

Numerical Simulations (HD and MHD) of Planetary Winds in the Vicinity of Stellar Winds Emanating from their Host Stars

Anselmo Falorca

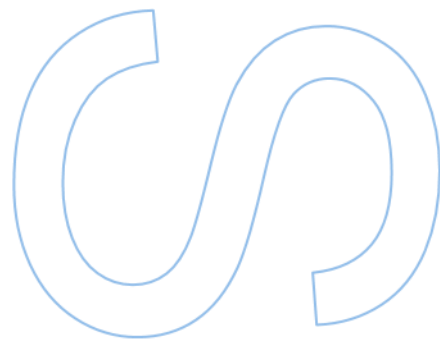
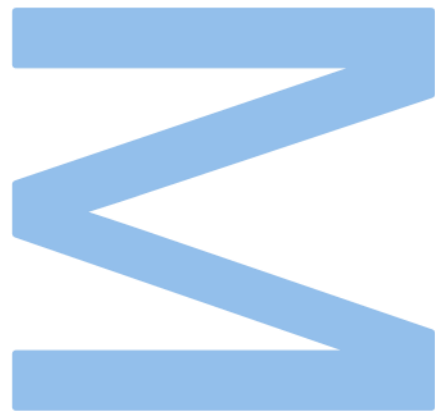
Master in Physics (Specialization in Theoretical Physics)
Departamento de Física e Astronomia
2023

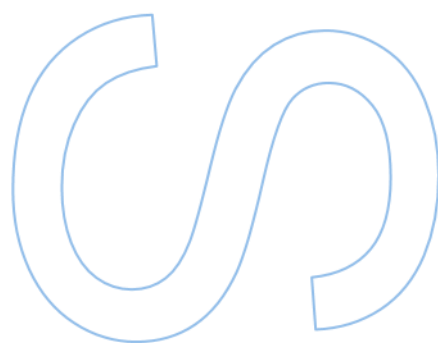
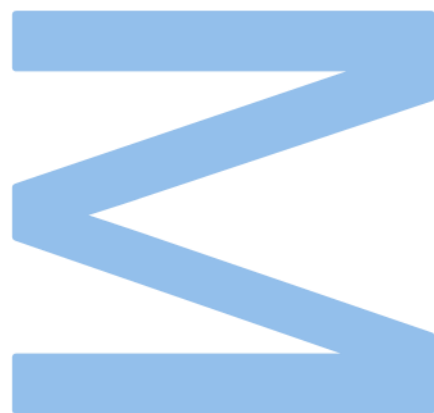
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“ We may brave human laws, but we cannot resist natural ones. ”

20000 Leagues Under the Sea, written by Jules Verne

Acknowledgements

First and foremost, I want to express my heartfelt gratitude to Professor João Lima, for he most generously attended my plead to embark on a new project inside the areas of HD and MHD. It was, without a doubt, a bold step to take with me, that ultimately ended up revealing myself as a researcher.

Secondly, to Professor Adam Frank, the privilege of having his wisdom closely monitoring several aspects of this project. The extremely stimulating talks and chats have had me smiling for what science is all about.

Almost as a co-supervisor, I must gratefully thank Professor Christophe Sauty for his important presence over the course of this dissertation. His expertise in the code's numerical elements and his bibliographic recommendations have been greatly valued.

To Manuel Monteiro, for accompanying me with precious attention throughout CAUP's computers details and frustrations. The words "this is all free, this knowledge" have taken a good toll in my *modus operandi*.

For the extremely precious presence of Doctor Jonathan Carroll-Nellenback, as many helpful tips have been crucial in this dissertation.

To my parents and sister, that have shown me that any path that I take is the correct path, as long as it makes me realize I feel good taking it.

To Cristina Cunha, as my short numerical outrages and long theoretical discussions have been very well appeased by your kindness. For many more balcony talks facing Ipanema.

To all my colleagues and close friends in Porto, for having your relaxing presence either at the coffee time or at a short time gap over a random day.

At last, but not least, to my close friends from Covilhã, whose friendship has never been forgotten, for having grown me to the happy fellow I am.

To everyone I mentioned and forgot to mention, thank you deeply.

UNIVERSIDADE DO PORTO

Abstract

Faculdade de Ciências da Universidade do Porto

Departamento de Física e Astronomia

MSc. Physics

Numerical Simulations (HD and MHD) of Planetary Winds in the Vicinity of Stellar Winds Emanating from their Host Star.

by [Anselmo FALORCA](#)

The aim of this dissertation is to explore and understand the behaviour and dynamics of planetary outflows which are originated by the physical process of photoevaporation. Ultimately, it is also intended to having these planetary winds interacting with the stellar host winds, as it is what the state-of-the-art numerical simulations on photoevaporation theory have reached.

In order to achieve such goals, it is used the astrophysical code **PLUTO** ([Mignone et al., 2007](#)). With this, the differential equations are solved in a Gudonov scheme, with the ability to implement specific initial conditions as well as boundary conditions that mimic the chosen physical system HD209458.

The physical strategy implemented follows the HD works of [Stone and Proga \(2009\)](#); [Carroll-Nellenback et al. \(2017\)](#) in which temperature anisotropy is inserted explicitly at the Hot Jupiter's (HJ) atmosphere. As such, the step-by-step model building that is implemented - in which the HJ is static, spinning, orbiting and having its outflow interacting with stellar winds - is applied both for an isothermal atmosphere and a realistic yet simplistic "dayside"/"nightside" case. In addition to this, such simulations are carried out in a hydrodynamic framework (HD) and in a magnetohydrodynamic (MHD), where the winds become magnetized by the presence of dipolar magnetic fields.

Results in either types of models show themselves quite compatible to the intended HD works or even with MHD results ([Matsakos et al., 2015](#)). While an isothermal atmosphere induces an outflow similar to a Parker Wind ([Parker, 1958](#)), having temperature anisotropy forces the dayside outflow to bend into the HJ's "nightside" due to lateral

pressures. As such, there exists an inflow of material at the nightside, which is strongly affected by the shock of the "dayside" and "nightside" winds. Although it was not possible to achieve a simulation of MHD with anisotropy, discussion shows the importance of magnetic pressure at the bow-shock of the interacting winds.

UNIVERSIDADE DO PORTO

Resumo

Faculdade de Ciências da Universidade do Porto

Departamento de Física e Astronomia

Mestrado em Física

Simulações Numéricas (HD e MHD) de Ventos Planetários Próximos de Ventos

Solares Emanados pela Estrela Anfitriã

por [Anselmo FALORCA](#)

O objectivo desta dissertação é explorar e entender o comportamento e dinâmica de ventos planetários que são originados pelo processo físico da fotoevaporação. Adicionalmente, também se pretende obter modelos de ventos planetários a interagir com ventos solares - que é o estado da arte em simulações numéricas dentro da teoria de fotoevaporação.

De forma a alcançar tais objectivos, é usado o código para gases astrofísicos **PLUTO** ([Mignone et al., 2007](#)). É com a ajuda deste software que as equações diferenciais são resolvidas por um esquema de Gudonov, com a versatilidade de implementar adicionalmente condições iniciais específicas, assim como condições fronteira, que imitam as características físicas do sistema HD209458.

A estratégia física implementada segue as obras de [Stone and Proga \(2009\)](#); [Carroll-Nellenback et al. \(2017\)](#) nos quais é explicitamente implementado um gradiente de temperatura na atmosfera do Hot Jupiter (HJ). Como tal, são construídos modelos camada por camada - onde se começa com um HJ estático, a rodar, a orbitar e, por fim, com os seus ventos a interagirem com ventos solares - que serão aplicados quer a uma atmosfera isotérmica, quer a uma atmosfera realística e ao mesmo tempo simples que segue um formato de "dia"/"noite". Para além disso, estas simulações serão realizadas com um cariz hidrodinâmico (HD) e magnetohidrodinâmico (MHD), onde os ventos se tornam magnetizados pela presença de campos magnéticos dipolares.

Os resultados em ambos os modelos mostram-se compatíveis com as obras seguidas, quer nas simulações HD, quer nas MHD ([Matsakos et al., 2015](#)). Enquanto que uma atmosfera isotérmica induz um vento planetário muito semelhante a um "Parker Wind"

([Parker, 1958](#)), um gradiente de temperature força a que o vento planetário na zona diurna seja redirecionado para a zona noturna devido a pressões laterais. Consequentemente, existe uma re-entrada de gás pela zona noturna do planeta que está fortemente ligada ao choque entre os ventos diários e noturnos. Infelizmente não foi possível simular um modelo MHD com gradiente de temperatura; no entanto, a discussão demonstra a importância das pressões magnéticas no limiar do choque dos ventos a interagirem.

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Physical Constants

Gravitational constant	G	$= 6.67430 \times 10^{-8} \text{ dyn} \cdot \text{g}^2 \cdot \text{cm}^{-2}$
Boltzmann constant	k_B	$= 1.38065 \times 10^{-8} \text{ erg} \cdot \text{K}^{-1}$
Proton's Mass	m_p	$= 1.67262 \times 10^{-24} \text{ g}$
Jupiter's Mass	M_J	$= (1.89813 \pm 0.00019) \times 10^{30} \text{ g}$
Sun's Mass	$M_\odot$	$= (1.98847 \pm 0.00007) \times 10^{33} \text{ g}$
Jupiter's Radius	R_J	$= 7.1492 \times 10^9 \text{ cm}$
Sun's Radius	$R_\odot$	$= (6.95660 \pm 0.00014) \times 10^{10} \text{ cm}$

Glossary

HD	Hydrodynamic
MHD	Magnetohydrodynamic
HJ	Hot Jupiter
UV	Ultra-violet
EUV	Extreme Ultra-violet
H	Hydrogen
He	Helium
SPI	Star-Planet Interaction
AMR	Adaptive Mesh Refinement
PPM	Piecewise Parabolic Method
EOS	Equation of State
RHS	Right Hand Side
LHS	Left Hand Side

Thank you God, for giving me a withstanding Windows laptop.

Chapter 1

Introduction

This dissertation explores the interactions of stellar winds with outflowing planetary winds which are a direct consequence of photoevaporation. The main goal is to compare Hydrodynamic (HD) wind interactions with Magnetohydrodynamic (MHD) ones, focusing on the important phenomena of temperature anisotropy. This anisotropy causes winds to shock around the planet, which may be one important aspect when gathering numerical results that can be confronted with observational data.

1.1 Motivation

Photoevaporation has shown itself as a useful theory to explore possible solutions to observational problems that scientists have dueled for a while. However, planetary photoevaporation offers mass loss and synthetic observations that do not match observations in several exoplanets.

Many important works have shed light into understanding the major phenomena behind Hot Jupiter's photoevaporation, such as [McCann et al. \(2019\)](#) and [Carroll-Nellenback et al. \(2017\)](#) for HD winds and [Matsakos et al. \(2015\)](#) and [Zhilkin and Bisikalo \(2019\)](#) for MHD winds interacting. Nevertheless, these morphology treatments require some fine-tuning to give factual information on the details of these systems.

Due to the computational demand of 3D simulations, which are required to capture the most of a system's asymmetric behaviour, only a certain amount of physical phenomena can be explored. Some 1D works have taken the opposite role: while far many

physical processes can be studied at once, these fall behind in reproducing the entirety of the system's complex behaviour.

The commonly implemented strategy has been to explore with good detail the phase-space of a given physical process by changing important parameters of the system. This may translate to upper limits of the explored phenomena, which can be enlightening to verify if a given physical process can become fundamental to be verified by observations. One such example can be the work of [Debrecht et al. \(2020\)](#), which understood that the effects of radiation pressure are not significant to reproduce the observed enhancement of stellar Lyman- α lines in the blue wing compared to the red wing, during a planetary transit.

As a result, there is still much to learn in this field of study.

1.2 Objectives

This dissertation focuses on the exploration of HD and MHD winds in a framework where cooling and heating phenomena at the planet's atmosphere is inserted explicitly. This follows an initial objective of reproducing the works of [Stone and Proga \(2009\)](#) and [Carroll-Nellenback et al. \(2017\)](#), where important results have been drawn from these. However, the main goal will be to understand what are the major consequences of inserting a magnetosphere in the planet. Magnetized stellar winds will also be modeled to interact with the magnetized planetary winds. This will be studied by a detailed analysis of the mass loss of the planet, as well as having a strong sense of the winds' dynamics with the use of physical plots and time-lapsing animations. The numerical results will be explored from the outputs of the astrophysical code used to carry out simulations, where differential equations are solved with enforced initial conditions and boundaries. Comparison with other studies should be able to support the models. Additionally, there is also the need to understand the consequences of inserting temperature anisotropy with magnetic fields. This would generalize the results of [Carroll-Nellenback et al. \(2017\)](#) from HD to MHD, something that has not been addressed up to now, as far as the author is aware.

1.3 Structure of the dissertation

In order to achieve the objectives stated above, this dissertation requires an effective framework. Thus, prior to anything, gathering articles and exploring the state-of-the-art

results is essential. Having a wide collection of examples to learn how a complex problem can be constructed from root allows for a more trusted understanding of physical (and non-physical) results. Thus, Section 2 paves the historical background of photoevaporation theory to the most recent and impacting works.

In order to skip the need of solving complex and computationally demanding differential equations with boundaries and grids, a established numerical code was used, **PLUTO**. Of course, this required some prior familiarization with the code itself, before applying it to our problem. Thus, after the code has been downloaded, several test problems were executed to verify that the final outputs match the known theoretical results. In this way, as several layers of the code were discovered, it was ensured that the used version was error free. Section 3 appears as a requirement for the reader to be fully engaged into the computational physics of these outflowing models.

After these, sections 4 and 5 are dedicated to modeling the photoevaporating systems in a HD and MHD framework, respectively. Section 4 tends to fall in a way of obtaining similar results to the selected referenced articles, but ever independently focused on understanding the reality of the chosen astrophysical system. Section 5, on the other hand, brings new highlights to the type of structured problem at hand. Both Sections are introduced by a review of major aspects and the theory itself and followed by a chapter for each new "structural block" that is inserted into the problem. Inside each chapter, the following structure is replicated: theoretical introduction, as the new "block" of complexity inserted can bear new features; framework simulation, where it is explained how it is numerically inserted and other details; results and important remarks, where it is discussed the outputs of the simulation.

At last, the ending Section 6 has the intention of summing up the main aspects retrieved in Sections 4 and 5, as well as proposing future works in order to complement any loose end of this dissertation.

1.4 Outputs

With the development of this dissertation, it was possible to contribute with additional works at several meetings:

- Two posters were presented: in IJUP (Investigação Jovem da Universidade do Porto) 2023, on the 12th may, with the title of "Modeling Hot Jupiter winds and their anisotropies using HD numerical simulations"; and in TOE (Towards Other Earths) III conference, from the 17th – 21st july, with the title of "Modeling Hot Jupiter winds with HD simulations: Investigating inflow with temperature anisotropy".
- Two oral presentations were also given: in ENAA (Encontro Nacional de Astronomia e Astrofísica) 2023 on the 7th september, with the title "Numerical Simulations (HD and MHD) of Planetary Outflows from Hot Jupiters"; and in CAUP on the 26th october, as a "Cookie Seminar", with the title: "Numerical Simulations (HD and MHD) of outflows from photoevaporating Hot Jupiters interacting with stellar winds".

Chapter 2

Literature Review

As space technology advances and our understanding of the Universe beyond Earth grows, an increasing number of publications are being written about exoplanets. In the year 1995, Mayor and Queloz found the first Hot Jupiter (HJ) [Mayor et al. \(1995\)](#). Since then, there has been an ongoing attempt to investigate planet formation and evolution. This class of exoplanets has a profile similar to Jupiter, a gas giant, but the key distinguishing feature being an extraordinarily short orbital period ($P_{orb} \lesssim 10$ days).

To test hypotheses on the existence of exoplanets, observations with extremely high resolution together with robust statistical tools are needed to identify patterns and thus detect those objects. Similarly, theoretical breakthroughs are critical in paving the way for new observational ground. The existing literature will be reviewed in this chapter as a means of understanding what has already been accomplished.

2.1 Introduction

Numerous questions about exoplanets are now within the grasp of researchers due to this rapidly changing research field.

A recent discovery named "radius valley" ([Fulton et al., 2017](#); [Fulton and Petigura, 2018](#); [Berger et al., 2018](#)) which refers to a deficit of exoplanets with a radius between 1.5 to 2.0 $R_{\oplus}$ - check Figure 2.1 - has had its developments due to the field of planetary photoevaporation ([Zhu and Dong, 2021](#)). Interestingly, this gap was predicted before its discovery ([Owen and Murray-Clay, 2018](#)). As a result, the significance of both theoretical and observational work in this field of research is recognized, as theoretical predictions can guide future observations.

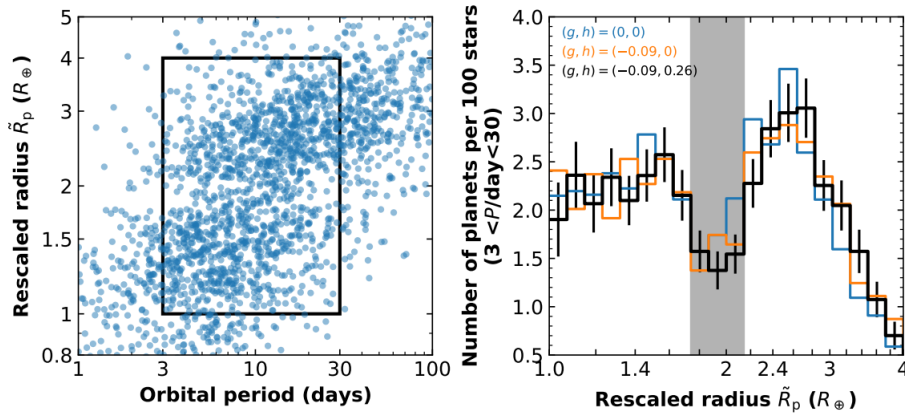


FIGURE 2.1: These plots were taken from [Zhu and Dong \(2021\)](#): (a) Zoom-in view of the Kepler planets from the discovery’s paper sample ([Van Eylen *et al.*, 2018](#)), were $\tilde{R}_p = R_p (P/10 \text{ days})^{-g} (M_*/M_\odot)^{-h}$; (b) Valley radius, highlighted by the gray area, with several best-fits for comparison.

2.1.1 Photoevaporation

Photoevaporation is the atmospheric evaporation driven by high-energy photons from the host star - for an historic overview, see the review paper of [Owen \(2019\)](#). Since hot-Jupiters’ atmospheres are dominated by Hydrogen (H) and Helium (He) and have relatively short periods (≈ 1.5 days ([Lanza, 2020](#))), it should raise questions of whether such planets with volatile atmospheres are stable, as did [Lammer *et al.* \(2003\)](#). For a H-rich atmosphere, energetically high UV flux from the host star can dissociate stable molecules, leading to an upper atmosphere mainly composed of atomic H. In these regions, continuous heating can result in gas reaching temperatures at the order of $(5 - 10) \times 10^3$ Kelvin, meaning that such a weakly bound gas shall most likely overcome gravitational energy and escape.

According to [Lecavelier Des Etangs \(2007\)](#), by means of diagrams of potential energy versus energetic flux, HJs go through an extensive exposure to such high UV fluxes. When taking into account the total time integrated energy flux, it ends up representing a significant part of its total gravitational binding energy.

Therefore, it becomes quite imperative, in a theoretical point of view, to be able to predict this atmospheric blow-off (also found in literature as hydrodynamic escape) and mass loss behaviour, since these can play quite a strong influence on planetary evolution. Evaporative winds and atmospheric escape may be critical in determining exoplanets’ habitability ([Lammer *et al.*, 2014](#); [Tarter *et al.*, 2007](#); [Seager *et al.*, 2009](#)), which can itself

lead to more far reaching consequences. The work of [Canet and Gómez de Castro \(2023\)](#) is a recent study on this type of idea: how Earth-like planets do in photoevaporation cases.

2.1.2 Important Concepts

As it has been discussed in several papers, with emphasis on [Murray-Clay et al. \(2009\)](#), that one important approximation to allow studying evaporative HJ outflows is modeling the bunch of escaping gases as a fluid (either HD or MHD, accordingly to the presence of magnetic considerations) rather than a collection of particles that undergo Jeans escape. This is because the gas (or the plasma) remains collisional beyond the flow's critical point. A shortened explanation on this is having the exobase lying above the sonic point. The exobase is the height at which the scale height $H \equiv (\nabla \log \rho)^{-1}$ equals the mean free path to collisions $\lambda_{\text{mfp}} \equiv 1/(n_H \sigma)$, where ρ is density, n_H is the H density in the plasma and σ is the particle's cross section. This quantity is commonly used in literature as Knudsen number Kn :

$$Kn = \frac{\lambda_{\text{mfp}}}{H} \quad (2.1)$$

For a gas with $Kn \ll 1$, it obeys fluid mechanics, while for gas with $Kn \gtrsim 1$, HD or MHD equations are no longer suitable to describe the gas. The sonic point is the point at which the plasma flow's velocity equals the speed of sound in the plasma.

$$c_s = \sqrt{\frac{k_B T}{\mu m_p}} \quad (2.2)$$

where T is temperature and μ is the mean particle mass.

Additionally, one other parameter of interest for atmospheric escape control is the ratio between the gravitational binding energy and the thermal energy of a gas particle λ - also know as the escape parameter:

$$\lambda = \frac{GM\mu m_p}{Rk_B T} \equiv \frac{1}{2} \frac{v_{\text{esc}}^2}{c_s^2} \sim 21.466\mu \left(\frac{M}{M_J}\right) \left(\frac{R}{R_J}\right)^{-1} \left(\frac{T}{10^4 \text{K}}\right)^{-1} \quad (2.3)$$

where M is the object's mass, R is the radius and:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \quad (2.4)$$

For $\lambda \lesssim 10$, the internal energy of the gas exceeds the gravitational binding energy and it is expected to flow out of the planet's atmosphere - "blow-off", as used in [Öpik \(1963\)](#).

Otherwise, the atmosphere is tightly bound and the only escape possible is by nonthermal processes, resulting in a subsonic breeze.

There is a sensitive transition of regimes in a photoevaporating atmosphere. For a standard exoplanet with the characteristics of the HJ: HD209458b, [Murray-Clay et al. \(2009\)](#) developed a summary of scales that is present in Figure 2.2. The photoionization base ($\tau = 1$ surface) is set at $\sim 1.1R_p$, where R_p is the planet's radius, which is where most of the extreme UV (EUV) radiation is considered to have been absorbed. Naturally, the atmospheric gas reaches the highest temperatures at the photoionization base, which consequently denotes where the outflow begins.

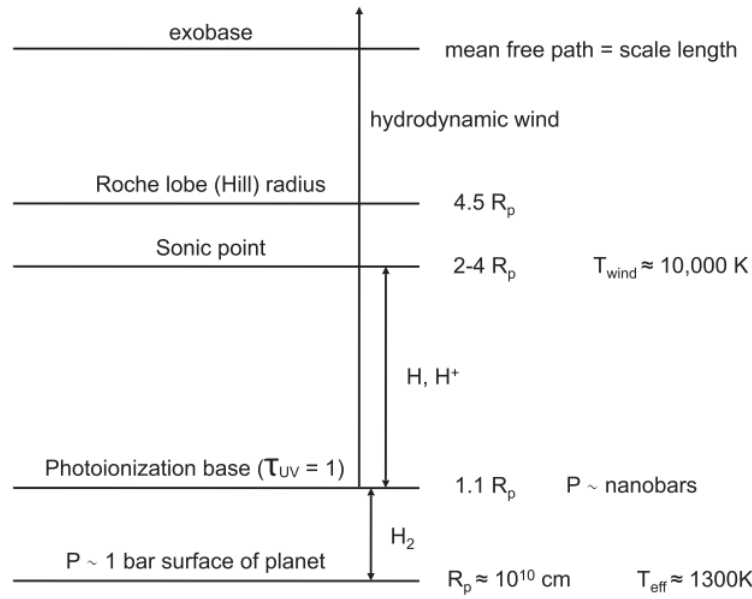


FIGURE 2.2: Scheme taken from [Murray-Clay et al. \(2009\)](#), derived according to the wind solution set up with the characteristics of an exoplanet, in order to evaluate the validity of the model.

A consequence drawn from considering the escaping gas as a fluid is that mass loss can no longer be based on Jeans escape, as shown in [Yelle \(2004\)](#), and should be considered a hydrodynamic wind. Let us introduce the "energy-limited" escape: a planet absorbs all the high-energy flux in a thin layer at R_{HE} around the point of optical depth equal to one. In the absence of any cooling and heating sources, it is the high-energy flux supply F_{HE} that balances the work done by escaping the gravitational potential. Hence, the mass-flux per steradian is given by:

$$\dot{M} = \frac{F_{HE} R_{HE}^2 R_p}{GM_p} \quad (2.5)$$

This concept was developed in [Watson et al. \(1981\)](#), where it was shown that planetary atmospheres can become hydrodynamically unstable when exposed to high-energy irradiation. However, a new form of calculating mass-loss has become popular, which bears in mind the gain of potential energy of the evaporated material and neglecting any kinematic contributions ([Erkaev et al., 2007](#)):

$$\dot{M} = \eta \frac{\pi F_{\text{HE}} R_p^3}{G M_p K_{\text{eff}}} \quad (2.6)$$

where this gives the total mass-loss. The new term K_{eff} accounts for the fact that the flow must not escape to infinity, but only to the Roche lobe, which stands approximately at the Hill radius:

$$R_{\text{H}} = a \sqrt[3]{\frac{M_p}{3M_*}} \quad (2.7)$$

where a stands for the orbital semi-major axis, M_p is the planet's mass and M_* is the host star's mass. The prefactor η varies in the literature, because it represents an efficiency parameter which incorporates the (lack of) knowledge of energy gain and loss processes. An important note must be made: equation (2.6) does not determinate if HD escape is the correct theory to use. However, it can be used to be compared at the end of the simulations, since Jeans escape bear much lower mass-loss rates.

Lastly, it is also essential to understand one observational technique to detect atmospheric escape: transit measurement of a Lyman- α line. The concept is similar to the transit method in that it calculates the ratio of an obscuring area to the area of the stellar disk by observing the absorption of stellar light when the planet transits in front of the star. The Lyman- α cross-section at line centre is $\sigma_{\alpha, c} = 5.9 \times 10^{-14} \text{cm}^2$ for 10^4K gas, thus, for an obscuring region with a scale size of order of the planet's radius, the number density of neutral H (HI) is:

$$n_{\text{HI}} = \frac{1}{\sigma_{\alpha, c} R_p} \sim 2.4 \times 10^2 \text{cm}^{-2} \left(\frac{R_p}{R_J} \right)^{-1} \quad (2.8)$$

This allows for a good detection of small quantities of HI at large distances from the planet. The Lyman- α transit was detected for the first transiting planet HD209458b shortly after its discovery, with an absorption depth of $\approx 15\%$ which compared to the broadband, was extremely good news ([Vidal-Madjar et al., 2003](#)).

2.2 Initial Models

Much effort has been made in a theoretical point of view; consequently, physical modelling has had a rising importance. Many questions have been debated through simulation and many are still to be uncovered: there has not yet appeared any complete numerical simulation that includes all the necessary radiative transfer, thermodynamics and chemistry even in 1D calculations, as discussed in [Owen \(2019\)](#). Naturally, many approximations are considered to simplify the computational cost of several ingredients, and many models focus solemnly on a close group of variables that affect wind structure and outflow rates.

Organization and methodical strategy is essential to build consistent models. As noted by [Owen \(2019\)](#), many 1D models begin in a similar way, which is based on the fact that HJ's are expected to be tidally locked, as discussed in [Matsumura et al. \(2010\)](#). By comparing the ratio of the gravitational acceleration to the acceleration arising from the Coriolis effect, it is seen for the observed exoplanets that gravitational force dominates inside the sonic point - this was numerically verified by [Murray-Clay et al. \(2009\)](#). As a consequence, the streamlines in the sub-sonic regime will not be strongly bent by the Coriolis effect. Therefore, the basic initial setup of many models is assuming a streamline with spherical divergence, which then leads to solving the HD problem on the coordinates of such streamlines.

Some of the first 1D HD models of energy-limited escape ([Lammer et al., 2003](#); [Baraffe et al., 2004](#)) predicted catastrophic evaporation for HJs - mainly due to assuming that all energy deposited by radiation ends up as heating, this is, it is blind to radiative cooling. [Yelle \(2004\)](#) studied atmospheric escape with HD209458b as well, taking into account H/He ionization and chemical networks. [García Muñoz \(2007\)](#) adopted [Yelle \(2004\)](#) strategy of constant heating efficiency, from which many later-acclaimed conclusions were drawn. One of which is that the planetary outflows do not need to take a transonic form, pointing out that the stellar wind may be able to pressure-confine the planet's outflow to a subsonic breeze.

In 2009, [Murray-Clay et al. \(2009\)](#) unveiled several important obstacles in their 1D model. By studying a model with realistic heating and cooling, ionization balance with a pure H atmosphere, tidal gravity and pressure-confinement, they showed there were two different regimes of HD escape from HJ. For low UV flux, $\dot{M} \propto F_{UV}^{0.9}$, whilst for high UV flux, $\dot{M} \propto F_{UV}^{0.6}$ - this results were mainly explained by radiative recombination.

Taking into account MHD considerations on photoevaporation of HJs arouse with the work of [Cohen et al. \(2011\)](#). These authors studied the 3D interaction of stellar magnetic fields and winds with the planet's magnetosphere and outflow. The concept of Alfvénic point (or Alfvén radius) takes the place of the HD equivalent sonic radius: it is the distance at which the accelerating stellar wind equals the Alfvén speed:

$$v_A = \frac{|\mathbf{B}|}{\sqrt{4\pi\rho}} \quad (2.9)$$

Beyond the Alfvén point, the flow is super-alfvénic and magnetized information cannot propagate back. Star-Planet Interactions (SPI) are understood to be increasingly more complex with the addition of magnetic fields.

The work of [Adams \(2011\)](#) goes a step further, exploring HJ flows in a regime where magnetic fields are strong enough to guide the flow, thereby determining the geometric nature of the outflow. This was achieved by reducing a complicated field structure to a 1D problem, by constructing an orthogonal coordinate system, where the only coordinate followed the magnetic field lines. Additionally, [Adams \(2011\)](#) also presented a theoretically well funded basis to show that the gas is well coupled to the magnetic field and discusses the background magnetic field (from the star), limiting the sphere of influence of the planet. By comparing length scales involved in the planetary outflow, it was shown that the launch of the outflow could be considered independent of the subsequent propagation of the flow, since:

$$H \ll R_p < r_s < r_H \sim r_X \sim R_* \ll a \quad (2.10)$$

where r_H is the Hill radius, which determines the gravitational influence of the planet and r_X is the X-point radius, which tells the magnetic influence of the planet.

Inherent asymmetries in atmospheric escape cannot be modeled by 1D simulations. Stellar wind's pressure confinement and Coriolis effect are two simple examples of such asymmetries. Thus, let us see how models focused on higher dimensions evolved over time. [Schneider et al. \(2007\)](#) developed one of the first simple 3D model, which intended to model HD209458b, and obtained similar results of mass-loss to [Vidal-Madjar et al. \(2003\)](#). [Stone and Proga \(2009\)](#) performed a 2D numerical simulation, where they included SPIs. The authors showed that a planet's permanent day-side and night-side bears a heating anisotropy on the upper atmosphere.

A 3D isothermal MHD model was proposed by [Trammell et al. \(2011\)](#) to study the structure of HJs' magnetospheres by including a "dead-zone" near the magnetic equator, which contained gas confined by the magnetic fields, estimating its size to be around several to tens of planetary radii. A consequence of their model was the appearance of transiting signal from magnetically confined gas, some of which was outside the Lagrangian L_1 point. This implies that gas outside L_1 does not imply mass-loss, which can be an aspect to take into consideration.

Time-dependent EUV flux was studied by [Lecavelier des Etangs et al. \(2012\)](#) to grasp wind structures and outflow rates which provides some of the first evidence of interaction between the exoplanet's atmosphere and stellar variations. Latter on, maintaining the same target exoplanet HD189733b characteristics in the model, these authors studied HI escape in [Bourrier et al. \(2013\)](#). By comparing with data from the Hubble space telescope, they concluded that it was required an additional mechanism of acceleration for the planetary winds besides the radiation pressure acceleration. They suggested that one strong candidate should be charge exchange between stellar wind protons and H atoms in the planetary wind.

Understanding how charge exchange could justify wind velocity measured during transit was possible with the work of [Tremblin and Chiang \(2012\)](#). Their 2D simulations showed that the contact discontinuity of the two winds is Kelvin-Helmholtz unstable, leading to a mixing of the winds (Figure 2.3). As a consequence, in the mixing layer, within a small time interval, chemical equilibrium is achieved between neutral H of the star (hot) and planet (cold), supporting observations.

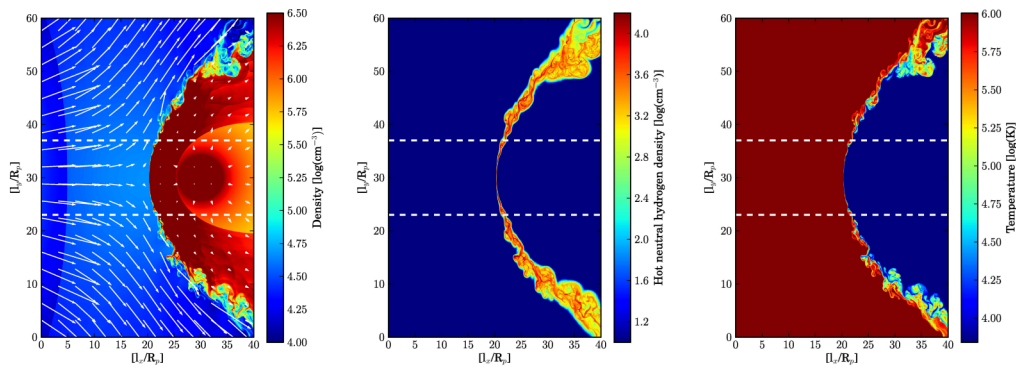


FIGURE 2.3: This plot was taken from [Tremblin and Chiang \(2012\)](#): Kelvin-Helmholtz discontinuity mixing layer formation and its presence in 3 different system parameters (resolution of 3200×4800).

[Matsakos et al. \(2015\)](#) is still considered with high regards due to its important discussion and conclusions: the authors intended to study the morphology of different type of interactions between HJs and their host stars. They performed 3D MHD simulations of SPIs with realistic outflows, and exploring a wide range of parameters. It was possible to classify SPIs into four different categories based on characteristic length scales that quantify many effects of the simulation:

$$R_m = \sqrt[6]{\frac{B_p^2 R_p^6}{8\pi P_{\text{amb}}}} \quad , \quad R_W = \sqrt{\frac{\dot{M} v_W}{4\pi P_{\text{amb}}}} \quad , \quad R_t = R_{\text{orb}} \sqrt[3]{\frac{M_p}{3M_*}} \quad (2.11)$$

where R_m defines the scale of dominance by the planet's magnetosphere (obtained by equating the dipole field in the magnetic pressure); R_W defines the stand-off point, *i.e.* the scale at which begins the outflow's supercriticality (obtained by equating the ambient pressure to the ram pressure) and v_W is the wind's velocity of the planet; R_t defines the scale of tidal influence (Hill radius) which defines the Lagrangian L_1 point to the center of the planet. Figure 2.4 shows the different morphologies:

$$\begin{aligned} \text{Type I: } (R_t > R_m > R_W) \quad ; \quad & \text{Type II: } (R_t > R_W > R_m) \\ \text{Type III: } (R_W > R_t) + (R_W > R_m) \quad ; \quad & \text{Type IV: } (R_m > R_t) + (R_m > R_W) \end{aligned} \quad (2.12)$$

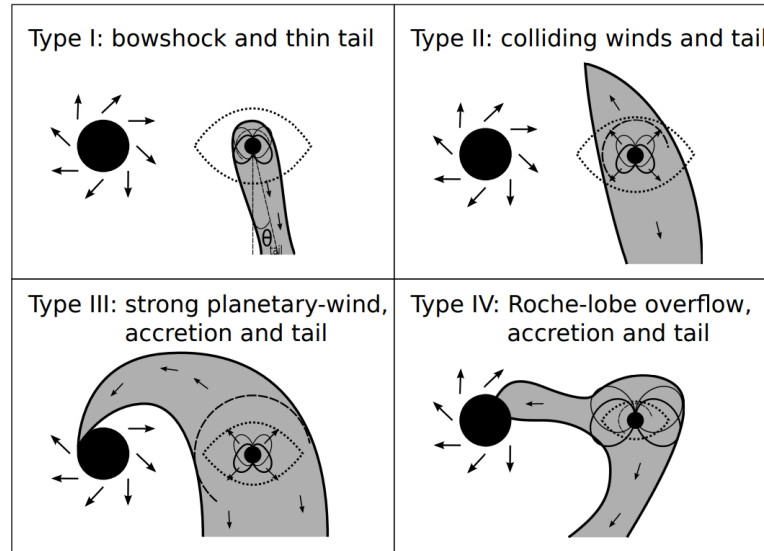


FIGURE 2.4: This plot was taken from [Matsakos et al. \(2015\)](#): Schematic of the four identified different morphologies. The proposed classification scheme is based on the relative ordering of R_m , R_W and R_t .

[Tripathi et al. \(2015\)](#) also presented a 3D numerical model, employing the method of [Murray-Clay et al. \(2009\)](#), but focusing on asymmetric heating on local outflow. Their

results indicate that the flow on the night-side is unsteady, most likely derived by Kelvin-Helmholtz instability.

2.3 State-of-the-Art Simulations

The purpose of this section, besides presenting the most recent results, is to give insight to how numerical simulations are performed. Additionally, some of the focus will be put on what types of code exist to solve the HD or MHD set of equations and evolve the initial conditions of a given setup.

Though there is a wide variety of codes for HD and MHD simulations, some of the most attractive are very lightly described in Table 2.1.

Name of Code	Authors	MHD	AMR	Citated Papers
AstroBEAR	Cunningham et al. (2009) ; Carroll-Nellenback et al. (2011)	Yes	Yes	Carroll-Nellenback et al. (2017) ; Debrecht et al. (2019)
PLUTO	Mignone et al. (2007)	Yes	Yes	Matsakos et al. (2015) ; Daley-Yates and Stevens (2018) ; Colombo et al. (2022)
PLUTO-CLOUDY	Salz et al. (2015)	Yes	No	Salz et al. (2016)
ZEUS	Hayes et al. (2006)	Yes	No	Stone and Proga (2009) ; Trammell et al. (2014) ; Owen and Adams (2014)
GUACHO	Esquivel et al. (2009)	Yes	No	D'Angelo et al. (2018) ; Schneider et al. (2016)
ATHENA	Stone et al. (2008)	Yes	Yes	Tripathi et al. (2015) ; McCann et al. (2019)
HERACLES	González et al. (2007)	No	No	Tremblin and Chiang (2012)
BATS-R-US	Powell et al. (1999) ; Tóth et al. (2005)	Yes	Yes	Carolan et al. (2021) ; Harbach et al. (2021)

TABLE 2.1: Table with most used codes in numerical simulations for the study of photo-evaporation.

Many of the codes are more than simple grids that are able to solve HD and MHD equations using different schemes. Many adopt an Adaptive Mesh Refinement (AMR), which is a code's ability to allow for a cell to split accordingly, thus gaining resolution. Additionally, most of all of the codes present in Table 2.1 allow the user to choose the

space domain, the entire model coordinate system (Cartesian, cylindrical, spherical), spatial dimensions (1D, 2D, 2.5D - essentially 3D but axis-symmetric - and 3D), the temporal and spatial solvers, divergence machinery (for example, to allow for $\nabla \cdot B = 0$), etc. It is up to the user to input the initial conditions of the system and boundary conditions.

With these powerful tools, it becomes more appealing to model 3D simulations and to explore specific traits or execute detailed analysis. Most papers written in the last 6 years do make use of available codes and aim for the most computationally intelligent form of simulating the pretended process. Unlike what may seem, many questions remain open due to the high number of mechanism that can affect the evolution of photoevaporated planetary winds:

- Stellar and planetary magnetic fields ([Owen and Adams, 2014](#); [Khodachenko et al., 2017](#); [Matsakos et al., 2015](#); [D'Angelo et al., 2018](#)).
- Time-dependent phenomena such as flares ([Bisikalo et al., 2018](#)); stellar variability ([Lecavelier des Etangs et al., 2012](#)) and atmospheric circulation ([Teyssandier et al., 2015](#)).
- Wind-Wind interaction phenomena: charge exchange ([Tremblin and Chiang, 2012](#); [Christie et al., 2016](#)); stellar wind confinement ([Schneider et al., 2016](#); [McCann et al., 2019](#)), etc.

From the extensive literature available, the following papers are examples of state-of-the-art research on these mechanisms.

[Carroll-Nellenback et al. \(2017\)](#) drew new intuition on wind-wind interactions on HD flows, including anisotropy of the planetary wind launched and explored the orbital dynamics of the resulting flows with great detail. The simulated system tries to mimic HD209458b, which alongside with synthetic observation of Lyman- α line profiles and transit light curves provide intuition for new observational signatures. Wind anisotropy follows a similar profile to [Stone and Proga \(2009\)](#), where the day-side has a angle of incidence dependent temperature, and the dark side is left at 100K. This paper confirmed that the wind generation falls into two regimes as seen in [Murray-Clay et al. \(2009\)](#): the energy limited case where the UV flux energy goes entirely into pdV work to launch the wind. The second regime is radiation-recombination limit, in which recombination and cooling take the flow's energy. Fallback was also one important feature observed (most works with isotropic winds such as [Matsakos et al. \(2015\)](#) cannot retrieve this feature) and

it could reach up to 40% of total outward flow (see also [Tripathi et al., 2015](#); [Christie et al., 2016](#)). The two-arm wind structure was also observed, mainly driven by the Coriolis and tidal effects in shaping planetary winds into upstream and downstream arms.

[Debrecht et al. \(2019\)](#) developed a study on how variable mass and energy flux would affect photoevaporative winds launched in a self-consistent radiative transfer. This work was also compared to the relevant physics of two previous works: [Tripathi et al. \(2015\)](#) and [Carroll-Nellenback et al. \(2017\)](#). Results show that low-mass planets produce potentially observable extended neutral tails being advected from within the night side of the planet.

Observed high velocities in the absorbing material of mass-loss systems, which remains unexplained, was studied by [Debrecht et al. \(2020\)](#), where these authors carried their 3D HD simulations with a planet similar to HD209458b. In this work, the planetary wind was launched self-consistently by calculating the ionization and heating results from incident high-energy radiation, and accounted for the effects of radiation pressure - which was proposed by [Schneider et al. \(2016\)](#) as a mechanism to drive the escaping atmosphere into a "cometary" tail, a result from their work of studying photoionization of planetary winds. The paper's simulations suggest that radiation pressure alone is insufficient to create a cometary tail in the outflow by itself, nor can it reproduce the observed absorption spectra of HD209458b. Therefore, since the simulations assumed that the Lyman- α photons would be entirely absorbed, ignoring Doppler shifts, it provides an upper-limit of efficacy for radiation pressure.

Continuing the search for the main factor in the origin of the high absorption line velocities in mass-loss systems, [Debrecht et al. \(2022\)](#) perceived the effects of charge exchange, which create energetic neutral atoms (ENAs). The paper works analytically the production of ENAs, which is later compared to the simulations - showing good agreement. However, this work end up concluding that there is insufficient ENA production in the wind interaction to explain the observed effects.

Some effort has also been put into exploring new observable features that rise from the intricate outflow structure or wind-wind interaction. One example is radio emission in the form of Electron Cyclotron Maser Instability (EMCI), and thus, in order to search for this potential observable feature, one should fathom magnetic Star-Planet Interactions (mSPI).

[Daley-Yates and Stevens \(2017\)](#) and [Weber et al. \(2017\)](#) started exploring MHD models in order to find if EMCI was possible to be detected. [Weber et al. \(2017\)](#) does not explain much of their model, simply stating that they follow the works of [Shaikhislamov et al. \(2014\)](#); [Khodachenko et al. \(2015\)](#). On the other hand, the work conducted in [Daley-Yates and Stevens \(2017\)](#) modeled two exoplanets, differing only by their mass-loss rate. This paper's simulations showed a magnetic field being ripped open by the stellar wind and planet outflow being swept back. As a consequence, and this is observed also in [Weber et al. \(2017\)](#), the increase in plasma density in the planet's magnetosphere blocks EMCI emission, explaining the lack of detected radio emission.

The works [Daley-Yates and Stevens \(2018, 2019\)](#) carried out the first 3D MHD simulation of wind-wind interaction in which the stellar surface evolution is studied in detail. More specifically, they simulate the outflow of HJ based on far UV (FUV) models, which leads to build-up of circumstellar and circumplanetary material that accretes to the stellar surface in form of "coronal rain". In [Daley-Yates and Stevens \(2019\)](#), it is shown that magnetic topology plays a major role in forming accretion streams between the star and the planet. Later on, [Colombo et al. \(2022\)](#) realized a similar study, with a slightly different model, with intent to gather knowledge whether there are observable features from material accretion.

A recent paper on possible new observable features is [MacLeod and Oklopčić \(2022\)](#), where the signature of He 1083nm line is studied in a 3D HD model (since, contrary to Lyman- α lines, these lines do not suffer absorption from the interstellar medium and can be observed with many ground-based instruments). This study concludes that the interaction of stellar winds with planetary winds can affect observable properties of He 1083nm.

Thus far, not much diversity has been set on planetary magnetic field directions. There has been studies on such idea of shocks between magnetic systems ([Li et al., 2013](#)), but [Zhilkin et al. \(2020\)](#) applied this concept to study the flow structure of extended envelopes of HJs with magnetic dipoles with different angle orientation. Results show that the mass-loss rate depends on the orientation of the planet's dipole magnetic field and increases with increasing angle between the direction to the star and the direction closest to the inner Lagrangian point's magnetic dipole.

[Carolan et al. \(2021\)](#) studied the double tail formation with the effect of magnetic fields with self-consistent radiation taken into consideration. Results show that for a certain

magnetic strength, a dead-zone near the equator appears due to low-velocity material trapped by the magnetic fields. Additionally, one important conclusion made from these authors is that atmospheric escape in magnetized planets occur through polar outflows, in contrast to the night-side escape in HD models.

2.4 Final Remarks

The stand-point of photoevaporation is essentially stagnated by the computational adversities. Nevertheless, a careful analysis of different physical processes has allowed for restricting upper or lower bounds on results. Thus, a more geometric and morphological analysis but complemented by quantitative aspects of the simulations has been the adopted and preferred path by the state-of-the-art research.

Chapter 3

PLUTO

PLUTO is a finite-volume / finite-difference, shock-capturing code designed to integrate a system of conservation laws:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{T}_h = \nabla \cdot \mathbf{T}_p + \mathbf{S}(\mathbf{U}) \quad (3.1)$$

where $\mathbf{U}$ represents a set of conservative quantities, $\mathbf{T}_h(\mathbf{U})$ is the (hyperbolic) flux tensor, $\mathbf{T}_p$ is the diffusion (parabolic) flux tensor and $\mathbf{S}(\mathbf{U})$ defines the source terms. These terms depend on the particular physics module selected.

PLUTO adopts a structured mesh approach for the solution of the system of conservation laws from equation (3.1). Flow quantities are discretized on a logically rectangular computational grid enclosed by a boundary and augmented with guard cells or ‘ghost points’ in order to implement boundary conditions on a given computational stencil.

As the flow evolves, the grid being used can be either *static* or dynamically *adaptive*:

- In the *static* grid version, the entire package is written in C programming language ([Mignone et al., 2007](#)).
- In the *adaptive* grid version, the code relies on the Chombo library for adaptive mesh refinement (AMR), written in C++ and Fortran ([Mignone et al., 2011](#)).

Parallelization is handled internally in each type of grid. Due to the high amount of computations required for typical 3D simulations, it will be favourable to execute simulations in the parallel mode. It is possible to define the number of processors for each direction of the grid.

3.1 Configuration in PLUTO

In order to configure and setup **PLUTO** for a particular problem, four main steps have to be followed. The resulting configuration will then be stored in 4 different files:

- *definition.h*: header file containing all problem-dependent preprocessor directives required at compilation time.
- *makefile*: needed to compile **PLUTO**.
- *pluto.ini*: startup initialization file containing run-time parameters, such as grid characteristics, output formats, etc.
- *init.c*: file written in C with the implementations for initial conditions, boundary conditions, extra analysis that surpasses the simple standard output of **PLUTO**, etc.

It is the Python script *setup.py* that is used for writting the problem's characteristics in the first two files. The remaining two files (*pluto.ini* and *init.c*) are edited by the user.

As an initial step, **PLUTO** presents several test-problems to guarantee that there is no apparent problem. This test-problems are present for all physics modules, however, for the scope of this project, only HD and MHD test-problems were made. Since **PLUTO** has a system documentation (*Doxygen*), it was possible to compare the results of the test problems with what is expected. After completing a safe amount of these, it was possible to assume that no problem would appear in the construction of the simulations.

3.2 Physics Modules

PLUTO presents several modules for the solution of the fluid equations under different regimes:

- HydroDynamics (HD) - Newtonian (classical) hydrodynamics.
- MagnetoHydroDynamics (MHD) - Ideal or resistive magnetohydrodynamics.
- Relativistic HD (RHD) - Special relativistic hydrodynamics.
- Relativistic MHD (RMHD) - Special (ideal) relativistic magnetohydrodynamics.
- Resistive RMHD (ResRMHD) - Special (ideal) relativsitic magnetohydrodynamics with resistivity.

In this section, it will be overviewed how the two used modules in the dissertation (HD and MHD) work. The reader should also be reminded that only first-order spatial derivatives accounting for the hyperbolic part of the equations are described. The diffusion terms such as thermal conduction, viscosity, magnetic resistivity and cooling will not take place in the discussion.

3.2.1 The HD Module

The HD module may be used to solve the Euler or the Navier-Stokes equations of classical fluid dynamics. With this module, **PLUTO** evolves in time the following system of conservation laws:

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ \mathbf{m} \\ E_t + \rho\Phi \end{pmatrix} + \nabla \cdot \begin{pmatrix} \rho\mathbf{v} \\ \mathbf{m}\mathbf{v} + \mathbf{1}p \\ (E_t + p + \rho\Phi)\mathbf{v} \end{pmatrix}^T = \begin{pmatrix} 0 \\ -\rho\nabla\Phi + \rho\mathbf{g} \\ \mathbf{m} \cdot \mathbf{g} \end{pmatrix} \quad (3.2)$$

where ρ is the mass density, $\mathbf{m} = \rho\mathbf{v}$ is the momentum density, $\mathbf{v}$ is the velocity, p is the thermal pressure and E_t is the total energy density:

$$E_t = \rho e + \frac{\mathbf{m}^2}{2\rho} \quad (3.3)$$

An equation of state provides the closure $\rho e = \rho e(p, \rho)$. The source term on the right includes contributions from body forces and is written in terms of the gravitational potential Φ and the acceleration vector $\mathbf{g}$.

Primitive variables are defined by $\mathbf{V} = (\rho, \mathbf{v}, p)^T$, where $\mathbf{v} = \mathbf{m}/\rho$ while $p = p(\rho, \rho e)$ depends on the equation of state, which will be discussed in a following subsection. The maps $\mathbf{U}(\mathbf{V})$ and its inverse are provided by the functions 'PrimToCons()' and 'ConsToPrim()', respectively.

Primitive variables are generally more convenient and preferred when assigning initial/boundary conditions and in the interpolation algorithms. The vector of primitive quantities $\mathbf{V}$ obeys the quasilinear form of the equations:

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v} = 0 \quad (3.4a)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{\nabla p}{\rho} = -\nabla \Phi + \mathbf{g} \quad (3.4b)$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \rho c_s^2 \nabla \cdot \mathbf{v} = 0 \quad (3.4c)$$

where $c_s = \sqrt{\Gamma p / \rho}$ is the adiabatic speed of sound for an ideal EOS.

