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Analytical Revenue Management in Rent-a-Car Company

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Abstract

The car rental industry is a worldwide business. The importance of fleet and price management processes is growing within organizations because the dynamic nature and volatility of the demand compromise their profits. In this type of business, market and management decisions impact revenue generation. When making these decisions, one can consider short and long-term goals, for example, by defining when the management of fleet inflows and exits will occur or deciding which pricing strategy to adopt. Thus, continuously and strategically developing a platform to support price, fleet, and revenue planning is vital.

This dissertation presents the *Rental Revenue Management* (RRM) project developed for a car rental company operating in Portugal. The RRM project aims to develop an analytical model for coordinating price management with fleet management to be implemented in the company in question. Apart from this, it allows the management team to understand the real-time problem and helps find the best possible decision for different periods and demand trends. Overall, the project supports the optimization of critical decision-making in order to maximize the benefits.

A mixed-integer programming model was developed to tackle price and fleet management. Every day, the goal is to define the price and fleet of each vehicle group, station, and length of rental (LOR), which will help optimize the fleet's occupation rate, taking into account the number of days until the pickup and the corresponding occupation target. The optimization model supports the car rental company by setting the prices for each station, vehicle group, reservation day, and LOR levels. In addition, it also recommends optimal fleet distribution across various stations and supports the setting of one-way rates. Overall, the objective is to maximize the company's total revenue. A relax-and-fix method was implemented because the optimal solution is generally deemed impractical due to the processing time required.

Furthermore, another optimization module was developed, which suggests vehicle repositioning between stations and pools. The model relies on the optimal fleet suggested by the previous solution and the fleet available at each moment to reposition the different groups of vehicles in each station. The main objective is to minimize the vehicle repositioning cost of the car rental company in question. The problem was formulated as a linear programming model and considered other practical details such as the possibility of one-ways taking place, the arrival of new cars (infleets), and the need to reallocate a car by an empty transfer.

Both models were implemented in a decision support system that allows the company to control prices and reposition the vehicles on a daily basis. This system is expected to improve the car rental company's revenue and reduce the repositioning cost compared to its current procedures. Moreover, it allows the company to increase control over its operation and capacity to adjust the decisions they make on a daily basis, which would otherwise require a larger team.

Resumo

A indústria rent-a-car é um negócio mundial. A importância dos processos de gestão da frota e dos preços está a crescer dentro das organizações, uma vez que a natureza dinâmica e a volatilidade da procura comprometem os seus lucros. Neste tipo de negócio, as decisões de mercado e de gestão têm impacto na geração de receitas. Ao tomar estas decisões, pode-se considerar objetivo a curto e longo prazo, por exemplo, definindo quando ocorrerá a gestão das entradas e saídas da frota ou decidindo qual a estratégia de preços a adotar. Assim, é vital desenvolver contínua e estrategicamente uma plataforma para apoiar o planeamento de preços, frotas e receitas.

Esta dissertação apresenta o projeto de *Rental Revenue Management* (RRM) desenvolvido para uma empresa de aluguer de automóveis a operar em Portugal. O projeto RRM visa desenvolver um modelo analítico para coordenar a gestão de preços com a gestão da frota a ser implementada na empresa em questão. Para além disso, permite à equipa de gestão compreender o problema em tempo real e ajuda a encontrar a melhor decisão possível para diferentes períodos e tendências da procura. Globalmente, o projeto apoia a otimização da tomada de decisões críticas, a fim de maximizar os benefícios.

Foi desenvolvido um modelo de programação matemática inteira mista para fazer face à gestão dos preços e da frota. Todos os dias, o objectivo é definir o preço e a frota de cada grupo de veículos, estação, e duração do aluguer (LOR), o que ajudará a otimizar a taxa de ocupação da frota, tendo em conta o número de dias até à recolha e o objetivo de ocupação correspondente. O modelo de otimização apoia a empresa de aluguer de automóveis, definindo os preços para cada estação, grupo de veículos, dia de reserva, e níveis de LOR. Além disso, recomenda também uma distribuição ótima da frota nas várias estações e suporta a definição de *one-ways*. Globalmente, o objetivo é maximizar as receitas totais da empresa. O modelo foi implementado baseando-se num método *relax-and-fix*, uma vez que a solução exata é geralmente considerada impraticável devido ao tempo de processamento necessário.

Além disso, foi desenvolvido outro módulo de otimização, que sugere o reposicionamento de veículos entre estações e *pools*. O modelo conta com a frota ideal devolvida pela solução anterior e a frota disponível em cada momento para reposicionar os diferentes grupos de veículos em cada estação. O principal objetivo é minimizar o custo de reposicionamento do veículo da empresa de aluguer de automóveis em questão. O problema foi formulado como um modelo de programação matemática linear e considerou outros detalhes práticos, tais como a possibilidade de se efetuar *one-ways*, a chegada de novos carros (*infleets*), e a necessidade de realocar um carro através de uma transferência em vazio.

Ambos os modelos foram implementados num sistema de apoio à decisão que permite à empresa controlar os preços e reposicionar os veículos diariamente. Espera-se que este sistema melhore as receitas da empresa de aluguer de automóveis e reduza o custo de reposicionamento em comparação com os seus procedimentos atuais. Além disso, permite à empresa aumentar o controlo sobre o seu funcionamento e a capacidade de ajustar as decisões que tomam diariamente, o que de outra forma exigiria uma equipa maior.

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Chega o fim de uma das etapas mais marcantes da minha vida. Mais um objetivo de vida atingido. Um marco com imenso significado para mim, que inclui um caminho permeado por inúmeros obstáculos, desafios, tristezas, alegrias e muita aprendizagem. Realizar este caminho só foi possível com apoio, energia e força de várias pessoas, a quem agradeço especialmente.

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“It always seems impossible until it’s done.”

Nelson Mandela

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List of Acronyms

ABT	Advance Booking Time
ADP	Approximation Dynamic Programming
ARAC	Association of Car Rental Industries
iRPD	Incremental Revenue Per Day
LOR	Lenght Of Rental
RM	Revenue Management
RPD	Revenue Per Day
RRM	Rental Revenue Management
T&M	Time & Mileage

Chapter 1

Introduction

This chapter briefly introduces the car rental business and the motivation for creating the Rental Revenue Management project as well as its objectives. Additionally, it presents the structure of the document.

1.1 Context and Motivation

The car rental industry is a worldwide business. It is a long-term, periodic, and seasonal industry (Lazov, 2017). Global companies are commonly known (e.g., Europcar, Sixt, Hertz, etc.), and local companies usually are either franchisings or small business of a particular location. Nowadays, the importance of fleet and price management processes is growing within organizations. Given the dynamism and volatility of the demand, such compromises their profits. It is crucial to have an integrated platform for revenue management to maximize the benefits.

In the car rental business, market and management decisions impact revenue generation. These decisions can consider short and long-term goals, for example, by defining fleet inflows and exits (e.g., new vehicles, breakdowns, sales, vehicle repositions, etc.), by deciding which pricing strategy to adopt. Suppose the company's decision-makers properly design the fleet. In that case, effective fleet management avoids concerns such as a low occupancy rate or inability to meet consumer demand, and, consequently, the company can achieve a substantial increase in revenue (Rani et al., 2021). The companies in this sector are affected by a considerable factor called uncertainty (Oliveira et al., 2017). For instance, it would affect the next customer when someone returns their car late (Rani et al., 2021). Thus, the platform to support decision-making needs to optimize critical decisions. It is vital to develop it to support price, fleet, and revenue planning continuously and strategically.

The profitability of a company is dependent on its fleet and its decisions. So, the Rental Revenue Management (RRM) project aims to develop an analytical model for coordinating price management with fleet management, to be applied in a company within the car rental sector. The car rental company in question does not currently have an automatic fleet and price management process.

The first part of the RRM project relies on data analysis to help predict the market demand. The second part helps the team support the tactical strategy. The third and fourth parts focus on developing a prescriptive model to maximize revenue and vehicle utilization. Overall, the RRM project allows the management team to understand the real-time problem and assists in finding the best possible solution for different periods and demand trends. This Master's thesis research will focus on the analytical module to optimize pricing decisions and fleet planning and on another optimization module that suggests vehicle repositioning between stations and pools, which are a group of stations. Apart from this, some control systems will be developed to monitor and warn if something unexpected occurs. Overall, the project intends to tackle some current challenges by developing crucial tasks, such as:

- integrate fleet size with price management;
- improve the analytical and predictive capacity;
- improve the reactivity and frequency of reviewing assumptions and pricing decisions;
- propose a fleet size;
- propose vehicle repositions between stations and pools.

1.2 Objectives

The Rental Revenue Management project is a fully integrated revenue management analytics platform, with granular optimization of crucial decisions to support the company's pricing and capacity management strategically and operationally.

From the point of view of the car rental business, three key results are expected:

1. increased revenue, considering the price optimization at each moment;
2. enhanced analytical support for decision-making;
3. increased performance and focus on value-added tasks.

The embedded analytical model needs to coordinate the price management with the fleet management to accomplish these goals, considering the available capacity and the price strategy at each moment. The research of this Master's thesis will focus on the analytical module to optimize pricing decisions and fleet planning, as well as on another optimization module that suggests vehicle repositioning. In addition, some control systems will be developed to monitor and warn of unexpected occurrences.

1.3 Dissertation Structure

The first chapter is the introduction, which contains essential aspects of the context, motivation, and objectives of the dissertation's problem. The second chapter presents the description of the problem and explores different phases of the RRM project. The third chapter focuses on the literature review, which includes: contextualization of the car rental business around the world including Portugal, how Covid-19 affected the sector; definition of the revenue management problem; and the critical analysis of the presented solutions. The fourth and fifth chapters focus on the methodology and include, among others, the optimization models for the time and mileage pricing and fleet planning model and the vehicle reposition model, respectively. Then, in the sixth chapter, the results for both models are discussed. Finally, the seventh chapter shows the conclusions of the RRM project and some future work to take into consideration.

Chapter 2

Problem Description

The RRM – Rental Revenue Management - project aims to implement a revenue management system based on the capacity of each station and its fleet availability and booking evolution in Portugal. The project was developed in LTPlabs. The car rental company in question is one of LTP’s clients. “LTPlabs is a boutique analytical-driven management consultancy firm that helps its clients achieve significant and sustainable performance improvements by combining business expertise with advanced analytics (e.g., prescriptive & predictive modelling, optimization, simulation, machine learning).” ([LTPlabs Website, 2022](#)).

The car rental company divides its internal structure into four main initiatives: pricing management, capacity management, reservations management, and monitor and alarm systems. The RRM project aims to address all the initiatives, except reservations management. Figure 2.1 shows the division of the project’s modules. This Master’s thesis will focus on the third module and show some examples of control systems to monitor the pricing implementation.

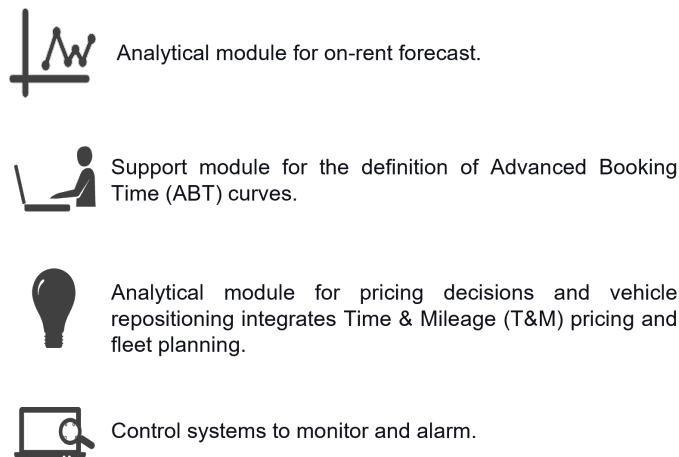


Figure 2.1: Modules’ Division

2.1 Car Rental Company - Main Processes

The project will necessarily lead to a process change. The three major processes affected by the project will be budgeting, a monthly review of the pricing committee, and operational planning (pricing and transfers).

In the process of budgeting revenue, four central aspects stand out. Firstly, the Chairman, the pricing committee, and the pricing team need to compile all relevant information, including the company's global and strategic vision, standard costs, Incremental Revenue Per Day (iRPD), assumptions and damages, and forecast of non-work segments (i.e., Retail and Leisure segments) on the RRM platform. Secondly, the same people have to introduce fleet and Revenue Per Day (RPD) targets as inputs that reflect the company's global and strategic vision. Thirdly, an iterative scenario definition and forecast generation process will run until the pricing team validates the information. Finally, the General Management and the Chairman must validate the results in terms of revenue.

In the monthly review of the pricing committee, the process is summed up to: update the revenue projections and analysis of the plan of each segment; adjust the targets through What-if analyses based on the generation of different scenarios; and analyze and validate the plans for the business segments, while also considering changes in fleet allocation.

Finally, the operational planning process includes updating pricing suggestions, analyzing the relocation of vehicles, setting thresholds for control alarms, and validating the model results. These results are validated automatically by the tool within the defined thresholds (when no alarm goes off) and by manually setting another revenue per day to implement.

2.2 Requirements

The analytics platform connects to the car rental company's systems to feed them the data needed to update the generated information directly. Also, the platform has multiple interfaces (tactical and operational) that allow interaction with users (e.g., committee, pricing, and fleet team members). As a general requirement for the development of the revenue management analytics platform, it needs to access the source of the company's historical and up to date data (where a pre-processing for the correction of inconsistencies has ideally already been incorporated), with the potential to automate the periodic extraction of python data and send it to the database. Besides this, the platform has some data requirements presented in table 2.1. Each requirement is presented with characteristics like the field name, the detail, the granularity, and the updated periodicity that the field will have. The concept regarding *Target ABT curves* will be explained in section 2.4.2.

Table 2.1: Data Requirements

Field and detail	Granularity	Update periodicity
<i>Stations</i> : name, code, and pool	Season	Weekly
<i>Vehicle Group</i> : name, code, and cluster	Group of vehicles	Weekly
<i>Reservations</i> : booking date, date and time of the pick-up station, date and time of the delivery station, group of vehicles, RPD T&M, LOR, business segment, point-of-sale, and status (confirmed, canceled, or no-show)	Reservation (code)	Daily, preferably hourly
<i>Contracts</i> : associated booking, driven vehicle group, date / time / effective pick-up station, effective date / time / delivery station, and business segment	Contract (code)	Daily, preferably hourly
<i>Fleet</i> : available, through consideration of the current fleet (in-fleet and de-fleet), in contract, in maintenance or with damage, for historical and planned data	<i>Day, station, and vehicle group</i>	Daily
<i>RPD competition</i>	<i>Day, season, LOR, and vehicle group</i>	Daily
<i>Target ABT curves</i>	-	Weekly

2.3 Decision Support System Overview

Considering the car rental company's leading processes and requirements, a decision support system architecture is presented in figure 2.2. Five primary connections exist between the company in question and the LTPlabs. Firstly, connections A and B update daily the information in the data warehouse located in LTP. Secondly, connections C and D create a flow for powering planning and control interfaces. The car rental company can define new scenarios, new planning and change particular parametrizations. The data warehouse RRM will deliver on interfaces all the output data from the different modules of the Revenue Management Analytics Platform. Then, the last connection, E, is needed to feed the reports of exploitation of results (reports controlled only by the RRM project).

2.4 Revenue Management Analytics Platform

The first part of the RRM project relies on data analysis to help predict the market demand. The second part helps the team support the tactical strategy. The third and fourth parts focus on developing a prescriptive model to maximize revenue and vehicle utilization. Overall, the RRM project allows the management team to understand the real-time problem and assists in finding the best possible solution for different periods and demand trends. This Master's thesis research will focus

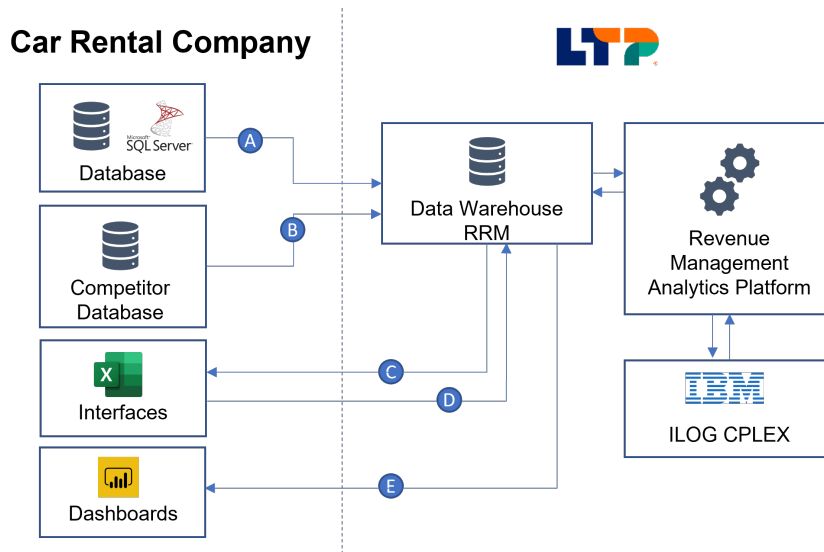


Figure 2.2: Decision support system architecture

on the analytical module to optimize pricing decisions and fleet planning and on another optimization module that suggests vehicle repositioning. Apart from this, some control systems will be developed to monitor and warn if something unexpected occurs.

2.4.1 On-Rent Forecast

The RRM project will start by forecasting the number of rentals predicted over time, depending on an operating context. It relies on data analysis to help improve the current on-rent forecast's reliability and granularity and predict market demand. This module does not provide any optimization of the tactical or strategic levels. It can only be used from a What-if scenario perspective based on inputs. Even though the forecast is not the focus of this Master's thesis, it is essential for project understanding. The three main company processes will use rental forecasts at three different times:

1. in the budgeting phase, with two distinct purposes: support the definition of the initial budget by calculating the on-rent forecast for each distinct context of operation (fleet and target RPDs), and disaggregate aggregated estimates made during validation processes;
2. in the process of a monthly review of the budget by the pricing committee, with two purposes: monthly performance of the on-rent forecast taking into account the relationship with the budget and the evolution of reserves, and validation and adjustment of the targets foreseen for the remaining months; and conducting What-if analyses to assess the impact of fleet changes or price strategy;
3. in the operational planning process, to assess the impact of the changes in the price and fleet availability in the forecasts as part of the strategy for price change. This process has a daily periodicity.

The generation of predictions will resort to machine learning models, which allow for the modeling of the impact of the variables present in rentals. This method was selected because on-rent forecasts depend on the temporal evolution of reservations (seasonality and trend), which are drastically affected by planning factors such as price and fleet availability.

The model will have both a long-term and a short-term version with distinct granularities: *business segment, station, vehicle group, and reservation day* for the long-term; and, for the short-term, *business segment, station, vehicle group, reservation day, and LOR*. The forecast period may be parameterizable, but will encompass a period of three months for the short-term and 12 months for the long-term. The two models will not simultaneously generate the same period.

The model outputs include direct and calculated outputs. The direct results include the number of rentals which are the reservations planned for the period in question and the incremental revenue planned for the period (iRPD).

