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OPTIMAL TIMING, LEVERAGE AND PREMIUM FOR LBOs – A REAL OPTIONS APPROACH

Marco Alessandro Leite Pires

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Professor Paulo Pereira

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Abstract

In this dissertation, I develop a theoretical model with the real options approach that aims to determine the optimal timing, leverage that should be used, and premium paid for leveraged buyout (LBO) deals. Simple answers that could increase the success rate of such deals that usually face serious difficulties when confronted to miscalculations about the potential gain from the acquisition, the right time to invest or about overpayment. The model appears as a useful tool, helping private equity firms to anticipate the target's reaction after the takeover offer and to initiate conversations about the acquisition. We will see that financing the acquisition with debt also provides the buyer an advantage over other potential bidders and increases the takeover activity.

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Abstract

Nesta dissertação, desenvolvo um modelo teórico usando a abordagem de opções reais que visa determinar o momento ideal, a alavancagem que deve ser usada para aquisições envolvendo elevados níveis de dívida (LBO), assim como o prêmio ótimo a ser oferecido. Respostas simples que podem aumentar a taxa de sucesso de tais aquisições que geralmente enfrentam sérias dificuldades quando confrontadas com erros de cálculo sobre o potencial ganho da transação, o momento certo para investir ou sobre o excesso de pagamento. O modelo aparece como uma ferramenta útil, ajudando as empresas de private equity a antecipar a reação do alvo após a oferta de aquisição e a iniciar conversas sobre a aquisição. Veremos que o financiamento da aquisição com dívida também oferece ao comprador uma vantagem sobre outros possíveis concorrentes e aumenta a atividade de aquisições.

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1. Introduction

When it comes to acquire a company, there is the possibility for the acquirer to alleviate the total amount invested from his pocket by issuing debt for the transaction. Private equity firms, most of the times appearing as the buyer, will take that debt issued to a significant higher level, appearing as one of the most know characteristics of leverage buyouts. The use of debt, usually cheaper than equity, will also provide great benefits for the success of the deal.

For private equity firms, there are some relevant questions about targeting the companies that they are willing to invest in and related features for the transaction. Knowing the company on the other side and trying to understand its business is the first step for a successful acquisition, but the main challenge in any of them will be the ability of the acquirer to not overpay for the target firm. Even the best due diligence, selection of underleveraged firms, with high agency costs, and other aspects that might be crucial for a successful deal, it will always exist uncertainty related to any kind of investment and it will represent a serious matter in terms of how much it is worth to pay for it. Overpayment can easily damage the transaction's performance and the mispricing of the target firm usually observed in this kind of deals can exacerbate the problem and lead to an early exit with huge losses for the investors. This is the main objective of this dissertation, avoid this situation to happen by developing a model able to determine, besides the timing and premium to be paid, the optimal level of debt in any acquisition. In addition, the model can be applicable to any LBO variants as long as the major source of funding the deal is debt.

Therefore, the model proposed in this dissertation will estimate three major aspects of every leveraged buyout deal that are crucial for the success or failure of it. The bidder will be given the opportunity to know the optimal timing to invest in a target company, how much should the premium be to close the deal, and the optimal amount of leverage to be used, with the application of the dynamic model that follows a real options approach. As we will see in the literature review, most of the models treat acquisitions as simple takeovers without making any differentiation about the way it is done. This will become the major contribution of the model I propose to develop in this dissertation, once it will be focusing on the aspect of debt used for the acquisition. The model will define the timing, the premium, the optimal leverage and even the optimal timing to abandon the company by incorporating some variables, such as the synergies expected to be created and the volatility related to the

improvement performance arising from the new management team chosen by the bidder and how it will affect the success of the transaction. The model will be aiming the maximization of the value of the deal and act as an intuitive tool that will support the decision of investing.

As stated above, many leveraged buyouts failed in the past due to bad estimations of the true potential of the deal. We must treat problems like overpayment, mispricing or bad timing of investment carefully to avoid them and this is one of the main goals of this model. However, we need to keep in mind that it is impossible to represent the reality with all the existing details and for that reason it is wise to develop a model as simple as possible, able to answer the most important features related to any acquisition, that can be used easily and able to adapt to the various deals. Another problem that comes with complex models is the difficulty of applying it in the real world. With a simple model, none of this will happen and it will be possible to apply to a larger range of potential deals.

After developing the model, I expect to answer the most critical questions that arise from leverage buyout deals, more precisely the issue related to the optimal debt. Key decisions such as when or how much to invest and how much premium to offer in the deal should be provided by the model that will present the optimal values for its maximization. It will also include the abandon option that the bidder acquires with the target company, allowing the private equity firm to leave the company if the deal starts going wrong.

The next section will make a review of the previous literature already existing related with what I am proposing to do, organized in a way that will provide a better understanding of each of the elements of my model. Section 3 will develop the model, presenting the optimal investment timing, the optimal debt level and the optimal premium. It will be followed by section 4 presenting a numerical example and the sensitivity analysis. Section 5 is destinated to introduce the effect of the target's mispricing on the model's outputs and finally, at section 6 I will present the most relevant conclusions.

2. Literature Review

There is some prior literature about mergers and acquisitions models that use a real options approach, but it becomes harder to find any of those models that include debt to finance the transaction. For that reason, we will need to see every acquisition deal as an investment opportunity for the bidder and therefore, a real option. The "real options" term was first proposed by Myers (1977) where he compares the option to invest in a project that a company might have with financial options. The analogy made in his paper provided an entirely different way of calculating the value of an investment opportunity. In fact, the work of Myers had a huge impact on the decisions of capital budgeting that were based on traditional methods like the net present value. According to the author, there is the possibility of acquiring an option linked to the investment that is made initially, turning the valuation of that option an important feature for the investment decision. In my model, the bidder will have an investment opportunity that represents the possibility to acquire another company through an acquisition that includes debt, a leveraged buyout.

The transaction will present the three basic ideas of Dixit and Pindyck (1994). The first one is the irreversibility of the initial investment that the private equity firm will incur. There is no way of recovering the costs of the transaction if the performance is lower than expected. The second idea is the uncertainty related to future benefits from the deal. In acquisitions where the bidder appeals to debt to reduce the financial effort of the investment, we could have an idea of how much would be the tax savings that the interest payments bring, but it will certainly not be the only source of value creation. There is a strong uncertainty linked to improvements in the performance of the acquired company that are also one of the measures taken by bidders to maximize its value. Waiting for new information will decrease the uncertainty related to the deal, but the benefits of the acquisition cannot be fully predicted. Finally, there is also flexibility in such deals, not only with the option to choose when to invest, by postponing the decision, but also by having the option to change anything needed during the post-acquisition period. Managers are not expected to be static and act as defined in the beginning of the deal, they will adapt their decisions as new information arise.