3.2.2 The MHD Module

The MHD module is suitable for the solution of ideal or resistive (non-relativistic) magnetohydrodynamical equations. With the MHD module, **PLUTO** solves the following system of conservation laws:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3.5a)$$

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot \left[\mathbf{m} \mathbf{v} - \mathbf{B} \mathbf{B} + \mathbf{I} \left(p + \frac{B^2}{2} \right) \right]^T = -\rho \nabla \Phi + \rho \mathbf{g} \quad (3.5b)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (c \mathbf{E}) = 0 \quad (3.5c)$$

$$\frac{\partial (E_t + \rho \Phi)}{\partial t} + \nabla \cdot \left[\left(\frac{\rho v^2}{2} + \rho e + p + \rho \Phi \right) \mathbf{v} + c \mathbf{E} \times \mathbf{B} \right] = \mathbf{m} \cdot \mathbf{g} \quad (3.5d)$$

where the same variables from HD module appear and with B as the magnetic field with a factor of $1/4\pi$ absorbed and E_t is the total energy density:

$$E_t = \rho e + \frac{m^2}{2\rho} + \frac{B^2}{2} \quad (3.6)$$

where an additional equation of state provides the closure $\rho e = \rho e(p, \rho)$.

In the third of equation (3.5), E is the electric field defined as:

$$c \mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{\eta}{c} \cdot \mathbf{J} + \frac{\mathbf{J}}{ne} \times \mathbf{B} \quad ; \quad (\mathbf{J} = c \nabla \times \mathbf{B}) \quad (3.7)$$

where the first term is the convective term, the second term is the resistive term (η denotes the resistivity tensor) while the third term is the Hall term. The speed of light c never enters in the equations but is kept for the sake of completeness. In the ideal case, the energy flux takes the form:

$$\frac{\partial (E_t + \rho \Phi)}{\partial t} + \nabla \cdot \left[(E_t + p_t + \rho \Phi) \mathbf{v} - \mathbf{B}(\mathbf{v} \cdot \mathbf{B}) \right] = \mathbf{m} \cdot \mathbf{g} \quad (3.8)$$

The primitive form of the equations, neglecting diffusion terms, is

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v} = 0 \quad (3.9a)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{1}{\rho} \mathbf{B} \times (\nabla \times \mathbf{B}) + \frac{1}{\rho} \nabla p = -\nabla \Phi + \mathbf{g} \quad (3.9b)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{B}(\nabla \cdot \mathbf{v}) - (\mathbf{B} \cdot \nabla) \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{B} = \mathbf{v}(\nabla \cdot \mathbf{B}) \quad (3.9c)$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \rho c_s^2 \nabla \cdot \mathbf{v} = 0, \quad (3.9d)$$

where the $(\nabla \cdot \mathbf{B})$ on the right hand side of the third equation is kept for reasons of convenience, although zero at the continuous level.

3.3 Extra Physics

Additional to understanding the fundamental physics, the number of methods that are required to compute the desired solutions is quite large. **PLUTO** offers a wide range of different tools, which the user can choose from, influencing aspects as important as accuracy and computation time. The following subsections present a simple discussion of several steps that need to be built in order to perform the next chapters' models.

3.3.1 Unit Scaling

PLUTO works in a non-dimensional set of units, so that flow quantities can be properly scaled to "reasonable" numbers. This scaling is recommended in order to avoid occurrences of extremely small ($\lesssim 10^{-9}$) or large ($\gtrsim 10^{12}$) numbers that may be misinterpreted by numerical algorithms.

Dimensionalization is strictly necessary when specific length, time or energy scales are introduced in the problem and they must compare to the dynamical advection scales. Therefore, three fundamental units need to be specified through the definitions of the following symbolic constants:

- ρ_0 (*UNIT_DENSITY*), sets the reference density in gr/cm^3 .
- L_0 (*UNIT_LENGTH*), sets the reference length in cm.
- v_0 (*UNIT_VELOCITY*), sets the reference velocity in cm/s .

All other units are derived from a combination of the previous three: time is measured in units of $T_0 = L_0/v_0$, pressure in units of $p_0 = \rho_0 v_0^2$, magnetic field in units of $B_0 =$

$\sqrt{4\pi\rho_0 v_0^2}$. Therefore, computations are now made in such a framework:

$$\rho = \frac{\bar{\rho}}{\rho_0} \quad , \quad \mathbf{v} = \frac{\bar{\mathbf{v}}}{v_0} \quad , \quad p = \frac{\bar{p}}{\rho_0 v_0^2} \quad , \quad \mathbf{B} = \frac{\bar{\mathbf{B}}}{\sqrt{4\pi\rho_0 v_0^2}} \quad (3.10)$$

where the parameters with a bar are meant to be dimensional.

Additionally, a direct relation between pressure and density in non-dimensional units and temperature in Kelvin units is provided:

$$\bar{T} = \frac{\bar{p}\mu m_p}{k_B \bar{\rho}} = \frac{\mu p}{\rho} \frac{m_p v_0^2}{k_B} \equiv \frac{\mu p}{\rho} \mathfrak{K} \quad (3.11)$$

where the conversion factor $\mathfrak{K}$ depends on v_0 . This relation makes the sound speed c_s to be defined by the following relation:

$$\bar{c}_s = \sqrt{\frac{\Gamma \bar{p}}{\bar{\rho}}} = \sqrt{\frac{\Gamma T}{\mathfrak{K} \mu}} \quad (3.12)$$

We can appreciate the dimensionless form of the equations for the HD case with the code fundamental units factored out:

$$\frac{\rho_0}{t_0} \frac{\partial \rho}{\partial t} + \frac{\rho_0 v_0}{L_0} \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3.13a)$$

$$\frac{\rho_0 v_0}{t_0} \frac{\partial \mathbf{m}}{\partial t} + \frac{\rho_0 v_0^2}{L_0} \nabla \cdot (\mathbf{m} \mathbf{v}) + \frac{\rho_0 v_0^2}{L_0} \nabla p = \frac{\rho_0}{L_0} (-\rho \nabla \Phi) + \rho_0 \rho \bar{\mathbf{g}} \quad (3.13b)$$

$$\frac{\rho_0 v_0^2}{t_0} \frac{\partial (E + \rho \Phi)}{\partial t} + \frac{\rho_0 v_0^3}{L_0} \nabla \cdot \left[\left(\frac{\rho v^2}{2} + \rho e + p + \rho \Phi \right) \mathbf{v} \right] = \rho_0 v_0 \mathbf{m} \cdot \bar{\mathbf{g}} \quad (3.13c)$$

Straightforward manipulation yields the dimensionless form of the equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3.14a)$$

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot (\mathbf{m} \mathbf{v}) + \nabla p = -\rho \nabla \Phi + \rho \mathbf{g} \quad (3.14b)$$

$$\frac{\partial (E + \rho \Phi)}{\partial t} + \nabla \cdot \left[\left(\frac{\rho v^2}{2} + \rho e + p + \rho \Phi \right) \mathbf{v} \right] = \mathbf{m} \cdot \mathbf{g} \quad (3.14c)$$

where:

$$\Phi = \frac{\bar{\Phi}}{v_0^2} \quad , \quad \mathbf{g} = \frac{\bar{\mathbf{g}}}{v_0^2/L_0} \quad (3.15)$$

Any other physical extension (thermal conduction and cooling, MHD, etc.) will most-likely require attention to correct dimensionalization.

3.3.2 Grid

In the static version of **PLUTO**, the grid defines the physical domain and generates the whole computation mesh. In the AMR version, it is used to specify the base grid. For each dimension, the number of non-overlapping, adjacent 1D grid patches that make up the computational domain can be defined. The grid type can take the following characteristics:

- *u* or uniform: A uniform grid patch is constructed; if x_L and x_R are the leftmost and rightmost point of the patch for a N number of cells, the grid spacing becomes:

$$\Delta x = \frac{x_R - x_L}{N} \quad (3.16)$$

- *s* or stretched: a stretched grid is generated. Stretched grids can be used only if at least one uniform grid is present. The stretching ratio r is computed as follows:

$$\Delta x (r + r^2 + \dots + r^N) = x_R - x_L \implies r \frac{1 - r^N}{1 - r} = \frac{x_R - x_L}{\Delta x} \quad (3.17)$$

where Δx is taken from the closest uniform grid, N is the number of points in the stretched grid and x_L and x_R are the leftmost and rightmost points of the patch.

- $l\pm$: a logarithmic grid is generated. When $l+$ is invoked, the mesh size is increasing with the coordinate:

$$\Delta x_i = (x_{i-\frac{1}{2}} + |x_L| - x_L) (10^{\Delta \xi} - 1), \quad \Delta \xi = \frac{1}{N} \log_{10} \left(\frac{x_R + |x_L| - x_L}{|x_L|} \right) \quad (3.18)$$

when $l-$ is invoked, the mesh size decreases with the coordinate:

$$\Delta x_i = (x_{i-\frac{1}{2}} - |x_L| - x_R) (10^{\Delta \xi} - 1), \quad \Delta \xi = -\frac{1}{N} \log_{10} \left(\frac{x_R + |x_L| - x_L}{|x_L|} \right) \quad (3.19)$$

3.3.3 Boundary Conditions

As mentioned before, the 'ghost zones' in **PLUTO** are used for specific boundary conditions to be applied. These extra cells are spammed over the borders of the grid, thus there are $2d$ boundary conditions to be applied, being d the number of dimensions. The only used boundary condition in **PLUTO** for this dissertation (besides the user-defined one) is *outflow*. This sets a zero gradient across the boundary. Assuming that q is a scalar quantity

and n is the coordinate direction orthogonal to the boundary plane, *outflow* is set as:

$$\frac{\partial q}{\partial n} = 0 \quad , \quad \frac{\partial \mathbf{v}}{\partial n} = 0 \quad , \quad \frac{\partial \mathbf{B}}{\partial n} = 0 \quad (3.20)$$

3.3.4 Equation of State (EOS)

PLUTO describes a thermally ideal gas obeying the thermal EOS:

$$p = nk_B T = \frac{\rho}{m_p \mu} k_B T \quad (3.21)$$

where p is the pressure, n is the total particle number density, T is the temperature, ρ is the density and μ is the mean molecular weight.

Another fundamental quantity is the specific internal energy e whose rate of change under a physical process is regulated by the first law of thermodynamics:

$$de = dQ - pd \left(\frac{1}{\rho} \right) \quad (3.22)$$

where Q represents the heat absorbed or released.

There are four options offered:

- Ideal: uses the perfect gas law with constant specific heat ratio.
- Isothermal: uses an isothermal equation of state without any energy equation included.
- *PVTE_LAW*: allows the user to employ more complex equations of state by specifying the caloric equation of state $e = e(\rho, T)$.
- *TAUB*: uses the Taub-Matthews equation of state which is only available for relativistic modules.

In this project it is only used the ideal EOS. This is described as how an ideal gas is defined. The ratio of specific heats Γ is constant and the internal energy can be written:

$$\rho e = \frac{p}{\Gamma - 1} \quad (3.23)$$

3.3.5 Reconstruction

There is a need for spatial integration and **PLUTO** offers several different orders of integration:

- Flat: first order reconstruction.
- Linear: this is the default mode. A piecewise TVD linear reconstruction to primitive variables and is accurate by second order.
- Other third order accurate options such as *WENO3* (Yamaleev and Carpenter, 2009), *LimO3* (Čada and Torrilhon, 2009).
- Parabolic: piecewise parabolic method (PPM) implemented by Mignone (2014). The order of this method can be defined between 3 and 5.
- *MP5*: fifth order accurate (Suresh and Huynh, 1997), only implemented for Cartesian regular meshes.

The most commonly used types of reconstruction used in this work is Linear and Parabolic. Non-uniform grid spacings and curvilinear coordinates are more correctly handled with these two. Note that although third order accurate reconstructions are available, the finite volume version of the code retains a global second order accuracy as fluxes are computed at the interface midpoint.

3.3.6 Time Stepping

The time increment between two consecutive steps $\Delta t^n = t^{n+1} - t^n$ and the discretized spatial operator $\mathcal{L}$ can be understood in several ways by PLUTO:

- *EULER*: uses the first Euler algorithm to evolve from $\mathbf{U}^n$ to $\mathbf{U}^{n+1}$:

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \Delta t^n \mathcal{L}^n \quad (3.24)$$

- *RK2*: second order TVD Runge Kutta. By defining:

$$\mathbf{U}^* = \mathbf{U}^n + \Delta t^n \mathcal{L}^n \quad (3.25)$$

it is possible to advance to the next time solution by interating:

$$\mathbf{U}^{n+1} = \frac{1}{2} (\mathbf{U}^n + \mathbf{U}^* + \Delta t^n \mathcal{L}^*) \quad (3.26)$$

- *RK3*: third order TVD Runge Kutta. By defining:

$$\mathbf{U}^{**} = \frac{1}{4} (3\mathbf{U}^n + \mathbf{U}^* + \Delta t^n \mathcal{L}^*) \quad (3.27)$$

it is possible to advance to the next time solution by iterating:

$$\mathbf{U}^{n+1} = \frac{1}{3} (\mathbf{U}^n + \mathbf{U}^* + \mathbf{U}^{**} + 2\Delta t^n \mathcal{L}^{**}) \quad (3.28)$$

- *CHARACTERISTIC_TRACING* and *HANCOCK*: both evolve $\mathbf{U}^n$ according to:

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \Delta t^n \mathcal{L} \left(\mathbf{V}^{n+\frac{1}{2}} \right) \quad (3.29)$$

where $\mathbf{V}^{n+\frac{1}{2}}$ is computed by suitable Taylor expansion. These are second order accurate in space and time and are less dissipative than the previous algorithms.

The time step is computed using the information available from the previous integration step and it can be controlled by the Courant-Friedrichs-Lewy (CFL) number C_a , by the limits present in the following table:

SCHEME		CFL Limit
<i>RK</i>	$\frac{\Delta t^n}{N_{\text{dim}}}$	$\left[\max_{ijk} \left(\sum_d \frac{ \lambda_d }{\Delta l_d} \right) + \max_{ijk} \left(\sum_d \frac{2\tau_d}{\Delta l_d^2} \right) \right] = C_a \leq \frac{1}{N_{\text{dim}}}$
<i>Hancock/ChTr</i>	$\Delta t^n \max_d$	$\left[\max_{ijk} \left(\frac{ \lambda_d }{\Delta l_d} \right) + \max_{ijk} \left(\frac{2\tau_d}{\Delta l_d^2} \right) \right] = C_a \leq \begin{cases} 1 & \text{in } 2D \\ 1/2 & \text{in } 3D \end{cases}$

TABLE 3.1: Table with CFL conditions used by **PLUTO** for different explicit time stepping methods. For a given direction d , Δl_d represents the cell physical length in that direction, λ_d provides the largest signal speed and τ_d accounts for diffusion processes. These limits are based on a stability analysis by [Beckers \(1992\)](#).

For a cell physical length Δ_l , the time step roughly scales as $\sim \Delta_l$ for hyperbolic problems and as $\sim \Delta_l^2$ when parabolic terms are included.

In order to ensure stability in whatever scheme, it is chosen $C_a = 0.25$ for the HD simulations and $C_a = 0.225$ for MHD.

3.3.7 Riemann Solver

Flux computation requires a Riemann solver. In the following points, the several solvers are displayed from the most accurate (least diffusive) to the least accurate (most diffusive). The solvers of interest are:

- *roe*: Linearized Roe Riemann solver based on characteristic decomposition of the Roe matrix ([Roe, 1981](#)).
- *hlld*: Approximate Riemann solver of [Miyoshi and Kusano \(2005\)](#).

- *hllc*: Harten, Lax, Van Leer approximate Riemann solver that restores with the middle contact discontinuity.
- *hll*: Harten, Lax, Van Leer approximate Riemann solver.
- *tvdlf*: simple Lax-Friedrichs scheme.

Note that under circumstances of high Mach number flows or low density plasmas, more diffusive solvers such as *hll* or *tvdlf* turn out to be more robust than accurate solvers. This is because they become better at preventing the occurrence of negative pressures or densities in such situations. But hybrid strategies can also be used in **PLUTO**, such as a control to escape from any solver to the *tvdlf* solver as a shock is encountered.

3.3.8 Divergence of the Magnetic Field

Numerical methods do not naturally preserve the condition $\nabla \cdot \mathbf{B} = 0$. For this reason, **PLUTO** allows for a control strategy to enforce this constraint:

- *NO*: divergence constraint is not enforced.
- *EIGHT_WAVES*: magnetic fields retain a cell average representation. This formulation was introduced by [Powell \(1994\)](#).
- *DIV_CLEANING*: magnetic fields retain a cell average representation and the mixed hyperbolic/parabolic divergence cleaning technique of [Dedner et al. \(2002\)](#) is used, making use of a new scalar variable, the generalized Lagrange multiplier ψ .
- *CONSTRAINED_TRANSPORT*: the magnetic field has a staggered representation and the constrained transport is used ([Balsara and Spicer, 1999](#)).

The commonly used mechanism to ensure $\nabla \cdot \mathbf{B} = 0$ is the *DIV_CLEANING* method. This method consists in solving a modified system of conservation laws, where the induction equation is coupled to a generalized Lagrange multiplier (GLM). Using the mixed GLM hyperbolic/parabolic correction, the induction equation and the solenoidal constraint are replaced, respectively, by

$$\begin{cases} \frac{\partial \mathbf{B}}{\partial t} + \nabla \times (c\mathbf{E}) + \nabla \psi &= 0 \\ \frac{\partial \psi}{\partial t} + c_h^2 \nabla \cdot \mathbf{B} &= -\frac{c_h^2}{c_p^2} \psi \end{cases} \quad (3.30)$$

where $c_h = C_a \times \Delta l_{\min} / \Delta t^n$ is maximum speed compatible with the step size, $c_p = \sqrt{\Delta l_{\min} c_h / \alpha}$ and $\Delta l_{\min}$ is the minimum cell length. The free parameter α controls the rate at which monopole are damped and its value is set by the user-defined constant *GLM_ALPHA* (default value 0.1). A number of tests suggests that the optimal range can be found for $0.05 \lesssim \alpha \lesssim 0.3$.

The advantage of this formulation (GLM-MHD) is that the equations retain a conservative form (no source terms are introduced), all variables (including magnetic fields) retain a cell-centered representation and standard 7-wave Riemann solvers (with a single value of the normal component of magnetic field) may be used.

3.3.9 Background Magnetic Field

An intrinsic background magnetic field may present, for example, when it is only of interest to study the planetary magnetosphere and there exists a stellar dipole field. Thus, it may be convenient to write the total magnetic field as $\mathbf{B}(x, t) = \mathbf{B}_0(x) + \mathbf{B}_1(x, t)$ where $\mathbf{B}_0$ is a background curl-free magnetic field and $\mathbf{B}_1(x, t)$ is a deviation. The background field must satisfy the following conditions:

$$\frac{\partial \mathbf{B}_0}{\partial t} = 0 \quad , \quad \nabla \cdot \mathbf{B}_0 = 0 \quad , \quad \nabla \times \mathbf{B}_0 = \mathbf{0} \quad (3.31)$$

In this case one can show that the MHD equations reduce to:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3.32a)$$

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot (\mathbf{m} \mathbf{v} - \mathbf{B}_1 \mathbf{B} - \mathbf{B}_0 \mathbf{B}_1) + \nabla p_t = \rho(-\nabla \Phi + \mathbf{g}) \quad (3.32b)$$

$$\frac{\partial (E_{t1} + \rho \Phi)}{\partial t} + \nabla \cdot \left[(E_{t1} + p_t + \rho \Phi) \mathbf{v} - (\mathbf{v} \cdot \mathbf{B}_1) \mathbf{B} \right] = \mathbf{m} \cdot \mathbf{g} \quad (3.32c)$$

$$\frac{\partial \mathbf{B}_1}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \quad (3.32d)$$

where

$$p_t = p + \frac{\mathbf{B}_1^2}{2} + \mathbf{B}_1 \cdot \mathbf{B}_0 \quad , \quad E_{t1} = \frac{p}{\Gamma - 1} + \frac{1}{2} (\rho v^2 + \mathbf{B}_1^2) \quad (3.33)$$

Thus the energy depends only on $\mathbf{B}_1$, a feature that turns out to be useful when dealing with low-beta plasma. The sets of conservative and primitive variables are the same as the original ones, with $\mathbf{B} \rightarrow \mathbf{B}_1$, $E \rightarrow E_{t1}$.

3.4 Final Remarks

This Chapter has reviewed the main machinery that is recurrent in the simulation setup. It should be appreciated the toll of computational physics inside **PLUTO**, which, as several other codes also do, are imprinted in all the results of works seen in [Chapter 2](#).

Chapter 4

Hydrodynamic Simulations

Atmospheric escape plays a key role in the evolution of planetary bodies, as seen in the chapter 2. Mass loss from highly-irradiated planets typically occurs through HD outflows, rather than via the kinetic loss mechanisms. It is of interest to study mass loss since it can be used as a platform to build firmer constraints. Additionally, UV observations of the HJ HD209458b found up to a 15% occultation in the wings of Lyman- α , with effective Doppler shifts up to $\pm 150 \text{ km s}^{-1}$ (Vidal-Madjar et al., 2003). This high occultation and large velocities are indicative of a fast and extended component of the atmosphere as an escaping planetary wind. As these close-in planets receive large amounts of external heating, they are believed to be undergoing HD escape with outflow structures similar to a Parker Wind (Parker, 1958). A wide variety of papers have made significant advances to comprehend several pieces of the entire problem:

- Dayside photoionization heating in 2D (Owen and Adams, 2014) and 3D (Tripathi et al., 2015).
- Photoionization starting at the Hill radius (Schneider et al., 2016).
- Stellar wind confinement in 2D (Stone and Proga, 2009; Tremblin and Chiang, 2012) and 3D (Bisikalo et al., 2013).
- Development of morphological structures such as a circumstellar torus from the planetary outflow in 3D global simulations (Carroll-Nellenback et al., 2017)

In all these simulations, several works identified potential enhancements of occultation in Lyman- α wings.

It is then necessary a “piece-by-piece” investigation to reach important conclusions in a global scale simulation. A bottom-up strategy will be applied to explore the effects of temperature anisotropy (with an input of heat, instead of having explicit heating and cooling effects at an atmospheric level).

- In Section 4.1, we start by a deep analysis of a Parker Wind model (Parker, 1958), both analytically and numerically, since this is the building block of the outflows.
- After this, Section 4.2 will explore the effects of temperature gradients at the launching boundary, which are extremely important. The latter is fundamental, since it will be a strong pillar to compare with other works (Stone and Proga, 2009; Carroll-Nellenback et al., 2017).
- As these are well settled, more sections will be dedicated to the analysis of rotation (Section 4.3) and orbiting of the HJ (Section 4.4) at a local scale.
- Section 4.5 will introduce stellar winds, which leads to the main goal of this first part of the dissertation.
- At the end of each section, there is a subsection dedicated to discussing the results, which will be summarized in the last section of this chapter (Section 4.6).

As a means to introduce all these Sections, a theoretical overview is lightly explored. Due to spherical symmetry of the problem, a mostly radial description is adopted. However, when adapting the important results to a cartesian grid, a transformation of coordinates is required. Thus, such a procedure attends to Appendix A.

4.1 Isothermal Outflowing Wind

In this section, a theoretical description of wind models will be quickly reviewed, following Lamers et al. (1999). Since most simulations will focus on similarly structured winds from the star and from the planet, the description will be directed for both.

Simple winds follow a set of equations which can be solved to infer properties such as velocity or density. The main goal is the study of an isothermal stellar wind in which the gas is subject to only two forces: the inward directed gravity and the outward directed gradient of the gas pressure. These are the main ingredients of the Parker wind model (Parker, 1958) which will be presented in this section.

The motion of the gas in a wind can be described by Newton's law:

$$f = \rho \frac{dv}{dt} \quad (4.1)$$

The velocity and location of a unit of mass that is accelerated in the wind depends on distance r and time t :

$$\frac{dv(r,t)}{dt} = \frac{\partial v(r,t)}{\partial t} + \frac{\partial v(r,t)}{\partial r} \frac{dr(t)}{dr} = v(r) \frac{dv}{dr} \quad (4.2)$$

where it is assumed that the wind is stationary, i.e. the velocity at a given distance does not change with time.

With no other forces acting on the wind, the equation of motion is:

$$v \frac{dv}{dr} + \frac{1}{\rho} \frac{dp}{dr} + \frac{GM}{r^2} = 0 \quad (4.3)$$

where the first terms of this *momentum equation* is the acceleration, produced by the pressure gradient (second term) and gravity (third term). If the flow behaves like a perfect gas, then:

$$p = \frac{k_B T \rho}{m_p \mu} \quad (4.4)$$

The force due to pressure gradient can then be written as:

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{k_B}{m_p \mu} \left(\frac{dT}{dr} + \frac{T}{\rho} \frac{d\rho}{dr} \right) = \left(\frac{k_B T}{m_p \mu} \right) \frac{1}{\rho} \frac{d\rho}{dr} \quad (4.5)$$

for an isothermal wind. Re-using the concept of sound speed c_s from equation (2.2) but in an isothermal frame of work ($\Gamma = 1$):

$$\frac{1}{\rho} \frac{dp}{dr} = c_s^2 \frac{1}{\rho} \frac{d\rho}{dr} \quad (4.6)$$

Since we are assuming that there is a constant outflow of wind, conservation of mass expresses:

$$\dot{M} = 4\pi r^2 \rho(r) v(r) = 4\pi \mathcal{F}_m \quad (4.7)$$

where $\mathcal{F}_m$ is the mass flux per steradian. The density gradient can be derived by making use of conservation of mass:

$$\frac{1}{\rho} \frac{d\rho}{dr} = -\frac{1}{v} \frac{dv}{dr} - \frac{2}{r} \quad (4.8)$$

So, inserting these results in equation (4.3), it yields:

$$v \frac{dv}{dr} + c_s^2 \left(-\frac{1}{v} \frac{dv}{dr} - \frac{2}{r} \right) + \frac{GM}{r^2} = 0 \quad (4.9)$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dr} = \frac{1}{v^2 - c_s^2} \left(\frac{2c_s^2}{r} - \frac{GM}{r^2} \right) \quad (4.10)$$

There is a lower boundary to equation (4.10), located at r_0 described by the photospheric radius. As can be noticed, there exists a singularity at $v(r) = c_s$. This singularity is important because it implies that the mass loss rate is fixed.

From the numerator of equation (4.10), a critical distance can also be calculated to being the point at which it goes to zero:

$$r = r_s \equiv \frac{GM}{2c_s^2} = \frac{1}{4} \frac{v_{\text{esc}}^2}{c_s^2} R \quad (4.11)$$

where R is the radius of the object relative to the escape velocity v_{esc} . This critical distance r_s is the sonic radius, which should not be strange by now, since:

$$r_s = \frac{\lambda}{2} R \quad (4.12)$$

In a nutshell, the velocity gradient at the critical distance will be zero. The critical solution is *transonic*, because it starts subsonic at small distances and reaches a supersonic velocity at large distances.

The topology of the solutions of equation (4.10) is shown in Figure 4.1 for various initial velocities $v_0 \equiv v(r_0)$:

- Curve 1: critical solution.
- Curve 2: passes through the critical point, but starts supersonically.
- Curve 3: solution in the case that the lower boundary velocity v_0 is too small to reach the critical point, so it remains subsonic with a maximum at r_c .
- Curve 4: solution for a supersonic wind.
- Curves 5 and 6: mathematical solutions which do not have physical meaning because they are multi-valued.

This discussion has shown that the critical solution occurs for only one particular value of the velocity at the lower boundary. This implies that an isothermal envelope with given density ρ_e at its bottom can only produce a transonic wind if:

$$\dot{M} = 4\pi r_0^2 \rho_e v_0 \quad (4.13)$$

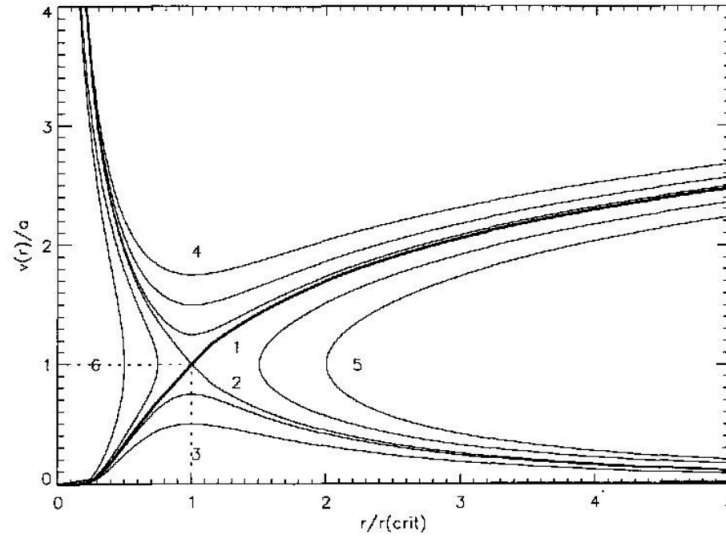


FIGURE 4.1: This plot was taken from [Lamers et al. \(1999\)](#): Solutions of the momentum equation of an isothermal wind. The different curves are described in the text. The thick curve is the transonic solution.

This is a very important results which shows that an isothermal wind with a given lower boundary can reach supersonic velocities for only one specific value of mass loss rate. Thus, the only solution is producing a HD flow, starting as subsonic, transitioned through the critical point of the sonic point and becoming supersonic. As the flow beyond r_s becomes supersonic, any information can no longer be propagated upstream.

More can be found in Appendix B, in particular, the explicit velocity and density law at which the stream abides to.

4.1.1 Simulation Framework

In order to initiate a simulation, it should be important to understand what important parameters should be first used. The HJ that is being modeled is supposed to have similar characteristics to HD290458b, used as main model in [Carroll-Nellenback et al. \(2017\)](#). Such parameters can be found in Table 4.1.

Additionally, it is required to specify the grid. Three models were produced with different resolutions, with fixed solvers:

- Cartesian grid of $[100 \times 100 \times 80]$, using *hllc*, *LINEAR* and *RK2*.
- Cartesian grid of $[200 \times 200 \times 160]$, using *hllc*, *LINEAR* and *RK2*.
- Cartesian grid of $[512 \times 512 \times 512]$, using *hllc*, *LINEAR* and *RK2*.

HJ Parameter	Symbol	Value	Units
Mass	M_p	0.69	M_J
Radius	R_p	1.39	R_J
Density	ρ_p	5×10^{-15}	g cm^{-3}
Orbital Distance	a	99.3	R_J
Rotation Period	Ω	3.5	days
Temperature	T_p	10^4	K
Mean Molecular Weight	μ	$\frac{1}{2}$	-
Outer Boundary	R_{launch}	1.1	R_p

TABLE 4.1: Table with used parameters in the models for the HJ.

This usage of several resolution grids can be seen as an initial instrument to verify at which order of resolution the effects of being a cartesian grid are lost. This is because, at a low order of accuracy, the nature of the grid should bear striking effects on the model at a low resolution.

During the entire simulation, it is specified a certain region of the grid not to evolve, i.e. the RHS of the conservative equations are set to zero in this region, thus there is no time evolution. This can be done using the command *INTERNAL_BOUNDARY* set to yes in the file *definitions.h* and then set the details of the region in the *init.c* file. Since our aim in the models rely outside of the objects, this mechanism can help us simplifying the interior of the planet or the star. Because this region is fixed, the least amount of required computations, the better. Therefore, it has been defined the following fixed structure for the HJ:

- From the center up to $0.75R_p \equiv R_0$, there is no wind $\mathbf{v} = \vec{0}$ and it resembles a constant-density sphere:

1. Density is fixed ρ_p
2. Gravity follows:

$$\mathbf{g}(r) = \frac{4\pi}{3}G\rho_p\mathbf{r} \quad (4.14)$$

3. This leads to a hydrostatic pressure that follows:

$$P(r) = P_p + \frac{2\pi}{3}G\rho_p^2(R_p^2 - r^2) \quad (4.15)$$

- From R_0 to R_p , a Parker wind is initiated while keeping density, gravity and pressure in the same condition.

- From R_p to $1.1R_p \equiv R_{\text{launch}}$, it is fixed the Parker wind. This choice of $1.1R_p$ is justified by the work of [Murray-Clay et al. \(2009\)](#), where the author shows that the outflow only truly starts at such height.

As seen before, the wind is a transcendental equation, thus, using a Newton-Raphson scheme to get velocity at each r is the appropriate way to address the question. In Appendix D it can be seen the code used. In this code, it can be seen that there has been used two different initial value branches according to being in a subsonic or supersonic position of the theoretical wind - justified by convergence strategies.

As for the initial condition for the entire grid, a Parker wind is set, respecting the boundaries set for the HJ. The model is left evolve during 25.0 **PLUTO** seconds, which in this case, given the natural units used, should correspond to:

$$T_{\text{run}} = 25T_0[s] = 25 \times \frac{L_0}{v_0}[s] = 25 \times \frac{R_J}{\sqrt{\frac{2GM_J}{R_J}}}[s] \sim 25 \times (7.546 \times 10^3)[s] \quad (4.16)$$

Another unit of time can be used in this context, which is the denominated 'sonic time', corresponding to the time that a particle launched by the surface of the planet takes to reach the sonic sound, this is:

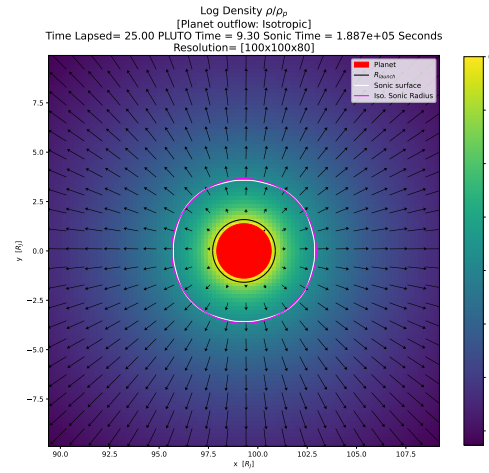
$$T_{\text{sonic}} = \frac{r_s}{c_s}[s] = \frac{\lambda R_p}{2c_s}[s] \sim 20.618 \times 10^3[s] \quad (4.17)$$

This means that $T_{\text{sonic}} \sim 2.732T_0$.

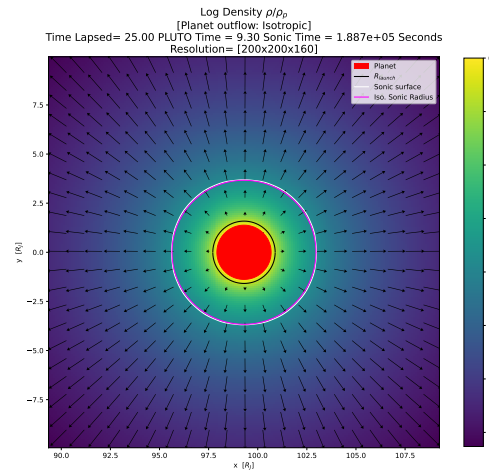
4.1.2 Results

From **PLUTO**, it is possible to select the desired outputs. It has been selected to get as an output the entire grid and all the variables at every 0.25 **PLUTO** seconds as a binary file with single precision and a binary file with double precision at every 5.0 **PLUTO** seconds. From these, it is then possible, with the tool **pyPLUTO**, to plot easily the desired grids without any extensive array work. Figure 4.2 presents three plots with the logarithm of the normalized density over the entire grid after the running time.

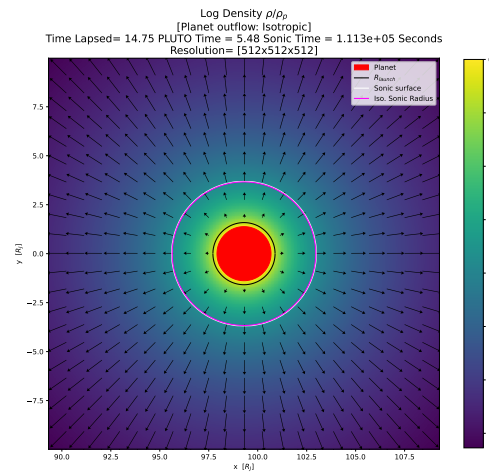
In these plots it is possible to see the typical direction of the flow with the arrows, which are proportional to the directional velocities of the plane. Additionally, the white and pink surface represents the sonic surface and the theoretical sonic radius. This combination is important in order to gain sense of where and whether the flow becomes supersonic.



(A) Lowest Resolution



(B) Intermediate Resolution



(C) Highest Resolution

FIGURE 4.2: Three plots of increasing resolution of the models' logarithmic normalized density. The black line denotes the launching radius, beyond which the grid is free to evolve. The white line denotes the sonic surface, where $|\mathbf{v}| = c_s$ and the pink denotes the theoretical isothermal sonic surface, with a radius equal to the sonic radius r_s .

Other plots are also possible to analyse, such as the logarithm of normalized density for xz or yz axis. However, due to spherical symmetry of this model, the results are equal at good enough resolution.

Figure 4.3 presents two plots from the Intermediate Resolution model: plot (A) shows the three directional velocities along each axis and plot (B) shows the logarithm of the normalized density along $(x, 0, 0)$. In each plot is also present in a dashed green color the

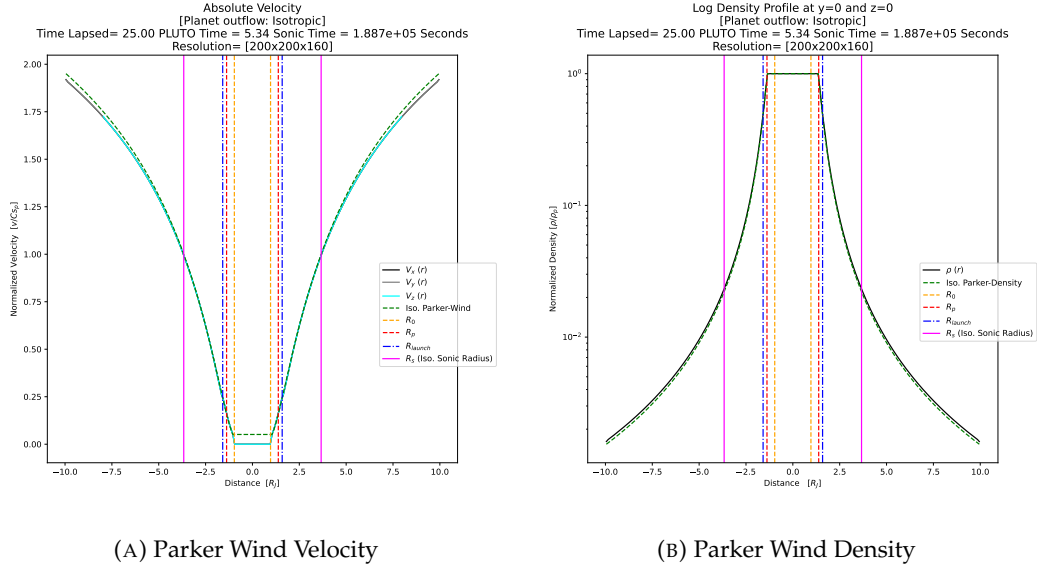


FIGURE 4.3: Plots from the Intermediate Resolution model: on the left is plotted the directional velocities at each axis compared to the expected Parker Wind profile normalized to the sonic speed; on the right is plotted the logarithm of the axial density $\rho(x, 0, 0)$ and the expected Parker Wind density profile. Vertical lines are fixed boundaries of the model.

expected value computed using a Newton-Raphson method. There exists a slight deviation from the true expected value, which could be expected since it was used a constant temperature, contrasting with the numerical EOS, with an adiabatic constant different from 1.0 ($\gamma = 1.01$).

Finally, we can see how the model behaves in an outflow point of view by measuring the mass loss of the planet. Figure 4.4 shows the time evolution of the mass loss at a given distance of the planet for the same model. This mass loss is calculated by summing the value of $r^2 \rho v$ in each cell inside a shell of thickness $2dr$, where $dr = \sqrt{dx^2 + dy^2 + dz^2}$ in that cell. Normalization refers to the theoretical value of expected mass loss: $\dot{M} = 4\pi r^2 \rho v$ at $r = R_p$, which goes as $\dot{M} \sim 1.231 \times 10^{12} \text{gs}^{-1}$ - this value is two orders of magnitude bigger from the value used in Carroll-Nellenback et al. (2017) due to them using a twice as large λ_p . From this plot, it is possible to observe some important aspects:

In the point of view of the model's resolution, it can be understood that the cartesian grid bears a heavy grip on the outputs in a inverse relation to the resolution: the higher the resolution, the least the cartesian characteristics are felt. Although it should be noted that these models were run with a *LINEAR* reconstruction and *RK2* time-stepping, thus meaning these results mark the lower bound range of this simple model.

Another important aspect to address is the initial conditions of the model and the convergence to the steady state. As it has been seen, depending on the size of the grid, there is an initial T_{run} where the model evolves due to an unstable set of initial configurations. Numerically, this evolution should behave at a speed no smaller than v_0 , being typically at the order of c_s . In other words, a perturbation starting at the launching radius of the outflow should take about $\sqrt[3]{V_{\text{Grid}}}/(c_s)_{\text{object}} \lesssim T_{\text{perturbation}} \sim \sqrt[3]{V_{\text{Grid}}}/v_0$, where V_{Grid} is the volume of the grid. Therefore, since the grids of the three models seen have a volume of $[20 \times 20 \times 16]L_0^3$, we have $\sqrt[3]{V_{\text{Grid}}} \sim 18.6L_0$. Since the perturbation begins in the origin of the grid, and it propagates radially, a factor of 1/2 appears, approximating $T_{\text{perturbation}} \sim 9.3T_0$, which is very close to what is verified.

It is expected that numerically the model does not reach a steady state perfectly matching the theoretical results. However, we must understand that there are also other important aspects taking place, such as the fact that the EOS is ideal but does not ensure that the temperature is constant. This is where an important discussion must be brought to attention: the effects of an adiabatic index close to 1.0. Figure 4.5 shows several contours of the temperature of the outflow. Although the HJ's surface temperature is fixed inside the launching radius to be $T = T_p = 10 \times 10^4 K$, pressure drives temperature to increase around 2.3%.

In Appendix B, section B.2, a simple scheme of polytropic winds is drawn in order to understand the effects of a polytropic relation (Γ dependency), which can be translated to heat loss or gain along the flow of the winds. However, in a scheme where $\Gamma = \gamma$, *i.e.* an isentropic case, there is heat exchange, although none is lost (see McCann et al., 2019, Appendix A.3). Let us expand the equations deduced in the Appendix such that $\Gamma = \gamma = 1.0 + \epsilon$ with $\epsilon \ll 1.0$. Starting with equation (B.12):

$$a(r) = \sqrt{\frac{(1 + \epsilon)p_0}{\rho_p} \left(\frac{v_0 r_0^2}{v r^2} \right)^\epsilon} \sim \sqrt{\frac{p_0}{\rho_p}} \left(1 + \frac{1}{2}\epsilon + \frac{1}{4} \left(\frac{2vr^2}{v_0 r_0^2} - 1 \right) \epsilon^2 \right) + \mathcal{O}(\epsilon^3) \quad (4.18)$$

Thus, the adiabatic sound speed increases relatively to c_s . The radial dependency on the adiabatic sound speed is only noticeable at an order of ϵ^2 .

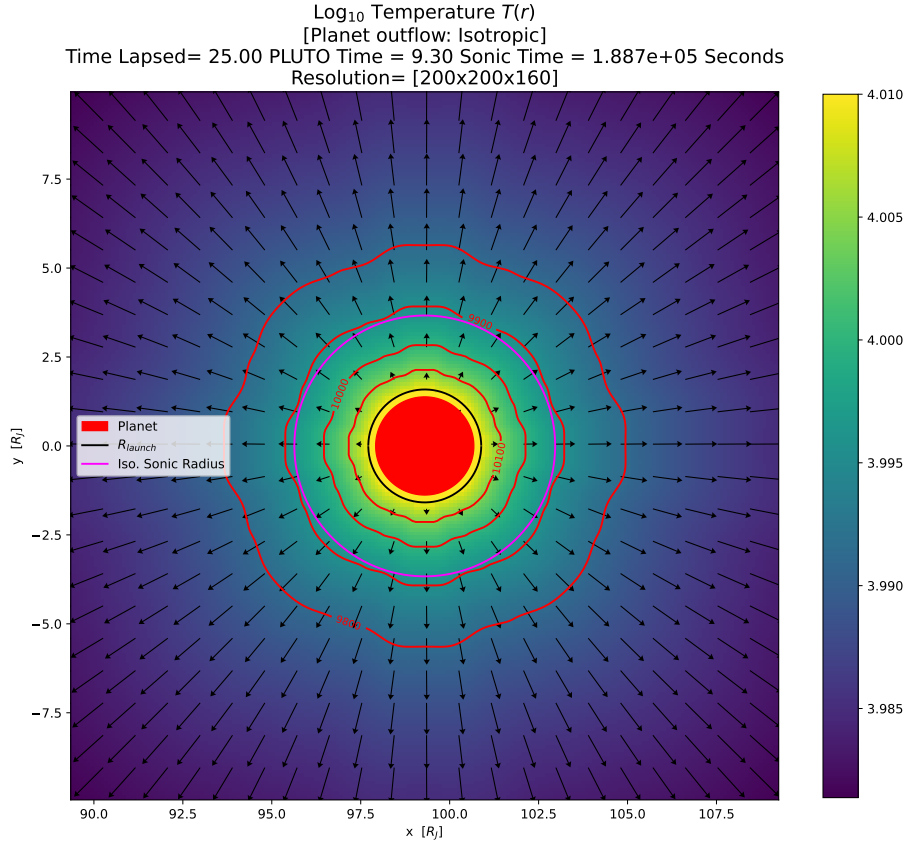


FIGURE 4.5: Temperature plot from the Intermediate Resolution Model, with several contours of constant temperature surfaces projected to the xy plane.

Finally, we can take a look at the mass loss in a polytropic wind under this approximation. Equation (B.28) yields:

$$\dot{M} = 4\pi\tau_0^2\rho_0\sqrt{\frac{k_B T_0}{m_p\mu}}\sqrt{1+\epsilon w_0^{\frac{2+\epsilon}{2}}}x_0^\epsilon = \dot{M}_0\sqrt{1+\epsilon w_0^{\frac{\epsilon}{2}}}x_0^\epsilon \quad (4.19)$$

where $\dot{M}_0 = 4\pi\tau_0^2\rho_0v_0c_s/v_c$ describes a characteristic mass loss totally dependent on the initial conditions. Taylor expansion yields:

$$\dot{M} \sim \dot{M}_0 \left[1 + \epsilon \left(\frac{1}{2} \right) + \frac{\epsilon^2}{2} \left(-\frac{1}{4} + \frac{w_0^{-1}}{2} + \frac{1}{x_0} \right) + \mathcal{O}(\epsilon^3) \right] \quad (4.20)$$

This result is slightly complex, because the characteristic mass loss depends on v_c , the critical velocity, which is also modified in perturbation theory. Nevertheless, the mass loss value is constant, because it depends only on the boundary conditions, as should be expected, since the outflow is isentropic.

Although an nearly isothermal wind can give rise to small deviations of the expected values, the mass loss plot reveals a variation $\sim 4\epsilon \sim 4\%$. Figure 4.4 shows that at the boundary of the launching radius there is a clear strong gradient. Thus it is expected a visible variation in the absolute ram pressure ρv^2 as well. Figure B.1 shows the plots of absolute ram pressure normalized to the corresponding one at the surface of the planet for each model.

Clearly, the resolution stands at the origin of this problem. Since the boundaries follow a spherical configuration, which the cartesian grid fails to adhere at lower resolutions, the model will evolve from the initial configurations to a steady state which tries to compensate this.

This presents a problem that is important to address in global models, where the stellar wind is present: the resolution of the grid around the outflowing objects should be an indicator of the mass outflow at a large scale model. Therefore, in order for an accurate analysis on the results, it should also be paid attention at the local mass loss of each object, which takes into account the resolution.

4.2 Planetary Outflowing Wind with Temperature Anisotropy

The structure of a wind in which there are temperature gradients will be different from an isothermal wind for two reasons:

- The presence of temperature gradients implies that extra pressure gradients enter into the momentum equation. Consequently, the velocity structure will change.
- The variation of temperature with distance implies a variation of the sound speed and Mach number of the flow. As such, the critical point will be affected, as will the mass loss.

A useful way of discussing non-isothermal winds is to use the energy per unit mass $e(r)$ which is the sum of the kinetic and gravitational energies and enthalpy:

$$e(r) = \frac{v(r)^2}{2} - \frac{GM}{r} + \frac{\gamma}{\gamma - 1} \frac{k_B T}{m_p \mu} \quad (4.21)$$

From this it can be understood that a wind can only reach super-escape velocities if energy is added to it or work is performed on the gas.

The addition of energy that is needed to drive a wind can be in the form of heat input or in the form of momentum input. Therefore the presence of a outward force is usually called momentum deposition, in contrast to energy deposition.

The effects of energy deposition on the wind can be expressed by means of the first law of thermodynamics:

$$du = dQ - pd\frac{1}{\rho} \quad (4.22)$$

where du is the change in the internal energy, dQ is the amount of heat added and $pd\frac{1}{\rho}$ is the work done by the gas.

The momentum equation in a wind with energy and momentum deposition does not contain the thermal energy position $q(r)$. The effect of energy input on the velocity can be derived from the momentum equation by taking into account its effects on the pressure gradient.

The pressure term: $\rho^{-1}dp/dr$ can be expressed:

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{d(p/\rho)}{dr} + \frac{p}{\rho} \frac{d \ln \rho}{dr} \quad (4.23)$$

The density gradient can be eliminated by means of the mass continuity equation:

$$d \ln \rho = -d \ln v - 2d \ln r \quad (4.24)$$

Thus, when substituting with the isothermal sound speed for a perfect gas $a(r)^2 = p/\rho$, yields:

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{da^2}{dr} - \frac{2a^2}{r} - a^2 \frac{d \ln v}{dr} \quad (4.25)$$

This results in the momentum equation in a general form that involves both the effects of a change in temperature and an external force:

$$v \frac{dv}{dr} - \frac{a^2}{v} \frac{dv}{dr} + \frac{da^2}{dr} - \frac{2a^2}{r} + \frac{GM}{r^2} = f \quad (4.26)$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dr} = \frac{1}{v^2 - a^2} \left(+ \frac{2a^2}{r} - \frac{da^2}{dr} - \frac{GM}{r^2} + f \right) \quad (4.27)$$

This equation contains the term $da^2/dr \propto dT/dr$.

From the energy equation, it can be derived that:

$$\frac{da^2}{dr} = \frac{k_B}{m_p \mu} \frac{dT}{dr} = \frac{\gamma - 1}{\gamma} \left(f + q - \frac{GM}{r^2} - v \frac{dv}{dr} \right) \quad (4.28)$$

Substituting for this term in the momentum equation, one finds that:

$$\frac{1}{v} \frac{dv}{dr} = \frac{1}{v^2 - c_s^2} \left(\frac{2c_s^2}{r} + f - (\gamma - 1)q - \frac{GM}{r^2} \right) \quad (4.29)$$

where $c_s = \sqrt{\gamma a}$, the adiabatic sound speed. This is the most general form of the momentum equation of a spherically symmetric stellar wind with energy input and momentum input. Notice the energetic dynamics inside this equation: the energy input, $q > 0$, produces an inward directed force $(\gamma - 1)q$ which counteracts the outward force. This is because the energy input heats the gas which reduces the outward temperature gradient and thus the outward force of the pressure gradient.

