
International Portfolio and Currency Risk Management

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Abstract:

This research focuses on the creation and management of international diversified investment portfolios, especially in relation to currency risk. It highlights the enduring problem of systemic and currency risks associated with global investments. The study analyzes the practical and theoretical aspects of hedging tools like forwards and options. An empirical analysis simulates a five-year investment plan for an investor, measuring returns, volatility, and Sharpe ratios for hedged and unhedged portfolios against benchmarks. The findings reveal that robust currency hedging fortifies risk-adjusted performance, which enhances portfolio outcomes and minimizes foreign exchange risk.

Keywords: International Portfolio, Portfolio Theory, Currency Risk, Hedging

Sumário:

Esta dissertação foca-se na criação e gestão de carteiras de investimento internacionalmente diversificadas, com especial atenção ao risco cambial. Destaca o problema persistente dos riscos sistémicos e cambiais associados aos investimentos a nível global. O estudo analisa as componentes práticas e teóricas dos instrumentos de cobertura, como contratos forward e opções. Uma análise empírica simula um plano de investimento a cinco anos para um investidor, medindo os retornos, a volatilidade e os rácios de Sharpe de carteiras com e sem cobertura cambial, em comparação com benchmarks. Os resultados sugerem que uma cobertura cambial robusta reforça o desempenho ajustado pelo risco, melhorando os resultados da carteira e minimizando o risco cambial.

Palavras-chave: Carteiras Internacionais, Teoria das Carteiras, Risco Cambial, Cobertura de Risco

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Introduction

The rapidly globalized world has rendered the effective management of international investment portfolios both a necessity and a challenge. Investor capital is now crossing borders on a larger scale due to the search for higher returns and better diversification. While international diversification can significantly reduce risk and improve performance, it also adds another layer of complexity, especially with regard to currency risk. This dissertation looks into the practical and theoretical aspects of international portfolio construction with special emphasis on techniques for managing currency risk to hedge adverse foreign exchange movements.

The cornerstone of this work is the Modern Portfolio Theory (MPT), which posits a mathematical basis for considering the optimal tradeoff between risk and return. MPT makes it very clear how an investor can reduce unsystematic risk through diversification by adding assets with low or negative correlations. However, diversification, based on MPT, raises other concerns when applied to the international perspective, including the systematic risk due to global politics and economy, as well as, more importantly, currency risk. Therefore, it is necessary to pay attention and tailor investment strategies, which will be addressed through both theoretical exploration and empirical application in this dissertation.

The dissertation is systematically arranged such that it progresses from basic ideas to specified methodologies. The first part offers an extensive theoretical exposition on the fundamentals of portfolio management, the mechanics of international diversification, and the systematic and unsystematic investment risks. The analysis of currency risk, for instance, incorporates forwards and options as well as their theoretical explanations and practical usage. Classical models like the Capital Asset Pricing Model (CAPM), the Markowitz optimization model, and alternative approaches such as the Treynor-Black Model are scrutinized for their limitations to reveal the constraints and possibilities of practical portfolio management.

The second part of this dissertation covers empirical research where an international investment portfolio is simulated and constructed for an investor with a conservative risk appetite over five years. This research uses an empirical strategy based on the hypothesis that

portfolio performance can be improved using historical returns. Efficient portfolios with and without the currency risk hedge are created and tested. Portfolio performance with no hedge and with hedge is measured by calculating the return, volatility, and the Sharpe ratio, and compared against major benchmarks, including the MSCI All Country World Index (ACWI) and the S&P 500. This comparative analysis provides insight into the effectiveness of international diversification and hedging strategies in enhancing risk-adjusted returns.

This dissertation aims to connect the two worlds of practical investment decision-making and theoretical foundations. Understanding how international investors can manage risk and enhance portfolio performance is advanced with analyses of currency exposure and quantitative evidence on hedging strategies. The results highlight the importance of tailored hedging policies and show that risk management, when implemented effectively, transforms from a protective measure into a value-enhancing strategy for global investment portfolios.

A. Theoretical Background and Literature Review

The dissertation starts with a theoretical background and a literature review. The theoretical background builds up a foundation for the practical part. It helps interpret and analyze empirical findings. Furthermore, some relevant literature review from empirical studies, articles, and reports will be introduced to provide insightful information to support the theories. There are several topics we can deal with in this part (portfolio management, international diversification, risks in international portfolios, and currency risk hedging tools).

1. Portfolio Management

The theoretical foundation for portfolio management includes two parts. First, the modern portfolio theory, along with different optimization models, will be introduced. Secondly, the following section discusses the difference between active and passive management. This part is essential for our dissertation as it will be a foundation to create an international portfolio in the practical part.

1.1. Modern Portfolio Theory

Modern Portfolio Theory (MPT) serves as a fundamental principle of modern financial economics, which provides a quantitative decision-making system for investments when facing uncertain conditions. Harry Markowitz presented the concept of MPT for the first time through his groundbreaking 1952 publication “Portfolio Selection”, which transformed asset management by moving attention from single security traits to portfolio asset interconnections. Markowitz demonstrated to investors that they should evaluate both asset return expectations and return variance together with asset covariance when building portfolios (Markowitz, 1952).

The fundamental tenet of MPT demonstrates that portfolio-wide risk reduction through diversification maintains returns at current levels. Markowitz established this fundamental understanding, which led to the development of the efficient frontier as a visual tool that maps optimal portfolios according to their highest expected return at specific risk levels and their lowest risk at defined return targets (Markowitz, 1959). Investors who choose assets

with low correlations can reduce unsystematic risk significantly while obtaining more consistent returns over time.

The evolution of MPT produced a risk management framework that separates systematic risk from unsystematic risk since systematic risk impacts the entire market without diversification potential, but unsystematic risk exists at the asset level, and diversification can reduce it (Bodie, Kane, & Marcus, 2024). The conceptual framework developed by MPT enabled the creation of subsequent asset pricing models, including the Capital Asset Pricing Model (CAPM), which built on MPT by incorporating market equilibrium and expected risk premiums (Sharpe, 1964).

MPT started with the assumption of a closed domestic market, yet practitioners now apply it to international investment because cross-border diversification brings additional benefits through lower correlations between global markets (Solnik B. H., 1995). However, the introduction of international diversification creates new challenges, including currency risk, transaction costs, and geopolitical uncertainties.

Modern finance relies heavily on MPT, yet faces various criticisms from the financial community. Research evidence shows that financial performance fails to match normal distribution assumptions and independent data characteristics, while investors fail to behave as predicted by rational utility-maximizing models (Barberis & Thaler, 2003). MPT serves as an essential foundation both in academic research and investment practice since it enables the creation of optimal portfolios based on investor preferences and risk tolerance, which forms the basis of this dissertation's methodology and empirical analysis.

The vast influence of MPT depends on fundamental assumptions that help build mathematical models that explain investor choices. Portfolio optimization and asset pricing models like the Capital Market Line (CML) and CAPM derive their theoretical basis from these assumptions. Real-world applications may not perfectly match these assumptions, but they establish important principles for analyzing investment risks and returns.

The fundamental principle of MPT assumes investors demonstrate rational behavior alongside risk-averse tendencies. Investors demonstrate a preference for increased returns when faced with the same level of risk because they will take on higher risk only when offered higher expected returns (Bodie, Kane, & Marcus, 2024). The decision-making process of

investors, according to MPT theory, depends only on portfolio return statistics, including mean and variance, under the assumption that returns follow a normal distribution, about which these two parameters provide a complete description for the investors to decide (Elton et al., 2014). It also indicates that all investors will possess identical expectations regarding market performance. Every investor shares the same estimates regarding asset expected returns, standard deviations, and correlations. Therefore, all investors achieve the same efficient frontier from the accessible set of assets and then pick their point based on their risk tolerance (Markowitz, 1959).

In addition, the fundamental market requirement for MPT is perfect competition coupled with no market friction. The theoretical framework features the complete elimination of all transaction costs and taxes along with unrestricted borrowing and lending capabilities. Investors can borrow or lend any amount of funds without restriction at the risk-free interest rate while maintaining unrestricted short-selling abilities for all assets (Sharpe, 1964). Given those capabilities, investors can develop their preferred risk-return combination by mixing a risk-free asset with their optimal risky portfolio. Another assumption is that investment choices take place across a single time period because investors seek to optimize their utility at the period's conclusion. Despite the fact that, in reality, investors work with multiple time periods; this assumption enables model simplification by treating investment decisions as non-dynamic (Bodie et al., 2024).

Last but not least, the portfolio theory also assumes that investors can divide their assets infinitely and conduct continuous portfolio adjustments with precise accuracy. Although the theoretical optimization problems require this condition, practical implementation faces challenges because of market constraints such as minimum investment requirements and trading liquidity issues.

1.1.1. Portfolio Risk and Return

Modern Portfolio Theory focuses its analysis on measuring risk and returns at the portfolio level instead of analyzing individual assets separately. The change in perspective allows investors to design portfolios that generate optimal expected performance based on risk levels, thus producing better results than holding individual securities independently (Markowitz, 1952).

A portfolio's expected return is calculated as the sum of individual assets' expected returns weighted according to their respective portfolio percentages. In equation (1) below, the portfolio's expected return, denoted as $E(R_p)$ returns from its constituent assets, is w_i the fraction of the portfolio invested in asset i and $E(R_i)$ is the expected return of asset i (Markowitz, 1959). A portfolio containing n assets produces an expected return calculated as follows:

$$E(R_p) = \sum_{i=1}^n w_i E(R_i) \quad (1)$$

On the other hand, the weighted average approach does not apply when calculating portfolio risk since it involves more than simply combining individual asset risks. The calculation of portfolio risk through variance or standard deviation requires the incorporation of covariance or correlation between asset returns to capture their mutual relationships (Elton et al., 2014). Portfolio risk levels depend heavily on return volatility since risk measurement relies on either variance or standard deviation analysis while considering the correlation or covariance between asset returns. The formula shows the need to combine assets with different risk profiles to achieve better volatility reduction than using individual investments alone. Equation (2) illustrates the formula to measure risk in the case of a portfolio including 2 assets. σ_p^2 is the variance of the 2-assets portfolio, and $\sigma_{1,2}$ is the covariance between asset 1 and asset 2.

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{1,2} \quad (2)$$

The calculation expands to include a variance-covariance matrix containing all possible asset pairwise covariances when considering n assets in the general case. The off-diagonal terms in this matrix show how assets co-move with each other, which directly affects portfolio diversification benefits (Elton et al., 2014). The formula for the variance of an n -asset portfolio is generalized in equation (3) below:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \quad (3)$$

The statistical measure of how two assets' returns correlate with each other through covariance. A positive covariance value shows that asset returns move together, but a negative covariance indicates that asset returns move against each other. The value of covariance has no straightforward interpretation because it varies according to the measurement scale. On the other hand, the correlation coefficient serves as a better alternative to covariance because it standardizes the measurement. The relationship between the correlation coefficient and covariance is described through the mathematical formula in equation (4). The correlation coefficient ρ_{ij} between assets i and j ranges between -1 to +1. The lower the correlation, the greater the potential benefit of combining the two assets. A correlation value of -1 between assets allows for complete risk elimination through a perfect set of specific combinations of weights. When $\rho_{ij} = 1$, the portfolio risk equals the weighted average of the individual risks because no diversification effect exists (Bodie, Kane, & Marcus, 2024).

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j \quad (4)$$

This fundamental relationship in portfolio theory demonstrates how portfolio risk depends on asset individual volatilities and their interaction patterns. The mathematical principle of diversification demonstrates that portfolio risk becomes lower than average individual risk when assets exhibit a negative correlation or less correlation with each other. Therefore, the construction of variance-covariance matrices requires this relationship because it serves as the basis for calculating portfolio variance and supporting mean-variance optimization algorithms that generate the efficient frontier. Building optimal portfolios that lie on the efficient frontier requires a thorough comprehension of how return expectations interact with risk measures, including variance and correlation. These portfolios maximize return at a specific risk level or minimize risk at a target return level, and this concept will be analyzed in detail in the following section (Bodie, Kane, & Marcus, 2024).

Modern Portfolio Theory relies heavily on diversification because it shows the benefit of using assets whose returns do not have a perfect correlation. The non-linear movements between uncorrelated assets create offsetting effects that reduce volatility without affecting expected returns proportionally. This mathematical framework establishes the essential

principle behind diversification which serves as fundamental practice in domestic and international portfolio management.

Empirical studies have reinforced the significance of this principle. Statman (1987) showed that adding more stocks to a portfolio leads to significant reductions in unsystematic risk (asset-specific risk) until reaching a point where additional stocks provide decreasing marginal benefits beyond 20 to 30 randomly chosen stocks. Solnik (1995), Glen, and Jorion (1993) discovered that international market correlations remain relatively low, thus providing greater risk reduction through global diversification, though these correlations increase during worldwide crises. Markowitz (1959) showed that effective diversification reduces unsystematic risk significantly so that only systematic risk (market-wide risk) affects overall portfolio volatility. Systematic risk, which represents macroeconomic shocks and many others, remains impossible to diversify in one market, but unsystematic risk from company-specific events can be substantially minimized. The strategic implication indicates that portfolio construction needs evaluation of assets through their portfolio interaction effects. The essential role of diversification leads to better risk-return outcomes, which produce maximum utility for investors.

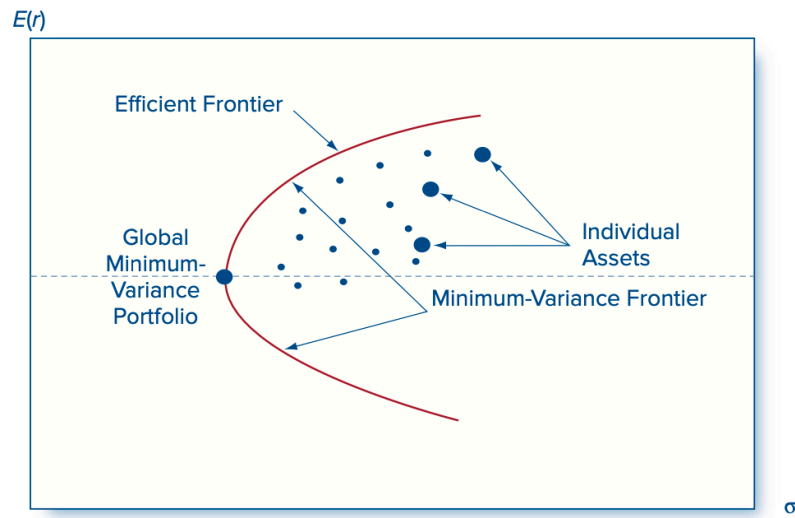
1.1.2. Efficient portfolios

A portfolio is efficient if it is not dominated by another portfolio in terms of both lower risk and higher expected return. The set of such portfolios forms the efficient frontier, which is upward-sloping and concave to the origin in the mean-standard deviation space (Bodie, Kane, & Marcus, 2024). Those portfolios that lie below the frontier are suboptimal because investors can find better alternatives that provide either higher returns with equivalent risks or identical returns with reduced risks. The efficient frontier is derived from combinations of risky assets (e.g., stocks and bonds) and visualized alongside the minimum variance portfolio (MVP), which represents the portfolio with the lowest possible risk. The minimum-variance frontier graphically shows the lowest variance given the portfolio's expected return.

Figure 1 below illustrates both the minimum-variance frontier and the efficient frontier. As we can see from Figure 1, the minimum-variance portfolio is the portfolio that possesses the lowest risk on the efficient frontier and also separates it from the suboptimal or inefficient portfolios, which lie on the lower frontier.

Investors can choose portfolios based on their risk tolerance through the efficient frontier construction process. Risk-averse investors will select portfolios from the lower end of the frontier, while risk-tolerant investors will choose higher-risk and higher-return combinations. Bodie et al. (2024) explain that investors base their selection on their utility function to create optimal asset allocation.

Figure 1: Minimum-Variance Frontier and Efficient Frontier



Source: (Bodie, Kane, & Marcus, 2024)

1.1.3. Capital Allocation Line

The Modern Portfolio Theory achieves more efficient portfolio selection through the addition of risk-free assets to the investment opportunity set. Due to the fact that the risk-free asset return mostly remains constant over time, its standard deviation equals zero, and its correlation with risky assets does not exist. The Capital Allocation Line (CAL), as presented in Figure 2, emerges from the combination of a risk-free asset with a specified risky portfolio as a straight line that shows all possible portfolio combinations. The CAL stands as a core investment management principle because it demonstrates portfolio diversification and risk-based investor decision-making under uncertain conditions (Bodie, Kane, & Marcus, 2024). The complete portfolio's expected return, $E(R_C)$, and standard

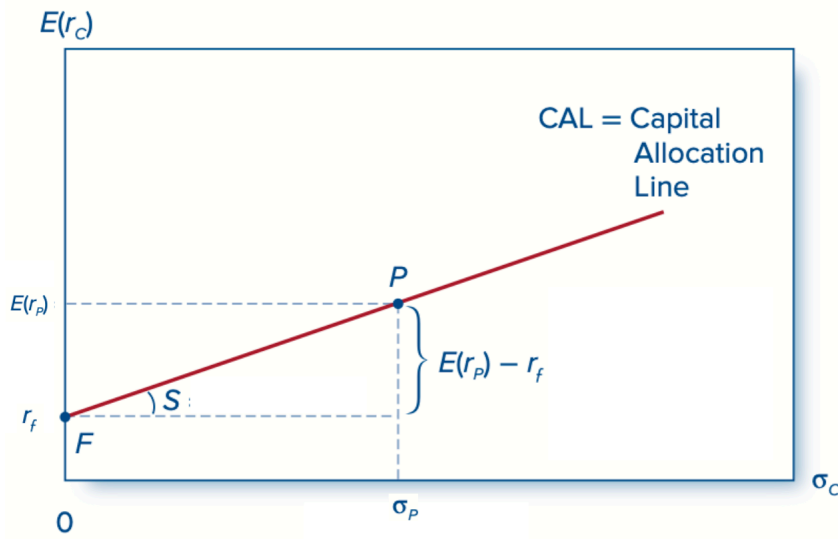
deviation, σ_C result from combining a risk-free asset R_f with a risky portfolio R_P according to the following equations:

$$E(R_C) = y E(R_P) + (1 - y)R_f \quad (5)$$

$$\sigma_C = y \sigma_P \quad (6)$$

The relationships between portfolio components establish a direct proportional relationship between risk and return. The Sharpe Ratio represents the slope of the CAL, which calculates excess return per unit of risk. A higher Sharpe Ratio indicates better risk-return performance. The optimal CAL exists as the tangent point to the efficient frontier of risky assets because it produces the highest Sharpe Ratio among all possible CALs.

Figure 2: Capital Allocation Line



Source: (Bodie, Kane, & Marcus, 2024)

The CAL enables investors to modify their total risk profile based on their personal risk tolerance. Risk-averse investors will put additional funds into the risk-free asset, whereas risk-tolerant investors will borrow at the risk-free rate to invest more than 100% in risky assets ($y > 1$). This decision can be written as a utility maximization problem. The investor's utility function uses a quadratic utility function to penalize variance, which is expressed as:

$$U = E(R_c) - \frac{1}{2} A \sigma_c^2 \quad (7)$$

The coefficient A determines risk aversion in this utility function. The optimal capital allocation results from maximizing this utility function with respect to the weight on the risky asset y :

$$y^* = \frac{E(R_P) - R_f}{A \sigma_P^2} \quad (8)$$

The equation (8) demonstrates that investors will take more risk exposure when the risk premium rises, but their exposure decreases when both their risk aversion and portfolio variance increase. The optimal complete portfolio exists on the CAL and represents the maximum achievable indifference curve from the investment opportunity set (Bodie, Kane, & Marcus, 2024).

