

Inventory Management in a Process Industry

Mariana Gonçalves Barrias

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Supervisor at FEUP: Prof. Bernardo Almada Lobo



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Abstract

Over time, BA Glass has grown dramatically and become increasingly noticeable. More and more customers use its products, requiring a greater degree of company's production strategy and reaction.

Being a Make-to-Stock industry and facing a demand greater than supply, it is vital to develop inventory management process that attempts to respond more correctly and efficiently to customer requests and requirements. Analogously to the growing competitiveness, it is essential to increase the degree of service offered to consumers in order to achieve maximum customer satisfaction and hence differentiate the business. Consequently, it is crucial to respond swiftly to demand fluctuations and optimize the supply chain process.

Through this dissertation, it is intended to develop a strategy for optimizing the stock leveling and production planning in order to meet a specified service level. To accomplish this, it was required to specify all key performance indicators (KPI) and variables necessary for the development of the optimization problem. To ensure customer satisfaction, it is relevant to take into account the manufacturing lead time and to respect the order size.

Initially, it was resorted a study and analysis of the most relevant inventory related metrics appropriate to assess the service level provided in a production environment. Subsequently, it was analyzed the current status of the company and calculated the quality of service provided through demand, sales plan, production and stocks. Then, the demand trend and seasonality of each product family was investigated. In a final stage, a heuristic for solving the optimization problem was developed in Python One of the heuristic's purpose is the calculation of the cycle and safety stock to be maintained during the year for each family of products, having as criteria a given fill rate to be maintained.

So, in general, the present project aims to develop an optimization problem that allows prioritising the service level provided to the most important customers. In other words, reduce the risk of stock-outs and meet the desired order capacity, ensuring a greater number of successful deliveries to priority customers to the detriment of lower segmentation customers, considering the associated costs.

Resumo

A BA Glass tem crescido exponencialmente no decorrer do tempo, tornando-se cada vez mais visível. Um maior número de clientes recorre aos seus produtos, o que, por sua vez, exige um maior grau de coordenação na estratégia de produção e resposta por parte da empresa.

Sendo uma indústria Make-to-Stock e a procura maior que a oferta, torna-se necessário a criação de um método de gestão de stocks que procure responder com maior rigor e eficiência aos pedidos e requisitos exigidos pelos clientes. Analogamente à crescente competitividade, revela-se então crucial elevar o nível de serviço prestado aos clientes, de modo a alcançar a máxima satisfação e, assim, destacar a empresa. Deste modo, é imprescindível agir rapidamente às variações da procura e otimizar o processo da cadeia de abastecimento.

Através desta dissertação, procura-se criar uma metodologia que otimize o nivelamento de stock e o plano de produção tendo como target um determinado nível de serviço estipulado. Para tal, foi necessário definir todos os KPIs e variáveis a entrar nas respetivas equações e restrições. De forma a garantir uma resposta eficiente à procura é necessário ter em conta o lead time de produção e respeitar as quantidades de produto requisitado.

Inicialmente, recorreu-se a um estudo e análise das métricas relacionadas com o inventário mais adequadas para avaliar o nível de serviço prestado em ambiente de produção. Posteriormente, procurou-se analisar o status atual da empresa e calcular a qualidade de serviço prestado tendo em conta a procura, plano de vendas, produções e stocks. De seguida, é realizado um estudo à tendência da procura e sazonalidade de cada família de produtos. Numa fase final, é desenvolvido o problema de otimização, através de uma heurística, em Python. Uma das variáveis de decisão é a quantidade de stock de ciclo e segurança a manter durante o decorrer do ano para cada família de produtos, tendo como critério o cumprimento de um determinado *fill rate*.

De um modo geral, o presente projeto visa então desenvolver um problema de otimização que permita priorizar o nível de serviço prestado aos clientes mais importantes. Ou seja, diminuir o risco de rotura de stock e cumprir a capacidade da encomenda desejada, assegurando maior número de entregas com sucesso para os clientes prioritários em detrimento dos clientes de menor segmentação, tendo em atenção os custos associados.

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“The secret of getting ahead is getting started.”

– Mark Twain

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Acronyms and Symbols

AM	Amber
AS	Strong Amber
AV	Avintes
BA	BA Glass S.A.
BG	Big
BR	Flint
BU	UV Flint
CV	Variance Coefficient
CO	Cobalt Blue
CS	Cycle Stock
CSL	Cycle Service Level
EOQ	Economic Order Quantity
ERP	Enterprise Resource Planning
FM	Dead Leaf
FR	Fill Rate
I	Iberia
KPI	Key Performance Indicator
LE	Léon
LT	Lead Time
MG	Marinha Grande
MI	Miniature
MTO	Make-to-Order
MTS	Make-to-Stock
NN	Narrow-Neck-Press-Blow
OOS	Out Of Stock
PR	Black
PS	Press-Blow
ROP	Reorder Point
SG	Single Gob
SKU	Stock Keeping Unit
SLA	Service Level Agreement
SM	Small
SS	Safety Stock
Ton	Tonne
UV	UV Green
VB	Antique Green
VF	VillaFranca de Los Barros
VG	Georgia Green
VN	Venda Nova
VR	Green

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Chapter 1

Introduction

This current report is part of the Dissertation Curricular Unit of the Master's Degree in Mechanical Engineering in the specialization of Production Management at the Faculty of Engineering of the University of Porto.

BA Glass manufactures a diverse set of glass products. The glass industry typically produces for stock and this company maintains a continuous production of glass packages and its ovens working on a 24 hour a day, seven day week basis. As a result, special attention must be paid to production planning in order to avoid creating excess inventory of low-turnover products and to avoid out-of-stocks (OOS) of high-turnover products requested, specially, for high-segmentation customers.

A solid degree of customer service is a critical aspect to retain in firms. This helps the firm to not only defend its market position, but also to reinforce its image in the eyes of the customers, allowing it to grow its market share (Kilger et al., 2008).

This chapter presents the context and motivation for the described project, the dissertation's stated aims, as well as all the methodology and steps used in its execution. Besides this, a brief background of the company and a succinct explanation of the production method are presented.

1.1 Project Motivation

This project is inserted in the Planning Department. This department is responsible for developing production plans taking into account each plant capacity, while aiming to achieve the highest Key Performance Indicators (KPIs) values.

In this particular company, the main goal of the production strategy is to forecast and anticipate the expected demand. In reality, adhering to the stock-keeping philosophy allows the acquisition of a lot of benefits at the production and customer level, Ahmadi et al. (2020).

In BA, it is well known that potential demand exceeds capacity, so, as a result, preventing OOS becomes incredibly challenging. The best course of action is to attempt to minimize this situation as much as possible, by calculating the optimal stock to maintain for each product's family, giving priority to the major customers.

To retain a strategic advantage, firms must cut costs while preserving customer service. As a result, a high degree of ordering flexibility and a quick reaction to consumer requests have become critical to a business's success (Tao et al., 2017).

Nowadays customers require more attention and this point service level should be higher than ever (Çelik et al., 2013). Thus, and within the Planning Department, it is necessary to implement a metric that evaluates the current service level offered and analyze whether the customer's needs are being truly satisfied. It is essential to maintain a high level of client satisfaction in order to enhance sales, Borchardt et al. (2018), since the quality of service is critical in ensuring client fidelity.

The purpose of this dissertation is to investigate a ideal stock levelling and production approach with the goal of providing a solution that avoids stock shortages, prioritizing the most segmented customers and increasing the service level offered, by defining an annual production plan.

1.2 BA Glass

BA is a Business-to-Business company that produces in twelve factories located within seven different countries. It also currently sells to over seventy countries, and produces over 23.000K tons a year.

In this section it will be explore the company history and its main milestones.

1.2.1 Background

- 1912** Raul da Silva Barbosa and Domingos de Almeida founded BA Glass. This business is dedicated to the manufacture of glass containers.
- 1930** Introduction of automatic technology.
- 1969** Start of operation at the Avintes industrial unit.
- 1993** BA acquires the CIVE (Companhia Industrial Vidreira), which is based in Marinha Grande.
- 1998** Start of operation at Villafranca de Los Barros industrial unit.
- 1999** Acquisition of Vidriera Leonesa.
- 2012** Expansion of the market to Eastern Europe.
- 2016** The corporation acquires a German company with its headquarters in Gardelegen.
- 2017** The company became a member of the Yioula Group, which comprises four factories located in three countries: Greece, Bulgaria and Romania, illustrated in Figure 1.1.
- 2020** BA won the sustainability award by World Finance for being the most sustainable company in the world in the glass industry (Glass, 2020).

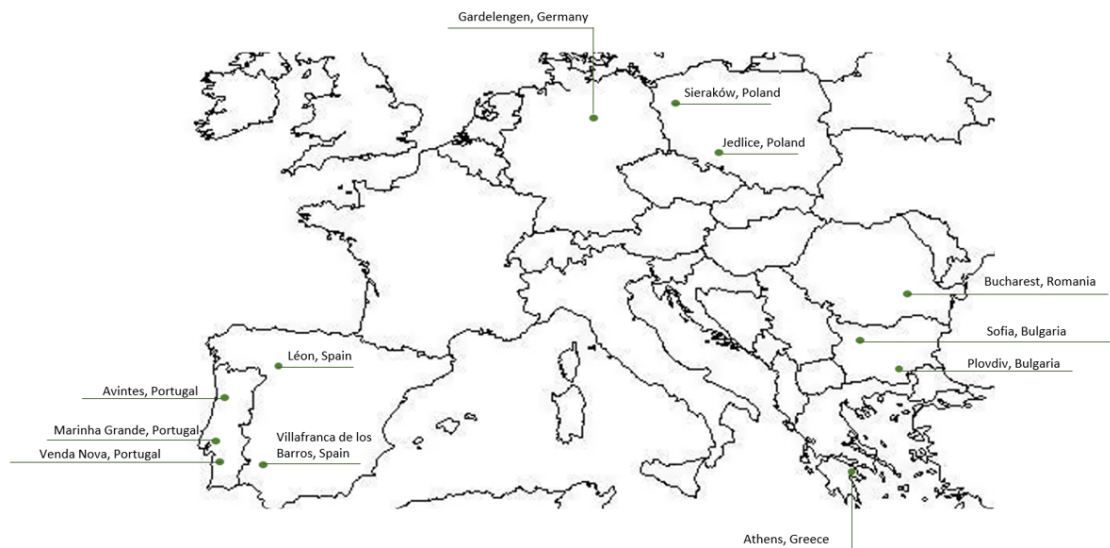


Figure 1.1: Geographic Expansion of BA Glass Factories

1.3 Project Goals

With the firm's expansion and the acquisition of new factories, furnaces and production lines in numerous countries throughout Europe, the company increased its customer base, diversified its product assortment and increased the density and complexity of its supply chain.

As the global market becomes more dynamic and competitive, businesses are compelled to strengthen their fundamental business competencies in order to maintain client satisfaction (Saputro et al., 2021). With this in mind, it is clear that there is a greater need for attention and organization at the production planning level in order to respond efficiently and quickly to all requests from various customers, i.e., to be able to meet demand in the right quantities, thereby strengthening the relationship established with the customer.

For the company's future success, it is critical for it to manage its relationship with the customers (Nandapala and Jayasena, 2020). As such, market transparency, regarding the level of service that the firm can provide, has become an increasingly important milestone for companies to work towards.

As a result, the necessity to evaluate whether the company is truly prioritizing the most segmented clients emerged. If this is not the case, it should act upon this information and attempt to increase the high profile customers' service level. Customers are categorized and divided, by the company, according to specific criteria, which will later be demonstrated.

The service level will be evaluated for each of the client segmentation using two distinct metrics.

Thus, this project intends to analyse the level of service that is currently being provided and develop an optimization problem, based on the company's historical data, that assists and supports the planning of production in Iberian factories. This aims to avoid stock-outs by increasing production and therefore stock coverage, while guaranteeing a minimum service level.

1.4 Methodology

The first several weeks were dedicated to multiple training sessions in various areas related to the project's theme. That is, the departments of Planning, Logistics, Order Management, Front Office and Production. This procedure was critical for the growth of this project since it has established who to ask for assistance in case of doubt and who to share ideas with.

After gaining a better understanding of the company organization and the project theme, the dissertation's methodology and chronology was planned. The integrated business management system used is SAP, which enabled the integration of all departments, (Gupta and Kohli, 2006). This is a database management system that connects all data and enables the collecting of necessary information for analysis.

Initially, it was investigated and determined which metrics would be most appropriate for evaluating the level of service supplied to clients in the production context. Following that, the current situation and the degree of service offered to customers over the last several months were analyzed. For the development of this study and data analysis, the products were divided into family groups and the customers by segmentation.

Subsequently, a stock optimization problem and the algorithms for solving it were developed and adapted by extensive reading and support of various studies. To do this, all variables, statistical functions, main objective function and constraints were defined to obtain a good quality solution in reasonable computation time. Clients with a higher segmentation will require a higher level of service.

The following stage was to conduct multiple simulations using Python language and verify and discuss the results' veracity.

The subsequent diagram in Figure 1.2 illustrates the development process in a more simplified and efficient manner.

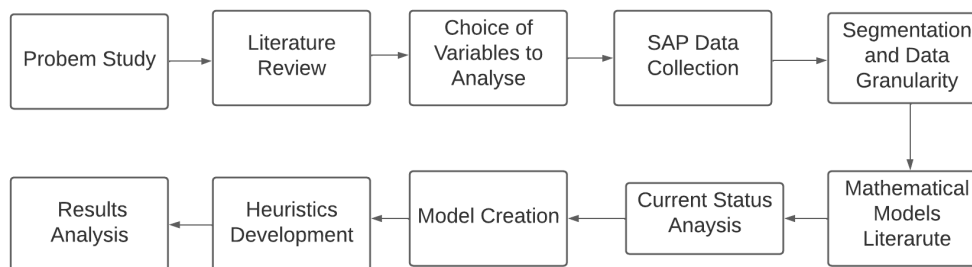


Figure 1.2: Task Execution Plan

1.5 Dissertation Structure

This dissertation is composed by six separate chapters.

The first chapter introduced the company, the project's purpose, its objectives and approach used.

Following that, in the 2nd Chapter, a literature review, that is, a study of the state of the art, was performed for a better understanding of the theoretical principles necessary for the development of this thesis.

Chapter 3 provides an overview of the company's current state, as well as a summary explanation of how clients are evaluated and some impasses that have arisen in the planning department. Then, in Chapter 4, a study and categorization are conducted regarding the fluctuation of materials demand for each client type and it is performed a study of the service level currently provided by the company.

Furthermore, Chapter 5 details the steps involved in the development of the optimization problem model, as well as data processing.

Then, in Chapter 6, the results from the heuristics are provided and discuss. And finally, in Chapter 7 a conclusion is made about the work completed and future directions for the theme are presented.

Chapter 2

Literature Review

2.1 Logistics System

The Logistics System's primary goal is to create value for the consumer. This component entails performing a series of operations and actions in order to make the appropriate product, at the right time, in the desired quantity and at the lowest possible cost (de Carvalho and Cardoso, 2002).

2.1.1 Supply Chain

Supply-Chain excellence is known to be the key to gaining competitive advantages (Çelik et al., 2013). One of the most significant shifts in the paradigm of contemporary company management is the fact that individual enterprises are no longer competing as stand-alone organizations, but as supply chains (Lambert et al., 1998b). Among the numerous modes of competition, supply chain management has emerged as one of the most competitive models for corporate success. It entails a number of interconnected processes, as illustrated in the Figure 2.1.

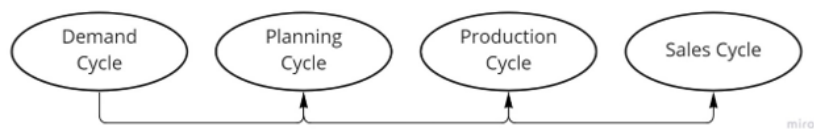


Figure 2.1: Supply Chain Processes

Each of these cycles is composed by a number of sub-processes, including customer relationship management, demand management, customer order fulfillment, production flow management and product creation (Almeida, 2020).

When the supply chain is complex, it is tough to know when and what to keep in stock and how to strike the right balance between overall inventory, related expenses and customer service (Graves and Willems, 2000). This kind of process needs a great deal of planning, research and analysis.

2.1.1.1 Production Strategy

When examining the output level of an organization, managers are confronted with a crucial choice: Setting Responsiveness versus Efficiency Trade-Offs (da Silva, 2008). Typically, organizations adopt either a push approach (Make-to-Stock) or a pull strategy (Make-to-Order). Some hybrid strategies may also be employed, such as Finish-to-Order (FTO).

Make to Order

Many forward-thinking businesses have switched to MTO systems, where companies keep no inventory and produce only when needed (Kaminsky and Kaya, 2009). Especially when inventory is found prior to the manufacturing process, the supply chain follows this pull logic in which commodities are created only on demand (Sousa, 2019). Therefore, there is no inventory at the end of the supply chain and all manufacturing is based on validated demand projections.

Although this strategy reduces the total cost of maintaining inventory, the lead time to deliver the goods to the client is longer, which can result in a loss of competitiveness, Kaminsky and Kaya (2009), and it may also trigger high setups.

Make to Stock

MTS is a production approach in which manufacturing planning and scheduling are determined by anticipated product demand. Products produced during one production period are utilized to meet orders placed during the subsequent manufacturing periods. So, the business produces in order to estimate or anticipate client sales.

Generally, in these cases there is a temporal lag between demand and supply (Lambert et al., 1998a). The existence of stock enables consumption to be isolated from production.

The existence of warehousing also allows for a faster response to the client, improving service level. However maintaining a high volume of stock is costly and it also introduces additional risks such as storage, expiration and price declines.

Whereas many companies view stocks as a cost, some of them view them as active components that contribute to the leadership of a certain market share through a high service level provided and the availability of items in the amounts required by clients.

The MTS approach has various advantages in terms of several metrics, Slack and Brandon-Jones (2019), such as, for example:

- **Demand Fluctuation:** Stock acts as a protective barrier against demand fluctuations, since it is not always easy to forecast future requirements;
- **Production Failures:** When the factory fails or a production line abruptly stops, prolonging production times and reducing available product quantities, stock, once again, serves as protection (de Carvalho and Cardoso, 2002).

