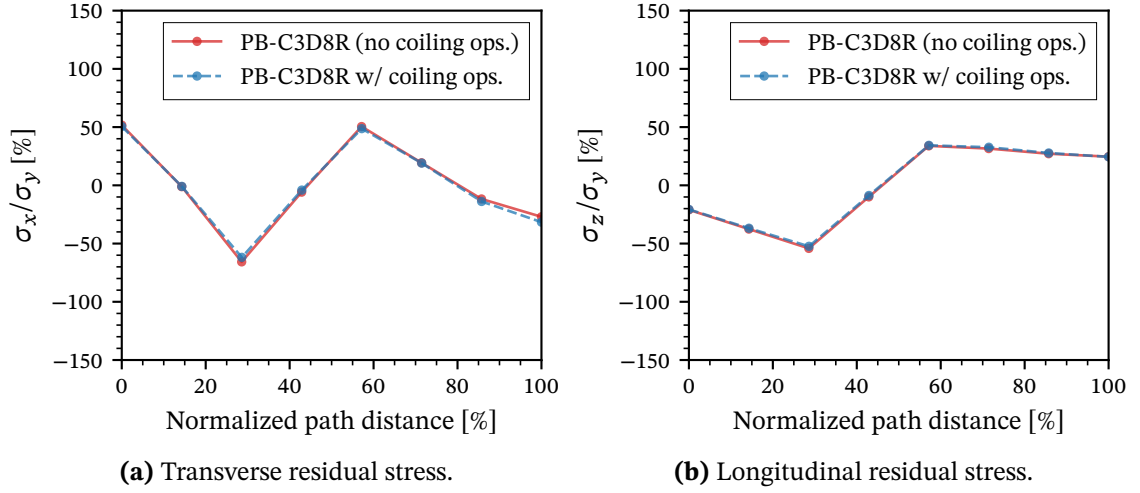


Figure 5.23: Effects of pre-forming coiling operations in the transverse and longitudinal residual stress distributions along path b (middle of the corner, across the thickness) for the investigated cold-formed corner.

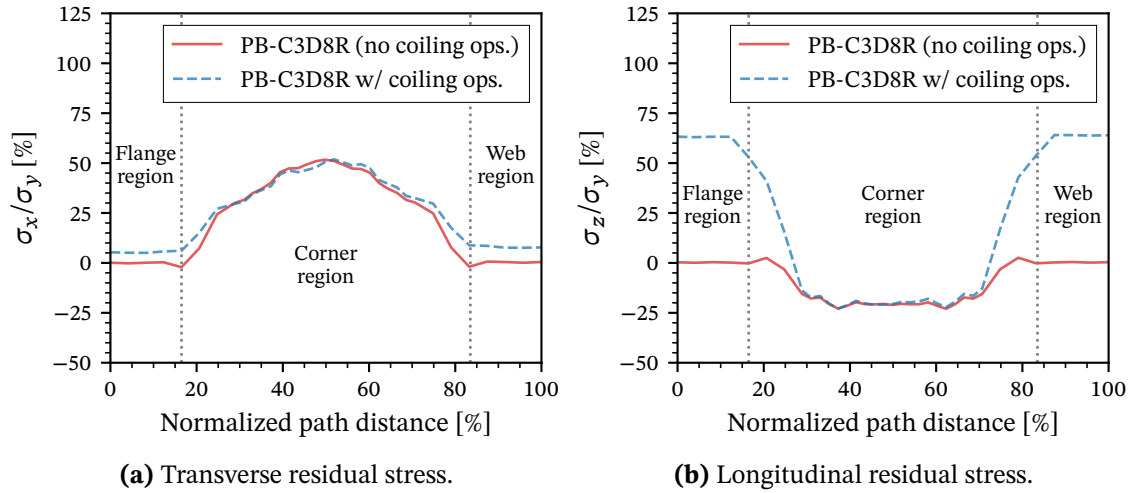


Figure 5.24: Effects of pre-forming coiling operations in the transverse and longitudinal residual stress distributions along path a (inside the corner, along the surface) for the investigated cold-formed corner.

5.6 Comparison against results found in the literature

The experimental determination of cold forming residual stresses is difficult to accomplish when considering thin-walled profiles, particularly when nonlinear distributions across the thickness are expected [27]. In this regard, when experimental results are presented in the literature, they are usually limited to the flat regions [13, 93, 94] or, when considering results at the cold formed corners, results are usually limited to the surfaces [13, 31, 93–97].

Weng and White (1990) [33, 98] were among the first researchers to provide a through-thickness distribution of residual stresses in press-braked steel corners. To this end, experimental hole drilling and sectioning techniques were used. However, the through-thickness distribution was only provided for the transverse component of residual stresses, whereas longitudinal residual stresses were only made available at the surfaces. Additionally, while the current work is focused on thin-walled profiles (material thickness of 3.5 mm) made of mild steel (S355MC steel grade), Weng and White considered thick plates (material thickness of 25.4 mm) made of high-strength steel (HY-80 steel grade). Nevertheless, a similar inside corner radius to material thickness ratio is considered in this work (ratio of 2.3) and by Weng and White (ratio of 2.5; among others considered).

Later, Quach et al. (2006) [30] numerically simulated the press braking process used to produce the specimens considered by Weng and White (1990) [33, 98]. These simulations were carried out using Abaqus, where the previously presented CPE4R finite element type was considered, assuming a 2D plane strain condition.

A comparison of cold forming residual stresses due to press braking along path b (middle of the corner, across the thickness; see Figure 5.12) is presented in Figure 5.25. Besides the results arising from the current work and those from Weng and White (1990) [33, 98] (experimental) and from Quach et al. (2006) [30] (numerical), the figure also shows, for reference, results via

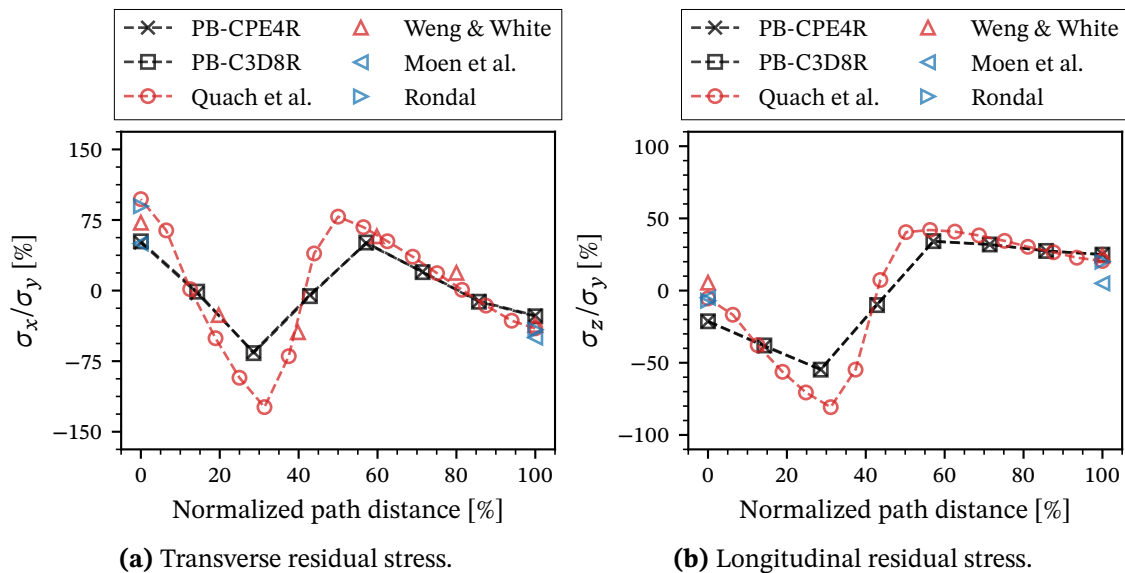


Figure 5.25: Comparison of cold forming residual stresses due to press braking along path b (middle of the corner, across the thickness) for the investigated cold-formed corners.

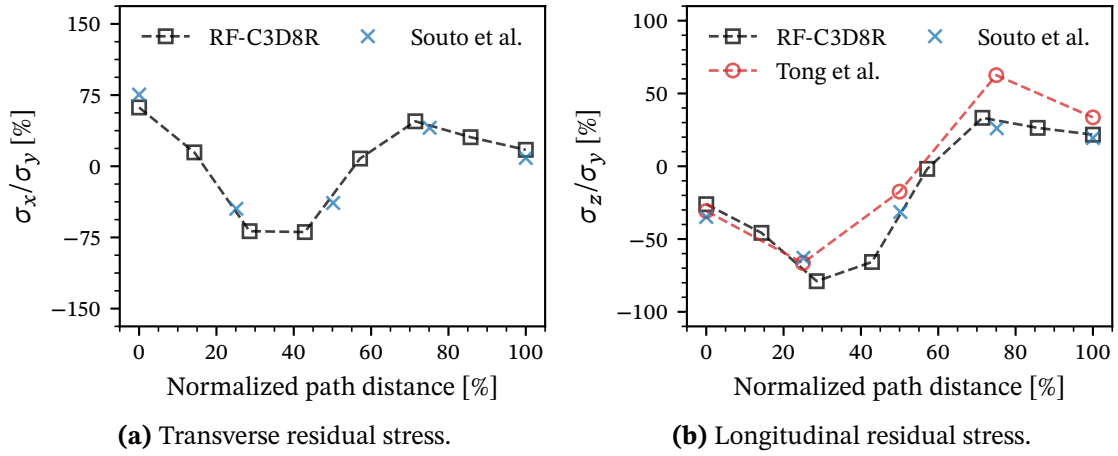


Figure 5.26: Comparison of cold forming residual stresses due to roll forming along path b (middle of the corner, across the thickness) for the investigated cold-formed corners.

the previously presented residual stress modelling approaches by Moen et al. (2008) [28] and Rondal (1987) [37] (see Section 2.1.4). As shown by Figure 5.25, a decent match is verified when considering the presented normalized results. Therefore, considering that the results from Weng and White and from Quach et al. are based on thicker plates and a higher-strength material, it would appear that the residual stress distribution is mainly a function of the corner radius to material thickness ratio and yield strength of the material, as expected.

Regarding roll forming, Tong et al. (2012) [99] used experimental hole drilling and X-ray diffraction methods to measure the longitudinal residual stress distribution across the thickness of cold roll-formed square and hollow mild steel sections. Moreover, Souto et al. (2022) [13] presented preliminary numerical simulations of the cold roll-forming process of the currently considered Z-profile (same case study). However, Souto et al. used the COPRA FEA RF software, a specific FEA package based on MSC Marc and aimed exclusively at roll forming. Due to the long simulation times encountered by the authors, course meshes were considered, relying on trilinear hexahedral elements; nevertheless, to account for volumetric locking, which is common in large strain plasticity analyses, a constant dilatation integration scheme was used (also known as the B-bar method for incompressible materials), as well as an assumed strain formulation to account for shear locking during corner bending.

A comparison of cold forming residual stresses due to roll forming along path b (middle of the corner, across the thickness; see Figure 5.12) is presented in Figure 5.26. Results arising from the current work and those from Tong et al. (2012) [99] (experimental) and from Souto et al. (2022) [13] (numerical) are shown. A decent match is verified when considering the presented normalized results, validating the obtained residual stress field at the crack initiation site.

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Chapter 6

Fatigue behaviour of cold-formed thin-walled mild steel members

Rack structures are characterized by the application of cold-formed thin-walled mild steel members joined via bolted connections. In this chapter, we look at specific applications in which rack structures serve as the structural framework for automated S&R solutions, e.g., pallet carrier shuttles and stacker cranes, which offer an efficient and continuous operation to satisfy the current demands of warehouse logistics. These structures are then subjected to fatigue loads arising from the cyclic storage of heavy palletized goods combined with the dynamic actions of moving S&R systems. This eventually leads to fatigue cracking, as explained in [Section 1.1](#) (see [Figure 1.3](#)), which raises safety concerns and is responsible for expensive maintenance and downtimes.

Although rack structures are commonly found in warehouses, distribution centres, retail stores, and other facilities where efficient storage and easy access to items are essential, cold-formed steel and thin-walled structures are completely missing from current versions of European standards for fatigue design (as of the time of writing). In fact, Eurocode 3 covers the static design of thick sections in EN 1993-1-1 [3] and that of cold-formed members and sheeting in EN 1993-1-3 [4]; however, in regard to fatigue design, EN 1993-1-9 [5] only covers thick, hot-rolled sections. Furthermore, the EN 15512 standard [6], dedicated to the structural design of adjustable pallet racking systems, acknowledges fracture due to fatigue as a form of ultimate limit state; however, the standard makes no further reference to fatigue, later stating that ‘it is therefore implicit that normal rack structures are not subject to fatigue...’. Nevertheless, there are North American standards, such as the AISI S100-16 standard [7], addressing structural members and connections made of cold-formed steel subject to cyclic loading within the elastic range of stresses of frequency and magnitude sufficient to initiate cracking and progressive failure, defining the limit state of fatigue.

Within this background, arises the FASTCOLD European research project, which aimed the development of new fatigue design guidelines for constructional cold-formed mild steel details commonly found in the rack industry, filling the identified gap in the currently available European design standards for fatigue. While the FASTCOLD project has been completed, it generated important fatigue data, supporting the research presented in this chapter.

Currently, fatigue research revolves around global and local total life approaches as well as damage-tolerant approaches based on fracture mechanics and stable fatigue crack growth

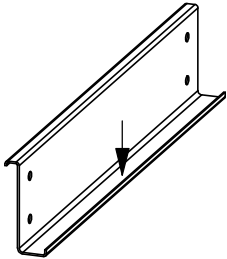
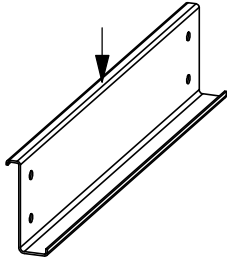
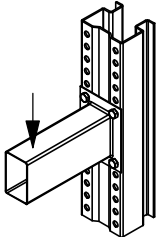
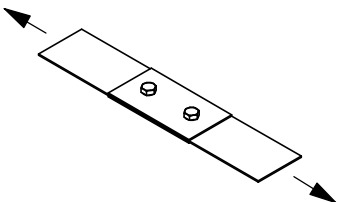
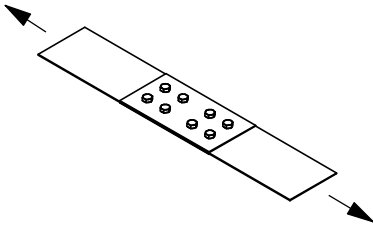
[2]. The stochastic nature of fatigue may also be included in structural integrity assessments via probabilistic fatigue life prediction models and frameworks. Nevertheless, in all cases, published research is usually supported by fatigue testing campaigns considering small-scale specimens tested in standard fatigue testing machines, where information regarding actual loading conditions during service and material hardening history due to manufacturing is not fully taken into account. In this regard, the research presented within this chapter elevates the current state of the art. In particular, this chapter presents the results of several fatigue testing campaigns performed during the FASTCOLD project, where experimental data is obtained from the testing of full-scale cold roll-formed mild steel profiles sampled directly from the manufacturer, besides small-scale specimens extracted from those profiles. The full-scale profiles are tested in purpose-built setups, aiming the recreation of actual service conditions, while small-scale specimens are used to reinforce the reliability of the generated $S-N$ data. Moreover, fatigue life predictions are made following two innovative approaches, i.e., a global $S-N$ approach based on numerically computed peak pseudo-elastic maximum principal stresses, which account for all sources of stress concentrations during loading, and a local multiaxial fatigue approach based on the mean stress corrected Basquin-Coffin-Manson model combined with elastoplastic FEA, where the results from cold forming simulations are used as the initial state in subsequent structural analyses, including in this way the effects of manufacturing residual stresses in the final structural and fatigue behaviours. Overall, these additional considerations should result in more reliable and trustworthy fatigue life predictions, which are made via a post-processing procedure.

6.1 Fatigue testing campaigns

The fatigue details investigated during the FASTCOLD project are shown in [Table 6.1](#). These details were investigated during distinct fatigue testing campaigns. Regarding details 4 and 5, the bolted joints were made of thin plates of zinc-coated S350GD steel. These should represent the bolted connections commonly used in the rack industry, which rely on preloaded and non-preloaded bolts in normal clearance holes, and are not subjected to load reversals. These details were tested under uniaxial loading conditions in servohydraulic machines, and a full fatigue testing campaign was performed, evaluating the effect of bolt size, bolt configuration, bolt preload level, and bolt hole production process, while considering distinct load levels and load ratios. See [Chapter 4](#) for a full description, results, and discussion regarding this topic.

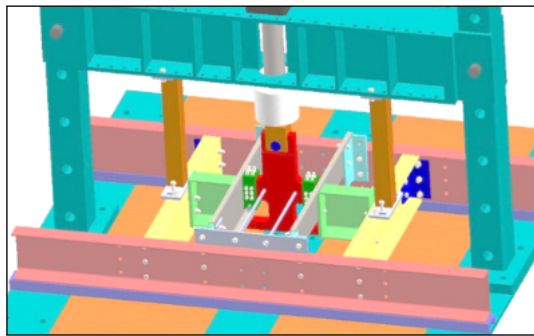
Fatigue details 1 and 2 pertain to thin-walled mild steel Z-profiles obtained via cold roll forming. The considered material is the S355MC steel grade (thickness of 3.5 mm), which is presented in [Chapter 3](#). These profiles are used as guide rails for pallet carrier shuttles (see [Figure 1.2](#)). In particular, detail 1 corresponds to the localized bending of the lower flange due to the passage of a loaded shuttle, whereas detail 2 corresponds to the localized bending of the upper flange due to the static weight of stored palletized goods. In both cases, the deflection of the flange creates a stress concentration at the cold-formed web-to-flange corners, which are seen as the critical spot. Details 1 and 2 were tested in a purpose-built setup during a full-scale fatigue testing campaign. The setup is shown in [Figure 6.1](#). Additionally, [Figure 6.2](#) shows how this setup can be used in a downward and an upward loading scenario with the intent of respectively representing details 1 and 2. Tests are performed under load control, and a load ratio of $R = 0.1$ is considered so that proper contact between the load actuator and the

Table 6.1: Fatigue details investigated during the FASTCOLD project.

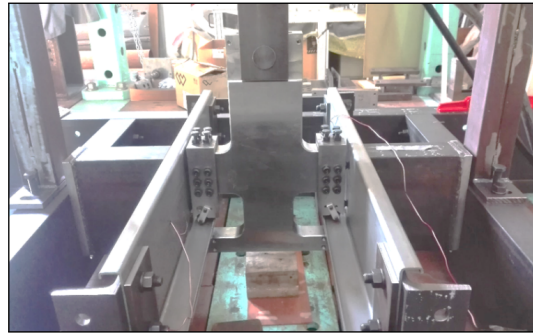
#	Fatigue detail	Description
1		Cold-formed thin-walled mild steel profile under local bending of the lower flange corresponding to the passage of a moving pallet carrier shuttle.
2		Cold-formed thin-walled mild steel profile under local bending of the upper flange corresponding to the static weight of stored palletized goods.
3		Bolted beam-to-upright connection of cold-formed thin-walled mild steel sections with punched perforations. Initiation site at the upright due to push-pull bolt forces.
4		Double lap joint under tension made of thin galvanized mild steel plates with preloaded or non-preloaded bolts. 1+1 bolt array with punched or drilled holes.
5		Double lap joint under tension made of thin galvanized mild steel plates with preloaded or non-preloaded bolts. 4+4 bolt array with punched or drilled holes.

profile's flange is maintained during testing. As shown in [Figure 6.3](#), a strain gauge is placed at a strategic location for the purpose of fatigue crack monitoring. As a crack develops in the corner during testing, the discontinuity in the stress field is captured by the strain gauge in the form of an apparent load reduction. The fatigue life of the profile is then considered when a

6: Fatigue behaviour of cold-formed thin-walled mild steel members

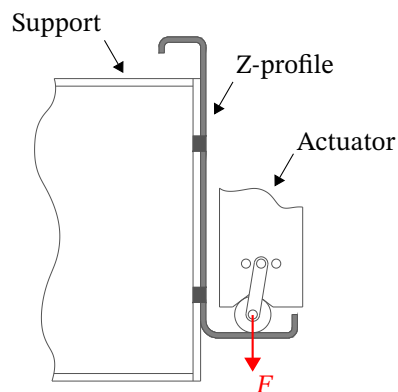


(a) CAD model.

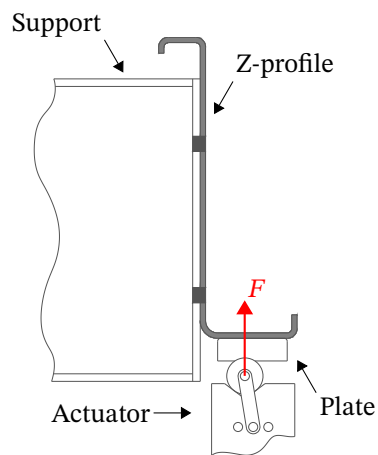


(b) Picture of the setup.

Figure 6.1: Purpose-built setup used during the FASTCOLD project (University of Porto) for the fatigue testing of full-scale Z-profiles.



(a) Load actuator placed on top of the lower flange (downward load case).



(b) Load actuator placed underneath the lower flange (upward load case).



Figure 6.2: Downward and upward load cases considered during the fatigue testing of full-scale Z-profiles.

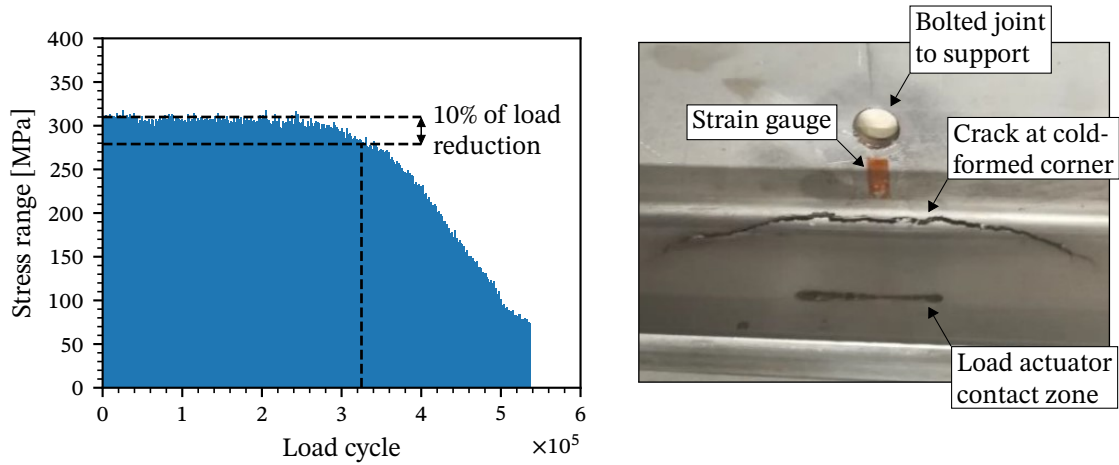


Figure 6.3: Strain gauge used for fatigue crack detection and monitoring.

10% load decrease is verified by the strain gauge, which should represent a through-thickness crack. More information regarding the development of the experimental setup can be found in reference [141]. A total of 36 profiles were tested considering distinct fatigue loads.

Fatigue detail number 3 represents a bolted beam-to-upright connection of cold-formed thin-walled mild steel sections, where the initiation site is located in the upright due to push-pull bolt forces. As shown in Figure 1.3b, the cold-formed corner is again seen as a critical region. While details 1, 2, 4, and 5 were tested at the University of Porto, detail 3 was tested by other partners of the FASTCOLD project (University of Pisa) in a full-scale testing campaign, following the same general approach as the one discussed above for details 1 and 2. The purpose-built setup is shown in Figure 6.4. The geometry consists of a cantilever beam

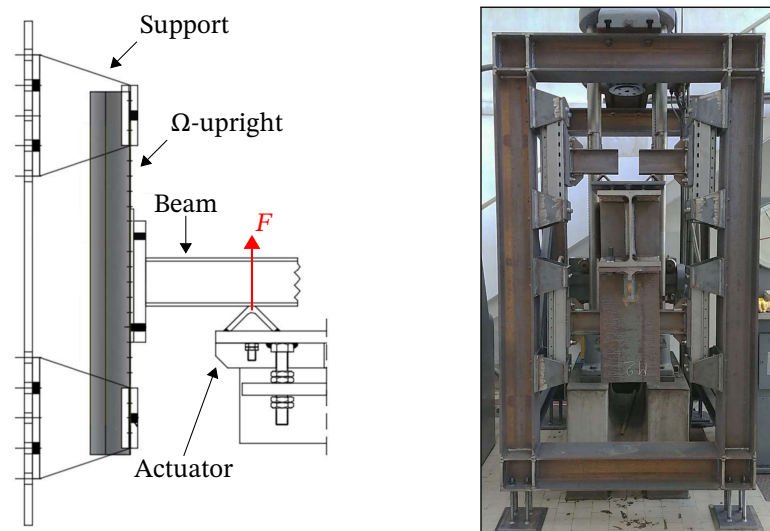


Figure 6.4: Purpose-built setup used during the FASTCOLD project (University of Pisa) for the fatigue testing of full-scale Ω -uprights [14, 142].

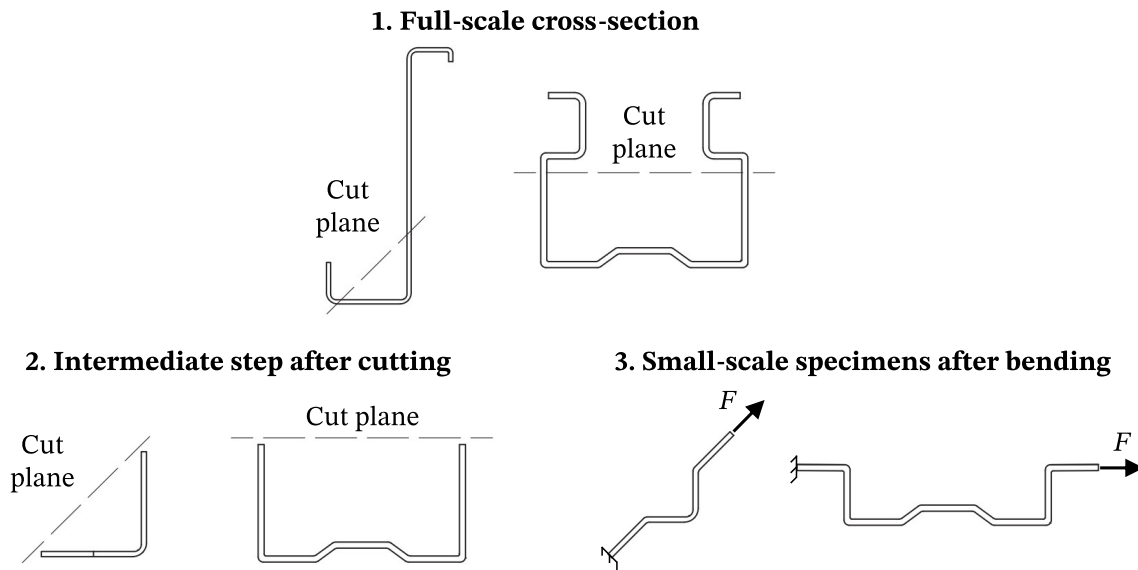


Figure 6.5: Extraction of small-scale specimens from full-scale sections [14, 101].

with a welded end-plate bolted to a fixed upright (column). The load actuator creates a bending moment that will result in localized deformation at the web of the upright, in the vicinity of the bolt slots and cold-formed corners. Four distinct Ω -upright sections were considered, combining different materials (S355 and S460 steel grades), section geometries, and manufacturers. More information regarding the development of the experimental setup can be found in reference [142]. A total of 56 uprights were tested considering distinct fatigue loads.

Besides the full-scale tests, details 1, 2, and 3 were also tested in small-scale experiments by other partners of the FASTCOLD project (Polytechnic University of Milan). For this purpose, the specimens were sampled from their full-scale counterparts, as shown in Figure 6.5. The small-scale specimens are then tested under uniaxial loading in servohydraulic machines; see reference [101] for the full details. However, small-scale specimens were also tested in a purpose-built device based on the principles of harmonic motion and resonance of single-mass oscillators. This setup is shown in Figure 6.6 (Aachen University; see reference [90]).

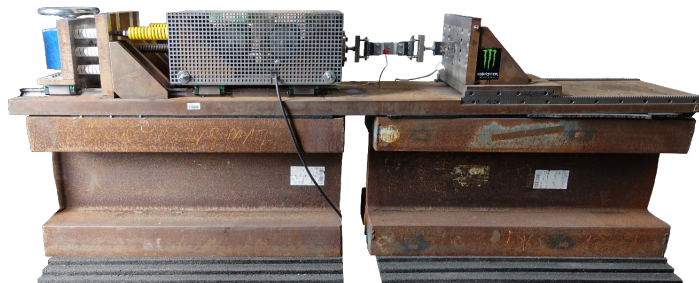


Figure 6.6: Purpose-built testing device used during the FASTCOLD project (Aachen University) in the fatigue testing of small-scale specimens [14, 90].

A full description of the aforementioned experimental fatigue testing campaigns can be found within the FASTCOLD project deliverables and final report, which are publicly available on request; moreover, a database containing all the experimental results is also available. A final summary of the FASTCOLD research is also presented by Souto et al. (2023) [14].

6.2 Numerical modelling of the experimental setups

Regarding the Z-profiles, i.e., details 1 and 2 from Table 6.1, finite element models simulating the aforementioned fatigue testing setup were built for determining the stress state of the profiles during structural and fatigue loading. The models are built within the Abaqus/CAE environment, sharing the same geometry and material definition from the cold forming numerical models presented in the previous chapter. The finite element mesh is also kept the same; however, for simplicity, only the C3D8R element is now considered. Nevertheless, some nuances are as follows:

- A static analysis is now performed for the structural simulations, whereas a dynamic implicit quasi-static stepping procedure is used for the cold forming simulations. This is because the simpler structural simulations do not require the increased solver stability of the latter approach.
- Contact is still considered frictionless, but a surface-to-surface contact discretization is considered and the resulting constraints are enforced via Lagrange multipliers. This is now the case since the structural simulations are much less demanding than the cold forming simulations, allowing for this preferred, more accurate contact modelling approach.
- Regarding the boundary conditions, the bolted connections to the testing setup supports are modelled by fixing the nodes corresponding to the supported locations, as shown in Figure 6.7. This simplified approach is deemed acceptable since the critical spot is found

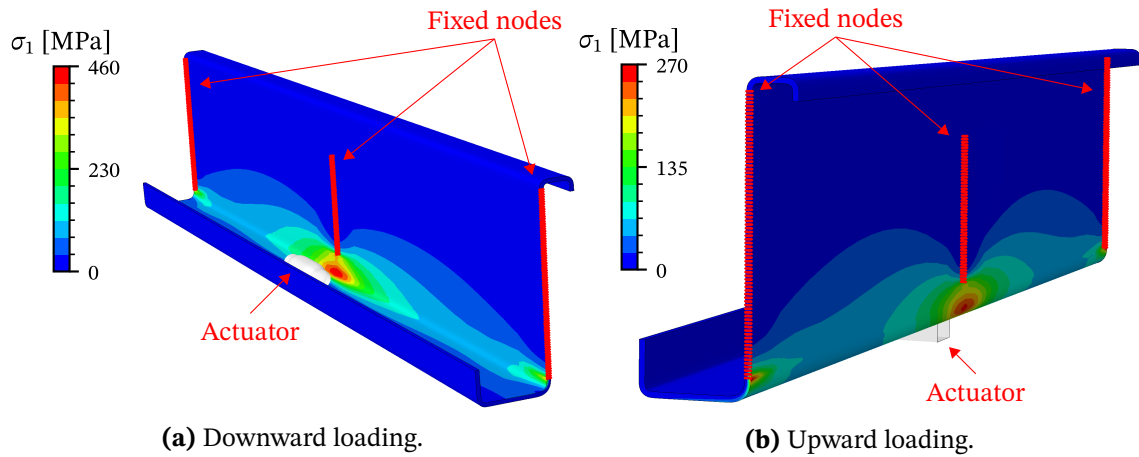


Figure 6.7: Pseudo-elastic maximum principal stresses on full-scale Z-profiles.

6: Fatigue behaviour of cold-formed thin-walled mild steel members

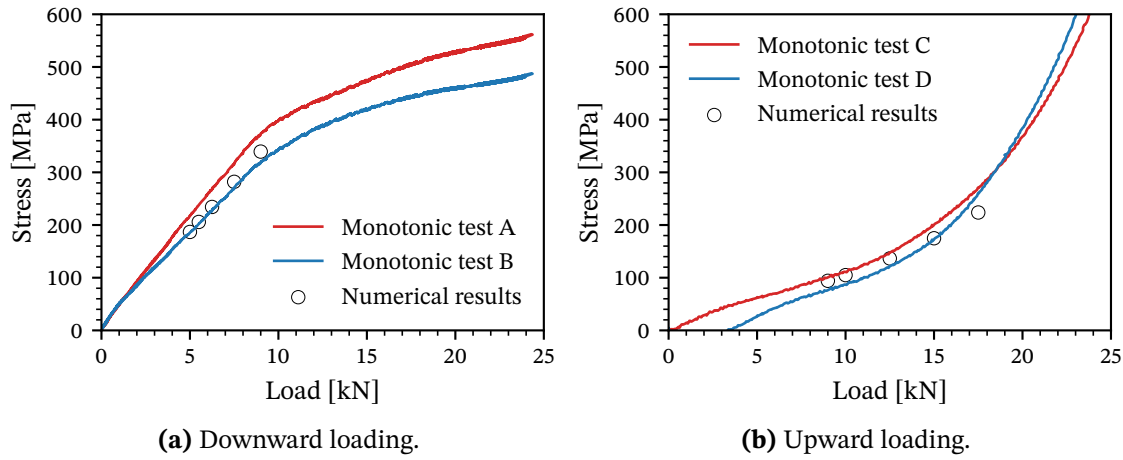


Figure 6.8: Validation of the finite element models pertaining to the full-scale Z-profiles.

away from the bolt holes; otherwise, a more accurate connection modelling approach could have been considered.