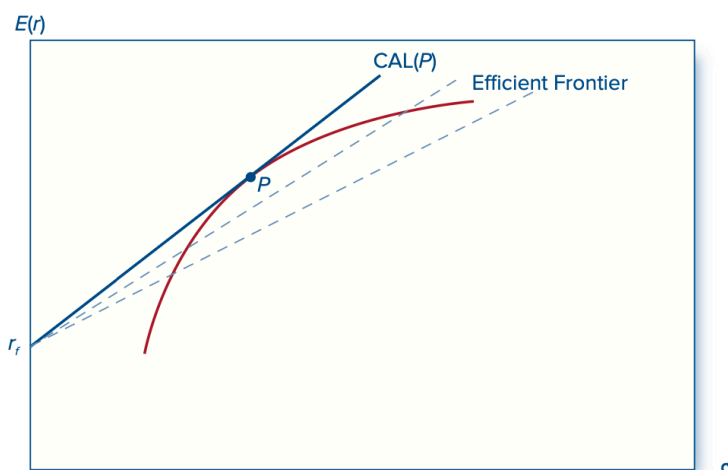
1.1.4. Markowitz Portfolio Optimization

The Markowitz Portfolio Optimization Model functions as the central computational instrument that supports MPT through its theoretical framework. Harry Markowitz created this model during the early 1950s to use quantitative methods that identify efficient asset combinations through expected return and variance and covariance analysis. The Markowitz model presents portfolio construction as a two-part process, which includes security selection followed by asset allocation. The selection of risky assets for a portfolio requires finding combinations that reduce portfolio volatility at a specified expected return level. A mathematical optimization framework controls this process by minimizing portfolio variance through specified constraints on expected return and asset weights (Bodie, Kane, & Marcus, 2024).

The security selection process is generally designed for a portfolio that includes a risk-free asset and many risky assets. The optimization plan first defines all the combinations of risks and returns from the pool of risky assets. These are computed based on equations (1), and (2). The next stage is capital allocation by introducing a risk-free asset to the portfolio. Here, what we need to do is figure out the optimal risky portfolio. Simply put, it can be done by

identifying the optimal weight for risky assets, which results in the CAL with the highest slope. To do so, we find the tangency point between the efficient frontier and CAL. This is also the point that will show the highest Sharpe Ratio; therefore, it asserts dominance against other CAL in broken lines as presented in Figure 3 (Bodie, Kane, & Marcus, 2024).

Figure 3: Efficient frontier of risky assets and the optimal CAL



Source: (Bodie, Kane, & Marcus, 2024)

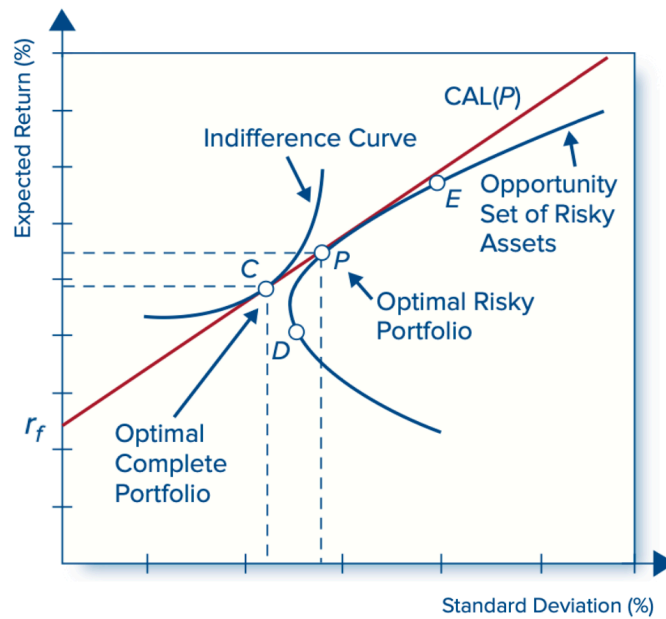
Once we have the optimal risky portfolio, the final step in the optimization process is to determine the allocation mix between the optimal risky portfolio and the risk-free asset. The main difference between this and the first two steps is that the investor's or the client's risk preference only exists here. It means that with the set of risky assets, portfolio managers will give the same optimal risky portfolio to all of their clients regardless of their risk appetite. Nonetheless, investors' risk tolerance will decide the appropriate asset allocation mix in the final stage. If they have low risk-tolerance or high risk-aversion, more investment will be assigned to risk-free assets and less to the optimal risky portfolio, and vice versa (Bodie, Kane, & Marcus, 2024).

1.1.5. Optimal Complete Portfolio and Utility Maximization

Investors can create different portfolios through the combination of a risk-free asset with an optimal risky portfolio, which extends along the Capital Allocation Line (CAL). Every investor selects their most suitable portfolio from available combinations based on their individual risk tolerance. The Optimal Complete Portfolio represents the point on the CAL that delivers the highest satisfaction to investors based on their risk versus return preferences.

Portfolio theory uses utility functions to model individual preferences because these functions measure the satisfaction that investors receive from various risk-return combinations. Bodie, Kane, and Marcus (2024) explain that investor preferences show up as risk-return indifference curves when graphically represented. Each curve shows the investor's utility-equivalent portfolio points that lie on the same utility level.

Figure 4: Optimal Complete Portfolio



Source: (Bodie, Kane, & Marcus, 2024)

The upward-sloping convex shape of indifference curves shows that investors want better returns when they accept similar risk levels and need more return compensation to take on risk increases. The highest indifference curve that touches the CAL determines the investor's optimal complete portfolio position (Bodie, Kane, & Marcus, 2024).

1.1.6. Alternative models

In portfolio optimization, while the Markowitz optimization framework has set the foundational principles for managing portfolio risk and return through diversification, alternative models have been developed to address some of its limitations, notably the Single-Index or Sharpe Model and the Treynor-Black Model.

The Single-Index Model, also known as the Sharpe Model, simplifies the Markowitz model significantly by assuming that returns of individual securities are correlated with a single factor, typically a market index. According to Sharpe (1963), this model captures the systematic risk shared by all assets through their individual beta coefficients, significantly reducing the complexity involved in estimating covariances among asset returns. The model implies that each security's expected return can be described by its specific alpha, the systematic component related to the market, and its specific risk, independent of market movements (Bodie, Kane, & Marcus, 2024).

The Multiple-Index Model extends the Single-Index framework by considering multiple systematic factors affecting security returns. By including various macroeconomic and financial market factors such as GDP growth, inflation rates, interest rates, and foreign exchange fluctuations, it provides a more comprehensive and accurate depiction of asset returns. The model recognizes the impact of several common influences rather than attributing all systematic risk to a single market factor, thereby enhancing precision in portfolio risk assessment and diversification strategies.

A key advantage of the Single-Index and Multiple-Index Models lies in their practical simplicity and lower computational requirements compared to Markowitz's full covariance matrix approach. However, these models may still oversimplify market dynamics, omitting potentially relevant factors not explicitly included in the indexes or failing to capture idiosyncratic variations accurately.

Another notable alternative, the Treynor-Black Model, integrates elements of both passive and active management strategies (these strategies will be discussed more in the following section). It aims to enhance portfolio performance by combining a broadly diversified passive portfolio, designed to replicate market performance, with active bets on selected securities that managers perceive as mispriced based on specific research and forecasts. Treynor and Black (1973) introduced a structured approach to balancing these active positions, optimizing the portfolio's risk-return trade-off by weighting securities in direct proportion to their expected alpha relative to their specific risks.

The strength of the Treynor-Black Model is in its dynamic flexibility, accommodating both market efficiency assumptions and the managers' abilities to identify and capitalize on

mispricing opportunities. It effectively manages the trade-off between diversification, through passive investing, and potential outperformance, leveraging the manager's forecasting ability. However, practical implementation can be complex and depends critically on accurate and reliable alpha forecasts, presenting challenges due to prediction errors or biases (Treynor & Black, 1973).

Despite these alternative models, Markowitz's Optimization Model remains highly relevant and useful. Markowitz's model explicitly accounts for the interactions among all securities within a portfolio, providing a nuanced view of risk management through diversification. Its strength lies in its comprehensive utilization of covariance matrices, allowing detailed portfolio tailoring based on precise investor risk-return preferences. Though computationally intensive, Markowitz's approach is advantageous due to its flexibility, theoretical rigor, and the ability to precisely manage portfolio risks at various scales. Furthermore, its broad applicability across different types of asset classes and global portfolios makes it indispensable for sophisticated investment management practices (Bodie, Kane, & Marcus, 2024).

However, the Markowitz Optimization Model is not without limitations. A significant issue is the complexity and computational intensity, particularly with large numbers of securities, which involves estimating an extensive number of covariances. Additionally, the accuracy and reliability of the model's results heavily depend on the quality of input data, which can lead to estimation errors and suboptimal portfolios if inputs are inaccurate. The model also tends to produce extreme asset weights, making practical implementation challenging unless specific constraints are applied (Bodie, Kane, & Marcus, 2024).

These alternative models have enriched portfolio theory, enhancing tools available beyond the classic Markowitz framework. Each model offers unique perspectives and practical solutions tailored to specific scenarios or constraints, yet the Markowitz model's depth, comprehensiveness, and theoretical soundness often provide superior portfolio optimization outcomes in practice.

1.2. Passive and Active Management

Portfolio management strategies typically fall within two broad categories: passive and active management. The choice between these strategies fundamentally depends on beliefs about market efficiency, managerial skills, and investment goals.

Passive management involves creating a portfolio designed to mirror the performance of a specific market index. It is based on the Efficient Market Hypothesis (EMH), which posits that market prices reflect all available information, making it exceedingly challenging to outperform the market consistently. Passive management, therefore, does not seek to beat the market but rather to replicate its performance at minimal cost. Common strategies include pure indexing and stratified or optimization sampling methods, each balancing tracking accuracy against transaction costs (Bodie, Kane, & Marcus, 2024).

In contrast, active management relies on the premise that skilled managers can identify undervalued or overvalued securities and strategically adjust portfolio allocations to outperform market benchmarks. Active strategies encompass security selection, tactical asset allocation, market timing, and sector rotation (Grinold & Kahn, 2000). However, active management carries higher transaction and management costs and demands considerable analytical resources to support decision-making.

Empirical evidence on the relative effectiveness of active versus passive management is mixed. Studies by Carhart (1997) indicate little consistent evidence of superior active management performance when accounting for management fees and transaction costs. Conversely, Berk and van Binsbergen (2015) argue that skilled active managers can generate value, although distinguishing skill from luck remains complex. Active management's effectiveness can vary significantly depending on market conditions, the manager's expertise, and the accuracy of investment predictions.

A critical issue in active management is manager oversight, which includes avoiding practices such as closet indexing and ensuring that active strategies are genuinely implemented. This can be managed through clear tracking error constraints and splitting portfolios into actively and passively managed components, facilitating more transparent oversight (Jorion, 2003).

Ultimately, the selection between passive and active portfolio management strategies should align with investor-specific considerations such as risk tolerance, investment horizon, costs, and the perceived ability of managers to deliver superior performance consistently.

2. International Diversification

International diversification is defined as the investment in financial instruments and assets beyond the confines of one's country with an aim of optimizing returns while managing risk. It extends the Modern Portfolio Theory, which states that portfolios consisting of assets that are not perfectly correlated with one another will yield better risk-adjusted returns. This principle, when taken to the international scale, allows investors to take advantage of distinct economic cycles, currency fluctuations, and market activity, which often results in better portfolio performance relative to investments confined to one's home country (Bodie, Kane, & Marcus, 2024).

An international portfolio is marked by cross-border diversification from both developed and emerging markets, access to multiple currencies, and varying economic and political contexts. This exposure presents the potential for augmented returns, but also introduces additional complexities such as currency risk and regulatory differences. The diversity of international financial markets enables investors to tap into sectors and growth opportunities that are not easily accessible within the country. For instance, emerging markets like India or Brazil often offer high-growth potential, while developed markets like the U.S. or Germany provide liquidity and stability. Collectively, these components create a synergistic and robust investment structure (Eun, Resnick, & Chuluun, 2021).

The advantages gained from international diversification stem from the fact that different national markets have low correlations. This implies that negative returns in one region are less likely to occur at the same time as declines in other regions, thereby mitigating aggregate portfolio volatility. According to studies by Glen and Jorion (1993), and further supported by Solnik (1995), the addition of foreign securities can reduce portfolio risk by up to 40%. These effects are most prominent during normal periods but may change during global financial crises when market correlations typically rise sharply (Stewart, Piros, & Heisler, 2019).

Regardless of being appealing in theory, the phenomenon known as 'home bias' illustrates that international diversification is not fully being utilized by investors. Home bias indicates the overarching tendency of investors to overly allocate funds to domestic securities, often to the detriment of much greater international diversification. This behavioral bias persists

for a number of reasons, including familiarity with domestic companies, perceived information asymmetry, aversion to risks associated with foreign currencies and politics, and even regulatory or tax constraints. Overconfidence and inertia are two behavioral explanations that have been widely discussed in the context of home-biased investment decisions (Stewart, Piros, & Heisler, 2019). Empirical data highlight this pattern starkly; U.S. investors, for instance, allocate less than 20% of their portfolios to international assets, despite the U.S. representing less than half of global market capitalization (Elton, Gruber, Brown, & Goetzmann, 2014).

International investment opportunities differ with respect to the classification of the markets. The United States, Germany, and Japan, as developed markets, offer high liquidity and a stable political climate. On the other hand, Vietnam, India, and South Africa, as emerging markets, provide demographic advantages and access to faster economic growth. However, emerging markets also bring heightened risks such as political instability and economic volatility. Not only are developed markets slower to evolve, but they are also more correlated. Emerging markets, however, enhance portfolio performance due to their unique growth trajectories and decoupled economies (Eun, Resnick, & Chuluun, 2021).

New emerging markets enable stronger portfolio returns. Simply put, international investments enhance diversification benefits. Emerging markets offer unique growth opportunities while developed markets carry greater risk at a measured return. However, the unhedged portfolio frame suffers deep currency risk, eroding value when foreign assets are converted back to the base currency. Additionally, changing policy risks tend to violate investor trust. Market liquidity is another concern, as smaller foreign markets may have wider bid-ask spreads and lower trading volumes. Further complications arise from varying accounting standards, which can hinder cross-country comparisons and distort financial analysis (Bodie, Kane, & Marcus, 2024).

Strategically, international portfolios can be managed through passive global allocation, regional focus, or thematic strategies targeting specific sectors like green energy or technology. Effective performance evaluation of international portfolios requires separating asset allocation effects, currency impacts, and security selection results. Research shows that strategic asset allocation accounts for the majority of portfolio performance variability over

time, underscoring the importance of carefully balancing exposures across regions and asset classes (Stewart, Piros, & Heisler, 2019).

3. Portfolio Risk

This chapter deals with an introduction to risk. In particular, we will classify it into unsystematic risk and systematic risk. It helps understand the type of risk international investors may have to face when considering investment globally.

3.1. Unsystematic risk

Unsystematic risk, also known as specific or idiosyncratic risk, is the portion of the total investment risk that is associated with individual assets or a small group of assets within a portfolio. It results from internal, firm-specific events such as changes in management, product recall, labor strikes, or regulatory actions that are not directly linked with market-wide effects. Unlike systematic risk, which is market or economy-wide, unsystematic risk is specific to a particular company or industry and can be diversified away through the right portfolio management.

Unsystematic risk contains multiple distinct categories of risk. The most prevalent type of unsystematic risk is business risk, which emerges from operational inefficiencies, poor strategic decisions, and consumer demand fluctuations that directly impact company revenues and profits. In addition, financial risk emerges directly from a company's capital structure, especially when it depends heavily on debt financing. Companies with significant leverage exposure face greater risks from interest rate fluctuations and credit market changes, which endanger their financial solvency (Brealey, Myers, & Allen, 2020). Another type of unsystematic risk is operational risk, which refers to the potential for failures in internal processes, systems, or human errors that disrupt the smooth functioning of a firm's operations. Firms face regulatory or legal risks when adverse legal actions or government policy changes threaten their operational profitability. Negative publicity, together with ethical breaches and corporate scandals, creates reputational risk, which damages both public image and investor confidence, thus leading to decreased stock value. Event risk encompasses the unpredictable effects of isolated occurrences such as natural disasters, labor strikes, and major lawsuits that affect specific companies, though they do not impact the market as a whole (Jorion, 2007).

The theoretical basis of unsystematic risk can be traced back to the Modern Portfolio Theory, which defined total portfolio risk as the sum of systematic and unsystematic risks. Markowitz showed that by pooling assets with low or negative correlations, investors can minimize unsystematic risk since the losses of some assets can be compensated for by the gains of other assets. As the number of uncorrelated assets in a portfolio increases, the only risk that remains is systematic risk, which cannot be diversified away, and it can reduce home unsystematic risk using the international portfolio.

This perspective is in agreement with Elton et al. (2014), who stressed that diversification is the key to reducing unsystematic risk. They state that as the portfolio size increases, particularly beyond 20 to 30 randomly picked stocks, the incremental decrease in total portfolio volatility is almost zero, which means that most unsystematic risk has been eliminated. This principle is used in a wide range of investment approaches, including index investing, where the exposure to the risk of a particular firm is reduced to a minimum.

Studies have also provided evidence of the diversification effect. For example, Statman (1987) found out that as the number of stocks in the portfolio increases, the volatility of the portfolio returns also decreases, but at a certain point, the addition of more stocks does not bring much further reduction in risk. Furthermore, research based on global investment has shown that not only does country-specific unsystematic risk decrease with international diversification, but also economic cycles in different regions are not correlated, thereby improving the risk-return trade-off.

Although it is diversifiable, unsystematic risk is still important for portfolio managers, especially in managed funds or concentrated portfolios. Investors who wish to beat the market take deliberate, unsystematic risks through security picking or market timing. However, this means that the potential gains may be higher, but so is the possibility of experiencing firm-specific events. Thus, identifying and managing unsystematic risk remains a key goal in active investment management.

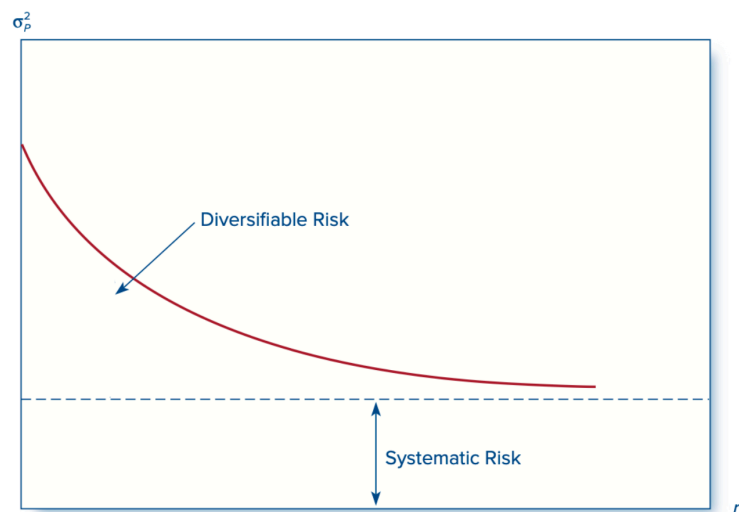
Therefore, although unsystematic risk is often neglected due to the fact that it can be diversified away, it is an important factor in determining the overall risk of a portfolio and in making investment decisions. Thus, the management of this risk through diversification is in

line with the concepts of modern portfolio theory and is beneficial for the overall efficiency and effectiveness of the portfolio in the long run.

3.2. Systematic risk

Systematic risk, which is also known as market risk or non-diversifiable risk, is that portion of total investment risk that is caused by wide macroeconomic factors that affect the whole market or economy. Unsystematic risk can be reduced by diversification, but systematic risk is a component of the financial system and cannot be reduced to zero as depicted by Figure 5. They include changes in interest rates, inflation rates, economic recession, political instability, and other financial crises that affect the entire market.

Figure 5: Systematic Risk vs Unsystematic Risk



Source: (Bodie, Kane, & Marcus, 2024)

This class of risk is very crucial in the development of MPT and CAPM. Markowitz, in his work on portfolio optimization, noted that although diversification can help reduce portfolio risk, no portfolio can be completely safe from systematic factors. Sharpe (1964) expanded this idea in CAPM by introducing the beta coefficient (β), which measures the sensitivity of a security to the market. A higher beta means higher sensitivity to market fluctuations, and so investors need to be compensated with higher returns for taking this inevitable risk.

