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New methodologies for dynamic monitoring of floating wind turbines validated with experimental data

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Ao meu Avô João

Abstract

As environmental concerns raise, so does the importance of finding alternative and renewable energy sources to replace the long used fossil fuels. In the last decades, wind energy has proved to be one of the most promising paths, in particular with the onset of offshore solutions in general, and floating platforms for deep waters in particular, until recently hold back because of their prohibitive cost. The seek of more efficient designs must rely, at least in its first steps, in numerical models, but monitoring strategies during their operation lifetime is also of paramount importance for their adequate maintenance in very harsh conditions, operation cost reduction and design optimisation. In this sense, the present work proposes to explore the highly complex and multidisciplinary field of wind turbines numerical models and develop both a numerical and theoretical framework where different designs and monitoring strategies can be tested, given special relevance to the evaluation of monitoring strategies for floating wind turbines.

After a brief overview of the theoretical background and fundamental concepts, structural analysis, aerodynamics and hydrodynamics numerical modelling of wind turbines is validated using two different case studies. To test the aerodynamic behaviour, experimental data from a onshore wind farm located in Tocha, Portugal will be used, while the hydrodynamic behaviour for the NREL 5MW floating platform will be compared using the NREL's OpenFAST software and ANSYS/AQWA®.

Once the numerical models are properly validated, they can be used as an artificial laboratory where numerically simulated time series that mimic the ones one would find in an experimental campaign can be used to test new methodologies. In particular, and seeking a deeper understanding of these structures dynamics, analytical considerations are tested and used to make predictions that are then validated with numerically simulated data. These models focus on different elements, starting from blades measurements and the relevant information they contain, all the way to the impact of placing a wind turbine on the top of a floating platform, and the frequency content of the forces transmitted to the rotor at the hub.

Finally, the results of the monitoring campaign of a full scale floating wind turbine are explored, a relevant landmark in the field as this is the first application available for academic purposes. Here, Operational Modal Analysis is successfully applied to identify all the relevant natural frequencies of the system, from the very low platform dominated modes, to the typical tower and rotor dominated ones. Additionally, the internal forces are reconstructed for the monitoring period, and a new methodology for fatigue estimation based on tower top accelerations is presented and validated. This is the first time new data processing methodologies for structural monitoring, specially tuned for floating wind turbines, are tested in experimental data collected in an utility-scale wind turbine.

Keywords: Floating offshore wind turbine. Operational Modal Analysis. Fatigue damage. Aeroelastic simulations. Analytical mechanics

Resumo

A transição para fontes de energia renováveis é vital para fazer face às cada vez maiores preocupações climáticas. Nas últimas décadas, a energia eólica tem-se revelado uma das soluções mais promissoras, em particular com o surgimento das explorações em contexto marítimo e com a cada vez maior viabilidade das soluções flutuantes necessárias para explorar zonas de maior profundidade. Se o desenvolvimento de estruturas mais eficientes assenta, numa primeira fase, em modelos numéricos, as estratégias de monitorização durante a sua operação são também fundamentais para definir as necessárias operações de manutenção, permitindo redução de custos de operação e melhoria dos processos de dimensionamento. Tendo isto em consideração, este trabalho propõe-se a explorar a complexa e multi-disciplinar área da modelação numérica de geradores eólicos, bem como o desenvolvimento de enquadramentos teóricos que permitam testar diferentes soluções e estratégias de monitorização, com especial ênfase nas aplicações para geradores eólicos flutuantes.

Depois de uma breve descrição dos conceitos teóricos fundamentais, a modelação estrutural, aerodinâmica e hidrodinâmica de geradores eólicos flutuantes é apresentada e validada a partir de dois casos de estudo. A modelação aerodinâmica será validada recorrendo aos resultados de uma campanha experimental a decorrer no parque eólico da Tocha, Portugal, enquanto a componente hidrodinâmica será validada comparando as respostas obtidas a partir dos programas de cálculo da NREL OpenFAST e ANSYS/AQWA® de um modelo do gerador de 5MW da NREL.

Depois de validados, os modelos numéricos podem ser utilizados como um laboratório artificial para gerar séries temporais que mimetizam as respostas que poderiam ser medidas no contexto de uma campanha experimental e que podem ser usadas para testar novas metodologias. Em particular, procurando um entendimento mais profundo sobre o comportamento dinâmico destas estruturas, são apresentadas previsões obtidas a partir de considerações analíticas, posteriormente validadas com as séries numéricas geradas. Estas formulações incidem sobre diferentes elementos, desde as medições nas pás e informação aí contida, até ao impacto de colocar um determinado gerador eólico no topo de uma plataforma flutuante, passando pela caracterização das forças transmitidas pelo rotor à torre.

Finalmente, os resultados de uma campanha de monitorização de um gerador eólico flutuante em operação são explorados, no que constitui a primeira aplicação deste tipo apresentada em contexto académico. São utilizadas metodologias de Identificação Modal Operacional para identificar as frequências naturais do sistema, desde aquelas associadas aos movimentos da plataforma, até às que são tipicamente dominadas pelo gerador. Para além disso, os esforços internos são reconstruídos para o período de monitorização e uma nova metodologia para estimativa de dano por fadiga, baseada nas acelerações registadas na torre, é apresentada e validada. Esta é a primeira vez em que novas metodologia desenvolvidas especificamente para a monitorização de geradores eólicos flutuantes é testada a partir de dados experimentais de um gerador em funcionamento.

Palavras-chave: Geradores eólicos flutuantes. Identificação Modal Operacional. Fadiga. Simulações aeroelásticas. Mecânica analítica

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Contents

List of Figures	xxiv
List of Tables	xxv
Nomenclature	xxvii
1 Introduction	1
1.1 Research Context	1
1.2 Objectives of the Thesis	2
1.3 Organisation of the Thesis	4
I Research background	9
2 Floating offshore wind turbines theoretical minimum	11
2.1 Introduction	11
2.2 Floating wind turbines environment	11
2.2.1 Wind characterisation	11
2.2.1.1 Wind physics	12
2.2.1.2 Wind prediction	15
2.2.2 Waves and sea states characterisation	18
2.2.2.1 Waves physics	19
2.2.2.2 Wave models	21
2.2.2.3 Waves statistical properties	22
2.3 Floating wind turbines components	26
2.3.1 Structural components	26
2.3.2 Control systems	27
2.3.3 Monitoring systems	29
2.4 Basic data processing tools for structural monitoring	30
2.4.1 Operational Modal Analysis	30
2.4.2 Fatigue estimation	34
2.4.3 Monitoring challenges in the offshore scenario	35
3 Basis for numerical modelling of floating offshore wind turbines	37
3.1 Introduction	37
3.2 Wind loading and aerodynamic response	37
3.2.1 Aerodynamic forces	37
3.2.1.1 Ideal flow	38
3.2.1.2 Viscous effects	40

3.2.1.3	Computational Fluid Dynamics	43
3.2.2	Rotor aerodynamics models	47
3.2.2.1	Linear momentum theory	48
3.2.2.2	Angular momentum theory	50
3.2.2.3	Blade Element/Momentum Theory	52
3.2.2.4	Unsteady aerodynamics effects	58
3.3	Waves loading and hydrodynamic response	63
3.3.1	Actions over free floating structures	63
3.3.1.1	Wave forces over slender structures	64
3.3.1.2	Wave forces over large bodies	66
3.3.1.3	Hydrostatic forces	72
3.3.2	Mooring forces	72
3.3.3	Platform motions	76
3.3.3.1	Frequency domain approach	76
3.3.3.2	Time domain approach	78
3.4	Final remarks	80
II	Research developments	81
4	Validation and calibration of numerical models illustrated with two case studies	83
4.1	Introduction	83
4.2	OpenFAST structure	84
4.2.1	Structural modelling	84
4.2.2	Wind field generation	85
4.2.3	Aerodynamic modelling	87
4.2.4	Control systems modelling	87
4.2.5	Hydrodynamic modelling	90
4.2.6	Mooring system modelling	90
4.3	Aerodynamic and structural modelling validation using experimental data	91
4.3.1	Experimental campaign	91
4.3.2	OpenFAST model	92
4.3.3	Response comparison	95
4.4	Hydrodynamic modelling validation	99
4.4.1	Structure description	99
4.4.2	OpenFAST model	100
4.4.3	ANSYS AQWA Model	101
4.4.4	Response comparison	106
4.4.5	Preliminary results of OMA	110
4.5	Final remarks	115
5	Analytical considerations regarding wind turbines dynamics	117
5.1	Introduction	117
5.2	Analysis of blades accelerations and stresses	118
5.2.1	Measurements description	118
5.2.1.1	Reference frames and rotation matrix	118
5.2.1.2	Accelerations measurements	119
5.2.1.3	Strain gauges measurements	120
5.2.2	Wind turbine geometry characterisation	124

5.2.3	Operating conditions estimates	128
5.2.4	Strain gauges calibration	130
5.3	Rotor dynamics	133
5.3.1	Equations of motion	133
5.3.2	Quasi-static conditions	136
5.3.3	Random vibrations	137
5.4	Floating structures dynamics	140
5.4.1	Equations of motion	140
5.4.1.1	Model definition	140
5.4.1.2	Free vibration	141
5.4.1.3	Forced motion	145
5.4.2	Relations between accelerations and bending moments	146
5.4.2.1	Relations without thrust force on the tower	147
5.4.2.2	Relations for the thrust force on the tower	148
5.4.3	Strain gauges calibration	150
5.5	Final remarks	154
6	Dynamic monitoring of a full scale floating offshore wind turbine	155
6.1	Introduction	155
6.2	Experimental campaign description	156
6.2.1	Structure description	156
6.2.2	Experimental layout	156
6.2.3	Monitoring components	158
6.2.4	Data description	160
6.3	Operational Modal Analysis	160
6.3.1	Platform analysis	160
6.3.2	Tower analysis	168
6.4	Internal forces analysis	173
6.4.1	Strains calibration	174
6.4.2	Determination of the FA and SS directions	178
6.4.3	Bending moments reconstruction	180
6.4.4	Fatigue results	182
6.5	Final remarks	186
7	Predictive model for fatigue damage evaluation of floating offshore wind turbines	187
7.1	Introduction	187
7.2	Evaluation of the lifetime fatigue consumption of wind turbines	188
7.3	Proposed methodology	189
7.3.1	Model definition	189
7.3.2	Model calibration	192
7.3.3	Bending moments reconstruction	195
7.4	Results	197
7.5	Final remarks	200
8	Conclusions	201
8.1	Summary	201
8.2	Main contributions	203
8.3	Limitations and future developments	204

Bibliography	207
A Aerodynamics	221
A.1 Aerofoil analytical formulation	221
A.2 Computational Fluid Dynamics	228
A.3 BEMT	231
B Hydrodynamics	233
B.1 Morison equation	233
B.2 Second order drift forces	235
B.3 Mooring lines	237
C Analytical mechanics	241
C.1 Lagrange method	241
C.1.1 Generalised coordinates and constraints	241
C.1.2 Lagrange's equations	243
C.2 Kane method	246
C.2.1 Kane's equations	246
C.2.2 A simplified wind turbine model	248

List of Figures

2.1	Wind spectrum obtained from 1Hz SCADA data from one of the ongoing FEUP experimental campaigns.	13
2.2	Probability density function following a Rayleigh distribution for different scale parameters. The mean value associated with each distribution is represented by the corresponding colour vertical dashed line.	16
2.3	Schematic representation of the different wind components. In red, a plot with the wind speed evolution over height for a given time instant, with the mean wind profile as a dashed black line. In blue and green, the time evolution of the wind speeds in two different points, that should respect the power spectrum on the left and the coherence function C_{ij}	18
2.4	Effect of the directional spreading for $s = 1$ (on the top right) and $s = 15$ (on the bottom right) on a Pierson-Moskowitz spectrum (on the left) considering $\mu = 0$. .	25
2.5	Schematic representation of different floating solutions. From left to right, tension-leg platform (TLP), semi-submersible, spar and barge (pontoon) type.	27
2.6	Typical operating regions of wind turbines constructed from the data points (blue points) of one of the ongoing FEUP experimental campaigns. The green line, proportional to the 3^{rd} of the wind speed, schematically indicates the wind energy available, while the red dashed line represents the maximum energy that can be captured (approximately 0.593 of the total energy available, as will be discussed in Section 3.2.2).	28
2.7	Schematic representation of the fatigue damage estimate based on SN curves. . .	35
3.1	Scheme of the different aerodynamic forces conventions.	38
3.2	Velocity analysis at the trailing edge for 2 different aerofoil shapes.	40
3.3	Air flow scheme due to the presence of a wind turbine. The dotted and dashed line schematically represent the pressure and velocity profile evolution, respectively. .	48
3.4	Evolution of both thrust and power coefficients and far wake to free stream wind speed ratio with the axial induction factor.	50
3.5	Evolution of the optimal induction coefficients (on the left) and of the power coefficient (on the right) as a function of the local speed ratio and tip speed ratio, respectively.	51
3.6	Forces acting on a blade element scheme and corresponding angles: α is the aerodynamic angle of attack, ϕ is the angle between the relative wind speed and rotor plane and θ is the angle between the chord and the rotor plane	53
3.7	Tip, root and total loss for a three-bladed rotor with a non dimensional hub radius of 0.05 and, for simplicity, assuming an uniform flow angle of 5°	56
3.8	Thrust coefficient considering a turbulent wake with Glauert's and Buhl's correction with (on the right) and without (on the left) considering a loss factor.	56

3.9	Angles convention for a blade with azimuth angle α and pre-cone β in a yawed (ψ) and tilted (θ) rotor.	59
3.10	Velocity field derived from a potential flow (on the left) and with a tower shadow model (on the right) for a unitary radius tower.	61
3.11	Rigid body degrees of freedom of a floating offshore wind turbine.	63
3.12	Schematic representation of the forces transmitted to the suspended cable segment.	73
3.13	Total force transmitted by the mooring system to a structure as a function of the displacement surge-pitch (on top) and sway-roll (on bottom).	74
4.1	OpenFAST degrees of freedom (partial representation). On the left panel, are represented the platform and nacelle yaw motions, while the platform pitch and roll motions are represented on the middle and right panel. The tower flexibility is represented by the tower top displacement q_{FA} and q_{SS} for the FA and SS directions and the rotor angular position by one of the blades position, q_{ψ}	85
4.2	Wind field generated by InFlowWind considering an uniform wind field.	86
4.3	Full wind field generated by TurbSim considering Taylor's frozen wake hypothesis.	87
4.4	Simplified torque control implemented in OpenFAST.	88
4.5	Simplified response obtained using a PID controller based on the wind speed.	89
4.6	Tocha wind farm location and layout (Pacheco, 2022).	91
4.7	Mode shapes and 6 th degree polynomial fitted to the configuration obtained for the FA direction.	93
4.8	Representation of the adopted blade sections (on the left) obtained from the blade 3D model (on the right).	94
4.9	Lift and Drag coefficients for the full range of angles of attack obtained for section S7.	94
4.10	Comparison between simulated values (red points) and theoretical data (blue line) for the thrust force (left panel) and the power output (right panel).	95
4.11	Power spectrum estimation (parked rotor) in the FA (blue) and SS (orange) directions from experimental and numerical data.	96
4.12	Comparison between numerical results (red points) and SCADA data (blue points). The horizontal axis scale has been omitted due to confidentiality constraints.	97
4.13	Comparison between numerical (orange) and experimental (blue) bending moments for selected time intervals.	97
4.14	Comparison between numerical and experimental/theoretical forces for 100 independent generated time series (x scale omitted due to confidentiality constraints).	98
4.15	Evolution of the response frequency content.	98
4.16	Schematic representation of the floating platform, based on the data from (Robertson et al., 2014).	99
4.17	Evolution along the span of blade's linear density, stiffness and structural twist (from left to right, respectively).	100
4.18	Tower mass distribution (on the left) and stiffness (on the right).	100
4.19	Ratio between diameter and wave length for the different elements represented in Figure 4.16. The different diameters correspond to the lower part of the eccentric columns (24 m), its top part (12 m), the central column (6.5 m) and the pontoons (1.60 m).	104
4.20	Lateral and 3D view of the model created in ANSYS/AQWA. The yy axis is aligned with one of the cables and the nacelle orientation, or, equivalently, orthogonal to the rotor plane.	104

4.21	Evolution of the hydrodynamic added mass (on top) and radiation damping (on bottom) with the frequency as described in OpenFAST and computed by ANSYS at the structure centre of gravity.	105
4.22	Evolution of the hydrodynamic added mass (on top) and radiation damping (on bottom) with the frequency as described in OpenFAST and computed by ANSYS at the intersection of the tower axis with the mean water level (after application of transformation matrices).	106
4.23	Comparison between OpenFAST and ANSYS response for the 6 still water simulations.	107
4.24	Comparison between OpenFAST and ANSYS cable forces for the 6 still water simulations (when several cables present the same response, only one of them is represented).	107
4.25	Comparison between OpenFAST and ANSYS for simulation 7 (on top) and 8 (on bottom) for the relevant degrees of freedom.	108
4.26	RAO operator obtained with a frequency analysis in ANSYS for the translation (on the left) and rotation (on the right) degrees of freedom.	109
4.27	Comparison of the RAO operator obtained with a frequency analysis in ANSYS for the translation degrees of freedom evaluated at the centre of gravity (on the left), the WRP (on the centre) and tower top (on the right).	109
4.28	Power spectrum obtained for surge (on the left), pitch (on the center) and heave (on the right) response from ANSYS and OpenFAST generated time series. . . .	109
4.29	RAO operator obtained for surge (on the left), pitch (on the centre) and heave (on the right) response from ANSYS and OpenFAST generated time series.	110
4.30	Stabilisation and damping diagrams for Simulation T1, T2 and T3.	111
4.31	Stabilisation and damping diagrams for Simulation T4, T5 and T6.	112
4.32	Relation between surge and sway modal coordinates for the translation mode (in red) and between the pitch and roll modal coordinates for the rotational mode (in blue).	113
4.33	Relation between surge and sway modal coordinates for the first (in red) and second (in black) modes and between the pitch and roll modal coordinates for the rotational mode (in blue) considering the damaged cable.	114
4.34	Stabilisation and damping diagrams for the different wave heights.	114
5.1	Schematic representation of the angles used to define the position of a sensor. It should be noted that, as represented, $\theta < 0$	119
5.2	Relation between different blades' measurements over a full revolution as a function of the nacelle tilt (θ), considering different combinations of the blades' pre-cone (β) and pitch (γ) angles. The x , y and z directions are defined by the blade-fixed reference frame ($\hat{e}_i$).	121
5.3	Relation between different blades' measurements over a full revolution as a function of the blade precone angle (β), considering different combinations of the nacelle tilt (θ) and blade pitch (γ) angles. The x , y and z directions are defined by the blade-fixed reference frame ($\hat{e}_i$).	121
5.4	Relation between different blades' measurements over a full revolution as a function of the blade pitch angle, considering different combinations of the blades' pre-cone (β) and nacelle tilt (θ) angles. The x , y and z directions are defined by the blade-fixed reference frame ($\hat{e}_i$).	122
5.5	Measured data (red points) and fitted ellipse (black dashed line) for parked time periods selected from WT5 Blade 2 data.	127

5.6	Rotor tilt, blade precone and pitch angle for different values of the rotor angular velocity for WT1 (on top) and WT5 (on bottom).	127
5.7	Reconstructed SCADA data for a single setup.	129
5.8	Reconstructed SCADA full data. On the left, the comparison between estimated (red) and recorded (blue) values. On the right, the pitch curve reconstructed against the SCADA one.	130
5.9	Original (on the left) and transformed (with the additional offset) strains time series for the blade in the upright position.	132
5.10	Estimated offset for each sensor. The distribution plots have been computed by applying the calibration expressions to each point in the time series, while the black dashed lines have been computed considering only their mean value. Both approaches are shown to be mostly equivalent.	133
5.11	Schematically representation of the degrees of freedom used to model a single blade dynamic response.	134
5.12	Power spectrum of the force transmitted to the tower at the hub in the FA (on left) and SS (on the centre), and a single blade shear force (on the right). The colour map has been saturated at the extreme 5% value for each plot. The black solid lines represent the $3k\Omega$ harmonics, while the black dashed lines represent $k\Omega$ harmonics, being k any integer.	137
5.13	Power spectrum of the force transmitted to the tower at the hub in the FA (on left) and SS (on the centre), and a single blade shear force (on the right). The time series used for creating the top and bottom panels have been simulated using two different pitch angles, differing 32.5° . The colour map has been saturated at the extreme 5% value for each plot. The black solid lines represent the $3k\Omega$ harmonics, while the black dashed lines represent $k\Omega$ harmonics, being k any integer.	139
5.14	Power spectrum of the force transmitted to the tower at the hub in the FA (on left) and SS (on the centre), and a single blade shear force (on the right) considering all blade and tower modes. The colour map has been saturated at the extreme 5% value for each plot. The black solid lines represent the $3k\Omega$ harmonics, while the black dashed lines represent $k\Omega$ harmonics, being k any integer.	139
5.15	Simplified model of a floating wind turbine, with the corresponding generalised degrees of freedom identified.	141
5.16	Stabilisation diagrams obtained for two different pitch angle values.	143
5.17	First pair of modes modal configuration. The simulation S1 and S2 have been computed considering different values of pitch angle, according to Figure 5.16	144
5.18	First modes natural frequency evolution as a function of the change in the platform inertia. The blue and red dots correspond to the 1 st FA and SS mode, respectively.	145
5.19	Numerical validation of the transfer functions between accelerations and bending moments neglecting the wind action for simulations S1 (on the left) and S2 (on the right). The points have been coloured as a function of the logarithm of the amplitude of the bending moments power spectrum. The accelerations power spectrum and bending moments power spectrum are presented in orange and blue, respectively.	148

5.20	Numerical validation of the transfer function between accelerations and bending moments for the wind action for simulations S3 (on the left) and S4 (on the right). The points have been coloured as a function of the logarithm of the amplitude of the bending moments power spectrum. The accelerations power spectrum and bending moments power spectrum are presented in orange and blue, respectively. The theoretical prediction from Equation 5.91 , considering a generalised structural transfer function considering a damping ratio of 1%, is plotted as a dashed black line.	150
5.21	Numerical validation of the transfer function between accelerations and bending moments for the combined action of wind and waves for simulations S5 (on the left) and S6 (on the right). The points have been coloured as a function of the logarithm of the amplitude of the bending moments power spectrum. The accelerations power spectrum and bending moments power spectrum are presented in orange and blue, respectively. The theoretical prediction from Equation 5.91 , considering a generalised structural transfer function considering a damping ratio of 1%, and the constant value from Equation 5.87 are plotted as dashed black lines. .	150
5.22	Evolution of the bending moments in a given direction as the nacelle yaws around the tower at the tower base (on the left) and a virtual instrumented section 20m above the tower base (on the right). The non-linear estimates have been obtained increasing the linear response by 7.5%, a value obtained from the FE model. . . .	154
6.1	Picture of one of the wind turbines from the Wind Float Atlantic wind farm taken during the installation of the monitoring system.	156
6.2	Instrumented turbine overall structure (on the left) and picture taken during the installation of the monitoring system, where the maintenance team boat can be seen for reference (on the right).	157
6.3	Instrumented sections' location. The location of the sensors is marked with a grey circle in each tower section.	157
6.4	Accelerometers' pictures taken during the installation operation (from left to right, section 1, 2 and 4). The FEUP wireless mode includes a tri-axial accelerometer developed by FEUP that together with the tri-axial force balance accelerometer are part of the complementary system used for validation and redundancy.	158
6.5	Strain gauges' and temperature sensors pictures taken during the installation operation. From left to right: uni-axial strain gauge glued to the prepared tower surface, strain gauge and temperature sensor with protective cover together with connection box and overview of the installation.	159
6.6	Monitoring cabinet elements and their location.	159
6.7	Data whitening process representation for one of the channels. The original, transformed and fitted spectrum correspond to the absolute value squared of $a_i(\omega)$, $a_i^*(\omega)$ and $f_i(\omega)$ terms in Equation 6.2	161
6.8	Comparison of the accelerometers readings at the tower top for different tilt/surge ratios, expressed in degrees per meter.	163
6.9	Ratio between the tower top and the tower base accelerometers readings for different modes combinations.	164

6.10	SSI-COV results with the raw time series (top plots) and the pre-processed ones (bottom plots). From left to right, the stabilisation diagrams considering different model orders, the corresponding damping diagrams of the stable poles only, the modal representation of the tower top and tower base modal coordinates in the horizontal, xy plane, and the corresponding vertical modal coordinate as a function of the mode frequency.	165
6.11	Poles fitted to all the data from the raw time series (top red plots) and transformed time series (bottom blue plots). On the left, the poles clusters frequency. On the centre panel the damping diagram against frequency. On the right panel the vertical modal coordinate as a function of frequency.	166
6.12	On the left, the damping diagram as a function of the mode frequency. On the centre and right plots, the tower top and tower base modal coordinates for the low frequency modes. For reference, the heave dominated poles are also represented in blue in the damping-frequency diagram.	166
6.13	On the left, the ratio η_2/η_1 evaluated for a significant range of effective heights for the frequency defined clusters identifiable in the centre plot (η_1 and η_2 computed for the lower and higher frequency clusters, respectively). On the centre and right panels, the ratio between the tower top and tower base modal coordinates and its distribution, respectively.	167
6.14	Tracked poles across the setups. On the left panel, the identified frequency. On the centre panel, the damping diagram. On the right panel the orientation of the different modes.	168
6.15	Stabilisation diagrams for two different setups under parked conditions.	169
6.16	First pair of modes for two different setups under parked conditions.	169
6.17	Identified clusters in all parked setups and corresponding damping coefficient. The points have been coloured as a function of the relative importance of the FA and SS components, parameterised by the orientation φ as defined by Equation 6.12, meaning that red and blue points are mostly FA and SS modes, respectively. . . .	169
6.18	Identified clusters in all parked setups as a function of the yaw direction. The points colour identifies the different pitch values found in these setups. The blue points correspond to the lowest value of the pitch angle associated with parked conditions and the red points to the highest value. The green points are for intermediate pitch values, closer to the upper limit than to the lower one.	170
6.19	Evolution of the first FA (red) and SS (blue) natural frequency with the nacelle orientation and corresponding fit.	171
6.20	Identified clusters in selected stable normal operation setups and corresponding damping coefficient.	172
6.21	Identified clusters in selected stable normal operation setups as a function of the rotor angular velocity for the FA (on the left) and SS (on the right) direction. The points have been coloured based on the estimated damping coefficient.	172
6.22	Axis convention for the bending moments positive direction. On the left panel, the axis are defined according to the sensors positions. On the right panel is schematically represented the bending moments associated to a reference frame rotated of θ with respect to sensors defined reference frame.	174
6.23	Mean bending moments and accelerations for all the setups. For the accelerations the accelerometers in the tower base have been used and projected into the strain gauges direction. Only the highlighted points have been used for the fit.	175

6.24	Untwist event bending moments (in blue) and accelerations (in red) time series. On the left plot are presented the strain time series (corrected for temperature and zeroed). In the middle panel are plotted the filtered time series used for the fitting and with the relevant quantities schematically represented. On the right panel, it is estimated the yaw relative direction, that can be seen to vary smoothly during the event.	176
6.25	Mean accelerations value as a function of the platform sensors measurements (on top) and the corresponding residues ordered in time (on bottom). The blue series correspond to S1-S7 sensors alignment and the red series correspond to the S3-S5 sensors.	176
6.26	Mean bending moments as a function of the mean accelerations measurements (on top) and the corresponding residues ordered in time (on bottom). The blue series correspond to S1-S7 sensors alignment and the red series correspond to the S3-S5 sensors.	177
6.27	Corrected accelerations a_0^* and bending moments M_0^* time series. In the residues plots the Gaussian distribution assuming a standard deviation of 1.5 MN.m (solid lines) and 0.75 MN.m (dashed lines) have been included. The blue series correspond to S1-S7 sensors alignment and the red series correspond to the S3-S5 sensors.	178
6.28	On the left panel is plotted the maximum mean stress orientation as a function of the nacelle yaw direction. The relative direction between the two quantities ($\Delta\phi$) is plotted as a function of the wind speed (in the middle). The distribution of the selected events, marked in red in the centre panel, is plotted on the right panel. . .	179
6.29	Estimated relative position of the strain gauges, reference yaw direction and the platform principal direction. To determine the angle between the alignment S1-S7 and the axis of symmetry of the platform, it was assumed that the tower door is in the plane of symmetry of the platform.	179
6.30	Bending moment along the S1-S7 and S3-S5 axis as a function of the yaw position. The dashed line represented the bending moments envelope for a relative orientation of 61.3° . The shaded region represent the same envelope considering the uncertainty in this estimate from Figure 6.28	180
6.31	FA (on the left) and SS (on the right) bending moments as a function of the mean wind speed.	181
6.32	FA (on the left) and SS (on the right) bending moments induced solely by the rotor thrust force as a function of the mean wind speed.	181
6.33	FA and SS bending moments frequency content for normal operation setups. . . .	182
6.34	Fatigue SN curve from the DNV-RP-C203 and EC3.	184
6.35	On the left panel it is plotted the critical direction for fatigue as a function of the nacelle yaw direction. The relative direction between the two quantities ($\Delta\phi$) is plotted as a function of the wind speed (in the middle). The distribution of the selected events, marked in red in the centre panel, is plotted on the right panel. . .	185
6.36	Fatigue damage dependency on the mean wind speed. The corresponding turbulence intensity is mapped into the plot colour. All setups on the left and corresponding mean value and standard deviation, on the right.	185
6.37	Fatigue damage dependency on the tower top accelerations standard deviation. The mean wind speed is mapped into the plot colour. All setups on the left and corresponding mean value and standard deviation, on the right.	186

7.1	Schematically representation of the fatigue damage extrapolation (forward and backwards in time) from monitoring data, where the fatigue damage is characterised from the mean wind speed and turbulence intensity.	189
7.2	Schematically representation of the proposed model for fatigue damage estimation of floating offshore wind turbines. The SCADA data is used to label the events as parked (P), normal operation (N) and transition (T) setups, that are then divided into a low frequency (L), a background (θ) and harmonic (H) component. The transformation to be applied to each is calibrated using the strain gauges measurements, leading to the 3 bending moments time series λ_i , that can be superimposed to estimate the total response λ and corresponding fatigue damage (D). The estimated and measured values can be compared for the training period (setups used for the calibration) and testing period (remaining setups).	192
7.3	Results for the calibration of C_θ . On the left panel, the bending moments (blue) and accelerations (orange) power spectrum densities (on bottom) and their ratio (on top) for a selected 10 minutes setup and the FA direction. The colours are proportional to the logarithm of the power spectrum amplitude. On the centre panel the estimated value of C_θ for the FA direction. On the right the same results for the SS direction.	193
7.4	Results for the calibration of the harmonic response parameters, C_r^h and C_ω^h . On the left panel, the bending moments (blue) and accelerations scaled by C_θ (orange) power spectrum densities (on bottom) and their ratio (on top) for a selected 10 minutes setup. The red dots were used for the definition of the parameters and have been identified as the frequency compatible with rotor angular velocity ranges where the response peaks. On the centre and right panels the estimated value of C_r^h and C_ω^h , respectively.	194
7.5	Results for the calibration of the harmonic response parameters, C_M^l and C_ω^l . On the left panel, the bending moments (blue) and accelerations scaled by C_θ (orange) power spectrum densities (on bottom) and their ratio (on top) for a selected 10 minutes setup. The black highlighted region identifies the frequency range used for the fit. On the centre and right panels the estimated value of C_M^l and C_ω^l , respectively.	194
7.6	Example of the bending moments estimation for a parked setup. On the top panels are plotted the time series and on the bottom panels the corresponding power spectra. On the left, are plotted the bending moments (blue) and accelerations (orange) raw time series. On the centre panels are plotted the different contributions: from the top to bottom, following the order in Figure 7.2, the low frequency, background and harmonics data series. On the right are plotted the measured (blue) and estimated from the accelerations (orange) bending moments.	195
7.7	Example of the bending moments estimation for a setup under normal operation conditions. On the top panels are plotted the time series and on the bottom panels the corresponding power spectra. On the left, are plotted the bending moments (blue) and accelerations (orange) raw time series. On the centre panels are plotted the different contributions: from the top to bottom, following the order in Figure 7.2, the low frequency, background and harmonics data series. On the right are plotted the measured (blue) and estimated from the accelerations (orange) bending moments.	196

7.8	Example of the bending moments estimation for a transition setup. On the top panels are plotted the time series and on the bottom panels the corresponding power spectra. On the left, are plotted the bending moments (blue) and accelerations (orange) raw time series. On the centre panels are plotted the different contributions: from the top to bottom, following the order in Figure 7.2 , the low frequency, background and harmonics data series. On the right are plotted the measured (blue) and estimated from the accelerations (orange) bending moments.	197
7.9	Comparison between the measured fatigue damage (red line) and the estimates obtained with the proposed methodology (green line) and the conventional approach (black line) for parked setups. Negative and positive values on the horizontal axis correspond to training and testing periods, respectively. Since all the curves have been forced to pass through the (0,0) point, the training period results should be read as the absolute value of the corresponding curve.	198
7.10	Comparison between the measured fatigue damage (red line) and the estimates obtained with the proposed methodology (green line) and the conventional approach (black line) for normal operation setups. Negative and positive values on the horizontal axis correspond to training and testing periods, respectively. Since all the curves have been forced to pass through the (0,0) point, the training period results should be read as the absolute value of the corresponding curve.	199
7.11	Comparison between the measured fatigue damage (red line) and the estimates obtained with the proposed methodology (green line) and the conventional approach (black line) for transition setups. Negative and positive values on the horizontal axis correspond to training and testing periods, respectively. Since all the curves have been forced to pass through the (0,0) point, the training period results should be read as the absolute value of the corresponding curve.	199
A.1	Distributed vorticity over an aerofoil.	222
A.2	Transformation of a circular section into an aerofoil through a conformal transformation by means of Equation A.18 , considering $a = -0.5$ and $\varepsilon = -0.25$	225
A.3	NACA0012 section normalised.	227
A.4	Dependence of ψ and ε on ϕ para a NACA0012.	227
A.5	Comparison of the pressure coefficients obtained (red dots) and the results from <i>software</i> XFOIL (blue line).	228
A.6	Scheme of the computational domain used in the numerical simulations (in chord units).	228
A.7	Evolution of the drag force (on the left) and number of mesh elements (on the right) as a function of the model order.	229
A.8	Final mesh used in the CFD simulations - global and detailed view.	229
A.9	Pressure (on the left) and velocity (on the right) over the airfoil at 0° angle of attack.	230
A.10	Lift and drag coefficients validation.	230
A.11	Blade's chord (on the left) and twist (on the right) evolution over the span.	231
A.12	Blade's induction (on the left) and force coefficients (on the right) evolution over the span.	232
B.1	Ansys model of a floating cylinder and its principal properties.	234
B.2	Comparison of the forces over 3 different diameter cylinders (on top) and for different wave length for the 20 [m] diameter cylinder (on bottom).	235
B.3	Drift motion (on the left) and speed (on the right) for the medium wave.	237

B.4	Total force transmitted by the mooring lines to the cylinder has function of the displacement <i>surge-pitch</i> (on top) and <i>sway-roll</i> (on bottom).	238
B.5	Total restoring moment transmitted by the mooring lines to the cylinder has function of the displacement <i>surge-pitch</i> (on top) and <i>sway-roll</i> (on bottom).	238
B.6	Surge displacement (on the left) and mooring system total force (on the right) overtime from a quasi static and a full dynamic simulation.	239
B.7	Force-displacement law for a single cable (on the left) and for the mooring system (on the right) from a quasi static and a full dynamic simulation.	239
B.8	Surge displacement (on the left) and a single cable force-displacement law (on the right) neglecting or not cables' bending stiffness.	239
C.1	Schematic representation of the gravity pendulum.	243
C.2	Schematic representation of the spinning pendulum.	246
C.3	Schematic representation of the wind turbine simplified model and respective degrees of freedom.	248
C.4	Tower mass distribution (on the left) and stiffness (on the right).	252
C.5	Tower top displacement response and its derivative in both time (on the left) and tower top displacement response on frequency (on the right) domain.	254
C.6	Tower top displacement (on the left) and rotor angular velocity (on the right). . .	255
C.7	Tower top displacement response spectrum for the unsteady simulation.	256

List of Tables

4.1	Tower properties.	102
4.2	Hub and nacelle properties with respect to their principal axis.	102
4.3	Single blade properties with respect to its principal axis.	103
4.4	Chacaracterization of the different simulations carried out.	106
4.5	Modal properties, including angular frequencies, natural frequencies, modal damping ratios and 6 mode shapes components obtained for the different simulations (X - surge; Y - sway; Z - heave; RX - roll; RY - pitch; RZ - yaw).	113
A.1	Iterative process for obtaining the axial and transverse induction coefficients. . .	232
B.1	Waves used for the simulations properties.	234
C.1	Rotor and nacelle properties with respect to their principal axis.	253
C.2	Modal Mass and Modal Stiffness associated with the first four modes, assuming the mode shape normalization presented in Equation C.46	253

Nomenclature

Abbreviations and Acronyms

CFD	Computational Fluid Dynamics
DNS	Direct Numerical Simulation
FEUP	Faculty of Engineering of University of Porto
FFT	Fast Fourier Transform
JONSWAP	Joint North Sea Wave Project
LES	Large Eddy Simulation
MEMS	Micro-Electro-Mechanical Systems
NREL	National Renewable Energy Laboratory
OMA	Operational Modal Analysis
PM	Pierson-Moskowitz
RANS	Reynolds Average Navier Stokes
RAO	Response Amplitude Operator
SCADA	Supervisory Control and Data Acquisition
SSI-COV	Covariance driven stochastic subspace Identification
SWL	Still Water Level
TLP	Tension leg platform

Lowercase Latin

a	Axial induction coefficient
a'	Tangential induction coefficient
c	Weibull scale parameter Wave propagation speed Blade chord
d	Water depth
f	Frequency
$\mathbf{f}$	Force
f_a	Acquisition rate
f_R	Root loss factor
f_T	Tip loss factor
$\mathbf{g}$	Gravitational acceleration
g	Gravitational acceleration magnitude
k	Weibull shape parameter Wave number Rigidity Turbulence kinetic energy
ℓ	Turbulent characteristic length
m	Mass per unit length
m_n	Distribution moment of order n
m_h	Horizontal linear shear
m_v	Vertical linear shear
n	Recorded number of cycles for fatigue evaluation
p	Fluid pressure
$\mathbf{r}$	Position
s	Spreading function cosine power parameter Distance between vortex sheets
t	Time
$\mathbf{u}$	Flow velocity
u_i	Flow velocity in the spatial direction i
u_r	Reference velocity
u_τ	Friction velocity
u^+	Dimensionless velocity
$\mathbf{v}$	State-space model noise vector for the observation equation
$\mathbf{w}$	State-space model noise vector for the state equation
w	Wake width
$\mathbf{x}$	State-space state vector Displacement
$\mathbf{y}$	State-space output vector
y^+	Dimensionless wall coordinate
z_0	Surface roughness
z_r	Reference height
z	Altitude

Uppercase Latin

A	State-space model state matrix
A_i	Flow cross section at section i
B	State-space model input matrix
C	State-space model output matrix
C_{ij}	Spatial coherence function between points i and j
C_i	Aerodynamic coefficient for the force F_i
C_P	Power coefficient
C_T	Thrust coefficient
D	State-space model direct transmission matrix
D	Cumulative fatigue damage
F_T	Thrust force
F_w	Wave forces
G	Spreading function Green function
G_0	Spreading function normalisation constant
G	Correlation matrix between state vector and response vector
$\mathcal{H}$	Structural transfer function
H_s	Significant wave height
KC	Keulegan-Carpenter number
$\mathcal{L}$	Lagrangian
L_T	Cable total length
L_0	Cable suspended length
M	Mass
N	Cycle limit for fatigue Number of blades
O	Observability matrix
P	Power
$\mathcal{R}$	Platform transfer function
R	Output correlation matrix Reynolds stress tensor
Re	Reynolds number
S	Output power spectrum matrix Deformation rate tensor
S	Stress variation
S_η	Wave amplitude power spectrum
T	Transfer function Toeplitz matrix
T	Wave period
T_D	PID Derivative characteristic time
T_I	PID Integral characteristic time
TI	Turbulence intensity
U	Wind speed
U_w	Mean wind speed (Weibull distribution) Wake wind speed
U_0	Free stream wind speed
U_d	Wind speed at the rotor plane
W	Relative wind speed Cable weight

Lowercase Greek

α	Power law exponent Pierson-Moskowitz and JONSWAP scale parameter Angle of attack Rotor azimuthal position
α_0	Zero-lift angle of attack
α_s	Sea-bed slope
β	Blade precone
γ	Peak-enhancement factor Vorticity per unit length Pitch angle
δ	Boundary layer thickness
ε	Perturbation expansion parameter Strain
η	Wave elevation
η_0	Maximum wave elevation
θ_P	Wave dominant direction
θ	Nacelle tilt Platform tilt
κ	Von Karman constant
λ	Wavelength Tip speed ratio Lagrange multiplier
λ_k	Continuous time k^{th} eigenvalue
λ_r	Local speed ratio
μ	Asymmetry parameter Dimensionless span position
μ_k	Discrete time k^{th} eigenvalue
ξ	Spreading function symmetry breaking parameter
ξ_k	Damping ratio of mode k
ρ	Fluid density
σ	Wind speed standard deviation Complex source strength Stress
τ	Shear stress tensor
τ	JONSWAP frequency dependent parameter Shear stress Torque Characteristic time
τ_P	Pitching moment
ν	Fluid kinematic viscosity
ν_T	Turbulent viscosity
ϕ	Latitude Flow velocity potential
ϕ_n	Modal shape n
χ	Wake-skew angle
χ_0	Angle between the rotor normal and the wind main direction
ψ	Rotor skew angle Yaw angle
ω	Angular frequency
ω_0	Spectrum peak angular frequency
ω_k	Natural angular frequency of mode k

Uppercase Greek

Γ	Stochastic controllability matrix
Γ	Circulation Gamma function
Δ	Maximum velocity deficit in the wake
Λ	Modified tip speed ratio
Φ_F	External force potential
Φ_r	Radiated wave potential
Φ_D	Diffacted wave potential
Φ_d	Scattered wave potential
Φ_0	Unperturbed wave potential
Ψ	Modal configurations matrix
Ω	Earth angular velocity
Ω	Earth angular velocity (magnitude) Rotor angular velocity
Ω_D	Objective rotor angular velocity

Special symbols and operators

$*$	Complex conjugate
$\dagger$	Moore-Pensore pseudo-inverse
$\sim$	Fourier transform
$\wedge$	Unitary vector Turbulent component
$\langle \rangle$	Expected value
$\parallel$	Parallel
$\perp$	Perpendicular
T	Transpose
H	Hermitian conjugation
D_t	Material derivative
∂_t	Partial derivative with respect to time
∂_i	Partial derivative with respect to the spatial coordinate i
∂_{tt}	Second partial derivative with respect to time
∂_{ii}	Second partial derivative with respect to spatial coordinate i
∇	Nabla operator
∇^2	Laplacian operator
$\square$	d'Alembert operator

(See [Section 1.3](#) for additional conventions on upper and lower indexes usage)

Chapter 1

Introduction

1.1 Research Context

The world energy needs have been increasing exponentially since the industrial revolution, a demand that has been mostly satisfied by the intensive use of fossil fuels. However, with the raise of environmental concerns, the search for sustainable energy sources has been increasing significantly over the last years, including wind powered energy devices. The total power produced by wind energy devices in Europe has become an important contribution for the overall electricity demand and has been steadily increasing its capacity for the last years ([WindEurope, 2023](#)). While onshore wind turbines have been already used for thousands of years (with a big technological jump in the last decades), offshore applications, although providing an optimal location due to higher mean speed and low turbulence winds, have been off the table until the last decades, either to lack of technical solutions or, more recently, to its prohibitive cost, specially for higher water depths. However, this has changed in the last decade, since more and more offshore farms have been installed and connected to the grid.

Although the offshore contribution is still small when compared to the onshore solutions, it is already responsible for a significant amount of energy in countries like Belgium, Denmark or the UK ([WindEurope, 2023](#)). Furthermore, until very recently these solutions were confined to lower depths and near shore waters, using bottom fixed solutions (e.g. gravity structures and monopiles) for the overall stability of the system. With the advent of floating platform solutions, this limitation was overcome and wind turbines have been installed in successively deeper waters and further away from the shore. In particular, the European governments plans for the REPowerEU 2030 now add up to an impressive 160 GW of installed capacity in offshore conditions until 2030, where Portugal has an important contribution by targeting at 10 GW on its own ([WindEurope, 2022](#)).

Focusing for now on the floating solutions alone, a particular relevant project is currently taking place in the northern of Portugal, named WindFloat Atlantic ([WindPlus, 2022](#); [Principle Power, 2022](#)). A preliminary 2 MW test unit has been designed, constructed and installed off the Portuguese coast, close to Aguçadoura, and it was connected to the grid at the end of 2011. After

successfully running for 5 years and surviving 17 m high waves, this prototype - WindFloat 1 - has been decommissioned, but from this successful demonstration version, a precommercial solution has already been installed and connected to the grid at 20 km off the shore of Viana do Castelo in 100 m deep waters. The Windfloat Atlantic Phase 1 consists of three V164-8.4 MW wind turbines, the first floating wind farm installed in the world, for a total of 25 MW of installed capacity.

Although it is indeed a turning point in the field by demonstrating the viability of offshore wind based on floating solutions, to scale up the 25 MW to the mentioned 10 GW by 2030 requires an important and combined investment in a wide range of areas, from permitting procedures and grid capacity bottlenecks, to supply chain and specialised staff short-comes, and it is in this particular last aspect that the academic world and research groups must most certainly play an important role. To try to contribute and answer this increase in demand, a research program has been initiated at Faculty of Engineering of University of Porto's (FEUP) CONSTRUCT research unit focused on the development of skills in numerical modelling and structural monitoring of this type of structures, by combining a strong theoretical background with important experimental campaigns. In particular and after experimental campaigns over full-scale onshore wind turbines with accelerometers and strain gauges and the development of several tools for the post processing of the obtained data, two PhD thesis have already resulted from this project (Oliveira, 2016; Pacheco, 2022). It is in this context that this thesis emerges, as an effort to improve the current know-how in floating wind turbines physical processes and numerical modelling, and to extend and adapt the monitoring methodologies to the floating offshore cases.

1.2 Objectives of the Thesis

From the last section, it is clear that offshore wind farms, in general, and floating solutions for deep waters, in particular, are an important part of the solution to the ongoing transition to cleaner sources of energy. If the very ambitious goals defined for 2030 are to be realisable, every design aspect counts and so modern wind turbines should be highly efficient, fine tuned designed and slender structures. Moreover, as important as its design, it is their operation and maintenance strategies, as small differences in the approach can lead to a substantial profit loss at the end of the year (for instance, as discussed by Pao and Johnson (2011), a 5% modelling error in the controller systems can lead to 1% to 3% energy loss and to over €100000 losses in a typical 100-MW wind farm). Because of this, the ability to correctly model and predict the structural behaviour and durability of a wind turbine, prior to its installation, as well as the definition of the most efficient monitoring strategies are vital for this competitive market to grow and eventually become the leading source of energy. These monitoring systems can also be used to identify that a damage is being initiated at a specific location, so that only the maintenance operations that are indeed needed can be executed and no damage is left unattended.

Due to several confidentiality issues in a highly competitive market, the availability of real, experimental data from floating wind turbines for academic purposes is almost nonexistent and so the methods developed for bottom fixed or onshore wind turbines are still mostly unexplored in

these scenarios. In fact, this thesis initiated under the assumption that these limitations would most probably still hold throughout the working period, and hence the initial objective was the development of reliable numerical models that could mimic the lacking experimental data. These models would not be the final goal themselves, but could be interpreted as an artificial laboratory to generate structural responses for testing and validation of monitoring strategies. If this was the context during the first 3 years of the thesis work, in 2022 emerged the very fortunate opportunity to install a monitoring system in a full scale floating wind turbine from the Wind Float Atlantic wind farm. Since this data is naturally much more interesting, as well as challenging, than any numerically simulated time series could eventually be, no matter how fine tuned and high-fidelity would be the model considered, the scope of the thesis was slightly deviated from a purely numerical basis.

Having set the time line of events clear, the motivation and the definition of the different thesis objectives can now be more easily understood. Firstly, in all the developed work, there is a background desire to better understand the complex dynamic response of these structures, an objective that is also imprinted in the document structure itself, that in some occasions assumes a more pedagogical perspective, as it tries to lay the basis for future works in this topic. More importantly, it leads to the development of different analytical models, with different implicit assumptions and ranges of applicability, as will be discussed later, that should not be read as the objective *per se*, but rather as a theoretical framework over which the robust definition of monitoring strategies and experimental data interpretation can be made. Secondly, and even considering the existence of experimental data, the development of the artificial laboratory concept was still kept as a relevant outcome of this work, by essentially two different reasons. On one hand, it allows to test new methodologies in a more controlled environment and under environmental conditions that might not have been faced in the experimental campaign yet. On the other hand, these simulations can also be used to test and validate any prediction based on the analytical approach just described. Finally, two very clear objectives were defined that extend conventional monitoring strategies to the floating offshore scenarios, that can now be tackled over the strong foundations provided by both the analytical and numerical approaches just described. On one hand, a very relevant capacity of any structural monitoring campaign is the identification of the system resonant frequencies in operation conditions, from where the structural integrity of the system may be evaluated indirectly. In this work, the Covariance driven stochastic subspace Identification (SSI-COV) method viability is shown, even for the identification of some very low frequency modes mostly associated with the platform motions. On the other hand, and noting that it is a critical aspect for the management and life extension decisions of these structures, it is of vital importance to have a methodology to estimate the fatigue damage accumulated during the structure lifetime, preferably without the need for the relatively complex installation of strain gauges and with an almost on demand prediction, such that the operation strategy can be steadily evaluated and adapted. This very clear objective lead to the definition, validation and testing with experimental data of a new methodology that satisfies all the requirements above.

From the discussion above, it can be said that the ultimate purpose of this thesis is to develop a strong theoretical background over which new methodologies can be devised, in an effort to

provide relevant tools that can aid the decision making process of floating offshore wind farms explorations.

1.3 Organisation of the Thesis

To answer the objectives above, for which the numerical models reliability is an important part, either for methodology testing or analytical predictions validation, an important part of the work was devoted to properly understand the physical processes behind any aeroelastic simulation of a wind turbine. To clearly separate these fundamental principles that had to be studied to properly develop the physical intuition that was needed to study and develop the new methodologies here presented, the thesis is divided in [Part I](#), that constitutes the Research Background, and [Part II](#), with the Research original developments.

In [Part I](#), the theoretical principles are described over two Chapters. Here, no original work, except for the illustrative examples that will be presented whenever considered to be relevant, is included, but it is still a fundamental part to be considered, specially for those less familiar with aerodynamic and/or hydrodynamic modelling. In [Chapter 2](#), the principal concepts needed to describe a wind turbine will be presented, as well as the environmental conditions that should be considered. In addition, the basis of structural monitoring strategies will be described, and the expected challenges in offshore scenarios briefly discussed. The objective of this chapter is not to provide a detailed description of any concept but rather to set the basis needed to a proper discussion of the more advanced topics addresses subsequently. In particular, it should be noted that the structure/fluid interaction is not considered in this Chapter and is left for [Chapter 3](#). There, the fundamental concepts regarding both aerodynamic and hydrodynamic behaviour and modelling will be addressed. For the former, the consequences of a given wind field, that at this point has already been presented and characterised, on the external forces experienced by different wind turbines due to their aerodynamic properties will be derived. To better understand this, the flow over a blade will be studied, starting from simplified, ideal flow models all the way to Computational Fluid Dynamics topics. Having studied the aerodynamics of a single blade, the larger scale behaviour of the rotor will be presented from first principles to their deviations from the ideal rotor formulation corrected through engineering models. Complementary, and focusing now on the hydrodynamic behaviour, a similar approach will be followed, where the sea states previously described will be translated into external forces acting over a floating body. Here, and unlike the aerodynamic behaviour, the hydrodynamic response of a floating body can be computed adequately from a potential formulation with minor corrections to include the viscous drag forces. Because the description of a wind turbine floating platform is not complete without the definition of a mooring system, the fundamental principles behind cables modelling will be also presented. Finally, the equations of motion of a moored floating body will be derived considering all the relevant actions in both time and frequency domain. Together with [Chapter 2](#), this concludes [Part I](#) of this work.

In [Part II](#) are then included all the relevant contributions of this thesis. Since numerically simulated data will be used as an artificial laboratory where some of the approaches discussed afterwards will be tested, it was considered that the numerical modelling itself should be firstly validated, ensuring that the time series generated are representative of the real behaviour of a floating wind turbine. This is the goal of [Chapter 4](#), where both aerodynamic and hydrodynamic concepts presented will be tested and evaluated with numerical models and compared against experimental data. A brief description of the OpenFAST software that will be extensively used in this work will be made and the link with the previously presented concepts established. A first model of a real wind turbine will be described and defined from design drawings and monitoring data. The model predictions (tower top accelerations, tower bending moments...) will then be tested against the experimental results available for the same wind turbine, that is currently being monitored by FEUP. Finally, a well documented certification test from OpenFAST of a floating wind turbine will be reproduced in ANSYS/AQWA and both results compared. The time series here generated will be used as a preliminary test of the viability of Operational Modal Analysis methods in the offshore environment.

Once the principal modelling concepts are properly validated, numerical simulations can be used to support and test the physical interpretation of experimental data measurements. In [Chapter 5](#), a theoretical framework based on simplified models and analytical considerations that contribute to a better interpretation of the experimental measurements in different structural elements is explored. Firstly, it is shown that the measurement of accelerations in the blades of a wind turbine include very valuable information regarding both the rotor geometric properties and the wind turbine operation conditions. This is particularly relevant since this information is often unavailable. In a different note, the effect of a spinning rotor in the forces transmitted to the tower, where accelerations and strains are often measured, is also studied in analytical grounds. This exercise is relevant as it allows for a better interpretation of the experimental data. Finally, and if the two tasks above are quite general to any wind turbine, this Chapter ends with a specific analysis of a floating solution, where the effect of a floating platform is studied and some very interesting relations concerning the natural frequencies dependency on the structure properties unveiled. Additionally, it is also shown that the tower top accelerations and the bending moments at its base follow relations that are clearly distinct of the ones expected in onshore scenarios, or any conventional civil engineering structure, in general.

Although all of the predictions of the analytical derivations just discussed are validated by numerical simulations, some of them will also be applied to the experimental data of a full scale wind turbine. The experimental campaign of one of the wind turbines of the WindFloat Atlantic wind farm is described in [Chapter 6](#), where Operational Modal Analysis is also used to identify the system natural frequencies and damping ratios. A particular relevant discussion is made regarding the very low frequency modes mostly associated with the platform motions, where a new data pre-processing methodology that greatly enhances the SSI-COV identification ability is presented. In this Chapter it is also discussed the reconstruction of the internal forces from strain gauges measurements, where the calibration procedure of these sensors based on some particular events used

is the onshore context is extended to floating wind turbines. This Chapter ends with a preliminary analysis of the fatigue damage obtained from the reconstructed stresses time series and its relation to both environmental and operation conditions. Finally, and based on the theoretical developments of [Chapter 5](#) and the fatigue damage estimates from [Chapter 6](#), a new methodology for the fatigue evaluation of floating wind turbines is presented in [Chapter 7](#). In this Chapter, the analytical considerations are extended to incorporate the oversimplifications made in their derivation and calibrated using a subset of the available experimental data. After this calibration, its response is compared against a conventional approach based on SCADA data only that is often applied in the analysis of onshore wind turbines, demonstrating a significantly better output.

In any case, and as it always happens in research activities, not only there are some working lines that have not been completely explored in this thesis, the theoretical frameworks here developed also opens up new possibilities for the definition of new and better monitoring strategies. Some of these are highlighted in [Chapter 8](#), where the principal contributions and main ideas to retain from this work are also summarised.

As a final note, and having in mind the pedagogical perspective mentioned in the last section, it should be mentioned that some concepts may appear in more than one place under different approaches (for instance, when describing the equations that govern some of the aerodynamic and hydrodynamic properties). Although this may not be ideal, this choice was made in order to keep the different subjects self-contained, so that a potential reader that seeks deeper understanding in a particular topic may do it with ease. There are, however, some general conventions that are present in most derivations and should be highlighted right away. Following a common convention, vectors will be represented by bold lower case letters while bold upper case letters will stand for matrices, while in contrast scalars are indistinctly represented by upper and lower case letters, but not bold ones. The functional dependence of any of these entities will be expressed between parenthesis after its symbol, but can be omitted whenever its explicit representation is not needed or physical interpretation is not meaningful. In the same spirit, some variables, when there is not a clear physical meaning, will be implicitly defined by the equations in which they appear firstly and not in the body of the text. For instance, in:

$$x(y;H) = \frac{3 \cdot H}{2} \cdot y = R \cdot y \quad (1.1)$$

$x(y;H)$ is a scalar function of a single variable y and R is implicitly defined as $R = \frac{3 \cdot H}{2}$ (note that H is not a variable but rather a scalar parameter for the definition of x , and hence the separation with the semi-colon). Additionally, Einstein notation over repeated indexes has been introduced ([Hashiguchi, 2020](#); [Einstein, 1916](#)) for most of the equations. In this convention when an (dummy) index is repeated in a single term a summation is implied, such that, the dot product between two vectors may be written as:

$$\mathbf{a} \cdot \mathbf{b} = \sum_i a_i b_i = a_i b^i \quad (1.2)$$

Finally, a particular convention has been systematically used for derivatives, where subscripts identify the derivatives that are being taken. For instance:

$$\partial_x \equiv \frac{\partial f}{\partial x} \quad (1.3a)$$

$$\partial_{xy} \equiv \frac{\partial^2 f}{\partial x \partial y} \quad (1.3b)$$

The conventions above can be used simultaneously, such that the divergence of a vector field can be written as:

$$\nabla \cdot \mathbf{u} = \sum_i \frac{\partial u_i}{\partial x_i} = \partial^i u_i \quad (1.4)$$

Part I

Research background

Chapter 2

Floating offshore wind turbines theoretical minimum

2.1 Introduction

In the XXth century the physicist Lev Landau developed an exam based on everything a student was supposed to know in order to work with him called *The Theoretical Minimum*. Although the definition was slightly relaxed, this concept led to a series of courses, from Classical Mechanics to General Relativity and Quantum Mechanics, lectured by Stanford's Professor Leonard Susskind. In his idea, the theoretical minimum was "just what you need to know in order to proceed to the next level" (Susskind and Hrabovsky, 2014). In this chapter, and following the latter interpretation, the main concepts that will be essential for the derivations and discussions that will be made in following chapters will be briefly presented. Firstly, a description of the environmental conditions that these structures need to face will be given, followed by an overall description of the most relevant components of any floating wind turbine. At the end of this chapter, the principal techniques used to monitor and evaluate its behaviour will be introduced, and the current challenges that conventional approaches face in offshore scenarios briefly discussed.

2.2 Floating wind turbines environment

2.2.1 Wind characterisation

Before being able to either understand the dynamic behaviour or develop a proper numerical model of these complex systems, the environmental conditions that need to be considered, and that will source the external forces acting on the wind turbine, should be studied.

Due to the ultimate purpose for which wind turbines are designed, the wind action is naturally a fundamental one to consider. In this section are laid the basis for a proper understanding of wind variations in time and space, which are fundamental to correctly assess the power output expected for a given site and the lifetime loads wind turbines will face.

2.2.1.1 Wind physics

Making correct predictions of the wind speed is not an easy task due to the large number of influencing factors and phenomenon, some of which still not fully understood. The dynamic behaviour of air masses can be mathematically described by the Navier Stokes' equations (NS), that, when applied to an incompressible flow, can be expressed in an Earth-bound rotating reference frame as:

$$D_t \mathbf{u} + 2 \cdot \boldsymbol{\Omega} \times \mathbf{u} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) = \nu \cdot \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{g} \quad (2.1)$$

where ρ , p and ν are the fluid density, pressure and kinematic viscosity, $\mathbf{u}$ is the flow velocity, $\boldsymbol{\Omega}$ the Earth-angular velocity, the operator $D_t = \partial_t + \mathbf{u} \cdot \nabla = \partial_t + u_i \partial^i$ represents the material derivative, $\nabla^2 = \partial_i \partial^i$ is the Laplacian operator and $\mathbf{r}$ is the position vector. The terms on the right hand side can be identified, following the order they appear, as the forces per unit mass arising from molecular viscosity, the pressure gradient and the Earth gravitational field ($\mathbf{g}$ is the gravitational acceleration).

If the friction force is neglected some global behaviour of the atmosphere can be explained. While the centrifugal pseudo-force (the $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$ term in Equation 2.1) is always present, even for resting air masses, and can be accounted for considering an effective gravitational pull, the Coriolis pseudo-force¹, should be taken into account as its effects are relevant. Assuming a reference frame in which the velocity vector can be expressed as $\mathbf{u} = (u_x, u_y, u_z)$ with its components along x and y pointing in the East-West and South-North direction, respectively, and z pointing away from the surface, the velocity components for a given latitude ϕ are:

$$D_t u_x - 2 \cdot \Omega \cdot \sin \phi \cdot u_y = -\frac{1}{\rho} \partial_x p \quad (2.2a)$$

$$D_t u_y + 2 \cdot \Omega \cdot \sin \phi \cdot u_x = -\frac{1}{\rho} \partial_y p \quad (2.2b)$$

$$D_t u_z = -\frac{1}{\rho} \partial_z p - g \quad (2.2c)$$

From the presented expressions, it is possible to conclude that a stationary state is associated with an hydrostatic balance in the vertical direction and with an equilibrium between the Coriolis force and pressure gradients in the horizontal direction. In fact, as the Coriolis force only appears with the relative velocity, a movement that is initially set by the pressure gradients suffers the influence of a force perpendicular to its movement. In equilibrium, these 2 forces lie on the same line and give rise to the so-called **geostrophic wind**. More rigorously, in this type of wind the isobars are implicitly assumed to be straight and uniformly spaced, but this is rarely the case. In fact, as the isobars bend due to localised areas of over and under pressure, an additional centrifugal force appears, leading to a **gradient wind**, that is still parallel to the isobars, but where the equilibrium is no longer achieved by the Coriolis force and pressure gradients alone.

¹From now on the prefix "pseudo" will be omitted whenever this forces are mentioned.

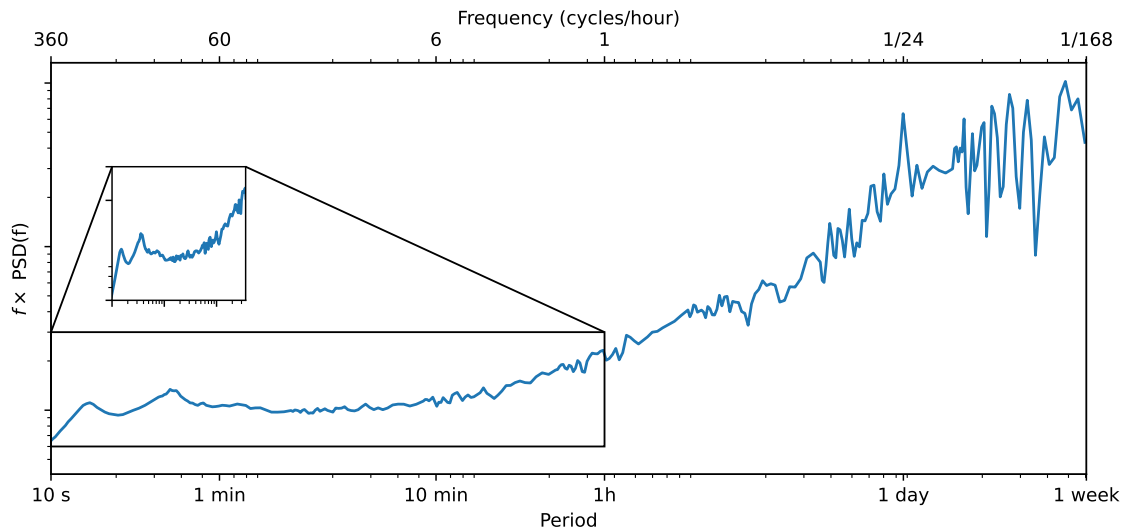


Figure 2.1: Wind spectrum obtained from 1Hz SCADA data from one of the ongoing FEUP experimental campaigns.

Without detailing any further on this matter, it may still be concluded that the global behaviour of wind is mainly explained by hydrostatic balance in the vertical direction and an equilibrium of the forces presented in the horizontal direction. However, if the wind intensity is to be described with more detail, shorter time intervals, shorter length scales and friction forces should be considered as they alter significantly the dynamics of the flow previously described.

In fact, in time periods not greater than a few days, there are 3 types of relevant occurrences that can be defined in a typical wind speed spectrum that may be grouped by their characteristic time scale:

- Periods of the order of a few days - **Synoptic Peak**;
- Periods of the order of a day - **Diurnal Peak**;
- Periods of the order of a few minutes - **Turbulent Peak**.

The synoptic peak, of the order of some days, is mainly related to large scale atmospheric phenomenon, while the diurnal peak is related to variations through out a day mainly due to a non homogeneous surface heating from the sun radiation. Because of its origin, the latter tends to disappear during night time, which produces frequencies of the order of half a day. The wind spectrum from one of the experimental campaign developed by FEUP is represented in [Figure 2.1](#), where the different peaks can be identified, although the turbulent component is relatively low in the period under analysis. In any case, an important aspect to retain is the spectral gap observed for frequencies between 10 minutes and a couple of hours with relatively low energy content in this region.

Having in mind this spectral gap, it is possible to apply Reynolds decomposition ([Müller, 2006](#)) to separate the turbulent high frequency component from a mean wind speed, which is more slowly varying. Here, it should be mentioned that in the Reynolds decomposition it is assumed that the mean quantities are time independent, but one may slightly relax this assumption by allowing

the mean flow to vary in longer time scales when compared to its turbulent counterpart. Under this assumption, the fully turbulent wind speed ($\mathbf{u}$) can now be expressed as a linear superposition of a mean speed ($\langle \mathbf{u} \rangle$) and a turbulent component ($\hat{\mathbf{u}}$) so that:

$$u_x = \langle u_x \rangle + \hat{u}_x \quad (2.3a)$$

$$u_y = \langle u_y \rangle + \hat{u}_y \quad (2.3b)$$

$$u_z = \langle u_z \rangle + \hat{u}_z \approx \hat{u}_z \quad (2.3c)$$

where the last equality in Equation 2.3c tacitly assumes the hydrostatic balance in the vertical direction already discussed. These definitions allow to take the ensemble average of the Navier Stokes equations to write the so called **Reynolds Averaged Navier Stokes** equations, more commonly named **RANS**:

$$\partial_i \langle u_i \rangle + \langle u_j \rangle \partial^j \langle u_i \rangle = -\frac{1}{\rho} \partial_i p - \partial^j (\hat{u}_i \hat{u}_j) + \nu \cdot \nabla^2 \langle u_i \rangle \quad (2.4a)$$

$$\partial^i \langle u_i \rangle = 0 \quad (2.4b)$$

where it has been assumed that the mean and derivative operators commute², while the equations averaging process is such that:

$$\langle \hat{u}_i \rangle = 0 \quad (2.5a)$$

$$\langle \langle u_i \rangle \cdot \langle u_j \rangle \rangle = \langle u_i \rangle \cdot \langle u_j \rangle \quad (2.5b)$$

$$\langle \langle u_i \rangle \cdot \hat{u}_j \rangle = \langle u_i \rangle \cdot \langle \hat{u}_j \rangle = 0 \quad (2.5c)$$

$$\langle \hat{u}_i \cdot \hat{u}_j \rangle \neq 0 \quad (2.5d)$$

where the 3 spatial directions have been represented as u_i with $i \in \{x, y, z\}$.

Having the RANS equations presented, it should be noted that it is still not possible to solve them, as new variables have been introduced and additional models are needed to relate the turbulent component to the mean flow velocity known as **closure relations**. Although these will be further explored in Chapter 3, in the context of characterising the wind profile evolution over height, a simple stationary shear flow $u_x = u_x(z)$ it will suffice to consider the eddy viscosity concept, where it is assumed that the turbulent dissipation can be described as a Laplace operator in the same way molecular viscosity can. This assumption can be mathematically expressed as:

$$-\partial^j (\hat{u}_i \hat{u}_j) = \nu_T \cdot \nabla^2 \langle u_i \rangle \quad (2.6)$$

If the turbulent viscosity parameter (ν_T) is assumed constant, then the obtained profile will follow the distribution associated with a Poiseuille flow, but a better approach is to use Prandtl's mixing length concept (Prandtl, 1928), where the Prandtl mixing length is assumed to be proportional to

²For any variable A , $\langle \partial_s A \rangle = \partial_s \langle A \rangle$

the distance to the surface such that:

$$\hat{u}_i = \ell \cdot \partial_z u_i \quad (2.7)$$

where ℓ is a characteristic length. For the shear flow in the x direction here considered, this can be seen to be equivalent to define the turbulent viscosity as (no summation implied):

$$\partial^z (\hat{u}_x \hat{u}_z) = (\ell^2 \partial_z u_x) \partial_{zz} u_z \longrightarrow \nu_T = \ell^2 \cdot \partial_z u_x \quad (2.8)$$

Assuming now a self-similar solution to estimate Prandtl's mixing length allows to solve [Equation 2.7](#) as:

$$\frac{du_x}{dz} = \frac{\hat{u}_x}{\ell} = \frac{u_\tau}{\kappa \cdot z} \longrightarrow u_x(z) = \frac{u_\tau}{\kappa} \cdot \ln(z) + C = \frac{u_\tau}{\kappa} \cdot \ln\left(\frac{z}{z_0}\right) \quad (2.9)$$

where κ is the Von Karman constant, u_τ is the friction velocity at the boundary of the flow and z_0 the surface roughness. This velocity profile is still a function of two distinct parameters, that must be fitted to different locations and conditions.

More complex models can and must be used to adequately model the flow over a given body (for example to model the flow over an aerofoil as will be presented in [Section 3.2](#)) but when it comes to understanding the Earth's boundary layer mechanics, the presented models be enough. Additionally, and mainly due to the high complexity of these models, an empirical law - the **Power law** - is frequently used to describe the wind speed variation with height at a given location as:

$$u(z) = u_r \left(\frac{z}{z_r} \right)^\alpha \quad (2.10)$$

As can be seen, the power law is controlled by a reference height (z_r) and a corresponding reference velocity (u_r), and the exponent parameter α , that can vary significantly with the time of day or the season ([Jung and Schindler, 2021](#)). As a reference, a typical value around 1/7 for open onshore sites is commonly used, but a value of 0.11 is often more appropriate for offshore applications ([Peterson and Hennessey, 1978](#); [Hsu et al., 1994](#)). Although being able to provide good results, as long as the exponent parameter is adequately chosen, it is important to understand that the power law is an empirical rule with no physical basis.

2.2.1.2 Wind prediction

Having the fundamental physics behind atmospheric motion described, it is now important to understand the distribution of wind speeds that are expected to occur in a given location. Firstly, the mean wind speed distribution over a given place will be characterised and only after the smaller and higher frequency fluctuations associated with turbulence will be explained.

Mean wind speed variations

Although wind variability over large periods of time (decades, for example) is still extremely dif-

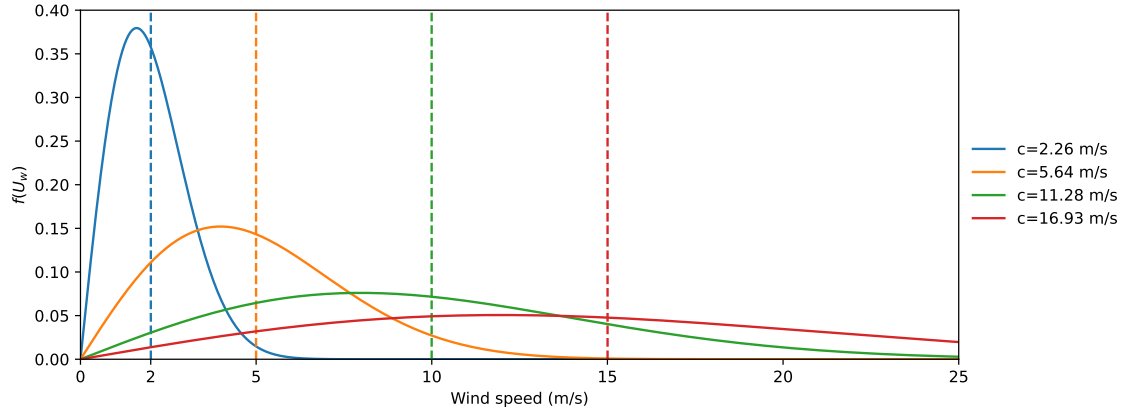


Figure 2.2: Probability density function following a Rayleigh distribution for different scale parameters. The mean value associated with each distribution is represented by the corresponding colour vertical dashed line.

difficult to predict (leading to a difficult evaluation process of a wind park rentability) on a given site, season variations can be reasonably predicted by the 2 parameters (a shape parameter k and scale parameter c) Weibull distribution. This distribution can be defined by the cumulative distribution function $F(U_w)$, that can be used to predict the probability of a given wind speed (U_w) being exceeded as:

$$F(U_w) = 1 - \exp \left[- \left(\frac{U_w}{c} \right)^k \right] \rightarrow P(U > U_w) = 1 - F(U_w) = \exp \left[- \left(\frac{U_w}{c} \right)^k \right] \quad (2.11)$$

with the corresponding probability density function given by:

$$f(U_w) = \frac{dF(U_w)}{dU_w} = k \cdot \frac{U_w^{k-1}}{c^k} \cdot \exp \left[- \left(\frac{U_w}{c} \right)^k \right] \quad (2.12)$$

This distribution cannot be fully characterized from the knowledge of the mean wind speed alone and some other information is needed to calibrate both of its parameters. The general expression for the n^{th} momentum of the distribution (m_n) is quite simply defined from the gamma function (Γ) as:

$$m_n = \langle U_w^n \rangle = c^n \cdot \Gamma \left(1 + \frac{n}{k} \right) \quad (2.13)$$

To obtain the two parameters needed, different methods can be applied depending on the purpose of the prediction or on the information available, being the most commonly used presented and compared by [Chang \(2011\)](#) and [Jourdi er and Drobinski \(2017\)](#). In some cases no more information besides the mean wind speed is available and a primary guess can be made adopting $k = 2$, which is usually accurate enough as in many sites this parameter is indeed around this value. This particular choice of k reduces the Weibull distribution to the Rayleigh distribution, represented in [Figure 2.2](#) for different mean wind speeds. If more information is available, more robust methods can be used

with different outcomes. One of the most commonly used is the so called **Methods of Moments** in which the distribution is fitted through 2 different statistical moments of the collected data. If the standard deviation is known, then this can be achieved with the first 2 moments. However, as the energy available is proportional to the 3rd power of the wind speed, it is eventually more important in the case of wind energy devices to take the 3rd moment into consideration, instead of the 2nd, and guarantee that the energy content is correctly described, in what is commonly referred to as the **Power Density method**³. Finally, it is important to name the method used in Wind Atlas Analysis and Application Program (Mortensen et al., 1993; Bowen and Mortensen, 1996). In this method, the energy content is correctly described as it also uses the distribution 3rd moment, but it is the probability of exceeding a given wind speed rather than the mean value that is used. This procedure allows a better definition of the tail of the distribution which is of great importance in the design of wind farms. In parallel with any of the presented methods, in regions with strong seasonal variations, a weighted combination of 2 different Weibull distributions may be needed, or even alternative distributions (Drobinski et al., 2015; Jourdir and Drobinski, 2017).

Turbulent fluctuations

Having the mean wind speed predicted, it is now important to devote some attention to the higher frequency fluctuations due to turbulent phenomenon. As this process is highly complex and essentially chaotic⁴, it is more adequate to analyse it using its statistical properties rather than adopting a deterministic approach. One of the relevant parameters for such characterisation is the so called **turbulence intensity** (TI). This is defined as the ratio between the standard deviation (σ) and the mean wind speed over time series of observations:

$$TI = \frac{\sigma_{u_i}}{\langle u_i \rangle} = \frac{\sqrt{\langle \hat{u}_i^2 \rangle}}{\langle u_i \rangle} \quad (2.14)$$

While the turbulent fluctuations amplitude is commonly characterised from a zero-mean Gaussian distribution with standard deviation proportional to the intended turbulence intensity, its frequency content ($S(f)$) can be described, for instance, from either a Von Karman or a Kaimal spectrum:

$$S_{\text{Von Karman}}(f) = \frac{4 \cdot \sigma_u^2 \cdot \ell_{\text{Von Karman}}}{U_w} \cdot \left[1 + 70.8 \cdot \left(f \cdot \frac{\ell_{\text{Von Karman}}}{U_w} \right)^2 \right]^{-\frac{5}{6}} \quad (2.15a)$$

$$S_{\text{Kaimal}}(f) = \frac{4 \cdot \sigma_u^2 \cdot \ell_{\text{Kaimal}}}{U_w} \cdot \left[1 + 6f \cdot \frac{\ell_{\text{Kaimal}}}{U_w} \right]^{-\frac{5}{3}} \quad (2.15b)$$

where $\ell_{\text{Von Karman}}$ and ℓ_{Kaimal} are two appropriate turbulence length scales. Due to Kolmogorov's law, which states that the turbulence intensity is asymptotically proportional to $f^{-\frac{5}{3}}$, it can be

³Note that although the wind energy **content** is correctly described, since it is proportional to the distribution's 3rd momentum, the energy **production** also depends on the wind turbine specific power curve. Since the latter is not constant for different wind speeds, the energy production is, in general, different between fitted and measured values.

⁴By a chaotic process it is meant that any minor change of boundary or initial conditions results in drastically different outcomes over a relatively short period of time.

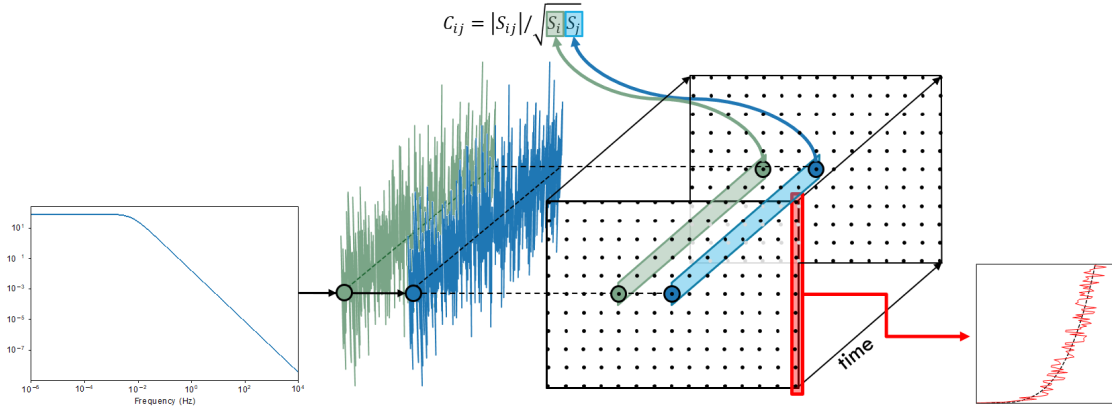


Figure 2.3: Schematic representation of the different wind components. In red, a plot with the wind speed evolution over height for a given time instant, with the mean wind profile as a dashed black line. In blue and green, the time evolution of the wind speeds in two different points, that should respect the power spectrum on the left and the coherence function C_{ij} .

easily shown that both length scales must be such that:

$$\ell_{\text{Kaimal}} = \left(\frac{36}{70.8} \right)^{-\frac{5}{4}} \cdot \ell_{\text{Von Karman}} \approx 2.39 \cdot \ell_{\text{Von Karman}} \quad (2.16)$$

If the turbulent fluctuations over time have been described for a given point in space, the turbulent fluctuations when two distinct points are compared are certainly correlated. The spatial coherence function C_{ij} between two different points i and j can be defined as:

$$C_{ij} = \frac{|S_{ij}|}{\sqrt{S_i \cdot S_j}} \quad (2.17)$$

The cross-spectra (S_{ij}) between any two points can then be obtained if the auto-spectrum and the coherence function are known. Different standards provide different descriptions (see for example [IEC, 2005](#)), with significant discrepancies, so a careful analysis must be undertaken.

A schematic representation of all the components described above and the relations between them is presented in [Figure 2.3](#).

2.2.2 Waves and sea states characterisation

While the last section presented and discussed the wind field properties, for offshore applications the sea states are also an important part, if the environmental conditions that the structure will face are to be properly described. Following a similar reasoning with respect to the wind description, in this section the equations that govern the waves motion will be firstly presented, and only then their statistical properties briefly described.

2.2.2.1 Waves physics

For the particular case of ocean waves, it is convenient to define some parameters that will be used throughout this section, namely:

- **Still Water Level (SWL)**: reference level for wave elevation;
- **Period (T)**: time interval to recover at a given location the same dynamic properties. The wave frequency $f = 1/T$ and angular frequency $\omega = 2\pi f$ will also be used interchangeably;
- **Wavelength (λ)**: spatial distance between two points with identical phase at a given time instant. The wave number $k = 2\pi/\lambda$ will also be used.
- **Water depth (d)**: distance measured between the seabed and the still water level;
- **Elevation ($\eta(x, t)$)**: distance measured between the free surface and the reference level at a given position (x) and at a given time (t);
- **Amplitude (η_0)**: maximum elevation.

If one may have a physical intuition regarding the problem under analysis, from a mathematical point of view, a wave can be defined as any scalar function, here the wave elevation η , of a time variable t and n spatial variables $x_1, x_2, x_3, \dots, x_n$ that satisfies the partial differential equation:

$$\square \eta = 0 \quad (2.18)$$

where $\square = \partial_{tt} - c^2 \nabla^2$, being c the wave propagation speed, is the d'Alembert operator. Naturally, if this equation governs the system evolution, the wave properties itself that are compatible with ocean waves must be obtained by defining proper boundary conditions and for that it is easier to work with a potential formulation.

Potential formulation

The equations that need to be solved to fully characterise a given ocean wave or waves share many of the same principles used in [Section 2.2.1.1](#). If the viscous effects are neglected, it follows that the fluid velocity can be expressed in terms of a velocity potential:

$$\mathbf{u} = \nabla \phi \quad (2.19)$$

If the fluid is assumed to be incompressible such that $\nabla \cdot \mathbf{u} = 0$, it immediately follows that the velocity potential must satisfy the Laplace equation:

$$\nabla^2 \phi = 0 \quad (2.20)$$

As viscous effects are neglected, the Navier-Stokes equations presented in [Section 2.2.1.1](#) collapse to the more treatable Euler equation:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{f} \quad (2.21)$$

where here $\mathbf{f}$ includes any external applied force per unit mass, that is equivalent to [Equation 2.4a](#) if the viscous terms and the turbulent fluctuations are neglected. If the external forces are assumed to be conservative, they may also be obtained from a potential Φ_F , and the Euler equation can be integrated to provide the Bernoulli's equation in its most general form ([Batchelor, 2000](#)):

$$\partial_t \phi + \frac{1}{2} \cdot \mathbf{u}^2 + \frac{p}{\rho} + \Phi_F = f(t) \quad (2.22)$$

where $\mathbf{u}^2 = \mathbf{u} \cdot \mathbf{u}$. Embedding $f(t)$ in the potential itself, and considering gravitational forces alone, the Bernoulli equation can be rewritten in a more convenient form that will be used in the next sections:

$$\partial_t \phi + \frac{1}{2} (\nabla \phi)^2 + \frac{p}{\rho} + gh = 0 \quad (2.23)$$

where h is the vertical distance from an arbitrary reference height. From [Equation 2.23](#) it is clear that once the potential is known, the pressure can readily be obtained, but doing so requires suitable boundary conditions.

Boundary conditions

Although a flow defined by a potential formulation satisfies the Laplace equation, not all the solutions of the latter are physically meaningful when it comes to ocean waves and additional conditions must be defined. For the remaining of this section, the flow will be assumed bi-dimensional, with the x coordinate oriented along the wave propagation direction and z along the vertical direction.

At the free surface, both kinematic and dynamic boundary conditions can be imposed. The first condition implies that a particle at the free water surface cannot escape from it and can be obtained by matching both particle's and surface velocities ([Chakrabarti, 1987](#)):

$$\partial_z \phi - \partial_t \eta - \partial_x \phi \partial_x \eta \Big|_{z=\eta} = 0 \quad (2.24)$$

The dynamic boundary condition follows directly from Bernoulli's equation, assuming the atmospheric pressure to be constant and hence included in the potential function:

$$\partial_t \phi + \frac{1}{2} (\nabla \phi)^2 + g\eta \Big|_{z=\eta} = 0 \quad (2.25)$$

Finally, the vertical velocity of a water particle at the sea bed must also vanish, since this is assumed to be impermeable:

$$\partial_z \phi \Big|_{z=-d} = 0 \quad (2.26)$$

This condition will eventually be generalised to express an impermeability condition at any solid body surface.

2.2.2.2 Wave models

Equation 2.24 and Equation 2.25 are clearly non-linear as they both involve either second derivatives or multiplicative terms of the unknown quantities. However, and under different assumptions, simplified models can be used that are valid in different conditions.

Linear waves

Perhaps the most obvious one is the so-called **linear theory**, in which the assumption that the wave length is very large by comparison to its height allows to neglect the quadratic terms in Equation 2.24 and Equation 2.25. Under this assumption, both expressions may also be evaluated at the mean water level, rather than on the free surface itself, yielding:

$$\partial_t \eta - \partial_z \phi = 0 \quad (2.27a)$$

$$\partial_t \phi + g\eta = 0 \quad (2.27b)$$

The equations above can be combined to provide a singular boundary condition on the potential function, correct to first order as:

$$\partial_{tt} \phi + g \partial_z \phi = 0 \quad (2.28)$$

Laplace equation, subjected to the boundary conditions defined by Equation 2.26 and Equation 2.27, can now be solved by separation of variables, leading to the solution (Chakrabarti, 1987):

$$\phi = \eta_0 \frac{g}{\omega} \frac{\cosh[k(d+z)]}{\cosh(kd)} \sin(kx - \omega t) \quad (2.29)$$

where it was assumed, without any loss of generality, $\phi(0,0,0) = 0$. With the potential function characterised, the wave characteristics and the velocity field can be obtained by Equation 2.19 and the pressure distribution from Equation 2.23.

Non-linear waves

As the wave height increases, the non-linear terms neglected when solving Equation 2.24 and Equation 2.25 need to be considered if an accurate solution is to be obtained. Maybe the most obvious way to include the non-linear effects neglected previously, is to define a new potential as a power series on a nondimensional perturbation (ϵ), an approximation usually identified as Stokes' expansion:

$$\phi = \sum_{n=1}^{\infty} \epsilon^n \cdot \phi_n \approx \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \dots \epsilon^i \phi_i \quad (\text{no summation implicit}) \quad (2.30)$$

If only the first component of the expansion is considered, then the linear theory expressions are recovered, but if higher and higher order are used, then different expressions are obtained for the

wave elevation and particle motions. Although this method allows for the consideration of higher order terms in the potential function, it is only applicable if the adimensional parameter is such that $\varepsilon < 1$, to ensure that every term of the expansion is lower than its predecessor. The number of terms used (i) defines the order of the theory used and the 5th order Stokes expansion is commonly used to study waves, mainly when the ratio between depth and wave length is higher than 0.01. Although Stokes' expansion is always an approximation there are cases where exact non-linear and periodic waves solutions are known to exist, as cnoidal or solitary waves. Due to the vast literature available related to this subject, no further details will be presented here, while more details can be found in (Chakrabarti, 1987) or (Fenton, 1990), for instance. Nevertheless, it should be mentioned that, for practical purposes, high order Stokes theory are often used as a reference frame.

2.2.2.3 Waves statistical properties

Once the potential is known, the wave configuration can be obtained from the boundary conditions already presented, being the simplest one the sinusoidal wave. A **regular wave** is then defined as any wave with a single, well defined frequency ω such that, considering the linear theory only for the sake of simplicity, its elevation can easily be obtained using Equation 2.29 in any of the first equalities from Equation 2.27:

$$\eta(x, t) = \eta_0 \cdot \cos(kx - \omega t) \quad (2.31)$$

which can easily be seen to be a solution to the wave equation, Equation 2.18, with wave propagation speed $c = k/\omega$. This type of waves are a particular important case to study, as any irregular, and thus more realistic, wave can be expressed as a Fourier series of superimposed sinusoidal waves. Furthermore, using the potential function in Equation 2.28, it is possible to obtain the so called **dispersion relation** between a wave's frequency and wavelength:

$$\omega^2 = kg \tanh(kd) \longrightarrow \lambda = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{\lambda}\right) \quad (2.32)$$

from where it is possible to conclude that the wave propagation speed is controlled by two different physical parameters in low and high water depths limits. While for the latter it is proportional to the wave period:

$$\lim_{d \rightarrow \infty} c = \lim_{d \rightarrow \infty} \frac{\omega}{k} = \lim_{d \rightarrow \infty} \frac{g}{\omega} \tanh(kd) = \frac{g}{\omega} = \frac{g}{2\pi} \cdot T \quad (2.33)$$

for very shallow waters, the wave propagation speed is proportional to the square root of the water depth:

$$\lim_{d \rightarrow 0} c = \lim_{d \rightarrow 0} \frac{g}{\omega} \tanh(kd) \approx \sqrt{gd} \quad (2.34)$$

If a particular set of potential functions is a solution of the Laplace equation and respects all the boundary conditions expressed, so does any linear combination of them. The superposition of the resulting waves (each a regular wave on its own) will define a more complex sea state that does not have a single, well defined frequency and hence will be referred to as **irregular waves**. In the following derivations, it will be assumed that each component is obtained from linear theory such that:

$$\eta(x, t) = \lim_{N \rightarrow \infty} \sum_{i=1}^N [\eta_{0,i} \cdot \cos(k_i x - \omega_i t + \delta_i)] \quad (2.35)$$

When only the statistical properties of the waves are of interest, and not the exact water level at a given time instant, the phase angle introduced in Equation 2.35 (δ_i) can be treated as a random variable and the amplitude ($\eta_{0,i}$) of a given frequency (ω_i) is associated with the corresponding energy content and hence to the respective Fourier coefficient according to:

$$S_\eta(\omega_i) = \frac{1}{2} \cdot \eta_{0,i}^2 \quad (2.36)$$

A final note should be made, by noting that when transforming a spectrum basis function (for instance, expressing the ordinate as a function of the frequency rather than the angular frequency) attention must be taken to preserve the energy content of equivalent infinitesimal intervals. Mathematically, the mentioned transformation can be written as:

$$S_\eta(\omega_i) \cdot d\omega = S_\eta(f_i) \cdot df \longrightarrow S_\eta(\omega_i) = S_\eta(f_i) \cdot \frac{df}{d\omega} = \frac{1}{2\pi} \cdot S_\eta(f_i) \quad (2.37)$$

If the description above is able to produce a spectrum estimate from a given wave record, for many cases, and in particular when performing the analysis of a given structure, it is the opposite that is intended: for a given spectrum (many times defined by design codes) how can a wave time series be generated? This process presupposes a spectrum is known and naturally throughout the years many attempts have been made to obtain a normalised spectrum that could be used to a wide variety of structures. For any given spectrum there are however some general relations that can already be defined, being the two more common the significant wave height (H_s) and the mean wave period ($\bar{T}$) as:

$$H_s = 4 \cdot \sqrt{m_0} \quad (2.38a)$$

$$\bar{T} = \frac{2\pi}{\bar{\omega}} = \frac{m_0}{m_1} \quad (2.38b)$$

both related to the spectrum moments of n^{th} order (m_n):

$$m_n = \int_0^\infty \omega^n \cdot S_\eta(\omega) d\omega \quad (2.39)$$

One of the most widely used spectrum for the design of floating and offshore structures is the **Pierson-Moskowitz** spectrum (Pierson Jr. and Moskowitz, 1964), completely defined by a single

parameter. If both wind direction and speed do not change significantly over a sufficiently large time interval and distance (fetch), than the full developed sea states are compatible with:

$$S_{PM}(\omega) = 0.0081g^2\omega^{-5} \exp \left[-0.74 \left(\frac{\omega U_w}{g} \right)^{-4} \right] = 0.0081 \cdot g^2\omega^{-5} \exp \left[-1.25 \left(\frac{\omega}{\omega_0} \right)^{-4} \right] \quad (2.40)$$

where in the last equality the spectrum has been rewritten as a function of the angular frequency associated with its peak amplitude (ω_0). From the presented expressions and the spectrum moments definition, the peak frequency can be related to the significant wave height by:

$$\omega_0^2 = \frac{0.161g}{H_s} \quad (2.41)$$

Another frequently used spectrum was developed under a joint investigation named **Joint North Sea Wave Project - JONSWAP**. In this project it was concluded that the sea state is never fully developed and additional parameters were introduced to account for this effect ([Hasselmann et al., 1973](#)). In this sense, the JONSWAP spectrum can be thought of a variation of the PM spectrum, and is now written as:

$$S_J(\omega) = \alpha g^2 \omega^{-5} \exp \left[-1.25 \left(\frac{\omega}{\omega_0} \right)^{-4} \right] \gamma^{\exp \left[-\frac{(\omega - \omega_0)^2}{2\tau^2 \omega_0^2} \right]} \quad (2.42)$$

Although this spectrum requires 5 parameters to be fully characterised, for practical purposes τ_a , τ_b and α are often kept constant. In the case of wind blowing at a fixed speed and fetch (X), the 5 parameters spectrum, already indicating the typical values for the mentioned variables, are often defined as:

$$\gamma \in [1, 7] \approx 3.30 \quad , \quad \omega_0 = 2\pi \cdot (g/U_w) \cdot X_0^{-0.33} \quad (2.43a)$$

$$\tau = \tau_a \approx 0.07 \text{ , if } \omega \leq \omega_0 \quad , \quad \tau = \tau_b \approx 0.09 \text{ , if } \omega > \omega_0 \quad (2.43b)$$

$$X_0 = g \cdot X / U_w^2 \quad , \quad \alpha = 0.076 \cdot X_0^{-0.22} \approx 0.0081 \quad (2.43c)$$

As the peak-enhancement factor (γ) can have a great effect on the spectrum, a design spectrum is often obtained through a weighted combination of different spectra computed with different parameters ([Ochi, 1978](#)).

Finally, it is important to introduce the concept of **directional spreading** of the wave packet. In the previously defined spectra, all the wave energy is found to progress along a well defined direction, which does not happen in reality. To simulate this effect, a spreading function ($G(\omega, \theta)$) was suggested by [Lee et al. \(2010\)](#), extending the original work of [Longuet-Higgins et al. \(1961\)](#) to include asymmetric spreading⁵, that transforms the previously defined, frequency only dependent,

⁵In the work of [Longuet-Higgins et al. \(1961\)](#), the symmetry breaking term ξ is not present in the spreading function.