Equation (4.29) might suggest that in most general cases, a wind with energy and momentum deposition will have the sonic point as critical point. However, this is not true if either q or f depend on the velocity gradient dv/dr . This dependence will produce additional terms in the LHS of equation (4.29) and move the critical point to another location.

It is then inferred that the addition of momentum or heat to the wind has the following effects on the mass loss rate and velocity:

- Adding momentum in the subsonic part of the wind results in an increase of the mass loss rate and higher velocities with a smaller gradient dv/dr in the subsonic region.
- Adding heat in the subsonic part of the wind results in an increase of the mass loss rates and in higher velocities in the subsonic part of the wind.

These can be understood most easily by realizing that the velocity and density structure in the subsonic part of the wind is essentially determined by hydrostatic equilibrium. The addition of both momentum by means of an outward force and the input of heat both work effectively as an increase of the scale height in the subsonic region. An increase of the subsonic scale height results in an increase of the density at the critical point and hence produces an increase of the mass loss rate.

4.2.1 Simulation Framework

In this problem, the fixed boundaries remain exactly the same as in the isothermal model. However, the temperature anisotropy will be turned on after $T_{\text{run}} > 10\text{s}$, which means that the fixed condition of $T(r = R_p) = T_p$ will now follow a different profile. The

temperature profile follows the one chosen in the works of [Stone and Proga \(2009\)](#) and [Carroll-Nellenback et al. \(2017\)](#):

$$T(r = R_p, \theta) = \begin{cases} T_p \cos \theta & , \cos \theta < 0.05 \\ 0.05 T_p & , \text{otherwise} \end{cases} \quad (4.30)$$

It is important to understand that θ represents the HJ's surface angle at which the stellar radiation is deposited.

Assume a configuration such as the scheme from Figure 4.6. Several scenarios can happen as to how the temperature gradient can appear inside the launching radius, such that it is purely θ dependent. These scenarios are depicted in the three different Heating Cases in Figure 4.7.

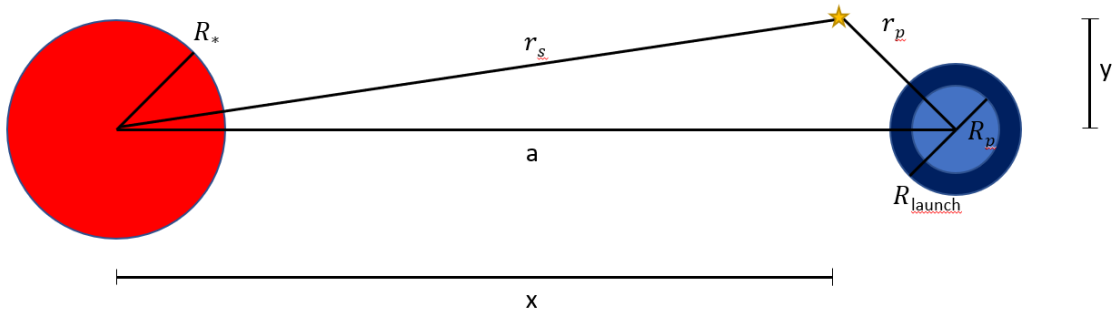


FIGURE 4.6: Schematic configuration for the possible temperature anisotropy configurations.

The case which papers [Stone and Proga \(2009\)](#) and [Carroll-Nellenback et al. \(2017\)](#) focus on is the third case of temperature anisotropy. Their work has a launching radius at the object's radius, which is the limit where Case 1 and Case 2 become the same. However, the point was never the configuration itself, but the simplicity of the model, in order to capture the wind interaction dynamics instead of the atmospheric behaviour.

Lastly, it is necessary to address the "switching on" of this temperature anisotropy. It is not numerically friendly to have in a consecutive time step a totally different atmospheric configuration. Therefore, a "slow frosting" of the HJ should take place. This also yields that a linear configuration from the initial temperature state to the final is not ideal, since this would behave as a discontinuous transition. The adopted solution for this is by having a hyperbolic tangent decrease from the initial state to the final state, which is

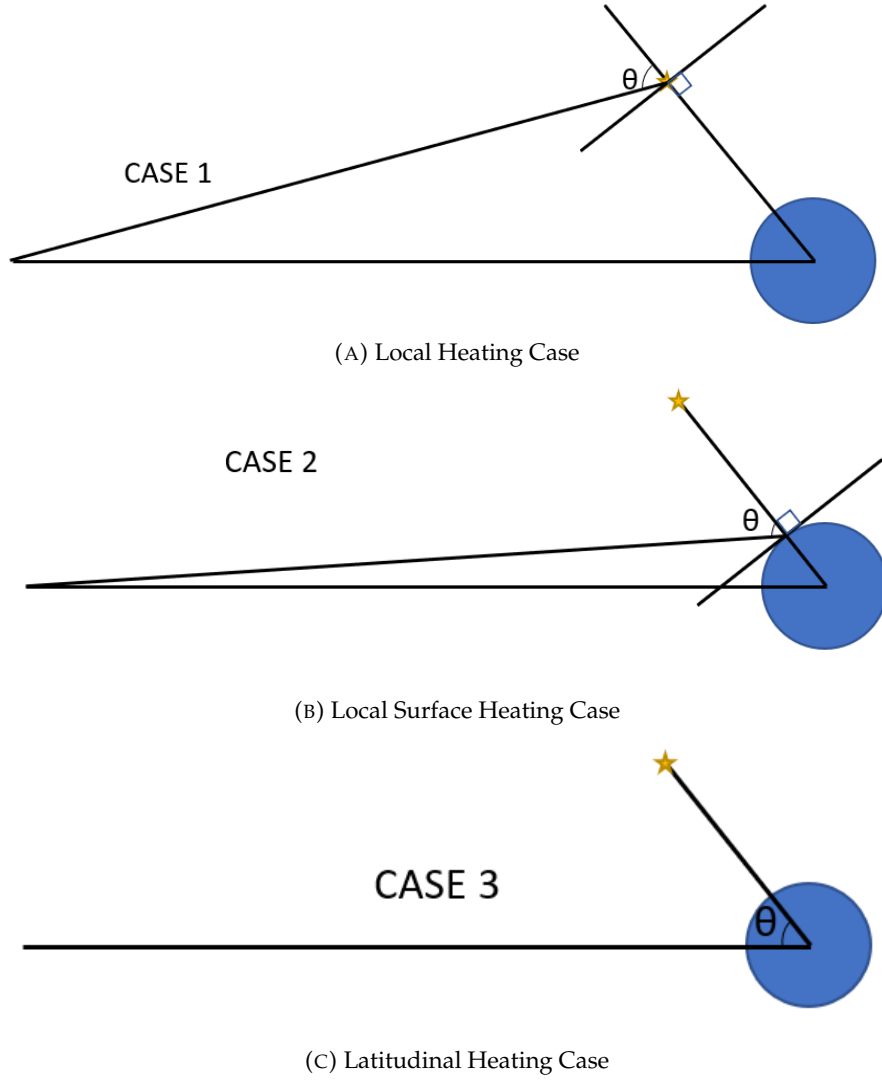


FIGURE 4.7: The different possible configurations for temperature anisotropy. The yellow star symbol is the desired position inside the launching radius. Case 1 is a local heating case; Case 2 follows a heating scheme in which the surface projection comprehends the global heating; Case 3, which is the least complex heating scheme and is the one chosen, translates in a temperature anisotropy that is latitudinally distributed.

written in equation (4.31):

$$T(t, \theta) = \begin{cases} T_p & , t < t_i \\ T_p + \frac{1}{2}(T_p - T_f(\theta)) \left(\tanh^m \left(-\alpha \left(t - \frac{t_i + t_f}{2} \right) \right) - 1 \right) & , t_i \leq t \leq t_f \\ T_f(\theta) & , t > t_f \end{cases} \quad (4.31)$$

where $T_f(\theta)$ is the final temperature configuration, t_i and t_f are the "switch on" time and the time to reach the final state, respectively. The power $\tanh^m$ is a mechanism to allow for an intermediate break on the temperature evolution (m must be an odd integer and

bigger than 1). When $t \ll (t_i + t_f)/2$ then $\tanh^m \rightarrow +1$, retrieving the value of T_p . When $t \gg (t_i + t_f)/2$ then $\tanh^m \rightarrow -1$, obtaining the value of $T_f(\theta)$. Of course, there is a discontinuity at t_i and t_f , which was the main purpose of focusing on a relaxation different from the linear one. However, numerically, this discontinuity can be controlled to be a much smaller step-function. It is, nonetheless, the smoothness of the relaxation that is important here. Additionally, the smaller the value of α , the closer the function becomes to a linear function away from $t = (t_i + t_f)/2$.

For this models it was used $m = 1, \alpha = 0.35, t_i = 10, t_f = 25$. Notice that for $t = t_i$, $\tanh(\alpha(t_f - t_i)/2) \sim 0.99$, which is a discontinuity smaller than the observed defects that resolution bears to the final steady state of the model.

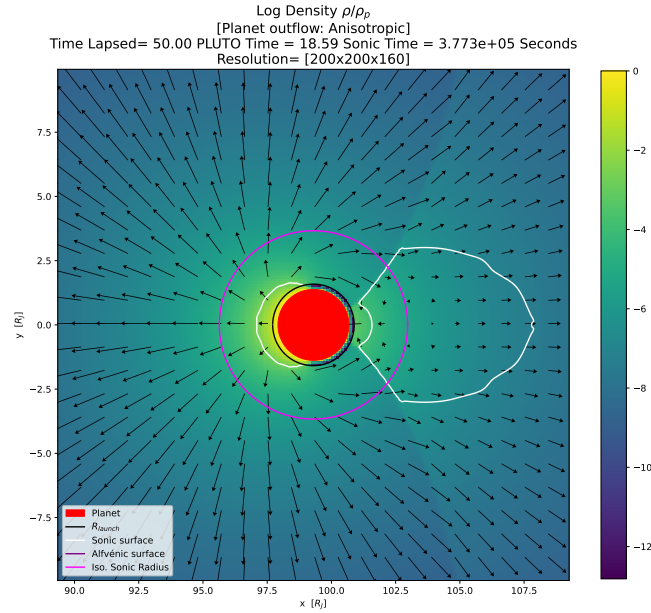
4.2.2 Results

This problem remains axissymmetric, therefore it is enough to plot one plane, which in this case is chosen to be the xy plane. Figure 4.8 shows two plots of the final steady state, following the same color scheme as before.

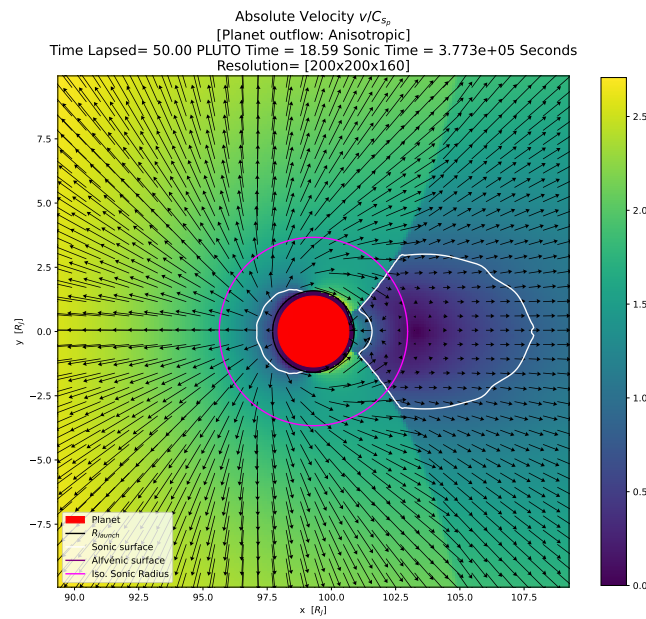
Several aspects must be enlightened here:

- In both plots it is possible to observe a discontinuity at the "night side" of the planet ($x > a$). This is a shock that is formed by the "day side" outflow at supersonic speeds being pushed by the ram pressure towards the "night side" outflow.
- The peculiar sonic surface: this surface follows approximately the resulting sonic surfaces from the works of [Stone and Proga \(2009\)](#) and [Carroll-Nellenback et al. \(2017\)](#). The pressure gradient, which is created by the temperature anisotropy in an angular configuration, accelerates the "day side" outflow, which shortens the sonic surface.
- It is possible to observe in good detail by the velocity plot from Figure 4.8 that an inflow takes place in the "night side". The directional arrows gives us a detailed sense of the flow's profile, which is complemented by the coloring of the image. Is is possible to observe that there exists a small region in which the velocity of the flow becomes null.

To help us understand how can this result be achieved it is crucial to verify the ram pressure of the steady state - Figure 4.9.



(A) Density Profile



(B) Absolute Velocity Profile

FIGURE 4.8: Density and velocity plots from the steady-state configuration of the model with Intermediate Resolution and with temperature anisotropy.

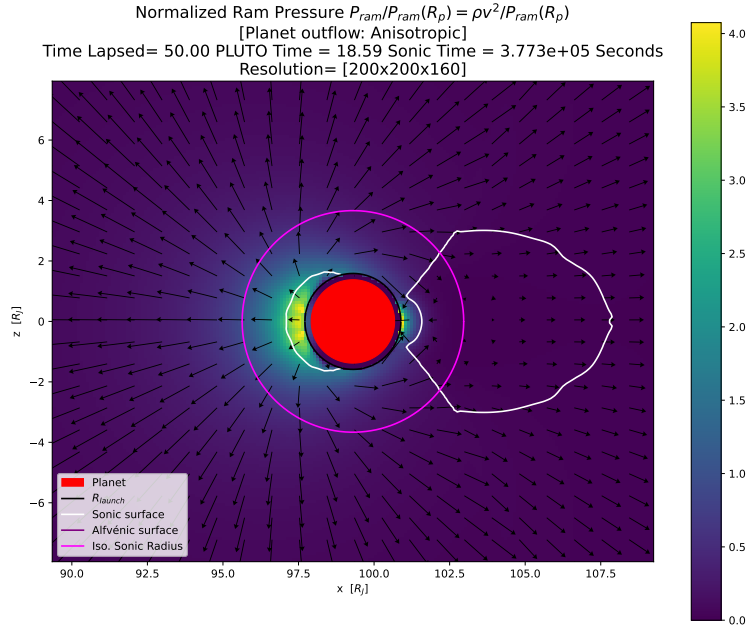


FIGURE 4.9: Ram Pressure plot from the steady-state configuration of the model with Intermediate Resolution and with temperature anisotropy.

It can be observed that the ram pressure is enhanced in the “day side” of the HJ but with a given tilt. Remembering that the problem is axisymmetric, this will form an enhanced “crown” of ram pressure, which will drive the winds to the back of the planet in supersonic speeds. This is the origin for the convergence of winds to the “night side” that then creates the stagnation point. This point separates material that is recaptured by the planet from that which is pressure driven outwards, eventually reaching a supersonic velocity.

In Figure 4.10 is plotted the temperature of the steady state with contours. By appreciating the geometry of this contours, it is possible to understand that the steady state is a highly angular dependent configuration. Another observation is the behaviour of the temperature of the gas at the “night side” of the planet. Although the shock creates noticeable jumps in the contours, there is a pronounced temperature “tail” in the x axis for $x > a$. This is a consequence of the evolution of the shock.

To have an understanding of how did the model evolve, Figure 4.11 shows the plots of the total mass loss over time and the inflow that happened.

Before the first $T_{run} = 10s$, the behaviour is exactly the same as the previous isotropic

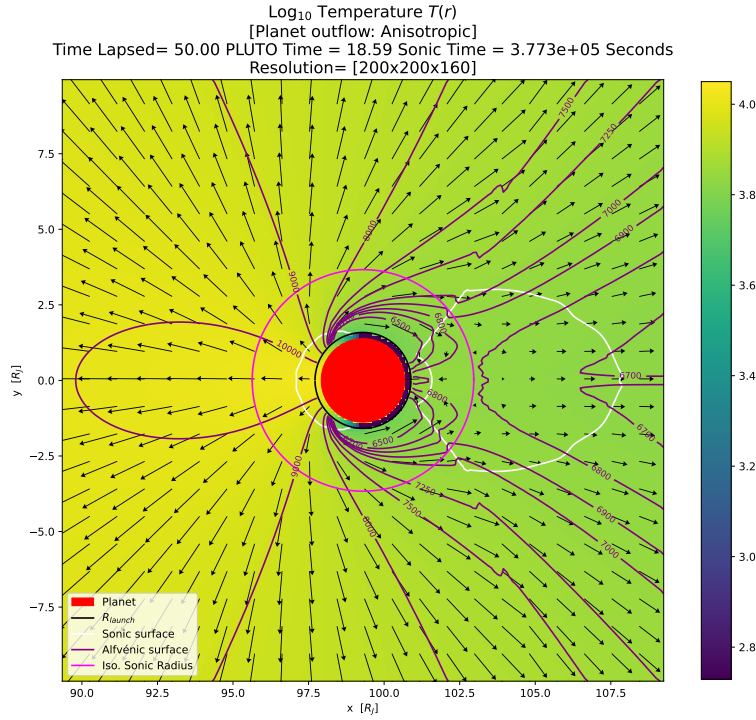


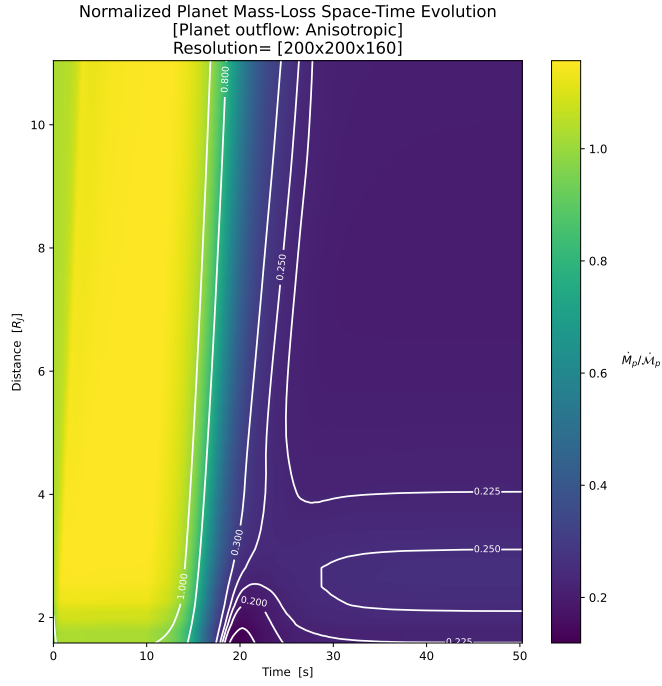
FIGURE 4.10: Logarithm of the temperature plot from the steady-state configuration of the model with Intermediate Resolution and with temperature anisotropy.

models. As the temperature anisotropy is switched on at t_i and reaches the final configuration at t_f with a frequency parameter α in the relaxation configuration (given by equation (4.31)), it is possible to observe a powerful inflow into the planet. Consequently, the total outflow falls strongly. The inflow plot also shows that at a certain distance it is null, which suggests that the local experience is quite different from a global configuration, where the HJ will appear to be an totally outflowing object.

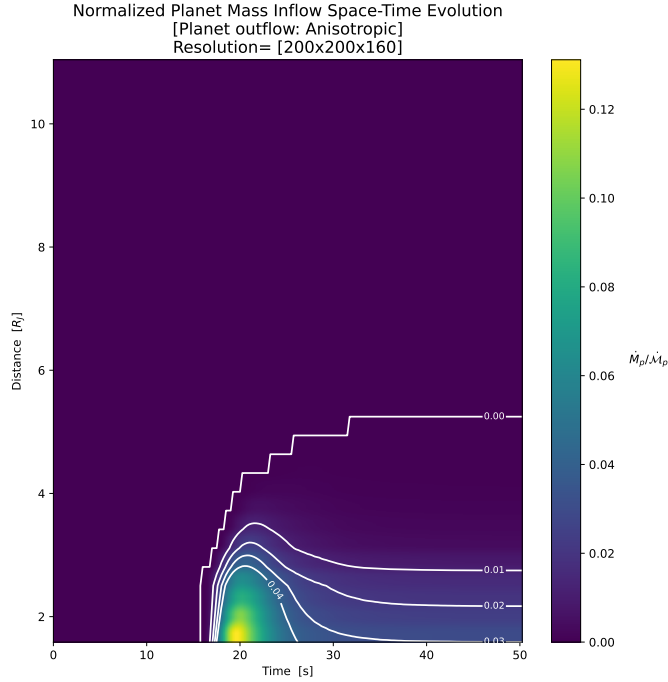
4.2.3 Important Remarks

Having an adiabatic index of $\gamma = 1.01$ creates a nearly isothermal wind, this is, the temperature is not significantly affected by geometric compression or expansion. This justifies the similarity of the temperature contours and the stream flow seen in Figure 4.10.

From what has been deduced in the analytical introduction, it is possible to confirm some results. Although this problem has a heat function that is highly θ dependent, the main radial effects are seen as well: the cooling effect in the subsonic part of the wind results in a decrease of the mass loss rate. From the inflow plot of Figure 4.11, at $T_{\text{run}} \sim 20s$ there is a drastic rise in the inflow. This corroborates what has been said: as the temperature gradient increased, so did the lateral flows. At this moment, from an animation



(A) Total Outflow Profile



(B) Inflow Profile

FIGURE 4.11: Mass Loss plots of the model with Intermediate Resolution and with temperature anisotropy.

analysis *, it is possible to observe the beginning of the separation between the "day side" sonic surface from the "night side" one. Which means that the flow becomes supersonic in this launching to the "night side" due to lateral pressures. At this stage, the stagnation point is so close to the planet that the "night side" material that is gravitationally bounded inflows towards the planet at an angle as big as the polar angles. It is the gradual distancing of this stagnation point that decreases the amount of inflowing gas. This movement is caused by the shock: supersonic flows push the stagnation point further away from the HJ. Thus, it is the evolution of the shock that dictates the inflow.

The total mass loss beyond the region where there is no inflow is $\sim 0.205\dot{M}_p$, corresponding to a decrease of $\sim 80\%$. These match with the results of [Stone and Proga \(2009\)](#), which report a reduction of a factor of 3.7. Although the paper does not explicit at which radius is calculated the mass loss, there is a general agreement of results given the different planet's parameters.

Regarding the work of [Carroll-Nellenback et al. \(2017\)](#), the decrease was of 37%, which differs greatly. One possible explanation is the fact that, although the HJ's parameters are exactly the same, the paper uses a value λ twice as big, which I suspect is due to considering $\mu = 1$. This will consequently increase the sonic radius by a factor of 2, and greatly affect velocity and density laws. Additionally, their chosen value of $\gamma = 1.0001$ most likely creates a temperature profile less flexible to strong gradients - which can be deduced by the large "night side" sonic surface following the shock that is present in the paper's Figure 3. Lastly, due to resolution issues and characteristic scales used in **PLUTO**, the current model's chosen minimum temperature is $0.05T_p$, whilst these works achieve a minimal temperature of $0.01T_p$. All these factors, combined with the possibility of using a different relaxation rate for the temperature anisotropy, can converge to such a difference. Nevertheless, the overall aspect of the results are in good agreement, although the total mass loss obtained in [Carroll-Nellenback et al. \(2017\)](#), when focusing on the radial behaviour, fails to retrieve the slight increase at around $r \sim 2R_p$ - possibly from the fact that $\gamma = 1.0001$.

Due to this, there is motivation for a small exploration of the degrees of freedom of the model's phase space. The relaxation imposed in the temperature anisotropy can become an important aspect for the shock velocity, which imposes a distinct initial inflow, since the stagnation point is pushed away differently. Thus, having fixed all other parameters,

* Link for paste with animations regarding the Anisotropic model: [HD_Aniso](#)

it was built a model with the following characteristics in the relaxation function: $m = 1, \alpha = 1.05, t_i = 10, t_f = 15$ - chosen so that the discontinuity remains quantitatively the same.

Figure B.2 and B.3 (in the Appendix B) show the previous discussed plots in order to have a comparison. As can be seen, the final total outflow is even smaller than before, and the initial inflow reaches higher values. From the temperature plot, a distinct "day side" and "night side" interaction takes place. Due to the fast temperature decay, the inflow at the night side captures higher velocity winds, thus explaining the initial values of inflow. In addition, the total outflow reaches a minimum at larger distances, which means that the strong lateral flows are being rapidly engulfed into the planet. Therefore, it is possible to extrapolate that the relaxation function used in Carroll-Nellenback et al. (2017) should have a smoother gradient, thus bearing higher total mass loss values.

Another important aspect is the fixed temperature of the "night side". This parameter dictates the strength of the lateral flows, since it is the gradient of the temperature that translates to the pressure gradients. The works Stone and Proga (2009); Carroll-Nellenback et al. (2017) fix $0.01T_p$, which is 5 times smaller than the studied model. As a consequence, the next model to study should be one in which the "night side" temperature is larger: $0.2T_p$ at the "night side".

Figure B.4 and B.5 (in the Appendix B) show the previous discussed plots in order to follow another comparison. As can be seen, the final total outflow is smaller than the main model, although the initial inflow does not reach higher values. A clear distinct sonic surface is formed: lateral flows are so slightly faster than the sound speed, which means that the "night side" temperature strongly affects the lateral flows, as predicted. This deceleration of wind speed compared to the main model will then originate a slower propagating shock. However, since the "night side" temperature is higher, the local sound speeds will be competitive enough to face the gravitational potential.

4.3 Planetary Outflowing Wind with Rotation

Facing a rotating object requires the introduction of extra components to the wind's velocity:

$$\mathbf{v} = V_r(r)\hat{\mathbf{e}}_r + V_\phi(r)\hat{\mathbf{e}}_\phi \quad (4.32)$$

Here, it is assumed that the wind can be separated into a radial component and one azimuthal component, which are radial functions, in order to gather similar characteristics to a Parker Wind.

The momentum equation:

$$(\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \nabla p + \frac{GM}{r^2} \hat{\mathbf{e}}_r = 0 \quad (4.33)$$

can be separated into the radial and azimuthal components:

$$\hat{\mathbf{e}}_r \left[V_r \frac{\partial V_r}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{V_\phi^2}{r} + \frac{GM}{r^2} \right] = 0 \quad (4.34a)$$

$$\hat{\mathbf{e}}_\phi \left[\frac{V_r}{r} \frac{\partial}{\partial r} (r V_\phi) \right] = 0 \quad (4.34b)$$

From conservation of mass, it can only be extracted the radial component (if assumed a density which behaves as $\rho = \rho(r, \theta)$):

$$\nabla \cdot (\rho \mathbf{v}) = 0 \implies \frac{1}{r^2} \frac{\partial}{\partial r} (\rho V_r r^2) = 0 \quad (4.35)$$

Therefore, the following solution appears:

$$\rho V_r r^2 = C_1 \quad (4.36)$$

which can be given physical meaning by equating C_1 to the object's boundary: $C_1 = \rho_p V_r(r = R_p) R_p^2$.

Additionally:

$$p(r) = \frac{k_B T(r) \rho(r)}{m_p \mu} \implies p(r) = c_s^2 \rho(r) \quad (4.37)$$

where it has been assumed that $T(r)$ is constant.

From equation (4.34b), it follows:

$$r V_\phi = C_2 \quad (4.38)$$

Again, physical interpretation bear $C_2 = V_\phi(r = R_p) R_p$.

Finally, by searching for the radial velocity equation which is derived from the radial part of the momentum equation (equation (4.34a)):

$$V_r \frac{\partial V_r}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{V_\phi^2}{r} + \frac{GM}{r^2} = 0 \quad (4.39)$$

The second component of this equation will be:

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = c_s^2 \frac{\partial}{\partial r} (\ln \rho) = c_s^2 \left(-\frac{1}{V_r} \frac{\partial V_r}{\partial r} - \frac{2}{r} \right) \quad (4.40)$$

such that:

$$V_r \frac{\partial V_r}{\partial r} + c_s^2 \left(-\frac{1}{V_r} \frac{\partial V_r}{\partial r} - \frac{2}{r} \right) - \frac{C_2^2}{r^3} + \frac{GM}{r^2} = 0 \quad (4.41)$$

$$\implies \frac{1}{V_r} \frac{\partial V_r}{\partial r} = \frac{1}{V_r^2 - c_s^2} \left(\frac{2c_s^2}{r} + \frac{C_2^2}{r^3} - \frac{GM}{r^2} \right) \quad (4.42)$$

It can be proceeded as what was done for the isothermal outflowing wind: change of variables $w = V_r/c_s$.

$$\implies \left(w - \frac{1}{w} \right) \frac{\partial w}{\partial r} = \frac{2}{r} + \frac{C_2^2}{c_s^2} \frac{1}{r^3} - \frac{GM}{c_s^2} \frac{1}{r^2} \quad (4.43)$$

This equation can be integrated with the condition that when $w = 1 \implies r = r_c$.

By simplifying this result, since it is understood that, in a first order approximation, the azimuthal component of the wind behaves as a solid body $V_\phi = \frac{R^2 \Omega}{r}$, which means: $C_2 = R^2 \Omega$. Thus:

$$\implies \left(w - \frac{1}{w} \right) \frac{\partial w}{\partial r} = \frac{2}{r} + \left(\frac{R^4 \Omega^2}{c_s^2} \right) \frac{1}{r^3} - \frac{GM}{c_s^2} \frac{1}{r^2} \quad (4.44)$$

Consequently:

$$\implies w^2 - \ln w^2 - 1 = 4 \ln \frac{r}{r_c} - \left(\frac{R^4 \Omega^2}{c_s^2} \right) \left(\frac{1}{r^2} - \frac{1}{r_c^2} \right) + \frac{2GM}{c_s^2} \left(\frac{1}{r} - \frac{1}{r_c} \right) \quad (4.45)$$

where r_c can be obtained by solving:

$$\frac{2}{r_c} + \left(\frac{R^4 \Omega^2}{c_s^2} \right) \frac{1}{r_c^3} - \frac{GM}{c_s^2} \frac{1}{r_c^2} = 0 \quad (4.46)$$

$$\iff r_c = \frac{GM}{4c_s^2} \left(1 \pm \sqrt{1 - 8 \left(\frac{R^2 \Omega c_s}{GM} \right)^2} \right) \quad (4.47)$$

from which a critical value of Ω appears:

$$\Omega_c = \sqrt{\frac{1}{8} \frac{GM}{R^2 c_s}} \sim 3.22 \times 10^{-3} \left(\frac{M}{M_J} \right) \left(\frac{R}{R_J} \right)^{-2} \left(\frac{T}{10^4 \text{K}} \right)^{-\frac{1}{2}} \sqrt{\mu} \text{ rad.s}^{-1} \quad (4.48)$$

In the limit $\Omega \longrightarrow 0$, the same solution as for the non-rotating isotropic case is retrieved. The model's parameters stand away from this critical limit: $\Omega = 2\pi/(3.5\text{days}) \sim 2.08 \times 10^{-5} \text{rad.s}^{-1}$.

If one wants to explore higher degrees of freedom, as in, considering a wind with r and θ dependent functions, other methods are needed to achieve final solutions - the paper of [Lima and Priest \(1993\)](#) is a good example to understand the difficulty an increased number of degrees of freedom.

Let us also define the ϵ parameter, which defines the ratio squared of the rotation speed and the thermal speed, *i.e.* the ratio between the tidal potential and sound speed:

$$\epsilon = \left(\frac{R\Omega}{c_s} \right)^2 \sim 0.2703 \left(\frac{1\text{day}}{P_{\text{rot}}} \right)^2 \left(\frac{R}{R_J} \right)^2 \left(\frac{10\text{kms}^{-1}}{c_s} \right)^2 \quad (4.49)$$

By prior definition, the critical value of Ω can be used to have an upper value of ϵ , defined by:

$$\epsilon_c = \left(\frac{R\Omega_c}{c_s} \right)^2 = \left(\frac{GM}{\sqrt{8}Rc_s^2} \right)^2 = \frac{\lambda^2}{8} \quad (4.50)$$

Notice that with the values of λ and ϵ it is possible to have a general characterization of the system at hand, since they arise naturally from Ω/Ω_c or ϵ/ϵ_c . In this way, the phase space could be richfully explored, as [Trammell et al. \(2011\)](#) did prior to the study of HJs' magnetospheres.

4.3.1 Simulation Framework

It is important to note that rotation influences the critical radius in a slight way, given the selected parameters. In this way, it can be explored how a rotating HJ with a Parker wind can converge into the deduced wind profile. Thus, it has been constructed two different models: one that follows the analytical considerations made above - designated as *TeoRot* - and one that starts as a Parker Wind but molded to the rotating star as a solid body - designated as *Rot*. However, the *Rot* model obeys conservation laws, this is, the azimuthal component of the wind still goes as $\propto r^{-1}$. This differs greatly from many works, which only take into account radial Parker Winds - and it will be discussed ahead the differences this aspect can create in a model. It will be through the analysis of these models that we can deduce if a model with planetary rotation and temperature anisotropy can indeed converge into a theoretical close enough steady state.

The wind modification now focuses mainly in the initial velocity profiles and inside the boundaries. Both models will incorporate an additional azimuthal component:

$$\mathbf{v}(r) = V_r(r)\hat{\mathbf{e}}_r + V_\phi(r)\hat{\mathbf{e}}_\phi = V_{\text{Parker}}\hat{\mathbf{e}}_r + \Omega R_p \frac{R_p}{r} \hat{\mathbf{e}}_\phi \quad (4.51)$$

The *Rot* model maintains a density and velocity profiles with respect to a radial Parker Wind, although an angular momentum conserved azimuthal wind is also inserted. The *TeoRot* model has an identical velocity profile, except that V_r now follows equation (4.45) and the density profile is obtained by the following relation:

$$\rho(r) = \frac{\rho_p R_p^2}{r^2} \frac{V_r(r = R_p)}{V_r(r)} \quad (4.52)$$

which is a consequence of the mass conservation equation seen above.

Additional to all these new parameters, we have chosen to follow the reference frame of the HJ's surface. This choice is purely optional, although is an initial strategy of adaptation to the global frame which, since we will consider a tidally locked planet, the reference frame of the system is the best option. Thus, a rotating frame goes as:

$$V_{\text{frame}}(r) = \mathbf{\Omega} \times \mathbf{r} \quad (4.53)$$

However, since the frame follows the planet's rotation, **PLUTO** is in reality fixing the grid, *i.e.* the planet and have the frame rotate opposite to it. Thus, it is important to carefully understand that the code's $\mathbf{\Omega}$ is defined clockwise to have an anti-clockwise spinning (orbiting) HJ, while the physical rotation frequency is $\hat{\mathbf{z}}$ directed.

Consequently, the wind's velocity profiles will behave as:

$$\mathbf{v} = \begin{cases} \mathbf{\Omega} \times \mathbf{r} & , r < R_0 \\ V_r \hat{\mathbf{e}}_r - (\mathbf{\Omega} \times \mathbf{r}) \left(\frac{R_p^2}{r^2} - 1 \right) & , R_0 < r < R_{\text{launch}} \end{cases} \quad (4.54)$$

Finally, the rotating frame will introduce fictitious forces such as Coriolis and centrifugal forces. These will be better explored in the next section, however a quick note can be elaborated. These forces yield:

$$\frac{1}{\rho} \mathbf{f} = -\mathbf{\Omega} \times \mathbf{v}' - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}') \quad (4.55)$$

where the primed variables are relative to the rotation's frame. In order to simplify the numerical problem, the HJ is placed at the origin in these next simulations.

4.3.2 Results

The aim of the present subsection is to compare the two models discussed before. Figure 4.12 shows the logarithm of normalized density of both models.

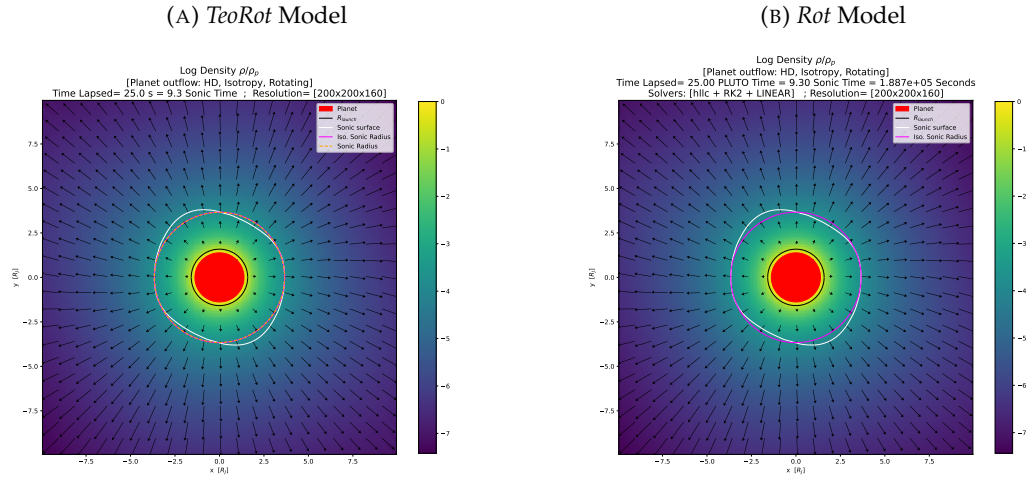


FIGURE 4.12: Density plots for both models in the rotating frame.

From an initial perspective, the final configuration of the model with initial conditions and fixed boundaries following the theoretical values (*TeoRot*) resembles the *Rot* model. It has also been plotted the theoretical sonic radius, which should be shrunk by the increase of the rotation speed, however nearly overplots the isothermal sonic radius.

Additionally, there is an elongation in both models of the sonic surface in a right upwards direction. This is a consequence of the fictitious forces derived from the rotating frame, with the Coriolis force $\propto -\Omega \times \mathbf{v}$ having a higher toll on the flow's dynamics, since the centrifugal force goes as $\propto \Omega^2 r$.

Figure 4.13 has plots with an increased density of direction arrows and the plots show the plane directions velocities normalized to the sound speed. From these, it is possible to notice the slight bending of the flow due to the azimuthal component in the planet's frame.

These results show that from distinct initial configurations, the less accurate model converges to the accurate model. Additional plots in Appendix B, section B.6 can be used to understand how quantitatively similar are the final configurations of these models.

Figure 4.14 shows some parameters profiles compared to what is theoretically expected, to understand if the theoretical model itself deviates from the expected results. As it can be seen by the dashed green light which expresses the theoretical expected configuration has a good agreement in both velocity and density profiles - remember that in the plot displaying velocities, the theoretical plot is the absolute velocity. The V_z component of velocity does not behave as the other components since the azimuthal wind profile does not introduce any contributions to this direction. However, as the azimuthal wind goes

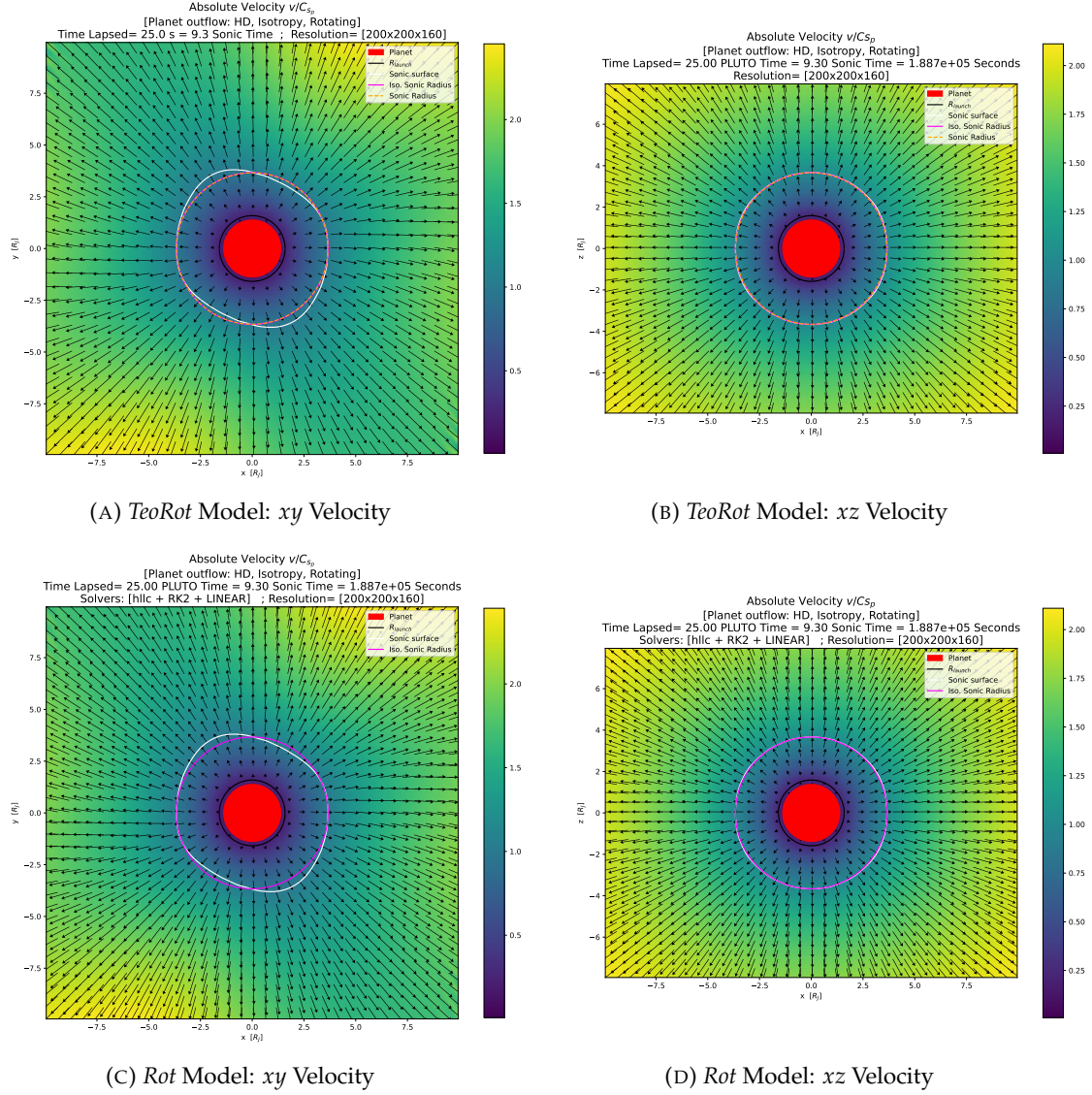


FIGURE 4.13: Velocity plots for both models in xy and xz planes, in the planet's rotating frame.

as r^{-1} , only the velocity profile close to R_p behaves drastically different. All components rapidly tend to the isothermal radial Parker Wind.

Summing all these aspects up, there is a positive validation for the simple strategy that will be implemented in the next models: having an isotropic Parker outflow in a rotating frame to evolve as perturbations are inserted in the model, as long as the simple model evolves long enough to converge.

Therefore, in order to explore temperature anisotropy, we can perform as follows:

1. Create a *Rot* model, in which the wind components respect angular momentum conservation inside the boundary regions, although the radial component is not

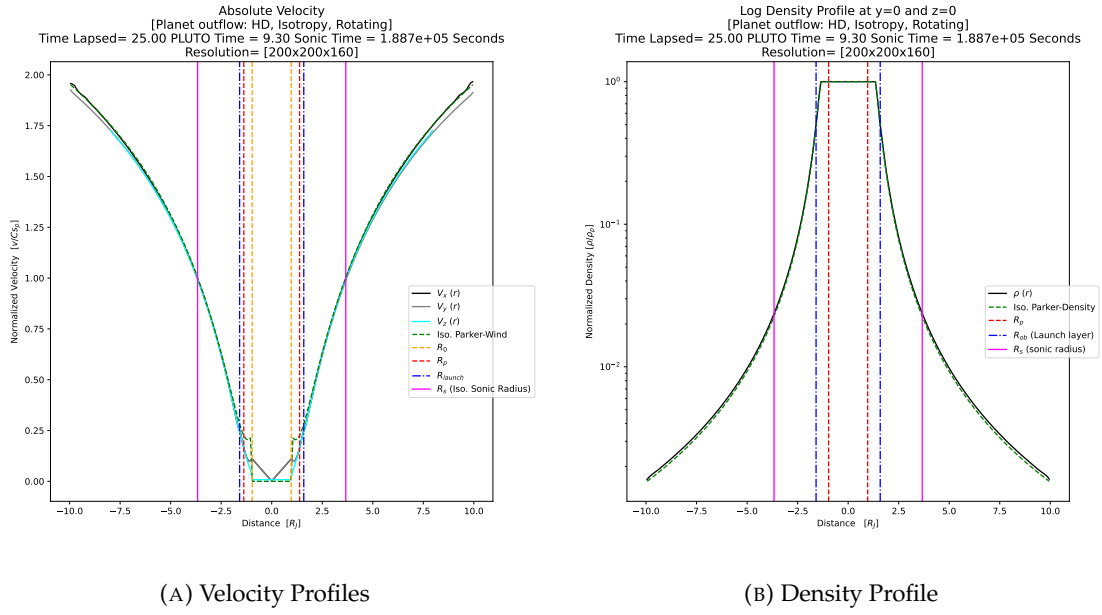


FIGURE 4.14: Plots comparing the density and velocity profiles to the predicted values: on the left is plotted the directional velocities at each axis compared to the expected radial velocity in a rotating isothermal Parker Wind; on the right is plotted the logarithm of the axial density $\rho(x, 0, 0)$ and the expected Parker Wind density.

explicitly calculated, only follows an isothermal Parker wind.

2. Let this initial configuration evolve until converge $T_{\text{run}} \sim 15s$, and apply the temperature relaxation as in the previous section.
3. Allow for the shock to evolve enough to notice if it stabilizes or repeats a pattern: $T_{\text{end}} \sim 120s$.

4.3.3 Outflow with Rotation and Temperature Anisotropy

It has been introduced the explicit heating technique seen before (Section 4.2) in this system. The temperature gradient is given by equation (4.31), with a similar relaxation method as before, in which the initial rotating system is allowed to evolve until a state close enough to the steady state. Thus, parameters for this model remain $m = 1$, $\alpha = 0.35$, $t_i = 10$ and $t_f = 25$. Due to the last subsection's results, it is safe to allow a non-explicit Parker Wind to be launched - *i.e.* the initial model is the same as the *Rot* Model. This will be denoted as *AnisoRot* Model and Figure 4.15 presents both planes' plots with normalized density in log scale and absolute velocities normalized to c_s .

At last, symmetry is broken: lateral shocks appear at different positions due to the tidal effect. It is important to take notice that symmetry at the z axis in the night-side

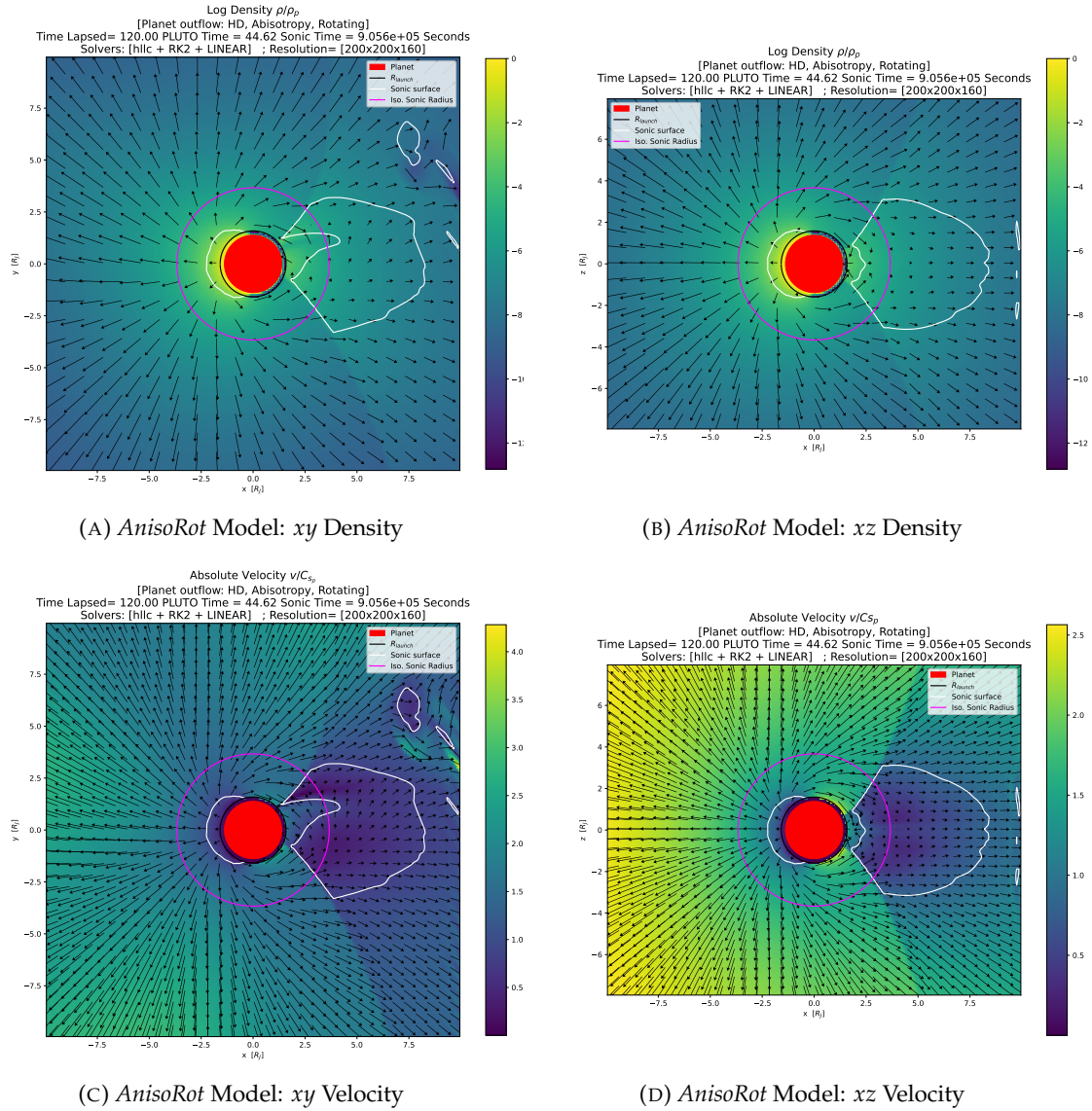


FIGURE 4.15: Density and velocity plots for *AnisoRot* model in xy and xz planes, in the planet's rotating frame.

remains, since a rotating body with $\Omega = \Omega \vec{z}$ preserves a z steady state. By analysis from Figure 4.15, polar flows maintain their velocity and eventually create a stagnation point, although remains with an equatorial non-null speed.

However, contrasting to prior steady-states, the final form of the system does not reach a steady-state, but a pattern of flows that do not seem too atypical in a long run. In order to observe this feature, the following animated analysis * allows to understand the existing cycle.

* Link for paste with animations regarding the Anisotropic model in the rotating frame: [HD_AnisoRot](#)

In Appendix B we present Figure B.8 for further visual analysis. However, it is with Figure 4.16 that we observe a quantitative relic of the cycle around the steady state. From the inflow profile plot is possible to see through the white contour of 0.0 value that there seems to exist a resistance to steadiness, which, combined with the fact that the other contours are rather stable at close distances to the HJ, a steady-state concept must be discussed.

