

# Determination of Age and Mass for Seismic Stars in the *Ariel* Input Catalog

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Mestrado em Astronomia e Astrofísica

Departamento de Física e Astronomia

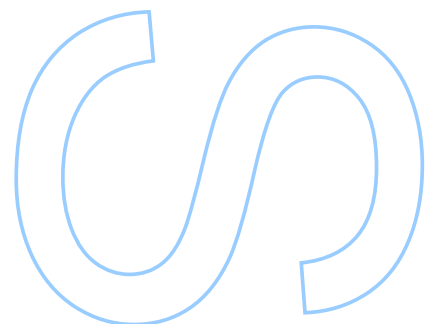
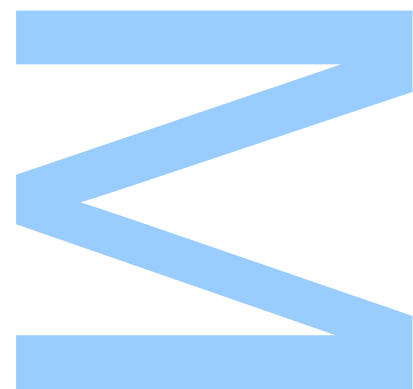
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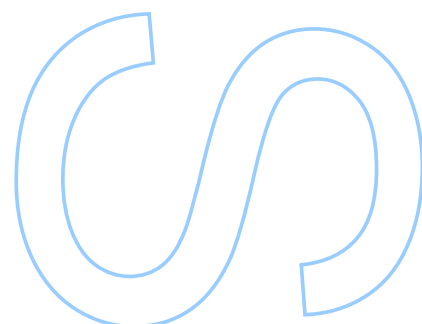
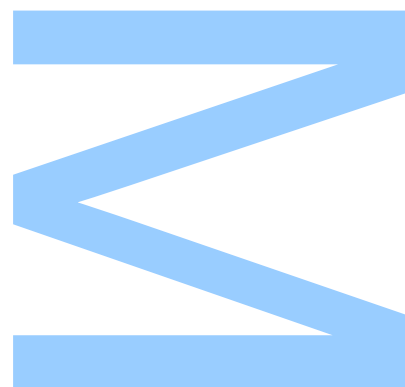




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Andreas Wilhelm de Barros da Silva Neitzel

30 de Setembro de 2022



UNIVERSIDADE DO PORTO

MASTERS THESIS

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# Determination of Age and Mass for Seismic Stars in the *Ariel* Input Catalog

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*“ If you want it, you do it. ”*

My father



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# *Abstract*

Faculdade de Ciências da Universidade do Porto

Departamento de Física e Astronomia

MSc. Astronomy & Astrophysics

## **Determination of Age and Mass for Seismic Stars in the *Ariel* Input Catalog**

by [Andreas Wilhelm de Barros da Silva NEITZEL](#)

Context: *Ariel* is a space telescope currently in development by the European Space Agency with the aim to study the chemical compositions in the atmospheres of known exoplanets. Accurate and precise planetary characterization is dependent on accurate and precise stellar characterization. Stellar characterization is in turn dependent on the homogeneous extraction of stellar data and on the imposed set of constraints. Asteroseismic data in the form of solar-like oscillations have been detected in various exoplanet-host stars within the *Ariel* input catalog, allowing to further constrain the age, mass and radius of the star.

Aims: The aim of this dissertation is to characterize main sequence F-G-K types stars in the *Ariel* input catalog that have been measured to have solar-like oscillations.

Methods: Stellar characterization was performed by comparing spectroscopic, photometric and seismic data of F-G-K type stars with a grid of computationally simulated stars. Codes were utilized to infer dust reddening, perform bolometric corrections on magnitudes, simulate grids of stellar models as well as stellar oscillations and finally perform isochrone-fitting with Bayesian methods. The Bayesian optimization code utilized, PARAM, was tested against its own input physics and against other stellar characterization codes to test its performance and to look for evidences of bias. The code was then ran on a set of 10 stars in the *Ariel* input catalog using different sets of constraints.

Results: This dissertation presents the results of PARAM self-consistency tests with a set of synthetic “cluster” stars and a set of stars taken along the same track. PARAM, using global seismic parameters as constraints, is tested against four other stellar characterization codes using individual frequencies as constraints to provide the ages and masses of 33 seismic stars. From the *Ariel* input catalog, 10 F-G-K seismic stars had their luminosities determined and had their ages and masses determined in PARAM via sixteen different sets of constraints, showing the different effects of the different constraints. These 10 stars later had their input uncertainties adjusted to match values common in the literature and tests were redone.

Conclusions: Internal consistency tests show that the relative errors of PARAM’s outputs are generally kept within their relative uncertainty values. Consistent with the literature, stellar age determination remains less precise than the determination of the mass, with PARAM showing less precision in the age estimation of stars with initial masses  $M_{\text{ini}} < 0.8 M_{\odot}$ , since even small variations in the input parameters lead to large uncertainties in the age determination. For the set of 33 seismic stars, comparisons between PARAM and other codes show agreement of precision values typically found in the literature. For the 10 *Ariel* input catalog stars, the inclusion of seismic data in the constraints is shown to increase general precision in the determination of age and mass when compared to constraints that do not make use of seismology. It is also shown that, when the input uncertainties are comparable to typical values in the literature, global seismic constraints fare better than luminosity alone in constraining the age. In addition, it is shown that the inclusion of both the seismic constraints and luminosity together constraint both the ages and masses even further. Stars with initial masses  $M_{\text{ini}} > 1.2 M_{\odot}$  are more prone to double solutions due to the emergence of a hook feature in the HR diagram, giving rise to degeneracies.

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## *Resumo*

Faculdade de Ciências da Universidade do Porto

Departamento de Física e Astronomia

Mestrado em Astronomia e Astrofísica

**Determination of Age and Mass for Seismic Stars in the *Ariel* Input Catalog.  
Determinação da Idade e da Massa de Estrelas Sísmicas no Catálogo de Entrada *Ariel***

por [Andreas Wilhelm de Barros da Silva NEITZEL](#)

**Contexto:** A Agência Espacial Europeia está a desenvolver o satélite espacial *Ariel* que tem como objetivo o estudo da composição química das atmosferas de exoplanetas. Uma avaliação precisa e exata das características destes exoplanetas está dependente de uma avaliação precisa e exata das estrelas que os albergam. A caracterização estelar está, por sua vez, dependente de uma extração homogênea dos dados estelares e também das constrições utilizadas. Dados asterossísmicos na forma de oscilações do tipo solar foram detetados em diversas estrelas que constituem o catálogo de entrada *Ariel*, permitindo uma constrição mais aprofundada da idade, da massa e do raio da estelar.

**Objetivos:** O objetivo desta dissertação é a caracterização de estrelas de sequência principal do tipo F-G-K do catálogo de entrada *Ariel* das quais foram medidas oscilações do tipo solar.

**Métodos:** A caracterização estelar foi feita através da comparação de dados espectroscópicos, fotométricos e sísmicos de estrelas do tipo F-G-K com uma grelha de estrelas simuladas por computador. Programas computacionais foram utilizados para inferir o avermelhamento causado pela poeira interestelar, realizar correções bolométricas nas magnitudes, simular grelhas de modelos estelares, assim como dados de oscilações estelares, e realizar ajuste de isócronos através de métodos de Bayes. No âmbito da pesquisa de tendências no comportamento e testagem do desempenho do código de optimização Bayesiano, PARAM, efetuaram-se comparações entre a física de referência deste e as estrelas sintéticas

provenientes dessa física, assim como comparações entre o PARAM e outros programas de caracterização estelar. O código foi, de seguida, executado num conjunto de 10 estrelas pertencentes ao catálogo de entrada *Ariel* usando diferentes constrições. Estas 10 estrelas tiveram de seguida as suas incertezas ajustadas de forma a condizer com valores tipicamente encontrados na literatura e testes foram refeitos.

**Resultados:** Nesta dissertação apresentam-se os resultados dos testes de auto-consistência do PARAM aplicados a um conjunto de estrelas sintéticas de um “aglomerado” estelar e a outro conjunto de estrelas com a mesma configuração inicial em diferentes etapas de vida. Dados de 33 estrelas associados a quatro outros programas de caracterização estelar foram comparados aos resultados no PARAM. Neste exercício o PARAM usou parâmetros sísmicos globais enquanto que os outros programas tiveram acesso a modos individuais. Do catálogo de entrada *Ariel* foram selecionadas 10 estrelas sísmicas do tipo F-G-K cujas luminosidades foram determinadas e, conseqüentemente, dezasseis execuções com constrições diferentes foram realizadas no PARAM para determinar os efeitos que diferentes constrições têm na determinação da idade e da massa.

**Conclusões:** Testes de autoconsistência demonstram que os erros relativos do PARAM são geralmente abrangidos pelas incertezas relativas. A determinação da idade estelar permanece mais difícil do que a determinação da massa, resultado que é consistente com a literatura. Para estrelas com massas iniciais de  $M_{\text{ini}} < 0.8 M_{\odot}$  o PARAM obtém resultados de menor precisão na estimação da idade. Este resultado deve-se ao facto de haver grandes incertezas na determinação da idade mesmo quando as variações dos parâmetros de entrada são pequenas. Para o conjunto das 33 estrelas sísmicas, comparações entre o PARAM e outros programas demonstram uma concordância com precisões tipicamente encontrados na literatura. Para o conjunto das 10 estrelas do catálogo de input *Ariel*, a inclusão de dados sísmicos nas constrições leva a um aumento da precisão na determinação da idade e da massa das estrelas em comparação com constrições que não incluem sísmologia. Quando as incertezas iniciais são comparáveis com valores típicos na literatura, a inclusão de parâmetros sísmicos globais são mais eficientes a apertar a precisão da idade do que a inclusão de luminosidade. Demonstra-se também que a inclusão conjunta dos parâmetros sísmicos globais com a luminosidade levam a uma maior precisão na estimativa das idades e das massas. Estrelas com massas iniciais  $M_{\text{ini}} > 1.2 M_{\odot}$  têm uma maior probabilidade de demonstrarem soluções duplas devido à emergência

de uma característica em forma de gancho no diagrama HR, o que leva ao surgimento de degenerações.



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# Symbols

|                  |                                     |                                                   |
|------------------|-------------------------------------|---------------------------------------------------|
| $\odot$          | Relative to the sun                 |                                                   |
| $\sigma(\cdot)$  | Uncertainty of $\cdot$              |                                                   |
| $M$              | Mass                                | $M_{\odot}$                                       |
| $R$              | Radius                              | $R_{\odot}$                                       |
| $L$              | Luminosity                          | $L_{\odot}$                                       |
| $T_{\text{eff}}$ | Effective temperature               | K                                                 |
| $m$              | Mass                                | g                                                 |
| $r$              | Radial coordinate                   | cm                                                |
| $g$              | Surface gravity                     | $\text{cm}\cdot\text{s}^{-2}$                     |
| $P$              | Pressure                            | Ba                                                |
| $\rho$           | Density                             | $\text{g}\cdot\text{cm}^{-3}$                     |
| $c$              | Speed of light                      | $\text{cm}\cdot\text{s}^{-1}$                     |
| $G$              | Gravitational constant              | $\text{cm}^3\cdot\text{g}^{-1}\cdot\text{s}^{-2}$ |
| $k$              | Opacity                             | $\text{cm}^2\cdot\text{g}^{-1}$                   |
| $\epsilon$       | Power per unit mass                 | $\text{erg}\cdot\text{s}^{-1}\cdot\text{g}^{-1}$  |
| $X$              | Hydrogen mass fraction              |                                                   |
| $Y$              | Helium mass fraction                |                                                   |
| $Z$              | Mass fraction of all other elements |                                                   |
| $[M/H]$          | Overall metallicity                 | dex                                               |
| $[Fe/H]$         | Iron abundance                      | dex                                               |
| $\mu$            | Mean molecular weight               | g                                                 |

---

|                       |                                    |                                                   |
|-----------------------|------------------------------------|---------------------------------------------------|
| $\nabla$              | Temperature gradient               |                                                   |
| $\nabla_{\text{ad}}$  | Adiabatic temperature gradient     |                                                   |
| $\nabla_{\text{rad}}$ | Radiative temperature gradient     |                                                   |
| $\nabla_{\mu}$        | Chemical gradient                  |                                                   |
| $H_p$                 | Pressure scale height              | cm                                                |
| $N$                   | Brunt-Väisälä frequency            | Hz                                                |
| $\nu$                 | Frequency                          | $\mu\text{Hz}$                                    |
| $\nu_{\text{max}}$    | Frequency of maximum power         | $\mu\text{Hz}$                                    |
| $\Delta\nu$           | Large frequency separation         | $\mu\text{Hz}$                                    |
| $\delta\nu$           | Small frequency separation         | $\mu\text{Hz}$                                    |
| $n$                   | Spherical harmonic radial order    |                                                   |
| $l$                   | Spherical harmonic angular order   |                                                   |
| $m$                   | Spherical harmonic azimuthal order |                                                   |
| $\nu_{n,l,m}$         | Spherical harmonic frequency mode  | $\mu\text{Hz}$                                    |
| $d$                   | Distance                           | pc                                                |
| $p$                   | Parallax                           | mas                                               |
| $F$                   | Flux                               | $\text{erg}\cdot\text{s}^{-1}\cdot\text{cm}^{-2}$ |
| $m$                   | Apparent magnitude                 | mag                                               |
| $M$                   | Absolute magnitude                 | mag                                               |
| $A$                   | Extinction                         | mag                                               |
| $E_{B-V}$             | Reddening                          | mag                                               |
| $\lambda$             | Wavelength                         | cm                                                |

# Glossary

|                   |                              |
|-------------------|------------------------------|
| <b>HR Diagram</b> | Hertzsprung-Russell diagram  |
| <b>MS</b>         | main sequence                |
| <b>ZAMS</b>       | zero-age main sequence       |
| <b>SG</b>         | subgiant phase               |
| <b>RGB</b>        | red-giant branch             |
| <b>pp chain</b>   | proton-proton chain          |
| <b>CNO cycle</b>  | carbon-nitrogen-oxygen cycle |
| <b>PSD</b>        | power spectral density       |
| <b>p-mode</b>     | pressure mode                |
| <b>g-mode</b>     | gravity mode                 |



# Chapter 1

## Introduction

In 2014, the European Space Agency (ESA) set out a call for a Medium-size opportunity to be implemented in the framework of its Cosmic Vision plan. Of the 27 proposals submitted, the chosen one was the Atmospheric Remote-sensing Infrared Exoplanet Large-survey, endearingly called *Ariel* (ARIEL, previously). Set to launch in 2029 on-board the Ariane 6.2 launch vehicle in Kourou, French Guiana, *Ariel* is a space telescope whose mission is the study of chemical compositions in the atmospheres of other worlds beyond our solar system. Its survey shall encompass roughly 1000 known transiting exoplanets, from gas giants to telluric planets, in the hot to temperate zones of main sequence stars of spectral types ranging from A to M. Such a survey shall be conducted with transit, eclipse and phase-curve spectroscopy in the 0.5-7.8  $\mu\text{m}$  range, divided over three photometric bands and three spectroscopic bands (see Table 1.1). The intent is to deliver a homogeneous catalogue of planetary spectra with refined molecular abundances, chemical gradients and atmospheric structures, among other properties. The space telescope shall reside in the second Lagrangian point (L2) where it will live out a 4 year nominal lifetime with the goal of being able to extend another 2 years. For a more detailed explanation of *Ariel*, the *Ariel* Red Book can be consulted online<sup>1</sup>.

The *Ariel* mission is under the supervision of the *Ariel* Consortium, an international team of scientists, experts and collaborators subdivided into various working groups, each with their own purposes and responsibilities. One of these is the Stellar Characterization working group and within it lies the Age/Mass/Radius sub-workgroup, of which my supervisors and myself are a part of<sup>2</sup>. As the name implies, this sub-workgroup focus

---

<sup>1</sup><https://www.cosmos.esa.int/web/ariel>

<sup>2</sup>The *Ariel* Age/Mass/Radius sub-workgroup is led by my co-supervisor, Tiago Campante.

| Instrument | Wavelength range<br>[ $\mu\text{m}$ ] | Resolving power<br>$\frac{\lambda}{\Delta\lambda}$ | Type         |
|------------|---------------------------------------|----------------------------------------------------|--------------|
| VISPhot    | 0.50-0.60                             | Integrated band                                    | Photometry   |
| FGS1       | 0.60-0.80                             | Integrated band                                    | Photometry   |
| FGS2       | 0.80-1.10                             | Integrated band                                    | Photometry   |
| NIRSpec    | 1.10-1.95                             | $R \geq 15$                                        | Spectroscopy |
| AIRS       | 1.95-3.9                              | $R \geq 100$                                       | Spectroscopy |
| AIRS       | 3.9-7.8                               | $R \geq 30$                                        | Spectroscopy |

TABLE 1.1: *Ariel* photometric and spectroscopic coverage.

on the determination of the ages, masses and radii of the stars which host the exoplanets of interest. Before we can ask how such a study is performed we must first ask why it is even necessary.

The *Ariel* mission, whilst specifically tasked to the study of exoplanets, would fail without comprehensive knowledge of the very same stars to which these exoplanets belong. This is because planetary characterization cannot be divorced from stellar characterization (Adibekyan, Sousa & Santos, 2018). Different stars possess different physical characteristics, differing from one another in mass, composition and evolutionary phase. Such properties affect the behavior of the star and with it the kinds of synergies that take place between planet and star. Whereas the Sun is calm and forgiving to our home-planet Earth, other stars may abuse their planetary system by belching flares, coronal mass ejections (CMEs) and winds of ionized particles, all of which directly affect the study of exoplanetary atmospheres. Such phenomena are determined by stellar magnetic activity, however some studies suggest that the magnetosphere of exoplanets may also play a role (McIvor, Jardine & Holzwarth, 2006; Lanza, 2008; Cohen et al., 2011a,b). Precise measurement of atmospheric signals and their chemical characterizations require exact knowledge of the spectrum of the host star, including its spectral variations due to stellar activity, otherwise these can lead to erroneous interpretations of the observations, such as confusion with planetary features (Robertson, Roy & Mahadevan, 2015). These challenges, among others, justify the need for a reliable stellar characterization, preferentially in a homogeneous manner. As detailed in Magrini et al. (2022), inhomogeneous methods lead to inconsistencies in the stellar parameters that hinder statistical studies, whereas with homogeneous methods we can expect to obtain a more robust statistical study with greater precision.

[Magrini et al. \(2022\)](#) also derived homogeneous stellar parameters for 187 main sequence F-G-K type stars in the *Ariel* input catalog, of which 5 are also studied in this dissertation, thereby emphasizing the relevance of this paper.

The work of [Edwards et al. \(2019\)](#) states that at the time of its writing, approximately 4000 exoplanets had been discovered, 3000 transited their stars and from these roughly 2000 had been identified as candidates whose atmospheres could potentially be characterized by *Ariel*. From these, an example Mission Reference Sample (MRS) of 1000 diverse exoplanets could be selected. Since then this work has been updated in [Edwards & Tinetti \(2022\)](#) to include exoplanets observed by the Transiting Exoplanet Survey Satellite (TESS, [Ricker et al., 2015](#)), a NASA space telescope capable of resolving seismic oscillations. This means extra stellar information which in turn means a greater scientific opportunity for the study and characterization of such stars. This is the field of asteroseismology, which delves into the topics of stellar oscillation. Other space telescopes such as CoRoT (CNES+ESA, [Baglin et al., 2006](#)) and *Kepler* (NASA, [Borucki et al., 2010](#)) have also provided valuable seismic data on a range of exoplanet-harboring stars. There is therefore at present a great opportunity to enhance our capabilities for stellar characterization which can in turn enhance our comprehension of exoplanets.

As of the writing of this dissertation, 561 exoplanets orbiting 502 host stars have been validated in the *Ariel* input catalog. Although *Ariel* is expected to handle exoplanets orbiting main sequence A-F-G-K-M stars, those that present detectable solar-like oscillations fall within the F-G-K spectral types. As a reference point, the Sun is classified as a G2V type star, meaning that it is a main-sequence star with a surface temperature around 5800 K. For the purpose of this dissertation we shall pay particular attention to the main sequence F-G-K spectral type stars within the *Ariel* input catalog. The usage of asteroseismology as an additional constraint has been done in the past in papers such as, among others, [Stello et al. \(2009\)](#), [Reese et al. \(2016\)](#), [Rodrigues et al. \(2017\)](#), [Cunha et al. \(2021\)](#).

Having determined the importance of stellar characterization, the next step is naturally the process by which one characterizes stars. Effective stellar characterization requires a robust understanding of stellar physics, comparisons of empirical observations with theoretical modelling, as well as the aid of computational resources. This is done by first

defining the parameter range of interest and then collecting spectroscopic and photometric data within such constraints. Secondly, theoretical models are constructed from current understanding of physics, for example in the form of models created by computer programs. Thirdly, compare empirical results with theoretical assumption. Fourthly, employ human analysis to determine whether or not these results make sense. More in-depth details are provided in the chapters to come.

Chapter 2 provides an overview of the key aspects of the structure and stellar evolution of F-G-K stars in the main sequence and subgiant phase phases, as well as introduce the reader to the stellar modelling software *Modules for Experiments in Stellar Astrophysics* (MESA, [Paxton et al., 2011](#)). Chapter 3 contains a non-exhaustive reminder of the theory of stellar oscillations, with a focus on solar-like oscillations. The oscillation modelling software GYRE ([Townsend & Teitler, 2013](#)) is also introduced. Chapter 4 is dedicated to the description of the stellar observed data used in this work and the methods used to collect them. Having gone through an explanation of the theory, tools and obstacles faced, Chapter 5 introduces the use of PARAM ([da Silva et al., 2006](#); [Rodrigues et al., 2014](#); [Rodrigues et al., 2017](#)), an in-house Bayesian optimization tool that performs the final step in stellar characterization. Results are provided in Chapter 6 and finally in Chapter 7 we have the summary and conclusions of this work.

## Chapter 2

# Stellar Structure and Evolution of F-G-K stars

This chapter provides a brief overview on the structure and evolution of stars with initial masses ranging between  $M_{\text{ini}} \in [0.5, 1.8]$ , starting from the main sequence (MS) and ending just above the luminosity bump on the red-giant branch (RGB). F-G-K stars have their spectral types defined indicatively by the values in Table 2.1. These spectral-type stars were chosen because, as we shall see, they are the only main sequence stars for which solar-like oscillations have been detected. The content of this chapter is by far not meant to be an exhaustive text. I refer the interested reader to the book *Stellar Structure and Evolution* by [Kippenhahn & Weigert \(1990\)](#), which will serve as the reference point and authoritative text for the details on the subject matter.

| Spectral Type | Surface Temperature<br>[K] |
|---------------|----------------------------|
| F             | 6000 - 7500                |
| G             | 5000 - 6000                |
| K             | 4000 - 5000                |

TABLE 2.1: Spectral classification of F-G-K stars according to the *Ariel* Red Book.

## 2.1 Equations of Structure

Stars are conceived through the gravitational collapse of gigantic gas clouds, gradually collecting matter into its center and heating up due to gravitational contraction until sufficiently high temperatures ignite the very first nuclear reactions at the core. Hydrogen, the most abundant of elements and the easiest one to fuse together, is burnt for the first time. When nuclear fusion has replaced gravitational collapse as the dominant form of energy and once both hydrostatic and thermal equilibrium have been reached, we say that a star was born. This marks the entry of the star into the main sequence, where it will calmly live out the majority of its life. The very beginning of this phase is called the zero-age main sequence (ZAMS).

The physics of the stars are governed by a set of fundamental equations that regiment the characteristics of its structure and how this evolves over time. These are Equations 2.1 to 2.6, taken from page 64 of [Kippenhahn & Weigert \(1990\)](#). Such equations assume spherical symmetry (which in turn imply they assume non-rotation). These also neglect the variability of stellar mass due to stellar winds, which would require extra considerations. Despite such simplifications, the following equations serve as the backbone for our understanding of stellar interiors and can still be relied on. First, we have the equation ensuring the conservation of mass within the star,

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}, \quad (2.1)$$

where  $r$  stands for the radial coordinate,  $m$  for the mass, and  $\rho$  for the density. Second, the equation of hydrostatic equilibrium which ensures the balances of forces between pressure,  $P$  and gravity is

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}, \quad (2.2)$$

where  $G$  is the gravitational constant and  $t$  is time. Third, the equation for energy conservation,

$$\frac{\partial L}{\partial m} = \epsilon_n - \epsilon_\nu + \epsilon_g, \quad (2.3)$$

which dictates that the expense of energy emanating from the star in the form of luminosity is dependent on the energetic phenomena occurring within it. In this case, there are three energies taken into consideration: the nuclear reactions  $\epsilon_n$ , the neutrino emission  $\epsilon_\nu$

and the exhaustion of energy caused by gravitational expansion,  $\epsilon_g$ . Fourth, we have an equation pertaining to the transport of energy within the star,

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla, \quad (2.4)$$

where the parameter  $\nabla$  stands for the temperature gradient and is defined as

$$\nabla \equiv \frac{d \ln T}{d \ln P}. \quad (2.5)$$

There are three main ways by which energy can be transported: conduction, convection and radiation. For the most part the latter two are the most efficient in the context of stars. Conduction, the energy transport mechanism pertaining to the kinetic collisions of particles, is relevant for the study of degenerate cores, but we neglect this in the following text. The parameter  $\nabla$  shall be a recurring point of interest since we can use it to determine which form of energy transport is dominant. This shall be explained more in-depth in the subsequent sections. Lastly, we have the equation for the change of mass fraction over time,

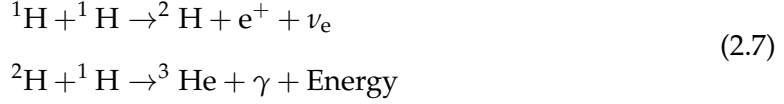
$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \left[ \sum_j r_{ji} - \sum_k r_{ik} \right], \quad (2.6)$$

where  $X_i$  is the mass fraction of some element  $i$  and  $r_{ab}$ , not to be confused with the radial coordinate, stands for the reaction rate where an element  $a$  is destroyed to create the element  $b$ .

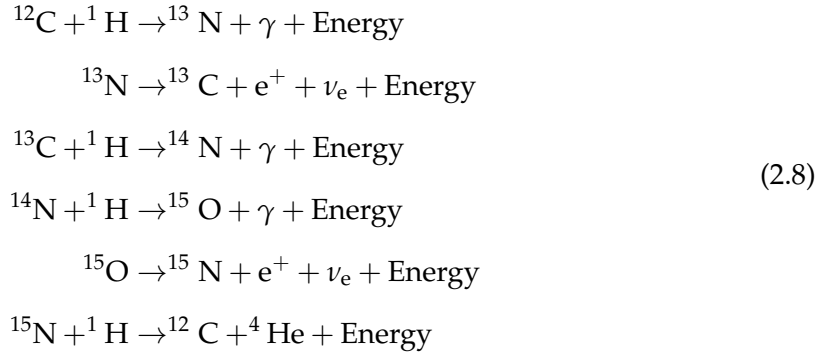
## 2.2 Stellar Nuclear Reactions

The main sequence is defined as a phase of hydrostatic and thermal equilibrium in which stellar energy production is dominated by the fusion of hydrogen at the core, leading to the production of helium. This fusion, often colloquially referred to as “burning”, is mostly commonly performed via two separate channels. The first of these channels, most common for stars masses  $M \lesssim 1.2 M_\odot$ , is the Proton-Proton chain (pp chain), given in Equation 2.7. Two hydrogen atoms  $^1\text{H}$  fuse together through the weak nuclear force, transforming one proton into a neutron to form the deuterium isotope  $^2\text{H}$  (conserving baryon number), releasing a positron  $e^+$  (conserving charge) and an electron neutrino  $\nu_e$  (conserving lepton number). This deuterium isotope fuses together with another hydrogen atom in a process mediated by the strong nuclear force, forming a helium-3 isotope

and releasing energy in the form of a gamma ray  $\gamma$  plus an extra expense of energy. The pp chain is triggered when temperatures reach the order of  $T \gtrsim 10^6$  K and it is the dominant form of energy in the case of our Sun.



If temperatures are sufficiently high, however, of  $T \gtrsim 1.5 \times 10^7$  K, then the carbon-nitrogen-oxygen cycle (CNO cycle) takes over as the dominant form of energy production. Due to the temperatures demanded, such a process becomes relevant only in stars of  $M \gtrsim 1.2 M_\odot$ . The CNO cycle, given in Equation 2.8, is a catalytic process more convoluted than the pp chain. It is called a catalytic process because, although there is fusion of carbon-12 into higher elements, the transformation is merely a temporary intermediate step which reverts back to the original element. What is actually spent is the hydrogen which is converted into helium, gamma rays and other forms of energy throughout the cycle.



In general only the core can sustain temperatures high enough to produce nuclear fusion and the manner in which this fusion takes place shapes the core and by extension the rest of the star. Nuclear fusion requires more than just sufficiently high temperatures, it also requires the existence of fuel. Simply put, if there is no hydrogen to burn then hydrogen is not burnt, no matter how high the temperatures. If the core exhausts itself of hydrogen but the star has not achieved temperatures high enough to ignite helium then the only manner in which it can survive is by burning hydrogen in a shell around the inert core.

## 2.3 The Stellar Life-Cycle

For F-G-K stars the main sequence lasts anywhere from a few giga-years to various dozen giga-years, for more massive to less massive stars respectively. The entire life-cycle of a F-type star (6000-7500 K) from birth to death is but a fraction of the time that a K-type star (4000-5000 K) will spend on its main sequence. In general the more massive a star is the hotter it will burn and therefore the faster it will consume its fuel, whereas the lower the mass the more calmly this fuel is spent. For K dwarf stars, the time necessary to expend its hydrogen fuel throughout its lifespan is predicted by theory to be greater than the age of the universe.

Slow evolution makes age determination a greater challenge, as a difference of various giga-years leads to barely perceptible changes in its structure. This can best be visualized in a Hertzsprung-Russell (HR) diagram. A HR diagram is a luminosity,  $L$ , to effective temperature diagram, where the luminosity is measured in terms of its logarithm relative to the Sun and the effective temperature,  $T_{\text{eff}}$ , is inverted out of convention.

A HR diagram is provided in Figure 2.1, which illustrates the stellar track of a star generated with initial mass  $M_{\text{ini}} = 1 M_{\odot}$  and initial metallicity  $[M/H]_{\text{ini}} = 0.0$ . The numbers help illustrate important points in the stellar life. The number 1 indicates the entry of the star into the main sequence, also called the zero-age main sequence. At this stage the star burns hydrogen in the core, progressing to the left (increase of effective temperature,  $T_{\text{eff}}$ ) and upwards (increase of luminosity,  $L$ ). When it exhausts all of the hydrogen fuel in the core it exits the main sequence (number 2), progressing almost horizontally to the right in what is called the subgiant phase (SG). In this phase hydrogen burns in a shell around the inert helium-rich core, causing it to contract and in turn for the star to expand in radius. At number 3 the hydrogen-burning shell thins out due to a lack of supply of hydrogen. Hydrostatic equilibrium demands that temperatures increase to meet up with energetic demands, forcing the core to contract and in turn for the outer shells to greatly expand. Surface temperature continues to plummet but the increase in radius causes a dramatic increase in luminosity. This is the transition from the subgiant phase into the red-giant branch. At number 4, still in the red-giant branch, we arrive at a the RGB bump, a sudden bump in luminosity that will be explained in the upcoming sections. At the top of the plot

the RGB is truncated at a value around  $\log L \approx 2 \log L_{\odot}$ , marking the end of the red-giant branch. Beyond this point the fate of the star depends widely on its mass and metallicity.

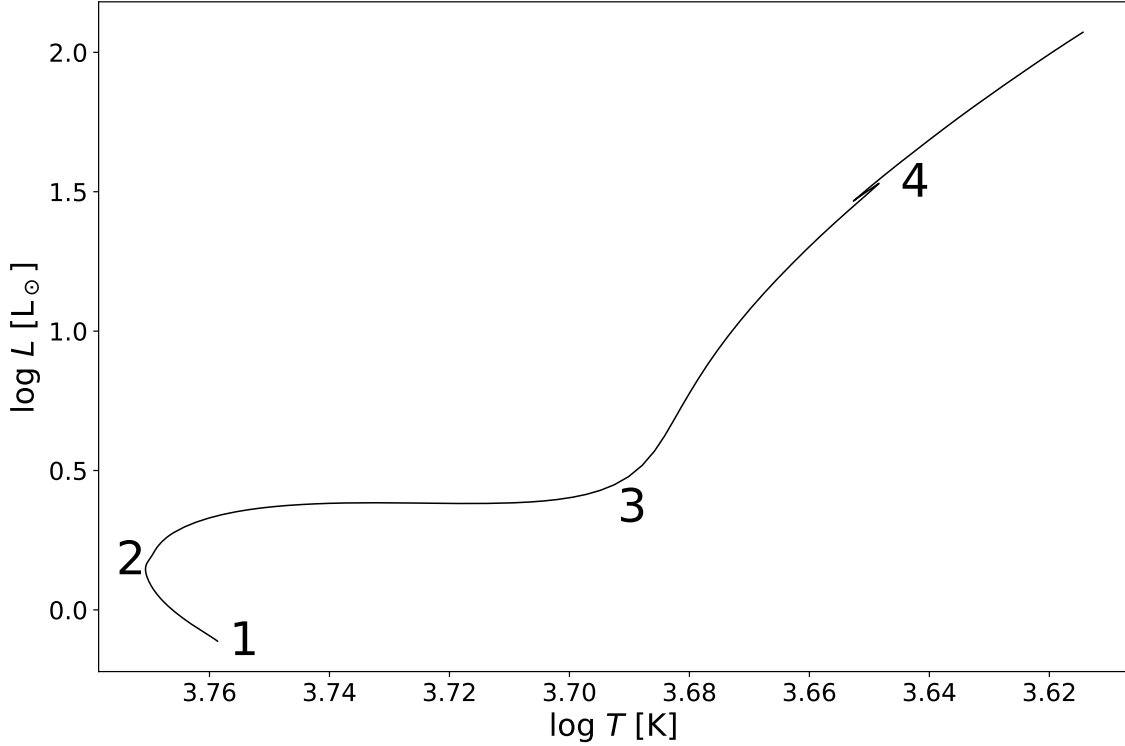


FIGURE 2.1: HR diagram of a MESA-generated star.  $M_{\text{ini}} = 1 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$ . (1) ZAMS (2) End of MS, beginning of SG (3) End of SG and beginning of the RGB (4) RGB Bump.

If we were to parametrize the stellar track curve with time, we would see that the vast majority of the stellar life-cycle is spent on the main sequence (as we shall see later in Figure 2.8). The main sequence occupies a relatively small portion of the HR diagram. As a result, even with small variations in the measurement of luminosity and effective temperature we may still obtain great uncertainties in the determination of age in the main sequence, whereas we can be fairly confident that the same uncertainties in the SG and RGB occur within a tighter timeframe. The lower the stellar mass the longer the star spends in the main sequence and therefore the greater the associated uncertainty.

