



High- and Low-Fidelity Damage Models for Composite Joints

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Abstract

Double-double laminates are a new family of composite materials that replace the conventional quad, which is limited to angles of $(0/90/\pm 45)$, and allow for layups without angle limitations. They consist of two pairs of ply angles $[\pm\phi, \pm\psi]$ and aim to revolutionise composite structures with the promise of being lighter, stronger, easier to understand and design, and faster and cheaper to manufacture [1]. While Double-Double laminates offer a transformative alternative to traditional composite designs, enabling broader angle configurations and promising enhanced mechanical performance, their full potential is still being explored. Early investigations suggest that these laminates could lead to significant structural weight savings, with industrial design strategies indicating cost reductions of up to 40% compared to conventional layups [2]. This has major implications for both aircraft manufacturers and operators seeking efficiency gains. However, despite these advantages, the application of Double-Double laminates at a structural level remains in a relatively early stage of development. Key uncertainties persist, particularly regarding their behaviour under bolted joint loading, still the standard method for assembling aeronautical composites. Limited studies have addressed how these laminates respond to localised fracture and how they absorb energy during failure, which are critical factors for their widespread adoption [1].

First, considering in-plane equivalence, verified through lamination parameters, and employing a homogenisation criterion based on the new stiffness parameter, trace, an equivalent double-double layup to the quad laminate studied in Ref. [3] is obtained. Then, the high-fidelity model previously developed for the study of combined bearing/pull-through loading, which was only prepared to obtain structured meshes for traditional quad laminates with conventional ply angles, was improved to allow the generation of aligned meshes for any ply angle. Using the novel meshing approach, several high-fidelity three-dimensional damage simulations that replicate the pure bearing, pure pull-through, and combined loading tests for several loading axes (longitudinal, transversal and off-axis) are run to obtain the first load drop of the joint. Through the ellipsoidal fitting of the simulation results, a failure envelope for the quad equivalent double-double laminate is developed. Since, in general, all the different simulations of the double-double laminate resulted in a higher failure load and a more sustained crack evolution, their failure envelope, when compared to the quad one, reveals a higher radius, leading to a higher volume.

With both the previously established QUAD-based failure envelope and the newly developed envelope for the Double-Double laminates, an extensive uncertainty quantification study was conducted using the simplified multi-bolt models introduced in Ref. [4]. These models were adapted to support the Double-Double layup and subjected to controlled variations in input parameters to evaluate their predictive accuracy and robustness under uncertainty. The models were tested by varying their material properties, including Young's modulus, Poisson's ratio, and shear modulus, as well as connector stiffnesses and the failure envelope radius. These changes were applied in three main directions: longitudinal, transversal, and pull-through. The results showed that the ultimate failure load was primarily influenced by the longitudinal bearing radius of the failure envelope. The slope of the load-displacement curve was mainly affected by variations in connector stiffness and Young's modulus in the longitudinal direction. Finally, the CTF1 bolt ratio was most sensitive to changes in connector stiffness in the pull-through direction.

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Chapter 1

Introduction

1.1 Motivation and objectives

Composite laminates have become increasingly prevalent in aerospace engineering due to their superior strength-to-weight ratio compared to traditional metallic materials. One of the most significant advantages of these materials is their contribution to lighter aircraft structures, which has translated into considerable reductions in fuel consumption and associated CO₂ emissions by some estimates, as much as 20% since their introduction into the industry [9].

Despite these environmental and structural benefits, the conventional approach of legacy quad laminates remains dominant. This method typically involves stacking plies at standard orientations (0°, 90°, ±45°), following strict industry guidelines that demand symmetry and balance in the laminate design. Although such rules are intended to ensure reliable performance under various loading conditions, they often lead to overly conservative structures that are limited more by prescriptive design practices than by true performance requirements [10].

In response to these limitations, a novel approach called Double-Double (DD) laminates has been proposed by Tsai et al. [10]. Unlike the traditional quad-stacking method, DD laminates use only two ply orientations per building block, arranged in a bi-angle configuration, defined as $[\pm\phi/\pm\psi]_n$, where ϕ and ψ can represent any angle from 0° to 90° and n represents the number of repetitions. This flexible stacking strategy allows for customised angles and fewer design constraints, resulting in improved resistance to damage mechanisms like delamination and micro-cracking. It also supports faster manufacturing and more efficient structural optimisation, which could lead to substantial weight reductions, potentially up to 40%, and lower production costs for aerospace components [2].

Conventional modelling approaches rely heavily on high-fidelity finite element simulations to accurately predict joint performance and failure mechanisms. While these models offer great accuracy, they are computationally expensive and often impractical for early-stage design for probabilistic analyses. This challenge is further compounded by the growing interest in recent composite layups such as Double-Double laminates.

There is thus a clear motivation to bridge the gap between accuracy and efficiency. On one hand, there is a need to develop reliable failure prediction methods for Double-Double laminates using high-fidelity simulations. On the other hand, there is an increasing demand for simplified models that retain predictive capability while being computationally affordable. Furthermore, to gain confidence in such simplified models, it is essential to evaluate their sensitivity to input parameters and quantify uncertainties in their predictions.

This thesis aims to evaluate the mechanical performance of double-double laminates against equivalent standard quad laminates through the use of high and low fidelity simulations. A first-ever 3D failure envelope for a double-double laminate stiffness equivalent to a traditional quad laminate will be generated with the usage of high fidelity modelling strategies previously developed

by Furtado et al. [3]. It has a dual objective of advancing the understanding of bolted joint failure in novel composite layups and enhancing the reliability of simplified design tools, previously developed in Ref. [4]. A detailed uncertainty quantification strategy will be performed on such simplified models, with the aim of assessing their robustness in predicting the failure load of composite bolted joints under combined load scenarios.

With this in mind, a newly developed modelling strategy that generates structured meshes for any ply orientation is proposed, as well as a new failure ellipsoid applied to the equivalent double-double layup. These contributions are expected to support faster, more reliable, and more cost-effective design workflows for next-generation composite structures.

1.2 Thesis outline

This thesis is structured into five main chapters. Chapter 2 provides a comprehensive literature review, covering the recently developed double-double laminates, uncertainty quantification techniques, high-fidelity modelling strategies, and simplified modelling approaches of composite bolted connections.

Chapter 3 outlines the process of deriving an equivalent double-double laminate corresponding to the quad laminate studied in earlier work, including the necessary theoretical foundations.

Chapter 4 details the previously established methodology for numerically simulating composite laminates under combined bearing and pull-through loading conditions. It also introduces a novel meshing approach that enables structured mesh generation for arbitrary ply angles. A comparison is presented between simulations using structured and unstructured meshes. High-fidelity results for the double-double layup are then analysed and contrasted with the failure loads previously obtained for quad laminates. The resulting failure envelope for the double-double laminate is also derived and compared to its quad counterpart.

In Chapter 5, the uncertainty quantification of the simplified multi-bolt models is presented for both layups. This includes the modelling strategy, sensitivity analysis, and a detailed discussion of the results obtained.

Finally, Chapter 6 summarises the main findings and contributions of the thesis and suggests directions for future research and potential improvements.

Details regarding the testing rig developed in Ref. [3] are shown in Appendix A, and a final test for the definition of the connector stiffness using plasticity is shown in Appendix B.

1.3 UN SDGs framework

This thesis aims to contribute to the industrial applicability and understanding of novel composite laminate configurations through a detailed comparison of double-double (DD) and quad laminates. By developing both high-fidelity and simplified models for failure prediction in bolted joints, this research supports more efficient and sustainable design strategies in composite structures. The implementation of the present work and the achievement of its defined objectives are aligned with the Sustainable Development Goals (SDGs) established by the United Nations (UN). As such, the outcomes of this work have the potential to positively impact progress toward the following goals:

- SDG 9 - Industry, Innovation and Infrastructure

This thesis contributes to SDG 9 by advancing the understanding and simulation of composite bolted joints, a critical component in aircraft and structural applications. The development of high-fidelity and simplified models for predicting failure in bolted joints directly

supports innovation in materials engineering and structural design. By comparing double-double (DD) laminates with traditional quad configurations, the research identifies alternative design strategies that have the potential for superior mechanical performance. These insights foster the design of more efficient, lightweight, and resilient infrastructure and promote sustainable industrial innovation in high-performance composite applications.

- **SDG 12 – Responsible Consumption and Production**

By investigating alternative laminate configurations and developing accurate models, this work contributes to optimising material use in structural components. The proposed simplified models reduce the need for extensive physical prototyping and testing, minimising material waste and energy consumption in the design phase. Additionally, understanding how DD laminates perform under load allows for more informed material selection, potentially extending the lifespan of composite structures and reducing over-engineering. These outcomes align with SDG 12 by supporting more sustainable production processes and resource-efficient engineering practices.

- **SDG 13 - Climate action**

Lightweight composite materials, such as those studied in this thesis, play a crucial role in reducing emissions across sectors like aviation, transport, and renewable energy. By improving the predictive capabilities for failure in bolted joints, this research enables safer and more optimised use of composites, encouraging their broader adoption. The improved structural efficiency enabled by DD laminates may lead to weight savings and lower fuel consumption in transport applications, directly contributing to emissions reduction and climate mitigation goals outlined in SDG 13.

Chapter 2

Literature Review

This chapter provides a comprehensive overview of the existing research relevant to the modelling and analysis of bolted joints in composite structures, with a particular focus on double-double (DD) laminates. The review begins by addressing the unique characteristics and mechanical advantages of DD laminates, including recent advances in their homogenisation and the role of the newly discovered property, trace. High-fidelity modelling approaches are then discussed, and subsequently, simplified modelling strategies for bolted joints are examined, highlighting trade-offs between computational efficiency and physical accuracy. Finally, the chapter examines current methods in uncertainty quantification (UQ), highlighting their relevance in predicting model performance and developing robust design methodologies. This review establishes the foundation for the subsequent methodological developments and analyses presented in this work.

2.1 Double-double laminates

Double-double (DD) laminates are a novel category of composite laminates designed as an alternative to the traditional quad configuration, which restricts ply angles to 0° , $\pm 45^\circ$, and 90° , as well as other so-called unconventional laminates, initially proposed by Tsai et al. [11, 12]. Traditional quad laminates have a series of rules imposed, such as mid-plane symmetry, to avoid warping, 10% rule where each of the 4 angles must account for 10% of the total plies of the laminates and balanced laminates where $A_{16} = A_{26} = 0$. As a result of the imposed rules, sub-laminates used to build the total laminates become thicker to achieve the desired directionality. To increase the shear rigidity, more $\pm 45^\circ$ plies are added, and similarly, more 0° plies are needed to increase the stiffness along the longitudinal axis. This results in an increase in the minimum thickness of the laminate. Ultimately, thicker sub-laminates result in an unoptimal and difficult blending of laminates among elements and zones of structures [12].

Unlike these conventional approaches, DD laminates employ an optimised ply stacking sequence aimed at simplifying manufacturing, improving performance, and enhancing design predictability. This innovation enables faster and more cost-effective production, reduces weight, and supports manufacturing strategies that are not feasible with conventional or unconventional laminate designs. In double-double laminates, instead of determining the percentage of ply orientations, two angles, ϕ and ψ , are found, and the building block is repeated until the desired thickness is reached. This block is composed of 4 plies, distributed in a $[\pm\phi/\pm\psi]$ configuration, where both sets of angles are continuous and form a field rather than a collection of discrete laminates and is 4-ply thick, whereas the basic building block of a traditional quad has a minimum of 8 plies thickness when mid-plane symmetry is imposed, as shown in Figure 2.1.

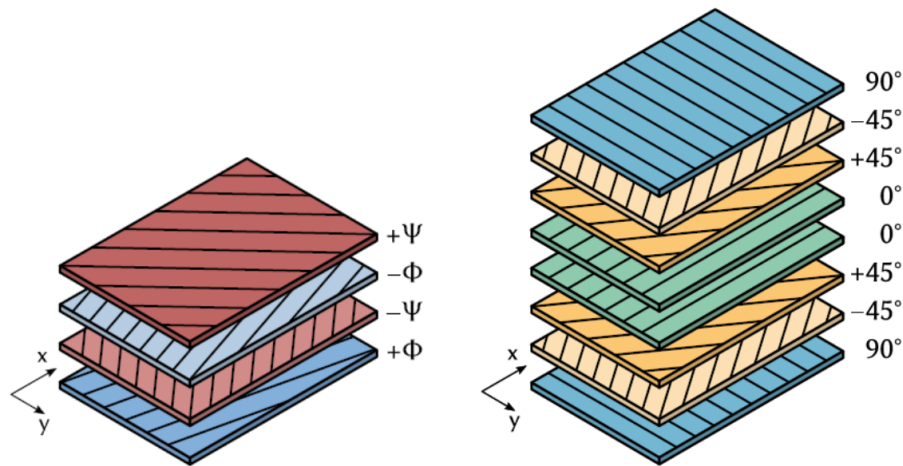


Figure 2.1: Double-double basic building block, left, vs quad-laminate quasi-isotropic (25/50/25) building block, right. Adapted from [5].

Many benefits arise from such simplifications. By not needing to comply with many of the traditional composite rules, double-double laminates are simpler to manufacture and easier to design. From a manufacturing perspective, the advantages over the traditional laminates are many. According to Tsai et al. [13], the basic building blocks can be remanufactured, which should enable a one-axis layup, quadrupling the layup speed now that one building block can be laid down at once. Additionally, since the laminate and the stacking sequence are largely the same, the likelihood of mistakes is reduced, as well as the manufacturing complexity.

Contrary to what is observed in the quad-laminates, in DD, plies at the external surfaces can be dropped, and the outer ply is not continuous, as shown in Figure 2.2 and, as suggested by Tsai et al. [13], single ply drops are possible in double-double laminates, and the risk of delamination is reduced by eliminating resin pockets and internal discontinuities. Recent developments by Kapel [14] and Gui and Nettles [15] state that even an entire building block can be dropped for faster tapering.

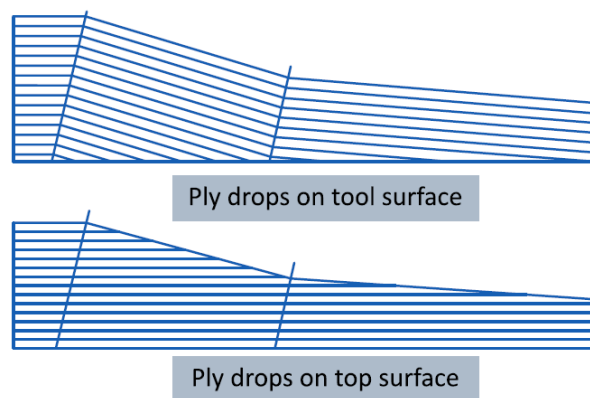


Figure 2.2: Surface ply drops for DD laminates. [5]

Finally, a key advantage of double-double laminates is that the complex set of traditional composite rules stated here can often be satisfied through alternative approaches, such as homogenisation [13], as shown and discussed later.

Experimental studies regarding the bolted joint behaviour under bearing and pull-through loading for double-double laminates were not found, due to the novelty of this concept. A recent overview of the current state of research on double-double laminates was presented by Kappel and Tsai [16], which references a study on single-shear and single-fastener bearing tension performance of DD

laminates conducted by Waruna Seneviratne at the National Institute for Aviation Research (NIAR), Wichita State University (WSU). However, the cited document is not publicly available, and its results could not be verified. Additionally, a study recently published by Li et al. [17] aims to investigate the effect of open holes on the failure behaviour and strength of DD laminates. The study consisted in four groups of open-hole tensile specimens, including quasi-isotropic and soft quad laminates as well as their equivalent double-double layups, which were designed to maintain equivalent in-plane stiffness, for progressive damage analysis and strength predictions. The results show that, while the quasi-isotropic quad laminate presented an expected behaviour, where the notched strength decreases as the hole diameter increases when the diameter to width ratio is maintained, DD shows the inverse effect, where the strength ends up exceeding the one for the respective quad, when the hole diameter is the largest. Additionally, quad laminates primarily failed under a fibre-dominated pull-out mode, whereas in the DD, a matrix-dominated pull-out mode was found for smaller holes and a brittle-like mode for larger ones. This study highlights that double-double laminates are not fully understood yet and require more research.

2.1.1 Homogenisation

Homogenisation is a laminate characteristic, achieved by repeating the basic building block a certain number of times, which means that its properties are the same across the thickness, and it can also be the same across a component in the longitudinal and transversal axis. According to Tsai et al. [13], homogenisation makes many of the rules governing laminate manufacturing and design unnecessary and prevents critical effects from coupling or warpage and twisting after cooling, usually associated with nonsymmetric laminates.

Starting with mid-plane symmetry, if the laminate is homogenised across the thickness, the need for mid-plane symmetry is removed as the extension-bending coupling is minimised, making the layup much simpler, faster and less prone to error. The stacking sequence no longer needs to stop at the mid-plane in order to mirror the laminate, making the manufacturing process continuous. Additionally, if a laminate is homogenised, instead of having to be in symmetric pairs, plies can be dropped in singles and on the outside, leaving the interior free of discontinuities and wrinkles, which produces better quality laminates.

Quad laminates are constrained by at least five key rules: mid-plane symmetry, balanced layups, the 10% rule, ply nesting limited to four or fewer plies, and inter-ply angles not exceeding 45 degrees. In contrast, Double-Double (DD) laminates offer a much simpler approach. They can achieve weight savings that neither conventional nor unconventional laminates can, due to the latter's inability to reach homogenisation. Homogenisation is essential for reducing weight in lightly loaded areas of a structure. Moreover, DD laminates are naturally resistant to delamination and facilitate straightforward calculations of interlaminar shear stress, buckling, and natural frequencies. With only two ply angles to manage, DD requires just a single-axis layup without the need for cross-plying, a level of simplicity that quads cannot provide [13]. The concept of homogenisation is key to Double-Double laminates, and it will be further addressed in Chapter 3.

2.1.2 One parameter stiffness: trace

A new invariant-based approach, initially proposed by Tsai and Melo [18], takes the trace of the plane stress stiffness matrix of a unidirectional ply, $\text{tr}(\mathbf{Q})$, as a material property to describe the stiffness of composite plies and laminates. This property is invariant concerning coordinate transformations, which means that the trace of the thickness-normalised in-plane ($\mathbf{A}^*$) and flexural ($\mathbf{D}^*$) stiffness matrices is also the same, further sustaining that apart from being a matrix algebraic relation, the trace is rightfully considered a fundamental property of the material, independent of coordinate system (and material symmetry), number of layers, layup sequence and loading condition (in-plane or flexural). It reflects the overall usable stiffness, influencing how laminated composites

are described and how their elastic behaviour is understood.

Based on experimental data, Tsai and Melo [18] were able to identify that when normalised by the trace of the stiffness matrix, carbon-fibre reinforced polymer (CFRP) laminates present common stiffness properties:

$$\begin{aligned} Q_{11}^* &= \frac{Q_{11}}{\text{tr}(\mathbf{Q})}, \\ Q_{22}^* &= \frac{Q_{22}}{\text{tr}(\mathbf{Q})}, \\ Q_{12}^* &= \frac{Q_{12}}{\text{tr}(\mathbf{Q})}, \\ Q_{66}^* &= \frac{Q_{66}}{\text{tr}(\mathbf{Q})} \end{aligned} \quad (2.1)$$

with

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \quad (2.2)$$

Specifically, the trace-normalised longitudinal stiffness component Q_{11}^* , which is fibre dominated, presented a variation of only 1.5% whereas the matrix-dominated transverse and shear components had a larger variation of up to 16.4%, which is not concerning since their contribution to the trace is very small in absolute terms. Additionally, a definition of a Master Ply was proposed using the median values of the trace-normalised stiffness components of a set of CFRP. This definition presents a universal relation between the trace and the components of the in-plane stiffness matrix of any CFRP, as shown in Equation 2.3. In unidirectional CFRP composites, the longitudinal stiffness accounts for roughly 88% of the trace, while matrix-related components contribute only 12%. As a result, despite their higher variability, the influence of the matrix-dominated components on the overall laminate properties is overshadowed by the fibre-dominated component, which exhibits low variability.

$$\begin{aligned} Q_{11} &= Q_{11}^* \cdot \text{tr}(\mathbf{Q}), \\ Q_{22} &= Q_{22}^* \cdot \text{tr}(\mathbf{Q}), \\ Q_{12} &= Q_{12}^* \cdot \text{tr}(\mathbf{Q}), \\ Q_{66} &= Q_{66}^* \cdot \text{tr}(\mathbf{Q}) \end{aligned} \quad (2.3)$$

With the trace-normalised compliance matrix $\mathbf{S}^* = (\mathbf{Q}^*)^{-1}$, the trace-normalised engineering constants can be determined as:

$$E_1^* = \frac{1}{S_{11}^*}, E_2^* = \frac{1}{S_{22}^*}, G_{12}^* = \frac{1}{S_{66}^*}, \nu_{12} = -\frac{S_{12}^*}{S_{11}^*} \quad (2.4)$$

with

$$E_1^* = \frac{E_1}{\text{tr}(\mathbf{Q})}, E_2^* = \frac{E_2}{\text{tr}(\mathbf{Q})}, G_{12}^* = \frac{G_{12}}{\text{tr}(\mathbf{Q})} \quad (2.5)$$

where E_1 , E_2 , G_{12} and ν_{12} are the ply engineering constants. The Master Ply definition is shown in Table 2.1.

Table 2.1: Master Ply definition.

	Q_{11}^*	Q_{22}^*	Q_{12}^*	Q_{66}^*
Master Ply	0.885	0.052	0.017	0.031
	E_1^*	E_2^*	G_{12}^*	ν_{12}^*
Master Ply	0.880	0.052	0.031	0.320

With the Master Ply trace-normalised plane stress stiffness components, E_1^* , E_2^* , G_{12}^* and ν_{12}^* , the stiffness matrix of a laminate A^* can be obtained and, from the compliance matrix $a^* = (A^*)^{-1}$, the trace normalised engineering constants for the laminate can be calculated as shown in Equation 2.6.

$$E_x^* = \frac{E_x}{\text{tr}(A^*)}, E_x^* = \frac{E_x}{\text{tr}(A^*)}, G_{xy}^* = \frac{G_{xy}}{\text{tr}(A^*)} \quad (2.6)$$

With the longitudinal Young's modulus of a 0° unidirectional laminate, E_1 , or with the longitudinal Young's modulus of a multi-directional laminate, E_x , the value of the trace, $\text{tr}(Q) = \text{tr}(A^*)$, can be determined dividing it by the corresponding trace-normalised laminate factor, E_1^* or E_x^* , as shown in Equations 2.7 and 2.8, respectively. This is made possible since the trace is dominated by the value of the longitudinal stiffness. Also, the elastic properties of any laminate can be obtained with the multiplication of the trace-normalised laminate factors by the trace value of the corresponding CFRP, as shown in Equation 2.9.

$$\text{tr}(Q) = \frac{E_1}{E_1^*} \quad (2.7)$$

$$\text{tr}(Q) = \text{tr}(A^*) = \frac{E_x}{E_x^*} \quad (2.8)$$

$$E_x = E_x^* \cdot \text{tr}(A^*), E_y = E_y^* \cdot \text{tr}(A^*), G_{xy} = G_{xy}^* \cdot \text{tr}(A^*) \quad (2.9)$$

A key simplification emerges in the otherwise intricate elastic characterisation of CFRPs, the trace of the in-plane stiffness matrix serves as the sole independent elastic parameter needed. This finding is very important, as it significantly reduces the complexity of mechanical testing required to determine orthotropic in-plane elastic properties. Traditionally, three distinct tests at the ply level are necessary to extract parameters like E_1 , E_2 , and G_{12} but with the usage of this novel concept, the process gets simpler. With just a single test to determine E_1 , $\text{tr}(Q)$ can be obtained through Equation 2.7, eliminating the need for the shear test, a procedure known for its non-linearity, high cost, and difficulty. An alternative approach shifts testing to the laminate level. Here, a single measurement of E_x is enough, through Equation 2.8, again $\text{tr}(Q)$ is obtained. This method inherently includes lamination effects, potentially rendering further laminate-level tests unnecessary.

2.2 High-fidelity simulation of composite joints

Composite bolted joints are widely used in aerospace, automotive, and other engineering applications due to their excellent strength-to-weight ratio and design flexibility. However, the complexity of load transfer, the anisotropic nature of composite materials, and the presence of stress concentrations around bolt holes present significant challenges for accurate structural analysis. High-fidelity modelling approaches, particularly those based on detailed finite element (FE) simulations, have emerged as powerful tools to capture the complex mechanical behaviour and failure

mechanisms of these joints [19]. Different models incorporate material anisotropy, nonlinear contact interactions, bolt preload effects, and progressive damage evolution, enabling more reliable prediction of joint performance under realistic service conditions.

One of the earliest approaches to the 3D numerical modelling of such bolted joints was proposed by McCarthy and McCarthy [19, 20]. In this study, the defined Finite Element model, consisting of a single-bolt, single-lap composite joint, was found to be in close relation to the experimental results, and it was also used to evaluate the effects of different parameters on the joint, such as bolt-hole clearance. It was demonstrated that greater bolt clearance results in increased bolt rotation, a reduction in the bolt-hole contact area, and a decrease in the overall joint stiffness. In single-lap joints, clearance induces three-dimensional variations in the stress distribution within the laminate, which are also influenced by the ply lay-up configuration. This effect is evidenced by the radial and tangential stress patterns observed in each ply. This model was then used by Egan et al. [21], to analyse how stress components are distributed through the layers of the laminate and by Olmedo and Santiuste [22], to apply a three-dimensional failure criteria that consider the distribution of shear stress through the thickness of the composite plates. A study conducted by Zhou et al. [23] successfully implemented a 3D finite element model which included the effects coming from delamination, cracking and plasticity, predicting damage evolution.

The use of fibre-aligned structured meshes is recommended in Refs. [24, 25]. Falcó et al [25] stated that the misalignment between crack band direction and mesh lines induces stress locking, which was mitigated by the alignment of the mesh lines with the orthotropic material directions. Additionally, it was concluded that the coupon simulation using a regular mesh, i.e., not aligned with material directions, yields a wrong and non-physical prediction of failure mechanisms.

A recent study [26] introduced a novel finite element formulation specifically designed for the structural analysis of composite bolted joints, aiming to strike a balance between accuracy and computational efficiency while maintaining a low number of degrees of freedom. This approach combines an analytical evaluation of the bolted region stiffness using the First-Order Shear Deformation Theory and the Ritz method to model the elasticity of composite plates around the bolt. Additionally, it incorporates bolt stiffness contributions from the shank, head, and bolt-hole bearing deformation, allowing precise assessment of friction, preload, and radial clearance effects. The resulting user-defined Composite Bolted Joint Element (CBJE) transfers laminate stiffness to radially arranged beam elements, integrating bolt behaviour through beam and spring components. Validation against experimental data demonstrated excellent agreement, and sensitivity analyses revealed comparable accuracy to detailed 3D FE models, confirming the method's effectiveness.

In bolted joints of unidirectional composite laminates, the dominant mesoscale (ply-level) failure mechanisms vary with the macroscopic failure mode, involving combinations of fibre kinking, matrix cracking, delamination, and ply-dependent tensile and shear failure. To effectively predict how composite bolted joints ultimately fail, analysis models need to represent the mesoscale failure mechanisms involved. If large ply blocks that trigger delamination are not included, 2D models may be used. However, bearing is a highly 3D phenomenon for which 3D damage models should be employed [24]. The intralaminar constitutive model employed in the high-fidelity models used in this thesis and developed in Ref. [3] is the 3D infinitesimal smeared crack model for the simulation of the intralaminar degradation mechanism of the composite ply developed in Ref. [27]. This constitutive model was selected, as opposed to other models available in the research group [28–30], for its ability to accurately represent 3D failure phenomena, including fibre kinking and through-thickness effects, using an embedded cohesive crack approach and invariant-based failure criteria, features that are essential for capturing the intralaminar damage observed in this study. Turon et al. [31, 32] introduced a cohesive zone model, and Alfano and Sacco [33] proposed the Coulomb-based friction model, both coupled into the interlaminar frictional cohesive-zone model used in the high-fidelity models in study, developed in Ref. [3]. This model accurately represents the delamination mechanism in the presence of compressive through-the-thickness stresses. Previous applications of this approach include successful analysis of delamination propagation in both transverse crack

tensile coupons and double-shear bolted joints in composites [24, 34], and it has been expanded for combined bearing/pull-through loading in Ref. [3]. The modelling strategies used in this work will be later detailed in Chapter 4.