- Regarding the loading conditions, the load actuator is modelled as an analytical rigid part, and it is displaced against the lower flange until the intended load is achieved as the magnitude of reaction force at the actuator's reference/control point.

Figure 6.7 shows the finite element models created in Abaqus/CAE for the downward and upward loading scenarios. In order to validate the models, results from monotonic tests are compared against numerical results. This comparison is shown in Figure 6.8, where a satisfactory match is verified. To clarify, numerical results are extracted from the mesh node deemed closest to the strain gauge location, and the relevant normal stress component is considered. Moreover, experimental values are obtained by converting registered strains into

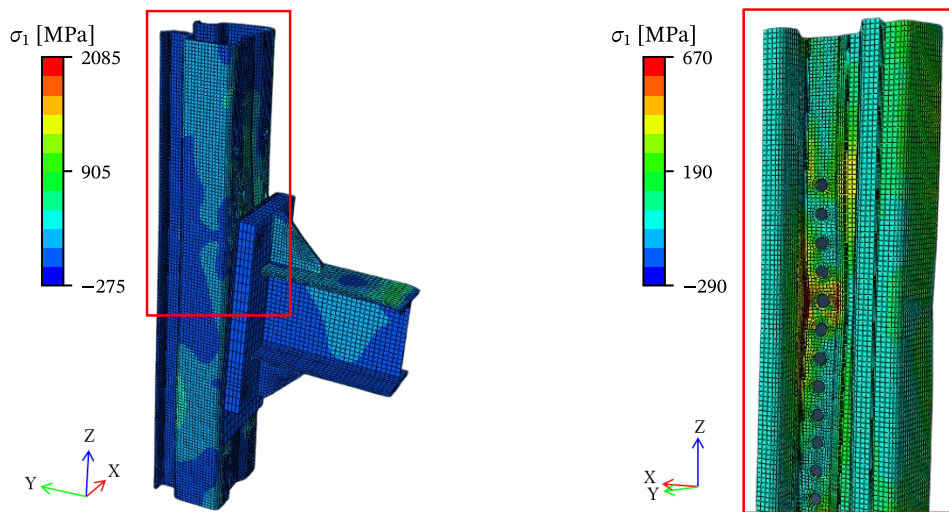


Figure 6.9: Pseudo-elastic maximum principal stresses on full-scale Ω -uprights.

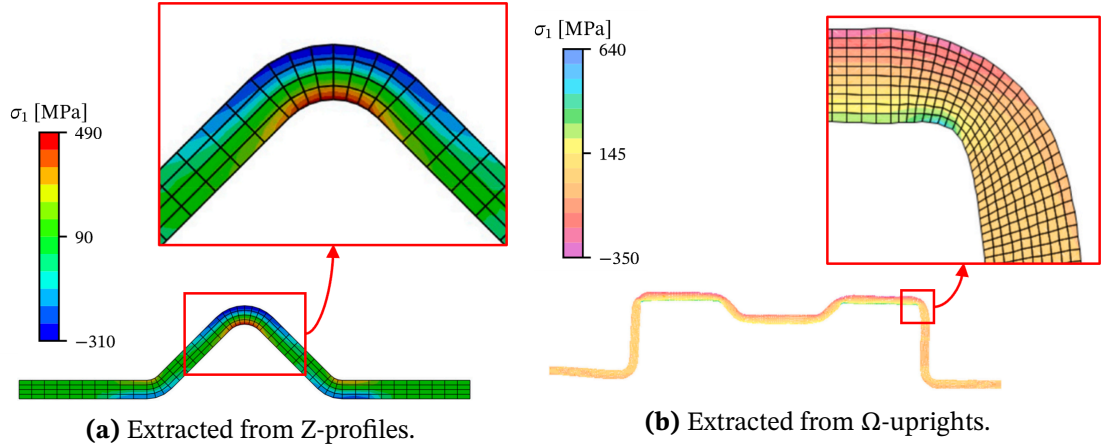


Figure 6.10: Pseudo-elastic maximum principal stresses on small-scale specimens (plane strain models).

stresses assuming a linear-elastic constitutive relationship ($\sigma = E\varepsilon$). Finally, the load pertains to the registered actuator's load divided by the two profiles that are tested simultaneously.

Regarding the bolted beam-to-upright connections (detail 3 from Table 6.1) and the supplementary small-scale specimens, finite element models were also created by the FASTCOLD partners who performed the corresponding fatigue tests. These models are shown in Figure 6.9 and Figure 6.10, and they are used for the numerical computation of pseudo-elastic maximum principal stresses in order to generate additional S - N data.

6.3 Fatigue life predictions based on a global approach

In the proposed global approach, the ASTM E739 standard procedure [66] is used to generate mean S - N curves for the obtained experimental data. The objective is to build master curves capable of capturing several of the tested conditions. This is accomplished by numerically evaluating the stress states of the specimens during fatigue testing. As previously discussed, the FEM is used for this purpose via commercially available software applications. Solid elements are considered in the numerical models, and S - N data is generated by considering peak pseudo-elastic maximum principal stresses (i.e., the largest eigenvalue of the stress tensor at the critical location), capturing in this way all sources of stress concentration and simplifying the analysis of complex multi-axial cases. Maximum principal stresses are also invariant and no particular coordinate system is required. Moreover, it is understood from fracture mechanics that cracks tend to initiate and propagate perpendicularly to the maximum principal stress direction since it usually represents the direction of tensile stresses. The term *pseudo-elastic* is used simply to denote that, in this case, stresses are based on a linear-elastic material model even if plasticity effects are present during testing. Such an approach is common for design purposes and is consistent with EN 1993-1-9 (Eurocode 3) [5].

The procedure mentioned above has already been successfully applied to the bolted joints, as presented in Chapter 4. However, for the bolted joints, this procedure was introduced as an alternative to the already well-established global approaches based on nominal stress meas-

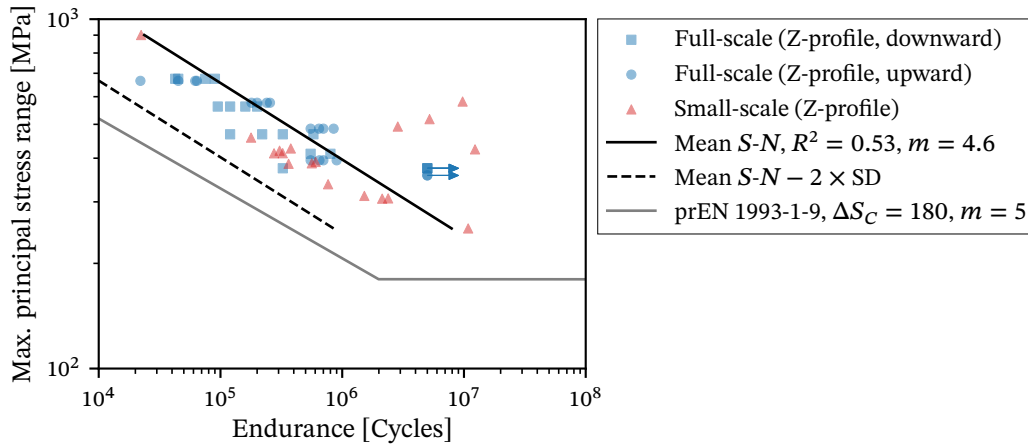


Figure 6.11: Overview of the generated S - N data for the tested full-scale Z-profiles and corresponding small-scale specimens.

ures. Now, this new procedure is seen as the only option for the Z-profiles and Ω -uprights since the application of classical approaches is no longer straightforward due to difficulties in defining a correct measure of applied nominal stress. Alternatively, load vs. endurance curves could have been used, but this would result in a less general detail categorization system.

Regarding the supplementary small-scale specimens, the actual specimen geometry was measured by other authors (see reference [101]) and considered in the finite element calculations. Additionally, the true inside corner radius to thickness ratio (r_i/t) was also determined for each specimen. As presented by Castiglioni and Menghini (2022) [101], small deviations were found between the measured geometry and the nominal one (i.e., the one found in drawings delivered by the manufacturer) for the specimens extracted from the Z-profiles, whereas specimens extracted from the Ω -uprights show higher scatter. Nevertheless, such deviations are expected, arising from the production process, and the fatigue data will be analysed taking into account the average r_i/t ratio.

Figure 6.11 shows the obtained S - N data for the tested Z-profiles and derived small-scale specimens, where the mean S - N curve includes all 54 data points. The dashed line represents a translation of the mean curve by twice the standard deviation, which can be seen as a lower bound for the S - N data (delimiting the 97.72% survival proportion). Although some scatter is present, this lower bound is very consistent with the upcoming prEN 1993-1-9 standard (Eurocode 3 revision) [8], which suggests a slope of $m = 5$ for rolled products. Hence, the category of $\Delta S_C = 180$ MPa with $m = 5$ for an endurance of $N_f \leq 2 \times 10^6$ load cycles may be recommended. Figure 6.11 also reveals the consistency between the full- and small-scale data, particularly within the finite life regime, highlighting the versatility of the presented approach based on maximum principal stresses.

Figure 6.12 presents the results regarding full- and small-scale tests from Z-profiles merged with the data pertaining to Ω -uprights (small-scale only). This figure gathers all the fatigue test data for cold-formed thin-walled mild steel profiles under local corner bending. The ASTM-derived mean S - N curve includes all 175 data points. The dashed lines again represent a translation of the mean curve by twice the standard deviation, revealing significant scatter. Nev-

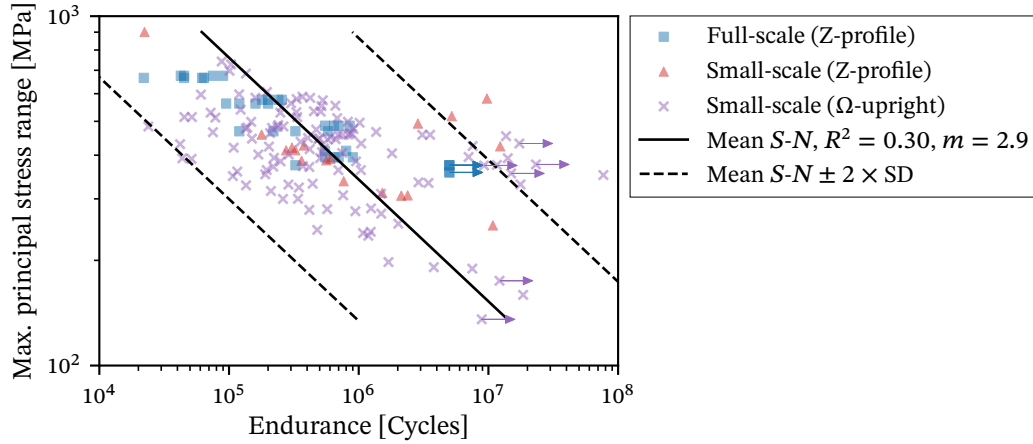


Figure 6.12: Overview of the generated S - N data for the tested full-scale Z-profiles and all small-scale specimens.

ertheless, the sheer amount of fatigue data should allow for the determination of a reliable fatigue design category.

Figure 6.13 presents the same data shown in Figure 6.12 categorized by the r_i/t ratio. When considering all the generated fatigue data for cold-formed thin-walled mild steel profiles under local corner bending, the category of $\Delta S_C = 160$ MPa with $m = 5$ for an endurance of $N_f \leq 2 \times 10^6$ load cycles introduced by the prEN 1993-1-9 revision [8] can be seen as a very consistent lower band, with just a single run-out point falling on the unsafe side. This category is again recommended by the standard for rolled products. Moreover, despite the significant scatter, Figure 6.13 also shows that points approaching the safe-unsafe border of the detail category (solid line) pertain mostly to the lower r_i/t ratio interval. Consequently, the geometrical deviations at the hot spot region due to manufacturing tolerances are important and representative geometries should be considered during the FEA. For this purpose, a statistical analysis of the manufacturing process in view of geometrical deviations is recommended. In fact, one was performed by Castiglioni and Menghini (2022) [101] for the considered small-scale spe-

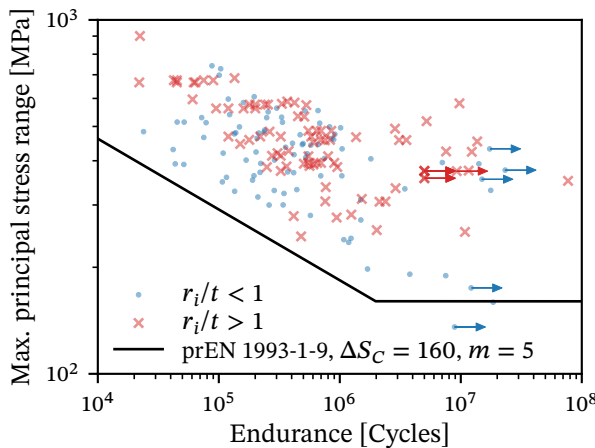


Figure 6.13: Overview of the generated S - N data for the tested full-scale Z-profiles and all small-scale specimens categorized by the r_i/t ratio.

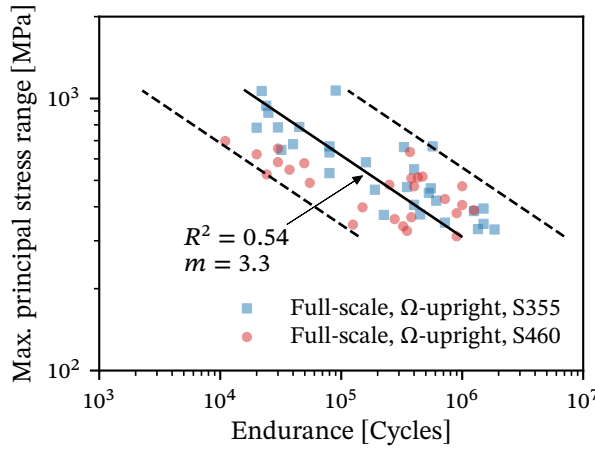


Figure 6.14: Overview of the generated S - N data for the tested full-scale Ω -uprights.

cimens.

Regarding the bolted beam-to-upright connections (detail 3 from Table 6.1), Figure 6.14 presents the obtained S - N data, where the mean S - N curve (solid line) includes all 56 data points. The dashed lines again represent a translation of the mean curve by twice the standard deviation. An acceptable fit is obtained, considering that different materials and Ω cross-sections (i.e., geometries) are included in the statistical analysis. Moreover, the S - N data is grouped by material, revealing a reasonable blend of the data, particularly for higher fatigue lives, which allows for a more practical single fatigue design curve.

Figure 6.15 shows the generated S - N data for the tested full-scale Ω -uprights grouped by their average r_i/t ratio. It is interesting to note that, again, the lowest fatigue strength points correspond to the lowest r_i/t ratio group. Furthermore, a slope of $m = 5$ has been demonstrated to be consistent with the previous rack details. Thus, the detail category of $\Delta S_C = 180$ MPa with $m = 5$ for an endurance of $N_f \leq 2 \times 10^6$ load cycles introduced by the prEN 1993-1-9 revision [8] for rolled products may be recommended, as shown in the figure.

Finally, Figure 6.16 presents a global overview of the generated S - N data for Z-profiles and Ω -uprights, including full- and small-scale results, grouped by their r_i/t ratio. This data pertains to details 1, 2, and 3 from Table 6.1, where the crack initiation site is within the cold-

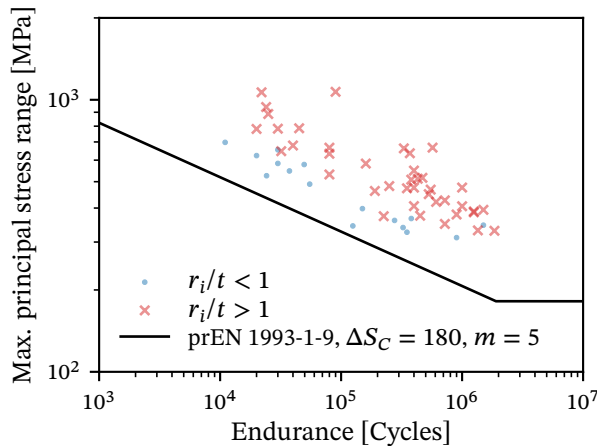


Figure 6.15: Generated S - N data for the tested full-scale Ω -uprights grouped by r_i/t ratio.

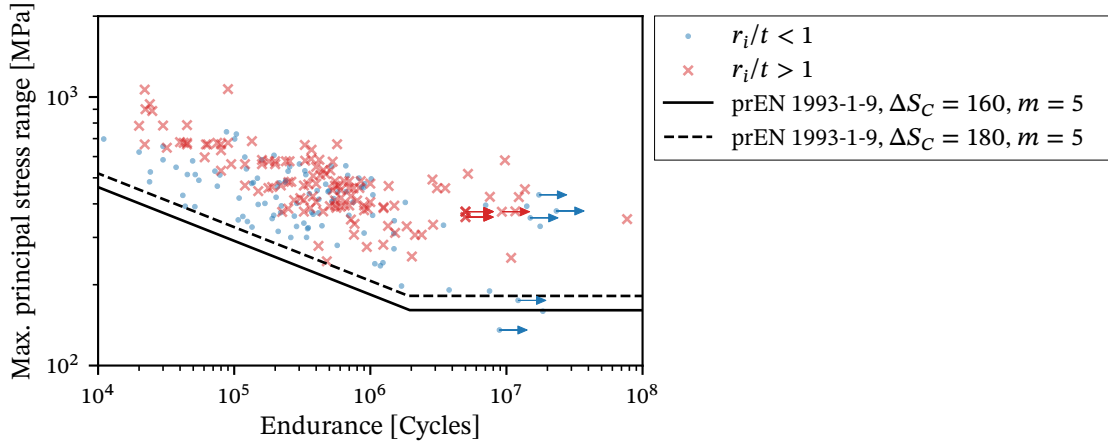


Figure 6.16: Global overview of the generated S - N data for Z-profiles and Ω -uprights (full- and small-scale) grouped by r_i/t ratio.

formed corner region. A total of 231 data points were gathered. The figure also shows that the detail category of $\Delta S_C = 160$ MPa with $m = 5$ for an endurance of $N_f \leq 2 \times 10^6$ load cycles introduced by the prEN 1993-1-9 revision [8] for rolled products can be seen as a very consistent lower band, with just a single run-out point falling on the unsafe side. Thus, this category could be used as a global fatigue design curve; however, such an approach could be too conservative for some of the previously presented cases. Additionally, the figure reveals that, in general, the lowest fatigue strength points correspond to geometries where the inside corner radius to thickness ratio is less than the unity ($r_i/t < 1$), and that the $\Delta S_C = 180$ MPa category may be used for $r_i/t > 1$. This fact supports the idea that actual or representative geometries should be accounted for, e.g., during the action evaluation (i.e., during the stress computation).

6.3.1 Simplified stress computation

For design purposes, simple to use formulae are generally preferred whenever possible. Considering the Z-profile case study as an example (fatigue detail #1 from Table 6.1), it is possible to fit a simple mathematical model to the numerical results generated by a parameterized finite element model. The quality of the fit then dictates the accuracy of the simplified model.

The finite element model shown in Figure 6.7a is considered, where the inside corner radius, the material thickness, and the magnitude of the applied load are updated via scripting using the Python interface to Abaqus/CAE. Considering these parameters, the inside corner radius was modelled with the values of $r \in \{2, 4, 6, 8, 10, 12\}$ mm, the material thickness was modelled with the values of $t \in \{1, 2, 3, 4, 5\}$ mm, and the load was modelled with the magnitude of $F \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ kN. Ultimately, this results in 300 distinct configurations (see Figure 6.17). It should be noted that the simplified model, presented below, should be used for interpolating within the ranges presented above, as extrapolation may yield inaccurate results.

For each of the 300 configurations generated, the peak value of the pseudo-elastic maximum principal stresses was determined at the critical cold-formed corner region, identified

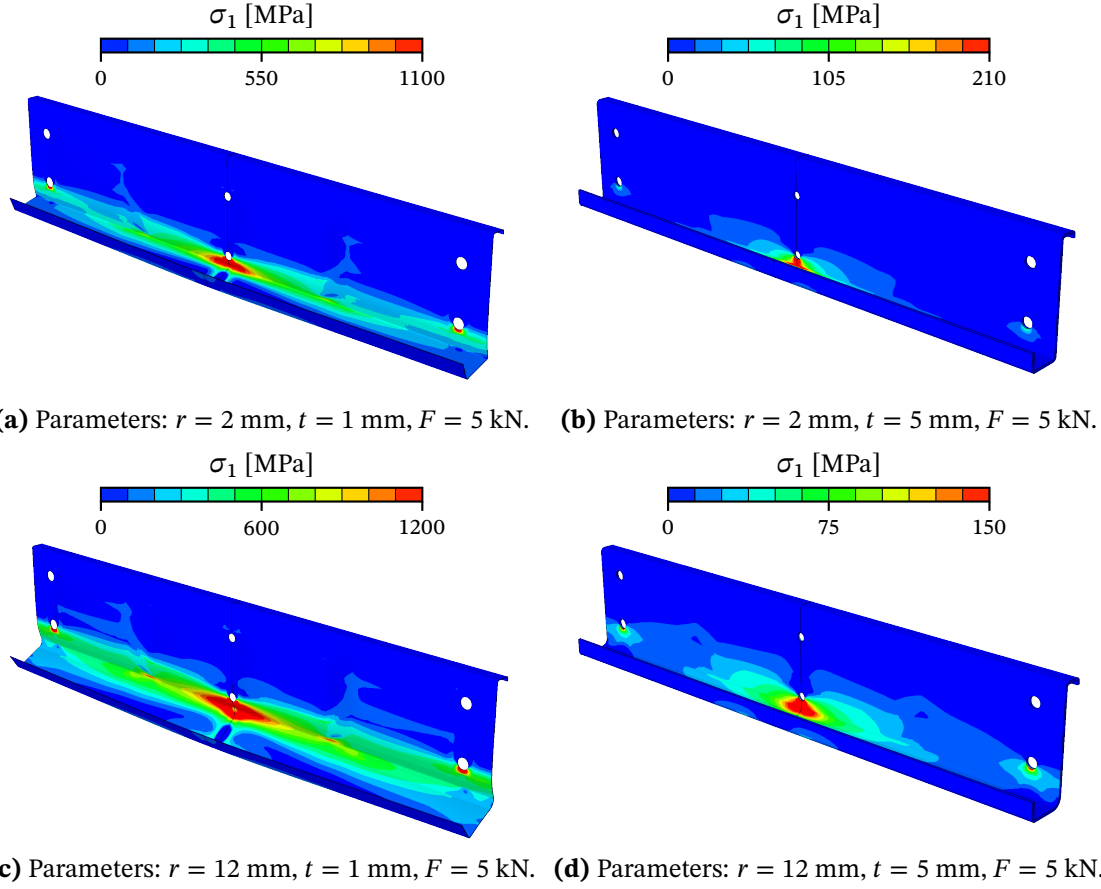


Figure 6.17: Example of 4 configurations generated by the parameterized finite element model.

as the crack initiation site. The objective is now to adequately fit a simplified mathematical model to the numerically generated results. To achieve this, one may first start by fixing two of the considered parameters, e.g., by setting $r = 8$ mm and $t = 3$ mm, which then enables the fit of σ_1 as a function of the remaining parameter F . As shown in Figure 6.18a, a linear relationship is verified, hence:

$$\sigma_1 = \alpha F, \quad (6.1)$$

in which α denotes the slope of the linear relationship. The objective is now to verify how α varies as a function of the next parameter, e.g., t . As shown in Figure 6.18b, a power law seems to be an adequate approximation for this relationship, hence:

$$\alpha = \beta t^\gamma, \quad (6.2)$$

in which β and γ must be determined as a function of the remaining parameter, r . The fitted approximation is shown in Figure 6.18c and Figure 6.18d, resulting in:

$$\beta = \beta_2 r^2 + \beta_1 r + \beta_0, \quad (6.3)$$

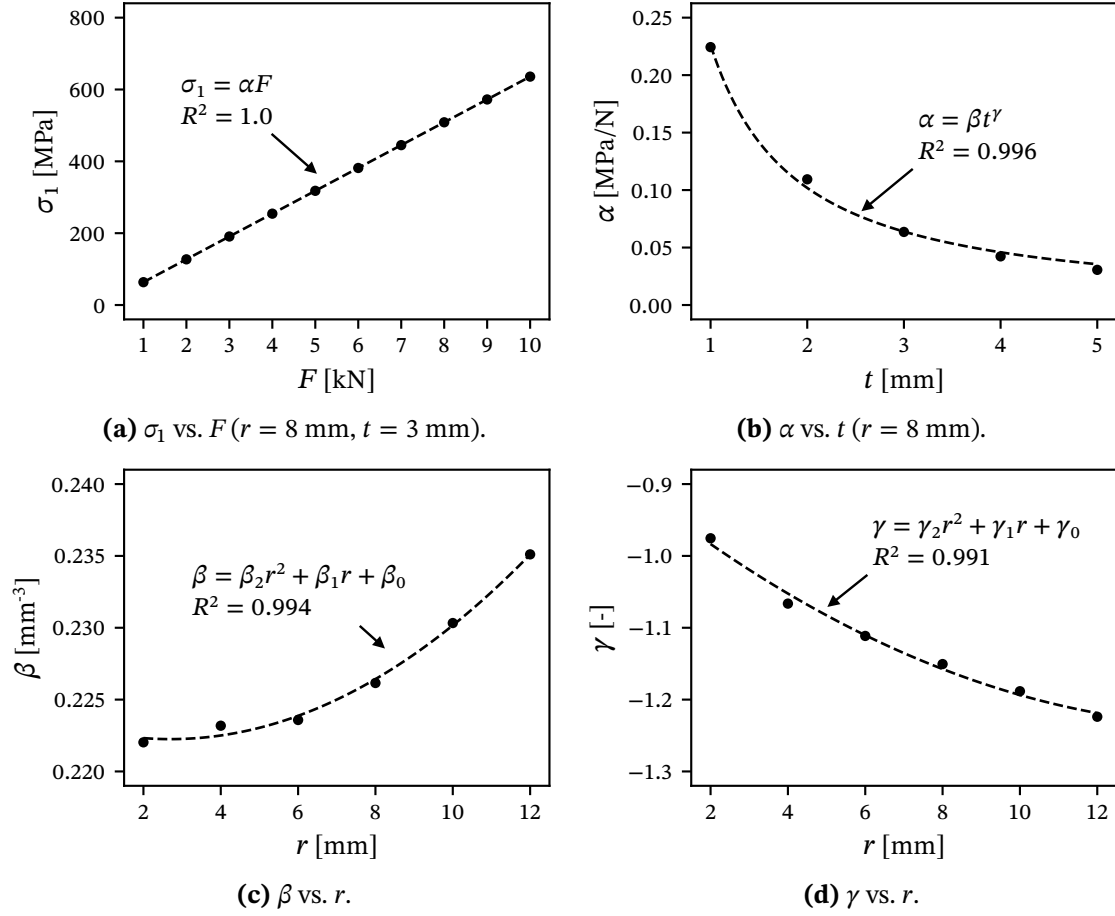


Figure 6.18: Fit of the simplified model for stress computation to the numerically generated results.

and

$$\gamma = \gamma_2 r^2 + \gamma_1 r + \gamma_0. \quad (6.4)$$

Because there are no more parameters to be considered, the β_i and γ_i coefficients are constant, given by:

$$\begin{aligned} \beta_0 &= +2.233 \times 10^{-1}, \quad \beta_1 = -7.986 \times 10^{-4}, \quad \beta_2 = +1.483 \times 10^{-4}, \\ \gamma_0 &= -9.036 \times 10^{-1}, \quad \gamma_1 = -4.266 \times 10^{-2}, \quad \gamma_2 = +1.366 \times 10^{-3}. \end{aligned} \quad (6.5)$$

In conclusion, for design purposes, in order to determine peak pseudo-elastic maximum principal stresses for the considered Z-profile case study (fatigue detail #1 from Table 6.1), the coefficients shown in (6.5) may be used to compute the β coefficient and γ exponent following the relationships shown in (6.3) and (6.4), respectively. Finally, the α coefficient may be determined using the expression shown in (6.2), allowing the computation of σ_1 via (6.1).

Comparing the numerically obtained results and those obtained by the fitted simplified

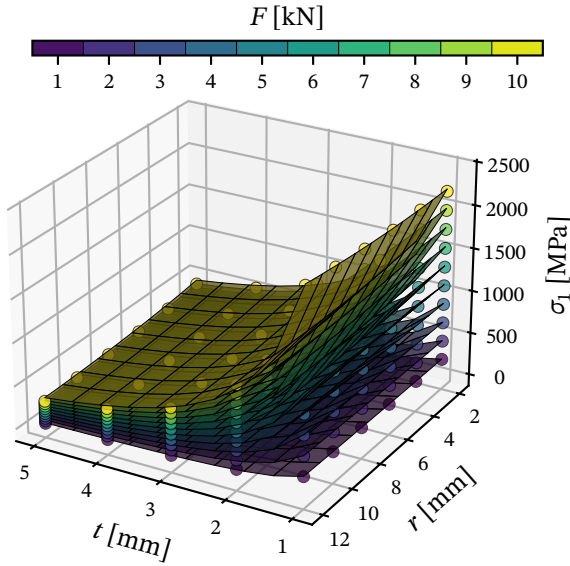


Figure 6.19: Peak pseudo-elastic maximum principal stress determination: fit of the simplified analytical model (surfaces) to the available numerical results (points).

model, an adjusted coefficient of determination of

$$\bar{R}^2 = 1 - (1 - R^2) \frac{n - 1}{n - p - 1} = 0.997 \quad (6.6)$$

is verified, in which R^2 is the coefficient of determination, n is the sample size, and p is the number of independent variables. Figure 6.19 presents a visualization of the corresponding fit. Of course, the simplified model is more convenient to use and should still be accurate for interpolating between the considered parameters, whereas a new finite element model would have been required instead.

6.3.2 Overview of the global approach

Figure 6.20 presents an overview of the framework for fatigue life predictions based on global approaches. As shown in this figure, in a classical global approach ($S-N$ approach), a fatigue testing campaign of the critical detail is performed followed by a statistical analysis of those results in which a load (usually in the form of an engineering stress) vs. endurance relationship is generated for the purpose of fatigue life prediction. While the classical approach has been applied to the bolted joints (see Chapter 4), it results in a convoluted detail categorization system, where the generated $S-N$ curve is applicable exclusively to the tested detail. Alternatively, in the proposed global approach, more accurate information regarding the stress state of the tested detail is considered by numerically simulating the structural and fatigue loading. This information is then considered in the subsequent statistical analysis in order to generate an equivalent stress vs. endurance relationship, in which peak pseudo-elastic maximum principal stresses were considered in this work. As shown in Chapter 4, this resulted in a master $S-N$ curve capable of capturing several of the tested conditions and joint parameters. In Section 6.3, it is shown that the proposed global approach is again successfully applied, this time to merge the data pertaining to distinct fatigue testing campaigns of several constructional details in which full- and small-scale specimens were considered, highlighting the versatility of this approach.

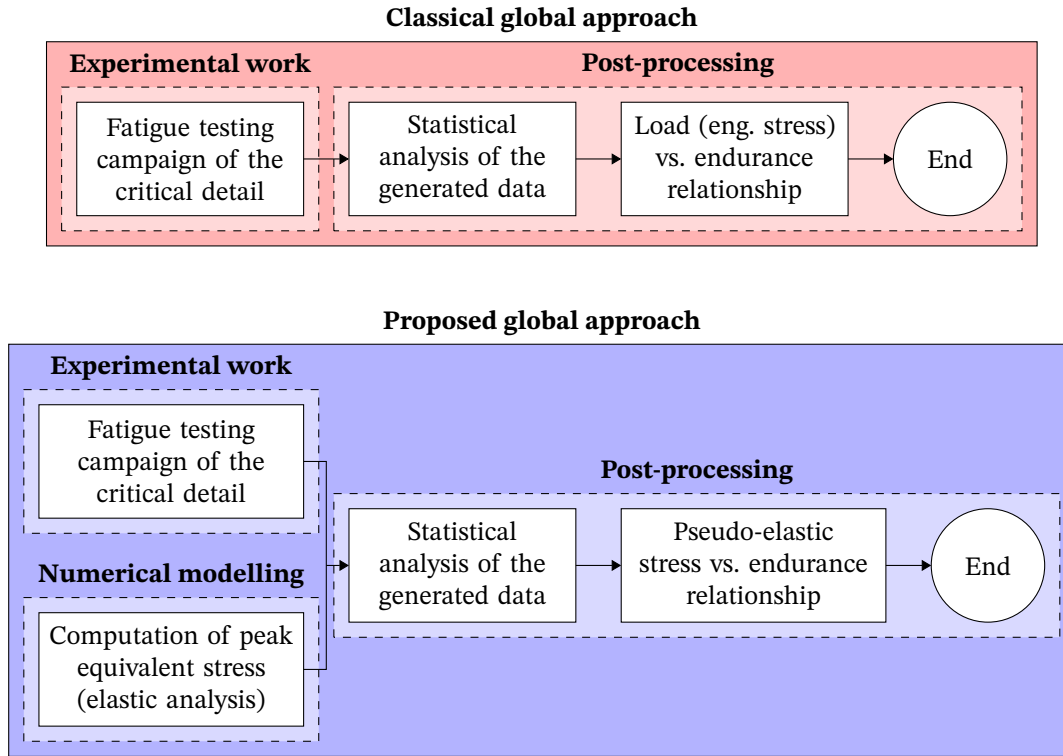


Figure 6.20: Overview of the framework for fatigue life predictions based on global approaches.

