ECI 5325 FOUNDATION DESIGN

TERM PAPER

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APPLICATION OF LIMIT STATE DESIGN THEORY TO PILES

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Introduction

1.1. Probabilities in Structural Safety

In the design and checking of the structures in the field of civil engineering the use of probabilistic principles and methodologies has now a large number of supporters. It is the object of several research projects and it is introduced into nearly all technical codes. It is a relatively young science that evolved in the same way as other new areas of study: began with theoretical studies that dictate the general principles with which all problems are systematically treated. However, the great practical difficulty in obtaining enough statistical data, the sophistication of the probabilistic methods involved and the length of the analytical processes to determine the structural reliability created, within the framework of structural stability, originated different working levels. These working levels depend on the problem considered and on the desired accuracy for the reliability evaluation. The basic levels in limit state design are three: Level 1, Level 2 and Level 3. The number increases with the sophistication of the method used. To describe what are the differences between these levels, the concepts of limit state design and probability of failure must be defined.

1.2. Limit State Design

The concept of limit state may be described as that that state beyond which a structure, or part of it, can no longer fulfill the functions or satisfy the conditions for which it was designed. Namely, the structure is said to reach a limit state when a specific response parameter attains a threshold value. As examples of ultimate limit states are the loss of equilibrium of a part or the whole of the structure considered as a rigid body, failure or excessive plasticity of critical sections due to static actions, transformation of the structure into a mechanism, buckling due to elastic or plastic instability, fatigue, excessive deflections and excessive cracking. The most modern codes divide the limit states into two main groups: ultimate limit states corresponding to the maximum load carrying capacity, and service limit states, related to the criteria governing normal use and durability. For each of these groups the importance of damages caused is different and that is represented by the respective probability of failure adopted.

1.3. Probability of Failure

The probability of failure envisaged in limit state designs vary with the risk of loss of human lives, the number of lives affected and the economic consequences. In ultimate limit states the range of probability of failure adopted is between 10^{-3} and 10^{-7} over a 50 years design life of structure. In serviceability limit states the probability of failure varies between 10^{-1} and 10^{-3}. The figure, for example, of 10^{-3}, means that on, the average, out of 1000 nominally identical buildings one will crack or deform excessively. Now, it is evident that in civil engineering 1000 identical buildings rarely occur, even neglecting the fact that a statistically significant number require samples at least 10 to 20 times larger. Moreover, the determination of these low probabilities requires extrapolations of statistical properties that are experimentally known only around the central values of the random quantities. For these reasons, the probabilities of failure in civil engineering have no real statistical significance and they are conventional comparative values.

1.4. The Use of Limit State Design Levels

In consequence of the above considerations the differences between the methods used in each of the three levels are rather operational than conceptual and there are no rigid boundaries between them. Level 3 methods require a complete analysis of the problem and also the integration of the joint distribution density of the random variables extended over the safety domain. They remain in the field of research and are used to check the validity of approximations, idealizations and simplifications performed in the other two levels. The Level 2 methods use random variables characterized by their known or assumed distribution functions, defined in terms of important parameters as means and variances. This avoids the multidimensional integration of the previous method. These methods may be used by engineers to solve problems of special technical and economical importance and by code committees engaged in drafting and revising standard codes of practice to evaluate the partial safety factors. In Level 1 methods the probabilistic aspect of the problem is represented by characteristic values of the random variables involved. With these characteristic values partial safety factors are derived using Level 2 methods. They are used by most engineers where these methods are the basis of their code provisions.

$$\sigma_x^2 = \sigma_r^2 + \sigma_m^2 - \cancelto{0}{2\rho\sigma_r\sigma_m}$$

0, because $\rho = 0$

Basic Problem Formulation

2.1.Definitions for a Two Dimensional Problem

Let R and S be two random variables, where R defines strength and S the actions. Then the limit state function, z, is defined as

$$z = r - s$$

In figure 1 the domain D (z > 0) is the safe domain and D' is the failure domain. The probability of failure, p_f, is the probability that a point (r,s) belongs to D'. Once the statistical distributions of the random variables R and S are known the numerical solution of the following equation will determine p_f.