## 2.4 Transport of Energy

In Section 2.1 we defined in Equation 2.5 the temperature gradient. It was mentioned at the time that conduction is an inefficient mechanism for the transport of energy in stellar

interiors, leaving only radiative and convective transport as viable mechanisms. Typically one of these types of transports dominates over the other, but in order to begin to ascertain this we must first define two types of temperature gradients: the adiabatic gradient

$$\nabla_{\text{ad}} \equiv \left( \frac{d \ln T}{d \ln P} \right)_{\text{ad}} , \quad (2.9)$$

and the radiative gradient

$$\nabla_{\text{rad}} \equiv \left( \frac{d \ln T}{d \ln P} \right)_{\text{rad}} . \quad (2.10)$$

The radiative gradient can also be written as (see page 32 of [Kippenhahn & Weigert 1990](#))

$$\nabla_{\text{rad}} = \frac{3}{16\pi c G} \frac{kLP}{mT^4} , \quad (2.11)$$

where  $a$  is the radiation density constant,  $k$  the opacity,  $c$  the speed of light,  $G$  is the gravitational constant,  $P$  the pressure,  $L$  is the luminosity and  $T$  the temperature. The adiabatic gradient stands for the rate of change in temperature in an adiabatic context for a given element of mass, whereas the radiative gradient stands for variation of temperature in regards to depth of the matter surrounding the element of mass, where the pressure  $P$  is a proxy for depth. What we are comparing in the end is whether a rising bubble of mass (presumed to be adiabatic) will cool down quickly enough relative to its surroundings to sink back down (radiative transport) or if it remains hot enough to rise without end (convective transport). Comparison between these two values gives us the Schwarzschild criterion for stability against convection,

$$\nabla_{\text{rad}} < \nabla_{\text{ad}} . \quad (2.12)$$

When this criterion holds then we have what we call stability against convection. No convection means radiative transport of energy. In the opposite case we have convective transport of energy. The Schwarzschild criterion is therefore key towards the understanding of the internal structure of stars. It is not, however, exhaustive. An equation of state  $\rho = \rho(P, T, \mu)$  depends on the mean molecular weight  $\mu$ , meaning that we ought to take into consideration the effects of chemical elements that have not been homogeneously

mixed in the surrounding material. A small variation of density  $d\rho$  leads to

$$\begin{aligned} d\rho &= \frac{\partial \rho}{\partial P} dP + \frac{\partial \rho}{\partial T} dT + \frac{\partial \rho}{\partial \mu} d\mu \\ &= \rho \left( \alpha \frac{dP}{P} - \delta \frac{dT}{T} + \phi \frac{d\mu}{\mu} \right), \end{aligned} \quad (2.13)$$

with

$$\alpha \equiv \left( \frac{\partial \ln \rho}{\partial \ln P} \right) \quad ; \quad \delta \equiv - \left( \frac{\partial \ln \rho}{\partial \ln T} \right) \quad ; \quad \phi \equiv \left( \frac{\partial \ln \rho}{\partial \ln \mu} \right). \quad (2.14)$$

The condition for stability against convection can be put in equation form. First, we enforce the assumption of pressure balance between the element and its surroundings,  $DP = 0$ . Secondly, we know that if the density of the element (subscript e) is lower than the density of its surroundings (subscript s) then it will rise and rise, giving convection. Stability is therefore represented by the relation

$$\left( \frac{d\rho}{dr} \right)_e - \left( \frac{d\rho}{dr} \right)_s > 0. \quad (2.15)$$

Implementing Equation 2.13 into Equation 2.15, taking into consideration  $DP = 0$ , the homogeneity of the chemical elements within the adiabatic mass element implies  $(D\mu)_e = 0$ . We can define the scale height of pressure as

$$H_P \equiv - \frac{dr}{d \ln P} \quad (2.16)$$

and use it to perform substitutions that will lead to the Ledoux criterion, expressed as

$$\nabla < \nabla_e + \frac{\phi}{\delta} \nabla_\mu, \quad (2.17)$$

with

$$\nabla \equiv \nabla_s \quad ; \quad \nabla_e \equiv \left( \frac{d \ln T}{d \ln P} \right)_e \quad ; \quad \nabla_\mu \equiv \left( \frac{d \ln \mu}{d \ln P} \right)_s. \quad (2.18)$$

If the transport of energy is radiative then  $\nabla = \nabla_{\text{rad}}$ . Furthermore if we presume that the element of mass changes adiabatically ( $\nabla_e = \nabla_{\text{ad}}$ ) then the Ledoux criterion in Equation 2.17 assumes the form:

$$\nabla_{\text{rad}} < \nabla_{\text{ad}} + \frac{\phi}{\delta} \nabla_\mu, \quad (2.19)$$

which degenerates into the Schwarzschild criterion if the region is chemically homogeneous ( $\nabla_\mu = 0$ ).

Figure 2.2 shows the Schwarzschild criterion in action for a MESA generated star of initial mass  $M_{\text{ini}} = 1 M_\odot$  and initial metallicity  $[M/H]_{\text{ini}} = 0.0$ . This image reveals how  $\nabla_{\text{rad}}$  and  $\nabla_{\text{ad}}$  behave along the interior of the star and how their interplay reveals which zones are dominated by radiative or convective transport of energy. We can infer that for such a star there exists a radiative interior surrounded by a convective envelope. This convective envelope shifts in size, from thin to thick throughout the evolution of the star.  $\nabla_{\text{ad}}$  maintains itself approximately constant due to the polytropic nature of the stellar interior, whereas  $\nabla_{\text{rad}}$  varies.

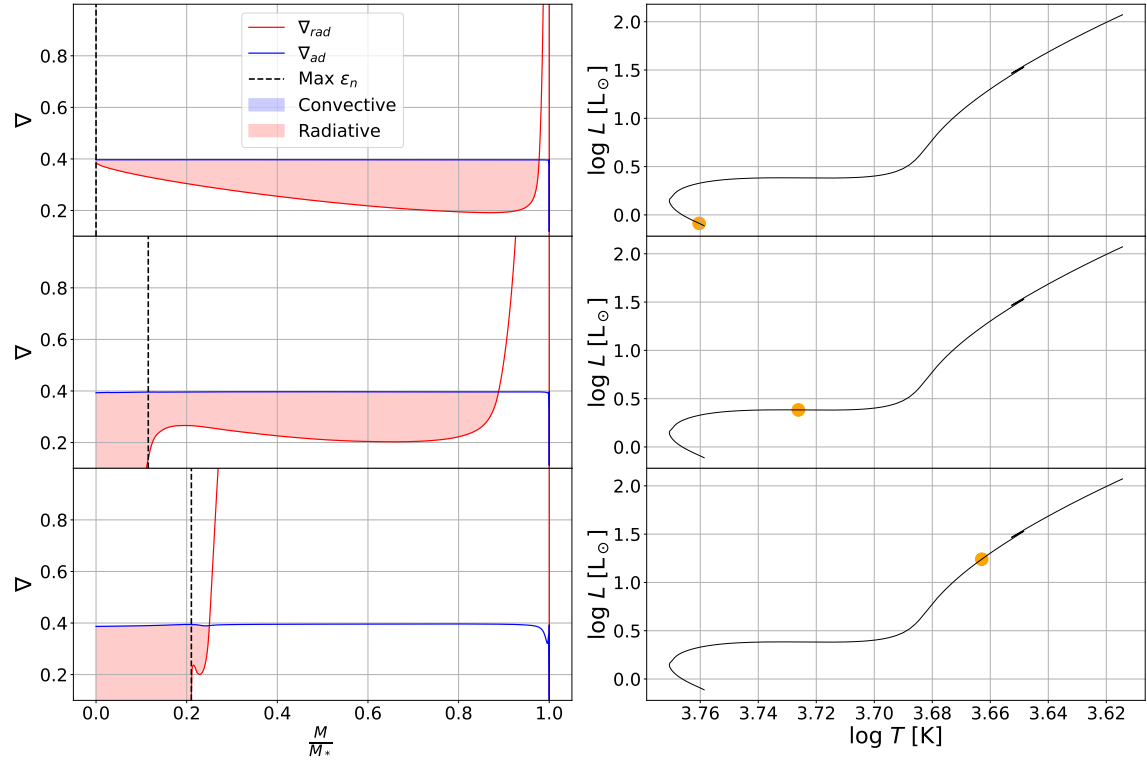


FIGURE 2.2: MESA-generated star with  $M_{\text{ini}} = 1 M_\odot$  and  $[M/H]_{\text{ini}} = 0.0$  at different stages of evolution. From top to bottom: early MS, middle of SG, middle of RGB before RGB Bump. **Left:** Temperature gradient comparison along the interior of the star with the Schwarzschild criterion shaded in, with the black line representing the location of maximum nuclear energy production. **Right:** Corresponding position in the HR diagram.

Figure 2.2 plots three different stages in the evolution of the star, with the left representing the internal structure of the star and the right representing the corresponding location of the star in the HR diagram. From the top plot to the middle plot, comparing MS to SG

phase, the  $\nabla_{\text{rad}}$  rises in value at deeper levels from the surface, resulting in an expansion of the convective envelope deeper inside the star. This is due to the sudden increase in opacity as the decreased temperatures near the surface fail to fully ionize the atoms. Also in this transition from top to middle plot the  $\nabla_{\text{rad}}$  plummets in the core. This is by no accident. Since the core is inert at that stage there is no production of luminosity, which occurs instead at a shell around the core (see the movement of the black line, which represents the point of maximum nuclear energy production). This shell can actually be observed in the middle and bottom plots as the slight bump in  $\nabla_{\text{rad}}$  between the core and the convective envelope. Finally, from top to bottom there is a noticeable increase of the depth of the envelope, reaching its maximum extent in the red-giant branch, which brushes against the borders of the burning shell. This extended envelope comes into contact with different chemicals near the core and spreads them up to the surface due to convective mixing. This is the first dredge-up. The RGB bump occurs due to the hydrogen shell reaching the chemical discontinuity left by the first dredge-up.

The study of the transport of energy is of vital importance for understanding stellar evolution and in the next section we shall see that different stars have different evolutionary behavior. Although Figure 2.2 provides insight into this, the next section will also introduce a more effective way to visualize the evolution of the transport of energy inside a star.

## 2.5 The Hook

As we have seen before,  $\nabla_{\text{rad}}$  as expressed in Equation 2.11 is dependent on opacity, which is great at the stellar surface due to partial ionization. We should therefore expect convective envelopes. Indeed this is what we observe, for example in the Sun, which is fully radiative except for a convective envelope. The internal structure of the Sun is not true for all stars, however. Stars of masses  $M \gtrsim 1.2 M_{\odot}$  possess not only a convective envelope but also a convective core. Furthermore, as the mass increases the convective envelope becomes thinner and thinner, until the star effectively represents a total inversion of the interior structure of the Sun: convective on the inside, radiative on the outside.

Stars of masses  $M \gtrsim 1.2 M_{\odot}$  fuse hydrogen into helium via the CNO cycle described in Equation 2.8. The energy production rates of the pp chain,  $\epsilon_{\text{pp}}$ , and of the CNO cycle,

$\epsilon_{\text{CNO}}$ , possess a dependence in temperature which can be approximated by the power laws given in Equations 2.20 and 2.21.

$$\epsilon_{\text{pp}} \propto T^4 \quad (2.20)$$

$$\epsilon_{\text{CNO}} \propto T^{17} \quad (2.21)$$

These dependencies vastly differ from one another in magnitude and thus give rise to completely different consequences. Due to the extremely high sensitivity on temperature of the CNO cycle, a small shift in temperature is enough to provoke a sizeable shift in energy production. This acts as a thermostat, allowing the star to regulate its equilibrium with small variations in temperature. This mechanism is also responsible for dramatic temperature gradients, seeing as a small deviation from the center of the star leads to a sizeable deviation in temperature which in turn leads to an even more pronounced deviation in nuclear energy production. As a result of these dramatic changes we get that  $\nabla_{\text{rad}} > \nabla_{\text{ad}}$  and therefore by the Schwarzschild criterion the core of the star becomes convective. As a consequence of this convection, chemical composition is mixed within the core, causing a homogeneous depletion of the hydrogen fuel in the convective core. Hertzsprung-Russell diagrams and Kippenhahn diagrams provide a useful window into the visualization of these effects.

Figure 2.3 illustrates two HR diagrams: one for a  $1 M_{\odot}$  star and another for a  $1.2 M_{\odot}$  star, both born with initial metallicity  $[M/H]_{\text{ini}} = 0.0$ . The figure illustrates stellar life-cycles from the moment of their birth at the ZAMS, at the end of the MS, evolving horizontally throughout the subgiant phase, rocketing up the red-giant branch, zig-zagging at the RGB Bump and finally reaching where the track has been truncated. These two different tracks are similar in behavior, with the exception of the formation of a hook-like feature during the main sequence of the  $1.2 M_{\odot}$  star. To further understand why this occurs we can have a look at Kippenhahn diagrams.

A Kippenhahn diagram is a diagram that shows the internal structure of a star as it evolves through time. We can make use of such diagrams in order to better understand the types of behaviors demonstrated in Figure 2.3, which is precisely what is illustrated in Figure 2.4. Here we can see the difference of internal evolution between the two stars,

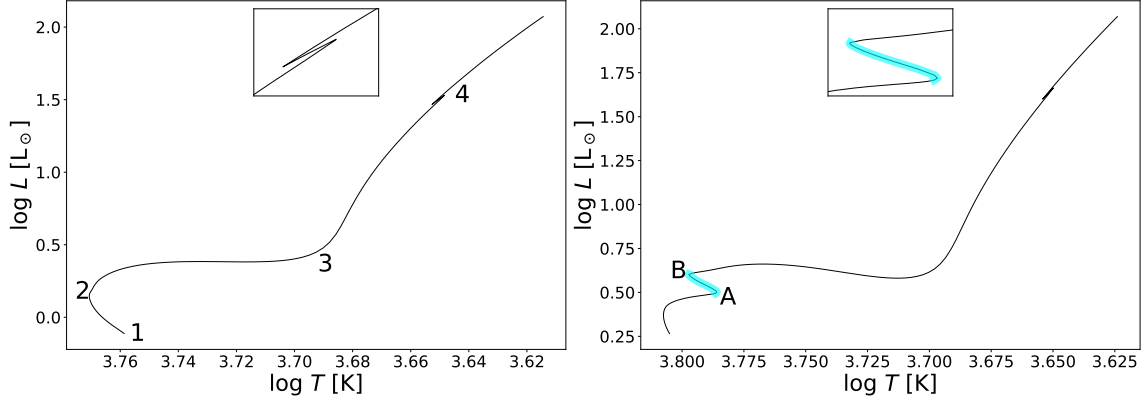


FIGURE 2.3: HR diagram of MESA-generated stars. **Left:**  $M_{\text{ini}} = 1 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$ . (1) ZAMS (2) End of MS, beginning of SG (3) End of SG and beginning of the RGB (4) RGB bump. Inset: zoomed in RGB Bump. **Right:**  $M_{\text{ini}} = 1.2 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$ . Hook feature has been highlighted in cyan. (A) Beginning of the hook feature (B) End of the hook feature and also the end of the main sequence. Inset: zoomed in hook feature.

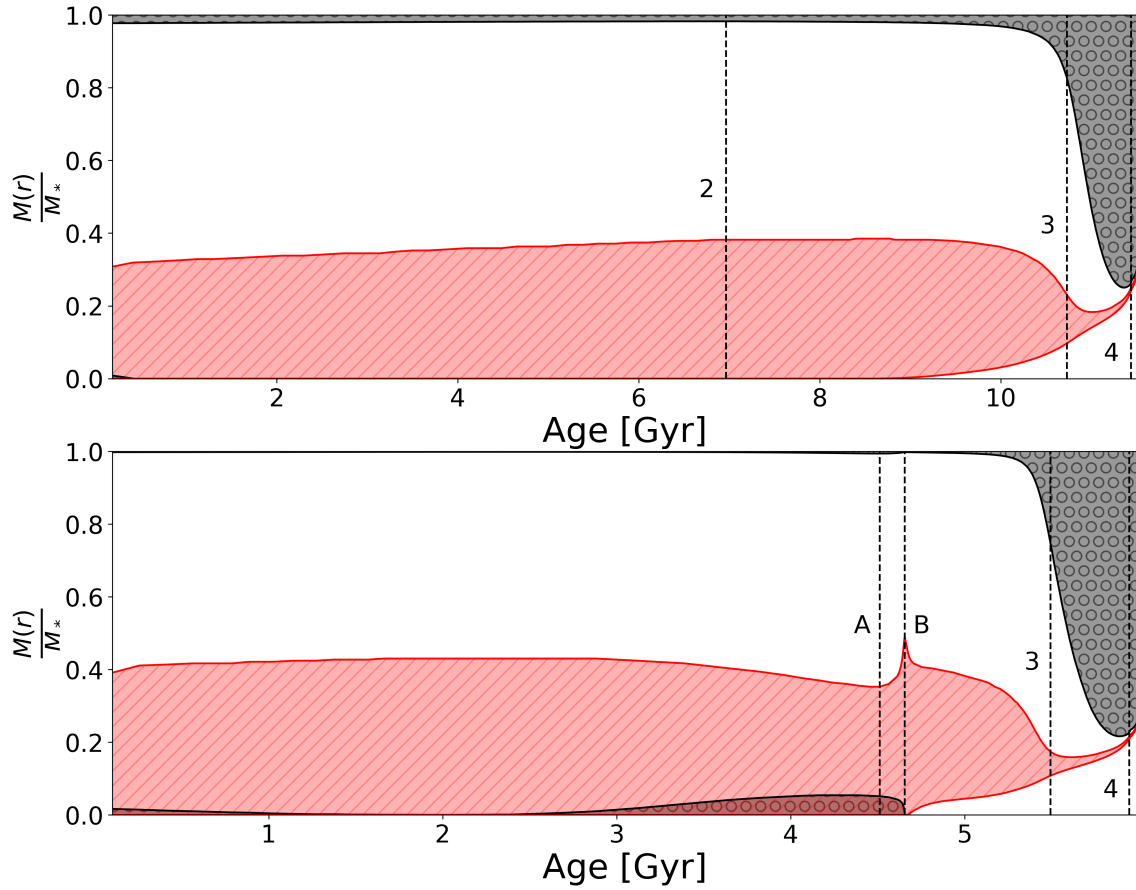


FIGURE 2.4: Kippenhahn diagram of MESA-generated stars. Gray zones with circles represent convective zones and red zones with lines represent nuclear fusion zones. The labelling is the same as in Figure 2.3. **Top:**  $M_{\text{ini}} = 1 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$ . **Bottom:**  $M_{\text{ini}} = 1.2 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$ .

making note of how one star possesses a convective core throughout the main sequence whilst the other does not. It is the star with the convective core that presents a hook-like feature in the HR diagram and we see that the occurrence of this hook (a surface phenomenon) is intrinsically linked to interesting structural evolution in the interior of the star, namely the disappearance of the convective core at the end of the main sequence. This hook feature forms because hydrogen becomes nearly depleted in the totality of the core, affecting nuclear reaction rates and in consequence throwing the star out of equilibrium. The star contracts and between points A and B the dominant source of energy is gravitational.

A star exits the main sequence when there is no longer nuclear reaction in the core. Having depleted itself of hydrogen fuel, the core becomes an inert accumulation of helium. Nuclear reactions resume in a hydrogen shell which burns around the core, causing the star to advance through the rather horizontal subgiant phase until it eventually enters the red-giant branch. It is around this point that the first dredge-up occurs, which can so plainly be seen in the Kippenhahn diagrams.

A robust understanding of the physics regimenting stellar structure and evolution is necessary to be able to explain various different phenomena and whether these pose opportunity or obstacle in the mission for stellar characterization. These illustrations were made possible due to computational modelling. The equations of stellar structure outlined in Equations 2.1, 2.2, 2.3, 2.4 and 2.6, among various others relevant for a series of physical nuances, can be computed numerically to create such models. Various programs have been developed for this very purpose and they are able to evolve these equations in time to simulate the evolution of a star from the zero-age main sequence all the way up to the red-giant branch, the asymptotic giant branch (AGB) and up until the inevitable death of the star. Key to this dissertation has been the program *Modules for Experiments in Stellar Astrophysics* (MESA), which deserves its very own section.

## 2.6 MESA

The *Modules for Experiments in Stellar Astrophysics* (Paxton et al., 2011), also known as MESA, provides a stellar evolution code which is capable of numerically processing physical formulae pertaining to the equations of state, nuclear reaction rates and others together to simulate a wide range of stellar evolution scenarios. This flexibility permits the simulation of the life-cycle of stars of different initial masses and chemical content, outputting the properties of a star as its internal structure evolves radially along time, including even data on the diffusion of elements. It is capable of adaptive time-step control in order to handle each evolutionary phase with satisfying resolution in a computationally sound manner.

It is with MESA where the first steps of testing theory with observational data take place. If the theories of astrophysics are correct then it should follow that these must agree with the observed data. Should this be the case then the door is opened for inference methods on stellar characterization. That is to say, given a set of known observed values, one may then compare this with the outputs of MESA to infer a set of other values that may not be discernible from observation alone. This is the job of PARAM, the program that will be described in Chapter 5. MESA must concern itself only with realistic stellar modelling. However, as ought to be expected, any model is inevitably a simplification of the real world and therefore exact correspondence cannot be completely achieved, only approximated. Furthermore, MESA is also limited at a computational level, sacrificing valuable rotational data by assuming spherical symmetry. Concessions must be made, however. Not all stellar processes are well understood and a single MESA output occupies memory in the order of hundreds of megabytes despite only having structural data in a single spatial dimension (the radial dimension). Although it is prudent to be aware of all nuances that can interfere with the scientific method in the same way that a chef must be knowledgeable of all the ingredients that go into the pot, how these obstacles are tackled is out of the scope of this work.

Despite such obstacles, MESA has proven itself to be a reliable program for the endeavour of stellar characterization and is capable of initializing stars with different masses and metallicities. Metallicity is defined through equation 2.22, where  $X$  is the hydrogen mass fraction,  $Y$  the helium mass fraction and  $Z$  the mass fraction of heavy element. Together

these add up to unity,  $X + Y + Z = 1$ . Metallicity, defined by

$$[M/H] = \log \left( \frac{Z}{X} \right) - \log \left( \frac{Z_{\odot}}{X_{\odot}} \right), \quad (2.22)$$

refers to the abundance of heavy elements at the surface of the star, as this is what can be observed. For this dissertation we consider the metallicity,  $[M/H]$ , to be similar enough to the iron content,  $[Fe/H]$ , within uncertainty of measurement. This is not always true, however, and in [Moedas et al. \(2022\)](#) it is shown that under certain conditions these values can become significantly different. For the grids used in this dissertation the difference between  $[M/H]$  and  $[Fe/H]$  is within 0.01 dex at maximum.

A single stellar track may not tell us much if it is completely offset from the star we wish to study, but a series of tracks spanning a wide range of initial masses and metallicities can do the trick. Such a collection of tracks is called a grid and its purpose is to span a range of masses and metallicities such that the star of interest falls within the confines of this grid. Since this work is interested in F-G-K stars, an appropriate grid must be computed. The chosen grid was built in MESA following the input physics described by the grid D1 in the work of [Moedas et al. \(2022\)](#), which computes atomic diffusion without radiative acceleration.

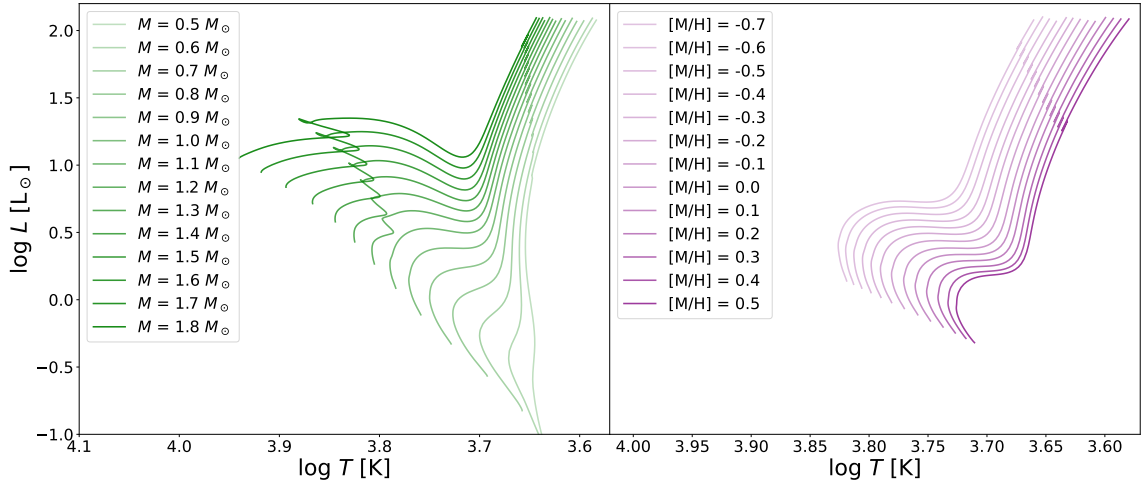


FIGURE 2.5: HR diagram of MESA-generated stars. **Left:**  $[M/H]_{\text{ini}} = 0.0$  with varying mass. **Right:**  $M_{\text{ini}} = 1 M_{\odot}$  with varying metallicity.

The stellar tracks within this grid spans initial masses of values  $M \in [0.5, 1.8] M_{\odot}$  with step  $\Delta M = 0.1 M_{\odot}$  and initial metallicities of values  $[M/H] \in [-0.70, +0.50]$  with spacing  $\Delta[M/H] = 0.10$ . Figure 2.5 illustrates the effect that both a difference in initial mass and

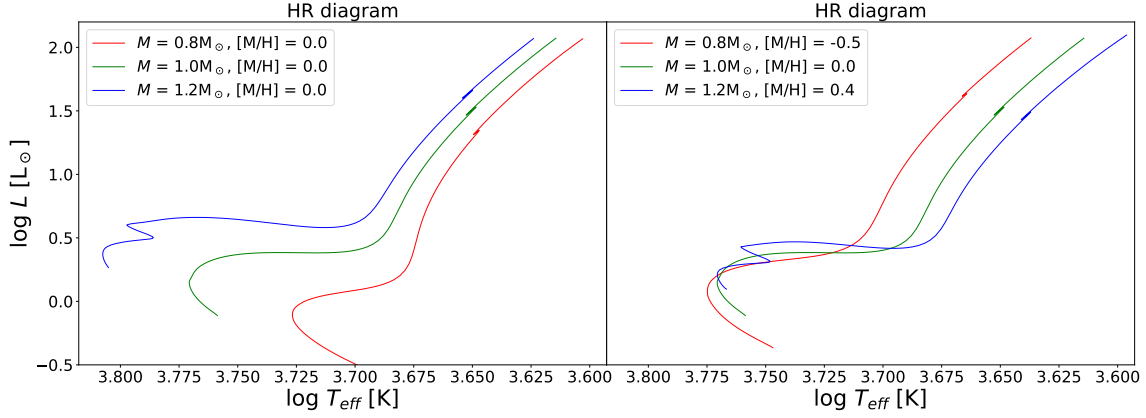


FIGURE 2.6: HR diagram of MESA-generated stars. **Left:** Stars of varying mass with the same metallicity showing no overlap. **Right:** Stars with the same initial mass as the other plot but with different values of metallicity. Note the overlap.

initial metallicity can do to the life-cycle of a star. An increase in initial mass leads towards a leftwards tendency in the HR diagram, whereas an increase in metallicity provides the opposite effect and diverges instead to the right of the HR diagram. Although these tracks tend to avoid touching one another, if only one of these parameters is varied, extreme overlapping becomes unavoidable when both are varied. Figure 2.6 illustrates this plainly. The protruding of the hook exacerbates the effects of overlapping and shall complicate the process of stellar characterization. Finally, in Figure 2.7 we have the full grid.

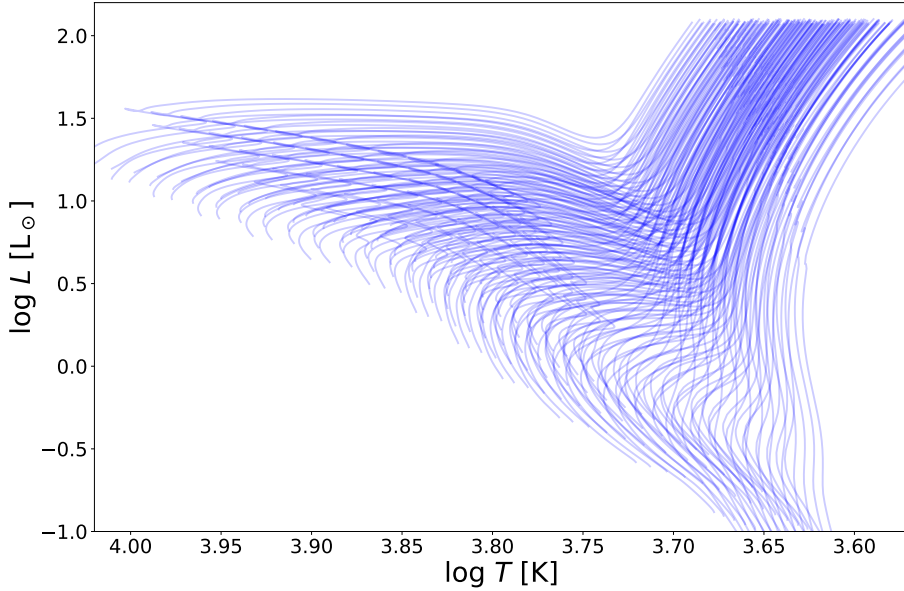


FIGURE 2.7: HR diagram of MESA-generated stars forming a grid ranging from  $M_{\text{ini}} \in [0.5, 1.8] M_{\odot}$  with a step of  $\Delta M = 0.1 M_{\odot}$  and initial metallicities of values  $[M/H]_{\text{ini}} \in [-0.70, +0.50]$  with spacing  $\Delta[M/H] = 0.10$ .

The stellar grid possesses more information than just the luminosity and effective temperature of the modelled stars. Each point in the grid possesses information on the age, radius, surface gravity and other parameters as well. Figure 2.8 exemplifies the distribution of ages across the grid, allowing therefore to infer the age of a star. The overlaps provoked by the hook do, however, remain an obstacle. This gives rise to degeneracies, which then begs the question as to how such degeneracies can be lifted. The answer is, of course, to expand the variables in the parameter space, for example through asteroseismic data.

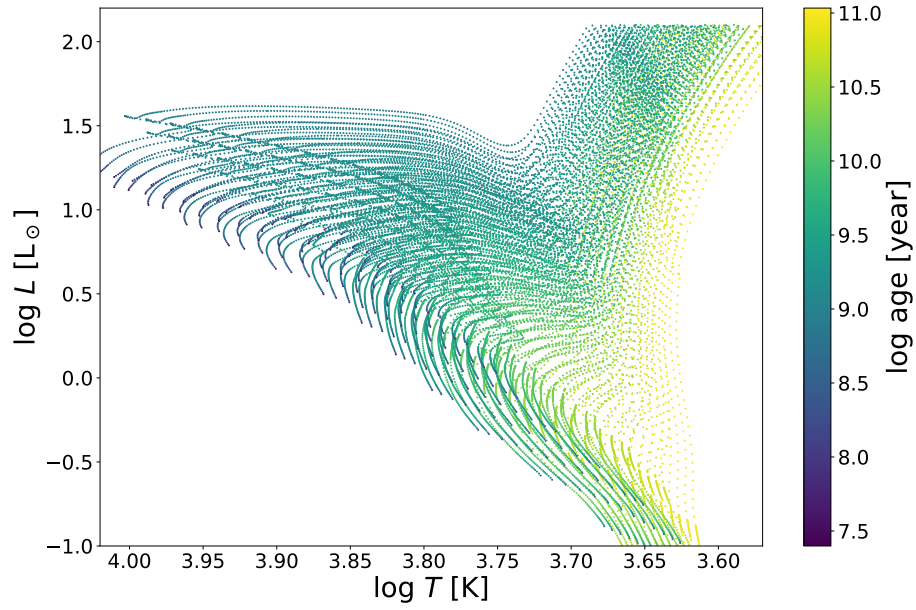


FIGURE 2.8: The same grid as in Figure 2.7 but colour-coded according to age.



## Chapter 3

# Theory of stellar oscillations

### 3.1 The Power Spectral Density

Helioseismology is the field of the oscillations occurring in the Sun. Asteroseismology expands this field to all stars. Such oscillations can be observed in two manners. First, by the radial velocity method. As material rises up and down due to oscillatory behavior, a redshift Doppler effect causes the Fraunhofer lines to shift beyond what would be expected. The second manner is the photometric method, which measures the variation in brightness of the star. Time series observation decomposed by Fourier transformation give rise to the power spectral density (PSD) over the frequency space. Figure 3.1 shows the PSD of the Sun obtained from the *Global Oscillations at Low Frequencies* (GOLF, (GOLF, Gabriel et al., 1995, 1997)) and compares it with the results obtained from the *Variability of solar IRradiance and Gravity Oscillations instrument* (VIRGO, Fröhlich et al., 1995), using the radial velocity with the photometric method, respectively. As shown, the convective background is greater in the photometric measurement (green) than in the spectroscopic measurement (black). This discrepancy occurs because the granulation of the Sun causes more noise in photometric data than in spectroscopic data. As a consequence, the signal-to-noise ratio (S/N) of the acoustic modes is greater for the spectroscopic method than it is for the photometric method.

As shown in Figure 3.1, a pronounced effect occurs for frequencies above  $\nu \gtrsim 1000\mu\text{Hz}$ . These are the solar-like oscillations of the Sun. Figure 3.2 provides a better view of these oscillations, obtained from Doppler observations by the *Birmingham Solar Oscillations Network* (BiSON, Hale et al., 2016). These spikes in the PSD are resonant modes occurring

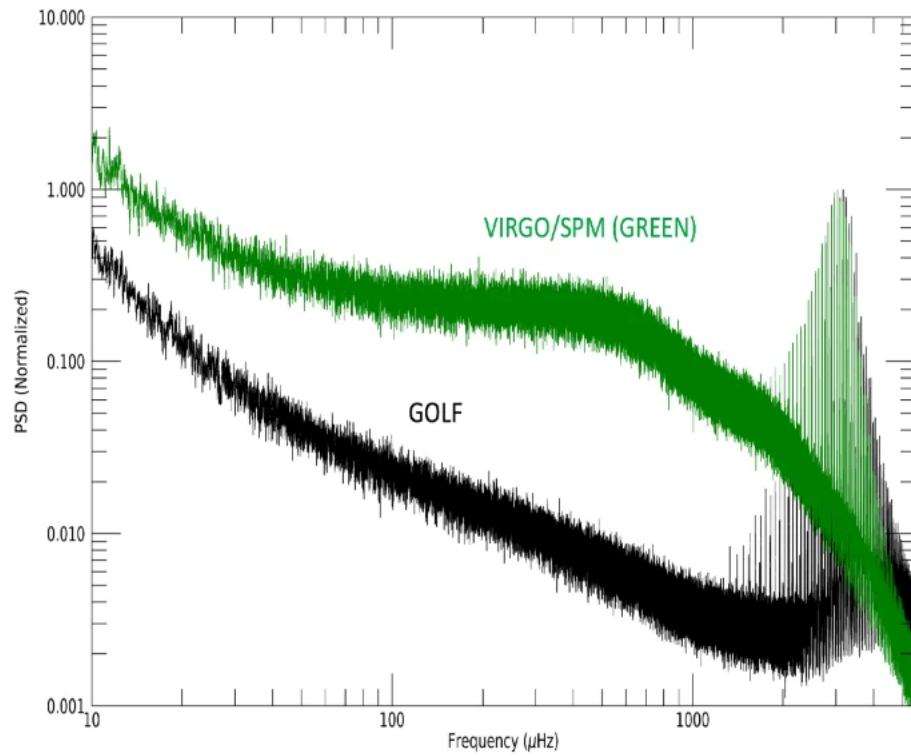


FIGURE 3.1: Power spectral density of the oscillations in the Sun, obtained from Doppler velocity by GOLF (black) and from photometric measurements by VIRGO/SPM (green). Values have been normalized such that the maximum of the p-mode bump coincides for both measurements. Image taken from [García & Ballot \(2019\)](#).

at specific frequencies. As we shall see, these can tell us much about the star. Enter the theory of stellar oscillations.