6.4 Fatigue life predictions based on a local approach

In the previous section, [Section 6.3](#), a global approach was used to correlate the fatigue life of several components. In the presented global approach, fatigue lives were determined via purpose-built setups during full-scale fatigue testing campaigns, with small-scale specimens included for increased reliability. Once fatigue lives were experimentally determined, peak pseudo-elastic maximum principal stresses were computed via finite element modelling in order to generate a stress vs. endurance relationship. This relationship can then be used for fatigue life prediction. While this approach is shown to be successful, it relies on extensive and complex fatigue testing campaigns, which are generally expensive and time-consuming. Therefore, an alternative procedure is proposed in the current section.

The aforementioned global approach relies on the post-processing, i.e., on a statistical analysis, of the experimental data generated during a large-scale fatigue investigation consisting of distinct testing campaigns. Since extensive fatigue testing of full-scale profiles is generally not practised in the industry, an alternative local approach is presented below and, in terms of experiments, only requires fatigue constants pertaining to the plain material, which are more readily available or obtainable, such as those determined in [Chapter 3](#). Therefore, results arising from the global and local approaches, presented in this work, are completely independent – this is an important aspect.

As a case study, fatigue detail #1 from [Table 6.1](#) is considered. Because the Z-profile is

cold-formed, a significant formation of residual stresses is found at the cold-formed corners, which impacts the overall fatigue behaviour of the profile. Therefore, the straining history of the material must be included in the fatigue analysis, as the proposed framework relies on the fatigue properties of the plain material. This is achieved by numerically simulating the manufacturing process of the profile in order to obtain a representative residual stress field, as shown in [Chapter 5](#). These results are then used as the initial state during the subsequent structural and fatigue analyses, as shown further. Importantly, this section also aims to bring some insight on the effect of residual stresses due to cold forming in the predictive capabilities of the considered finite element models.

6.4.1 Overview of the local approach

[Figure 6.21](#) presents an overview of the framework for fatigue life predictions based on a local approach. In this proposed framework, a short experimental testing campaign may be required in order to determine empirical constants pertaining to the plain material. The objective is to obtain the Basquin-Coffin-Manson relationship via low- and high-cycle fatigue testing, and the strain hardening behaviour via monotonic testing. This experimental work has already been performed for the investigated material and is detailed in [Chapter 3](#). Alternatively, these constants may be extracted from the literature, but this may introduce further uncertainties if the material is not exactly the same.

As explained above, it may be necessary to consider the strain hardening history of the material since fatigue constants pertain to its plain state. To this end, once a strain hardening model has been fitted to the experimental data, it may be used as an input during the numerical modelling of forming and pre-forming manufacturing processes. The objective is to generate representative residual stress and strain fields via the numerical simulation of those processes. Of course, the representativeness of those residual fields is dependent on several parameters, such as those arising from the processes themselves (such as tooling geometries, order of operations, etc.) and those arising from the finite element modelling aspect (such as mesh refinement level, finite element formulation, etc.). Nevertheless, when complex manufacturing processes are considered, such as roll forming, the resulting numerical work is inevitably challenging and extensive. In this work, the press braking and roll forming processes are considered and, in regard to pre-forming processes, the effects of coiling operations are also investigated. See [Chapter 5](#) for the numerical determination of residual stress and strain fields.

In regard to the material's strain hardening history, it was shown in [Chapter 5](#) that coiling operations, as a pre-forming process, can have a significant effect on the final residual stress field of a cold-formed profile. However, in [Chapter 5](#), it is also discussed how these effects are only relevant within the flat (i.e., non-formed) regions of the profile; besides, coiling effects are most relevant when considering the longitudinal residual stress component. Because the considered critical region corresponding to the crack initiation site is the middle of the cold-formed corner, in which cold forming is the dominant process, contributing the most to the residual stresses found at that location, and because the transverse stresses are the most relevant for the subsequent structural and fatigue analysis, coiling effects are discarded in this case study, and only residual stresses due to cold forming are considered. Nevertheless, it must be noted that effects arising from pre-forming processes may, sometimes, be relevant, depending

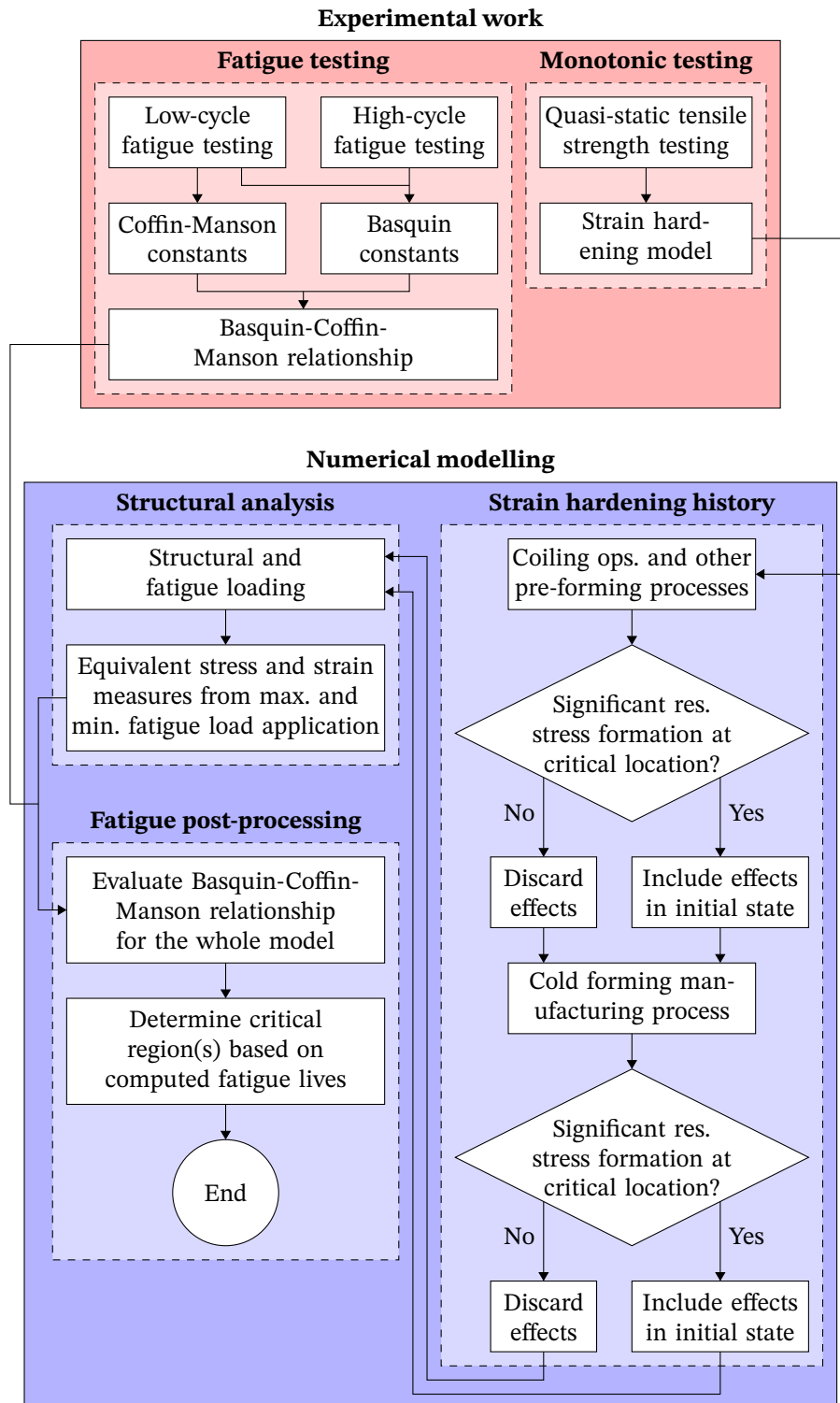


Figure 6.21: Overview of the framework for fatigue life predictions based on a local approach.

on the considered case study.

Once the strain hardening history has been obtained, it is used as an input in the subsequent structural analysis in which the fatigue loads are applied. Equivalent stress and strain measures resulting from the fatigue load application are then used to evaluate the Basquin-Coffin-Manson relationship for the whole model. The critical life region is determined based on the computed fatigue lives (the lowest value in the model). This region should coincide with the crack initiation site.

6.4.2 Application of the local approach

In regard to the modelling of the structural fatigue loads, numerical models rely on the geometry and mesh resulting from the previous cold forming simulations. Moreover, the inclusion of the corresponding residual stress fields in the initial state of the analyses is investigated. In particular, the objective is to verify how the inclusion or omission of residual stresses may impact the structural response and subsequent local fatigue life predictions. Note that, as explained above, coiling effects are disregarded in this case study. Additionally, since the obtained profile geometry does not include holes for the bolted connections to the supports, boundary conditions are simplified: nodes located within the supported regions have null prescribed displacements. A concentrated load is then applied to the rigid load actuator, in which the load magnitude corresponds to the ones tested during the full-scale fatigue testing campaign. [Figure 6.22](#) presents the finite element models used for the structural fatigue loading simulations. The figure reveals the residual stress effect on the structural response for loaded press-braked and roll-formed Z-profiles considering an example 9 kN load.

Fatigue loading is simulated by performing two load steps, each corresponding to the application of the maximum and minimum loads of the constant amplitude fatigue load cycle. From the resulting stress state, which may include previous residual effects, the mean-stress corrected Basquin-Coffin-Manson relationship, as shown in [Section 2.2.3](#), is evaluated. Note that, to account for the general 3D stress state, an equivalent stress and strain measure is required. In this study, major principal stresses are used, defined as

$$\sigma_{\text{major}} = \begin{cases} \sigma_1 & \text{if } |\sigma_1| \geq |\sigma_3|, \\ \sigma_3 & \text{otherwise,} \end{cases} \quad (6.7)$$

in which σ_1 and σ_3 are the maximum and minimum principal stresses, respectively, corresponding to the local direction of tension and compression. Major principal strains are also required and defined analogously. Once stresses and strains have been computed for each load step, the ranges and corresponding amplitudes and mean values are easily obtained, which are then used as an input for the Basquin-Coffin-Manson model alongside the material's fatigue constants determined in [Chapter 3](#).

The mean-stress corrected Basquin-Coffin-Manson model is then solved for each node of the finite element mesh, as this allows for the direct computation of fatigue life predictions at the surfaces, which, in fatigue, are viewed as the critical regions. The nonlinear equation is solved using the Newton-Raphson method via a post-processing Python script. An example of the generated output is shown in [Figure 6.23](#), from which a detrimental residual stress effect on the fatigue life prediction field may be visualized.

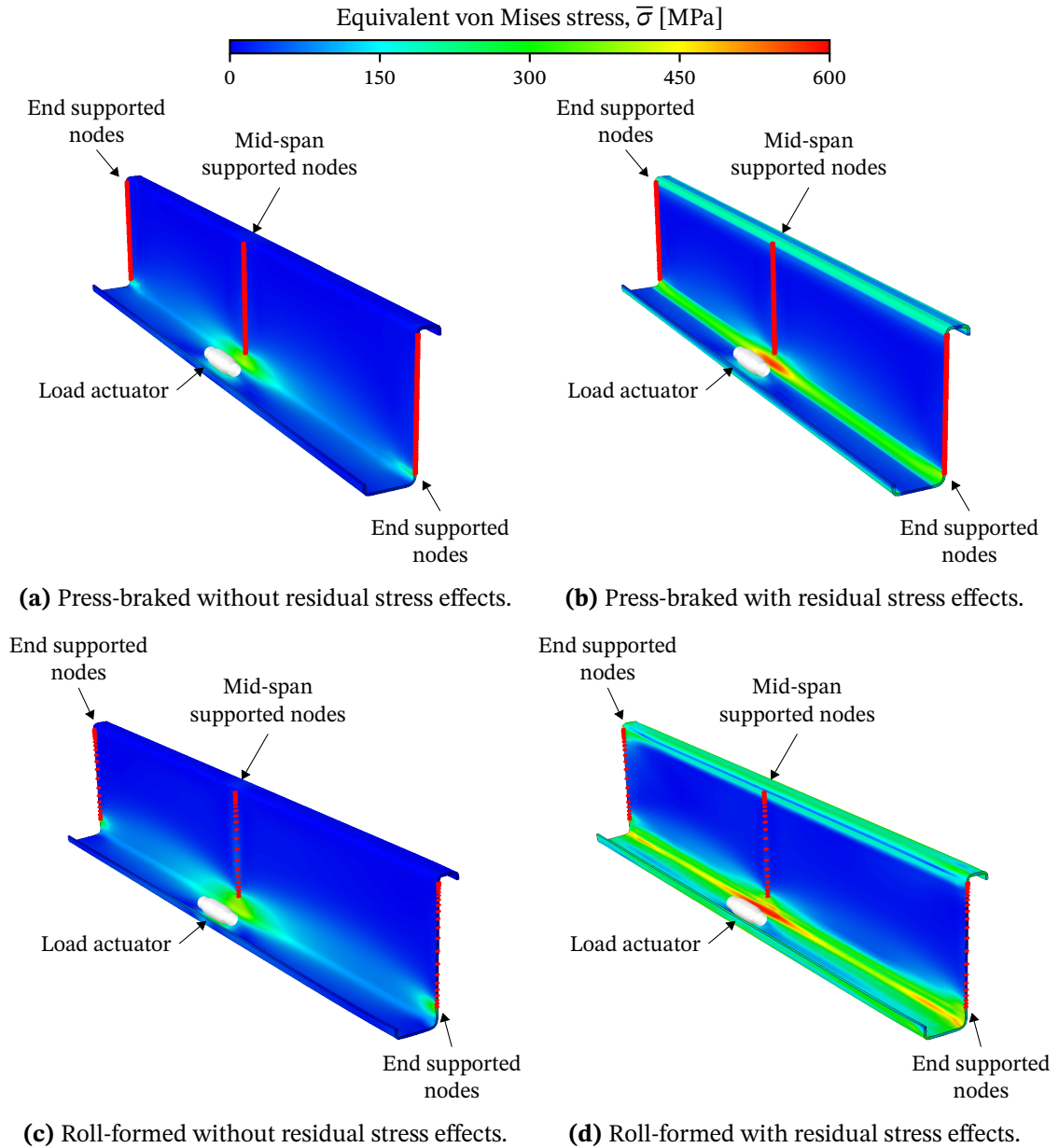


Figure 6.22: Residual stress effect on the structural response of a loaded Z-profile (9 kN load example).

Figure 6.24 presents the numerically computed fatigue life predictions. In this figure, predictions are directly compared against the experimental data, since the latter is available. To simplify the presentation, experimental fatigue life is averaged for each of the tested fatigue loads. Note that experimental data pertains to roll-formed Z-profiles only; however, numerical predictions based on the press-braked profile are also presented. The figure finally reveals the residual stress effect on the considered local fatigue approach. In summary, predictions without considering residual stresses are not adequate; however, when considering residual

6: Fatigue behaviour of cold-formed thin-walled mild steel members

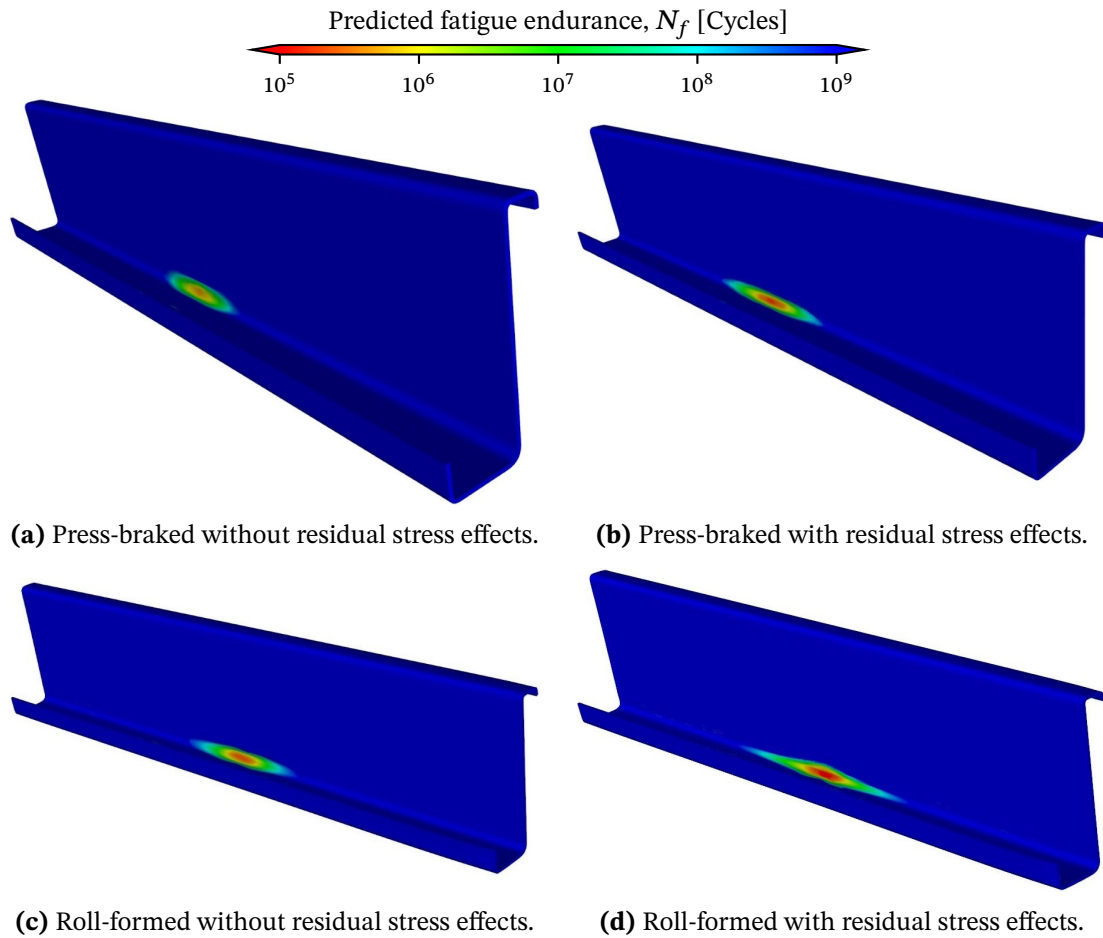


Figure 6.23: Residual stress effect on the fatigue life prediction field for the investigated Z-profile (9 kN fatigue load example).

stresses, the predicted fatigue life decreases, and these become more consistent with the obtained experimental data. Numerically, residual stresses have a clear detrimental effect on the fatigue life predictions, in the considered case study. Importantly, the inclusion of residual stresses results in much more accurate predictions. Note that numerical predictions tend to lie on the unsafe side of the plot shown in Figure 6.24; hence, the residual stress effect is very relevant in this case.

Considering the most accurate fatigue life prediction model, which is based on the roll-formed Z-profile, its geometry, and residual stress field, Figure 6.25 presents the considered experimental data alongside the numerical predictions in the form of an applied actuator force vs. obtained fatigue endurance relationship. While results are not perfect, they are satisfactory once one takes into account the high variability and uncertainty related to full-scale fatigue testing alongside some uncertainties regarding the manufacturing process due to its complexity. Importantly, predictions are completely independent from what was observed during the full-scale experiments, i.e., the predictive model is not based on the post-processing of the results arising from the full-scale testing campaign nor is it based on a statistical analysis of

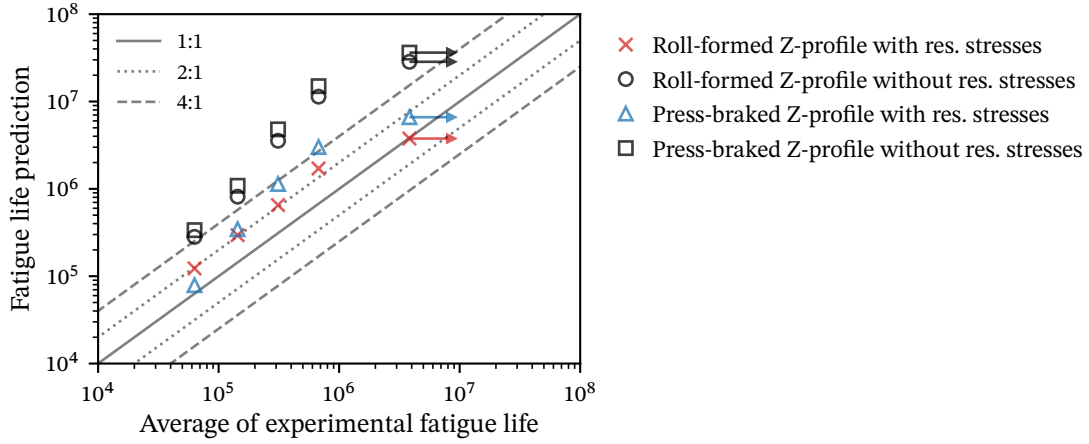


Figure 6.24: Fatigue life predictions vs. experimental data.

those results; instead, the proposed framework for fatigue life prediction relies only on the fatigue properties of the plain material, which are more easily available or obtainable, and on general FEA. For finite fatigue lives, predictions are deemed valid as they are within twice the standard deviation (SD) from the mean of the experimental data, as shown in the figure. Approaching the infinite fatigue life region (run-out region), predictions are still consistent and more conservative in comparison to the observed run-outs. Hence, Figure 6.25 shows that, when full-scale testing is not possible, which is common in the industry, the proposed framework is a viable alternative that should be subjected to higher safety margins due to its possible non-conservativeness. Finally, it should be noted that fatigue properties were derived from polished small-scale specimens of the plain material, whereas the full-scale testing campaign used unpolished cold-formed profiles sampled directly from the manufacturer. Therefore, a possible detrimental surface effect, not captured by the numerical analyses, may be the subject of further investigation for refining the proposed framework.

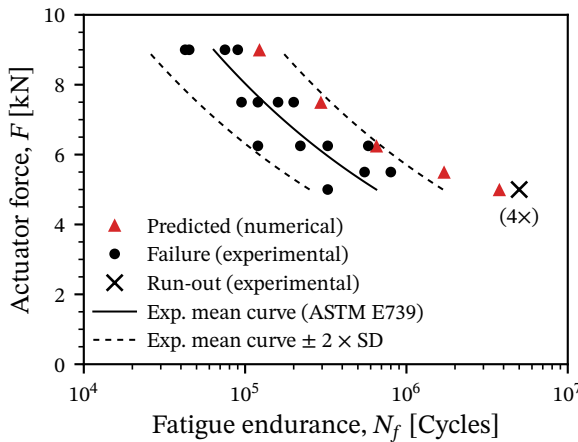


Figure 6.25: Endurance of cold roll-formed Z-profiles: experimental data vs. numerical predictions.

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Chapter 7

Development of a damage-tolerant approach

Nowadays, it is common for clients to outline their project requirements and invite contractors to submit proposals. Contractors then compete by offering the lowest cost, innovative solutions, or added value, while balancing quality, timeliness, and regulatory compliance. The client then selects the contractor that best meets their needs, often prioritizing affordability.

Contractors are then motivated to propose thinner structural members in order to reduce costs and win bids, but structural integrity and long-term durability must still be ensured. Therefore, advanced engineering techniques often play a role in achieving such a balance. Ultimately, the residual fatigue life of a profile in which a visible fatigue crack has already started to develop, may be the determining factor to ensure the structure's service life.

The goal of this chapter is to develop a damage-tolerant approach that can be applied to real-world scenarios, extrapolating from well-known engineering principles based on LEFM and stable fatigue crack growth. As a case study, the Z-profile is considered, which has been used during full-scale fatigue testing. Having experimental data from full-scale testing allows for further insight in regard to the reliability and applicability of the damage-tolerant approach under development. In regard to these experiments, refer to [Section 6.1](#), [Figure 6.1](#), [Figure 6.2](#), and [Figure 6.3](#), as well as reference [141].

This chapter begins with a preliminary study to find the best suited numerical approach for stress intensity factor determination, followed by a parametric study on crack parameters, such as sensitivity to mesh resolution, crack shape (crack depth and rectangular vs. elliptical cracks), mixed-mode influences, as well as load and boundary conditions. Once a critical and representative scenario has been identified, a parameterized finite element model is created, based on which a simplified analytical model is fitted to the available numerical data, which can then be used for engineering design purposes, where simple formulae are generally preferred. Nevertheless, [Appendix B](#) presents an alternative surrogate neural network model.

Furthermore, given the fracture toughness of the material, the aforementioned simplified model allows for the computation of critical crack lengths and corresponding loads, allowing for the static design of a cracked profile and delimiting the domain of stable fatigue crack growth. Then, by relying on Paris' law, the residual fatigue life of a cracked profile may be estimated. Finally, exploratory research is also performed to account for residual stresses in the estimation of the residual fatigue life.

7.1 Numerical determination of stress intensity factors

In this section, several approaches for the numerical determination of stress intensity factors are tested. For this purpose, the problem shown in Figure 7.1a is considered, consisting of an uniaxially loaded rectangular plate with a central crack. The geometry of the plate is defined by a total width of $2w = 200$ mm, a total height of $2h = 200$ mm, an out-of-plane thickness of $t = 4$ mm, and a total crack length of $2a = 20$ mm. The material is considered linear-elastic defined by a Young's modulus of $E = 210$ GPa and a Poisson's ratio of $\nu = 0.3$. The applied load has a magnitude of $\sigma_\infty = 100$ MPa. A plane stress condition is assumed since $t/w = t/h = 0.04$, and the crack is assumed to be small since $a/w = a/h = 0.1$. Figure 7.1b presents the used modelling approach, where only 1/4 of the plate is modelled due to symmetry, and the crack is modelled via the boundary conditions. The model is solved via the FEM using Abaqus and by performing a linear-elastic static analysis, where the CPS4 element is used throughout, which is a fully-integrated pure-displacement formulation bilinear element. The stress intensity factor is then determined as a post-processing procedure.

For this simple problem, an analytical solution for the stress intensity factor (mode I) exists [2] and is given by

$$K = \sigma_\infty \sqrt{\pi a} \Rightarrow K = 560.5 \text{ MPa} \cdot \text{mm}^{1/2}. \quad (7.1)$$

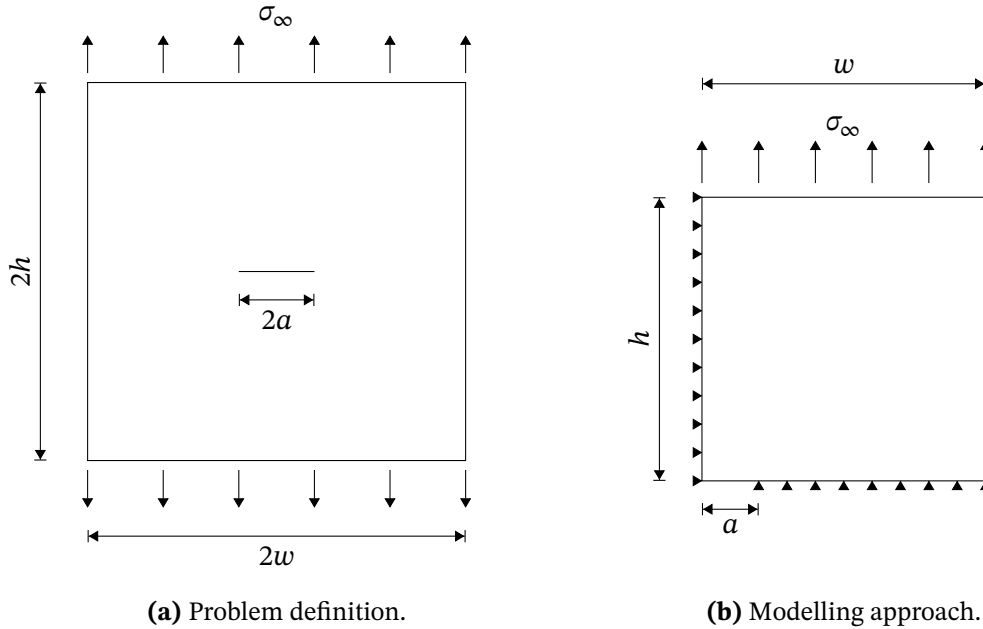


Figure 7.1: Uniaxially loaded rectangular plate with a central crack.

7.1.1 Force-based extrapolation approach

For the considered problem, and considering a coordinate system aligned with the crack tip, as shown in Figure 2.23, the stress field in Cartesian coordinates is given by [2]

$$\begin{aligned}\sigma_{xx} &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\ \sigma_{yy} &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\ \sigma_{xy} &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(\sin \frac{\theta}{2} \cos \frac{3\theta}{2} \right).\end{aligned}\quad (7.2)$$

Additionally, for the current planar problem, one may write that

$$F_y = t \int \sigma_{yy} dr, \quad (7.3)$$

where F_y , the internal force, is available at the boundary that is constrained due to vertical symmetry, i.e., for $\theta = 0$, in the form of a reaction force; hence,

$$\theta = 0 \Rightarrow \sigma_{yy} = \frac{K}{\sqrt{2\pi r}} \Leftrightarrow F_y = t \int \frac{K}{\sqrt{2\pi r}} dr \Leftrightarrow F_y = tK\sqrt{\frac{2r}{\pi}}. \quad (7.4)$$

Considering the finite element discretization, we can write that

$$K_n = \frac{1}{t} \sqrt{\frac{\pi}{2r}} \sum_{i=1}^n F_{y,i}, \quad (7.5)$$

where $F_{y,i}$ is the nodal force at node i , and the nodes are numbered as shown in Figure 7.2. Moreover, a singularity is expected as $r \rightarrow 0$, i.e., at the crack tip; therefore, for extrapolation purposes, the results for some nodes near the crack tip are discarded, as well as for nodes that are too far away, improving the accuracy of the method. For consistency, extrapolation is performed considering the results for $r \in [a, 2a]$.

Figure 7.3a presents stress intensity factors determined via the force-based extrapolation approach for distinct mesh refinement levels. The final values are obtained via extrapolation

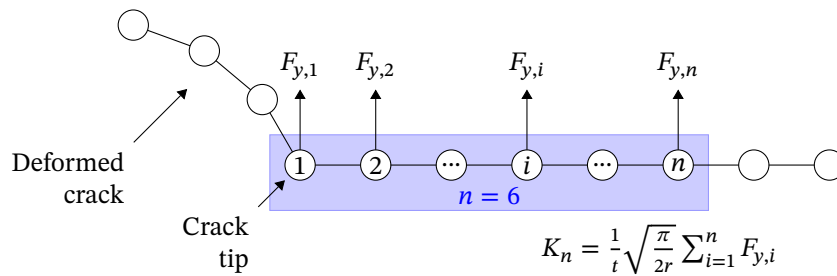


Figure 7.2: Node numbering used for the force-based extrapolation approach.

7: Development of a damage-tolerant approach

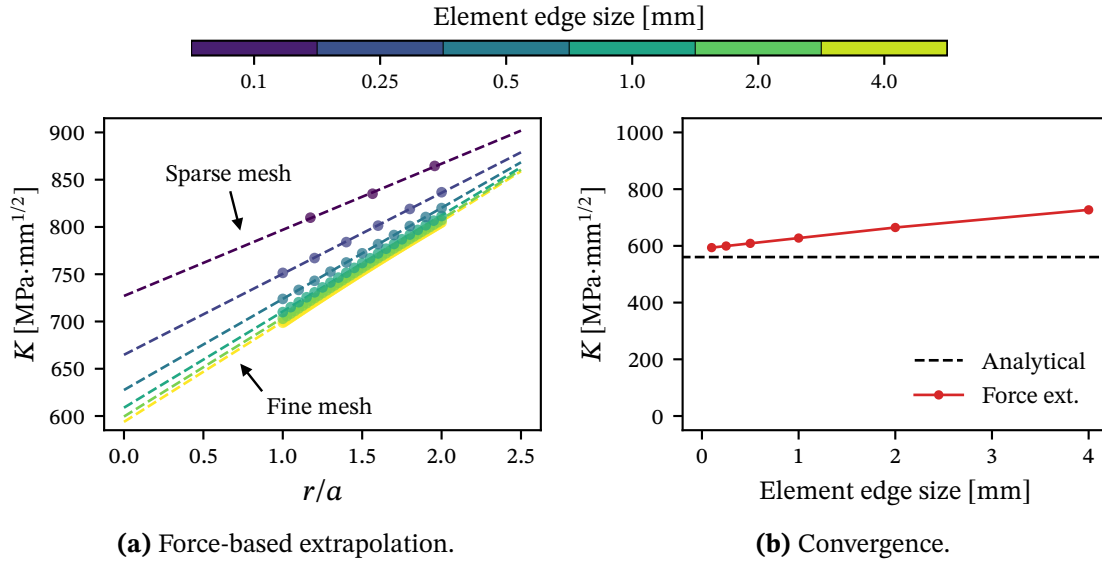


Figure 7.3: Stress intensity factors determined via the force-based extrapolation approach for distinct mesh refinement levels.

from $r \in [a, 2a]$ to $r = 0$, corresponding to the vertical intercept of the fitted linear regression line. Additionally, Figure 7.3b shows how the final computed value changes with the considered mesh refinement level, revealing that results are mesh dependent but seem to converge to the analytical solution as an increasingly finer mesh is considered. Considering the force-based extrapolation approach, and depending on the considered mesh refinement level, relative differences against the analytical solution range from 5.9% to 29.7%.