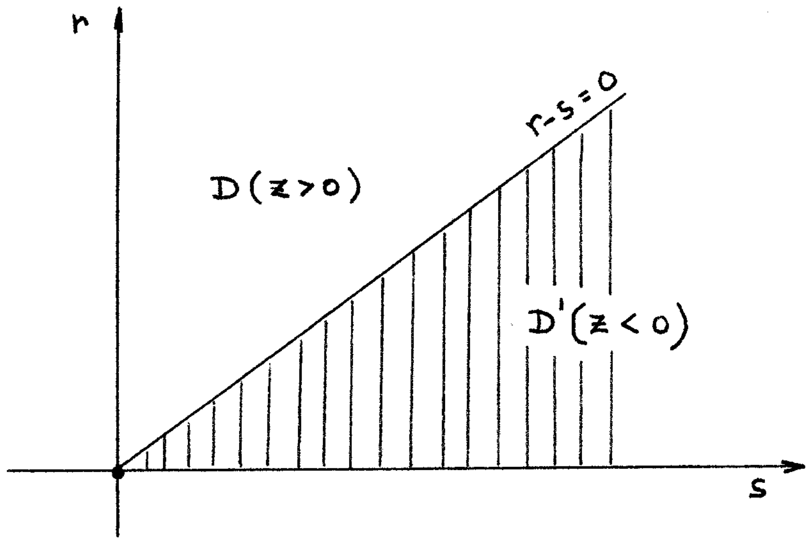
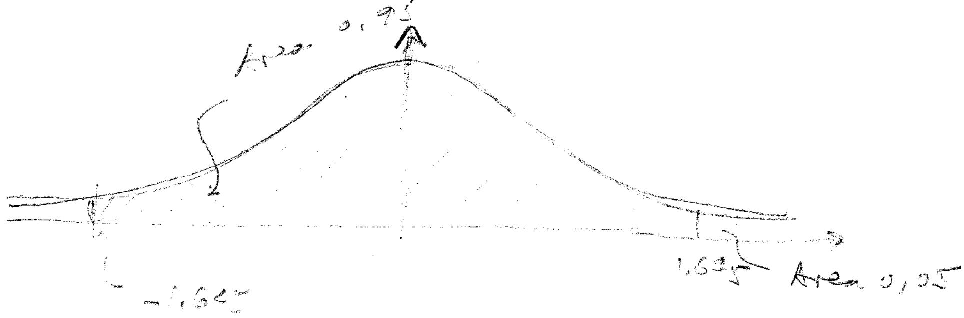


Figure 1.Geometric representation of safety margin z.

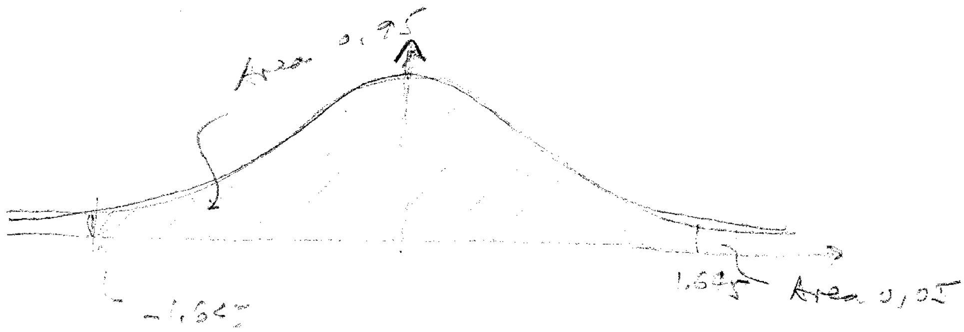
2.2.Case of Normal Distribution

Assuming both variables, R and S, have a Gaussian distribution, defining r^m and s^m as the mean values and σ_R and σ_S as standard deviations, the random variable Z will also be normal with parameters

$$z^m = r^m - s^m \quad \text{and} \quad \sigma_z = (\sigma^2{}_r + \sigma^2{}_s)^{1/2}$$



Area 0,95
-1,645
1,645
Area 0,05


Area 0,95
-1,645
1,645
Area 0,05

Then, with F_z the cumulative normal distribution function

$$p_f = P\{Z < 0\} = F_z(0)$$

Introducing the standardized variable

$$u = (z - z^m) / \sigma_z$$

and

$$\beta = z^m / \sigma_z = (r^m - s^m) / (\sigma^2_r + \sigma^2_s)^{1/2}$$

then

$$p_f = F_u(-z^m / \sigma_z) = F_u(-\beta)$$

2.3.Central Safety Factor

The central safety factor is defined as

$$\mu_o = r^m / s^m$$

Denoting the coefficients of variation of R and S by

$$c_r = \sigma_r / r^m \quad \text{and} \quad c_s = \sigma_s / s^m$$

and rearranging the equations the central safety factor may be expressed in a convenient form

$$\mu_o = \frac{1 + [\beta^2(c^2_r + c^2_s) - \beta^4 c^2_r c^2_s]}{1 - \beta^2 c_r^2}$$

This correlation between β and μ_o is often used by several authors for different values of c_r and c_s.

