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Perspectives on Modelling Airline Integrated Scheduling Problem: a Review on State-of-the-Art Methodologies

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A R T I C L E I N F O

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A B S T R A C T

Operating on volatile profit margins, it is imperative for the airline industry to utilize its capacity and resources via sophisticated schedules for efficient operation. While the airline scheduling problem consisting of schedule design, fleet assignment, aircraft routing, and crew scheduling subproblems were extensively studied and successfully applied in real practice, the recent decade has witnessed a growing interest in integrated scheduling problems that capture the interdependencies between various decisions across different subproblems and airline resources with significantly reduced operation costs. This paper aims to provide insights for modelling the airline integrated scheduling problem, indicating a roadmap for future research in this domain. Multiple perspectives concerning demand-supply interaction, operational consistency, and schedule robustness are first provided. State-of-the-art mathematical formulations and coupling relationships for representative integrated problems are summarized additionally. In the end, we conclude with a series of themes that can be further addressed in the future.

1. Introduction

Having strongly recovered from the unprecedented hit brought by COVID-19 (Sun et al., 2023; Wu et al., 2023), the total revenue of the airline industry is expected to grow by 9.7% in 2023[1]. In the meantime, fierce competition and high operating costs have always been a pressing need for airlines to organize their services with high efficiency to increase the net profits (Barnhart and Cohn, 2004). As a critical indicator of its business strategy, the flight schedule of an airline directly determines the served markets and potential revenues based on various practical considerations such as fleet composition, crew eligibility and operating constraints (Belobaba et al., 2015). Therefore, constructing a successful flight schedule is at the heart of airline operations (Xu et al., 2023a) and is referred to as the airline scheduling problem.

In brief, the airline scheduling problem tries to optimize decisions concerning which markets to serve with what fleet types and what frequencies. The specific flight departure/arrival time also has to be determined to arrange individual aircraft and cabin/cockpit crews' working schedules. Since large carriers typically feature thousands of daily flights and hundreds of aircraft with varying fleet types, the airline scheduling problem is of very high complexity. Besides, complex operating constraints related to maintenance, crews and airport slots further contribute to the problem intractability (Eltoukhy et al., 2017). Considering its internal complexity and decision sequence, the complete scheduling problem is divided into the following four subproblems: schedule design problem (SDP) (Jiang and Barnhart, 2013), fleet assignment problem (FAP) (Sherali et al., 2006), aircraft (maintenance) routing problem (ARP) (Desaulniers et al., 1997) and crew scheduling problem (CSP) (Gopalakrishnan and Johnson, 2005). As shown in Figure 1, these subproblems are interrelated and cover the complete planning horizon.

To be specific, the SDP determines the flight frequency and departure/arrival times for various airport pairs. Based on the generated timetable, FAP aims to allocate the appropriate fleet types to each flight considering the differences in operating cost and number of seats. Subsequently, routes of aircraft belonging to the same fleet type are set by ARP whilst satisfying maintenance requirements. Finally, the crew schedules are planned in CSP considering a number of pairing/duty rules. Such decomposition strategy provides overall suboptimal solutions and may also fail to satisfy feasibility constraints of an otherwise feasible integrated problem (Papadakos, 2009). Moreover, frequently encountered uncertain irregularities such as airport closure, crew unavailability and demand fluctuation are likely to result in flight delay/cancellation, profit reduction and increase in operation cost (Ma et al., 2022). These operational risks can be mitigated, to some extent, by enhancing the robustness of flight schedules using collaborative decision-making among different airline departments. In fact, the integrated scheduling approach was widely reported to achieve significant potential savings and revenues (millions of dollars) when compared to the sequential optimization approach (Jiang and Barnhart, 2013; Lohatepanont and Barnhart, 2004; Papadakos, 2009). Consequently, increasing research interests are

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[1] https://www.iata.org/en/pressroom/2023-releases/2023-06-05-01/.

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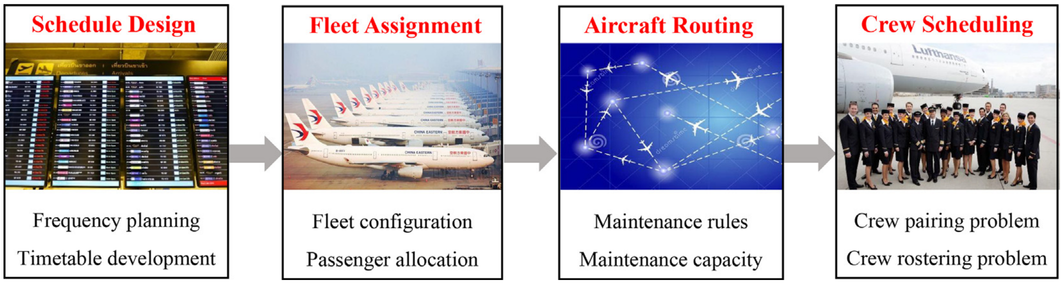

Fig. 1. Categories of airline scheduling subproblems and the corresponding decisions/objectives

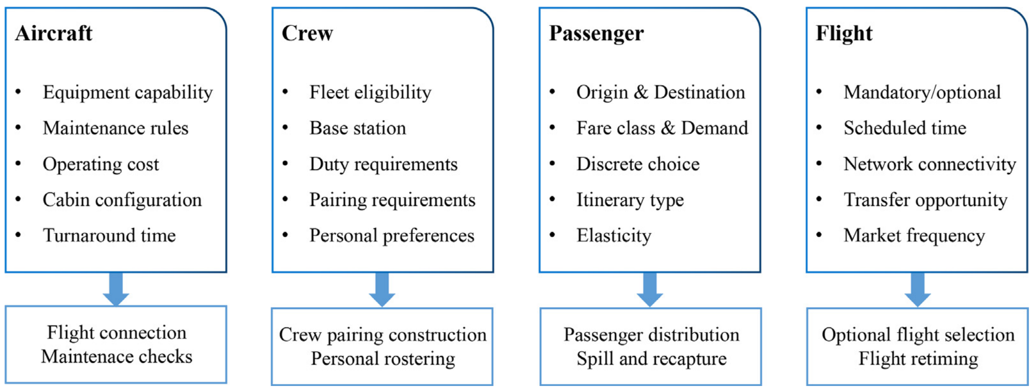

Fig. 2. Taxonomy of airline integrated scheduling problem according to operating resources

devoted to airline integrated scheduling problems where two or three subproblems are combined for consistent and collaborative decisions on critical resources/assets of airlines (Wandelt et al., 2024).

While multiple reviews and surveys have offered a comprehensive survey of past research on airline schedule planning problems (Eltoukhy et al., 2017; Wen et al., 2021; Xu et al., 2024; Zhou et al., 2020) in terms of integration schemes or strategies, our paper concentrates more on exploring underlying motivations for integration and robustness. To provide a framework for the broad array of integrated airline scheduling problems, a taxonomy is presented in Figure 2 concerning the critical operating resources (aircraft, crew, passenger and flight). When optimizing aircraft-related decisions such as flight connections and maintenance checks, certain constraints like the available number of seats and equipment capability are correlated with passenger demand and optional flights, respectively. On the other hand, fleet eligibility and duty/pairing rules of crew members are also associated with aircraft routes and selected flight connections. Lastly, there exist interrelationships between passenger demand and flight supply (i.e. network effects) which need to be captured via modelling techniques such as spill and recapture.

Based on the introduced taxonomy, the remainder of this article begins with a historical perspective about airline scheduling subproblems in Section 2. Then three perspectives for demand and supply interaction, operational consistency and schedule robustness are presented in Sections 3-5 to provide an explanation of the motivation or benefits of integrated scheduling problems. Finally, the article concludes with a summary pinpointing key areas for future research.

2. A historical perspective

Being one of the most successful stories for the application of operations research methodologies, the airline schedule planning problem was proposed and developed after the deregulation of the U.S. domestic airline industry. During that period, existing carriers were threatened by intensive market competition induced by a large number of new entrant carriers (Morrison and Winston, 2010; Sun et al., 2024). As a result, schedule planning and revenue management approaches helped to sustain the competitive ability of airlines. Previous research efforts mainly focus on FAP, ARP, and CSP (including crew pairing and crew rostering problems) and will be introduced as follows.

