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INTERLACING ISLAMIC ART WITH THE TEACHING OF SYMMETRY IN PORTUGUESE SCHOOLS

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Abstract: *After five centuries of Muslim presence in Portugal, it would be unthinkable to assume that, apart from the naming of regions, towns, and villages (Algarve, Ourém, Alcabideche), Islamic culture has not left its mark on Portuguese society. It is not possible to ignore the impact of Islamic art on Portuguese art and architecture, especially in decorative arts such as tiles or crockery. That rich and long cultural heritage can be used to promote the teaching of Mathematics in schools. Geometry can be explored through the analysis of decorative art found in Portuguese historical buildings, in particular through the analysis of ceramic tile panels and facades used extensively throughout several centuries. Through the analysis of decorative art found using ceramic tiles in two Portuguese monuments, we propose several didactical activities to explore the topics of isometry and symmetry in schools. We give some examples of rosettes, friezes and wallpapers found in the monuments, classified according to their symmetry groups. We also give some examples of applied activities using GeoGebra and exploring Islamic motives/patterns.*

Keywords: Symmetry; Wallpaper patterns; Islamic Art; Mudejar Art; Mathematics; Tiles.

INTRODUCTION

In literature there are several articles about teaching symmetry in the classroom with the use of everyday life objects, nature images, art, etc. (Knuchel, 2004; Seah & Horne, 2019; Stylianou & Grzegorczyk, 2005). In this paper our aim is to illustrate that it is possible, and useful, to interlace Islamic and Mudejar Art with the teaching of some geometric concepts in Mathematics. More specifically, we will use geometric patterns of Islamic influence existing in Portuguese monuments, to approach and motivate students for the study of isometry and symmetry at school.

There exists a difference between Islamic Art and Mudejar Art. The first one is produced by Muslims in territories under Muslim rule dominion (7th century to the present) while the second, called Mudejar art, is produced by Moors in Christian dominated territories (in the Iberian Peninsula, in general, from the 12th to the 16th century, and in Portugal with greatest intensity in the 15th to 16th centuries); the term Mudejar Art also applies to later Iberian architectural and decorative works influenced by Islamic Art (Rita, 2021). It is not one of the objectives of this work, to study the history of Islamic and Mudejar Art, but to use examples of patterns found in our historical/cultural heritage of Islamic influence, such as tile panels and carved wood ceilings of churches and/or palaces, to explore specific Geometry topics at school. Another objective is to propose a set of didactical activities, related to the geometric characteristics of Islamic patterns that enhance the Portuguese heritage.

Several software programs like *iOrnament* (Richter-Gebert, 2021) or *The Geometer's Sketchpad* can be used to work with symmetries (Thangamani *et al*., 2018). We have chosen to propose some activities in GeoGebra because it is free and very used by Portuguese teachers and students. Any of the proposed activities can be completed using a dynamic geometry tool (like *GeoGebra*) or using instruments such as ruler, compass, set square and protractor. One way to engage students in learning is to approach the contents from an interdisciplinary perspective, and symmetry is a topic where teachers can build bridges between Mathematics, Art Education, History and Geography.

MUDEJAR ART AND THE SYMMETRIES OF WALLPAPER PATTERNS

A geometric transformation $\mathcal{T}$ on the plane is a bijective function in $\mathbb{R}^2$, symbolically defined by:

$$\begin{aligned} \mathcal{T} : \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ P &\rightsquigarrow \mathcal{T}(P) = P' \end{aligned}$$

Translations, rotations, reflections, glide reflections, dilations, spiral dilations, and elongations are examples of geometric transformations. Special interest lies on the first four examples which constitutes the set of isometries on the plane and are the base for the definition of symmetry on the plane. A geometric transformation $\mathcal{T}$ is an *isometry* if for any two points P and Q we have $\overline{P'Q'} = \overline{PQ}$, where $P' = \mathcal{T}(P)$ e $Q' = \mathcal{T}(Q)$. The set of all isometries on the plane together with the operation composition forms a group.

Symmetry can be found almost anywhere around us: in Greek vases, in snowflakes, in buildings and even in alphabet letters. In everyday language, it is a concept associated with the harmony of proportions that makes objects and figures visually appealing. From a mathematical perspective, symmetry can be used to organize and classify figures in decorative art. A flat figure $\mathcal{F}$ is a set of points on the plane. A symmetry of $\mathcal{F}$ is any isometry i which leaves $\mathcal{F}$ invariant, $i(\mathcal{F}) = \mathcal{F}$. The set of symmetries of a figure $\mathcal{F}$ together with the operation composition forms a group which is known as the symmetry group of $\mathcal{F}$. In the plane there are only three categories of discrete symmetry groups: rosette groups (they have a finite number of symmetries which can only be rotations or reflections); frieze groups (they have translation symmetries in only one direction); and wallpaper groups (they have translation symmetries in two directions, spreading over the plane). There are two types of rosette groups (cyclic groups, C_n, and dihedral groups, D_n,), seven types of frieze groups and 17 types of wallpaper groups.