2.1.1.2 Lead Time

Lead time, from the perspective of the customer, is the period between placing an order and receiving the product. On the other hand, from the company's perspective, it is something more complex, involving an examination of all phases of the company's process and the duration of each until the product is delivered to the customer.

So, lead time decision-making process involves interactions between the supply and demand sides of a supply chain, each having a unique set of interests (Song et al., 2013). This is a critical performance indicator that denotes the amount of time required to complete a process.

There are four primary types of LT:

- Customer Lead Time: time frame between when a customer places an order and when the items are delivered to him;
- Material Lead Time: length of time it takes to receive materials from a supplier after the placement of the order;
- Production Lead Time: period of time it takes a manufacturer to fulfill an order placed by a retailer;
- Cumulative Lead Time: longest amount of time required to complete the activity in question.

2.2 Stock

Usually, company's products are identifiable by their Stock Keeping Unit code (SKU). This code is a critical component in controlling and managing the warehouse's inventory. It is the product's unique reference number as it appears in the company's system.

A manufacturing business may have a variety of stock types (nibusiness, 2016):

- Raw materials: materials ready to be use in production;
- Work in Progress: stocks of unfinished goods in production;
- Consumable: like fuel, for example;
- Finished Goods: products ready for sale.

In finished goods inventory there are two types of stock, represented in the graph of Figure 2.2, important to take into account:

- **Cycle Stock:** quantity of inventory required to meet projected demand during a particular order cycle. The magnitude of the cycle inventory is due to the production, transportation or acquisition of vast quantities of material, Chopra and Meindl (2007). The following equation demonstrates how to calculate this type of stock:

$$CS = D \times LTr, \quad (2.1)$$

being:

D : average daily demand;

LTr : replenishment lead time.

- **Safety Stock**: reserve of inventory used to mitigate risk associated with demand's unpredictability and guarantee that the needed service is maintained (Almada-Lobo, 2021a).

$$SS = z \times \sigma_L, \quad (2.2)$$

where:

z : number of standard deviations for a specific service level;

σ_L : standard deviation of demand during replenishment lead time.

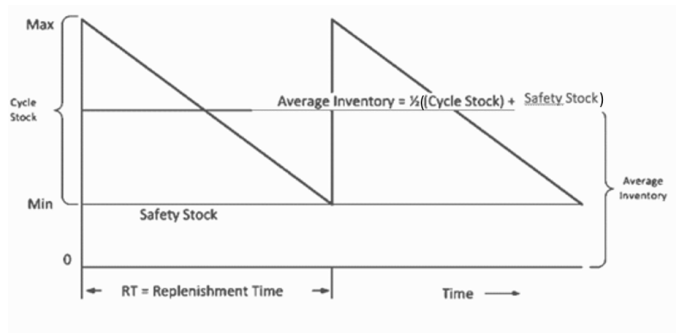


Figure 2.2: Cycle Stock Vs Safety Stock (Source:Loucka (2013))

2.2.1 Safety-Stock

The term *Safety Stock* acts as a buffer against above-average demand during the replenishing period (Silver et al., 2016), as can be seen in Figure 2.3. If the stock value falls below the safety stock limit, there is a risk of a shortage. To determine this value it is vital to understand the probability of stock-out and the quality of service desired. Choosing a specific level of safety stock requires achieving a compromise between the service wanted to provide to a particular type of customer and the associated cost of holding that additional stock (Fleischmann et al., 2005).

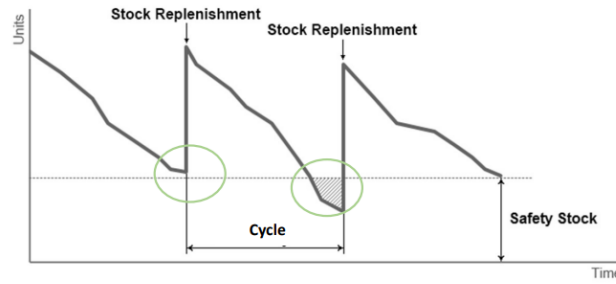


Figure 2.3: Stock Level Variance (Source: Consulting (2019))

It is widely considered in inventory management that safety stocks are an appropriate method for dealing with demand and supply volatility (Gonçalves et al., 2020) and protect against unforeseeable events, Çelik et al. (2013). It's crucial to note, though, that SS determinations are not meant to eliminate all stock outs, rather, they are meant to remove the majority of them (de Carvalho and Cardoso, 2002). Safety Stock is, in reality, the best approach to maintain a high level of service.

So, it proves to be essential to keep a SS in order to meet needs when demand, replenishment timings and volumes are erratic (Schmidt et al., 2012). There are several ways to calculate SS, depending on how demand and lead time varies:

- **Variability in Demand**

When only demand varies, the safety stock required to provide a specific degree of protection is calculated by the following equation (King, 2011):

$$SS = Z \times \sqrt{\frac{LT}{T}} \times \sigma_d, \quad (2.3)$$

where:

Z : safety factor depending on service level;

LT : total lead time;

T : time interval considered to calculate the demand standard deviation;

σ_d : standard deviation of demand over T .

- **Variability in Lead Time**

In this case, the calculation will be as follows:

$$SS = Z \times \sigma_{LT} \times D_{avg}, \quad (2.4)$$

being:

σ_{LT} : standard deviation of lead time;

D_{avg} : average demand.

- **Demand and Lead Time Variability are Independent**

When both are variable and independent:

$$SS = Z \times \sqrt{\left(\frac{LT}{T} \times \sigma_d^2\right) + (\sigma_{LT} \times D_{avg})^2} \quad (2.5)$$

- **Demand and Lead Time Variability are not Independent**

Finally, when both are variable and not independent, the equation used is as follows:

$$SS = \left(Z \times \frac{LT}{T} \times \sigma_d\right) + \left(Z \times \sigma_{LT} \times D_{avg}\right) \quad (2.6)$$

2.2.2 ABC and XYZ Analyses

ABC and XYZ analyses are carried out for the inventory items to determine the production strategy of each item class (Scholz-Reiter et al., 2012).

Pareto analysis, also known as ABC analysis (dos Santos, 2017) is a relatively simple categorization method that enables significant cost control over a business's complete inventory. On the other hand, XYZ analysis is advantageous for optimizing ordering processes in which demand or product price changes (Pandya¹ and Thakkar, 2016). A combination of ABC and XYZ analysis ensures optimal outcomes in inventory management.

ABC analysis is a straightforward categorization of materials that categorizes products based on their frequency of use and their value. Pareto principle is based on the observation that a small number of components dominate the accomplishment of desired goals in a lot of situations, Stojanović and Regodić (2017). This categorization enables the determination of which products are more critical to the business and therefore require additional attention when monitoring stock levels (Maxitrans, 2020). Table 2.1 summarizes ABC classification and actions to take.

Table 2.1: ABC Classification Strategy, (Pandya¹ and Thakkar, 2016)

Class Items	Percentage of Items	Percentage of Sales	Actions
A	About 20%	About 80%	Close day to day control
B	About 30%	About 15%	Periodic review
C	About 50%	About 5%	Infrequent review

The graph in Figure 2.4 describes what was previously shown in the preceding table in a more visual way:

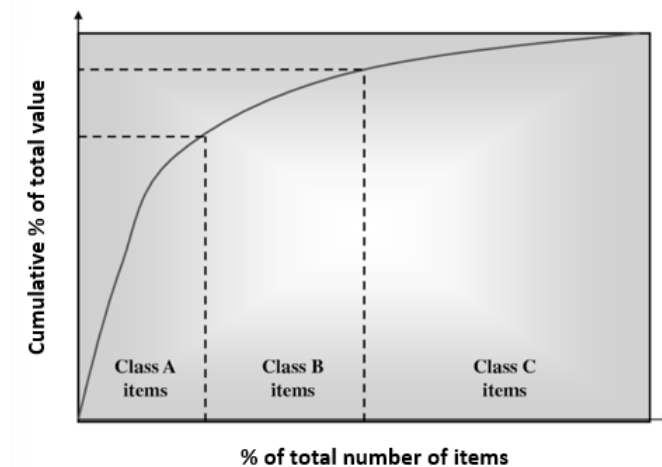


Figure 2.4: Cumulative Curve of ABC Analysis

However, this type of classification has some shortcomings regarding the rigorousness of inventory management. Not only because the segmentation into just three groups is rather simplistic, but also because ABC analysis provides just a picture of the current reality. So, this analysis alone is insufficient for optimizing stocking.

In turn, the XYZ process is generally referred to manage the changes in product demand frequency. This curve is essential in demand planning and inventory management since it enables the study of items based on their criticality, providing additional information for planning, replenishment and management procedures.

In Figure 2.5, by combining these two classifications, the black field indicates high-volume products, as well as those with the highest frequency of orders (standard items), da Costa (2012). Apart from being the most commonly consumed products, they are also the most regularly ordered by suppliers.

	X	Y	Z
A			
B			
C			

Figure 2.5: ABC/XYZ Analysis for Identifying Standard Articles.

The XYZ categorization is used to determine fluctuations in the demand and the coefficient of variation is calculated for each item. This analysis classifies items into three categories based on their consumption characteristics (Stojanović and Regodić, 2017). This classification and division regarding the criticality of materials is done as follows (EazyStock, 2020):

- **X Category:** materials whose consumption is very consistent or varies rarely. The ability to plan or make accurate predictions is extremely high;

- **Y Category:** materials whose demand fluctuates significantly due to seasonal factors or product use trends. The capacity to plan or forecast is medium;
- **Z Category:** materials whose usage is very erratic and whose ability to plan or forecast is reduced.

The variation coefficient (relative measure of the probability distribution's dispersion) must be determined, which is defined as the ratio of the standard deviation to the average sales:

$$CV = \frac{\sigma}{\bar{x}} \quad (2.7)$$

The integrated ABC-XYZ technique is used to determine the activities associated with each of the identified categories of articles in a paired comparison matrix, as seen in the resume Table 2.2:

Table 2.2: ABC and XYZ Matrix, CGMA (2022)

Class	A	B	C
X	high value high predictability continuous demand	medium value high predictability continuous demand	low value high predictability continuous demand
Y	high value medium predictability fluctuating demand	medium value medium predictability fluctuating demand	low value medium predictability fluctuating demand
Z	high value low predictability irregular demand	medium value low predictability irregular demand	low value low predictability irregular demand

2.2.2.1 Stock Turnover and Coverage

By analyzing inventory rotation/ turnover and coverage, it is possible to establish better control of outflows. These two indicators are essential in inventory management decisions (ISABELLE, 2019).

$$Inventory\ Turnover = \frac{Sales}{AIV}, \quad (2.8)$$

where:

Sales: consumption made over that same period;

AIV: average inventory available over a period.

In essence, the rotation rate measures the speed with which stock is replenished in a given period of time. It is noticeable that a rise in sales without a commensurate increase in inventory will result in a higher inventory turnover.

The A-rated items will have a higher turnover than the B and C.

Also, controlling the range of days of stock coverage is essential to meet customers' needs in full. It is known that:

$$Coverage\ Rate = \frac{1}{Inventory\ Turnover} = \frac{AIV}{Sales} \times Period, \quad (2.9)$$

where:

Period : the number of days in the period covered.

Because items Z are more irregular, they should have longer days of stock. In this way, it is possible to calculate the need based on average demand and supply the right quantity to achieve maximum efficiency and desired service level. This indicator is very important, because it allows the company to provide a good picture of its sales.

2.3 Service Level

Customer relationship management has evolved into a critical factor in the success company strategies in today's highly competitive corporate world (Kim et al., 2006). Customer satisfaction is proportional to the quality of service delivered.

The degree of service can be described as the probability of satisfying all clients, i.e. the probability of not having any out-of-stock items.

2.3.1 Service Level Agreement (SLA)

An SLA is a component of a service contract in which a service is specified based on the supplier and customer's agreement. The agreement specifies the amount of service a customer expects from a provider, the metrics by which the service is assessed and the remedies or consequences for failing to meet the agreed-upon service levels, (Chana and Singh, 2014). This kind of arrangements are often between corporations and external suppliers, but may also be between divisions inside the same organization.

So, SLA refers to the contractual service and its performance. It is an agreement that describes the warranties and actions in the event of a breach.

2.3.1.1 Customer Service Metrics

Currently, the terminology *Customer Service* is used in the area of logistics management to refer to a variety of distinct but crucial operations (Tucker, 1983).

The customer service and associated measures are used in inventory management to evaluate the success of inventory policies and indicate the likelihood of not being out of stock and losing revenues. According to research, stock outs pose a significant danger to client happiness. This occurrence may affect the customer database in the long term, since customers are likely to give up and seek alternative options in the competition if this problem occurs frequently (Füller et al., 2008).

Customer segmentation has practically limitless potential as a tool for leading companies toward more effective marketing and product development strategies, Cooil et al. (2008). This method focuses on the most powerful and adaptable analytical techniques, such as clustering (Wu and Lin, 2005) and statistical models (Qiuru et al., 2012), to identify the different types of clients. « To retain a competitive edge over other markets, especially in today's customer-oriented market, a growing number of organizations are offering high levels of service to delight the most important consumers and drive sales in order to obtain market share via better service and product availability.

It is becoming increasingly clear that businesses who provide outstanding customer service pose a stronger competitive threat and are ultimately more profitable, Radasanu et al. (2016). As a result, companies are under pressure to design a strategy for meeting client expectations completely and on time, in order to compete in the same market.

The high degree of service aids businesses in collecting more revenue, however it is also accompanied by larger inventory concerns (Schalit and Vermorel, 2014). Table 2.3 represents some ways of evaluating the service provided to customers regarding the existing inventory:

Table 2.3: Some Options for Calculating the Service Level

Criterion	Formula
Stock Out	n° of quantities delivered / total quantity of the demand
Stock Out Period	n° of days with stock out / total n° of days
Frequency of Stock Outs	n° of order delivered / total quantity of customer sales orders

Beside these, Fill Rate and Cycle Service Level are the most commonly used metrics in conventional inventory management to establish the amount of safety stock required to meet the target fulfillment percentage (Van Donselaar and Broekmeulen, 2013).

Type I: Cycle Service Level

The item cycle service level (or simply service level) is probably the most used customer service metric for the supply chain. This parameter measures the frequency with which the customer's demand can be filled (Albrecht, 2017). It indicates which portion of the customer's demand can be met on an item-by-item basis. It is straightforward to monitor observed service levels and compare them to the target.

Figure 2.6 represents the relation of the normal distribution curve with the service level type I and OOS probability:

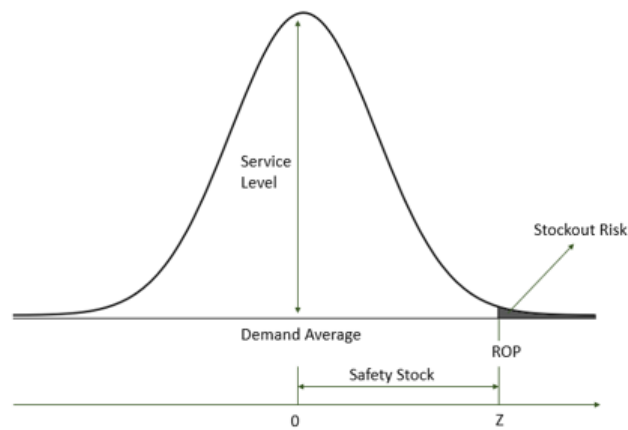


Figure 2.6: Normal Distribution of Demand According with Gauss (Source: Silva (2018))

The service factor (Z-score) is based on the statistical Table 2.4:

Table 2.4: Z-Score Given Depending on Service Level Required

Service Level	Service Factor (Z)
60%	0,254
80%	0,842
90%	1,282
95%	1,645
97%	1,880
98%	2,055
99,9%	3,100
99,99%	3,620

CSL represents the probability of no out-of-stock condition occurring and α is defined as the probability of demand not meeting production during the manufacturing lead time. So in other words, the chance of OOS is equivalent to the risk of $(1-\text{CSL})$ or α .

Rupture will exist if the demand during the lead time is higher than the defined order point. Then:

$$P(X > R) = \alpha, \quad (2.10)$$

where:

X : demand during the lead time (units);

R : order point (units);

α : probability of rupture (%).

Consequently, the probability of the demand during the delivery period to be less or equal than to the defined order point will correspond to the cycle service level.

$$P(X \leq R) = 1 - \alpha, \quad (2.11)$$

And,

$$(1 - \alpha) = \text{Cycle Service Level}$$

However, if a corporation focuses on whole order fulfillment, it is not sufficient to measure the item (Albrecht, 2017).

Type II: Fill rate

The fill rate is perhaps the most critical performance metric. It is defined as the average percentage of demand that is met instantly by inventory (Tan et al., 2017). In other words, the fill rate is the percentage of demand met directly by on-hand inventory (Hegedus and Hopp, 2001).

This method, like the cycle service level method, takes demand and lead time unpredictability into account.

The periodic calculation of the fill rate basis enables the identification of supply chain weak points and modifications. This service metric is denoted by β and it is the ratio between the total number of units fulfilled by existing inventory and the total number of units requested within a certain time period (Hopp and Spearman, 2001):

$$FR = \frac{\text{Total Units Shipped}}{\text{Total Units Placed}} \times 100 \quad (2.12)$$

2.3.1.2 Metrics Comparison

These two criteria are measuring different things and, naturally, demonstrate some discrepancies in the value of safety stocks. The essence of these definitions is to evaluate whether the demand or set of requests can be satisfied from the inventory (Shivsharan, 2012).

To clarify the distinction between these two service indicators in terms of finished product inventories, illustration 2.7 can be considered:

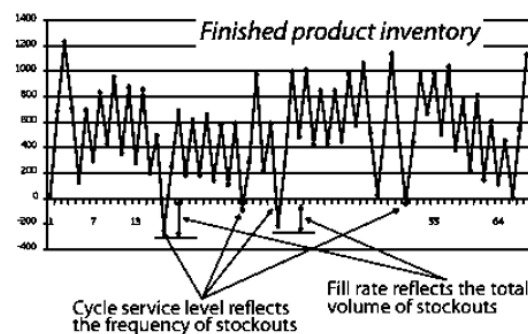


Figure 2.7: Cycle Service Level Vs Fill Rate (Source: King (2011))

So, as noticed before, type 1 is employed in situations when the occurrence of a shortfall has the same implications regardless of the time or amount involved.