4.3.4 Important Remarks

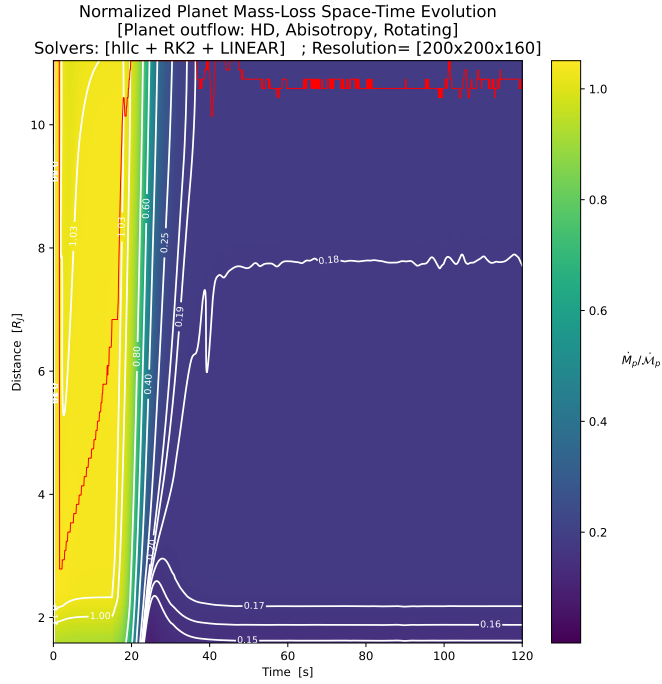
Focusing on the results of Figure 4.15, several aspects can be discussed.

Firstly, the lost of symmetry in the shock zones, which can be explained by taking into account the jump conditions. For a revision of these concepts quickly, the reader is addressed to Appendix B, section B.3. Previously, when inserting a temperature anisotropy in a non-rotating body, the relative velocities prior and after the shock region would have developed the three directional values symmetrically:

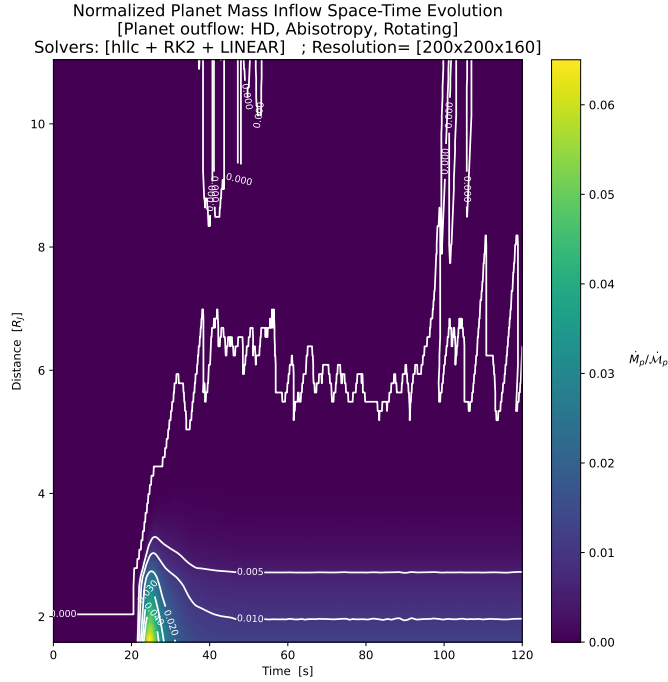
$$\left(u_{0,x}(t), u_{0,y}(t), u_{0,z}(t)\right)^{Q_1} \longrightarrow \left(u_{0,x}(t), -u_{0,y}(t), -u_{0,z}(t)\right)^{Q_4}$$

- where it is used the notation of Q_i for the respective quadrants relative to the planet, $i = 1, 2, 3, 4$ in both xy and xz planes. However, in a rotating frame, the only component that preserves this symmetry is the one that is parallel to the angular velocity vector. Both x and y components of the relative velocity will have distinct behaviours over space and time, which leads to the asymmetric evolution of the shock boundary. Another way to confirm this set of ideas is by noticing that the crown-shaped sonic region at the "day side" of the planet has been deformed, unlike the previous model, where it was symmetric. As a consequence, lateral flows are now favoured in the rightmost equatorial zone ($x = 0, y = +R_p, z = 0$), allowing for an enhanced supersonic flow until reaching the shock region in contrast to the opposite zone.

Secondly, the loss of the xy stagnation point leads to a sonic surface that tends to split in two regions. The sonic surface that stands at negative y values remains mostly stable (denoted as ssA), and the region that stands at positive y values presents a more perturbed character (denoted as ssB). This perturbative feature can be explained by the fact that there is a supersonic flow coming from ssA and encountering a shock region right above ssB . This forces the shock region and ssB to adjust over space and time, eventually leading to a launch of subsonic flows from the tail of ssB out to the boundaries of the



(A) Total Outflow Profile



(B) Inflow Profile

FIGURE 4.16: Mass Loss plots of the *AnisoRot* Model. The plot above presents a red line which tracks the maximum value over time.

computational grid and having both ssA and ssB to join together. This means that the strong flows originated by the centrifugal effect create a region of subsonic and supersonic transition.

In a final remark, it is interesting to address the fact that the inflow profile for a close distance to the planet ($r < 4R_J$) resembles closely the profile for the non-rotating anisotropic model - compare figures 4.11 and 4.16. Since the relaxation function used for the temperature gradient follows the same behaviour and the same parameters, it should be expected such behaviour. However, an important discussion must follow to explain the fact that it seems to appear an inflow from the outtermost radius. The mass loss calculation is executed by integrating a shell of thickness $4dr$. Therefore, the mass loss at distances of $r \sim 10R_J$ is calculated over a shell that can be cut by the boundaries of the grid. As the animations present in the link* show, there are sudden appearances of small sonic surfaces at the "night side" boundary of the planet. These can be explained by the boundary conditions used in these models: the outflow boundary condition (which has been explained in Section 3). The forcing of any quantity's gradient normal to the boundary equal to zero gives rise to "tangential problems". For example, the tangent velocity to the x -rightmost boundary can survive the outflow condition, which over time leads to a fictitious subsonic flow created by the slight shock of the gas reaching the boundary and these tangential ghost velocities. Although, it should be expected for these "ghost flows" to disappear over a certain time since the ghost velocities should be able to reach their respective boundary or at least being redirected to the normal direction by the shock - as it is observed in the inflow profile. Thus, the distant negligible inflows have been explained: they are formed by shocks of boundary-normal velocities with tangential ghost velocities. This aspect could have been minimised if a grid size and resolution had been previously fine-tuned to avoid these "ghostly" features.

4.4 Planetary Wind from an Orbiting and Rotating Planet

Although a rotating planet introduces interesting fictitious forces (Coriolis), an orbiting and rotating planet exhibits even stronger effects. Right now, our focus will be on the ingredients of the momentum equation:

$$(\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \nabla p + \frac{1}{\rho} \mathbf{f} = 0 \quad (4.56)$$

where $\mathbf{f}$ is the total force applied. This includes gravitational terms and fictitious terms, such that it can be expressed as so:

$$\frac{1}{\rho} \mathbf{f} = -\nabla \Phi_p - \nabla \Phi_s - \frac{d\mathbf{\Omega}}{dt} \times \mathbf{r}' - 2\mathbf{\Omega} \times \mathbf{v}' - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}') \quad (4.57)$$

where the primed variables r', v' are the radius and velocity of the object in respect to the rotating reference frame, respectively. The first two terms are the gravitational potentials from the planet and star, respectively. The third term is the Euler force, which makes an appearance when the rotating frame is non-uniformly rotating. The fourth term and fifth are the Coriolis and the centrifugal forces, respectively.

For simplicity, we assume that the planet is rigidly rotating around the star, with no eccentricity, this is, describing a circular trajectory at constant speed. Additionally, if one chooses to follow the same strategy of [D'Angelo et al. \(2018\)](#), *i.e.* taking into account the radiation pressure from the star, then in the term $\nabla \Phi_s$ will be inserted a factor of $(1 - \xi)$, where ξ is the ratio between the stellar radiation pressure and the gravitational force. Literature assumes a similar Lyman- α flux of the star HD209458 to the Sun, which corresponds to $\xi \sim 0.7$ ([Bzowski et al., 2008](#)).

Thus, the momentum equation translates to:

$$(\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \nabla p + \frac{GM_p}{r^2} \hat{\mathbf{e}}_r + (1 - \xi) \frac{GM_*}{r_*^2} \hat{\mathbf{e}}_{r_*} - 2\mathbf{\Omega} \times \mathbf{v} - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_*) = 0 \quad (4.58)$$

where $\mathbf{r}_* = (x + a, y, z)$ represents the position with respect to the star. Here, it is already being taken into account the planet's rotation, since it is tidally locked. Lastly, let us understand how can $\vec{\mathbf{r}}_*$ be translated in terms of $\vec{\mathbf{r}}$ in order to have a sense of how symmetry is lost. As can be seen in [Appendix A](#):

$$\hat{\mathbf{x}} = \sin \theta \cos \phi \hat{\mathbf{e}}_r + \cos \theta \cos \phi \hat{\mathbf{e}}_\theta - \sin \phi \hat{\mathbf{e}}_\phi \quad (4.59)$$

and since $\vec{\mathbf{r}} = \vec{\mathbf{r}}_* - a\vec{\mathbf{x}}$, we have:

$$\vec{\mathbf{r}}_* = \vec{\mathbf{r}} + a \left(\sin \theta \cos \phi \hat{\mathbf{e}}_r + \cos \theta \cos \phi \hat{\mathbf{e}}_\theta - \sin \phi \hat{\mathbf{e}}_\phi \right) \quad (4.60)$$

Consequently, radial symmetry breaking becomes an important feature as long as stellar effects are taken into account.

4.4.1 Simulation Framework

Having a global grid rotating, with a star - which in this model is not yet emanating any stellar wind - and a planet orbiting is quite computationally expensive. Thus, as a first order approximation, the local model can have the fictitious forces inserted explicitly (and have $\xi = 0$ for simplicity). Maintaining the rotating frame preserves the planet's rotational effects, though the orbiting velocity and distance to the star must now be calculated. Let us assume:

$$\mathbf{r}' = \mathbf{r}_p + a\hat{\mathbf{x}} \quad (4.61)$$

where $\mathbf{r}_p$ is the distance in the planet's frame and

$$\mathbf{v}' = \mathbf{v}_p - a\Omega\hat{\mathbf{y}} \quad (4.62)$$

where $\mathbf{v}_p$ is the velocity in the planet's frame and $-a\Omega\hat{\mathbf{y}}$ is the circular orbit of the planet in the rotating frame. These yield:

$$-2\Omega \times \mathbf{v}' - \Omega \times (\Omega \times \mathbf{r}') = -2\Omega \times (\mathbf{v}_p - a\Omega\hat{\mathbf{y}}) - \Omega \times (\Omega \times (\mathbf{r} + a\hat{\mathbf{x}})) \quad (4.63)$$

which become:

$$\implies -2\Omega \times \mathbf{v}_p - \Omega \times (\Omega \times \mathbf{r}_p) + 2a\Omega\Omega \times \hat{\mathbf{y}} - a\Omega \times (\Omega \times \hat{\mathbf{x}}) \quad (4.64)$$

i.e., we recover the local fictitious forces from the previous model with two extra terms.

Since these forces are due to the frame's rotation, the correct vector to apply is $-\Omega\hat{\mathbf{z}}$, thus, the total force becomes:

$$\frac{1}{\rho}\mathbf{f} = -\nabla\Phi_p - \nabla\Phi_s - 2\Omega \times \mathbf{v}_p - \Omega \times (\Omega \times \mathbf{r}_p) + a\Omega^2\hat{\mathbf{x}} \quad (4.65)$$

Therefore, in these local orbiting models, besides having a rotating frame to have **PLUTO** derive the local Coriolis and centrifugal forces, it is inserted the gravitational effect of the star and an extra force given by the last term of equation (4.65). These two terms are complementary, since we are assuming a tidally locked planet with a circular orbit. The orbital period has been calculated to be $\Omega = \sqrt{\frac{G(M_* + M_p)}{a^3}} \sim \sqrt{\frac{GM_*}{a^3}}$, thus:

$$-\nabla\Phi_s + a\Omega^2\hat{\mathbf{x}} = -\frac{GM_*}{r_*^2}\hat{\mathbf{e}}_{r_*} + \frac{G(M_* + M_p)}{a^2}\hat{\mathbf{x}} \quad (4.66)$$

Only at a negligible distance to the center of the planet at the x -axis, this two components cancel each other.

Now that there are stellar effects presents, it follows the table with stellar parameters - Table 4.2.

Stellar Parameter	Symbol	Value	Units
Mass	M_*	1.148	$M_\odot$
Radius	R_*	1.203	$R_\odot$
Density	ρ_p	5×10^{-13}	g cm^{-3}
Temperature	T_p	10^6	K
Mean Molecular Weight	μ	$\frac{1}{2}$	-

TABLE 4.2: Table with used parameters in the models for the HJ's host star.

4.4.2 Results

Here will be presented the results of the numerical models without and with an explicit temperature gradient. As explained in the previous subsection, the simulations are done in a local grid, centered on the planet, with the acting forces inserted explicitly in **PLUTO**. Figure 4.17 shows plots of the density profile for xy and xz plane in the isothermal case - *IsoCor* Model - and with the temperature gradient - *AnisoCor* Model.

Figure 4.18 shows plots with the temperature profile for xy and xz plane, which with the help of the overplotted contours, it is possible to see the bending of the isosurfaces.

In Figures 4.17 and 4.18 is plotted the approximate Hill radius - yellow line - which determines the sphere inside of which gravitational influence is mainly due to the planet. This is an important reference, since we are studying models in which the sonic radius is smaller than the Hill radius $r_s < r_H$. In other words, the sonic surface is mainly influenced by the planet's gravitational force.

4.4.3 Important Remarks

Perhaps the best starting point is to understand the forces present that influence the flow. Due to the presence of pressure gradients and the planet's gravitational force, an acceleration that describes the Parker wind's behaviour of the flow - now denominated as Parker acceleration a_p - goes as:

$$a_p(r) = \frac{1}{2} \frac{dv^2}{dr} = \frac{1}{2} \frac{c_s^2}{r_s} \frac{dw^2}{d\Xi} \quad (4.67)$$

where $w = v/c_s$ and $\Xi = r/r_s$. From this equation, we can extract a characteristic acceleration: $\alpha_s = \frac{c_s^2}{r_s}$, which expresses the typical acceleration a particle is bound to in an

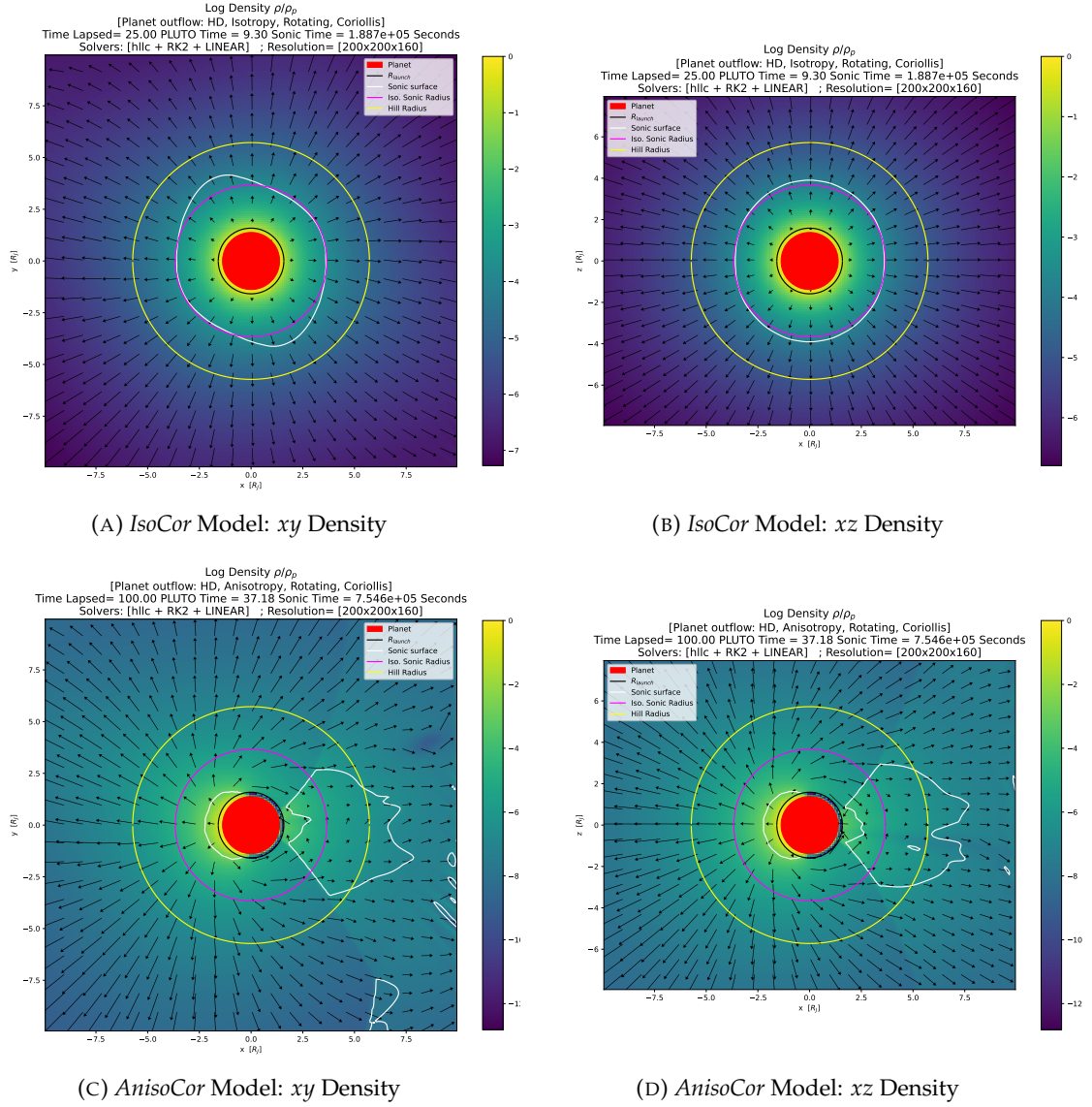


FIGURE 4.17: Density plots for models *IsoCor* and *AnisoCor* in xy and xz planes, in the planet's rotating frame.

isothermal Parker Wind. Thus, normalization brings:

$$\alpha_P(\Xi) = \frac{1}{2} \frac{dw^2}{d\Xi} \quad (4.68)$$

Additionally, the Coriolis fictitious force is acting under the Coriolis acceleration a_C :

$$a_C(r) = 2\Omega v = 2 \frac{c_s^2}{r_s} \Omega \tau_s w \quad (4.69)$$

where the characteristic Parker time $\tau_s \equiv r_s/c_s$ appears, which can be understood as the typical time for a ballistic particle launched from the planet to reach the sonic radius r_s .

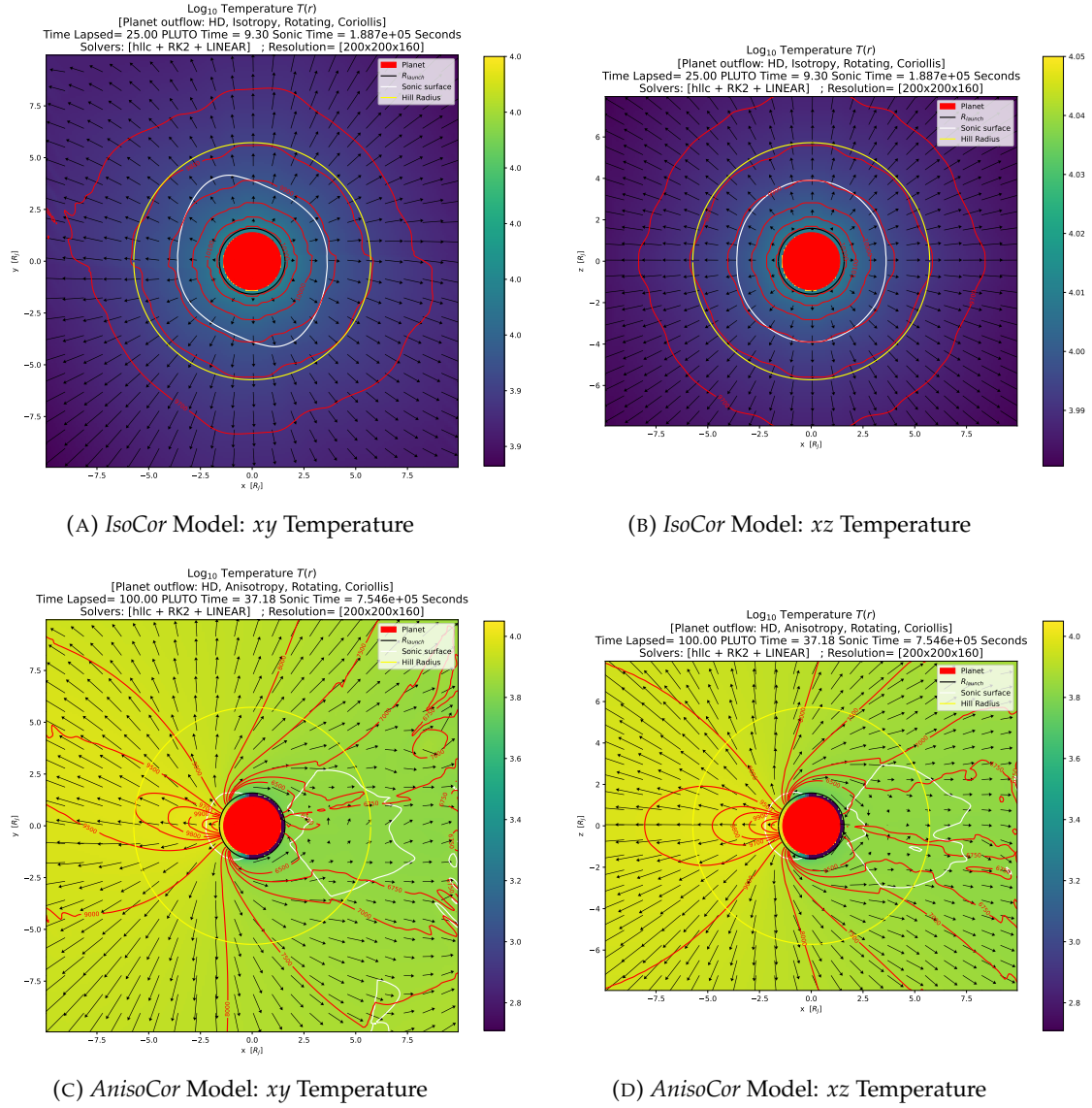


FIGURE 4.18: Temperature plots for models *IsoCor* and *AnisoCor* in xy and xz planes, in the planet's rotating frame.

When normalized to the characteristic Parker acceleration, it yields:

$$\alpha_C(\Xi) = 2\Omega\tau_s w \quad (4.70)$$

Finally, the effects of centrifugal and stellar gravitational influence, which, as seen before, relate with each other at first order, give rise to a tidal acceleration a_T :

$$\mathbf{a}_T(\mathbf{r}) = -\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{a}) - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) - \frac{GM_*}{|\mathbf{r}_*|^3} \mathbf{r}_* \quad (4.71)$$

where the first term accounts for the rigid orbit of the planet. Substituting $\mathbf{r}_* = \mathbf{r} + \mathbf{a}$, it follows:

$$\mathbf{a}_T(r) = \Omega^2(\mathbf{a} + \mathbf{r}) - \frac{GM_*}{(a^2 + 2\mathbf{a} \cdot \mathbf{r} + r^2)^{\frac{3}{2}}}(\mathbf{r} + \mathbf{a}) \quad (4.72)$$

Expanding around $r \ll a$, it yields:

$$\mathbf{a}_T(r) \sim \Omega^2(\mathbf{a} + \mathbf{r}) - (GM_*) \left(\frac{1}{a^3} - \frac{3\mathbf{a} \cdot \mathbf{r}}{a^5} + \mathcal{O}(r^2) \right) (\mathbf{r} + \mathbf{a}) \quad (4.73)$$

and we can now also use $\Omega^2 = G(M_* + M_p)/a^3$:

$$\begin{aligned} \Rightarrow \mathbf{a}_T(r) \sim & \left[-\frac{GM_*}{a^3} + \frac{G(M_* + M_p)}{a^3} \right] \mathbf{r} + \\ & + \left[\left(\frac{GM_*}{a^3} \right) \left(-1 + \frac{3}{a^2}(\mathbf{a} \cdot \mathbf{r}) \right) + \frac{G(M_* + M_p)}{a^3} \right] \mathbf{a} + \mathcal{O}(r^2) \end{aligned} \quad (4.74)$$

$$\mathbf{a}_T(r) \sim \frac{GM_p}{a^3} \mathbf{r} + \left(\frac{GM_*}{a^3} \right) [3\hat{\mathbf{a}} \cdot \mathbf{r}] \hat{\mathbf{a}} + \frac{GM_p}{a^3} \mathbf{a} \quad (4.75)$$

which can be transformed into:

$$\mathbf{a}_T(r) = \left(\frac{G(M_* + M_p)}{a^3} \right) [3\hat{\mathbf{a}} \cdot \mathbf{r}] \hat{\mathbf{a}} + \frac{GM_p}{a^3} [\mathbf{r} - 3(\hat{\mathbf{a}} \cdot \mathbf{r}) \hat{\mathbf{a}} + \mathbf{a}] \quad (4.76)$$

Thus, at first order:

$$a_T(r) \sim 3\Omega^2 r \quad (4.77)$$

which can be normalized to:

$$\alpha_T(\Xi) = 3\Omega^2 \tau_s^2 \Xi \quad (4.78)$$

It is worth noting that the product $\Omega\tau_s$ is non-dimensional, which may alternatively be understood as the ratio between the sonic time and the orbital time $1/\Omega$. These combined accelerations distinguish different profiles in the flow's dynamics: as $\Omega\tau_s \rightarrow 0$, the Parker isothermal case prevails, since a free particle reaches sonic speed considerably faster than the planet executes one orbit. This ratio between the time a particle takes to reach sonic speed in comparison to the orbital time defines the bending of the streamlines. Thus, as $\Omega\tau_s \gtrsim 1$, tidal and Coriolis forces deform the streamlines to a point where the Parker behaviour is confined to narrow bands. Bending by tidal or Coriolis forces can be understood by $\Xi_p = \frac{R_p}{r_s} = \frac{2}{\lambda_p}$: if $\Xi_p \lesssim 1 \iff \alpha_C \ll \alpha_T$, then the bending follows predominantly a spinning model. If $\Xi \ll 1 \iff \alpha_C \gg \alpha_T$, implying that the sonic radius is distant, the bending happens by the Coriolis force influences in the streamline.

In this model, the parameters used translate into $r_s = 26.4731 \times 10^9 \text{cm} = 3.703L_0$, $c_s = 12.8487 \text{kms}^{-1} = 0.2158v_0$, $\Omega = 2\pi/(3.5\text{days}) = 2.0778 \times 10^{-5} \text{rads}^{-1} = 0.0250t_0$ meaning $\Omega\tau_s = 0.4281$, with $\lambda_p = 5.3280$ - the reader is advised to check Figure 6 of [Carroll-Nellenback et al. \(2017\)](#). Thus, as the models show, the results should bear a quite strong resemblance to the spinning model. The previously defined critical value of Ω shows that:

$$\Omega_c \tau_s = \sqrt{\frac{1}{8} \frac{GM_p r_s}{R_p^2 c_s}} = \sqrt{\frac{1}{8} \frac{GM_p \lambda}{2R_p c_s^2}} = \frac{\lambda^2}{\sqrt{32}} \quad (4.79)$$

Since the planet is tidally locked, we can find critical values of a desired variable by having:

$$\Omega_c^2 = \frac{G(M_* + M_p)}{a^3} \bigg|_c = \frac{1}{8} \left(\frac{GM_p}{R_p^2 c_s} \right)^2 \bigg|_c = \frac{1}{8} \frac{\mu m_p}{k_b T_p} \left(\frac{GM_p}{R_p^2} \right)^2 \bigg|_c \quad (4.80)$$

thus:

$$\frac{T_p (M_* + M_p) R_p^4}{\mu M_p^2 a^3} \bigg|_c = \frac{Gm_p}{8k_b} \quad (4.81)$$

this hyper-dimensional surface limits the possible parameter phase-space for a tidally-locked HJ emanating a wind. It is possible to derive a critical distance to the host star, having all other variables fixed:

$$\frac{1}{a_c^3} = \frac{1}{8} \frac{G\mu m_p}{(M_* + M_p) k_b T_p} \left(\frac{M_p}{R_p^2} \right)^2 \implies a_c = \left(\frac{1}{8} \frac{GM_p}{R_p^4 c_s^2} \frac{M_p}{(M_* + M_p)} \right)^{-\frac{1}{3}} \sim 19.15R_J = 1.6R_* \quad (4.82)$$

which is ~ 5 times closer to the star than the present observable distance.

4.5 Planetary Wind from an Orbiting and Rotating Planet embedded in a Stellar Wind

Two interacting winds can demonstrate a wide range of phenomena. The appearance of a bow shock around a planet is formed when there exists a supersonic relative motion between the planet and the stellar wind. In this framework, it is possible to derive three main cases of shock configuration - as derived in the work of [Vidotto et al. \(2010\)](#):

- "Dayside-shock", which occurs when the stellar flow encounters the planet radially. This condition is met when the local radial stellar wind $v_{*,r}$ is greater than the planetary sound speed: $v_{*,r} > c_s$.

- "Ahead-shock", which occurs when the stellar flow encounters the planet from the relative azimuthal direction. This condition is met when the keplarian velocity of the planet $v_{p,K}$, relative to the azimuthal velocity of the stellar wind, is greater than the planetary sound speed: $\Delta v = |v_{p,K} - v_{*,\phi}| > c_s$.
- These two limits comprise an intermediate case, where the contribution of both radial and relative azimuthal velocities of the wind contribute to the formation of a shock. As a consequence, a deflection angle θ_W can be derived:

$$\theta_W = \arctan \frac{v_{*,r}}{\Delta v} \quad (4.83)$$

Notice that from this derivation, it is possible to find an interesting space-phase constraint on the orbital period in order to achieve a bow shock.

$$\Delta v = |v_{p,K} - v_{*,\phi}| = |\Omega a - v_{*,\phi}| \quad (4.84)$$

$$\implies 0 \leq \Delta v \leq \left| \Omega a - \Omega \frac{R_*^2}{a} \right| \quad (4.85)$$

where the limit $\Delta v \rightarrow 0$ translates the solid rotation of the wind $v_{*,\phi} \rightarrow \Omega a$, and the RHS translates the total angular momentum conserved case. Since the condition for shock formation follows that $(\Delta v^2 + v_{r,*}^2)^{\frac{1}{2}} > c_s$, then:

$$\Omega_c = \frac{\sqrt{v_{r,*}^2 - c_s^2}}{a - \frac{R_*^2}{a}} \quad (4.86)$$

If the mass of the objects becomes fixed, then Ω will only depend on a , the distance of the planet to the host star, which can itself translate to a new constraint.

[McCann et al. \(2019\)](#) studied the morphology of HD SPIs, and distinguished three interaction scenarios: *strong*, *intermediate* and *weak* cases of confinement of the planetary wind by the stellar wind. The bow-shock case is predominantly encountered in the *strong* case, where the compression done by the stellar wind forces the shock to be below the sonic surface. Naturally, there exists a strong suppression of the outflow rate - which is seen in [Carolan et al. \(2021\)](#) and [Christie et al. \(2016\)](#). The *intermediate* and *weak* cases can give rise to a region of "unshocked" planetary wind.

Because the ram pressure balance dictates the location and morphology of the wind-wind interaction ([Matsakos et al., 2015](#)), a useful order of magnitude comparison can be

to compare the ram pressures at their sonic points:

$$\frac{\rho_p c_{s,p}^2}{\rho_* c_{s,*}^2} = \left(\frac{\rho_p}{\rho_*} \right) \left(\frac{M_p}{M_*} \right) \left(\frac{R_*}{R_p} \right) \left(\frac{\lambda_*}{\lambda_p} \right) \exp(\lambda_* - \lambda_p) \quad (4.87)$$

where it has been used the definition of the sound speed and equation (B.7) for the density. Substituting all the parameters by the ones considered for the system HD209458, it follows that the ratio in equation (4.87) is ~ 0.03 . However, as it is debated by MacLeod and Oklopčić (2022), for a ratio $\lesssim 1$, it is encountered some limitations, since there might be cases where the planetary wind might not even reach sound speed - as in the *strong* case. For such cases, comparing the explicit ram pressures should be a good enough approximation. Taking the equation (B.6), two limits may appear:

- A gravitationally bound subsonic wind with $v \ll c_s < v_{\text{esc}}$, close the the object's radius R behaves as:

$$v \sim c_s \left(\frac{r_s}{R} \right)^2 \exp \left(-\lambda + \frac{3}{2} \right) \quad (4.88)$$

- For large distances $r \gg R$, where a sonic wind exists, a close description yields:

$$v \sim 2c_s \sqrt{\log \left(\frac{r}{R} \right)} \quad (4.89)$$

Thus, for a case where the two limits are present as the planetary wind is subsonic and the stellar wind has passed the sonic speed by far, which should closely resemble the *strong* case, it yields that:

$$\frac{\rho_p v_p^2}{\rho_* v_*^2} \sim 2 \exp \frac{3}{2} \lambda_p^2 \left(\frac{\rho_p}{\rho_*} \right) \left(\frac{c_{s,p}}{c_{s,*}} \right) \frac{\exp \left[\lambda_p \left(\frac{R_p}{r_p} - 2 \right) \right]}{\exp \left[\lambda_* \left(\frac{R_*}{r_*} - 1 \right) \right]} \frac{\exp \left[-\frac{1}{2} \left(\left(\frac{r_s}{R_p} \right)^2 \exp \left(-\lambda_p + \frac{3}{2} \right) \right)^2 \right]}{\left(\frac{r_*}{R_*} \right)^2 \sqrt{\log \left(\frac{r_*}{R_*} \right)}} \quad (4.90)$$

and by taking $r_* \sim a$ and $r_p \sim R_p$, it follows:

$$\frac{\rho_p v_p^2}{\rho_* v_*^2} \sim 2 \exp \frac{3}{2} \lambda_p^2 \left(\frac{\rho_p}{\rho_*} \right) \left(\frac{c_{s,p}}{c_{s,*}} \right) \exp(\lambda_* - \lambda_p) \frac{\exp \left[-8 \lambda_p^4 \exp(-2\lambda_p + 3) \right]}{\left(\frac{a}{R_*} \right)^2 \sqrt{\log \left(\frac{a}{R_*} \right) \exp \left(\lambda_* \left(\frac{R_*}{a} \right) \right)}} \quad (4.91)$$

becoming extremely small, meaning that the bow shock should be found very close to the planet. Again, substituting the parameters, it follows that equation (4.91) yields a ratio of ~ 0.0093 .

The work of [Matsakos et al. \(2015\)](#) describes the morphology of wind interaction in an MHD framework by using relative magnitudes of characteristic radii. However, in this HD case, the same applies with the exception that the magnetic pressure is not taken into consideration. As discussed in [Matsakos et al. \(2015\)](#), a quasi-stationary morphological structure in the interaction between a star and a planet entails the establishment of an interface where pressure equilibrium is reached. This interface is the shock structure between the winds, and can be described by radial considerations when equating specific planetary pressures to the ambient pressure defined by the stellar wind. This ambient pressure is a sum of three main contributions:

$$P_{\text{amb}} = P_{\text{th}} + P_{\text{ram}} + P_{\text{orb. ram}} \quad (4.92)$$

where:

$$P_{\text{th}} = \frac{\rho_w k_b T_w}{\mu m_p} \quad (4.93)$$

where the subscript w is referent to the wind, locally.

$$P_{\text{ram}} = \rho_w v_w^2 \quad (4.94)$$

and

$$P_{\text{orb. ram}} = \rho_w v_{\text{orb}}^2 \quad (4.95)$$

4.5.1 Simulation Framework

Due to computational demands, the applied strategy to simulate these models takes only a larger local grid. A global grid, where the star is inserted alongside with the planet becomes extremely memory heavy, and the available cores do not allow for more than a distribution of 4 cores for each direction at the same time. Thus, the chosen grid follows:

- x -direction: stretched grid from $x = -20$ to $x = -6$ with a resolution of 72 cells, and from $x = +6$ to $x = 40$ with a resolution of 110 cells; and a regular grid between $-6 \leq x \leq +6$ with a resolution of 120 cells.
- y -direction: stretched grid from $x = -40$ to $x = -6$ with a resolution of 110 cells, and from $x = +6$ to $x = 20$ with a resolution of 72 cells; and a regular grid between $-6 \leq x \leq +6$ with a resolution of 120 cells.

- z-direction: stretched grid from $x = -10$ to $x = -6$ and from $x = +6$ to $x = 10$ with equal resolutions of 30 cells; and a regular grid between $-6 \leq x \leq +6$ with a resolution of 120 cells.

The local grid had the stellar wind inserted at the beginning of the x boundary and at the end of the y boundary. As a result, the previously defined boundary conditions of "outflow" have now been altered to "userdef". This boundary choice is related to the fact that the stellar wind is being injected with an azimuthal component determined by the angular momentum and rotation from the star. Furthermore, because the reference frame is centered on the spinning planet, the relative orbital velocity is incorporated into the stellar wind that is entering along the boundaries. Thus, this model comprises the previous scenario of an orbiting planet - with fictitious forces explicitly applied - with the inclusion of a stellar-like wind.

Due to previous tests, under the system's initial parameters as present in [Carroll-Nellenback et al. \(2017\)](#), the inserted stellar wind would comprise almost entirely the planet's wind. Thus, a small fine-tuning of stellar parameters was done in order to observe two distinct cases: *strong* and *intermediate* cases. The table 4.3 shows the distinct values used.

Stellar Parameter	Case A	Case B	Units
Density	1.0	0.2	ρ_p
Temperature	0.8	0.75	T_p
λ_*	13.78	14.7	-
$\rho_p c_{s,p} / \rho_* c_{s,*}$	0.586	7.837	-

TABLE 4.3: Table with used parameters in the different models.

Both scenarios with isothermal wind launch will be examined and addressed first. The models with temperature anisotropy follow after these. Temperature anisotropic cases are not run from the beginning; rather, it is used an useful mechanism of PLUTO, which is an "upload" of a grid with a double precision *dbl* file with the parameters. Following that, the code runs by starting at the end of the previous isotropic model. The temperature gradient in the planet follows the same specifications as in previous models, with the exception that it now occurs 1.0s after commencing, but having the same relaxation (decay time of 15s with $\alpha = 0.35$).

4.5.2 Results for the Isothermal Models

4.5.2.1 Case *Iso-A*

As present in table 4.3, Case A should capture a well defined bow shock. Figure 4.19 plots the logarithm (in base 10) of the normalized density for both xy and xz planes.

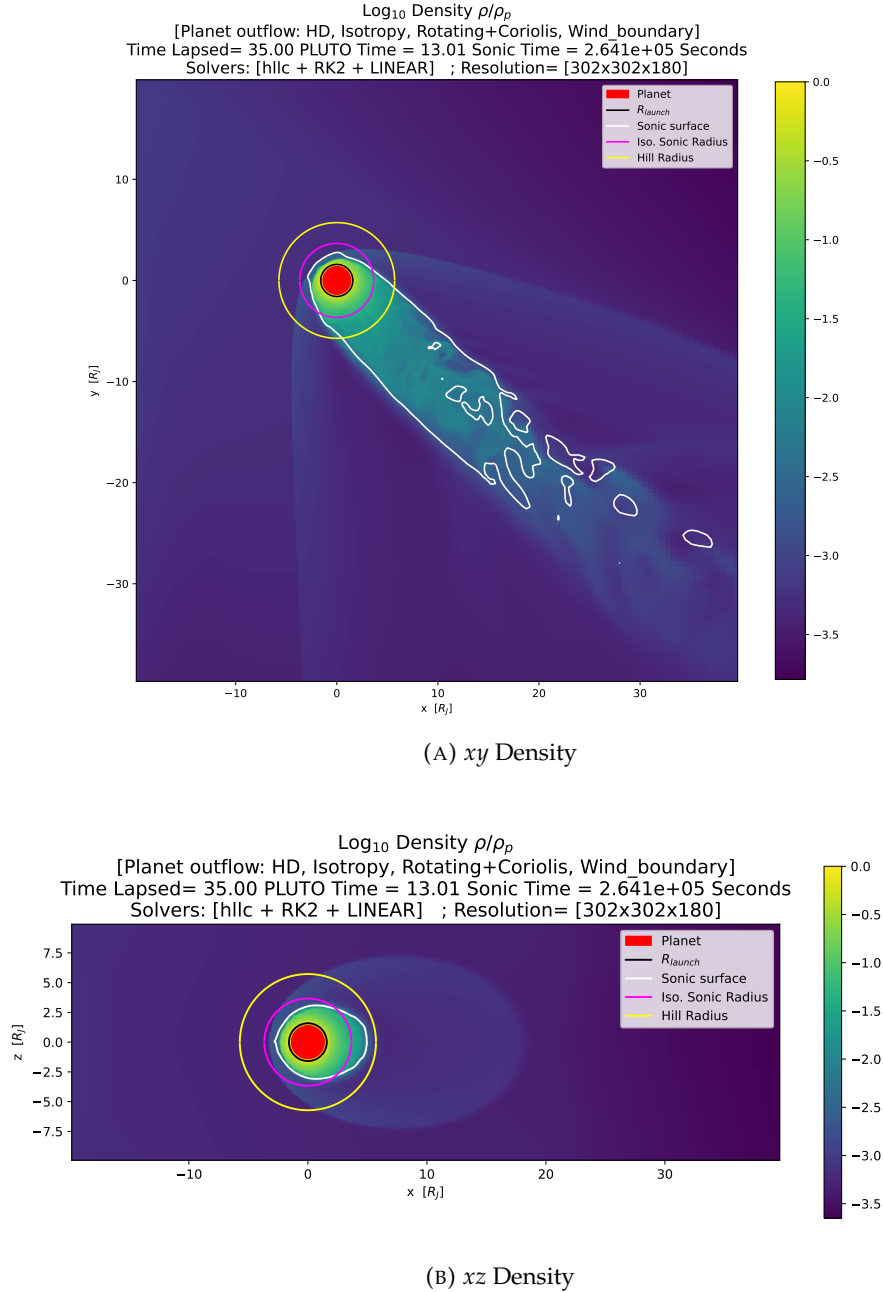


FIGURE 4.19: Density plots for model *Iso-A* in xy and xz planes, in the planet's rotating frame.

It is possible to observe the bow-shock in either plane, and the clear shrunk sonic surface, very close to the planet. Additionally, there exist clear instabilities over the tail of the planet.

In the Appendix B, section B.8, Figure B.9 shows two pair of plots of this same case, with ram pressure and temperature.

4.5.2.2 Case Iso-B

As previously discussed, Case B should capture an *intermediate* type case of wind-wind interaction. Figure 4.20 plots the logarithm of the normalized density for both xy and xz planes.

The planetary wind is now comparable to the stellar wind, such that there is an out-flow up to the Hill radius. As a result, the bow-shock is disturbed and the "burping phenomenon" described in McCann et al. (2019) for the *intermediate* case is captured. Similarly, the sonic surface close to the planet is lost, leaving an irregular sonic surface located outside the Hill radius, where heating and velocity increase. The planetary wind now becomes what is known in the literature as a Hot Jupiter breeze.

In the Appendix B, section B.9, Figure B.10 shows two pair of plots of this same case, with ram pressure and temperature.

4.5.2.3 Important Remarks for Iso Cases

As demonstrated in the theoretical introduction of this section, it can be determined the angle of deflection of the tail in case *Iso-A*. The inserted stellar wind has a velocity relative to the planet, thus:

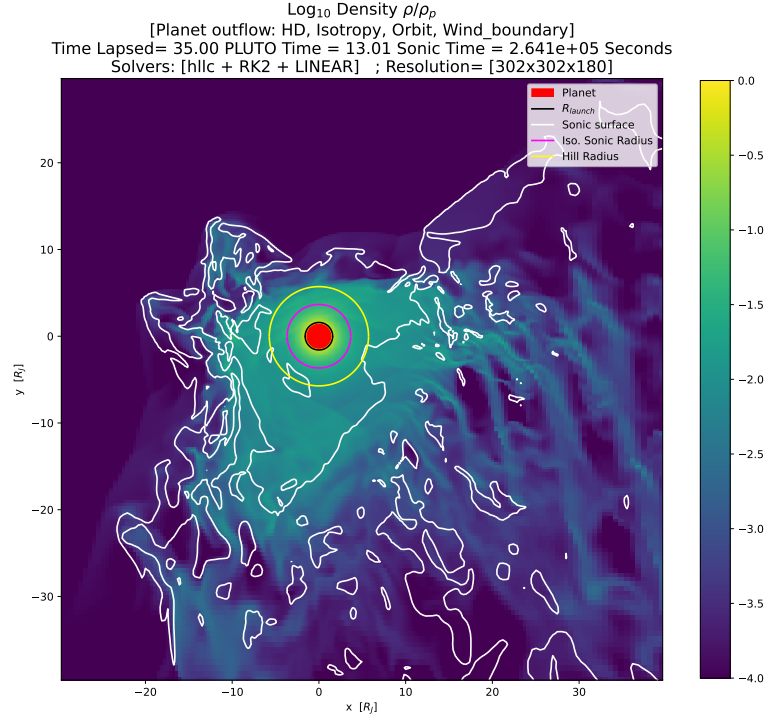
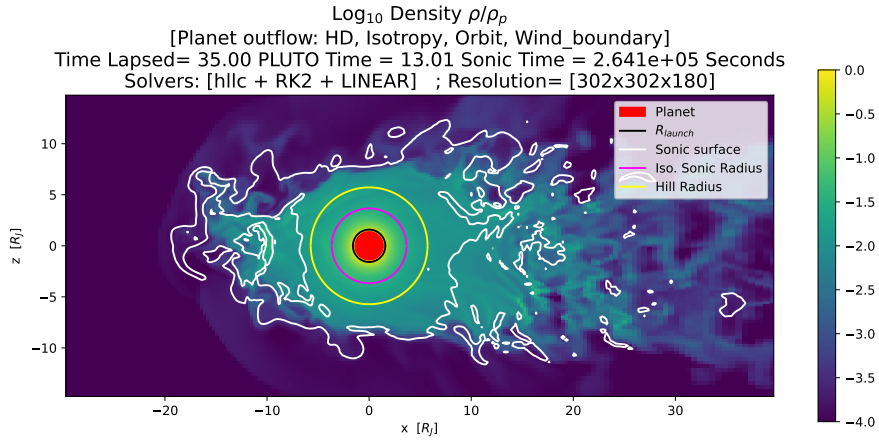
$$\Delta v \sim \Omega a - \Omega \frac{R_*^2}{a} \quad (4.96)$$

$$\implies \theta_W \sim \arctan \frac{14.85 v_0}{\Omega a \left(1 - \frac{R_*^2}{a^2}\right)} \sim 43.85^\circ \quad (4.97)$$

where the radial velocity was computed explicitly at $r = a$ using the Newton-Raphson method. By visual inspection, the estimated deflection angle is quite close to what was obtained.

In order to understand the scale of the ambient pressure, let us:

$$P_{\text{amb}} = P_{\text{th}} + P_{\text{ram}} + P_{\text{orb. ram}} = \rho_w \left(\frac{1}{\Gamma} c_{s,*}^2 \frac{T_w}{T_*} + v_w^2 + v_{\text{orb}}^2 \right) \quad (4.98)$$

(A) xy Density(B) xz DensityFIGURE 4.20: Density plots for model *Iso-B* in xy and xz planes, in the planet's rotating frame.

Because the wind is inserted within the box's boundary, the wind's temperature T_w has not changed significantly because there has been no transfer of energy over a vast distance. Thus T_w will correspond to the temperature of the star $T_w/T_* \sim 1.0$:

$$P_{\text{amb}} \sim \rho_* \exp \left(\lambda_* \left(\frac{R_*}{a} - 1 \right) - \frac{1}{2} \left(\frac{v_w(r=a)}{c_{s,*}} \right)^2 \right) \left(\frac{1}{\Gamma} c_{s,*}^2 + v_w^2(r=a) + \frac{GM_*}{a} \right) \quad (4.99)$$

where the explicit wind density profile was inserted. Thus, it can be re-written as:

$$\implies P_{\text{amb}} \sim \rho_* c_{s,*}^2 \left(\frac{1}{\Gamma} + W^2 + \lambda_* \frac{R_*}{a} \right) \exp \left(\lambda_* \left(\frac{R_*}{a} - 1 \right) - \frac{1}{2} W^2 \right) \quad (4.100)$$

where $W = v_w(r=a)/c_{s,*}$ is the normalized stellar wind to the stellar sound speed at the planet's position. In an axiomatic manner, the change of density and temperature (sound speed) at the base of the wind is enough to change strongly the ambient pressure - and these are the commonly altered parameters (Matsakos et al., 2015; MacLeod and Oklopčić, 2022; McCann et al., 2019). Additionally, changing the planet's distance to the host star, a - as Matsakos et al. (2015) does - or the change of the parameter λ - as Christie et al. (2016) - are possible approaches to explore wind-wind interactions.

Following a similar description as Matsakos et al. (2015), the characteristic lengths of action that interest us in these models are:

- The Hill radius r_H , previously defined, and since the mass of the objects of the system is not altered, so is not this quantity.
- For a planetary outflow colliding with the stellar gas at a (super-)sonic speed, the following length can be defined: R_w , otherwise known by "standoff distance". Equating the ram pressure of the HJ's outflow with the ambient pressure, it appears if the definition of mass loss rate is used instead of the density:

$$\dot{M}_p = 4\pi r^2 \rho_{w,p} v_{w,p} \iff \rho_{w,p} v_{w,p}^2 = \frac{\dot{M}_p v_{w,p}}{4\pi r^2} \quad (4.101)$$

Thus yielding:

$$R_w = \sqrt{\frac{\dot{M}_p v_{w,p}}{4\pi P_{\text{amb}}}} \quad (4.102)$$

So, two cases can be identified and associated to the models studied:

- Case 1: $R_w < r_H$, which corresponds to having a gravitational influence greater than the outflow's strength, since it may have not reached a supercritical velocity.

The shock sweeps back the planetary outflow, leading to the formation of a thin tail, which remains confined to the vicinity of the planet's orbit.

- Case 2: $R_w \gtrsim r_H$ which involves a strong HJ's wind. In this case, as the shock happens, part of the outflow is swept back to a tail. However, since this shock happens beyond the Hill radius, the remaining part of the outflow becomes gravitationally bound to the star. As a consequence, a stream is formed that may spiral and accrete onto the star. This stream should exhibit a complex and fragmented morphology, which arises from Kelvin-Helmholtz and Rayleigh-Taylor instabilities (due to the tangential velocity shear and by radial acceleration exerted on the denser planetary material, respectively) (Matsakos et al., 2015).

So, Case 1 applies to *Iso-A* and Case 2 applies to *Iso-B*.

Quantitatively, this is yielded by the following: $\dot{M}_p \sim 25.424 \rho_0 v_0 R_J^2$ and $r_H \sim 5.72 R_J$. Without any change in parameters, and as a first order approximation, a calculation of the ambient pressure for the HD209458 system follows. Figure 4.21 shows the plot of the analytical isothermal ambient pressure derived in equation (4.100), using the same Newton-Raphson method to compute the Parker wind.

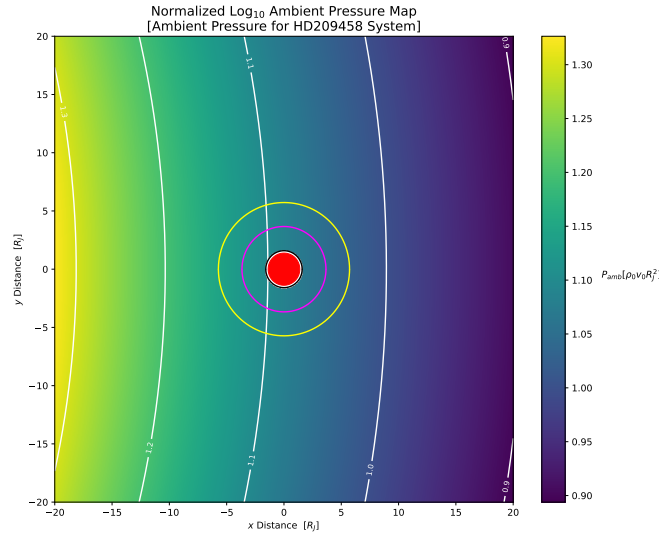


FIGURE 4.21: Ambient Pressure from the isothermal stellar Parker Wind centered on the planet's position. The y axis can be used either for the y or z direction.