2.4.2 Advance Booking Time Curve

Before diving into the other aspects of the project, it is crucial to understand a key relevant concept used. Advance Booking Time (ABT) curves are used throughout the project to help the team support the tactical strategy at the level of the evolution of the occupancy rate with the advance of the reservation or to help compare the historical and theoretical or real and expected curves.

Figure 2.3 shows an example of an Advance Booking Time (ABT) curve and compares the historical fleet occupation values (orange curve) with the theoretical values obtained. The X-axis represents the number of weeks in advance towards a given date, and the Y-axis represents the fleet occupation. So, empirically, in this example, the ABT curve reflects the reservations' evolution as a function of weeks in advance. There is a significant difference between theoretical and historical fleet occupation values because this curve is transversal to all vehicle groups and stations. Currently, the car rental company in question does not use the price elasticity concept in the price strategy to fulfill the ABT curve. Therefore, applying this concept and adjusting the theoretical curve to each vehicle group and station is crucial. The RRM project will propose a solution by applying the price elasticity concept. This concept enables us to find a better solution by measuring the percentual change in the number of reservations when there is a price change.

The generation of ABT curves uses machine learning models, which allow for the modeling of the impact of certain variables on the resulting curves. These curves need to be evaluated, adjusted, and approved by the company in question as a tactical tool. The ABT generation is not discussed in this document due to it not being part of the scope of this Master's thesis.

2.4.3 Time and Mileage Price and Fleet Planning

From this section on, the focus of the Master's thesis will be the pricing and fleet model, which is an optimization model that should consider different decisions and constraints. Every day, the goal is to define the price and fleet of each vehicle group, station, and LOR which will help to achieve the occupation target for the fleet, taking the advance for the day of the rental into account. The

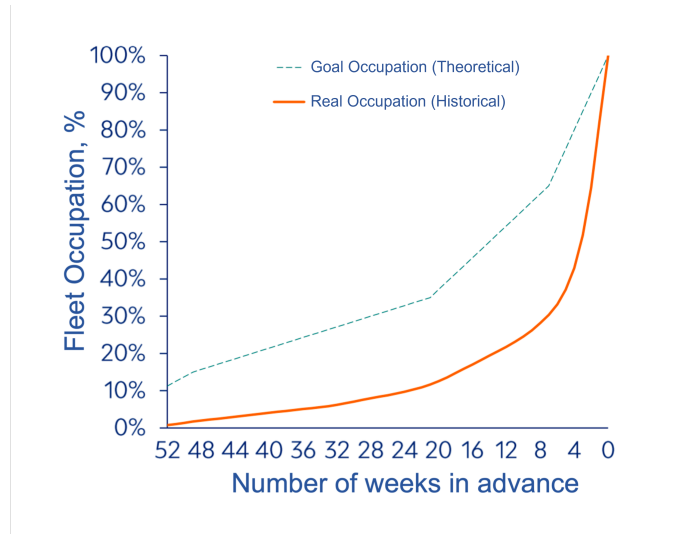


Figure 2.3: Example of Advance Booking Time Curve

advance here is the number of days between “today” and the day of the rental. The main goal considers three essential decisions:

- support the pricing team in setting the prices to be applied at each moment at the *station*, *group of vehicles*, *reservation day* and *LOR* levels;
- recommend optimal fleet distribution across the various stations to safeguard the maximization of total revenue (or profitability), considering the rules laid out in the previous section;
- support the definition of one-way rates by identifying the repositioning needs of vehicles derived from the optimal fleet distribution.

A set of constraints will also ensure beacons for the assigned values, the relationship between groups and LORs, and operational capacity. Some constraints to take into consideration are:

- relationship between the prices of the various vehicle groups and LORs;
- compliance with the expected occupancy rate, while allowing for the exploration of the remaining objectives;
- minimum and maximum price to apply;
- minimum and maximum fleet at each station and pool;
- respecting the operational capacity.

Overall, the three decisions and the constraints mentioned above must comply with the target occupancy rate and maximize the revenue. The purpose of pricing should be to follow the strategy defined in the budget and reviewed by the pricing committee, based on daily target revenue and

target occupancy rate (through the ABT curve) inputs. Therefore, the problem should be configured as follows: maximizing the revenue earned daily while ensuring that increasing/decreasing the price leads to a decrease/increase in reservations. Revenue maximization is limited by an upper and lower limit of the target occupancy rate.

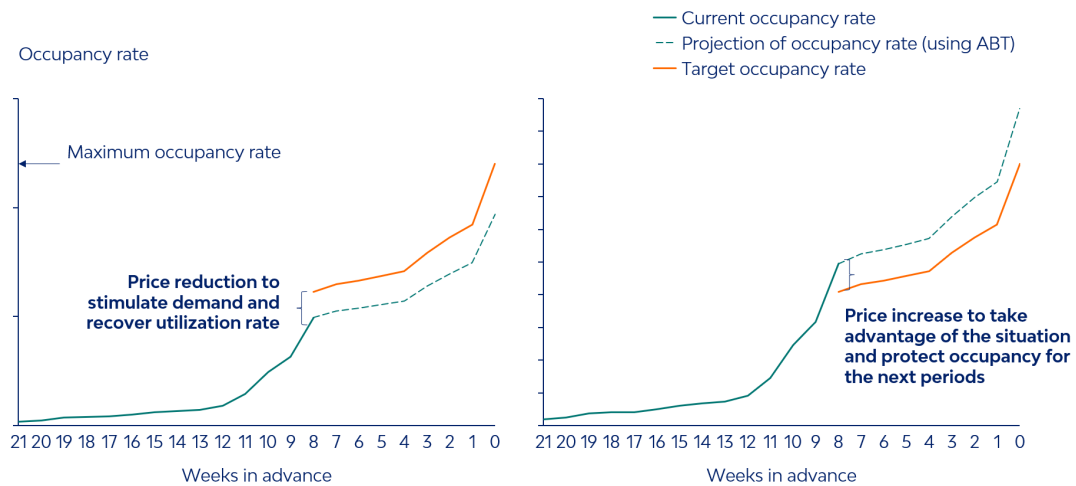


Figure 2.4: Increase or decrease price to encourage the target occupation rate

Figure 2.4 shows how the model should react to encourage the maximum occupancy rate. Let us suppose the forecast ABT curve shows that the company has a projection of an occupancy rate which is lower than the target. In that case, the model must reduce the price to stimulate demand and recover the utilization rate. The opposite would happen as well: if the occupancy rate is higher than the target, the model must increase the price to take advantage of the situation and protect its occupancy weeks in advance.

The pricing model will use the forecasting model to determine the impact of each price on the expected demand. Additionally, as expressed in subsection 2.4.2, it is crucial to apply the price elasticity concept to the RRM project. This concept is a way to measure the percentage change in demand when there is a price increase. Figure 2.5 shows a graph which illustrates the impact of price change on reservation change. It is possible to argue that reducing the price will lead to an increase in reservations with the elasticity curve.

In addition, it is possible to see two elasticity curves in figure 2.5: one that represents the prices higher than the competitors of the company in question and another for lower prices. In both cases, reducing the price from P1 to P2 increases client demand from R1 to R2. Another conclusion from the graph is that if the price applied is higher/lower than the competitors' prices, it will decrease/increase the demand and consequently the revenue per day. Finally, it only compensates for price change when the additional revenue generated by increased demand compensates for the loss of revenue by reducing the price.

Furthermore, the costs of repositioning must be subtracted from revenue by considering a repositioning or holding cost, depending on the pick-up station of the reservation, in order not to let the model make high-cost operations in the fleet planning process. To transfer vehicles between



Figure 2.5: Impact of price changes

stations or pools is costly for the car rental company. Therefore, the model only suggests moving a car when the repositioning or holding cost is assured, and the company would thus make more money by having that car in a different station or pool. If the model did not incorporate these costs, it would move more cars daily. That is why ensuring that the model has a cost sensitivity implemented is crucial.

Each day, the prices and repositioning suggested by the model must respect the existing fleet and the expected occupancy rate limits (given by the ABT curve). The prices suggested by the model are limited to a set of constraints: the relationship between vehicle groups, since more economical vehicles need to have a lower (or equal) price to a higher group; minimum and maximum price to apply per *vehicle group*, *LOR* - the minimum is based on the costs supported by the company in question; and the minimum price per *LOR*.

2.4.4 Fleet Repositioning

The information that will feed the fleet repositioning can come from two sources. First, in generating the scenarios, the availability of the fleet is manually input, being controlled by the planner during the budgeting/monthly review. However, during the daily execution, fleet availability information should come from the systems directly (data warehouse). It is vital to safeguard the correct updating of this information for the entire analysis period.

The repositioning of vehicles can be performed between pools and stations. The fleet repositioning between pools is a tactical decision and an operational one between stations. Both deployment values are validated at a pricing committee meeting. The team must block the validated value until the date defined by the team in that meeting. After validation, the repositioning period

should have a daily granularity, with the constraint of blocking changes between pools. By default, the repositioning between stations is not blocked for any subsequent period. However, if the company in question wants to block a certain period for transfers of vehicles between stations, it can be parametrizable by them. Figure 2.6 shows an example of how periods can be blocked.

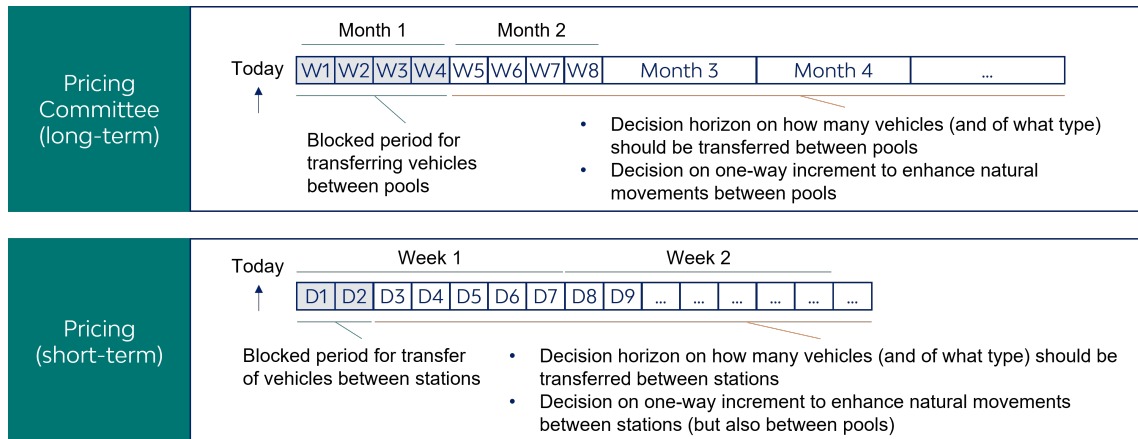


Figure 2.6: Fleet Reposition Decisions

In addition, the fleet repositioning model should also decide where to place the infleets - the new cars that the car rental company buys. The repositioning of the infleets is a tactical decision, and the result retrieved by the model is validated at the pricing committee meeting. At the end of this validation process, the committee team must define a blocking date specific to block the infleets reposition suggestions during the day of the meeting and the given date defined. Defining the block date in advance is essential to operationalize the transfers between stations and pools.

The fleet available at each moment for price optimization and visualization during planning should always come from the warehouse data. Therefore, the fleet available at any given time is equal to the sum of the fleet outlook - the number of vehicles the company predicts to have on a particular day in the future - plus the movements made by the RRM project plus the predicted one-ways that can happen in the future.

There are some constraints to be aware of:

- minimum and maximum limits should be set for the fleet of each station. The RRM platform needs to set the target fleet between the interval set;
- fleet repositioning should be limited to a maximum number of repositions per day and per pool (or global);
- the fleet and infleet deployment should consider the cost of repositioning and the blocked dates defined by the committee team.

Overall, the fleet and infleet deployment model proposes a set of car moves. Weekly, the fleet planning team validates the empty transfers for the next period, defines a new blocked date for manual repositioning, and creates the respective reservations manually. Monthly, the same team

validates and adjusts the infleets reposition. They define the next blocked date and manually adjust the distribution of the infleets on their platform.

2.5 Objectives

From the point of view of the car rental business, three key results are expected:

1. increased revenue, considering the price optimization at each moment;
2. enhanced analytical support for decision-making;
3. increased performance and focus on value-added tasks.

In sum, the project team needs to develop a fully integrated revenue management analytics platform, with granular optimization of crucial decisions to strategically and operationally support the company's pricing and capacity management. The team needs to reshape the company's vehicle pricing and repositioning processes in order to ensure more informed and supported decision-making. The research of this Master's thesis will focus on sections 2.4.3 and 2.4.4. Suggesting price and fleet deployment, implementing new prices, and controlling the information shared with the company in question will increase the monthly and annual revenue.

Chapter 3

Literature Review

This chapter has five main sections. The first section contextualizes the car rental business and its behavior during the past few years. The second section focuses on the revenue management and the relevant topics to consider when dealing with a problem within the car rental business sector. The third section discusses pricing management and fleet planning studies that rely on research operations and computational intelligence methodologies. The fourth section explores other solutions for the companies to get the attention of their customers even though they do not offer a scientific contribution. Finally, the fifth section presents a critical reflection on the problem by showing improvement opportunities and the Rental Revenue Management project requirements.

3.1 Contextualization

This section presents a contextualization regarding the car rental business. It starts by highlighting an overview of the sector, including a brief history and its behavior during the past few years. In addition, this section explains how the business is implemented in Portugal and analyzes the impact of Covid-19.

3.1.1 Overview of the Car Rental Business

A car rental is a business where customers rent a car, use it during their rental time, and only pay for the period they use it in. The companies in this business provide customers with a vast selection of vehicles at reasonable prices ([Research Dive, 2020](#)). This activity began as a simple business - the companies provided a standard car for a fixed rate per day, plus a fee per kilometer traveled. Then, it evolved into a multi-variant business as it is known today due to increased demand, technological evolution, and increased customer information ([Carroll and Grimes, 1995](#)).

The car rental companies in the market can be worldwide or local. The first includes well-known, reputable, and large corporations. The second one aggregates the smaller companies located in a local business. These smaller corporations are sometimes franchise businesses. This

project will only focus on the first type. However, the local companies can apply the same algorithm to their management plan.

This business model originated in the United States, and with Europe only adopting it at a later time. In the United States, the activity began over a century ago, but the car rental industry only noticed a significant increase in business volume after the 1930s and 1940s. In the 1950s, following the Second World War, and with the expansion of airlines, Europe expanded this practice. However, it only reached the same level and volume as the United States between the 1960s and 1970s (Carroll and Grimes, 1995).

The car rental sector has seen significant expansion in recent years, owing primarily to an increase in consumers' disposable income. Car rental companies such as Europcar, Hertz Corporation, and Sixt have also connected their fleet and pricing management with newer digital technologies to maintain and meet their company's objectives. These are the primary driving forces behind the markets (Research Dive, 2020).

Nowadays, how society sees a car differs from the traditional concept from the late twentieth century. Currently, society views the car as a mobility service rather than a valuable possession. There are many car rental options, which are available 24 hours a day, seven days a week (Quintanilha, 2020). By 2027, the worldwide car rental industry expects to reach \$1.48.129,4 million in revenue, up from \$92.252,4 million in 2019, a 15.5 percent increase. Innovating car rental services and strategic alliances among market competitors are likely to accelerate the growth of the car rental market (Research Dive, 2020).

Companies and people are increasingly opting turning to rent a vehicle on an irregular basis or for long periods rather than purchasing, maintaining, and continually charging them. Car rental serve customers that need to rent a car for several reasons, for instance when the car is being repaired or damaged, when people are on holiday and need a car to visit more places, or even when people or companies prefer to use rental cars rather than their own (Carroll and Grimes, 1995).

The companies in this sector define themselves by the choices they make. Price and fleet are essential factors that change the company's outcome. When designing an integrated model between pricing management and capacity control, there are several difficulties to consider such as which price strategy to apply to a reservation or uncertainties in business because there are always phenomena that the company cannot control, like cancellations or late returns. These will be explored later in this document.

A rental company includes multiple stations to serve a more significant number of individuals. Rental cars are delivered and picked up at these stations. Typically, the company places these stations in strategic zones like airports or urban areas. There are numerous car types to meet the diverse demands and interests of the customers. Companies provide customers with a selection of compact, mini-vans, and premium economy cars. It is possible to make a reservation with a car rental in advance or to rent a car on the spot (Wu, 2010).

When renting a car, the customer must select the vehicle category, the location from which it will be collected and returned, the length of time they will rent the vehicle for, and the type of

insurance that will be included. As the number of options grows, the complexity of the decisions grows as well. Due to this complexity, in the 1990s, the business implemented management and performance systems that allowed price and availability control (Carroll and Grimes, 1995).

There are characteristics of larger organizations that may bring them competitive advantages over local companies. Larger organizations have more stations and cars, which allows them to lower operating and maintenance expenses, allowing them to provide a more convenient, less expensive service that fits the needs of their customers (Wu, 2010).

3.1.2 Car Rental Business in Portugal

In Portugal, tourism has been the primary factor in increasing demand since 2018, so it directly influenced the growth of the activity (Quintanilha, 2020). During the years before Covid-19, tourism was the main driving force behind the growth of the car rental sector. Compared to 2018, in the 2019 year, there was a growth of 44%. To face this growth, in September 2019, rent-a-car companies bought 2.636 new cars, from which 2.339 were light passenger vehicles and 297 were light freight vehicles. According to the Association of Car Rental Industries (ARAC), tourism supports nearly 60% of the turnover of rent-a-car companies in Portugal. This sector is the largest buyer of new vehicles (Fleet Magazine, 2019).

In 2019, the rental companies had 110 thousand vehicles available, 10% more than in 2018. Despite the global increase, the number of companies registered in ARAC has barely increased in a decade, from 122 to 132. The number of people working in this sector has grown significantly (54%), from 3390 workers in 2008 to 5203 in 2018. In terms of average rental days, it went from six to nine. From 2020 to now, the sector has struggled to survive. The next section illustrates the impact of Covid-19.

3.1.3 Impact of Covid-19 on the Car Rental Market

The widespread COVID-19 pandemic has caused negativities in worldwide economies, with companies across industries feeling the brunt of the damage. Transportation services have almost stopped. In many locations, the challenges due to worldwide lockdowns are vast and untraceable. Due to global travel restrictions to control the spread of the virus, demand for rental cars has slowed at airports as a result of lower worldwide air traffic (Mordor Intelligence, 2022).

According to the World Tourism Organization (UNWTO), because of the spread of Covid-19, global tourism declined by 60-80% in 2020, leading to a revenue loss of \$910 billion to \$1.2 trillion. Furthermore, leading auto rental companies such as Avis are implementing cost-cutting measures such as a 20% reduction in fleet size and a 70% reduction in staff. Nevertheless, numerous car rental companies, such as Revv and Europcar, are stepping forward with strategic plans to assist society during this difficult time. Other factors, such as the risk of contracting a virus while using public transportation and public transportation constraints, also drive up the demand for rental car companies (Research Dive, 2020).