2.3 Simplified models to simulate composite bolted joints

Bolted structures can be modelled at varying degrees of detail and computational expense. Among the available approaches, three-dimensional solid modelling is recognised for its high accuracy [35]. However, this comes with significant computational and data requirements, which limit its practicality. As a result, such detailed 3D models are often used in a localised manner, concentrating on specific regions of the structure to capture detailed material responses, such as load versus displacement behaviour, which are then integrated into global models using bolt representations. The process of simplifying existing 3D solid bolt joint models in Abaqus has been extensively explored in the existing literature, as outlined below.

Tanlak et al. [36] used Timoshenko beams, connector beams and different combinations of deformable and rigid shell elements as a simplified approach to serve as a basis for comparison with a fully-solid bolted joint under impact loading, which resulted in an overall saving of 80-90% of computational time for the same mesh density. However, among the various alternatives to 3D solid bolts, connector elements are widely recognised in the literature as a reliable, lightweight, and convenient solution with one of the main difficulties associated with connector elements lying in the requirement to define their flexibility and stiffness in advance.

For several connector stiffness equations namely Boeing's 2 equations [37, 38], Grumman [37, 39], Huth [40], Swift [39], and Tate and Rosenfeld [37, 39], Eremenin et al. [37], found that the first Boeing equation offers the greatest suitability for plates of small and medium thickness while Huth's formula remains as the most indicated for large thickness laminates. Among the models examined, Huth and Grumman fasteners provided the most accurate predictions of fastener flexibility and load distribution when simulated using Abaqus-defined bushing connectors, as identified by Chandregowda [39].

Volpi [4] developed the simplified multibolt models used in this work, and later detailed, recurring to connector-based models. Initial tests to develop the simplified models were conducted between a model employing connector elements to represent bolts and the global bolted joint model proposed by Gray et al. [41]. For the connector models, both bushing and cartesian connectors were tested. The bushing connectors were found to behave like rubber and the bolt ratio was inconsistent across the simulation having too much of longitudinal bearing load whereas the cartesian connectors were able to correctly maintain a steady load ratio across the simulation, as desired, concluding that the cartesian connectors were the best connector type to predict the failure load of composite bolted joints. The author found that the connector model offers advantages in terms of faster computation time and reduced memory usage, with validation completed in seconds using 10 CPUs. However, it has limitations, such as the inability to analyse the impact of clearance on composite plate bearing capability and a less accurate representation of joint deformation due to the lack of physical modelling of the bolt. Preload effects must be accounted for in the fastener flexibility definition rather than directly in Abaqus. There are also uncertainties regarding the accuracy of the connector stiffness definition and its applicability to multi-bolt cases. Furthermore, accurate results require not only joint stiffness prediction but also correct load distribution across fasteners.

Additionally, the author from Ref. [4] also concluded that the global bolted joint model (GBJM), while more complex, provides a more accurate representation of joint deformation, including secondary and tertiary bending, bolt displacement and deformation, and bolt-hole distortion. It allows for adjustments to clearance and pre-tension to be made directly in Abaqus, and bolt parameters are linked to the bolt's material properties, eliminating the need for an additional pre-determination step. However, the major drawback of this model is its runtime, which takes approx-

imately 55 minutes with 10 CPUs to achieve a 1 mm displacement in the loading direction. Despite this, it still offers a 93.5% time savings compared to the 3D multi-bolt model. It's worth noting that this time could be improved, as the single-bolt high-fidelity case only reaches a maximum displacement of 0.45 mm. However, due to the lack of robustness of the generative Python script developed for the creation of GBJM models in the definition of contact and clearance for the GBJM models and due to their longer run time, the connector models were the ones chosen for the studies related to the application of the failure criteria. The modelling strategy used in this work will be further detailed in Section 5.1.

2.4 Uncertainty quantification

Engineering structures, namely bolted composite joints for aerospace and mechanical applications, must meet strict safety standards. When modelling structures, real-world parameters, such as bolt preload, material properties, and contact conditions, are never perfectly known, which makes computation models only reliable up to a certain degree [42].

Assessment of uncertainties in simulation results is performed in the context of verification, validation and uncertainty quantification. It allows the assessment of the accuracy of a model and the quantification of its predictive capability. By systematically identifying and quantifying sources of uncertainty, engineers can better understand the limitations of their models and improve their reliability.

By inducing precisely measured and controlled changes in the input parameters and evaluating the specimen response, it is possible to quantify the variations on the outputs and increase the confidence in simulation results in a probabilistic way. This enables not only more efficient designs but also the development of methodologies that optimise structural performance while ensuring compliance with safety standards. Furthermore, integrating UQ into engineering workflows aids in risk assessment and decision-making, allowing engineers to determine the probability of failure, refine safety margins, and avoid excessive conservatism in design [43].

In recent years, many different methods, including the Monte Carlo, polynomial chaos and perturbation method, have been employed in uncertainty quantification of engineering problems [42]. Taylor series-based perturbation methods depend on higher-order derivatives to achieve greater precision, which are computationally expensive and, additionally, their effectiveness diminishes with large perturbations, becoming inaccurate for large changes, limiting their applicability in practical engineering contexts.

Initially derived by Wiener [44], for Gaussian distributed uncertainties and further generalised for any given distribution by Xiu and Karniadakis [45], polynomial chaos expansion is proposed in the literature as one of the most efficient uncertainty quantification methods [42]. However, it faces one problem referred to as the curse of dimensionality, where if a large number of input uncertainties is given, the computational time and number of simulations needed increase exponentially. A solution for this problem based on adaptive sparse polynomial chaos, based on the Least Angle Regression algorithm, is proposed in Reference [42].

In composite structures, two of the most commonly used approaches for uncertainty quantification are the Monte Carlo-based approach and the perturbation-based approach [46]. The Monte Carlo approach consists of generating a large number of random samples of the uncertain input parameters according to their probability distributions and evaluating the model for each sample to build a statistical picture of the output. The Perturbation approach, on the other hand, involves systematically varying the input parameters within predefined bounds to explore all possible combinations and assess their impact on the output, without relying on probabilistic assumptions. A key limitation of the perturbation approach lies in its reliance on complex analytical derivations and its inability to provide a full probabilistic characterisation of the response variables. Moreover, its validity is generally restricted to cases with low levels of input uncertainty. The drawbacks as-

sociated with the Monte Carlo approach are the requirements of a large set of simulations, which for complex and computationally expensive models, becomes very complicated in terms of needed time and data [46]. Nonetheless, it remains a widely adopted strategy due to its conceptual simplicity, ease of implementation, and ability to provide a comprehensive probabilistic characterisation of model responses, particularly in cases where computational demands are manageable. This topic will be later addressed in Chapter 5.

Chapter 3

Laminate Selection

In this chapter, an equivalent double-double laminate to the quad configuration studied in Ref. [8] is derived, based on in-plane stiffness equivalence. The quad laminate is a directional $[45/90/-45/0/0/45/0/0/-45/0]_S$ manufactured in Hexcel's IM7/8552 with a cured ply thickness of 0.125 mm, and the properties are detailed in Tables 4.1 and 4.2. Classical Laminate Theory and the concept of laminate homogenisation are introduced to provide the necessary theoretical background. The methodology used for establishing equivalence is detailed, and its relevance to subsequent modelling and analysis is discussed.

The process of obtaining an equivalent laminate begins by plotting the domain of double-double laminates in terms of in-plane lamination parameters, in order to identify the ply angles that correspond to those of the quad laminate. Once these angles are determined, the optimal layup is selected based on homogenisation criteria, resulting in the final equivalent laminate configuration.

3.1 Classical laminate theory

Classical Laminate Theory (CLT) provides a framework for predicting the mechanical behaviour of fibre-reinforced composite laminates based on the properties and orientations of individual plies and is detailed here. It is worth noting that the coordinate system in use is shown in Figure 3.1.

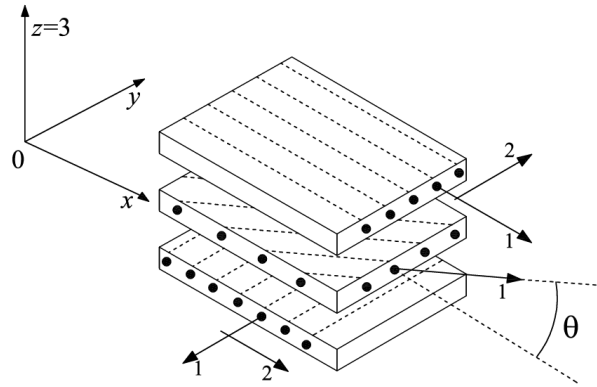


Figure 3.1: Coordinate system in use. [6]

The stress-strain relationships for a lamina made of an orthotropic material under plane stress, expressed in the principal material coordinates, are given by:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (3.1)$$

where the Q_{ij} are the reduced stiffnesses, determined in terms of the engineering constants as

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} & Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} & Q_{66} &= G_{12} \end{aligned} \quad (3.2)$$

In any other coordinate system in the plane of the lamina, the stresses are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (3.3)$$

where the transformed reduced stiffnesses, $\bar{Q}_{ij}$ are given in terms of the reduced stiffnesses by

$$[\bar{Q}] = [T]^{-1} [Q] [T] \quad (3.4)$$

with

$$[T] = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & m^4 + n^4 & -4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2 - n^2)^2 \\ m^3n & -mn^3 & m^3n - mn^3 & 2(m^3n - mn^3) \\ mn^3 & -m^3n & m^3n - mn^3 & 2(m^3n - mn^3) \end{bmatrix}, \quad m = \cos\theta, \quad n = \sin\theta \quad (3.5)$$

where θ is the ply orientation. Both these equations can be thought of as stress-strain relations for the k_{th} layer of a laminate, and Equation 3.3 can be rewritten as

$$\{\sigma\}_k = [\bar{Q}]_k \{\varepsilon\}_k \quad (3.6)$$

The laminate is assumed to be composed of perfectly bonded plies, and these bonds are considered to be infinitesimally thin and incapable of shear deformation. As a result, displacements remain continuous across the boundaries between plies, preventing any relative slipping between adjacent layers. Thus, if the laminate is thin, a line normal to the middle surface, when the laminate is deformed, such line is assumed to remain straight and perpendicular to the middle surface. This means that the shearing strains in planes perpendicular to the middle surface are 0, $\gamma_{xz} = \gamma_{yz} = 0$, with z being the direction of the normal to the middle surface. Additionally, it is assumed that the normals maintain a constant length, implying that the strain in the direction perpendicular to the mid-surface is neglected, $\varepsilon_z = 0$.

The effects of these assumptions on the laminate displacements u , v , and w in the x , y , and z directions are derived using the laminate cross-section in the x - z plane, as shown in Figure 3.2.

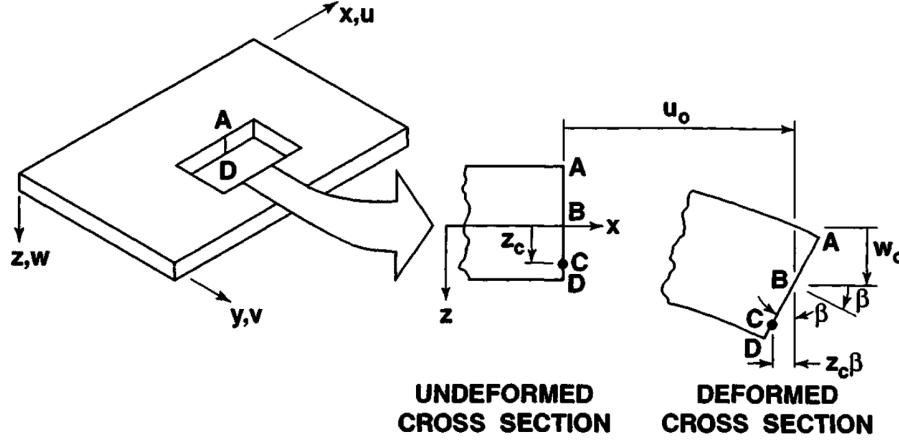


Figure 3.2: Geometry of Deformation in the x-z Plane. [7]

The displacement in the x-direction of point B from the undeformed middle surface to the deformed middle surface is u_0 , where 0 is used to designate middle-surface values of a variable, and, since the line ABCD remains straight under deformation of the laminate, the displacement at point C is

$$u_c = u_0 - z_c \beta \quad (3.7)$$

However, since line ABCD always remains perpendicular to the middle surface, β is the slope of the laminate middle surface in the x-direction, that is,

$$\beta = \frac{\partial W_0}{\partial x} \quad (3.8)$$

and the displacement, u , at any point z through the laminate thickness is

$$u = u_0 - z \frac{\partial W_0}{\partial x} \quad (3.9)$$

while similarly, the displacement, v , in the y-direction becomes

$$v = v_0 - z \frac{\partial W_0}{\partial y} \quad (3.10)$$

As shown earlier, with the assumptions made, the strains have been reduced to ϵ_x, ϵ_y and γ_{xy} which, for small strains, are defined in terms of displacements as:

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{aligned} \quad (3.11)$$

which, with the displacements shown in Equations 3.9 and 3.10, become:

$$\begin{aligned} \epsilon_x &= \frac{\partial u_0}{\partial x} - z \frac{\partial^2 W_0}{\partial x^2} \\ \epsilon_y &= \frac{\partial v_0}{\partial y} - z \frac{\partial^2 W_0}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 W_0}{\partial x \partial y} \end{aligned} \quad (3.12)$$

or

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.13)$$

where the middle surface strains are

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix} \quad (3.14)$$

and the middle surface curvatures are

$$\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = - \begin{bmatrix} \frac{\partial^2 W_0}{\partial x^2} \\ \frac{\partial^2 W_0}{\partial y^2} \\ 2 \frac{\partial^2 W_0}{\partial x \partial y} \end{bmatrix} \quad (3.15)$$

With the substitution of Equation 3.13 in 3.6, the stresses in the k^{th} layer can be expressed in terms of the laminate middle-surface strains and curvatures as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.16)$$

The resulting forces and moments on a laminate are determined by integrating the stresses within each individual layer or lamina across the laminate's thickness:

$$N_x = \int_{-t/2}^{t/2} \sigma_x dz \quad M_x = \int_{-t/2}^{t/2} \sigma_x z dz \quad (3.17)$$

N_x represents the in-plane force per unit width of the laminate's cross section, as illustrated in Figure 3.3a, while M_x denotes the bending moment per unit width, as shown in Figure 3.3b. For simplicity, both will be referred to ahead simply as forces and moments, respectively.

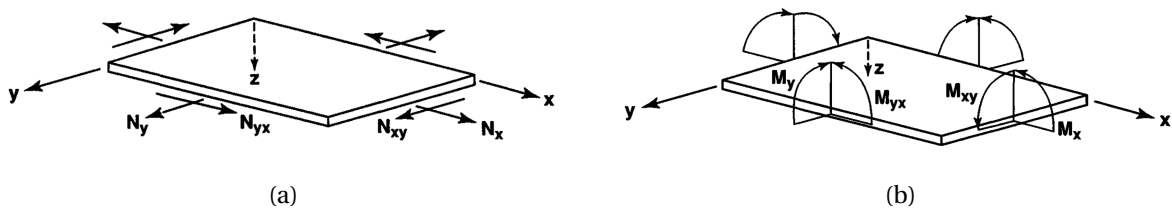


Figure 3.3: In-plane forces (a), and in-plane moments (b), of a flat laminate. [7]

The total forces and moments for an N-layered laminate are defined as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-t/2}^{t/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz = \sum_{k=1}^N \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k dz \quad (3.18)$$

and

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-t/2}^{t/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z dz = \sum_{k=1}^N \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z dz \quad (3.19)$$

where z_k and z_{k-1} are defined in Figure 3.4. These force and moment resultants do not depend on z after integration, but are functions of x and y , the coordinates in the plane of the laminate middle surface.

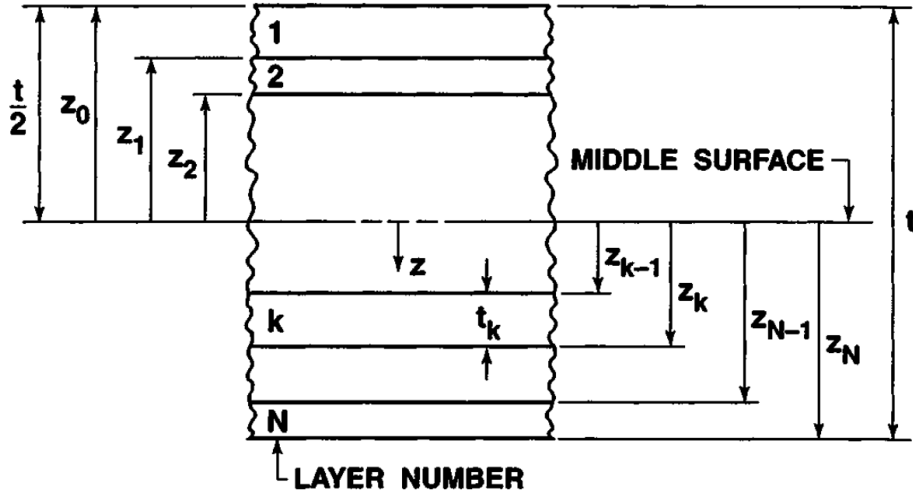


Figure 3.4: Geometry of an N-layered laminate. [7]

To take advantage of the fact that the stiffness matrix is often constant for each ply, Equations 3.18 and 3.19 can be rearranged with Equation 3.16 and become

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^N \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \left[\int_{z_{k-1}}^{z_k} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} z dz \right] \quad (3.20)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^N \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left[\int_{z_{k-1}}^{z_k} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} z dz + \int_{z_{k-1}}^{z_k} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} z^2 dz \right] \quad (3.21)$$

As stated above, ϵ_x^0 , ϵ_y^0 , γ_{xy}^0 , κ_x , κ_y and κ_{xy} are not functions of z , but are middle-surface values so they can be removed from within the summation signs, for which Equations 3.20 and 3.21, can be rewritten as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.22)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.23)$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \end{aligned} \quad (3.24)$$

with A_{ij} being the extensional stiffnesses, B_{ij} the bending-extension coupling stiffnesses, and D_{ij} the bending stiffness, matrices further used and referred to in this thesis.

3.2 Homogenisation

In the analysis and design of composite laminates, homogenisation is a very important characteristic. When a laminate is homogeneous, it means that its effective stiffness properties are uniformly distributed across the thickness and along the longitudinal axis, leading to predictable mechanical responses under loading. This assumption is often used to simplify numerical modelling and analytical formulations. According to Tsai et al. [13], a laminate is considered homogeneous when two specific conditions are met:

- $[A^*] = [D^*]$
- $[B^*] = 0$

with

$$[A^*] = \frac{1}{h} [A] \quad (3.25)$$

$$[D^*] = \frac{12}{h^3} [D] \quad (3.26)$$

$$[B^*] = \frac{2}{h^2} [B] \quad (3.27)$$

where h is the laminate thickness, $[A^*]$ is the normalised in-plane stiffness matrix, $[B^*]$ is the normalised coupling stiffness matrix, and $[D^*]$ is the normalised out-of-plane (bending) stiffness matrix. These conditions imply that the laminate exhibits identical stiffness characteristics in tension and bending, and there is no coupling between extension and bending behaviours. Such conditions are typically satisfied in symmetric and balanced laminates, which are commonly used in practical composite design to ensure mechanical stability and to reduce warping or unexpected deformation under load.

The homogenisation of double-double laminates, comprising only four plies arranged as $[\pm\phi/\pm\psi]$, is significantly simplified [13]. Even when conventional design guidelines such as mid-plane symmetry, laminate balance, the 10% rule, and maximum inter-ply angles of 45 degrees are disregarded, these novel laminates can still achieve homogenisation more efficiently. In contrast to a homogeneous laminate, as defined earlier, a laminate is considered homogenised if it satisfies the following criteria, based on the maximum absolute values of the components of $[B^*]$ and $[A^*] - [D^*]$:

$$\begin{aligned} |[A^*_{ij}] - [D^*_{ij}]| &< 2\% \text{ Tr} \\ |[B^*_{ij}]| &< 2\% \text{ Tr} \\ \text{with Tr} &= A^*_{11} + A^*_{22} + 2A^*_{66} \end{aligned} \quad (3.28)$$

It is important that both concepts are clearly defined, as they represent distinct ideas that can easily be conflated. Without this clarification, there is a risk of misinterpretation in analysis or design, potentially leading to incorrect assumptions about the material behaviour or performance criteria.

3.3 Equivalent laminate

In this section, the equivalent laminate is obtained. The aim is to find a Double-Double laminate which is in-plane equivalent to an industry used Hard laminate, [45/90/-45/0/0/45/0/0/-45/0]_S manufactured in Hexcel's IM7/8552, studied in Ref. [4]. To do so, lamination parameters are analysed to obtain the equivalent ply angles, and the best layup is obtained regarding homogenisation criteria.

3.3.1 Lamination parameters

Lamination parameters, initially proposed by Tsai et al. [47], make it possible to geometrically define any laminate by only needing twelve parameters and a thickness variable, without the need to consider the ply number. The in-plane, $\zeta_{\{1,2,3,4\}}^A$, coupled, $\zeta_{\{1,2,3,4\}}^B$ and out-of plane, $\zeta_{\{1,2,3,4\}}^D$, parameters can be calculated as:

$$\zeta_{\{1,2,3,4\}}^A = \frac{1}{h} \sum_{i=1}^N \begin{Bmatrix} \cos(2\theta_i) \\ \cos(4\theta_i) \\ \sin(2\theta_i) \\ \sin(4\theta_i) \end{Bmatrix} (z_i - z_{i-1}) \quad (3.29)$$

$$\zeta_{\{1,2,3,4\}}^B = \frac{2}{h^2} \sum_{i=1}^N \begin{Bmatrix} \cos(2\theta_i) \\ \cos(4\theta_i) \\ \sin(2\theta_i) \\ \sin(4\theta_i) \end{Bmatrix} (z_i^2 - z_{i-1}^2) \quad (3.30)$$

$$\zeta_{\{1,2,3,4\}}^D = \frac{4}{h^3} \sum_{i=1}^N \begin{Bmatrix} \cos(2\theta_i) \\ \cos(4\theta_i) \\ \sin(2\theta_i) \\ \sin(4\theta_i) \end{Bmatrix} (z_i^3 - z_{i-1}^3) \quad (3.31)$$

where θ_i is the fibre orientation at height $z \in [z_{i-1}, z_i]$, h is the laminates thickness and N the number of plies of the laminate. To find in-plane equivalence, we only need to consider ζ^A parameters and since we are dealing with balanced laminates, $\zeta_{3,4}^A = 0$, making $\zeta_{1,2}^A$ the only relevant lamination parameters.

By first plotting the domain of the double-double lamination parameters, where a step of 2° was used for ϕ and ψ , we observe in Figure 3.5 that there is no direct equivalence between the studied in Ref. [4] quad laminate, a hard [45/90/-45/0/0/45/0/0/-45/0]_S manufactured in Hexcel's IM7/8552, and the $[\pm\phi/\pm\psi]$ domain.

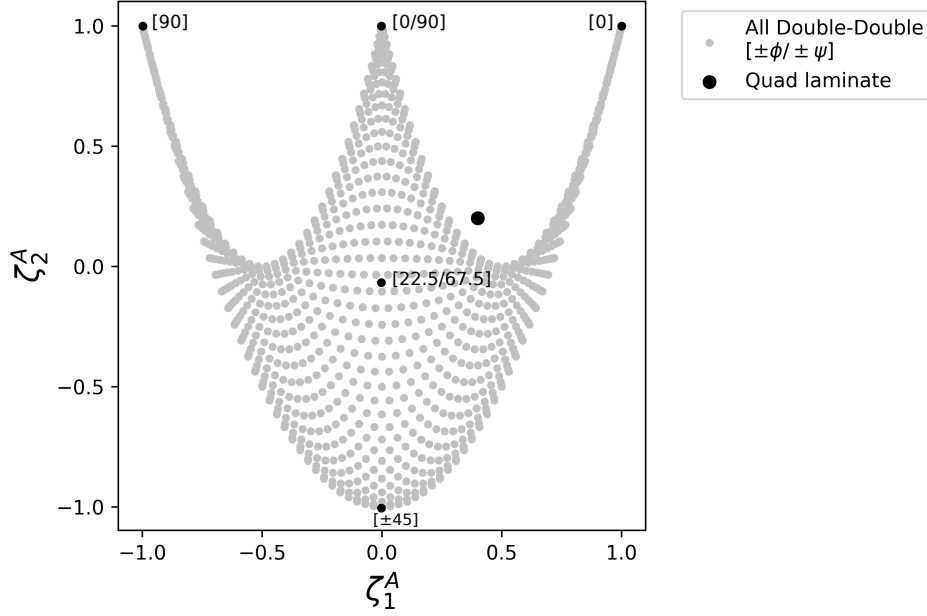
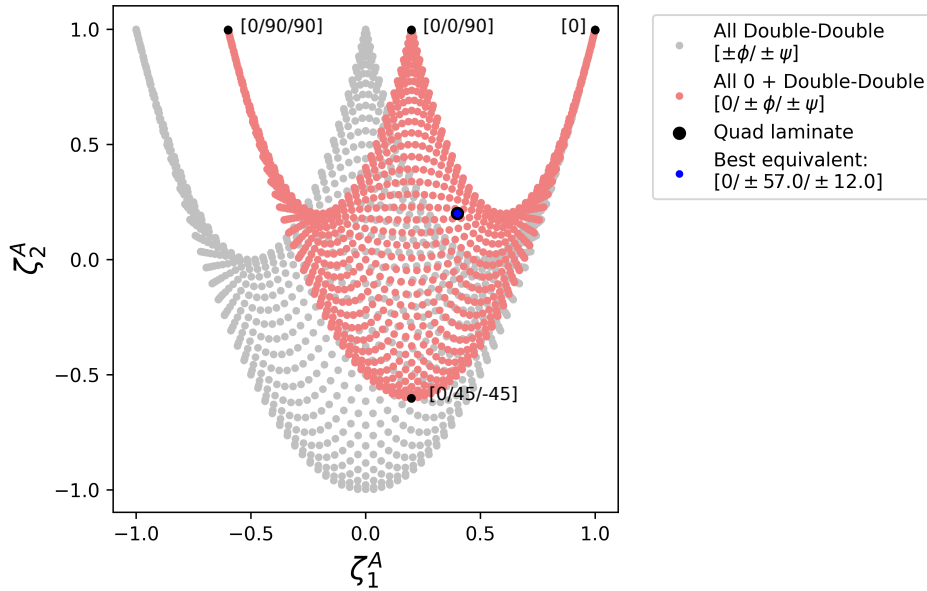


Figure 3.5: Double-Double lamination parameters domain.

However, with the addition of a 0° ply, as shown in Figure 3.6 and suggested in Ref. [13], a direct equivalence emerges for a laminate with 57° and 12° plies. In both cases, the total number of plies was maintained at 20, consistent with the original quad laminate. Having defined the double-double layup equivalent to the quad laminate defined in Ref. [8], it is time to select the stacking sequence to use. We will aim at defining a homogenised layup that follows some additional manufacturing constraints.

Figure 3.6: Double-Double with added 0° ply lamination parameters domain.

3.3.2 Double-double homogenisation

As previously stated and thoroughly discussed in Tsai et al. [13], double-double laminates offer the significant advantage of having virtually no design restrictions. This flexibility allows the stack-

ing sequence to follow any arbitrary order, which is particularly beneficial when aiming to achieve homogenisation of the laminate properties. Homogenisation, as mentioned earlier, is assessed based on the largest absolute components of the normalised stiffness matrices $[B^*]$ and $[A^*] - [D^*]$. The criterion employed to evaluate the degree of homogenisation is defined as follows:

$$\begin{aligned} |[A_{ij}^*] - [D_{ij}^*]| &< 2\% Tr \\ |[B_{ij}^*]| &< 2\% Tr \\ \text{with } Tr &= A_{11}^* + A_{22}^* + 2A_{66}^* \end{aligned} \quad (3.32)$$

where $[A^*]$, $[B^*]$, and $[D^*]$ represent the normalised in-plane, coupling, and bending stiffness matrices, respectively, and Tr denotes the trace of the in-plane stiffness matrix. This criterion was applied to every possible stacking sequence generated using the previously selected ply angles of 12° , 57° , and 0° . The five stacking sequences that yielded the best homogenisation performance, according to this criterion, are summarised in Table 3.1. It is important to note that, although not mandatory, in order to maintain consistency with the laminate studied in Ref. [8], the number of plies was fixed at 20 for all the different configurations.