2.1. Fleet assignment problem

FAP aims to maximize airline profits by assigning varying types of aircraft to the scheduled flights given different seat capacities, operating costs, and other factors. In one of the pioneering works, Abara (1989) introduced a three-index model to derive assignment solutions based on a connection network. As shown in Figure 3 (a), edges between two flight nodes represent connecting opportunities if spatial and temporal continuity conditions are satisfied. Decision variables concerning valid flight connections can thus be constructed. Using a similar three-index formulation, Rushmeier and Kontogiorgis (1997) introduced preprocessing techniques to partition connections into different connecting complexes to isolate connections outside one complex.

In contrast to the connection network, the time-space network with relatively lower density was also adopted in the literature (Berge and Hopperstad, 1993; Hane et al., 1995) by aggregating aircraft ground movement. As shown in Figure 3(b), an airport is associated with a timeline consisting of a sequence of departure and arrival events in increasing order of event time. Accordingly, every vertex corresponds to either a departure or arrival event in the flight schedule. Flight edges and ground edges are also introduced to indicate the movements of aircraft. To further ease the computation burden of solving the FAP, preprocessing techniques like node aggregation and isolation are introduced in Hane et al. (1995) to eliminate dummy ground arcs and vertices.

Except for the basic model formulation, successive research presented more generic models. For instance, Ioachim et al. (1999) extended the daily schedule to a weekly schedule by incorporating synchronization constraints to limit the changes in departure/arrival times of flights with the same flight numbers. To more accurately model the passenger dynamics, Barnhart et al. (2002) proposed a passenger mix model to take passenger spill and recapture into account. Demand un-

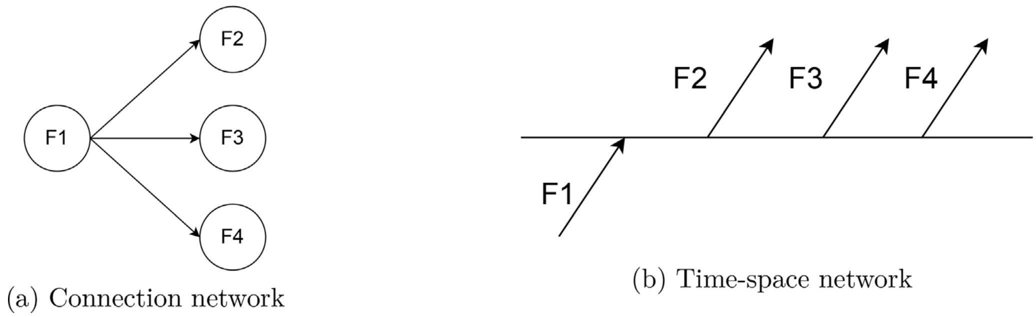

Fig. 3. Network structures widely used in the airline scheduling literature for a four flights example

certainty was considered in Pilla et al. (2012) with a two-stage stochastic programming model which was solved with multivariate adaptive regression splines cutting plans. To enhance the integration of revenue management decisions within FAP, Barnhart et al. (2009) introduced a subnetwork fleet assignment model which offers a more accurate approximation to the itinerary revenue while maintaining efficient computation. A solution approach is devised after model reformulation by employing fare product partitioning and decision variable enumeration. To improve the thorough flight design for better passenger transfers which cannot be directly derived using time-space networks, Ahuja et al. (2007) presented an optimization model defined with flight connections. The model was solved via the very large neighborhood search algorithm, leveraging a proposed enhancement graph to articulate local search operators.

2.2. Aircraft routing problem

Based on the assigned fleet types, aircraft of the same type are consecutively allocated to a subset of flights for the fleet family whilst maintenance requirements are satisfied. While these requirements can be classified as short-term maintenance checks (Type A, Type B) and long-term ones (Type C and Type D), short-term maintenance is more frequently studied in airline routing-related research and may be split into several phase checks to meet tight flight schedules. Three types of maintenance check conditions are widely adopted: the maximum flying time before maintenance, the maximum total elapsed time before maintenance and the maximum elapsed days before maintenance Barnhart et al. (1998); Liang and Chaovalitwongse (2013).

Similar to FAP, ARP-related models are able to be constructed from connection network or time-space network-based decision variables. In an early work of Clarke et al. (1997), a connection network-based ARP model was developed to find maintenance-feasible aircraft rotations. To model the aforementioned three maintenance conditions, a compact non-linear model was developed by Haouari et al. (2013) and reformulated for computational tractability. In time-space network-based models, maintenance checks cannot be explicitly arranged in flight connections due to the aggregation nature of ground arcs. As a result, related works (Liang and Chaovalitwongse, 2013; Liang et al., 2011) assumed the occurrence of maintenance events at night which is directly associated with the requirement of *maximum elapsed days* before maintenance.

Different from the introduced compact models, set partition formulation is also extensively studied by incorporating route-based decision variables. Specifically, an aircraft route is a sequence of maintenance-feasible connected flights that can be executed by the same aircraft consecutively. By embedding maintenance constraints in the variable construction process, the overall formulation of the corresponding mathematical model is simplified. One representative work by Barnhart et al. (1998) developed a model consisting of augmented flight strings that originate and terminate at airports with maintenance capabilities. By concatenating two or more augmented strings, a complete maintenance feasible aircraft route is generated. To generate these flight strings, the branch-and-price algorithm was applied with the follow-on strategy to branch based on fractional values of connections. As the ARP model is also applicable to solving the tail assignment problem which features a shorter planning horizon, Sarac et al. (2006) developed an operational model that takes into account the legal remaining hours of various aircraft and resource constraints at maintenance stations. To accommodate these resource constraints, they developed two additional branching schemes alongside the existing follow-on branching scheme. In Maher et al. (2018), a daily operational aircraft routing problem was solved with day two and day three maintenance requirements via look-ahead constraints.

2.3. Crew scheduling problem

When the aircraft routes are determined, the CSP is solved to obtain effective crew schedules and satisfy a number of working rules. Crew-related cost constitutes a large proportion of airline operating cost (see Figure 4) and is extensively studied concerning its two subproblems: crew pairing problem, and crew rostering problem Kasirzadeh et al. (2017). Since the pairing cost is determined by adding up the duty cost, which is contingent on factors such as maximum flying hours, elapsed time, and minimum guaranteed time, the crew rostering problem and pairing problem are commonly tackled as set partition/covering problems. Based on the planning horizon, the CSP can be categorized into daily, weekly, and dated (monthly) problem variants.

In the early stages of research on the crew pairing problem, Lavoie et al. (1988) treated pairings as decision variables and established set partition/covering constraints in the model to ensure each flight is covered by at least one pairing (including deadhead chances). Such set partition models can be efficiently solved using the Branch-and-Price algorithm introduced by Hoffman and Padberg (1993) to find promising pairings. Building upon a similar formulation, Barnhart et al. (1995) incorporated deadhead flight decisions to enhance crew utilization and bring reductions in crew costs. In Makri and Klabjan (2004), different types of pruning methods were introduced to solve the pricing subproblem, which employed label-setting algorithms to identify critical bounds and eliminate unpromising labels during label extension. In addition to the set partition model, a novel compact model was introduced by Haouari et al. (2019), utilizing non-linear constraints. This led to the application of reformulation and linearization techniques to enhance computational performance, with the inclusion of valid inequalities to tighten the model. A notable benefit of this model is its direct implementation using commonly available commercial solvers, eliminating the need for complex algorithms.

In contrast to the crew pairing problem, the crew rostering problem focuses more on assigning specific pairings to individual crew members considering personal preferences and fairness criteria. While the fundamental crew rostering model primarily involves crew counting constraints and the minimum required crew number constraints, numer-

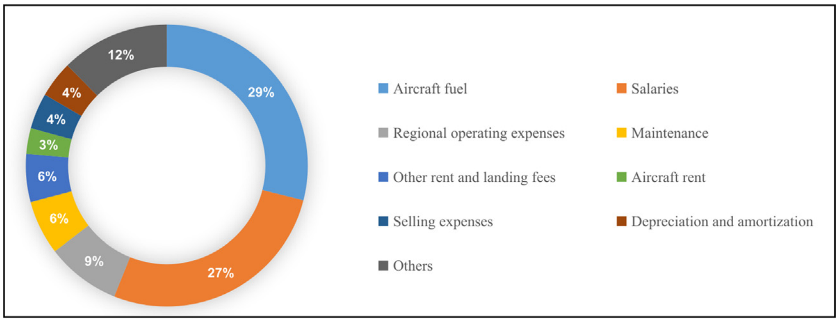

Fig. 4. Operating cost structure of a U.S. legacy carrier

Table 1
Summary for demand and supply interaction related publications.