Systematic risk encompasses various interrelated types. Interest rate risk is the risk that comes with all financial assets due to the change in monetary policy and the decisions made by the

central bank as far as the cost of borrowing and asset value are concerned. Inflation risk is the decrease in the purchasing power of money over time, which means that the nominal returns on investment will be lower in real terms (Eakins & Mishkin, 2018). Economic risk is the risk that is related to business cycles and includes expansion, contraction, and stagflation that affect employment rates, consumption, and investment income (Bodie, Kane, & Marcus, 2024). Political and geopolitical risks, which include elections, wars, or changes in the government, can lead to a lot of uncertainty in the market across the world (Jorion, 2007). In the case of international business, currency risk is also systemic in that exchange rate volatility will affect the returns on foreign assets regardless of the performance of the specific company (Bekaert & Hodrick, 2017).

These risks, with that being said, lie under the umbrella of systematic risks. However, they are implicitly expressed as unsystematic risks as well. For instance, with interest rates being higher, firms that possess a high level of leverage can be affected. In the same way, companies with different cost structures and pricing power cope with the impact of inflation risk. Political and geopolitical risks, while influencing the global markets, can simultaneously show direct impact towards some specific markets, sectors, industries, or firms, thus making it a concern for unsystematic risk as well. Exchange rate risks, as said earlier, clearly affect the global markets and international investors, creating many troubles in managing cross-border cash flows. At the same time, specific businesses or investors who have higher exposure to the risk of currency fluctuations might face greater foreign exchange risk. As a result, we should always count those risks discussed above as both systematic and unsystematic risks. By understanding this, subjects which confront such risks are able to manage them.

These wide-ranging and persistent risks require a risk premium. CAPM captures this relationship in the equation:

$$E(R_i) = R_f + \beta_i \times [E(R_m) - R_f] \quad (9)$$

where $E(R_i)$ is the expected return on asset i , R_f is the risk-free rate, β_i is the beta of the asset, and $E(R_m)$ is the expected return on the market portfolio (Sharpe, 1964). According to Elton et al. (2014), the existence of systematic risk means that investors can only get higher returns if they are willing to take a higher level of systematic risk.

While international diversification can be used to reduce unsystematic (country-specific) risks, it cannot eliminate systematic shocks. In the case of the 2008 global financial crisis or the COVID-19 pandemic, for instance, the correlations between international markets increased; hence, portfolio losses were observed across the board. These events also go to show that even well-spread-out portfolios are not safe from the effects of macro factors (Glen & Jorion, 1993).

In conclusion, systematic risk is a real factor that cannot be ignored in investment as it is related to the impact of the macroeconomic and geopolitical factors on all types of assets. Even though it cannot be diversified away, it can be defined, measured, and managed through the right asset allocation and hedging. Thus, it is very important to understand and to take into consideration systematic risk in order to make the right investment decisions, to create stable portfolios, and to achieve maximum return on investment.

3.2.1. Exchange rate risk

International investors need to consider exchange rate risk because it represents a fundamental factor that affects their portfolio risk. The value of foreign currency investments becomes vulnerable because exchange rates can fluctuate in ways that generate adverse effects on investments. The exchange rate movements affecting global asset allocation decisions result in unpredictable outcomes that strongly influence investment returns for investors who operate worldwide. Traditional financial theory focuses on market or systematic risk, but currency risk stands as a crucial separate risk management dimension in an era of rising capital mobility and financial globalization (Solnik & McLeavey, 2008).

Currency risk exists in three distinct categories, which include transaction risk and translation risk, along with economic risk. Foreign currency exposure exists in transactions that include purchasing foreign assets and receiving foreign dividends, as well as repatriating profits. Translation risk impacts multinational corporations because it shows how currency exchange rates affect financial statements when foreign subsidiary accounts need to be combined into the parent company's reporting currency. Economic or operating exposure stands as the most extensive category within currency risk. The effect of currency changes on long-term business competitiveness, together with pricing abilities and market value, represents the third category (Madura, 2020).

The primary concern for international investors regarding transaction risk directly affects the actual returns they earn from foreign securities. A local currency investment profit becomes negative compared to the investor's base currency due to unfavorable currency exchange rates, even though the investment returned positively. On the other hand, the benefit of currency movements can increase investment profits through speculative gains despite investors' intentions. Not only does it affect the returns of the portfolio, but it also impacts the portfolio's overall volatility. This is due to the fact that the return of the foreign assets inherently combines the return of the assets themselves in the local currency and the return realized from the exchange rate. Thus, the exchange rate volatility increases the volatility of the investment return. Consequently, this will impact the risk-adjusted performance of the international portfolio.

Modern Portfolio Theory faces challenges because it assumes a normal distribution of asset returns while ignoring added complexities from currency risk when seeking international diversification. Exchange rates often display unpredictable and asymmetric patterns because political events combine with central bank actions and differences between economies (Santis & Gérard, 1998). Such features violate the fundamental assumption that risk measurement depends solely on standard deviation. However, such non-normal distributions like asymmetry and fat tails are not limited to only exchange rate risk, but all other risks that exist. Therefore, it is part of the critique of the mean-variance framework and influences both international and domestic portfolios.

Research evidence shows that currency volatility generates major fluctuations that affect total portfolio risk levels in international investments. Solnik (1995) conducted pioneering research into international diversification by showing that unhedged portfolios exhibited higher volatility than hedged portfolios. At the same time, a hedged portfolio enhances the risk-return trade-off. This is measured by the Sharpe ratio, which evaluates the risk-adjusted performance. Glen and Jorion (1993) found that although international diversification provides better risk-adjusted performance, it creates currency-related volatility, which investors should address.

Investors use different hedging methods to protect themselves from currency risk. The main hedging strategies for managing risk include passive hedging through constant hedge ratios or full hedging, and active dynamic hedging, which involves adjusting hedge ratios using

economic indicators and market forecasts. Currency forwards, along with futures, options, and swaps, represent common hedging instruments.

Black (1989) developed the theory of universal hedging, which presents an optimal hedging method that considers exchange rate return covariance with asset returns. The approach of full hedging does not provide the best results because investors need to determine specific hedge ratios to optimize their returns based on risk factors. De Roon et al. (2003) demonstrate that selective hedging techniques, which adjust based on interest rates or economic factors, generate superior Sharpe ratios compared to unconditional hedging strategies.

Hedging comes with certain financial expenses. The advantages of hedging are diminished by the expenses of transactions, along with the costs of passing up beneficial currency movements and the intricate process of managing derivatives. According to Adler and Dumas (1983), investors should determine their hedging level based on their risk tolerance and time span, as well as exchange rate movement correlations with their assets.

Since the breakdown of Bretton Woods in 1971, the significance of currency risk has risen substantially as major economies have moved from fixed to floating exchange rates. The floating exchange rate system leads to higher exchange rate volatility because currency values depend on market forces such as interest rate differences, inflation projections, and political stability. Investors face elevated exchange rate risk when they invest in countries whose currencies experience volatility, particularly among emerging markets.

Currency volatility rates vary across different currency pairs while showing increased volatility during economic stress periods (Bekaert & Hodrick, 2017). Such situations lead to increased correlations between asset classes as well as between domestic and international markets, which makes diversification less effective, while currency fluctuations grow more powerful. During the 2008 financial crisis and the COVID-19 pandemic, the correlation breakdown appeared when global markets linked together while exchange rates experienced sharp movements.

Portfolio managers should recognize that currency exposure delivers both risk protection and revenue generation potential. Certain portfolio managers analyze currencies as their own distinct asset class by implementing active currency management that leads to improved

investment results. Studies have demonstrated that currency performance derives from three main factors: interest rate spread-based carry trades, as well as momentum and value-based assessment methods (Burnside, Eichenbaum, Kleshchelski, & Rebelo, 2011). The currency carry trade requires investors to borrow money from low-interest currencies and then use that capital to invest in higher-interest currencies. Although currency carry trade strategies produce additional profits, they may face rapid reversals during market declines, which reveals major tail risks. Therefore, for investors with low risk aversion, active currency management can improve the performance of the portfolio and provide diversification benefits beyond traditional equity or bond investments.

International portfolio management requires exchange rate risk to be treated as an essential factor. The unique volatility layer can both diminish investment returns and function as a diversification tool and generator of alpha. Investors need to adjust their hedging strategies according to their goals, risk tolerance, and investment constraints in order to properly handle currency risk. Currency risk management stands as a critical component for capital preservation and portfolio enhancement within our growing interconnected global market.

4. Currency Risk Hedging

International investment brings exposure to foreign exchange risk, which is commonly known as currency risk. The inherent risk from exchange rate movements undoubtedly leads to investment return reductions during foreign earnings conversion into the investor's home currency. Hence, internationally diversified portfolios experience substantial return volatility because of currency fluctuations, which demonstrates the need for efficient currency risk management practices (Madura, 2020). Currency hedging exists to shield investments from unpredictable exchange rate changes. Investment companies implement hedging strategies to decrease portfolio volatility and stabilize return performance while potentially improving risk-adjusted investment results. In order to achieve those objectives, investors use derivative instruments (Hull, 2022). Foreign exchange risk management depends heavily on derivatives because they offer flexible and effective solutions through forwards, futures, options, and swaps.

Modern Portfolio Theory provides a foundation for currency risk hedging practices, despite the fact that its main focus remains domestic asset diversification. MPT demonstrates how diversification works by using assets that show low or negative correlations. The diversification advantages become harder to achieve because currency volatility correlations rise during financial crises (Solnik & McLeavey, 2008). Effective currency risk management becomes essential for internationally diversified portfolios because of its critical role in protecting investment returns.

Hedging strategies are classified into passive and active categories. The passive hedging approach requires fixed hedge ratios that maintain stable protection levels without the necessity for regular modifications. Nevertheless, it comes with a trade-off. This strategy provides complete or substantial currency risk protection but gives up potential benefits from positive currency fluctuations. On the contrary, active hedging strategies require dynamic changes to hedge positions according to projections of future exchange rates combined with economic indicators and market analysis (Brigham & Ehrhardt, 2019). Although active portfolio management generates better returns, it demands both significant financial resources and advanced skills in making forecasts.

The theoretical foundation of optimal hedge ratio analysis began with Fisher Black's initial discussion (Black, 1989). In the paper, Black defines that the optimal hedge ratio depends on three essential variables: international portfolio returns expectations, portfolio volatility, and exchange rate volatility. The process of choosing an optimal hedge ratio demands weighing hedging costs against risk reduction benefits because higher hedge ratios generate diminishing marginal returns (Black, 1989).

The empirical literature has demonstrated the effectiveness and strategic value of currency hedging in international portfolios. Glen and Jorion (1993) demonstrate through their research that portfolio managers who implement currency hedging strategies achieve reduced volatility without compromising their risk-adjusted returns. The authors also highlighted that active currency hedging through conditional approaches performed better than fixed hedging methods. The research by De Roon and his co-authors (2003) builds upon earlier findings to examine currency hedging using mean-variance optimization methods. The study showed that investors who employed conditional hedging strategies achieved superior portfolio performance, especially when hedge ratios changed dynamically according to interest rate differences and macroeconomic indicators. Unconditional hedging proved less successful at delivering improved returns because it failed to demonstrate consistent performance across longer time periods.

Current research shows that hedging operations come with certain limitations. De Roon, Eiling, Gerard, and Hillion (2012) reveal that currency hedging may produce unfavorable effects on portfolio characteristics by reducing skewness and kurtosis, which increases the risk of extreme losses. Hedges may sometimes return lower results than unhedged investments because derivative transactions lead to additional costs and missed currency benefits.

Yu et al. (2020) demonstrate through empirical data that currency derivatives, particularly currency options, serve as highly efficient tools to reduce risk exposure in international investment portfolios. Options deliver enhanced portfolio performance through risk limitation and provide flexible protection beyond traditional forwards and futures instruments.

The research evidence shows divergent outcomes when evaluating different hedging instruments against each other. Xu (2014) analyzed international portfolio investments that combined developed markets with emerging markets by utilizing forwards and options, resulting in improved risk-adjusted returns. According to Xu, different market conditions and volatility levels recommend unique hedging strategies for investors. Emerging markets benefited most from deep out-of-the-money put options, whereas moderately out-of-the-money options generated superior results within developed markets.

Topaloglou, Vladimirov, and Zenios (2007) used stochastic programming to evaluate forwards and options. It was determined that strategically chosen forward contracts achieved better results than basic put options in professional portfolio management. But it also supports advanced options approaches that include multiple strike prices and expiration dates, which have generated superior results than basic forward contracts, especially when market volatility is high.

The empirical research about currency swaps as a tool for long-term risk management remains relatively limited. Kolb and Overdahl (2007) highlight swaps as beneficial instruments because they enable the matching of long-term cash flows, which provides institutional investors with predictable management of their international obligations. The long-term predictability of currency exposure makes currency swaps ideal for pension funds, together with multinational corporations and similar entities that face steady currency risks.

The literature indicates that successful currency risk management for international portfolios requires investors to strategically choose and deploy derivatives according to their risk tolerance and market expectations. With that being said, derivative instruments provide investors with innovative tools to manage currency risk better in the globalized financial environment.

4.1. Forwards

Currency forward contracts serve as the primary derivative instrument for international portfolio currency risk management. A forward contract represents a personalized agreement between two parties who agree to exchange particular currency amounts at a predetermined forward rate on a designated future date (Hull, 2022). Forward contracts differ from futures

contracts because they operate through over-the-counter (OTC) transactions, which enable counterparties to create customized terms that suit their particular requirements (Witzany, 2020). Forward contracts gain their main appeal because they can be tailored to meet individual needs. Investors and firms select currency amounts, settlement dates, and exchange rates to create precise hedging solutions. Given that attraction, forward contracts offer valuable flexibility, helping users manage transaction exposure and translation exposure (Hull, 2022).

The pricing mechanism for forward contracts operates theoretically through interest rate parity (IRP). The forward exchange rate should exhibit the interest rate differences between two currencies according to the interest rate parity principle. The covered interest rate parity condition allows mathematical representation of forward currency pair prices through the following formula (Hull, 2022):

$$F = S \times \frac{1 + r_d \times (T)}{1 + r_f \times (T)} \quad (10)$$

In the equation (10), F is the forward exchange rate, S is the current spot exchange rate, T is the time to maturity of the forward contract, r_d illustrates the interest rate in home currency, whereas, r_f represents interest rate in foreign currency. The failure to maintain covered interest rate parity creates opportunities for arbitrage activities. Because of that, the forward prices show strong alignment with theoretical IRP conditions in financial markets that operate effectively.

The profit from forward contracts emerges from the price difference between the agreed forward price K and the actual spot price S_T at maturity. Forward contracts allow investors to take either a long or short position (Hull, 2022):

$$\begin{aligned} \text{Payoff (Long)} &= S_T - K \\ \text{Payoff (Short)} &= K - S_T \end{aligned} \quad (11)$$

A long forward position requires investors to purchase currency at the agreed price K in the future. The investor earns profits when the spot price at maturity S_T surpasses the forward

price K because the currency value exceeds initial expectations. The investor who takes a short position in forward contracts must sell the currency at price K in the future. The short position becomes profitable when currency depreciation occurs because the spot price at maturity S_T drops below the agreed forward price K (Hull, 2022).

Forward contracts provide substantial flexibility to users, but they come with major risks, including counterparty risk. The lack of clearinghouse guarantees in forward contracts means that one party has the potential to default on their obligations, which could result in financial losses for the other party (Witzany, 2020). This risk can be managed through thorough counterparty selection and credit evaluation and by establishing collateral agreements.

Research evidence demonstrates that forward contracts function as effective risk management tools for international portfolios to reduce foreign exchange exposure. The effectiveness of forward contracts relative to other derivatives, such as futures and options, shows substantial differences based on market conditions and investor goals. Glen and Jorion (1993) investigated how forward contracts function as hedging tools for international equity portfolios through their research and found that currency forwards effectively reduced portfolio volatility. The research highlighted that investors could use forwards to secure exchange rates and stabilize their future cash flows, leading to predictable returns. The research conducted by Celebuski et al. (1990) showed that forward contracts cut international portfolio volatility by 40%, which proved most beneficial for investors who want stable returns.

The research by De Roon, Nijman, and Werker (2003) showed that forward contracts deliver strong results across multiple market conditions by improving the risk-adjusted performance of international portfolios. The study showed that dynamic or conditional forward strategies, which modify hedging positions according to interest rate differentials and economic indicators, produce better portfolio results than static or unconditional forward hedging approaches.

Maurer and Valiani (2007) used the mean-lower partial moment (LPM) method to compare currency forwards and options within international portfolios. Their results showed that forward contracts were generally more effective than options in terms of risk-adjusted returns when optimal hedging of portfolios was considered. However, they also noted that under

highly volatile market conditions, put options may offer better downside protection. Thus, their study indicated that the choice of hedging instruments depends heavily on the market environment.

Alvarado-Vargas and Kessakorn (2013) employed the Conditional Value-at-Risk (CVaR) model to assess the performance of forward contracts versus put options in portfolios that included Chinese Yuan and Indian Rupee-denominated assets. The results showed that forward contracts were effective in improving portfolio performance in different scenarios. They also noted that no single hedging instrument was universally superior, and that hedging strategies should be tailored to specific currencies and market contexts. Yin and Han (2013) used stochastic modeling to compare forwards with options. They found that forwards were superior in stable economic periods because they are easier to implement and more effective at fixing exchange rates, thus providing better hedging results.

Forward contracts have been shown to be useful and effective in managing currency risk, especially in stable or predictable market conditions. Although forwards may not be as effective in extreme volatility as options, their simplicity, customizability, and ability to lock in rates for specific future transactions continue to make them indispensable tools for international portfolio managers.

4.2. Futures

Currency futures contracts serve as standardized derivative instruments that financial institutions use to manage currency risk when handling international portfolios. A currency futures contract functions as an exchange-traded agreement that requires participants to exchange a set amount of currency at a predetermined exchange rate known as the futures price before a specific settlement date (Hull, 2022). The strict regulatory oversight and standardized features of futures contracts, such as fixed contract sizes, expiration dates, and delivery procedures, lead to higher market transparency and liquidity than forward contracts (Witzany, 2020).

The main difference between currency futures and forwards emerges from their trading frameworks. For instance, the Chicago Mercantile Exchange (CME) operates as a regulated exchange where standardized futures contracts enable simple trading and lower transaction

expenses. This centralized trading system of futures markets provides investors with high market liquidity because it enables smooth position entry and exit without causing major price fluctuations or market disturbances (Hull, 2022).

The daily mark-to-market settlement process in futures contracts requires traders to calculate and settle position gains and losses every trading day. Investors need to open clearinghouse margin accounts to meet their daily settlement requirements. This margin system is a protective mechanism because the clearinghouse acts as a middleman to guarantee that both parties fulfill their obligations. The clearinghouse takes responsibility for covering obligations when one party defaults, which protects the other party from facing default risk. This structure not only reduces counterparty risk, which is faced when dealing with forwards, but also lowers credit risk to maintain stability and financial integrity. Given that standardized scheme, it can, however, limit investors who want to hedge with specific amounts or particular settlement dates (Witzany, 2020).

Multiple studies have assessed the effectiveness of currency futures contracts for exchange rate risk management in international investment portfolios and demonstrated their practical application and hedging capabilities. Gastineau and Kritzman (1999) pointed out that currency futures possess standardized features and daily marking-to-market procedures that enhance risk mitigation and financial stability better than forward contracts. The authors also emphasized that futures contracts work well for short-term hedging and speculation because they offer high liquidity and simple trading processes.

The research by DeMaskey (1995) investigated how currency futures and options compare for foreign exchange risk management. According to DeMaskey, futures contracts work best for covered hedging strategies because they require exact knowledge of future currency requirements. The research established that futures contracts provide the most suitable solution for situations requiring straightforward payoff structures and predictable outcomes.