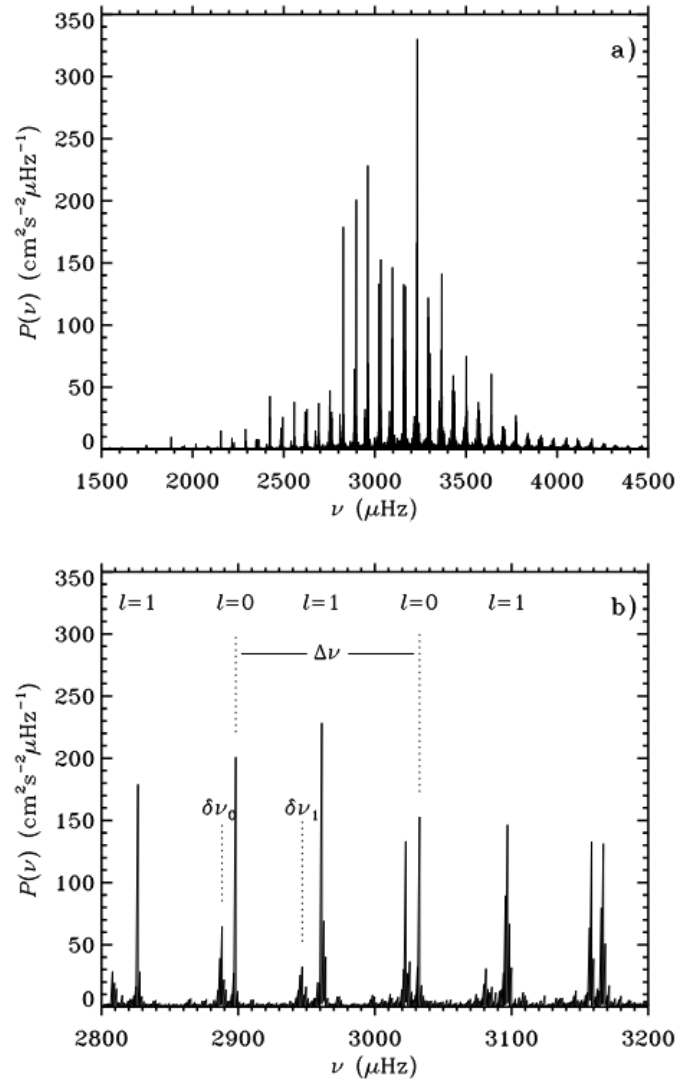


FIGURE 3.2: Power spectral density of the oscillations in the Sun, obtained from Doppler observations by BiSON. Some modes have been labelled by their degree,  $l$ , and the large and small frequency separations,  $\Delta\nu$  and  $\delta\nu_l$ . Image taken from book *Lecture Notes on Stellar Oscillations* [Christensen-Dalsgaard \(2014\)](#).

## 3.2 Solar-like Oscillations

The theory of stellar oscillations is the theory that tries to explain observed stellar pulsations. These pulsations are thought to be the consequence of perturbations from states of equilibrium that are under the effect of restoring forces that seek to eliminate such deviations. When you combine a perturbation with a restoring force you may either obtain an exponentially damped motion back to equilibrium or an overcompensating effect that goes beyond the point of equilibrium. The latter gives rise to an oscillatory state.

Solar-like oscillations, so called because they have characteristics and excitation mechanisms similar to the Sun, are oscillations excited in the same way as in the sun. That is to say, through turbulent convection. Stars exhibiting solar-like oscillations are dubbed solar-like stars. These solar-like stars are defined as cool main sequence dwarfs, as well as red giants, with near-surface convection zones. These include F-G-K type stars. Chapter 2 reviewed the types of equations that govern the stars. Stars seek to bring these equations into equilibrium, meaning that deviations from equilibrium are met with restoring forces. If these deviations are small enough then they can be considered linear perturbations. Linear perturbations under the effect of a restoring force generate waves that travel throughout the stellar body. In the case of solar-like oscillations, these give rise to two types of modes.

Most famous of the solar-like oscillation modes are the p-modes, pressure modes (also known as acoustic modes) driven by the pressure gradient. These are the modes responsible for the bump in Figure 3.1 and which can so plainly be seen in Figure 3.2. P-modes may travel throughout the entire stellar body and are believed to be stochastically excited by convective motions near the surface, driving the modes to follow a pronounced bell-shaped curve which is called the Gaussian envelope (see Figure 3.5). This theory is entertained in Goldreich & Keeley (1977), for example.

The other type of modes are called the g-modes, the mechanism of which shall be briefly explained. As said previously, oscillations are due to linear perturbations. By linearizing the stellar equations we give rise to the differential equation (see chapter 6 of Kippenhahn & Weigert 1990 for a derivation)

$$\frac{\partial^2(\Delta r)}{\partial t^2} = -\frac{g\delta}{H_p} \left[ \nabla_e - \nabla + \frac{\phi}{\delta} \nabla_\mu \right] \Delta r, \quad (3.1)$$

where  $\Delta r$  is a radial perturbation of an element of mass away from its state of equilibrium and  $g$  is the gravitational acceleration. If we presume that the element is adiabatic ( $\nabla_e = \nabla_{ad}$ ), then the solution to this differential equation takes the form

$$\Delta r = \Delta r_0 e^{iNt}, \quad (3.2)$$

where  $\Delta r_0$  is the amplitude and  $N$  is the Brunt–Väisälä frequency, defined as

$$N \equiv \frac{g\delta}{H_p} \left( \nabla_{ad} - \nabla + \frac{\phi}{\delta} \nabla_\mu \right), \quad (3.3)$$

If the Ledoux criterion, Equation 2.17, (Schwarzschild criterion if  $\nabla_\mu = 0$ , Equation 2.12) holds then we have  $N^2 > 0$ , which provides an oscillatory behavior in the radiative zone. If it is violated then  $N^2 < 0$ , which leads to an imaginary  $N$ . The exponent in equation 3.2 becomes real and as such no oscillation occurs and instead the element rises or sinks indefinitely. These waves, the ones confined to radiative zones, are called internal gravity waves and their modes are called g-modes. If they enter convective zones they dissipate as an evanescent wave, however they too are believed to be stochastically excited at the border of convection motion.

As stars evolve their structure changes. At the end of the main sequence the radiative gradient falls considerably in core (see Figure 2.2). By Equation 3.3 this means a sharp increase in the Brunt–Väisälä frequency in the core. With the rise of the g-mode frequency domains, these can intertwine with the p-modes, coupling together to form mixed modes with characteristics from both. Figures 3.3 and 3.4 illustrates the different structures and mode domains within a main-sequence  $1 M_\odot$  star and a main-sequence F-type star, respectively.

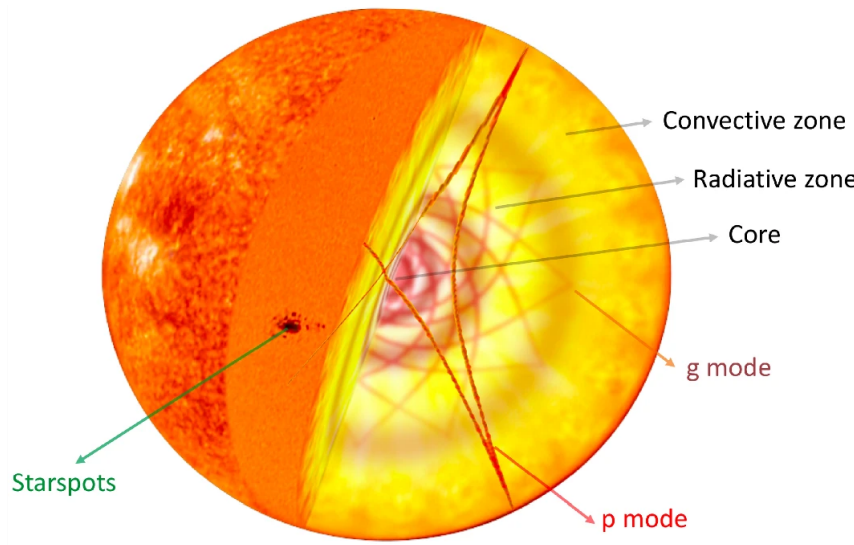


FIGURE 3.3: Artist's representation of the interior of a  $1 M_\odot$  main-sequence solar-like pulsating star. Its structure is composed by a radiative interior and a convective envelope. Image taken from [García & Ballot \(2019\)](#).

Modes are characterized by a discrete set of eigenfrequencies which arise as the valid solutions to the differential equations governing stellar oscillations. If we assume spherical symmetry then we can describe the non-radial modes in terms of spherical harmonics,  $Y_l^m$ , and fully characterize the modes in terms of three integer numbers:

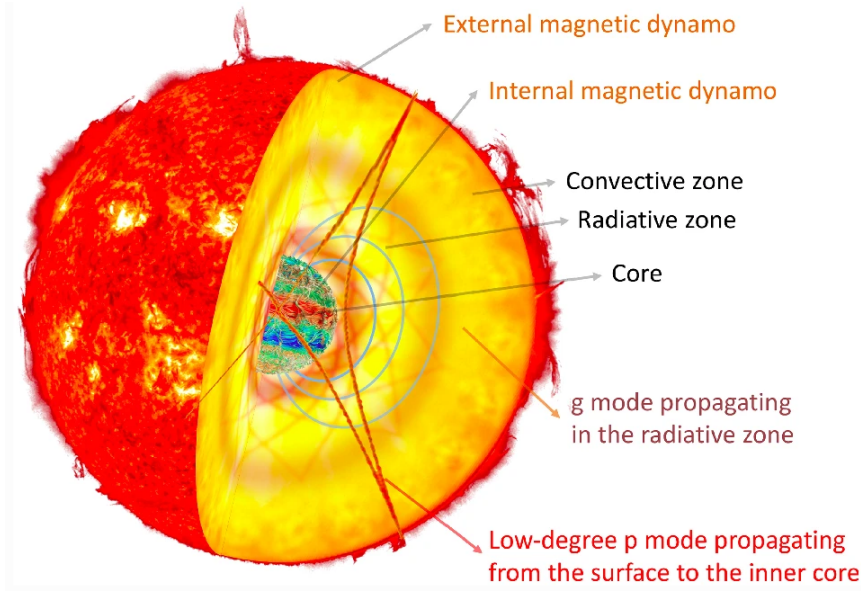


FIGURE 3.4: Artist's representation of the interior of a main-sequence F-type pulsating star. Its structure is composed by a convective core, a radiative interior and a convective envelope. Image adapted from [García & Ballot \(2019\)](#).

- $n$ : the radial order, which is positive for p-modes and negative for g-modes
- $l$ : the angular degree, which describes radial modes for  $l = 0$ , dipole modes for  $l = 1$ , quadrupole modes for  $l = 2$  and so on.
- $m$ : the azimuthal order, which ranges from  $-l$  to  $l$  and is axisymmetric when  $m = 0$ .

The frequency of a mode,  $\nu_{n,l,m}$ , is expressed in Hz and since  $m = 0$  in cases of spherical symmetry then, by our prior assumptions, we ignore the effects of stellar rotation and consider only the simplified case of azimuthal degeneracy,  $\nu_{n,l,m} = \nu_{n,l}$ .

### 3.3 Seismic Scaling Relations

Solar-like oscillations can have their properties summarized by their global seismic parameters. Figure 3.5 illustrates the power spectral density of a solar-like star. The observed p-modes, excited by stochastic turbulence in the convective envelope, form a bell-curve shape that is also called the Gaussian envelope (dotted blue Gaussian shape). Such a curve is fitted on top of the modes and the location of its peak is the frequency of maximum power,  $\nu_{\max}$  (not to be confused with the mode of highest amplitude in the PSD). In the inset of the figure we have the large frequency separation,  $\Delta\nu$ , defined as

the separation between consecutive modes of same angular degree  $l$  and consecutive radial order  $n$ ,  $\Delta\nu_{n,l} = \nu_{n,l} - \nu_{n-1,l}$ . The small frequency separation,  $\delta\nu_{n,02}$ , is defined as  $\delta\nu_{n,l,l+2} = \nu_{n,l} - \nu_{n-1,l+2}$ . Many different  $\Delta\nu_{n,l}$  and  $\delta\nu_{n,l,l+2}$  exist, so an average is taken across all available  $n$  to acquire singular values. Furthermore, since we are interested in the case where  $l = 0$  (and also  $l = 2$  for the small frequency separation), these subscripts can be simplified to  $\Delta\nu$  and  $\delta\nu_{02}$ . These three seismic parameters,  $\nu_{\max}$ ,  $\Delta\nu$  and  $\delta\nu_{02}$ , are strongly correlated with valuable information on the internal structure of the star.

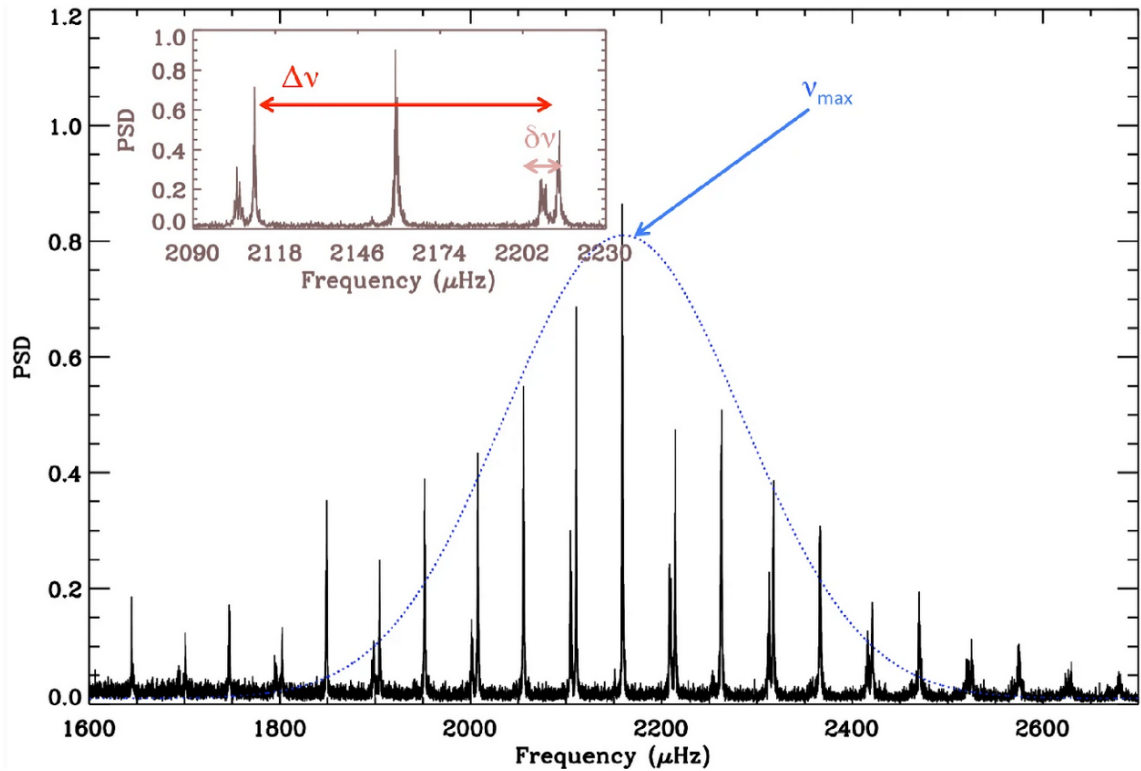


FIGURE 3.5: Power spectral density (in arbitrary units) of the *Kepler* target 16 Cyg A. Represented in this image is the Gaussian envelope (blue dotted line) and three global asteroseismic parameters:  $\nu_{\max}$ ,  $\Delta\nu$  and  $\delta\nu_{02}$ . Image taken from [García & Ballot \(2019\)](#).

Homology relations are relations which allow the conversion of a set parameters into another set of parameters via a transformation that conserves the same underlying physics. For instance, the higher the mass of a star the greater its temperature ought to be, in general, presuming that they are for example both in the main sequence. Since stars are complex and have various different states depending, one cannot simply upscale and downscale stars at a whim. Despite this, it may still prove useful to find analytical expressions that transform one valid stellar model into another. These take the form of Equation 3.4, which relates the set of arbitrary parameters  $\{A, B\}$  with the set  $\{A', B'\}$

$$\frac{A}{A'} = \frac{B}{B'} \quad (3.4)$$

So long as two stars possess a relation of homology between them, it is possible to infer the properties of an unknown star behave based on the properties of a known star. Such relations exist for the global seismic parameters, with which the Sun can be used the reference for solar-like oscillators, particularly for  $\Delta\nu$  and  $\nu_{\max}$ . These are the seismic relations 3.6 (see [Kjeldsen & Bedding 1995](#); [White et al. 2011](#); [Huber et al. 2011](#)) and are given by

$$\Delta\nu \approx \Delta\nu_{\odot} \left( \frac{M}{M_{\odot}} \right)^{1/2} \left( \frac{R}{R_{\odot}} \right)^{-3/2}, \quad (3.5)$$

$$\nu_{\max} \approx \nu_{\max,\odot} \left( \frac{M}{M_{\odot}} \right) \left( \frac{R}{R_{\odot}} \right)^{-2} \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{-1/2}. \quad (3.6)$$

These equations can be rewritten to provide the scaling relations for the mass and radius, like so

$$R \approx R_{\odot} \left( \frac{\Delta\nu}{\Delta\nu_{\odot}} \right)^{-2} \left( \frac{\nu_{\max}}{\nu_{\max,\odot}} \right) \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{1/2}, \quad (3.7)$$

$$M \approx M_{\odot} \left( \frac{\Delta\nu}{\Delta\nu_{\odot}} \right)^{-4} \left( \frac{\nu_{\max}}{\nu_{\max,\odot}} \right)^3 \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{3/2}. \quad (3.8)$$

What these last two equations provide is a window into stellar characterization. Both the mass and radius are difficult parameters to determine, but with seismology we are able to form homology relations that allow us to estimate these quantities.

As an addendum, we may also exploit these equations with other relations, such as the Stefan-Boltzmann law for black-bodies

$$L = 4\pi\sigma\epsilon R^2 T^4, \quad (3.9)$$

where  $\sigma$  is the Stefan-Boltzmann constant,  $R$  is the radius of the black-body (assuming it is spherical with surface area  $A = 4\pi R^2$ ),  $\epsilon$  is the emissivity and  $T$  is the thermodynamical temperature of the black-body. Provided robust enough homology relation, the Stefan-Boltzmann law can be used to bridge the seismic relations with the luminosity of the star.

### 3.4 GYRE

Asteroseismology plays an important role in stellar characterization due to the seismic relations given by Equations 3.7 and 3.8, serving as constraints to the determination of the masses and radii of stars. Since we already employ the help of MESA to construct a stellar grid full of points containing values in a wide range of parameters, it would be useful to expand the parameter space with the inclusion of the global seismic parameters.

Knowing what the global seismic parameters are at any given moment in the evolution of the star requires first knowledge on the solar-like oscillations displayed by the star at that given step in its evolution. For this we resort to GYRE (Townsend & Teitler, 2013).

GYRE is an oscillation code capable of solving differential equations for stellar pulsations in the adiabatic and non-adiabatic regimes. In doing so the code is able to produce mode eigenfrequencies, calculating the p-modes, g-modes and the mixed modes. As an output it lists the modes in terms of the radial order  $n$ , the angular degree  $l$ , the frequency  $\nu$  and to which category these modes belong to (p, g or mixed).

The outputs generated by GYRE can be combined with the MESA outputs, providing a grid with an enhanced parameter space that includes  $\Delta\nu$  and  $\nu_{\max}$ . This is done by making use of a toolkit that automatically processes MESA data with GYRE modes to return the seismic parameters. It is possible to also include  $\delta\nu_{02}$ , but the final program in the puzzle, PARAM (Chapter 5), does not yet consider the small frequency separation as a valid input.



## Chapter 4

# Observational constraints

This chapter delves into the data collection of relevant parameters, the instruments that collect such data, how said instruments collect this data, the challenges and limitations posed, and the ways in which such obstacles can be handled. To answer such questions we must begin by defining the types of stars we are interested in, which are F-G-K exoplanet-harboursing stars serving as potential candidates in the Ariel input catalog.

### 4.1 Astrometry and Photometry

#### 4.1.1 Parallax

Parallax is here referenced in the context of stellar parallax, which is the apparent shift of the position in the sky of a star when compared to a background of extremely distant reference points, usually other stars, whose greater distances result in negligible parallax of their own. The parallax,  $p$ , is a means to measure the distance that a star is from us. The expression of distance is given by

$$d(\text{pc}) = \frac{1(\text{AU})}{p('')}, \quad (4.1)$$

where the reciprocal of the parallax in miliarcseconds (mas) gives the distance in parsecs, roughly 3.26 light-years.

What is actually measured is therefore not the distance itself, but the parallax. The measurement of this parallax comes with its associated uncertainties and it should be noted that a Gaussian uncertainty in parallax will result in an asymmetric non-Gaussian uncertainty in the distance. This may grow to unwieldy values. In such a case the inverse

of the parallax fails to provide the distance and other methods must be sought (Bailer-Jones et al., 2021). Because of this we define the fractional parallax uncertainty, fpu, as

$$\text{fpu} = \frac{\sigma_p}{p}, \quad (4.2)$$

where  $\sigma_p$  stands for the uncertainty on the parallax. The determination of the parallax is subject to noise and systematic errors, which become more impactful the greater the distance of the star. Since stars become fainter with distance the fpu increases. Equation 4.1, a geometric inference of the distance, may provide extremely incorrect results if we aren't careful with our parallaxes. So long as the distance is not too large ( $\lesssim 1$  kpc, for example) and the fpu not too large ( $\lesssim 0.1$ , for example), geometric inference suffices. Alternatively, photogeometric methods may need to be used, in which the difference in extinction for different bands is taken into consideration as a means to constrain the distance estimation.

The *Gaia* survey (Gaia Collaboration et al., 2018) provides highly precise astrometric and photometric data. Since the stars analyzed in this dissertation are nearby and have a low fpu, the reciprocal of the parallax remains a good estimator of the distance and no extra considerations need to be taken into account (shown later in Table 6.5).

#### 4.1.2 Photometric surveys

One of the most important parameters of a star is its luminosity. If we know both the luminosity and the effective temperature then we can plot the star on the HR diagram, comparing it with others to get a first idea on what type of star it is. Luminosity is not something that can be directly observed, however. As a star shines it emits light in the form of photons, which are collected by photosensitive instruments as discrete packets of energy. These packets of energy come in different frequencies, requiring the usage of filters that are optimized to be as sensitive as possible to certain ranges of frequencies. These are the bands, of which various exist to capture different frequencies of light with different sensitivities. Collections of bands operating under the same methodology are called photometric systems. One such system is, for example, the Johnson-Cousins UB-VRI photometric system (Landolt, 2009). Figure 4.1 illustrates an example of different photometric bands belonging to different photometric systems.

Different instruments possess different photometric specifications, thereby restricting themselves to the study of a select few bands. Multiple surveys may therefore be needed. This dissertation makes use of two surveys:

- The *Gaia* survey (DR2: [Gaia Collaboration et al. 2018](#), eDR3: [Gaia Collaboration et al. 2021](#), DR3: [Gaia Collaboration et al. 2022](#)), with interest in the G (green), G\_BP (*Gaia* Blue Pass) and G\_RP (*Gaia* Red Pass) bands, covering a portion of the visible spectrum.
- The Two-Micron All Sky Survey (2MASS, [Skrutskie et al., 2006](#)), using the J, H and  $K_s$  bands, which lie in the near-infrared.

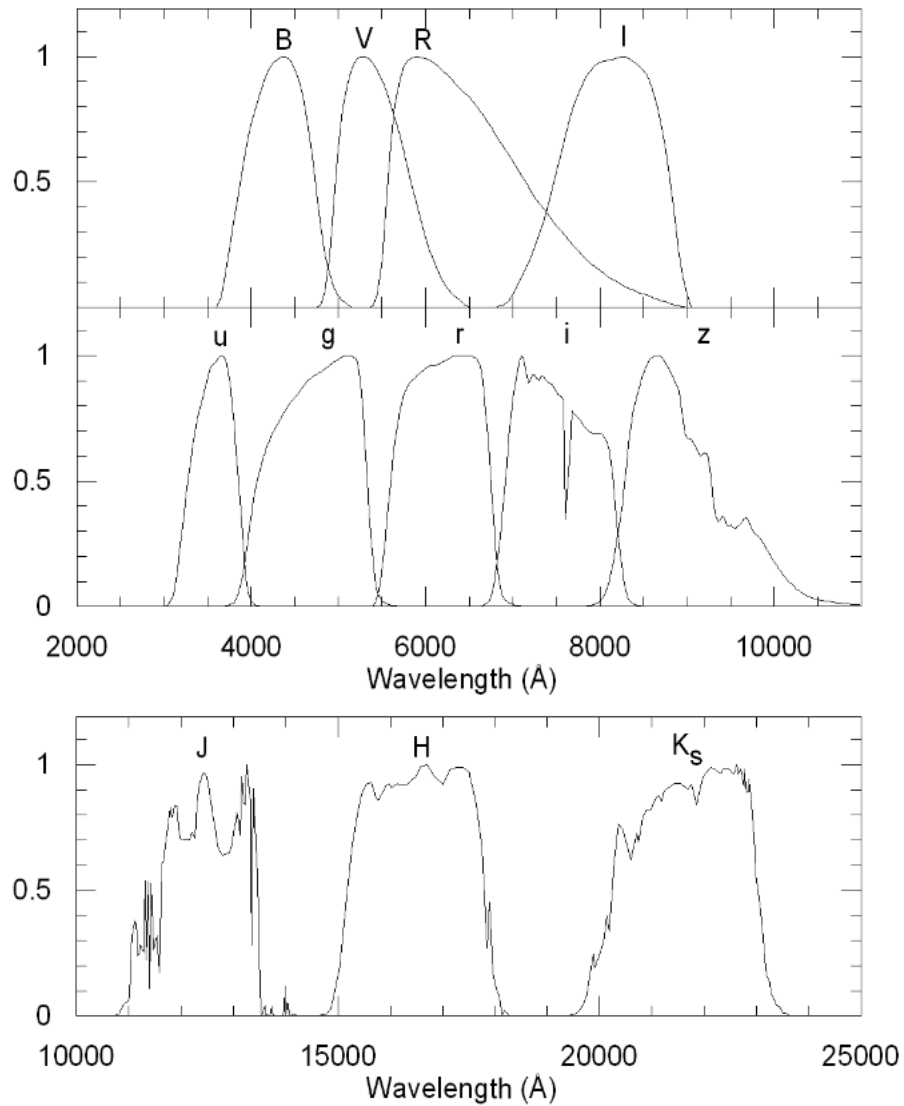


FIGURE 4.1: Normalized photometric systems. Top to bottom: Johnson-Cousins BVRI filters, SDSS ugriz filters and 2MASS filters. Image taken from [Bilir et al. \(2008\)](#).

### 4.1.3 Reddening and Extinction

A measure by which astronomers reference the brightness of a star is the *magnitude*. Let the subscript  $\zeta$  represent an arbitrary spectral band. The apparent magnitude of a star in a given spectral band,  $m_\zeta$ , is related to the flux in the same band,  $F_\zeta$  by the equation

$$m_\zeta = -2.5 \log_{10} \left( \frac{F_\zeta}{F_{\zeta,0}} \right), \quad (4.3)$$

where  $F_{\zeta,0}$  is the zero-point reference flux for that band. It is important to specify which zero-point reference is utilized. An example of a commonly used reference is the star Vega.

As stated,  $m_\zeta$ , refers to the magnitude of a star at a given band  $\zeta$ . When the entire electromagnetic radiation is taken into consideration, the magnitude is referred to as the *bolometric magnitude*, which can be represented either by  $m_{\text{bol}}$  or simply as  $m$ . The apparent magnitude is not the most useful for objective comparisons between stars, thereby bringing into necessity the concept of *absolute magnitude*,  $M$ , which is defined as the magnitude the star would present if it were at a distance of 10 parsecs away from the observer. For a given band  $\zeta$ , the absolute magnitude for a star at a distance of  $d$  parsecs away is

$$M_\zeta = m_\zeta - 5 \log_{10} \left( \frac{d}{10\text{pc}} \right). \quad (4.4)$$

The absolute magnitude is useful when comparing the luminosity,  $L$ , of different stars. Whereas the flux is a measure of the photons received by our instruments, the luminosity is an intrinsic quantity associated to the star. The flux at a distance  $d$  from the star is associated with the luminosity by the expression  $L = 4\pi d^2 F$ .

Within the vastness of space that separates us from the stars lies interstellar gas and dust which serves as an obstacle to the passage of light. The combined effects of photon absorption and scattering induced by these clouds is given the name of *extinction*, as it extinguishes parts of the light which would have otherwise made its way to our instruments. In terms of magnitude, the extinction at a given band is represented by  $A_\zeta$  and can

be related with Equation 4.4 to give

$$\begin{aligned} m_{\zeta} &= M_{\zeta} + 5 \log_{10} \left( \frac{d}{10 \text{pc}} \right) + A_{\zeta} \\ &= m_{\zeta,0} + A_{\zeta}, \end{aligned} \quad (4.5)$$

where  $m_{\zeta,0}$  refers to the apparent magnitude that we should expect to observe in the absence of extinction, also called the intrinsic apparent magnitude. Equation 4.5 requires us to know the distance,  $d$ , of the star, which may not always be obvious and be subjected to errors. Instead what we can do is define the difference between the extinction in two different bands,  $\zeta$  and  $\xi$ . This turns Equation 4.5 into its distance-independent form,

$$A_{\zeta} - A_{\xi} = (m_{\zeta} - m_{\xi})_{\text{observed}} - (m_{\zeta} - m_{\xi})_{\text{intrinsic}}, \quad (4.6)$$

which defines the colour excess due to extinction,

$$E_{\zeta-\xi} = A_{\zeta} - A_{\xi}, \quad (4.7)$$

and therefore the *reddening*,  $E_{B-V}$ , where  $B$  and  $V$  refers to the blue and visible electromagnetic bands, respectively. Rather than to write  $m_B$  or  $m_V$ , it is customary to write the apparent magnitudes in these bands as  $B$  and  $V$ . We can therefore join Equations 4.6 and 4.7 to define reddening as

$$E_{B-V} = (B - V)_{\text{observed}} - (B - V)_{\text{intrinsic}}. \quad (4.8)$$

Reddening is a phenomenon associated with extinction where the interstellar dust causes non-uniform attenuation of light over its electromagnetic spectrum. Shorter wavelengths tend to be more strongly attenuated than longer wavelengths, causing more extinction in the blue wavelengths than in the red wavelengths. This results in the star appearing redder than it should. Reddening shall not be confused with redshift, which is a spectrum-wide shift towards longer wavelengths.

#### 4.1.4 MWDust

Since extinction is caused by interstellar dust it becomes necessary to have knowledge on the column density of dust between our instruments and an observed star. For this purpose we can use MWDust (Bovy et al., 2016), which is a code freely available online<sup>1</sup>

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<sup>1</sup><https://github.com/jobovy/mwdust>

that possesses a set of different dust maps of the Milky Way galaxy. These dust maps can be 2D (e.g. [Schlegel, Finkbeiner & Davis, 1998](#)) or 3D (e.g. [Green et al., 2019](#)) and can even be combinations of different dust maps conjoined together. For example, we take the previous 3D map with another map to fill in missing values ([Drimmel, Cabrera-Lavers & López-Corredoira, 2003](#)) and a substitute higher resolution disk map ([Marshall et al., 2006](#)), as shown in Figure 4.2.

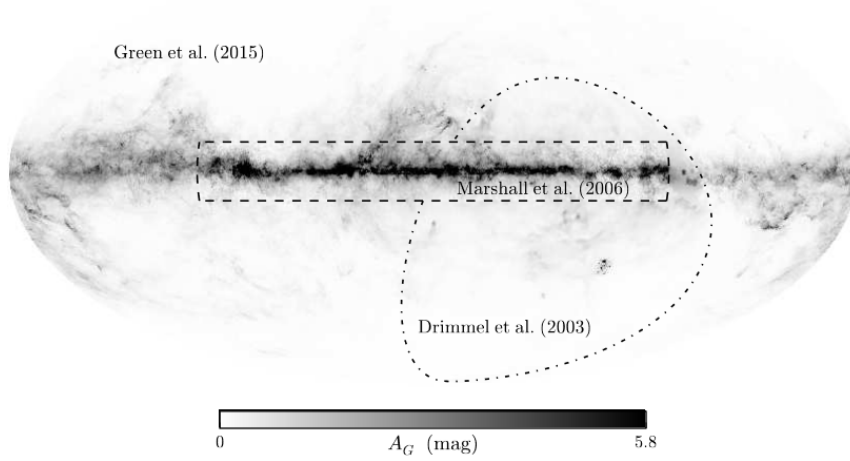


FIGURE 4.2: Mollweide plot of the 3D Milky Way dust using dust maps from [Green et al. \(2019\)](#), [Drimmel, Cabrera-Lavers & López-Corredoira \(2003\)](#) and [Marshall et al. \(2006\)](#). Colour scheme indicates the G-band extinction,  $A_G$ , to 5kpc from the Sun. Image taken from [Bovy et al. \(2016\)](#).

A 3D dust map is a dust map that can provide the value of dust in three dimensions: galactic longitude,  $l$ , galactic latitude,  $b$ , and distance,  $d$ , whereas a 2D dust map integrates the entire radial column, providing only the integrated variation of the dust in  $l$  and  $b$ . Figure 4.3 demonstrates how a 3D dust map changes over distance.

Since we are interested in the dust columns between our instruments and a star within our galaxy, integrated 2D dust maps are of little use. Instead, 3D dust maps are favored. For the sake of homogeneity, a dust map from a single source using a single methodology, rather than the combination of several using different methodologies, is favored. The dust map from [Green et al. \(2019\)](#) is 3-Dimensional, relatively recent and covers a wide field of view of the Milky Way. So long as the stars we observe fall within the scope of this dust map then it can be used.

