

**UNIVERSIDADE DO PORTO**



**Liquefaction of mine tailing storage facilities:  
material characterization and numerical  
analysis**

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**DOUTOR EM ENGENHARIA CIVIL**

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To my parents



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## ABSTRACT

This investigation begins with the collapse of the Brumadinho B1 dam, which resulted from the occurrence of static liquefaction of the stored tailings. Understanding flow liquefaction involves analyzing the inherent properties of the material and evaluating the in-situ soil state. To achieve this objective, an extensive experimental study is presented, focusing on defining the stress-strain behavior of four types of materials identified by the post-mortem campaign conducted by FEUP. Particularly, this study extensively utilized triaxial devices to establish the critical state locus and examine soil behavior across a range of initial states, either loose or dense, and various stress paths.

Modeling the behavior of these materials is a complex task that requires a constitutive model capable of simulating a wide range of geomaterial behaviors. To this end, a unified critical state model for clays and sands (CASM) has been employed. This model is capable of simulating the undrained brittleness behavior that underlies the phenomenon of flow liquefaction. Different methodologies for calibrating the constitutive model are discussed, with a particular focus on deriving the specific model parameters  $n$  and  $r$ . These parameters can be determined using standard laboratory and in situ tests, as they are linked to specific soil properties such as peak and residual undrained shear strength, as well as the initial state parameter. Susceptibility to flow liquefaction is assessed in light of the current advances in numerical simulation of cone penetration in liquefiable materials using advanced constitutive models. Those criteria have been applied to CPTu tests conducted before the failure of the Brumadinho B1 dam.

Finally, the numerical simulation of the Merriespruit dam failure through the CASM model was chosen as a benchmark for this thesis. This choice was made because it serves as a prime example of mechanical liquefaction triggering, features relatively simple geometry, and has been extensively researched, including laboratory and in-situ characterization. This section explores various factors crucial to dam stability, including the effects of material viscosity and the impact that anisotropic consolidation during triaxial tests has on constitutive model calibration. Additionally, it examines the sensitivity of triggering to material parameters controlling undrained brittleness, as well as the sensitivity of triggering to spatial location.



## RESUMO

O insumo para esta pesquisa é o colapso da barragem B1 de Brumadinho, resultante da liquefação estática dos rejeitos armazenados. O estudo da liquefação por fluxo requer a análise das propriedades inerentes do material e a avaliação do estado do solo in situ. Para alcançar este objetivo, apresenta-se um estudo experimental amplo, focando na definição do comportamento tensão-deformação de quatro tipos de materiais identificados na campanha post-mortem realizada pela FEUP. Este estudo utilizou predominantemente ensaios triaxiais para determinar os parâmetros de estado crítico do solo e analisar o comportamento do solo em uma gama de estados iniciais, desde solto até compacto, sob diferentes caminhos de tensão.

A modelagem do comportamento desses materiais é uma tarefa complexa que demanda um modelo constitutivo capaz de simular uma vasta gama de comportamentos de geomateriais. Nesse sentido, empregou-se um modelo de estado crítico unificado para argilas e areias (CASM), capaz de simular o comportamento frágil não drenado que fundamenta o fenômeno de liquefação por fluxo. Discutem-se diferentes metodologias para a calibração do modelo constitutivo, com ênfase especial na derivação dos parâmetros específicos do modelo,  $n$  e  $r$ . Estes parâmetros podem ser determinados por meio de testes de laboratório, além de testes in situ, estando associados a propriedades específicas do solo, como a resistência ao corte não drenada, tanto de pico quanto residual, bem como o parâmetro de estado inicial. A susceptibilidade à liquefação por fluxo é avaliada à luz dos avanços recentes na simulação numérica de penetração de cone em materiais susceptíveis à liquefação usando modelos constitutivos avançados. Estes critérios foram aplicados aos testes CPTu realizados antes do colapso da barragem B1 de Brumadinho.

Finalmente, a simulação numérica do colapso da barragem de Merriespruit, utilizando o modelo CASM, foi selecionada como um referencial para esta tese. Isso se deve ao fato de que Merriespruit representa um exemplo muito claro de liquefação por fluxo, apresenta uma geometria relativamente simples e foi extensivamente investigada, incluindo caracterizações laboratoriais e in-situ. Esta seção aborda diversos fatores críticos para a estabilidade da barragem, incluindo os efeitos da viscosidade do material e o impacto da consolidação

anisotrópica durante os testes triaxiais na calibração do modelo constitutivo. Adicionalmente, examina-se a sensibilidade do trigger a parâmetros que controlam a fragilidade não drenada, bem como a sensibilidade do trigger à localização espacial.

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## TABLE OF ABBREVIATIONS AND SYMBOLS

|                   |                                                                                  |
|-------------------|----------------------------------------------------------------------------------|
| $\alpha$          | parameter controlling the relation between $M$ and $\theta$ (Sheng et al., 2000) |
| $B$               | Skempton's coefficient                                                           |
| $BE$              | bender element                                                                   |
| BP                | back pressure                                                                    |
| $\beta$           | parameter employed to scaling the plastic potential in CASM                      |
| CASM              | clay and sand model                                                              |
| CIMNE             | International Centre for Numerical Methods in Engineering                        |
| CID               | isotropically consolidated drained triaxial test                                 |
| CIU               | isotropically consolidated undrained triaxial test                               |
| CP                | cell pressure                                                                    |
| CPTu              | cone penetration test with pore pressure measurement                             |
| CSL               | critical state line                                                              |
| CSSM              | critical state soil mechanic                                                     |
| $D_{min}$         | maximum dilation                                                                 |
| $e$               | void ratio                                                                       |
| $e_{cs}$          | critical state void ratio                                                        |
| $E_m$             | represent the elastic modulus of the membrane                                    |
| EoT               | end of test method                                                               |
| $\varepsilon_q^e$ | elastic deviatoric strains                                                       |
| $\varepsilon_q^p$ | shear plastic strain                                                             |
| $\varepsilon_v^e$ | elastic volumetric strains                                                       |
| $\varepsilon_v^p$ | volumetric plastic strain                                                        |
| $F_e$             | void ratio function                                                              |
| FC                | fine content                                                                     |
| FEUP              | Faculty of Engineering University of Porto                                       |

|                |                                                              |
|----------------|--------------------------------------------------------------|
| $f$            | yield surface                                                |
| $g$            | plastic potential function                                   |
| $G$            | shear modulus                                                |
| $g_M$          | plastic potential (Miguel Manica, 2020)                      |
| $g_R$          | plastic potential (Rowe, 1962)                               |
| $g_Y$          | plastic potential (Yu, 2006)                                 |
| $G_{max}$      | elastic shear modulus                                        |
| $G_{ref}$      | reference shear modulus                                      |
| $G_s$          | specific gravity                                             |
| $\Gamma$       | intercept of the critical state line at 1 kpa                |
| $\gamma_d$     | dry unit weight                                              |
| $\gamma_{wet}$ | wet unit weight                                              |
| $H$            | hardening modulus                                            |
| HTC            | hollow cylinder tests                                        |
| ICMM           | International Council on Mining & Metals                     |
| ICOLD          | International Commission on Large Dams                       |
| $k$            | permeability                                                 |
| $K$            | bulk modulus                                                 |
| $k$            | slope of the swell line in the $v - \ln p'$ space            |
| $k_{kc}$       | Kozeny-Carman permeability                                   |
| $K_0$          | coefficient of earth pressure at rest                        |
| $\lambda$      | slope of the critical state line in the $e - \log(p')$ space |
| $m$            | Miguel Manica flow rule parameter                            |
| $M$            | critical friction ratio                                      |
| $M_i$          | zero-dilatancy stress ratio (image point in NorSand)         |
| $M_{tc}$       | critical friction ratio in triaxial compression              |

|                |                                                                        |
|----------------|------------------------------------------------------------------------|
| $M_{\theta}$   | critical friction ratio as function of the lode angle                  |
| MCC            | modified cam clay                                                      |
| MT             | moist tamping                                                          |
| MTWRM          | Management of Tailings and Waste Rock in Mining Activities             |
| N              | volumetric coupling factor                                             |
| NCL            | normal consolidation line                                              |
| n              | yield surface shape parameter in CASM model (stress-state coefficient) |
| OCC            | original cam clay                                                      |
| $p'$           | mean effective stress                                                  |
| $p'_0$         | isotropic pre-consolidation stress                                     |
| $p'_{cs}$      | mean effective stress at the critical state                            |
| PN             | pore pressure                                                          |
| $p_{ref}$      | reference stress level                                                 |
| $PSD$          | particle size distribution                                             |
| q              | deviatoric stress                                                      |
| r              | yield surface shape parameter in CASM model (spacing ratio)            |
| $S_p$          | peak undrained strength ratio normalised with $p'_{ini}$               |
| $S_{\sigma v}$ | peak undrained strength ratio normalised with $\sigma'_{v,ini}$        |
| $S_r$          | degree of saturation                                                   |
| $S_s$          | specific surface                                                       |
| SS             | cyclic simple shear test                                               |
| $T_0$          | initial thickness of the membrane                                      |
| TSF            | tailings storage facility                                              |
| $\tau$         | shear stress                                                           |
| $U_n$          | overcompactation ratio                                                 |
| UPC            | Universidad Polit cnica de Catalu a                                    |

|                    |                               |
|--------------------|-------------------------------|
| $V_P$              | compression wave velocity     |
| $V_S$              | shear wave velocity           |
| $w$                | moist content                 |
| $\chi_{tc}$        | state-dilatancy parameter     |
| $\psi$             | state parameter               |
| $\psi_r$           | reference state parameter     |
| $\sigma$           | total stress                  |
| $\sigma'$          | effective stress              |
| $\sigma_1$         | major principal stress        |
| $\sigma_2$         | intermediate principal stress |
| $\sigma_3$         | minor principal stress        |
| $\sigma_{r(corr)}$ | correction for radial stress  |
| $\sigma_{a(corr)}$ | correction for axial stress   |
| $\theta$           | Lode angle                    |
| $\eta'$            | effective stress ratio        |
| $\mu$              | poisson's ratio               |

# **1. Introduction**

## **1.1. Background**

On a global scale, mining has undergone an unprecedented expansion. One of the reasons of this steady growth is the increasingly vital role that ores play in modern society. Minerals are valuable for construction and the high-tech industry, particularly in the manufacture of mobile devices, computers, and ceramics. Moreover, the transition to clean energy instead of fossil fuel-based systems has increased the demand for batteries, solar panels, and other energy devices, with a consequent rising request for minerals.

In the process of mining operations, most of the crushed rock represents waste material, and only a small percentage of it can be safely repurposed for other uses, such as building materials. The volume of waste generated varies based on several factors, including the mining technique used and the rock's composition. For example, in the gold industry, the production process yields only a few grams of gold per ton of dry tailings. Additionally, the quality of metal ore which can be economically extracted has generally decreased over the years (ICOLD, 2001). The copper ore concentration has decreased from 34% in the 1900s to just 0.3% at the end of the past century, resulting in a corresponding surge in waste mining production. Despite the recent advances in the recycling of tailing resources, the volume of tailings produced every year is increasing. As Robertson (2011) reported, every 30 years the volume of tailings worldwide increases about ten-fold, the area covered by tailing storage facilities increases five-fold and average heights doubles.

Developing a strategy for sustainable management of mine waste is becoming a more critical challenge. Currently, the practice involves storing mine waste in ponds or impoundments behind large dams, known as tailings storage facilities (TSFs). Although these geotechnical structures share several features with other types of retention dams, their construction methods and operational needs are different. Typically, construction occurs in stages over several years. Indeed, intermediate retaining embankments are initially constructed and then raised as storage demand increases (Lottermoser, 2007). Therefore, given the challenges associated with tailings dams and the lack of legislation in several countries, the mining industry has seen a rising number

of dam failures, with the risk being ten times higher than that for conventional retention dams (Davies, 2002). Furthermore, an analysis of databases summarizing incidents over the last few decades reveals that information is only satisfactorily reported for a few countries. Many failure events remain unreported, especially in countries with less stringent regulations.

The chronology of major tailings dam failures (<https://www.wiseuranium.org/mdaf.html>) shows that more than 150 accidents have been registered during the period from 1960 to 2022, with almost 50% occurring in the last 20 years. The Fundão and Brumadinho accidents, which occurred in Brazil in November 2015 and January 2019, respectively, are likely the two most significant accidents in terms of both environmental damage and human loss. However, these two events are just the latest in a long series that have affected not only Brazil but many other countries as well. Notable examples include the Val di Stava disaster in Italy (1985), the Merriespruit dam failure in South Africa (1994), the Aznalcollar in Spain (1998), the Mount Polley in Canada (2014), and the Cadia in Australia (2018).

These events have led to stricter mining legislation, especially in countries with vast natural resources. For instance, Brazil's government has decided to ban tailings dams built by the upstream method. However, most upstream dams have performed well, and storage dams built using other construction methods have also failed (Santamarina et al., 2019). Among the various failure modes that have occurred in TSFs, static liquefaction is likely the most common (Davies et al., 2012). While media attribute these failures to mining companies, the root issue lies in a gap in engineering education (Jefferies, 2022). The complexity of this issue has spurred increased research into the causes of tailings dam failures. This has led to extensive research, including both laboratory tests and field studies. Moreover, numerical modeling, especially using the finite element method, has proven to be the most effective approach for understanding the behavior of these geotechnical structures.

## **1.2. Motivation and main objectives**

The mechanisms by which tailings dams fail are diverse, ranging from slope instability, overtopping, foundation failure, seismic activity, and static liquefaction. Static liquefaction stands out as the most frequent cause of such failures (Davies et al., 2012) and is often considered the most severe since it is followed by a fast-moving mudflow.

Static liquefaction has been known to dam engineers for almost a century, but the tools to address it from a mechanics-based perspective instead of from a purely empirical one have been developed only recently (Jefferies & Been, 2006). At the heart of this perspective are critical state soil mechanics models. While this is now generally accepted in principle there are numerous practical difficulties to overcome in applications. This thesis aims to contribute to the mechanistic understanding of static liquefaction risk by addressing some of those difficulties.

The application of critical state mechanics to static liquefaction analyses is impossible without good quality experimental data. The thesis thus begins with an example of laboratory-based characterization of mine tailings, including basic geotechnical properties, stress-strain behaviour, and critical state parameters that are key to simulate liquefaction. This part of the investigation was developed in the context of the FEUP collaboration in a post-mortem campaign concerning the Brumadinho tailings dam failure (Arroyo & Gens, 2021).

The observed experimental behaviour of the Brumadinho tailings was then examined in depth against the framework given by a critical state based constitutive soil model, with a focus on strain softening in undrained conditions. To apply critical state constitutive models in practice it is not enough with laboratory data, the in situ state also needs to be identified. This identification is also useful to judge if liquefaction risk is present. This aspect is addressed in the thesis by means of systematic comparison of available methods to assess the in-situ soil state from CPTu tests, considering both drained and undrained penetration scenarios.

Investigations of dam stability using critical state models require numerical simulation, typically based on the finite element method. This work examines critical state based numerical simulation of static liquefaction focusing on the sensitivity of results to material and geometric

variability. We also look at the effect of constitutive assumptions, in particular that of a viscous component of soil deformation on liquefaction triggering. The study evaluates dam stability by calibrating the constitutive model using isotropically or  $K_0$  consolidated triaxial tests and performs sensitivity analyses on specific CASM parameters under diverse geotechnical conditions. Additionally, the research investigates the spatial sensitivity of liquefaction triggering within the dam.

All these numerical simulations are performed using the Merriespruit dam failure as a benchmark. This choice was made because it serves as a prime example of mechanical liquefaction triggering, features relatively simple geometry, and has been extensively researched, including laboratory and in-situ characterization.

### **1.3. Thesis layout**

This thesis is structured into nine chapters, with the main contents of each chapter as follows: After a brief introduction, Chapter 2 reviews the physical and mechanical properties of tailings and their storage methods. The chapter initially describes the techniques used to separate valuable minerals from rocks, as well as the classification of tailing types based on water content. Finally, it provides an overview of the primary construction methods used in tailings dams.

Chapter 3 introduces the fundamental principles of critical state soil mechanics, emphasizing the relationship between shear strength, mean pressure, and the void ratio at the critical state. Moreover, the concept of the state parameter is introduced. This chapter also includes a brief review of two critical state models, the CASM (Yu, 1998) and NorSand (Jefferies, 1993), highlighting their ability to reproduce undrained strain softening.

Chapters 4 presents the laboratory methods used to reconstitute and test laboratory soil samples, to identify the critical state soil parameters, with particular emphasis on reconstituted triaxial specimens. Following this, Chapter 5 presents results from an experimental campaign on Brumadinho B1 dam tailings, starting with basic geotechnical characterization, followed by triaxial tests, permeability tests, and bender element tests.

Chapter 6 presents a detailed description of the different methods available to calibrate the model CASM, with a special focus on the shape parameters  $n$  and  $r$  that control the shape of the yield surface. Afterwards, the CASM model is calibrated using laboratory data for the Brumadinho B1 materials tested in Chapter 5.

In Chapter 7, the phenomenon of flow liquefaction is introduced, describing different methodologies that have been developed to predict it. Special emphasis is given to the state parameter criteria to identify soil susceptibility to liquefaction, with a focus on those derived from CPTu. These criteria are then applied to two representative CPTu soundings. Finally, a review of methods for estimating peak and residual shear strength using CPTu data is presented.

Chapter 8 employs the numerical simulation of the Merriespruit dam failure using CASM to explore various factors that affect finite element-based assessments of liquefaction triggering. The factors examined include: a) the influence of possible material viscous behavior; b) the influence of calibration procedures based on isotropic or anisotropic consolidated triaxial tests; c) the influence of material undrained brittleness, as described by the difference between undrained residual and peak strength; and d) the sensitivity of triggering to spatial location.

In Chapter 9, the main findings of this study are summarized, and potential future developments are outlined.



## 2. Origin of tailings and TSF

### 2.1. Introduction

In this chapter, we describe the process by which tailings are originated and later stored in dedicated facilities. The overall mining process comprises several key steps, starting with exploration and planning, followed by extraction, processing, and waste disposal. Techniques such as drilling, blasting, crushing, and separation are employed to extract and process the ore. The extracted minerals are then refined through techniques such as smelting, leaching, or flotation. Finally, the waste material or tailings is disposed of in an environmentally responsible manner.

### 2.2. Processing steps and waste management

#### 2.2.1. General considerations

Given that the chemical, mechanical, and physical properties of tailings are closely related to the processing to which they are subjected, it is convenient to briefly review the main steps of that process prior to storing the tailing slurry in the impoundment. Solid waste from mining operations is generated in a wide range of sizes, from rock fragments around a few tens of centimeters down to clay size particles. Although the extraction of mineral values requires procedures as diverse as the ores processed, some fundamental steps in the processes are common to many ores ( Vick, 1990; Krizek, 2000).

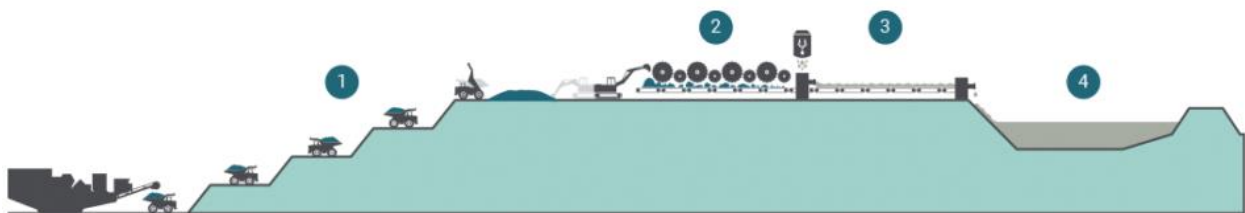


Figure 1 – Stages of the mining cycle (globaltailingsreview.org)

The first step in the mining cycle commonly involves drilling and blasting. In some cases, millions of tons of waste rock need to be displaced before valuable ore-containing rocks can be found. Once the ore body has a high concentration, the rock is crushed and ground to break it down into

smaller sizes, increasing its surface area and aiding in the extraction of the desired material (ICOLD, 1996). The valuable minerals can then be separated from the waste particles that will form the tailings. The term "tailings" typically refers to fine rock particles (sand-silt size particles) that are deposited in slurry form. Finally, some of the water can be removed from the tailings-water slurry before it is discharged into the disposal area. These steps are outlined in Figure 2.

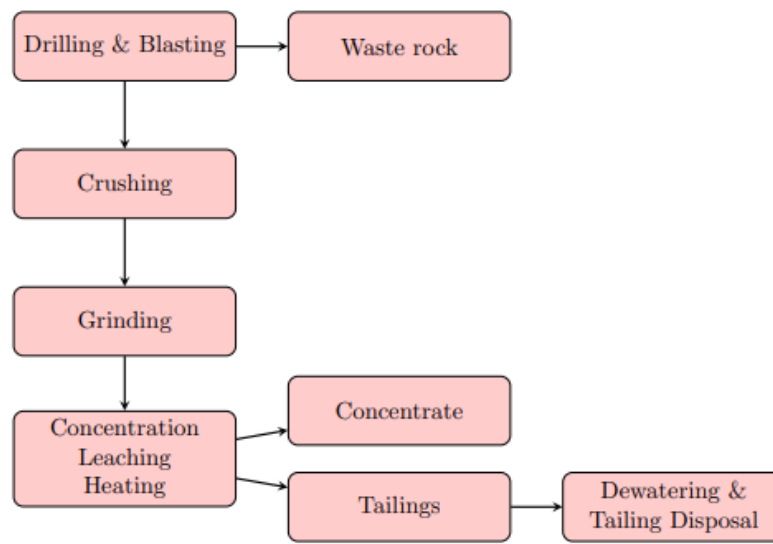


Figure 2 – Typical flow chart in tailing production (adapted from Raposo, 2016)

### 2.2.2. Crushing and Grinding

Crushing is typically a dry operation that involves breaking down the ore by compressing it against rigid surfaces or impacting it against hard surfaces in a controlled motion flow (MTWRM, 2009). This is often done in stages to reduce rock fragments from the size mined to a size that can be accepted as feed to grinding equipment (Vick, 1990). Grinding, which is primarily done using rod mills or ball mills, further reduces the size of particles produced by crushing. Ball mills typically produce smaller particles and are often used after rod mills to obtain a particle size gradation of approximately 0.5 mm or less. The gradation obtained largely depends on the parent rock used, although the material derived is usually very angular due to the severe physical violence inflicted during the process (Pettibone & Kealy, 1971).

### **2.2.3. Separating minerals from waste rock**

The resulting material of value need to be separated from the waste rock. For this purpose, a number of different separation techniques are used depending on the mineral properties. Since different minerals are often found together, a combination of techniques may be used to separate the minerals from the waste and then the minerals from each other.

#### **Concentration**

This is usually accomplished by gravity separation, magnetic separation or froth flotation. *Gravity separation* requires that the mineral and its host rock have considerably different specific gravities it relies on water flow and gravity to separate target value minerals and gangues. *Magnetic separation* is often used for the extraction of minerals with magnetic properties, thanks to strong electromagnets which remove the valuable mineral (e.g. iron) from the non-magnetic waste. Froth flotation, a highly complex physiochemical process, is the most widely used concentration method (Vick, 1990). Chemicals are added to change the surface properties of the valuable minerals so that air bubbles can attach to them. Since the air bubbles are much less dense than the solution they rise to the top where the minerals can be scraped off easily.

#### **Leaching**

This is a chemical process for extracting valuable minerals from ore by direct contact with a solvent. This process is carried out when the given ore may be dissolved in a solvent. The chief alkaline leach reagents are hydroxides and carbonates of sodium or ammonium, for example sodium cyanide is the common leaching reagent for extraction of gold and silver (Vick, 1990). The leaching process can be used alone or combined with other concentration methods.

#### **Heating**

When rock is subjected to heating, stresses are developed within the rock matrix that leads to cracking as result of thermal expansion and contraction (Sefiu & Hussin, 2020). This pre-treatment is useful as it can reduce the strength of rocks before the crushing and grinding steps.

#### **2.2.4. Dewatering tailing classification**

Because the material resulting from the mining process is usually in slurry form, disposal of slurry tailings in impoundments has been the most common method employed to date. However, a growing trend in the mining industry is to eliminate ponded water on top of tailings and to thicken tailings to conserve and minimize the amount of water that has to be managed (Li et al., 2009). Thanks to suitable flocculant and thickener technology (Australian Government, 2016), thickened tailings transport and disposal is now possible.

Flocculants are typically added to the slurry mixture of tailings and water to promote the aggregation of small particles into larger, heavier ones, which can then settle more easily. The use of flocculants in tailings management can improve the efficiency of water recovery and reduce the volume of water required for disposal. Flocculants can also aid in the thickening of tailings, which can further reduce water content and simplify tailings disposal. This technology has resulted in a safer and more environmentally friendly method of tailings disposal, reducing the risks associated with tailings dam failures and contamination of water resources. The advantages of dewatering tailings disposal include:

- achieve safer disposal;
- reduce tailing storage volume;
- reduce the amount of water (essential where water resources are scarce).

According to the classification proposed by Jewell & Fourie (2015), the mixture of water and solid particles that form the tailings can be divided into conventional slurry, thickened slurry, paste and cake, depending on the consistency of the mix. The only difference between the four groups is the amount of water that is mixed with the tailings particles, being the cake the mixture with the lower water content. Figure 3 illustrates the classification system presented above, including four images representing the consistency of the different mixtures. The horizontal axis represents the tailings' consistency while the vertical axis is a measure of strength.

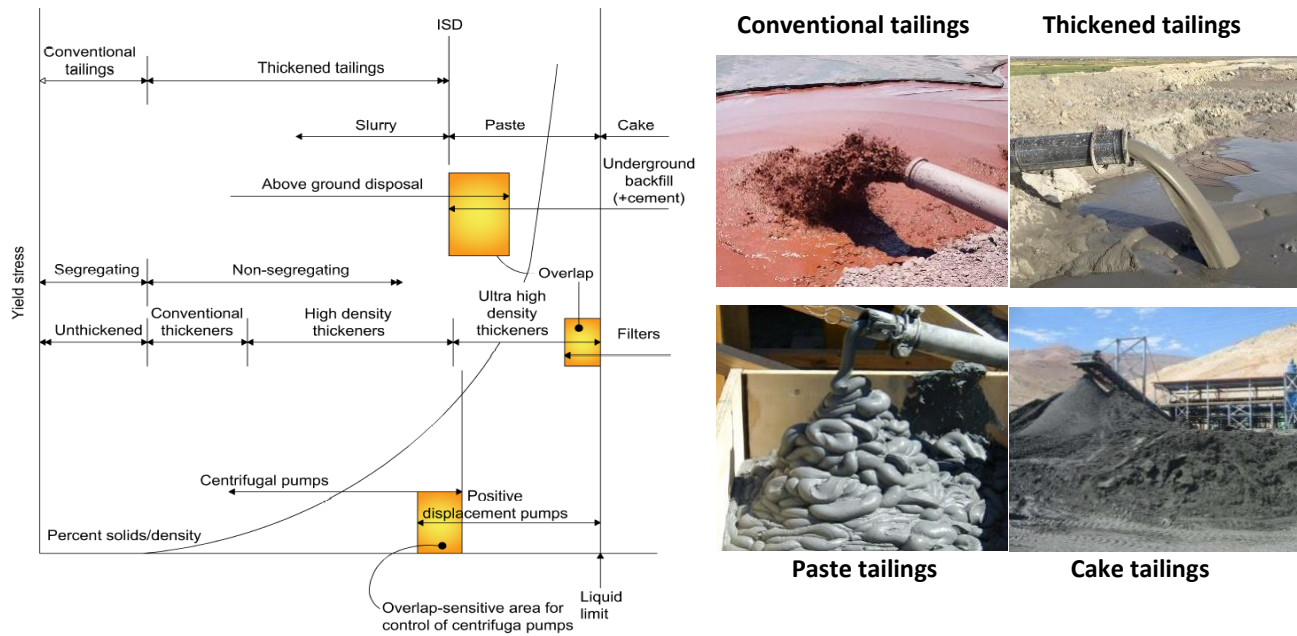


Figure 3 – Classification system for tailings (adapted from Jewell & Fourie, 2015).

### Conventional tailings slurry

In this mixture, the solid particles are suspended in a fluid. Thus, when the slurry is discharged into the impoundment, it always segregates. As a result, the coarser particles tend to settle near the discharge point, while the finer particles (slimes) are transported in the flowing slurry and conveyed farther away. As shown in Figure 4, this slurry is usually deposited hydraulically by multiple spigots using centrifugal pumps, obtaining a very flat beach with an average slope lower than 1%. The conventional tailings slurry is taken either directly from the processing plants or may be slightly densified before discharging into the storage.

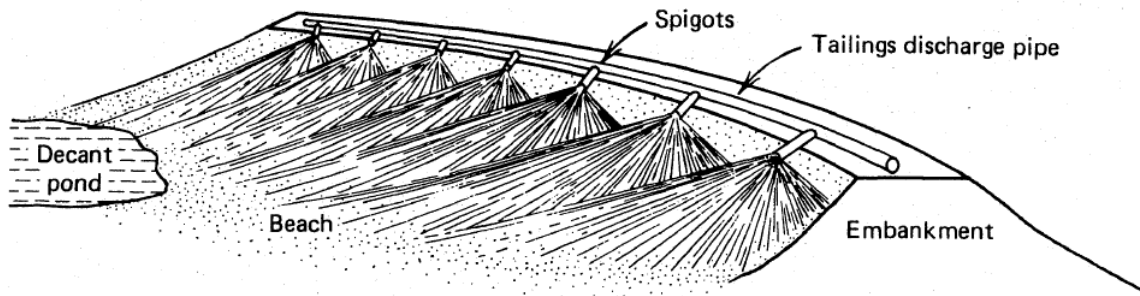


Figure 4 – Multiple spigots discharge method (Vick, 1990)

### **Thickened tailing slurry**

The consistency of thickened tailings ranges from the segregation threshold up to the initial settled density, that is the point at which the discharge tailings will not bleed water anymore (Jewell & Fourie, 2015). They are directly recovered in the underflow from the thickener, being centrifugal pumps often sufficient to transport the material to the discharge point. When deposited into the impoundment, they will form a homogeneous non-segregated mass, creating a beach with a slope ranging between 1 and 3 %.

### **Paste tailings**

As described in the classification system, the upper consistency threshold of paste tailing is the liquid limit. Since this material presents high viscosity, it is not easy to be transported with centrifugal pumps, thus positive displacement pumps need to be considered for transporting the material. Its consistency, similar to that of toothpaste, makes it difficult to store above-ground tailings. Hence, during deposition, the material beach is steep, and the discharge point needs to be moved regularly. For this reason, paste tailings are mainly used in underground mine fill, usually as the fine fraction in a cement paste backfill.

## **Cake tailings**

Tailings filtered cake, commonly called dry-stack, is a saturated soil resulting from a dewatering process using high pressure or vacuum force. One of the advantages of this method is the water recovery before the deposition, essential in areas where water resources are scarce. Another advantage associated with this technique is that the resulting filter cake can be stacked in a stockpile, making it safer from a stability standpoint (Davies et al., 2012). This cake material is not pumpable due to its low moisture content and needs to be transported by conveyor or truck to its storage site. Even if dry stacking has many benefits, it is also the most expensive tailings deposition alternative due to dewatering, transport, and deposit configuration costs.

### **2.3. Conventional tailings storage facilities**

In past centuries, it was a common practice to discharge the tailings from mining and industrial operations directly into the nearest water course (ICOLD, 1996). This method of disposal had catastrophic environmental effects, as the tailings contained toxic substances and heavy metals that contaminated the soil, water, and air. However, as society became more aware of the detrimental effects of this practice, regulations were implemented to prevent the direct discharge of tailings into waterways. Today, the mining industry is committed to responsible tailings management practices, including the safe storage and disposal of tailings in specially designed facilities that meet strict environmental standards. Indeed, following the latest dam failures in Brazil, the International Council on Mining & Metals (ICMM) promulgated a Global Industry Standard on Tailings Management (ICMM, 2020), addressing the crucial role of robust design and well-managed construction. This standard has been followed by another guidance with the publication of the Tailings Management: Good Practice Guide (ICMM, 2021).

At present, tailings transport by pumping in a pipeline remains the most used method. The transported material, usually slurry tailings, could be stored in different ways depending on safety, economy, prevailing topographical, climatological and seismic factors. Figure 5 illustrates

three common layouts of impoundment suitable to contain the discharge of tailings slurry, these include ring-dike (paddock dam), valley tailings impoundment and pit storage. However, the disposal method and impoundment layout can differ when the stored material has a higher consistency, such as for paste and cake tailings.

Embankments for conventional storage facilities are designed to retain tailings and water (Vick, 1990), whereas embankments employed in dry stack facilities retain only saturated or partially saturated soils.



a

b

c

Figure 5 – Conventional tailings storage facilities: a) ring dike - Mogalakwena mine, South Africa; b) valley tailing impoundment - Fundão dam, Brazil; c) in-pit storage – Cadia pit, Australia.

### Ring dykes

The ring dyke impoundment layout (also known as a paddock) is an appropriate option when the topography of the selected area is flat, and in absence of natural valleys. As shown in Figure 5a, all sides of the impoundment are enclosed to contain the tailings. This means that a large amount of material is required for the perimetral embankment construction. Unlike a valley impoundment, in this layout, the quantity of water arriving at the impoundment is limited to that transported with the tailings and any precipitation (Branch, 1994). According to Ritcey & Silver (1987), the ring-dike system has two main drawbacks. Firstly, the expense of constructing the embankment is relatively high compared to the amount of storage it provides. Secondly, as the

length of the embankment walls is increased, the likelihood of a weak point that may lead to failure increases.

### **Valley impoundment**

When the mine operation is located close to a valley, it is common practice to dam the natural depression and utilize the area for storing tailings. In this layout, the benefit is related to the reduced dam size, since a portion of the valley is used to store the tailings. Therefore, this type of impoundment system has the highest ratio of storage volume to confining embankment volume, which makes it the most utilized and least expensive option to build. However, the valley dam design is particularly sensitive to overtopping by flood-water, to erosion near the intersection of the dam and the valley hillside, and to suffer drainage problem due to a higher inflow of water from the natural catchment basin and from high precipitation runoff/runoff (Branch, 1994). Figure 5b shows the Fundão valley tailings facility in Brazil.

### **In-pit storage**

At the end life of an open-pit mine an attractive option is to backfill the mining hole with the previously mined tailings. The main advantage of this layout is that the confinement of the deposited material will be affected largely or entirely by the existing ground, making for cheap disposal (ICOLD, 1996). However, an unlined in-pit tailings containment system can put groundwater at risk, as the liquids slowly seep down through the soil. Figure 5c shows how the old Cadia Hill open pit has been used as tailings storage dam.

## **2.4. Dam Raising alternatives**

Tailings surface impoundment structure commonly consists of raised embankments, which differ from conventional water-retention type structures in that construction of the embankment is staged over the life of the impoundment (Vick, 1990). Before to raise embankments, a starter dike is formed usually with the waste rock produced by mining process. Occasionally, the starter dam may also include an impervious clay core capable of retaining water without the assistance

of the tailings seal. Each raised embankment may assume many configurations. They may be built upstream, downstream or at the centerline in relation to its starter dam position. Figure 6 shows schematic diagrams of the three main approaches listed above.

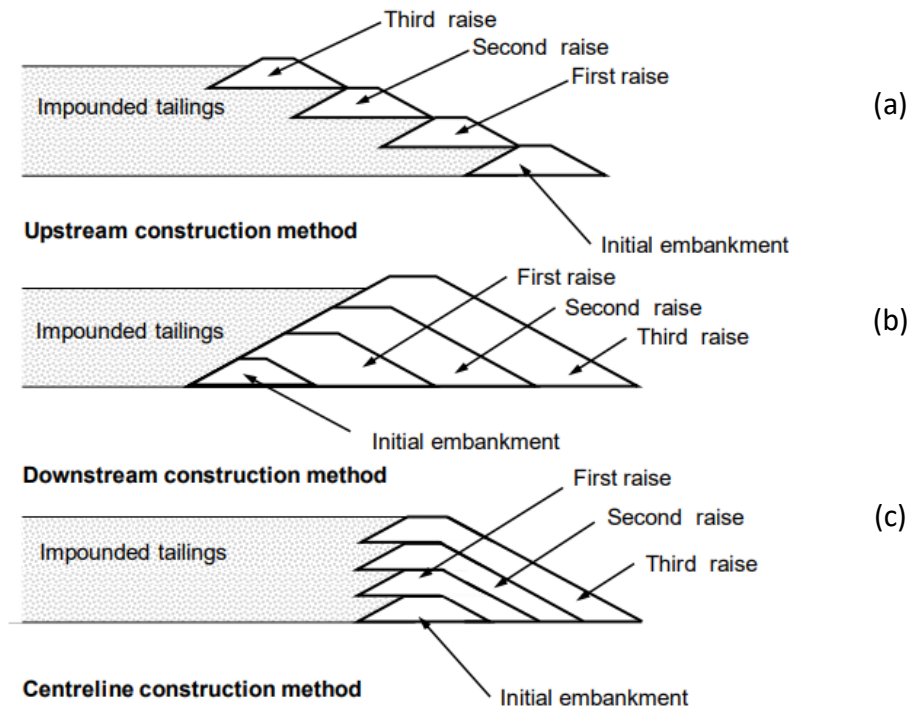


Figure 6 – Rising methods for tailings dam perimeter embankments: a) upstream method; b) downstream method; c) centreline method (adapted from Victorian Department of Primary Industries, 2004).

The advantages of raised embankment designs are many. The embankment is raised at certain time intervals to increase the available volume for the storage of tailings and water, thus this type of design has a lower initial capital cost than retention dams, because fill material and placement costs are distributed over the life of the impoundment. Moreover, a variety of materials can be used for the embankment construction, including natural soil, tailings, and waste rock in any combination (Vick, 1990). Tailings are frequently selected as construction material since provide the most economical alternative. Nevertheless, waste rock is a favorable alternative if the tailings dam is located close to the mine. Among the three construction methods, the upstream technique has been the most used until now. This is due to its low cost since the perimeter dikes require the least volume of fill material for construction.

After the embankment has been raised, tailings are released from the crest of the dam, forming a long and wide beach until they almost fill the starter dam capacity. Thus, in the upstream construction method, the beach becomes the foundation of the next dike. As a result, a key aspect of this configuration is to achieve a wide beach, through segregation of the coarse fraction of the discharged tailings next to the crest. Typically, the discharge material should contain no less than 40 to 60 percent sand (Vick, 1990). Upstream tailings dams may be riskier than other design techniques because they are more conducive to have close to the slope saturated loose tailings materials, i.e. material that can liquefy and lose their strength under both static and dynamic loading (McLeod & Bjelkevik, 2017). This danger was made particularly evident by the catastrophic failures that occurred in Fundão in 2015 and Brumadinho in 2019. Consequently, some countries such as Chile and Brazil have currently banned upstream tailings dams.

On the other hand, tailings facilities constructed using the downstream method are totally supported on the foundation soil and tailings only act as a load against the retaining embankment (Fourie et al., 2022). The centerline method may be seen as a compromise between the downstream and upstream methods, considering the advantages of both methods. As illustrated in Figure 6, centreline tailings dams are raised progressively while maintaining the original centreline of the starter dam. However, the centreline and upstream method are not the panacea. Although tailings dams constructed with these methods have been stable under strong seismic loading conditions they are not free of risk. In particular, when assessing the failure rate of upstream, downstream, and centerline-raised tailings dams, since the 2000s, the upstream failure rates have decreased more than those for the other two raising methods on a global scale (Stark et al., 2022). The exception is Brazil, where the upstream failure rate has experienced a fourfold increase since the 2000s.



### 3. Critical state soil mechanics applied to tailings

#### 3.1. Introduction

Soil is a naturally occurring complex material composed of mineral particles, organic matter, and water. Its behaviour is influenced by various factors such as particle size and shape, moisture content, and chemical composition. Moreover, when subjected to external forces, the soil responds differently depending on its current state. For instance, two samples of soil consolidated under the same mean effective stress will exhibit different shear strengths if one is denser than the other. This important aspect of soil behaviour can be understood within the framework of critical state soil mechanics.

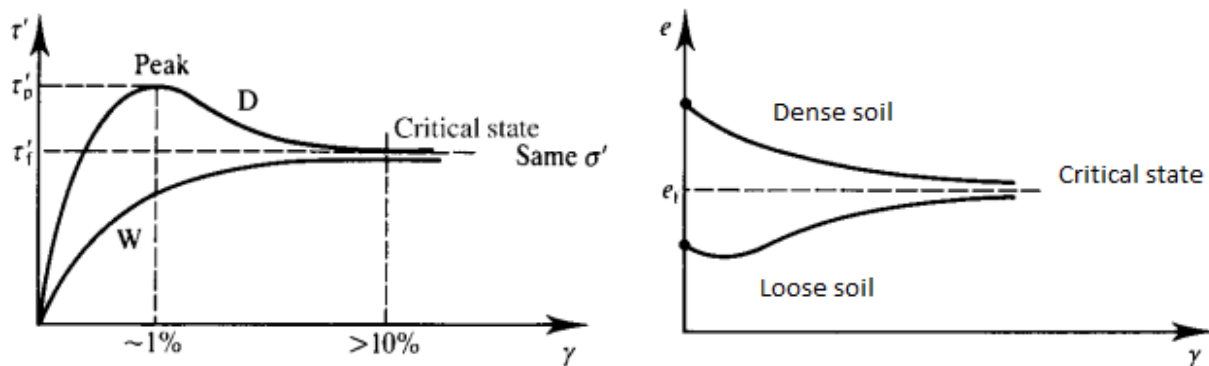


Figure 7 – Typical behavior of loose and dense specimen of a sandy soil

For a loosely packed sand specimen, the soil progressively decreases in volume and approaches an ultimate state, reached at large strains, where distortion can occur without further changes in volume or effective stress ratio. In contrast, a denser packed sand is characterized by shearing with an increase in volume, attaining a peak condition at the maximum rate of dilation. After reaching this peak, further deformations show that this peak condition is unstable, as the stress ratio gradually decreases until an ultimate condition is achieved, where distortions again occur without any change in volume or effective stress ratio.

Casagrande first investigated this concept in 1936. Using the shear box test, he observed that loose sands contracted while dense sands dilated, both eventually reaching a similar "critical void ratio" at large strains (see Figure 7). Later, Taylor expanded Casagrande observations regarding

the critical void ratio, founding that the critical void ratio depended on the effective confining stress. Hence, the fundamental concept of the critical state is that, under sustained uniform shearing, there exists a unique relationships among void ratio  $e_{cs}$ , mean effective pressure  $p'_{cs}$ , and deviator stress  $q_{cs}$  (Mitchell & Soga, 2005). This state is represented by a curve in a three-dimensional space defined by three axes ( $p', q, e$ ), towards which every soil state tends for large deformations. Thus, the critical state line (CSL) can be defined using the stress variable ( $p'$  and  $q$ ) and void ratio ( $e$ ). If  $p', q, e$  are considered as three orthogonal axes, the combinations of stresses and volume can be plotted in the three-dimensional space defined by these axes. The critical state line can be represented as a single curved line, with its projections onto the  $p' - q$  plane ( $e = 0$ ) and  $p' - e$  plane ( $q = 0$ ), respectively.

The critical state concept was popularized by Schofield & Wroth (1968), who formalized it by means of the Cam Clay constitutive model. One important trait of critical state models is the emphatical distinction made in soil between true properties that are invariant with density and measures of the soils state. Since any specific soil can exist across a wide range of densities it is unreasonable to treat any particular density as having its own properties (Jefferies & Been, 2015).

### 3.2. Defining the critical state line

In soil mechanics the compression triaxial test is widely used to explore soil behaviour. In classical soil mechanics, data recorded during a triaxial test (drained or undrained), are plotted in a  $\sigma - \tau$  plane. The major principal stress  $\sigma_1$  and minor principal stress  $\sigma_3$  defining the Mohr circle diameter at failure. By constructing a line that is tangent to at least two Mohr circles, the failure envelope is defined. Cohesion and friction angle can be determined by calculating the slope of the failure envelope and the intercept of the failure envelope with the  $\tau$  axis.

The conventional triaxial test is also employed to determine the locus of critical states in  $p', q, e$  space. For convenience, the critical state line is often described in terms of its 2D projection on the  $p', q$  space using the strength parameter  $M$  and its 2D projection on the  $e - \log p'$  space. The equations that describe the CSL in  $p', q$  is:

$$q = Mp' \quad (1)$$

Where  $q$  is the deviatoric stress at failure,  $M$  is a frictional constant, and  $p'$  is the mean effective stress at failure. For conventional triaxial tests, the specimen is subjected to a compressive stress state, thus  $M = M_{tc}$ . It should be noted that the slope of the critical state line in the  $p', q$  space depends on the stress conditions.

The CSL in the  $p', e$  plane is traditionally described by semilogarithmic equation, including as parameters slope  $\lambda$  and intercept  $\Gamma$ :

$$e = \Gamma - \lambda \ln p' \quad (2)$$

where  $\Gamma$  is the value of the specific volume at the reference mean effective pressure  $p' = 1$  kPa. Thus, three parameters  $M, \lambda$  and  $\Gamma$  are required to determine the CSL. These critical state lines obtained from triaxial compression tests are illustrated in Figure 8.

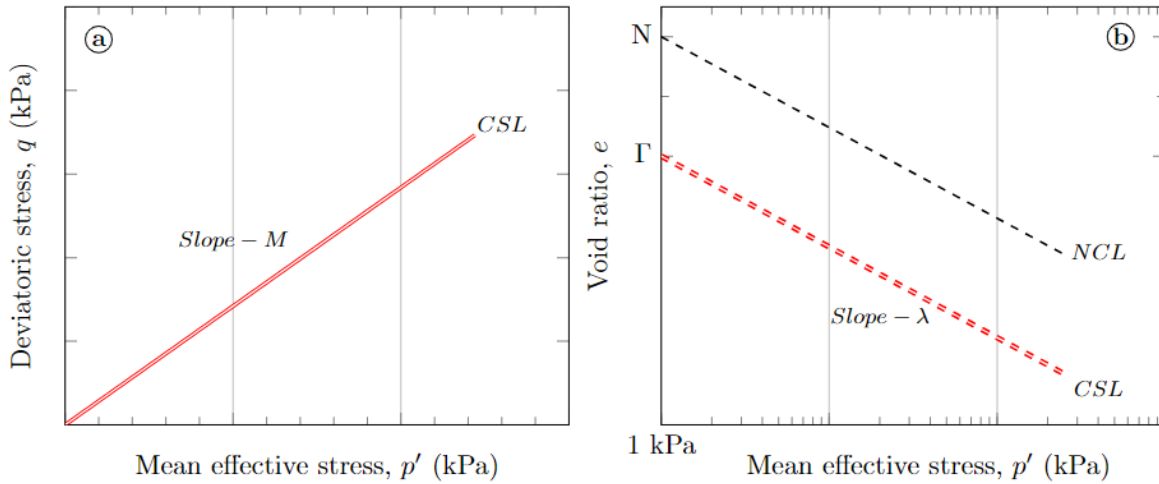


Figure 8 – Critical state line of soil.

Critical state line parameters are example of “true properties” that depend only on the physico-chemical nature of the soil (grain size and shape distributions, etc).

### 3.3. State parameter

The mechanical behaviour of soil depends not only on its density but also on the stress level applied. Within the critical state framework, the behaviour of soil may be anticipated using the state parameter,  $\psi$ . The state parameter, as defined by Been & Jefferies (1985), is the difference between the current void ratio and the void ratio at the same mean effective stress on the critical state line (see Figure 9).

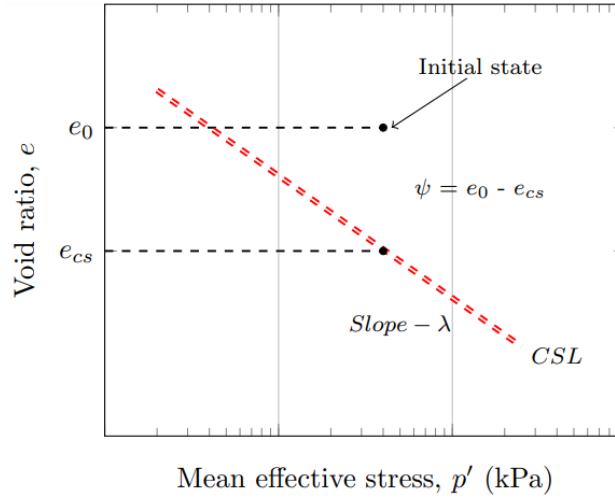


Figure 9 – Definition of state parameter.

As reported in the previous paragraph, when a soil sample is sheared two situations can occur. For a specific stress level soil dilate or contract depending on the initial void ratio position. This means that the value of state parameter  $\psi$  will increase the greater the distance from the critical state line. Therefore, when the state of a soil is above the critical state line, it corresponds to a positive  $\psi$ , whereas  $\psi$  is negative if it is located below the critical state line. This parameter is then defined by the following equation:

$$\psi = e - e_c \quad (3)$$

where  $e$  is the current void ratio of the soil and  $e_c$  is the void ratio at the critical state for the same mean stress.

### **3.4. Critical state constitutive models**

#### **3.4.1. Introduction**

A proper constitutive model, rather than capturing all the facets of soil behavior, should pay attention to the essential features of a problem and leave aside features that are irrelevant. The aim of a constitutive soil model is not to give a mirror image of the real behavior of the soil but to provide the key to understanding reality (Wood, 1990).

Over the past fifty years many constitutive models have been developed for modelling the stress-strain behavior of soils. Simple constitutive soil models like perfect plasticity, had much use, in part because its simplicity frequently allowed to obtain analytical solutions. However, soil response is not linearly elastic and perfectly plastic for the entire range of loading, but rather undergoes both elastic and plastic deformation when subjected to loading. The introduction of critical state soil mechanics (Roscoe et al., 1958; Schofield & Wroth, 1968) offered a framework to formulate a new set of constitutive models that took this into account. The earliest Cam clay model (Roscoe & Schofield, 1963) and later the Modified Cam clay model (Roscoe & Burland, 1968) represent the first models incorporating the critical state concept itself. In order to find a model that fits better to a wide range of soils, in subsequent years, multiple models based on the original Cam Clay model were proposed. However, the critical state models were mostly limited to saturated clays and silts; the yield surfaces overestimated the failure stresses on the 'dry side'; and granular materials were rarely modelled by critical state models (Gens & Potts, 1988; Yu, 1998, 2006).

A further step in the prediction of stress-strain soil behavior was taken with the inclusion of the state parameter concept (Been & Jefferies, 1985) in constitutive models. NorSand (Jefferies, 1993) was the first constitutive model to integrate both the state parameter and critical state theory. Afterwards, other advanced constitutive models have been proposed based on the same concept, including models targeting the realistic simulation of cyclic liquefaction phenomena (Manzari & Dafalias, 1997; Taiebat & Dafalias, 2008; Yu, 1998). In this investigation the Clay and Sand Model CASM proposed by (Yu, 1998) has been selected since has the advantage of being relatively simple to calibrate and has been proven as a useful tool in flow liquefaction triggering

studies (Arroyo & Gens, 2021; Manica et al. 2021). A brief description of CASM is presented in the next section.

### 3.4.2. Clay and Sand model - main features

The clay and sand model (CASM; Yu, 1998) can be regarded as an extension of classical critical state models such as the Original Cam clay (OCC) or the Modified Cam clay (MCC). CASM employs a total of seven model parameters, five of which are common to the classical Cam clay models. The yield function can be expressed in term of the conventional triaxial stress invariants as follows:

$$f = \left( \frac{q}{p' M_\theta} \right)^n + \frac{1}{\ln r} \ln \frac{p'}{p'_0} \quad (4)$$

where,  $p'$  is the mean effective stress,  $q$  is deviatoric stress,  $M_\theta$  is the slope of the CSL expressed as a function of the Lode angle  $\theta$ ,  $p'_0$  is the isotropic pre-consolidation stress controlling the size of the yield surface, and the remaining two parameters  $n$  and  $r$  control the shape of the yield function as shown in Figure 10.

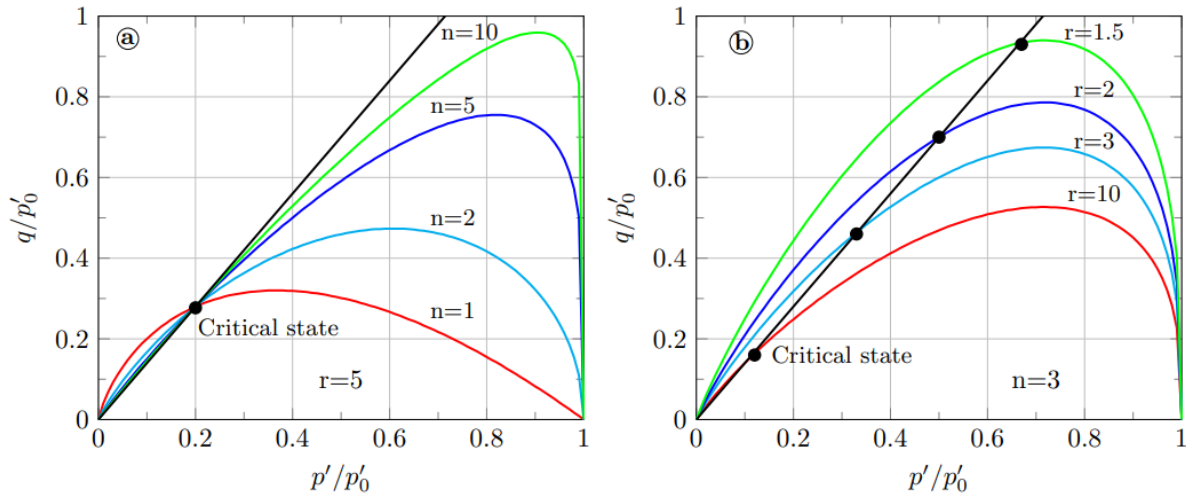


Figure 10 – Influence of  $n$  and  $r$  in the shape of the yield surface.

Particularly,  $r$  is defined as the spacing ratio and define the distance between the NCL and the CSL in the compression plane, since  $\psi_r = (\lambda - k) \ln r$ . Moreover,  $r$  controls the intersection between the CSL and the yield surface. In Figure 10a, it can be observed that all the yield surfaces cross the CSL at a point where the mean effective stress ( $p'/p'_0$ ) equal to  $1/r$ , i.e at 0.2. This already indicate that  $r$  is in some way linked to the residual (or critical state) undrained strength; that relation is made explicit below. On the other hand, the maximum deviatoric stress that can be attained is influenced by both  $n$  and  $r$  parameters.

It is also noteworthy that the yield function of CASM can reproduce the classical Cam clay surfaces by selecting specific shape parameters. For the OCC the conjunction of parameters are  $n=1$  and  $r=2.7$  whereas for the MCC are  $r=2$  and  $n$  around 1.5-2.

Before yielding, the soil behavior is assumed isotropic and purely elastic, with the increments of volumetric and deviatoric strains being defined as follow:

$$d\varepsilon_v^e = \frac{dp'}{K} \quad (5)$$

$$d\varepsilon_q^e = \frac{dq}{3G} \quad (6)$$

where  $(K)$  is the bulk modulus and  $(G)$  is the shear modulus defined as:

$$K = \frac{vp'}{k} \quad (7)$$

$$G = \frac{3(1-2\mu)K}{2(1+\mu)} = \frac{3(1-2\mu)}{2(1+\mu)} \frac{vp'}{k} \quad (8)$$

where  $\mu$  denotes the Poisson's ratio, and  $k$  is the slope of the swell line in the  $v - \ln p'$  space. The full elastic stiffness matrix can be defined using a constant value of  $\mu$  or alternatively by a constant shear modulus  $G$ . While this is expedient, it should be borne in mind that as is the case

for the Cam-clay models this assumption has some problems, as a constant value of  $\mu$  lead to thermodynamics violation in cyclic tests while a constant value of shear module can lead to negative values of Poisson's ratio at low stresses. Experimental evidence suggests, however, that  $G$  varies with stress level (Kurtulus & Stokoe, 2008; Wroth et al., 1979). The alternative of using a constant shear modulus in numerical calculations may therefore lead to significant errors (Houlsby, 1985). Therefore, it is more convenient to specify a constant value of  $\mu$ , even though this approach can introduce inaccuracies, particularly for cyclic loading. Considering that cyclic loading is out of the scope of this work, a constant  $\mu$  is employed.

The hardening law is governed by the pre-consolidation pressure  $p'_0$  which control the yield surface size. Thus, the plastic hardening modulus  $H$  can be expressed as follows:

$$H = -\frac{df}{dp'_0} \frac{dp'_0}{d\varepsilon_v^p} \frac{dg}{dp'} = \frac{1}{\ln r} \frac{v_i}{\lambda - k} \frac{dg}{dp'} \quad (9)$$

Where  $g$  represents the plastic potential function.