Literature	Scheduling decision			Revenue evaluation		Algorithm	Feature
	Retiming	Optional	Scratch	S&R	Choice		
Rexing et al. (2000)	✓					B&B, Iterative method	Flight arc copies
Lohatepanont and Barnhart (2004)	✓	✓		✓		column-and-row generation	Demand correction term
Yan et al. (2008)	✓				✓	B&B	Stochastic demand
Sherali et al. (2013)		✓		✓		BD	Polyhedral analysis
Pita et al. (2013)	✓	✓	✓	✓		B&B	Aircraft/passenger delay
Atasoy et al. (2014)		✓			✓	BONMIN solver	Pricing decision
Cadarso and de Celis (2017)		✓		✓		BD	Demand scenario
Kenan et al. (2018a)		✓				CG	Stochastic demand and delay
Wei et al. (2020)	✓	✓	✓		✓	Rule based heuristic	General attraction model
Xu et al. (2021)	✓	✓		✓		Variable neighborhood search, CG	Passenger miss-connection
Birolini et al. (2021a)		✓			✓	B&B	Demand generation modelling
Birolini et al. (2021b)		✓	✓			Outer approximation	Embedding demand estimation model
Yan and Ghate (2022)		✓			✓	Iterative partition method	Subnetwork decomposition
Justin et al. (2022)		✓	✓	✓		B&B	Carbon emission optimization
Xu et al. (2023a)	✓	✓	✓		✓	Large neighborhood search, CG	Dynamic competition, prospect theory

B&B: Branch-and-Bound, **BD**: Benders decomposition, **B&B**: Branch & bound, **CG**: Column generation, **S&R**: Spill and recapture.

ous researchers have expanded the scope of this problem from various angles. For instance, in the work of Dawid et al. (2001), an enhanced rostering model was introduced that permits crew members to be downgraded to meet demand. Doi et al. (2018) considered equitable working hours for crew members by penalizing deviations from standard assigned values. The model's solution algorithm utilizes a metaheuristic approach. Initially, the original model is decomposed into a subproblem and a master problem, subsequently restructured into a generalized set partition model.

3. Demand and supply interaction perspective

It is of paramount importance for airlines to operate their critical assets with high utilization rates and match with the market demand so as to maximize the overall profits under network effects where both supply (frequency, network structures) and demand are mutually correlated (Kawasaki, 2008). To properly handle such effects, decisions including but not limited to schedule design, and fleet assignment must be appropriately combined. This section thus tries to investigate the existing integrated works that handle this issue.

While early FAP research such as Hane et al. (1995) assumed that the revenue of the entire network can be expressed at individual flight level by aggregating demand, such assumption caused inaccuracy stemming from ignorance of connecting passengers. To capture network dependencies, studies were carried out to investigate itinerary-based demand, passenger spill and recapture and discrete choice models. Relevant literature is summarized in Table 1. Column *Scratch* indicates whether an optimization model is able to build a comprehensive flight schedule from scratch. In contrast, the incremental design approach relies on some existing schedules for further adjustment (flight retiming or optional flight selection).

3.1. Spill and recapture

As for works dealing with spill and recapture effects, Lohatepanont and Barnhart (2004) developed an integrated schedule design and fleet assignment model where a demand correction term was tailored to adapt the market demand in response to variations in the flight schedule (e.g. demand increase when new flights are operated). The recapture rates were predicted and utilized to illustrate the case that spilled passengers who are not accommodated on their desired itineraries due to insufficient capacity may be recaptured on alternative itineraries. Similar spill and recapture effects were considered in Sherali et al. (2013) where the recapture rates were estimated using the quality of service index and the integrated model includes scheduling, fleeting and routing decisions. Cadarso and de Celis (2017) estimated the recapture probability per market, based on both travel time and current market share of the spilling airline. To optimize passenger spill cost, Khanmirza et al. (2020) developed a parallel master-slave genetic algorithm with two passenger redirection modes to solve the integrated fleet assignment and schedule design problem. In particular, multi-leg itineraries are redirected to available single-leg ones with a higher priority. In Xu et al. (2021), a multinomial logit model was applied to construct an integrated model for profit maximization and delay propagation minimization.

In general, the spill and recapture approach is typically adopted at itinerary level and relies on the estimation of recapture rates between different airline sales products considering the variations such as departure time and fares. It overlooks the possible changes and stochastic nature due to the absence of market-level information. Advancements with additional complexity involve the introduction of demand correction terms as applied in Lohatepanont and Barnhart (2004) to indicate the exogenous effects of frequencies (Birolini et al., 2021a). One representative formulation of spill and recapture approach from

Sherali et al. (2013) is stated as follows.

$$\pi_{pqh} \leq \beta_{pqh}(\mu_{ph} - \pi_{pph})z_q \qquad \forall p \in \Pi, h \in H_p, q \in \Pi_{ph} \tag{1}$$

$$\pi_{pph} \leq \mu_{ph} z_p \qquad \forall p \in \Pi^o, h \in H_p \tag{2}$$

$$\sum_{q \in \Pi_{ph}} \pi_{pqh} \leq \mu_{ph} \qquad \forall p \in \Pi, h \in H_p \tag{3}$$

Here, the continuous decision variable π_{pqh} denotes the number of passengers in fare class h redirected from itinerary p to q. The binary decision variable z_q takes value one if itinerary q is selected for inclusion. Constraints (1) computes the number of recaptured passengers in fare class h and itinerary q given the recapture rate parameter β_{pqh}, demand μ_{ph} if the itinerary is selected and there remain passengers not allocated to their original itinerary p. Constraints (2) and (3) compute the upper bound for redirected passenger number.

3.2. Passenger discrete choice models

While the spill and recapture approach assumes that the demand for passenger itineraries is deterministic, the discrete choice model-based approach treats the demand and supply interaction from a market viewpoint. Atasoy et al. (2014) devised an integrated model that incorporates fleeting, scheduling, and pricing decisions jointly. Therefore, demand dynamics can be directly modelled via a multinomial logit model in a monopolistic market. However, this approach yields a highly nonlinear model, presenting computational challenges for efficient solving. In Wei et al. (2020), a generalized attraction model addressed some of the inaccuracy of the multinomial logit model, stemming from the independence of irrelevant alternative issues. Yan and Ghate (2022) utilized reformulation and approximation methodologies to integrate choice-based demand and developed a partitioning technique to ensure computational feasibility. Birolini et al. (2021a) addressed the integrated schedule design and fleet assignment issue by incorporating endogenized demand generation and allocation. They linearized the corresponding nonlinear choice model using a two-segment piecewise linear function. In the work of Birolini et al. (2021b), an airline network planning problem was solved. By considering fleet assignment constraints, the model selected served routes and flight frequency based on a nonlinear demand estimation model. Then, the non-convexity was solved using a cutting-plane-based outer approximation method. Using a basic recapture scheme, Justin et al. (2022) formulated a half-leg half-itinerary MIP model using a hierarchical multi-objective approach (including carbon emission cost). In this way, high aircraft utilization rates and passenger itineraries with a short travel time are obtained. More recently, Xu et al. (2023a) developed a competitive schedule design and fleet assignment model that takes dynamic market competition into account. Passenger dynamics were captured via a prospect theory-based model which enables a better depiction of irrational risk-aversion attitudes when building the discrete choice model.

In a word, by embedding passenger choice behaviours, the integrated model is able to effectively capture market-level demand dynamics induced by executing optional flights and changing departure times. To illustrate the idea of discrete choice, part of the model from Yan and Ghate (2022) is presented as follows

$$\sum_{p \in P(m)} \frac{w_p - \tilde{w}_p}{w_p} s_p + \frac{w_m + \sum_{p \in P(m)} \tilde{w}_p}{w_m} s_m = \Lambda_m \qquad \forall m \in M \tag{4}$$

$$\frac{s_p}{w_p} - \frac{s_m}{w_m} \leq 0 \qquad \forall m \in M, p \in P(m) \tag{5}$$

Here, continuous decision variables s_p and s_m represent ticket sales volume for product p and the outside option in market m, respectively. Constraints (4) ensure the sale allocation is restricted to the total market demand. Scaling Constraints (5) require the sale of a product p is bounded by $\frac{w_p s_m}{w_m}$ where parameter w and $\tilde{w}$ denotes the attractiveness and shadow attractiveness of a fare product p which are defined in the context of the general attractive model.

4. Operational consistency perspective

Except for improving the demand-supply interaction modelling, integrated scheduling models are also expected to significantly improve the consistency of decisions among different resources in terms of various coupling operating constraints. As illustrated in Table 2, two promising directions are being studied to enhance the consistency: *aircraft-crew route synchronization* and *fleet assignment with crew resources consideration*.