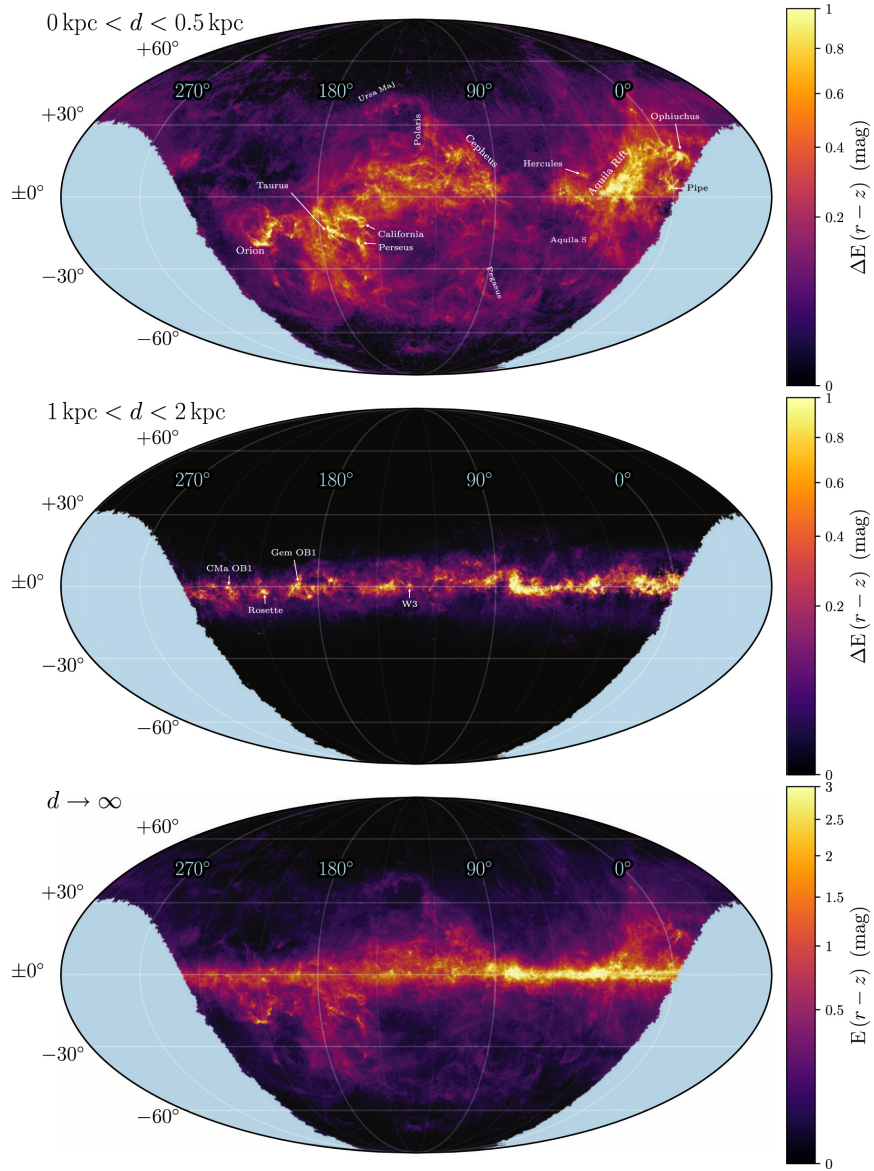


FIGURE 4.3: Mollweide plots of the 3D Milky Way dust using dust maps from [Green et al. \(2019\)](#) showing how the cumulative reddening,  $\Delta E(r-z)$  (where  $r$  and  $z$  are bands in the Pan-STARRS 1 grizy system), in the Milky way changes over different ranges of distance. Image taken from the same paper.

Once a dust map has been selected, MWDust requires only the  $l$ ,  $b$  and  $d$  of the star (or stars) to estimate what values of reddening we ought to expect. Knowing both the magnitude and the reddening of the stars opens the doors for the next step in determining the luminosity.

## 4.2 Luminosity and Bolometric Correction

It is in our interest to infer the bolometric luminosity,  $L$ , of a star, though it is not immediately obvious how this can be achieved when observations for different bands result in different magnitudes. In other words,  $m_\zeta \neq m_{\bar{\zeta}} \neq m_{\text{bol}}$ . The correction which allows us to infer the bolometric magnitude from the band magnitude is called *bolometric correction*. The bolometric correction for a given band,  $BC_\zeta$ , is defined as

$$\begin{aligned} BC_\zeta &= m_{\text{bol}} - m_\zeta \\ &= M_{\text{bol}} - M_\zeta . \end{aligned} \tag{4.9}$$

Once the absolute bolometric magnitude of the star,  $M_{\text{bol}}$ , is found, we can compare it to that of the Sun,  $M_{\text{bol},\odot}$ , to determine its luminosity

$$M_{\text{bol},\odot} - M_{\text{bol}} = -2.5 \log_{10} \left( \frac{L}{L_\odot} \right) . \tag{4.10}$$

A nuance that must be taken into account is the fact that the bolometric correction for two different bands,  $BC_\zeta$  and  $BC_{\bar{\zeta}}$ , for the same star, can due to uncertainty result in slightly different bolometric magnitudes,  $M_{\text{bol}}$ . For this reason, this dissertation shall employ the notation of  $M_{\text{bol}}^\zeta$  and  $M_{\text{bol}}^{\bar{\zeta}}$  to specify the bolometric magnitudes acquired by applying the bolometric corrections  $BC_\zeta$  and  $BC_{\bar{\zeta}}$  to the magnitudes  $M_\zeta$  and  $M_{\bar{\zeta}}$ , respectively. Likewise, since the luminosity is computed from these bolometric magnitudes, so too will the band be used in superscript to denote where this luminosity originated from. In equation form this means

$$\frac{L^\zeta}{L_\odot} = 10^{0.4(M_{\text{bol},\odot} - M_{\text{bol}}^\zeta)} . \tag{4.11}$$

### 4.2.1 BCCode and YBC

The luminosity of a star provides a measure of the power which it emits, giving insight into the stellar properties. To capture the total power we must measure the entire spectrum of electromagnetic radiation, rather than that of a single band. This is why clean bolometric magnitudes, unaltered by the effects of extinction, are so important. With the study of dust maps and the usage of tools like MWDust we can take extinction into account, however we have not yet obtained what we wanted. As we've seen, photometric

instruments do not measure the entire electromagnetic spectrum however, but are rather optimized for the study of particular bands occupying a fixed range of wavelengths.

Bolometric corrections (BCs) are the bridge which relate the observed magnitude at a given, band  $m_\zeta$ , with the bolometric magnitude,  $m_{\text{bol}}$ . From the apparent magnitude we can infer the absolute magnitude which in turn gives us the luminosity. Bolometric corrections can be obtained through codes like BCCode (Casagrande & Vandenberg 2014, 2018a,b) and YBC (Chen et al., 2019), both of which are freely available online<sup>12</sup>. The way these codes work is by interpolating libraries of synthetic stellar spectra to figure out the theoretically expected flux from a spectrum with a certain  $T_{\text{eff}}$ ,  $[\text{Fe}/\text{H}]$  and  $\log g$ , parameters which are necessary to include as input. In addition to this, they take into account the reddening  $E(B - V)$ . This theoretical flux is compared with the observed apparent magnitude of a real star in the same band. The bolometric correction is therefore simply the difference given by Equation 4.9.

Figure 4.4 shows the sensitivity of BC in BCCode when a parameter is free to vary whilst keeping all others fixed. Different bands have been tested with solar values used as reference. What is shown is that the  $T_{\text{eff}}$  provides the most significant variance in BC, followed by  $E(B - V)$  and by  $[\text{Fe}/\text{H}]$  and  $\log g$ . Note how the band which shows the least variance as  $E(B - V)$  is the 2MASS  $K_s$  band. This is because the 2MASS bands are in the infra-red, therefore being the least affected by reddening. The 2MASS  $K_s$  band is the band with the highest wavelength and therefore, in situations where  $E(B - V)$  is difficult to ascertain, we ought to trust more a luminosity calculated through the bolometric correction  $\text{BC}_{K_s}$ , giving  $L^{K_s}$ , than a luminosity calculated through the bolometric correction of the G band,  $L^G$ .

Since our input parameters have uncertainties associated with them then we ought to expect an uncertainty associated with the determination of BC and by extension of the luminosity. We can run several Monte-Carlo simulations into BCCode to determine how  $\text{BC}_\zeta$  varies and how the final calculation of the luminosity varies. Figure 4.5 shows  $N=2000$  Monte-Carlo simulations for a star in the *Ariel* input catalog (see Chapter 6), with

<sup>1</sup><https://github.com/casaluca/bolometric-corrections>

<sup>2</sup><http://stev.oapd.inaf.it/YBC/index.html>

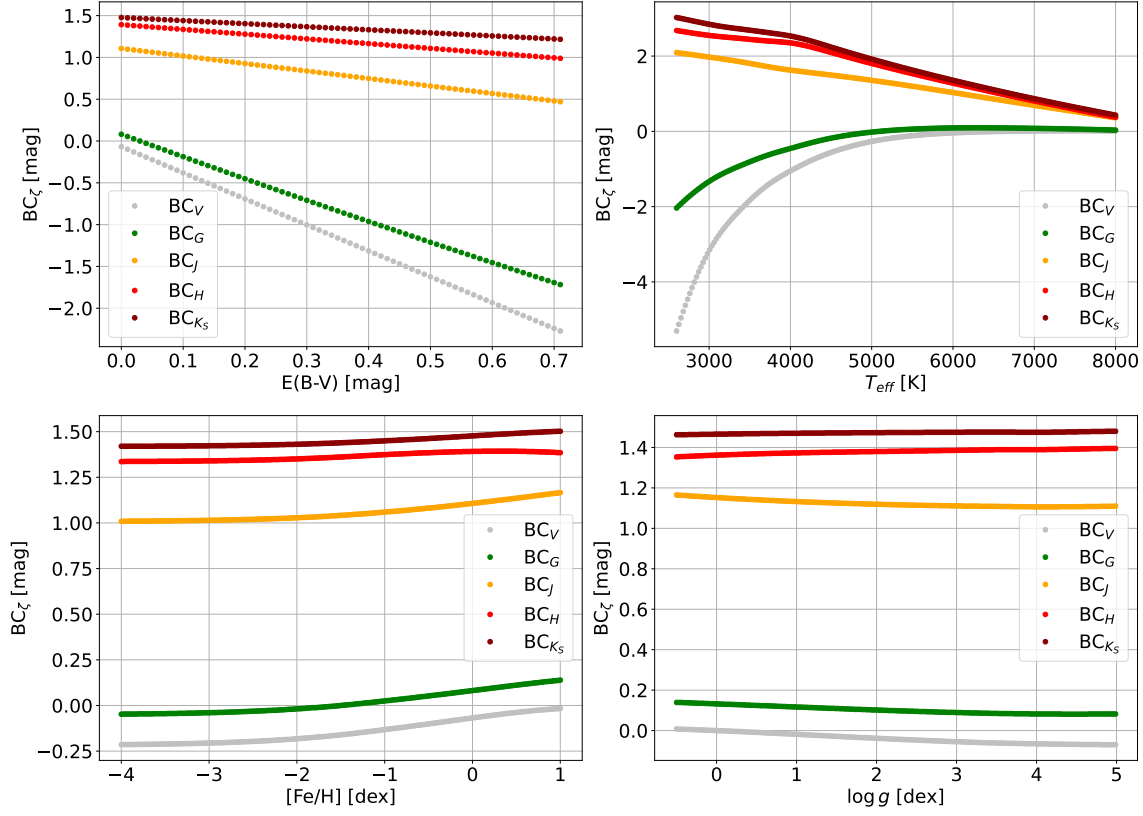


FIGURE 4.4: Sensitivity of bolometric correction values by BCCode when a given parameter is free to vary whilst keeping the others fixed. Solar values are used as reference.

the simulations following a gaussian distribution with standard deviation equal to the uncertainty. Figure 4.6 shows the BCCode results and in Figure 4.7 we get  $L^\zeta$ , the luminosity calculated if we choose the bolometric correction  $BC_\zeta$  applied to the band  $\zeta$ .

The fact that the luminosities in Figure 4.7 do not exactly coincide is evidence of discrepancies being at play, such as the propagation of systematic errors. We notice nonetheless that the measurements are in agreement with their uncertainties, hinting at a general agreement despite the fact that these arise from two different photometric surveys. It is evident that, whichever luminosity is correct, this particular example belongs to a star less luminous than the Sun and as a result our intuition tells us that this ought to be a star less massive than the Sun. Since we have to choose between one of these luminosities to be the canonical one used for the purposes of investigation,  $L^{Ks}$  is selected due to the presumption that this shall be the least affected by reddening and therefore the one with the greatest degree of accuracy. This standard is kept for all stars.

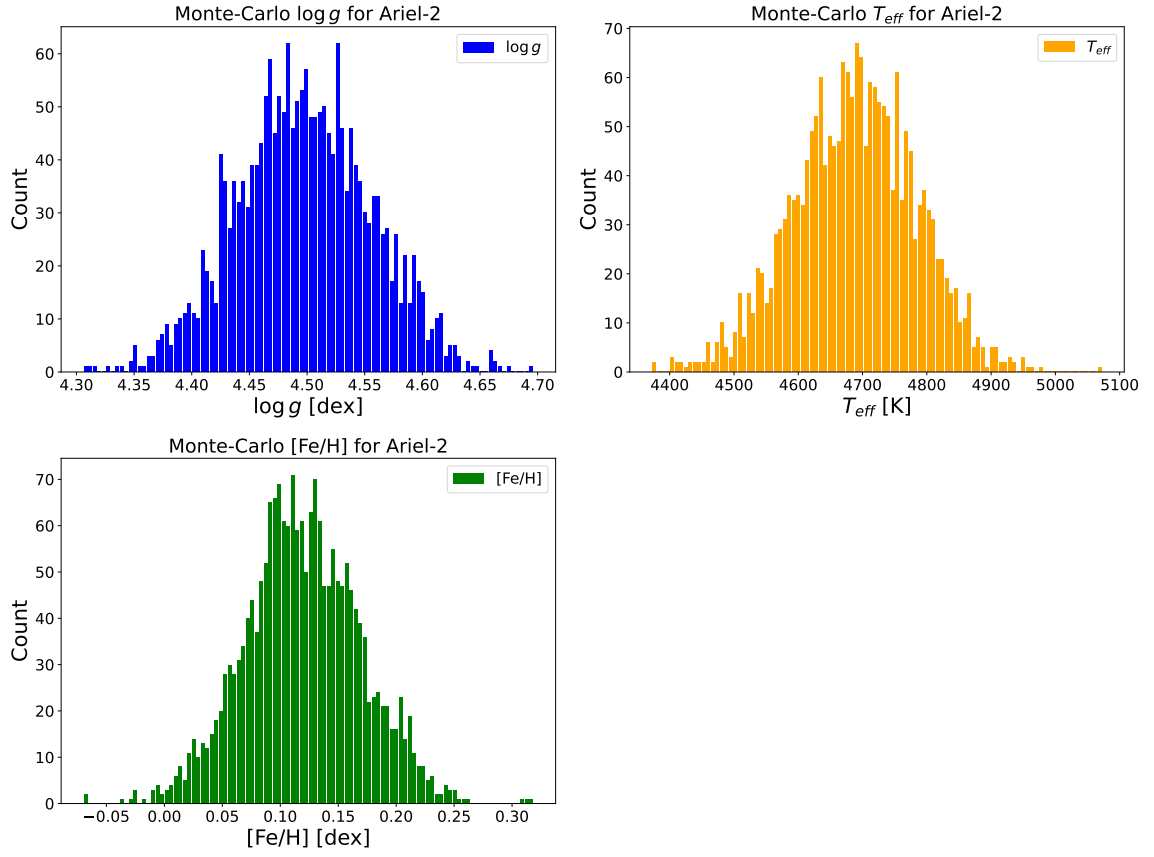


FIGURE 4.5: Monte-Carlo (N = 2000) simulations of the input parameters for the star Ariel-2.

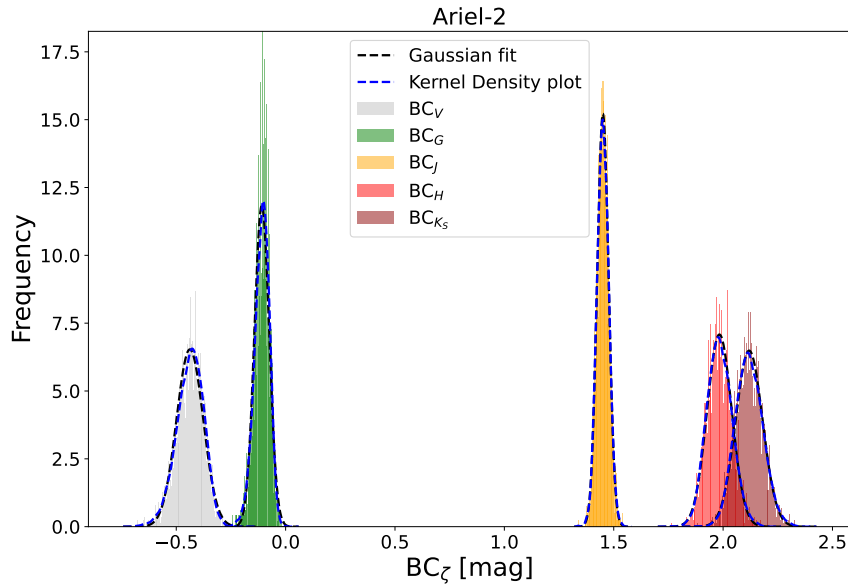
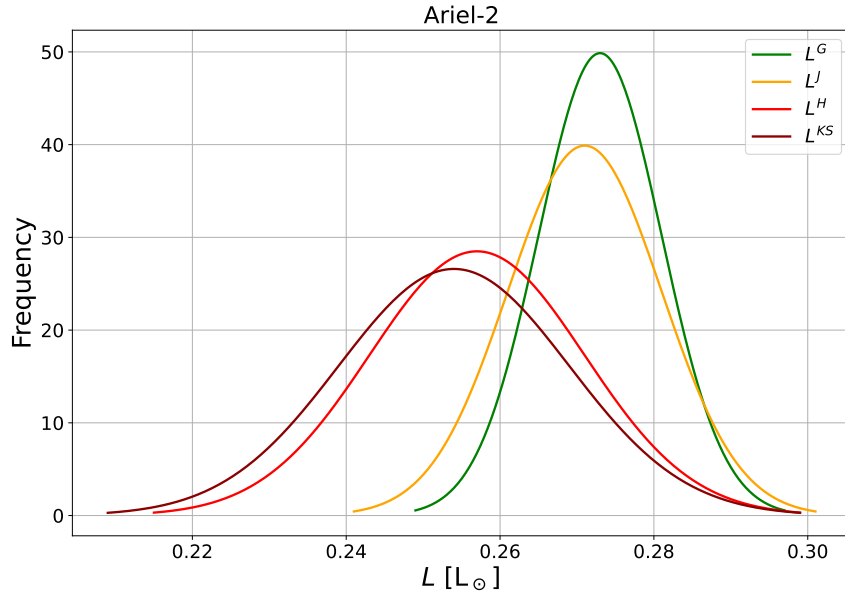


FIGURE 4.6: BCCode BC results for the different Monte-Carlo simulations of Ariel-2.

FIGURE 4.7: Calculated luminosity using different  $BC_z$  for Ariel-2.

### 4.3 Spectroscopic Constraints

The focus of this dissertation is the study of main sequence F-G-K stars in the *Ariel* input catalog, making it necessary to understand the parameter space of interest. The work of [Magrini et al. \(2022\)](#) provides homogeneous stellar parameters on 187 main sequence exoplanet-harboring F-G-K type stars in the *Ariel* input catalog, using an iterative spectro-photometric method to derive  $[Fe/H]$ ,  $T_{\text{eff}}$  and  $\log g$ . First, Stellar spectra was collected from the *UV-Visual Echelle Spectrograph* (UVES, [D’Odorico, 1997](#)) and from the *High Accuracy Radial velocity Planet Searcher* (HARPS-N, [Cosentino et al., 2012](#)) at the Northern hemisphere, the latter being supplemented with data from the *Potsdam Echelle Polarimetric and Spectroscopic Instrument* (PEPSI, [Strassmeier et al., 2015](#)). Second, photometric and astrometric data was then collected from the *Gaia* mission ([Gaia Collaboration et al., 2018](#)) and from the Two Micron All Star Survey (2MASS, [Skrutskie et al., 2006](#)). Together, this data was utilized to derive and constrain spectroscopic parameters.

The methods utilized for the spectral analysis in [Magrini et al. \(2022\)](#) is based on the absorption line equivalent widths (EWs), which were measured using DAOSPEC ([Stetson & Pancino, 2008](#)). A radiative transfer code computed the stellar parameters using MARCS model atmospheres ([Gustafsson et al., 2008](#)), which serve as a prior. This prior

set  $\{T_{\text{eff},0}, \log g_0, [\text{Fe}/\text{H}]_0\}$  was obtained by deriving the iron abundances from the absorption lines of Fe I and Fe II, establishing the excitation equilibrium which in turn allows to infer  $T_{\text{eff}}$ . Meanwhile the ionization balance helped derive  $\log g$ .

From this first set they used isochrone fitting to determine the mass,  $M$ . This, together with the bolometric magnitude,  $M_{\text{bol}}$ , and with  $T_{\text{eff},0}$  allowed the estimation the photometric surface gravity,  $\log g_{\text{phot}}$ . The spectral analysis was repeated and the values updated until a final set of parameters,  $\{T_{\text{eff}}, \log g, [\text{Fe}/\text{H}]\}$ , was obtained.

In order to obtain the luminosity, the paper made use of the relation

$$\log L = -0.4 \left( K_S + BC_{K_S} - \left( -5 + 5 \log \left( \frac{1}{p} \right) \right) - M_{\text{bol},\odot} - A_{K_S} \right), \quad (4.12)$$

where the parallax  $p$  is taken from Gaia eDR3 and  $A_{K_S}$  is the absorption in the  $K_S$  band. The bolometric correction and absorption were calculated using YBC. Figure 4.8 shows the distribution of the 187 F-G-K stars analyzed in that work overlayed on-top of a MESA generated grid (see Figure 2.7). Figures 4.9 and 4.10 show the distribution of stellar parameters and their respective uncertainties, respectively. Due to their asymmetric distributions, the median, rather than the average, may prove to be a better estimate of the standard uncertainty that we ought to expect.

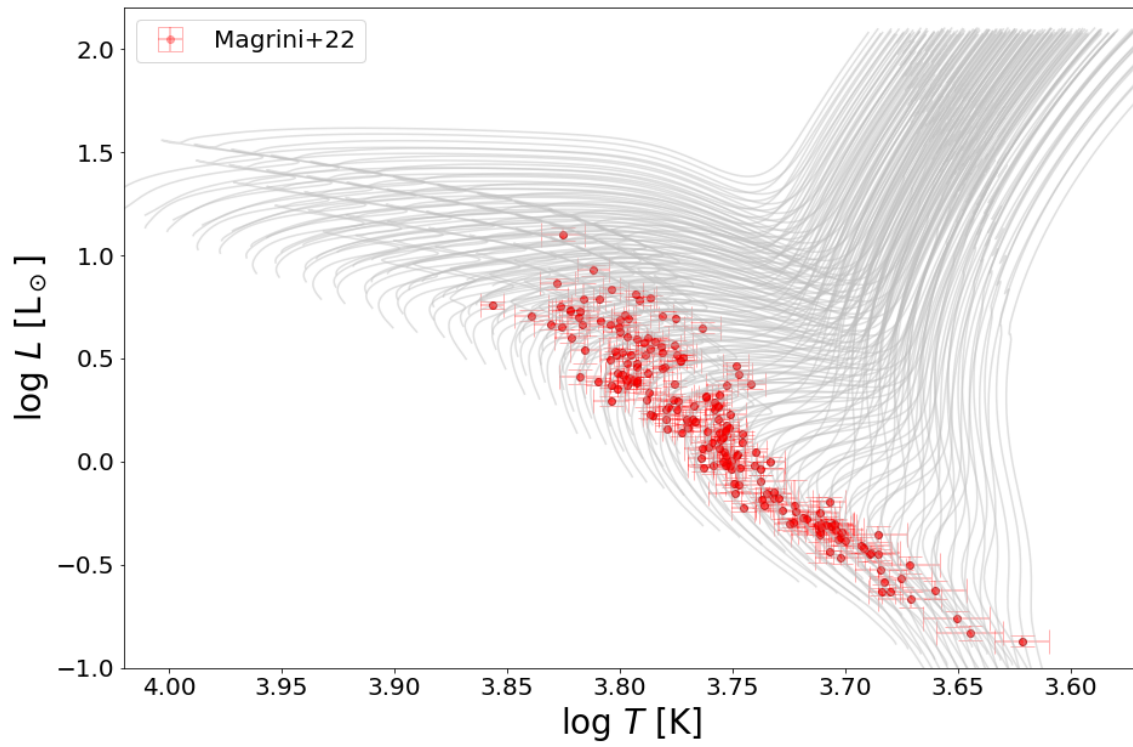


FIGURE 4.8: HR diagram with the position of 187 F-G-K stars with the central values and uncertainties obtained from [Magrini et al. \(2022\)](#) (red). The gray tracks is the MESA-generated grid shown in Figure 2.7.

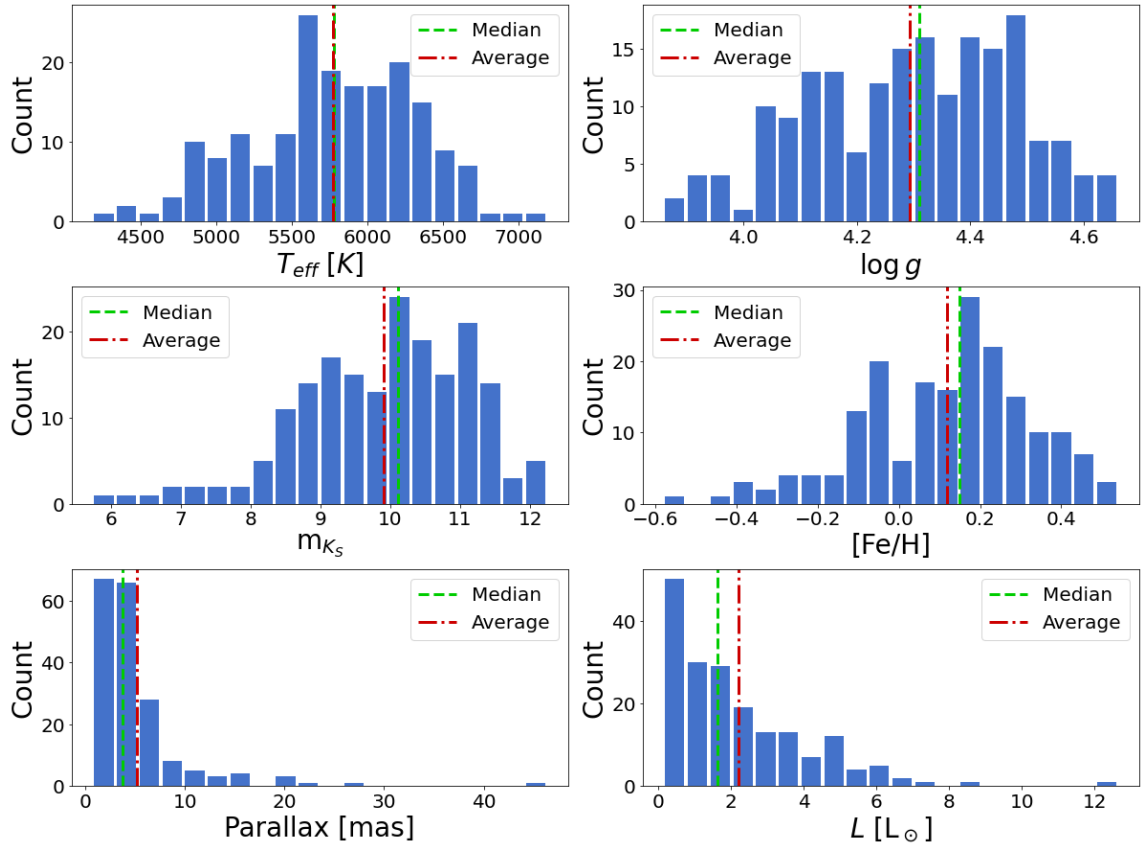


FIGURE 4.9: Stellar parameters of the 187 F-G-K stars from [Magrini et al. \(2022\)](#) with their median (green) and average (red) values.

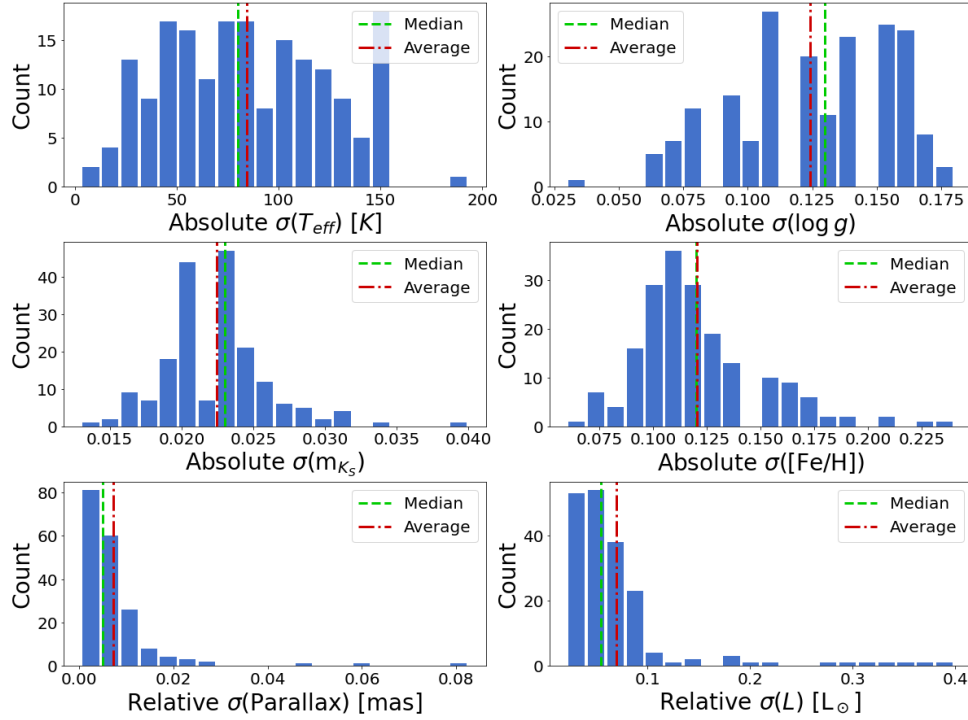


FIGURE 4.10: Stellar parameter uncertainties of the 187 F-G-K stars from [Magrini et al. \(2022\)](#) with their median (green) and average (red) values.

## 4.4 Asteroseismology

### 4.4.1 Power Spectral Density

As was shown in Figure 3.1, asteroseismic data can be collected either by spectroscopic radial velocity methods or by photometric time-series light-curve methods. Space telescopes like CoRoT, *Kepler* and TESS have provided a great deal of asteroseismic data on various stars, with future missions such as PLATO (Rauer et al., 2021) on the horizon to further broaden our knowledge on the seismology of stars. The data provided can be plotted as a power spectral density plot in the frequency domain. An example is given in Figure 4.11, which shows the PSD for the CoRoT target HD52265. On the right-hand side of this plot we can see, above the 1000  $\mu\text{Hz}$  frequency mark, a prominence which takes an approximately bell-curve shape. These are the acoustic modes and their shape is owed to the stochastic excitation induced by turbulent motion in the photosphere.

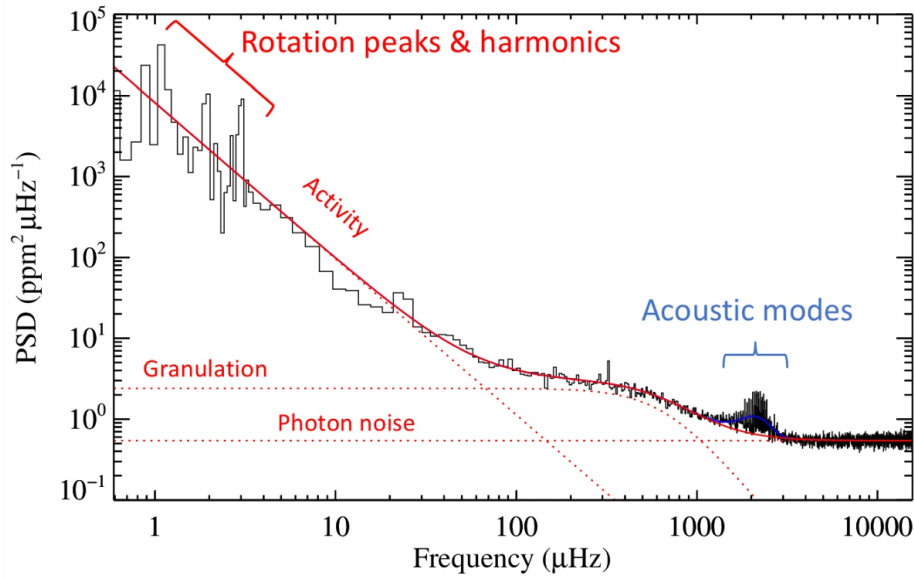


FIGURE 4.11: Power spectral density (PSD) of the CoRoT target HD52265. Acoustic modes represent the domain of interest in asteroseismology. Image taken from García & Ballot (2019).

This image shows the amount of components that go into the formation of the PSD, making it difficult to directly analyze the region of interest. Back in Chapter 3 we saw what a close-up of this portion of PSD would look like, such as in Figure 3.2 for the Sun and in Figure 3.5 for the *Kepler* target 16 Cyg A. Due to stochastic excitation in the convective envelope, the modes become pronounced according to a bell-curve shape that we call the Gaussian envelope. Modes close to the frequency of maximum power,  $\nu_{\text{max}}$ , are more

excited than modes further away from it. This difference of excitation results in difference of uncertainties for each mode, making it not as trivial to determine the global seismic parameters,  $\nu_{\max}$ ,  $\Delta\nu$  and  $\delta\nu_{02}$ .

Asteroseismic analysis is therefore a three step process involving first the estimation of the position of the power excess in the PSD, after which fitting and correction for the background can be performed and only then can the global seismic parameters be estimated. The paper [Huber et al. \(2013\)](#) describes the application of two types of time series filters: one to eliminate the effects resulting from the transit of exoplanets and another filter to eliminate the effects of stellar activity and instrument variability. More in-depth technical explanations for the acquisition of asteroseismic data are given in [Huber et al. \(2009\)](#), which shows how the precision of  $\nu_{\max}$  is a function of  $\nu_{\max}$  itself, whereas the precision for  $\Delta\nu$  is roughly constant with itself. This, combined with the need for a higher signal-to-noise ratio (S/N) for  $\nu_{\max}$ , leads to a greater difficulty in the determination of  $\nu_{\max}$  than in the determination of  $\Delta\nu$ . This explains why some of the *Ariel* stars analyzed in Chapter 6 have information on  $\Delta\nu$  but not on  $\nu_{\max}$ .



## Chapter 5

# PARAM

PARAM (Rodrigues et al., 2017) is a Bayesian method isochrone fitting tool that enables the implementation of observed stellar inputs such as the effective temperature,  $T_{\text{eff}}$ , surface metallicity,  $[M/H]$ , surface gravity,  $\log g$ , luminosity,  $L$ , along with their respective  $1\sigma$  uncertainties, to provide reliable estimates for the stellar mass,  $M$ , radius,  $R$ , and age. Among these input parameters, PARAM is also capable of accepting global asteroseismic values as input, specifically the large separation,  $\Delta\nu$ , and the frequency of maximum power,  $\nu_{\text{max}}$ . Of the various inputs that PARAM can accept, only the metallicity,  $[M/H]$ , is compulsory, meaning that other inputs are optional. This allows for many different input permutations, allowing for instance to explore the sensitivity that each input or combination of inputs have in the accuracy and precision of the outputs of the codes.