### Flow rule

Historically, different plastic potential formulations have been employed with CASM. To define the plastic potential Yu (1998) adopt a non-associated flow rule based on the stress-dilatancy relation of Rowe (1962):

$$\frac{d\varepsilon_v^p}{d\varepsilon_q^p} = \frac{9(M - \eta')}{9 + 3M - 2M\eta'} \quad (10)$$

where  $\eta'$  is the effective stress ratio,  $d\varepsilon_v^p$  is the volumetric plastic strain and  $d\varepsilon_q^p$  is the shear plastic strain. When integrated, the stress-dilatancy relation take the form of the following  $g_R$  plastic potential:

$$g_R = 3M \ln \frac{p'}{\beta} + (3 + 2M) \ln(2\eta' + 3) - (3 - M) \ln(3 - \eta') \quad (11)$$

where  $\beta$  is a parameter employed to scaling the plastic potential for any given stress state ( $p'$ ,  $q$ ). Although Rowe's stress-dilatancy relation provides satisfactory results for many practical problems it does not correctly reproduce the plastic strain increment in an isotropic stress path (or with low stress ratio) since the plastic potential has a vertex when  $\eta' = 0$ . This imply that Rowe flow rule predicts deviatoric and volumetric plastic strain increments al low stress ratio, while the experimental evidence suggests that only volumetric plastic strain is generated along the isotropic axis. To overcome these limitations Yu (2006) suggested an alternative flow rule, with a similar shape to the CASM yield function but with null volumetric strain increments at critical states. Yu (2006) non-associated flow rule is given by:

$$\frac{d\varepsilon_v^p}{d\varepsilon_q^p} = \frac{(M^n - \eta'^n)}{m\eta'^{n-1}} \quad (12)$$

which gives the following  $g_Y$  plastic potential:

$$g_Y = \ln \frac{p'}{\beta} + \frac{m}{n(m-1)} \ln \left[ 1 + (m-1) \left( \frac{\eta'}{M} \right)^n \right] \quad (13)$$

where  $m$  is an additional CASM parameter controlling the shape of the plastic surface. Note that by setting  $n=2$  and  $m=2$  the plastic flow rule of the MCC is derived, whereas using  $n=1$  and  $m=1$  the OCC flow rule is recovered.

Mánica et al. (2021) implemented a new flow rule also similar to the CASM yield function. This function has shown improved convergence properties in the implicit integration procedure (Arroyo & Gens, 2021b) implemented in the version of CASM available as a user-defined soil model in Plaxis. The new stress dilatancy equation is given by:

$$\frac{d\varepsilon_v^p}{d\varepsilon_q^p} = - \frac{m-1}{m} \frac{(\eta'^m - M^m)}{\eta'^{m-1}} \quad (14)$$

or:

$$\frac{d\varepsilon_v^p}{d\varepsilon_q^p} = \left[ \frac{M}{m} - M \right] \left[ \frac{\eta}{M} \right] + \left[ M - \frac{M}{m} \right] \left[ \frac{\eta}{M} \right]^{1-m} \quad (15)$$

The flow rule may be integrated to give the following  $g_M$  plastic potential:

$$g_M = \left( \frac{q}{p' M_\theta} \right)^m + m - \frac{(m-1)}{p'} p_c - 1 \quad (16)$$

For a direct comparison, all the flow rules above mentioned are illustrated in Figure 11.

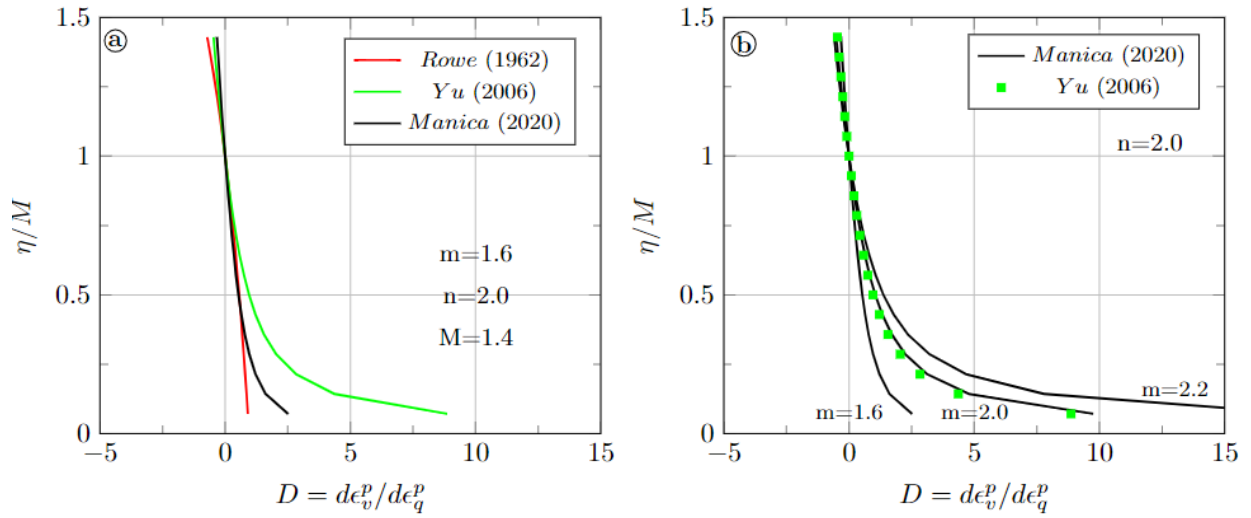


Figure 11 – A comparison of different flow rules employed in different CASM formulations

### 3.4.3. NorSand

Estimating the in situ state parameter,  $\psi$ , is a crucial aspect of characterizing tailings storage facilities for liquefaction screening and stability assessments. The cone penetration test, supplemented by laboratory testing, is the current standard practice for assessing this parameter. Recently, cavity expansion-based CPTu inversion methods, particularly the NorSand Widget, have been routinely adopted for this purpose (Smith et al., 2021). Although the NorSand model is not the primary focus of this investigation, it will be due to its widespread use, especially

for estimating the in situ soil state. Chapter 8 will detail the main procedures available for estimating  $\Psi$ . For now, the NorSand constitutive model is introduced.

NorSand is a critical state constitutive model, formulated by Jefferies (1993). It is a general-purpose model but has been particularly used in the field of tailings geotechnics for static liquefaction problems (Vivek & Macedo, 2023). It is capable of capturing the softening and dilatancy behaviour of sands by introducing the state parameter concept and by postulating an infinity of normal consolidation loci (NCLs). The concept of an infinite number of NCL was first proposed by Ishihara et al. (1975), based on results of laboratory tests. This means that soil does not have a unique NCL; rather, each NCL depends on the initial void ratio at deposition. This phenomenon is not merely a laboratory artifact; similar behaviour has been observed in large-scale hydraulic fills (Jefferies & Been, 2015).

A key feature of the NorSand model is the image point, which corresponds to the highest point of the yield surface (see Figure 12a). The mean stress at the image state,  $p_i$ , describes the size of the yield surface exactly, similar to how  $p_c$  is used in Cam Clay to link the yield surface to the CSL. Because NorSand is an associative model, this point defines the zero-dilatancy stress ratio,  $M_i$ ; therefore, the dilatancy at this image point is zero. The Yield surface is derived from the normality condition and the stress-dilatancy relationship as follows:

$$f = \frac{q}{p'} - M_i \left( 1 - \ln \frac{p}{p_i} \right) \quad (17)$$

This particular transient condition at the image point has long been recognized and is commonly referred to as phase transformation or pseudo-steady state.

Regardless of the initial soil condition, whether loose or dense, when the stress reaches the critical state,  $M_i$  is equal to  $M_{tc}$ . Because the yield surface is decoupled from the CSL, an additional soil property is required to describe the rate at which the yield surface responds to distortional strain: the hardening modulus,  $H$ . The growth or shrinkage of the yield surface is governed by the hardening rule, which control the yield surface's shrinking and expansion. As

highlighted in Figure 12b, when  $p_{i,max} - p_i > 0$  the yield surface grows, otherwise, it shrinks. This behavior is described by the following equation:

$$\frac{\dot{p}_{im}}{p} = H \left( \frac{p}{p_{im}} \right)^2 \left[ \exp \left( \frac{-\chi_i \psi_i}{M_{im,tc}} \right) \frac{p_{im}}{p} \right] \dot{\varepsilon}_p^q = H \left( \frac{p}{p_{im}} \right)^2 \left[ \frac{p_{im,max} - p_{im}}{p} \right] \dot{\varepsilon}_p^q \quad (18)$$

where the hardening modulus  $H$  is expressed by:

$$H = H_0 - H_\psi \psi \quad (19)$$

with  $H_0$  and  $H_\psi$  being material constants. These properties  $H_0$  and  $H_\psi$  are dimensionless and both are needed because most soils show a dependence of hardening on the state parameter. As can be seen in Figure 12b, the difference  $p_{im,max} - p_{im}$  governs the hardening behaviour. If this difference exceeds 0, the yield surface expands. In contrast, if it is less than 0, the yield surface reduces in size.

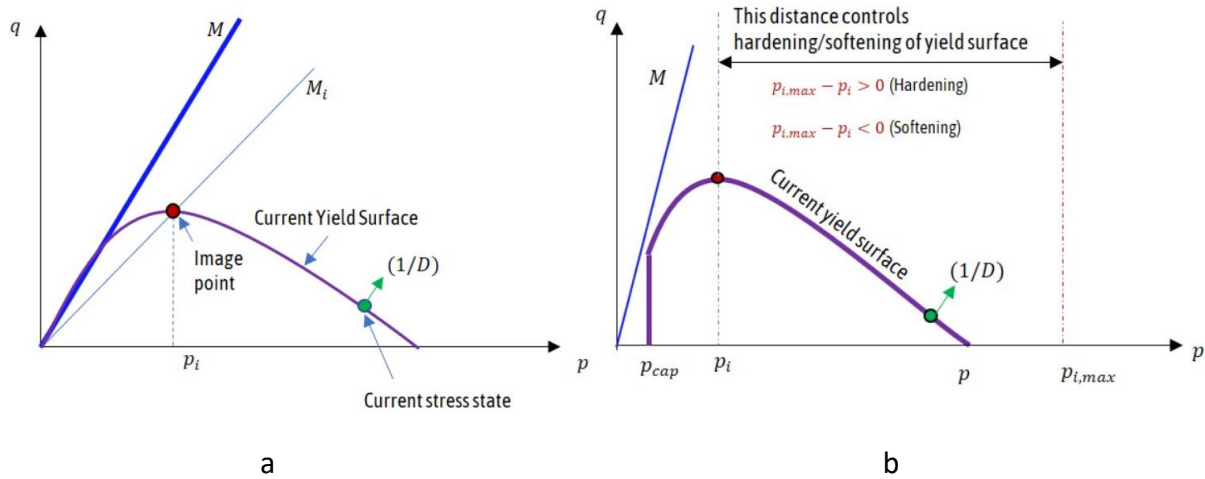


Figure 12 – Illustration of some key features of NorSand from (Vivek & Macedo, 2023)

## Elasticity

Soil elasticity usually shows a dependence of both bulk and shear moduli on mean effective stress (i.e.,  $K$  and  $G$ , respectively), as well as some dependence on void ratio. A constant Poisson's ratio is adopted to relate  $G$  and  $K$  to avoid unconservative elastic idealization. NorSand anchors

elasticity to  $G_{ref} = G_{100}$ , which is the value of the mean effective stress at the reference pressure  $p_{ref}$  equal to 100 kPa (a widespread convention), combined with an exponent  $n_G$  for a power-law trend, thus introducing the stress-dependency on this modulus (Plaxis, 2020). Thus, elasticity is:

$$G = G_{ref} \left( \frac{p}{p_{ref}} \right)^{n_G} \quad (20)$$

$$K = 2G \left( \frac{1+\nu}{3-6\nu} \right) \quad (21)$$

Geophysical measurements, whether in situ (both seismic CPT and DMT) or in the laboratory (bender elements), have come to dominate geotechnical engineering, making the elastic shear modulus the fundamental elastic input. However, in most cases, the modulus is affected by the confining stress level, meaning each layer of soil should be modeled with its own  $G$ . Using Equation (20), with two material properties  $G_{ref}$  and  $n_G$ , NorSand is capable of defining an elastic shear modulus that is dependent on the mean stress. Note that  $n_G$  takes values between 0 and 1, and when it is set to 0, it corresponds to a constant elastic shear modulus.

### **Critical state locus: parameters $\Gamma$ , $\lambda_e$ or $A$ , $B$ and $C$**

Two options are currently available to define the critical state void ratio  $e_c$ : the traditional semi-log form and the power-law form. In the first case, the CSL in the  $e$ - $p'$  plane is defined using  $\Gamma$  and  $\lambda_e$ . Conversely, when employing a power-law formulation, the three parameters  $A, B, C$  are necessary. It is worth noting that care is needed to avoid choosing void ratios above the CSL when modelling undrained loading with the power-law CSL, as the specified state may imply zero critical state strength and thus cause numerical instability. In this thesis, the traditional semi-logarithmic form of the CSL was employed, since the NorSand model, as implemented in Widget, employs this idealization of the CSL.

**Dilation limit: parameter  $\chi_{tc}$** 

NorSand limits dilation using a first-order rate approach; which is equivalent to what is often referred to as the Hvorslev surface. Due to the compression-positive convention, the "maximum" dilation is represented by  $D_{min}^p$  and is defined under triaxial compression. The dilation limit is given by:

$$D_{min}^p = \chi_{tc}\psi \quad (22)$$

The soil property  $\chi_{tc}$  is determined by carrying out dense drained tests and then plotting the measured dilatancy limit versus the state parameter at that limit (i.e., not the initial state parameter of the test). Although this parameter also affects the predicted response for loose soils, it is difficult to separate from other model properties influencing loose soil behaviour which prevents its direct determination from simple plotting. Thus, even if the focus of interest is loose soil some tests on dense samples are helpful. However, silts can be challenging with present laboratory procedures sometimes being unable to produce dense samples of some silts; in this situation  $\chi_{tc}$  has to be inferred from modelling loose soils by iteratively adjusting  $\chi_{tc}$  to best-fit the test data after all other properties have been determined.

**Strength parameter:  $M_{tc}$  and N**

NorSand uses a dilatancy-related expression derived from Nova (1982), that introduce the volumetric coupling coefficient N:

$$\eta_{max} = M_{tc} - 1(1 - N)D_{min}^p \quad (23)$$

$M_{tc}$  represent the critical friction ratio in compression and it is usually estimated through triaxial drained and undrained tests, plotting the results of these tests on the  $p'$ - $q$  plane, considering end-point of their stress-paths. However, the method originated by Bishop is considered also a standard approach for determining  $M_{tc}$  and N, plotting  $D_{min}^p$  and  $\eta_{max}$  from drained triaxial tests on samples of varying density. Each test is reduced to a value of peak measured dilatancy  $D_{min}^p$

at peak strength  $\eta_{max}$ . Plotting the results of several tests and projecting the result to zero dilation indicates  $M_{tc}$ , whereas the slope of the best-fit trend line through these points correspond to  $(1 - N)$ . If no dense tests are available,  $N \approx 0.3$  is a plausible initial estimate.

### 3.5. Summary

This chapter presents the main concepts of critical state soil mechanics that are useful for the work developed in this thesis. It is illustrated that when soil is sheared under a specific mean effective stress, it reaches a unique critical void ratio, independent of its initial density. This is the basic concept that defines the Critical State Line, providing a fundamental framework for understanding soil behaviour.

Special focus is given to constitutive models that incorporate CSSM principles, focusing on the CASM and NorSand models that are suitable for reproducing undrained strain softening. The mathematical formulation of CASM is presented in this chapter. It includes a brief description of the CASM parameters and graphically illustrates how varying these parameters can reproduce a wide range of soil behaviours.

The NorSand model is also described. This model applies the concept of an infinite of normal consolidation loci, suggesting that soil does not have a unique NCL but varies depends on the initial void ratio at deposition. This feature is combined with an associative flow rule. The discussion includes an overview of the parameters that make up the NorSand model, emphasizing the importance of the image point, defining the zero-dilatancy stress ratio,  $M_i$ . The NorSand model will be employed in chapter 7 to simulate cavity expansion solutions to determine the in situ state of the soil.



## **4. Laboratory methods to determine critical state properties of tailings**

### **4.1. Introduction**

The incorporation of a critical state soil mechanics framework in the design of tailings storage facilities is quickly becoming a standard practice within the consulting industry (Smith et al., 2021). Soils that exist in situ above the critical state line (CSL) or steady-state line in the  $e$ - $\ln(p')$  plane, exhibiting positive state parameters, are identified as more susceptible to liquefaction. Recent studies on tailings storage facility failures have indeed utilized this framework (Morgenstern et al., 2016; Jefferies et al., 2019; Robertson et al., 2019; Arroyo & Gens, 2021).

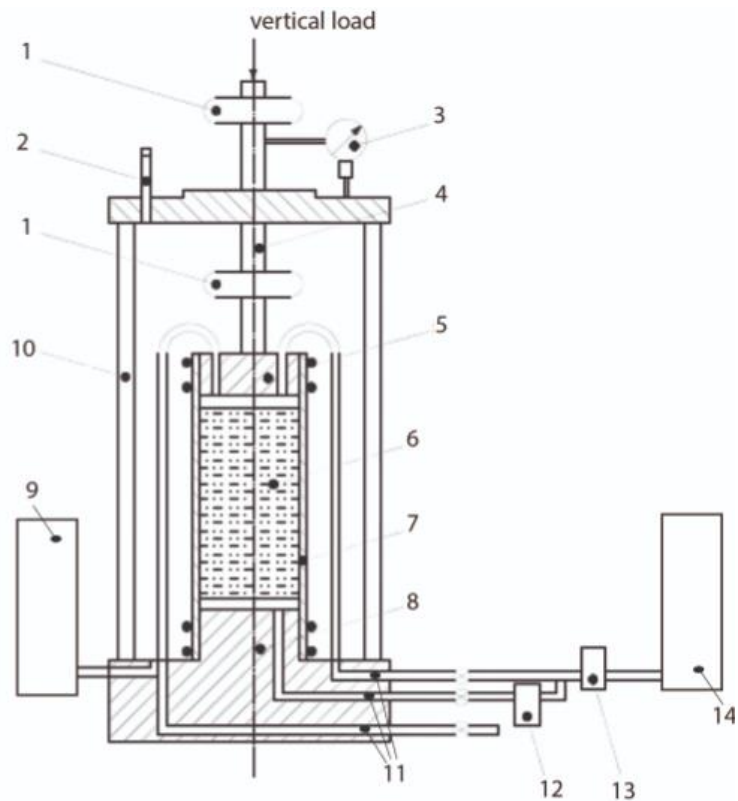
The critical state involves two aspects: defining the Critical State Line (CSL) in terms of void ratio–mean effective stress and the ratio between the stresses at the critical state. Although direct shear tests were initially used to define the CSL (Middlebrooks, 1942), the current preferred method involves a series of triaxial compression tests. Among all available techniques, reconstituted triaxial test using moist tamping (MT) gives the possibility to replicate samples of homogeneous material with target densities, useful to achieve a reliable measurement of the CSL. Furthermore, samples that are generally much looser than the critical state are used because they do not form shear planes. Avoiding localization is important, because when triaxial specimens localize in shear at failure it is difficult to interpret the boundary measurements in terms of effective stress at the failure plane and is almost impossible unless special observation techniques such as computed tomography are employed (Desrues et al., 1996) to recover the void ratio on the shear plane.

Even when localization is avoided measurement of void ratio at critical state in triaxial specimens is difficult (Jefferies & Been, 2015; Reid et al., 2021) and the determination of CSL in the laboratory requires a number of specialized experimental technologies. This chapter aims to present the apparatus and experimental procedures used in this thesis to obtain the CSL from triaxial tests. This is essential because constitutive soil models employed in the subsequent numerical analyses will be calibrated using laboratory test data. Reliable results from high-quality laboratory tests can significantly influence the accuracy of the model calibration.

## 4.2. Experimental procedure to determine the CSL

### 4.2.1. Triaxial equipment

The triaxial test is perhaps the most widely used method for studying the geomechanical behaviour of soils. The combination of a well-controlled device, the versatility of the apparatus, and the relative ease of use makes the triaxial cell a preferred choice among other devices. Moreover, compared to other sophisticated devices such as HTC (Hollow Cylinder Tests) and SS (cyclic simple shear), the triaxial cell is the most cost-effective option, even though it does not allow for the rotation of principal stresses or control of principal stresses other than the vertical and horizontal ones (Soares, 2015). Figure 13 includes both a schematic and an image of the triaxial device used in this investigation.



- 1.- alternative positions for load measuring device
- 2.- air bleed
- 3.- vertical compression measuring device
- 4.- piston
- 5.- top cap
- 6.- soil specimen
- 7.- membrane

- 8.- pedestal
- 9.- device for measurement and control of cell pressure
- 10.- triaxial cell
- 11.- drainage tube
- 12.- pore pressure sensor
- 13.- volume change sensor
- 14.- device for measurement and control of back pressure

Figure 13 – Triaxial testing device at LabGeo

Most of the triaxial cells employed in this investigation had a maximum operating pressure of 1700 kPa, while the limiting capacity of the pressure transducers was 2000 kPa. LabGeo's triaxial devices are equipped with three types of pressure transducers: cell pressure (CP), back pressure (BP), and pore pressure (PN), along with an internal submersible load cell and a linear displacement transducer. Pressure control is managed using a GDS standard level pressure-volume controller, capable of reaching up to 2000 kPa. Additionally, auxiliary master control panels were employed to maintain stable water pressure throughout the tests. The load frame employed had a capacity of 50 kN.

#### **4.2.2. Moist tamping using under-compaction technique**

With the moist tamping technique, specimens are typically prepared at a target moisture content ranging from 5% to 15%. The moist condition creates capillary forces (suction) between the soil grains, allowing the material to be placed inside the mould even at low densities, which may not be possible with other specimen preparation methods (Viana da Fonseca et al., 2021). The main steps in this methodology are shown in Figure 14. The under-compaction technique (R. Ladd, 1978) allows compacting each layer to a selected percentage of the target dry unit weight, producing a specimen of approximately uniform density throughout the entire preparation.

Starting from a specific void ratio and considering the size cylindrical mould, the dry density and dry weight of the sample are calculated by the following formula:

$$\gamma_d = G_s / (1 + e) \quad (24)$$

Then, given the initial moisture content, the wet density and weight of sample could be estimated:

$$\gamma_{wet} = \gamma_d / (1 + w) \quad (25)$$

Finally, if the sample is prepared in layers of equal thickness, the weight of a single layer is calculated by the following equation:

$$W_i = \frac{Vol G (1+w)}{(1+e_0)n} \left( \frac{2U_n(1-1)}{1-n} + 1 + U_n \right) \quad (26)$$

where:

$Vol$  = volume

$G_s$  = Specific Gravity

$w$  = moisture content

$e_0$  = void ratio

$n$  = number of layers

$U_n$  = Overcompaction ratio

Once the wet weight of each layer has been computed, the procedure for specimen remoulding is in accordance with Soares (2015):

1. guiding marks are first drawn on the membrane;
2. two discs of latex membrane and a filter paper are placed in the lubricated end platens.
3. the membrane is attached to the base of the chamber by applying two "o-rings";
4. a cylindrical metallic bipartite mould is placed on the base, and the membrane is then adjusted, 20 kPa of vacuum pressure is applied between the mould and the membrane for a perfect stretch;
5. the soil is poured into the mould with a spoon, then gently tamped using the compaction device with an adjustment stop. Before tamping, the stop is adjusted so that the tamping foot stops at exactly the stop of the layer;
6. after compacting all layers, the lubricated top cap is placed, and the membrane wrapped using two "o-rings";
7. a small vacuum (negative pressure between 10 kPa to 15 kPa) is applied to the top of the sample for a safe remove of the mould, preventing a soil collapse. The final dimensions of the specimen are then measured using the caliper;

8. finally, the acrylic chambers are mounted on the base and then properly closed and filled with water; the internal pressure is replaced by the surrounding water confining pressure (positive 10 to 15 kPa).

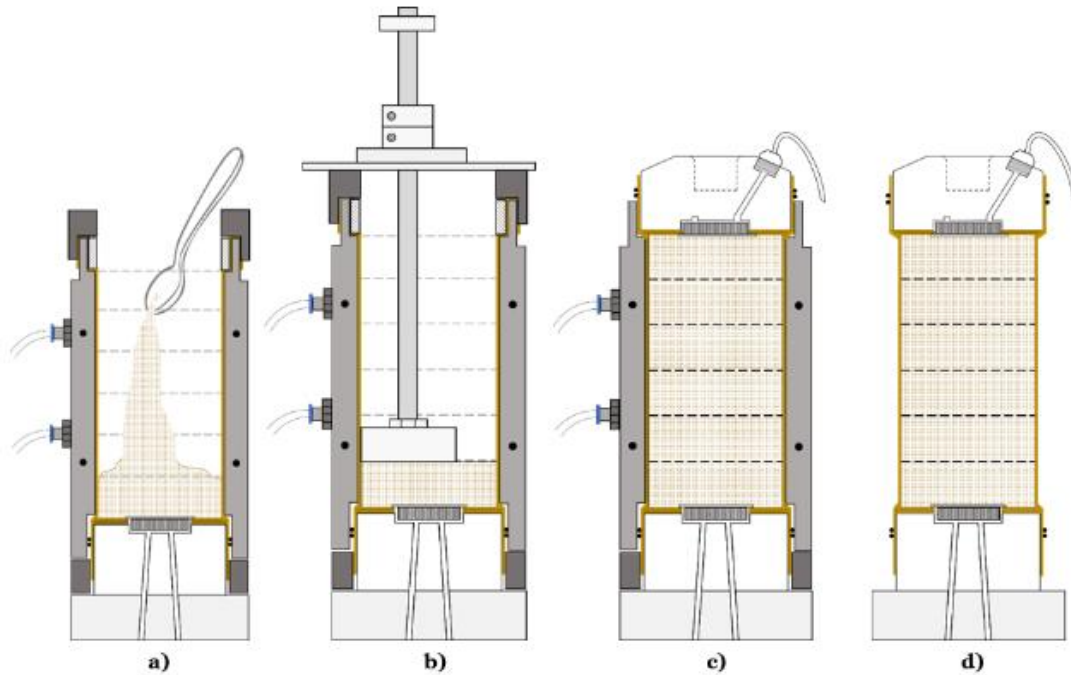


Figure 14 – Reconstitution of soil specimens: (a) layer deposition; (b) tamping process; (c) isolation of the sample; (d) fully reconstituted sample (Viana da Fonseca et al., 2021).

#### 4.2.3. Sample saturation

Saturation of soil samples refers to the complete filling of the void spaces within the soil matrix with water. The degree of saturation is a critical factor in determining the behavior of soil during laboratory tests, especially in triaxial testing, where it can influence the measurement of excess pore pressure in undrained conditions. In this case, the degree of saturation is a key parameter since the development of excess pore-water pressure strongly depends on the stiffness of the pore fluid (Viana da Fonseca et al., 2021). Additionally, obtaining complete saturation during a drained test is important to ensure that the measurement of volume change is not affected. Therefore, ensuring full saturation is central to a proper experimental assessment of the CSL.

There are several techniques available to aid in the saturation of remoulded soil samples. The standard saturation process for dry or moist stamped specimens involves flushing deaerated water through the specimen to displace any trapped air bubbles. To achieve higher degrees of saturation, it is recommended to use carbon dioxide ( $CO_2$ ) before the water flushing. Subsequently, the water pressure is gradually increased until a back pressure equal to 300 kPa is reached, and the degree of saturation is checked using the coefficient  $B$ , as defined by (Skempton, 1954). Alternatively, or simultaneously, the compression wave velocity  $V_p$  can be used to corroborate the full saturation condition of the soil.

### **Carbon dioxide treatment**

Carbon dioxide, being much more soluble in water than air, is introduced through the specimen before the water percolation stage. The gas is introduced through the bottom drain line, thus pushing the lighter air into the sample through the upper drain line. If the pedestal of the cell has a lower secondary drainage line, it is suggested to flow the carbon dioxide through this line, to saturate the bottom of the cell. The  $CO_2$  is vented through a thin tube from the top platen, which is placed inside a water bottle, turned upside down in a container of water to observe the bubbles and the volume change inside of the bottle. A bubble rate of 1 to 5 bubbles per second is about right, with the process lasting 1 to 2 hours (Jefferies & Been, 2015). This method works well for specimens that are initially dry or slightly wet and relatively permeable.

### **Percolation with water**

To avoid chemical reactions between fluid and soil, it is recommended to use deionized water for specimen saturation. In the flushing stage, the water is flushed through the specimen from the bottom to the top under a low hydraulic gradient. Usually, the water volume flowing through the specimen is about twice the specimen void ratio. Percolation took place with a cell pressure of 20 kPa, applying a water pressure on the bottom of 10 kPa. Once the required water volume has been reached, it is good practice to check that there are no small air bubbles in the drainage tube. The time-lapse of the entire process is greatly influenced by soil permeability.

## Application of back pressure

In the last stage, saturation is affected by raising the pore pressure to a level high enough for the water to absorb into solution all the air originally in the void spaces. At the same time the confining pressure is raised to maintain a small positive effective stress in the specimen, in these tests equal to 10 kPa. Saturation by the application of back pressure not only dissolves air contained in the specimen, but also eliminates any air bubbles in the drainage line and pore pressure connections which could not be flushed out. In these tests saturation process was obtained by increasing the back pressure up to 300 kPa and cell pressure to 310 kPa.

### 4.2.4. Evaluation of the degree of saturation

In the laboratory, the soil specimen saturation can be measured through the pore pressure coefficient  $B$ . Traditionally, a value of  $B$  equal to 0.95 is accepted as a criterion of saturation (Head, 1998). This is in accordance with standard ISO/TS 17892-9 (2009), which specifies a  $B$ -value greater than 0.95 for triaxial tests. However, a high  $B$ -value does not always guarantee full saturation for every type of soil (Lade, 2016). Moreover, the commonly accepted saturation requirement ( $B=0.95$ ) is easily obtainable in soft soils, but in rigid soil this may not be high enough to ensure full saturation.

Another indirect method that can be used to indicate the degree of saturation during a triaxial test is the measurement of the compression wave velocity  $V_p$ . It is well known that compressional waves, unlike shear waves, propagate through saturated soils with a velocity that is strongly affected by the water filling the interstices of soil grains. According to Domenico (1974), when the initial saturation degree is low, the value of  $V_p$  decreases as saturation increases because the fluid makes the sample density larger. However, when water saturation reaches a high value,  $V_p$  shows a noticeable increase. Additionally, Ferreira (2008) notes that while parameter  $B$  increases slowly with increasing total stress, the P-wave velocities remain almost constant until the final stages of stress, under which they undergo a sharp increase, quickly reaching values close to the speed of sound propagation in water (approximately 1500 m/s).

#### 4.2.5. Lubricated end platens and embedded top cap

Another limitation of the conventional triaxial equipment is the frictional forces developing between the end of the specimen and the device, both over the top caps and the bottom plate. These forces inevitably restrict the specimen ends' free lateral movement (Bishop & Green, 1965). This drawback can be overcome by applying lubricated ends on the specimen, enabling uniform strains and stresses, and thus ensuring an accurate soil response.

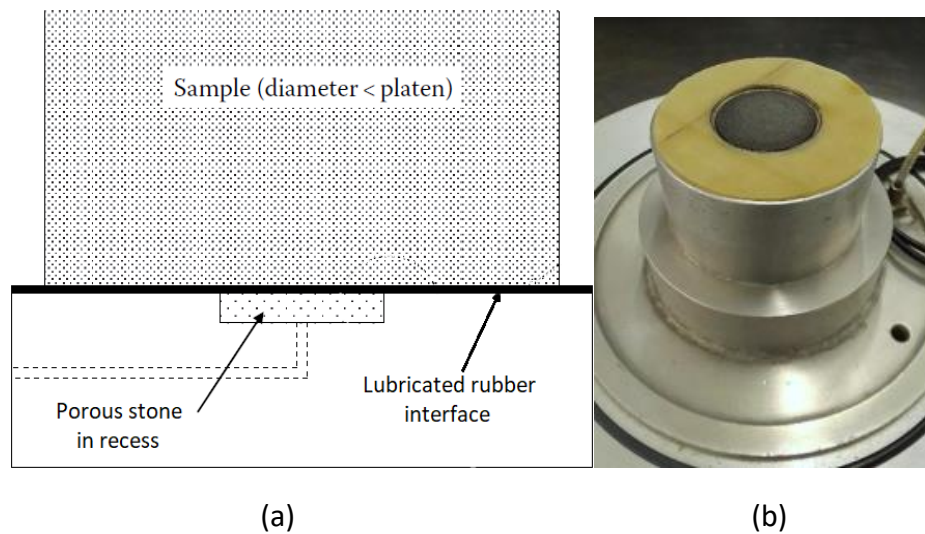


Figure 15 – (a) Lubricated end platen (Jefferies & Been, 2015); (b) Detail of porous stone and disc of latex.

The lubricated end consists of two discs of standard triaxial latex membrane, with a thin layer of silicone grease sandwiched between them (Jefferies & Been, 2015). As shown in Figure 15, the platens in contact with the specimen are oversized. This design is deliberate to accommodate any radial expansion of the specimen's volume, avoiding any overhang and ensuring uniform radial strains across all samples.

Furthermore, in order to accommodate lubricated ends, a full-sized porous stone cannot be used. As shown in Figure 15, the porous stone used in the LabGeo is considerably smaller in diameter than the platen, with the platen measuring around 76 mm and the porous stone 33 mm. An important task when using this oversized platen is to ensure at the beginning of the test that the

drainage paths are completely unobstructed. This is crucial because the reduce size of the porous stone, combined with the use of silicon grease and fine soil particles, could potentially clog the drainage. The fundamental principle behind employing lubricated ends is that the specimen deforms as a right cylinder (Lade, 2016), indicating that the specimen deforms away from its central axis. Therefore, the surface roughness of the disc does not adversely affect the test.

Another key aspect to consider during the shearing phase of the triaxial test is maintaining the verticality of the specimen. This can be particularly critical for remoulded silt or sand specimens that, during water flushing and consolidation, can become misaligned, especially when considering specimen in loose state. This issue can be overcome using an embedded top-cap guided ram connection in the triaxial. This solution, originally introduced by Hettler & Vardoulakis (1984), effectively reduces load eccentricities during triaxial shearing. As illustrated in Figure 16, this approach was also adopted by the geotechnical laboratory at Porto.

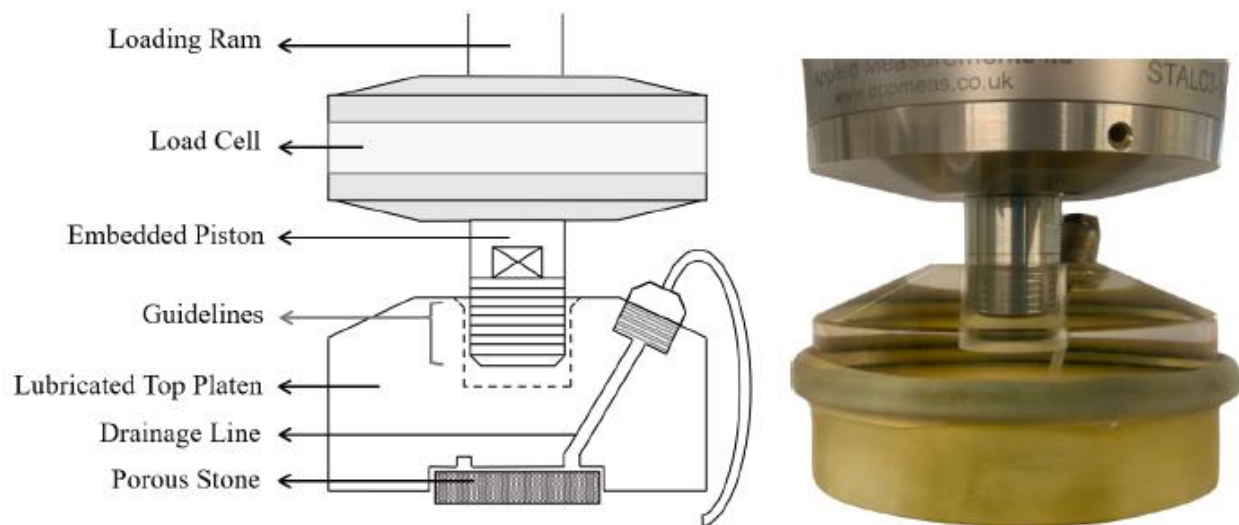


Figure 16 – Embedded top-cap loading ram connection (Viana da Fonseca et al., 2021)

Employing an embedded top-cap loading ram connection ensures central and vertical loading, leading to uniform shearing behavior and volume changes in the sample (Reid et al., 2021). These are critical factors for accurately assessing the Critical State Line (CSL).

#### 4.2.6. Membrane effects

When performing triaxial tests, the latex membrane can introduce systematic errors, particularly due to its penetration into the voids between particle grains and its stiffness. Under loading paths involving changes in effective confining stress, membrane penetration can affect the measured volume changes in drained tests or the excess pore pressures in undrained tests (Sivathayalan & Vaid, 1998). The degree of impact from membrane penetration largely depends on the average particle size, the void ratio of the specimen, variations in the confining pressure, and the total surface area of the membrane. For tests involving samples of 71 mm diameter and materials ranging from sand to silt, the membrane compliance effect can be neglected (Nicholson et al., 1993). Consequently, the correction for membrane penetration was not accounted for in this investigation.

Regarding the impact of membrane stiffness on experimental results, such stiffness leads to lateral confinement of the specimen during shearing. As deformation occurs, the membrane tends to constrain the specimen laterally, resulting in an actual radial stress that is higher than the one that is measured (Viana da Fonseca et al., 2021). To account for these effects, several corrections have been proposed. Among all the approaches, the one proposed by Duncan & Seed (1967) is particularly notable. It offers versatile adjustments that are suitable for a broad range of scenarios, including large strains under both drained and undrained conditions, as well as during compression and extension tests. Due to its versatility, this method was employed in this research. The correction for axial and radial stresses ( $\sigma_a$ ,  $\sigma_r$ ) are outlined as follows:

$$\sigma_{a(corr)} = \sigma_a - \frac{2}{3} E_m \left( 2\varepsilon_a + 1 - \sqrt{\frac{1-\varepsilon_v}{1-\varepsilon_a}} \right) \left( \frac{4T_0}{D_0(1-\varepsilon_v)} \right) \quad (27)$$

$$\sigma_{r(corr)} = \sigma_r - \frac{2}{3} E_m \left( \varepsilon_a + 2 - 2 \sqrt{\frac{1-\varepsilon_v}{1-\varepsilon_a}} \right) \left( \frac{2T_0}{D_0(1-\varepsilon_v)} \right) \quad (28)$$

Here,  $T_0$  refers to the initial thickness of the membrane,  $E_m$  represent the elastic modulus of the membrane, and  $D_0$  is the specimen's initial diameter.

#### 4.2.7. Void ratio determination by end-of-test soil sample freezing

Measurement of void ratio at the end of a triaxial test is crucial since a small error in void ratio estimation could lead to an improper determination of the critical state line. While it is a relatively simple matter to determine initial sample dimension and the dry weight of the sample, it is the volume change during sample saturation and consolidation that can lead to large errors if they are ignored (Jefferies & Been, 2015). At the beginning of the shearing stage, when the specimen is completely saturated, it is possible to measure both axial and volumetric strains. However, over the percolation and saturation stages, it is not possible to measure any deformation as the sample is not saturated and the piston is not yet in contact with the top cap.

A good way to estimate the total volume change including any possible change during the initial stages is by freezing the soil sample and associated cell parts after the test. Once it is frozen, they are carefully removed from the base so that the water content can be determined. The void ratio at the end of the shear stage can be easily calculated using the following formula:

$$e = \frac{G_s \left( \frac{w}{100} \right)}{S_r} \quad (29)$$

where  $G_s$  is the specific gravity of the soil,  $w$  is the moisture content and  $S_r$  is the degree of saturation. With this final void ratio, it is possible to determine the soil sample volume at the end of the test using the following formula:

$$V = \frac{(1+e)W_{dry}}{G_s} \quad (30)$$

The formula can be used to calculate the soil sample volume  $V$  at the end of the test, based on the earlier calculated final void ratio and the dry weight of the specimen  $W_{dry}$ . By measuring the continuous changes in volume, the void ratio at the end of both consolidation and shear stages can be easily determined. Although the freezing method provides the most accurate void ratio measurements for CSL determination, it is beneficial to compare this measurement with another technique. The loading ram is equipped with guidelines spaced 2 mm apart. With the embedded

top-cap loading ram connection, it is possible to obtain an approximate value of the sample's axial shortening during saturation and consolidation by counting how many lines the top cap has moved downward (since the position of the loading ram is fixed during saturation and consolidation). This configuration is shown in Figure 17.

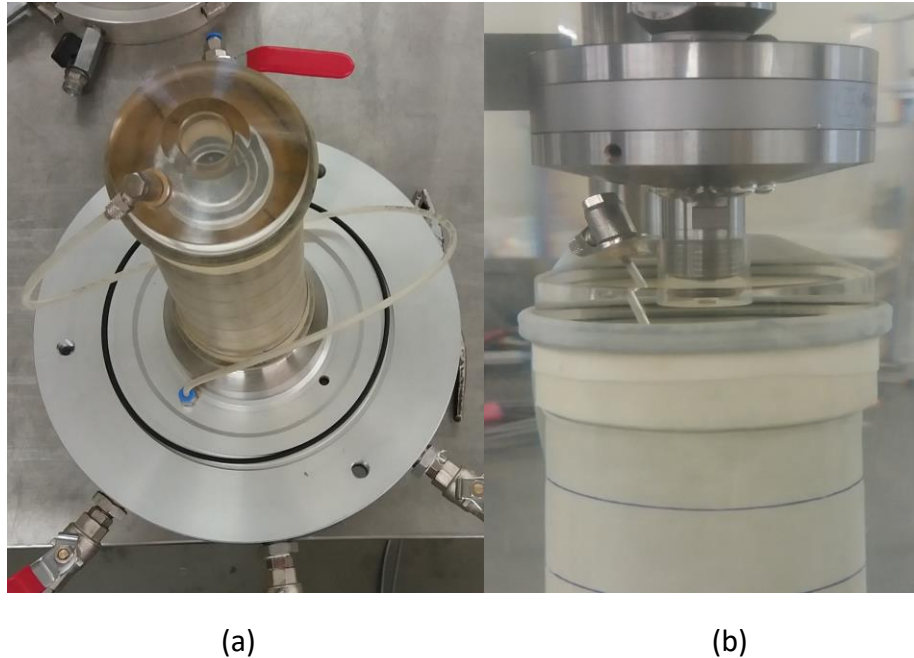


Figure 17 – Detail of the cup cylindrical guide hole.

### 4.3. Summary

This chapter presents the experimental techniques employed to determine the critical state line. It underscores the importance of achieving uniform sample densities and thorough saturation to ensure accurate CSL measurements, especially when undrained triaxial tests are performed.

Additionally, the chapter discusses the limitations of conventional triaxial equipment and methods, showcasing modifications made to traditional equipment to enhance laboratory results. In this context, the benefits of the embedded top-cap loading ram connection and lubricated end platens are detailed. Membrane penetration correction is also addressed. Given that the critical void ratio is the most challenging parameter to estimate in assessing the CSL, the determination of void ratio by soil sample freezing is discussed.

## **5. Laboratory tests on tailings material from dam B1 in Brumadinho**

### **5.1. Introduction**

The experimental campaign reported in this chapter was motivated by the need to provide experimental support to the post-mortem numerical studies of Brumadinho dam B1 failure carried out at CIMNE-UPC (Arroyo & Gens, 2021). Even though a first set of laboratory tests was conducted as part of a technical report (Robertson et al., 2019) commissioned by Vale to investigate the causes of the failure, the geotechnical information obtained from those studies has been considered not exhaustive for the numerical investigation that was to be performed. It was therefore necessary to conduct a new geotechnical campaign, and the experimental work described herein corresponds to those tests carried out by the author as part of the FEUP GEOLab team.

The post-mortem campaign conducted by FEUP identified four different materials to represent the various types of tailings presented in the dam, namely Types 1, 2, 3, and 4. Results on Types 1, 2 and 3 were already included in the experimental Appendix of Arroyo & Gens (2021); however, those results have been re-elaborated and are presented anew below. Results on Type 4 material are here presented for the first time; they were not considered in Arroyo & Gens (2021) as the experimental campaign on this material took place after that report was finished.

The experimental programme carried out on the tailings starts with the particle size distribution and particle soil density. A series of oedometer tests with different initial void ratio were carried out to identify the compressibility of the material. Isotropically and anisotropically consolidated triaxial tests were executed to define the critical state locus and the soil behaviour under a range of initial soil states (loose or dense) and stress paths. Some triaxial devices include Bender Element (BE) to investigate the shear modulus at very small values of strain  $G_0$  at different mean effective stress. Additionally, the hydraulic conductivity was measured in parallel to the seismic wave measurements using the constant head permeability method. All these tests were performed on the four distinct material types that were mentioned above.

## 5.2. Sampling

Material types in a tailing dam generally consist in sand and silt-size particles. While thickened tailings shows uniform gradation after deposition, conventional tailing slurry tend to segregate when hydraulically discharge, with the sand fraction settles close to the spigot and the silt or sandy silt particles moving further down and settling at the tailings pond. This implies at least two materials should be investigated. However, change in spigot location, in solid contents on the tailings stream and in composition of the suspended tailings material frequently lead in practice to a far more complex picture, with strong lateral variation and mesostructural features such as subcentimetric layering of sands and silts. As a result, it is an open question how many different soil types should be selected as representative of a tailings dam impoundment. The selection and definition of representative materials is made more difficult when it is realized that small changes in fines content may significantly affect the CSL position (Fourie et al., 2002; Carrera et al., 2011; Jefferies & Been, 2015)



Figure 18 – Example of layered block sample of Brumadinho tailing Dam (Arroyo & Gens, 2021)

Obtaining undisturbed samples of tailings material is not an easy task. Sand is especially troublesome to sample, even with advanced techniques such as in-situ freezing or gel-push methods. On the other hand, silt can be sampled with standard devices, but there is a considerable densification when the specimen is extruded, trimmed and consolidated to the in-situ stress level (Been, 2016). Moreover, another issue arises with undisturbed soil samples, since

tailings are frequently layered, as shown in Figure 18, making it challenging to recover similar samples.

Thus, a common practice is to carry out a bulk sampling campaign in which material is collected and shipped to the laboratory for testing. Later, the bulk sample can be combined, sieved, and mixed to guarantee the homogeneity of the material and to reproduce the representative gradation for laboratory tests. This post-processed material is then characterized again in terms of gradation, color and specific gravity.

Details on the sampling campaign carried out on Brumadinho B1 tailings can be found in Viana da Fonseca et al. (2022). The sampling locations were selected based on data from CPTu soundings (pre-failure) in conjunction the physical properties determined in the laboratory. This analysis included assessments of color variability, specific gravity, grain size analysis and iron content related to mineral morphology.

### **5.3. Basic properties**

The methods routinely employed for the evaluation of particle size distribution of soil follow the ISO 17892-4 (2016) standard. In this method, the fine particles are preliminary removed by washing the material over a fine sieve, followed by dry sieving of particles coarser than 75  $\mu\text{m}$ ; whereas the distribution of particles finer than 75  $\mu\text{m}$  is determined using the hydrometer method.

Figure 19 illustrates the particle size distribution of tailings from the Brumadinho B1 tailings dam. The grain size distribution depicted includes the four types of tailings employed in this thesis, as well as those considered in the Expert Panel Technical Report (Robertson et al., 2019). The shaded area represents the upper and lower grading limits of the soil samples collected during the EPR campaign. The variability of grain size distribution within the B1 tailings dam was therefore large, with the fine content varying approximately from 20% to 90%.

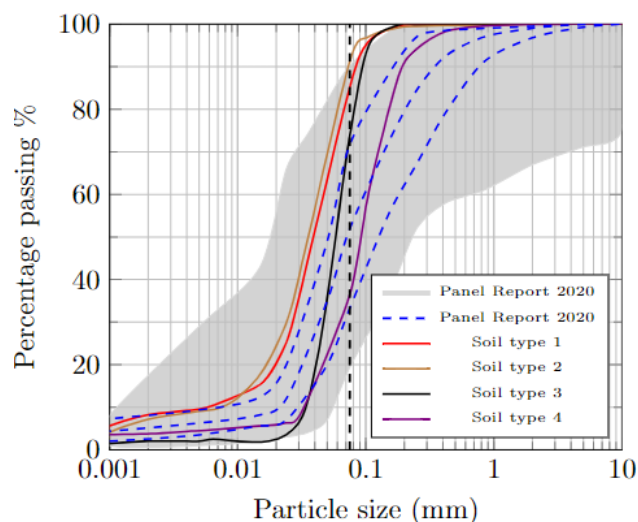


Figure 19 – Particle size distribution from the Brumadinho B1 tailing dam

For natural soils,  $G_S$  is often between 2.65 and 2.70, being unusual values outside this range. Tailings, on the contrary, routinely exhibit values well outside this range. The wide range of specific gravity in tailings is related to the concentration of each constituent mineral. Because iron minerals possess high specific gravity, an increase in their content correlates with an increase in the specific gravity of the tailings. Moreover, older tailings deposits tend to have a higher iron content because the process of extracting iron ore from the rocks sourced was less efficient. Considering that construction of the dam started in 1977 and continued operations until 2001, the broad spectrum of  $G_S$  values identified through laboratory testing, ranging from 3.94 to 5.00, can be attributed to this wide operational period.

Table 1  $G_S$  of some tailings

| Ore      | Specific Gravity ( $G_S$ ) | Reference                |
|----------|----------------------------|--------------------------|
| Gold     | 2.7 to 3.1                 | (Torres-Cruz, 2016)      |
| Iron     | 3.14 to 5.00               | (Robertson et al., 2019) |
| Gold     | 3.10 to 2.89               | (Bedin et al., 2012)     |
| Fluorite | 2.72 to 2.83               | (Carrera et al., 2011)   |

Tailings are usually composed of two parts: the finer fraction smaller than 0.075 mm, also called 'tailing slimes', and the coarser fraction. While the coarser part is generally non-plastic, slimes tailings may exhibit low plasticity. Therefore, the purpose of conducting the Atterberg limits tests is to determine the basic nature of the fine-grained fractions of the material. The plastic limit ( $w_p$ ) is identified as the water content below which the soil no longer exhibits plastic behaviour. According to the laboratory test results, all the materials tested in this study has been classified as non-plastic.

#### **5.4. One dimensional and isotropic compressibility**

One-dimensional consolidation and isotropic consolidated triaxial tests were carried out on reconstituted specimens of the four types of tailings investigated to examine their compressibility and transitional behaviour. Transitional behavior can easily be identified by assessing the convergence of the normal consolidation lines to a unique intrinsic consolidation line through one-dimensional consolidation tests in soil samples with different initial void ratios ( $e_0$ ). Oedometer tests were conducted on specimens with a 50 mm diameter using a very gentle moist tamping method (spoon deposition) across a range of initial void ratios.

All the oedometer specimens were prepared following the same methodology. The material was carefully spooned into the oedometer ring in three layers, and a wood tamp was used to compact the material. To improve the contact between the soil particle, the first layer was scarified before compacting the next soil layer. Samples were saturated by inundating the oedometer cell with deaired water after placing the top cap provided with porous stone. For all samples, incremental loading and unloading steps were applied, and the maximum vertical effective stress reached during the test was 6400 kPa. The voids ratio throughout the test were calculated from the sample's initial height and the measured axial strain for each stage. Non-significant differences resulted when the void ratio is calculated from final water content of the retrieved sample. For oedometer tests, the mean effective stress  $p'$  was estimated by assuming a coefficient of earth pressure  $K$  equal to  $k_{cs} = (1 - \sin \varphi'_{cs})$ . Although this assumption could be affected by some

uncertainties, the inaccuracy of the K value would not change the slope of the compressibility curve, only its relative position in the x- axis.

As a complement to the oedometer consolidation tests, a series of isotropic compression triaxial tests were carried out on loose samples formed using the procedures recommended in Chapter 4. All the samples were consolidated through several isotropic loading stages with measurements of the volumetric change over time until stabilization. Initial void ratios were calculated using the final gravimetric water content (GWC), while the intermediate void ratios were computed through the recorded volumetric strain.

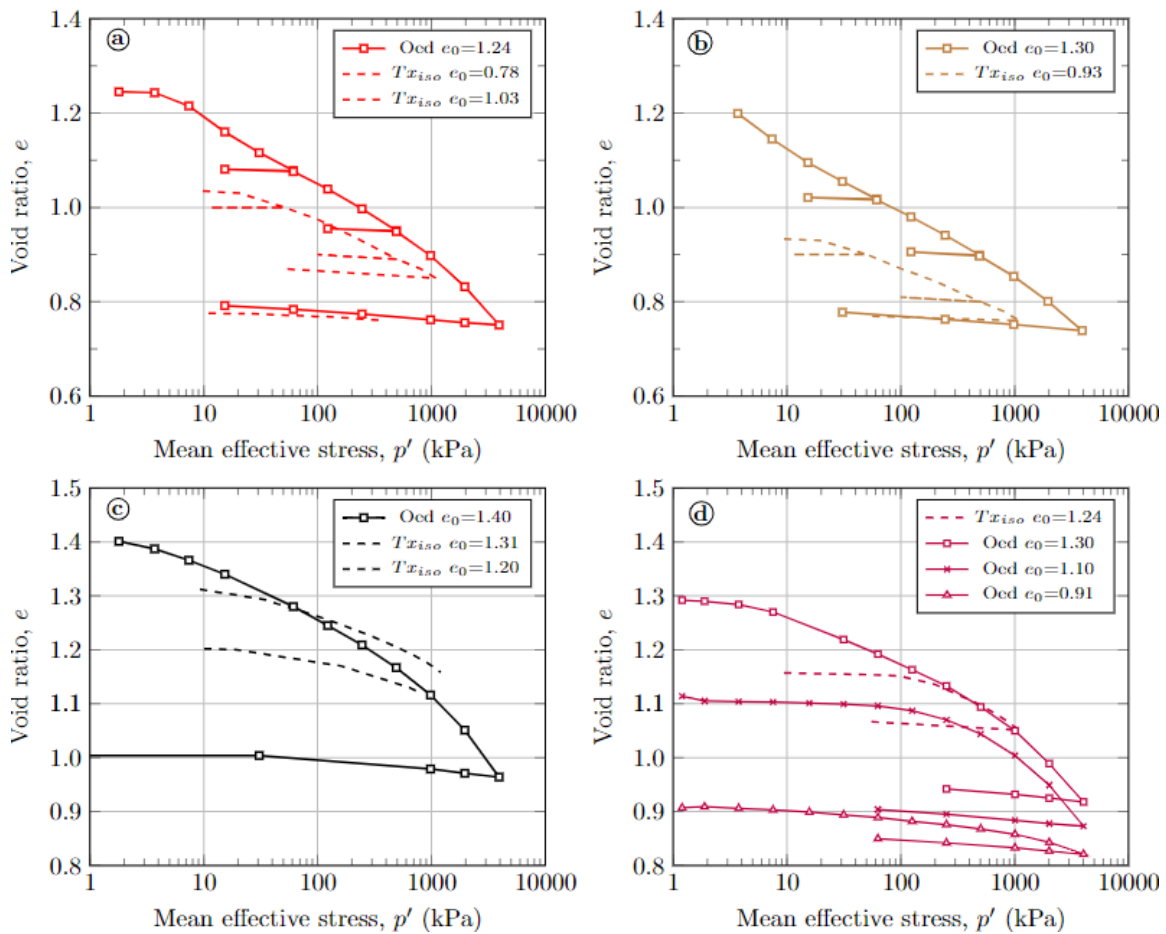


Figure 20 – Comparison of compression curves and critical state lines for soil types 1 to 4

The oedometer compression data are shown in Figure 20. For each soil type, almost one test was prepared to ensure a loose soil state condition (Viana da Fonseca et al., 2022). The deviation in

void ratio observed for low confining pressures is attributed to volumetric collapse during saturation. The elastic modulus  $\kappa$  was evaluated through a series of loading and unloading steps. As reported in Table 2,  $\kappa$  values increase for the unloading performed at the highest stress. This may signal an evolution in the soil morphology, namely the particle shape or size. Nevertheless, the effect of confining pressure on  $\kappa$  is little, and the four types of tailings exhibit a broadly similar elastic compressibility.

On Figure 20d, oedometer samples prepared with different initial void ratios do not seem to converge to a unique 1D-NCL. Similar behaviour was identified by Nocilla et al. (2006) in soil with low fines content. Another possible reason is that the specimens have not reached the 1D-NCL and a yield point cannot be detected for the stress range investigated. A similar observation was made by Carrera et al. (2011), where convergence was noted for all tailing gradations tested, with convergence not being reached until 10 MPa. For denser specimens, the magnitude of compaction stresses may exceed the consolidation stresses achieved during the test. Similar effects have been observed by Chang et al. (2011) and Frost & Park (2003) for remolded high dense samples using the moist tamping technique. However, it is not clear if the behaviour observed refers to a likely transitional material or if the soil will converge to a unique NCL for higher pressures.