4.1. Aircraft-crew route synchronization

Research that combines CSP with ARP offers the advantage of generating feasible and efficient schedules for aircraft routing and crew pairings jointly, as opposed to the sequential modelling approach. This integration is particularly beneficial in reducing operational costs resulting from improper crew routing, such as crew deadhead, and crew aircraft switching. Diverging from ordinary tactical scheduling approaches, Ruther et al. (2016) introduced a model to dynamically adjust aircraft routes and crew pairings, particularly in response to daily operational disruptions by separating the complete weekly planning horizon into two phases to preserve rescheduling flexibility. To efficiently and accurately tackle the integrated model, an enhanced column generation approach is devised incorporating superimposed pricing problems and early branching. Using a polynomial-sized compact model, Ahmed et al. (2018) solved the daily aircraft routing and crew scheduling problem with robust and cost-effective solutions. The integration was achieved by enforcing short and critical crew connections. Addressing operational hurdles specific to Air France, Parmentier and Meunier (2020) proposed two optimization problems (i.e. aircraft routing and crew pairing) and implemented a dynamic cut generation method to effectively generate feasible aircraft routes and crew pairings with reduced cost. Similarly, Shao et al. (2017) formulated a compact model encompassing fleeting, routing, and pairing decisions. The solution utilized an accelerated Benders decomposition method, supplemented by column generation along with a perturbed Lagrangian dual operator. As part of a broader crew scheduling context, Zeighami et al. (2020) integrated crew pairing and assignment subproblems to optimize crew pairings, particularly in accommodating crew vacation requests. Computational complexities were managed through an alternating Lagrangian decomposition algorithm.

Specifically, the integration of routing and pairing decisions relies on the coordination between aircraft turnaround and crew switching aircraft, further reducing joint propagated delays pose a higher requirement on route synchronization. For instance, the coupling constraints of the integrated model from Ruther et al. (2016) are shown as follows.

$$\sum_{b \in B} \sum_{p \in P_b} h^{pb}_{f_1 f_2} y^{pb} - \sum_{a \in A} \sum_{r \in R_a} h^{ra}_{f_1 f_2} x^{ra} \leq 0 \qquad \forall (f_1, f_2) \in C_{Sh} \tag{6}$$

$$\sum_{b \in B} \sum_{p \in P_b} h^{pb}_{f_1 f_2} y^{pb} - \sum_{a \in A} \sum_{r \in R_a} h^{ra}_{f_1 f_2} x^{ra} - z_{f_1 f_2} \leq 0 \qquad \forall (f_1, f_2) \in C_{Re} \tag{7}$$

By introducing binary variables x_{ra} and y_{pb} to represent the selection of route r and pairing p for aircraft a and crew block b respectively, the linking Constraints (6) stipulate that a short connection may be utilized by crews only if it is also covered by an aircraft. Here, parameters $h^{ra}_{f_1 f_2}$ and $h^{pb}_{f_1 f_2}$ take a value of 1 if connection (f_1, f_2) is part of route r or pairing p. Constraints (7) additionally consider aircraft changes on restricted connections where the binary variable $z_{f_1 f_2}$ takes a value of one to impose the corresponding penalty.

Table 2
Summary of operational consistency related publications.

Literature	FAP	ARP	CSP	Flights	Aircraft	Model	Algorithm	Feature
Sandhu and Klabjan (2007)	✓		✓	942	-	MIP	Lagrangian relaxation, BD	Essential ground arc
Papadakos (2009)	✓	✓	✓	705	167	MIP	BD, CG	Crew short connection
Salazar-González (2014)	✓	✓	✓	156	16	MIP	B&B, four-step heuristic	Two step routing and rostering problem.
Ruther et al. (2016)		✓	✓	1,130	50	MIP	CG, Diving	Comprehensive comparison on CG techniques.
Cacchiani and Salazar-González (2017)	✓	✓	✓	172	17	MIP	Branch-and-Price	Bounding cuts for acceleration.
Shao et al. (2017)	✓	✓	✓	676	192	MIP	BD, CG	Crew eligibility restriction.
Parmentier and Meunier (2020)		✓	✓	1,766	59	MIP	CG, B&B	Two-stage dynamic optimization of aircraft and crew routes.
Ahmed et al. (2022)	✓	✓	✓	646	202	MIP	Proximity search, B&B	Node-arc flow network representation
Wang et al. (2022)	✓		✓	1,572	380	MIP	BD, CG	Monthly/Yearly crew flight time limit.

MIP: Mixed Integer Programming, **MINP**: Mixed Integer Nonlinear Programming, **CG**: Column generation; **BD**: Benders decomposition, **B&B**: Branch & bound, **-**:not reported.

4.2. Fleet assignment with crew resources consideration

Different from models integrating aircraft routing and crew pairing decisions for schedule synchronization purposes, optimizing fleet assignment and crew pairing together leads to efficient and balanced allocation of crew resources across all flights. In contrast, the sequential optimization approach may lead to the absence of crew resources in the long term. In a study conducted by Sandhu and Klabjan (2007), a model was introduced to integrate fleet and crew pairing decisions. Additionally, plane-counting constraints were incorporated to ensure feasibility. To handle the complex coupling constraints, a technique combining Benders decomposition and Lagrangian relaxation was developed. Targeting a small regional airline and short planning horizon, Cacchiani and Salazar-González (2013) presented an optimization model aiming at minimizing crew routing time, number of crew/aircraft routes, and aircraft changes in crew rotations through decisions on fleet, aircraft, and crew resources. Salazar-González (2014) proposed a compact arc-based integrated model, merging fleet assignment, aircraft routing, and crew pairing problems without considering daylight aircraft maintenance. In another work by Cacchiani and Salazar-González (2017), two mathematical models were presented to integrate fleet assignment, aircraft routing, and crew scheduling. To derive exact solutions for these models, a multi-phase solution algorithm that combines bounding cuts and branch-and-price was proposed. Also working on integrating three subproblems, Ahmed et al. (2022) addressed the integrated robust optimization problem by developing an arc-based multi-commodity flow model. This model ensured robustness by penalizing short connections and incentivizing crews to adhere to aircraft routes. To cope with the inherent complexity, the model was decomposed and solved based on different aircraft families, and an improvement was applied using a proximity search method.

In brief, jointly optimizing crew schedules and fleet assignments can be categorized into microscopic and macroscopic ways under varying planning horizons. For instance, in the work of Wang et al. (2022), a tactical fleet assignment and crew pairing problem with crew flight time allocation were solved under macroscopic crew flight time constraints. Three key linking constraints are shown as follows:

$$\sum_{p\in P_b^m} \delta_{lp} z_{pb}^m = \sum_{f\in F_b} x_{lf}^m \qquad \forall m\in M, l\in L, b\in B \tag{8}$$

$$\sum_{p\in P_b^m} t_b^p z_{pb}^m \le t_b^m \qquad \forall m\in M, b\in B \tag{9}$$

$$\sum_{m\in M}\sum_{p\in P_b^m} t_b^p z_{pb}^m \le t_b \qquad \forall b\in B \tag{10}$$

Let binary variables x_{lf}^m take value 1 if flight leg $l \in L_m$ in month m is assigned to fleet f, the binary variable z_{pb}^m take value 1 when pairing $p \in P_b^m$ for fleet family b is included. Then Constraints (8) ensure that a pairing exactly covers a flight leg assigned to aircraft in the same fleet family ($\delta_{lp} = 1$ if leg l is included in pairing p). Constraints (9) and (10) pose the upper bounds for the total monthly and yearly available crew flight time, respectively.

5. Schedule robustness perspective

In contrast to integrated models that focus on improving revenue level and reducing operation inconsistency, robustness-related models cast light on schedule stability or resilience against potential disturbances. Actually, the airline industry is highly vulnerable to external factors like fluctuating demand, adverse weather conditions, and airspace congestion. Subtle disruptions in the original schedule may lead to cascading failures and reduced efficiency due to the complex coupling relationship among operating resources. Accordingly, the objective of robustness-based studies is to minimize potential financial setbacks resulting from disruptions through proactive planning measures. The related publications are summarized in Table 3 concerning their involved subproblems and robustness features.

5.1. Demand fluctuation

As a pivotal factor shaping airline flight schedules, demand is typically assessed months prior to operations, often in tandem with FAP, to inform decisions regarding aircraft and crew scheduling. Nonetheless, it's crucial to acknowledge the inherent uncertainty surrounding market demand during this planning period, as it can greatly impact the profitability of flight schedules. To address these challenges, there is a need to be flexible, and decisions may need to be re-evaluated within a relatively short planning horizon (Jiang and Barnhart, 2013), relying on updated and highly reliable demand predictions. Furthermore, various methodologies for handling robustness and uncertainty have been actively explored (Kenan et al., 2018a; Sherali and Zhu, 2008).