With this metric, the stock out for a single unit is assigned with the same weight as stock out for a thousand units, making it a poor performance indicator (Shivsharan, 2012).

When the focus is on filling complete customer orders, fill rate would be the preferred service level measure, Larsen and Thorstenson (2014).

So, fill rate and cycle service level respond differently and choosing one way of establishing safety stock over the other has a significant impact on the quantity of safety stock maintained on hand.

2.3.2 Seasonality

Seasonality may be seen in a variety of industries, including agriculture, tourism, and manufacturing (Koenig-Lewis and Bischoff, 2005). Seasonal variations in commercial businesses are defined as regular fluctuations in a time series during a certain period of year, according to a broad definition.

Customer demand is generally seasonal, meaning that it varies based on the time of year. Stock-outs and excess inventory are frequently caused by supply chain economies of scale, poor retail performance and demand volatility, (Ehrental et al., 2014).

The estimate for sales growth in certain time periods would necessitate increased expenditures in existing assets, including inventories, as well as changes in organizational structure, including the employment of new personnel and the construction of facilities, among other factors, de Mesquita et al. (2016).

Not taking into consideration seasonal pattern causes systematic demand and supply mismatches at the item-store level, resulting in higher-than-necessary costs.

2.4 Inventory Management

The term *Inventory* refers to a collection of items, customers and information that happens throughout a certain activity (Safran et al., 2007). It is, in essence, a collection of non-immediately consumable sources or resources employed in a certain organization.

Inventory management effectiveness is critical for a manufacturing business to remain competitive, Buliński et al. (2012). Thus, optimizing it significantly improves the quality of the output of company performance. It is, indeed, a critical component of a business, especially when it comes to fulfilling high client demand (Ahmad and Zabri, 2018).

According to some prior research, the most common issues that organizations experience include underproduction, overproduction, stock-out situations and inventory discrepancy (Chan et al., 2017). So, accurate management is crucial to supply chain success. The main problem in inventory management is to maintain a balance between supply, demand and associated costs (Agu et al., 2016).

2.4.1 Inventory Models

When inventory is limited and there are insufficient quantities to satisfy demand within a specific time frame, the organization can select from a variety of options to face this problem, such as backorders or missed sales, that result from unmet demand (Thomopoulos et al., 2004).

Backordering

At the moment that a consumer puts an order or purchases merchandise that is not yet available, this can be known as an inventory backorder. This may arise when items have sold out due to unexpectedly strong demand and all of the safety stock has been sold.

A backorder is when the client waits for the supply to be refilled before the demand is fulfilled (Thomopoulos et al., 2004). In these cases, unsatisfied demand is delayed to the subsequent cycle and will be met as soon as the system gets an order to deliver a sufficient quantity, Estellés-Miguel et al. (2014).

Lost-Sales

By definition, a lost sale is a missed sale opportunity due to out-of-stock merchandise, Thomopoulos et al. (2004). In contrast to backordering inventory models, which assume that the client is willing to wait for his order to be completed, inventories that assume lost sales presume that the consumer is not content to wait. So, when a buyer refuses to wait for his purchase to arrive or the company follows this policy, a sale is lost.

These systems need distinct replenishment methods to reduce costs (Bijvank and Vis, 2011). Figure 2.8 represents an example of the inventory leveling of these two approaches, where R denotes the review instant:

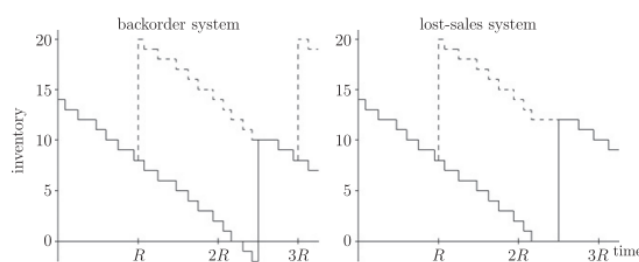


Figure 2.8: Backorder System Vs Lost-Sales Approach (Source: Bijvank and Vis (2011))

2.4.2 Inventory Costs

Stocks are a substantial investment in practically every sector of the commercial activities, da Costa (2012). Inventory management is generally concerned with decreasing the expenses associated with material inventories. It is only worthwhile to analyze the expenses influenced by management decisions, specifically those associated with:

- Material Provisioning;
- Stock Holding;
- Stock-Outs, (Pandya¹ and Thakkar, 2016).

Provisioning costs are linked with material acquisition. While holding costs include financial costs, maintenance costs and devaluations. Finally, the costs associated with stock-outs are related to an item that is required but is not immediately available and so occurs a shortage. This type of cost is difficult to quantify. A disruption can result in lost revenue, a deterioration of the company's reputation, production disruptions and emergency supplies, among other consequences (Vasconcelos, 1991).

2.4.3 Stock Management Methods

The goal of selecting a stock management policy is to address three critical issues: "How Much to Order?", "When to Order?" and "Which Products to Keep on Hand?" - to make the process affordable, the consumers satisfied and to maintain a targeted service level, de Carvalho and Cardoso (2002).

When the parameters change over time, demand is probabilistic or several products are managed together, the results of the stock's policy adopted turn out to be the main components in decision making. Furthermore, the goal is to provide decision support for a planner rather than to have an automatic replenishment system.

The stock management policy chosen requires understanding about the nature, behavior and uncertainty associated with supply and demand (de Carvalho, 2019). The two most known methods used are the deterministic and stochastic models. On the demand side, that is, the customer's side, if the amounts ordered are known, the demand is deterministic. If demand is unclear and fluctuating, the scenario is random.

2.4.3.1 Deterministic Models

The deterministic models most often used are the Economic Order Quantity Models, Economic Order Quantity Models with Quantity Discounts and Economic Order Quantity Models without Instantaneous Stock Replenishment (Carvalho, 2013). This type of model assumes continuous lead times and an unlimited supply capacity with no contingency for stock-outs (Arslan and Turkey, 2013). The EOQ challenge is to determine the ordered quantity Q (or the amount to be produced) that minimizes the total average cost of replenishment under certain assumptions.

Thus, in the EOQ model, the economic order quantity, or optimum Q , is defined as follows:

$$Q = \sqrt{\frac{2kC}{H}}, \quad (2.13)$$

where:

k : annual demand/consumption rate (units/year);

C : unit order/ production cost (€/order);

H : unit cost of holding stock (€/unit/year).

An economic order quantity corresponds to an economic period between orders (da Costa, 2012), also known as the optimal time between replenishment (T^*) and it is calculated by the following equation:

$$T^* = \sqrt{\frac{2C}{Hk}} \quad (2.14)$$

The following chart on Figure 2.9 describes the inventory level fluctuations in this kind of model:

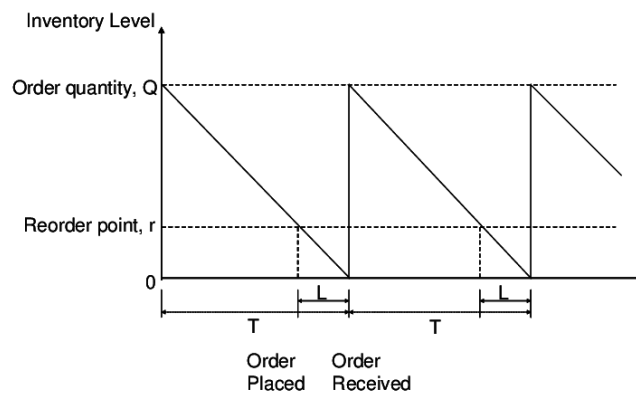


Figure 2.9: Inventory Level in EOQ Model (Source: Herrera et al. (2009))

It is also important to mention, as seen in the picture above, that:

$$r = ROP = \text{Demand During Lead Time} + \text{Safety Stock} \quad (2.15)$$

This parameter, named reorder point (ROP) marks when inventory has to be replenished. To avoid OOS, this option suggests when to place an order (Kao and Hsu, 2002).

Due to the assumptions imposed by conventional deterministic inventory management models, implausible solutions are produced, compromising their validity. These models do not implicate that the desired service levels will be achieved (Stock, 1990). Because of this models' fragility in adapting to the realities of businesses, stochastic models are used, which are suitable to situations in which demand and lead time are unpredictable and uncertain.

2.4.3.2 Stochastic Models

Stochastic models are used when the behavior of demand and/or supply is random. This aspect raises the difficulty of inventory management since it allows stock-outs. For this reason, it becomes vital to maintain a safety stock that can absorb deviations above the typical levels recorded (Clark and Scarf, 1960).

The higher the desired level of service, the larger the safety stock that should be maintained. This value will be affected by the inventory management model used. The optimal quantity to order in each period requires a proper demand analysis. So, when demand and production exhibit more random and uncertain behavior, this type of models are appropriate.

There are two primary types of models in this category: Continuous Revision Models and Periodic Revision Models.

Continuous Review

In this kind of models, stock levels are constantly monitored. This step is important because it is necessary to restart production when the stock level of a particular item reaches a preset quantity (order point). Otherwise, the chance of stock-outs increases.

This model is a modification of the economic order quantity model for situations in which demand is random. The most well-known policy in this paradigm is the Order Level Policy (s, Q), which stipulates a fixed quantity to be produced but a variable period between productions.

Therefore, this policy requires a continuous knowledge of on-hand stock and on-order stock, implying a control of all the movements. So, as mentioned before, a fixed quantity Q is requested whenever the stock position reaches the order level s . Figure 2.10 illustrates this type of stock policy:

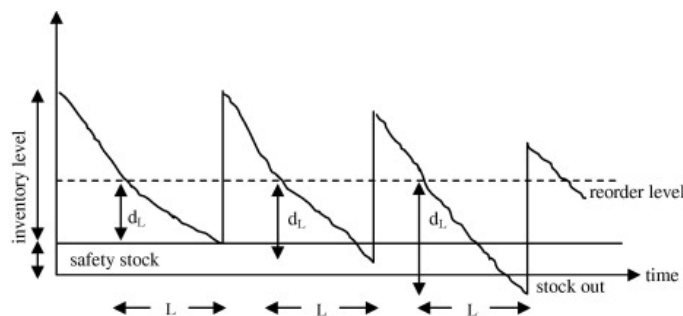


Figure 2.10: Continuous Review Inventory Model (Source: Dutta et al. (2007))

And:

L : delivery time;

dL : demand during the delivery time.

Periodic Review

This model follows a periodic review cycle (Tao et al., 2017). In this type of method, the day for placing an order with the supplier or scheduling a production is established with a predetermined periodicity. The quantity to order or produce will correspond to the difference between the stock needed for the following period and the existing stock (Carvalho, 2013).

Stock levels are monitored on a periodic basis rather than continuously and the time interval between orders is fixed, but the quantity to be ordered is variable. The following Figure 2.11 illustrates the methodology of this stock policy:

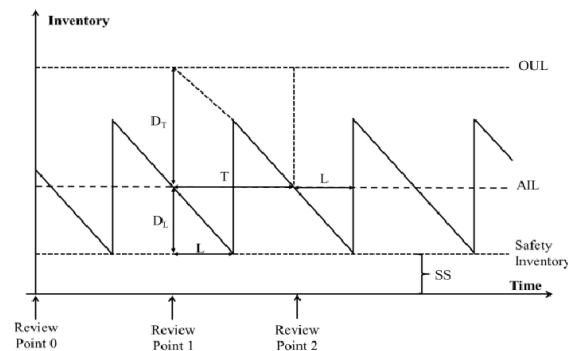


Figure 2.11: Inventory Profile Using Periodic Review (Source: Setyaningsih and Basri (2013))

Where:

T : review period;

OUL : order up to level;

AIL : average inventory level;

DT : mean demand during T ;

DL : mean demand during L .

2.4.3.3 Hybrid Models

Static control strategies, in general, are insufficient to address issues such as demand unpredictability. They are often utilized as a springboard for developing new policies across all approaches. In a continuous revision system, the static control policy (r, Q) serves as a foundation for developing or modifying additional policies, such as (s, S) , (r, S) , (s, Q) , (s, nQ) , and (r, s, Q) , Babai et al. (2009). Similarly, the hybrid periodic revision policies (T, r, Q) and (T, r, S) are derivations of the static control policies (T, S) and (r, Q) , respectively.

Figure 2.12 illustrates the most extensively researched control strategies and their adjustments, since they are more realistic and complex methods. Note that s stands for the resupply level or minimum stock, S refers to the order up to level and n is the number of shipments in a supplier's cycle.

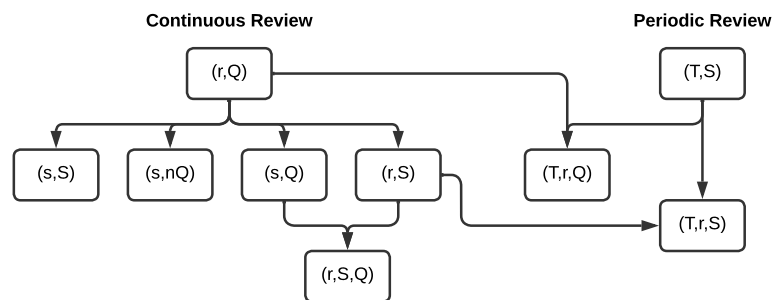


Figure 2.12: Stock Control Policies

2.5 Final Considerations

This chapter's objective was to explore the different subjects and concepts necessary for the development of this dissertation, digging into the most frequently employed methods and terms within this theme. That is, regarding product classification, stock management processes and service level.

This in-depth article analysis was essential in assisting with the formulation and resolution of the optimization problem.

Chapter 3

Problem Description

BA's expansion throughout the years has resulted in numerous benefits, including improved exposure and recognition, resulting in the acquisition of new customers. On the other hand, this growth has disrupted the entire production and supply planning process, which must be adapted to the new reality.

In this circumstances, the need for the development of this project arises. To maintain a prosperous and respectable image in the eyes of consumers, it is necessary to study the optimal stock to keep and produce, in order to meet a certain service level to customers based on their value. The strict control of the inventory becomes a convenience for companies, allowing them to serve their consumers more efficiently and quickly.

In this context it would be important to assess and analyse the level of customer satisfaction currently provided. In other terms, to determine whether the consumers with the higher segmentation rank are given priority over others. This priority can be determined by the inventory availability: cycle stock and safety stock.

This chapter will explain essential ideas and some important information for the comprehension of the subsequent sections. Figure 3.1 shows the performance metrics that must be considered when approaching this subject and determining the optimal inventory method to employ.



Figure 3.1: Inventory Strategy (Source: Almada-Lobo (2021a))

3.1 Planning Department

The planning team is responsible for developing all production plans, taking into consideration numerous criteria and KPIs. The department maintains constant communication with the surrounding departments, as can be seen in the flowchart of Figure B.1 in the Appendix B.

BA operates on a Make-to-Stock production method. That is, it produces in order to create stock. This strategy is most commonly when demand is uncertain and forecasts are used to calculate the quantity to stock. Currently, the department neither accounts for SS nor has a method for calculating it for each customer type.

It is already known that service level is inherent to the existence of OOS, which is closely linked with parameters such as the production time, lead time and stock coverage. To address market needs, any business must plan properly and establish a product strategy that satisfies client expectations. The department's main concerns are to answer questions such as "How Much to Produce?", "When to Produce?", "Is Safety Stock Required? And How Much?".

Nowadays, the department does not benefit from an algorithm for planning support and does not have a statistic for measuring the service quality provided relative to production. In other words, it is difficult to assess and account for whether the company is indeed responding with greater rigor and attention to higher segmentation clients to the detriment of lower segmentation customers, regarding manufacturing and existing inventory.

3.1.1 Production Process

The company is equipped with many plants and furnaces that supply multiple production lines with glass packaging in a wide range of sizes, colors and shapes. Each factory consists of several furnaces and each furnace has multiple production lines. Also, each furnace is usually intended to make products of a specific color and each line produces products with similar forms and using the same process production. So, each plant/ factory normally produces the same kind of product families.

There are 3 different ways to produce a given material, depending on the process used. The Blow-Blow (SS) method utilizes pneumatic technology, whereas the Press-Blow (PS) and Narrow-Neck-Press-Blow (NN) processes utilize a punch that occupies a specific volume within the mass of glass, producing the desired preform (Manoel et al., 2010).

To cluster products based on their attributes, the company groups them into distinct products families. So, each product is associated with a family with a particular code. Figure 3.2 represents an example of the constitution of the family's code. In Tables A.1, A.2 and A.3 in the Appendix A it is possible to see all the possible colors, processes and sizes of the different product families. This classification will be several times throughout this thesis.

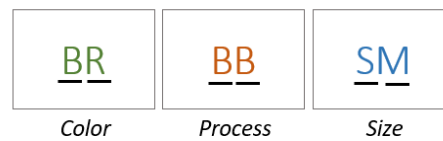


Figure 3.2: Product Family Code

It is generally convenient to plan successive production of similar articles (campaign), since this procedure is inherently associated to a setup time, which is related to the complexity of the job/production change. The more complex the change, the more time is lost and the more defects can arise in production because the machines are more difficult to tune.

3.1.2 Key Performance Indicators

The main KPIs to take into account in this department are to:

- Avoid exceeding or going below furnace **Pull** limit values;
- Reach the best **Pack to Melt** (net glass tons packed, saleable, over the melted glass of the furnace);
- Get the higher possible **Job Change Index**. It measures the efficiency of the job changes, taking into account the number of complete pallets produced after 12 hours since the machine has stopped for the job change;
- Reduce the quantity of stock as much as possible.

While planning a production, it is also necessary to respect the limited amount of job changes of each factory within a week and within a day and, also, plan similar articles in sequence campaigns, so that the change is not excessively time-consuming and risky.

3.2 Customer Segmentation

The company established a criteria for the segmentation of clients into quadrants (from A to F) with a greater degree of similarity in terms of service level provided.