As can be expected, Case 1 and Case 2 will have local ambient pressures increasingly smaller - in Appendix B, section B.10, Figure B.11 shows a pair of plots with the local P_{amb}

for each case. From these, it is possible to calculate the value $R_w(r)$ if following the exact Parker solutions - check Figure 4.22.

Thus, Case 1 clearly shows R_w that does not reach the value of r_H before it. However, Case 2 does have a $R_w < r_H$ for the exception of the most "dayside" zone, the one closest to the star. According to the simulations, there is a clear escape of material at the "nightside" of the planet, but there is also some thrusts of matter directed towards the star. This can be due to the fact that the rise of ram pressure in the planetary wind which collides at the rightmost zone of the equator transfers momentum to the "dayside" wind, increasing the value of R_w and eventually overcoming the Hill radius. It is important to mention that the computed values present in Figure 4.22 are for a clear isothermal and static case. The orbital influence, as well as variable mass loss $\dot{M}_p \rightarrow \dot{M}_p(r, \theta)$, should also be incorporated in the Parker solution of both winds.

4.5.3 Results for the Anisotropic Models

4.5.3.1 Case *Aniso-A*

Figure 4.23 plots the logarithm (now it is in base 10) of the normalized density for both xy and xz planes.

From what can be understood now, the bow shock has shrunk in the nightside, which is a direct consequence of the temperature gradient.

In the Appendix B, section B.11, Figure B.12 shows two pair of plots of this same case, with ram pressure and temperature.

4.5.3.2 Case *Aniso-B*

Figure 4.24 plots the logarithm in base 10 of the normalized density for both xy and xz planes for model B with temperature gradient.

In the Appendix B, section B.12, Figure B.13 shows two pair of plots of this same case, with ram pressure and temperature.

For any of the previous 4 models, it is recommended the reader to have an inspection at the animations made *.

* Link for paste with animations regarding the Wind-Wind Interaction models: [HD.SPI](#)

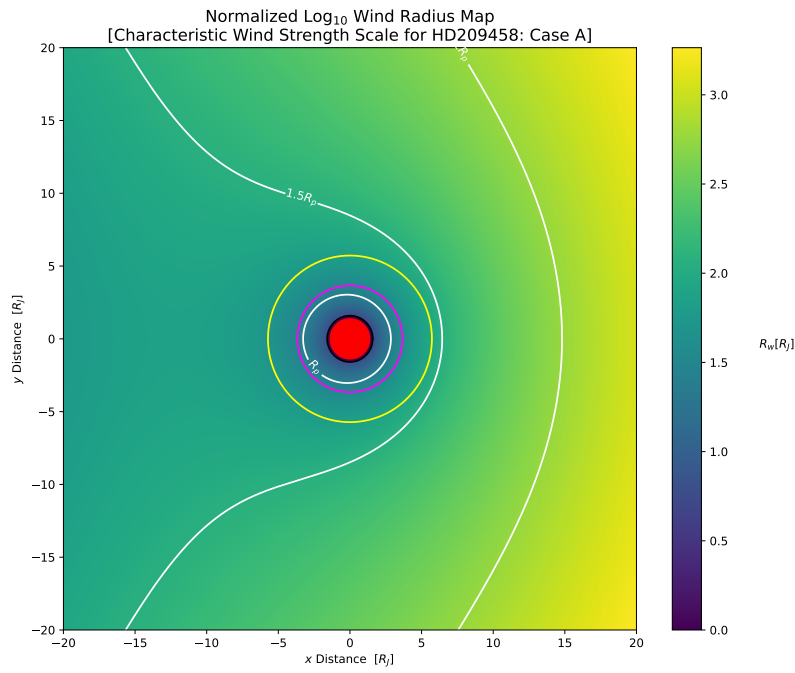
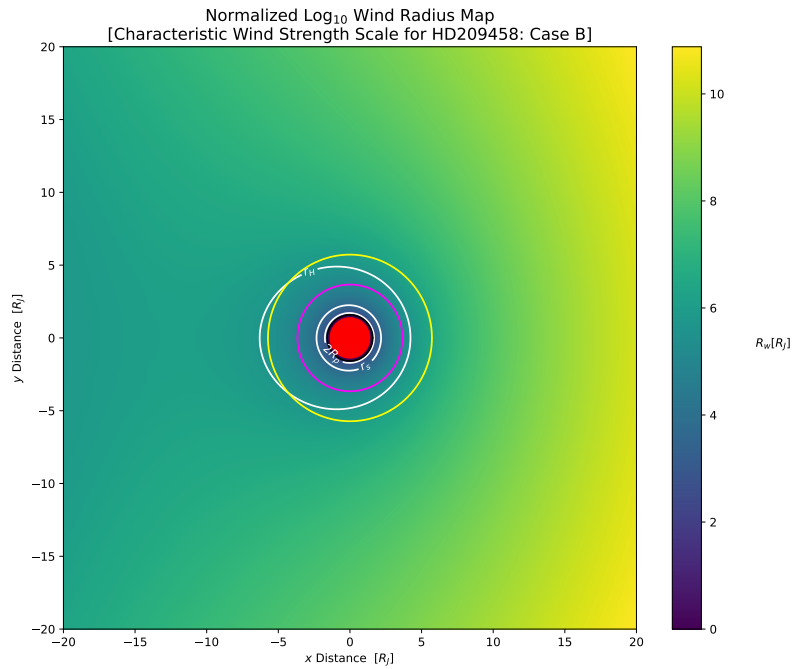
(A) *Iso-A* Model(B) *Iso-B* Model

FIGURE 4.22: Characteristic wind strength length R_w , following equation (4.102), using explicit Parker solutions.

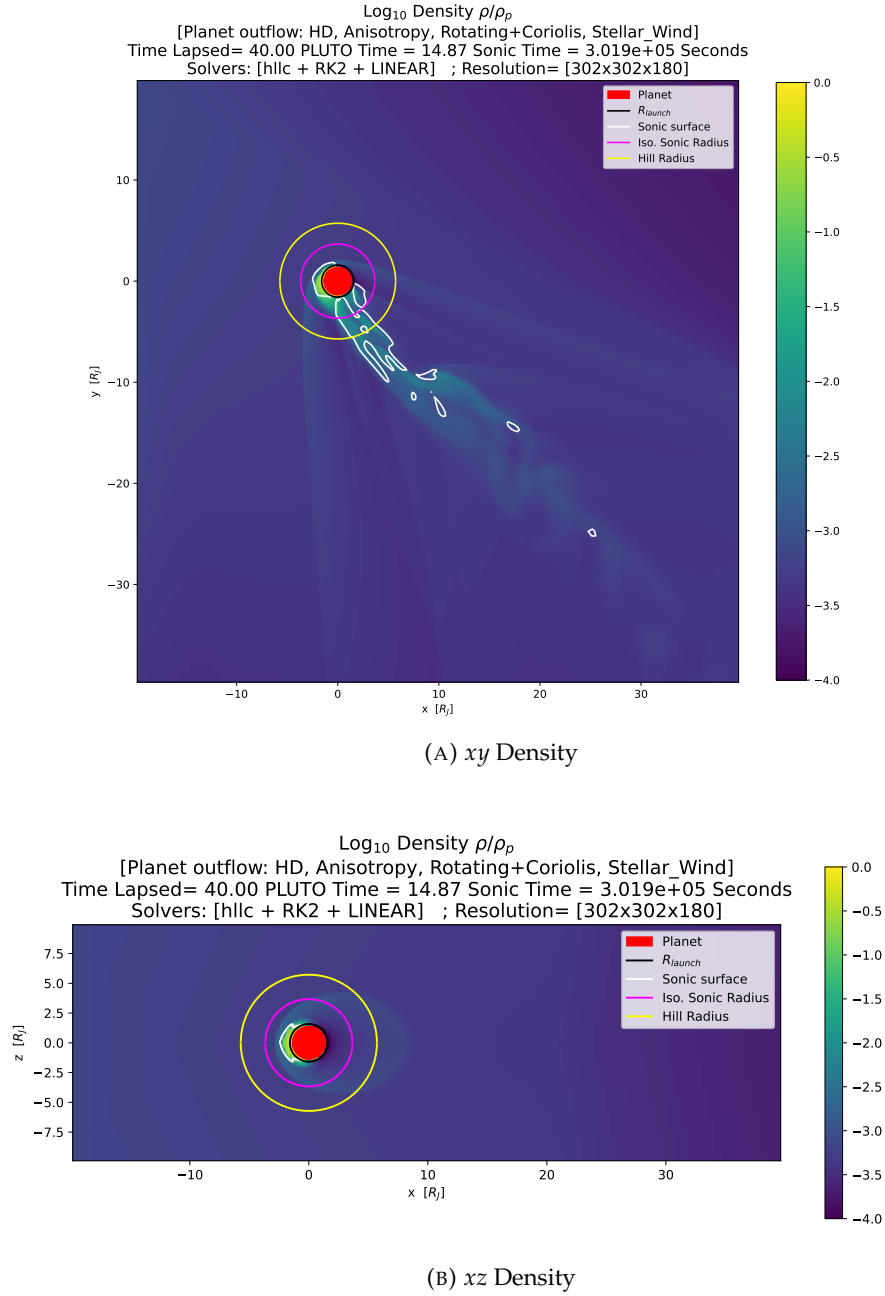
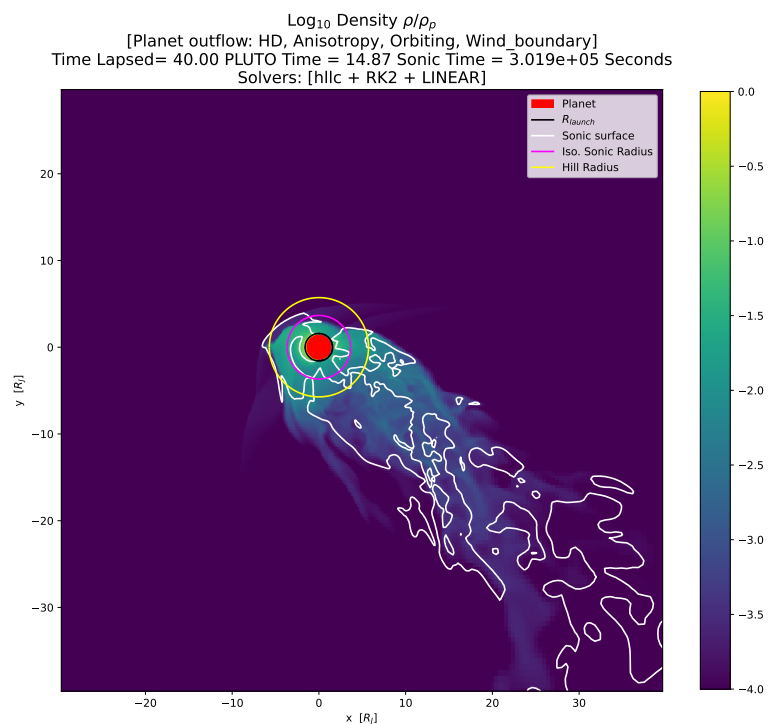
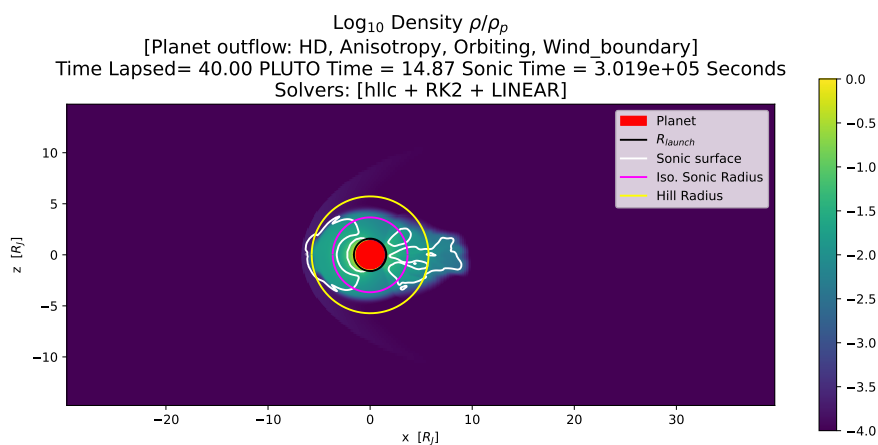


FIGURE 4.23: Density plots for model *Aniso-A* in xy and xz planes, in the planet's rotating frame.

(A) xy Density(B) xz DensityFIGURE 4.24: Density plots for model *Aniso-B* in xy and xz planes, in the planet's rotating frame.

4.5.3.3 Important Remarks for *Aniso* Cases

The major difference present in these cases is the lack of symmetry of the HJ's wind launch. On the one hand, this allows for the "nightside" stellar wind to get closer to the planet. This has the consequence of increasing the ram pressure around the rightmost equator zone the *strong* case. As a result, the low ram pressure (relative to the stellar wind's ram pressure) at the zone of wind-wind impact of the planet is now forced to be only at the "dayside" - this is observed by comparing the Ram Pressure plots of Figures B.9 and B.12. Thus, relatively strong lateral flows at the "nightside" of the planet will lead to an increase of ram pressure impacting the tail, changing its angle of deflection.

On the other hand, as the stellar wind hardly reaches the isothermal sonic radius - such as in the *intermediate* case - the temperature gradient forces the breeze to compensate for the "nightside". The same principle of the *AnisoRot* case is retrieved as the "dayside" ram pressure rises due to non radial pressure gradients: supersonic lateral flows travel to the "nightside". Consequently, large energy transfers take place, decreasing the momentum of the front flow, which allows for the stellar wind to dominate up to a distance of around the Hill radius to the planet.

The sonic surface formed by the bow-shock gives crucial information: the stellar wind shocks with the planetary wind at relative supersonic speed. This can be understood by the xz plane of the Ram Pressure plots present in Figure B.13. Furthermore, by inspection of the same Figure's xy plot, it can be deduced that this shock causes lateral flows that are enhanced at the rightmost zone of the HJ's equator, shocking again with the "nightside" flow. This enhancement is understood by verifying the temperature xy plane plot in Figure B.13. As a result, the bow-shock that would enter by the "nightside" of the planet is maintained at a distance such that the inflow to the planet becomes stable - as can be seen in Figure 4.25.

This plot entangles the inflow of the leftmost zone of the HJ's equator, which should quantitatively resemble the *AnisoRot* case, and the rightmost zone, which has the bow-shock contribution.

Because the mass loss of the planet is θ dependent, it becomes analytically impossible to compute the characteristic radius of wind influence in these models. However, it is qualitatively clear that as $|\theta|$ grows, $R_w(\theta)$ becomes smaller. Thus, at small angles, R_w remains close to the values of *Iso* cases, which goes hand in hand with the animation analysis*. From time to time, a small clump of matter is launched beyond the Hill radius.

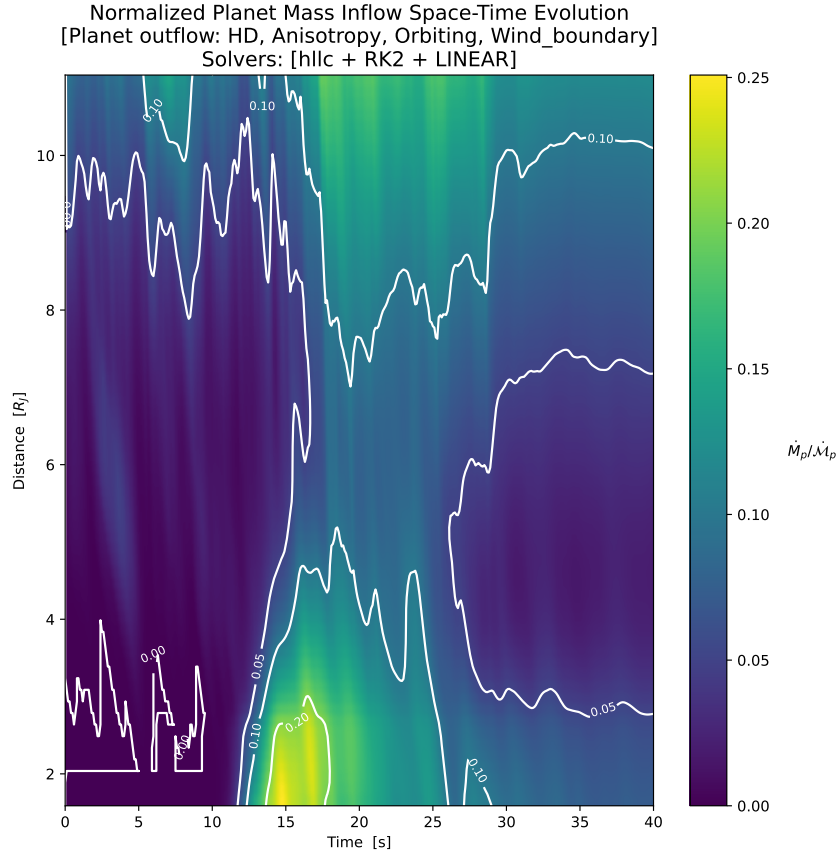


FIGURE 4.25: Inflow plot for model *Aniso-B* with relation to time and distance.

However, with the lack of lateral pressure, it's momentum is lost after the bow-shock and is pushed away by the stellar wind.

4.6 Discussion

This section is meant to provide an overview of the physical and numerical outcomes of this HD Chapter.

The study of the Parker Wind had its beginning with [Parker \(1958\)](#) and yet, work can still be done in the present days. Although the Parker Wind is primarily of stellar significance, its applications are not limited. HJs enduring photoevaporation can, at a good order of approximation, be recreated by having an outflowing atmosphere in form of a Parker Wind. Other works ([Owen and Adams, 2014](#); [Tripathi et al., 2015](#)) explicitly explore several layers of the atmosphere, since they impose implicit heating. However, due to the contributions of [Murray-Clay et al. \(2009\)](#), it can be allowed to explore planetary

outflows without the numerical necessity of implementing a code which conducts the energy heating of the atmosphere. Thus, [Carroll-Nellenback et al. \(2017\)](#) - which delve into the details of [Stone and Proga \(2009\)](#) - was able to describe the local and global scales of an HD photoevaporating system with commendable analytical support. This dissertation thus far was able to retain many important aspects present in [Carroll-Nellenback et al. \(2017\)](#) and [Stone and Proga \(2009\)](#):

- The step-by-step study of starting with a static planet, introducing spin and orbit, then repeating the procedure with the introduction of temperature anisotropy, as is found in [Carroll-Nellenback et al. \(2017\)](#), was reproduced with success. Although the forces generated by the orbital trajectory of the planet are explicitly inserted, instead of considering a local box orbiting, can make a significant difference. One important aspect that was left aside in this work is the ballistic analysis - which beside being done in [Carroll-Nellenback et al. \(2017\)](#), so is done in [McCann et al. \(2019\)](#). The description of a point-like particle being launched by the planet to understand how the outflow evolves is additionally helpful. However, contrasting with these papers, the parameters of the system are such that the centrifugal and Coriolis forces do not influence strongly the outflow - as seen before in Section 4.4.
- A major disadvantage is the slight differences in the problem setup of these works: [Stone and Proga \(2009\)](#) is only 2D and has a free parameter - varies the γ coefficient - although follows the planetary values; [Carroll-Nellenback et al. \(2017\)](#) does not vary any coefficient, but fails to keep with the parameters, leading to a λ for the HJ which is twice the expected value. Nevertheless, the static HJ with temperature gradient shows a completely compatible steady-state, with the main difference being the calculated inflow of material.
- Resolution is perhaps the "elephant in the room": resolution and computation time were highly dependent on the available resources at Centro de Astrofísica da Universidade do Porto (CAUP). Most models were run with a number of cores between 20 and 48. For a grid of $[200 \times 200 \times 200]$ using 40 cores, each core would process around $\sim 8 \times 10^5$ cells. The introduction of a third dimension can indeed impact deeply the resolution - for example [Tremblin and Chiang \(2012\)](#) is able to capture the Kelvin-Helmholtz instabilities at very high resolution in the bow-shock, since the work is 2D. And for most of the parts, a 2.5D would be enough (2D and an

axis-symmetric direction). However, when orbit and fictitious forces are introduced, along with an impacting stellar wind, the asymmetries appear clearly.

- The wind-wind interaction models become hard to compare with [Carroll-Nellenback et al. \(2017\)](#), due to the paper's different λ_p value. [Stone and Proga \(2009\)](#) studies an impacting stellar wind with deflection angle of $\theta_W = 90^\circ$ and distinct energy and density parameters at the base of the host star. Similar sonic surfaces for the *Aniso-B* model are found at the "dayside" of the planet - lending support to the discussion. The works of [McCann et al. \(2019\)](#) and [Matsakos et al. \(2015\)](#); [Daley-Yates and Stevens \(2018\)](#) (even though these last two are entirely MHD), corroborate the theoretical justification of the used characteristic scales.

4.6.1 Concluding Remarks

In this Chapter, it was possible to understand each layer that a simple photoevaporation problem may have. It was analysed and discussed several models: static, spinning, orbiting models centered on the HJ, with temperature gradient switched off and on. Lastly, the interaction of boundary-inserted stellar winds with the outflowing planetary winds in two cases: strong and intermediate stellar winds.

The structure of this Chapter is nearly complete, since a Weak stellar wind case could also have been introduced, although, it would only escape the main focus of this thesis. All these results show themselves worthy of comparison with the works of [Carroll-Nellenback et al. \(2017\)](#), [McCann et al. \(2019\)](#) and [Stone and Proga \(2009\)](#), even with quite a shortened computational power.

The time has come to introduce a new impacting parameter: magnetic fields.

Chapter 5

Magnetohydrodynamic Simulations

Host stars of close-in planets typically drive magnetized stellar winds that are accelerated to speeds of a few hundred km s^{-1} ([Adams, 2011](#)). The magnetized plasma that is driven out in these winds is now the medium into which the evaporative outflows from close-in exoplanets expand. Naturally, a close-in planet and its host star may interact through their respective outflows and magnetic fields. Several observable effects can appear:

- Bow shocks formed by supersonic stellar wind in front of the planet ([Llama et al., 2013](#)).
- Cometary-type tail associated with the swept-up planetary wind that trails the planet in its orbit ([Kulow et al., 2014](#)).
- [Cohen et al. \(2011\)](#) suggested that the interaction of stellar magnetized wind with a HJ's magnetosphere can lead to a reduction in the mass loss rate, and even more strongly in the angular momentum loss rate from the host star.

Gas giants have H as the most abundant element and heating is dominated by photoionization in the thermosphere ([Yelle, 2004](#)). Photoionization leads to significantly ionized gas at high altitudes that is subject to magnetic forces. Due to dynamo action in the planet's core, the planetary magnetic field is possibly dynamically important and can affect the density and velocity profiles in the upper atmosphere. Additionally, the stellar wind limits the size of the magnetosphere, determined by stress balance at the magnetopause ([Preusse et al., 2007](#)). Reconnection between field lines in the stellar wind and magnetosphere may lead to the magnetospheric convection, limiting the high density region to be inside a plasmapause - as happens with Earth.

[Adams \(2011\)](#) discussed that magnetic fields of both planets and host stars are generally strong enough to guide the flow. When the outflow follows the magnetic field lines, the geometry of the flow pattern is set by the field structure. Due to this complexity, outflows depart significantly from spherical symmetry and the existing models fail to be applicable.

Near the planet, the dynamo-generated, roughly dipole field from the planets core is expected to dominate. Moving outward, currents generated in the magnetosphere comb the field lines into a nearly radial direction beyond the dead zone radius. Such a geometry has been used before in the context of stellar wind ([Mestel, 1968](#)). [Trammell et al. \(2011\)](#); [Trammell et al. \(2014\)](#); [Khodachenko et al. \(2015\)](#) also found that atmospheric material can be supported within closed magnetic field lines in what is known as a wind "dead zone".

From this introduction, it can be understood that magnetic fields play a vital role in the morphology of stellar and planetary outflows. In this chapter, each section will address the work and models developed with increasingly more detailed structures, as a means to achieve results which can be comparable to the work of [Matsakos et al. \(2015\)](#) in the same framework of [Carroll-Nellenback et al. \(2017\)](#). The sections are displayed in a quite similar way as in Chapter 4:

- As before, Section 5.1 will regard the study of magnetic outflows from HJs. A large variety of models will be discussed in order to access the best solvers which to use in this Chapter. Additionally, the important feature of the opening of polar field lines will also be explored.
- Following this understanding of the building blocks, an explanatory discussion will be given towards the failure of studying the temperature anisotropy in a magnetically shielded HJ.
- Sections 5.2 and 5.3 focus on rotation and orbiting, respectively.
- The final goal stands at this Section 5.4: inserting magnetic stellar winds with the MHD planetary outflow case. This will prove to be a quite important stand-point of work developed, since no other work has combined the works of [Stone and Proga \(2009\)](#); [Carroll-Nellenback et al. \(2017\)](#) with a MHD framework. Additionally, comparison to other works allows the understanding of the high relevance of this dissertation.
- Finally, Section 5.5 will summarize and discuss the main ideas of this chapter.

5.1 Planetary Winds with Magnetic Fields

We can quickly follow the initial discussion of [Adams \(2011\)](#) to see how the plasma is well coupled to the magnetic field: having the cyclotron frequency w_C larger than the collision frequency w_λ . The cyclotron frequency is given by:

$$w_C = \frac{Bq}{mc} \quad (5.1)$$

and the collision rate:

$$w_\lambda = \frac{v}{\lambda_{\text{mfp}}} = n\sigma v \quad (5.2)$$

Using the continuity equation, we can write the following ratio of frequencies:

$$\frac{w_C}{w_\lambda} = \frac{\lambda q B}{cmv} = \frac{qB}{cnm\sigma v} = \frac{4\pi r^2 q B}{c\sigma \dot{M}} \quad (5.3)$$

where it was used that $\rho = nm$. This ratio behaves as $Br^2 \propto r^{-3}r^2 = r^{-1}$. If we substitute by typical values, this equation yields:

$$\frac{w_C}{w_\lambda} \sim 10^4 \left(\frac{B}{1\text{G}} \right) \left(\frac{\sigma}{2 \times 10^{-13}\text{cm}^2} \right)^{-1} \left(\frac{\dot{M}}{10^{10}\text{g.s}^{-1}} \right)^{-1} \quad (5.4)$$

Thus, the ratio formally exceeds unity at a radius of $r \sim 10^4 R_p$, which is well beyond the sonic radius of the wind r_s . Therefore, it is safe to assume that the outflow is well coupled to the magnetic field.

Another necessary condition for the magnetic field to guide the outflow is that the magnetic pressure must be larger than the ram pressure of the flow. Starting by finding the radius at which the two pressures are equal, it needs to be computed $\rho v^2 \sim B^2/8\pi$. Again, we express density in terms of the outflow rate, however, it is useful to express this at a known velocity: $\dot{M} = \eta_m 4\pi r^2 \rho c_s$, where η_m is a simple normalization constant. With a dipolar relation for the magnetic field $B = B_p (R_p/r)^3$, we can solve for the radius:

$$\frac{r}{R_p} = \left(B_p R_p \right)^{\frac{1}{2}} \left(2\dot{M} c_s \eta_m \right)^{-\frac{1}{4}} \sim 26 \left(\frac{B_p}{10\text{G}} \frac{R_p}{10^{10}\text{cm}} \right)^{\frac{1}{2}} \left(\frac{\dot{M}}{10^{10}\text{g.s}^{-1}} \frac{c_s}{10\text{km.s}^{-1}} \right)^{-\frac{1}{4}} \quad (5.5)$$

At a radius of $r \sim 26 R_p$ we find a stellar magnetic field generally stronger than the planetary field. As a consequence, where it would be expected for the ram pressure to start dominating, the stellar wind magnetic pressure has long been exceeding. Therefore, it can be concluded with certainty that the outflow must be magnetically controlled.

5.1.1 Simulation Framework

By considering a dipolar magnetic field configuration, it means we are dealing with a magnetic dipole $\mathbf{m} = m\hat{\mathbf{z}}$. This means that the HJ's magnetic field $\mathbf{B}$ is given by:

$$\mathbf{B}(r, \theta) = \frac{|\mathbf{m}|}{4\pi r^3} (2 \cos \theta \hat{\mathbf{e}}_r + \sin \theta \hat{\mathbf{e}}_\theta) \quad (5.6)$$

Notice that this initial configuration will be let free to evolve, which requires that at a certain boundary region it should be fixed. Additionally, this fixed region must relax any complex configurations, *i.e.* field singularities at the origin. Thus, the chosen configuration follows:

- A constant $\hat{\mathbf{z}}$ oriented field is fixed until $0.5R_p$.
- From $0.5R_p$ to R_p the field is fixed in a dipolar configuration.
- In an initial configuration, the dipolar field configuration is present in all grid, but is left free to evolve.
- The previous HD-Static model's wind fixed regions is applied.

As discussed in chapter 3, a possible background field can be inserted in the simulations. Since it will be important to understand in some initial configurations if the planetary field will have its lines opened, it will also be explored, with different configurations.

- Models with no background field, their intent is to study the dipole evolution by itself.
- Models with a constant $\hat{\mathbf{z}}$ background field as:

$$H_{\text{bckgd}} = B_s \left(\frac{R_s}{a} \right)^3 \hat{\mathbf{z}} \quad (5.7)$$

which can be controlled by a global parameter known as β which translates the ratio between the planetary and stellar fields:

$$\beta = \frac{B_p}{B_s} \left(\frac{R_p}{R_s} \right)^3 \quad (5.8)$$

so that $H_p(r) = \beta H_s(r)$. This models will help investigating at first order the effects of a stellar host. Notice that this configuration shows xy spatial symmetry.

- Models with a constant dipolar background field, following equation (5.6), with the β parameters as a controller as well. Spatial symmetry is locally lost, which can be regarded as a more realistic model, although globally fixed.

At a first objective, it will be given more importance to the opening of the field lines than to the simple outflow in an MHD framework. This can be justified by the fact that, as stated in many works [Adams \(2011\)](#); [Trammell et al. \(2011\)](#); [Zhilkin and Bisikalo \(2019\)](#), it is imperative to account for rotation, orbit and centrifugal terms to open the field lines in a natural way. Since this section focuses on the simple configurations, and as the starting point of the new ground of exploration, it is required to check for all important expected details. The opening of the field lines will be primarily given by the discussed background fields.

Additionally, the solvers of Riemann flux, spatial integration and time-stepping are also crucial in this aspect. In the previous chapter, the collective configuration of *hllc* + *LINEAR* + *RK2* was the mainly used one. Now, due to incompatibility problems of **PLUTO** of combining *hllc* with the *DIV_CLEANNING* method for imposing the $\nabla \cdot \mathbf{B} = 0$ or with *hlld* with the existence of background fields, the need to find the best solver configuration should be explored as well.

5.1.2 Results

In order to address most models built, it should be better to firstly detail which solvers in each model were used and their success. It goes without saying that resolution plays a major role in these, however, for a strategic manage of time and memory, it has been given more relevance to those models which did not fail with lower resolution.

# Model	Resolution	Time	Spatial	Flux	$\nabla \cdot \mathbf{B} = 0$	Success
1.1	Low	ChTr	<i>LINEAR</i>	<i>hlld</i>	<i>DIV_CLEANNING</i>	No
1.2	Low	ChTr	<i>PARABOLIC</i>	<i>hlld</i>	<i>DIV_CLEANNING</i>	No
1.3	Low	RK3	<i>PARABOLIC</i>	<i>hlld</i>	<i>DIV_CLEANNING</i>	Yes
1.4	Mid	RK3	<i>PARABOLIC</i>	<i>hlld</i>	<i>DIV_CLEANNING</i>	Yes
1.5	Mid	RK3	<i>PARABOLIC</i>	<i>hll</i>	<i>DIV_CLEANNING</i>	No
1.6	Mid	RK3	<i>PARABOLIC</i>	<i>hllc</i>	8-WAVE	No
1.7	Mid	RK3	<i>LINEAR</i>	<i>hlld</i>	<i>DIV_CLEANNING</i>	Yes

TABLE 5.1: Table with used solvers in the different models.

The different applied resolutions are generally:

- Low: $[146 \times 146 \times 136]$

- Mid: $[220 \times 220 \times 200]$

As expected, the best group of solvers is the one that are used in [Matsakos et al. \(2015\)](#): RK3 time-stepping, parabolic spatial reconstruction, hlld Riemann flux solver and *DIV_CLEANING* for the Maxwell's law.

Thus, hereinafter, these will be the usually used ones if nothing else is said. Additionally, several stellar background fields were superimposed in the simulations, however it has been found that does not lead to good results. On the one hand, it is being imposed a background field on a region of the grid which should not evolve. On the other, physically is the wind that is magnetized, and not the space itself. Thus, these models can be said to have the finality of fine-tuning the intermediate resolution of the grid. It was possible to discern the effects of symmetry when imposing a radial background field and the asymmetric shape of the magnetosphere when imposing a dipolar field. As well, the opening of the field lines with an inverted background field (one which bears the same charge as the planet's poles) was successfully observed. The resolution fine-tuning concluded that at least, the $[5R_J \times 5R_J \times 5R_J]$ grid around the planet's center must have a minimum resolution of $dx = dy = dz = 0.1R_J \longrightarrow dV \sim 0.001R_J^3$.

With this, it was possible to proceed to an Isothermal Static Case - denoted as **M-IsoStatic** - with enough confidence regarding the final results. This model has $B_p = 1\text{G}$ as the HJ's magnetic field strength at the surface. Figure 5.1 shows the density plots of the model. Additionally, Figure 5.2 plots the magnetic field strength in logarithmic scale with field lines overplotted, which were computed using an interpolation method.

The **PLUTO's** *pyPLUTO* module comes with a variety of plotting-adds, including a tool for plotting field lines. This tool, however, had some limitations, regarding cartesian grids. Besides these, it was very simplistic, since it would gather the largest infinitesimal step in the entire grid between the pretended directions ($\max(dx, dy)$). It was thus implemented a technique to search for the smallest infinitesimal step $\min(dx, dy)$ if the standard deviation of the arrays dx or dy is larger than a given threshold ϵ_d . Otherwise, it uses the smallest infinitesimal step of the pretended directions closest to the interpolated cell. This means that, inside a good enough threshold, it can be considered that the grid is uniform and interpolation should not depend on the infinitesimal step - if the contrary happens, then the local spatial resolution is used.

Now that magnetic fields are being explored, the alfvénic velocity given by $v_A = |B|/\sqrt{4\pi\rho}$ is a rich property of the outflow to be analysed. Already inserted in Figures

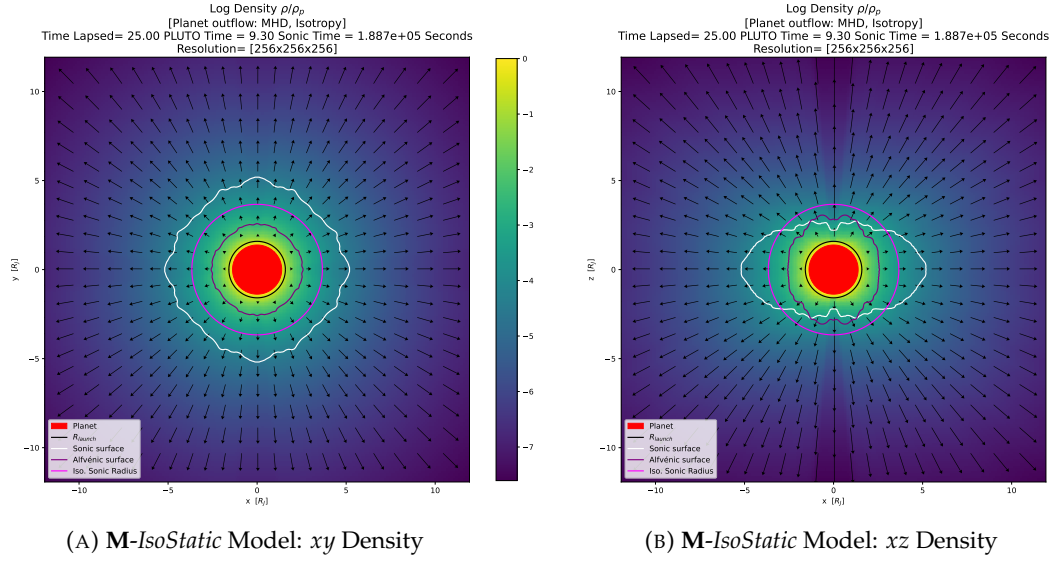


FIGURE 5.1: Density plots for M-IsoStatic Model in xy and xz planes, normalized to the planet's base density and displayed in a logarithmic scale.

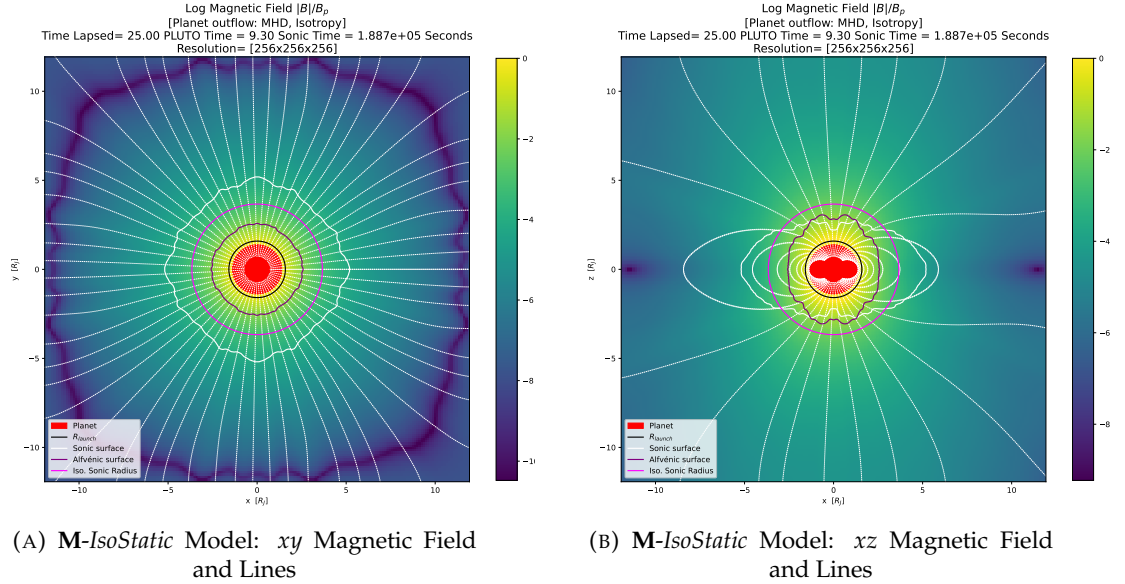


FIGURE 5.2: Magnetic field strength and line plots for M-IsoStatic Model in xy and xz planes, normalized to the planet's base magnetic strength B_p and displayed in a logarithmic scale.

5.1 and 5.2, the alfvénic surface is overplotted in a purple line. This is the surface where the flow’s velocity is equal to the flow’s alfvénic speed: $v = v_A$. Figure 5.3 plots the alfvénic speed for both planes.

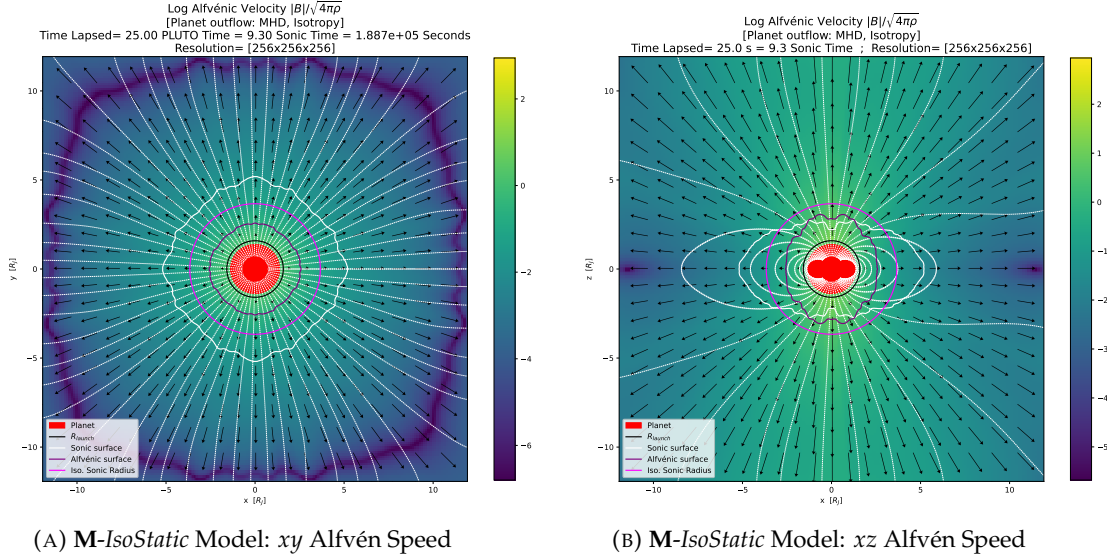


FIGURE 5.3: Alfvénic speed plots for M-IsoStatic Model in xy and xz planes, normalized to the planet’s base alfvénic speed $v_{\text{alf},p} = \frac{B_p}{\sqrt{4\pi\rho_p}}$ and displayed in a logarithmic scale, with the sonic and alfvénic surfaces overplotted, and the magnetic field lines and flow’s velocity direction.

At last, Figure 5.4 shows the mass loss plot over time and distance. It is possible to observe quite a slight change in mass loss behaviour from the previous HD Iso Case.

5.1.3 Important Remarks

This model has been ran in several resolutions: the results from the simulation shown follow the following grid:

- any-direction: stretched grid from $x = -12$ to $x = -5$ with a resolution of 64 cells, and from $x = -5$ to $x = +5$ with a resolution of 128 cells; an equal stretched grid from $x = +5$ to $x = +12$, being x representative of any x , y or z direction.

However, another simulation was ran where the grid had the following aspect:

- x and y -directions: stretched grid from $x = -15$ to $x = -6$ with a resolution of 64 cells, and from $x = -6$ to $x = +6$ with a resolution of 120 cells; an equal stretched grid from $x = +6$ to $x = +15$.

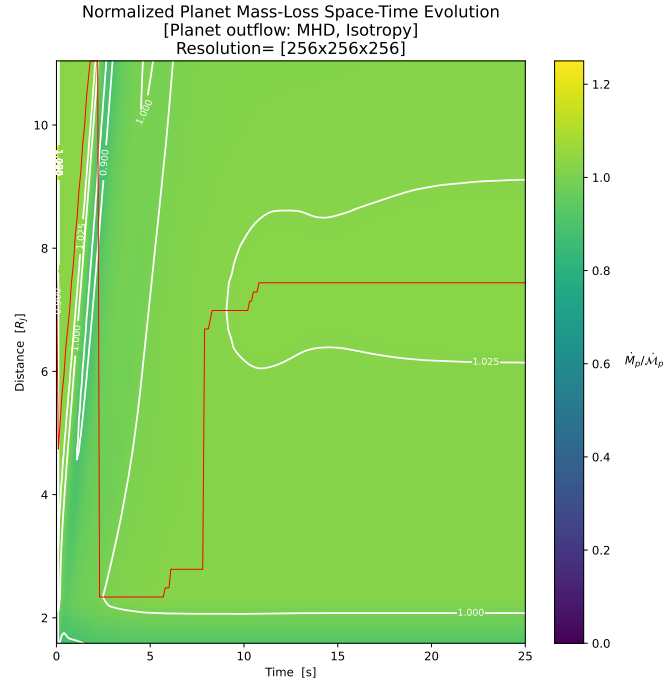


FIGURE 5.4: Normalized mass loss plot for **M-IsoStatic** Model over distance and times. The red line is the maximum value over distance for each time step.

- z-direction: stretched grid from $x = -12$ to $x = -6$ with a resolution of 50 cells, and from $x = -6$ to $x = +6$ with a resolution of 120 cells; an equal stretched grid from $x = +6$ to $x = +12$.

the latter is displayed in Figure 5.5, where significant differences in the sonic and alfvénic surfaces are detectable.

Thus, the grid's characteristics bear a strong effect on the magnetic field behaviour and flow's direction as well. It is possible to notice that the cartesian effect appears in both ran simulations, with a striking effect on the non-smooth and geometric surface's behaviour.

Another significant factor to consider is the magnetic field's behaviour near the $z = 0$ plane, which is common in most simulations: the absolute magnitude of the field tends to fall and then rise close to the borders. Despite the fact that it is in absolute value, there is definitely a signal change. This is only observed in the equator region, which is compatible to the expected consequence of a magnetically controlled flow and a "dead zone". However, when comparing both simulations, where both grids have different sizes at the equator, it seems to fail. This could be a numerical result of the boundary conditions used. Since it is being forced at the ghost cells that all quantities go to zero as their gradients, there appears a rise of "numerically spurious" currents due to the chosen

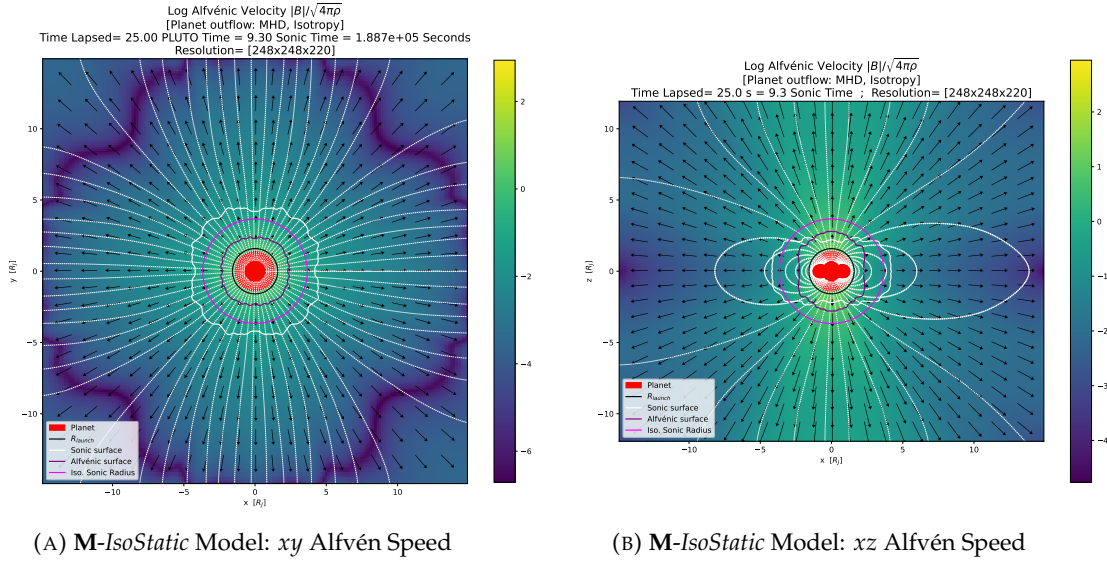


FIGURE 5.5: Alfvénic speed plots for a lower resolution **M-IsoStatic** Model in xy and xz planes, normalized to the planet’s base alfvénic speed.

$\nabla \cdot \mathbf{B} = 0$ controller and the boundary conditions. These “currents” force the magnetic field to drop strongly before it reaches the boundaries, for whatever size the grid may take. The bending of the field lines closer to the corners are another proof of boundary condition’s misbehaviour.

As an additional consequence, the HJ’s mass loss does not grow the further the outflow goes. This seen property (purely numerical) now is also influenced due to the numerical conditions imposed. The imposition of magnetic fields creates a new form of energy transfer, which in a static dipolar configuration, it becomes (r, θ) dependent. As can be seen in Figure 5.1, the HJ’s poles have a deficit in outflow strength. This can be explained by the energy exchange between the gas, which is pushing the closed magnetic lines, increasing the magnetic stress. As a consequence, the structure of the subsonic region, which is mainly determined by the hydrostatic density structure and not by the velocity law (Lamers et al., 1999), is altered. As a result of the energy exchange and density structure variation, the sonic surface is pushed away, and the outflow’s initial structure is determined by the alfvénic critical behaviour. This, joined with the fact that the outflow now feels strongly the boundary conditions, creates a highly delicate energy dynamics, giving rise to a “sweet spot” for mass loss where it is maximum.

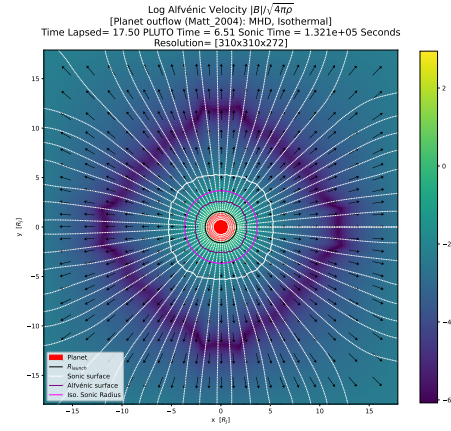
As will be deeply discussed in the next section, the steady state of this model has the sonic surface beyond the alfvénic surface, which is a first indication that the final

state is not theoretically predicted. The sonic surface is defined by a quasi sonic-radius, which expresses a "slow wave" length for perturbations. This should be smaller than the alfvénic radius at the equator, which expresses the "fast wave" length for perturbations. Although [Matsakos et al. \(2015\)](#) results do present this characteristic of a sonic surface outside the alfvénic surface, discussion must be done after the theoretical introduction of the next Section.

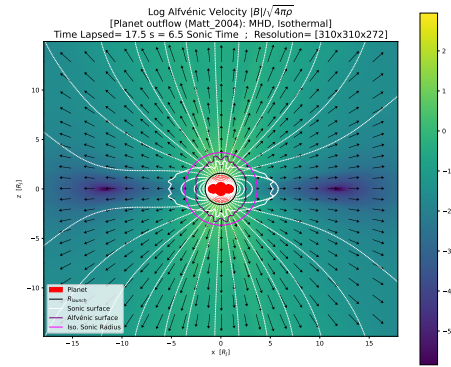
[Matt and Balick \(2004\)](#) have developed a "rule-of-thumb" in order to retrieve a magnetically controlled flow in a typical magnetic rotator problem. Nonetheless, it can also be applied in this case of a static magnetic body. The grid has been extended, in order to see if the change of sign in the magnetic field at the equator is observed closer to the planet, which is also an indicator to how far the "ghostly currents" only affect up to a certain distance of the grid. This technique will be discussed with broader details in the next Section, however, the main idea is to freeze the wind at the launch boundary, *i.e.* the direction of the wind is parallel to the field. Numerically, it will be understood as a magnetically controlled flow. Figure 5.6 plots the alfvénic velocities and ram pressures for a *M-Static* model. In Appendix C, Section C.3, Figure C.2 shows the outflow over time and distance, which is strikingly lower than seen in the models that do not have the magnetically controlled wind launched at the boundaries.

From the several impacting differences present, the magnetic behaviour of the flow originates the so called "dead zone" near the equator. This zone is denoted by the grid volume where the field remains close and the gas velocity is quite low. Thus, this zone corresponds to the squashed torus of slowly outflowing gas around the HJ's equator until the zone where the field changes sign. However, the cartesian effects from the grid are still present in the parameters. Additionally, the opening of the field lines occur at a higher footpoint angle, which will also be defined below. As a concluding remark on this short model, the method defined by [Matt and Balick \(2004\)](#) is applicable with the computational physics of this problem.