Given a set of input values,  $\vec{X}$ , with their respective  $1\sigma$  uncertainties,  $\sigma(\vec{X})$ , PARAM works by first loading a pre-built stellar grid, such as one constructed with MESA (see Section 2.6). From this grid it determines isochrones, hyperplanes that trace out equal stellar age across the parameter space. The program inspects these isochrones, making use of Bayesian statistics to trace out multivariate probability distributions,  $P(\cdot)$ , to look for the most probable matches between the input observation values,  $\vec{X}^{\text{In}}$ , and the pre-calculated models within the grid,  $\vec{X}^{\text{Grid}}$ . Once finished, PARAM returns marginalized univariate and bivariate probability distribution functions,  $P(\vec{Y})$ , on the output parameters,  $\vec{Y}$ , such as the age, mass and radius. These probability distributions provide the mode, median and their respective 68th percentile uncertainties, giving a wealth of insight into the effort of stellar characterization. These distributions are often asymmetric and sometimes display more than one dominant solution. Multiple pronounced peaks are

interpreted as multiple solutions and such cases are referred to as degeneracies. Precision is therefore not the only obstacle, but also accuracy and it is not at all trivial whether a star ought to be characterized by the mode of this distribution or by its median.

## 5.1 Global Seismic Parameters

There are three main global seismic properties:  $\nu_{\max}$ ,  $\Delta\nu$  and  $\delta\nu_{02}$ . The frequency of maximum power,  $\nu_{\max}$ , is not the frequency of the mode with the highest peak, but rather the frequency in which the stochastic turbulence has maximum exciting power. To infer  $\nu_{\max}$  we must therefore infer the stochastic excitation itself. The stochastic excitation can be modeled by fitting a Gaussian to the salient modes, given by

$$w = A \exp \left\{ -\frac{(\nu - \nu_{\max})^2}{2\sigma_w^2} \right\}, \quad (5.1)$$

where  $A$  is the amplitude,  $\nu_{\max}$  is the mean and  $\sigma_w$  is the Gaussian envelope width. As a result, to find  $\nu_{\max}$  one must simply find the mean of the Gaussian fit. This Gaussian envelope has further uses. Recall that the large and small frequency separations,

$$\Delta\nu_n \equiv \nu_{n,l=0} - \nu_{n-1,l=0}, \quad (5.2)$$

$$\delta\nu_{n,02} \equiv \nu_{n,l=0} - \nu_{n-1,l=2}, \quad (5.3)$$

are given by comparing the modes with one another. What these equations tell us is that due to the existence of many pairs of modes, many  $\Delta\nu_n$  and  $\delta\nu_{n,02}$  exist. We would like to have just one  $\Delta\nu$  and one  $\delta\nu_{02}$  to characterize the global seismic properties of the star.

The simplistic approach would be to compute all possible  $\Delta\nu_n$  and  $\delta\nu_{n,02}$  and then perform the average, however one must consider that not all modes are excited with the same power and as such they have varying levels of prominence. By association these modes have varying degrees of uncertainty, requiring them to be weighed differently from one another. We can make use of the Gaussian envelope in Equation 5.1 to provide the weights for the modes. PARAM calculates  $\Delta\nu$  using a linear regression,

$$\Delta\nu = \frac{\sum w \sum (w \cdot n_0 \cdot \nu_0) - \sum (w \cdot \nu_0)}{\sum w \sum (w \cdot n_0 \cdot n_0) - (\sum (w \cdot n_0))^2}, \quad (5.4)$$

where  $n_0$  stands for the modes  $n$  with  $l = 0$ ,  $\nu_0$  stands for the frequency at the mode  $(n, l = 0)$  and  $w$  is the weight of the corresponding mode.

As an example of this method we can use MESA to generate a  $M_{\text{ini}} = 1 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$  star and with GYRE generate the p-modes, as is plotted in Figure 5.1. There we can see the distribution of the  $l = 0$  and  $l = 2$  modes highlighted from the remaining GYRE modes.

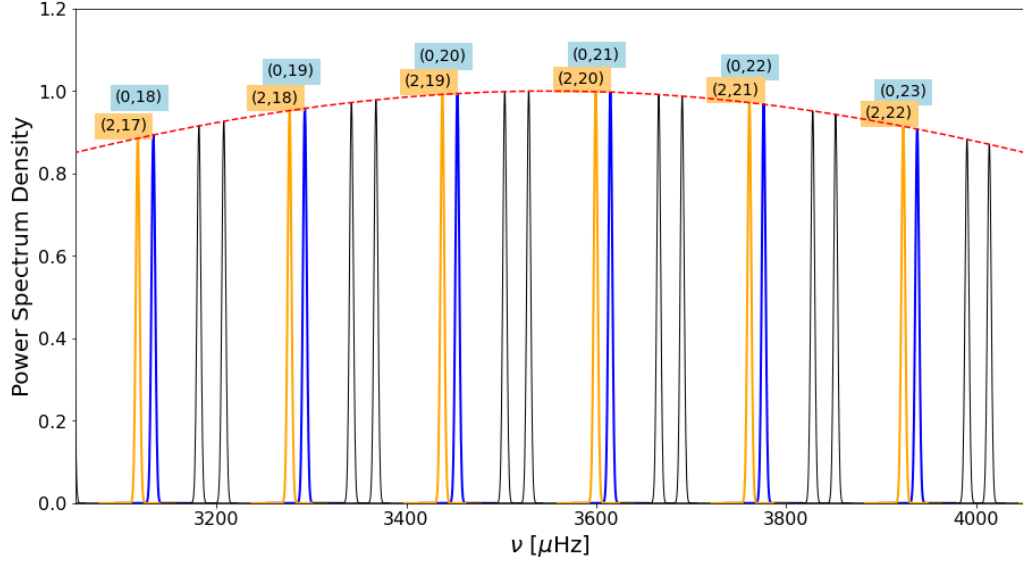


FIGURE 5.1: Power spectral density of p-modes generated by GYRE for a MESA generated  $M_{\text{ini}} = 1 M_{\odot}$  and  $[M/H]_{\text{ini}} = 0.0$  star at the start of the main sequence. Gaussian envelope plotted in red,  $l = 0$  modes plotted in blue,  $l = 2$  modes plotted in orange and the remaining modes in black. All modes represented as Gaussian curves with an artificial standard deviation of  $2 \mu\text{Hz}$ . Text shows  $(l, n)$  of the modes of interest.

In order to simulate the Gaussian behavior of the GYRE modes the Gaussian envelope was instead inferred. Using the scaling relation in Equation 3.6,  $\nu_{\text{max}}$  was calculated so that  $\sigma_w$  could be extracted from the empirical relation (see: Mosser et al., 2012) given by

$$\sigma_w = 0.66\nu_{\text{max}}^{0.88}, \quad (5.5)$$

Both  $\nu_{\text{max}}$  and  $\sigma_w$  were plugged into Equation 5.1 to create the Gaussian envelope with an arbitrary re-scaling such that the maximum is at unitary value. The modes have their maximum values adjusted accordingly and are treated as Gaussian shapes with an artificial standard deviation of  $2 \mu\text{Hz}$  for visualization purposes, since GYRE does not provide the profile of each mode.

Using Equation 5.4 we obtain a linear regression for the  $l = 0$  modes, from which we can extract the  $\Delta\nu$ . Figure 5.2 illustrates this. In this example,  $\nu_{\max} = 3551.2 \mu\text{Hz}$  and  $\Delta\nu = 162.5 \mu\text{Hz}$ .

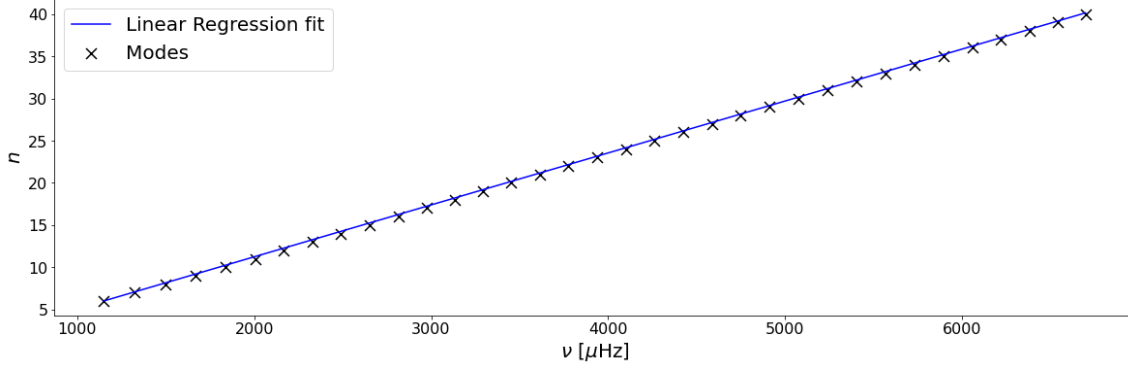


FIGURE 5.2: Linear regression of the  $l = 0$  modes to determine  $\Delta\nu$ .

Alternative weighing methods can be implemented. For example, although it is not utilized as of yet as an input parameter in PARAM, the determination of  $\delta\nu_{02}$  is already done in the PARAM toolkit by extracting the weight for each  $\delta\nu_{n,02}$ ,  $w(\delta\nu_{n,02})$ , and then taking the average of the weight between both modes,

$$w(\delta\nu_{n,02}) = \frac{w(\nu_{n,l=0}) + w(\nu_{n-1,l=2})}{2}, \quad (5.6)$$

followed by taking the weighed average,  $\delta\nu_{02} = \sum(w(\delta\nu_{n,02}) \cdot \delta\nu_{n,02})$ .

## 5.2 Assembling the PARAM grid

Due to the taxing memory requirements of MESA output files, as well as the need to process stellar oscillation data computed by GYRE, a toolkit was used to transform MESA and GYRE outputs into a format that is optimal for PARAM to read. This toolkit eliminates irrelevant spatial information, discards unwanted parameter data and then computes seismic parameters from the GYRE modes.  $\nu_{\max}$  is computed from scaling relations (Equation 3.6), whereas  $\Delta\nu$  is calculated by weighing the GYRE modes with an inferred Gaussian envelope width centered on  $\nu_{\max}$ , as discussed in Section 5.1.

All of this information is combined and the toolkit tries also to guess the various stages of stellar evolution presented in these tracks. If these guesses are not satisfying then the critical points which determine various points of interest can be altered manually. These

critical points are important to assist PARAM in interpolating the stellar tracks, particularly where ambiguity arises, such as between a track that possesses a hook and one which does not. Figure 5.3 illustrates the critical points defined by PARAM. These are:

1. MS\_BEG: The beginning of the main sequence
2. MS\_PT2: The beginning of the hook (if it exists)
3. MS\_PT3: The end of the main sequence
4. RG\_BASE: The end of the subgiant phase branch
5. RG\_B1: The beginning of the red-giant branch bump
6. RG\_B2: The end of the red-giant branch bump
7. RG\_TIP: Where the red-giant branch had been truncated ( $\log L \approx 2L_{\odot}$ ), not to be confused with the physical red-giant tip.

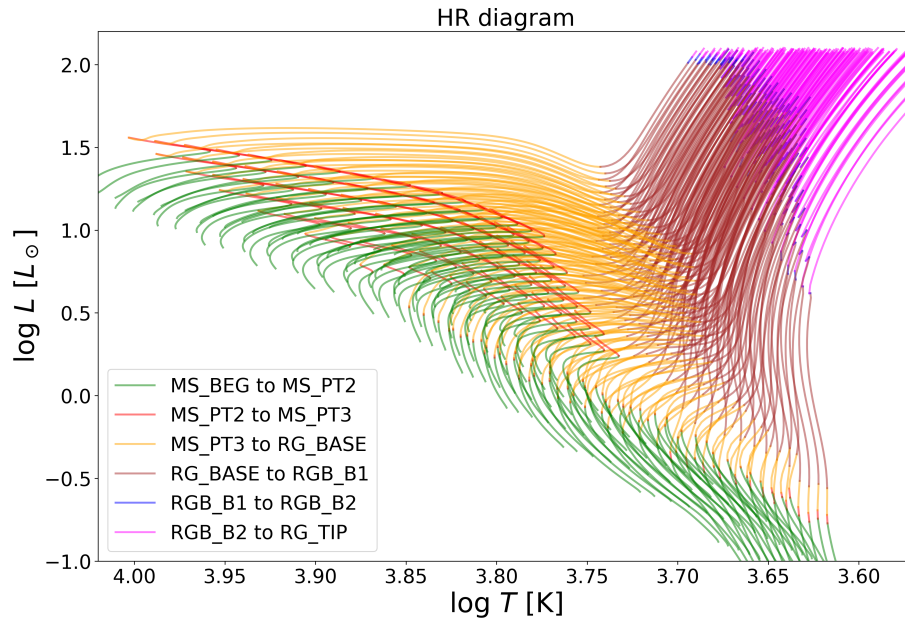


FIGURE 5.3: HR diagram of MESA-generated stars forming a grid ranging from  $M_{\text{ini}} \in [0.5, 1.8] M_{\odot}$  with a step of  $\Delta M = 0.1 M_{\odot}$  and initial metallicities of values  $[M/H]_{\text{ini}} \in [-0.70, +0.50]$  with spacing  $\Delta[M/H] = 0.10$ . Colour-coded according to different interpolative phases.

### 5.3 Internal Consistency Tests

To review the internal consistency of PARAM, two sets of synthetic stars were generated based on the exact values provided by the MESA grid which PARAM uses. One of

these sets is composed of 8 “cluster” stars (same initial age and metallicity), with masses ranging from  $M_{\text{ini}} \in [0.7, 1.4] M_{\odot}$ . Another set is composed of 5 stars taken along the same track (same initial mass and metallicity, different age). Figure 5.4 illustrates these sets, with their respective stellar tracks highlighted.

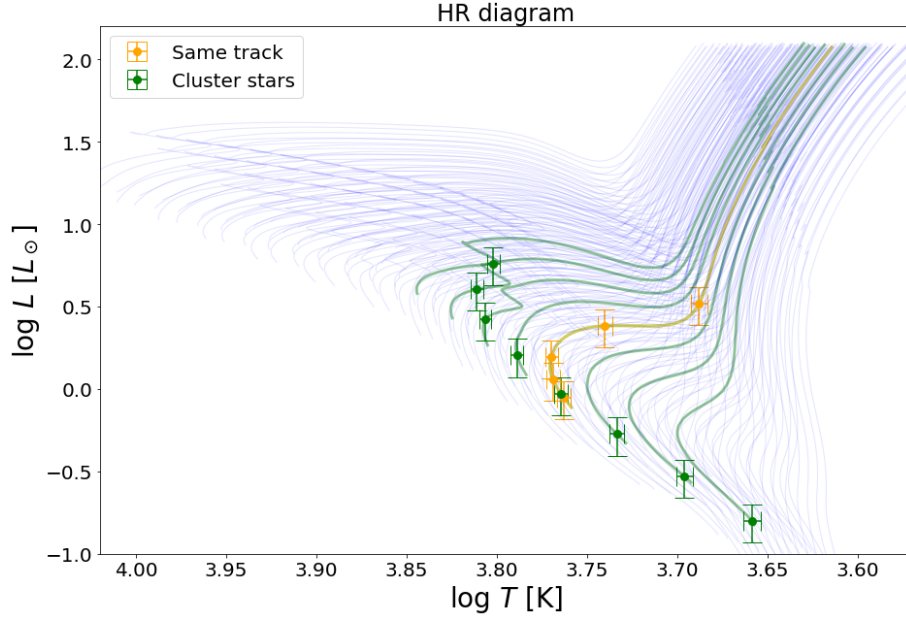


FIGURE 5.4: HR diagram of MESA-generated stars ranging from  $M_{\text{ini}} \in [0.7, 1.4] M_{\odot}$  with a step of  $\Delta M = 0.1 M_{\odot}$  and initial metallicities of values  $[M/H]_{\text{ini}} = 0.0$ . Green stars are “cluster” stars (same age and metallicity). Orange stars are part of the same track  $M_{\text{ini}} = 1.0$  and  $[M/H]_{\text{ini}} = 0.0$ . Corresponding stellar tracks highlighted.

The “cluster” stars, consisting of main sequence F-G-K stars with effective temperature and luminosities ranging from  $[4556, 6473]$  K and  $[0.158, 5.742]$   $L_{\odot}$ , respectively, were chosen to occupy a large range within the expected parameter space of the *Ariel* mission. For comparison, the 187 main sequence F-G-K stars in Magrini et al. (2022) occupy effective temperature and luminosities ranging from  $[4184, 7187]$  K and  $[0.13, 12.65]$   $L_{\odot}$ , respectively.

The central values of these synthetic stars are kept the same as the exact values from the grid. To test the self-consistency of PARAM, these synthetic stars will be fed into the program with artificial uncertainties on the input parameters,  $\{T_{\text{eff}}, [Fe/H], L, \Delta\nu, \nu_{\text{max}}\}$ . These uncertainties were generated as such:

- $T_{\text{eff}} \pm 50$  K, a constant uncertainty
- $[Fe/H] \pm 0.1$  dex, a constant uncertainty

- $\Delta\nu \pm 0.02\Delta\nu$ , a 2% uncertainty
- $\nu_{\max} \pm 0.02\nu_{\max}$ , a 2% uncertainty
- $L \pm \max\left(\left|L - 10^{\log_{10}(L) \pm 0.1}\right|\right)$ , where a symmetric uncertainty is considered for  $\log_{10} L$  and the uncertainty chosen for  $L$  is the largest of the asymmetric uncertainty when converted to linear.

Figures 5.5 and 5.6 exemplify what a summarized view of a PARAM output looks like. Two outputs are shown. First, the output for the synthetic star cluster-4, which was initialized with  $M_{\text{ini}} = 1.0$  and  $[M/H]_{\text{ini}} = 0.0$ . Second, the output for the synthetic star cluster-8, which was initialized with  $M_{\text{ini}} = 1.4$  and  $[M/H]_{\text{ini}} = 0.0$ . PARAM outputs show a corner plot of the univariate and bivariate probability distributions on mass, age and radius. Green values indicate the median, blue the mode and red the exact values. PARAM output corner plots of the Age/Mass/Radius of all the stars considered in this dissertation can be found in an online repository<sup>1</sup>.

Figure 5.6 is interesting because, unlike Figure 5.5, a second somewhat prominent peak emerges. This is what we would call a double-solution, a feature that occurs for stars of masses  $M \gtrsim 1.2 M_{\odot}$  due to the emergence of the hook feature in the HR diagram, causing degenerate solutions for stars inhabiting such zones in the HR diagram.

Figures 5.7 and 5.8 show the relative accuracies on mass, age and radius for “cluster” and same track synthetic stars, respectively, as well as their uncertainties. Here accuracy is defined as the distance between the guessed value and the true value (zero-point in the plot), whereas the uncertainty is represented by the bars with caps. We can use either the mode or the median (and their respective uncertainties) to characterize the star, with results tending to be in general more favorable to the median. The advantage of using the median, rather than the mode, is that it will encompass both double-solutions when these arise, rather than insisting on one of them.

Table 5.1 shows the second quartile (Q2) relative accuracy and uncertainties for the median estimates. For synthetic “cluster” stars, the relative accuracy are in the order of 1% for mass, 8% for age and 1% for radius. For synthetic same track stars the relative

<sup>1</sup><https://github.com/AndreasWNeitzel/MSc-Thesis.PARAM.Outputs>

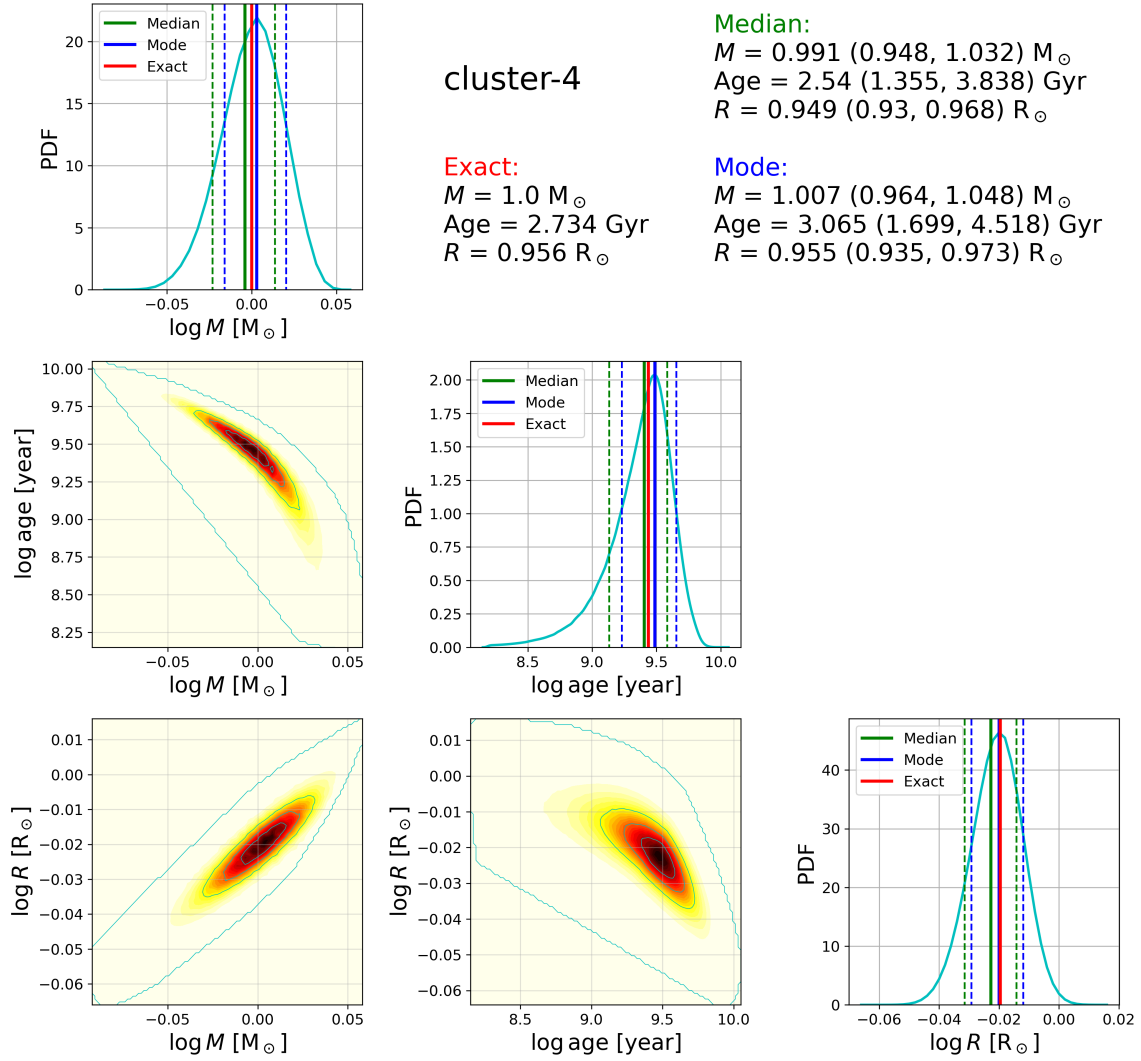


FIGURE 5.5: PARAM corner plot output of a synthetic “cluster” star with  $M_{\text{ini}} = 1.0$  and  $[M/H]_{\text{ini}} = 0.0$ . Values in parenthesis show asymmetric uncertainties.

accuracies are in the order of less than 1% for the mass, 3% for the age and less than 1% for the radius. We note the following characteristics. First, the relative accuracies generally stay well within the relative uncertainties for both the cluster and same track sets. From this result we can conclude that PARAM has a high degree of accuracy and that we should trust that the real results are encapsulated with the uncertainties. Second, from the inspection of the Figures 5.7 and 5.8 we can see that stars of higher mass have very good results for the determination of age, whereas low mass stars, as well as stars of low age, show a highly pronounced relative uncertainty of age, with stars of low mass in particular being more difficult to constrain accuracy in age. Figure 2.8 shows why this effect occurs. Both the main sequence and the tracks of low mass stars HR diagram show a far greater variation in age than in other regions. As a result, a tiny deviation in the

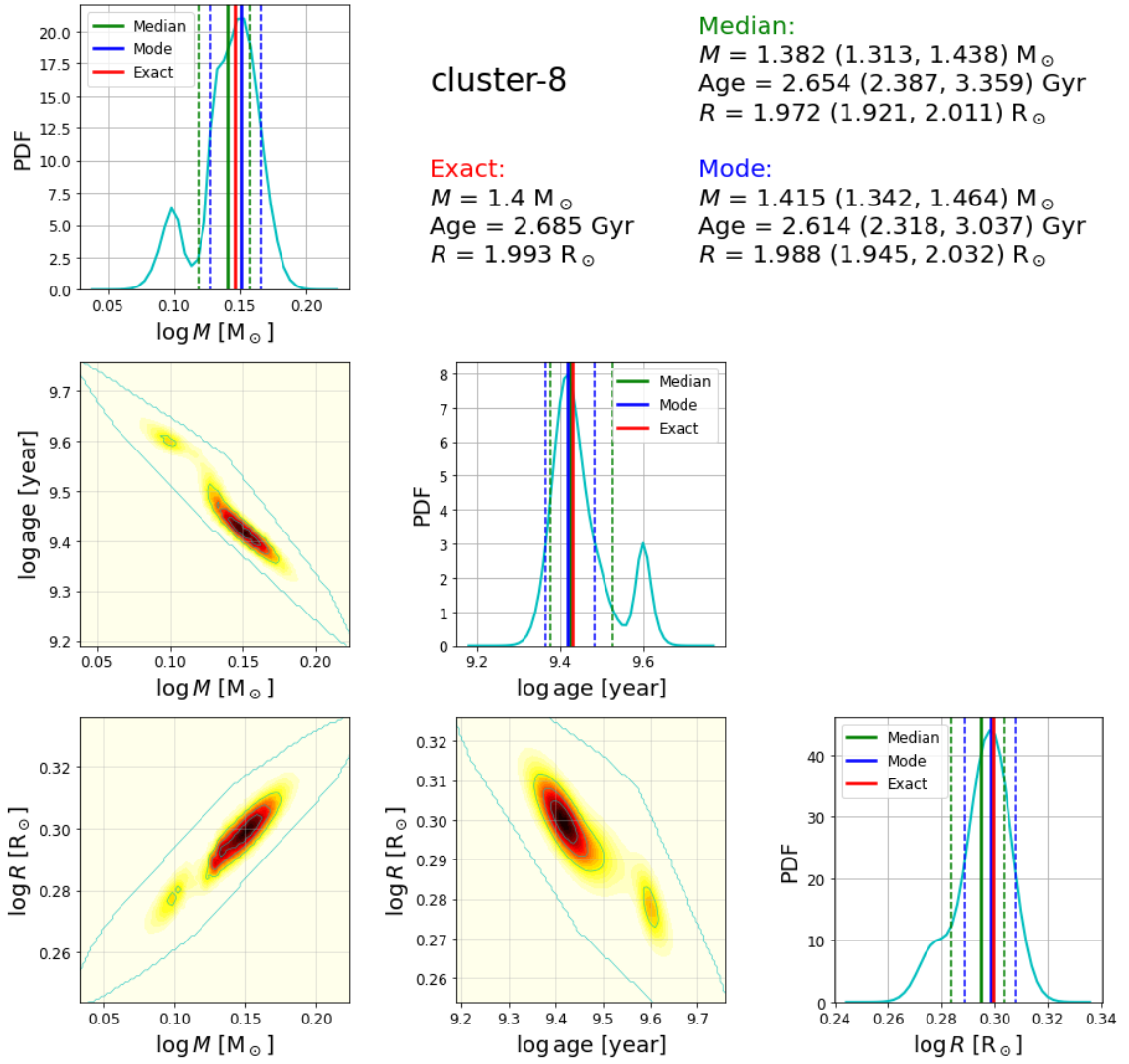


FIGURE 5.6: PARAM corner plot output of a synthetic “cluster” star with  $M_{\text{ini}} = 1.4$  and  $[M/H]_{\text{ini}} = 0.0$ . Values in parenthesis show asymmetric uncertainties.

input parameters leads to great difficulty in the age estimation.

The results from PARAM are self-consistent and valid. One must remember, however, that this synthetic stars exercise consists of an ideal situation, since in the real world we deal not only with problems in precision but also with problems in accuracy. Realistically, due to biases in the instrumentation for example, the central values of the real world data do not coincide with their real values. Furthermore, since PARAM makes use of computationally simulated theoretical data, the results are dependant on our understanding of the physics that regiment the lifecycle of stars. This exercise serves therefore as a first order estimate of the competences of PARAM.

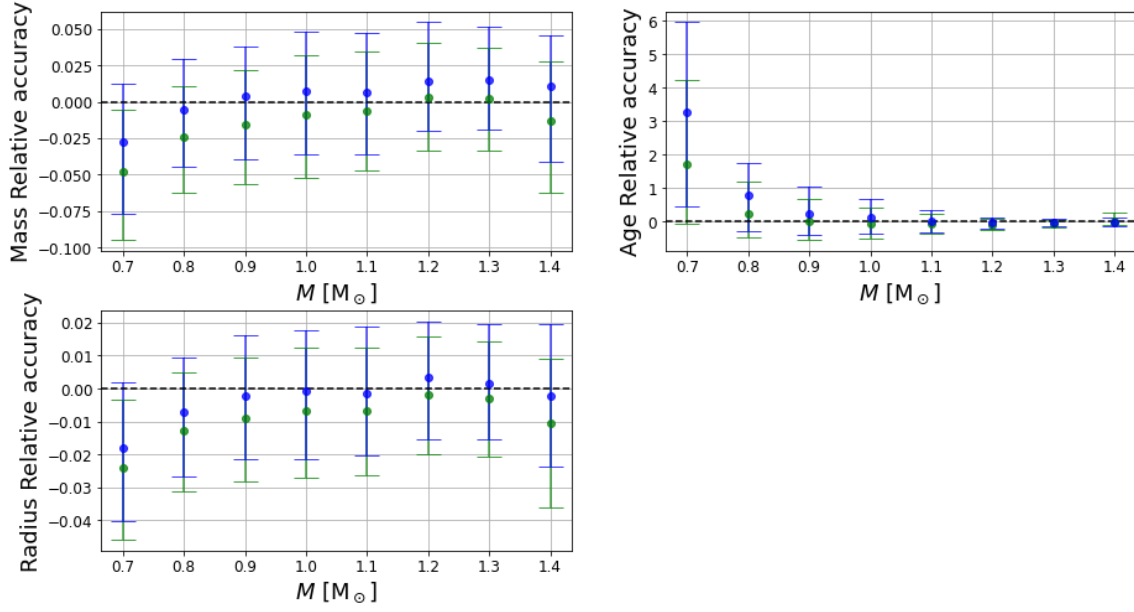


FIGURE 5.7: Relative accuracy of synthetic "cluster" stars in PARAM. Green: median. Blue: mode.

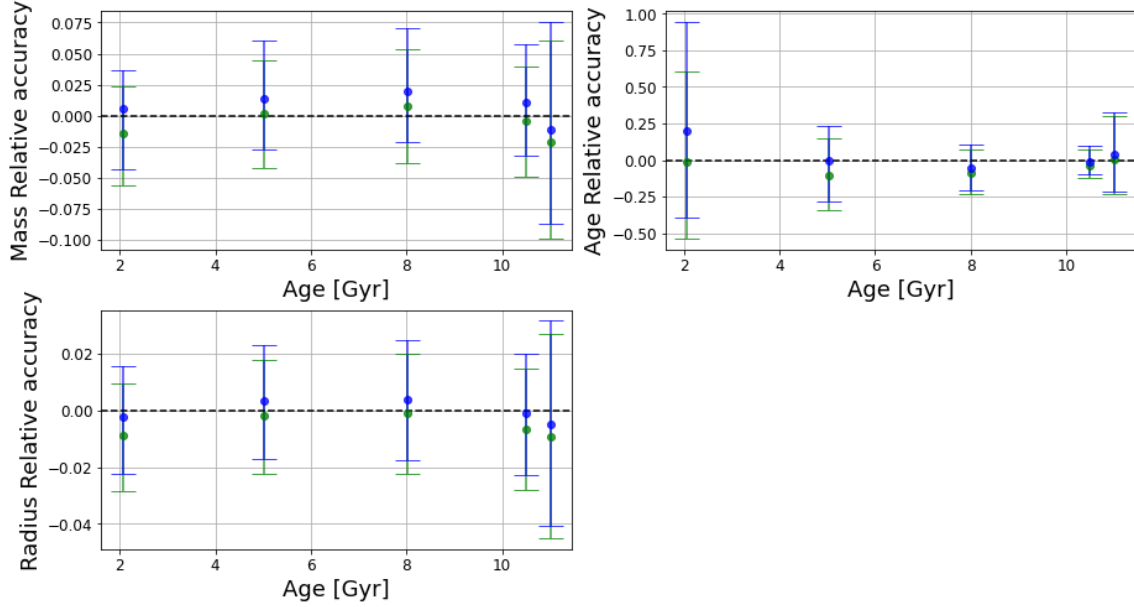


FIGURE 5.8: Relative accuracy of synthetic same track stars in PARAM. Green: median. Blue: mode.

| Set     | $\frac{M - M_r}{M_r}$ | $\frac{\sigma(M)}{M}$ | $\frac{\text{Age} - \text{Age}_r}{\text{Age}_r}$ | $\frac{\sigma(\text{Age})}{\text{Age}}$ | $\frac{R - R_r}{R_r}$ | $\frac{\sigma(R)}{R}$ |
|---------|-----------------------|-----------------------|--------------------------------------------------|-----------------------------------------|-----------------------|-----------------------|
|         | $M_r$                 | $M$                   | $\text{Age}_r$                                   | $\text{Age}$                            | $R_r$                 | $R$                   |
|         | [%]                   | [%]                   | [%]                                              | [%]                                     | [%]                   | [%]                   |
| Cluster | 1.1                   | 4.1                   | 7.9                                              | 43.6                                    | 0.8                   | 2.0                   |
| Track   | 0.7                   | 4.5                   | 3.4                                              | 28.2                                    | 0.6                   | 2.1                   |

TABLE 5.1: Table of the second quartile (Q2) relative accuracies and relative uncertainties of the median estimates for the sets of cluster and same track synthetic stars. The subscript r refers to the reference (real) value, whereas  $\sigma$  refers to the uncertainty.



# Chapter 6

## Results

### 6.1 PARAM vs Other Codes

In order to determine just how faithful PARAM is, and whether it can stand up to other stellar characterization programs, comparative tests were done on a set of 33 *Kepler Objects of Interest* (KOIs) exoplanet-harboring seismic stars taken from table 3 of [Silva Aguirre et al. \(2015\)](#), which provides as input parameters the  $\Delta\nu$ ,  $\nu_{\max}$ ,  $T_{\text{eff}}$  and  $[\text{Fe}/\text{H}]$  from [Huber et al. \(2013\)](#). In [Silva Aguirre et al. \(2015\)](#) these stars were tested on a variety of different stellar characterization programs to determine the age and mass, using individual frequencies as a constraint. The results can be found in the paper’s online repository (Strasbourg Astronomical Data Center). Figure 6.1 plots the distribution of the input parameters,  $\{\nu_{\max}, \Delta\nu, T_{\text{eff}}, [\text{Fe}/\text{H}]\}$ , which are given in full in Table 6.1.