Distinct service levels must be established for each type of client in order to satisfy those who truly add value to BA. So, the primary criteria is the revenue generated. Figure 3.3 summarizes the way that the customer division is done:

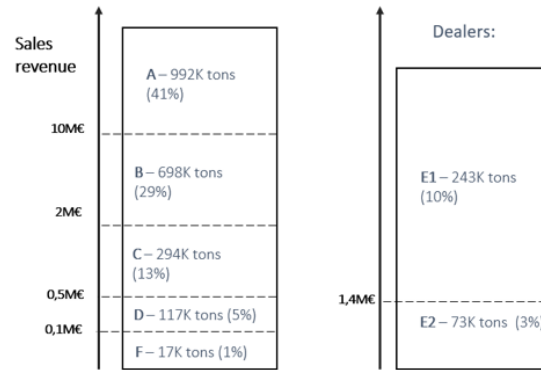


Figure 3.3: Current Customer Classification

Dealers are considered intermediate customers. That is, they buy from BA and then sell to other companies, so they are not final direct customers and are not associated with the same significance.

Having established this, it is crucial to select the service level to provide for each of these customers groups, strengthening relationships and intensifying loyalty, especially with the most indispensable ones.

3.2.1 Service Level Metrics

Since demand exceeds capacity, preventing out-of-stocks becomes incredibly challenging. The ideal plan of action is to attempt to avoid or decrease the OOS situation, as well as to reduce the quantity of products missing from orders for more segmented clients.

Currently, the production and storage of safety stock in the supply chain must be well thought out, to provide with a buffer in case the demand exceeds the foretasted. (Ghadimi and Aouam, 2021). The final objective will then be to study and analyze how to improve the service level provided to the highest segmentation customers based on the existence of OOS. Thus, and within the Planning Department, arises the necessity to develop a metric that evaluates the current service level offered.

In this situation, two intriguing choices emerge, represented in the Figure 3.4 and explained in Chapter 2, that take demand uncertainty and the safety stock into account: **Cycle Service Level** (α) and **Fill Rate** (β).

In summary, it can be stated that the Cycle Service Level takes into account how frequently OOS happens and the Fill Rate accounts for missing quantities.

It is self-evident that especially when demand is erratic, these two measurements act quite differently and have a significant value discrepancy. However, they also have a strong bond. Increasing the cycle service level of a business also raises the fill rate, (Chopra and Meindl, 2007).

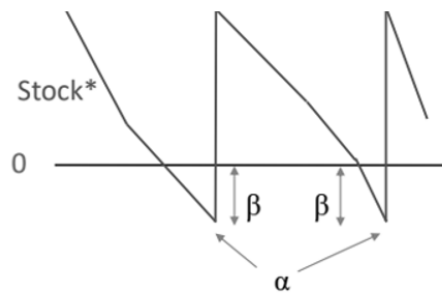


Figure 3.4: CSL Vs FR

Both measures account for the trade-off between inventory costs (stock on hand) and out-of-stock costs (stock-outs).

The goal will be to create a model that helps to maintain sufficient inventory to meet the majority of consumer demand, taking into account parameters such as the cycle stock, safety stock, the associative costs and service metrics. Figure 3.5 distinguishes between these two types of stock and mentions the parameters on which they are dependent.

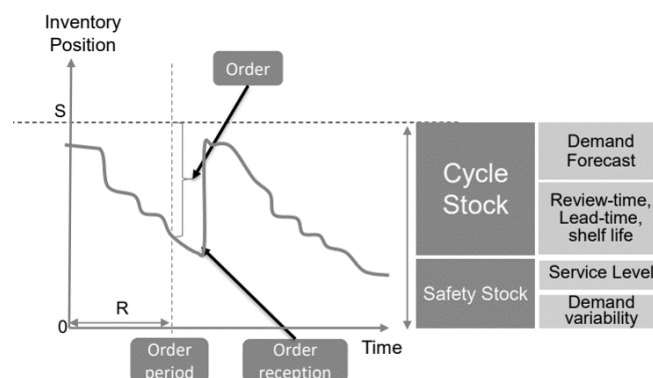


Figure 3.5: Total Stock (Source: Almada-Lobo (2021b))

being:

S : Target Stock;

R : Review-Time.

3.3 Costs

In this section, the types of costs that must be considered while discussing production and sales will be described.

3.3.1 Industrial Cost

Industrial cost or production cost encompasses all expenses incurred during the manufacturing of a product and is estimated using two parameters:

$$\text{Industrial Cost} = \text{Variable Cost} + \text{Fixed Cost} \quad (3.1)$$

Variable Cost

The parameters associated to this attribute are:

- Raw Material;
- Consumable Materials;
- Moulds Repairs;
- Packaging Costs;
- Returnable Packaging.

The raw material, for example, differs from family to family, since it is determined by the material's color.

Fixed Cost

This parameter encompasses the following terms:

- Personnel Costs;
- Depreciation and Amortization;
- Provisions;
- Third Party Supplies and Services Rendered;
- Subcontracting Costs;
- Diesel;
- Water;
- Rentals;
- External Warehouse Rents;
- Internal Transportation;
- Communications;

- Cleaning Costs;
- Temporary Job;
- Other External Costs.

3.3.2 OOS Cost

This cost represents the profit that was not obtained by not selling a certain product. In other words, it is the margin between the sales cost and the industrial cost:

$$OOS\ Cost = Selling\ Price - Industrial\ Cost \quad (3.2)$$

Chapter 4

Service Level Analysis

In a first analysis, the focus was applied on the customers. A research was conducted to determine the degree of service offered, since last year, to each category of customer in order to determine if the most important customers are, indeed, being prioritized. For this purpose, both the Cycle Service Level and Fill Rate were used.

The software SAP was utilized and multiple transactions were used in order to extract the data needed for the problem analysis. The data used was from the months between May 2021 and May 2022 for the plants in Iberia: Avintes (AV), Venda-Nova (VN), Marinha Grande (MG), Léon (LE) and VillaFranca (VF).

4.1 Customer Demand

The biggest internal production planning concern is the variability of demand, Casimiro (2015). The greater the variability of demand, the greater the standard deviation, which requires a higher stock to ensure the service level given.

It is common to have successive extra requests, i.e. after the needs that are part of the annual plan have been identified, new requests for materials frequently arise. The existence of extra requests is even more serious when the acquisition processes are already underway.

In this company, the demand forecast is made by the salesperson using forecasts sent by the customers in a certain period of time and the person's own experience in dealing with different customers.

Clustering

The customers are already divided into 7 classes according to their importance and sales volume, as mentioned in Chapter 3.

Another interesting classification would be to divide each customer segmentation according to the family's demand variability, that is, to make an XYZ classification for product's families (concept explained in section 3.1.1) This approach permits determining whether the demand for each family is regular, sporadic or irregular, within each client category.

The demand patterns identified by this methodology are: smooth (X), erratic (Y) and intermittent (Z). This analysis facilitates the study and analysis of the service level, as well as the days of coverage to be provided to each customer type.

Following on from this idea, a study was made of the demand signal, consequently evaluating the criticality of each family of products within each type of customer. To make this classification, it is necessary to calculate the coefficient of variation:

$$CV = \frac{\text{Standard Deviation of Demand}}{\text{Mean of Demand}} \times 100 \quad (4.1)$$

For the classification of the demand size variability (Almada-Lobo, 2021a), the following criteria was used:

Group X	20% of products families can be forecast accurately (lowest CV value)
Group Y	30% of the families have fluctuate demand (medium CV value)
Group Z	50% of the families present sporadic demand (highest CV value)

Through the calculating of the CV of the demand of each family belonging to a customer type using the Excel tool and the criteria showed above, a table like the one represented in Figure 4.1 was obtained:

FAMILY (i)	Avrg Demand/ month	Stdev Demand	Vol. Sales	CV	SEGM	XYZ	Comb
VBNNBG	24	5	0,02%	19%	B	X	BX
VRBBBBG	204	39	0,19%	19%	B	X	BX
UVBBSM	299	57	0,27%	19%	B	Y	BY
BRPBSM	4 476	864	4,07%	19%	A	Y	AY
BRPBSM	138	28	0,13%	20%	F	Y	FY

Figure 4.1: CV Calculation for Each Family's Demand

As a rule, the intention is to offer a more qualified service to those clients who require large volumes on a more constant basis.

The graph in Figure 4.2 shows the proportionality between the CV and the number of families involved:

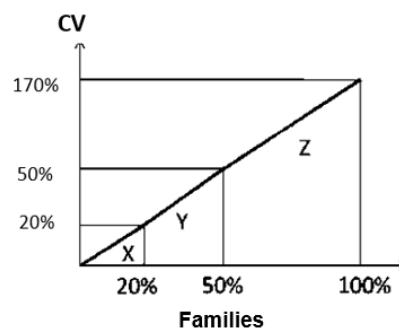


Figure 4.2: Demand Pattern

As it is possible to notice, according to last year's data, around 20% of the product families have a coefficient of variation between 0% and 20%, 30% of the families have a CV between 20% and 50%, and finally, approximately 50% of the families have a CV between 50% and 170%.

To obtain a better understanding of the cluster with the most associated products, the matrix in Table 4.1 was created:

Table 4.1: Number of Associated Families to Each Cluster

Classification	X	Y	Z
A	4	5	15
B	7	11	8
C	5	8	13
D	4	6	11
E1	2	4	9
E2	5	6	9
F	4	6	13

Finally, Table 4.2 shows the sales volume associated with each classification.

Table 4.2: Sales Volume

XYZ	Sales Volume
X	57%
Y	33%
Z	10%

It is easily noticeable that consistent demand is obviously more recurring. Families from the Y and Z classifications should have more stock coverage days for the same service level provided and the same manufacturing lead time.

Seasonality

During the course of the year, the demand for each family varies depending on the season. Seasonality research is vital for analyzing trends and patterns as well as forecasting demand, enabling more precise and accurate production planning.

Using Microsoft Excel and the sales plan's historical data from past years, the demand pattern for the various material families was investigated. The graph in Figure 4.3 represents an example of the study performed. In this case, for example, it is shown how the demand for family UVBBSM varies over the year, through values generated by averaging the monthly demand averages over several years.

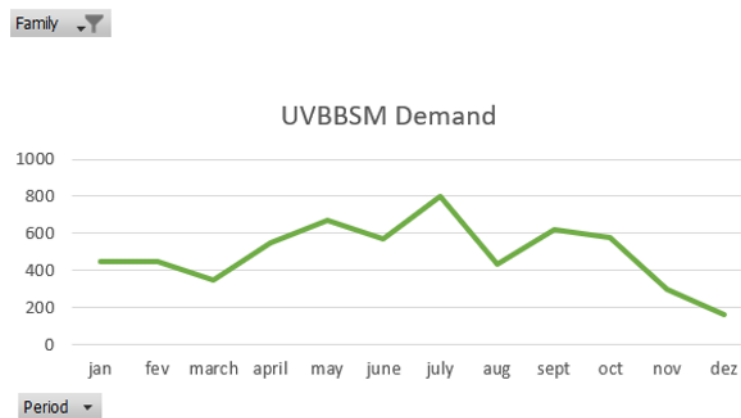


Figure 4.3: Study of the Demand Variation Throughout the Year for Family UVBBSM

The graph suggests, for instance, that the demand for this family peaks in July and declines over the winter season.

4.2 Cycle Service Level and Fill Rate Evaluation

In a second study, the degree of service offered to each customer type each month, based on the two metrics mentioned above, was analyzed and calculated.

The initial monthly sales plan (expected demand) was extracted from SAP, as well as the monthly sales plan that include the production plan and current stock (expected delivery). Also, the production records were extracted.

It is important to remember that:

- Each customer is only associated to one segment;
- Each segment has many customers;
- Depending on the color, process and size of the glass container, a product family may include numerous products references;
- A product reference belongs to only one family;
- A customer is associated to several families;
- A family is associated to several customers;
- A given family and client is associated a given demand (in tons);
- The demand is assumed to follow a normal distribution with a mean of d and a standard deviation of σ .

After data treatment, the table from Figure 4.4 demonstrates the relation between families, customers and expected demand:

Material	Costumer	Both	Family	Segmentation	Expected Demand (FXX)
1666L129UV	SOGEVINUS FINE WINES,SA	1666L129UV SOGEVINUS FINE WINES,SA	UVBBBG	C	114
1793P151UV	SOGEVINUS FINE WINES,SA	1793P151UV SOGEVINUS FINE WINES,SA	UVBBBG	C	27
1807P201VB	SOGEVINUS FINE WINES,SA	1807P201VB SOGEVINUS FINE WINES,SA	VBBBBG	C	11
1808L129VB	SOGEVINUS FINE WINES,SA	1808L129VB SOGEVINUS FINE WINES,SA	VBBBBG	C	45
1809L129VB	SOGEVINUS FINE WINES,SA	1809L129VB SOGEVINUS FINE WINES,SA	VBBBBG	C	28
1036C002BR	ADEGA COOP. FAVAIOS CrI	1036C002BR ADEGA COOP. FAVAIOS CrI	BRPBMI	B	151
1829P151BR	ADEGA COOP. FAVAIOS CrI	1829P151BR ADEGA COOP. FAVAIOS CrI	BRBBBG	B	10
1829P151VB	ADEGA COOP. FAVAIOS CrI	1829P151VB ADEGA COOP. FAVAIOS CrI	VBBBBG	B	108

Figure 4.4: Data Aggregation

Then, the second stage was to research the service level given by the company to each client segmentation. For this purpose, the table from Figure 4.5 was prepared. The final column corresponds to a binary value taking the value "1" in case of OOS, i.e., when the expected demand is greater than the expected delivery.

Both	Family	Segmentation	Expected Demand (FXX)	Expected Delivery (AXX)	OOS
9753T634BR NESTLE PORTUGAL, Unipessoal, Lda	BRPBBG	A	504	504	0
9292T194BR H.J. HEINZ SUPPLY CHAIN EUROPE B.V.	BRPBMI	A	372	320	1
9418T271BR H.J. HEINZ SUPPLY CHAIN EUROPE B.V.	BRPBBG	A	360	256	1
5605T183BR Unilever Europe BV	BRPBBG	A	6	6	1
6426T636BR Unilever Europe BV	BRPBBG	A	313	313	0
6622T205BR Unilever Europe BV	BRPBBG	A	254	165	1
6705T248BR Unilever Europe BV	BRPBBG	A	68	68	0

Figure 4.5: OOS Evaluation

Through this data, which provides the expected demand and the quantities that were actually expected to be sold (taking into account production and stocks), were measured two separate metrics: **Cycle Service Level (CSL)** and **Fill Rate (FR)**. The following formulas were used to calculate that metrics, in a given period:

$$CSL_{i,k} = 1 - \frac{ncr_{i,k}}{nc_{i,k}}, \quad (4.2)$$

with:

$CSL_{i,k}$: service level calculated for family i to customer k ;

$ncr_{i,k}$: n° of resupply cycles with stock out of family i to customer k ;

$nc_{i,k}$: total n° of resupply cycles of family i customer k .

$$FR_{i,k} = \frac{\sum_{i,k} QF_{i,k}}{\sum_{i,k} QD_{i,k}}, \quad (4.3)$$

where:

$FR_{i,k}$: fill rate calculated of family i to customer k ;

$QF_{i,k}$: n° of units supplied of family i to customer k ;

$QD_{i,k}$: n° of demanded units of family i from customer k .

These two measures were calculated monthly for each family of each type of customer, resulting in Figure 4.6.

The colours shown in the table are relative to the degree of service level provided. Green represents the optimal values, yellow the tolerable levels, orange the values closest to critical, and red the critical values, which are too low.

Cluster	Expected Demand	Expected Delivery	nº cycles	OOS	P(OOS)	Fill Rate	Service Level	Volume Sales
A	37 325	29 445	134	56	41,79%	78,89%	58,21%	42,47%
B	32 769	25 028	257	104	40,47%	76,38%	59,53%	36,10%
C	9 807	8 301	181	62	34,25%	84,64%	65,75%	11,97%
D	3 377	2 901	101	32	31,68%	85,92%	68,32%	4,18%
ASBBBG	11	11	1	0	0,00%	100,00%	100,00%	0,02%
ASNNSM	34	34	1	0	0,00%	100,00%	100,00%	0,05%
BABBBG	62	44	3	2	66,67%	70,97%	33,33%	0,06%
BABBSM	10	7	1	1	100,00%	69,43%	0,00%	0,01%
BRBBBG	422	309	13	9	69,23%	73,10%	30,77%	0,45%
BRBBSG	80	64	2	1	50,00%	80,27%	50,00%	0,09%
BRBBSM	68	68	1	0	0,00%	100,00%	100,00%	0,10%
BRNNSM	40	20	1	1	100,00%	50,40%	0,00%	0,03%
BRPBBG	870	794	23	5	21,74%	91,25%	78,26%	1,15%

Figure 4.6: Fill Rate and Cycle Service Level for Each Family and Customer

The overall average quality of service delivered in each month and for each customer segmentation was then calculated. To do this, for each of the seven types of customers, the weighted average service level was determined, taking into account sales volume and giving more weight to the families with the highest demand.

The table in Figure 4.7 represents a portion of the acquired results. Table B.1, that summarizes all the results, is attached in the Appendix B.

		A	B	C	D	E1	E2	F	AVERAGE
may	Fill Rate	97,10%	98,15%	93,56%	97,44%	99,19%	98,76%	98,20%	97,16%
	Service Level	94,15%	93,97%	94,22%	95,50%	94,81%	96,10%	93,75%	94,24%
	Volume Sales	44,90%	30,18%	11,77%	5,39%	4,85%	1,83%	1,08%	
june	Fill Rate	98,03%	89,80%	96,97%	97,44%	92,64%	96,93%	95,82%	95,21%
	Service Level	93,01%	94,74%	93,33%	95,05%	93,51%	94,87%	94,20%	93,71%
	Volume Sales	47,58%	29,21%	12,03%	4,46%	4,33%	1,55%	0,84%	
		...			...				
may	Fill Rate	93%	94%	94%	95%	87%	93%	99%	93%
	Service Level	79%	86%	92%	92%	90%	85%	95%	84%
	Volume Sales	46%	28%	11%	6%	3%	3%	2%	
april	Fill Rate	97%	94%	95%	96%	85%	98%	97%	96%
	Service Level	88%	88%	95%	92%	85%	91%	98%	89%
	Volume Sales	45%	27%	11%	7%	4%	4%	2%	
	Fill Rate	95,46%	93,24%	93,87%	94,44%	88,93%	94,38%	95,55%	94,29%
	Service Level	87,22%	89,89%	92,49%	93,10%	87,91%	89,52%	92,72%	89,09%
	Volume Sales	45,64%	30,94%	11,63%	5,08%	3,48%	2,27%	0,96%	

Figure 4.7: Current Service Level

It is easily deducible that the number of times that OOS occurs in a manufacturing business like this one is quite high. On the other hand, the magnitude of each stock-out is rather minimal. That is, despite not producing the specific number required, the quantity produced is high and not far from the one ordered, which explains the values obtained in the table, where $CSL < FR$.