4.3. Options

Currency options grant holders the right to purchase (call) or sell (put) foreign currency at specified strike prices on or before their expiration date. It, however, requires investors to pay an upfront amount, so-called premium. The initial premium paid for options creates a

protective cap on losses, allowing investors to benefit from positive market conditions while safeguarding against currency market declines (Hull, 2022). Investors can use protective puts and covered calls as sophisticated strategies to create custom hedging solutions that match their risk tolerance and market expectations. The option premium highly depends on several factors, including the volatility of the market, the strike price, and the interest rates. The option premium is the maximum loss for the buyer, though they are capable of gaining substantially. On the other hand, although the seller receives the premium, it can leave their position at great loss in case the market moves against them. Given this asymmetric payoff of currency options, it enables investors to control foreign exchange risks and capitalize on market opportunities while protecting against negative market movements. Nonetheless, the implementation of currency options needs advanced financial modeling expertise together with ongoing market surveillance which leads to elevated operational complexity and increased costs (Witzany, 2020).

The payoff structure for the buyer of the option is mathematically described in the equation (12), in which K is the predetermined strike price, S is the spot exchange rate at expiration. In a call position, the buyer gains profits if S exceeds K , signifying that they bought the currency at low price and sell it later at higher price. In contrast, K surpassing S will favors the buyers in put options, as they can sell the currency at a higher price than the market.

$$\begin{aligned} \text{Call Payoff} &= \max(S - K, 0) - \text{Premium} \\ \text{Put Payoff} &= \max(K - S, 0) - \text{Premium} \end{aligned} \tag{12}$$

The two main types of currency options are European and American options. European-style options allow exercise only on the expiration date, whereas American-style options enable investors to exercise their options at any point before expiration (Hull, 2022). The selection between these styles depends on investor expectations, together with strategy and risk profile.

Currency options are also classified into three types: At-The-Money (ATM), In-The-Money (ITM), and Out-Of-The-Money (OTM) options, which are based on the difference between the strike price and the spot price. ATM options have a strike price equal to or very close to the current spot price of the underlying currency pair. ATM options are particularly popular

among investors because they exhibit the highest sensitivity, measured as "vega", to changes in market volatility (Hull, 2022). Due to this high volatility sensitivity, ATM options are extensively used in currency hedging strategies, as they provide balanced risk exposure and an efficient hedge against moderate currency movements (Witzany, 2020).

Additionally, there are two other primary categories of currency options based on the strike price. As for In-The-Money (ITM) options, the spot price is above the strike price in call options, and the spot price is below the strike price when it is put options. ITM options have intrinsic value and provide immediate payoff if exercised, thus typically being more expensive than ATM or OTM options. On the other hand, Out-Of-The-Money (OTM) options have no intrinsic value at the time of purchase. For call options, OTM means the strike price is higher than the spot price; for put options, it implies the strike price is below the spot price. Furthermore, OTM options are less expensive but have a lower probability of finishing profitably. Investors often use OTM options for speculative purposes or to hedge against extreme currency movements at a lower initial cost (Hull, 2022). Each type of currency option plays a unique role in currency risk management strategies. Therefore, investors must carefully choose options based on their specific hedging objectives, market outlook, and volatility expectations

The pricing of currency options relies on advanced mathematical models, with the Black-Scholes model standing as one of the most widely used. The Black-Scholes model calculates European-style currency call option prices (C) and put option prices (P) through the following mathematical formula (Hull, 2022):

$$\begin{aligned} C &= S_0 e^{-r_f T} N(d_1) - K e^{-r_d T} N(d_2) \\ P &= K e^{-r_d T} N(-d_2) - S_0 e^{-r_f T} N(-d_1) \end{aligned} \tag{13}$$

In equation (13), S_0 is the current spot exchange rate of the underlying currency, K is the strike price or exercise price, r_f and r_d are foreign and domestic risk-free interest rate, and $N(\cdot)$ is cumulative normal distribution function. The variables d_1 d_2 are given by the following formula, in which σ is the exchange rate volatility and T is the time until expiration of the option (Hull, 2022).

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_d - r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (14)$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The price of options reacts to three main elements, which include the volatility of the currency pair, interest rate differences, and time until option expiration. Option values tend to rise when volatility increases because higher uncertainty creates more potential payoff opportunities. Therefore, options become more valuable when volatility rises, which makes them particularly useful during periods of high currency market uncertainty. Furthermore, the time until option expiration works against option value because it reduces the available period for profitable price movements (Witzany, 2020).

The effectiveness of currency options in FX risk management has been well established through research that shows these instruments offer distinct advantages relative to forwards and futures. Topaloglou Vladimirou Zenios (2007) evaluated currency forwards and options through stochastic optimization in active portfolio management scenarios. Forward contracts delivered reliable protection against market risks, but carefully designed currency options strategies using multiple strike prices and maturities generated better results, especially when markets experienced high volatility. The researchers showed that options require careful application to reach their optimal hedging performance. Yin and Han (2013) studied currency options through stochastic modeling to determine their effectiveness relative to forwards and futures. Currency options demonstrated better performance than forwards and futures in volatile markets because they offered adaptable hedging solutions. Successful use of options demands both thorough market analysis and accurate forecasting methods, according to their research.

Yu et al. (2020) demonstrate that currency options serve as effective tools for strategic risk management of international portfolio downside risks. Investors who use currency options protect their investments from negative currency shifts but maintain the advantage of benefiting from beneficial currency movements. The one-sided payout design of options works best during unpredictable market fluctuations.

Xu (2014) expanded existing research by showing that currency options deliver successful results across various market environments. Xu demonstrated through Conditional Value-at-Risk (CVaR) analysis that currency options produced superior returns in emerging markets that experience elevated currency volatility. The research showed that options work well as risk management instruments when exchange rates exhibit substantial volatility.

Through their research using Lower Partial Moment (LPM) Lien and Tse (2001) demonstrated that currency options delivered better protection than other derivatives against major negative risks. Options prove especially valuable for uncertain currency positions because they protect against variable future exchange amounts and timing.

The study of Liu and Jacobsen (2021) investigates how currency options perform in dynamic hedging methods across international equity portfolios. The study showed that active portfolio management benefited from dynamic hedge ratio adjustments based on volatility predictions because this method produced better risk-adjusted returns than static and passive hedging approaches.

4.4. Swaps

Currency swaps enable parties to exchange principal amounts along with periodic interest payments in different currencies throughout a defined time period. The parties conduct a principal amount exchange at the beginning using a predetermined spot exchange rate. The parties then conduct periodic interest payments based on fixed or floating rates before re-exchanging the principal amounts at maturity using the original exchange rate. Swaps operate similarly to forward contracts because they act as OTC derivatives, which enable customized terms to fulfill the precise requirements of both counterparties regarding amounts, maturities, payment intervals, and interest rate structures. This agreement creates exposure to default risks for both parties involved. It needs proper credit evaluation and effective collateral handling for its mitigation (Hull, 2022).

The main purpose of currency swaps is to handle long-term currency exposure while helping companies and institutional investors manage their debt obligations and create predictable cash flow patterns when operating across multiple currency markets. Through swaps, institutions can transform their current debt obligations into better currency and interest rate

terms, which leads to improved financial standing and reduced debt servicing expenses. Swaps function better than forwards, futures, and options for extended periods of time, and they cover exposures from one year up to more than ten years. Nevertheless, currency swaps require extensive long-term financial commitments that restrict both liquidity and flexibility, so they are not appropriate for short-term or unpredictable exposures (Witzany, 2020).

Research on currency swaps through academic and empirical studies examines their ability to control long-term foreign exchange and interest rate risks in international financial management systems. Giddy and Dufey (1995) studied currency swaps because they serve as a fundamental tool for multinational corporations to control cross-border financial risks. The research demonstrates that currency swaps enable companies to manage interest rate risk and currency risk simultaneously which leads to reduced funding expenses and improved cash-flow stability. Kumar and Rabinovitch (2013) examined both the valuation processes and risk management techniques of currency swaps. The authors stressed that proper valuation and risk assessment techniques form the foundation for successful currency swap implementation. The research showed how swaps enable corporations and financial institutions to handle complex currency and interest rate exposures, but require advanced management abilities and credit-risk evaluation skills.

Popper (1993) conducted a detailed analysis and found that currency swaps reduced debt costs for firms more effectively than domestic borrowing or separate currency hedging instruments. The matching of debt obligations with revenue streams through swaps allowed companies to effectively hedge their long-term currency risks while reducing their total financial exposure.

B. Practical Part and Empirical Findings

The investment process starts by defining an *investment policy*. Firstly, portfolio managers must know their clients' objectives when selecting securities and assets. What is the current wealth of the investors, what are the current wealth goals, and what is the return on investment they demand? These are some of the questions we need to answer. Their use of money could also be a relevant topic for investment motives, whether the investors invest for retirement savings, to purchase a house in the future, or to prepare for urgent events such as sickness. Furthermore, when it comes to investor preference, there are also some restrictions regarding liquidity, investment time horizon, etc. An investor's risk profile is another important factor in deciding which assets to consider. Investors can be risk-averse, risk-loving, or risk-neutral. This information is usually obtained through a questionnaire given to the investor.

The next step is *security analysis*. In this stage, the investable assets such as equities, bonds, real estate, and commodities,... are chosen and managed according to the investor's risk profile and needs that are pre-defined in the previous step. The portfolio manager acquired the adjusted price of the securities, which serve as inputs for the return and risks of each asset and the correlation of those assets.

Once we have all the inputs of the securities, a *portfolio analysis* is followed to define a set of efficient portfolios by applying mean-variance optimization and modern portfolio theory. The optimal portfolio is then selected considering the investors' preferences according to their utility function. This is called *portfolio selection*. A periodic portfolio review, or rebalancing, is necessary to reflect the market conditions.

After the portfolio analysis and selection, we need to assess the performance; this process is called *portfolio performance evaluation*. In this step, we try to evaluate the risk and return measures as well as compare them to the appropriate benchmarks. Market indices can be considered as benchmarks as they usually determine the success of the investment strategy. Overall, the benchmarks should align with the investment policy and strategy, which are determined at the very beginning of the investment process.

5. Portfolio Construction

To start with the portfolio construction, we imagine ourselves as portfolio managers and try to help our investor who wants to establish an international portfolio. First, we start off with an investment policy. This is a very crucial step of the analysis as it sets strong foundations and establishes assumptions for security analysis, portfolio analysis, and selection.

5.1. Investment Policy

Mr. Bennett is a 40-year-old American citizen who wants to invest to have some funds for his retirement. He is seeking help for building a portfolio in which he demands that stocks are globally diversified. Because he knows that this portfolio would be exposed to the exchange rate risk, he would like to know to what degree the unhedged portfolio performs, and if he embraces hedging instruments, how well the portfolio would turn out to be at the end of the holding period. We have the description and preferences of Mr. Bennett as follows:

Table 1: Investor and Portfolio Characteristics

Age	40
Nationality	American
Objective of investing	Retirement savings
Investment period	1.1.2020 – 31.12.2024
Investment amount	100.000 USD
Active or Passive Portfolio	Active
Short-Selling	Yes (Risk-free assets), No (Risky assets)
Rebalancing	Annually
Tax rate	15%
Management and Performance fee	1%, 10%

Mr. Bennett does not specify any limitations regarding the weights of securities in the portfolio. He would also allow short selling, which would result in negative percentage numbers. The capital gain tax rate applied to the investor is 15%. In our portfolio, dividend tax will not be used because the adjusted close price, which we use in the analysis later, already reflects the dividends and stock splits that may happen. Therefore, it is taxed by the capital gain tax. Moreover, there is a management fee of 1% and a performance fee of 10%. These will be used to compare portfolio performance with and without taxes and fees.

We choose active management for the portfolio. We do not try to replicate market returns or follow a strategy of the passive portfolio, as we believe that, together with the good choice of securities, optimization models will give us a better performance than the market offers. To accomplish that, we will discuss the security selection process in the following chapters. We also compare our portfolio with the benchmark at the end to see if our confidence in the active strategy is really justified. By doing active management, we expect to have some performance and management fees from managing the portfolio actively, which is shown in Table 1.

Regarding the rebalancing of the portfolio, we decided to do this annually. The portfolio will not be rebalanced towards its initial weights at the beginning, but instead, the new data from the year passed will be included to estimate the new optimal portfolio weight. In order to do that, data from 5 years prior to the date of portfolio selection will be used and decide the portfolio selection. Simply put, portfolio optimization for the year 2020 will use data from 2015 to 2019. This process continues until it stops in 2024 as the investment period ends and our clients withdraw money upon fund liquidation.

5.1.1. Investor's Risk Aversion

Identifying the risk profile of the investor before building the portfolio is essential as the strategy applied to the portfolio construction might vary a lot due to the risk tolerance level. To be able to understand that, we let Mr. Bennett answer a questionnaire which is developed by Hanna and Lindamood (2004). Risk aversion understanding stands as a critical factor because it directly affects how people invest their money and their overall financial health.

Our investor first replies to the questions in Appendix 1, from which risk aversion levels will be determined. Mr. Bennett answered questions 1 – A, questions 2 – A, and questions 3 – D. It is then categorized investor for Mr. Bennett as a low risk-tolerant investor. A low risk tolerance level is synonymous with a high grade of risk aversion and vice versa. According to the risk aversion level – denoted as A – in Appendix 2, the score for Mr. Bennett would range between 7.5 and 9.3. We will employ a lower boundary, $A = 7.5$, as it sounds more reasonable for such a defensive investor as Mr. Bennett. This number will be used later as a risk aversion coefficient to build the optimal portfolio as part of our portfolio construction

process. It is, therefore, important to know the client's risk profile so that portfolio managers can start building a portfolio for them.

5.1.2. Security Selection

From the questionnaire, we understand more about our clients' needs and preferences for the portfolio. It seems that Mr. Bennett's goal for his retirement savings is to look for value stocks. Value stocks are seen to have lower market prices and undervalued shares of companies compared to their intrinsic value. On the contrary, growth stocks focus more on future prospects and potential, which might favor the investors greatly later, or it can turn out to be a sign of overestimation. Value stocks perform better than growth stocks due to the value premium, according to some research, particularly in the US and Japan (Fama & French, 1998). Instead of betting on stocks coming from technology, healthcare, or innovative sectors that hold high levels of risk, picking on neglected and undervalued stocks of companies that come from mature industries makes much more sense at the table for a defensive investor like Mr. Bennett.

Once we have the portfolio and the investor's characteristics, the next step is security analysis. Firstly, we will choose securities for our investor, Mr. Bennett. Because he is a very risk-averse investor, as mentioned above, we will follow a stock-picking strategy that suits him the best. The screening process is thoroughly done to filter out the most suitable securities. A stock screener by Trading View is used for this purpose. We based the approach of selecting adequate financial ratios for the screening process on Benjamin Graham's book (The Intelligent Investor). Through his value investing philosophy, Benjamin Graham established the foundation for stock selection by recommending a structured approach that focuses on financial health and price-to-book ratio analysis. The Intelligent Investor contains Graham's essential screening criteria, which assist conservative investors in finding strong fundamental companies that sell at prices lower than their actual worth. His evaluation process examines a company's market size together with its financial health and its historical earnings performance and dividend payments, and its price-to-earnings (P/E) and price-to-book (P/B) ratio values. The investment criteria Graham recommends include a minimum current ratio of 2, ten consecutive years of earnings stability, and twenty years of dividend payments without interruption. Investors should stay away from stocks that cost more than

their worth by following two rules: P/E ratios below 15 and P/B ratios below 1.5 or by confirming that the product of these two values does not reach 22.5 (Graham, 2006). The systematic screening method enables investors to reduce their risk exposure by excluding speculative choices while maintaining value investing principles.

For the purpose of an international portfolio, we will choose stocks from different countries, including ones from both developed and emerging markets. The markets chosen are the U.S., China, Japan, Europe, and the UK. In each country, we will use the market index (S&P 500 for the U.S., Nikkei 225 for Japan, Hang Seng Index for Hong Kong, China, FTSE 100 for the UK, and STOXX Europe 600 for the Eurozone) to apply financial ratios and screen out preferred stocks for our portfolio. As we mentioned earlier, the approach of screening is based on the instructions from Benjamin Graham; however, we neither attempt to duplicate nor apply all of the book's lessons (such as utilizing the margin of safety or determining fair value).

For each index mentioned above, we first look at the financial health by filtering out the current ratio to be higher than 2. Another indicator to take into consideration is the price-to-book value (P/B), which we set to be lower than 1.5. This reflects the fundamental characteristics of our client, as Mr. Bennett is a risk-averse investor and has an interest in value stocks. Then, we look at the price-to-earnings ratio (P/E), which we deemed to be lower than 15. We try to choose businesses using this ratio that are not yet attracted by the market but could gain traction once investors show interest in them. The goal, as we explained to Mr. Bennett, is to identify businesses that are undervalued, purchase them, and then wait for the market to adjust their price so that we may profit. Furthermore, we only included companies with a continuous dividend payout of at least 10 years. To ensure the earnings of the firms are healthy and stable, we set net income to stay positive. Companies' sectors will serve as the basis for other fundamentals, whereby we choose the stocks derived from different industries so that they follow the diversification rule.

After applying all the rules above, we have come up with the finalists as follows:

Table 2: Stock selection

Market	Symbol	Company	Industry	Sector
Hong Kong, China	1088	China Shenhua Energy	Coal	Energy minerals
U.S.	STT	State Street Corporation	Investment banks / Brokers	Finance
U.K.	BKG	The Berkeley Group Holdings	Homebuilding	Consumer durables
Europe	NDA	Aurubis	Other metals / Minerals	Non-energy minerals
Japan	1928	Sekisui House	Engineering & construction	Industrial services

Let's have a short description of each of the companies to understand their business and financial performance before the investment period. Firstly, China Shenhua Energy is one of the leading and largest coal producers in China. It plays a significant role in the coal supply chain in mainland China. Its business focuses on many areas, from producing and selling coal to transportation services for coal-related products, to generating and selling power. Besides its broad core business areas, the company also has extensive operations ranging from coal mines, seaports, and railway networks. Furthermore, the company also shifts its focus towards renewable energy, such as wind or solar systems, to diversify its portfolio and be sustainable, as well as stimulate ESG standards. Before 2020, many disruptions affected the company, such as pressure from low interest rates, fluctuations in coal prices, or fierce competition. Despite such pressures, the company maintained its stable growth in revenue and profit margins (China Shenhua Energy, 2020).

State Street Corporation is a financial provider in the U.S. that serves institutional investors, including mutual funds, pension funds, insurance companies, etc. Their main services are asset management, securities trading, and fund administration. It also focuses on digitalization and innovations as a result of regulatory and technological challenges. These initiatives, however, enabled STT to enhance its financial services provided to its clients, helping them keep consistent revenue growth. As a result, the stock price of State Street Corporation has increased from 50.88 USD in 2015 to 66.56 USD by the end of 2019 (State Street Corporation , 2020).

The third stock chosen is Berkey Group Holdings (BKG). It is a real estate developer mainly focusing on developing residential projects around London and the south of the UK. The

company also operates in developing commercial and community projects. However, its revenue mostly comes from projects such as high-quality homes and mixed-use real properties. Before COVID-19 hit, the company experienced high demand for premium and luxurious houses. The real estate market also demonstrated stable growth in the UK. Given the company's expertise in land acquisition and property development, it has shown its excellent and resilient performance by providing strong earnings growth regardless of market uncertainty, such as Brexit (Berkey Group Holdings, 2020).

Aurubis (NDA), coming from Germany, is the biggest copper producer in Europe and also one of the top leaders in the non-ferrous metals sector in the world. They recycle and produce high-quality copper materials and products from smelters and refineries, which are based around Europe and North America. These products are then supplied to customers coming from various industries, such as construction, engineering, electronics, and automotive. Lots of investment is put into recycling technologies to make their business competitive in the industry. The copper prices, however, have witnessed some turbulence during the last five years before the global pandemic. It caused the share price of the company to shake and fall in 2015 and 2016. Nevertheless, given the company's innovative technology and strong position in the market, it was able to normalize it later (Aurubis, 2020).