7.1.2 Displacement-based extrapolation approach

For the considered problem, and considering a coordinate system aligned with the crack tip, as shown in Figure 2.23, the displacement field in Cartesian coordinates is given by [2]

$$\begin{aligned} u &= \frac{K(1+\nu)}{2E} \sqrt{\frac{r}{2\pi}} \left[(2\kappa - 1) \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right], \\ v &= \frac{K(1+\nu)}{2E} \sqrt{\frac{r}{2\pi}} \left[(2\kappa + 1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right], \end{aligned} \quad (7.6)$$

where $\kappa = (3 - \nu)/(1 + \nu)$ for the current plane stress case. Considering the vertical displacements of the free crack surface at $\theta = 180^\circ$, the following is obtained:

$$\theta = 180^\circ \Rightarrow v = \frac{4K}{E} \sqrt{\frac{r}{2\pi}} \Leftrightarrow K = \frac{\nu E}{4} \sqrt{\frac{2\pi}{r}}. \quad (7.7)$$

Figure 7.4a presents stress intensity factors determined via the displacement-based extrapolation approach for distinct mesh refinement levels. To avoid the singularity at the crack tip, the final computed values are obtained via extrapolation from $r \in [a/4, a]$ to $r = 0$, corresponding to the vertical intercept of the fitted linear regression line. Additionally, Figure 7.4b

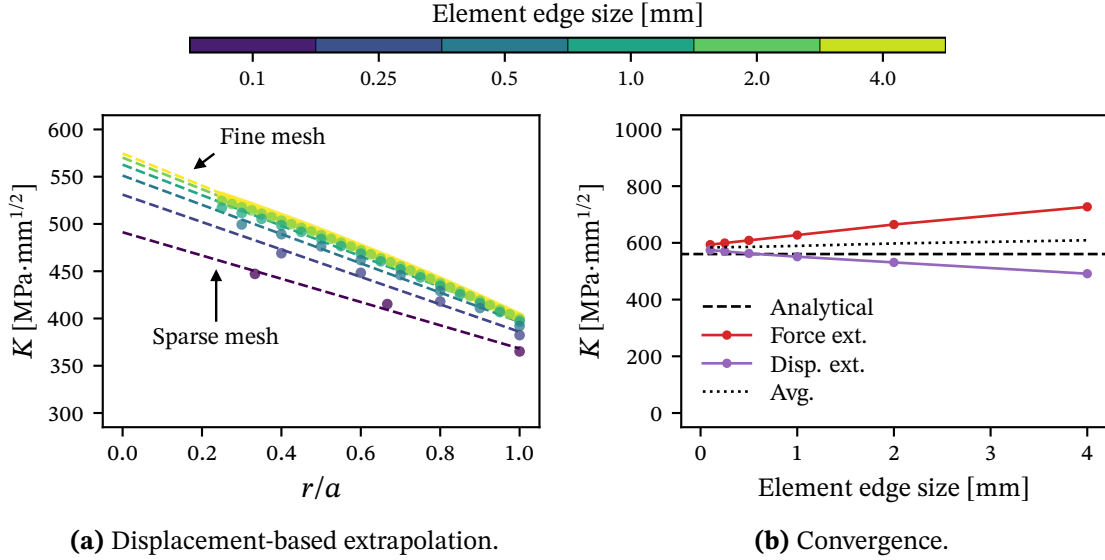


Figure 7.4: Stress intensity factors determined via the displacement-based extrapolation approach for distinct mesh refinement levels.

shows how the final computed value changes with the considered mesh refinement level, revealing that results are mesh dependent but seem to converge to the analytical solution as an increasingly finer mesh is considered. Considering the displacement-based extrapolation approach, and depending on the considered mesh refinement level, relative differences against the analytical solution range from -12.4% to 2.5% .

Interestingly, if one assumes that the force- and displacement-based extrapolation approaches are complementary, a new approach can be used by taking the average of the results. In this case, Figure 7.4b reveals that this new approach is much less sensitive to the mesh refinement level, where relative differences against the analytical solution range from 4.2% to 8.7% .

7.1.3 Virtual crack extension method

The virtual crack extension method (VCEM), as presented in Section 2.2.4.3, is based on the following finite difference approximation for computing the strain energy release rate G :

$$G \approx -\frac{\Delta\Pi}{\Delta A} = -\frac{{}^a\Pi - {}^{a+\Delta a}\Pi}{\Delta A} \quad \text{with} \quad \Delta A \sim 0, \quad (7.8)$$

where ${}^a\Pi$ is the total potential energy of the system considering the initial crack surface area A , whereas ${}^{a+\Delta a}\Pi$ is the total potential energy of the system considering the initial size of the crack plus an additional arbitrarily small extension of ΔA . Hence, unlike the previous methods, the VCEM approach requires performing two distinct finite element analyses. Once G has been computed, the expression shown in (2.92) can be used to compute K for the current plane stress problem.

Figure 7.5a shows, for an element edge size of 0.1 mm, how the choice of the ΔA parameter influences the obtained results. As expected due to the finite difference approximation, results

7: Development of a damage-tolerant approach

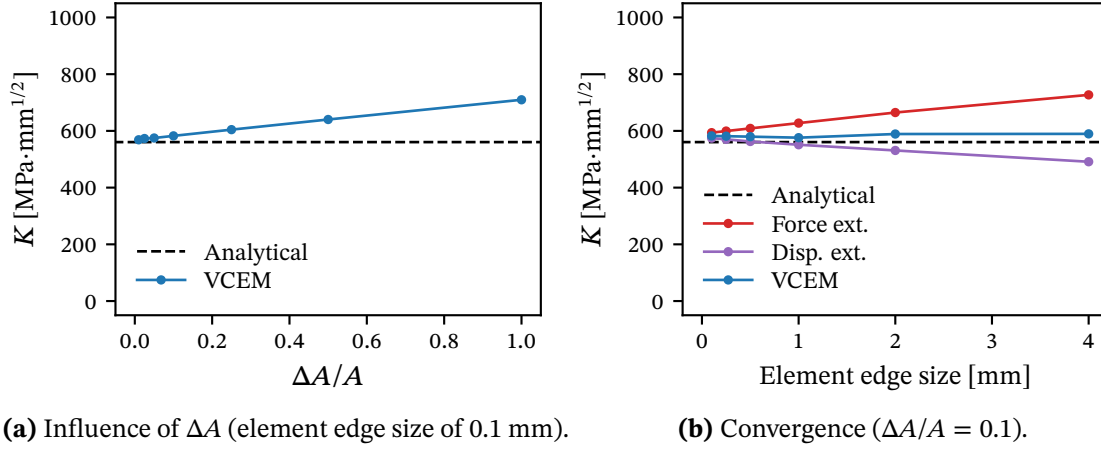


Figure 7.5: Stress intensity factors determined via the virtual crack extension method.

are better for smaller values of the considered crack extension, and a value of $\Delta A \leq 0.1A$ seems to be an appropriate choice for the current case.

Figure 7.5b shows, for $\Delta A = 0.1A$, how results using the VCEM approach are almost independent of the chosen mesh refinement level, which is a strong advantage over the two previous methods. Considering the VCEM approach, a relative difference of less than 5% is verified when comparing against the analytical solution.

7.1.4 Virtual crack closure technique

The virtual crack closure technique (VCCT), as presented in Section 2.2.4.2, is applied in a very similar way as the VCEM approach, where a two-step finite element analysis is performed, as shown in Figure 2.24. Again, the objective is to first compute the strain energy release rate G , this time using the following expression:

$$G = -\frac{1}{2t\Delta a}F_y\Delta v, \quad (7.9)$$

where the nodal force F_y is extracted from the crack tip node during the first analysis step, where the crack is modelled with the considered initial length, and the relative nodal displacement Δv is extracted from the same node during the second analysis step, where a crack extension is considered. However, while the VCEM approach can be used with an arbitrarily small virtual crack extension, in the VCCT approach, the virtual crack extension/closure is linked to the considered finite element edge size. Finally, once G has been computed, the expression shown in (2.92) can be used to compute K for the current plane stress problem.

Figure 7.6 shows the results obtained via the VCCT approach, which are very similar to the ones obtained via the VCEM approach, i.e., results obtained via both approaches seem to be very stable even while considering distinct mesh refinement levels. A similar relative difference against the analytical solution is also obtained.

Additionally, a modified version of the virtual crack closure technique (mVCCT) is also tested, in which it is assumed that the stress state near the crack tip does not change significantly if a sufficiently small virtual crack extension/closure is assumed. Therefore, the nodal

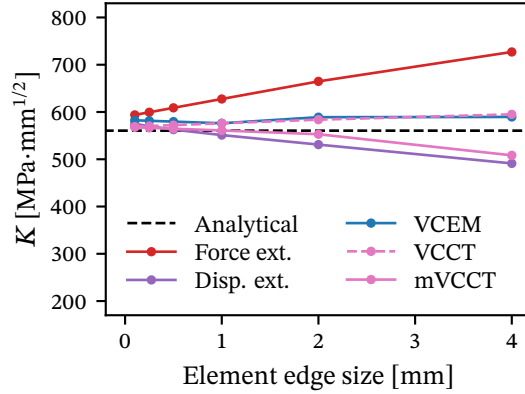


Figure 7.6: Stress intensity factors determined via the virtual crack closure techniques.

displacements, which on the original approach are obtained from the second analysis step, can instead be obtained from the first analysis step, considering the free finite element node closest to the crack tip node. The obvious advantage of the mVCCT approach is that there is no longer a need for a two-step analysis, effectively cutting the computational cost in half.

Results based on the mVCCT approach are also shown in Figure 7.6, revealing that this approach is more sensitive to the mesh refinement level than the original version, with relative differences ranging from -9.3% to 1.3% when comparing against the analytical solution.

7.1.5 The J-integral

As presented in Section 2.2.4.4, in LEFM, the J-integral is a path-independent contour integral that can be used to compute the strain energy release rate, which in turn can be used to compute the stress intensity factor. The proprietary implementation employed by Abaqus is used for the evaluation of the J-integral, which is based on virtual crack extensions and equivalent domain integration methods. Details can be found in the works of Parks (1977) [143] and Shih et al. (1986) [144]; see also [145–147]. For the current problem, the contours used for the J-integral evaluation are defined by circular rings of nodes centred and surrounding the crack tip. The radius of these rings is denoted r .

Figure 7.7a reveals that stress intensity factors computed based on the J-integral evaluation are indeed independent of the chosen contour. Additionally, Figure 7.7b shows how the method is also mostly stable when considering distinct mesh refinement levels, where an average relative difference of 0.4% is verified when comparing results against the analytical solution.

Ultimately, the determination of stress intensity factors based on the J-integral evaluation appears to be the best approach among those tested, where results are accurate and mostly stable regardless of the chosen integration contour and mesh refinement level. Additionally, unlike the VCEM and VCCT approaches, which are also good and stable alternatives, the J-integral evaluation made conveniently and directly by Abaqus does not require a two-step finite element analysis, making it more computationally efficient, which is an important aspect when applying these methods in larger, more realistic problems. Therefore, the approach based on the J-integral evaluation is considered hereinafter.

7: Development of a damage-tolerant approach

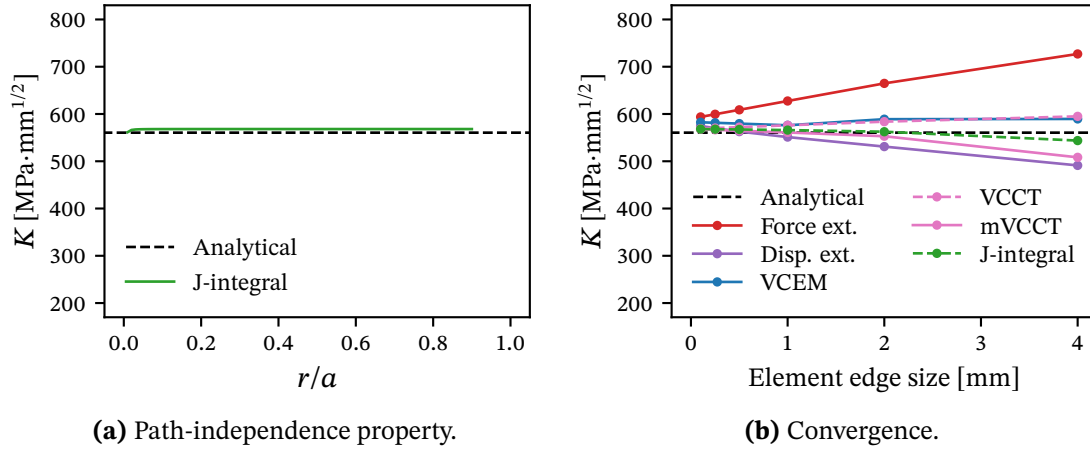


Figure 7.7: Stress intensity factors determined via the J-integral approach.

7.2 Parametric study on crack parameters

A parametric study on crack parameters is performed in order to better understand what is the critical condition that should be considered during the numerical modelling of the Z-profile case study. Moreover, this parametric study is performed in view of the determination of stress intensity factors and how sensitive they are to the considered parameters.

The following parameters are considered: (a) mesh resolution near the crack tip; (b) crack shape (length, depth, and rectangular vs. semi-elliptical crack); (c) crack location with respect to profile's supports; and (d) magnitude of applied load.

The finite element modelling is carried out within the Abaqus/CAE environment. As shown in Figure 7.8, the models consist of a Z-profile and a load actuator, where the latter is modelled as an analytical rigid part. A symmetry condition is considered and only half the profile's length is modelled. To further reduce the computational cost, the majority of the Z-profile is modelled as a shell; however, a portion of the mid-span region, where the crack is located, is modelled as a solid. For connecting both shell and solid domains, a shell-to-solid coupling is used, which is available in Abaqus. Regarding the material, i.e., the S355MC mild steel, it is considered homogeneous, isotropic, and linear-elastic, and no residual stresses are considered, as stress intensity factors are generally used in the context of LEFM.

For simplicity, a static analysis is performed and nonlinear geometry effects are discarded, i.e., small displacements and deformations are assumed, which is typical in LEFM. The Z-profile is clamped at the bolt hole surfaces (all nodal degrees of freedom are constrained in these regions). The load actuator is placed at the mid-span of the Z-profile, which is then displaced against the top surface of the lower flange, creating a stress concentration at the web-to-flange corner, where a crack is modelled. Regarding contact interactions, tangential contact is considered frictionless and a hard pressure-overclosure is considered for the normal contact; additionally, constraints are enforced via the penalty method, which has been used successfully throughout this work.

Regarding the finite element mesh, the C3D8R and S4R element types, both of which have been presented previously, are used to discretize the solid and shell domains, respectively.

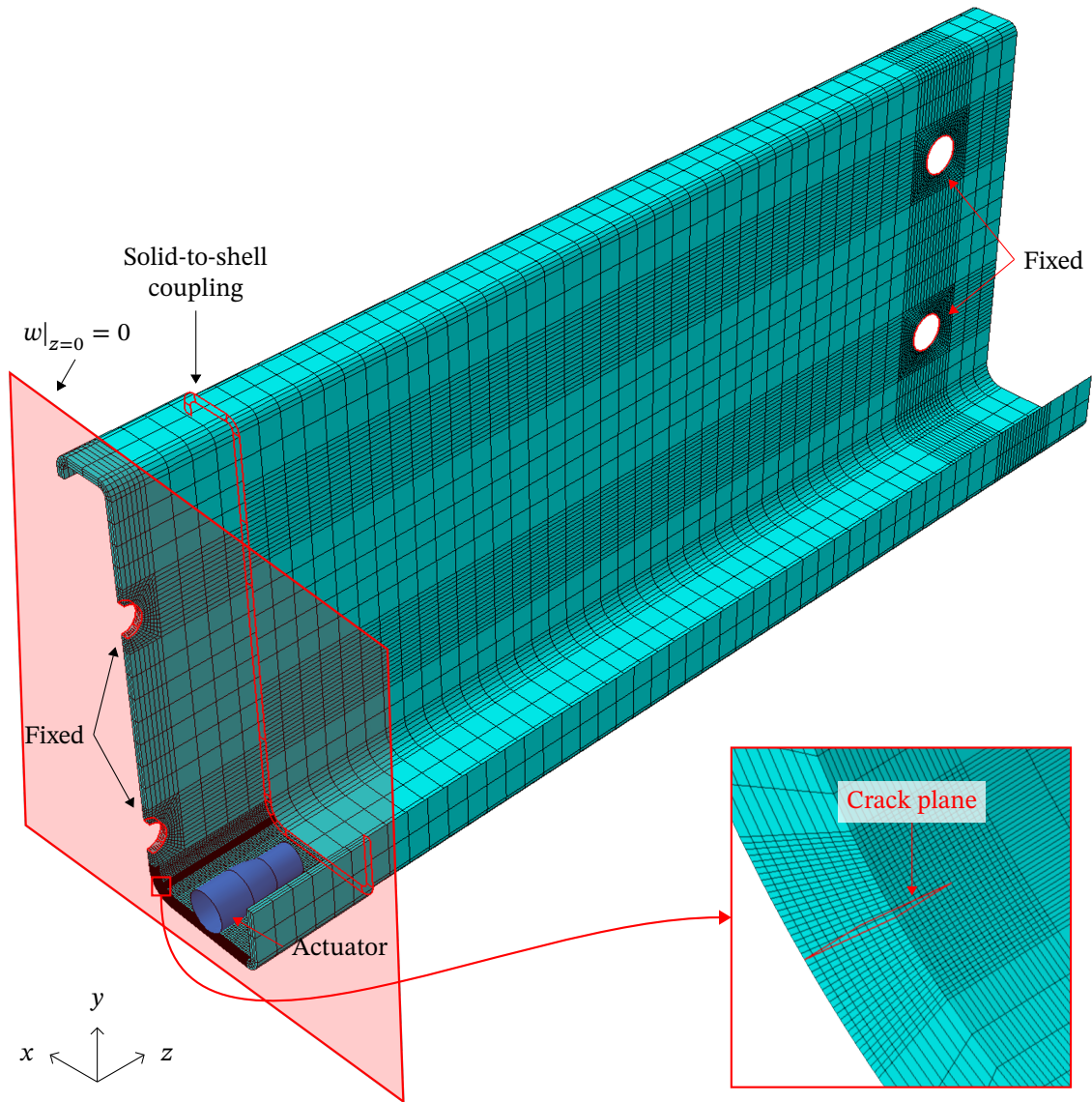


Figure 7.8: Finite element model for crack modelling: mesh and boundary conditions.

Surrounding the crack location, the C3D8 element type is used (fully integrated). In regard to the mesh resolution, a global edge size of around 10 mm is specified for the regions away from the stress concentration and crack locations, whereas a value of 1 mm is specified for the actuator-flange contact zone. Moreover, within curved regions, i.e., bend and hole regions, a local edge seed of around 0.75 mm is specified, whereas at the crack location, a value of 0.1 mm is specified. This local refinement allows keeping the numerical model relatively lightweight while focusing computational efforts into the crack region. Ultimately, the mesh consists of 93,142 nodes and 82,248 elements for the base model (1 mm crack length), with increasing node and element counts for increasing crack lengths.

As for the crack modelling approach, the crack is modelled as a seam, meaning that nodes

7: Development of a damage-tolerant approach

coinciding with the crack plane are duplicated in order to break element connectivity, allowing for the crack to open under loading. Consequently, the crack is modelled as a geometrical discontinuity or a notch with null radius, which is consistent with what is commonly found in LEFM. Moreover, contact is modelled as to prevent penetrations between crack surfaces, allowing for compressive loads to be transmitted during crack closure. Stress intensity factors are determined via the J-integral, as presented in [Section 2.2.4.4](#), using Abaqus built-in capabilities (details can be found in [\[145–147\]](#)). For the J-integral evaluation, 10 contours are considered, corresponding to rings of nodes surrounding the crack tip, from closest and shortest ring to increasingly larger rings. In theory, only a single contour is required, as the J-integral should be path-independent; however, in practice, various contours are used to verify its path-independence property because of the approximate nature of the finite element solution, as a gradual variation may indicate that a finer mesh is required or, when plasticity is included, that the contour integral domain does not completely include the plastic zone. Based on LEFM, the J-integral can be used to compute the stress intensity factors. The relationship used by Abaqus is as follows [\[146, 148–150\]](#):

$$J = \frac{1}{\bar{E}} (K_I^2 + K_{II}^2) + \frac{1}{2G} K_{III}^2, \quad (7.10)$$

where $\bar{E} = E$ for plane stress and $\bar{E} = E/(1 - \nu^2)$ for plane strain, axisymmetry, and three dimensions. As for mode decomposition, the well-known within LEFM maximum tangential stress criterion is considered. The mode decomposition is possible when considering a given direction of a virtual crack extension, which is assumed to be straight along the middle of the web-to-flange corner, consistent with the observed failure modes (see [Figure 1.3a](#)).

7.2.1 Base model results and sensitivity to mesh resolution

For the initial base model, a through-thickness crack is considered with a total length of 1 mm. The crack is located at the middle of the web-to-flange corner, aligned with the fixed web-to-upright bolt holes at the mid-span of the profile. The load actuator is aligned with the crack. A 5 kN load is considered, which corresponds to the lowest tested fatigue load. The model is shown in [Figure 7.8](#) and [Figure 7.9](#).

[Figure 7.10](#) presents how the values of K_I , K_{II} , and K_{III} change for each contour. In this figure, contour 1 corresponds to the shortest possible ring of nodes enclosing the crack tip, and contour 10 corresponds to the longest of the considered rings. Moreover, as a through-thickness crack is considered, stress intensity factors are obtained across the thickness, for each node coinciding with the crack front, and warm (red) colours correspond to stress intensity factors near the inner surface of the web-to-flange corner (subjected to tensile stresses), whereas cool (blue) colours correspond to values near the outer surface (subjected to compressive stresses). Based on [Figure 7.10](#), one may immediately realize that mode I is the dominant mode, with minor influences from modes II and III. Considering the results for mode I, one can verify that K_I is positive above the neutral axis and negative below it, as expected, as the corner is subjected to a bending load, and negative values of K are found at the regions under compression. A clear path independence is found for positive values of K_I , which are the most important values for fatigue, corresponding to the tensile region where the crack is opening. Path dependence is found within the compressive region for contours 1, 2, 3, and 4;

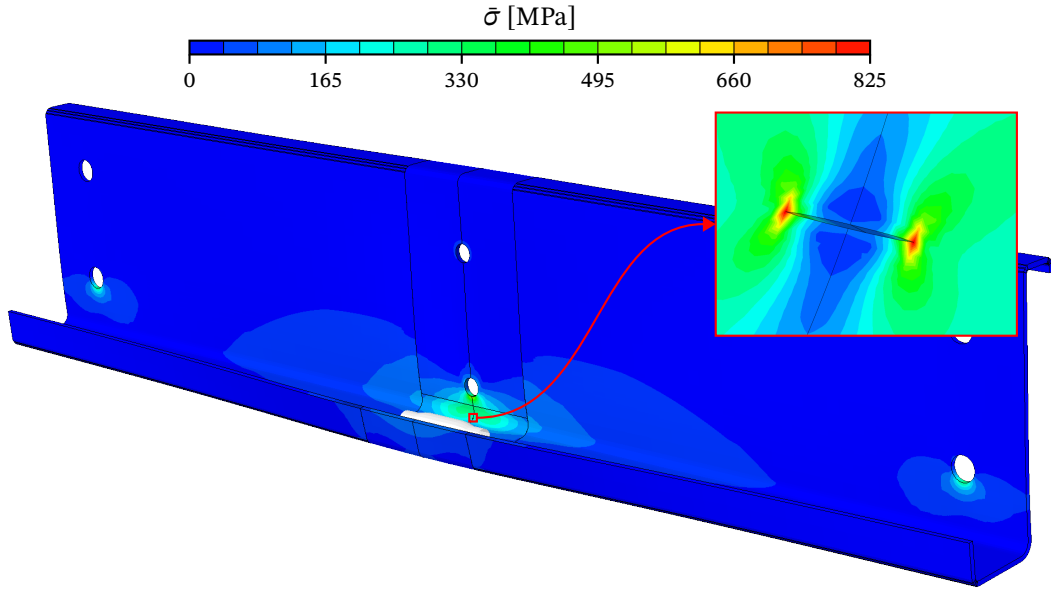


Figure 7.9: Loaded crack in base model (deformation scale factor of 10).

however, the value of K_I immediately stabilizes for further contours, and this initial dependence is not surprising since the first contours are very close to the crack tip and the associated singularity. Regarding modes II and III, a minor path dependence is found for K_{II} for paths 1, 2, 3, and 4, whereas mode K_{III} seems fully path-independent. In conclusion, a strong and general path independence is found for K_I , K_{II} , and K_{III} if disregarding contours 1, 2, 3, and 4 (i.e., the contours that are perhaps too close to the crack tip singularity). Consequently, further results will consider the average value of K_I , K_{II} , and K_{III} for contours 5 to 10.

Figure 7.11 shows the distribution of stress intensity factors across the thickness, along the crack front. The figure reiterates how mode I is the dominant mode, with minor influences

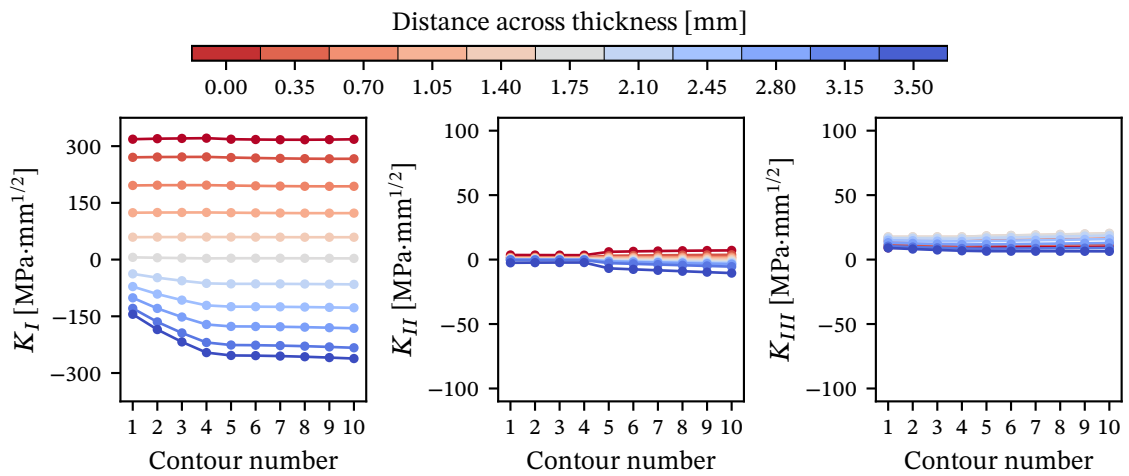


Figure 7.10: Path-(in)dependence property of stress intensity factors (base model).

7: Development of a damage-tolerant approach

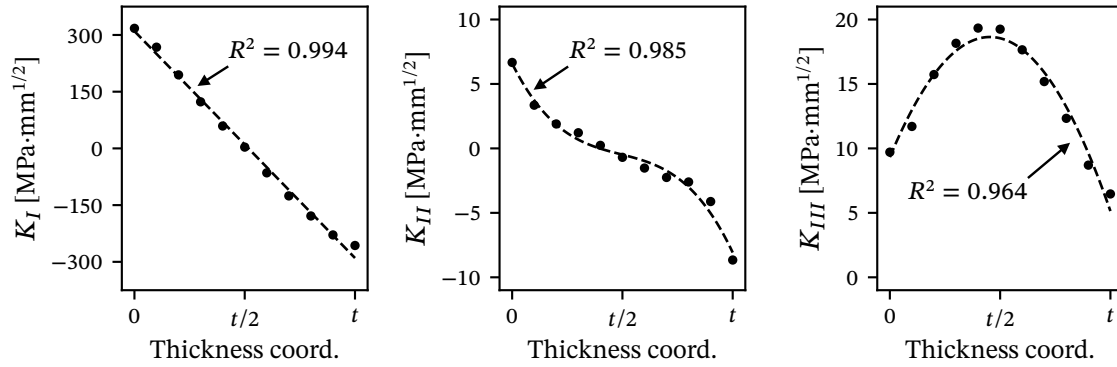


Figure 7.11: Distribution of stress intensity factors across the thickness (base model).

from modes II and III. A linear distribution is found for mode I, whereas modes II and III have a nonlinear distribution. Based on Figure 7.10 and Figure 7.11, it is clear that the critical spot corresponds to the crack front at the inner surface of the web-to-flange corner, which is under tension and the crack is opening, whereas the outer surface is under compression and K_I is negative (crack closing). Additionally, a critical K_I value of 317.5 MPa·mm^{1/2} is verified.

Figure 7.12 reveals that stress intensity factors computed based on the J-integral evaluation are stable for a wide range of mesh resolutions, showcasing the appeal of approaches based on a contour integral evaluation, such as the J-integral approach. A K_I variation of around 1% is verified for edge seeds ranging from 0.05 mm to 0.8 mm. Consequently, and as previously stated, a mesh edge seed of 0.1 mm is considered hereinafter. Furthermore, Figure 7.13 shows the influence of the finite element type choice in the computed values of stress intensity factors. For the considered mesh resolution, which is sufficiently refined, this variation becomes negligible.

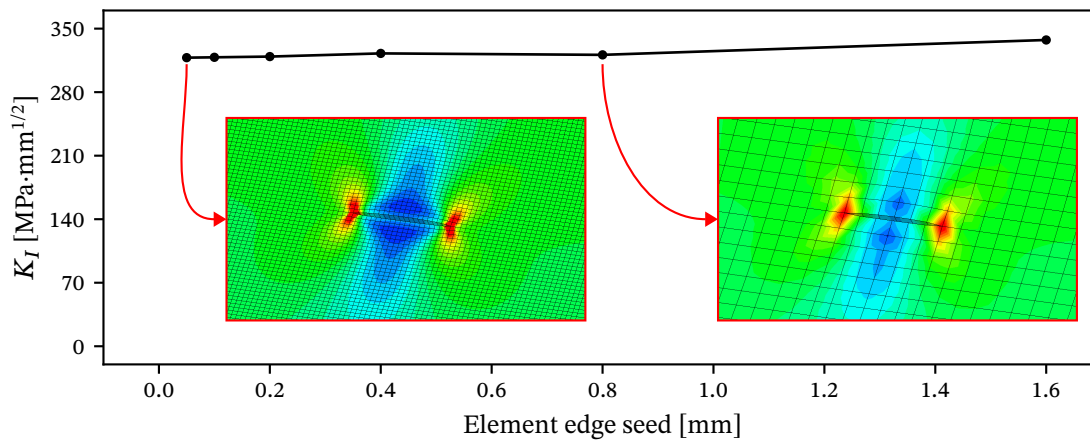


Figure 7.12: Stress intensity factor sensitivity to mesh resolution.

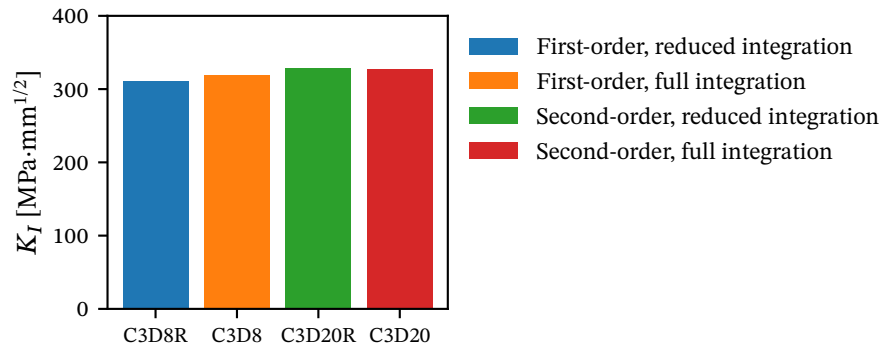


Figure 7.13: Influence of the finite element type choice in the computed values of stress intensity factors.