For works involving uncertain demand scenarios, Cadarso and de Celis (2017) presented a two-stage stochastic programming model that optimizes fleet assignment and optional flight selection decisions to improve the expected profits over a range of itinerary demand scenarios. Benders decomposition is applied to solve this model with a master model and a range of sub-models. Besides, in the work of Şafak et al. (2018), a three-stage stochastic nonlinear programming model was presented to simultaneously optimize flight retiming, fleeting and routing decisions. By incorporating variable passenger demand and non-cruise time, expected operational costs including delays, spilled

Table 3
Summary for schedule robustness related publications.

Literature	SDP	FAP	ARP	CSP	Method	Flight	Robust feature
Rosenberger et al. (2004)		✓			B&B	1,200	Short cycles
Yen and Birge (2006)				✓	B&P	79	Crew switching planes
Sherali and Zhu (2008)		✓			BD	900	Demand uncertainty
Weide et al. (2010)			✓	✓	CG, Iterative algorithm	750	Delay propagation
Dück et al. (2012)			✓	✓	CG, Dynamic programming	272	Propagated delays
Dunbar et al. (2012)			✓	✓	CG, Iterative algorithm	54	Delay propagation
Aloulou et al. (2013)	✓		✓		B&B	1,278	Delay propagation
Jiang and Barnhart (2013)	✓	✓			CG	1,000	Demand variability
Gürkan et al. (2016)	✓	✓	✓		Multi-stage search	114	Delay propagation
Ahmed et al. (2017)	✓		✓		B&B	1,195	Delay and passengers
Cadarso and de Celis (2017)	✓	✓			BD	-	Demand uncertainty
Kenan et al. (2018a)	✓	✓	✓		CG	228	Demand uncertainty, delay propagation
Şafak et al. (2018)	✓	✓	✓		SOCP, Cutting plane	102	Demand and non-cruise time uncertainty
Cacchiani and Salazar-González (2020)		✓	✓	✓	CG, pre-selection heuristic	150	Connection time
Xu et al. (2021)	✓	✓	✓		CG, Variable neighborhood search	1,607	Connection time
Ahmed et al. (2022)		✓	✓	✓	Proximity search	676	Crew follow aircraft
Şimşek and Aktürk (2022)		✓	✓		Discretized approximation	50	Uncertain non-cruise time

CG: Column generation, **BD**: Benders decomposition, **B&B**: Branch & bound, **B&P**: Branch & price, -:not reported.

passengers and misconnection are reduced. To solve the model with a large decision tree, a scenario group-wise decomposition and a cutting plane algorithm are developed. The relevant objective function terms and constraints are shown as follows:

$$\min \sum_{\omega\in\Omega}\left(\sum_{i\in J}\sum_{k\in K}\max\{0, demand_i(\omega) \\ -CAP_k\}\left(\sum_{j\in U_i} z^2_{jik}(\omega) + y^2_{ik}(\omega)c^i_{sp}\right)\right)p(\omega) \tag{11}$$

$$\sum_{k\in K}\left(\sum_{j\in U_i} z^2_{jik}(\omega) + y^2_{ik}(\omega)\right) = 1 \qquad \forall i \in J, \omega\in\Omega \tag{12}$$

$$z^2_{jik}(\omega) = z^2_{jik}(\bar\omega) \qquad \forall i\in J, j\in U_i, k\in K, \omega\in\Omega, \bar\omega\in\bar J(\omega) \tag{13}$$

$$y^2_{jik}(\omega) = y^2_{jik}(\bar\omega) \qquad \forall i\in J, j\in U_i, k\in K, \omega\in\Omega, \bar\omega\in\bar J(\omega) \tag{14}$$

Here, binary variable $z^2_{ijk}(\omega)$ takes one if both flights $i\in J$ and $j\in U_i$ are consecutively operated by aircraft type $k\in K$ under scenario $\omega\in\Omega$. Binary variable $y^2_{ik}(\omega)$ takes one if flight i is the first flight operated by aircraft type k under scenario ω. Then, given flight-aggregated demand $demand_i$, fleet capacity CAP_k and cost of spilled passenger c^i_{sp}, the objective function (11) tries to optimize the expected spilled cost based on a set of scenarios with probability $p(\omega)$. Constraints (12) ensure each flight is covered in every scenario. Non-anticipativity constraints (13) and (14) ensure that fleeting variables take equal values for the scenarios $\bar\omega\in\bar J(\omega)$ which share the same realization of demands.

5.2. Propagated delays

Focusing more closely on operational-level stability, another extensively explored aspect of robustness within airline scheduling pertains to flight delays, stemming from inefficient allocation of aircraft and crew turnaround times. To alleviate the considerable costs associated with delays, including crew wages, passenger discontent (Czerny and Lang, 2023), growing interests have been devoted to integrated robust scheduling works (Aloulou et al., 2013; Lan et al., 2006; Liang et al., 2015).

In general, airline scheduling studies for propagated delays typically fall into two main categories: recoverability and stability-based approaches. As for the recoverability approach, Rosenberger et al. (2004) proposed fleet assignment solutions featuring shorter aircraft cycles, enhancing flexibility in cancellation decisions. Froyland et al. (2014) further tackled the aircraft recovery subproblem by introducing a tail assignment model, minimizing weighted recovery costs through stochastic programming techniques. By proposing an integrated approach to a resilient scheduling model with chance constraints, Şimşek and Aktürk (2022) minimized the variability of a system based on aircraft characteristics, fleeting, and scheduling decisions. Specifically, parameters for the distribution of non-cruise time are derived using a logistic regression-based data-driven model and chance constraints are reformulated consequently using value-at-risk measure.

By integrating the aircraft recovery subproblem, Froyland et al. (2014) presented a tail assignment model (operational aircraft routing) that minimizes the weighted recovery cost using a stochastic programming approach. Concerning the stability approach, Liang et al. (2015) presented a robust model based on a weekly line-of-flight network and optimized the expected propagated delays of aircraft routes to derive robust weekly aircraft routing solutions. Due to the non-linear nature when computing (expected) propagated delays, column generation was adopted as the solution methodology. In the work of Xu et al. (2021), two MIP models were presented to maximize the overall profits which consist of itinerary revenue, operation cost, and expected delay cost. Passenger misconnections were also considered under the expected departure/arrival time of the corresponding connecting flights. The key linking constraints and decision variables associated with propagated delays are displayed as follows:

$$\max \sum_{i\in I} Fare_i h_i - \sum_{a\in A}\sum_{r\in R_a} c_r x_{a,r} \tag{15}$$

$$edt_f = STD_f + \sum_{a\in A}\sum_{r\in R_a} dev_{r,f} x_{a,r} \qquad \forall f\in F \tag{16}$$

$$tf_i = \sum_{a\in A}\sum_{r\in R_a}\theta^3_{r,i} x_{a,r} \qquad \forall i\in I^C \tag{17}$$

$$edt_{f_2} - edt_{f_1} - FT_{f_1} \geq (-MPT + MTT)tf_i \\ + MPTz_i - M_{f_1,f_2}(1-z_i) \qquad \forall i\in I^C, f_1, f_2\in F(i) \tag{18}$$

Here, binary variable x_{ar} takes value one if aircraft a execute route r in which various operational constraints, such as minimum turnaround times, maintenance check requirements, and initial aircraft positions, are implicitly considered. The objective function (15) seeks to maximize the overall profits, which encompass ticket sales h_i of itinerary i and aircraft route operation costs. The operation cost, denoted as c_r, encompasses both the direct expenses, such as fuel consumption and penalties associated with expected propagated delays. Constraints (16) derive the expected departure time edt_f of flight f based on the scheduled time of departure STD_f and deviation $dev_{r,f}$(including flight retiming and expected delay time) in route r. Through connection tf_i for a one-stop itinerary $i \in I^C$ is captured in Constraints (17) if the connecting flights are included in the same aircraft route ($\theta^3_{r,i} = 1$). Defining FT_f, MPT, MTT as the flight time, minimum passenger connection time, and minimum turnaround time, respectively. Binary variable z_i takes the value of one if itinerary i is selected for inclusion. Then, the big-M constraints (18) require that the connection time of every one-stop itinerary must be no less than the minimum passenger connection time if the two component flights are not through flights.