In this dissertation, the model considers a common takeover at the begging and then, it will be added the optimal debt that should be used for a successful transaction that maximizes the potential profit. Seeing the acquisition in that way, as a simple takeover, we can find some real options' models that have a similar goal of the one of this paper. Morellec

and Zhdanov (2005) presents a dynamic model able to determine the timing and the terms of a takeover based on the stock market valuations of merging firms. It relies on two key elements, the analogy between takeover opportunities and exchange options, and the imperfect information that investors can access. The first one relates to the common takeover deal where the participants have an option to exchange one asset for another, usually the shares in the initial firm for the shares in the new firm or cash. According to the paper, it will have an impact on the timing and the terms of takeovers once each company of the deal will exercise its option taking into consideration the strategy that the company on the other side is expected to do. The second element that the model considers is the information set of investors. This element implies that although outside investors only have incomplete information about the deal, they can anticipate the timing and the terms of the takeover by observing the behavior of the participating firms that have complete information. So, the stock market valuations of the participating firms will be influenced by the takeover surplus generated that can be captured by the public information. The higher the probability of the takeover happening, the higher the fraction of the takeover surplus that will be incorporated in the stock market valuations. The authors, through their model, predict that the returns to target shareholders should be larger than returns to bidding shareholders or the fact that competition speeds up the acquisition process and decreases returns to bidding shareholders.

There is also a similar model about mergers than can be related to the paper described above. Lambrecht (2004) proposes a model that can determine the timing and terms of mergers motivated by economies of scale. Taking a closer view in his work, the number of mergers seems to significantly increase in periods of economic expansions and the main benefits arising from them is the synergies related to the deal, as higher output generation without having to increase production costs as much as they would if both firms remain operating individually. Even if merging will bring some costs such as legal fees, fees to investment banks and other merger promotors, and costs of restructuring and integration the two companies, both firms forgo higher profits by not merging. In a real options point of view, each firm has the right, but not the obligation to merge, and this higher profitability that can be reached together acts like and incentive to exercise the option. On the other hand, the irreversibility of doing it, at least partially, will act as an incentive to delay the merger. Both incentives will be pushing the decision of merging on opposite directions and the optimal merger timing will be obtained through a balance between the two. The way the

merger gains will be distributed in a merger, or in a hostile takeover is expected to be quite different. Mostly because the payoff to each firm defined by the post-merger ownership share will be different when we are talking about those two types of deals and will have an impact on the timing of the restructuring. The author also assumes market efficiency and incorporates the possibility of increasing the market power through the merger, finding that this market power increase tends to speed up merger activity as it creates a second reason to merge, in addition to synergies.

The other key topic that I will treat in this dissertation, is the optimal premium paid to the target to conclude the transaction. Alvarez and Stenbacka (2006) designed a model able to determine the timing of a takeover and the distribution of the surplus created through the asset restructuring. The optimal price offered to the target will correspond to its value under current ownership plus a fraction of the surplus generated by the transaction, leading us to the optimal premium. The real option related to the takeover decision will have a substantial degree of implementation uncertainty, that will have an impact on the timing and surplus generated. Their model also incorporates a divestment option, that the acquiring firm obtains with the transaction and allows to divest in non-core activities.

Lukas and Welling (2012) also explains how the surplus generated by an acquisition is shared between the buyer and the seller. The optimal premium will depend on which party places the offer. They show two alternatives for the determination of the optimal premium if the bid is placed by the seller, or if it is the buyer that is the offering party. However, the optimal timing for investing will be independent of whether the buyer or the seller is the reacting party. Only the fraction of the surplus generated by the deal is shared differently between the buyer and the seller, and dependent of which is the first mover, the offering party. They show the first-mover advantage where the expected profit for being the offering party is greater than for being the accepting party. In this case, uncertainty is the more pronounced factor with an impact on the first-mover advantage. An increase in uncertainty will increase the net gain because it pays to wait, and the offering party will profit with the postponement. Consequently, the bargaining power of the reacting party will be lowered once it will have less control regarding the exercise of the real option, and the gain associated with the first-mover advantage increases.

Van Bekkum, Smit, and Pennings (2011) studied if the takeover activity is driven by new growth opportunities and synergies that arise from the deal or by irrational mispricing

in financial markets. On the first hand, we have the value creation generated by the deal, also valid for any acquisition that target firms with specific characteristics. The point of doing so is the chance to bring improvements to those aspects, such as agency problems or underleverage, and increase the value of the company that will be acquired. Note that we are not expected to see as many synergies as it happens with mergers, but the tax benefits arising from the acquisition will be impactful. Bidders do not seek for new synergies as main goal, they will primarily focus on maximizing the value of firms targeted and benefit with the use of debt. We could say that great part of the synergies created can be reached by reducing the difference between both the current performance of the target and its true potential. On the other side, acquisitions are enhanced by mispriced target that show different market values comparing to the fundamental value. For this kind of acquisitions, this phenomenon can be easily observed too, undervalued firms become attractive targets, and overvalued firms can lead to unsuccessful deals. According to them both explanations coexist, bidders are usually overvalued but also have strong growth potential and targets' equity is less overpriced than bidders. Since target growth potential is less mispriced than bidders, its value is more strongly based on fundamentals. This is why bidders buy smart and time smart. More, since targets also are undervalued relative to bidders when the bids are made, by buying fundamental growth may cushion bidders against a future drop in market value. Mispricing, without any doubt, will have an impact on the deal and it is important to incorporate this feature in the model as well.

All the literature above can be related to the first two outputs that my model will determine, i.e. the optimal timing and the optimal premium paid in acquisitions. However, we have another crucial point to take into consideration in such deals before introducing the optimal leverage, that are not approached in most of previous models, the abandon strategy. In fact, the success of the acquisition will not rely only on the improvement of the operational performance, debt repayment or the tax shield, there is also a great impact of the optimal timing for abandoning the company if something went wrong, that is as important as the surplus generated. Not all acquisitions end as planned, there is external factors that can have a huge impact on the deal's success and cannot be fully predicted neither controlled. Having a way out if it happens appears as a valuable option for the bidder, influencing its investment decision. Then, we will see it as an abandon option's exercise due to the underperformance of the deal, as we might see in models about divestment options, providing to the bidder the possibility of leaving the company without necessarily letting all his initial investment on the

table. This option of abandoning the target company has value that needs to be considered while defining the other optimal triggers. Dixit (1989) shows the optimal trigger to invest in a project and its optimal abandon trigger. The paper examines the influence of the output price that follows a random walk, something that can also happen with the price of the acquired firm's product or service in any acquisition. This uncertainty will have an impact on the abandon strategy of an unsuccessful transaction.