2.4.Characteristic Safety Factor

In addition to the central safety factor another typical parameter is used to classify the structural safety

$$\mu_k = r_{0.05} / s_{0.95}$$

where $r_{0.05}$ is the value of r exceeded in 95% and $s_{0.95}$ is the value of s exceeded in only 5%. This characteristic safety factor is also defined in other ways

$$\mu_k = (r^m - 1.645.r^m.c_r) / (s^m + 1.645.s^m.c_s)$$

$$\mu_k = (\mu_o - 1.645.\mu_o.c_r) / (1 + 1.645.c_s)$$

3.1. Lind-Hasofer Method

With Level 2 methods, safety checks are made at a finite number of points of the safety domain boundary. In the case where this check is made at only one point, the parameter to be determined is the minimum distance between the origin of the system of coordinates of the random variables to the boundary of the safety domain. It is possible to associate with this distance a precise meaning in terms of reliability. A technique derived from this concept is the Lind-Hasofer Minimum Distance method described in reference 6, 10 and 11.

Let $\underline{X}$ [$X_1, X_2, \ldots, X_n$]be the vector of the basic random variables, assumed to be uncorrelated, involved in a given structural problem. Let $z = g\ (x_1, x_2, \ldots, x_n) = 0$ be the boundary of the safety domain. The values of $\underline{X}$ belonging to the failure domain will satisfy the inequality

$$z = g\ (\ \underline{x}\ ) < 0$$

The method consists in projecting z in the space of standardized variables defined as

$$u_i = (\ x_i - x^m{}_i\ )\ /\ \sigma x_i$$

and measuring, in this space, the minimum distance β of the transformed surface $g\ (\ u_1, u_2, \ldots, u_n\ )$ from the origin of the axes. A design is regarded reliable if $\beta \geq \beta^*$, β^* prescribed by an appropriate code provision. In geometrical terms, the hypersphere having radius β^* and with center at the origin of the axes u_i is required to lie within the transformed safety domain. The justification for such method is that most of the joint probability density of the variables involved will be concentrated in the hypersphere having radius β^* and that consequently it will be associated with values of vector $\underline{X}$ belonging to the safety domain. Mathematically, the problem to be solved is

$$\beta = \min\ (\ \Sigma\ u^2{}_i\ )^{1/2}$$

and

$$g\ (\ u_1, u_2, \ldots, u_n\ ) = 0$$

3.2.Case of Linear Safety Margin Function

In a great number of cases the safety boundary domàin is linear and then

$$z = g\ (\ x_1,\ x_2, \ldots,\ x_n\ ) = b + \Sigma\ a_i . x_i .$$

Then, β can be immediately determined as follows

$$g\ (\ u_1,\ u_2, \ldots,\ u_n\ ) = b + \Sigma\ a_i . x^m{}_i + \Sigma\ a_i . \sigma x_i . u_i = 0$$

and the distance of this hyperplane to the origin of the is

$$\beta = (\ \Sigma\ a_i . x^m{}_i + b\ )\ /\ [\ \Sigma\ a^2{}_i . \sigma x^2{}_i\ ]^{1/2}$$

A geometric interpretation for a two dimensional case is given in figure 2. Expressing in terms of the standardized variables is the same as replacing the hypersurface by the hyperplane passing through P*, point of minimum distance.