The sector has struggled in Portugal. According to ARAC, 2019 was the best year, and the expectations for 2020 were even higher. However, with the pandemic, the occupation rates of the vehicles decreased by 17%. The same tourism that propelled the vehicle rental industry forward is now forcing it to slow down. Rent-a-car companies had their best year ever in 2019, with a turnover of 700 million Euros. ARAC said that the value might have dropped by half (Sanlez, 2020).

Overall, Covid-19 impacted this sector dramatically. ARAC showed that occupation has declined, fleets have been reduced, and prices have decreased. Consequently, it is crucial for car rental companies to have a revenue management system that can integrate the pricing with the fleet management. An analytical model can help companies by doing the reverse of what the Covid-19 virus did. For instance, it can increase efficiency and fleet occupation with better price margins.

3.2 Rental Revenue Management

The main objective of this project is to dynamically control the availability of the fleet over time to maximize overall profits. To maximize the revenue, firstly, it is necessary to understand revenue management and the relevant topics (or settings) to consider within the car rental business. This section will explain these two topics further.

3.2.1 Revenue Management

Fleet and price management are typical problems in revenue management (RM), where the company aims to maximize revenue. The airline industry deregulation in the 1970s and 1980s made airline companies apply RM in their business (Haensel et al., 2011). After witnessing its successful implementation, organizations started implementing a revenue management system in their companies. As people may know, the car rental sector has similarities with other businesses like airlines and hotels (Guillen et al., 2019). Usually, it applies to industries with the following characteristics:

- **time-perishable inventory** - if a customer does not rent a car for this day, the company loses the opportunity to rent it;
- **time-variable demand** - there are peak periods and off-season periods. For example, on holidays, the number of reservation requests is higher because there is more demand from tourists;
- **low cost of sale as compared to fixed costs** - the cost to rent a car is low compared to the company's operating costs.

Since the early 1990s, the car rental industry has also given attention to revenue management (Anderson et al., 2004) since it is heavily dependent on its fleet and its price strategy. The decisions span across all strategic levels of the company (Oliveira et al., 2017). RM has evolved

into a universal and necessary methodological framework over the past few decades, encompassing techniques for actively managing demand and increasing profitability in different businesses (Klein et al., 2020). Parallel to that, revenue management has multiple applications like airplane, car rental, hotel, and home delivery business (Klein et al., 2020).

In Klein et al. (2020), the authors focus on methodological breakthroughs in choice-based theory and approaches for controlling availability over time. They assume that a general company offers cars, insurance and additional features. For this project, this must be an assumption as well. In addition to that, they also mention that production innovations, upgrades, overbooking, personalization, and risk-aversion are some generalizations of the basic RM settings discussed in the second part. Besides that, they only focus on a quantity-based RM rather than a price-based RM. For this project, it is required to focus on a price-based RM problem as well.

Revenue management encompasses three types of demand-management decisions: structural, pricing, and quantity (van Ryzin and Talluri, 2005). A quantity-based approach considers prices for a different product and then optimizes quantity allocation decisions. Littlewood's model is a famous example of a quantity-based RM framework (Lobel, 2020). A price-based approach tries to optimize price decisions. Overall, it employs inventory, capacity-allocation choices, or pricing as the principal tactical tool for managing demand. In that case, it is a quantity-based RM or price-based RM, respectively (van Ryzin and Talluri, 2005).

3.2.2 Relevant Topics to Consider

The following subsections' objective is to briefly explain each relevant topic to consider in the Rental Revenue Management (RRM) project.

3.2.2.1 Network

It is crucial to state that a company that intends to maximize the generated revenue for the whole rental network will not achieve such goals by considering an optimal control for a single rental station (Haensel et al., 2011). Almost all global car rental companies have rental stations across a large region (e.g., a country). Usually, some stations are aggregated in pools - groups of stations that share the same fleet (Oliveira et al., 2017). The repositioning of vehicles challenges capacity management. Sometimes there is a need to transfer vehicles between stations to maximize their utilization. It can also happen between pools.

Generally, pools are distinct from other administrative divisions (such as regional divisions), although they may overlap; as a result, they have some flexibility in terms of alteration and adjustment (Oliveira et al., 2017). The pool design may be subject to change during review times.

3.2.2.2 Fleet

The particular car placement within the pool is determined by its status, whether it is completing a reservation, idle at one specific station, or undergoing maintenance (Oliveira et al., 2017). A car rental fleet comprises many different vehicle types (rental groups). Firstly, the cars are split

into classes, for example, between economic or premium classes. Secondly, they are divided into combustion, hybrid, or electric, depending on their engine.

The company's strategic orientation primarily defines the size of the fleet and available capital (Oliveira et al., 2017). This decision is outside the scope of the project. Nevertheless, it is needed to buy new cars in some cases and increase the fleet size (Fleet Magazine, 2019). Some car rental companies tend to buy and sell the vehicle at a deadline defined for each vehicle to increase profitability.

Another important choice is distributing the existing fleet among the rental stations in a pool. This reposition is an essential consideration because most car rental operational expenditures are tied to idle fleets (Oliveira et al., 2017).

Overall, the decisions related to the fleet definition and utilization must remain flexible, as the available fleet changes due to pick-ups, returns, or substitutions within a station. The transfer between pools and stations is costly, so the RRM project must plan to reduce transfer costs (Oliveira et al., 2017).

3.2.2.3 Demand Periods and Reservations

Rental requests are accepted as long as the booking limit for the car set is not reached (Haensel et al., 2011). Typically, the rent-a-car in question takes reservations for as long as a year. During that window period, the unreserved cars represent a lost revenue opportunity (Anderson et al., 2004). The main attributes of a booking request are the desired vehicle group and segment, the pick-up date and station, and the return date and station (Oliveira et al., 2017). If the pick-up station is equal to the return station, the extra *one-way price* will not be charged. It is important to mention that rent-a-car companies can decide whether they will accept or reject the reservation request. The decision considers the strong relationship between demand and fleet occupation and the impact that occupation has on the company's operating efficiency and cost structure.

3.2.2.4 Uncertainty

There are often cancellations and late returns. Moreover, market demand is uncertain as well. Even with rental forecast methods, there is always a percentage associated with uncertainty. Although some businesses may fulfill all booking requests where the order is received as long as capacity is available, others manage demand by reserving capacity for the uncertainty (Oliveira et al., 2017).

Other factors add to the problem's unpredictability, significantly impacting fleet management. When it comes to purchasing new vehicles and retiring existing ones from the fleet, the costs and profits associated with these decisions are unpredictable, impacting the final decision. Uncertainties such as unplanned vehicle maintenance and repairs and delayed car returns significantly impact the fleet's availability (Oliveira et al., 2017).

3.2.2.5 Price Strategy

Car rental companies change the prices within fleet classes daily (Anderson et al., 2004). Price changes generate subtle changes in demand. The pricing against its competitors can be a decisive factor (Anderson et al., 2004). Generally, people tend to go with the cheaper option when comparing the same car to different rent-a-cars. Hence, some of these companies resort to strategies such as web-scraping to verify the competitor's prices monitor.

Price elasticity of demand is a measurement of the change in consumption of a product in relation to a change in its price (Anderson et al., 2004). Usually, the price elasticity makes it easy to understand how supply and demand for a vehicle change when its price changes. For instance, if a causal relationship can establish that a 10% discount will lead to higher revenue, the rent-a-car should lower the final price. Therefore, if the price change causes a substantial change in demand or vehicle utilization, the car rental company must consider it in its revenue management problem. The price elasticity formula is:

$$\text{Price Elasticity of Demand} = \frac{\text{Change in Quantity Demanded, \%}}{\text{Change in Price, \%}} \quad (3.1)$$

3.3 Methodology

This section will present different research findings and discuss if they apply to the Rental Revenue Management project and how to apply them. The first section will expose the difference between a predictive and a prescriptive model. Afterward, some improvement opportunities are presented based on the literature.

3.3.1 Predictive and Prescriptive Model

The Rental Revenue Management project covers a predictive and a prescriptive model. The first one includes predictions and forecasts. Predictive modeling analyzes current and historical data using machine learning algorithms and statistical techniques to predict future trends, behavior, and activity (Deshpande et al., 2019). The credibility of any prediction is based on the model's ability to interpret the past. Understanding previous behavioral patterns and future forecasting trends will not be enough to capitalize on business opportunities. Prescriptive analytics determines a possible solution (in some cases an optimal solution) or outcome among various options given the known parameters (Deshpande et al., 2019). It provides organizations with adaptive, automated, and time-bound operational action sequences. It provides answers to questions such as "What," "Why," "When," and, most importantly, "How." (Delen and Demirkan, 2013). The prediction must be good to warrant the prescription quality.

For instance, car rental management team A predicts that the vehicle's market search will be high. Based on the number of the available rental cars and the competitor's price, they define a revenue per day of X Euros. Nonetheless, after some time, the number of rented cars does not match what was expected. Thus, applying the prescriptive model suggests a range of actions and

the potential outcome of each action. For example, imagine a possible action detected by the prescriptive analyses to establish that a 10% discount will increase revenue. Company A should lower the final price. Providing such a discount, car rental A will meet the business goal of maximizing revenue (Liu, 2019).

Overall, the first part of the RRM project is the on-rent forecast (explained in subsection 2.4.1), which is a predictive model. In contrast, the focus of the Master's thesis is on two prescriptive models. One for the time and mileage pricing and fleet planning model (presented in subsection 2.4.3) and other for the vehicle reposition model (presented in subsection 2.4.4).

3.3.2 Pricing and Fleet Management

Currently, the car rental in question does not consider the relationship between vehicle repositioning and price management. It affects them significantly, and the problems described in this section relate to this lack of consideration. Because of this, the maximum vehicle utilization could be compromised, and consequently, so could maximum revenue.

The fleet has a dynamic behavior. When a customer rents a car, the available capacity decreases. Only when the customer returns does the vehicle have the capacity to be stored. Also, to meet the demand, the company generally uses tools, such as upgrades, vehicle transfers between stations and/or pools, or the temporary rental of additional products. The fleet is available at any given moment, and any related concerns such as pricing, vehicle purchases and decommissioning impact of the profitability of the car rental company (Guillen et al., 2019). The capacity to determine competitors' rates in real-time and a company's ability to quickly adjust its prices online are becoming crucial in the vehicle rental market, with prices therefore becoming more dynamic (Guillen et al., 2019).

A car rental company must decide on the number and type of vehicles in its fleet to meet demand while planning a new season. Because demand for rental cars is unpredictable and price-sensitive, fleet and pricing decisions are linked (Oliveira et al., 2019). In Oliveira et al. (2017), the article's authors introduced an integrated framework between fleet and revenue management. Firstly, their conceptual framework has the authority to define the pool design: number of pools, number of stations in each pool, and pool logistics center. The RRM project will not define the pool division. Instead it is an input, which is parameterized by the company in question. Secondly, for each pool, they decide on the number of vehicles per group (*fleet size/mix*) and per "age" (length of time the vehicles have been in the fleet), the number of vehicles in each station per group (*fleet deployment*), the potential earnings per vehicle (*pricing*), and if a vehicle is available or not (*fleet assignment*). Finally, the model can decide on which prices to charge per reservation type and which bookings to serve under the revenue management framework.

Continuing their investigation in Oliveira et al. (2019), the same authors developed a meta-heuristic framework based on a co-evolutionary genetic algorithm. It seeks to assist decision makers with various risk profiles in planning a season by providing valuable solutions to decide on how many and which groups of vehicles to have in the fleet in order to meet demand. Besides that, it also assists car rental companies by outlining their ability to deal with uncertainty when there is

limited information available. The RRM project must incorporate a method like this one, since it is a requirement to provide decision-makers with a representative variety of possible scenarios.

In Soares (2020), the author follows a similar approach as the previous authors. He created a Variable Neighbourhood Search (VNS) metaheuristic. Nevertheless, the results were surprising. The author demonstrates how important it is to select an appropriate metaheuristic, especially in the RRM project where finding a globally optimal solution is complex. In addition, he presents a mathematical model for a rent-a-car business published earlier by Oliveira et al. (2018b). It describes all decisions regarding fleet management and price strategy for the different types of reservations. The goal of this model is to maximize the company's profit.

The paper, published in the *INFORMS Journal On Applied Analytics* (Guillen et al., 2019), proposes a revolutionary decision-making system. The system approaches forecasting, discrete-event simulation, and optimization to integrate revenue and capacity management. Up to six months ahead of time, it forecasts future demand for Europcar's vehicle fleet, optimizing fleet capacity management (vehicle acquisitions, decommissions, and transfers). It also allows Europcar to maximize its revenue management and rental pricing strategies, considering its present fleet, competitive information, and predicted vehicle demand.

Furthermore, unlike previously proposed solutions, Europcar considers free upgrades (Guillen et al., 2019). When a transfer or vehicle acquisition is more expensive than an upgrade in meeting demand, the latter solution is automatically implemented via the proposed solution (Guillen et al., 2019). This feature is directly modeled in the costs included in the objective function.

3.3.2.1 Pricing Management

Rental payments account for the majority of revenue (Lazov, 2017). Nevertheless, based on uncertain events, the car rental business has some penalties and fees that the customer needs to be aware of (Guillen et al., 2019), such as:

- Late Return - a customer returns a car later than the arranged check-out date. The customer will face an additional penalty fee;
- One-way - the one-way fee is the cost of picking up a car in one station and returning it to another;
- Paid Upgrade - a customer agrees to pay an additional charge to rent a car of a higher class before the rental period begins (i.e. an upgrade);
- Extension - the length of a rental agreement can be extended if the customer agrees to pay an additional fee;
- Cancellation - some companies might charge a fee if a customer cancels a reservation;
- No Show - the customer does not show up to claim the reservation.

The research in [Kourentzes et al. \(2019\)](#) shows how to improve a company's revenue by 0.5% - 1.4%. These empirical results are based on data from a car rental company. Increases in revenue on this scale are noteworthy due to low marginal costs. They propose three key improvements. Firstly, a trended exponential smoothing technique generally captures better non-homogeneous booking curves. Secondly, a novel method for optimizing model parameters improves the performance of the proposed methods. Finally, they optimize smoothing parameters over groups of constrained booking curves to improve robustness without sacrificing information.

The authors in [Lei et al. \(2017\)](#) started by examining various elements holistically. Then, they developed a multi-task learning-based regression approach to predict the demand of rental cars. Their examination is based on policymakers performing the "what if" analysis. This method may understand not only essential traits but also relationships between regions. This approach forecasts the future demand, and as such it is a predictive method.

The study by [Yu et al. \(2018\)](#) reveals that per-use rental services interact with sales when vertical differentiation occurs by investigating the optimal pricing problem for per-use rental services and sales. The sheer difference has an impact on a company's profitability. Their findings also reveal that, in per-use rental services, delivering products of relatively high quality is highly profitable for product categories with a substantial pooling effect or when there are high company-side benefits from ownership. Even though this study is not specific to a car rental business, it has the same profit maximization problem as the Rental Revenue Management project.

The authors in the article by [Li and Pang \(2017\)](#) formulated, over an unlimited horizon, a dynamic booking control model as a discrete-time stochastic dynamic program. Due to the intractable nature of this formulation, they offer a decomposition strategy to address the computational issue. Firstly, they use an Approximation Dynamic Programming (ADP) heuristic, which uses the value functions of the decomposed problems to approximate the value functions. However, because the best booking limits are achieved by solving a nonlinear program in the periodic-review context, such an ADP approach is only applicable to small-scale situations. The second heuristic is constructed directly from the optimal booking limits applied to a real-world problem. Overall, the results show that both heuristics are better than the Probabilistic Non-Linear Programming (PNLP) approach, which is commonly used.

3.3.2.2 Fleet Management

Several connected sub-problems compose fleet planning at the strategic, tactical, and operational decision levels (see [3.1](#)), all of which are managed by many decision-makers. Those decision levels can be applied to all transportation systems rather than only in the car rental business. The fleet planning components, such as fleet sizing/composition, fleet deployment, fleet inventory control, and empty vehicle relocation problems, have a significant interaction ([Baykasoğlu et al., 2019](#)). In other words, fleet allocation and vehicle inventories on the logistics network can substantially impact the flow within a pool, which closely links to fleet size optimization.

Fleet expansion/reduction strategies have to deal with vehicle acquisition (buying new or used cars), the sales of second-hand vehicles, car rentals, and the disposal of end-of-life cars ([Tran and](#)

Haasis, 2015; Tsai et al., 2012). These decisions are vital for the company's policymakers. As the article by Oliveira et al. (2017) highlighted, the acquisition and reduction of number of vehicles to and from a pool fleet are flexible. Vehicle allocation and reallocation issues aim to assign the various types of cars in the fleet to a set of routes on the transportation network.

Fleet planners should also identify, when required, which types of vehicles are necessary, how many are needed, and whether they are required to replace or expand the fleet size. In addition to focusing on age/mileage, repair/maintenance expenses, and the residual value of the vehicles, an economic life cycle analysis should be used to make fleet replacement decisions (Baykasoğlu et al., 2019).

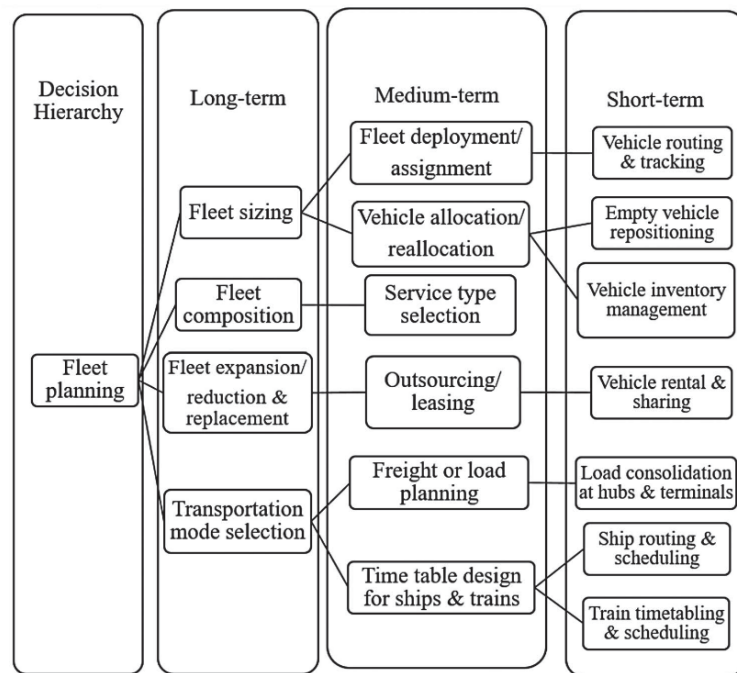


Figure 3.1: Strategic, Tactical, and Operational Decision Levels (Baykasoğlu et al., 2019)

Throughout Oliveira et al. (2014), the authors discuss three crucial decisions related to vehicle transportation. Firstly, *fleet sizing*, deciding on how many vehicles should be available in a specific station at a particular period. Secondly, *fleet deployment*, which is related to the short and mid-term decision of how many vehicles the company has to move between stations at a specific time. Thirdly, *fleet assignment*, deciding on which vehicle should fulfil each reservation, and whether or not to take the cost impact of the empty transfers into account. A relax-and-fix method is implemented to solve this model. This method is a constructive heuristic, which means that each iteration solves a partial problem by optimally fixing a set of integer variables while taking previous iterations' variables into account. This procedure aims to find a good solution rather than an optimal solution within a reasonable amount of time. The objective function of the heuristic is to maximize profit, and the authors tackled the problem as a network flow one. This approach improved 33% when compared to the company's current implementation.