Table 2 Compressibility parameters.

| Test      | Soil type | Initial void ratio | k           |
|-----------|-----------|--------------------|-------------|
| Oedometer | Type 1    | 1.24               | 0.003-0.007 |
| Oedometer | Type 2    | 1.30               | 0.006 0.009 |
| Oedometer | Type 3    | 1.40               | 0.008       |
| Oedometer | Type 4    | 1.30               | 0.006 0.008 |
| Oedometer | Type 4    | 1.10               | 0.006 0.008 |
| Oedometer | Type 4    | 0.91               | 0.005 0.007 |

## 5.5. Elastic stiffness properties

The elastic shear modulus ( $G_0$  or  $G_{max}$ ) of the soil is associated with shear strains levels below 0.001 %. The small strain behaviour of the soil is an important property as it can reflect the soil fabric, density, and the shear stress low deformation. By itself, it is a parameter widely used in geotechnical analysis for tasks such as evaluating liquefaction potential, controlling soil improvement quality, and designing soil-structure interactions

Several techniques have been developed for measuring  $G_{max}$  by in situ methods and laboratory testing. Direct determination of seismic wave velocities in the laboratory is becoming common practice worldwide given its great potential in the definition of the stiffness at very small strains (Ferreira et al., 2014). Among others, the resonant column and the bender element (BE) test have been increasingly included in geotechnical laboratories as standard devices. Particularly, bender elements (BE) and other piezoceramic transducers have opened a new window in laboratory testing due to the high precision and effectiveness in measuring the small strain properties of soil. This allows for the performance of standard mechanical, static, or cyclic tests simultaneously with seismic wave dynamic tests (Consoli et al., 2012).

The elastic shear modulus was estimated using shear wave velocities measured at different stages of isotropic consolidation in a triaxial chamber equipped with bender elements. The estimation was based on the shear wave velocity  $V_s$  and density, as follows:

$$G_{max} = \rho V_s^2 \quad (31)$$

where  $\rho$  represent the bulk density of the soil and  $V_s$  is the velocity of shear wave. Since  $G_{max}$  is strongly affected by mean effective stress, the idealization of the shear modulus can be expressed as:

$$G_{max} = G_{ref} \left( \frac{p'}{p_{ref}} \right)^n \quad (32)$$

where  $G_{ref}$  is a reference shear modulus at the reference stress level  $p_{ref}$  of 100 kPa.

Figure 21 shows the variation of the elastic shear modulus with the void ratio and mean effective stress for the four types of tailing investigated. It is interesting to note the tendency of  $G_{max}$  in soil type 3 and 4, despite them having they have similar properties in term of CSL,  $G_s$  and permeability, and even though the void ratio was similar during isotropic consolidation,  $G_{max}$  shows a different pattern. This can be attributed on the effect of fine content, ranging from 36% in soil type 4 to 71% in soli type 3. The role of fine in modifying  $G_{max}$  has been investigated in recent works (Salgado et al., 2000; J. Yang & Liu, 2016) showing that the small strain stiffness decreases with increasing fine content. Nevertheless, the relation between FC and  $G_{max}$  is not monotonous since there is a specific FC value above which do not cause measurable changes in  $G_{max}$ .

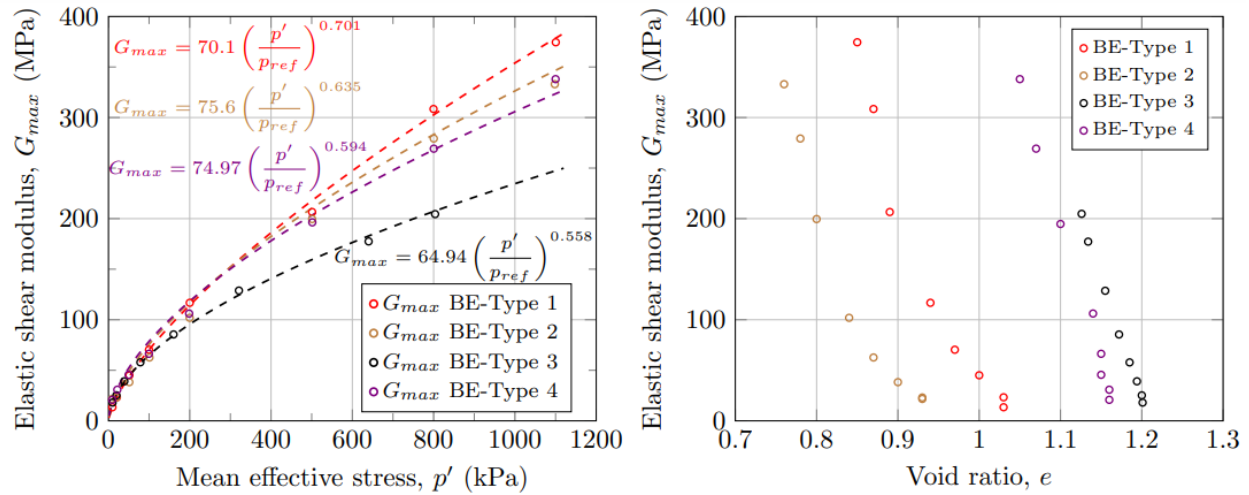


Figure 21 – Results of Bender element tests: (a) relation curves between  $G_{max}$  and mean effective stress; (b) relation curves between  $G_{max}$  and void ratio.

However, the magnitude of  $G_{max}$  is influenced not only by the mean effective stress, as captured in equation (32), but also by void ratio and other properties. Many empirical correlations have been developed to provide the variation of  $G_{max}$  with soil properties and state (Hardin & Richart, 1963; Jamiolkowski, 1991) which can be generalized with the general form:

$$G_{max} = A F_e \left( \frac{p'}{p_{ref}} \right)^n \quad (33)$$

where  $A$  is a constant depending on the characteristics of soil;  $F_e$  is a void ratio function reflecting the relation of soil density,  $p_{ref}$  is a reference stress (usually 100 kPa) and  $n$  is an empirical fitting constant. Various forms of the  $F_e$  function have been proposed, which can be applied to specific types of soils. To explore the impact of void ratio and confining pressure on  $G_{max}$ , our test program included an isotropic consolidation test complemented by a series of loading and unloading consolidation paths, along with measurements of S-wave velocities. The  $F_e$  function adopted this study is outlined in Shuttle & Jefferies (2016):

$$G_{max} = \frac{A}{(e - e_{min})} \left( \frac{p'}{p_{ref}} \right)^n \quad (34)$$

In equation (34),  $e_{min}$  should be calculated according to the test procedures ASTM D-4254. However, in this study the  $e_{min}$  was obtained from oedometer tests by accounting for the lowest void ratio at the highest applied pressure (see Figure 20d). The black dashed lines depicted in Figure 22b, which represent specific confining stresses from 50 to 500 kPa, are derived using equation (34). The specific soil parameters have been calibrated using laboratory measurements where  $G_{max}$  was recorded across different densities under the same confining pressure, marked with squares and triangles in Figure 22. The calibrated material parameters for equation (34)  $A = 21.7$ ,  $e_{min} = 0.85$  and  $n = 0.48$ , and the estimated data clearly match the observations from laboratory tests

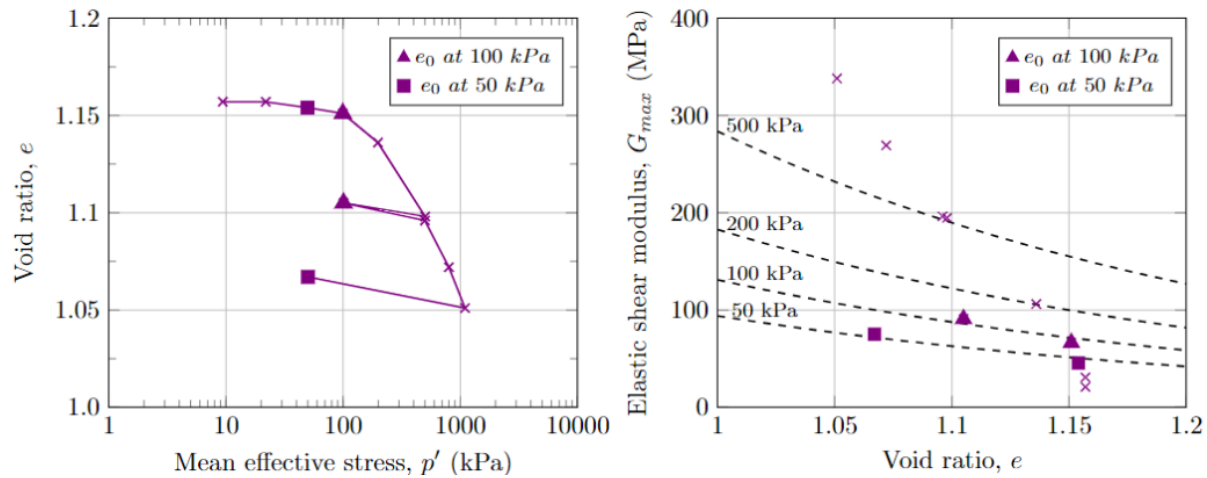


Figure 22 – Results of Bender element tests: (a) isotropic consolidation test; (b)  $G_{max}$  measurement at different void ratio and coincident confining stresses.

## 5.6. Hydraulic conductivity

The permeability of tailing deposits is a key property to evaluate drainage conditions during the geotechnical design. More than any other engineering property, tailings permeability is challenging to generalize (Vick, 1990) since it's affected by many factors such as grain size, plasticity, depth within the deposit, void ratio, and saturation among others. Permeability can be obtained from several in situ and laboratory tests with specific advantages or disadvantages related to the test selected.

In the laboratory, one of the common methods for the direct measurement of permeability is by conducting constant head tests in a standard triaxial equipment. This set up eliminate the uncertainties of conventional equipment, such as the effect of water flow along a rigid confining container (Patiño et al., 2019). Moreover, an advantage associated with the triaxial equipment is the possibility of guaranteeing the full saturation of the specimen and expressing the variation of the permeability with the mean effective stress and the void ratio. In this investigation, the hydraulic conductivity was measured in four different steps of the isotropic consolidation, usually between 20 and 1000 KPa. A constant head was kept by raising the back pressure through the lower drainage lines so as to generate an upward flow through the specimen (e.g., 302.5 kPa at

the bottom and 297.5 kPa at the top). The results shown in Figure 23 present the coefficient of hydraulic conductivity  $k$  as a function of the effective stresses and void ratio conditions.

Alternatively, the permeability soils can be estimated using the equation proposed by Kozeny-Carman (KC):

$$\log(k_{kc}) = 0.5 + \log \left( \frac{e^3}{S_s^2 G_s^2 (1+e)} \right) \quad (35)$$

In the KC equation above the value of permeability is obtained in m/s and  $e$  is the void ratio,  $G_s$  is the specific gravity of the soil and  $S_s$  is the value of specific surface area (in  $m^2/kg$ ). The latter parameter can be measured directly in laboratory using the gas adsorption method or estimated by means of correlations that vary significantly depending on soil type and mineralogy (Lambe & Whitman, 1991). For non-plastic soils like tailings, the specific surface area can be estimated from grain size distribution curves using the Chapuis & Legare (1992) equation:

$$S_s = \frac{6}{\rho_s} \left[ \sum \frac{[P(d_{i+1}) - P(d_i)]}{d_i} \right] \quad (36)$$

where  $\rho_s$  is the specific weight of the particle and  $P_{(di)}$  can be calculated from the PSD and represents the percentage in weight smaller than size diameter  $d_i$ . For particle smaller than the minimum measured size  $d_{min}$ , an equivalent size diameter  $d_{eq}$  must be defined to compute the contribution of  $S_s$  for the smaller particles. It should be noted that the contribution related to  $d_{eq}$  can greatly affect the result. The equivalent diameter is given by:

$$d_{eq} = \frac{d_{min}}{\sqrt{3}} \quad (37)$$

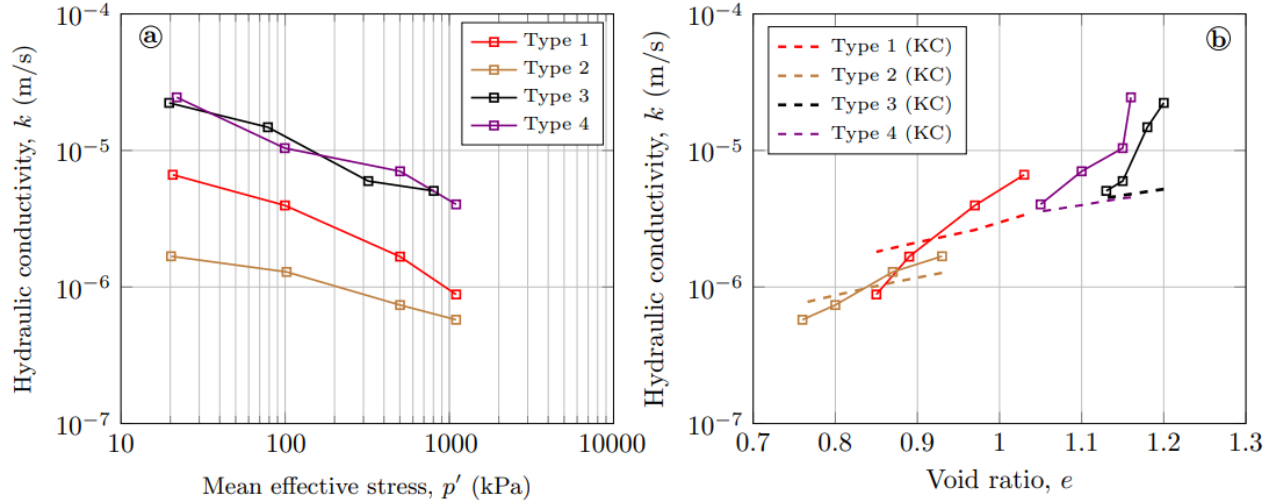


Figure 23 – Variation of the coefficient of hydraulic conductivity for the three types as a function of: (a) effective stress conditions; (b) void ratio.

Figure 23b reports the predicted (dashed lines) and computed  $k$  value, showing a good agreement of the Kozeny-Carman equation with experimental data. Somewhat surprisingly, the KC equation seems to predict very well the permeability of the finer tailings (type 2), while it significantly underestimates the permeability measured for the coarser materials (type 3 and type 4).

## 5.7. Critical state parameters and compressibility

### 5.7.1. Overview

Isotropically consolidated triaxial compression tests have been used to identify the CSL in both  $e - p'$  and  $q - p'$  projection. The projection of the critical state line in the  $p' - q$  plane reflect the shape and mineralogy of the grains (Cho et al., 2006) and, if the tailing come from the same ore bodies it is expected similar values of friction ratio ( $M$ ) (Torres-Cruz, 2016). However,  $PSD$  and in particular the percentage of fine content largely influence the position and slope of the critical state line in the  $e - p'$  plane (Carrera et al., 2011). Given that, four different types were used in the testing program to investigate the critical state locus.

### 5.7.2. Isotropically consolidated triaxial tests

Drained and undrained triaxial tests on loose specimens were performed in a traditional triaxial chamber following the procedure described in chapter 4.

To define the CSL we utilize both drained and undrained tests. Indeed, defining the CSL using only undrained triaxial tests with specimens in a loose state may require achieving consolidation pressures that could exceed 2 MPa. Such pressures not only surpass the capabilities of conventional triaxial chambers but can also lead to grain crushing effects (Jefferies & Been, 2015).

Thus, contractive drained tests can be useful for initially identifying the pattern of the Critical State Line, whether it is linear or curved. Indeed, these tests consistently reach the CSL. Additionally, drained tests at higher consolidation pressure (above 1000 kPa) have been conducted to potentially capture the curvature of the CSL at elevated mean effective stresses. Similarly, undrained tests have been performed to define the CSL curvature at lower pressure levels. A list of all isotropically consolidated triaxial tests performed, along with details about the testing procedures followed, is summarized in Table 3.

Table 3 List of isotropically consolidated triaxial tests.

| Soil type | Test Type | Equipment       | Preparation Phase |       | Consolidation Phase |
|-----------|-----------|-----------------|-------------------|-------|---------------------|
|           |           |                 | Method            | State | $p'$                |
| Type 1    | CID       | Conventional Tx | MT                | Loose | 200                 |
|           | CID       | Conventional Tx | MT                | Loose | 500                 |
|           | CID       | Conventional Tx | MT                | Loose | 800                 |
|           | CIU       | Conventional Tx | MT                | Loose | 400                 |
|           | CIU       | Conventional Tx | MT                | Loose | 800                 |
|           | CIU       | Conventional Tx | MT                | Loose | 800                 |
|           | CID       | Conventional Tx | SC                | Dense | 100                 |
|           | CID       | Conventional Tx | SC                | Dense | 200                 |
|           | CID       | Conventional Tx | SC                | Dense | 400                 |
| Type 2    | CID       | Conventional Tx | MT                | Loose | 50                  |
|           | CID       | Conventional Tx | MT                | Loose | 100                 |
|           | CID       | Conventional Tx | MT                | Loose | 400                 |

|        |     |                 |    |       |      |
|--------|-----|-----------------|----|-------|------|
|        | CID | Conventional Tx | MT | Loose | 800  |
|        | CIU | Conventional Tx | MT | Loose | 400  |
|        | CID | Conventional Tx | SC | Dense | 200  |
| Type 3 | CID | Conventional Tx | MT | Loose | 30   |
|        | CID | Conventional Tx | MT | Loose | 100  |
|        | CID | Conventional Tx | MT | Loose | 200  |
|        | CID | Conventional Tx | MT | Loose | 400  |
|        | CID | Conventional Tx | MT | Loose | 800  |
|        | CIU | Conventional Tx | MT | Loose | 600  |
|        | CIU | Conventional Tx | MT | Loose | 800  |
|        | CIU | Conventional Tx | MT | Loose | 1200 |
|        | CID | Conventional Tx | SC | Dense | 100  |
|        | CID | Conventional Tx | SC | Dense | 100  |
|        | CID | Conventional Tx | SC | Dense | 100  |
|        | CID | Conventional Tx | SC | Dense | 200  |
| Type 4 | CID | Conventional Tx | MT | Loose | 50   |
|        | CID | Conventional Tx | MT | Loose | 100  |
|        | CID | Conventional Tx | MT | Loose | 200  |
|        | CID | Conventional Tx | MT | Loose | 400  |
|        | CID | Conventional Tx | MT | Loose | 800  |
|        | CID | Conventional Tx | MT | Loose | 1600 |
|        | CIU | Conventional Tx | MT | Loose | 750  |
|        | CIU | Conventional Tx | MT | Loose | 800  |
|        | CIU | Conventional Tx | MT | Loose | 1600 |
|        | CID | Conventional Tx | SC | Dense | 200  |
|        | CID | Conventional Tx | SC | Dense | 200  |
|        | CID | Conventional Tx | SC | Dense | 200  |
|        | CID | Conventional Tx | SC | Dense | 200  |
|        | CID | Conventional Tx | SC | Dense | 400  |

#### 5.7.2.1. Critical state line of tailing types 1 and 2

The behaviour of soil types 1 and 2 is summarized in Figure 24 and Figure 25 in term of stress path in the  $q - p'$  space and state plot in the  $e - p'$  space. To identify the CSL only samples with

negligible change in stresses, volume change or shear-induced pore pressure at the end of the test have been considered. In this set of tests, the critical friction ratio  $M_{tc}$  is well defined using both contractive and dilative specimens. Tests result in Figure 24a and Figure 25a indicate a strength parameter  $M_{tc}$  equal to 1.42 and 1.40 for the soil types 1 and 2. Excluding dense tests from the dataset did not substantially modify the critical friction ratio; indeed,  $M_{tc}$  for soil type 1 experienced a slight change to 1.41, while for soil type 2 it remained constant.  $M_{tc}$  parameter was evaluated according to the End of the Test Method EoT (Ghafghazi & Shuttle, 2006; Sladen & Handford, 1987) since tests reach the critical state condition in terms of critical friction ratio.

In Figure 24b and Figure 25b the critical state locus is projected on the  $e$ -log  $p'$  space. Specimens that exhibited dilative behaviour, even if included in the plots, were not employed to define the CSL, as dense samples are prone to shear localization causing non-uniform sample void ratio (Jefferies & Been, 2015; Reid et al., 2018). These tests show a consistent approximation of the CSL with linear a semi-log idealization at stresses above 100 kPa. For soil type 1, the undrained test consolidated at 400 kPa, which was the lowest consolidation stress for this material shows considerable strain softening. This suggest that at low stresses, the critical state line may flatten out and may be better represented with a power law function. This characteristic was not featured in the tests performed on soil type 2, for which, as illustrate in Figure 25b, the end of the test point lies on a straight idealization of the CSL.

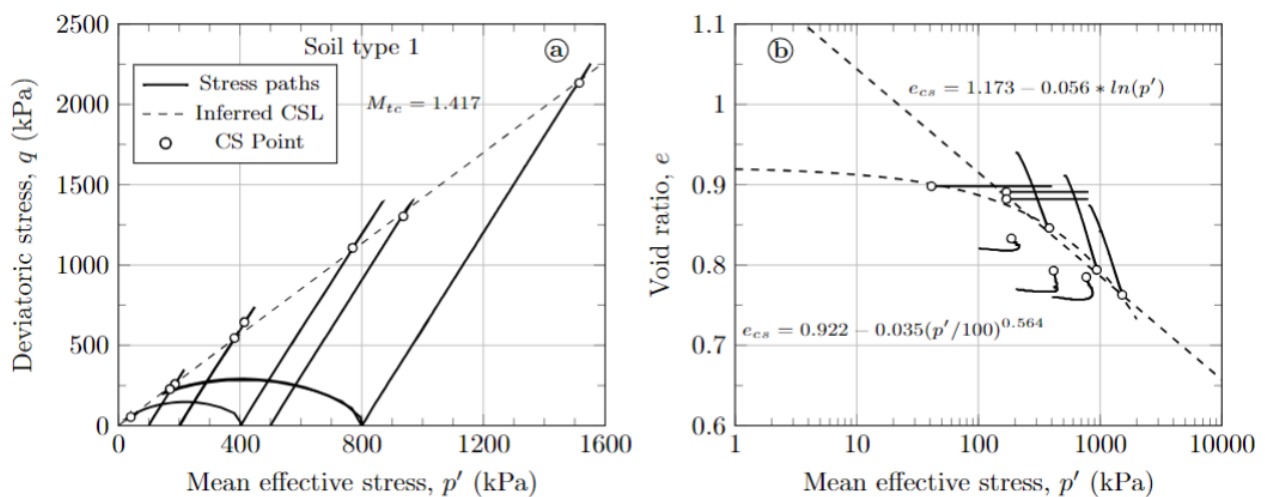


Figure 24 – Triaxial tests results and inferred CSL Soil type 1: (a) stress paths, (b) state diagram.

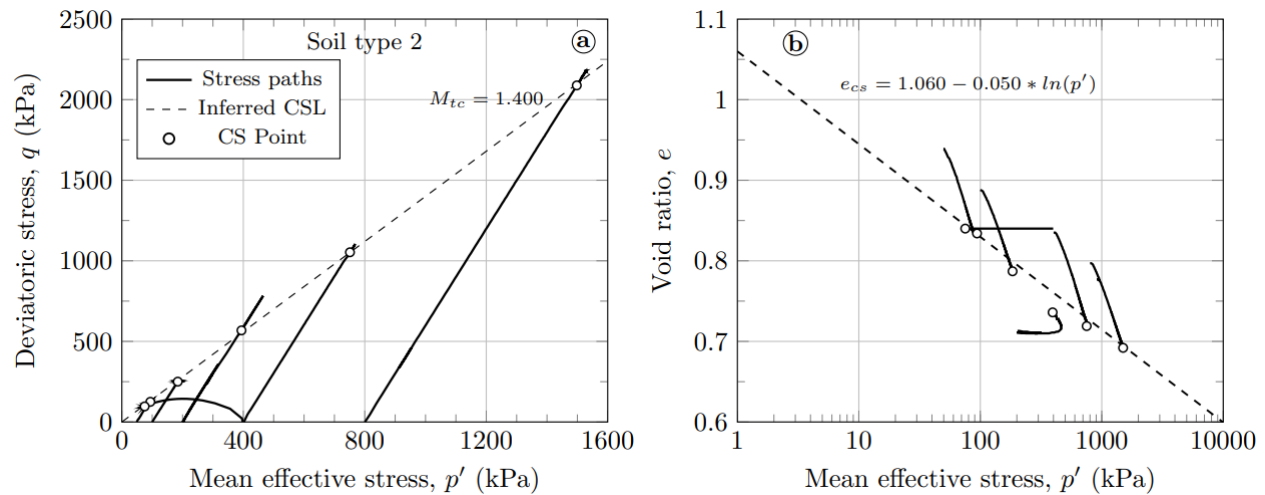


Figure 25 – Triaxial tests results and inferred CSL Soil type 2: (a) stress paths, (b) state diagram.

#### 5.7.2.2. Critical state line of tailing types 3 and 4

Figure 26 and Figure 27 summarises the results obtained from the tests conducted on iron tailings type 3 and 4. As before, only the results of drained and undrained loose triaxial tests considered to have reached the critical state by the EoT method were used to define the critical state parameters.

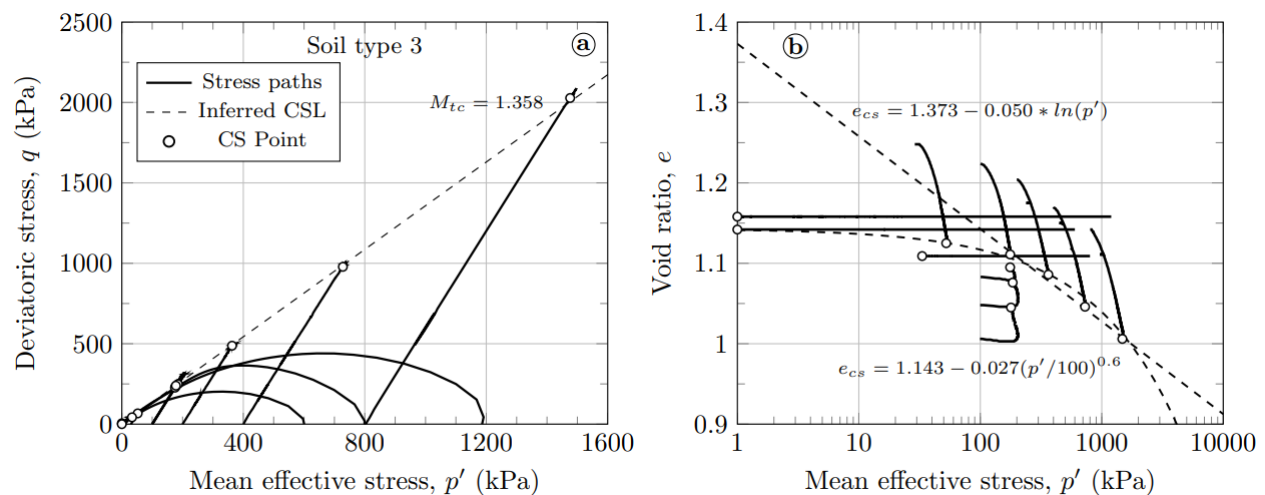


Figure 26 – Triaxial tests results and inferred CSL Soil type 3: (a) stress paths, (b) state diagram.

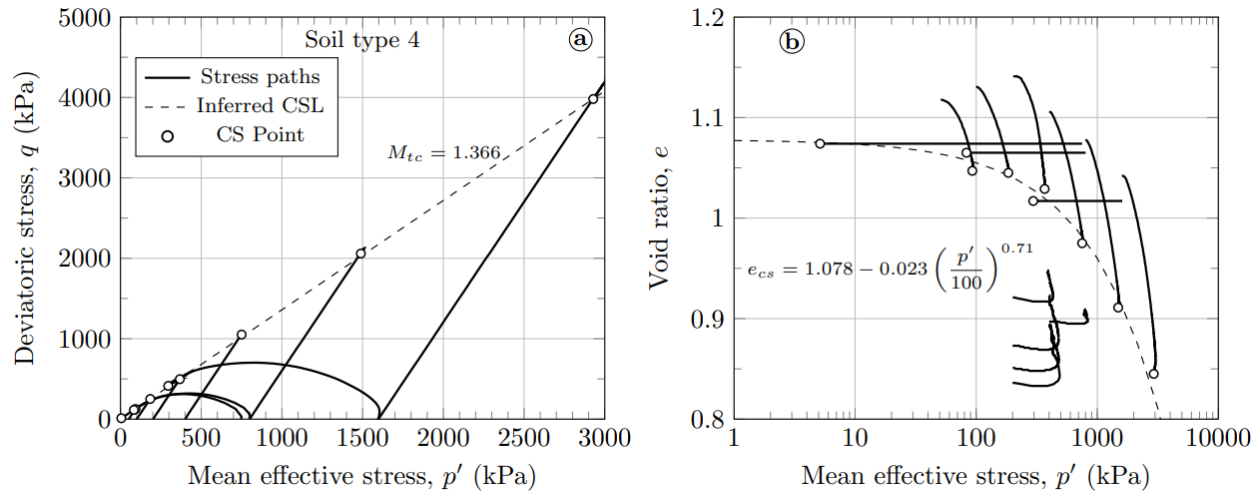


Figure 27 – Triaxial tests results and inferred CSL Soil type 4: (a) stress paths, (b) state diagram.

The gradient of the critical state line on the  $e - \log p'$  space shows in this case a very significant curvature. Several authors have explained the curvature in the CSL of granular soils by reference to either to flow instabilities (at the specimen scale) at low-stress states and/or to shear induced particle evolution (size and/or shape) at higher-stress states (Carrera et al., 2011; Bedin et al., 2012; Soares & Viana Da Fonseca, 2016).

Carrera introduce the terminology true liquefaction to describe triaxial behaviour in which  $p'$  and therefore  $q$  is observed to attain values close to zero at critical state, whereas the term strain softening was reserved for stress paths attaining critical states with non-zero effective stress (Carrera et al., 2011). PSD tests were performed after triaxial tests, and no appreciable particle breakage can be observed when compared to the original soil. Indeed, the CSL of both tailing for pressure above 200 kPa remains straight. The curvature of critical state line at low stresses was rather caused by undrained triaxial tests at high void ratios. For soil type 3, two undrained triaxial tests consolidated to 800 and 1200 kPa underwent true liquefaction, whereas in soil type 4, this behaviour was not observed and undrained tests consolidated to 750, 800 and 1600 kPa exhibited moderate to considerable strain softening.

Furthermore, undrained tests may occasionally fail to reach the critical state. This is illustrated in Figure 26b, where the test consolidated at a mean effective stress of 1200 kPa (as loosely as possible) do not intersect the CSL, resulting in complete liquefaction. This means that in the

laboratory, it is possible to achieve void ratios that exceed the maximum critical void ratio defined by the CSL. This issue is particularly pronounced for a highly curved CSL, as observed with soil type 4, where it is likely that undrained tests conducted with the loosest possible packing and a mean effective stress below 750 kPa will result in complete liquefaction.

### 5.7.2.3. Data Synthesis

For a linear idealization of the CSL, parameters  $M_{tc}$ ,  $\lambda$ ,  $\Gamma$  are presented in Table 4 along with coefficients A, B, C when the CSL exhibits curvature. Moreover, the index parameters  $FS$  and  $G_s$  are included.

Table 4 Inex tailings properties and CSL parameters for Soil type 1 to 4

| Name        | $G_s$ | $k$ (m/s)         | FC | $M_{tc}$ | $\lambda$                | $\Gamma$ | $R^2$ |
|-------------|-------|-------------------|----|----------|--------------------------|----------|-------|
| Soil type 1 | 3.94  | $5 \cdot 10^{-6}$ | 85 | 1.42     | 0.056                    | 1.173    | 0.99  |
| Soil type 2 | 4.55  | $1 \cdot 10^{-6}$ | 91 | 1.40     | 0.050                    | 1.060    | 0.98  |
| Soil type 3 | 5.00  | $1 \cdot 10^{-5}$ | 71 | 1.36     | 0.050                    | 1.373    | 0.98* |
|             |       |                   |    |          | A=1.137; B=0.024; C=0.62 |          | 0.97  |
| Soil type 4 | 4.99  | $1 \cdot 10^{-5}$ | 36 | 1.37     | 0.073                    | 1.441    | 0.96* |
|             |       |                   |    |          | A=1.078; B=0.023; C=0.71 |          | 0.99  |

\* These R-squared values were calculated based on tests that reached the critical state condition at mean effective stresses exceeding 100 kPa.

To synthesize the results of this section, the four CSLs investigated are illustrated in Figure 28. In terms of the critical friction ratio  $M_{tc}$ , the variability is minimal, being nearly identical for the first two soil types. Notably, although soil type 3 has a high fine content (71%), the  $M_{tc}$  of soil type 4 is nearly identical, even though its fine content is considerably lower (36%).

In Figure 28b, the CSL of soil types 1 and 2 follows a linear relationship, while the CSL of soil types 3 and 4 can be idealized as curved. Notably, the materials with a CSL that can be approximated

by a power law function have the lowest fine content and the highest specific gravity among the four soil types analyzed. Smith et al. (2021) investigated the CSL of 29 different tailings and found that only four exhibit a curved CSL. The fine content of these four tailings ranges from 3% to 54%, positioning them at the lower end of the spectrum among the 29 tailings studied.

To describe the locus of the critical state in the compression plane for soil types 1 and 2, a semi-log linearization fits the laboratory data well across the investigated pressure range. In fact, the R-squared value of the fitted expression is close to unity. For soil types 3 and 4, the best fit is achieved with a power law equation, as the R-squared values are also close to unity. Additionally, a linear trend for the CSL can be observed for these soil types when considering a pressure range (at the critical state) exceeding 100 kPa.

Figure 28b also includes the results of a one-dimensional consolidation test and an isotropically consolidated triaxial test. It can be observed that when the specimen void ratio at the beginning of consolidation is close to the critical void ratio, the CSL runs almost parallel to the oedometer curve. Specifically, in Figure 28b, the compression curve for soil type 4, with an initial void ratio  $e_0$  of 1.1, lies parallel to the CSL. A similar observation is also shown for soil type 3. Although this observation warrants further investigation, it may be interesting because it provides a preliminary approach to the CSL at low stress levels, which is crucial for assessing the risk of liquefaction (Carrera et al., 2011).

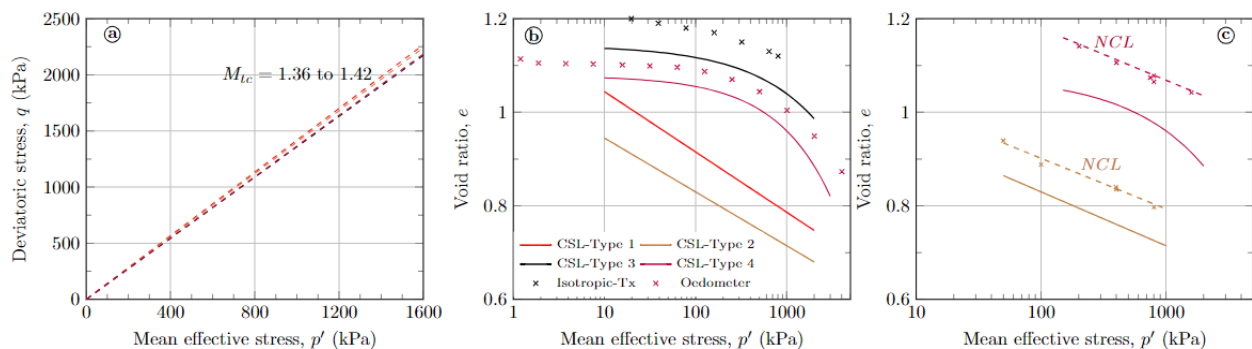


Figure 28 – Critical state line idealization for soil type 1, 2, 3 and 4.

Figure 28c shows a comparison of the CSLs and NCLs for soil types 2 and 4, which have the lowest and highest fine content, respectively. The NCL and CSL tend to be parallel in the finer material. A similar trend was also observed by Macedo & Vergaray (2022).

## 5.8. Summary

Laboratory tests on tailings from Dam B1 in Brumadinho have shown considerable variability in particle size distribution and specific gravity. The four non-plastic materials tested displayed a fines content ranging from 36% to 91% and specific gravities between 3.94 and 5.00. Measurements of the elastic shear modulus at various mean effective stress levels indicated a broad spectrum of results, which aligns with values found in previous investigation on tailings (Macedo & Vergaray, 2022; Shuttle & Jefferies, 2016).  $G_0$  was shown to be influenced by both the mean effective stress and the void ratio, and this soil behavior can be accurately captured using Equation (34).

In terms of critical state parameters, laboratory data show two different patterns. Soil types 1 and 2 exhibit a CSL in the  $e-\ln(p')$  plane that can be represented as linear, while soil types 2 and 3 present a pronounced curvature. The critical friction ratio was found to range from 1.36 to 1.42, so the values vary slightly despite differences in fines content among the soils investigated. This is probably due to the common origin of the material investigated. To assess the compressibility the testing program includes a series of one-dimensional and isotropic consolidation tests. The elastic modulus  $k$ , evaluated through a series of loading and unloading steps, ranges from 0.003 to 0.009. The results show that  $k$  increases when unloading occurs at higher stress levels. This trend suggests potential changes in the soil's morphology, particularly in terms of particle shape or size. Finally, direct measurements of hydraulic soil conductivity were taken at four different stages of isotropic consolidation. It was found that its variability spans two orders of magnitude, ranging from  $10^{-7}$  to  $10^{-5}$  m/s. Interestingly, the KC equation accurately predicts the permeability of the finer tailings but falls short in estimating the permeability of coarser materials.



## **6. Exploring CASM performance for tailings**

### **6.1. Introduction**

The aim of calibrating a constitutive model for a specific soil is to obtain a consistent set of material parameters that adequately represent the soil behavior across a range of pressures and states. Therefore, goodness of fit for any individual test is not so important as achieving a more accurate representation of the overall soil response.

This chapter introduces the constitutive model parameters and the procedures used to derive them. It begins with an overview of the parameters that can be estimated through conventional laboratory tests, which are the critical state and elasticity parameters. Then, it explores the various methods used to assess specific soil parameters,  $n$  and  $r$ , and discusses how these parameters are connected to the state parameter and undrained shear strength.

Following this, CASM is calibrated using the results of triaxial tests on the Brumadinho Dam B1 dataset, as reported in Chapter 5. Comparisons of simulated results and laboratory data for each soil type are presented in terms of stress-strain response, stress path, volumetric strain, and state diagrams, including the inferred NCL and CSL.

### **6.2. CASM implementation**

The CASM model has been selected for the subsequent analysis not only for its capability to accurately represent a broad spectrum of soil behaviors but also because it has been implemented into the small-strain finite element code PLAXIS (Manica et al., 2021) and into the large strain particle finite element code G-PFEM (Carbonell et al., 2022). Additionally, these implementations have been successfully applied in simulating relevant geotechnical problem such as tailings dam stability (PLAXIS) and the cone penetration test (Monforte et al., 2021).

In this Thesis we have worked with CASM using both codes. The G-PFEM implementation, in conjunction with MATLAB, was employed for the calibration and element test simulation work described in this chapter; while the PLAXIS implementation, using python scripting, was selected

for the numerical simulations of tailings dam described in chapter 8. It was verified that both CASM implementations produced very similar results.

As an example, an isotropic, normally consolidated undrained triaxial compression test is presented in Figure 29, showing a perfect match.

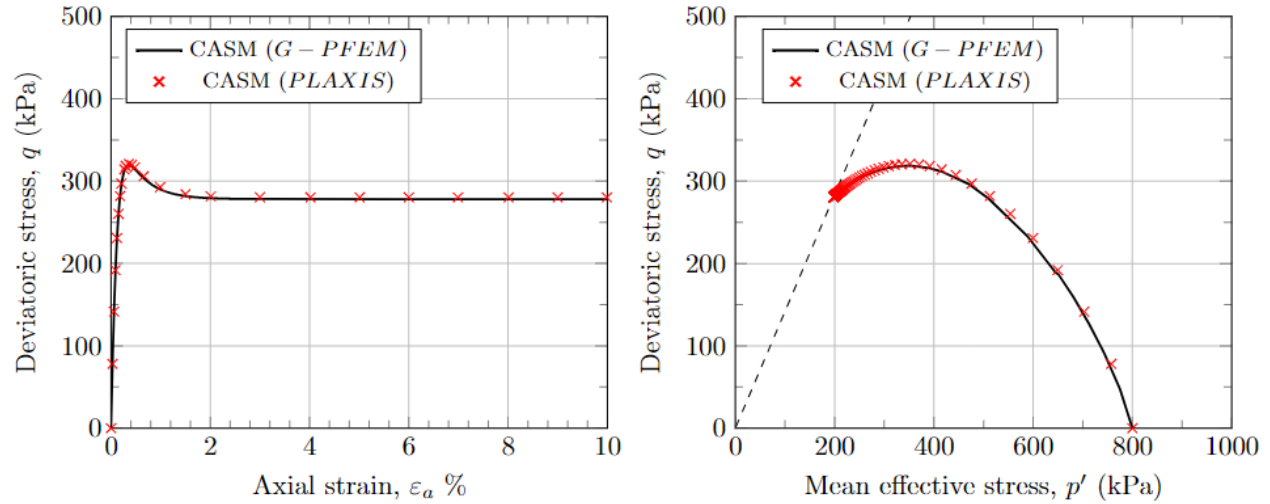


Figure 29 – Simulation of an undrained isotropic triaxial test using PLAXIS and G-PFEM.

### 6.3. CASM calibration procedure

#### 6.3.1. Critical state parameter: $M$ , $\lambda$ and $\Gamma$

The critical state parameters  $M$ ,  $\lambda$  and  $\Gamma$  are common to most elasto-plastic models such as MCC, NorSand or CASM. As shown in chapter 5, these soil properties are usually determined by triaxial compression tests on loose samples.  $\lambda$  and  $\Gamma$  represent, respectively, the slope and intercept (at a reference pressure of 1 kPa) of the CSL in  $v - \ln p'$  space, whereas  $M$  is the stress ratio at critical state. It is related to the critical state friction angle as follows:

$$M = \frac{6 \sin \varphi'}{3 - \sin \varphi'} \quad (38)$$

Originally, CASM was implemented with a constant slope of the CSL (YU). In the version of CASM used in this thesis,  $M$  is expressed as a function of the Lode angle  $\theta$ , influencing the shape of the

failure surface in the deviatoric plane. The relationship between  $M$  and  $\theta$  employed here, as proposed by (Sheng et al., 2000), is as follows:

$$M_{\theta} = M \left[ \frac{2\alpha^4}{1 + \alpha^4 + (1 - \alpha^4) \sin 3\theta} \right]^{1/4} \quad (39)$$

where  $\alpha$  is a parameter controlling the shape of the function as well as the difference of  $M$  between triaxial compression and extension.

A frequently applied way to select  $M$  values is by plotting the stress path of loosed drained and undrained triaxial compression tests in the  $q - p'$  plane. The average of all the stress ratio  $\eta$  ( $q/p'$ ) at critical state is selected as  $M$ . This procedure is commonly referred to as “*End of Test Method*” (Ghafghazi & Shuttle, 2005).

However, there are alternative procedures to obtain the critical state friction ratio from triaxial tests.  $M$  can be extrapolated using the stress-dilatancy method, as referenced in Ghafghazi & Shuttle (2005), by plotting the stress ratio versus dilatancy of a drained test. At the critical state, where there is no volumetric plastic deformation, the stress ratio at  $D = 0$  equals  $M$ . This is evident in Figure 30a, where the stress paths for some loose specimens are illustrated. Theoretically, it would be more appropriate to use the plastic dilatancy instead of the total dilatancy. However, the difference between plastic and total dilatancy at the critical state and at the peak is very small as elastic strain also are small compared to plastic strains. Thus, it is assumed  $D = D_p$ .

Alternatively, Bishop (1971) suggested that critical friction ratio could also be obtained using results from several dilative drained tests. Figure 30b shows the single value of the stress ratio  $\eta_{max}$  measured when the sample reach the minimum dilatancy  $D_{min}$  of each individual test. Using this plot  $M$  is then determined by extrapolating to zero dilatancy the trend-line through all the tests.

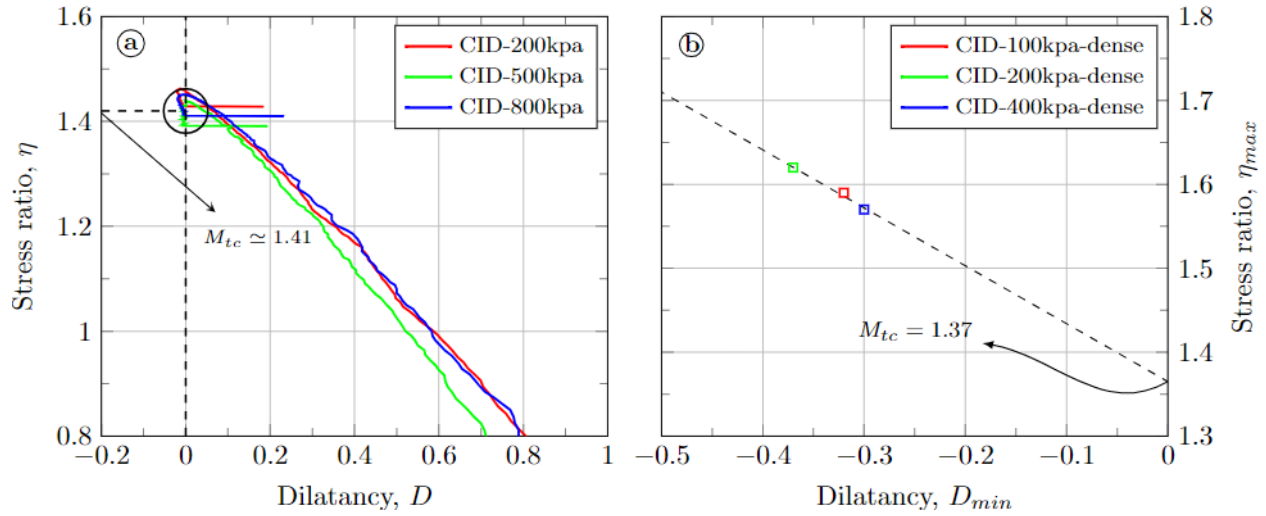


Figure 30 – Application of stress-dilatancy methods and Bishop method to estimate  $M_{tc}$  in soil type 1

The results obtained using the stress-dilatancy method are consistent with those from the End of Test Method, which included both drained and undrained tests. Specifically, the critical friction ratio is 1.42 with the stress-dilatancy method (see Table 4) and 1.41 with the End of Test Method.

For the example shown in Figure 30b, the critical friction ratio determined by Bishop method is slightly lower compared to that obtained by the End of Test methods. In general, the Bishop method faces difficulties if the minimum dilatancy of each test is fairly similar as this can result in a wrong estimation of the trendline.

### 6.3.2. Elasticity: $K$ and $\mu$

The bulk modulus  $K$ , dependent on mean effective stress  $p'$  (equation (7)), is usually obtained using the swelling index  $k$  defined in the semi-logarithmic compression plane. In practice, it is calculated fitting an unload/reload cycle from oedometric tests or isotropic compression tests. The second elastic parameter, Poisson's ratio  $\mu$ , describes the relationship between radial and axial strain in a drained triaxial compression test. Due to the relatively low sensitivity of soil behavior to Poisson's ratio, this is not typically directly measured. Instead, it is often assumed to fall within the range of 0.15 to 0.35.

As discussed in Chapter 3, the elastic stiffness matrix can be defined either by a constant value of  $\mu$  or by using a constant shear modulus  $G_0$ . If  $G_0$  is selected as the input, then  $\mu$  will be determined based on  $G_0$ . In this case, the shear modulus  $G_0$  can be accurately measured in the laboratory using bender elements.

### 6.3.3. Parameters $n$ and $r$

CASM parameters include two,  $n$  and  $r$ , that control the shape and size of the yield surface. These parameters can be estimated from laboratory tests and may be related to several soil properties, such as the peak and residual undrained shear strength and the initial state parameter. The main links between different soil properties and parameters  $n$  and  $r$  are discussed in this section.

#### Insights from Yu (1998)

Parameter  $r$  simply represents the distance between the NCL and the CSL expressed in terms of a reference state parameter:

$$\psi_r = (\lambda - k) \ln r \quad (40)$$

where:

$$r = e^{\left(\frac{\psi_r}{\lambda - k}\right)} \quad (41)$$

According to Yu (1998), experimental data indicates that for clays,  $r$  typically lies in the range of approximately 1.5 – 3.0, while for sands, the value of  $r$  is generally much larger. Since for clays the NCL is easily located, there are no difficulties to determine  $r$ . In contrast, locating the NCL for sands is more challenging, as it requires test device capable of applying very high pressures.

To identify  $n$ , Yu (1998) suggests plotting the stress paths of drained and undrained triaxial tests with different initial conditions in term of stress ratio  $\eta$  against the state parameter  $\psi$ . Using the general stress strain relation adopted in CASM, the parameter  $n$  may be link to a straight line in the plot of  $\ln (1 - (\psi/\psi_r))$  vs  $\ln (\eta/M)$ . In this specific log-log plot, the stress-state coefficient  $n$  is identified by the slope of the state boundary surface.

### Manica et al. (2021) approach

Mánica et al. (2021) proposed a calibration procedure for normally consolidated materials linking  $n$  and  $r$  to the undrained peak shear strength  $S_p$  and the initial state of the material  $\psi$ . It was hypothesized that when the undrained peak strength is reached in triaxial undrained compression, the stress path is at the highest point on the yield surface (see Figure 31).

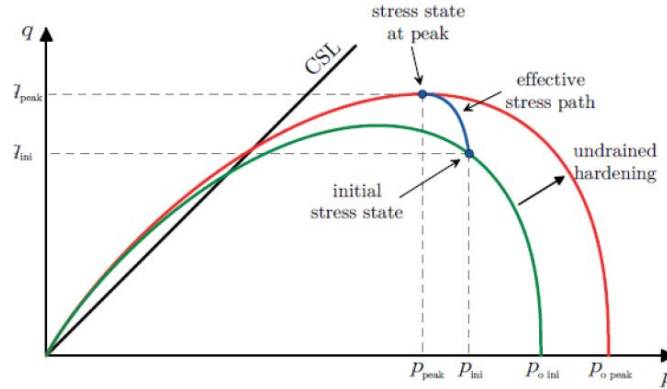


Figure 31 – Hardening during undrained loading from Mánica et al. (2021).

An analytical expression for the highest point on the yield surface may be obtained considering it as a maximum and working out the derivative expression, as follows:

$$f = \left( \frac{q}{p' M_\theta} \right)^n + \frac{1}{\ln r} \ln \frac{p'}{p'_0} \quad (42)$$

$$f = \left( \frac{1}{p'} \right)^n \left( \frac{q}{M_\theta} \right)^n + \frac{1}{\ln r} \ln \frac{p'}{p'_0} \quad (43)$$

$$f'_{(p')} = -n(p')^{(-n-1)} \left( \frac{q}{M_\theta} \right)^n + \frac{1}{p' \ln r} = 0 \quad (44)$$

Substituting the peak point coordinate into the general expression of the derivative, the result is:

$$f'_{(pk)} = -n(p'_{pk})^{-n-1} \left( \frac{q_{pk}}{M_\vartheta} \right)^n + \frac{1}{p'_{pk} \ln r} = 0 \quad (45)$$

From which the  $q_{pk}$  can be written as:

$$q_{pk} = p'_{pk} \frac{M_\vartheta}{\sqrt[n]{n \ln r}} \quad (46)$$

Thus, the stress ratio at this point is given by:

$$\eta_{in} = \frac{M_\vartheta}{\sqrt[n]{n \ln r}} \quad (47)$$

Employing the definition of the undrained peak strength ratio  $S_p$ , which is the ratio of the undrained peak strength to the mean effective stress at consolidation, the deviatoric stress at the peak can be written as:

$$q_{pk} = 2S_p p'_{in} \quad (48)$$

From this, the mean effective consolidation pressure can be dexpressed as:

$$p'_{in} = \frac{q_{pk}}{2S_p} \quad (49)$$

When the material is normally consolidated, from the yield surface (equation (42)) the initial value of preconsolidation pressure  $p'_{0in}$  is directly related with the initial mean stress by:

$$p'_{0in} = p'_{in} e^{\left[ \left( \frac{\eta_{in}}{M_\vartheta} \right)^n \ln r \right]} \quad (50)$$

A similar relation to Equation (50) can be written for peak strength condition:

$$p'_{0pk} = p'_{pk} e^{\left[\left(\frac{\eta_{pk}}{M_{\vartheta}}\right)^n \ln r\right]} \quad (51)$$

Because the total volume changes during an undrained loading are nil, elastic and plastic volumetric deformation are equivalent but with opposite sign:

$$(\lambda - k) \ln \frac{p'_{0pk}}{p'_{oin}} = -k \ln \frac{p'_{pk}}{p'_{in}} \quad (52)$$

Note that equation (52) is identical for CASM and Cam Clay models. Finally, substituting equations (47), (49), (50) and (51) into equation (52) gives the relation between  $S_p$  and the parameters  $n$  and  $r$ :

$$e^{\left[\left(\frac{1}{\sqrt[n]{n \ln r}}\right)^n \ln r\right]} = \frac{M_{\vartheta}}{2S_p \sqrt[n]{n \ln r}} e^{\left[\left(\frac{\eta_{in}}{M_{\vartheta}}\right)^n \ln r\right]} e^{\left[\frac{k}{\lambda - k} \ln \left(\frac{M_{\vartheta}}{2S_p \sqrt[n]{n \ln r}}\right)\right]} \quad (53)$$

Another equation is required to determine the two parameters  $n$  and  $r$ . For this purpose, the yield surface (equation (42)) can be described using the state parameter  $\psi$  as follow:

$$f = \left(\frac{q}{p' M_{\vartheta}}\right)^n + \frac{\psi}{\psi_r} - 1 \quad (54)$$

Now the second relation between  $\psi$  and the parameters  $n$  and  $r$  is obtained by substituting in the previous expression the reference state parameter as defined in equation (40), to obtain:

$$\psi = (\lambda - k) \ln r \left[1 - \left(\frac{\eta}{M_{\vartheta}}\right)^n\right] \quad (55)$$

### Other useful relation between $r$ and $S_{u,res}$

It is possible to establish an additional relation by introducing the over-consolidation ratio  $R$  and the residual undrained strength, i.e., when the critical state is achieved. Further detail of this relationship can be found in Arroyo & Gens (2021):

$$r = R \left( \frac{M}{2} \frac{p'}{S_{u,res}} \right)^{\frac{1}{\Lambda}} = R \left[ \frac{M}{6} (1 + 2K_0) \frac{\sigma'_v}{S_{u,res}} \right]^{\frac{1}{\Lambda}} \quad (56)$$

Finally, equation (56) can be employed to estimate  $r$  whereas through equation (53) or (55) the parameter  $n$  can be calculated. However, it should be noted that equation (56) is only valid if shearing starts from a normally consolidated state.

#### 6.3.4. Plastic potential: $m$

The parameter  $m$  is a material constant controlling the plastic flow ratio  $m$  can be estimated noting that, during oedometric conditions, the flow rule should predict a stress ratio consistent with Jaky's expression. Jaky's proposal for  $K_0$  reads:

$$K_0 = 1 - \sin \varphi = \frac{6-2M}{6+M} \quad (57)$$

In  $p - q'$  space,  $K_0$  condition induces the following expression in stress ratio:

$$\eta_{k_0} = \frac{3(1-K_0)}{1+2K_0} = \frac{3M}{6-M} \quad (58)$$

In oedometric conditions, since the volumetric deformation equals the axial one, the relation between volumetric and deviatoric deformation is given by:

$$\frac{\varepsilon_v}{\varepsilon_q} = \frac{\varepsilon_a}{\frac{2}{3}\varepsilon_a} = \frac{3}{2} \quad (59)$$

Neglecting the elastic shear strain but considering volumetric elastic strains:

$$\dot{\varepsilon}_q^p \simeq \dot{\varepsilon}_q = \frac{2}{3} \dot{\varepsilon}_v = \frac{2}{3} \lambda \frac{\dot{p}}{p} \quad (60)$$

The dilatancy ratio under  $K_0$  condition is defined as:

$$D_{k_0} = \frac{d\varepsilon_v^p}{d\varepsilon_q^p} = \frac{3}{2} \frac{\lambda - k}{\lambda} = \frac{3}{2} \Lambda \quad (61)$$

Applying the flow rule (equation (15)), developed under the  $K_0$  condition, results in the following expression:

$$\frac{3}{2} \Lambda = \left[ \frac{M}{m} - M \right] \left[ \frac{\eta_{k0}}{M} \right] + \left[ M - \frac{M}{m} \right] \left[ \frac{\eta_{k0}}{M} \right]^{1-m} \quad (62)$$

Therefore, the value of  $m$  can be obtained solving equation (62), once parameters  $M$  and  $\Lambda$  are known. As show in Figure 32, it is also recommended to plot some drained triaxial tests in terms of stress ratio versus dilatancy to inspect the fit between laboratory tests and the flow rule adopted (equation (14) and (15)). If needed, the value of  $m$  can be slightly modified to get a better match to the measured data.