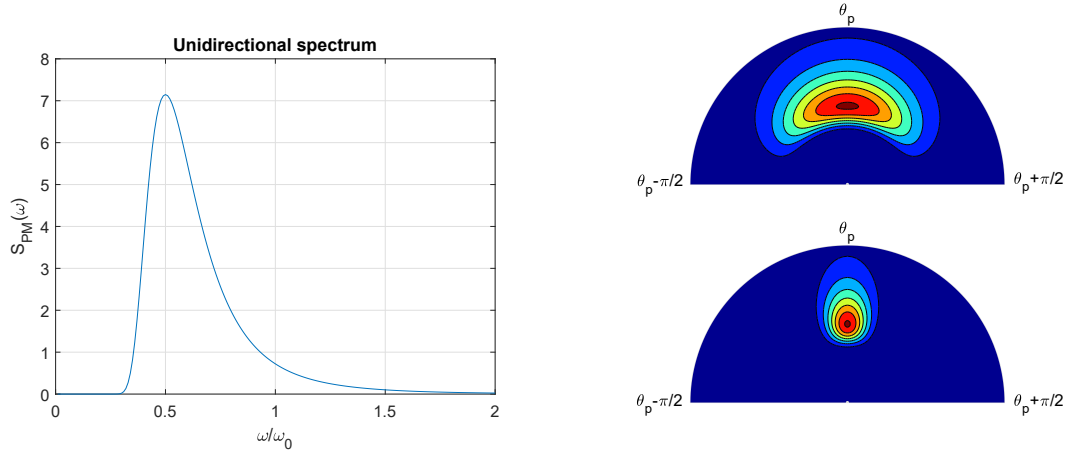


Figure 2.4: Effect of the directional spreading for $s = 1$ (on the top right) and $s = 15$ (on the bottom right) on a Pierson-Moskowitz spectrum (on the left) considering $\mu = 0$.

spectrum ($S_\eta(\omega)$) to:

$$S_\eta(\omega) \rightarrow S_\eta(\omega, \theta) = G_0 \cos^{2s} \left(\frac{\theta - \theta_p}{2} \xi \right) S_\eta(\omega) \quad (2.44)$$

where $\theta \in [-\pi, \pi]$, θ_p is the dominant wave direction, and G_0 is a normalisation constant defined as:

$$G_0 = \int_{\theta_{min}}^{\theta_{max}} \cos^{2s} \left(\frac{\theta - \theta_p}{2} \xi \right) d\theta \quad (2.45)$$

that ensures that the energy content associated with a given frequency is kept constant, or:

$$\int S_\eta(\omega, \theta) d\theta = 1 \quad (2.46)$$

Finally, the symmetry breaking term is defined as:

$$\xi = \begin{cases} e^{-\mu}, & \theta > \theta_p \\ e^{\mu}, & \theta \leq \theta_p \end{cases} \quad (2.47)$$

where μ is the asymmetry parameter. Although the cosine power parameter s is usually frequency dependent, if this dependence is neglected, a value of $s = 15$ is commonly accepted (Mitsuyasu et al., 1975). A representation of this effect on a Pierson-Moskowitz spectrum for two different values of s is presented in Figure 2.4.

2.3 Floating wind turbines components

Wind turbines, in general, are highly complex systems involving several different engineering fields. Having described the environmental conditions that floating offshore wind turbines will face, it is now important to provide a brief overview of the different elements that constitute the structure itself and that play an essential role in capturing the available wind energy. The interaction of all these elements has important influence on the forces observed in the main structural components, so all of them need to be taken in account in the modelling and monitoring data processing. Here, the fundamental components of wind turbines, and in this concept are included not only the structural, physical elements themselves, but also the control and monitoring systems will be described (further details can be found in [Manwell et al., 2010](#); [Burton et al., 2001](#); [Oliveira et al., 2018](#)).

2.3.1 Structural components

To capture the available wind power, a wind turbine relies on a spinning **rotor** made of a finite number of **blades** (nowadays the 3-bladed rotors are the most commonly found solutions). Due to the blades aerodynamic properties, that depend mostly on their cross section exterior shape (usually referred to as the **aerofoils**), a lift force is generated along the span that sources the aerodynamic torque acting over the rotor axis of rotation. Because the generated power is proportional to the acting torque and the generator angular velocity, a **gearbox** is used between the **low speed shaft**, connected to the rotating blades at the **hub**, and the **high speed shaft**, connected to the **generator** itself, that is then connected to the grid. All of these components are placed inside a protecting structure called the **nacelle**. All these elements need to be supported by a properly designed structure, typically materialised by a steel **tower**, that allows the nacelle to be placed at a sufficient height so that the rotor may rotate without hitting the ground, while at the same time sampling higher wind speeds.

The wind turbine itself (tower and rotor), must then be supported by a foundation, in the onshore cases. In the offshore scenario, however, the solution chosen to support the wind turbine is highly dependent on the water depth of the specific site of installation. While for shallow waters (less than 60 m deep according to [Jimenez et al., 2016](#)), the tower can be connected to the sea bed with **fixed bottom solutions**, for deeper waters these solutions are simply not feasible and **floating solutions** appear as the best candidates. In these cases, the displacements of the structure induced by the wind, waves and sea currents loading, need to be controlled and a **mooring system** must also be provided. Different designs have been studied for these structures, that are often divided into 4 major types, schematically represented in [Figure 2.5](#) as described by [Wang et al. \(2010\)](#):

- Tension-leg platform, a solution that includes a floating device that is kept at a equilibrium position where the buoyancy exceeds the structure self weight and the remaining stability is guaranteed by tensions cables.
- Semi-submersible type, a solution where the stability is guaranteed from some large steel columns connected to each other by tubular members. This is the floating solution adopted

in the numerical models develop in [Chapter 4](#) and in the wind farm whose experimental campaign will be discussed in [Chapter 6](#).

- Spar-buoy, a solution where the stability of the wind turbine is guaranteed by the inertia for horizontal axis rotations of a massive cylinder whose draft is at least equal to the wind turbine hub height and that is ballasted with water for better stability.
- Pontoon-type (or barge-type), a solution where a massive structure is used to support one or groups of wind turbines, achieving its stability by its own distributed buoyancy.

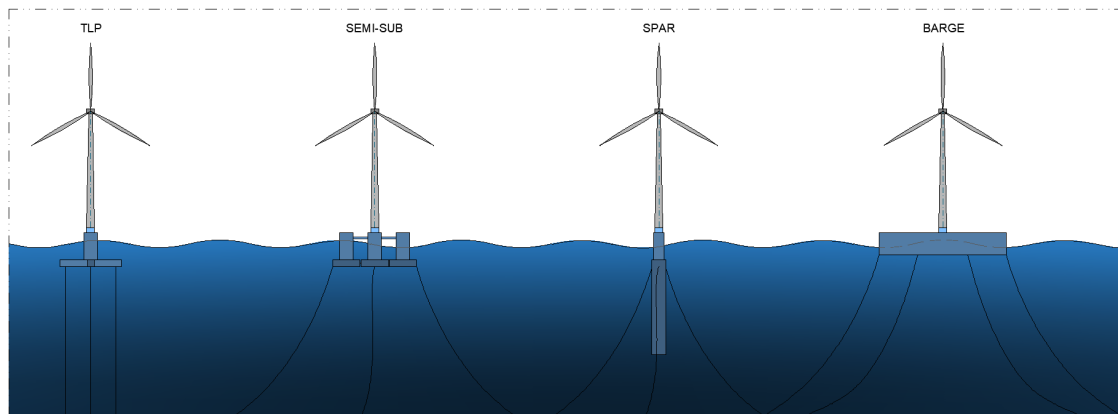


Figure 2.5: Schematic representation of different floating solutions. From left to right, tension-leg platform (TLP), semi-submersible, spar and barge (pontoon) type.

2.3.2 Control systems

The main components that allow a wind turbine to correctly operate have been described by now. However, as the wind resource presents strong variations over time in either direction and intensity, different control mechanisms are included in a wind turbine detailed project, such that power production can be optimised and structural integrity guaranteed. The latter objective, in particular, concerns not only the control of the peak loads over the structure, but also the minimisation of the accumulated fatigue damage for different operation conditions.

Firstly, and because wind direction changes over time, it is important that a wind turbine can adjust its rotor orientation accordingly. To do so, the nacelle's connection to the tower must be able to allow rotations around the yaw axis (vertical axis) and so this first control mechanism will from now on be referred to as **yaw control**. However, keeping aligned with the wind principal direction is not sufficient to guarantee an optimised power production. In fact, almost all modern wind turbines rely on **pitch control** to capture as much available energy as possible for different mean wind speeds. This control refers to the capability of a blade to rotate around its longitudinal axis, changing its aerodynamic properties accordingly. The pitch control is also useful to reduce the aerodynamic loads on the blades under extreme wind speeds, a procedure usually known as feathering out the blades. However, and because for a given wind speed the rotor angular velocity should not increase indefinitely, the aerodynamic torque must be balanced by a resistant torque

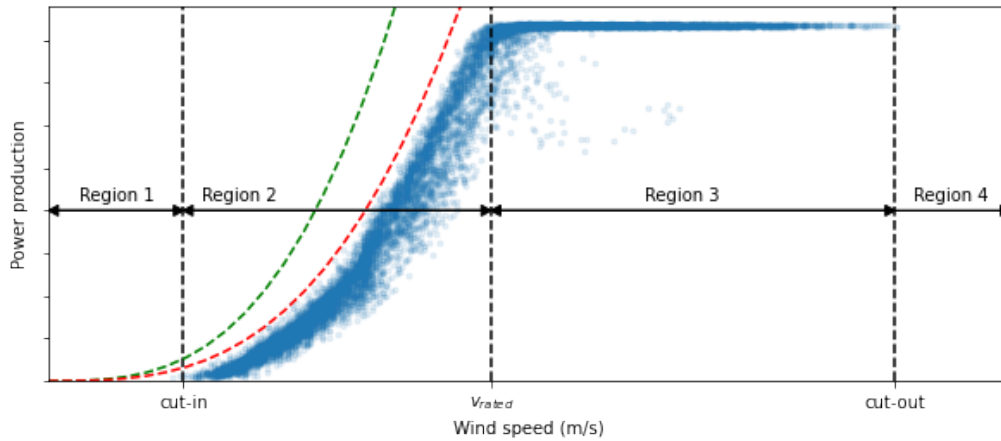


Figure 2.6: Typical operating regions of wind turbines constructed from the data points (blue points) of one of the ongoing FEUP experimental campaigns. The green line, proportional to the 3rd of the wind speed, schematically indicates the wind energy available, while the red dashed line represents the maximum energy that can be captured (approximately 0.593 of the total energy available, as will be discussed in [Section 3.2.2](#)).

applied at the generator. This **torque control** can be achieved by means of an electromagnetic field and hence the kinetic energy of the rotating shaft is converted into electrical energy.

The control systems described above, namely pitch and torque control, divide a typically wind turbine's power curve in different regions, as represented in [Figure 2.6](#). While in **Region 1** the wind speed is simply too low to allow the operation of the wind turbine (commonly referred to as the **cut-in** limit), in **Region 2** the principal goal of the controls is to obtain maximum economic benefit and capture as much available energy in the wind field as possible. Typically, the pitch angle is kept approximately constant and the power output increases systematically with the wind speed until the generator capacity is achieved ([Horiuchi and Kawahito, 2001](#)). At this point, the pitch control is needed to keep the loads and power output within the wind turbine capacity in **Region 3**. Because the power curve from Region 2 hardly matches the rated power output from Region 3, it is usually defined a **Region 2– $\frac{1}{2}$** where the transition is made using both torque and pitch control. As the wind speed increases, so do the loads acting on the structure until the wind turbine can no longer operate under safety conditions and the **cut-out** wind speed is reached. At this point the blades must be feathered out, in what may be identified as a **Region 4**.

If the control systems described until now are a fundamental part of any wind turbine, in the particular case of floating solutions, additionally systems are often implemented. Due to the relatively low stiffness of the floating systems, at least when compared against any bottom-fixed or onshore case, the combined action of wind and waves induces rotations that can be significant (some degrees). To mitigate the consequences of this effect on the wind turbine response and power output, most platforms are designed with a **ballast distribution control**, that, in the particular case of the semi-submersible solutions, is achieved by partly transferring the ballast between the different floater columns, to keep the structure in optimal position for different environmental conditions.

Finally, it should be mentioned that if the controls described are mainly concerned with power optimisation, they can also be used to limit the structural loads that wind turbines must withstand and even incorporate different mechanisms specifically designed for emergency situations. Control theory will not be detailed here as, it would imply an excessively long document, but further details and discussions of different types of control and their implications can be found in (Bossanyi, 2003; Njiri and Söffker, 2016; Menezes et al., 2018).

2.3.3 Monitoring systems

Condition and structural monitoring of wind turbines is essential to maximise the economic benefit of a given wind farm, as this information is used to optimise its behaviour as well as to predict necessary maintenance actions (see, for instance Tchakoua et al., 2014; Straub et al., 2017; Kamariotis et al., 2022). In this section the systems installed within any modern operating wind turbine, as well as some additional monitoring strategies, will be briefly presented.

Every installed and operating wind turbine is provided with its own **Supervisory Control and Data Acquisition** system, more commonly referred to as **SCADA**. Over the last years, the huge amount of information made available by the SCADA systems and the systematic development of artificial intelligence and deep learning techniques have been used for wind turbine condition monitoring (see, for instance Tautz-Weinert and Watson, 2017), mainly concerning the mechanical and electrical parts of the rotor. Although there are some applications of neural networks applied to SCADA information, namely in the analysis of the power curve to identify unhealthy wind turbines within a wind farm (see, for instance, the work by Papatheou et al. (2014) for neural networks application or the work by Gottschall and Peinke (2008) for a stochastic approach of wind turbines' power curve) when it comes to structural damage (fatigue loads cycles are a major concern for this type of structures) these information may just not be enough, or, at least, not the most relevant one. Because of this, the use of complementary monitoring systems may be relevant and hence should be briefly described.

The most obvious way to access the load history of a given wind turbine is to measure it directly, but this is both time consuming and cost expensive, especially if applied to a large number of turbines. However, there have been some developments in which only some wisely chosen turbines are monitored and then SCADA data is used to extrapolate the results for the remaining, not monitored ones. This procedure is detailed by Obdam et al. (2009a) and an application to offshore scenarios can be found in the work by Obdam et al. (2009b) or Weijtens et al. (2016). To apply this procedure it is necessary that at least some of the wind turbines are monitored. In the work by Pacheco et al. (2020) a detailed explanation and comparison of different instrumentation layouts can be found and so only a brief description will be presented here.

One of the most commonly used structural instrumentation devices are **accelerometers**, whose main objective is to provide a time history of the instrumented point accelerations. Their sensibility and background noise level can differ depending on the measuring principle, and so does the price. In typical civil engineering structures as bridges or buildings, the use of high resolution accelerometers with very low noise floors is important as the vibration levels are relatively low,

but this is clearly not the case of wind turbines and so cheaper solutions may be adopted (MEMS based sensors have demonstrated good performance in several applications (Moutinho and Cunha, 2019; Moutinho et al., 2020), including the one described in Chapter 6). Finally, considering the low values of the natural frequencies that may be found in floating offshore solutions, as will become clear in this work, it is important to assure that the selected sensors are able to measure acceleration from DC (0 Hz). This is ensured by MEMS based sensors.

The most direct way to track the internal forces in a wind turbine is to install **strain gauges** in a given section. Although these devices provide direct high quality information about the stresses in the instrumented section, their installation is laborious and intrusive: the tower (or blades) surface must be prepared and the results must be corrected to account for the temperature effect, that must be tracked as well. In Chapter 6, a monitoring system comprising accelerometers and strain gauges for floating offshore wind turbines monitoring will be described, while examples of structural monitoring layouts for onshore wind turbines developed at FEUP and corresponding data analysis may be found in Pacheco et al. (2020, 2021, 2023).

Finally, when it comes to offshore applications, the monitoring of the platform motions, translation and rotations, is also relevant. The latter, in particular, are a fundamental input for the ballast redistribution controls mentioned above. Because of this, the platforms are usually equipped with GPS sensors and clinometers, often installed in more than one column.

2.4 Basic data processing tools for structural monitoring

The last section ended with a brief description of different possible monitoring devices that may be used for structural health monitoring of any structure, and wind turbines in particular. However, having the data collected is only half the story as this needs to be processed before any useful information is obtained. In the previous section two types of measurements, suitable for different analysis, were introduced: accelerations and stresses. While the latter can be used as a (almost) direct input for **fatigue load cycle counting** methods, described in Section 2.4.2, the former are the typical input for **operational modal analysis** methods and so these will be briefly described next.

2.4.1 Operational Modal Analysis

Operational modal analysis (OMA) based on stochastic processes is a widely documented topic (Magalhães and Cunha, 2011, for instance), but in the context of this "theoretical minimum" approach it is convenient to highlight some principal points. In Magalhaes et al. (2009); Magalhães (2010), some of the presented methods (and others that will not be addressed here) are combined with statistical tools to allow the onset of **automated** operational modal analysis.

Perhaps the most simple method that can be applied to a given structure is the Peak Picking method, firstly applied in this context by Felber (1994), in which the natural frequencies are identified from the peaks of the response spectrum. The response spectrum matrix can be obtained from the Fast Fourier Transform (FFT), organised on a column vector $\tilde{\mathbf{y}}(\omega)$, of the time signals

($\mathbf{y}(t)$) recorded in several instrumented points during a certain time length with a sampling rate of f_a as (Bendat and Piersol, 2011):

$$\mathbf{S}(\omega) \propto \tilde{\mathbf{y}}(\omega) \cdot \tilde{\mathbf{y}}(\omega)^H \quad (2.48)$$

where H represents the Hermitian conjugate. Under the influence of a white noise type of loading⁶, the response spectrum matrix is maximised at the structural natural frequencies and may be written, in the modal space, as:

$$\mathbf{S}(\omega_k) \propto \phi_k \phi_k^T \quad (2.49)$$

where ϕ_k is a column vector with the modal configuration associated with the natural frequency ω_k . In this approximation, the **transfer function** (T_{ij}), defined as the ratio between the cross-spectrum between two degrees of freedom i and j and the auto-spectrum of the degree of freedom j , is a direct estimation of a modal coordinates ratio⁷:

$$T_{ij}(\omega) = \frac{S_{ij}(\omega)}{S_{jj}(\omega)} \longrightarrow T_{ij}(\omega_k) = \frac{S_{ij}(\omega_k)}{S_{jj}(\omega_k)} \approx \frac{\phi_k^i}{\phi_k^j} \quad (2.50)$$

where ϕ_k^i is the i^{th} modal coordinate of the k^{th} modal configuration. Although these analyses of the different spectra already provide useful information, it requires significant user interaction to properly select the relevant frequencies, and hence it is not suitable for extensive analyses, that are intended to be mostly automated.

The SSI-COV method (**CO**Variance-driven-**Stochastic Subspace Identification**) is one of the most reliable methods for output-only modal identification. As this method is fully described by Peeters and De Roeck (1999), here only the most relevant steps are described. It is a parametric method in the time domain that tries to identify a discrete state space model from the recorded response (Van Overschee and De Moor, 2012):

$$\mathbf{x}_{k+1} = \mathbf{A} \cdot \mathbf{x}_k + \mathbf{B} \cdot \mathbf{u}_k + \mathbf{w}'_k = \mathbf{A} \cdot \mathbf{x}_k + \mathbf{w}_k \quad (2.51a)$$

$$\mathbf{y}_k = \mathbf{C} \cdot \mathbf{x}_k + \mathbf{D} \cdot \mathbf{u}_k + \mathbf{v}'_k = \mathbf{C} \cdot \mathbf{x}_k + \mathbf{v}_k \quad (2.51b)$$

where $\mathbf{x}_i$ and $\mathbf{y}_i$ are the state vector containing the structural displacements, $\mathbf{u}_k$, and corresponding velocities, and the response observations vector evaluated at time instant i/f_a , respectively, and $\mathbf{w}'_k$ and $\mathbf{v}'_k$ are included to model disturbances or modelling inaccuracies as noise sources. Furthermore, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are commonly identified as the state matrix, the input matrix, the output

⁶While this is a commonly valid assumption in the context of bridges and buildings, it may be arguable that the excitation over a wind turbine is not compatible with the white noise assumption. For instance, if the rotor is rotating at a given frequency, then it is expected to detect an harmonic action over the structure.

⁷It is important to note that the estimates of the cross-spectrum are, in general, complex entities and so is the obtained modal configuration. If the modes are real, however, the amplitude of the transfer function is the modal amplitude and its phase will be either 0 rad, if the displacement has the same direction of the reference, or π rad, if it is in opposite direction.

matrix and the direct transmission matrix, respectively. For OMA applications, since the system inputs are now known, the terms associated with $\mathbf{u}$ are included in the noise vectors, $\mathbf{w}_k$ and $\mathbf{v}_k$, justifying the last equality.

Having defined the state-space model, the state matrix $\mathbf{A}$ contains all the relevant dynamic information of the system and can be computed from the correlation matrix ($\mathbf{R}_y$) calculated from the response of the instrumented degrees of freedom (DOF), defined as:

$$\mathbf{R}_y(j) = E \left[\mathbf{y}_{k+j} \cdot \mathbf{y}_k^T \right] = \mathbf{C} \cdot \mathbf{A}^{j-1} \cdot \mathbf{G} = \mathbf{C} \cdot \mathbf{A}^{j-1} \cdot E \left[\mathbf{x}_{k+1} \cdot \mathbf{y}_k^T \right] \quad (2.52)$$

where $\mathbf{G}$ is the correlation matrix between the state vector $\mathbf{x}$ in $i+1$ and the response vector $\mathbf{y}$ at time i . The correlation matrices can be distributed in a Toeplitz matrix such that:

$$\mathbf{T}_{(1|j_i)} = \begin{bmatrix} \mathbf{R}_{j_i}^{ref} & \mathbf{R}_{j_i-1}^{ref} & \cdots & \mathbf{R}_1^{ref} \\ \mathbf{R}_{j_i+1}^{ref} & \mathbf{R}_{j_i}^{ref} & \cdots & \mathbf{R}_2^{ref} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{2 \cdot j_i-1}^{ref} & \mathbf{R}_{2 \cdot j_i-2}^{ref} & \cdots & \mathbf{R}_{j_i}^{ref} \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C} \cdot \mathbf{A} \\ \vdots \\ \mathbf{C} \cdot \mathbf{A}^{j_i-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{A}^{j_i-1} \cdot \mathbf{G} & \cdots & \mathbf{A} \cdot \mathbf{G} & \mathbf{G} \end{bmatrix} \quad (2.53)$$

It should be mentioned that when the number of points of interest exceeds the number of available sensors, only some of them can be measured during all the setups, implying that only the corresponding columns of the correlation matrix are fully characterised. This limitation leads to the definition of the reference degrees of freedom (and hence the superscript *ref*), but it is not present in continuous monitoring campaigns, as the ones intended for wind turbines applications, since the instrumented points are not modified after the installation of the sensors. In any case, the Toeplitz matrix from Equation 2.53 can be decomposed according to:

$$\mathbf{T}_{(1|j_i)} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} \quad (2.54)$$

where $\mathbf{S}$ is a diagonal matrix with the singular values, $\mathbf{U}$ and $\mathbf{V}$ contain the left and right singular vectors, respectively, and the index 1 is identify the sub-representations associated to non zero singular values. Defining the extended observability matrix $\mathbf{O}$ and the reversed extended stochastic controllability matrix $\mathbf{\Gamma}$ as:

$$\mathbf{O}_{j_i} = \mathbf{U}_1 \cdot \mathbf{S}_1^{\frac{1}{2}} \quad (2.55a)$$

$$\mathbf{\Gamma}_{j_i} = \mathbf{S}_1^{\frac{1}{2}} \cdot \mathbf{V}_1^T \quad (2.55b)$$

such that:

$$\mathbf{T}_{(1|j_i)} = \mathbf{O}_{j_i} \cdot \mathbf{\Gamma}_{j_i} \quad (2.56)$$

it can easily be concluded that the matrix $\mathbf{C}$ can be directly obtained from the first n lines of the

observability matrix, being n the number of degrees of freedom. The state matrix can then be obtained as the solution of the optimisation problem:

$$\begin{bmatrix} \mathbf{C} \\ \mathbf{C} \cdot \mathbf{A} \\ \vdots \\ \mathbf{C} \cdot \mathbf{A}^{j_i-2} \end{bmatrix} \cdot \mathbf{A} = \begin{bmatrix} \mathbf{C} \cdot \mathbf{A} \\ \mathbf{C} \cdot \mathbf{A}^2 \\ \vdots \\ \mathbf{C} \cdot \mathbf{A}^{j_i-1} \end{bmatrix} \longrightarrow \mathbf{A} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C} \cdot \mathbf{A} \\ \vdots \\ \mathbf{C} \cdot \mathbf{A}^{j_i-2} \end{bmatrix}^\dagger \cdot \begin{bmatrix} \mathbf{C} \cdot \mathbf{A} \\ \mathbf{C} \cdot \mathbf{A}^2 \\ \vdots \\ \mathbf{C} \cdot \mathbf{A}^{j_i-1} \end{bmatrix} \quad (2.57)$$

where $\dagger$ stands for the Moore-Pensore pseudo-inverse matrix. Having the state matrix identified, the modal configurations, natural frequencies and modal damping ratios follow directly from its eigendecomposition (Peeters and De Roeck, 1999):

$$\mathbf{A} = \mathbf{\Psi} \cdot \mathbf{\Lambda}_D \cdot \mathbf{\Psi}^{-1} \quad (2.58)$$

where $\mathbf{\Psi}$ contains the modal configurations in its columns and $\mathbf{\Lambda}_D$ is a diagonal matrix with the discrete time eigenvalues (μ_k), that are related to their continuous counterparts by:

$$\mu_k = \exp(\lambda_k \Delta t) \quad (2.59)$$

being $\Delta t = 1/f_a$ the acquisition time step. These eigenvalues in turn are related to the natural frequencies (ω_k) and modal damping coefficients (ξ_k) by:

$$\omega_k = |\lambda_k| \quad (2.60a)$$

$$\xi_k = -\frac{\Re(\lambda_k)}{|\lambda_k|} \quad (2.60b)$$

Although this method provides good results for natural frequencies and modal damping ratios even for closely spaced modes, the number of relevant singular values to retain after the Toeplitz matrix decomposition, or model order, is not previously known. A common practice is to consider different model orders and built stabilisation diagrams, where the structural natural frequencies consistently present for several model orders can be easily identified.

Having presented the main characteristics of the SSI-COV method, it is important to provide a brief description of how it can be applied to large volumes of data. As described by Magalhaes et al. (2009), it is possible to use a cluster analysis, grouping the similar modes together, by defining the distance between two modes obtained in two different model orders as:

$$d_{ij} = \alpha \cdot \left| \frac{f_i - f_j}{f_j} \right| + \beta \cdot (1 - MAC_{ij}) \quad (2.61)$$

where f_i and f_j are the modes frequencies, MAC_{ij} is the modal assurance criterion (Allemang, 2003; Pastor et al., 2012) between the mode shapes, which evaluates their similarity, and α and β are weighting coefficients to be tuned in each application. The defined criteria allow the construction of a hierarchical tree that must be provided with some stoppage criteria with respect to the

differences inside a given cluster. Evidently, the more restrict the criteria, the higher the number of clusters obtained, which can make the distinction between real and spurious modes more difficult. Finally, having the clusters defined, the tracking of a specific mode shape can be made comparing the different cluster mean value with a reference frequency and mode shape. This reference values should be tuned in each application from either theoretical values or good quality experimental setup.

2.4.2 Fatigue estimation

If the load history of a given section is known (from numerical models or after adequate pre-processing of measured strains as described in [Chapter 6](#)), then fatigue damage and remaining fatigue life can be directly evaluated. As fatigue itself is a topic widely reported and there are numerous quality works on this matter ([Fatemi and Yang, 1998](#); [Suresh, 1998](#); [Dowling, 2012](#); [Ma et al., 2019](#); [Horas, 2021](#)), in this work only the very basis of this phenomenon will be detailed.

The principle of fatigue is that a given structure or structural component can only withstand a finite number of cycles of a given amplitude, even if the static failure load⁸ is never achieved. Intuitively, it can be argued that this resistance should depend on the variation amplitude, but also on the mean stress state, and so it does⁹. One of the most widely used approached is based on the so called **S-N curves**, which essentially express the number of cycles (N) a given structural detail can withstand under a given stress variation (S). Once this curve is defined, the cumulative fatigue damage (D) associated to a particular stress history can be computed using the **Palmgren-Miner** rule ([Miner, 1945](#)):

$$D = \sum_{i=1}^J \frac{n_i}{N_i} \quad (2.62)$$

where n_i are the recorded number of cycles and N_i are the number of cycles that would lead to failure according to the S-N curve for a given stress amplitude s_i . Under this rule, fatigue is said to occur whenever $D > 1$. The only question that remains to be answered it how to count the load cycles.

The rainflow method has been widely used since its introduction in 1968 ([Matsuishi and Endo, 1968](#)) to provide a basis for the cycle amplitude counting. In this methodology, the cycle counting is obtained by analogy with the path a rain drop would take if the stress time history was plotted in the vertical. By comparison with simplified approaches, the rainflow methodology allows the identification of important stress variations in time series with significant noise or high frequency fluctuations (not rarely induced by numerical instabilities or measurement noise) that could be lost otherwise. A schematic representation of this procedure is presented in [Figure 2.7](#).

A final remark must be made here to a particular relevant point. When a path is not interrupted and full cycle is not identified, [Matsuishi and Endo \(1968\)](#) assumed it could be treated as an half

⁸By static failure load it is meant the maximum load that could be applied under static conditions.

⁹Although the mean stress state does impact the fatigue damage experienced by a structure, in many analysis this effect is not taken into account.

cycle, which may not always be conservative. A generalised treatment of this residue processing can be found in (Amzallag et al., 1994) and a comparison of different techniques, including an application to an offshore wind turbine, is available in (Marsh et al., 2016).

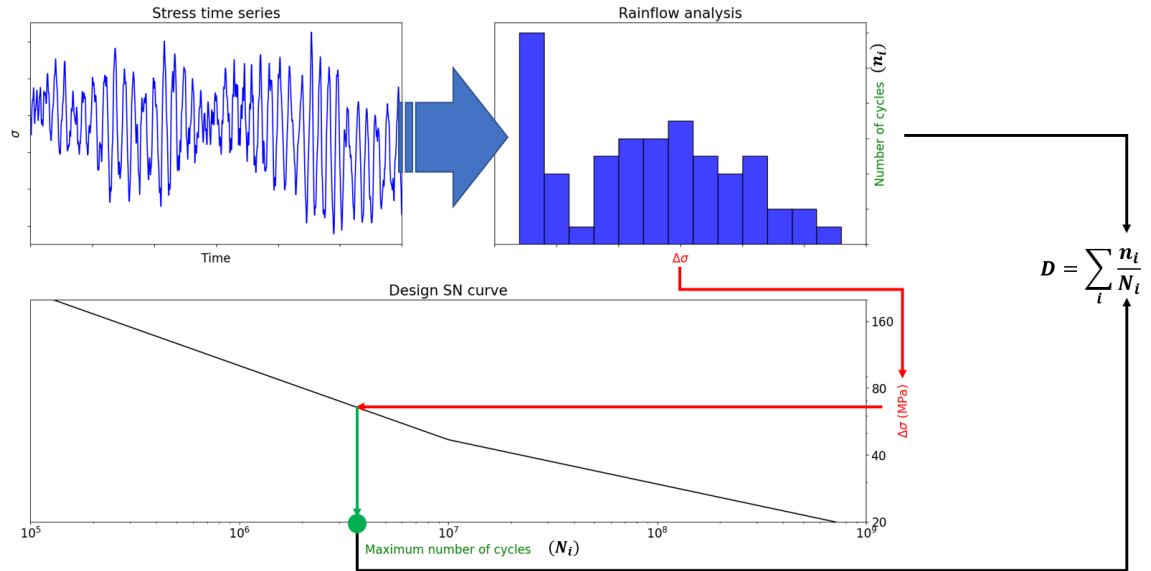


Figure 2.7: Schematic representation of the fatigue damage estimate based on SN curves.

2.4.3 Monitoring challenges in the offshore scenario

Output only operational modal analysis methods rely on some baseline assumptions regarding either the structure under analysis and the external loads properties. In particular, the covariance-driven stochastic subspace identification method just discussed requires the system to be under stationary conditions and tacitly assumes that the excitation is compatible with a white noise spectrum. Although these assumptions are hardly ever strictly verified, this is not an issue for conventional civil engineering structures, where there is a long history of successful application of such techniques. The same methods have also been extended to onshore wind turbines and SSI-COV has been shown to provide good results, even considering that the white noise power spectrum assumption is not met, as will be discussed in Chapter 5. The identification of structural natural frequencies in floating offshore structures based on output-only OMA methods, however, is mostly unexplored, but is expected to present additional challenges.

On one hand, the expected range of the platform very low frequencies requires long monitoring periods where stationary is hardly guaranteed. On the other hand, the waves loading is not compatible with a white noise spectrum, but rather has a frequency range with very significant energy. This effect is more challenging than, and cumulative with, the aforementioned harmonic content of the onshore wind turbines, since it is spread across a relatively wide range of frequencies, hence overpopulating any SSI-COV stabilisation diagram. For most of this work, new analytical developments will be used to extend and adapt the data analysis to provide the exploration of floating wind farms with powerful tools that can aid the daily basis decision making processes.

Chapter 3

Basis for numerical modelling of floating offshore wind turbines

3.1 Introduction

The main goal of [Chapter 2](#) was to introduce the different environmental conditions that must be considered in any floating offshore wind turbine numerical model, as well as the key elements that may influence its dynamic response. However, to construct such a model, it is vital not only to characterise the wind or waves field, but also to understand how these conditions are reflected in the external forces that act over the structure.

In this Chapter, the aerodynamic forces, fundamental for the analysis of any wind turbine, offshore or not, will be firstly presented, starting from the analysis of the flow over 2D bodies and only then extended to the full rotor aerodynamic. After that, a brief overview of the hydrodynamic properties of a floating structure will be presented, having in mind its application to the semi-submersible solutions that will be further explored in the next Chapters. When analysed together with [Chapter 2](#), the concepts here introduced are important to present most of the numerical models implemented in the software adopted in [Chapter 4](#).

3.2 Wind loading and aerodynamic response

3.2.1 Aerodynamic forces

Since some of the rotor aerodynamics models that will be introduced in [Section 3.2.2](#) require the determination on the aerodynamic forces variation over the blade length, a preliminary discussion of the latter is needed.

For 2D bodies, in general, and aerofoils, in particular, the aerodynamic forces are commonly decomposed in a **drag force** and a **lift force**, along and orthogonal to the relative wind direction, respectively, and a **pitching moment** computed at a $c/4$ distance from the aerofoil nose, being c its chord. Alternatively, these forces are sometimes replaced by a normal and tangential force, defined in relation to the aerofoil chord, that is deviated from the wind defined direction by the

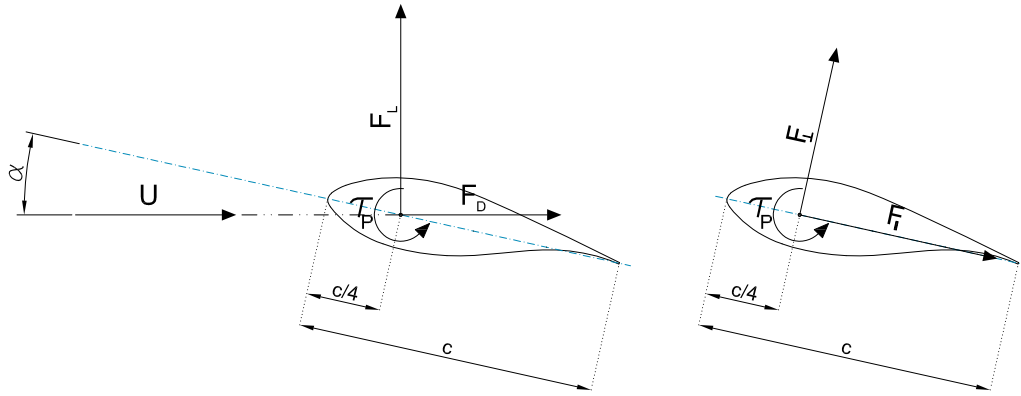


Figure 3.1: Scheme of the different aerodynamic forces conventions.

angle of attack, α . All these forces and geometrical parameters are schematically represented in [Figure 3.1](#).

In any case, all of these forces are usually expressed from its adimensional coefficient, normalised by the dynamic pressure¹ and a reference area, here assumed to be given, per unit length, by the aerofoil chord. In the same spirit, the pitching moment can be normalised by the dynamic pressure, the same reference area and the aerofoil chord:

$$F_i = \left(\frac{1}{2} \rho U^2 c \right) C_i \quad (3.1a)$$

$$\tau_P = \left(\frac{1}{2} \rho U^2 c^2 \right) C_\tau \quad (3.1b)$$

where U is the wind speed, and i may stand for L (lift), D (drag), $\perp$ (normal) or $\parallel$ (tangential). With the computational power available nowadays, Computational Fluid Dynamics (CFD) models may be used to explicitly solve the Navier Stokes equations for an arbitrary shape. However, for thin aerofoils and under some simplifying assumptions, it is also possible to develop analytical models that may be used to estimate the same quantities and gain deeper insight into the relevant phenomenon. In this section, both approaches will be briefly presented.

3.2.1.1 Ideal flow

Firstly, the flow will be assumed to be irrotational, inviscid and incompressible. While for some applications reasonable approximations can be achieved with this model, neglecting viscous effects will lead to some inconsistencies as will be seen, which in turn will demand a proper analysis of these effects.

One of the main advantages of the ideal flow formulation is that it can be characterised through a velocity potential formulation ϕ , an approach already introduced in [Section 2.2.2.1](#) and here

¹For an incompressible flow, the dynamic pressure is defined as $\frac{1}{2} \rho U^2$, being ρ the fluid density and U the flow speed.

recovered as:

$$\mathbf{u} = \nabla \phi \quad (3.2)$$

If the velocity profile is known, the pressure and the lift forces can be obtained by integration taken over all the aerofoil perimeter. For this approach, it is useful the vortex formulation presented for example in [Anderson \(2001\)](#), where the aerofoil is assumed to be an object carrying a vorticity per unit length $\gamma(s)$, where s is measured along the aerofoil contour. The generalised formulation proposed by Prandtl *et al* in the beginning of the 20th century (see for example [Munk, 1923](#)) consists in determining the vorticity function such that the aerofoil contour becomes a stream line, meaning that the velocity vector is oriented along the aerofoil boundary in every point.

Although no analytical solution is known for the majority of the situations, the problem is still solvable by numerical means. Once the vorticity function is known, the lift force (F_L) and coefficient (C_L) result directly from the application of the **Kutta-Joukowski theorem**:

$$F_L = \rho \cdot U \cdot \Gamma = \rho \cdot U \cdot \oint \gamma(s) \cdot ds \quad (3.3a)$$

$$C_L = \frac{F_L}{\frac{1}{2} \rho U^2 \cdot c} = \frac{2}{U \cdot c} \cdot \Gamma \quad (3.3b)$$

where the circulation Γ has been implicitly defined as the vorticity integral along the aerofoil perimeter and the problem has now been reduced to identifying the vorticity distribution that will respect the boundary conditions at the aerofoil contour. A final remark must be made, however, since this procedure is not totally defined and different solutions can be obtained associated with different circulation around the blade and hence different lift forces. In order to obtain an unequivocal solution, it is necessary to consider the **Kutta condition**, which states that, as the flow cannot have two different velocities at the trailing edge², it must either come to a stop (stagnation point) or leave the aerofoil smoothly for a finite angle or cusped aerofoil, respectively, as sketched in [Figure 3.2](#). For the limiting case where the aerofoil thickness is neglected, the lift coefficient can be shown to be given by:

$$C_L = 2\pi \cdot (\alpha - \alpha_0) \quad (3.4)$$

where α_0 can be interpreted as the zero-lift angle of attack, which can be determined exclusively from the aerofoil geometry. The generalisation for an arbitrary shape, where the aerofoil thickness is explicitly considered, does not provide a closed form solution, but can still be solved numerically, as presented in [Section A.1](#), where [Equation 3.4](#) is also carefully derived and further detailed.

²The flow cannot have two different velocities at any point of the aerofoil, but only at the trailing edge new conditions can be imposed on the problem due to the encounter of both top and bottom surfaces.

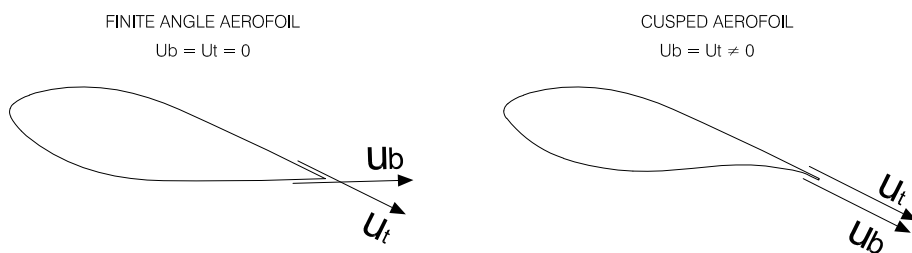


Figure 3.2: Velocity analysis at the trailing edge for 2 different aerofoil shapes.

3.2.1.2 Viscous effects

In the previous section, no viscous effects were considered, which would be a valid assumption if the diffusion term could be neglected in the Navier-Stokes equations. Although the characteristics of many fluids, namely air and water, support this hypothesis, as their dynamic viscosity is, indeed, very small, it can be shown that, under these conditions, a solid body moving at constant speed through a fluid at rest would experience no drag force. This **d'Alembert paradox** is strongly contradicted by any experiment with real fluids and reveals the importance of considering the viscous effects more carefully.

In fact, an ideal fluid will experience slip near the wall of the solid body, as it will fail to match its velocity, which means that, even for fluids with very low viscosity, to respect the boundary condition at the wall, there will be a region with such a strong velocity shear that the diffusion term can no longer be neglected. For a streamline body, this influence is confined to a thin near wall layer in which the relative velocity increases from 0 (at the wall) to the value predicted by the ideal fluid model. This behaviour allowed Prandtl to propose the **boundary layer** concept in the beginning of the 20th century (see [Prandtl \(1904\)](#), or [Prandtl \(1928\)](#) for a later translation). For some particular conditions, mainly in the presence of blunt bodies as a cylinder, but also for aerofoils for high angles of attack, the boundary layer thickness can increase so greatly that zones of reversed flow near the body are created. In these cases, there is a region of strongly decelerated flow (wake of the body) that is associated with large energy losses and great deviations from the ideal fluid approach on the pressure distribution, which will lead to a sudden decrease of the lift force and a significant drag increase. The first of these phenomenon, known as **skin friction drag**, usually has a negligible effect on the lift coefficient, while the latter (**form drag**) will be important to both lift and drag forces. For the remaining of this section, both viscous effects will be briefly presented and characterised with respect to a streamline body like an aerofoil.

Skin friction drag

The skin friction drag effect can be qualitatively studied by analysing the order of the different terms in the Navier Stokes and the continuity equations that, for high Reynolds numbers, may be approximated inside the boundary layer by (Kundu et al., 2015):

$$u_i \partial^i u_x = -\frac{1}{\rho} \partial_x p + \nu (\partial_{xx} u_x + \partial_{yy} u_x) \approx -\frac{1}{\rho} \partial_x p + \nu \partial_{yy} u_x \quad (3.5a)$$

$$u_i \partial^i u_y + \frac{1}{\rho} \partial_y p - \nu (\partial_{xx} u_y + \partial_{yy} u_y) \approx \frac{1}{\rho} \partial_y p = 0 \quad (3.5b)$$

$$\partial^i u_i = 0 \quad (3.5c)$$

where in this 2D description the dummy index on the summations may be taken as $i = x, y$, with x and y being measured along and orthogonally to the aerofoil contour, respectively. Equation 3.5b immediately reveals why a potential flow formulation gives good results for the lift coefficient: as the pressure acting on the aerofoil surface is the same as the one experienced at the edge of the boundary layer ($\partial_y p = 0$ inside the boundary layer), where the potential flow model is valid, so does the lift force³. The momentum equation in the x direction and the continuity equation can be solved subject to the no slip condition at the wall and a smooth transition to the region outside the boundary layer, previously solved with the ideal fluid model. A first approximation of the tangential force acting on a body per unit length can be obtained for a laminar boundary layer once more through a dimensional analysis as:

$$f_D = \rho \nu \partial_y u \sim \frac{\rho \nu U}{\delta} \quad (3.6)$$

being δ the boundary layer thickness that may taken to be such that $\sqrt{\text{Re}} \sim c/\delta$, being $\text{Re} = Uc/\nu$ the Reynolds number of the flow (Kundu et al., 2015). The corresponding drag coefficient follows directly as:

$$c_D = \frac{f_D}{\frac{1}{2} \rho U^2} \sim \frac{c}{\delta} \frac{2}{\text{Re}} \sim \frac{2}{\sqrt{\text{Re}}} \quad (3.7)$$

where it should be noted that the aerofoil chord is absent from the denominator since the force in the numerator was computed per unit length, a fact that is made explicit by using lower case letters for both the drag force and coefficient. The exact solution of some problems, as the one given by Blasius (1907) for a laminar flow over a semi-infinite flat plate, shows that this is indeed the correct order of the friction force, while the multiplying factor is case dependent. This can be better understood by noting that one important step of the dimensional analysis described above comes from the identification of the boundary layer thickness (δ) with the inverse of the Reynolds number, that carries the implicit assumption that the calculations have been made over the entire aerofoil. If this identification is made locally, the height of the boundary layer increases

³As mentioned in Anderson (2001) this conclusion is not valid in the cases where there is flow separation or where the curvature of the surface is not large when compared to the boundary layer thickness as happens in blunt bodies.

downstream proportionally to the square root of the distance to the edge (x). The drag coefficient obtained by integration along all the surface for the case of the aforementioned flat plate leads to the exact solution given by Blasius (1907):

$$C_D = \frac{\int_0^c c_D(x) dx}{c} \approx \frac{1.328}{\sqrt{\text{Re}}} \quad (3.8)$$

Another consequence of the discussion just made is that a boundary layer is always laminar at the beginning of the body and becomes more and more unstable as it progresses downstream, eventually becoming fully turbulent. In this case, no analytic solution is known and the full Navier Stokes equations must be solved, although it should be noted that the fully turbulent boundary layer thickness increases faster than in the laminar scenario and so does the wall shear stress and corresponding drag force (Kundu et al., 2015). In a real solid body, the boundary layer is neither fully laminar or fully turbulent. In fact, as already pointed out, it always starts as laminar until a given critical Reynolds number is achieved, transiting then, over a certain length, properly called transition region, to a fully turbulent phase. The knowledge of this critical Reynolds number allows the identification of the point where the transition to fully turbulent is achieved, but this insight must come from semi-empirical methods or wind tunnel testing as there is no analytical solution to it⁴. There is however another effect that comes with viscous effects and perhaps harder to estimate from simple expressions: flow separation and corresponding form drag. In the following sub-section, this problem will be briefly discussed and explained without pretending to provide a full description of its effects.

Form drag

Flow separation introduces a new source of drag in a solid body moving relatively to a fluid that would not exist if the boundary layer was fully attached to it. For a typical aerofoil, at low angles of attack, the pressure coefficient is unitary at the leading edge and decreases drastically in the first segment of the chord. From this point on, the pressure increases gradually creating a region of moderate **adverse pressure gradient** that slows down the flow. If this effect is often negligible in streamline bodies, for blunt bodies or aerofoils at higher angles of attack, this adverse pressure gradient is so severe that the inner particles of the fluid do not have enough kinetic energy to overcome it and come to a point when they stop (separation point). Further downstream, these reverse their motion, causing the point of zero velocity and all the boundary layer to separate from the surface.

This detachment of the boundary layer has two major consequences for the forces acting on a solid body. On one hand, the pressure over the aerofoil pushing it "down" is significantly higher in the separated flow case, which will inevitably lead to a drastic reduction in the vertical direction net force: there is a significant reduction in lift. On the other hand, the separated flow pressure near the trailing edge of the aerofoil is smaller than for the ideal fluid, which leads to a lower force balancing the over pressure on the leading edge. As the drag force is the net result of the pressure

⁴As a reference, a critical Reynolds number of the order of 5×10^5 can be used (Kundu et al., 2015).

along all the surface, higher net forces are experienced as the flow detaches from the body. The two major effects of flow separation are then a significant reduction in lift force and increase in drag force. In real fluids there is always a point of separation, but for streamline bodies this region is so small that its effect is negligible, and these elements experience very low form drag forces. The same does not happen for blunt bodies or for aerofoils for angles of attack greater than a critical value referred to as stall angle that is associated with the maximum lift coefficient and may be taken to be $\sim 15^\circ$.

The main effects of the viscosity of real fluids on the forces acting over a solid body have now been highlighted, at least in a qualitative way, while further details about boundary layers can be found for instance in the work by [Schlichting and Gersten \(2016\)](#). In the next section the Navier-Stokes will finally be tackled with the currently available computational power.

3.2.1.3 Computational Fluid Dynamics

Until this point several simplifications have been made so that closed form analytical solutions could be found for the flow over an aerofoil, but with the computational power available nowadays, the numerical resolution of the full Navier-Stokes equations has become less time consuming than ever before, making them a valid alternative. Although different models can be used, these are all based in the conservation laws already mentioned and here recovered for an incompressible flow⁵:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \mathbf{g} - \frac{1}{\rho} \nabla p + \nu \cdot \nabla^2 \mathbf{u} \quad (3.9a)$$

$$\nabla \cdot (\rho \mathbf{u}) = 0 \quad (3.9b)$$

While a full description of CFD capabilities and limitations is not intended as it would lead to an extensive document by itself, some main topics will be briefly addressed and some references given for further details, so that the reader can either follow the principal points and difficulties when building a CFD model or find more advanced techniques and theoretical background if desired. The following sub-sections will mainly follow Professor's Bert Blocken "Sports and Buildings Aerodynamics" open course ([Blocken, 2019](#)).

Modelling approaches

Although the conservation laws are in the same number of the unknown variables, and the system is therefore of closed form, they create a system of non linear coupled differential equations with no known solution for an arbitrary geometry. Because of this, different approaches can be followed, being the most commonly used:

- **Direct Numerical Simulation (DNS)**, where the full Navier-Stokes are solved numerically;
- **Large Eddy Simulation (LES)**, where filtered Navier Stokes equations are solved such that only larger scale eddies are solved and the minor ones modelled ([Blocken, 2018](#));

⁵If the flow is not incompressible, then an additional energy equation and a state equation are needed to provide a full dynamic theory.

- **Reynolds Averaged Navier Stokes (RANS)**, where only the mean flow equations are solved and all the turbulent fluctuations are modelled, in an extension of the procedure followed in [Section 2.2.1.1](#).

Because LES, and specially DNS, imply higher computational efforts, for the purpose of this work, special attention will be given to the RANS simulations, a commonly used approach in the wind engineering field ([Blocken, 2014, 2018](#)). The assumptions from which the RANS equations were derived have already been presented in [Section 2.2.1.1](#) and are now recovered:

$$\partial_i \langle u_i \rangle + \langle u_j \rangle \partial^j \langle u_i \rangle = -\frac{1}{\rho} \partial_i p - \partial^j (\hat{u}_i \cdot \hat{u}_j) + \nu \partial_j \partial^j \langle u_i \rangle = -\frac{1}{\rho} \partial_i p + \partial^j (\tau_{ij} - \hat{u}_i \hat{u}_j) \quad (3.10a)$$

$$\partial^i \langle u_i \rangle = 0 \quad (3.10b)$$

where the momentum equations has been written in a perhaps more enlightening manner for a Newtonian incompressible fluid by making use of the usual shear stress tensor (τ) definition:

$$\tau_{ij} = \nu (\partial_j \langle u_i \rangle - \partial_i \langle u_j \rangle) \quad (3.11)$$

In this notation, it is clear the turbulence effects appear in an equivalent way to the shear stress tensor and the quantities defined as $\hat{u}_i \hat{u}_j$ are usually referred to as the components of the **Reynolds stress, $\mathbf{R}$** . The Reynolds stress diagonal components give the kinetic energy of the turbulent fluctuations:

$$\hat{k} = \frac{1}{2} \sum_{i=1}^3 R_{ii} = \frac{1}{2} R_i^i = \frac{1}{2} (\hat{u}_x^2 + \hat{u}_y^2 + \hat{u}_z^2) \quad (3.12)$$

Turbulence models

It is important to emphasize that while the original Navier-Stokes equations form a closed set of equations (4 unknown to 4 equations for an incompressible fluid or 6 to 6 in a compressible one), RANS equations do not, because of the introduced Reynolds stress. Because of this, an exact solution for the mean flow variables is not possible and the so-called turbulence models must be used once more ([Ferziger and Perić, 2002](#)). In this section, only a very small number of models will be introduced and more information can be found in ([Wilcox, 1994](#); [Bardina et al., 1997](#); [Roy and Blottner, 2006](#)).

The simplest consistent approach is directly based on the Boussinesq or eddy-viscosity hypothesis, where the Reynolds stress is written as a function of the mean flow gradients:

$$R_{ij} = \frac{2}{3} \hat{k} \delta_{ij} - 2\nu_T S_{ij} \quad (3.13)$$

where the first term (δ_{ij} is the Kronecker delta) is introduced so that the trace of the stress tensor, and hence its energy content, is not modified, and the second term is obtained from the deformation

rate tensor of the mean flow as:

$$S_{ij} = \frac{1}{2} \left(\partial_j \langle u_i \rangle + \partial_i \langle u_j \rangle - \frac{2}{3} \partial^k \langle u_k \rangle \delta_{ij} \right) = \frac{1}{2} (\partial_j \langle u_i \rangle + \partial_i \langle u_j \rangle) \quad (3.14)$$

where the last equality holds for an incompressible flow. The ultimate goal of any turbulence model is to provide expressions for the turbulent viscosity (ν_T) and this is normally achieved by means of one-equation models, as the Spalart-Allmaras model (Spalart and Allmaras, 1992), or two-equation models, as the $k - \omega$ (Wilcox, 2008), the SST $k - \omega$ (Jones and Launder, 1972) or the SST $k - \varepsilon$ (Launder and Sharma, 1974) and corresponding variations.

For the simple 2D flow around an aerofoil that is the object of this work, the already mentioned methods provide good results for small angles of attack. However, as the flow detaches from the surface the SST $k - \omega$ is usually considered the best starting option (Eleni et al., 2012).

Near wall behaviour

Another important aspect of a CFD model is the near wall approach. As these regions typically exhibit very strong velocity gradients at low Reynolds numbers, very fine grids are needed to provide adequate solutions. Furthermore, most of the turbulence models, namely the ones indicated in the previous section, were developed for high Reynolds number flows where the viscous force are mainly due to turbulent phenomenon, whereas in these regions the molecular viscosity does take an important role. In fact, this region where the viscous effects are relevant, can still be divided in 3 distinct sub-regions: a linear sub-layer immediately after the boundary, where the flow is laminar and dominated by viscous effects, a log-law layer immediately before the outer, fully turbulent layer, where the turbulent terms dominate although the flow is still laminar, and a buffer, in between, layer where viscous and turbulent forces are of the same order. Because these layers present different dominant features, different models must be used to adequately deal with each one. According to Von Kármán (1931), the velocities profile near a given wall exhibit a self similar solution - **universal law of the wall** - and can be expressed by two adimensional terms:

$$u^+ = \frac{u}{u_\tau} \quad (3.15a)$$

$$y^+ = \frac{y \cdot u_\tau}{\nu} = \frac{y}{\nu} \frac{u}{u^+} \quad (3.15b)$$

where $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ is the friction velocity, y^+ is the dimensionless wall coordinate and u^+ is the dimensionless velocity. In the viscous sub-layer, where $\tau_w = \rho \cdot \nu \partial_y u$, the law of the wall simply reads:

$$u^+ = y^+ \quad (3.16)$$

while for the turbulent dominated layer, and using the Prandtl mixing length once more, the profile will follow a log law much in the same way the wind speed over height in the atmospheric

boundary does:

$$u^+ = \frac{1}{\kappa} \cdot \ln y^+ + C \quad (3.17)$$

To correctly treat each of these sub-layers, two major approaches can be used:

- Low Reynolds number modelling in the near wall region;
- Wall functions.

The first approach - Low Reynolds number modelling - basically consists in providing a more and more refined mesh as it approaches the boundary wall. As have been seen before, the thickness of the boundary layer decreases with increasingly Reynolds number and so, for highly turbulent flows, like most cases that will be analysed, this approach will require extremely fine mesh and hence will be highly computationally demanding. Furthermore, it is also assumed the wall to be perfectly smooth, which is always an approximation⁶. Because of these, attention will be devoted to the other option: wall functions. In wall functions, the viscous sub layer is neglected and only its effect on a point already in the log-law region is considered through an empirical, or semi-empirical, function⁷. If a cell centre is pretended in the log-layer, then, according to [Chanson \(2009\)](#), its dimensionless height should be between 30 to 1000 (preferably in the lower region from 30 and up to 60). Because of its empirical nature, the wall functions can easily incorporate surface roughness on their expressions.

Best Practice Guidelines: Errors, verification and validation

The large number of choices that must be made when building a CFD model (which turbulence model to used, for instance) can have a strong impact on the obtained results ([Blocken, 2015](#)). According to [Casey and Wintergerste \(2000\)](#), the errors and uncertainties in a CFD simulation can be categorised by their nature. Because the exact flow equations are not solved, but only a simplified version of it (either by using RANS with a given turbulence model, or assuming an incompressible flow, between others), a first kind of error arise as **model uncertainties**. Furthermore, even these simplified equations are not analytically solved so only a numerical solution with a finite resolution in time and space is obtained, and so **discretisation or numerical errors** appear. Due to the iterative numerical procedure usually used to solve differential equations, the solution is not usually fully converged over the entire integration domain leading to **iteration or convergence errors**. In fact, even if the solution is fully converged for a given computer with a finite accuracy, **round-off errors** can still be present. Until now, the errors presented are mainly due to computational limitations, but even before the solution is initialized there are already some **application errors** inherent to the problem that is being tackled, as a precise knowledge of the geometry, boundary conditions or available data is only accurate to a certain degree. Finally, it must not be forgotten that the user or the code itself can introduce sources of errors by either carelessness of use or development bugs, leading to **user and code errors**.

⁶If rough surfaces are to be studied with this approach, then all the details of it should be modelled.

⁷Because of this, heat and mass transfer phenomenon are not correctly treated by this approach, as the largest resistance to these transfers is precisely in the laminar sub-layer, which is not modelled at all.

As can be seen, a CFD solution is quite dependent on several parameters. Many times, the solution obtained is sufficiently believable to be considered a good solution when it can be significantly flawed and so arises the need for a careful **verification** and **validation** of the procedure used⁸. An extensive description of these errors, as well as some measures that can be taken to minimise them, can be found in literature, for example in ([American Institute of Aeronautics and Astronautics, 1998](#); [Casey and Wintergerste, 2000](#); [Oberkampf and Trucano, 2002](#)). Some main aspects are highlighted by [Blocken \(2015\)](#), from which only four will be mentioned here:

- **Create an adequate computational domain:** in many cases only the bottom of the computational domain corresponds to a physical boundary and in the particular case of the flow over an aerofoil, not even this is true. To avoid any artificial acceleration originated by flow contraction, it is vital to model a large enough computational domain;
- **Create a high-quality grid:** in fact, this aspect should consume most of the time devoted to the entire simulation as it can drastically influence the results. In order to obtain such a grid, the cells shape and orientation are as important as a sufficient grid resolution. This last aspect can be properly checked by a grid-convergence study where three systematically and substantially refined grids are used;
- **Select higher-order discretization schemes:** it is recommended to use at least second-order accurate spatial discretizations as first-order schemes are extremely vulnerable to numerical diffusion errors;
- **Perform appropriate validation studies:** until this point all the techniques mentioned aim at making sure the solution obtained is as independent as it can be from the modelling choices, but it is not assured that it does indeed adequately represent reality. To do so, it is important to use as much as possible experimental data from either experimental campaigns or wind tunnel tests, at least to sub divisions of the full computational domain/parameters, as it is expected that the combination of different previously validated entities will still provide an accurate result.

For reference, the development and validation of a CFD model of a NACA 0012 aerofoil is presented in [Section A.2](#). This example not only serves as an illustration of the process described above, but also prepares and validates the numerical model that will be used to compute the aerodynamic properties, necessary inputs for a full aeroelastic simulation of a real wind turbine (see [Chapter 4](#)).

3.2.2 Rotor aerodynamics models

In the previous section, the theoretical basis needed to compute the forces acting on a static aerofoil provided the wind speed is known have been presented, but additional considerations are needed if one wants to predict the aerodynamic loading over a wind turbine. The reasons for this can be easily understood: firstly, the wind turbine blades are not static but moving with a velocity

⁸In the words of Professor Bert Blocken, "verification is solving the equations right while validation is about solving the right equations" ([Blocken, 2019](#)).

controlled by the rotor angular velocity. While this issue could be intuitively solved by considering a relative wind speed (that happens to not be precisely true) a perhaps more relevant issue should be considered. The expected wind speed conditions are defined with respect to the so called unperturbed wind field, but the mere presence of the wind turbine modifies the wind field perceived at the rotor itself, that in turn impacts on the loading and, eventually, on the rotor angular velocity. In the following sections, the main theories to estimate the power production capacity and forces to be withstand by the structure, namely rotor blades and tower, will be briefly presented. In all the theories it will be assumed that only the air mass passing directly through the rotor is affected by its presence.

3.2.2.1 Linear momentum theory

From the discussion of the previous section, it is clear that the aerodynamic properties of the blades must play an important roll in the response of a wind turbine, and so they do. However, simpler considerations regarding the variation of the linear and angular momentum of the air mass can be used to provide a first insight into the expected forces acting on these structures.

The first and simpler method to estimate the energy that can be extracted from a given airflow lies its basis on the conversion of the air mass linear momentum into energy at the rotor plane. Because of the presence of the rotor, the airflow near a wind turbine suffers some important modifications. Denoting by the indexes 0, d and w a section far upstream, at the rotor plane and in the far wake, according to [Figure 3.3](#), respectively, the continuity equation implies that:

$$\rho \cdot A_0 \cdot U_0 = \rho \cdot A_d \cdot U_d = \rho \cdot A_w \cdot U_w \quad (3.18)$$

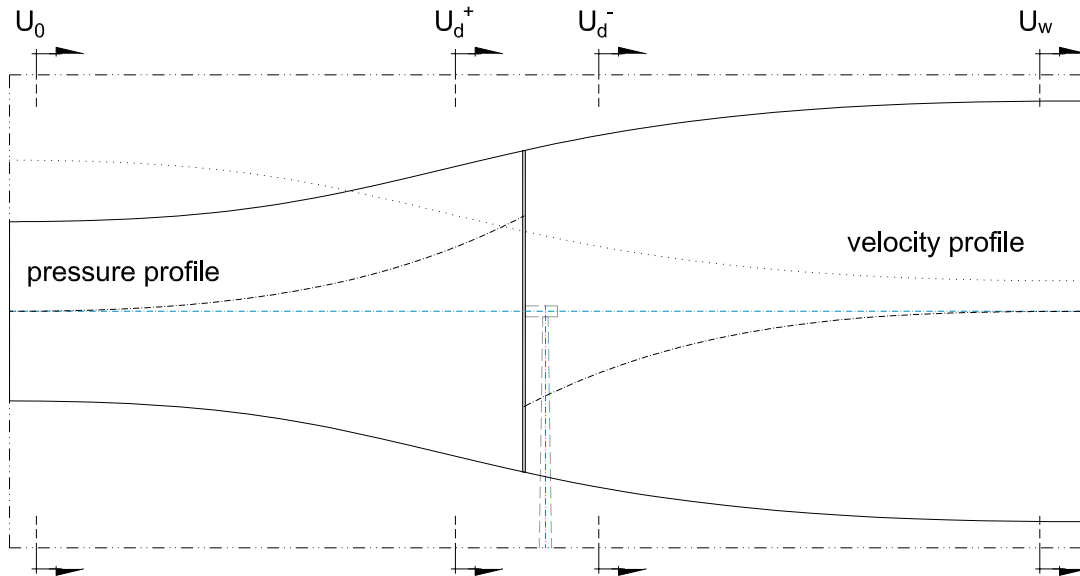


Figure 3.3: Air flow scheme due to the presence of a wind turbine. The dotted and dashed line schematically represent the pressure and velocity profile evolution, respectively.

To relate the wind speed at the vicinity of the rotor with the free stream velocity, the rotor can be regarded as an element inducing a wind speed variation expressed by an **axial induction factor** (a), that will be assumed to be an input parameter for now:

$$U_d = U_0 - \Delta U_0 = U_0 - a \cdot U_0 = U_0 \cdot (1 - a) \quad (3.19)$$

Due to the energy loss at the disk, Bernoulli's equation cannot be directly applied between the far upstream and far wake sections, but it can be used to relate each of these to the section immediately before and after the rotor plane, leading to:

$$\frac{\rho \cdot U_0^2}{2} + p_0 = \frac{\rho \cdot U_d^{+2}}{2} + p_d^+ \quad (3.20a)$$

$$\frac{\rho \cdot U_w^2}{2} + p_w = \frac{\rho \cdot U_d^{-2}}{2} + p_d^- \quad (3.20b)$$

A simple integration of the pressure drop defined by Equation 3.20 across all its area leads to the thrust force (F_T) acting on the rotor, which can be identified with the global change of axial momentum of the stream tube and used to relate the far wake wind speed with the free stream velocity:

$$F_T = \Delta p_d \cdot A_d = \frac{1}{2} \cdot \rho \cdot (U_0^2 - U_w^2) \cdot A_d = (U_0 - U_w) \cdot \rho \cdot U_d \cdot A_d \longrightarrow U_w = (1 - 2a) \cdot U_0 \quad (3.21)$$

The generated power (P) may also be obtained from the air flow rate and pressure drop. Both the generated power and acting force are often referred to in their non-dimensional form, the **power coefficient** (C_P) and the **thrust coefficient** (C_T), respectively:

$$C_P = \frac{P}{P_0} = \frac{2\rho \cdot A_d \cdot U_0^3 \cdot a \cdot (1 - a)^2}{\left(\frac{1}{2} \cdot \rho \cdot U_0^2\right) \cdot A_d \cdot U_0} = 4 \cdot a \cdot (1 - a)^2 \quad (3.22a)$$

$$C_T = \frac{F_T}{F_0} = \frac{2\rho \cdot A_d \cdot U_0^2 \cdot a \cdot (1 - a)^2}{\left(\frac{1}{2} \cdot \rho \cdot U_0^2\right) \cdot A_d} = 4 \cdot a \cdot (1 - a) \quad (3.22b)$$

that may only be computed as long as the axial induction factor is known or prescribed. However, and from Equation 3.21 it is clear that this cannot be higher than 0.5, as it would lead to reverse flow in the far wake. The evolution of both factors is represented in Figure 3.4, where it is clear that while the thrust coefficient is strictly growing with the axial induction factor, the power coefficient reaches a well defined maximum for $a = 1/3$ given by:

$$C_P \left(a = \frac{1}{3} \right) = \frac{16}{27} \quad (3.23)$$

This maximum power coefficient is named after Betz (1919) as the **Betz limit** and sets an upper limit to the amount of energy any wind turbine is able to extract, no matter how efficient, as this limitation is not due to any design inefficiency.

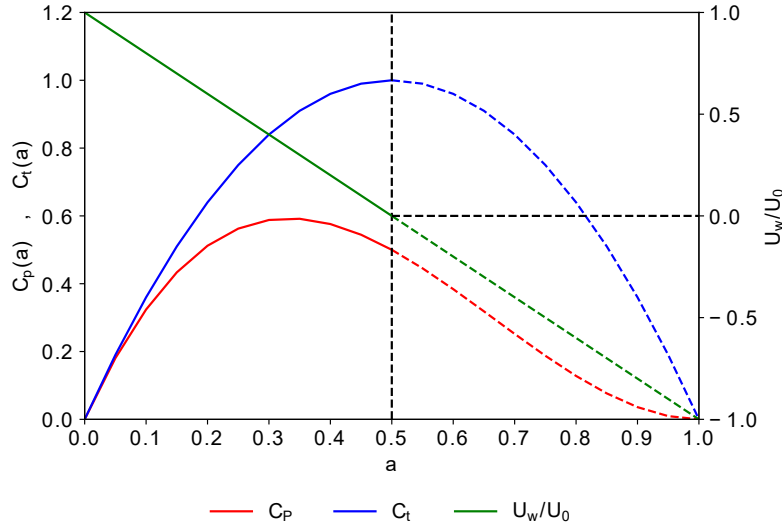


Figure 3.4: Evolution of both thrust and power coefficients and far wake to free stream wind speed ratio with the axial induction factor.

3.2.2.2 Angular momentum theory

In the previous derivations, the airflow was assumed to be purely axial (without any rotation), which is not true for a real wind turbine rotating with a finite angular velocity (Ω) and with a discrete number of blades. One theory that allows to take care of the first limitation is based on **Euler's turbine equation** (Euler, 1754).

In fact, the rotating rotor induces a tangential acceleration on the flow that acquires an angular velocity (ω) itself, with a direction that is opposite to the rotor direction. In the same spirit as before, this allows the definition of a **tangential induction factor** (a') as half of the ratio between rotor and wake angular velocities. From this definition, the tangential velocity acquired by the air flow can be written as a function of the distance to centre of rotation as:

$$u_t(r) = \omega \cdot r = 2 \cdot a' \cdot \Omega \cdot r \quad (3.24)$$

The total power extracted from the airflow can be computed by simply integrating the infinitesimal contributions across the disk area as:

$$P = \int_{A_d} \delta P = \int_{A_d} \delta \tau \cdot \Omega = 4\pi \cdot \Omega^2 \cdot \rho \cdot U_0 \int_0^R (1-a) \cdot a' \cdot r^3 dr \quad (3.25)$$

where the infinitesimal torque variation $\delta \tau$ as been obtained from the rate of variation of the angular momentum (L) of the air mass crossing the rotor as:

$$\begin{aligned} \delta \tau &= \delta \left(\frac{dL}{dt} \right) = \delta \left(\frac{m \times \Delta u_t \times r}{\Delta t} \right) = \underbrace{\rho \cdot 2\pi r \cdot \delta r \times U_0 \cdot (1-a)}_{\delta m / \Delta t} \times \underbrace{2 \cdot a' \cdot \Omega \cdot r}_{\Delta u_t} \times r = \\ &= 4\pi r^3 \cdot \rho \cdot U_0 \cdot (1-a) \cdot a' \cdot \Omega \cdot \delta r \end{aligned} \quad (3.26)$$

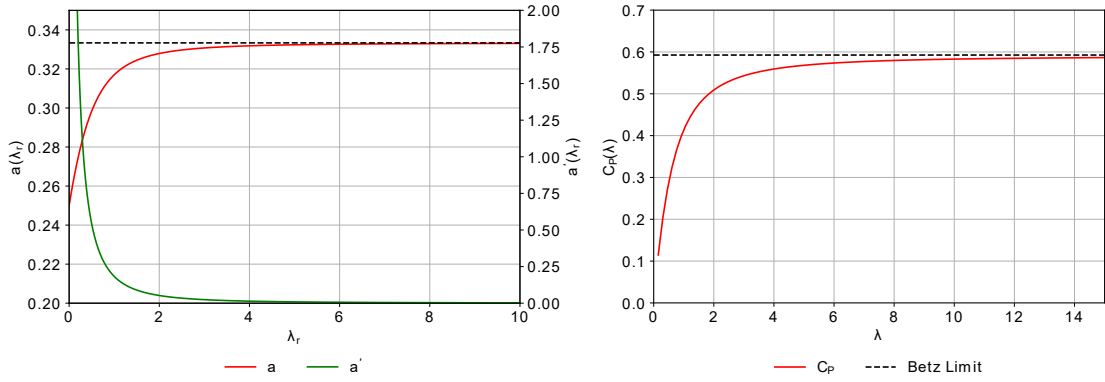


Figure 3.5: Evolution of the optimal induction coefficients (on the left) and of the power coefficient (on the right) as a function of the local speed ratio and tip speed ratio, respectively.

The corresponding power coefficient that incorporates the angular motion of the wake is now given by:

$$C_P = \frac{8}{\lambda^2} \int_0^\lambda a' \cdot (1 - a) \cdot \lambda_r^3 \cdot d\lambda_r \quad (3.27)$$

where the **local speed ratio** (λ_r) or **tip speed ratio** (λ), when evaluated at the tip of a blade, is defined as the ratio between the tangential speed at a given position and the free stream wind speed:

$$\lambda_r(r) = \frac{\Omega \cdot r}{U_0} \quad (3.28)$$

From Equation 3.27, it is clear that to maximise the power extracted from the wind, it is necessary to maximise the integrand subject to the constraint:

$$\lambda_r^2 \cdot a' \cdot (1 + a') = a \cdot (1 - a) \quad (3.29)$$

This is a classical constrained minimisation problem that may be solved applying the Lagrange multipliers method and that results in:

$$a' = \frac{1 - 3 \cdot a}{4 \cdot a - 1} \quad (3.30)$$

Equation 3.29 and Equation 3.30 provide all the information needed to evaluate the induction coefficients that maximise the power output and characterise the power coefficient as a function of the tip speed ratio alone. The results are plotted in Figure 3.5, where it may be seen that, as the local speed ratio increases, the axial induction coefficient tends to the 1/3 value predicted by Betz theory, while the tangential coefficient acquires negligible values. These conclusions are consistent with the axial momentum theory, as a zero tangential coefficient means there is no rotation of the air flows at the wake of the generator (Equation 3.24). Furthermore, as the tip speed

ratio increases, the power loss due to the rotating flow is also negligible and so a variable speed wind turbine is able to keep its efficiency independently of the mean wind speed, as long as the angular velocity is such that the tip speed ratio is higher than a given threshold value:

$$\Omega = \frac{U_0}{R} \cdot \lambda \sim 6 \cdot \frac{U_0}{R} \quad (3.31)$$

3.2.2.3 Blade Element/Momentum Theory

Until now, all the theories presented rely on the prescription of the induction factor coefficients, and hence do not allow for a closed form definition of the wind loading and power output of a given wind turbine. However, the estimated forces and power output must be compatible with the local conditions of the aerofoils and their relative motion with respect to the air masses.

The Blade Element Momentum Theory (BEMT) provides a framework where this coupling is achieved and will allow to estimate all the dynamics based on the unperturbed wind field and the wind turbine properties. This widely used formulation is indeed a combination of two different major assumptions. Firstly, it is assumed that the forces acting on a blade element can be calculated from the aerodynamic properties of the aerofoil under a bi-dimensional flow with an angle of attack determined by the inflow relative velocity to each blade. Secondly, it is assumed that the forces over a given infinitesimal ring with thickness δr are only responsible by the momentum variation of the air mass that flows through that ring.

When both these assumptions hold, this ideal rotor scenario can be used for predicting the global response of the wind turbine and this will be the topic of the first part of this section. However, and depending on both environmental and operating conditions, this is not always the case and additional corrective models, that will be presented in the second part of this section, should be considered.

The ideal rotor

As three-dimensional effects of the flow and the velocity component along the span direction are ignored, the forces acting in an infinitesimal blade element follow directly from the corresponding aerofoil aerodynamic lift and drag coefficients, which depend on the angle of attack, as:

$$\delta F_L = \frac{1}{2} \rho \cdot W^2 \cdot c \cdot C_L \cdot \delta r \quad (3.32a)$$

$$\delta F_D = \frac{1}{2} \rho \cdot W^2 \cdot c \cdot C_D \cdot \delta r \quad (3.32b)$$

where W is the relative velocity between the aerofoil and the air mass that may be expressed from the induction coefficients as:

$$W = \sqrt{U_0^2 \cdot (1 - a)^2 + \Omega^2 \cdot r^2 \cdot (1 + a')^2} \quad (3.33)$$

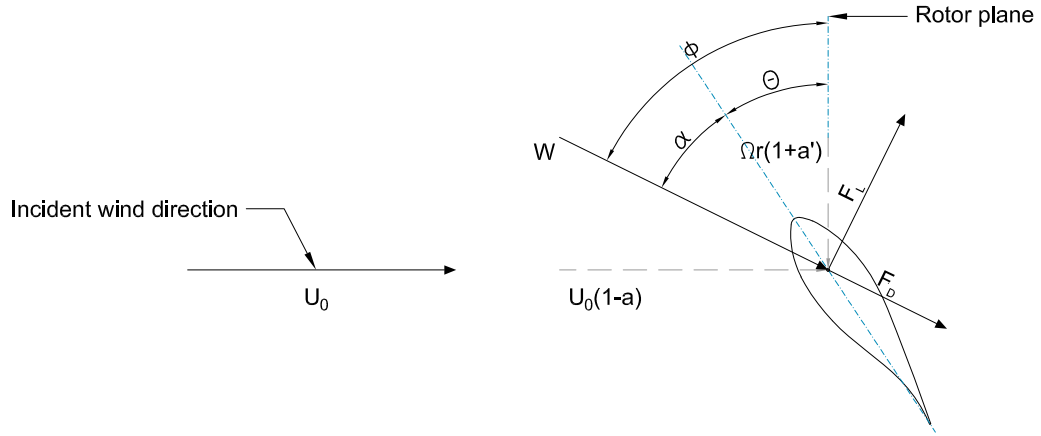


Figure 3.6: Forces acting on a blade element scheme and corresponding angles: α is the aerodynamic angle of attack, ϕ is the angle between the relative wind speed and rotor plane and θ is the angle between the chord and the rotor plane

These forces can be used to express the momentum and forces equilibrium along the rotor rotation axis as:

$$\delta F_x = \delta F_L \cos(\phi) + \delta F_D \sin(\phi) = \frac{\rho}{2} W^2 c [C_L \cos(\phi) + C_D \sin(\phi)] \delta r \quad (3.34a)$$

$$\delta M_x = [\delta F_L \sin(\phi) - \delta F_D \cos(\phi)] r = \frac{\rho}{2} W^2 c [C_L \sin(\phi) - C_D \cos(\phi)] r \delta r \quad (3.34b)$$

where ϕ is the angle between the relative velocity and the rotor plane, as represented in [Figure 3.6](#), and given by:

$$\tan \phi = \tan(\alpha + \theta) = \frac{U_0 \cdot (1 - a)}{\Omega \cdot r \cdot (1 + a')} \quad (3.35)$$

where α is the angle of attack and θ is the angle between the aerofoil chord and the rotor plane.

On the other hand, the second BEMT assumption implies that any radial interaction is not taken into consideration, which is only valid if the axial flow induction factor is not varying over the blade span. Although this does not happen for a real wind turbine, the error is typically small for tip speed ratios higher than 3 ([Burton et al., 2001](#)). In this scenario, the axial and angular momentum obtained from a local analysis of the aerofoils aerodynamics must equal the axial and angular momentum obtained in the previous sections, here defined for a finite number of blades (N):

$$N \delta F_x = \Delta U_d \cdot \rho \cdot U_d \cdot \delta A_d + \Delta p \cdot \delta A_d = \left[2aU_0 \cdot \rho \cdot (1 - a)U_0 + \frac{1}{2} \cdot \rho \cdot (2a'\Omega r)^2 \right] 2\pi r \delta r \quad (3.36a)$$

$$N \delta M_x = \rho \cdot U_d \cdot \Delta u_t \cdot r \cdot \delta A_d = \rho \cdot U_0 \cdot (1 - a) \cdot 2a' \cdot \Omega \cdot r \cdot 2\pi r \cdot r \cdot \delta r \quad (3.36b)$$

Equating the matching terms of [Equation 3.34](#) and [Equation 3.36](#), an expression for the torque

and thrust force as a function of the remaining parameters can finally be obtained:

$$\frac{W^2}{U_0^2} \cdot N \cdot C_{\parallel} \cdot \frac{c}{R} = 8\pi \cdot [a \cdot (1-a) + (a' \cdot \lambda \cdot \mu)^2] \cdot \mu \quad (3.37a)$$

$$\frac{W^2}{U_0^2} \cdot N \cdot C_{\perp} \cdot \frac{c}{R} = 8\pi \cdot \lambda \cdot \mu^2 \cdot a' \cdot (1-a) \quad (3.37b)$$

where the tip speed ratio (λ) definition is recovered, $\mu = r/R$ is the adimensional span position and the tangential and normal coefficients are related to their lift and drag counterparts by:

$$C_{\parallel} = C_L \cdot \cos(\phi) + C_D \cdot \sin(\phi) \quad (3.38a)$$

$$C_{\perp} = C_L \cdot \sin(\phi) - C_D \cdot \cos(\phi) \quad (3.38b)$$

As the forces depend on the induction coefficients (through the angle of attack) that in turn depend on the forces and momentum balance expressed in [Equation 3.37](#), an iterative process must be carried out that usually converges in a few iterations. An illustrative example of this exercise is detailed in [Section A.3](#).

Once the induction coefficients evolution across the blade span are determined, the total torque and thrust force can be obtained integrating the infinitesimal quantities given by [Equation 3.36](#) along all the blade length, while the total power is obtained by the product of the angular velocity and the aerodynamic torque:

$$F_T = 4\pi \cdot \rho \cdot U_0 \cdot \Omega \cdot \int_0^R (1-a) \cdot a' \cdot r^3 \cdot dr = 4\pi \cdot R^3 \cdot \rho \cdot U_0^2 \cdot \lambda \cdot \int_0^1 (1-a) \cdot a' \cdot \mu^3 \cdot d\mu \quad (3.39a)$$

$$P = \Omega \cdot M_x \longrightarrow C_P = \frac{P}{\frac{1}{2} \cdot \rho \cdot U_0^3 \cdot \pi \cdot R^2} = 8\lambda^2 \cdot \int_0^1 (1-a) \cdot a' \cdot \mu^3 \cdot d\mu \quad (3.39b)$$

Tip and root losses

In the ideal rotor model it was assumed, for the momentum variation of the air mass, an infinite number of blades, meaning that any fluid particle is equally affected by them. However, due to a finite number of blades, the axial induction factor is not azimuthally uniform, being higher near the blades. From [Equation 3.39b](#), with the increase of the axial induction coefficient comes an inevitable reduction of the aerodynamic torque. In parallel to this, the flow over the blades creates an under pressure downstream of the rotor and tends to progress around the tip of the blade rather than over it. While the solution to this problem can be obtained in terms of Bessel functions ([Goldstein, 1929](#)), an approximated solution with a simple analytical formula was proposed by [Prandtl \(1923\)](#) where a tip loss function is defined in its most general form as:

$$f\left(\frac{r}{R}\right) = f(\mu) = \frac{2}{\pi} \cdot \cos^{-1} \left[\exp \left(-\pi \frac{R-r}{s} \right) \right] \quad (3.40)$$

where the $R - r$ is the distance from the tip and s is the distance between successive vortex sheets. For a rotor with N blades, this distance can simply be expressed as (Ramdin, 2017):

$$\frac{1}{s} = \frac{N}{2\pi r} \frac{1}{\sin \phi(r)} = \frac{N}{2\pi r} \frac{\sqrt{U_{\perp}^2(r) + U_{\parallel}^2(r)}}{U_{\perp}(r)} = \frac{N}{2\pi r} \sqrt{1 + \frac{U_{\parallel}^2(r)}{U_{\perp}^2(r)}} \quad (3.41)$$

Different uses of Prandtl's generalised tip-loss factor can be made and a systematic review can be found in (Branlard, 2011). If, for example, the implicit definition of the velocity components present in Equation 3.32 is used in Equation 3.41, then:

$$f_T(\mu) = \frac{2}{\pi} \cdot \cos^{-1} \left[\exp \left(-\frac{N}{2} \cdot \frac{\lambda - \lambda_r}{\lambda} \cdot \sqrt{1 + \lambda^2 \left(\frac{1 + a'(r)}{1 - a(r)} \right)^2} \right) \right] \quad (3.42)$$

which is the "exact" correction suggested by Prandtl in his work. Another common solution, which has the advantage of computing the loss factor at different positions without knowing the flow solution at the tip, is to consider $R \cdot \sin \phi(R) \approx r \cdot \sin \phi(r)$ in Equation 3.41, leading to (Wilson and Lissaman, 1974):

$$f_T(\mu) = \frac{2}{\pi} \cos^{-1} \left[\exp \left(-\frac{N}{2} \cdot \frac{R - r}{r \sin \phi} \right) \right] \quad (3.43)$$

If the expressions above have been presented to describe the impact on the lift forces near the blade tip, a vortex based model can be used to show that a similar process occurs near the blade root. A common practice is to modify Equation 3.43 such that the blade root loss is given by:

$$f_R(\mu) = \frac{2}{\pi} \cos^{-1} \left[\exp \left(-\frac{N}{2} \cdot \frac{r - R_{\text{Hub}}}{r \sin \phi} \right) \right] \quad (3.44)$$

and once more, different approaches can be used to determine this loss factor. The total loss taking into account both tip and root losses, whatever assumption is made in its derivation, that should be applied to the momentum equations may then be written as:

$$f_{\text{Total}}(\mu) = f_T(\mu) \cdot f_R(\mu) \quad (3.45)$$

An example of this function evaluated for an arbitrary case is presented in Figure 3.7.

Turbulent wake

Another important correction concerns a limitation already mentioned during the linear momentum theory description. For heavily loaded turbines, and consequently high axial induction coefficients, Equation 3.21 predicts flow reversal at the far wake. However, as the axial induction coefficient increases, the wake becomes more and more turbulent and its flow is much more complex than the one predicted by the BEMT, leading to much greater thrust forces as experienced by the rotor. To overcome this lack of agreement between theoretical and experimental values,

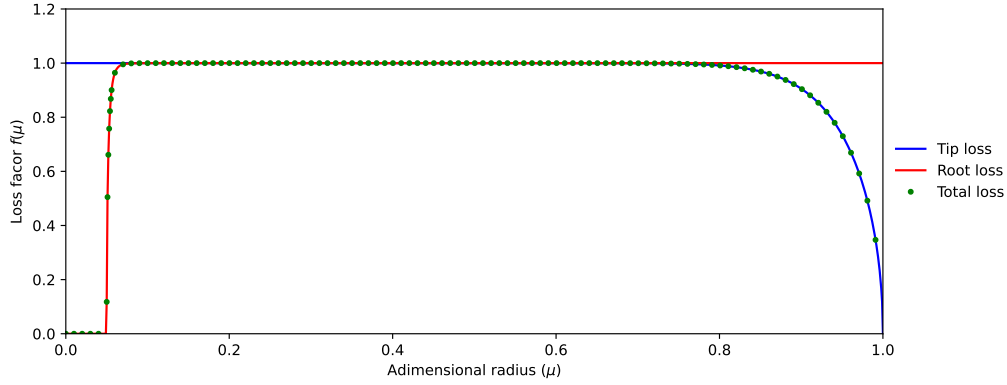


Figure 3.7: Tip, root and total loss for a three-bladed rotor with a non dimensional hub radius of 0.05 and, for simplicity, assuming a uniform flow angle of 5° .

Glauert proposed an empirical expression, documented by [Eggleston and Stoddard \(1987\)](#), that alters the total thrust rotor coefficient predicted by [Equation 3.22](#) according to:

$$a = 0.143 + \sqrt{0.0203 - 0.6427 \cdot (0.889 - C_T)} \longrightarrow C_T = \frac{0.55106 + (a - 0.143)^2}{0.6427} \quad (3.46)$$

Since this correction was empirically obtained considering the total thrust, it already includes the tip losses effect, but it is common practice to consider it still holds for local thrusts coefficients. However, this leads to a discontinuity in the thrust curve, that can be mitigated by dividing [Equation 3.46](#) by the loss factor (f):

$$a = \frac{1}{f} \cdot \left[0.143 + \sqrt{0.0203 - 0.6427 \cdot (0.889 - C_T)} \right] \quad (3.47)$$

This modification does indeed shorten the gap between the curves, but it does not eliminate it, as plotted in [Figure 3.8](#) for non corrected BEMT and for a 0.9 corrective factor according to [Equation 3.47](#). Furthermore, as Glauert's empirical correction is modified, it does not represent the best

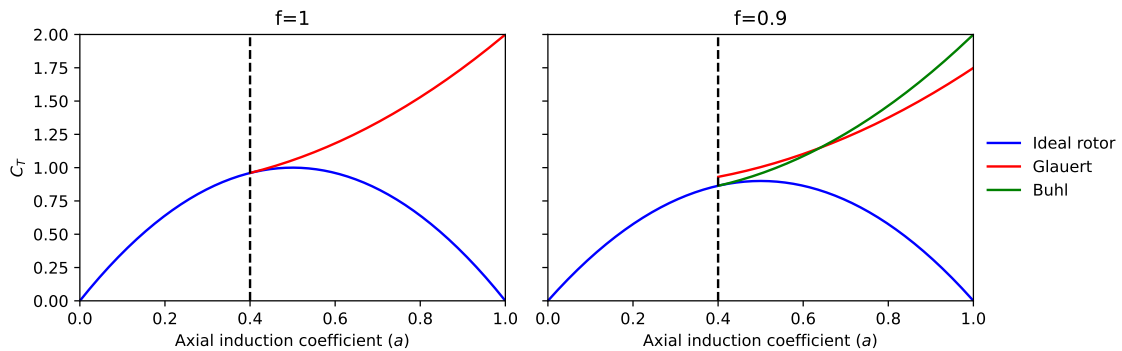


Figure 3.8: Thrust coefficient considering a turbulent wake with Glauert's and Buhl's correction with (on the right) and without (on the left) considering a loss factor.

fit to experimental data anymore. Assuming the curve was derived in order to guarantee continuity at the intersection point as well as a thrust coefficient of 2 for an unitary axial induction coefficient, [Buhl Jr \(2005\)](#) proposed to adopt a more generalised parabola, also included in [Figure 3.8](#), that respects all these conditions and given by:

$$C_T = \frac{8}{9} + \left(4 \cdot f - \frac{40}{9}\right) \cdot a + \left(\frac{50}{9} - 4 \cdot f\right) \cdot a^2 \quad (3.48)$$

Stall delay

A well known phenomenon firstly noticed in propellers is that significantly greater lift coefficients can be obtained for a rotating blade by comparison with the same blade at rest, providing equivalence of relative speeds ([Himmelskamp et al., 1947](#)). Although no definite physical explanation of this has been obtained, it is well established that the blade solidity, defined as the ratio between blades' area and the rotor ring area, is a key parameter to predict the phenomenon ([Burton et al., 2001](#)). Naturally, and mainly based on numerical simulations and experimental results, different models have been proposed over the years to represent this effect, being one of the simplest the one proposed by [Snel et al. \(1993\)](#), where the 2D statically obtained lift coefficients are modified according to:

$$C_L^{3D} = C_L^{2D} + \left(\frac{\sqrt{3} \cdot c}{r}\right)^2 \Delta C_L \quad (3.49)$$

where ΔC_L is the difference between the obtained static lift curve and the values obtained by extrapolating its linear region beyond stall, that has the important property of keeping the region before stall unaffected. More recently, other models have been proposed where both lift and drag coefficients should be corrected. Most models that correct the lift and drag directly (see [Breton et al., 2008](#), for a review and comparison) start by generalising the Snel model by replacing the $(\sqrt{3}c/r)^2$ factor by a generic function $f_{L/D}$ such that:

$$C_L^{3D} = C_L^{2D} + f_L \cdot \Delta C_L \quad (3.50a)$$

$$C_D^{3D} = C_D^{2D} + f_D \cdot \Delta C_D \quad (3.50b)$$

As an example of these models, the corrective functions proposed by [Du and Selig \(1998\)](#) are given by:

$$F_L = \frac{1}{2\pi} \cdot \left[\frac{1.6}{0.1267} \cdot \frac{c}{r} \cdot \frac{a_s - (c/r)^{\frac{d_s}{\Lambda} \frac{1}{\mu}}}{b_s + (c/r)^{\frac{d_s}{\Lambda} \frac{1}{\mu}}} - 1 \right] \quad (3.51a)$$

$$F_D = \frac{1}{2\pi} \cdot \left[\frac{1.6}{0.1267} \cdot \frac{c}{r} \cdot \frac{a_s - (c/r)^{\frac{d_s}{2\Lambda} \frac{1}{\mu}}}{b_s + (c/r)^{\frac{d_s}{\Lambda} \frac{1}{\mu}}} - 1 \right] \quad (3.51b)$$

where $\Lambda = \Omega R / \sqrt{U_0^2 + (\Omega R)^2}$ is a modified tip speed ratio and a_s , b_s and d_s are model constants that may be taken as unitary. It can be noted that the $\frac{1}{2\pi}$ factor in both lift and drag weighting functions are the slope of the lift coefficient obtained for an inviscid flow over a thin aerofoil and may be replaced by the inverse of the slope obtained from numerical simulations with viscous flows or experimental data.

In a different approach, [Eggers et al. \(2003\)](#) proposed to correct the normal and tangential coefficients rather than the lift and drag coefficients directly (see [Schmitz, 2019](#)). The corrected values are given by:

$$C_n^{3D} = C_n^{2D} + \Delta C_n^{3D} \quad (3.52a)$$

$$C_t^{3D} = C_t^{2D} + 0.12 \cdot \Delta C_n^{3D} \quad (3.52b)$$

where:

$$\Delta C_n^{3D} = \frac{1}{2} \cdot \frac{(R^2 - r^2) \cdot \lambda^2}{(1 - a)^2 + (1 + a')^2 \cdot \lambda^2} \quad (3.53)$$

Because both normal and tangential corrections are based on the same parameter (ΔC_n), a general expression relating the lift and drag corrections may be derived as:

$$C_D^{3D} = C_D^{2D} + (C_L^{3D} - C_L^{2D}) \cdot \frac{\sin \alpha - 0.12 \cdot \cos \alpha}{\cos \alpha + 0.12 \cdot \sin \alpha} \quad (3.54)$$

where C_L^{3D} can be obtained, for instance, from the Selig and Du model.

3.2.2.4 Unsteady aerodynamics effects

In the previous section, all the theories presented, in general, and BEMT expressions, in particular, have implicitly been derived based on a steady inflow, namely when the axial and angular momentum were introduced in [Equation 3.36](#). However, this is clearly not the condition a real wind turbine will experience, and some additional corrections must be used with BEMT expressions to obtain accurate results.

Skewed wake

An important effect takes place when the wind inflow is not normal to the rotor plane due to the nacelle tilt, yaw error (even a well designed turbine cannot instantaneously follow the wind rapidly changing direction and so the rotor is not perfectly aligned during relevant periods of time), vertical wind components or non uniform inflow across the rotor area, which leads to higher wind loads ([Hansen, 1992](#)). Obviously, an easy correction can be applied to the wind components, implicit in [Equation 3.32](#), that can be transformed for a tilted (θ) and skewed (ψ) rotor and pre-coned blades

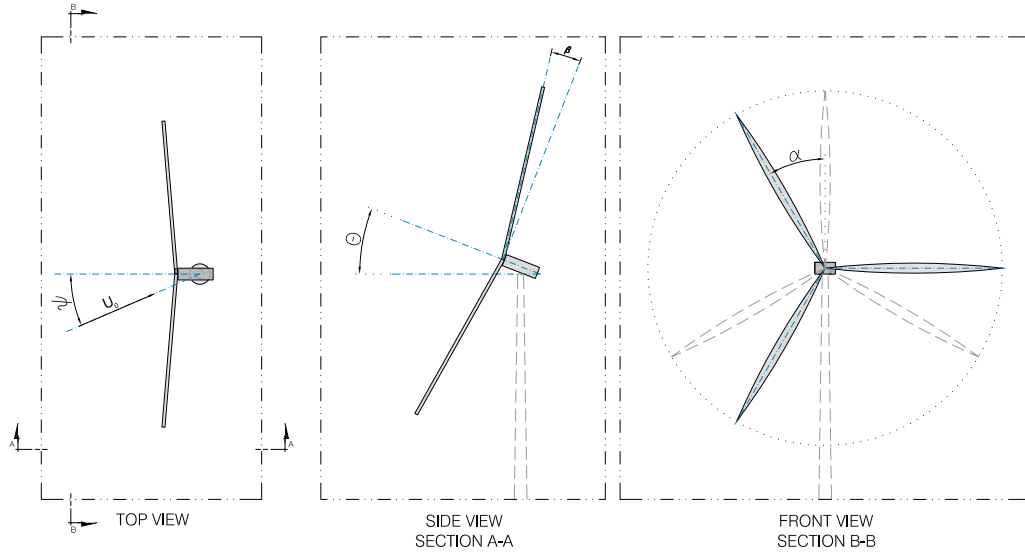


Figure 3.9: Angles convention for a blade with azimuth angle α and pre-cone β in a yawed (ψ) and tilted (θ) rotor.