7.2.2 Effects of crack shape

In order to analyse the effects of crack shape on the resulting stress intensity factors, several cracks are modelled considering distinct crack lengths, depths, and overall shape (rectangular vs. semi-elliptical cracks). Initially, rectangular cracks are considered, with crack lengths ranging from 1 mm to 10 cm, as shown in Figure 7.14. Two crack depths are compared: a through-thickness crack (depth of $t = 3.5$ mm) vs. a half-thickness crack (depth of $t/2$). Results are shown in Figure 7.15a, where a denotes the total crack length, revealing that, for shorter crack lengths, through-thickness and half-thickness cracks share similar stress intensity factors. This is unsurprising because the corner is in bending; hence, from 0 to $t/2$ the

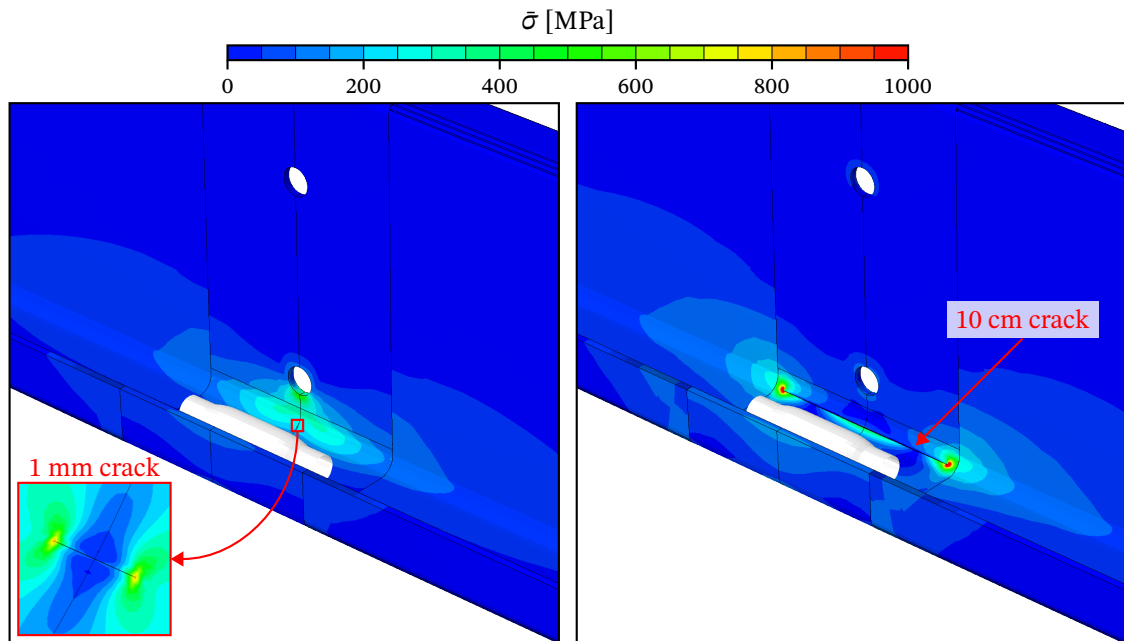


Figure 7.14: Crack length: 1 mm vs. 10 cm crack (true deformation).

7: Development of a damage-tolerant approach

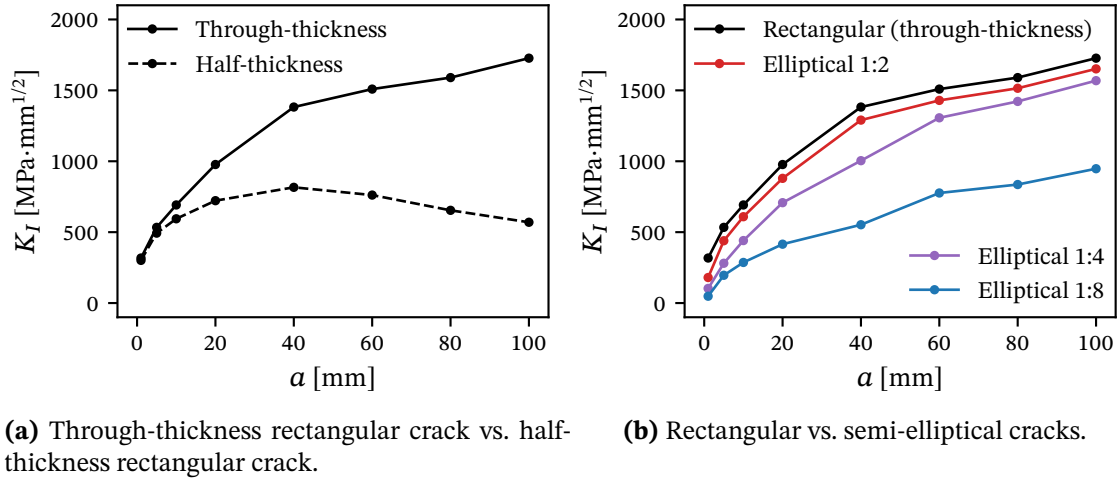


Figure 7.15: Effects of crack shape on the resulting stress intensity factors.

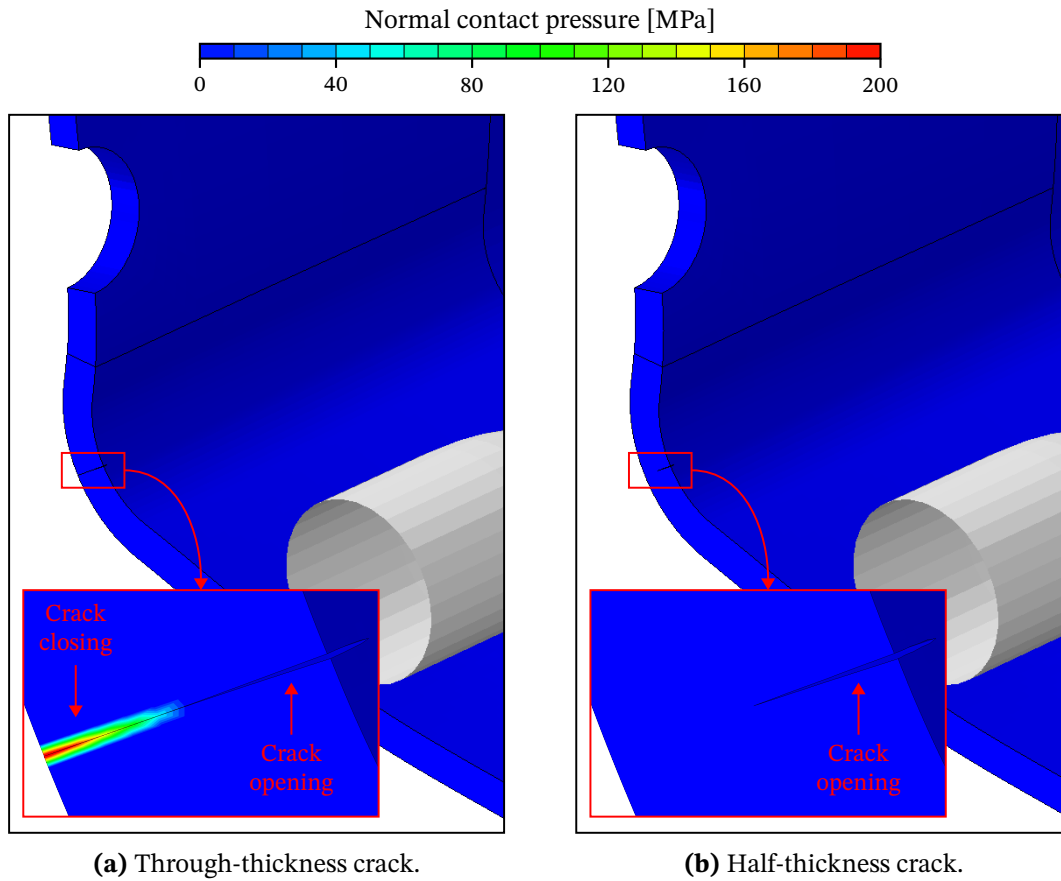


Figure 7.16: Through-thickness vs. half-thickness rectangular crack with a crack length of 1 mm (deformation scale factor of 20).

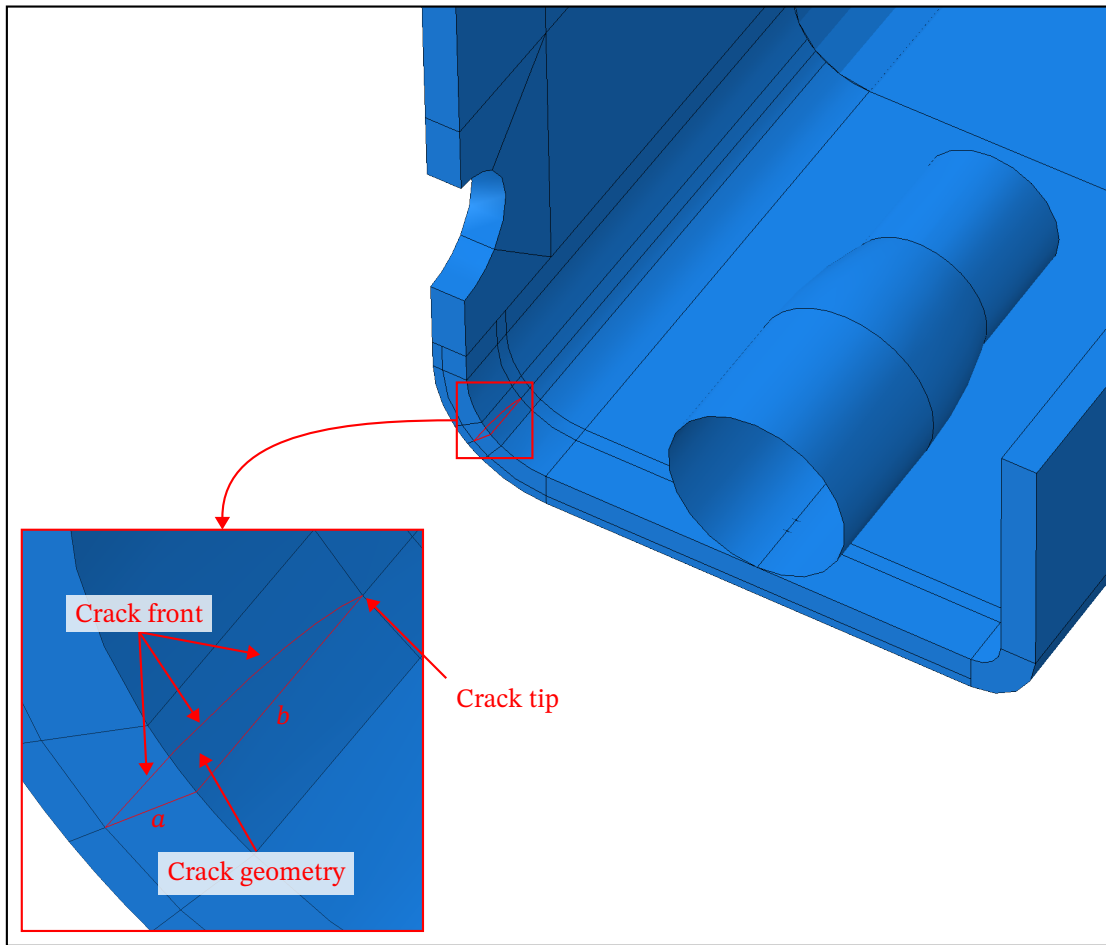


Figure 7.17: Example of a semi-elliptical crack (aspect ratio of 1:2 and crack length of 10 mm).

corner is under tension, allowing for the crack to open, but from $t/2$ to t the corner is under compression, forcing the crack to close. Since contact is enabled between the crack surfaces, the load is still transmitted through the regions where the crack is closing, resulting in very similar behaving cracks regardless if a through-thickness or half-thickness crack is considered. This effect is shown in Figure 7.16. For longer cracks, it is clear that a through-thickness crack results in a higher stress intensity factor; however, results for long half-thickness cracks are merely for reference, since it is not realistic to assume that cracks do not grow across the thickness (i.e., long cracks are most likely also through-thickness cracks).

Semi-elliptical cracks were also investigated. In this regard, three distinct aspect ratios (1:2, 1:4, and 1:8), defined as the ratio of the through-thickness axis (a) to the longitudinal axis (b), were considered for the crack geometry. An example crack is shown in Figure 7.17. Results are shown in Figure 7.15b, where it becomes clear that stress intensity factors are increasingly similar for rectangular and semi-elliptical cracks as the aspect ratio of the ellipse increases. This is expected, as a semi-elliptical crack with an ∞ :1 aspect ratio would share the same geometric crack front as a through-thickness rectangular crack.

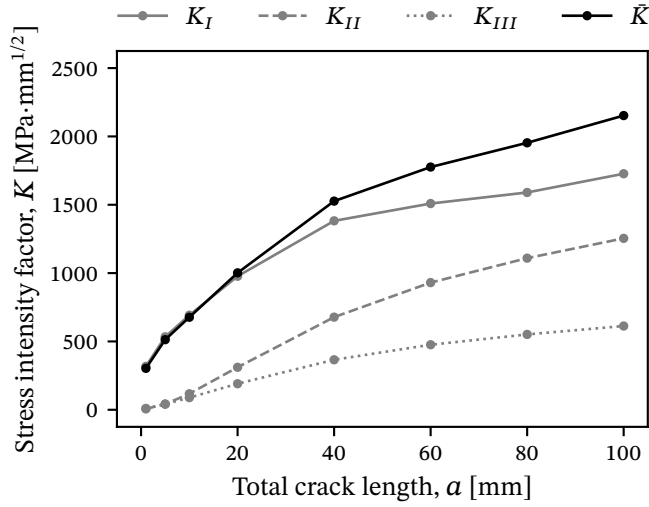


Figure 7.18: Influence from modes II and III on stress intensity factor values.

7.2.3 Influence from modes II and III

For short crack lengths ($a \sim 1$ mm), mode I is the dominant crack opening mode, with very minor influences from modes II and III (see Figure 7.10 and Figure 7.11). However, for increasing crack lengths, modes II and III begin to exert a more prominent effect. For the case at hand, Figure 7.18 shows the stress intensity factors K_I , K_{II} , and K_{III} as a function of the total crack length a . Additionally, to account for a mixed-mode condition, the classical formula

$$\bar{K}^2 = (1 - \nu^2)(K_I^2 + K_{II}^2) + (1 + \nu)K_{III}^2 \quad (7.11)$$

is used [2], in which $\bar{K}$ denotes an equivalent or effective stress intensity factor. This formula is compatible with the one shown in (7.10), yielding the classical $\bar{K} = \sqrt{JE}$ relationship.

As discussed below, a conservative value of $K_c = 1200$ MPa·mm^{1/2} seems to be appropriate for the fracture toughness of the S355 steel [20, 151, 152], which results in a critical crack length of $a_c = 31.0$ mm if considering only K_I or $a_c = 27.6$ mm if considering $\bar{K}$, which is a function of K_I , K_{II} , and K_{III} ; hence, for the case at hand, a maximum relative difference of ~10% can be attributed to the influence from modes II and III in the computation of the critical crack length parameter. This relative difference is expected to increase for increasing crack lengths, but the crack is already expected to propagate in an unstable manner for $a > a_c$.

7.2.4 Effects of load and boundary conditions

In regard to the boundary conditions, the usage of mid-span supports in the form of bolted beam-to-upright connections is also investigated. Figure 7.19a shows a model where the holes located at the mid-span are fixed, representing the bolted connection, which is absent from the model shown in Figure 7.19b. Figure 7.20 reveals that stress intensity factors are lower in magnitude for the case where no mid-span support is modelled, which can be attributed to a more flexible mid-span region resulting in a more favourable stress field. Since higher stress intensity factors relate to fast-growing fatigue cracks, the case where a mid-span support is included is deemed as the critical case. Even if a mid-span support is not used in reality,

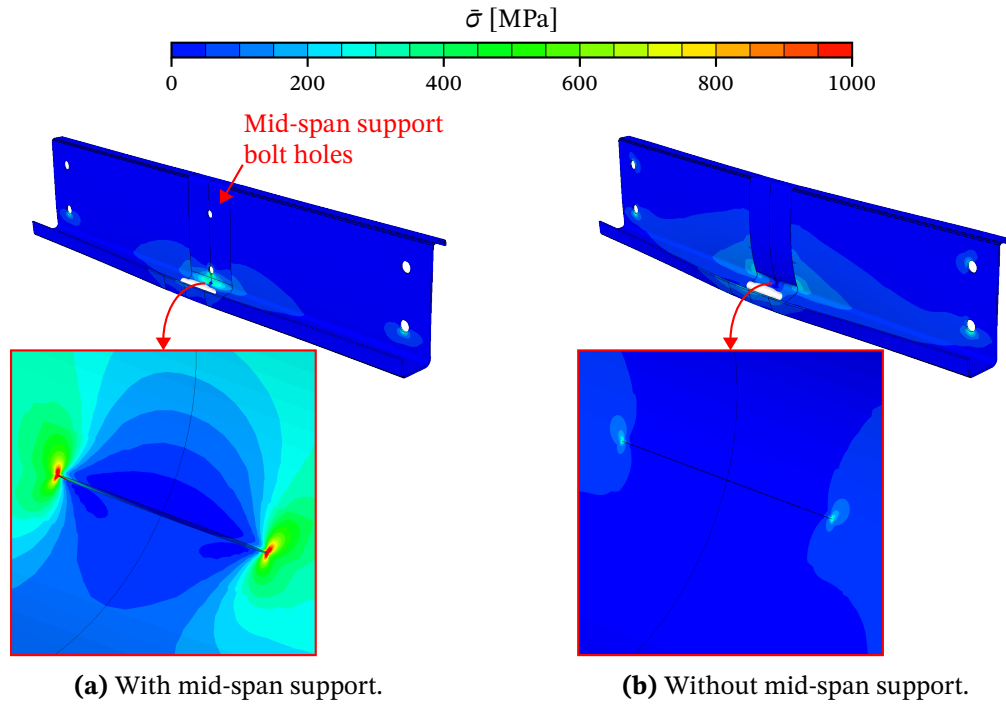


Figure 7.19: Effect of mid-span support on the resulting stress field (crack of 10 mm; deformation scale factor of 10).

this case could also represent a fatigue crack located near the ends of the profile, where end supports are generally required and in the form of bolted connections. Note that Figure 7.20 accounts for K_I , K_{II} , and K_{III} via the expression shown in (7.11).

In regard to the magnitude of the applied load, values ranging from 1 to 10 kN are considered. Results are shown in Figure 7.21a, where a clear linear relationship between $\bar{K}$ and F is verified. Additionally, the value of the slope of the linear relationship for each of the considered crack lengths is plotted in Figure 7.21b.

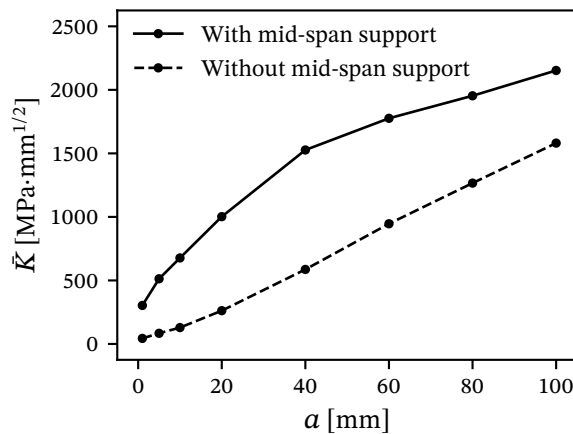


Figure 7.20: Effect of mid-span support on stress intensity factors.

7: Development of a damage-tolerant approach

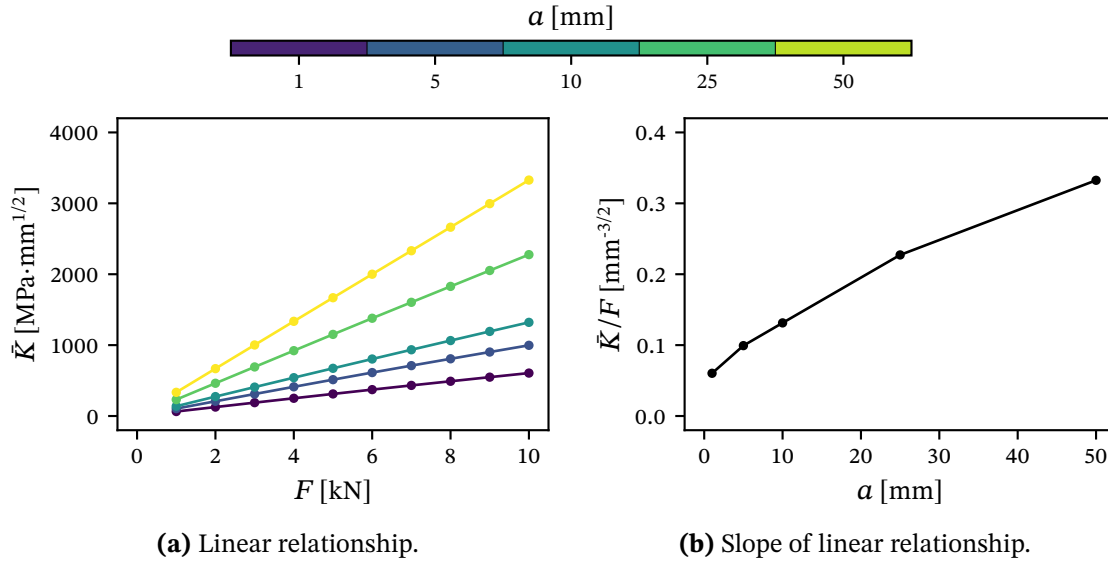


Figure 7.21: Effect of load magnitude on stress intensity factors.

7.3 Simplified approach for design purposes

For design purposes, simple to use formulae are generally preferred whenever possible. To generate such a formula for the current case study, a parameterized finite element model is created in order to perform several finite element analyses while considering distinct values for the considered parameters. A mathematical model is then fitted to the obtained numerical results. Nevertheless, [Appendix B](#) presents an alternative surrogate neural network model.

Regarding the finite element model, it has been presented in [Section 7.2](#), where a sufficiently refined mesh is used to maintain the path-independence property of the J-integral which is used for the stress intensity factor determination. Additionally, a through-thickness rectangular crack aligned with an existing mid-span support is considered, as it was shown in the previous section that it represents the worst-case scenario. Moreover, to account for the influence from modes II and III, the classical formula shown in (7.11) is used.

Regarding the considered parameters, the inside corner radius was modelled with the values of $r \in \{6, 7, 8, 9, 10\}$ mm, the material thickness was modelled with the values of $t \in \{2.5, 3, 3.5, 4, 4.5\}$ mm, the crack length was modelled with the values of $a \in \{1, 5, 10, 25, 50\}$ mm, and the load was modelled with the magnitude of $F \in \{2, 4, 6\}$ kN. Ultimately, this results in 375 finite element models, which were generated via scripting using the Python interface to Abaqus/CAE. Still, it would be beneficial to consider wider ranges for the values of the parameters, which would allow for a more robust analysis of the results. Nevertheless, the choice of the presented values is primarily limited by the computational resources that were available during this work. Additionally, it should be noted that the simplified model, presented below, should be used for interpolating within the ranges presented above, as extrapolation may yield inaccurate results.

Regarding the simplified model, one may start by consider the classical equation for stress intensity factor determination shown in (2.90) from [Section 2.2.4.1](#). As it is shown, the stress

intensity factor is related to an applied remote stress σ_∞ ; however, for the current case study, it is not clear on how to quantify the load in this manner, since the load actuator theoretically applies a line load, which, like a point load, results in an infinitely large stress at the actuator-flange contact zone. Therefore, the applied remote stress σ_∞ is replaced by a local stress measure. In this case, the maximum principal stress σ_1 at the middle of the web-to-flange corner (crack initiation site) is considered, which is determined beforehand and by not including the presence of a crack. Conveniently, a simplified model for the determination of σ_1 has already been developed and presented in [Section 6.3.1](#), which will be reused here. Hence, the simplified model for the determination of the equivalent or effective stress intensity factor $\bar{K}$ assumes the following form:

$$\bar{K}(r, t, F, a) = Y(r, t) \cdot \sigma_1(r, t, F) \cdot \sqrt{\pi a}, \quad (7.12)$$

which accounts for the inside corner radius r , the material thickness t , the magnitude of the applied load F , and the total crack length a . Additionally, the geometric correction factor Y is assumed to be a function of r and t only, a fact that is shown below.

The objective is now to adequately fit the geometric correction factor Y to the obtained numerical data. In this work, Y is fitted for each of the considered values of r , t , and F . For example, for $r = 8$ mm, $t = 3.5$ mm, and $F = 4$ kN, [Figure 7.22a](#) shows the resulting fit, which is treated as a nonlinear least squares problem and solved by unconstrained minimization using the Levenberg-Marquardt algorithm [[120](#), [121](#)] via the SciPy library for scientific computing [[119](#)].

Once the geometric correction factor Y has been fitted to the numerical results, and considering every combination of r , t , and F , one may continue to develop the simplified model by analysing how the value of Y varies by changing one of the considered parameters, e.g., the inside corner radius r . This is shown in [Figure 7.22b](#), where a quadratic relationship is verified. Therefore, the geometric correction factor Y , used in (7.12), is given by

$$Y = A_2 r^2 + A_1 r + A_0. \quad (7.13)$$

The process then repeats, that is, once A_2 , A_1 , and A_0 have been fitted to the available data, one may continue to develop the simplified model by analysing how these coefficients change by varying the next parameter among those considered, e.g., the material thickness t . This is shown in [Figure 7.22c](#), [Figure 7.22d](#), and [Figure 7.22e](#), where quadratic relationships are verified. Hence, the A_2 , A_1 , and A_0 coefficients, used in (7.13), are given by

$$\begin{aligned} A_2 &= A_{22} t^2 + A_{21} t + A_{20}, \\ A_1 &= A_{12} t^2 + A_{11} t + A_{10}, \\ A_0 &= A_{02} t^2 + A_{01} t + A_{00}. \end{aligned} \quad (7.14)$$

Finally, the A_{ij} coefficients are fitted to the available data. Additionally, after verifying how these coefficients change when varying the final parameter among those considered, i.e., the magnitude of the applied load F , it was found that these coefficients remain essentially constant. This is a satisfying result since it means that the geometric correction factor Y is a function of geometric parameters only, namely, the inside corner radius r and the material

7: Development of a damage-tolerant approach

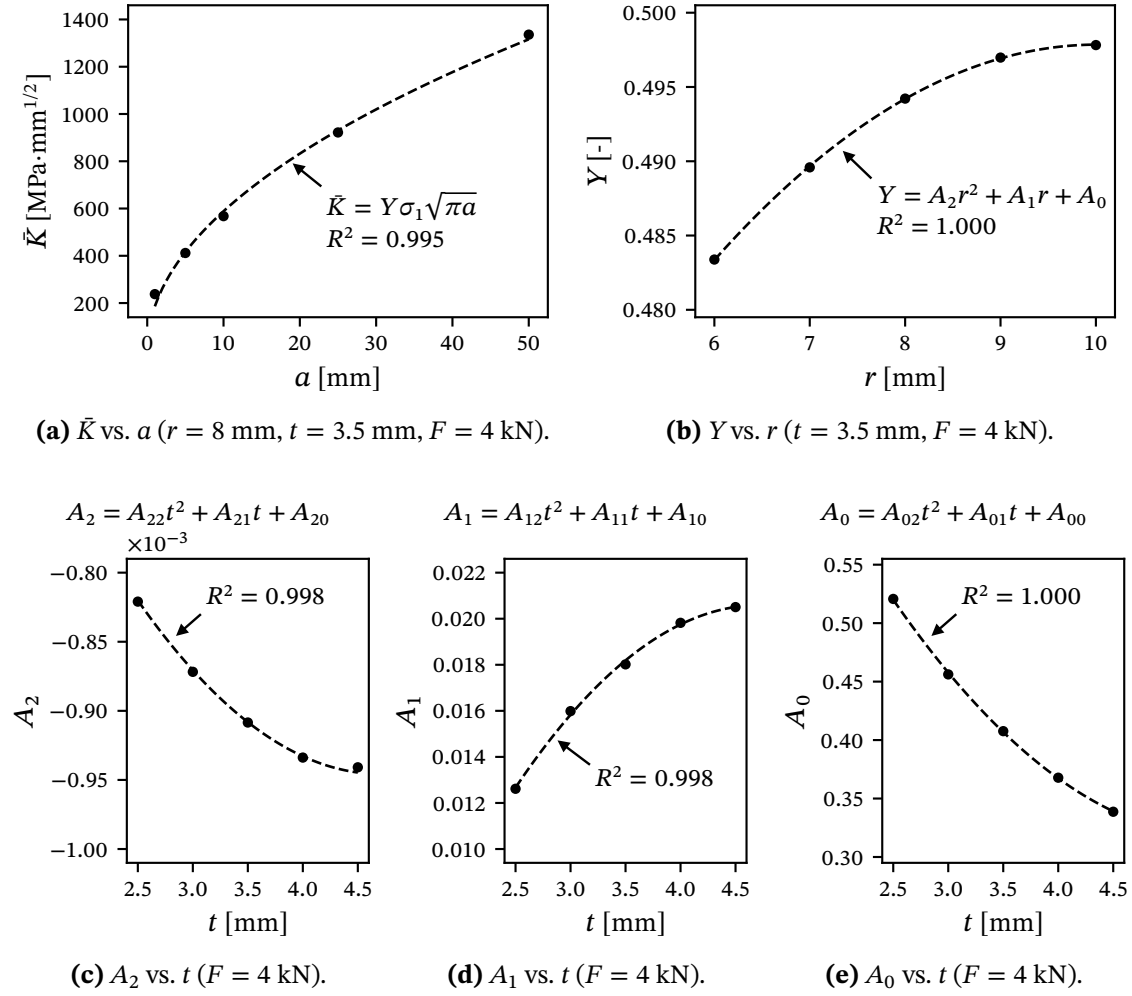


Figure 7.22: Fit of the simplified model for stress intensity factor determination based on the available numerical results.

thickness t . The obtained values are

$$\begin{aligned} A_{22} &= +2.474 \times 10^{-5}, \quad A_{21} = -2.362 \times 10^{-4}, \quad A_{20} = -3.779 \times 10^{-4}, \\ A_{12} &= -1.497 \times 10^{-3}, \quad A_{11} = +1.429 \times 10^{-2}, \quad A_{10} = -1.352 \times 10^{-2}, \\ A_{02} &= +2.385 \times 10^{-2}, \quad A_{01} = -2.593 \times 10^{-1}, \quad A_{00} = +1.025. \end{aligned} \quad (7.15)$$

In conclusion, the constants shown in (7.15) are used to compute the A_2 , A_1 , and A_0 coefficients using the expressions shown in (7.14). Then, the A_2 , A_1 , and A_0 coefficients are used to compute the geometric correction factor Y using the expression shown in (7.13). In parallel, the maximum principal stress σ_1 at the crack initiation site is computed using the expression shown in (6.1), as discussed in Section 6.3.1. Finally, the equivalent stress intensity factor $\bar{K}$ is given by the expression shown in (7.12).

Comparing the numerically obtained results and those obtained by the fitted simplified

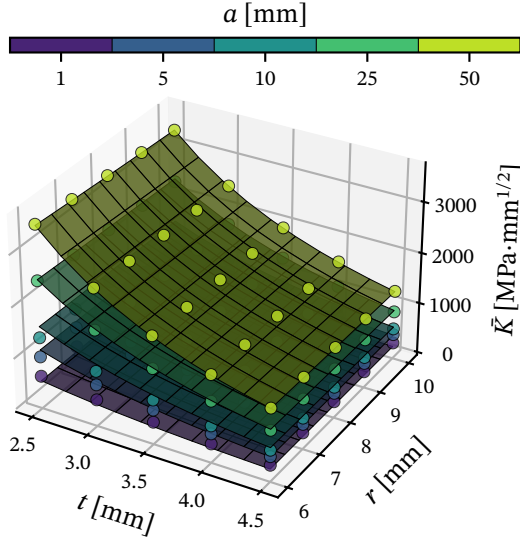


Figure 7.23: Determination of stress intensity factors: fit of the simplified analytical model (surfaces) to the available numerical results (points); example for $F = 6$ kN.

model, an adjusted coefficient of determination of

$$\bar{R}^2 = 1 - (1 - R^2) \frac{n - 1}{n - p - 1} = 0.994 \quad (7.16)$$

is verified, in which R^2 is the coefficient of determination, n is the sample size, and p is the number of independent variables. Figure 7.23 presents a visualization of the corresponding fit. Additionally, an average relative difference of just 2.4% is verified between the available numerical results and those computed via the simplified model. Hence, the simplified model is both convenient and accurate, eliminating the need for a new finite element model when interpolating between the given parameters.

From the literature, one can find that the S355 steel has a fracture toughness K_c between 1194.7 MPa·mm^{1/2} and 1277.6 MPa·mm^{1/2} [151]. Additionally, from reference [152], one can infer a K_c value between 1264.9 MPa·mm^{1/2} and 1581.1 MPa·mm^{1/2}, which is consistent with the data from reference [20]. Therefore, a conservative value of $K_c = 1200$ MPa·mm^{1/2} seems to be appropriate for the S355 steel. Considering this value and the current case study, in which we have an inside corner radius of $r = 8$ mm and a material thickness of $t = 3.5$ mm, using the simplified model presented above, Figure 7.24 presents the critical load magnitude F_c , for which the stress intensity factor $\bar{K}$ is equal to the fracture toughness K_c , as a function of the crack length a . Such a curve delimits a safe/unsafe region, i.e., the crack of length a is expected to propagate in an unstable manner if the load magnitude F is larger than F_c . Interestingly, the figure also highlights three points in which the final crack has been measured after fatigue testing of the Z-profile (see Chapter 6). Of course, these points lie within the unsafe region of the F vs. a relationship, as fatigue testing was stopped shortly after some erratic crack propagation was verified, which should coincide with the point in time after which the crack is considered to be propagating in an unstable manner. These points then validate the conservativeness of the curve.

Finally, Figure 7.25 shows the critical load magnitude F_c , for which the stress intensity factor $\bar{K}$ is equal to the fracture toughness K_c , as a function of the crack length a for various material thicknesses while considering an inside corner radius of $r = 8$ mm.