Last but not least, Sekisui House (1928) is one of the largest homebuilders in the world. Compared to the Berkey Group Holdings, Sekisui House's main core business lies in residential housing; it is famous for its prefabricated homes, which are homes that come from parts already built in the factories. They provide real estate services not only in their home-based country, Japan, but also serve customers, clients, and buyers from all around the world. Its geographically diversified portfolio enables the firm to broaden its revenue streams, thus improving global financial stability. Their stock performance experienced strong and stable growth from 2015 to 2019, thanks to the strong brand reputation, emphasis on innovation, customer satisfaction, and high demand in housing markets (Sekisui House, 2020).

Besides the risky assets, we will utilize the portfolio by including a risk-free asset. The risk-free asset chosen is the U.S. Treasury Securities at 10-year maturity, so that we will use this as the yield for a risk-free asset for our portfolio covering the 2020 – 2024 period. Combining the risk-free assets with the risky assets allows us to balance between capital preservation and

capital growth. The 10-year U.S. bill is retrieved from Federal Reserve Economic Data (FRED). FRED gives two types: FII10, which shows the real yield reflecting the yield after considering the expected inflation, and GS10, which demonstrates the nominal yield. We chose GS10 for our risk-free rate as we work with nominal cash flows, and Modern Portfolio Theory and Capital Allocation Line also assume nominal yield for the risk-free return.

We also introduced the idea of short-selling the risky assets to Mr. Bennett. However, it did not really convince or sway his thoughts on accepting this idea for two main reasons. Firstly, our client knows that he would need to set up and maintain a margin account, from which money would be borrowed, and interest would be paid. He also does not like the idea that in case of shorted securities unexpectedly going up instead of down, the loss incurred can hit our client hard, and he thinks that the volatility of the risky assets is unpredictable, so that it might not go as planned. Given the risk tolerance analysis we conducted on Mr. Bennett earlier, these conclusions do make sense. As a result, Mr. Bennett disagreed with us on this and decided that no short-selling techniques would be applied to the risky assets.

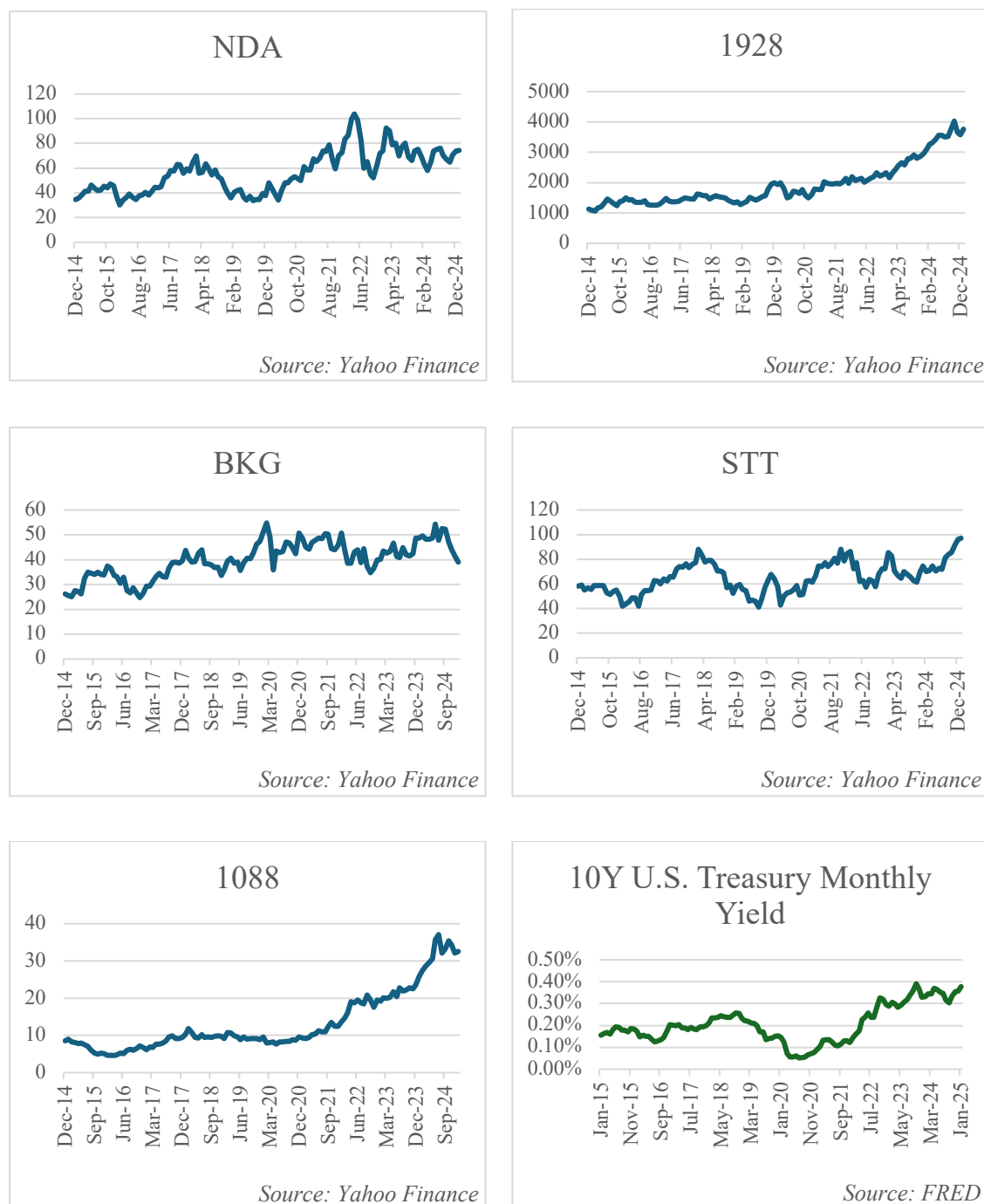
On the other hand, Mr. Bennett was persuaded and liked the plan of a short position on the risk-free asset. This idea allows our client to borrow at a risk-free rate that is more stable and predictable. Because of this, the client would know how much he owed in interest. Unlike betting in volatile securities, shorting in riskless assets ensures no surprise costs or margin calls in the event that the market moves against the plan. Shorting in risk-free assets allows investors to use leverage, which means borrowing at the risk-free rate and subsequently investing in riskier assets, leading to an increased size of the investment portfolio. Thus, the weight allocated to the risky portfolio can go beyond 100% in the efficient complete portfolio. In case that happens, there will be a limit of 20% leverage to boost the potential returns, meaning that completed portfolio maximum weights will not exceed 120% for the risky portfolio and trail below -20% for the risk-free asset. Therefore, Mr. Bennett agrees to this plan if our portfolio optimization suggests a short position on the 10-year Treasury bill.

5.2. Security Analysis

Up until now, we have chosen five stocks. We will start the security analysis by collecting data for those securities. The monthly value of five stocks is collected from Yahoo Finance. The adjusted close prices are used instead of close prices as they take into account factors

that influence the stock price, such as dividends paid and stock splits. Our analysis uses the adjusted close price at the beginning of every month. However, in case of holidays or weekends, the next day's price or the first trading day of the month will be used instead.

Figure 6: Assets' price development in 2014 - 2024



The line graphs depict the stock prices and the risk-free monthly yield. Although we witnessed fluctuation with different variations, the general trajectory supports an upward

trend of the stock prices in the last ten years. Understanding the price and return development lets us make a thorough decision in the later parts, such as asset allocation and portfolio selection.

After collecting the raw data, we compute each stock's simple monthly and logarithmic monthly returns. From the simple monthly returns, we can derive the monthly average return, monthly variances, and monthly standard deviations, which can obtain annual returns and annual standard deviations. These two data together with the returns serve as foundations for the Markowitz optimization model later on portfolio analysis.

From the inputs of returns of five risky assets, we derive the correlation matrix for five years as follows:

Table 3: Correlation Matrix - Historical data 2015-2019

Correlation Matrix					
	NDA	1928	BKG	STT	1088
NDA	1	0.2663	0.1613	0.4562	0.3758
1928	0.2663	1	0.3228	0.3456	-0.0762
BKG	0.1613	0.3228	1	0.2094	-0.1125
STT	0.4562	0.3456	0.2094	1	0.2285
1088	0.3758	-0.0762	-0.1125	0.2285	1

Table 4: Correlation Matrix - Historical data 2016-2020

Correlation Matrix					
	NDA	1928	BKG	STT	1088
NDA	1	0.3900	0.4293	0.5871	0.4074
1928	0.3900	1	0.4165	0.5166	-0.1058
BKG	0.4293	0.4165	1	0.4831	-0.0823
STT	0.5871	0.5166	0.4831	1	0.1688
1088	0.4074	-0.1058	-0.0823	0.1688	1

Table 5: Correlation Matrix - Historical data 2017-2021

Correlation Matrix					
	NDA	1928	BKG	STT	1088
NDA	1	0.3870	0.5577	0.5313	0.3381
1928	0.3870	1	0.3832	0.5767	0.0123
BKG	0.5577	0.3832	1	0.5448	0.1395
STT	0.5313	0.5767	0.5448	1	0.2469
1088	0.3381	0.0123	0.1395	0.2469	1

Table 6: Correlation Matrix - Historical data 2018-2022

Correlation Matrix					
	NDA	1928	BKG	STT	1088
NDA	1	0.2978	0.5283	0.5506	0.2608
1928	0.2978	1	0.3466	0.5781	0.1002
BKG	0.5283	0.3466	1	0.5164	0.0366
STT	0.5506	0.5781	0.5164	1	0.2163
1088	0.2608	0.1002	0.0366	0.2163	1

Table 7: Correlation Matrix - Historical data 2019-2023

Correlation Matrix					
	NDA	1928	BKG	STT	1088
NDA	1	0.2510	0.5191	0.5770	0.2143
1928	0.2510	1	0.3515	0.5633	0.1566
BKG	0.5191	0.3515	1	0.5630	0.0130
STT	0.5770	0.5633	0.5630	1	0.2021
1088	0.2143	0.1566	0.0130	0.2021	1

Table 3 - Table 7 depict the correlation matrices for every 5-year period, which will be used to build the variance-covariance matrices. The correlation matrix becomes handy for understanding the benefits of diversification in an internationally diversified portfolio. Overall, the pairwise relationship among the five international stocks is positive from 2015 to 2023. Even though these stocks are both internationally diversified and industry diversified, the result shows that these stocks usually move in the same direction. However, the correlations do not remain the same over the period; they fluctuate. It means some changes and developments in macroeconomics, geopolitics, or industry-specific happenings have impacted the instability.

There are some negative correlations witnessed in Table 3 and Table 4, specifically between BKG and 1088, and between 1928 and 1088. It suggests the potential benefit for diversification between these pairs. It also remains close to the level of no correlation for other periods. The tendency is observed between these pairs during five periods, with the correlations staying almost at 0. In fact, 1088 has shown relatively low or negative correlations with other securities across all periods. This stresses how important and valuable 1088 is regarding capital allocation and the construction of the efficient frontier. A consistent and low correlation of a stock with others can prove the significant benefit of diversification in reducing unsystematic risk. Given the risk-averse framework, like with Mr. Bennett, where

lowering risk or minimizing variance is crucial, 1088 might have a high chance of being allocated more weight in the portfolio if it also has a very good risk-return profile.

On the other hand, it can be seen that higher correlations exist among NDA, STT, and BKG, reflecting the interconnection and synchronization that prevail between the markets of these stocks. The benefit of diversification, however, diminished due to the rising correlation of these stocks as shown in Table 5, and Table 6. This indicates the influence of global events, such as trade tensions and the pandemic, during this period. It showed some improvement with a slight reduction in correlations of some pairs in Table 7, highlighting the recovery effect from global distresses during the post-pandemic time.

5.3. Portfolio Analysis

After we had conducted the security analysis, the investment process continued with the portfolio analysis. The Markowitz optimization model will be selected to use in the portfolio analysis. Besides the annual returns and annual standard deviations, we computed for five stocks from the previous section, the covariance matrix and the risk-free rate are also needed. With these inputs, we can find the optimal weights for risky assets, which produce the highest Sharpe ratio. Solver tool is employed to determine these optimal weights. A few constraints are applied: maximizing the Sharpe ratio, changing the weights of risky assets so that they have to be non-negative numbers (indicating no short-selling allowance in the risky portfolio), and summing the weights so that they equal 1. This optimization is repetitive for every year from 2020 to 2024, using the historical data of the five years before the year of analysis.

Regarding rebalancing, it is the process of changing a portfolio's asset allocations to match predefined levels specified in an investment plan, taking into account the investor's preferred rewards and risk tolerance. Allocations may be impacted over time by changes in asset values brought about by shifts in market performance. In order to restore the original desired allocation, rebalancing entails buying or selling assets on a regular basis (Manning, 2022).

5.3.1. Initial Portfolio in 2020

In this context, we start the portfolio analysis by evaluating the initial portfolio in 2020. This serves as our base portfolio, from which we rebalance the portfolio over the next four years. The data below in Table 8 illustrates all the necessary inputs for the Markowitz optimization in 2020, including annual return, annual standard deviation, and variance-covariance matrix of the five securities, as well as the risk-free rate. These historical data span from 2015 to 2019. In this period, we can see that most of the risky securities have very high variance, except for 1928, which has the lowest variance (0.03) and the second highest return (0.13). This gives 1928 many advantages over its fellow in the optimization model. In addition, BKG has the highest return (0.17) and the second-lowest standard deviation (0.24). The lowest return of 0.06 is witnessed from 1088, despite having relatively low or negative covariance and correlation with other securities.

Table 8: Portfolio Construction inputs – Historical data 2015-2019

Variance - Covariance Matrix					
	NDA	1928	BKG	STT	1088
NDA	0.1051	0.0155	0.0125	0.0416	0.0327
1928	0.0155	0.0324	0.0139	0.0175	-0.0037
BKG	0.0125	0.0139	0.0573	0.0141	-0.0072
STT	0.0416	0.0175	0.0141	0.0792	0.0172
1088	0.0327	-0.0037	-0.0072	0.0172	0.0719

Annual Returns	NDA	1928	BKG	STT	1088
	0.12	0.13	0.17	0.07	0.06

Annual Standard Deviation	NDA	1928	BKG	STT	1088
	0.32	0.18	0.24	0.28	0.27

Annual Risk-Free Rate	2.24%
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Given the above data, the Solver tool gives the optimized weights for the risky portfolio as depicted in Table 9. No capital will be allocated to STT, whereas 2% is allocated to NDA, 15% to 1088, 38% to BKG, and 46% to 1928. As we mentioned earlier, this huge distribution was achieved in both 1928 and BKG thanks to their superior characteristics regarding return and variance over the last five years. On the contrary, poor performance from NDA or BKG shows little or no attraction to investing in a risky portfolio.

Table 9: Optimized risky portfolio weights - Markowitz Optimization for 2020

Optimized Weights (Risky Portfolio)	NDA	1928	BKG	STT	1088	Total
	2%	46%	38%	0%	15%	100%

The next step is to introduce the risk-free asset in the complete portfolio and determine the optimal weights for the risky portfolio as well as the risk-free asset. To do that, we have to consider Mr. Bennett's risk preference represented by the risk aversion coefficient to maximize our client's utility function given by equation (7) in the theoretical part. By doing this, the Sharpe ratio neither increased nor decreased; instead, it remained the slope of the Capital Allocation Line and moved along it according to our client's risk appetite. To calculate the risky portfolio weight, we use equation (8), which gives us a substantial allocation of 74%. We distribute the rest of the complete portfolio weight, 26%, to the risk-free asset so that the total weight stays 100%. This year, there is no short position in risk-free asset weight.

Table 10: Optimal weights of the complete portfolio - Markowitz optimization for 2020

Risk Aversion Coefficient		7.50
Optimal Complete Portfolio		
Risky Portfolio Weight		74%
NDA		1%
1928		34%
BKG		28%
STT		0%
1088		11%
Risk-free Asset Weight		26%
Total Weight		100%

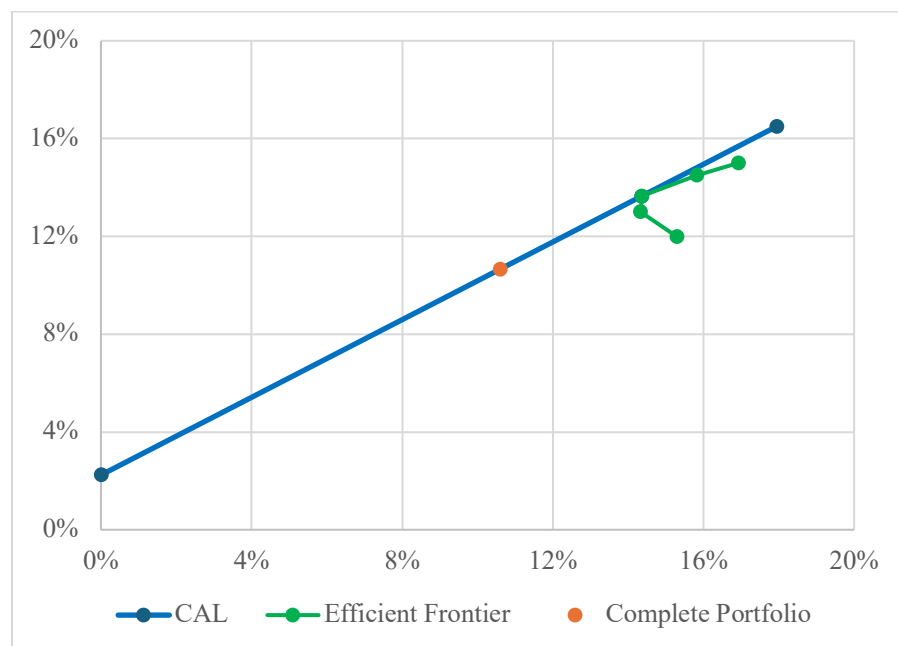
The complete portfolio's characteristics are demonstrated in Table 11. Portfolio return is computed using equation (1), whereas portfolio risk is calculated using equation (2). As a result, the complete portfolio presents a Sharpe ratio of 0.79. Although the Sharpe ratio is below 1, it still implies a well-optimized portfolio, which balances the risk and return of the portfolio given the portfolio assumptions of the client's needs and preferences.

Table 11: Optimal Complete Portfolio - Markowitz optimization for 2020

Annual Return	10.66%
Annual Risk	10.60%
Annual Sharpe Ratio	0.79

As a result, based on the inputs of historical data we collected from 2015 to 2019, we can draw the efficient frontier and capital allocation line graphically in the figure below to show the tangency point. This point is the optimal risky portfolio that serves as the one with the highest Sharpe ratio (0.79). The furthest point to the left in the efficient frontier is the minimum-variance portfolio. The Sharpe ratio also serves as the slope of the CAL, which starts at the point of the risk-free rate (2.24%). The orange dot presents the optimal complete portfolio, which maximizes our investor's utility function, corresponding with the return and risk as shown in Figure 7.

Figure 7: Efficient Frontier and Capital Allocation Line



5.3.2. Portfolio Rebalance in 2021

The procedure for 2021 will be similar to the one we carried out for portfolio optimization in the previous year. For portfolio rebalancing, we are not using the same historical data as used in the optimization of the initial year, but instead, including the historical data of the year we have invested. This means that the portfolio rebalancing for 2021 will be based on the historical data spanning from 2016 to 2020 (same time horizon of five years, but distinct periods). The rebalancing helps preserve portfolio efficiency across different markets. From this data set, we can derive annual returns, standard deviations, and a covariance matrix as

illustrated in Table 12 below. The risk-free rate, which is derived from the 10-year U.S. Treasury note, is also updated accordingly.