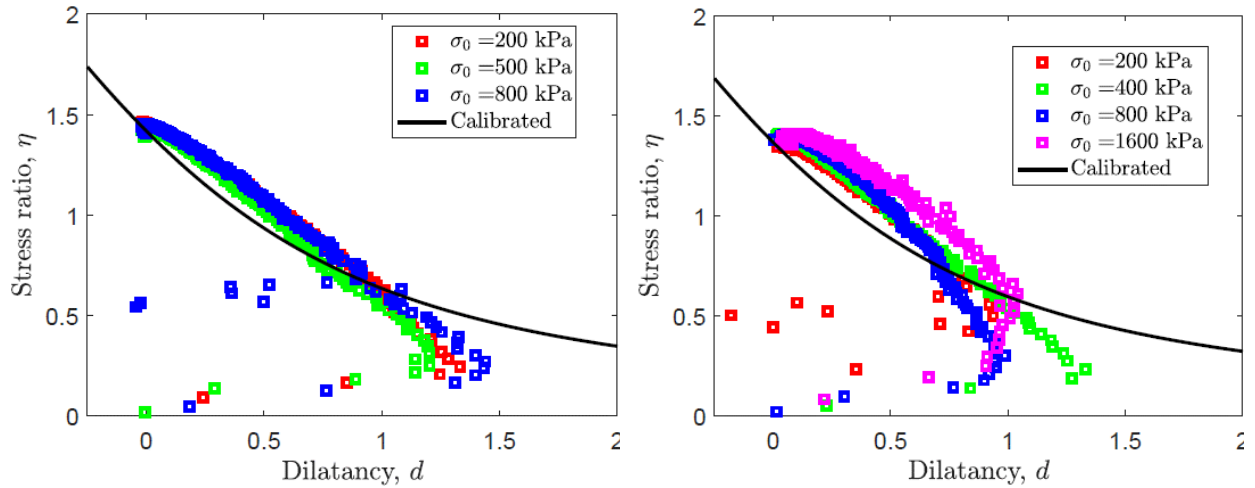


Figure 32 – Comparison of stress-dilatancy data to CASM flow rule (a) Soil type 1; (b) Soil type 4.

### 6.3.5. Summary of CASM calibration formulas

Table 5 summarizes some of the expressions presented above that are useful for the calibration of  $n$  and  $r$  based on the residual undrained shear strength, peak undrained shear strength and the state parameter. These relations particularized considering a) the cases of isotropic or anisotropic consolidation paths, and b) if shearing starts from a normally consolidated or an overconsolidated state.

Table 5 List of monotonic triaxial tests.

| Parameters  | Isotropic                                         |                                                   | Anisotropic                                                            |                                                           |
|-------------|---------------------------------------------------|---------------------------------------------------|------------------------------------------------------------------------|-----------------------------------------------------------|
|             | NC                                                | OC                                                | NC                                                                     | OC                                                        |
| $S_{u,res}$ | $\frac{M}{2} p' \left(\frac{1}{r}\right)^\Lambda$ | $\frac{M}{2} p' \left(\frac{R}{r}\right)^\Lambda$ | -                                                                      | $\frac{M}{2} p' e^{\left(-\frac{\psi_0}{\lambda}\right)}$ |
| $S_p$       | Equation (53)                                     | -                                                 | Equation (53)                                                          | -                                                         |
| $\psi$      | $(\lambda - k) \ln r$                             | -                                                 | $(\lambda - k) \ln r \left[1 - \left(\frac{\eta}{M_g}\right)^n\right]$ | -                                                         |

## 6.4. Application of CASM to dam B1 datasets

### 6.4.1. Summary

Calibration is an important step in the development of a soil model because it allows the model to be adjusted to represent the behaviour of a particular soil. To do this, the actual stress-strain curve is compared to a modelled curve that uses the material parameters as inputs. The calibration process is iterative, with the material parameters adjusted until the modelled curve closely matches the actual curve. This procedure is repeated until the material parameters yield a satisfactory match.

To evaluate the performance of CASM for tailings, the model has been calibrated using the specific set of triaxial tests outlined in Chapter 5. Since tailing are usually found in a normally

consolidated condition, most of these tests were conducted with samples prepared as loosely as practicable, using the moist tamping technique. In this case, the model calibration relied solely on laboratory test results. To further investigate the performance of CASM with dilative materials, two triaxial specimens prepared in a dense state were also simulated.

The CIMNE report (Arroyo & Gens, 2021) also addressed the calibration of CASM for Brumadinho Dam B1 material. The report presented two key aspects. Firstly, a comprehensive set of CASM parameters was inferred, not only for the reference tailings tested in the laboratory, but also considering the peak strength ratio and state parameter value estimated by the CPTu test. Thanks to these soil parameters, it is possible to determine the specific CASM parameters  $n$  and  $r$ . Furthermore, the report included a series of anisotropically consolidated undrained triaxial tests to explore how strain rates influence the behavior of tailing materials, employing CASM-visco for laboratory tests simulations.

The subsequent sections report the simulations performed and details the selection of parameters for each material tested.

#### **6.4.2. CASM calibration for soil types 1 and 2**

Experimental results from drained and undrained isotropically consolidated triaxial tests were used to calibrate CASM model. For soil types 1 and 2, a linear critical state line is approximately correct, as shown in Figure 24 and Figure 25 in Chapter 5. The model assigns a unique initial state parameter for all tests in the consolidation line; this is 0.09 for soil type 1 and 0.07 for soil type 2. Figure 33 and Figure 34 show that the tests employed for calibration involve only contractive samples. These figures also compare the experimental data and model predictions across various aspects of soil behavior: stress-strain, state diagrams, axial strain versus deviatoric stress and volumetric strain for drained tests, alongside axial strain versus deviatoric stress and excess pore pressure for undrained tests.

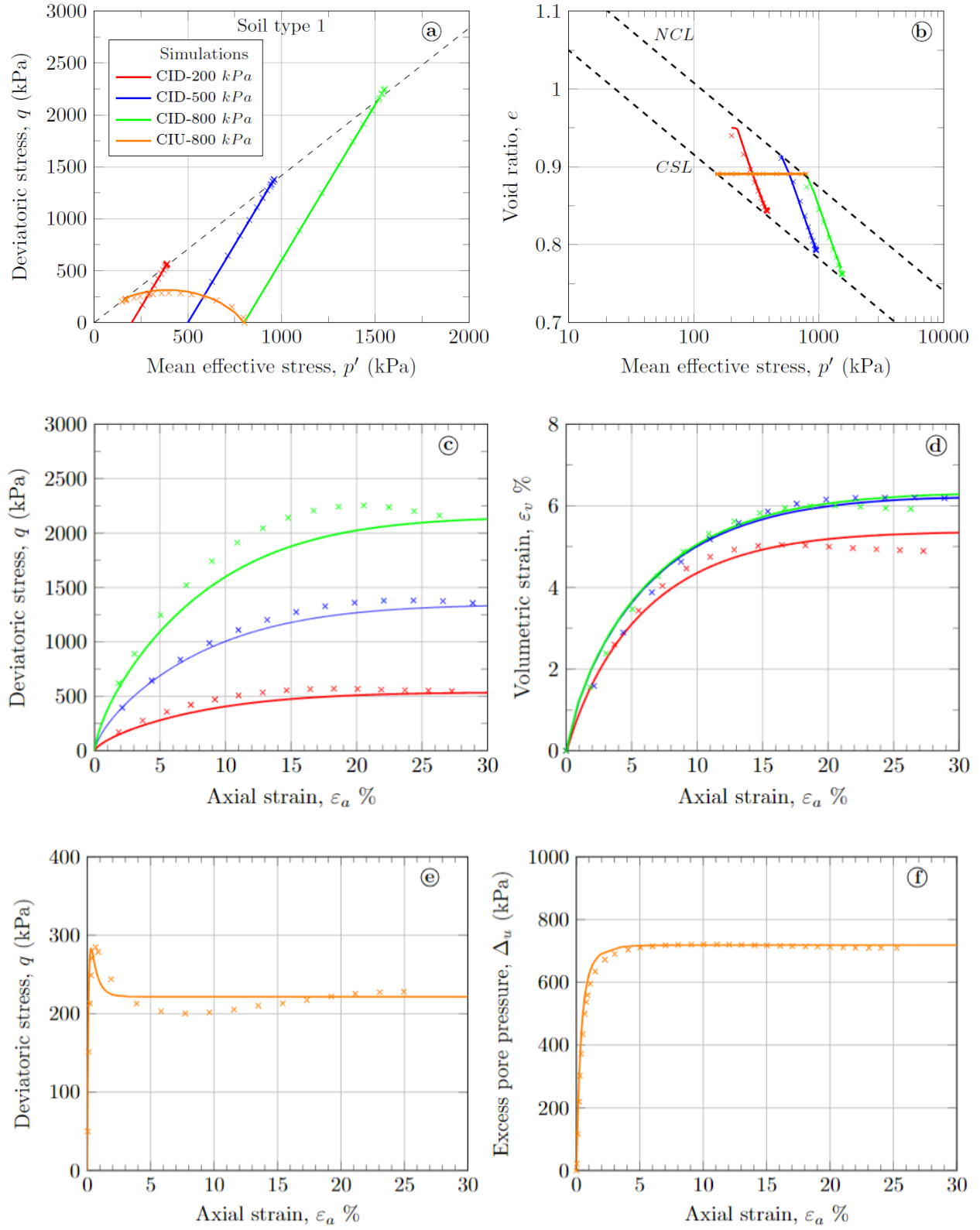


Figure 33 – Comparison of simulated results (continuous line) with laboratory data (cross marks) for Soil Type 1.

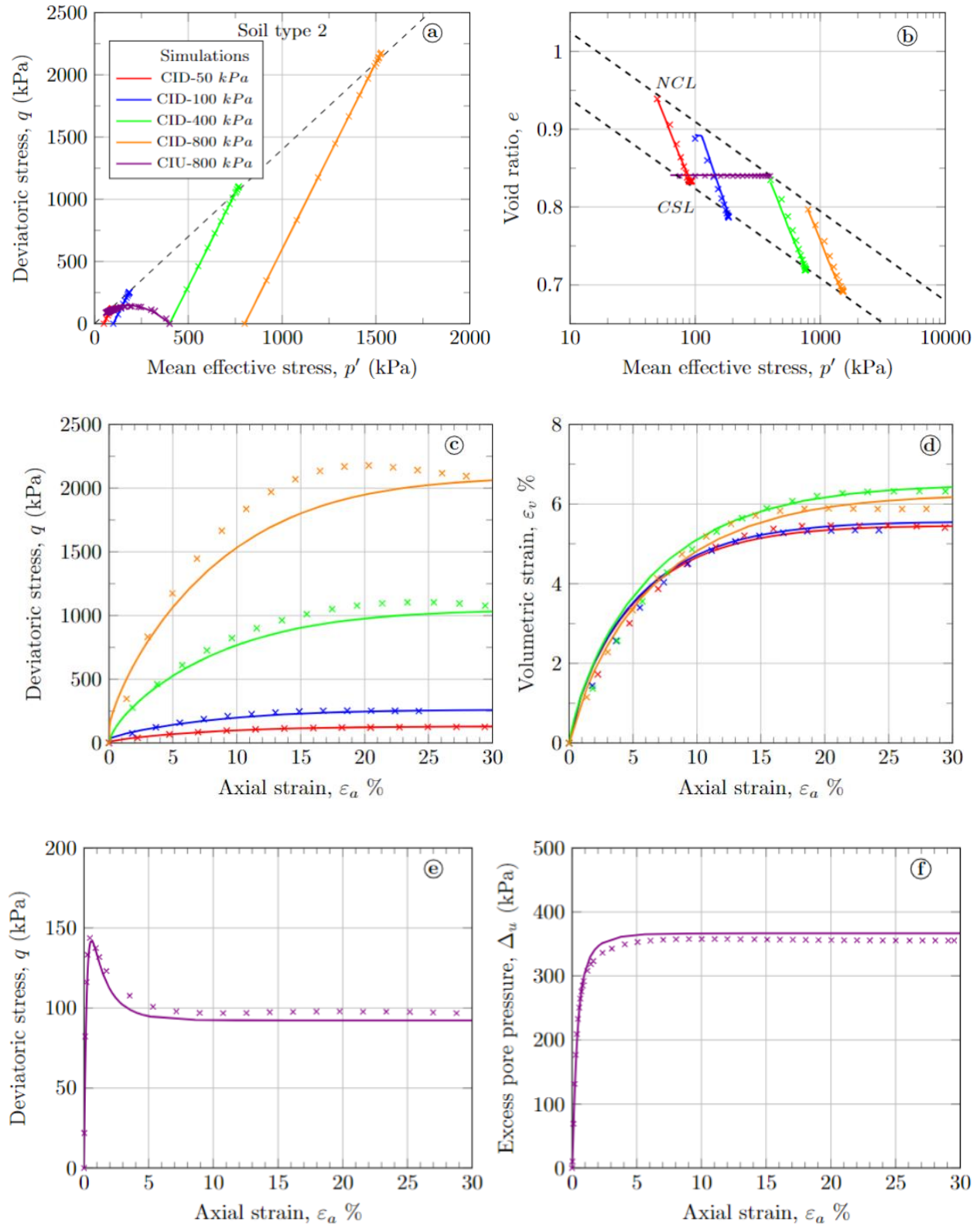


Figure 34 – Comparison of simulated results (continuous line) with laboratory data (cross marks) for Soil Type 2.

The response observed in the normally consolidated drained tests is accurately represented within the investigated pressure range. However, it is noticeable that for both soil types 1 and 2, the CID 800 kPa tests exhibit a slight peak in deviatoric stress before approaching the critical state condition, at an axial strain of about 20%. Laboratory tests were conducted using a fixed-top cap setup, as this method likely reduces significant tilting of the top cap, thereby minimizing peak deviatoric stress and subsequent strain softening (Reid et al., 2021). Additionally, lubricated end platens were used to minimize the development of non-homogeneous deformation (Bobei et al., 2013). Although this setup may lessen the likelihood of nonuniform stress-strain in triaxial samples, it does not entirely eliminate it. In fact, the effectiveness of these measures can vary based on several factors, including the characteristics of the lubricated end platens, stress levels, and the physical properties of the material (Zhang & Garga, 1997).

In these materials the undrained tests on loose specimens exhibit peak resistance at relatively small axial strains, quickly softening to the critical state, a behavior typical of soil undergoing static liquefaction. The CASM model captures this behavior reasonably well, as evidenced by the axial strain vs deviatoric stress simulations shown in Figure 33e and Figure 34e, successfully simulating both the peak and the residual shear strength. The results of the numerical analyses demonstrate also the capability of the critical state model in producing pore pressures that are consistent with laboratory observations. Consistent with laboratory results, simulations for drained tests achieve the critical state condition at an axial strain of about 30%, whereas for undrained tests, it is reached at approximately 5%. In terms of volumetric strain, the constitutive model accurately replicates it for both soil types 1 and 2.

Table 6 summarizes the parameters used for simulating soil types 1 and 2. The first five parameters were derived directly from laboratory test results, as detailed in Chapter 5. The shape parameters  $n$  and  $r$  were initially estimated using the procedure outlined by Mánica et al. (2021), as reported in Section 6.3.3. Following an initial estimation of these parameters, a trial-and-error approach was employed, slightly adjusting  $n$  and  $r$  to better fit simulation curves with experimental data. The over-consolidation ratio for all tests was set between 1 and 1.5,

depending on the distance of their end of consolidation point to the CSL. Indeed, as seen in Figure 33b and Figure 34b, the starting void ratio for all tests lies close to or on the NCL.

Table 6 Calibrated properties of soil type 1 and 2.

| PROPERTY    | Soil type 1 | Soil type 2 |
|-------------|-------------|-------------|
| $\Gamma^*$  | 1.173       | 1.060       |
| $\lambda_e$ | 0.056       | 0.050       |
| $M_{tc}$    | 1.42        | 1.40        |
| $k$         | 0.006       | 0.006       |
| $\nu$       | 0.3         | 0.3         |
| $n$         | 1.2         | 1.4         |
| $r$         | 6.3         | 8.2         |
| $m$         | 1.8         | 1.8         |
| $R$         | 1.0 to 1.5  | 1.0 to 1.5  |

\*  $\Gamma$  represents the void ratio where the CSL cross the 1 kPa pressure on the  $e - \ln p'$  plane.

#### 6.4.3. CASM calibration for soil types 3 and 4

This section discusses the application of CASM to represent the drained and undrained isotropically consolidated triaxial tests of soil types 3 and 4. For these materials, the laboratory tests cover a wider effective stress range than in the previous cases. The results indicate that the relationship between  $e - \log p'$  at the critical condition would be best represented by a curved line in the semilogarithmic compression plane (refer to Figure 24 and Figure 25). Additionally, the ICL appears to mirror the trend of the CSL. However, as currently formulated, the CASM model does not allow for a curved idealization of the CSL or ICL.

Despite this limitation, using the linear idealization of the CSL still provides satisfactory outcomes when simulating drained tests within a limited stress range. For soil type 3, reasonably approximate results are observed for specimens consolidated at confining pressures above 100 kPa, as illustrated in Figure 35b. For soil type 4, a similar effectiveness is achieved with pressures above 200 kPa, as shown in Figure 36b. Indeed, the simulations reach the same critical conditions defined by laboratory tests.

Figure 35c Figure 36c offers a comparative analysis between experimental data and model predictions, focusing on the relationship between axial strain and deviatoric stress. The CASM model simulates the hardening observed in normally consolidated tests, although a minor discrepancy is noted in tests consolidated at pressures above 800 kPa. The laboratory tests have higher soil stiffness. Ideally, replicating this stiffness in simulations could be achieved by simply increasing the elasticity parameter,  $k$ . However, this change impacts the overall behavior of the soil. This is due to the dependency of parameter  $r$  on  $k$ , as detailed in equation (41).

With the positions selected for the CSL and ICL the model cannot represent correctly the tests at lower confining pressures. This is visible for soil type 3, particularly in the test conducted at a confining pressure of 30 kPa. This problem is even more pronounced for soil type 4, as demonstrated in the state diagram of drained tests under 50 and 100 kPa confining pressures, shown in Figure 36a, and marked by teal and red crosses, respectively. With the assumed calibration whose parameters are reported in Table 7, the initial point of the experiments would be located below the CSL. This suggests a dilative soil behavior, contrary to what was observed in laboratory tests.

Regarding undrained tests, most intersect the CSL where it flattens, specifically when the consolidation pressure is below 1000 kPa. For undrained tests, the peak shear resistance is reached at a relatively small axial strain of approximately 0.35%, followed by strain softening. This is illustrated in Figure 35 and Figure 36 (e and f), which compare the experimental stress-strain relationships to the simulated ones. While the model accurately predicts peak conditions, the decrease in shear strength following the peak is more pronounced in laboratory tests than in the simulation. The CASM simulation is unable to capture the strong post-peak softening evident in the experimental response, although the pore pressure response is better approximated, particularly for Type 4. However, for specimens consolidated at pressures exceeding 1000 kPa, the linear idealization of the CSL is more effective. Specifically, for soil type 4, a better result is clearly achieved for the specimen consolidated at 1200 kPa.

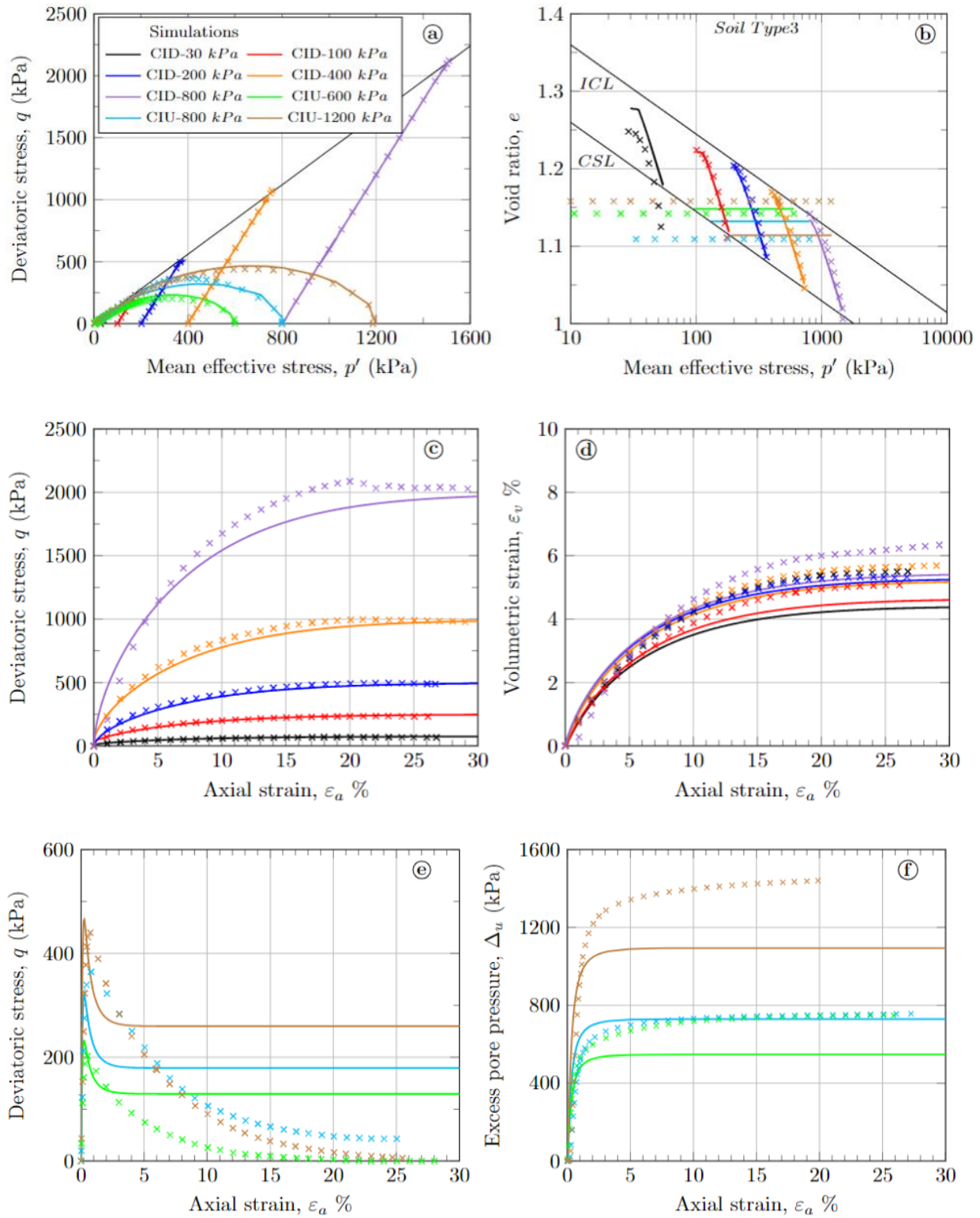


Figure 35 – Comparison of simulated results (continuous line) with laboratory data (cross marks) for Soil Type 3.

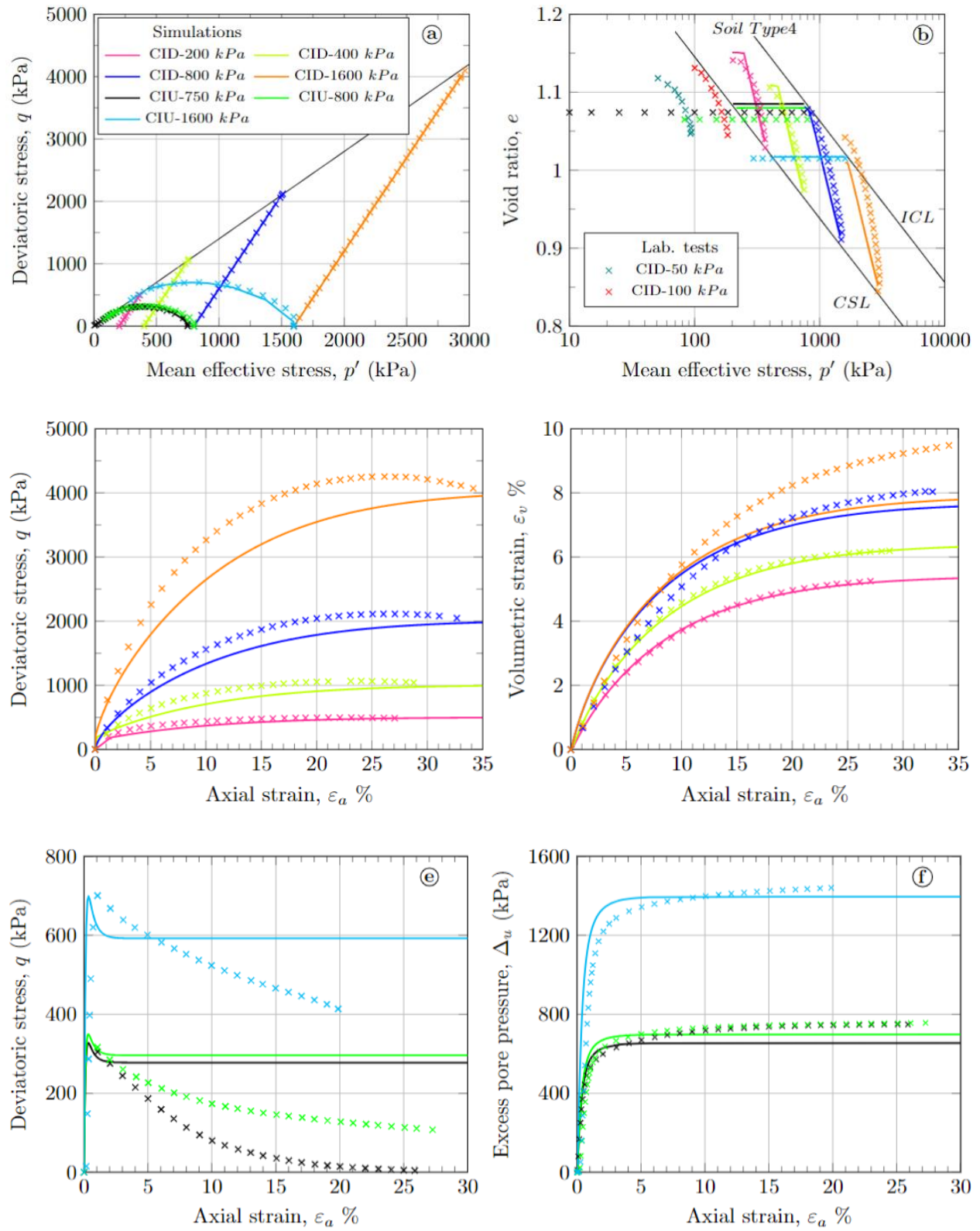


Figure 36 – Comparison of simulated results (continuous line) with laboratory data (cross marks) for Soil Type 4.

Table 7 Calibrated properties of soil type 3 and 4.

| PROPERTY    | Soil type 3 | Soil type 4 |
|-------------|-------------|-------------|
| $\Gamma^*$  | 1.373       | 1.060       |
| $\lambda_e$ | 0.05        | 0.051       |
| $M_{tc}$    | 1.36        | 1.37        |
| $k$         | 0.006       | 0.006       |
| $\nu$       | 0.3         | 0.3         |
| $n$         | 1.6         | 1.6         |
| $r$         | 8.0         | 4.8         |
| $m$         | 1.8         | 1.8         |
| $R$         | 1.0 to 1.5  | 1.0 to 1.5  |

\*  $\Gamma$  represents the void ratio where the CSL cross the 1 kPa pressure on the  $e - \ln p'$  plane.

In the undrained simulations, critical state is attained at higher mean effective stress than in the corresponding laboratory tests. To fit the simulation with experimental results using the assumed CSL, a possible work-around will be is to raise the isotropic consolidation line. This ensures that the simulations maintain the same initial stress as the experimental data. However, to match the initial void ratio, it is necessary to consider some specimens as over-consolidated. This method requires using a relatively high reference state parameter while keeping the CSL slope and the elasticity parameter  $k$  constant. As a result, the parameter  $r$  takes on large values.

Another approach that can be explored to improve fitting at low pressures is to change the position  $\Gamma$  and the slope  $\lambda$  of the CSL to accurately represent tests at lower confining pressures. In this scenario, the CSL is flat with  $\lambda$  set to 0.02. To maintain the same separation between the CSL and ICL, as in the previous simulation, the shape parameter  $r$  is set equal to 100. Additionally, the parameter  $n$  was set to 2.3 to achieve a good agreement between experimental and numerical results for undrained peak deviatoric stress. Table 8 summarizes the CASM parameters for this alternative calibration. Figure 37 shows the results of these adjustments, demonstrating a good fit in drained tests at low confining pressures. In undrained tests, the model predictions align well with the experimental data in terms of peak and residual deviatoric stress, as well as excess pore pressure. However, experimental results show more limited rate of softening after the maximum resistance is attained, meaning the rate at which deformations occur is much

slower. While these parameter sets may perform better at low pressures compared to those reported in Table 7, the results at higher pressures are unsatisfactory. A flattened CSL causes loose samples sheared under undrained conditions to behave as if they are overconsolidated, as their void ratios fall below the CSL.

Table 8 Alternative calibration parameters for low-pressure fit in soil type 4

| $\Gamma^*$ | $\lambda_e$ | $M_{tc}$ | $k$   | $\nu$ | $n$ | $r$ | $m$ | $R$        |
|------------|-------------|----------|-------|-------|-----|-----|-----|------------|
| 1.080      | 0.020       | 1.37     | 0.006 | 0.3   | 2.3 | 100 | 1.8 | 1.0 to 1.5 |

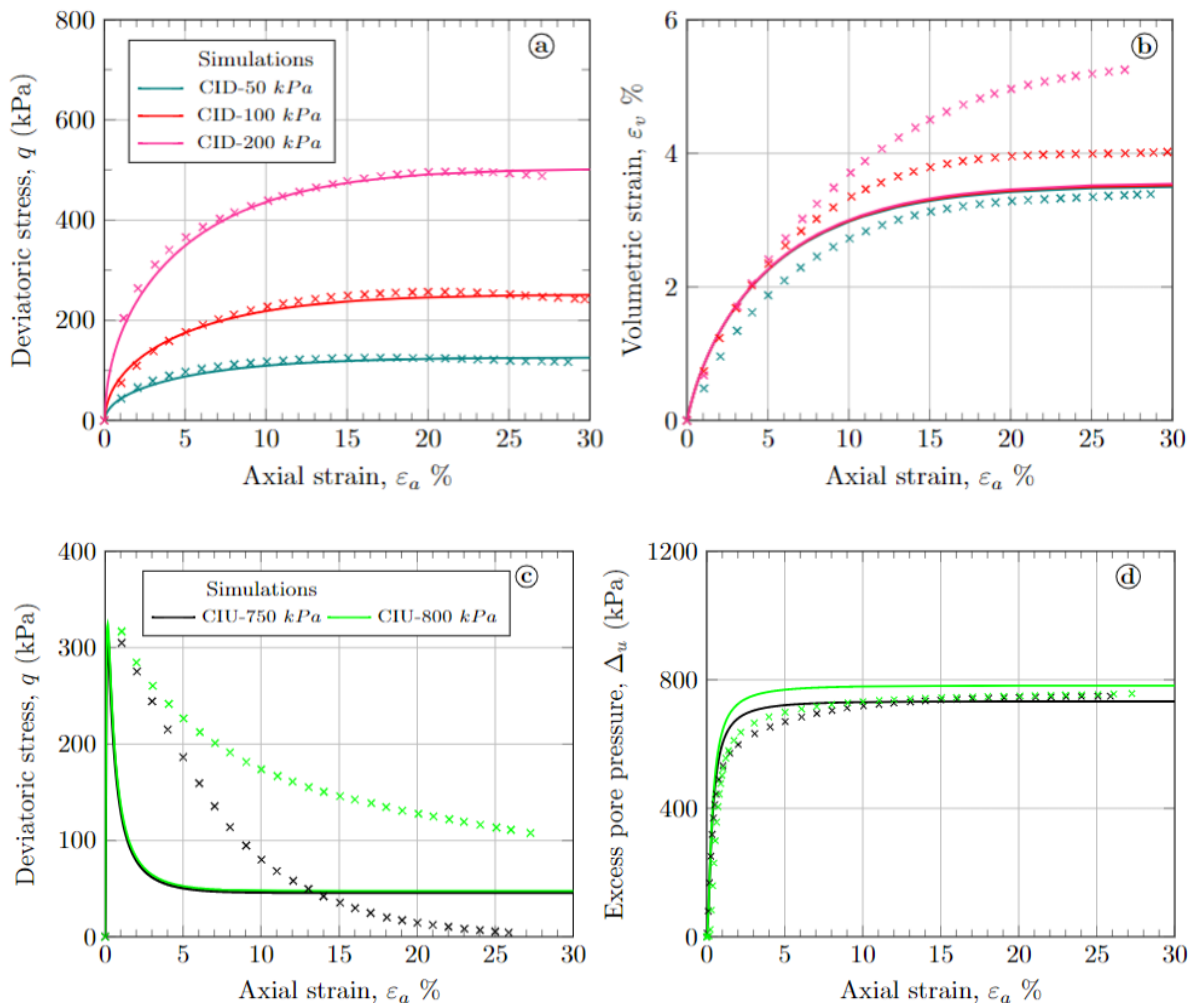


Figure 37 – Comparison of simulated results (continuous line) with laboratory data (cross marks) for low-pressure fit in soil type 4.

#### 6.4.4. Simulation of dilative triaxial sample

"The simulation of two triaxial tests conducted on overconsolidated materials were carried out to assess CASM performance for these conditions. The soil used in the tests was of type 1, and the model parameters are the same as those listed in Table 6, except for  $R$ , which was estimated to be 30 for the CID-100 kPa test and 25 for the CID-200 kPa test.

Figure 38 illustrates the result of heavily overconsolidated specimens. It should be noted that the initial stiffness predicted by CASM, in the elastic region, is satisfactory. However, the model tends to overpredict the peak resistance. Additionally, there is a discrepancy in the predicted volumetric strain, which may be caused by the laboratory specimens not reaching the critical state condition during shearing. It can be observed a lack of smooth transition from elastic to plastic behaviour on the stress-strain curve, this is because CASM does not allow any plastic deformation to develop within the state boundary surface. Such sharp change from elastic to plastic behaviour is far from the observed behavior of most soils.

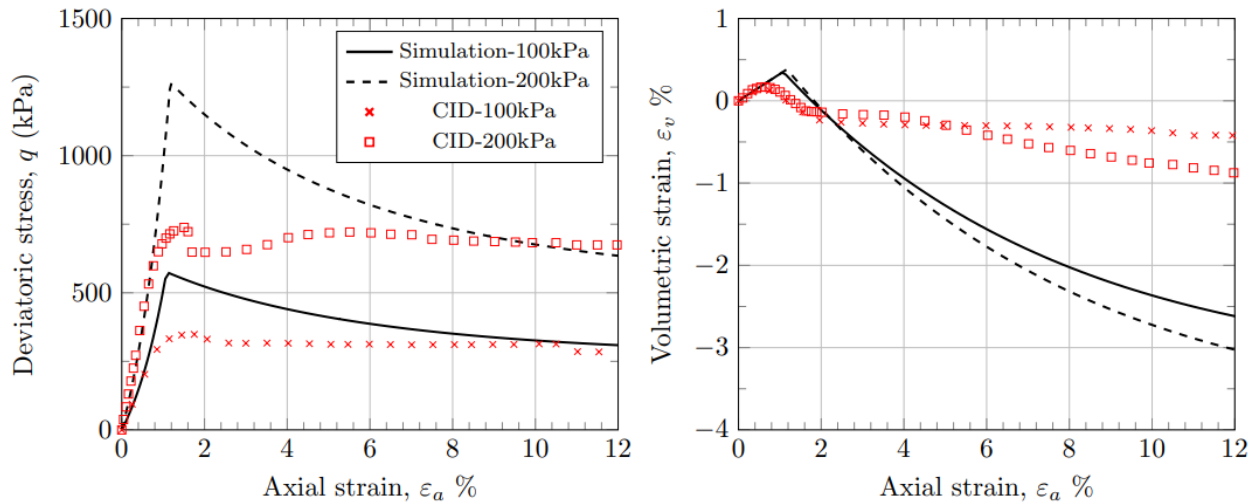


Figure 38 – Comparison of simulated results and measured laboratory data on soil type 1: (a) Stress-strain; (b) volumetric strain.

A possible explanation for these discrepancies is non-uniformity of stress and strain on the experiments. Dilative triaxial tests often end in localization planes during shearing, which leads

to non-uniform distribution of stresses and strains on the sample (Desrues et al., 1996). In fact, localized strains were observed in these dense samples, manifesting as localized bands typically during the initial increment of deviatoric stress. This mismatch between simulation and laboratory tests can be attributed to this effect.

## **6.5. Exploring the effect of consolidation on undrained instability**

### **6.5.1. Pre-consolidation path and the instability line: previous work**

Undrained triaxial tests on remolded samples are commonly used to investigate the liquefaction and instability behavior of tailings. These tests are characterized by a rapid decrease in the undrained stress path towards the critical state condition once the maximum deviatoric strength is reached. While the final state following undrained softening is reasonably well understood based on critical state considerations, there are fewer conceptual constraints regarding the point of maximum (peak) strength, which marks the onset of instability (Gens, 2019). This phenomenon can be described within the framework of the instability line or the undrained peak strength ratio.

The idea of an instability line was originally introduced by Lade (1992). In loose soil samples, there is typically a linear relationship between the peak deviatoric strength and the mean effective stress at the peak. This correlation enables the identification of the instability locus by drawing a straight line passing through the origin and the peak points of undrained stress paths. When the stress falls above the instability line, static liquefaction may be immediately triggered by any incremental undrained load. Alternatively, additional correlation constraints can be established for the undrained peak strength. In 1960, Henkel and Parry explored the shear strength properties of remoulded clay samples, normally consolidated and over-consolidated. Their findings revealed that samples with the same over-consolidation ratios but varying effective consolidation pressures exhibit a relatively constant value for the undrained peak strength when normalized by the effective consolidation pressure. These observations have been repeatedly confirmed by subsequent studies (Castro, 1969; Hanzawa, 1980; C. C. Ladd & Foott, 1974;

Mohamad & Dobry, 1986). This normalized parameter is referred to as the undrained peak strength ratio  $S_p$ .

However, it has been recognized that both the instability line and undrained strength ratio are influenced by various factors, including soil density and, of particular interest here, the specific consolidation paths followed before shearing (Fotovat et al., 2022; Imam et al., 2005; Riemer & Seed, 1997). Although the standard approach for triaxial testing involves isotropic consolidation before shearing, this does not fully replicate the behavior of natural soil deposits. Under natural conditions, soil consolidation follows a stress path under zero lateral strain conditions, with the ratio of horizontal to vertical effective stresses equal to the coefficient of lateral earth pressure at rest ( $K_0$ ) (Bishop & Henkel, 1962). Consequently, tests on anisotropically consolidated specimen can lead to a distinct material fabric compared to isotropically consolidated sample (Konrad, 1993; Kato et al., 2001; Yang et al., 2018). This idea has motivated the use of anisotropically consolidated triaxial tests, to measure the undrained shear strength of contractive brittle granular soils.

#### **6.5.2. Observations on Brumadinho B1 tailings**

For the Brumadinho tailings, anisotropic triaxial tests were conducted under stress-controlled shearing, at different strain rates, as illustrated in Figure 39. The primary objective of these tests was to investigate any potential viscous effects on the tailing response. It is evident in the tests that the viscous effect is negligible, as anisotropic triaxial tests at different stress rates practically overlap (Arroyo & Gens, 2021a; Viana da Fonseca et al., 2022).

The endpoints of the anisotropically consolidated tests align with the CSL determined in previous section for isotropically consolidated tests. This alignment corroborates prior findings (Fourie & Tshabalala, 2005; Riemer & Seed, 1997) that the critical state line remains unaffected by the consolidation stress path chosen before shearing. In Figure 39, the instability line for both isotropic and anisotropic triaxial tests is illustrated, connecting the peak deviatoric stress points from the tests on loose soil samples.

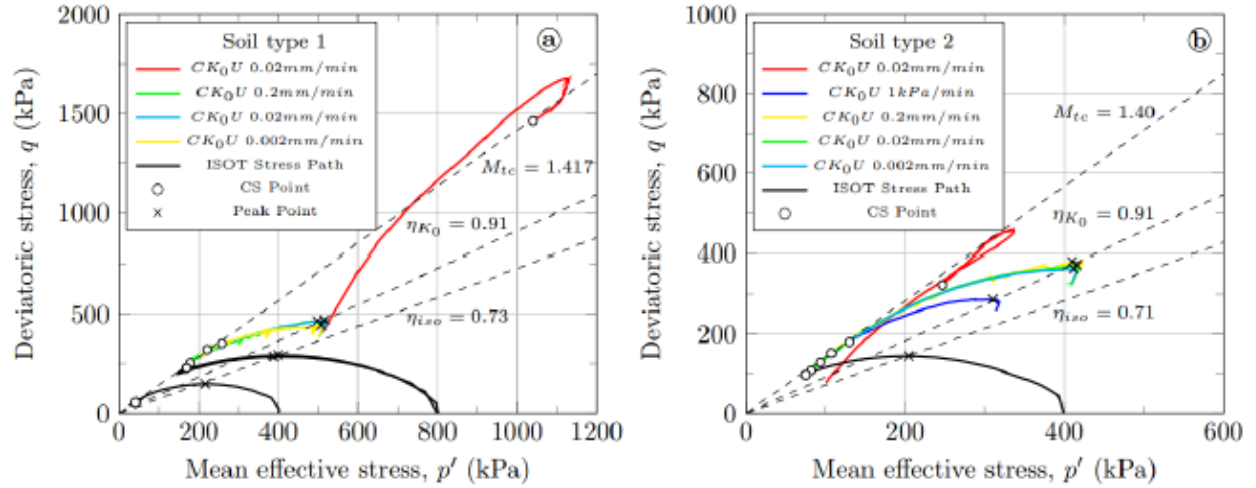


Figure 39 – Undrained stress path of isotropically and anisotropically consolidated triaxial tests; (a) Soil type 1; (b) Soil type 2.

Data presented in Figure 39 clearly shows that, for both soil types 1 and 2, the peak strength measured after isotropic consolidation define an instability line that is distinct from the one defined by anisotropically consolidated specimens. In fact, that was inevitable, as the anisotropic triaxial specimens underwent consolidation at a stress ratio higher than that of the instability line derived from isotropic triaxial tests. As shown in Figure 39, anisotropic tests reach the instability condition at stress ratio of 0.91, in contrast to isotropic tests, which achieved this condition at a stress ratio of 0.73 in soil type 1 and 0.71 in soil type 2.

### 6.5.3. CASM simulations

We used the CASM model to explore how instability under undrained conditions is affected by the previous consolidation history. To this end a series of triaxial tests were simulated. These simulations used the constitutive model parameters detailed in Table 6 of Section 6.4.2 to represent soil types 1 and 2.

In the first set of simulation specimens were consolidated to the same effective confining stress of 800 kPa but following different consolidation stress path, with a stress ratio of  $K_c = \sigma'_h / \sigma'_v$  ranging from 1.0 to 0.3. This is illustrated in Figure 40a and Figure 41a. It can be observed that all undrained tests consolidated along paths where  $K_c$  was lower than 0.5 have their own different instability line. Conversely, for all tests consolidated with  $K_c > 0.5$ , the instability line

is unique. Interestingly the  $K_c$  value of 0.5, which represents the threshold where a different instability line can be interpreted, corresponds to the coefficient of earth pressure at rest ( $K_0$ ). This  $K_0$  value was used as an input for estimating the CASM parameter  $m$  (as reported in section 6.3.4), which governs the plastic flow ratio.

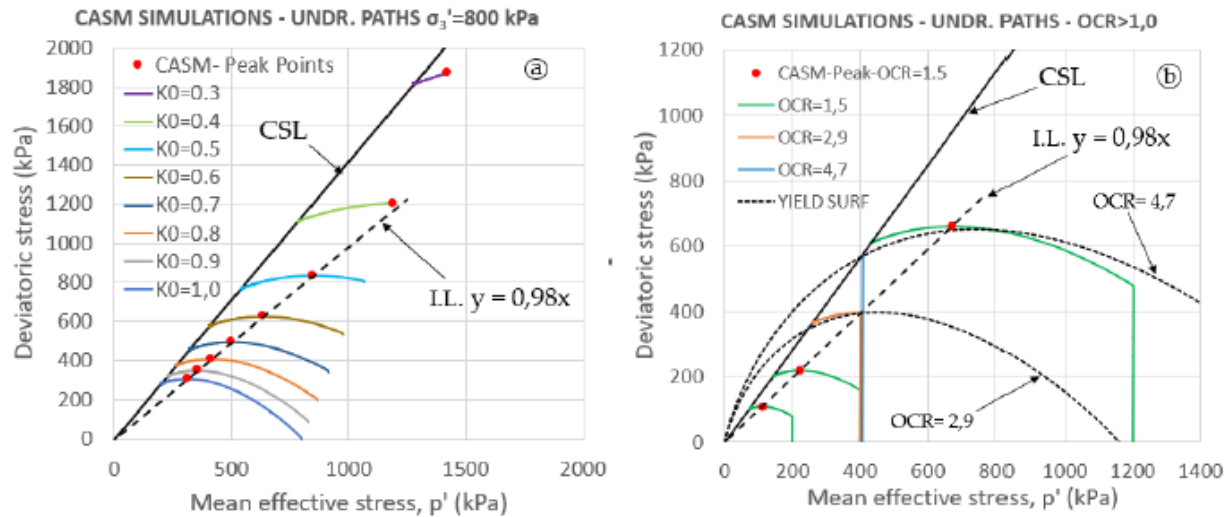


Figure 40 – Undrained stress path simulations in soil type 1; a) isotropically and anisotropically consolidated triaxial tests; b) isotropically consolidated triaxial tests  $OCR>1.0$ .

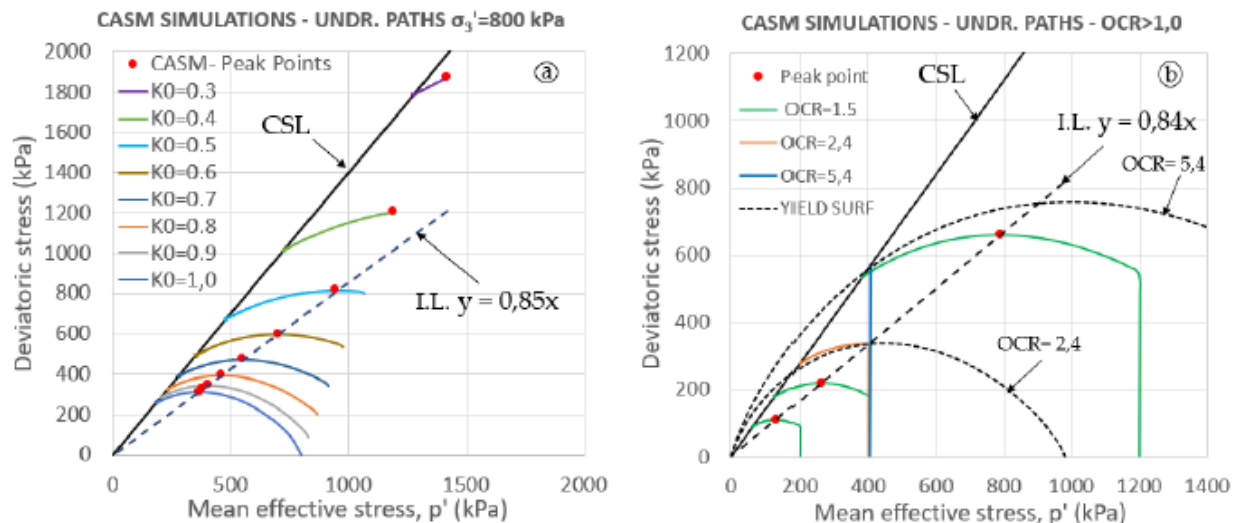


Figure 41 – Undrained stress path simulations in soil type 2; a) isotropically and anisotropically consolidated triaxial tests; b) isotropically consolidated triaxial tests  $OCR>1.0$ .

Actually, examining the stress paths for cases where  $K_c < 0.5$ , reveals that peak strength is not easily discernible; instead, these paths demonstrate a consistent decline from the onset of the shear phase. In other words, the undrained stress paths are unstable right from the start of the shear phase. It seems then that CASM is able to adequately capture the effect of varying consolidation paths on undrained instability, at least qualitatively.

In a second set of simulations, illustrated in Figure 40b and Figure 41b, the effect of overconsolidation on defining the instability locus is explored. Simulations were carried out using different consolidation mean effective stresses of 200, 400, and 1200 kPa, firstly considering sample with an OCR of 1.5. The resulting stress paths are depicted in green, with their corresponding peak points marked with red dots.

The results reveal that the instability line is consistent with that observed in the previous simulation, being defined for stress ratios of 0.98 and 0.85, as shown in Figure 40a and Figure 41a. Subsequently, simulations were conducted using a consolidation mean effective stress of 400 kPa while slightly increasing the OCR. With OCR values of 2.9 for soil type 1 and 2.4 for soil type 2, respectively, the peak points of the stress paths were found to align with the previously established instability line (IL). Conversely, for higher OCR values (4.7 for soil type 1 and 5.4 for soil type 2), the IL no longer applies. These cases, represented by the cyan stress paths in Figure 40b and Figure 41b, show that the stress path remain in the elastic domain even after crossing the IL (as it is still within the yield surface depicted by a dashed black line), making the attainment of peak strength unfeasible. Consequently, the behavior of the soil shifts from contractive to dilative when the OCR values surpass 4.7 and 5.4 for soils 1 and 2, respectively.

If the instability line is defined for a specific set of soil parameters, it becomes feasible to delineate the spectrum of over-consolidation ratios, within which the instability line transitions from its minimal to its maximal extent. Employing the CASM yield function (equation (42)) and specifying that the yield surface intersects the instability line at a deviatoric stress equivalent to  $p'\eta IL$ , enables the formulation of the following equation:

$$OCR = \frac{1}{e^{-\left[\left(\frac{\eta_{IL}}{M_{\theta}}\right)^n \ln r\right]}} \quad (63)$$

Therefore, a range of OCR values can be established by substituting either the lower slope of the instability line (derived from isotropically consolidated triaxial tests with OCR=1) or the higher slope of the instability line, specifically when it coincides with the critical state line, into Equation (63). Similarly, data can be analyzed in terms of the state parameter. By applying Equation (55) and assuming a stress ratio  $\eta$  of zero, the maximum value of the reference state parameter is obtained. Conversely, using a stress ratio corresponding to that of the instability line yields a threshold state parameter, beyond which the slope of the instability line begins to increase.

Consequently, there is a range of state parameters for which, under undrained conditions, the instability line determined for isotropic conditions will apply. Once this threshold is exceeded, the instability line starts shifting towards the critical state line. This phenomenon is illustrated in Figure 42, where undrained triaxial tests were simulated under a consistent confining pressure of 400 kPa but with varying state parameters.

Chu et al. (2012) experimental results showed that the instability line (*IL*) was highly dependent on the initial state parameter ( $e_0 - e_{crit}$ , for the same  $p'$ ). In their research, they conducted undrained tests under identical effective confining stresses on specimens with varying void ratios. Their observations indicate that, for a given effective confining stress, a smaller void ratio results in a higher IL. Furthermore, these tests revealed that the IL slope remains consistent once the state parameter surpasses a specific threshold.

The experimental results of Chu et al. (2012) are compared in Figure 42 with CASM predictions for the Brumadinho Soils Type 1 and 2. The comparison is favorable. It is evident that the instability line remains unchanged until the state parameter attains a specific value, which is of around 0.05 in simulation of soil type 1 (red dashed line). Once this value is exceeded, the instability line slope begins to increase until it intersects the CSL. The area spanning from the lower instability line to the failure line is identified as the zone of potential instability (Lade, 1993). Consequently, the stability of a specimen within this zone is closely linked to its state

parameter. Furthermore, the laboratory data (cross marks) closely align with the lines derived by CASM simulations.

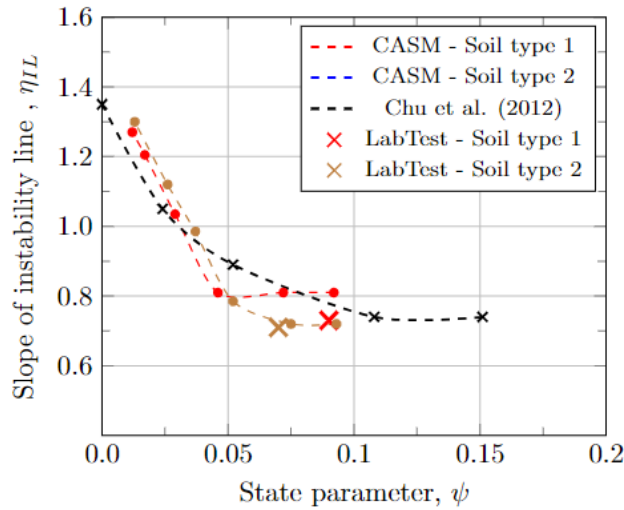


Figure 42 – Variation of  $\eta_{IL}$  with the state parameter in soil type 1 and 2

## 6.6. Summary

This chapter presents the constitutive model parameters and the methods used to derive them from laboratory tests on the Brumadinho Dam B1 dataset. It details the procedures for determining critical state and elasticity parameters and discusses the correlation of shape parameters  $n$  and  $r$  with peak and residual undrained shear strength, as well as the initial state parameter.

The application of CASM to simulate drained and undrained isotropically consolidated triaxial tests using the tests described in Chapter 5 is outlined. CASM effectively replicates the overall soil behavior for soil types 1 and 2. Normally consolidated drained tests are accurately represented within the investigated pressure range. The CASM model successfully simulates undrained tests on loose specimens exhibiting static liquefaction, capturing peak and residual shear strength and consistent pore pressure with laboratory observations. Additionally, the model accurately replicates volumetric strain for both soil types.

For soil types 3 and 4, triaxial test results indicate that the relationship between  $e - \log p'$  at the critical condition is best represented by a curved CSL in the semilogarithmic compression plane. Although CASM does not allow for a curved CSL idealization, using a linear CSL adequately simulates drained tests within a limited stress range, with good results for soil types 3 and 4 at confining pressures above 100 kPa. However, discrepancies occur at lower pressures. To improve fitting at low pressures, CASM was recalibrated using a flattened CSL, resulting in a satisfactory fit between simulation and laboratory tests. Nevertheless, a flattened CSL causes loose samples sheared under undrained conditions to be dilative, thereby compromising the overall simulation accuracy at higher pressures.

Additionally, we used the CASM model to explore the effect of consolidation history on undrained instability. To this end, a series of triaxial tests were simulated using the same constitutive model parameters previously employed for soil types 1 and 2. The results suggest that CASM can qualitatively capture the effects of varying consolidation paths on undrained instability.

## **7. Flow liquefaction risk assessment**

### **7.1. Introduction**

The term liquefaction has been historically associated to sandy materials subjected to cyclic loading, usually due to seismic events. The term 'spontaneous liquefaction' was introduced by (Terzaghi et al., 1957), who described it as the rapid transformation of loose sandy material into a state resembling a viscous fluid. Later, Mogami & Kubu (1953) also recognize this phenomenon following the investigation of the Fukui (Japan) earthquake in 1948. However, it is only after the 1964 Alaska and Niigata (Japan) earthquakes that this phenomenon captures the interest of the geotechnical engineering community. Actually, the concept of liquefaction encompasses a broad array of phenomena involving soil deformations. These deformations are the result of monotonic, transient, or repeated disturbances in saturated, cohesionless soils under undrained conditions (Kramer, 1996). The tendency for volumetric contraction in loose, saturated materials leads to an increase in pore water pressure, resulting in a decrease in effective stresses turning in loss of shear strength.

While there was initially some confusion when utilizing the terminology liquefaction, it is now widely common to distinguish between the phenomena of flow liquefaction and cyclic mobility (Gens, 2019). The first type, known as flow liquefaction, can cause considerable damage as it happens quickly and without warning. This occurs when the undrained strength attained during liquefaction falls below that necessary to support the soil own weight, leading to significant ground deformation. The second type, referred to as cyclic mobility, although less dramatic, can still cause substantial damage. Cyclic mobility may lead to significant ground deformation, but it is not as severe as the effects caused with flow liquefaction.

Cyclic mobility, closely associated with earthquake events, can also be triggered by other dynamic vibration sources, such as piling and blasting operations. It is important to note that this phenomenon can also occur in soils with a negative state parameter; in other words, soils that are in either loose or dense states. On the other hand, flow liquefaction can be triggered by both dynamic and static loading.

## **7.2. Criteria for assessing liquefaction potential**

There are many criteria for evaluating whether a specific soil is susceptible to liquefaction. Often, these criteria are applied indistinctly to both flow liquefaction and cyclic mobility, although it is more appropriate to distinguish between them. The susceptibility to liquefaction can be judged based on various criteria, including historical data, geological context, composition, and the state of the soil (Kramer, 1996).

The purpose of some of these criteria lies in providing an initial screening step before evaluating liquefaction risks using detailed in-situ and laboratory testing. Interpretations based solely on empirical data from field case histories, without the support of a more rigorous framework, may not adequately clarify under what conditions these empirical relationships are valid.