This table also shows, for instance, that on average, C customers have a higher service level than B customers and D customers have a higher service level than C customers, which is not the more appropriate practice to maintain.

To evaluate in more detail the service level provided, it was also calculated the FR and CSL for each cluster, discussed in section 4.1, and Table 4.3 was generated:

Table 4.3: Service Level by Cluster

CLUSTER	FR	CSL
AX	96,0%	88,4%
AY	95,0%	87,2%
AZ	97,3%	92,0%
BX	95,3%	90,7%
BY	93,1%	89,2%
BZ	90,6%	75,6%
CX	95,6%	94,5%
CX	91,6%	89,4%
CZ	94,3%	89,6%
DX	96,0%	94,3%
DY	88,7%	88,4%
DZ	95,6%	95,0%

This table demonstrates that within the same segmentation, the service level is not effectively distributed. For example, cluster AZ provides a higher degree of service than cluster AY. Or also analysing different customer segmentation, cluster DX has a better service level than BX, which should not happen.

In this way, by examining these tables and data, it is evident that service level enhancements should be made so that A customers are distinguished and receive more individualized attention.

This type of service level is determined by the quantity of missing product, thus, it is closely associated with the production process. To attain a specified service level, an optimization problem and the respective heuristics for solving it will be constructed based on the family's production that permits the computation of suitable stock levels.

Chapter 5

Data Treatment and Optimization Problem

This chapter describes the essential data treatment for the development of the optimization problem and the heuristics used, as well as the equations and contextualization of the problem itself.

Furthermore, this section also discusses and describes the operation of the two algorithms created to solve the problem.

5.1 Heuristics for Stock Leveling Based on Service Metrics

In continuation of the prior investigation, it was decided to solve the stock optimization problem based on last year's consumption using Microsoft Excel for data treatment and then Python for data simulation, obtaining results close to the optimum.

The service metric used will be the fill rate, since in addition to taking into account demand variability, it considers missing amounts and allows analysis of sales strategy, permitting the estimation of the proportion of demand that is converted into sales. That being stated, it makes more sense to assess and focus on this service metric in a manufacturing company.

For the optimization problem developed it makes sense to work with product's families, since the model is intended for production. Through a certain value of production days for each family and days until the next production, the heuristic allows to calculate the stock to be kept, that is, how much to produce in each replenishment cycle, imposing a certain desired service level and prioritizing the most significant product families. In addition, it allows for the calculation of the stock coverage and anticipate a sales plan, as well as the lost sales cost, due to OOS.

5.1.1 Family's Importance

A product's family can be destined for various types of customers, as already explained. As a result, for the development of the model and its inputs, a particular degree of service was assigned based on the family's significance.

In this way, the most relevant families for the company were identified and calculated based on those whose type A customers generate the most revenue.

$$\text{Revenue} = \text{Average Sales Price (per ton)} \times \text{Number of Tons Sold} \quad (5.1)$$

Table 5.1 demonstrates the considered information and the outcomes:

Table 5.1: Families Organized by Level of Importance

<i>index</i>	Fam	Customer A Sales (Tons)	Sales Price/ Ton	Revenue
1	AMNNSM	166 139	417	69 265 903
2	BRPBBG	62 824	422	26 492 457
3	VRNNSM	52 053	442	22 983 376
4	BRPBSM	43 503	516	22 445 926
5	BRNNSM	46 274	452	20 933 026
6	BRBBBBG	32 564	459	14 934 152
7	AMNNBG	26 709	386	10 300 918

In the preceding optimization problem, the family will be arranged from the most significant to the least significant. Thus, the family with the lowest index ($i=1$) will be the most valuable.

5.1.2 Data Treatment

In order to implement this model, it was necessary to extract various data from SAP and treat them according to the objective. Note that all of the presented tables are incomplete, their main purpose is to demonstrate the approach done.

Target and Critical Fill Rate

Initially, it was determined to set a target fill rate close to 100 % for all the families and a certain critical service level value, depending on the degree of significance of each family, obtaining the following Table 5.2:

Table 5.2: Initial Fill Rate Values Associated with Each Family

Family	Target FR	Critical FR
AMNNSM	>99,9%	93%
BRPBBG	>99,9%	93%
VRNNSM	>99,9%	93%
BRPBSM	>99,9%	93%
BRNNSM	>99,9%	93%
BRBBBBG	>99,9%	93%
AMNNBG	>99,9%	93%
BRPBMI	>99,9%	93%
UVBBBBG	>99,9%	93%
BUNNSM	>99,9%	90%
BRBBSM	>99,9%	90%

Demand

Firstly, through the information regarding the expected sales plan, the materials were grouped by family and the demand and standard deviation of demand were calculated for each family. This was done for each month, since several families present different demand trends depending on the period of the year.

Assuming that each family's daily demand follows a normal (Gaussian) distribution, the demand distribution for one month equals the sum of the daily distributions, so:

$$d_i(\text{per day}) = \frac{d_i(\text{per month})}{30} \quad (5.2)$$

$$\sigma d_i(\text{per day}) = \frac{\sigma d_i(\text{per month})}{\sqrt{30}} \quad (5.3)$$

Table 5.3 displays the structure obtained relative to demand information for each family:

Table 5.3: Demand and Standard Deviation

Family	Month	Demand	Stdev Demand	Daily Demand	Daily Stdev Demand
BRPBBG	9	25 398	1 270	847	232
BRPBBG	6	24 386	1 219	813	223
BRPBBG	10	24 061	1 203	802	220
AMNNSM	8	23 654	1 183	788	216
BRPBBG	8	22 787	1 139	760	208
BRPBBG	1	22 440	1 122	748	205

Production

As explained in section 3.1.1, normally each plant produces the same particular families.

A study was made to understand in which plant the different families of products were produced. Figure 5.1 shows that the majority of the families are manufactured at only one plant. There are, however, some families that are processed in two or three distinct locations (marked with dark green):

AV	LE	MG	VF	VN
AMBBBG	BRBBBG	BRNNBG	AMNNBG	BABBBG
AMBBSM	BRBBSM	BRNNSM	AMNNSM	BABBSM
AMNNBG	BRNNSM	BRPBBG	VBBBBG	BRBBBG
AMNNSM	BRPBBG	BRPBMI	VBBBSG	BRBBSG
ASBBBG	BRPBSM	BRPBSM	VBBBSM	BRPBBG
ASBBSG	VGBBBG		VRBBBG	BRPBMI
ASBBSM	VGBBSM		VRBBSM	BRPBSG
ASNNSM			VRNNSM	BRPBSM
BABBBG				
BABBSM				
BRBBBG				
BRNNSM				
BUBBBG				
BUNNSM				
UVBBBG				
UVBBSM				
UVNNBG				
UVNNSM				
VBBBBG				
VBBBSG				
VBBBSM				
VBNNSM				
VRBBBG				

Figure 5.1: Families Produced by Each Plant

Then, to calculate the manufacturing lead time, also known as time between productions (tbp), the materials were divided by factory and the duration between productions for the same materials was calculated. After that, each material was assigned to its own family and, for the final calculation of the average lead time, a weighted average was calculated using the produced quantity, giving greater weight to the manufacturing lead time of the materials that are produced in more quantities within each family.

For families produced in multiple plants, a weighted average of each family's average lead time in each plant was also calculated based on the quantity produced in each plant.

In the following Table 5.4 , it can be seen how the initial data taken from SAP was arranged.

Table 5.4: Production Dates of each Material by Plant

Material	Plant	Begin Date	End Date	Prod. Days	Tons
6435T194BRA	LE	15/03/2021	04/05/2021	50	5 633
6018Y232BRZ	MG	19/03/2021	18/06/2021	91	4 196
9255T568BRA	LE	13/04/2021	05/05/2021	22	2 046
1128C002AMA	VF	13/04/2021	20/05/2021	37	4 786
2998C021AMA	VF	14/04/2021	05/05/2021	21	3 670

The production days (pd) were calculated in the same way.

That is, the different materials were grouped by family and a weighted average of the respective production days was made, giving more weight to the value of the production days of materials produced in larger quantities, making the average value more accurate.

Subsequently, through these two parameters, the time of the replenishment lead time was calculated, as can be seen in Table 5.5. Also, with this information it is possible to calculate the number of replenishment cycles in a year. This concept will be better explained further in the subsection 5.1.3.1.

Table 5.5: Lead Time by Family

Family	thp (days)	pd (days)	lt
ASBBBSM	160	8	168
BUNNSM	45	41	86
AMNNBG	36	82	118
BRNNSM	71	43	114
AMNNSM	46	151	197
VRNNSM	53	78	131
BABBSM	186	11	197

$$n^{\circ} \text{ of Replenishment Cycles} = \frac{365}{TBP + PD} = \frac{365}{\text{Replenishment Lead Time}} \quad (5.4)$$

Additionally, through production data from Table 5.4, it was possible to calculate the percentage of production of each family in the different Iberian plants, as can be seen in Table 5.6:

Table 5.6: Production in each Plant

Family	AV	LE	MG	VF	VN
AMBBBG	100%	0%	0%	0%	0%
AMBBBSM	100%	0%	0%	0%	0%
AMNNBG	15%	0%	0%	85%	0%
AMNNSM	46%	0%	0%	54%	0%
ASBBBG	100%	0%	0%	0%	0%
ASBBSG	100%	0%	0%	0%	0%
ASBBBSM	100%	0%	0%	0%	0%
ASNNSM	100%	0%	0%	0%	0%
BABBBG	44%	0%	0%	0%	56%

In addition, with the help of historical data, the maximum capacity was calculated. Through the register of production days and tons produced in previous years, the maximum tons that can be produced in each plant annually were calculated. The data used take into account the scrap losses due to defects and rejection in production.

Through this information, Table 5.7 was obtained:

Table 5.7: Capacity Production by Plant

Plant	Tons/ year
AV	332 624
LE	214 956
MG	214 764
VF	389 798
VN	210 046

Costs

The industrial cost and sales prices (in €) were gathered through communication with the Production Management and Sales departments. Subsequently, each family was associated with its respective cost. Tables 5.8 and 5.9 provide the result:

Table 5.8: Industrial Cost

Family	Variable Cost/ Ton	Fix Cost/ Ton
ASBBSM	111,7769518	91,19
BUNNSM	124,2005545	91,19
AMNNBG	125,8481963	91,19
BRNNSM	147,3176517	91,19
AMNNSM	125,8481963	91,19
VRNNSM	151,5994442	91,19
BABBSM	114,6966805	91,19

Table 5.9: Sales Price

Family	Sales Price/ Tons
ASBBSM	510,83
BUNNSM	506,87
AMNNBG	385,67
BRNNSM	452,37
AMNNSM	416,92
VRNNSM	441,54
BABBSM	618,86
VRBBSM	513,35

Income statements describe the flow of revenues and costs over a certain period and are calculated as follows:

$$\text{Operating Profit} = \text{Revenue} - \text{Operating Expenses} \quad (5.5)$$

The established model, instead of calculating the profit, will calculate how much the company does not profit from losing a certain number of sales, due to OOS situations.

5.1.3 Contextualization

The optimization problem resolution will support inventory planning and management. The following ideas were taken into account:

- Regarding the constraints of the optimization problem, it was decided to use a production capacity value rather than a warehouse capacity value, since it would be harder to account for and compare the storage value with production, as sales and production are in a constant cycle. When sales happen, stock goes out, and in the same way, when production happens stock comes in constantly, so it would be difficult to predict the free space in the warehouses at the time of each production;
- Analogously to reality, it is assumed that the production is always planned in order to achieve the maximum pull of each furnace and the highest possible efficiency and cadence. In this way, production planning is always done at maximum production capacity, producing as much as possible daily. So, the problem will use this parameter as a constraint;
- Regarding the production budget, the value used will be from this year (2022), since it predicts more realistic results, and it gets closer to the existing financial issues, as for example, the brutal rise in prices for electricity and gas;
- Concerning the total amount of inventory to be maintained (reorder point), it is defined as the sum of the cycle stock and the safety stock. As previously seen, the cycle inventory is the average quantity of inventory utilized to supply demand between the time while the production is not yet available, therefore, it is equivalent to demand during lead time. On the other hand, safety stock is the inventory stored in case demand exceeds expectations.

So, to determine the amount of inventory to hold for each family, it is necessary to know the replenishment lead time for each family and the respective demand in each cycle during the year. The replenishment cycle is defined as the time between inventory supply. It will be explored and better explained further.

The main purpose of the heuristics will be to maximize the service level, fill rate, provided to the most important families to the detriment of the least significant families, calculating the quantity of inventory to hold while satisfying the constraints explained above. The following parameters will serve as inputs and outputs for the heuristic:

<i>Input</i>	<i>Output</i>
Demand	Safety stock
Demand variability	Cycle stock
Manufacturing lead time	Total stock
Production days	Stock days
Costs	Replenishment lead time
Sales price	N° of cycles annually
% of production in each plant	
Budget	
Production capacity	

The input will be the families manufactured annually in Iberia and all related parameters mentioned above, as well as a fill rate target value and a critical value for each family.

The output will be the value of all those parameters and the final fill rate associated with each family.

Heuristics Assumptions:

For the development of the heuristics, the following assumptions were considered:

- All replenishment cycles for all families start on January 1 (changeable);
- Production becomes inventory only at the end of each replenishment cycle;
- Each replenishment cycle starts with zero initial stock;
- The duration of the manufacturing lead time and production period is constant along the year.

5.1.3.1 Replenishment Cycle

Since the time horizon chosen was one year, it is necessary to calculate how many replenishment cycles there are annually. The replenishment cycle or restocking cycle takes into account the days that family i is being produced and the manufacturing lead time, that is the time between the start of the next production of family i and the end of the previous production of family i (time between productions).

In this approach, the amount of stock required to satisfy demand is considered while that family is not being produced as well as during production, since the available stock is only considered at the end of each production period. Figure 5.2 represents the duration of a replenishment cycle and the respective cycle stock, according to the definition of a Gaussian curve (normal distribution):

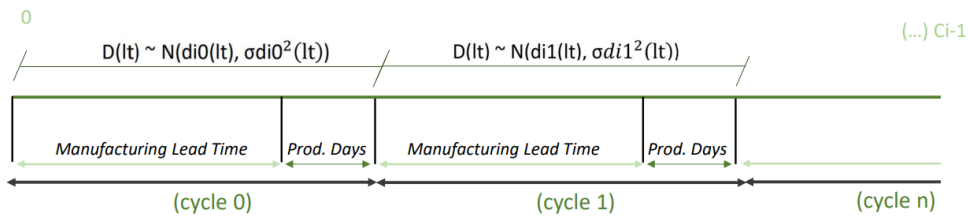


Figure 5.2: Illustration of the Replenishment Cycle of One Family

Regarding the fluctuation of the demand trend, the optimization problem model contains a seasonality component since each cycle is denoted by the start and end day and the average daily demand for each item throughout the year is known. So, knowing the months that the replenishment cycle covers, it is known how much stock needs to be produced depending on the demand at that time of the year.

5.1.4 Optimization Problem Model

Indices

- i family $i \in [1, 2, \dots, F]$
- j plants $j \in [1, 2, 3, 4, 5]$
- n cycle $n \in [0, 1, \dots, C_i - 1]$

Parameters

- tfr_i target fill rate of family i
- cfr_i critical fill rate of family i
- cpd_j annual production capacity at plant j
- p_{ij} percentage of the amount of family i produced at plant j
- bud annual industrial cost budget
- vc_i variable industrial cost associated with each family i
- fc industrial fixed cost
- bc_i breakage cost associated with each family i
- sp_i average selling price associated with family i
- C_i number of cycles of family i during the year
- $d_i(t)$ demand on day t for family i (in tons)
- pd_i production days of family i
- $\sigma d_i(t)$ standard deviation of demand on day t for family i
- tbp_i manufacturing lead time (or time between productions) of family i
- lt_i replenishment lead time related with family i (in days)
- σl_{in} standard deviation of demand during cycle n of the family i (in days)
- f_s standard normal density function
- F_s standard normal cumulative distribution function

Decision Variables

- SS_{in} safety stock of family i during the cycle n (in tons)
- BKG_{in} expected shortage associated with family i in the cycle n (in tons)
- fr_i fill rate of family i

Auxiliary Variables

- SC_{in} cycle stock of family i during the cycle n (in tons)
- ST_{in} total stock needed for family i during the cycle n (in tons)
- Sl_{in} sales of family i in the cycle n (in tons)
- COV_{in} stock coverage of family i during the cycle n (in days)
- IC_i industrial cost associated with family i (in €)

Constraints

$$lt_i = tbp_i + pd_i \quad \forall i \quad (5.6)$$

$$SC_{in} = \sum_{t=n.lt_i+1}^{(n+1).lt_i} d_i(t) \quad \forall i, n \quad (5.7)$$

$$\sigma l_{in} = \sqrt{\sum_{t=n.lt_i+1}^{(n+1).lt_i} \sigma d_i(t)^2} \quad \forall i, n \quad (5.8)$$

$$FR_{in} = \frac{ST_{in}}{(ST_{in} + BKG_{in})} \quad \forall i, n \quad (5.9)$$

$$BKG_{in} = -SS_{in} \times (1 - F_s(\frac{SS_{in}}{\sigma l_{in}})) + \sigma l_{in} \times f_s(\frac{SS_{in}}{\sigma l_{in}}) \quad \forall i, n \quad (5.10)$$

$$ST_{in} = SS_{in} + SC_{in} \quad \forall i, n \quad (5.11)$$

$$IC_i = cv_i + cf \quad \forall i \quad (5.12)$$

$$cb_i = BKG_{in} \times (sp_i - IC_i) \quad \forall i \quad (5.13)$$

$$Sl_{in} = ST_{in} \quad \forall i, n \quad (5.14)$$

$$COV_{in} = \frac{ST_{in} \times lt_i}{(ST_{in} + BKG_{in})} \quad \forall i, n \quad (5.15)$$

$$\sum_{i=1}^F \sum_{n=0}^{C_i-1} (ST_{in} \times IC_i) \leq bud \quad (5.16)$$

$$\sum_{i=1}^F x_{ij} (\sum_{n=0}^{C_i-1} ST_{in}) \leq cpd_j \quad \forall j \quad (5.17)$$

$$cfr_i \leq fr_i \leq tfr_i \quad \forall i \quad (5.18)$$

In accordance with the statistical premise that the standard deviation of a series of independent events is equal to the square root of the sum of the variations, the standard deviation of the demand during the replenishment period can be defined as the square root of the sum of the daily variations, as illustrated in the Equation (5.8).