Another model that determines the optimal exit that firms must follow for their investment projects is Alvarez (1999). The author considered the valuation and optimal exit strategy of a competitive price-taking firm facing demand uncertainty and how to deal with the excess supply that the project might generate due to an overestimation of the total demand. The underperformance related to the excess supply is similar to an underperformance related to the lack of demand for a product. In acquisitions, the new management team will need to improve the performance of the target firm for a successful deal and in the case that the observed improvement is lower than the expected one, it is possible to assist to an early exit. By using a real options approach, we will see how the uncertainty of the performance improvement can influence the optimal exit strategy. We can also find some useful knowledge in McDonald and Siegel (1985) with his paper showing how option-pricing techniques can be used to study the investment problem of a firm which has the option to shut down production if variable production costs exceed revenues. Again, the investment project will be the acquired firm that will generate revenues optimized by the private equity firm. We can expect uncertainty in those revenues because one of the value creation factors of the deal that we cannot know for sure in advance, is the performance improvement.

Lambrecht and Myers (2007) developed a real options model of takeovers and disinvestments that can be somehow related to the abandon option that my model will introduce, giving a closer approach about the way the investors of the private equity firm is paid back, as well as the debtholders. Their model presents a view where the managers of an underleveraged firm always close too late and cannot maximize the investors´ overall value. In fact, this inefficient phenomenon also occurs in management buyouts and mergers of equals. It is then possible to relate the disinvestment part of this model with the model I propose to do, that will integrate an exit strategy for the investors, so they can reduce their losses with the perfect timing for the abandon option supposing something went wrong. In

this case, the model will focus on mergers and acquisitions that seek for greater efficiency through layoffs, consolidation and disinvestment. They first analyze the behavior of a manager that has the option to abandon the business, without any takeover threat, if the demand of the product decreases until a specific low level, and then what happens if the threat is real and managers are forced to exit.

Until now, we have been assisting to models that do not take into consideration the optimal level of debt for a takeover, mainly because they deal with mergers and acquisitions in a more general way. Then, we will need to focus in something more than just the optimal timing and premium. Indeed, the optimal leverage used for the transaction is a crucial point for any leveraged buyout and the other part of the model I am developing is related with that, how to maximize the value that the target firm can provide, through the optimal level of debt and therefore, the return of such deal for the investors. Leland (1998) developed a model examining optimal firm decisions, providing quantitative guidance on the amount and maturity of debt, on financial restructuring, and on the firm's optimal risk strategy. The choice of the capital structure and investment risk is controlled by equityholders, and to maximize the value of their claims, they will choose strategies that reduce the value of other claimants, including the government (tax collector), external claimants in default and debtholders. The paper considers all claimants in the determination of the capital structure and investment risk, in contrast with Modigliani and Miller (1963) that support the optimal level of debt is a tradeoff between tax deductions provided by interest payments and the default costs that it brings, and Jensen and Meckling (1976) that emphasize the importance of bondholders' claims in determining risk. Equityholders might extract value from debtholders by increasing investment risk, after already existing debt: the "asset substitution" problem.

Mauer and Triantis (1994) treat both the capital budgeting and capital structure decisions as one single dynamic model where the firm has the flexibility to manage those decisions over time. The investment project, in the case of the paper, a production facility, can be closed and reopened according to price fluctuations. The flexibility that the firm has relies on the possibility of issuing both debt and equity and change its capital structure over time by paying recapitalization costs. The tradeoff that we already described earlier is also present in this model aiming the firm value maximization. Mauer and Sarkar (2005) have a similar approach where a firm that has an option to invest is partially financed with debt and

that the timing of the exercise is at the discretion of equityholders. The bondholders will have the chance to set the terms of the debt issue to protect themselves against the possibility of an exercise policy chosen by equityholders that maximizes the equity value instead of the total firm value. Their model will determine the optimal size of debt issue through a tradeoff between interest tax shields, expected bankruptcy costs, and those agency costs arising from a stockholder-bondholder conflict explained above. However, my dissertation will present some different assumptions regarding the optimal capital structure from the previous ones. In leveraged buyouts, the investment opportunity for the bidder will always be a company previously targeted for its underperformance and the main difference that it will present comparing with another kind of project opportunity, like the option to invest in a production facility, is the agency costs related to it. In fact, the agency problems that the target company is facing before the transaction, are significantly reduced after the deal. So, the model of my dissertation needs to take this into consideration while developing the tradeoff between the benefits of using debt, the increasing bankruptcy costs of using it and the agency costs present after the transaction, to determine the optimal leverage to be used.

Still about the optimal capital structure used for acquisitions that can easily lead to high bankruptcy costs, Lyandres and Zhdanov (2013) modeled a real options model taking into consideration the investment opportunities that a firm may have, and not only the current state of the assets in place like most of previous models predict. It states that the mix of investment opportunity and assets in place is an important determinant of the probability of default. Shareholders are willing to wait more and willing to inject more cash into a struggling firm before defaulting, if the growth options' value is high. In the case of bankruptcy, where the firm is surrendered to the debtholders, shareholders will not only lose the cash flows generated by the assets in place but also the right to exercise the investment option that has certainly value, turning the default even more costly than what we could think about. On the other hand, the more valuable the firm's investment opportunity, the higher the value of the residual claims on the firm's assets. The main contribution of their model is proving that there is a negative relationship between the value of investment opportunities and the likelihood of bankruptcy, influencing the timing of default as well. The paper provides both the optimal investment strategy and the optimal default strategy.

Sundaresan and Wang (2006) developed a dynamic investment, financing and default decisions model of a firm with growth options. Basically, growth options will be optimally

exercised over time, with the firm adjusting its capital structure with new debt or equity issue to finance them while considering the tradeoff between tax benefits, distress and agency costs associated to the use of debt. Their paper shows that existing debt may distort investment decisions, firms with more growth options usually have lower leverages to anticipate those distortions and the seniority structure of debt also has an impact on the firm's default, leverage and investment decisions.