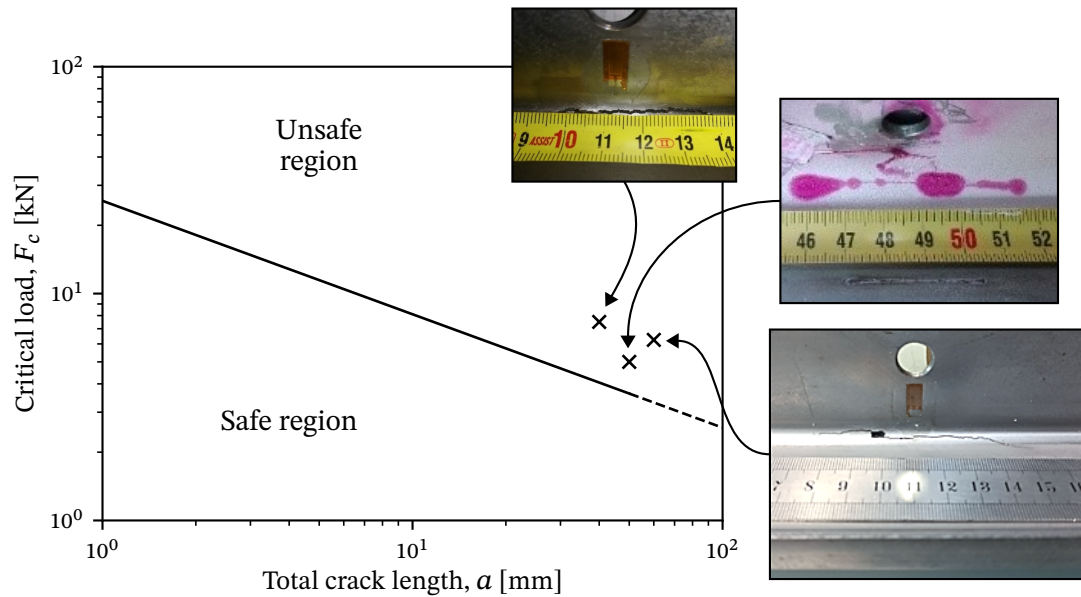


Figure 7.24: Critical load F_c , for which the stress intensity factor $\bar{K}$ is equal to the fracture toughness K_c , as a function of the crack length a . Example for $r = 8$ mm and $t = 3.5$ mm.

7.4 Estimating the residual fatigue life of a cracked profile

In [Section 7.3](#), the simplified model shown in (7.12) was developed by fitting a classical expression to the numerical results generated by a parameterized finite element model, resulting in a straightforward approach for computing equivalent stress intensity factors for the current Z-profile case study, without losing significant accuracy in comparison to direct numerical modelling. This simplified model allows for a static damage-tolerant design which, for a given

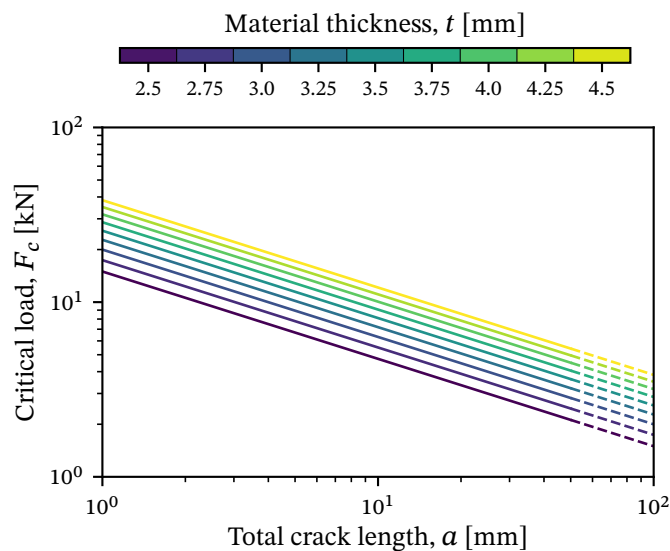


Figure 7.25: Critical load as a function of the crack length for various material thicknesses ($r = 8$ mm).

static load, returns the critical crack length after which unstable propagation is expected, or vice versa (as shown in Figure 7.24 and Figure 7.25).

We can further develop this damage-tolerant approach by using Paris' law, which has been discussed in Section 2.2.4.5, in order to estimate the residual fatigue life of a cracked profile during which stable fatigue crack growth is assumed. In reference [20], the Paris' law coefficient and exponent have been experimentally determined for the S355 steel grade via fatigue crack propagation tests of compact tension specimens under load-controlled conditions and in accordance with the ASTM E647 standard [85], resulting in $C = 2.5893 \times 10^{-15} \text{ mm}^{1/2}/(\text{MPa} \cdot \text{cycle})$ and $m = 3.5622$ for $R_\sigma = 0$. These empirical constants are used in the Paris-Erdogan equation, as shown in (2.98). Besides the Paris' law coefficient and exponent, the stress intensity factor range may be obtained from (7.12), resulting in

$$\begin{aligned} \Delta \bar{K}(r, t, F_{\max}, R_\sigma, a) &= Y(r, t) \cdot \Delta \sigma_1(r, t, F_{\max}, R_\sigma) \cdot \sqrt{\pi a} \\ &= Y(r, t) \cdot \sigma_{1, \max}(r, t, F_{\max}) \cdot (1 - R_\sigma) \cdot \sqrt{\pi a}, \end{aligned} \quad (7.17)$$

where $F_{\max}$ denotes the maximum load value within the constant amplitude fatigue load cycle and R_σ is the stress ratio.

For the current Z-profile case study, the inside corner radius has a value of $r = 8 \text{ mm}$ and a material thickness of $t = 3.5 \text{ mm}$. Additionally, as discussed above, a fracture toughness of $K_c = 1200 \text{ MPa} \cdot \text{mm}^{1/2}$ has been considered for the S355 mild steel grade [20, 151, 152]. To estimate the residual fatigue life of the cracked component, Paris' law must be integrated, as shown in (2.99), from an initial crack length a_i to a critical crack length a_c . As an example, we may consider $F_{\max} = 5 \text{ kN}$ and $a_i = 1 \text{ mm}$. Using the Newton-Raphson method we can then find the critical crack length so that the expression shown in (7.12) results in the fracture toughness K_c , resulting in $a_c = 26.3 \text{ mm}$. Performing the numerical integration, e.g., via the trapezoidal rule, yields a residual fatigue life of $N_f = 1.7 \times 10^6$ load cycles.

By performing the above calculations for distinct maximum values of the fatigue load ($F_{\max}$) and initial crack lengths (a_i), fatigue design curves for estimating the residual endurance of a cracked profile (N_f) can be built in the form of $F_{\max}$ vs. N_f relationships, as shown in Figure 7.26. Additionally, for an initial crack length of $a_i = 1 \text{ mm}$, Figure 7.27 highlights the effect of the profile's thickness in the estimation of residual endurance.

Furthermore, De Jesus et al. (2012) [20], while considering a similar S355 mild steel, studied the effect of stress ratio in the fatigue crack propagation rate during testing of compact tension specimens under load-controlled conditions (see Figure 7.28). It is clear that fatigue crack propagation rates increase for positive stress ratios (R_σ); however, crack propagation rates are similar for the considered positive levels of R_σ . The authors attribute this interesting behaviour to a 'crack closure effect' occurring for $0 \leq R_\sigma < 0.25$, in which the applied stress intensity factor range is not fully effective, becoming so for $R_\sigma \geq 0.25$. Ultimately, the Paris' law coefficient and exponent are influenced by the value of R_σ . For the current case study, a load ratio of $R = 0.1$ is considered and, because a linear load-stress response is assumed during the computation of the stress intensity factors (consistent with LEFM), $R_\sigma = R = 0.1$. As shown in Figure 7.28, one may infer the values of $C = 2.3980 \times 10^{-15} \text{ mm}^{1/2}/(\text{MPa} \cdot \text{cycle})$ and $m = 3.6352$ for $R_\sigma = 0.1$ by interpolating between the available data.

In order to test the predicting capabilities of the proposed approach, specimens 20 and 22 used during the full-scale testing campaign of the Z-profiles are considered (refer to Section 6.1, Figure 6.1, Figure 6.2, and Figure 6.3, as well as reference [141]). The strain gauge

7: Development of a damage-tolerant approach

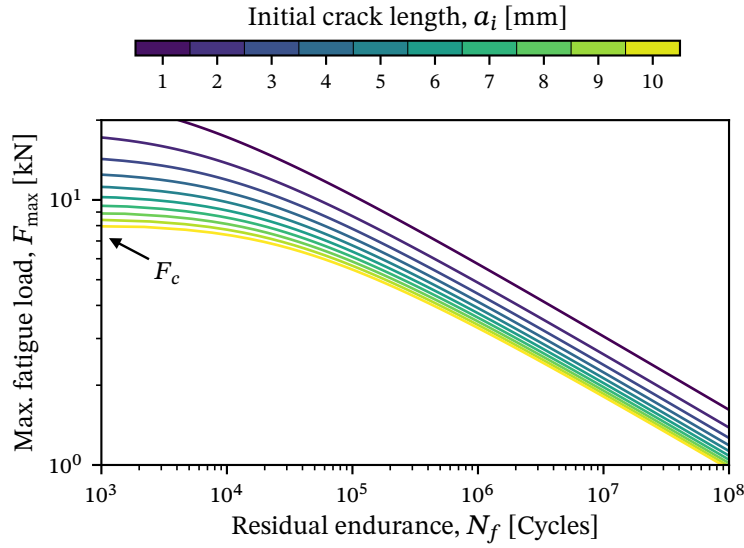


Figure 7.26: Fatigue design curves for estimating the residual endurance of a cracked profile ($r = 8$ mm, $t = 3.5$ mm, $R_\sigma = 0$).

data (converted to stress via $\sigma = E\varepsilon$) is shown in Figure 7.29. As discussed in Chapter 6, the strain gauge is placed at a strategic location for the purpose of fatigue crack monitoring (see Figure 6.3); additionally, as a crack develops in the corner during testing, the discontinuity in the stress and strain fields is captured by the strain gauge in the form of an apparent load reduction. In this case, it is assumed that this initial moment corresponds to a significant total crack length of $a = t/2 = 1.75$ mm.

Ultimately, for the current case study we have an inside corner radius of $r = 8$ mm and a material thickness of $t = 3.5$ mm. Moreover, the maximum fatigue load value is $F_{\max} = 6.25$ kN for specimen 20 and $F_{\max} = 9$ kN for specimen 22, with $R_\sigma = R = 0.1$ resulting in $C = 2.3980 \times 10^{-15}$ mm^{1/2}/(MPa·cycle) and $m = 3.6352$. Paris' law is then integrated, as shown in (2.99), from an initial total crack length of $a_i = t/2 = 1.75$ mm to a critical

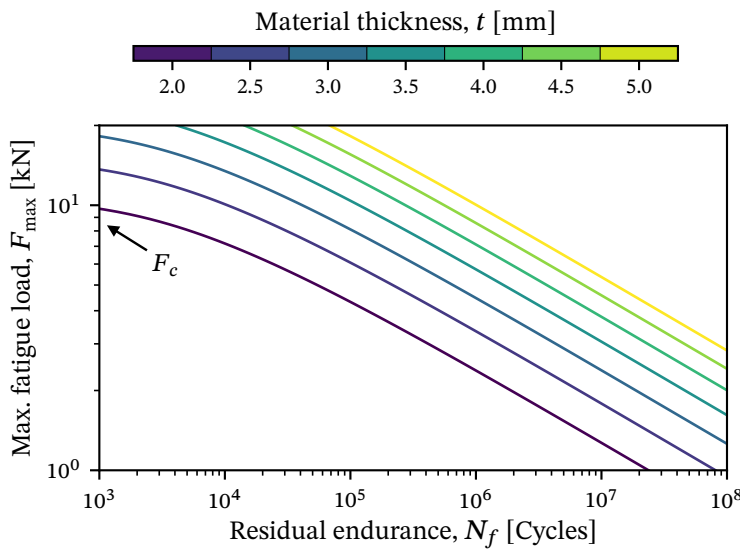


Figure 7.27: Effect of material thickness in the estimation of residual endurance ($r = 8$ mm, $a_i = 1$ mm, $R_\sigma = 0$).

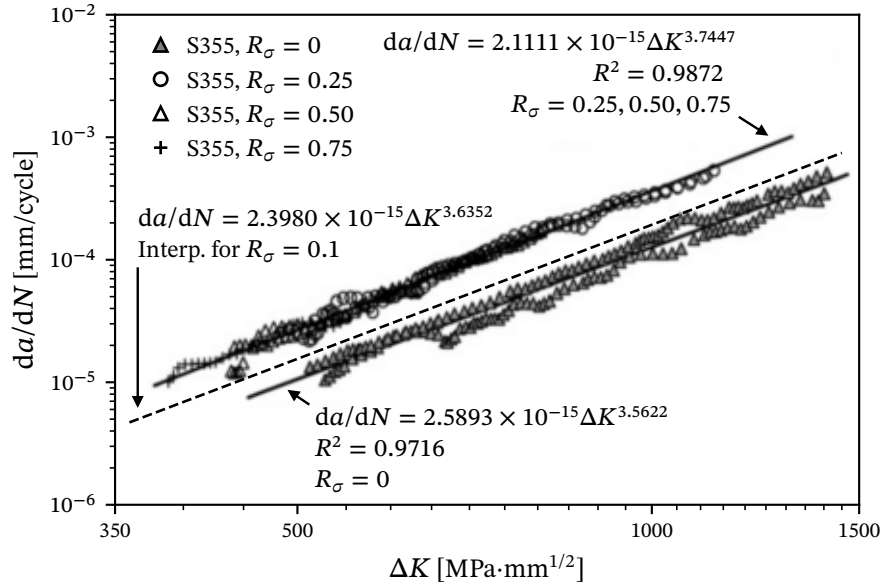


Figure 7.28: Effect of stress ratio in the fatigue crack propagation rate of S355 mild steel [20].

total crack length a_c computed from (7.12) considering a fracture toughness of $K_c = 1200 \text{ MPa}\cdot\text{mm}^{1/2}$. Results are shown in Figure 7.29, which can be directly compared against the strain gauge data obtained during full-scale testing. At the time, these tests were not aimed at the determination of residual fatigue life; therefore, the exact residual life is unavailable because testing was stopped shortly after the crack propagation path started to deviate from the profile's corner into the lower flange (see Figure 6.3). Nevertheless, Figure 7.29 reveals a qualitative interpretation of the results, where predicted residual fatigue lives appear to be overestimated, which is not surprising since residual stresses are not yet taken into account.

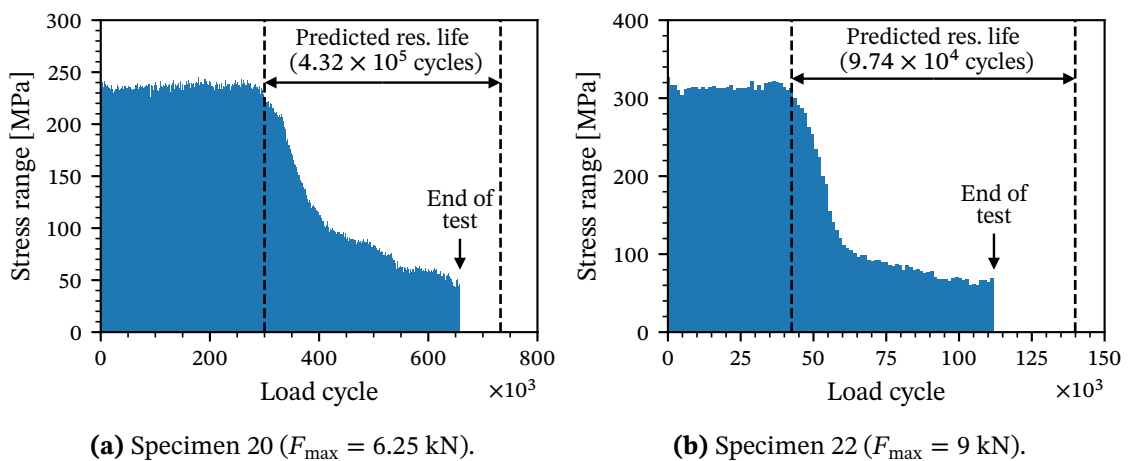


Figure 7.29: Residual fatigue life: prediction vs. experimental data.

7.5 Accounting for residual stresses in the estimation of residual fatigue life

In order to account for residual stresses in the estimation of residual fatigue life in a way that is still compatible with LEFM, it is assumed that residual stresses due to cold forming can be added to the pseudo-elastic stresses arising from fatigue loading (superposition principle). To confirm that this approach is adequate, Table 7.1 presents the cold forming residual stress at the considered fatigue crack initiation site (middle of the cold-formed corner) obtained via elastoplastic FEA for the roll-formed (RF) and press-braked (PB) Z-profiles (refer to Chapter 5) and via the simplified approach due to Moen et al. (2008) [28] (refer to Section 2.1.4). Note that, for consistency with the damage-tolerant approach under development, the maximum principal stress component is considered for the FEA-based results, whereas the simplified approach due to Moen et al. outputs the transverse stress component; however, this is a non-issue since FEA confirms that the maximum principal stress direction closely coincides with the transverse direction. Additionally, Table 7.1 also presents the final maximum principal stress at the considered fatigue crack initiation site, which has been determined via fully elastoplastic FEA for the roll-formed and press-braked Z-profiles (refer to Chapter 6), but also via linear superposition, in which manufacturing residual stresses obtained via the simplified approach due to Moen et al. are added to the pseudo-elastic stresses obtained via the simplified model developed in Section 6.3.1. Ultimately, comparing the results obtained via fully elastoplastic FEA vs. those obtained via stress superposition based on simplified models confirms that relying on the superposition principle is an adequate choice for the considered fatigue load levels, resulting in an average relative difference of -0.4% when comparing against results obtained for the roll-formed profile and $+2.4\%$ for the press-braked profile.

It has been shown above that the superposition principle can be applied to account for cold forming residual stresses during the fatigue loading of a Z-profile, resulting in a pseudo-elastic analysis that is consistent with the principles of LEFM. Consequently, the simplified model for the computation of the equivalent or effective stress intensity factor $\bar{K}$ assumes the following updated form:

$$\bar{K}(r, t, F, a, \sigma_y) = Y(r, t) \cdot [\sigma_1(r, t, F) + \sigma_{\text{res}}(\sigma_y)] \cdot \sqrt{\pi a}, \quad (7.18)$$

in which σ_{res} corresponds to $\sigma_x^{\text{forming}}$ from (2.42) (refer to Section 2.1.4) and $\sigma_y = 394.5$ MPa

Table 7.1: Cold forming residual stress and final maximum principal stress at fatigue crack initiation site.

Fatigue load [N]	Cold forming res. stress [MPa]			Final max. prin. stress [MPa]		
	FEA RF	FEA PB	Simplified (Moen et al.)	FEA RF	FEA PB	Simplified (Linear superpos.)
5000	236.3	198.9	197.5	478.2	461.3	463.1 (-3.2% $+0.4\%$)
5500	236.3	198.9	197.5	506.4	487.4	489.6 (-3.3% $+0.5\%$)
6250	236.3	198.9	197.5	548.7	526.4	529.5 (-3.5% $+0.6\%$)
7500	236.3	198.9	197.5	597.8	588.3	595.9 (-0.3% $+1.3\%$)
9000	236.3	198.9	197.5	625.2	618.4	675.5 ($+8.0\%$ $+9.2\%$)

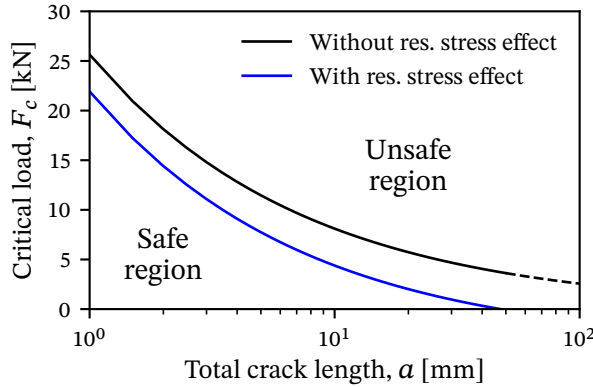


Figure 7.30: Residual stress effect on the critical load F_c as a function of the crack length a .

for the S355MC material (see Figure 3.4). Figure 7.30 shows the residual stress effect on the critical load magnitude F_c as a function of the total crack length a .

Furthermore, in order to compute the residual fatigue life while accounting for residual stresses at the crack initiation site, the stress intensity factor range may be obtained by the following expression:

$$\Delta \bar{K}(r, t, F_{\max}, F_{\min}, a, \sigma_y) = \bar{K}(r, t, F_{\max}, a, \sigma_y) - \bar{K}(r, t, F_{\min}, a, \sigma_y), \quad (7.19)$$

where $\bar{K}$ is obtained from (7.18). The value of $\Delta \bar{K}$ is then used when integrating Paris' law, as shown in (2.99), from an initial crack length a_i to a critical crack length a_c . Note that a_c is computed from (7.18) by optimizing the value of a so that the condition $\bar{K} = K_c$ is verified; hence, a_c is influenced by the residual stress value at the crack initiation site. Additionally, residual stresses also have an effect on the stress ratio R_σ , which is no longer equal to the load ratio $R = F_{\min}/F_{\max} = 0.1$; instead, the stress ratio may be determined as

$$R_\sigma = \frac{\sigma_1(r, t, F_{\min}) + \sigma_{\text{res}}(\sigma_y)}{\sigma_1(r, t, F_{\max}) + \sigma_{\text{res}}(\sigma_y)}, \quad (7.20)$$

where σ_1 has been derived in Section 6.3.1. Consequently, the Paris' law coefficient and exponent should also be updated based on the computed stress ratio (see Figure 7.28).

As an application example, for the current case study we have an inside corner radius of $r = 8$ mm and a material thickness of $t = 3.5$ mm. Moreover, the load ratio is $R = F_{\min}/F_{\max} = 0.1$ and the maximum fatigue load value is $F_{\max} = 6.25$ kN for specimens 20 and 23 and $F_{\max} = 9$ kN for specimens 21 and 22 (in regard to the experiments, refer to Section 6.1, Figure 6.1, Figure 6.2, and Figure 6.3, as well as reference [141]). Due to the presence of residual stresses, the stress ratio is $R_\sigma = 0.44$ for specimens 20 and 23 and $R_\sigma = 0.36$ for specimens 21 and 22, resulting in $C = 2.1111 \times 10^{-15}$ mm^{1/2}/(MPa·cycle) and $m = 3.7447$ for both cases (see Figure 7.28). Paris' law is then integrated, as shown in (2.99), from an initial total crack length of $a_i = t/2 = 1.75$ mm to a critical total crack length a_c computed from (7.18) considering a fracture toughness of $K_c = 1200$ MPa·mm^{1/2} and the residual stress value at the crack initiation site. Results are shown in Figure 7.31, which can be directly compared against the strain gauge data obtained during full-scale testing. A clear decrease in the computed residual fatigue life is verified when considering the effects of residual stresses. However, since the exact residual life is unavailable because testing was stopped shortly after the crack propagation path started to deviate from the profile's corner into the lower flange (see Figure 6.3),

7: Development of a damage-tolerant approach

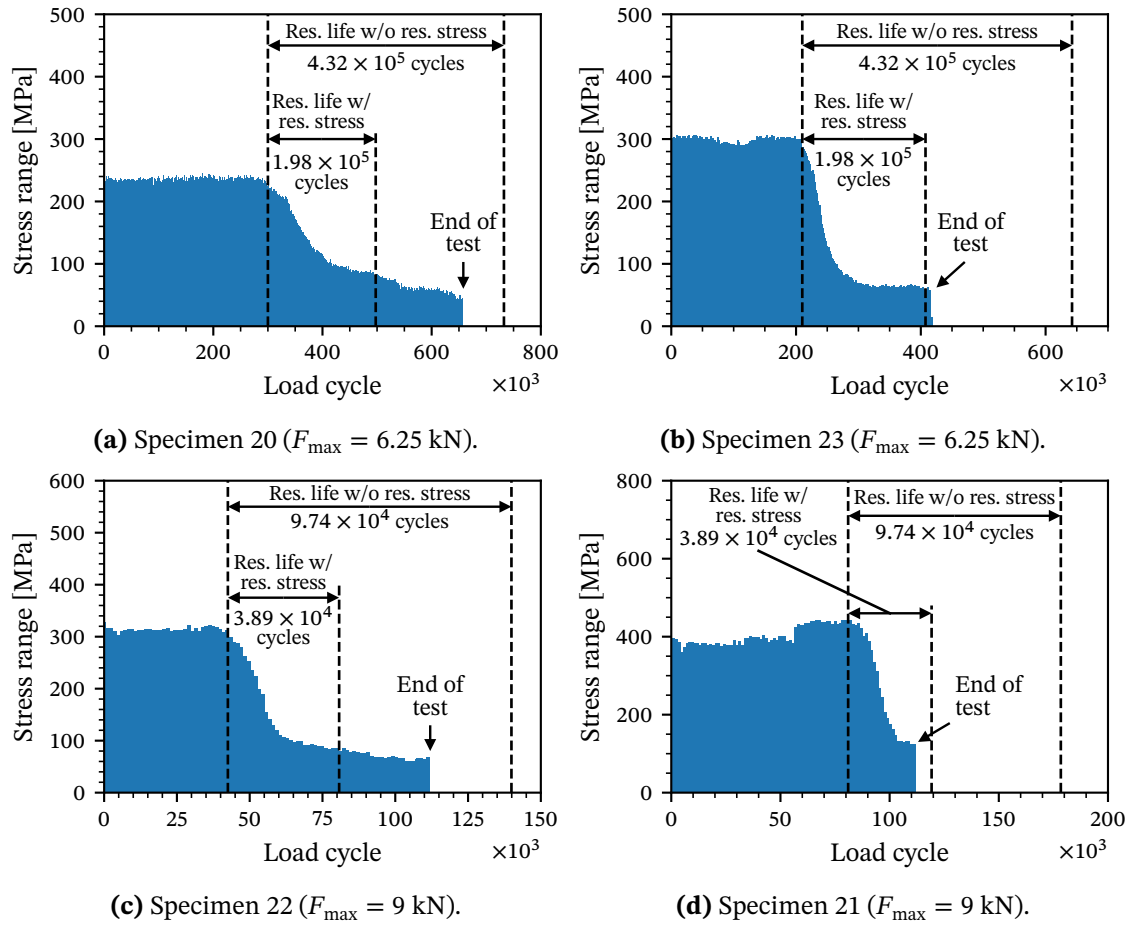


Figure 7.31: Predicted residual fatigue life with and without residual stress effect vs. experimental data.

Figure 7.31 only reveals a qualitative interpretation of the results, in which it is clear that the inclusion of residual stresses in the analysis is favourable, since predictions may be unsafe otherwise.

Chapter 8

Conclusions

The major conclusions of this PhD thesis are summarized below by chapter. For further insight, readers should refer to the corresponding discussions in those chapters. Suggestions for potential future work are also provided.

8.1 Concluding remarks

Chapter 1: Introduction – In this chapter, we discussed how thin-walled rack structures often experience fatigue loads that can lead to fatigue cracks and expensive downtimes for repairs. Because these structures rely on cold-formed mild steel members, it was deemed necessary to investigate the effect of manufacturing residual stresses due to cold forming on the fatigue behaviour of the produced parts. Additionally, these types of structures tend to rely on non-preloaded bolted connections, resulting in limited shear slip resistance joints, an effect that is further exacerbated by low friction zinc coatings in galvanized materials. These characteristics require special considerations that should be considered in the fatigue design; however, current EU regulations do not comprehensively cover these aspects. Besides, detail categorization schemes commonly used for design purposes are becoming more convoluted in an attempt to cover an increasing number of constructional details and specific cases. Hence, there is also a need for more general fatigue assessment procedures that can be applied to a wider range of scenarios or, because large-scale fatigue testing is not always possible in the industry, that can be proposed as an alternative to regression-based approaches. Finally, the importance of the topic at hand is highlighted by the timing and efforts made by the CEN to update the current version of Eurocode 3 via the prEN 1993-1-9 revision [8], which is currently under approval and includes new considerations that are relevant within the current work, and also by research projects, such as the FASTCOLD project funded by the RFCS of the EU (project no. 745982), which aimed the development of specific fatigue assessment procedures for the rack industry, generating important fatigue data that made the current thesis possible.

Chapter 2: Literature review – Within the above background, this chapter presented a comprehensive overview of the theoretical background that sustains this thesis, while also situating the current work within existing research on fatigue, referencing both historically significant publications and recent studies that define the current state of the art.

Chapter 3: Mechanical testing of the plain materials – The mechanical characterization of the S350GD (zinc-coated) and S355MC (uncoated) mild steels has been presented in this chapter, with both materials being representative of current industry applications. Specifically, quasi-static tensile strength testing was performed to assess the yield behaviour of the materials. Mathematical strain hardening models were successfully fitted to the experimental data, which were then used for the material definition in subsequent numerical models. Both steel grades showed the typical ductile yield behaviour of low-carbon steel. However, because the S350GD is cold-rolled, it did not exhibit the typical yield plateau of mild steels, while the S355MC did, since it is hot-rolled. The S355MC mild steel was further tested in cyclic strain-controlled low-frequency tests and load-controlled high-frequency tests to respectively evaluate the material's low- and high-cycle fatigue behaviours. Through low-frequency testing, it was found that the material softens during strain-controlled load cycling. Additionally, from the resulting pseudo-stabilized stress-strain hysteresis loops, it was possible to derive a cyclic stress vs. plastic strain curve for the material, as well as to define a plastic strain vs. endurance (ϵ_p-N_f) relationship. In regard, to high-cycle fatigue, complementary load-controlled high-frequency tests showed that it was adequate to extrapolate the low-cycle fatigue behaviour of the material to the high-cycle fatigue region, resulting in a stress vs. endurance ($\sigma-N_f$) relationship. Because an accurate determination of the ϵ_p-N_f and $\sigma-N_f$ relationships is of utmost importance for the subsequent fatigue analyses, the obtained experimental data was complemented with results extracted from compatible works found in the literature, increasing the reliability of the fitted fatigue constants. Finally, the derived empirical models were summarized in [Table 3.5](#).

Chapter 4: Fatigue behaviour of bolted joints used in rack structures – In this chapter, the topic of bolted joints is thoroughly investigated based on an extensive fatigue testing campaign. Three approaches for the fatigue design of such joints are applied: a classical global $S-N$ approach based on the categorization of the bolted joints, a modified regression-based approach based on numerically computed pseudo-elastic effective stresses and on a master stress-life curve, and an alternative approach based on elastoplastic finite element analysis and on local fatigue damage models relying on empirical properties pertaining to the plain material. The following concluding remarks are made:

- Two distinct failure modes were observed during testing, based on whether shear slip occurs between the joint's plates, which may or may not be prevented, depending on the bolt preload level and applied fatigue loads. If shear slip occurs, a stress concentration at the pin-loaded bolt hole is the leading fatigue damage factor and cracks initiate within the net cross-sectional area. When shear slip is prevented, fretting fatigue seems to be the main fatigue damage factor and cracks initiate within the gross cross-sectional area.
- A classical global $S-N$ approach is successfully applied, resulting in the categorization of the bolted joints based on several joint parameters, such as bolt arrangement and bolt preload level, among others. In this classical approach, a categorization scheme is required because load vs. endurance relationships are considered, where load is described by an engineering measure of stress.
- In order to simplify the above categorization scheme, at the expense of a more complex stress analysis, a modified regression-based approach was successfully applied in

which an effective stress vs. endurance relationship was generated based on a simplified pseudo-elastic material, where stresses were computed via finite element modelling. The resulting master stress-life curve successfully captured the effects of several of the considered joint parameters. Additionally, it was shown that the numerical models were capable of predicting whether shear slip occurs and the corresponding crack initiation sites. The determination of the failure mode during numerical modelling is an improvement over the classical approach, in which the category for preloaded joints may assume that shear slip is prevented; however, in this work, for high enough fatigue loads, it was shown that preloaded joints may behave very similarly to non-preloaded (snug-tight) joints.

- While regression-based approaches were successfully applied, an alternative procedure for fatigue life prediction using locally evaluated fatigue damage models was also investigated, based on a more representative elastoplastic material model and on fatigue properties pertaining to the plain material. In this study, distinct sources for the fatigue constants were considered; later, these parameters were optimized in an inverse analysis. Ultimately, the developed FEA-based local approach is presented as a viable option for fatigue life prediction; however, it was also shown that it was not adequate when fretting fatigue is the leading damage factor.

Chapter 5: Numerical determination of cold forming residual stresses – In this chapter, finite element modelling was used to investigate the processes of press braking and roll forming in view of resulting manufacturing residual stress and strain fields due to cold forming, in which a specific thin-walled Z-profile was considered as a case study. The effects due to pre-forming coiling operations were also included in this investigation. The following concluding remarks are made:

- Distinct finite element formulations were tested: this study included the modelling of the base sheet material as a 2D plane strain solid, as a general 3D solid, or as a general shell. Moreover, shear and volumetric locking effects in solid elements were accounted for via the incompatible mode and hybrid incompressible solid formulations or, alternatively, via reduced integration in pure-displacement formulation elements, and finite element resolutions were kept consistent between the finite element models, so that results could be directly compared, which revealed some variability in the resulting residual fields depending on the chosen methodology. Nevertheless, all approaches effectively produced representative results, achieving the target Z-profile geometry with minimal deviations from the nominal cross-section. Ultimately, pure-displacement formulation elements relying on a reduced integration scheme proved to be the most efficient, offering computational savings while capturing key details, such as the shift in the neutral axis towards the inner corner due to plasticity effects.
- While press braking is a simple bending process, where a plane strain condition can be assumed and residual stresses are limited to the cold-formed corners, roll forming is a much more complex process, in which redundant deformations result in a more intricate residual stress field. In particular, the obtained roll-formed profile showed large longitudinal residual stress values located within the flat regions, near the cold-formed

corners. However, within the cold-formed corner regions, the residual stress fields due to press braking and roll forming are similar. In regard to the modelling process itself, when considering a complex process such as roll forming, the required numerical work was both challenging and time-consuming, resulting in extremely long simulation times.