The customer's orders, when OOS occurs, are not replenished. That means that when there is a out-of-stock, it is assumed a lost sale. In this environment, the expected total demand per cycle is no longer just ST , but also the expected loss per supply cycle is added to this value, as approached by Silver et al. (2016) in 6A3.2. This can explain the Equation (5.9). The variable BKG is the predicted number of missing units from a normalized table where the mean is equal to zero and $\sigma = 1$.

Equation (5.10) was adapted from Equation (12.7) from the book *Supply Chain Management Strategy, Planning and Operation* (Chopra and Meindl, 2007) and its development can be clarified in the Appendix C. Thus, Equations (5.9) and (5.10) constitute a system of equations, being BKG_{in} and SS_{in} the unknowns. The total stock needed, Equation (5.11), is calculated to cover the anticipated demand at the replenishment cycle plus a safety stock proportional to the desired service level pretended.

Regarding the restrictions, Constraint (5.16) is related to costs. It stipulates that the overall manufacturing expenses cannot exceed the annual budgeted amount. On the other hand, Constraint (5.17) delimits the maximum number of tons that can be produced annually in each plant, according to the maximum pull of the respective furnaces. Finally, Constraint (5.18) stipulates a required minimum service level. If these constraints are not satisfied, the fill rate is adjusted (given priority to the most significant families) until they are satisfied. Decreasing the fill rate means a reduction in inventory.

Assuming that the stock is zero at the beginning of each restocking cycle, the total stock (stock required to meet demand) will be the stock that must be produced. If the initial stock was taken into account in each cycle, the picture would be different. The subsequent equations and Figure 5.3 illustrate the modifications that could be made to the model if this factor was taken into account. In a deeper model, the equation 5.19 concerning the stock to be produced should be:

$$(Stock\ to\ Produce)_{in} = (ST)_{in} - (Initial\ Stock)_{in}, \quad (5.19)$$

where:

$$(Initial\ Stock)_{in} = (Final\ Stock)_{i(n-1)}, \quad (5.20)$$

with:

$$(Final\ Stock)_{i(n-1)} = (Existing\ Stock\ before\ Production)_{i(n-1)} - (Demand\ during\ Production)_{i(n-1)} \quad (5.21)$$

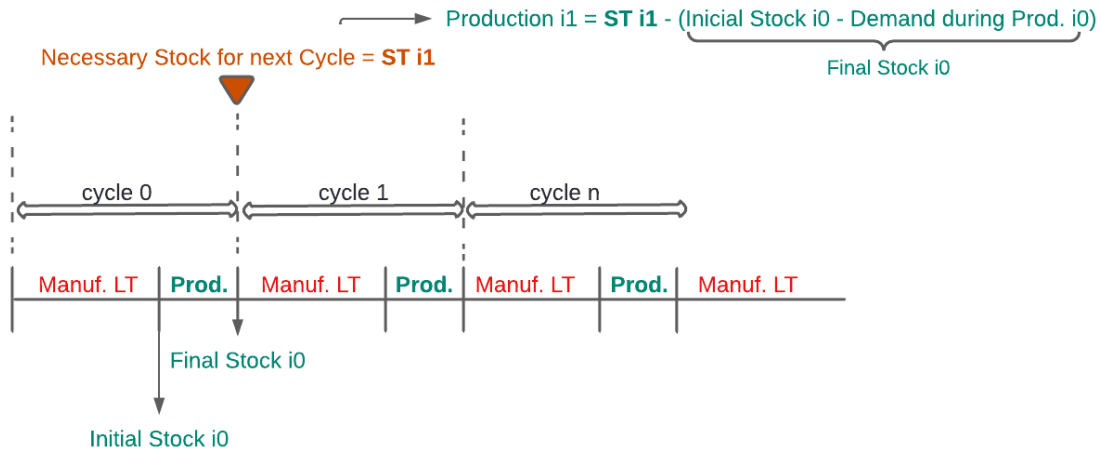


Figure 5.3: Replenishment Cycle Taking Into Account Initial Stock

5.1.5 Heuristic Algorithms Flowchart

Initially, a certain critical fill rate number was assigned to each family, according to their importance. Before developing the heuristics in Python, it was verified whether the selected numbers for the critical fill rate complied with the Constraints 5.16 and 5.17. Following this test and after confirming the values, the heuristics were running in Python.

After the variables calculation, if the stock produced does not fulfill all the constraints, then it will be necessary to lower the fill rate supplied and, consequently, the total stock produced.

First, Constraint (5.16) is evaluated. If the budget constraint is not met, the program automatically lowers the service level provided to the less important family until the restriction is met or the critical fill rate is reached. If that happens, it moves on to the next least important family, and so on, until the restriction is met.

Regarding the Constraint (5.17), if only this one is not fulfilled, then it is not worth changing the service level of the least important family to be produced in Iberia, because that family may not even be produced in the plant that does not meet the production capacity. So, this case is more complex.

To solve this condition, two different theories emerged in the course of the dissertation and were, then, examined. The results were subsequently compared in Chapter 6. These two problem-solving strategies gave rise to two algorithms: Algorithm 1 and Algorithm 2. Next, in Figures 5.4 and 5.5, it is demonstrated the flowchart of the algorithms associated with the heuristics developed and a brief explanation.

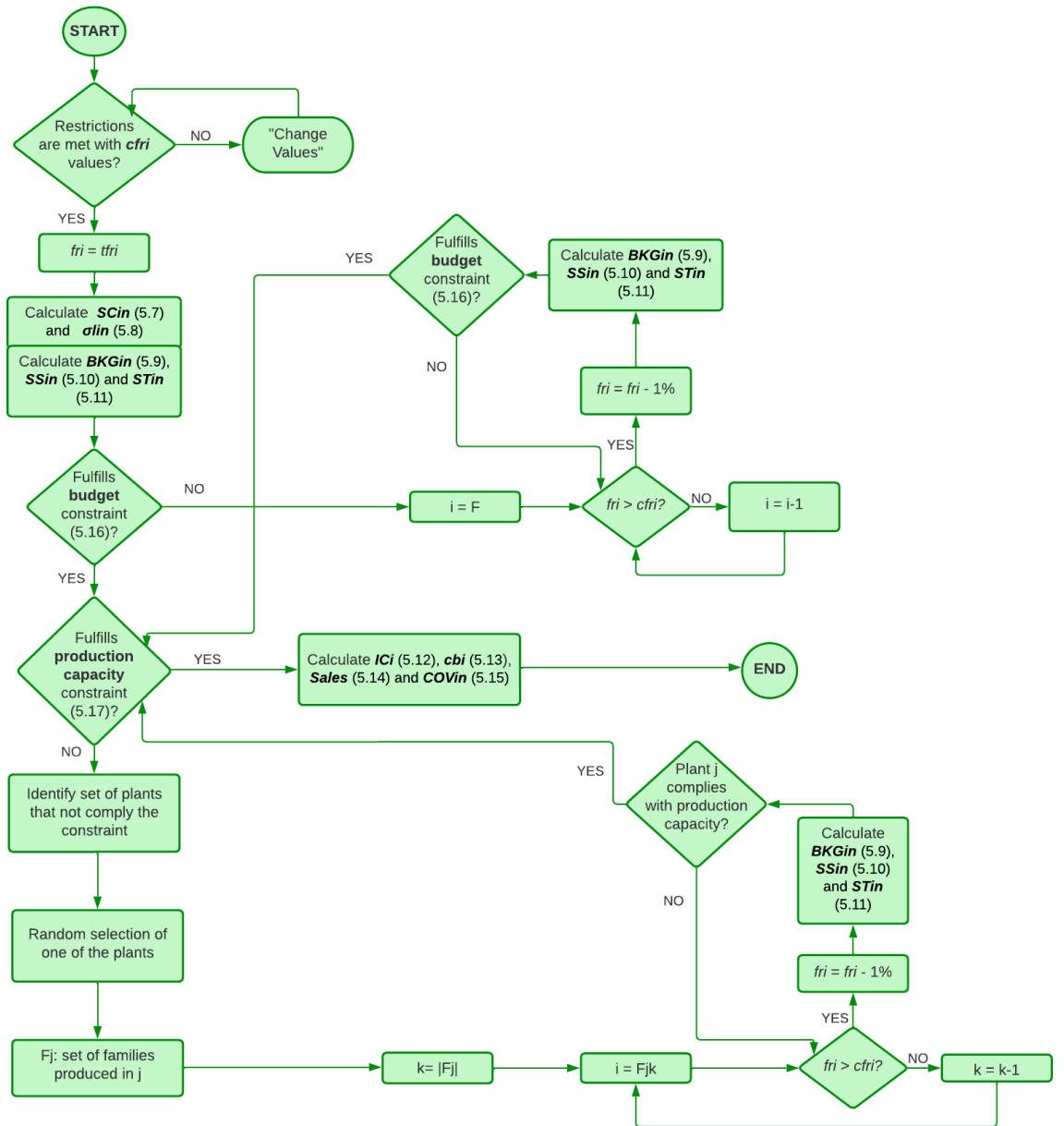


Figure 5.4: Algorithm 1

This first hypothesis, simpler, utilizes a random procedure to identify the plant at which to begin modifying the fill rate of the least significant family. That is, randomly select one of the factories that does not satisfy the restriction and then decrease the fill rate value of the family with the highest index in the plant until it satisfies the Constraint (5.17) or reaches the critical fill rate. If that happens, the program will automatically proceed to the next less relevant family and repeat the process. Thus subsequently, until Restriction (5.17) is satisfied.

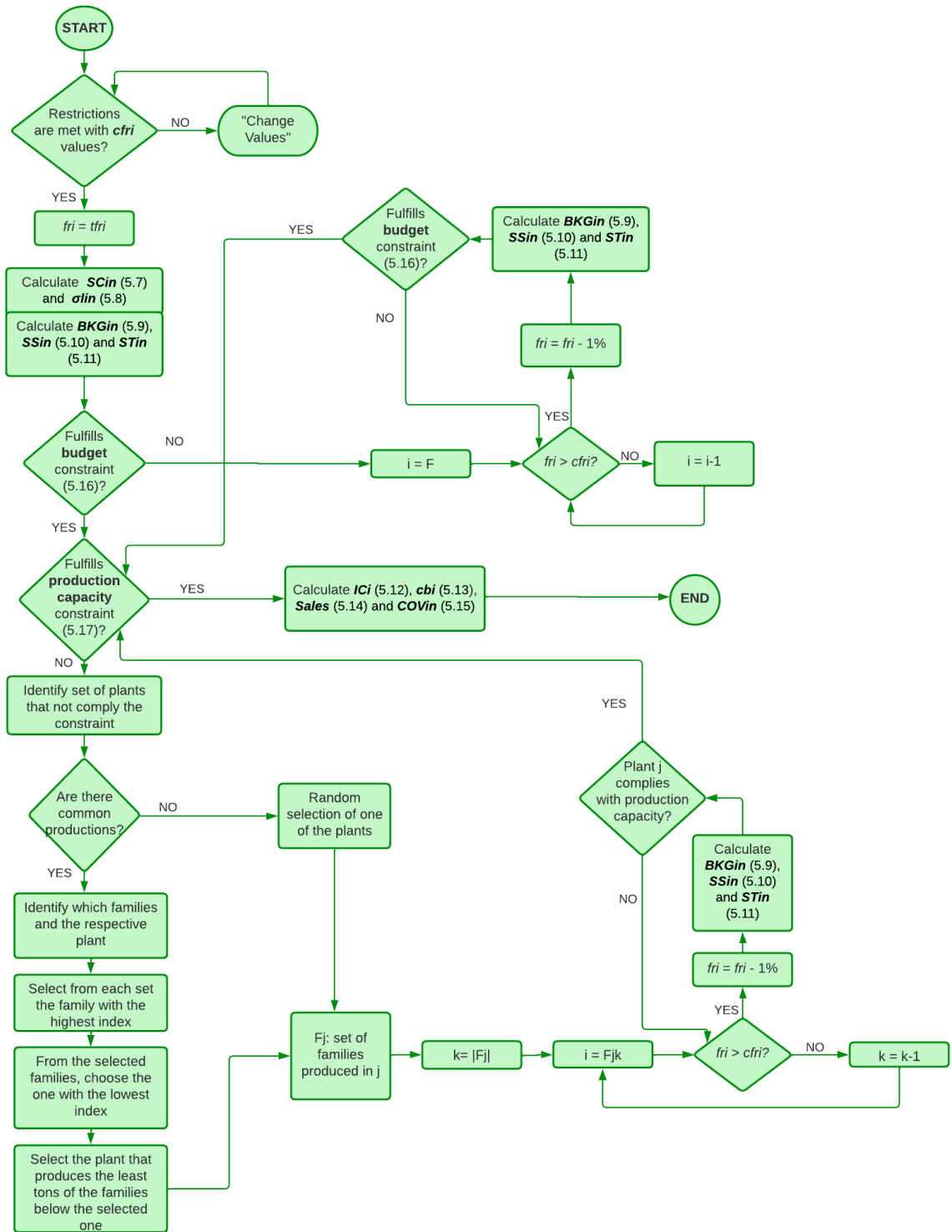


Figure 5.5: Algorithm 2

This second hypothesis, more complex, is developed in order to optimize the production, i.e. increase the quantity produced, maintaining the same ideology of prioritising the most segmented customers.

In this way, the choice of the plant is no longer totally random and it takes into account common production, i.e. families produced in more than one factory.

Firstly, if there are productions of the same family in plants that do not comply with the Restriction 5.17, these families are selected by plant. Subsequently, from these sets, the families with the highest index (less important) are removed and then, from this new set, the family with the lowest index (more important) is chosen. The plant that produces this reference family will be the one chosen to start the fill rate decrease cycle.

Essentially, this approach prevents from interfering with less significant families when, inevitably, there is the need to interfere with a more significant family in order to comply with the Restriction (5.17). So, with this algorithm, relative to the other, it should be produced the same from the most important families and more from the least important ones, making it more optimal.

On the other hand, if there is no common production, a plant is randomly selected, as happens in algorithm 1.

Concluding, the heuristics, using any of the flowcharts, recalculates the best fill rate to achieve for each family in each cycle until it complies with all the constraints such as total budget cost of production, capacity production of each plant and a minimum service level to maintain for each family group, based on its importance.

Chapter 6

Results

The heuristics were created in Python. The goal of this code is to create a tool that might be used and adapted to future problems. Part of it is shown in the Appendix D.

The user of the program just needs to choose the parameters, add the information in the excel sheet and run the algorithm implemented in the code ¹.

Appendix E contains the output provided by an execution of Algorithm 1 for the year of 2022. Each entry in the table contains a value for the family identifier, the cycle, the cycle's start and end date, the resulting fill rate (FR), total stock (ST), cycle stock (SC), safety stock (SS), breakages (BKG) and the associated lost sales, replenishment lead time and the stock coverage. Table 6.1 is a subset of these results, to be further analysed throughout this section.

Table 6.1: Heuristic Results

Family	Cycle	FR	ST	SC	SS	BKG	BKG Cost	Replen. LT	Cov. Days
AMNNSM	0	100%	134782,91	128846,37	5936,55	8,28	1655	197	197
AMNNSM	1		102579,10	97147,15	5431,96	3,38	676	197	197
BRPBGG	0	99%	90364,38	90464,01	-99,63	912,77	167207	135	134
BRPBGG	1		102184,48	102299,70	-115,22	1032,17	189078	135	134
BRPBGG	2		67220,68	67044,19	176,48	679,00	124383	135	134
VRNNSM	0	100%	30909,80	25967,25	4942,54	0,00	0	131	131
VRNNSM	1		27638,23	23108,70	4529,54	0,00	0	131	131
VRNNSM	2		25573,32	21143,56	4429,76	0,00	0	131	131
BRPBSM	0	93%	56309,62	60547,49	-4237,87	4238,36	1175976	143	133
BRPBSM	1		65701,19	70645,81	-4944,62	4945,25	1372110	143	133
BRPBSM	2		32086,03	34496,03	-2410,00	2415,08	670088	143	133

When analyzing the numbers, it can be noticed that the optimization problem achieves its main goal. In fact, when examining the final data in Appendix E, it is possible to conclude that the most important families are related with a higher Fill Rate than the less significant families.

In situations where this is not the case, such as with the family BRPBGG and VRNNSM, it is certainly due to the process of complying with the Constraint (5.17), explained in Chapter 5.

¹model source code: <https://github.com/MarianaBarrias/model>

Also, when comparing the current fill rate values for each family, as shown in Table B.2 of Appendix B, it reveals that there have been changes in the prioritizing of important families and the quality of services offered. For instance, the AMNNSM and BRPBGG families, which are more significant than BUNNSM and VRPBSM families, were receiving inferior quality of service. By using these heuristics, it is feasible to avoid and prevent such occurrences.

Beyond this, detailing the results of Table 6.1, it can be observed that with a $FR = 100\%$, the losses are zero, i.e., the customer receives everything he orders and the days of stock coverage are equal to the number of days of the replenishment cycle (LT). However for a FR less than 100% , $COV\ Days < LT$, because there are lost sales and SS may be negative. This happens because there are losses when $FR < 100\%$ and the SC itself provides a higher fill rate than the one imposed, and therefore the SS is used to discount stock from SC ($ST < SC$), which in turn reduces the service level further until it reaches the supposed value.