(β), according to Figure 3.9, as (Ning et al., 2015):

$$U^\perp = U_0 \cdot [(\cos \psi \cdot \sin \theta \cdot \cos \alpha + \sin \psi \cdot \sin \alpha) \cdot \sin \beta + \cos \psi \cdot \cos \theta \cdot \cos \beta] \quad (3.55a)$$

$$U^\parallel = U_0 \cdot (\cos \psi \cdot \sin \theta \cdot \sin \alpha - \sin \psi \cdot \cos \alpha) \cdot \sin \beta + \Omega \cdot r \cdot \cos \beta \quad (3.55b)$$

This correction only impacts the "Blade element" part of the equations and further corrections can be applied to the momentum component as well. From Glauert (1926) autogyro studies, this can be achieved by modifying the axial induction coefficient according to a function (K) of the wake-skew angle (χ) as:

$$a_{yaw} = a \cdot \left(1 + K(\chi) \cdot \frac{r}{R} \cdot \sin \alpha \right) \quad (3.56)$$

Note that χ is the skew angle of the wake, and not of the rotor. A frequently used model to estimate it is based on the work of Pitt and Peters (1981) with a coefficient chosen accordingly to Snel and Schepers (1995) and a skew-angle estimated from the angle between the vector normal to the rotor plane and the wind vector (χ_0)⁹ as:

$$\chi = (0.6 \cdot a + 1) \cdot \chi_0 \quad \longrightarrow \quad a_{yaw} = a \cdot \left(1 + \frac{15\pi}{64} \cdot \tan \frac{\chi}{2} \cdot \frac{r}{R} \cdot \sin \alpha \right) \quad (3.57)$$

In the presented corrections it is clear that only the axial component of the velocity is being affected. A more complete analysis can be made considering the effects of a skewed wake in both angular and axial directions. As described by Burton et al. (2001), when this approach is followed, the two BEMT equations become fully coupled and must be solved simultaneously.

⁹Once more, note that χ_0 is only given directly by γ for $\theta = 0$.

Tower Shadow

While this effect is more relevant for downwind rotors, the presence of the tower also impacts the wind velocity field in the upwind region, as discussed in the numerical study of the tower shadow effect on blade loads presented by [Kim et al. \(2011\)](#). Based on a potential flow solution over a cylinder, the velocity field is most easily expressed in polar coordinates that can then be transformed to Cartesian coordinates by ([Reiso, 2013](#)):

$$u_r = \frac{U_r}{U_0} = \left(1 - \frac{R^2}{r^2}\right) \cdot \cos \theta \longrightarrow \frac{u_x}{U_0} = 1 - \frac{x^2 - y^2}{(x^2 + y^2)^2} \cdot \left(\frac{D}{2}\right)^2 \quad (3.58a)$$

$$u_\theta = \frac{U_\theta}{U_0} = -\left(1 + \frac{R^2}{r^2}\right) \cdot \sin \theta \longrightarrow \frac{u_y}{U_0} = -\frac{2 \cdot x \cdot y}{(x^2 + y^2)^2} \cdot \left(\frac{D}{2}\right)^2 \quad (3.58b)$$

where U_r and U_θ are the radial and tangential components of the velocity on an horizontal plane and u_r and u_θ their dimensionless counterpart. Furthermore, x and y the Cartesian coordinates of a reference frame within that plane and centred at the tower axis, with x aligned with the main wind direction, and D is the tower diameter. If the viscous effects are to be considered, then the velocity field can be generalised making use of drag coefficient as well ([Bak et al., 2001](#)):

$$u_x = \frac{U_x}{U_0} = 1 - \frac{x_c^2 - y^2}{(x_c^2 + y^2)^2} \cdot \left(\frac{D}{2}\right)^2 + \frac{C_d}{2\pi} \cdot \frac{x_c}{x_c^2 + y^2} \cdot \frac{D}{2} \quad (3.59a)$$

$$u_y = \frac{U_y}{U_0} = -\frac{2 \cdot x_c \cdot y}{(x_c^2 + y^2)^2} \cdot \left(\frac{D}{2}\right)^2 + \frac{C_d}{2\pi} \cdot \frac{y}{x_c^2 + y^2} \cdot \frac{D}{2} \quad (3.59b)$$

where U_x , U_y , u_x , and u_y are the velocity and its dimensionless representation along the x and y direction and the critical distance, x_c , is given by:

$$x_c = x + 0.1 \cdot \frac{D}{2} \quad (3.60)$$

While for a upwind rotor this description may be directly applied, for a downwind rotor, the potential solution is not enough to characterise the flow and one must once more rely on some kind of model to capture the wake complex behaviour. Different models can be used (see [Reiso, 2013](#), for instance) but in this work attention will be devoted to Powles' model based solutions ([Powles, 1983](#)). In his original work, Powles suggested a tower shadow model based on experimental data in which the dimensionless velocity deficit in the wake (δ_w) is a function of its width (w), maximum velocity deficit (Δ) and position:

$$U_w = U_0 \cdot (1 - \delta_w) = U_0 \cdot \left[1 - \Delta \cdot \cos^2 \left(\frac{y \cdot \pi}{2 \cdot w}\right)\right] \quad (3.61)$$

in which the model parameters (w and Δ) must be obtained from experimental data or empirical models (in [Powles, 1983](#), these quantities are given for different tower radius by graphical means). Because the wake width increases with the distance to the tower, an expression for the wind speed deficit based on Powles' model and on the radial distance to the tower centre $d = \sqrt{x^2 + y^2}$ can

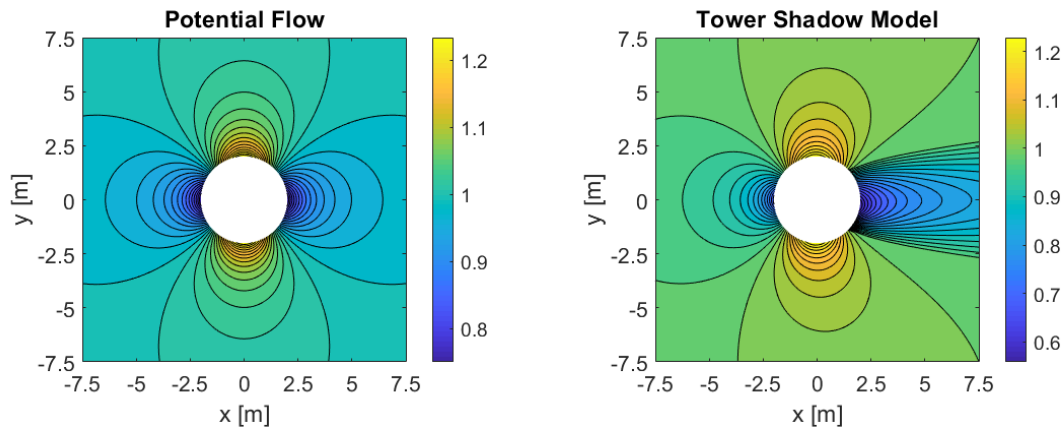


Figure 3.10: Velocity field derived from a potential flow (on the left) and with a tower shadow model (on the right) for a unitary radius tower.

be written as (Moriarty and Hansen, 2005):

$$\delta w = \frac{C_D}{\sqrt{d}} \cos^2 \left(\frac{\pi}{2} \cdot \frac{y}{\sqrt{d}} \right) \quad , \text{ if } |y| \leq \sqrt{d} \quad (3.62)$$

The final local wind velocity for the downwind region is then obtained combining Bak's model with Equation 3.62 as:

$$U_x = (u_x - \delta_w) \cdot U_0 \quad (3.63a)$$

$$U_y = (u_y - \delta_w) \cdot U_0 \quad (3.63b)$$

where u_x and u_y are obtained from Equation 3.59. The velocity field derived from a potential flow and considering the described tower shadow model can be seen in Figure 3.10, where it was considered a drag coefficient of 0.5.

Dynamic stall

As was already seen, even in steady flows, stall occurs in conditions that are significantly different from the ones obtained in static conditions. Stall effects are a rather complex phenomenon but with very important consequences on the loads experienced by blades in wind turbines. Either because the wind field itself is rapidly changing or because the blade crosses different regions of a non uniform field, rapid changes in angle of attack as perceived by the blade cause a sudden detachment and then reattachment of the flow along the aerofoil, leading to strong transient effects and greater fatigue loads. As detailed by McCroskey (1981), although there is a small amount of unsteadiness for any flow over an aerofoil, specially at higher angles of attack, when the aerofoil motion itself is unsteady, this phenomenon is much more complex and, once more, as no theoretical solution for dynamic stall is known, engineering models are called to scene, usually expressed as an ordinary

differential equation in time such that (Snel, 2004):

$$\tau \cdot \frac{dC_L}{dt} + f(\alpha) \cdot C_L = g\left(C_L, \alpha, \frac{d\alpha}{dt}, \frac{d^2\alpha}{dt^2}\right) \quad (3.64)$$

being τ is a characteristic time.

A comparison of different dynamic stall models can be found in (Björck, 1996; Johansen, 1999; Holierhoek et al., 2013; Khan, 2018), but only the formulation introduced by Beddoes (1976), later systematized by a cooperative paper between Leishman and Beddoes (1986) and revised in (Leishman and Beddoes, 1989), from where it become known as **Leishman-Beddoes** dynamic stall model, will be detailed here. Although the Leishman-Beddoes model was originally developed for helicopter applications, where compressible effects do take an important role, it has been successfully adapted and applied to incompressible conditions by Pierce (1996). This model requires the static aerofoil data for normal force and pitching moment to be known and can be decomposed in three distinct sub-models (Pereira, 2010):

- A model for the fully attached, inviscid flow;
- A model for the separated flow, including leading and trailing edge flow separation;
- A model for the detachment and convection of vortex.

The first step towards obtaining the unsteady lift force is to compute the dynamic attached flow response based on the potential solution derived by Theodorsen (1949). This response is modified according to the estimated separation point from Kirchoff flow theory (Thwaites and Meyer, 1960) and the vortex buildup and shedding effects are finally included (Pierce, 1996). Without further details, it should be mentioned that this model relies on the definition of different parameters that have been studied and optimised by Björck et al. (1999).

Dynamic Inflow

Finally, the wind inflow itself changes over time, even at a rotor scale, and so does the induced velocity fields. As this process does not occur instantaneously, there is a time lag between both phenomenon (Yu, 2018). On one hand, it can be assumed that the wake remains unaltered (frozen) in accordance to the mean wind conditions. On the other hand, it could be considered that the wake reacts instantaneously to the changes of the wind speed and so do the induced velocities. Having in mind these two scenarios, it is clear that the real situation falls somewhere in the middle and, to overcome this difficulty without turning into more complex and computationally demanding models, some dynamic inflow models have been developed over the years that can be used alongside with BEM theory. A comparison of several methods, including the Pitt and Peters (1981) model, the Øye (1986) dynamic model and ECN differential equation model (Schepers, 2012), can be found in (Snel and Schepers, 1992) and further detailed in (Snel and Schepers, 1995). As a final note, it should be mentioned that the platform motions of floating wind turbines introduce a new challenge in the definition of these dynamic inflow models, but recent studies have shown that even in these cases the BEMT framework may be extended and still provide accurate results (Ferreira et al., 2022).

3.3 Waves loading and hydrodynamic response

3.3.1 Actions over free floating structures

In Chapter 2, the theoretical characterisation of a marine flow was presented in its general form. However, the ultimate purpose of that derivation is to provide the tools to evaluate the forces acting on a floating body and its consequent motions, that will be described in accordance with Figure 3.11. Because these will be used intensively for the rest of this work, it is convenient to associate each degree of freedom to its most common designation. The translation movements along the x_b , y_b and z_b axis, defined in Figure 3.11 for a semi-submersible solution, will be referred to as **surge**, **sway** and **heave** motions, respectively, while the rigid body rotations over the same orthogonal axis will be referred to as **roll**, **pitch** and **yaw** motions.

To gain some insight into the relevance of the different wave parameters in the wave forces felt by a floating body, Chakrabarti (1987) argued that, in general, waves force (F_w) acting on a structure of characteristic dimension D can be defined as a function of time, wave period, wave length and maximum water particle velocity (u_0)¹⁰, $F_w = F_w(D, t, T, \lambda, u_0; \nu, \rho)$. This can also be written in a non-dimensional form using a dimensionless time, the Keulegan-Carpenter parameter KC , the Reynolds number (Re) and a diffraction parameter:

$$\frac{F_w}{\rho u_0^2 D} = F_w \left(\frac{t}{T}, \frac{u_0 T}{D}, \frac{u_0 D}{\nu}, \frac{\pi D}{\lambda} \right) = F_w \left(\frac{t}{T}, KC, Re, \frac{\pi D}{\lambda} \right) \quad (3.65)$$

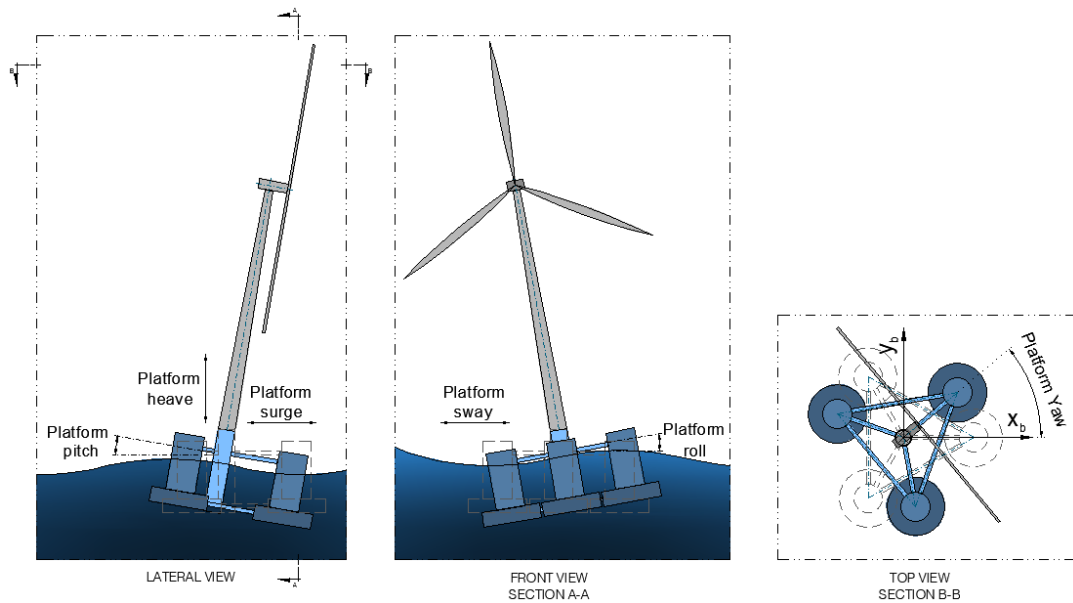


Figure 3.11: Rigid body degrees of freedom of a floating offshore wind turbine.

¹⁰In the mentioned work it is considered that the wave force also depends on the fluid density (ρ) and viscosity (ν), but as only water in incompressible conditions will be considered, these will be assumed constant and so as input parameters of the function rather than variables.

The KC parameter and the diffraction parameters are useful to evaluate the importance of drag forces and diffraction forces, respectively, and, for a given sea state, they can be seen to be inversely proportional. The relative values of these parameters also determines the best way to tackle wave forces for a given structure, and there are 3 principal approaches that are usually followed:

- When the drag forces are relevant and the characteristic dimension is very small by comparison with the wave length (slender structures), the Morison equation is used;
- When the drag forces are small, and the inertia forces dominate, but the characteristic dimension is still reasonably small by comparison with the wave length, Froude-Krylov forces may be used;
- When the structure size is not negligible anymore in relation to the wave length, then the wave field itself is expected to be affected and a full potential theory considering the radiation and diffraction effects must be taken into account.

These different situations will be addressed as the wave forces for small and large structures are characterised in the following sections.

3.3.1.1 Wave forces over slender structures

If the characteristic dimension of a given body is very small when compared to the wave length, the flow is mainly unperturbed by its presence. Recovering [Equation 2.29](#), the water particle velocities can be obtained by [Equation 2.19](#) and simplified using the dispersion relation as:

$$u_x = \frac{\eta_0 g k}{\omega} \cdot \frac{\cosh(kd + kz)}{\cosh(kd)} \cdot \cos(kx - \omega t) = \eta_0 \cdot \omega \cdot \frac{\cosh(kd + kz)}{\sinh(kd)} \cdot \cos(kx - \omega t) \quad (3.66a)$$

$$u_z = \frac{\eta_0 g k}{\omega} \cdot \frac{\sinh(kd + kz)}{\cosh(kd)} \cdot \sin(kx - \omega t) = \eta_0 \cdot \omega \cdot \frac{\sinh(kd + kz)}{\sinh(kd)} \cdot \sin(kx - \omega t) \quad (3.66b)$$

If the structure is small enough, the flow variations over its characteristic length are negligible, implying that the argument of the non hyperbolic trigonometric function can be treated as an explicit time dependency and a phase angle. Since it carries no physical meaning, the latter can be removed from the analysis. Furthermore, assuming the structure to be in the upright position, no vertical forces will develop, besides the buoyancy and the structure self-weight, and the kinematics of the unperturbed flow are fully defined by:

$$u_x(z, t) = \eta_0 \cdot \omega \cdot \frac{\cosh(kd + kz)}{\sinh(kd)} \cdot \cos(\omega t) = u_a(z) \cdot \cos(\omega t) \quad (3.67a)$$

$$\dot{u}_x(z, t) = -\omega \cdot u_a \cdot \sin(\omega t) \quad (3.67b)$$

where the elevation dependence has been implicitly included in $u_a = u_a(z)$.

Although the exact expressions of fluid velocities and accelerations are not important for the proceeding derivations, the assumption that the flow can be described by its value at a given point representing the slender body will be held. For the sake of simplicity, a cylinder with diameter

$D = 2R$ will be assumed to be the structure under study. If the pressure gradient were not affected by the body's presence, than the force acting per unit length would be given by:

$$\begin{aligned} F_{I,1}(t) &= 2 \cdot \int_0^\pi p(R, \theta, t) \cdot R \cdot \cos(\theta) \cdot d\theta = 2 \cdot \int_0^{\frac{\pi}{2}} \frac{dp}{dx} \delta x \cdot R \cdot \cos(\theta) \cdot d\theta = \\ &= 2R \cdot \rho \cdot \dot{u} \cdot \int_0^{\frac{\pi}{2}} \delta x \cdot \cos(\theta) \cdot d\theta = 2R \cdot \rho \cdot \dot{u} \cdot \int_0^{\frac{\pi}{2}} 2R \cdot \cos^2(\theta) \cdot d\theta = \\ &= \rho \pi R^2 \cdot \dot{u} \end{aligned} \quad (3.68)$$

where the position increment has been written as $\delta x = 2R \cos \theta$ and the pressure gradient has been obtained from the Bernoulli equation as:

$$\frac{dp}{dx} = \rho \cdot \dot{u} \quad (3.69)$$

This force, that is present as long as there is a pressure gradient, is, by definition, nothing more than a Froude-Krylov force¹¹. However, as the body itself is not permeable, the fluid particles must accelerate and decelerate in the vicinity of the body, a process that leads to an additional force. By comparison of perturbed and unperturbed flow kinetic energies, it can be shown (Lamb, 1993; Choudhuri, 1998) its magnitude to be numerical equal to the Froude-Krylov force so that the total inertia force for a cylinder is given by:

$$F_I = F_{I,1} + F_{I,2} = 2 \times [\pi \cdot R^2 \cdot \rho \cdot \dot{u}(t)] \quad (3.70)$$

Equation 3.70 can be extended and generalised to an arbitrary shape, assuming it holds for different conditions, by introducing an inertia coefficient (C_M) as the combination of a Froude-Krylov coefficient (generally it is considered the exposed derivations hold for the Froude-Krylov forces, even for viscous fluids) and an added mass term (C_a) that must be carefully chosen:

$$C_M = C_{FK} + C_a = 1 + C_a \longrightarrow F_I = C_M \times [\pi \cdot R^2 \cdot \rho \cdot \dot{u}(t)] \quad (3.71)$$

Inertia forces however, are not the only forces felt by a submerged body. For a potential, inviscid flow, a solid body within it would not experience any drag force, leading to the d'Alembert paradox already introduced in Section 3.2. For a real viscous flow, experiments have shown that for a steady current with velocity U_c , a cylinder with diameter D will also experience an inline force with intensity given by:

$$F_D = \frac{1}{2} \cdot \rho \cdot |U_c| U_c \cdot C_D \cdot D \quad (3.72)$$

where C_D is, as in Section 3.2, the drag coefficient and the quadratic term has been made explicit to guarantee the drag force has always the same direction as the velocity.

¹¹Froude-Krylov are by definition calculated from the pressures of a non perturbed flow and can be defined in general by the integral of the pressure field over the submerged area (S): $f_{FK} = - \int_S p \cdot n \cdot dS$.

The general formulation that is widely used for slender bodies was proposed by Morison assuming that the inertia forces obtained from the potential model and the drag forces measured in real fluids could be superimposed and still provide accurate results. With this assumption, the total force on a submerged body with characteristic dimension D and cross sectional area S in a moving flow is given by the so-called **Morison equation**:

$$F(t) = F_D(t) + F_I(t) = \underbrace{\frac{1}{2} \cdot \rho \cdot C_D \cdot D \cdot u(t) \cdot |u(t)|}_{\text{Drag term}} + \underbrace{\rho \cdot C_M \cdot S \cdot \dot{u}(t)}_{\text{Inertia term}} \quad (3.73)$$

It should be noted that although the added mass force happens to be numerically equal to the Froude-Krylov counterpart in this example, their relative importance actually depends on the dynamics of the body, a distinction that becomes clear when two apparently similar situations with drastically different outcomes are considered:

- Forces acting on a cylinder hold still in an accelerated flow;
- Forces acting on a moving cylinder in still waters.

While for the first scenario both components are present and the inertia coefficient is, in theory at least, equal to 2, in the second scenario the Froude-Krylov is identically equal to 0 as there is no pressure gradient. Because of this different nature of the two components in the inertia force, [Equation 3.73](#) must be adapted when the body itself is moving in the inline direction with a given velocity (u_b) and rewritten as:

$$F(t) = \underbrace{\frac{1}{2} \cdot \rho \cdot C_D \cdot D \cdot (u - u_b) \cdot |u - u_b|}_{\text{Drag term}} + \underbrace{\rho \cdot C_a \cdot S \cdot (\dot{u} - \dot{u}_b)}_{\text{Added mass term}} + \underbrace{\rho \cdot S \cdot \ddot{u}}_{\text{FK term}} \quad (3.74)$$

where the temporal dependence of the fluid velocity and acceleration has now been omitted.

Finally it is important to remember that the Morison equation is an empirical formulation with some limitations as it assumes the inertia and drag forces obtained for the limit cases of very small and very high KC numbers, respectively, may be superimposed to intermediate conditions ([Sarpkaya, 2010](#)). Additionally, it also implicit assumes that the structure characteristic dimension is small when compared to the wavelength, such that the potential flow is mostly unperturbed. When this is not case, the Morison equation is not capable of providing adequate estimates for the hydrodynamic forces, as becomes clear in the numerical example presented in [Section B.1](#). In the next section, this assumptions will be relaxed.

3.3.1.2 Wave forces over large bodies

If in the previous section expressions for the force acting on slender structures were derived based on the assumption that the presence of the body does not affect significantly the wave field, this is no longer true if the structure dimensions are not small when compared to the wave length, and a full potential formulation is required.

Full potential formulation

If the body's motions are restrained, then a scattered potential (Φ_d) must be added to the unperturbed incident potential (Φ_0) such that:

$$\partial_n \Phi_D = \partial_n \Phi_0 + \partial_n \Phi_d = 0 \longrightarrow \partial_n \Phi_0 = -\partial_n \Phi_d \quad (3.75)$$

which is nothing more than the generalisation of the sea bed impermeability condition (Equation 2.26) to any solid body boundary defined by its normal direction n and where $\Phi_D = \Phi_0 + \Phi_d$ is usually referred to as diffraction potential. Because the unperturbed wave potential does not, in general, satisfy this condition, a diffracted wave is induced.

These potentials can still be superimposed with the effect of the body motions in still water and provide the most general potential formulation. In this case, the wave field will result from the superposition of an incident wave, a diffracted wave and a **radiated wave** due to motion of the body. The corresponding potential may then be written as:

$$\Phi = \Phi_0 + \Phi_d + \sum_{j=1}^6 \Phi_j = \Phi_0 + \Phi_d + \Phi_r \quad (3.76)$$

where Φ_0 is the incident unperturbed wave potential, Φ_d is the scattered wave potential about the **restrained** body and Φ_r is the radiation potential from the oscillatory body's motions in otherwise **still** water, written from the superposition of the potential associated with each one of the degrees of freedom identified in Figure 3.11, Φ_j .

In the context of linear theory, these potentials can be decomposed by separation of variables in a space dependent term and an harmonic term, leading to the more compact representation introduced by Tuck (1970):

$$\Phi(x, y, z, t) = -i\omega \sum_{j=0}^7 \phi_j \cdot a_j \cdot e^{-i\omega t} \quad (3.77)$$

where a_j is the complex amplitude of the j^{th} potential, the 0 and 7 indexed terms are associated with the unperturbed and diffracted waves, respectively, and indexes 1 to 6 are the same as in Equation 3.76. From this, it is implied that the velocity potentials inside the summation operator are defined for unitary wave amplitude or body displacement. The incident and unperturbed wave can be generalised from Equation 2.29 as¹²:

$$\Phi_0 = -i\omega \cdot \eta_0 \cdot \phi_0 \cdot e^{i\omega \cdot t} = -i\omega \cdot \eta_0 \frac{g}{\omega^2} \cdot \frac{\cosh(k \cdot d + k \cdot z)}{\cosh(k \cdot d)} \cdot e^{ik \cdot (x \cdot \cos \mu + y \cdot \sin \mu)} \cdot e^{-i\omega t} \quad (3.78)$$

where μ is the incident wave direction. Expressing the body motions in the complex plane ($\hat{x}(t) = \mathbf{x}e^{-i\omega t}$), the pressure can be obtained directly from the Bernoulli equation applied to the total

¹²The incident complex potential is written in such a way that its real part recovers Equation 2.29 for $\mu = 0$.

potential:

$$p(x, y, z, t) = -\rho \partial_t \Phi = \rho \omega^2 \left[\eta_0 \cdot (\phi_0 + \phi_7) + \sum_{j=1}^6 \phi_j \cdot x_j \right] \cdot e^{-i\omega t} \quad (3.79)$$

The forces acting on the structure can then be separated into first order wave exciting forces¹³ (X_k) and the oscillating hydrodynamic forces (F_k) originated by the moving body:

$$X_k = - \int_S p \cdot n_k \cdot dS = -\rho \omega^2 \cdot \eta_0 \cdot e^{-i\omega t} \int_S (\phi_0 + \phi_7) \cdot n_k \cdot dS \quad (3.80a)$$

$$F_k = - \int_S p \cdot n_k \cdot dS = -\rho \omega^2 \cdot e^{-i\omega t} \cdot \sum_{j=1}^6 \left(x_j \int_S \phi_j \cdot n_j \cdot dS \right) \quad (3.80b)$$

where the integrations are carried over all the wetted surface and n_i are the direction cosines of the normal surface vector $\mathbf{n}$ and the different axis unitary vectors:

$$n_1 = \hat{\mathbf{n}} \cdot \hat{\mathbf{x}}, \quad n_2 = \hat{\mathbf{n}} \cdot \hat{\mathbf{y}}, \quad n_3 = \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \quad (3.81a)$$

$$n_4 = \hat{\mathbf{n}} \times \mathbf{r}|_x, \quad n_5 = \hat{\mathbf{n}} \times \mathbf{r}|_y, \quad n_6 = \hat{\mathbf{n}} \times \mathbf{r}|_z \quad (3.81b)$$

where $\mathbf{r}$ is the position of a given point when expressed in a reference frame centred at the structure centre of gravity. If the radiation potential is expressed from its real and imaginary part in Equation 3.80, the forces can be decomposed in a term in phase with the body acceleration and a term proportional to the body's velocity, which suggests a definition of an **added mass** (a_{kj}) and **damping** (b_{kj}) coefficients, respectively given by:

$$a_{kj} = -\Re \left[\rho \int_S \phi_j \cdot n_k \cdot dS \right] \quad (3.82a)$$

$$b_{kj} = -\Im \left[\omega \cdot \rho \int_S \phi_j \cdot n_k \cdot dS \right] \quad (3.82b)$$

As a final note, it should be mentioned that the description above tacitly assumes a vanishing forward speed of the structure. If this is not case, but if the velocities are still small, an approximated solution can be found by replacing the time dependence of the potential functions by an effective frequency of encounter ω_e given by:

$$\omega_e = \omega - kV \cos \beta \quad (3.83)$$

where k is the wave number of the incident wave, V is the structure forward speed, deviated of an angle β from the wave incident direction. Further details of an implementation of this approach can be found in (ANSYS, 2017), but in the derivations that will now be presented it will be assumed that $V \sim 0$.

¹³The term forces is used as a generalised concept and can be referring to both forces as to torques.

Solving the potentials

From Equation 3.80 and Equation 3.82 it is clear that the forces can be calculated as long as the different potentials are known. While the incident wave potential has already been introduced, the remaining potential functions can be obtained from a continuous distribution of sources over the wetted body surface according to Lamb (1993):

$$\phi_j(x, y, z) = \frac{1}{4\pi} \int_S \sigma_j(\hat{x}, \hat{y}, \hat{z}) \cdot G(x, y, z, \hat{x}, \hat{y}, \hat{z}) \cdot dS \quad (3.84)$$

where the coordinates identified by an $\hat{\cdot}$ are integration variables, meaning that σ_j is the complex **source strength** of a generic point $(\hat{x}, \hat{y}, \hat{z})$ and G is **Green's function** (or influence function) of the source function with respect to the potential evaluated on another arbitrary point (x, y, z) and that satisfies Laplace's equation, the linearized boundary conditions on both the sea bed and on the free surface and the radiation condition at infinity (see, for instance John, 1950; Wehausen et al., 1960, for alternative representations of the frequency domain pulsating Green's function in finite depth water). The source strengths can be obtained based on the impermeability condition on the body surface and taking the derivative of Equation 3.84 with respect to the normal direction, it is possible to write:

$$\partial_n \phi_j = -\frac{1}{2} \sigma_j + \frac{1}{4\pi} \int_S \sigma_j \cdot \partial_n G \cdot dS \quad (3.85)$$

For the 6 degrees of freedom, Equation 3.85 must correspond to the body motions evaluated at each point, which is nothing more than the direction cosines defined in Equation 3.81. For the diffraction potential, use can be made of Equation 3.75 to provide the final relation from the known incident potential. Once the potential functions are known, Bernoulli's equation can be used to provide the pressure at any given point of space.

Most available software, like ANSYS or WAMIT, use the so-called **Boundary Element Method** (BEM). In this case, the mean wetted surface is divided into quadrilateral (or triangular) panel elements and, assuming a constant source strength within each panel equal to its average value, the integral Equation 3.85 must be replaced by their discrete counterparts, where the contribution of the m^{th} panel is (ANSYS, 2017):

$$\partial_{n_m} \phi_j = -\frac{1}{2} \sigma_{j,m} + \frac{1}{4\pi} \sum_{n \neq m} \sigma_{j,n} \partial_{n_m} G_{mn} S_n \quad (3.86)$$

where N is the number of panels modelled, S_n is the area of the n^{th} panel and G_{mn} is evaluated between the points (x_m, y_m, z_m) and (x_n, y_n, z_n) .

A final remark must be made concerning a particular relevant numerical issue. Equation 3.86 can be written in matrix form as:

$$\begin{bmatrix} A_{11} & \dots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{N1} & \dots & A_{NN} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{1j} \\ \vdots \\ \sigma_{Nj} \end{bmatrix} = \begin{bmatrix} n_{1j} \\ \vdots \\ n_{Nj} \end{bmatrix} \quad (3.87)$$

If the determinant of matrix $\mathbf{A}$ is zero then the solution of Equation 3.87 is undetermined, which does happen for a finite number of frequencies. Many of this **irregular frequencies** are related to the formation of stationary waves within the solid bodies that, although possess no physical meaning, respect all the boundary conditions already defined (Du et al., 2011). Refining the model with smaller length scales can help reduce this problem, but will not eliminate it. A practical solution can however be obtained by modelling a planar surface - lead - at the still water level within the solid body that will not alter the solution outside it, but will prevent the formation of these stationary waves.

Viscosity effect

Although potential flow models can provide accurate results for a wide range of phenomenon, they are not able to fully reproduce a solid body behaviour in a real viscous flow. This effect impacts not only the experienced drag forces, but also the added mass coefficients obtained from potential theory alone (see, for instance, Ramesh Reddy et al., 2022). However, and in the same spirit of the Morison equation derivation, it is possible to mitigate these differences by explicitly including additional coefficients associated to different elements. For instance, the viscous damping forces are often modelled by adding a linear or quadratic damping matrix relating all the degrees of freedom, that may be written as:

$$\mathbf{f}_{Drag} = -\mathbf{B}_{Drag} \cdot |\dot{\mathbf{x}}_r| \cdot \dot{\mathbf{x}}_r \quad (3.88)$$

where the subscript r has been intentionally added as a reminder that this forces arise whenever there is a relative motion between the structure and the fluid. The quadratic damping matrix coefficients must be carefully chosen, most of the times, based on experimental data or empirical expressions and their value can have some dependency on structural motions and derivatives, leading to non linear viscous forces.

Second order wave drift forces

Until now only the first order effects resulting from linear theory have been considered, but it can be shown that even from a first order potential, some second order phenomenon can be exactly obtained. The total force over a structure is once more given by:

$$\mathbf{f} = \int_S p \hat{\mathbf{n}} dS \quad (3.89)$$

where the domain of integration S is the instantaneous wetted surface and extends from the sea bed to the still water level (S_0) and includes an oscillatory part up to the free surface elevation (S_1) (Pinkster, 1980). The normal direction vector and pressure forces must be considered up to second order expansions based on a perturbation theory to guarantee a correct second order solution is also

obtained:

$$\mathbf{f} = \int_{S_0 + \varepsilon S_1} (p_0 + \varepsilon p_1 + \varepsilon^2 p_2) \cdot (\hat{\mathbf{n}} + \varepsilon \hat{\mathbf{n}}_1) dS = \mathbf{f}_0 + \varepsilon \mathbf{f}_1 + \varepsilon^2 \mathbf{f}_2 + \mathcal{O}(\varepsilon^3) \quad (3.90)$$

If the forces components are expanded, the still not introduced hydrostatic (0^{th} order), first order oscillatory fluid force and the intended second order fluid force are obtained as:

$$\mathbf{f}_0 = \int_{S_0} p_0 \cdot \hat{\mathbf{n}} dS \quad (3.91a)$$

$$\varepsilon \mathbf{f}_1 = \int_{S_0} p_0 \cdot \varepsilon \hat{\mathbf{n}}_1 dS + \int_{S_0} \varepsilon p_1 \cdot \hat{\mathbf{n}} dS + \int_{\varepsilon S_1} p_0 \cdot \hat{\mathbf{n}} dS \quad (3.91b)$$

$$\varepsilon^2 \mathbf{f}_2 = \int_{S_0} \varepsilon p_1 \cdot \varepsilon \hat{\mathbf{n}}_1 dS + \int_{S_0} \varepsilon^2 p_2 \cdot \hat{\mathbf{n}} dS + \int_{\varepsilon S_1} p_0 \cdot \varepsilon \hat{\mathbf{n}}_1 dS + \int_{\varepsilon S_1} \varepsilon p_1 \cdot \hat{\mathbf{n}} dS \quad (3.91c)$$

Assuming now that the motion is dominated by first order forces, Bernoulli's equation provides an expression for the different order pressure terms:

$$p_0 = -\rho g z_0 \quad (3.92a)$$

$$p_1 = -\rho g z_1 - \rho \partial_t \phi_1 \quad (3.92b)$$

$$p_2 = -\frac{1}{2} \rho (\nabla \phi_1)^2 - \rho \partial_t \phi_2 - \rho [\mathbf{x} \cdot \nabla (\partial_t \phi_1)] \quad (3.92c)$$

Although the complete derivation of the forces presented can be found in [M J Journée and W Massie \(2001\)](#), only an important result will be detailed here. If all the perturbations are supposed to be of the form $e^{i\omega t}$, then it follows that all the mean value of the quantities dependent on the second order potential vanish and the second order mean forces can be correctly computed from the linear theory potential and pressures. For simplicity, it will be assumed that body motions are restrained, at least heave, pitch and roll motions, such that the expansion of the normal vector is not needed. Under this assumption, and considering an infinite draft stationary wall, then the only non zero component of the drift force is in the surge direction and is given by:

$$\mathbf{f}_2 = \left[-\rho \int_{-\infty}^0 \partial_t \phi_2 \cdot dz - \frac{1}{2} \cdot \rho \int_{-\infty}^0 (\nabla \phi_1)^2 \cdot dz - \rho \int_0^\eta (\partial_t \phi_1 + g \cdot z) \cdot dz \right] \hat{\mathbf{x}} \quad (3.93)$$

Taylor expanding the last integral and taking its mean value, the second order drift force is given by:

$$\overline{\mathbf{f}_2} = \left[-\frac{1}{2} \rho \int_{-\infty}^0 (\nabla \phi_1)^2 \cdot dz + \frac{1}{2} g \rho \eta^2 \right] \hat{\mathbf{x}} \quad (3.94)$$

where the boundary condition from [Equation 2.28](#) has been used. Because of this second order effect from the linear theory potential alone, a floating body drifts away over time, as can be found in the example presented in [Section B.2](#) for a particular simple case.

3.3.1.3 Hydrostatic forces

Until this point special attention was given to the wave forces acting over a given structure but an important component has been systematically left out of the analysis, that is present even in still waters, and must now be recovered. In a stationary fluid, it follows directly from either the Euler equation or Bernoulli principle that the pressure evolution is such that:

$$p_{hs} = p_0 - \rho \cdot g \cdot z \quad (3.95)$$

where z is positive in the upward direction and the hs underscript stands for hydrostatic. For a given arbitrary displacement $\mathbf{x}$ a restoring force appears such that $\mathbf{f} = -\mathbf{K}_{hs}\mathbf{x}$. The hydrostatic stiffness matrix components are given for a rigid body whose wet surface is S_b and buoyancy centre has coordinates (x_b, y_b, z_b) by:

$$K_{i1} = K_{i2} = K_{i3} = K_{i4} = K_{i5} = K_{i6} = 0 \quad , \quad \text{for } i \in [1, 2] \quad (3.96a)$$

$$K_{33} = \rho g \int_{S_b} n_3 dS_b \quad (3.96b)$$

$$K_{34} = \rho g \int_{S_b} y \cdot n_3 dS_b \quad , \quad K_{35} = -\rho g \int_{S_b} x \cdot n_3 dS_b \quad (3.96c)$$

$$K_{44} = \rho g \int_{S_b} y^2 \cdot n_3 dS_b + \rho g V z_b - mg z_g \quad (3.96d)$$

$$K_{45} = -\rho g \int_{S_b} x \cdot y \cdot n_3 dS_b \quad , \quad K_{46} = -\rho g \cdot V \cdot x_b + mg x_g \quad (3.96e)$$

$$K_{55} = \rho g \int_{S_b} x^2 \cdot n_3 dS_b + \rho g V z_b - mg z_g \quad (3.96f)$$

$$K_{54} = -\rho g \int_{S_b} x \cdot y \cdot n_3 dS_b \quad , \quad K_{56} = -\rho g \cdot V \cdot y_b + mg y_g \quad (3.96g)$$

If the submerged platform has one or more planes of symmetry, the presented equations can be simplified but the important aspect to retain is that the hydrostatic stiffness only provides stability to heave, pitch and roll motions, leaving surge, sway and yaw motions unrestrained.

3.3.2 Mooring forces

The previous section ended with the conclusion that the hydrostatic action over a floating body only provides stability for certain degrees of freedom. As surge, sway motions and yaw are left unrestrained, a proper mooring system should be designed to control these motions. Although the currently available computational power does not seem to justify the use of simplified models, these can be useful to understand the global behaviour of a cable element and so a brief description will be made before introducing the full dynamic equations.

Quasi static formulation

A simple way to obtain the forces transmitted by a mooring system to a floating platform from the knowledge of the latter motions is to apply the so called **quasi static** formulation. In this context,

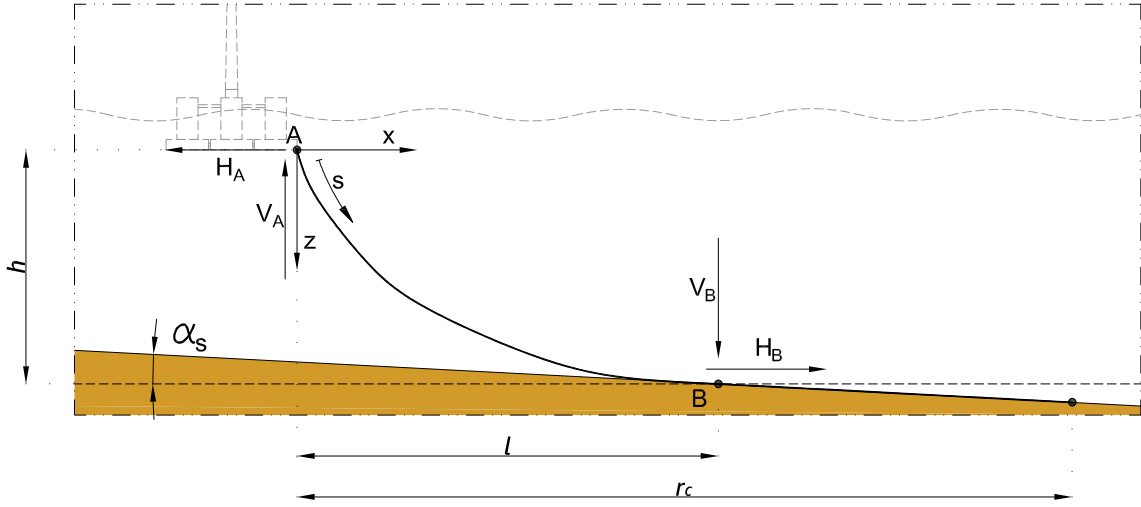


Figure 3.12: Schematic representation of the forces transmitted to the suspended cable segment.

the cable is assumed to be in static equilibrium at every time step and so no inertia or damping terms (that depend on the acceleration and velocity) are considered.

Although different cable models can be applied with different degrees of refinement, from a simple linear (or non linear) spring to the elastic catenary, only the latter will be briefly explored. The parametric equations of such a cable contained in the xz plane, being the x -axis an horizontal axis, z -axis a vertical oriented one, and the associated internal axial force (T), can be written as a function of the generalised coordinate s measured along the cable axis as (Caetano, 2007):

$$x(s) = \frac{H_A s}{EA_0} + \frac{H_A}{mg} \left[\operatorname{arcsinh} \left(\frac{V_A}{H_A} \right) - \operatorname{arcsinh} \left(\frac{V_A - mgs}{H_A} \right) \right] \quad (3.97a)$$

$$z(s) = \frac{s}{EA_0} \left(V_A - \frac{mgs}{2} \right) + \frac{H_A}{mg} \left[\sqrt{1 + \left(\frac{V_A}{H_A} \right)^2} - \sqrt{1 + \left(\frac{V_A - mgs}{H_A} \right)^2} \right] \quad (3.97b)$$

$$T(s) = \sqrt{H_A^2 + (V_A - mgs)^2} \quad (3.97c)$$

where EA_0 is the cable's stiffness, m the cable's mass per unit length, and V_A and H_A the reaction forces at the upper fairlead, as represented in Figure 3.12. Assuming the vertical distance (h) between the seabed and the top fairlead is known, a new set of equations can be derived as a function of the tension force components in the horizontal and vertical directions at the fairlead, in the platform, and the horizontal distance to the contact point l (Caetano, 2007):

$$l = x(s = L_0) = L_0 \left\{ \frac{H_A}{EA_0} + \frac{H_A}{W} \left[\operatorname{arcsinh} \left(\frac{V_A}{H_A} \right) - \operatorname{arcsinh} \left(\frac{V_A - W}{H_A} \right) \right] \right\} \quad (3.98a)$$

$$h = z(s = L_0) = L_0 \left\{ \frac{2V_A - W}{2EA_0} + \frac{H_A}{W} \left[\sqrt{1 + \left(\frac{V_A}{H_A} \right)^2} - \sqrt{1 + \left(\frac{V_A - W}{H_A} \right)^2} \right] \right\} \quad (3.98b)$$

where the suspended cable unrestrained length L_0 and total weight W have been introduced. It

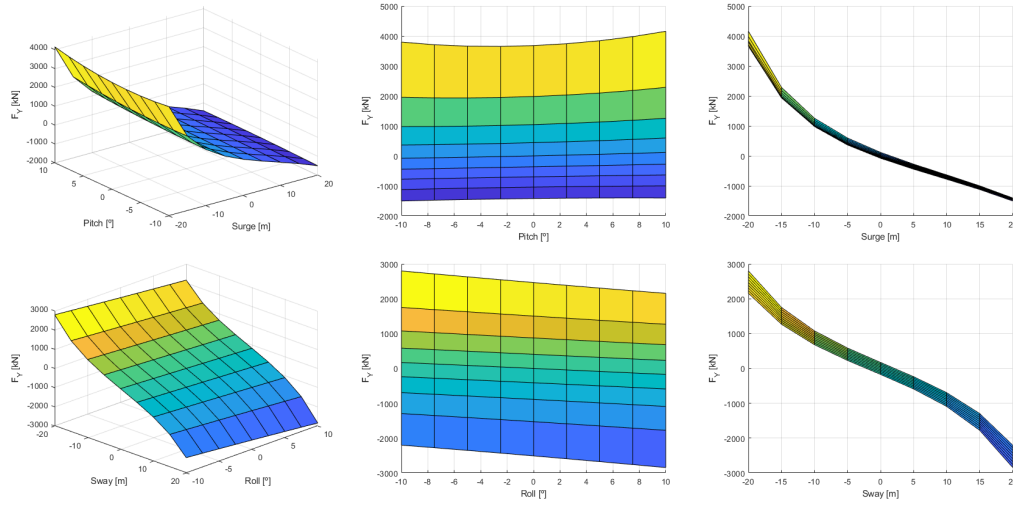


Figure 3.13: Total force transmitted by the mooring system to a structure as a function of the displacement **surge-pitch** (on top) and **sway-roll** (on bottom).

should be noted that in the case of mooring lines, a significant part of the cable is laid directly on the seabed, implying that l cannot be known *a priori*, and an additional closure relation is needed given by the compatibility of slopes at the seabed:

$$\alpha_s = \arctan\left(\frac{V_B}{H_B}\right) = \arctan\left(\frac{V_A - W}{H_A}\right) \quad (3.99)$$

where α_s is the sea bed slope, as defined in Figure 3.12. This condition only holds if the cable is not completely suspended, which may be verified by noting that:

$$(r_c - l) \cos \alpha_s = L_T - L_0 > 0 \longrightarrow L_T - L_0 > 0 \quad (3.100)$$

where L_T is the cable total length and r_c is the horizontal distance measured between the cable end points. If this condition is not met, then the slope compatibility is not applicable, since the cable is fully suspended, and should be replaced by $L_0 = L_T$.

An example of the typical behaviour of the quasi static force transmitted by a mooring system to a floating platform as a function of the floating structure motions is presented in Figure 3.13, from where it may be seen that the mooring system response is clearly non linear for high amplitude motions, but may be relatively well approximated by a linear spring in the vicinity of a given equilibrium point.

Full dynamic formulation

Although the simplified approach of the previous section can be used as a first approximation to predict platform mooring forces, it is not able to capture the dynamic effects of the problem. While this may not be of the greatest importance when it comes to the platform motions *per se*, it can have drastic effects on predicting the forces along the cable and corresponding fatigue loads

(Ansari and Khan, 1986), which is of great importance, specially in a market that struggles to be cost effective.

The governing equation of a cable segment acted on by a given exterior force ($\mathbf{f}_{ext}$) and considering only the axial stiffness can be expressed from the generalised position vector ($\mathbf{r}$), mass per unit length (m), strain (ε) and axial force ($N = |\mathbf{n}|$) as described by Aamo and Fossen (2000) or Montano et al. (2007):

$$\partial_{tt}\mathbf{r} = \frac{1}{m} \cdot \partial_s \mathbf{n} + \frac{1+\varepsilon}{m} \cdot \mathbf{f}_{ext} = \frac{1}{m} \cdot \partial_s \left(\frac{N}{1+\varepsilon} \partial_s \mathbf{r} \right) + \frac{1+\varepsilon}{m} \cdot \mathbf{f}_{ext} \quad (3.101)$$

Considering the cable segment as a Morison type element according to Section 3.3.1.1, the external forces can be decomposed in 3 distinct components - self weight ($\mathbf{f}_B$), inertia ($\mathbf{f}_m$) and drag forces ($\mathbf{f}_D$):

$$\mathbf{f}_{ext} = \mathbf{f}_B + \mathbf{f}_m + \mathbf{f}_D = \mathbf{f}_B + \mathbf{f}_m + \underbrace{\mathbf{f}_{d_t} + \mathbf{f}_{d_n}}_{\mathbf{f}_D} \quad (3.102)$$

where the drag force has been explicitly decomposed into its transversal and longitudinal components, identified by the indexes t and n , respectively.

If the bending stiffness is also taken into account, which can be important to overcome some numerical stability issues, Equation 3.101 must be extended to include the shear forces and a momentum equilibrium equation must be provided. Representing the transverse forces and bending moments at the segment nodes by $\mathbf{f}_t$ and $\mathbf{m}$ respectively and assuming a distributed momentum applied along its length $\mathbf{m}_d$, the new equilibrium equations are:

$$\partial_{tt}\mathbf{r} = \frac{1}{m} \cdot \partial_s \mathbf{n} + \frac{1+\varepsilon}{m} \cdot \mathbf{f}_{ext} + \frac{1}{m} \cdot \partial_s \mathbf{f}_t \quad (3.103a)$$

$$\partial_s \mathbf{m} + \partial_s \mathbf{r} \times \mathbf{f}_t = -\mathbf{m}_d \quad (3.103b)$$

In which the internal efforts can be related to material properties from well known elasticity theory expressions (Silva, 2013):

$$\mathbf{m} = EI \cdot \partial_s \mathbf{r} \times \partial_{s,s} \mathbf{r} = -EI \cdot \mathbf{c} \quad (3.104a)$$

$$N = EA \cdot \varepsilon \quad (3.104b)$$

being $\mathbf{c}$ the cable curvature. Equation 3.103 can finally be solved providing adequate boundary conditions on the whole cable. For most cases, pinned connections are considered at both cable end points, whose position is also known, leading to:

$$\mathbf{r}(s = \{0, L\}) = \mathbf{r}_{\{0, L\}} \quad (3.105a)$$

$$\partial_{ss} \mathbf{r}(s = \{0, L\}) = \mathbf{0} \quad (3.105b)$$

Although the presented boundary conditions and mathematical formulation provide the means

to solve cable motions, obtaining these solutions is, in most cases, a complex procedure, so a numerical approach must be used. One of the most commonly used is the **lumped-mass model**, where the cable is modelled not as a single element but as group of connected elements of reduced dimensions, to whom different material properties can be attributed. For illustrative purposes, a comparison of the response of a simple structure considering a quasi static model, a full dynamic formulation considering only the cables' axial stiffness and a full dynamic formulation considering both axial and bending stiffness is presented in [Section B.3](#)

3.3.3 Platform motions

Over the previous sections both hydrodynamic forces as well as external forces transmitted by a mooring system have been described, but the platform motions themselves, fundamental for a proper modelling of the dynamic response of a floating wind turbine, are yet to be obtained. For a moored floating rigid body with 6 degrees of freedom, the generalised equations of motion according to Newton's 2nd law can be expressed from all the previously defined forces as:

$$\mathbf{M} \cdot \ddot{\mathbf{x}}(t) = \mathbf{f}_f + \mathbf{f}_{pe} + \mathbf{f}_m + \mathbf{f}_{ext} \quad (3.106)$$

where $\mathbf{M}$ is the structural inertia matrix, and the $\mathbf{f}_f$, $\mathbf{f}_{pe}$ and $\mathbf{f}_m$ represent the friction, pressure and mooring forces, respectively, and $\mathbf{f}_{ext}$ any other external applied forces. The latter, that may include for instance the wind loading on the rotor of a floating wind turbine, will not be considered for the time being.

[Equation 3.106](#) must be solved by numerical integration in time domain if all the non linear phenomenon are to be captured. However, a first analysis can be carried out in an almost straightforward way from the already introduced concepts provided the non linear terms are linearized or neglected. Although this approach may not deliver a full solution, it is useful to identify some aspects of motion and will be presented briefly as these follow almost directly from the potentials developed in the previous sections.

3.3.3.1 Frequency domain approach

If all the terms in [Equation 3.106](#) could be expressed as a superposition of oscillatory components, then it could be readily transformed to the frequency domain. The frequency domain representation of a given quantity, for instance the body motions $\mathbf{x}(t)$, will be represented as $\tilde{\mathbf{x}}(\omega)$, where the dependency on either t or ω may be omitted. The pressure forces can be separated in two different components, being one of them the hydrostatic restoring stiffness ($\mathbf{f}_{hs}$), and hence proportional to the body displacements:

$$\mathbf{f}_{hs}(t) = -\mathbf{K}_{hs} \cdot \mathbf{x}(t) \longrightarrow \tilde{\mathbf{f}}(\omega) = -\mathbf{K}_{hs} \cdot \tilde{\mathbf{x}}(\omega) \quad (3.107)$$

The other component that are sourced by the hydrodynamic loads ($\mathbf{f}_{hd}$) and can still be obtained from the 3 different potentials discussed. While the unperturbed and diffracted potentials are

proportional to the wave height, represented by the complex amplitude for unitary waves $\tilde{\mathbf{f}}_e$, the radiation force can be expressed by an added mass and damping coefficient (see [Section 3.3.1.2](#)). From this idea and the Bernoulli equation it follows:

$$\mathbf{f}_{hd}(t) = \rho \int_{S_b} \frac{\partial \phi}{\partial t} \mathbf{n} dS_b \longrightarrow \tilde{\mathbf{f}}_{hd} = \underbrace{i\omega\rho \int_{S_b} (\phi_0 + \phi_d) \cdot \mathbf{n} \cdot dS_b}_{\eta_0 \cdot \tilde{\mathbf{f}}_e} - \underbrace{i\omega\mathbf{B}\tilde{\mathbf{x}} + \omega^2\mathbf{A}\tilde{\mathbf{x}}}_{\tilde{\mathbf{f}}_r} \quad (3.108)$$

From both [Equation 3.107](#) and [Equation 3.108](#) the total hydrodynamic force is found to be expressed in the frequency domain as:

$$\hat{\mathbf{f}}_{pe} = -\mathbf{K}_{hs}\tilde{\mathbf{x}} + \eta_0 \cdot \tilde{\mathbf{f}}_e - i\omega\mathbf{B}\tilde{\mathbf{x}} + \omega^2\mathbf{A}\tilde{\mathbf{x}} \quad (3.109)$$

The viscous forces, however, do not provide a simple relation as were defined proportional to the velocity squared and are no longer linear in the displacements. If a frequency domain analysis is to be carried out some linear approximation is needed and the most commonly used is to assume a viscous damping matrix such that:

$$\mathbf{f}_f(t) = -\mathbf{B}_f \cdot \dot{\mathbf{x}}(t) \longrightarrow \tilde{\mathbf{f}}_f(\omega) = -i\omega\mathbf{B}_f \cdot \tilde{\mathbf{x}}(\omega) \quad (3.110)$$

Finally, the mooring forces are also strongly non linear so a frequency domain analysis of a cable component is not always an easy task - in frequency domain the mooring force turns out to be a linear force that acts on the platform as an additional restoring force. The origin of the non linearities can be separated according to its nature. On one hand, and considering an offshore scenario, platform motions change the boundary conditions of the cable and so the incremental force transmitted to the platform due to an unitary displacement (an equivalent instantaneous stiffness, so to speak). This problem is present even if there were no external forces acting on the cable itself, but in this scenario it would eventually be possible to linearise the response in the vicinity of a certain equilibrium position. Another kind of non linear phenomenon appears because of the nature of the forces acting on the cable due to the fluid and this is once again due to the drag force being proportional to the velocity squared. This problem arises even if the cable extreme points had fixed positions overtime and so, once more, if a frequency analysis is intended, the response must be approximated. One way to do so is described by [Borgman \(1967\)](#) and substitutes a non linear term by a weighting coefficient (u_{rms}) such that the drag force can be written as:

$$F_i = \frac{1}{2}\rho_w C_d \cdot d \cdot l \cdot |u_f - u_s| \cdot (u_f - u_s) \approx \frac{1}{2}\rho_w C_d \cdot d \cdot l \cdot \alpha \cdot u_{rms} \cdot (u_f - u_s) \quad (3.111)$$

where the fluid (u_f) and nodal (u_s) velocity components have been made intentionally explicit. From the presented equation, it is possible to identify a term proportional to the fluid velocity, which can be included as an external force, and a term proportional to the nodal velocity and so the damping matrix is now linearised. Although this techniques allow a reasonable characterisation of the structural response, they become less and less reliable as the displacements increase, mainly

for the correct assessment of the cable forces (Khan and Ansari, 1986).

For the time being, only a quasi static approach will be considered and linearised around a given equilibrium position such that:

$$\mathbf{f}_m(t) = -\mathbf{K}_m \mathbf{x}(t) \longrightarrow \tilde{\mathbf{f}}_m = -\mathbf{K}_m \cdot \tilde{\mathbf{x}}(\omega) \quad (3.112)$$

Using Equation 3.109, Equation 3.110 and Equation 3.112 in Equation 3.106 the motion complex amplitude is given by:

$$\tilde{\mathbf{x}}(\omega) = \eta_0 \tilde{\mathbf{f}}_e \left[-\omega^2 (\mathbf{M} + \mathbf{A}) + i\omega (\mathbf{B} + \mathbf{B}_f) + (\mathbf{K}_{hs} + \mathbf{K}_m) \right]^{-1} \quad (3.113)$$

Because Equation 3.113 relates the response directly to the wave amplitude, it is commonly to define the **Response Amplitude Operator**, or **RAO**, as:

$$\frac{\tilde{\mathbf{x}}}{\eta_0}(\omega) = \tilde{\mathbf{f}}_e \left[-\omega^2 (\mathbf{M} + \mathbf{A}) + i\omega (\mathbf{B} + \mathbf{B}_f) + (\mathbf{K}_{hs} + \mathbf{K}_m) \right]^{-1} \quad (3.114)$$

where $\tilde{\mathbf{f}}_e$ is the complex amplitude force for a wave with unitary amplitude and can be obtained from the incident and diffracted potentials alone (Equation 3.108)¹⁴. The RAO is hence a complex, frequency dependent function and can be expressed by its real amplitude and phase. Recovering the spectrum definition from Equation 2.36, the response amplitude can be written, for a given degree of freedom, as:

$$S_z(\omega) = \frac{1}{2} \cdot z(\omega)^2 = \frac{1}{2} \cdot \left| \frac{z}{\eta_0}(\omega) \right|^2 \cdot \eta_0^2(\omega) = \left| \frac{z}{\eta_0}(\omega) \right|^2 \cdot S_\eta \quad (3.115)$$

where the last equality defines the so-called **transfer function** between the wave and response spectrum that can be identified with the RAO operator from Equation 3.114 squared. An important consequence of Equation 3.115 is that the body's response reaches a local maximum for both wave spectrum (S_η) and RAO peak frequencies.

3.3.3.2 Time domain approach

In the previous section only linear or linearised terms were considered to provide the basis for a frequency domain analysis. However, there is often the need to correctly consider the neglected non linearities and so a time domain simulation must be carried out.

To obtain the forces needed to solve Equation 3.106 in time domain from the already defined potentials, the classic formulation derived by Cummins (1962) may be used. Assuming a linear model, and dividing the potential due to an impulse action in two separated contributions (one valid during the duration of the impulse and one valid after the impulse extinguished), the equations of

¹⁴This force can be thought of as “the force a sinusoidal wave of unit amplitude produces on a fixed body or as the force that we would need to apply in a freely floating body to keep it fixed” (Alves, 2012).

motion in time domain can be written as (Cummins, 1962):

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}}(t) + \int_0^\infty \mathbf{B}(\tau) \cdot \dot{\mathbf{x}}(t - \tau) \cdot d\tau + \mathbf{K}_{hs}\mathbf{x}(t) = \mathbf{f}_{nl}(x, \dot{x}, \ddot{x}, t) \quad (3.116)$$

which is the so called **Cummins equation** and all the non linear effects, as quadratic drag forces, mooring forces, externally applied loads or second order wave forces, have been included in $\mathbf{f}_{nl}(t)$. Note that the integral can be expressed, by making the substitution $\tau \rightarrow t - \tau$, as:

$$\int_0^\infty \mathbf{B}(\tau) \cdot \dot{\mathbf{x}}(t - \tau) \cdot d\tau \rightarrow \int_{-\infty}^t \mathbf{B}(t - \tau) \cdot \dot{\mathbf{x}}(\tau) \cdot d\tau \quad (3.117)$$

where its physical interpretation is now more clear: it represents the convolution between the velocity and the radiation impulse response function $\mathbf{B}(t)$. This term implies an impulsive displacement at a given time influences the motions during all the subsequent periods, or, alternatively, the motions in a given time interval are influenced by all the motions before that time interval. Because of this interpretation, it can be argued that the system has a sort of memory, and so $\mathbf{B}(t)$ is commonly referred to as the **memory function**. The added mass matrix and the memory function must still be determined and a direct approach is not an easy task to be carried. A simpler method was defined by Ogilvie (1964) by using the frequency dependent results for the added mass and damping coefficients. If the body is assumed to carry out a harmonic motion such that $\mathbf{x}(t) = \mathbf{1} \cos(\omega t)$, the Cummins equation reads (M J Journée and W Massie, 2001):

$$\begin{aligned} \mathbf{F}_{nl}(t) - \mathbf{K}_{hs} \cos(\omega t) = & -\omega^2 \left(\mathbf{M} + \mathbf{A} - \frac{1}{\omega} \int_0^\infty \mathbf{B}(\tau) \sin(\omega \tau) d\tau \right) \cos(\omega t) - \\ & - \omega \left(\int_0^\infty \mathbf{B}(\tau) \cos(\omega \tau) d\tau \right) \sin(\omega t) \end{aligned} \quad (3.118)$$

Alternatively, Equation 3.106, as have already been seen, can be expressed in the frequency domain as:

$$-\omega^2 \cdot (\mathbf{M} + \mathbf{A}_\omega(\omega)) \cos(\omega t) - \omega \mathbf{B}_\omega(\omega) \sin(\omega t) + \mathbf{K}_{hs} \cos(\omega t) = \Re(\tilde{\mathbf{f}}_{nl}(\omega)) \quad (3.119)$$

where $\mathbf{A}_\omega$ and $\mathbf{B}_\omega$ are now the frequency dependent added mass and damping matrices to which the index ω has been added for clarity. Comparing the corresponding terms of Equation 3.118 and Equation 3.119, one can conclude that:

$$\mathbf{A}_\omega(\omega) = \mathbf{A} - \frac{1}{\omega} \int_0^\infty \mathbf{B}(\tau) \sin(\omega \tau) d\tau \quad (3.120a)$$

$$\mathbf{B}_\omega(\omega) = \int_0^\infty \mathbf{B}(\tau) \cos(\omega \tau) d\tau \quad (3.120b)$$

The presented expression for $\mathbf{B}_\omega(\omega)$ can be identified with the first order Fourier transform coefficients and hence $\mathbf{B}(\tau)$ can be obtained by an inverse Fourier transformation as:

$$\mathbf{B}(\tau) = \frac{2}{\pi} \int_0^\infty \mathbf{B}_\omega(\omega) \cos(\omega \tau) d\omega \quad (3.121)$$

Finally, an expression for the time independent added mass matrix can be obtained as:

$$\mathbf{A} = \mathbf{A}_\omega(\omega) + \frac{1}{\omega} \int_0^\infty \mathbf{B}(\tau) \sin(\omega\tau) d\tau \quad (3.122)$$

and, as it must be valid for any value of ω , it can be defined from the limiting case of $\omega \rightarrow \infty$ as:

$$\mathbf{A} = \lim_{\omega \rightarrow \infty} \mathbf{A}_\omega(\omega) + \frac{1}{\omega} \int_0^\infty \mathbf{B}(\tau) \sin(\omega\tau) d\tau \longrightarrow \mathbf{A} = \mathbf{A}_\infty = \lim_{\omega \rightarrow \infty} \mathbf{A}_\omega(\omega) \quad (3.123)$$

3.4 Final remarks

In this Chapter the physics behind aerodynamics and hydrodynamics that are needed to provide the theoretical background for numerical modelling of floating offshore wind turbines have been presented.

This description started from properties of 2D flow models over static aerofoils, and proceeded all the way to the derivation of the fundamental equations that govern the rotor, and the wind turbine, in general, aerodynamics, making use of empirical or semi-empirical models to by pass some oversimplifications regarding the conditions that will be faced by an operating wind turbine.

Additionally, and once this task was concluded, a step further was taken towards the floating offshore conditions, that required a proper revision of the hydrodynamic behaviour of floating structures, including the mooring systems impact. Here, most derivations were made based on a potential solution of the flow, but particular attention was given to the different modelling choices that can and should be made for the different elements, depending on the particular environmental conditions that are under analysis. This description ended with the description of both frequency and time domain analysis. It is important to note that while the latter are fundamental to properly capture the non linear effects that are intrinsic to this type of structures, frequency analyses also play an important role as they are able to quickly provide relevant information regarding the critical frequencies of the system, which is particularly relevant for design purposes.

Together with the discussions of the environmental conditions presented in [Chapter 2](#), that may be now transformed into external forces and motions of the floating offshore wind turbines, this Chapter concludes [Part I](#) of this work, leaving for [Part II](#) the presentation and discussion of the original developments that justify this thesis. Firstly, the concepts just discussed will be applied to two numerical case studies. This will be useful for two main reasons. On one hand, it will provide a proper modelling validation that will ensure that the response being simulated is representative of a full scale floating offshore wind turbine. On the other hand, and assuming the first objective is fulfilled, the very same models can and will be used as a preliminary validation of the further investigations regarding the dynamic behaviour of floating offshore wind turbines the will be made on analytical grounds in [Chapter 5](#). Finally, with the numerical validation concluded, the experimental data of a full scale structure, and all the additional challenges that come with it, will be finally tackled.

Part II

Research developments

Chapter 4

Validation and calibration of numerical models illustrated with two case studies

4.1 Introduction

After having presented and discussed the key aspects and equations that govern the aerodynamic and hydrodynamic phenomenon, they must be combined in order to perform a full aeroelastic simulation of a floating wind turbine, where the flexible nature of the tower and blades must be also considered, as it impacts the dynamic of the all system. Additionally, and if these concepts are to be used to develop a truly digital twin, as conceptually introduced by [Gelernter \(1991\)](#) and named by [Piascik et al. \(2012\)](#), it is vital that the numerical model is capable of actually reproducing the behaviour of the real structure, so that it can interact with it and condition its operation.

Since in the development of any numerical model several assumptions are made and significant sources of uncertainties are present, a proper model validation is needed. In this Chapter, an example of model calibration and validation process will be presented from two different perspectives. Firstly, the aerodynamic and structural modelling validation will be made by comparing a model performance with the results from one of the experimental campaigns currently being carried out by FEUP. Once this step is concluded, the hydrodynamic modelling validation will be made based on a code to code comparison between two different softwares, that will then be used for a preliminary validation of the application of OMA techniques to the identification of the floating body low frequency modes.

The numerical models will be mainly developed using OpenFAST - where FAST stands for **F**atigue, **A**erodynamics, **S**tructures and **T**urbulence - a software firstly developed by [Jonkman \(2007\)](#) from NREL - **N**ational **R**enewable **E**nergy **L**aboratory - during his PhD work. FAST code has been updated ever since and recently transited to OpenFAST ([NREL, 2020](#)). Since it was thought to be important to adequately explain and illustrate all the information needed to obtain a reliable model in OpenFAST, its code and structure will be briefly presented in the beginning of this Chapter, and only then the case studies themselves presented.

4.2 OpenFAST structure

OpenFAST works with a multibody generalised degree of freedom approach, solving Kane's equations of motion to provide a full time history description of the complex dynamic problem that is an offshore wind turbine. For completeness and illustrative purposes, a brief overview of analytical mechanics approaches including the application of Kane's method to a very simplified wind turbine model is presented in [Appendix C](#).

4.2.1 Structural modelling

The structural modelling of the different components is mainly achieved combining rigid and flexible body dynamics with appropriate constraints. The tower and blades stiffness are explicitly considered using a modal space representation of these elements. While the tower fore-aft (FA), side-to-side (SS) and the blades' flapwise displacements are represented by two modal configurations each, blades' edgewise displacements use a single modal configuration. The hub, nacelle and floating platform are considered to be rigid bodies and hence their equations of motion are fully characterised from their centre of mass position and rotations about axis with origin on it.

Having identified all the relevant bodies, it could be thought a total of 55 degrees of freedom¹ would be needed, but it must be noted that these are not fully independent and so a smaller set of variables can be used, as long as the adequate motion constraints are considered. Without further proof, it can be easily concluded that the full structural dynamics characterisation is obtained considering the following 21 degrees of freedom, partially represented in [Figure 4.1](#), where it has been tacitly assumed all the connections, but the generator/rotor one, to be rigid.:

- 6 platform rigid body motions;
- 4 modal coordinates for the tower;
- 3 modal coordinates for each blade;
- 1 nacelle rotation about the yaw axis;
- 1 generator angular displacement.

The assumed constraints that allow the determination of position, velocity and acceleration of any given point are then:

- Equivalence between tower base and platform kinematics at the connection point;
- Equivalence between tower top and nacelle (except for the yaw position) kinematics at the connection point;
- Equivalence between nacelle and hub (except for rotations around the rotor axis) kinematics at the connection point;
- Relation between the generator and hub angular positions and velocities defined from the gearbox ratio and the axis/drive train flexibility;
- Equivalence between hub and blades' kinematics at the connection point.

¹4 for the tower and 3×3 for the blades' stiffness considerations plus 7×6 for the tower, floating platform, 3 blades, hub and nacelle rigid body motions.

It can be noted that the 21 degrees of freedom plus the 34 constraints defined² recover the previously identified 55 degrees of freedom.

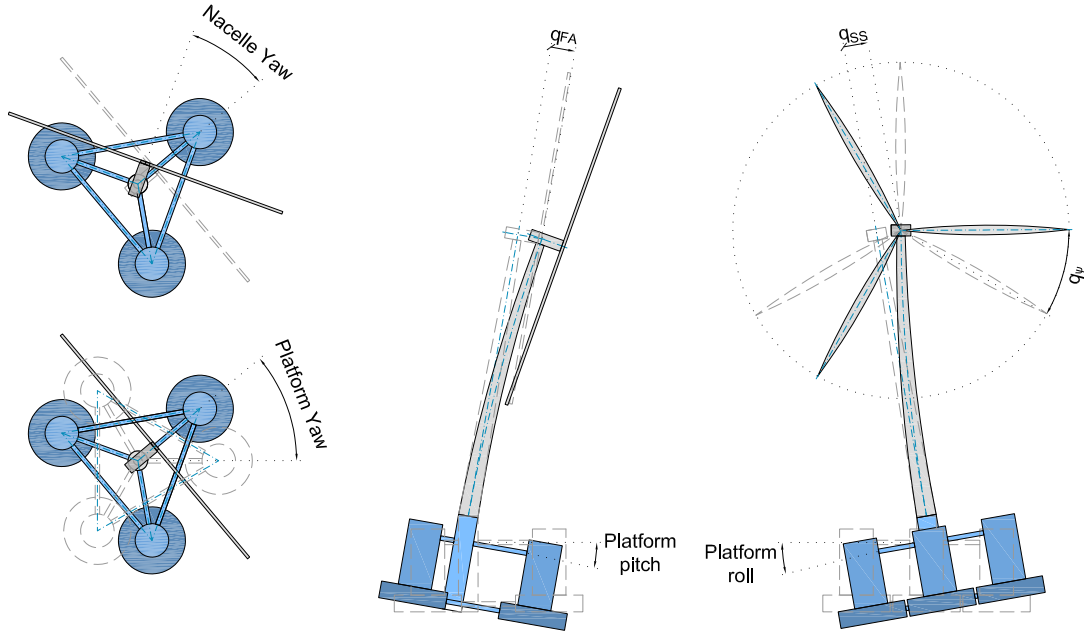


Figure 4.1: OpenFAST degrees of freedom (partial representation). On the left panel, are represented the platform and nacelle yaw motions, while the platform pitch and roll motions are represented on the middle and right panel. The tower flexibility is represented by the tower top displacement q_{FA} and q_{SS} for the FA and SS directions and the rotor angular position by one of the blades position, q_{ψ} .

4.2.2 Wind field generation

Having defined the principal degrees of freedom, it is necessary to provide the corresponding wind field to be used in the calculations. The first step is to generate an unperturbed wind field, which can be done using OpenFAST **InFlowWind** module (Platt et al., 2016). The simplest way of doing so is to create a **steady wind field** that follows the power law presented in Section 2.2.1.1, and that in this module nomenclature reads:

$$v_h(z) = V^{ref} \cdot \left(\frac{z}{H^{ref}} \right)^{\alpha} \quad (4.1)$$

where the horizontal wind speed (v_h) for a given height is computed from a reference wind speed (V^{ref}) and height (H^{ref}) and a power law coefficient (α).

However, this is a very simplified approach and a step further can be taken to create an unsteady, but still **uniform wind field**³. In this case, the horizontal ($v_h = u + w$) and vertical (v_v)

²The constraint involving both the hub and the generator does not eliminate a degree of freedom, as the generator itself has not been explicitly considered as a rigid body.

³By uniform, in this context, OpenFAST assumes that any point wind speed can be computed from a reference point, which does not happen in a truly random process.

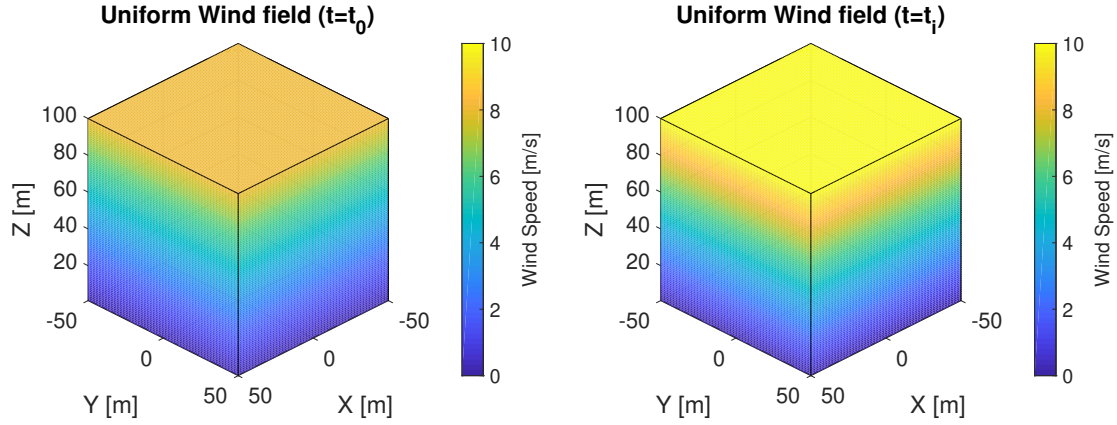


Figure 4.2: Wind field generated by InFlowWind considering an uniform wind field.

components of the wind speed are given by:

$$v_h = V_h^{ref}(t) \left[\underbrace{\left(\frac{z}{H^{ref}} \right)^\alpha}_{\text{Power law term}} + \frac{m_h}{\ell} (x \sin \theta(t) + y \cdot \cos \theta(t)) + \frac{m_v}{\ell} (z - H^{ref}) \right] + V_{gust}(t) \quad (4.2a)$$

$$v_v = V_v^{ref}(t) \quad (4.2b)$$

where m_h and m_v are the linear shear in the vertical and horizontal directions, respectively, ℓ is a characteristic length, θ is the wind direction and the remaining symbols represent the same as in the previous case. In order to include a random component, a V_{gust} term is added to the horizontal wind speed in every point, but this does not prevent the wind field to be fully described, at any time step, from a single point values. In Figure 4.2 are plotted the wind fields obtained for two time steps, only considering the power law term for $V_h^{ref}(t_0) = 8$ m/s and $V_h^{ref}(t_i) = 10$ m/s.

Finally, a more general approach can be adopted where the full 3D wind field is generated, but this is more easily said than done, since the generated field is pretended to respect a given spectrum and coherence functions (see Section 3.2). A full description of the procedure will not be given here, but the interested reader can find a pioneer algorithm capable of generating the longitudinal time series only in (Veers, 1988), that was later extended to the lateral and vertical components by Kelley (1992). More recently, NREL adapted and used these procedures to compile the **TurbSim** wind generator, whose outputs are ready to be read by OpenFAST (Jonkman and Buhl Jr, 2006). TurbSim generates multiple plane grids in accordance with a prescribed turbulence intensity and mean speed spectrum. Note that a spectrum is associated to every single grid point, and all of these must respect the spatial coherent structure. The obtained wind components thus represent a single plane configuration, or more formally, $u_i = u_i(y, z, t)$. In order to transform this into a full, 3D, wind field, FAST InFlowWind module uses Taylor's frozen wake hypothesis (Taylor, 1938; Veers, 1984). In this assumption, the turbulent coherent structures are assumed to propagate in the main wind direction with the mean wind speed and so the previously defined wind speeds $u_i(y, z, t)$ generate a full wind field $u_i = u_i(x, y, z, t)$. In Figure 4.3 is represented the wind field generated by

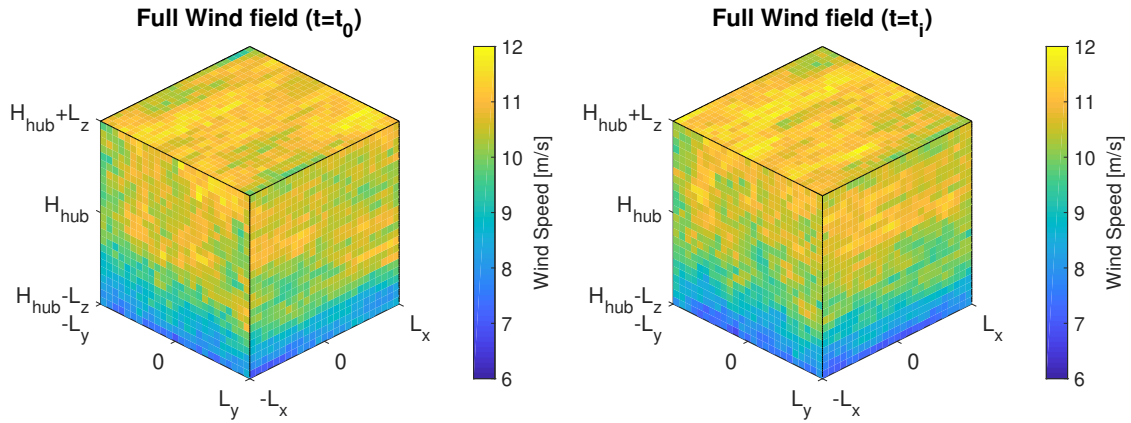


Figure 4.3: Full wind field generated by TurbSim considering Taylor's frozen wake hypothesis.

TurbSim at two time instants separated by $\Delta t = \frac{L_x}{U}$, such that during the considered time interval a given point travels half the grid length along the wind principal direction (x).

4.2.3 Aerodynamic modelling

Having defined the wind field to be used in the simulation, it is necessary to convert it to forces acting on the different structural components and in OpenFAST this is accomplished through its aerodynamic module **AeroDyn**. This module is mainly based on BEMT equations and additional corrections presented in [Section 3.2.2.3](#), as well as all the unsteady aerodynamic effects presented in [Section 3.2.2.4](#), most of these optionally included. The blade is divided in a user specified number of segments and a normalised aerofoil shape should be assigned to each one. In turn, every specified aerofoil must be characterised in a separated file containing all the relevant aerodynamic properties, namely the unsteady aerodynamic parameters needed to apply dynamic stall corrections and the lift, drag and momentum coefficients for the full angles of attack range. As all these aspects have been extensively detailed in [Section 3.2](#), no further details will be presented here.

4.2.4 Control systems modelling

Control systems design and modelling is a full field on its own that has not been really developed in this work and so a description covering the most relevant aspects should now be given. OpenFAST has a module entirely devoted to this modelling, named **ServoDyn**, that incorporates the pitch, torque, yaw controls and emergency breaks mentioned in [Section 2.3.2](#). The brief description of controls provided there is not enough to provide a full control system for the model to be built and hence the basic model presented by [Malcolm and Hansen \(2006\)](#) will be used for pitch control, while a simplified, rotor angular velocity dependent function will be used as torque control.

Torque Control

The torque control implemented is a very simple function of the rotor angular speed. In accordance to [Section 2.3.2](#), 3 distinct regions should be defined as in [Figure 4.4](#). For practical purposes,

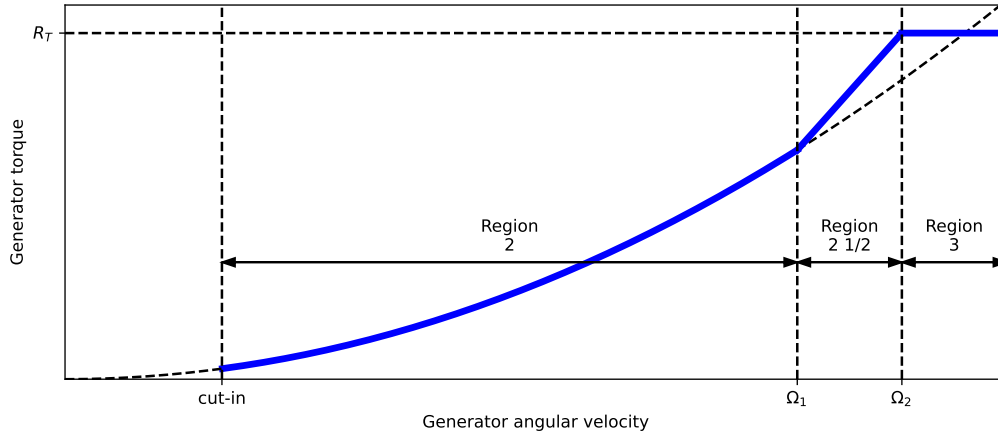


Figure 4.4: Simplified torque control implemented in OpenFAST.

and using the nomenclature included in Figure 4.4, the generator speed is an independent, input variable for the torque control algorithm and hence Region 2 is defined from a single coefficient, VS_Rgn2K . On the other hand, Region 3 must be defined from the rated torque value (R_T) and a rated generator speed (Ω_2).

Having these two regions defined, Region 2– $\frac{1}{2}$ (representing the transition from the other two operating regimes) can be specified with a single user defined parameter (VS_SLPc). The full torque control function is then:

$$\begin{cases} G_T = a \cdot \Omega_G^2 & , \text{ if } \Omega_G \leq \Omega_1 \\ G_T = G_T(\Omega_1) + (\Omega - \Omega_1) \cdot m & , \text{ if } \Omega_1 \leq \Omega_G \leq \Omega_2 \\ G_T = R_T & , \text{ if } \Omega_G > \Omega_2 \end{cases} \quad (4.3)$$

where Ω_1, Ω_2 are the limiting values of Region 2– $\frac{1}{2}$ and m is related with the slip ratio defined in OpenFAST by:

$$VS_SLPc = 100 \cdot \left(\frac{\Omega_2}{\Omega_1 - G_T(\Omega_1)/m} - 1 \right) = 100 \cdot \left(\frac{\Omega_2}{\Omega_1 - a\Omega_1^2/m} - 1 \right) \quad (4.4)$$

Pitch Control

The pitch control described by Malcolm and Hansen (2006) is a **Proportional-Integral-Derivative** controller - usually known as a PID (see, for example, Marlin, 1995). This type of controllers earned their name because the correction to be applied is based on 3 terms computed from the error between the measured and desired values of a given variable ($e(t)$):

- A term proportional to $e(t)$ itself: $P = K_P \cdot e(t)$;
- A term proportional to the integral of $e(t)$: $I = K_I \cdot \int_0^t e(\tau) \cdot d\tau$;
- A term proportional to the derivative of $e(t)$: $D = K_D \cdot \frac{de(t)}{dt}$.

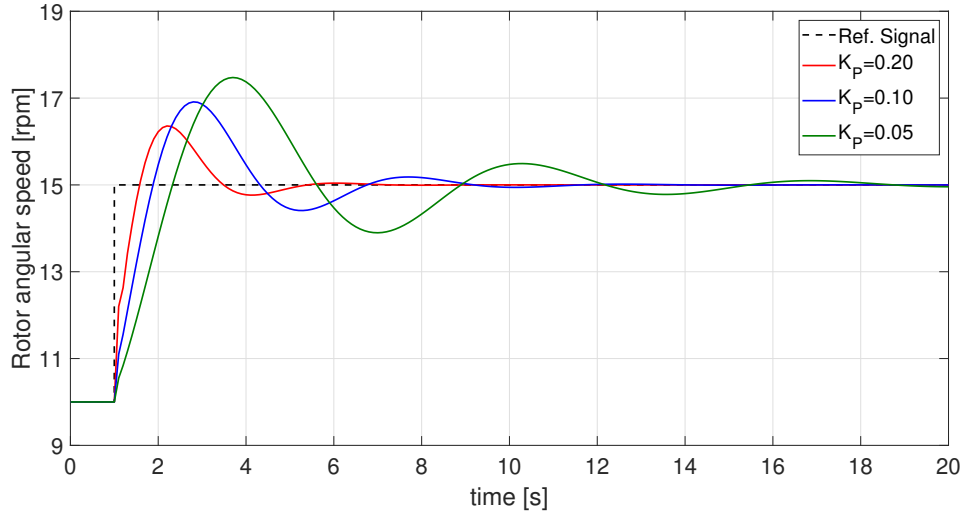


Figure 4.5: Simplified response obtained using a PID controller based on the wind speed.

The correction to be passed into the system is given by:

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt} = K_P \left(e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau + T_D \frac{de(t)}{dt} \right) \quad (4.5)$$

where the integral and derivative gain constants have been replaced by the integral and derivative characteristic times (T_I and T_D , respectively). Equation 4.5 can alternatively be written in the Laplace domain⁴ as (Ang et al., 2005):

$$U(s) = K_P \cdot \left(1 + \frac{1}{T_I \cdot s} + T_D \cdot s \right) \cdot E(s) = G(s) \cdot E(s) \quad (4.6)$$

where $G(s)$ is the gain function. Note that the integral term reduces the steady-state error as it compensates for the accumulated error over time, while the derivative term improves the response for transient phenomenon (Ang et al., 2005). An example of the response of a simple PID controller that tracks the rotor angular speed is represented in Figure 4.5 for different values of K_P , keeping the integral and derivative time constant and equal to 0.5 s and 0.1 s, respectively.

In the currently used model, the pitch control error function is obtained comparing the rotor speed with the pretended value (Ω_D) and the derivative term includes an additional time scale τ (small when compared to all the other time scales) to improve numerical stability. The pitch (θ) is assumed equal for the 3 blades:

$$e(t) = \Omega_D - \Omega(t) \longrightarrow \theta(s) = \left(K_P + K_D \cdot \frac{s}{1 + \tau \cdot s} + \frac{K_I}{s} \right) \cdot E(s) \quad (4.7)$$

In the control algorithm that will be used, the rotor set point is defined from the rotor angular speed

⁴The Laplace transform, $F(s) = \int_0^\infty f(t) \cdot e^{-s \cdot t} \cdot dt$, implies that the time domain derivative and integration correspond to multiplying and dividing the original transform by the Laplace variable s in Laplace's domain, respectively.

at rated power and the pitch demand is limited to an interval between a minimum value (θ_{min}), that is kept when the rotor speed is below the set point, and 90° (Malcolm and Hansen, 2006).

4.2.5 Hydrodynamic modelling

Until this point, all the relevant modules needed to create an onshore wind turbine model have been presented and attention must now turn to the specificity of offshore environments. The **HydroDyn** (Jonkman et al., 2014) module controls everything that has to do with the floating platform, including the incident waves kinematic model, except the mooring systems. The exciting waves can be either regular or irregular with any user defined spectrum, including the ones presented in Section 2.2.2.3, eventually considering currents and 2^{nd} order waves. The incident waves time series can be created from a prescribed spectrum and the surface elevation is obtained from an inverse Fourier transform as (Emmerhoff, 1994):

$$\eta(t) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} W(\omega) \cdot \sqrt{2\pi \cdot S_\eta(\omega)} \cdot e^{i\omega t} \quad (4.8)$$

where $W(\omega)$ is the Fourier transform of a standard normal distribution (white Gaussian noise time-series with zero mean and unit variance). Having the wave elevation time series created, the forces acting on the structure can easily be computed as described in Section 3.3.3.2.

Although OpenFAST itself does not solve the potential formulation for large bodies, it does solve the time domain equations from Section 3.3.3.2 as long as the frequency dependent added mass and damping and the frequency independent added mass matrices are provided. In parallel, the platform can be modelled as a superposition of linear Morison elements with given characteristic dimensions, hydrodynamic coefficients and known start and end points. This approach can be used to either model slender bodies with negligible refraction effects, or to include the viscous effects, absent from potential theory. Finally, additional stiffness, linear damping and/or quadratic drag matrices can be directly prescribed.

4.2.6 Mooring system modelling

Finally, a floating structure model is not complete if the mooring system is not considered. In OpenFAST this can be achieved by either a quasi-static approach (MAP++ code described by Masciola, 2013) or with a full dynamic formulation of every cable, which in turn can be obtained from a coupled simulation between OpenFAST and the commercial software OrcaFLEX (Masciola et al., 2011) or with a lumped mass model, in accordance with Section 3.3.2, considering two different codes: **FEAMooring** (Bae, 2014) or **MoorDyn** (Hall, 2015). For the reasons presented in Section 3.3.2, only the latter approach, namely the MoorDyn code, will be used, where the cables' geometry is defined from the coordinates of their end points and their mechanical and dynamic properties from the equivalent axial stiffness (the bending and torsional stiffness are not considered) and the intended damping coefficient. In accordance with the lumped mass model, the forces are computed based on Morison-type elements and hence the corresponding hydrodynamic

coefficients must be defined for both longitudinal and transverse directions. Although only MoorDyn will be used for the rest of this chapter, a comparison of the response obtained considering different mooring systems codes can be found in (Andersen et al., 2016).

4.3 Aerodynamic and structural modelling validation using experimental data

In this first case study the aerodynamic loads and structural behaviour obtained with OpenFAST will be tested against a real life operating wind turbine. The detailed description of the experimental layout and results (see, for example, Pacheco et al., 2020) is not the principal objective of this work, but as the latter will be used to validate the numerical model, a brief description is needed. In any case, the content of this section mostly follows the work by Pimenta et al. (2020a).

4.3.1 Experimental campaign

The selected wind farm is located in Tocha, Portugal, and the monitoring systems were installed in the context of the WindFarmSHM project. It includes 5 wind turbines as represented in Figure 4.6, each a VESTAS V100 1.8MW wind turbine.

Since the main objective of this campaign was to study different monitoring layouts to then propose an optimised solution to be implemented at the wind farm level, it includes 3 different levels of instrumentation, from a very complete layout to a very simple one using a single accelerometer. To validate the numerical model only the more intensive layout will be used, including:

- 2 alternative systems to characterise the tower's accelerations: a commercial system of high quality accelerometers and low-cost designed and assembled at FEUP MEMS accelerometers based system;
- Strain gauges at approximately 6.5 m from the tower base to record the stress time history under different operating conditions;
- A set of fibre optic strain gauges at the blades roots;
- Tri-axial MEMS accelerometers placed 10 m from the blades root.



Figure 4.6: Tocha wind farm location and layout (Pacheco, 2022).

4.3.2 OpenFAST model

In order to accurately reproduce the dynamic behaviour of the wind turbine, both structural properties and loads acting on the structure must be correctly defined. While the first point can be obtained without great difficulty, the latter is mainly influenced by the aerodynamic properties of the blades. To accurately model both effects, OpenFAST software with the previously described modules was used.

Structural properties

The static and dynamic response of the structure to any loads that are to be applied are mainly dependent on the behaviour of the tower and blades. As OpenFAST uses a generalised degree of freedom approach based on the modal configurations of the tower main modes, a finite element model was made to obtain these configurations.

In this initial FEM model the tower was simulated providing the linear mass density and SS/FA stiffness in 37 points distributed along its height. The blades were discretised in 49 different sections and to each section were associated 2 directions stiffness, density and structural twist.

In an initial phase, only the external geometry of the blades and its global mass were known so the stiffness properties and variation of linear weight were scaled from the 5MW baseline turbine (see [Section 4.4.1](#)). The chord scale (k_C) was calculated as the ratio between the chord of the VESTAS V100 and the reference blade. Assuming these can be treated as thin wall hollow sections, then the mass scale can be obtained directly from the product of the chord and thickness scale (k_t). Matching the global mass obtained for both allowed to calculate the latter. With this assumption, the inertia scale (k_I) for each section can finally be calculated as the product of the 3rd power of the chord scale by the global thickness scale:

$$k_I = (k_C)^3 \times k_t = (k_C)^3 \times \frac{k_m}{(k_L)^2} \quad (4.9)$$

Although with the income of additional design information, it was possible to relax this assumptions and use the adequate blade stiffness, the base model will still be presented here as an example of how experimental data may be used to overcome the lack of missing structural data. The pile supported foundation slab was modelled with shell elements with properties taken from design drawings. Based on this information, a first FE model was built in ROBOT Structural Analysis software (RSA) assuming a rigid connection between the nacelle and the hub.

As the eigenfrequencies of this model did not match the experimental ones identified by [Pacheco et al. \(2020\)](#), two main sources of uncertainty were identified: the blades stiffness and the drivetrain flexibility. While the first can be adjusted by changing the blades material properties (equivalent to an additional global inertia scale factor), the latter can be introduced in the model by means of an elastic connection between the rotor and the hub. Both this parameters were chosen in order to minimise the differences between numerical and experimental eigenfrequencies.