6. Conclusion and outlook

The challenges and complexities inherent in airline integrated scheduling problems have long captivated the attention of both scholars and industry practitioners. While considerable strides have been made in addressing integrated problems consisting of multiple subproblems over the past two decades, many existing integrated models still rely on assumptions that may not fully align with real-world operational dynamics.

The overarching goal for integrated scheduling research lies in the development of models that enable cohesive decision-making processes, resulting in enhanced profitability and decreased operation costs. Therefore, striking a balance between the complexity of solutions and the quality of models is imperative for devising practical decision-support frameworks. While previous endeavors have concentrated on formulating scalable algorithms, tackling medium to large-scale integrated models remains formidable, especially when employing near-exact decomposition methods such as column generation. Heuristic algorithms have garnered less favor due to their inability to provide optimality guarantees and their reliance on intricate problem-specific tuning procedures. Besides, existing studies have delved into robust scheduling problems to enhance the resilience and flexibility of flight schedules against unexpected disturbances. To incorporate these features efficiently, domain-knowledge-based evaluation metrics and data-driven methodologies have been proposed and aim for a balance between data mining ability and the complexity of blending in existing optimization models.

Our review of existing models and literature reveals ample opportunities for further exploration in the realm of airline integrated schedule planning.

First, as a complex planning process with multiple stakeholders, airline scheduling involves a large number of parameters and input data with inherent uncertainty such as demand (Kenan et al., 2018b), fuel price (Geursen et al., 2023), crew availability (Bayliss et al., 2017) and flight block time (Sohoni et al., 2011). Ignoring the potential fluctuation impact of these critical parameters and their inner correlations is likely to lead to low actual productivity and efficiency of a *static optimal* schedule. Considering the possible evolution and impact degree of various uncertainties during the complete planning horizon, future research may focus on developing data-driven multi-stage approaches based on adaptive robust optimization with integer recourse (Subramanyam et al., 2020) and stochastic programming (Zhang et al., 2023), to enhance operational agility and responsiveness against changing market conditions.

Secondly, by seamlessly integrating air travel with other transportation modes such as high-speed rail (Xu et al., 2023b), shuttle bus (Di Vito et al., 2022), and urban air mobility (Roy et al., 2022), great connectivity and flexibility of passenger door-to-door travel are provided. Additionally, multimodal transportation opens up new market opportunities for airlines by extending their reach to regions that may not be directly served by air routes. Resource utilization can also benefit from the increased connectivity of the overall operation network (e.g. crew deadhead). While synchronizing schedules is expected to improve the overall system efficiency, passenger passenger-oriented collaborative decision-making system with various stakeholders and priorities ought to be established via timely communication and information sharing (Marzuoli et al., 2016; Sun et al., 2017). Corresponding data and analytical models for mode choice (Nielsen et al., 2021) and transfer location (Jin et al., 2016), are also needed to more accurately capture the demand-and-supply interaction.

Thirdly, to improve the sustainability of flight operation, schedule decisions concerning demand and speed-dependent fuel burn (Krömer et al., 2024), crew pairing based greenhouse gas emissions (Wen et al., 2023), and carbon footprint induced by fleet composition can be jointly optimized either under certain regulatory requirements or over conflicting objectives (e.g. high load factor, profits). In parallel, research efforts have also been devoted to schedule planning based on sustainable aircraft fleet families, encompassing electric aircraft (Kinene et al., 2023; Vehlhaber and Salazar, 2023) and aircraft with sustainable aviation fuels (SAF) (Cabrera and de Sousa, 2022). Future research can be cast into the operation of hybrid sustainable fleet families and contribute to the long-term viability of the industry.

Finally, concerning the computation efficiency, to handle the inherent complexity of large-scale integrated models, there will be a growing need for scalable solution algorithms, with an increasing emphasis on machine learning-based approaches. Leveraging advanced data analytics and predictive modelling techniques, further research can utilize machine-learning techniques to either refine existing optimization algorithms such as Branch-and-Bound (Balcan et al., 2018) and column generation (Tahir et al., 2021) or reduce the solution space by identifying critical flight connections (Huang et al., 2023; Rashedi et al., 2023), ultimately striking a balance between optimality and scalability.

CRediT authorship contribution statement

Yifan Xu: Writing – original draft, Methodology, Investigation, Conceptualization.

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References

Abara, J. (1989). Applying integer linear programming to the fleet assignment problem. *Interfaces, 19*(4), 20–28.
Ahmed, M. B., Ghroubi, W., Haouari, M., & Sherali, H. D. (2017). A hybrid optimization-simulation approach for robust weekly aircraft routing and retiming. *Transportation Research Part C: Emerging Technologies, 84*, 1–20.
Ahmed, M. B., Hryhoryeva, M., Hvattum, L. M., & Haouari, M. (2022). A matheuristic for the robust integrated airline fleet assignment, aircraft routing, and crew pairing problem. *Computers & Operations Research, 137*, 105551.
Ahmed, M. B., Mansour, F. Z., & Haouari, M. (2018). Robust integrated maintenance aircraft routing and crew pairing. *Journal of Air Transport Management, 73*, 15–31.
Ahuja, R. K., Goodstein, J., Mukherjee, A., Orlin, J. B., & Sharma, D. (2007). A very large-scale neighborhood search algorithm for the combined through-fleet-assignment model. *INFORMS Journal on Computing, 19*(3), 416–428.
Aloulou, M. A., Haouari, M., & Mansour, F. Z. (2013). A model for enhancing robustness of aircraft and passenger connections. *Transportation Research Part C: Emerging Technologies, 32*, 48–60.
Atasoy, B., Salani, M., & Bierlaire, M. (2014). An integrated airline scheduling, fleeting, and pricing model for a monopolized market. *Computer-Aided Civil and Infrastructure Engineering, 29*(2), 76–90.
Balcan, M.-F., Dick, T., Sandholm, T., & Vitercik, E. (2018). Learning to branch. In *International conference on machine learning* (pp. 344–353). PMLR.
Barnhart, C., Boland, N. L., Clarke, L. W., Johnson, E. L., Nemhauser, G. L., & Shenoi, R. G. (1998). Flight string models for aircraft fleeting and routing. *Transportation Science, 32*(3), 208–220.
Barnhart, C., & Cohn, A. (2004). Airline schedule planning: Accomplishments and opportunities. *Manufacturing & service operations management, 6*(1), 3–22.
Barnhart, C., Farahat, A., & Lohatepanont, M. (2009). Airline fleet assignment with enhanced revenue modeling. *Operations Research, 57*(1), 231–244.
Barnhart, C., Hatay, L., & Johnson, E. L. (1995). Deadhead selection for the long-haul crew pairing problem. *Operations Research, 43*(3), 491–499.
Barnhart, C., Kniker, T. S., & Lohatepanont, M. (2002). Itinerary-based airline fleet assignment. *Transportation Science, 36*(2), 199–217.