Aside from the solutions explored above, in the article by Fontem (2016), the author analyzes the optimal time to sell a car through a stochastic model application. Selling a car with low mileage prevents a significant drop in the car's residual market value. On the other hand, delaying the sale of a rental car is problematic because the more rental customers use a vehicle, the more its mileage will increase.

The authors in Guillen et al. (2019) modeled the fleet management problem as a linear programming problem, by solving it with a meta-heuristic algorithm based on genetic algorithms. At the same time, the algorithm optimizes the fleet's composition and utilization. Their approach considered almost all possible scenarios in the real world. Apart from all of the decisions presented here, the authors included a "decommission optimization" where the model defines when Europcar should remove a vehicle. The optimizer even determines if paying additional fees for keeping it longer than expected is profitable or not. Furthermore, none of the previously proposed solutions account for the time needed to generate a solution. Indeed, the Europcar solution is generated within a few hours, providing the company with a tool to use in its day-to-day operations.

A different approach is presented in Haensel et al. (2011), where the research focuses on a stochastic programming approach for a network with flexible capacities. The authors' method optimizes the fleet distribution and the capacity control at the station level. Nevertheless, their solution is oversimplified because they only consider one car type and only one rental per car, per day. Furthermore, they overlook cancellations and late returns.

The problem of fleet management is also a question in matters of the electric vehicles (EVs) sharing service. The paper by Jiao et al. (2017) proposes two methodologies for developing robust fleet repositioning plans that reduce rental demand uncertainty and imbalances. Like the car rental business, this service allows the clients to pick up the electric vehicles and then return them to the original stations (round-way EVs) or to another station (one-way EVs). More specifically, they use a dynamically assignable robust optimization model (AROM) and a joint chance constraints model (CCM) to assign and distribute EVs. Both models make decisions for future stages based on information about the realized demand from previous periods. The model's parameters and decision variables are like the variables the RRM will use for the fleet management model. The research concludes that both AROM and CCM improve the redistribution significantly and offer a stable solution for the company's service. The AROM performs at a lower cost than CCM when comparing both solutions.

Julian Pachon, in Pachon et al. (2006), exhibits a sequential hierarchical structure composed of three phases for the entire fleet planning process: (i) pool segmentation, (ii) strategic fleet planning, and (iii) tactical fleet planning. As previously mentioned, the RRM project will not handle the pool segmentation. In the second phase, the author determines the total fleet size for each pool over a long period. This phase involves decisions that the RRM project will not tackle, such as fleet acquisitions and returns to and from the contracted car manufacturer. The last phase is the daily managing of the business, similar to what has been presented during this section.

3.4 Other Solutions

This section aims to explore existing solutions and prototypes on the market to apply in the future. Even if the solutions do not have a scientific contribution, they help car companies develop further interaction with the customer and assist with the pricing and fleet management.

Firstly, in [Bakar and Yuspani \(2020\)](#), the authors developed a prototype for an online system where customers can book a car on their website. The system has six sequential steps: plan, analyze, design, development, test, and evaluate. The prototype aims to have many features already implemented in other e-services and online websites. They help the customer by providing a user-friendly booking system, but they also help the rent-a-car company face the demand. The author's work is more related to reservation management, and because of that, his proposed solution cannot be applied to Rental Revenue Management project.

In the same direction, the article by [Kesrarat et al. \(2017\)](#) highlights the development of an intelligent matching system. It primarily assists the process of vehicle matching in a search function to provide the most satisfying result for customers and avoid rejection of unavailable cars for rent by suggesting alternative products that meet the customer's requirements. The "Smart Matching for Car Rental" project does not consider either fleet management or pricing management. A similar investigation is presented in [Huamani-Avendaño et al. \(2019\)](#), where the main objective is to identify the ideal car for a customer according to the selected characteristics. The solution includes building a Data Warehouse that helps to improve the analysis process. Once the system generated the data, they applied a Random Forest algorithm to find the best products for the customer.

The solution from [Costa \(2019\)](#) includes designing and implementing an integrated system that collects all necessary prices, fleet, and occupation data from the company's many files and stores it in a database. It allows the connection to other tools for data analysis afterward.

Finally, the study by [Rani et al. \(2021\)](#) uses a car rental simulation technique to structure and organize car fleet management in order to optimize the number of cars in the ABC Car Rental Company. Their principal objective is to find alternatives for its low vehicle utilization. The car rental company will use the outcomes of this study to help them manage the number of cars they have in their fleet. The car rental company needs to optimize its fleet to improve its business operations and effectively manage its car inventory.

3.5 Critical Reflection

The literature review helps to create a base to tackle the Rental Revenue Management project. The first section enables an understanding of how the car rental business is structured and how it has behaved since its creation. The second section presents revenue management and identifies relevant topics to consider in the project. Finally, the third section illustrates the leading study which the project will follow.

Firstly, car rental companies have the capability and potential to combine their dynamic pricing and capacity decisions. Demand is influenced by pricing and capacity, which translates to fleet

size, acquisition planning, and fleet deployment across the network, which can be employed to meet this price-sensitive demand (Oliveira et al., 2018a). The rent-a-car company in question does not currently have an automatic state-of-the-art model for fleet and price management.

Secondly, the project must follow a quantity-based revenue management approach and price-based approach. After reviewing the literature from other authors, some improvement opportunities were found to help the company. The network must be well-designed to maximize vehicle utilization in this type of business. The fleet definition and utilization decisions must remain flexible since pick-ups, returns, or substitutions within a station constantly change the available fleet. Transfer costs between stations and pools are high. Additionally, the price within fleet classes adjusts daily. Because of this, the project must implement the concept of price elasticity.

Thirdly, a review of the remaining literature provides with the knowledge of different proposed solutions for the car rental business. The RRM project will rely on the ideas shared in theoretical to practical implementations. It will cover a prescriptive model to coordinate pricing and fleet management integration.

Last but not least, the Rental Revenue Management project must consider the strategic, tactical and operational decision levels. The main objective is to answer some questions, such as (Oliveira et al., 2018a): “How many cars should there be in the fleet?”; “When should they be acquired or sold?”; “How should they be distributed among rental stations and pools over time?”; “How should these rentals be priced?”.

Overall, the project intends to tackle some current challenges by developing crucial tasks:

- integrate fleet size with price management;
- improve the analytical and predictive capacity;
- improve the reactivity and frequency of reviewing assumptions and pricing decisions;
- propose a fleet size;
- propose vehicle repositions between stations and pools.

Chapter 4

Time and Mileage Pricing and Fleet Planning

This chapter explores the optimization model related to time and mileage pricing and fleet planning. The first section explains the company's parameters and some crucial constraints that the model must consider. The second highlights the mathematical model by presenting the sets, subsets, indices, parameters, decision variables, constraints, objective function, and the constraints' description. The third section explains how the model is executed based on a relax-and-fix approach and why it was needed to add gaps to the mathematical model. It also presents how the performance issues were resolved. Finally, the last section focuses on the operational interface where the results are available for validation.

4.1 Context, Assumptions, and Inputs

The optimization model works with two major decision groups: one which suggests a price for each day, based on the vehicle group, station, and LOR; and another decision which suggests the daily fleet size, based on the vehicle group, and station. The model will be multi-period to ensure that a decision made on the previous day also affects the following day. This model will not differentiate price according to business segment, point of sale, sales channel, or payment type.

Additionally, the model decisions per station will be aggregated to their respective pool - a set of stations in a particular region defined by the company. For instance, a pool could be located in the North of Portugal, so all stations that belong to the North will be aggregated to this pool, independent of its location. Due to the fact that there are stations inside a pool that share the same parking lot, the setting of the price and fleet to be assigned to a station s must be considered simultaneously for its respective pool p . In those cases, the maximum available fleet for station s is equal to the maximum available fleet for pool p . The sum of the available fleet inside a pool must not be higher than the maximum bound defined by the company.

Moreover, the car rental company has some limitations on the structure of the prices charged. Their prices are defined through unique codes (which may also be called price steps). The company establishes a code Y with a specific price for a specific vehicle group, LOR and station. From one code to another, there is a price step. The model maps the recommended price based on the different price steps and the original code defined by the company in question. For example, the company specifies that the three following codes, Y_1 , Y_2 , and Y_3 , have a price of six, ten, and fourteen Euros, respectively. The model concludes that the price to be considered should be fourteen Euros, code Y_3 . The car rental company will implement this new code and replace the original one. When comparing two different vehicle groups, they know that if they have implemented an equal code (not equal prices), the vehicle groups are at a similar level. This processing reduces the necessary steps and, consequently, the time required for the operation of defining prices.

Regarding the fleet, on each day, the prices and fleet size suggested by the model must respect the existing fleet and the expected occupancy rate bounds (given by the ABT curve). Additionally, the model must consider the maximum operational capacity per station, which defines the maximum number of bookings per day.

The definition of the fleet to be allocated to each station shall be considered simultaneously with the pricing. Both decisions are dependent on each other. The car rental company parameterizes maximum and minimum fleet limits per station and pool. The maximum number of repositions per period should consider that a part of the repositions can be performed through one-way rentals. The model receives the fleet outlook as input, which is the total number of vehicles to be rented at a certain point in time.

An additional relevant concept for this model are the types of sets of days. There are two different sets: reservations and pick-up days. A particular reservation with pick-up day u and LOR l can have multiple reservation days r . For instance, a reservation which has a LOR 2 and is being picked up today, will have two reservation dates: today and tomorrow. To convert pick-up days into reservation days, a parameter $\alpha_{r|lu}$ was created, which is a boolean parameter that indicates whether a particular reservation with LOR l and pick-up day u is active or not on the reservation day r .

Furthermore, it is essential to understand how the model reaches its occupation goal. Figure 4.1 represents the occupation rate behavior during the weeks in advance. Firstly, the on-rent forecast model predicts the occupation rate. Secondly, those predictions are distributed in advance based on the ABT curve. Thirdly, the model divides the predictions that should occur until a given date (represented by the confirmed reservations in the chart, orange curve) and the predicted reservations that can happen in the future (represented by the predicted reservations in the chart, green curve). Once the division is clear, the model adds the predicted reservations to the confirmed reservations for a final prediction. For instance, imagine that the model is in week 8. It knows the real occupation rate and the predicted rate curve for that week. In order to reach the target fleet occupation for the weeks in advance, the model analyzes multiple forecasts based on several different prices, and chooses the sum of the future forecast plus the confirmed reservations that are closer to the occupation goal.

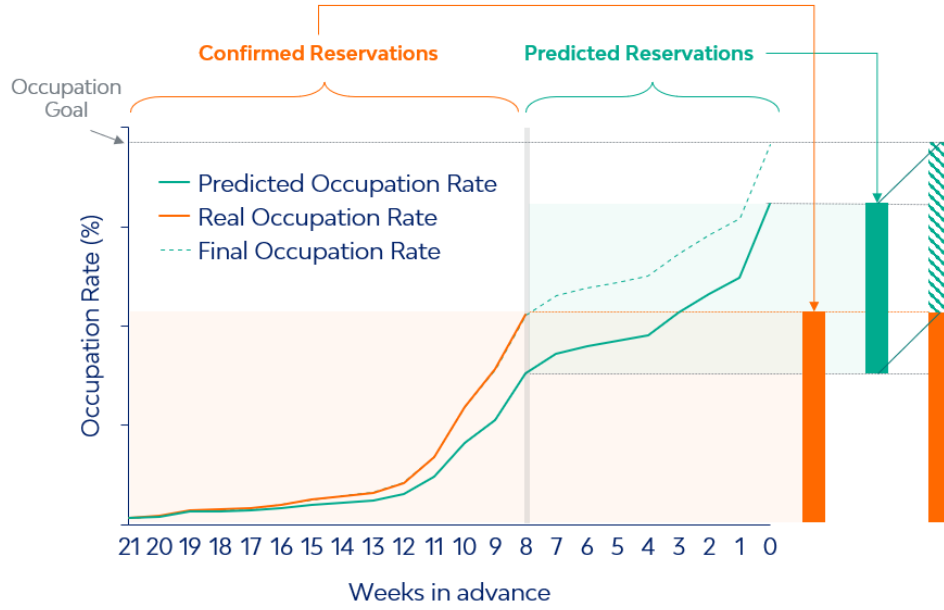


Figure 4.1: Occupation rate behavior during weeks in advance

Last but not least, the available fleet should consider in advance the number of one-ways that can still happen in the future. A script was created to estimate two probabilities considering the last twelve months. One probability of a one-way of vehicle group v from origin s to destination b happening, and another probability of a drop-off (term given to the returning of a car) in station s of a vehicle group v happening. Only the drop-off probability varies depending on the day of the week. It is expected that the possibility of a weekend drop-off is higher than a drop-off during the week. When the estimations are calculated, the script will tell us how many drop-offs happen from a group of reservations and how many of them are one-ways. This group of reservations includes all the reservations of occupied cars, whether on pick-up day, drop-off day, or somewhere in the middle. For instance, in station s on reservation day r , there are 50 confirmed reservations, and both probabilities are of 25%. Therefore, the number of drop-offs is 12, and the number of one-ways is 4. From those 4 one-ways, the script (based on the twelve predictions made) knows that 50% are from s to b . Finally, the script will recalculate the fleet outlook by adding the number of entries (2) in station b and subtracting the number of exits (2) in station s . Overall, the available fleet parameter considers the possibility of one-ways happening in advance by adding the number of entries or subtracting the number of exits in station s .

4.2 Mathematical Model

This section explores the mathematical model by presenting the sets, subsets, indices, parameters, decision variables, constraints, objective function, and the constraints description.

Sets and Subsets

K	Price Steps
L	LORs
V	Vehicle groups
P	Pools
S	Stations
$S_p \subset S$	Subset of stations that belong to pool p
R	Set of reservation days in the time horizon
U	Set of pick-up days in the time horizon

Indices

k	Indices for price steps
l	Indices for LORs
v	Indices for vehicle groups
p	Indices for pools
s	Indices for stations
r	Indices for reservation days
u	Indices for pick-up days

Parameters

AF_{vsr}	Available fleet of the vehicle group v in station s on reservation day r . The available fleet is the fleet outlook minus the fleet that is currently unavailable (due to maintenance or car accidents) and rented, minus/plus the number of one-way exits / entries that will occur in station s .
CPD_{vsluk}	Costs per day for vehicle group v in station s with LOR l and price step k , on pick-up day u . The cost per day is the sum of the holding, running, and other costs for the car rental company. The holding cost only depends on the vehicle group v and station s . The running cost depends on all indices. The other cost is a fixed rate.
CP_p	Cost to reposition fleet on pool p . This cost is a weighted average value, which considers the cost to transport a car from any pool to pool p .
CS_s	Cost to reposition fleet on station s . This cost is a weighted average value, which considers the cost to transport a car from any station to station s .
FP_p	Minimum fleet for pool p .

FS_s	Minimum fleet for station s .
M_1	High enough coefficient, related to the fleet repositioning cost.
OC_s	Operational capacity in station s , which defines the maximum number of bookings for station s .
OG_{vsr}^-	Occupation minimum bound for vehicle group v in station s on reservation day r .
OG_{vsr}^+	Occupation maximum bound for vehicle group v in station s on reservation day r .
OP_{vsu}	Occupation goal for vehicle group v in station s on pick-up day u . The variable domain is in the range between 0 and 1.
OR_{vsr}	Occupation goal for vehicle group v in station s on reservation day r . The variable domain is in the range between 0 and 1.
PD	Maximum number of daily movements to transfer vehicles inside a pool.
PF_p	Maximum available fleet for pool p .
RC_{vslu}	Number of confirmed reservations by the company for vehicle group v in station s , with LOR l , on pick-up day u .
RP_{vsluk}	Number of predicted reservations for vehicle group v in station s , with LOR l and price step k , on pick-up day u .
RPD_{vsluk}	Revenue per day for a reservation with LOR l and price step k of a vehicle group v in station s , on pick-up day u .
SF_s	Maximum available fleet for station s .
TD	Maximum number of daily movements between stations.
VH_{vslu}	First vehicle group immediately higher in comparison to v in station s with LOR l , on pick-up day u .
VL_{vslu}	First vehicle group immediately lower in comparison to v in station s with LOR l , on pick-up day u .
α_{rlu}	Boolean parameter that indicates whether a certain reservation with LOR l and pick-up day u is active or not on reservation day r .
δ_r	Boolean parameter that indicates if transfers between stations on reservation day r are allowed.
ρ_r	Boolean parameter that indicates if transfers between pools on reservation day r are allowed.

Decision Variables

d_{vsluk}	Price step k selected for vehicle group v in station s , with LOR l , on pick-up day u . Each price step has a given revenue per day price. If price step k is selected, then $d_{vsluk} = 1$. If not, $d_{vsluk} = 0$.
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- f_{vsr} Fleet level for vehicle group v in station s , on reservation day r . This decision variable will be an input for the vehicle repositioning model. This decision variable defines the target fleet of vehicle group v to have in station s on reservation day r .
- w_{vpr} Fleet decrease for vehicle group v in pool p , on reservation day r . This variable represents the number of vehicles v that the car rental company should remove from pool p .
- x_{vpr} Fleet increase for vehicle group v in pool p , on reservation day r . The variable represents the number of vehicles v that the car rental company should bring to pool p .
- y_{vsr} Fleet decrease for vehicle group v in station s , on reservation day r . This variable represents the number of vehicles v that the car rental company should remove from station s .
- z_{vsr} Fleet increase for vehicle group v in station s , on reservation day r . The variable represents the number of vehicles v that the car rental company should bring to station s .