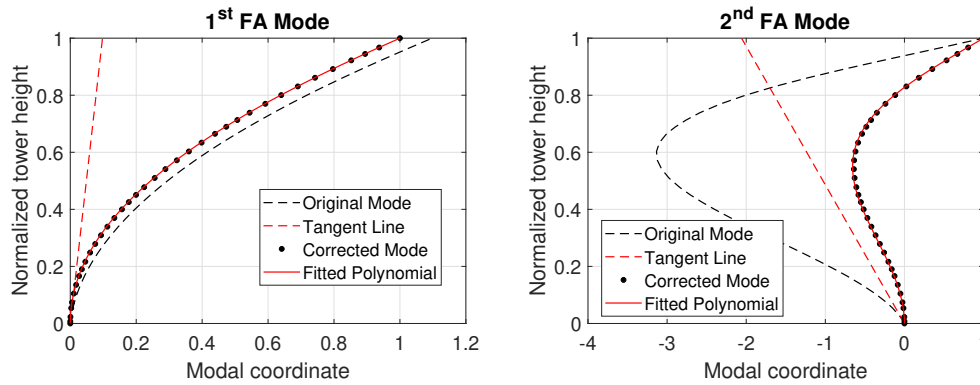


Figure 4.7: Mode shapes and 6th degree polynomial fitted to the configuration obtained for the FA direction.

Having the dynamic behaviour of the structure tuned, the two principal modes configuration for the tower in each direction were deducted from the tangent line at the tower base, so that they respect OpenFAST assumption of 0 slope in this section, and fitted to a 6th degree polynomial with unitary displacement on the top of the tower, as required (Jonkman, 2007). This procedure is illustrated in Figure 4.7 for the FA direction. In the OpenFAST model, the foundation was assumed to have infinite stiffness to translations and rotation about the tower yaw axis and the remaining components of the stiffness matrix were evaluated from the bending moment and rotation of the tower base of the described FEM model. To simulate this effect, it was used the HydroDyn module with an imposed linear stiffness matrix only.

Aerodynamic properties

Having the structural properties defined, it is now important to provide means to correctly predict the loads acting on the structure and one of the first tasks is the computation of the aerodynamic lift and drag coefficients along the blade. The blade was divided in 10 different sections and the NACA validated model from Section A.2 was used (in Figure 4.8 the different sections can be seen as well as the original 3D model gently provided by VESTAS).

However, as the simulations for angles of attack higher than stall angle become more and more complex, a previous analysis was made to predict the values that the different sections are expected to experiment during the various wind turbine operating conditions. If a linear behaviour near the 0° angle of attack is assumed (see Section 3.2), then the lift coefficient is given by:

$$C_L = m \times (\alpha - \alpha_0) \quad (4.10)$$

where the slope (m) and zero lift angle of attack (α_0) were obtained from CFD simulations between -5° and 5°. Furthermore, if the drag coefficient is assumed to be identical to that of a flow over a flat plate with a fully turbulent boundary layer, it can be estimated from Equation 3.8, here recovered

for simplicity:

$$C_D = \frac{1.328}{Re} \quad (4.11)$$

Finally, BEMT theory equations can be used together with the structural twist and choosing the pitch angle in accordance to the SCADA system records to provide the expected value for the angle of attack for all sections and operating conditions. From these analyses, aerodynamic coefficients for angles of attack in the range $[-10, 12]$ were obtained for all analysed aerofoils.

After having the drag and lift coefficients for these angles of attack, two important corrections should be considered according to [Section 3.2.2.3](#). Firstly, the 3D stall delay was considered by the corrections proposed by [Du and Selig \(1998\)](#) and by [Eggers et al. \(2003\)](#) for the lift and drag coefficients, respectively. Furthermore, as only angles of attack in a restricted domain were tested and a full description for any other value may be needed during a simulation, it is important to extrapolate the values for higher angles of attack. This was made using the extrapolation method proposed by [Viterna and Janetzke \(1982\)](#). The representation of lift and drag coefficients for one of the aerofoils can be seen in [Figure 4.9](#).

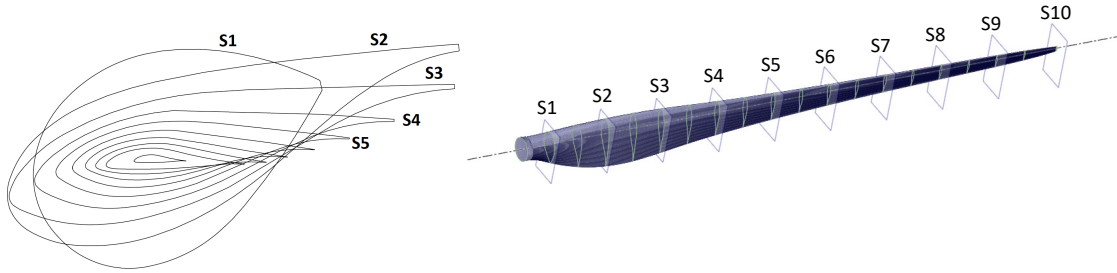


Figure 4.8: Representation of the adopted blade sections (on the left) obtained from the blade 3D model (on the right).

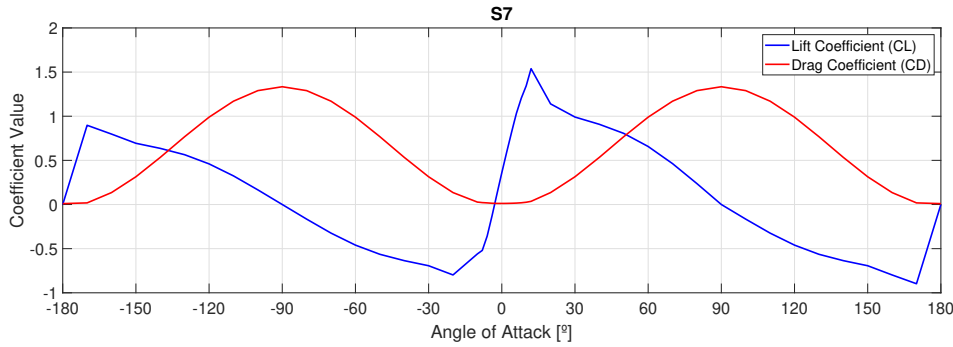


Figure 4.9: Lift and Drag coefficients for the full range of angles of attack obtained for section S7.

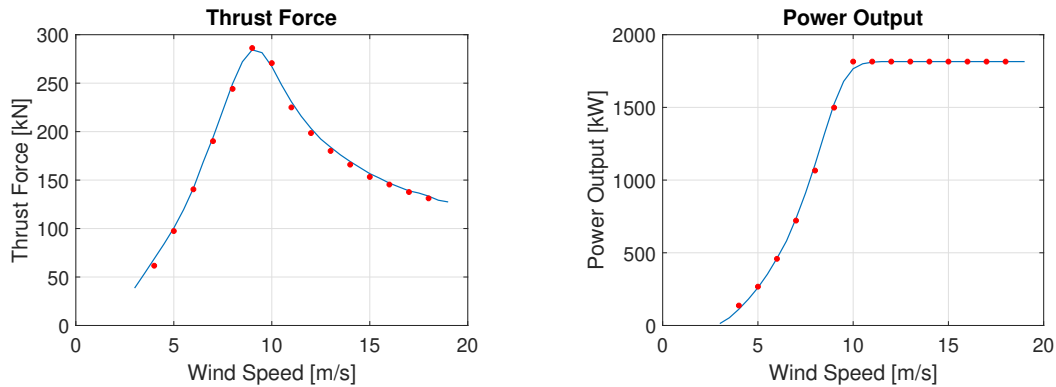


Figure 4.10: Comparison between simulated values (red points) and theoretical data (blue line) for the thrust force (left panel) and the power output (right panel).

Control systems properties

The control system properties were defined using the simplified routines presented by [Malcolm and Hansen \(2006\)](#). As there was no available information about the pitch angles below rated speed, it was necessary to look into the SCADA data and define an average minimum pitch angle that was fed into the pitch control routine for this operating regime.

The torque control was defined so that the power output would match the theoretical values, setting the appropriate values of gear box efficiency, power law coefficient of Region 2 and rated torque value of Region 3⁵. As Region 2 is assumed to have constant pitch angle (θ_{min} from the pitch control defined in [page 88](#)), an infinite number of equilibrium values between aerodynamic torque and resistant torque can be defined associated with different rotor angular speeds and hence different thrust forces. Having this in mind, the power law coefficient was defined by an iterative process in order to minimise the differences between theoretical and numerical thrust forces. The gear box efficiency can then be calibrated matching the theoretical power output with the generator speed obtained in the last iteration of the previous step. The rated torque value can be defined from the already determined gear box efficiency and rated speed so that the theoretical rated power is achieved for wind speed above 12 m/s. The power and thrust curves obtained from the simulations at steady states for different wind speeds can be seen in [Figure 4.10](#), and a good match with theoretical data is found.

4.3.3 Response comparison

With the model fully characterised, it is now possible to compare the numerically simulated and experimental data available. In order to validate the simulated response, a comparison of some characteristic curves can be made with the SCADA data and further validations can be obtained comparing the internal forces and dynamic properties derived from simulated and measured datasets.

⁵The mentioned regions are in accordance with the nomenclature introduced in [Section 4.2.4](#). Region's 2 torque is assumed to be proportional to the 2nd power of the generator speed and Region's 3 torque is assumed to be constant.

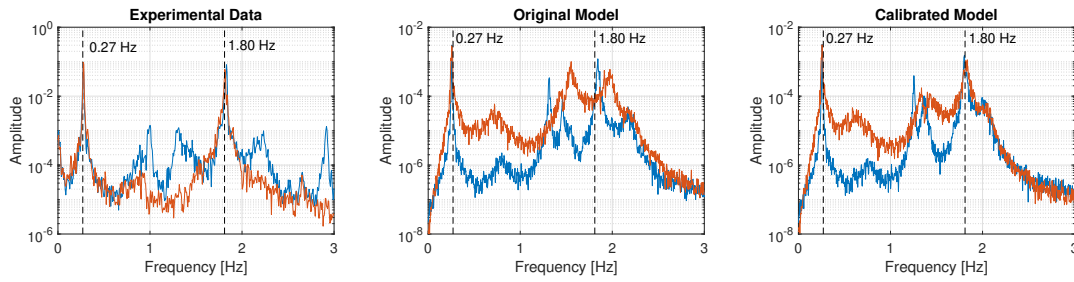


Figure 4.11: Power spectrum estimation (parked rotor) in the FA (blue) and SS (orange) directions from experimental and numerical data.

Single realisations comparison

To check that the modelled dynamic behaviour is the pretended one, both experimental and numerical natural frequencies can be compared using operational modal analysis techniques. The response of the structure was simulated during 30 minutes with the rotor parked and then divided in shorter series of 300 s. The power spectrum was calculated for each 5 minutes dataset separately and then averaged. The spectra obtained from both experimental and simulated accelerations at 75% of the tower height can be seen in Figure 4.11, where it is marked a reference value for the resonant frequencies obtained from experimental data. The non calibrated model is also presented. While the 1st mode could be accurately predicted from the original model, the 2nd mode presents higher deviations from experimental data due to its strong dependence on the interaction between tower and blades, only accounted for in the calibrated model (the model is the model obtained after calibrating the drivetrain flexibility and blades' stiffness, as described in the structural modelling section in page 92). Although the principal resonant frequencies are correctly identified in the calibrated model, other peaks of smaller amplitude are not correctly obtained. Based on the experimental data analysis and the FE model results, these frequencies are associated with mode shapes strongly dependent on the blades behaviour and so these less accurate results can be explained by all the simplifications made in the definition of the structural and mechanical properties of these elements.

The numerical power, rotor speed and pitch curves can be superimposed with the ones recorded by the SCADA system as represented in Figure 4.12. Analysing the rotor speed and pitch angle curve, it is clear that the implemented control algorithm reproduced adequately the values obtained from the SCADA system for most wind speeds. The results associated with lower wind speeds are not a major concern, as they are associated with lower forces⁶. The power output seems to be in good agreement with the expected value for all the tested scenarios.

Finally, and to compare the internal forces of the model with the ones measured in the field, a selection of some 10 minutes intervals with wind speed oscillating around an approximately constant mean value were used. To select such intervals, the SCADA data from February 2019

⁶A remark must be made here. Due to the simplified pitch control algorithm, the rotor speed obtained for lower wind speeds is dangerously close to the structural resonant frequencies and so significant loads and displacements can be induced. Analysing the experimental data, it is clear that this problem does not arise in the real structure, where the loads for low wind speeds are not to relevant.

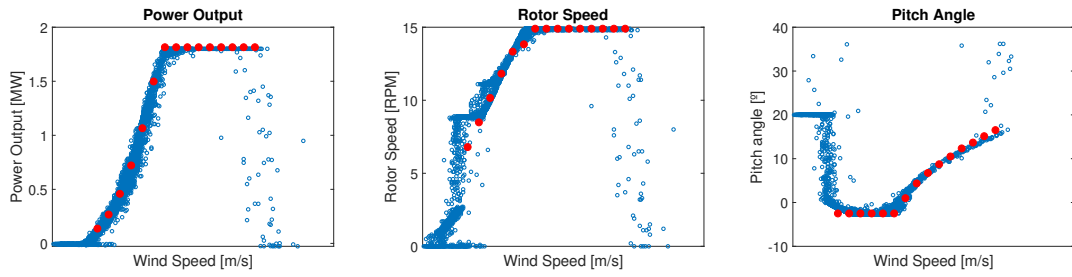


Figure 4.12: Comparison between numerical results (red points) and SCADA data (blue points). The horizontal axis scale has been omitted due to confidentiality constraints.

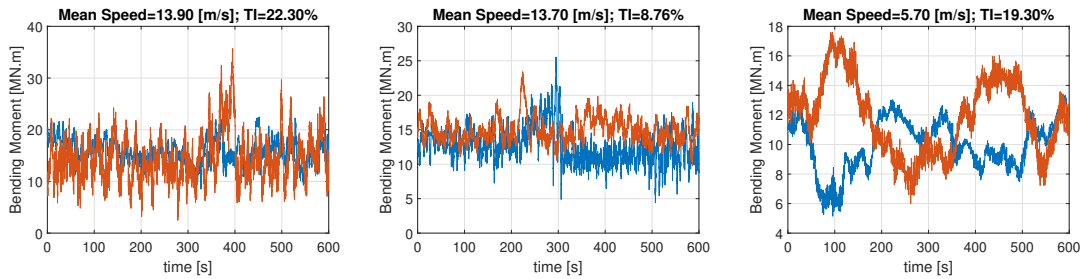


Figure 4.13: Comparison between numerical (orange) and experimental (blue) bending moments for selected time intervals.

was analysed and 10-minute time intervals were selected. Based on the wind characteristics, it was possible to construct 3 dimensional wind grids with equivalent mean speed and turbulence intensity with TurbSim preprocessor (see [Section 4.2.2](#)). For these series, it was assumed a power law mean wind profile with shear coefficient of 0.20 for the rotor disk and a log law with a surface roughness of 0.03 m elsewhere. The time series of bending moments at the tower section instrumented with strain gauges (6.5 m above tower base) of both experimental and numerical data can be seen in [Figure 4.13](#). Although numerical and experimental statistical properties comparison is only possible with a large number of simulations, a visual comparison of the series offers a first evaluation of the quality of the numerical results that are in good agreement with the experimental ones. It should be noted that while both numerical and experimental wind data share some common statistical properties, they do not represent the same time series, implying that only the comparison of some general properties of the results, like the mean bending moment and the magnitude of the variations, is meaningful.

Extended results

To provide a more robust validation of the model, 100 independent 10 minutes simulations, considering a 10% turbulent intensity, for different mean wind speeds were computed. The thrust force acting on the rotor, and corresponding bending moment at the control section, obtained for each computation with a given prescribed mean wind speed is plotted in [Figure 4.14](#) considering a 0.5 and 1 tower drag coefficient (both limiting values taken from [Kawai et al., 2008](#)). The error bars identify the maximum and minimum mean values obtained in the simulations and the experi-

mental values are averaged values of all the datasets that matched the prescribed mean wind speed. As may be checked, the lower drag coefficient leads to a better agreement with experimental data.

In the same way, the response power spectrum was computed for each independent data set and then averaged for a given wind speed, resulting in the 3D array represented in the colour maps of [Figure 4.15](#). From the analysis of the obtained results for a fixed mean wind speed, it can be seen that the principal frequencies are stable for all simulations and while some of the values can easily be matched with the ones from [Figure 4.11](#), it is clear that some previously absent frequencies are now important for the response content. The nature of this peaks can be understood when the frequencies evolution with the mean wind speed is analysed. In these diagrams, the natural frequencies of the tower can be seen to be stable over all the simulations, but the newly identified peaks do increase with the mean wind speed until a given value near rated wind speed. In fact, this peak is due to each blade passing through the non uniform turbulence field and hence are associated with a frequency equal to 3 times the rotor angular speed (0.745 Hz for the 14.9 rpm associated with rated wind speed).

As a final note, it should be mentioned that this model was used as a preliminary validation for a new methodology to estimate the bending moments in the tower from accelerations based on a Bayesian approach to OMA. Although this work is outside the scope of this thesis, the discussion can be found in ([Pimenta et al., 2022](#)).

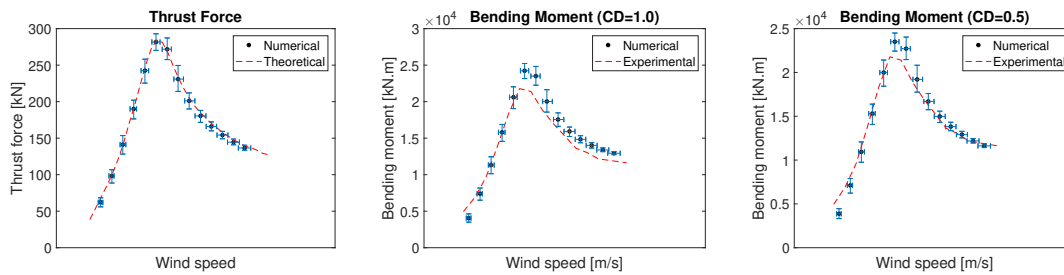


Figure 4.14: Comparison between numerical and experimental/theoretical forces for 100 independent generated time series (x scale omitted due to confidentiality constraints).

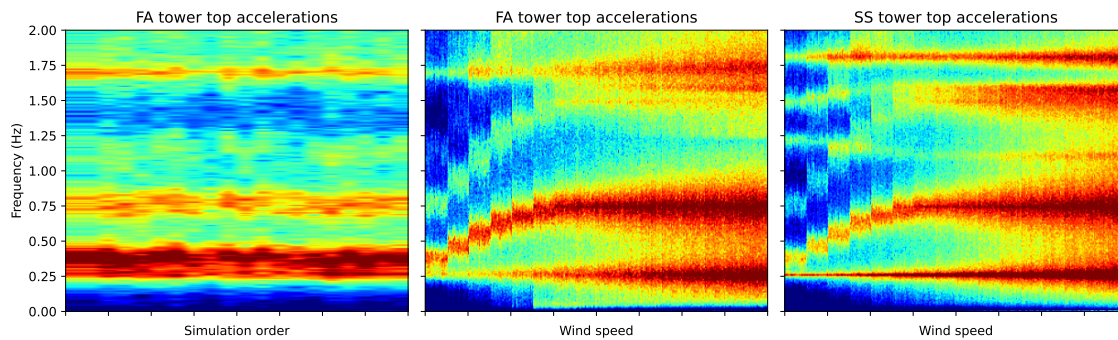


Figure 4.15: Evolution of the response frequency content.

4.4 Hydrodynamic modelling validation

Having the aerodynamic model validated, it's now time to take a step further to the offshore scenario. However, due to lack of real experimental data simultaneously with detailed information that would allow to develop a reliable numerical model, the NREL 5MW reference model, that has been used over the years to test and support the OpenFAST code development and has also been tested by comparison with both numerical and reduced scale physical models, was used. In order to validate the OpenFAST model and the methods to create any model with a different geometry if needed, OpenFAST results will be compared with the ones obtained with ANSYS AQWA®. As this ANSYS module does not model the complex dynamic behaviour of the rotor and the main objective of this process is to validate the hydrodynamic behaviour of a floating platform, the following analysis will be performed in parked conditions only.

4.4.1 Structure description

Since the wind turbine main properties are extensively detailed by [Jonkman et al. \(2009\)](#) and the relevant quantities will be mentioned in the model description, only a general overview of the structure will be provided here. The structure initially designed for the Offshore Code Comparison 3 (OC3) has been adapted to floating cases and a platform together with the corresponding mooring system defined. The structure is constituted by a central cylinder and 3 eccentric cylinders placed at the vertexes of a 50 m side equilateral triangle. While the first one connects to the tower 10m over the still water level (SWL), the remaining elevate themselves 12m above SWL. All the cylinders are connected by a tubular structure and the structure total draft is 20m. The eccentric columns are partially filled with water to provide the required stability to pitch and roll motions such that the platform total mass is 1.3473×10^7 kg, from which 3.8522×10^6 kg are due

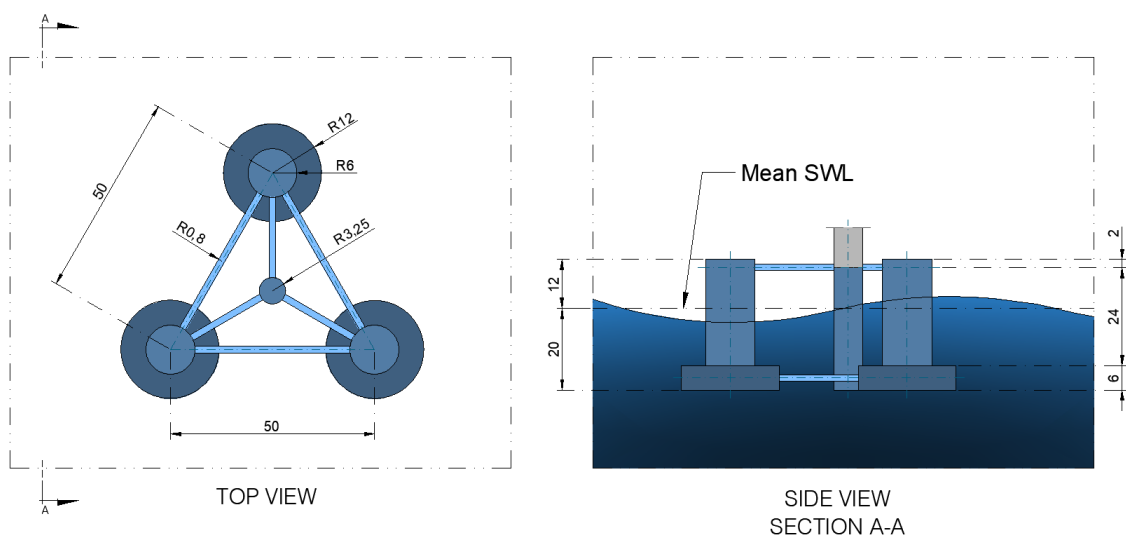


Figure 4.16: Schematic representation of the floating platform, based on the data from ([Robertson et al., 2014](#)).

to the steel self-weight and the remaining 9.6208×10^6 kg are due to the ballast. A schematic representation of the semi-submersible structure is presented in Figure 4.16 while for additional information, a detailed description was given by Robertson et al. (2014).

4.4.2 OpenFAST model

The blades are defined from their density and stiffness in both principal directions evaluated in 49 different sections. As the blade chord line rotates along the span, there is the need to additionally specify the structural twist. The evolution of all these properties can be seen in Figure 4.17. The tower stiffness and mass are defined in 11 different sections and their height dependency is represented in Figure 4.18. The hydrodynamic properties of the floating platform must be given to OpenFAST as an input and in this model a mixed potential flow and strip theory approach was used. For the former, the added mass and radiation damping were obtained from WAMIT⁷, while the latter is used to capture the viscous forces acting on the structure, considering non zero coefficients for the drag forces only.

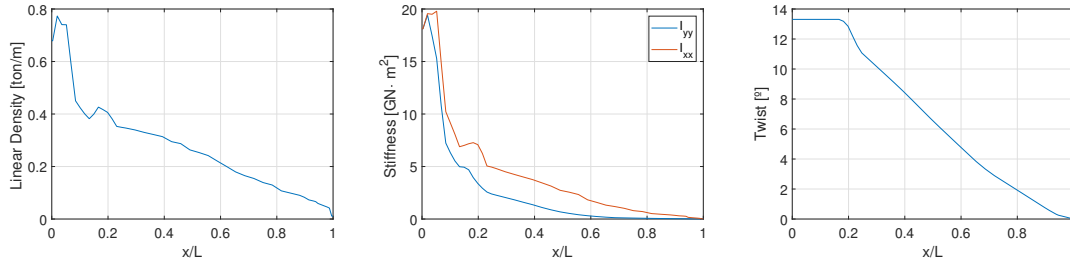


Figure 4.17: Evolution along the span of blade's linear density, stiffness and structural twist (from left to right, respectively).

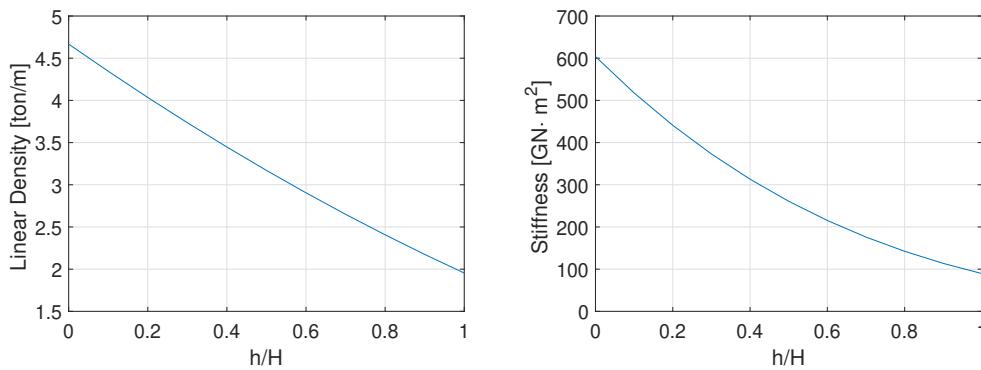


Figure 4.18: Tower mass distribution (on the left) and stiffness (on the right).

⁷It is important to understand that different radiation damping and added mass matrices can be obtained for a given structure if different reference points, and corresponding generalised displacements, are considered. In OpenFAST, this reference point is defined as the intersection between the tower axis and the still water free surface plane. Finally, the WAMIT output files are normalised by the water density and a characteristic length, here unitary.

Finally, the mooring lines are modelled with MoorDyn and are constituted by 3 independent 0.0766 m diameter cables, each 835.5 m long, and weighting 113.35 kg per unit length. The cables, connected to the structure 14 m below still water level over a circumference with 40.868 m radius, are placed uniformly at every 120° and anchored to the sea bed over a 837.6 m radius. The cables' axial rigidity is 753.5 MN, while their added mass is assumed to be 0.8, and the normal and transversal drag coefficients are assumed to be 2 and 0.4, respectively.

4.4.3 ANSYS AQWA Model

Having the OpenFAST model studied, it is possible to reproduce the same structure in ANSYS AQWA®. However, as both softwares use different modelling approaches some adaptations and assumptions must be made. The modelling description will be divided in wind turbine (tower and rotor) and platform (mooring lines included).

Wind Turbine

Although the wind turbine itself does not directly affect the hydrodynamic properties of the platform, having the mass distribution correctly modelled is vital to achieve realistic results. Because ANSYS AQWA® assumes all the bodies to be infinitely stiff, then it suffices to provide the total mass and moments of inertia with respect to the centre of mass for every member modelled. As the tower and blades are discretised in OpenFAST, and assuming that both can be represented by linear elements, the needed values in the local axis system can be computed as:

$$M_t = \int_0^h m(z) \cdot dz \quad (4.12a)$$

$$x_g = y_g = 0 \quad (4.12b)$$

$$z_g = \frac{\int_0^h m(z) \cdot z \cdot dz}{M_t} \quad (4.12c)$$

$$I_{xx} = \int_{-z_g}^{h-z_g} I_x(z) \cdot dz + \int_{-z_g}^{h-z_g} m(z) \cdot z^2 \cdot dz = \int_{-z_g}^{h-z_g} \frac{k_x(z)}{E} \cdot \rho(z) \cdot dz + \int_{-z_g}^{h-z_g} m(z) \cdot z^2 \cdot dz \quad (4.12d)$$

$$I_{yy} = \int_{-z_g}^{h-z_g} I_y(z) \cdot dz + \int_{-z_g}^{h-z_g} m(z) \cdot z^2 \cdot dz = \int_{-z_g}^{h-z_g} \frac{k_y(z)}{E} \cdot \rho(z) \cdot dz + \int_{-z_g}^{h-z_g} m(z) \cdot z^2 \cdot dz \quad (4.12e)$$

$$I_{zz} = \int_{-z_g}^{h-z_g} \frac{k_x(z) + k_y(z)}{E} \cdot \rho(z) \cdot dz \quad (4.12f)$$

where k_x , k_y and m are the bending rigidity in the x and y directions and the linear mass density, plotted in [Figure 4.17](#) and [Figure 4.18](#). It should be noted that the quantities obtained from [Equation 4.12](#) are expressed in the body local axis and must be expressed in an axis system parallel to ANSYS global reference frame. This can be achieved by transforming the inertia tensor as ([Goldstein et al., 2014](#)):

$$\mathbf{I}_{ANSYS} = \mathbf{T} \mathbf{I}_g \mathbf{T}^t \quad (4.13)$$

where $T_{i,j} = \cos(\theta_{i,j})$ are the cosine directions between the original and transformed reference frames' axis.

Tower properties

The tower distributed properties presented in [Figure 4.18](#) can be numerically integrated leading to the properties in [Table 4.1](#) and no further correction is needed as long as the global zz axis coincides, as it does, with the tower longitudinal axis.

Hub and Nacelle Properties

Although representing a relative small portion of the structure's total mass, the characterisation of these elements is not a difficult task to be carried out. The hub can be modelled as a spherical shell so that the knowledge of the inertia along one arbitrary direction and total mass, both values obtained from OpenFAST model, is sufficient to fully describe it. For the nacelle, its inertia around the wind turbine's axis of rotation and the global mass are known and it was assumed it could be modelled as a cylindrical element with 1.75 m radius. The properties for both these elements in their principal axis are presented in [Table 4.2](#). The hub properties do not need any transformation but the nacelle does, as its longitudinal axis is tilted upwards 5° . According to [Equation 4.13](#), the inertia tensor must then be transformed by:

$$\mathbf{T}_{Nacelle} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-5^\circ) & \cos(-95^\circ) \\ 0 & \cos(85^\circ) & \cos(-5^\circ) \end{bmatrix} \quad (4.14)$$

Blades Properties

The procedure followed to obtain the overall properties of the blades was similar to the one used in the tower, but using the properties from [Figure 4.17](#). Because of the structural twist, blades' sections principal axis are rotating along the span and so a first correction must be made before applying [Equation 4.12](#). However, as one of the body dimensions (its length) is much greater than the remaining ones, the total inertia is mainly dominated by the second term from the second

Table 4.1: Tower properties.

Prop.	M_t [kg]	z_g [m]	I_{xx} [kg m ²]	I_{yy} [kg m ²]	I_{zz} [kg m ²]
Value	249718	33.24	122.534×10^6	122.534×10^6	1.682×10^6

Table 4.2: Hub and nacelle properties with respect to their principal axis.

Element	M_t [kg]	(x_g, y_g, z_g) [m]	I_{xx} [kg m ²]	I_{yy} [kg m ²]	I_{zz} [kg m ²]
Hub	56780	(0,5,90)	115926	115926	115926
Nacelle	240000	(0,-1.9,89.35)	1741490	367500	1741490

Table 4.3: Single blade properties with respect to its principal axis.

Prop.	M_t [kg]	z_g [m]	I_{xx} [kg m ²]	I_{yy} [kg m ²]	I_{zz} [kg m ²]
Value	17609	20.507	429.482×10^4	429.482×10^4	2.274×10^4

equality of the inertia around xx and yy axis and so a reasonable approximation can be obtained considering a circular geometry with inertia given by the mean of both directions. The global results for a single blade with reference to its principal axis are given in [Table 4.3](#). The transformation matrix can be obtained by considering two sequential rotations: one around blade's local xx axis, corresponding to the pre-cone angle (α), followed by a rotation around the local yy axis of $\beta = n \cdot 120^\circ$, with $n = \{0, 1, 2\}$ (one value for each blade) defined by two transformation matrices:

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \cos(90^\circ + \alpha) \\ 0 & \cos(90^\circ - \alpha) & \cos(\alpha) \end{bmatrix} \quad (4.15a)$$

$$\mathbf{T}_2 = \begin{bmatrix} \cos(-n \cdot 120^\circ) & 0 & \cos(-n \cdot 120^\circ + 90^\circ) \\ 0 & 1 & 0 \\ \cos(n \cdot 120^\circ - 90^\circ) & 0 & \cos(n \cdot 120^\circ) \end{bmatrix} \quad (4.15b)$$

The overall transformation matrix is obtained multiplying both matrices defined in [Equation 4.15](#).

Floating Platform

The platform elements can be modelled as diffracting or non diffracting elements and this decision was based on the ratio between the element diameter and the wavelength, since it is generally accepted that diffraction plays an important role whenever this value is higher than 0.2 ([Sumer and others, 2006](#)). In [Figure 4.19](#) this ratio is represented for the different elements as a function of the incident wave frequency, that can be related to its wavelength by [Equation 2.32](#). As can be easily seen, the pontoons diffraction effect will only be relevant for high frequency waves, usually associated with lower wave amplitudes, and so these were modelled as Morison type elements. Furthermore, and to include the viscous effect on the diffraction elements, an additional equivalent quadratic drag matrix was introduced, as given by [Robertson et al. \(2014\)](#) in consistent units:

$$\mathbf{B}_{\text{Drag}} = \begin{bmatrix} 3.95 \times 10^5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.95 \times 10^5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.88 \times 10^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.70 \times 10^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.70 \times 10^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.08 \times 10^4 \end{bmatrix} \quad (4.16)$$

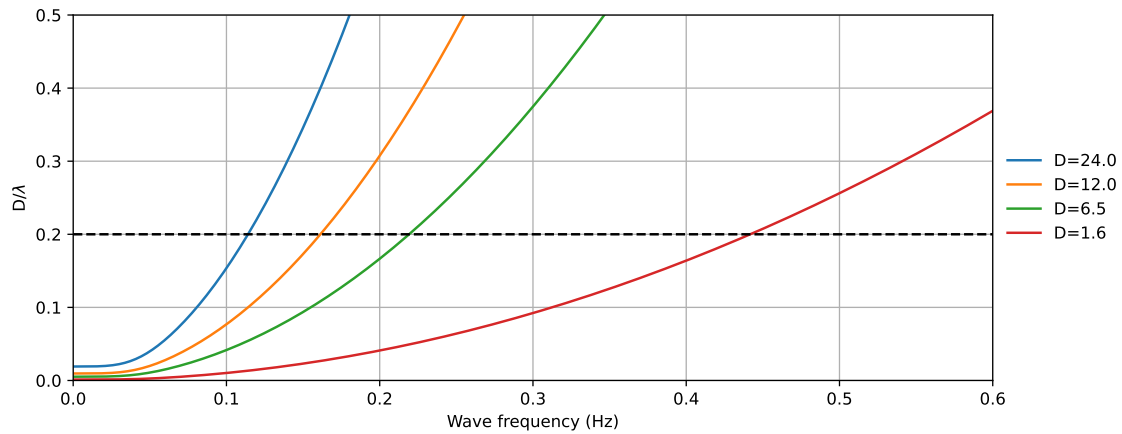


Figure 4.19: Ratio between diameter and wave length for the different elements represented in Figure 4.16. The different diameters correspond to the lower part of the eccentric columns (24 m), its top part (12 m), the central column (6.5 m) and the pontoons (1.60 m).

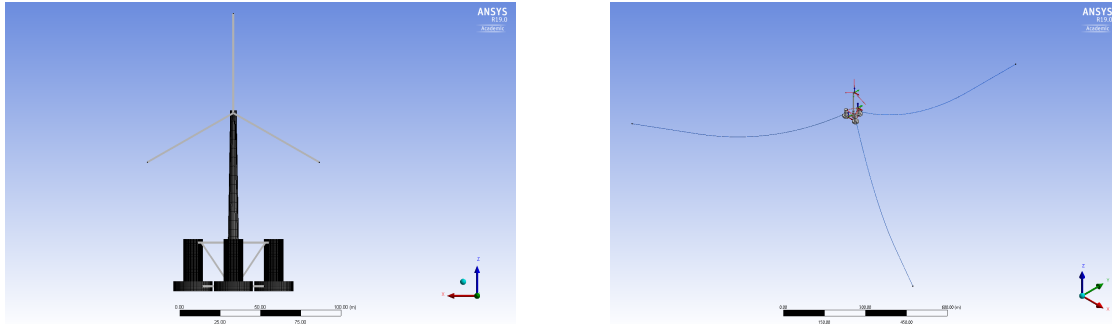


Figure 4.20: Lateral and 3D view of the model created in ANSYS/AQWA. The yy axis is aligned with one of the cables and the nacelle orientation, or, equivalently, orthogonal to the rotor plane.

The mooring system cables were modelled using non linear catenary elements with the properties described in Section 4.4.2, each discretized in 100 segments of equal length to correctly capture its dynamic behaviour. The final model can be seen in Figure 4.20.

Hydrodynamic coefficients validation

A preliminary validation can be made right away, as the hydrodynamic coefficients computed with ANSYS can be compared to the ones used in the OpenFAST model and provided by NREL, with the results presented in Figure 4.21.

At first sight, it appears to exist a significant difference between both results, but it must be noted, as mentioned earlier, that the WAMIT results adopted in OpenFAST use the still water level as a reference point for the calculations, while ANSYS uses the structure centre of gravity. Because of this, the results cannot be directly compared. There are two ways to overcome this difficulty: one is to artificially alter the position of the centre of gravity of the structure modelled in ANSYS (this procedure will not change the diffraction results here under analysis, but no simulation can be computed with this model), another is to note that the generalised displacements of

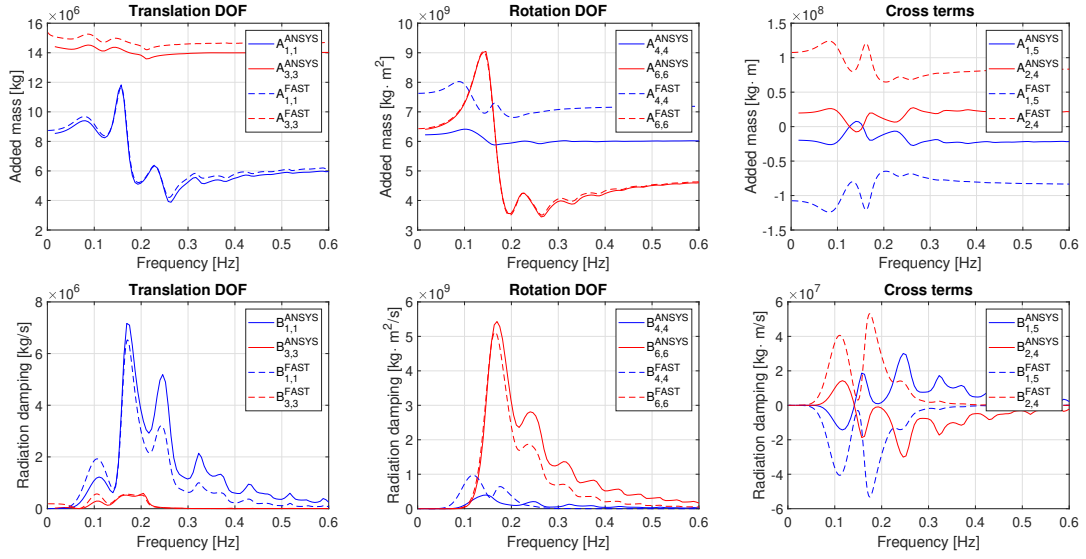


Figure 4.21: Evolution of the hydrodynamic added mass (on top) and radiation damping (on bottom) with the frequency as described in OpenFAST and computed by ANSYS at the structure centre of gravity.

the OpenFAST model ($\mathbf{u}_F$) can be related with the ones of the ANSYS model ($\mathbf{u}_A$) by:

$$\mathbf{u}_F = \begin{bmatrix} 1 & 0 & 0 & 0 & z_g & -y_g \\ 0 & 1 & 0 & -z_g & 0 & x_g \\ 0 & 0 & 1 & y_g & -x_g & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \mathbf{u}_A = \begin{bmatrix} \mathbf{1} & \mathbf{R} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \cdot \mathbf{u}_A = \mathbf{T} \cdot \mathbf{u}_A \quad (4.17)$$

On the other hand, it may also be written:

$$\mathbf{f}_A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -z_g & y_g & 1 & 0 & 0 \\ z_g & 0 & -x_g & 0 & 1 & 0 \\ -y_g & x_g & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \mathbf{f}_F = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{R}^T & \mathbf{1} \end{bmatrix} \cdot \mathbf{u}_A = \mathbf{T}^T \cdot \mathbf{f}_F \quad (4.18)$$

where $\mathbf{f}$ stand for the forces acting on the body and the minus sign is present because the centre of gravity position vector has been previously defined from the ANSYS reference point to OpenFAST reference point. Combining these definitions one can easily write:

$$\mathbf{f}_A = \mathbf{T}^T \cdot \mathbf{f}_F = \mathbf{T}^T \cdot \mathbf{K}_F \cdot \mathbf{u}_F = \mathbf{T}^T \cdot \mathbf{K}_F \cdot \mathbf{T} \cdot \mathbf{u}_A = \mathbf{K}_A \cdot \mathbf{u}_A \quad (4.19)$$

and hence both matrices can be converted with the transformation matrix $\mathbf{T}$ obtained from the

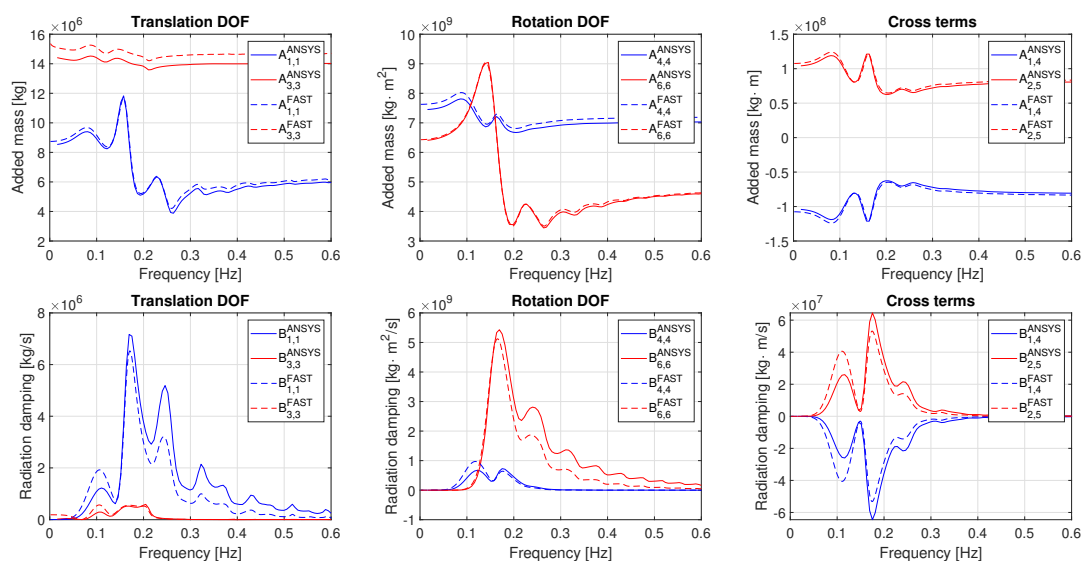


Figure 4.22: Evolution of the hydrodynamic added mass (on top) and radiation damping (on bottom) with the frequency as described in OpenFAST and computed by ANSYS at the intersection of the tower axis with the mean water level (after application of transformation matrices).

coordinates of the centre of gravity only. Either way, the hydrodynamic coefficients transformed to the same reference point as in WAMIT are plotted in Figure 4.22. As can be seen, while the main trends of all the coefficients are correctly obtained, some variations can still be detected, which can be explained by different considerations of the diffracting and non diffracting elements.

4.4.4 Response comparison

Having both models briefly described and the hydrodynamic coefficients validated, a set of equivalent tests were conducted with the properties summarised in Table 4.4.

Table 4.4: Characterization of the different simulations carried out.

Simulation ID	Wave type	Wave direction	Initial displacement
1		Still waters	(25 m,0,0,0,0)
2		Still waters	(0,25 m,0,0,0)
3		Still waters	(0,0,5 m,0,0)
4		Still waters	(0,0,0,10°,0)
5		Still waters	(0,0,0,0,10°,0)
6		Still waters	(0,0,0,0,0,10°)
7	Regular	0°	-
8	Regular	90°	-
9	Irregular	45°	-

Free motion simulations

Firstly, the response after displacing the structure from its equilibrium position and considering still waters can be seen in Figure 4.23, where only the degree of freedom directly related to the simulated initial displacement is presented. For the same simulations, the cable forces can be seen in Figure 4.24. The first set of results proves that for both models the floating structure presents the same main natural frequencies and modal damping coefficient. The results associated with the mooring lines prove the consistency on the overall behaviour, but the results of OpenFAST, associated with the imposition of rotations, present higher frequency fluctuations.

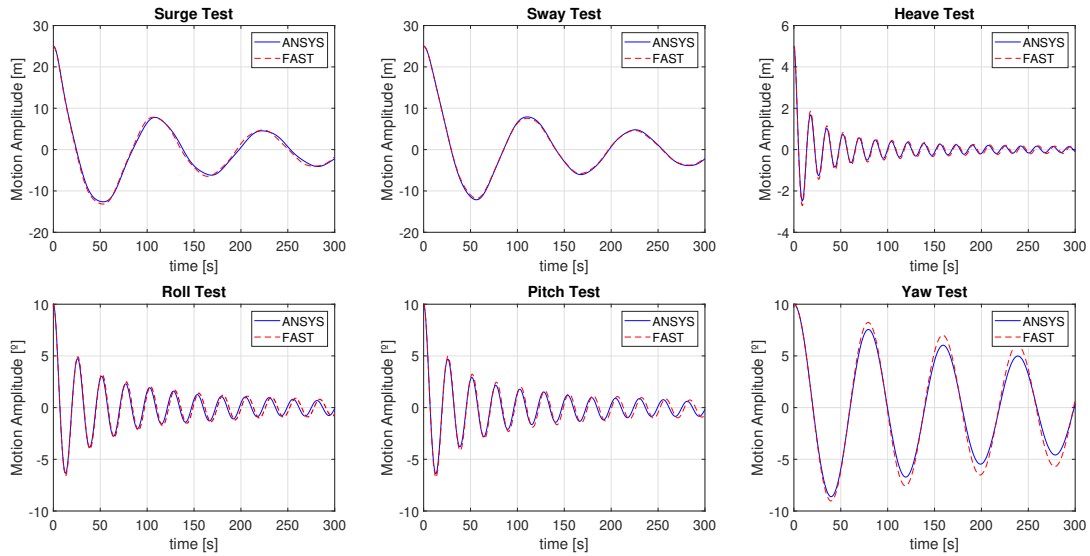


Figure 4.23: Comparison between OpenFAST and ANSYS response for the 6 still water simulations.

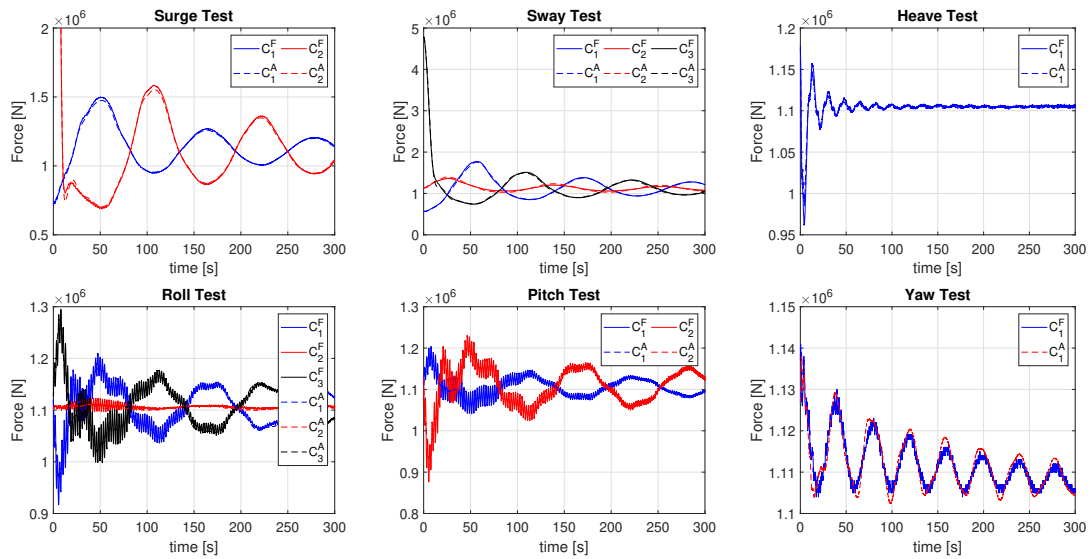


Figure 4.24: Comparison between OpenFAST and ANSYS cable forces for the 6 still water simulations (when several cables present the same response, only one of them is represented).

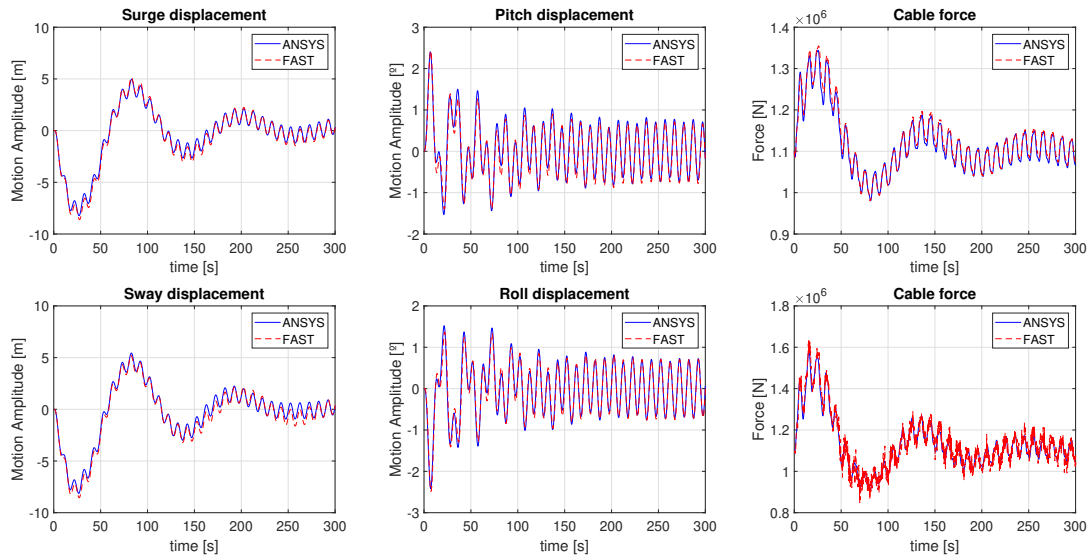


Figure 4.25: Comparison between OpenFAST and ANSYS for simulation 7 (on top) and 8 (on bottom) for the relevant degrees of freedom.

Regular wave simulations

From the previous section, it was clear that the structure response is essentially the same for all the simulations tested, but no waves have been considered yet. To extend the obtained results to a more realistic wave scenario, a set of regular, 2 m high and 10 s period waves have been generated in two orthogonal directions (simulations 7 and 8 from Table 4.4) and the motions and cables' forces computed. Figure 4.25 compares the results obtained for the more relevant directions in each simulation. From this and the previous section results, it is clear that surge and sway motions are essentially equivalent, as they should be due to the structure axial symmetry (neglecting the (very small) rotor induced eccentricity of the centre of mass).

Irregular wave simulations

Finally, the simulations were taken a step further and the response under a random set of irregular waves computed. The identification of the most relevant frequencies can be made from a frequency analysis and the corresponding RAO operator, presented in Figure 4.26, where a second, smaller, peak can be seen in the translation RAO in accordance with the dominant peaks from rotation RAO. This additional peak appears because the response is being analysed in a point that is not the centre of gravity, and hence translation motions do appear due to rigid body rotations. To illustrate this effect, the RAO were computed with reference to the structure centre of gravity (CG), the WAMIT reference point (WRP) and the tower top, with the results presented in Figure 4.27.

Having identified the relevant frequencies, waves were generated from a white noise spectrum of limited range between 0.005 Hz and 0.1 Hz. Due to its random nature, a direct comparison of the response for a given time interval is pointless, but its frequency content can still be validated. However, and trying to take a first step into experimental data analysis, this will be done based on the time response generated by either ANSYS and OpenFAST rather than from a direct frequency

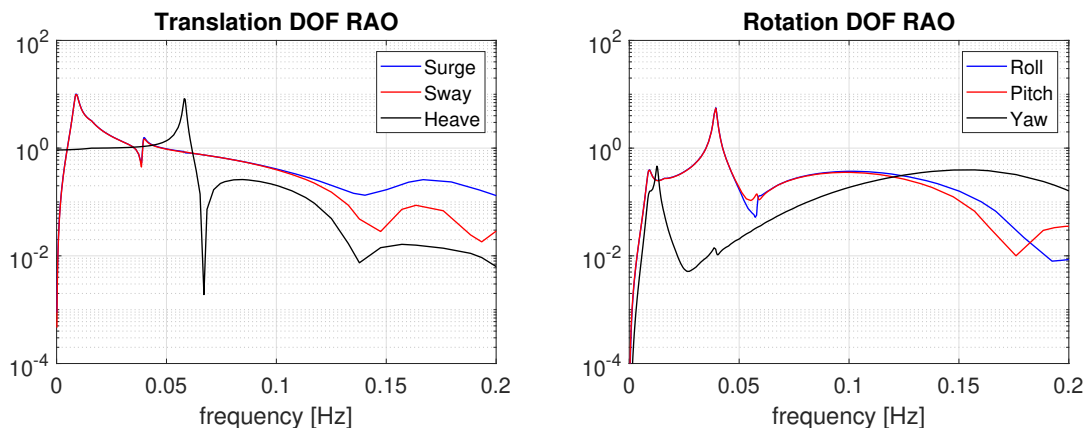


Figure 4.26: RAO operator obtained with a frequency analysis in ANSYS for the translation (on the left) and rotation (on the right) degrees of freedom.

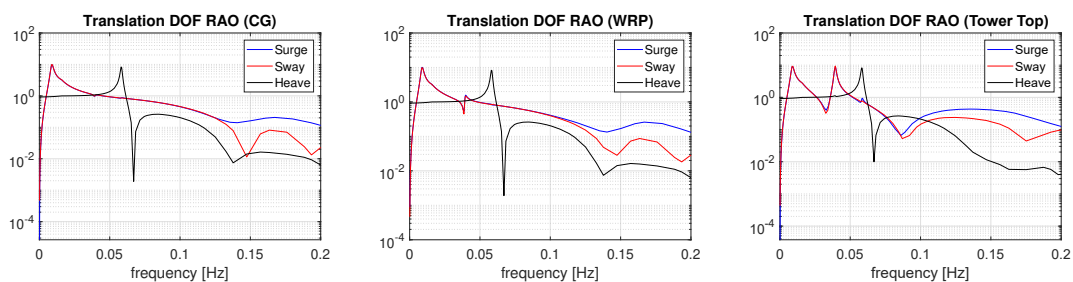


Figure 4.27: Comparison of the RAO operator obtained with a frequency analysis in ANSYS for the translation degrees of freedom evaluated at the centre of gravity (on the left), the WRP (on the centre) and tower top (on the right).

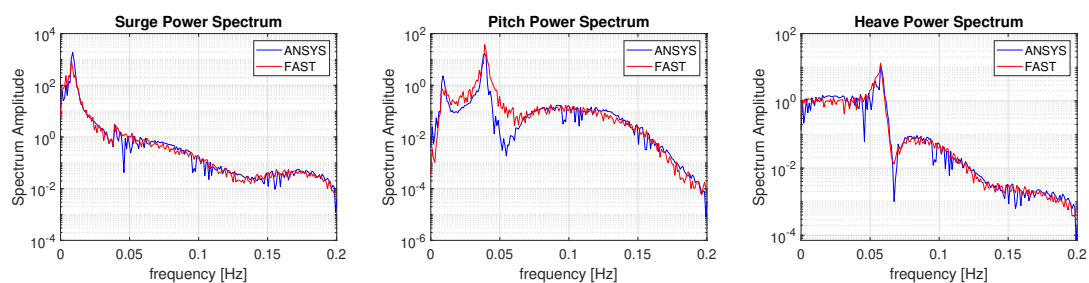


Figure 4.28: Power spectrum obtained for surge (on the left), pitch (on the center) and heave (on the right) response from ANSYS and OpenFAST generated time series.

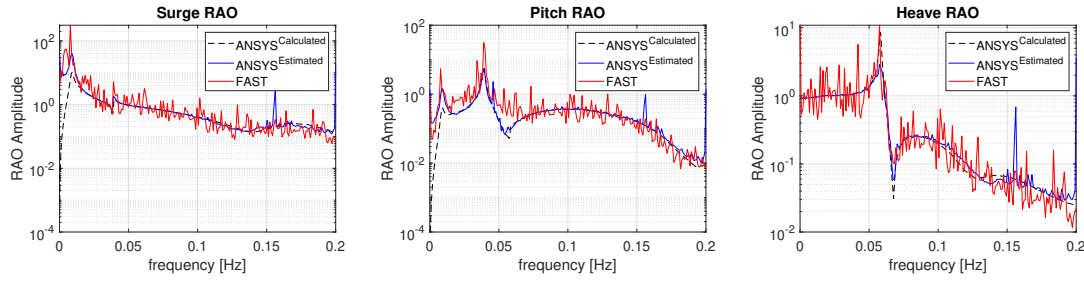


Figure 4.29: RAO operator obtained for surge (on the left), pitch (on the centre) and heave (on the right) response from ANSYS and OpenFAST generated time series.

analysis, with the implied linearizations. Because a longer observation period allows for a better frequency resolution (this is important due to the very low frequencies that are being studied), a set of waves with a total duration of 7200 s has been generated in both softwares. Due to the equivalent response in the 2 principal directions, only the results associated to surge, pitch and heave motions are presented. The power spectra obtained from both simulations are presented in [Figure 4.28](#), where the different plots represent the spectrum estimation obtained from the time response alone. However, and even considering that a white noise spectrum has been imposed, there is no guarantee that the generated waves have the same energy content in all considered frequencies (due to the limited time simulation, they probably do not) and, even if they do, in a real scenario this would not certainly be the case. A more relevant analysis is to obtain an estimation for the RAO, which can be obtained from the auto-spectrum of the wave elevation (S_{xx}) and the cross-spectrum between the wave elevation and the measured response (S_{xy}) ([Jonkman et al., 2013](#)):

$$\text{RAO}(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} \quad (4.20)$$

The estimations obtained using [Equation 4.20](#) are presented in [Figure 4.29](#), where the all frequency analysis results have been included for comparison.

4.4.5 Preliminary results of OMA

The application of OMA to the identification of the floating platform motions is a field where there is very few experience. Before the analyses that are presented in this document, some numerical applications had already shown that output-only methods can provide valuable results ([Ruzzo et al., 2016, 2017](#)). Here, the NREL 5MW floating platform model just described was used as a preliminary test of the SSI-COV algorithm application to these type of structures. This section mostly follows the discussion published by [Pimenta et al. \(2020b\)](#), but further developments will be presented in [Chapter 6](#).

To better understand the dynamics of the system, the response over 7200s (7500 – 300 s to damp out any transient component of the response) was simulated with the following properties:

- T1 - Yaw, sway and roll DOF blocked; Waves in the surge direction;
- T2 - Yaw DOF blocked; Waves in the surge direction;
- T3 - All DOF active; Waves in the surge direction;
- T4 - Yaw DOF blocked; Waves at 45°;
- T5 - All DOF active; Waves at 45°;
- T6 - All DOF active; Waves at 45°; Cable 1 stiffness reduced to 10% of its original value;

The stabilisation and damping diagrams obtained with the SSI-COV method are presented in [Figure 4.30](#) for Simulation T1, T2 and T3, and in [Figure 4.31](#) for Simulation T4, T5 and T6. In both cases, only the stable poles are plotted and a pole was considered stable when the frequency and damping coefficient estimates from consecutive model orders did not change more than 1% and 5%, respectively, and the MAC was at least 0.99.

Simulation scenario T1 is a very simple configuration, where the structure motion can be described by only 3 degrees of freedom, which is confirmed from the corresponding stabilisation and damping diagram, where 3 independent clusters can be easily identified. In Simulation T2, it is already possible to identify all the rigid body motions, except for the yaw DOF, still blocked. Although it may not be clear from the stabilisation diagram, the presence of the sway and roll modes is easily identified in the damping diagram, where additional clusters can now be seen, associated with lower damping coefficients. The lower value of the damping coefficient of these modes is justified as the waves incident direction is still perfectly aligned with surge motion and hence these motions are associated with poorly excited modes. The same reasoning applies to Simulation T3, where it is now possible to identify the yaw motion frequency for higher model orders at around 0.012 Hz. For all the surge aligned waves the SSI-COV method is able to correctly identify both frequency and damping coefficients of all the relevant DOF.

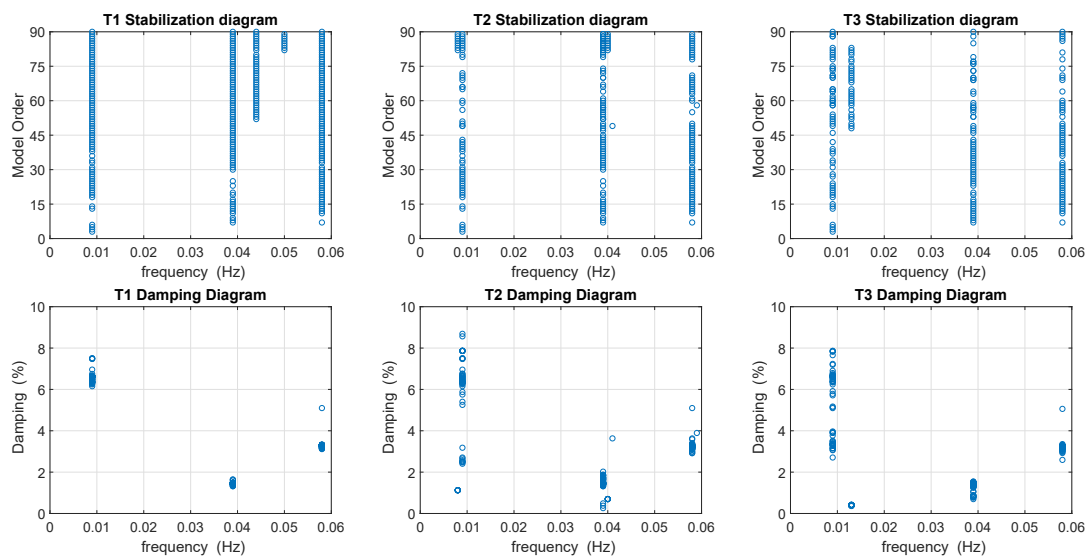


Figure 4.30: Stabilisation and damping diagrams for Simulation T1, T2 and T3.

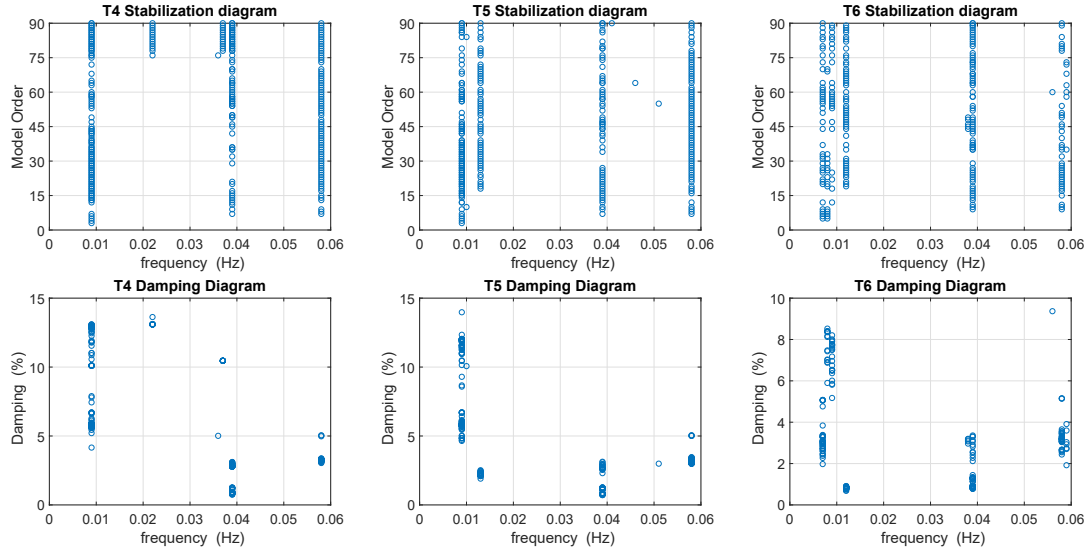


Figure 4.31: Stabilisation and damping diagrams for Simulation T4, T5 and T6.

As can be seen in [Figure 4.31](#), both surge-sway and roll-pitch modes are identified, even in Simulation T4, where their frequency is essentially the same. Once more, the damping diagram reveals the presence of two distinct modes, associated with different damping coefficients. When yaw motions are not restrained (Simulation T5 and T6), an additional peak appears at around 0.012 Hz, consistent with the previously identified yaw frequency in Simulation T3. Because this mode is poorly excited, its corresponding hydrodynamic damping is very low. Finally, when the Cable 1 stiffness is modified in Simulation T6, the structure symmetric behaviour collapses and surge and sway frequencies can be identified separately. All the modal properties, including the best estimation for the amplitude of the complex mode shape configurations obtained, are summarised in [Table 4.5](#), where for Simulation T1, T2 and T3 only the most excited modes have been presented.

From the results presented in [Table 4.5](#) it can be seen that the frequencies and damping coefficients are consistently obtained from all the simulations. Furthermore, the effect of the Cable 1 stiffness modification can be seen in the surge and sway dominated modes of Simulation T6. This effect is much more effective in decoupling surge-sway then pitch-roll motions because while the former stiffness is essentially provided by the mooring system, the latter has a strong dependency in hydrodynamic restoring forces, that have not been modified, and so the platform oscillates around an axis orthogonal to the waves' direction.

In order to explore this effect, a new set of simulations were carried out, where the waves' heading direction was gradually changed between 0° and 90° degrees, measured in relation to surge direction. The relation between surge-sway modal ordinates in the first mode ($\Phi_{1,1}/\Phi_{2,1}$) and roll-pitch modal ordinates in the third mode ($\Phi_{4,3}/\Phi_{5,3}$) is presented in [Figure 4.32](#). As can be seen, the response is explained by a translation in the wave's direction and a rotation around an axis orthogonal to it, as expected considering the indetermination of the mode shapes in the case of an axisymmetric structure.

Table 4.5: Modal properties, including angular frequencies, natural frequencies, modal damping ratios and 6 mode shapes components obtained for the different simulations (X - surge; Y - sway; Z - heave; RX - roll; RY - pitch; RZ - yaw).

Simulation	Mode	ω	f	ξ	Mode shapes components					
		[mrad/s]	[mHz]	[%]	X	Y	Z	RX	RY	RZ
T1	1	55.3±0.1	8.81±0.02	6.6±0.2	1	0	-0.04	0	0.04	0
	2	245.8±0.1	39.11±0.02	1.47±0.05	0.32	0	0.32	0	1	0
	3	365.6±0.2	58.19±0.04	3.3±0.2	-0.57	0	1	0	-0.08	0
T2	1	55.4±0.2	8.81±0.03	6.5±0.7	1	0	-0.04	0	0.04	0
	2	245.2±0.9	39.0±0.2	1.5±0.4	0.32	0	0.32	0	1	0
	3	365.6±0.9	58.2±0.2	3.3±0.3	-0.56	0	1	0	-0.08	0
T3	1	56.1±0.2	8.92±0.03	6.3±0.8	1	0	-0.03	0	0.04	0
	2	245±1	39.1±0.2	1.3±0.3	0.31	0	0.30	0	1	0
	3	365.4±0.5	58.2±0.1	3.2±0.3	-0.58	0	1	0	-0.07	0
T4	1	55.1±0.3	8.77±0.04	5.9±0.4	0.93	1	-0.04	0.03	-0.04	0
	2	58.0±0.2	9.23±0.04	11.7±1.5	1	0.86	-0.06	-0.03	0.04	0
	3	244±0.4	38.9±0.1	2.9±0.1	-0.46	-0.47	-0.58	1	-0.94	0
	4	246.2±0.3	39.2±0.1	1.0±0.2	-0.21	0.21	0.31	-0.95	1	0
	5	365.5±0.4	58.17±0.06	3.3±0.4	-0.40	-0.40	1	0.03	-0.04	0
T5	1	55.6±0.2	8.84±0.03	5.8±0.5	0.96	1	-0.04	-0.04	0.04	0
	2	58.9±0.3	9.37±0.04	11.2±0.4	1	0.76	-0.07	-0.03	0.04	0.01
	3	79.95±0.09	12.73±0.02	2.2±0.1	0.73	0.77	-0.05	-0.03	0.03	1
	4	244.5±0.6	38.92±0.09	2.8±0.2	-0.51	-0.51	-0.61	1	-0.93	-0.01
	5	246.2±0.3	39.18±0.05	1.0±0.2	-0.20	-0.20	0.30	-0.95	1	-0.01
	6	365.4±0.6	58.15±0.09	3.3±0.4	-0.40	-0.40	1	0.03	-0.04	-0.02
T6	1	46.5±0.2	7.40±0.03	3.3±0.8	1	0.60	-0.02	-0.02	0.03	-0.01
	2	54.2±0.2	8.62±0.03	7.0±0.9	0.78	1	-0.05	-0.03	0.02	-0.02
	3	74.59±0.03	11.87±0.01	0.82±0.05	-0.07	-0.21	-0.04	0.01	-0.01	1
	4	244±2	38.8±0.2	2.9±0.4	0.42	0.42	0.54	-1	1	0
	5	245.7±0.6	39.1±0.1	1.0±0.2	-0.24	-0.20	0.29	-0.93	1	-0.01
	6	366.4±0.9	58.3±0.2	3.3±0.6	-0.47	-0.46	1	0.04	-0.04	-0.02

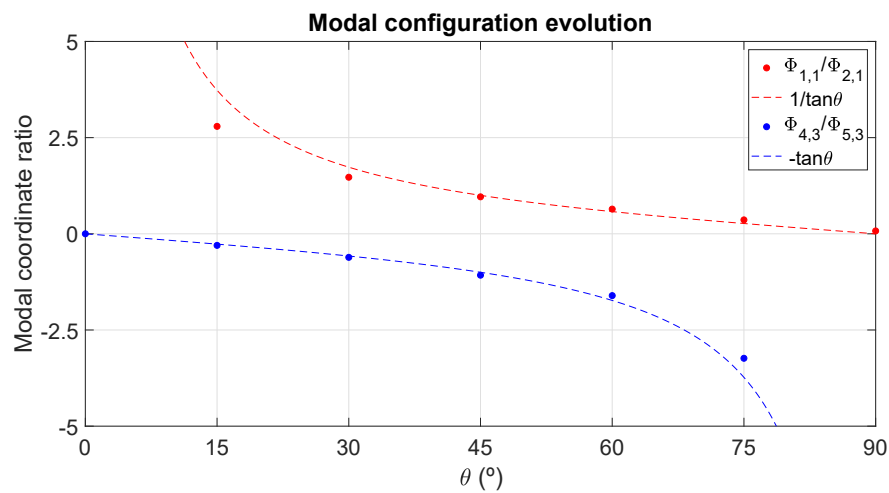


Figure 4.32: Relation between surge and sway modal coordinates for the translation mode (in red) and between the pitch and roll modal coordinates for the rotational mode (in blue).

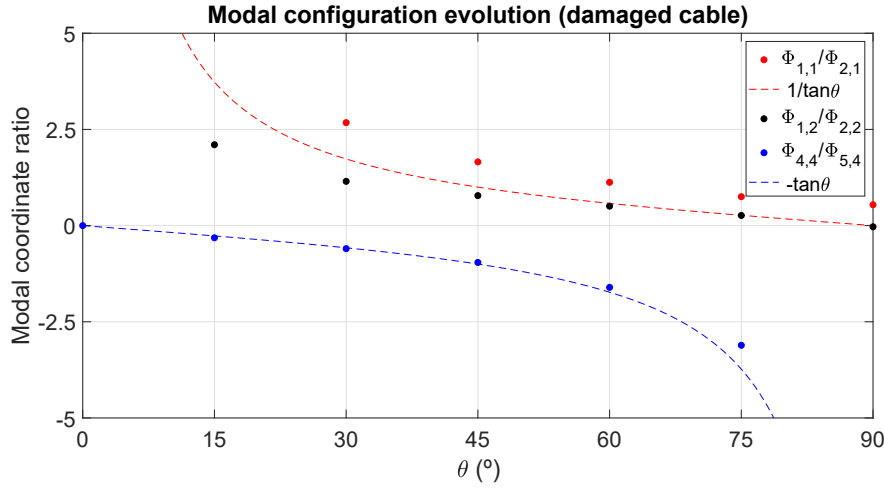


Figure 4.33: Relation between surge and sway modal coordinates for the first (in red) and second (in black) modes and between the pitch and roll modal coordinates for the rotational mode (in blue) considering the damaged cable.

If the same procedure is carried out with the modified cable structure, the translation components evolution reveals now significant deviations from the reference scenario, as can be seen in Figure 4.33, where the red and black dots represent the relation between the surge and sway modal coordinates for the first two modes, respectively, and present higher deviations from the wave incident direction.

Finally, the wave amplitude effect was evaluated. Although a white noise spectrum has been used in the presented simulations, the spectrum amplitude does depend on the significant wave height. Two additional wave series have been generated, where the spectral amplitude has been divided and multiplied by 4, being the results presented in Figure 4.34. While the natural frequen-

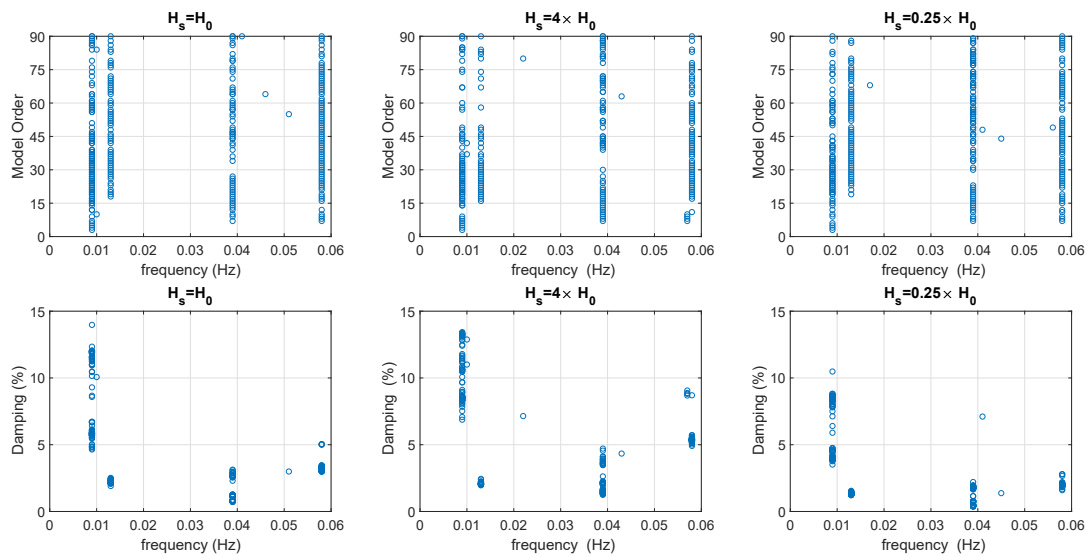


Figure 4.34: Stabilisation and damping diagrams for the different wave heights.

cies do not appear to change, it seems that the effective damping coefficients are higher for higher wave significant heights, validating the potential of OMA to identify variations of the damping coefficients with the operating conditions of the platform.

4.5 Final remarks

The presented case studies allowed to test the background concepts introduced in [Section 3.2](#) and [Section 3.3](#), as well as to explore the software that will be used in later works.

At this moment, the OpenFAST modelling approach is validated from an aerodynamic, structural and hydrodynamic point of view. Although these validations have been done separately, it may still be argued that, once used together, the model will still be able to properly reproduce the behaviour of a full scale floating offshore wind turbine. Additionally, some of the monitoring challenges mentioned in [Section 2.4.3](#) can already be identified, both in the onshore and in the offshore model. In the first case, it was clear the presence of the rotor induced harmonic response (see [Figure 4.15](#)), while in the second situation the very low frequency of the floating platform induced motions, as well as the non uniform waves frequency content could also be identified.

Finally, this Chapter ended with a preliminary application of the SSI-COV algorithm as a tool to identify and track the floating platform natural frequencies. In this case, the use of the term "preliminary" is made by noting that no special modification has been made to the usual methodology, besides a natural adaptation of the time series and correlation function length. Even in this case, the fundamental properties of the structure could be identified. However, and since experimental data is expected to be significant more challenging than this numerical, and controlled, scenario, further investigation is needed to better understand the intrinsically complex dynamic coupling of these structures. This will be made in the next Chapter, where the wind frequency sampling induced by the rotor motion will also be explored, as well some interesting relations between measurements and structural properties, in both onshore and offshore cases.

Chapter 5

Analytical considerations regarding wind turbines dynamics

5.1 Introduction

In this section, the dynamics of wind turbines, in general, and floating wind turbines, in particular, will be briefly explored from an analytical perspective. The main objective of such an analysis is to provide additional insight into the dynamic properties of the response that may not be trivial or obvious at first sight, while at the same time relating them to the measured quantities in a given experimental campaign, unveiling new possibilities for monitoring strategies. Naturally, some simplifications are needed and different simplified models will be used to explore different properties of these complex systems. Whenever suitable, the derived relations will be compared with experimental data or numerical simulations.

Firstly, the accelerations and stresses observed on the blades will be explored as a function of the different geometrical angles of the rotor. This analysis provides two relevant outputs: a new methodology to either calibrate strain gauges, or numerical models, from accelerations measurements and a procedure to obtain the time variation of the rotor rotation speed and the blades pitch, without the need for additional SCADA information. On a different approach, the dynamics of the blades will be briefly explored, namely the forces that are transmitted to the tower at the hub, that play an important role in the frequency response observed in the tower and corresponding interpretation. Finally, the dynamics of a FOWT will be explored by considering a simplified mass-spring model. This model will be essential to characterise the dynamics of the system and its consequences for Operational Modal Analysis procedures, while at the same time providing the needed theoretical framework that will sustain the proposal presented in [Chapter 7](#) for fatigue damage estimation, by relating accelerations and bending moments time series.

For most of the referred analysis, a Lagrangian formulation will be used. The Lagrangian of the system is defined from its kinetic T and potential V energy functions as:

$$\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) = T - V \tag{5.1}$$

where $\mathbf{q}$ and $\dot{\mathbf{q}}$ are the generalised degrees of freedom, used to parameterise the motion, and corresponding generalised velocities. This formulation easily incorporates the constraints by defining additional degrees of freedom and corresponding constrain equations, that are said to be holonomic if these can be written as a function of position, and eventually time, alone:

$$f_j(\mathbf{q}, t) = 0 \quad (5.2)$$

where the index j identifies a given constraint. The corresponding Euler-Lagrange equations of motion, one for each generalised degree of freedom i , are given by:

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) = \sum_j \lambda_j \frac{\partial f_j}{\partial q_i} + \sum_k \mathbf{F}_k \cdot \frac{\partial \mathbf{r}_k}{\partial q_i} \quad (5.3)$$

where the sum over j runs over all the constraint equations $f_j(q_i) = 0$ and λ_j are the Lagrange multipliers, that are equivalent to the interaction force of the corresponding constraint. The sum over k runs over all the non conservative forces $\mathbf{F}_k$, applied on the point $\mathbf{r}_k$. As a final note, it is recalled that an example of application of Lagrange's method is available in [Appendix C](#).

5.2 Analysis of blades accelerations and stresses

5.2.1 Measurements description

The experimental characterisation of the blades kinematics is commonly based on the use of tri-axial accelerometers, fixed to a inner point of the blades at a certain distance from the hub (normally constrained by accessibility). For a more completed description of the motion, it is desirable that the adopted sensors are sensitive to gravity acceleration (as it is the case of force balance and MEMS sensors) and that high pass filters are not applied to eliminate this quasi-static component. This type of sensors will be assumed in the following analysis. On the other hand, the experimental characterisation of the blades stresses is commonly preformed with strain gauges installed in the vicinity of the blade root. In a typical installation, two pairs of diametrically opposed sensors, oriented along the blade axis, are adopted for estimation of the bending moments in two orthogonal directions. Once more, this layout will be assumed in the following analysis (a monitoring project that includes these two layouts in described in [Section 4.3.1](#)).

5.2.1.1 Reference frames and rotation matrix

The position of the sensors in the blades with respect to a fixed reference frame can be defined from 4 different angles:

- The inclination of the nacelle θ
- The angular position of the blade α
- The precone angle β
- The pitch plus structural twist over the blade axis γ

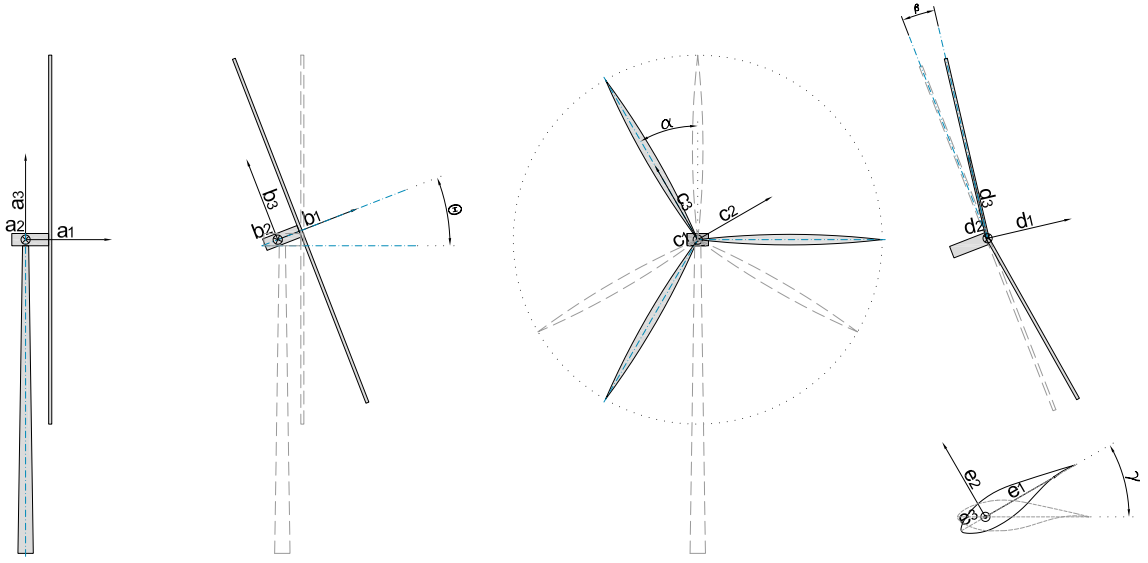


Figure 5.1: Schematic representation of the angles used to define the position of a sensor. It should be noted that, as represented, $\theta < 0$.

In the following calculations, the angles will be used in this order to transform vector quantities from one reference frame (RF) to another. The relevant reference frames, as represented in Figure 5.1, are a tower bounded RF (define by the unitary vectors $\hat{\mathbf{a}}_i$), a nacelle bounded RF ($\hat{\mathbf{b}}_i$), a RF rotated over the rotor rotation axis by the azimuth angle of a reference blade ($\hat{\mathbf{c}}_i$), a RF obtained by rotating the previously defined RF by the blade cone angle ($\hat{\mathbf{d}}_i$) and a blade-axis bounded RF that pitches with the blade ($\hat{\mathbf{e}}_i$).

Since the accelerations and strain gauges should be expressed in the blades' moving reference frame, the rotation matrices that relate a given reference frame to the next are needed, namely:

$$\mathbf{R}_1 = \begin{bmatrix} c_\theta & 0 & -s_\theta \\ 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{bmatrix}; \mathbf{R}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix}; \mathbf{R}_3 = \begin{bmatrix} c_\beta & 0 & -s_\beta \\ 0 & 1 & 0 \\ s_\beta & 0 & c_\beta \end{bmatrix}; \mathbf{R}_4 = \begin{bmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5.4)$$

where the convention $s_\delta \equiv \sin \delta$ and $c_\delta \equiv \cos \delta$ have been introduced.

5.2.1.2 Accelerations measurements

The gravitational force is more easily expressed in the $\hat{\mathbf{a}}_i$ reference frame such that, in the absence of any rotation, the measured acceleration of a given accelometer is simply:

$$\mathbf{a}_g = \mathbf{g} = g \cdot \mathbf{a}_3 = g \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}_a^T \quad (5.5)$$

where the index in the explicit vector representation has been included to clearly identify the reference frame. It is important to note that the accelerometer measurements (of sensors that are sensitive to gravity acceleration, as is being assumed to be the case) are recorded with respect to a

free falling reference frame, such that a standing still accelerometer at the Earth's surface pointing upwards will measure 9.81 m/s^2 , while a free falling one will register 0 m/s^2 .

On the other hand, the centripetal and tangential accelerations due to the rotor rotation are more easily expressed in the reference frame that moves with the blade $\hat{\mathbf{c}}_i$:

$$\mathbf{a}_c = -\Omega^2 \cdot R \cdot \cos \beta \cdot \mathbf{c}_3 = -\Omega^2 \cdot R \cdot \cos \beta \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}_c^T \quad (5.6a)$$

$$\mathbf{a}_t = -\frac{d\Omega}{dt} \cdot R \cdot \cos \beta \cdot \mathbf{c}_2 = -\frac{d\Omega}{dt} \cdot R \cdot \cos \beta \cdot \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}_c^T \quad (5.6b)$$

The value measured by an accelerometer can then be obtained considering and transforming the different contributions as:

$$\mathbf{a}_m|_e = \mathbf{R}_4 \cdot \mathbf{R}_3 \cdot (\mathbf{R}_2 \cdot \mathbf{R}_1 \cdot \mathbf{a}_g|_a + \mathbf{a}_c|_c + \mathbf{a}_t|_c) \quad (5.7)$$

The sensitivity of these measurements to variations of the different parameters is represented in [Figure 5.2](#), [Figure 5.3](#) and [Figure 5.4](#), where two out of the nacelle tilt, blade cone and blade pitch angles have been fixed and the remaining one varied within a certain range. It can be seen that the accelerations time series, when any combination of 2 channels is considered, describe an ellipsis that may be written, in its most general form, as:

$$\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) (x - x_c)^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) (y - y_c)^2 + 2c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) (x - x_c)(y - y_c) = 1 \quad (5.8)$$

where x_c and y_c are the centre point coordinates, μ is the rotation angle as measured from the positive x semi-axis and a and b are the ellipsis semi-axis associated to the axis obtained after the rotation by the angle $-\mu$. As will be seen, these parameters, that can be measured in the field, can be related with the rotor geometrical properties.

5.2.1.3 Strain gauges measurements

Once the accelerations time series have been explored, it may be noted that the same model can be used to relate the blades' position with its internal forces. In this case, and seeking to provide the theoretical basis for the sensors calibration procedures usually carried out for blades' monitoring ([IEC, 2015](#)), it will be assumed that the rotor is parked. In any case, the axial force due to the blades' self-weight is better described as:

$$N_{sw} = \mathbf{P} \cdot \mathbf{e}_3 = -P \cdot \mathbf{a}_3 \cdot \mathbf{e}_3 \quad (5.9)$$

where P is the blade weight from the instrumented section to the blade's tip. Neglecting the wind forces, the corresponding bending moments may be written as:

$$\boldsymbol{\tau} = \int_{r_i}^{r_{tip}} \mathbf{r} \times \mathbf{p}(r) \cdot dr = -r_{cg} P (\mathbf{e}_3 \times \mathbf{a}_3) \quad (5.10)$$

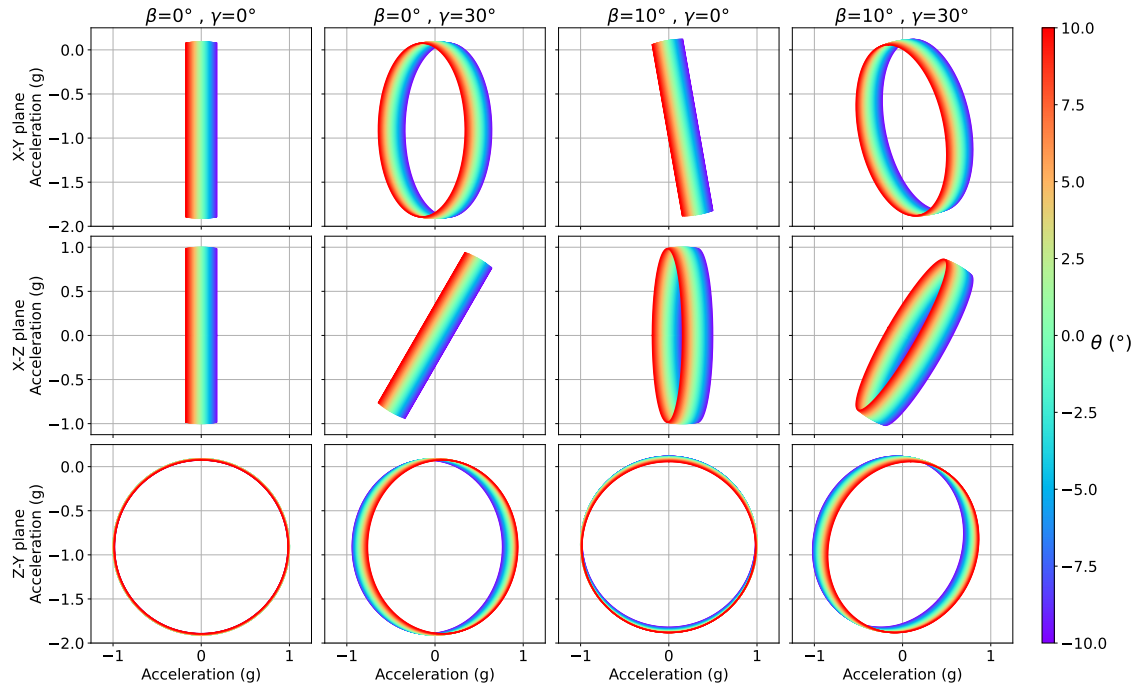


Figure 5.2: Relation between different blades' measurements over a full revolution as a function of the nacelle tilt (θ), considering different combinations of the blades' pre-cone (β) and pitch (γ) angles. The x , y and z directions are defined by the blade-fixed reference frame ($\hat{e}_i$).

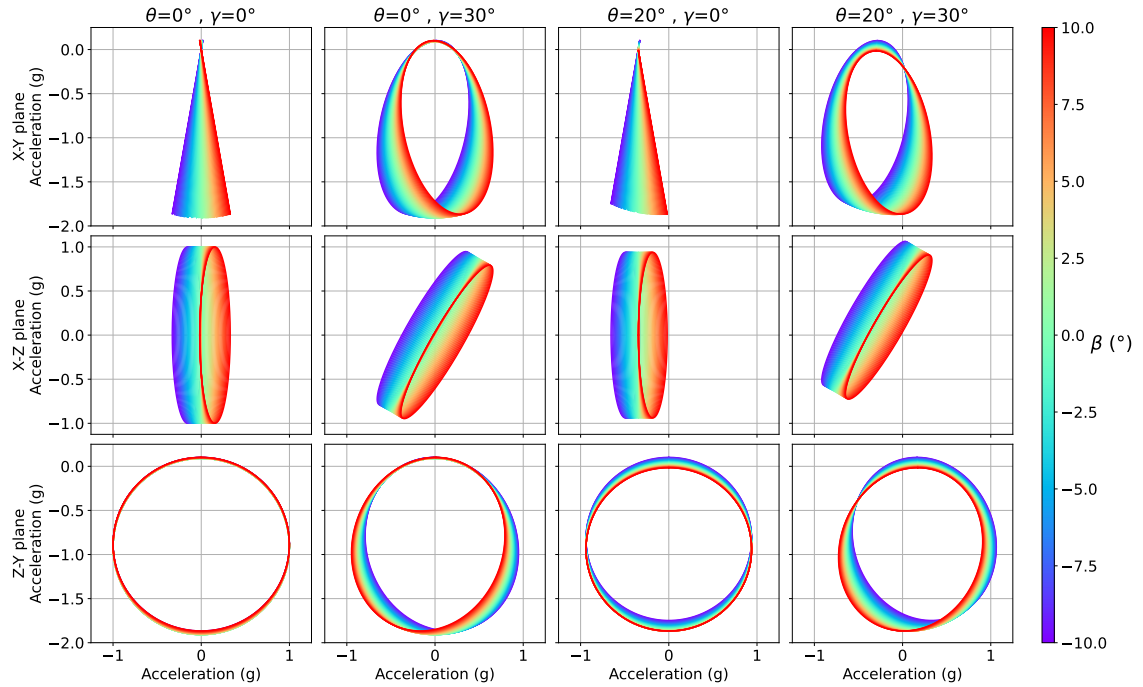


Figure 5.3: Relation between different blades' measurements over a full revolution as a function of the blade precone angle (β), considering different combinations of the nacelle tilt (θ) and blade pitch (γ) angles. The x , y and z directions are defined by the blade-fixed reference frame ($\hat{e}_i$).

