Considering the introduction to Limit State Design presented as Term Paper in the "Foundation Design" course, fall. 86 two different types of problems are presented.:

The first one consists of the application of level II methods for the design of a pile.

The second is an attempt to evaluate characteristic coefficients to be used as level I method. These coefficients, if conveniently obtained, may be used as code parameters to be included in a future Design Code.

PART I

Suppose that we have a pile in sand where the physical soil properties are statistically defined and the dimensions of the pile are given. Suppose that the probability of failure is also given, corresponding to a certain reliability index. Suppose that the type of load is also given. We want to know what is the maximum load the pile can resist.

Hypothesis:

1 - The strength of the pile is given by

$$Q_{tip} + Q_{friction} = Q_r$$

$$Q_{tip} = \pi R^2 (0.6\, \gamma R N_\gamma + \gamma D_f N_q)$$

$$Q_{friction} = \int_0^L f(L)\, \pi\, 2R\, dL$$

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where, R - diameter of pile (ft)

γ - soil weight (pcf)

N_γ, N_q, — soil parameters

$f(L)$ - lateral friction ($\#/ft^2$)

D_f - depth of tip (ft)

2 - Assume that ϕ, angle of internal friction of the soil, has been analysed for that site. Adopting Meyerhoff equations (1955) and Hansen equations (1970) we have:

$$N_q = \tan^2\left(45^\circ + \frac{\phi}{2}\right) e^{\pi \tan\phi}$$

$$N_\gamma = 1.5\,(N_q - 1)\tan\phi$$

The lateral friction, $f(L)$ may also be found from ϕ, at least through a regression analysis:

$$f_{e_i} = a_1 \phi_i + b_i \qquad \begin{cases} \Sigma f_i = a_1 \Sigma \phi_i + n b_1 \\ \Sigma f_i \phi_i = a_1 \Sigma \phi_i^2 + b_1 \Sigma \phi_i \end{cases}$$

With these relations and the statistical relation between two independent variables, we can find the standard deviation of N_γ and N_q in function of ϕ.

Proof: $U = g(\underset{\sim}{x})$, $\underset{\sim}{x} = (x_1, x_2, x_3, \ldots x_n)$, then

$$\sigma_U = \sqrt{\left[\sum^{n} \frac{\partial U}{\partial x_i} \cdot \sigma_{x_i}\right]^2}$$

Benjamin & Cornell (1970)
McGraw-Hill Book Co.

If $U = g(x)$ then $\sigma_U = \dfrac{dU}{dx} \cdot \sigma_x$, case of N_γ and N_q.

For example, suppose $\phi = 30°$ and $\sigma_\phi = 2.5°$

Then,

$N_q = 18.40$ and $\dfrac{dN_q}{d\phi} = 119.57$ – Appendix VI of Smith (1986)

$N_\gamma = 15.07$ and $\dfrac{dN_\gamma}{d\phi} = 138.36$ – Appendix VII (same)

$$\sigma_{N_q} = 119.57\left(\frac{2.5\pi}{180}\right) = 5.22$$

$$\sigma_{N_\gamma} = 138.36\left(\frac{2.5\pi}{180}\right) = 6.03$$

As no data is available for the regression analysis of $f(L)$ and ϕ let us assume

$k \tan\delta\, \gamma\, \pi = 0.95(0.45)(128)\pi$

$$f(L) = \begin{cases} 171.9\,(2R)\,L & (L \leq 20\,(2R)) \\ 171.9\,(2R)\,20\,(2R) & (L > 20\,(2R)) \end{cases}$$

critical depth

Fixing $k = 0.5$ ft then

$$f(L) = \begin{cases} 171.9\,L & (L \leq 20') \\ 3438\ \# & (L > 20') \end{cases}$$

Suppose that the coefficient of variation of f is 10%. So $\sigma_f = 0.1 f$

Fix total length at 25 ft.

3- The reliability index is fixed at $4 (=\beta)$. The loads are dead load and live loads and the coefficient of variation is 15%. So, $\sigma_P = 0.15\,P$.