Table 3.1: Summary of the homogenisation criterion for the five best layups.

Layup	$ [B_{ij}^*] < 2\% Tr$ [% of Tr]	$ [A_{ij}^*] - [D_{ij}^*] < 2\% Tr$ [% of Tr]
$[-12/57/0/-57/12]_{4T}$	0.98	0.75
$[0/57/-12/-57/12]_{4T}$	1.03	0.92
$[0/-57/12/57/-12]_{4T}$	1.03	0.92
$[-12/12/57/-57/0]_{4T}$	1.11	1.34
$[12/-12/57/-57/0]_{4T}$	1.11	1.34

Plyies with 0° orientation in the outer parts of a laminate present several mechanical and manufacturing risks in composite materials. These plies, aligned with the primary load direction, have low transverse strength and are especially vulnerable to surface damage and delamination under impact or out-of-plane loads [48]. When placed at the laminate surface, they reduce resistance to interlaminar stresses and can initiate cracking that propagates through the laminate [49]. Furthermore, such configurations can increase residual thermal stresses due to mismatched coefficients of thermal expansion and are more sensitive to moisture ingress during service [50]. For these reasons, design guidelines often recommend placing more damage-tolerant orientations, such as $\pm 45^\circ$ or 90° , at the surfaces, reserving 0° plies for interior regions where they can most effectively contribute to axial stiffness and strength.

With the restriction of not having outer plies with an orientation of 0° , both outer plies in this configuration are oriented at 12° , which raises concerns due to their similarity to a 0° orientation. This may limit their effectiveness in addressing the challenges typically associated with having outer plies at such a low angle. Since the aim is to place the 12° and 0° plies on the inside and keep the 57° plies on the outside, a similar study was developed restricting the outer plies to be either oriented at 57° or -57° which, resulted in the values presented in Table 3.2. Additionally, the entire layup permutations were also evaluated for both criteria, with the results shown in Figure 3.7, and ordered by value of $|[B_{ij}^*]|$.

Table 3.2: Summary of the homogenisation criterion for the layups that follow the restrictions.

Layup	$ B_{ij}^* < 2\% Tr$ [% of Tr]	$ A_{ij}^* - D_{ij}^* < 2\% Tr$ [% of Tr]
$[57/-12/0/12/-57]_{4T}$	2.10	1.75
$[57/0/-12/12/-57]_{4T}$	2.13	1.71
$[57/-12/12/0/-57]_{4T}$	2.13	1.71
$[57/12/-12/0/-57]_{4T}$	2.17	1.71
$[57/0/12/-12/-57]_{4T}$	2.17	1.71
$[57/12/0/-12/-57]_{4T}$	2.20	1.75

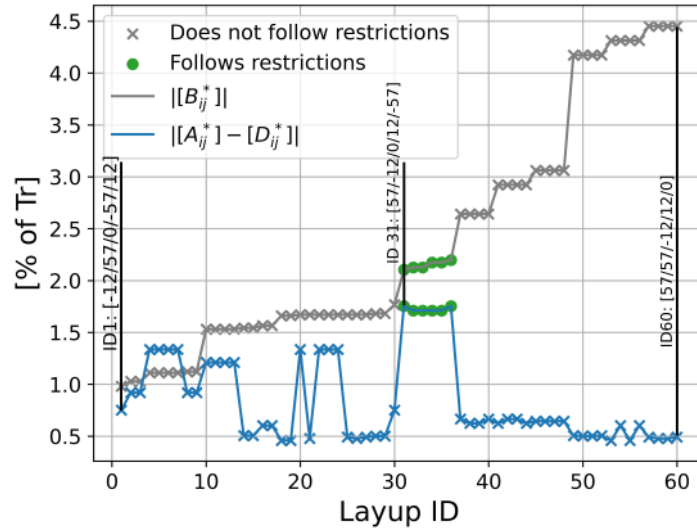


Figure 3.7: Homogenisation criteria evaluation for double-double layups.

After evaluating all possible stacking sequences using the homogenisation criterion defined by Tsai et al. [13], the layup $[57/-12/0/12/-57]_{4T}$ was ultimately selected, as it was the layup with the lower $||B_{ij}^*||$ that follows the imposed constraints. Although certain unconstrained stacking sequences, such as $[-12/57/0/-57/12]_{4T}$, demonstrated slightly better criteria values, practical design considerations and structural integrity requirements were pivotal in the final decision. Specifically, it was essential to avoid 0° and 12° plies on the outer surfaces of the laminate, ensuring compliance with the manufacturing restrictions. Among the layups satisfying this constraint, $[57/-12/0/12/-57]_{4T}$ exhibited the best performance, with both $||B_{ij}^*||$ and $||A_{ij}^*|| - ||D_{ij}^*||$ remaining below 2.2% of the trace of A^* , thus almost satisfying the homogenisation threshold. Therefore, this layup results in an effective balance between theoretical homogenisation, structural robustness, and alignment with practical design guidelines, justifying its selection as the final laminate configuration. However, this selection requires experimental verification. In particular, the flatness of the plates post-cure needs to be confirmed. This double-double layup will be the subject of study in the following chapters. Its performance will be compared to that of the equivalent quad laminate using numerical simulations.

Chapter 4

Determination of the Failure Envelope of an Equivalent Double-Double Laminate Through High-Fidelity Simulation

In this chapter, the failure envelope for the Double-Double (DD) laminate under study is established using a high-fidelity modelling approach. The aim is the development of a robust failure criterion that can be later applied in simplified modelling strategies. While previous efforts have focused on quad laminates with conventional stacking sequences, this chapter addresses the double-double configuration previously determined.

The high-fidelity modelling strategy used here is similar to the one earlier proposed by Volpi [4] for the quad laminates. It focuses on the high-fidelity models previously developed to replicate the behaviour of bolted composite joints under combined bearing/pull-through loading conditions. These models are used as a foundation to obtain the failure criteria later employed in the simplified multi-bolt modelling framework and are adapted for the double-double layup studied in this work.

To achieve this, the first step involves adapting the high-fidelity models, originally developed in Ref. [3], which include pure bearing, pure pull-through, and combined loading configurations to simulate DD laminates. Given that the structured meshing approach used in the original work is only applicable to conventional quad fibre orientations, 0°, 45°, and 90°, a new meshing strategy was developed in this work to allow mesh alignment with any arbitrary fibre direction, making it suitable for any kind of layup.

Once the new meshing methodology is established, a study was conducted to evaluate the effect of mesh alignment by comparing the results of simulations using aligned and non-aligned meshes for identical loading cases. In the literature, the usage of an aligned mesh is recommended [24], yet it is important to assess the impact of such a strategy.

Subsequently, high-fidelity simulations were performed for double-double and compared with quad laminates. The failure criteria for the quad laminate was already established in previous work, and was obtained with the first significant load drop (>10%). This chapter also aims to extend the methodology to the DD case. The resulting failure loads obtained from the DD simulations are then compared with those of the quad laminate to highlight key differences in failure behaviour. Finally, a failure criterion specific to the double-double laminate is formulated and compared to the one employed for the quad laminates, enabling its use in the simplified modelling strategy.

These high-fidelity models are developed based on the experimental testing rig developed by Furtado et al. [3]. For the sake of completeness, the rig is fully detailed in Appendix A and the material properties are detailed in Tables 4.1 and 4.2. The quad laminate is a directional [45/90/-45/0/0/45/0/0/-45/0]_S manufactured in Hexcel's IM7/8552 with a cured ply thickness of 0.125 mm, and the properties are detailed in Tables 4.1 and 4.2.

4.1 High-fidelity models

The high-fidelity models used to obtain the failure criteria for both double-double and quad laminates were developed by Furtado et al. in Ref. [3] with the goal of replicating the experimental rig test conditions. This section provides a brief overview of these models, including their modelling strategy and key characteristics.

4.1.1 Combined bearing/pull-through model

Starting with the combined loading models, they aim to be able to correctly predict the behaviour of composite bolted joints under realistic loading scenarios with varying ratios of bearing (in-plane) and pull-through (out-of-plane), allowing the characterisation of the behaviour of such joints. Those ratios, defined by modifying the dimensions a and b of the rig part, are obtained as shown in Equation 4.1, with P_B and P_{PT} being the bearing and pull-through loads, respectively. This equation is derived from the configuration of the experimental rig, detailed in Appendix A.

$$\eta = \frac{P_B}{P_B + P_{PT}} \quad (4.1)$$

As shown in Figure 4.1, the combined-loading model is composed of six parts: one block, one bolt, one rig, one washer, an inner laminate (under analysis) and an outer laminate with different loading ratios of bearing/pull-through being defined by changing a and b of the rig part.

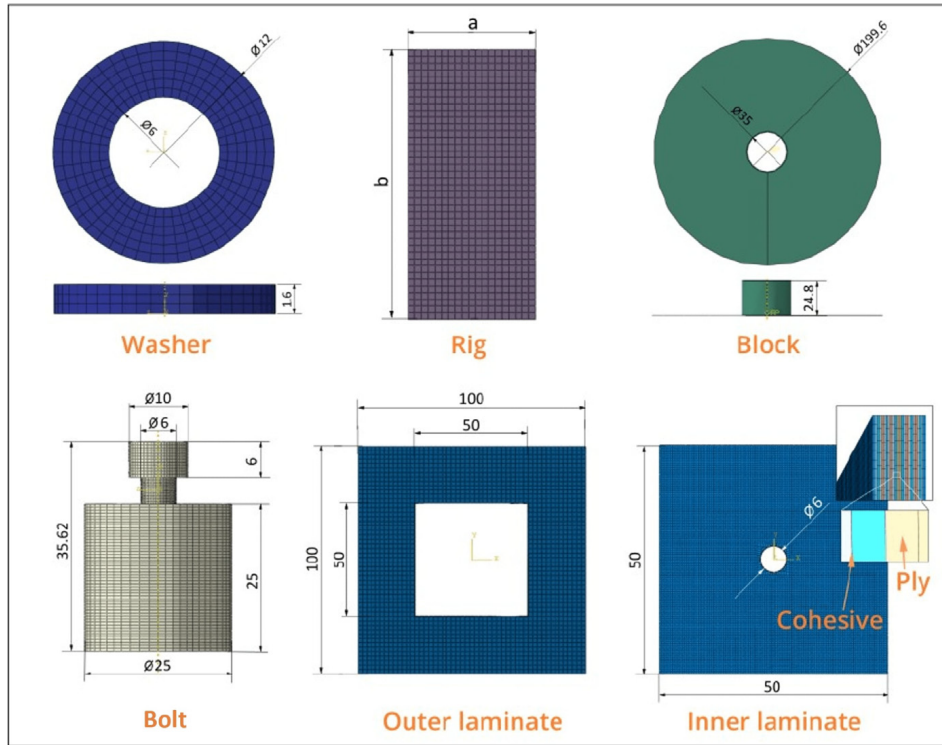


Figure 4.1: Model components. [3]

The inner laminate consists of a $50 \times 50 \text{ mm}^2$ composite layup that can be damaged, comprising solid plies linked to the adjacent cohesive layer through tie constraints. The outer laminate is modelled as a homogenised transversely isotropic linear elastic solid, with a coarser mesh used to reduce computational time. The final assembly is shown in Figure 4.2. To replicate the constraint effects of the four fastening bolts used in the experimental rig of Reference [3], the top edge of the outer laminate was fixed. The applied load was introduced through a smooth velocity imposed

along the y-direction at point A of the rig assembly, while point B was constrained in both the x and z directions to replicate the effect of a slotted pin support. Load transfer between the rig and the bolt was handled using coupling constraints. Specifically, a pull-through coupling connected the base of the bolt to point O on the rig, drawing the bolt through the laminate thickness. Simultaneously, bearing loads were transmitted through a separate coupling to the upper portion of the bolt shank, inducing a downward force. Surface interactions were governed by a contact definition that used 'Hard' contact in the normal direction and frictional behaviour tangentially, and finally, to emulate the experimentally applied fingertight torque of 2.2 Nm, a temperature differential step was introduced on one of the washers to simulate the preload effect.

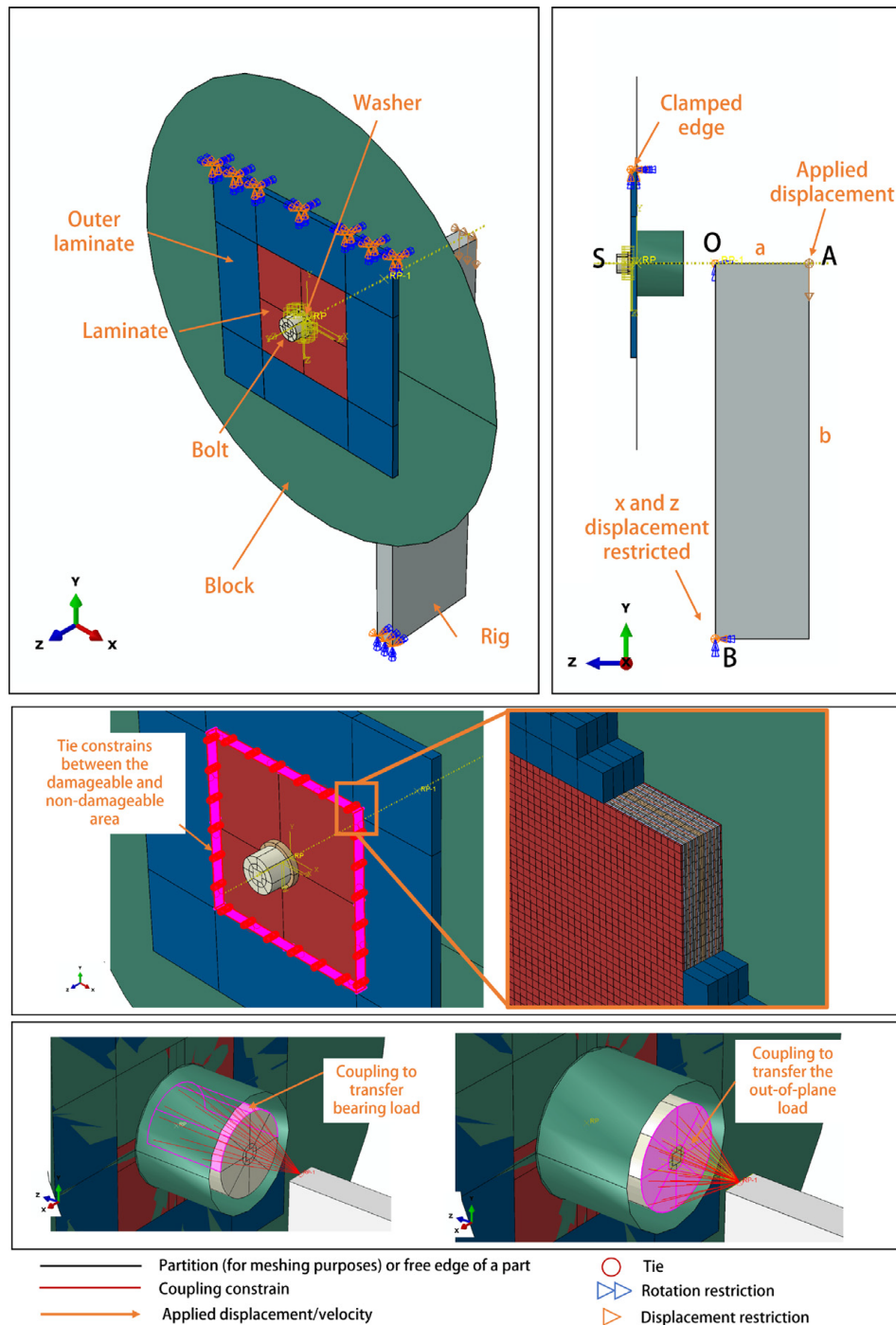


Figure 4.2: Assembly, boundary conditions and constraints of the mixed mode model. [3]

Table 4.1: IM7/8552 material properties.

Elastic			
E_1	[MPa]	171420	Refs. [51, 52]
$E_2 = E_3$	[MPa]	9080	Refs. [51–53]
$G_{12} = G_{13}$	[MPa]	5290	Refs. [51–53]
G_{23}	[MPa]	3290	Refs. [54, 55]
$\nu_{12} = \nu_{13}$	[-]	0.32	Refs. [51–53]
ν_{23}	[-]	0.487	Refs. [51, 52, 56]
Fracture			
G_{XT}	[kJ/m ²]	133.3	Ref. [56]
f_{GT}	[-]	0.3	Ref. [28]
G_{XC}	[kJ/m ²]	61	Ref. [57]
G_{YT}	[kJ/m ²]	0.28	Refs. [51, 52]
G_{SL}	[kJ/m ²]	0.79	Refs. [51, 52]
Strength			
X_T	[MPa]	2323.5	Refs. [51, 52]
f_{XT}	[-]	0.4	Ref. [58]
X_C	[MPa]	1200.1	Refs. [51, 52]
f_{XC}	[-]	0.2	Ref. [28]
Y_T (UD)	[MPa]	62.3	Refs. [51, 52]
Y_T (inner, t=0.125mm)	[MPa]	161	Note (1)
Y_T (inner, t=0.250mm)	[MPa]	114	Note (1)
Y_T (inner, t=0.125mm)	[MPa]	102	Note (1)
Y_C (UD)	[MPa]	253	Refs. [51, 52]
Y_C (inner, t=0.125mm)	[MPa]	312	Note (1)
Y_C (inner, t=0.250mm)	[MPa]	217	Note (1)
Y_C (inner, t=0.125mm)	[MPa]	266	Note (1)
S_L (UD)	[MPa]	92.3	Refs. [59–61]
S_L (inner, t=0.125mm)	[MPa]	118	Note (1)
S_L (inner, t=0.250mm)	[MPa]	114	Note (1)
S_L (inner, t=0.125mm)	[MPa]	97	Note (1)
S_T (UD)	[MPa]	92.3	Ref. [59]
S_T (inner, t=0.125mm)	[MPa]	161	Note (2)
S_T (inner, t=0.250mm)	[MPa]	114	Note (2)
S_T (inner, t=0.125mm)	[MPa]	101	Note (2)
Y_{BT} (UD)	[MPa]	38.7	Refs. [59–62]
Y_{BC} (UD)	[MPa]	600	Refs. [59–62]

(1) Calculated based on Ref. [63]

(2) Calculated based on Ref. [59]

Table 4.2: Interlaminar material properties.

Elastic			
K_n	[N/mm ³]	10 ⁶	Ref. [64]
Strengths			
τ_n	[MPa]	62.3	Refs. [51, 52]
τ_{sh}	[MPa]	92.3	Refs. [59–61]
Fracture			
G_{IC}	[kJ/m ²]	0.28	Refs. [51, 52]
G_{IIC}	[kJ/m ²]	0.79	Refs. [51, 52]
η_{B-K}	[-]	1.45	Refs. [65, 66]
Friction			
μ	[-]	0.4	Ref. [24]

4.1.2 Pure bearing model

To evaluate the pure bearing response, a numerical model replicating a double-lap shear test was developed. Although the general layout of the laminate, bolt, and washers remains consistent with that of the combined loading case, the loading and boundary conditions are adapted specifically for bearing. A smooth velocity is applied to two rigid blocks positioned on either side of the laminate, which restricts the bolt's motion to the x-direction only, as illustrated in Figure 4.3.

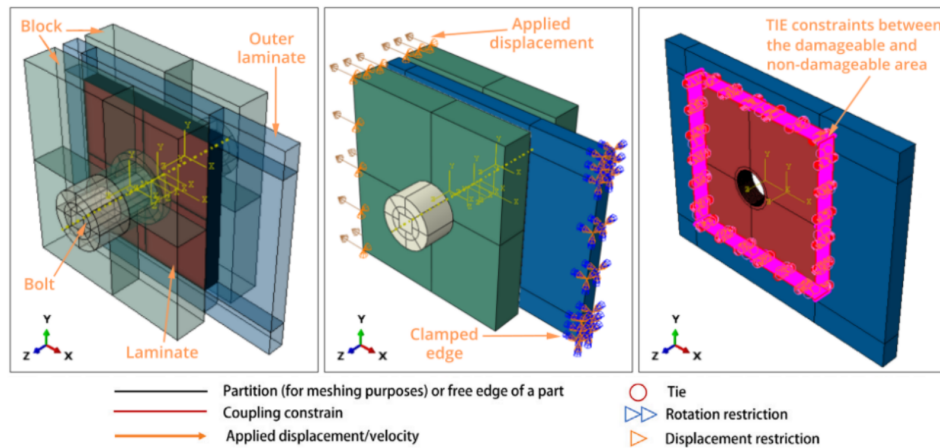


Figure 4.3: Pure bearing model, boundary conditions and constraints. [3]

The model preserves the mesh density, material properties, and geometric features used previously to ensure comparability across loading scenarios. The central region of the laminate, subject to damage, measures 27×27 mm². Around this, a 4.5 mm margin is included, with an additional 7 mm outer strip to enable edge clamping. The surrounding blocks measure 36×36×6 mm³ and contain bolt holes with a diameter of 6 mm, aligned with those in the laminate. Washers are placed between the blocks and the laminate on both sides, retaining the same dimensions as the combined model. The bolt head and nut are modelled with a diameter of 10 mm and a thickness of 6 mm.

4.1.3 Pure pull-through model

The numerical model developed for the pure pull-through case builds upon the same framework as the combined loading model, with a few key modifications. Most notably, the rig component was removed to simplify the setup, and the loading was applied directly to the bolt along the z-axis using a smooth velocity input, as shown in Figure 4.4.

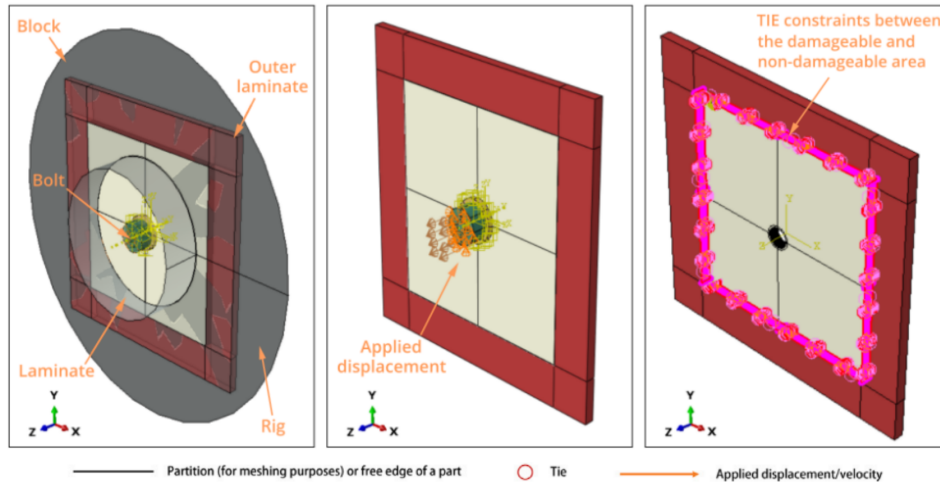


Figure 4.4: Pure pull-through model, boundary conditions and constraints. [3]

The inner laminate used in the model measures $60 \times 60 \text{ mm}^2$, while the surrounding outer laminate extends to $84 \times 84 \text{ mm}^2$. A 50 mm clearance is incorporated into the setup to accommodate the movement of the bolted connection during loading. Washers are placed on both sides of the laminate, and their dimensions are consistent with those used in previous configurations. Similarly, the bolt head and nut maintain the same geometry as in the double-lap shear test model.

The high-fidelity models were validated for an IM7/8552 quasi-isotropic laminate, $[90/0/\pm 45]_3S$, with the material properties detailed in Tables 4.1 and 4.2, against experimental data to ensure their accuracy in predicting the mechanical behaviour of bolted joints under various loading conditions [3]. By evaluating the loading ratios during the simulations, the authors in Ref. [3] found that the imposed ratio value was achieved after an initial stabilisation process, which indicated that the boundary conditions were correctly defined. Additionally, the ultimate failure loads were found to be in error ranges from -10.7% to 9.9%, which revealed excellent agreement for all test cases. However, modelling a 3D model with damage in Abaqus leads to excessive computation times, which motivated the authors from Reference [8] to develop simplified multi-bolt models in an attempt to reduce computational time while obtaining accurate results. It is worth highlighting that although the models were verified for the here mentioned quasi isotropic laminate, the quad laminate in study and used to obtain double-double in-plane equivalence is the one mentioned in Chapter 3, a directional $[45/90/-45/0/0/45/0/0/-45/0]_S$.

4.2 Structured meshing

Using a structured mesh that is aligned with the fibre orientation is crucial in composite modelling, particularly when damage initiation and propagation are key outputs [24]. Structured meshes help ensure uniform element sizes and shapes, which improves numerical stability and reduces mesh-induced variability in the results. Additionally, aligning the mesh with the fibre direction enhances the accuracy of stress and strain predictions along the primary load-bearing paths, which is essential for correctly capturing anisotropic behaviour and intralaminar damage mechanisms in composite materials [24].

The original high-fidelity models developed in Ref. [3] relied on structured meshing techniques that were only feasible for laminate orientations of 0° , 45° , and 90° , as shown in Figure 4.5.

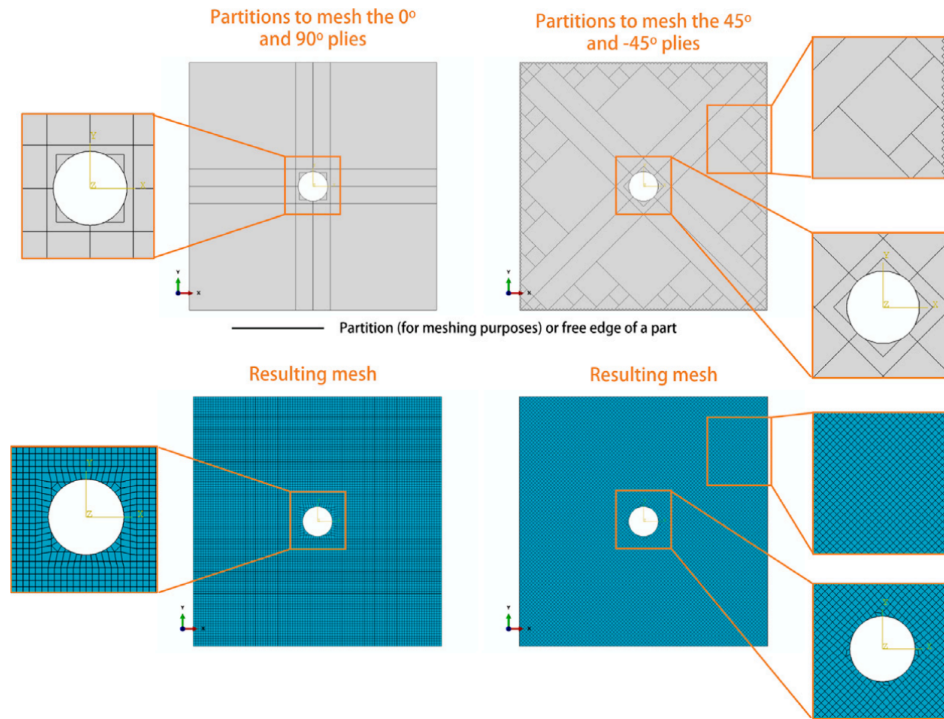


Figure 4.5: Previously developed meshing strategy. [3]

However, for arbitrary angles, the geometry and mesh alignment become more complex, and the existing modelling strategy cannot produce a structured mesh while maintaining mesh quality and consistency across different simulations. To overcome this limitation, a new methodology was developed to enable structured meshing for any fibre orientation angle.

The concept of this approach is based on a chessboard technique. In this method, an initial square region is created concentrically around the notched hole, serving as the innermost partition, then, from this central square, a series of successive square layers are constructed outwards. Each of these squares can be individually partitioned and seeded, allowing full control over element distribution and alignment. The partitioning is shown in Figure 4.6, and the resulting mesh can be seen in Figure 4.7.

This new meshing strategy offers an improvement over the previous approach by enabling the generation of high-quality structured meshes for any fibre orientation angle. By leveraging the flexibility of the chessboard-based partitioning method, it is now possible to maintain mesh alignment with the fibre direction regardless of the ply angle, ensuring consistency across simulations. This enhancement not only preserves the benefits of structured meshing, such as numerical stability and accurate stress prediction, but also extends the applicability of the high-fidelity modelling framework to a wider range of laminate configurations.