Bayliss, C., De Maere, G., Atkin, J. A., & Paelinck, M. (2017). A simulation scenario based mixed integer programming approach to airline reserve crew scheduling under uncertainty. *Annals of Operations Research, 252*, 335–363.
Belobaba, P., Odoni, A., & Barnhart, C. (2015). *The global airline industry*. John Wiley & Sons.
Berge, M. E., & Hopperstad, C. A. (1993). Demand driven dispatch: A method for dynamic aircraft capacity assignment, models and algorithms. *Operations Research, 41*(1), 153–168.
Birolini, S., Antunes, A. P., Cattaneo, M., Malighetti, P., & Paleari, S. (2021a). Integrated flight scheduling and fleet assignment with improved supply-demand interactions. *Transportation Research Part B: Methodological, 149*, 162–180.
Birolini, S., Jacquillat, A., Cattaneo, M., & Antunes, A. P. (2021b). Airline network planning: Mixed-integer non-convex optimization with demand–supply interactions. *Transportation Research Part B: Methodological, 154*, 100–124.
Cabrera, E., & de Sousa, J. M. M. (2022). Use of sustainable fuels in aviation-a review. *Energies, 15*(7), 2440.
Cacchiani, V., & Salazar-González, J.-J. (2013). A heuristic approach for an integrated fleet-assignment, aircraft-routing and crew-pairing problem. *Electronic Notes in Discrete Mathematics, 41*, 391–398.
Cacchiani, V., & Salazar-González, J.-J. (2017). Optimal solutions to a real-world integrated airline scheduling problem. *Transportation Science, 51*(1), 250–268.
Cacchiani, V., & Salazar-González, J.-J. (2020). Heuristic approaches for flight retiming in an integrated airline scheduling problem of a regional carrier. *Omega, 91*, 102028.
Cadarso, L., & de Celis, R. (2017). Integrated airline planning: Robust update of scheduling and fleet balancing under demand uncertainty. *Transportation Research Part C: Emerging Technologies, 81*, 227–245.
Clarke, L., Johnson, E., Nemhauser, G., & Zhu, Z. (1997). The aircraft rotation problem. *Annals of Operations Research, 69*, 33–46.
Czerny, A. I., & Lang, H. (2023). Modeling airport networks for theoretical policy analyses. *Journal of the Air Transport Research Society, 1*(1), 54–62.
Dawid, H., König, J., & Strauss, C. (2001). An enhanced rostering model for airline crews. *Computers & Operations Research, 28*(7), 671–688.
Desaulniers, G., Desrosiers, J., Dumas, Y., Solomon, M. M., & Soumis, F. (1997). Daily aircraft routing and scheduling. *Management Science, 43*(6), 841–855.
Di Vito, V., Montaquila, R. V., Cerasuolo, G., Dziugieł, B., Maczka, M., Mazur, A., Meincke, P. A., Naser, F., Mota, M. M., Bagamanova, M., et al., (2022). An outline of a concept of operations for integration of atm and air transport into multimodal transport system for door-to-door travel. In *2022 integrated communication, navigation and surveillance conference (icns)* (pp. 1–12). IEEE.
Doi, T., Nishi, T., & Voß, S. (2018). Two-level decomposition-based matheuristic for airline crew rostering problems with fair working time. *European Journal of Operational Research, 267*(2), 428–438.
Dück, V., Ionescu, L., Kliewer, N., & Suhl, L. (2012). Increasing stability of crew and aircraft schedules. *Transportation research part C: emerging technologies, 20*(1), 47–61.
Dunbar, M., Froyland, G., & Wu, C.-L. (2012). Robust airline schedule planning: Minimizing propagated delay in an integrated routing and crewing framework. *Transportation Science, 46*(2), 204–216.
Eltoukhy, A. E., Chan, F. T., & Chung, S. H. (2017). Airline schedule planning: a review and future directions. *Industrial Management & Data Systems*.
Froyland, G., Maher, S. J., & Wu, C.-L. (2014). The recoverable robust tail assignment problem. *Transportation Science, 48*(3), 351–372.
Geursen, I. L., Santos, B. F., & Yorke-Smith, N. (2023). Fleet planning under demand and fuel price uncertainty using actor–critic reinforcement learning. *Journal of Air Transport Management, 109*, 102397.
Gopalakrishnan, B., & Johnson, E. L. (2005). Airline crew scheduling: State-of-the-art. *Annals of Operations Research, 140*, 305–337.
Gürkan, H., Gürel, S., & Aktürk, M. S. (2016). An integrated approach for airline scheduling, aircraft fleeting and routing with cruise speed control. *Transportation Research Part C: Emerging Technologies, 68*, 38–57.
Hane, C. A., Barnhart, C., Johnson, E. L., Marsten, R. E., Nemhauser, G. L., & Sigismondi, G. (1995). The fleet assignment problem: Solving a large-scale integer program. *Mathematical Programming, 70*(1), 211–232.
Haouari, M., Shao, S., & Sherali, H. D. (2013). A lifted compact formulation for the daily aircraft maintenance routing problem. *Transportation Science, 47*(4), 508–525.
Haouari, M., Zeghal Mansour, F., & Sherali, H. D. (2019). A new compact formulation for the daily crew pairing problem. *Transportation Science, 53*(3), 811–828.
Hoffman, K. L., & Padberg, M. (1993). Solving airline crew scheduling problems by branch-and-cut. *Management science, 39*(6), 657–682.
Huang, L., Xiao, F., Zhou, J., Duan, Z., Zhang, H., & Liang, Z. (2023). A machine learning based column-and-row generation approach for integrated air cargo recovery problem. *Transportation Research Part B: Methodological, 178*, 102846.
Ioachim, I., Desrosiers, J., Soumis, F., & Bélanger, N. (1999). Fleet assignment and routing with schedule synchronization constraints. *European Journal of Operational Research, 119*(1), 75–90.
Jiang, H., & Barnhart, C. (2013). Robust airline schedule design in a dynamic scheduling environment. *Computers & Operations Research, 40*(3), 831–840.
Jin, J. G., Teo, K. M., & Odoni, A. R. (2016). Optimizing bus bridging services in response to disruptions of urban transit rail networks. *Transportation Science, 50*(3), 790–804.
Justin, C. Y., Payan, A. P., & Mavris, D. N. (2022). Integrated fleet assignment and scheduling for environmentally friendly electrified regional air mobility. *Transportation Research Part C: Emerging Technologies, 138*, 103567.
Kasirzadeh, A., Saddoune, M., & Soumis, F. (2017). Airline crew scheduling: models, algorithms, and data sets. *EURO Journal on Transportation and Logistics, 6*(2), 111–137.
Kawasaki, A. (2008). Network effects, heterogeneous time value and network formation in the airline market. *Regional Science and Urban Economics, 38*(4), 388–403.
Kenan, N., Jebali, A., & Diabat, A. (2018a). The integrated aircraft routing problem with optional flights and delay considerations. *Transportation Research Part E: Logistics and Transportation Review, 118*, 355–375.
Kenan, N., Jebali, A., & Diabat, A. (2018b). An integrated flight scheduling and fleet assignment problem under uncertainty. *Computers & Operations Research, 100*, 333–342.
Khanmirza, E., Nazarahari, M., & Haghbeigi, M. (2020). A heuristic approach for optimal integrated airline schedule design and fleet assignment with demand recapture. *Applied Soft Computing, 96*, 106681.
Kinene, A., Birolini, S., Cattaneo, M., & Granberg, T. A. (2023). Electric aircraft charging network design for regional routes: A novel mathematical formulation and kernel search heuristic. *European Journal of Operational Research, 309*(3), 1300–1315.
Krömer, M. M., Topchishvili, D., & Schön, C. (2024). Sustainable airline planning and scheduling. *Journal of Cleaner Production, 434*, 139986.
Lan, S., Clarke, J.-P., & Barnhart, C. (2006). Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions. *Transportation Science, 40*(1), 15–28.
Lavoie, S., Minoux, M., & Odier, E. (1988). A new approach for crew pairing problems by column generation with an application to air transportation. *European Journal of Operational Research, 35*(1), 45–58.
Liang, Z., & Chaovalitwongse, W. A. (2013). A network-based model for the integrated weekly aircraft maintenance routing and fleet assignment problem. *Transportation Science, 47*(4), 493–507.
Liang, Z., Chaovalitwongse, W. A., Huang, H. C., & Johnson, E. L. (2011). On a new rotation tour network model for aircraft maintenance routing problem. *Transportation Science, 45*(1), 109–120.
Liang, Z., Feng, Y., Zhang, X., Wu, T., & Chaovalitwongse, W. A. (2015). Robust weekly aircraft maintenance routing problem and the extension to the tail assignment problem. *Transportation Research Part B: Methodological, 78*, 238–259.
Lohatepanont, M., & Barnhart, C. (2004). Airline schedule planning: Integrated models and algorithms for schedule design and fleet assignment. *Transportation Science, 38*(1), 19–32.
Ma, H.-L., Sun, Y., Chung, S.-H., & Chan, H. K. (2022). Tackling uncertainties in aircraft maintenance routing: A review of emerging technologies. *Transportation Research Part E: Logistics and Transportation Review, 164*, 102805.
Maher, S. J., Desaulniers, G., & Soumis, F. (2018). The daily tail assignment problem under operational uncertainty using look-ahead maintenance constraints. *European Journal of Operational Research, 264*(2), 534–547.
Makri, A., & Klabjan, D. (2004). A new pricing scheme for airline crew scheduling. *INFORMS Journal on Computing, 16*(1), 56–67.
Marzuoli, A., Boidot, E., Colomar, P., Guerpillon, M., Feron, E., Bayen, A., & Hansen, M. (2016). Improving disruption management with multimodal collaborative decision-making: A case study of the asiana crash and lessons learned. *IEEE Transactions on Intelligent Transportation Systems, 17*(10), 2699–2717.
Morrison, S., & Winston, C. (2010). *The economic effects of airline deregulation*. Brookings Institution Press.
Nielsen, O. A., Eltved, M., Anderson, M. K., & Prato, C. G. (2021). Relevance of detailed transfer attributes in large-scale multimodal route choice models for metropolitan public transport passengers. *Transportation Research Part A: Policy and Practice, 147*, 76–92.
Papadakos, N. (2009). Integrated airline scheduling. *Computers & Operations Research, 36*(1), 176–195.
Parmentier, A., & Meunier, F. (2020). Aircraft routing and crew pairing: updated algorithms at air france. *Omega, 93*, 102073.
Pilla, V. L., Rosenberger, J. M., Chen, V., Engsuwan, N., & Siddappa, S. (2012). A multivariate adaptive regression splines cutting plane approach for solving a two-stage stochastic programming fleet assignment model. *European Journal of Operational Research, 216*(1), 162–171.
Pita, J. P., Barnhart, C., & Antunes, A. P. (2013). Integrated flight scheduling and fleet assignment under airport congestion. *Transportation Science, 47*(4), 477–492.
Rashedi, N., Sankey, N., Vaze, V., & Wei, K. (2023). A machine learning approach for solution space reduction in aircraft disruption recovery. *Available at SSRN 4444548*.
Rexing, B., Barnhart, C., Kniker, T., Jarrah, A., & Krishnamurthy, N. (2000). Airline fleet assignment with time windows. *Transportation Science, 34*(1), 1–20.
Rosenberger, J. M., Johnson, E. L., & Nemhauser, G. L. (2004). A robust fleet-assignment model with hub isolation and short cycles. *Transportation Science, 38*(3), 357–368.
Roy, S., Kotwicz Herniczek, M. T., Leonard, C., German, B. J., & Garrow, L. A. (2022). Flight scheduling and fleet sizing for an airport shuttle air taxi service. *Journal of Air Transportation, 30*(2), 49–58.
Rushmeier, R. A., & Kontogiorgis, S. A. (1997). Advances in the optimization of airline fleet assignment. *Transportation Science, 31*(2), 159–169.
Ruther, S., Boland, N., Engineer, F. G., & Evans, I. (2016). Integrated aircraft routing, crew pairing, and tail assignment: branch-and-price with many pricing problems. *Transportation Science, 51*(1), 177–195.
Şafak, Ö., Çavuş, Ö., & Aktürk, M. S. (2018). Multi-stage airline scheduling problem with stochastic passenger demand and non-cruise times. *Transportation Research Part B: Methodological, 114*, 39–67.
Salazar-González, J.-J. (2014). Approaches to solve the fleet-assignment, aircraft-routing, crew-pairing and crew-rostering problems of a regional carrier. *Omega, 43*, 71–82.
Sandhu, R., & Klabjan, D. (2007). Integrated airline fleeting and crew-pairing decisions. *Operations Research, 55*(3), 439–456.
Sarac, A., Batta, R., & Rump, C. M. (2006). A branch-and-price approach for operational aircraft maintenance routing. *European Journal of Operational Research, 175*(3), 1850–1869.
Shao, S., Sherali, H. D., & Haouari, M. (2017). A novel model and decomposition approach for the integrated airline fleet assignment, aircraft routing, and crew pairing problem. *Transportation Science, 51*(1), 233–249.