These stars were ran in PARAM to determine its mass and age. Results are shown in Table 6.2. The outputs are shown as a histogram in Figure 6.2 and as a scatter plot in Figure 6.3, with the Sun as a reference. As is the case here, PARAM may estimate stellar ages that are older than the universe. Although such results ought not to be trusted (no star can be older than the universe), PARAM has no built-in age weighting system or other such checks to prevent these results. This is intentional, as the purpose is to come as close as possible to reality without tampering with the statistics. Inflated ages are expected for lower mass stars (see Figure 5.7) due to their parameter neighbourhood encompassing an extremely wide range of ages (see Figure 2.8).

| Star           | $\Delta\nu$<br>[ $\mu\text{Hz}$ ] | $\sigma(\Delta\nu)$<br>[ $\mu\text{Hz}$ ] | $\nu_{\text{max}}$<br>[ $\mu\text{Hz}$ ] | $\sigma(\nu_{\text{max}})$<br>[ $\mu\text{Hz}$ ] | [Fe/H]<br>[dex] | $\sigma([\text{Fe}/\text{H}])$<br>[dex] | $T_{\text{eff}}$<br>[K] | $\sigma(T_{\text{eff}})$<br>[K] |
|----------------|-----------------------------------|-------------------------------------------|------------------------------------------|--------------------------------------------------|-----------------|-----------------------------------------|-------------------------|---------------------------------|
| False-positive | 70.00                             | 1.00                                      | 1303                                     | 30                                               | -0.11           | 0.11                                    | 5784                    | 98                              |
| HAT-P7         | 59.22                             | 0.59                                      | 1115                                     | 110                                              | 0.26            | 0.08                                    | 6350                    | 80                              |
| KOI-268        | 92.60                             | 1.50                                      | 2038                                     | 60                                               | -0.04           | 0.10                                    | 6343                    | 85                              |
| KOI-269        | 88.60                             | 1.30                                      | 1895                                     | 73                                               | 0.09            | 0.11                                    | 6463                    | 110                             |
| KOI-276        | 107.90                            | 1.90                                      | 2381                                     | 95                                               | -0.02           | 0.10                                    | 5982                    | 82                              |
| KOI-280        | 128.80                            | 1.30                                      | 2928                                     | 97                                               | -0.24           | 0.10                                    | 6134                    | 91                              |
| KOI-281        | 77.20                             | 1.30                                      | 1458                                     | 57                                               | -0.40           | 0.11                                    | 5622                    | 106                             |
| KOI-288        | 53.54                             | 0.32                                      | 1008                                     | 21                                               | 0.22            | 0.10                                    | 6174                    | 92                              |
| KOI-319        | 52.20                             | 0.70                                      | 938                                      | 72                                               | 0.17            | 0.08                                    | 5924                    | 50                              |
| KOI-5          | 61.98                             | 0.96                                      | 1153                                     | 76                                               | 0.17            | 0.05                                    | 5945                    | 60                              |
| KOI-75         | 38.90                             | 0.80                                      | 644                                      | 8                                                | -0.22           | 0.08                                    | 5824                    | 50                              |
| KOI-974        | 60.05                             | 0.27                                      | 1115                                     | 32                                               | -0.13           | 0.10                                    | 6253                    | 75                              |
| Kepler-10      | 118.20                            | 0.20                                      | 2730                                     | 280                                              | -0.15           | 0.10                                    | 5647                    | 74                              |
| Kepler-100     | 77.00                             | 1.10                                      | 1502                                     | 31                                               | 0.02            | 0.10                                    | 5825                    | 75                              |
| Kepler-103     | 81.50                             | 1.60                                      | 1663                                     | 56                                               | 0.07            | 0.11                                    | 5845                    | 88                              |
| Kepler-109     | 91.10                             | 2.30                                      | 1745                                     | 117                                              | -0.08           | 0.10                                    | 5952                    | 75                              |
| Kepler-126     | 92.85                             | 0.35                                      | 1983                                     | 37                                               | -0.26           | 0.07                                    | 6205                    | 100                             |
| Kepler-128     | 68.80                             | 0.64                                      | 1324                                     | 39                                               | -0.11           | 0.07                                    | 6105                    | 100                             |
| Kepler-129     | 69.20                             | 1.40                                      | 1395                                     | 40                                               | 0.29            | 0.10                                    | 5770                    | 83                              |
| Kepler-145     | 61.80                             | 0.76                                      | 1133                                     | 81                                               | 0.13            | 0.10                                    | 6144                    | 106                             |
| Kepler-21      | 60.86                             | 0.55                                      | 1153                                     | 32                                               | -0.03           | 0.10                                    | 6305                    | 50                              |
| Kepler-25      | 98.27                             | 0.57                                      | 2106                                     | 50                                               | -0.04           | 0.10                                    | 6270                    | 79                              |
| Kepler-36      | 67.90                             | 1.20                                      | 1250                                     | 44                                               | -0.24           | 0.06                                    | 5960                    | 80                              |
| Kepler-37      | 178.70                            | 1.40                                      | 4660                                     | 50                                               | -0.32           | 0.07                                    | 5417                    | 75                              |
| Kepler-4       | 74.40                             | 1.10                                      | 1436                                     | 42                                               | 0.09            | 0.10                                    | 5781                    | 76                              |
| Kepler-408     | 103.20                            | 0.63                                      | 2193                                     | 48                                               | -0.20           | 0.10                                    | 6104                    | 74                              |
| Kepler-409     | 153.18                            | 0.14                                      | 3546                                     | 119                                              | 0.08            | 0.10                                    | 5460                    | 75                              |
| Kepler-410     | 94.50                             | 0.60                                      | 2014                                     | 32                                               | 0.01            | 0.10                                    | 6325                    | 75                              |
| Kepler-444     | 179.64                            | 0.76                                      | 4538                                     | 144                                              | -0.37           | 0.09                                    | 5046                    | 74                              |
| Kepler-50      | 75.71                             | 0.31                                      | 1496                                     | 56                                               | -0.00           | 0.08                                    | 6225                    | 75                              |
| Kepler-65      | 89.56                             | 0.48                                      | 1880                                     | 60                                               | 0.09            | 0.08                                    | 6169                    | 50                              |
| Kepler-68      | 101.57                            | 0.10                                      | 2154                                     | 13                                               | 0.12            | 0.07                                    | 5793                    | 74                              |
| Kepler-92      | 66.60                             | 1.10                                      | 1299                                     | 53                                               | 0.17            | 0.11                                    | 5871                    | 94                              |
| Kepler-93      | 145.77                            | 0.45                                      | 3366                                     | 81                                               | -0.18           | 0.10                                    | 5669                    | 75                              |
| Kepler-95      | 83.60                             | 1.40                                      | 1677                                     | 90                                               | 0.30            | 0.10                                    | 5699                    | 74                              |

TABLE 6.1: Table of the 33 stars input parameters with their respective uncertainties.

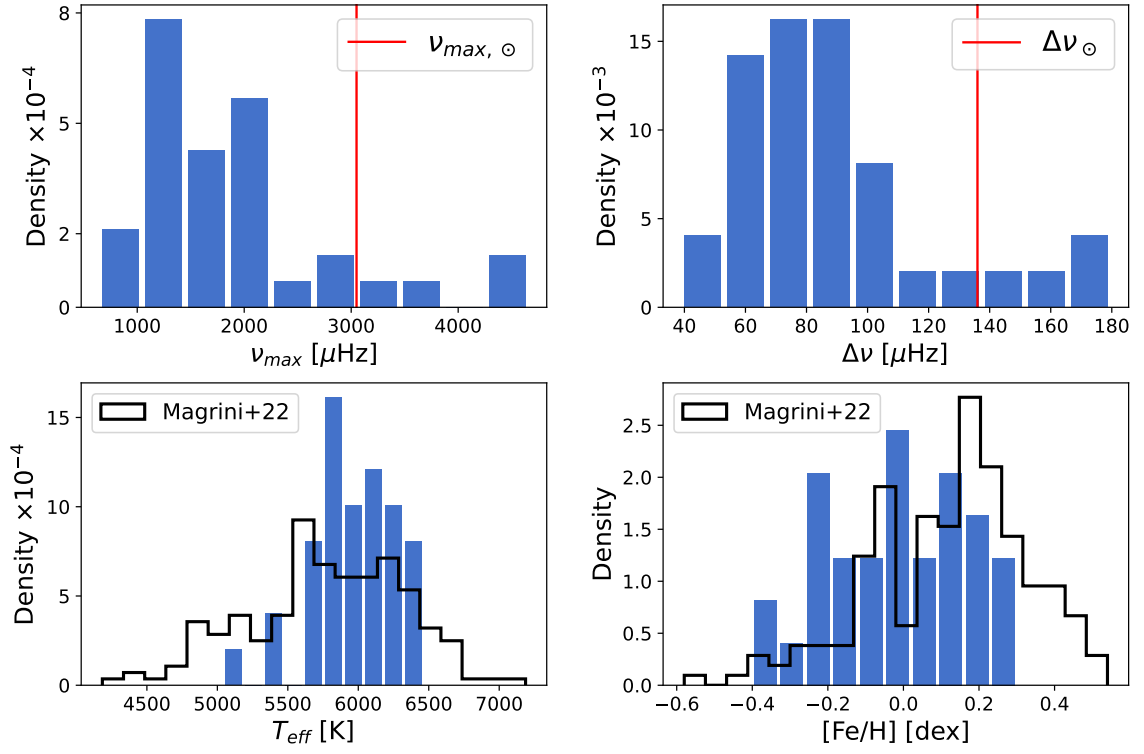


FIGURE 6.1: Histogram of PARAM inputs. Red vertical lines are solar values. Black represents the distribution in [Magrini et al. \(2022\)](#).

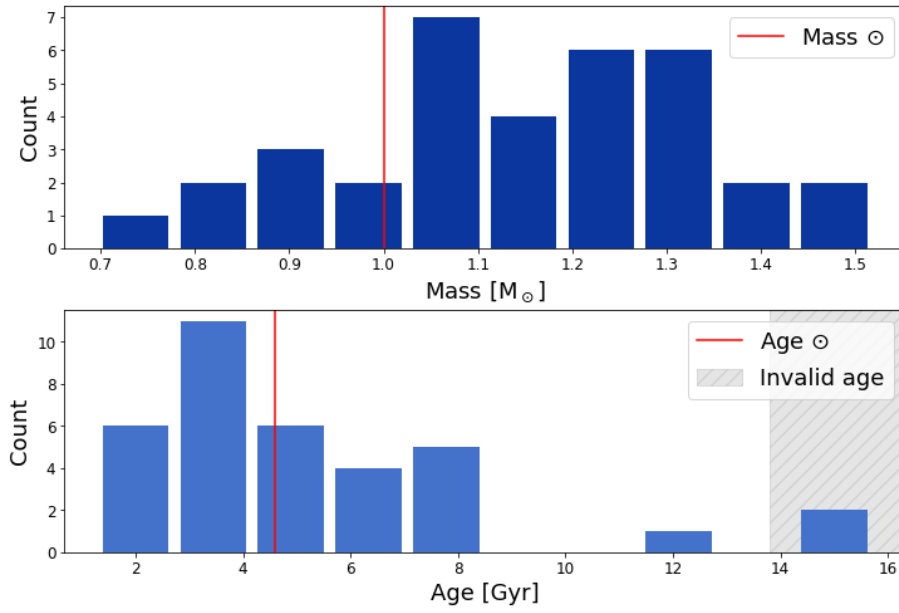


FIGURE 6.2: Histogram of PARAM mass and age output with the Sun as reference. Shaded region shows ages older than the universe.

| Star           | Age<br>[Gyr]               | $M$<br>[ $M_{\odot}$ ]    |
|----------------|----------------------------|---------------------------|
| False-positive | $8.426^{+1.466}_{-1.335}$  | $1.029^{+0.057}_{-0.055}$ |
| HAT-P7         | $2.041^{+0.239}_{-0.224}$  | $1.518^{+0.041}_{-0.042}$ |
| KOI-268        | $2.625^{+0.639}_{-0.58}$   | $1.218^{+0.049}_{-0.049}$ |
| KOI-269        | $1.927^{+0.609}_{-0.528}$  | $1.306^{+0.057}_{-0.057}$ |
| KOI-276        | $5.124^{+1.452}_{-1.29}$   | $1.056^{+0.053}_{-0.052}$ |
| KOI-280        | $3.389^{+1.429}_{-1.258}$  | $1.007^{+0.049}_{-0.049}$ |
| KOI-281        | $14.911^{+2.712}_{-2.339}$ | $0.832^{+0.05}_{-0.047}$  |
| KOI-288        | $2.346^{+0.301}_{-0.251}$  | $1.516^{+0.045}_{-0.05}$  |
| KOI-319        | $4.332^{+0.325}_{-1.315}$  | $1.296^{+0.163}_{-0.042}$ |
| KOI-5          | $3.88^{+1.494}_{-0.503}$   | $1.321^{+0.043}_{-0.119}$ |
| KOI-75         | $3.845^{+0.338}_{-0.275}$  | $1.247^{+0.046}_{-0.046}$ |
| KOI-974        | $3.043^{+1.167}_{-0.3}$    | $1.329^{+0.041}_{-0.113}$ |
| Kepler-10      | $11.88^{+2.531}_{-2.308}$  | $0.872^{+0.049}_{-0.047}$ |
| Kepler-100     | $7.234^{+1.29}_{-1.905}$   | $1.083^{+0.094}_{-0.053}$ |
| Kepler-103     | $6.073^{+1.912}_{-1.282}$  | $1.127^{+0.067}_{-0.079}$ |
| Kepler-109     | $7.298^{+1.528}_{-1.48}$   | $1.026^{+0.062}_{-0.056}$ |
| Kepler-126     | $3.695^{+0.541}_{-0.486}$  | $1.137^{+0.027}_{-0.024}$ |
| Kepler-128     | $4.014^{+1.654}_{-0.528}$  | $1.226^{+0.041}_{-0.106}$ |
| Kepler-129     | $4.475^{+1.601}_{-0.567}$  | $1.284^{+0.042}_{-0.1}$   |
| Kepler-145     | $3.14^{+1.481}_{-0.494}$   | $1.358^{+0.064}_{-0.131}$ |
| Kepler-21      | $2.755^{+0.331}_{-0.25}$   | $1.366^{+0.046}_{-0.051}$ |
| Kepler-25      | $2.963^{+0.697}_{-0.641}$  | $1.177^{+0.046}_{-0.046}$ |
| Kepler-36      | $7.582^{+1.012}_{-0.932}$  | $1.021^{+0.041}_{-0.039}$ |
| Kepler-37      | $1.278^{+0.995}_{-0.695}$  | $0.851^{+0.018}_{-0.018}$ |
| Kepler-4       | $7.3^{+1.305}_{-1.895}$    | $1.095^{+0.098}_{-0.054}$ |
| Kepler-408     | $5.164^{+1.144}_{-1.051}$  | $1.049^{+0.047}_{-0.045}$ |
| Kepler-409     | $5.555^{+2.303}_{-2.121}$  | $0.903^{+0.044}_{-0.042}$ |
| Kepler-410     | $2.42^{+0.542}_{-0.486}$   | $1.236^{+0.041}_{-0.041}$ |
| Kepler-444     | $15.723^{+4.231}_{-3.711}$ | $0.696^{+0.031}_{-0.031}$ |
| Kepler-50      | $3.484^{+0.583}_{-0.436}$  | $1.249^{+0.041}_{-0.05}$  |
| Kepler-65      | $3.407^{+0.483}_{-0.478}$  | $1.214^{+0.038}_{-0.036}$ |
| Kepler-68      | $6.278^{+0.963}_{-0.946}$  | $1.072^{+0.028}_{-0.027}$ |
| Kepler-92      | $4.252^{+2.052}_{-0.647}$  | $1.284^{+0.055}_{-0.135}$ |
| Kepler-93      | $6.924^{+2.04}_{-2.016}$   | $0.885^{+0.043}_{-0.041}$ |
| Kepler-95      | $6.607^{+1.719}_{-1.37}$   | $1.133^{+0.067}_{-0.061}$ |

TABLE 6.2: Age and mass PARAM output of the 33 stars.

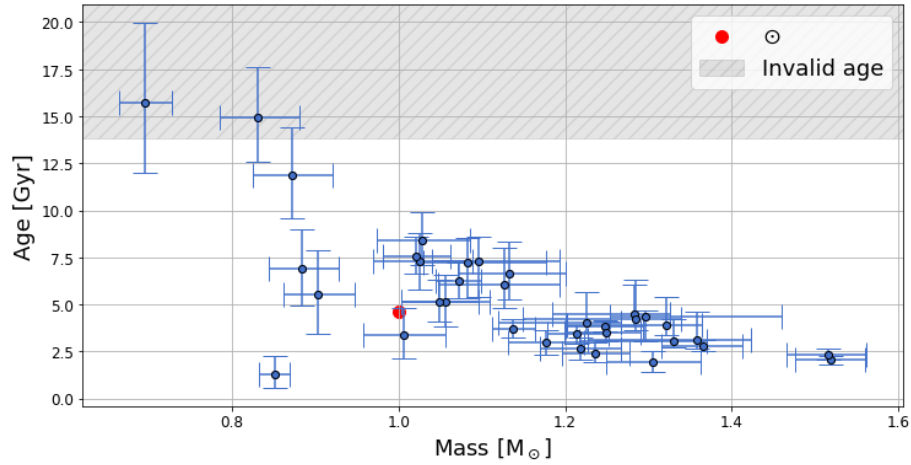


FIGURE 6.3: Scatter plot of PARAM mass and age output with the Sun as reference. Shaded region shows ages older than the universe.

These 33 stars characterized by PARAM were compared with the results of the stellar characterization codes provided in paper [Silva Aguirre et al. \(2015\)](#), of which there are four:

- The *B*ayesian *S*tellar *A*lgorithm (BASTA, [Serenelli et al., 2013](#)), taking as input physics stellar tracks modelled by the *G*Arching *S*tellar *E*volution *C*ode (GARSTEC, [Weiss & Schlattl, 2008](#))
- The *A*arhus *S*tellar *E*volution *C*ode (ASTEC, [Christensen-Dalsgaard, 2008b](#)) together with the *A*arhus *a*diabatic *o*scillation *p*ackage (ADIPLS, [Christensen-Dalsgaard, 2008a](#)), forming the *ASTEC Fitting* (ASTEC) method
- The *Y*ale *R*otating *s*tellar *E*volution *C*ode (YREC, [Demarque et al., 2008](#)) with its pulsation code ([Antia & Basu, 1994](#)), forming the *YALE Monte-Carlo Method* (YMCM)
- The *A*steroseismic *M*odeling *P*ortal (AMP, [Metcalf, Creevey & Christensen-Dalsgaard, 2009](#)), which uses the same pulsation code as in ASTFIT.

All of the aforementioned stellar characterization codes made use of individual seismic mode frequencies when taking into account seismic input data. Comparisons between PARAM and these codes are given by Figures 6.4 & 6.5 versus BASTA, Figures 6.6 & 6.7 versus ASTFIT, Figures 6.8 & 6.9 versus YMCM and Figures 6.10 & 6.11 versus AMP.

These plots compare the mass and age determination of each program as well as their relative residuals. The points are color-coded according to the PARAM output value of

the other variable (age for mass, mass for age). The bias was calculated by taking the weighed average of the relative residuals (weighed according to the inverse of the largest of their asymmetric uncertainties) and the red shaded regions signify the weighed standard deviation of the bias divided by  $\sqrt{N}$  where  $N$  is the number of points. If this shaded region did not cross the zero point (meaning the standard deviation is smaller than the bias) then this would be an indication of significant bias. These values are given in Table 6.3.

From these plots we can see a clear tendency for high mass stars to occupy low ages and for low mass stars to occupy high ages. Furthermore we can infer that PARAM, when tested against BASTA and ASTFIT, holds relatively low bias values for the determination mass ( $0.2 \pm 0.7\%$  and  $0.6 \pm 0.7\%$ ) but a non-negligible bias for the determination of age ( $-5.2 \pm 3.6\%$  and  $-12.0 \pm 3.3\%$ ). Against AMP the calculated bias stays within the standard deviation in both the mass ( $0.9 \pm 1.0\%$ ) and age ( $-3.8 \pm 4.1\%$ ). Against YMCM, however, the results are flipped, with a passing grade of  $0.0 \pm 3.8\%$  bias in the age but a rather pronounced underestimation bias of  $-2.2 \pm 0.6\%$  in the mass.

Table 6.4 shows the relative uncertainties for the various codes. In both the estimation of age and mass PARAM is outperformed by the other codes, with PARAM having a median uncertainty of around 5% for the mass and 26% for the age. These results are consistent with Chaplin et al. (2014), which analyzed over 500 main sequence stars using six different grid-based codes and obtained uncertainties in the mass and age in the order of 5.4% and 25%, respectively, when global seismic parameters are used as constraints.

| PARAM<br>vs | Bias(M)<br>[%] | Bias(Age)<br>[%] |
|-------------|----------------|------------------|
| BASTA       | $0.2 \pm 0.7$  | $-5.2 \pm 3.6$   |
| YMCM        | $-2.2 \pm 0.6$ | $0.0 \pm 3.8$    |
| ASTFIT      | $0.6 \pm 0.7$  | $-12.0 \pm 3.3$  |
| AMP         | $0.9 \pm 1.0$  | $-3.8 \pm 4.1$   |

TABLE 6.3: Comparison of the bias in age and mass determination between PARAM and other codes.

|        | $\frac{\sigma(M)}{M}$<br>[%] | $\frac{\sigma(\text{Age})}{\text{Age}}$<br>[%] |
|--------|------------------------------|------------------------------------------------|
| PARAM  | 4.9                          | 26.0                                           |
| BASTA  | 2.7                          | 11.6                                           |
| ASTFIT | 2.8                          | 14.6                                           |
| YMCM   | 3.4                          | 9.6                                            |
| AMP    | 3.0                          | 12.8                                           |

TABLE 6.4: Table of the median relative uncertainties of the 33 stars characterized by PARAM and other codes.

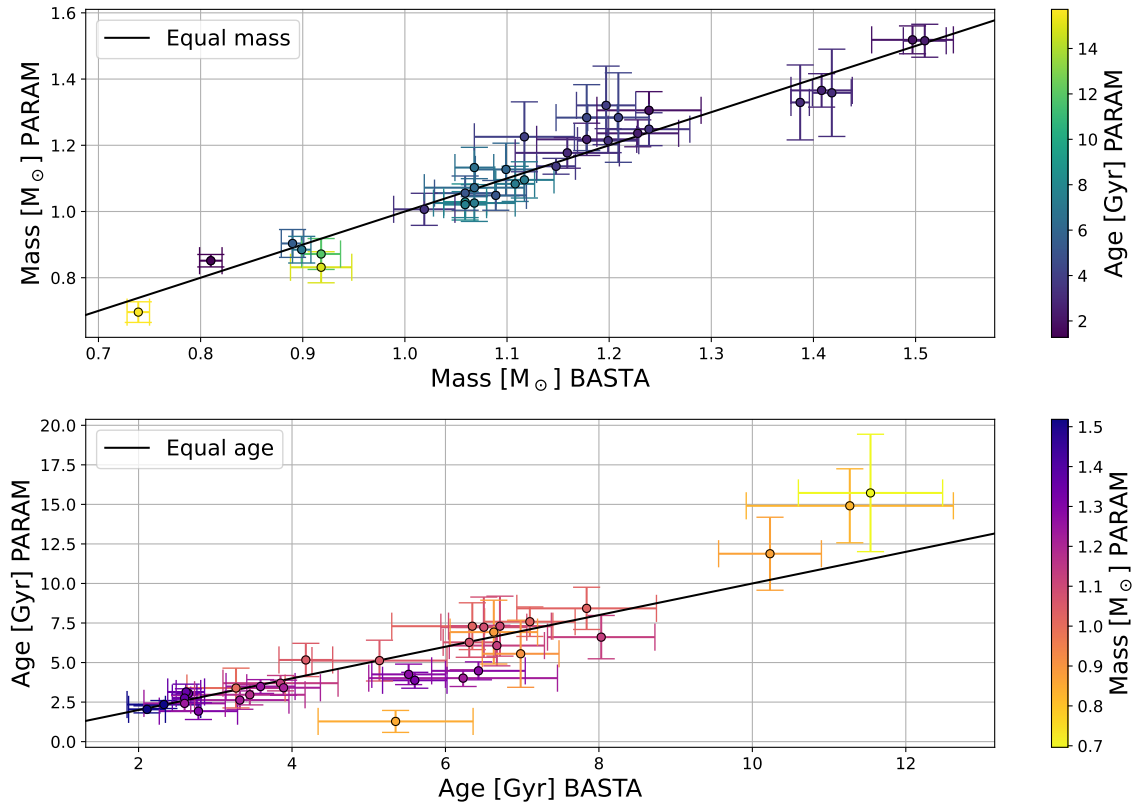


FIGURE 6.4: PARAM vs BASTA mass and age determination comparison.

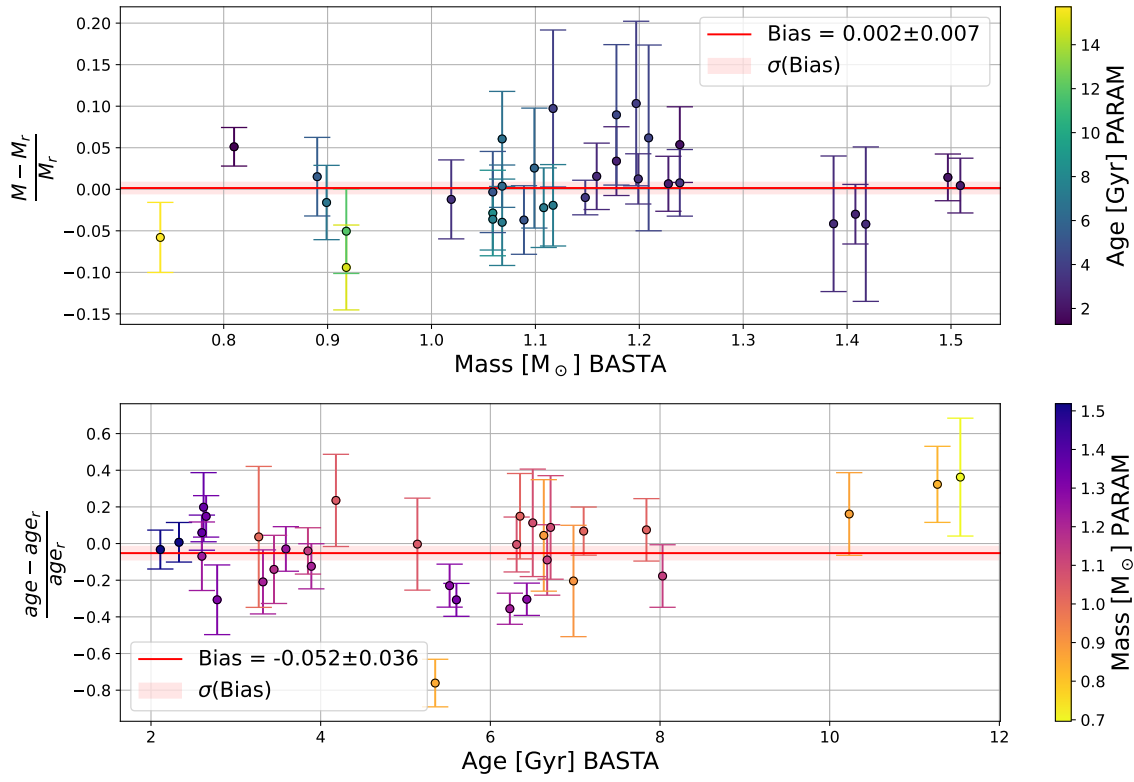


FIGURE 6.5: PARAM vs BASTA mass and age determination relative residual comparison.

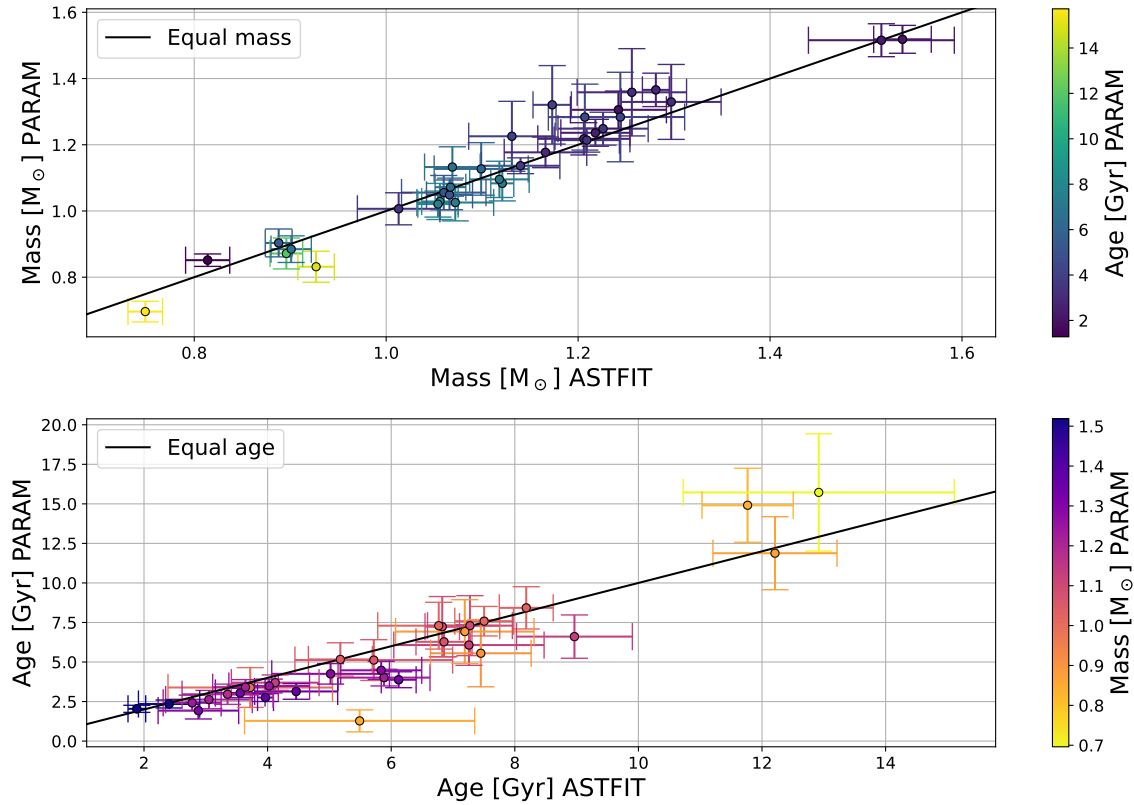


FIGURE 6.6: PARAM vs ASTFIT mass and age determination comparison.

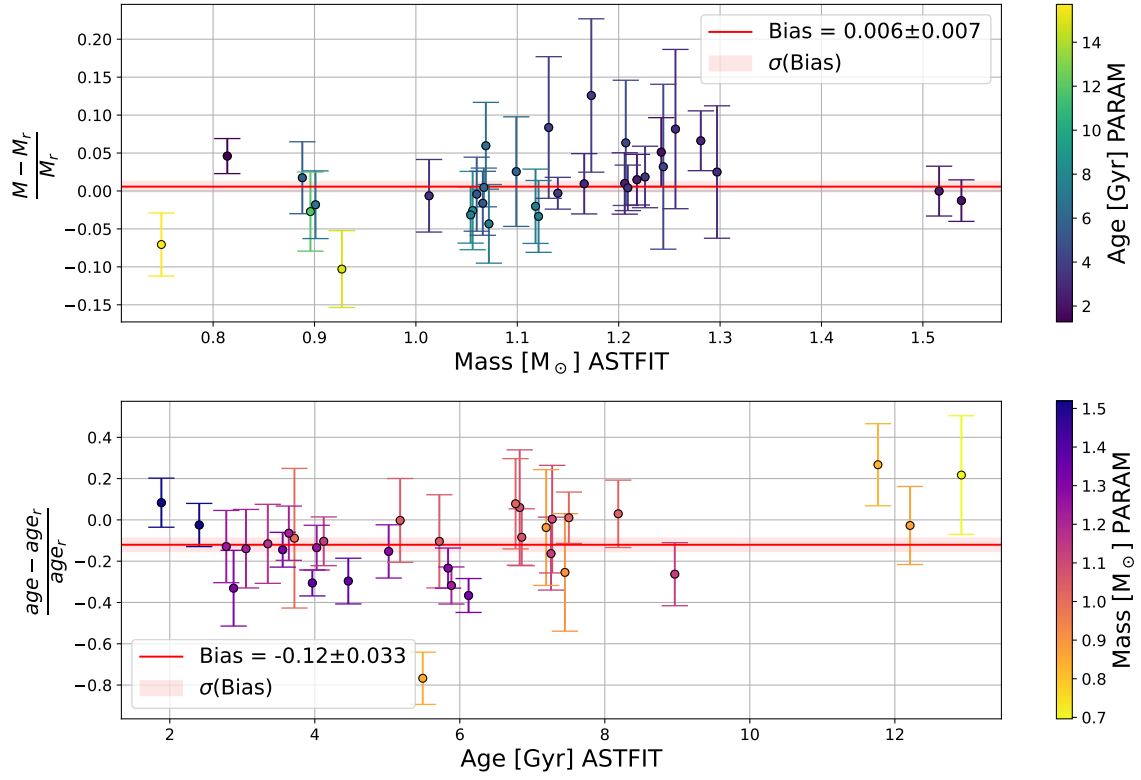


FIGURE 6.7: PARAM vs ASTFIT mass and age determination relative residual comparison.

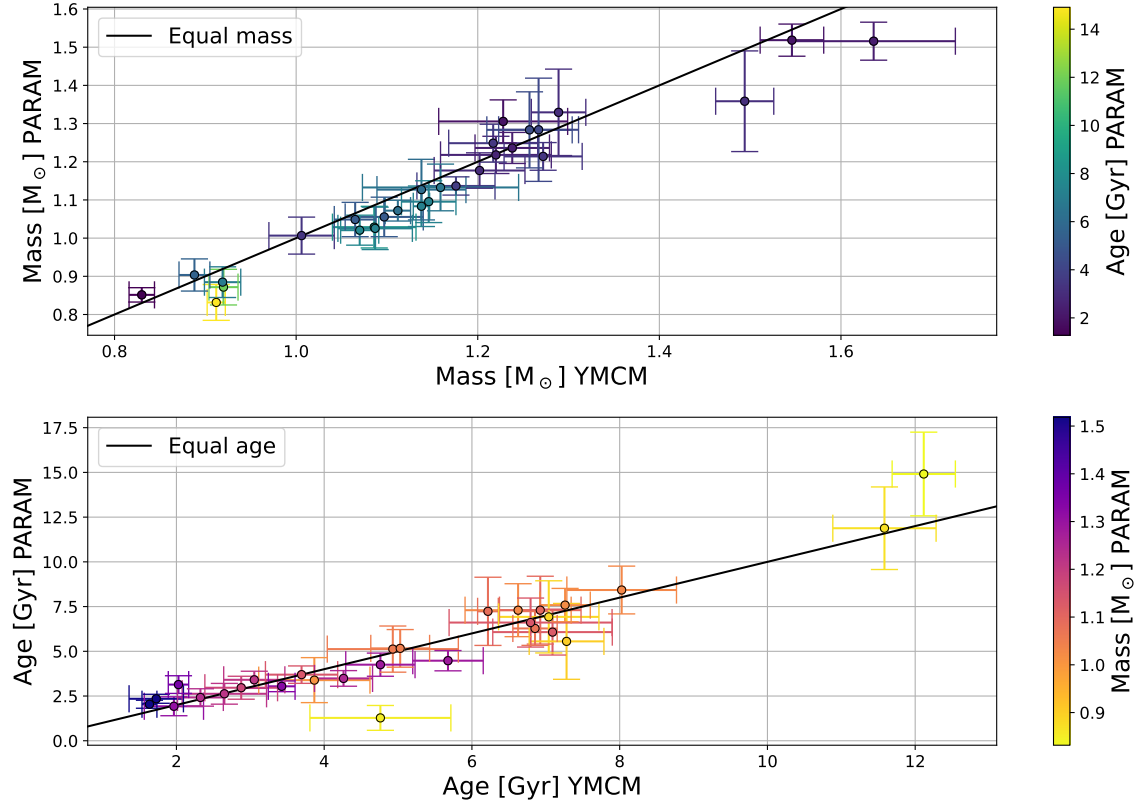


FIGURE 6.8: PARAM vs YMCM mass and age determination comparison.