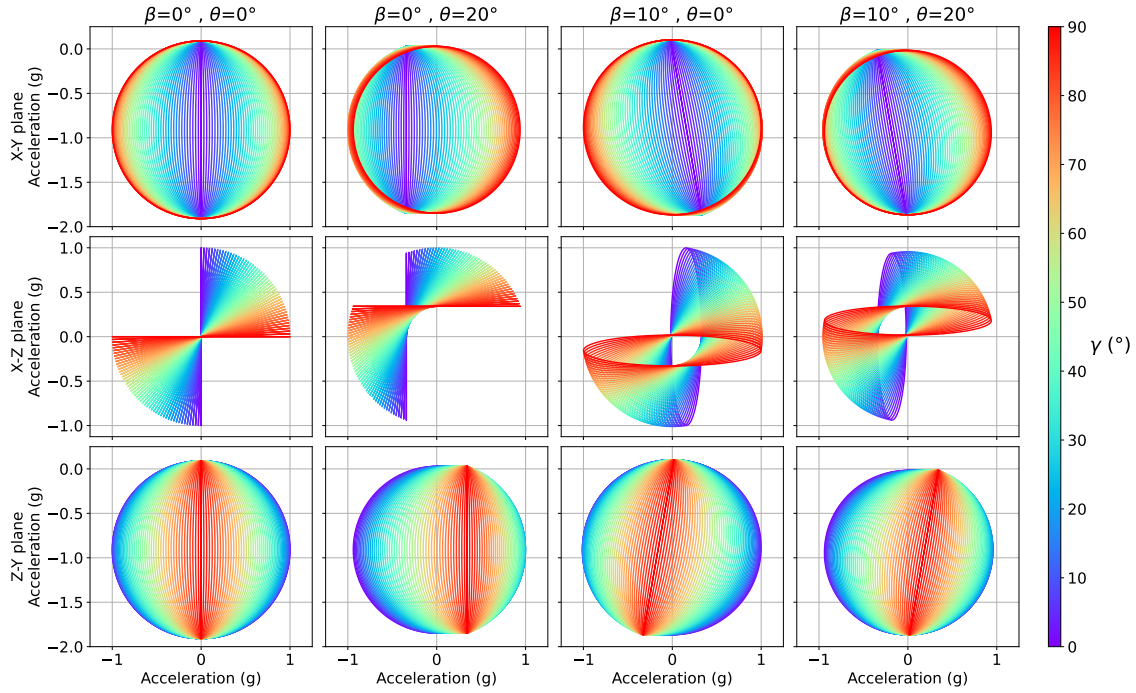


Figure 5.4: Relation between different blades' measurements over a full revolution as a function of the blade pitch angle, considering different combinations of the blades' pre-cone (β) and nacelle tilt (θ) angles. The x , y and z directions are defined by the blade-fixed reference frame ($\hat{\mathbf{e}}_i$).

where $\mathbf{p}(r) \equiv p(r)\mathbf{a}_3$ is the blade's weight per unit length and the blade's centre of gravity r_{cg} has been implicitly defined. Having defined the internal forces acting in the section, the scalar and cross products needed to express them as a function of the rotor angles are:

$$\mathbf{a}_3 \cdot \mathbf{e}_3 = -s_\beta s_\theta + c_\beta c_\alpha c_\theta \quad (5.11a)$$

$$\mathbf{e}_3 \times \mathbf{a}_3 = [c_\theta c_\gamma s_\alpha + s_\gamma (c_\beta s_\theta + s_\beta c_\theta c_\alpha)] \mathbf{e}_1 + [c_\gamma (c_\beta s_\theta + s_\beta c_\theta c_\alpha) - c_\theta s_\gamma s_\alpha] \mathbf{e}_2 \quad (5.11b)$$

Substituting back in Equation 5.9 and Equation 5.10, the total axial force and bending moment are given by:

$$\frac{N}{P} = s_\beta s_\theta - c_\beta c_\alpha c_\theta \quad (5.12a)$$

$$\frac{\tau}{r_{cg}P} = [c_\theta c_\gamma s_\alpha + s_\gamma (c_\beta s_\theta + s_\beta c_\theta c_\alpha)] \mathbf{e}_1 + [c_\gamma (c_\beta s_\theta + s_\beta c_\theta c_\alpha) - c_\theta s_\gamma s_\alpha] \mathbf{e}_2 \quad (5.12b)$$

Before proceeding any further, it is convenient to define some particular angular position of the blade that will play a central role in the calibration method proposed. Let $^i, ^{ii}, ^{iii}$, denote the internal forces for $\alpha = 0$, $\alpha = \frac{2\pi}{3}$ and $\alpha = \frac{4\pi}{3}$, respectively. The corresponding axial forces are given by:

$$N^i = P(s_\beta s_\theta - c_\beta c_\theta) \quad , \quad N^{ii} = P\left(s_\beta s_\theta + \frac{1}{2}c_\beta c_\theta\right) \quad , \quad N^{iii} = P\left(s_\beta s_\theta + \frac{1}{2}c_\beta c_\theta\right) \quad (5.13)$$

while the corresponding bending moments are by:

$$\frac{M_x^i}{r_{cg}P} = s_\gamma (c_\beta s_\theta + s_\beta c_\theta) \quad \frac{M_y^i}{r_{cg}P} = c_\gamma (c_\beta s_\theta + s_\beta c_\theta) \quad (5.14a)$$

$$\frac{M_x^{ii}}{r_{cg}P} = s_\gamma \left(c_\beta s_\theta - \frac{1}{2} s_\beta c_\theta \right) + \frac{\sqrt{3}}{2} c_\theta c_\gamma \quad \frac{M_y^{ii}}{r_{cg}P} = c_\gamma \left(c_\beta s_\theta - \frac{1}{2} s_\beta c_\theta \right) - \frac{\sqrt{3}}{2} c_\theta s_\gamma \quad (5.14b)$$

$$\frac{M_x^{iii}}{r_{cg}P} = s_\gamma \left(c_\beta s_\theta - \frac{1}{2} s_\beta c_\theta \right) - \frac{\sqrt{3}}{2} c_\theta c_\gamma \quad \frac{M_y^{iii}}{r_{cg}P} = c_\gamma \left(c_\beta s_\theta - \frac{1}{2} s_\beta c_\theta \right) + \frac{\sqrt{3}}{2} c_\theta s_\gamma \quad (5.14c)$$

where the subscript x and y have been used to identify the bending moment component along the corresponding axis.

Having expressed the loads acting on a given section from the blade's self-weight and the geometrical angles defined in [Figure 5.1](#), these may now be related to the strains measurements, that, for a given sensor, are given by:

$$\varepsilon = \varepsilon(x, y) = \frac{M_x}{EI_x} y + \frac{M_y}{EI_y} x + \frac{N}{EA} \quad (5.15)$$

where x and y are the sensor location in the section plane. For an homogeneous and circular cross section with radius r , the expression above can be reduced to:

$$\varepsilon = \frac{M}{EI} r + \frac{N}{EA} \quad (5.16)$$

where now $M = \sqrt{M_x^2 + M_y^2}$ is the total bending moment acting on the instrumented section.

On the other hand, the pitch angle γ introduced before is defined by the sensors position, in such a way that a pair of sensors will only detect M_x , while the other pair will measure M_y . Naturally, all of them measure the axial force. Finally, each one of the sensors may exhibit an unknown offset, such that the strain measurements are given in its most general form by:

$$\varepsilon_{1/4} = C_{1/4} + \frac{N}{EA} - \frac{M_{y/x}}{E \omega_{y/x}} \quad (5.17a)$$

$$\varepsilon_{2/3} = C_{2/3} + \frac{N}{EA} + \frac{M_{y/x}}{E \omega_{y/x}} \quad (5.17b)$$

which implies that, for any given time instant:

$$\varepsilon_{1/3} + \varepsilon_{2/4} = C_{1/3} + C_{2/4} + 2 \frac{N}{EA} \quad (5.18a)$$

$$\varepsilon_{2/3} - \varepsilon_{1/4} = C_{2/3} - C_{1/4} + 2 \frac{M_{y/x}}{E \omega_{y/x}} \quad (5.18b)$$

Relating these quantities to experimental measurements will be the goal of [Section 5.2.4](#).

5.2.2 Wind turbine geometry characterisation

The first contribution based on the blades accelerations measurements that will be presented is the possibility of determining the rotor geometrical angles (nacelle tilt, blades precone and pitch) from experimental measurements alone. This is particularly relevant for the development of numerical models of structures that are being instrumented when confidentiality issues limit the amount of data available. For this particular analysis, only setups with particular conditions (that may be identified from the measurements alone) will be used, a restriction that will then be relaxed.

By requiring the rotor to rotate with a constant angular velocity, a corresponding constant term is originated by the centripetal acceleration, given by:

$$\mathbf{a}_c = -\Omega^2 R \begin{bmatrix} -c_\gamma s_\beta c_\beta & s_\gamma s_\beta c_\beta & c_\beta^2 \end{bmatrix}^T \quad (5.19)$$

where from now on the index in the vector explicit representation is omitted since it is implicitly assumed a representation in the local sensors reference frame. On the other hand, an additional constant term and a time varying component have their origin in the gravitational pull since:

$$\mathbf{a}_g = g \begin{bmatrix} -c_\gamma c_\beta s_\theta + (s_\gamma s_\alpha - c_\gamma s_\beta c_\alpha) c_\theta & s_\gamma c_\beta s_\theta + (c_\gamma s_\alpha + s_\gamma s_\beta c_\alpha) c_\theta & -s_\beta s_\theta + c_\beta c_\alpha c_\theta \end{bmatrix}^T \quad (5.20)$$

Defining the constant terms, a_i^0 , as:

$$a_x^0 = -c_\gamma c_\beta s_\theta + \frac{\Omega^2 R}{g} c_\gamma s_\beta c_\beta, \quad a_y^0 = s_\gamma c_\beta s_\theta - \frac{\Omega^2 R}{g} s_\gamma s_\beta c_\beta, \quad a_z^0 = -s_\beta s_\theta - \frac{\Omega^2 R}{g} c_\beta^2 \quad (5.21)$$

the ellipsis equation for the different directions measurements combinations can be derived. In what follows, this derivation will be detailed for the xy plane, while the equivalent expressions for the remaining planes will only be presented for completeness, without further derivations. For the xy plane, it may be written:

$$\begin{aligned} & \left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) (a_x - a_x^0)^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) (a_y - a_y^0)^2 + 2c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) (a_x - a_x^0) (a_y - a_y^0) = \\ & = c_\alpha^2 \left\{ c_\theta^2 \left[\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) c_\gamma^2 s_\beta^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) s_\gamma^2 s_\beta^2 - 2c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) c_\gamma s_\gamma s_\beta^2 \right] \right\} + \\ & + s_\alpha^2 \left\{ c_\theta^2 \left[\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) s_\gamma^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) c_\gamma^2 + 2c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) s_\gamma c_\gamma \right] \right\} + \\ & + c_\alpha s_\alpha \left\{ 2c_\theta^2 s_\beta \left[s_\gamma c_\gamma \left[\left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) - \left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) \right] + c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) (1 - 2c_\gamma^2) \right] \right\} \end{aligned} \quad (5.22)$$

Matching the different terms with the ones in [Equation 5.8](#), the different parameters can be constrained by requiring that the coefficients multiplying $\sin^2 \alpha$ and $\cos^2 \alpha$ must equal 1 independently

and that the coefficients multiplying $\sin \alpha \cos \alpha$ must vanish, leading to:

$$x_c = a_x^0 = \Omega^2 R c_\gamma s_\beta c_\beta - g (c_\gamma c_\beta s_\theta) \quad (5.23a)$$

$$y_c = x_y^0 = -\Omega^2 R s_\gamma s_\beta c_\beta + g (s_\gamma c_\beta s_\theta) \quad (5.23b)$$

$$\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) c_\gamma^2 s_\beta^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) s_\gamma^2 s_\beta^2 - 2c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) c_\gamma s_\gamma s_\beta^2 = \frac{1}{g^2 c_\theta^2} \quad (5.23c)$$

$$\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) s_\gamma^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) c_\gamma^2 + 2c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) s_\gamma c_\gamma = \frac{1}{g^2 c_\theta^2} \quad (5.23d)$$

$$s_\gamma c_\gamma \left[\left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) - \left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) \right] + c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) (s_\gamma^2 - c_\gamma^2) = 0 \quad (5.23e)$$

At this point a special remark should be made regarding the accelerometers uncertainties. The response of such an equipment can be parameterised as:

$$a_m = S \times a_r + O \quad (5.24)$$

where S is the sensitivity and O the offset that map the real acceleration a_r to the measured value a_m . While it may be assumed that $S \sim 1$ for well calibrated devices, it might not be that $O \sim 0$ (as discussed by [Hoffmann, 2012](#), it is common the existence of small variations of the sensor offset over time). From the equations above, it can be seen that the offset condition impacts the coordinates of the ellipsis centre, while the sensitivity impacts both the centre and the ellipsis parameters. Since only the errors on the sensitivity are being neglected, it can be seen that although the centre point coordinates could theoretically be used to estimate the needed parameters, these are not reliable, so only differential measurements should be used. Having set this clear, it can be noted that [Equation 5.23e](#) can readily be used to provide a direct estimate for γ since:

$$s_\gamma c_\gamma \left(\frac{1}{a^2} - \frac{1}{b^2} \right) (s_\mu^2 - c_\mu^2) = -c_\mu s_\mu \left(\frac{1}{a^2} - \frac{1}{b^2} \right) (s_\gamma^2 - c_\gamma^2) \longrightarrow \mu = -\gamma \quad (5.25)$$

The pitch angle (plus the structural twist) is then simply the inclination of the ellipse when evaluated in the xy plane. Using this conclusion, [Equation 5.23c](#) and [Equation 5.23d](#) above can be rewritten as:

$$s_\beta^2 \left[\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) c_\mu^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) s_\mu^2 + 2c_\mu^2 s_\mu^2 \left(\frac{1}{a^2} - \frac{1}{b^2} \right) \right] = \frac{s_\beta^2}{a^2} = \frac{1}{g^2 c_\theta^2} \quad (5.26a)$$

$$\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) s_\mu^2 + \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) c_\mu^2 - 2c_\mu^2 s_\mu^2 \left(\frac{1}{a^2} - \frac{1}{b^2} \right) = \frac{1}{b^2} = \frac{1}{g^2 c_\theta^2} \quad (5.26b)$$

that can be combined to yield another two direct estimates, this time for β and θ :

$$s_\beta^2 = \frac{a^2}{b^2} \quad (5.27a)$$

$$c_\theta^2 = \frac{b^2}{g^2} \quad (5.27b)$$

The expressions above show that the precone angle can be obtained by the ratio between the major and minor axis of the xy plane ellipse, while the tilt can be computed from the ratio between one of the semi-axis and the gravitational acceleration.

As mentioned before, for completeness and without further derivation, the ellipsis parameters in the xz are related to the geometrical angles by:

$$\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) = \frac{1}{g^2 c_\theta^2 s_\gamma^2}, \quad \left(\frac{c_\mu^2 b^2 + s_\mu^2 a^2}{c_\mu s_\mu b^2 - c_\mu s_\mu a^2} \right) = \frac{c_\beta}{c_\gamma s_\beta}, \quad \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) = \frac{s_\gamma^2 + c_\gamma^2 s_\beta^2}{g^2 c_\theta^2 c_\beta^2 s_\gamma^2} \quad (5.28)$$

while for yz plane it may shown that:

$$\left(\frac{c_\mu^2}{a^2} + \frac{s_\mu^2}{b^2} \right) = \frac{1}{g^2 c_\theta^2 c_\gamma^2}, \quad \left(\frac{c_\mu^2 b^2 + s_\mu^2 a^2}{c_\mu s_\mu b^2 - c_\mu s_\mu a^2} \right) = -\frac{c_\beta}{s_\gamma s_\beta}, \quad \left(\frac{s_\mu^2}{a^2} + \frac{c_\mu^2}{b^2} \right) = \frac{c_\gamma^2 + s_\gamma^2 s_\beta^2}{g^2 c_\theta^2 c_\beta^2 c_\gamma^2} \quad (5.29)$$

respectively. As can be seen, both sets of equations are more cumbersome and do not provide as direct estimates as the ones in the xy plane. From this point on, only the latter will be used.

Validation

To apply and test the method proposed above that makes use of the accelerations time series, the data from the Tocha's wind farm ([Magalhães et al., 2019](#)) was used. This includes data from 5 accelerometers distributed in two wind turbines (WT1 with the 3 blades instrumented and WT5 with two blades instrumented) and 10-min averaged SCADA data. Firstly, time periods where the rotor was parked have been selected by identifying regions where the accelerations measurements were constant with a time length of at least 1 minute. Additionally, it should be noted that, as long as the pitch angle is the same, all the selected time periods should be possible to fit within a single ellipse. The results are presented in [Figure 5.5](#), while from the fitted parameters, the geometrical angles can be immediately estimated:

$$s_\beta^2 = \left(\frac{0.763}{9.443} \right)^2 \longrightarrow \beta = 4.7^\circ \quad (5.30a)$$

$$c_\theta^2 = \left(\frac{9.479}{9.806} \right)^2 \longrightarrow \theta = -14.8^\circ \quad (5.30b)$$

$$\mu = -\gamma = 35.2^\circ \quad (5.30c)$$

The pitch angle obtained above is not particularly relevant since it only represents the sensors particular position on the blade and should not be read as the SCADA equivalent pitch value.

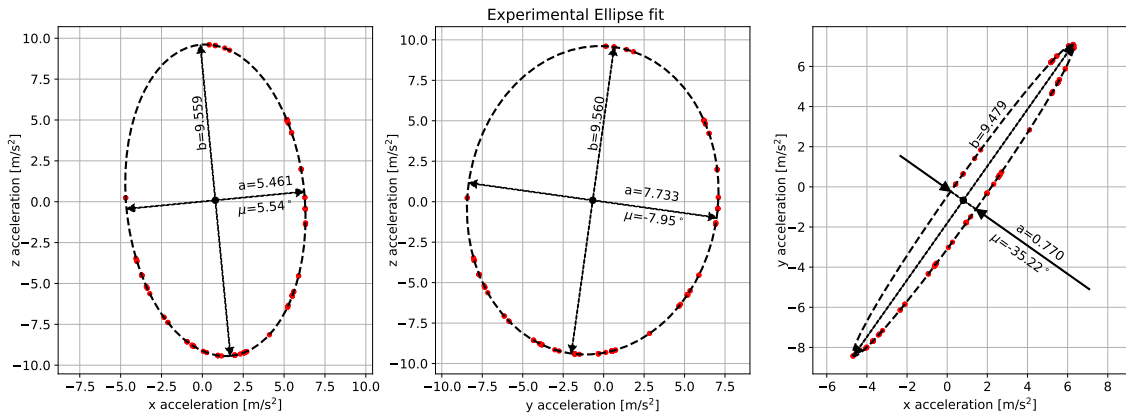


Figure 5.5: Measured data (red points) and fitted ellipse (black dashed line) for parked time periods selected from WT5 Blade 2 data.

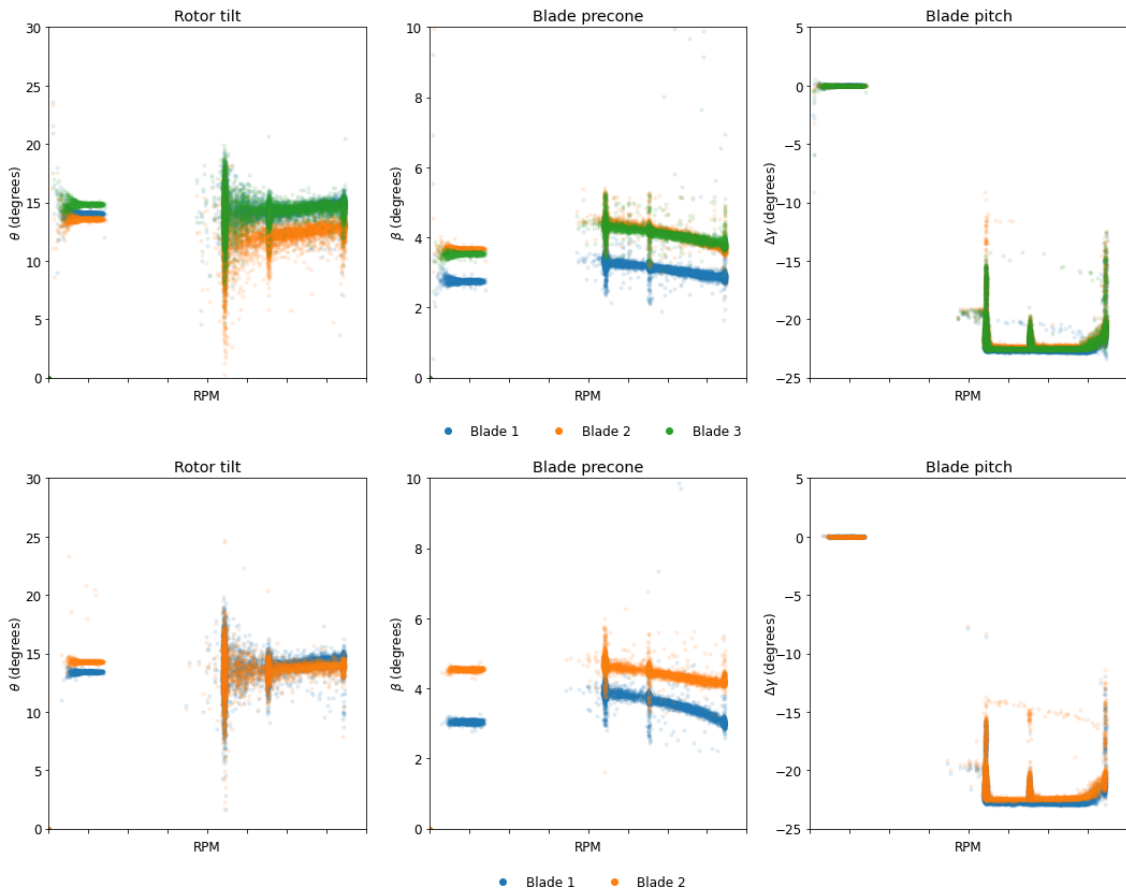


Figure 5.6: Rotor tilt, blade precone and pitch angle for different values of the rotor angular velocity for WT1 (on top) and WT5 (on bottom).

To study how these parameters evolve with the operating conditions, the parked rotor scenario has been dropped, while still demanding for constant angular velocities. The evolution of each angle with the rotor angular velocity are plotted in Figure 5.6, where have been included results for all the blades of both wind turbines and where have been plotted not the pitch angle value, but its variation, since the reference value has no particularly significant meaning as it is related to the sensors location on the blade. Although the expected pitch curve is successfully reproduced, the tilt and precone dependencies still need further explanations. In fact, it can be seen that for regions of constant pitch angle, the tilt increases with the rotor speed, and hence with the wind speed, while the precone decreases. Both these results are expected. However, some coupling between the pitch angle and both predictions can also be identified, in particular in the discontinuity between the estimations for very low wind speeds and operating conditions. This may be explained since the measurements are not being made on the blades centre, but on its surface. Nevertheless, the parameters needed for a numerical model can still be identified as the almost constant values obtained for parked conditions, since the wind induced behaviour should be properly reproduced when the structural response under different loading conditions is considered. Furthermore, small variations of about 1° of the averaged pre-cone angle observed for different blades might be explained by the sensor installation (the sensor longitudinal axis might not be perfectly parallel with the blade longitudinal axis) and this will most certainly not have an important impact on the modelling results.

5.2.3 Operating conditions estimates

Once the geometrical angles are constrained (or known *a priori*), every time series may be used to recover the angular velocity and pitch angle. This information is once more hardly available (the more easily available SCADA data is limited to a time resolution of 10 minutes), but can be used to gain additional insight into the operation conditions of the wind turbine that can significantly enhance the interpretation of the monitoring data.

Firstly, let the data be cleaned such that only the component directly related to the rotor rotation is kept, which may be easily done by applying a calibrated low pass filter to the data. After this step, it can be noted that the following relations hold:

$$a_x = g [-c_\gamma c_\beta s_\theta + (s_\gamma s_\alpha - c_\gamma s_\beta c_\alpha) c_\theta] + \Omega^2 R c_\gamma s_\beta c_\beta - a_t s_\gamma c_\beta \quad (5.31a)$$

$$a_y = g [s_\gamma c_\beta s_\theta + (c_\gamma s_\alpha + s_\gamma s_\beta c_\alpha) c_\theta] - \Omega^2 R s_\gamma s_\beta c_\beta - a_t c_\gamma c_\beta \quad (5.31b)$$

$$a_z = g [-s_\beta s_\theta + c_\beta c_\alpha c_\theta] - \omega^2 R c_\beta^2 = g [c_\beta c_\alpha c_\theta - s_\beta s_\theta] - \Omega^2 R c_\beta^2 \quad (5.31c)$$

The difference between consecutive records measured at times t_i and $t_j = t_i + \delta t$ are given by:

$$a_{x_j} - a_{x_i} \approx g c_\theta s_\gamma \Omega \delta t c_\alpha + g c_\theta c_\gamma s_\beta \omega \delta t s_\alpha \quad (5.32a)$$

$$a_{y_j} - a_{y_i} \approx g c_\theta c_\gamma \Omega \delta t c_\alpha - g c_\theta s_\gamma s_\beta \omega \delta t s_\alpha \quad (5.32b)$$

$$a_{z_j} - a_{z_i} \approx g c_\theta c_\beta (c_{\alpha_j} - c_{\alpha_i}) = -g c_\theta c_\beta \Omega \delta t s_\alpha \quad (5.32c)$$

where it has been used that $\delta\alpha = \Omega\delta t$ and:

$$\sin(\alpha + \delta\alpha) = \sin(\alpha)\cos(\delta\alpha) + \cos(\alpha)\sin(\delta\alpha) \approx \sin\alpha + \delta\alpha\cos(\alpha) \quad (5.33a)$$

$$\cos(\alpha + \delta\alpha) = \cos(\alpha)\cos(\delta\alpha) - \sin(\alpha)\sin(\delta\alpha) \approx \cos(\alpha) - \delta\alpha\sin(\alpha) \quad (5.33b)$$

The expressions above can all be combined to give an estimate for the rotor angular velocity (Ω) and the blade's pitch angle (γ):

$$\Omega = \frac{1}{gc_\theta\delta t} \sqrt{(a_{x_j} - a_{x_i})^2 + (a_{y_j} - a_{y_i})^2 + (a_{z_j} - a_{z_i})^2} \quad (5.34a)$$

$$c_\gamma(a_{x_j} - a_{x_i}) - s_\gamma(a_{y_j} - a_{y_i}) = gc_\theta\omega\delta ts_\beta s_\alpha = -t_\beta(a_{z_j} - a_{z_i}) \quad (5.34b)$$

Finally, it should be noted that Equation 5.34a can be used to identify constant RPM setups, or even parked rotor scenarios, even before fitting the value of any of the geometrical angles at play.

Validation

Once more, the experimental data from Tocha wind farm has been used to illustrate the relations and SCADA recovery process derived above.

Firstly, a single setup has been selected where varying operating conditions could be easily identified. For this particular setup, the 10 minutes resolution SCADA data registered a rotor angular velocity between 2.3 and 10.1, with a mean value of 4.3 RPM, and a pitch angle between -3.6° and 20.1° , with a mean value of 13.6° . After applying Equation 5.34a and Equation 5.34b, the full pitch and rotor angular velocity time series have been reconstructed as represented in Figure 5.7, where it should be recalled that the absolute value of the pitch value is not significant, only its variation. The distinct operation conditions are easily identifiable in the accelerations time series, and are also present in the reconstructed SCADA time series. The maximum and minimum values of the RPM are validated, the total pitch variation is validated and the mean RPM value is also confirmed.

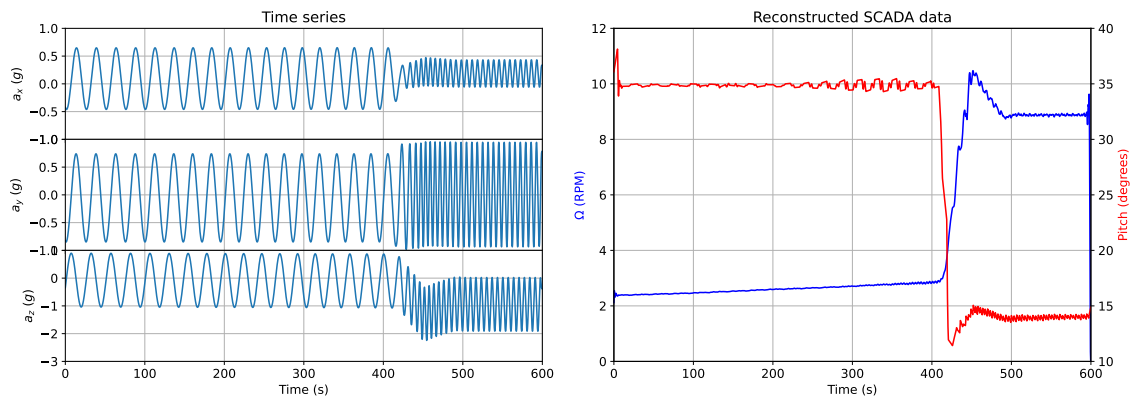


Figure 5.7: Reconstructed SCADA data for a single setup.

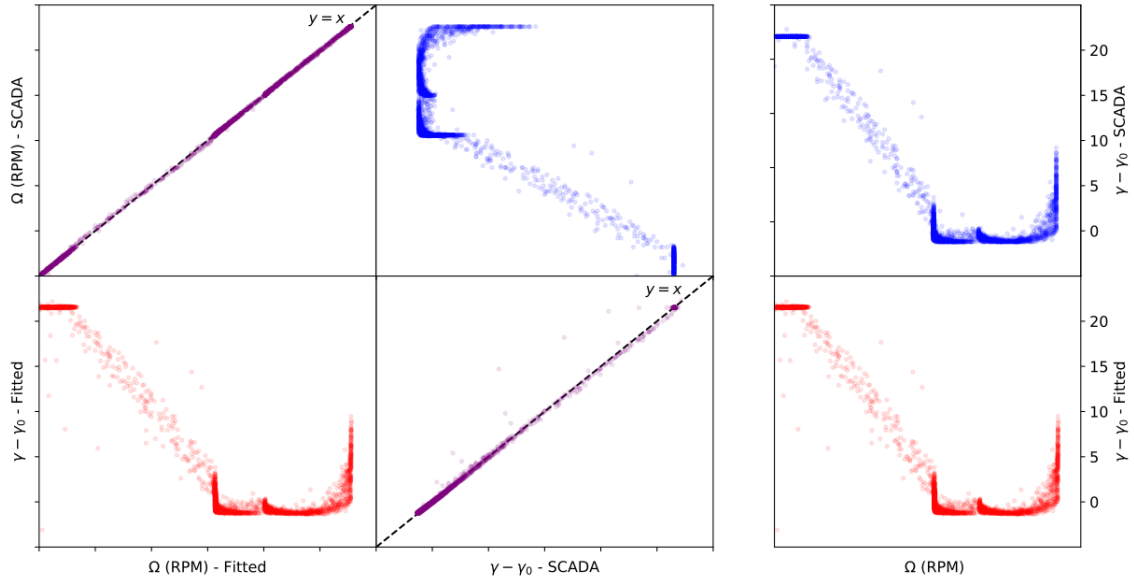


Figure 5.8: Reconstructed SCADA full data. On the left, the comparison between estimated (red) and recorded (blue) values. On the right, the pitch curve reconstructed against the SCADA one.

Finally, a full month of data has been analysed and the estimates for the mean value of the rotor angular velocity and pitch value compared with the SCADA ones. The results are plotted in Figure 5.8. As can be seen, the predictions closely match the registered values in the SCADA system and hence both the expressions and the methodology are validated.

5.2.4 Strain gauges calibration

Finally, another important issue that is present in most experimental campaign that involve the direct estimate of internal forces from strain gauges measurements is the need to calibrate these sensors, for two main reasons:

1. The stress state at the installation is not known, which introduces an arbitrary offset in the strains reading;
2. Strain gauges tend to exhibit some low frequency drifts over time that are not meaningful and not related to any relevant physical process (Hoffmann, 2012).

If the calibration of the tower sensors, as will be described later on, makes good use of the nacelle full rotations for cable untwist events, the same reasoning cannot be applied to the blades sensors. Most often, the blades sensors calibrations are made based on the measurements made at particular blades positions and assumptions for needed blades properties, but a more robust theoretical framework is needed to make this procedure more reliable.

Combining Equation 5.13 and Equation 5.14 with Equation 5.18 allows to effectively determine the section properties. Firstly, it should be noted that:

$$(\epsilon_1 + \epsilon_2)^i - \frac{1}{2}(\epsilon_1 + \epsilon_2)^{ii} - \frac{1}{2}(\epsilon_1 + \epsilon_2)^{iii} = \frac{1}{EA} (2N^i - N^{iii} - N^{iii}) = -\frac{3P}{EA} c_\beta c_\theta \quad (5.35)$$

which may be inverted to yield the first predictive equation for the section axial stiffness:

$$EA = -\frac{6}{2(\varepsilon_1 + \varepsilon_2)^i - (\varepsilon_1 + \varepsilon_2)^{ii} - (\varepsilon_1 + \varepsilon_2)^{iii}} P c_\beta c_\theta \quad (5.36)$$

One the other hand, it should also be noted that:

$$(\varepsilon_2 - \varepsilon_1)^{ii} - (\varepsilon_2 - \varepsilon_1)^{iii} = \frac{2}{E \omega_y} (M_y^{ii} - M_y^{iii}) = -\frac{\sqrt{12}}{E \omega_y} r_{cg} P c_\theta s_\gamma \quad (5.37a)$$

$$(\varepsilon_3 - \varepsilon_4)^{ii} - (\varepsilon_3 - \varepsilon_4)^{iii} = \frac{2}{E \omega_x} (M_x^{ii} - M_x^{iii}) = \frac{\sqrt{12}}{E \omega_x} r_{cg} P c_\theta c_\gamma \quad (5.37b)$$

where it has been tacitly assumed that the pitch angle does not vary significantly between the 2 positions. The expressions above can be used to obtain the value of $E \omega_x$ and $E \omega_y$ independently, as long as the pitch angles of the sensors location is known. If this is not the case, but now assuming a circular section (as it is usual at the blade root), summing the square of both expressions in [Equation 5.37](#) allows to remove the pitch dependency:

$$\left[(\varepsilon_2 - \varepsilon_1)^{ii} - (\varepsilon_2 - \varepsilon_1)^{iii} \right]^2 + \left[(\varepsilon_3 - \varepsilon_4)^{ii} - (\varepsilon_3 - \varepsilon_4)^{iii} \right]^2 = \frac{12}{E^2 \omega^2} r_{cg}^2 P^2 c_\theta^2 \quad (5.38)$$

that may once more be inverted to yield another predictive equation, this time for the section bending stiffness:

$$E \omega = \sqrt{\frac{12}{\left[(\varepsilon_2 - \varepsilon_1)^{ii} - (\varepsilon_2 - \varepsilon_1)^{iii} \right]^2 + \left[(\varepsilon_3 - \varepsilon_4)^{ii} - (\varepsilon_3 - \varepsilon_4)^{iii} \right]^2}} r_{cg} P c_\theta \quad (5.39)$$

If until this point a way to obtain the instrumented section properties from experimental data and global mass properties of the blade have been derived, it may be noted that the same expressions can be used to provide a new method to properly calibrate the strain gauges measurements. Firstly, as already mentioned, the measurements in any section at any given time instant should respect:

$$\varepsilon_1 + \varepsilon_2 - (C_1 + C_2) = \varepsilon_3 + \varepsilon_4 - (C_3 + C_4) \longrightarrow C_1 + C_2 = C_3 + C_4 + \varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 \quad (5.40a)$$

Additionally, and now considering the different time instants defined by the sections in [Equation 5.13](#) and [Equation 5.14](#), it may also be written:

$$\frac{1}{2} \left(\varepsilon_{1/4}^{ii} + \varepsilon_{2/3}^{ii} + \varepsilon_{1/4}^{iii} + \varepsilon_{2/3}^{iii} \right) = C_{1/4} + C_{2/3} + \frac{1}{EA} (2s_\beta s_\theta + c_\beta c_\theta) P \quad (5.41)$$

On the other hand, it is also true that:

$$\frac{1}{2} (\varepsilon_2^{iii} - \varepsilon_1^{iii} + \varepsilon_2^{ii} - \varepsilon_1^{ii}) = C_2 - C_1 + \frac{1}{E \omega_y} r_{cg} P c_\gamma (2c_\beta s_\theta - s_\beta c_\theta) \quad (5.42a)$$

$$\frac{1}{2} (\varepsilon_3^{iii} - \varepsilon_4^{iii} + \varepsilon_3^{ii} - \varepsilon_4^{ii}) = C_3 - C_4 + \frac{1}{E \omega_x} r_{cg} P s_\gamma (2c_\beta s_\theta - s_\beta c_\theta) \quad (5.42b)$$

Since the mechanical properties of the section have already been expressed from the experimental data and may be conveniently rewritten as:

$$\frac{P}{EA} (2s_\beta s_\theta + c_\beta c_\theta) = \frac{(\varepsilon_1 + \varepsilon_2)^{ii} + (\varepsilon_1 + \varepsilon_2)^{iii} - 2(\varepsilon_1 + \varepsilon_2)^i}{6} (1 + 2t_\beta t_\theta) \quad (5.43a)$$

$$\frac{r_{cg} P s_\gamma}{E \omega} (2c_\beta s_\theta - s_\beta c_\theta) = \frac{1}{\sqrt{12}} \left[(\varepsilon_2 - \varepsilon_1)^{iii} - (\varepsilon_2 - \varepsilon_1)^{ii} \right] (2c_\beta t_\theta - s_\beta) \quad (5.43b)$$

$$\frac{r_{cg} P c_\gamma}{E \omega} (2c_\beta s_\theta - s_\beta c_\theta) = \frac{1}{\sqrt{12}} \left[(\varepsilon_3 - \varepsilon_4)^{ii} - (\varepsilon_3 - \varepsilon_4)^{iii} \right] (2c_\beta t_\theta - s_\beta) \quad (5.43c)$$

the expressions above can also be rearranged to yield:

$$C_{1/2} = \frac{1}{2} (\varepsilon_{1/2}^{ii} + \varepsilon_{1/2}^{iii}) - \frac{(\varepsilon_1 + \varepsilon_2)^{ii} + (\varepsilon_1 + \varepsilon_2)^{iii} - 2(\varepsilon_1 + \varepsilon_2)^i}{12} (1 + 2t_\beta t_\theta) + \pm \frac{1}{\sqrt{48}} \left[(\varepsilon_3 - \varepsilon_4)^{ii} - (\varepsilon_3 - \varepsilon_4)^{iii} \right] (2c_\beta t_\theta - s_\beta) \quad (5.44a)$$

$$C_{3/4} = \frac{1}{2} (\varepsilon_{3/4}^{ii} + \varepsilon_{3/4}^{iii}) - \frac{(\varepsilon_1 + \varepsilon_2)^{ii} + (\varepsilon_1 + \varepsilon_2)^{iii} - 2(\varepsilon_1 + \varepsilon_2)^i}{12} (1 + 2t_\beta t_\theta) - \mp \frac{1}{\sqrt{48}} \left[(\varepsilon_2 - \varepsilon_1)^{iii} - (\varepsilon_2 - \varepsilon_1)^{ii} \right] (2c_\beta t_\theta - s_\beta) \quad (5.44b)$$

The expressions above are the predictive equations for the strain gauges offset that can be obtained from the measurements in 3 different blade positions under parked conditions.

Validation

To test the methodology proposed for the sensors calibration, numerically simulated data was generated from the model described in (Pimenta et al., 2020a), while it is left for future developments to apply the methodology to experimental data. To do that, numerically simulated bending mo-

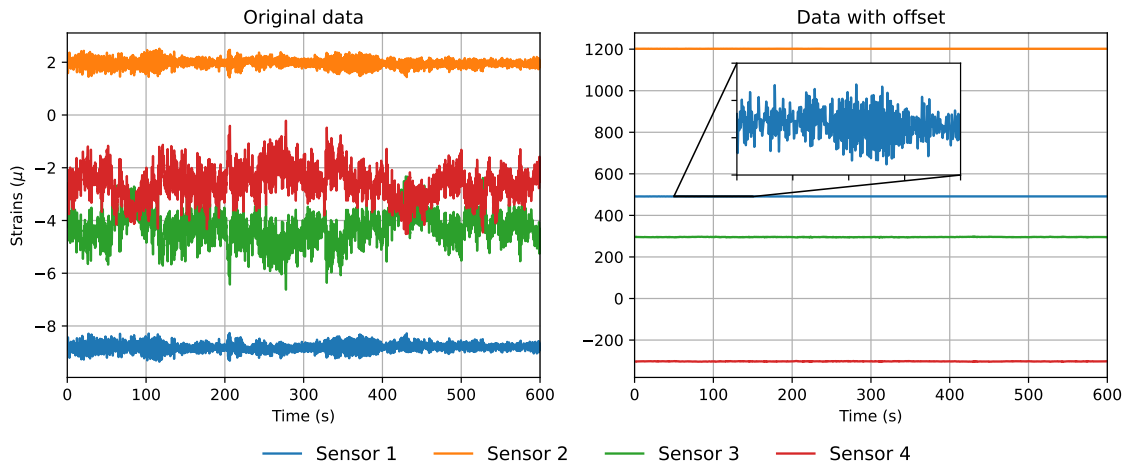


Figure 5.9: Original (on the left) and transformed (with the additional offset) strains time series for the blade in the upright position.

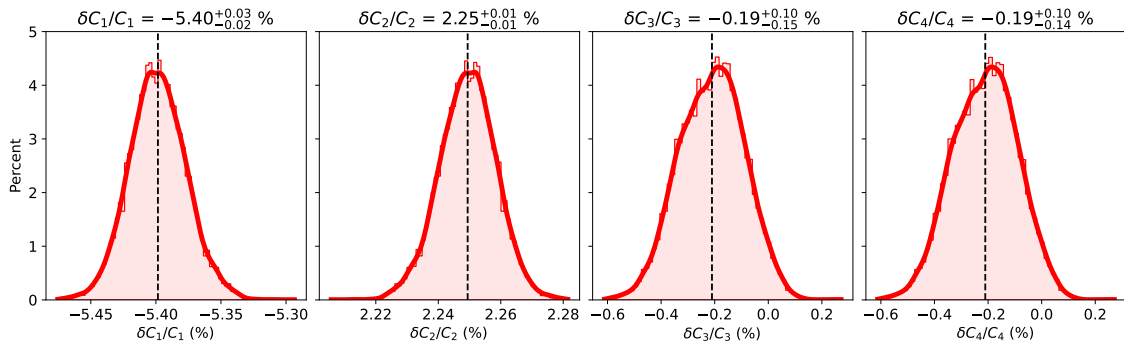


Figure 5.10: Estimated offset for each sensor. The distribution plots have been computed by applying the calibration expressions to each point in the time series, while the black dashed lines have been computed considering only their mean value. Both approaches are shown to be mostly equivalent.

ments and axial forces time series computed considering a 4.5 m/s mean wind speed and 10% turbulence intensity have been transformed into strains using Equation 5.15, assuming the classical instrumentation layout based on 4 sensors. An offset of 500, 1200, 300 and -300 μ has been artificially added to the sensors 1, 2, 3 and 4, respectively. The generated and transformed (with the offset) strains time series are plotted in Figure 5.9 for one of the blade's position. From these time series, the calibration expressions for the offset constants were used to estimate the different C_i and these estimates compared with the initial, known, value. The results for each sensor offset value are presented in Figure 5.10, where it can be seen that the uncertainties are of the order of the few percent.

5.3 Rotor dynamics

In the previous section the measurement of the accelerations in the blades of a wind turbine were shown to contain valuable information regarding the geometry of the rotor and the operating conditions, that may be used to either calibrate numerical models and even strain gauges or to gain deeper insight on the operating conditions that cannot be obtained from 10-minutes averaged SCADA data information. In this section, the forces transmitted by the blades to the tower, that will play an important role in the analyses of the measurements performed at the tower itself, will be briefly analysed. For that, a simple analytical model based on a Lagrangian formalism will be used.

5.3.1 Equations of motion

To analyse the forces transmitted by the rotating rotor to the tower, a simplified model with a fixed support will be used. Defining x_b and y_b as the support FA and SS motions, that will be required

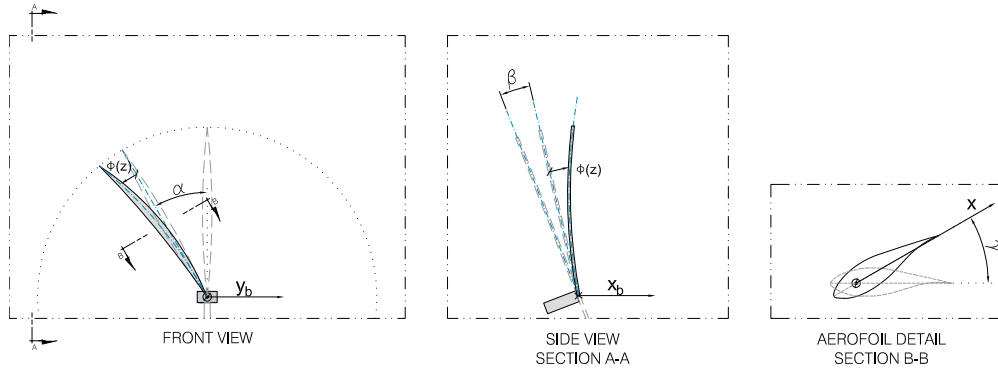


Figure 5.11: Schematically representation of the degrees of freedom used to model a single blade dynamic response.

to vanish at the end by imposing the constraint:

$$f(x, y, x_b, y_b, t) = x_b = y_b = 0 \quad (5.45)$$

the velocity at a distance z from the blade root for a flexible blade with a given angular velocity Ω is given by:

$$\mathbf{v}(z) = \dot{x}_b \hat{\mathbf{a}}_1 + \dot{y}_b \hat{\mathbf{a}}_2 + \Omega z c_\beta \hat{\mathbf{c}}_2 + \dot{\phi} \hat{\mathbf{e}}_1 \quad (5.46)$$

where the different reference frames are the same as before, defined in Figure 5.1, and ϕ is the modal configuration under analysis. All the defined degrees of freedom are represented in Figure 5.11. The corresponding velocity squared is then:

$$\begin{aligned} \mathbf{v}^T \mathbf{v} &= [\dot{x}_b \hat{\mathbf{a}}_1^T + \dot{y}_b \hat{\mathbf{a}}_2^T + \Omega z c_\beta \hat{\mathbf{c}}_2^T + \dot{\phi} \hat{\mathbf{e}}_1^T] [\dot{x}_b \hat{\mathbf{a}}_1 + \dot{y}_b \hat{\mathbf{a}}_2 + \Omega z c_\beta \hat{\mathbf{c}}_2 + \dot{\phi} \hat{\mathbf{e}}_1] = \\ &= \dot{x}_b^2 + \dot{y}_b^2 + (\Omega z c_\beta)^2 + (\dot{\phi})^2 + \\ &\quad + 2\dot{x}_b \Omega z c_\beta \hat{\mathbf{a}}_1^T \hat{\mathbf{c}}_2 + 2\dot{x}_b \dot{\phi} \hat{\mathbf{a}}_1^T \hat{\mathbf{e}}_1 + 2\dot{y}_b \Omega z c_\beta \hat{\mathbf{a}}_2^T \hat{\mathbf{c}}_2 + 2\dot{y}_b \dot{\phi} \hat{\mathbf{a}}_2^T \hat{\mathbf{e}}_1 + 2\Omega z c_\beta \dot{\phi} \hat{\mathbf{c}}_2^T \hat{\mathbf{e}}_1 \end{aligned} \quad (5.47)$$

where the missing scalar products can be evaluated noting that:

$$\hat{\mathbf{a}}_1^T \hat{\mathbf{c}}_2 = s_\theta s_\alpha \quad (5.48a)$$

$$\hat{\mathbf{a}}_1^T \hat{\mathbf{e}}_1 = c_\theta c_\beta c_\gamma + s_\theta s_\alpha s_\gamma - s_\theta c_\alpha s_\beta c_\gamma \quad (5.48b)$$

$$\hat{\mathbf{a}}_2^T \hat{\mathbf{c}}_2 = c_\alpha \quad (5.48c)$$

$$\hat{\mathbf{a}}_2^T \hat{\mathbf{e}}_1 = c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \quad (5.48d)$$

$$\hat{\mathbf{c}}_2^T \hat{\mathbf{e}}_1 = s_\gamma \quad (5.48e)$$

The corresponding kinetic energy can now be obtained by integrating over all the blade:

$$\begin{aligned} T &= \frac{1}{2} \int \rho v^2 dz = \frac{1}{2} M (\dot{x}_b^2 + \dot{y}_b^2) + \frac{1}{2} M_x \dot{x}^2 + \frac{1}{2} M_\Omega \Omega^2 + \\ &\quad + M_{x_b, \Omega} \dot{x}_b \Omega + M_{x_b, x} \dot{x}_b \dot{x} + M_{y_b, \Omega} \dot{y}_b \Omega + M_{y_b, x} \dot{y}_b \dot{x} + M_{x, \Omega} \dot{x} \Omega \end{aligned} \quad (5.49)$$

while the potential energy to include the blade flexibility may be written as:

$$V = \frac{1}{2}k_x x^2 + MgL_g (c_\theta c_\alpha c_\beta - s_\theta s_\beta) \quad (5.50)$$

being L_g the centre of mass position, measured along the blade axis. For the blades deflection, the equation of motion is simply:

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) = -k_x x - \frac{d}{dt} (M_x \dot{x} + M_{x,x_b} \dot{x}_b + M_{x,y_b} \dot{y}_b + M_{x,\Omega} \Omega) = -k_x x - M_x \ddot{x} = 0 \quad (5.51)$$

where the replacement $x_b = \dot{x}_b = \ddot{x}_b = \dot{\Omega} = 0$ has already been used. This is just the equation for an harmonic oscillator. The equations for the reaction forces due to a single blade are:

$$\lambda_x = \frac{\partial \mathcal{L}}{\partial x_b} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}_b} \right) = -(\dot{M}_{x_b,x} \dot{x} + \dot{M}_{x_b,\Omega} \Omega) - (M_{x_b,x} \ddot{x}) \quad (5.52a)$$

$$\lambda_y = -(\dot{M}_{y_b,x} \dot{x} + \dot{M}_{y_b,\Omega} \Omega) - (M_{y_b,x} \ddot{x}) \quad (5.52b)$$

It should be noted that the generalised mass terms result from integrating the different functions along the blade length, and hence inherit its time dependence:

$$M_{x_b,x} \propto \hat{\mathbf{a}}_1^T \hat{\mathbf{e}}_1 = c_\theta c_\gamma c_\beta + s_\theta s_\alpha s_\gamma - s_\theta c_\alpha s_\beta c_\gamma \longrightarrow \dot{M}_{x_b,x} \propto \Omega s_\theta (c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma) \quad (5.53a)$$

$$M_{x_b,\Omega} \propto \hat{\mathbf{a}}_1^T \hat{\mathbf{e}}_2 = s_\theta s_\alpha \longrightarrow \dot{M}_{x_b,\Omega} \propto \Omega s_\theta c_\alpha \quad (5.53b)$$

$$M_{y_b,x} \propto \hat{\mathbf{a}}_2^T \hat{\mathbf{e}}_1 = c_\gamma s_\beta s_\alpha + s_\gamma c_\alpha \longrightarrow \dot{M}_{y_b,x} \propto \Omega (c_\gamma s_\beta c_\alpha - s_\gamma s_\alpha) \quad (5.53c)$$

$$M_{y_b,\Omega} \propto \hat{\mathbf{a}}_2^T \hat{\mathbf{e}}_2 = c_\alpha \longrightarrow \dot{M}_{y_b,\Omega} \propto -\Omega s_\alpha \quad (5.53d)$$

while the reaction forces should also be summed up over all the 3 blades, leading to the full equations for the reaction forces:

$$\lambda_x \propto - \sum_i [\Omega s_\theta (c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma) \dot{x}_i + (c_\theta c_\gamma c_\beta + s_\theta s_\alpha s_\gamma - s_\theta c_\alpha s_\beta c_\gamma) \ddot{x}_i] \quad (5.54a)$$

$$\lambda_y \propto - \sum_i [(c_\gamma s_\beta c_\alpha - s_\gamma s_\alpha) \Omega \dot{x}_i + (c_\gamma s_\beta s_\alpha + s_\gamma c_\alpha) \ddot{x}_i] \quad (5.54b)$$

If an additional external force is to be considered, an additional term appears in the equations given by:

$$Q_{q_j} = \sum_i \mathbf{F}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} \quad (5.55)$$

By writing the blades' tip position as:

$$\mathbf{r}_i = x_b \hat{\mathbf{a}}_1 + y_b \hat{\mathbf{a}}_2 + L \hat{\mathbf{e}}_3 \quad (5.56)$$

and assuming for simplicity an external force of the type $\mathbf{F} = f_x \hat{\mathbf{b}}_1 + f_y \hat{\mathbf{b}}_2 + f_z \hat{\mathbf{b}}_3$ acting on the blade tip only, the generalised forces, that should be added to the reaction forces in Equation 5.54a and Equation 5.54b, are:

$$Q_{x_b} = \sum_i f_x \quad (5.57a)$$

$$Q_{y_b} = \sum_i f_y \quad (5.57b)$$

5.3.2 Quasi-static conditions

Since the Lagrange multiplier λ can be interpreted as the reaction force on the support, the different contributions to this term can now be further analysed. For the particular case where $\mathbf{F} = f_x \hat{\mathbf{b}}_1$, being $f_x \propto z^\kappa \propto (1 + \cos \alpha_i)^\kappa$, the generalised force can be expanded as:

$$\begin{aligned} f_{x_i} &= 1 + \kappa \cos \alpha_i + \frac{\kappa(\kappa-1)}{2} \cos^2 \alpha_i + \frac{\kappa(\kappa-1)(\kappa-2)}{6} \cos^3 \alpha_i + \dots = \\ &= 1 + \kappa \cos \alpha_i + \frac{\kappa(\kappa-1)}{4} (1 + \cos 2\alpha_i) + \frac{\kappa(\kappa-1)(\kappa-2)}{24} (3 \cos \alpha_i + \cos 3\alpha_i) + \dots \end{aligned} \quad (5.58)$$

Assuming by now a quasi-static approach, meaning setting $\dot{x}_i = \ddot{x}_i = 0$, the terms proportional to $\sum c_{\alpha_i}$ will vanish, while the remaining ones are given by:

$$\sum_i \cos \left(n\alpha_0 + \frac{2\pi}{3} ni \right) = \cos(n\alpha) \sum_i \cos \left(n\frac{2\pi}{3} i \right) - \sin(n\alpha) \sum_i \sin \left(n\frac{2\pi}{3} i \right) \quad (5.59)$$

The expression above is only non null if $n = 3k$, with k any integer, in which case the corresponding term is proportional to a periodic function with frequency $3k\omega$. It should be noted, however, that the force transmitted by a single blade exhibits all the harmonics with frequency $k\Omega$. It is only when all the contributions are considered that the $3k$ response is highlighted.

Validation

To illustrate the phenomenon above, the numerical model presented and validated in Section 4.3 has been modified and the flexible nature of the different terms not considered. Additionally, simulations with constant pitch value were run for different, constant rotor angular velocities. The power spectrum of the FA and SS hub forces and the force transmitted by a single blade are presented in Figure 5.12, where the rotor harmonics are easily identifiable. Additionally, it is also clear that while the $3k\Omega$ harmonics are present in all plots, the $k\Omega$ are visible in the blades' shear force results, but completely absent in the hub forces diagrams. A real wind turbine will only follow this result if the 3 blades present exactly the same properties.

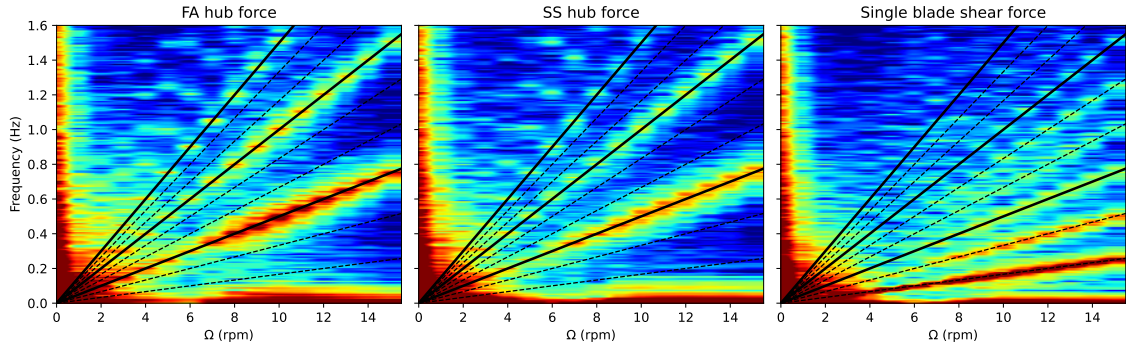


Figure 5.12: Power spectrum of the force transmitted to the tower at the hub in the FA (on left) and SS (on the centre), and a single blade shear force (on the right). The colour map has been saturated at the extreme 5% value for each plot. The black solid lines represent the $3k\Omega$ harmonics, while the black dashed lines represent $k\Omega$ harmonics, being k any integer.

5.3.3 Random vibrations

Having analysed the quasi-static nature of the wind action, it is now time to look into the random contribution that impacts each one of the blades due to random fluctuations with frequency similar to the rotor angular motion. Firstly, let any of the blades motion follow:

$$x_i \propto \cos(\omega_0 t + \delta_i); \quad \dot{x}_i \propto -\omega_0 \sin(\omega_0 t + \delta_i); \quad \ddot{x}_i \propto -\omega_0^2 \cos(\omega_0 t + \delta_i) \quad (5.60)$$

where ω_0 is the blade natural frequency for the mode under analysis and the positions, α_i , of the blades are related, without any loss of generality, by:

$$\alpha_i = \Omega t + \frac{2\pi}{3}i \quad (5.61)$$

It can now be seen that the SS reaction forces will not exhibit a frequency of ω_0 since only terms of the type:

$$\cos \Omega t \sin(\omega_0 t + \delta) \propto \sin(\Omega t + \omega_0 t + \delta) - \sin(\Omega t - \omega_0 t - \delta) = \sin^+ - \sin^- \quad (5.62a)$$

$$\cos \Omega t \cos(\omega_0 t + \delta) \propto \cos(\Omega t + \omega_0 t + \delta) + \cos(\Omega t - \omega_0 t - \delta) = \cos^+ + \cos^- \quad (5.62b)$$

$$\sin \Omega t \sin(\omega_0 t + \delta) \propto \cos(\Omega t - \omega_0 t - \delta) - \cos(\Omega t + \omega_0 t + \delta) = \cos^- - \cos^+ \quad (5.62c)$$

$$\sin \Omega t \cos(\omega_0 t + \delta) \propto \sin(\Omega t + \omega_0 t + \delta) + \sin(\Omega t - \omega_0 t - \delta) = \sin^+ + \sin^- \quad (5.62d)$$

will appear. For the FA direction the same terms appear, but the blades' natural frequency is also expected to be present due to the term proportional to $c_\theta c_\gamma c_\beta \ddot{x}_i \propto \cos(\omega_0 t + \delta_i)$ in [Equation 5.54a](#). The reaction force will experience a periodic variation that is not the same as the blades' natural frequency, but shifted by $\pm\Omega$. Since only the case of random stochastic actions is under analysis, each blade is assumed to be uncorrelated to each other. For a particular blade motion in the local

x direction, the corresponding reaction force is:

$$\lambda_x \propto s_\theta R^+ \sin [(\Omega + \omega_0)t + \delta_x^+] + s_\theta R^- \sin [(\Omega - \omega_0)t + \delta_y^-] \quad (5.63a)$$

$$\lambda_y \propto R^+ \sin [(\Omega + \omega_0)t + \delta_y^+] + R^- \sin [(\Omega - \omega_0)t + \delta_y^-] \quad (5.63b)$$

where, assuming $\omega_0 > \Omega$:

$$R^\pm = (\omega_0^2 \pm \Omega\omega_0) \sqrt{\cos^2 \gamma^2 \sin^2 \beta^2 + \sin^2 \gamma^2} \quad (5.64)$$

and hence it can be concluded that the positive offset is favoured since:

$$\frac{R^+}{R^-} = \frac{\omega_0^2 + \Omega\omega_0}{\omega_0^2 - \Omega\omega_0} = \frac{\omega_0 + \Omega}{\omega_0 - \Omega} \geq 1 \quad (5.65)$$

As a final note, it should be mentioned that the approach above is clearly an oversimplification of the problem under study. For instance, due to the blades' geometry and interaction with the wind field or with the tower flexible nature, the modes are often coupled and the distinction between effects that only appear in the FA or SS direction is not real. In any case, it is still able to qualitative explain the interaction between the dynamics of the different elements in these structures.

Validation

In order to illustrate some of the derived phenomenon above, the numerical model was once more used to generate some hub forces time series that should reproduce some of the expected features.

Firstly, the simulations were conducted considering a single blade edgewise mode. The power spectrum for two different pitch angle values, that should impact how this particular mode is perceived in the FA and SS directions are presented in [Figure 5.13](#). As can be seen, when the pitch angle favours the SS direction, the blade natural frequency is not detected in the tower, but is consistently found when analysing the shear force of a single blade. Additionally, it can be seen that by modifying the pitch angle such that it also impacts the FA forces, the natural frequency of the blades can also be identified in the tower.

Finally, and for completeness, the complete simulations where all the model degrees of freedom enabled have been analysed, with the results presented in [Figure 5.14](#). Although the results are clearly more complex, the same overall trends may be identified regarding the modes that are mostly perceived in the FA and SS directions and the presence of additional harmonics in the blades shear forces. These results are in accordance to the ones obtained by [Pereira et al. \(2023\)](#) from experimental data and Operational Modal Analysis methodologies.

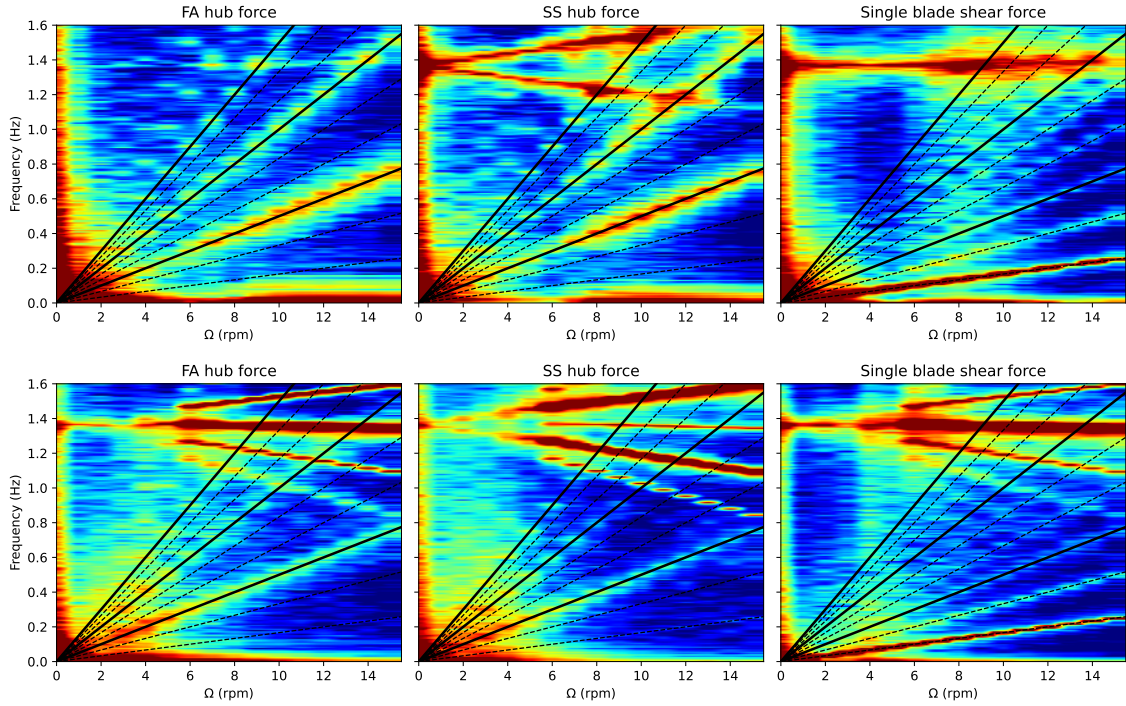


Figure 5.13: Power spectrum of the force transmitted to the tower at the hub in the FA (on left) and SS (on the centre), and a single blade shear force (on the right). The time series used for creating the top and bottom panels have been simulated using two different pitch angles, differing 32.5° . The colour map has been saturated at the extreme 5% value for each plot. The black solid lines represent the $3k\Omega$ harmonics, while the black dashed lines represent $k\Omega$ harmonics, being k any integer.

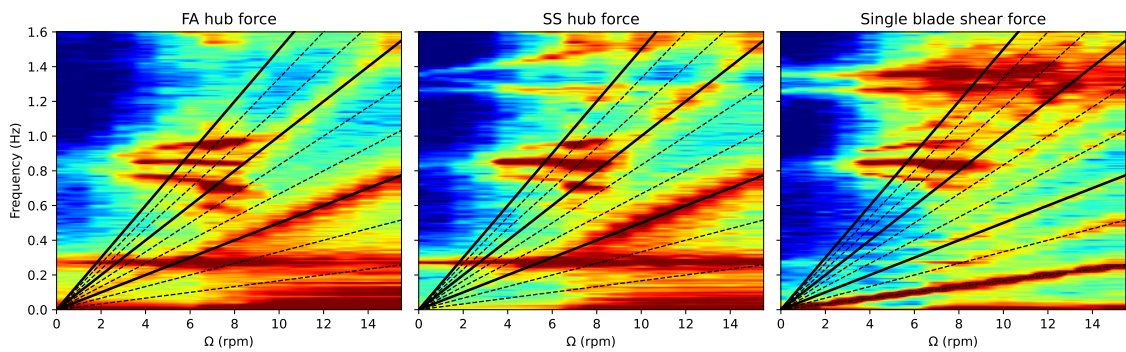


Figure 5.14: Power spectrum of the force transmitted to the tower at the hub in the FA (on left) and SS (on the centre), and a single blade shear force (on the right) considering all blade and tower modes. The colour map has been saturated at the extreme 5% value for each plot. The black solid lines represent the $3k\Omega$ harmonics, while the black dashed lines represent $k\Omega$ harmonics, being k any integer.

5.4 Floating structures dynamics

Once the rotor dynamics have been properly understood, it is now important to understand the dynamic behaviour of a floating wind turbine. This study will seek to characterise some critical aspects of the design and monitoring of these structures. On one hand, the effect of placing a given wind turbine on the top of a floating structure, to which are typically associated very low frequency modes, and its impact in the natural frequencies of the overall system will be studied, since it is relevant for both design procedures as well as monitoring campaigns data analysis. On the other hand, the equations that will be presented will unveil some important correlations between accelerations measurements and the tower internal forces, that are drastically different from the behaviour of onshore wind turbines. The derived relations, that will be the base of the work developed and presented in [Chapter 7](#), will be validated through numerical simulations.

5.4.1 Equations of motion

5.4.1.1 Model definition

A wind turbine may be regarded, in a very simplified approach, as a localised mass representing the nacelle supported by a flexible tower. If the tower flexible response is characterised by the first mode alone, the system described above is a single DOF dynamic system, that can be extended to explicitly include the foundation/floating platform stiffness by including an additional DOF as a rotation stiffness. It should be noted that the horizontal displacement of the platform has not been considered. Although this is clearly an over simplification, it may be argued that in experimental campaigns this motions is hardly identifiable in accelerations measurements, as will be discussed in [Chapter 6](#). Such a model is schematically represented in [Figure 5.15](#), where:

- the platform stiffness is materialised by a rotation spring with constant k_θ
- the tower flexible behaviour is materialised by a spring with constant k_x
- H_m is the distance between the top mass and the centre of gravity of the system
- H_b is the distance between the tower base and the centre of gravity of the system
- $\Delta H = H_m - H_b$ is the tower height, or, equivalently, the distance between the top mass and the tower base
- M is the localised mass on the tower top

The degrees of freedom of this system are the platform rotation θ and the tower deflection, parameterised from the tower top deflection x_t and the first mode modal configuration. Since it will be relevant to obtain an explicit expression for the tower base bending moment, an additional, virtual, degree of freedom (α) was included, with the corresponding constraint $\alpha = \dot{\alpha} = \ddot{\alpha} = 0$. The top mass velocity can now be expressed as:

$$v_M = \dot{x}_t + \dot{\theta}H_m + \dot{\alpha}(H_m - H_b) = \dot{x}_t + \dot{\theta}H_m + \dot{\alpha}\Delta H \quad (5.66)$$

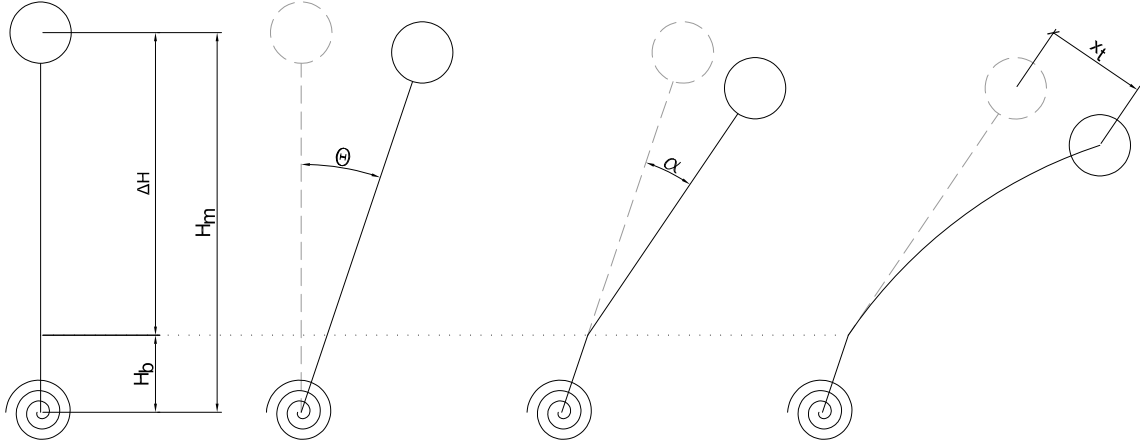


Figure 5.15: Simplified model of a floating wind turbine, with the corresponding generalised degrees of freedom identified.

leading to the system kinetic (T) and potential (V) energies:

$$T = \frac{1}{2}M(\dot{x}_t^2 + \dot{\theta}^2 H_m^2 + \dot{\alpha}^2 \Delta H^2) + M(\dot{x}_t \dot{\theta} H_m + \dot{x}_t \dot{\alpha} \Delta H + \dot{\theta} \dot{\alpha} H_m \Delta H) + \frac{1}{2}I_\theta \dot{\theta}^2 \quad (5.67a)$$

$$V = \frac{1}{2}k_x x_t^2 + \frac{1}{2}k_\theta \theta^2 - Mg \left(H_m \frac{\theta^2}{2} + \Delta H \alpha \theta + x_t \theta + \Delta H \frac{\alpha^2}{2} + x_t \alpha \right) \quad (5.67b)$$

where the platform inertia around the system centre of gravity (I_θ) has been introduced. The corresponding Lagrangian $\mathcal{L} = T - V$ follows directly as:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}M\dot{x}_t^2 + \frac{1}{2}\tilde{I}\dot{\theta}^2 + \frac{1}{2}\tilde{I}_\alpha\dot{\alpha}^2 + M(\dot{x}_t \dot{\theta} H_m + \dot{x}_t \dot{\alpha} \Delta H + \dot{\theta} \dot{\alpha} H_m \Delta H) - \\ & - \frac{1}{2}k_x x_t^2 - \frac{1}{2}\tilde{k}_\theta \theta^2 + \frac{1}{2}\tilde{k}_\alpha \alpha^2 + Mg(\Delta H \alpha \theta + x_t \theta + x_t \alpha) \end{aligned} \quad (5.68)$$

where, by convenience, $\tilde{I} = I_\theta + MH_m^2$, $\tilde{I}_\alpha = M\Delta H^2$, $\tilde{k}_\theta = k_\theta - MgH_m$ and $\tilde{k}_\alpha = Mg\Delta H$ have been defined.

5.4.1.2 Free vibration

For a system without any additional external force, the equations of motion follow directly as:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{\partial \mathcal{L}}{\partial x} = \frac{k_x}{\omega_x^2} (\omega_x^2 x_t + \ddot{x}_t) + MH_m \ddot{\theta} - Mg \theta = 0 \quad (5.69a)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = \frac{\tilde{k}_\theta}{\omega_\theta^2} (\omega_\theta^2 \theta + \ddot{\theta}) + MH_m \ddot{x}_t - Mg x_t = 0 \quad (5.69b)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} \right) - \frac{\partial \mathcal{L}}{\partial \alpha} = M\Delta H \ddot{x}_t + MH_m \Delta H \ddot{\theta} - Mg(\Delta H \theta + x_t) = -\lambda \quad (5.69c)$$

where λ is the Lagrange multiplier and represents the bending moments in the tower base, and where the tower and platform natural frequencies, when considered independently, have been

defined as:

$$\omega_x^2 = \frac{k_x}{M} \quad (5.70a)$$

$$\omega_\theta^2 = \frac{\tilde{k}_\theta}{\tilde{I}} \quad (5.70b)$$

The system representation in the frequency domain is given by:

$$\frac{k_x}{\omega_x^2} (\omega_x^2 - \omega^2) \tilde{x}_t = M (H_m \omega^2 + g) \tilde{\theta} \quad (5.71a)$$

$$\frac{\tilde{k}_\theta}{\omega_\theta^2} (\omega_\theta^2 - \omega^2) \tilde{\theta} = M (H_m \omega^2 + g) \tilde{x}_t \quad (5.71b)$$

$$\tilde{\lambda} = M g (\Delta H \tilde{\theta} + \tilde{x}_t) + \omega^2 M \Delta H \tilde{x}_t + \omega^2 M H_m \Delta H \tilde{\theta} \quad (5.71c)$$

The expressions above, in particular [Equation 5.71a](#) and [Equation 5.71b](#), can be combined to yield:

$$\frac{k_x}{\omega_x^2 (\omega_\theta^2 - \omega^2)} \left[(\omega_x^2 - \omega^2) (\omega_\theta^2 - \omega^2) - \frac{M H_m^2}{\tilde{I}} \omega^4 - \frac{2 M H_m g}{\tilde{I}} \omega^2 - \frac{M g^2}{\tilde{I}} \right] \tilde{x}_t = 0 \quad (5.72)$$

which implies, for non vanishing $\tilde{x}_t$, that:

$$\omega^4 r_I - \omega^2 \left(\omega_\theta^2 + \omega_x^2 - \frac{2 M H g}{\tilde{I}} \right) + \omega_x^2 \omega_\theta^2 - \frac{M g^2}{\tilde{I}} = 0 \quad (5.73)$$

where the ratio between the platform inertia and the full system inertia, $r_I = \frac{I_\theta}{\tilde{I}}$, has been defined.

The expression above has the solution:

$$\omega^2 = \frac{1}{2 r_I} \left[\left(\omega_\theta^2 + \omega_x^2 - \frac{2 M H g}{\tilde{I}} \right) \pm \sqrt{\left(\omega_\theta^2 + \omega_x^2 - \frac{2 M H g}{\tilde{I}} \right)^2 - 4 r_I \left(\omega_x^2 \omega_\theta^2 - \frac{M g^2}{\tilde{I}} \right)} \right] \quad (5.74)$$

that, by expressing the platform and tower structural frequencies in such a way that $\omega_\theta = \kappa \omega_x$, may be rewritten as:

$$\omega^2 = \frac{\omega_x^2}{2 r_I} \left[\left(1 + \kappa^2 - \frac{2 M H_m g}{\omega_x^2 \tilde{I}} \right) \pm \sqrt{\left(1 + \kappa^2 - \frac{2 M H_m g}{\omega_x^2 \tilde{I}} \right)^2 - 4 r_I \left(\kappa^2 - \frac{M g^2}{\omega_x^2 \tilde{I}} \right)} \right] \quad (5.75)$$

Taking now the limit where $\kappa \ll 1$, and assuming that $\omega_x^2 \tilde{I} = \omega_x^2 (I + M H_m^2) \gg M H_m g$, the expression above can still be approximated by:

$$\begin{aligned} \omega^2 &\approx \frac{\omega_x^2}{2 r_I} \left[\left(1 + \kappa^2 - 2 \frac{M H_m g}{\omega_x^2 \tilde{I}} \right) \pm \sqrt{1 + 2 \kappa^2 (1 - 2 r_I) - 4 \frac{M H_m g}{\omega_x^2 \tilde{I}} \left(1 - \frac{g r_I}{H_m} \right)} \right] \approx \\ &\approx \frac{\omega_x^2}{2 r_I} \left[\left(1 + \kappa^2 - 2 \frac{M H_m g}{\omega_x^2 \tilde{I}} \right) \pm \left(1 + \kappa^2 (1 - 2 r_I) - 2 \frac{M H_m g}{\omega_x^2 \tilde{I}} \left(1 - \frac{g r_I}{H_m} \right) \right) \right] \end{aligned} \quad (5.76)$$

with the solutions:

$$\omega_+^2 \approx \frac{\omega_x^2}{r_I} \left[1 + \kappa^2 (1 - r_I) + \frac{Mg}{\omega_x^2 \bar{I}} (gr_I - 2H_m) \right] \approx \frac{\omega_x^2}{r_I} \quad (5.77a)$$

$$\omega_-^2 \approx \omega_x^2 \left(\kappa^2 - \frac{Mg^2}{\omega_x^2 \bar{I}} \right) \approx \omega_\theta^2 - \frac{Mg^2}{\bar{I}} \quad (5.77b)$$

From the expression above, it can be easily seen that the platform natural frequencies are mostly unaffected by the presence of the tower, while the tower structural frequency is amplified by a factor of $r_I^{-1} > 1$.

An additional conclusion can be read from the equations of motion by analysing the behaviour for $\omega = \omega_+$. In this case, and substituting back into Equation 5.71a, it can be shown that:

$$\tilde{x}_t = -\frac{H_m + r_I g / \omega_x^2}{1 - r_I} \tilde{\theta} \quad (5.78)$$

that, since $r_I < 1$, implies that x_t and θ have opposite signals.

Validation

To validate the frequency coupling between the tower structural frequencies and the platform inertia, as well as the prediction regarding the relative direction between tower deflection and platform motions several simulations with the 5MW NREL numerical model have been conducted. Since the goal of this task is to validate the dynamic response, the simplest possible scenario has been considered and all the simulations have been carried out under parked conditions and still waters.

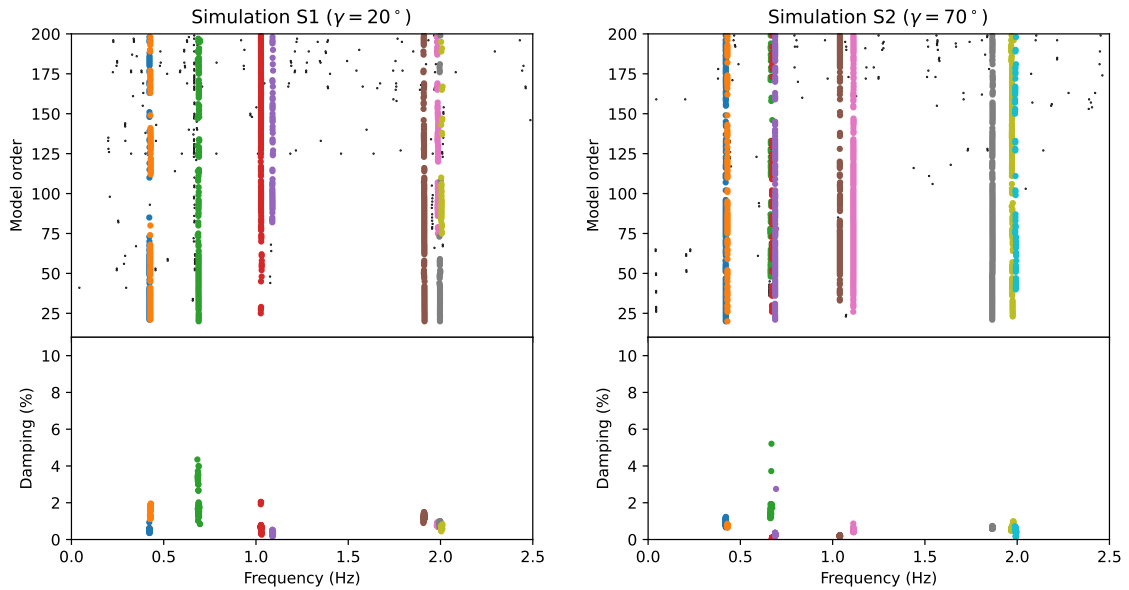


Figure 5.16: Stabilisation diagrams obtained for two different pitch angle values.

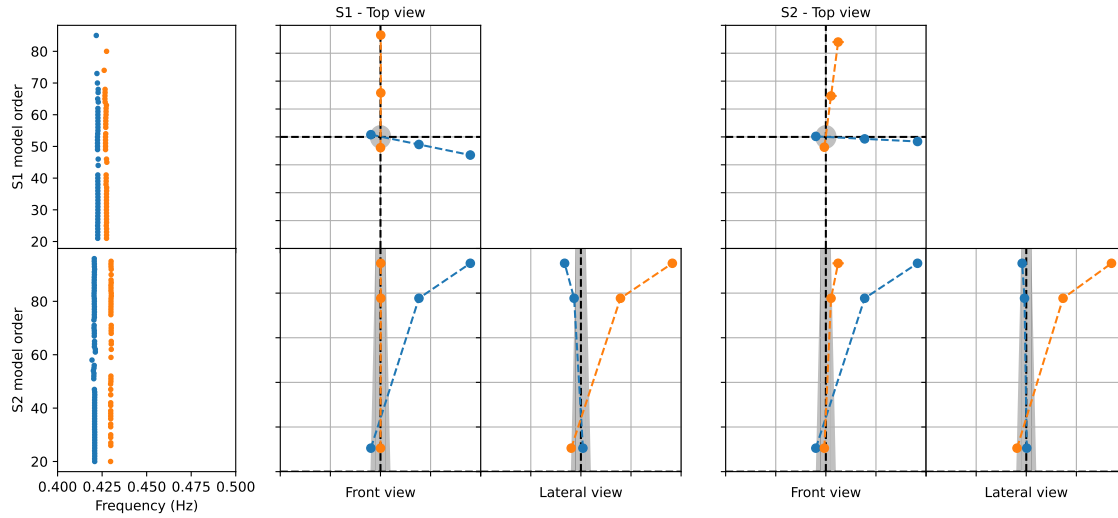


Figure 5.17: First pair of modes modal configuration. The simulation S1 and S2 have been computed considering different values of pitch angle, according to Figure 5.16.

Firstly, the original model has been used to simulate the structural response considering two different pitch values: simulation S1, with $\gamma = 20^\circ$ and simulation S2, with $\gamma = 70^\circ$, being γ the blades' pitch angle. The numerically simulated accelerations time series at the tower base, 75% of the tower height and at the tower top have been used as input for the SSI-COV algorithm and the stabilisation diagrams presented in Figure 5.16 obtained.

As may be seen, a significant number of stable alignments can be found, but there are also some relevant differences. In fact, and looking more closely to the first pair of modes, associated with the first tower mode in each direction, it can be seen that there is a subtle shift in at least one of the modes frequency. These modes are detailed in Figure 5.17, where the modal configuration has also been included. As it is now clear, the two alignments correspond to two modes in mostly orthogonal directions. Additionally, both frequencies appear to be slightly deviated between the two simulations, being this effect much more pronounced in the FA modes (orange modes in Figure 5.17). The negative modal coordinate at the tower base also demonstrates the prediction of Equation 5.78, assuming that the platform translations in the vicinity of this frequency are suppressed with respect to the rotations.

Having identified this coupling between the blades' pitch and the tower natural frequency, the predicted frequencies coupling given by Equation 5.77a is still to be proven. To do so, the platform inertia associated with the pitch motion has been modified and the SSI-COV algorithm once again applied to the different simulations output to identify the first tower mode frequency, and hence its dependency on the modified inertia. However, and since the platform inertia is being partly modified (the ballast contribution, for instance, is not affected), it is convenient to rewrite

Equation 5.77a as:

$$\omega_+^2 = \omega_x^2 \left(\frac{I_\theta + \Delta I + MH_m^2}{I_\theta + \Delta I} \right) = \omega_x^2 \left(1 + \frac{MH_m^2}{I_\theta + \Delta I} \right) \rightarrow \frac{\omega_+^2}{\omega_x^2} = \frac{I_\theta}{MH_m^2} + \frac{1}{MH_m^2} \Delta I \quad (5.79)$$

implying that plotting the natural frequency squared against the inverse of the platform inertia should exhibit a linear dependency. To determine the tower frequency for an infinitely stiff foundation, a simulation was conducted with the same tower in onshore conditions and a value of $f_x = \omega_x/2\pi = 0.37$ Hz obtained¹. The results are presented in Figure 5.18, where the dependency on the platform inertia can be confirmed. For clarity, the natural frequencies can be reconstructed from the linear fit parameters from Equation 5.79:

$$\omega_+ = \omega_x \sqrt{1 + \frac{1}{b + m\Delta I}} \quad (5.80)$$

where $b = I/(MH_m^2)$ and $m = 1/(MH_m^2)$ were obtained from the linear fit, a result also presented in Figure 5.18.

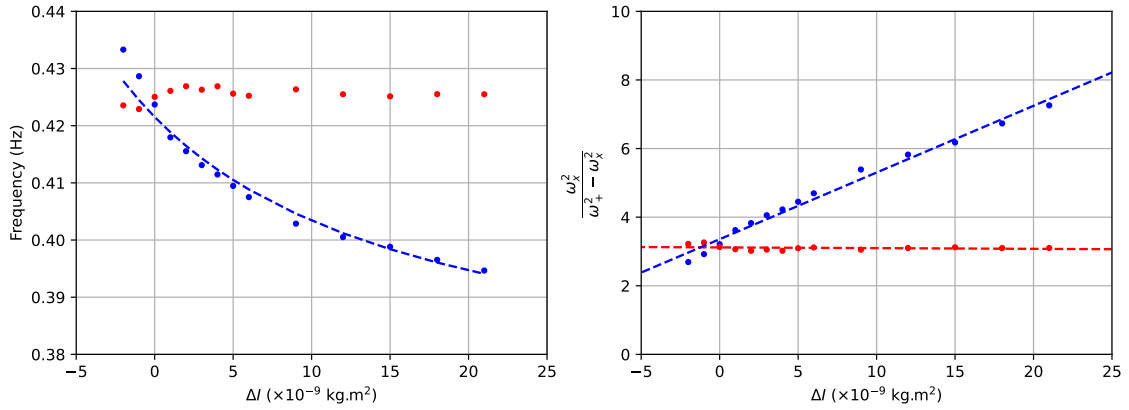


Figure 5.18: First modes natural frequency evolution as a function of the change in the platform inertia. The blue and red dots correspond to the 1st FA and SS mode, respectively.

5.4.1.3 Forced motion

Having analysed the dynamical properties, external forces must now be explicitly considered if the model is intended to provide a suitable description of a FOWT. For this purpose, two different sources of external loading will be considered: a wave induced torque applied in the floating platform (τ_{waves}) and a force applied at the tower top (F_t). The corresponding generalised force

¹It should be noted that this frequency slightly deviates from the NREL report, since the tower height has been modified to be compatible with the offshore simulations.

that is needed for each equation is now:

$$Q_{x_t} = F_t \quad (5.81a)$$

$$Q_\theta = \tau_{\text{waves}} + H_m F_t \quad (5.81b)$$

$$Q_\alpha = \Delta H F_t \quad (5.81c)$$

leading to the equations of motion, here represented in the frequency domain only:

$$\frac{\tilde{x}_t}{\mathcal{H}} = \tilde{F}_t + M (H_m \omega^2 + g) \tilde{\theta} \quad (5.82a)$$

$$\frac{\tilde{\theta}}{\mathcal{R}} = \tilde{\tau}_{\text{waves}} + H_m \tilde{F}_t + M (H_m \omega^2 + g) \tilde{x}_t \quad (5.82b)$$

$$\tilde{\lambda} = M (\omega^2 \Delta H + g) \tilde{x}_t + M \Delta H (\omega^2 H_m + g) \tilde{\theta} + \Delta H \tilde{F}_t = M \left(\omega^2 \Delta H + g + \frac{\Delta H}{M \mathcal{H}} \right) \tilde{x}_t \quad (5.82c)$$

where the tower ($\mathcal{H}$) and platform ($\mathcal{R}$) transfer functions have been defined as:

$$\mathcal{H} = \frac{\omega_x^2}{k_x (\omega_x^2 - \omega^2)} = \frac{1}{M (\omega_x^2 - \omega^2)} \quad (5.83a)$$

$$\mathcal{R} = \frac{\omega_\theta^2}{k_\theta (\omega_\theta^2 - \omega^2)} = \frac{1}{\tilde{I} (\omega_\theta^2 - \omega^2)} \quad (5.83b)$$

As can be seen, for a static action, the bending moments are given by $\lambda = Mg (\Delta H \theta + x_t) + \Delta F_t$, as one would expect.

Finally, it is important to note that these transfer functions could be easily generalised to include the damping effect, either structural, aerodynamic or hydrodynamic. However, it would not alter qualitatively the derivations that will follow, and hence the simplified versions defined in [Equation 5.83](#) will be used.

5.4.2 Relations between accelerations and bending moments

Having derived the equations that describe the dynamic response of a FOWT, it is now important to use them to relate the different quantities of interest. In particular, how easily measured variables, like the tower top accelerations, relate to the bending moments at the tower base. Firstly, it should be noted that the acceleration measurements at the tower top provided by an accelerometer can be expressed in terms of the variables x_t and θ as:

$$a(t) = -g \sin \theta + H_m \ddot{\theta} + \ddot{x}_t \approx -g \theta + H_m \ddot{\theta} + \ddot{x}_t \quad (5.84)$$

where the first term is present due to the sensitivity of these virtual sensors (simulating the currently used MEMS based sensors) to the gravitational pull. Once more, this may be written in the

frequency domain as:

$$\tilde{a} = -g\tilde{\theta} - \omega^2 H_m \tilde{\theta} - \omega^2 \tilde{x}_t = -(g + H_m \omega^2) \tilde{\theta} - \omega^2 \tilde{x}_t = \frac{\tilde{F}_t}{M} - \left(\omega^2 + \frac{1}{M\mathcal{H}} \right) \tilde{x}_t \quad (5.85)$$

5.4.2.1 Relations without thrust force on the tower

Having formalised the accelerations readings as a function of the same variables as the bending moments, both quantities may be related. Firstly, if the tower force is neglected, it follows immediately that:

$$\tilde{a} = - \left(\omega^2 + \frac{1}{M\mathcal{H}} \right) \tilde{x}_t \approx -\omega_x^2 \tilde{x}_t \quad (5.86)$$

implying:

$$\tilde{\lambda} = M \left(\omega^2 \Delta H + g + \frac{\Delta H}{M\mathcal{H}} \right) \tilde{x}_t \approx M (\omega_x^2 \Delta H + g) \tilde{x}_t \approx -\tilde{a} \frac{M (g + \Delta H \omega_x^2)}{\omega_x^2} \approx -M \Delta H \tilde{a} \quad (5.87)$$

where the last equality is strictly only valid for $\Delta H \omega_x^2 \gg g$. However, and even if this is not the case, it should be noted that the bending moments are still proportional to the accelerations time series.

Validation

Before proceeding any further, it is important to validate the relations just derived. To do that, the NREL 5MW numerical model already presented and validated has been used to generate accelerations and tower base bending moments time series that could be analysed and compared with the predictions above. For the sake of clarity it is important to mention that the accelerations provided by OpenFAST are not fully equivalent to the ones that would be recorded in an experimental campaign, since the gravitational pull projection is not included, which is vital for the arguments presented above. However, this issue can be easily solved by requesting as output the platform roll and pitch motions and adding *a posteriori* the corresponding projection in the relevant directions.

Since at this point the relations that are being checked assume the absence of wind action, two different 10 minute simulations have been carried out considering a JONSWAP power spectrum with 4.8s (simulation S1) and 10s (simulation S2) peak period. After running the simulations the power spectrum of bending moments and (corrected) accelerations were computed and their relation compared, as presented in [Figure 5.19](#). As can be seen, the relation between accelerations and bending moments power spectra are mostly constant in both simulations, specially in the frequency range where the bending moments are more relevant.

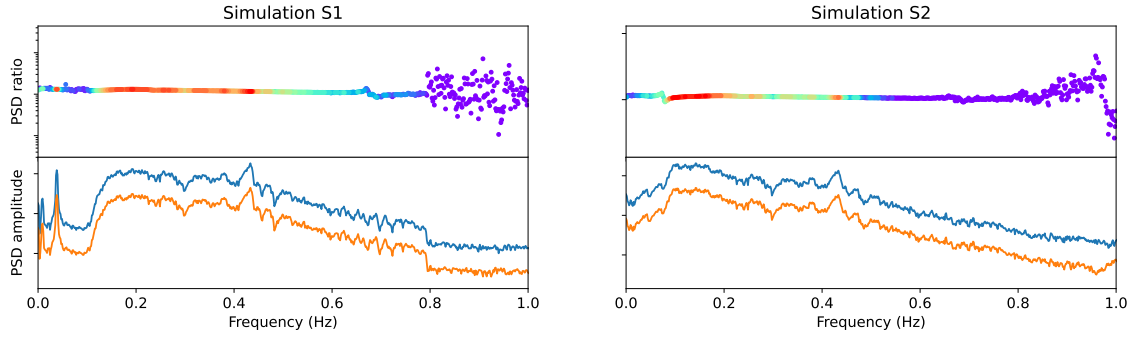


Figure 5.19: Numerical validation of the transfer functions between accelerations and bending moments neglecting the wind action for simulations S1 (on the left) and S2 (on the right). The points have been coloured as a function of the logarithm of the amplitude of the bending moments power spectrum. The accelerations power spectrum and bending moments power spectrum are presented in orange and blue, respectively.

5.4.2.2 Relations for the thrust force on the tower

In the previous point the thrust force on the tower top has been neglected and a trivial relation between accelerations and bending moments has been derived. On the other hand, by setting $\tau_{\text{waves}} = 0$, a different relation is obtained. In this case, the platform motion is given by:

$$\tilde{\theta} \approx \mathcal{R} [H_m \tilde{F}_t + M (H_m \omega^2 + g) \tilde{x}_t] \quad (5.88)$$

that may be substituted in the remaining expressions to obtain:

$$\tilde{x}_t \approx \mathcal{H} [\tilde{F}_t + M (H_m \omega^2 + g) \tilde{\theta}] = \mathcal{H} \frac{1 + \mathcal{R} (MH_m \omega^2 + Mg) H_m}{1 - \mathcal{H} \mathcal{R} (MH_m \omega^2 + Mg)^2} \tilde{F}_t \quad (5.89a)$$

$$\tilde{\lambda} = M (g + \Delta H \omega_x^2) \tilde{x}_t = \mathcal{H} M (g + \Delta H \omega_x^2) \frac{1 + \mathcal{R} (MH_m \omega^2 + Mg) H_m}{1 - \mathcal{H} \mathcal{R} (MH_m \omega^2 + Mg)^2} \tilde{F}_t \quad (5.89b)$$

while the accelerations are now given by:

$$\begin{aligned} \tilde{a} &= \frac{\tilde{F}_t}{M} - \left(\omega^2 + \frac{1}{M \mathcal{H}} \right) \tilde{x}_t = \\ &= -\frac{1}{M} \left[\frac{\mathcal{H} \mathcal{R} (MH_m \omega^2 + Mg)^2 + \mathcal{R} (MH_m \omega^2 + Mg) (1 + M \omega^2 \mathcal{H}) H_m + M \omega^2 \mathcal{H}}{1 - \mathcal{H} \mathcal{R} (MH_m \omega^2 + Mg)^2} \right] \tilde{F}_t \end{aligned} \quad (5.90)$$

The relation between bending moments and accelerations is now far from trivial and is giving, in its most general form, as:

$$\tilde{\lambda} = -\frac{M^2 (g + \Delta H \omega_x^2) \mathcal{H} [1 + \mathcal{R} (MH_m \omega^2 + Mg) H_m]}{\mathcal{H} \mathcal{R} (MH_m \omega^2 + Mg)^2 + \mathcal{R} (MH_m \omega^2 + Mg) (1 + M \omega^2 \mathcal{H}) H_m + M \omega^2 \mathcal{H}} \tilde{a} \quad (5.91)$$

This relation is not easily interpreted, except for some particular cases. On one hand, for very low frequencies, it simplifies to:

$$\tilde{\lambda} \approx -M\Delta H \left[1 + \frac{\tilde{I}}{MgH_m} (\omega_\theta^2 - \omega^2) \right] \tilde{a} = - \left[M\Delta H + \frac{\tilde{I}\Delta H}{gH_m} \omega_\theta^2 \left(1 - \frac{\omega^2}{\omega_\theta^2} \right) \right] \tilde{a} \quad (5.92)$$

This behaviour is significantly different from the one observed in onshore structures, where this relation decays with ω^2 , making the estimation of bending moments from accelerations in the very low frequency range extremely difficult. This difficulty is not present in the floating offshore scenarios, since the platform motions can be used as a direct measurement of the actions in this frequency range. A particular interesting conclusion is that for $\omega = \omega_\theta$ the relation obtained by neglecting the wind force is recovered. On the other hand, and noting that ω_θ is typically very small, another limit can be obtained when $\omega \gg \omega_\theta \sim 0$:

$$\tilde{\lambda} = M(g + \Delta H \omega_x^2) \frac{r_I}{\left(\omega_x^2 + 2\frac{g}{H_m} \right) (1 - r_I) - \omega^2 r_I} \tilde{a} \approx -M\Delta H \frac{1}{\left(1 - \frac{1}{r_I} \right) + \frac{\omega^2}{\omega_x^2}} \tilde{a} \quad (5.93)$$

From the expression above, and if assumed $r_I \sim 1$ or, alternatively, $\omega \gg 1$, it can be easily concluded that the typical decay with ω^2 is recovered.