3. The Model

Consider an investor that has the option to acquire an all equity firm with management underperformance and a current capital structure that does not capture the true potential of the company's value. We assume the target as a private company, unrelated to potential market mispricing[1]. We further assume the investor as the bidder, for instance a private equity firm created for that purpose. The bidder will issue new debt to finance the acquisition and the target will receive a premium over its current value. I assume that the new debt is issued immediately before the acquisition and the management skills of the acquirer has an immediate impact on the cash flow generation. The probability distribution of the cash flow generated by the target firm, *X*, is assumed to follow the geometric Brownian motion (gBm):

$$dX = \alpha X dt + \sigma X dz, \quad (1)$$

where α is the constant expected growth rate of the future cash flows, σ is the volatility associated to it, and dz is the increment of a standard Wiener process. With the dividend yield δ, being the opportunity cost that the bidder will bear when deferring the decision to invest, we compute α as r - δ, where r corresponds to the risk-free rate.

The investment trigger is firstly defined by the target firm, depending to the premium they require so they accept the offer from the bidder and sell the company, like it happens in Lukas and Welling (2012)[2], and according to that trigger we will be able to determine the optimal coupon and the optimal premium paid to the target. The main contribution of my model is the addition of debt on the equation. In real life, we would see the acquiring firm pay the premium to the target for the transaction and only then start making any necessary change in the capital structure to maximize the new firm value. However, for this model, we will need to consider the capital structure restructuration before any investment decision. Therefore, the optimal future capital structure will be defined first, through the optimal coupon payment, and it will give us the possibility to determine the optimal premium that the bidder optimally chooses to pay. This method will allow the premium determined by the model to capture not only the synergies created by the transaction, but also the potential

[1] As shown by DePamphilis (2009), transactions that are highly leveraged consist on acquisitions of private firms.

[2] An adapted version of Lukas and Welling (2012) model introducing taxes was created to make the comparison.

synergies arising from the new capital structure, as future tax savings, or even the reduction of inefficiencies imposed by the bidder to discipline the management team to repay the debt.

The private equity firm will acquire the target by paying an amount of capital, corresponding to the optimal investment trigger determined by this model, that considers the premium over the target's current value before the acquisition. With the debt issued for the acquisition, the acquirer will be able to invest in the company and benefit with all the synergies created by the deal. It is assumed that the post-acquisition capital structure, will be kept unchanged over time for simplicity. Two main types of synergies are expected to be created with this kind of transaction. The first type of synergies is related to the ones arising from the management skills brought by the bidder. The new management team can, among other changes, impact the production efficiency by eliminating futile processes, increase overall margins, improve its product distribution, all of them able to increase the cash-flows generation after the acquisition[3]. The other type of synergies created come from the leverage used for the acquisition and capital structure restructuration. Commonly associated to such leveraged deals, we know the future tax savings that arise as the main benefit. However, related to the new management team, the use of debt also could increase the potential growth of the target, opening the doors to a wide range of possible investments opportunities and not restricted by the previous investment capability[4]. This potential growth is boosted by the new debt, that also forces the management team brought by the bidder to discipline its decisions to meet all his duties towards the debtholders, increasing the company's efficiency.

The debt is borrowed under a continuous coupon payment of c per year, that is tax deductible at a constant corporate tax rate t. The optimal coupon payment, that already considers the optimal investment trigger, is determined through the maximization of the deal. The bidder will acquire the target by negotiating with debtholders a contract for that purpose. The model can maximize the value that the bidder will receive as function of the present value of the target firm, considering the tax shield, the market value of debt, something that will be explained when developing the model, and the option to abandon the company. The abandon option is acquired by the bidder with the acquisition of the target

[3] As comparison, Parrino and Harris (1999) found that almost all mergers that succeeded had replaced the CEO of the target firm. When the top management remains on board, there is no outperformance of operating returns over industry averages.

[4] Boucly, Sraer, and Thesmar (2011)

firm which allows him to leave the company to debtholders, defaulting on its debt payments and reducing the losses if the deal turns to be unsuccessful. On the other side, the bidder will have to pay the optimal premium to the target, considering that the present value of the future cash flows generated by the target before the acquisition meets the optimal timing and therefore defines the amount that the bidder will have to pay. In addition, the bidder still has to support part of the transaction costs as the usual fees and all the costs related to the integration process.

The model determines all the optimal triggers in the shareholders perspective. The fact that all the decision makers are the shareholders, whom define the first goal as the maximization of the equity's value of the new firm restructured, regardless of the amount borrowed from the debtholders, may lead to potential agency problems between both. However, debtholders already agree on having zero control over the new firm's decisions before the deal is made and are willing to increase their exposure to the risk involved in such transactions. This agreement comes with the fact that the bidder, usually a private equity firm, will not reveal a higher default rate than other similarly leveraged firms. There is, in fact, evidence that show that private equity firms are well prepared to deal with the high leverage after the acquisition, ensuring the fulfillment of all the agreements when comparing with similar companies equally leveraged[5]. Even the private equity firm's reputation and relationships with the banks will have a role on the deal, providing capital at lower cost than what they usually are negotiated[6]. That's why private equity firms are willing to invest in companies that do not seem very glamour to acquire but have a huge potential, taking advantages of their particular skills.

[5] For more details about evidence: Dittmar, Li, and Nain (2012); Kaplan and Stromberg (2009)

[6] Demiroglu and James (2010)

3.1 Definition

3.1.1 The investment timing

Consider first the situation of the target, whether it is willing to sell its company or not, it has the possibility of leaving it by selling to a potential acquirer, a private equity firm. In fact, it is the target that will decide at which point in time will accept the bidder's offer, thus defining the investment trigger for the acquisition. Note that this is relevant for the bidder's point of view, that will try to anticipate the move of the other party and take advantage of the situation. Therefore, according to the bidder's estimation, the target will accept the offer at which the generated cash flows reach the investment trigger. The function $f(X)$ shows how the target defines the investment trigger. The appropriated premium $(\phi > 1)$ and the amount paid by the target for the transaction costs (K_T) will have an impact on the optimal investment trigger, that only considers the actual potential of the firm and not an eventual value creation through the acquisition synergies, at least not directly. Note that the tax benefits and synergies are calculated by the bidder to define the offer, having an impact on the premium. Therefore, the acquirer is able to know in advance how much has to pay so that the offer will be accepted by the target, once his reaction is known before any offer is made. To answer that question, the optimal investment timing, we set the investment trigger as the solution for the following maximization problem:

$$f(X) = \max_{X^*} \left[(\phi - 1) \frac{X^*(\phi)}{\delta} (1 - t) - K_T \right] \left(\frac{X}{X^*(\phi)} \right)^{\beta_1} \tag{2}$$

with β_1 being determined by

$$\beta_1 = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{3}$$

The investment trigger is then determined in function of the optimal premium and defined by the following equation:

$$X^* = \frac{\beta_1}{\beta_1 - 1} \frac{\delta K_T}{(1 - t)(\phi - 1)} \tag{4}$$

3.1.2 The optimal coupon

Consider now the bidding company, that will choose to finance the deal through a determined amount of debt while paying a fixed amount of capital each year (c). The main objective here is to calculate the optimal payment that the new company, that is the target firm post acquisition, will have to pay to maximize the shareholders' wealth. As stated above, the right premium will depend on many factors and one of them is the leverage used for the acquisition. Therefore, we first need to know how much the optimal coupon payment is, as well as all the benefits associated, so we can truly understand how it will impact the deal performance. Knowing the optimal coupon payment, we can then define the optimal premium. Remember that we already know the optimal investment trigger, set by the target. The function $h(c)$ represents the value created by the bidder when it is optimal to acquire the target company.