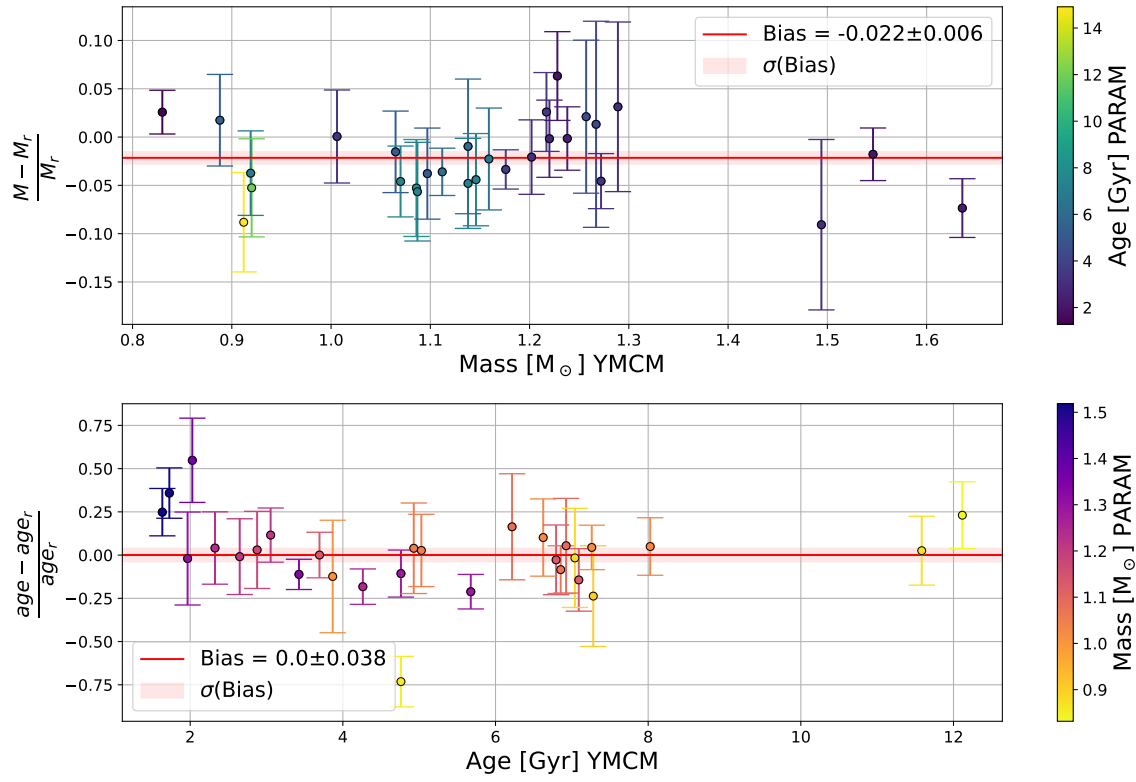


FIGURE 6.9: PARAM vs YMCM mass and age determination relative residual comparison.

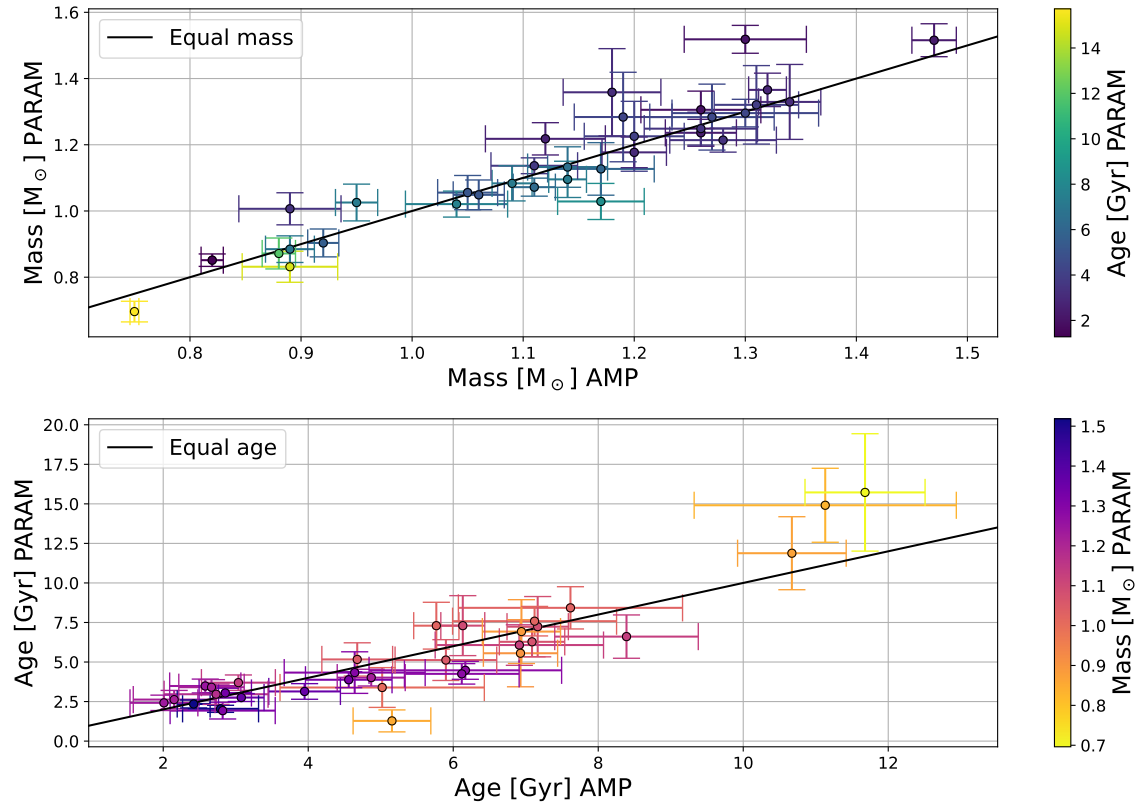


FIGURE 6.10: PARAM vs AMP mass and age determination comparison.

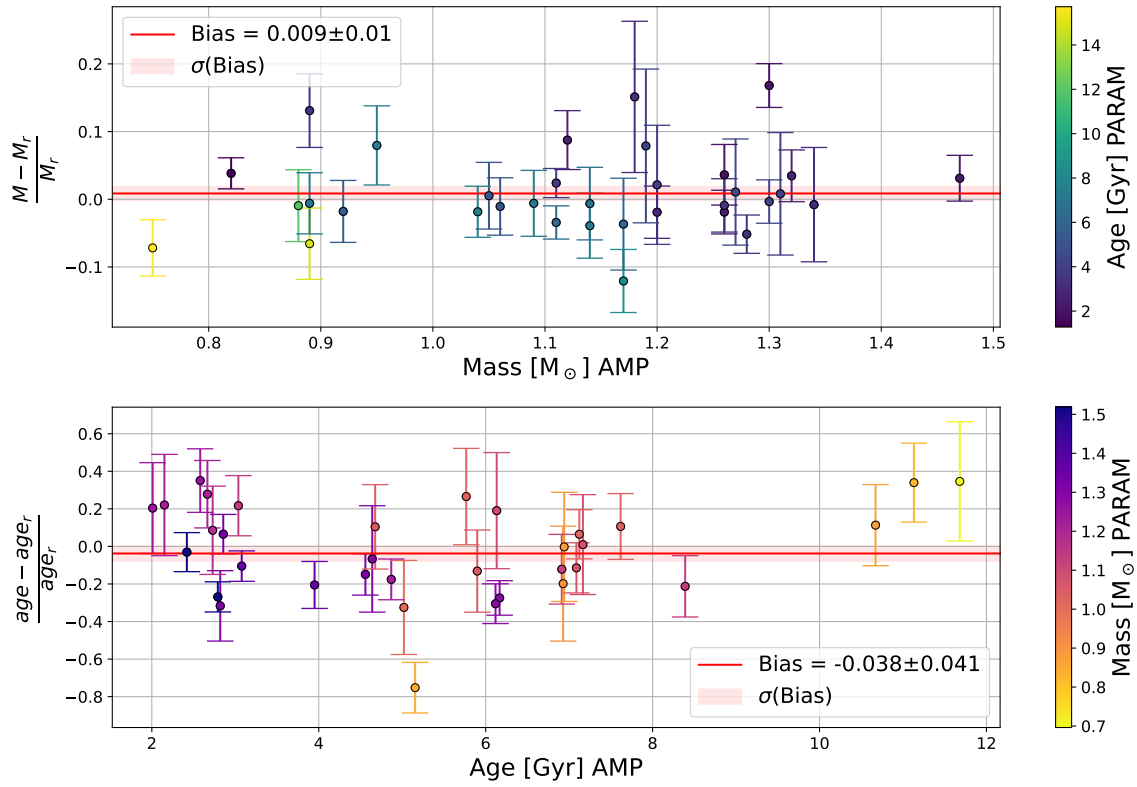


FIGURE 6.11: PARAM vs AMP mass and age determination relative residual comparison.

## 6.2 *Ariel* input catalog

Ten exoplanet-harboring stars from the *Ariel* input catalog, potential target candidates for the *Ariel* space mission, were selected to be characterized by PARAM, five of which were studied in [Magrini et al. \(2022\)](#). These stars have their identifications concealed due to an embargo by the *Ariel* Consortium, thereby being renamed as Ariel-1 to Ariel-10. Also due to this embargo the methodology used to describe the measurement of their parameters is omitted.

The luminosity of these 10 stars was derived by taking the 2MASS  $K_S$  band with the *Gaia* parallax and applying bolometric correction software. These are given in table 6.5. The seismic and spectroscopic input parameters for are given in Tables 6.6 and 6.7. Seismic data was gathered from various literature pieces whereas metallicity, effective temperature and  $\log g$  were taken from SWEETCat ([Sousa et al., 2021](#)). Of the two different  $\log g$  provided by SWEETCat, the one from *Gaia* was utilized.

These stars were executed in PARAM in sixteen different runs, each comprising its own set of constraints. These constraints are organized according to following terminology:

- spec:  $\{[\text{Fe}/\text{H}], T_{\text{eff}}, \log g\}$
- nolg\_spec:  $\{[\text{Fe}/\text{H}], T_{\text{eff}}\}$ , meaning  $\log g$  is not used.
- phot: Includes  $L$
- $+\Delta\nu, +\nu_{\text{max}}$  or  $+\Delta\nu+\nu_{\text{max}}$

The PARAM outputs of the constraints that don't use  $\log g$  are given in Tables 6.9, 6.10, 6.11 and 6.12 whereas the PARAM outputs of the constraints using  $\log g$  are given in Tables 6.13, 6.14, 6.15 and 6.16. Violin plots of the PARAM outputs for age and mass are shown in Figures 6.12 and 6.13 together with the indication of the mode and 16th and 84th uncertainty percentile, with the abscissa indicating different sets of constraints. The violin plots were chosen as a visual medium because the probability distribution function show much more than a mere error bar. Different colours are used for constraints with and without  $\log g$  and a red cross indicates constraints that weren't tested due to missing  $\nu_{\text{max}}$  data.

There is a general trend for the inclusion of  $\log g$  to result in wildly different solutions than in runs where  $\log g$  had been excluded as an input when neither seismic nor photometric inputs are used, such as in Ariel-4 and Ariel-6. When these seismology and/or photometry are used, however, there appears to be a convergence of agreement in the results. Different input parameters lead not only to different precision but also to different solutions, such as in Ariel-9 and Ariel-10. Ariel-6 shows a nice example of a converging solution as more inputs are used, whereas Ariel-4 shows the exact opposite phenomena, witnessing the emergence of a double solution. When different constraints lead to conflicting answers, doubts may be raised on the validity of extracted data for that particular star.

In Figures 6.14 and 6.15 we see the comparison of the relative uncertainties for the age and mass in the constraint sets without luminosity and with luminosity, respectively, without the use of  $\log g$ , whereas in Figures 6.16 and 6.17 we see the comparison when  $\log g$  is used as an input. We observe an overall improvement in the precision for virtually

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all stars when seismology is used as an input parameter, except for Ariel-4 due to the emergence of a double solution. Median relative uncertainties are given in Table [6.8](#).

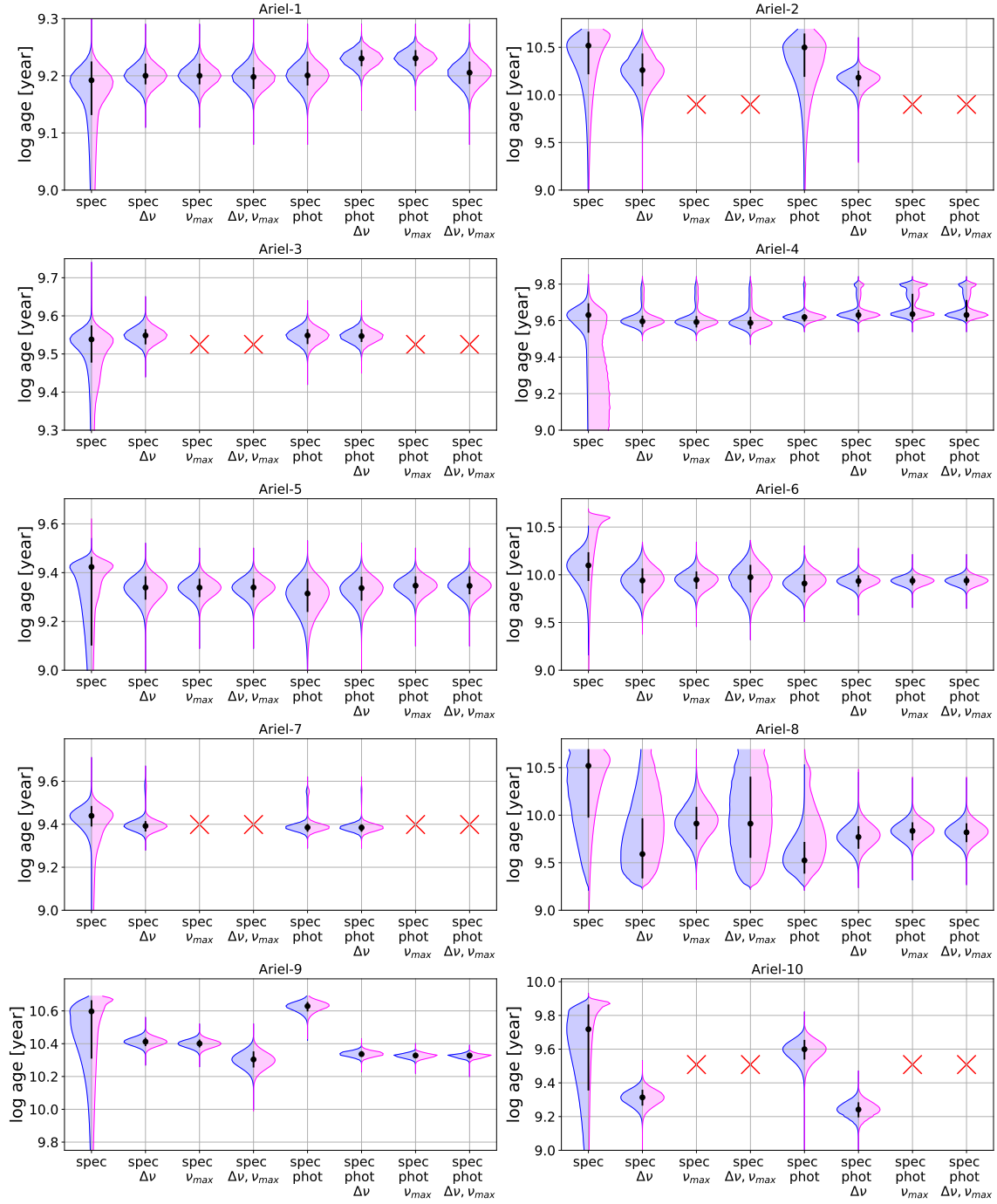


FIGURE 6.12: Violin plots for the age estimation of the ten *Ariel* target stars with different sets of constraints. Blue (left half) represents use of  $\log g$  as an input and purple (right half) without. Black dots and lines represent the mode and the 16th and 84th percentile uncertainty for the case with  $\log g$ . Red crosses represent plots that could not be generated due to missing input data.

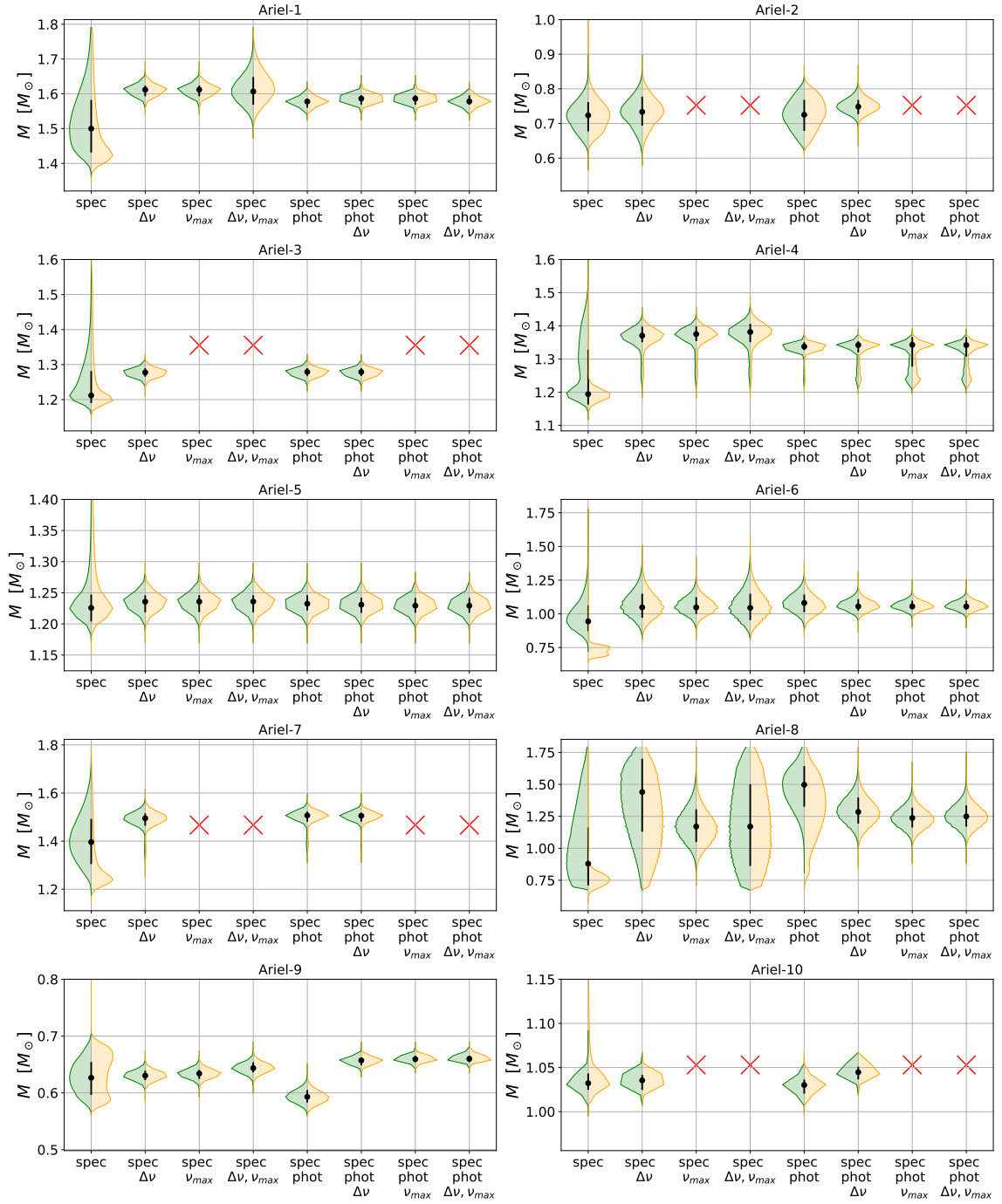


FIGURE 6.13: Violin plots for the mass estimation of the ten *Ariel* target stars with different sets of constraints. Green represents use of  $\log g$  and orange without. Black dots and lines represent the mode and the 16th and 84th percentile uncertainty for the case with  $\log g$ . Red crosses represent plots that could not be generated due to missing input data.

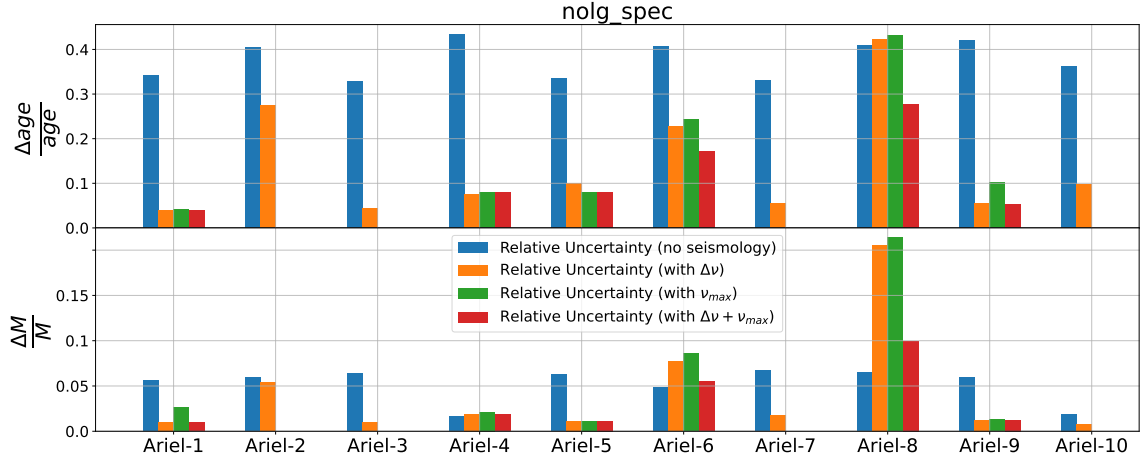


FIGURE 6.14: Comparison of the relative uncertainties for age and mass for the 10 *Ariel* stars for different input sets, without  $\log g$  and without photometry.

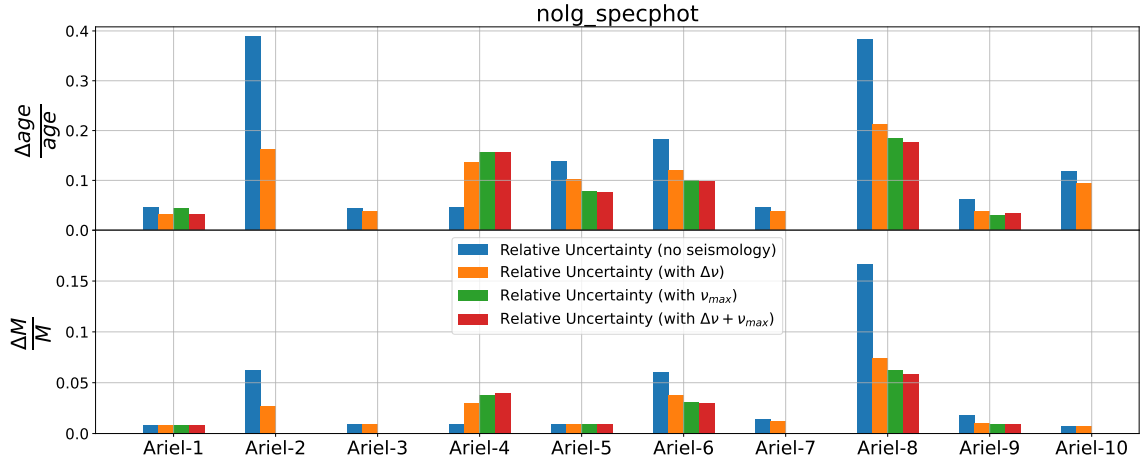


FIGURE 6.15: Comparison of the relative uncertainties for age and mass for the 10 *Ariel* stars for different input sets, without  $\log g$  and with photometry.

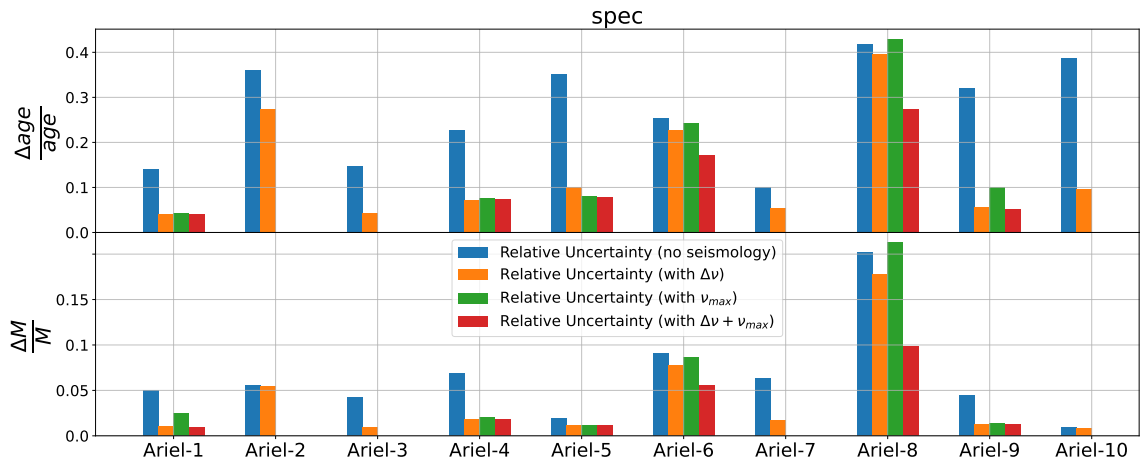


FIGURE 6.16: Comparison of the relative uncertainties for age and mass for the 10 *Ariel* stars for different input sets, with  $\log g$  and without photometry.

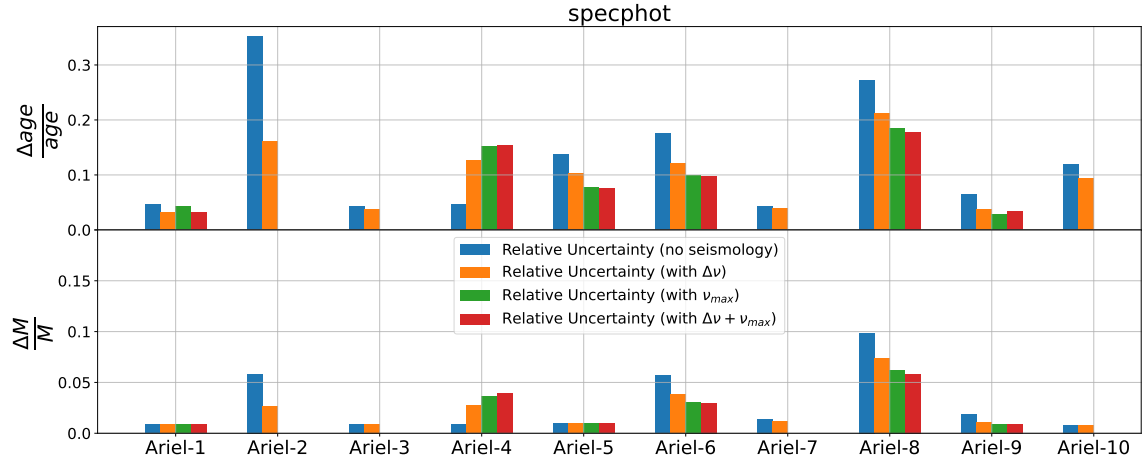


FIGURE 6.17: Comparison of the relative uncertainties for age and mass for the 10 *Ariel* stars for different input sets, with  $\log g$  and with photometry.

| star     | $L$<br>[ $L_{\odot}$ ] | $\sigma(L)$<br>[ $L_{\odot}$ ] | $K_S$<br>[mag] | $\sigma(K_S)$<br>[mag] | Parallax<br>[mas] | $\sigma(\text{Parallax})$<br>[mas] |
|----------|------------------------|--------------------------------|----------------|------------------------|-------------------|------------------------------------|
| Ariel-1  | 6.201                  | 0.146                          | 9.334          | 0.018                  | 2.999             | 0.011                              |
| Ariel-2  | 0.254                  | 0.015                          | 7.009          | 0.020                  | 26.427            | 0.011                              |
| Ariel-3  | 2.763                  | 0.068                          | 6.763          | 0.024                  | 12.914            | 0.018                              |
| Ariel-4  | 2.812                  | 0.074                          | 7.721          | 0.018                  | 7.612             | 0.017                              |
| Ariel-5  | 2.482                  | 0.078                          | 7.722          | 0.031                  | 9.436             | 0.021                              |
| Ariel-6  | 7.162                  | 0.278                          | 9.540          | 0.023                  | 1.622             | 0.021                              |
| Ariel-7  | 5.838                  | 0.207                          | 11.535         | 0.020                  | 1.032             | 0.012                              |
| Ariel-8  | 15.675                 | 1.157                          | 10.136         | 0.021                  | 0.790             | 0.010                              |
| Ariel-9  | 0.396                  | 0.009                          | 6.703          | 0.020                  | 27.358            | 0.012                              |
| Ariel-10 | 1.149                  | 0.025                          | 9.846          | 0.022                  | 4.631             | 0.012                              |

TABLE 6.5: Photometric input parameters of the 10 *Ariel* target stars.

| star     | $\nu_{\max}$<br>[ $\mu\text{Hz}$ ] | $\sigma(\nu_{\max})$<br>[ $\mu\text{Hz}$ ] | $\Delta\nu$<br>[ $\mu\text{Hz}$ ] | $\sigma(\Delta\nu)$<br>[ $\mu\text{Hz}$ ] |
|----------|------------------------------------|--------------------------------------------|-----------------------------------|-------------------------------------------|
| Ariel-1  | 1115                               | 110                                        | 59.20                             | 0.60                                      |
| Ariel-2  | -                                  | -                                          | 180.60                            | 0.60                                      |
| Ariel-3  | -                                  | -                                          | 83.44                             | 0.15                                      |
| Ariel-4  | 1300                               | 58                                         | 67                                | 1.87                                      |
| Ariel-5  | 2114                               | 38                                         | 99.86                             | 2.48                                      |
| Ariel-6  | 245.65                             | 3.51                                       | 18.46                             | 0.26                                      |
| Ariel-7  | -                                  | -                                          | 56.40                             | 1.70                                      |
| Ariel-8  | 107                                | 3.20                                       | 9.40                              | 0.10                                      |
| Ariel-9  | 4538                               | 144                                        | 179.60                            | 0.80                                      |
| Ariel-10 | -                                  | -                                          | 141.20                            | 0.40                                      |

TABLE 6.6: Seismic input parameters of the 10 *Ariel* target stars.

| star     | [Fe/H]<br>[dex] | $\sigma([\text{Fe}/\text{H}])$<br>[dex] | $T_{\text{eff}}$<br>[K] | $\sigma(T_{\text{eff}})$<br>[K] | $\log g$<br>[dex] | $\sigma(\log g)$<br>[dex] |
|----------|-----------------|-----------------------------------------|-------------------------|---------------------------------|-------------------|---------------------------|
| Ariel-1  | 0.28            | 0.02                                    | 6575                    | 34                              | 4.085             | 0.011                     |
| Ariel-2  | 0.12            | 0.05                                    | 4692                    | 97                              | 4.496             | 0.056                     |
| Ariel-3  | 0.22            | 0.02                                    | 6069                    | 22                              | 4.197             | 0.001                     |
| Ariel-4  | 0.53            | 0.03                                    | 5761                    | 41                              | 4.130             | 0.017                     |
| Ariel-5  | 0.04            | 0.02                                    | 6377                    | 27                              | 4.318             | 0.010                     |
| Ariel-6  | -0.01           | 0.02                                    | 4828                    | 33                              | 3.396             | 0.019                     |
| Ariel-7  | 0.19            | 0.03                                    | 6213                    | 41                              | 4.029             | 0.016                     |
| Ariel-8  | 0.35            | 0.06                                    | 4619                    | 113                             | 3.090             | 0.060                     |
| Ariel-9  | -0.56           | 0.02                                    | 5067                    | 25                              | 4.490             | 0.012                     |
| Ariel-10 | 0.01            | 0.01                                    | 5875                    | 13                              | 4.450             | 0.001                     |

TABLE 6.7: Spectroscopic input parameters of the 10 *Ariel* target stars.

| Constraint                                    | $\sigma(\text{Age})$ | $\sigma(M)$ |
|-----------------------------------------------|----------------------|-------------|
|                                               | Age                  | M           |
|                                               | [%]                  | [%]         |
| nolg_spec                                     | 38.3                 | 5.9         |
| spec                                          | 28.8                 | 5.3         |
| nolg_spec+ $\Delta\nu$                        | 8.6                  | 1.5         |
| spec+ $\Delta\nu$                             | 8.4                  | 1.5         |
| nolg_spec+ $\nu_{\text{max}}$                 | 9.1                  | 2.4         |
| spec+ $\nu_{\text{max}}$                      | 9.1                  | 2.3         |
| nolg_spec+ $\Delta\nu + \nu_{\text{max}}$     | 7.9                  | 1.6         |
| spec+ $\Delta\nu + \nu_{\text{max}}$          | 7.7                  | 1.5         |
| nolg_specphot                                 | 9.1                  | 1.2         |
| specphot                                      | 9.2                  | 1.2         |
| nolg_specphot+ $\Delta\nu$                    | 9.8                  | 1.2         |
| specphot+ $\Delta\nu$                         | 9.8                  | 1.1         |
| nolg_specphot+ $\nu_{\text{max}}$             | 8.9                  | 2.0         |
| specphot+ $\nu_{\text{max}}$                  | 8.9                  | 2.0         |
| nolg_specphot+ $\Delta\nu + \nu_{\text{max}}$ | 8.7                  | 2.0         |
| specphot+ $\Delta\nu + \nu_{\text{max}}$      | 8.7                  | 2.0         |

TABLE 6.8: Table of the median relative uncertainties of the 10 *Ariel* stars characterized by PARAM.