Validation

Once more, the NREL 5MW numerical model has been used to test the expressions derived. Since the objective of this analysis is to validate the wind response, two additional simulations have been done, in still waters, and considering a wind speed of 4.5 m/s (simulation S3) and 12.5 m/s (simulation S4), with 12% turbulence intensity for both cases. The power spectra of accelerations and bending moments, as well as their ratio, are presented in [Figure 5.20](#), as well as the estimates from [Equation 5.91](#), where the structural transfer function has been generalised and computed considering a damping ratio of 1%.

In this case, it can be seen that there is an important contribution for the bending moments in the low frequency range, due to the wind action induced by the platform motions, and near the tower structural frequency, specially in simulation S3, where the harmonic forces transmitted by the rotor are near the tower coupled natural frequency.

Finally, and for completeness, the wind action associated with the mean wind speed of 12.5 m/s was combined with wave loads simulated by a JONSWAP power spectrum with peak period of 4.8s (simulation S5) and 10s (simulation S6). The power spectra and their ratio are presented in [Figure 5.21](#). As may be seen, in the frequency ranges more relevant for the bending moments, the relation is well approximated by one of the theoretical expressions (two dashed lines). This conclusion will motivate the fatigue prediction model presented in [Chapter 7](#).

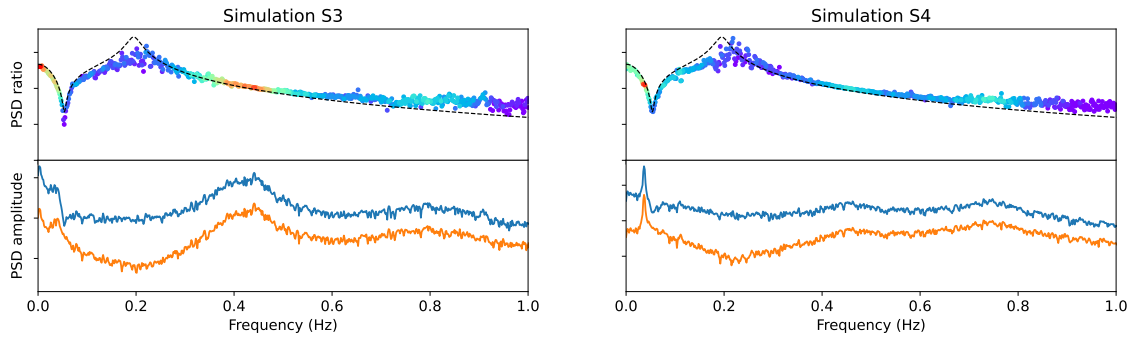


Figure 5.20: Numerical validation of the transfer function between accelerations and bending moments for the wind action for simulations S3 (on the left) and S4 (on the right). The points have been coloured as a function of the logarithm of the amplitude of the bending moments power spectrum. The accelerations power spectrum and bending moments power spectrum are presented in orange and blue, respectively. The theoretical prediction from Equation 5.91, considering a generalised structural transfer function considering a damping ratio of 1%, is plotted as a dashed black line.

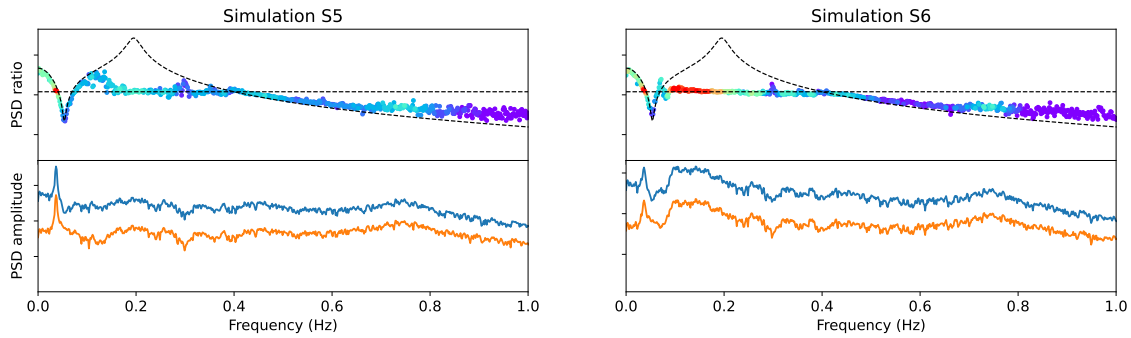


Figure 5.21: Numerical validation of the transfer function between accelerations and bending moments for the combined action of wind and waves for simulations S5 (on the left) and S6 (on the right). The points have been coloured as a function of the logarithm of the amplitude of the bending moments power spectrum. The accelerations power spectrum and bending moments power spectrum are presented in orange and blue, respectively. The theoretical prediction from Equation 5.91, considering a generalised structural transfer function considering a damping ratio of 1%, and the constant value from Equation 5.87 are plotted as dashed black lines.

5.4.3 Strain gauges calibration

The relations derived also unveil another issue that must be considered in floating offshore wind turbines monitoring campaigns that is not present in the onshore cases. As already discussed, an important part of any monitoring campaign, if the bending moments are to be properly estimated, is the regular calibration of the strain gauges. The usual procedure applied in onshore wind turbines relies on the nacelle full rotation for cables untwist events and starts by noting that the total strains measured by a strain gauge in a wind turbine tower can be written as the superposition of 4 major

effects (Loraux, 2018):

$$\varepsilon = \varepsilon_m + \varepsilon_{T,s} + \varepsilon_{T_{nl}} + \varepsilon_{\text{drift}} \quad (5.94)$$

where the different terms stand for the mechanical strains induced by the wind, nacelle offset and other external loads, the thermal effects on the strain gauge itself, the strains induced by a non linear temperature profile and the sensors drift overtime. Since most of these effects are unknown, some assumptions must be placed on their periodic nature over a full revolution. If the axial force variations over time are neglected, the mechanical loads will originate symmetric readings in diametrically opposite sensors. Assuming that the same holds for the non linear temperature profile induced strains, as extensively discussed by Loraux (2018), the relevant strains can obtained by:

1. Compensate the thermal strains from the temperature sensors readings in each sensor
2. Remove the low frequency filtered mean value of opposing sensors

Once the procedure above is properly conducted, diametrically symmetric sensors should exhibit a symmetric behaviour, but the zero value of each sensors still does not have any physical meaning. In onshore wind turbines, each sensors can be zeroed by using the measurements during the nacelle full rotations for cables untwist events with low wind speed, where each time series, when considered over a full revolution, should average to 0, independently. However, the offshore scenario has its own rules and is significantly different from the onshore situations. To better understand this issue, it is important to understand how a non vertical position of the tower impacts both accelerations and strains measurements. Since these events are usually associated with time periods of the order of ~ 15 minutes, a quasi static approach will be used here, and not the dynamic formulation described in the previous subsection, but at the same time the tower top mass will be more detailed to explicitly consider its eccentricity with respect to the tower axis.

Platform rotation effect on the accelerometers

Firstly, and assuming a quasi-static behaviour during a nacelle full rotation event, the accelerometers readings over a full revolution can be written, in units of g , as:

$$a_x = \sin[\theta_0 + \Delta\theta(t)] = \sin[\theta_0 + k \cos \psi] \approx \sin \theta_0 + k \cos \psi \cos \theta_0 \quad (5.95)$$

where θ_0 and $\Delta\theta$ are the tower inclination at equilibrium and the variation induced over the full revolution, that will be assumed to be proportional to the nacelle orientation cosine, $\cos \psi$, through the proportionality factor k , and where the last equality holds as long as $k \ll 1$. Under this assumption, the accelerometers mean value and half-amplitude over a full revolution are given by:

$$\bar{a}_x = \sin \theta_0 \quad (5.96a)$$

$$\Delta a_x = k \cos \theta_0 \quad (5.96b)$$

Assuming that the induced $\Delta\theta$, and hence k , is the same for any nacelle untwist event, independently of the equilibrium position, it is expected that part of the identified drift should be proportional to the accelerometers readings and should not be removed from the raw signals since it has a clear physical meaning. The expression above can be extended to explicitly consider a linear trend related to platform adjustments as:

$$a_x \approx \sin(\theta_0 + k_p t) + k \cos \psi \cos \theta_0 \approx k \cos \psi \cos \theta_0 + \sin \theta_0 + k_p t \cos \theta_0 \quad (5.97)$$

where once more it was assumed that the adjustment during a full nacelle revolution is made at sufficient low rates such that $k_p t$ is always small and second order terms on the k 's have been dropped. It can be concluded that the net effect of assuming such a behaviour is adding a linear trend to the measurements, leading to the new predictions for the mean value and amplitude over a full revolution:

$$\bar{a}_x = \sin \theta_0 + \frac{k_p T}{2} \cos \theta_0 \quad (5.98a)$$

$$\Delta a_x = k \cos \theta_0 \quad (5.98b)$$

where T is the untwist period and where the amplitude should be computed after removing the linear trend from the filtered signal.

Platform rotation effect on bending moments

The low frequency (periods of the order of 15 minutes) variations on the bending moments (M) during a nacelle full rotation event is due to the eccentricity (L) of the nacelle weight (P) and once more the conventional approach must be generalised to properly account for a non vertical equilibrium position of the tower. For the measurements in a given direction, the bending moment can be approximated by:

$$\begin{aligned} M &= P [L \cos \psi \cos \theta + H \sin \theta] + H_t P_t \sin \theta = PL \cos \psi \cos \theta + (PH + P_t H_t) \sin \theta = \\ &= PL \cos \psi \cos \theta + PH (1 + \beta) \sin \theta \end{aligned} \quad (5.99)$$

where β has been introduced to account for the tower self-weight and that conveniently reduces to the usual expression used in onshore wind turbines in the limit $\theta \rightarrow 0$. The bending moment may now be estimated considering the already presented expansion for θ :

$$\begin{aligned} \frac{M}{HP} &= \frac{L}{H} \cos \psi \cos (\theta_0 + k \cos \psi + k_p t) + (1 + \beta) \sin (\theta_0 + k \cos \psi + k_p t) = \\ &= \frac{L}{H} \cos \psi \cos \theta_0 + (1 + \beta) (\sin \theta_0 + k_p t \cos \theta_0 + k \cos \psi \cos \theta_0) = \\ &= \left(\frac{L}{H} + (1 + \beta) k \right) \cos \psi \cos \theta_0 + (1 + \beta) \sin \theta_0 + (1 + \beta) k_p t \cos \theta_0 \end{aligned} \quad (5.100)$$

where second order terms have once more been left out. The expression above implies that the bending moment, and hence the strain gauges readings, will experience the following mean values

and amplitude:

$$\frac{\bar{M}}{HP} = (1 + \beta) \left[\sin \theta_0 + \frac{k_p T}{2} \cos \theta_0 \right] = (1 + \beta) \bar{a}_x \quad (5.101a)$$

$$\frac{\Delta M}{HP} = \left(\frac{L}{H} + (1 + \beta) k \right) \cos \theta_0 \approx \frac{L}{H} + (1 + \beta) \Delta a_x \quad (5.101b)$$

where the bending amplitude should be computed once more after removing the linear trend over one revolution.

To avoid the linear trends that may be present in both accelerations and strains time series, it may still be noted that an auxiliary quantity can be defined as:

$$\frac{M}{HP} - a_x = \left(\frac{L}{H} + \beta k \right) \cos \psi \cos \theta_0 + \beta \sin \theta_0 + \beta k_p t \cos \theta_0 \approx \left(\frac{L}{H} + \beta k \right) \cos \psi \cos \theta_0 \quad (5.102)$$

where now the last two terms may be safely neglected. This means that, even if the platform is actively controlled, the quantity above can be used to obtain two important quantities: firstly, to constrain the value of $L/H \cos \theta_0$, and, hence, of θ_0 itself (or, assuming that θ_0 is small, to obtain the ratio L/H). Secondly, to map the yaw position of the nacelle, through the periodic nature of the signal. Knowing the angular position of the nacelle effectively removes the dependency on the yaw untwisting be carried out at a constant pace, since the expressions above can be fitted already knowing both t and $\cos \psi(t)$. Once this angular position is known, the different terms in Equation 5.97 and Equation 5.100 can be obtained.

Finally, it should be noted that for a given nacelle revolution, oppositely placed sensors will still experience symmetric physical responses, and hence, just like in (Loraux, 2018), the mean value can still be removed at any time instant. The subtle difference in the offshore scenario is that it cannot be assumed that the sensors will have a zero mean over a revolution.

Validation

To validate this model, a non-linear analysis over a finite element model (FEM) has been developed in the commercial software ROBOT Structural Analysis® (RSA) of the NREL 5MW tower. For simplicity, the nacelle was represented as 350 ton mass with an eccentricity of 0.40m, values compatible with the tower defined in OpenFAST model.

For the analysis, a 0.5 degrees tilted reference position with a tilt amplitude (k) of 0.3 degrees in the nacelle direction has been considered. The results are plotted in Figure 5.22 for the tower base and at a tower section at 20m from its base, where it can be seen that the non linear bending moments estimates increase the predictions from Equation 5.100 by $\sim 7.5\%$. Additionally, and most importantly, not only the model validates the proposed relations for the estimates, but also highlights the fundamental role of considering the tower contribution, that affects both the mean value as well as the amplitude.

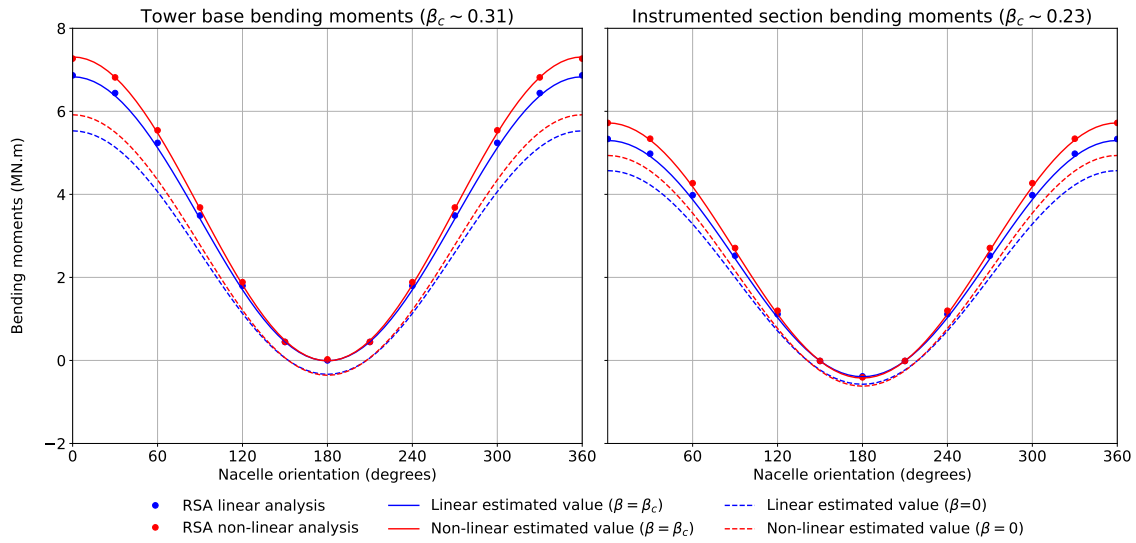


Figure 5.22: Evolution of the bending moments in a given direction as the nacelle yaws around the tower at the tower base (on the left) and a virtual instrumented section 20m above the tower base (on the right). The non-linear estimates have been obtained increasing the linear response by 7.5%, a value obtained from the FE model.

5.5 Final remarks

In this chapter a detailed analytical analysis of the dynamics of a floating offshore wind turbine has been presented, covering different phenomenon, from the blades motion to the platform impact on the structural response, and their effect on the interpretation of the experimental campaigns measurements of these structures.

The first two section have been made quite general, in the sense that they both apply to either onshore and offshore wind turbines. Firstly, In [Section 5.2](#) the main focus was to highlight the relevant information that is hidden in the monitoring data of the blades. With this analysis, it was shown that it can be used to better constrain some rotor geometrical properties, an information that is often not available, while in parallel a new strain gauges calibration was introduced, that still needs experimental validation. Secondly, [Section 5.3](#) was mostly designed to provide a better interpretation of the tower dynamic response, and in particular the spinning rotor effect on it.

On the other hand, [Section 5.4](#) is clearly oriented for floating offshore solutions. By using a simplified model, some important properties of the tower/platform coupling were highlighted and validated by numerical simulations. Additionally, the relation between accelerations and bending moments derived are clearly different from the onshore cases. This is particularly relevant since it opens a new range of possibilities that can be used to estimate the bending moments in the tower from accelerations measurements alone, with implications for the monitoring campaigns that will be further explored in [Chapter 7](#).

Chapter 6

Dynamic monitoring of a full scale floating offshore wind turbine

6.1 Introduction

In [Chapter 5](#) and [Chapter 4](#) the response of wind turbines, in general, and floating solutions, in particular, has been studied from an analytical and numerical point of view. However, this work would not be complete if these developments were not put at test against the experimental data of a full scale floating offshore wind turbine. A paradigmatic example of a set of these structures is the floating wind farm Wind Float Atlantic (WFA), located 18 km of the coast of Viana do Castelo, Portugal. This wind farm, the first pre-commercial application to wind turbines of the floating concept, includes 3 wind turbines, one of which presented in [Figure 6.1](#), with a total installed capacity of 25 MW that powers the Portuguese national grid since 2020 ([WindPlus, 2022](#)). Due to a collaboration between FEUP and Ocean Winds, a structural monitoring campaign is currently taking place over one of the wind turbines of this wind farm, that will be the basis for this Chapter.

In recent years, structural health monitoring of wind turbines has been successfully extended to bottom-fixed offshore solutions, where its application has been proven effective against experimental data ([Devriendt et al., 2014](#)) or reduced scale physical models ([Vidal et al., 2020](#)). However, experimental data regarding floating solutions, of any kind, is mostly unavailable for academic purposes. To by pass this bottle neck, numerical models have been used to develop, test and compare the performance of different monitoring strategies ([Kim et al., 2019](#); [Chung et al., 2021](#); [Sakaris et al., 2021](#)), an approach that has also been used in [Chapter 5](#). Having this in mind, the analysis and results presented in this Chapter are a turning point in this field and, together with the methodology that will be presented in [Chapter 7](#), one of the most important contributions of this work, by providing a first evaluation of the viability of structural monitoring to this type of structures.

Firstly, a brief description of the structure and of the monitoring system itself will be provided, including the sensors location and the description of the acquisition system properties. After that, Operational Modal Analysis will be applied to identify and track the system natural frequencies,



Figure 6.1: Picture of one of the wind turbines from the Wind Float Atlantic wind farm taken during the installation of the monitoring system.

including the platform dominated and the coupled tower/platform modes. Finally, the internal forces at the tower and the corresponding fatigue damage will be analysed, tasks that will prepare the path for the more detailed fatigue analysis conducted in [Chapter 7](#).

6.2 Experimental campaign description

6.2.1 Structure description

In this work, only Wind Turbine 1, one of the 3 wind turbines that integrate the Wind Float Atlantic wind farm, is currently being monitored. The wind turbine tower is $\sim 90\text{m}$ high and supports a 82m radius rotor, making the blade tip reach a maximum height of 207m ([WindPlus, 2022](#)). This massive structure is supported by a floating platform, developed and patented by Principle Power® made of 3 vertical columns, one of them receiving the tower directly, located at the vertexes of an equilateral triangle with a side of around 50m , whose stability is ensured by a combined static and dynamic ballast distribution system ([WindPlus, 2022](#)) and that is anchored to the 100m deep seabed by 3 catenary mooring lines ([Principle Power, 2022](#)). A schematic representation of this structure is presented in [Figure 6.2](#).

6.2.2 Experimental layout

The monitoring campaign at Wind Turbine 1 of Wind Float Atlantic wind farm comprehends the instrumentation of different tower sections with accelerometers, strain gauges and temperature sensors. The instrumentation layout is constituted by a total of 7 accelerometers distributed along 3 different tower heights, and 4 strain gauges and 4 temperature sensors placed in a single tower section. All of these sensors are labelled and oriented according to the representation in [Figure 6.3](#).

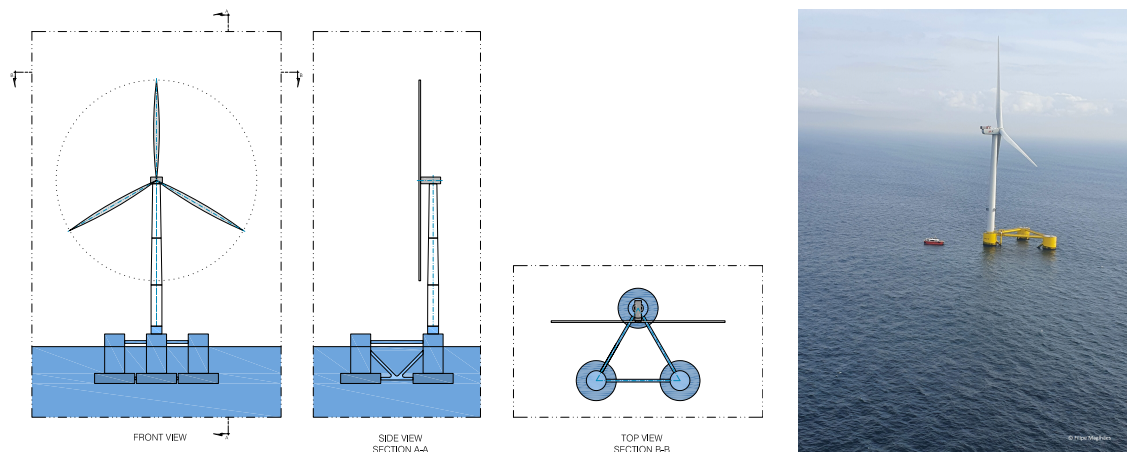


Figure 6.2: Instrumented turbine overall structure (on the left) and picture taken during the installation of the monitoring system, where the maintenance team boat can be seen for reference (on the right).

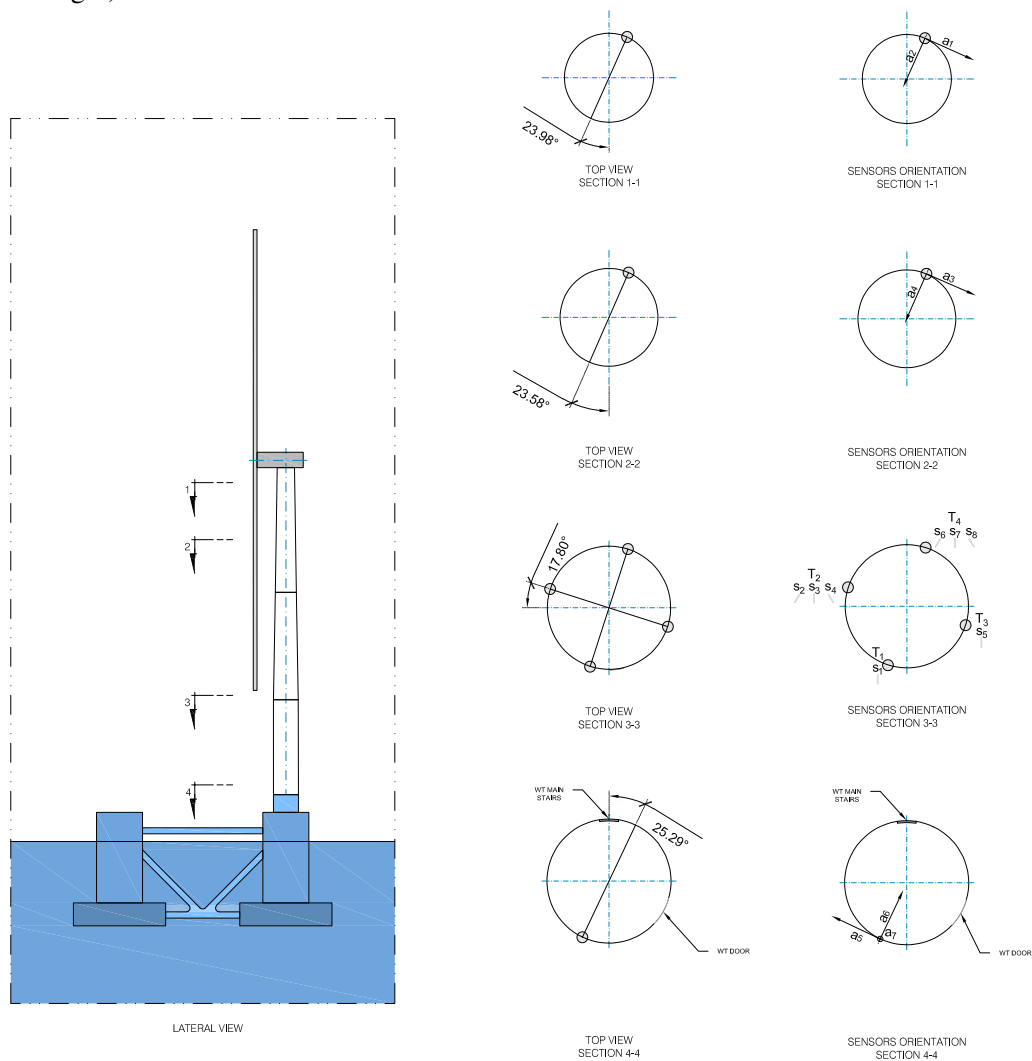


Figure 6.3: Instrumented sections' location. The location of the sensors is marked with a grey circle in each tower section.

In addition to the data that is being collected by the sensors described above, floating platform monitoring data obtained from clinometers and GPS sensors and SCADA data was also made available by Wind Plus. The latter includes, but is not limited to, the wind mean speed, main direction and turbulence intensity, power production and rotor angular velocity and blades' pitch angle, while the former mostly includes translation and rotations of the different columns of the floating platform. For reference of the environmental conditions verified during the monitoring period, it should be mentioned that the wind exhibits a predominant direction mostly aligned with the North-South orientation.

6.2.3 Monitoring components

Accelerometers

In all the sections with accelerometers (1, 2 and 4) two uniaxial accelerometers oriented along two orthogonal horizontal directions are installed. Additionally, in Section 4 (near the tower base), an additional uniaxial accelerometer to measure the vertical acceleration has also been installed, while two extra tri-axial accelerometers were installed in sections 4 and 2. The latter were included for validation and redundancy purposes, and so the data analysis that will be presented will not rely on it. The adopted accelerometers are low noise ($20\mu\text{g}/\sqrt{\text{Hz}}$) MEMS based sensors that allow the characterisation of the structural response from DC (meaning that they can measure the gravity acceleration and the dynamic component from 0 Hz). The sensitivity is 2V/g in a measuring range $\pm 2\text{g}$. Some pictures of the installation are presented in [Figure 6.4](#).

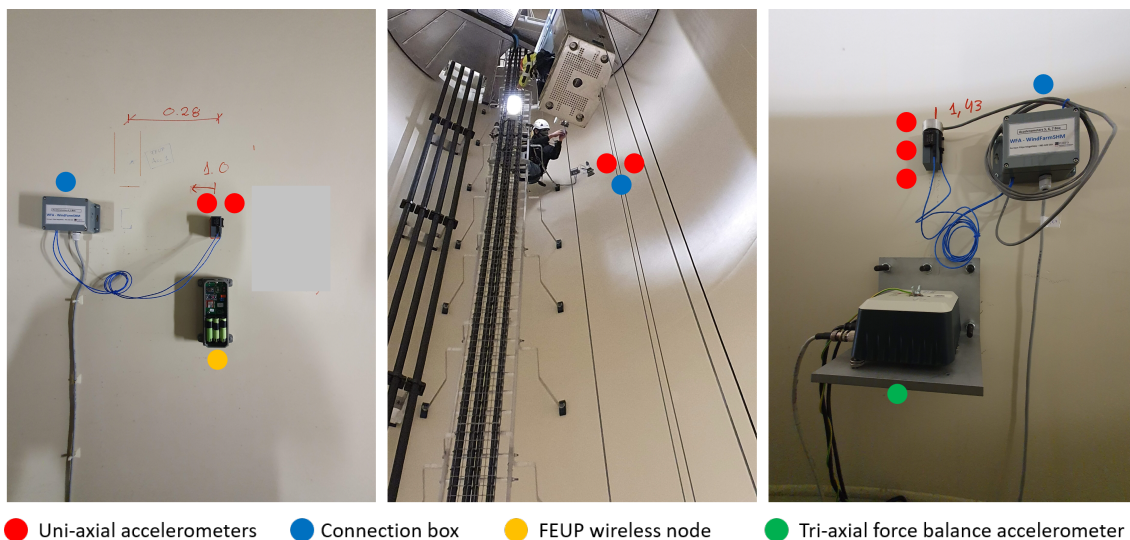


Figure 6.4: Accelerometers' pictures taken during the installation operation (from left to right, section 1, 2 and 4). The FEUP wireless mode includes a tri-axial accelerometer developed by FEUP that together with the tri-axial force balance accelerometer are part of the complementary system used for validation and redundancy.

Strain gauges and temperature sensors

In Section 3 there are 4 high performance temperature compensated electrical strain gauges and 4 temperature sensors (PT100). As depicted in [Figure 6.3](#) two of these sensors are uni-axial, whereas other two are rosettes with 3 measuring directions. In the present work only the measurements aligned with the tower axis are analysed. Some pictures of the installation are presented in [Figure 6.5](#).

Monitoring central

All the sensor are connected to a monitoring cabinet placed at the bottom of the tower. As depicted in [Figure 6.6](#), this includes the digitizer and field processor (in blue), a modem for data retrieving to a FEUP server, power supply and a backup battery to ensure continuous operation.



Figure 6.5: Strain gauges' and temperature sensors pictures taken during the installation operation. From left to right: uni-axial strain gauge glued to the prepared tower surface, strain gauge and temperature sensor with protective cover together with connection box and overview of the installation.

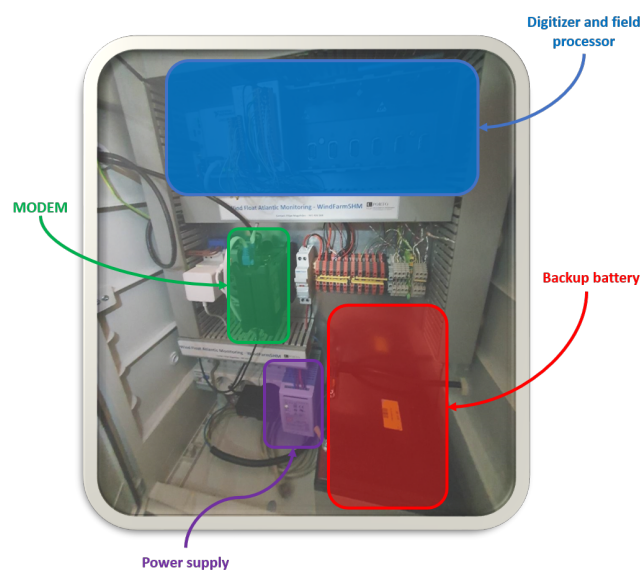


Figure 6.6: Monitoring cabinet elements and their location.

6.2.4 Data description

The field processor is programmed to continuously generate 1 hour long files. These files are transmitted to a FEUP server and processed there. Each file name is encoded as XYYYYMMD-DHH0000, where X stands for the sensor type¹, and YYYYY, MM, DD and HH are the year, month, day and hour at the starting of the 1 hour file. Each file has the data records for each sensors distributed by columns, following the order defined in [Figure 6.3](#). The accelerations and strains are recorded at 50 samples per second and the temperatures at 1 sample/second. To reduce the data volume under analysis, and taking into account the typical frequency range of these structures, all the data series, except the temperature data, have been decimated by a factor of 5, leaving a downsampled data for analysis with a frequency of 10 Hz. The data units of the different files are the following:

- Accelerometers: data in volts (V), reading 2V per g , being g the gravitational acceleration.
- Strain gauges: data in μ -strains
- Temperature: data in Celsius

The system started recording on February 3rd 2022, and has been working continuously without any interruption ever since.

6.3 Operational Modal Analysis

The sensors described include measurements of the structure accelerations and strains, that are naturally conditioned by its dynamical properties and the external loads that are being applied to it. Since the latter is not known, output-only methods are needed to take the most out of the available data. In this section, the accelerations measurements will be explored and the SSI-COV algorithm used to infer about the dynamical properties of the structure under analysis.

Due to their different nature and frequency range, the identification and characterisation of the platform motions and tower structural response will be presented separately. In any case, it is convenient to recall that the accelerations measurements in a given direction may be written as:

$$a(t) \approx -g\theta + H\ddot{\theta} + \ddot{s} + \ddot{x} \quad (6.1)$$

where θ is the platform tilt in that direction, assumed to be small such that $\sin \theta \approx \theta$, and s and x are the platform horizontal translation and the displacement associated with the tower deflection projected along the sensor direction, respectively.

6.3.1 Platform analysis

For the platform analysis, only two levels of sensors, corresponding to Section 1 and 4 in [Figure 6.3](#), have been used. It should be noted that an important contribution for the measured response is due to the structural displacement with the wave field, with the implication that the very

¹A for accelerations, T for temperature and S for strains.

relevant energy content in the waves frequency range is expected to make the identification of the very low frequency platform modes more difficult, leading to the definition of the preprocessing data strategy described below to mitigate this issue.

Data preprocessing

Before any analysis, a data whitening procedure was followed, that may be expressed as:

$$a_i(\omega) \rightarrow a_i^*(\omega) = \frac{a_i(\omega)}{f_i(\omega)} \quad (6.2)$$

where $a_i(\omega)$ and $a_i^*(\omega)$ are the representation of the measured data series and the transformed data series in the frequency domain, and $f_i(\omega)$ is the fitted background, obtained by passing the measured power spectrum through a low pass filter. An example of this procedure is represented, for one of the channels, in [Figure 6.7](#). The transformed time series will be used as input for the SSI-COV algorithm.

A word of caution is needed here, though, since the data whitening described above also impacts part of the response that is associated with the natural frequencies of the structure. Although this does require further investigation, it is not expected to have a great impact on the final results. In fact, and provided the low pass filter is suitably calibrated, the sharp shape of the natural frequencies peaks ensures that the normalisation is smooth enough to maintain most of the dynamic properties unaffected, unless, eventually, for very damped modes.

Modal shape reconstruction

An immediate consequence of the presented transformations is the distortion of the modal configurations, since each component is being transformed independently. However, it should be noted that the measured (ϕ) and transformed (ϕ^*) modal coordinates, obtained after the application of

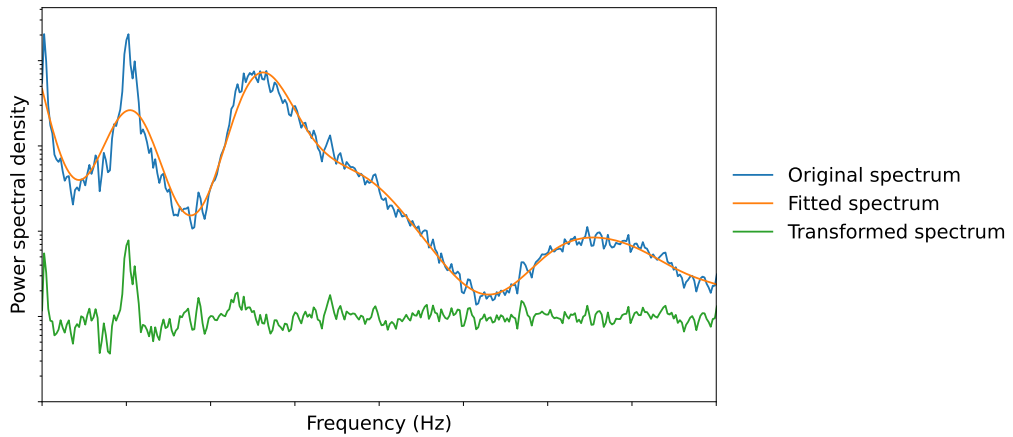


Figure 6.7: Data whitening process representation for one of the channels. The original, transformed and fitted spectrum correspond to the absolute value squared of $a_i(\omega)$, $a_i^*(\omega)$ and $f_i(\omega)$ terms in [Equation 6.2](#).

any OMA algorithm, are related by:

$$\phi_j^i \equiv \phi^i(\omega_j) = f_i(\omega_j) (\phi^i(\omega_j))^* = f_i(\omega_j) (\phi_j^i)^* \quad (6.3)$$

where ω_j is the j^{th} mode eigenfrequency and where the i component of the mode j has been defined as ϕ_j^i . It should be noted that although the expressions above hold for any $f_i(\omega)$, if this is not smooth enough around a given pole, the damping estimates will be affected.

Modes characterisation

After fitting and transforming back the mode shapes to the real modal space, it is important to understand how they can be characterised in terms of the typical rigid body motions of a floating structure, namely an horizontal translation along each principal direction of the platform (surge/sway), a vertical translation (heave) and a rotation around the horizontal axis associated with the translations (pitch/roll).

While the heave motions are expected to be identifiable as configurations with relevant modal coordinate associated with the vertical sensor, the remaining ones are more intrinsically coupled. In fact, the horizontal accelerations measured by a sensor that is sensitive to g (as is the case of the adopted MEMS sensors) at a given height H above the system centre of gravity can be approximated, in the frequency domain, by:

$$\tilde{a} = (g + H\omega^2)\tilde{\theta} + \omega^2\tilde{s} \quad (6.4)$$

where θ is the platform tilt (assumed to be small such that $\sin \theta \approx \theta$), s the platform translation (named after the usual identification as surge or sway motions) and the tower deflection has been neglected (reasonable assumption in the the frequency range under analysis, much lower than the first natural frequency with important tower deflection). The expression above has the immediate consequence of distorting the platform tilt and translation relative contributions and in fact two different mechanisms amplify the rotation motion, as perceived at the tower top. Firstly, the tower height amplification induced by H , that may be taken of order ~ 100 m. Additionally, the sensitivity to the gravitational pull, that is actually sampling the rotation itself, and not the acceleration, further enhances the rotation effect by comparison with the translation. An example of the evaluation of the accelerometers measurements at the tower top for different frequencies and tilt to surge ratio is presented in [Figure 6.8](#), while the ratio between tower top and base measurements for different combinations of translation and rotational modes is presented in [Figure 6.9](#). In both cases it may be seen that even a very small tower tilt can be dominant when compared with a given displacement, leading to a wrong identification of a rotation mode, when in fact it is mostly dominated by the translation motion. To solve this issue, one should look for additional metrics that may be used to separate between both. An immediate metric for θ can be defined as:

$$\Delta\tilde{a} = \tilde{a}_t - \tilde{a}_b = \omega^2\Delta H\tilde{\theta} \quad (6.5)$$

where $\tilde{a}_t$ and $\tilde{a}_b$ are the tower top and tower base measurements. On the other hand, a metric for the translation motion of the same mode can be obtained by defining the auxiliary variable $\tilde{a}$ as:

$$\tilde{a}' = \tilde{a} - \frac{g + H\omega^2}{\Delta H\omega^2} \Delta \tilde{a} = \tilde{a} - h(\omega; H, \Delta H) \Delta \tilde{a} = \omega^2 \tilde{s} \quad (6.6)$$

From these definitions, it is clear that a metric for the ratio between the translation and rotation amplitudes can be defined as:

$$\frac{\tilde{s}}{\tilde{\theta}} = \Delta H \frac{\tilde{a}'}{\Delta \tilde{a}} \quad (6.7)$$

The expression above allows the definition of a new variable that can be evaluated for each identified mode shape as:

$$\eta_j = \Delta H \frac{\phi_j^t - h(\omega_j) (\phi_j^t - \phi_j^b)}{\phi_j^t - \phi_j^b} = \left(\frac{\tilde{s}}{\tilde{\theta}} \right)_j \quad (6.8)$$

It should be noted that the auxiliary variable η defined above has dimensions, and hence should not be read as a direct measurement of whether a given mode is dominated by rotations or translations, except for the limiting cases where $\eta = 0$ or $\eta \rightarrow \infty$, but these are not expected to actually occur. However, it can still be used to compare different modes, noting that η_j/η_i is dimensionless and does provide a relative comparison of the translation/rotation dominance between modes i and j . Finally, it should be noted that while ΔH is well determined from the geometry of the monitoring layout, the same does not hold for H , defined with respect to the system centre of gravity, whose position in the absence of a numerical model is usually unknown. Nevertheless, if accurate information for H is not available, the expression above can be evaluated for a wide range of H and used to compare the relative contribution from rotations and translation to two different modes.

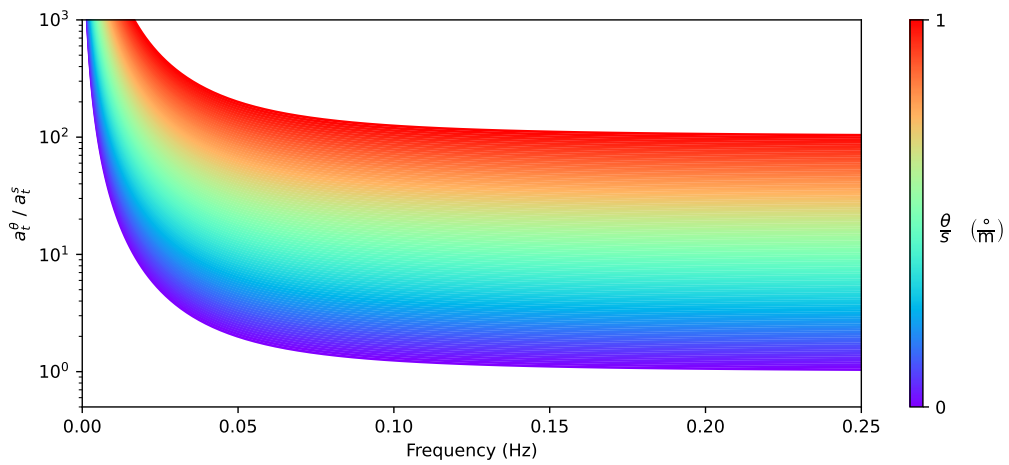


Figure 6.8: Comparison of the accelerometers readings at the tower top for different tilt/surge ratios, expressed in degrees per meter.

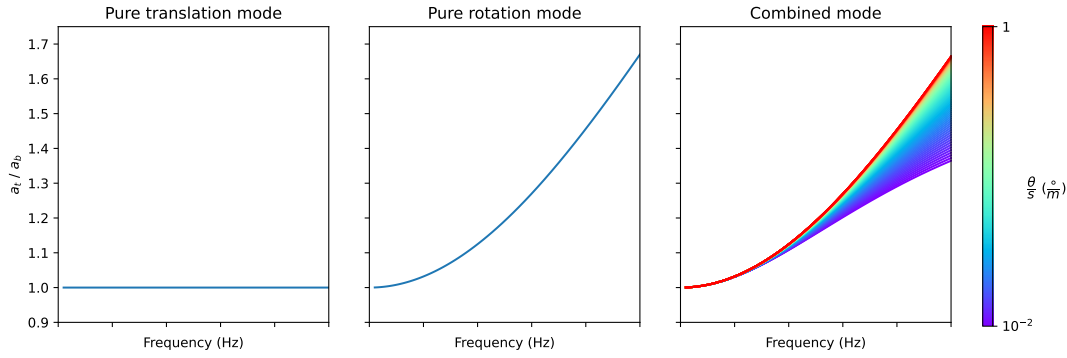


Figure 6.9: Ratio between the tower top and the tower base accelerometers readings for different modes combinations.

Results

In this section, the methodology proposed in the last section will be applied to one month of data obtained from the experimental campaign already described and compared with the SSI-COV results when applied to the raw data series. For all the results, 1 hour setups have been used.

In Figure 6.10 are presented the stabilisation diagrams (SD) for a given setup, where a pole was considered stable if its frequency and damping variation over consecutive model orders were less than 5% and 10%, respectively, and if the modal assurance criteria (MAC) was above 0.8. For the clusters definition, the distance between two given poles was computed as:

$$d_{ij} = 2 \frac{|f_i - f_j|}{f_i + f_j} + [1 - \text{MAC}(\phi_i, \phi_j)] \quad (6.9)$$

where ϕ_i is the modal configuration of the mode under comparison, and should not be read as the mode component, ϕ^i . In both analysis, the mode shapes have been divided by the maximum absolute value modal coordinate, which is equivalent to a rotation in the complex plane where the modal configurations live, and only after normalised by the modal amplitude, $\phi\phi^*$. As can be seen, the SD obtained with the transformed time series is cleaner than its raw counterpart and the lowest frequency modes are more easily constrained. Additionally, not only two pairs of relatively orthogonal modal configurations can be identified, but it may also be seen that the Z dominated modes appear to be easily identifiable from the value of the modal coordinate associated with the vertical sensors. A final remark should be made, however, by noting that the setup presented in Figure 6.10 was selected between those where the identification was cleaner and more complete and should not be considered as representative of an arbitrary setup. Nevertheless, and although most setups are not as clean as the one presented, it reveals the ability of the proposed methodology to highlight some very low frequency modes that are mostly undetected by the conventional procedure.

Applying now the same procedure for a full month of data results in the poles represented in Figure 6.11, where the vertical modal coordinate has also been included. Although the scale of the frequency and damping cannot be disclosed, it can be said that the modes under analysis present

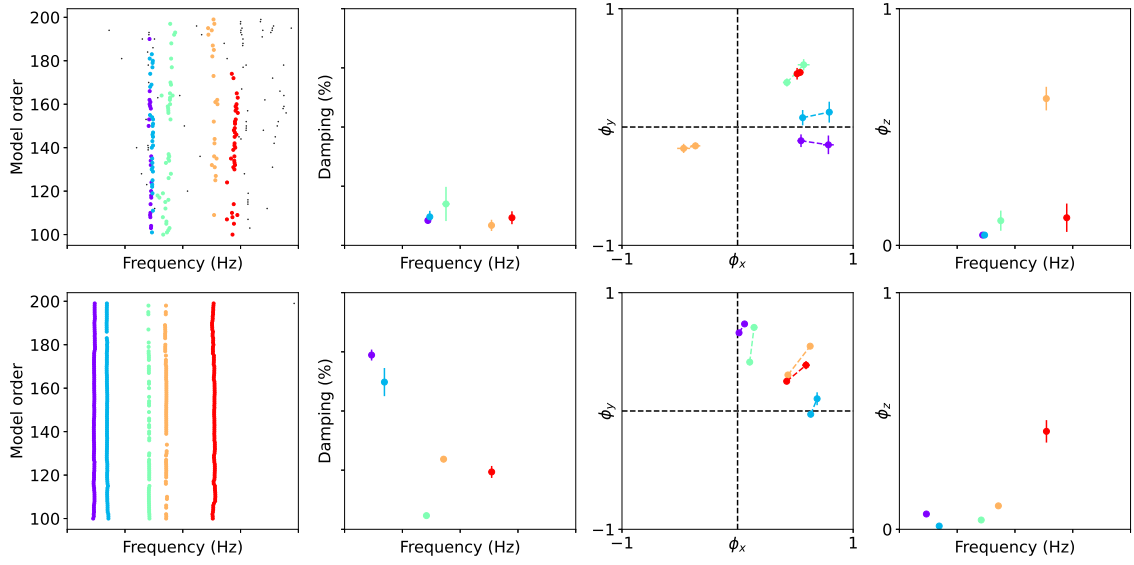


Figure 6.10: SSI-COV results with the raw time series (top plots) and the pre-processed ones (bottom plots). From left to right, the stabilisation diagrams considering different model orders, the corresponding damping diagrams of the stable poles only, the modal representation of the tower top and tower base modal coordinates in the horizontal, xy plane, and the corresponding vertical modal coordinate as a function of the mode frequency.

natural frequencies below 0.08 Hz and damping coefficients that are generally lower than 20%, although this may not always be the case. For the remaining analysis only the poles obtained by the proposed pre-processing methodology will be used.

Looking more closely into the fitted poles, it is clear that the z direction measurements can be used to provide a first separation between heave-dominated modes and non-heave dominated modes. Although the highest frequency region presents some dispersion, it can certainly be argued that a mode with $\phi_z > 0.2$, is most likely to be heave dominated. On the other hand, and referring now to the remaining, non-heave dominated, modes, the tower top and tower base modal coordinates, defined as:

$$\phi_i^{t/b} = \sqrt{(\phi_i^{t/b,x})^2 + (\phi_i^{t/b,y})^2} \quad (6.10)$$

are represented in Figure 6.12, where it is clear that the two clusters of points (presented in brown and orange) defined considering the values of the estimated natural frequency have different statistical properties for the tower base and top modal ordinates.

Following the discussion in Section 6.3.1, Equation 6.8 may be evaluated to compare the two low frequency defined clusters for a wide range of values of H . The quantities η_2 and η_1 were computed from the higher frequency and lower frequency modes modal coordinates distributions presented in Figure 6.12 mean value, respectively. The results for the ratio η_2/η_1 , are presented in Figure 6.13, where the ratio ϕ^t/ϕ^b of the estimated modes has also been included. As can be seen

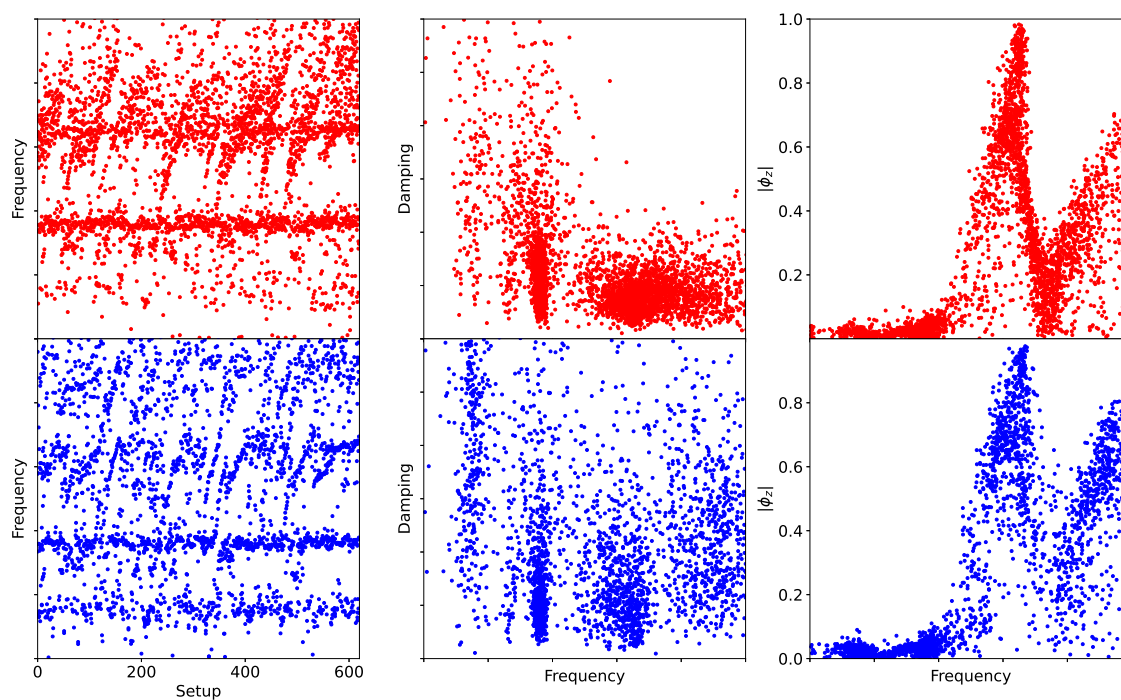


Figure 6.11: Poles fitted to all the data from the raw time series (top red plots) and transformed time series (bottom blue plots). On the left, the poles clusters frequency. On the centre panel the damping diagram against frequency. On the right panel the vertical modal coordinate as a function of frequency.

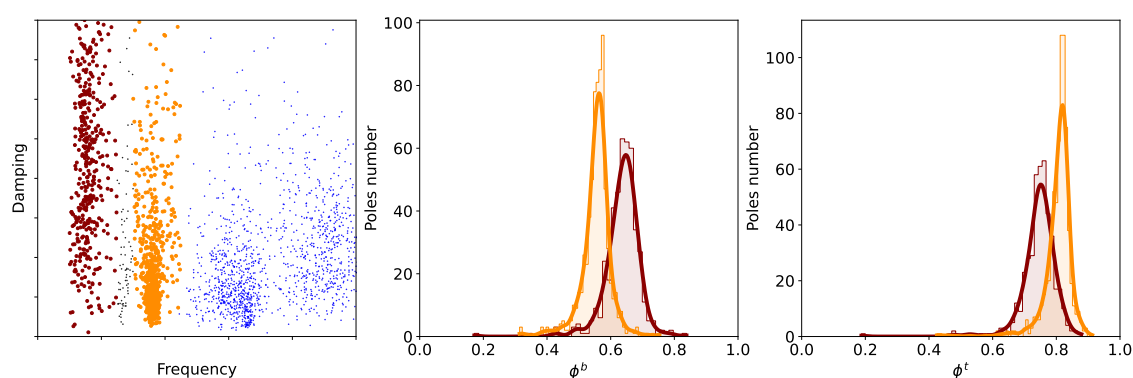


Figure 6.12: On the left, the damping diagram as a function of the mode frequency. On the centre and right plots, the tower top and tower base modal coordinates for the low frequency modes. For reference, the heave dominated poles are also represented in blue in the damping-frequency diagram.

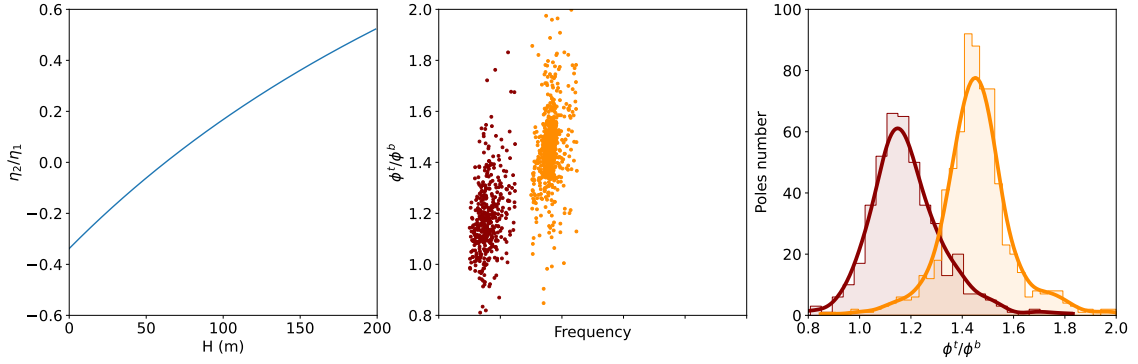


Figure 6.13: On the left, the ratio η_2/η_1 evaluated for a significant range of effective heights for the frequency defined clusters identifiable in the centre plot (η_1 and η_2 computed for the lower and higher frequency clusters, respectively). On the centre and right panels, the ratio between the tower top and tower base modal coordinates and its distribution, respectively.

from the proposed ratio, the lower frequency group of poles is expected to be more dominated by translations than the higher frequency ones.

The full set of poles can now be divided in 3 distinct groups. Firstly, the z -modal coordinate is used to identify heave-dominated modes. Only after, the remaining modes are identified according to the ratio ϕ^t/ϕ^b being higher or lower than 1.25, a value that was considered representative from the distributions in Figure 6.13. All the poles coloured according to these conventions are represented in Figure 6.14, where the mode orientation in the xy plane has been defined as:

$$\varphi_i = \arctan \frac{\phi_i^{t,y}}{\phi_i^{t,x}} \quad (6.11)$$

It should be noted that in this plot (in opposition to the ones presented in Figure 6.13) the poles labelling does not consider the frequency directly, but rather the different physics-based metrics previously defined. This option provides a natural interpretation for the modes as surge/sway, roll/pitch and heave dominated. Additionally, as represented in the right panel of Figure 6.14, and following one of the conclusions of Pimenta et al. (2020b), it appears that these modes can be mobilised in different directions. A possible explanation for this behaviour is that the nearly symmetric configuration of these structures does not allow for a proper decomposition of the modes, but rather mobilises a single direction governed by the waves excitation. Alternatively, it may also be that this method is still not refined enough to make this separation properly, a question that is left for future works and enhanced models.

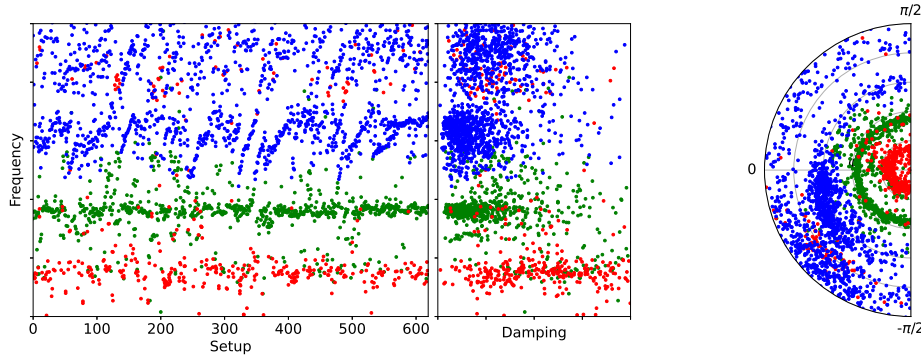


Figure 6.14: Tracked poles across the setups. On the left panel, the identified frequency. On the centre panel, the damping diagram. On the right panel the orientation of the different modes.

6.3.2 Tower analysis

Focusing now on the tower response, the SSI-COV conventional approach was found to provide good results. For this particular analysis, the 3 levels of accelerometers time series have been used (Section 1, 2 and 4 in Figure 6.3), but the vertical sensor near the tower base was not considered. Additionally, all the time series were projected along the FA and SS direction, as defined by the nacelle orientation and determined by the analysis that will be presented in Section 6.4.

To compute the stabilisation diagrams, a pole was considered stable if the frequency and damping coefficient variation between consecutive orders was lower than 1% and 5%, respectively, and the MAC was at least 95. Although these criteria were used to define a stable pole, the different clusters were computed using the following metric between poles i and j :

$$d_{ij} = 2 \frac{|f_i - f_j|}{f_i + f_j} + [1 - \text{MAC}(\phi_i, \phi_j)] \quad (6.12)$$

where ϕ_i is the modal configuration of the mode under comparison. Additionally, only clusters with a minimum of 20 elements were accepted. In Figure 6.15 are represented the stabilisation diagram obtained for two different setups. In both cases the wind turbine is parked, but the yaw and pitch angles differ between them.

Although both diagrams share some common trends, they also reveal some important differences. Focusing for now on the first pair of modes, represented in Figure 6.16 for the same setups, it can be seen that they are compatible with the tower first mode in both directions, where the different distribution around the SS and FA direction can be partially accounted for from the uncertainties in the relative position of the sensors with respect to the yaw reference direction.

It should be noted that the modal coordinate at the tower base, as discussed before, is proportional to the platform tilt. From Figure 6.16 it is clear that the tower top and tower base coordinates have opposite signals, as predicted in Section 5.4. Additionally, the modes, and in particular the FA dominated one (in blue), present a frequency shift in the two setups under analysis that is not irrelevant. Referring back to Figure 6.15, it also appears that some well defined alignments are present only in one of them, or alternatively must also exhibit a significant frequency shift.

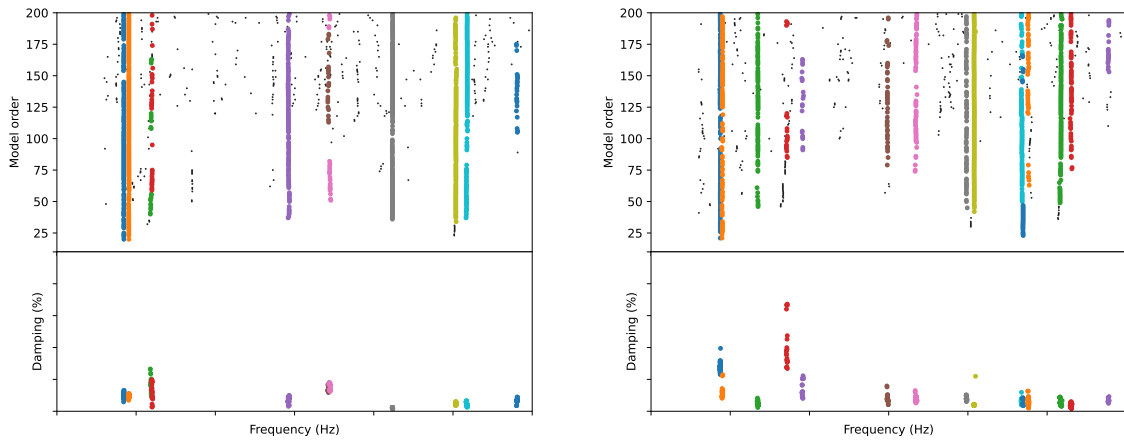


Figure 6.15: Stabilisation diagrams for two different setups under parked conditions.

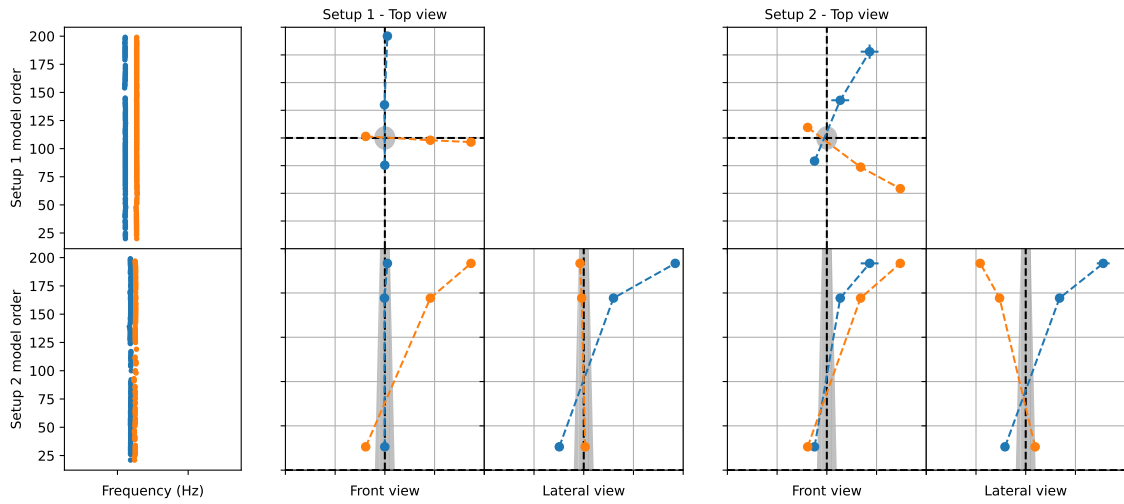


Figure 6.16: First pair of modes for two different setups under parked conditions.

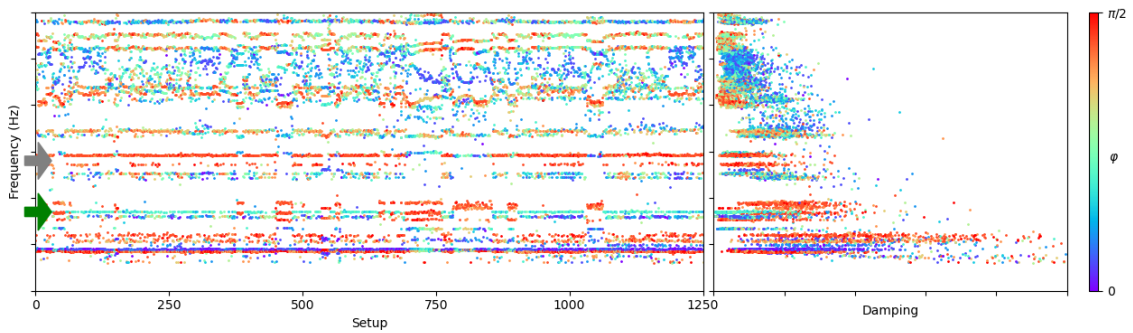


Figure 6.17: Identified clusters in all parked setups and corresponding damping coefficient. The points have been coloured as a function of the relative importance of the FA and SS components, parameterised by the orientation φ as defined by Equation 6.12, meaning that red and blue points are mostly FA and SS modes, respectively.

To understand if these differences are only observed in these particular setups or if they are related with additional structural parameters, a more systematic approach is needed and a full month of data was analysed, leading to the results presented in Figure 6.17. There, are represented the identified clusters for all the parked setups within the full data set, where the points have been coloured based on the corresponding mode orientation, defined as:

$$\varphi = \arctan \frac{\max \phi_x}{\max \phi_y} \quad (6.13)$$

being ϕ_x and ϕ_y the modal coordinates in the FA and SS direction, respectively.

From the results presented, it seems clear that some physically meaningful frequencies are consistently obtained across the setups, but not in all of them. For instance, the triple clusters around the frequency identified by the green arrow is only identified in the setups where the stable alignments near the grey arrow are not seen, and vice-versa.

To solve this issue, the FA and SS dominated modes have been analysed separately and correlated with different operating conditions and it was found that this behaviour could be explained from the yaw direction of the rotor and the pitch angles of the blades, as represented in Figure 6.18. It should be noted that the periodicity of the identified natural frequencies is expected to be related to the relative orientation between the nacelle (or wind) direction and the platform principal directions, implying that the observed trend should be well explained by a variation law of the type:

$$f_i(\theta) = f_0 + A |\sin(\psi + \psi_0)| \quad (6.14)$$

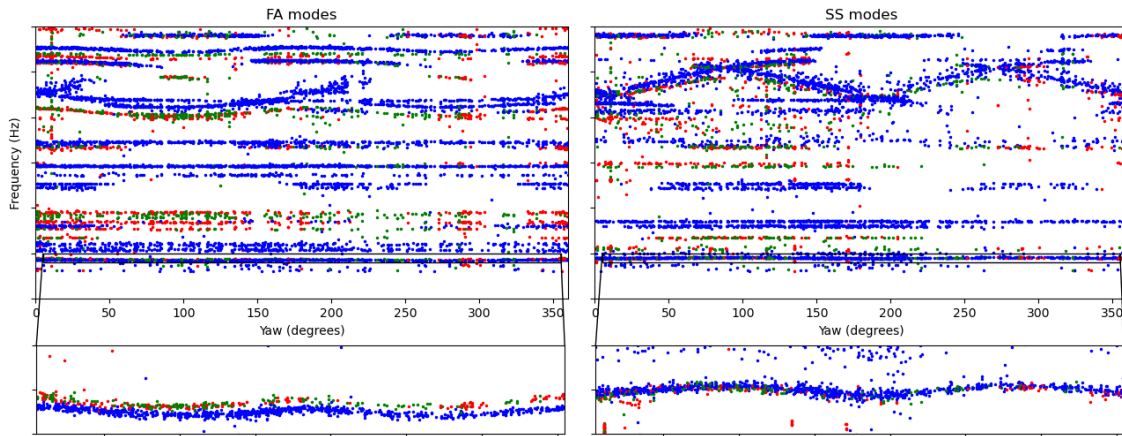


Figure 6.18: Identified clusters in all parked setups as a function of the yaw direction. The points colour identifies the different pitch values found in these setups. The blue points correspond to the lowest value of the pitch angle associated with parked conditions and the red points to the highest value. The green points are for intermediate pitch values, closer to the upper limit than to the lower one.

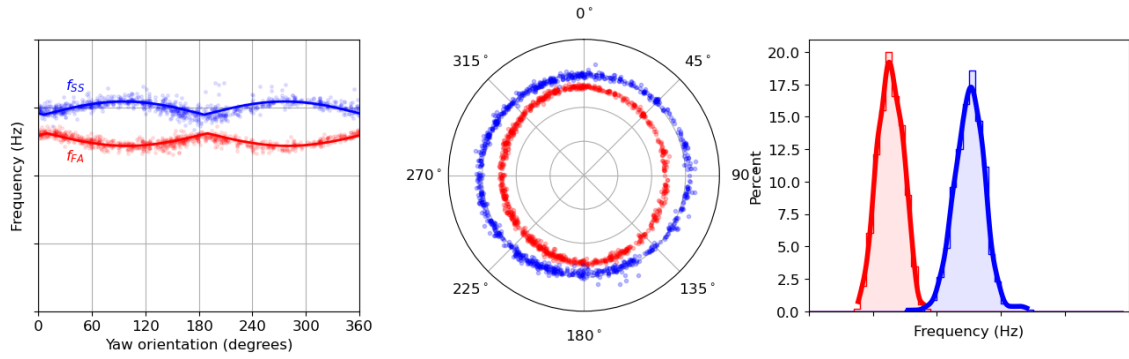


Figure 6.19: Evolution of the first FA (red) and SS (blue) natural frequency with the nacelle orientation and corresponding fit.

where f_i is the observed frequency identified for a yaw orientation ψ , f_0 a constant with frequency units and ψ_0 the relative difference between the reference axis for the nacelle yaw measurements and one of the platform principal directions. Focusing for now on the first pair of modes and restricting the setups to those corresponding to the lower pitch values (the blue points in Figure 6.18), it is possible to determine the parameters needed for Equation 6.14, as represented in Figure 6.19. It should be said that the expression above is expected to be particularly suitable for the SS direction, since in the FA direction the orientation of the rotor may induce a non equivalent response for orientations separated by 180° due to the non symmetric location of the tower with respect to the platform. For this reason, the SS fitted parameters will be used as reference to define the relative orientation between the nacelle and the platform main directions.

Finally, and looking now for the setups where the wind turbine is under normal operation conditions, the stabilisation diagrams are more cumbersome, mainly in the FA direction, due to the appearance of the rotor harmonics. To aid the visual interpretation of the results, for this analysis only particularly stable setups have been used. To avoid the nacelle direction degeneracy identified under parked conditions, but still using the highest number of setups possible, only setups where the wind direction was in the range $\psi_{\text{peak}} \pm 25$, where ψ_{peak} is the dominant wind direction identified from the SCADA data, were selected. Additionally, to ensure that the rotor angular velocity as registered by the SCADA system was representative of the operation conditions, an extra filter was considered where only setups with variations over 10 minutes of this parameter lower than 0.6 RPM have been used. The evolution over time of all the poles identified in these setups is presented in Figure 6.20.

Some relevant aspects should be highlighted right away based on Figure 6.20. Firstly, it can be seen that some alignments are stable throughout all the analysed setups, while others seem to fluctuate over time. Additionally, and focusing now on the damping estimates, it may be seen that the redder alignments (predominantly FA dominated modes) tend to exhibit higher damping than their bluer counterpart (SS dominated modes). This aspect is particularly evident in the first pair of modes, where both directions peak at similar frequencies. In any case, the results are better understood if analysed separately in the FA and SS directions, and, for normal operation

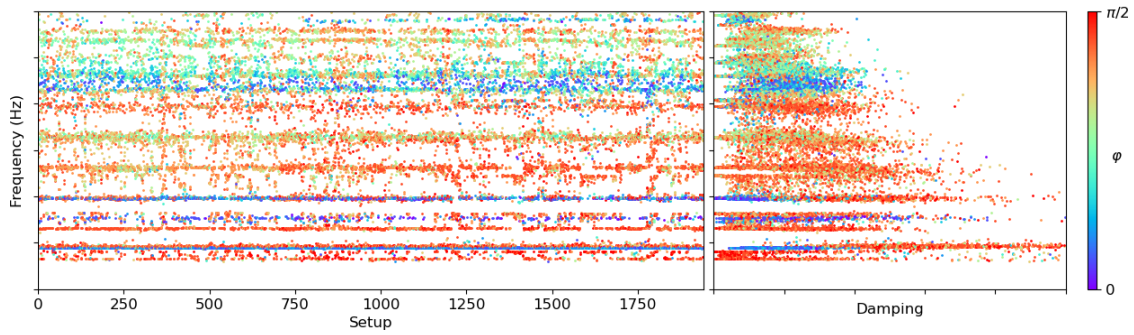


Figure 6.20: Identified clusters in selected stable normal operation setups and corresponding damping coefficient.

conditions, if ordered by the rotor angular velocity. There is, however, a data driven limitation to produce such a plot, since the rotor angular velocities are only known at some discrete values, space by 0.3 rpm. A possible solution, although with its own limitations, is to order the setups within each Ω bin according to the power production.

The results, organised based on the rotor angular velocity and on the criteria just mentioned are presented in Figure 6.21, where the points have been coloured as a function of the identified damping coefficient. As can be seen, the FA direction exhibits a very pronounced dependency on the rotor angular velocity, while the SS direction appears to be much less affected by it, meaning that the rotor induced motion is mostly associated with the former direction. Looking more closely to the FA results, it can also be seen that the different rotor harmonics, and the 3Ω in particular, present very low damping coefficients, as one would expect for forced harmonic motion. On the other hand, the frequencies that are constant over the different operation conditions exhibit much

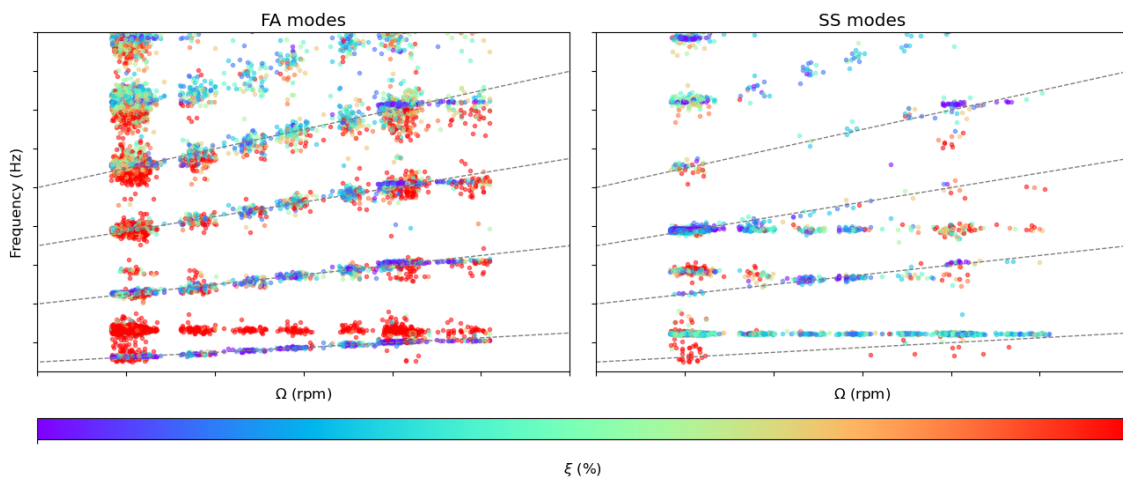


Figure 6.21: Identified clusters in selected stable normal operation setups as a function of the rotor angular velocity for the FA (on the left) and SS (on the right) direction. The points have been coloured based on the estimated damping coefficient.

higher damping coefficients when compared with either the rotor harmonics or their SS counterparts. This may be accounted for noting that the aerodynamic drag force is more pronounced in this direction. Finally, these results could still be used to automatically track a given mode over time in a more quantitative approach, similar to conventional procedures in onshore wind turbines (see, for instance [Pereira et al., 2023](#)). Here, these results are no further explored, since the relevant aspect to retain is the capability of producing meaningful and coherent over time stabilisation diagrams for floating wind turbines.

6.4 Internal forces analysis

The stresses in the instrumented fibres can be obtained by Hook's law as $\sigma = \varepsilon \times E$, being E the Young modulus of steel, taken to be 210 GPa, and ε the corresponding measured strains, after corrected for temperature effects according to the sensor data sheet and the temperatures measured in the vicinity of each sensor. Since the sensors are oppositely placed in the cross section, the internal forces can be related to the measured strains in the direction defined by the sensors as:

$$M_{\perp} = \frac{\sigma_+ - \sigma_-}{2} \frac{I_{\perp}}{r} = \frac{\varepsilon_+ - \varepsilon_-}{2} \frac{EI_{\perp}}{r} \quad (6.15a)$$

$$N = \frac{\sigma_+ + \sigma_-}{2} A = \frac{\varepsilon_+ + \varepsilon_-}{2} EA \quad (6.15b)$$

where σ_+ (ε_+) and σ_- (ε_-) are the stresses (strains) in the positive and negative semi-axis, respectively, and N and $M_{\perp}$ are the axial force and the bending moment in a direction orthogonal to the direction defined by the sensors, taken to be positive if it induces tension in the positive semi-axis, as represented in [Figure 6.22](#). In the same representation are also included the relevant parameters to transform the bending moments to any relevant direction characterised by its angular position ψ and $\psi + \pi/2$ through:

$$M_{x'x'} = M_{xx} \cos \psi + M_{yy} \sin \psi \quad (6.16a)$$

$$M_{y'y'} = M_{xx} \cos (\psi + \pi/2) + M_{yy} \sin (\psi + \pi/2) = -M_{xx} \sin \psi + M_{yy} \cos \psi \quad (6.16b)$$

where M_{xx} and M_{yy} are the bending moments in the sensors defined directions, as computed from [Equation 6.15a](#).

Assuming the axial forces do not vary significantly and that non linear thermal effects produce symmetric effects in oppositely placed sensors, a first correction can be applied to time series, by imposing that the mean of two opposite sensors is zero. From here on, all the references to stresses are hence related to the stresses induced by the bending moments alone and should not be read as an absolute value. However, and before proceeding to the computation of the bending moments, the strain gauges must be properly calibrated, as discussed in [Chapter 5](#).

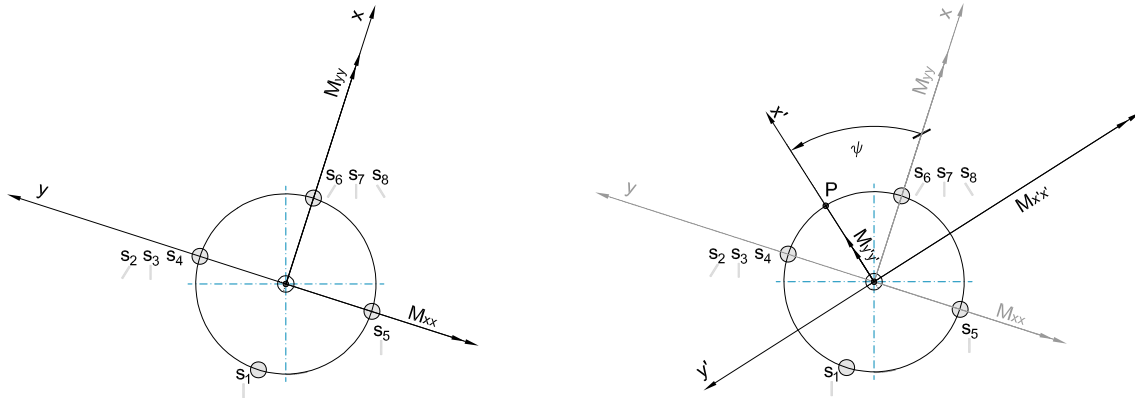


Figure 6.22: Axis convention for the bending moments positive direction. On the left panel, the axis are defined according to the sensors positions. On the right panel is schematically represented the bending moments associated to a reference frame rotated of θ with respect to sensors defined reference frame.

6.4.1 Strains calibration

Since the tower non vertical position effect on the accelerations and bending moments is by now properly understood from the discussion in [Chapter 5](#), the estimates from both sensors can be compared and used to calibrate the strain gauges. Noting that the accelerations and bending moments amplitude is not impacted by the eventual sensor drifts, [Equation 5.101b](#) can be used to constraint the important quantity $PH + P_t H_t = HP(1 + \beta)$ that maps accelerations to bending moments. However, it should be noted that the derived expressions tacitly assume that the bending moments at the instrumented section are only due to the nacelle eccentricity, which is not the case since the wind action over the structure still has to be considered. Although the nacelle full rotations for power cable untwist events are usually carried out under relatively low wind speed conditions, this fact implies that the relatively small number of occurrences do not provide strong enough constraints on the quantity above. A better approach is to use all the 10 minutes setups and note that, if the wind action is mostly suppressed, it should still be true that:

$$\frac{\bar{M}}{HP} = \frac{L}{H} \cos \psi \cos \theta_0 + (1 + \beta) \sin \theta_0 \quad (6.17a)$$

$$\bar{a}_x = \sin \theta_0 \quad (6.17b)$$

Assuming that environmental conditions are uniformly distributed over the monitoring period, plotting the raw bending moments against the mean acceleration for all 10 minutes setups, would still provide a linear fit with slope proportional to $PH + P_t H_t$, if the yaw did not vary. Since it does vary, the ψ - θ_0 degeneracy may still be solved by restricting the events to be used to those that have similar values for ψ , or, alternatively, to look for setups where the eccentric load is expected to dominate (large amplitude setups).

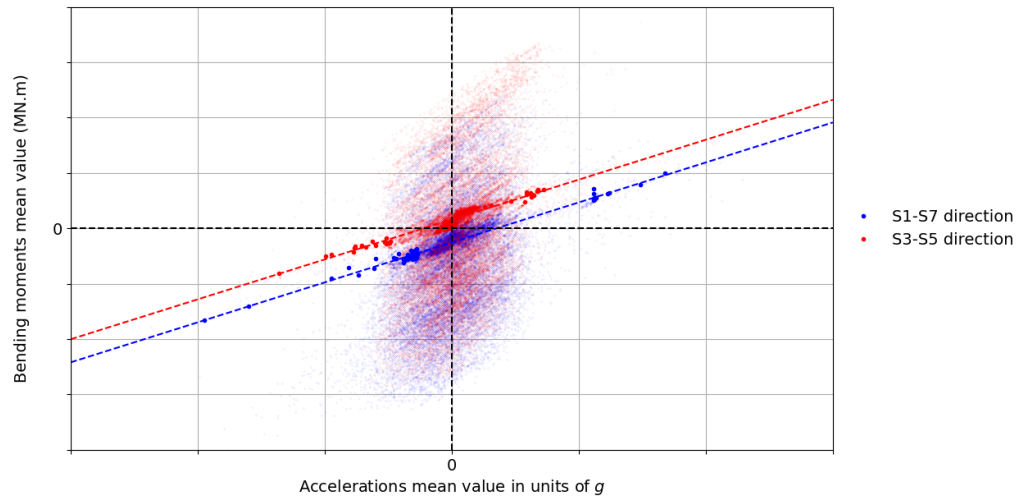


Figure 6.23: Mean bending moments and accelerations for all the setups. For the accelerations the accelerometers in the tower base have been used and projected into the strain gauges direction. Only the highlighted points have been used for the fit.

In Figure 6.23 are plotted the mean acceleration reading and computed bending moments, where only the extreme 5% tilt events under parked rotor conditions have been used for the fit. An immediate conclusion is that even without the wind loading on the structure, the platform motions can still induce significant bending moments. Once this quantity is obtained, the nacelle rotation for cable untwist events can be analysed independently. The bending moments and accelerations time series are represented in Figure 6.24 for one of those events. For the fitting procedure, the data was filtered to remove higher frequencies contribution, as represented in the middle panel of Figure 6.24, where the mean values and amplitude have been schematically represented. Finally, as a side result of fitting Equation 5.102, the relative yaw position to each alignment is also obtained, as represented in the right panel of Figure 6.24.

Applying the same reasoning to all the untwist events, different mean values and amplitude for both accelerations and bending moments are obtained that can be related and used to calibrate the strain gauges, as long as the accelerations readings are to be trusted for evaluation of absolute rotations². To analyse this issue the platform sensors and accelerometers measurements have been compared, with the results presented Figure 6.25, where the sensors readings have been projected into the platform sensors directions.

Even considering the uncertainty in the relative orientation, two major conclusions may be drawn from Figure 6.25. Firstly, when properly ordered in time, the readings are consistent for all the setups, hence ruling out the existence of any significant drift. Secondly, and assuming that the platform sensors are properly calibrated³, an additional offset should be considered for the accelerations mean values in the platform x_p and y_p directions of -0.50×10^{-2} and 1.24×10^{-2} ,

²The differential measurements over any time period are always reliable.

³It may be argued that the offset could be applied in any of the measurements. However, it should be noted that this will only translate in an offset in the bending moments estimate, but that it is not relevant for drift correction purposes.

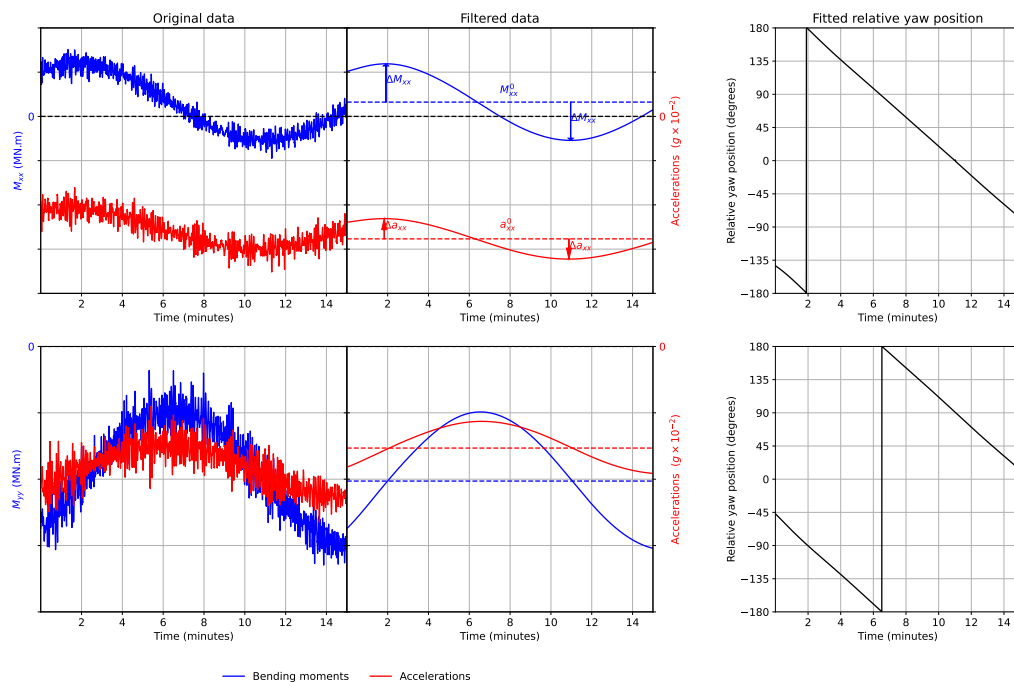


Figure 6.24: Untwist event bending moments (in blue) and accelerations (in red) time series. On the left plot are presented the strain time series (corrected for temperature and zeroed). In the middle panel are plotted the filtered time series used for the fitting and with the relevant quantities schematically represented. On the right panel, it is estimated the yaw relative direction, that can be seen to vary smoothly during the event.

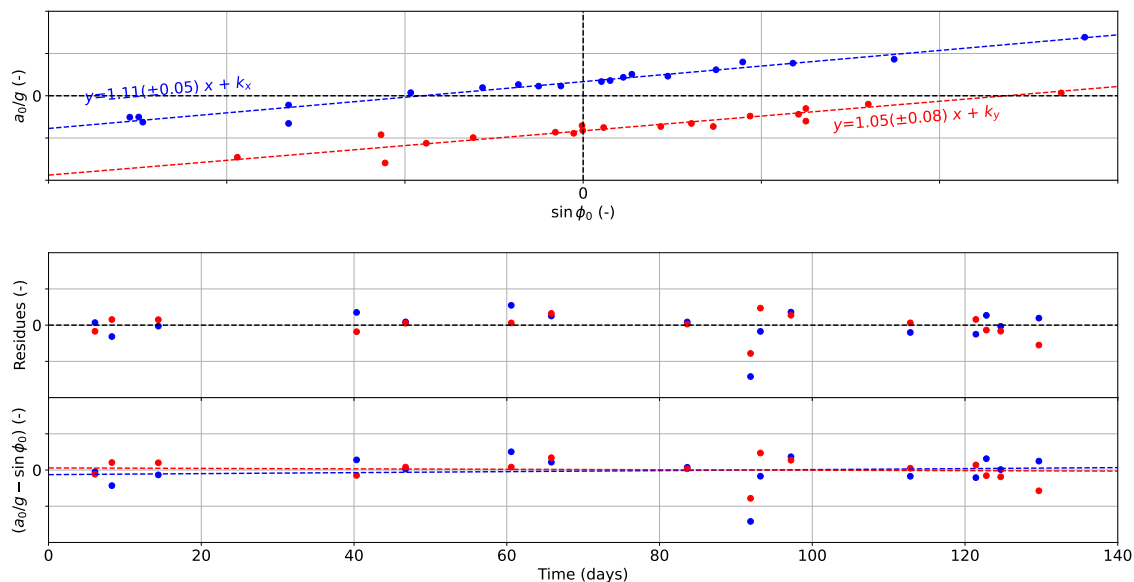


Figure 6.25: Mean accelerations value as a function of the platform sensors measurements (on top) and the corresponding residues ordered in time (on bottom). The blue series correspond to S1-S7 sensors alignment and the red series correspond to the S3-S5 sensors.

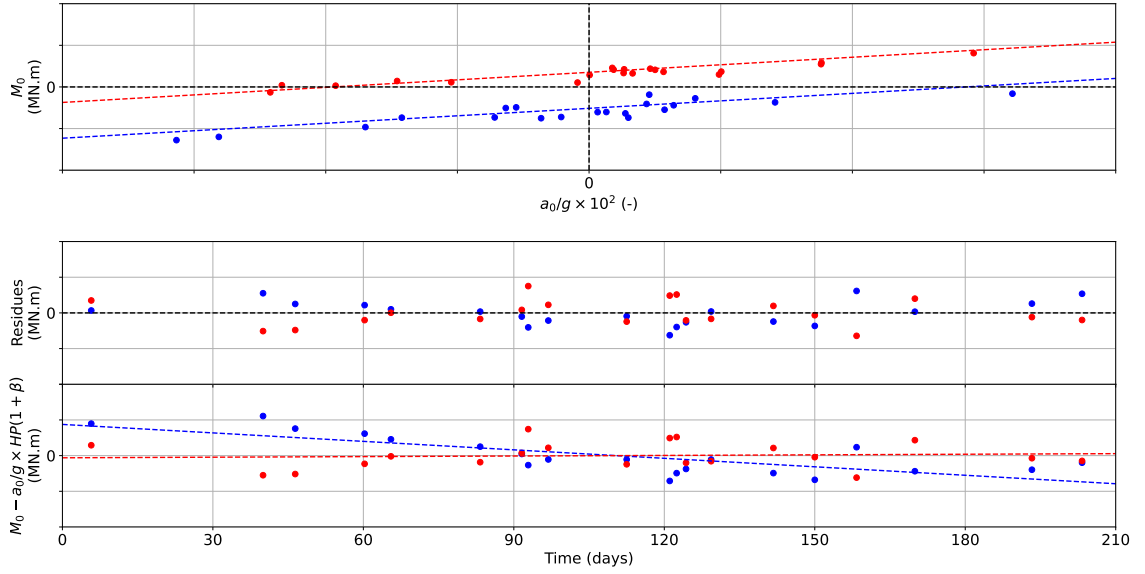


Figure 6.26: Mean bending moments as a function of the mean accelerations measurements (on top) and the corresponding residues ordered in time (on bottom). The blue series correspond to S1-S7 sensors alignment and the red series correspond to the S3-S5 sensors.

both in units of g , corresponding to a proper correction of -1.27×10^{-2} and -0.40×10^{-2} , now in the sensors x and y direction. The need for such corrections might be justified by the fact that the accelerometers are not perfectly aligned with the horizontal direction.

Once the accelerometers have shown to be reliable, at least when it comes to their drift over time, their measurements may now be compared to the bending moments estimates obtained from the strain gauges themselves during all the observed untwist operations as represented in [Figure 6.26](#). The differences between both estimates have been computed and the residues ordered in time, where it is indeed observed a linear long term drift of the strain gauges measurements and can be used to define the correction to be applied to each sensor. The results after applying the corrections to both the accelerations and the bending moments time series are presented in [Figure 6.27](#), where an uncertainty of ~ 1 MN.m in each direction remains, which is quite acceptable considering the values of the observed bending moments. It should still be mentioned that the identified trend may vary over time, so this calibration should be continuously evaluated.

Finally, if the analysis above has relied on the registered mean values, the bending moments and acceleration measurements amplitudes can be used to obtain from [Equation 5.101b](#) and the slope constrained in [Figure 6.23](#) an estimate for the nacelle eccentric loading on the tower, a quantity that will be relevant for the internal forces analysis presented in [Section 6.4.3](#).

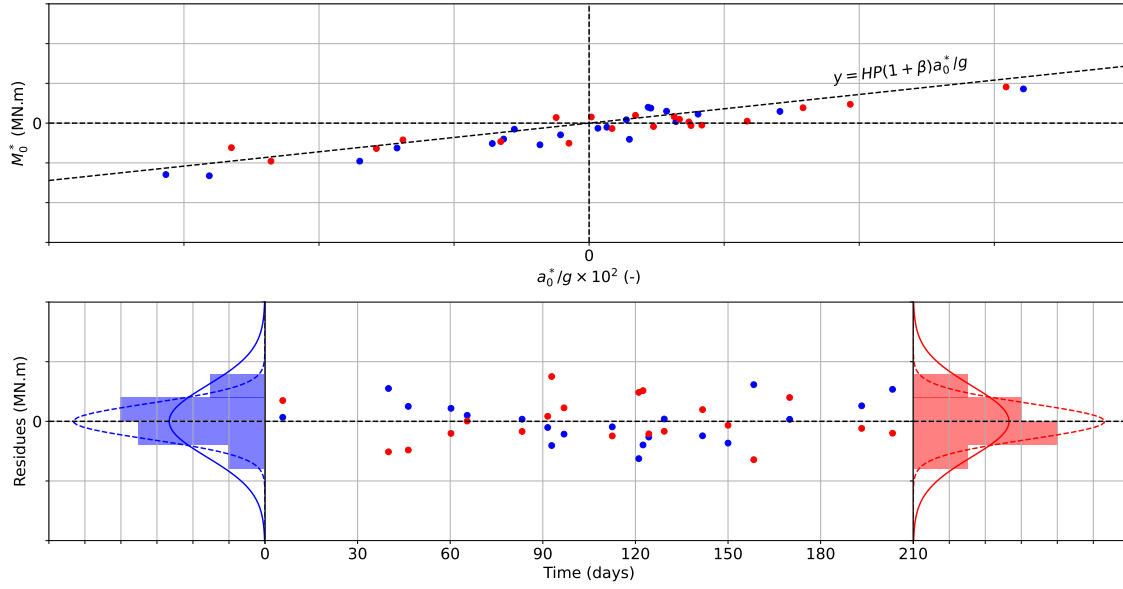


Figure 6.27: Corrected accelerations a_0^* and bending moments M_0^* time series. In the residues plots the Gaussian distribution assuming a standard deviation of 1.5 MN.m (solid lines) and 0.75 MN.m (dashed lines) have been included. The blue series correspond to S1-S7 sensors alignment and the red series correspond to the S3-S5 sensors.