Mathematical Model

$$\begin{aligned}
 \max \sum_{v \in V} \sum_{s \in S} \sum_{r \in R} & \left[\sum_{l \in L} \sum_{u \in U} \sum_{k \in K} (d_{vsluk} \times RP_{vsluk} \times (RPD_{vsluk} - CPD_{vsluk}) \times \alpha_{rlu}) \times (1 - OR_{vsr}) \right] \\
 & - \sum_{v \in V} \sum_{s \in S} \sum_{r \in R} ((M_1 \times \delta_r + CS_s) \times z_{vsr}) \\
 & - \sum_{v \in V} \sum_{p \in P} \sum_{r \in R} ((M_1 \times \rho_r + CP_p) \times x_{vpr})
 \end{aligned} \tag{4.1}$$

$$\sum_{k \in K} d_{vsluk} = 1 \quad \forall v \in V, s \in S, l \in L, u \in U \tag{4.2}$$

$$\begin{aligned}
 AF_{vsr} - AF_{v,s,r-1} + f_{v,s,r-1} + z_{vsr} - y_{vsr} &= f_{vsr} \quad \forall v \in V, s \in S, r \in R \tag{4.3} \\
 \sum_{s \in S_p} (AF_{vsr} - AF_{v,s,r-1}) + f_{v,p,r-1} + x_{vpr} - w_{vpr} &=
 \end{aligned}$$

$$\sum_{s \in S_p} f_{vsr} \quad \forall v \in V, p \in P, r \in R \tag{4.4}$$

$$\sum_{s \in S} AF_{vsr} = \sum_{s \in S} f_{vsr} \quad \forall v \in V, r \in R : \delta_r = 1 \tag{4.5}$$

$$FS_s \leq \sum_{v \in V} f_{vsr} \quad \forall s \in S, r \in R : \delta_r = 1 \tag{4.6}$$

$$FP_p \leq \sum_{v \in V} \sum_{s \in S_p} f_{vsr} \quad \forall p \in P, r \in R : \delta_r \times \rho_r = 1 \tag{4.7}$$

$$\begin{aligned}
& \sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \\
& (1 - OR_{vsr}) + \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) \geq \\
& f_{vsr} \times OG_{vsr}^- \quad \forall v \in V, s \in S, r \in R \quad (4.8)
\end{aligned}$$

$$\begin{aligned}
& \sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \\
& (1 - OR_{vsr}) + \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) \leq \\
& f_{vsr} \times OG_{vsr}^+ \quad \forall v \in V, s \in S, r \in R \quad (4.9)
\end{aligned}$$

$$\begin{aligned}
& \sum_{v \in V} \left[\sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \right. \\
& \left. (1 - OR_{vsr}) \right] + \sum_{v \in V} \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) + \\
& SF_s \geq \sum_{v \in V} f_{vsr} \quad \forall s \in S, r \in R \quad (4.10)
\end{aligned}$$

$$\begin{aligned}
& \sum_{v \in V} \sum_{s \in S_p} \left[\sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \right. \\
& \left. (1 - OR_{vsr}) \right] + \sum_{v \in V} \sum_{s \in S_p} \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) + \\
& PF_p \geq \sum_{v \in V} \sum_{s \in S_p} f_{vsr} \quad \forall p \in P, r \in R \quad (4.11)
\end{aligned}$$

$$\sum_{v \in V} \sum_{s \in S} z_{vsr} \leq TD \quad \forall r \in R \quad (4.12)$$

$$\sum_{v \in V} \sum_{s \in S_p} z_{vsr} \leq PD \quad \forall p \in P, r \in R \quad (4.13)$$

$$\begin{aligned}
& \sum_{v \in V} \left[\sum_{l \in L} \sum_{k \in K} (RP_{vsluk} \times d_{vsluk}) \times (1 - OP_{vsu}) \right] + \\
& \sum_{v \in V} RC_{vslu} \leq OC_s \quad \forall s \in S, u \in U \quad (4.14)
\end{aligned}$$

$$\begin{aligned}
& \sum_{k \in K} d_{v_L sluk} \times RPD_{v_L sluk} \leq \\
& \sum_{k \in K} d_{vsluk} \times RPD_{vsluk} \quad \forall v \in V, v_L = VL_{vslu}, \\
& s \in S, l \in L, u \in U \quad (4.15)
\end{aligned}$$

$$\begin{aligned}
& \sum_{k \in K} d_{vsluk} \times RPD_{vsluk} \leq \\
& \sum_{k \in K} d_{v_H sluk} \times RPD_{v_H sluk} \quad \forall v \in V, v_H = VH_{vslu}, \\
& s \in S, l \in L, u \in U \quad (4.16)
\end{aligned}$$

$$d_{vsluk} \in (0, 1) \quad \forall v \in V, s \in S, l \in L, u \in U, k \in K \quad (4.17)$$

$$f_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.18)$$

$$w_{vpr} \in \mathbb{R}_0^+ \quad \forall v \in V, p \in P, r \in R \quad (4.19)$$

$$y_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.20)$$

$$x_{vpr} \in \mathbb{R}_0^+ \quad \forall v \in V, p \in P, r \in R \quad (4.21)$$

$$z_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.22)$$

The model's objective is to maximize the car rental company's revenue. The first part of objective function 4.1 calculates all the future profits by subtracting the cost per day from the revenue per day and then multiplying it by all the reservations that will happen in the future. This last value equals all the predicted reservations multiplied by one minus the occupation goal for each vehicle group v , station s , and reservation day r . For instance, see figure 4.2, and suppose that the model is in advance on day 20. Additionally, it is known that on day 0, the total predicted reservations (100%) would be filled (represented by a blue dot), on day 12, the occupation goal is 55% (represented by an orange dot), and on day 4 it is 80% (represented by a yellow dot). So future reservations that can deliver future profits are 45% on day 12 and 20% on day 4. Therefore, one minus the occupation goal equals the percentage of the total predicted reservations that can happen in advance until day 0. Finally, the second and third part of the objective function considers the cost of repositioning vehicles between stations and pools, respectively.

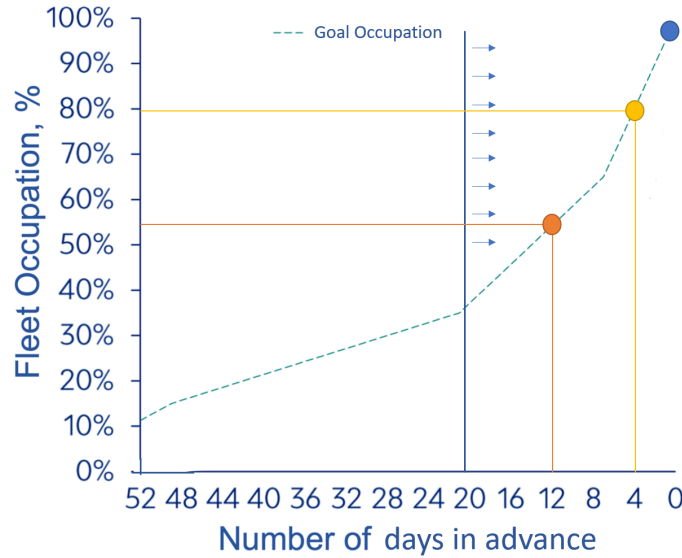


Figure 4.2: First part of the objective function explanation

The model starts with 4.2 by choosing only one price step. Then, the constraints shown in 4.3 and 4.4 indicate the fleet size in station s or pool p on day d must equal all transfers made until a given date plus the available fleet on that day. Constraints 4.5 ensure that the model respects the

global fleet. Apart from this, the model must respect maximum and minimum limits in stations and pools - constraints 4.6 and 4.7 guarantee the minimum station and pool fleet, respectively. 4.8 and 4.9 are for the model to respect the minimum and maximum station occupation goals. 4.10 and 4.11 respect the maximum available fleet for each station and pool, respectively.

Furthermore, there are transfer limits. Constraints 4.12 safeguard that the total number of daily movements does not exceed its limit. The same happens for pool movements in constraints 4.13. Also, the model must respect the daily operational capacity of a station s with constraints 4.14.

Finally, a higher vehicle group must have a price higher than a lower vehicle group. The last two group of constraints, 4.15 and 4.16, respect the lower and the higher vehicle groups' price orders.

4.3 Solution Approach

This section addresses the solution method developed to tackle the challenges in the implementation of the model. The first subsection approaches a relax-and-fix solution for complexity management. The second subsection presents the transformations required to the model to handle issues in real data. The third subsection presents a new mathematical model with the adjustments mentioned before. Finally, the last subsection discusses the performance issues that the model had and how they were fixed.

4.3.1 Complexity Management

One of the car rental company's requirements was to set up daily new prices for the next twelve months. Due to the processing time required, the optimal solution was deemed impractical. The solution proposed is to implement the model based on a relax-and-fix method. The lengths of rentals will be considered for up to 30 days (parameterizable) for complexity management, defining each model's periodic moment. So, the whole model is divided into 24 smaller models, henceforth called submodels. Each submodel has the first fifteen days fixed, with the remaining fifteen as a variable, which means the first fifteen have a decision variable binary.

In contrast, the second part is continuous. This way, it is ensured that the following month will be dependent on the previous month. The background idea is to create a similar approach to a relax-and-fix solution. The solution fixes the first fifteen days and relaxes the remaining fifteen. The fixed part is optimized in each submodel, and the relaxed part is only optimized in the following submodel. The following image helps to demonstrate this concept. The blue rectangle is the fixed part, and the white one is the relaxed part.

By relaxing and fixing the model, the parameters related to reservations need to be adapted. In section 4.2, a parameter RF_{vslu} , which represents the fixed reservations, must be added. Figure 4.4 helps understand how the model pre-processes these parameters. Firstly, parameter RC_{vslu} is the number of reservations guaranteed to happen. This value comes directly from the tabular car rental. Secondly, one of the outputs from the forecast process is the reservations predicted to happen, the parameter RP_{vsluk} . Every time a new iteration (represented in figure 4.3) starts, the

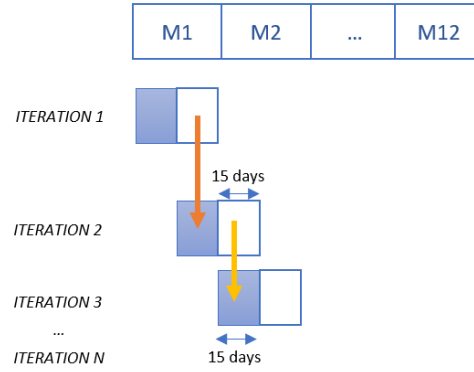


Figure 4.3: Model Execution

model will fix all predicted reservations that have a pick-up starting on the current date and fifteen days after this, which is shown in figure 4.4. Parameter RF_{vslu} represents the reservations that fulfill that condition (represented in blue). The RP_{vsluk} represents the other reservations (shown in green) that do not fit the condition and only start after the final date to be fixed. This date (represented by DEF) is equal to the sum of the date when the submodel starts plus fifteen days thereafter.

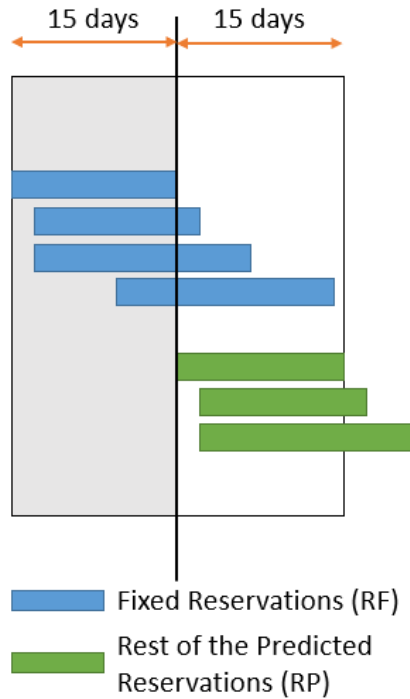


Figure 4.4: Explanation of the Fixed and Unfixed Reservations

4.3.2 Data Issues Management

After running the mathematical model presented in section 4.2 with real data from the company, the output message was “*Infeasible - No solution exists*”. While researching into what could be wrong, there was evidence that some issues appeared due to the data retrieved directly from the car rental company. For instance, the company in question shares the same fleet and price for three vehicle groups: *A*, *B*, and *C*. For the lower vehicle group, *A*, there are no vehicles available in the system. In that case, the company will deliver a car *B* or *C* to the client, and the reservation will have a vehicle group *A* requested. When the model runs, the operational capacity and fleet are higher than the maximum bound for operational capacity and the fleet has been surpassed. Consequently, the maximum occupation will be surpassed as well.

After identifying the “soft constraints” that could be violated at some point, seven gaps were added to the model in each of the constraints with a high cost in the objective function. The parameter for this cost, M_2 , is ten points percentual lower than the parameter M_1 defined in the previous mathematical model ($M_2 = 0.1 \times M_1$). At the beginning of the process, all gaps are fixed to 0. The model only uses them when the solution for the original model is infeasible. Iteratively, the model unfixes a gap variable according to a defined priority line. It does that until the solution becomes feasible, where the unfixed gaps take values greater than zero if the data requires so.

- a_{pr} Maximum available fleet gap for pool p on reservation day r .
- b_{pr} Minimum fleet gap for pool p on reservation day r .
- g_{sr} Maximum available fleet gap for station s on reservation day r .
- h_{sr} Minimum fleet gap for station s on reservation day r .
- m_{vsr} Maximum occupation gap for vehicle group v in station s , on reservation day r .
- n_{vsr} Minimum occupation gap for vehicle group v in station s , on reservation day r .
- o_{su} Operational capacity gap for station s on pick-up day u .

4.3.3 Adapted Mathematical Model

This subsection presents an adapted mathematical model. The changes related to the subsection 4.3.1 are shown in blue. The changes which occur due to the addition of gaps are shown in red.

$$\begin{aligned}
 \max \sum_{v \in V} \sum_{s \in S} \sum_{r \in R} & \left[\sum_{l \in L} \sum_{u \in U} \sum_{k \in K} (d_{vsluk} \times RP_{vsluk} \times (RPD_{vsluk} - CPD_{vsluk}) \times \alpha_{rlu}) \times (1 - OR_{vsr}) \right] \\
 & - \sum_{v \in V} \sum_{s \in S} \sum_{r \in R} (M_2 \times n_{vsr} + M_2 \times m_{vsr} + (M_1 \times \delta_r + CS_s) \times z_{vsr}) \\
 & - \sum_{v \in V} \sum_{p \in P} \sum_{r \in R} (M_2 \times b_{pr} + M_2 \times a_{pr} + (M_1 \times \rho_r + CP_p) \times x_{vpr}) \\
 & - M_2 \times \sum_{s \in S} \sum_{r \in R} (h_{sr} + g_{sr})
 \end{aligned} \tag{4.23}$$

$$\sum_{k \in K} d_{vsluk} = 1 \quad \forall v \in V, s \in S, l \in L, u \in U \quad (4.24)$$

$$AF_{vsr} - AF_{v,s,r-1} + f_{v,s,r-1} + z_{vsr} - y_{vsr} = f_{vsr} \quad \forall v \in V, s \in S, r \in R \quad (4.25)$$

$$\sum_{s \in S_p} (AF_{vsr} - AF_{v,s,r-1}) + f_{v,p,r-1} + x_{vpr} - w_{vpr} = \sum_{s \in S_p} f_{vsr} \quad \forall v \in V, p \in P, r \in R \quad (4.26)$$

$$\sum_{s \in S} AF_{vsr} = \sum_{s \in S} f_{vsr} \quad \forall v \in V, r \in R : \delta_r = 1 \quad (4.27)$$

$$FS_s - \mathbf{h}_{sr} \leq \sum_{v \in V} f_{vsr} \quad \forall s \in S, r \in R : \delta_r = 1 \quad (4.28)$$

$$FP_p - \mathbf{b}_{pr} \leq \sum_{v \in V} \sum_{s \in S_p} f_{vsr} \quad \forall p \in P, r \in R : \delta_r \times \rho_r = 1 \quad (4.29)$$

$$\begin{aligned} & \sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \\ & (1 - OR_{vsr}) + \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) + \\ & \left(\sum_{l \in L} \sum_{u \in U} RF_{vslu} \times \alpha_{rlu} \right) \times (1 - OR_{vsr}) \geq \\ & f_{vsr} \times OG_{vsr}^- - \mathbf{n}_{vsr} \quad \forall v \in V, s \in S, r \in R \end{aligned} \quad (4.30)$$

$$\begin{aligned} & \sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \\ & (1 - OR_{vsr}) + \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) + \\ & \left(\sum_{l \in L} \sum_{u \in U} RF_{vslu} \times \alpha_{rlu} \right) \times (1 - OR_{vsr}) \leq \\ & f_{vsr} \times OG_{vsr}^+ + \mathbf{m}_{vsr} \quad \forall v \in V, s \in S, r \in R \end{aligned} \quad (4.31)$$

$$\begin{aligned} & \sum_{v \in V} \left[\sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \right. \\ & \left. (1 - OR_{vsr}) \right] + \sum_{v \in V} \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) + \\ & \sum_{v \in V} \left[\left(\sum_{l \in L} \sum_{u \in U} RF_{vslu} \times \alpha_{rlu} \right) \times (1 - OR_{vsr}) \right] + \\ & SF_s + \mathbf{g}_{sr} \geq \sum_{v \in V} f_{vsr} \quad \forall s \in S, r \in R \end{aligned} \quad (4.32)$$

$$\begin{aligned} & \sum_{v \in V} \sum_{s \in S_p} \left[\sum_{l \in L} \sum_{k \in K} \sum_{u \in U} (d_{vsluk} \times RP_{vsluk} \times \alpha_{rlu}) \times \right. \\ & \left. (1 - OR_{vsr}) \right] + \sum_{v \in V} \sum_{s \in S_p} \sum_{l \in L} \sum_{u \in U} (RC_{vslu} \times \alpha_{rlu}) + \\ & \sum_{v \in V} \sum_{s \in S_p} \left[\left(\sum_{l \in L} \sum_{u \in U} RF_{vslu} \times \alpha_{rlu} \right) \times (1 - OR_{vsr}) \right] + \\ & PF_p + \mathbf{a}_{pr} \geq \sum_{v \in V} \sum_{s \in S_p} f_{vsr} \quad \forall p \in P, r \in R \end{aligned} \quad (4.33)$$

$$\sum_{v \in V} \sum_{s \in S} z_{vsr} \leq TD \quad \forall r \in R \quad (4.34)$$

$$\sum_{v \in V} \sum_{s \in S_p} z_{vsr} \leq PD \quad \forall p \in P, r \in R \quad (4.35)$$

$$\sum_{v \in V} \left[\sum_{l \in L} \sum_{k \in K} (RP_{vsluk} \times d_{vsluk}) \times (1 - OP_{vsu}) \right] + \sum_{v \in V} (RC_{vslu} + RF_{vslu} \times (1 - OP_{vsu})) \leq OC_s + o_{su} \quad \forall s \in S, u \in U \quad (4.36)$$

$$\begin{aligned} \sum_{k \in K} d_{v_Lsluk} \times RPD_{v_Lsluk} &\leq \\ \sum_{k \in K} d_{vsluk} \times RPD_{vsluk} &\quad \forall v \in V, v_L = VL_{vslu}, \\ &\quad s \in S, l \in L, u \in U \end{aligned} \quad (4.37)$$

$$\begin{aligned} \sum_{k \in K} d_{vsluk} \times RPD_{vsluk} &\leq \\ \sum_{k \in K} d_{v_Hsluk} \times RPD_{v_Hsluk} &\quad \forall v \in V, v_H = VH_{vslu}, \\ &\quad s \in S, l \in L, u \in U \end{aligned} \quad (4.38)$$

$$d_{vsluk} \in (0, 1) \quad \forall v \in V, s \in S, l \in L, u \in U : u \leq DEF, k \in K \quad (4.39)$$

$$d_{vsluk} \in [0, 1] \quad \forall v \in V, s \in S, l \in L, u \in U : u > DEF, k \in K \quad (4.40)$$

$$f_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.41)$$

$$w_{vpr} \in \mathbb{R}_0^+ \quad \forall v \in V, p \in P, r \in R \quad (4.42)$$

$$y_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.43)$$

$$x_{vpr} \in \mathbb{R}_0^+ \quad \forall v \in V, p \in P, r \in R \quad (4.44)$$

$$z_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.45)$$

$$a_{pr} \in \mathbb{R}_0^+ \quad \forall p \in P, r \in R \quad (4.46)$$

$$b_{pr} \in \mathbb{R}_0^+ \quad \forall p \in P, r \in R \quad (4.47)$$

$$g_{sr} \in \mathbb{R}_0^+ \quad \forall s \in S, r \in R \quad (4.48)$$

$$h_{sr} \in \mathbb{R}_0^+ \quad \forall s \in S, r \in R \quad (4.49)$$

$$m_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, \forall s \in S, r \in R \quad (4.50)$$

$$n_{vsr} \in \mathbb{R}_0^+ \quad \forall v \in V, s \in S, r \in R \quad (4.51)$$

$$o_{su} \in \mathbb{R}_0^+ \quad \forall s \in S, u \in U \quad (4.52)$$

4.3.4 Performance Issues

After implementing the changes above and running the model, it retrieves a solution for the first submodel in one hour and thirty minutes. Therefore, for it to run for a year, it would take 36 hours. A requirement from the company in question is that this model should be run daily. With that high of a processing time, this requirement cannot be met. As such, the focus was on reducing this time.