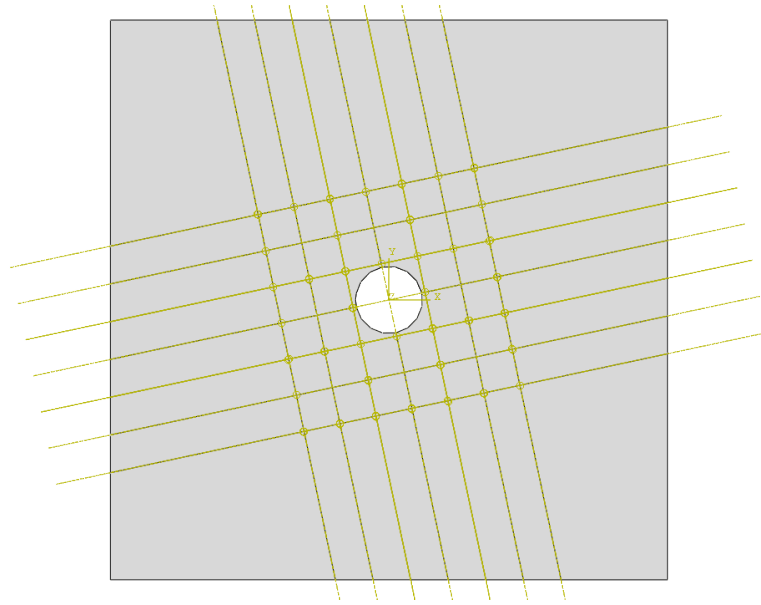


Figure 4.6: Example of the partitioning for a 12° ply.

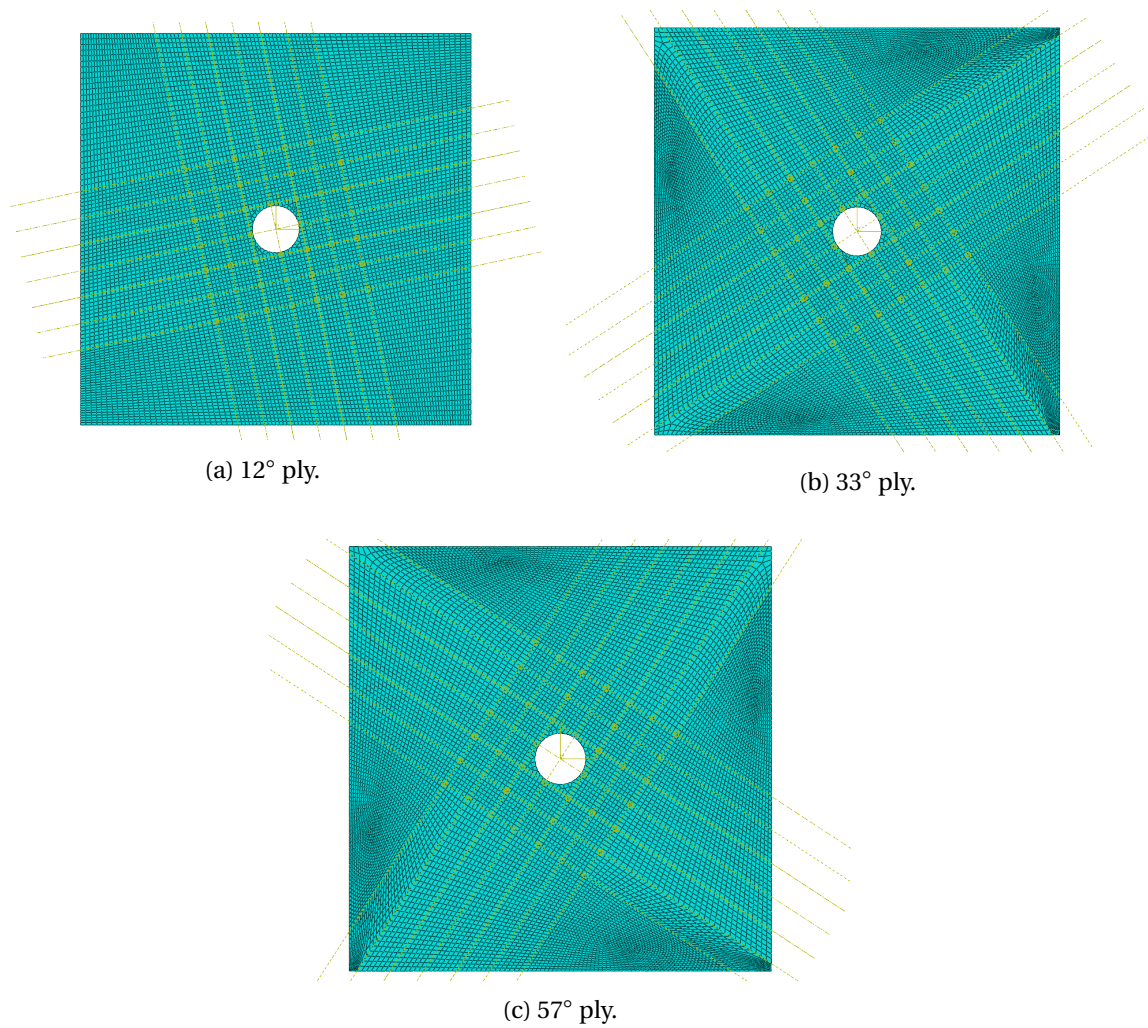


Figure 4.7: Example of the resulting meshes for different ply angles.

4.2.1 Comparison of the results with and without structured meshing

Although the literature recommends using a structured mesh, the difference between structured and unstructured meshing strategies is assessed here by comparing two failure load values for the double-double layup. A new simulation under pure bearing loading was performed using an unstructured mesh, and its load-displacement curve was compared to that of the structured mesh, as shown in Figure 4.8.

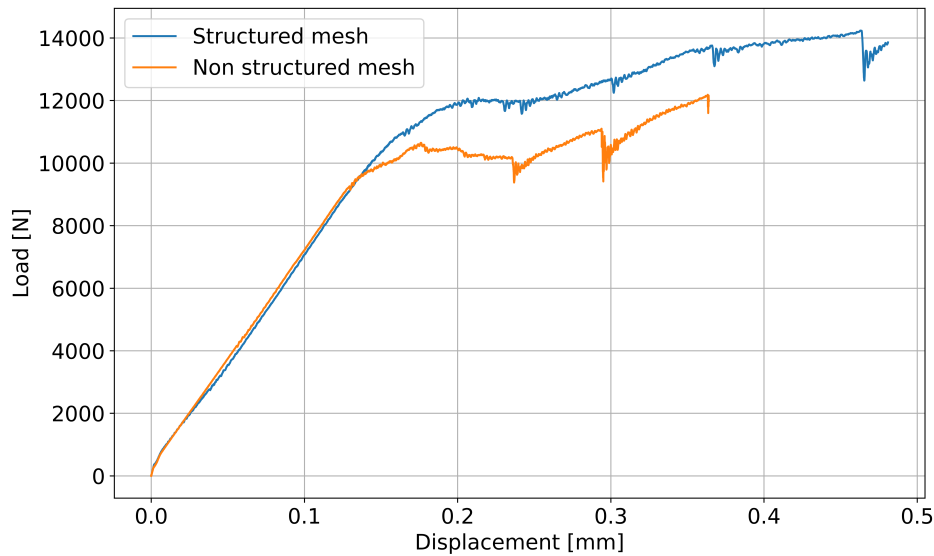


Figure 4.8: Load-displacement curve comparison for a pure bearing simulation using structured and unstructured meshes.

The first load drop observed in the unstructured mesh model was 11.88% lower than that of the structured mesh model. While there is no experimental data for the double-double laminate, studies on the impact of mesh alignment on quad laminate behaviour indicate that unstructured meshes tend to underpredict the first failure load. As a similar trend is observed for the double-double laminate and aligned meshing is recommended in the literature, all subsequent models in this study will adopt the newly developed aligned meshing strategy.

4.3 High fidelity results for both quad and double-double layups

As a final step before obtaining the failure envelope, all the high-fidelity simulations were run. This included pure loading cases of bearing and pull-through, as well as four different combined loading ratios, ranging from 0.2 to 0.8. The anisotropy of composite laminates leads to different mechanical behaviour in the two principal in-plane directions. The highest stiffness and strength is achieved by loading in the direction of the fibre due to their superior properties, while the lowest is obtained when the load is primarily carried by the matrix, transversal bearing. Therefore, a 3D envelope is needed to account for the laminate behaviour in the 3 main directions, longitudinal bearing, transversal bearing and pull through. This was obtained by rotating the layup definition. This resulted in a total of 17 different simulations, 6 for the default laminate, 6 for the 90° rotated layup, and 3 additional simulations were run with the layup rotated 45° to cover the load ratios from 0.2 to 0.8, thereby obtaining the points in the middle of the envelope. The 0.2 load ratio case with the 45° rotation, however, encountered numerical issues and could not be completed, preventing its inclusion. Additionally, the pure bearing model was also rotated 30° and 60° to obtain the connecting points between the longitudinal and transversal bearing cases. The first load drop was defined as

the point when the load decreased by more than 10%. The results of the load-displacement curves for the double-double simulations plotted against the quad laminate runs can be seen in Figure 4.9 and the final layups after rotation, together with their respective load ratios, are shown in Table 4.3.

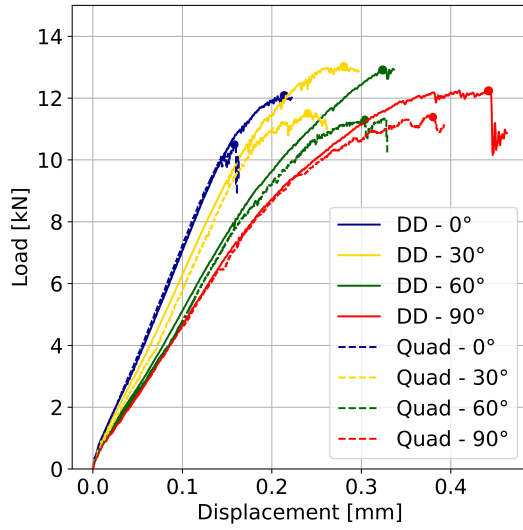
Table 4.3: Directional layups and respective loading conditions. In the "Plane" column, "B₀" designates longitudinal bearing, "B₉₀" represents transversal bearing, and "PT" stands for pull-through.

Rotation angle	Layup	Load ratios	Plane
Original layup	[57/-12/0/12/-57] _{4T}	0.0, 0.2, 0.4, 0.6, 0.8, 1.0	B ₀ /PT
30°	[87/18/30/42/-27] _{4T}	1.0	B ₀ /B ₉₀
45°	[102/33/45/57/-12] _{4T}	0.4, 0.6, 0.8	-
60°	[117/48/60/72/3] _{4T}	1.0	B ₀ /B ₉₀
90°	[147/78/90/102/33] _{4T}	0.0, 0.2, 0.4, 0.6, 0.8, 1.0	B ₉₀ /PT

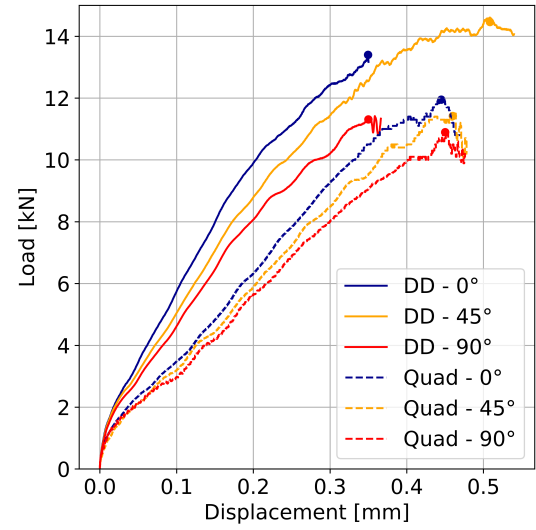
Starting with the pure-bearing load case, $\eta = 1.0$, we can observe a similar trend for all four layup rotations. Compared to the quad, the double-double layup achieves a relatively higher first load drop for larger displacement values, and the stiffness is similar to that of the quads for all rotations. For load ratios of 0.8 and 0.6, the stiffness for the double-double laminates is higher, with the peak load being also relatively higher. However, this is achieved at lower displacement values than those obtained for the traditional material. This may be explained due to the fact that, as previously explained, the layup was obtained for in-plane equivalence, and, as pull-through loading begins, a higher disparity is found between quad and DD simulations as there is no out-of-plane equivalence. However, when the loading scenarios become more pull-through dominated, at the 0.4 and 0.2 ratios, the curves and peak loads are almost identical to those of the quad layup, with almost no difference in peak load being identified. Finally, in the pure pull-through scenario, the stiffness is the same, although the double-double simulation achieved a higher peak load at a higher displacement value. This suggests that the double-double layup may offer advantages under bearing-dominated conditions, but its benefits diminish as the loading becomes more pull-through dominated. It is worth noting that bearing failure for bolted joints is preferable since it is a more progressive failure mode as opposed to pull-through failure, which is catastrophic.

In addition to the trends observed in stiffness and peak load, it is worth highlighting that the double-double (DD) laminates exhibit a more gradual and sustained crack evolution when compared to the traditional quad layup. Across all loading scenarios, the damage in the DD configurations tends to propagate more stably, with fewer sudden load drops and less pronounced structural degradation. This indicates that failure in DD laminates is generally less catastrophic, allowing for better energy absorption and offering increased damage tolerance.

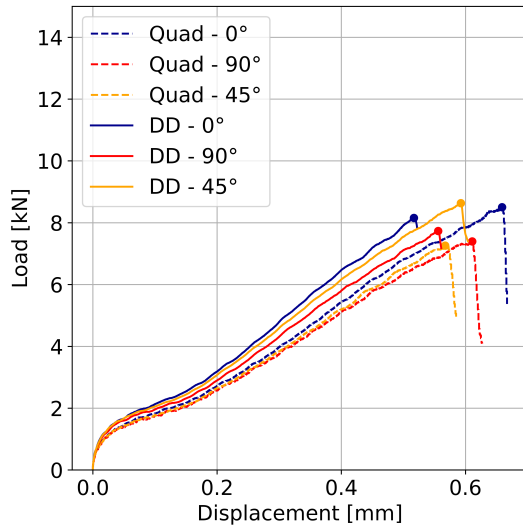
A bar plot with a comparison between the different first ply drop load values between quad and dd layup is shown in Figures 4.10 and 4.11. As stated above, the double-double laminate experienced a high load value for every case apart from $\eta = 0.6$ with the default rotation. However, for $\eta = 0.4$ and $\eta = 0.2$, both also with the default rotation, the difference between the loads is marginal. Since there was a numerical problem with the $\eta = 0.2$ model rotated 45°, the load will not be compared here.



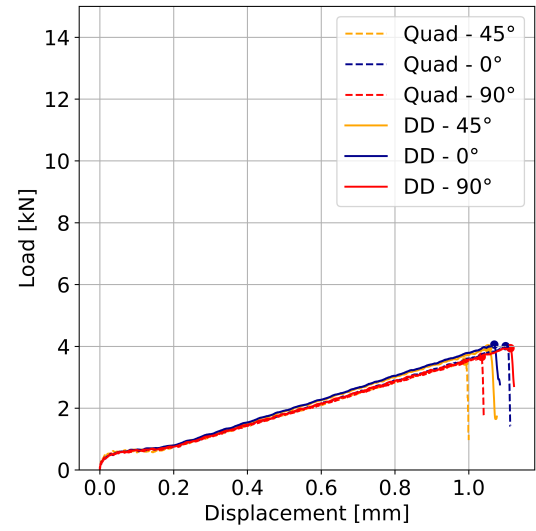
(a) $\eta = 1.0$



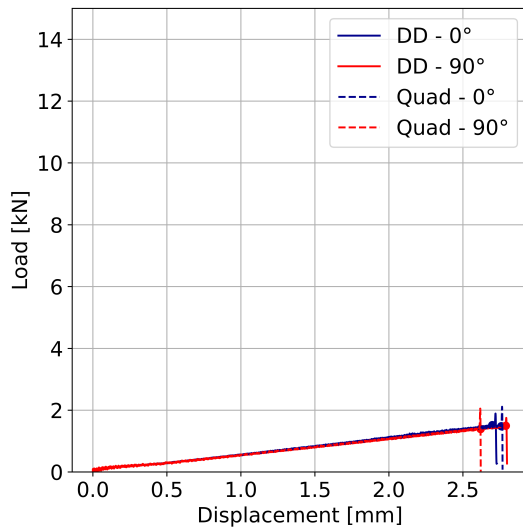
(b) $\eta = 0.8$



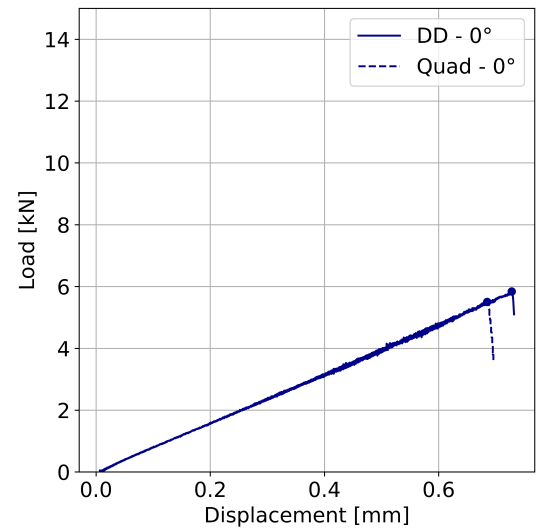
(c) $\eta = 0.6$



(d) $\eta = 0.4$



(e) $\eta = 0.2$



(f) $\eta = 0.0$

Figure 4.9: Numerical load-displacement curves for all load ratios, comparing both Quad [4], and DD layups.

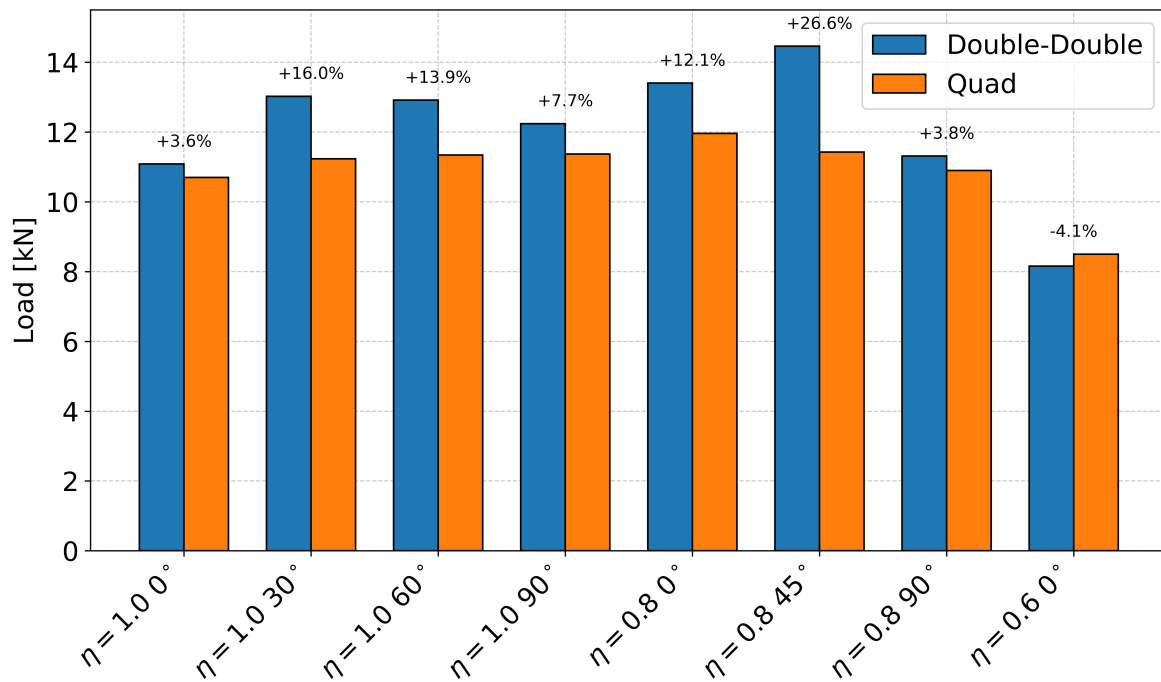


Figure 4.10: Critical load comparison part 1.

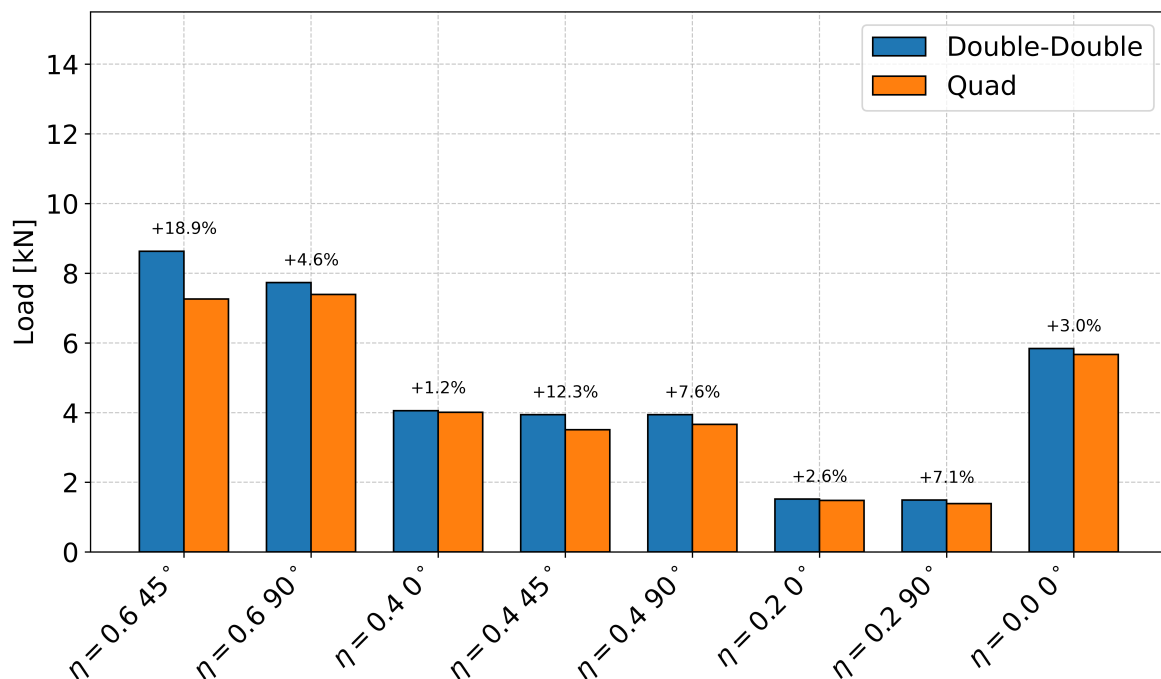


Figure 4.11: Critical load comparison part 2.

4.4 Failure ellipsoid

Using the procedure and the load values detailed in Section 4.3, the failure ellipsoid of the double-double laminate can be defined. This failure envelope will be used later applied in the simplified

models to aid composite design by providing a prediction of the failure load. In this section, the previously developed in Ref. [4] quad laminate failure envelope is shown, followed by the one obtained here for the DD layout, as well as a comparison between them.

4.4.1 Quad laminate failure envelope

By using the high-fidelity models referenced in Section 4.1 to simulate 18 loading cases for a directional $[45/90/-45/0/0/45/0/0/-45/0]_S$ laminate manufactured in Hexcel's IM7/8552 carbon fibre/epoxy material with a cured ply thickness of 0.125 mm and with varying bearing/pull-through load ratios and bearing load directions, the authors obtained the failure load predictions at first significant load drop ($>10\%$). The 18 load cases were obtained by the 6 different load ratios, from 0.0 to 1.0 and by rotating the layout 30° , 45° , 60° and 90° , as shown in Table 4.3.

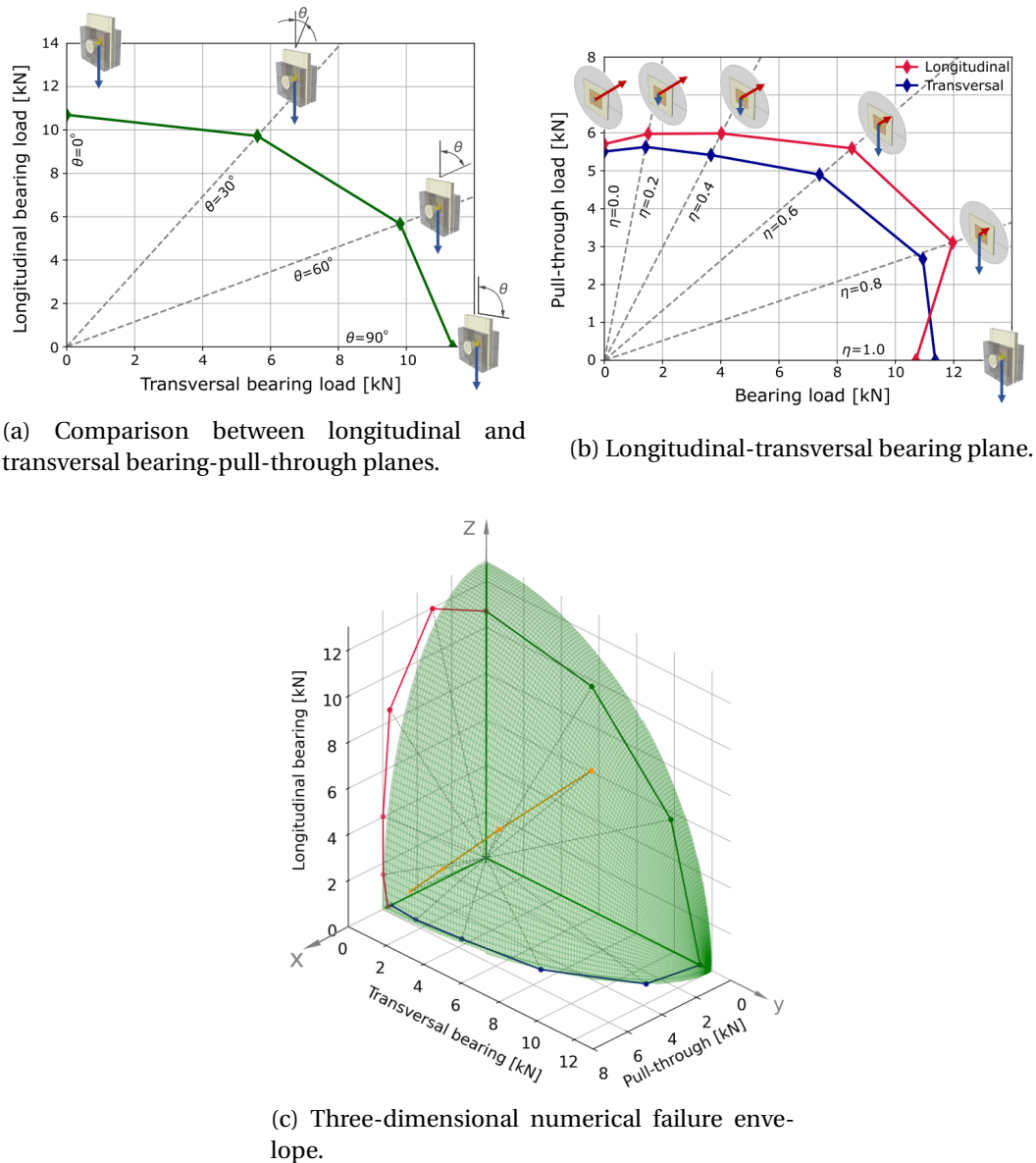


Figure 4.12: Numerically determined failure envelope of the $[45/90/-45/0/0/45/0/0/-45/0]_S$ IM7/8552 laminate.