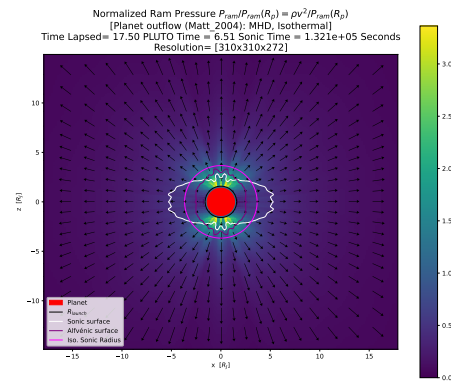
As an unfortunate consequence of all this discussion, inserting a temperature gradient becomes unbearably difficult due to a shortage of computer power, memory, and time. The impact of a "dayside"- "nightside" temperature difference will bend the magnetic fields, creating stresses and transferring energy from the field to the gas. Consequently, at first, the gas increases its velocity near the poles, followed by the bending of the outflow's streamlines around the equator. However, the CFL number - which dictates the Δt



(A) **M-Static Model: xy Alfvén Velocity**



(B) **M-Static Model: xz Alfvén Velocity**



(C) **M-Static Model: xz Ram Pressure**

FIGURE 5.6: Ram pressure and alfvénic velocity plots for **M-Static** model following the paper of [Matt and Balick \(2004\)](#)'s framework.

stepping - becomes too high, forcing $\Delta t \rightarrow \Delta t < 10^{-6}\text{s}$. To have an idea, the average $\Delta t \sim 3 \times 10^{-4}\text{s}$, and fluctuations do not cross $\Delta t \gtrsim 10^{-5}\text{s}$. Thus, such a model becomes exceedingly long, and any *M-Aniso* simulation will not be possible to be presented in this dissertation.

5.2 MHD Planetary Outflow with Rotation

Theoretical development in magnetized rotating outflows have been quite debated over some time, since, in a good approximation, they describe the overall picture of Earth and the Sun. For any loose end from the following considerations, it is recommended for the reader to see [Lamers et al. \(1999, Chapter 9\)](#) and [Priest \(2014, Chapter 13.6\)](#).

Naturally, the rotation of an object causes the magnetic field and velocity field to develop azimuthal components. Using (r, θ, ϕ) as spherical polar coordinates, the equatorial flow ($\theta = \pi/2$) will have the following:

$$\mathbf{v} = V_r(r)\hat{\mathbf{e}}_r + V_\phi(r)\hat{\mathbf{e}}_\phi \quad (5.9a)$$

$$\mathbf{B} = B_r(r)\hat{\mathbf{e}}_r + B_\phi(r)\hat{\mathbf{e}}_\phi \quad (5.9b)$$

Because it is being assumed rotational symmetry, the components V_r , V_ϕ , B_r and B_ϕ become functions of radius only. It may become convenient to use Alfvén velocities in order to express the components of the magnetic field in velocity units:

$$|\mathbf{A}|^2 = A_r^2 + A_\phi^2 = \frac{B_r^2}{4\pi\rho} + \frac{B_\phi^2}{4\pi\rho} \quad (5.10)$$

To visualize the differences between the velocity and magnetic structures, consider a purely radial outflow. Because a magnetic field is rooted on the object, its rotation causes the magnetic field to twist into a spiral pattern. For a rotating object, the magnetic field lines leave the star radially, rather than at constant pitch angle of record grooves. This radial field causes the gas to co-rotate with the star out to some distance called the ‘co-rotation radius’. The plasma velocity does not follow a radial path inside the co-rotation radius, but has a V_ϕ component that increases linearly with r . The tendency of the field to cause co-rotation leads to an outward transfer of angular momentum from the field to the object. The reaction of that torque, which is transmitted along the magnetic field lines, removes angular momentum from the star.

The vector form of the momentum equation is:

$$\rho (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla p + \rho \frac{GM}{r^2} \hat{\mathbf{e}}_r - \frac{1}{c} \mathbf{J} \times \mathbf{B} = 0 \quad (5.11)$$

where the last term is the Lorentz force in Gaussian units.

In the stationary frame of reference, the current density $\mathbf{J}$ satisfies Ohm's law:

$$\mathbf{J} = \sigma_c \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \quad (5.12)$$

where σ_c is the coefficient of conductivity. Since this coefficient is extremely large in a plasma, assume a 'frozen-in field' so that the current remains finite:

$$\mathbf{E} = -\frac{\mathbf{v} \times \mathbf{B}}{c} \quad (5.13)$$

Using Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (5.14)$$

and assuming steady conditions ($\partial \mathbf{B} / \partial t = 0$), we have:

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \quad (5.15)$$

Both the radial and azimuthal components of this vector are zero. From the ϕ component, we get:

$$\frac{1}{r} \frac{d}{dr} (r V_r B_\phi - r V_\phi B_r) = 0 \quad (5.16)$$

Now let Ω be the angular speed of the object at r_0 . The tangential velocity $V_\phi(r = r_0) = r_0 \Omega$ is large and $V_r(r = r_0) \ll V_\phi(r = r_0)$ because the radial velocity at the base of the wind is small. Since the field lines that leave the object surface are radial: $B_\phi(r = r_0) \ll B_r(r = r_0)$. Thus $V_r B_\phi \ll V_\phi B_r$ at $r = r_0$, we have one condition to solve equation (5.16):

$$r(V_r B_\phi - V_\phi B_r) = \text{constant} = -r_0^2 \Omega B_r(r_0) \quad (5.17)$$

Making use of Maxwell's equation:

$$\nabla \cdot \mathbf{B} = 0 \quad (5.18)$$

provides for a conserved quantity in spherical coordinates:

$$\mathcal{F}_B = r^2 B_r = r_0^2 B_r(r_0) \quad (5.19)$$

since the magnetic field components are radial functions.

Thus, the ratio follows:

$$\frac{B_\phi}{B_r} = \frac{V_\phi - r\Omega}{V_r} \quad (5.20)$$

Additionally, using Ampère's law:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} \quad (5.21)$$

to express the Lorentz force as:

$$\frac{1}{c} \mathbf{J} \times \mathbf{B} = \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} \quad (5.22)$$

This vector expression can be expanded to provide terms in the three component directions, which can be simplified by using the assumptions associated with the equatorial flow:

1. $\partial/\partial\phi$ and $\partial/\partial\theta$ are zero.
2. Also in the equatorial plane: $B_\theta = 0$ and $\sin\theta = 1$.

Thus, the Lorentz force in the equatorial plane yields:

$$\frac{1}{c} \mathbf{J} \times \mathbf{B} = -\frac{1}{4\pi} \left[\frac{B_\phi}{r} \frac{d}{dr} (rB_\phi) \right] \hat{\mathbf{e}}_r + \frac{1}{4\pi} \left[\frac{B_r}{r} \frac{d}{dr} (rB_\phi) \right] \hat{\mathbf{e}}_\phi \quad (5.23)$$

Using a vector transformation to polar coordinates, the first term of the equation (5.11) can be rewritten as:

$$(\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{1}{2} \nabla v^2 - \mathbf{v} \times (\nabla \times \mathbf{v}) \quad (5.24a)$$

$$= \left(\frac{1}{2} \frac{d}{dr} (V_r^2 + V_\phi^2) - \frac{V_\phi}{r} \frac{d}{dr} (rV_\phi) \right) \hat{\mathbf{e}}_r + \left(\frac{V_r}{r} \frac{d}{dr} (rV_\phi) \right) \hat{\mathbf{e}}_\phi \quad (5.24b)$$

Combining equations (5.24), (5.23) and (5.11), we can separate the radial and azimuthal terms. The obtained equation of motion in the radial direction is:

$$V_r \frac{dV_r}{dr} + \frac{1}{\rho} \frac{dp}{dr} + \frac{GM}{r^2} - \frac{V_\phi^2}{r} + \frac{B_\phi}{4\pi\rho r} \frac{d}{dr} (rB_\phi) = 0 \quad (5.25)$$

The first three terms are identical to the non-rotating wind momentum equation. The fourth term corresponds to centrifugal acceleration. The last term is the radial acceleration produced by the slinging effect of the rotating magnetic field. This produces an outward acceleration if B_ϕ decreases with radius more gradually than r^{-1} .

The equation of motion in the azimuthal direction is:

$$\rho V_r \frac{d}{dr} (rV_\phi) = \frac{B_r}{4\pi} \frac{d}{dr} (rB_\phi) \quad (5.26)$$

This equation states that the rate of change of the angular momentum per unit volume is the result of the torque per unit volume produced by the magnetic field $r(\mathbf{J} \times \mathbf{B})_\phi$.

In addition to the momentum equations, conservation of mass becomes:

$$\mathcal{F}_m = \rho V_r r^2 = \text{constant} \quad (5.27)$$

Since the azimuthal and radial equations of motion can be expressed as perfect differentials, they can provide with two constants of motion.

Looking at the azimuthal equation of motion first, we can use the fact that:

$$\frac{\mathcal{F}_B}{\mathcal{F}_m} = \text{constant} = \frac{r^2 B_r}{\rho V_r r^2} \quad (5.28)$$

therefore, when dividing equation (5.26) by ρV_r , it yields:

$$\frac{d\mathcal{L}}{dr} = 0 \quad , \quad \mathcal{L} = rV_\phi - \frac{rB_r B_\phi}{4\pi\rho V_r} = \text{constant} \quad (5.29)$$

this is the 'constant of angular momentum per unit mass'. The first term is the angular momentum per unit mass that is carried by the gas. The second term is the angular momentum carried by the magnetic stresses.

The effect of the magnetic term in the conservation of angular momentum per unit mass can be understood by considering a marble being flung by a flexible tube. Part of the angular momentum of the system would be in stress in the tube, and so even if the rotation and the base of the tube were stopped, the unflexing of the tube would continue to transfer angular momentum to the marble during the straightening of the tube.

Using equation (5.20) to eliminate B_ϕ from equation (5.29), we find:

$$V_\phi = r\Omega \frac{V_r^2 \mathcal{L}}{r^2 \Omega} - \frac{A_r^2}{V_r^2 - A_r^2} \quad (5.30)$$

From this equation, a singular point appear where the denominator goes to zero. This occurs at the Alfvén radius r_A :

$$V_r(r_A) = A_r(r_A) = \sqrt{\frac{B_r^2(r_A)}{4\pi\rho(r_A)}} \equiv V_A \quad (5.31)$$

The vanishing of the numerator determines the value of the Alfvén radius and leads to the requirement that:

$$\mathcal{L} = r_A^2 \Omega \quad (5.32)$$

Since the product $r^2\Omega$ corresponds to the specific angular momentum of matter co-rotating with the object, this now states that the angular momentum per unit mass behaves as if the object rotates as a solid body out to the Alfvén radius. As if because the near solid body co-rotation actually ceases at a radius well within r_A . Besides the centrifugal contribution, the stress of the magnetic field also contributes to the angular momentum of the rotating wind.

It now follows also:

$$\frac{B_\phi}{B_r} = \frac{r\Omega \left(V_r \frac{\mathcal{L}}{r^2\Omega} - V_r \right)}{V_r^2 - A_r^2} \quad (5.33)$$

Let us now consider the radial momentum equation. The derivative of B_ϕ appearing in the momentum equation can be found using B_ϕ/B_r :

$$rB_\phi = \left(r^2 B_r \right) \Omega \left(4\pi\rho V_r r^2 \right) \frac{\mathcal{L}/\Omega - r^2}{(4\pi\rho V_r r^2) - (r^4 B_r^2)} \quad (5.34)$$

where the parentheses in this equation contain conserved quantities.

$$\Rightarrow \frac{d}{dr} (rB_\phi) = \frac{-2V_r \left(V_r B_\phi + r\Omega B_r \right) - rB_\phi V_r \frac{dV_r}{dr}}{V_r^2 - A_r^2} \quad (5.35)$$

The magnetic force is proportional to $d(rB_\phi)/dr$. This will introduce another term to the radial momentum equation that contains $V_r dV_r/dr$. We obtain:

$$\left(V_r^2 - c_s^2 - \frac{A_\phi^2 V_r^2}{V_r^2 - A_r^2} \right) \frac{r}{V_r} \frac{d}{dr} V_r = 2c_s^2 - \frac{GM}{r} + V_\phi^2 + 2 \frac{A_r A_\phi V_r V_\phi}{V_r^2 - A_r^2} \quad (5.36)$$

This is a general expression for the velocity gradient in isothermal magnetic rotator theory. It contains the modifications introduced by the centrifugal and Lorentz forces.

Figure 5.7 shows the typical space-phase solution of a polytropic magnetized wind - equation (5.36) is only slightly changed.

According to [Weber and Davis \(1967\)](#), the description takes:

- u_{a1} gives the behaviour of a supersonic, super-Alfvénic wind at infinity.
- u_{a2} gives the behaviour of a super-Alfvénic and subsonic flow, as it passes through the critical point again - as can be checked by the second panel in Figure 5.7.
- u_β are solutions with non-zero pressures at infinity.

These singularities denote critical points of the flow, where the flow's velocity equals a characteristic wave disturbance in the fluid. At the sonic radius r_s , which is now slightly

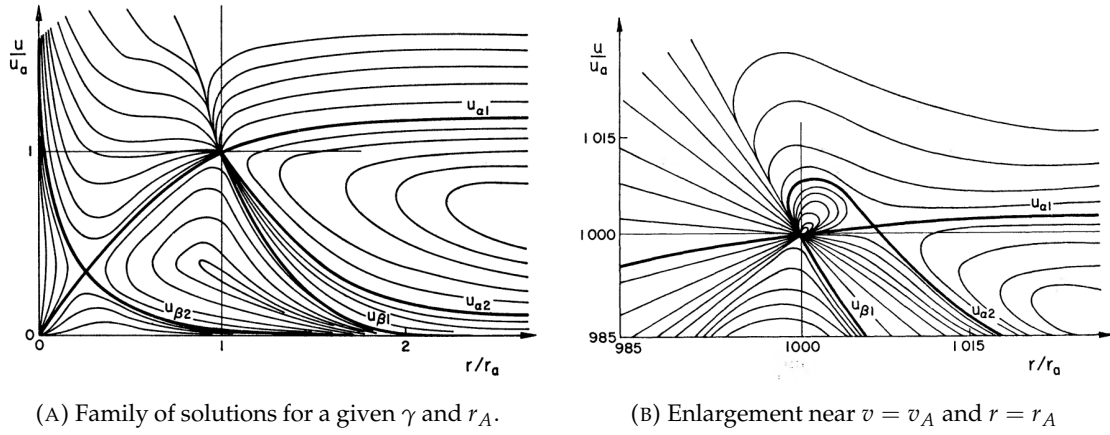


FIGURE 5.7: Plots taken from the work of [Weber and Davis \(1967\)](#), which sketch the topologic solutions of equation (5.36).

lower than the HD r_s , the fluid velocity equals the velocity of the "slow" wave - sonic wave. At r_f , the flow's velocity equals the "fast" wave velocity, which is slightly higher than the alfvénic speed. This gives rise to a quite interesting disturbance propagation:

- All disturbances originated past r_f will not propagate backwards, and thus cannot grow and produce local instabilities.
- All disturbances originated between r_s and r_f can be carried back towards the object, however, these will travel in terms of "fast" wave modes. The "slow" wave disturbances will be carried away.
- As before, any disturbance located prior to r_s will propagate the information backwards the flow by both "fast" and "slow" modes.

Figure 5.8 shows the typical profiles of these velocities which are derived from the radial momentum equation. As a matter of fact, [Zhilkin and Bisikalo \(2021\)](#) was able to translate equation (5.36) into a explicit v_r and r dependent differential equation, with only v_A , r_A and c_s as dependent parameters.

5.2.1 Simulation Framework

Inserting a magnetic rotating dipolar field is itself a quite complex layer-by-layer problem. From a theoretical point of view, the initial wind configuration should be released from the static regions of the planet (or star) as equation (5.36) states. However, this equation is only appreciated in the equatorial plane, where $\theta = 0$. For a dipolar configuration, B_θ

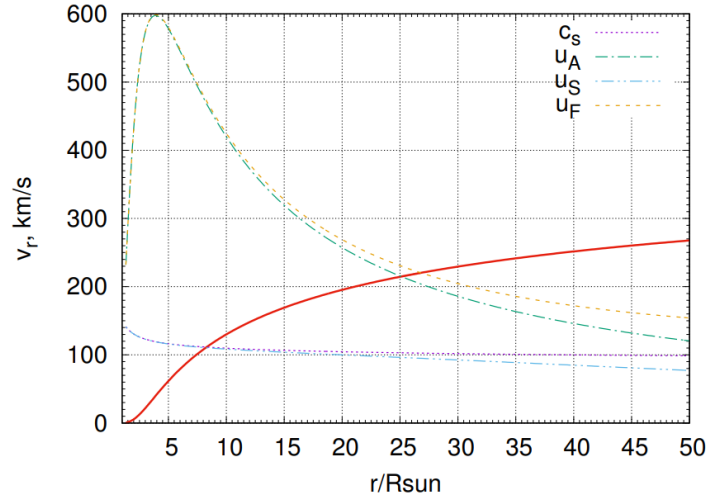


FIGURE 5.8: Plot taken from [Zhilkin and Bisikalo \(2021\)](#), which sketches the behaviour of the several velocity profiles following the HD209458 system. The red solid line is the radial velocity profile v_r , the sound speed c_s , the alfvén velocity u_A , slow u_S and fast u_F magnetosonic velocities.

contributions appear which modify drastically the $V_r(r, \theta)$. In fact, poloidal and toroidal field components can be separated and work then on, as does [Spruit \(1996\)](#) by assuming that a flux function ψ can be used to express the poloidal field such that

$$\mathbf{B}_p = \frac{1}{\omega} \nabla \psi \times \hat{\mathbf{e}}_\phi \quad (5.37)$$

where the cylindrical coordinates are used (ω, ϕ, z) , assuming that the poloidal coordinates are defined by $p \equiv (\omega, z)$. Nevertheless, as is developed in Appendix C, Section C.1, a static poloidal solution is analytically complicated.

Thus, the methodology used for setting up the simulation takes some ideas from the work of [Matt and Balick \(2004\)](#):

1. The first fixed region, defined by $r_{\text{static}, B} = 0.5R_p$, fixes a constant vertical magnetic field: $B = 16B_p \hat{\mathbf{z}}$ and no radial wind, except a ϕ component co-rotating with the star.
2. The second fixed region, defined in $r_{\text{static}, B} < r < r_{\text{static}, W} = 0.75R_p$ fixes a constant dipolar field, with no rotation, maintaining the wind conditions.
3. The third fixed region, defined in $r_{\text{static}, W} < r < R_p$ takes the same wind configuration as in the spinning case, with a dipolar magnetic field with a B_ϕ component defined by equation (5.20).

4. The last layer $R_p < r < r_{\text{static}} = 1.1R_p$, fixes the thermal wind to behave as in the Parker case, with the previous dipolar configuration. Until this final layer, pressure and density behaved as in the previous HD models, following the hydrostatic configuration of a constant density sphere.

The strongest deviation from [Matt and Balick \(2004\)](#) is the fact that from the third layer to the last, the wind does not follow the same direction as the magnetic field, *i.e.*, $v_p \parallel B_p$ is not applied.

Beyond these boundaries, the wind follows the behaviour of the last layer prior to the computations, however, it is completely free to evolve.

5.2.2 Results

As seen before, the most informative characteristic of the outflow is the alfvénic speed. Figure 5.9 plots the alfvénic speed for both planes.

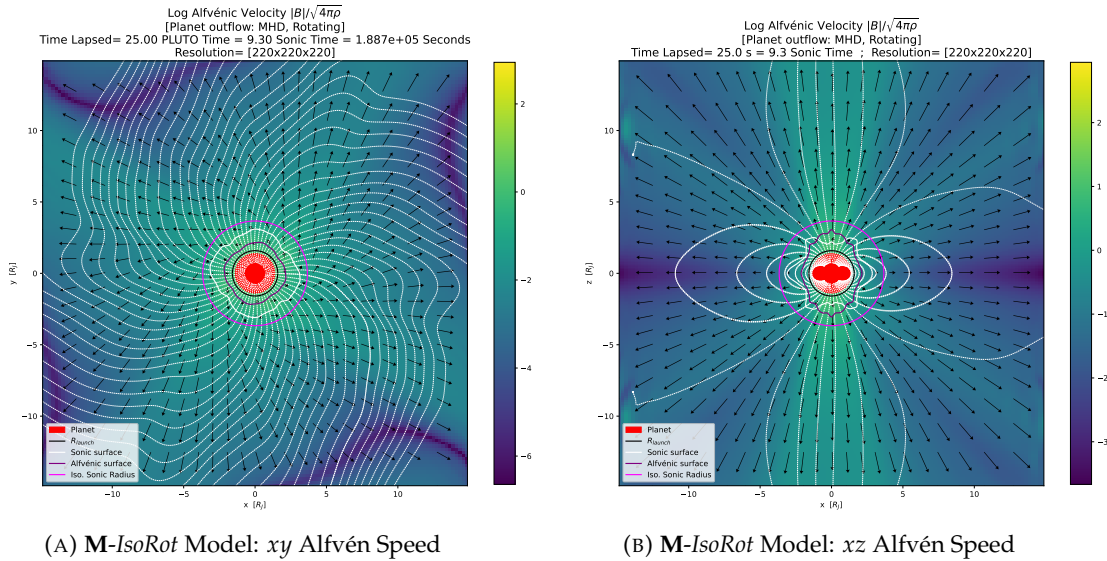


FIGURE 5.9: Alfvénic speed plots for M-IsoRot Model in xy and xz planes, normalized to the planet's base alfvénic speed $v_{\text{alf},p} = \frac{B_p}{\sqrt{4\pi\rho_p}}$ and displayed in a logarithmic scale, with the sonic and alfvénic surfaces overplotted, and the magnetic field lines in dotted white and flow's velocity direction with arrows.

Additionally, in order to understand how angular momentum is behaving in such a system, Figure 5.10 plots the angular momentum as defined in equation (5.29).

Lastly, Figure 5.11 shows the mass loss of the outflow over time and space.

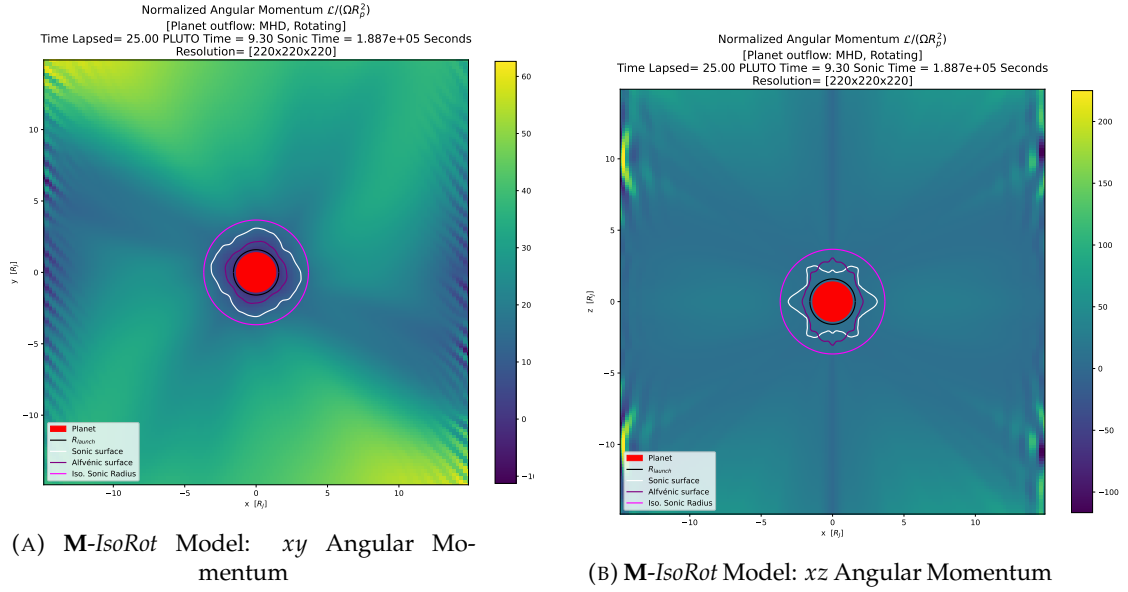


FIGURE 5.10: Angular momentum plots for **M-IsoRot** Model in xy and xz planes, normalized to the HJ's base angular momentum $\mathcal{L}_0 = R_p^2 \Omega$, with the sonic and alfvénic surfaces overplotted

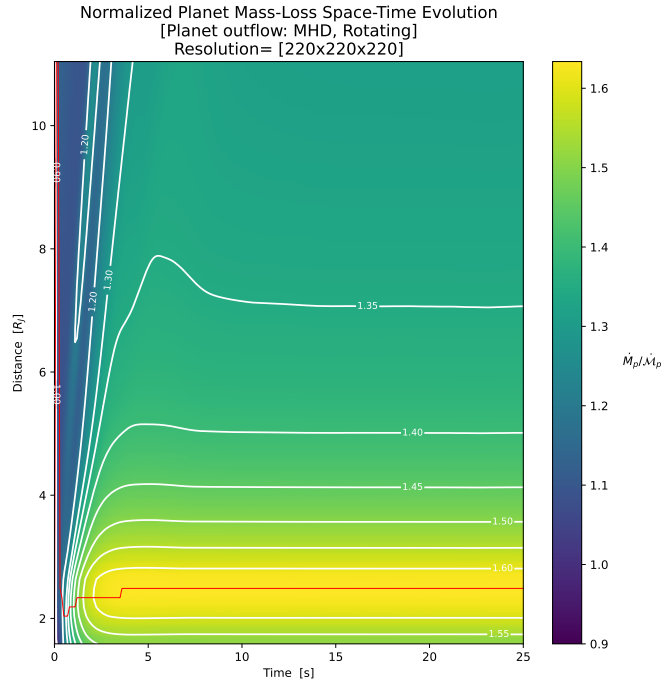


FIGURE 5.11: Normalized mass loss plot for **M-IsoRot** Model over distance and times. The red line is the maximum value over distance for each time step.

5.2.3 Important Remarks

A quite chaotic behaviour that is captured in the outputs of this model is the fact that the streamlines of the gas cross several magnetic field lines over the grid. This is far from expected, since the gas was assumed to be magnetically controlled. It seems that the boundary effects keep interacting with the outflow in a peculiar manner at the equator plane: notice how the minimum of the absolute value of the alfvén speed - which is due to the diminishing of the magnetic field - only follows the field lines that do not bend near the boundaries. When the lines are captured by the outflow boundary condition, the previously discussed “fictitious currents” appear and force the magnetic field to change sign. However, for the bent field lines that fail to be captured by the boundaries, the alfvén speed continues decreasing without change of sign. Thus, near the equator, it should be expected that momentum conservation fails, which is what is observed in Figure 5.10. Additionally, with the change of sign, the increase of angular momentum at the corners is also justified. Because of this unnaturally adjusted magnetic steady state, the flow ceases to be magnetically controlled, thus allowing for the gas streamlines to have a different direction from that of the field.

In the polar plane (xz plane), angular momentum is in fact closer to a constant value. However, towards the corners, there is an alternation of minima and maxima of angular momentum. This corresponds to where the outflow boundary condition captures magnetic field lines, leading to opening or stressing. Additionally, the poles of the HJ also struggle to keep a value close to the one seen in the equator: as polar orbital velocity is smaller and with the opening of the field lines, the absolute value of the field increases and consequently, $\mathcal{L}$ decreases.

The corners in the xz plane do contribute to the angle at which magnetic field lines open up. It is possible to approximately find the value for this angle by the plots of Figure 5.12: at a radius of $1.5R_p \lesssim r \lesssim 2.0R_p$ it is found an enhanced outflow at $\theta \sim \frac{\pi}{4}$. Thus, by the work seen in Adams (2011) where the magnetic field lines follow a constant value $q = x^{-1} \sin^2 \theta = \sin^2 \theta_b$, where $x = r/R_p$, it is possible to extract $\theta_b \sim \arcsin \left(\sqrt{\frac{1}{2} \sin^2 \frac{\pi}{4}} \right) = 0.524$.

In Appendix C, Section C.2 it is possible to see how this angle can be captured by studying the pressure along a magnetic loop. By solving equation (C.22), substituting the values of the system at place, it is obtained $\theta_b \sim 0.2$. The fact that the numerical state presents a wider footpoint angle compared to what is expected can be related to the HJ's

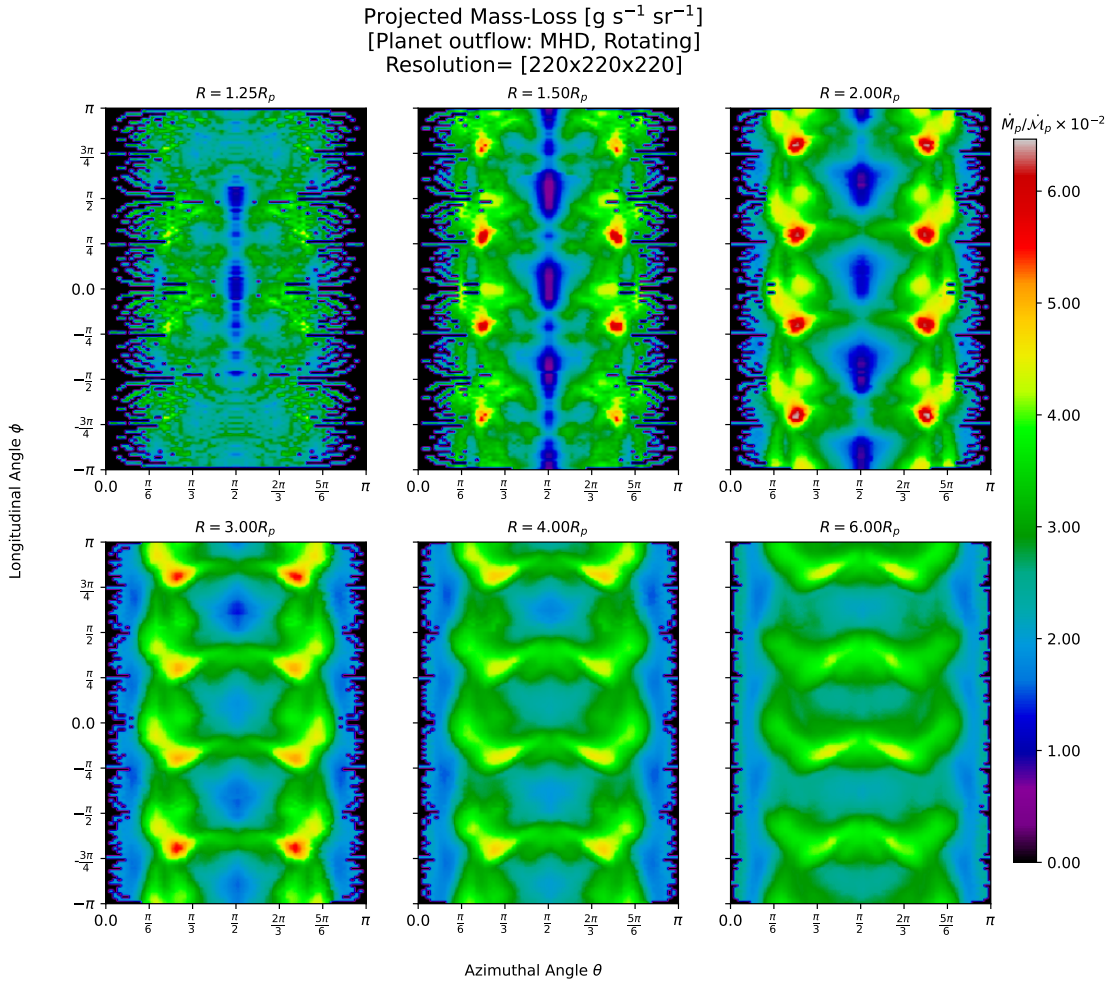


FIGURE 5.12: Normalized mass loss per steradian plot for **M-IsoRot** Model for 6 fixed distances. The overload of null values at the angular edges is due to the cartesian geometry of the grid - which can be seen to have a deep effect on the symmetry of the simulation. This plots were obtained by selecting a sphere with a fixed radius and a thickness of 0.1 of the radius.

boundary condition that affects the behaviour of the magnetic field. Furthermore, as the plots of Figure 5.12 display, the mass loss is remarkably impacted by the cartesian grid.

Thus, this model requires a re-evaluation. Some already beaten up possible causes for this strange state regard the computational power: a wider grid to have neglected boundary effects, higher resolution in order to have more natural bent magnetic lines. However, as stated previously, when [Matt and Balick \(2004\)](#) prepared their fixed boundaries for a stellar magnetic outflow, it was forced that the poloidal components of the velocity be parallel to the poloidal components of the magnetic field. The authors justify their choices by stating that in a case of a strong dipole, the magnetic loops that form the dead zone

require the gas to have a poloidal velocity nearly null in order to corotate with the star. Indeed, when forcing this parallel velocity, the x and y components will behave as $\propto xz/r^2$ and $\propto yz/r^2$ respectively, as the field does, which by definition, is a case of magnetically controlled outflow. Thus, close to the equator $z \sim 0$, the dead zone is as if it was numerically and magnetically forced to appear. On the other hand, as many MHD wind-wind interaction papers do (Matsakos et al., 2015; Carolan et al., 2021; Zhilkin et al., 2021), the simplistic touch of imposing an overlapped Parker wind with a dipole field works fine. As a result of this discussion, it is concluded that in order to analyse and study a “correct” and well-structured a fast magnetic rotator’s outflow, a higher degree of attention for these subtleties is required.

5.3 MHD Planetary Wind from an Orbiting and Rotating Planet

For close-in, tidally locked planets, tides and centrifugal forces play a key role in the upper atmosphere structure. Three main forces are present as before: the HJ’s gravity itself, tidal gravity from the star and centrifugal forces due to rotation. Since there is tidal locking, then centrifugal and tidal forces are comparable in strength, although the angular dependence is quite different. Following the theoretical introduction of Trammell et al. (2011), it is possible to approximate (Kopal, 1978) the sum of the three accelerations in the x direction as:

$$U(\mathbf{x}) \sim -\frac{GM_p}{r} - \frac{1}{2}\Omega^2 r^2 (f \sin^2 \theta - 1) \quad (5.38)$$

where $f = 1 + 3 \cos^2 \phi$. In order to verify how the magnetic field alter the radius beyond which the net gravity points outwards, it will be necessary to project the acceleration along the field lines. From this, it will be the parallel component that will be studied. Thus, the explicit components of the acceleration $\mathbf{a}(\mathbf{x}) = -\nabla U(\mathbf{x})$ yield:

$$a_r = -\frac{\partial U}{\partial r} = -\frac{GM_p}{r^2} + \Omega^2 r (f \sin^2 \theta + 1) \quad (5.39a)$$

$$a_\theta = -\frac{1}{r} \frac{\partial U}{\partial \theta} = f \Omega^2 r \sin \theta \cos \theta \quad (5.39b)$$

$$a_\phi = -\frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} = -3 \Omega^2 r \sin \theta \cos \theta \cos \phi \quad (5.39c)$$

$$(5.39d)$$

For a dipole geometry:

$$\mathbf{B} = B_p \left(\frac{R_p}{r} \right)^3 (2 \cos \theta \hat{\mathbf{e}}_r + \sin \theta \hat{\mathbf{e}}_\theta) \quad (5.40)$$

whose unit vector is:

$$\mathbf{b} = \frac{\mathbf{B}}{|\mathbf{B}|} = \frac{1}{N} (2 \cos \theta \hat{\mathbf{e}}_r + \sin \theta \hat{\mathbf{e}}_\theta) \quad (5.41)$$

where:

$$N = \sqrt{4 \cos^2 \theta + \sin^2 \theta} \quad (5.42)$$

So, the parallel projection of the acceleration along the field lines is given by:

$$a_{||} = -\mathbf{b} \cdot \nabla U = \frac{2 \cos \theta}{N} \left[- \left(\frac{GM_p}{r^2} + \Omega^2 r \right) + \frac{3}{2} f \Omega^2 r \sin^2 \theta \right] \quad (5.43)$$

It is required for the quantity in brackets to be positive, since the net acceleration should be away from the planet. In order to achieve this, it can be used the concept of magnetic footpoint θ_b (Jardine et al., 2006; Trammell et al., 2011). This represents the angle at which the wind opens the field lines to infinity - Figure 5.13 should help to understand the importance of this relation.

$$r(\theta) = R_p \frac{\sin^2 \theta}{\sin^2 \theta_b} \equiv r_{eq} \sin^2 \theta \quad (5.44)$$

From other derivations in Trammell et al. (2011), it is possible to approximate:

$$\sin^2 \theta_b \sim \left(\frac{\dot{M} v_\infty}{R_p^2 B_p^2} \right)^{\frac{1}{4}} \quad (5.45)$$

Thus, the "magnetic Roche-lobe radius" at which the projected acceleration $a_{||} = 0$ is given by the solution of the equation:

$$\frac{GM_p}{r_{KB}^2} + \Omega^2 r_{KB} = \frac{3}{2} f \Omega^2 \frac{r_{KB}^2}{r_{eq}} \quad (5.46)$$

which can be described in terms of global parameters in the following way:

$$\frac{\lambda}{x^2} + \epsilon x = \frac{3}{2} \epsilon f \sin^2 \theta_b x^2 \quad (5.47)$$

where $x = r_{KB}/R_p$.

It is also possible to derive an equation to estimate the sonic point r_s . However, the problem must be simplified so that the sonic point is close enough to the planet for the magnetic stresses to dominate over hydrodynamic stresses - which is also known as rigid

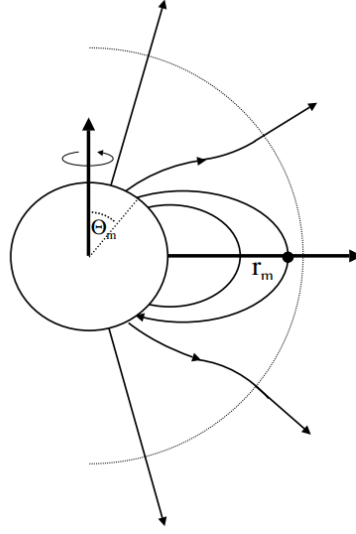


FIGURE 5.13: Scheme taken from [Jardine et al. \(2006\)](#), depicting the magnetic structure of a dipole field that has been distorted by the outward pressure of an outflowing gas trapped on the field lines. Beyond some radius, the pressure of the gas is large enough to force open the magnetic field lines. The last closed field line that intersects the equatorial plane at a radial distance r_m (which in this discussion is r_{eq}) and emerges at a base latitude Θ_m (θ_b).

field line approximation. In this situation, the fluid nearly corotates with the planet, and can be accelerated by the magnetic field. Thus, the velocity can be described as a parallel velocity: $\mathbf{v} = v\mathbf{b}$. Additionally with the no monopoles condition, the continuity equation assume the following form:

$$\mathbf{B} \cdot \nabla \left(\frac{\rho v}{B} \right) = 0 \quad (5.48)$$

which means that $\rho v / B$ is a constant along field lines and can be interpreted as the mass loss rate per unit of magnetic flux along a flux tube of area $\propto B^{-1}$.

The Euler equation along a field line s is written as:

$$v \frac{dv}{ds} = -c_s^2 \frac{d \ln \rho}{ds} - \frac{dU}{ds} \quad (5.49)$$

and when inserting the mass conservation equation, it yields:

$$\left(v - \frac{c_s^2}{v} \right) \frac{dv}{ds} = -c_s^2 \frac{d \ln B}{ds} - \frac{dU}{ds} \quad (5.50)$$

So, at the sonic point, where $v = c_s$:

$$-c_s^2 \frac{d \ln B}{ds} = \frac{dU}{ds} \quad (5.51)$$

The term on the LHS represents the pressure gradient due to the geometry set by the magnetic field, and the term on the RHS is the net acceleration along the field line.

Taking the dipolar geometry:

$$-\frac{d \ln B}{ds} = \frac{3 \cos \theta}{Nr} \left(2 + \frac{\sin^2 \theta}{N} \right) \quad (5.52)$$

and from what has been derived previously:

$$-\frac{dU}{ds} = \frac{\cos \theta}{Nr} \left(\frac{GM_p}{r} + \Omega^2 r^2 \left(\frac{3}{2} f \sin^2 \theta - 1 \right) \right) \quad (5.53)$$

it yields:

$$-c_s^2 \frac{d \ln B}{ds} = \frac{dU}{ds} \implies 3c_s^2 \left(2 + \frac{\sin^2 \theta}{N} \right) = \left(\frac{GM_p}{r} + \Omega^2 r^2 \left(\frac{3}{2} f \sin^2 \theta - 1 \right) \right) \quad (5.54)$$

Using the field line geometry where $r(\theta)$, it is possible to eliminate any θ dependency with $\sin^2 \theta = r/r_{\text{eq}}$. This gives:

$$3c_s^2 \left(2 + \frac{1}{2} \frac{r}{r_{\text{eq}}} \frac{1}{\sqrt{1 - \frac{3}{4} \frac{r}{r_{\text{eq}}}}} \right) = \frac{GM_p}{r} + \Omega^2 r^2 \left(\frac{3}{2} f \frac{r}{r_{\text{eq}}} - 1 \right) \quad (5.55)$$

which is all taken in $r = r_s$. This equation can be re-written such that only depends on global parameters:

$$6 + \frac{3x}{2\sqrt{1 - \frac{3}{4}x}} = \frac{\lambda \sin^2 \theta_b}{x} + \frac{\epsilon}{\sin^4 \theta_b} x^2 \left(\frac{3}{2} f x - 1 \right) \quad (5.56)$$

where $x = r_s/r_{\text{eq}}$ and all other variables have been previously derived. From the square root, it appears the condition that:

$$r_s < \frac{4}{3 \sin^2 \theta_b} R_p \quad (5.57)$$

There is the important detail that both r_{KB} and r_s depend on λ , ϵ and θ_b , however, the latter is defined by the mass loss at the base of the HJ, which itself depends on the λ parameter. Thus, when trying to recover these radii (equations (5.47) and (5.57)), this must be taken into account on the parameters' space phase. If the intend is to explore solutions varying λ with fixed ϵ^* , then $\epsilon^* \longrightarrow \epsilon \left(c_{s,p}/c_{s,p}^* \right)^2$, where $c_{s,p}^*$ is the sonic sound speed which is altered due to parameter's self consistency.

Figure 5.14 shows plots done in order to appreciate the behaviour of the radii with respect to ϕ , the azimuthal angle.

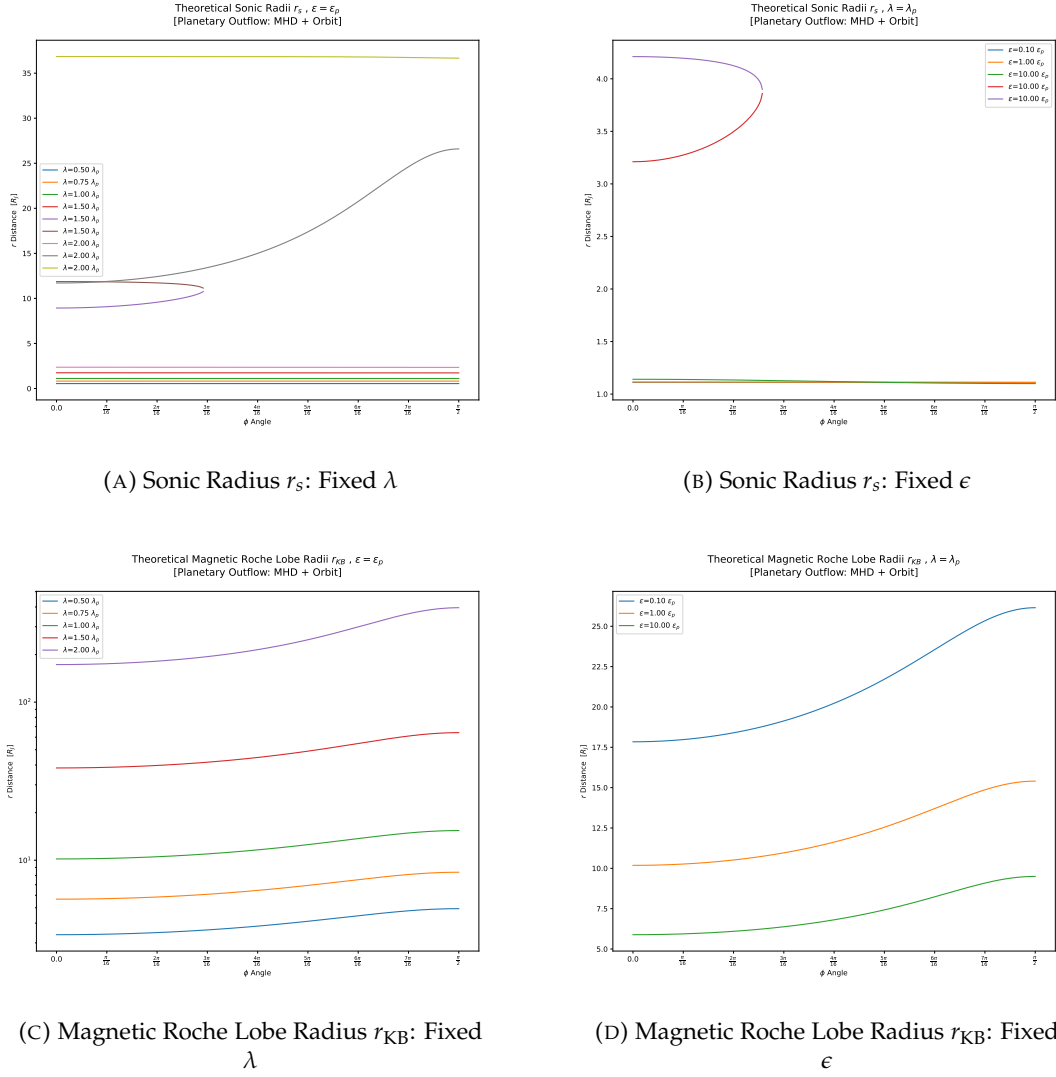


FIGURE 5.14: Plots for theoretical sonic radius and magnetic Roche Lobe radius by varying $\lambda(c_s)$ or $\epsilon(\Omega)$ parameter.

As would be expected, the increase of λ , which translates to a weaker outflow with stronger gravitational pull, enlarges the sonic surface. However, due to having ϵ fixed with relation to λ , degeneracy starts appearing. The appearance of a triplet of solutions can give rise to interesting concepts: for example, a local torus of supersonic flows followed by another torus of subsonic flow around the $x = 0$ plane (as $\lambda = 1.5\lambda_p$) or even an entire blob around the HJ with the same dynamic ($\lambda = 2.0\lambda_p$). Since the flow was initially considered to be parallel to the magnetic field, with $\rho v / B$ being constant along field lines, this degeneracy will be strongly related to rotation as angular momentum conservation takes place.

The magnetic Roche Lobe radius is somewhat similar by the same physical principles:

the increase of λ increases the gravitational influence of the planet on the magnetically controlled flows. Additionally, varying ϵ seems to be inversely proportional, as tidal influence is also taken into account in a Roche Lobe formulation.

5.3.1 Simulation Framework

The simulation strategy is quite similar to Section 5.2: the grid and initial conditions are the same. However, the main change is related to the forces that the flow endures: there is the insertion of the stellar gravitational potential and the fictitious forces derived from the orbiting planet in the local frame - Section 4.4. Due to the same reasons as before, the anisotropic model will be left as future work. Thus, this section will only regard the *M-IsoCor* Model.

5.3.2 Results

Due to high Mach numbers that form close to the boundaries as tidal effects start to affect strongly the weakly bounded gas, the simulation had to be stopped. The final output was at $t = 6.00s$, which is generally when the slow perturbations start propagating outside the sonic radius. This means that inside $r \sim r_s$ is present the configuration close to the numerical steady-state.

Figure 5.15 shows the alfvén speed with magnetic field lines of the last output.

Additionally, Figure 5.16 shows the temperature profile for the same model.

5.3.3 Important Remarks

As can be seen in Figure 5.16, the simulation is still far from a steady state as the equatorial plane shows a clover-like region distorted by tidal effects of higher local temperature. This region is originated by a energy transfer between the magnetic stress to the gas. Since the field lines are opening up, the ram pressure and consequently, the gas' temperature increase close to the planet where these lines begin. Thus, the heating originated in the poles is then transferred to the equatorial plane. The heated gas is then launched by Coriolis forces until reaching the boundaries of the grid. These boundaries are highly disturbed since the boundary conditions of outflow originate some negative pressure zones which is replaced by **PLUTO** with a previously defined minimum value of pressure.

Until not having a *M-IsoRot* Model which is at least comparable to theoretical results, this present simulation is also required to be revised in future works. One possible way to

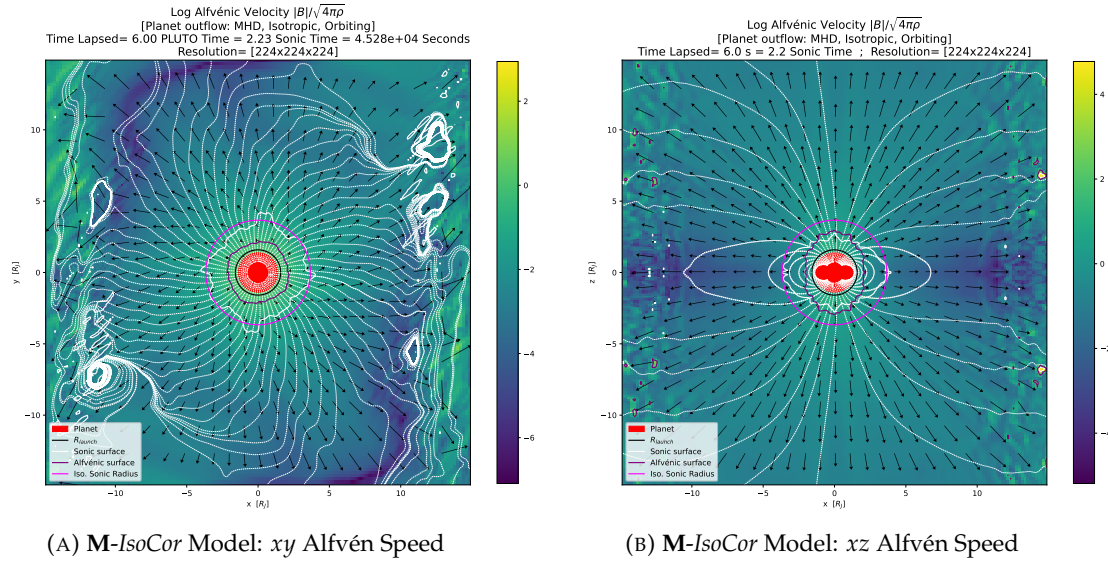


FIGURE 5.15: Alfvénic speed plots for **M-IsoCor** Model in xy and xz planes, normalized to the planet's base alfvénic speed $v_{\text{alf},p} = \frac{B_p}{\sqrt{4\pi\rho_p}}$ and displayed in a logarithmic scale, with the sonic and alfvénic surfaces overplotted, and the magnetic field lines in dotted white and flow's velocity direction with arrows.