Firstly, the high coefficient in the objective function due to the soft constraints was reevaluated to stay as close as possible to the maximum value that the objective function would take without the gap. Secondly, the gaps had no maximum values. Adding maximum values to the gaps was expected to help solve the model. However, in this case, adding it did not change anything. To solve the model with *CPLEX*, the problem without maximum values for gaps took 883.39 seconds to root, plus branch and cut the solution tree, and it took 882 seconds with the maximum values.

Thirdly, the parameters of the *CPLEX* were finetuned to get a good solution within a reasonable time. The parameter gap tolerance was increased from 0.005 to 0.05, and the number of threads was reduced from 8 to 3. These two adjustments improved the solution time by a few seconds. The gap tolerance parameter instructs *CPLEX* to terminate when a feasible integer solution is proven to be within five percent of optimality. This parameter affects the optimal solution. By increasing the parameter, the further the optimal solution will be. The solution for the car rental company does not need to be the ideal. After a careful analysis (presented in chapter 6), a feasible solution with five percent of optimality is better than the current procedures of the company in question.

After three iterations without significant time improvements and thinking again about the model, another change was implemented. Instead of considering the decision variable d_{vsluk} integer during the first fifteen days, it was deemed to be continuous all the time. It was known that the model would therefore retrieve an approximate solution instead of an optimal solution.

$$d_{vsluk} \in [0, 1] \quad \forall v \in V, s \in S, l \in L, u \in U, k \in K \quad (4.53)$$

After running the whole model for one month and analyzing the output data, it is possible to conclude that the model still prefers to select one price step rather than decimal price steps. This result took place in 93% of all the cases. The root and branch & cut time decreased from 883.39 seconds to 363.50 seconds, which represents a reduction of 58.85%. With this adjustment, instead of running the first submodel in one hour and thirty minutes, it only runs for about forty minutes, which would mean 16 hours in a year.

Even with the latest change, the server's specifications were improved by adding more RAM memory and CPU capacity to process the information faster. This improvement further reduced the time to twenty-six minutes, less than 35% of the time to run the first submodel. During a year, it would take about 11 hours to run the entire model.

Finally, a last change was implemented. Instead of reading the static information at the beginning of each iteration, the model only reads it once. Additionally, there is a query in the pre-

processing data to read and process the forecasts that have price curves for multiple elasticities. That query uses a large amount of data, around 350GB for fifteen days. By improving the query and the pre-processing data, the execution time for the first submodel was reduced to 19 minutes. This execution time is equal to the reading of the data and transforming it into data frames (pre-processing time), plus the time to add the decision variables, the objective function, and the model's constraints (pre-processing time), plus the time to find an approximate/optimal solution (processing time), plus the time to export the output data to a data warehouse (post-processing time).

After showing the promising results to the car rental company, it was discussed that they only needed daily information for the next six months and once a week for the next twelve months. Overall, the daily processing time was reduced from 36 hours to 4 hours, which means a reduction of 89%. In the end, the final solution is not optimal. Nevertheless, the model retrieves an approximate solution, which is still better than the actual procedures of the company in question.

4.4 Interface

Two interfaces were developed for the company in question. The first one that was created was the tactical interface where the car rental company can create new scenarios, define new ABT curves, parameterize specific data, and trigger the forecast model, among other things. Additionally, an operational interface was designed for the committee team to validate new prices and control prices. In the chapter Time and Mileage Pricing and Fleet Planning, the Master's thesis only focuses on the operational interface. All images in this section with hidden sections do so for reasons of confidentiality on behalf of the car rental company.

Operacionalização do Pricing

powered by **LTP**

1. Seleccionar a Zona para análise

Atualizar Estações

Zona: Lisboa

2. Seleccionar a Estação para análise

Estação: Aeroporto Lisboa

3. Definir horizonte de análise

Prazo: Curto-Prazo

Início: 5/30/2022 **Fim:** 6/29/2022

4. Avançar

Figure 4.5: Operational Interface - First Page

Figure 4.5 shows the first page of the operational interface, where the company in question can select a pool, a station, and which time zone to look at. Then, by clicking on the “next” button the results related to that station and time zone are presented. Figures 4.6, 4.7, 4.8, and 4.9 show the second page of the operational interface for each *pick-up day*, *LOR*, and *vehicle group*. The first part compares the actual and implemented prices to the prices charged by competitors. Then, it is possible to compare the occupation rate goal with the company’s at that moment. Apart from this, it is possible to see the price for the revenue per day which is currently being implemented, the suggested price, and the target defined in the tactical interface.

Estação Aeroporto Lisboa			PVP Concorrência	
Data de PickUp	LOR	Grupo	Implementado	A Implementar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar

Figure 4.6: Operational Interface - Second Page Part 1

Taxa de Ocupação				RPD T&M		
Tx. Objetivo à data	Real MIN	Real MED	Real MAX	Implementado	Sugestão	Objetivo Planeamento à data
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar
Bar	Bar	Bar	Bar	Bar	Bar	Bar

Figure 4.7: Operational Interface - Second Page Part 2

The last two parts of this second page of the interface are related more to the control systems and alarms that have been created. If no alarm is triggered, the price will be validated automatically and appear green in the last column. If one alarm is triggered, a yellow flag appears. If two or three alarms are triggered, a red flag shows up. Apart from this, it is possible to control the fleet on the interface at any moment. In the Status column, it is possible to control whether the company has available cars. If it has, then the status will be “Free”. If not, the status column appears “OR”, meaning the fleet is on request.

The first alarm is related to the occupation growth rate. If the actual occupation is higher than the target occupation and the growth during the previous week is higher than the expected growth, it triggers an alarm. If the actual occupation is lower than the target occupation and the growth

during the previous week is lower than expected, it also triggers an alarm. The second alarm is related to the suggested RPD variation compared to the target RPD. If the suggested RPD is lower than 20% or the suggested RPD is higher than 80%, an alarm is triggered. Finally, the third alarm is the absolute difference between the company's public sale price (PVP) and its competitors. If that difference is higher or equal to 25%, an alarm is triggered.

Alarmísticas		
Crescimento Tx. Ocupação ▼	Variação do RPD sugerido face ao objetivo à data ▼	Variação do PVP a implementar face à concorrência ▼
0	1	1
0	0	0
0	1	0
0	1	1
0	1	1
0	1	1
0	1	1
0	1	1

Figure 4.8: Operational Interface - Second Page Part 3

Frota		Resultado						
Disponível ▼	Status ▼	Validar ▼	Prioridade ▼		A Implementar ▼			Validado ▼
	Free	▶						
	Free	▶						
	OR	▶						
	Free	▶						
	Free	▶						
	OR	▶						
	Free	▶						

Figure 4.9: Operational Interface - Second Page Part 4

Chapter 5

Fleet Repositioning

This chapter explores two different approaches to the vehicle repositioning model: a single-period and a multi-period approach. Before running the model, it is necessary to read the model's inputs first and perform pre-data processing. The repositioning of vehicles is a process that takes place after the time and mileage pricing and fleet planning model. Based on the differences between the target fleet and the existing fleet, the repositioning model will suggest fleet transfers by minimizing repositioning costs.

5.1 Context, Assumptions, and Inputs

After reviewing multiple results from the last chapter, two approaches were implemented and compared. The first approach is a single-period, which offers a new solution each day without considering the transfers made until that day. The second is a multi-period approach, which considers all transfers (i.e., empty, one-ways, and infleets) until a given date.

As discussed in chapter 2, the company in question has to parameterize two blocked dates: one to stop the model from retrieving new suggestions for empty transfers between pools and another date for infleets. In terms of business, when the company blocks movements from happening, it means that between a given date (i.e., “today”) and the date defined, the company will implement empty transfers based on the previous decision made during the committee meeting. Defining the block date in advance is essential to operationalize the transfers between stations and pools. For instance, transferring a car between Porto and Lisbon takes one day, whereas a transfer between Porto and Madeira takes seven days.

Furthermore, this model receives input information from the T&M pricing and fleet planning model. In order to get the latest information, firstly, the vehicle reposition model will read the latest run ID code from the T&M model. For instance, imagine that the previous chapter model fails, then the repositioning model has to run regardless of whether or not the previous model ran. Taking this into consideration, this model has to get the values from the data warehouse, which were referred to during the run that includes the ID code with valid results. The following subsections show some of the models' inputs.

Stations

The car rental company can define whether a station is active or inactive in business. After they upload this information, the model will incorporate it. Additionally, a dummy station was created to receive all data regarding new cars which will arrive in the future and reposition these infleets to different stations via this dummy station. Before requesting this model, the company in question will put the vehicles in each airport station based on the demand for each airport. Then, whenever a station needs a car, they would transfer it from the closest airport station to that station. The model output will let the company know where the infleets are more cost-effective by transferring them this way.

Possible Transfers

Before defining the mathematical model, it is mandatory to establish the possible transfers between stations on a particular day. The model will start by gathering data pertaining to all distinct vehicle groups, stations, and dates. Then, performing a day-by-day cycle, the model will calculate all possible combinations to transfer vehicle group v from station i to station j on date d . Additionally, the model has been programmed to only suggest transfers between pools after thirty days, and to shut down after ninety days. Algorithm 1 shows how this was implemented. Also, the algorithm assumes that the number of days equals the time difference between the date the model is looking for and today's date.

Costs

Afterward, the model needs to get the repositioning cost to transfer a vehicle group v from station i to station j , also parameterized by the car rental company. In addition, it needs to add a cost *null* for the dummy station.

Fleet Size

Finally, the vehicle repositioning model needs to access the fleet size information. Firstly, it will read the output from the T&M pricing and fleet planning model where it defines the target fleet size for vehicle group v in station i , DEM_{iv} . Secondly, the available fleet of vehicle group v in station i , A_{iv} , should consider in advance the number of one-ways that can still happen in the future. For that, we use the exact implementation explained in the last paragraph of section 4.1.

Algorithm 1 Possible transfers outlook

F = dummy station
 E = date to block empty transfers between pools – “today”
 I = date to block infleets – “today”
 B = number of days to block transfers between stations
 $dates$ = Set of Dates to Implement
 $stations$ = Set of All Stations
 $vehicles$ = Set of All Group Vehicles
 $n = 0$ ▷ n = number of days
 $i = 0$ ▷ i = origin station
 $j = 0$ ▷ j = destination station
for each $d \in dates$ **do**
 if $n \leq B$ **then**
 continue ▷ no possible transfers
 else if $n > 90$ **then**
 break ▷ finish run
 end if
 for each $i \in stations$ **do**
 if $n < I$ & $i \in F$ **then**
 continue ▷ no infleets
 end if
 for each $j \in stations$ **do**
 if $j \in F$ **then**
 continue ▷ no transfers to dummy station
 else if $i = j$ **then**
 continue ▷ same station
 else if $n \geq 30 \wedge pool(i) = pool(j)$ **then**
 continue ▷ same pool
 else if $n < E \wedge n < 30 \wedge pool(i) \neq pool(j)$ **then**
 continue ▷ no transfers outside pool
 else if $n < E \wedge n < 30 \wedge pool(i) = pool(j)$ **then**
 for each $v \in vehicles$ **do**
 transfer v from i to j in day d
 end for
 else if $n \geq E \wedge n < 30$ **then**
 for each $v \in vehicles$ **do**
 transfer v from i to j in day d
 end for
 else if $n \geq E \wedge n \geq 30$ **then**
 for each $v \in vehicles$ **do**
 transfer v from $pool(i)$ to $pool(j)$ in day d
 end for
 end if
 end for
 end for
 $n = n + 1$
end for

5.2 Single-Period Approach

This section presents the mathematical model related to the single-period approach by presenting the sets, subsets, indices, parameters, decision variables, constraints, objective function, and the description of constraints.

Sets and Subsets

- S Stations
- F Dummy station for infleets
- V Vehicle groups

Indices

- i Indices for origin stations in sets S, F
- j Indices for destination stations in sets S, F
- v Indices for vehicle groups in set V

Parameters

- A_{iv} Available fleet of vehicle group v in station i . The available fleet is the fleet outlook minus the fleet that is currently unavailable (due to maintenance or car accidents) and rented, minus/plus the number of exits/entries by one-ways that will occur in station i .
- C_{ijv} Reposition cost to transfer a vehicle group v from station i to station j , where $i \neq j$.
- F_{iv} Target fleet for vehicle group v in station i .

Decision variables

- x_{ijv} Number of vehicles of group v to move from origin i to destination j , where $i \neq j$.

Mathematical model

$$\min \sum_{i \in S \cup F} \sum_{j \in S} \sum_{v \in V} x_{ijv} \times C_{ijv} \quad (5.1)$$

$$A_{iv} + \sum_{j \in (S \cup F)} x_{jiv} - \sum_{j \in S} x_{ijv} \geq F_{iv} \quad \forall i \in S, v \in V \quad (5.2)$$

$$\sum_{j \in S} x_{ijv} \leq A_{iv} \quad \forall i \in S, v \in V \quad (5.3)$$

$$\sum_{j \in S} x_{ijv} = A_{iv} \quad \forall i \in F, v \in V \quad (5.4)$$

$$\sum_{j \in S} x_{jiv} = 0 \quad \forall i \in F, v \in V \quad (5.5)$$

$$x_{ijv} \in \mathbb{R}_0^+ \quad \forall i \in S \cup F, j \in S, v \in V \quad (5.6)$$

The model's objective is to minimize the repositioning cost. It multiplies the total number of vehicles from group v to move from station i to station j by the corresponding reposition cost from i to j . The balancing of all repositions must result in the lowest total cost possible.

The model starts by guaranteeing the target fleet of vehicle group v in origin i , with constraints 5.2. These constraints respect the target fleet by adding the difference between entries and exits to the currently available fleet. Then, with constraints 5.3, the model knows that the number of exits of vehicle v in station s must not surpass the fleet available at that moment. Furthermore, when a station i corresponds to the dummy station, the model needs to respect two constraints. Firstly, all available vehicles must exit that station (constraints 5.4). Secondly, the dummy station only has exits. Therefore, the number of entries in the dummy station is zero (constraints 5.5).

5.3 Multi-Period Approach

This section presents the mathematical model related to the multi-period approach by presenting the sets, subsets, indices, parameters, decision variables, constraints, objective function, and the description of constraints. In contrast to the previous approach, the multi-period considers all the transfers made up to a given date. For instance, a transfer made “today” will affect the transfers of tomorrow. As a result, the model will make more prudent decisions regarding long-term costs because current decisions will inevitably affect future decisions.

Sets and Subsets

- S Stations
- F Dummy station for infleets
- V Vehicle groups
- D Days in the time horizon

Indices

- i Indices for origin stations in sets S, F
- j Indices for destination stations in sets S, F
- v Indices for vehicle group in set V
- d, t Indices for days in set D

Parameters

- A_{ivd} Available fleet of vehicle group v in station i on day d . The available fleet is the fleet outlook minus the fleet that is currently unavailable (due to maintenance or car accidents) and rented, minus/plus the number of exits/entries by one-ways that will occur in station i .
- C_{ijv} Cost to transfer a vehicle group v from station i to station j , where $i \neq j$.
- F_{ivd} Target fleet for vehicle group v in station i on day d .

Decision variables

- x_{ijvd} On day d , number of vehicles of group v to move from origin i to destination j , where $i \neq j$.

Mathematical model

$$\min \sum_{i \in (S \cup F)} \sum_{j \in S} \sum_{v \in V} \sum_{d \in D} x_{ijvd} \times C_{ijv} \quad (5.7)$$

$$A_{ivd} + \sum_{j \in (S \cup F)} \sum_{t=0}^d x_{jivt} - \sum_{j \in S} \sum_{t=0}^d x_{ijvt} \geq F_{ivd} \quad \forall i \in S, v \in V, d \in D \quad (5.8)$$

$$\sum_{j \in S} x_{ijvd} \leq A_{ivd} + \sum_{j \in (S \cup F)} \sum_{t=0}^{d-1} x_{jivt} - \sum_{j \in S} \sum_{t=0}^{d-1} x_{ijvt} \quad \forall i \in S, v \in V, d \in D : d > 0 \quad (5.9)$$

$$\sum_{j \in S} x_{ijv0} \leq A_{iv0} \quad \forall i \in S, v \in V \quad (5.10)$$

$$\sum_{j \in S} x_{ijvd} = A_{ivd} \quad \forall i \in F, v \in V, d \in D \quad (5.11)$$

$$\sum_{j \in S} x_{jivd} = 0 \quad \forall i \in F, v \in V, d \in D \quad (5.12)$$

$$x_{ijvd} \in \mathbb{R}_0^+ \quad \forall i \in S \cup F, j \in S, v \in V, d \in D \quad (5.13)$$

The model's objective is to minimize the repositioning cost. It multiplies the total number of vehicles from group v to move from station i to station j by the corresponding reposition cost from i to j for all days in consideration. The balancing of all repositions must result in the lowest total cost possible.

The model starts by guaranteeing the target fleet of vehicle group v in origin i , with constraints 5.8. These constraints respect the target fleet by adding to the currently available fleet the difference between entries and exits up to a given day. Then, with constraints 5.9 and 5.10, the model knows that the number of exits of vehicle v in station s must not surpass the fleet available at that moment, plus the difference between the number of entries and exits up to the previous day.

Furthermore, when a station i corresponds to the dummy station, the model needs to respect two constraints. Firstly, on day d , all available vehicles must exit that station (constraints 5.11). Secondly, the dummy station only has exits. Therefore, on day d , the number of entries in the dummy station is zero (constraints 5.12).

5.4 Interfaces

As discussed in the previous chapter, two interfaces were developed for the company in question. The tactical interface shows some results related to the pool regarding this model. Another operational interface dedicated to the validation of vehicle repositions by the committee and fleet planning team is contemplated for the analysis and adjustment of values. In contrast to the previous interface, the vehicle reposition interface will not implement controls and alarms. All images in this section with hidden sections do so for reasons of confidentiality on behalf of the car rental company.