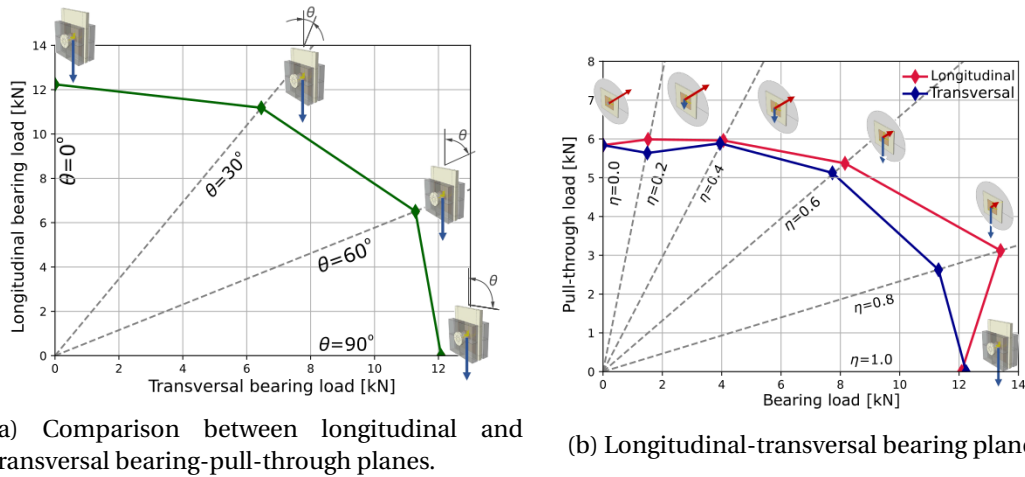
These predictions served as the basis for deriving the two-dimensional failure envelopes in the

three principal planes, ultimately leading to the formulation of the three-dimensional ellipsoidal failure criterion shown in Figure 4.12 with the equation that fits the numerical data given by Equation 4.2 where the pull-through, longitudinal bearing, and transversal bearing failure loads are represented by f_{PT} , f_{PB_0} and $f_{PB_{90}}$, respectively. This failure envelope will be used as a failure criterion for the quad simplified multi-bolt models.

$$\left(\frac{f_{PB_{90}}}{11.786}\right)^2 + \left(\frac{f_{PT}}{5.799}\right)^2 + \left(\frac{f_{PB_0}}{12.737}\right)^2 = 1 \quad (4.2)$$

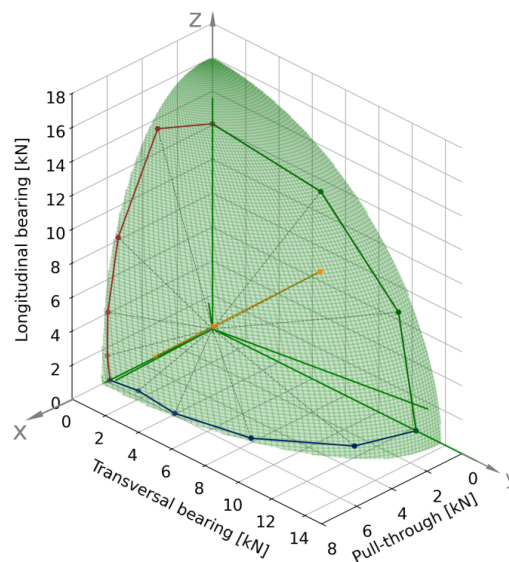
4.4.2 Double-double laminate failure envelope

For the double-double laminate, using the same methodology previously established and with the results detailed in Section 4.3, the failure envelope resulted in Equation 4.3, used as failure criterion for the double-double simplified models, and plotted on Figure 4.13. As expected, since the failure loads are generally higher, the parameters of the ellipse are also higher when compared to the ones from the quad failure envelope.



(a) Comparison between longitudinal and transversal bearing-pull-through planes.

(b) Longitudinal-transversal bearing plane.



(c) Three-dimensional numerical failure envelope.

Figure 4.13: Numerically determined failure envelope of the $[57/-12/0/12/-57]_{4T}$ IM7/8552 laminate.

$$\left(\frac{f_{PB_{90}}}{13.600}\right)^2 + \left(\frac{f_{PT}}{6.121}\right)^2 + \left(\frac{f_{PB_0}}{15.965}\right)^2 = 1 \quad (4.3)$$

Since in a three-dimensional view the differences may not be so evident, the three different projections are shown in Figure 4.14, compared against the quad envelope projections. Figure a) represents the red curve, b) the green and c) the blue curve. In general, the first load drop of the double-double laminate has higher values of longitudinal bearing, transversal bearing and pull through. It is worth mentioning that this new failure ellipsoid will be used to assess the failure of the simplified multi-bolt models, further in this thesis.

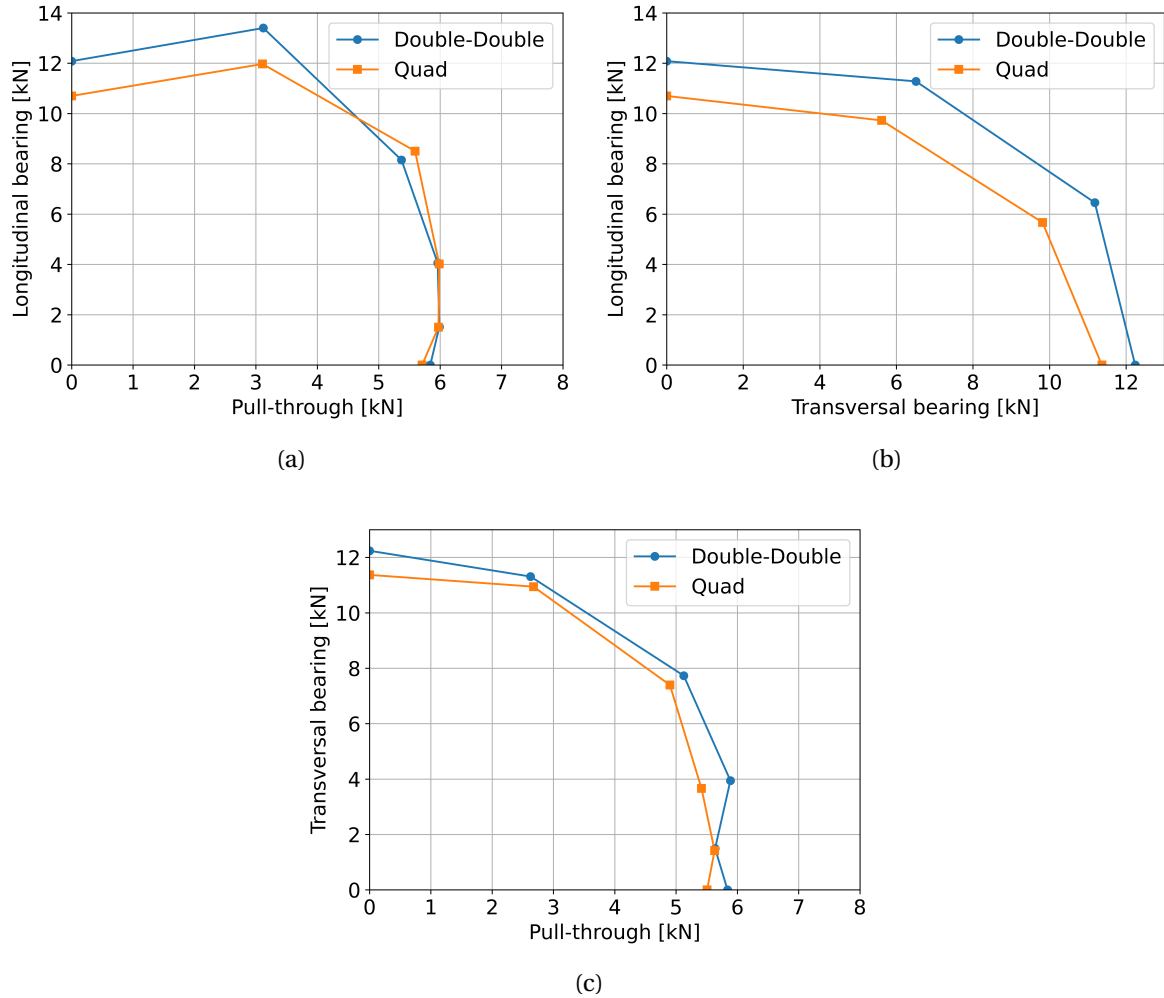


Figure 4.14: Different failure envelope projections.

Chapter 5

Uncertainty Quantification Method for Failure Prediction in Simplified Multi-Bolt Models

This chapter presents a methodology for uncertainty quantification applied to simplified multi-bolt models, aiming to predict joint failure under different input conditions. The purpose of this approach is to assess the reliability and predictive capability of these models when subject to variations in the inputs. This methodology is required to validate the simplified multi-bolt models previously developed in Ref. [4]. These models aim to correctly predict the failure load of composite material joints while significantly reducing the computational power required. Not only in terms of time but also regarding memory, high-fidelity simulations are very computationally expensive; thus, the need to find simplified yet reliable ways to model multi-bolt joints is crucial.

It begins with an explanation of the modelling strategy and an analysis of the models for both quad and double-double layups with deterministic properties, followed by a baseline sensitivity analysis to evaluate the effects of different properties and determine the required number of simulations for statistical representation. Finally, the chapter discusses the results of the uncertainty quantification process for both quad and double-double laminates, highlighting the key sources of variability and quantifying their impact on predicted failure loads. The findings are used to assess the robustness and practical applicability of the simplified models under uncertain conditions, thus providing confidence in their use.

5.1 Simplified multi-bolt modelling strategy

As shown in Figure 5.1, seven different multi-bolt configurations were used [4], including a single-bolt case, 2 and 3 bolts in length, 2 bolts in width, and 2 bolts in length and width, as well as pure bearing and pure pull-through configurations.

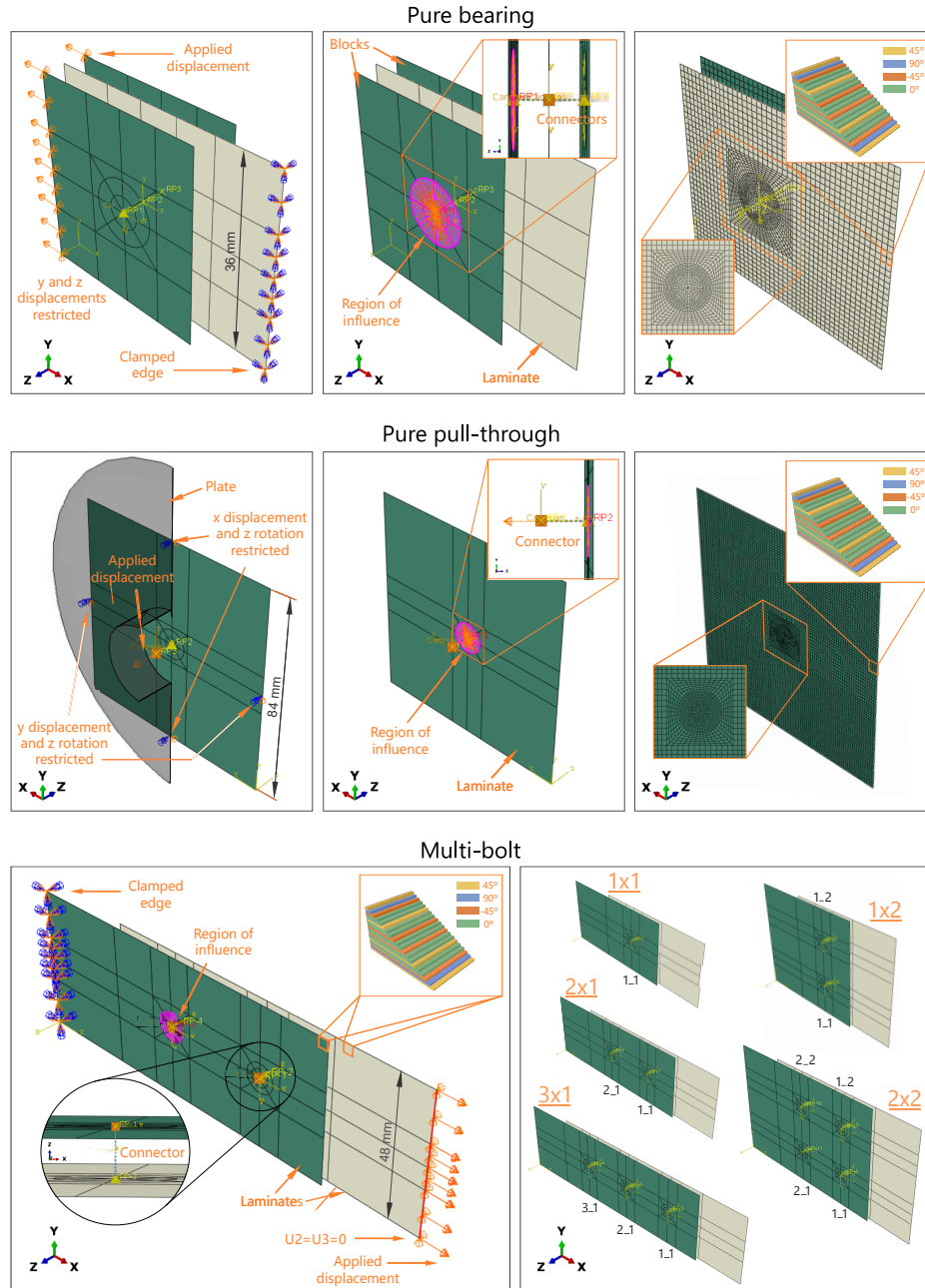


Figure 5.1: Simplified multi-bolt connector models. [8]

The simplified pure bearing model consists of a composite laminate measuring $36 \times 43 \text{ mm}^2$, along with two steel plates of $36 \times 36 \times 6 \text{ mm}^3$. All components are represented using four-node shell elements (S4R) with reduced integration, each having a size of $1 \times 1 \text{ mm}^2$, resulting in a total of 5903 elements. The composite layup is defined based on the elastic properties of IM7/8552, which are summarised in Table 4.1. As in the high-fidelity model, the composite laminate's outer edge is fully constrained, while a displacement of 1 mm is imposed on both steel plates in the direction of the joint, exceeding the maximum displacement observed in high-fidelity simulations from the previously developed work in Reference [3]. A value of 1 mm was chosen to exceed the maximum displacement observed in the high-fidelity simulations. This facilitates post-processing and ensures that the connector force values intersect the failure criteria. Contact between overlapping laminate regions is modelled with a 'Hard' contact normal behaviour. Unlike in the high-fidelity model, the

steel-composite friction coefficient was omitted from this analysis and instead incorporated during the connector stiffness calibration step. The steel plates are linked to the composite laminate using Abaqus-defined connector elements attached to circular regions of influence, with a diameter corresponding to the washer area from the high-fidelity model, ensuring an accurate representation of the clamping effect.

The model designed to simulate pure pull-through conditions features a rigid analytical shell rig with a 35 mm clearance hole and a shell laminate measuring $84 \times 84 \text{ mm}^2$, discretised into 7372 S4R elements. To prevent unintended movement, the rig is fully clamped, while the composite plate is constrained at the midpoints of its edges to restrict rotation around the z-axis and translation in the x- and y-directions. A 1 mm out-of-plane displacement is applied along the axis of the connector element at its free edge.

Finally, the simplified multi-bolt laminates consist of two composite laminates arranged in the specified multi-bolt configurations. These are modelled as planar shells using between 7872 and 18,248 linear elastic S4R elements, depending on the configuration (1×1 or 2×2 , respectively).

Since these models are using Abaqus-defined cartesian connector elements, an a priori determination of the connector stiffness is required. In Ref. [8], the authors concluded that the available formulas for different laminates and joint geometries were poorly understood, so they obtained the connector stiffnesses through an iterative calibration procedure until the slopes of the high-fidelity simulations for pure bearing and pure pull-through were matched. This resulted in 49500 N/mm and 18500 N/m for the pure bearing and pure-pull through, respectively. The authors from Ref. [8] compared these values against high-fidelity multi-bolt models, and the results were found to be in great agreement. In this thesis, the simplified models using the double-double layup were also compared in terms of stiffness against the DD high-fidelity results, revealing great agreement, so the connector stiffness values were maintained. However, the definition connector stiffness was made considering a linear elastic behaviour, which is a simplification. A test using a plastic behaviour is shown in Appendix B.

Using 8 CPUs, simulation times ranged from 60 seconds for the simpler 1×1 model to 370 seconds for the more complex 2×2 simplified model. Once all simulations were completed, the dataset was processed to extract key quantities, including load, displacement, and connector forces (CTF1, CTF2, and CTF3), used to obtain the bolt ratio, computed as $CTF1/(CTF1 + CTF2 + CTF3)$. Since the value of transversal bearing load is negligible, this ratio is comparable to the η ratio shown for the high-fidelity simulations. Additionally, the intersection with the failure ellipsoid was identified, marking the point of failure. The full post-processing procedure is shown in Figure 5.2. It starts with the extraction of the connector force curves, which are then used to intersect the previously obtained failure envelope so that bolt loads at failure are obtained and then, with the displacement at failure, the force in the load direction at failure, RF1, is obtained.

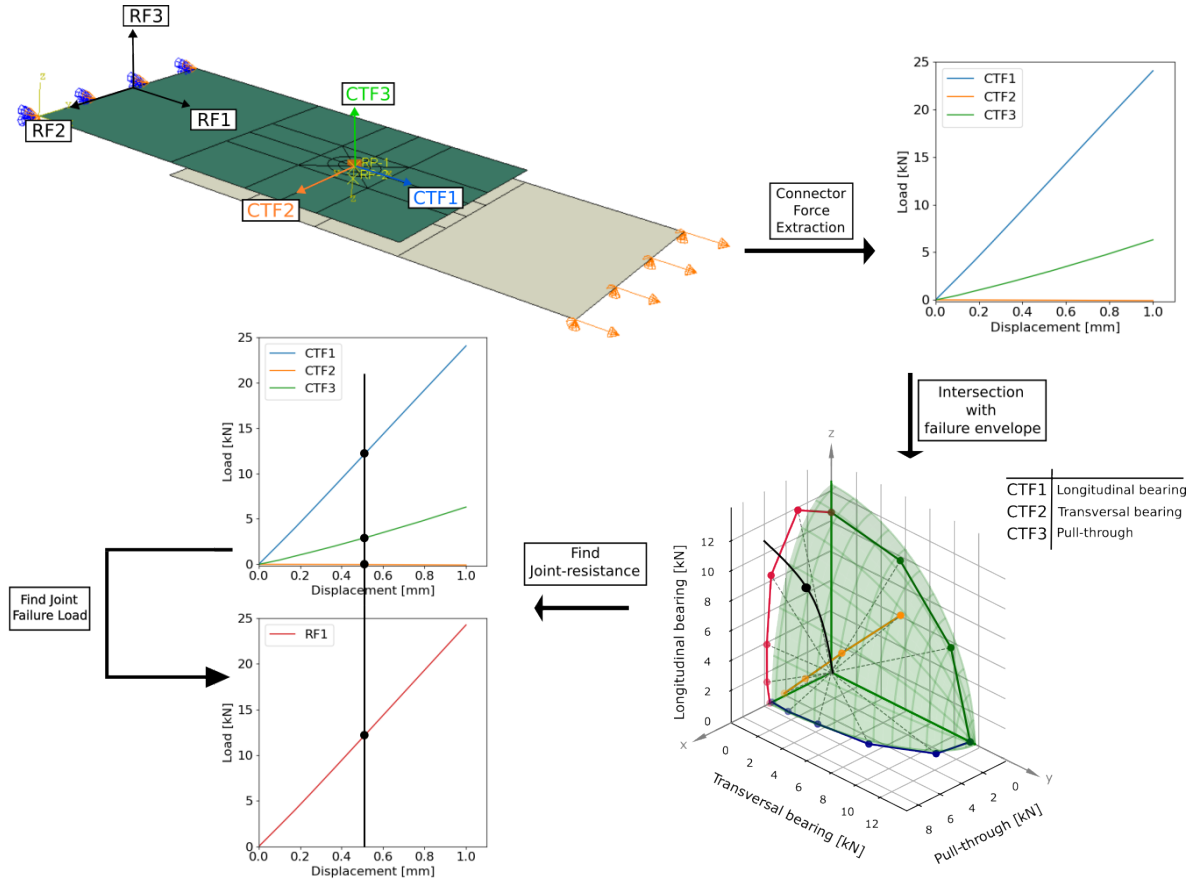


Figure 5.2: Post-process procedure.

5.2 Simplified multi-bolt models with deterministic properties

Before starting the process of uncertainty quantification, the simplified multi-bolt models were tested with the default material properties to compare the differences in the ultimate failure load induced by the layup. Figure 5.3 presents the load-displacement curves and corresponding failure loads for each model, with percentage comparisons between the layups.

For the same model, the stiffness remained consistent between layups. As expected, the double-double (DD) layup exhibited a higher failure load across all bolt configurations. The largest difference was observed in the 2×1 model, where the DD layup achieved a failure load 26.2% higher. In contrast, the smallest margin was found in the pure pull-through model, with the DD layup being only 5.5% superior.

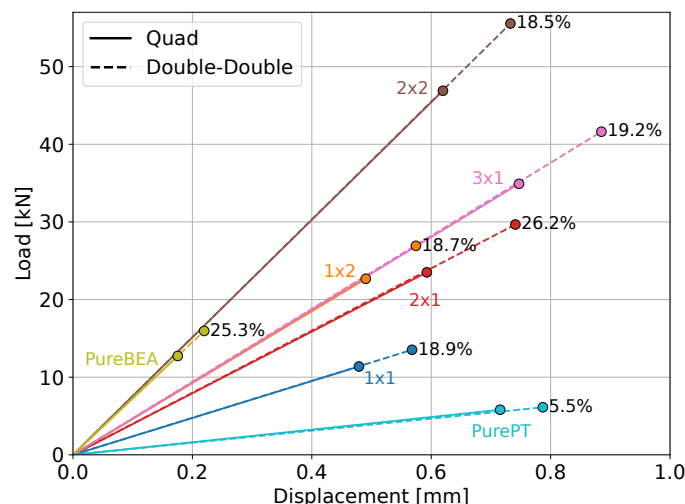


Figure 5.3: Load-displacement curves for the different simplified multi-bolt models with quad and double-double layouts.

5.3 Sensitivity analysis

As a first step in the uncertainty quantification process, a baseline sensitivity analysis was performed on the 1x1 model with the quad layout, to assess the influence of different parameters on the system response, summarised in Figures 5.4, 5.5 and 5.6. This study is very important since it provides an insight into what to expect in terms of the dominant parameters influencing failure, helping to refine the focus of subsequent analyses.

5.3.1 Individual parameters contribution

This study was conducted using the quad laminate layout, which is then intersected with the previously obtained in Ref. [4] failure envelope, and it served as a baseline comprehension on which of the 12 model properties that can change, Table 5.1 has a bigger effect on the final result.

To develop this study, 3 different models were run with only one of the properties changing within a range of $\pm 20\%$ from its base value, as shown in Table 5.1, with all of the different parameters remaining in the default value. The effects on the failure load, stiffness curve and bolt ratio are shown in Figures 5.4, 5.5 and 5.6, respectively. It is worth mentioning that the connector stiffnesses C_1 , C_2 and C_3 are the values in the longitudinal, transversal and pull-through directions, respectively. The $radii_1$, $radii_2$ and $radii_3$ are the failure envelope radii in the transversal, pull-through and longitudinal directions, respectively.

Table 5.1: Properties variation for the baseline sensitivity analysis.

	C_1 [N/mm]	C_2 [N/mm]	C_3 [N/mm]	E_1 [MPa]	E_2 [MPa]	G_{12} [MPa]
-20%	39600	39600	14800	137136	7264	4232
Base value	49500	49500	18500	171420	9080	5290
+20%	59400	59400	22200	205704	10896	6348

	G_{23} [MPa]	$\text{rad}ii_1$ [-]	$\text{rad}ii_2$ [-]	$\text{rad}ii_3$ [-]	ν_{12} [-]	ν_{23} [-]
-20%	3136	9.4288	4.6392	10.1896	0.256	0.3896
Base value	3920	11.786	5.799	12.737	0.32	0.487
+20%	4704	14.1432	6.9588	15.2844	0.384	0.5844

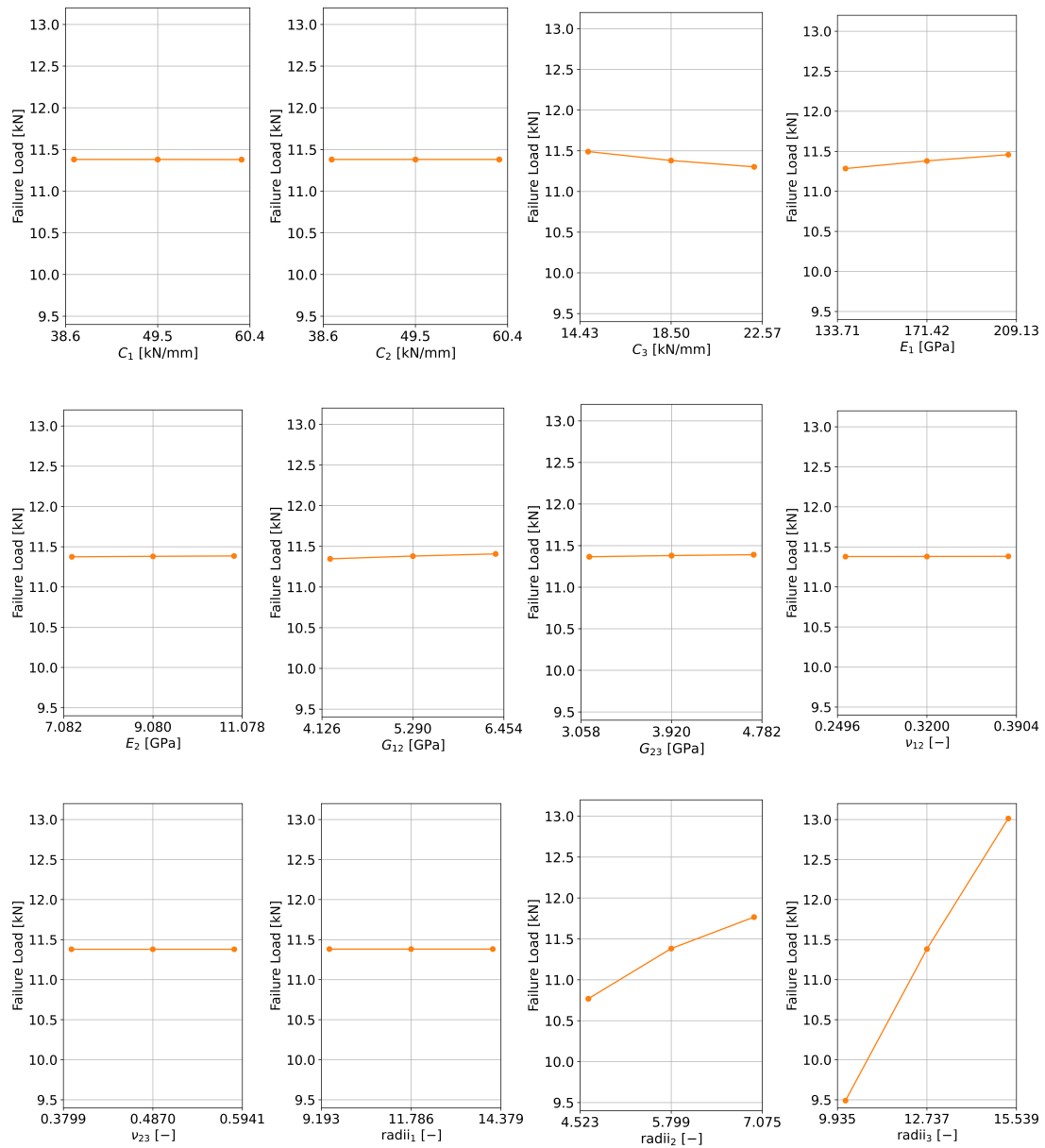


Figure 5.4: Influence of the different parameters on the failure load.