6.4.2 Determination of the FA and SS directions

Since the bending moments are most significant when expressed in the FA and SS direction, as defined by the nacelle orientation, an estimate for the relative orientation between the yaw reference direction and the sensors location is needed. To define it, it may be noted that the maximum mean tension is expected to be located along the wind direction. Since the reasoning above tacitly assumes that the bending moments are mostly related to the wind action, the setups where the wind turbine is fully operating will be used. For all these setups the bending moments principal direction has been computed as:

$$\phi_{\max} \approx \arctan \frac{\bar{M}_{yy}}{\bar{M}_{xx}} \quad (6.18)$$

and compared to the yaw defined direction through the relative orientation distribution, represented on the right panel for a lower range of wind speeds (red points), as:

$$\Delta\phi = \phi_{\max} - \psi_{\text{yaw}} \quad (6.19)$$

The results are plotted in Figure 6.28, where it can be seen that the difference between both is mostly constant for all setups with an uncertainty $\sim 5^\circ$ at the 1σ level.

It should be noted that due to the presence of both FA and SS bending moments, the direction above may be deviated from the actual yaw reference direction by some degrees. Additionally, for the region where the pitch control is not active, it is expected that both quantities above (FA

and SS bending moments) are mostly proportional to the wind speed squared, which leaves the deviation constant. However, when the pitch control starts to actively reduce the loads, the FA bending moments are expected to increase at a lower rate than its SS counterpart. This effect is clearly seen in the middle plot in Figure 6.28. The corresponding estimates for the sensors location with respect to the platform main directions are presented in Figure 6.29.

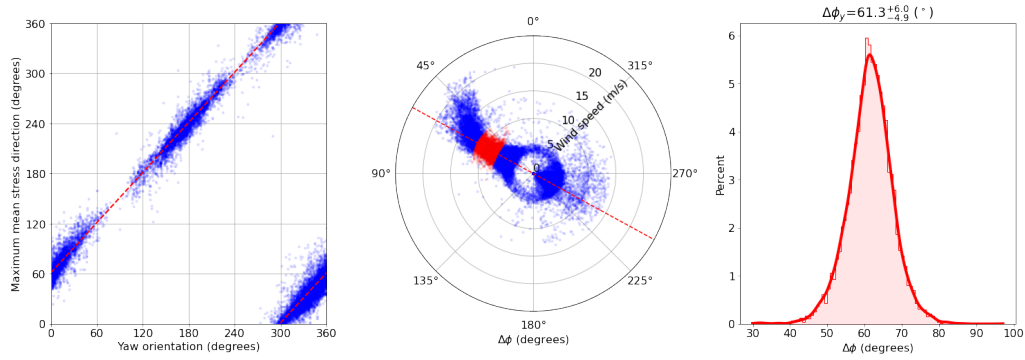


Figure 6.28: On the left panel is plotted the maximum mean stress orientation as a function of the nacelle yaw direction. The relative direction between the two quantities ($\Delta\phi$) is plotted as a function of the wind speed (in the middle). The distribution of the selected events, marked in red in the centre panel, is plotted on the right panel.

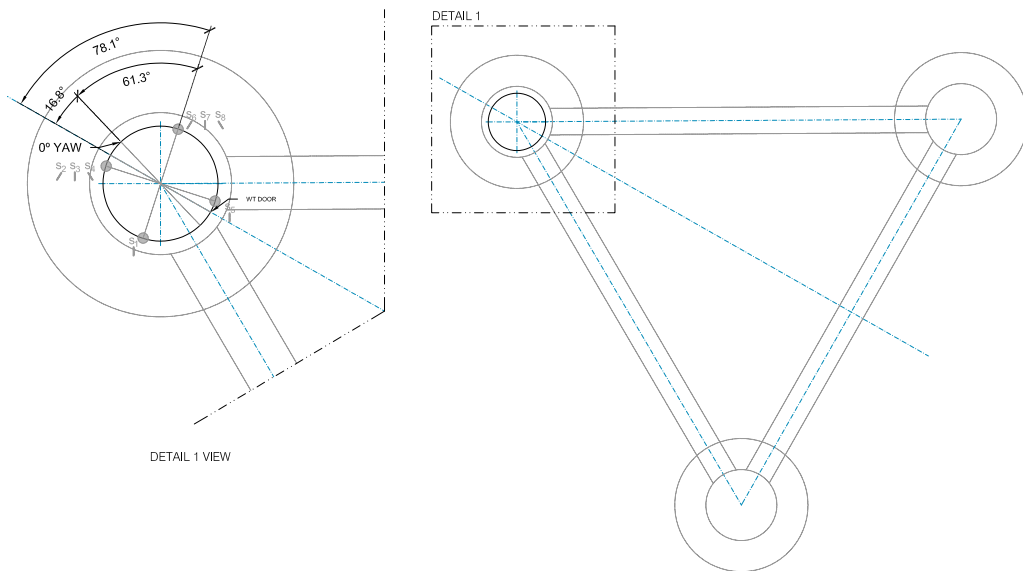


Figure 6.29: Estimated relative position of the strain gauges, reference yaw direction and the platform principal direction. To determine the angle between the alignment S1-S7 and the axis of symmetry of the platform, it was assumed that the tower door is in the plane of symmetry of the platform.

6.4.3 Bending moments reconstruction

With the strain gauges calibrated and the relative orientation between the nacelle reference direction and the strain gauges position determined, the FA and SS bending moments may finally be reconstructed for all the monitoring period.

To check that both calibration and relative direction between the sensors reference frame and the yaw position are properly defined, the bending moments along the two sensors axis are plotted in [Figure 6.30](#) as a function of the wind direction. For the purposes of this verification, only setups where the relative orientation between the nacelle direction and the wind main direction was small and in which the wind turbine was either operating or non operating, but not shutting down or starting up, have been used. It can clearly be seen that the internal forces peak is compatible with the previously determined relative direction obtained with the estimated angle and expected maximum thrust force at the rotor (gray shaded region).

Finally, in [Figure 6.31](#) are plotted the FA and SS bending moments for all setups as a function of the mean wind speed, where the absolute maximum value envelope has also been included. These results have been computed by distributing all the setups in 1 m/s wide bins based on wind speed. It can be seen that, although the results are essentially compatible with the theoretical thrust curve, some important dispersion is also identifiable. This issue is better understood by noting that three additional loading sources, other than the wind directly, should be considered and removed, namely:

1. The nacelle eccentric mass
2. The effect of the tower tilt
3. The effect of the wind loading acting over the tower

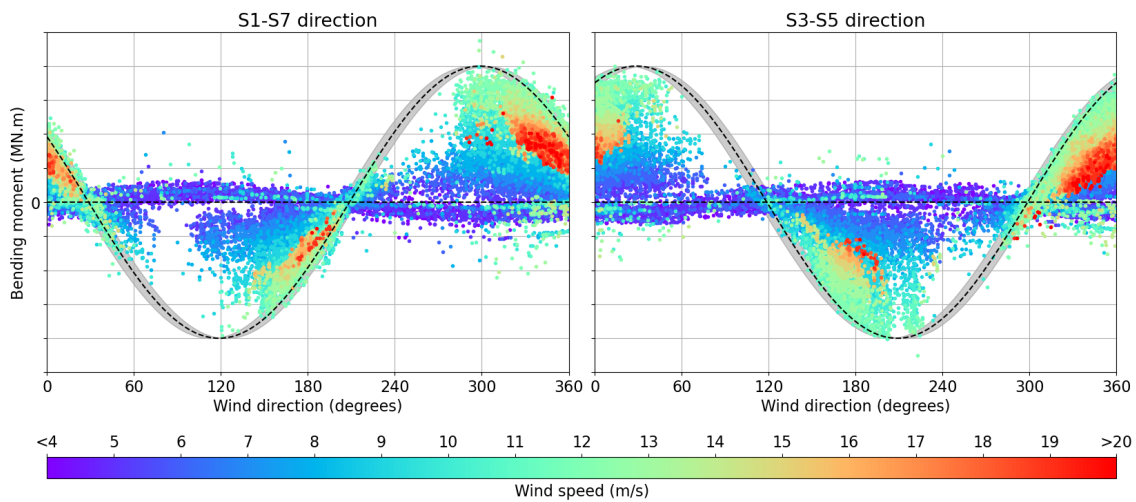


Figure 6.30: Bending moment along the S1-S7 and S3-S5 axis as a function of the yaw position. The dashed line represented the bending moments envelope for a relative orientation of 61.3° . The shaded region represent the same envelope considering the uncertainty in this estimate from [Figure 6.28](#).

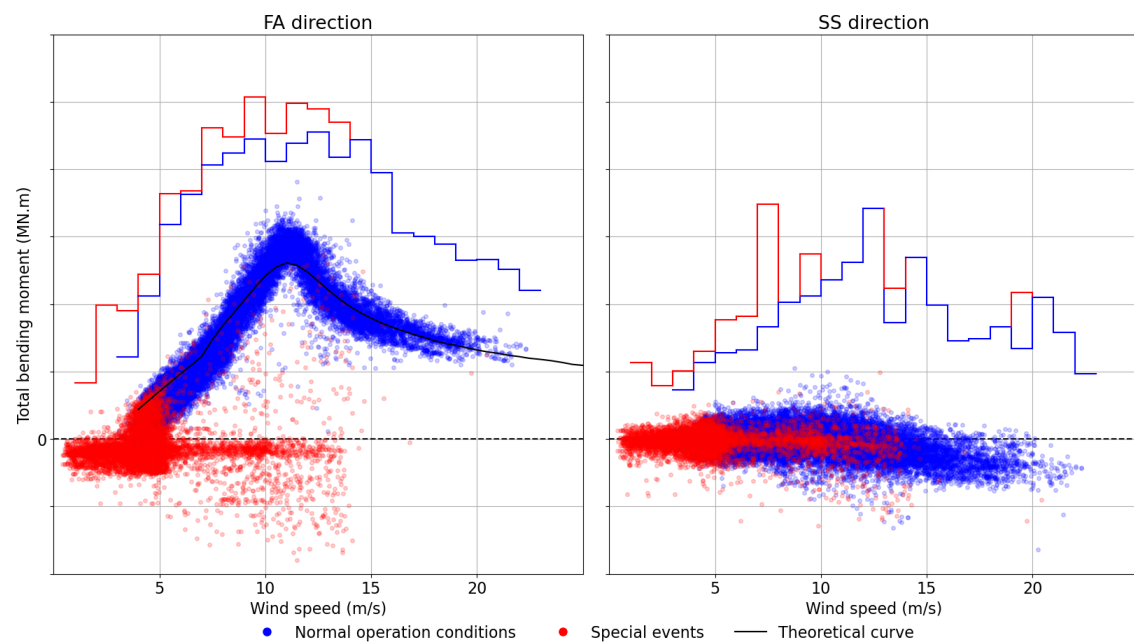


Figure 6.31: FA (on the left) and SS (on the right) bending moments as a function of the mean wind speed.

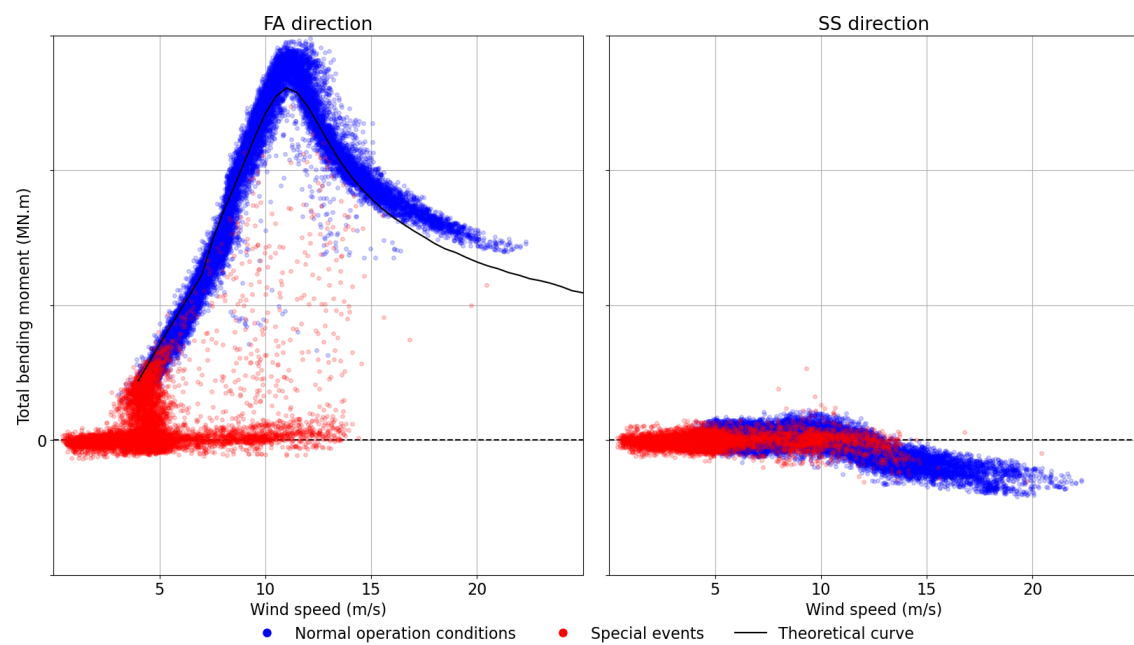


Figure 6.32: FA (on the left) and SS (on the right) bending moments induced solely by the rotor thrust force as a function of the mean wind speed.

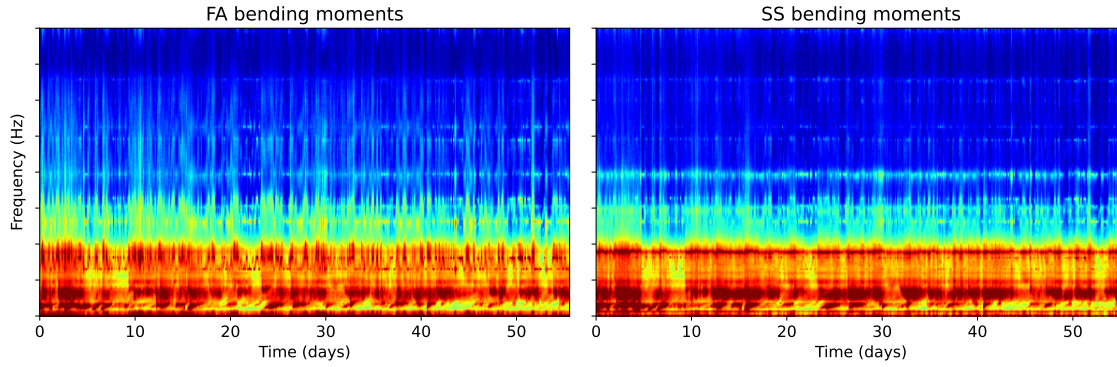


Figure 6.33: FA and SS bending moments frequency content for normal operation setups.

For the first effect, it may simply be considered the previously obtained estimate for the nacelle eccentric loading in the FA component. For the second effect, the accelerometers and the calibrated relation between the mean acceleration value and the mean bending moments may be used. Although no rigorous expression exists for the last effect, the corresponding bending moments at the instrumented section may still be approximated by:

$$M_{\text{tower}} = \int_h^H \frac{\rho}{2} \cdot C_D \cdot v(z)^2 \cdot D(z) \cdot (z - h) \cdot dz \approx 2.728 \times v_{\text{hub}}^2 \quad \text{kN.m} \quad (6.20)$$

being ρ the air density, $C_D \sim 0.5$ the tower drag coefficient, D the tower diameter, and $v(z)$ and v_{hub} the wind speed at height z and at the hub, respectively. The rotor wind loading induced bending moments after removing all the effects above are plotted in Figure 6.32, where it can be seen that the dispersion around the mean value is now significantly reduced.

For completeness, the frequency content of the bending moments time series for normal operation setups is presented in Figure 6.33. As can be seen, most of the significant energy content is found for the lower frequency range, that, for reference, can be taken as frequencies below 1.5 Hz.

6.4.4 Fatigue results

Although the reconstruction of the bending moments has its inherent interest and may be compared against numerical models predictions and used to calibrate them, a perhaps more important aspect of including strain gauges in an experimental campaign of a wind turbine is the possibility to directly evaluate the fatigue damage consumption over time.

Before proceeding to the fatigue computations, it should be noted that the previously discussed uncertainties in the FA-SS bending moment projections do not impact the fatigue estimates. This can be easily understood by decomposing the measured strains, and corresponding stresses, in their drift (d) and mechanical (m) components as:

$$\sigma = \varepsilon E = E (\varepsilon^d + \varepsilon^m) = \sigma^d + \sigma^m \quad (6.21)$$

With this decomposition, at any time instant the stresses at any point in the instrumented section are given by:

$$\sigma(\theta) = E \left[(\varepsilon_x^d \cos \psi + \varepsilon_y^d \sin \psi) + (\varepsilon_x^m \cos \psi + \varepsilon_y^m \sin \psi) \right] \quad (6.22)$$

having ψ , ε_x and ε_y the same meaning as before. It may be argued that the low frequency strains drifts of the sensors readings over time do not impact the fatigue estimations, at least not when 10 minutes averaged time periods are considered and it is followed the classical procedure for steel structures where the contribution of the average stress is neglected, since these only impact the mean stress installed in any fibre in the section. Once the stress time history is reproduced at any point in the section, the corresponding fatigue damage can be computed. A final note should be made here, however, by noting that the purpose of this section and of the discussion in [Chapter 7](#) is to develop, validate and present a methodology to adequately correlate different environmental variables with the experienced fatigue damage of the instrumented section, and not the element fatigue itself. Having this in mind, and tacitly assuming that the conclusions and relative results obtained from a simplified fatigue approach will also be observed if more sophisticated analysis are carried out, the analysis will be based in a typical S-N curves approach.

Having set this point clear, the S-N curves from DNV-RP-C203 have been considered for the fatigue evaluation. In these curves, the predicted number of cycles to failure (N) for a given stress range ($\Delta\sigma$) is given by:

$$\log N = \log \bar{a} - m \log \left[\Delta\sigma \left(\frac{t}{t_{ref}} \right)^k \right] \quad (6.23)$$

where the power term between the square brackets includes the plate thickness (t) dependence of the damage for plates thicker than the reference thickness (t_{ref}). The total fatigue damage (D) is computed assuming the cumulative damage Palmgren-Miner rule as:

$$D = \sum \frac{n_i}{N_i} = \frac{1}{\bar{a}} \sum n_i \Delta\sigma_i^m \left(\frac{t}{t_{ref}} \right)^{k \cdot m} \quad (6.24)$$

where the last term only is used for plates thicker than the reference thickness. In this work it has assumed a S-N curve type E of the DNV-RP-C203, corresponding to the EC3 detail 80, as represented in [Figure 6.34](#). This curve includes two linear regions, with negative inverse slope $m = [3, 5]$ and a thickness exponent of 0.20. The stress amplitudes follow two different linear regimes depending on the number of cycles to failure, with a change at 10^7 cycles⁴. For this purpose, the estimated maximum number of cycles can be written as:

$$\begin{cases} N = \frac{10^{12.010}}{\Delta\sigma^3} & , \quad \text{if } \Delta\sigma \geq 46.78 \text{ MPa} \\ N = \frac{10^{15.350}}{\Delta\sigma^5} & , \quad \text{if } \Delta\sigma < 46.78 \text{ MPa} \end{cases} \quad (6.25)$$

⁴Note that the EC3 makes the separation at 5×10^6 cycles.

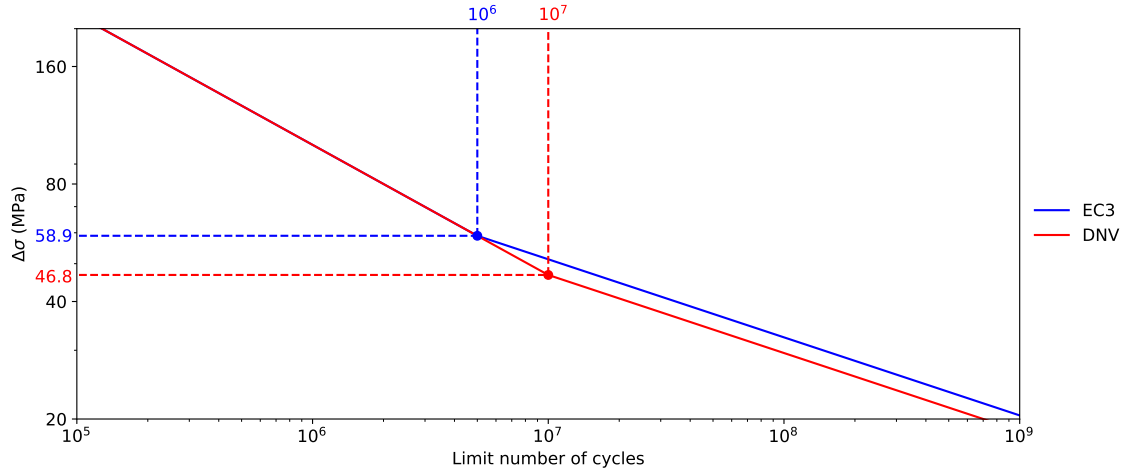


Figure 6.34: Fatigue SN curve from the DNV-RP-C203 and EC3.

Assuming a design lifetime of T years for the structure and damage evaluated in datasets of 10 minutes, a new metric can be defined as:

$$\tilde{D} = \log(6 \times 24 \times 365 \times T \times D) \approx 4.72 + \log T + \log(D) \quad (6.26)$$

that will be 0 for an equivalent fatigue damage of 1, corresponding to a 10-minutes damage of $\sim 10^{-6}$ for the typical design lifetimes assumed for these structures. In a simple reasoning, it can be said that setups with negative $\tilde{D}$ contribute to a fatigue life longer than T year, while setups with positive $\tilde{D}$ contribute to a fatigue life lower than T years.

Since the fatigue damage can be estimated at any fibre in the instrumented section, the critical direction for fatigue can be compared with the already identified maximum bending moment direction. In Figure 6.35 it is plotted the critical fatigue direction and its dependency on the yaw angle. As can be seen the critical direction for fatigue exhibits a lot more dispersion than the bending moments one, and is consistently deviated $\sim 7^\circ$ from it.

Having identified this overall trends, and preparing the path for the analysis that will be presented in Chapter 7, other correlations between the fatigue damage and other monitoring quantities or environmental conditions were searched for. The standard approach for fatigue damage characterisation is to identify setups of similar mean wind speed and turbulence intensity and define a probability distribution for the fatigue damage for that particular wind conditions, as represented in Figure 6.36 where the evolution of the fatigue damage for different wind speeds is plotted, being the turbulence intensity mapped into the series colours. For the offshore case, this does not seem to be the best approach due to the importance of the stresses induced by the platform motions. A better approach seems to be to also use the tower accelerations as a metric, as represented in Figure 6.37, where not only appears to be an universal law for every wind speed, but it should also be pointed out that accelerations actually provide a continuous metric, and hence can be more informative for a given setup. This close relation between accelerations and fatigue was already mentioned in Section 5.4, but will be further explored in the next Chapter.

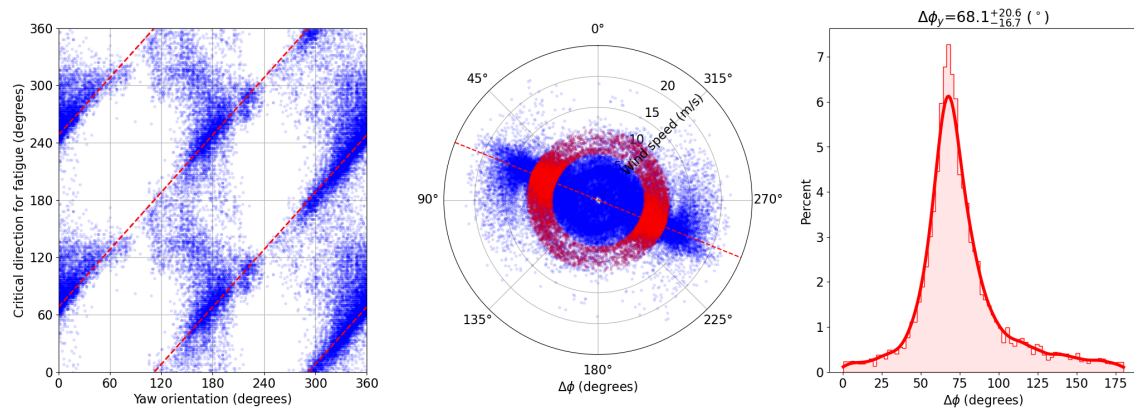


Figure 6.35: On the left panel it is plotted the critical direction for fatigue as a function of the nacelle yaw direction. The relative direction between the two quantities ($\Delta\phi$) is plotted as a function of the wind speed (in the middle). The distribution of the selected events, marked in red in the centre panel, is plotted on the right panel.

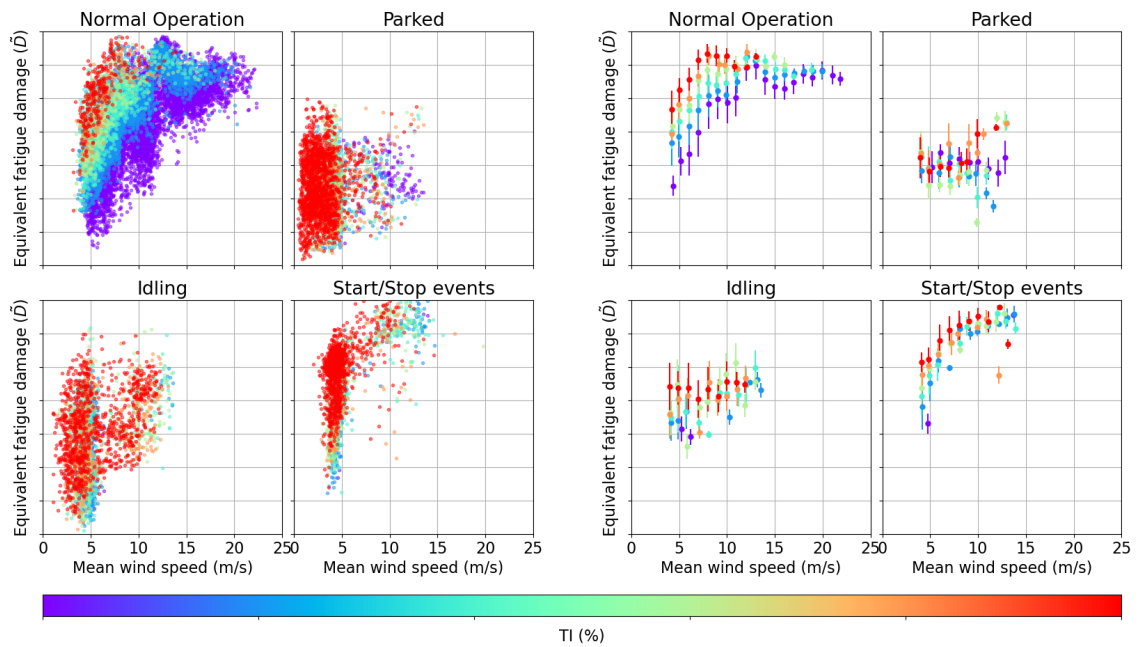


Figure 6.36: Fatigue damage dependency on the mean wind speed. The corresponding turbulence intensity is mapped into the plot colour. All setups on the left and corresponding mean value and standard deviation, on the right.

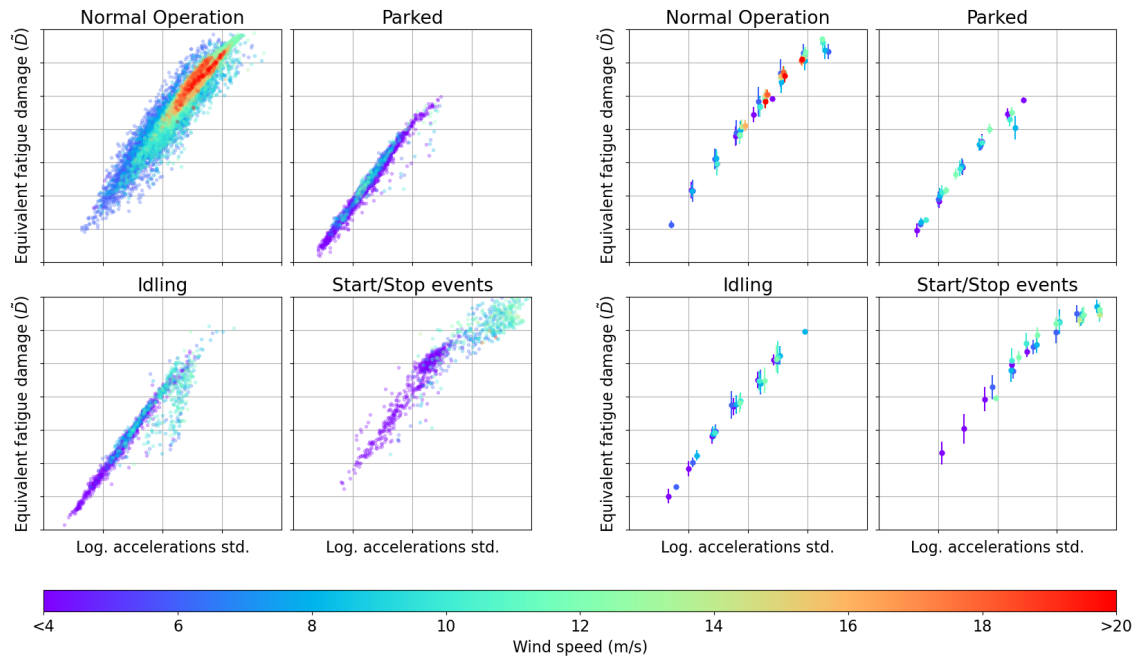


Figure 6.37: Fatigue damage dependency on the tower top accelerations standard deviation. The mean wind speed is mapped into the plot colour. All setups on the left and corresponding mean value and standard deviation, on the right.

6.5 Final remarks

In this Chapter the monitoring data from the experimental campaign taking place over one of the wind turbines of the Wind Float Atlantic wind farm have been used as input for Operational Modal Analysis and fatigue estimate procedures.

For a proper identification of the low frequency platform dominated motions, a new pre-processing methodology was introduced that tries to take into account the existence of frequency ranges that are clearly more excited than others. Based on a data whitening approach, this methodology was able to produce much cleaner stabilisation diagrams, hence allowing for an easier identification of the relevant frequencies and corresponding damping coefficient. When applied to the higher frequency range, mostly dominated by the tower properties, the SSI-COV algorithm was shown to be an efficient approach. Based on these two important results, it was finally shown that the SSI-COV can be applied to the floating offshore field and deliver robust and meaningful results, a conclusion that may be eventually true to Operational Modal Analysis methods, in general.

Finally, the monitoring of the internal forces of these structures was shown to be a natural extension of the conventional applications in onshore, or even bottom-fixed, scenarios. As discussed, however, there is a subtle, but rather relevant, distinction between them when it comes to sensors calibration, that is more challenging in floating offshore wind turbines. Once the internal forces were properly reconstructed, the fatigue damage can be estimated and some interesting relations have been found between this and the tower top accelerations. This last conclusion confirms the results from [Chapter 5](#) and prepares the path for the methodology to be presented in [Chapter 7](#).

Chapter 7

Predictive model for fatigue damage evaluation of floating offshore wind turbines

7.1 Introduction

If in [Chapter 6](#) the monitoring data was used for identification purposes, in this Chapter a new methodology to estimate the internal forces and, perhaps more important, the corresponding fatigue damage without the need to install any additional sensors but the ones that are typically present in any wind turbine will be presented.

As discussed in [Section 5.4](#), the presence of a floating platform with natural frequencies that are significantly lower than the tower structural ones provides an effective way to sample the very low frequency region from accelerations measurements, a region that is typically suppressed in onshore wind turbines. In fact, and if there were no wind action, the bending moments could be trivially recovered from the tower top accelerations alone. Since this is clearly not a reasonable assumption, the model was extended to properly account for the wind action, and a far from trivial relation obtained, mainly if structural properties of the tower and floating platform are not known. However, it was also argued, and shown, that it may not be needed to recover the full transfer function for wind loading, but rather to approximate its value at very precise frequency values. If in [Chapter 5](#) these relations were seen to hold in numerically simulated data series, here the same relations are put to test against the much more challenging data of the WFA wind farm described in [Chapter 6](#) and form the physical theoretical background over which a new methodology for fatigue monitoring of floating offshore wind turbines is developed. However, and before presenting and testing the method itself, a brief overview of an approach that is typically used in onshore wind turbines and that will be used as benchmark will be presented.

7.2 Evaluation of the lifetime fatigue consumption of wind turbines

The fatigue damage is typically evaluated based on relatively short time periods, at least when compared with the design lifetime of the structure. Since this approach does not account for very low frequency amplitude cycles, a proper treatment of the fatigue residues over longer time periods can still be used to estimate their contributions (Marsh et al., 2016; Sadeghi et al., 2022). It should be noted that estimating the fatigue damage based on the full stress time history at once would neither be feasible or desirable. It would not be feasible, since the rainflow count methodology would be computationally very expensive due to the enormous number of cycles that would need to be accounted for, and it would not be desirable since the usage of a single time series could not be used to relate the fatigue damage experienced with the environmental or operation conditions of the wind turbine, which would prevent any extrapolation both forward or backwards in time. It might be argued that this last point is not really relevant, since the fatigue damage for the full period is properly obtained, but it should be noted that this would not be the case if the monitoring system was not installed since the first day and that any remaining lifetime evaluation could not be done properly.

An approach that has already been proved effective in onshore wind turbines is to correlate the 10 minutes time series fatigue equivalent damage with different events labelled from the SCADA system recordings according to design standards (Pacheco, 2022). Typically, this process is based on the operating regime (for instance, normal operation, start up or shutdown manoeuvres and non operating periods) and in the wind field properties (mean wind speed and turbulence intensity), hence defining j different type of events to which a given fatigue damage D_j can be associated with a given probability distribution that characterises its probability of occurrence. If it is assumed that a given event will occur n_j times over the wind turbine lifetime, the total fatigue damage can be obtained:

$$D_{\text{total}} = \sum_j D_j \times n_j \quad (7.1)$$

When applied during the design phase, the different D_j are obtained from numerical simulations, while the n_j are estimated from the site specific characteristics and the wind turbine expected performance. However, when results from a monitoring campaign that is initiated with the wind turbine already operating for a given period of time are available, two different situations can be distinguished: the reconstruction of the fatigue damage that occurred during the monitoring period and the extrapolation forward or backward in time. For the first case the fatigue damage in the instrumented section can be rigorously determined, since strains are being directly monitored, but for the extrapolation a design-like approach must be followed. In onshore wind turbines, the same design bins are often used to define the j events, but the D_j are now obtained from a capture matrix estimated from the experimental data, and eventually completed with numerical simulations for missing type of events. This process is schematically represented in Figure 7.1 for an arbitrary operating regime.

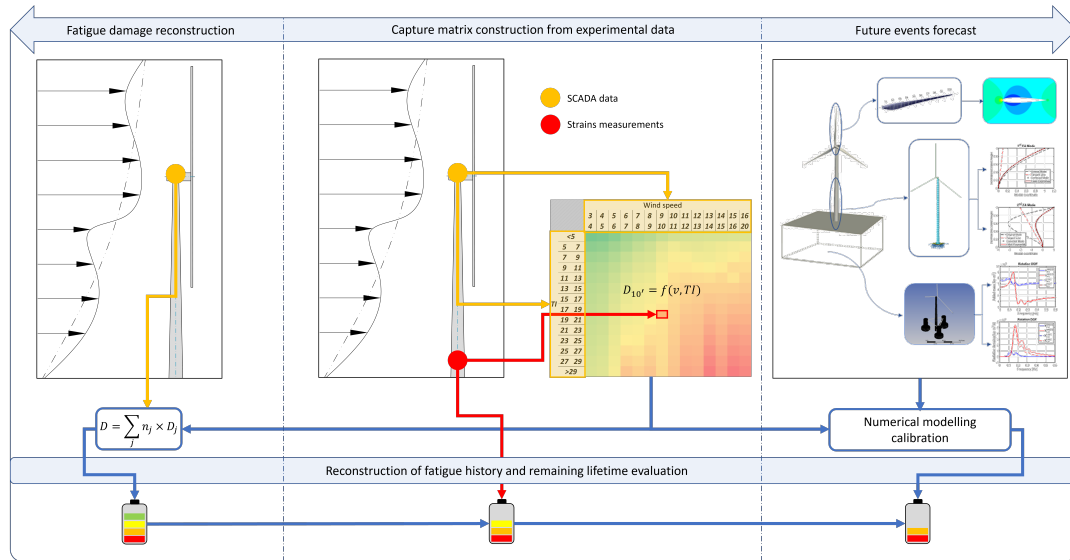


Figure 7.1: Schematically representation of the fatigue damage extrapolation (forward and backwards in time) from monitoring data, where the fatigue damage is characterised from the mean wind speed and turbulence intensity.

Although already proven to deliver good results in onshore wind turbines, this approach has some important limitations. In the onshore cases, perhaps the most significant one is the tacit assumption that the structural response will be equivalent for equivalent ambient conditions, which prevents a robust extrapolation between periods where the control and operating strategies have been modified by any reason, which often occurs in software updates. Additionally, and because this approach relies on an average value obtained for a significant number of similar events, between which significant differences may exist, its predictions for any single event are not meaningful. This limitation implies that long time periods are needed and hence this approach cannot be used to evaluate the effect of different control strategies on demand. In the offshore case, there is still an additional limitation and source of uncertainty since the capture matrix would most probably need to be extended to accommodate different sea states alongside with different wind fields, hence increasing the complexity and the number of events that need to be characterised from experimental data. For all the reasons above, the application of such method to offshore wind turbines is far from trivial and is mostly untested. In this section, it is suggested that a mere extension to accommodate the different sea states is not the most efficient approach, but rather a completely different methodology that explicitly incorporates accelerations measurements is suggested.

7.3 Proposed methodology

7.3.1 Model definition

The starting point for this proposal is the obvious relation between the bending moments time series and the accumulated fatigue damage, meaning that the knowledge of the former yields the

required value of the latter. It should be noted that in the methodology described in the previous section, the fatigue damage is correlated directly with SCADA data and the bending moments (or, equivalently, the stress) time series are never obtained. In this new methodology, the goal will be to provide estimates for the bending moments directly, that can then be used to compute the corresponding fatigue damage, hence providing significant more insight into the structural response. The application of the proposed methodology implies the continuous measurement of accelerations (in two orthogonal horizontal directions) in the tower top, which is already performed by the SCADA systems of modern wind turbines. Its calibration with experimental data implies also the measurement of strains in a selected cross section of the tower. Following the discussion in [Section 5.4](#), the accelerations time series at the tower top and the bending moments at the tower base can be written as:

$$a(t) = a_\theta(t) + a_{\text{wind}}(t) \quad (7.2a)$$

$$\lambda(t) = \lambda_\theta(t) + \lambda_{\text{wind}}(t) \quad (7.2b)$$

where the indexes θ and **wind** stand for the non-wind induced (that will also be referred to as background data series, since it will be relevant for most of the frequency range under analysis), that may include the waves action directly or any imposed rotation of the platform since their effects are indistinguishable from the perspective of the tower top accelerations, and the wind induced components, respectively. The terms in the equation above can be related in the frequency domain by:

$$\tilde{\lambda}_\theta(\omega) = -\tilde{a}_\theta \frac{M(g + \Delta H \omega_x^2)}{\omega_x^2} \approx -M \Delta H \tilde{a}_\theta \quad (7.3a)$$

$$\tilde{\lambda}_{\text{wind}}(\omega) = - \frac{\mathcal{H} M^2 (g + \Delta H \omega_x^2) [1 + \mathcal{R} (M H_m \omega^2 + M g) H_m]}{\mathcal{H} \mathcal{R} (M H_m \omega^2 + M g)^2 + \mathcal{R} (M H_m \omega^2 + M g) (1 + M \omega^2 \mathcal{H}) H_m + M \omega^2 \mathcal{H}} \tilde{a}_{\text{wind}} \quad (7.3b)$$

where it should be recalled that ω_x is the tower natural frequency when considered independently with a rigid foundation, M is the rotor mass, ΔH and H_m are the tower height and the distance between the top mass and the centre of gravity of the system, and $\mathcal{H}$ and $\mathcal{R}$ are the tower and platform frequency dependent transfer functions. The wind action can still be decomposed into the very low frequency part $\tilde{\lambda}_{\text{wind}}^{\text{lf}}$, approximated by [Equation 5.92](#), and a rotor harmonic response $\tilde{\lambda}_{\text{wind}}^{\text{h}}$, where [Equation 5.93](#) is expected to hold:

$$\tilde{\lambda}_{\text{wind}} = \tilde{\lambda}_{\text{wind}}^{\text{lf}} + \tilde{\lambda}_{\text{wind}}^{\text{h}} \quad (7.4)$$

Although these relations have already been validated from a numerical point of view, two additional challenges need to be tackled to apply them successfully as a tool to aid the decision making process regarding any FOWT. On one hand, some of the parameters that are needed to transform accelerations into bending moments are not easily determined or properly constrained

from experimental data alone. On the other hand, the wind induced and non-wind induced response must be properly weighted for a given monitoring period with particular sea, wind and turbine conditions, and these weights should also be possible to obtain from the experimental data alone. For the first issue, it should be noted that a small number of parameters are needed to apply any of the expressions, and so experimental data can be used to calibrate the models. The second issue solution is more subtle, since there is no *a priori* method to determine the different components weight. However, and by making reasonable assumptions regarding the expected frequency where each of the action is expected to be dominant, the problem can be solved not by computing the weights directly, but rather by applying a frequency dependent transformation, based on the operating regime identified from the SCADA data.

The first event type to be defined correspond to the cases where the wind turbine is not producing energy. In these cases, the wind action is almost completely suppressed through the action of the pitch control algorithms, implying that the internal forces can be reconstructed by applying Equation 7.3a directly. These setups can be used to constraint the quantity $M\Delta H$ using measured accelerations and bending moments, hence setting its estimated value for the next cases.

Once this quantity is constrained, the remaining setups can be analysed. For this purpose, it is important to separate between normal operation conditions and transition events, where the wind turbine is either starting up or shutting down. Focusing for now on the former, it should be noted that the wind action in the tower is expected to present very well defined and significant energy content for frequencies in the vicinity of the rotor harmonics, which are expected to be well approximated by the high frequency limit of Equation 7.3b. For the remaining range, the stochastic nature of the action is expected to be governed by Equation 7.3a, with an important exception: the very low frequency range, where the induced wind speed due to the platform motions and eventually the pitch response to it may trigger an important contribution from the wind field. Additionally, the low frequency limit of Equation 7.3b dominates over Equation 7.3a, and hence the wind effect is expected to dominate the response.

Finally, the transition setups need to be analysed, but it should be noted that these can be described as a superposition of the previous two cases, with the additional caveat that the identification of the rotor harmonics is made considering only the angular velocity of the rotor during its operating phase. For the remaining of the setup, Equation 7.3a is expected to be applicable.

The overall picture of the model is hence to use the SCADA system data to separate between different event types. For each of these events, the very low frequency component and the harmonic components based on the recorded rotor angular velocity are filtered out of the time series and transformed based on the appropriate limit of Equation 7.3b, while the remaining, background, signal is transformed according to Equation 7.3a. After this independent transformation, the components are superimposed to provide the bending moments estimates that can then be used to compute the fatigue damage. To test the predictive capacity of this model, the setups have been divided in equal parts in a training period (setups used for the calibration) and a testing period (remaining setups). In general, it may be expected that a given model response differs between both periods. This process is schematically illustrated in Figure 7.2.

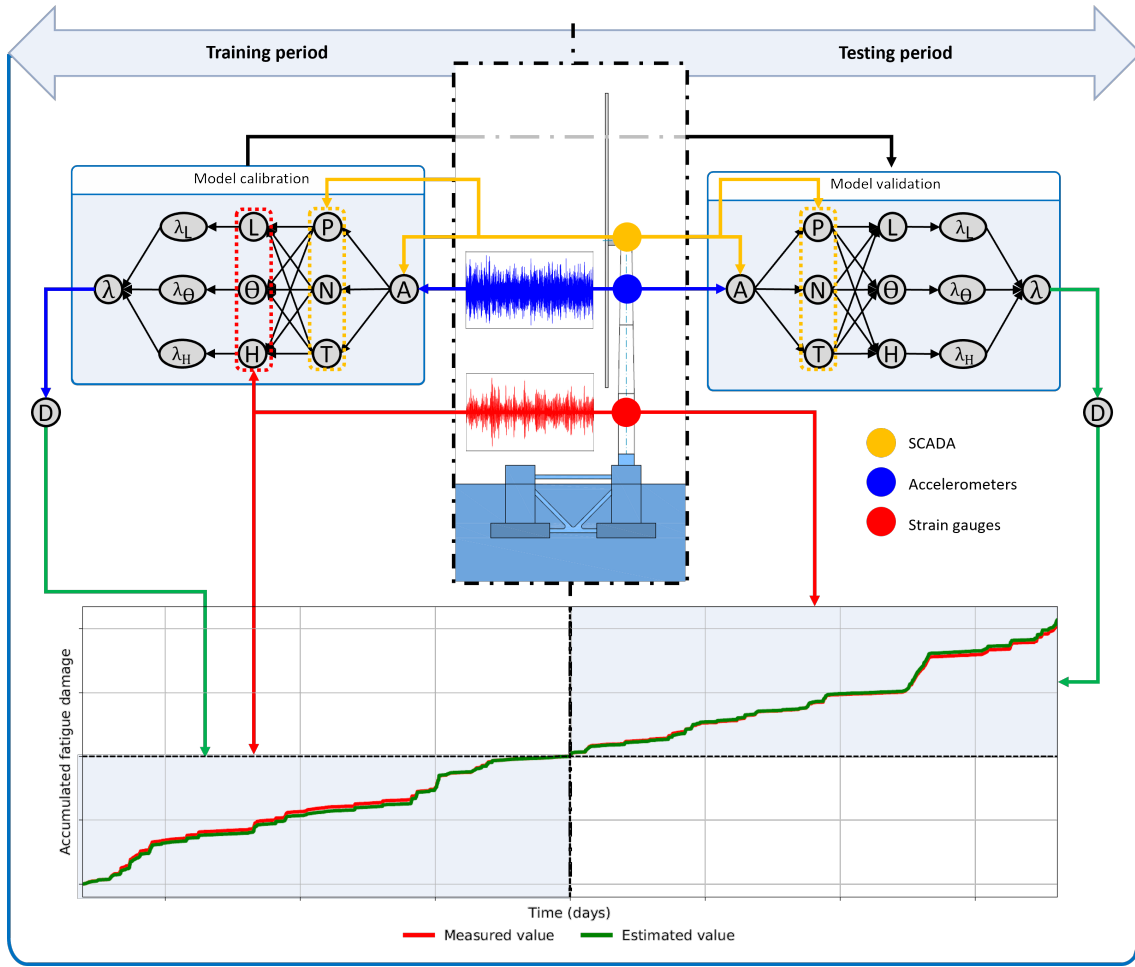


Figure 7.2: Schematically representation of the proposed model for fatigue damage estimation of floating offshore wind turbines. The SCADA data is used to label the events as parked (P), normal operation (N) and transition (T) setups, that are then divided into a low frequency (L), a background (θ) and harmonic (H) component. The transformation to be applied to each is calibrated using the strain gauges measurements, leading to the 3 bending moments time series λ_i , that can be superimposed to estimate the total response λ and corresponding fatigue damage (D). The estimated and measured values can be compared for the training period (setups used for the calibration) and testing period (remaining setups).

7.3.2 Model calibration

As discussed in the point above, the needed transformations from accelerations to bending moments can either be obtained through numerical models or calibrated from the experimental data. Here, the latter approach is followed considering the availability of the experimental data characterised in [Chapter 6](#). Furthermore, the use of a numerical model without any experimental validation would probably be associated with large uncertainties. For the calibration, all the events have been labelled and only half of them have been used. The remaining have been used afterwards for the model validation process.

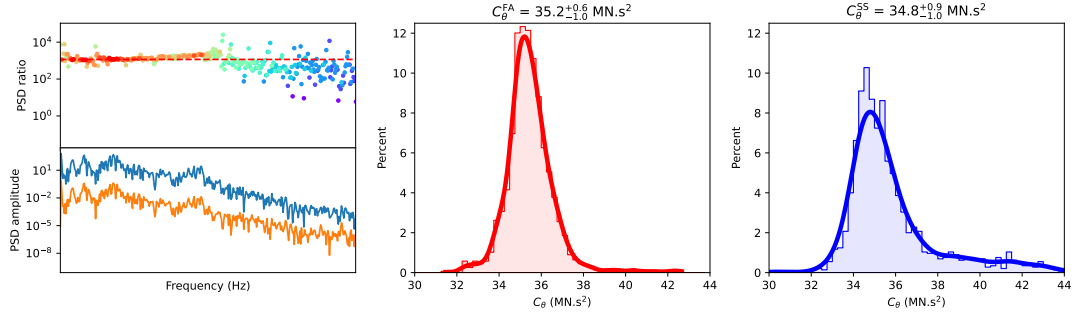


Figure 7.3: Results for the calibration of C_θ . On the left panel, the bending moments (blue) and accelerations (orange) power spectrum densities (on bottom) and their ratio (on top) for a selected 10 minutes setup and the FA direction. The colours are proportional to the logarithm of the power spectrum amplitude. On the centre panel the estimated value of C_θ for the FA direction. On the right the same results for the SS direction.

The first quantity to constrain is the factor relating accelerations and bending moments in the absence of wind loading. For this matter, it is convenient to rewrite Equation 7.3a as:

$$\tilde{\lambda}_\theta = C_\theta \tilde{a}_\theta \quad (7.5)$$

where C_θ is a parameter with units of MN.s^2 that also takes into account the model oversimplification of considering a single mass on the top of the tower. Although this transformation is mostly independent of the relative direction between the nacelle and the floating platform, it will be computed independently in the FA and SS directions, since these will be the relevant directions for the remaining events. The results are presented in Figure 7.3, where it can be seen that the coefficients associated with each direction are consistent at the 1σ level. Additionally, it should be noted that the conclusions obtained in the numerical study also hold for the experimental data, where the fit is particularly good for the frequencies that most contribute for the bending moments time series (indicated by the redder points in the left panel).

After constraining the conversion factor for the parked setup, that will be assumed to hold for the non-wind induced behaviour, the relation must now be fitted for the remaining components. Starting from the harmonic response, it is convenient to rewrite Equation 5.93 as:

$$C_\theta \frac{\tilde{a}_{\text{wind}}^h}{\tilde{\lambda}_{\text{wind}}^h} = 1 - \frac{1}{r_I} + \frac{\omega^2}{\omega_x^2} = C_r^h + C_\omega^h \omega^2 \quad (7.6)$$

where once more the theoretical expression has been generalised for an arbitrary function, that is now expected to be a function of the rotor angular velocity. Due to this expectancy, the normal operation setups have been labelled according to their RPM value and the harmonic response in the vicinity of its 2 first harmonics (at frequencies 3Ω and 6Ω) filtered out for the FA direction alone. It should be noted that the adimensional parameter C_r^h is expected to be negative, while C_ω^h , with units of s^2/rad^2 , is expected to be positive. The results are presented in Figure 7.4. Although

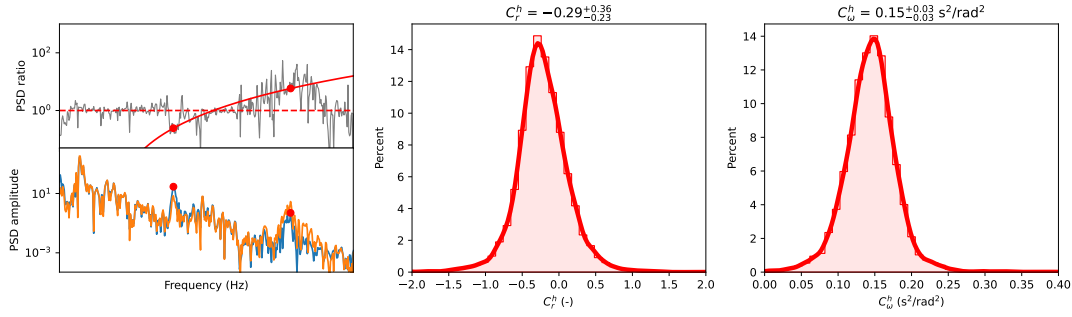


Figure 7.4: Results for the calibration of the harmonic response parameters, C_r^h and C_ω^h . On the left panel, the bending moments (blue) and accelerations scaled by C_θ (orange) power spectrum densities (on bottom) and their ratio (on top) for a selected 10 minutes setup. The red dots were used for the definition of the parameters and have been identified as the frequency compatible with rotor angular velocity ranges where the response peaks. On the centre and right panels the estimated value of C_r^h and C_ω^h , respectively.

C_r is not as well constrained as C_ω , the quantity of interest is given by Equation 7.6, and this is mostly dominated, in the relevant frequency range, by the ω^2 dependency.

Following the same reasoning, the very low frequency limit defined by Equation 5.92 may also be conveniently rewritten as:

$$\frac{1}{C_\theta} \frac{\tilde{\lambda}_{\text{wind}}^{\text{lf}}}{\tilde{a}_{\text{wind}}^{\text{lf}}} = C_M^l + C_\omega^l \omega^2 \quad (7.7)$$

where the model has been once more generalised. To fit these quantities, the frequency power spectrum below 0.05 Hz have been compared for all the normal operation setups. The results are presented in Figure 7.5

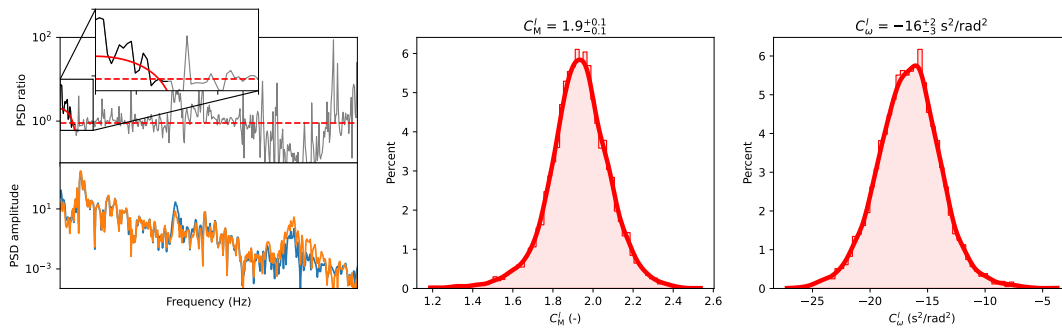


Figure 7.5: Results for the calibration of the harmonic response parameters, C_M^l and C_ω^l . On the left panel, the bending moments (blue) and accelerations scaled by C_θ (orange) power spectrum densities (on bottom) and their ratio (on top) for a selected 10 minutes setup. The black highlighted region identifies the frequency range used for the fit. On the centre and right panels the estimated value of C_M^l and C_ω^l , respectively.

7.3.3 Bending moments reconstruction

Once the parameters above have been properly calibrated, 3 different contributions can be in general defined, separated and transformed independently to reproduce the bending moments time series from the accelerations measurements. Before proceeding to the extensive computations and extrapolations of the fatigue damage, a single setup of each event type has been analysed and the measured and estimated bending moments compared. To better track all the components across the different setups presented, the 3 contributions will be always presented, even if some of them are null for a given operation condition.

Parked setups

For this particular condition, the wind action is not expected to be significant, and all the bending moments time series are expected to be recovered from the trivial transformation provided by C_θ . The results for one of these setups is presented in Figure 7.6, where it may be clearly seen that both the time series as the frequency power spectrum are properly recovered.

Normal operation setups

With the methodology validated for a parked setup, here the same methodology will be illustrated for a normal operation setup. Since the quantity C_θ appears in all expressions, the preliminary

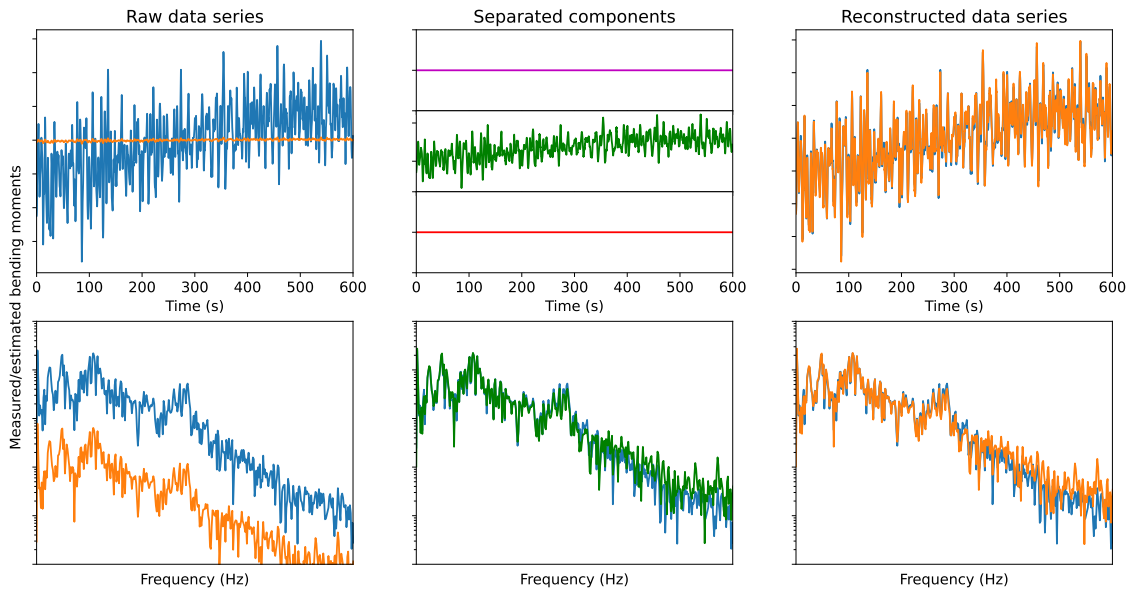


Figure 7.6: Example of the bending moments estimation for a parked setup. On the top panels are plotted the time series and on the bottom panels the corresponding power spectra. On the left, are plotted the bending moments (blue) and accelerations (orange) raw time series. On the centre panels are plotted the different contributions: from the top to bottom, following the order in Figure 7.2, the low frequency, background and harmonics data series. On the right are plotted the measured (blue) and estimated from the accelerations (orange) bending moments.

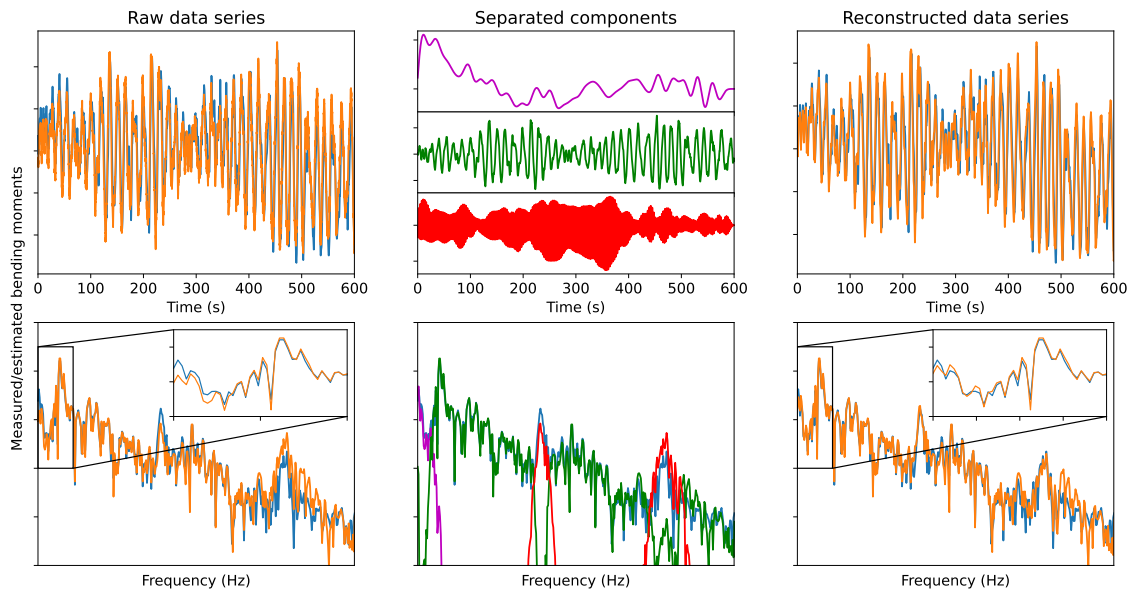


Figure 7.7: Example of the bending moments estimation for a setup under normal operation conditions. On the top panels are plotted the time series and on the bottom panels the corresponding power spectra. On the left, are plotted the bending moments (blue) and accelerations (orange) raw time series. On the centre panels are plotted the different contributions: from the top to bottom, following the order in Figure 7.2, the low frequency, background and harmonics data series. On the right are plotted the measured (blue) and estimated from the accelerations (orange) bending moments.

estimates for the remaining setup types for the bending moments will be obtained multiplying the accelerations time series by this factor, hence allowing for an easier comparison between time series that would be roughly a order of magnitude apart otherwise. The results are presented in Figure 7.7. From the raw data series power spectrum, it is now clear that no single multiplying constant may be used to properly reconstruct the full time series and two particular aspects are worthy to detail further. Firstly, particular attention should be paid to the zoomed frequency range, where it becomes clear that the relation between accelerations and bending moments here is different from the frequency range where the waves are expected to dominate. On the other hand, the very distinct harmonic response and its frequency dependency can also be easily spotted. In fact, it is quite evident that the first harmonic lies below the background prediction, while the second rotor harmonic lies above the same reference level. These conclusions justify and demonstrate that these contributions must be treated independently, as expected. After properly selecting and transforming the different components, the bending moments time series are successfully reconstructed, as may be more evident from the power spectrum analysis.

Transition setups

Finally, the only event type that is still to be presented is the transition setup, where the wind turbine starts or stops its operation. An example of such a setup is presented in Figure 7.8, where

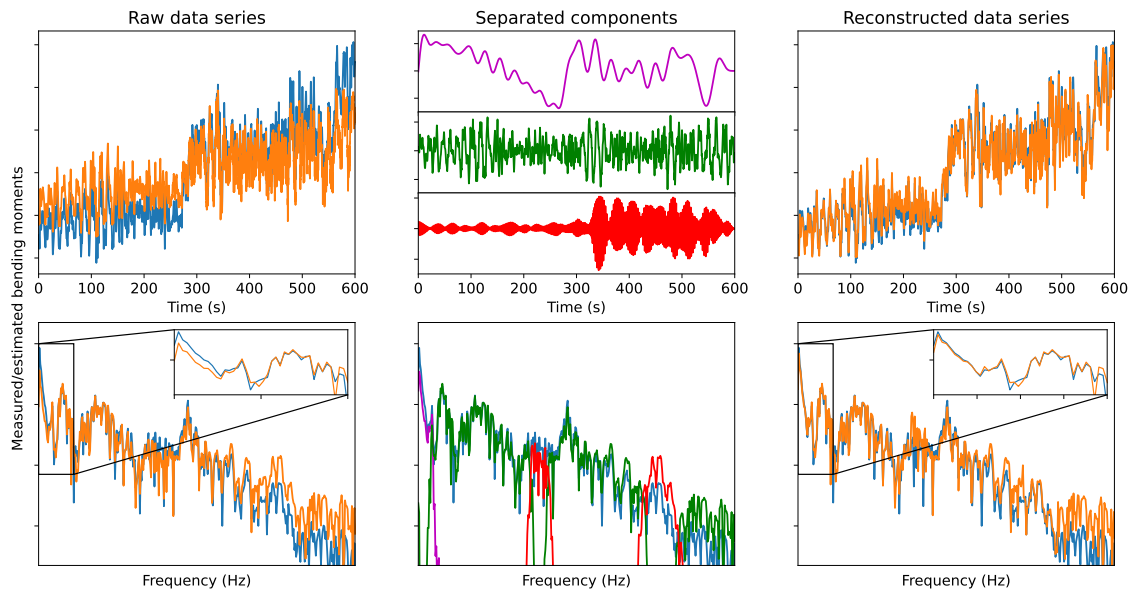


Figure 7.8: Example of the bending moments estimation for a transition setup. On the top panels are plotted the time series and on the bottom panels the corresponding power spectra. On the left, are plotted the bending moments (blue) and accelerations (orange) raw time series. On the centre panels are plotted the different contributions: from the top to bottom, following the order in Figure 7.2, the low frequency, background and harmonics data series. On the right are plotted the measured (blue) and estimated from the accelerations (orange) bending moments.

the same low frequency and harmonic behaviour that were identified in the normal operation setup are also present. It is very interesting to observe that even in these more challenging setups, the reconstitution of moments from acceleration measurements is excellent.

7.4 Results

Once the model is properly calibrated and it has been shown that it is capable to properly estimate the bending moments time series, the natural next step is to compute the fatigue damage from the estimated time series and compare these predictions against the ones obtained with the measured strains. For this analysis, the different event types used in the last section will be used as training setups, for the calibration, and the testing setups, that were left out of that process, will be analysed independently, so that the ability of this methodology to exceed its training range may be evaluated and discussed. Finally, and as a reference base, the results will be compared with the estimates obtained with the conventional approach for onshore wind turbines based on the wind properties recorded in the SCADA data and organised in a capture matrix, as described in the beginning of this chapter.

Parked setups

The results for all the setups identified as parked conditions during the period under analysis are

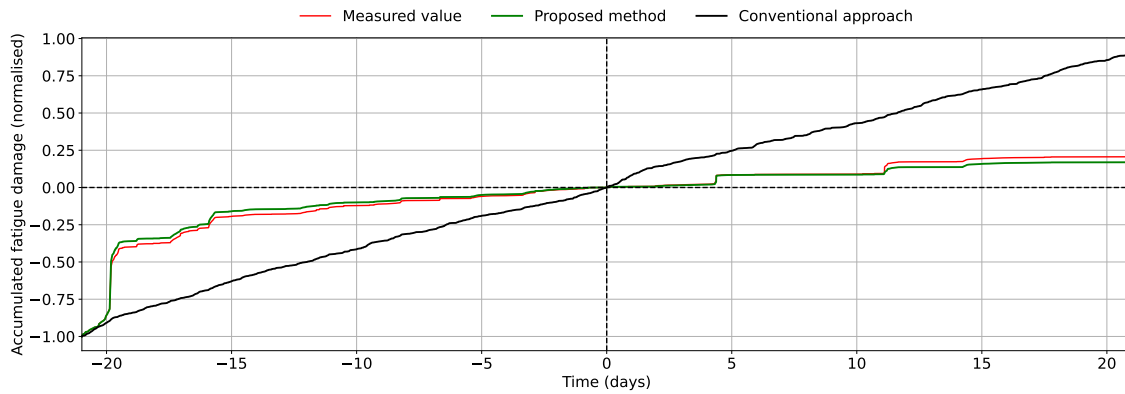


Figure 7.9: Comparison between the measured fatigue damage (red line) and the estimates obtained with the proposed methodology (green line) and the conventional approach (black line) for parked setups. Negative and positive values on the horizontal axis correspond to training and testing periods, respectively. Since all the curves have been forced to pass through the (0,0) point, the training period results should be read as the absolute value of the corresponding curve.

presented in Figure 7.9. For convenience the accumulated fatigue damage has been represented in such a way that the separation between training and testing periods ($t = 0$ days) corresponds to the absence of any damage (meaning that each and every curve passes through the origin of the reference frame). This choice prevents the propagation of the accumulated error during the training period to the testing period, without disrupting the visual interpretation of fatigue damage evolution. Additionally, it allows for an easy evaluation of the total damage in both periods, that for the training period should be read as the absolute value of the corresponding curve. Finally, and for confidentiality issues, the fatigue damage has been properly normalised such that the measured accumulated fatigue damage during the training period equals one. As may be seen, the proposed methodology behaviour is clearly better than the standard approach, that estimates a total fatigue damage significantly higher than its measured counterpart. This fact may be easily explained by noting that for these setups the structural response is much more dependent on the sea states and wave loading than on the very suppressed wind action. Because of this, the usual approach based on the wind field properties, obtained from the SCADA data, labels as similar events that may be significantly distinct from one another. The new methodology, by incorporating explicitly the accelerations measurements, does not suffer from this limitation.

Normal operation setups

Repeating the same analysis for the normal operation events, provides the results presented in Figure 7.10, where the same conventions for training/testing results separation and fatigue normalisation adopted for the parked events has been used.

Once more, the proposed methodology is seen to provide good results, but in this case the same can be said regarding the conventional approach. In fact, the same argument that justified the differences for the parked setups may now be used to understand why both approaches correspond

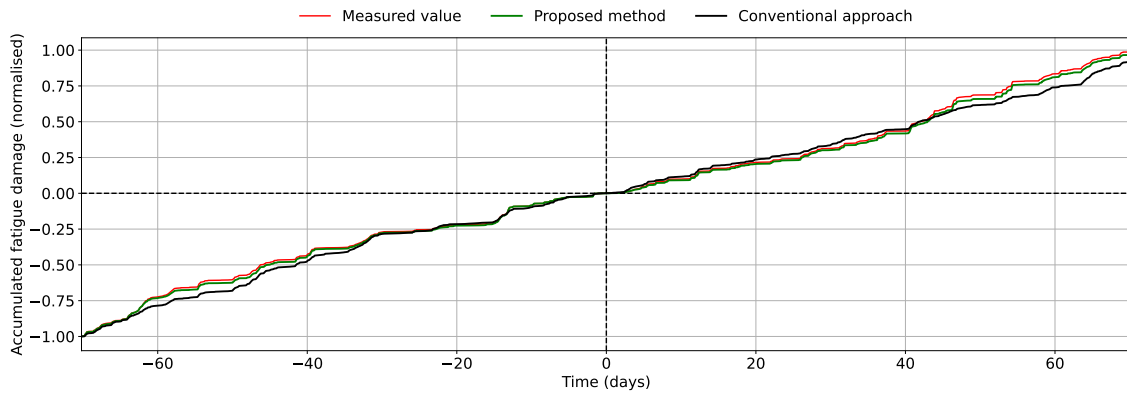


Figure 7.10: Comparison between the measured fatigue damage (red line) and the estimates obtained with the proposed methodology (green line) and the conventional approach (black line) for normal operation setups. Negative and positive values on the horizontal axis correspond to training and testing periods, respectively. Since all the curves have been forced to pass through the (0,0) point, the training period results should be read as the absolute value of the corresponding curve.

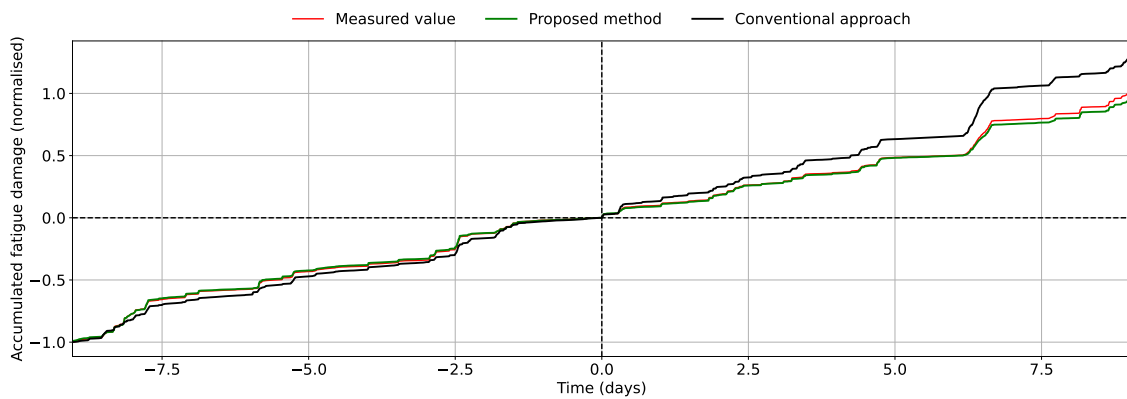


Figure 7.11: Comparison between the measured fatigue damage (red line) and the estimates obtained with the proposed methodology (green line) and the conventional approach (black line) for transition setups. Negative and positive values on the horizontal axis correspond to training and testing periods, respectively. Since all the curves have been forced to pass through the (0,0) point, the training period results should be read as the absolute value of the corresponding curve.

to similar results since for these setups the structural dynamic response is expected to be much more dependent on the wind field properties than before, and these are precisely the parameters of the conventional approach. Still, it is clear that the proposed approach is able to more precisely characterise the variations associated with each fatigue increase (the black line is smoother).

Transition setups

Finally, the analysis is now extended to the transition setups, with the results presented in [Figure 7.11](#). Once more, not only the proposed methodology is capable of providing accurate predictions, as it clearly outperforms the conventional approach, specially in the testing period, where

the latter once more overestimates the real fatigue consumption. Again, the explanation for these differences can be justified by the explicitly incorporation of the accelerations measurements.

7.5 Final remarks

In this Chapter a new methodology that explicitly incorporates tower accelerations for the fatigue estimate of floating offshore wind turbines has been presented.

If this methodology was based on the theoretical developments presented in [Chapter 5](#) and validated by numerical applications, in order for it to be taken as a robust alternative to aid on the decision making process regarding floating wind farms exploration, a proper validation using experimental data of a full scale structure was needed. When applied to 6 months of data collected on the still ongoing experimental campaign of Wind Float Atlantic wind farm, the proposed method was shown simultaneously more robust and reliable than conventional approaches based solely on the environmental and operation conditions.

A particular relevant aspect is the ability of the new methodology to outperform outside its training region, an important limitation of the SCADA-only based approach. The importance of this cannot be stressed enough mostly due two different implications. On one hand, it allows for a proper and robust reconstruction of the fatigue damage history prior to the onset of the monitoring campaign, independently of whether on not the operation strategies have been modified over time, which is a common practice in the never ending search for optimal efficiency of every wind farm exploration. On the other, this new tool, by incorporating the accelerations that have been registered in the period under analysis, allows to evaluate almost instantaneously the internal forces and fatigue damage consumed during the same period, which may be used to fine tune management and operation modifications, as for instance, alterations on the controls algorithms of both tower and platform. On the contrary, on the conventional approach this is clearly not possible. Since it is based on statistical values of a significant number of events that are taken to be similar, but that may present significant dispersion between them, its descriptive power regarding a single realisation is extremely suppressed.

If until this point the most relevant features of the proposed methodology have been highlighted, there is also room for improvement and important future developments to better suit the needs of the offshore wind industry. Firstly, with the increase of the data available from the still ongoing monitoring campaign, the model training and testing will naturally become reinforced, eventually allowing to disentangle new subtleties of the response that are still not fully understood or modelled. Furthermore, combining these results with the development of high fidelity models of the structure under analysis may allow to bypass some of the calibration steps, that have been used to experimentally determine the relevant parameters for the transfer functions between accelerations and bending moments. In particular, it may be possible to *a priori* characterise these entities without the need, or at least with important reduction, of experimental data, eventually allowing for the development of a full wind farm monitoring system with no need for installation of complementary monitoring systems, in general, and strain gauges in particular.

Chapter 8

Conclusions

8.1 Summary

This thesis has focused on the development and implementation of new methodologies for the dynamic monitoring of floating offshore wind turbines, specially aiming at the identification of the relevant natural frequencies and damping ratios of the system, and at a reliable estimation of the fatigue damage consumption. If these developments could, in principal, be made from purely data analysis methodologies, it was considered that a deeper understanding could not be possible without developing a proper physical intuition of these structures behaviour, aided by confronting its predictions against the results from controlled, but reliable, numerical models outputs. Both objectives and the path followed to achieve them defined the structure of the thesis.

In [Part I](#) of this work the fundamental principles and commonly used conventions regarding the analysis of these structures have been presented. Although no relevant original developments are included here, this Chapter was fundamental to better understand the different relevant phenomenon at play. In [Chapter 2](#), the characterisation of the environmental conditions and the description of the fundamental components of wind turbines, in general, and floating solutions, in particular, have been introduced. Additionally, a preliminary introduction to the relevant outputs of structural monitoring campaigns is included. However, describing the wind and waves fields is not enough to develop a numerical model and these must be converted into external forces that act over the structure, a task that has been reserved for [Chapter 3](#). There, an overview of both aerodynamic and hydrodynamic physical principles and modelling approaches is presented. For the former, the aerodynamic forces have been firstly studied in the context of bidimensional bodies, considering both analytical as numerical (CFD) approaches, and only then extended to the full rotor. In this case, the macroscopic description of the process that is commonly used to the aeroelastic simulations and that constitutes the backbone of the OpenFAST software used later was introduced, firstly for the ideal rotor, and then extended to non-ideal conditions by including different engineering models. Complementary, the hydrodynamic description was based on a potential formulation only.

With the description and general overview above, the main principles that allow a proper

understanding of the relevant dynamic phenomenon and to build a numerical model have been presented, but if the latter are intended to provide a reliable artificial laboratory where new methodologies can be tested, the model itself must be somehow validated. [Chapter 4](#) tries to provide this validation using two different approaches. On one hand, and by making good use of the experimental campaigns conducted by FEUP over the last years in onshore wind farms, in particular using those where the detailed geometry of the structure was also known, the aerodynamic modelling was validated and it was also shown how experimental results can be used to bypass some data driven limitations due to confidentiality issues. On the other hand, and as the same overlap between experimental results and structure details were not available for the offshore cases, the hydrodynamic validation was based on a code to code comparison. Here, a preliminary application based on the natural extension of conventional OMA methodologies to the offshore environment has been applied to the identification of the very low frequency modes associated with the floating platform motions. At this point, the numerical models had been validated from both an aerodynamic and hydrodynamic perspective, and, assuming that when properly combined, the results would still be reliable, the artificial laboratory for methodology testing was available for time series generation.

In [Chapter 5](#), a robust theoretical framework where the response is predicted on physics based and simplified analytical models was presented, and different predictions compared to the full numerical model outputs. Firstly, the blades accelerations and internal forces interpretation from the blade perspective are analysed and then reinterpreted from the external forces transmitted to the tower point of view. If these results are applicable to any wind turbine, the implications of including a floating platform on a given tower are then explored and its implications for the relevant resonance behaviour that should be accounted for in the design highlighted.

In [Chapter 6](#), the gained insight from the previous chapter, validated against numerical simulations, is put at test against the much more challenging data of a full scale floating offshore wind turbine. After a description of the monitoring system itself, that includes accelerometers, strain gauges and temperature sensors, OMA methodologies are extended to the identification of the coupled platform/wind turbine natural frequencies. If for the tower dominated modes conventional approaches have been shown to provide accurate results, and the analytical predictions regarding the relation between tower top and platform motions in this frequency range confirmed, a proper identification of the very low frequency modes lead to the proposal of a new pre-processing methodology of the experimental time series, that enhanced significantly the identification power of the SSI-COV algorithm. Furthermore, the internal forces have also been analysed here and the particular additional challenges regarding strain gauges calibration when compared to onshore applications addressed. In this case, an extension of the calibration procedures based on the nacelle rotations for cable untwist events has been presented and validated with the experimental data. After successfully recovering the bending moments at the instrumented section, the strain gauges readings have also been used to briefly characterise the fatigue damage as a function of the environmental and operation conditions, preparing the way to more detailed discussion regarding fatigue estimates presented in the following Chapter.

Finally, in [Chapter 7](#) the analytical developments of [Chapter 5](#) are combined with the experimental data and processing strategies from [Chapter 6](#) to define and validate a new methodology to characterised the fatigue damage of a floating offshore wind turbine based on tower top accelerations, only. Here, the experimental data has been divided into a training a testing data set. While the former have been used to calibrate a new predictive model based on an extension of one of the simplified analytical frameworks, the latter is then used to compare predicted and measured fatigue damages at the instrumented section. A conventional approach based on SCADA data outputs has also been used for benchmark purposes, and it is shown that the new methodology outperforms it for any environmental and/or operation conditions, a conclusion that is expected to hold for any floating wind turbine.

8.2 Main contributions

If in the previous section a brief overview of the work developed was given, in what may be seen as a summary of the concluding remarks provided in each section, some particular aspects and outcomes of the work should be particularly emphasised.

Firstly, the analytical approach detailed in [Chapter 5](#) clearly highlighted how very simplified models can still exhibit the major properties of the full, more complex, system that they try to mimic. If this is particularly interesting, if one considers the level of over simplification that each one presents, it also unveils the power of a proper problem formulation and exploration of its consequences as a support for the definition of data analysis methodologies and experimental data interpretation. In particular, it should be noted that this approach leads to a more clear interpretation of the measurements carried out at the blades, that were shown to contain very relevant information that can be used for the development of more reliable numerical models. Additionally, the floating platform low rotation stiffness effect on the coupled modes of the structure were also fully derived and validated, a result that can be used directly for design purposes of the different elements.

On the other hand, Operational Modal Analysis methodologies were also successfully extended, and for the first time, to floating wind turbines. In particular, it was shown that conventional approaches are able to sample the higher frequency range associated with tower dominated modes, while a new pre-processing methodology seems to be better at constraining the low frequency range of the response. If this latter aspect was not a mere extension of the conventional approaches applied in onshore cases, the same was shown to happen for the internal forces reconstruction. It was shown that the more relevant rotations experienced by these structures effectively prevents the assumption that a given strain gauge will experience a zero-mean signal over a full revolution of the nacelle during the cable untwist events, a condition that can and should be replaced by information regarding the tower tilt. This information can be obtained from accelerations measurements, as long as the accelerometers can measured all the way from DC (0 HZ), hence highlighting that these type of sensors are clearly a better option than others for the monitoring of floating wind turbines. Once these aspects are properly accounted for, the reconstruction of

the bending moments time series follows the same path as their onshore counterparts and exhibits similar trends when expressed as a function of environmental and operation conditions.

If this is true for the internal forces expected value, the same does not hold for the fatigue estimates, that exhibit a very particular correlation with the tower top accelerations, specially in the low frequency range. This is a particularly interesting output of this work, since it provides a new methodology to estimate the accumulated fatigue damage, specially suitable for floating offshore solutions, as it solves two important limitations of the conventional approach, used in onshore cases. On one hand, by explicitly incorporating the tower top accelerations, its enhanced predictive power regarding a single event provides an almost instantaneously feedback regarding eventual modifications of the control algorithms. In the conventional approach, this cannot be done not only because its predictions regarding a single event are not meaningful (its only when a significant number of similar events is considered that the method is applicable), but also because modifying the controls strategies would require a new calibration period. This latter point also makes that the reconstruction of the full time history, prior to the onset of the experimental campaign, is also more robust than before, no matter the evolution of operation strategies over time. On the other hand, due to the strong theoretical background of this new methodology, it may also be applicable without the need to install any strain gauges. Once more, this is a quite distinct feature that is not present in the conventional approach, where there is always the need for a calibration period using experimental strains data, and that may allow to easily extend this new methodology to the full wind farm without any significant increase in cost, since most of the wind turbines are already equipped with an accelerometer on their top.

8.3 Limitations and future developments

If the methodology in [Chapter 7](#) concludes the outputs contained in this thesis, as always there are some open questions that should be explored in the future, as well as new possibilities that the presented results will most certainly open for the continuous monitoring of floating wind turbines. Without the prejudice for any other application, from the author perspective, there are clearly some research approaches where the work here presented may proven to be very useful.

On one hand, there are some natural extensions of the already developed methodologies. Firstly, the blades strain gauges procedure presented in [Section 5.2](#) has only been tested with numerical simulations and in future works it should be compared with other approaches using experimental results. Additionally, it would be very interesting in future works to test the fatigue methodology presented in [Chapter 7](#) in a fleet wide sense. Here, the explicit need of the strain gauges could be tested, revealing if the response is mostly defined by the structural properties, in which case a robust numerical model would suffice.

On the other hand, there are some future developments, that require a different level of information regarding the different floating wind turbine components, where any of the approaches here presented would be part of a more integrated approach in the context of a truly digital twin development, from at least two different perspectives. Firstly, and with a high-fidelity numerical

model, some of the calibrations of the fatigue damage methodology could be bypassed, completely removing the need for the installation of strain gauges. If in this case it would be the numerical model to feed the data analysis being conducted, the opposite is also true, and the rotor characterisation methodologies from blades accelerations can be used to improve the numerical model itself. Finally, there is a deeper level of integration, where the OMA results or the instantaneously fatigue damage experienced by the wind turbine tower may condition directly the controls and operation strategies. This would be a turning point in the field from both an economical and structural point view, as it would allow to manage the structural elements with much more confidence and support life extension decisions, but also to optimise the efficiency of the exploration by better combining high and low energy demands periods with the corresponding structural accumulated fatigue damage. As a final note, but still in this digital twin concept, it would also be important to explore new methodologies to extrapolate the internal forces and corresponding fatigue damage between different tower sections.

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Appendix A

Aerodynamics

A.1 Aerofoil analytical formulation

For illustrative purposes, a full analytical formulation for the computation of the lift force induced over a given aerofoil is presented here. For that a 2D ideal flow described by the potential ϕ will be assumed. Under these assumptions, the two components of the velocity in the flow plane xz (u_x and u_z) are given by:

$$u_x = \partial_x \phi \quad (\text{A.1a})$$

$$u_z = \partial_z \phi \quad (\text{A.1b})$$

General Formulation

For the derivation of the flow over an aerofoil it will be useful the vortex formulation presented for example by [Anderson \(2001\)](#). If the aerofoil is assumed to be an object carrying a vorticity per unit length $\gamma(s)$ as shown in [Figure A.1](#), the induced velocity by the infinitesimal vortex segment is given by¹:

$$du_P = -\frac{\gamma(s) \cdot ds}{2\pi r} \quad (\text{A.2})$$

where s identifies the position on the contour. As the velocity direction varies along the contour, it is easier to work with the velocity potential rather than the velocity itself and to write it as a function of the distributed vorticity around the aerofoil as:

$$\phi(x, y) = -\frac{1}{2\pi} \cdot \int_a^b \theta(s) \cdot \gamma \cdot ds \quad (\text{A.3})$$

where θ is the the normal to the surface makes with the horizontal direction, as represented in [Figure A.1](#). Formally, the condition that is required for the aerofoil contour to be a stream line can

¹By definition the circulation Γ is numerically equal to the symmetric of the line integral of the velocity taken over a closed path and using Stoke's theorem this can be written as $\Gamma = -\oint_C \mathbf{u} \cdot d\mathbf{s} = -\iint_S (\nabla \times \mathbf{u}) \cdot d\mathbf{s}$. Applying this concept to a vortex driven flow in which the stream lines are concentric circles with velocity inversely proportional to its radius, the tangential component is given by $\Gamma = -u_t \cdot 2\pi r \longrightarrow u_t = -\frac{\Gamma}{2\pi r}$.

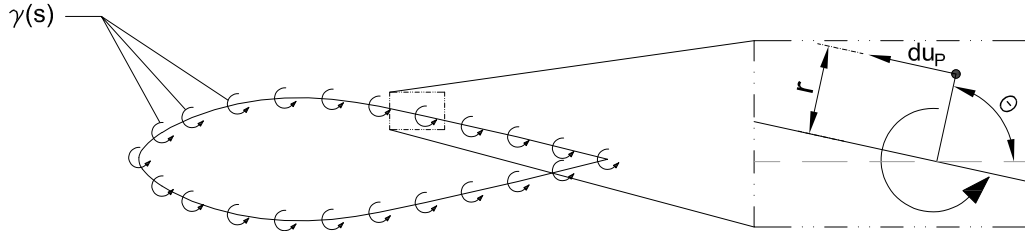


Figure A.1: Distributed vorticity over an aerofoil.

be mathematically expressed as:

$$U_0^\perp + u^\perp(s) = 0 \quad (\text{A.4})$$

where U_0 and $u(s)$ are the free stream velocity and the vortex induced velocity at the contour of the aerofoil, respectively, and $\perp$ is used to identify the velocity component orthogonal to it.

Flow over thin wing sections

If the aerofoil is thin enough, then it can be defined by its centre line parameterised as $z = z(x)$. For an incident flow with an angle of attack (α), the orthogonal component of the free stream velocity at the wall can be written without loss of generality as:

$$U_0^\perp = U_0 \cdot \sin \left[\alpha + \arctan \left(-\frac{dz}{dx} \right) \right] \approx U_0 \cdot \left(\alpha - \frac{dz}{dx} \right) \quad (\text{A.5})$$

where the last equality of is only valid if both the angle of attack and the thickness of the aerofoil are small, such that $\sin \theta \approx \tan \theta \approx \theta$.

Still considering a thin aerofoil, the orthogonal component of the induced velocity can also be written considering a vortex sheet distributed along the mean camber line, such that the velocity induced in a specific point is now given by:

$$u^\perp(s) \approx u(x) = - \int_0^c \frac{\gamma(\epsilon) \cdot d\epsilon}{2\pi \cdot (x - \epsilon)} = \int_0^c \frac{\gamma(\epsilon) \cdot d\epsilon}{2\pi \cdot (\epsilon - x)} \quad (\text{A.6})$$

where ϵ is now measured along the aerofoil chord. The combination of Equation A.5 and Equation A.6 leads to the **fundamental equation of thin aerofoil theory** (Anderson, 2001):

$$U_0 \cdot \left(\alpha - \frac{dz}{dx} \right) = \int_0^c \frac{\gamma(\epsilon) \cdot d\epsilon}{2\pi \cdot (\epsilon - x)} \quad (\text{A.7})$$

Equation A.7 is not totally defined however, as different solutions can be obtained associated with different circulation around the blade and hence different lift forces. In order to obtain an unequivocal solution, it is necessary to consider the **Kutta condition**, which states that, as the flow cannot have two different velocities at the trailing edge (see Figure 3.2 and the discussion there for more details).

Finally, if the fundamental equation of thin aerofoils is to be solved, it is convenient to express it with respect to a new set of variables and parameters as:

$$\frac{1}{2\pi} \int_0^\pi \frac{\gamma(\theta) \cdot \sin(\theta) \cdot d\theta}{\cos(\theta) - \cos(\theta_0)} = U_0 \cdot \left(\alpha - \frac{dz}{dx} \right) \quad (\text{A.8})$$

where:

$$\varepsilon = \frac{c}{2} \cdot [1 - \cos(\theta)] \quad (\text{A.9a})$$

$$d\varepsilon = \frac{c}{2} \cdot \sin(\theta) \cdot d\theta \quad (\text{A.9b})$$

$$x = \frac{c}{2} \cdot [1 - \cos(\theta_0)] \quad (\text{A.9c})$$

The integral equation written in the new variable θ has indeed the analytical solution ([Hildebrand, 1962](#)):

$$\gamma(\theta) = 2 \cdot U_0 \cdot \left[A_0 \cdot \frac{1 + \cos(\theta)}{\sin(\theta)} + \sum_{n=1}^{\infty} (A_n \cdot \sin(n \cdot \theta)) \right] \quad (\text{A.10})$$

where:

$$A_0 = \alpha - \frac{1}{\pi} \cdot \int_0^\pi \frac{dz}{dx} \cdot d\theta_0 \quad (\text{A.11a})$$

$$A_n = \frac{2}{\pi} \cdot \int_0^\pi \frac{dz}{dx} \cdot \cos(n\theta_0) \cdot d\theta_0 \quad (\text{A.11b})$$

Finally, using [Equation A.11](#), one may now write the circulation as:

$$\begin{aligned} \Gamma &= \frac{c}{2} \int_0^\pi \gamma(\theta) \cdot \sin(\theta) \cdot d\theta = \\ &= c \cdot U_0 \cdot \left[A_0 \int_0^\pi (1 + \cos(\theta)) d\theta + \sum_{n=1}^{\infty} \left(A_n \int_0^\pi \sin(\theta) \cdot \sin(n\theta) \cdot d\theta \right) \right] \end{aligned} \quad (\text{A.12})$$

The integrals that appear in the lift expression are easily solvable and it can be shown that the one associated to A_0 is numerically equal to π while the ones present in the infinite series have non zero value if and only if $n = 1$, in which case the integral is numerically equal to $\pi/2$. With this information the circulation can finally be expressed with only two coefficients as:

$$\Gamma = c \cdot U_0 \cdot \left(\pi \cdot A_0 + \frac{\pi}{2} \cdot A_1 \right) \quad (\text{A.13})$$

The lift force per unit length and corresponding coefficient can now easily be obtained substituting the circulation in [Equation 3.3](#):

$$F_L = \rho U_0^2 c \pi \left(A_0 + \frac{1}{2} \cdot A_1 \right) \longrightarrow C_L = \frac{F_L}{\frac{1}{2} \rho U_0^2 \cdot c} = 2\pi \left[\alpha + \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} \cdot (\cos \theta_0 - 1) \cdot d\theta_0 \right] \quad (\text{A.14})$$

The expression obtained for the lift coefficient shows it varies linearly with the angle of attack with slope equal to 2π and that the lift force for 0 [rad] angle of attack depends on the shape of the aerofoil itself². Finally, the expressions obtained are all expressed as a function of the slope of the top and bottom surfaces of the aerofoil, but this is not always useful. An alternative way of expressing the same forces with respect to the mean camber line coordinate is presented in (Abbott and Von Doenhoff, 1959):

$$C_L = 2\pi \cdot (\alpha + \beta) = 2\pi \cdot \left[\alpha + \int_0^1 \frac{z}{c^2} \cdot \frac{1}{\pi \cdot (1-x/c) \cdot \sqrt{x/c \cdot (1-x/c)}} \cdot dx \right] \quad (\text{A.15})$$

Generalisation to an arbitrary shape

The expressions presented until this point assumed the angle of attack and the aerofoil thickness could be considered small, which allowed to apply some important simplifications, namely in Equation A.11. However, it is sometimes important to calculate the lift force in situations that do not guarantee these conditions, being the flow around a cylinder and the force at high angles of attack examples of both situations, respectively. One of the techniques that can be used to determine the desired quantities is to map a given aerofoil into a cylinder, with a well known solution, by means of conformal mapping. To do so, it is convenient to work with the complex velocity potential from which the complex velocity (w) can be obtained as (Choudhuri, 1998):

$$f(z) = f(x + i \cdot y) = \phi(x, y) + i \cdot \psi(x, y) \longrightarrow w = -\frac{df}{dz} = u_x - i \cdot u_y \quad (\text{A.16})$$

where ψ is the stream function, that relates to the flow velocities in the xy plane as:

$$u_x = -\partial_y \psi \quad (\text{A.17a})$$

$$u_y = \partial_x \psi \quad (\text{A.17b})$$

If $f(z)$ represents a given flow in the original xy plane, then it is possible to write a complex potential $g(\zeta) = g(\xi + i \cdot \eta)$ representing the same flow in a different plane such that $f(z) = g(\zeta)$, as long as the relation $\zeta = \zeta(z)$ is known. The transformation presented in Equation A.18 allows to transform the original cylinder section into a shape that resembles a typical aerofoil, as illustrated in Figure A.2. In this case, the lift coefficient can be easily computed based on the 2 transformation parameters and the angle of attack as:

$$\zeta = z + \varepsilon + \frac{(a + \varepsilon)^2}{z + \varepsilon} \longrightarrow C_L = 2\pi \cdot \left(1 + \frac{\varepsilon}{a} \right) \cdot \sin(\alpha) \quad (\text{A.18})$$

Although the proposed transformation seems to be able to transform a cylinder into an aerofoil shape section, obtaining the parameters to do so may not always be an easy task if the geometry is indeed arbitrary. There is however an efficient way to find a suitable transformation noticing that

²In the limit case of a symmetric wing, where $\frac{dz}{dx} = 0$ the lift depends exclusively on the angle of attack as the only non zero coefficient is A_0 .