- This investigation revealed a clear effect due to pre-forming coiling and uncoiling operations on the final residual stress field due to cold forming. In particular, when considering the obtained press-braked profile with and without the effect of coiling and uncoiling, large longitudinal residual stress values are found within the flat regions of the profile based on coiled material. However, within the cold-formed corner regions, cold forming was found to be the dominant process, contributing almost exclusively to the residual stresses found at those locations. Consequently, for the considered case study, in which the crack initiation site is located within the cold-formed corner regions, and because coiling effects are not relevant at these locations, coiling effects are subsequently disregarded in view of simplifying the models.
- The obtained numerical results in view of the final manufacturing residual stress fields were shown to match against previous numerical and experimental findings extracted from the literature, demonstrating that residual stress distributions are mainly a function of the corner radius to material thickness ratio and yield strength of the material.
- Ultimately, this chapter serves as a basis for further studies, providing the accurate determination of manufacturing residual stresses due to cold forming via the numerical simulation of those processes. Such results are later used as the initial state in subsequent structural and fatigue analyses in order to investigate any possible residual stress effect on the fatigue performance of the produced part.

Chapter 6: Fatigue behaviour of cold-formed thin-walled mild steel members – A summary of the main fatigue data generated during the FASTCOLD project is presented in this chapter, which includes results derived from full- and small-scale fatigue testing campaigns that total in approximately 290 *S-N* data points. Based on this large dataset, a unified regression-based detail categorization procedure is proposed where fatigue actions are evaluated via finite element modelling considering a simplified pseudo-elastic material. Regarding this unified approach, the following concluding remarks are made:

- Through numerical modelling, it has been shown that the locations of stress concentration correlate extremely well with the verified crack initiation sites. This allowed for a precise determination of the critical spot, which can vary with several model parameters. In turn, this allows for a unified approach, removing the need for complex detail categorization systems adopted by standards that are becoming increasingly more convoluted.
- The use of numerically computed peak maximum principal stress ranges in solid elements allowed for the assessment of general multiaxial cases that are generally not foreseen by design standards as this invariant measure of stress accounts for all sources of

stress concentration and represents the direction of tensile stresses, simplifying the analysis.

- In general, the proposed detail categorizations converge with the ongoing Eurocode 3 revision, representing several innovations in regard to the existing rules. [Table 8.1](#) summarizes the proposed fatigue detail categories. In total, 7 detail categories are proposed, including the bolted joints analysed in a previous chapter.

Besides the presented advantages and innovations, the proposed unified regression-based approach relies on finite element calculations, which may introduce some disadvantages:

- This type of analysis usually relies on the engineer's technical know-how, expertise, and experience. It is also more complex, expensive, and less straightforward than using simple formulae. However, this approach was first intended to be applied in multiaxial cases, usually not foreseen by design codes, where analytical solutions are simply not available.
- An uncertainty is introduced regarding the most adequate finite element technology and mesh resolution to consider. Regarding the latter, it is common practice to perform a parametric study on the mesh parameters and search for convergent results. Additionally, solid elements are preferred; however, to avoid shear locking, full integration schemes should be avoided in first-order elements based on a pure-displacement formulation when bending is a major part of the static response (i.e., reduced integration or alternative formulations may be preferred), and higher-order elements may be unfavoured as it is well known that they tend to suffer from inaccuracies in contact problems [113]. Still, any numerical study would benefit from experimental data for validation. Throughout the FASTCOLD project, numerical models and subsequent results have been validated via correlation with experimental data arising from monotonic tests. This increased the reliability of the numerical models, ensuring that repeatable and representative results were being achieved.

Despite some possible disadvantages, it seems that the usage of numerical tools is becoming more commonly adopted for design purposes, as hinted by the upcoming prEN 1993-1-14 standard [153] entitled 'Eurocode 3: Design of steel structures - Part 1-14: Design assisted by finite element analysis'. For the numerical stress analyses, each case should be analysed for possible nuances and particularities. However, the following general guidelines can be given:

- For simplicity, a linear-elastic stress vs. strain relationship may be considered. As shown in this work, this results in fatigue design curves based on pseudo-elastic stresses. However, this does not mean that the system's response is linear.
- Solid elements are preferred and maximum principal stresses may be considered, ensuring that all sources of tensile stress concentration are captured via an invariant stress measure. A mesh refinement should be considered in the hot spot regions. To reduce the computational cost, shell or beam elements may be employed away from the hot spot region if coupling technologies are available.

8: Conclusions

- A static or quasi-static analysis may be considered. The maximum and minimum loads should be simulated in succession, resulting in corresponding maximum and minimum stresses. The critical spot may be defined as the mesh node showing the largest maximum principal stress range within the load cycle. The critical spot should match with observed crack initiation sites.
- The external load and boundary conditions should be modelled as close to reality as possible. Hence, a contact simulation is most likely required, either for the load application or boundary conditions in the form of bolted connections. For the latter, representative bolt preload levels should be included, and tangential contact should be modelled with appropriate friction coefficients if shear slip is expected. The geometrical deviations at the cold-formed corners due to manufacturing tolerances are relevant; on effect, actual or representative geometries should be considered.
- Finally, in the case of high fatigue loads resulting in large displacements or rotations, the consideration of nonlinear geometry effects may be investigated.

Besides the aforementioned unified regression-based procedure, an alternative state-of-the-art framework for fatigue life prediction in full-scale thin-walled and cold-formed mild steel profiles was presented. The framework may be summarized as follows (also see [Figure 6.21](#)):

1. A local fatigue damage model is required. In this work, the Basquin-Coffin-Manson model with Morrow's mean-stress correction was considered.
2. The corresponding fatigue properties pertaining to the plain material (i.e., the S355MC mild steel) were evaluated via strain- and load-controlled experiments of smooth and polished specimens, evaluating the material's low- and high-cycle fatigue behaviours.
3. Considering the elastoplastic behaviour of the plain material, the manufacturing process of the full-scale profile was simulated via finite element modelling in order to obtain representative residual stress and strain fields. In this work, the press braking and roll forming processes were considered. Moreover, the effects due to pre-forming coiling operations were investigated, which were found to affect mainly the longitudinal component of residual stress within the flat regions, away from the initiation site at the cold-formed corners; hence, coiling effects were subsequently discarded for simplicity.
4. A structural analysis was then performed using the geometry and residual stress state computed during the cold forming simulations as the initial state. Fatigue loads were then applied and equivalent stress and strain measures were computed. In this work, major principal values of stress and strain were considered.
5. Finally, the considered local fatigue model was solved for each node present in the finite element mesh, allowing for the direct computation of fatigue life predictions at the critical surfaces of the cold-formed corners.

In regard to the residual stress effect on fatigue life predictions, a clear detrimental effect was verified for the considered case study since, locally, tensile residual stresses aligned with tensile structural stresses during fatigue loading, increasing the mean-stress level. Importantly, however, is to note that fatigue life predictions were found to be much more accurate

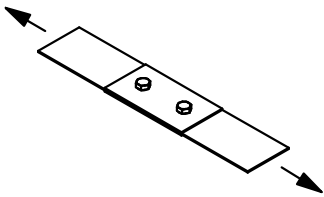
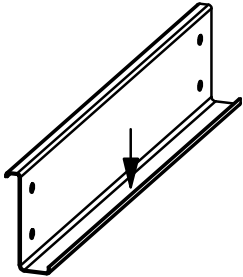
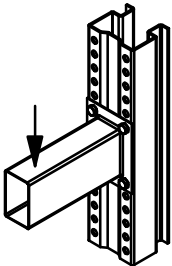
when residual stresses were considered during the analyses, accounting in this way for the material's straining and hardening history. In the considered case study, this is particularly important as fatigue lives were excessively overestimated prior to the inclusion of residual stress effects, resulting in unsafe predictions. With this new framework, it was also possible to generate load vs. endurance points (see [Figure 6.25](#)) that were consistent with the fatigue lives verified during full-scale testing, despite both approaches being independent. Therefore, this alternative, more numerically intensive framework was shown to be a viable option for cases in which full-scale fatigue testing is not viable, relying solely on fatigue properties of the plain material, which are more easily available or obtainable, and on general finite element analysis.

Chapter 7: Development of a damage-tolerant approach – In this chapter, the development of a damage-tolerant approach has been presented. The development process started off with a preliminary study to find the best suited numerical approach for the determination of stress intensity factors. In this study, the J-integral is ultimately chosen due to its path-independence and low sensitivity to mesh resolution. Furthermore, a parametric study on crack parameters, such as mesh resolution, crack shape (crack depth and rectangular vs. semi-elliptical cracks), mixed-mode influences, as well as load and boundary conditions was performed. A critical scenario was identified, in which a rectangular crack is aligned with the mid-span supports of the profile. Additionally, it was found that, for small enough crack lengths (up to ~ 20 mm), the crack front is dominated almost exclusively by mode I. However, as the crack grows, or if the mid-span support is removed, a mixed-mode condition is verified. After this initial analysis, a parameterized finite element model was created, allowing for the fit of a subsequent simplified analytical model for the purpose of stress intensity factor determination, which was then used for computing critical crack lengths and corresponding loads within the static design of a cracked profile, delimiting the domain of stable fatigue crack growth based on the fracture toughness of the material. By considering Paris' law, the residual fatigue life of the cracked Z-profile was determined, resulting in design curves for distinct profile thicknesses and initial crack lengths. Finally, exploratory research was also performed to account for manufacturing residual stresses in the estimation of the residual fatigue life. Throughout this analysis, using the experimental data arising from previous full-scale fatigue testing, a qualitative comparison has been presented to verify the predicting capabilities of the derived approach. It was shown that predicted residual fatigue lives may be overestimated, so the inclusion of residual stresses, which have a clear detrimental effect, are key to generate more representative results.

Ultimately, to reduce costs and win bids for modern-day projects, contractors are motivated to propose thinner structural members that balance affordability, structural integrity, and regulatory compliance. In this context, advanced engineering techniques, such as the damage-tolerant approach developed for predicting the residual fatigue life of a profile in which a visible fatigue crack has already started to develop, may be crucial for ensuring or extending a structure's service life.

8: Conclusions

Table 8.1: Fatigue categories for structural details found in rack structures. Notes: ΔS_C is the characteristic reference value of fatigue resistance at 2×10^6 load cycles; m is the slope parameter of the fatigue resistance curve; and N_D is the corresponding fatigue limit.

Detail	Description	Supplementary requirements	ΔS_C [MPa]	m [-]	N_D [Cycles]
	Double covered symmetrical joint with normal bolts in holes with normal clearances subjected to normal stress. Joint made of thin galvanized mild steel plates ($t \sim 2$ mm). Punched holes or drilled with equivalent or superior quality.	With preloaded bolts. Nominal stress range based on the gross cross-section of the member with potential fatigue cracking.	100	5	2×10^6
		With non-preloaded bolts. Nominal stress range based on the net cross-section of the member with potential fatigue cracking.	90	5	2×10^6
		With preloaded or non-preloaded bolts. Numerically computed peak pseudo-elastic maximum principal stress range accounting for actual friction and bolt preload. Critical location to be determined by a detailed stress analysis.	160	3	5×10^6
	Thin-walled cold-formed mild steel rail profile under localized web-to-flange corner bending ($t \sim 3$ mm).	For $r/t > 1$. Numerically computed peak pseudo-elastic maximum principal stress range based on the actual quality-controlled geometry. Critical location to be determined by a detailed stress analysis.	180	5	2×10^6
		For $r/t \leq 1$. Numerically computed peak pseudo-elastic maximum principal stress range based on the actual quality-controlled geometry. Critical location to be determined by a detailed stress analysis.	160	5	2×10^6
	Bolted beam-to-upright joint of thin-walled cold-formed mild steel sections ($t \sim 3$ mm). Initiation site at the upright due to push-pull bolt forces. Punched perforations.	For $r/t > 1$. Numerically computed peak pseudo-elastic maximum principal stress range based on the actual quality-controlled geometry. Critical location to be determined by a detailed stress analysis.	180	5	2×10^6
		For $r/t \leq 1$. Numerically computed peak pseudo-elastic maximum principal stress range based on the actual quality-controlled geometry. Critical location to be determined by a detailed stress analysis.	160	5	2×10^6

8.2 Future work

Some suggestions for potential future work are provided as follows:

- Regarding the characterization of the materials ([Chapter 3](#)), empirical constants related to the Basquin and Coffin-Manson models were obtained for the S355MC mild steel grade via low- and high-cycle fatigue testing ([Table 3.5](#)). This analysis could be extended to include the S350GD material, enabling a comparative assessment of both materials' fatigue behaviour. Additionally, within the context of the damage-tolerant approach, the Paris' law coefficient and exponent were extracted from the literature for a similar S355 mild steel grade. However, it would be preferable to determine these constants experimentally for the exact materials in question and to compare them with the literature values.
- In regard to the investigated bolted joints ([Chapter 4](#)), damage due to fretting fatigue could be explored and incorporated into the finite element modelling, possibly enhancing the accuracy of the considered local fatigue life prediction approach, particularly for higher fatigue lives approaching the infinite fatigue life region. Moreover, the application of distinct multiaxial fatigue criteria could also be investigated.
- Regarding the numerical determination of cold forming residual stresses ([Chapter 5](#)), this study could benefit from further refinements, such as an increased mesh resolution, which was primarily limited by the computational resources that were available. Additionally, a more representative roll forming model could be explored, incorporating factors such as friction, roll rotational speed, fully dynamic analysis, extended sheet length, and deformable solid rolls with deformable shafts modelled using beam elements.
- As for the local fatigue life prediction approach applied to the Z-profiles ([Chapter 6](#)), a possible surface effect could be investigated, since specimens used to determine the Basquin and Coffin-Manson constants were polished, whereas the tested full-scale Z-profiles were sampled directly from the manufacturer without any surface treatment. Additionally, a statistical analysis could be conducted to evaluate the actual cold-formed corner geometry and its deviations from the nominal profile. Furthermore, during numerical modelling, the impact of different fatigue load stepping procedures could be investigated and, in regard to fatigue life prediction, alternative local models and various methods for determining effective stress and strain measures could also be explored.
- Regarding the damage-tolerant approach ([Chapter 7](#)), this topic would benefit from further experimental validation. Additionally, it was clear from the literature that the Paris' law coefficient and exponent are affected by the applied stress ratio; however, a deeper understanding of this effect in the fracture toughness of the material is still required. Besides, the application of fatigue crack propagation laws that account for the three distinct propagation regimes could also be investigated.

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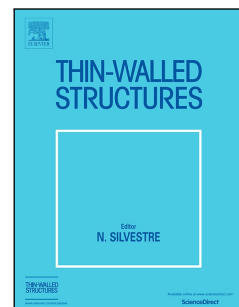
Appendix A

List of publications

This appendix lists 10 research papers published within the scope of this thesis. Most are peer-reviewed articles published in international scientific journals, with some shorter papers published in conference proceedings. These publications have received a total of 208 citations across 198 documents, according to Scopus at the time of writing.

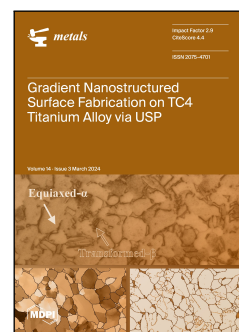
C. Souto, M. Parente, J. Correia, and A. de Jesus. ‘The effect of cold forming residual stresses in local fatigue approaches: A numerical perspective’. *Thin-Walled Structures* 205A (2024) 112476.
[doi:10.1016/j.tws.2024.112476](https://doi.org/10.1016/j.tws.2024.112476)

CiteScore: 9.6 · Impact Factor: 5.7



V. M. G. Gomes, **C. D. S. Souto**, J. A. F. O. Correia, and A. M. P. de Jesus. ‘Monotonic and Fatigue Behaviour of the 51CrV4 Steel with Application in Leaf Springs of Railway Rolling Stock’. *Metals* 14.3 (2024) 266.
[doi:10.3390/met14030266](https://doi.org/10.3390/met14030266)

CiteScore: 4.9 · Impact Factor: 2.6

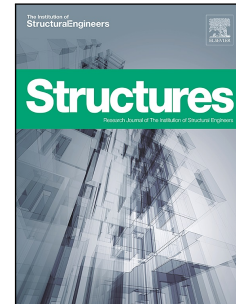


A: List of publications

C. Souto, C. Castiglioni, A. Menghini, F. Morelli, A. Piscini, B. Hoffmeister, T. Geers, H. Degée, P. Tzintzos, O. Kraus, J. Frederiks, S. Sesana, M. Figueiredo, V. Gomes, J. Correia, and A. de Jesus. 'A unified approach for fatigue detail categorization applied to rack structures'. *Structures* 56 (2023) 104948.

[doi:10.1016/j.istruc.2023.104948](https://doi.org/10.1016/j.istruc.2023.104948)

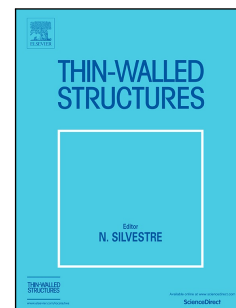
CiteScore: 5.7 · Impact Factor: 3.9



C. D. S. Souto, A. Menghini, A. Díaz, J. M. Manso, A. M. P. de Jesus, and C. A. Castiglioni. 'Determination of manufacturing residual stresses in cold-formed thin-walled steel profiles'. *Thin-Walled Structures* 180 (2022) 109945.

[doi:10.1016/j.tws.2022.109945](https://doi.org/10.1016/j.tws.2022.109945)

CiteScore: 9.6 · Impact Factor: 5.7



C. D. S. Souto, V. M. G. Gomes, L. F. R. C. da Silva, M. V. Figueiredo, J. A. F. O. Correia, G. Lesiuk, A. A. Fernandes, and A. M. P. de Jesus. 'Global-local fatigue approaches for snug-tight and preloaded hot-dip galvanized steel bolted joints'. *International Journal of Fatigue* 153 (2021) 106486.

[doi:10.1016/j.ijfatigue.2021.106486](https://doi.org/10.1016/j.ijfatigue.2021.106486)

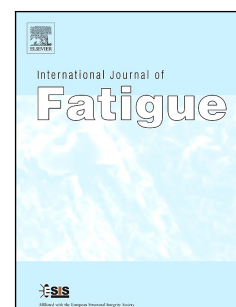
CiteScore: 10.7 · Impact Factor: 5.7



C. D. S. Souto, V. M. G. Gomes, M. Figueiredo, J. A. F. O. Correia, G. Lesiuk, A. A. Fernandes, and A. M. P. de Jesus. 'Fatigue behaviour of thin-walled cold roll-formed steel sections'. *International Journal of Fatigue* 149 (2021) 106299.

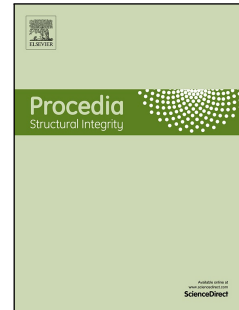
[doi:10.1016/j.ijfatigue.2021.106299](https://doi.org/10.1016/j.ijfatigue.2021.106299)

CiteScore: 10.7 · Impact Factor: 5.7



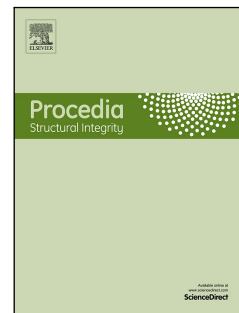
C. D. S. Souto, S. M. O. Tavares, J. A. F. O. Correia, and A. M. P. de Jesus. 'Numerical determination of stress intensity factors: J-integral and modified virtual crack closure technique'. *Procedia Structural Integrity* 28 (2020) 146-154.
[doi:10.1016/j.prostr.2020.10.019](https://doi.org/10.1016/j.prostr.2020.10.019)

CiteScore: 1.7



C. D. S. Souto, V. M. G. Gomes, M. Figueiredo, J. A. F. O. Correia, G. Lesiuk, A. A. Fernandes, and A. M. P. de Jesus. 'Fatigue behavior of cold roll-formed rail profiles for rack structures'. *Procedia Structural Integrity* 28 (2020) 139-145.
[doi:10.1016/j.prostr.2020.10.018](https://doi.org/10.1016/j.prostr.2020.10.018)

CiteScore: 1.7



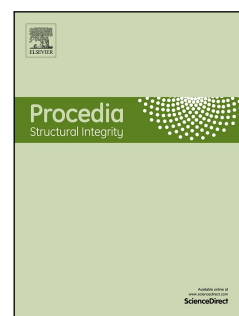
Y. Ai, S. P. Zhu, D. Liao, J. A. F. O. Correia, **C. Souto**, A. M. P. de Jesus, and B. Keshtegar. 'Probabilistic modeling of fatigue life distribution and size effect of components with random defects'. *International Journal of Fatigue* 126 (2019) 165-173.
[doi:10.1016/j.ijfatigue.2019.05.005](https://doi.org/10.1016/j.ijfatigue.2019.05.005)

CiteScore: 10.7 · Impact Factor: 5.7



C. D. S. Souto, J. A. F. O. Correia, A. M. P. de Jesus, and R. A. B. Calçada. 'Fatigue Damage Tool (FDT) - A tool for fatigue damage assessment according to design codes'. *Procedia Structural Integrity* 22 (2019) 376-385.
[doi:10.1016/j.prostr.2020.01.047](https://doi.org/10.1016/j.prostr.2020.01.047)

CiteScore: 1.7



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Appendix B

Using machine learning for stress intensity factor determination

In [Chapter 7](#), a damage-tolerant approach was developed to estimate the residual fatigue life of a full-scale Z-profile exhibiting a visible fatigue crack. This approach is based on Paris' law and assumes stable fatigue crack growth from an initial crack length up to a critical crack length, beyond which it is assumed that unstable propagation will lead to failure of the structural member. In order to apply this approach, it is necessary to compute the stress intensity factor, which is used to determine the critical crack length (based on the fracture toughness of the material) and the applied stress intensity factor range, since both parameters are required for integrating the Paris' law equation.

The stress intensity factor depends, directly or indirectly, on several variables, including the profile's geometry, use case (i.e., boundary and loading conditions), material properties, and crack characteristics (e.g., crack length and shape). In [Chapter 7](#), a critical scenario was identified, in which a rectangular crack is aligned with the mid-span supports of the profile; consequently, this condition was investigated further. Additionally, it was found that a mixed-mode condition must be assumed for longer crack lengths and, to account for this, an effective stress intensity factor is considered, as defined in [\(7.11\)](#).

Initially, the stress intensity factor was determined via finite element modelling and numerical evaluation of the J-integral, as described throughout [Chapter 7](#). In this context, the finite element model was parameterized using the Python-based interface to Abaqus/CAE in order to enable rapid adjustments to key variables, including the inside radius of the Z-profile's cold-formed corner (r), material thickness (t), crack length (a), and applied load magnitude (F). This parameterization facilitated the development of a simplified analytical model, as shown in [\(7.12\)](#), fitted to the numerical results. The formula was adapted from a well-known expression used in engineering for stress intensity factor determination, shown in [\(2.90\)](#).

The fitted analytical model enabled a more efficient and straightforward determination of stress intensity factor values. Importantly, it also supported interpolation across the considered r , t , a , and F parameters without requiring additional computationally intensive simulations. In turn, this allowed for the generation of design curves for a full-scale Z-profile exhibiting a visible fatigue crack. Two types of curves were generated: one type aimed for static design, relating a critical load magnitude to the crack length, and another curve type for estimating the residual fatigue life as a function of the applied maximum fatigue load. Both

B: Using machine learning for stress intensity factor determination

curve types can generate curves for varying r and t values and, in the case of residual fatigue life curves, also for distinct initial crack lengths and stress ratios.

Although the damage-tolerant approach was developed in [Chapter 7](#), and the simplified analytical model was shown to be successful in replacing the more computationally demanding parameterized finite element model, there may be cases without a suitable closed-form expression for fitting, posing a fundamental challenge in regard to the methodology applied during the development of the damage-tolerant approach. To address this, machine learning can be used as an alternative to the aforementioned analytical model, offering a flexible and data-driven solution.

Within this context, and assuming that the effective stress intensity factor varies continuously with respect to the inside radius of the Z-profile's cold-formed corner, material thickness, crack length, and applied load magnitude, the objective is to replace the computationally expensive, high-fidelity and physics-based finite element model with a surrogate model: a fully connected neural network trained using machine learning techniques, enabling faster evaluations of the stress intensity factor across the parameter space. To this end, the open-source PyTorch deep learning framework is used via the Python programming language.

Parameter space and data normalization

As shown in [Figure B.1](#), for the purposes of machine learning, the finite element model can be treated as a “black box” that accepts four scalar inputs, i.e., inside radius of the Z-profile's cold-formed corner (r), material thickness (t), crack length (a), and applied load magnitude (F), and returns a single scalar output, i.e., the effective stress intensity factor ($\bar{K}$). Additionally, a dataset of 375 computed $\bar{K}$ values was generated by evaluating the finite element model over the parameter grid defined by $r \in \{6, 7, 8, 9, 10\}$ mm, $t \in \{2.5, 3, 3.5, 4, 4.5\}$ mm, $a \in \{1, 5, 10, 25, 50\}$ mm, $F \in \{2, 4, 6\}$ kN

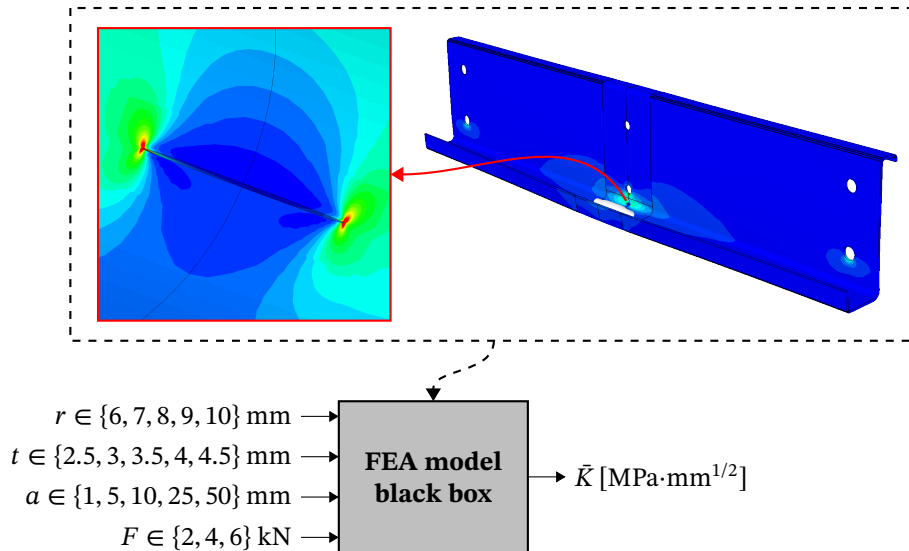


Figure B.1: Parameterized finite element model for stress intensity factor determination viewed as a “black box”.

$\{1, 5, 10, 25, 50\}$ mm, and $F \in \{2, 4, 6\}$ kN.

Moreover, machine learning models typically perform best when parameters are on similar scales or have been normalized, as this helps to mitigate issues such as the vanishing/-exploding gradient problem [154]. To this end, min-max normalization is used to rescale the parameter space by normalizing each parameter to the $[0, 1]$ range:

$$\mathbf{x}' = \frac{\mathbf{x} - \min(\mathbf{x})}{\max(\mathbf{x}) - \min(\mathbf{x})}. \quad (\text{B.1})$$

The $[0, 1]$ range is considered since all parameters are naturally non-negative; otherwise, the $[-1, 1]$ range, for example, could have been considered instead.

Training and validation subsets

In order to evaluate the performance and generalization capability of the neural network under development, the dataset was divided into a training set and a validation set. Specifically, 75% of the total 375 numerically generated samples were allocated to the training set, while the remaining 25% were used for validation. The splitting was performed using uniform random sampling, ensuring that each sample was used in exactly one of the two subsets. This allows the neural network to learn from the majority of the data while being evaluated on a separate, unseen subset to assess the predictive accuracy.

Neural network architecture

The surrogate model for stress intensity factor prediction is implemented as a fully connected feedforward neural network, also known as a multilayer perceptron. As previously mentioned, the implementation relies on the PyTorch deep learning framework. In particular, a PyTorch sequential container is used, which provides a convenient way to build neural network models by stacking layers in a given order, and the output of each layer is passed to the next layer, forming a simple feedforward structure with no branching or skipped connections. Since four normalized scalar input parameters are necessary (corresponding to r , t , a , and F), the first layer consists of four input neurons. The final layer consists of a single output neuron, representing the predicted and normalized value of $\bar{K}$. Additionally, between the input and output layers, the neural network contains three hidden layers, each with 64 neurons. The hidden layers represent the core of the neural network where learning occurs through the adjustment of weights and biases. Moreover, each layer is linear, meaning that an affine transformation is applied:

$$\mathbf{x}_j = \mathbf{A}\mathbf{x}_i + \mathbf{b}, \quad (\text{B.2})$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ denote the input and output vectors of the layer, $\mathbf{A}$ is a matrix of weights, and $\mathbf{b}$ is a vector of biases. The weights and biases are the model parameters to be adjusted during the training process. Finally, linear layers alone can only represent linear functions. Consequently, each linear transformation is followed by a rectified linear unit (ReLU) activation

B: Using machine learning for stress intensity factor determination

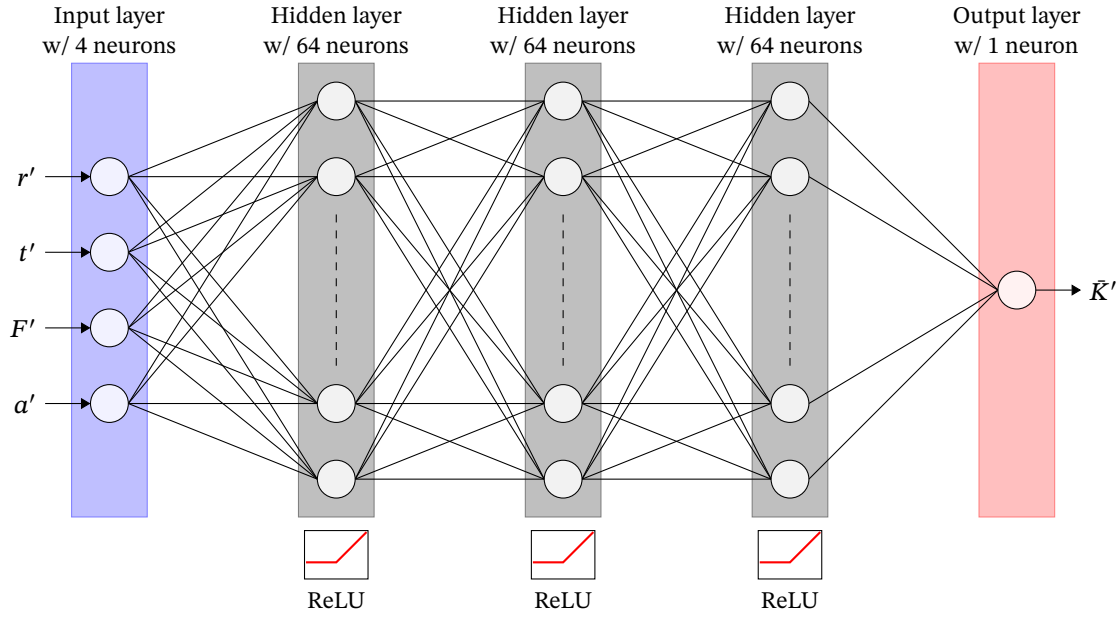


Figure B.2: Surrogate model: the fully connected feedforward neural network under development for stress intensity factor determination.

function, which is a standard choice to introduce nonlinearity. A ReLU is computationally inexpensive as it represents a simple element-wise thresholding operation:

$$\text{ReLU}(x) = \max(0, x). \quad (\text{B.3})$$

Figure B.2 illustrates the architecture of the neural network described above. Ultimately, the number of layers (depth) and the number of neurons per layer (width) were selected based on experimentation to balance model complexity and predictive performance.

Loss function and optimizer

To fit or train the neural network under development, a loss function that quantifies the discrepancy between predicted and numerically obtained values of the normalized effective stress intensity factor is required. Since the surrogate model is designed for a regression task with a continuous scalar output, the mean squared error (MSE) is selected:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n \left(\bar{K}_i' - \hat{K}_i' \right)^2, \quad (\text{B.4})$$

where $\bar{K}_i'$ denotes the normalized and numerically obtained effective stress intensity factor, $\hat{K}_i'$ is the corresponding predicted value from the neural network, and n is the number of training samples. The MSE is a standard choice in regression problems and is well-suited for gradient-based optimization.

In order to minimize the loss function, the Adam (adaptive moment estimation) gradient-based algorithm for stochastic optimization is considered, which is available through PyTorch.

Developed by Kingma and Ba (2017) [155], Adam's current popularity stems from its robustness, computational efficiency, and the use of individual adaptive learning rates for different parameters, which can lead to faster convergence compared to optimizers with a single fixed learning rate.