### **Historical and geological criteria**

Mapping where soil is likely to liquefy can be very helpful in identifying areas prone to liquefaction. This process can be achieved using historical information or evidence that indicates where liquefaction has occurred, which can then inform inferences about the properties of the affected deposits. Furthermore, numerous studies have demonstrated that liquefaction typically occurs within a specific distance from a seismic event's origin. Therefore, it is possible to estimate the maximum distance from an earthquake's epicenter at which soil is likely to liquefy, based on the earthquake's magnitude.

This information indicates that soil deposits vulnerable to liquefaction typically originate from specific geological settings. Soils with uniform grain sizes in loose conditions, such as those in fluvial, colluvial, and aeolian environments when saturated, are highly susceptible to liquefaction. Generally, newer deposits are more susceptible to liquefaction than older ones. For instance, artificially created soil deposits, particularly those that are loosely placed, face a high risk of liquefaction. This group includes the loosely settled soils found in hydraulic fills and mine tailings. The likelihood of liquefaction is also closely linked to soil saturation levels and the depth of groundwater, with the risk decreasing as groundwater depth increases.

## **Compositional criteria**

The vulnerability of clean sandy soils to liquefaction has long been recognized and extensively investigated. Regarding gravel soils, although there are numerous case histories where gravelly soils undergoing liquefaction (Andrus, 1994; Evans & Seed, 1987), their susceptibility is notably lower. This is attributed to the free drainage of the material and the tendency of the particles to be deposited in dense states, rather than loosely. Silty materials are also susceptible to liquefaction, with the mechanical properties of silts with low plasticity being very similar to those of sand. Indeed, Song et al. (2004) tested three materials with a silt content above 60% but different plasticity index, finding that only the non-plastic material experienced complete liquefaction. Previously, Ishihara (1984) showed that non-plastic silts can liquefy, suggesting that plasticity, rather than grain size alone, influence liquefaction susceptibility. Beyond laboratory tests, numerous instances of geotechnical structures predominantly composed of silt, such as tailings dams, have been observed to undergo liquefaction, underscoring the material's vulnerability.

Many criteria have been developed to determine whether soils liquefy, focusing on basic soil properties such as grain size and Atterberg limits. Tsuchida (1970) compiled the grain size distributions of cohesionless soils that either liquefied or remained stable during specific earthquake events. This investigation resulted in the identification of grain size curves limits that distinguish soils susceptible to liquefaction from those that are not. Seed (1982) expanded the 'Chinese Criteria' established by Wang (1979) for identifying fine-grained soils susceptible to liquefaction. This criterion, known as the Modified Chinese Criteria, considers not only grain size but also includes liquid limit and water content in the analysis. In 2003, Seed et al. introduced a new framework designed to evaluate the liquefaction potential for soils with a notable amount of fine content, serving as an alternative to the 'Modified Chinese Criteria'. According to Figure 43, soils located within Zone A are considered potentially susceptible to 'classic' cyclically-induced soil liquefaction. Those in Zone B are within a transitional area, indicating that they might be prone to liquefaction under certain conditions. Conversely, soils outside these two zones are not generally susceptible to 'classic' liquefaction.

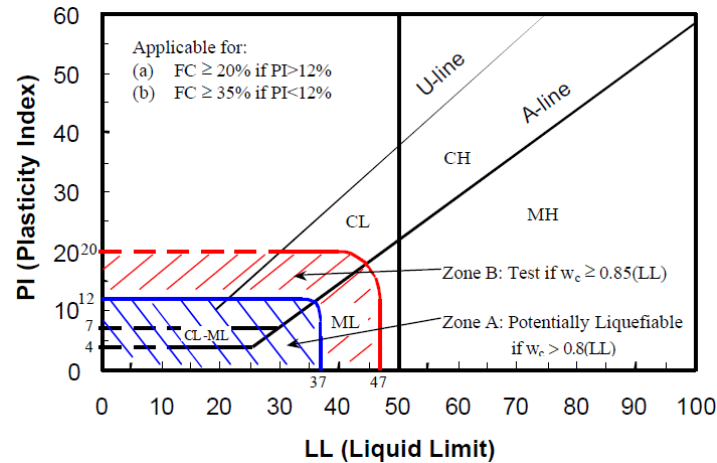


Figure 43 – Assessment of liquefiable soil types (Seed et al., 2003)

### State Criteria

The criteria outlined in the preceding section are intended to serve as qualitative guidelines. While on one hand, they are useful for identifying the potential for liquefaction based on basic soil characterization, on the other hand, they do not constitute a comprehensive framework for evaluating susceptibility to liquefaction. Susceptibility to liquefaction is closely related to the state of the soil, which is defined by the void ratio at a specific stress level. Indeed, a soil in a loose state can generate excess pore pressure when sheared, consequently reducing the effective stress. Therefore, the concept of relative density alone is inadequate for describing the soil's state, as soils with identical relative density but subjected to different confining stresses may either contract or dilate. The state parameter approach is crucial for defining the soil's state. Hence, the state parameter approach is essential for accurately defining the soil's state. While determining the soil state in the laboratory is relatively straightforward, defining this parameter in situ is more challenging. A variety of in situ tests, including CPTu, DMT, and the pressuremeter test, have been utilized for this purpose. The following section will outline some of the most significant and recent methodologies used to calculate the state parameter from CPTu.

### 7.3. Methods for evaluating in-situ state parameter using CPTu

#### 7.3.1. Summary of previous findings

The response of soil varies with both mean effective stress and void ratio. This aspect can be captured through the concept of state parameter  $\Psi$  (Been & Jefferies, 1985), defined as the difference between the current void ratio  $e$  and critical state void ratio  $e_c$ , at the same mean stress. A positive value of  $\Psi$  leads to a contractive soil response instead negative values of  $\Psi$  imply a dilative soil response.

Due to specific laboratory procedures (Jefferies & Been, 2015; Viana da Fonseca et al., 2021), it is possible to determine critical state soil parameters with an acceptable scatter between different laboratories (Reid et al., 2021). Nevertheless, assessing the in-situ state remains a challenge. A first screening of the in-situ void ratio with respect to the critical state line can be performed using the soil sample retrieved below the water table. If the sample is fully saturated, the void ratio can be easily calculated by using the specific gravity of the soil particles and the water content. Frequently, the retrieved samples are also subjected to triaxial compression tests once consolidated to a stress condition that is representative of its in-situ stress. However, this methodology has two issues. Firstly, silty tailings can experience significant disturbance during the sampling process, leading to unreliable void ratio measurements. Secondly, due to the thin layering presented in many tailing samples, the question of which critical state line should be used to infer  $\Psi$  is problematic. Thus, in the last 30 years, deducting in-situ state from the results of in-situ tests has become increasingly popular. The CPTu is undoubtedly the most commonly used tool for this purpose, but other field investigation techniques, such as the standard penetration test (SPT), the flat dilatometer test (DMT) and the self-boring pressuremeter test (SBP), have also been employed.

The starting point for deriving  $\Psi$  in sand tailings was a series of calibration chamber tests using a full-size cone (Been et al., 1987). Subsequently, many empirical correlations involving the normalized tip resistance  $Q_p$  and  $\Psi$  have been developed. While there have been many previous studies on CPTs for sands, it has been noted by Jefferies & Been (2015) and Shuttle & Jefferies (2016) that there is a lack of comprehensive and appropriate calibration studies for silty tailings.

Setting up a soil sample in the calibration chamber is not trivial in terms of cost and time-consuming since over two tons of material are involved. Recently, numerical simulation of cone penetration in liquefiable materials has been identified as a useful tool for addressing the lack of calibration chamber tests in silty materials (Pezeshki & Ahmadi, 2022). A brief description of the primary contributions to assessing the state parameter is presented in the following sections.

### 7.3.2. Screening approach: Plewes et al. (1992)

Plewes et al. (1992), following the investigation of drained CPTu tests results in calibration chamber performed by (Been et al. (1987), slightly modify the empirical correlation between  $Q_p$  and the initial state parameter  $\Psi$  to adapt the equation for drained and undrained conditions:

$$Q_p(1 - B_q) + 1 = k e^{(-m\psi_0)} \quad (64)$$

or:

$$\Psi = \frac{\ln(Q_p(1 - B_q)/k)}{m} \quad (65)$$

Since equation (64) is valid under both drained and undrained condition, it was necessary to introduce the dimensionless parameter  $Q_p(1 - B_q)$  instead of  $Q_p$ . It should be noted that when  $B_q$  is close to 0, i.e. drained conditions, equation 9 turns in to the original equation for sandy soils. Here, the effective coefficient  $k$  and  $m$  are function of the soil compressibility  $\lambda$  and of the critical state friction ratio  $M_{tc}$  and are defined by:

$$\bar{k} = M_{tc} \left( 3 + \frac{0.85}{\lambda_{10}} \right) \quad (66)$$

and

$$\bar{m} = 11.9 - 13.3\lambda_{10} \quad (67)$$

To estimate  $\lambda_{10}$  from the CPT data, Plewes's method used the followings empirical correlation:

$$\lambda_{10} = \frac{F}{10} \quad (68)$$

The approach of Been & Jefferies (1992), which is based on equations (64) and (65), differs only in the way as  $\lambda_{10}$  is estimated, being this parameter related to the soil behavior type index  $I_c$ . Nevertheless, as pointed out by Shuttle & Cuning (2007), the friction ratio  $F$  appears to be a better indicator of soil properties than  $I_c$ . Figure 44 shows the trend line between  $\lambda_{10}$  and  $F$  proposed by Plewes et al. (1992) with additional data from Reid (2012) and from Robertson et al. (2019) using Brumadinho tailings.

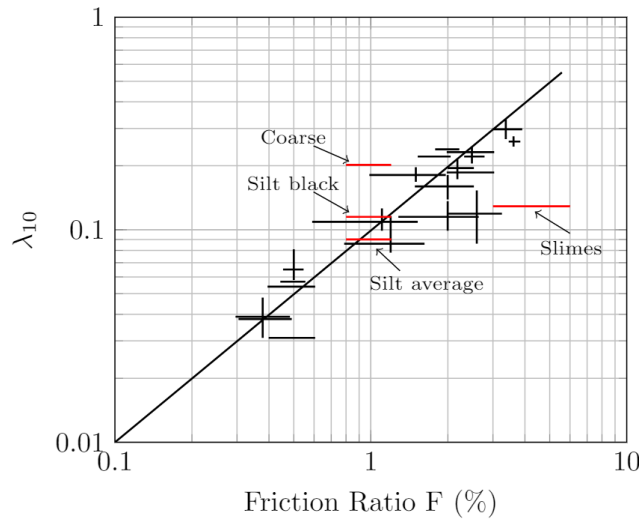


Figure 44 - Relationship between  $\lambda_{10}$  and  $F$  reported in (Jefferies & Been, 2015) with additional data.

Since  $F$  varies along the investigated layer, the red lines reported in Figure 44 correspond to the 10<sup>th</sup> percentile. As illustrated on Figure 44, there is a substantial scatter between  $\lambda_{10}$  estimated from triaxial tests in slime material and the relationship proposed by Plewes et al. (1992). However, the specific parameters  $M_{tc}$  and  $\lambda_{10}$  are usually available from laboratory triaxial tests. Thus, use of such material-specific parameters is likely to increase the accuracy of the Plewes's method if it is applied to relevant material in situ (Reid, 2015).

### 7.3.3. Methods for drained conditions

#### 7.3.3.1. Shuttle and Jefferies (1998) method

The initial efforts to determine  $\psi$  values from CPT data date back to the mid-80s, utilizing the results of calibration chambers tests and deriving the critical state line of the material through triaxial tests. This data, obtained from drained materials, demonstrates that the dimensionless CPT parameter  $M_{tc}$  is related to the in-situ state using the following equation:

$$Q_p = k e^{(-m\psi_0)} \quad (69)$$

where the coefficients  $k$  and  $m$  are specific constants that depend on soil parameters. However, the trend line obtained from each specific sand in the calibration chamber indicates a significant bias with stress level (Sladen, 1989). To better understand the reason for this bias, Shuttle & Jefferies (1998) conducted a large-scale numerical examination of equation (69) using cavity expansion analysis and the NorSand model. The advantage of this numerical simulation lies in the ability to explore the effect of various material parameters on the CPT resistance (Jefferies & Been, 2015). Remarkably, the study by Shuttle & Jefferies (1998) revealed that the bias observed could be attributed to the soil rigidity,  $I_r = G/p_0$ , which was not considered (or measured) in the earlier investigations. In fact, changing  $G$  while keeping the ratio  $G/p_0$  constant, the simulations consistently produced identical state parameters. However, when solely modifying the soil rigidity, while keeping all other soil parameters constant, distinct state parameters values were observed. This finding highlights the significance of considering the soil rigidity parameter in accurately interpreting CPT data.

Therefore, the methodology proposed by Shuttle and Jefferies requires performing multiple numerical simulations, similar to the ones they did, to determine the values of  $k$  and  $m$  as a function of both the soil properties and rigidity. However, this methodology can be time-consuming. Hence, to estimate the values of  $m$  and  $k$  more efficiently, they provide a closed-form solution to the problem:

$$k = (f_1(G/p')f_2(M)f_3(N)f_4(H)f_5(\lambda)f_6(v))1.45 \quad (70)$$

$$m = (f_7(G|p')f_8(M)f_9(N)f_{10}(H)f_{11}(\lambda)f_{12}(v))1.45 \quad (71)$$

This approach required the critical state parameters  $M_{tc}$  and  $\lambda_{10}$ , the soil rigidity  $I_r = G/p_0$  the poison ration  $\nu$  and two others specific Norsand parameter  $N$  and  $H$ . Nevertheless, it was found that the plastic hardening modulus  $H$  and the critical friction ratio  $M_{tc}$  have a strong influence on these two parameters while Poisson's ratio has essentially no effect (Jefferies & Shuttle, 2011). A detailed representation of variables  $f_1$  to  $f_{12}$  have been provided by Jefferies & Been (2015).

### 7.3.3.2. Robertson (2010) method

Robertson (2010) proposed a method, built upon previously published data and case histories, that links the state parameter with  $Q_{tn,cs}$  in predominately sand-like soils, characterized by a soil behavior type index  $I_c < 2.60$ . This method is defined by:

$$\psi = 0.56 - 0.33 \log Q_{tn,cs} \quad (72)$$

This relation depends on the  $Q_{tn,cs}$  which is the normalized cone resistance in silty sands to an equivalent clean sand (Robertson & Wride, 1998) defined by:

$$Q_{tn,cs} = Q_{tn}K_c \quad (73)$$

where  $K_c$  is a correction factor which considers grain characteristics, fines content, mineralogy, and plasticity of the soil;  $K_c$  can be estimated using the soil behavior index ( $I_c$ ) as follows:

$$K_c = 1 \quad (74)$$

$$if I_c \leq 1.70 \quad (75)$$

$$K_c = -0.403I_c^4 + 5.58I_c^3 - 21.63I_c^2 + 33.75I_c - 17.88 \quad (76)$$

$$\text{if } I_c > 1.70 \quad (77)$$

However, in transition soils with  $2.6 < I_c < 3.0$ , Robertson (2022) recommends an update to the previous equation to take into consideration the partially drained cone penetration that results in a small excess pore pressure. The update  $K_c$  is as follows:

$$K_c = 1.8346I_c^5 - 23.673I_c^4 + 124.02I_c^3 - 320.616I_c^2 + 405.821I_c - 199.97 \quad (78)$$

#### 7.3.4. Methods for undrained conditions

The determination of the state parameter plays a crucial role in assessing the susceptibility of a material to liquefaction. To do this, the best approach consist to correlate the state parameter to CPTu results from calibration chamber (CC). However, most of these correlations are derived from CC data using sandy materials, and thus, they primarily account for fully drained scenarios. This presents a notable discrepancy when considering tailing storage facilities, which are frequently characterized by silt deposits where cone penetration often occurs under undrained conditions. Recent studies (Ayala et al., 2020, 2022) have utilized a smaller-scale CC setup to investigate the CPT response in silty soil. However, due to the considerably lower penetration speed compared to standard CPT, achieving undrained conditions during penetration was not possible.

This lack of undrained CC data has led to alternative ways to determine the state parameter, involving numerical simulations of cone penetration or cavity expansion simulations. The advantage of these methodologies is the ability to perform many simulations, which allows for a better understanding of how each individual model parameter influences the state parameter. However, as depicted in Figure 45, cavity expansion serves only as an analogous representation of CPT penetration. Thus, it is necessary to quantify the scaling between CC and numerical

simulation through the denoting scaling factor  $C_q$ . To determine this specific shift, a set of undrained CC and cavity expansion tests would be ideal. Given that pairing CC and cavity expansion simulation are only available for drained conditions, the approach adopted thus far has been to gain a better understanding of the  $C_q$  factor in sand and then assume that the developed understanding can be transported to silts (Shuttle & Jefferies, 2016).

The approximation of cavity expansion has proven appealing due to the initial challenges in simulating cone penetration. However, recent advancements in numerical methods have made it possible to achieve realistic cone simulations using various techniques (Ceccato & Simonini, 2017; Monforte et al., 2021; Pezeshki & Ahmadi, 2022) and advanced constitutive models. The next section describes some of these techniques valid for undrained conditions.

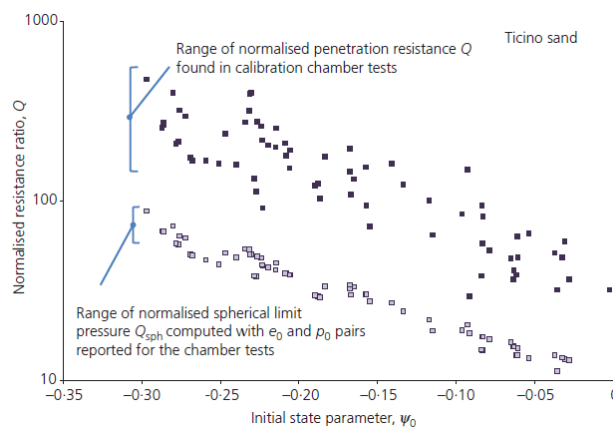


Figure 45 - Comparison of numerical results for spherical cavity expansion in Ticino sand with data from calibration chamber tests (Shuttle & Jefferies, 2016).

### Shuttle and Jefferies (2016)

Although the screening method proposed by Plewes et al. (1992) is also applicable to undrained conditions, the approach introduced by Shuttle and Cunning in 2007 represents one of the earliest comprehensive attempts to estimate the state parameter in undrained materials using the NorSand model. In line with the findings of (Ladanyi & Foriero, 1998), they adopted a scaling factor of approximately two between cavity expansion results and cone. Subsequently, Shuttle & Jefferies (2016) evaluated a new scaling expression  $C_q$  based on four suitable CC programs in sand and assumed that the insights gained could be extrapolated to silts. To this end, they

conducted a series of simulations encompassing a range of effective initial mean pressure ( $p'_0$ ), state parameter ( $\psi_0$ ), and rigidity index ( $I_r$ ), while keeping the NorSand soil properties constant. Using the spherical cavity finite element code and the new scaling factor, the corrected tip resistance was calculated as follow:

$$q_t = C_q p_{lim} + u_1 \quad (79)$$

$$C_q = 3.2e^{(-2.4\psi)} \quad (80)$$

where  $C_q$  is the mapping factor obtained from calibration chamber test in sand and  $u_1$  is the numerical pore pressure derived from the radial expansion. Subsequently the dimensionless parameter  $Q_p$  and  $B_q$  could be estimated as follows:

$$Q_p = (q_t - p_0)/p'_0 \quad (81)$$

$$B_q = (u_2 - u_0)/(q_t - p_0) \quad (82)$$

where  $u_2$  is the excess pore pressure at the CPT tip computed by the software and  $u_0$  is the initial pore pressure. Similar to equation (64), the dimensionless variables  $Q_p(1 - B_q) + 1$  has been related to  $\psi$  as follows:

$$Q_p(1 - B_q) + 1 = \bar{k} e^{(-\bar{m}\psi_0)} \quad (83)$$

The coefficients  $\bar{k}$  and  $\bar{m}$  were determined by fitting trend lines through the simulation results, as demonstrated in Figure 46. Notably, under undrained conditions, the relationship between the state parameter and effective stress, represented by the tip resistance  $Q_p(1 - B_q) + 1$ , appears unaffected by the elastic rigidity of the soil. Shuttle & Jefferies (2016) noted that this is due to the numerical procedure, where the soil close to the cavity is brought to its critical state, rendering soil properties such as  $N$ ,  $\chi$ ,  $H$  and  $G_{max}$  irrelevant in the critical state. The data plotted

on the figure show a slight bias when considering  $I_r = 400$  and  $p_0 = 500$ . Finally, determining the coefficients  $\bar{m}$  and  $\bar{k}$  proved challenging in the undrained case, as no closed form solution could be found. Therefore, to derive these coefficients, it is necessary to conduct the spherical cavity expansion using the specific NorSand parameters of the material investigated.

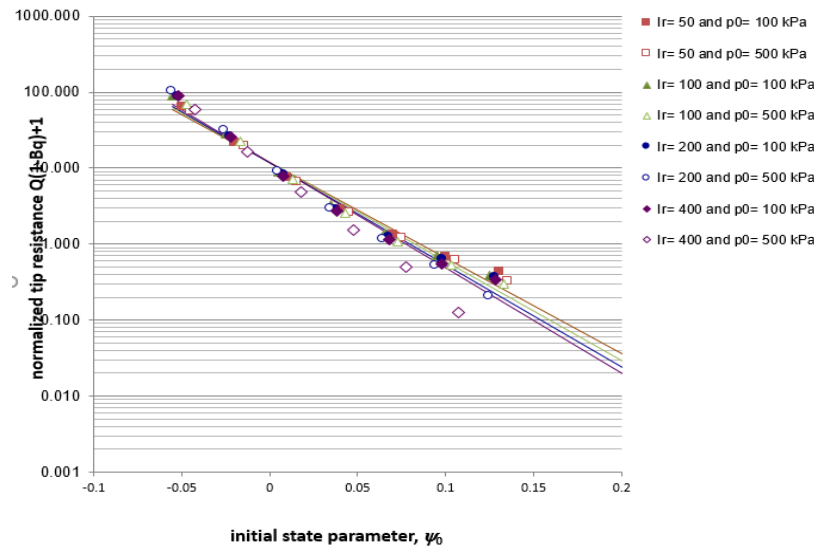


Figure 46 - Relationship between  $\Psi_0$  and  $Q_p(1 - B_q) + 1$  for a set of mean effective stress and rigidity index.

### Pezeshi and Ahmadi (2022)

In their study, Pezeshki & Ahmadi (2021) developed a large-strain numerical model of CPTu using the Norsand constitutive model, which is accessible through the commercial Fast Lagrangian Analysis of Continua (FLAC) software. The numerical model was built with an axially symmetric geometry, and a penetration rate of 20 mm/s was employed, matching the rate used in the standard device. Given the lack of specific CC data for silty soils, Pezeshki & Ahmadi (2021) conducted a comparative analysis, mapping the excess pore water pressure and tip resistance at specific depths of penetration using both CPTu measurements and simulated CPTu data. The

comparison between field measurements and numerical predictions revealed a satisfactory agreement within an error range of  $\pm 20\%$ .

Subsequently, they employed the inverted form of equation (83) to compare the state parameter obtained from the CPTu simulation with that derived from the scaled cavity expansion method. This comparison revealed notable discrepancies between the two approaches. To gain insight into these discrepancies, they conducted several numerical analyses of CPTu to investigate the impact of  $M$  and  $\lambda$  parameters on the dimensionless CPTu resistance  $Q_p(1 - B_q) + 1$ . As a result of their investigations, they proposed the following equation to estimate the values of  $\bar{m}$  and  $\bar{k}$ :

$$\bar{m} = 6 + \frac{0.4}{\lambda_{10}} \quad (84)$$

$$\bar{k} = \left(3.3 - \frac{0.08}{\lambda_{10}}\right) M \quad (85)$$

Based on equation (84), it is evident that for a constant value of compressibility ( $\lambda_{10}$ ), the extrapolated line maintains a consistent slope. Nevertheless, an increase in  $M$  causes the line to shift upward, indicating a corresponding increase in the dimensionless CPT resistance. Therefore, Pezeshki & Ahmadi (2021) suggest that the dimensionless CPT resistance increases with an increase in compressibility, which contradicts the prediction made by Plewes et al. (1992).

### **Monforte et al. (2023)**

Monforte et al. (2021) successfully showed that the Particle Finite Element Method (PFEM) could be used to model the penetration of a cone into brittle soil. This achievement was made possible by adopting a fully coupled hydromechanical formulation and effectively incorporating the CASM model, which well replicates the brittle undrained behavior of the soil. Building on the knowledge acquired from their previous study, Monforte et al. (2023) focused on the correlation between the state parameter and cone metrics under undrained conditions. To achieve this, the cone

penetration simulations were conducted at the standard velocity of 2 cm/s, while employing a hydraulic conductivity value of  $k = 10^{-10}$  m/s to ensure the attainment of undrained conditions.

Their research shows that the patterns observed in spherical/cylindrical cavity expansion are consistent with those obtained from CPTu simulations, despite a notable quantitative difference between the two outcomes emerges due to distinct geometrical and boundary conditions. A remarkably finding from the simulations is that the normalized effective cone resistance, calculated using pore pressure measurements at the  $u_1$  position, aligns more closely with the results of the cavity expansion simulation than those derived using pore pressure measurement at the  $u_2$  position. The result of this simulations are depicted in Figure 47.

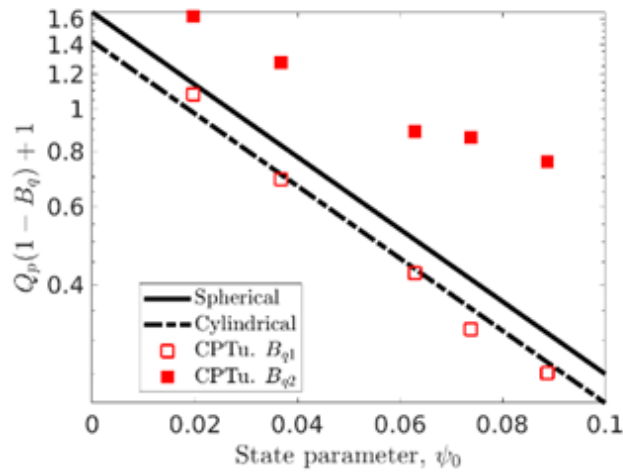


Figure 47 - Relationship between  $\Psi_0$  and  $Q_p(1 - B_q) + 1$  (Monforte et al. 2023)

Consequently, the ratio between these two cone metrics yields the mapping factor  $c_q$ , similar to those described in previous works:

$$\frac{Q_p(1-B_{q1})+1}{Q_p(1-B_{q2})+1} = C_q \quad (86)$$

The study also highlights that the correction factor  $c_q$  is affected not only by soil properties, such as the soil friction angle  $M$ , but also by the effect of cone roughness. When the soil-cone interface

is assumed to be smooth, this parameter is equal to 1. However, using a rough cone, which more closely mirrors real conditions, the  $c_q$  coefficient varies from 1.3 to 1.4. Thus, the authors proposed the following formula to estimate the state parameter from cone readings:

$$Q_p(1 - B_{q1}) + 1 = c_q \left(1 + \frac{2}{3}M\right) \exp\left(-\frac{\psi_0}{\lambda}\right) \quad (87)$$

However,  $B_{q1}$  in Equation (87) is calculated using  $u_1$  pressure, but the common location of the pressure sensor is just above the cone tip, called the  $u_2$  location. Therefore, Equation (87) should be rewritten to consider a factor  $\beta$  that accounts for the difference between the pore pressure measured at the  $u_1$  and  $u_2$  positions. Field observations of CPTu (Peuchen et al., 2010) are consistent with numerical simulations (Monforte et al., 2023), indicating that the shift between the two measurement is approximately 20%, with the  $u_1$  pressure being greater. Thus, the equation can be rewritten in the following form.

$$Q_p(1 - \beta B_{q2}) + 1 = c_q \left(1 + \frac{2}{3}M\right) \exp\left(-\frac{\psi_0}{\lambda}\right) \quad (88)$$

Interestingly, Equation (88) retains the form of the classical equation for estimating the state parameter (Equation (64)), wherein  $c_q \left(1 + \frac{2}{3}M\right)$  is analogous to the parameter  $\bar{k}$ , and  $\frac{1}{\lambda}$  corresponds to the variable  $\bar{m}$ . Diverging from others approaches, Monforte et al. (2023) assert that parameter  $\bar{k}$  remains unaffected by  $\lambda$  but exhibits significant dependence on the critical friction ratio,  $M$ . Indeed, increasing the soil's friction angle from  $M = 0.98$  to  $M = 1.33$  led to a significant upward shift in normalized cone resistance. Additionally, the role of elasticity constant  $k$  appears to be minimal in the simulations. Similarly, the influence of elastic stiffness appears to have no effect on the effective CPTu resistance, supporting the assertion made by Shuttle & Jefferies (2016).

#### **7.4. Interpretation of state parameter for Brumadinho B1 dam (section 3-3)**

The collapse of the tailings dam at the Córrego do Feijão iron ore mine in Brumadinho, southeastern Brazil, in 2019, underscores the criticality of accurately estimating soil states for

liquefaction susceptibility assessment. Figure 48, shows views of the dam before and after the failure. Caused by flow liquefaction, this event led to significant environmental damage and numerous human fatalities, prompting detailed investigations (Arroyo & Gens, 2021b; Robertson et al., 2019; Viana da Fonseca et al., 2022) to determine the factors that may have contributed to the failure of this critical geotechnical structure. This research includes both laboratory and field tests, as well as numerical analysis.

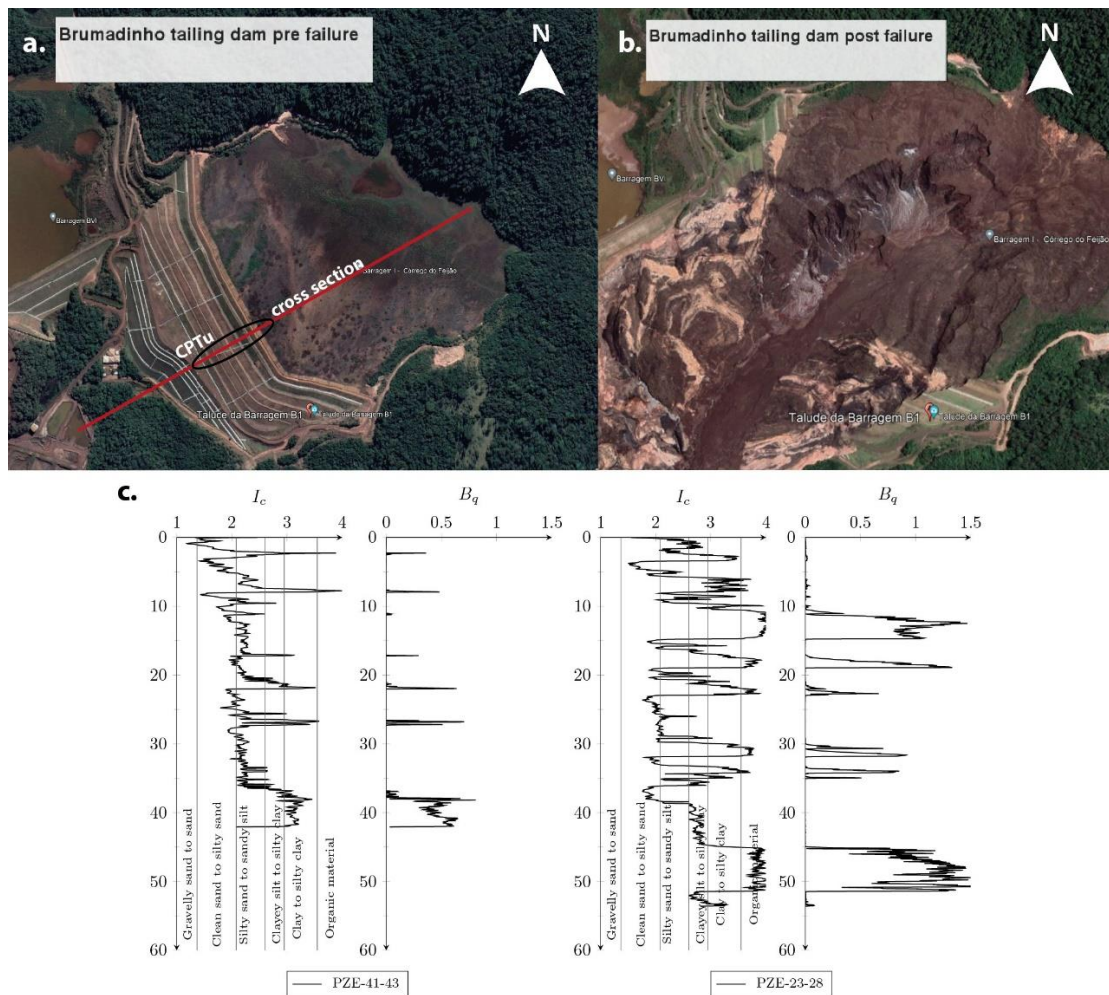


Figure 48 – Brumadinho tailings dam (a) before failure and (b) after failure (sourced from Google Earth in 2018 and 2020, respectively). (c) Results of two representative CPTu tests.

Three CPTu campaigns were conducted between 2005 and 2018, before the failure occurred. This data includes 28 CPTu tests. For this analysis, two representative CPTu tests were selected, identified as CPTu PZE-41-43 and PZE-23-28. Both tests were conducted in 2018, i.e., shortly

before the collapse. As illustrated in Figure 48c, both CPTu tests clearly show a layered profile. In terms of drainage characteristics, there are layers that exhibit fully drained behavior, alongside others that indicate drained behavior. This variation allows to apply the previously discussed methodology to estimate the state parameter. CPTu PZE-41-43 was executed in the flat area situated between the third and fourth risings, as documented in the historical reconstruction timeline by Robertson et al. (2019). Conversely, CPTu 23-28 was performed close the dam's crest, a few meters upstream from the last raising.

Using the normalized cone parameters, the data from the CPTu tests were plotted on the soil behavior type classification charts (Robertson, 2016). The two charts displayed in Figure 49 represent a three-dimensional classification system that integrates all three pieces of CPTu data. The chart on the left can be used without measurements of excess pore pressure ( $u_2$  reading), whereas the right-hand chart requires both  $q_c$  and  $u_2$  readings. In addition to serving as a guide for soil classification, these charts offer a comprehensive overview of other soil properties, including over-consolidation ratio (OCR), soil sensitivity, and drainage conditions.

The data presented in Figure 49 cover a broad area of the chart, emphasizing the variability of this tailings. When examining the data using the normalized friction ratio and normalized cone resistance, most data points cluster near the boundary that defines contractive and dilative soil behaviors, with a noticeable tendency towards the contractive region. In terms of soil behavior type classification, a significant number of points fall into the area indicative of a mixture of silt and sand, specifically zones 4 and 5. Insight into the drainage condition is provided by Figure 49, which examines the normalized excess pore pressure index  $B_q$ . It can be observed that majority of CPTu has a  $B_q$  close to zero (as indicated by the points marked in red on the chart), suggesting drained behavior. However, a notable number of points are also located in the area where  $B_q$  is greater than 0, indicating undrained (or partially drained) behavior.

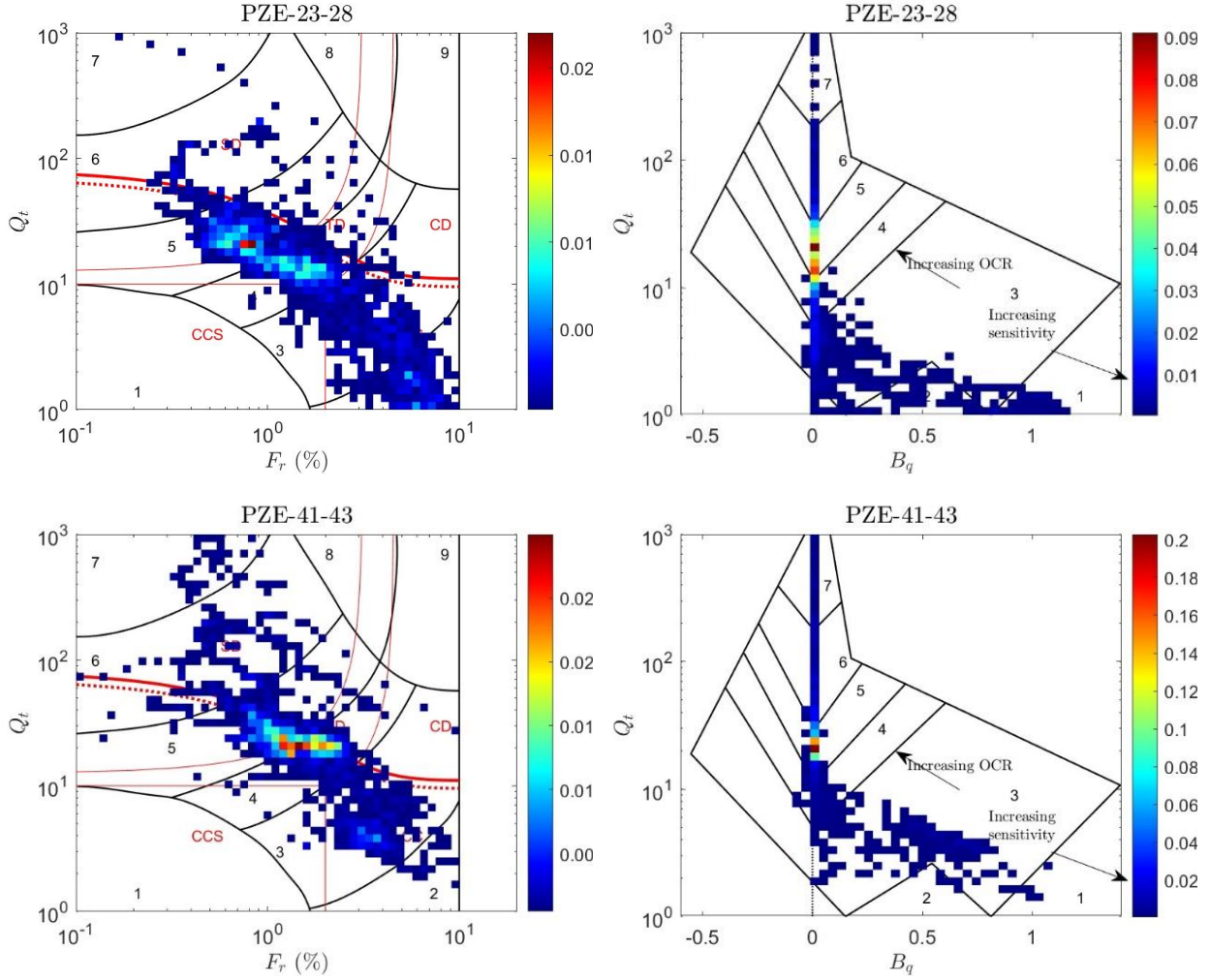


Figure 49 – CPTu classification chart for Brumadinho data.

Considering the wide range of hydraulic conductivity ( $0 < B_q < 1.5$ ) observed across soundings, this section aims to illustrate the application of the methodologies previously outlined for both undrained (Monforte et al., 2023; Pezeshki & Ahmadi, 2021; Shuttle & Jefferies, 2016) and drained conditions (Plewes et al., 1992; Robertson, 2022). The first step is to define when CPTu penetration can be considered as either drained or undrained. Regarding the undrained methodology, it seems appropriate to define the undrained penetration by simply observing the  $B_q$  parameter. This approach is justified as the three criteria employ the classic inverse formula to infer soil state from basic CPTu readings, with each method incorporating the  $B_q$  parameter. However, there is no universal agreement on a cutoff point that defines undrained penetration

in CPTu. While Senneset et al. (1989) set this threshold at  $B_q > 0.4$ , Monforte et al. (2023) suggest a limit of  $B_q > 0.5$ . In this investigation, a threshold of  $B_q = 0.4$  has been selected as the standard for identifying the drained undrained conditions.

Regarding drained conditions, Robertson (2022) employs the material index  $I_c$  to determine whether the CPT penetration process is predominantly drained, suggesting that an  $I_c$  value below 2.6 indicates the soil behaves mostly as drained. Conversely,  $I_c$  values between 2.6 and 3 are indicative of partially drained conditions, where slight excess pore pressures might be detected. On the other hand, the Plewes et al. (1992) method can be employed for both drained and undrained conditions. However, it has been shown that this method carries significant uncertainty (Monforte et al., 2023; Torres-Cruz, 2021), especially when deducing the state parameter for undrained conditions (Smith et al., 2021). For this reason, the Plewes method has been restricted to cases where  $I_c$  does not exceed 3.0.

Additionally, these methods depend on the calibration of specific material parameters, meaning the results apply only to the specific soil layers for which they are calibrated. Thus, the presence of alternating fine and coarse layers identified in each CPTu, along with the horizontal variability found in tailings deposits, presents a significant challenge. A common approach in the industry is to test both a coarser and a finer tailings fraction (Smith et al., 2021). However, there is no assurance that intermediate gradations will display properties bounded by these two extremes. Addressing this issue, an extensive geotechnical characterization of the B1 tailings was undertaken, as detailed in reports by Robertson et al. (2019) and (Arroyo & Gens, 2021a). The Robertson et al. report identify three materials with varying gradations to depict the range of materials found within the dam, categorized as Fine, Coarse, and Average. The FEUP post-mortem campaign (included in the Arroyo & Gens (2021) report) further identified three ideal materials, named Type 1, Type 2, and Type 3. In terms of the critical state parameters, the ranges reported in the two investigations are not significantly different, with the critical state line's slope  $\lambda$ , falling within the same range. However, the CSL intercept at 1 kPa exhibits significantly more variability.

As described in Chapter 6, this thesis uses the three materials studied in the Geo-Lab in FEUP, supplemented by additional triaxial tests. Indeed, a fourth material, identified as Type 4, collected in a later campaign, was also examined. For the undrained analysis, Type 2 material was chosen as the most representative due to having the lowest permeability among the tailings studied. Interestingly, even though Type 1 material has a permeability that is an order of magnitude higher, its critical state parameters  $\lambda$  and  $M$  are remarkably similar. Therefore, given that the estimation of the state parameter, primarily depends on  $\lambda$  and  $M$ , selecting either material Type 2 or Type 1 would yield virtually identical results. On the other hand, the approach proposed by Shuttle & Jefferies (2016) does not provide a closed formula to calculate the CPTu inversion parameters  $k'$  and  $m'$ . Instead, these parameters need to be derived from the cavity expansion results using the Widget, after calibrating the NorSand model for the specific type of soil being analyzed. A summary of the parameters used is presented in the following table:

Table 9 Summary of formulas and parameters employed to estimate  $\psi$  for undrained condition

| <i>Parameter</i> | (Pezeshki & Ahmadi, 2021)                 | (Monforte et al., 2023)                     | (Jefferies & Shuttle, 2011)        |
|------------------|-------------------------------------------|---------------------------------------------|------------------------------------|
| $M_{tc}$         | 1.4                                       | 1.4                                         | 1.4                                |
| $\lambda_e$      | 0.050                                     | 0.050                                       | 0.050                              |
| $m'$             | $9.28 = [6 + 0.4/\lambda_{10}]$           | $18.87 = [1/\lambda_e]$                     | 18.5 (Widget)                      |
| $k'$             | $3.7 = [(3.3 - 0.08/\lambda_{10})M_{tc}]$ | $2.61 = [c_q(1 + \alpha_m)]$                | 11.5 (Widget)                      |
| $\psi$           | $-\frac{\ln(Q_p(1 - B_q)/k')}{m'}$        | $-\frac{\ln(Q_p(1 - \beta B_{q2})/k')}{m'}$ | $-\frac{\ln(Q_p(1 - B_q)/k')}{m'}$ |

When considering drained and partially drained conditions, Robertson method only rely on direct CPTu readings to estimate the in-situ soil state. On the other hand, Plewes method typically requires laboratory tests due to its reliance on CSL parameters  $\lambda$  and  $M$ . Although several

attempts have been made to estimate  $\lambda$  from CPT readings the uncertainty introduced by estimating  $\lambda$  from these correlations can be significant (Reid, 2015).

Given these challenges, this investigation employed the  $\lambda$  value obtained from direct laboratory testing. In this context, the remolded laboratory samples best representing drained conditions correspond to Material Types 3 and 4. These materials exhibit similar permeability rates, falling within the range of  $10^{-5}$  m/s, characteristic of sand. In term of CSL parameters, both materials exhibit nearly identical critical friction ratios in compression ( $M_{tc} \approx 1.36$ ) along with noticeable non-linearity in the  $e$  versus  $p'$  plane. This non-linearity observed would prevent to assign a single  $\lambda$  parameter across the entire range of pressure. Nonetheless, the CSL on the  $e$  versus  $p'$  plane for both materials appear linear within a specific pressure range of  $200 \text{ kPa} < p' < 2000 \text{ kPa}$  (see Figure 26 and Figure 27). As a result, the slope of the CSL have been determined as  $\lambda = 0.05$  for soil type 3 and  $\lambda = 0.074$  for soil type 4. When applying the Plewes method,  $\lambda$  was selected as 0.5, since it is the most frequently identified value from lab tests; in fact, 3 out of the 4 materials studied have values very close to this. Figure 50 illustrate the results of this analysis across two soundings, for both drained and undrained scenarios.

The state parameter estimates for the undrained layers show a consistently contractive soil behavior across different criteria. Specifically,  $\psi$  ranges from 0.140 to 0.089 for CPTu PZE-23-28 and from 0.086 to 0.028 for CPTu PZE-41-43. According to the mean values presented in Figure 50, the methodologies by Pezeshki & Ahmadi (2021) and Shuttle & Jefferies (2016) establish the upper limit of the state parameter, while the Monforte criterion marks the lower boundary. The right side of Figure 50 compares the criteria of Robertson (2022) and Plewes et al. (1992) for drained and partially drained layers, also displaying the distributions of the state parameter  $\psi$ , along with their means and standard deviations. The data from both methodologies are consistent, indicating dilative soil behavior, on average. Additionally, for both soundings, Plewes criterion sets the lower limit of  $\psi$  in the range of -0.073 to -0.122. For the Robertson method,  $\psi$  ranges from -0.034 to -0.051, providing an estimate for the minimum state parameter.

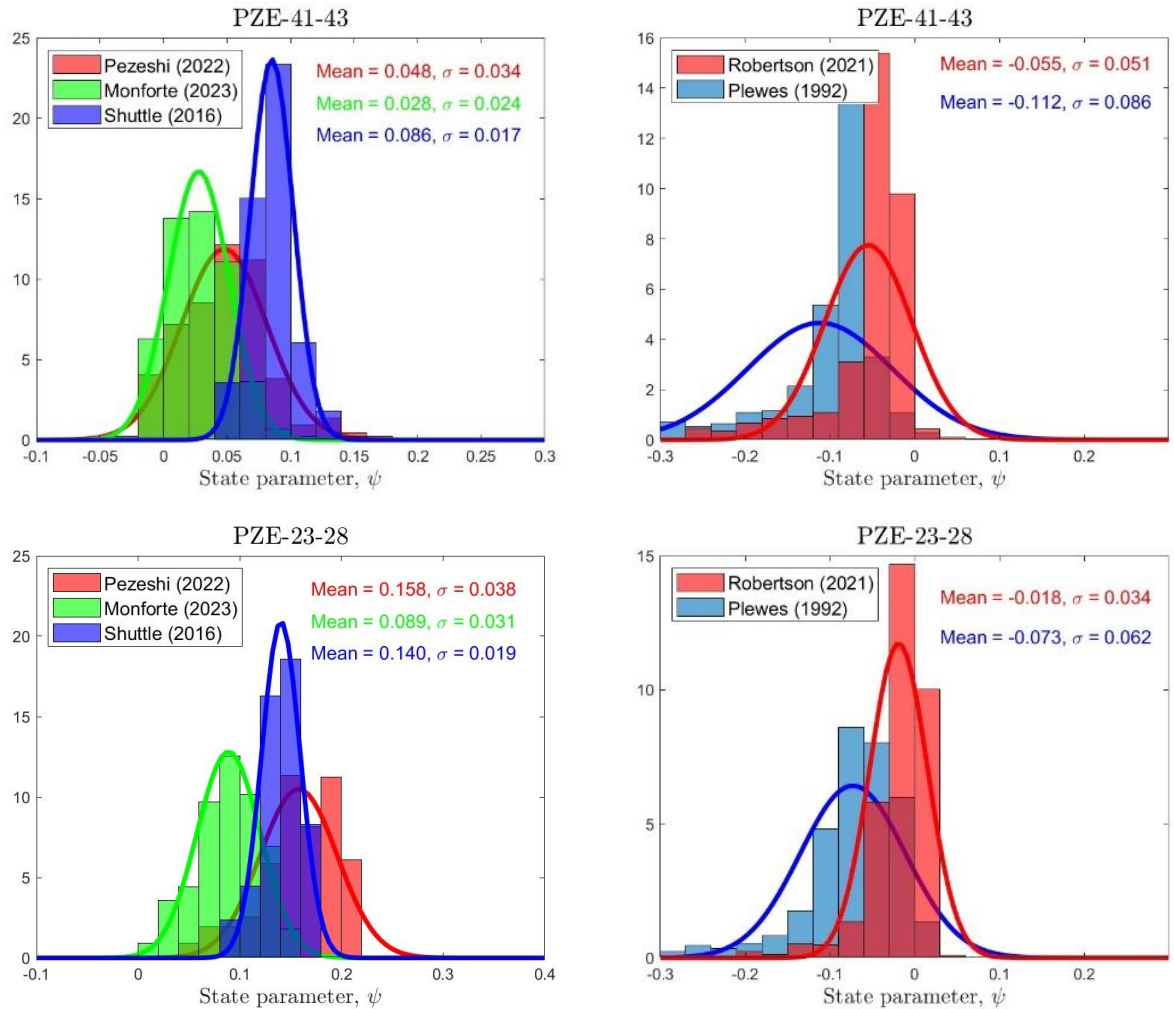


Figure 50 – Comparison of state parameter estimation In CPTu PZE-41-43 and PZE-23-28

## 7.5. Assessment of undrained peak and residual shear strength

### 7.5.1. Estimating CASM parameters using yield and residual undrained shear strength

Chapter 5 presents a variety of approaches for determining the shape CASM parameters  $n$  and  $r$ . Of particular interest is the incorporation of peak and residual shear strength in CASM calibration, given the existence of multiple methodologies for determining these values. Some of the methodologies, using both in situ and laboratory tests, are discussed below. Illustrating them with a dataset from Merriespruit.

### 7.5.2. $S_p$ from undrained triaxial tests

When a saturated, loose, cohesionless soil specimen undergoes undrained loading, it experiences undrained instability at a relatively low level of strain. This phenomenon is characterized by a rapid drop of the undrained stress path toward the critical state condition once the maximum deviator strength is attained. Whereas the final state following undrained softening is reasonably well-established based on critical state considerations, there are fewer conceptual constraints regarding the point of maximum (peak) strength that signals the onset of instability (Gens, 2019).

The concept of the instability line is considered the most remarkable attempt to rationalize the undrained peak strength in materials that display static or flow liquefaction. This idea was first proposed by Lade in 1993. Typically, a linear correlation between the mean effective stress and peak deviatoric strength can be observed in loose soil samples. This correlation makes it possible to define the locus of instability by plotting a straight line through the origin and the peak points along undrained stress paths. In addition, further constraints can be established to determine undrained peak strength. In 1960, Henkel and Parry explored the shear strength properties of remolded clay samples, whether normally consolidated or overconsolidated. Their research revealed that samples of remolded clay, having identical overconsolidation ratios but consolidated at varying mean effective stresses, show a linear relationship between the undrained peak strength normalized for the mean effective stress at consolidation, defining this parameter as the peak undrained strength ratio.

$$S_p = S_{u,pk}/p'_{ini} \quad (89)$$

Based on the undrained triaxial test results reported by Fourie & Tshabalala (2005) depicted in Figure 58, the undrained peak strength ratios for Merriespruit tailings, determined using isotropic and  $K_0$ -consolidated triaxial tests are 0.24 and 0.37, respectively, indicating a dependence on the stress path of consolidation.

### 7.5.3. $S_p$ from CPTu using the Olson and Stark (2003) method

The correlations proposed by Olson & Stark (2003) between normalized cone tip resistance  $q_{c1}$  and normalized peak and liquefied undrained strength are based on back-analyses of embankment and dam liquefaction case histories, both dynamic and static. They presented 33 liquefaction flow failure case histories from the literature, many of which involved silty sand material, and classified them into three categories based on the type of failure: static loading-induced, deformation-induced, and seismically induced. However, only the loading-induced failures, i.e., eight of them, were used with confidence to assess the yield shear strength because the shear strength back-calculated from the pre-failure geometry corresponds directly to the yield strength envelope Olson & Stark (2003). While CPT results were used to determine  $q_{c1}$  values in two of the eight case histories, the  $q_{c1}$  values for the other six were either inferred from SPT results. Despite the potential uncertainties in  $q_{c1}$  values and given the implicit presence of several complicating factors in the analysed case-histories, such as material variability, anisotropic stress effects, partial drainage, among others, a trend of increasing yield strength ratio with increasing penetration resistance was observed. The equation proposed by Olson and Stark to describe this trend is as follows:

$$\frac{S_{u,pk}}{\sigma'_{v0}} = 0.205 + 0.0143 q_{c1} \pm 0.04 \quad (90)$$

Where:

$$q_{c1} = q_c \frac{1.8}{0.8 + \left(\frac{\sigma'_v}{p_a}\right)} \quad (91)$$

The term  $\pm 0.04$  represents the upper and lower boundaries suggested to account for data scatter; however, only the mean values have been employed for evaluating the normalized peak undrained strength. These methodologies have been applied in this study for the Merriespruit tailings dam using the available (CPTu) data. Out of the total number of CPTu tests conducted, only one was excluded from the analysis as the probe crossed the zone above the water table

during penetration, making it challenging to interpret the results. The distribution of the normalized peak undrained strength using the Olson and Stark method are illustrated in Figure 51.

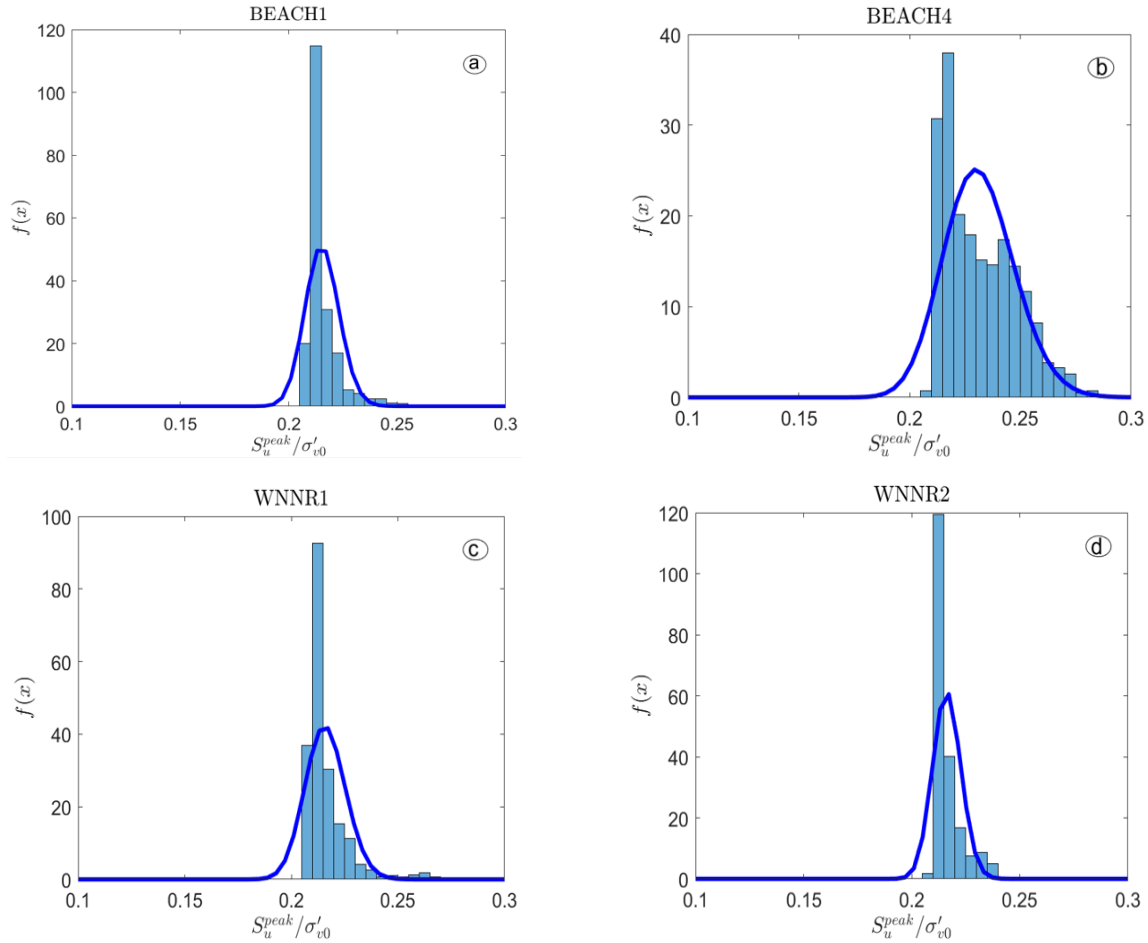


Figure 51 – Distributions for the Merriespruit CPTu of implied normalized peak strength using the Olson and Stark (2003) method.