Table 12: Portfolio Construction inputs – Historical data 2016-2020

Variance - Covariance Matrix					
	NDA	1928	BKG	STT	1088
NDA	0.1316	0.0287	0.0440	0.0707	0.0388
1928	0.0287	0.0412	0.0239	0.0348	-0.0056
BKG	0.0440	0.0239	0.0798	0.0453	-0.0061
STT	0.0707	0.0348	0.0453	0.1101	0.0147
1088	0.0388	-0.0056	-0.0061	0.0147	0.0690

Annual Returns	NDA	1928	BKG	STT	1088
	0.12	0.06	0.12	0.08	0.17

Annual Standard Deviation	NDA	1928	BKG	STT	1088
	0.36	0.20	0.28	0.33	0.26

Annual Risk-Free Rate	1.97%
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A few remarks are noted in the historical data from 2016 to 2020. Firstly, the security 1088 has significantly improved its returns compared to the last data set, from 0.06 to 0.17. Except for BKG and 1088, all other risky assets' standard deviation has moderately risen, implying a probable impact from the volatile markets caused by COVID-19 in 2020. Moreover, 1928 reduced its return substantially from 0.13 to 0.06, and slightly increased in the standard deviation, making it less dominant in the optimized risky portfolio weight. Instead of 46% as before, Markowitz optimization suggests only 12%, as shown in Table 13, of the capital allocated to 1928 for 2021. On the other hand, given the advancement of 1088, the results highlighted that the proportion occupied by 1088 should be more than half of the risky portfolio, with 57%. Despite the slight return improvement, the system again proposed no allocation to either NDA or STA for 2021 due to its weak risk-return profile.

Table 13: Optimized risky portfolio weights - Markowitz Optimization for 2021

Optimized Weights (Risky Portfolio)	NDA	1928	BKG	STT	1088	Total
	0%	12%	31%	0%	57%	100%

By applying the risk aversion coefficient of 7.5, we can now estimate the risky portfolio and risk-free asset weight in the optimal complete portfolio as depicted in Table 14. In particular, unlike last year, there is more balance in 2021, with 53% in the risky portfolio and 47% in

the risk-free asset. The significant drop from 74% last year to only 53% this year in the risky portfolio indicates that the investor should bear in mind the volatile environment that could boost market risk and thus hurt them. As a result, an increased share of a safer option, such as a 10-year Treasury note, can enhance the investor's position under the erratic market movements.

Table 14: Optimal weights of the complete portfolio - Markowitz optimization for 2021

Risk Aversion Coefficient		7.50
Optimal Complete Portfolio		
Risky Portfolio Weight		53%
	NDA	0%
	1928	6%
	BKG	17%
	STT	0%
	1088	30%
Risk-Free Asset Weight		47%
Total Weight		100%

Table 15 demonstrates the complete portfolio's return, risk, and Sharpe ratio. This year witnessed a reduction in all parameters. Portfolio return decreased from 10.66% to 8.24%, portfolio risk dropped from 10.60% to 9.14%, and the Sharpe ratio fell from 0.79 to 0.69. The lowering of ratios stresses again the defensive position that the investor takes for 2021, considering the adverse effect from market downturns like the pandemic and global trade constraints.

Table 15: Optimal Complete Portfolio - Markowitz optimization for 2021

Annual Return	8.24%
Annual Risk	9.14%
Annual Sharpe Ratio	0.69

5.3.3. Portfolio Rebalance in 2022

The process for 2022 is more or less the same as last year; however, this time we use the historical data from 2017 to 2021. The Markowitz optimization needs inputs – returns, standard deviations, risk-free rate, and a variance-covariance matrix – to construct the complete portfolio for 2022. These data are shown below:

Table 16: Portfolio Construction inputs – Historical data 2017-2021

Variance - Covariance Matrix					
	NDA	1928	BKG	STT	1088
NDA	0.1300	0.0295	0.0535	0.0597	0.0304
1928	0.0295	0.0446	0.0215	0.0380	0.0007
BKG	0.0535	0.0215	0.0707	0.0452	0.0092
STT	0.0597	0.0380	0.0452	0.0973	0.0192
1088	0.0304	0.0007	0.0092	0.0192	0.0622

Annual Returns	NDA	1928	BKG	STT	1088
	0.22	0.12	0.18	0.11	0.19

Annual Standard Deviation	NDA	1928	BKG	STT	1088
	0.36	0.21	0.27	0.31	0.25

Annual Risk-Free Rate	1.91%
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During the period 2017-2021, it can be seen that the annual returns from NDA and 1928 have increased significantly. Additionally, risky securities' standard deviation remains stable or has dropped slightly. 1088 still offers the most favorable risk-return profile as compared to other stocks. Thus, this stock is a primary contributor, with a share of 45%, to the risky portfolio in 2022. Given the better performance of NDA and 1928 than in the last period, the system indicates an increase in allocation for these two stocks. Notably, NDA has witnessed an allocation of 2%, whereas 1928 also gave a larger share (27%) in the risky portfolio. Slight decrease from 31% to 27% in the allocation for BKG. A more balanced contribution of 1928, BKG, and 1088 demonstrates a greater diversification benefit for the portfolio. On the other hand, with the outperformance of those stocks and an underperformed risk-return profile of STT, STT still does not contribute any part to the risky portfolio for 2022.

Table 17: Optimized risky portfolio weights - Markowitz Optimization for 2022

Optimized Weights (Risky Portfolio)	NDA	1928	BKG	STT	1088	Total
	2%	27%	27%	0%	45%	100%

With the risk aversion coefficient of 7.5, the utility function maximization results in a recommendation of 72% allocation to the risky portfolio weight and 28% allocation to the risk-free asset weight. A more detailed breakdown of stocks within the risky portfolio is also

presented in Table 18. Compared to the caution shown last year, the investor will invest more in risky assets given their lower variance and higher returns.

Table 18: Optimal weights of the complete portfolio - Markowitz optimization for 2022

Risk Aversion Coefficient		7.50
Optimal Complete Portfolio		
Risky Portfolio Weight		72%
	NDA	1%
	1928	19%
	BKG	19%
	STT	0%
	1088	32%
Risk-Free Asset Weight		28%
Total Weight		100%

Because of more allocation to the risky portfolio, we have seen an increase in the complete portfolio's return, risk, and Sharpe ratio. After successfully adapting to the changing risk condition along with the client's risk appetite, the Sharpe ratio has grown from 0.69 to 0.90, indicating a healthier risk-return profile in contrast to last year's results.

Table 19: Optimal Complete Portfolio - Markowitz optimization for 2022

Annual Return	12.60%
Annual Risk	11.94%
Annual Sharpe Ratio	0.90

5.3.4. Portfolio Rebalance in 2023

To rebalance the portfolio for 2023, we use the historical data from 2018 to 2022 to compute the returns, standard deviation, and covariance matrix. As we can see from the table below, there have been many changes across risky securities. For this period, a substantial plummet in NDA, 1928, BKG, and STT returns is observed. BKG's return dived from 0.18 in the last period to just 0.05 this time, whereas 1928, BKG and STT's returns fell by approximately 40 to 50%. In addition, the standard deviations of the stocks moderately increased in those stocks. The decreased returns and increased standard deviations highlighted some disturbing events for investors, such as post-COVID slowdown, rising interest rates in late 2021 and early 2022, and geopolitical risk arising from the Russian invasion of Ukraine. Despite this tendency, 1088 is the only one that maintains its return and risk level.

Table 20: Portfolio Construction inputs – Historical data 2018-2022

Variance - Covariance Matrix					
	NDA	1928	BKG	STT	1088
NDA	0.1639	0.0255	0.0640	0.0768	0.0272
1928	0.0255	0.0448	0.0219	0.0421	0.0055
BKG	0.0640	0.0219	0.0894	0.0532	0.0028
STT	0.0768	0.0421	0.0532	0.1186	0.0192
1088	0.0272	0.0055	0.0028	0.0192	0.0661

Annual Returns	NDA	1928	BKG	STT	1088
	0.14	0.08	0.05	0.05	0.19

Annual Standard Deviation	NDA	1928	BKG	STT	1088
	0.40	0.21	0.30	0.34	0.26

Annual Risk-Free Rate	2.09%
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Because of many negative shifts in the stock's performance, there are supposed to be significant changes in the optimized weights in the risky portfolio. This is reflected in Table 21. The unattractiveness of NDA, 1928, BKG, and STT's risk-return profile saw the reduced allocation to these stocks. We anticipate a long position with 5% in NDA and 29% in 1928, while there will be no allocation in BKG and STT. This is also the first time BKG received no weights in the risky portfolio. As opposed to the reduction in allocation of these stocks, this year witnessed a dominant share of 66% from 1088 to the risky portfolio.

Table 21: Optimized risky portfolio weights - Markowitz Optimization for 2023

Optimized Weights (Risky Portfolio)	NDA	1928	BKG	STT	1088	Total
	5%	29%	0%	0%	66%	100%

We then maximize the client's utility function by combining a risky portfolio with the risk-free asset. This year, the optimization indicates that only 47% of the capital is allocated to the risky portfolio as opposed to last year, which occupied 72% of the complete portfolio. In contrast to the risky portfolio, with a slight increase in the 10-year Treasury note from 1.97% to 2.09%, thus it is reasonable for the investor to be cautious and distribute a little more proportion, 53%, to the risk-free asset weight in 2023, considering all the disruptions that happened in the market in 2022.

Table 22: Optimal weights of the complete portfolio - Markowitz optimization for 2023

Risk Aversion Coefficient		7.50
Optimal Complete Portfolio		
Risky Portfolio Weight		47%
NDA		2%
1928		14%
BKG		0%
STT		0%
1088		31%
Risk-Free Asset Weight		53%
Total Weight		100%

Due to the major share of risk-free asset weight in the complete portfolio, the annual return decreases to 8.40%. The standard deviation also declined to 9.18%. As a result, the Sharpe ratio is expected to reduce from 0.90 to 0.69.

Table 23: Optimal Complete Portfolio - Markowitz optimization for 2023

Annual Return	8.40%
Annual Risk	9.18%
Annual Sharpe Ratio	0.69

5.3.5. Portfolio Rebalance in 2024

Portfolio optimization continues with the rebalancing for the last year of the investment period. Similar to previous years and assumptions, historical data from 2019 to 2023 were used for the portfolio construction inputs, including annual returns, standard deviation, variance-covariance matrix, and an updated risk-free rate. The results are as follows:

Table 24: Portfolio Construction inputs – Historical data 2019-2023

Variance - Covariance Matrix					
	NDA	1928	BKG	STT	1088
NDA	0.1663	0.0216	0.0638	0.0823	0.0211
1928	0.0216	0.0446	0.0224	0.0416	0.0080
BKG	0.0638	0.0224	0.0909	0.0594	0.0009
STT	0.0823	0.0416	0.0594	0.1223	0.0170
1088	0.0211	0.0080	0.0009	0.0170	0.0581
Annual Returns					
	NDA	1928	BKG	STT	1088
	0.21	0.20	0.13	0.11	0.24
Annual Standard Deviation					
	NDA	1928	BKG	STT	1088
	0.41	0.21	0.30	0.35	0.24
Annual Risk-Free Rate		2.30%			

Generally, the annual returns have strongly improved across all the risky assets. This is reinforced further by unchanged standard deviations. In spite of such improvement, still no allocation assigned to STT. On the other hand, an increase in the share of 1928 from 29% last year to 46% this year makes 1088 less potent in the risky portfolio, but still accounts for a substantial amount, with 66%. A minor adjustment from NDA, from 5% to 3%, was seen in optimization for 2024. BKG received a 2% share in the risky portfolio.

Table 25: Optimized risky portfolio weights - Markowitz Optimization for 2024

Optimized Weights (Risky Portfolio)	NDA	1928	BKG	STT	1088	Total
	3%	46%	2%	0%	48%	100%

The attractiveness of the risky portfolio increased to 92% of the complete portfolio is assigned here. The shifts towards risky portfolios imply that more confidence is shown by the investors due to the recovery of stock returns over the last five years. As a result, a smaller share of only 8% is allocated to the risk-free asset.

Table 26: Optimal weights of the complete portfolio - Markowitz optimization for 2024

Risk Aversion Coefficient		7.50
Optimal Complete Portfolio		
Risky Portfolio Weight		92%
	NDA	3%
	1928	43%
	BKG	2%
	STT	0%
	1088	44%
Risk-Free Asset Weight		8%
Total Weight		100%

Table 27 depicts the properties of the complete portfolio in 2024. The annual return rose sharply from 8.40% to 20.33%. The standard deviation for the first time has shown to be much lower than the return, residing at 15.51%. Given this excellent performance, the Sharpe ratio peaked at 1.16, compared to the previous years.

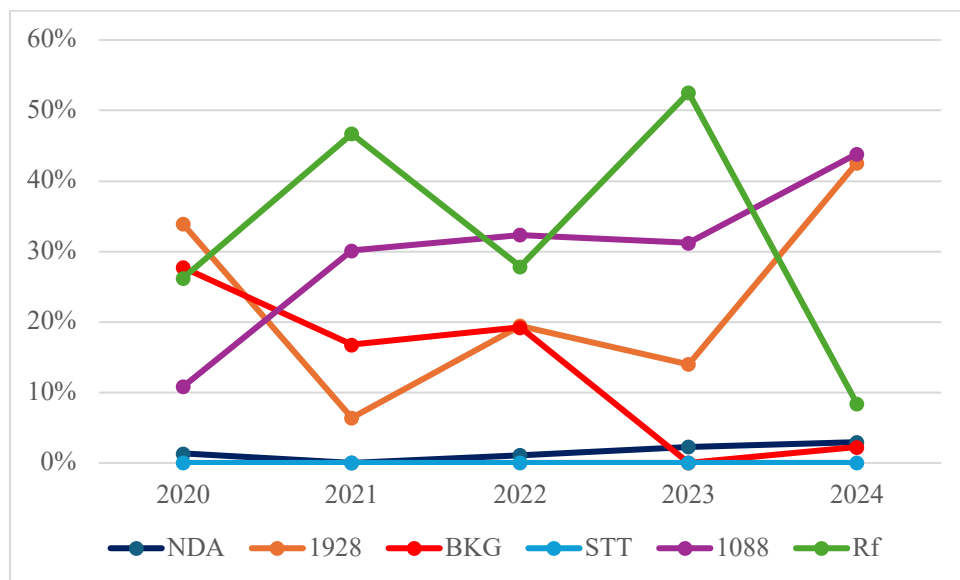
Table 27: Optimal Complete Portfolio - Markowitz optimization for 2024

Annual Return	20.33%
Annual Risk	15.51%
Annual Sharpe Ratio	1.16

6. Portfolio Performance

Figure 8 summarizes the optimal complete portfolio weights over five years of investment. We have maintained no investment in STT over the years, whereas small investments have been distributed to NDA. We gradually increased the allocation to 1088 and, conversely, reduced the share of BKG in the portfolio. Both 1928 and the risk-free asset see fluctuations of weights in the complete portfolio. Even though we have introduced the possibility of shorting in the risk-free assets, it turned out that we will not short in this asset. This is due to the rising risk-free return year on year, as a result of the FED's monetary policy, in which interest rates were raised to fight against the inflation spike. Along with the low risk tolerance level of our client, the system favors more risk-free assets, as we can see its dominance in some years of the investment period. Now, let's base on these optimized weights and analyze the actual performance of the portfolio. But before doing so, the exchange rate movements between 2020 and 2024 are examined to understand their impact on our portfolio performance.

Figure 8: Optimal Complete Portfolio weights



6.1. Exchange rate performance

The line graph below illustrates the movement of exchange rates of EUR/USD and GBP/USD between 2020 and 2024. It can be seen that both exchange rates declined over five years. Because these exchange rates are quoted in direct quotations, the reduction in these rates means a depreciation of the foreign currency (EUR and GBP), or an appreciation of our domestic currency (USD). As we mentioned, our international portfolio consists of a large share of GBP currency through the stock BKG, and a small proportion of EUR currency through the stock NDA. Thus, the depreciation of these foreign currencies against the home currency will be unfavorable for international investors like Mr. Bennett. Conversely, when the foreign currencies appreciate against the dollar, our investor will benefit. In general, we can see that both exchange rates move in the same direction. The foreign currencies appreciated in 2020 and 2023, and depreciated in the other years.

Figure 9: Exchange rate of EUR/USD and GBP/USD during 2020-2024

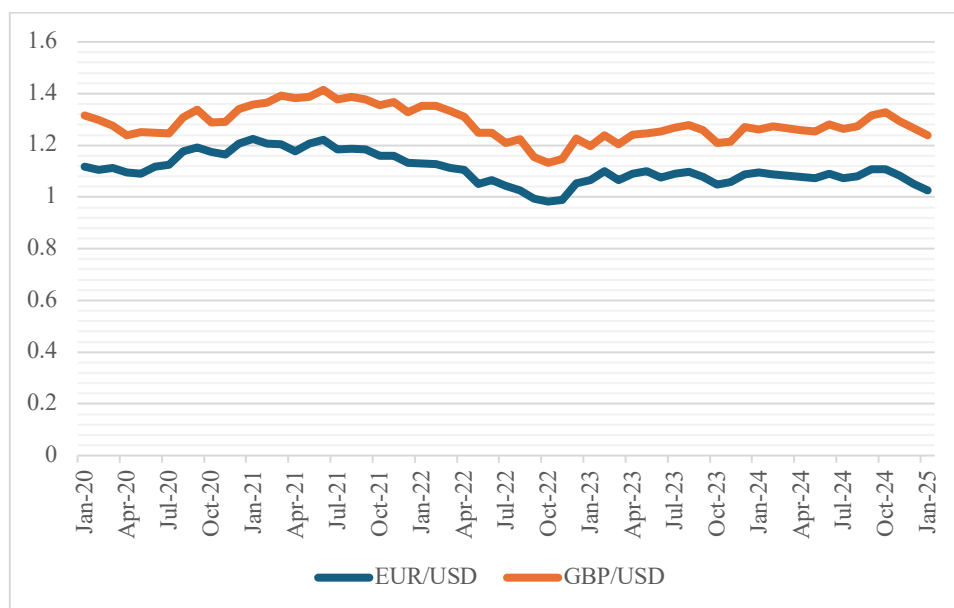
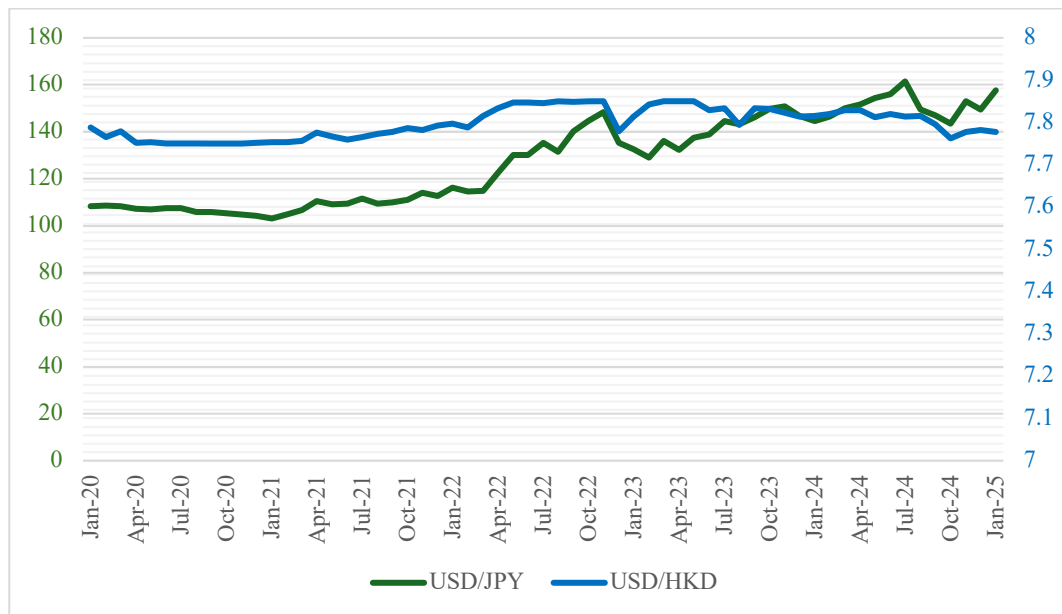


Figure 10 depicts the changes in USD/JPY and USD/HKD exchange rates during the five years of investment. Particularly, both exchange rates declined in 2020 and slowly picked up the pace to rise in the next four years, with the exception of the decrease in USD/HKD in the last year. Overall, both exchange rates increased, but at different rates. On one hand, the JPY started the year 2020 at 108 JPY for a USD and skyrocketed to nearly 158 JPY for 1 USD by the end of 2024. On the other hand, USD/HKD also shows a growth but much

calmer development. This is due to the fact that the Hong Kong dollar follows a US dollar-pegged currency system, meaning the HKD ranges between 7.75 and 7.85 to USD (Reuters, 2025). Therefore, the variance of this pair is minor and indicates much lower currency risk for American investors holding HKD. The quotations shown here are indirect; the quoted currency is the foreign currency instead. It means if the exchange rates go up, either foreign currencies (JPY and HKD) depreciate or the domestic currency (USD) appreciates. Like the previous case with EUR and GBP, this will have a negative effect on the international portfolio since a significant allocation is given to both JPY-denominated stock (1928) and HKD-denominated stock (1088) over five years.