In this paper we shall focus on wallpaper groups. Wallpaper patterns are figures with have translation symmetries in two directions each of which with a minimum modulus, $\mathcal{T}_{\vec{u}}$ and $\mathcal{T}_{\vec{v}}$. These translations generate all translation symmetries of the pattern and to find all other symmetries it suffices to find those related to a fundamental region of the pattern. More details on wallpaper symmetry groups may be found in Veloso (2012). Several notations have been proposed for the wallpaper groups, and in this paper we shall use the crystallographic one: *p1, p2, pm, pg, pmm, pmg, pgg, cm, cmm, p4, p4m, p4g, p3, p3m1, p31m, p6, p6m*. The classification of wallpaper patterns can be done using the flowchart proposed by Washburn and Crowe (1988).

The first author of this work explored wallpaper patterns in two particular Portuguese monuments where several Mudejar artworks can be found: Palácio Nacional de Sintra and Sé Velha de Coimbra. Of the 17 existing types of patterns, only six of them were found and one of the most frequent is group p4m. Figure 1 shows two examples of such patterns.

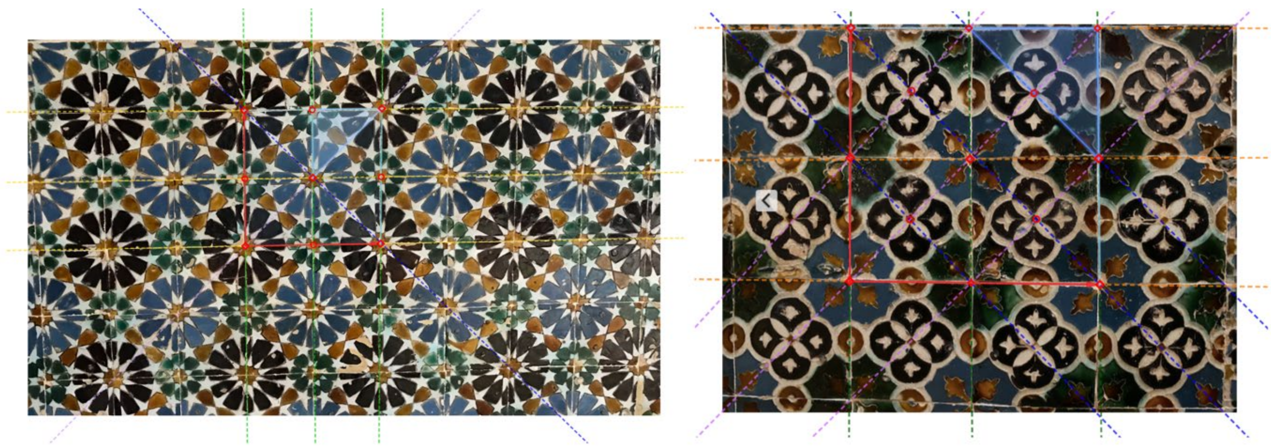

Figure 1 Wallpaper patterns with symmetry group *p4m* found in tile panels of two Portuguese monuments (Palácio Nacional de Sintra on the left and Sé Velha de Coimbra on the right).

ACTIVITIES

The activities proposed in this paper can be used, with the proper adaptation, for students in elementary school (grades 1- 9), secondary school (grades 10 - 12) and vocational school (grades 10 - 12), as part of the study of the geometry contents.

The first activity is related with the already mentioned fact that in Palácio Nacional de Sintra we can find several Mudejar artworks. The rosette shown in Figure 2 can be found not only in the palace but also in several panels from other buildings and monuments in Portugal.

Figure 2 Tile rosette from Palácio Nacional de Sintra.

Inspired by the rosette given in Figure 2, one of the proposed didactical activities challenges students to construct, using Geogebra, the following panel, given some guidelines:

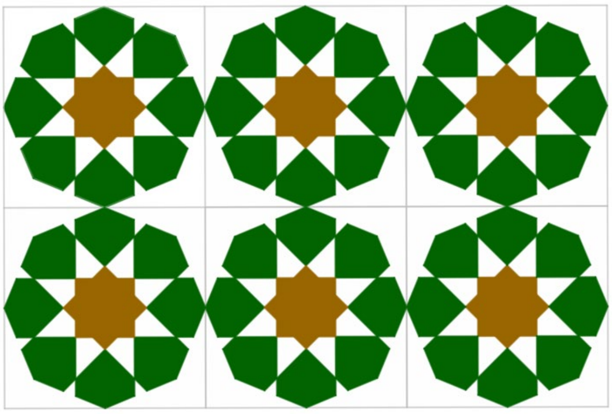

Figure 3 Result of a pattern construction.

To construct the base of this pattern, students need to apply previous learned concepts like, for example, parallel and perpendicular lines, circumferences, squares, and regular octagons.

The second activity consists of asking students to identify and characterize all kinds of symmetries they can find in a given pattern, such as the one in Figure 4.