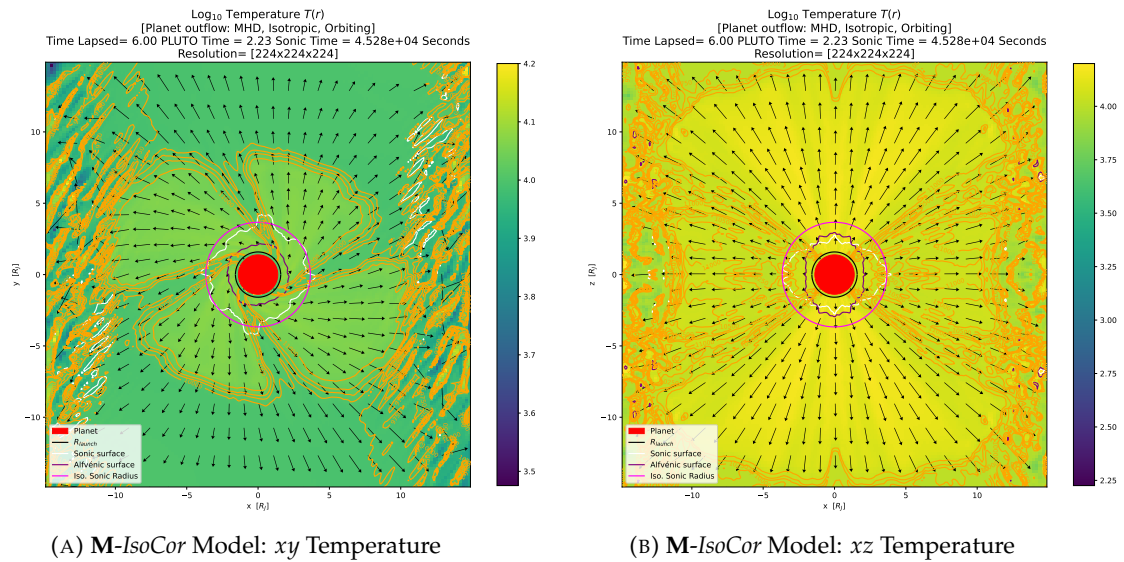


FIGURE 5.16: Temperature plots for **M-IsoCor** Model in xy and xz planes, in logarithmic scale of base 10.

allow for less boundary problems is by relaxing the tidal forces that are due to the stellar host, in a similar way as the temperature anisotropy. In an initial instance, the steady state would be an isotropic spinning planet and would slowly bend the streamlines in an up-orbit and down-orbit manner as seen in the HD chapter. As a consequence of orbit, the furthest point at which the field lines are still close in the equator- r_{eq} distance - would shrink, meaning a slightly wider footpoint angle θ_b , by equation (5.44).

5.4 MHD Planetary Wind from an Orbiting and Rotating Planet embedded in a Stellar Wind

For such a complex dynamical system, it is required to retrieve characteristic lengths, as in the work of Matsakos et al. (2015). Now, at a MHD level, magnetic pressure is must be taken into account. Thus, the interface of the shock structure is described by radial considerations when equating specific planetary pressures to the ambient pressure defined by the stellar wind, which now carries an extra term:

$$P_{\text{amb}} = P_{\text{th}} + P_{\text{ram}} + P_{\text{orb. ram}} + P_{\text{mag}} \quad (5.58)$$

where the first three terms have already been discussed (equations (4.93), (4.94), (4.95) respectively) and

$$P_{\text{mag}} = \frac{B^2}{8\pi} = \frac{B_*^2}{8\pi} \left(\frac{R_*}{r} \right)^6 \quad (5.59)$$

if only radial terms from a dipolar configuration are taken into account.

As a direct consequence, a standoff magnetic distance can be expressed, by equating the magnetic pressure of the planetary wind to the ambient pressure:

$$P_{\text{amb}} = \frac{B_p^2}{8\pi} \left(\frac{R_p}{r} \right)^6 \implies R_m = \left(\frac{B_p^2}{8\pi P_{\text{amb}}} \right)^{\frac{1}{6}} R_p \quad (5.60)$$

The behaviour of the magnetic standoff radii has a very distinct behaviour from the wind strength characteristic radius: the magnetic pressure drops with the sixth power of the distance, while the ram pressure has a drop that goes exponentially due to density times the square of the velocity contribution. This again, puts the subsonic outflow zone being magnetically dominated.

Additional to this discussion, it is possible to evaluate the typical distance where $R_m \sim R_w$:

$$\left(\frac{B_p^2}{8\pi P_{\text{amb}}} \right)^{\frac{1}{6}} R_p = \left(\frac{\dot{M}_p v_p}{4\pi P_{\text{amb}}} \right)^{\frac{1}{2}} \implies \frac{(\dot{M}_p v_p)^3}{B_p^2 R_p^6}(r) = 8\pi^2 P_{\text{amb}}^2(r) \quad (5.61)$$

5.4.1 Simulation Framework

The typical *Iso* model in the HD framework would take, at most, 6 hours to compute. Now, each 0.25s parameter file *flt* in the MHD framework takes between 45 minutes to 2 hours, if the time stepping Δt does not fall from its maximum value (if it does, it may take 12 hours). As a consequence, this section will only take into discussion one case, which is the case where most of the parameters follow the main characteristics of the HD209458 system. Thus, the chosen grid follows:

- *x*-direction: stretched grid from $x = -15$ to $x = -5$ with a resolution of 120 cells, and from $x = +5$ to $x = 30$ with a resolution of 180 cells; and a regular grid between $-5 \leq x \leq +5$ with a resolution of 125 cells.
- *y*-direction: stretched grid from $x = -30$ to $x = -5$ with a resolution of 180 cells, and from $x = +5$ to $x = +15$ with a resolution of 120 cells; and a regular grid between $-5 \leq x \leq +5$ with a resolution of 125 cells.
- *z*-direction: stretched grid from $x = -14$ to $x = -5$ and from $x = +5$ to $x = +14$ with equal resolutions of 64 cells; and a regular grid between $-5 \leq x \leq +5$ with a resolution of 125 cells.

This grid has an increased resolution in comparison with the previous grids used for the HD wind-wind models due to the fact that several magnetic reconnections will most likely take place at the shock zone. This requires a high enough cell density to capture the parabolic spatial integration without any parameter "explosion" - negative densities or very high Mach numbers.

The local grid has the stellar wind inserted at the beginning of the *x* boundary and at the end of the *y* boundary, as before, although now it is magnetized. The magnetic wind follows the theoretical description of a magnetic rotator, with explicit momentum conservation.

The only model studied will be the **M-Strong** Wind Case, which follows the same parameter choice as in the HD case (Section 4.5), with a stellar base magnetic field such

that:

$$B_*(R_s) = 2G \implies \beta \sim 9.55 \times 10^{-4} \quad (5.62)$$

as defined by equation (5.8).

The HJ's outflow is not inserted as [Matt and Balick \(2004\)](#)'s configuration, where the planetary wind is parallel to the field's direction, since the shock will bend the field to a magnetospheric configuration. Careful analysis would have to be done in such a model, which is clearly one of the future objectives. For the time being, a Parker Wind hydrostatic launch configuration with an overlapped dipolar field is used (density law follows an HD framework).

5.4.2 Results

The model took around 16 days to run on 45 CPUs in two different nodes. Figure 5.17 displays the plots of the logarithm in base 10 of normalized alfvén velocity of the final state of the simulation.

In order to observe the dynamics of this interaction, there have been displayed animations in the [link*](#). These animations also include a close-up on the HJ's dynamics.

5.4.3 Important Remarks

This model characterizes a system where magnetic pressure appears to have a role. This parameter is inserted both in the ambient pressure from the stellar wind and in the HJ's outflow. At first order, it is possible to understand the magnetic influence by comparing it with the typical pressure of each object:

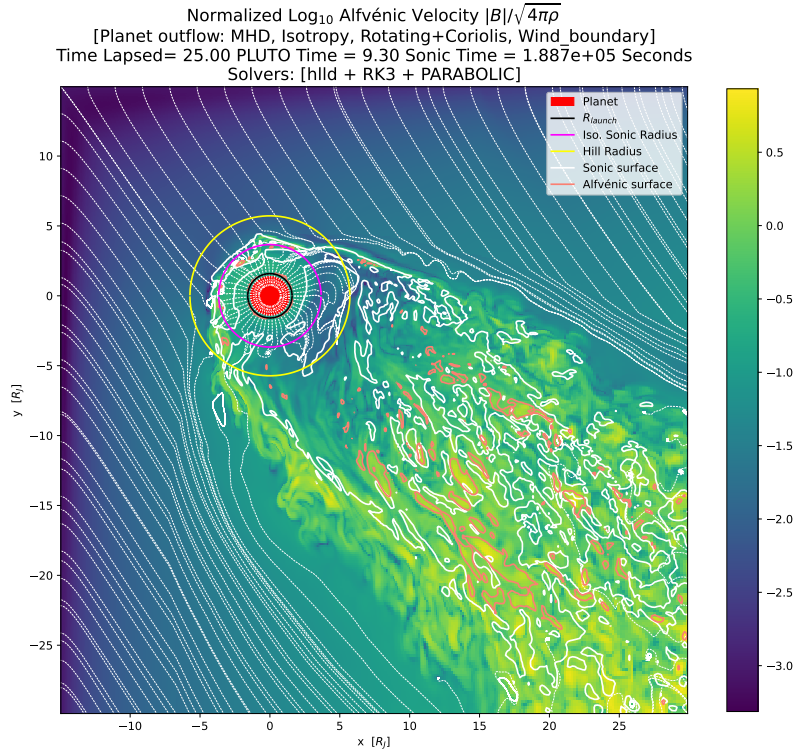
$$\left. \frac{P_{\text{mag}, p}}{P_{\text{ram}, p}} \right|_{r=r_s} \sim \frac{B_p^2}{8\pi} \left(\frac{R_p}{r_s} \right)^6 (\rho_p c_{s,p}^2)^{-1} = \frac{B_p^2}{8\pi} \left(\frac{2}{\lambda_p} \right)^6 (\rho_p c_{s,p}^2)^{-1} \quad (5.63a)$$

$$= \frac{B_p^2}{8\pi} \left(\frac{4c_{s,p}^2}{v_{\text{esc}, p}^2} \right)^6 (\rho_p c_{s,p}^2)^{-1} = \frac{512}{\pi} \frac{B_p^2 c_{s,p}^4}{\rho_p v_{\text{esc}, p}^6} \quad (5.63b)$$

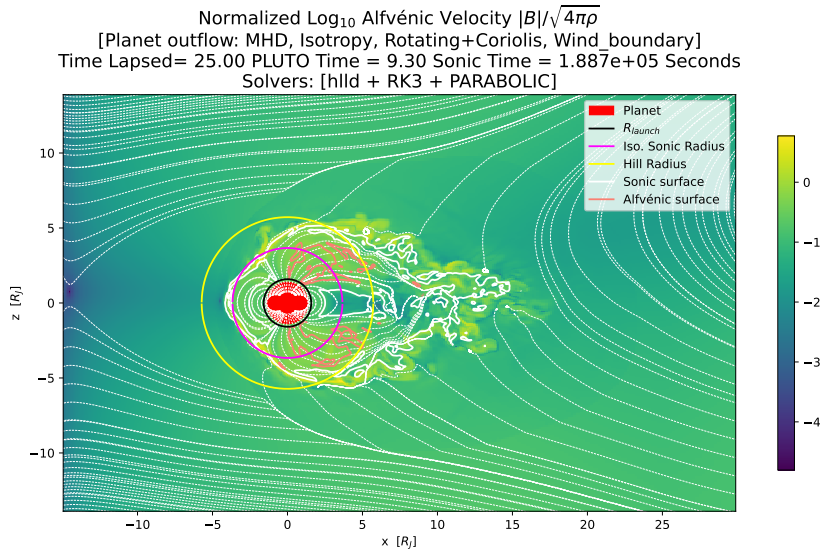
$$\sim 0.499 \left(\frac{B_p}{1G} \right)^2 \left(\frac{\rho_p}{5 \times 10^{-15} \text{gcm}^{-3}} \right)^{-1} \left(\frac{T_p}{10^4 \text{K}} \right)^2 \left(\frac{R_p}{R_J} \right)^3 \left(\frac{M_p}{M_J} \right)^{-3} \mu^2 \quad (5.63c)$$

Thus, the magnetic pressure is critical at close distances, preferentially, for $r \lesssim r_s$. Which means that the ambient pressure, which is stellar dominated, is magnetically weak, since the magnetic pressure decays with the sixth power of distance, whilst other pressures

* Link for paste with animations regarding the Wind-Wind Interaction models: [MHD_SPI](#)



(A) *M-Strong* Wind Model: xy Alfvén Speed



(B) *M-Strong* Wind Model: xz Alfvén Speed

FIGURE 5.17: Alfvénic speed plots for *M-Strong* Wind Model in xy and xz planes, normalized to the planet's base alfvénic speed $v_{\text{alf},p} = \frac{|B_p|}{\sqrt{4\pi\rho_p}}$ and displayed in a logarithmic scale, with the sonic and alfvénic surfaces overplotted, and the magnetic field lines in dotted white.

prevail. This means that the bow-shock from the magnetized winds is quite sensitive to the magnetic stresses close to the planet.

The magnetic standoff distance R_m , along with the ram standoff distance R_w can now be compared in a simple static and isothermal model. Figure 5.18 shows the plots from both planes that compare these characteristic scales.

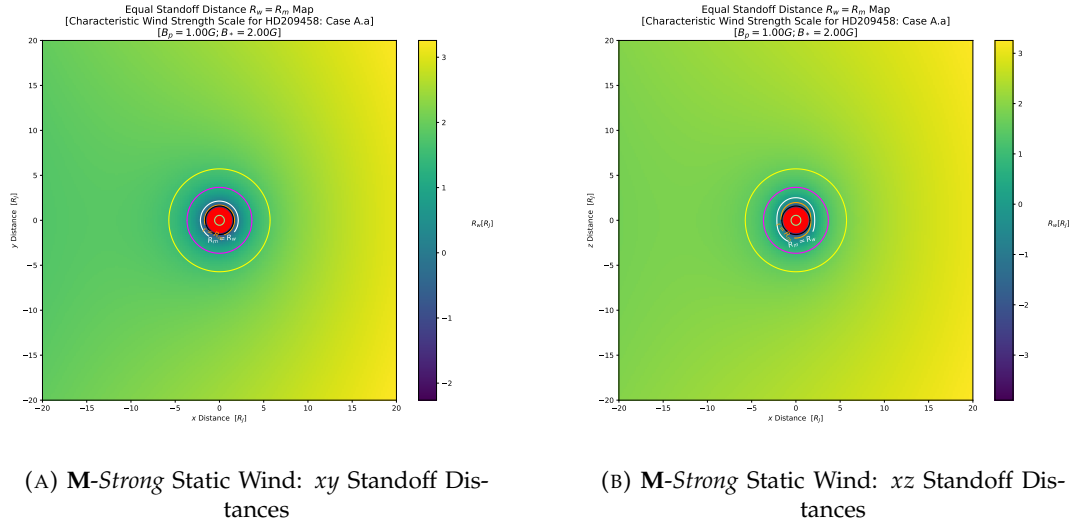


FIGURE 5.18: Standoff Distances in a static isothermal model, both magnetic R_m and ram R_w . The lines plot the region where the distance of the planet to the pressure equilibrium match (greenish line represents $r = R_w$), as well as where both scales match.

The simulations seem to have a higher R_m , which may be due to momentum conservation of the stellar magnetized wind and hydrostatic launch of a magnetic planetary wind - as seen before, is stronger than expected.

The xz plane in Figure 5.17 shows a geometry similar to the Earth's magnetosphere. According to several works (Bisikalo et al., 2021; Carolan et al., 2021; Daley-Yates and Stevens, 2018; Matsakos et al., 2015), this configuration is very natural to appear in this type of systems. The magnetized stellar wind shocks with the front of the HJ's magnetic field, shortening the latter's extension. In addition to this, the polar lines which were open, are now bent drastically such that they reconnect at the tail of the bow-shock. As a consequence, the magnetic stresses and ram pressure are such that the polar regions are super-alfvénic and super-sonic. Another characteristic trait of this wind-wind interaction is the appearance of the "alfvén wings" as known in the literature. These are the bent stellar field lines that reconnect at the back of the tail, as magnetic perturbations have already been decayed.

Kelvin-Helmholtz and Rayleigh-Taylor instabilities are present, as they previously did as well. Shear velocities and strong density differences give rise to a complicated dynamic, which now has an additional energy transfer due to magnetic stresses. It is perhaps due to this additional mechanism that the bow-shock has a comparative enlarged envelope to the HD simulations. Several interesting consequences can be taken from this enlargement, although this peculiarity, which is not commonly captured in other works, may point to new discoveries under synthetic observations.

Nonetheless, this model also requires some re-doing. For example, the grid was initiated with the HJ's magnetic flow in every cell, and the stellar wind was inserted at the boundary, in the "ghost cells". As a consequence, the stellar wind lost the expected spiral curvature, and became bounded to the boundaries by the "fictitious currents", affecting it to become almost oblique. This can indeed enlarge the bow-shock, in an undesired way, since the orbiting planet is now facing a front which is symmetric in a sense. One possible way to amend this is by inserting the planetary wind inside a sphere big enough for the lines to open, before the stellar influences are heavily felt. In a totally distinct idea, another possible future necessity is to fine-tune the parameters inside the $\nabla \cdot \mathbf{B} = 0$ solver such that it becomes "boundary-friendly".

5.5 Discussion

The MHD Chapter has a realistic touch to its main objective: the interaction between winds with the inclusion of magnetic fields. Understanding the dynamics is rather important, since observations with neutral atoms seem to deviate highly from the theoretical predictions. And with magnetized winds, the free electrons and polarized particles endure effects that are not visible in the HD framework. Works like [Carolan et al. \(2021\)](#) and [D'Angelo et al. \(2018\)](#) explore the planetary outflow with a null field and with a dipolar field interacting with a always magnetized wind. These are known in the literature by 'unipolar interactions' or 'dipolar interactions' ([Strugarek et al., 2012](#)). Although being an interesting concept, as it bridges smoothly between the non-magnetized and magnetized interacting winds, it escapes slightly the purpose of this thesis. Nevertheless, comparison between the HD and MHD frameworks is now possible - with the exception of the anisotropic models. The results presented have proven important to retain the following aspects:

- A static magnetic object with a dipolar configuration with outflow has never been regarded with much interest. In fact, the need to develop the explicit momentum equation for (v_r, v_θ) still exists. As most theoretical developments of magnetized outflows use a monopole field configuration, several aspects appear in a simple way. However, with a higher angular momentum configuration, the magnetically controlled configuration needs to be explicitly inserted - as [Matt and Balick \(2004\)](#) discussed. Nevertheless, it is possible to compare the behaviour of the static magnetized outflow with a limiting case of a spinning magnetic object forced to have $\Omega \rightarrow 0$. Above all, the opening of the field lines at the poles was obtained, either in a configuration where the magnetic field is superimposed in the Parker Wind, or in a model where the launching wind is parallel to the direction of the magnetic field (which follows the encouraged numerical proceeding [Matt and Balick \(2004\)](#)).
- The theory of a magnetic rotor has seen developments since the work of [Parker \(1958\)](#). Tidal effects of the HJ itself or due to the stellar companion can affect several aspects from the angle at which field lines open, the footpoint angle θ_b , which is related to the r_{eq} distance in the equatorial plane by equation (5.44), angular momentum conservation, and so on. However, due to computationally heavy necessities, the analysis of these models falls behind of expectations. From the works of [Trammell et al. \(2011\)](#); [Trammell et al. \(2014\)](#); [Zanni and Ferreira \(2009\)](#), a comparison by eye can be made to allow verify the main outflow properties.
- Although the parameter space-phase has increased, the wind-wind interaction section did not exploit enough models. This is because the time a simulation takes to run is too extensive for the availability. However, the model seen which corresponds to a strong-stellar wind case engulfing the planetary outflow out to a tail has characteristic analogous to other works ([Daley-Yates and Stevens, 2018](#); [Carolan et al., 2021](#); [Matsakos et al., 2015](#); [Zhilkin and Bisikalo, 2019](#)). The necessity to quantitatively compare the bow-shock width and mass loss remains, as these are rarely addressed. Interestingly, the observed enlargement of the bow-shock compared to the HD case proves the importance of inserting the magnetic fields. As observations can retrieve the main characteristics of a transit of a planet, a larger darkening or higher redshift/blueshift of absorption lines can flag the existence of magnetic fields. The works of [Carolan et al. \(2021\)](#) and [D'Angelo et al. \(2018\)](#) explore this phenomena: they produce these models and then take what are known

as "synthetic observations" - simulate the information that would be retrieved in a real observation - in order to study the influence of magnetic presences.

- The main reason why it is possible to compare the latter and most complex model to the existing literature, instead of the most simple can be mainly justified by the computational physics behind the models. Firstly, the resolution and dimensionality - which also relates to memory and time issues - can fall behind compared to most works. Unfortunately, from the last part of the semester, the available processors decreased due to hardware update and software transfer, which also bear a heavy burden. Secondly, as **PLUTO** behaves as a Gudonov scheme, the combination of the type of grid (cartesian) with the spatial integration, Riemann flux solver - which also takes into account diffusivity - and the boundary conditions cause quite an impact. This was discussed inside several subtle difficulties, for example, as the "ghostly currents" at the "ghost cells" of the boundaries of the grid and the fact that the magnetic field has an accelerated decrease around the borders. Clearly, a need to entail all these numerical hazards into a beneficial tool grows as the complexity of the problem becomes important. The work of [Daley-Yates and Stevens \(2019\)](#) discusses this observation as well - since the authors also use **PLUTO**.

5.5.1 Concluding Remarks

In this Chapter, it was possible to evaluate the impacting differences when magnetic fields are employed in the models. Photoevaporation models need to grow in sophistication when distinguishing the presence or absence of magnetically controlled outflows. As the concept of habitability for astrophysics becomes a growing topic, Earth-like conditions entangled in observations are worthy of attention. As seen in Section 5.4, the model showed a not so strange magnetosphere, which can be essential to target habitable exoplanets. As the several layers behind the shock of two magnetized winds were discussed, from the static to a tidally-locked orbiting HJ, important aspects were raised. Notice that, although some works have already employed **PLUTO** for MHD photoevaporation such as [Matsakos et al. \(2015\)](#) and [Daley-Yates and Stevens \(2017, 2018, 2019\)](#), new aspects have been unlocked with this dissertation. As a title of example, the importance of respecting the outflow natural conditions has been discussed. Numerical difficulties have not allowed for the intended need to construct an explicit anisotropic temperature around the planet with magnetic dipolar configurations, which would be claimed as the first work with such

characteristics. The additional advantage of the latter point would be to complement the road that [Carroll-Nellenback et al. \(2017\)](#) and [Stone and Proga \(2009\)](#) had paved. This would bridge these HD papers to the MHD with implicit temperature at the HJ's atmosphere, allowing to understand how a simple model can bring understanding to complex observational aspects.

Chapter 6

Summary and Conclusion

In this Chapter, a brief summary regarding the main aspects of this dissertation is presented, alongside the missing objectives that would complement positively the discussion.

6.1 Summary

The main purpose of this dissertation has been to develop a solid ground-base study of photoevaporation theory applied to a physical existing system: HD209458. Supported by the works of [Stone and Proga \(2009\)](#) and [Carroll-Nellenback et al. \(2017\)](#), inside a HD framework, a HJ's outflow was developed with increasing complexity.

Starting with a simple static and isolated planetary outflow, we were able to reproduce, in Chapter 4, the main aspects of a Parker wind. Discussion allowed to verify the consistency of the base outflow, either quantitatively as qualitatively, with special emphasis on the numerical sonic surface compared to the theoretically predicted one, as well as mass loss behaviour. As it is intended to take the simulation the closest possible to a simple yet rather realistic case, temperature anisotropy on the heating layer is inserted explicitly, as a way to reproduce a "dayside"/"nightside" behaviour. As a consequence, the outflow's behaviour changes strongly, with the appearance of a "nightside" shock due to the bending of the streamlines derived by the temperature gradient and lateral pressure. Additionally, inside the shock region, gas inflow back to the planet is also verified, which in terms of atmospheric circulation and habitability can become a quite important aspect.

As comparison to other works where temperature is explicitly inserted ([Stone and Proga, 2009](#); [Carroll-Nellenback et al., 2017](#)) or implicitly calculated ([McCann et al., 2019](#))

allowed us to verify the latter simplistic model, while additional layers of physical phenomena were implemented: inserting rotation to the planet, and orbital trajectory originate new steady-states. These models are explored both in an isothermal and anisotropic framework.

In the final part of Chapter 4, we studied the interactions between the stellar wind and planetary wind. These final models are, at first order, the closest to what observations eventually detect: bow-shocks when the stellar wind is strong enough to push all the planetary outflow backwards into a comet-like tail and less-well defined structures for when the pressure strength of both winds become comparable at higher distances. When the temperature gradient is switched on, the symmetric aspect of the bow-shock disappears as the HJ's backside outflow is not strong enough to face the extreme ram pressures. However, in an intermediate case, the observed "burping" phenomena is lost as the gas cannot reach the Hill radius since the "dayside" outflow as a given amount of gas is bent to the "nightside".

In Chapter 5, magnetic fields were inserted - enabling us to move from HD to the MHD description. This, in turn, lead to more complex models, taking our simulations one step-closer to more realistic cases. The same structure as the prior chapter was used, starting of with a simple static and isothermal outflow model and improving with the addition of new layers. As discussed, temperature anisotropy models become extremely sensitive and time demanding, since the bending of the field lines creates an energy transfer to the gas which surpasses the typical Mach numbers $M \sim 3.5$, reaching values of $M \gtrsim 250$, which force the RK3 time-stepping to decrease too harshly. Nonetheless, it was possible to reach the final model of a strong and magnetized stellar wind case interacting with the magnetized HJ's outflow. This latter model showed an increased bow-shock with highly complex magnetic dynamics along the tail due to Kelvin-Helmholtz and Rayleigh-Taylor instabilities. Additionally, the planet developed a Earth-like magnetosphere, where the polar lines reconnected at the back of the planet.

In summary, our results clearly show the importance of adding temperature anisotropy, on one hand, and magnetic field on the other. Both are crucial ingredients that should be included in any photoevaporation models that aim to describe planetary winds surrounded by winds from their host stars.

6.2 Future Directions

The discussion along the Chapters 4 and 5 made it clear that several aspects still need additional research. Relative to the HD framework:

- Although several aspects achieved checked the main goal of obtaining similar results to [Carroll-Nellenback et al. \(2017\)](#), a ballistic understanding of the gas particles would characterize with much higher comprehension the outflow's behaviour. In order to execute such an analysis, it would be necessary to propagate a point-like particle over the grid following the velocity field - this can also be acquired by creating a new scalar variable inside **PLUTO**.
- The stellar wind was required to be altered in order to observe two distinct morphological cases, however, one other model can still be obtained: the weak wind case. This case is present in [McCann et al. \(2019\)](#); [Debrecht et al. \(2020\)](#), however, from what has already been tested, a larger grid would be required. Consequently, higher computational power and time windows would follow as needs.
- The calculations of mass loss and mass inflow also require some adjustments: as it is being used a shell with small width and varying radius, the result becomes blunt to the latitude and longitude of flow relative to the planet. The construction of shells with mass-loss per solid angle ($\text{gs}^{-1}\text{sr}^{-1}$) of varying radius can shed some light into the effects of lateral pressures on inflow - as Figure 5.12 does, though lacks resolution. This can allow for a much more precise analysis on the temperature relaxation function used, specially shock-formation relation. Additionally, it would allow to confirm existing symmetries inside the model.

On the MHD numerical simulation, some urgent requirements are also in need for a near-future plan:

- Perhaps the most important aspect to achieve is having a MHD model with temperature anisotropy. This would not only open new outflow behaviours which are in need of being quantified and understood, as well as would create a complete scheme between the works of [Stone and Proga \(2009\)](#); [Carroll-Nellenback et al. \(2017\)](#) and [Matsakos et al. \(2015\)](#); [Zhilkin and Bisikalo \(2021\)](#). The requirements to achieve this are simple: computational time.

- Understanding how solvers, grid and boundary aspects can be modified in order to have a positive outcome in the simulations instead of disrupting the physical behaviour of the evolving system. For example, the compatibility with some solvers on the divergence of the field $\nabla \cdot \mathbf{B} = 0$ with the type of spatial integration methods or the diffusivity present in the Riemann flux solvers must be better understood as a whole. Thus, it is proposed as future works to evoke the same type of scheme done in Chapter 5, Section 5.1 in Table 5.1. However, instead of being a Boolean scheme, quantifying some important aspects would be the main goal.
- Exploring the parameter space-phase would also be a crucial task. Several examples are: the intensity of the stellar wind - as a means to compare all cases with the HD framework; the strength of the stellar and planetary field, as well as direction and polarity; higher order momentum configurations (quadrupolar, octapolar, etc.). The morphological diversity which can be obtained (Matsakos et al., 2015) may be of major interest to justify observations, as well as shed light into physical correlations between parameters.
- Lastly, grasping the more subtle details regarding the opening of field lines can also come at hand for more diverse models. As the flow is assumed magnetically controlled, the need to construct a correct magnetic behaviour in the models is always present, specially when focused on mass inflow.

6.3 Closing Remarks

As a closing statement, there is a wide diversity of modelling and numerical simulation on photoevaporation theory. This dissertation should present itself as an initial step on such an interesting and physically fruitful path of knowledge.

Part of the work on this dissertation was supported by Fundação para a Ciência e a Tecnologia (FCT) through the research grants UIDB/04434/2020 and UIDP/04434/2020.

Appendix A

Mathematical Expressions

A.1 Spherical Coordinates Considerations

Spherical Polar Coordinates (r, θ, ϕ)

A.1.1 Algebra

$$\nabla A = \frac{\partial A}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial A}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial A}{\partial \phi} \hat{\phi} \quad (\text{A.1})$$

$$\nabla \cdot \mathbf{B} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 B_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta B_\theta) + \frac{1}{r \sin \theta} \frac{\partial B_\phi}{\partial \phi} \quad (\text{A.2})$$

$$\begin{aligned} \nabla \times \mathbf{B} = & \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} (\sin \theta B_\phi) - \frac{\partial B_\theta}{\partial \phi} \right) \hat{r} + \left(\frac{1}{r \sin \theta} \frac{\partial B_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r B_\phi) \right) \hat{\theta} + \\ & + \left(\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) - \frac{1}{r} \frac{\partial B_r}{\partial \theta} \right) \hat{\phi} \end{aligned} \quad (\text{A.3})$$

$$\nabla^2 A = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial A}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 A}{\partial \phi^2} \quad (\text{A.4})$$

$$\begin{aligned} (\mathbf{B} \cdot \nabla) \mathbf{B} = & \left(B_r \frac{\partial B_r}{\partial r} + \frac{B_\theta}{r} \frac{\partial B_r}{\partial \theta} - \frac{B_\theta^2 + B_\phi^2}{r} + \frac{B_\phi}{r \sin \theta} \frac{\partial B_r}{\partial \phi} \right) \hat{r} \\ & + \left[B_r \frac{\partial B_\theta}{\partial r} + \frac{B_\theta}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{B_r B_\theta}{r} + \frac{B_\phi}{r \sin \theta} \left(\frac{\partial B_\theta}{\partial \phi} - \cos \theta B_\phi \right) \right] \hat{\theta} \\ & + \left(B_r \frac{\partial B_\phi}{\partial r} + \frac{B_r B_\phi}{r} + \frac{B_\phi}{r \sin \theta} \frac{\partial B_\phi}{\partial \phi} + \frac{B_\theta}{r \sin \theta} \frac{\partial}{\partial \theta} (B_\phi \sin \theta) \right) \hat{\phi} \end{aligned} \quad (\text{A.5})$$

A.1.2 Transformation to Cartesian

Since most times it is the spherical behaviour of the model that is analytically used, it is then required to translate it to a cartesian grid.

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \theta \\ \phi \end{bmatrix} \quad (\text{A.6})$$

Appendix B

HD Outflowing Winds

B.1 Isothermal Wind

B.1.1 Transcendental Equation and Explicit Solutions

Equation (4.10) can be re-written into a non-dimensional form. Having $w = \frac{v}{c_s}$, it follows:

$$\frac{1}{v} \frac{dv}{dr} = \frac{1}{v^2 - c_s^2} \left(\frac{2c_s^2}{r} - \frac{GM}{r^2} \right) \quad (\text{B.1})$$

$$\iff \frac{1}{w} \frac{dw}{dr} = \frac{2c_s^2}{c_s^2} \frac{1}{w^2 - 1} \left(\frac{1}{r} - \frac{r_c}{r^2} \right) \quad (\text{B.2})$$

$$\implies \left(w - \frac{1}{w} \right) \frac{dw}{dr} = \frac{2}{r} \left(1 - \frac{r_c}{r} \right) \quad (\text{B.3})$$

Since the desired solution must pass across the sonic point such that: $w = 1$ at $r = r_c = \frac{\lambda R}{2}$. Integrating the last equation and having the constant of integration follow this condition, the continuous solution yields:

$$w^2 - \ln w^2 + 3 = -4 \ln \frac{r}{r_c} + 4 \frac{r_c}{r} \quad (\text{B.4})$$

$$w^2 - \ln w^2 + 3 = -4 \ln \frac{2}{\lambda} + 4 \ln \frac{r}{R} + 2\lambda \frac{R}{r} \quad (\text{B.5})$$

A velocity solution appears in the form:

$$w \exp \left(-\frac{w^2}{2} \right) = \left(\frac{r_c}{r} \right)^2 \exp \left(-\frac{2r_c}{r} + \frac{3}{2} \right) \quad (\text{B.6})$$

The density structure is given by the mass continuity equation:

$$\frac{\rho(r)}{\rho_e} = \exp \left[\lambda \left(\frac{R}{r} - 1 \right) - \frac{1}{2} w^2 \right] \quad (\text{B.7})$$

where the subscript e denotes the object's (star or planet) parameters.

B.2 Polytropic Winds

Since there is no detailed energy deposition into the planetary atmosphere, we can overcome the use of an adiabatic index $\gamma = 5/3$, and use one that is $\gamma = 1.01$. This is because there a "hidden" energy injection built into the adiabatic index, such as an isothermal Parker Wind with $\Gamma \sim 1$. The work of [McCann et al. \(2019\)](#) works with $\Gamma = \gamma = 5/3$ since their intent was to have a refined description of the HJ's atmosphere. Therefore, the outflowing winds in this dissertation are modeled as spherical isentropic winds: a "polytrope" with $\Gamma = \gamma$. By assuming that Γ is equal to the ratio of specific heats, this is, the adiabatic index, we are considering no energy loss in heat or cooling. Consequently, it should possible to recover the isothermal wind at first order.

To investigate the effects of a temperature gradient on the structure of the wind, we consider $T(r) \sim \rho^{\Gamma-1}(r)$, also known as a polytropic relation. The procedure that follows assumes a polytropic index that is constant and $\Gamma > 1$.

We can relate temperature and pressure of a polytropic flow as:

$$T = T_0 \left(\frac{\rho}{\rho_0} \right)^{\Gamma-1} \quad (\text{B.8})$$

and

$$p = \frac{\rho k_B T}{\mu m_p} = p_0 \left(\frac{\rho}{\rho_0} \right)^{\Gamma} \quad (\text{B.9})$$

Now, the pressure gradient in the momentum equation can be eliminated by the equation of mass conservation and the ideal gas law:

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{p}{\rho} \frac{d \ln \rho}{dr} = \frac{k_b T}{\mu m_p} \Gamma \frac{d \ln p}{dr} = \frac{k_b T}{\mu m_p} \Gamma \left(-\frac{1}{v} \frac{dv}{dr} - \frac{2}{r} \right) \quad (\text{B.10})$$

The momentum equation is now written as:

$$\implies \frac{1}{v} \frac{dv}{dr} = \frac{1}{v^2 - a^2} \left(\frac{2a^2}{r} - \frac{GM}{r^2} \right) \quad (\text{B.11})$$

where we have a as the isothermal sound speed, but now is varying with r since we are dealing with a polytropic wind ($a(r) = \sqrt{\Gamma} c_s(r)$). We can see this by using the definition of the isothermal sound speed, the polytropic relation of pressure and density (equation (B.9)) and a consequence of the steady state mass continuity equation $\rho v r^2 = \rho_p v_0 r_0^2$ (where the RHS is all evaluated at $r = R_p$):

$$a(r) = \sqrt{\frac{\Gamma p_0}{\rho_p} \left(\frac{v_0 r_0^2}{v r^2} \right)^{\gamma-1}} \quad (\text{B.12})$$

A solution that starts subsonic at the base of the wind and becomes supersonic further outward must pass through the critical point:

$$r_c = \frac{GM}{2\Gamma a^2(r_c)} \quad (\text{B.13})$$

In the case of an isentropic wind ($\Gamma = \gamma$) presents the advantage that there is no longer an isothermal sound speed, but an adiabatic sound speed:

$$\Gamma c_s^2 \equiv a^2 = \gamma c_s^2 \quad (\text{B.14})$$

The momentum equation can be written in an alternative form by making use of polytropic relations $p \sim (p/\rho)^{\Gamma/(\Gamma-1)}$:

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{p}{\rho} \frac{d \ln p}{dr} = \frac{\Gamma}{\Gamma-1} \frac{p}{\rho} \frac{d \ln \left(\frac{p}{\rho} \right)}{dr} = \frac{\Gamma}{\Gamma-1} \frac{d \left(\frac{p}{\rho} \right)}{dr} \quad (\text{B.15})$$

which gives:

$$v \frac{dv}{dr} + \frac{\Gamma}{\Gamma-1} \frac{d \left(\frac{p}{\rho} \right)}{dr} + \frac{GM}{r^2} = 0 \quad (\text{B.16})$$

$$\implies \frac{d}{dr} \left[\frac{v^2}{2} + \frac{\Gamma}{\Gamma-1} \frac{k_B T}{m_p \mu} - \frac{GM}{r} \right] = 0 \quad (\text{B.17})$$

This implies that a polytropic wind has a constant value of the parameter:

$$e_\Gamma \equiv \frac{v^2}{2} + \frac{\Gamma}{\Gamma-1} \frac{k_B T}{m_p \mu} - \frac{GM}{r} \quad (\text{B.18})$$

for $\Gamma \neq 1$. This constant energy must be positive for the wind to escape. The range of allowed values of e_Γ can be derived from the condition that the velocity law of the wind should be transonic with a critical point. Thus, we find:

$$e_\Gamma = \left(\frac{5-3\Gamma}{\Gamma-1} \right) \frac{\Gamma a_c^2}{2} \quad (\text{B.19})$$

holding true for $1 < \Gamma < 5/3$.

The velocity structure of a polytropic wind can be derived by solving the momentum equation (B.11) and rewriting the e_Γ equation:

$$e_\Gamma = \frac{v^2}{2} - \frac{v_{\text{esc}}^2(r_c)}{2} \frac{r_c}{r} + \frac{\Gamma}{\Gamma-1} a_c^2 \left(\frac{r}{r_c} \right)^{2(1-\Gamma)} \left(\frac{v}{v_c} \right)^{1-\Gamma} \quad (\text{B.20})$$

where subscript c refers to the respective values at the critical point. In this equation it was used another polytropic relation:

$$a^2 = a_c^2 \left(\frac{T}{T_c} \right) = a_c^2 \left(\frac{\rho}{\rho_c} \right)^{\Gamma-1} \quad (\text{B.21})$$

We can make use of a convenient form with the following dimensionless parameters:

$$x = \frac{r}{r_c}, \quad w = \frac{v}{v_c} \quad (\text{B.22})$$

so that:

$$e_\Gamma = \frac{v_c^2}{2} w^2 - \frac{2v_c^2}{x} + \frac{1}{\Gamma-1} v_c^2 x^{2(1-\Gamma)} w^{1-\Gamma} \quad (\text{B.23})$$

Multiplying by $w^{\Gamma-1}$ yields:

$$w^{\Gamma+1} - w^{\Gamma-1} \left(\frac{4}{x} + \frac{5-3\Gamma}{\Gamma-1} \right) + \frac{2}{\Gamma-1} x^{2(1-\Gamma)} = 0 \quad (\text{B.24})$$

This equation describes the dependence of w on x in an implicit way.

If we understand that: $T \sim \rho^{\Gamma-1} \sim v^{1-\Gamma} r^{2(1-\Gamma)}$, then:

$$\frac{T}{T_c}(r) = \left(\frac{r}{r_s} \right)^{2(1-\Gamma)} \left(\frac{v}{v_c} \right)^{1-\Gamma} \quad (\text{B.25})$$

The mass loss rate of a polytropic wind with a given density ρ_0 at its lower boundary r_0 , can be derived from the mass continuity equation if the velocity v_0 at r_0 is known. The temperature at the critical point can then be written as:

$$T_c = T_0 w_0^{\Gamma-1} x_0^{2(\Gamma-1)} \quad (\text{B.26})$$

because $x = 1$ and $w = 1$ at the critical point.

Thus:

$$\frac{\Gamma k_B T_c}{m_p \mu} = \frac{GM}{2r_c} \implies \frac{\Gamma k_B T_0}{m_p \mu} w_0^{\Gamma-1} x_0^{2(\Gamma-1)} = \frac{GM}{2r_0} x_0 \quad (\text{B.27})$$

Since $v_c = \sqrt{\Gamma k_B T_c / m_p \mu}$, we know the velocity at the lower boundary: $v_0 = w_0 v_c$, thus:

$$\dot{M} = 4\pi r_0^2 \rho_0 \sqrt{\frac{\Gamma k_B T_0}{m_p \mu}} w_0^{\frac{1+\Gamma}{2}} x_0^{\Gamma-1} \quad (\text{B.28})$$

B.3 HD Shocks: Jump Conditions

Here is followed the work of Walter Maciel (Hydrodynamics and Stellar Winds).

When applying the basic HD equations to a 1D shock wave, it is common to assume that the unperturbed gas has pressure P_0 , density ρ_0 and relative velocity u_0 to the shock. The corresponding properties P_1, ρ_1, u_1 for the gas after the shock.

Mass conservation equation considers that the mass flux of gas approaching the shock must be equal to the mass flux of the gas leaving the shock - there is no creation or destruction of matter in the shock zone. Thus, the mass conservation equation is written:

$$\rho_0 u_0 = \rho_1 u_1 \quad (\text{B.29})$$

The momentum conservation equation follows the same idea: the momentum flux of gas reaching the shock is $(\rho_0 u_0) u_0 = \rho_0 u_0^2$ and the gas leaving the shock is $(\rho_0 u_0) u_1$. Assuming that momentum change must be equal to the impulse of the resulting force, then the momentum flux variation corresponds to the pressure difference in the two regions:

$$\rho_0 u_0 u_1 - \rho_0 u_0^2 = P_1 - P_0 \quad (\text{B.30})$$

which, when having equation (B.29) inserted, it yields:

$$P_0 + \rho_0 u_0^2 = P_1 + \rho_1 u_1^2 \quad (\text{B.31})$$

In order to obtain the energy equation, recall that there may be some energy transformation from one form to another at the shock front, although total energy is conserved. For an adiabatic shock, the internal energy of the unperturbed gas is:

$$e_0 = \frac{1}{\gamma - 1} \frac{P_0}{\rho_0} \quad (\text{B.32})$$

which translates to a total energy flux given by:

$$E_0 = \rho_0 u_0 \left(\frac{1}{2} u_0^2 + e_0 \right) \quad (\text{B.33})$$

and the analogous follows for the gas leaving the shock. The difference between the energy before and after the shock must be equal to the work done by the gas, result from the pressure difference:

$$\begin{cases} E_1 - E_0 &= P_0 u_0 - P_1 u_1 \\ E_0 + P_0 u_0 &= E_1 + P_1 u_1 \end{cases} \quad (\text{B.34})$$

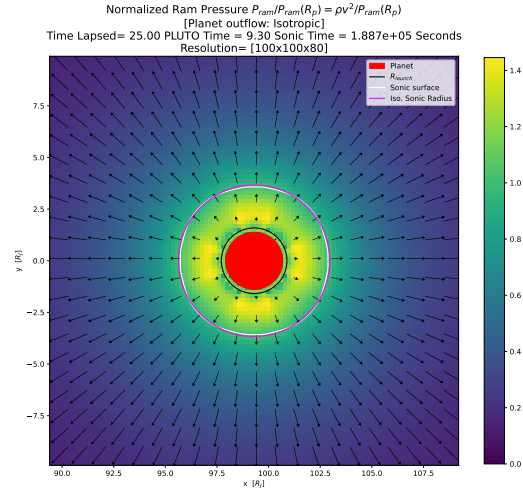
Consequently, conservation of energy yields:

$$\frac{1}{2} u_0^2 + \frac{\gamma}{\gamma - 1} \frac{P_0}{\rho_0} = \frac{1}{2} u_1^2 + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} \quad (\text{B.35})$$

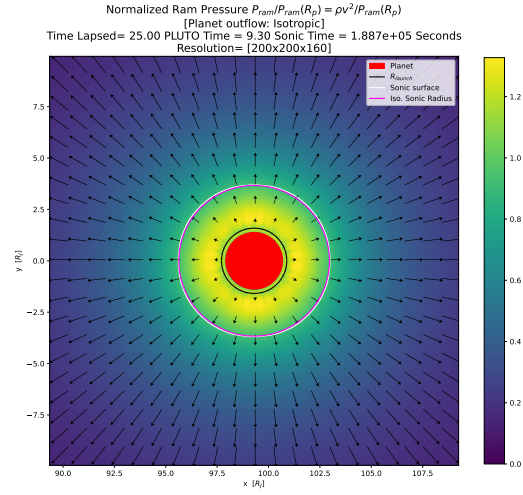
Equations (B.29), (B.31) and (B.35) express the physical conditions at the boundary region, and are so-called Rankine-Hugoniot conditions, or jump conditions.

B.4 Planetary Wind: Isotropic Models

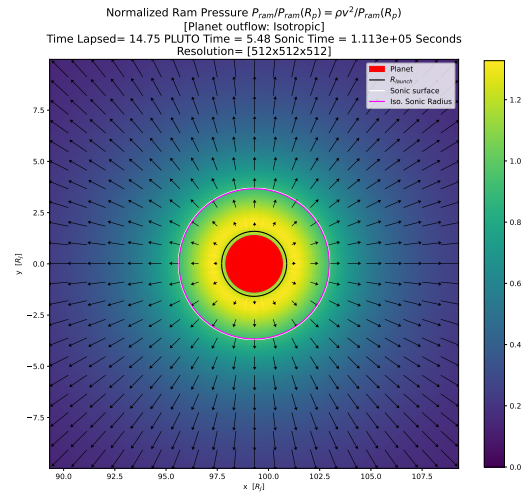
This section is used to display figures that have important aspects for discussion.



(A) Lowest Resolution



(B) Intermediate Resolution



(C) Highest Resolution

FIGURE B.1: Three plots of increasing resolution of the models' normalized Ram pressure. The lines have been described in Figure 4.2.

B.5 Planetary Wind: Anisotropic Models

This section is used to display figures that have important aspects for discussion.

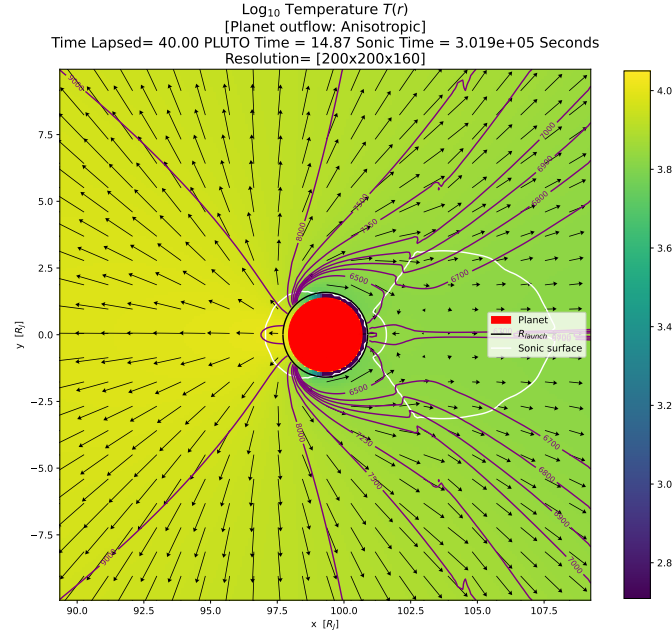


FIGURE B.2: Logarithm of the temperature plot from the steady-state configuration of the model with Intermediate Resolution and with temperature anisotropy and a faster relaxation function.

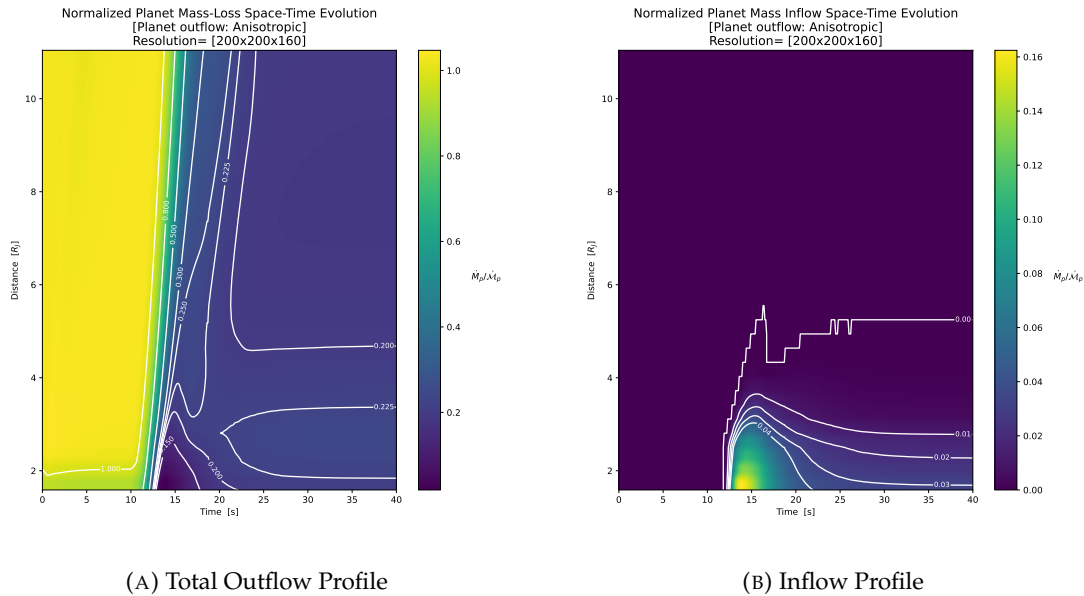


FIGURE B.3: Mass Loss plots of the model with Intermediate Resolution and with temperature anisotropy and a faster relaxation function.

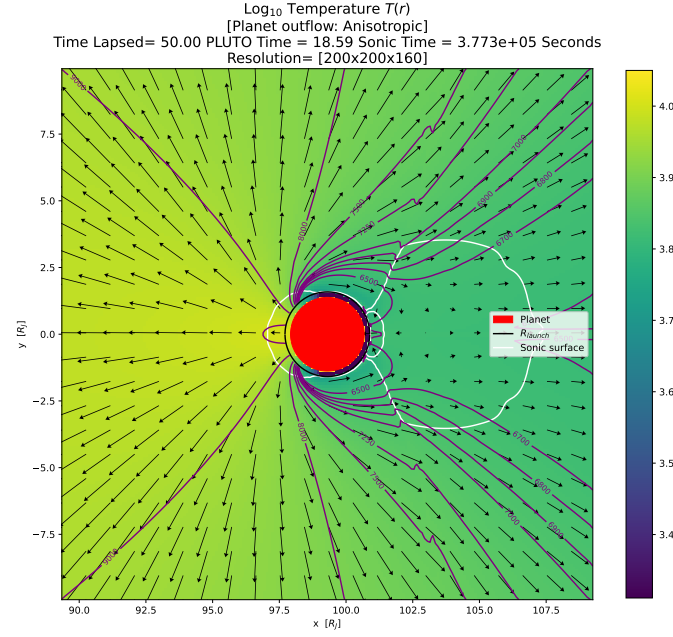


FIGURE B.4: Logarithm of the temperature plot from the steady-state configuration of the model with Intermediate Resolution and with temperature anisotropy and "night side" temperature 4 times larger.