Figure 10: Exchange rate of USD/JPY and USD/HKD during 2020-2024



Because all foreign currencies are weakening against the USD, Mr. Bennet should have his international portfolio hedged against those unfavorable events in the exchange rates. This will bring our investor a better portfolio value and return at the end of the investment. The next two sections will therefore examine how much an unhedged portfolio value is compared to the hedged portfolio with forward and options.

Table 28 demonstrates the correlation matrix of the four major exchange rates from 2020 to 2024, which are retrieved from Yahoo Finance. This matrix examines how the currency pairs moved together during this investment period to understand the diversification benefit from investing in foreign currencies and the currency risk exposure in the international portfolio.

In general, all currency pairs are positively correlated, with EUR/USD and GBP/USD showing the highest correlation among others. Given their geographic context and European background, this stresses the interconnection between the euro and the British pound relative to the dollar. Although the GBP/USD exhibits lower correlation with JPY/USD and HKD/USD, it is still relatively high. One of the reasons that the correlations are high is that many distressed events happened during 2020-2024, such as COVID-19 and the Russia-Ukraine war. Therefore, the diversification benefits are minimal, even though they exist.

Table 28: Correlation Matrix of Exchange Rates between 2020 and 2024

Correlation Matrix				
	EUR/USD	GBP/USD	JPY/USD	HKD/USD
EUR/USD	1			
GBP/USD	0.8930	1		
JPY/USD	0.7701	0.5983	1	
HKD/USD	0.7281	0.6177	0.7034	1

6.2. Unhedged portfolio value

This section will discuss the overall returns and performance of the client's portfolio for a 5-year investment period. As we introduced in the investment policy part, the investors holding an investment of 100,000 USD in the beginning invested in six securities, including four foreign stocks, one domestic stock, and a risk-free asset. The annual simple return of each security (e.g., in column 4 of Table 29) is calculated using monthly logarithmic returns of that stock, which are implicitly embedded by both the stock return and the foreign exchange return. In this sense, we have the profit or loss from exchange rate fluctuation reflected directly in the portfolio. Using the portfolio allocation at the beginning of the year and the annual return, we will have the portfolio value at the end of the year. Note that there are also management fees and performance fees. A management fee of 1% is taken out from the portfolio value at the end of each year. Furthermore, as the name suggests, a performance fee of 10% is collected only from the performance of the portfolio during that year. A concept of High-water mark is implemented to compute this fee, indicating that the performance fee is only paid when the portfolio value after management fee is higher than the highest historical value. This enables clients to pay only if the portfolio performance was positive and generated new profits. Additionally, it will be taxed at 15% on the capital gains

only at the end of the 5-year period. Below is a description and some insightful information about each year's performance.

Table 29 demonstrates the actual performance of the portfolio during the first year. Unlike the forecast at the beginning of 2020, when we tried to calculate the expected attributes of the optimal complete portfolio, which showed the expected return of 10.66%, the actual return in 2020 was realized at -1.29%, resulting in a net return of -2.28% after fees. This is understandable since this year has experienced great turbulence as a result of the global pandemic. The mismatch stresses the limitation of the optimization models, which are based on historical data to predict the future, thus ruling out the possibility of recession.

The errors made are the allocation to 1928, BKG, and 1088, which suffered a return of -3.37%, -2.02%, and -2.36%, respectively. As discussed by Glen and Jorion (1993), although diversification exists through geographic and sectoral factors, the effect was reduced during the global recession, making the decline across securities more harmonized. On the other hand, we unexpectedly witnessed a substantial return of 33% from NDA; however, we couldn't capitalize much on this due to a small allocation on this asset. The loss was also eased by the risk-free asset, which showed small positive returns. It, however, was not enough to offset the damage inflicted by 1928. It is worth mentioning that the appreciation of the foreign currencies during the first year created a cushion for the investor to reduce the damage inflicted by the worldwide economic slowdown.

Table 29: Portfolio Performance 2020

2020	Allocation (%)	Allocation (\$)	Return (%)	Return (\$)	Position at the end
NDA	1.33%	\$ 1,333.47	33.30%	\$ 444.11	\$ 1,777.57
1928	33.90%	\$ 33,896.76	-3.37%	\$ (1,141.50)	\$ 32,755.27
BKG	27.72%	\$ 27,718.80	-2.02%	\$ (559.02)	\$ 27,159.78
STT	0.00%	\$ -	-7.40%	\$ -	\$ -
1088	10.82%	\$ 10,822.80	-2.36%	\$ (255.89)	\$ 10,566.92
10-Year US Treasury	26.23%	\$ 26,228.17	0.84%	\$ 219.59	\$ 26,447.75
Total	100.00%	\$ 100,000.00	-1.29%	\$ (1,292.71)	\$ 98,707.29

Portfolio Value (End 2020)	\$ 98,707.29
Management Fees	\$ 987.07
Performance Fees	\$ -
Total Fees	\$ 987.07
Portfolio Value (Beginning 2021)	\$ 97,720.21
Net Return	-2.28%

Moving on to the second year, the NDA's return continued showing off with a significant return. However, based on the high variance it showed in the past, we didn't allocate any capital here, making the portfolio lose the opportunity to exploit its advantage. Similar to NDA, STT also surprisingly presented a high yield, and we could not capitalize on that. We made a good decision by investing in the rest, as all stocks indicate positive returns, especially 1088, which paid off the greatest with over 45% in 2021, as compared to -2.36% last year. In fact, the fueled coal price surges along with the production of coal-made products during 2021 explained such a huge turnaround from 1088 (Xin, 2021). The rebound of stock returns highlighted the recovery during the post-pandemic period, in which markets from many countries are stimulated by governments' policies on stimulus packages. It helped businesses get over such poor performance as experienced in 2020.

Nevertheless, exchange rates did go against the investor's favor in 2022, as we analyzed earlier. This then minimizes the returns and thus underscores the necessity to hedge against unfavorable exchange rate risk. Overall, we still managed to surpass the estimated value of return (8.24%) as shown in Table 15. In 2021, we gained 13.36%, resulting in a portfolio value of 110,773.19 USD. The client had to pay a performance fee this year as the portfolio started making a profit.

Table 30: Portfolio Performance 2021

2021	Allocation (%)	Allocation (\$)	Return (%)	Return (\$)	Position at the end
NDA	0.00%	\$ -	31.04%	\$ -	\$ -
1928	6.39%	\$ 6,246.13	9.36%	\$ 584.74	\$ 6,830.87
BKG	16.79%	\$ 16,406.64	4.15%	\$ 680.13	\$ 17,086.78
STT	0.00%	\$ -	34.05%	\$ -	\$ -
1088	30.14%	\$ 29,449.47	45.61%	\$ 13,432.30	\$ 42,881.77
10-Year US Treasury	46.68%	\$ 45,617.96	1.50%	\$ 683.84	\$ 46,301.80
Total	100.00%	\$ 97,720.21	15.74%	\$ 15,381.01	\$ 113,101.22

Portfolio Value (End 2021)	\$ 113,101.22
Management Fees	\$ 1,131.01
Performance Fees	\$ 1,197.02
Total Fees	\$ 2,328.03
Portfolio Value (Beginning 2022)	\$ 110,773.19
Net Return	13.36%

Portfolio performance in 2022 experienced a big turmoil in comparison to the previous year. After the short bounce back of the economy, we again entered another economic slump due to geopolitical tension, such as Russia's invasion, rising interest rates, and the aftermath of the global pandemic. Another example of a year in which we can point out the fault in the optimization model. From 13.36%, the portfolio's return plunged to only 3.64% in 2022, also far below the projected return of 12.6% estimated at the beginning of the year. Another factor contributing to the loss is the depreciation of foreign currencies, which worsens the foreign currency-denominated stocks' returns.

A surprisingly high return from NDA in previous years suddenly disappears this year as a result of global supply chain disruption and thus lower demand for industrial metals (Home, 2022). The drop in returns is also observed from NDA, 1928, and STT with -16.48%, -14.11%, and -14.17%, respectively. Of course, we made mistakes as we distributed major capital to NDA, 1928, and BKG, resulting in a loss of nearly 10,000 USD. Thankfully, a huge allocation in 1088 plus the substantial return in this stock, makes enough profit to offset the deficit suffered. This proved how valuable it is to have a value stock like 1088 in the portfolio. It is again thanks to the high coal price and high demand for coal production in China (Nagle & Temaj, 2022). Together with the profit gain from the risk-free asset, 5,641 USD is the earnings we have in 2022.

Table 31: Portfolio Performance 2022

2022	Allocation (%)	Allocation (\$)	Return (%)	Return (\$)	Position at the end
NDA	1.12%	\$ 1,236.12	-16.48%	\$ (203.76)	\$ 1,032.35
1928	19.46%	\$ 21,557.18	-14.11%	\$ (3,041.59)	\$ 18,515.59
BKG	19.23%	\$ 21,300.91	-29.95%	\$ (6,379.75)	\$ 14,921.16
STT	0.00%	\$ -	-14.17%	\$ -	\$ -
1088	32.35%	\$ 35,837.61	39.93%	\$ 14,311.27	\$ 50,148.88
10-Year US Treasury	27.84%	\$ 30,841.37	3.10%	\$ 955.25	\$ 31,796.62
Total	100.00%	\$ 110,773.19	5.09%	\$ 5,641.42	\$ 116,414.60

Portfolio Value (End 2022)	\$ 116,414.60
Management Fees	\$ 1,164.15
Performance Fees	\$ 447.73
Total Fees	\$ 1,611.87
Portfolio Value (Beginning 2023)	\$ 114,802.73
Net Return	3.64%

Portfolio performance for 2023 is depicted in the tables below. Different from last year, most of the returns have shown great improvement, with the exception of NDA having a negative return of -3.54%. Some stocks, such as 1928, BKG, and 1088, demonstrate significant annual returns; however, only 1928 and 1088 are taken advantage of. A huge payoff from 1928 is owed to the effect of global economic recovery, government stimulus, or weakened Yen, which boosts the real estate sector and Japanese exports. The depreciation of the Yen removes the potential for a higher gain due to the foreign exchange effect. It once more emphasizes the need to use hedging tools to protect our clients from exposure to this risk. Moreover, 1088 maintained its high return with 34.10% over the year 2023. Despite the Hong Kong dollar depreciation as well, its variance is smaller due to the pegged currency, making it less severe than the case of 1928.

Because of last year's catastrophic performance of the risky assets, apart from 1088, we can understand why a defensive position is taken this year. It increases the share of the risk-free asset to more than half of the portfolio. This simultaneously prevents the client from capturing massive gains from no-allocation security, such as BKG. In general, the portfolio

still exhibits a very high return of 14.20% after deducting all fees. This gives us a portfolio value of 131,220 US dollars invested in the last year of the investment period.

Table 32: Portfolio Performance 2023

2023	Allocation (%)	Allocation (\$)	Return (%)	Return (\$)	Position at the end
NDA	2.28%	\$ 2,613.93	-3.54%	\$ (92.51)	\$ 2,521.42
1928	14.00%	\$ 16,075.62	30.65%	\$ 4,927.24	\$ 21,002.86
BKG	0.00%	\$ -	27.80%	\$ -	\$ -
STT	0.00%	\$ -	2.83%	\$ -	\$ -
1088	31.20%	\$ 35,818.96	34.10%	\$ 12,214.99	\$ 48,033.96
10-Year US Treasury	52.52%	\$ 60,294.22	4.00%	\$ 2,412.34	\$ 62,706.56
Total	100.00%	\$ 114,802.73	16.95%	\$ 19,462.06	\$ 134,264.79

Portfolio Value (End 2023)	\$ 134,264.79
Management Fees	\$ 1,342.65
Performance Fees	\$ 1,811.94
Total Fees	\$ 3,154.59
Portfolio Value (Beginning 2024)	\$ 131,110.20
Net Return	14.20%

Table 33 highlights the performance of our client's portfolio in the last year 2024. Strong confidence in the risky assets lets us reap profits greatly from 1928 and 1088. Small blunder from the decision to invest in BKG, as it lost 21.51% in 2024. We fortunately occupy a minor proportion in BKG, therefore, an insignificant amount of 625 US dollars was lost. Again, the return could have been higher in the returns of NDA, 1928, and BKG, as the returns shown in the table below already accounted loss return from foreign exchange due to the weakened foreign currencies. In contrast, 1088's return has benefited further, although minimally, as a result of the slight JPY appreciation. Another remark worth mentioning is that despite the negative returns in the years 2020 and 2022, STT has been showing strong performance with an approximate return of around 30-35% in 2021 and 2024.

Overall, we have maintained approximately the same portfolio's return as last year. It, however, is less than the projected return of 20.33% predicted at the beginning of 2024. By the end of the year, the portfolio value after fees is 149,591 USD, which represents the total

5-year portfolio return of 49.59%. On an annual basis, this translates to 8.39% of yearly return. When the client closes the account at the end of the investment period and realizes the capital gains from the portfolio, the tax of 15% will be applied. It leaves the 5-year portfolio value after fees and taxes to be 142,152 USD, resulting in the total return of 42.15% or 7.29% annually.

Table 33: Portfolio Performance 2024

2024	Allocation (%)	Allocation (\$)	Return (%)	Return (\$)	Position at the end
NDA	2.94%	\$ 3,859.33	0.57%	\$ 21.92	\$ 3,881.25
1928	42.57%	\$ 55,817.77	12.05%	\$ 6,723.39	\$ 62,541.16
BKG	2.22%	\$ 2,906.85	-21.51%	\$ (625.38)	\$ 2,281.47
STT	0.00%	\$ -	30.44%	\$ -	\$ -
1088	43.85%	\$ 57,495.02	26.92%	\$ 15,476.34	\$ 72,971.36
10-Year US Treasury	8.41%	\$ 11,031.23	4.26%	\$ 469.44	\$ 11,500.67
Total	100.00%	\$ 131,110.20	16.83%	\$ 22,065.71	\$ 153,175.91

Portfolio Value (End 2024)	\$ 153,175.91
Management Fees	\$ 1,531.76
Performance Fees	\$ 2,053.40
Total Fees	\$ 3,585.15
Portfolio Value after fees	\$ 149,590.75
Net Return	14.10%

6.3. Hedged portfolio value

The previous chapter has introduced the unhedged portfolio, and we acknowledged the need to implement some derivatives to hedge against currency risk. We decided to use only forwards and options in our international portfolio and disregard the use of futures and swaps. As for futures, due to their similarity to forwards, their performance will not differ much from forward contracts. And swaps are more useful when it comes to long-term debt management, suitable for multinational companies that are exposed to the risk of long-term external financing. Since our portfolio is rebalanced annually, it makes more sense to use forwards and options, which give us more flexibility to manage over the short period of time.

This chapter will begin by analyzing the profit and loss when we use these hedging tools throughout the whole portfolio investment period. Then we can conclude whether the forwards and options actually increase and by how much the new portfolio value is for Mr. Bennett.

6.3.1. Forwards Performance

First off, let us analyze the payoff from forward contracts. As we already know from the theoretical part, equation (11) enables us to compute the profit or loss realized from entering the forward contracts. For our clients' portfolio, as he holds stocks in their local currencies, short forward contracts will be entered to sell these currencies at the end of their maturity. Furthermore, as we change the weights of our securities every year, the maturity of the forwards we decided to use is one year. This accordingly reflects the payoff from the corresponding allocations to the foreign currency-denominated assets at the beginning of the year.

As we entered short forward contracts, the profit is made when the future spot price is lower than the forward price set at the beginning of the year. The 1-year forward rate and spot rates are taken from Refinitiv Eikon. We lock in a higher rate when the foreign currencies weaken – lower directly quoted exchange rates or higher indirectly quoted exchange rates – make the forwards contract lucrative. The profit or loss from the forwards will be added or subtracted from the year-end portfolio value, producing a new portfolio value to be allocated for the following year. The value of forwards is described in the table below:

Table 34: Forwards value from 2020 to 2024

2020		Portfolio Value (beginning)	$K - S_T$	P/L (USD)
	NDA	€ 1,091.84	-0.0752	\$ (82.14)
	1928	¥ 3,499,501.80	-0.0003	\$ (1,026.57)
	BKG	£ 20,272.65	-0.0301	\$ (609.42)
	1088	HK\$ 83,901.62	-0.0009	\$ (73.07)
Total				\$ (1,791.20)

2021		Portfolio Value (beginning)	$K - S_T$	P/L (USD)
	NDA	€ -	0.0869	\$ -
	1928	¥ 705,760.81	0.0010	\$ 740.09
	BKG	£ 11,906.95	0.0168	\$ 200.05
	1088	HK\$ 225,416.07	0.0007	\$ 164.11
Total				\$ 1,104.25

2022		Portfolio Value (beginning)	$K - S_T$	P/L (USD)
	NDA	€ 1,146.22	0.0720	\$ 82.48
	1928	¥ 2,804,806.28	0.0011	\$ 3,098.94
	BKG	£ 17,474.14	0.1344	\$ 2,348.00
	1088	HK\$ 277,739.39	0.0003	\$ 70.26
Total				\$ 5,599.69

2023		Portfolio Value (beginning)	$K - S_T$	P/L (USD)
	NDA	€ 2,453.42	-0.0156	\$ (38.32)
	1928	¥ 2,348,883.83	0.0010	\$ 2,264.67
	BKG	£ -	-0.0614	\$ -
	1088	HK\$ 289,718.46	0.0003	\$ 87.21
Total				\$ 2,313.55

2024		Portfolio Value (beginning)	$K - S_T$	P/L (USD)
	NDA	€ 3,919.93	0.0869	\$ 340.72
	1928	¥ 9,225,761.46	0.0011	\$ 10,138.46
	BKG	£ 2,443.22	0.0223	\$ 54.45
	1088	HK\$ 469,629.17	0.0001	\$ 56.48
Total				\$ 10,590.11

The second column shows the initial portfolio value in foreign currencies, whereas the third column represents the difference between the forward price at the beginning of the year and the spot price at the end of the year. The product of these two columns brings about the forward return at the year-end as depicted in the last column. Overall, the performance of the forward return varies significantly each year. This stressed the volatility of the foreign exchange market. As we can see from the results for 2020, foreign currencies are weakened against the dollar, leading to the loss incurred in forward value. In total, it causes the loss of 1,791 US dollars. The largest contributions to this toll come from 1928 and BKG due to their immense weights in the portfolio.

In contrast to the disastrous performance of the forward contracts due to the weakening of the home currency last year, 2021 marked the beginning of a path in which foreign currencies depreciated. As a result, we gained 1,104 USD from the forwards in the second year. This trend was witnessed again in 2022, this time there is a higher gain, 5,599 USD. Given the EUR and GBP appreciation in 2023, it thus showed negative returns from both assets, which are denominated in these currencies. Regardless of that, the forward return from others consumes the loss and turns it into the total gain of 2,313 USD at the end of 2023. In the last year of our client's investment period, we saw a great total forward return of 10,590 USD. The main contributor was 1928, when major allocations plus the weakened Yen helped produce such a gain. The profit and loss from USD/HKD 1-year forwards is relatively small and can be negligible due to its pegged currency system.