The analysis of the data reveals that the mean value goes from 0.21 to 0.23 across all the CPTu. However, a linear regression analysis of the data performed by Torres-Cruz (2016) using the data presented by Olson & Stark (2003), showed a low fit with an  $R^2$  value of 0.15. The results suggest that establishing a direct correlation between the normalized peak undrained strength and  $q_{c1}$ , based solely on these data points, is not feasible.

#### 7.5.4. $S_p$ from CPTu using the dimensionless bearing capacity cone factor $N_{kt}$

Piezocone penetration tests offer a variety of empirical correlations to calculate undrained shear strength ( $s_u$ ). These correlations can be classified into three main categories, depending on whether they use net cone resistance ( $q_t - \sigma_{v0}$ ), effective cone resistance ( $q_t - u_2$ ), or excess pore water pressure ( $u_2 - u_0$ ). Among these, the most widely used methodology retrieves undrained shear strength from the net cone resistance ( $q_{net}$ ), which is calculated using the following equation:

$$s_u = \frac{q_t - \sigma_{v0}}{N_{kt}} = \frac{q_{net}}{N_{kt}} \quad (92)$$

where  $N_{kt}$  is the dimensionless bearing capacity cone factor. To select the appropriate value of  $N_{kt}$ , a site-specific calibration of the cone penetration test is recommended, which may involve laboratory testing on high-quality samples and/or field vane shear testing (VST) (Mayne & Peuchen, 2018). However, this approach can be both time-consuming and expensive, and may not be suitable for certain materials such as tailings, which can be difficult, if not impossible, to sample adequately and which are difficult to test undrained with VST. Theoretical solutions for  $N_{kt}$  are readily available, including expressions based on limit plasticity (Konrad & Law, 1987), cavity expansion theory (Mayne, 2016) and the relation between  $N_{kt}$  and the Bishop Brittleness index has been investigated by means of advance numerical modelling (Monforte et al., 2021).

Based on the results reported by Mayne (2014), Mayne & Peuchen (2018) presented a correlation between the CPTu pore pressure ratio  $B_q$  and  $N_{kt}$ , using a total of 407 paired sets of undrained shear strengths obtained from triaxial compression versus the net cone resistance. The correlation is defined as follow:

$$N_{kt} = 10.5 - 4.6 \ln (B_q + 0.1) \quad (93)$$

where  $B_q$  is defined as  $(u_2 - u_0)/q_{\text{net}}$ . Since most of the overall trend was obtained for  $B_q$  values greater than 0.4 in this study, it has been decided to limit formula 1 only for values of  $B_q$  greater than 0.4. The distribution of undrained strength estimated in this way is shown in Figure 52.

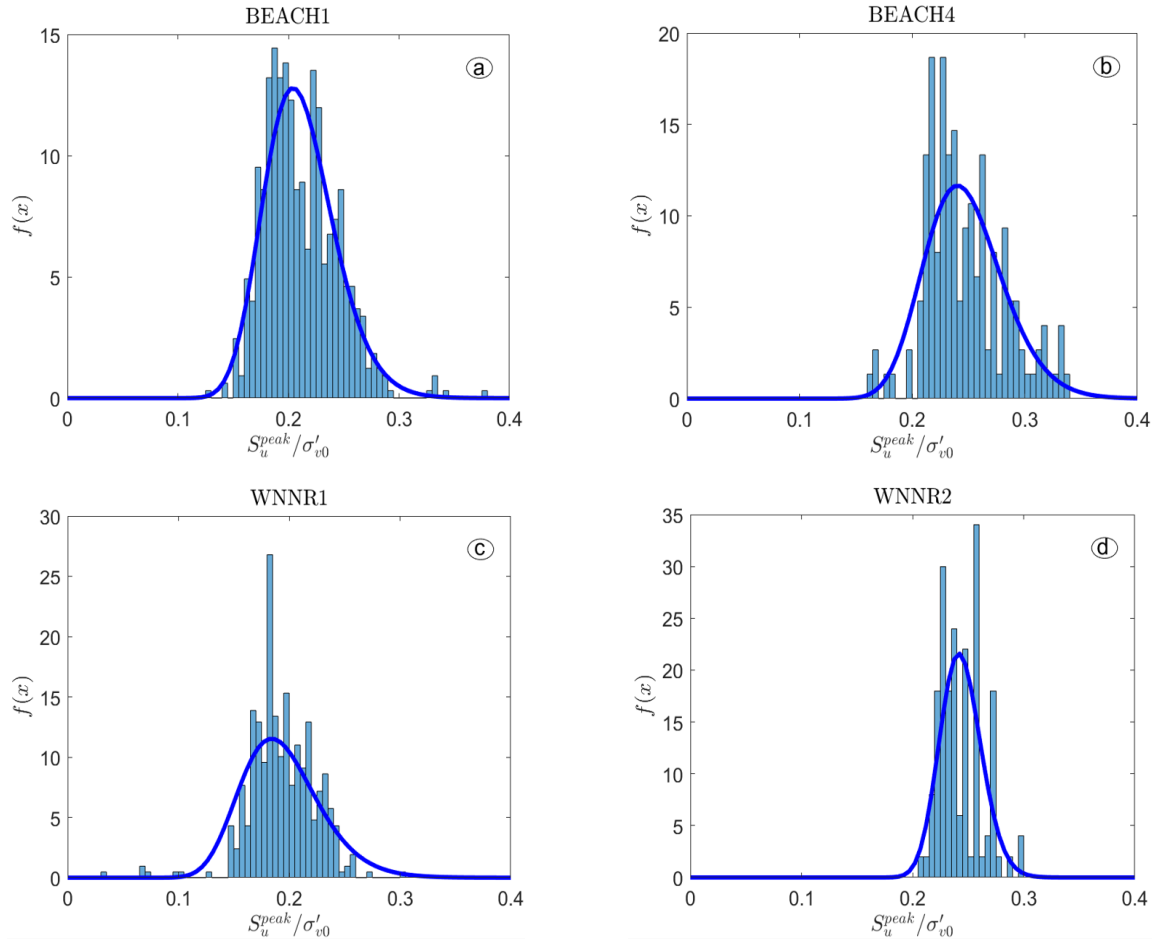


Figure 52 – Distributions for the Merriespruit CPTu of implied normalized peak strength using the  $N_{kt}$  method by Mayne & Peuchen (2018).

The analysis of the data reveals that the mean value goes from 0.21 to 0.24 across the 4 CPTu.

#### 7.5.5. $S_{res}$ from CPTu using the state parameter

When loose soil undergoes undrained shearing, it may experience strain softening, whereby the undrained residual or liquefied strength is achieved at the critical state condition under low (or close to zero) levels of stress. This behavior can be analyzed at the light of critical state soil mechanics, as the residual strength corresponds to the critical state condition. Granular

materials, such as loose sand, mine tailings, and hydraulic fills, as well as fine materials like quick clays, can exhibit this behavior. A schematic representation of this phenomenon is depicted in Figure 53.

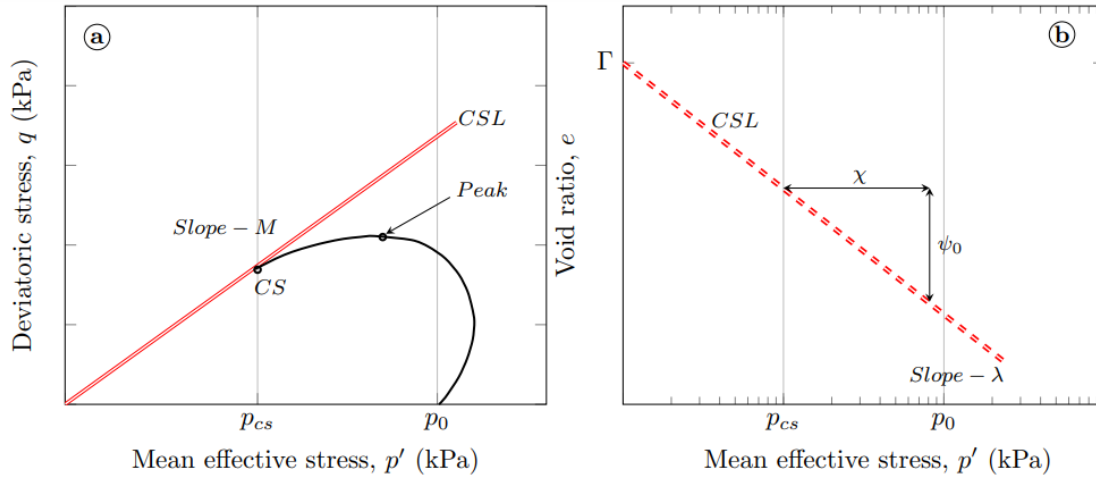


Figure 53 – Undrained softening: a) effective stress path; b) brittleness parameter  $\chi$ .

When analyzing materials that exhibit a straight critical state line, it is possible to obtain the brittleness parameter, denoted as  $\chi$  (Monforte et al., 2021), as illustrated in Figure 53a. This parameter can be obtained using equation (94), which relates  $\chi$  to the ratio of the initial effective pressure,  $p_0$ , and the critical state effective pressure,  $p_{cs}$ :

$$\chi = \ln \left( \frac{p'_0}{p'_{cs}} \right) \quad (94)$$

Furthermore, as depicted in Figure 53b, the parameter  $\chi$  is related to the slope of the CSL in the compression plane  $\lambda$  and the state parameter  $\psi$  by the following equation:

$$\chi = \frac{\psi}{\lambda} \quad (95)$$

By combining equations (94) and (95), and considering that  $q_{cs} = p_{cs} \cdot M$ , it is possible to derive the undrained shear strength at the critical state according to equation (96)

$$S_u^{res} = \frac{M}{2} \cdot p'_0 \cdot e^{\left(\frac{-\psi}{\lambda}\right)} \quad (96)$$

By using the critical soil parameters listed in Table 10 are known PON VALOR UTILIZADO O TABLA, it is possible to derive the ultimate undrained shear strength by means of the state parameter related to a specific initial mean effective stress. A broad range of techniques is available to estimate the state parameters from CPTu data. Among these, the (Plewes et al., 1992) screening technique can be used to estimate the state parameter for all drainage conditions.

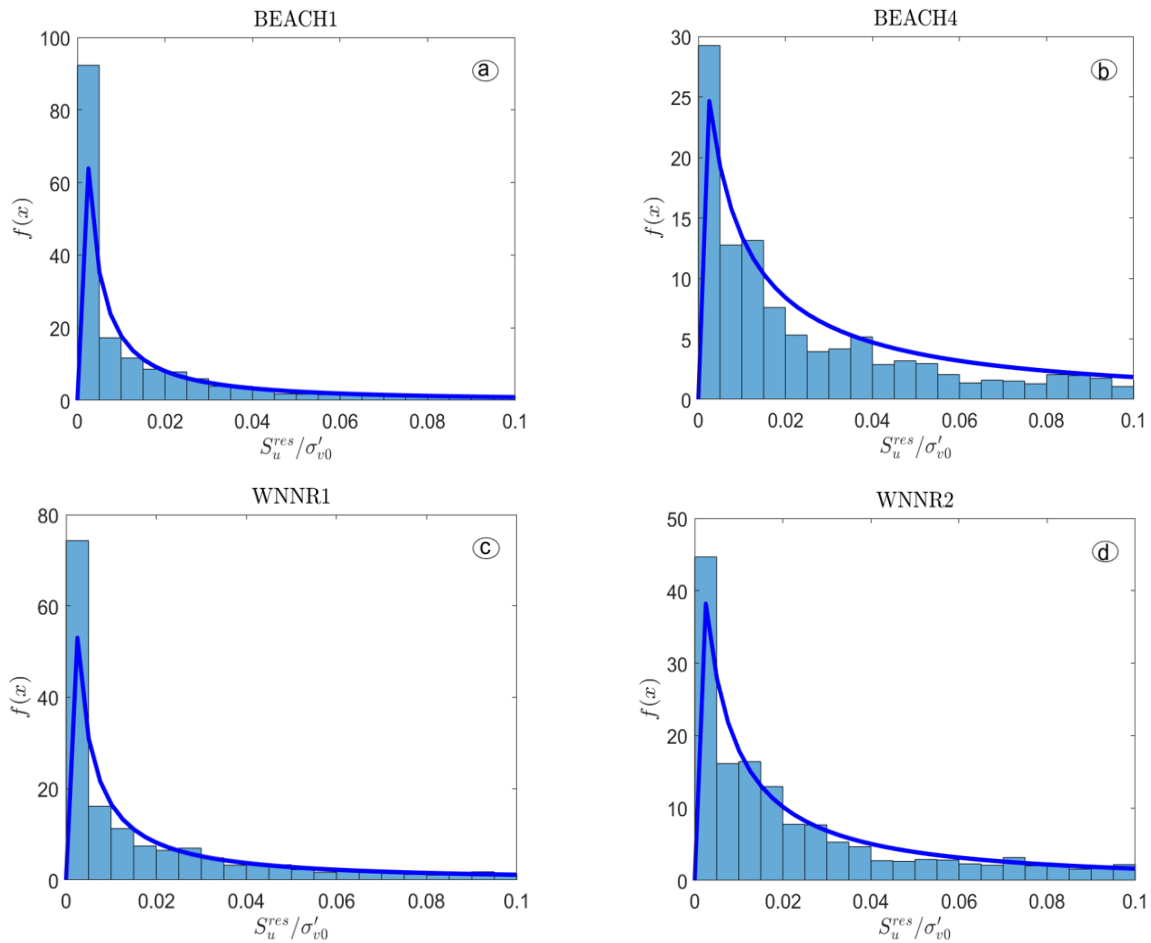


Figure 54 – Distributions for the Merriespruit CPTu of implied normalized residual strength.

Therefore, in Figure 54, the ultimate undrained shear strength is reported according to equation 81 using the state parameter derived from the Plewes methodology. It can be observed that very

low values of residual undrained strength ratio, lower than 0.01, are obtained from the analyzed CPTu data.

#### 7.5.6. $S_{res}$ from CPTu using the Olson and Stark (2003) method

The methodology described in the previous section results quite robust because is directly related to the critical state soil framework. However, this approach is not entirely impervious to errors, owing to its heavy reliance on the state parameter.

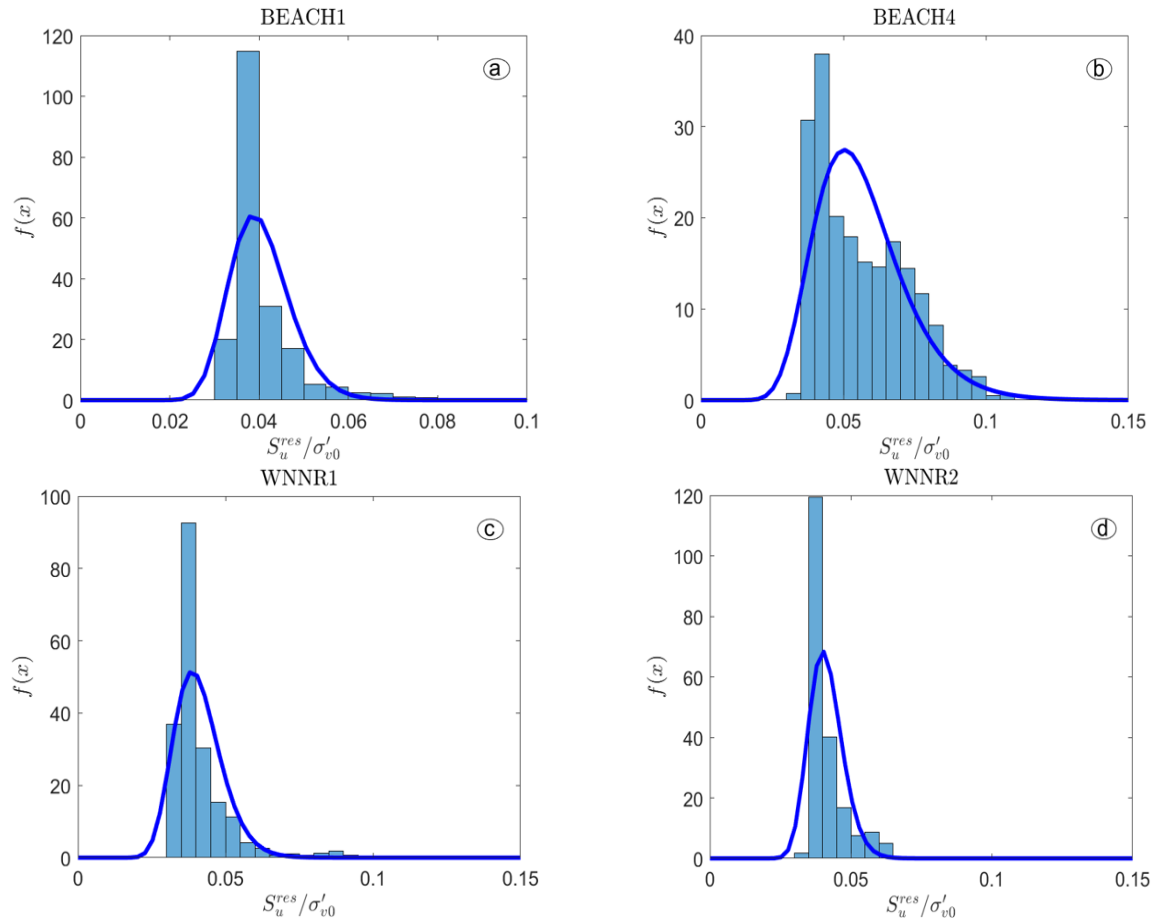


Figure 55 – Distributions for the Merriespruit CPTu of implied normalized residual strength

Despite several methodologies exist for estimating state parameters, the potential for variations in results exists, and the establishment of a clear consensus on the most appropriate methodology it is still under discussion. Empirical observations have revealed a strong correlation between CPTu sleeve friction and residual undrained strength in clay (Lunne et al., 1997; Robertson, 2010). This relationship has also been observed in numerical simulations of CPTu

penetration in brittle soils (Monforte CPT22). However, the CPTu test conducted in Merriespruit did not measure sleeve friction, and therefore this approach was not utilized.

Instead, the Olson & Stark (2003) method, which involves back-calculation of undrained shear strength from case histories, was used to estimate residual undrained strength. This approach has previously been employed for estimating peak undrained strength.

## 8. Flow liquefaction numerical studies for Merriespruit TSF

### 8.1. Introduction

Static liquefaction of tailings shows brittle behaviour. For soils with brittle stress-strain behavior (shear resistance decreases significantly as strain increases beyond the peak), progressive failure is a distinct possibility. This poses problems for classical slope stability analyses based on limit equilibrium, where the choice of the input shear resistance to employ is unclear, with peak values being too unconservative and residual values perhaps too conservative. In the case of liquefaction the problem is aggravated because the need to use undrained shear strength complicates the choice, as other aspects beyond strain (e.g. local permeability variations, loading speed) affect the choice of the strength value.

For these reasons the use of stress-strain numerical simulation to analyze dam vulnerability to liquefaction is gaining more acceptance; unfortunately there are no clear guidelines for application and many aspects on the set-up of the problem and the analysis of the result are open to interpretation. The purpose of this chapter is to contribute to this debate by examining several of those aspects in a realistic but simplified setting.

The Merriespruit case study stands as a key example of the mechanical triggering of liquefaction, having been subject to thorough research encompassing laboratory testing, in-situ characterization, and numerical modeling. For this reason, the Merriespruit dam failure has been chosen as a benchmark for this chapter. This case will be used to evaluate dam stability under different scenarios involving liquefaction.

To this end, the CASM model has been selected to represent the tailings due to its capability in simulating flow liquefaction. The different conditions explored include:

1. Assessing the impact of viscous effects on the stability of the dam upon triggering.
2. Examining the impact on dam stability by calibrating the constitutive model using isotropically or  $K_0$  consolidated triaxial tests.

3. Undertaking a sensitivity analysis on specific CASM parameters, using different geotechnical conditions, such as whether the material is viscous or not, and considering the analysis as undrained or fully coupled.
4. Investigating the sensitivity of triggering to spatial location.

## 8.2. Review of previous investigations

The Merriespruit tailings dam (MTD) was constructed with the ring-dykes technique, a common method for the construction of TSF in South Africa, due to the predominantly flat topography of the mining areas. During the construction of the MTD in 1978, two compartments, including the northern compartment (4A) and southern compartment (4B), were built for efficient tailings storage. The failure of the northern compartment in 1994 was a catastrophic event, triggered by a 30-minute rainstorm. This led to a 150-meter wide breach of the 31 meter high wall, as detailed by Wagener (1997). The breach caused an extensive volume of tailings to pour out, flowing several hundred meters towards the Merriespruit village, resulting in loss of life and considerable damage. Figure 56 show the flow path of the liquefied tailings and the geometry of the dam.

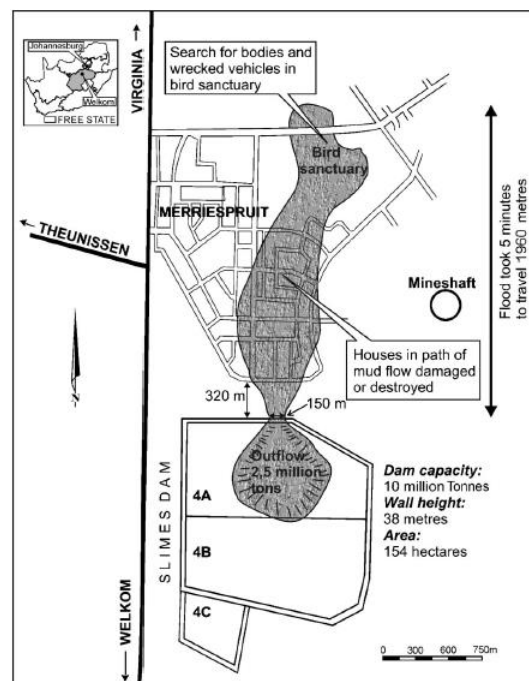


Figure 56 –Flow path on the Merriespruit village (Van Niekerk & Viljoen, 2005).

After the collapse of compartment (4A), multiple investigations were conducted, revealing the unacceptable conditions in which the dam was, due to inadequate management. It was noted that even though tailings deposition in this compartment was suspended in March 1993, unauthorized deposition still took place (Fourie et al., 2001). Furthermore, Strydom & Williams (1999) reported that Landsat satellite images taken three weeks prior to the dam failure showed free water on top of the impoundment and abutting the outer rim shortly before the event. In fact, the freeboard was insufficient, and when the thunderstorm occurred, it caused overtopping on the northern wall. The spilling water start eroding the outer wall, leading to a series of progressive failures that moved up the slope, ultimately resulting in the massive overall slope failure.

The failure of the Merriespruit tailings dam was unexpected, especially given the South African industry extensive experience with tailings dam operations; an experience that included several occurrences of local overtopping that had not led to catastrophic failure. The post-failure investigation made minimal reference to static liquefaction as a potential cause of the flow failure. This motivated (Fourie & Papageorgiou, 2001) to undertake a laboratory testing program on the Merriespruit gold tailings, using isotropically consolidated triaxial tests, sheared undrained, with four different grain size distributions. Following this intensive testing, Fourie et al. (2001) proposed that static liquefaction could explain the flow slide that occurred at Merriespruit.

Later, Fourie & Tshabalala (2005) investigated the role of  $K_0$  consolidation on the behavior of triaxial specimens from the Merriespruit tailings dam. They found that while the critical state conditions were similar to those defined by isotropically consolidated specimens, the collapse line differed. In fact,  $K_0$  consolidation resulted extremely important for correctly identifying the stresses that will trigger liquefaction. Torres-Cruz (2016) further investigated the performance of this tailing dam using different methods relying on CPTu data to evaluate the potential for liquefaction. The results indicate that, no matter which method was used to examine the CPTu data, the tailings were predicted to be predominantly liquefiable.

A recent investigation conducted by Mánica et al. (2021) presented a stress-deformation analysis of Merriespruit carried out using the CASM model. The results of the analysis revealed that even a minor erosion of the lower slope was sufficient to trigger liquefaction, thus confirming the argument put forward by (Fourie et al., 2001).

### 8.3. Description of the base model

#### 8.3.1. Geometry

The base model for the numerical studies that follow is similar to the one presented by Manica et al. (2021). It represents a transversal section of the dam wall using a mesh comprising 1171 triangular elements, with the geometry and primary boundary conditions illustrated in Figure 57. The model also incorporates the eroded part of the slope, which was subsequently eliminated to simulate the overtopping-induced erosion. The position of the phreatic surface utilized in the model was derived from the calculation performed by Wagener (1997). However, the tailings were presumed to be saturated even above the phreatic surface, taking into account the existence of pool water adjacent to the inner paddock wall in the days leading up to the failure, as reported by Strydom & Williams (1999).

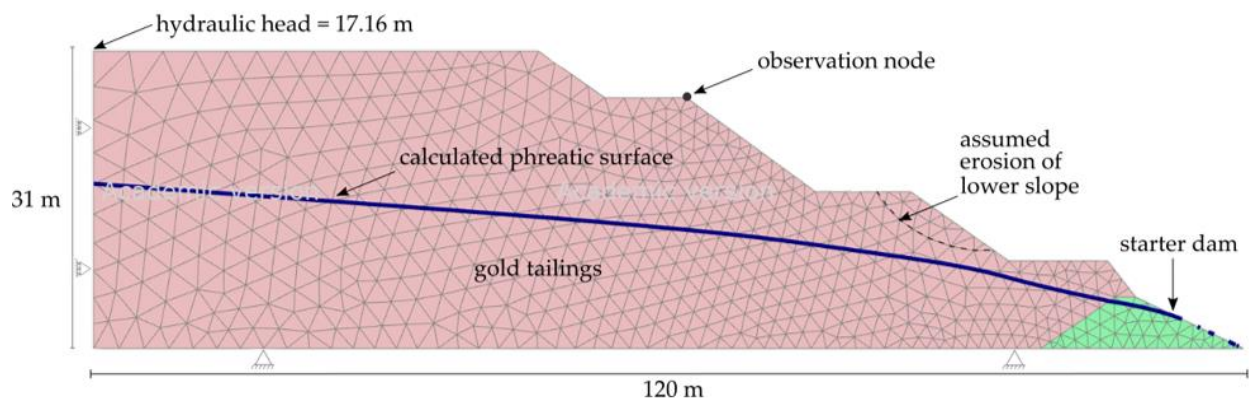


Figure 57 – Geometry, boundary conditions, and FE mesh.

Following Mánica et al. (2021), the model was restricted to two materials, namely the gold tailings and the starter dam. The starter dam was presumed to be composed of a competent

granular material that does not liquefy and was modeled utilizing an elastic-perfectly plastic Mohr-Coulomb model. On the other hand, the gold tailings were modeled utilizing CASM.

### 8.3.2. Material parameters

In 2001, Fourie & Papageorgiou studied the Merriespruit gold tailing testing four different grain size distributions. While the bulk sample collected had a 60% fine content, they created other materials in the laboratory with different grain size distribution to examine the effect of fine content on the critical state line location. These additional samples had fine contents of 0%, 20%, and 30%, in addition to the original collected sample distribution. Fourie & Tshabalala (2005) analyzed again these materials using a series of isotropically and  $K_0$ -consolidated triaxial tests. They employed the moist tamping technique for specimen reconstitution, as did Fourie & Papageorgiou. The objective of their investigation was to determine whether the stress path followed during consolidation could influence the instability line and critical state line location. They concluded that the critical state line obtained from both isotropically and  $K_0$  consolidated specimens are very similar. Therefore, it is adequate to define the critical state line using only isotropically consolidated specimens. Manica et al. (2021) calibrated CASM using only tests series presented by Fourie & Tshabalala (2005) for the material with a fine content (FC) of 60%. This decision was justified by the fact that samples collected near the failure area showed a fine content (FC) of approximately 60%, as reported by Fourie et al. (2001).

The undrained stress paths from isotropic and  $K_0$  consolidated triaxial tests shown in Figure 58 were digitized from Fourie & Tshabalala (2005). Comparison of the instability line derived from isotropic and  $K_0$  consolidated triaxial tests shows that the lower limit of the instability line was defined by isotropically consolidated specimens, whereas the upper limit was defined by  $K_0$  consolidated triaxial tests. The slopes of the instability lines were 0.76 and 0.96, corresponding to  $\phi_{flow}$  values of 24.5 and 19.9, respectively. Fourie and Tshabalala also assessed the effect of consolidation on the location of the CSL. They concluded that the CSL was unaffected by whether the specimens were isotropically or  $K_0$  consolidated, indicating that either series of tests could be used to define the CSL. However, examining the data in Figure 58, the CSL in the q-p' plane

appears slightly different between the two series of tests, with  $M_{tc} = 1.56$  for  $K_0$  consolidation and  $M_{tc} = 1.35$  for isotropically consolidated specimens. The cause for this discrepancy is unclear.

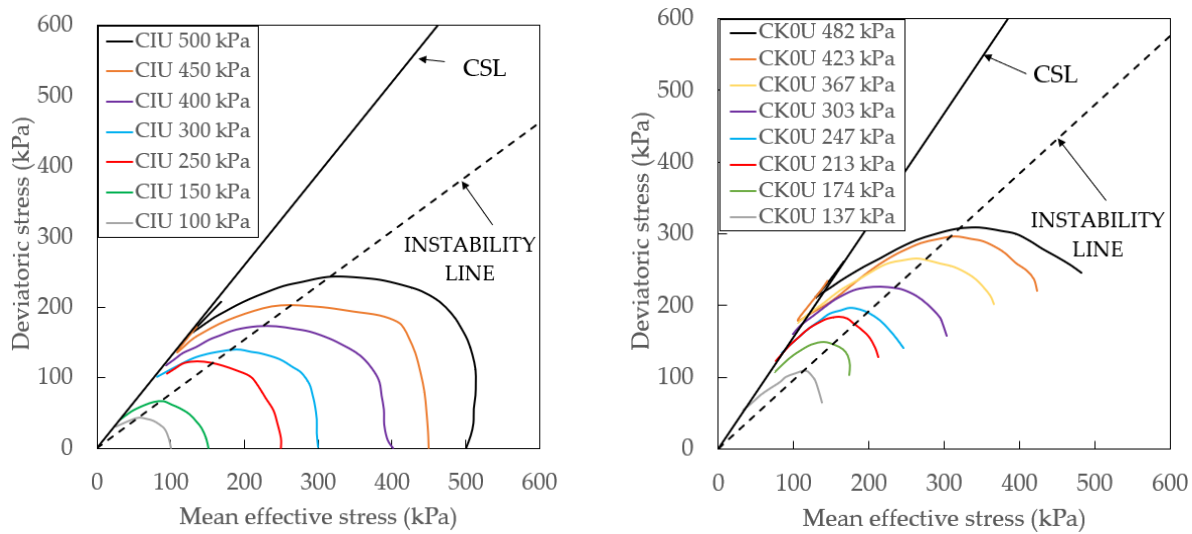


Figure 58 – Undrained triaxial tests on (a) isotropically consolidated specimens; (b)  $k_0$  consolidated specimens (Fourie and Tshabalala, 2005).

Nonetheless, the critical state parameters employed in the subsequent analysis are derived from  $K_0$  consolidated specimens, in alignment with the methodology used by Manica et al. (2021). Tables 10 and 11 provide a comprehensive summary of the majority of the model parameters for both the starter dam and the tailing material utilized in the analysis. In Table 10, some CASM parameters are indicated as variable because the different scenarios investigated require specific adjustments to those model parameters. Detailed explanations and justifications for these adjustments will be presented in the forthcoming sections of the study.

Table 10 CASM parameters for tailings in the Merriespruit base model.

| Employed parameters |             |          |      |       |       |        |                      |
|---------------------|-------------|----------|------|-------|-------|--------|----------------------|
| $\Gamma^*$          | $\lambda_e$ | $M_{tc}$ | $k$  | $\nu$ | $K_0$ | $\psi$ | $n, r, S_p, N, \eta$ |
| 0.89                | 0.03        | 1.56     | 0.01 | 0.3   | 0.6   | 0.1    | Variable             |

Table 11 Mohr-Coulomb parameters for the starter dam in the Merriespruit base model.

| Employed parameters |       |           |         |
|---------------------|-------|-----------|---------|
| E (kPa)             | $\nu$ | $\varphi$ | c (kPa) |
| 100 E <sup>-3</sup> | 0.3   | 40        | 20      |

### 8.3.3. Stress initialization

According to Wagener (1997), the construction of the dam started in 1978, employing the "upstream semi-dry paddock" construction method. Following the formation of the outer wall, a slurry mixture was systematically placed in layers with thicknesses ranging from 150 to 200 mm. After a section of the exterior walls had sufficiently dried, the paddock walls were raised once more, and the subsequent layer of slurry was deposited. Regrettably, a comprehensive chronology delineating the distinct phases of dam construction is unavailable. This lack of historical records precluded the possibility of generating a realistic simulation of the dam construction using Plaxis. Hence, the approach adopted in this study aligns with the methodology employed by Manica et al. (2021), wherein an initial equilibrium state is directly implemented on the pre-failure geometry. To this end a Mohr-Coulomb material model with nil cohesion is employed to represent the tailings. Friction is chosen large enough to avoid plastic states and Poisson ratio is chosen to impose the assumed coefficient of earth pressure at rest  $\mu = K_0/(1 + K_0)$ .

The selection of the friction angle ( $\varphi$ ) for the fictitious Mohr-Coulomb material took into account the undrained response of the material that was to be explored later with the model. To this end the friction angle was chosen as  $\varphi = 24.5^\circ$  which was the threshold of undrained instability derived from the  $K_0$ -consolidated triaxial tests (Fourie & Tshabalala, 2005), corresponding to the instability line ( $\varphi_{flow}$ ) depicted in Figure 58b. The influence of this choice was examined through two distinct scenarios. In the first scenario, initial stress state calculations incorporated a friction of  $33^\circ$  (as illustrated in Figure 59a), while in the second scenario a friction angle of  $26.2^\circ$  was employed (as depicted in Figure 59b).

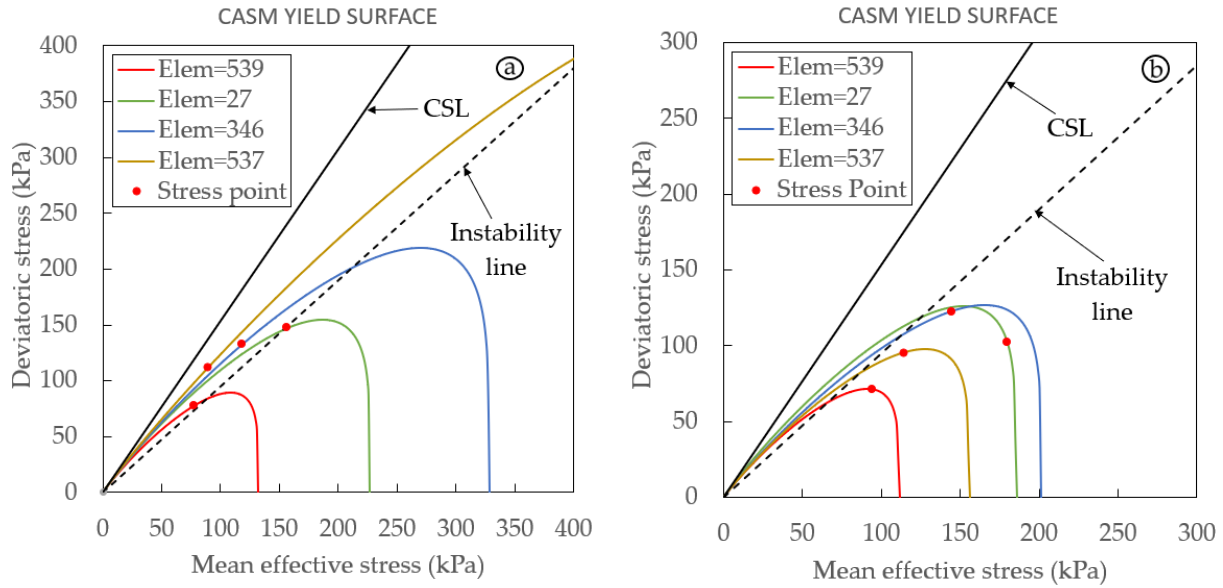


Figure 59 – Initial stress state using (a) a friction angle greater than the slope of the effective stress instability; (b) a friction angle equal to the slope of the effective stress instability.

Figure 59 illustrates four distinct stress points, marked with red dots, each associated with its respective yield surface in a normally consolidated state. These stress points are within elements 539, 537, 346, and 27 of the mesh, as shown in Figure 60. Constraining the friction angle to the threshold of  $26.2^\circ$  results in a stress state that is always within the stability zone. As depicted in Figure 59a, when the friction angle exceeds  $26.2^\circ$ , the stress points are situated above the instability line, indicating an unstable stress region.

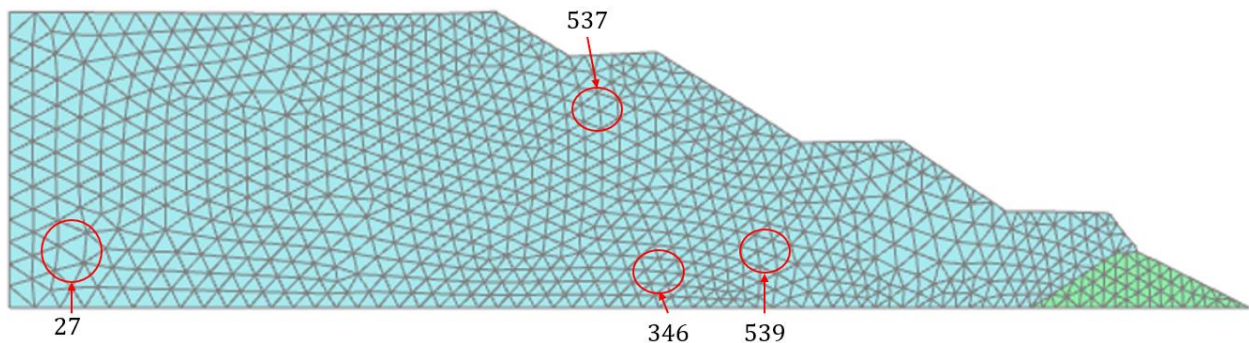


Figure 60 – Mesh of the Merriespruit base model, showing the stress points being investigated

## 8.4. Viscous effects on tailing dam stability upon triggering

### 8.4.1. CASM-Visco

In soil mechanics and other branches of engineering, it is sometimes the case that the relevant behavior of materials includes both plasticity and creep. Indeed, experimental observations for most soils shows more or less pronounced viscous properties. In ideally plastic materials plastic strains develop instantaneously, without time being a direct factor. However, creep strains develop gradually over time. To address that kind of behavior, a broad spectrum of visco-plastic constitutive models has been introduced. One of them is Perzyna's (1966) visco-plastic theory, which has been frequently applied to reproduce time-dependent effects of geomaterials (Desai et al., 1995; Yin et al., 2010).

A key concept that distinguishes viscoplastic theory from inviscid plasticity theory is that stress states beyond the yield surface are allowed in viscoplasticity. If the external loading remains constant, the stresses state would return to the yield surface as a function of time. Because of this feature Perzyna's viscoplastic theories are commonly called overstress laws. Figure 61 graphically illustrates the overstress theory as proposed by Perzyna (1966).

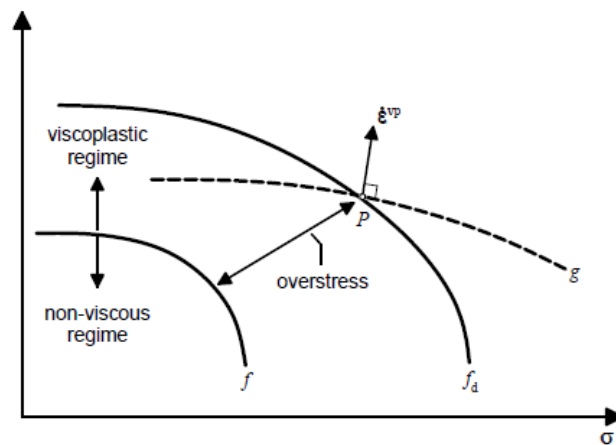


Figure 61 –Inviscid and viscoplastic yield Surfaces according to Perzyna's theory

Similar to the rate-independent theory the strain rate is decomposed into an elastic and a viscoplastic strain rate:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{vp} \quad (97)$$

The viscoplastic strain rate evolves via a flow rule:

$$\dot{\varepsilon}^{vp} = \dot{\lambda} m \quad (98)$$

where  $\dot{\lambda}$  is a non-negative parameter, known as the consistency parameter (Simo & Hughes, 1998). This parameter specifies the magnitude of  $\dot{\varepsilon}^{vp}$ , while  $m = dg/d\sigma$  determines the direction of  $\dot{\varepsilon}^{vp}$ . CASM-visco, initially introduced in Arroyo & Gens (2021), is a viscoplastic extension of CASM, obtained through the overstress theory of Perzyna (1966). Strains rates in CASM-visco are given by:

$$\dot{\varepsilon}^{vp} = \frac{\langle \Phi(f) \rangle}{\eta} \frac{dg}{d\sigma'} \quad (99)$$

Where  $\eta$  is a parameter controlling viscous effect,  $\Phi$  the overstress function that depends on the rate-independent yield surface, and  $\langle \cdot \rangle$  are the Macaulay brackets such that:

$$\langle \Phi(f) \rangle \begin{cases} \Phi(f) & \text{if } \Phi(f) \geq 0 \\ 0 & \text{if } \Phi(f) < 0 \end{cases} \quad (100)$$

To define  $\Phi$ , the following equation was employed

$$\Phi(f) = \left( \frac{f}{f_0} \right)^N \quad (101)$$

where  $f_0$  is a reference normalization stress and  $N$  is another parameter that control the viscosity. While  $N$  is dimensionless,  $\eta$  has units of  $m^2 skN^{-1}$ . In section 8.4.2, a series of simulations will be conducted using CASM-visco to understand how these two parameters control viscosity.

### 8.4.2. Triaxial test simulations using CASM-visco

To demonstrate the capabilities of the CASM-visco, a series of anisotropically consolidated triaxial compression tests were simulated using the soil test module of Plaxis, applying various combinations of viscosity parameters ( $N$  and  $\eta$ ) at different strain rates ranging from  $1.00E-5$  (1/min) to  $1.00E-2$  (1/min). Figure 62 clearly illustrates the effect of these different strain rates and parameters. Overall, it is evident that higher strain rates correspond to higher maximum deviatoric stress, while the test simulated using inviscid material show the lowest deviatoric stress value. It is also clear that as  $N$  increases the additional strength due to this viscous effect is more marked.

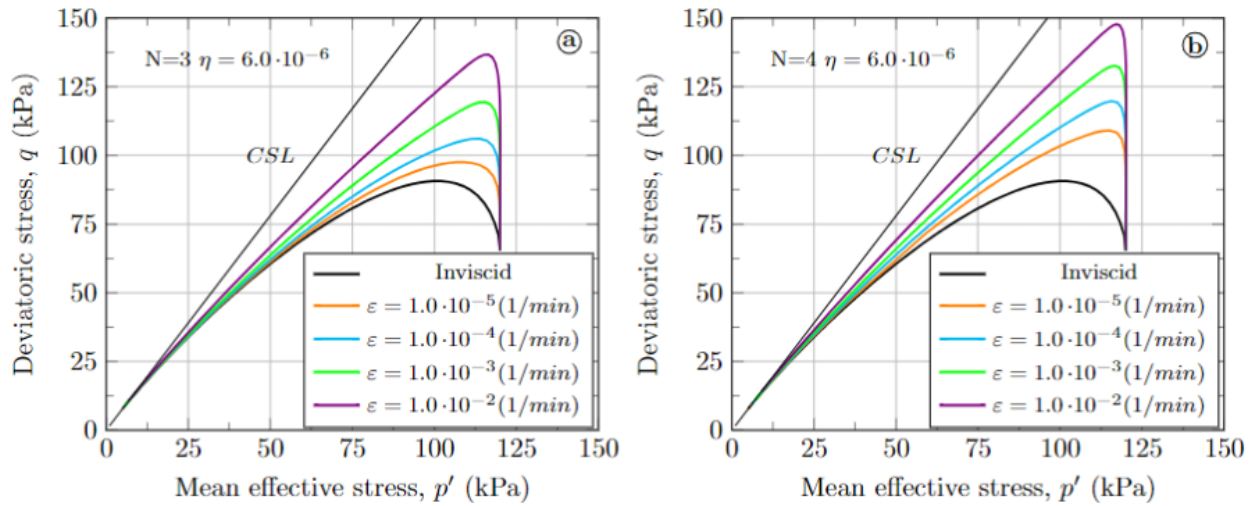


Figure 62 – Undrained stress paths of triaxial test simulations with varying strain rates and viscosity parameters.

To gain further deeper insights into the impact of each viscous parameter,  $N$  and  $\eta$ , additional simulations were conducted using the same strain rate range as before but over a wider range of viscous parameter values. As shown in Figure 63a, when keeping  $\eta$  equal to  $6.0E^{-6}m^2dayKN^{-1}$ , a considerable increase in peak undrained stress is noticed when  $N$  increases from 1 to 3, but no significant variation is observed when  $N$  is further increased to 4. At the same time, as outlined in Figure 63b, the peak undrained stress increases at a similar rate with strain rate for  $\eta$  values of 1.5, 3 and  $6.0E^{-6}m^2dayKN^{-1}$ . It is also worth noting that various combinations of  $N$  and  $\eta$  may result in similar rate effects on the undrained peak stress.

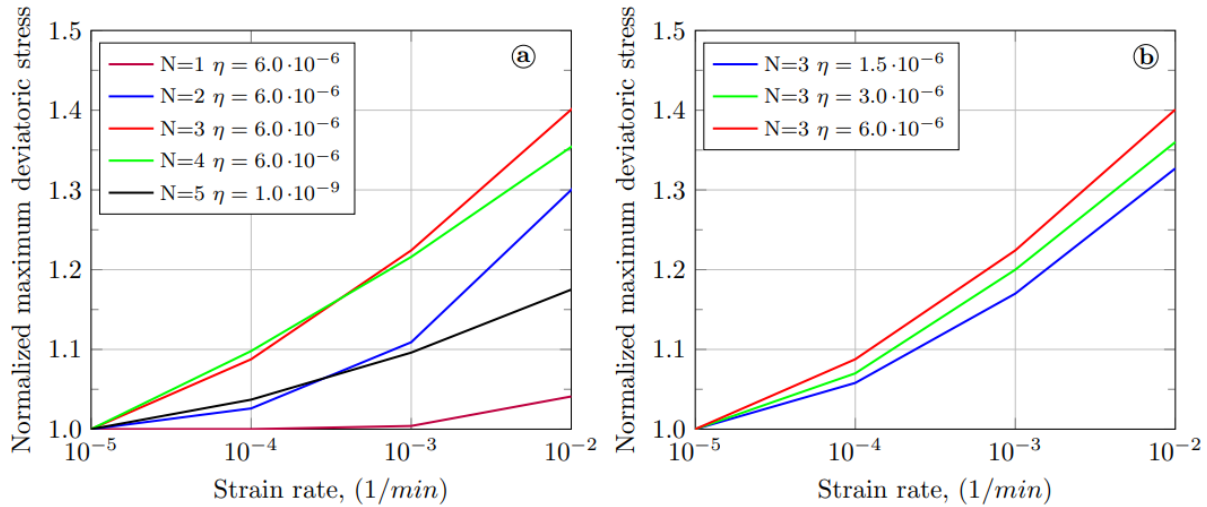


Figure 63 – Peak undrained strength increases with strain rate: (a) effect of  $N$ , (b) effect of  $\eta$ .

#### 8.4.3. Undrained creep in triaxial conditions

Experimental evidence shown that while drained creep can stabilize granular materials, undrained creep can lead to sharp pressure increases and may lead to instability due to the shift of stress states into regions of potential instability (Lade et al., 1997). Leong & Chu (2002) further demonstrated experimentally that loose sand may become unstable due to creep under undrained conditions, even when the stress states before creep are below the traditional instability threshold. This creep-induced instability is marked by increased pore-water pressure and the establishment of a creep-induced instability line, which delineates conditions for potential failure. The results of these tests are presented in Figure 64.

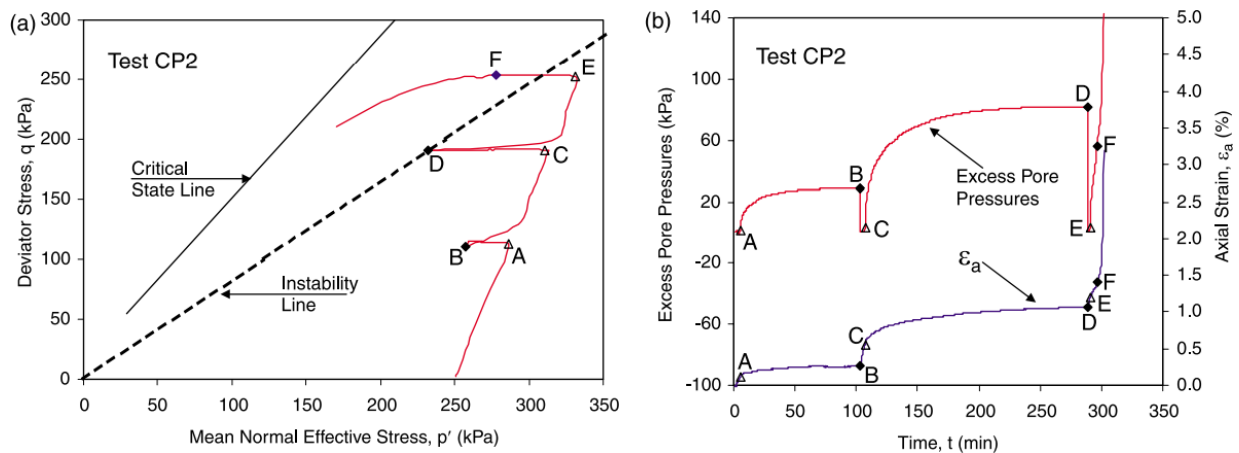


Figure 64 – Undrained creep observed in triaxial test conducted by Leong & Chu (2002)

Complementary findings by Marinelli et al. (2018) provided a mechanical interpretation using a viscoplastic model, revealing that creep strain acceleration precedes the sharp increases in pore pressure, acting as a precursor to failure.

We examined if CASM-visco could reproduce this creep-induced instability. Unlike previous simulations using the element test tool of Plaxis, these employed a one square meter domain with assumed symmetry around the central axis. The domain was divided into 32 triangular elements with 15 nodes each. This approach offers greater versatility for controlling the test in Plaxis, allowing for multiple phases and the ability to switch from drained to undrained conditions. Indeed, the test was executed in two stages: drained triaxial compression and undrained creep, following the protocol applied in laboratory experiments (Leong & Chu, 2002). The drained triaxial compression phase began from an isotropic state, loading at a rate of 0.1 kPa/min up to a deviatoric stress of 100 kPa. During the undrained creep phase, the deviatoric stress was held constant for a period of 10 days.

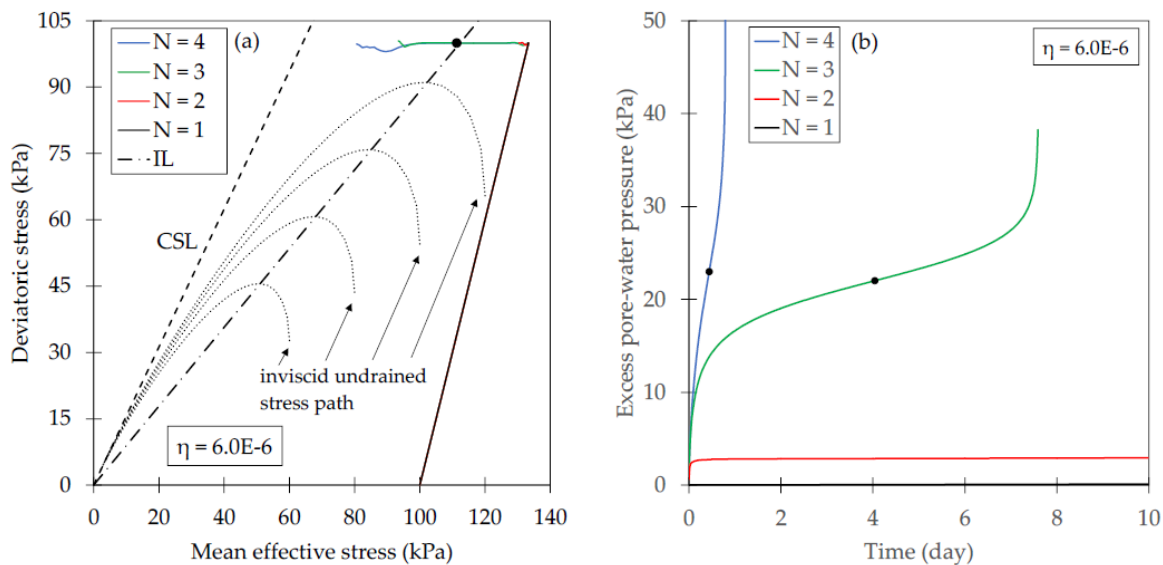


Figure 65 – Comparison of simulated results and measured laboratory data on soil type 1:

(a) Stress path; (b) volumetric strain.

Figure 65a presents the stress paths for various  $N$  values during the undrained phase. At  $N=1$  and  $N=2$ , the material exhibits minimal creep, i.e., a short undrained path ( $< 1$  kPa and less than 3 kPa

respectively). Conversely, when creep is substantial ( $N = 3$  and  $N = 4$ ), it can cause instability (i.e. liquefaction) which is observable as an increase in the excess pore pressure generated during creep. Notably, the onset of creep instability occurs when crossing the instability line determined using inviscid material.

Additional undrained creep simulations were conducted at loading rates of 0.1 and 0.01 kPa/min to study the effect of stress rate during the precedent drained shearing. The simulation procedure consisted of two phases: a drained loading phase where the deviatoric stress was gradually increased at a constant rate, followed by an undrained loading phase where the deviator load and cell pressure were held constant. The results of these simulations are shown in Figure 66 and Figure 67, showing that at lower stress levels, the pore water pressure initially increases but eventually stabilizes. However, at higher stress levels, the behavior becomes unstable as indicated by the sudden increase in pore pressure shown in Figure 66b and Figure 67b. Additionally, it is observed that higher stress rates during the drained phase lead to larger pore water pressure generation when undrained condition are applied. Specifically, when the deviatoric stress is 100 kPa and  $\dot{\epsilon}_q = 0.01 \text{ kPa/min}$ , undrained creep does not result in instability as the pore water pressure stops increasing over time. However, at a higher value of  $\dot{\epsilon}_q = 0.1 \text{ kPa/min}$ , the behavior becomes unstable. Thus, the undrained creep is influenced by the loading rate applied during the drained phase.

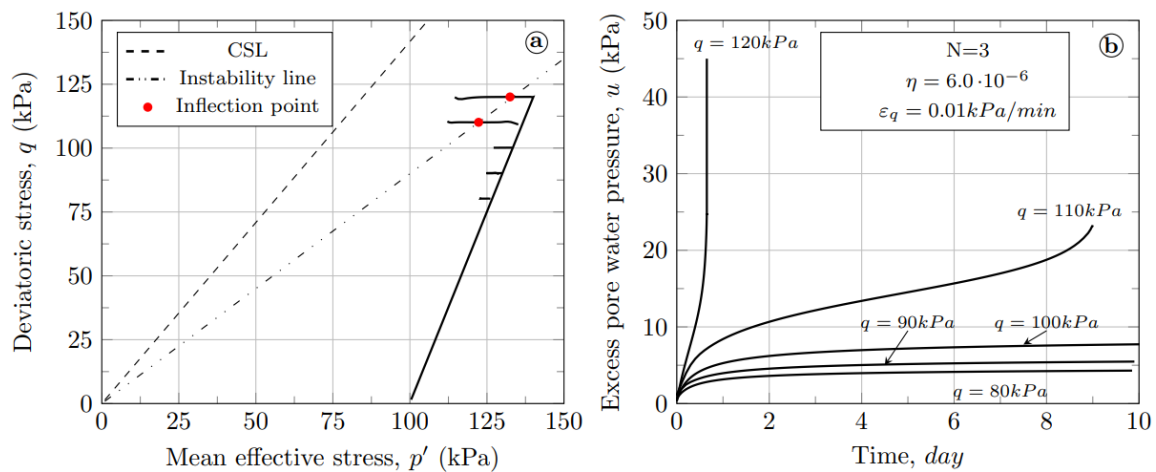


Figure 66 – Simulations of triaxial tests at a stress rate of 0.01 kPa/min. (a) Effective stress path and (b) excess pore water pressure.