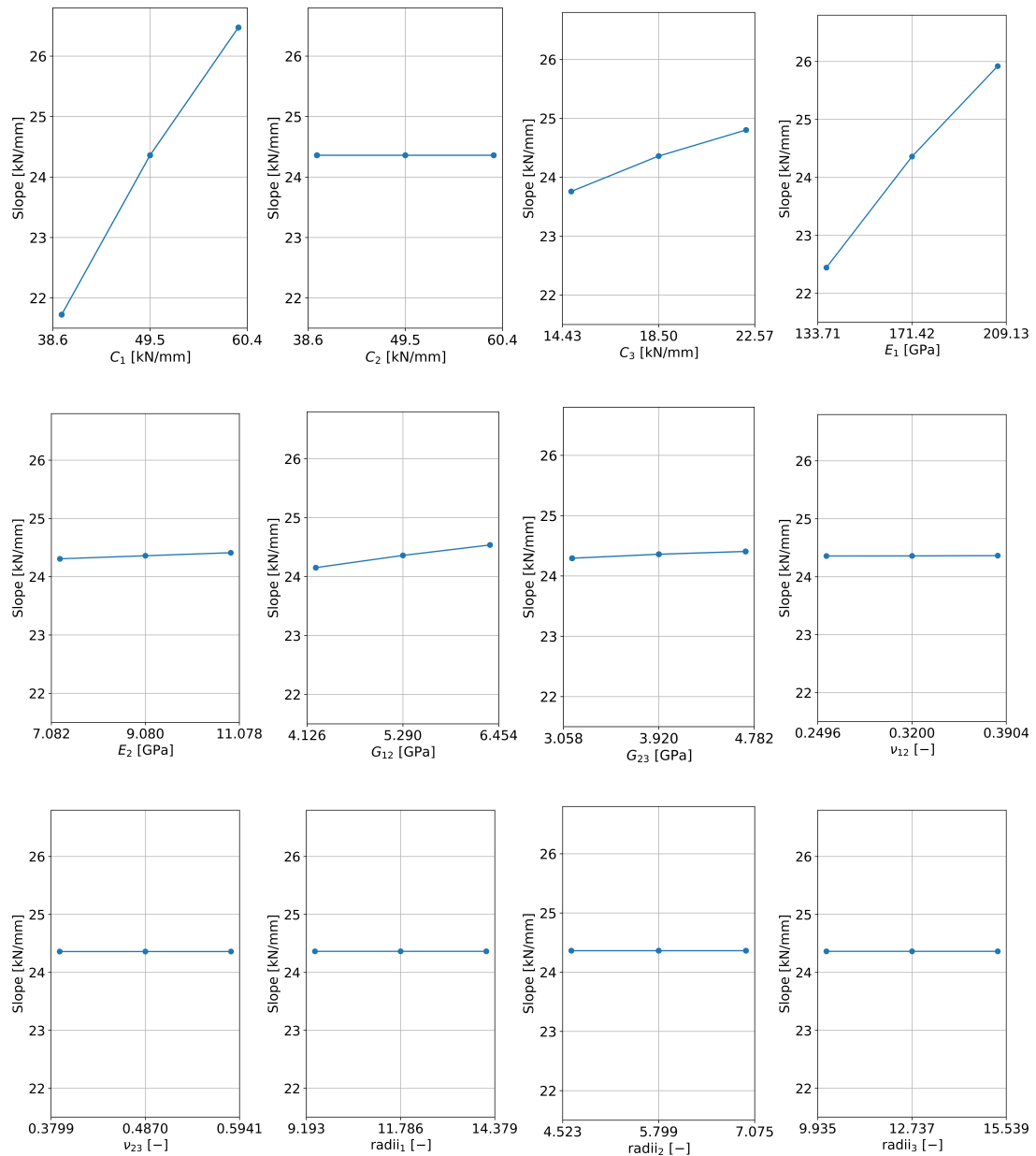


Figure 5.5: Influence of the different parameters on the slope of the load-displacement curve.

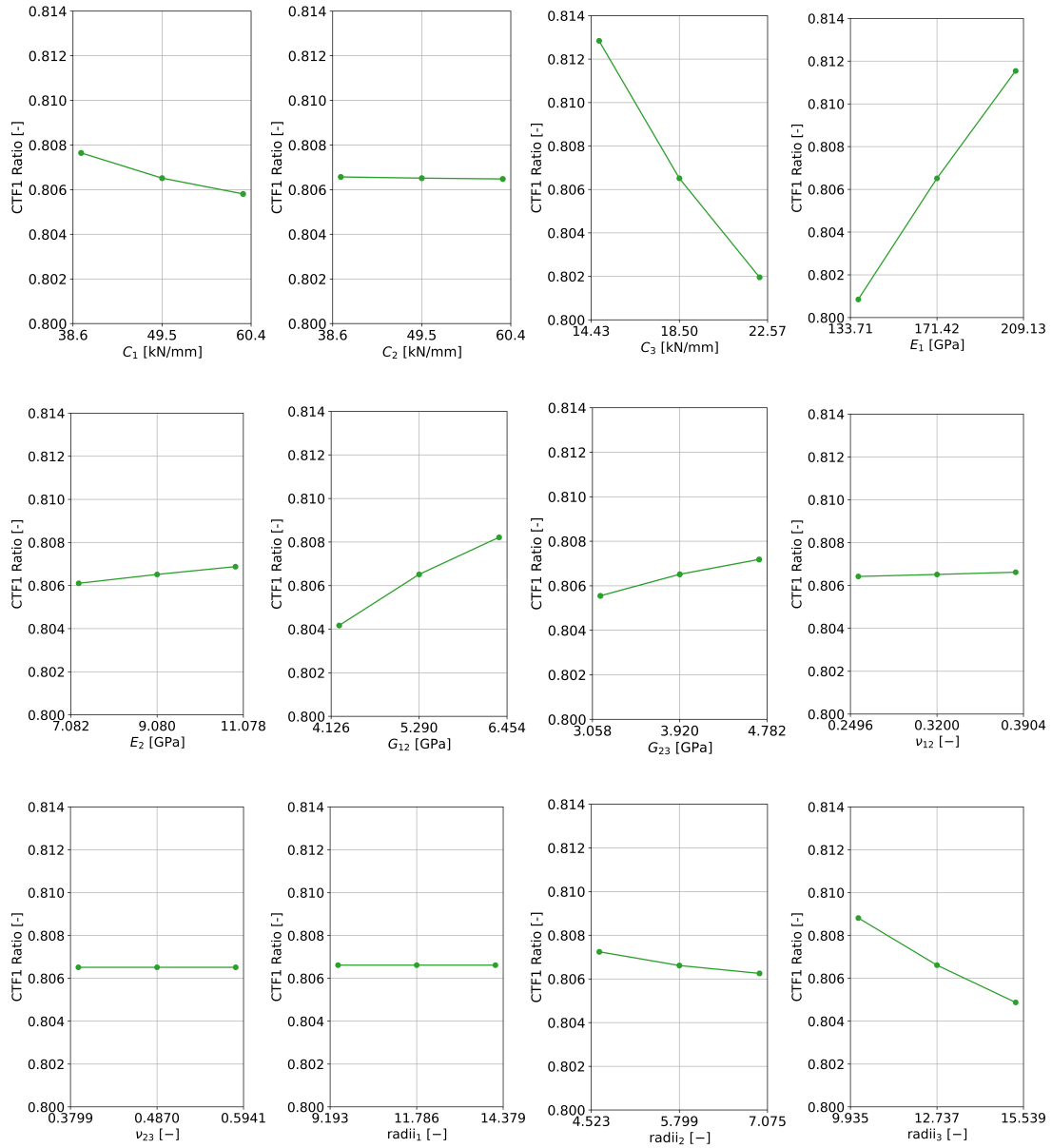


Figure 5.6: Influence of the different parameters on the CTF1 bolt ratio, obtained as $CTF1/(CTF1 + CTF2 + CTF3)$.

Regarding the failure load, the most significant variations are driven by the longitudinal bearing (LB) radius, a parameter from the failure ellipsoid equation, which corresponds to the load-carrying capacity under a pure longitudinal bearing condition. In addition to $radii_3$, the connector elasticity C_3 , the pull-through radius, $radii_2$, and the longitudinal Young's modulus E_1 also influence the failure load, albeit to a lesser extent.

When examining the slope of the load-displacement curve, the most influential parameters are the connector elasticity C_1 and Young's modulus E_1 , both of which govern the initial stiffness of the system.

As for the CTF1 ratio, it is primarily affected by the connector elasticity C_3 and the longitudinal modulus E_1 . Other properties such as C_1 , G_{12} , G_{23} , and $radii_3$ also contribute to variations in the ratio, although their influence is comparatively smaller. A detailed analysis of these variations and their implications will be presented in the following section. Finally, a summary of the influence of each parameter is shown in Figure 5.7, where they are organised by descending order regarding the absolute value of the curve slope.

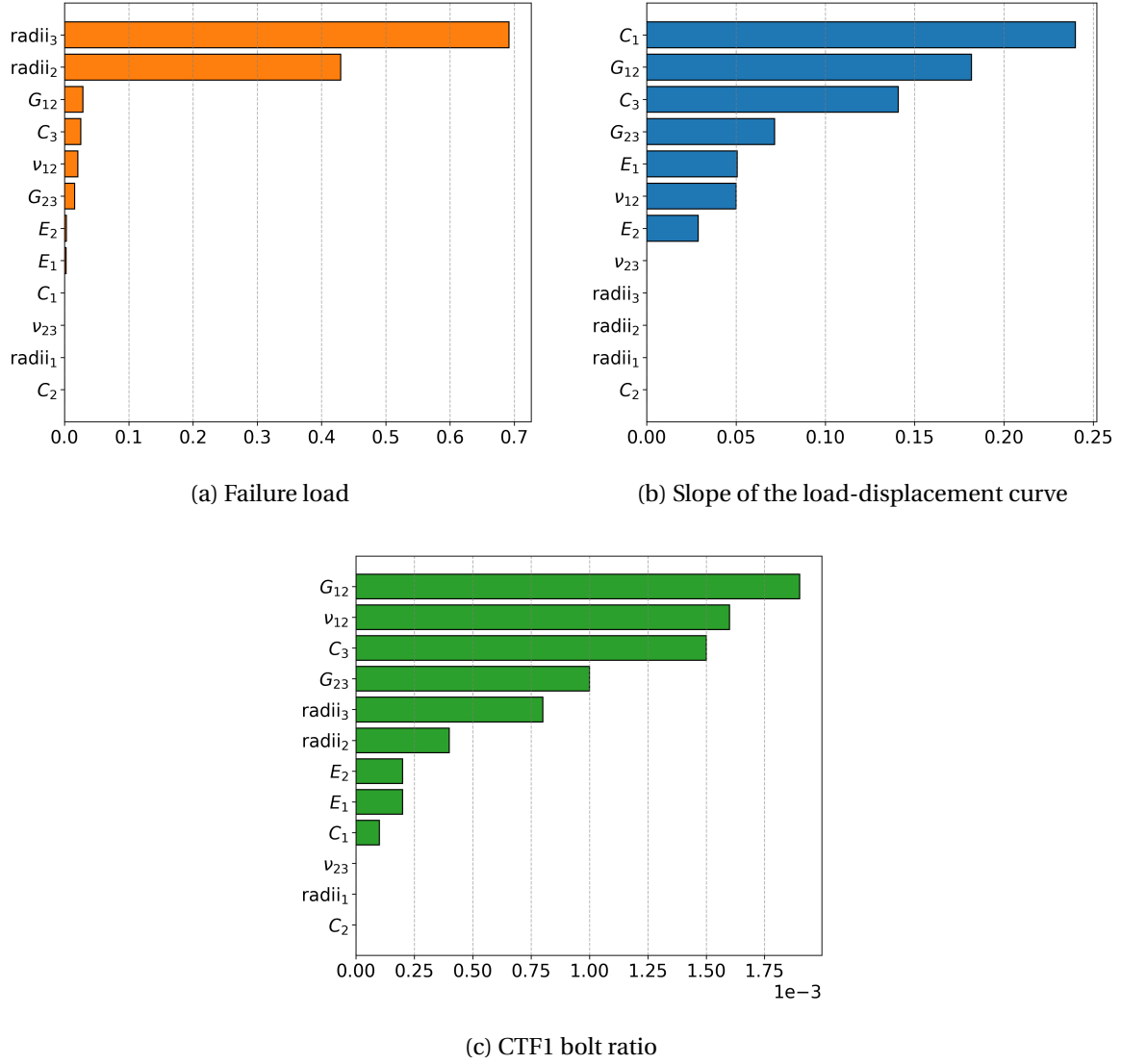


Figure 5.7: Summary of the influence of each property on the different parameters organised by absolute value of the curve slope.

5.3.2 Connectors' elasticity influence

To assess the influence of the different connector elasticity properties on the failure load and CTF1 bolt ratio, namely C_1 and C_3 , 16 pairs of intervals of both properties over a whole range within $\pm 20\%$ of the base value and subsequent simulations were performed. The connector forces CTF1 and CTF3 were obtained, and the CTF1 ratio or bearing ratio was calculated. The connector force in the transverse bearing direction (90°) was discarded as it is close to 0.

The failure load for each configuration was determined by identifying the intersection between the load path and the failure criterion, detailed previously in Section 4.4. This approach ensured a consistent evaluation of failure onset across different parameter variations.

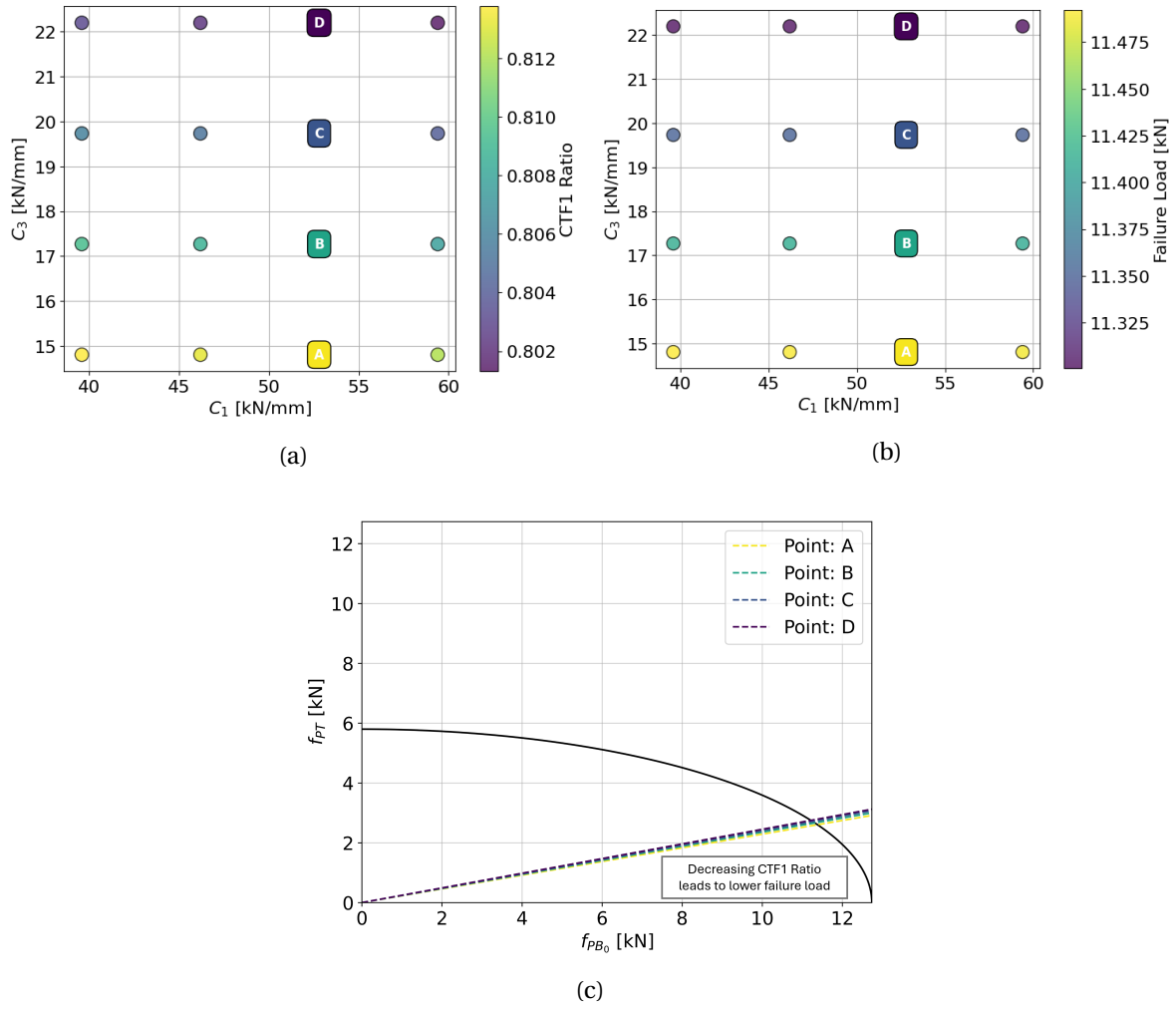


Figure 5.8: Connectors' elasticity influence on the failure load and CTF1 ratio.

In Figure 5.8a, the variations in the CTF1 ratio are clearly depicted, highlighting the differential impact of each connector elasticity property. Among them, the C_3 property, the one in the pull-through direction, emerges as the most influential, significantly driving the changes in the ratio. This indicates that the value of the ratio is primarily governed by the elasticity of C_3 , suggesting its failure role in the overall system response. The same trend is highlighted in Figure 5.8b, with C_3 governing the variation in the failure load and C_1 having a negligible influence. Although the influence of C_1 may not be immediately apparent, this becomes clearer in Figure 5.8c, which presents a 2-dimensional projection of the failure criterion. In this figure, the longitudinal bearing value is plotted on the x-axis, while the pull-through failure load is shown on the y-axis. If the ratio is closer to 1, meaning that the load is almost entirely longitudinal bearing, the failure load will increase, which is the opposite of what is happening. The decreasing CTF1 ratio as the C_3 increases results in the load path curves shifting "upwards" on the failure ellipsoid, signifying a decrease in the failure load.

This analysis highlighted the crucial role of the C_3 property in influencing both the CTF1 ratio and the failure load, and the clear dominance of C_3 over C_1 in governing system behaviour underscores its importance as a key parameter for predicting failure.

5.4 Uncertainty quantification

In this section, the uncertainty quantification process is performed for both quad and double-double layups. It starts with an explanation of the strategy, followed by the evaluation of the required number of simulations for statistical representation and finally, the results are shown, compared and discussed.

5.4.1 Uncertainty quantification strategy

As part of the uncertainty quantification of the simplified multi-bolt models described above, a series of simulations were conducted for each model. To better understand the influence of the material properties, connector elasticity, and ellipse parameters on the results, three separate sets of simulations were run:

- Run A - Material properties variation
- Run B - Connector elasticity variation
- Run C - Ellipse parameters variation
- Run D - All parameter variation

Each parameter followed a normal distribution, as detailed in Table 5.2. The standard deviation for material properties was based on Reference [67], while for connector properties and ellipse parameters, due to the lack of available data, a deviation of 10% from the base value was assumed. It is worth noting that the normal values were maintained throughout the different runs, so there was no uncertainty associated with data variation.

Table 5.2: Parameters of the properties' normal distribution.

Property		Base value	Standard deviation	Ref. [67]
E_1	[GPa]	171.420	2.38	
$E_2 = E_3$	[GPa]	9.08	0.09	
$G_{12} = G_{13}$	[GPa]	5.29	0.13	
$\nu_{12} = \nu_{13}$	[-]	0.32	0.02	
ν_{23}	[-]	0.487	-	
$C_1 = C_2$	[N/mm]	49500	10% of the base value	
C_3	[N/mm]	18500		
Radius _{TB}	[-]	11.786		
Radius _{PT}	[-]	5.799		
Radius _{LB}	[-]	12.737		

It is also worth highlighting that the uncertainty quantification procedure assumes normally distributed input parameters, as outlined in Table 5.2. For the material properties, this assumption leads to a reduced impact on the output variability, since the standard deviations reported in Reference [67] are considerably smaller than the wider parameter ranges explored in the baseline analysis. As a result, the contribution of material property uncertainty to the overall variability is inherently limited.

5.4.2 Study on the required number of simulations

To determine an appropriate number of simulations for the uncertainty quantification procedure, a preliminary analysis was conducted using a larger set of 1000 simulations, also for the 1x1

model. This analysis aimed to evaluate how smaller subsets of simulations captured the overall behaviour of the system. Empirical cumulative distribution functions (CDFs) were generated for progressively larger subsets and compared with the CDF obtained from the full 1000-simulation set. The convergence of the results was assessed by calculating the area between the CDFs of the subsets and that of the full dataset, as illustrated in Figure 5.9.

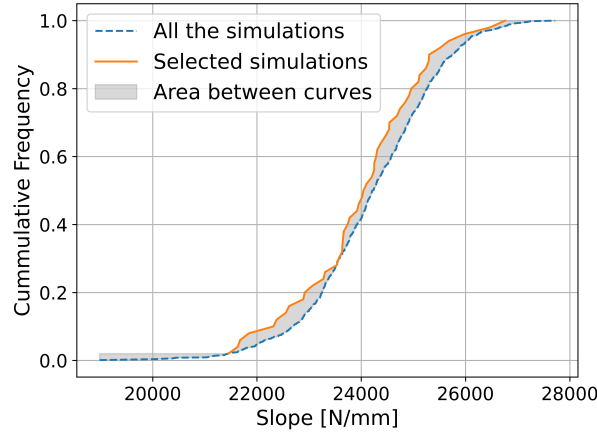


Figure 5.9: Example of the area between CDFs.

As shown in Figure 5.10, the area difference between the CDF corresponding to the full dataset and that derived from 100 simulations is minimal and shows signs of stabilisation. Based on this analysis, it was concluded that using 100 simulations would provide a sufficiently accurate representation of the uncertainty in the model.

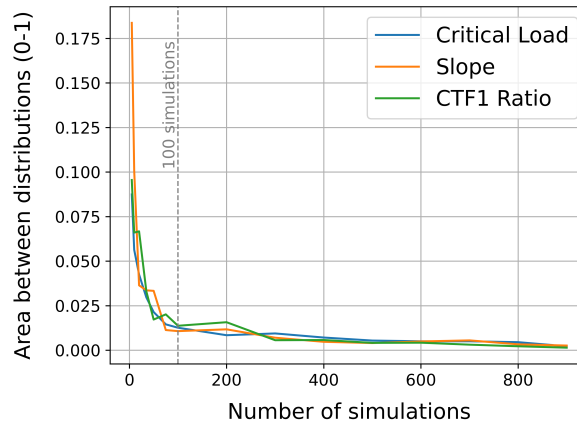


Figure 5.10: Evolution of the difference between the cumulative areas by number of simulations.

5.4.3 Results

Quad laminates

The first step in the uncertainty quantification was performed for the simplified models with the quad laminate layup, $[45/90/-45/0/0/45/0/0/-45/0]_S$, by doing 100 simulations for each of the aforementioned run A, B, C and D. The results highlight that the failure load is primarily affected by changes in the failure criterion, while material properties and connector elasticity play a minor role, except in specific configurations. The load-displacement slope remains largely unaffected, with most variations attributed to connector elasticity. Additionally, the CTF1 ratio is shown to be highly stable across models, reinforcing the dominance of bearing loading.

In Figure 5.11, the load-displacement curves from 100 simulations of the 1×1 model are shown for runs A, B, C, and D. Each curve includes the corresponding failure load, and an additional curve represents the average slope and average failure load across all runs. The same procedure is applied in Figure 5.12 for the remaining seven models, with results shown for run D under the condition where all properties vary. These plots indicate that the failure load is more significantly affected when all properties are varied simultaneously, whereas changes in material properties alone have only a minor influence on the ultimate failure load.

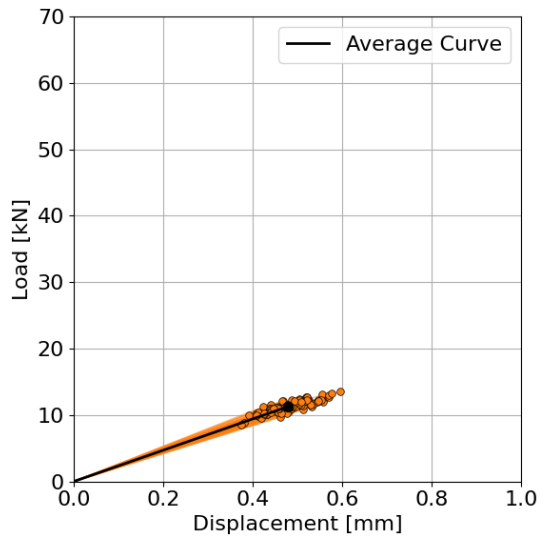
Table 5.6 summarises the results of the detailed uncertainty quantification analysis. This analysis considered variations in both the previously identified key properties and all other properties. Initially, the failure ellipse, connector, and material properties were varied individually, followed by a simultaneous variation of all properties and, for each model, the impact of these property variations was assessed using the CTF1 ratio, which inherently determines the CTF3 ratio due to their inverse relationship. Changes in slope and failure load were also evaluated. To visualise these effects, dashed trend lines were used to represent each individual variation case, while a solid line indicates the scenario where all properties were varied concurrently.

Regarding the failure load, the plots indicate that its value is primarily influenced by variations in the ellipsoid, which aligns with expectations since modifying the failure criterion directly affects its limits. As for the variation of the material properties or the connector elasticity, both have a similar reduced role across all models, being almost 0 in the pure bearing/pull-through scenarios. The standard deviation of the failure load across all models ranges from 0.40 to 2.79 kN, as shown in Table 5.3, with the highest deviation observed in the 2×2 model. Moreover, this configuration exhibits the highest load-bearing capacity, as anticipated. This is further supported by comparing the curve where all properties varied simultaneously with the one where only the failure criterion changed, as they are nearly identical.

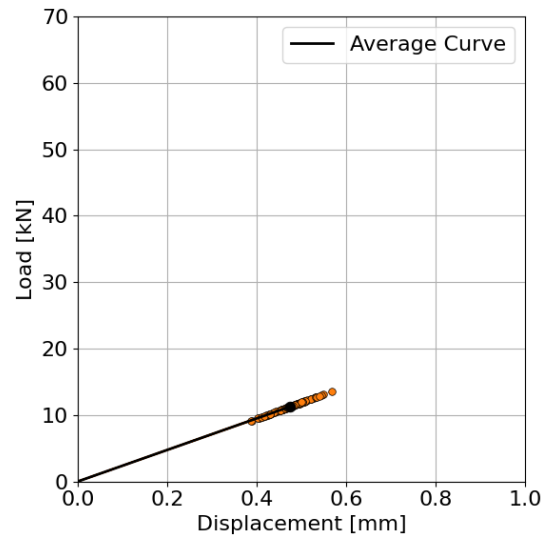
Since modifying the ellipsoid does not affect the material properties, the slope of the load-displacement curve was expected to remain unchanged, which was confirmed by the plots. Additionally, the changes in the properties had a relatively low effect on the slope, and almost all the variation comes from the changes in the connector elasticity. From Table 5.4 we can see that the highest standard deviation was in the pure bearing model and the lowest on the pure pull through.

Finally, for the CTF1 ratio, although by looking at the plots it seems that it is governed by connector elasticity, Table 5.5 confirms that changes are marginal with the standard deviation ranging from 0 to 0.05. This means that in every case, the loading is around 80% dominated by loading in the transversal bearing direction, with the remaining 20% coming from the pull through direction. In the 2x1, 2x2 and 3x1 models, we can see that the lines have a different shape, which occurs due to the fact that the bolt that is failing is changing, however, the ratio variations remain very small. As for the pure bearing and pure pull-through models, the ratios are respectively 1 and 0, as expected.

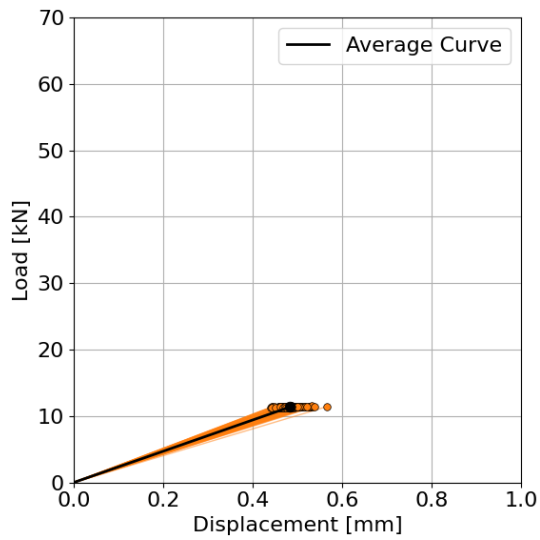
This study provides a comprehensive uncertainty quantification analysis of simplified multi-bolt models, assessing the influence of varying property sets on key failure parameters. The findings are crucial for further developing and understanding the behaviour of these models under various circumstances.