5.4.1 Operational Interface

Figure 5.1 shows the first part of the interface. The user can see information related to every combination of between day, origin, destination, and vehicle group. Apart from that, the user can filter results by origin, destination, and vehicle group on the left side. The second and third parts (represented in figures 5.2 and 5.3, respectively) show the results. Firstly, it shows the initial and final fleet size before and after the model run. In between these values, the information regarding vehicle transfers is presented. These transfers include guaranteed and predicted one-ways, empty transfers, infleets, and the total. The next column calculates the final cost to manage the total transfers. For example, for infleets and empty transfers, the cost is zero and fifty euros, respectively. Suppose the model suggests that one infleet goes to a station, and additionally, it is needed to perform two empty transfers; the final cost would be one-hundred euros. Finally, the last two columns indicate the values to implement (empty and infleets).

Reposicionamento de Veículos

Origem: [List of origins with search and filter icons]

Destino: [List of destinations with search and filter icons]

Grupo: [List of groups with search and filter icons]

Grid Headers: Dia, Origem, Destino, Grupo

Figure 5.1: Operational Interface - Part 1

Frota		Movimentações ARM				
Inicial Origem	Inicial Destino	OneWays em Carteira	OneWays Previstos	Trans. Manuais	Infleets	Total Movimentações
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	1	0	1
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	1	0	1
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	3	0	3
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	2	0	2
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	3	0	3
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	1	0	1
[Icon]	[Dropdown]	[Dropdown]	[Dropdown]	1	0	1

Figure 5.2: Operational Interface - Part 2

Frota		Custo		
Final Origem	Final Destino	Custo Final	A Implementar Manuais	A Implementar Infleets
[Icon]	[Dropdown]	[Dropdown]	1	0
[Icon]	[Dropdown]	[Dropdown]	1	0
[Icon]	[Dropdown]	150	3	0
[Icon]	[Dropdown]	100	2	0
[Icon]	[Dropdown]	150	3	0
[Icon]	[Dropdown]	50	1	0
[Icon]	[Dropdown]	50	1	0

Figure 5.3: Operational Interface - Part 3

5.4.2 Tactical Interface

Firstly, the user can navigate through the two-page menu (shown in figure 5.4) related to the vehicle reposition model. The first page of the tactical interface is similar to the operational interface. When comparing the two interfaces, only two main differences are visible. The first is that this interface only sees information regarding the pool, whereas the previous one shows the station. The second is that the filters appear on the right side above a chart. Figures A.1, A.2, and 5.5 represent the first, second, and third parts of the tactical interface, respectively. In figure 5.5, it is possible to see two dates in the upper right-hand corner, where the user can define the two blocked dates. Figure A.3 highlights how the filters are disposed of and a chart where it is possible to see how many empty transfers and infleets will occur each day.

Moreover, the second page (shown in figure 5.6) is the overview of the previous page, in which the committee and fleet planning teams can have an overview represented on a pivot table. The origin pools are on the vertical axis, whereas the destination pools are on the horizontal axis. Apart from that, the user can filter the information for the day, vehicle group, and which type of transfer he/she wants to see (i.e., total, empty, or infleets transfers).



Figure 5.4: Tactical Interface - Menu

Chapter 6

Results

In this chapter, both models' results are analyzed, from which several conclusions are drawn in regards to their behavior, as well as the positive results they bring to the car rental company in question. The first section presents the T&M pricing and fleet planning model, explained in chapter 4. The second section analyzes the vehicle repositioning model presented in chapter 5.

6.1 Time and Mileage Pricing and Fleet Planning

This section analyzes the results related to price and fleet management, which is divided into two categories: the time and mileage pricing and the fleet planning. Table 6.1 shows one car example for each group of vehicles that will be presented here. Many vehicle groups are not represented in this section. The acronym for each car represented here does not correspond to the actual acronym given by the company in question for confidentiality reasons.

Table 6.1: Acronyms and car examples for each vehicle group

Acronym	Example Car
RM	Renault Megane
RC	Renault Clio
B3SA	BMW 3-series Automatic
RK	Renault Kadjar
RGS	Renault Grand Scenic
MBEA	Mercedes-Benz E-Class Automatic
B4SC	BMW 4-series Cabrio

The data presented in this section results from a model run on the 22nd of June. Table 6.2 presents the characteristics of that run. Even though the 5% gap tolerance is a high number for considering an optimal solution, in the practical setting of this problem's application, it made sense to halt the model in this tolerance, forming a trade-off between solution quality and computing time.

Moreover, from reading and processing data time to exporting time, those values are an average of all the submodels. In chapter 4, section 4.3.4, it was referred that the daily processing time was reduced from 36 hours to 4 hours. With this test, it is possible to prove that the whole model can run in 2 hours and 36 minutes, which means a reduction of 92.8% of the initial time of 36 hours. In addition, with the same table, it is possible to see the vast model size. For instance, just for one month, there are 4 446 000 possible combinations for the decision variable d_{vstluk} . On average, 35.6% of those combinations have values different than zero for each submodel. Even if d_{vstluk} is now continuous, from the previous percentage, 9.5% are integer solutions.

Table 6.2: Run characteristics on the 22th of June for the time and mileage and fleet planning model

Characteristic	Value
Date Run	22nd June
Date Start	22nd June
Date End	18th December
Number of Days	180
Number of Months	6
Number of Submodels	12
Number of Stations	19
Number of Vehicle Groups	26
Number of Price Steps	20
Number of LORs	15
Reading Static Data Time (only at the beginning of the whole process)	35s
Reading and Processing Data Time	116s
Adding Decision Variables Time	15s
Adding Objective Function Time	100s
Adding Constraints Time	292s
CPLEX Number of Threads	3
CPLEX Gap Tolerance	5%
CPLEX Resolution Time	176s
Exporting Time	80s
Total Time Per SubModel	13 min
Total Time	2h36min

For confidentiality reasons, the absolute value of the objective function will not be analyzed. The rest of the section will be dedicated to analyzing different behaviors of the time and mileage pricing and fleet planning model. The analysis will be divided into the time and mileage pricing and the fleet planning. Several examples are portrayed to study the behavior of the different parts of the model.

Time and Mileage Pricing

As explained in chapter 4, figure 6.1 shows how the model should react to ensure the maximum occupancy rate. Suppose the forecast ABT curve shows that the company has a projection of an occupancy rate lower than the target. In this case, the model must reduce the price to stimulate demand and recover the utilization rate. A comparison between the output from the model and the objective defined by the committee team was studied. For instance, at the airport station of Porto, a reservation with pick-up day of the 24th of June and drop-off day of the 27th of June was selected.

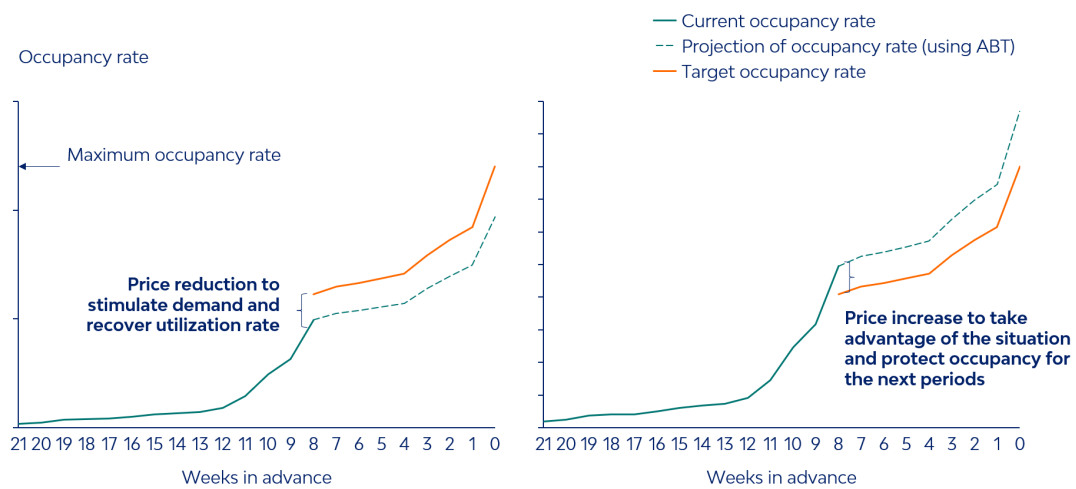


Figure 6.1: Increase or decrease price to encourage the target occupation rate

As table 6.3 shows, whenever there is a negative difference between the real and the objective occupation rate of the vehicles RM, B3SA, RK, RGS, MBEA, the model decreases the price of the revenue per day to stimulate demand and recover the utilization rate. When there is a positive difference (RC), the model will increase the price to take advantage of the situation and protect the occupancy for the subsequent periods. Additionally, the model will also increase the RPD when the difference is none, or the car rental does not have a fleet available (B4SC).

Table 6.3: Behavior when target occupation rate varies

Vehicle Group	Difference between real and objective occupation rate	Difference between suggested and objective RPD
RM	-6%	-25%
RC	+13%	+29%
B3SA	-7%	-26%
RK	-2%	-21%
RGS	-6%	-25%
MBEA	-6%	-24%
B4SC	0%	+26%

As the tourism rate increases, the occupancy rate will continue to grow during the holidays. The model will increase the revenue per day in all vehicle groups to take advantage of the holiday season. Figure 6.2 shows how the average revenue per day will behave during three months in the Lisbon airport. The data represented in the graph are outputs of the time and mileage pricing and fleet planning model. From the beginning of July to the middle of August, there is a significant increase in the RPD price of nearly 250%.

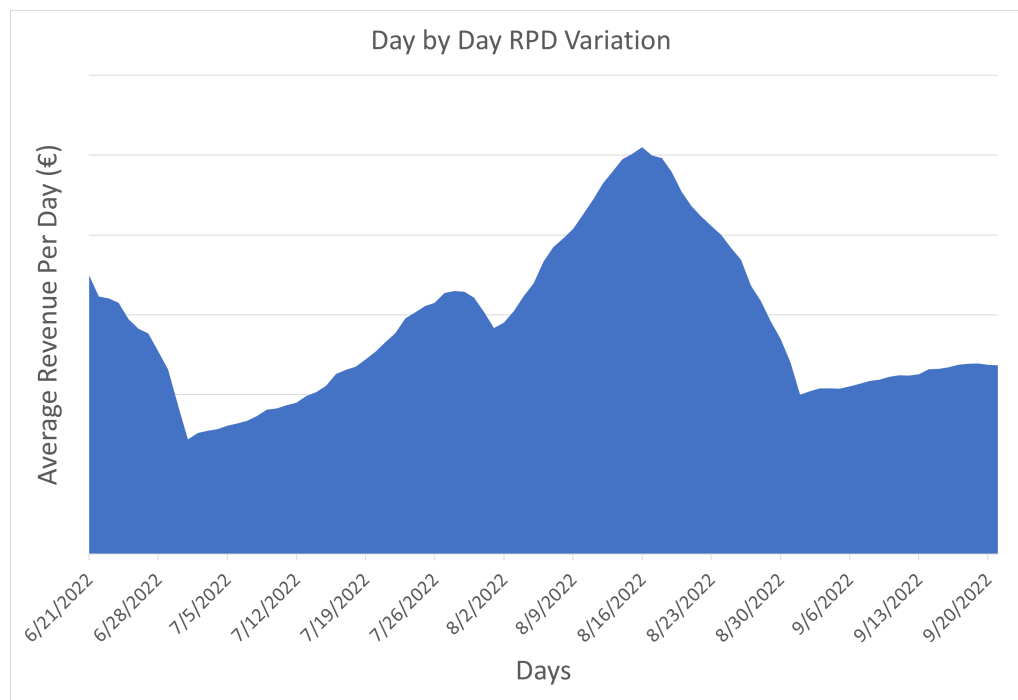


Figure 6.2: Average Revenue Per Day (RPD) per Day

While planning and setting objective values, the project management team of the car rental company can define how they want the average RPD price to vary across LORs. For instance, for economy cars, the average price per day should be lowered in a manner proportional to the rental period. However, it is different for a sports car, which should also consider mileage. After a particular LOR, the management team wants the daily revenue to increase to prevent the clients from increasing vehicle mileage. Table 6.4 presents an analysis regarding the evolution of the RPD when the length of rental is increased. The data represented is the model's results. The percentage referred is a relative difference between the previous price and the price with a higher LOR. Figure 6.3 shows the change of the daily revenue when the length of rental increases. For an economic car (e.g., RC), the RPD tends to lower as length of rental increases. In the sports car (e.g., B4SC) column, it is possible to see that the RPD has a significant drop on LOR 2 and stays on that price point during 9 LORs. Then, on LOR 11, the RPD increases by 70.07%.

As the project rollout will happen soon, an analysis comparing the model outputs with the real values is not yet possible. However, a quick analysis was made using a period during which the car rental business was at an all-time high in 2019. In some situations, the model suggests

Table 6.4: Evolution of the revenue per day (%) suggested by the time and mileage and fleet planning model when increasing the length of rental. Incremental of a particular LOR is compared to the previous LOR.

LOR	Economic Car	Sports Car
1	-	-
2	0.05%	-3.91%
3	1.20%	-4.83%
4	2.32%	-3.91%
5	-2.65%	-2.66%
6	-19.89%	3.50%
7	-27.98%	-2.74%
8	-7.38%	-3.91%
9	4.81%	-1.45%
10	-1.50%	-1.47%
11	-1.43%	70.07%
12	-1.55%	-3.91%
13	-1.57%	-1.54%
14	4.79%	-3.91%
15	-1.61%	-1.55%
16	0.00%	0.00%
25	-14.94%	-15.02%

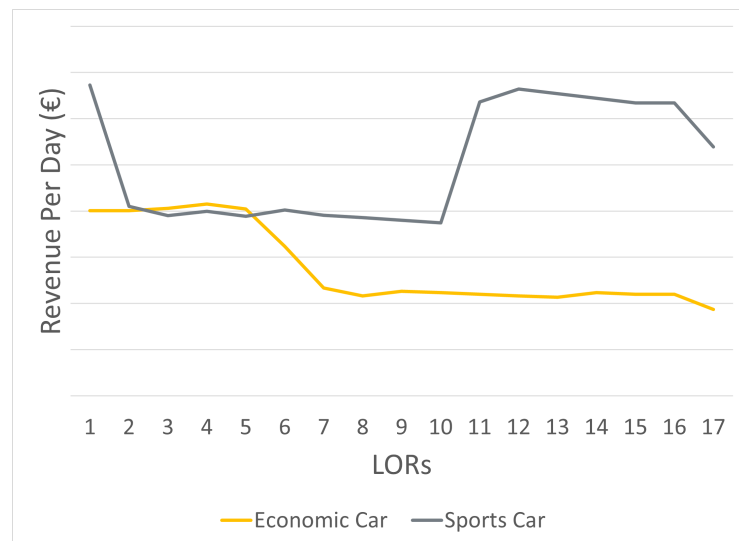


Figure 6.3: Change of the revenue per day as a result of increasing the length of the car rental

raising the prices to protect the occupancy rate, which would reduce the number of reservations and, consequently, the length of rentals measured in days. For an example, see figure 6.4. On week 46 of 2019, the model suggested increasing the price of the revenue per day by 43.3% in

the Lisbon Airport station and for the RM vehicles. As expected, it lowered the rental period days by 452 (12.9%) when compared to what happened that same year. When calculating the expected revenue (rental days multiplied by RPD), the profits increased by 2.5%. Overall, even if the number of rentals decreased, the company in question would still increase its revenue with this model.

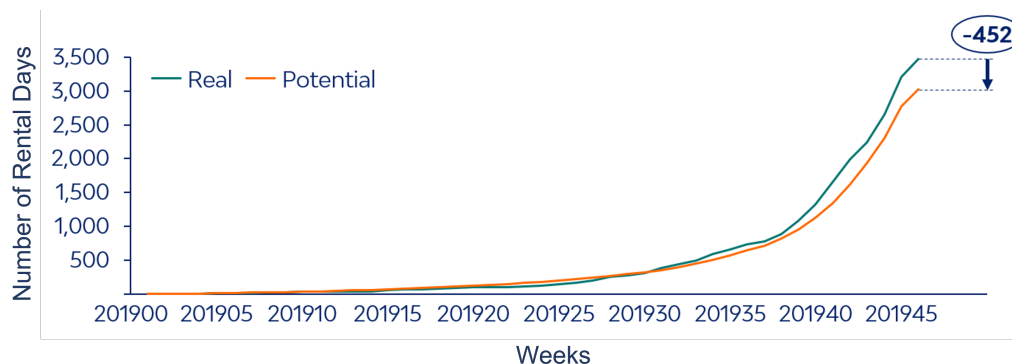


Figure 6.4: Application Example - Price suggestion higher than the implemented amount

In other situations, a price reduction can maximize the occupancy rate and, therefore, the expected revenue. Figure 6.5 demonstrates/illustrates this. On week 35 in 2019, in the Lisbon Airport station using the vehicle group B3SA, the model suggested lowering the price by 17.4%. It increased by 15.5% of the rental period, which resulted in an increase of 4.6% in revenue.

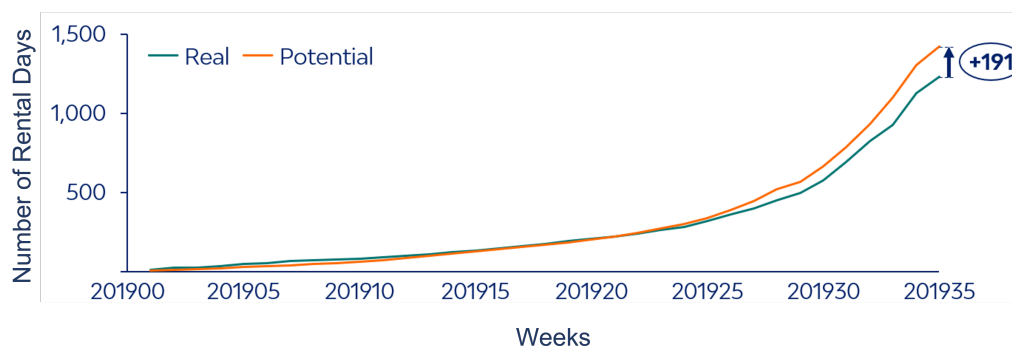


Figure 6.5: Application Example - Price suggestion lower than the implemented amount

Fleet Planning

As presented in chapter 2, the model must define the fleet size for each station, vehicle group, and pick-up day. Table 6.5 shows the percentage change in that station compared to the original fleet size. It is an example of the model performing the fleet size for two different vehicle groups and real stations. Globally, the daily sum of the target fleet for each vehicle equals the sum of the real available fleet. By analyzing the data from table 6.5, it is possible to conclude that the model increases and decreases depending on the occupancy rate of each station. If a station has more

vehicles than needed, the model will suggest moving them to another station. That way, the model will stimulate the utilization rate of each vehicle. Apart from this, there are more changes to the fleet of the RC vehicles. This means that the Renault Clio has a great potential for growth inside the company. Relocating this car to another station boosts the rental of the car in a way previously not seen by the company.