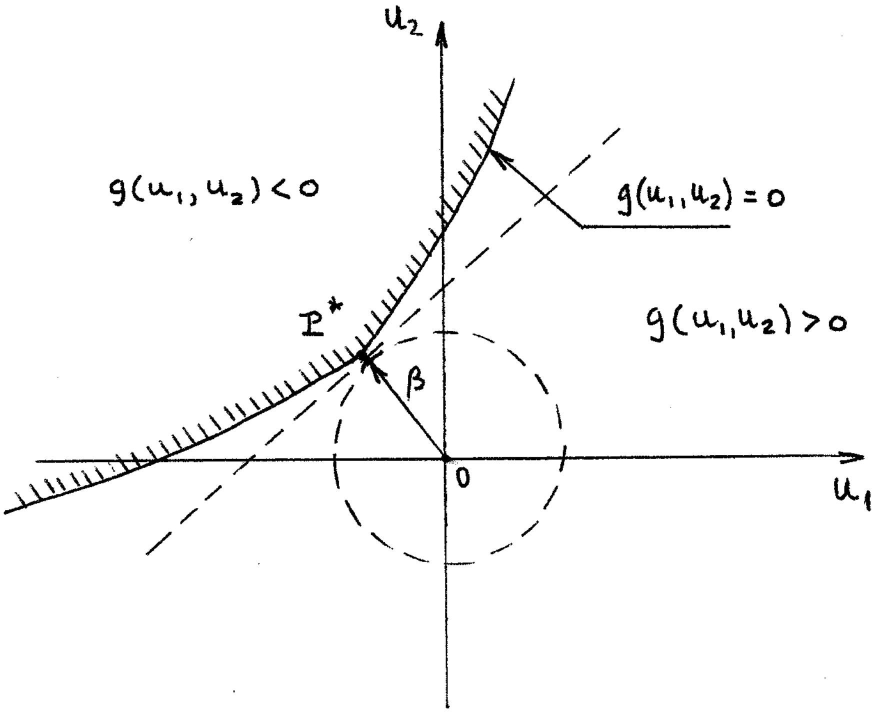


Figure 2.Geometric interpretation of reliability index β.

3.3. Algorithm for the General Problem

For the general problem it can be shown that the direction cosines α_i in reference to x_i of the point P^* are given by

$$\alpha_i = - [(\delta g / \overset{\partial x_i'}{\delta \cancel{y}_i}) \sigma x_i] / \{ \Sigma [(\delta g / \delta x_j) \sigma x_j]^2 \}^{1/2}$$

and that $u^*_i = \beta \, \alpha_i$.

Then a typical design problem, where an unknown deterministic parameter θ is to be found such that a target safety level is obtained, is solved iteratively:

1) fix a value of θ,
2) estimate $\underline{x}^*$,
3) compute α_i using given equation,
4) calculate $x_i = x^m_i + \alpha_i . \beta . \sigma x_i$,
5) determine θ by solving $g (\underline{x}) = 0$ and
6) repeat steps 3 to 5 until stable value of θ and $\underline{x}^*$ are obtained.

Level 1 Method

4.1. Characteristic Values

The nominal parameters used in these techniques are the characteristic values, defined as fractiles of a predetermined order of the respective distribution functions. They are low for the mechanical material properties, e. g. 0.05, and high for the actions, e. g. 0.95. The failure criterion is expressed in terms of the following expression

$$g (x_1 , \ldots , x_h , x_{h+1} , \ldots , x_n) < 0$$

and safety analysis is checked by the verification of the following inequality

$$g (x_{1k} / \mu_{m1} , \ldots , x_{hk} / \mu_{mh} , x_{(h+1)k} . \mu_{f(h+1)} , \ldots , x_{nk} . \mu_{fn}) \geq 0$$

where
X_r ($r = 1, 2, \ldots, h$) are the resisting variables
X_s ($s = h+1, \ldots, n$) are the actions
x_{rk} ($r = 1, 2, \ldots, h$) are the characteristic values of the resisting variables
x_{sk} ($s = h+1, \ldots, n$) are the characteristic

values of the actions

γ_{mr} are the partial safety factors for the resisting variables (≥ 1)

γ_{fm} are the partial safety factors for the actions (≥ 1)

4.2. General Considerations

These are the basis of present day codes that use probabilistic concepts. These methods could be in the future replaced by the Level 2 methods if an agreement was obtained in the following issues: selection of basic random variables for each specific problem, their distribution types and relative statistical parameters; form of the various limit state equations and choice of models; operational reliability levels to be adopted in different design situations. However, it must be underlined that the advantage of Level 1 schemes over Level 2 are their great operational simplicity due to the use of fixed and constant partial safety factors for a given class of design situation. The main disadvantage of Level 1 is the selection of partial safety factors for a given structural class in such a way that the efficiency of the method proposed is satisfactory. It must assure that the deviation of the reliability of a design made on the basis of the adopted coefficients from the desired reliability level laid down in the code should be acceptable, sacrificing accuracy.