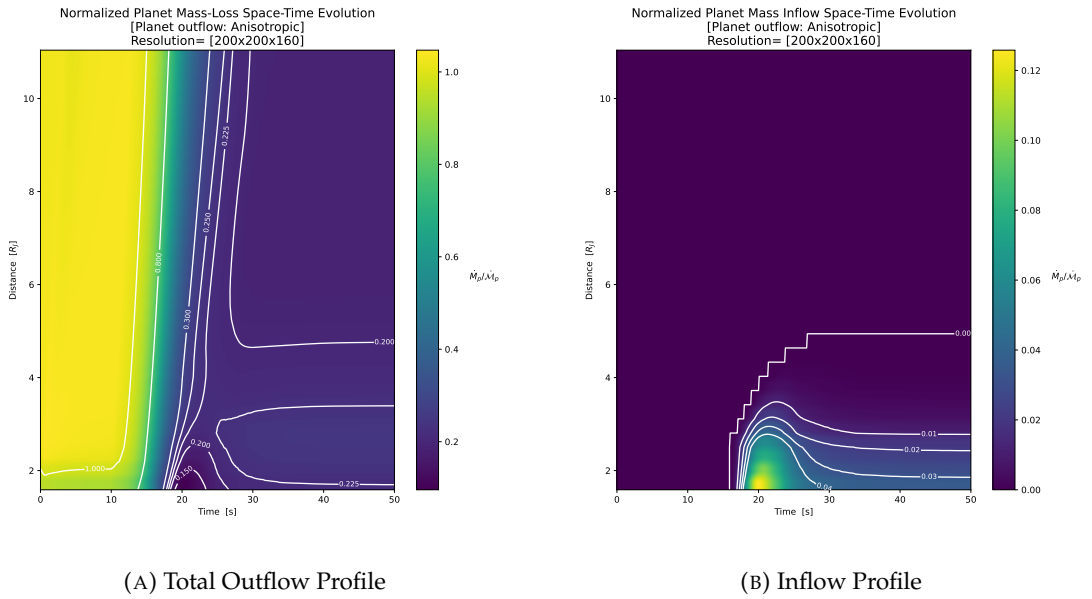
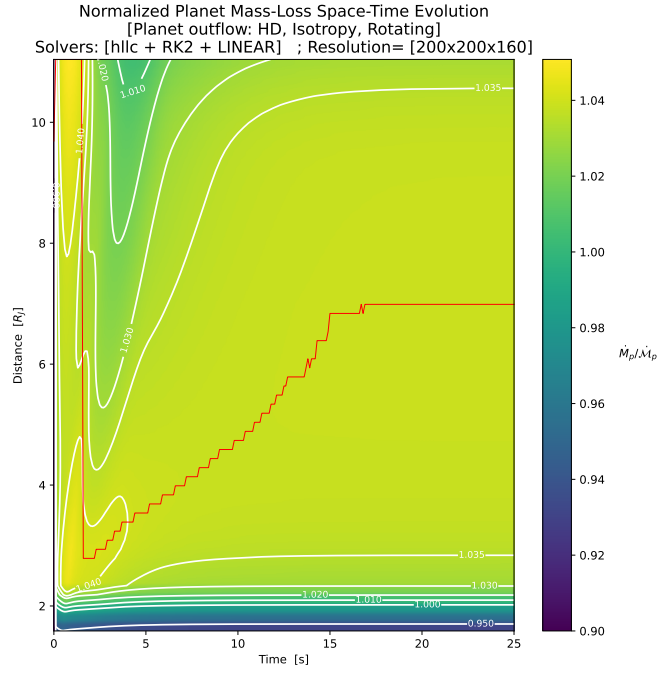


FIGURE B.5: Mass Loss plots of the model with Intermediate Resolution and with temperature anisotropy and "night side" temperature 4 times larger.

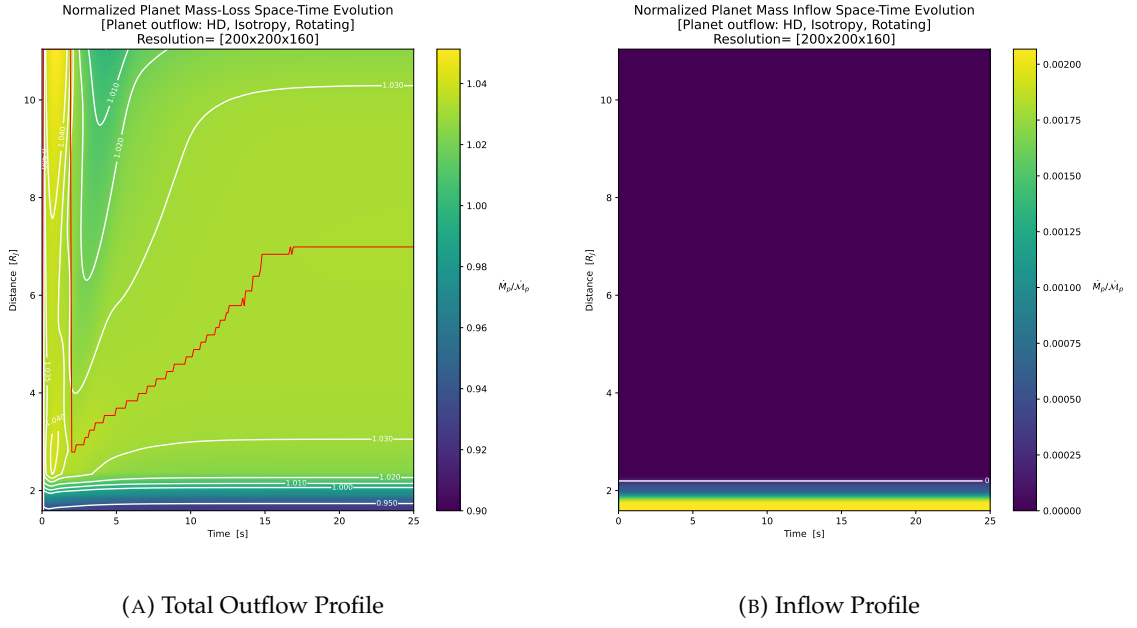
B.6 Planetary Wind: *TeoRot* and *Rot* Models

The following Figures are important to understand the convergence of two different although similar models.



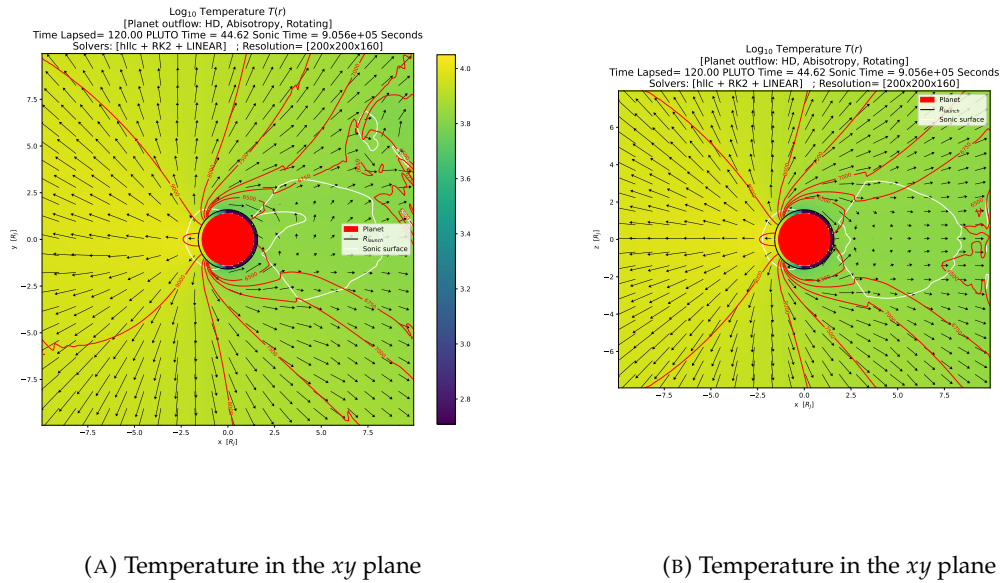
(A) Total Outflow Profile

FIGURE B.6: Mass Loss plot of the *Rot* model - there was no inflow.

FIGURE B.7: Mass Loss plots of the *TeoRot* model.

B.7 Planetary Wind: *AnisoRot* Models

The following Figure is important to understand the temperature behaviour of the final step run taken into account for the *AnisoRot* Model.

FIGURE B.8: Temperature plots of the *AnisoRot* model.

B.8 Planetary Wind: *Iso-A* Model

The following Figure is important to understand the other characteristics behind the interaction of the stellar and planetary wind.

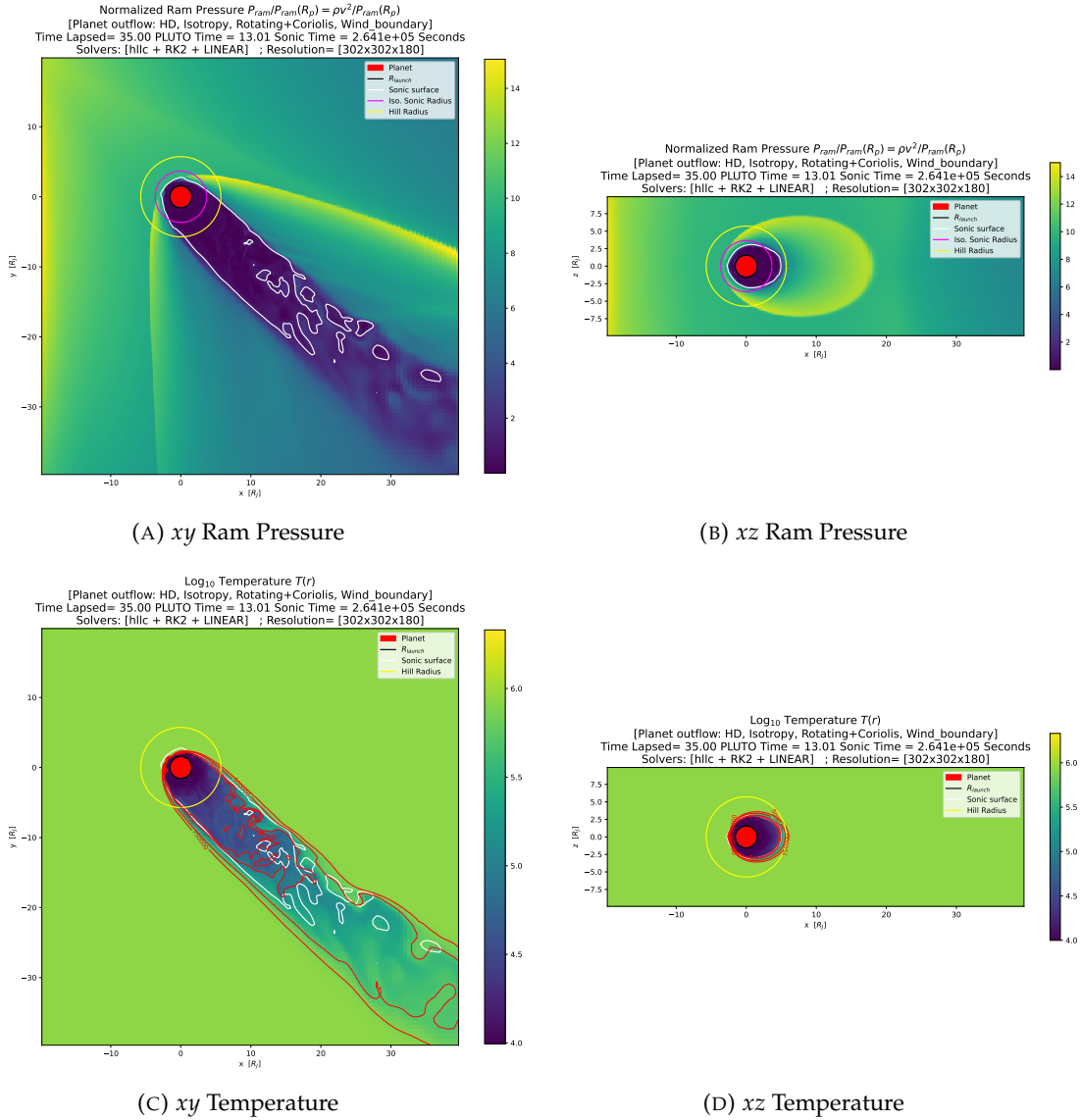


FIGURE B.9: Ram Pressure and Temperature plots for model *Iso-A* in xy and xz planes, in the planet's rotating frame.

B.9 Planetary Wind: *Iso-B* Model

The following Figure is important to understand the other characteristics behind the interaction of the stellar and planetary wind.

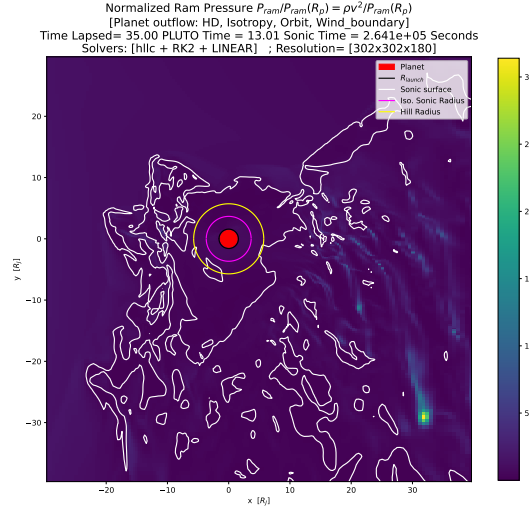
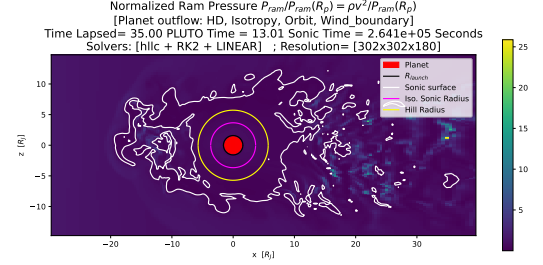
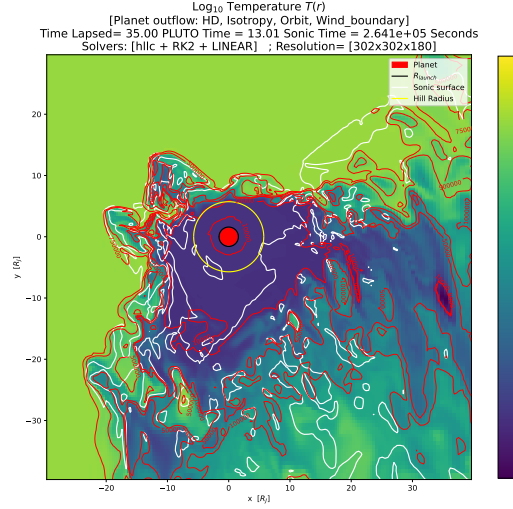
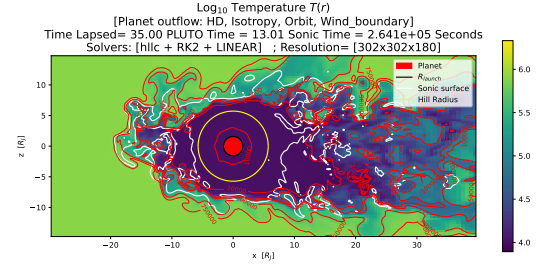
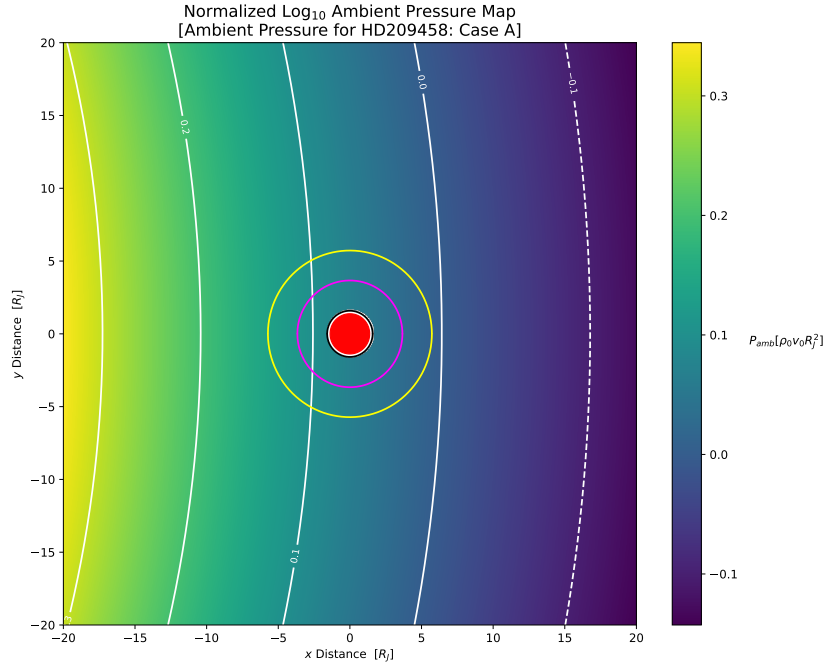
(A) xy Ram Pressure(B) xz Ram Pressure(C) xy Temperature(D) xz Temperature

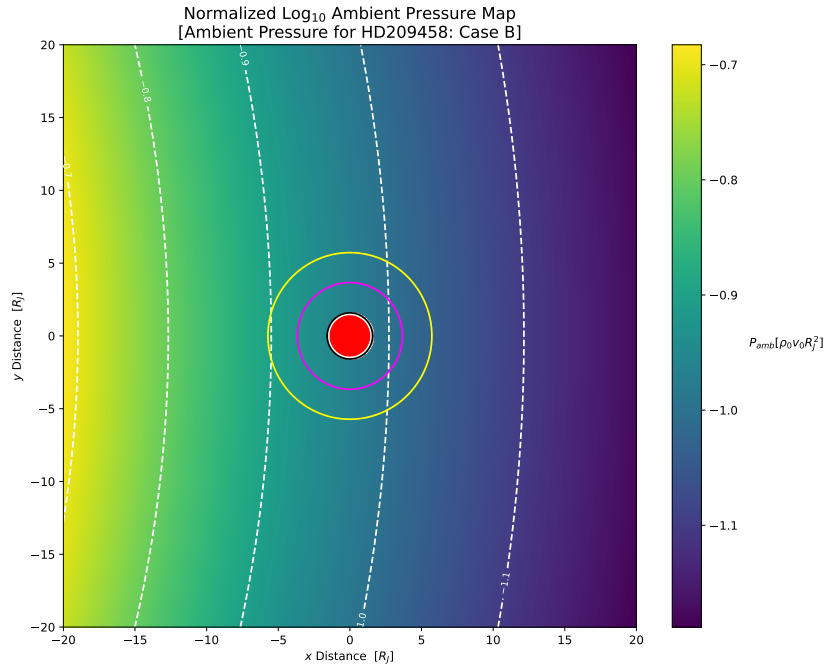
FIGURE B.10: Ram Pressure and Temperature plots for model *Iso-B* in xy and xz planes, in the planet's rotating frame.

B.10 *Iso* Models for Wind-Wind Interaction

The following Figure serves to show the quantitative aspect of the ambient pressure around the HJ for each studied Model.



(A) *Iso-A* Model



B.11 Planetary Wind: *Aniso-A* Model

The following Figure is important to understand the other characteristics behind the interaction of the stellar and planetary wind.

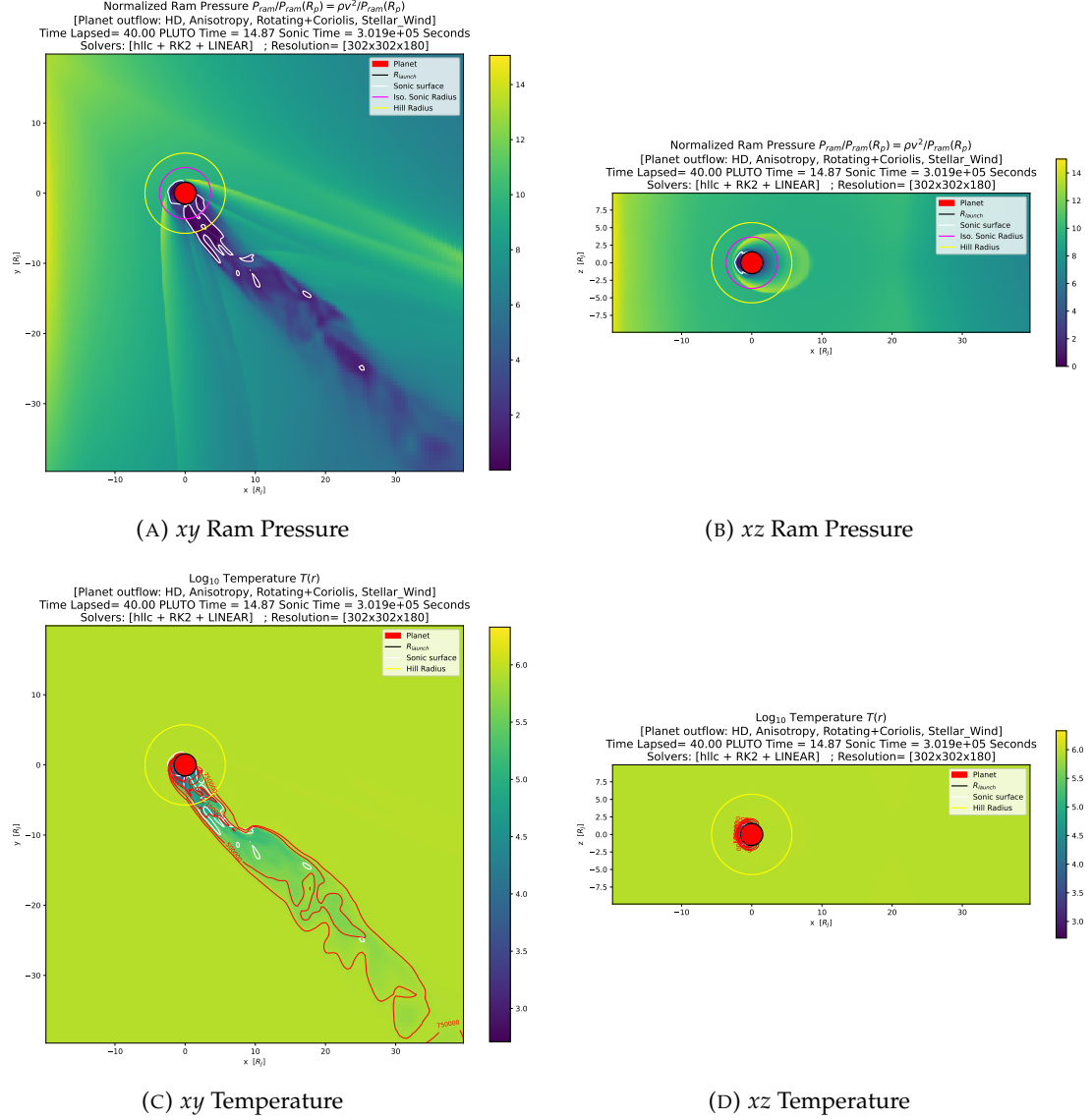
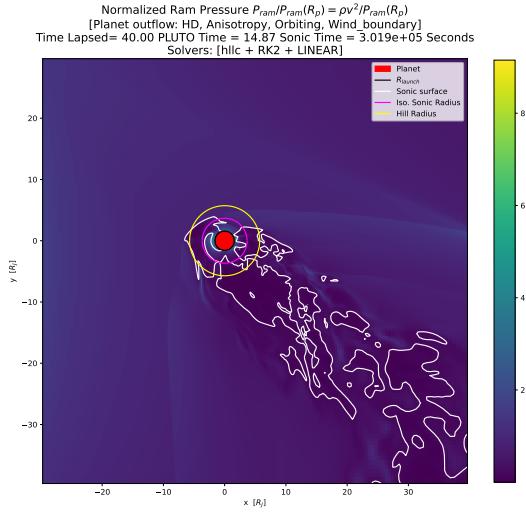


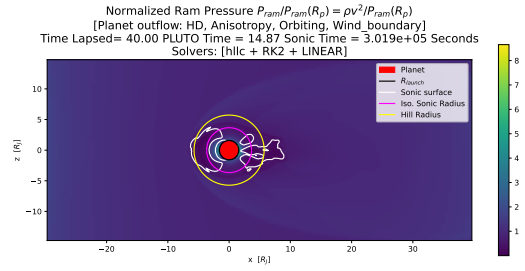
FIGURE B.12: Ram Pressure and Temperature plots for model *Aniso-A* in xy and xz planes, in the planet's rotating frame.

B.12 Planetary Wind: *Aniso-B* Model

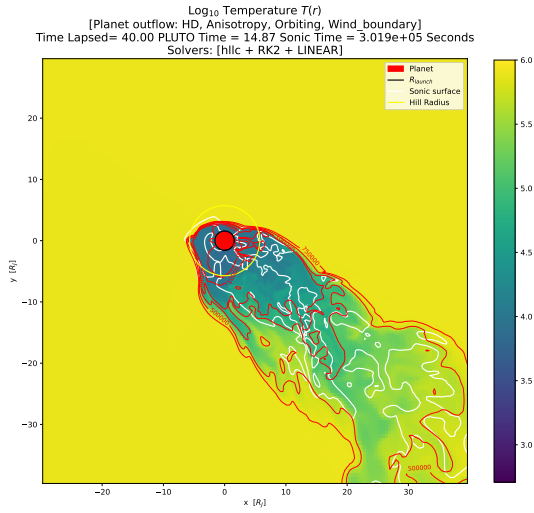
The following Figure is important to understand the other characteristics behind the interaction of the stellar and planetary wind.



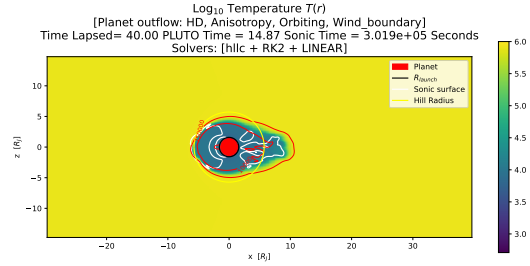
(A) xy Ram Pressure



(B) xz Ram Pressure



(C) xy Temperature



(D) xz Temperature

FIGURE B.13: Ram Pressure and Temperature plots for model *Aniso-B* in xy and xz planes, in the planet's rotating frame.

Appendix C

MHD Outflowing Winds

C.1 Magnetic Isothermal Wind: Dipolar configuration

This was done with intent of describing an outflowing wind with a magnetic dipolar configuration.

Let us assume that the object's magnetic field $\mathbf{B}$ is given by:

$$\mathbf{B}(r, \theta) = \frac{|\mathbf{m}|}{4\pi r^3} (2 \cos \theta \hat{\mathbf{e}}_r + \sin \theta \hat{\mathbf{e}}_\theta) \quad (\text{C.1})$$

In a general case:

$$\mathbf{v} = V_r(r, \theta) \hat{\mathbf{e}}_r + V_\theta(r, \theta) \hat{\mathbf{e}}_\theta \quad (\text{C.2a})$$

$$\mathbf{B} = B_r(r, \theta) \hat{\mathbf{e}}_r + B_\theta(r, \theta) \hat{\mathbf{e}}_\theta \quad (\text{C.2b})$$

There is no need to compute the divergence's entity, because the chosen configuration respects it. However, taking a look at the induction equation:

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \quad (\text{C.3})$$

which yields:

$$\hat{\mathbf{e}}_r \left[\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta (V_r B_\theta - V_\theta B_r)) \right] = 0 \quad (\text{C.4a})$$

$$\hat{\mathbf{e}}_\theta \left[-\frac{1}{r} \frac{\partial}{\partial r} (r (V_r B_\theta - V_\theta B_r)) \right] = 0 \quad (\text{C.4b})$$

which translates in conserved quantities in either directions.

$$\hat{\mathbf{e}}_r \left[\frac{|\mathbf{m}|}{4\pi r^4 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta (2V_r \cos \theta - V_\theta \sin \theta)) \right] = 0 \quad (\text{C.5a})$$

$$\hat{\mathbf{e}}_\theta \left[-\frac{|\mathbf{m}|}{4\pi r} \frac{\partial}{\partial r} \left(\frac{1}{r^2} (2V_r \cos \theta - V_\theta \sin \theta) \right) \right] = 0 \quad (\text{C.5b})$$

where it has been used the dipolar configuration definition to simplify the conserved quantities.

Since there is no ϕ dependency in this model, one can derive that:

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \quad (\text{C.6})$$

which will greatly simplify the momentum equation.

Let us pay attention to the equation of mass conservation:

$$\nabla \cdot (\rho \mathbf{v}) = 0 \quad (\text{C.7})$$

$$\implies \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho V_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho V_\theta) = 0 \quad (\text{C.8})$$

$$\iff \rho V_r + \frac{r}{2} \frac{\partial}{\partial r} (\rho V_r) + \frac{1}{2 \tan \theta} \rho V_\theta + \frac{1}{2} \frac{\partial}{\partial \theta} (\rho V_\theta) = 0 \quad (\text{C.9})$$

Finally, the momentum equation can be written in each direction as:

$$\hat{\mathbf{e}}_r \left[V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + c_s^2 \frac{1}{\rho} \frac{\partial \rho}{\partial r} + \frac{GM}{r^2} \right] = 0 \quad (\text{C.10a})$$

$$\hat{\mathbf{e}}_\theta \left[V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} - \frac{V_r V_\theta}{r} + c_s^2 \frac{1}{r \rho} \frac{\partial \rho}{\partial \theta} \right] = 0 \quad (\text{C.10b})$$

The goal now should be to obtain an expression of a differential equation of V_r dependent on r .

Introducing the work [Lima et al. \(2001\)](#), as an example, one possible method in order to reach this final objective is to use separation of variables, this is, assuming that:

$$V_r(r, \theta) = V_0 X(r) w(\theta) \quad ; \quad V_\theta(r, \theta) = V_1 Y(r) k(\theta) \quad (\text{C.11})$$

Additionally, it may be necessary to increase the number of degrees of freedom, which may be possible by eliminating the pressure terms with differentiation.

C.2 Magnetic Isothermal Wind: Pressure on Field Lines

It is interesting to evaluate the pressure along a magnetic loop in this rotating system. Following some analytical steps from [Owen and Adams \(2014\)](#), let us assume that the plasma is in hydrostatic equilibrium, which means:

$$P(s) = P_p \exp \left[\frac{1}{c_s^2} \int_0^s \mathbf{g} \cdot \hat{\mathbf{s}} ds \right] \quad (\text{C.12})$$

where s follows the magnetic loop, and $\mathbf{g}$ is the effective gravity of the system:

$$\mathbf{g} = (g_r, g_\theta, g_\phi) = \left(-\frac{GM_p}{r^2} + \Omega^2 r \sin^2 \theta, \Omega^2 r \sin \theta \cos \theta, 0 \right) \quad (\text{C.13})$$

Notice that, by definition $\hat{\mathbf{s}} = \mathbf{B}/|\mathbf{B}|$, yielding:

$$\mathbf{g} \cdot \hat{\mathbf{s}} ds = g_r dr + \frac{B_\theta}{B_r} g_\theta dr \quad (\text{C.14})$$

which translates to a pressure integral of the form:

$$P(s) = P_p \exp \left[\frac{1}{c_s^2} \int_{R_p}^r \left(g_r + \frac{B_\theta}{B_r} g_\theta \right) dr \right] \quad (\text{C.15})$$

which translate into three distinct integrals:

$$I_1 = \frac{1}{c_s^2} (-GM_p) \int_{R_p}^r \frac{dr}{r^2} = \lambda \left(\frac{1}{x} - 1 \right) \quad (\text{C.16a})$$

$$I_2 = \frac{1}{c_s^2} \Omega^2 \int_{R_p}^r r \sin^2 \theta dr = \epsilon \int_1^x x \sin^2 \theta dx \quad (\text{C.16b})$$

$$I_3 = \frac{1}{c_s^2} \Omega^2 \int_{R_p}^r \frac{B_\theta}{B_r} r \sin \theta \cos \theta dr = \epsilon \int_1^x \frac{B_\theta}{B_r} x \sin \theta \cos \theta dx \quad (\text{C.16c})$$

where $x = \frac{r}{R_p}$, with λ and ϵ the parameters previously defined. Important to notice is that the angle θ depends on the dimensionless radius x due to being along a given field line.

For a dipolar field configuration, the ratio:

$$\frac{B_\theta}{B_r} = \frac{B_p x^{-3} \sin \theta}{B_p x^{-3} 2 \cos \theta} = \frac{\sin \theta}{2 \cos \theta} \quad (\text{C.17})$$

which makes $I_2 + I_3 = \frac{3}{2} I_2$. The next step is to translate θ into an x dependence. It happens that this work has already been done by [Adams \(2011\)](#), which stated that magnetic field lines follow lines of constant values of $q = x^{-1} \sin^2 \theta = \sin^2 \theta_b$, which represents a constant determined by the polar angle θ_b where the footpoint is located - angle at which

the outflow opens the field lines. Therefore:

$$I_2 = \epsilon \int_1^x q x^2 dx = \frac{1}{3} q (x^3 - 1) \quad (\text{C.18})$$

making:

$$P(x) = P_p \exp \left[\lambda \left(\frac{1}{x} - 1 \right) + \frac{1}{2} \epsilon (x^3 - 1) \right] \quad (\text{C.19})$$

Now, the planetary surface supports both closed field lines and open field lines, with a critical magnetic field line delineating this boundary. Such critical streamline can be denoted as q_m , and is evaluated at the point where the ambient pressure equals the magnetic pressure:

$$P(x) = \frac{B^2}{8\pi} \iff P_p \exp \left[\lambda \left(\frac{1}{x} - 1 \right) + \frac{1}{2} \epsilon (x^3 - 1) \right] = \frac{B_p^2}{8\pi} x^{-6} (4 \cos^2 \theta + \sin^2 \theta) \quad (\text{C.20})$$

The maximum for the radius $x = 1/q$, with $\theta = \pi/2$ (along the equator). This make the above expression such that:

$$P_p \exp \left[\lambda (q - 1) + \frac{1}{2} \epsilon (q^{-3} - 1) \right] = \frac{B_p^2}{8\pi} q^6 \quad (\text{C.21})$$

$$\implies \lambda (q - 1) + \frac{1}{2} \epsilon (q^{-3} - 1) = \log \left(\frac{B_p^2}{8\pi P_p} \right) + 6 \log q \quad (\text{C.22})$$

the solution for q determines q_m , which itself determines the critical footpoint angle at the surface θ_m .

C.3 Magnetic Planetary Wind: M-Static Model

This section is used for plots which can enrich the discussion.

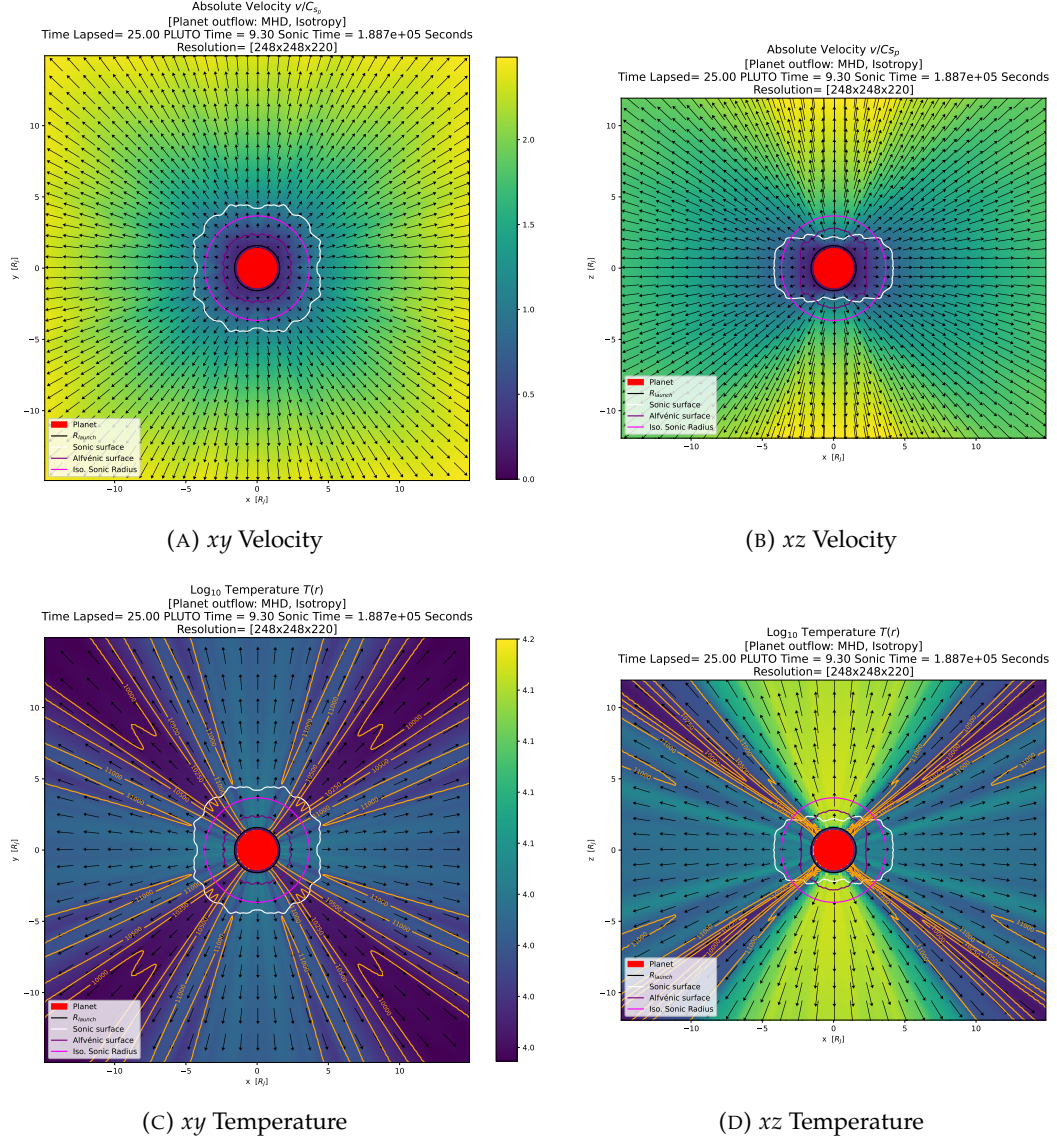


FIGURE C.1: Normalized Velocity and Temperature plots for *M-Static* Model in *xy* and *xz* planes.

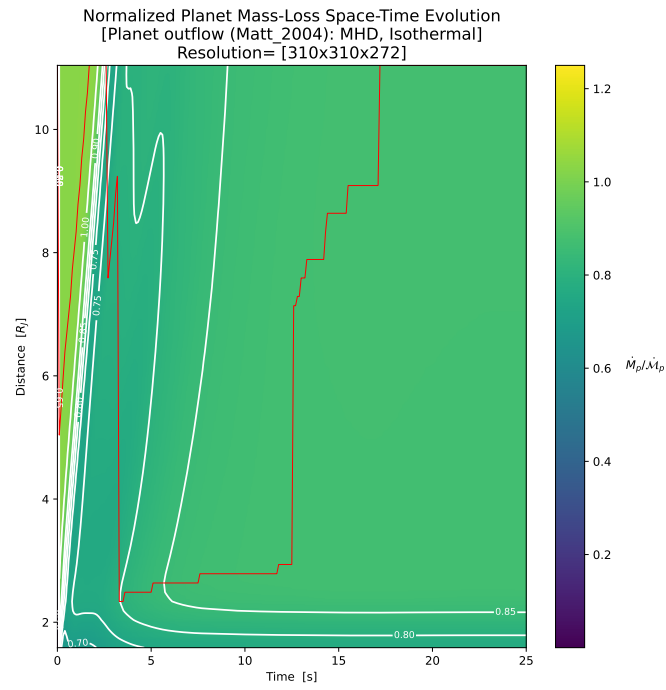


FIGURE C.2: Outflow plot normalized to base mass-loss of the HJ, with several contours and the red line maintains its meaning of representing the local maximum at each time step.

C.4 Magnetic Planetary Wind: M-*IsoRot* Model

This section is used for plots which can enrich the discussion.

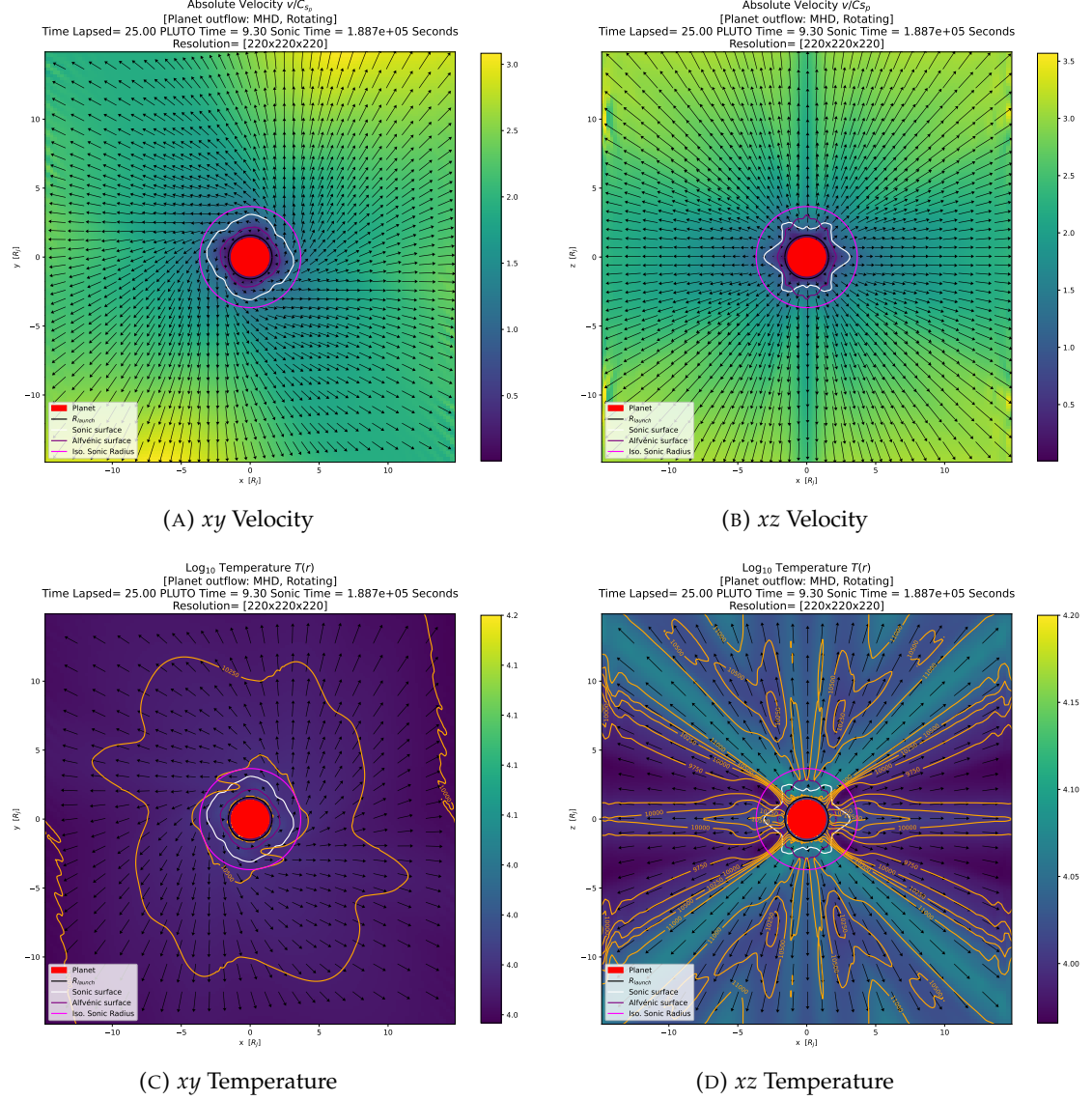


FIGURE C.3: Normalized Velocity and Temperature plots for M-*IsoRot* Model in *xy* and *xz* planes, in the rotating frame.

C.5 Magnetic Planetary Wind: M-*IsoCor* Model

This section is used for plots which can enrich the discussion.

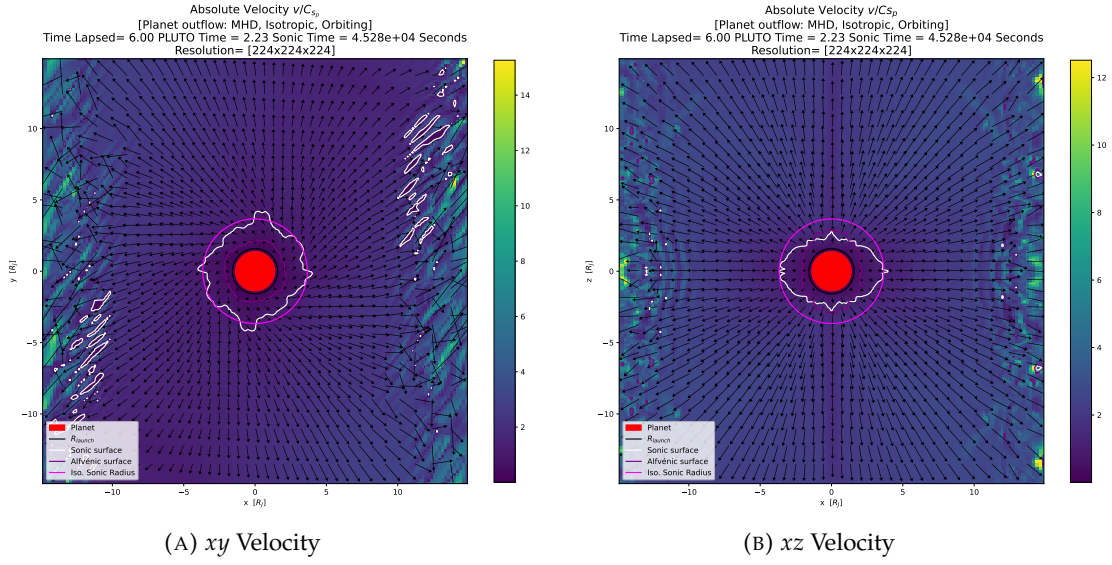


FIGURE C.4: Normalized Velocity plots for M-IsoCor Model in xy and xz planes, in the rotating frame.

C.6 Magnetic Planetary Wind: M-Strong Wind Model

This section is used for plots which can enrich the discussion.

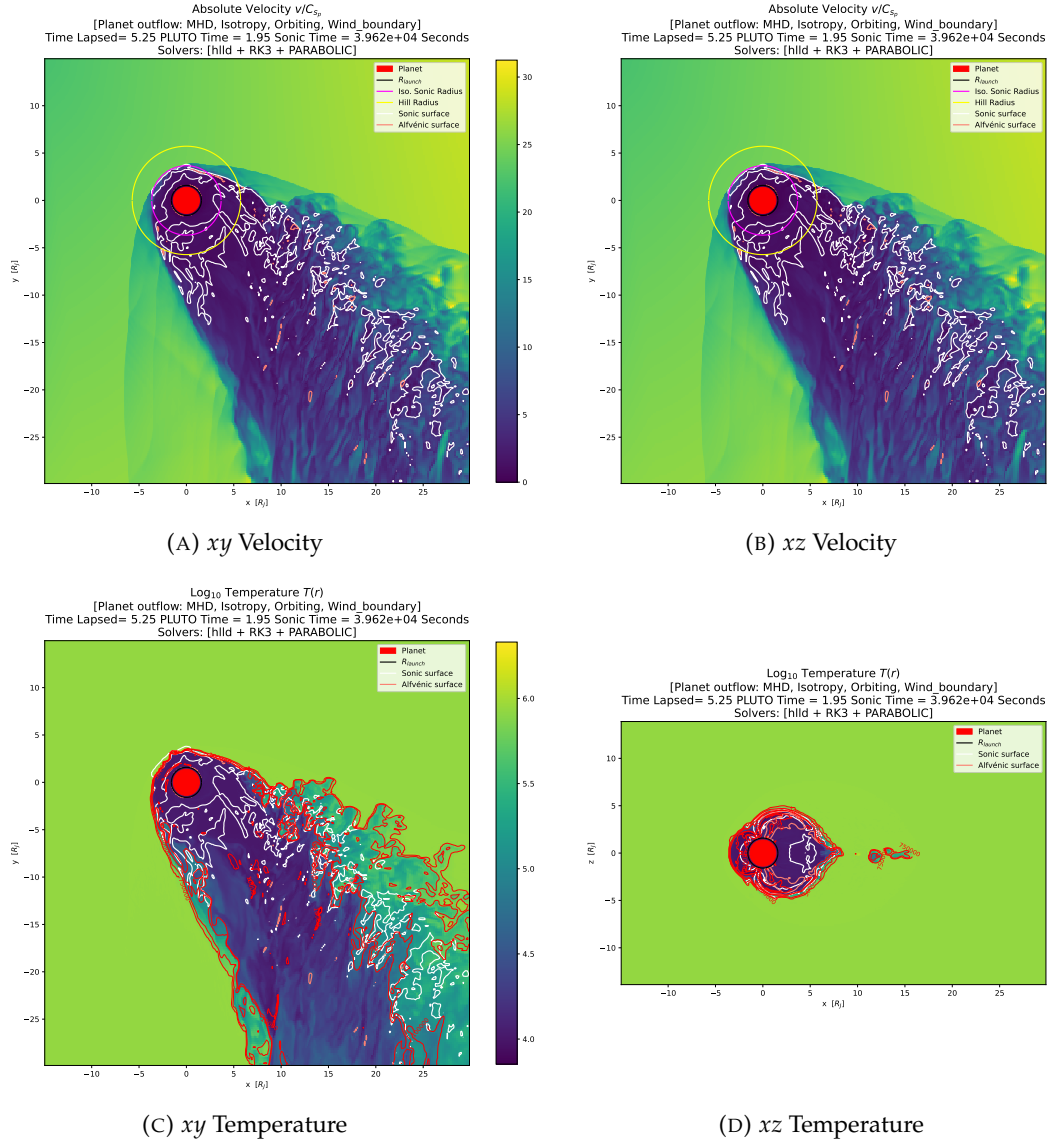


FIGURE C.5: Normalized Velocity and Temperature plots for **M-Strong** Wind Model in xy and xz planes, in the rotating frame.

Appendix D

Code Scripts

D.1 Parker Wind with Newton-Raphson Method

This code is written in C.

```
/* ***** PARKER WIND ***** */

/* define the derivative of the RHS eq of Parker wind:*/
double der_RHS( double v, double cs) {
    return 2.0/cs/cs*v-2.0/v;
}

/* define the Newton-Raphson Method for the Parker wind:*/
double newton_raphson(double (*f)(double, double, double),
    double (*df)(double, double), double v0, double eps, double cs, double lhs) {
    double v1 = v0;
    double fv0, df0;
    int i=0;
    do {
        i+=1;
        v0 = v1;
        fv0 = f(v0, cs, lhs);
        df0 = df(v0, cs);
        v1 = v0 - fv0 / df0;
    } while (fabs(v1 - v0) > eps && i<1024);
    return v1;
}

/* define the entire eq of Parker wind:*/
double Parker_eq( double v, double cs, double lhs) {
    double w=v*v/cs/cs;
    double rhs= w-log(w)-1.0;
```



```
    return rhs-4.0*lhs;
}

/* define the Parker wind:*/
double Parker( double r, double cs, double r_sonic) {
    double v_ini;
    double xo=r/r_sonic;
    double lhs = 1.0/xo-log(1.0/xo)-1.0;

    if (xo<1.0) {
        v_ini=pow(0.1, 1.1/xo);
    }
    else if (xo >= 1.0) {
        v_ini=6.0*cs;
    }

    return newton_raphson(Parker_eq, der_RHS, v_ini, 1e-5, cs, lhs);
}
```

For all simulations, codes are available in the following [link_for_codes](#).

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