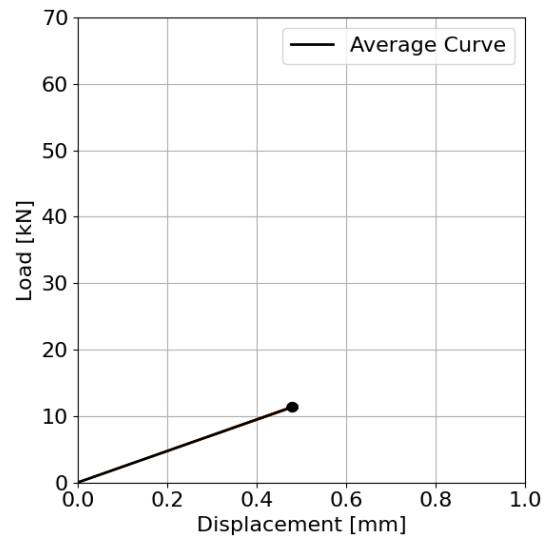
(a) All properties variation



(b) Failure envelope variation

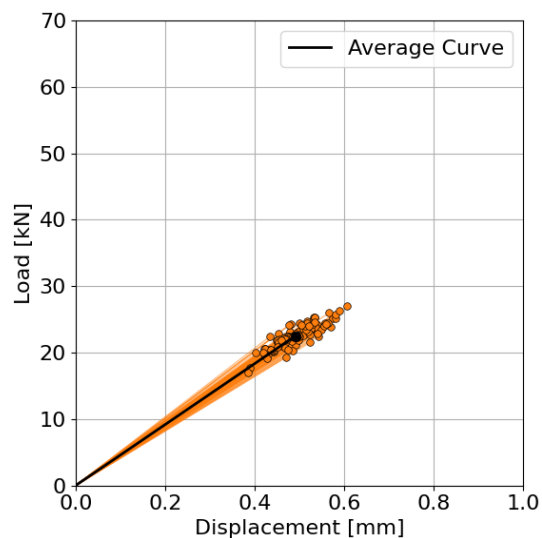


(c) Connector stiffness variation

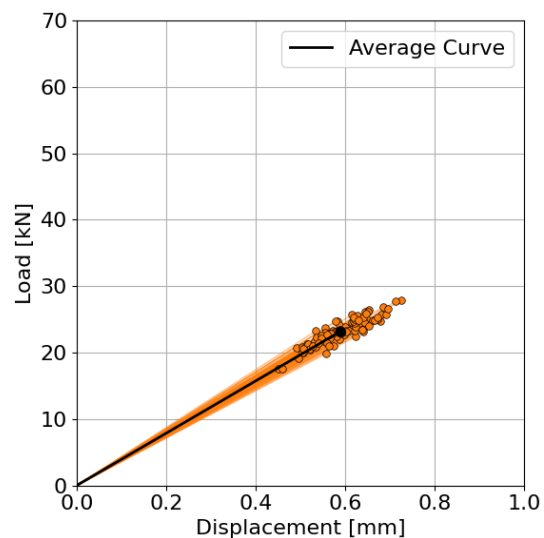


(d) Material properties variation

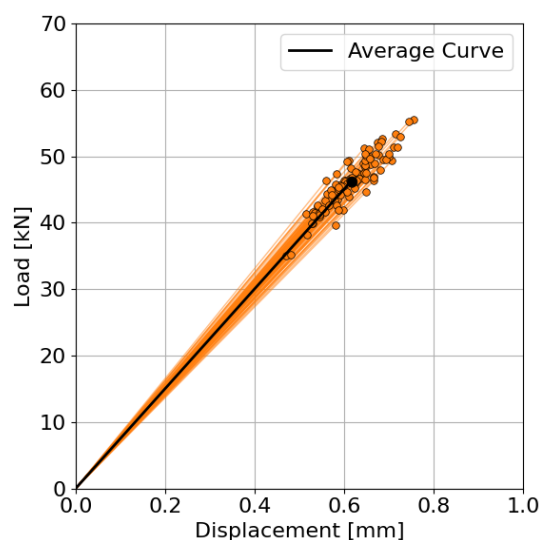
Figure 5.11: 100 load-displacement curves until failure for the 1x1 quad simplified model regarding the four different runs, A, B, C and D.



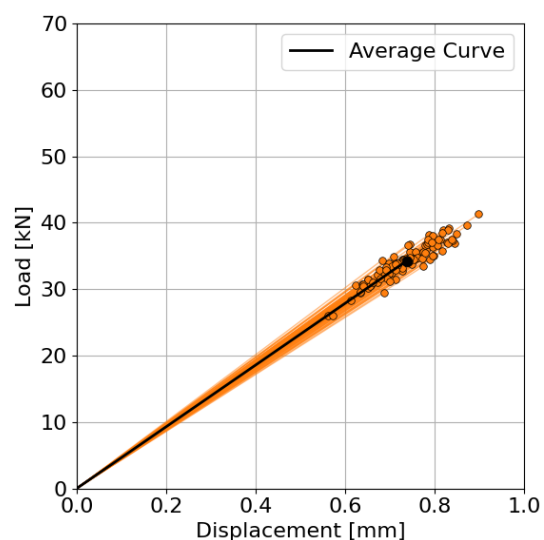
(a) 1x2 model



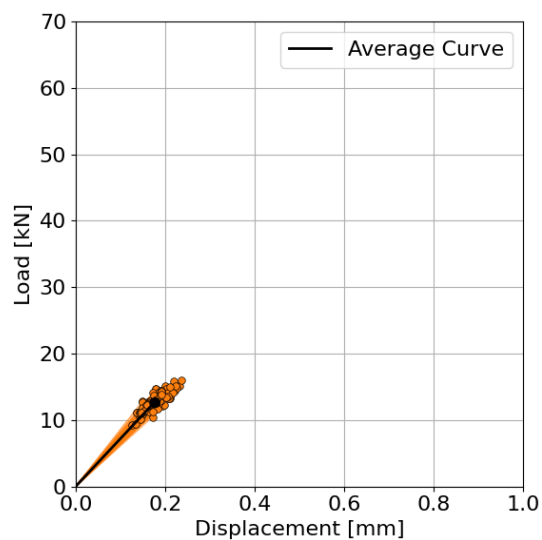
(b) 2x1 model



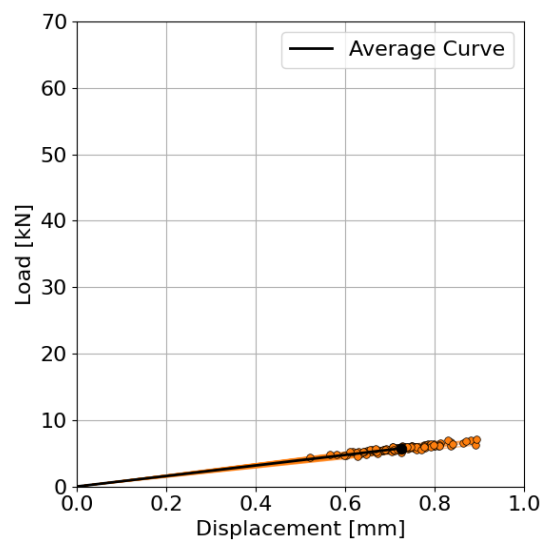
(c) 2x2 model



(d) 3x1 model



(e) Pure bearing model



(f) Pure pull-through model

Figure 5.12: 100 load-displacement curves until failure for the remaining quad simplified models regarding the run D, where all properties are being changed.

Table 5.3: Statistical Summary of Failure Load for Quad Simplified Models.

Model	Failure Load				
	Minimum	Maximum	Mean	Median	Standard deviation
	[kN]				
1x1	8.55	13.56	11.33	11.36	0.65
1x2	17.04	27.01	22.58	22.65	1.29
2x1	17.58	28.05	23.33	23.47	1.40
2x2	35.05	55.94	46.54	46.81	2.79
3x1	25.97	41.72	34.58	34.84	2.09
Pure Bearing	9.30	16.01	12.73	12.74	0.93
Pure Pull-through	3.69	7.13	5.79	5.81	0.40

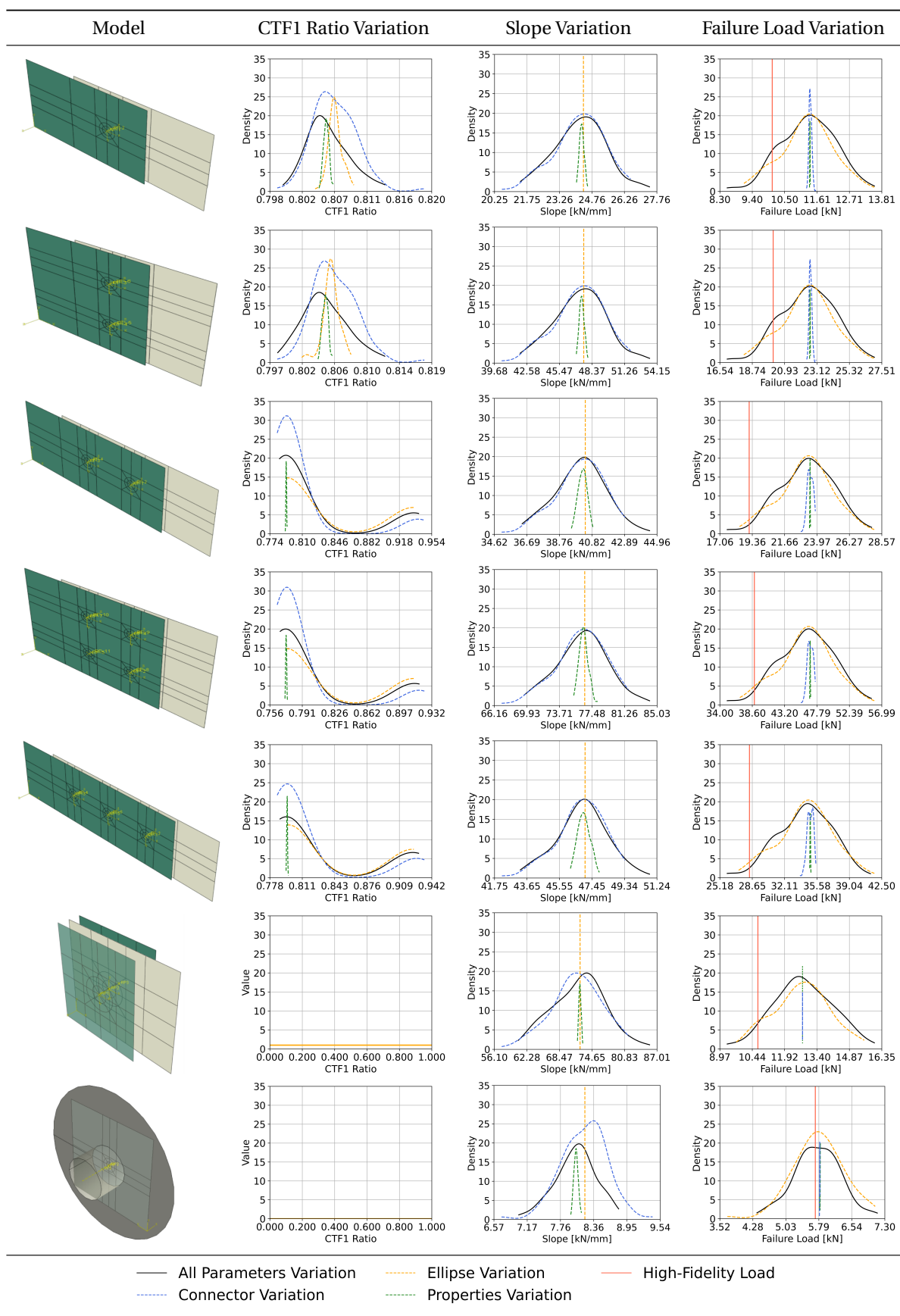
Table 5.4: Statistical Summary of Slope for Quad Simplified Models.

Model	Slope				
	Minimum	Maximum	Mean	Median	Standard deviation
	[kN/mm]				
1x1	20.59	27.42	24.26	24.36	0.85
1x2	40.34	53.50	47.41	47.63	1.63
2x1	35.09	44.49	40.20	40.40	1.15
2x2	67.02	84.17	76.44	76.64	2.11
3x1	42.18	50.81	46.92	47.04	1.04
Pure Bearing	57.51	85.61	72.17	72.39	3.52
Pure Pull-through	6.71	9.41	8.11	8.13	0.29

Table 5.5: Statistical Summary of CTF1 Ratio for Quad Simplified Models.

Model	CTF1 Ratio				
	Minimum	Maximum	Mean	Median	Standard deviation
	[-]				
1x1	0.80	0.82	0.81	0.81	0.00
1x2	0.80	0.82	0.81	0.80	0.00
2x1	0.78	0.95	0.81	0.79	0.05
2x2	0.76	0.92	0.80	0.77	0.05
3x1	0.79	0.93	0.82	0.80	0.05
Pure Bearing		1.00			0
Pure Pull-through			0		

Table 5.6: Uncertainty quantification results for quad laminates.



Double-double laminates

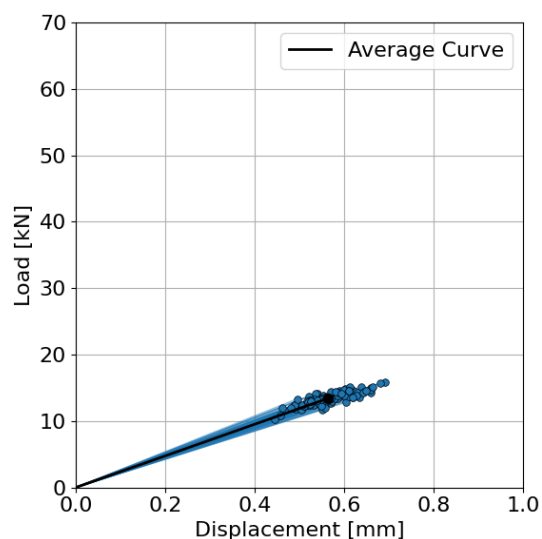
Finally, the process described above is repeated in full for the simplified multi-bolt models, this time using the previously selected double-double layup, $[57/-12/0/12/-57]_{4T}$. A statistical summary of the results is presented in Tables 5.7 to 5.9, while the detailed cumulative distributions of the evaluated properties are provided in Table 5.10. As with the quad layup, the impact of input variability is assessed with respect to the failure load, stiffness, slope of the load-displacement curve, and CTF1 ratio. The process was again performed for 100 simulations for each of the runs A, B, C and D.

Similarly to what was previously done for the quad layup models, in Figure 5.13, the resulting load-displacement curves are plotted alongside a curve showing the mean value of slope and failure load for the 1x1 model in runs A, B, C and D. In Figure 5.14, the curves are also displayed only for run D for the remaining simplified multi-bolt models. The results are again similar, with the changes in the failure load being more evident for the run D where are properties are simultaneously changed and run A, with the material properties changing, is the one where the failure load remains more stable. However, as expected, the higher failure load of the double-double layup models is evident in these plots.

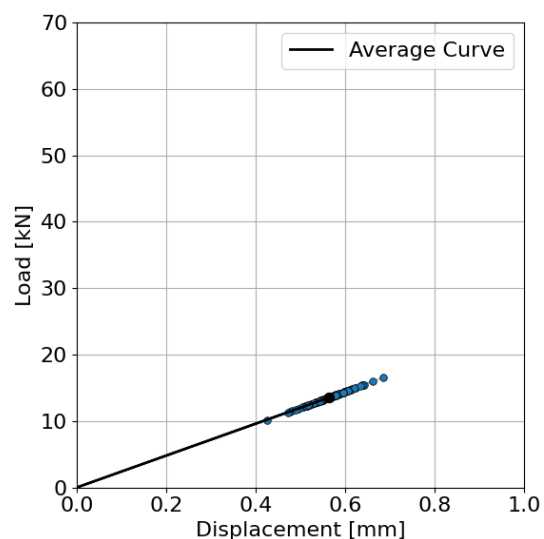
Regarding the failure load, the results behave as expected. Variation is primarily driven by changes in the failure criterion, with variations in material and connector properties having minimal influence. The standard deviation of the failure load ranges from 0.41 to 3.04 kN, with the highest value again observed in the 2x2 model, which is expected, given its superior load bearing capacity. When comparing the variation curves, the one driven solely by the failure criterion aligns closely with the curve incorporating all properties, further reinforcing the dominant influence of this parameter.

For the slope of the load-displacement curve, the variation is mainly influenced by changes in the connector's elastic properties. This is evidenced by the close agreement between the slope variation curve and the curve representing variation across all properties. Standard deviation values range from 0.84 to 3.52 kN/mm, with the Pure Bearing model showing the highest variability, consistent with the findings for the quad layup. As anticipated, the stiffest model is again the 2x2 configuration.

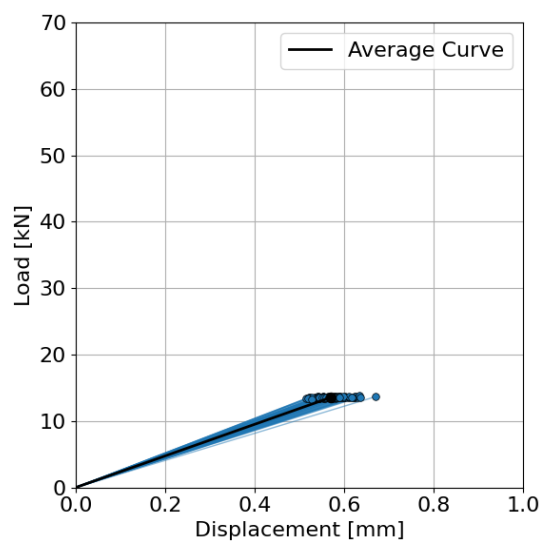
Finally, concerning the CTF1 ratio, variability remains minimal. The maximum observed standard deviation is 0.04. All models failed under loads where the CTF1 ratio was approximately 80% bearing-dominated, with the Pure Bearing and Pure Pull-Through models showing ratios of 1 and 0, respectively, as expected.



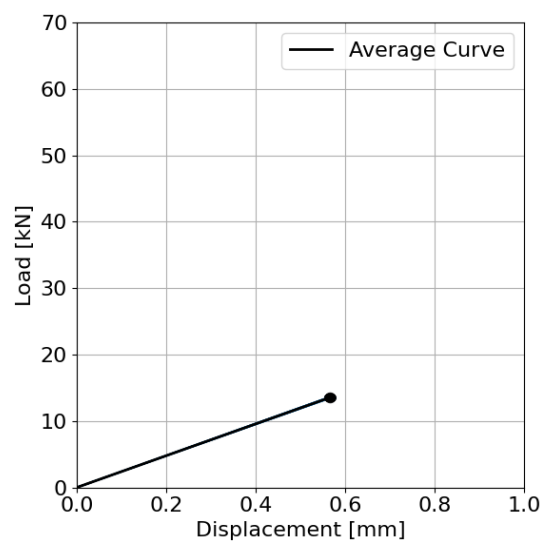
(a) All properties variation



(b) Failure envelope variation

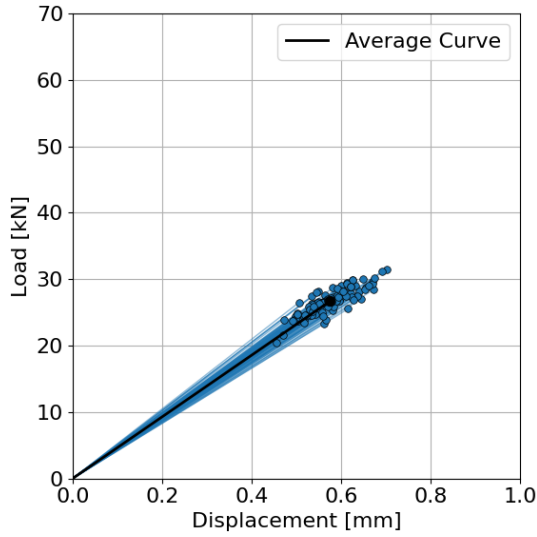


(c) Connector stiffness variation

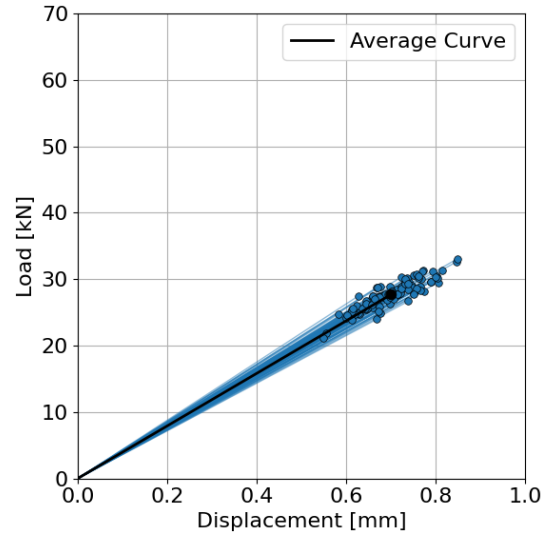


(d) Material properties variation

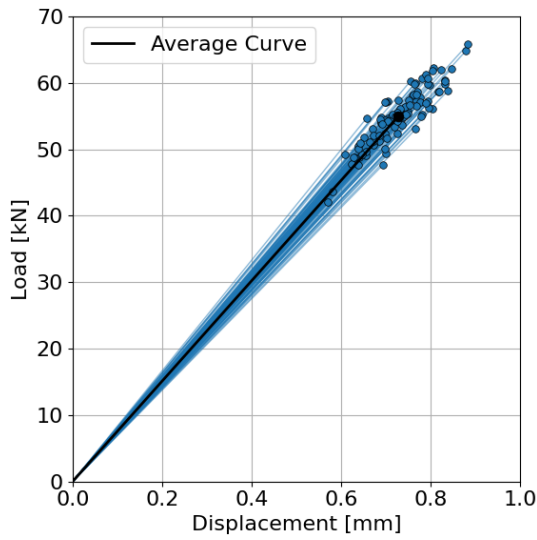
Figure 5.13: 100 load-displacement curves until failure for the 1x1 double-double simplified model regarding the four different runs, A, B, C and D.



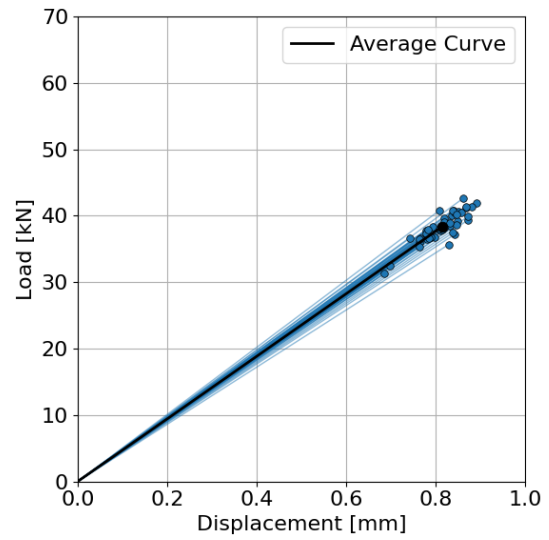
(a) 1x2 model



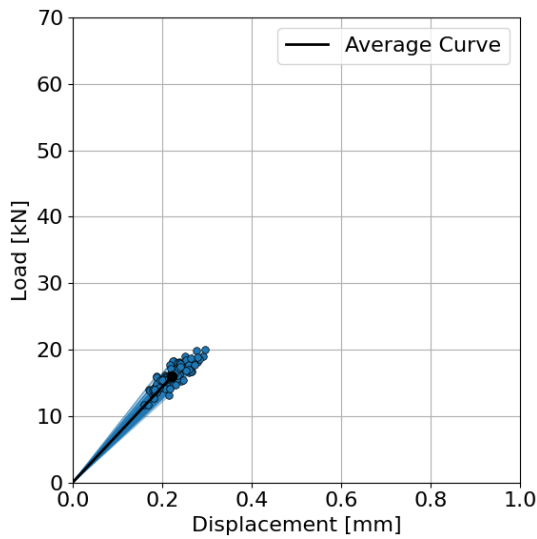
(b) 2x1 model



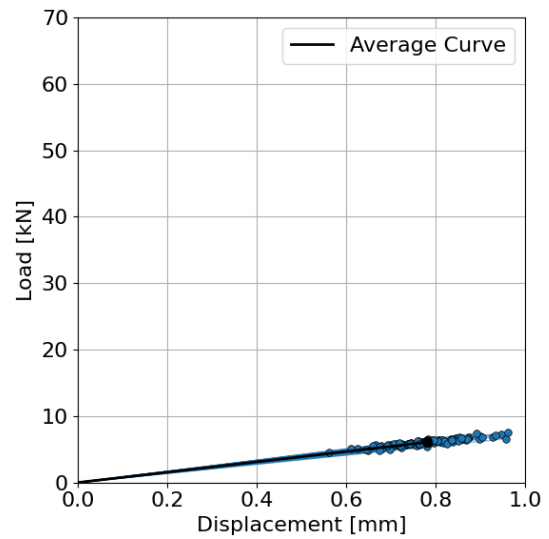
(c) 2x2 model



(d) 3x1 model



(e) Pure bearing model



(f) Pure pull-through model

Figure 5.14: 100 load-displacement curves until failure for the remaining double-double simplified models regarding the run D, where all properties are being changed.

Table 5.7: Statistical Summary of Failure Load for Double-Double Simplified Models.

Model	Failure Load				
	Minimum	Maximum	Mean	Median	Standard deviation
	[kN]				
1x1	10.17	16.52	11.52	13.54	0.74
1x2	20.29	32.70	26.87	26.92	1.46
2x1	20.47	33.97	27.90	27.95	1.57
2x2	40.82	65.83	55.43	55.56	3.04
3x1	30.21	46.59	41.26	41.57	2.09
Pure Bearing	10.79	22.09	15.99	15.96	1.21
Pure Pull-through	4.61	7.79	6.12	6.14	0.41

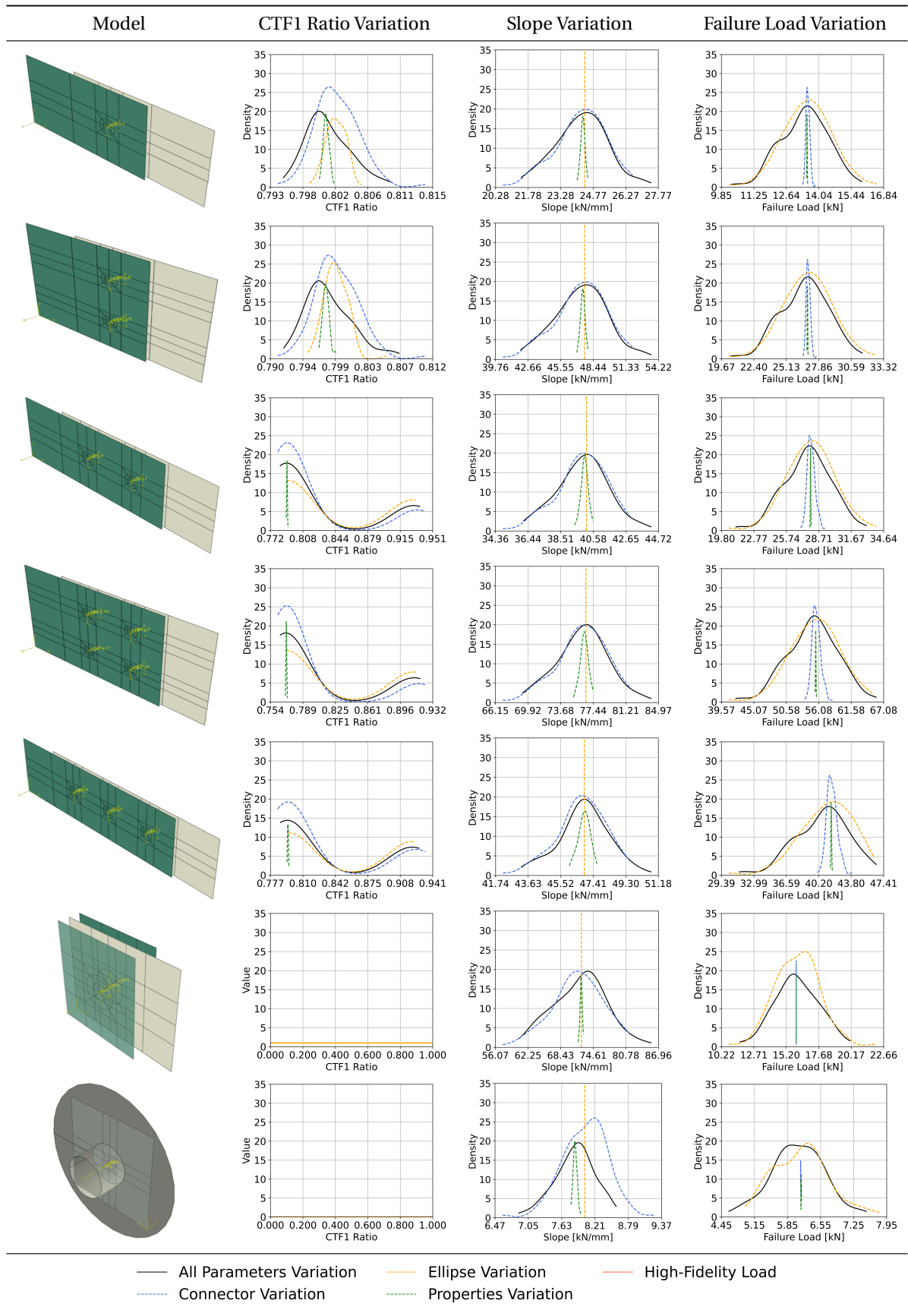
Table 5.8: Statistical Summary of Slope for Double-Double Simplified Models.