From reference 11, the probability of failure, p_f, and the reliability index, β, are within certain approximations related by

$$p_f = 1 - \phi (\beta)$$

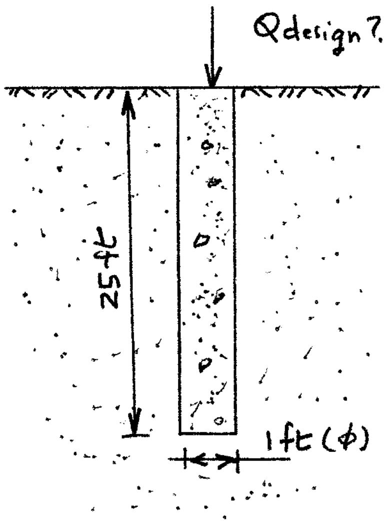
where ϕ is the function of standardized cumulative normal distribution. With this relation and expressing γ_m and γ_f in terms of functional relations of the random variables, these partial safety factors may be derived from a Level 2 problem as shown in an example of next chapter.

There are other procedures to find partial safety factors depending on the decisions of the code committees. To illustrate these differences some examples may be analysed from the ones presented in references 5,8 and 9 applied to the design of reinforced and prestressed concrete structures.

Application to Pile Design

5.1. Classical Design of a Pile

The example of class given October, 31 is used to illustrate traditional design. For the pile described in figure 3 find Q_{design} with a safety factor of 2.



Medium dense sand

weight (w) = 128 pcf

$N_g = 65$

$N_q = 50$

$D_c = 20.\text{diam}$

$k = 0.95$

$\tan\delta = 0.45$

Figure 3.Pile parameters.

$$Q_u = Q_{tip} + Q_{friction}$$

$$Q_{tip} = \pi\, r^2\ (0.3.w.\text{diam}.N_g + w.D_f.N_q) =$$

$$= \pi(0.5)^2[(0.3)(128)(1)(65) + (128)(20)(50)] =$$

$$=102{,}491 \text{ lb}$$

$$Q_{friction} = \Sigma\ k.\tan\delta.\pi.\text{diam}.L =$$

$$= (0.95)(0.45)\pi(1)(20)(128)[20 / 2 + 1 / 2] =$$

$$= 51{,}546 \text{ lb}$$

$$Q_u = 102{,}491 + 51{,}546 = 154{,}037 \text{ lb}$$

$$Q_{design} = Q_u / F.S. = 154{,}037 / 2 = 77{,}018 \text{ lb}$$

5.2. Determination of Probability of Failure

Some assumptions must be made to shorten the solution and to face the lack of reliable data. The type of distribution is considered normal for the basic random variables. The formula used to define the ultimate load is the described in 5.1, although other more accurate expressions may be found in current literature, e. g. reference 10. The variables are assumed to be independent from each other. The adopted basic random variables are

$$X_1 = N_g\,;\ X_2 = N_q \text{ and } X_3 = Q_{design}$$

The mean values are assumed to be the specified values. The standard deviations for the soil parameters were derived from references 2,3 and 7, although with uncertainty, and for the load, assumed as dead, from reference 4.

$$\sigma_{x1} = 13,\ \sigma_{x2} = 10 \quad \text{and} \quad \sigma_{x3} = 7{,}702 \text{ lb}$$

$$x^m{}_1 = 65,\ x^m{}_2 = 50 \quad \text{and} \quad x^m{}_3 = 77{,}018 \text{ lb}$$

The safety margin, z, is linear on the variables but the process described can be used with any type of function as shown in example 5.5 of reference 12.

$$z = r - s = b + a_1.x_1 + a_2.x_2 - a_3.x_3$$

$$b = 51{,}546 \text{ lb},\ a_1 = 30.144,\ a_2 = 2009.6 \quad \text{and} \quad a_3 = 1$$