To compute the optimal coupon payment, we need to consider the impact of the new management team that the bidder will bring with the acquisition. It is expected to see an increase in the management performance $(\theta > 1)$, once the target is initially chosen for its lack of efficiency in that matter[7]. That management performance also captures all potential synergies created with the acquisition, mainly from the experience of previous similar deals. Other benefits come with the acquisition, like the tax shield and an additional post-deal option[8]. The new shareholders will have the possibility to default on its debt payment and transfer the ownership to the debtholders, supposing that the value creation was overestimated, at the abandonment trigger $({X_A}^*)$. This will be the abandon option that the bidder will get, along with the target firm, with the acquisition. Once we know the value of

[7] A good target selection is one the most critical factors that have a tremendous impact on the acquisition's success. The possibility of improving the performance comes necessarily as a good indicator to choose good candidates. However, it will always heavily rely on the bidder's ability to predict the future operating cash-flows.

[8] This post-deal option refers to the option to abandon the company if the deal's performance does not meet the expectations. It is highly valuable for the investors and it can be done by selling the company to a strategic buyer, another financial buyer, an IPO, or even through a leveraged recapitalization where additional debt funds are raised to repurchase the shareholders' stocks. In this model, the exit strategy is seen as an underperformance of the company, something that is not always what happens in reality.

the abandonment option, we can determine the optimal coupon payment that will maximize the deal from the bidder's perspective, through the following maximization problem:

$$h(c) = \max_{c(X^*)} \left[\left(\frac{\theta X}{\delta} - \frac{c}{r} \right)(1-t) - \frac{\phi X}{\delta}(1-t) + \left(\frac{c}{r} - \frac{X_A^*}{\delta} \right)(1-t) \left(\frac{X^*}{X_A^*} \right)^{\beta_2} + \left(\frac{c}{r} - \left(\frac{c}{r} - (1-b)\frac{X_A^*}{\delta} \right) \left(\frac{X^*}{X_A^*} \right)^{\beta_2} \right) - K_B \right] \left(\frac{X}{X^*(\phi)} \right)^{\beta_1} \quad (5)$$

The first term on the right hand side of the equation represents the new value of the target firm that was upgraded by the various changes brought by the new management team with the acquisition. There is also the impact of the new debt issued on the firm value, since the new firm will be able to operate with more flexibility, without the previous problems from capital restrictions. Note that the whole acquisition process will be led by the new shareholders only, that want to maximize their wealth, i.e. the new equity value, and not the global firm value[9]. The commonly observed agency conflicts between the new shareholders and debtholders that could eventually arise from the fact that the decision makers are the new shareholders, are assumed to be none as explained above. Just like the fact that even with a higher risk exposure, the required rate of return of debtholders is likely to remain the same at a low level due to the private equity firm reputation.

The second term represents the amount that will be paid to the target. It corresponds to a premium over the value of the target firm before the acquisition. By accepting the offer, the target will receive that amount, but will lose the value of the company as shown on equation (2). Note that the totality of the payment goes to the target's shareholders, once it is assumed that the company is all-equity.

The third term of the equation stands for the option to abandon that the bidder gets with the acquisition. The new shareholders have the possibility to exercise their option and let the debtholders keep the target company if the cash flow generation was overestimated. By defaulting on the coupon payment, the new shareholders will be exempt of paying the

[9] It is assumed that the new management team follows a policy that maximizes the shareholders wealth. Shareholders bring to the target's board the managers that they know will not act for their own behalf, acting accordingly to shareholders' expectations.

remaining debt but, on the other hand will lose the company, valued by the present value of the future cash flows generated by the company at the abandon trigger.

The fourth term of the equation represents the market value of the debt related to the acquisition. It is divided into two major elements, the first one standing for the value of debt equaling a payment of the coupon for perpetuity. This would only happen when assisting the cash flows generation tending to increase, leading the probability of default going to zero. Therefore, the coupon payment would remain the same to keep an unchanged capital structure. The other element shows what debtholders will receive in case of default. They will not receive anything from the shareholders under the form of coupon payments, transforming the amount lent as a loss but, they will receive the present value of future cash flows generated by the company net of bankruptcy costs (b). This term ensures that the model captures the fair price of the debt once rational debtholders will value their contract differently knowing that the shareholders are expected to maximize the equity value and not the firm value.

Finally, the last term is the transaction costs supported by the bidder for the acquisition. Besides the usual transaction fees, these costs also include the costs associated to the integration process, among others.

After standard calculations, we get the optimal coupon payment, including the optimal investment trigger defined above, as:

$$c^* = \frac{K_T\, r\, \beta_1\, (\beta_2 - 1)\, \theta \left(- \frac{t\theta}{b\beta_2 - t\beta_2 + t(-1+\beta_2)\theta} \right)^{-1/\beta_2}}{(1-t)(\beta_1 - 1)\beta_2(\phi - 1)} \tag{6}$$

Note that we still need to compute the optimal premium that should be offered to the target shareholders so that it is possible to know both the investment trigger and the optimal coupon payment.