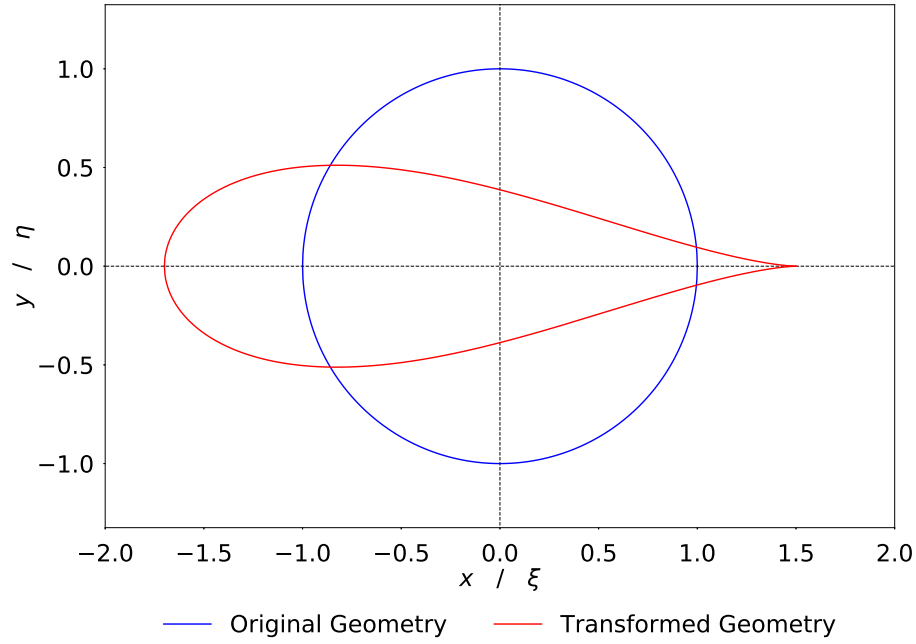


Figure A.2: Transformation of a circular section into an aerofoil through a conformal transformation by means of Equation A.18, considering $a = -0.5$ and $\varepsilon = -0.25$.

a simpler transformation with $\varepsilon = 0$ will transform a typical aerofoil section in a nearly circular curve as discussed by Theodorsen (1933). Based on this, it is possible to define the following complex planes:

- The true circle plane $z = a \exp(\psi_0 + i\phi_0) = r(\cos \phi_0 + i \sin \phi_0)$
- The quasi-circular plane $z' = a \exp(\psi + i\phi)$
- The aerofoil plane $\xi = x + iy = z' + \frac{a^2}{z}$

where ϕ and ϕ_0 can be interpreted as polar variables in the range $[0, 2\pi]$, and ϕ and ψ are defined from the original xy coordinates as:

$$2 \cdot \sin(\phi)^2 = p + \sqrt{p^2 + \left(\frac{y}{a}\right)^2} \quad (\text{A.19a})$$

$$2 \cdot \sinh(\psi)^2 = -p + \sqrt{p^2 + \left(\frac{y}{a}\right)^2} \quad (\text{A.19b})$$

where:

$$p = 1 - \left(\frac{x}{2a}\right)^2 - \left(\frac{y}{2a}\right)^2 \quad (\text{A.20})$$

Since the ultimate goal is to determine the transformation parameters ψ_0 and ϕ_0 that minimise the differences between z and z' . To do so, it is convenient to write:

$$z' = a \exp(\psi + i\phi) = z \exp[(\psi - \psi_0) + i(\phi - \phi_0)] = z \exp \left[\sum_{n=1}^{\infty} (A_n + i \cdot B_n) / z^n \right] \quad (\text{A.21})$$

implying:

$$\psi - \psi_0 + i \cdot (\phi - \phi_0) = \sum_n (A_n + i \cdot B_n) \cdot \frac{1}{z^n} \quad (\text{A.22})$$

Equation A.22 can be written using De Moivre theorem³ and matching the corresponding terms with z expressed in polar coordinates:

$$\psi - \psi_0 = \sum_n \left[\frac{A_n}{r^n} \cdot \cos(n \cdot \phi_0) + \frac{B_n}{r^n} \cdot \sin(n \cdot \phi_0) \right] \quad (\text{A.23a})$$

$$\phi - \phi_0 = \sum_n \left[\frac{B_n}{r^n} \cdot \cos(n \cdot \phi_0) - \frac{A_n}{r^n} \cdot \sin(n \cdot \phi_0) \right] \quad (\text{A.23b})$$

While the first equality can be used to obtain expressions for the coefficients as well as the value of ψ_0 , the second equality presented in Equation A.23 plays an important roll in the calculations that are to be made, so further simplifications are needed and a new variable $\varepsilon = \phi - \phi_0$ is defined. Having in mind hyperbolic functions properties, it is possible to establish the following relations:

$$\psi_0 = \frac{1}{2\pi} \cdot \int_0^{2\pi} \psi \cdot d\theta \quad (\text{A.24a})$$

$$\varepsilon(\phi_0) = \phi - \phi_0 = \frac{1}{2\pi} \cdot \int_0^{2\pi} \psi \cdot \cot \left[\frac{\gamma - \phi_0}{2} \right] \cdot d\gamma \quad (\text{A.24b})$$

The velocity magnitude in the aerofoil plane is then given by Equation A.25, where ε_T is the value of ε at the trailing edge (Abbott and Von Doenhoff, 1959):

$$|U| = U_0 \cdot \frac{[\sin(\alpha + \phi_0) + \sin(\alpha + \varepsilon_T) \cdot [1 + d\varepsilon/d\phi] \cdot e^{\psi_0}]}{\sqrt{[\sinh^2(\psi) + \sin^2(\phi)] \cdot [d\psi/d\phi]^2}} \quad (\text{A.25})$$

From the obtained velocity expression, the pressure distribution along the surface of the aerofoil can finally be calculated from Equation A.26, where Equation A.24 has been used, and the lift force obtained from simple integration over all the surface.

$$\frac{U}{U_0} = \frac{[\sin(\alpha + \phi_0 + \varepsilon) + \sin(\alpha + \varepsilon_T) \cdot [1 + d\varepsilon/d\phi] \cdot e^{\psi_0}]}{\sqrt{[\sinh^2(\psi) + \sin^2(\phi)] \cdot [d\psi/d\phi]^2}} \longrightarrow \frac{P}{P_0} = 1 - \left(\frac{U}{U_0} \right)^2 \quad (\text{A.26})$$

Until this point the fundamental equations have been presented, but there are still some issues that must be taken care of before effectively solving the problem. Although Equation A.26 provides a way to compute the pressure in any point of the aerofoil assuming one knows all the variables and the variation of ε and ψ with ϕ , Equation A.24 only provides these quantities with respect to ϕ_0 . To overcome this difficulty, a first iteration can be used considering $\phi \approx \phi_0$ and, if greater accuracy is required, an iterative process can be started where in every iteration ϕ is updated to $\phi_0 + \varepsilon$.

³ $(\cos(x) + i \cdot \sin(x))^n = \cos(nx) + i \cdot \sin(nx)$

Example A.1 - In order to illustrate the procedure that needs to be carried out to calculate the flow around an aerofoil, the NACA 0012 geometry will be used (McDevitt and Okuno, 1985). This geometry, normalised to the chord length, is represented in Figure A.3.

Given the geometry, the ψ and ϵ dependence on ϕ can easily be calculated by Equation A.19 and Equation A.24 considering $\phi \approx \phi_0$. The resultant curves are presented in Figure A.4.

The calculated curves are sufficient to compute the pressure coefficients in every point of the surface (as can be seen there is a discontinuity of the transfer functions at the trailing edge, but this point is not of great importance as the Kutta-Joukowski condition ensures this is a stagnation point). The evolution of the pressure coefficient for an angle of attack of 0° and 10° degrees obtained from Equation A.26 can be compared with the results of XFOIL, a MIT developed software for the analysis of subsonic flows over aerofoils (Drela, 1989) - Figure A.5.

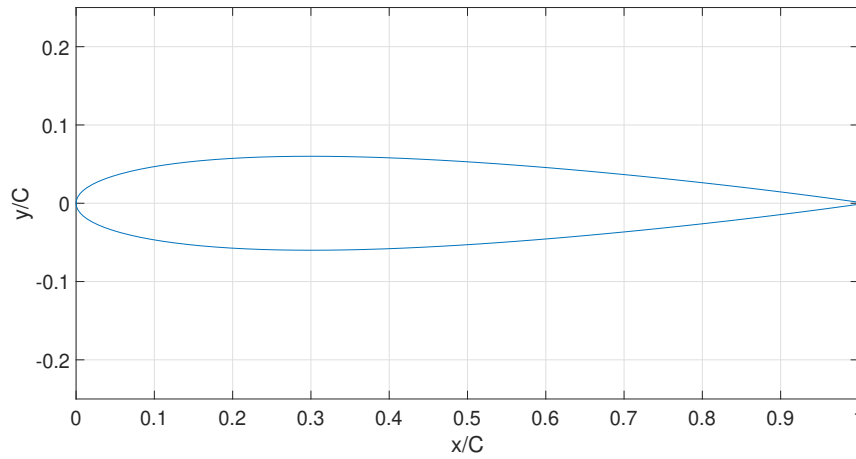


Figure A.3: NACA0012 section normalised.

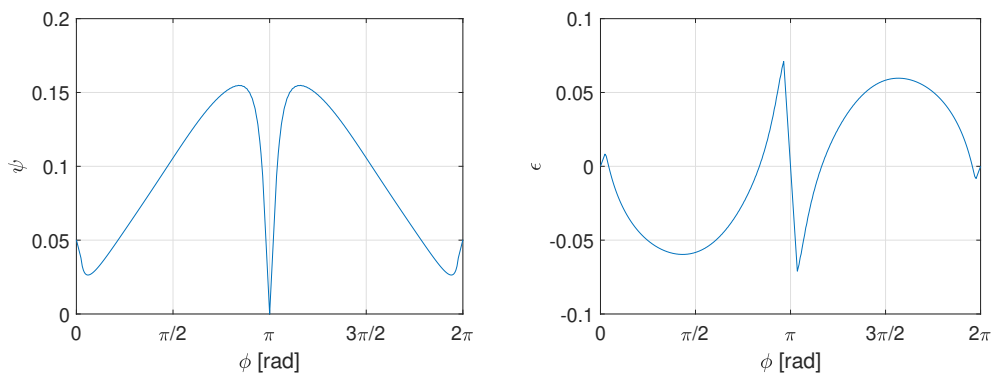


Figure A.4: Dependence of ψ and ϵ on ϕ para a NACA0012.

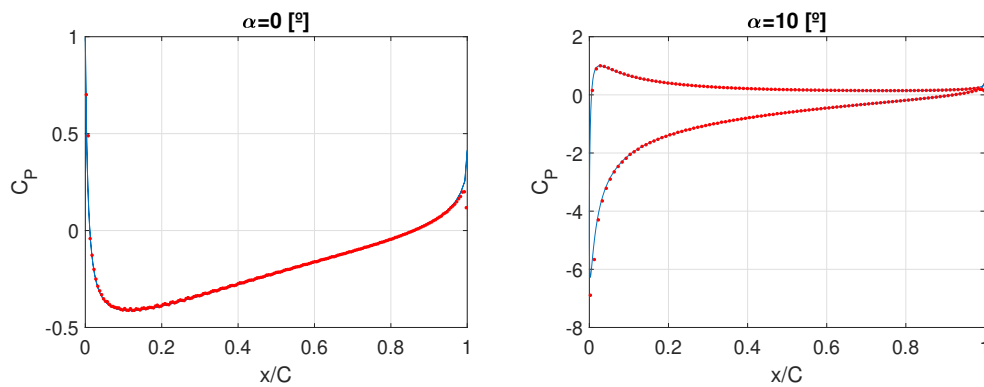


Figure A.5: Comparison of the pressure coefficients obtained (red dots) and the results from *software* XFOIL (blue line).

A.2 Computational Fluid Dynamics

In order to provide both a simple example for the exposed procedure, as well as a validated model to calculate different aerofoil geometries, a CFD model was built for the largely tested and studied NACA 0012 aerofoil geometry, already presented in Example A.1 (page 227). In order to create a suitable grid, the computational domain was divided according to Figure A.6 (the center of the circular region is 10 [cm] to the right of the airfoil nose).

In order to fulfil the criteria defined earlier for the near wall region at the lowest computational cost, a non uniform spacing was adopted, where each element is r times greater than its predecessor. If a line segment with dimension L is divided in N parts with a bias factor of ε , then the first

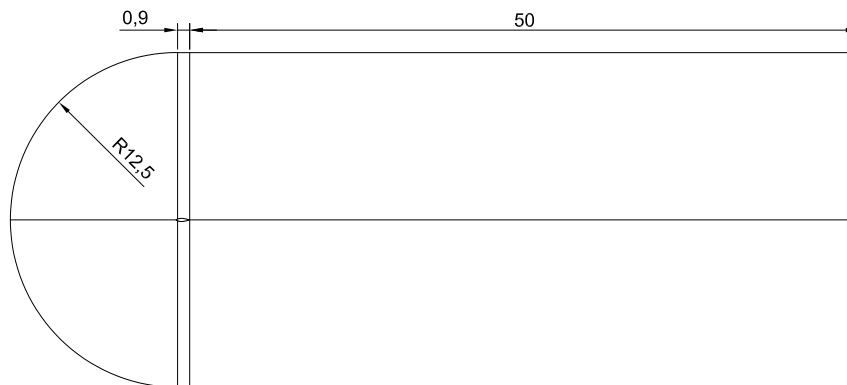


Figure A.6: Scheme of the computational domain used in the numerical simulations (in chord units).

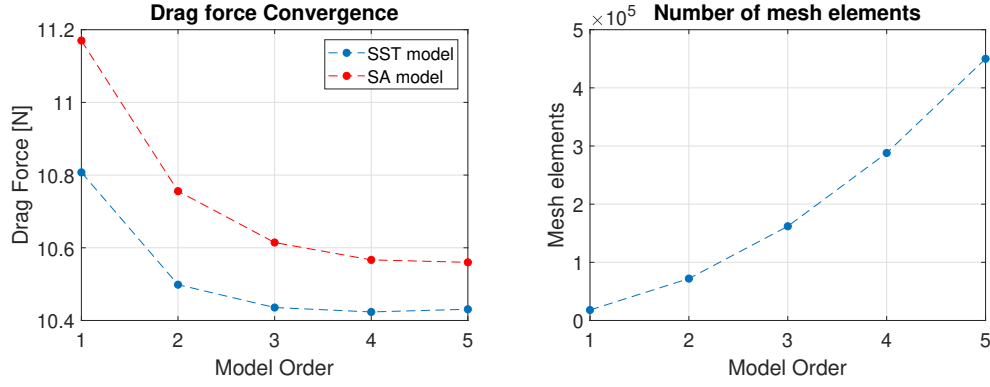


Figure A.7: Evolution of the drag force (on the left) and number of mesh elements (on the right) as a function of the model order.

element dimension can be estimated as:

$$r = \varepsilon^{\frac{1}{N-1}} \longrightarrow y^+ = \frac{L}{\frac{1-r^{N+1}}{1-r}} = \frac{L \times (1-r)}{1-r^{N+1}} \quad (\text{A.27})$$

Before proceeding to the actual computations, it was necessary to define the mesh dimension, so 5 different models were used. These models' mesh were uniformly increased such that the height of the domain was divided in $200 \cdot i$, being the i the order of the model. The CFD simulations were then carried out with ANSYS/Fluent code based on the RANS equations. As described in (Rumsey, 2019) the Spalart-Allmaras (SA) turbulence model (Spalart and Allmaras, 1992) has been systematically tested over the years and provided accurate results at least for the precision assumed necessary in this work. It was also used the SST $k-\omega$ turbulence model as it captures both the accurate formulation of standard $k-\omega$ model in the near-wall region and free stream independence of the $k-\varepsilon$ model in the far field (Menter, 1993). The airfoil was treated as a no-slip wall and all the equations were solved using second order schemes only. The evolution of the drag force obtained for both turbulence models considering a 1 [m] chord NACA 0012 airfoil and 43.82 [m/s] free stream wind speed (corresponding to a 3×10^6 Reynolds number flow) can be found in Figure A.7. As can be seen, the last 3 models provide consistent results and hence the

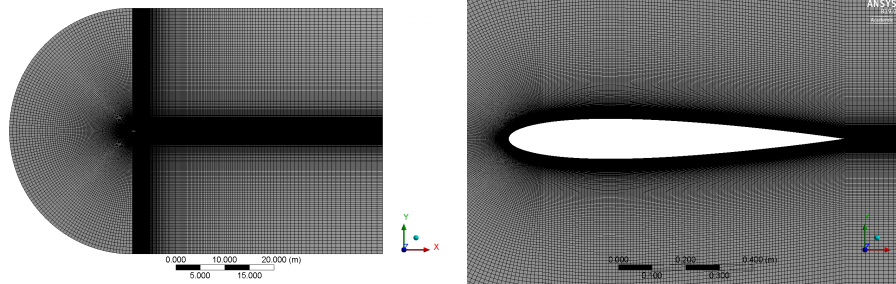


Figure A.8: Final mesh used in the CFD simulations - global and detailed view.

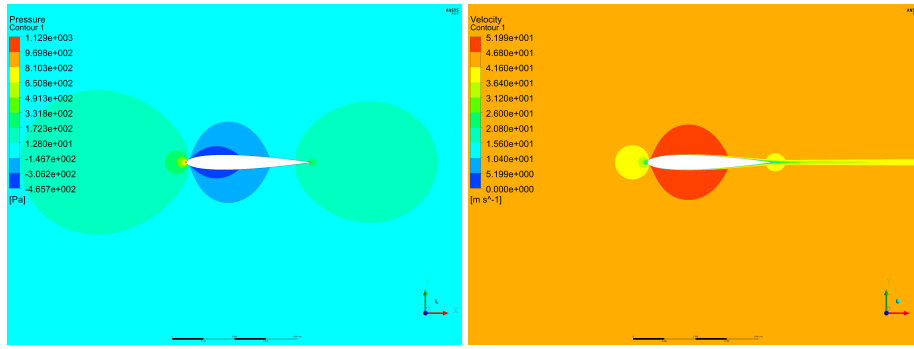


Figure A.9: Pressure (on the left) and velocity (on the right) over the airfoil at 0° angle of attack.

mesh is properly validated. As simulations for higher angles of attack will be, in theory at least, more computational demanding, the 4th order model was chosen for the remaining calculations. In Figure A.8 the chosen mesh is represented while in Figure A.9 the computed pressure and velocity plots can be seen.

Although the numerical simulations do appear to provide consistent results, it is still not guaranteed that these match the experimental ones (in fact, the 2 turbulence models gave slightly different drag forces already). To ensure that CFD results are of any use, the lift and drag forces were computed for several angles of attack with both turbulence models and compared with the experimental results obtained by Ladson in (Ladson, 1988) (the model properties and flow already mentioned guarantee both close Reynolds and Mach numbers similarity with the experimental results as well as a representative values for wind turbine operating conditions). Figure A.10 represents the results obtained for all the angles of attack tested. As can be seen, the CFD results are in good agreement with the experimental values and so the model is validated.

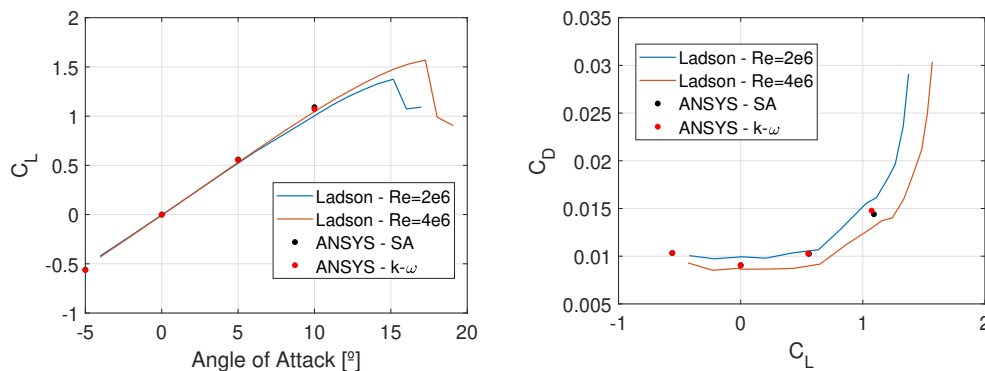


Figure A.10: Lift and drag coefficients validation.

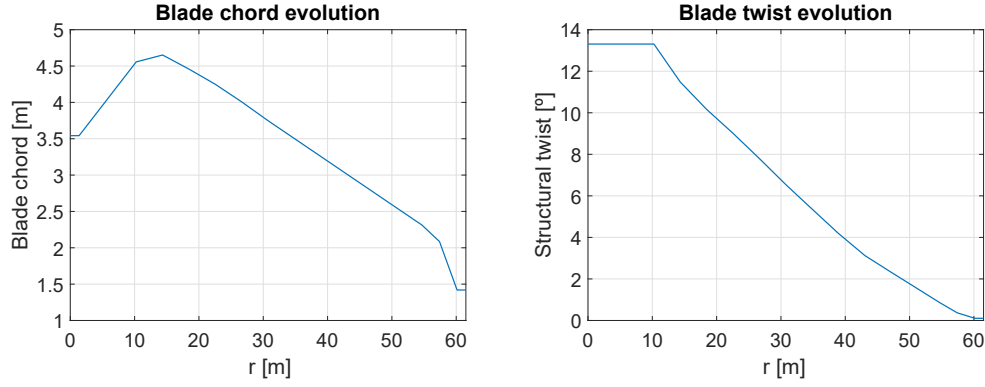


Figure A.11: Blade's chord (on the left) and twist (on the right) evolution over the span.

A.3 BEMT

The expressions of interest are more easily solved introducing the **blade solidity** (σ_r) as the ratio between blades' area and a rotor ring area:

$$\sigma_r = \frac{N}{2\pi} \cdot \frac{c}{r} = \frac{N}{2\pi\mu} \cdot \frac{c}{R} \quad (\text{A.28})$$

and rearranging Equation 3.37 as:

$$\frac{a}{1-a} = \frac{\sigma_r}{4 \cdot \sin^2(\phi)} \cdot \left[C_{\parallel}(a) - \frac{\sigma_r}{4 \cdot \sin^2(\phi)} \cdot C_{\perp}^2(a) \right] \quad (\text{A.29a})$$

$$\frac{a}{1+a'} = \frac{\sigma_r \cdot C_{\perp}(a)}{4 \cdot \sin(\phi) \cdot \cos(\phi)} \quad (\text{A.29b})$$

In order to illustrate the described procedure, a 3 blades rotor with a diameter of 123m is analysed. For the blades, an aerofoil adapted from one of the OpenFAST certification tests with the structural twist and chord length evolution over its 61.50 m span according to Figure A.11 was used. As a simplification, and because the purpose of this example is not to provide a full aerodynamic calculation but to illustrate the method, according to the results presented in Section 3.2, the lift and drag coefficient were assumed to be given by:

$$C_L = \frac{\pi^2}{90} \cdot (\alpha - 4.432), \text{ with } \alpha \text{ given in degrees} \quad (\text{A.30a})$$

$$C_D = \frac{1.328}{\sqrt{\frac{\rho \cdot W \cdot c}{\mu}}} \quad (\text{A.30b})$$

Starting from null induction coefficients for every section, its real values can be obtained by successive iterations of Equation A.29 until an equilibrium is reached. This process is illustrated in Table A.1 for the mid span section, with a 3.748 m chord and a structural twist of 6.544° , where it can be seen it does converge after few iterations. Applying the same procedure for all the other

Table A.1: Iterative process for obtaining the axial and transverse induction coefficients.

Iteration	a	a'	ϕ [°]	$C_{\parallel}$	$C_{\perp}$
0	0	0	17.260	0.658	0.204
1	0.097	0.011	15.513	0.480	0.133
2	0.088	0.008	15.704	0.499	0.140
3	0.090	0.008	15.678	0.497	0.139
4	0.089	0.008	15.681	0.497	0.139
5	0.089	0.008	15.681	0.497	0.139

section, allows to construct Figure A.12, where the evolution of both the induction, normal and tangential force coefficients are represented, assuming the first 10 m of the blade to be a circular section.

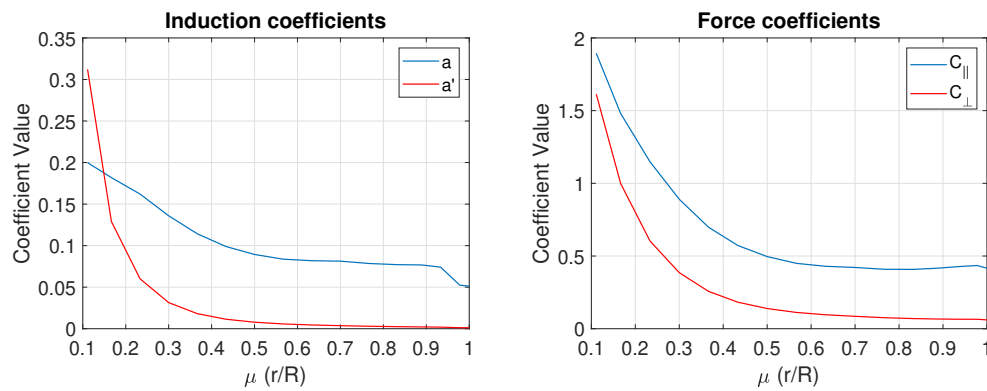


Figure A.12: Blade's induction (on the left) and force coefficients (on the right) evolution over the span.

Appendix B

Hydrodynamics

B.1 Morison equation

To illustrate the effect of the body characteristic dimension when compared to the incident wavelength, the inertia term in the Morison equation can be compared to the potential flow solution. Taking the potential limit in deep water from [Equation 2.29](#), the pressure evolution is given by:

$$\phi_0 = \frac{\eta_0 \cdot g}{\omega} \cdot e^{k \cdot z} \cdot \sin(kx - \omega t) \longrightarrow p \approx \rho \partial_t \phi = \underbrace{-\eta_0 \cdot \rho \cdot g \cdot e^{k \cdot z}}_{K_z} \cdot \cos(kx - \omega t) \quad (\text{B.1})$$

and, at a given distance y from the axis, the change in pressure is:

$$\Delta p = K_z \cdot \left[\cos(kx - \omega t) - \cos(-kx - \omega t) \right] = 2 \cdot K_z \cdot \sin kx \cdot \sin \omega t \quad (\text{B.2})$$

implying:

$$dF = 4 \cdot K_z \cdot \sin \omega t \cdot \underbrace{\int_0^{\frac{\pi}{2}} \sin(k \cdot R \cos \theta) \cdot R \cdot \cos \theta \cdot d\theta}_{F_1} = 4 \cdot K_z \cdot F_1 \cdot \sin \omega t \quad (\text{B.3})$$

where the x dependency, in general, can no longer be omitted (as was done in [Equation 3.68](#))¹. If the wave field were not affected by the body, in the same spirit that led to [Equation 3.70](#), the total force on the cylinder would be given by:

$$F = 2 \cdot 4 \cdot F_1 \cdot \sin \omega t \cdot \int_{-d}^0 K_z dz = 8 \cdot a \cdot g \cdot \rho \cdot \frac{1 - e^{-d \cdot k}}{k} \cdot F_1 \cdot \sin \omega t = F_0 \cdot \sin \omega t \quad (\text{B.4})$$

that should provide the force for any cylinder, if the potential field was not affected by the body's presence. A simple model of a 40 m height and 20 m diameter cylinder was built in ANSYS AQWA that will serve to illustrate the accuracy of this assumption. The cylinder was assumed

¹In the derivation presented F_1 is the value of the identified integral and hence can be evaluated from the geometry alone. In ([Chakrabarti, 1987](#)) it is shown that it can also be expressed in terms of the Bessel functions of the first kind.

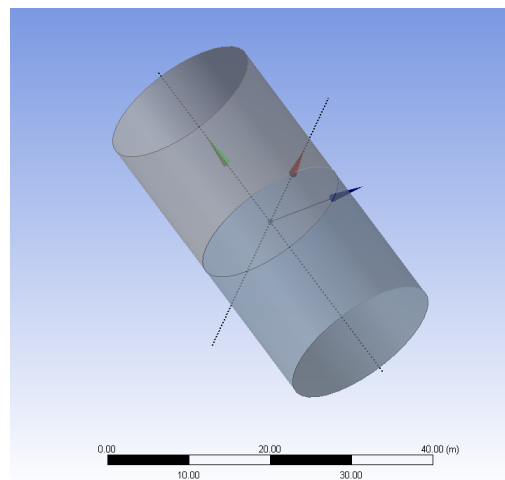


Figure B.1: Ansys model of a floating cylinder and its principal properties.

to have a relative density of 0.5 from which it can readily be concluded that static equilibrium is achieved when only the lower half of the cylinder is submerged, as represented in Figure B.1. The remaining inertial properties can be obtained from classical equations, such as in (Tipler and Mosca, 2007) or even, for more complex geometries, from a CAD software. In this case, the total mass is $M = 6283.2$ ton, the radius of gyration relative to any horizontal axis through the centre of mass is $R_{xx} = R_{yy} = 12.58$ m and the radius of gyration with respect to the longitudinal axis is $R_{zz} = 7.07$ m.

In parallel, two additional cylinders were modelled with a diameter of 5 m and 80 m, respectively and the forces computed for a unidirectional 6 m height wave and a period of 10 s, a common value for the Portuguese coast (Capitão et al., 2014). The 20 m diameter cylinder was also excited by two additional regular waves, a very short and a very long wave, both within the limits of wind generated ocean gravity waves (Toffoli and Bitner-Gregersen, 2017). The properties of all the waves tested is summarised in Table B.1, where it was assumed 1000 m water depth to relate wave period and length. The time evolution of the force computed with ANSYS and from Equation B.4 can be seen in Figure B.2 for the 6 cases, where it becomes clear that Equation B.4 losses its validity as the ratio of the wave length and diameter increases.

Table B.1: Waves used for the simulations properties.

H_s [m]	T [s]	λ [m]	$\frac{5}{\lambda}$	$\frac{20}{\lambda}$	$\frac{80}{\lambda}$
6	10	156.1	0.032	0.128	0.513
9.14	13	263.8	0.019	0.076	0.303
0.67	4.8	35.96	0.139	0.556	2.225

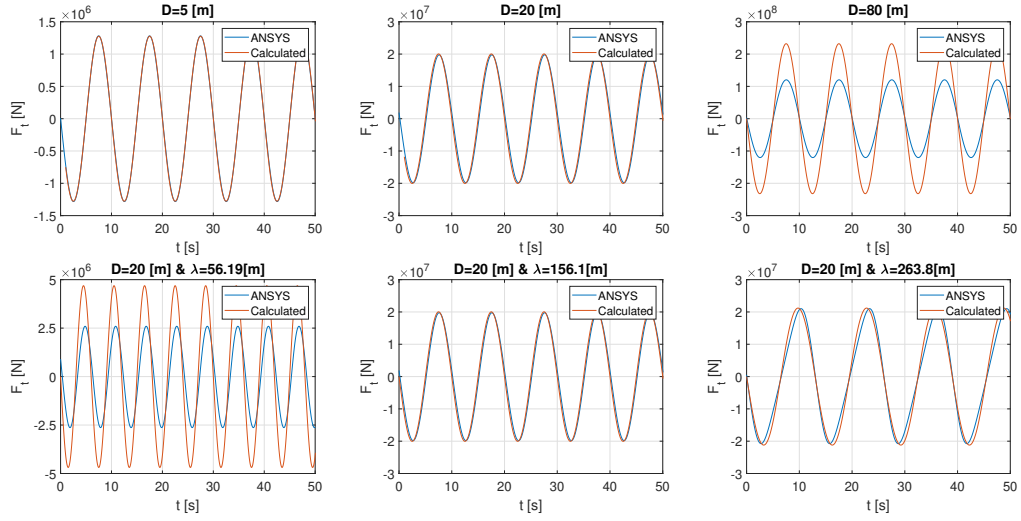


Figure B.2: Comparison of the forces over 3 different diameter cylinders (on top) and for different wave length for the 20 [m] diameter cylinder (on bottom).

B.2 Second order drift forces

For the case of an infinite stationary wall, and considering the deep water depth limit of the incident potential, the diffraction potential is easily obtained from Equation 3.75 as:

$$\phi_0 = \frac{\eta_0 \cdot g}{\omega} \cdot e^{k \cdot z} \cdot \sin(kx - \omega t) \longrightarrow \phi_D = \frac{\eta_0 \cdot g}{\omega} \cdot e^{k \cdot z} \cdot \sin(-kx - \omega t) \quad (\text{B.5})$$

Adding both potentials and using the deep water relation $\omega^2 = k \cdot g$ from Equation 2.33 leads to:

$$\int_{-\infty}^0 (\nabla \phi_1)^2 \cdot dz = \int_{-\infty}^0 (\partial_z \phi_1)^2 \cdot dz \left(\frac{2 \cdot \eta_0 \cdot g \cdot k \cdot \sin(\omega t)}{\omega} \right)^2 \cdot \int_{-\infty}^0 e^{2kz} dz = 2 \cdot \eta_0^2 \cdot \sin^2(\omega t) \cdot g \quad (\text{B.6})$$

where it has been assumed that the wall is located at $x = 0$. The surface elevation is given by:

$$\eta(x, t) = -\frac{1}{g} \partial_t \phi \Big|_{z=0} = 2 \cdot \eta_0 \cdot \cos(kx) \cos(\omega t) \longrightarrow \eta(0, t) = 2 \cdot a \cdot \cos(\omega t) \quad (\text{B.7})$$

and hence, the mean second order force in the x direction is given by:

$$\overline{F_2^x} = -\frac{1}{2} \rho g \eta_0^2 + g \rho \eta_0^2 = \frac{1}{2} \rho g \eta_0^2 \quad (\text{B.8})$$

or, if the lower limit of the integral is explicitly considered:

$$\overline{F_2^x} = -\frac{1}{2} \rho g \eta_0^2 \cdot (1 - e^{-2kh}) + g \rho \eta_0^2 = \frac{1}{2} \rho g \eta_0^2 \cdot (1 + e^{-2kh}) \quad (\text{B.9})$$

and the mean drift force is constant in time.

Although the cylinder from Section B.1 is not an infinite draft stationary wall, it can act like

one if the wavelengths are short enough. In that case, it is possible to show that the mean force in a cylinder is given by (Faltinsen, 1993):

$$\overline{F}_2^x = \frac{2}{3} \rho \cdot g \cdot \eta_0^2 \cdot R \cdot \left(1 + e^{-2kh}\right) \quad (\text{B.10})$$

where the correction factor to account for a non infinite draft has been included. If the cylinder is allowed to move, the total force acting on it is not as easily computed, but it has a constant component overtime that for long waves is almost exclusively explained by Equation 3.94 see (see Gupta, 2010; Choi et al., 2001, for further details). However, and because the wave is not small by comparison with the characteristic length, it is not given by Equation B.10 anymore. If there is a constant mean force over time, then, from Newton's 2nd law, the cylinder equation of motion is:

$$m \cdot \ddot{x}(t) = F(\omega) \longrightarrow x(t) = \frac{F(\omega)}{2m} \cdot t^2 \quad (\text{B.11})$$

which is the classical parabolic motion. As the body accelerates, a drift velocity appears changing the frequency of encounter from ω to $(\omega - k \cdot V)$. Because of this, the frequency dependent force is not really constant overtime, but Equation B.11 may be applied over short periods of time. According to (Faltinsen, 1980), the drift force for a body with forward speed V may be expressed as:

$$\overline{F}_2^x = \overline{F}_2^x \Big|_{V=0} \cdot \left(1 + \frac{2\omega V}{g}\right) \quad (\text{B.12})$$

and a component proportional to the velocity appears - an additional resistance (b_{add}). Applying once more Newton's 2nd law, it may be written:

$$m \cdot \ddot{x}(t) + b_{add} \cdot \dot{x}(t) = F(\omega) \longrightarrow x(t) = \frac{m}{b_{add}} \cdot e^{-\frac{b_{add}}{m} \cdot t} + \frac{F(\omega)}{b_{add}} \cdot t - \left(\frac{m}{b_{add}} - x_0\right) \quad (\text{B.13})$$

Because the exponential will decay overtime, based on Equation B.11 and Equation B.13, an initially parabolic motion should be followed by an uniform velocity motion that, once achieved, should remain stable, as the frequency of encounter is no longer changing. Although no closed form expression has been provided for the cylinder motion, some general properties have been identified that can be checked against the computed response motion and velocity represented in Figure B.3 for the medium wave from Section B.1, where both regions are clearly visible.

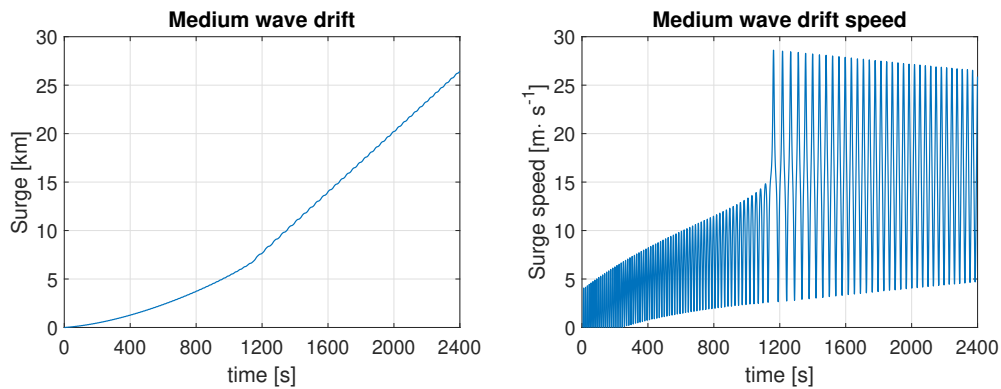


Figure B.3: Drift motion (on the left) and speed (on the right) for the medium wave.

B.3 Mooring lines

The cylinder from the previous examples was provided with a mooring system composed by 3 independent 0.0766 m diameter cables, each 835.5 m long, and weighting 113.35 kg/m per unit length. The cables, connected to the structure 14 m below still water level, are placed uniformly at every 120° and anchored to the sea bed over a 796.7 m radius.

Considering a quasi static approach, the forces were computed for all the pairs of degrees of freedom motion having obtained the force-displacement law represented in [Figure B.4](#) (the same as [Figure 3.13](#), but here recovered for simplicity), where it is clear the almost linear behaviour of the system for small amplitude motions around an equilibrium position. Doing the same for the restoring moments, [Figure B.5](#) is obtained. Although the behaviour is almost linear for a given equilibrium position, it seems to have a stronger dependency on the initial position than before.

In order to illustrate the non linear effects of the mooring system, the structure was displaced 25 [m] over surge direction and its motion restrained over all other degrees of freedom. In [Figure B.6](#) are presented the body displacements and total mooring force over a 300 [s] simulation.

If the mooring forces are plotted against body motions instead, then the almost linear region around different equilibrium positions can also be identified in [Figure B.7](#). Finally, and in order to investigate the possible effect of considering the cables bending stiffness, the simulations were repeated considering $EI = 1 \times 10^9$ [N.m²] for all the cables. The body motions and the force-displacement law for the mooring system are represented in [Figure B.8](#). The floating body motions and cable mean tension forces are reasonably predicted from a quasi static simulation. However, when it comes to evaluate fatigue loads on cables, a time simulation must be conducted, as the currently available computational power does not seem to justify any other approach.

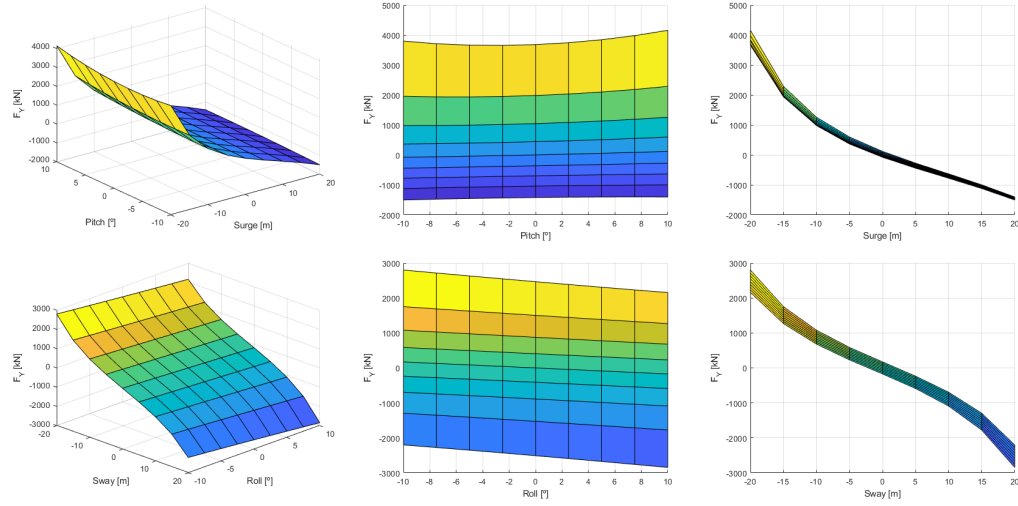


Figure B.4: Total force transmitted by the mooring lines to the cylinder has function of the displacement *surge-pitch* (on top) and *sway-roll* (on bottom).

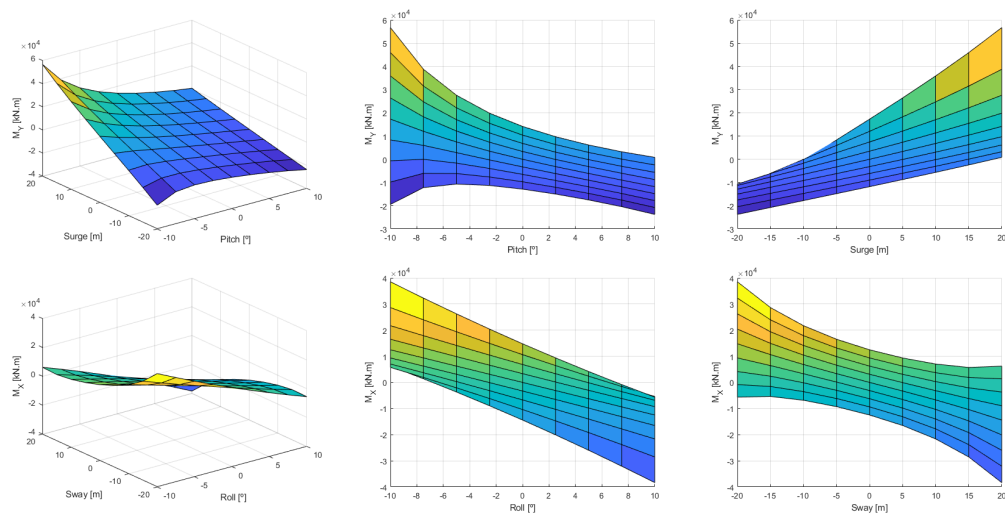


Figure B.5: Total restoring moment transmitted by the mooring lines to the cylinder has function of the displacement *surge-pitch* (on top) and *sway-roll* (on bottom).

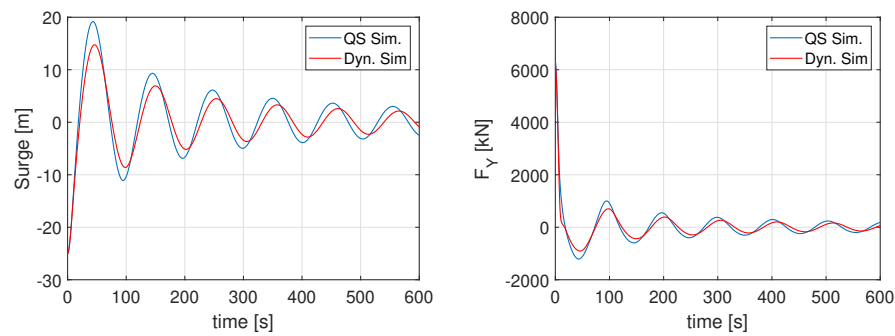


Figure B.6: Surge displacement (on the left) and mooring system total force (on the right) overtime from a quasi static and a full dynamic simulation.

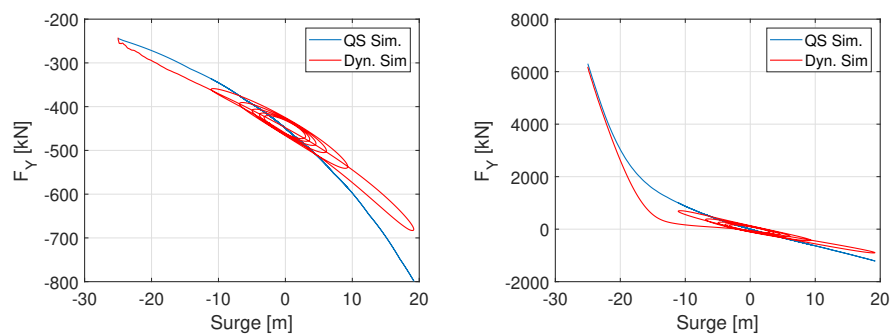


Figure B.7: Force-displacement law for a single cable (on the left) and for the mooring system (on the right) from a quasi static and a full dynamic simulation.

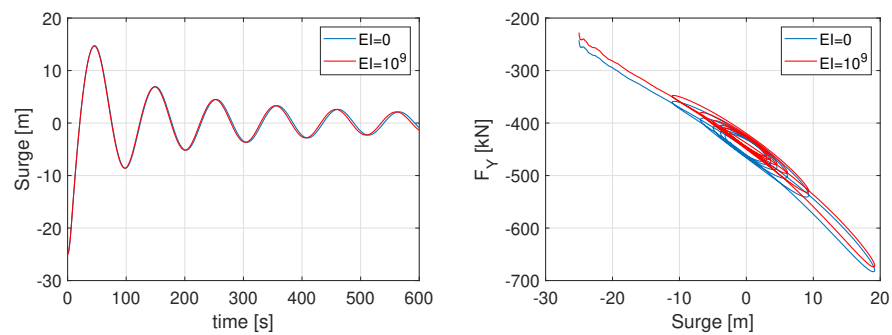


Figure B.8: Surge displacement (on the left) and a single cable force-displacement law (on the right) neglecting or not cables' bending stiffness.

Appendix C

Analytical mechanics

C.1 Lagrange method

C.1.1 Generalised coordinates and constraints

Generalised coordinates

A particle position can easily be located in any tri-dimensional coordinate system based on the 3 unit vectors $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}_3$ by a linear combination of its components in the direction along the basis vectors. In the very familiar Euclidean space and using the Cartesian coordinate system¹, this can be simply expressed as:

$$\mathbf{r}_P = x_1(t) \cdot \hat{\mathbf{e}}_1 + x_2(t) \cdot \hat{\mathbf{e}}_2 + x_3(t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.1})$$

By definition, the scalars in [Equation C.1](#) also constitute the coordinates of the point in the same reference frame as they locate it relative to the origin (or any other point). Although this distinction seems unnecessary, the Cartesian coordinate system is indeed the only one in which the coordinates of a vector are numerically equal to the position magnitudes ([Kasdin and Paley, 2011](#)). If this concept is generalised to any three independent scalar functions q_i that can be used to locate the point in a given reference frame, it can be written:

$$\mathbf{r}_P = a_1(q_1, q_2, q_3, t) \cdot \hat{\mathbf{e}}_1 + a_2(q_1, q_2, q_3, t) \cdot \hat{\mathbf{e}}_2 + a_3(q_1, q_2, q_3, t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.2})$$

For the polar coordinate system, for example, and considering the generalised coordinates as the distance from a given point r and a direction angle θ , the position vector is simply written as $\mathbf{r}_P = r \cdot \cos \theta \cdot \hat{\mathbf{e}}_x + r \cdot \sin \theta \cdot \hat{\mathbf{e}}_y = r \cdot \hat{\mathbf{e}}_r$. The time derivative of this more general expression is not as straightforward as in the Cartesian coordinate system, but some problems almost demand its solution to be expressed in alternative coordinate systems if they are to be understood properly.

¹ In this section special attention must be given to the difference between a reference frame and a coordinate system. The former defines the background to all the measurements and observations that are to be made, while the latter is a set of scalars that can be used to identify a position of a given point in the previously defined reference frame.

The velocity of a particle is then given by:

$$\mathbf{v}_P = \underbrace{\left(\sum_{i=1}^3 \frac{\partial a_1}{\partial q_i} \cdot \frac{\partial q_i}{\partial t} + \frac{\partial a_1}{\partial t} \right)}_{\frac{da_1}{dt}} \cdot \hat{\mathbf{e}}_1 + \underbrace{\left(\sum_{i=1}^3 \frac{\partial a_2}{\partial q_i} \cdot \frac{\partial q_i}{\partial t} + \frac{\partial a_2}{\partial t} \right)}_{\frac{da_2}{dt}} \cdot \hat{\mathbf{e}}_2 + \underbrace{\left(\sum_{i=1}^3 \frac{\partial a_3}{\partial q_i} \cdot \frac{\partial q_i}{\partial t} + \frac{\partial a_3}{\partial t} \right)}_{\frac{da_3}{dt}} \cdot \hat{\mathbf{e}}_3 \quad (\text{C.3})$$

Constraints

Because the purpose of a dynamic analysis is to identify the position of a particle over time and it takes, in general, 3 independent coordinates to do so, any free particle is associated with the same number of degrees of freedom. However, some limitations on the allowed motions can reduce the dimension of the problem. To simplify the explanation, a gravity pendulum of length l will now be considered moving in the xz plane. In this case, the otherwise free particle, is now limited to move in a given plane, such that $y(t) = 0$, and with a fixed distance from the anchor point such that $x^2 + z^2 = l^2$. The introduction of these two constraints on the equations of motion reduces the degrees of freedom of the system and, in this case, a single generalised coordinate can be used to characterise the motion (the angle with the vertical, for instance). The gravity pendulum is a very simple case, but this notion can indeed be generalised such that whenever K constraints can be expressed as in [Equation C.4](#) - an **holonomic constraint** - the $3N$ initial degrees of freedom (being N the number of particles) of the system are reduced by the same amount.

$$f(q_1, q_2, \dots, q_i, t) = 0 \quad (\text{C.4})$$

However, not all the constraints can be expressed in this way, as is the case of inequality constraints (for example, a particle rolling down a spherical shell) or, maybe more important for the present work, non integrable differential constraints involving the partial derivatives of the generalised coordinates. The last ones can be expressed as:

$$f(q_1, q_2, \dots, q_i, \dot{q}_1, \dot{q}_2, \dots, \dot{q}_i, t) = 0 \quad (\text{C.5})$$

The constraints in the form of [Equation C.5](#) are referred to as **nonholonomic** and, because they are non integrable, the number of generalised coordinates is not reduced, although the degrees of freedom are.

Example C.1 - The gravity pendulum, represented in [Figure C.1](#), is one of the most used systems in any physics book to illustrate some basic concepts. Its equations of motion can simply be written as:

$$m \frac{d^2 \mathbf{r}}{dt^2} = \mathbf{f}_g + \mathbf{f}_t \quad (\text{C.6})$$

where $\mathbf{f}_g$ and $\mathbf{f}_t$ stand for the gravitational and tension force, respectively.

However, a simpler solution can be obtained considering as generalised coordinates the azimuthal angle θ and the distance to the anchor point r . In this coordinate system, the equations

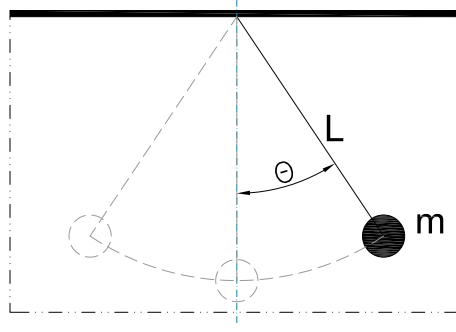


Figure C.1: Schematic representation of the gravity pendulum.

of motion now read:

$$\left\{ \begin{array}{l} \mathbf{r} = r \cdot \hat{e}_r \\ \mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \\ \mathbf{a} = \frac{d\mathbf{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta \end{array} \right. \longrightarrow \left\{ \begin{array}{l} m \cdot (\ddot{r} - r\dot{\theta}^2) = -F_t + mg \cos \theta \\ m \cdot (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -mg \sin \theta \end{array} \right. \quad (\text{C.7})$$

If the pendulum is supposed to be inextensible, then an holonomic constraint $r = L$ can easily be defined² and so only a generalised coordinate is needed, leading to the classical pendulum equation:

$$m \cdot L \cdot \ddot{\theta} + mg \sin \theta = 0 \longrightarrow \ddot{\theta} + \frac{g}{L} \sin \theta = 0 \quad (\text{C.8})$$

and centripetal force (F_c) for circular motion:

$$-m \cdot r \dot{\theta}^2 = -F_t + mg \cos \theta = -F_c \longrightarrow F_c = m \cdot r \cdot \dot{\theta}^2 \quad (\text{C.9})$$

It may seem a waste of time to develop this example when the forces equilibrium could be easily obtained from geometric considerations and the complex derivative of the generalised coordinates would not be needed, but the ability to describe a physical problem by the right choice of coordinates subjected to specific constraints will be very useful in more complex models.

C.1.2 Lagrange's equations

Although Newton's mechanics offers a comprehensive description of a given body equations of motion, it can become cumbersome as the dimension of the system, and so the number of degrees of freedom, increases. Before proceeding to the Lagrange method itself, it is important to introduce what become known as **d'Alembert principle**. In his formulation, d'Alembert shown that the work done by all the forces acting on a system over an infinitesimal displacement $\delta \mathbf{r}_j$ compatible

²Note that this constraint implies $\dot{r} = \ddot{r} = 0$.

with the constraints of the problem vanishes if the inertia forces are included³:

$$\sum_{j=1}^N \left(\mathbf{f}_j - m_j \cdot \frac{d\mathbf{v}_j}{dt} \right) \cdot \delta \mathbf{r}_j = 0 \quad (\text{C.10})$$

At first glance, Equation C.10 can be satisfied imposing $\mathbf{f}_j = m_j \cdot \frac{d\mathbf{v}_j}{dt}$ which is nothing more than Newton's 2nd law. However, and because the virtual displacements of every node are not independent, further analysis can be made. In fact, the first term in the Equation C.10 is identified as a **generalised force** Q_i while the second can be shown to be related to the kinetic energy T of the system (see Goldstein et al. (2014) or Kasdin and Paley (2011) for further details on the derivations) and the Euler-Lagrange equations are obtained for a system subject to holonomic constraints:

$$\left\{ \begin{array}{l} Q_i = \sum_{j=1}^N \mathbf{f}_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_i} \\ \sum_{j=1}^N m_j \cdot \frac{d\mathbf{v}_j}{dt} = \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} \end{array} \right. \longrightarrow \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i \quad (\text{C.11})$$

Note that although they are equivalent to Newton's law of motion in the sense that no new physics has been introduced, this formulation allows the determination of the equations of motion based on the generalised coordinates only, and these can be significantly less than the degrees of freedom of the system. If the forces are conservative, then they can be derived from a scalar potential $\mathbf{f} = -\nabla V$ and defining the Lagrangian of a system as $L = T - V$, Equation C.11 can be rewritten as:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i^{nc} \quad (\text{C.12})$$

where Q_i^{nc} are now only the non conservative generalised forces eventually present. While the derivations made assumed holonomic constraints such that the system could be treated as non restrained within a reduced domain, if reducing the Lagrangian to the independent generalised coordinates is not easily achieved or if it is useful to obtain the constraint forces, then it can be written (Goldstein et al., 2014):

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = - \sum_{\alpha=1}^m \lambda_{\alpha} \cdot \frac{\partial f_{\alpha}}{\partial q_k} \longrightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \sum_{\alpha=1}^m \lambda_{\alpha} \cdot \frac{\partial f_{\alpha}}{\partial q_k} = 0 \quad (\text{C.13})$$

in which the λ_{α} are the magnitude of the forces of constraint and the m constraint functions (f_{α}) are given by Equation C.4. A reduced dimension model of a wind turbine mainly devoted to blades' vibrations control using the Lagrangian approach is described in detail in (Staino et al., 2012) and (Fitzgerald et al., 2013).

³Note that because the displacements must be consistent with the constraints, the applied forces of Equation C.10 do not need to include the constraint forces as its work is automatically null.

Example C.2 - A spinning pendulum is a much more complex system than the simple gravity pendulum but is also a powerful one to illustrate the potential of the Lagrangian formulation. However, and before proceeding further in this example, this approach will be applied to gravity pendulum of Example C.1. As all the forces are conservative, it can be written:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad (\text{C.14})$$

where the Lagrangian is simply given by:

$$L = T - V = \underbrace{\frac{1}{2}m\left[\dot{r}^2 + (r\dot{\theta})^2\right]}_T + \underbrace{mgr \cos \theta}_{-V} \quad (\text{C.15})$$

where the inextensible condition was intentionally not included. The corresponding equations of motion are:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr \sin \theta = 0 \quad (\text{C.16a})$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} + \lambda = m\ddot{r} - mr\dot{\theta}^2 - mg \cos \theta + \lambda = 0 \quad (\text{C.16b})$$

Making use of the constraint equations, the Lagrange equations of motion simply reduce to:

$$\ddot{\theta} + \frac{g}{L} \sin \theta = 0 \quad (\text{C.17a})$$

$$\lambda = mr\dot{\theta}^2 + mg \cos \theta \quad (\text{C.17b})$$

where Equation C.8 can easily be identified and the force of constraint is directly obtained when solving the equations. Note that if it was assumed from the beginning that $r = L$, then a simplified Lagrangian would be obtained with a single equation of motion:

$$L = \frac{1}{2}m(r\dot{\theta})^2 + mgr \cos \theta \quad \longrightarrow \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = mr^2\ddot{\theta} + mgr \sin \theta = 0 \quad (\text{C.18})$$

where the force of constraint does not appear in the equations anymore.

Having the simple pendulum revisited with a Lagrangian formulation, the spinning pendulum problem will be addressed (see Example 13.8 from (Kasdin and Paley, 2011)). If a gravity pendulum whose plane is rotating around the vertical axis with a fixed angular speed Ω as in Figure C.2⁴ is considered, it can be noted that the mass velocity is now given by:

$$\mathbf{v} = l\dot{\theta} \cdot \hat{\mathbf{e}}_x + l \cdot \Omega \sin \theta \cdot \hat{\mathbf{e}}_y \quad (\text{C.19})$$

leading to the Lagrangian and single equation of motion:

$$L = \frac{1}{2} \cdot m \cdot l^2 \cdot (\dot{\theta}^2 + \Omega^2 \sin^2 \theta) + mgl \cos \theta \quad \longrightarrow \quad \ddot{\theta} + \left(\frac{g}{l} - \Omega^2 \cos \theta \right) = 0 \quad (\text{C.20})$$

⁴Note that this is still a one degree of freedom system and not a spherical pendulum.

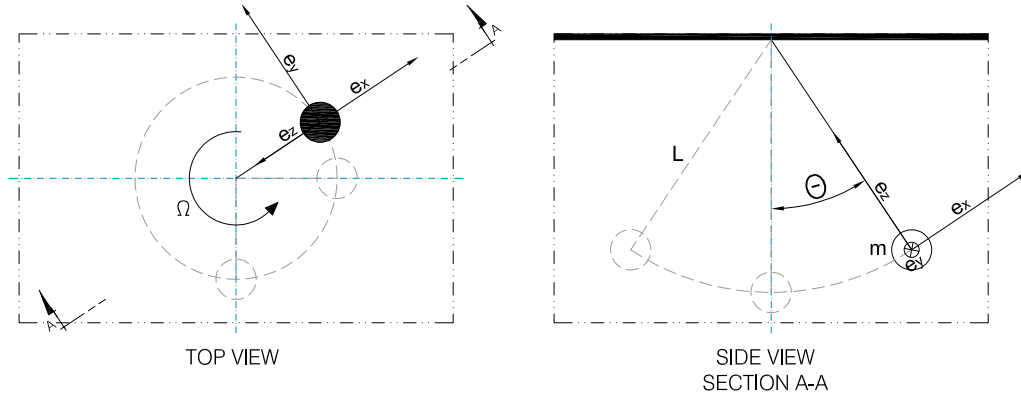


Figure C.2: Schematic representation of the spinning pendulum.

With this example the power of Lagrange's formulation is supposed to be illustrated, noting that the derivations with the traditional Newton's method would be much more laborious.

C.2 Kane method

C.2.1 Kane's equations

Although Lagrange's formulation leads to very simple expressions for complicated systems as the constraint can be easily included, it requires the scalar functions for potential and kinetic energy to be expressed, and this may not be an easy task. In this last section, it is presented a different approach to overcome this problem and provide first order only differential equations that can be easily solved by numerical means. To do so, it is necessary to introduce the **generalised speed** (u_r) concept as:

$$u_r = \sum_{s=1}^M Y_{rs}(q_1, \dots, q_M, t) \cdot \dot{q}_s + Z_r(q_1, \dots, q_M, t) \quad (\text{C.21})$$

where the nomenclature from (Kasdin and Paley, 2011) has been followed and Y_{rs} and Z_r are functions of the M generalised coordinates and time. If the generalised speed conventions are inverted, then an expression for the generalised coordinate derivative is obtained:

$$\dot{q}_s = \sum_{r=1}^M W_{sr}(q_1, \dots, q_M, t) \cdot u_r + X_s(q_1, \dots, q_M, t) \quad (\text{C.22})$$

where W_{sr} and X_s are also functions of the generalised coordinates and time.

The base of Kane's method is to provide the M first order differential equations that combined with Equation C.22 give a full description of the solution. With this formulation, the velocity of a

given particle P can be expressed from a combination of the partial velocities defined by:

$$\mathbf{v}_r^P = \frac{\partial \mathbf{v}_P}{\partial u_r} \longrightarrow \mathbf{v}_P = \sum_{r=1}^M \mathbf{v}_r^P \cdot u_r + \mathbf{v}_t^P \quad (\text{C.23})$$

In the same way, the **generalised active forces** and the **generalised inertia forces** are defined and Kane's equations of motion obtained⁵:

$$\begin{cases} F_r = \sum_{i=1}^N \mathbf{v}_r^i \cdot \mathbf{f}_i \\ F_r^* = - \sum_{i=1}^N \mathbf{v}_r^i \cdot m_i \cdot \mathbf{a}_i \end{cases} \longrightarrow F_r + F_r^* = 0 \quad (\text{C.24})$$

Until this point only point masses have been considered, but the formulation can easily be extended to rigid body motions. The generalised forces for every rigid body (B) must then be included in Equation C.24 and are given by:

$$F_r = \omega_r^B \cdot \mathbf{m}_Q + \mathbf{v}_r^Q \cdot \mathbf{f}_Q \quad (\text{C.25a})$$

$$F_r^* = \omega_r^B \cdot \left[-I_Q \cdot \alpha^B - \omega^B \times (I_Q \cdot \omega^B) + \mathbf{r}_{Q/G} \times m_G \cdot \mathbf{a}_Q \right] + \mathbf{v}_r^Q \cdot (-m_G \cdot \mathbf{a}_Q) \quad (\text{C.25b})$$

where Q is a given point, not necessarily the centre of mass G , around which the moment ($\mathbf{m}_Q$) and inertia (I_Q) must be evaluated and α is the angular acceleration of the body.

Example C.3 - The simple pendulum revisited. As before, a single generalised coordinate $q_1 = \theta$ will be chosen and its derivative $u_1 = \dot{q}_1 = \dot{\theta}$ the generalised speed. From Equation C.23 and Equation C.24, the partial velocity and the generalised active forces are obtained:

$$\begin{cases} \mathbf{v}_1^P = \frac{\partial}{\partial \dot{\theta}} (l \cdot \dot{\theta} \cdot \hat{e}_\theta) = l \cdot \hat{e}_\theta \\ \mathbf{f}_1 = -m \cdot g \cdot \hat{e}_z \end{cases} \longrightarrow F = -m \cdot g \cdot l \cdot \sin \theta \quad (\text{C.26})$$

The generalised inertia force can also be obtained from particle's acceleration as:

$$\begin{cases} \mathbf{v}_1^P = l \cdot \hat{e}_\theta \\ \mathbf{a}^P = l \cdot \ddot{\theta} \cdot \hat{e}_\theta - l \cdot \dot{\theta}^2 \cdot \hat{e}_r \end{cases} \longrightarrow F^* = -m \cdot l^2 \cdot \ddot{\theta} \quad (\text{C.27})$$

Kane's equation can be readily built and provide, with the generalised speed definition (note that the equations are written in terms of the generalised coordinates and speed, q_1 and u_1 , although in this simple example q_1 is identical to the angular coordinate θ):

$$\dot{q}_1 = \dot{\theta} = u_1 \quad (\text{C.28a})$$

$$F + F^* = -m \cdot g \cdot l \cdot \sin \theta - m \cdot l^2 \cdot \ddot{\theta} = 0 \longrightarrow \ddot{\theta} = -\frac{g}{l} \sin q_1 \quad (\text{C.28b})$$

⁵Note that the scalar product of the 2nd equation is between the r^{th} partial velocity and a point acceleration.

It can easily be seen that these are fully equivalent to the previously derived pendulum equations, but Kane's method has the advantage of producing a set of first order differential equation that can be easily solved by numerical means. An illustration of Kane's method applied to the spinning pendulum of Example C.2 can be seen in the Example 13.17 from (Kasdin and Paley, 2011).

C.2.2 A simplified wind turbine model

In (Sarkar et al., 2018) a simplified model of a wind turbine is presented where a total of 16 degrees of freedom have been considered:

- For the tower, two modes in the fore-aft and side-to-side directions - 4 degrees of freedom;
- For the blades, two modes in the flapwise direction and one mode in the edgewise direction - 3 degrees of freedom per blade;
- Nacelle rotation around the yaw axis - 1 degree of freedom;
- Generator azimuth angle - 1 degree of freedom;
- Drive train torsion - 1 degree of freedom.

In this work, in order to illustrate the presented concepts, an even more simplified, 2D model will be used, where the nacelle is prevented from yawing and the tower is characterized by a single degree of freedom in the top. Furthermore, it will be assumed that the tower vertical displacements are very small and can be neglected⁶ and the total rotor mass is located at the hub center. From this assumptions, two distinct reference frames can be defined as represented in Figure C.3 - one Earth bound reference frame, whose unit vectors do not change over time, and a reference frame moving with the rotor, and hence with unit vectors changing over time.

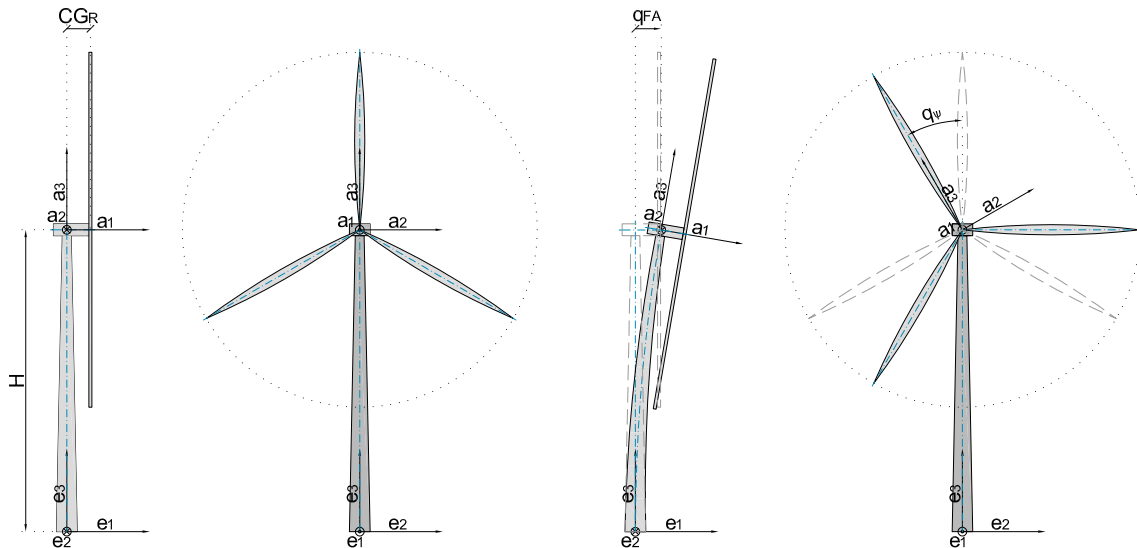


Figure C.3: Schematic representation of the wind turbine simplified model and respective degrees of freedom.

⁶It wouldn't add any additional difficulty to consider this displacement, but it would make the equations somewhat cumbersome and the explanatory purpose could be lost.

In order to describe the dynamic behaviour of the structure, only 2 degrees of freedom are needed: the tower top displacement (q_{FA}) and the rotor azimuth angle (q_ψ) (third and fourth picture in [Figure C.3](#), respectively). The generalised speeds will simply be the time derivatives of the generalised degrees of freedom:

$$u_1 = \frac{d}{dt} \left(q_{FA} \right) = \dot{q}_{FA} \quad (\text{C.29a})$$

$$u_2 = \frac{d}{dt} \left(q_\psi \right) = \dot{q}_\psi \quad (\text{C.29b})$$

The first step is to identify the relevant points of the structure. In this case, one can easily identify the tower nodes (defined by the number of the divisions of the element) and the rotor centre of mass. Having these identified, it is necessary to write the velocities and accelerations in the Earth-bound reference frame⁷. Because it will be necessary to take several derivatives along the process, the functional dependency of the variables will be written explicitly until the all kinematics are properly described. The position of any tower point (T) with respect to the origin of the reference frame (O) using a modal formulation normalised to have unitary displacements at the tower top is:

$$\mathbf{r}^{OT} = \phi_{FA}(h) \cdot q_{FA}(t) \hat{\mathbf{e}}_1 \quad (\text{C.30})$$

with the corresponding velocity and acceleration:

$${}^E \mathbf{v}^T = \phi_{FA}(h) \cdot \dot{q}_{FA}(t) \hat{\mathbf{e}}_1 \quad (\text{C.31a})$$

$${}^E \mathbf{a}^T = \phi_{FA}(h) \cdot \ddot{q}_{FA}(t) \hat{\mathbf{e}}_1 \quad (\text{C.31b})$$

The orientation of the rotor reference frame with respect to the Earth bound reference frame is obtained from the deflection at the top of the tower (R):

$$\theta(t) = \left. \frac{d\phi_{FA}}{dh} \right|_{h=H} \cdot q_{FA} = R \cdot q_{FA}(t) \quad (\text{C.32})$$

with the corresponding generalised velocity and acceleration:

$$\dot{\theta}(t) = R \cdot \dot{q}_{FA}(t) = R \cdot u_1(t) \quad (\text{C.33a})$$

$$\ddot{\theta}(t) = R \cdot \ddot{q}_{FA}(t) = R \cdot \dot{u}_1(t) \quad (\text{C.33b})$$

Applying the same procedure to the rotor centre of mass, and taking advantage of the adopted

⁷Note, however, that although velocities and angular velocities are always in relation to the Earth bound system, they may be written in a different basis.

mode normalisation, one obtains:

$$\mathbf{r}^{OR} = q_{FA}(t) \cdot \hat{\mathbf{e}}_1 + CG_R \cdot \hat{\mathbf{a}}_1(t) = [q_{FA}(t) + CG_R \cdot \cos \theta(t)] \cdot \hat{\mathbf{e}}_1 - CG_R \cdot \sin \theta(t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.34a})$$

$${}^E \mathbf{v}^R = [\dot{q}_{FA}(t) - CG_R \cdot \dot{\theta}(t) \cdot \sin \theta(t)] \cdot \hat{\mathbf{e}}_1 - CG_R \cdot \dot{\theta}(t) \cdot \cos \theta(t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.34b})$$

$${}^E \mathbf{a}^R = [\ddot{q}_{FA}(t) - CG_R \cdot (\ddot{\theta}(t) \sin \theta(t) + \dot{\theta}^2(t) \cos \theta(t))] \cdot \hat{\mathbf{e}}_1 - CG_R [\ddot{\theta}(t) \cos \theta(t) - \dot{\theta}^2(t) \sin \theta(t)] \cdot \hat{\mathbf{e}}_3 \quad (\text{C.34c})$$

If θ is assumed to be small, then a small angle approximation to first order may be used:

$$\mathbf{r}^{OR} \approx [q_{FA}(t) + CG_R] \cdot \hat{\mathbf{e}}_1 - CG_R \cdot \theta(t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.35a})$$

$${}^E \mathbf{v}^R \approx [\dot{q}_{FA}(t) - CG_R \cdot \dot{\theta}(t)] \cdot \hat{\mathbf{e}}_1 - CG_R \cdot \dot{\theta}(t) \cdot \hat{\mathbf{e}}_3 \approx \dot{q}_{FA}(t) \cdot \hat{\mathbf{e}}_1 - CG_R \cdot \dot{\theta}(t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.35b})$$

$$\begin{aligned} {}^E \mathbf{a}^R &\approx [\ddot{q}_{FA}(t) - CG_R \cdot (\ddot{\theta}(t) + \dot{\theta}^2(t))] \cdot \hat{\mathbf{e}}_1 - CG_R \cdot [\ddot{\theta}(t) - \dot{\theta}^2(t)] \cdot \hat{\mathbf{e}}_3 \approx \\ &\approx \ddot{q}_{FA}(t) \cdot \hat{\mathbf{e}}_1 - CG_R \cdot \ddot{\theta}(t) \cdot \hat{\mathbf{e}}_3 \end{aligned} \quad (\text{C.35c})$$

where 2nd order terms have been consistently left out. The relevant angular velocity and acceleration of the rotor are simply:

$${}^E \boldsymbol{\omega}^R = u_2(t) \cdot \hat{\mathbf{a}}_1(t) = u_2(t) \cdot \cos \theta(t) \cdot \hat{\mathbf{e}}_1 - u_2(t) \cdot \sin \theta(t) \cdot \hat{\mathbf{e}}_3 \approx u_2(t) \cdot \hat{\mathbf{e}}_1 - u_2(t) \cdot \theta(t) \cdot \hat{\mathbf{e}}_3 \quad (\text{C.36a})$$

$$\begin{aligned} {}^E \boldsymbol{\alpha}^R &= [\dot{u}_2(t) \cos \theta(t) - u_2(t) \cdot \dot{\theta}(t) \cdot \sin \theta(t)] \cdot \hat{\mathbf{e}}_1 - [\dot{u}_2(t) \sin \theta(t) + u_2(t) \cdot \dot{\theta}(t) \cdot \cos \theta(t)] \cdot \hat{\mathbf{e}}_3 \approx \\ &\approx [\dot{u}_2(t) - u_2(t) \cdot \dot{\theta}(t)] \cdot \hat{\mathbf{e}}_1 - [\dot{u}_2(t) \theta(t) + u_2(t) \cdot \dot{\theta}(t)] \cdot \hat{\mathbf{e}}_3 \approx \\ &\approx \dot{u}_2(t) \cdot \hat{\mathbf{e}}_1 - [\dot{u}_2(t) \theta(t) + u_2(t) \cdot \dot{\theta}(t)] \cdot \hat{\mathbf{e}}_3 \end{aligned} \quad (\text{C.36b})$$

Having the kinematics properly defined, the time dependency can be dropped and, using [Equation C.32](#) to write all the relevant velocities in terms of the generalised coordinates and speeds, the partial linear velocities and partial angular velocities are obtained from [Equation C.23](#):

$$\left\{ \begin{array}{l} {}^E \mathbf{v}^T = \phi_{FA}(h) \cdot u_1 \cdot \hat{\mathbf{e}}_1 \\ {}^E \mathbf{a}^T = \phi_{FA}(h) \cdot \dot{u}_1 \cdot \hat{\mathbf{e}}_1 \\ {}^E \mathbf{v}^R = u_1 \cdot \hat{\mathbf{e}}_1 - CG_R \cdot R \cdot u_1 \cdot \hat{\mathbf{e}}_3 \\ {}^E \mathbf{a}^R = \dot{u}_1 \cdot \hat{\mathbf{e}}_1 - CG_R \cdot R \cdot \dot{u}_1 \cdot \hat{\mathbf{e}}_3 \\ {}^E \boldsymbol{\omega}^R = u_2 \cdot \hat{\mathbf{e}}_1 - R \cdot u_2 \cdot q_{FA} \cdot \hat{\mathbf{e}}_3 \\ {}^E \boldsymbol{\alpha}^R = \dot{u}_2 \cdot \hat{\mathbf{e}}_1 - R \cdot [\dot{u}_2 \cdot q_{FA} + u_2 \cdot \dot{q}_{FA}] \cdot \hat{\mathbf{e}}_3 \end{array} \right. \longrightarrow \left\{ \begin{array}{l} {}^E \mathbf{v}_1^T = \phi_{FA}(h) \cdot \hat{\mathbf{e}}_1 \\ {}^E \mathbf{v}_2^T = \mathbf{0} \\ {}^E \mathbf{v}_1^R = \hat{\mathbf{e}}_1 - CG_R \cdot R \cdot \hat{\mathbf{e}}_3 \\ {}^E \mathbf{v}_2^R = \mathbf{0} \\ {}^E \boldsymbol{\omega}_1^R = \mathbf{0} \\ {}^E \boldsymbol{\omega}_2^R = \hat{\mathbf{e}}_1 - R \cdot q_{FA} \cdot \hat{\mathbf{e}}_3 \end{array} \right. \quad (\text{C.37})$$

Having the generalised degrees of freedom and partial velocities, the generalised active and inertia forces must be calculated. The generalised inertia forces for each degree of freedom can be written as:

$$F_r^* = F_r^* \Big|_{Tower} + F_r^* \Big|_{Rotor} \quad (\text{C.38})$$

While the rotor will be treated as a rigid body, and so the generalised inertia forces are com-

puted directly from Equation C.25, the tower forces must be integrated over its height considering explicitly its mass distribution $\mu(h)$:

$$F_r^* \Big|_{Tower} = - \int_0^H \mu(h) \cdot {}^E \mathbf{v}_r^T(h) \cdot {}^E \mathbf{a}^T dh \quad (C.39a)$$

$$F_r^* \Big|_{Rotor} = -m_R \cdot {}^E \mathbf{v}_r^R \cdot {}^E \mathbf{a}^R - {}^E \boldsymbol{\omega}_r^R \cdot \left[\mathbf{I}_R \cdot {}^E \boldsymbol{\alpha}^R + {}^E \boldsymbol{\omega}_r^R \times (\mathbf{I}_R \cdot {}^E \boldsymbol{\omega}_r^R) \right] \quad (C.39b)$$

And, substituting the relevant values:

$$\left\{ \begin{array}{l} F_1^* \Big|_{Tower} = -\dot{u}_1 \cdot \int_0^H \mu(h) \cdot \phi_{FA}^2 \cdot dh = -M_1 \cdot \dot{u}_1 \\ F_2^* \Big|_{Tower} = 0 \\ F_1^* \Big|_{Rotor} = -m_R \cdot \dot{u}_1 \cdot \left(1 + CG_R^2 \cdot R^2 \right) = -M_2 \cdot \dot{u}_1 \\ F_2^* \Big|_{Rotor} = -I_R \cdot \dot{u}_2 \end{array} \right. \longrightarrow \begin{array}{l} F_1^* = -(M_1 + M_2) \cdot \dot{u}_1 \\ F_2^* = -I_R \cdot \dot{u}_2 \end{array} \quad (C.40)$$

Until now, the generalised inertia forces do not depend on the actions over the structure and can be calculated from its geometry only. The generalised active forces can be divided as:

$$F_r = F_r \Big|_{Aero} + F_r \Big|_{Elastic} + F_r \Big|_{Gravity} + F_r \Big|_{Generator} \quad (C.41)$$

If a real wind turbine is to be modelled, these forces may not be easily expressed in closed form, but for this simplified example, it will be enough to consider a constant aerodynamic torque T_A acting on the rotor and an almost constant thrust force F_A (the torque will be assumed to be oriented along the axis of rotation $\hat{\mathbf{a}}_1$ but the force will be purely horizontal). In this case, it follows:

$$F_r \Big|_{Aero} = T_A \cdot {}^A \boldsymbol{\omega}_r^R \cdot \hat{\mathbf{a}}_1 - {}^A \mathbf{v}_r^R \cdot F(t) \cdot \hat{\mathbf{e}}_1 \quad (C.42a)$$

$$F_r \Big|_{Elastic} = -\frac{\partial V^T}{\partial q_r} = -q_{FA} \int_0^H EI \cdot \left(\frac{d^2 \phi_{FA}}{dh^2} \right)^2 \cdot dh = -K_1 \cdot q_{FA} \quad (C.42b)$$

$$F_r \Big|_{Gravity} = \int_0^H \mu(h) \cdot {}^E \mathbf{v}_r^T(h) \cdot \mathbf{g} \cdot dh + {}^E \mathbf{v}_r^R \cdot m_R \cdot \mathbf{g} = -g \cdot m_R \cdot {}^E \mathbf{v}_r^R \cdot \hat{\mathbf{e}}_3 \quad (C.42c)$$

$$F_r \Big|_{Generator} = -T_G \cdot {}^A \boldsymbol{\omega}_r^R \cdot \hat{\mathbf{a}}_1 \quad (C.42d)$$

And, substituting the relevant values:

$$F_1 = -F(t) - K_1 \cdot q_{FA} + g \cdot m_R \cdot CG_R \cdot R \quad (C.43a)$$

$$F_2 = T_A - T_G \quad (C.43b)$$

From the derived expressions, Kane's equations of motion can be finally obtained that, due to the simplicity of the model used, are fully uncoupled between tower displacement and rotor rotation⁸:

⁸Note that this is generally not the case. If the yaw degree of freedom was considered, for instance, the equations would be coupled.

$$\dot{q}_{FA} = u_1 \quad (\text{C.44a})$$

$$\dot{q}_{\psi} = u_2 \quad (\text{C.44b})$$

$$\dot{u}_1 = \frac{-F(t) + K_1 \cdot q_{FA} + m_R \cdot g \cdot CG_R \cdot R}{M_1 + M_2} \quad (\text{C.44c})$$

$$\dot{u}_2 = \frac{T_A - T_G}{I_R} \quad (\text{C.44d})$$

A detailed derivation considering all the degrees of freedom identified is presented in (Jonkman, 2003) or (Wilson et al., 1999) and a floating wind turbine model where platform motions appear as additional degrees of freedom is detailed in (Chen et al., 2017). In any case, the equations obtained are ready to be solved by numerical means. There are, of course, different techniques to do it efficiently and, although they will not be detailed here, (Newman, 2013) is a good reference on this matter.

Example C.4 - To convenient illustrate the presented concepts with an example that is both realistic and simple, an adapted model of 5MW NREL wind turbine fully detailed in Section 4.4 will be used.

The tower is 90 m high and has the stiffness and mass distribution over height presented in Figure C.4. This tower supports a 240 ton nacelle and a 110 ton rotor (blades included) with the (adapted) properties presented in Table C.1, where the xx axis is the axis of rotation.

If a tower with these properties is divided in several segments, the modal configurations can easily be obtained. The results for the first 4 modes are:

$$\mathbf{f} = \begin{bmatrix} 0.797 \\ 3.873 \\ 10.025 \\ 19.204 \end{bmatrix} \text{ [Hz]}; \quad \Phi = \begin{bmatrix} 0.077 & -0.276 & 0.495 & -0.563 \\ 0.356 & -0.555 & -0.114 & 0.606 \\ 0.773 & 0.275 & -0.189 & -0.566 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (\text{C.45})$$

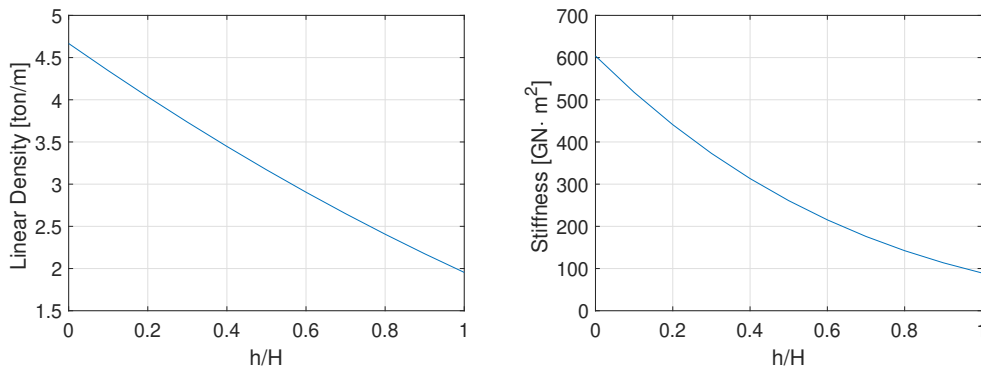


Figure C.4: Tower mass distribution (on the left) and stiffness (on the right).

Table C.1: Rotor and nacelle properties with respect to their principal axis.

Element	M_t [kg]	(x_g, y_g, z_g) [m]	I_{xx} [kg m ²]
Rotor	110000	(5, 0, 90)	22.33×10^6
Nacelle	240000	(-1.90, 0, 90)	-

where the nodal coordinate are only presented for the coordinates $z = [22.5, 49.5, 76.5, 90]$. Note that the top mass was not considered in these results. If it is taken into account, while still assuming for the sake of simplicity a nodal mass at the tower top without any eccentricity, then the natural frequencies and modal configurations are:

$$\mathbf{f} = \begin{bmatrix} 0.319 \\ 2.776 \\ 8.290 \\ 16.993 \end{bmatrix} \text{ [Hz]}; \quad \Phi = \begin{bmatrix} 0.065 & -1.858 & 6.716 & -11.746 \\ 0.319 & -5.161 & 1.463 & 12.633 \\ 0.746 & -2.766 & -8.909 & -15.198 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (\text{C.46})$$

Although there are significant differences for higher order modes, the fundamental mode is similar and, because of the frequency range of this type of structure, it will provide an acceptable first order approximation of the response. Using the modal configuration from Equation C.46, the modal mass and stiffness values can be computed and are presented in Table C.2. For instance, for a point load at the top of 1000 kN, the static equilibrium with the characteristic values from Table C.2 implies:

$$\mathbf{K}_m \mathbf{u}_m = \mathbf{f}_m \longrightarrow \mathbf{u}_m = \begin{bmatrix} 0.59934 \\ 0.00072 \\ 3 \times 10^{-5} \\ 3 \times 10^{-6} \end{bmatrix} \quad (\text{C.47})$$

The top displacement, because of the adopted mode shape normalisation, can be easily obtained as the components sum of $\mathbf{u}_m$. It can also be noted that the principal contribution comes from the first mode with an error of the order of 0.1%. From the modal configuration obtained, the constants previously defined as M_1 and K_1 can also be obtained from either numerical means or by fitting a polynomial function to the mode shape and then analytically integrating them. For

Table C.2: Modal Mass and Modal Stiffness associated with the first four modes, assuming the mode shape normalization presented in Equation C.46.

Mode	Mass [ton]	Stiffness [kN/m]
1	414.1	1668.5
2	4590.0	1.396×10^6
3	14024.5	3.805×10^7
4	30309.7	3.4555×10^8

instance, if a parabolic function is fitted, the expressions become:

$$M_1 = \int_0^H \mu(h) \cdot \phi_{FA}^2 \cdot dh = \int_0^H \mu(h) \cdot (a \cdot h^2 + b \cdot h + c)^2 \cdot dh \quad (C.48a)$$

$$K_1 = \int_0^H EI(h) \cdot \left(\frac{d^2 \phi_{FA}}{dh^2} \right)^2 \cdot dh = 4 \cdot a^2 \cdot \int_0^H EI(h) \cdot dh \quad (C.48b)$$

$$R = \left. \frac{d\phi_{FA}}{dh} \right|_{h=H} = 2 \cdot a \cdot H + b \quad (C.48c)$$

It can be noted that the expression for K_1 is indeed the modal stiffness k_k but the same cannot be applied, at least directly, to M_1 . However, it is true that:

$$M_1 + M_2 - m_R \cdot CG_R^2 \cdot R^2 = m_k \quad (C.49)$$

And hence, referring back to [Table C.2](#):

$$R = \left. \frac{d\phi_{FA}}{dh} \right|_{h=H} = 2 \cdot a \cdot H + b = 0.02166 \text{ [m}^{-1}\text{]} \quad (C.50a)$$

$$M_1 + M_2 = m_k + m_R \cdot CG_R^2 \cdot R^2 \approx m_k = 414.1 \text{ [ton] [kN/m]} \quad (C.50b)$$

$$K_1 = 1668.5 \text{ [kN/m]} \quad (C.50c)$$

where R has been estimated by a parabolic fit. Because only a point mass is being considered, it will be defined an equivalent position for the 350 [ton] total mass given by (see [Table C.1](#)):

$$CG_R = \frac{5 \cdot 110 - 240 \cdot 1.90}{110 + 240} = 0.269 \text{ [m]} \quad (C.51)$$

Having defined all the relevant quantities, Kane's equations of motion can now be solved for different operating conditions. For instance, if the rotor is parked and there is a constant wind force defined by a 300 [kN] step function at $t = 0$ [s], then the relevant equations are simply:

$$\dot{q}_{FA} = u_1 \quad (C.52a)$$

$$\dot{u}_1 = \frac{-300 - K_1 \cdot q_{FA} + m_R \cdot g \cdot CG_R \cdot R}{M_1 + M_2} \quad (C.52b)$$

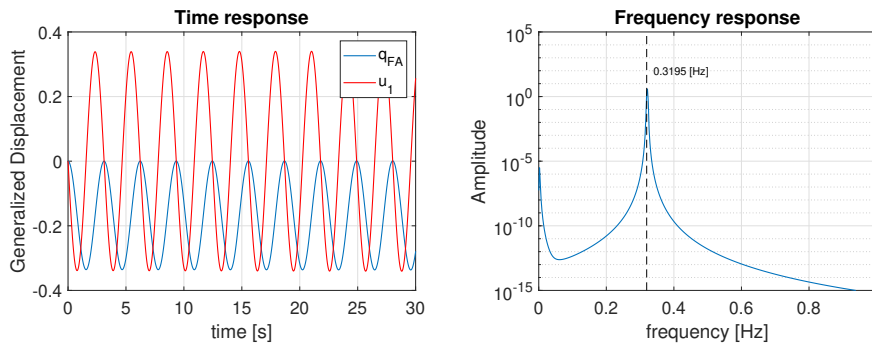


Figure C.5: Tower top displacement response and its derivative in both time (on the left) and tower top displacement response on frequency (on the right) domain.

that can be solved numerically to provide the time evolution represented in Figure C.5 in both time and frequency domain.

Taking into account a time varying thrust force and the aerodynamic torque, if the full equations of motion are to be solved, then an expression for the generator torque must be provided. In general, this is a function of the generator angular velocity (ω_G), which in turn is obtained from the rotor angular velocity and the gear box ratio (GB). In this case, it will be assumed it can be expressed as a parabolic function:

$$T_G = k_G \cdot \omega_G^2 = k_G \cdot (u_2 \cdot GB)^2 = k_G \cdot GB^2 \cdot u_2^2 = k_R \cdot u_2^2 \quad (C.53)$$

Substituting in the previously derived equations one obtains:

$$\dot{q}_{FA} = u_1 \quad (C.54a)$$

$$\dot{q}_{\psi} = u_2 \quad (C.54b)$$

$$\dot{u}_1 = \frac{-F(t) - K_1 \cdot q_{FA} + m_R \cdot g \cdot CG_R \cdot R}{M_1 + M_2} \quad (C.54c)$$

$$\dot{u}_2 = \frac{T_A - k_R \cdot u_2^2}{I_R} \quad (C.54d)$$

Once more, this can be solved numerically. The obtained results are plotted in Figure C.6, where it was assumed $k_R = 0.0029 \cdot 113^2 \approx 37$, $T_A = 1000$ [kN.m] and $F(t) = 300 + 20 \cdot [\cos(2\pi \cdot 0.10 \cdot t) + \varepsilon(t)]$, where ε is a random uniformly distributed number in the interval $[-1, 1]$. The frequency content of the tower response is represented in Figure C.7, where it is clear the additional local maximum for the frequency of 0.10 [Hz].

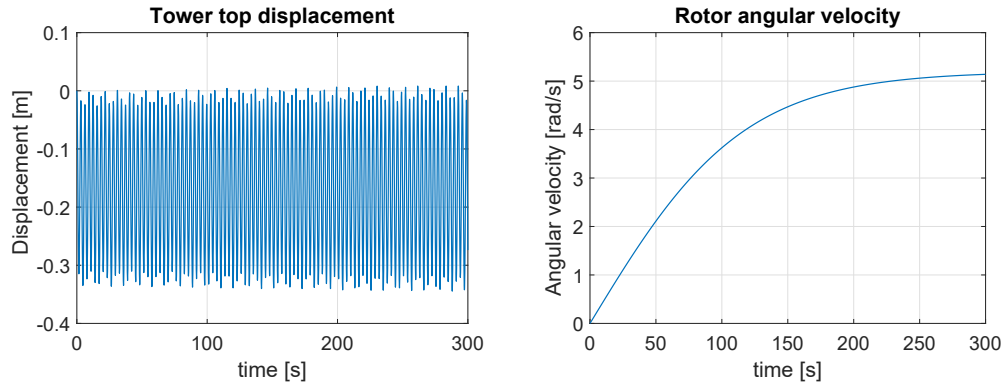


Figure C.6: Tower top displacement (on the left) and rotor angular velocity (on the right).

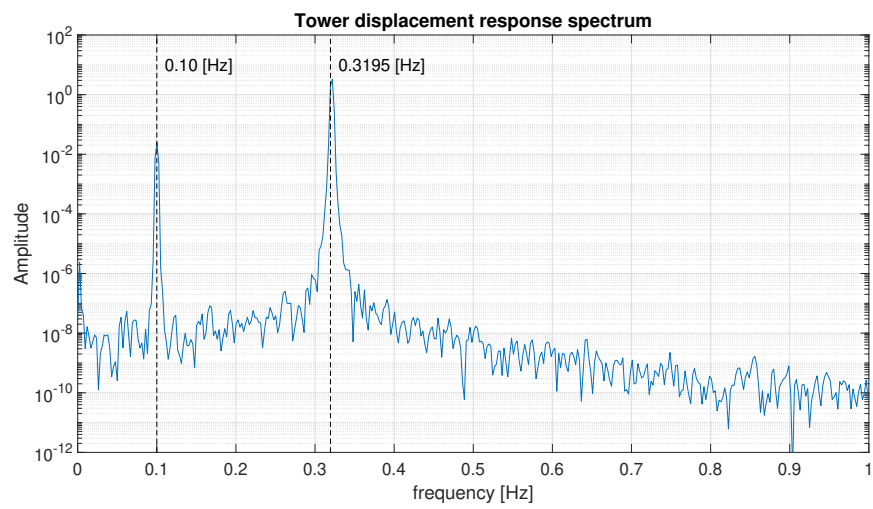


Figure C.7: Tower top displacement response spectrum for the unsteady simulation.