Table 6.5: Fleet increase and decrease compared to the original fleet size on the 24th of June

Station	Vehicle Group	Change in the fleet (%)	Vehicle Group	Change in the fleet (%)
A	RM	0%	RC	-0.22%
B	RM	-0.85%	RC	-6.88%
C	RM	-5.45%	RC	7.98%
D	RM	5.23%	RC	2.99%
E	RM	-0.21%	RC	0%
F	RM	20.74%	RC	-49.43%
G	RM	23.24%	RC	0%
H	RM	0%	RC	19.11%
I	RM	22.06%	RC	0.99%
J	RM	3493.44%	RC	-35.71%
K	RM	-62.09%	RC	-0.42%
L	RM	0%	RC	-78.19%
M	RM	0%	RC	133.22%
N	RM	7.64%	RC	-5.83%
O	RM	157.3%	RC	-4.7%
P	RM	-20.94%	RC	-22.41%
Q	RM	0%	RC	-35.85%
R	RM	0%	RC	-3.29%
S	RM	0%	RC	-28.76%
T	RM	0%	RC	0%
U	RM	0%	RC	-15.79%
V	RM	0%	RC	7.25%
W	RM	0%	RC	0.01%
X	RM	0%	RC	-28.41%

Another analysis was studied during a period of high demand. The results from table 6.6 are an illustrative example of what the model returns in the output. The model suggests fewer station transfers and consequently fewer pool movements during this period. It is possible to conclude that RM vehicles are closer to the objective occupancy rate than the RC vehicles, where there are some changes in the different stations.

Table 6.6: Fleet increase and decrease compared to the original fleet size on the 16th of August

Station	Vehicle Group	Change in the fleet (%)	Vehicle Group	Change in the fleet (%)
A	RM	0%	RC	5.69%
B	RM	0%	RC	1.15%
C	RM	0%	RC	0%
D	RM	-2.79%	RC	0%
E	RM	0%	RC	-35.12%
F	RM	0%	RC	-0%
G	RM	0%	RC	2.86%
H	RM	0%	RC	1.09%
I	RM	0%	RC	0%
J	RM	0%	RC	-3.02%
K	RM	0%	RC	-2.94%
L	RM	0%	RC	-0.28%
M	RM	0%	RC	0%
N	RM	0%	RC	0%
O	RM	2.35%	RC	0%
P	RM	0%	RC	0%
Q	RM	0%	RC	0%
R	RM	0%	RC	-3.26%
S	RM	0%	RC	-2.25%
T	RM	0%	RC	0%
U	RM	0%	RC	0%
V	RM	137.53%	RC	0%
W	RM	0%	RC	1.34%
X	RM	-8.18%	RC	0%

By looking at the results more carefully, it is possible to conclude that the model respects the minimum and maximum limits of each station and pool and the maximum number of daily movements between stations and pools.

Finally, in the end, an analysis regarding the data issues management was performed. Some conclusions:

1. the operational capacity gap was only used once on the 23rd of July for one station;
2. minimum fleet gap for stations appeared on 7% of all constraints that need this gap;
3. maximum available fleet gap for stations has an average residual value of nearly 0.7% of all the fleet in the constraints that need this gap;
4. minimum fleet gap for pools appeared on 4.4% of all constraints that need this gap;

5. maximum available fleet gap for pools has an average residual value of nearly 7.7% of all the fleet in the constraints that need this gap;
6. minimum and maximum occupation gaps are commonly used in the constraints that need these gaps.

This analysis shows that the car rental company needs to refactor the data shared with LTPlabs. Apart from this, the committee team must redefine the minimum and maximum occupation rates for each station and pick-up day. By working on these two critical points, the use of gaps in the model will reduce.

6.2 Vehicle Reposition

This section will analyze the behavior of the optimization model known as vehicle reposition. In addition, multiple comparisons between the two approaches will be analyzed in this section. The data presented in this section results from a model run on the 22nd of June. Table 6.7 presents the characteristics of that run. The single-period approach takes only 13 minutes to run daily, whereas the multi-period approach takes 37 minutes. This time difference is considerably large since some multi-period constraints in a given date depend on other constraints until that date, making the data frames heavier to process.

Table 6.7: Run characteristics on the 22th of June for the vehicle reposition model

Characteristic	Single-Period Approach	Multi-Period Approach
Date Run	22nd June	22nd June
Date Start	22nd June	22nd June
Date End	20th September	20th September
Number of Days	90	90
Number of Months	3	3
Number of Stations	19	19
Number of Vehicle Groups	26	26
Reading and Processing Data Time	10min 22s	11min 40s
Adding Decision Variables Time	15s	17s
Adding Objective Function Time	19s	18s
Adding Constraints Time	58s	15min 8s
CPLEX Number of Threads	3	3
CPLEX Gap Tolerance	0%	0%
CPLEX Resolution Time	38s	8min
Exporting Time	49s	94s
Total Time	13min 21s	37min

The rest of the section will be dedicated to analyzing the two different approaches: single and multi-period. Several examples are portrayed to study the behavior of the different parts of the model. The acronym for each station represented here does not correspond to the actual acronym given by the company in question for confidentiality reasons. Station DS is the dummy station where all the infleets will be placed and is virtually situated in the Lisbon pool. Apart from this, when it is referred that an empty transfer is performed, it means that the car exits from a particular origin and enters a specific destination. In addition, uppercase letters refer to stations, and locations (e.g., city names) refer to pools.

Firstly, the analysis will focus on the transfers between stations on the 30th of June. This date is chosen due to being the end of the month and all the most recent days when new cars (infleets) will arrive. As a result, in this date, it is possible to see both infleets and empty transfers happening between stations. Table 6.8 shows the daily suggestion for the RM vehicles. Station D receives vehicles from six stations: 14 cars from empty transfers and 24 from infleets. The model could transfer the 14 cars from the same station, but such would affect the system's equilibrium. The model considers the number of vehicles in each station and how many cars it can take from each station without affecting the station's equilibrium and respecting the minimum and maximum available fleet.

Table 6.8: Application Example - Single-Period Approach - Vehicle Reposition from a station to another on the 30th of June

Origin	Destination	Vehicle Group	Empty Transfers	Infleets
A	B	RM	8	0
C	D	RM	1	0
E	B	RM	14	0
E	F	RM	20	0
G	F	RM	10	0
H	I	RM	11	0
H	D	RM	1	0
J	D	RM	1	0
K	D	RM	9	0
K	L	RM	1	0
M	D	RM	2	0
N	O	RM	4	0
DS	D	RM	0	24
DS	P	RM	0	85
DS	Q	RM	0	32

In contrast to the single approach, the multi-period approach, on the same day, will remove cars from station D instead of adding them, which is shown in table 6.9. This happens because the multi-period approach considers all the transfers made up to that day. In total, the multi-period

on the 30th of June transfers 35 vehicles, whereas, in the single-period, it transfers 82 vehicles by empty movements. The amount of infleets is the same using both approaches due to the constraints 5.4 and 5.11.

Table 6.9: Application Example - Multi-Period Approach - Vehicle Reposition from a station to another on the 30th of June

Origin	Destination	Vehicle Group	Empty Transfers	Infleets
D	I	RM	27	0
D	H	RM	2	0
D	R	RM	3	0
D	J	RM	2	0
O	H	RM	1	0
DS	A	RM	0	46
DS	B	RM	0	35
DS	P	RM	0	18
DS	Q	RM	0	5
DS	F	RM	0	24
DS	S	RM	0	1
DS	E	RM	0	3
DS	T	RM	0	5
DS	G	RM	0	4

As algorithm 1 shows, after thirty days, the model switches from suggesting inter-station transfers to suggesting only inter-pool movements. Table 6.10 proves that event. The data comes from a run made on the 22nd of June, so after thirty days, the model retrieves pool transfers instead of giving station results.

Table 6.10: Application Example - Two consecutive days

Date	Origin	Destination	Vehicle Group	Empty Transfers	Infleets
2022-07-22	A	B	RK	1	0
2022-07-23	Lisbon	Porto	RK	1	0

Furthermore, a similar analysis like the one presented for vehicle repositioning between stations is made for pool transfers. Tables 6.11 and 6.12 show a part of the results on the 31st of July. Again, it is clear that both approaches utilize different methodologies. Firstly, the single-period approach suggests the movement of 79 new cars to only three stations, whereas the other suggests moving them to eight stations. Secondly, the vehicle group B4SC no longer needs to make an empty transfer on the 31st of July on the multi-period approach because it had already been transferred days before.

Table 6.11: Application Example - Single-Period Approach - Vehicle Reposition from a pool to another the 31st of July

Origin	Destination	Vehicle Group	Empty Transfers	Infleets
Lisbon	Lisbon	RC	0	4
Lisbon	Porto	RC	0	73
Lisbon	LM North	B4SC	1	2

Table 6.12: Application Example - Multi-Period Approach - Vehicle Reposition from a pool to another on the 31st of July

Origin	Destination	Vehicle Group	Empty Transfers	Infleets
Lisbon	Lisbon	RC	0	7
Lisbon	Madeira	RC	0	8
Lisbon	Porto	RC	0	41
Lisbon	Leiria	RC	0	2
Lisbon	LM North	RC	0	8
Lisbon	LM South	RC	0	2
Lisbon	Faro	RC	0	9
Lisbon	Porto	B4SC	0	2

On the whole, the multi-period approach suggests the same number of transfers via infleets and less than 96.7% movements via empty transfers compared to the single-period approach. In terms of cost, this means a reduction of 94.9%. With these numbers, it is easier to conclude that the multi-period approach is the better option and meets the needs of the car rental company in question.

Chapter 7

Conclusion and Future Work

This dissertation presents the *Rental Revenue Management* (RRM) project, which was developed for a car rental company operating in Portugal. The RRM project aimed to develop an analytical model for coordinating price management with fleet management to be implemented in the company in question. Apart from this, the project supports the optimization of critical decision-making in order to maximize the benefits for different periods and demand trends.

An explanation regarding the problem is presented in chapter 2. This chapter explains how the car rental company in question is divided, the project's modules, the focus of the Master's thesis, and the requirements that need to be fulfilled. The research of this Master's thesis was focused on the analytical module to optimize pricing decisions and fleet planning and another optimization module which suggests vehicle repositioning. In addition, some control systems were developed to monitor and warn the car rental company of unexpected occurrences.

Chapter 3 refers to the Master Thesis' state-of-the-art. Firstly, an overview of the car rental business is presented. It includes information about how the sector performed in Portugal last year and how Covid-19 impacted the car rental industry. Secondly, there is a discussion regarding the revenue management issues of the problem, as well as relevant topics to consider regarding the project, which was requested by the company in question. After reviewing other works and comparing them to this project, the following sections explore a discussion regarding previous pricing and fleet management studies. At the end of this chapter, a critical reflection is presented. The project has answered the following questions:

- how many cars should be in the fleet?
- how should they be distributed among rental stations and pools over time?
- how should these car rentals be priced?

Chapter 4 discusses a mixed-integer programming problem, which was developed to tackle the price and fleet management. This proposed integration will assist the company in question with any decisions that it makes, where the focus is to maximize revenue and vehicle utilization. Moreover, the project can be applied to global and local car rental companies and other industries

that share similar characteristics. Every day, the goal is to define the price and fleet of each vehicle group, station, and length of rental (LOR), which will help optimize the fleet's occupation rate, taking the number of days until the pickup and the corresponding occupation target into account. Due to the processing time required, a relax-and-fix method was implemented, which reduced the time to deliver the needed information by 89%.

Another optimization module is presented in chapter 5. It suggests vehicle repositioning between stations and pools. This model is based on the previous solution's data for the optimal fleet and relies on the fleet currently available at each moment, to determine the reposition of the different groups of vehicles in each station. The main objective is to minimize the vehicle repositioning cost of the car rental company in question. The model was formulated as a linear programming problem. Apart from this, two approaches were implemented and compared. The multi-period approach reduces the total repositioning cost by 94.9% when compared to the single-period approach.

Both models were implemented in a decision support system which allows the company to control prices and reposition the vehicles daily. This system has improved the car rental company's revenue and reduced the repositioning cost in comparison to the company's former procedures. Moreover, it has allowed the company to increase control over both their operation and their capacity to adjust the decisions made on a daily basis, which would otherwise require an immensely larger team.

Overall, a fully integrated revenue management analytics platform was developed, with granular optimization of the crucial decisions made to strategically and operationally support the company's pricing and capacity management. Moreover, the project has reshaped the company's vehicle pricing and repositioning processes to ensure that more informed and supported decision-making takes place. Suggesting price and fleet deployment, implementing new prices, and controlling the information shared with the company will increase the monthly and annual revenue. From the point of view of the car rental business, the three key objectives were fulfilled:

1. increased revenue, considering the price optimization at each moment;
2. enhanced analytical support for decision-making;
3. increased performance and focus on value-added tasks.

Finally, there are some next steps to improve in the future. Firstly, the time and mileage pricing model must integrate the competitors' prices, so the model will react immediately when a particular competitor changes the price of the revenue per day. Secondly, the car rental company wants to change the model from a daily to an hourly perspective. Last but not least, the model will go live soon, at which point it will be possible to compare the output data with actual data, which is naturally more accurate.

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Appendix A

Vehicle Reposition Tactical Interface

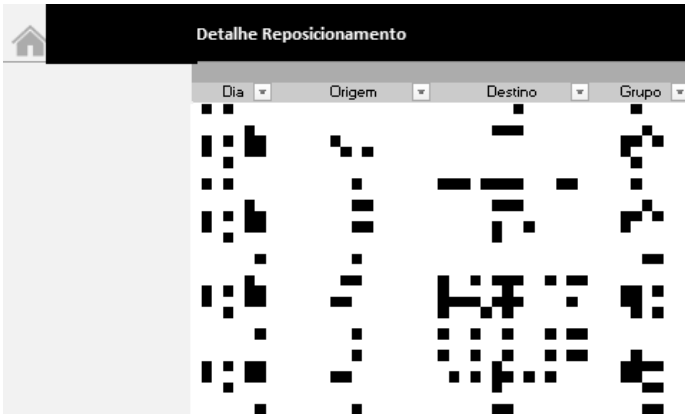


Figure A.1: Tactical Interface - Part 1

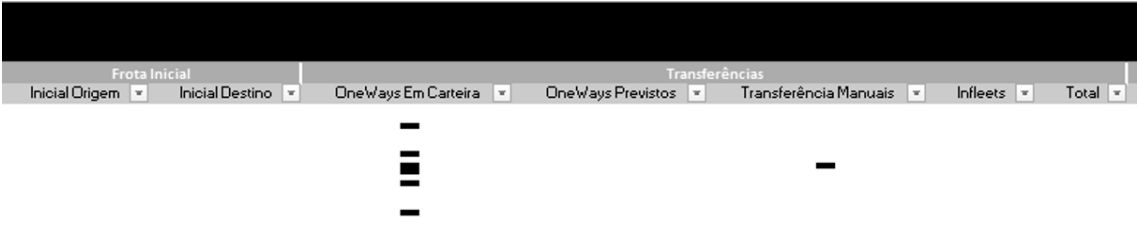


Figure A.2: Tactical Interface - Part 2

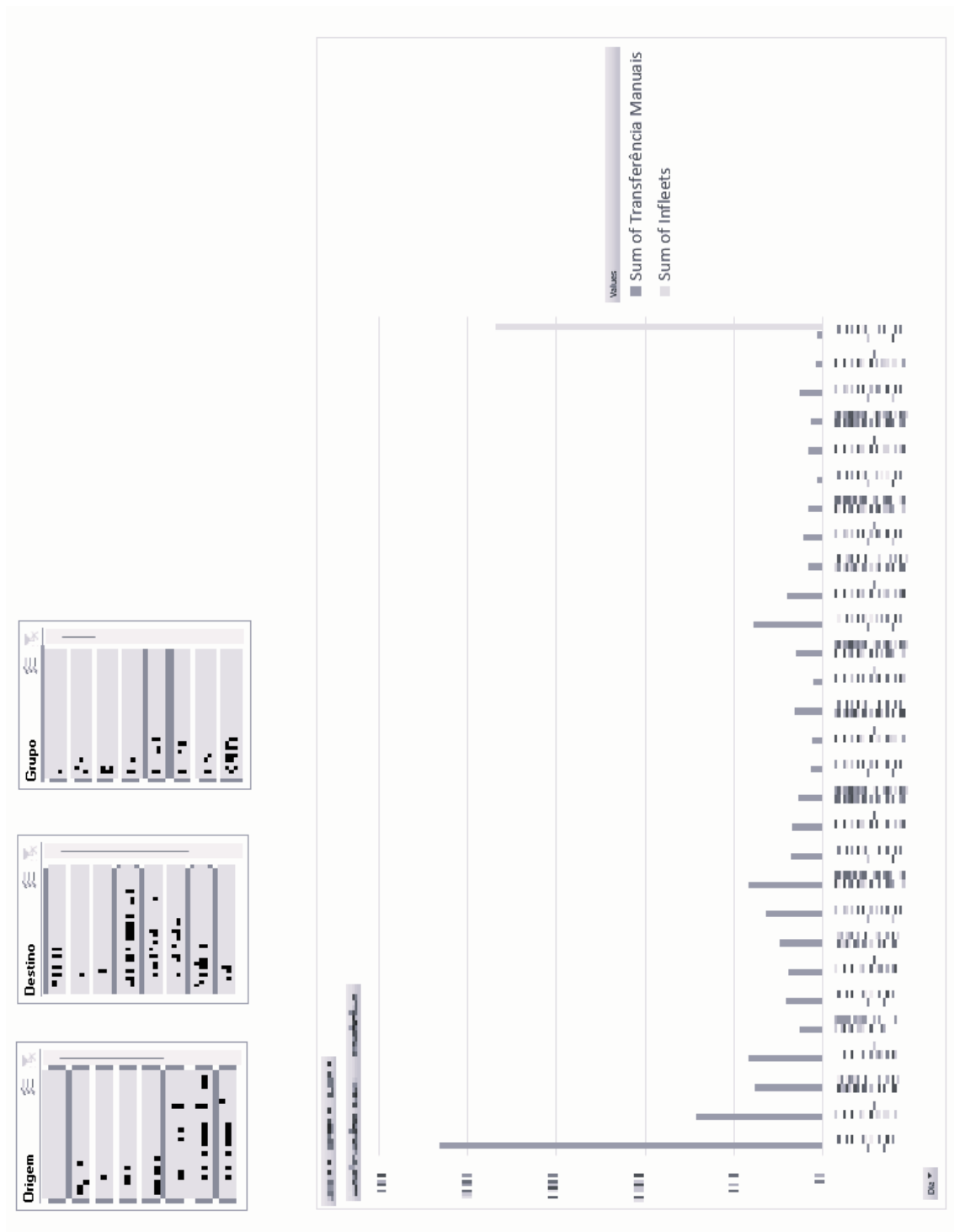


Figure A.3: Tactical Interface - Part 4