3.1.3 The optimal premium

The bidder, knowing what its investment timing and optimal leverage will be for the acquisition, is now able to define the optimal premium that it is willing to pay to acquire the target firm. Note that the premium will capture all the synergies created with the acquisition and the benefits of using debt to finance the deal. Therefore, consider the option to acquire

a firm that is underperforming, struggling to reach its true potential and far from its optimal capital structure. Those factors are just some of the most common elements that bidding firms are looking for when choosing their target. The bidder will have to pay a sufficiently high premium over the current target value so that it will be accepted by the target while considering the sum of the total potential benefits arising from the acquisition. Remember that the bidder already knows what to expect from the target about the investment trigger. The goal is to determine the premium according to what benefits the bidder when investing with the optimal timing and the optimal leverage for the deal. Therefore, the bidding firm will choose ϕ to offer to the target such that it maximizes:

$$h(\phi) = \max_{\phi} \left[\left[\left(\frac{\theta X}{\delta} - \frac{c^*}{r} \right)(1-t) - \frac{\phi X}{\delta}(1-t) + \left(\frac{c^*}{r} - \frac{{X_A}^*}{\delta} \right)(1-t) \left(\frac{X^*}{{X_A}^*} \right)^{\beta_2} \right. \right. \\ \left. \left. + \left(\frac{c^*}{r} - \left(\frac{c^*}{r} - (1-b)\frac{{X_A}^*}{\delta} \right) \left(\frac{X^*}{{X_A}^*} \right)^{\beta_2} \right) - K_B \right] \left(\frac{X}{X^*(\phi)} \right)^{\beta_1} \right] \tag{7}$$

The solution of the previous equation gives us the following optimal premium:

$$\phi^* = \frac{K_B\,(\beta_1 - 1) + K_T \left(1 + (\beta_1 - 1)\,\theta \left(1 - \frac{t \left(-\frac{t\theta}{b\beta_2 - t\beta_2 + t(\beta_2 - 1)\theta} \right)^{-1/\beta_2}}{t - 1} \right) \right)}{K_B\,(\beta_1 - 1) + K_T\,\beta_1} \tag{8}$$

3.1.4 Optimal investment outputs

We are now able to determine any of the three outputs initially proposed by the model. Substituting the solution of equation (8) on both equation (4) and (6) we get, respectively:

$$X^* = \frac{\beta_1\,(K_B\,(\beta_1 - 1) + K_T\,\beta_1)\,\delta \left(-\frac{t\theta}{b\beta_2 - t\beta_2 + t(\beta_2 - 1)\theta} \right)^{\frac{1}{\beta_2}}}{(\beta_1 - 1)^2 \left(t\theta - (t-1)(\theta - 1) \left(-\frac{t\theta}{b\beta_2 - t\beta_2 + t(\beta_2 - 1)\theta} \right)^{\frac{1}{\beta_2}} \right)} \tag{9}$$

$$c^* = \frac{r\,\beta_1\,(K_B\,(\beta_1 - 1) + K_T\,\beta_1)\,(\beta_2 - 1)\,\theta}{(\beta_1 - 1)^2\,\beta_2 \left(t\theta - (t-1)(\theta - 1) \left(-\frac{t\theta}{b\beta_2 - t\beta_2 + t(\beta_2 - 1)\theta} \right)^{\frac{1}{\beta_2}} \right)} \tag{10}$$

4. Numerical Example

4.1 The base case scenario

A more intuitive way to understand how the deal could eventually be done and what should be the bidder investment decision is shown through the following base-case parameter values. To illustrate the model, imagine a firm that has the option to acquire a given private company, with annual generated cash flows of $ 80M. The following table shows the remaining variables:

Input	*Description*	*Value*
θ	Synergies created with the acquisition	1.3
σ	Volatility related to the deal performance	20 %
r	Risk free rate	3 %
δ	Opportunity cost for deferring the acquisition	6 %
t	Corporate tax	20 %
b	Bankruptcy costs[10]	20 %
K_T	Transaction costs supported by the target	$ 120M
K_B	Transaction costs supported by the bidder	$ 180M

Table 1: Base case parameters.

Note that transaction costs already include costs associated with the integration process, supported by the bidder. Applying the model, it is given the following outputs:

Output	*Description*	*Value*
X^*	Optimal investment timing	$ 91.12M
c^*	Optimal annual coupon payment	$ 78.98M
ϕ^*	Optimal premium offered	14.82 %
X_A^*	Optimal abandonment trigger	$ 40.5M

Table 2: Optimal investment outputs for the acquisition.

[10] Altman and Hotchkiss (1993) and Andrade and Kaplan (1998) estimated average bankruptcy costs around 20% of firm value.

The decision to acquire the company immediately or not, relies on the comparison of the optimal timing from the Table 2 and the current value of the cash flow of the target firm. We already knew the target's current cash flow generation of $ 80M, making the delay of the acquisition as the right decision at that moment, once the investment trigger of $ 91.12M has not been reached yet.

Suppose now that the bidder is willing to wait for the acquisition at the optimal timing of that target for its potential value creation[11]. In a scenario where the target firm is able to increase its cash flow until the optimal trigger, the bidding firm would make an offer and acquire the target company. The total amount of cash needed for the acquisition would be around $ 1.575B, with the private equity firm funding itself with a market value of debt near $ 1.237B for the acquisition. It would represent an annual coupon payment of $ 78.98M, and it would make the acquisition leveraged at 78.57%. The optimal premium paid to the target's shareholders would be around 14.82%. Note that the premium will also be partially financed by the new debt issued. If by any chance, the annual cash flow would drop below values around $ 40.5M, its abandonment trigger, the private equity firm would exercise its option to abandon and default on its debt, leaving the company to the debtholders.

[11] Private equity firms may be bidding for multiple targets at a time. Sometimes, having a target that it is still not optimal to invest in can dismiss the idea of investing definitely, once the bidder is likely to give priority to those targets where he can invest right away.

4.2 Sensitivity analysis

To truly understand the role of debt in the timing and the premium of the previous base case scenario and for a leveraged acquisition we will compare this model to the one of Lukas and Welling (2012). Note that their model does not consider any debt to finance acquisitions neither taxes. Therefore, their model needed to be somehow adapted to be comparable with this one and taxes were included. We will treat it as the unlevered scenario. Before starting to compare both investment timing or premium of each model, let's look on the optimal coupon payment drift in this model and how it reacts to input changes.

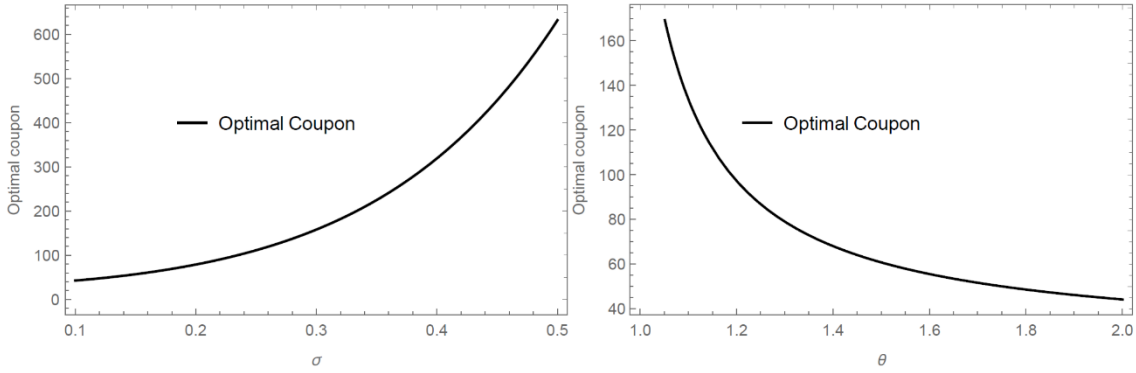

Figure 1: Optimal annual coupon payments variations with different values for volatility and synergies estimates.