Standardizing the variables

$$u_1 = (x_1 - 65)\ /\ 13$$

$$u_2 = (x_2 - 50)\ /\ 10$$

$$u_3 = (x_3 - 77{,}018)\ /\ 7{,}702$$

and replacing in the function z, the equation of the failure hyperplane is

$\alpha_1\sigma_{x_1}$ $\alpha_2\sigma_{x_2}$ $\alpha_3\sigma_{x_3}$

$$391.87u_1 + 20{,}096u_2 - 7{,}702u_3 + 76{,}967 = 0$$

Applying the formula for the cosine directors, α_i, in function of the standardized function z, the values are

$$\alpha_1 = -\ 391.87\ /\ k,\ \alpha_2 = -\ 20{,}096\ /\ k \text{ and } \alpha_3 = 7{,}702\ /\ k$$

The constant k is found since $\alpha^2{}_1 + \alpha^2{}_2 + \alpha^2{}_3 = 1$.

$$k = 21{,}525$$

The reliability index is

$$\beta = [(76,967)\ (21,525)]\ /\ (391.87^2 + 20,096^2 + 7,702^2)$$

$$\beta = 3.58$$

Associated with this value is a probability of failure of

$$p_f = 3.4 * 10^{-4}$$

The procedure here described is valid for all types of failure functions and the simplified formulas for hyperplanes were not deliberately used. In the case of z being a nonlinear function β and α are found iteratively or using any algorithm for unconstrained minimization.

5.3.Evaluation of Partial Safety Factors

Assuming that a reliability index of 4, corresponding to a $p_f = 6.3*10^{-5}$, and the parameter N_q may be modified to obtain this reliability index, find the adequate N_q and also the partial safety factors to be used in analogous conditions.

Unknowns: N_q, γ_{mNq}, γ_{mNg} and γ_{fQ}. Assume $N_q = 1$. The sensitivity coefficients are derived in accordance with the method described in reference 12, problem 11.3.

$$\alpha_{Nq} > \alpha_Q > \alpha_{Ng}$$

$$\alpha_{Ng} = -\ 0.331\ ;\ \alpha_{Nq} = -\ 0.8\ ;\ \alpha_Q = 0.7$$

The critical design point, corresponding to β_{min}, is

$$N_g{}^* = 65 - (0.331)\ (4)\ (13) = 47.79$$

$$N_q{}^* = 1.0 - (0.8)\ (4)\ (0.2) = 0.36$$

$$Q^* = 77,018 + (0.7)\ (4)\ (7,702) = 98,584 \text{ lb}$$

So the partial safety factors are

$$\gamma_{mNg} = 65\ /\ 47.79 = 1.36$$

$$\gamma_{mNq} = 1.0\ /\ 0.36 = 2.78$$

$$\gamma_{fQ} = 98,584\ /\ 77,018 = 1.28$$

N_q may found from the equation of the failure hyperplane

$$(1.28)(77,018) = 2,009.6N_q/2.78 + (30.14)(65)/1.36 + 51,546$$

$$N_q = 63$$

Using Level 2 simplified method for the linear case, the present value of β is evaluated.

$$\beta = \frac{-77,018 + (2,009.6)(63) + (30.14)(65) + 51,546}{[(2,009.6^2)(12.6^2) + (30.14^2)(13^2) + 7,702^2]}$$

$$\beta = 3.89 \approx 4 \text{ (ok)}$$

5.4. Final Considerations

As already pointed out in class the sensitivity of the strength of the pile strength is higher for the parameter N_q.

The choice of the adequate probability of failure for each type of designed structure has been the object of research. The method cited in reference 1 is the one proposed by the Construction Industry Research and Information Association, London published in 1977.

$$p_f = 10^{-5}\ U\ T\ /\ L$$

where - U depends on the type of building

Places of public assembly, dams........0.005
Domestic, office, travel & industry.....0.05
Bridges..................................0.5
Towers, masts, offshore structures.........5

- T is the life period of the structure,

- L is the number of people involved.

The methods to calibrate the partial safety factors to be used in codes are described in chapter 11 of reference 12. A particular flowchart for the coefficients used in BS 5400 is presented. To derive the partial safety factors to be used as Level 1 method sufficient data must be obtained from a complete survey of soil parameters. This will drastically increase the number of basic random variables. For this reason and also due to the correlation of most of these parameters, the complexity of the solution will rise substantially. After collecting the data, statistical and probabilistic analysis of these values will follow to transform it in partial safety factors. However, if for a particular problem, the significant statistical parameters of the soil and load are known, Level 2 methods may be used to find the reliability of that pile on that particular situation.

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