Note that negative SS does not mean negative inventory. The Reorder Point/ ST is calculated as the safety stock plus the usage during the lead time (cycle stock). Then a replenishment is scheduled when the on-hand reaches the Reorder Point. It is perfectly acceptable to have a Reorder Point calculated from a negative SS . The formula of this variable works regardless of the signal of the Safety Stock, as can also be noted in the article *The Safety Stock Paradox* by Snyder (1987).

The graph in Figure 6.1, built from the data in Table 6.2, illustrates that given a particular fill rate, the larger the demand variability the higher the related stock breakages:

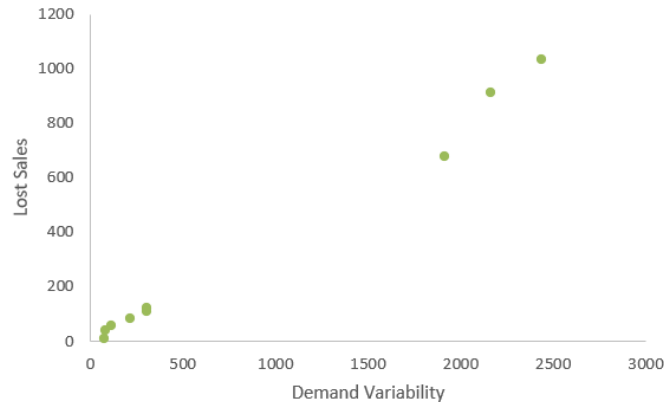


Figure 6.1: Demand Variability Vs Lost Sales

Table 6.2: Data Relating to Different Families with the Same FR

Family	FR	Demand Variab.	BKG
BRPBGG	99%	2160,816	912,771
BRPBGG	99%	2440,132	1032,166
BRPBGG	99%	1915,056	678,997
AMNNBG	99%	306,622	110,032
AMNNBG	99%	307,931	119,775
AMNNBG	99%	216,801	82,525
AMNNBG	99%	75,213	9,631
BABBBG	99%	114,257	56,857
BABBBG	99%	82,339	37,832

Lastly, after developing and executing the program according to Algorithm 2, it is proven that, indeed, there is an increase in the annual production, a better overall service level, fewer stock breakages and a greater profit. Table 6.3 presents the total sum of the tons produced in a year, a fill rate value weighted according to the importance/revenue of each family and the total sum of the costs associated with the lost sales, for both algorithms.

Table 6.3: Comparison of the Results of Both Algorithms

	ST (tons)	FR (%)	BKG Cost (€)
Algorithm 1	1326357	97%	10926248
Algorithm 2	1342257	98%	8669452
<i>Gain</i>	15900		2256796

The conclusion is that algorithm 2 optimizes inventory management, providing a gain in production and in the average service level provided.

Note, as observed in the flowcharts, that the heuristics denote a certain randomness regarding the fulfillment of Constraint (5.17), so in different runs of the program, some small discrepancies of values may arise.

Chapter 7

Conclusion and Future Works

This dissertation was developed within the Planning department. For its development, it was also necessary the communication and help of the Front Office, Production Management, Pricing and Logistics departments. In Appendix B, it is possible to see some of the interactions between certain departments through a workflow.

Through this project, it was possible to evaluate the level of service currently provided to customers in each segment, by analyzing two distinct service metrics: Cycle Service Level and Fill Rate. After analyzing the results, it was reasonable to conclude that it is necessary to better manage production in order to give priority to the most important customers.

In this way, through the consolidation of all the information, it was conceivable to elaborate a model that helps in the inventory management and stock leveling, always based on the level of service provided.

To measure the service level, the fill rate metric was chosen. The main objective is to assist the production planning, prioritizing the production of the most important families, that is, the families that comprise more revenue from A-customers. The optimization problem developed helps to understand what can be changed in terms of stock and production of each family, while always keeping costs in mind, benefiting the most important customers and respective product's family, ensuring market transparency.

The acquired results were as expected, since there is an increase of the service level for priority clients. Also, it is proven that Algorithm 2 permits a 1 % rise in service level and an increase in productivity. Even if the rise is minimal, it represents an improvement.

So, with the implemented solution, it was feasible to acquire stronger control over inventory and to estimate production plans for an entire year based on demand projections.

In this way, the heuristics will provide information regarding on how much stock will be needed to be produced during the replenishment cycle, not considering the possibility of having initial stock. However, there might be initial stock due to sales and demand forecast errors and to several uncontrollable and unpredictable factors that affect actual sales, such as crises, strikes, transportation, among many other factors.

As a future approach, it would be worthwhile to program the problem incorporating the equation 5.19 to make this heuristic more precise.

Also, it would be a wise decision to incorporate a better demand forecasting algorithm into this model to increase its accuracy and relevance at any moment.

Finally, the optimization problem developed is, however, fairly general in that it works with product families, each of which has a large number of different materials. So, it would be interesting to establish parameters for each family and agglomerate the data further and, in this way, adapt the problem to more specific products.

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Appendix A

Family Code

Table A.1: Color Code

Glass Color	
AM	Amber
AS	Strong Amber
BA	Light Blue
BR	Flint
BU	UV Flint
CO	Cobalt Blue
FM	Dead Leaf
PR	Black
UV	UV Green
VB	Antique Green
VG	Georgia Green
VR	Green

Table A.2: Process Code

Process	
SS	Blow-Blow
PS	Press-Blow
NN	Narrow-Neck-Press-Blow

Table A.3: Size Code

Size	
SM	Small
BG	Big
SG	Single Gob
MI	Miniature

Appendix B

Additional Information

B.1 Company Flowchart

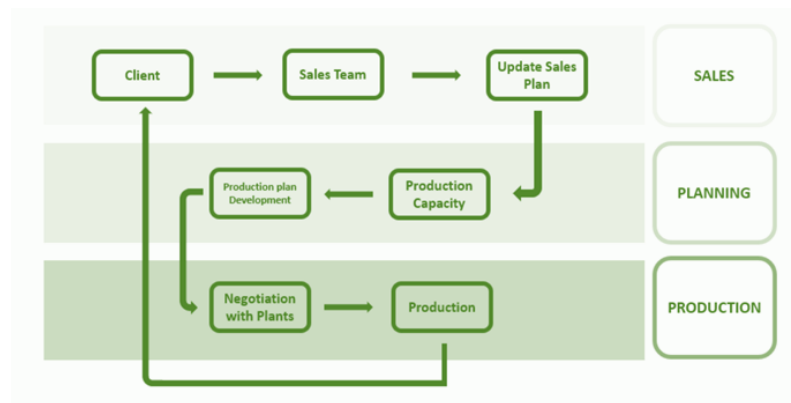


Figure B.1: Workflow Around the Planning Team

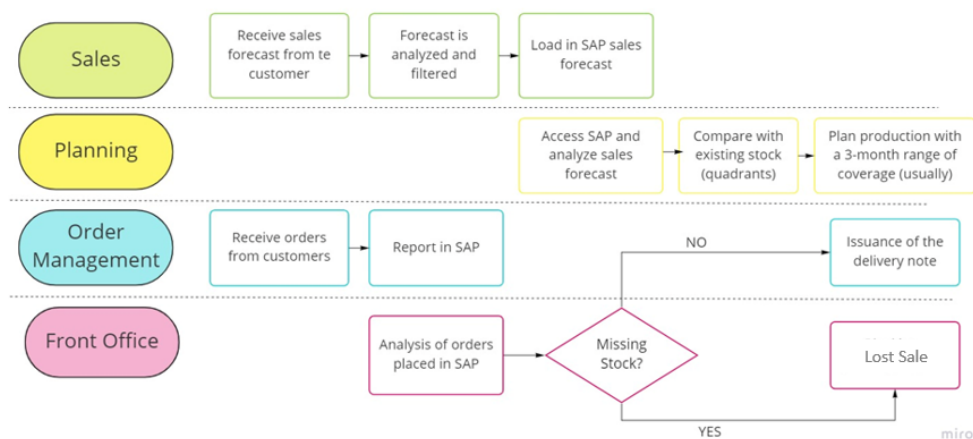


Figure B.2: Workflow Between Departments

B.2 Service Level

Table B.1: Level of Service Provided by the Company

		A	B	C	D	E1	E2	F	AVERAGE
may	Fill Rate	97,1%	98,2%	93,6%	97,4%	99,2%	98,8%	98,2%	97,2%
	Service Level	94,1%	94,0%	94,2%	95,5%	94,8%	96,1%	93,8%	94,2%
	Volume Sales	44,9%	30,2%	11,8%	5,4%	4,8%	1,8%	1,1%	
june	Fill Rate	98,0%	89,8%	97,0%	97,4%	92,6%	96,9%	95,8%	95,2%
	Service Level	93,0%	94,7%	93,3%	95,0%	93,5%	94,9%	94,2%	93,7%
	Volume Sales	47,6%	29,2%	12,0%	4,5%	4,3%	1,5%	0,8%	
july	Fill Rate	97,3%	98,6%	95,3%	90,1%	89,5%	96,3%	98,1%	96,8%
	Service Level	89,6%	94,6%	94,1%	95,6%	88,0%	94,8%	95,7%	92,0%
	Volume Sales	47,7%	29,8%	11,7%	4,7%	3,4%	1,7%	1,0%	
aug	Fill Rate	93,2%	94,5%	97,4%	94,5%	88,5%	96,1%	94,1%	94,1%
	Service Level	86,5%	93,8%	95,9%	95,1%	90,6%	93,0%	95,3%	90,4%
	Sales Volume	49,6%	31,0%	11,1%	3,9%	2,1%	1,4%	0,9%	
sept	Fill Rate	97,1%	95,6%	94,6%	97,2%	92,0%	93,7%	98,3%	96,1%
	Service Level	85,9%	93,8%	93,0%	95,3%	91,4%	90,0%	97,4%	90,3%
	Volume Sales	43,6%	35,0%	11,9%	4,5%	2,2%	1,7%	1,1%	
oct	Fill Rate	88,3%	80,9%	88,1%	86,3%	86,6%	91,9%	88,6%	85,6%
	Service Level	64,0%	71,5%	82,6%	88,0%	83,0%	81,5%	86,0%	71,6%
	Volume Sales	39,6%	35,0%	12,6%	5,0%	4,7%	2,0%	1,2%	
nov	Fill Rate	98,0%	96,2%	94,1%	98,4%	99,2%	99,5%	95,4%	97,0%
	Service Level	95,4%	94,1%	91,8%	95,9%	95,2%	97,5%	91,9%	94,6%
	Volume Sales	42,6%	31,2%	12,6%	5,4%	4,2%	3,0%	1,0%	
dec	Fill Rate	93,2%	87,5%	89,8%	92,6%	65,7%	76,6%	84,6%	90,0%
	Service Level	89,1%	82,0%	89,4%	89,1%	62,8%	75,0%	72,7%	86,1%
	Volume Sales	47,3%	29,1%	13,1%	5,4%	2,3%	2,3%	0,5%	
jan	Fill Rate	96,3%	91,8%	91,8%	94,0%	82,7%	94,9%	98,0%	93,9%
	Service Level	93,7%	91,7%	92,9%	90,7%	86,4%	86,1%	96,9%	92,3%
	Volume Sales	47,0%	29,4%	9,2%	6,0%	3,8%	3,5%	1,1%	
fev	Fill Rate	96,7%	98,5%	95,8%	94,5%	98,5%	96,8%	99,6%	97,1%
	Service Level	88,9%	94,1%	95,2%	93,5%	94,6%	89,2%	96,4%	91,6%
	Volume Sales	46,6%	29,5%	10,2%	6,0%	3,0%	3,7%	1,0%	
may	Fill Rate	92,9%	93,5%	93,7%	94,8%	87,2%	92,7%	98,7%	93,2%
	Service Level	78,9%	86,4%	92,3%	91,6%	89,7%	85,2%	94,6%	84,1%
	Volume Sales	46,2%	28,1%	11,1%	6,5%	3,0%	3,1%	1,9%	
april	Fill Rate	97,4%	93,8%	95,3%	96,1%	85,4%	98,4%	97,3%	95,7%
	Service Level	87,6%	88,2%	95,1%	91,9%	84,8%	91,0%	97,7%	89,1%
	Volume Sales	45,4%	26,9%	10,7%	7,2%	3,5%	4,2%	2,0%	
	Fill Rate	95,5%	93,2%	93,9%	94,4%	88,9%	94,4%	95,6%	94,3%
	Service Level	87,2%	89,9%	92,5%	93,1%	87,9%	89,5%	92,7%	89,1%
	Volume Sales	45,6%	30,9%	11,6%	5,1%	3,5%	2,3%	1,0%	

Table B.2: Current Fill Rate Level For Each Family

Family	Demand	Sales	FR
AMNNSM	225 993,5	214 326,6	95%
BRPBBG	259 808,9	244 891,7	94%
VRNNSM	70 219,5	66 123,7	94%
BRPBSM	165 689,3	154 022,5	93%
BRNNSM	63 414,9	59 414,7	94%
BRBBBG	118 591,4	107 565,9	91%
AMNNBG	31 777,3	31 047,5	98%
BRPBMI	20 941,4	19 810,9	95%
UVBBBG	103 849,1	99 332,8	96%
BUNNSM	6 518,8	6 518,8	100%
BRBBSM	13 029,6	12 515,2	96%
VBBBBG	165 882,8	160 290,9	97%
VRBBSM	5 771,8	4 946,6	86%
AMBBBG	5 112,1	4 174,5	82%
BABBBG	9 404,7	9 302,7	99%
BABBSM	1 685,4	1 652,1	98%
VRBBBG	7 688,8	7 301,9	95%
VBBBSG	4 752,8	4 463,8	94%
ASBBSM	755,1	731,9	97%
UVBBSM	5 656,5	5 519,5	98%
BRBBSG	2 987,1	2 842,6	95%
ASBBBG	2 143,3	1 733,1	81%
VBBBSM	5 617,7	5 194,0	92%
AMBBSM	719,0	595,6	83%
ASNNSM	2 352,8	2 050,7	87%
BRNNBG	1 212,1	1 159,0	96%
BRPBSG	889,5	733,3	82%
UVNNBG	6 375,1	5 625,0	88%
UVNNSM	4 717,5	4 469,6	95%
VBNNBG	334,7	318,6	95%
VGBBBG	613,5	543,8	89%
VGBBSM	115,7	102,1	88%
VRPBSM	3 832,7	3 832,7	100%

Appendix C

Normal Probability Distribution

C.1 The Normal Probability Distribution

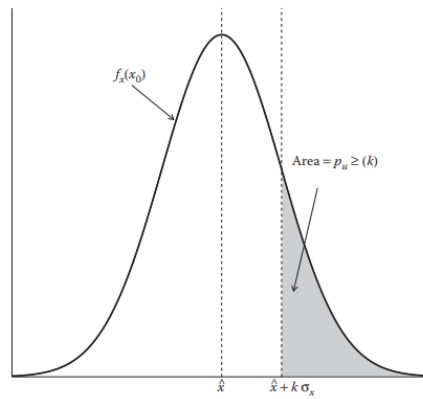


Figure C.1: The Normal p.d.f. (Silver et al., 2016)

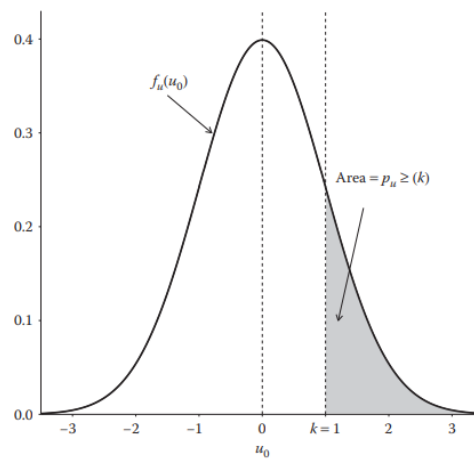


Figure C.2: The Unit (Or Standard) Normal p.d.f. (Silver et al., 2016)

$$G_u(k) = \int_k^{\infty} (u_0 - k)f_u(u_0)du_0$$

Note that the derivative of the unit normal p.d.f. can be expressed as follows:

$$\frac{df_u(u_0)}{du_0} = \frac{d}{du_0} \left(\frac{1}{\sqrt{2\pi}} e^{-u_0^2/2} \right) = -u_0 \left(\frac{1}{\sqrt{2\pi}} e^{-u_0^2/2} \right) = -u_0 f_u(u_0)$$

Using this relationship, we can write $G_u(k)$ as follows:

$$\begin{aligned} G_u(k) &= \int_k^{\infty} (u_0 - k)f_u(u_0)du_0 \\ &= \int_k^{\infty} u_0 f_u(u_0)du_0 - \int_k^{\infty} k f_u(u_0)du_0 \\ &= \int_k^{\infty} -\frac{df_u(u_0)}{du_0} du_0 - \int_k^{\infty} k f_u(u_0)du_0 \\ &= f_u(k) - k p_{u \geq}(k) \end{aligned}$$

Figure C.3: Functions Taken From Appendix II of The Book by Silver et al. (2016)

Another quantity of interest is the expected shortage per replenishment cycle.

$$\begin{aligned} \text{ESPRC} &= \int_{\hat{x} + k\sigma_x}^{\infty} (x_0 - \hat{x} - k\sigma_x)f_x(x_0)dx_0 \\ &= \int_{\hat{x} + k\sigma_x}^{\infty} (x_0 - \hat{x} - k\sigma_x) \frac{1}{\sigma_x \sqrt{2\pi}} \exp[-(x_0 - \hat{x})^2/2\sigma_x^2] dx_0 \end{aligned}$$

The key again is to substitute

$$u_0 = (x_0 - \hat{x})/\sigma_x$$

The result, after considerable simplification, is

$$\begin{aligned} \text{ESPRC} &= \sigma_x \int_k^{\infty} (u_0 - k)f_u(u_0)du_0 \\ &\quad \downarrow \\ \text{ESPRC} &= \sigma_x G_u(k) \end{aligned}$$

Figure C.4: Forms Relative to Breakages (Silver et al., 2016)