The Figure 1 shows the coupon payment that the bidder will negotiate with the debtholders for the acquisition and how it reacts to both volatility and created synergies increases. The left side represents the movement of the optimal coupon when subjected to volatility changes. As the variable increases, the optimal coupon payment also increases, just like it happens with the investment trigger as we will see. Note that the optimal coupon already includes the investment trigger, that tends to be higher, i.e. the private equity firm will delay the investment decision and the transaction will occur later, when the uncertainty about the potential gross project value increases[12]. This uncertainty is shared between the bidder and the debtholders, since a high amount of debt will be used to finance the acquisition, the new shareholders will rationally try to reduce their exposure by increasing

[12] The volatility of the potential gain with the acquisition may be one of the hardest inputs to calculate. Furthermore, its impact on the optimal coupon payment is way more pronounced than the synergies. Both are crucial for the optimal investment triggers and the deal's success, but we can see a higher range for the optimal coupon that uncertainty levels can bring. A slightly miscalculation of the volatility related to the potential gains from the acquisition can have a huge impact on the coupon payment. On the other side, the calculation of the potential synergies created have a lower impact on the coupon payment.

the annual coupon, and consequently the debt-to-equity ratio. The right side will have the symmetric explanation. The private equity firm, estimating high value creation through synergies, will try to reduce the annual coupon payment so they can almost entirely benefit from it with the acquisition. In the same way the bidder shares the exposure to the acquisition's volatility, it will also share the potential profits arising from the synergies created by the acquisition, since it will have to pay the annual coupon. It is then rational to expect that the bidder, in the case it estimates high synergies value creations, will reduce its debt-to equity ratio to maximize its profit. Their goal is to maximize their wealth and not debtholders'.

After analyzing the optimal coupon payment, we are able to understand the various optimal capital structures[13] used for the acquisition, influenced by those same variables. The annual coupon payment will influence the ratio but also the trigger and the premium as we will see. Results shown on the following graphs:

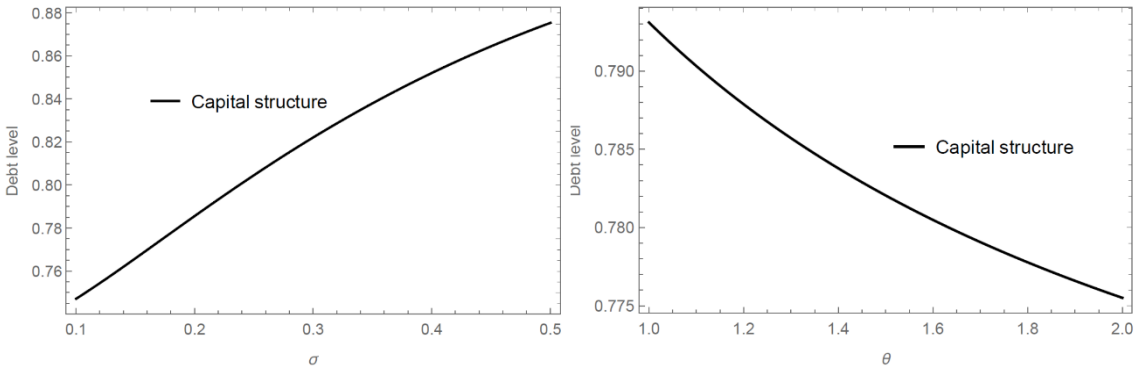

Figure 2: Optimal capital structure used for the acquisition depending on different values for volatility and synergies estimates.

On the left we can see the capital structure used by the bidder for the acquisition, according to different levels of uncertainty. Easy to understand that as the volatility increases, the debt level for the optimal capital structure will increase mainly due to the high coupon payment shown above. Note that the volatility also has an impact on the investment trigger and the premium, that will be shown below, making the total acquisition investment varying

[13] Capital structure shown on Figure 2 will always represent the fraction of debt used by the private equity firm for the acquisition.

with uncertainty[14]. However, as we will see, the increase on the investment timing due to uncertainty will be so high that will offset the reduction on the premium paid and increase the total acquisition payment. Consequently, as we increase the uncertainty levels, we see a slower increase of debt in the capital structure than the optimal coupon increase. Assuming moderate levels for the volatility estimated, the optimal leverage level would be always between 74% and 80%[15].

On the right side of the Figure 2, the results present the opposite trend explained above as we increase the value of the synergies created. On that scenario, the synergies created by the acquisition will induce the bidder to decrease the amount of debt used to finance the transaction, through the reduction of the optimal coupon payment. However, the optimal investment timing and the optimal premium will also be influenced by the value of synergies, making the global acquisition payment different. As we will see with more detail next, an increase of synergies will induce the bidder to invest way sooner, since it reduces the optimal investment trigger. Therefore, we will see a reduction on both the acquisition payment and the optimal annual coupon. The fact that the reduction on the optimal coupon payment is significantly sharper than the acquisition payment, the debt level for the acquisition will start to decrease. Note that these reductions do not mean that the bidder is willing to invest less when estimating high potential synergies, it would make no sense. It does mean that it is willing to invest way sooner in the acquisition, once those are optimal investment triggers.