Sherali, H., Bish, E., & Zhu, X. (2006). Airline fleet assignment concepts, models, and algorithms. *European Journal of Operational Research, 172*, 1–30. 10.1016/j.ejor.2005.01.056.
Sherali, H. D., Bae, K.-H., & Haouari, M. (2013). An integrated approach for airline flight selection and timing, fleet assignment, and aircraft routing. *Transportation Science, 47*(4), 455–476.
Sherali, H. D., & Zhu, X. (2008). Two-stage fleet assignment model considering stochastic passenger demands. *Operations Research, 56*(2), 383–399.
Şimşek, D., & Aktürk, M. S. (2022). Resilient airline scheduling to minimize delay risks. *Transportation Research Part C: Emerging Technologies, 141*, 103734.
Sohoni, M., Lee, Y.-C., & Klabjan, D. (2011). Robust airline scheduling under block-time uncertainty. *Transportation Science, 45*(4), 451–464.
Subramanyam, A., Gounaris, C. E., & Wiesemann, W. (2020). K-adaptability in two-stage mixed-integer robust optimization. *Mathematical Programming Computation, 12*, 193–224.
Sun, X., Wandelt, S., & Zhang, A. (2023). Aviation under the covid-19 pandemic: A synopsis from normalcy to chaos and back. *Journal of the Air Transport Research Society, 1*(1), 136–151.
Sun, X., Zhang, Y., Wandelt, S., et al., (2017). Air transport versus high-speed rail: an overview and research agenda. *Journal of Advanced Transportation, 2017*.
Sun, X., Zheng, C., Wandelt, S., & Zhang, A. (2024). Airline competition: A comprehensive review of recent research. *Journal of the Air Transport Research Society*, 100013.
Tahir, A., Quesnel, F., Desaulniers, G., El Hallaoui, I., & Yaakoubi, Y. (2021). An improved integral column generation algorithm using machine learning for aircrew pairing. *Transportation Science, 55*(6), 1411–1429.
Vehlhaber, F., & Salazar, M. (2023). Electric aircraft assignment, routing, and charge scheduling considering the availability of renewable energy. *IEEE Control Systems Letters*.
Wandelt, S., Antoniou, C., Birolini, S., Delahaye, D., Dresner, M., Fu, X., Gössling, S., Hong, S.-J., Odoni, A. R., Zanin, M., et al., (2024). Status quo and challenges in air transport management research. *Journal of the Air Transport Research Society, 2*, 100014.
Wang, D., Guo, S., Wang, W., & Liang, Z. (2022). Tactical fleet assignment and crew pairing problem with crew flight time allocation. *arXiv preprint arXiv:2209.13904*.
Wei, K., Vaze, V., & Jacquillat, A. (2020). Airline timetable development and fleet assignment incorporating passenger choice. *Transportation Science, 54*(1), 139–163.
Weide, O., Ryan, D., & Ehrgott, M. (2010). An iterative approach to robust and integrated aircraft routing and crew scheduling. *Computers & Operations Research, 37*(5), 833–844.
Wen, X., Chung, S.-H., Ma, H.-L., & Khan, W. A. (2023). Airline crew scheduling with sustainability enhancement by data analytics under circular economy. *Annals of Operations Research*, 1–27.
Wen, X., Sun, X., Sun, Y., & Yue, X. (2021). Airline crew scheduling: Models and algorithms. *Transportation research part E: logistics and transportation review, 149*, 102304.
Wu, X., Fu, X., Lei, Z., & Wang, K. (2023). Impact of the covid-19 pandemic on multi-airport systems worldwide. *Journal of the Air Transport Research Society, 1*(1), 117–135.
Xu, Y., Adler, N., Wandelt, S., & Sun, X. (2023a). Competitive integrated airline schedule design and fleet assignment. *European Journal of Operational Research*.
Xu, Y., Wandelt, S., & Sun, X. (2021). Airline integrated robust scheduling with a variable neighborhood search based heuristic. *Transportation Research Part B: Methodological, 149*, 181–203.
Xu, Y., Wandelt, S., & Sun, X. (2023b). Immuner: Integrated multimodal mobility under network disruptions. *IEEE Transactions on Intelligent Transportation Systems, 24*(2), 1480–1494.
Xu, Y., Wandelt, S., & Sun, X. (2024). Airline scheduling optimization: literature review and a discussion of modelling methodologies. *Intelligent Transportation Infrastructure, 3*, liad026.
Yan, C., & Ghate, A. (2022). A dynamic model for airline fleeting and scheduling. *Available at SSRN*.
Yan, S., Tang, C.-H., & Fu, T.-C. (2008). An airline scheduling model and solution algorithms under stochastic demands. *European Journal of Operational Research, 190*(1), 22–39.
Yen, J. W., & Birge, J. R. (2006). A stochastic programming approach to the airline crew scheduling problem. *Transportation Science, 40*(1), 3–14.
Zeighami, V., Saddoune, M., & Soumis, F. (2020). Alternating lagrangian decomposition for integrated airline crew scheduling problem. *European Journal of Operational Research, 287*(1), 211–224.
Zhang, W., Wang, K., Jacquillat, A., & Wang, S. (2023). Optimized scenario reduction: Solving large-scale stochastic programs with quality guarantees. *INFORMS Journal on Computing, 35*(4), 886–908.
Zhou, L., Liang, Z., Chou, C.-A., & Chaovalitwongse, W. A. (2020). Airline planning and scheduling: Models and solution methodologies. *Frontiers of Engineering Management, 7*(1), 1–26.