6.3.2. Options Performance

In this section, we will discuss the payoff from options, which is another derivative besides forwards, that is considered to hedge against currency risk. Compared to forwards, options are not an obligation, but you can choose to exercise them or not based on the favorable positions from the currency movement. Because of this, an option premium is paid. Options protect investors better when the exchange rate goes against the hedging plan. Thus, for instance, if the foreign currency appreciates, options can protect the portfolio by the option premium, resulting in less loss than in the case of forwards. It, however, simultaneously makes the option less profitable than forwards when the foreign currency depreciates. Similar to the forwards, a maturity of 1 year is taken into consideration for ATM options. A new option is entered at the beginning of the following year after it matures at the end of the current year. We buy EUR, GBP, JPY, and HKD put options, which allow us to sell the foreign currency at the year-end.

The option premium is calculated by using the application FX Options Calculator provided by Refinitiv Eikon. Because ATM options are used, the strike price will be equal to or very close to the current spot price. Applied this to the put payoff formula in equation (12), the P/L for options equals the difference between the current spot price and the spot price at the time of commencement. Moreover, the ATM implied volatility is used in the option premium calculation. It is highly relevant, as we know from the theoretical part, that the higher volatility may increase the option value given the potential of a higher payoff. As a result, we have the options premiums as shown in the table below, which have already been adjusted according to the portfolio value at the outset.

Table 35: Options value from 2020 to 2024

2020		Portfolio Value	$S_0 - S_T$	Premium (USD)	P/L (USD)
NDA	€	1,091.84	-0.1003	\$ 17	\$ (17)
1928	¥	3,499,501.80	-0.0005	\$ 541	\$ (541)
BKG	£	20,272.65	-0.0419	\$ 719	\$ (719)
1088	HK\$	83,901.62	-0.0006	\$ 65	\$ (65.00)
Total					\$ (1,342)

2021		Portfolio Value	$S_0 - S_T$	Premium (USD)	P/L (USD)
NDA	€	-	0.0844	\$ -	\$ -
1928	¥	709,032.00	0.0010	\$ 173	\$ 537
BKG	£	11,962.14	0.0142	\$ 508	\$ (338)
1088	HK\$	226,460.87	0.0007	\$ 112	\$ 51
Total					\$ 250

2022		Portfolio Value	$S_0 - S_T$	Premium (USD)	P/L (USD)
	NDA	€ 1,143.07	0.0592	\$ 22	\$ 46
	1928	¥ 2,797,079.66	0.0010	\$ 545	\$ 2,377
	BKG	£ 17,426.00	0.1374	\$ 746	\$ 1,648
	1088	HK\$ 276,974.28	0.0002	\$ 94	\$ (38)
Total					\$ 4,033

2023		Portfolio Value	$S_0 - S_T$	Premium (USD)	P/L (USD)
	NDA	€ 2,418.14	-0.0374	\$ 57	\$ (57)
	1928	¥ 2,315,106.96	0.0006	\$ 372	\$ 922
	BKG	£ -	-0.0684	\$ -	\$ -
	1088	HK\$ 285,552.32	-0.00001	\$ 171	\$ (171)
Total					\$ 694

2024		Portfolio Value	$S_0 - S_T$	Premium (USD)	P/L (USD)
	NDA	€ 3,823.38	0.0691	\$ 79	\$ 185
	1928	¥ 8,998,514.33	0.0007	\$ 1,127	\$ 5,503
	BKG	£ 2,383.04	0.0212	\$ 90	\$ (39)
	1088	HK\$ 458,061.36	-0.0007	\$ 93	\$ (93)
Total					\$ 5,556

In 2020, because all currencies appreciated against the U.S. dollar, options limited the loss to its option premium paid. As a result, we can see that the total loss is 1,342 USD, which is lower than the 1,791 USD lost from forward contracts. However, since the foreign currencies' depreciation happened in the following years, the options produced lower returns. Due to the high option premium, it can be seen that the option return results in negative returns, even though foreign currencies do not strengthen against the dollar. For example, losses from the GBP/USD option in 2021 and 2024, which are 338 USD and 39 USD, respectively, or a loss of 38 USD from the USD/HKD option in 2022.

Overall, both forwards and options have shown their own advantages and disadvantages. Although the exchange rate moved against the hedging plan in the initial year, it has turned out to follow the hedging strategy favorably in the following years. As a result, the portfolio value when hedged with either forwards or options becomes higher than the unhedged one. Compared to the unhedged portfolio value of 149,591 USD at the end of 2024, the portfolio value increased to 158,379 USD when hedged with options and to 166,739 USD when hedged with forwards. A more detailed comparison of these portfolios' attributes will be discussed in the next chapter, where we also introduce a benchmark such as the S&P 500.

6.4. Performance Comparison

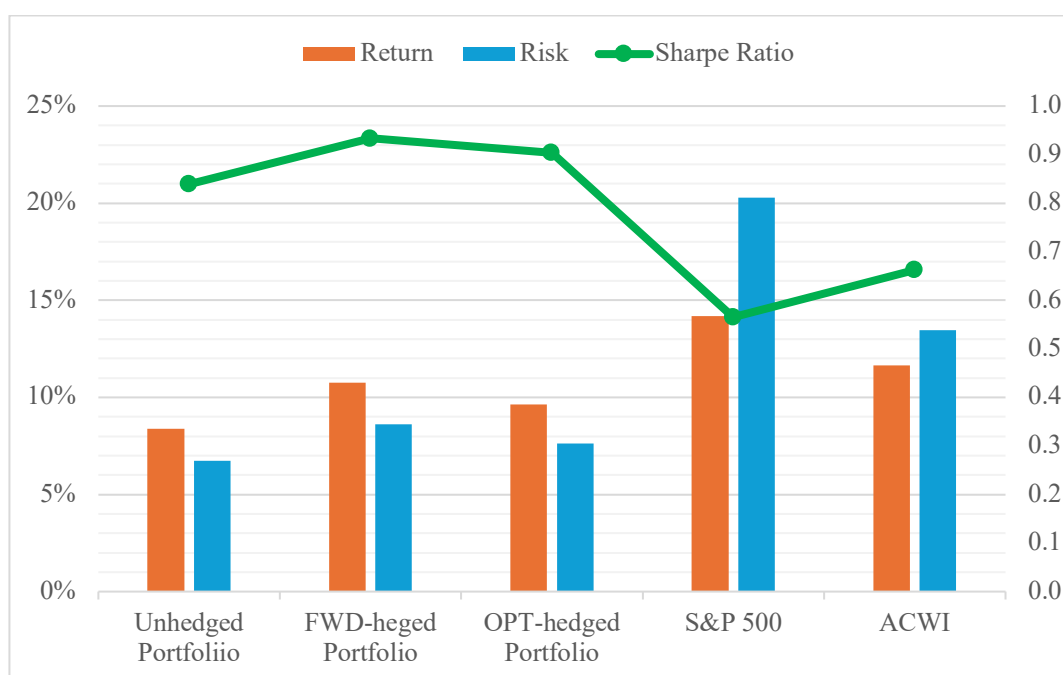
After we obtained all the portfolio properties from the unhedged portfolio, the portfolio hedged with forwards and options, we will put them together and compare their performance in this chapter. We also evaluate the performance of these portfolios by comparing them to relevant benchmarks of market return. To this end, we will understand if our investment is superior to the benchmark or whether we should replicate the benchmark performance to be more successful in case the market is more outstanding. For our portfolio, we choose two index benchmarks for the portfolio performance comparison with the market. The first one is the MSCI All Country World Index (MSCI ACWI). This is an international equity index that tracks stocks coming from both developed (90%) and developing countries (10%). It is a relevant measure to evaluate the performance of our international portfolio as it consists of stocks from emerging markets like China, and from advanced economies such as the U.S., the U.K., Japan, or Germany (representing Europe).

In addition, we select the S&P 500, which tracks the performance of the top 500 publicly traded companies in the United States, as the second benchmark for several reasons. Firstly, one of our chosen securities is listed in the S&P 500 index. Secondly, this is a common measure and a reliable financial indicator for investors to assess the stock market in the U.S. due to its solid historical long-term returns. Moreover, considering Mr. Bennett is of American nationality, the S&P 500 acts as a domestic and diversified index benchmark. By choosing it for comparison, we can understand how our internationally diversified portfolio differentiates from a domestically diversified benchmark. S&P 500 historical data from 2020 to 2024 is extracted from Yahoo Finance under the ticker symbol SPX.

Figure 11 below illustrates the returns and risks of the S&P 500, ACWI, and the international portfolio with 3 scenarios. The unhedged portfolio has an annual return of over 8% and a standard deviation of 7%. On the other hand, when the hedging tools are introduced to the portfolio, they moderately increase the portfolio's return and risk. The market return, measured from the S&P 500, has a significantly higher return but simultaneously much higher risk compared to our client's portfolio. The ACWI has returns close to the level of the forward-hedged portfolio's return; however, the market presents greater risk.

The Sharpe ratios are determined to assess the risk-adjusted performance. The best performance is observed from the portfolio using forwards. Its Sharpe ratio amounted to 0.93. The option-hedged portfolio comes very close to the second-highest Sharpe. 0.84 is the ratio for a portfolio not using any protection against exchange rate risk. MSCI ACWI demonstrated much lower performance compared to Mr. Bennett's portfolio, with a Sharpe ratio of 0.66. The worst result stems from the domestic benchmark, S&P 500, which has only 0.57 for this ratio.

Figure 11: Performance of International Portfolio compared to benchmarks



There are a few remarks we can discuss after seeing the results from the performance comparison. First and foremost, our results prove that, by using hedging tools, such as forwards and options, we see better risk-adjusted returns. It emphasizes the necessity and benefit of utilizing derivatives in an international portfolio to reduce the exposure to currency risk. Without proper currency risk management in an internationally diversified portfolio, investors are exposed to exchange rate risks, thus damaging the risk-adjusted performance of the portfolio. When we compare the performance between options and forwards, we found that the portfolio that used forwards to manage currency risk achieved better results than that from options. Moreover, our portfolio, as agreed with the client at the beginning, is actively managed to beat the market portfolio. The results showed that by proper

management, an active portfolio outperformed a passive portfolio represented by an index such as the S&P 500 and MSCI ACWI.

Conclusion

In the context of a globalized financial system, managing cross-border investment portfolios involves a multitude of risks, with foreign exchange risk being the most critical. In particular, this dissertation aimed to develop the theory and practice of international portfolio construction with emphasis on foreign exchange risk and its management through hedging techniques. Systematic exploration of modern portfolio theory, empirical portfolio simulations, and comparative studies of hedging instruments provided strong evidence for the advantages of controlled risk exposure and risk management achieved through diversification.

The study reinforced the strength of MPT guides for constructing efficient portfolios, especially in international settings, where the lower correlation between asset classes can be optimally utilized. However, it also revealed the inadequacy of traditional models when confronted with reality's exchange rate unpredictability, non-normal return distribution, and market crash. This dissertation incorporated MPT with global portfolios, bringing in forwards and options, thereby demonstrating the extent to which theory can be used to mitigate actual financial risks.

The empirical aspect of the study was centered around creating a comprehensive portfolio for a conservative US investor lasting five years. The simulation showed how the asset allocation changes over time due to market conditions, risk-return profiles, and macroeconomic shifts, all optimized through yearly restructuring, which went along with a set of optimization criteria. A major insight was the unstopped and stopped portfolios' performance gap. The portfolios that were hedged with currency forwards and options outshone the unhedged counterparts with regard to the Sharpe ratio, thus affirming the strategy's effectiveness in capital preservation and returns enhancement.

In particular, the forward-hedged portfolio displayed the greatest risk-taking Sharpe ratio, proving its usefulness for capital institutions and conservative investors. Although the options provided advantageous downside protection and flexibility during periods of extreme volatility, their positioning within the highly currency-dependent context made them secondary due to their depressed value and profits in favorable periods. Regardless, both

instruments significantly outperformed the unhedged strategy, solidifying the claim that currency hedging is a fundamental aspect of an international investment policy.

The dissertation presents a strategic conclusion that additional risk is not only meant to be mitigated, but that it can also serve as an opportunity for better decision making in a more uncertain environment. Having a proactive policy towards controlling exchange rate can, at times, turn portfolio liability currency risk into a controllable and sometimes, exploitatively favorable factor.

Nevertheless, the research has its own shortcomings. Simulation of this work was based on assumptions about the investors' behaviors, their taxes, and the fees involved in buying and selling stocks, all of which are bound to be different in real life. Also, while the application of derivatives was in an idealistic theoretical context, in the practical realm, one faces additional restrictions such as limited capital resources, legislative controls, and risk associated with the models used. The work also relies mainly on the exposure of currency risk; however, in reality, many other risks need to be dealt with that demand attention from investors. Moreover, data has to be amended for analysis as different stock exchange has different holidays and opening hours.

Subsequent work can explore policy constraints and their impact on systematically dynamic hedge with expectations of pricing impact and machine learning algorithms for evaluating these, along with numerous other classes of assets. Additionally, the effects of new macroeconomic threats, including but not limited to climate change or political fragmentation, on international portfolios could be analyzed. Another investment strategy, such as passive management or short-term investment, can also be a suggestion for later research.

In summary, this dissertation provides insights for investment practice by showing that international portfolio management coupled with currency risk strategies can significantly enhance financial performance. Investors who maintain a disciplined approach towards globalization and use appropriate analytical and hedging strategies gain access to greater markets as well as improved agility and performance amidst global financial instability.

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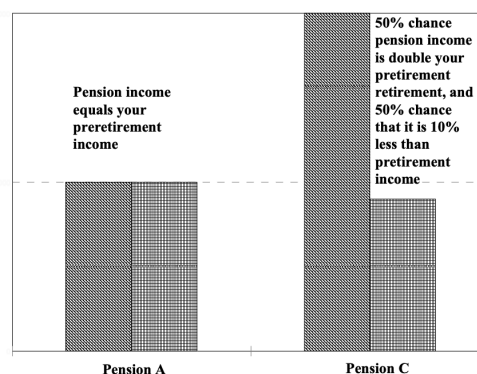
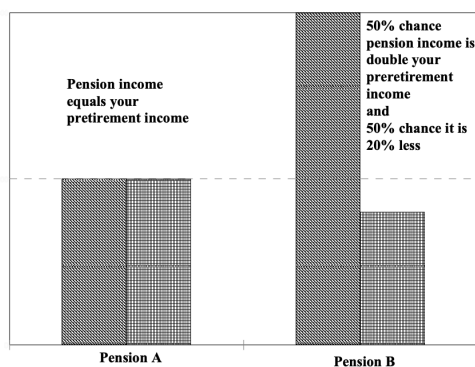
Appendix

Appendix 1: Risk Tolerance Questions - Subjective Risk Tolerance

Subjective Risk Tolerance Questions

- Suppose that you are about to retire, and have two choices for a pension
 Pension A gives you an income equal to your preretirement income.
 Pension B has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **20%** less than your preretirement income.
 You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future.
 All incomes are aftertax.
 Which pension would you choose?
 If A, go to #2.
 If B, go to #5.

- Suppose that you are about to retire, and have two choices for a pension
 Pension A gives you an income equal to your preretirement income.
 Pension C has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **10%** less than your preretirement income.
 You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future.
 All incomes are aftertax.
 Which pension would you choose?
 If A, go to #3 on the next page
 If C, your subjective risk tolerance is **Moderate**

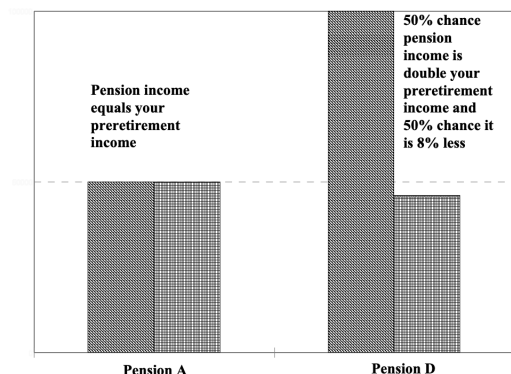


3. Suppose that you are about to retire, and have two choices for a pension. Pension A gives you an income equal to your preretirement income. Pension B has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **8%** less than your preretirement income. You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.

Which pension would you choose?

If A, go to #4

If D, your subjective risk tolerance is **Low**

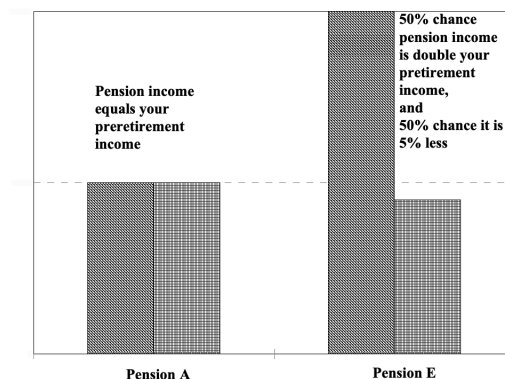


4. Suppose that you are about to retire, and have two choices for a pension. Pension A gives you an income equal to your preretirement income. Pension B has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **5%** less than your preretirement income. You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.

Which pension would you choose?

If A, your subjective risk tolerance is **Extremely Low**

If E, your subjective risk tolerance is **Very Low**

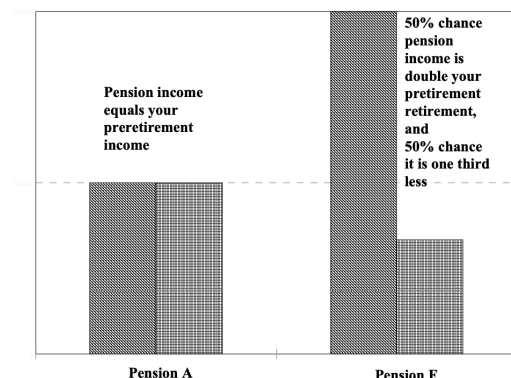


5. Suppose that you are about to retire, and have two choices for a pension. Pension A gives you an income equal to your preretirement income. Pension F has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **one third** less than your preretirement income. You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.

Which pension would you choose?

If A, your subjective risk tolerance is **Moderately High**

If F, go to #6

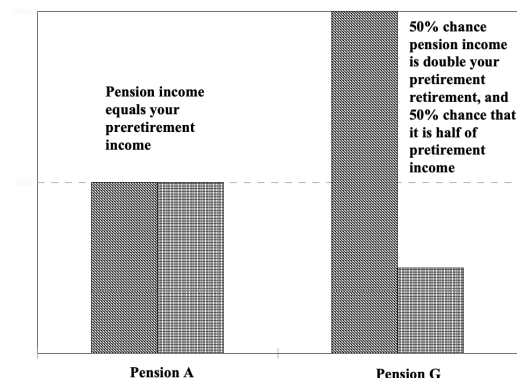


6. Suppose that you are about to retire, and have two choices for a pension. Pension A gives you an income equal to your preretirement income. Pension G has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **half** of your preretirement income. You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.

Which pension would you choose?

If A, your subjective risk tolerance is **Very High**

If G, your subjective risk tolerance is **Extremely High**



Risk Aversion Level*

Extremely Low Risk Aversion ($A < 1.0$)

Very Low Risk Aversion ($1.0 \leq A < 2.0$)

Moderately Low Risk Aversion ($2.0 \leq A < 3.8$)

Moderate Risk Aversion ($3.8 \leq A < 7.5$)

High Risk Aversion ($7.5 \leq A < 9.3$)

Very High Risk Aversion ($9.3 \leq A \leq 14.5$)

Extremely High Risk Aversion ($A > 14.5$)

*Lower Risk Aversion = Higher Risk Tolerance

A = Risk Aversion Level