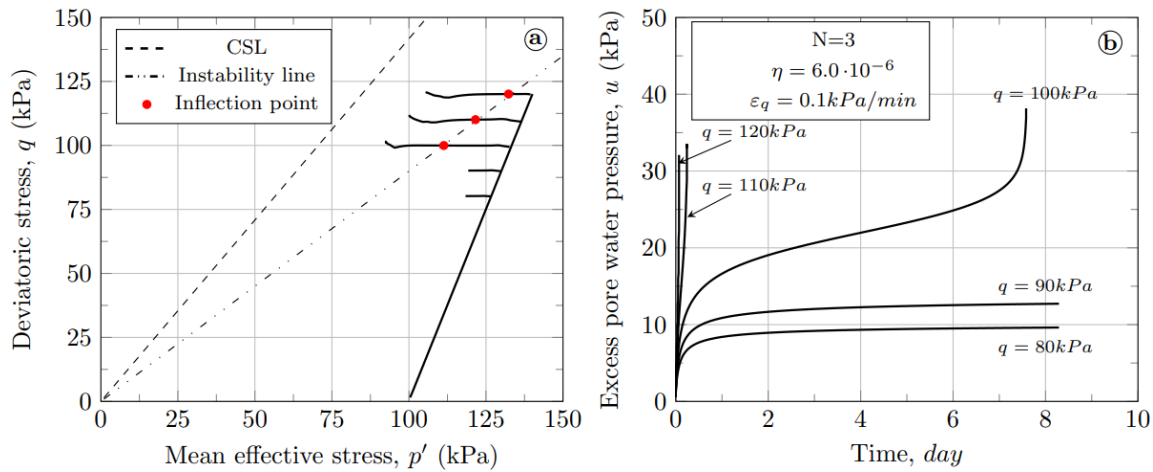


Figure 67 – Simulations of triaxial tests at a stress rate of 0.1 kPa/min. (a) Effective stress path and (b) excess pore water pressure.

The viscous parameters ( $N$  and  $\eta$ ) only determine the rate at which visco-plastic strains develop over time, not their magnitude. When the loading rate is faster during the drained phase, less visco-plastic deformation occurs during that phase, thus the deformation accumulates and develops during the undrained phase, causing further excess pore water pressure. Lastly, when the excess pore pressure generated during undrained creep exceeds the instability line, the evolution of pore water pressure becomes irreversible. This kind of results has also been observed in the laboratory (Leong & Chu, 2002). It suggests that all other things being equal, a steeper dam will be more prone to creep (Pinedo et al., 2022).

#### 8.4.4. Viscous and consolidation effects upon triggering

At the Merriespruit tailings dam (MTD) in South Africa (Strydom & Williams, 1999; Wagener, 1997a), dam overtopping led to erosion of the lower slope, which subsequently triggered a large failure that evolved into a destructive flowslide. Fourie et al. (2001) identified static liquefaction as a key factor in these events. Manica et al. (2021) demonstrated through stress-deformation analysis using the CASM model that even minimal erosion of the lower slope could trigger liquefaction. No data suggests that the Merriespruit tailings (PI between 1% and 8%, Fourie et al. (2001)) were actually viscous but, as shown by Mánica et al. (2021b), the case offers an interesting platform to explore the combined effects of consolidation and creep in triggering.

To gain a deeper understanding of these effects, the analyses utilized a coupled hydromechanical approach based on CASM-visco, assuming a tailings permeability of 3.0 m/year, as determined by Fourie et al. (2001). For the starter dam, a permeability value twice as high as that of the tailings was employed. The analysis was conducted in two phases: the first for creating the initial stress state and steady state hydraulic conditions, while the second phase simulated the erosion of the lower slope. We present results for alternative triggering hypothesis, in which the slope erosion at the dam is assumed to take place at different rates. Results are represented as the force reduction factor (FRF) versus horizontal displacement at a specific node located on the slope (Figure 57). The erosion phase is assumed to take place in 105 minutes (Figure 68), whereas the erosion phase shown in Figure 69 lasted after 1 day. All the simulations were performed using a value of  $\eta = 6.0E^{-6}m^2dayKN^{-1}$ .

In the first series, the parameter N was varied to represent different levels of viscous response. As shown in Figure 68a and Figure 69a, failure during erosion only occurred in the lower viscosity materials, with N values of 1 and 2. On the other hand, highly viscous materials with N values of 3 and 4 did not experience any global instability during the erosion phase. The reason is that the extra undrained strength resulting from viscous effects delays load transfer across any shear surface, thus reducing the spread of pore pressure generation during the erosion phase.

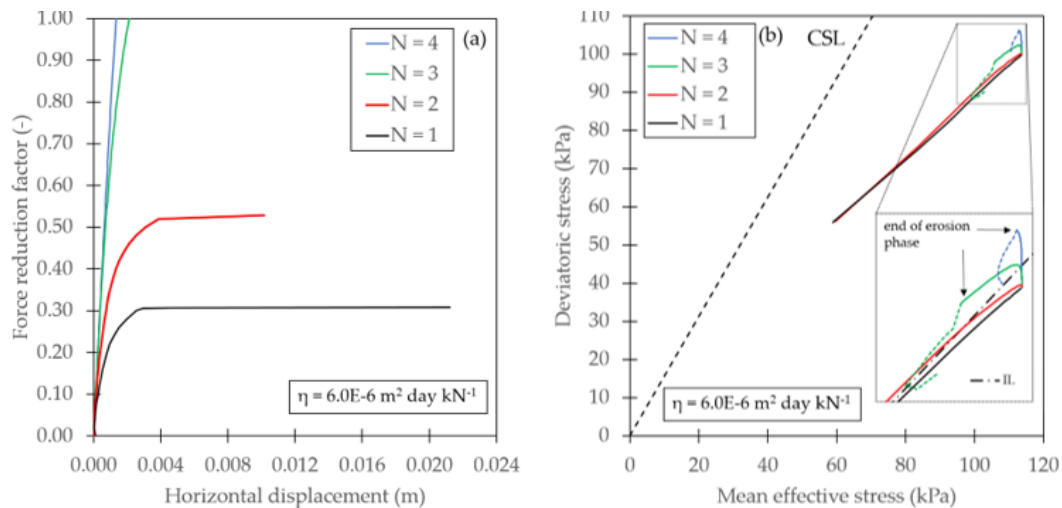


Figure 68 –Simulation results at the observation node assuming an erosion phase duration of 105 minutes (a) Force reduction factor vs. horizontal displacements(b) Stress path.

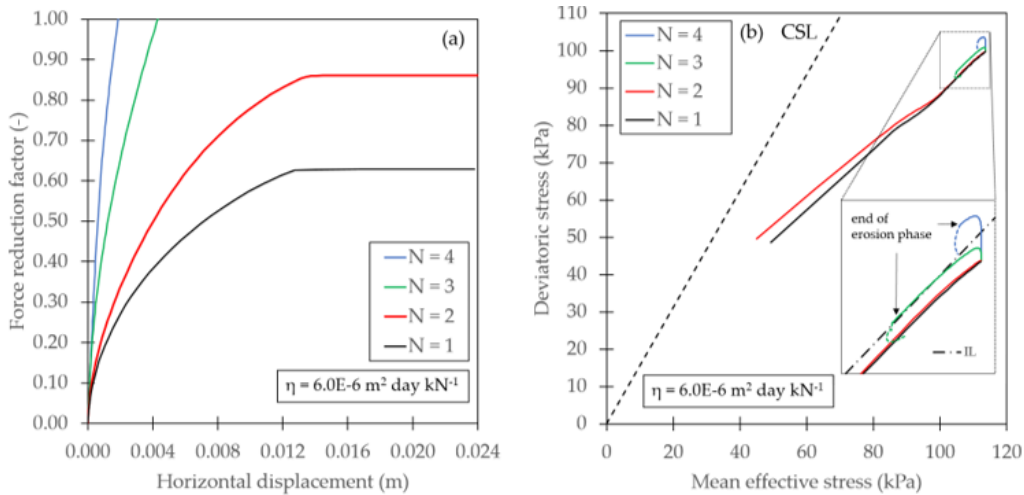


Figure 69 –Simulation results at the observation node assuming an erosion phase duration of 1 day (a) Force reduction factor vs. horizontal displacements(b) Stress path.

The benefit of lesser pore pressure generation due to the extra viscous strength is compounded when more time is allowed for consolidation. Stress paths at the most stressed Gauss points can cross the stability line (see stress path for  $N = 4$ ), but this does not imply overall failure because, contrary to what happens in the triaxial experiment, undrained behavior is not forced here. By comparing the force reduction factor at various erosion rates, it becomes clear that a larger amount of material can be eroded before reaching failure if the erosion period lasts 1 day instead of 1h45'.

Figure 70 illustrate the contours of incremental deviatoric strain using different values of  $N$  ranging from 1 to 4 while keeping  $\eta$  constant. When  $N$  is set to 1 or 2, a failure surface is clearly visible through a shear band at the bottom of the tailings, which aligns with the results found by Manica et al. (2021) . However, when the  $N$  value is increased to 3 and 4, as shown in Figure 70c-d, the erosion phase is completed without any signs of global instability and the onset of liquefaction is not observed at the lowermost part of the tailings.

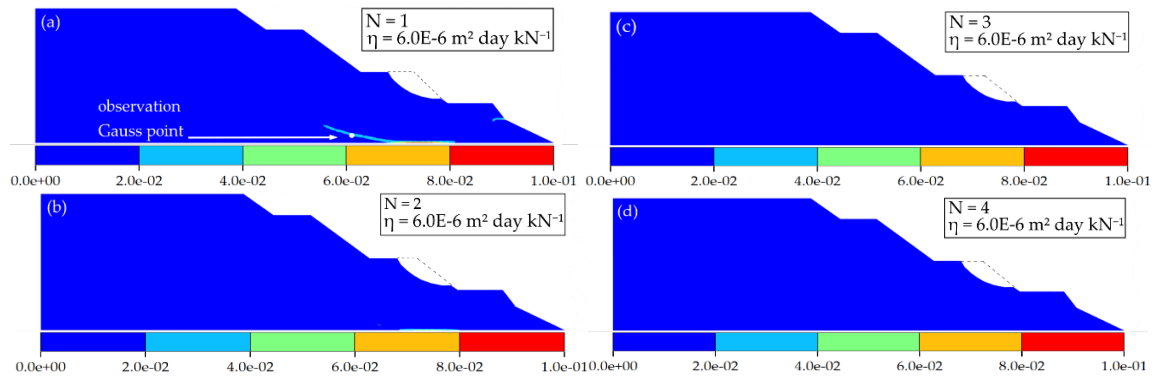


Figure 70 – Incremental deviatoric strain at the last converged increment for different  $N$  values.

Figure 71 depicts the contours of excess pore pressures at the end of the erosion phase. The position of the shear bands generated (Figure 71a-b) closely resemble those obtained in incremental deviatoric strains for  $N = 1$ , however in this case, there is a more widespread distribution throughout the tailings. Conversely, Figure 71c-d exhibit a minimal generation of excess pore pressures for  $N = 3$  and  $N = 4$  respectively.

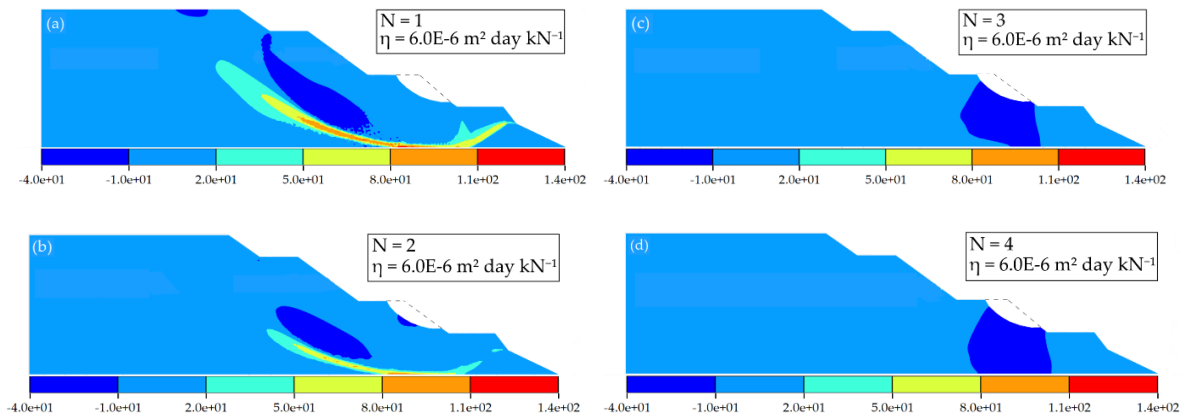


Figure 71 – Excess pore-water at the last converged increment for different  $N$  values.

The relevance of the erosion period for failure was further examined in a new simulation series conducted to study the effect of erosion rate and drainage conditions on viscous response. In these simulations, five different erosion period durations were selected while the viscous parameters  $N$  and  $\eta$  were kept constant, as shown in Figure 72. As expected, when erosion occurs over a short time frame, high pore water pressures are generated. Specifically, the excess pore pressure was found to range between 90 and 105 kPa, assuming that the erosion phase lasted

between 10 and 105 minutes. This suggests that when erosion occurs quickly, the soil behaves as if it is undrained. However, as the erosion phase is prolonged, the excess pore pressure tends to dissipate. This is evident from the erosion phase of 2 days where negligible excess pore pressure was generated, as shown in the contours of excess pore pressures in Figure 73d.

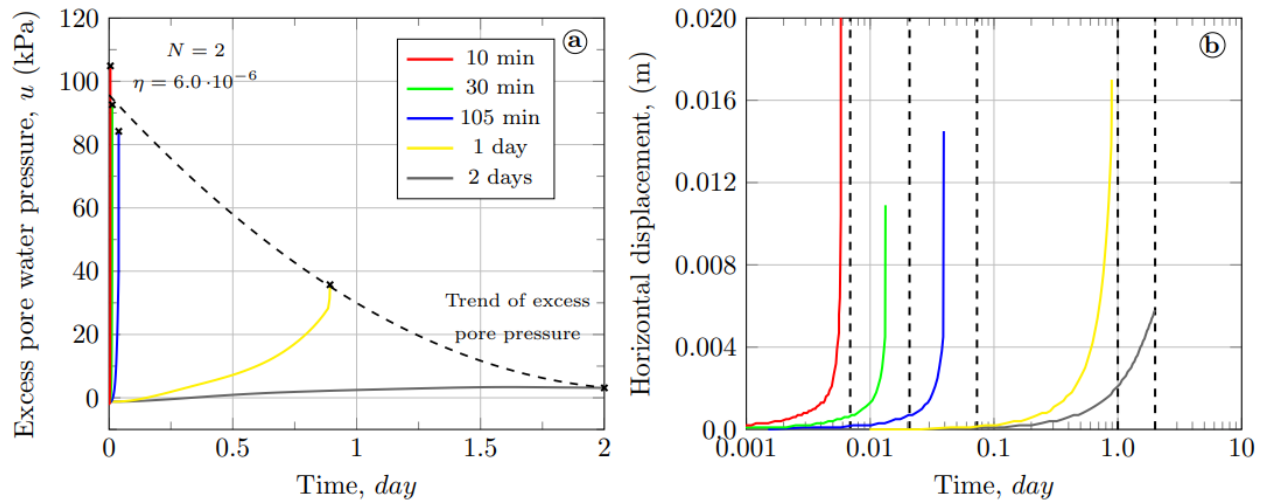


Figure 72 – Simulation results at the observation node. Trend of excess water pressure and (b) horizontal displacements for different duration of erosion phase for a single value of  $N$  and  $\eta$

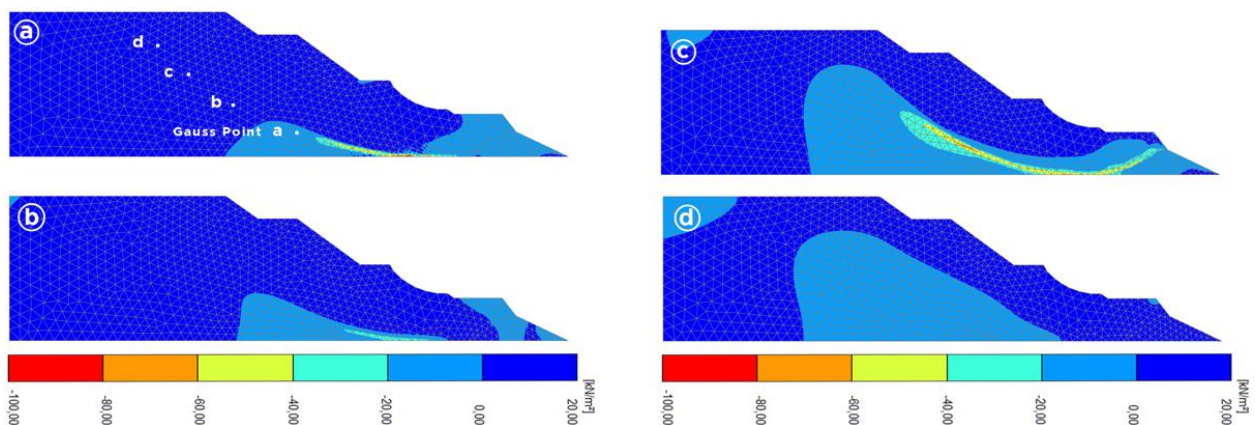


Figure 73 – Excess pore-water at the last converged increment using specific viscous parameter but with different erosion periods.

Incorporating a visco-plastic model in hydromechanical analyses requires consideration of key factors, namely permeability, load rate, and viscosity. The viscosity of soil contributes to a temporary extra undrained peak strength for a certain duration. However, applying load at a high rate generates higher excess pore pressures, leading to reduced soil strength. Additionally, permeability plays a crucial role in the dissipation of excess pore pressure, with highly permeable materials showing faster dissipation compared to less permeable materials. These findings provide a deeper understanding of the interplay between these factors and their impact on stability.

## **8.5. Impact of anisotropic consolidation on CASM calibration and triggering analysis**

### **8.5.1. Undrained peak strength ratio.**

The calibration of CASM through  $S_p$ , as detailed in Section 6.3, appears attractive due to the multiple options available for determining this parameter. CPTu tests have been extensively used to determine the maximum undrained strength of soils. In this respect, numerous researchers have suggested links between the undrained strength and CPTu parameters. Among the most prevalent methodologies is the correlation of the corrected cone resistance  $q_t$  with the undrained peak strength, by means of the cone factor  $N_{kt}$  as follows:

$$q_t - \sigma_v = s_u N_{kt} \quad (102)$$

Moreover, numerical simulation of CPTu employing the Particle Finite Element Method (PFEM) can offer new insights into obtaining the peak and residual strength of soil (Monforte et al., 2022). However, to directly evaluate the undrained behavior of soil, triaxial compression tests are typically conducted. Although most investigations have been executed on samples that have been isotropically consolidated, it has been recognize that anisotropic consolidation prior to the shear can potentially impact the soil fabric and consequently modify the undrained behavior of the soil. As example, Figure 74 shows undrained triaxial peak strength envelopes for tailings obtained after either isotropic and anisotropic consolidation. These linear envelopes, passing

through the origin, are fitted to the undrained peak strength derived from the loose undrained triaxial compression tests, plotted against their respective mean effective stress at the end of consolidation.

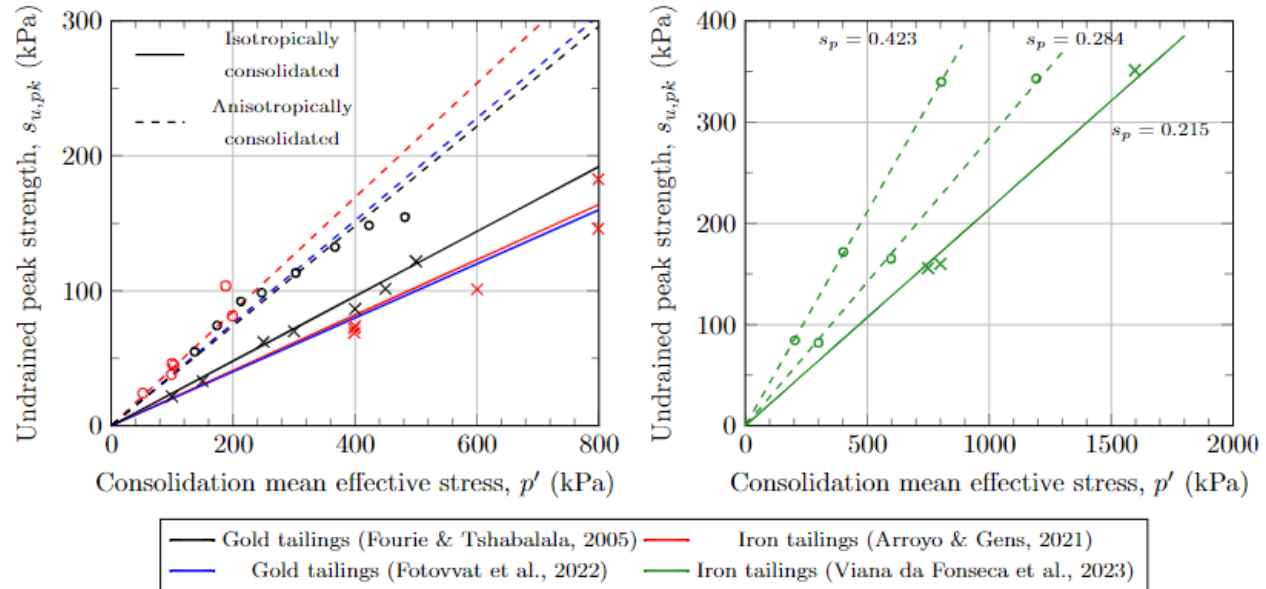


Figure 74 – Effect of consolidation path on the peak undrained strength of tailings observed on triaxial compression.

The interesting aspect is that the normalized undrained strength ratio depends on the consolidation path, presenting much larger values of  $S_p$  for anisotropic consolidation than for isotropic consolidation. Specifically, Viana Da Fonseca (2023) reported that when tailings were consolidated with  $K = \sigma'_h/\sigma'_v$  of 1, 0.7, and 0.5, the material exhibits  $S_p$  values ranging from 0.215 to 0.423, as depicted in Figure 74b. This finding highlights the potential influence of anisotropic consolidation on the undrained behavior of soil and underscores the importance of considering this condition in geotechnical design and analysis. A similar observation was reported in Chapter 6.5 for Brumadinho B1 tailings. Although discussed in terms of the instability line, it was noted that triaxial tests conducted along a  $K_0$  consolidation stress path exhibited a steeper instability line compared to specimen consolidated isotropically.

### 8.5.2. Predicting undrained shear strength with CASM

In 2001, Fourie & Papageorgiou conducted a series of undrained triaxial tests on Merriespruit gold tailings using the moist-tamping technique. These tests were performed on isotropically consolidated specimens with varying percentages of fines, including 0%, 20%, 30%, and 60%. Fourie & Tshabalala (2005) reanalyzed this material with the aim of investigating the effects of an anisotropic consolidation path on the fabric of the specimens. For this purpose, they conducted a series of isotropic and  $K_0$ -consolidated triaxial tests on specimens with the same four grading ranges. The results indicated that these specimens exhibited markedly more brittle behavior compared to the isotropically consolidated ones. According to Fourie et al. (2002), most of the samples obtained from areas near the failure section had a fines content (FC) greater than 50%. Therefore, it was decided to utilize only samples with a FC of 60% for the following analysis.

As previously discussed, one potential approach for calibrating CASM is to use the state parameter and the peak undrained strength ratio as inputs. Based on previous studies, the initial state parameter was set to 0.1 (Manica et al., 2021), even though this choice may be considered conservative compared to the laboratory derived state parameter reported by Fourie & Tshabalala (2005). Fourie et al. (2001) reported significantly higher  $\psi$  values from undisturbed samples obtained during post-failure investigations. Notably, over 85% of samples with an FC > 60% had  $\psi$  of 0.1 or greater, reaching up to 0.8. However, these values were estimated under the assumption that the CSL position for 60% FC tailings would also apply to tailings with higher FC, which is not always the case.

We analyzed CPTu soundings conducted 22 to 35 days after the failure by Wagener Consulting Civil Engineers (J&W), a firm tasked with investigating the causes of the failure. This information is available in Torres-Cruz (2016). CPTu data was analyzed to derive the state parameter using the Plewes et al. (1992) method. Since this CPTu data does not include  $f_s$  readings, the Plewes method was applied using the lambda derived from laboratory tests instead of estimating it using the friction ratio. The results confirm that a state parameter close to 0.1 may be appropriate.

By applying equations (53) and (55), and assuming a value of  $S_p = 0.37$  for  $K_0$  conditions, the resulting shape parameters were determined to be  $n = 5.16$  and  $r = 151.5$ , as reported by

Manica et al. (2021). Instead using the value  $S_p = 0.24$ , as determined for specimens that were isotropically consolidated, the shape parameters were determined as  $n = 2.81$  and  $r = 148.4$ . Notably, while  $r$  remains nearly constant, there is a significant variation observed in the parameter  $n$ . Since  $n$  determines the maximum deviatoric stress, and  $r$  governs the intersection point between CSL and the yield surface, it is reasonable to expect that, given the stability of  $r$ , a specimen with  $n = 5.16$  would display a more brittle behavior. A comprehensive summary of all the calibrated parameters can be found in Table 12.

Table 12 Calibrated properties of Merriespruit tailings

| Parameters  | Isotropic | Anisotropic |
|-------------|-----------|-------------|
| $\Gamma^*$  | 0.89      | 0.89        |
| $\lambda_e$ | 0.03      | 0.03        |
| $M_{tc}$    | 1.56      | 1.56        |
| $k$         | 0.01      | 0.01        |
| $\nu$       | 0.3       | 0.3         |
| $n$         | 2.81      | 5.16        |
| $r$         | 148.4     | 151.5       |
| $m$         | 1.75      | 1.75        |
| $R$         | 1.0       | 1.0         |

$\Gamma^*$  represents the void ratio where the CSL cross the 1 kPa pressure on the  $e - \ln p'$  plane.

It should be noted that for isotropically consolidated specimens, a state parameter value of 0.1 aligns with the reference state parameter. This reference state parameter essentially represents the distance between the NCL and the CSL. On the other hand, when employing a state parameter value of 0.1 in equation 76 for  $K_0$  consolidated specimens, it represents the distance between the normal consolidation line under  $K_0$  conditions and the critical state line. This implies that in the latter scenario, the reference state parameter is slightly greater than 0.1.

An effect of consolidation path on peak strength is actually predicted by CASM. To show this, a series of undrained triaxial tests was simulated using the element test tool available in Plaxis. These simulations were conducted using parameters detailed in table 12, corresponding to the isotropic conditions. The specimens were initially consolidated at mean effective stresses of 200, 400, 800, and 1200 kPa, following different consolidation paths, corresponding to different stress

ratios. Figure 75a shows that samples consolidated at different mean effective stresses but following the same consolidation path exhibit a linear relationship between the peak undrained strength and the mean effective stress at the peak. The results of these simulations are consistent with the laboratory triaxial test results. Additionally, the numerical simulations reveal that the undrained strength ratio increases for specimens that are anisotropically consolidated. It is evident that a lower stress ratio  $K = \sigma'_h / \sigma'_v$  during consolidation leads to a higher undrained strength ratio.

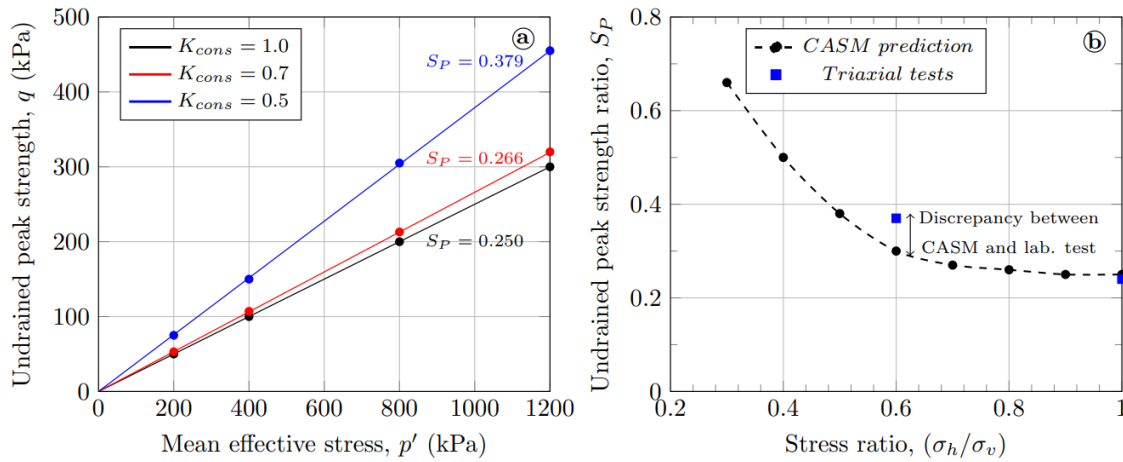


Figure 75 – Undrained strength behaviour of simulated triaxial tests,  $S_p = 0.24$ .

However, the effect of the consolidation path predicted by CASM is smaller than the one observed in experiments. Figure 75b presents a comparison between the simulation results (black markers) and the outcomes of laboratory tests (blue markers). Since CASM was calibrated with the result deduced from isotropic tests ( $S_p = 0.24$ ) the constitutive model closely reproduces that result, predicting an undrained strength ratio value of 0.25. On the other hand, the prediction for triaxial tests conducted along a  $K_0$  consolidation stress path of 0.6 show a discrepancy with measurements. Particularly, when the effective stress ratio reaches 0.6, this underestimation is almost 20%. This discrepancy may be attributed to soil fabric changes during consolidation that are not represented in the model (Manica et al., 2021).

The error is also present if the constitutive model is calibrated using as input the  $S_p$  value of 0.37, corresponding to  $K_0$  conditions. Again, good agreement between the model and the experiments is obtained for this condition, but, when considering isotropic consolidation conditions, as shown in Figure 76b, a significant discrepancy emerges. In this case, there is a substantial deviation of approximately 40% between the model's predictions and the laboratory values.

In the next section we explore how this model limitation may affect slope stability simulation results.

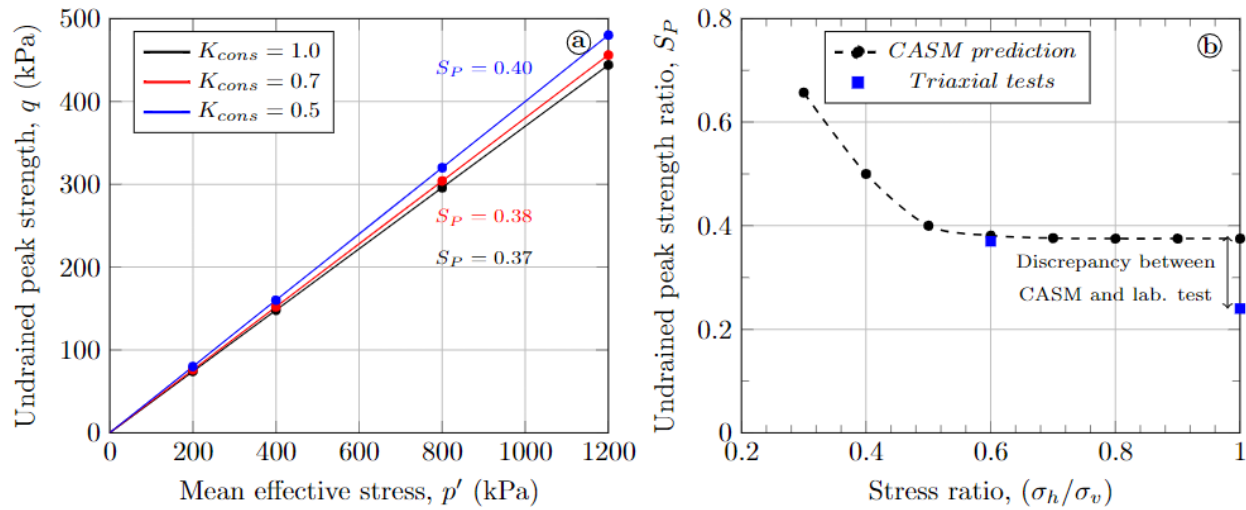


Figure 76 – Undrained strength behavior of simulated triaxial tests,  $S_p = 0.37$ .

### 8.5.3. Numerical analysis of the dam

The present simulations involved the same model geometry, material, and triggering mechanism for liquefaction as those utilized in the previous section. However, in the current analysis, the material was considered inviscid. For comparison purposes, the same model was subjected to two distinct analyses. In case A, the model used the constitutive parameters corresponding to an  $S_p$  value of 0.24, while case B employed parameters derived from  $S_p$  value of 0.37. The results, as presented in Figure 77a, shown that failure during undrained erosion occurred in both simulations. However it took place slightly later (with an FRF of 0.097) for the material calibrated

using the value deduced from isotropic tests ( $S_p$  value of 0.24), than for the material calibrated with an  $S_p$  value of 0.37, for which an unloading factor of FRF of 0.073 was obtained.

The distinct undrained soil response between the two simulations is illustrated in Figure 77b, where the reported degree of brittleness was computed using the Bishop's (1973) criterion. The results showed that the material with an  $n$  value of 2.81 (calibrated using isotropic triaxial tests) exhibited a less brittle behavior. Thus, if CASM is calibrated using the undrained shear strength obtained from isotropically consolidated triaxial tests, the model underestimates the undrained peak strength (see Figure 77b). As a result a smaller amount of soil can be removed before the onset of global instability.

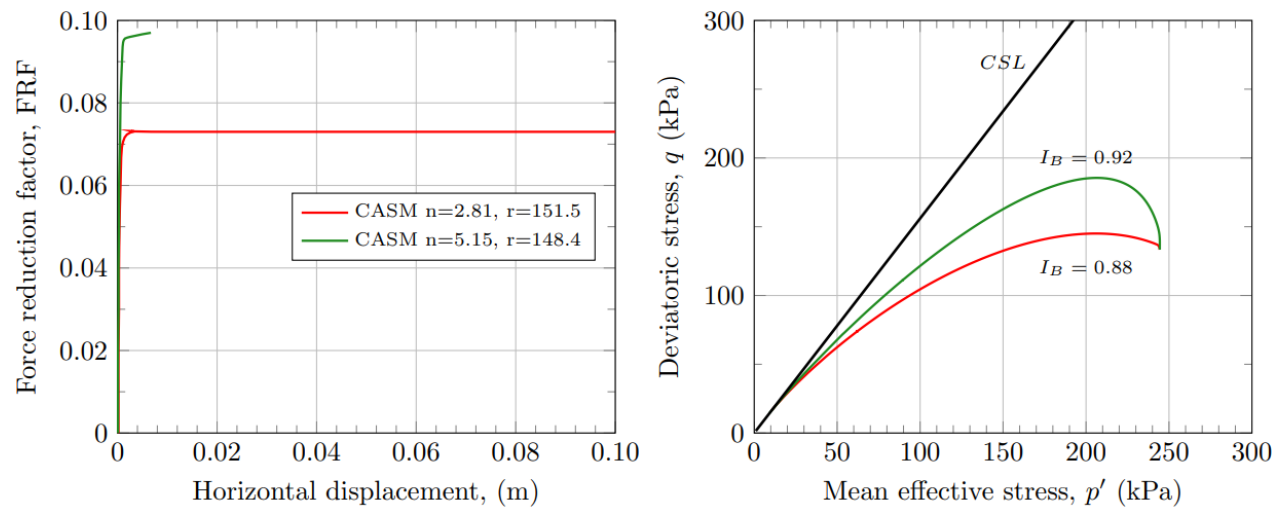


Figure 77 – a) Force reduction factor vs. horizontal displacements at the observation node; b) undrained triaxial test simulations.

The incremental deviatoric strains are illustrated in Figure 78, revealing the emergence of a shear band at the base of the eroded area in both scenarios. However, in the case calibrated with  $S_p = 0.37$  (i.e. less brittle), a more pronounced shear band is observed compared to case b. Moving on to Figure 79, the excess pore pressures is depicted at the end of the erosion phase. It is evident that these excess pore-water pressures closely trace the paths of the shear bands. Notably, in the less brittle material (Figure 79a), the excess pore pressures are considerably higher.

The results presented are indicative of the influence of calibrated undrained brittleness in the triggering simulation response. For a more thorough understanding of this issue further analysis will be undertaken in the forthcoming section.

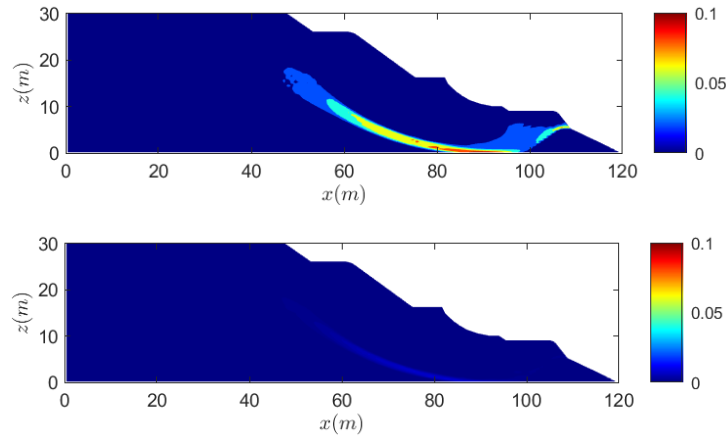


Figure 78 – Incremental deviatoric strains at the last converged increment (a)  $S_p = 0.24$  and (b)  $S_p = 0.37$ .

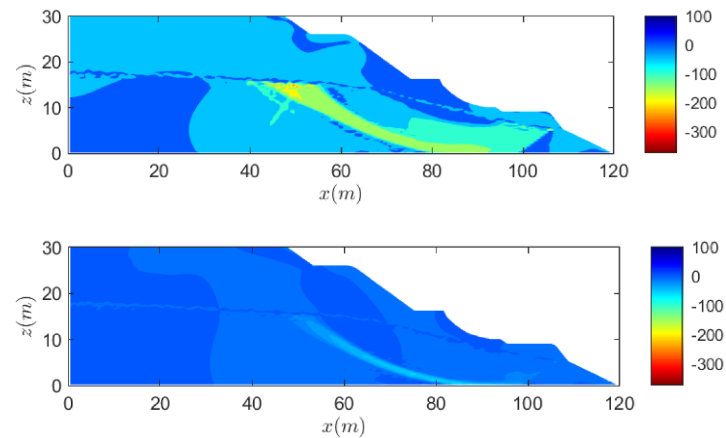


Figure 79 – Excess pore-water at the last converged increment (a)  $S_p = 0.24$  and (b)  $S_p = 0.37$ .

## 8.6. Sensitivity of triggering to material parameters controlling undrained brittleness

### 8.6.1. Introduction

Chapter 6 provides a comprehensive overview of the calibration procedure for CASM. Specifically, it explores an alternative approach to determining the parameters  $n$  and  $r$  which

influence the shape of the yield function. This approach involves the incorporation of both the undrained peak and residual shear strength as input parameters during the calibration process. Measuring undrained shear strength inherently involves some uncertainty, particularly with tailings. Obtaining a perfectly undisturbed sample for laboratory testing is virtually impossible. Therefore, an alternative approach is to reconstitute soil samples in the laboratory. However, the primary challenge lies in precisely replicating the loose in-situ deposition conditions. On the other hand, in-situ tests offer the advantage of testing soil in its natural condition and without the influence of specimen sampling and handling. Nonetheless, during the installation of in-situ testing equipment, natural soil fabric is destroyed, and despite the progress in numerical simulation (Monforte et al. 2023) interpretation of the test remains difficult and interpreted parameters uncertain. Indeed, occasional discrepancies emerge between laboratory and in-situ estimations. For example, Robertson (2022) showed that the laboratory peak strengths are significantly higher than empirical estimates based on CPTu data, while the residual strengths obtained from the laboratory are markedly lower than empirical predictions. Therefore, any numerical analysis of triggering events for a tailing dam will inevitably face a certain degree of material uncertainty that will affect the evaluation of risk implied by the analysis results. To explore these issues, we perform a sensitivity analysis of the erosion trigger in the Merriespruit TSF to the value of undrained residual and peak strength.

As explained in section 6.3.3 undrained residual and peak strength are not CASM parameters on their own, but through CASM calibration their value may be used to derive the constitutive parameters  $n$  and  $r$ . In the parametric study, the undrained peak strength ratio will be varied within the range of 0.2 to 0.33, while the undrained residual strength ratio will be adjusted between 0.02 and 0.33. This range of undrained shear strength ratios includes the values determined from laboratory tests and CPTu estimations for the Merriespruit tailings. Furthermore, this range of values is also congruent with what has been reported in TSF failure studies. For example, Jefferies and co-workers, in the technical report of the Cadia dam failure, posited that the lower limit for the peak strength ratio  $S_u^{pk}/\sigma'_{v0}$  was approximated 0.27, whereas for the residual strength, the lower limit was  $S_u^{res}/\sigma'_{v0}$  0.06. Robertson et al. (2019), in their report on the Brumadinho dam failure, documented values of peak undrained strength ratios

spanning 0.25-0.34 and 0.005-0.01 for the residual strength. Figure 80 illustrates the range of peak and residual undrained strength values used in the parametric analysis, highlighting values relevant to other significant tailings dam failures.

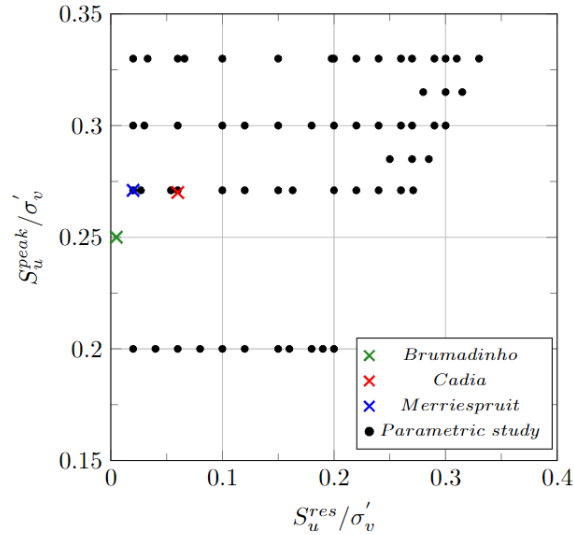


Figure 80 – Range of peak and residual undrained strength used in the sensitivity analysis.

The different combinations of peak and residual undrained strength cases studied in the parametric analysis imply different degrees of brittleness under undrained conditions. We present the different cases using, the brittleness index, as proposed by Bishop in 1976, which is:

$$I_b = \frac{(S_u)_p - (S_u)_{res}}{(S_u)_p} \quad (103)$$

CASM was thus calibrated, to obtain values of  $n$  and  $r$  equivalent to the normalized undrained strengths jus described; as additional inputs in the calibration we employed parameter reported in Table 10. The results are illustrated Figure 81, revealing that parameter  $r$  falls within a range between 2 to 156, while parameter  $n$  ranges from 1 to 7.9. Considering that  $r$  determines the point where the yield surface intersects with the critical state line (indicating the material's ultimate strength), it is reasonable to expect that higher values of  $r$  are associated with a lower undrained residual shear strength and, consequently, a higher degree of brittleness in the

material. Conversely,  $n$  governs the peak of the yield surface, meaning that higher  $n$  values correspond to a greater undrained peak strength ratio.

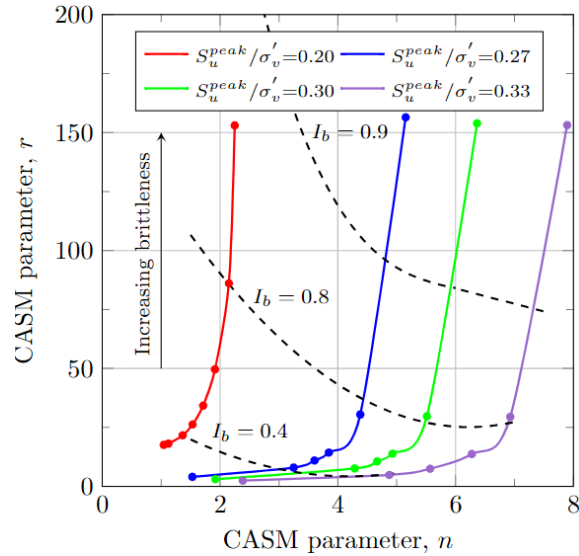


Figure 81 – Range of CASM parameters  $n$  and  $r$  used in the sensitivity analysis.

In the forthcoming section, a series of numerical analyses will be conducted, employing the range of parameters  $n$  and  $r$  depicted in Figure 81, while keeping all other constitutive parameters unchanged. This analysis will closely follow the previous one, in which the mechanism triggering liquefaction was the erosion of a specific section of the slope. Moreover, besides material brittleness, other specific soil properties that could influence the dam stability will also be considered. To address this, the study will explore a range of drainage conditions, covering both undrained and partially drained scenarios, as well as material viscosity.

### 8.6.2. Analysis under undrained conditions using an inviscid material.

In this section, the analysis attempts to replicate the failure resulting from the erosion of a portion of the slope due to overtopping. The analysis follows the same model conditions as discussed in section 8.5, which means that, in terms of drainage, the analyses are undrained and

the material is considered as inviscid. The assumption of undrained condition can be considered valid since it has been observed that the failure occurred within a short time frame, and CPTu tests suggest the presence of impermeable layers near the section where failure occurred. However, as the analysis is conducted considering undrained conditions, and the material is considered inviscid, the timing of erosion holds no significance for this analysis. This scenario can be considered the most demanding condition among all the analyses conducted.

Figure 82 shown the results of the sensitivity analysis carried out using various combinations of  $n$  and  $r$  parameters. In this figure, empty circles represent simulations in which complete erosion of the soil mass is successfully achieved without any failure. Conversely, the remaining circles vary within the range indicated in the color bar, where the red color indicates that almost 100% of the soil mass could be eroded with no failure, while the blue color represents the opposite condition.

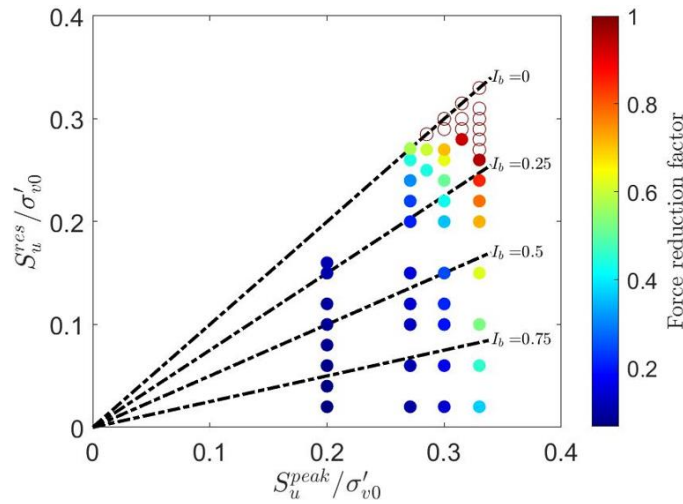


Figure 82 –Sensitivity analysis of slope erosion dam failure based on shear strength ratios and brittleness index, inviscid undrained conditions.

The dataset showcased in Figure 82 underscores the feasibility of achieving the full removal of a soil section through various combinations of residual and peak shear strength ratios. This occurs within the range where the brittleness index falls between 0% and 20%. However, to avoid liquefaction, it is worth noting that whenever the undrained peak strength ratio is below 0.27, any residual undrained shear strength results in liquefaction.

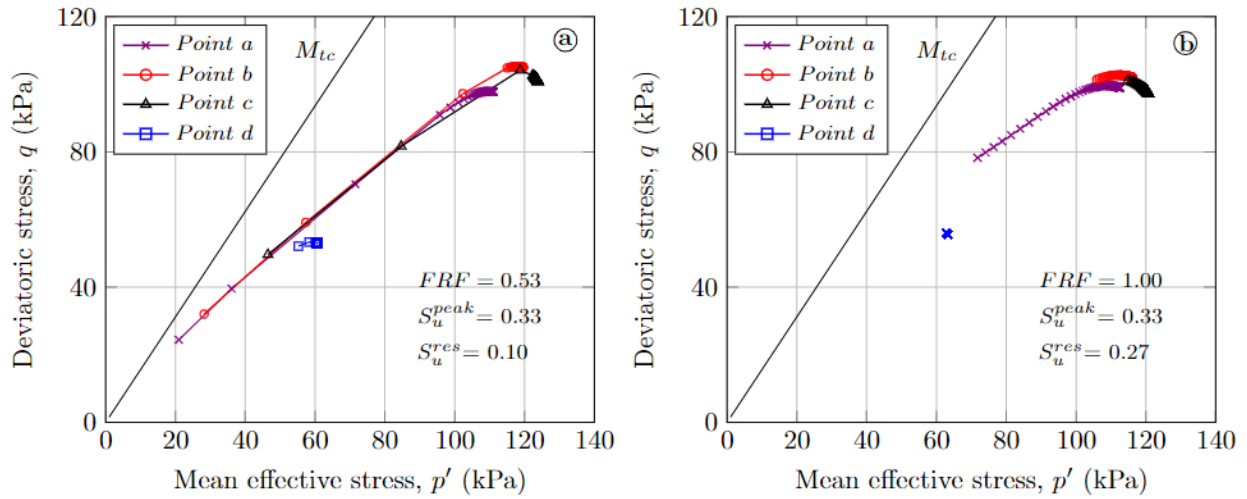


Figure 83 – Effective stress path at the 4-gauss point.

To gain some insights into the performance of the simulation, an analysis of the stress path was conducted at four strategically located gauss points within the shear band. Two distinct scenarios were analyzed: in the first scenario, dam failure was successfully prevented, enabling the complete removal of eroded soil (refer to Figure 83b); while in the second scenario, depicted in Figure 83a, dam failure occurred with an associated FRF of 53%.

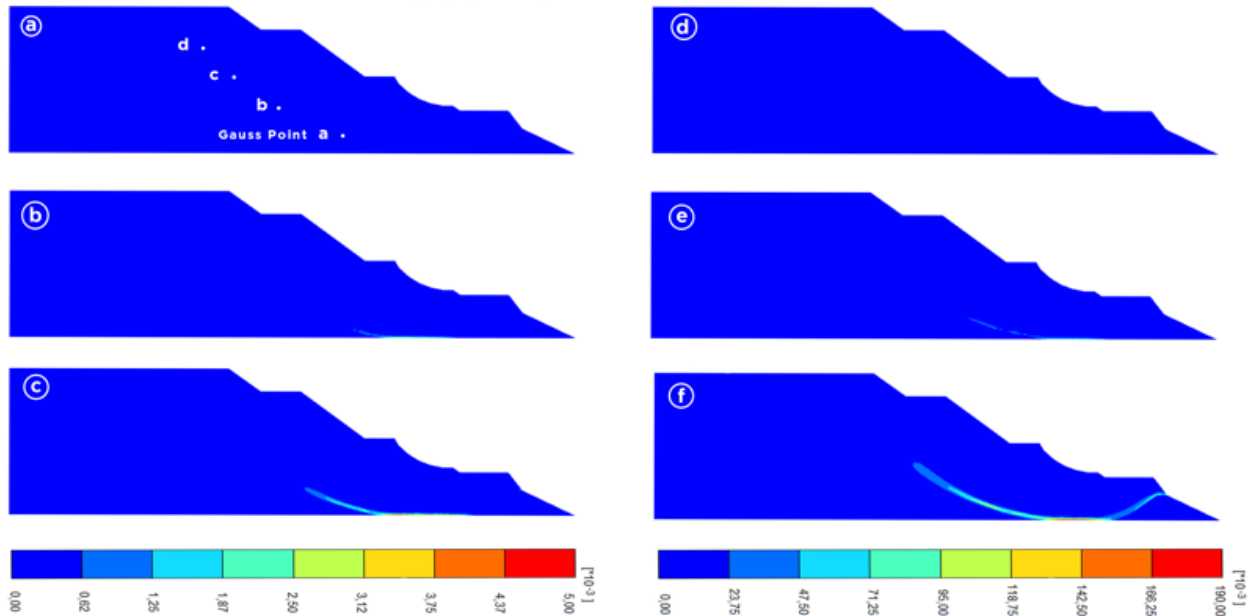


Figure 84 – Evolution of Incremental deviatoric strains: with force reduction factor at 0.53 (a-b-c-d) and force reduction at 1.00 (e-f-g-h).

The findings in Figure 83a show distinct behaviors among the four Gauss points. Points A, B, and C, which are near the onset of erosion, exhibited a gradual softening trend right from the beginning of the erosion process. In contrast, point D, located further from the initial instability zone, reached peak deviatoric stress without attaining the critical state condition. As highlighted in Figure 84c, the shear band's propagation across the dam is not uniform in the final simulation step. Although stress point D shows instability by exceeding the deviatoric peak, it has not yet reached critical state conditions.

In Figure 83b, it is evident that points A and B exceed the peak conditions, with point B attaining critical state conditions. However, it's important to note that localized instability at a few points does not necessarily imply global instability, and the gauss point that remains stable contributes to the overall stability of the dam. Moreover, it can be noted that, when stress points ultimately attain critical state conditions, they do so with values that are notably higher in comparison to the conditions depicted in Figure 83a. This distinct behavior can be attributed to higher undrained residual shear strength to which the model is calibrated in that case.

### **8.6.3. Analysis under undrained conditions using a viscous material.**

Chapter 8.4 explored the influence of viscosity on static liquefaction using various combinations of viscous parameters within a coupled hydromechanical analysis. In this section, the focus remains on the undrained condition, with a specific emphasis on tailings as a material displaying viscous properties. From a geotechnical perspective, viscosity has traditionally been studied in soft clays; however, in certain cases, tailings can also exhibit this behavior. For instance, this phenomenon is observed in oilsand tailings or when tailings processing involves some form of polymer or chemical addition that generate significant viscous deformation over time. Additionally, certain high-plasticity tailings may exhibit time-dependent compression even in the absence of additional chemical treatments intended to create a flocculated structure (Fourie et al., 2022).

Given the limited historical data concerning the erosion duration, it has been assumed, following Manica et al., (2021) that the erosion occurred over a period of 105-minute. Additionally, the parameters governing the visco-plastic behavior are set with  $N$  equal to 4, and  $\eta$  is established at

$1.0E^{-9}m^2 day KN^{-1}$ . These values represent a material with moderate viscosity, simulating real conditions that can occasionally occur in tailings deposits. It is worth noting that when incorporating a visco-plastic model into the analysis, both the viscous parameters and the time interval must be considered in the analysis.

Figure 85 illustrates the results of the simulations. As expected, a greater number of simulations were successfully completed in comparison to the inviscid analysis. Particularly, when evaluating simulations within the brittleness index ranging from 0.0 to 0.25, it is evident that, although complete removal of the soil portion without failure has not been achieved, the volume of soil that must be removed before triggering liquefaction has significantly increased compared to the inviscid condition. The reason for this performance can be attributed to the incorporation of a visco-plastic material in the analysis. Notably, the visco-plastic material has the capability to attain a higher undrained peak strength when subjected to higher loading rates, as opposed to the inviscid state. However, due to viscosity allowing for deformation under constant stress, the visco yield surface gradually reverts to the inviscid yield surface over time, resulting in the accumulation of plastic deformation even when the stress is maintained constant. The higher the applied loading rate, the more plastic deformation occurs at the end of the loading phase.

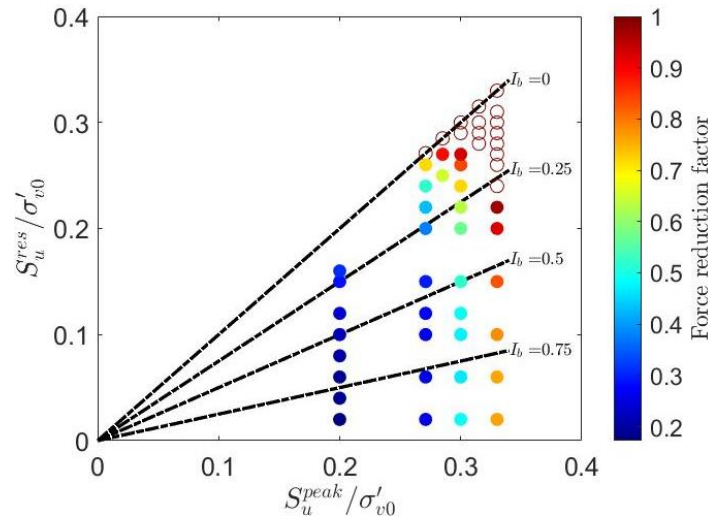


Figure 85 –Sensitivity analysis of slope erosion dam failure based on shear strength ratios and brittleness index, viscos undrained conditions.

Given this, it is advisable to conduct a thorough analysis of post-erosion events, as residual plastic deformations may occur based on the chosen visco parameters. While considering the erosion phase as undrained is justified due to its short duration, the longer post-erosion phase requires considering material drainage in accordance with its permeability. This analysis will be addressed in the next section, incorporating a fully coupled flow-deformation analysis from the beginning.