Procedure :

1- <u>Definition of variables</u> (all assumed normal distributions, whic is not true or certain because tests must be conducted to determine the respective type of distribution)

Variable	Meaning	Mean value	Stand. devia.
X_1	N_γ	15.07	6.03
X_2	N_q	18.40	5.22
X_3	$\int_0^L f(L)$	51,570 (*)	5157
X_4	P (design load)	?	$0.15\,P$

$$(*)\ \int_0^L f(L)\,dL = 171.9\left(\frac{20^2}{2}\right) + 3438\,(5) = 51{,}570\ \text{lb}$$

2- <u>Normalization of variables</u>

$$U_1 = \frac{X_1 - 15.07}{6.03} \Rightarrow X_1 = 6.03\,U_1 + 15.07$$

$$X_2 = 5.22\,U_2 + 18.40$$

$$X_3 = 5157\,U_3 + 51{,}570$$

$$X_4 = 0.15\,P\,U_4 + P$$

3 – Definition of failure function

$$Z = r - s$$

$$Z = b + a_1 x_1 + a_2 x_2 + a_3 x_3 - a_4 x_4$$

where $b = 0$, $a_1 = 30.16$, $a_2 = 2513$, $a_3 = 1$, $a_4 = 1$

Replacing the normalized variables U_i we have for $z = 0$:

$$30.16(6.03\,U_1 + 15.07) + 2513(5.22U_2 + 18.4) +$$
$$+ 5157U_3 + 51{,}570 - 0.15\,P U_4 - P = 0$$
$$181.86U_1 + 13118U_2 + 5157U_3 - 0.15PU_4 + 98264 - P = 0$$

The cosine directors of the vector giving the closest point are

$$\alpha_1 = \frac{\partial z}{\partial U_1} / k = 181.86/k \qquad \alpha_3 = 5157/k$$

$$\alpha_2 = 13118/k \qquad \alpha_4 = 0.15P/k$$

By geometry $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 = 1.$

$$181.86^2 + 5157^2 + 13118^2 + (0.15P)^2 = k^2$$

$$\beta = \frac{(98264 + P)\,k}{k^2} = \frac{98264 - P}{\sqrt{181.86^2 + 5157^2 + 13118^2 + (0.15P)^2}}$$

↳ distance between origin and failure function.

P is found iteratively since $\beta = 4$

P	β
50,000	3.02
40,000	3.80
38,000	3.96
37,500	4.00 (ok!)

P is 37,500 lb with a prob. of failure of $\Phi(4)$

Part II

To use Level I methods for less experienced designers or quicker design it is necessary to make research studies. Several methods are proposed. One is to use sensitivity coefficients from Level III methods. This method is described in reference 12. Another one, possible for similar problems, let us say in the case of pile design, sand deposits with same characteristics and identical type of loadings. This method may be applied in a possible code if a study is made for the characteristic sands and loadings in the state.

The justification for this method is the definition of characteristic values and is presented in reference 6, pages 85 - 87.

The values are

$$\gamma_{rmr}^{-1} = \frac{1 + \alpha_r \beta c_{xr}}{1 - k_r c_{xr}}$$

$$\gamma_{fs} = \frac{1 + \alpha_s \beta C_{xs}}{1 + K_s C_{xs}}$$

For example in problem of part I and for similar situations we can derive the partial safety factors for designs with $\beta = 4$.

Let us choose characteristic values as fractiles of 0.95 and 0.05. Then $k_r = 1.645$ and $K_s = 1.645$.

α_r and α_s are the cosine directors and have the following values:

$K = 15177$

$\alpha_1 = 0.0120$ $\qquad C_1 = 0.40$

$\alpha_2 = 0.8643$ $\qquad C_2 = 0.28$

$\alpha_3 = 0.3398$ $\qquad C_3 = 0.10$

$\alpha_4 = 0.3706$ $\qquad C_4 = 0.15$

$$\gamma_1^{-1} = \frac{1 + 0.012\,(4)\,(0.4)}{1 - 1.645\,(0.4)} = \frac{2.612}{0.342} = 7.64$$

$$\gamma_2^{-1} = \frac{1 + 0.8643\,(4)\,(0.28)}{1 - 1.645\,(0.28)} = \frac{1.968}{0.539} = 3.65$$

$$\gamma_3^{-1} = \frac{1 + 0.3398\,(4)\,0.1}{1 - 1.645\,(0.1)} = \frac{1.136}{0.836} = 1.36$$

$$\gamma_4 = \frac{1 + 0.3706\,(4)\,0.15}{1 + 1.645\,(0.15)} = \frac{1.222}{1.247} = 0.980$$

A global systematic procedure for complicated problems (lots of variables and correlations) is presented in reference 6, pages 93-94.