| Star     | Age                                          | Age                                        | Age                                         | Age                                        |
|----------|----------------------------------------------|--------------------------------------------|---------------------------------------------|--------------------------------------------|
|          | [Gyr]                                        | [Gyr]                                      | [Gyr]                                       | [Gyr]                                      |
|          | spec                                         | spec+ $\Delta\nu$                          | spec+ $\nu_{\text{max}}$                    | spec+ $\Delta\nu + \nu_{\text{max}}$       |
| Ariel-1  | 1.222 <sup>+0.304</sup> <sub>-0.736</sub>    | 1.55 <sup>+0.067</sup> <sub>-0.063</sub>   | 1.538 <sup>+0.069</sup> <sub>-0.066</sub>   | 1.55 <sup>+0.067</sup> <sub>-0.063</sub>   |
| Ariel-2  | 24.298 <sup>+15.447</sup> <sub>-16.634</sub> | 16.997 <sup>+7.971</sup> <sub>-5.78</sub>  | -                                           | -                                          |
| Ariel-3  | 2.776 <sup>+0.613</sup> <sub>-1.606</sub>    | 3.456 <sup>+0.164</sup> <sub>-0.157</sub>  | -                                           | -                                          |
| Ariel-4  | 1.115 <sup>+1.764</sup> <sub>-0.735</sub>    | 3.896 <sup>+0.401</sup> <sub>-0.246</sub>  | 3.826 <sup>+0.426</sup> <sub>-0.253</sub>   | 3.872 <sup>+0.437</sup> <sub>-0.241</sub>  |
| Ariel-5  | 2.124 <sup>+0.509</sup> <sub>-1.255</sub>    | 2.108 <sup>+0.231</sup> <sub>-0.236</sub>  | 2.127 <sup>+0.189</sup> <sub>-0.183</sub>   | 2.126 <sup>+0.187</sup> <sub>-0.182</sub>  |
| Ariel-6  | 21.858 <sup>+14.925</sup> <sub>-15.002</sub> | 8.715 <sup>+2.966</sup> <sub>-2.329</sub>  | 8.761 <sup>+3.313</sup> <sub>-2.567</sub>   | 8.644 <sup>+1.98</sup> <sub>-1.67</sub>    |
| Ariel-7  | 2.254 <sup>+0.557</sup> <sub>-1.303</sub>    | 2.421 <sup>+0.153</sup> <sub>-0.128</sub>  | -                                           | -                                          |
| Ariel-8  | 21.636 <sup>+16.249</sup> <sub>-14.781</sub> | 8.632 <sup>+14.318</sup> <sub>-5.119</sub> | 10.034 <sup>+16.621</sup> <sub>-6.321</sub> | 8.82 <sup>+4.455</sup> <sub>-2.867</sub>   |
| Ariel-9  | 24.243 <sup>+18.263</sup> <sub>-17.483</sub> | 25.239 <sup>+1.549</sup> <sub>-1.447</sub> | 19.331 <sup>+2.273</sup> <sub>-2.147</sub>  | 24.537 <sup>+1.371</sup> <sub>-1.328</sub> |
| Ariel-10 | 5.331 <sup>+1.829</sup> <sub>-3.369</sub>    | 2.001 <sup>+0.221</sup> <sub>-0.213</sub>  | -                                           | -                                          |

TABLE 6.9: Age PARAM output of the 10 *Ariel* stars without photometry as an input (without log g).

| Star     | Age<br>[Gyr]<br>spec                         | Age<br>[Gyr]<br>spec+ $\Delta\nu$          | Age<br>[Gyr]<br>spec+ $\nu_{\max}$         | Age<br>[Gyr]<br>spec+ $\Delta\nu + \nu_{\max}$ |
|----------|----------------------------------------------|--------------------------------------------|--------------------------------------------|------------------------------------------------|
| Ariel-1  | 1.548 <sup>+0.075</sup> <sub>-0.075</sub>    | 1.659 <sup>+0.054</sup> <sub>-0.054</sub>  | 1.566 <sup>+0.071</sup> <sub>-0.072</sub>  | 1.66 <sup>+0.053</sup> <sub>-0.054</sub>       |
| Ariel-2  | 20.095 <sup>+14.573</sup> <sub>-12.382</sub> | 14.302 <sup>+2.797</sup> <sub>-2.773</sub> | -                                          | -                                              |
| Ariel-3  | 3.448 <sup>+0.155</sup> <sub>-0.158</sub>    | 3.442 <sup>+0.135</sup> <sub>-0.133</sub>  | -                                          | -                                              |
| Ariel-4  | 4.077 <sup>+0.217</sup> <sub>-0.184</sub>    | 4.262 <sup>+1.311</sup> <sub>-0.206</sub>  | 4.342 <sup>+1.597</sup> <sub>-0.263</sub>  | 4.609 <sup>+1.415</sup> <sub>-0.458</sub>      |
| Ariel-5  | 1.969 <sup>+0.305</sup> <sub>-0.323</sub>    | 2.088 <sup>+0.236</sup> <sub>-0.239</sub>  | 2.163 <sup>+0.185</sup> <sub>-0.181</sub>  | 2.167 <sup>+0.182</sup> <sub>-0.177</sub>      |
| Ariel-6  | 8.541 <sup>+2.162</sup> <sub>-1.758</sub>    | 8.461 <sup>+1.255</sup> <sub>-1.101</sub>  | 8.462 <sup>+0.98</sup> <sub>-0.906</sub>   | 8.467 <sup>+0.959</sup> <sub>-0.881</sub>      |
| Ariel-7  | 2.386 <sup>+0.129</sup> <sub>-0.098</sub>    | 2.372 <sup>+0.099</sup> <sub>-0.092</sub>  | -                                          | -                                              |
| Ariel-8  | 5.931 <sup>+7.58</sup> <sub>-2.761</sub>     | 6.021 <sup>+1.909</sup> <sub>-1.456</sub>  | 6.532 <sup>+1.671</sup> <sub>-1.349</sub>  | 6.836 <sup>+1.662</sup> <sub>-1.348</sub>      |
| Ariel-9  | 41.716 <sup>+2.645</sup> <sub>-2.937</sub>   | 21.249 <sup>+0.818</sup> <sub>-0.83</sub>  | 20.684 <sup>+0.544</sup> <sub>-0.689</sub> | 20.716 <sup>+0.694</sup> <sub>-0.739</sub>     |
| Ariel-10 | 3.826 <sup>+0.511</sup> <sub>-0.523</sub>    | 1.705 <sup>+0.182</sup> <sub>-0.17</sub>   | -                                          | -                                              |

TABLE 6.10: Age PARAM output of the 10 *Ariel* stars with photometry as an input (without log  $g$ ).

| Star     | $M$<br>[ $M_{\odot}$ ]<br>nolg_specphot   | $M$<br>[ $M_{\odot}$ ]<br>nolg_specphot+ $\Delta\nu$ | $M$<br>[ $M_{\odot}$ ]<br>nolg_specphot+ $\nu_{\max}$ | $M$<br>[ $M_{\odot}$ ]<br>nolg_specphot+ $\Delta\nu + \nu_{\max}$ |
|----------|-------------------------------------------|------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------------------|
| Ariel-1  | 1.443 <sup>+0.131</sup> <sub>-0.045</sub> | 1.593 <sup>+0.014</sup> <sub>-0.018</sub>            | 1.599 <sup>+0.047</sup> <sub>-0.041</sub>             | 1.593 <sup>+0.014</sup> <sub>-0.017</sub>                         |
| Ariel-2  | 0.712 <sup>+0.047</sup> <sub>-0.043</sub> | 0.725 <sup>+0.039</sup> <sub>-0.043</sub>            | -                                                     | -                                                                 |
| Ariel-3  | 1.207 <sup>+0.146</sup> <sub>-0.027</sub> | 1.262 <sup>+0.012</sup> <sub>-0.013</sub>            | -                                                     | -                                                                 |
| Ariel-4  | 1.18 <sup>+0.021</sup> <sub>-0.018</sub>  | 1.352 <sup>+0.023</sup> <sub>-0.029</sub>            | 1.364 <sup>+0.027</sup> <sub>-0.032</sub>             | 1.357 <sup>+0.02</sup> <sub>-0.031</sub>                          |
| Ariel-5  | 1.231 <sup>+0.144</sup> <sub>-0.029</sub> | 1.221 <sup>+0.014</sup> <sub>-0.014</sub>            | 1.221 <sup>+0.014</sup> <sub>-0.014</sub>             | 1.221 <sup>+0.014</sup> <sub>-0.014</sub>                         |
| Ariel-6  | 0.719 <sup>+0.037</sup> <sub>-0.036</sub> | 1.039 <sup>+0.094</sup> <sub>-0.081</sub>            | 1.037 <sup>+0.106</sup> <sub>-0.09</sub>              | 1.041 <sup>+0.064</sup> <sub>-0.057</sub>                         |
| Ariel-7  | 1.252 <sup>+0.148</sup> <sub>-0.04</sub>  | 1.477 <sup>+0.026</sup> <sub>-0.027</sub>            | -                                                     | -                                                                 |
| Ariel-8  | 0.746 <sup>+0.054</sup> <sub>-0.05</sub>  | 1.135 <sup>+0.325</sup> <sub>-0.275</sub>            | 1.087 <sup>+0.351</sup> <sub>-0.264</sub>             | 1.128 <sup>+0.131</sup> <sub>-0.12</sub>                          |
| Ariel-9  | 0.627 <sup>+0.037</sup> <sub>-0.042</sub> | 0.623 <sup>+0.008</sup> <sub>-0.008</sub>            | 0.637 <sup>+0.008</sup> <sub>-0.009</sub>             | 0.627 <sup>+0.008</sup> <sub>-0.008</sub>                         |
| Ariel-10 | 1.025 <sup>+0.029</sup> <sub>-0.011</sub> | 1.024 <sup>+0.009</sup> <sub>-0.008</sub>            | -                                                     | -                                                                 |

TABLE 6.11: Mass PARAM output of the 10 *Ariel* stars without photometry as an input (without log  $g$ ).

| Star     | $M$                       | $M$                        | $M$                         | $M$                                   |
|----------|---------------------------|----------------------------|-----------------------------|---------------------------------------|
|          | $[M_{\odot}]$             | $[M_{\odot}]$              | $[M_{\odot}]$               | $[M_{\odot}]$                         |
|          | nolg_specphot             | nolg_specphot+ $\Delta\nu$ | nolg_specphot+ $\nu_{\max}$ | nolg_specphot+ $\Delta\nu+\nu_{\max}$ |
| Ariel-1  | $1.56^{+0.013}_{-0.014}$  | $1.568^{+0.014}_{-0.013}$  | $1.56^{+0.014}_{-0.013}$    | $1.568^{+0.014}_{-0.013}$             |
| Ariel-2  | $0.721^{+0.046}_{-0.05}$  | $0.742^{+0.021}_{-0.02}$   | -                           | -                                     |
| Ariel-3  | $1.264^{+0.01}_{-0.013}$  | $1.263^{+0.011}_{-0.012}$  | -                           | -                                     |
| Ariel-4  | $1.32^{+0.012}_{-0.013}$  | $1.322^{+0.011}_{-0.07}$   | $1.317^{+0.016}_{-0.085}$   | $1.308^{+0.024}_{-0.081}$             |
| Ariel-5  | $1.218^{+0.012}_{-0.012}$ | $1.217^{+0.012}_{-0.012}$  | $1.216^{+0.013}_{-0.011}$   | $1.216^{+0.013}_{-0.011}$             |
| Ariel-6  | $1.044^{+0.071}_{-0.063}$ | $1.047^{+0.043}_{-0.04}$   | $1.047^{+0.035}_{-0.032}$   | $1.047^{+0.033}_{-0.031}$             |
| Ariel-7  | $1.488^{+0.02}_{-0.023}$  | $1.488^{+0.018}_{-0.02}$   | -                           | -                                     |
| Ariel-8  | $1.261^{+0.24}_{-0.26}$   | $1.256^{+0.105}_{-0.097}$  | $1.228^{+0.085}_{-0.079}$   | $1.21^{+0.078}_{-0.074}$              |
| Ariel-9  | $0.585^{+0.011}_{-0.011}$ | $0.649^{+0.007}_{-0.007}$  | $0.653^{+0.007}_{-0.005}$   | $0.652^{+0.007}_{-0.005}$             |
| Ariel-10 | $1.018^{+0.007}_{-0.009}$ | $1.034^{+0.009}_{-0.007}$  | -                           | -                                     |

TABLE 6.12: Mass PARAM output of the 10 *Ariel* stars with photometry as an input (without log g).

| Star     | Age                          | Age                        | Age                        | Age                            |
|----------|------------------------------|----------------------------|----------------------------|--------------------------------|
|          | [Gyr]                        | [Gyr]                      | [Gyr]                      | [Gyr]                          |
|          | spec                         | spec+ $\Delta\nu$          | spec+ $\nu_{\max}$         | spec+ $\Delta\nu + \nu_{\max}$ |
| Ariel-1  | $1.462^{+0.113}_{-0.329}$    | $1.549^{+0.067}_{-0.063}$  | $1.542^{+0.07}_{-0.067}$   | $1.55^{+0.066}_{-0.063}$       |
| Ariel-2  | $23.513^{+12.674}_{-13.423}$ | $17.362^{+8.171}_{-5.838}$ | -                          | -                              |
| Ariel-3  | $3.229^{+0.266}_{-0.765}$    | $3.456^{+0.163}_{-0.157}$  | -                          | -                              |
| Ariel-4  | $3.91^{+0.587}_{-1.469}$     | $3.895^{+0.365}_{-0.242}$  | $3.833^{+0.397}_{-0.249}$  | $3.871^{+0.399}_{-0.239}$      |
| Ariel-5  | $1.766^{+0.753}_{-1.017}$    | $2.102^{+0.23}_{-0.236}$   | $2.125^{+0.189}_{-0.183}$  | $2.124^{+0.187}_{-0.182}$      |
| Ariel-6  | $11.58^{+4.355}_{-3.75}$     | $8.408^{+2.877}_{-2.242}$  | $8.729^{+3.303}_{-2.558}$  | $8.577^{+1.966}_{-1.656}$      |
| Ariel-7  | $2.64^{+0.279}_{-0.304}$     | $2.422^{+0.151}_{-0.127}$  | -                          | -                              |
| Ariel-8  | $15.012^{+17.81}_{-9.629}$   | $5.331^{+7.909}_{-2.598}$  | $9.486^{+16.075}_{-5.906}$ | $8.111^{+4.012}_{-2.61}$       |
| Ariel-9  | $26.808^{+11.573}_{-13.085}$ | $25.256^{+1.551}_{-1.45}$  | $19.557^{+2.278}_{-2.155}$ | $24.55^{+1.368}_{-1.33}$       |
| Ariel-10 | $3.399^{+2.269}_{-2.118}$    | $2.002^{+0.221}_{-0.213}$  | -                          | -                              |

TABLE 6.13: Age PARAM output of the 10 *Ariel* stars without photometry as an input (with log g).

| Star     | Age<br>[Gyr]<br>specphot     | Age<br>[Gyr]<br>specphot+ $\Delta\nu$ | Age<br>[Gyr]<br>specphot+ $\nu_{\max}$ | Age<br>[Gyr]<br>specphot+ $\Delta\nu + \nu_{\max}$ |
|----------|------------------------------|---------------------------------------|----------------------------------------|----------------------------------------------------|
| Ariel-1  | $1.547^{+0.075}_{-0.075}$    | $1.658^{+0.054}_{-0.054}$             | $1.564^{+0.071}_{-0.071}$              | $1.659^{+0.054}_{-0.054}$                          |
| Ariel-2  | $22.346^{+12.703}_{-11.985}$ | $14.387^{+2.792}_{-2.764}$            | -                                      | -                                                  |
| Ariel-3  | $3.447^{+0.154}_{-0.158}$    | $3.442^{+0.135}_{-0.134}$             | -                                      | -                                                  |
| Ariel-4  | $4.07^{+0.209}_{-0.182}$     | $4.245^{+1.18}_{-0.197}$              | $4.307^{+1.541}_{-0.239}$              | $4.49^{+1.498}_{-0.35}$                            |
| Ariel-5  | $1.963^{+0.302}_{-0.32}$     | $2.083^{+0.236}_{-0.237}$             | $2.161^{+0.185}_{-0.181}$              | $2.165^{+0.182}_{-0.176}$                          |
| Ariel-6  | $7.943^{+1.912}_{-1.546}$    | $8.365^{+1.235}_{-1.086}$             | $8.441^{+0.981}_{-0.902}$              | $8.44^{+0.958}_{-0.878}$                           |
| Ariel-7  | $2.381^{+0.114}_{-0.096}$    | $2.37^{+0.098}_{-0.091}$              | -                                      | -                                                  |
| Ariel-8  | $3.678^{+1.998}_{-1.097}$    | $5.729^{+1.804}_{-1.376}$             | $6.415^{+1.64}_{-1.322}$               | $6.654^{+1.611}_{-1.309}$                          |
| Ariel-9  | $41.231^{+2.65}_{-2.975}$    | $21.257^{+0.816}_{-0.829}$            | $20.686^{+0.544}_{-0.688}$             | $20.721^{+0.693}_{-0.738}$                         |
| Ariel-10 | $3.808^{+0.508}_{-0.519}$    | $1.705^{+0.183}_{-0.17}$              | -                                      | -                                                  |

TABLE 6.14: Age PARAM output of the 10 *Ariel* stars with photometry as an input (with  $\log g$ ).

| Star     | $M$<br>[ $M_{\odot}$ ]<br>spec | $M$<br>[ $M_{\odot}$ ]<br>spec+ $\Delta\nu$ | $M$<br>[ $M_{\odot}$ ]<br>spec+ $\nu_{\max}$ | $M$<br>[ $M_{\odot}$ ]<br>spec+ $\Delta\nu + \nu_{\max}$ |
|----------|--------------------------------|---------------------------------------------|----------------------------------------------|----------------------------------------------------------|
| Ariel-1  | $1.5^{+0.09}_{-0.068}$         | $1.593^{+0.014}_{-0.018}$                   | $1.591^{+0.042}_{-0.038}$                    | $1.593^{+0.014}_{-0.017}$                                |
| Ariel-2  | $0.71^{+0.041}_{-0.043}$       | $0.722^{+0.04}_{-0.043}$                    | -                                            | -                                                        |
| Ariel-3  | $1.227^{+0.071}_{-0.038}$      | $1.262^{+0.012}_{-0.013}$                   | -                                            | -                                                        |
| Ariel-4  | $1.252^{+0.112}_{-0.075}$      | $1.352^{+0.022}_{-0.028}$                   | $1.362^{+0.026}_{-0.031}$                    | $1.356^{+0.021}_{-0.028}$                                |
| Ariel-5  | $1.217^{+0.029}_{-0.02}$       | $1.221^{+0.014}_{-0.014}$                   | $1.221^{+0.014}_{-0.014}$                    | $1.221^{+0.014}_{-0.014}$                                |
| Ariel-6  | $0.959^{+0.111}_{-0.084}$      | $1.049^{+0.096}_{-0.082}$                   | $1.038^{+0.107}_{-0.09}$                     | $1.043^{+0.065}_{-0.057}$                                |
| Ariel-7  | $1.388^{+0.102}_{-0.087}$      | $1.476^{+0.026}_{-0.026}$                   | -                                            | -                                                        |
| Ariel-8  | $0.974^{+0.322}_{-0.201}$      | $1.3^{+0.266}_{-0.292}$                     | $1.105^{+0.347}_{-0.272}$                    | $1.155^{+0.131}_{-0.122}$                                |
| Ariel-9  | $0.62^{+0.03}_{-0.028}$        | $0.623^{+0.008}_{-0.008}$                   | $0.636^{+0.009}_{-0.009}$                    | $0.627^{+0.008}_{-0.008}$                                |
| Ariel-10 | $1.021^{+0.011}_{-0.008}$      | $1.024^{+0.009}_{-0.008}$                   | -                                            | -                                                        |

TABLE 6.15: Mass PARAM output of the 10 *Ariel* stars without photometry as an input (with  $\log g$ ).

| Star     | $M$<br>[ $M_{\odot}$ ]<br>specphot | $M$<br>[ $M_{\odot}$ ]<br>specphot+ $\Delta\nu$ | $M$<br>[ $M_{\odot}$ ]<br>specphot+ $\nu_{\max}$ | $M$<br>[ $M_{\odot}$ ]<br>specphot+ $\Delta\nu + \nu_{\max}$ |
|----------|------------------------------------|-------------------------------------------------|--------------------------------------------------|--------------------------------------------------------------|
| Ariel-1  | $1.56^{+0.013}_{-0.014}$           | $1.568^{+0.014}_{-0.013}$                       | $1.56^{+0.014}_{-0.013}$                         | $1.568^{+0.014}_{-0.013}$                                    |
| Ariel-2  | $0.714^{+0.044}_{-0.044}$          | $0.741^{+0.021}_{-0.02}$                        | -                                                | -                                                            |
| Ariel-3  | $1.264^{+0.01}_{-0.013}$           | $1.263^{+0.011}_{-0.012}$                       | -                                                | -                                                            |
| Ariel-4  | $1.321^{+0.011}_{-0.013}$          | $1.323^{+0.011}_{-0.062}$                       | $1.32^{+0.013}_{-0.083}$                         | $1.312^{+0.021}_{-0.083}$                                    |
| Ariel-5  | $1.218^{+0.012}_{-0.012}$          | $1.217^{+0.012}_{-0.012}$                       | $1.216^{+0.013}_{-0.011}$                        | $1.216^{+0.013}_{-0.011}$                                    |
| Ariel-6  | $1.066^{+0.067}_{-0.062}$          | $1.05^{+0.043}_{-0.04}$                         | $1.047^{+0.035}_{-0.031}$                        | $1.048^{+0.033}_{-0.031}$                                    |
| Ariel-7  | $1.488^{+0.02}_{-0.022}$           | $1.487^{+0.019}_{-0.018}$                       | -                                                | -                                                            |
| Ariel-8  | $1.44^{+0.149}_{-0.165}$           | $1.275^{+0.105}_{-0.098}$                       | $1.234^{+0.085}_{-0.079}$                        | $1.219^{+0.079}_{-0.073}$                                    |
| Ariel-9  | $0.587^{+0.011}_{-0.011}$          | $0.649^{+0.007}_{-0.007}$                       | $0.653^{+0.007}_{-0.005}$                        | $0.652^{+0.007}_{-0.005}$                                    |
| Ariel-10 | $1.018^{+0.008}_{-0.009}$          | $1.034^{+0.009}_{-0.007}$                       | -                                                | -                                                            |

TABLE 6.16: Mass PARAM output of the 10 *Ariel* stars with photometry as an input (with  $\log g$ ).

### 6.3 Artificially Inflated Input Uncertainties

When using only  $T_{\text{eff}}$ ,  $[\text{Fe}/\text{H}]$ ,  $\Delta\nu$  and  $\nu_{\text{max}}$  as constraints, the precision obtained in the analysis of the 33 stars (4.9% in mass and 26% in age) differs significantly from the precision obtained in the analysis of the 10 *Ariel* stars (1.6% in mass and 7.9% in age). This might be because the 10 *Ariel* stars present much smaller uncertainties in the input parameters than the 33 stars. The median of these uncertainties are compared in Table 6.17 which shows similar uncertainties for the global seismic parameters but very different uncertainties for the spectroscopic parameters. Also represented in the same table are the uncertainties used in the Magrini et al. (2022) paper, which studied 187 F-G-K stars in the *Ariel* input catalog. While this work did not make use of seismology, it shows that the uncertainties for both temperature and metallicities are closer to the 33 stars than they are to the uncertainties used for this dissertation. This reflects an evolution in the parameters of the *Ariel* input catalog.

In order to make a more comprehensive comparison, the tests made with the 10 *Ariel* stars in this dissertation were repeated but this time with the input uncertainties of  $T_{\text{eff}}$  and  $[\text{Fe}/\text{H}]$  inflated to 75 K (giving a median relative uncertainty of 1.3%) and 0.1 dex, respectively, so as to keep it in line with the uncertainties of the 33 stars and of the Magrini et al. (2022) work. Results are given in Table 6.18 and illustrated in Figures 6.18 and 6.19.

From Table 6.18 we can see that after inflating the uncertainties of the input parameters to values comparable to the set of 33 star, PARAM will provide uncertainties in the order of 19% and 4% for the age and mass, respectively. These values are much higher than before and comparable to the uncertainties obtained with the set of 33 stars, showing that the precision of the input parameters directly affects the precision of the output parameters in PARAM. Furthermore, when dealing with such uncertainties we observe that global seismic parameters ( $\Delta\nu$  and  $\nu_{\text{max}}$ ) are more effective in constraining the age than the luminosity (from 22-24% to 19%). We also see that the inclusion of both the luminosity and the global seismic parameters further constrain the age and mass when compared with using just one or the other (age uncertainty of 14% and mass uncertainty from 4% to 3.3%).

| Set                   | $\sigma(T_{\text{eff}})$<br>[K] | $\sigma([\text{Fe}/\text{H}])$ | $\frac{\sigma(\Delta\nu)}{\Delta\nu}$<br>[%] | $\frac{\sigma(\nu_{\text{max}})}{\nu_{\text{max}}}$<br>[%] |
|-----------------------|---------------------------------|--------------------------------|----------------------------------------------|------------------------------------------------------------|
| 33 stars              | 75                              | 0.1                            | 1.0                                          | 3.2                                                        |
| Magrini et al. (2022) | 80                              | 0.12                           | -                                            | -                                                          |
| 10 <i>Ariel</i> stars | 33.5                            | 0.02                           | 1.0                                          | 3.1                                                        |

TABLE 6.17: Comparison between the median relative uncertainties of  $\Delta\nu$  and  $\nu_{\text{max}}$ , and the median absolute uncertainty of  $T_{\text{eff}}$  and  $[\text{Fe}/\text{H}]$  between the set of 33 stars, the [Magrini et al. \(2022\)](#) article and the set of 10 *Ariel* stars.

| Constraint                                    | $\frac{\sigma(\text{Age})}{\text{Age}}$<br>[%] | $\frac{\sigma(M)}{M}$<br>[%] |
|-----------------------------------------------|------------------------------------------------|------------------------------|
| nolg_spec                                     | 37.2                                           | 6.5                          |
| nolg_specphot                                 | 24.1                                           | 4.1                          |
| nolg_spec+ $\Delta\nu + \nu_{\text{max}}$     | 19.1                                           | 4.3                          |
| nolg_specphot+ $\Delta\nu + \nu_{\text{max}}$ | 14.1                                           | 3.3                          |
| spec                                          | 34.1                                           | 5.9                          |
| specphot                                      | 22.0                                           | 3.9                          |
| spec+ $\Delta\nu + \nu_{\text{max}}$          | 19.0                                           | 4.1                          |
| specphot+ $\Delta\nu + \nu_{\text{max}}$      | 14.1                                           | 3.3                          |

TABLE 6.18: Table of the median relative uncertainties of the 10 *Ariel* stars characterized by PARAM when their input parameters have their uncertainties inflated to match the set of 33 stars.

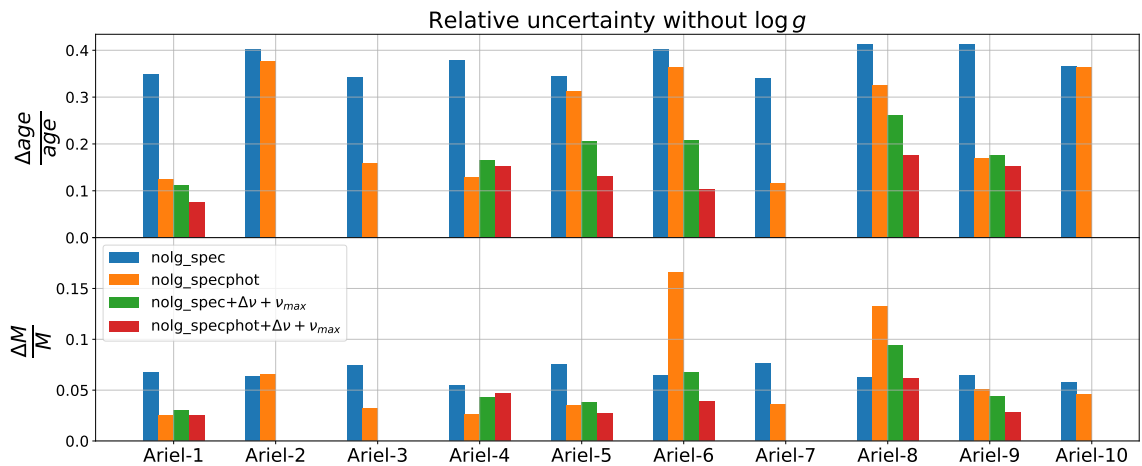


FIGURE 6.18: Comparison of the relative uncertainties for age and mass for the 10 *Ariel* stars for different input sets, without  $\log g$ .

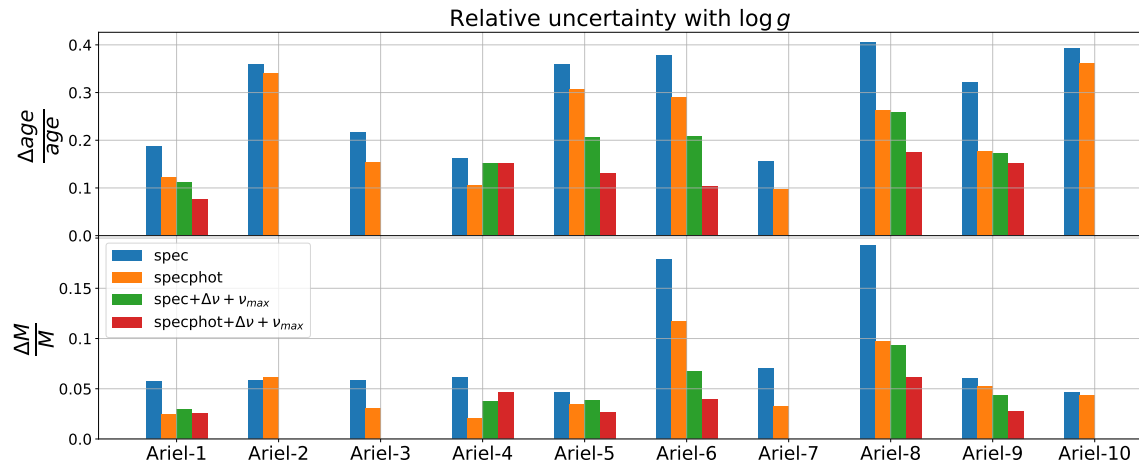


FIGURE 6.19: Comparison of the relative uncertainties for age and mass for the 10 *Ariel* stars for different input sets, with  $\log g$ .

## Chapter 7

# Summary and conclusions

Stellar observations are limited to superficial characteristics, which in turn are dependant on the unobservable internal structure. Through an understanding of theoretical physics we can infer the internal physical properties that give rise to the observable parameters. This is the purpose of software like MESA, GYRE and PARAM, which put together allow for comparisons between real-world observations and computationally simulated physical models. MESA generates stellar models, GYRE generates individual seismic modes and PARAM applies an isochrone-fitting method that takes into account spectroscopic, photometric and global seismic parameters as inputs to produce outputs that provide information on age, mass and radius.

Internal consistency checks were made with PARAM with the use of synthetic stars possessing artificial input uncertainties in the order of those typically found in the literature. From these consistency tests, PARAM produced outputs with relative accuracies that are well captured within the relative uncertainty. These tests also showed that it is more difficult to constrain the age for stars of initial masses  $M_{\text{in}} \lesssim 0.8 M_{\odot}$ , as well as for stars that are early in their main sequence.

PARAM was then compared against four other stellar characterization codes that make use of individual frequencies as input parameters. These stellar codes provided relative uncertainties in the order of 3% for mass and 10% for age, whereas PARAM obtained in the order of 5% and 26%, consistent with uncertainties for the mass and age of main sequence stars studied in [Silva Aguirre et al. \(2015\)](#) when using only  $T_{\text{eff}}$ ,  $[\text{Fe}/\text{H}]$ ,  $\Delta\nu$  and

$\nu_{\max}$  as constraints. Curiously, the precision obtained for the age also align with the uncertainties of red giants studied in the work of [Rodrigues et al. \(2017\)](#) when using the same constraints (in the order of 25%). There is no evidence to contradict the conclusion reached in [Silva Aguirre et al. \(2015\)](#), which states that the inclusion of individual modes as constraints provide a two-factor increase in the precision of the mass estimation (from the order of 5% to the order of 2%) and of the age estimation (from the order of 25% to the order of 10%). If PARAM is one day upgraded to handle individual modes as constraints then tests could be made to verify this. Comparisons between the results from PARAM with the other codes hint at the existence of non-negligible bias in the mass and age determination, however the results vary from code to code and it is worth remembering that the input physics is different and these codes used individual frequencies whereas PARAM used global seismic parameters. This dissertation concludes that PARAM is a reliable tool for stellar characterization of F-G-K type stars, however it would be in the interest of rigour to keep testing it for systematics.

Following this, PARAM was tested against 10 stars in the *Ariel* catalog, candidate observable targets for the ESA mission. Sixteen different input constraints were tested, some with the usage of luminosity, some with seismology and some with both. It was found that, in this case, the usage of global seismic parameters as constraints provide values of precision comparable to highly accurate photometric constraints, with both cases resulting in the relative mass uncertainty going from the order of 5% to 1% and the age from 29% to 9%. This shows that the inclusion of luminosity derived through accurate photometry and parallax for nearby stars is as effective as seismic constraints. The combination of both seismic and photometric constraints does not, in this case, lead to significant differences in precision.

It is important to note that the conclusions derived in the previous paragraph are conditioned by the uncertainties in the input parameters. The relative uncertainty for  $T_{\text{eff}}$  was 0.6% and the uncertainty for  $[\text{Fe}/\text{H}]$  was 0.02 dex, which contrasts the uncertainties typically found in the literature, such as the ones for the 33 stars (1.3% and 0.1 dex, respectively). When the uncertainties of the input parameters is increased to match those of the 33 stars, we obtain estimations in ages and masses that are comparable to the 33 stars. The results obtained contradict the previous paragraph, where it becomes evident that global seismic parameters are more effective than luminosity as a constrain for the

age. The second contradiction is that the combination of both seismology and luminosity together further constrain the estimate in age and mass.

An increase in precision is not always guaranteed, as certain input parameters could be wrong and therefore contradictory to the rest of the set. Rather than removing degeneracy in the output solutions (which is the case for the majority), it may indicate that another solution might be more appropriate (as is the case with Ariel-4), which in turn decreases the precision.

Double solutions are most common for F-type stars due to the formation of the hook feature in the main sequence, causing severe intersection of the stellar tracks which give rise to degeneracies. PARAM consistently predicts that Ariel-4 possesses a mass in the order of  $1.3 M_{\odot}$ , placing it in a region where the effect of the hook is significant. This could explain the observed rise of degenerate solution when luminosity and seismology inputs are provided, but whether this is truly the case requires a more in-depth and comprehensive study.

Uncertainties remain greatest in the age determination than in mass or radius. The reason for this has to do with the broad span of ages captured within a relatively small volume of the input parameter space. In other words, small variations in input still lead to large uncertainties in age outputs. This effect becomes exacerbated for K-type stars. Such challenges need to be addressed. As a result, this dissertation concludes that stellar characterization of K-type stars presents large uncertainties in age. Furthermore, stellar characterization of F-type stars might be prone to double solutions due to the emergence of a hook feature.



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