In Figure 86, a comparative analysis of stress paths is performed at the same four Gauss points previously discussed in Figure 84. These evaluations encompass both inviscid and viscous simulations, where the material parameters  $n$  and  $r$  are held constant, resulting in identical residual and peak shear strength properties for both material models. Figure 86 highlights a specific scenario in which the inclusion of a viscous material in the analysis does not lead to failure. In contrast, the corresponding inviscid analysis allows for the removal of only 85% of the soil segment, as indicated by the FRF.

Specifically, Figure 86a depicts the stress path of the viscous material. As observed, the force reduction factor equal to 1 indicates that no failure occurred at the end of the erosion phase. The results shown that during the erosion stage, the stress paths within the viscous material follow notably shorter compared to those observed in the inviscid simulations presented in Figure 86b. Notably, except for point 'a,' which is located near the trigger region, the remaining stress points do not reach their peak conditions, indicating a state of relative stability.

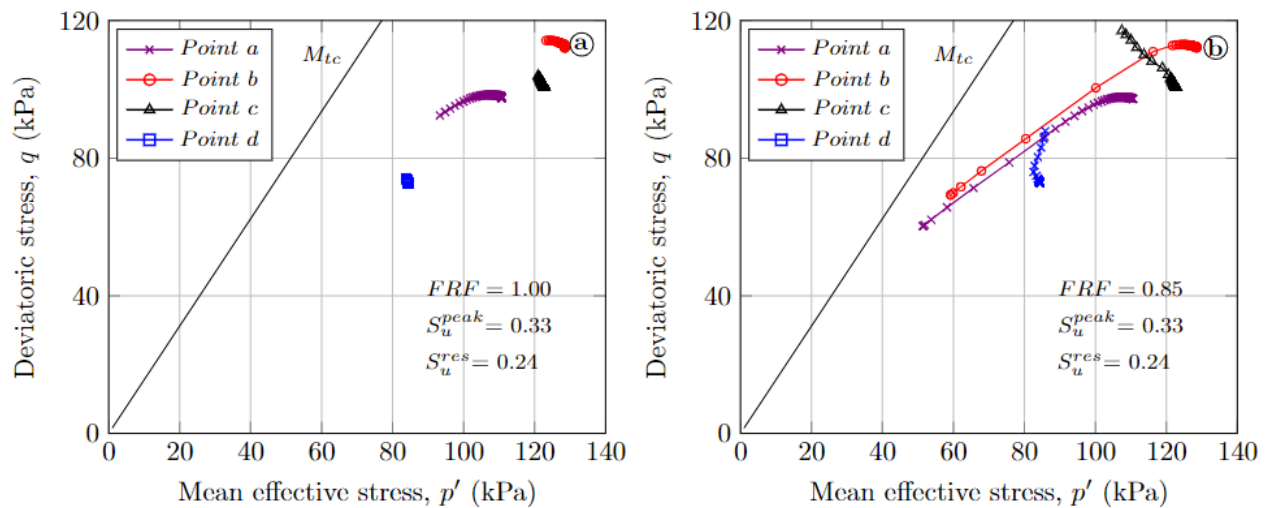


Figure 86 – Effective stress path at the 4-gauss point

#### 8.6.4. Fully coupled analysis using a viscous material.

In the preceding analysis, the emphasis was on examining how creep influences the likelihood of liquefaction triggers in tailing dams under undrained conditions. However, the influence of drainage can introduce an additional level of complexity to the problem. In this context, the analysis undertaken in this section will consider the impact of soil permeability using a fully coupled analysis. Both the geometry and the boundary conditions utilized here align with those from earlier analysis, as do the chosen sets of constitutive model parameters.

Creep behavior in soils can be assessed within two distinct limits: drained and undrained conditions. In the case of drained conditions, creep effects do not lead to pore pressure increase, maintaining the effective stress consistently over time. This phenomenon is commonly referred to as "aging" and is recognized for enhancing soil strength (Schmertmann, 1991). In contrast, undrained creep involves limited drainage coupled with creep deformation. In this scenario, the material experiences undrained conditions as it undergoes gradual deformation under a constant load, resulting in an increase in pore pressure. This shift in water pressure leads to a change in effective stress.

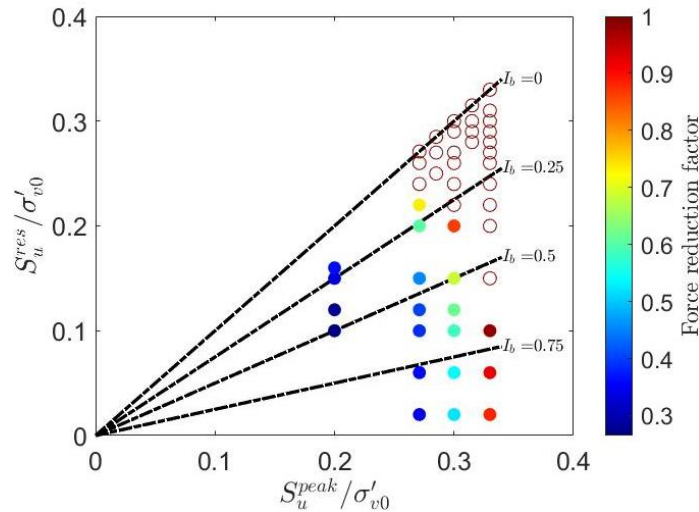


Figure 87 – Sensitivity analysis of slope erosion dam failure based on shear strength ratios and brittleness index, viscos and fully coupled conditions.

Thus, in conditions where only partial drainage is allowed, the changes in pore water pressure can significantly influence the soil's creep behavior. The rate at which this excess pressure dissipates, due to limited drainage, prominently influences how the soil's strength evolves during creep. As pore water pressure decreases, the effective stress within the soil change, resulting in an increase in soil strength. Figure 87 graphically depicts these simulation results. By the end of the erosion phase, in many simulations, denoted by hollow circles, the soil mass was fully eroded without failure, notably those within the 0 to 0.25 brittleness interval. In addition, the filled circles reveal that a larger fraction of the soil's weight was removed prior to failure compared to the undrained simulations. In analyzing a viscous material through a fully coupled analysis, it's crucial to consider the post-erosion phase, focusing on the dissipation of pore pressure following erosion. To this end, a subsequent one-day phase was incorporated immediately after the erosion process concluded, in all simulation of Figure 87 where a FRF of 100% was achieved. The findings from the simulations indicate that the dam remains stable even one day after the conclusion of the erosion phase.

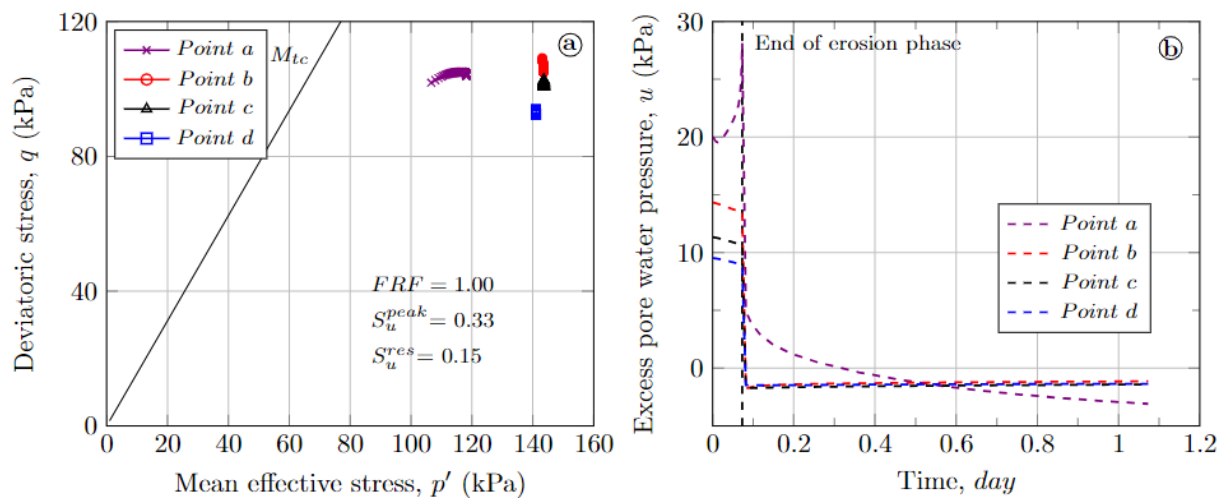


Figure 88 – Effective stress path at the 4-gauss point highlighted in Figure 84.

Figure 88a presents the simulation results in terms of stress paths and excess pore water pressure over time (Figure 88b). It is evident that most stress paths are notably short. Except for the purple dashed line (Point A), which exceeds the peak condition threshold, the remaining stress points are considered stable. This trend is further emphasized in Figure 88b. Point A, located near the

area where soil erosion occurred, experiences an increase in excess water pressure. This rise continues until the erosion phase concludes, after which it gradually returns to near-zero excess pore pressure. In contrast, the other stress points display an opposite trend, with excess pore pressure diminishing from the onset of the erosion phase. Eventually, all excess pore pressures stabilize at values close to zero.

Figure 89 displays a sequence of plots of the variation in excess pore pressure during and after the erosion phase corresponding to the same simulation presented in Figure 88. The left set of plots focuses on the erosion phase, with each plot corresponding to a different FRF, from 1% to complete erosion of 100 %. Notably, the red zones near eroded sections underscore a rise in excess pore pressure. This consistent pattern across all plots emphasizes the direct relationship between soil erosion and increased pore pressure.

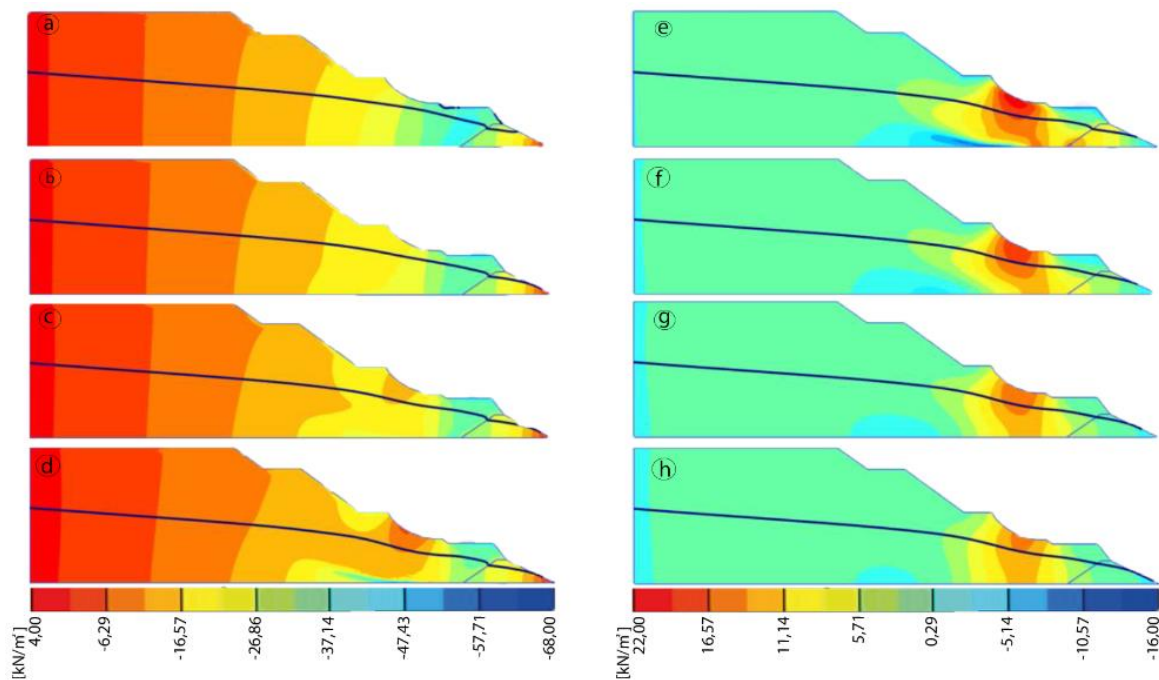


Figure 89 –Evolution of excess pore water pressure: During the Erosion Phase (a-b-c-d) and over one day post erosion (e-f-g-h).

At the end of the erosion phase, excess pore pressure attains its peak value, followed by a gradual decrease in the post-erosion phase. Clearly, the ability to dissipate excess pore pressure is directly

related to the soil's permeability. If the stress state is semi-stable when erosion ceased, the material can continue to accumulate deformation over time due to undrained creep, particularly if it is more impermeable. Consequently, the dam can still fail after the erosion has ended Mánica et al. (2021b).

## **8.7. Sensitivity of triggering to spatial location**

### **8.7.1. Introduction**

As highlighted in the preceding chapters, recent static liquefaction failures of tailings dams have thrust the issue of dam stability into the global spotlight (Jefferies et al., 2019; Robertson et al., 2019). In response to these events, the Australian National Committee on Large Dams (ANCOLD) published its 2019 guidelines, underscoring the challenges in identifying static liquefaction trigger mechanisms. While several triggers, such as rapid load changes, shifts in drainage state, and structural deformations, are well-documented and understood, the assessment of trigger mechanisms for static liquefaction remains challenging. Consequently, when evaluating the stability of materials susceptible to static liquefaction, it is now recommended to assume that triggering can indeed occur (ANCOLD, 2019).

That recommendation, however it is typically associated with a procedure to verify stability based on limit equilibrium analyses, such as the one describe by the International Commission on Large Dams, ICOLD (2022). When the stability analyses is based on more advanced procedures like critical-state based finite element simulation it is unclear if that recommendation still makes sense or how it should be interpreted.

The more systematic exploration of this issue to date is that carried out by Sfriso and co-workers. They proposed a simplified procedure to numerically evaluate a tailing dam's vulnerability to flow liquefaction using numerical modelling. Specifically, they utilized Plaxis to model an upstream-raised TSF and applied a horizontal contraction at the starter dam as the trigger mechanism for static liquefaction (Sottile et al., 2020). Similarly, Ledesma et al. (2022) applied a series of deteriorating events to assess the vulnerability of the Fundão Dam. These events, which included

applying a surface load to the crest of the upstream raise, inducing horizontal deformation at the toe of the setback, and raising the water level within the tailings, led to an increase in the  $q/p'$  ratio, each following different stress paths. These procedure for evaluating liquefaction vulnerability are not considered a triggering analysis, as the conditions leading to liquefaction onset are often rare and unpredictable. Indeed, the actions employed are not intended to reflect realistic scenarios for any specific dam but should be viewed as numerical excursions around a stable configuration to assess the stability of such equilibrium (Ledesma et al., 2022).

Localized liquefaction triggers within a dam have been postulated in several important cases: Arroyo & Gens (2021) argued that the Brumadinho B1 liquefaction failure was initiated by overpressure around a borehole. The triggering event for the Stava failure was identified as the collapse of a conduit passing through the dam (Chandler & Tosatti, 1995) . However, the presence of a localized triggering event does not need to cause structural failure: sometimes liquefaction is contained in space and does not propagate around the structure. An example of this was also identified in the analysis of Brumadinho by Arroyo & Gens (2021): there was ample evidence that a failed deep horizontal drain installation six months before failure initiated liquefaction, but that did not propagate through the dam.

It is also noteworthy in this context that tailings may be very variable within the same structure. This may be addressed systematically in finite element simulation for instance by the introduction of a random field description of spatial variability (Baecher & Christian, 2008). In this vein, Hicks & Onisiphorou (2005) investigate the propensity for liquefaction in dilative hydraulic sand fills by analyzing the case history of the Nerlerk artificial island in the Beaufort Sea. Their investigations suggest that pockets of loose material might have played a central role in the sand fills' collapse. This is because a sand fill, even if is predominantly dilative, may be prone to liquefaction if there are semi-continuous loose areas formed by the deposition-induced anisotropy.

Building on this consideration, this chapter embarks into an extensive set of numerical simulations using PLAXIS to investigate the influence spatially localized liquefaction triggering events on tailings dam stability. This is done selecting specific elements within the mesh and

assigning them reduced strength properties, and finally discerning how the location of one or several weak elements, coupled with the degree of strength reduction, can lead to flow liquefaction.

### **8.7.2. Integrating the weak element tool in Plaxis**

In this study, localized triggering was implemented through weak elements. This feature was based on the Fortran script of the CASM model developed by Miguel Manica et al. in 2021, implementing as a user-defined soil model within the Plaxis software. Comprehensive details about the model, including its parameters and the calibration methodology, were provided in Chapter 4. For incorporating the weak element feature, the code needs to be compiled into a 64-bit dynamic-link library (DLL), compatible with a Fortran compiler. This DLL consists of four Fortran files: "call.for," "main.for," "perm.for," and "param.for." The "main.for" file is pivotal as it contains the 'const\_law' subroutine that executes the calculation program's specified tasks, supported by a series of supplementary subroutines.

The code is structured into a set of arguments defined as IDTask, with every argument responsible for performs specific tasks. The specified IDTasks are listed below:

IDTask = 0 Check consistency of parameters

IDTask = 1 Initialize state variables

IDTask = 2 Calculate constitutive stresses

IDTask = 3 Create effective material stiffness matrix

IDTask = 4 Return the number of state parameters

IDTask = 5 Return matrix attributes

IDTask = 6 Create elastic material stiffness matrix

A novel parameter, identified as number 21 and named 'weak elem', has been introduced to the model, forming part of the "Props" vector. Activating this parameter by setting it to -1 enables the weak element feature. The integration of the weak element into the script occurs before initializing the state variables at IDTask1. Part of the script that was incorporated into the "main.for" file is shown below:

```
!-----!
! Weak element from file9 (n,r,R reduction)      !
!-----!

xn = Props(5)
r  = Props(6)

If (Props(21) .Eq. -1) Then
  File9 = 'C://Users//User//Desktop//Prova_output//file9.txt'
  Open(Unit= 9,File=File9, iostat=ios, Status='Old')
  Read(9,*) ingauss2(1,:)
  Do i=1,iel
    Read(9,*) strg_factor(:)
  End do
  xn = strg_factor(2)
  r  = strg_factor(1)
  R2 = strg_factor(3)
  close(9)
end if
Props(22) = xn
Props(23) = r
Props(24) = R2
```

This modification applies a reduction of the yield surface size at specific gauss points. To achieve this, a txt file named "file9.txt" must be created and stored in a directory path specified in the "main.for" file before performing any phase calculations in Plaxis. This file enables modification of the shape and size of the yield surface at any given gauss point using a new set of parameters:  $n_{CASM}$ ,  $r_{CASM}$ , and  $R2$ . Consequently, the file is structured into three columns, with the total number of rows corresponding to the number of stress points composing the model. The first column should contain the value of  $n_{CASM}$ , the second for  $r_{CASM}$ , and the third for the  $R2$  factor. If the  $R2$  value is set to 1 and the  $n_{CASM}$  and  $r_{CASM}$  values remain at their default settings, the yield surface at each Gauss point will not change. These parameters are incorporated as state variables

in Plaxis, which allows for their visual display on the model's mesh. This feature can be seen in the Plaxis output, as highlighted by the red box in Figure 90.

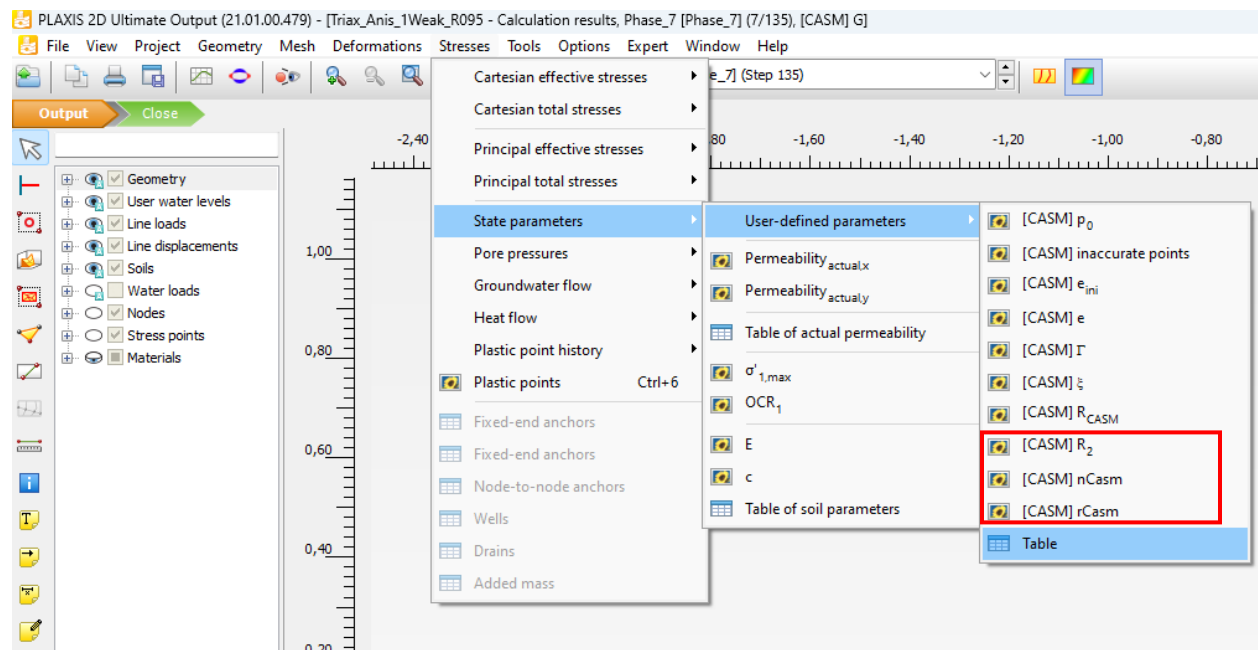


Figure 90 –User-defined parameters for the CASM model in Plaxis

In summary, setting parameter 21 to 0 results in no changes, preserving the original functionality of the constitutive model. On the other hand, setting parameter 21 to -1 necessitates defining three new parameters at each Gauss point using "file9.txt." While file9 allows for modifications to the yield surface, including changes to its shape and size, the forthcoming simulation will concentrate exclusively on adjusting the isotropic yield stress  $p'_0$  using the  $R_2$  factor, while keeping the  $n_{CASM}$  and  $r_{CASM}$  parameters constant.

### 8.7.3. Incorporating the weak element in triaxial test simulations

Prior to employing the weak element tool in the Merriespruit model, it's essential to understand the concept of a weak element and its application within Plaxis. The software allows users to choose between a 6-node and a 12-node triangular mesh for simulations. The Plaxis workflow start with the definition of soil layers or zones, establishment of boundary conditions and flow conditions. Once a suitable constitutive model and its associated properties have been selected,

materials are systematically assigned to their corresponding layers or zones and finally the mesh can be generated.

Upon mesh generation, it is unfeasible to specify different soil parameters for individual elements or Gauss points. If, during computation, it becomes necessary to change parameters for a specific zone of the mesh, a new material and its corresponding soil zone must be established a priori. In such a case, a new mesh is required, which will likely differ from the mesh in the initial configuration. When analysing problems of static liquefaction, in which a liquefied material condition propagates through the mesh it is not clear “a priori” if the results obtained are fully mesh-independent. This makes comparisons of analyses based on different meshes problematic.

The weak element technique that is described below has been developed as a workaround to avoid this problem. Using this approach, it's possible to assign unique constitutive attributes either to a specific Gauss point or, as implemented in this study, to an individual element comprising 12 Gauss points, all while avoiding the creation of a new mesh.

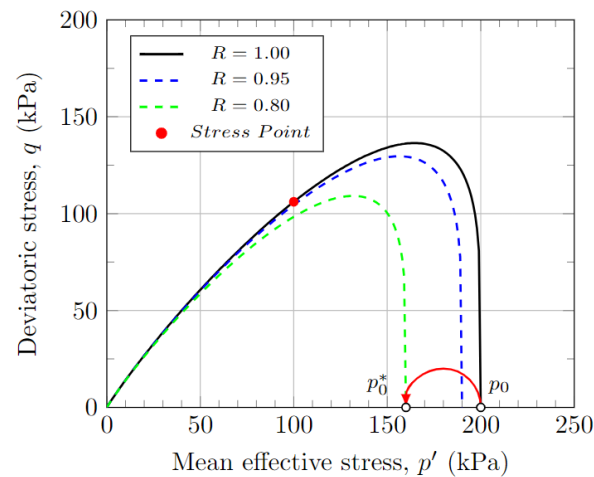


Figure 91 – Evolution of yield surface size with R2 parameter variation.

A Finite Element calculation is driven by out-of-balance forces in the model. These unbalanced forces can arise from changes in loads/stresses  $\Delta\sigma$  or changes in displacements and strains  $\Delta\epsilon$  (Plaxis, 2021). The weak element technique relies on the creation of unbalanced forces at specific points of the mesh. To address this within CASM, an additional parameter R2 has been introduced. The R2 parameter is designed to control the size of the yield surface at specific points

where stress is applied. By setting R2 to a value below one, the yield surface is effectively reduced in size. Consequently, R2 is the ratio between the adjusted preconsolidation pressure of the yield surface, denoted as " $p_0^*$ ", and the initial pressure " $p_0$ ". Figure 91 illustrate how the yield surface evolve when R2 values are set to 1.00, 0.95, and 0.80, respectively.

When the model is initialized under normally consolidated conditions, all stress points lie on the yield surface. However, setting the R2 value below one, certain stress points temporarily deviate from the yield surface. Consequently, the software checks whether these stress points violate the failure criterion. If they do, the stress at the affected point is corrected to maintain a valid stress state. This adjustment can result in an imbalance of forces, leading to additional strains.

To understand the effect of the R2 parameter on the behavior of a numerical model, a series of triaxial test simulations were carried out as a benchmark. Instead of using the Plaxis soil test tool, where the test is simulated at a single Gauss point, the test was performed using a model composed of 165 elements with axial symmetric conditions in a sample of 100 cm radius and 100 cm height. The modeling stages are composed of three phases: the initial phase, where the initial stress equilibrium is calculated; the first phase, where the material is anisotropically consolidated; and the second phase, where the horizontal and vertical stresses are maintained constant, and a weak element is embedded in the model's mesh.

In the first set of simulations, a consolidation stress ratio corresponding to  $K = \sigma'_h / \sigma'_v = 0.4$  was applied. Following the consolidation phase, three simulations were executed, each activating a weak element in a different position within the mesh, as illustrated in Figure 92. The R2 parameter is set to 0.95, indicating a temporary violation of the stress state in the selected element. As the second phase begin, Plaxis attempts to balance the sum of the external forces applied to the model with the integral of the internal stresses. Figure 92 displays the stress paths at two Gauss points, whose positions are indicated in the figure. These paths show that the initial stress state was below the instability line identified by  $K_0$ -consolidated triaxial tests, as discussed in section 8.3.2. However, an examination of these stress paths reveals a decline immediately after the weak element is activated, indicating that the undrained stress paths are unstable from the onset of the phase.

In the second series of simulations, the R2 parameter was consistently held at 0.95, but the number of activated weak elements was increased to three in each simulation (Figure 93). Additionally, the consolidation phase was executed with a vertical to horizontal stress ratio of  $K = \sigma'_h / \sigma'_v = 0.45$ . As a result, at the end of the consolidation phase, the stress state was slightly below that of the first set of simulations. Even with the activation of three weak elements, this perturbation was insufficient to induce global instability. Indeed, the stress path depicted in Figure 93 indicates that the material does not exhibit signs of instability. This is further evidenced by the contour of total shear strain in Figure, where the shear band is not fully formed along the entire specimen.

The third set of simulations retained the same setup as the previous one, with a consolidation stress ratio of 0.45 and three weak elements embedded in the mesh at the same locations. However, in this scenario, the R2 parameter was set to 0.9, resulting in a significantly greater reduction of the yield surface at the stress points within the three weak elements compared to the previous case. This indicates a greater perturbation in the stress state to achieve equilibrium between external and internal stresses. As shown in Figure 94, the stress paths of the two gauss points reveal that, upon activation of the weak elements, the material exhibits unstable behavior, as it cannot sustain the prescribed stress state, resulting in runaway deformation. This is confirmed by observing the contour of total shear strain, where a more clearly defined shear band is seen across the entire sample.

The series of simulations conducted in this section has provided insights into soil behavior under varying stress conditions using the weak element tool implemented in Plaxis. The R2 parameter is capable of inducing instability through the diminution of the yield surface at specific stress points. Instability can be triggered depending on the R2 value, the number of stress points affected by this reduction, and the initial stress condition. It has also been observed that shear bands, when formed, tend to pass through the areas where the weak element was activated. These results not only validate the weak element tool within the user-defined constitutive models in Plaxis, but they also offer a way for further studies into more complex geotechnical scenarios.

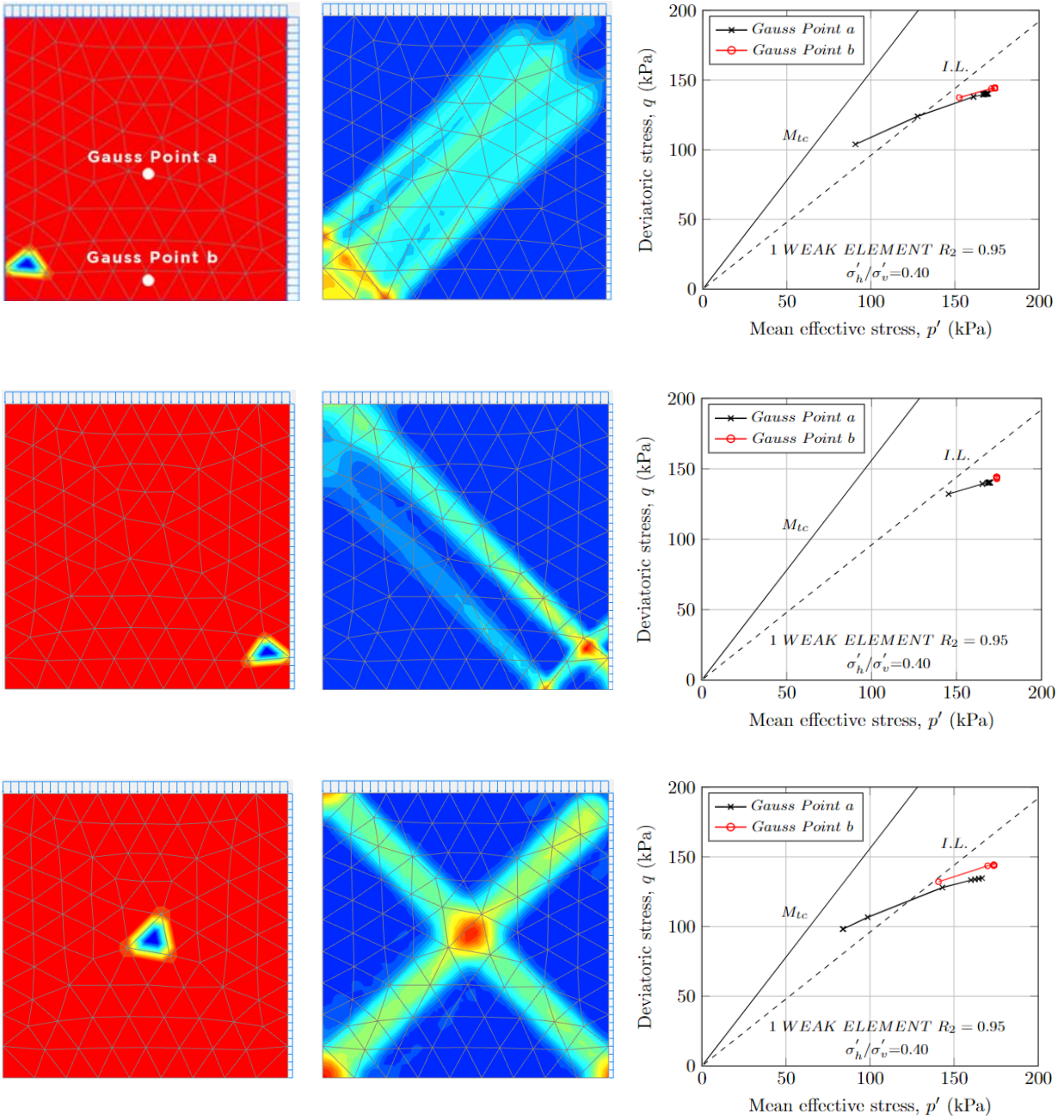


Figure 92 – The first column shows the 2D geometry of a triaxial test with 1 blue-highlighted Weak Element using  $R_2=0.95$  and  $\sigma'_h/\sigma'_v = 0.40$ ; the second column displays the contours of total shear-strain; and the third column presents the stress-paths of two gauss points.

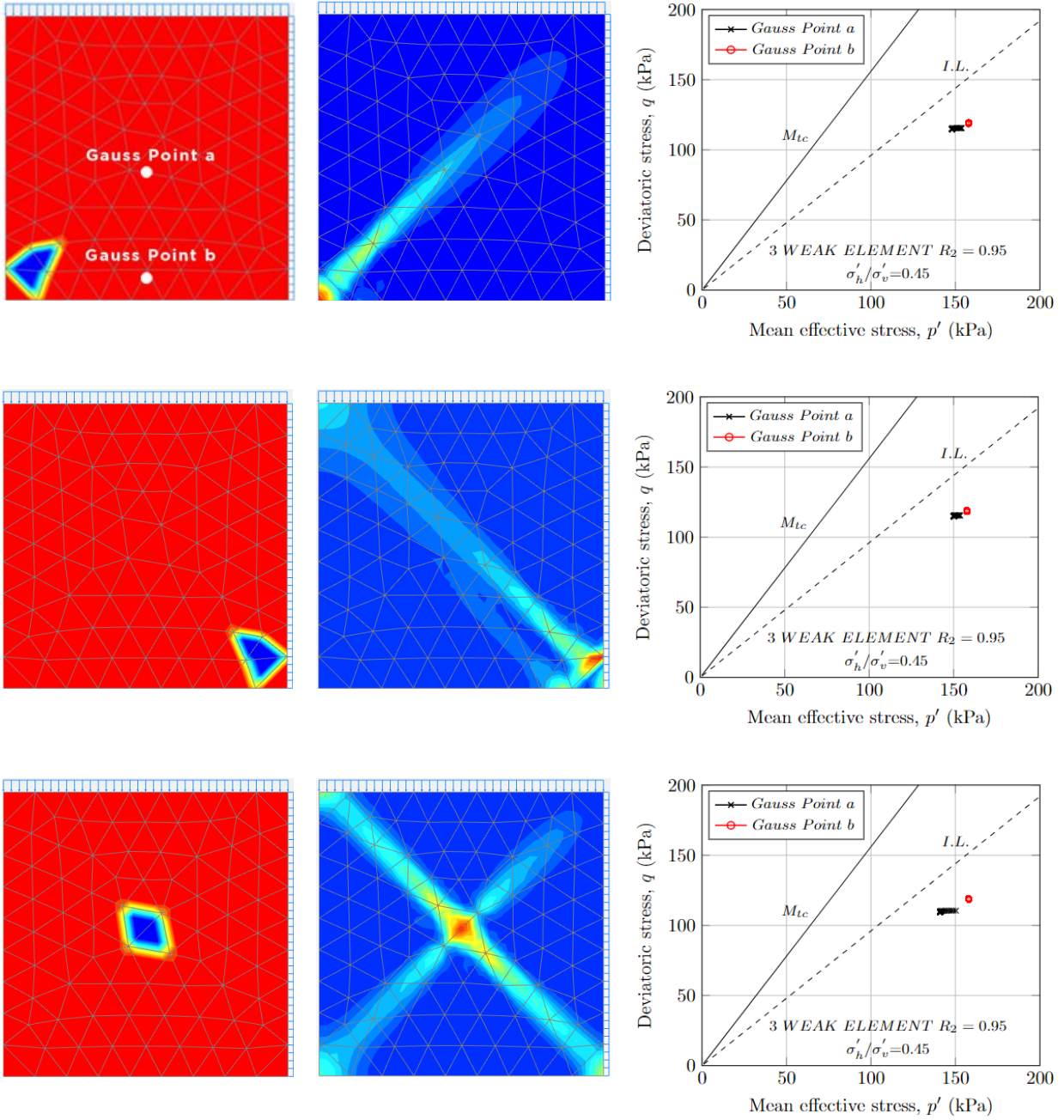


Figure 93 – The first column shows the 2D geometry of a triaxial test with 3 blue-highlighted Weak Element using  $R_2=0.95$  and  $\sigma'_h/\sigma'_v = 0.45$ ; the second column displays the contours of total shear-strain; and the third column presents the stress-paths of two gauss points.

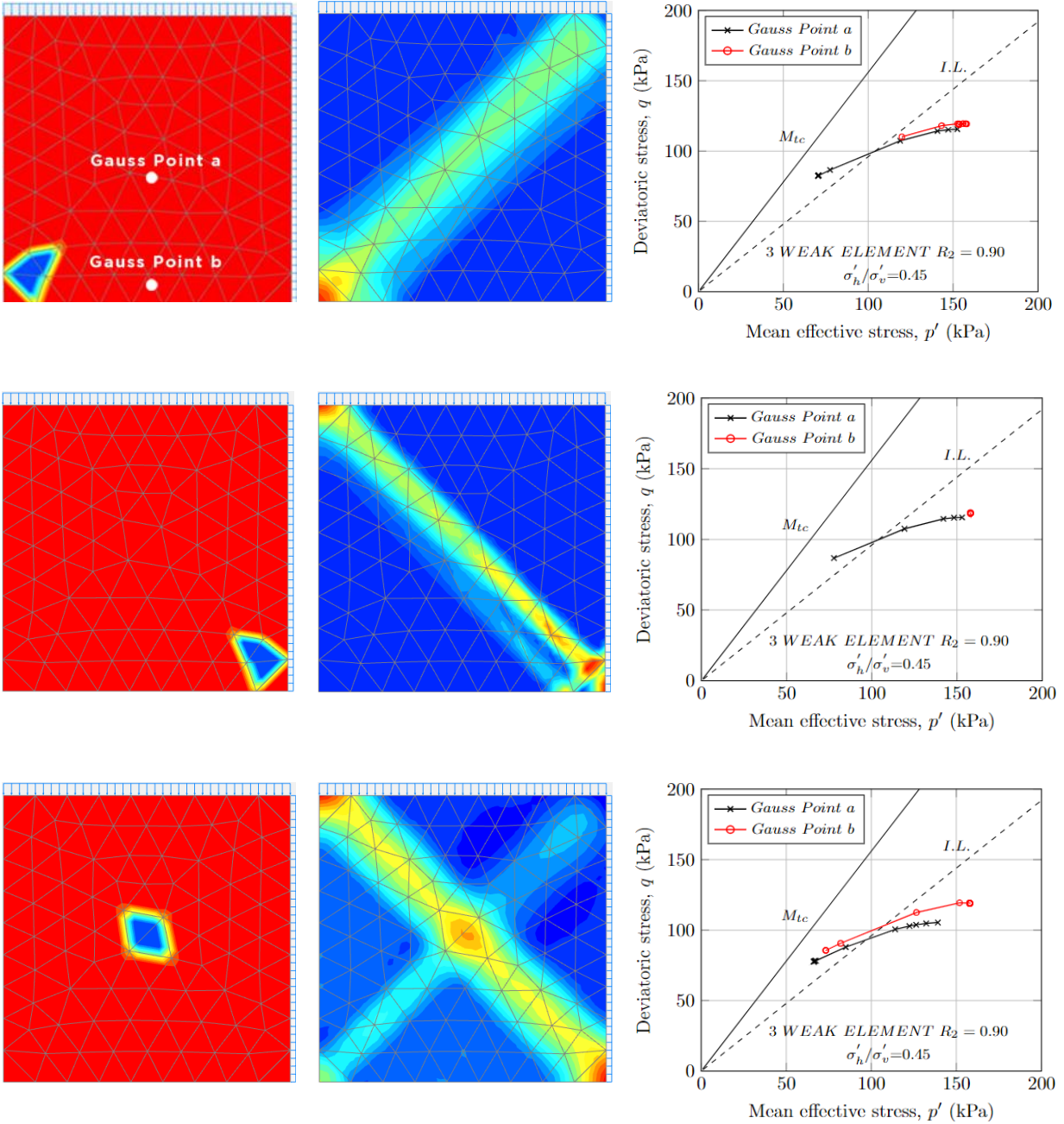


Figure 94 – The first column shows the 2D geometry of a triaxial test with 3 blue-highlighted Weak Element using  $R_2=0.90$  and  $\sigma'_h/\sigma'_v = 0.45$ ; the second column displays the contours of total shear-strain; and the third column presents the stress-paths of two gauss points.

#### 8.7.4. Triggering flow liquefaction in the Merriespruit dam through a weak element.

This section presents a method for simulating flow liquefaction by incorporating a 'weak element' trigger within the model's geometry. The effectiveness of this technique was demonstrated through its application to the Merriespruit tailings dam, which is described in detail in the preceding section. In practice, this involves defining a target area within the model, represented by a polygon, where this weak element can be activated. However, this activation is limited to regions where the material is characterized using the CASM model. This encompasses the entire dam, except for the starter dam, which was modeled using a Mohr-Coulomb material.

The simulation focuses on the dam's condition before failure, thereby omitting the slope erosion that was the trigger condition for liquefaction in the previous analysis. The fundamental idea behind the weak element tool is to examine how the spatial location of a trigger impacts liquefaction failure development. The weak element is positioned randomly within the numerical model, using Python to automate Plaxis. This process is executed by generating multiple random points, each assigned an x and y coordinate. These coordinates are randomly selected within the minimum and maximum limits of the polygon's area. The script then utilizes the 'contains' method for each point to ascertain whether it lies within the polygon's boundaries, where the CASM model is employed. Points that are verified to be inside the polygon have their coordinates included in a collection. This process continues until the collection comprises a predetermined number of points set by the user.

Figure 95 illustrates the dam geometry, whose mesh comprises 4th order, 15-node triangular elements. Excluding the green area, the geometry was modeled using the CASM model. In this numerical model, the blue and red elements denote the locations where the weak element feature was activated. A blue element indicates that when the weak element was activated in that position, the dam remained stable. On the other hand, a red element indicates that activation of a weak element in that location resulted in dam failure. Figure 95 presents two sets of simulations, illustrating changes in the spatial pattern; Figure 95a uses  $R2=0.95$ , and Figure 95b uses  $R2=0.85$ . As expected, the second set of simulations ( $R2=0.85$ ) shows a higher number of weak elements resulting in dam failure.

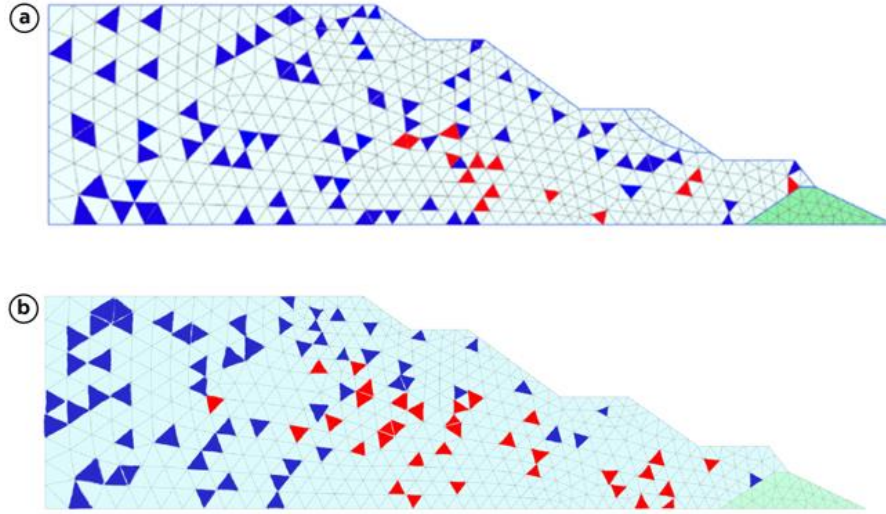


Figure 95 –Random distribution of weak elements within the mesh: (a)  $R_2=0.95$ ; (b)  $R_2=0.85$

It is insightful to examine the installed stress state before the activation of the trigger in the elements highlighted in Figure 95. As detailed in Section 8.3, the initial stress state was configured to ensure that all gauss points maintained the initial stress beyond the threshold defined by the instability line. In Figure 96, the red dots represent stress points within elements prone to failure upon weak element activation (i.e., the stress points within the red elements in Figure 95). Meanwhile, the blue dots represent stress points within elements that remained stable, not failing even after activation. This aligns with the expectation that activating the trigger in elements that are already in a semi-stable state of equilibrium, specifically those in the proximity to the instability line, may result in a sudden loss of strength due to relatively minor disturbances.

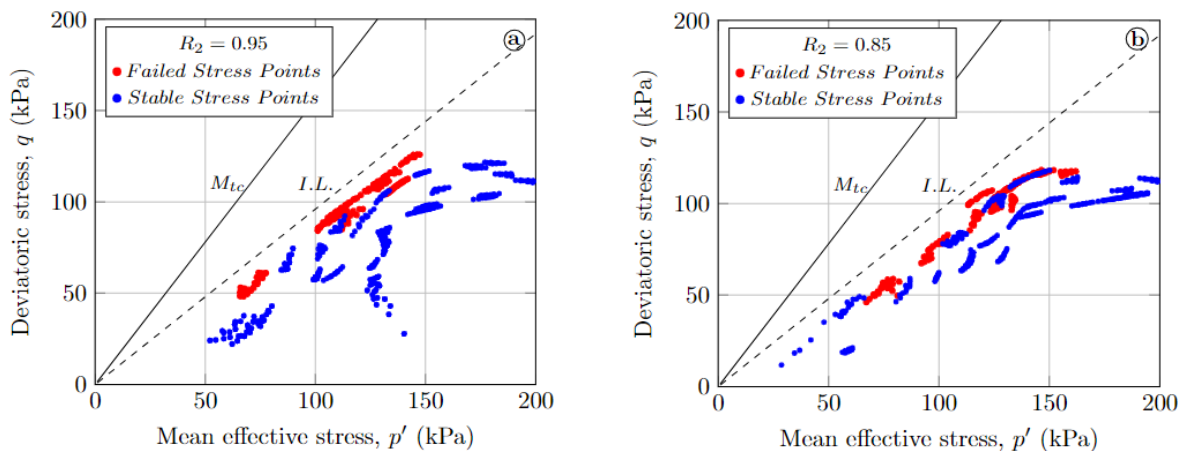


Figure 96 – Installed stress state before the activation of the trigger within the dam

While the weak element impact the stability of the dam, the exact implications for soil behavior when these elements are activated have not yet been discussed. Thus, examining reactions at specific stress points is insightful. To this end, two the stress points within the weak element were analyzed, focusing on volumetric shear strain and excess pore pressure over time. The findings are illustrated in Figure 97, corresponding to one of the weak elements that lead to failure (indicated by red dots in Figure 96a) with an  $R2 = 0.95$ .

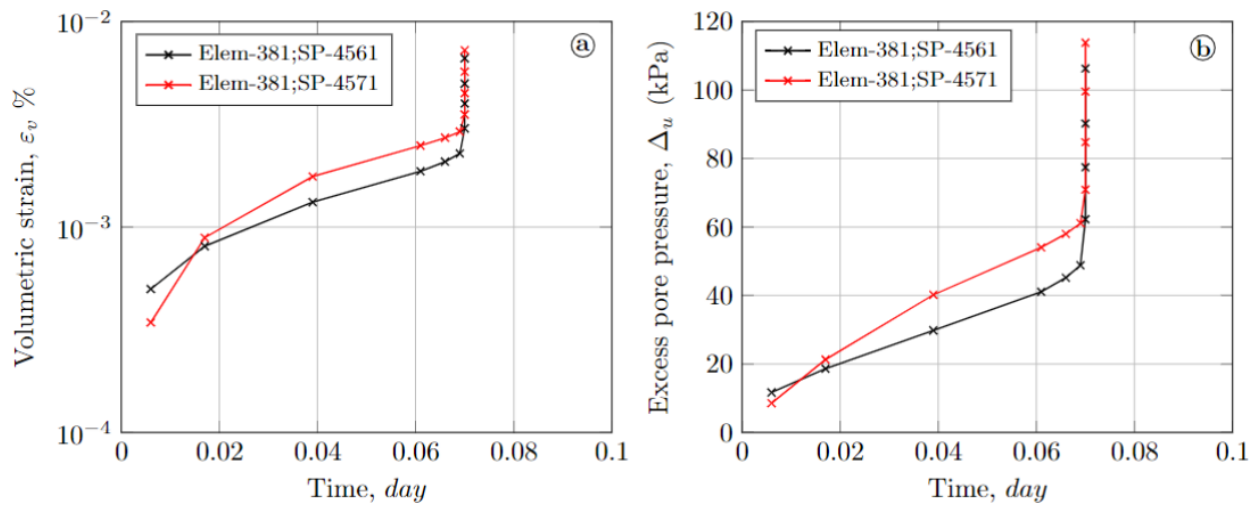


Figure 97 – (a) Evolution of volumetric strain at two Gauss points within the activated element;  
(b) Evolution of excess pore pressure at two Gauss points within the activated element.

Flow liquefaction is associated with a state of instability, characterized by abrupt increases in both strain and pore pressures. Substantial field evidence indicates a rapid decrease in shear strength, typically linked to a significant rise in pore water pressure, especially in loosely compacted, non-cohesive soil deposits.

Observations from Figure 97 reveal that both stress points exhibit similar response patterns. Since the analysis was conducted under undrained conditions, the volumetric strain is almost negligible, around 0.001%, as shown in Figure 97a. The pore pressure progressively builds up

once the weak element is activated. After a few steps, there is a sudden jump in pore pressure, which quickly spreads and triggers the onset of liquefaction.

While activating these weak areas within the dam's structure is not intended to precisely mimic a potential flow liquefaction trigger, the volumetric shear strain and excess pore pressure over time, show a similar pattern with previous analyses where liquefaction was induced through the erosion of a portion of the slope. This process begins locally within the weak element and eventually spreads, forming a shear band. This gradual progression is depicted in Figure 98, which illustrates four distinct steps of the simulation. It clearly shows the initial increase in pore pressure at a specific point, followed by its propagation until failure occurs.

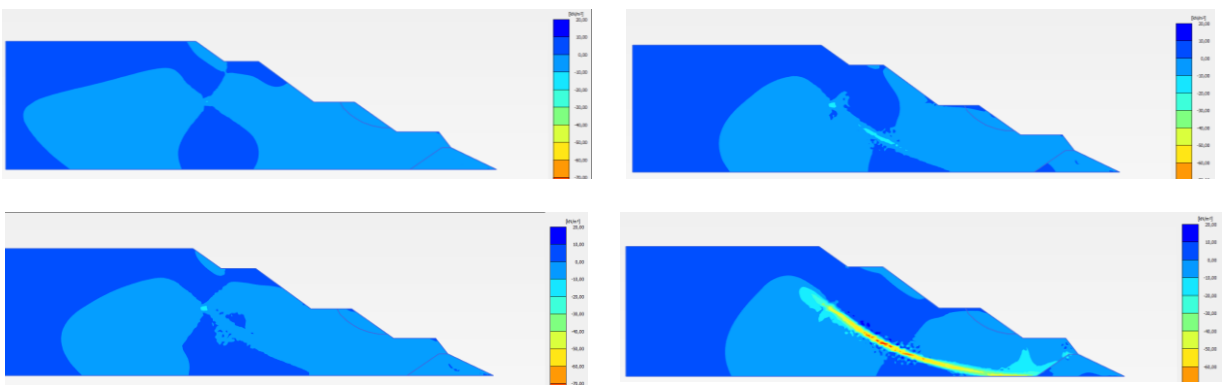


Figure 98 –Localized propagation of excess pore water pressure resulting in liquefaction

Another crucial aspect to consider is the percentage of weak elements that cause failure and the number of simulations needed to obtain a reliable result. This is particularly relevant for finite element simulations, which can be significantly time-consuming, depending primarily on the type of calculations and the refinement of the mesh. An example of this is illustrated in Figure 99, which plots the number of simulations against the percentage of simulations that reach failure. Even though 200 simulations were conducted, a noticeable convergence was observed after around 100 iterations. Consequently, the upcoming simulations will be conducted up to a number of 100 times for each set of parameters used.

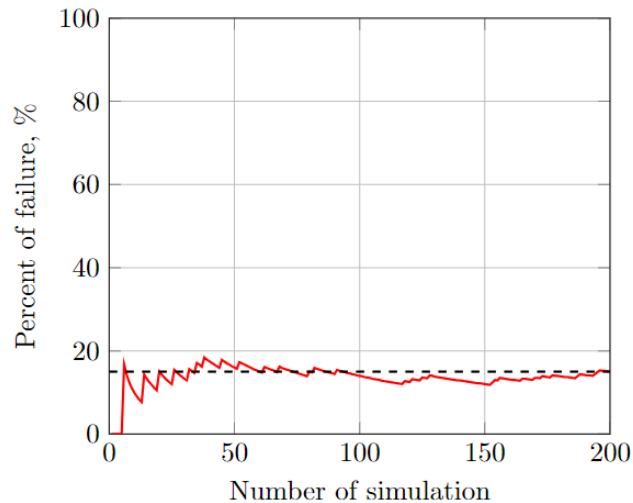


Figure 99 – Plot of the cumulative probability of failure against the number of simulations needed to reach a stable value.

The analysis shows that within the parameters of the current numerical model, introducing a random weak element results in an estimated 17% risk of failure when  $R_2$  equals 0.95, and a 36% risk when  $R_2$  equals 0.85. This is highlighted in Figure 96, where activation of a weak element near a point of instability significantly increases the likelihood of liquefaction.

It was also found that the susceptibility of a model to failure is also linked to the specific  $R_2$  values applied at the weak element. Figure 100 summarise the outcomes of 100 individual simulations, each involving a distinct weak element randomly placed within the dam and subjected to variable  $R_2$  values. It can be seen that reducing  $R_2$  values have a direct correlation with an increase in the probability of failure. Figure 100 indicates a clear trend: as the  $R_2$  value decreases, simulating a larger reduction in the isotropic yield stress  $p'_0$ , the percentage of failure within the simulations rises. This trend is consistent across the range of  $R_2$  values tested, underscoring the influence of this parameter on the stability of the dam, when considering the activation of weak elements.

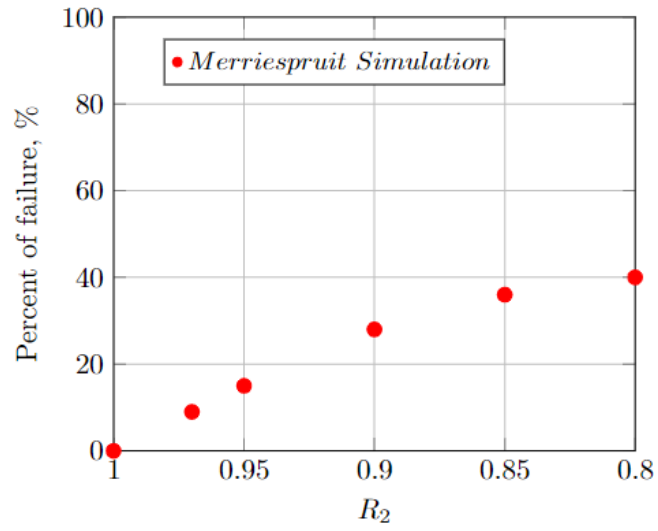


Figure 100 – Percentage of failure using one weak element with different  $R_2$  within the Merriespruit dam.



## 9. Conclusions

The investigation presented in this thesis provides a comprehensive analysis of the liquefaction potential of mine tailing storage facilities (TSFs) through material characterization and numerical modeling. The study was motivated by the failure of Brumadinho B1 dam, highlighting the critical need for a deeper understanding of static liquefaction mechanisms in tailing materials.

This study involved conducting laboratory tests on four types of tailings from the dam, namely Types 1, 2, 3, and 4. These tests provided significant insights into the tailings stress-strain behavior, critical state parameters, and susceptibility to flow liquefaction. The use of triaxial tests was particularly valuable in defining the critical state locus and understanding soil behavior under various initial states and stress paths. Laboratory tests offer a comprehensive understanding of how tailings respond to different loading conditions, which is crucial for developing accurate predictive models. The high-quality data obtained from these tests are essential for calibrating and validating critical state models, ensuring their reliability in predicting liquefaction.

Several methods for evaluating in situ state parameters using CPTu data were analyzed, with a focus on the Plewes screening method and techniques tailored for both drained and undrained conditions. For undrained conditions, methods based on cavity expansion theory were employed, using critical state parameters derived from laboratory tests.

The application of an elastoplastic model based on critical state theory, the Clay and Sand Model (CASM), proved effective in simulating the undrained brittleness of tailings. CASM calibration using experimental data allowed for accurate predictions of tailing behavior under various loading conditions, emphasizing the importance of precise and robust calibration procedures. The different conditions explored include assessing the impact of viscous effects on the stability of the dam upon triggering, examining the impact on dam stability by calibrating the constitutive model using isotropically or  $K_0$  consolidated triaxial tests, undertaking a sensitivity analysis on specific CASM parameters under different geotechnical conditions (such as whether the material is viscous or not), and considering the analysis as undrained or fully coupled, and investigating the sensitivity of triggering to spatial location. The versatility of CASM in modeling a wide range

of materials, coupled with its robustness in capturing the critical state behavior of soils, as well as undrained creep, makes it an excellent tool for assessing the stability of tailings dams and predicting their response to different stress scenarios.

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