Training strategy

The neural network was trained using mini-batch gradient descent, a widely used approach in deep learning. In this method, the training and validation datasets are divided into smaller subsets (mini-batches) of fixed size. For each training mini-batch, a forward pass is made to generate predictions, based on which the loss function (MSE) is evaluated. Using back-propagation, the gradient of the loss function with respect to the surrogate model parameters is computed via automatic differentiation. While symbolic differentiation requires manipulating mathematical expressions, and numerical differentiation approximates derivatives using finite differences, automatic differentiation computes exact derivatives via the chain rule during program execution [156]. Subsequently, the optimizer (Adam) uses the computed gradients to adaptively adjust the surrogate model parameters to minimize the loss function. Finally, additional forward passes are made using the validation mini-batches to monitor the neural network's performance and generalization capability (this is a diagnostic measure and has no effect on training).

Since a single full pass through the entire training dataset (i.e., one epoch) is typically not enough for a surrogate model to learn complex patterns, the neural network was trained over multiple epochs. Additionally, early stopping was implemented as a safeguard against overfitting and unnecessary computation. A straightforward approach was considered, in which training was stopped when both the training and validation losses simultaneously dropped below a pre-defined tolerance, indicating that the model had converged to a satisfactory level of predictive accuracy.

The batch size (32), maximum number of epochs (1000), and loss tolerance (5×10^{-6}) are known as hyperparameters, which are set before training and affect how it proceeds. While these hyperparameters can be fine-tuned using optimization techniques, these values were obtained after some experimentation to find a reasonable trade-off between training speed and training stability.

Figure B.3 shows the training and validation losses of the neural network during the fit process. The figure shows some training instability. Nevertheless, an early exit was verified on epoch 364 due to both losses being below the pre-defined tolerance.

Evaluating the surrogate model

Initially, the surrogate neural network model is evaluated using the full dataset of 375 computed $\bar{K}$ values, which was generated by evaluating the parameterized finite element model over the parameter grid defined by $r \in \{6, 7, 8, 9, 10\}$ mm, $t \in \{2.5, 3, 3.5, 4, 4.5\}$ mm, $a \in \{1, 5, 10, 25, 50\}$ mm, and $F \in \{2, 4, 6\}$ kN. Figure B.4a presents the results arising from the fitted surrogate model compared against those generated by the physics-based model. A very good fit is verified. Additionally, Figure B.4b presents the corresponding residuals, where it

B: Using machine learning for stress intensity factor determination

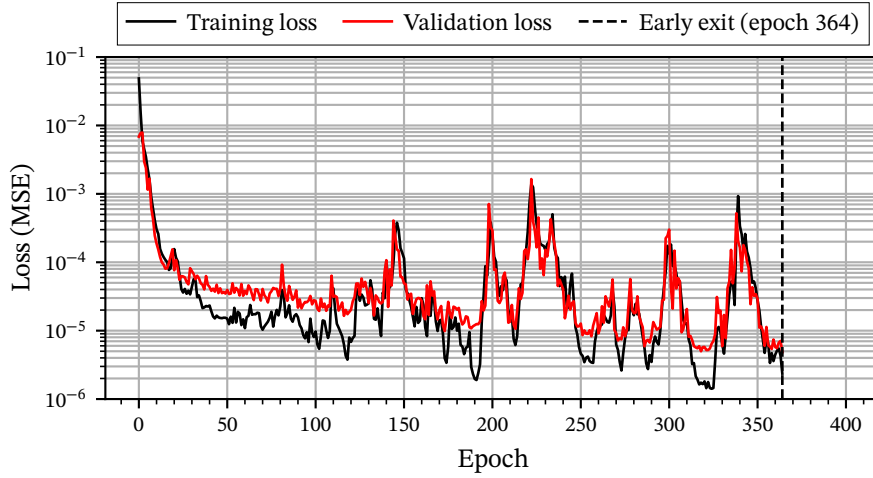


Figure B.3: Neural network training and validation losses.

is shown that an absolute relative difference of less than 5% is verified between the surrogate model and the physics-based model. Ultimately, these figures reveal a very good level of predictive accuracy for the trained neural network. Consequently, the surrogate model can be used to replace the parameterized finite element model. Moreover, the surrogate model is computationally inexpensive when compared against the numerical alternative.

While it has been demonstrated that the trained neural network effectively reproduces the results of the parameterized finite element model, it is also important to assess how well it aligns with the simplified analytical model developed in [Chapter 7](#), as the initial objective was to replace the latter with the former. [Figure B.5](#) presents a final comparative plot showing the outputs of the trained neural network vs. those predicted by the simplified analytical model. Both models were fitted based on the same dataset. To ensure a consistent basis for compar-

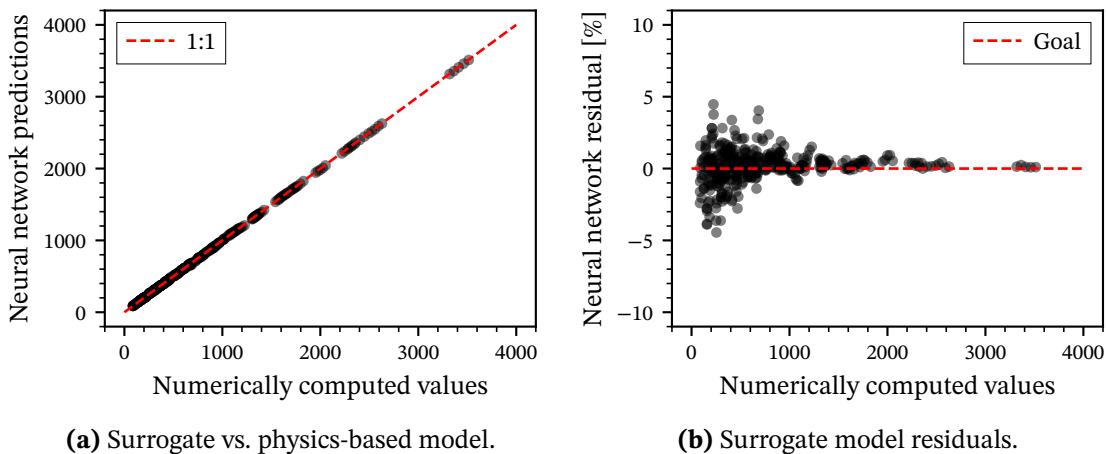


Figure B.4: Surrogate neural network model vs. physics-based finite element model for stress intensity factor determination.

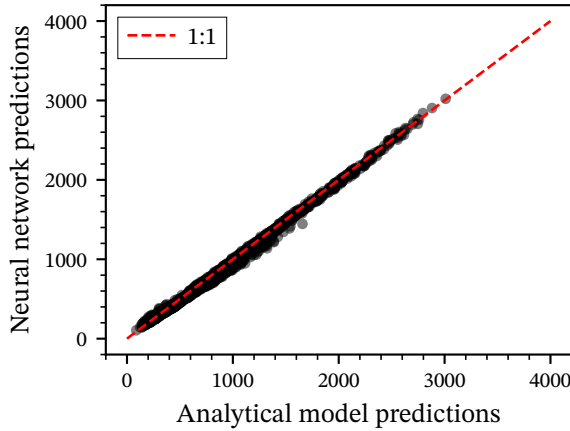


Figure B.5: Trained neural network vs. fitted analytical model for stress intensity factor determination.

ison, 2000 random input samples were generated within the defined parameter bounds and passed through both models. As shown in the figure, a good agreement is found between both approaches for stress intensity factor determination, indicating that the data-driven surrogate model is not only capable of learning complex mappings from finite element modelling but also yields results that are in good agreement with simplified closed-form expressions derived from LEFM principles. Ultimately, the developed neural network may replace the aforementioned analytical solution, without jeopardizing any of the following analyses shown in [Chapter 7](#), such as those applied for residual fatigue life prediction.

Full Python-based machine learning code

```

1  # required packages
2  import numpy as np
3  import torch as pt
4  import matplotlib.pyplot as plt
5
6  # manual seeds for reproducibility
7  SEED = 314
8  np.random.seed(SEED)
9  pt.manual_seed(SEED)
10
11 # load the raw (unnormalized) dataset generated by the finite element model
12 # raw_data is a matrix with shape (375, 5)
13 # each row contains [r, t, F, a, K]
14 raw_data = np.loadtxt(fname='dataset.csv', delimiter=',', dtype=np.float32)
15
16 # split data into inputs/outputs and convert to PyTorch tensors
17 raw_X = pt.tensor(raw_data[:, 0:4], dtype=pt.float32) # features: r, t, F, a
18 raw_Y = pt.tensor(raw_data[:, [-1]], dtype=pt.float32) # targets: K
19
20 # min-max normalization to [0, 1]
21 min_X = raw_X.min(dim=0).values
22 min_Y = raw_Y.min(dim=0).values
23 max_X = raw_X.max(dim=0).values
24 max_Y = raw_Y.max(dim=0).values
25 norm_X = (raw_X - min_X)/(max_X - min_X)
26 norm_Y = (raw_Y - min_Y)/(max_Y - min_Y)
27
28 # dataset split

```

B: Using machine learning for stress intensity factor determination

```
29 dataset = pt.utils.data.TensorDataset(norm_X, norm_Y)
30 train_size = int(0.75*len(dataset)) # 75% for training
31 valid_size = len(dataset) - train_size # remaining for validation
32 train_dataset, valid_dataset = pt.utils.data.random_split(
33     dataset, (train_size, valid_size))
34
35 # neural network architecture
36 nn_model = pt.nn.Sequential(
37     pt.nn.Linear(4, 64),
38     pt.nn.ReLU(),
39     pt.nn.Linear(64, 64),
40     pt.nn.ReLU(),
41     pt.nn.Linear(64, 64),
42     pt.nn.ReLU(),
43     pt.nn.Linear(64, 1)
44 )
45
46 # loss function and optimizer
47 loss_func = pt.nn.MSELoss()
48 optimizer = pt.optim.Adam(nn_model.parameters(), lr=0.01)
49
50 # training
51 loss_tol = 5e-6
52 batch_size = 32
53 num_epochs = 1000
54 train_losses = []
55 valid_losses = []
56 train_loader = pt.utils.data.DataLoader(train_dataset, batch_size, shuffle=True)
57 valid_loader = pt.utils.data.DataLoader(valid_dataset, batch_size, shuffle=False)
58 for epoch in range(num_epochs):
59
60     # train
61     nn_model.train()
62     train_losses.append(0.0)
63     for batch_X, batch_Y in train_loader:
64
65         # forward pass
66         pred_Y = nn_model(batch_X)
67         loss = loss_func(pred_Y, batch_Y)
68
69         # backward pass and optimization
70         optimizer.zero_grad()
71         loss.backward()
72         optimizer.step()
73
74         # track training losses
75         train_losses[-1] += loss.item()*batch_X.size(0)/train_size
76
77     # validate
78     nn_model.eval()
79     valid_losses.append(0.0)
80     with pt.no_grad():
81         for batch_X, batch_Y in valid_loader:
82
83             # forward pass
84             pred_Y = nn_model(batch_X)
85             loss = loss_func(pred_Y, batch_Y)
86
87             # track validation losses
88             valid_losses[-1] += loss.item()*batch_X.size(0)/valid_size
89
90     # print current progress
91     if epoch % 50 == 0 or epoch == num_epochs - 1:
92         print(
```



```

93         f'Epoch {epoch:4d} | ' +
94         f'Train loss: {train_losses[-1]:10.8f} | ' +
95         f'Validation loss: {valid_losses[-1]:10.8f}'
96     )
97
98     # simple early exit
99     if train_losses[-1] <= loss_tol and valid_losses[-1] <= loss_tol:
100         print(f'Early exit on epoch {epoch} (losses within tolerance)')
101         break
102
103 # loss plot
104 plt.plot(train_losses, label='Training loss')
105 plt.plot(valid_losses, label='Validation loss')
106 plt.title('Training and Validation Loss')
107 plt.xlabel('Epoch')
108 plt.ylabel('MSE')
109 plt.legend()
110 plt.show()
111
112 # fit plot
113 nn_model.eval()
114 with pt.no_grad(): norm_pred = nn_model(norm_X)
115 raw_pred = norm_pred*(max_Y - min_Y) + min_Y
116 plt.scatter(x := raw_Y.flatten(), y := raw_pred.flatten())
117 plt.plot((0, 4000), (0, 4000), '--r')
118 plt.xlabel('FEA')
119 plt.ylabel('Neural network')
120 plt.title('Surrogate Model Fit')
121 plt.axis('equal')
122 plt.show()
123
124 # residuals plot
125 res = 100*(x - y)/y
126 plt.scatter(x, res)
127 plt.plot((0, 4000), (0, 0), '--r')
128 plt.xlabel('FEA')
129 plt.ylabel('Neural network residual [%]')
130 plt.title('Residuals')
131 plt.show()

```

Flat-file database for use with the Python code

1	6.0,2.5,2000.0,1.0,204.2	27	6.0,3.0,6000.0,5.0,760.3
2	6.0,2.5,2000.0,5.0,338.1	28	6.0,3.0,6000.0,10.0,1025.7
3	6.0,2.5,2000.0,10.0,464.0	29	6.0,3.0,6000.0,25.0,1827.6
4	6.0,2.5,2000.0,25.0,827.9	30	6.0,3.0,6000.0,50.0,2629.4
5	6.0,2.5,2000.0,50.0,1177.3	31	6.0,3.5,2000.0,1.0,128.2
6	6.0,2.5,4000.0,1.0,399.8	32	6.0,3.5,2000.0,5.0,208.7
7	6.0,2.5,4000.0,5.0,662.8	33	6.0,3.5,2000.0,10.0,277.7
8	6.0,2.5,4000.0,10.0,910.4	34	6.0,3.5,2000.0,25.0,477.1
9	6.0,2.5,4000.0,25.0,1645.8	35	6.0,3.5,2000.0,50.0,686.3
10	6.0,2.5,4000.0,50.0,2348.7	36	6.0,3.5,4000.0,1.0,254.0
11	6.0,2.5,6000.0,1.0,582.9	37	6.0,3.5,4000.0,5.0,413.7
12	6.0,2.5,6000.0,5.0,968.0	38	6.0,3.5,4000.0,10.0,547.6
13	6.0,2.5,6000.0,10.0,1344.3	39	6.0,3.5,4000.0,25.0,952.2
14	6.0,2.5,6000.0,25.0,2448.0	40	6.0,3.5,4000.0,50.0,1371.8
15	6.0,2.5,6000.0,50.0,3514.8	41	6.0,3.5,6000.0,1.0,377.9
16	6.0,3.0,2000.0,1.0,159.0	42	6.0,3.5,6000.0,5.0,615.7
17	6.0,3.0,2000.0,5.0,260.4	43	6.0,3.5,6000.0,10.0,814.4
18	6.0,3.0,2000.0,10.0,351.1	44	6.0,3.5,6000.0,25.0,1425.0
19	6.0,3.0,2000.0,25.0,614.8	45	6.0,3.5,6000.0,50.0,2053.6
20	6.0,3.0,2000.0,50.0,879.5	46	6.0,4.0,2000.0,1.0,106.2
21	6.0,3.0,4000.0,1.0,314.0	47	6.0,4.0,2000.0,5.0,172.5
22	6.0,3.0,4000.0,5.0,514.4	48	6.0,4.0,2000.0,10.0,226.9
23	6.0,3.0,4000.0,10.0,691.3	49	6.0,4.0,2000.0,25.0,383.0
24	6.0,3.0,4000.0,25.0,1226.6	50	6.0,4.0,2000.0,50.0,553.0
25	6.0,3.0,4000.0,50.0,1757.1	51	6.0,4.0,4000.0,1.0,210.9
26	6.0,3.0,6000.0,1.0,463.7	52	6.0,4.0,4000.0,5.0,342.6

B: Using machine learning for stress intensity factor determination

53	6.0,4.0,4000.0,10.0,447.8	140	7.0,4.5,2000.0,50.0,451.1
54	6.0,4.0,4000.0,25.0,764.4	141	7.0,4.5,4000.0,1.0,176.6
55	6.0,4.0,4000.0,50.0,1105.3	142	7.0,4.5,4000.0,5.0,289.0
56	6.0,4.0,6000.0,1.0,314.5	143	7.0,4.5,4000.0,10.0,372.4
57	6.0,4.0,6000.0,5.0,511.0	144	7.0,4.5,4000.0,25.0,619.8
58	6.0,4.0,6000.0,10.0,666.9	145	7.0,4.5,4000.0,50.0,901.9
59	6.0,4.0,6000.0,25.0,1145.2	146	7.0,4.5,6000.0,1.0,263.8
60	6.0,4.0,6000.0,50.0,1656.6	147	7.0,4.5,6000.0,5.0,431.6
61	6.0,4.5,2000.0,1.0,90.1	148	7.0,4.5,6000.0,10.0,555.0
62	6.0,4.5,2000.0,5.0,146.2	149	7.0,4.5,6000.0,25.0,928.6
63	6.0,4.5,2000.0,10.0,190.3	150	7.0,4.5,6000.0,50.0,1352.0
64	6.0,4.5,2000.0,25.0,315.9	151	8.0,2.5,2000.0,1.0,203.2
65	6.0,4.5,2000.0,50.0,457.1	152	8.0,2.5,2000.0,5.0,337.9
66	6.0,4.5,4000.0,1.0,179.2	153	8.0,2.5,2000.0,10.0,458.7
67	6.0,4.5,4000.0,5.0,290.9	154	8.0,2.5,2000.0,25.0,799.9
68	6.0,4.5,4000.0,10.0,375.9	155	8.0,2.5,2000.0,50.0,1142.7
69	6.0,4.5,4000.0,25.0,630.6	156	8.0,2.5,4000.0,1.0,397.6
70	6.0,4.5,4000.0,50.0,913.9	157	8.0,2.5,4000.0,5.0,661.8
71	6.0,4.5,6000.0,1.0,267.5	158	8.0,2.5,4000.0,10.0,899.3
72	6.0,4.5,6000.0,5.0,434.4	159	8.0,2.5,4000.0,25.0,1588.1
73	6.0,4.5,6000.0,10.0,560.3	160	8.0,2.5,4000.0,50.0,2280.3
74	6.0,4.5,6000.0,25.0,944.8	161	8.0,2.5,6000.0,1.0,580.0
75	6.0,4.5,6000.0,50.0,1370.0	162	8.0,2.5,6000.0,5.0,966.8
76	7.0,2.5,2000.0,1.0,203.6	163	8.0,2.5,6000.0,10.0,1326.7
77	7.0,2.5,2000.0,5.0,338.0	164	8.0,2.5,6000.0,25.0,2360.0
78	7.0,2.5,2000.0,10.0,461.1	165	8.0,2.5,6000.0,50.0,3412.2
79	7.0,2.5,2000.0,25.0,813.2	166	8.0,3.0,2000.0,1.0,157.5
80	7.0,2.5,2000.0,50.0,1159.8	167	8.0,3.0,2000.0,5.0,259.9
81	7.0,2.5,4000.0,1.0,398.5	168	8.0,3.0,2000.0,10.0,347.1
82	7.0,2.5,4000.0,5.0,662.2	169	8.0,3.0,2000.0,25.0,595.2
83	7.0,2.5,4000.0,10.0,904.7	170	8.0,3.0,2000.0,50.0,855.6
84	7.0,2.5,4000.0,25.0,1615.5	171	8.0,3.0,4000.0,1.0,310.8
85	7.0,2.5,4000.0,50.0,2314.1	172	8.0,3.0,4000.0,5.0,513.4
86	7.0,2.5,6000.0,1.0,581.3	173	8.0,3.0,4000.0,10.0,683.4
87	7.0,2.5,6000.0,5.0,967.3	174	8.0,3.0,4000.0,25.0,1186.6
88	7.0,2.5,6000.0,10.0,1335.2	175	8.0,3.0,4000.0,50.0,1709.3
89	7.0,2.5,6000.0,25.0,2402.0	176	8.0,3.0,6000.0,1.0,459.0
90	7.0,2.5,6000.0,50.0,3463.3	177	8.0,3.0,6000.0,5.0,758.1
91	7.0,3.0,2000.0,1.0,158.1	178	8.0,3.0,6000.0,10.0,1013.6
92	7.0,3.0,2000.0,5.0,260.0	179	8.0,3.0,6000.0,25.0,1767.3
93	7.0,3.0,2000.0,10.0,348.9	180	8.0,3.0,6000.0,50.0,2557.8
94	7.0,3.0,2000.0,25.0,604.3	181	8.0,3.5,2000.0,1.0,126.3
95	7.0,3.0,2000.0,50.0,867.2	182	8.0,3.5,2000.0,5.0,207.7
96	7.0,3.0,4000.0,1.0,312.1	183	8.0,3.5,2000.0,10.0,274.0
97	7.0,3.0,4000.0,5.0,513.6	184	8.0,3.5,2000.0,25.0,462.1
98	7.0,3.0,4000.0,10.0,686.9	185	8.0,3.5,2000.0,50.0,668.2
99	7.0,3.0,4000.0,25.0,1205.3	186	8.0,3.5,4000.0,1.0,250.3
100	7.0,3.0,4000.0,50.0,1732.4	187	8.0,3.5,4000.0,5.0,411.8
101	7.0,3.0,6000.0,1.0,460.9	188	8.0,3.5,4000.0,10.0,540.7
102	7.0,3.0,6000.0,5.0,758.8	189	8.0,3.5,4000.0,25.0,921.9
103	7.0,3.0,6000.0,10.0,1018.8	190	8.0,3.5,4000.0,50.0,1335.8
104	7.0,3.0,6000.0,25.0,1795.5	191	8.0,3.5,6000.0,1.0,372.3
105	7.0,3.0,6000.0,50.0,2592.5	192	8.0,3.5,6000.0,5.0,612.8
106	7.0,3.5,2000.0,1.0,127.1	193	8.0,3.5,6000.0,10.0,804.1
107	7.0,3.5,2000.0,5.0,208.1	194	8.0,3.5,6000.0,25.0,1379.0
108	7.0,3.5,2000.0,10.0,275.6	195	8.0,3.5,6000.0,50.0,2000.1
109	7.0,3.5,2000.0,25.0,469.0	196	8.0,4.0,2000.0,1.0,104.1
110	7.0,3.5,2000.0,50.0,676.9	197	8.0,4.0,2000.0,5.0,171.2
111	7.0,3.5,4000.0,1.0,251.8	198	8.0,4.0,2000.0,10.0,223.5
112	7.0,3.5,4000.0,5.0,412.5	199	8.0,4.0,2000.0,25.0,371.1
113	7.0,3.5,4000.0,10.0,543.7	200	8.0,4.0,2000.0,50.0,538.8
114	7.0,3.5,4000.0,25.0,935.9	201	8.0,4.0,4000.0,1.0,206.8
115	7.0,3.5,4000.0,50.0,1353.1	202	8.0,4.0,4000.0,5.0,340.1
116	7.0,3.5,6000.0,1.0,374.6	203	8.0,4.0,4000.0,10.0,441.4
117	7.0,3.5,6000.0,5.0,613.8	204	8.0,4.0,4000.0,25.0,740.5
118	7.0,3.5,6000.0,10.0,808.5	205	8.0,4.0,4000.0,50.0,1077.2
119	7.0,3.5,6000.0,25.0,1400.3	206	8.0,4.0,6000.0,1.0,308.4
120	7.0,3.5,6000.0,50.0,2025.8	207	8.0,4.0,6000.0,5.0,507.2
121	7.0,4.0,2000.0,1.0,105.0	208	8.0,4.0,6000.0,10.0,657.4
122	7.0,4.0,2000.0,5.0,171.7	209	8.0,4.0,6000.0,25.0,1109.1
123	7.0,4.0,2000.0,10.0,225.0	210	8.0,4.0,6000.0,50.0,1615.3
124	7.0,4.0,2000.0,25.0,376.6	211	8.0,4.5,2000.0,1.0,87.8
125	7.0,4.0,2000.0,50.0,545.7	212	8.0,4.5,2000.0,5.0,144.6
126	7.0,4.0,4000.0,1.0,208.6	213	8.0,4.5,2000.0,10.0,187.0
127	7.0,4.0,4000.0,5.0,341.1	214	8.0,4.5,2000.0,25.0,305.9
128	7.0,4.0,4000.0,10.0,444.2	215	8.0,4.5,2000.0,50.0,445.4
129	7.0,4.0,4000.0,25.0,751.7	216	8.0,4.5,4000.0,1.0,174.7
130	7.0,4.0,4000.0,50.0,1090.8	217	8.0,4.5,4000.0,5.0,287.7
131	7.0,4.0,6000.0,1.0,311.0	218	8.0,4.5,4000.0,10.0,369.6
132	7.0,4.0,6000.0,5.0,508.8	219	8.0,4.5,4000.0,25.0,610.5
133	7.0,4.0,6000.0,10.0,661.6	220	8.0,4.5,4000.0,50.0,890.6
134	7.0,4.0,6000.0,25.0,1126.0	221	8.0,4.5,6000.0,1.0,260.9
135	7.0,4.0,6000.0,50.0,1635.3	222	8.0,4.5,6000.0,5.0,429.6
136	7.0,4.5,2000.0,1.0,88.8	223	8.0,4.5,6000.0,10.0,550.9
137	7.0,4.5,2000.0,5.0,145.3	224	8.0,4.5,6000.0,25.0,914.5
138	7.0,4.5,2000.0,10.0,188.4	225	8.0,4.5,6000.0,50.0,1335.1
139	7.0,4.5,2000.0,25.0,310.5	226	9.0,2.5,2000.0,1.0,202.9

B: Using machine learning for stress intensity factor determination

227	9.0,2.5,2000.0,5.0,337.9	302	10.0,2.5,2000.0,5.0,338.1
228	9.0,2.5,2000.0,10.0,456.6	303	10.0,2.5,2000.0,10.0,454.8
229	9.0,2.5,2000.0,25.0,788.1	304	10.0,2.5,2000.0,25.0,777.5
230	9.0,2.5,2000.0,50.0,1126.9	305	10.0,2.5,2000.0,50.0,1112.2
231	9.0,2.5,4000.0,1.0,396.9	306	10.0,2.5,4000.0,1.0,396.5
232	9.0,2.5,4000.0,5.0,661.6	307	10.0,2.5,4000.0,5.0,661.8
233	9.0,2.5,4000.0,10.0,894.9	308	10.0,2.5,4000.0,10.0,891.1
234	9.0,2.5,4000.0,25.0,1563.7	309	10.0,2.5,4000.0,25.0,1541.7
235	9.0,2.5,4000.0,50.0,2248.7	310	10.0,2.5,4000.0,50.0,2219.3
236	9.0,2.5,6000.0,1.0,579.2	311	10.0,2.5,6000.0,1.0,578.7
237	9.0,2.5,6000.0,5.0,966.6	312	10.0,2.5,6000.0,5.0,966.7
238	9.0,2.5,6000.0,10.0,1319.6	313	10.0,2.5,6000.0,10.0,1313.5
239	9.0,2.5,6000.0,25.0,2322.5	314	10.0,2.5,6000.0,25.0,2288.7
240	9.0,2.5,6000.0,50.0,3364.3	315	10.0,2.5,6000.0,50.0,3319.5
241	9.0,3.0,2000.0,1.0,157.0	316	10.0,3.0,2000.0,1.0,156.7
242	9.0,3.0,2000.0,5.0,259.9	317	10.0,3.0,2000.0,5.0,260.0
243	9.0,3.0,2000.0,10.0,345.6	318	10.0,3.0,2000.0,10.0,344.3
244	9.0,3.0,2000.0,25.0,587.0	319	10.0,3.0,2000.0,25.0,579.6
245	9.0,3.0,2000.0,50.0,844.7	320	10.0,3.0,2000.0,50.0,834.5
246	9.0,3.0,4000.0,1.0,310.0	321	10.0,3.0,4000.0,1.0,309.4
247	9.0,3.0,4000.0,5.0,513.3	322	10.0,3.0,4000.0,5.0,513.5
248	9.0,3.0,4000.0,10.0,680.5	323	10.0,3.0,4000.0,10.0,678.0
249	9.0,3.0,4000.0,25.0,1169.8	324	10.0,3.0,4000.0,25.0,1154.6
250	9.0,3.0,4000.0,50.0,1687.4	325	10.0,3.0,4000.0,50.0,1667.1
251	9.0,3.0,6000.0,1.0,457.5	326	10.0,3.0,6000.0,1.0,456.6
252	9.0,3.0,6000.0,5.0,758.0	327	10.0,3.0,6000.0,5.0,758.0
253	9.0,3.0,6000.0,10.0,1009.1	328	10.0,3.0,6000.0,10.0,1005.2
254	9.0,3.0,6000.0,25.0,1741.9	329	10.0,3.0,6000.0,25.0,1719.0
255	9.0,3.0,6000.0,50.0,2524.8	330	10.0,3.0,6000.0,50.0,2493.8
256	9.0,3.5,2000.0,1.0,125.7	331	10.0,3.5,2000.0,1.0,125.2
257	9.0,3.5,2000.0,5.0,207.6	332	10.0,3.5,2000.0,5.0,207.5
258	9.0,3.5,2000.0,10.0,272.7	333	10.0,3.5,2000.0,10.0,271.6
259	9.0,3.5,2000.0,25.0,455.9	334	10.0,3.5,2000.0,25.0,450.3
260	9.0,3.5,2000.0,50.0,660.1	335	10.0,3.5,2000.0,50.0,652.4
261	9.0,3.5,4000.0,1.0,249.1	336	10.0,3.5,4000.0,1.0,248.2
262	9.0,3.5,4000.0,5.0,411.4	337	10.0,3.5,4000.0,5.0,411.2
263	9.0,3.5,4000.0,10.0,538.3	338	10.0,3.5,4000.0,10.0,536.2
264	9.0,3.5,4000.0,25.0,909.3	339	10.0,3.5,4000.0,25.0,898.0
265	9.0,3.5,4000.0,50.0,1319.4	340	10.0,3.5,4000.0,50.0,1304.0
266	9.0,3.5,6000.0,1.0,370.5	341	10.0,3.5,6000.0,1.0,369.1
267	9.0,3.5,6000.0,5.0,612.2	342	10.0,3.5,6000.0,5.0,611.8
268	9.0,3.5,6000.0,10.0,800.4	343	10.0,3.5,6000.0,10.0,797.3
269	9.0,3.5,6000.0,25.0,1359.9	344	10.0,3.5,6000.0,25.0,1342.6
270	9.0,3.5,6000.0,50.0,1975.8	345	10.0,3.5,6000.0,50.0,1953.0
271	9.0,4.0,2000.0,1.0,103.4	346	10.0,4.0,2000.0,1.0,102.8
272	9.0,4.0,2000.0,5.0,170.8	347	10.0,4.0,2000.0,5.0,170.6
273	9.0,4.0,2000.0,10.0,222.3	348	10.0,4.0,2000.0,10.0,221.2
274	9.0,4.0,2000.0,25.0,366.1	349	10.0,4.0,2000.0,25.0,361.6
275	9.0,4.0,2000.0,50.0,532.4	350	10.0,4.0,2000.0,50.0,526.2
276	9.0,4.0,4000.0,1.0,205.4	351	10.0,4.0,4000.0,1.0,204.3
277	9.0,4.0,4000.0,5.0,339.4	352	10.0,4.0,4000.0,5.0,338.8
278	9.0,4.0,4000.0,10.0,439.1	353	10.0,4.0,4000.0,10.0,437.1
279	9.0,4.0,4000.0,25.0,730.5	354	10.0,4.0,4000.0,25.0,721.4
280	9.0,4.0,4000.0,50.0,1064.3	355	10.0,4.0,4000.0,50.0,1051.9
281	9.0,4.0,6000.0,1.0,306.3	356	10.0,4.0,6000.0,1.0,304.5
282	9.0,4.0,6000.0,5.0,506.1	357	10.0,4.0,6000.0,5.0,505.3
283	9.0,4.0,6000.0,10.0,654.0	358	10.0,4.0,6000.0,10.0,651.0
284	9.0,4.0,6000.0,25.0,1094.0	359	10.0,4.0,6000.0,25.0,1080.1
285	9.0,4.0,6000.0,50.0,1596.0	360	10.0,4.0,6000.0,50.0,1577.4
286	9.0,4.5,2000.0,1.0,87.1	361	10.0,4.5,2000.0,1.0,86.4
287	9.0,4.5,2000.0,5.0,144.1	362	10.0,4.5,2000.0,5.0,143.7
288	9.0,4.5,2000.0,10.0,185.8	363	10.0,4.5,2000.0,10.0,184.8
289	9.0,4.5,2000.0,25.0,301.7	364	10.0,4.5,2000.0,25.0,297.9
290	9.0,4.5,2000.0,50.0,440.2	365	10.0,4.5,2000.0,50.0,435.0
291	9.0,4.5,4000.0,1.0,173.2	366	10.0,4.5,4000.0,1.0,171.8
292	9.0,4.5,4000.0,5.0,286.7	367	10.0,4.5,4000.0,5.0,285.8
293	9.0,4.5,4000.0,10.0,367.3	368	10.0,4.5,4000.0,10.0,365.4
294	9.0,4.5,4000.0,25.0,602.1	369	10.0,4.5,4000.0,25.0,594.5
295	9.0,4.5,4000.0,50.0,880.1	370	10.0,4.5,4000.0,50.0,869.8
296	9.0,4.5,6000.0,1.0,258.6	371	10.0,4.5,6000.0,1.0,256.6
297	9.0,4.5,6000.0,5.0,428.1	372	10.0,4.5,6000.0,5.0,426.9
298	9.0,4.5,6000.0,10.0,547.6	373	10.0,4.5,6000.0,10.0,544.7
299	9.0,4.5,6000.0,25.0,902.0	374	10.0,4.5,6000.0,25.0,890.5
300	9.0,4.5,6000.0,50.0,1319.3	375	10.0,4.5,6000.0,50.0,1303.9
301	10.0,2.5,2000.0,1.0,202.7		

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