Figure 4 Tile panel from Palácio Nacional de Sintra with symmetry group *cmm.*

To give a concrete example that the same activity can be used with students of different grades we refer that in Figure 4, students up to the seventh grade are only asked to identify reflection symmetries of vertical and horizontal axis and half-turn rotations. However, students from the eighth grade onwards can also identify translation symmetries in various directions and glide reflection symmetries. The last activity presented here asks students to apply different geometric transformations to construct a tile panel. First students will open a GeoGebra file where they find the polygon showed in Figure 5. Then they must create a tile using only geometric transformations on the polygon. Two possible tile construction are given in Figure 6, using only rotations around a vertex of the polygon.

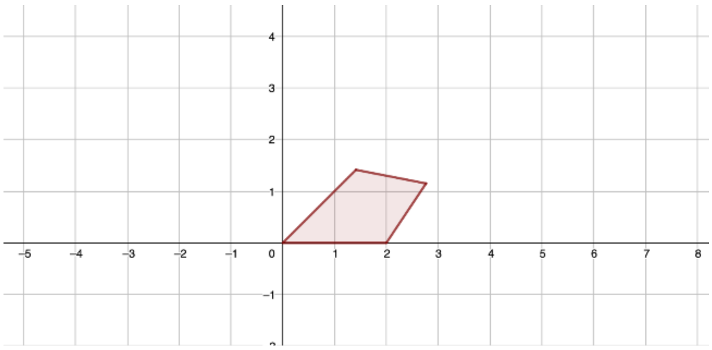

Figure 5 Polygonal motive for a wallpaper pattern construction.

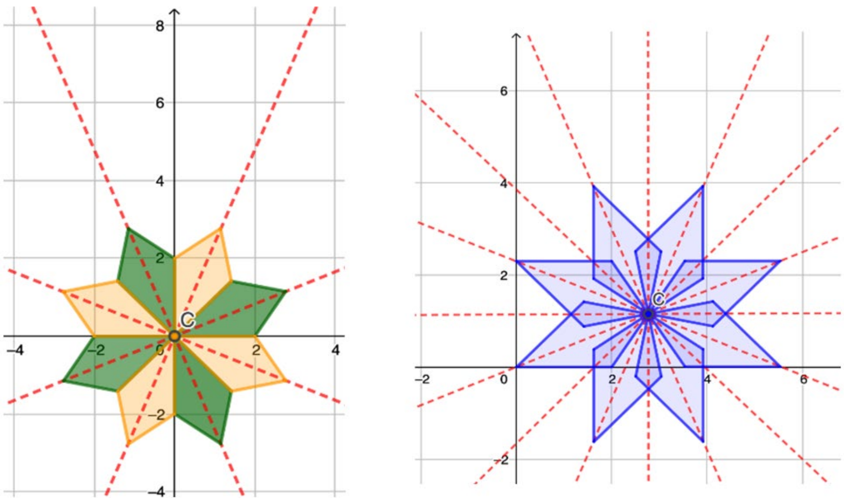

Figure 6 Two possible tile constructions from the motive in Figure 5.

Finally, students are asked to create several panels (with at least two tiles) applying only reflections, only rotations, only translations and several different kinds of geometric transformations sequentially, to the tile they previously created.

CONCLUSION

With this work we intend to use Islamic and Mudejar Art to motivate students to the learning of Geometry. To do this it is necessary to first understand what these two forms of art consist of and which of their main characteristics can be explored by teachers with their students in a multidisciplinary way. One such characteristic may be the variety of motives found in the artworks. In the case of the Mudejar patterns found in Palácio Nacional de Sintra and Sé Velha de Coimbra, the prevailing motives are geometric or vegetal. The tile panels found in these monuments are good examples to explore isometries and symmetries with the students. They are also a good source for the generation of didactical activities which combine mathematical topics with artistic creativity.

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Paula Cristina Correia Santa Rita obtained her degree in Mathematics - Operations Research at the University of Coimbra. After graduating she became interested in teaching and carried out her in service professionalization at the Higher Education School of Viana do Castelo. Being interested in statistics, she took a Specialization Course in Data Analysis at the Polytechnic Institute of Viseu. She is currently more interested in linking mathematics education with the arts and has completed a master's degree at the University of Aveiro, intitled "Islamic Art and the teaching of Geometry: a perfect marriage". She currently teaches mathematics at the Frei Rosa Viterbo Secondary School, Sátão.

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Nuno Bastos finished, in early 2012, his PhD in Mathematics. He is, currently, an Associate Professor at the Polytechnic Institute of Viseu and a researcher at CIDMA - Centre for Research & Development in Mathematics and Applications. In the beginning, his interests were fractional calculus, calculus of variations, but nowadays he is mostly interested in mathematical education, educational technology, and the popularization of mathematics. Regularly gives sessions in schools about mathematical magic that have the aim of increasing the interest in mathematics in students. In 2015, he received the Best Paper Award for the Signal Processing Journal by the European Association for Signal Processing (EURASIP) with the paper "Discrete-time fractional variational problems". He published academic works in different international scientific journals and was a member of organizing committees of national and international conferences.

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