Model	Slope				
	Minimum	Maximum	Mean	Median	Standard deviation
	[kN/mm]				
1x1	20.62	27.43	24.28	24.38	0.84
1x2	40.42	53.56	47.47	47.67	1.63
2x1	34.84	44.25	39.99	40.15	1.15
2x2	67.04	84.12	76.34	76.61	2.09
3x1	42.17	50.76	46.82	46.89	1.05
Pure Bearing	57.48	85.56	72.13	72.35	3.52
Pure Pull-through	6.60	9.24	7.95	7.95	0.28

Table 5.9: Statistical Summary of CTF1 Ratio for Double-Double Simplified Models.

Model	CTF1 Ratio				
	Minimum	Maximum	Mean	Median	Standard deviation
	[-]				
1x1	0.79	0.81	0.80	0.80	0.00
1x2	0.79	0.81	0.80	0.80	0.00
2x1	0.78	0.93	0.82	0.79	0.02
2x2	0.76	0.92	0.77	0.77	0.02
3x1	0.78	0.93	0.81	0.80	0.04
Pure Bearing	1.00				0
Pure Pull-through	0				

Table 5.10: Uncertainty quantification results for double-double laminates.



Comparison

When comparing the quad and double double results, statistically, the parameters under study, the failure load, the CTF1 ratio, and the slope of the load-displacement curve, exhibit very similar overall behaviour across all the analysed models.

Focusing initially on the 1x1 and 1x2 layup configurations, the ellipse parameter has a more pronounced effect on the double-double layup, particularly impacting both the CTF1 ratio and the failure load. In these models, the distributions of failure load and CTF1 are narrower, with values more centralised. For the 2x1 and 2x2 configurations, the ellipse continues to influence the double-double layup more strongly, although its effect on failure load and CTF1 ratio is less significant compared to the smaller layups.

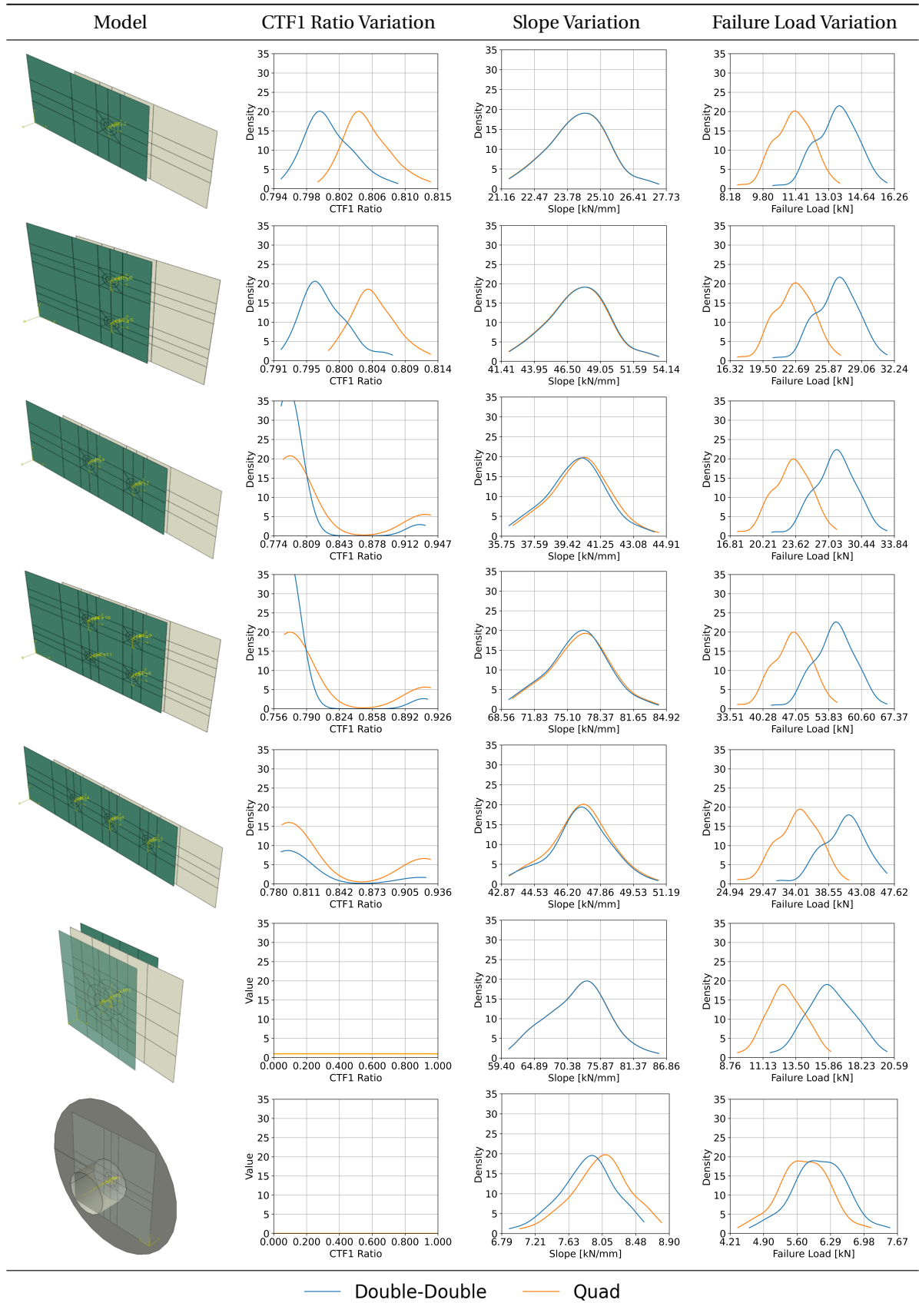
The 3x1 model exhibits a similar trend to the 2x1 and 2x2 configurations; however, its results closely resemble those of the quad models. In the Pure Bearing model, the ellipse parameter exerts the greatest influence on failure load across all models, while the other parameters show comparable effects for both quad and double-double layups. Lastly, the Pure Pull-Through model yields very similar results for both layups.

Statistically, differences between the two layups are minimal. For failure load, the quad layup's standard deviation ranges from 0.65 to 2.59 kN, lowest in the 1x1 and highest in the 2x2 model, with maximum and minimum values of 55.94 kN and 3.69 kN, respectively. The double-double layup shows a standard deviation range of 0.41 to 3.04 kN, with corresponding load extremes of 65.83 kN and 4.61 kN, this time obtained in the 2x2 and pure pull-through model, respectively. Regarding the slope of the load-displacement curve, the quad layup's standard deviation varies from 0.85 to 3.52 kN/mm, and the double-double layup's from 0.84 to 3.52 kN/mm, both obtained in the 1x1 and Pure Bearing models, respectively. The Pure Pull-Through model exhibits the lowest stiffness values for both layups, 6.71 kN/mm for quad and 6.60 kN/mm for double-double. Finally, for the CTF1 ratio, across both layups, the failure load is achieved for all the multibolt configurations with a value of around 0.80, indicating that the failure load is 80% bearing dominated.

Table 5.11 presents the results for run D, where all properties vary, comparing both the quad and double-double (DD) layups. These curves confirm that the distributions of the evaluated parameters are very similar for both laminates. However, a clear rightward shift in the DD curve for the failure load indicates a higher ultimate failure load compared to the quad layup, which is expected since the failure envelope previously obtained revealed higher radius values resulting in a higher volume. In contrast, the slope and the CTF1 ratio exhibit minimal differences between the layups, with some curves nearly superimposed, emphasising the marginal nature of these variations.

The comparison between quad and double-double layups reveals that failure load, CTF1 ratio, and load-displacement slope behave similarly across all tested models, although DD reveals a higher failure load across all models. The ellipse parameter has a stronger influence on double-double layups, particularly affecting failure load and CTF1 ratio. Statistical variations between layups are minimal, with both showing comparable distributions and stiffness values. Notably, the CTF1 ratio consistently indicates that failure is predominantly bearing dominated, with about 80% contribution across all configurations.

Table 5.11: Uncertainty quantification results comparison between quad and double-double laminates.



Chapter 6

Conclusions and Future Work

6.1 Conclusions

The main objectives of this thesis were to extend the application of an existing failure criterion to the newly studied double-double (DD) laminates and to evaluate the performance of previously developed simplified multi-bolt models for both conventional and DD materials.

First, an equivalent double-double layup to the previously studied quad laminate was selected. This was achieved using lamination parameters and the homogenisation criterion proposed in the literature, with a focus on in-plane equivalence. Initial results indicated that ply angles of 57° and 12° , supplemented by a 0° ply, were necessary to achieve equivalence, as a direct match was otherwise not possible. The optimal layup, based on the homogenisation criterion, was determined to be $[-12/57/0/-57/12]$. However, due to concerns about placing 0° plies near the laminate surfaces, and the fact that 12° was insufficiently misaligned to mitigate related effects, a modified layup, $[57/-12/0/12/-57]$, was selected for the remainder of the study.

The high-fidelity modelling strategy to simulate composite bolted joints, initially only able to generate structured meshes for 0° , $\pm 45^\circ$ and 90° plies, was successfully adapted to generate structured meshes for arbitrary ply angles. This new chessboard-based meshing approach proved effective, consistently producing well-aligned meshes regardless of orientation. Results were benchmarked against simulations without aligned meshes. While no experimental data was obtained for the DD laminates, comparisons with the conventional quad showed that unstructured meshes tend to underpredict the first ply load drop. These observations validated the use of the structured meshing strategy. Subsequently, results from high-fidelity simulations of DD laminates were compared to those of the quad. In general, the DD layup exhibited a higher initial load drop, leading to a failure envelope with a larger area and higher parameter values.

Finally, a preliminary sensitivity analysis was conducted to support the quantification of uncertainty in the simplified multi-bolt models. Focus was given to the influence of the material properties, connector stiffnesses and failure ellipsoid definition on the slope, bolt ratios and ultimate failure loads of the joints. It revealed that the most influential parameters on failure load variability were the radius of the failure envelope and the connector elasticity C_3 . Regarding stiffness, the dominant factors were the longitudinal Young's modulus E_1 and connector elasticity C_1 . A separate study determined that 100 simulation runs with varied parameters were sufficient to capture the model's variability. Results for both the quad and DD laminates were comparable, with low standard deviations in critical load, stiffness slope, and CTF1 ratio. Moreover, the findings from the baseline sensitivity analysis were confirmed, reinforcing the conclusion that the failure envelope and connector elasticity are the most critical parameters affecting the model's results.

6.2 Future work

While this study has provided valuable insights into the behaviour of double-double laminates and the performance of simplified multi-bolt models, it remains entirely numerical. A crucial next step is to perform experimental tests to validate the findings presented in this thesis.

Experimental testing of double-double laminates, particularly with the $[57/-12/0/12/-57]_{4T}$ layup, would allow for the verification of the predicted failure loads and failure envelopes. It would also provide essential data to assess the accuracy of the structured meshing approach used in the high-fidelity models, especially regarding the first ply load drop. Moreover, conducting experimental tests on the initially proposed layup, $[-12/57/0/-57/12]_{4T}$, would be important to verify whether the placement of 12° plies on the outer layers poses any manufacturing challenges in practice.

Finally, experimental data would offer a basis for extending the uncertainty quantification beyond the current numerical parameter space, incorporating real-world variability such as manufacturing tolerances and material defects. This would significantly enhance the confidence and applicability of the simplified models in design and certification workflows.

Appendix A

Experimental Rig for Combined Bearing/Pull-through Testing

Although this thesis focuses on the numerical approach of the previously developed work from Reference [3], the experimental setup used for the test of combined bearing/pull-through loading is relevant, since some key concepts depend on this understanding and it was fundamental since it served as basis for the numerical models and validation of the quad failure envelope. In this chapter, all the details regarding the mixed-mode multi-axial experimental rig are explained.

A.1 Conceptual design

To create a controlled multiaxial loading scenario, the input from the testing machine is decomposed into two orthogonal components, an in-plane and an out-of-plane load. These components are linked by a fixed load ratio, which is defined by the geometric configuration illustrated in the static free body diagram from Figure A.1. In this setup, point A marks where the vertical load P , measured via a load cell, is introduced. Point B corresponds to the application of the horizontal reaction force R . The specimen's ideal bolted connection, located at point S, experiences both bearing P_B and pull-through P_{PT} forces. The constant load ratio is governed by the distances a and b .

By applying the equilibrium conditions for forces and moments to this system, it is possible to derive relationships among the loads and distances. Specifically, from the force balance along the x and y directions, one can determine how the applied loads and reaction forces relate to the internal forces acting on the specimen.

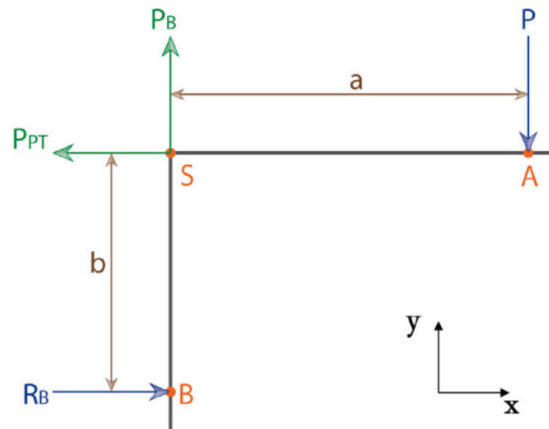


Figure A.1: Free body diagram. [3]

The forces obtained are:

$$P_{PT} = R_B \quad , \quad P_B = P \quad (\text{A.1})$$

meaning that the bearing load P_B is equal to the load P applied by the testing machine and is always known. The pull-through load P_{PT} , as a function of P , is obtained by determining the moment equilibrium in point B :

$$\sum M_B = 0 \Leftrightarrow P_{PT} \cdot b - P \cdot a = 0 \Leftrightarrow P_{PT} = \left(\frac{a}{b}\right) \cdot P \quad (\text{A.2})$$

With both P_B and P_{PT} , the load ratio η is obtained as shown in Equation A.3.

$$\eta = \frac{P_B}{P_B + P_{PT}} = \frac{P}{P + \frac{a}{b}P} = \frac{1}{1 + \frac{a}{b}} \quad (\text{A.3})$$

The η ratio, which ranges from 0 to 1, defines the loading condition and a value of $\eta = 0$ corresponds to a pure pull-through load case, occurring when the bearing load P_B is zero. On the other hand, when the pull-through load P_{PT} is absent, η reaches 1, indicating a pure bearing scenario. In both extreme cases, the results are expected to align with those obtained from standardised tests as specified in ASTM D7332 [68], and ASTM D5961 [69], respectively. Due to the mechanical limitations of the test rig, only certain mixed load conditions could be imposed. As a result, the study in Ref. [3] employed η values of 0.2, 0.4, 0.6, and 0.8 to represent a range of combined loading configurations, as summarised in Table A.1.

Table A.1: Values of a and b and corresponding η .

η	a [mm]	b [mm]
0.8	50	200
0.6	100	150
0.4	150	100
0.2	200	50

A.2 Design of the combined loading mechanism

Building upon the theoretical concept shown before, component M2, shown in Figure A.2, was developed. With point S defined as the specimen's fixed location, a grid of pin holes was incorporated into M1 at 50 mm intervals. This configuration enables the application of the load ratios presented in Table A.1.

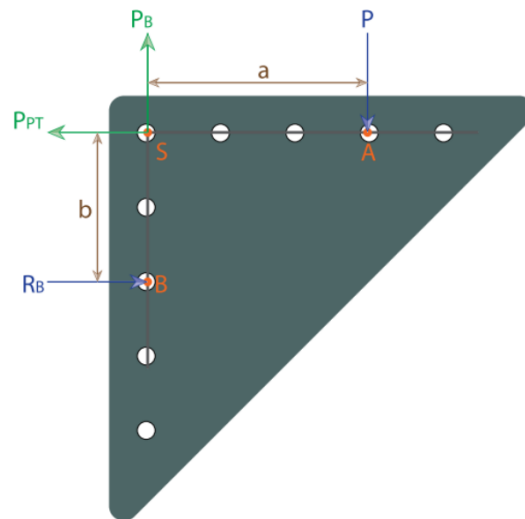


Figure A.2: Schematic representation of component M1. [3]

To accommodate functional constraints and ensure sufficient clearance for the relative movement of the rig's components, the specimen was repositioned to point S. This relocation was achieved using part M2, which shifts the specimen to a more accessible location without introducing additional moments. To prevent rotation of M2 and absorb any resulting moments, part M3 was added to the setup. As shown in Figure A.3, M2 is pinned to both M1 and M3, enabling a stable configuration. Additionally, component M3 is coated with a Teflon sleeve to reduce the appearance of unwanted loads and premature component degradation.

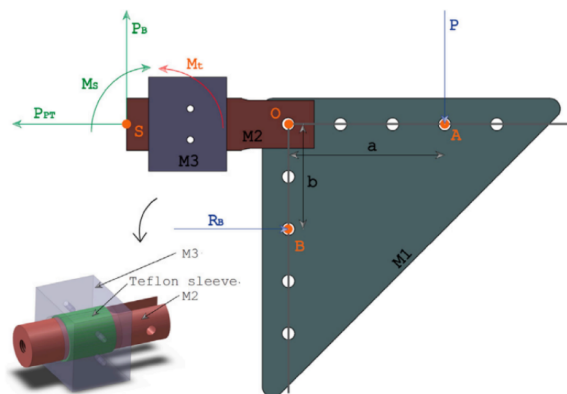


Figure A.3: Schematic of the ‘M1’, ‘M2’ and ‘M3’ components showing the forces and moments applied to the sample in point S. [3]

Component I2 transmits the vertical load to M1 via a pin connection that permits rocker-like rotation. Simultaneously, I1 is free to slide along the bottom adapter, accommodating any rotational movement of M1. The remaining elements, R1, R2, L1, L2, and F2, function as structural supports and alignment guides, as illustrated in Figure A.4. Additionally, M4 acts as an adapter, featuring an internal thread to accommodate an M6 (6 mm diameter) bolt.

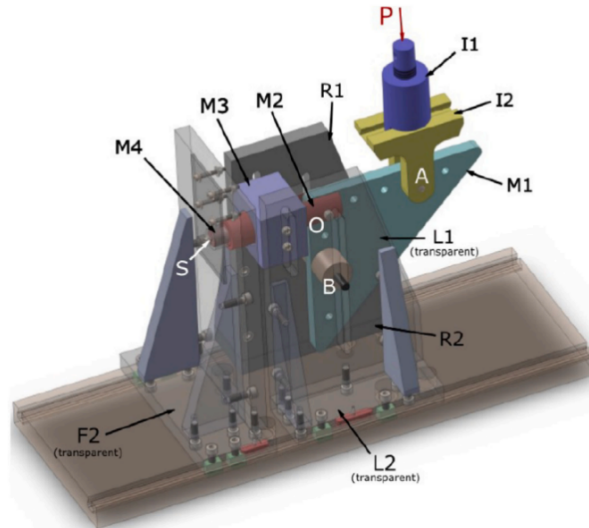


Figure A.4: Fully assembled configuration of the combined-loading rig. [3]

The final assembled rig is shown in Figure A.5. The specimen is secured using four fixation bolts, and a 35 mm diameter hole facilitates the loading of the bolted connection. Experimental testing was conducted with an Instron 4208 servo-mechanical testing machine, and to minimise undesired loads and ensure accurate load cell measurements, appropriate lubrication and surface finishing were applied. All components of the test fixture were fabricated from DIN CK45 steel.

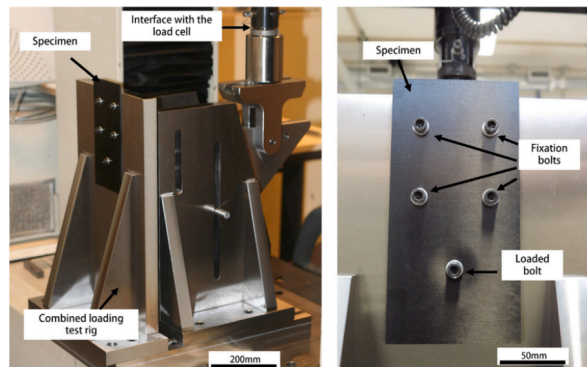


Figure A.5: Assembled combined-loading rig integrated with the Instron 4208 testing machine, left, and representative specimen under test conditions, right. [3]

Appendix B

Modelling of the Simplified Multi-Bolt Models using Connector Element Plasticity

In this chapter, a test will be conducted to define the connector stiffness using plasticity. In the models previously detailed in Chapter 5, the connectors were defined as linear elastic. Although modelling the connectors as elastic-plastic remains a simplification, since actual damage mechanisms are not captured, it may result in failure loads closer to those obtained with the high-fidelity models. It is also worth noting that this test was performed using the quad laminate layup.

B.1 Plasticity definition

To define the plasticity curve for the connector stiffness, a load-displacement curve from the pure bearing high-fidelity simulation was utilised, since the pull-through simulation presented no plasticity and sudden failure, only the pure bearing was considered. Beginning at the first ply load drop, a linear polynomial approximation was fitted to capture the slope of the non-linear portion of the simulation, as illustrated in Figure B.1.

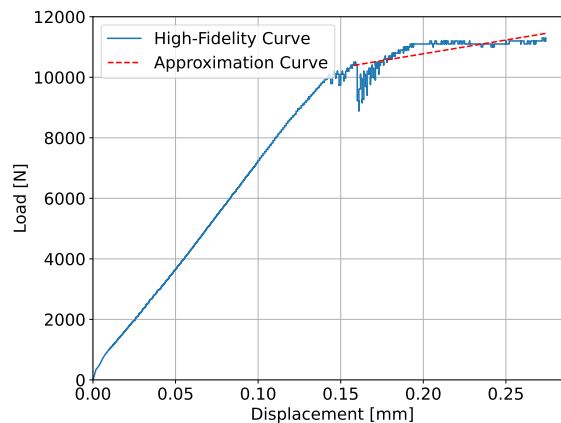


Figure B.1: Linear approximation of the non-linear part of the high-fidelity simulation.

The connector elements in the simplified models were then defined in Abaqus as elastic-plastic using the derived curve. This procedure was applied to all simplified models, and the resulting outcomes are presented in the following section.

B.2 Results

The different load-displacement curves are illustrated in Figure B.2 alongside the high-fidelity load-displacement curve as well as the linear elastic curve. For both simplified cases, post-processing was carried out using the previously obtained failure envelope, and the resulting differences in ultimate failure load are presented in Table B.1.

Table B.1: Failure load comparison between the simplified models using linear elastic and elastic plastic connector definition

Model	Linear elastic failure load [N]	Elastic plastic failure load [N]
1x1	11380	11424
1x2	22680	22783
2x1	23504	22459
2x2	46895	45018
3x1	34908	33686
PureBEA	12736	12074
PurePT	5808	5798

As shown in Table B.1, except for the 1x1 and 1x2 models, the failure loads of the models with the connector defined as elastic-plastic are lower than those obtained using the linear elastic definition, and the error % can be seen in Figure B.2. This outcome is encouraging, as Ref. [8] reported that all models tended to overpredict the failure load compared to the high-fidelity model. Although the models analysed here still overpredict the failure load, the reduction observed with the elastic-plastic definition shows promising signs. Nonetheless, further analysis is required to confirm these findings and to study them in greater detail.

Another important observation is presented in Table B.2, which reports the CTF1 bolt ratio at failure for the elastic-plastic models. These models successfully maintained the expected ratio at failure, further indicating the potential of this modelling approach.

Table B.2: CTF1 bolt ratio for the elastic-plastic simplified models at failure.

Model	CTF1 bolt ratio at failure
1x1	0.8113
1x2	0.8081
2x1	0.7963
2x2	0.7777
3x1	0.7992
PureBEA	1.0
PurePT	0.0

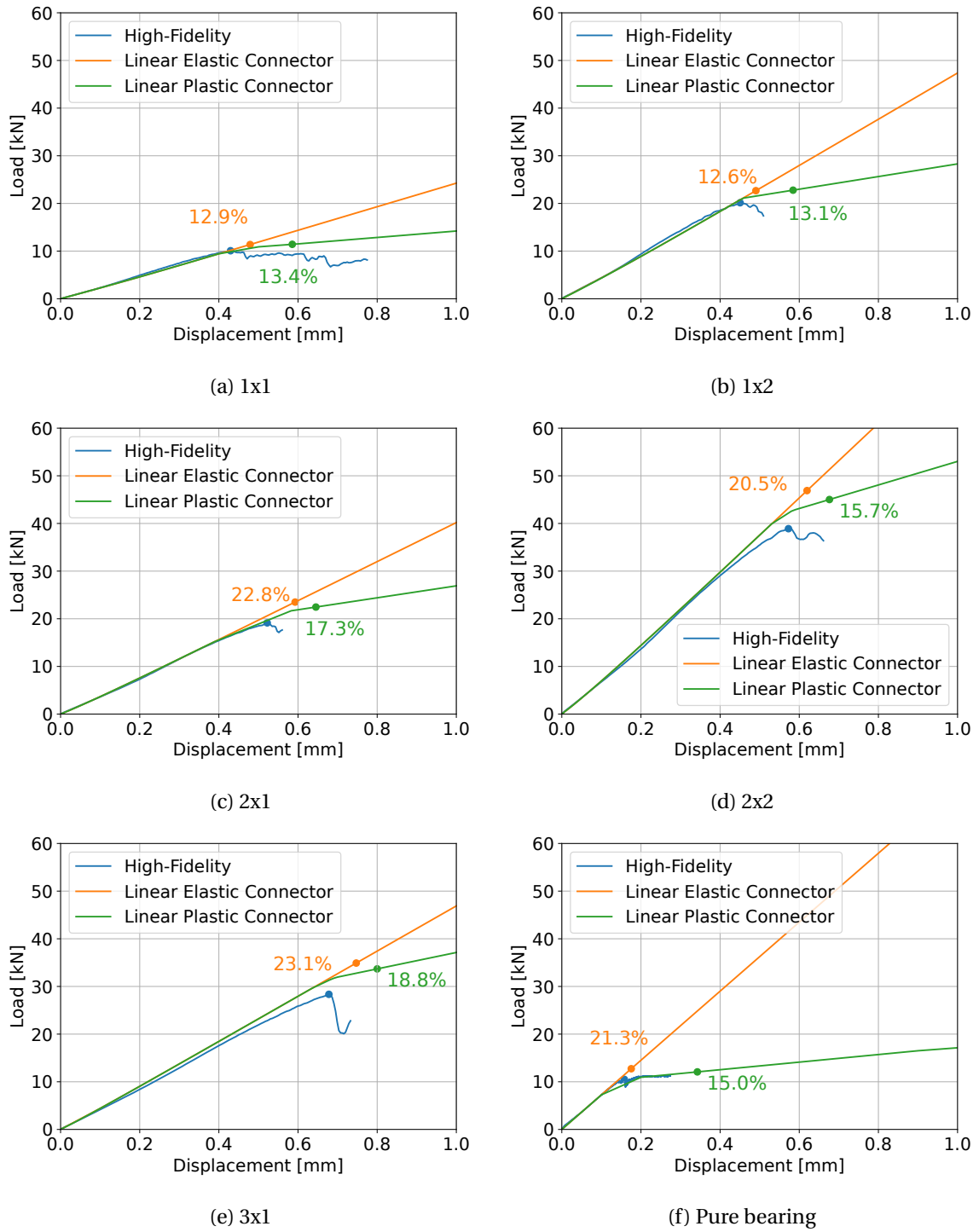


Figure B.2: Comparison between the load-displacement curves for the high fidelity, linear elastic and linear plastic simplified models alongside the failure load error.

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