Appendix D

Part of the Python Code

Panda was used as a way to link the excel data to the program.

```
1 import functools
2
3 from typing import List, Optional
4 from dataclasses import dataclass
5 from scipy.optimize import fsolve
6 from statistics import NormalDist
7 from datetime import datetime
8
9 from exc import MissingContext
10
11 normdist = NormalDist(mu=0, sigma=1)
12
13
14 @functools.total_ordering
15 class Family:
16
17
18     def __init__(
19         self,
20         family_id: str,
21         daily_demand: List[int],
22         daily_sigma: List[int],
23         dp: int,
24         tbp: int,
25         lt: int,
26         tfr: int,
27         cfr: int,
28         ind_cost: int,
29         revenue: float,
30         sp: float,
31         fr: Optional[int] = None,
32     ):
33         self.family_id = family_id
34         self.daily_demand = daily_demand
35         self.daily_sigma = daily_sigma
36         self.tbp = int(tbp)
37         self.dp = int(dp)
38         self.lt = int(lt)
39         self.tfr = tfr
40         self.cfr = cfr
41         self.ind_cost = ind_cost
42         self.revenue = revenue
```

```

43         self.sp = sp
44         self.__fr = fr
45         self.cycles = []
46         self.set_cycles()
47
48     def __hash__(self):
49         return hash(self.family_id)
50
51     def __eq__(self, other):
52         if not isinstance(other, self.__class__):
53             return False
54         return self.family_id == other.family_id
55
56
57     def __lt__(self, other):
58         if not isinstance(other, self.__class__):
59             raise Exception()
60         if self.revenue != other.revenue:
61             return self.revenue < other.revenue
62         else:
63             return self.family_id > other.family_id
64
65     def __repr__(self):
66         return f"{self.family_id}"
67
68     @property
69     def total_cost(self):
70         return self.ST*self.ind_cost
71
72     @property
73     def ST(self):
74         return functools.reduce(
75             lambda acc, cycle: acc+(cycle.SS + cycle.SC),
76             self.cycles, 0
77         )
78
79     @property
80     def fr(self):
81         return self.__fr
82
83     @fr.setter
84     def fr(self, value: float):
85         self.__fr = value
86         for i, cycle in enumerate(self.cycles):
87             cycle.fr = self.__fr
88
89     def set_cycles(self):
90         start_day = 1
91         while start_day <= 365:
92             cycle = Cycle(self.daily_demand, self.daily_sigma, start_day, self.lt)
93             self.cycles.append(cycle)
94             start_day += self.lt
95
96     def get_cycle_list(self):
97         return [
98             [
99                 self.family_id,
100                 i,
101                 datetime.strptime(
102                     "2022-" + str(cycle.start_day), "%Y-%j"

```

```

103         ).strftime("%Y-%m-%d"),
104         datetime.strptime(
105             "2022-" + str(min(cycle.start_day + cycle.lt, 365)), "%Y-%j"
106         ).strftime("%Y-%m-%d"),
107         self.fr,
108         cycle.ST,
109         cycle.SC,
110         cycle.SS,
111         cycle.BKG,
112         (self.sp - self.ind_cost)*cycle.BKG,
113         self.lt,
114         (cycle.ST)/((cycle.ST + cycle.BKG)/cycle.lt),
115         self.revenue
116     ]
117     for i, cycle in enumerate(self.cycles)
118 ]
119
120 class Cycle:
121
122     def __init__(
123         self,
124         daily_demand,
125         daily_sigma,
126         start_day,
127         lt,
128         fr = None
129     ):
130         self.start_day = start_day
131         self.lt = lt
132         self.SC = None
133         self.calculate_SC(daily_demand, start_day, lt)
134         self.sigma = None
135         self.calculate_sigma(daily_sigma, start_day, lt)
136         self.__fr = None
137         self.BKG = None
138         self.SS = None
139         if fr != None:
140             self.__fr = fr
141             self.calculate_SS_and_BKG()
142
143     @property
144     def fr(self):
145         return self.__fr
146
147     @property
148     def ST(self):
149         return self.SS + self.SC
150
151     @fr.setter
152     def fr(self, value: float):
153         self.__fr = value
154         self.calculate_SS_and_BKG()
155
156     def calculate_SC(self, daily_demand, start_day, lt):
157         self.SC = sum(daily_demand[start_day-1: start_day-1+lt])
158
159     def calculate_sigma(self, daily_sigma, start_day, lt):
160         self.sigma = functools.reduce(
161             lambda acc, val: acc+val**2,
162             daily_sigma[start_day-1: start_day-1+lt], 0

```

```

163         )**(1/2)
164
165     def calculate_SS_and_BKG(self):
166         self.SS, self.BKG = fsolve(
167             self.solve_for_ss_and_bkg,
168             [0, 0],
169             args=(self.sigma, self.SC, self.fr),
170             maxfev=10_000
171         )
172
173     @staticmethod
174     def solve_for_ss_and_bkg(xv, sigma, sc, fr):
175         global normdist
176         if sigma == 0:
177             return (0, 0)
178         return (
179             (xv[0] + sc)/(xv[0] + sc + xv[1]) - fr,
180             - xv[0]*(1 - normdist.cdf(xv[0]/sigma)) + sigma*normdist.pdf(xv[0]/
sigma) - xv[1]
181         )
182
183
184 class Plant:
185
186
187     def __init__(
188         self, plant_id, capacity, families, production_rv
189     ):
190         self.plant_id = plant_id
191         self.capacity = capacity
192         self.production_rv = production_rv
193         self.families = families
194         self.family_set = set()
195         for family, prod_rate in zip(families, production_rv):
196             if prod_rate > 1e-6:
197                 self.family_set |= set([family])
198
199     def __hash__(self):
200         return hash(self.plant_id)
201
202     def __eq__(self, other):
203         if not isinstance(other, self.__class__):
204             return False
205         return self.plant_id == other.plant_id
206
207     def __repr__(self):
208         return f"{self.plant_id}"
209
210     def tons_below(self, f: Family):
211         ton_list = [family.ST * prod_rate
212                     for family, prod_rate in zip(self.families, self.production_rv)
213                     if ((family in self.families)
214                         and (family < f)
215                         and not (family == f)
216                     )
217         ]
218         return sum(ton_list)

```

Listing D.1: Module *dstructures.py* Containing the Classes Required for the Developed Model

Appendix E

Heuristics's Results for 2022

Family	Cycle	Start Date	End Date	FR	ST	SC	SS	BKG	BKG Cost	Replen. LT	Cov. Days
AMNNSM	0	2022-01-01	2022-07-17	100%	134782,9	128846,4	5936,5	8,3	1654,7	197,0	197,0
AMNNSM	1	2022-07-17	2022-12-31		102579,1	97147,1	5432,0	3,4	675,6	197,0	197,0
BRPBBG	0	2022-01-01	2022-05-16		90364,4	90464,0	-99,6	912,8	167207,0	135,0	133,6
BRPBBG	1	2022-05-16	2022-09-28	99%	102184,5	102299,7	-115,2	1032,2	189078,5	135,0	133,7
BRPBBG	2	2022-09-28	2022-12-31		67220,7	67044,2	176,5	679,0	124382,7	135,0	133,7
VRNNSM	0	2022-01-01	2022-05-12		30909,8	25967,3	4942,5	0,0	0,0	131,0	131,0
VRNNSM	1	2022-05-12	2022-09-20	100%	27638,2	23108,7	4529,5	0,0	0,0	131,0	131,0
VRNNSM	2	2022-09-20	2022-12-31		25573,3	21143,6	4429,8	0,0	0,0	131,0	131,0
BRPBSM	0	2022-01-01	2022-05-24		56309,6	60547,5	-4237,9	4238,4	1175975,9	143,0	133,0
BRPBSM	1	2022-05-24	2022-10-14	93%	65701,2	70645,8	-4944,6	4945,3	1372110,4	143,0	133,0
BRPBSM	2	2022-10-14	2022-12-31		32086,0	34496,0	-2410,0	2415,1	670087,9	143,0	133,0
BRNNSM	0	2022-01-01	2022-04-25		15441,3	16603,0	-1161,7	1162,3	248564,0	114,0	106,0
BRNNSM	1	2022-04-25	2022-08-17	93%	24047,1	25856,1	-1809,0	1810,0	387094,3	114,0	106,0
BRNNSM	2	2022-08-17	2022-12-09		16448,9	17686,4	-1237,5	1238,1	264783,5	114,0	106,0
BRNNSM	3	2022-12-09	2022-12-31		3051,6	3269,5	-217,9	229,7	49122,7	114,0	106,0
BRBBBBG	0	2022-01-01	2022-05-08	96%	47746,4	49709,1	-1962,7	1989,4	437885,6	127,0	121,9
BRBBBBG	1	2022-05-08	2022-09-12		37985,2	39542,8	-1557,6	1582,7	348364,7	127,0	121,9
BRBBBBG	2	2022-09-12	2022-12-31		28188,0	29339,6	-1151,6	1174,5	258513,9	127,0	121,9
AMNNBG	0	2022-01-01	2022-04-29	99%	10893,2	10867,8	25,4	110,0	18554,9	118,0	116,8
AMNNBG	1	2022-04-29	2022-08-25		11857,7	11851,5	6,2	119,8	20197,8	118,0	116,8
AMNNBG	2	2022-08-25	2022-12-21		8170,0	8162,0	8,1	82,5	13916,3	118,0	116,8
AMNNBG	3	2022-12-21	2022-12-31		953,5	896,1	57,5	9,6	1624,2	118,0	116,8
BRPBMI	0	2022-01-01	2022-05-30	93%	8083,5	8691,9	-608,4	608,4	280569,5	149,0	138,6
BRPBMI	1	2022-05-30	2022-10-26		8135,3	8747,6	-612,3	612,3	282367,7	149,0	138,6
BRPBMI	2	2022-10-26	2022-12-31		3257,7	3501,9	-244,2	245,2	113069,5	149,0	138,6
UVBBBBG	0	2022-01-01	2022-05-28	100%	57922,7	54236,2	3686,5	0,5	106,8	147,0	147,0
UVBBBBG	1	2022-05-28	2022-10-22		41008,4	34500,1	6508,3	0,0	0,0	147,0	147,0
UVBBBBG	2	2022-10-22	2022-12-31		18940,0	15112,8	3827,3	0,0	0,0	147,0	147,0
BUNNSM	0	2022-01-01	2022-03-27		813,3	798,6	14,6	8,2	2394,5	85,0	84,1
BUNNSM	1	2022-03-27	2022-06-20		1760,0	1740,4	19,5	17,8	5181,8	85,0	84,2

BUNNSM	2	2022-06-20	2022-09-13	99%	3506,3	3491,4	14,9	35,4	10323,4	85,0	84,2
BUNNSM	3	2022-09-13	2022-12-07		507,9	488,3	19,6	5,1	1495,4	85,0	84,2
BUNNSM	4	2022-12-07	2022-12-31		0,0	0,0	0,0	0,0	0,0	85,0	84,2
BRBBSM	0	2022-01-01	2022-09-10	100%	9412,6	8266,6	1145,9	0,0	0,0	252,0	252,0
BRBBSM	1	2022-09-10	2022-12-31		5733,5	4763,0	970,5	0,0	0,0	252,0	252,0
VBBBBG	0	2022-01-01	2022-05-12		59271,5	59761,9	-490,4	852,9	176822,4	131,0	129,1
VBBBBG	1	2022-05-12	2022-09-20	90%	50077,4	55641,5	-5564,1	5564,2	1153525,4	131,0	117,9
VBBBBG	2	2022-09-20	2022-12-31		45431,5	50479,4	-5047,9	5047,9	1046506,7	131,0	117,9
VRBBSM	0	2022-01-01	2022-07-14	90%	2726,9	3029,9	-303,0	303,0	81975,3	194,0	174,6
VRBBSM	1	2022-07-14	2022-12-31		2467,7	2741,9	-274,2	274,2	74183,3	194,0	174,6
AMBBBBG	0	2022-01-01	2022-08-03	90%	2397,8	2664,3	-266,4	266,4	76815,9	214,0	192,6
AMBBBBG	1	2022-08-03	2022-12-31		2203,1	2447,9	-244,8	244,8	70577,4	214,0	192,6
BABBBG	0	2022-01-01	2022-07-19	90%	5084,9	5649,8	-565,0	565,0	185403,2	199,0	179,1
BABBBG	1	2022-07-19	2022-12-31		3379,4	3754,9	-375,5	375,5	123218,3	199,0	179,1
BABBSM	0	2022-01-01	2022-07-17	90%	811,7	901,9	-90,2	90,2	37245,5	197,0	177,3
BABBSM	1	2022-07-17	2022-12-31		705,2	783,5	-78,4	78,4	32357,4	197,0	177,3
VRBBBBG	0	2022-01-01	2022-07-08	90%	3723,5	4137,2	-413,7	413,7	78023,8	188,0	169,2
VRBBBBG	1	2022-07-08	2022-12-31		3196,4	3551,6	-355,2	355,2	66980,3	188,0	169,2
VBBBSG	0	2022-01-01	2022-06-15		2510,9	2789,9	-279,0	279,0	120183,0	165,0	148,5
VBBBSG	1	2022-06-15	2022-11-27	90%	1373,1	1525,7	-152,6	152,6	65723,4	165,0	148,5
VBBBSG	2	2022-11-27	2022-12-31		393,5	437,2	-43,6	43,7	18836,7	165,0	148,5
ASBBSM	0	2022-01-01	2022-06-18		220,5	245,0	-24,5	24,5	7542,7	168,0	151,2
ASBBSM	1	2022-06-18	2022-12-03	90%	324,5	360,5	-36,0	36,1	11098,9	168,0	151,2
ASBBSM	2	2022-12-03	2022-12-31		134,7	149,6	-14,9	15,0	4607,1	168,0	151,2
UVBBSM	0	2022-01-01	2022-06-19		2621,3	3013,0	-391,7	391,7	281723,4	169,0	147,0
UVBBSM	1	2022-06-19	2022-12-05	87%	2139,4	2459,1	-319,7	319,7	229932,0	169,0	147,0
UVBBSM	2	2022-12-05	2022-12-31		160,5	184,5	-24,0	24,0	17249,6	169,0	147,0
BRBBSG	0	2022-01-01	2022-06-27		1262,9	1451,6	-188,7	188,7	57943,1	177,0	154,0
BRBBSG	1	2022-06-27	2022-12-21	87%	1279,4	1470,6	-191,2	191,2	58701,7	177,0	154,0
BRBBSG	2	2022-12-21	2022-12-31		56,6	64,9	-8,3	8,5	2597,0	177,0	154,0
ASBBBBG	0	2022-01-01	2022-12-31	87%	1864,7	2143,3	-278,6	278,6	132039,1	384,0	334,1

VBBBSM	0	2022-01-01	2022-06-11		2196,5	2524,7	-328,2	328,2	95396,5	161,0	140,1
VBBBSM	1	2022-06-11	2022-11-19	87%	1954,7	2246,7	-292,1	292,1	84894,0	161,0	140,1
VBBBSM	2	2022-11-19	2022-12-31		736,3	846,3	-110,0	110,0	31977,7	161,0	140,1
AMBBSM	0	2022-01-01	2022-05-21		374,8	430,8	-56,0	56,0	13124,8	140,0	121,8
AMBBSM	1	2022-05-21	2022-10-08	87%	192,8	221,7	-28,8	28,8	6753,5	140,0	121,8
AMBBSM	2	2022-10-08	2022-12-31		57,9	66,6	-8,7	8,7	2028,7	140,0	121,8
ASNNSM	0	2022-01-01	2022-12-31	87%	2047,0	2352,8	-305,9	305,9	79029,4	386,0	335,8
BRNNBG	0	2022-01-01	2022-06-14		455,6	523,7	-68,1	68,1	10993,7	164,0	142,7
BRNNBG	1	2022-06-14	2022-11-25	87%	498,9	573,5	-74,6	74,6	12039,4	164,0	142,7
BRNNBG	2	2022-11-25	2022-12-31		100,1	115,0	-14,9	15,0	2414,4	164,0	142,7
BRPBSG	0	2022-01-01	2022-11-14		656,2	772,0	-115,8	115,8	21326,2	317,0	269,5
BRPBSG	1	2022-11-14	2022-12-31	85%	99,8	117,4	-17,6	17,6	3243,7	317,0	269,5
UVNNBG	0	2022-01-01	2022-05-18		796,9	937,5	-140,6	140,6	36378,2	137,0	116,4
UVNNBG	1	2022-05-18	2022-10-02	85%	3696,5	4348,8	-652,3	652,3	168747,9	137,0	116,5
UVNNBG	2	2022-10-02	2022-12-31		925,4	1088,7	-163,3	163,3	42246,6	137,0	116,5
UVNNSM	0	2022-01-01	2022-04-23	85%	1049,0	1234,1	-185,1	185,1	41960,2	112,0	95,2
UVNNSM	1	2022-04-23	2022-08-13	85%	1263,5	1486,5	-223,0	223,0	50543,3	112,0	95,2
UVNNSM	2	2022-08-13	2022-12-03	85%	1355,0	1594,1	-239,1	239,1	54200,2	112,0	95,2
UVNNSM	3	2022-12-03	2022-12-31	85%	342,5	402,9	-60,4	60,4	13698,7	112,0	95,2
VBNNBG	0	2022-01-01	2022-10-28		245,4	288,7	-43,3	43,3	6330,4	300,0	255,0
VBNNBG	1	2022-10-28	2022-12-31	85%	39,1	46,0	-6,9	6,9	1008,4	300,0	255,0
VGBBBG	0	2022-01-01	2022-12-31	100%	693,3	613,5	79,7	0,0	0,0	773,0	773,0
VGBBSM	0	2022-01-01	2022-09-12		155,2	115,7	39,5	0,0	0,0	254,0	254,0
VGBBSM	1	2022-09-12	2022-12-31	100%	0,0	0,0	0,0	0,0	0,0	254,0	
VRPBSM	0	2022-01-01	2022-05-04		1243,7	1463,2	-219,5	219,5	48684,6	123,0	104,6
VRPBSM	1	2022-05-04	2022-09-04	85%	970,2	1141,4	-171,2	171,2	37978,1	123,0	104,6
VRPBSM	2	2022-09-04	2022-12-31		1043,9	1228,1	-184,2	184,2	40861,4	123,0	104,6