The following figures will show the impact of debt on the remaining outputs provided by this model and compare them with those from the unlevered scenario model, the adapted model of Lukas and Welling (2012). The first one will be the optimal investment trigger. We know from previous works that the investment timing tends to occur later when there is high uncertainty associated to the project, forcing the company that has the option to undertake

[14] The capital structure used is obtained through the ratio between the debt value and the total acquisition payment. The debt value is determined as shown on equation (5) from the model development. The total acquisition payment (AP) is calculated with the following equation:

$$AP = \frac{\phi^* X^*}{\delta}(1-t) + K_B$$

[15] Target companies are usually chosen by private equity firms for their cash flow predictability, leading to low or moderate levels of uncertainty related to the deal.

the project to delay the investment and wait for more information. The way it happens with acquisitions will be identical, with the variations induced by known variables shown below:

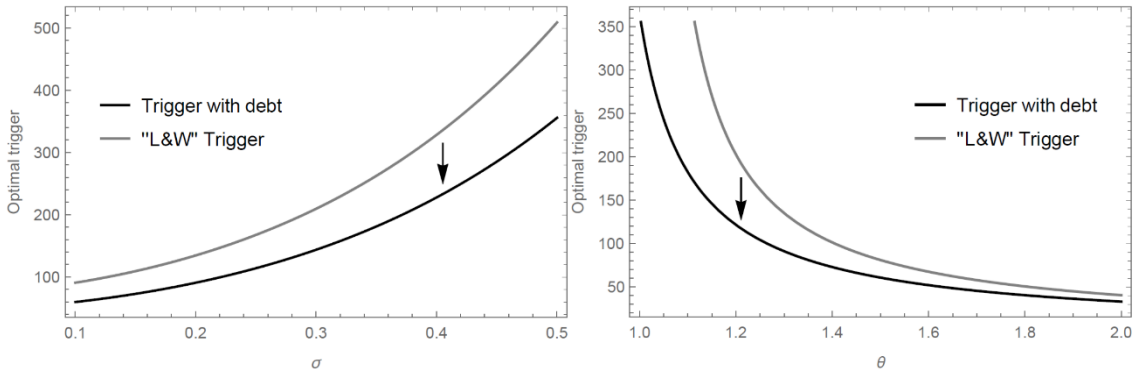

Figure 3: Optimal investment triggers for both the model from this dissertation and the adapted model from Lukas and Welling (2012), influenced by volatility and synergies changes.

Looking to the left side of Figure 3, the conclusion is straightforward. As the uncertainty level starts to increase, we will see an investment trigger tending to high values in both models. However, my model, where acquisition have debt involved, shows a constantly lower investment timing. But why? The reason is that leverage will provide the bidder the opportunity to share this uncertainty related to the acquisition's success or failure, with the debtholders. We know that leveraged buyouts have a tremendous number of elements that could impact the success of the deal. And this works for both sides, the deal could be a complete disaster and the investment made would be loss, at least partially, or it can be a total win. By reducing its exposure, investing less of its own money and more of debtholder's one, the bidder will be able to anticipate its timing decision, and invest sooner on a potential target, when comparing with the unlevered scenario, where there is no debt involved and the investors are forced to bear all the risk by themselves. In other words, we could say that the bidder reduces the influence that total value of the deal's volatility has on the investment timing, allowing the bidder to anticipate the decision by financing the deal with debt. By allowing the entry of debtholders in such deals, the new shareholders have the opportunity to undertake risky projects, without necessarily increase the potential downside associated to them. It is true that the upside will also be limited, once part of the synergies created are used to pay the debt issued, but with capital restrictions, one bidder could make an offer to multiple suitable companies and acquire more than one without having to pay all the amount from his pocket. Bidders are more likely to diversify their investments in that

way, and with club deals where multiple bidder invest together on the same target[16]. It clearly shows their concerns on minimizing their risk exposure but keeping the reward as high as possible.

On the right side we can see the impact of the synergies created by the acquisition on the investment trigger. We first need to understand that the trigger is higher with low synergies, but even higher with the unlevered scenario. Low synergies usually delay the investment timing, forcing the bidder to collect information that could help him to increase its synergies estimates, for instance, through the development of new technologies able to improve the target's efficiency significantly. Otherwise, the trigger could be never reached and there would be no deal at all. The investment will occur later, in both models, if synergies are low. Again, we can see a different impact on each of the models from the synergies created. Both triggers decrease with increasing synergies, but in the unlevered scenario the investment will occur constantly later. It shows that private equity firms take profits from the debt benefits empowered by the deal beyond the profits from the synergies created. Note that, when using debt to finance the acquisition, it is needed a lower synergy increase to anticipate the investment decision. This happens because of debt, again. In addition, remember that the target will appear as good candidate after the target selection with more facility, so that the bidder is willing to acquire the company, as long as they do it alongside with debtholders, showing us that it no longer needed to wait that much to reach that initial trigger defined by the model on the unlevered scenario. Therefore, there will be way more good targets that will provide the potential to succeed if acquired and the bidder will be capable to invest in most of those opportunities, something that would hardly happen if the bidder went all-equity in those deals.

In the case of unlevered model, the new shareholders will be more dependent on the reliability of their estimates, both uncertainty and synergy creation, making it a crucial element for the success of the investment decision. With a bad project evaluation, that could lead to an unsuccessful deal, the bidder will bear bigger losses if it invests alone, even if it could sell the company at a lower price, something that is not included on its model. In my model, those losses can be transferred, at least partially, to the debtholders if something went

[16] Club deals usually allow private equity firms to access cheaper capital when comparing with investing alone. Note that this collaboration happens when the acquisition requires an investment too high, making it difficult for a single bidder to raise enough of its own equity capital without going all-in on that deal.

wrong. Furthermore, the likelihood of finding the good target for investing is also way bigger in my model since it values the option to abandon the company if needed at some point.

We will see now how the premium that the bidder is willing to offer reacts to some input changes.

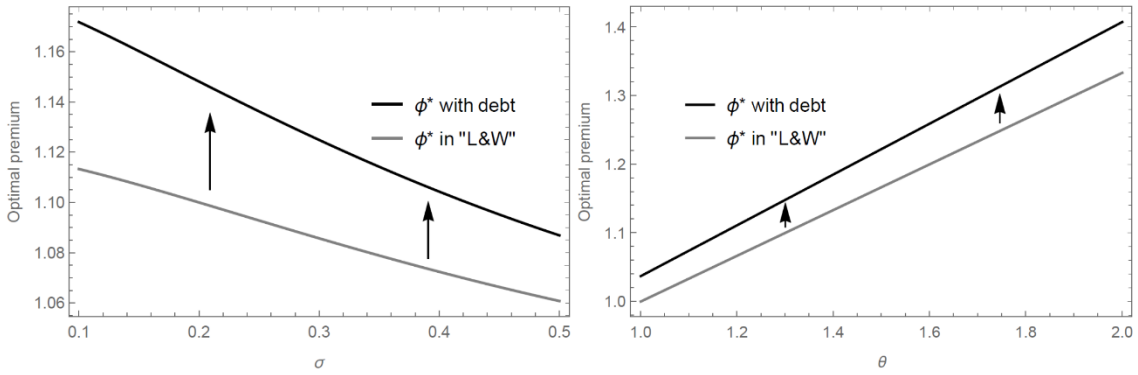

Figure 4: Optimal premium offered to the target for both the model from this dissertation and the adapted model from Lukas and Welling (2012), influenced by volatility and synergies changes

On the left side of Figure 4, we observe that the premium paid will start to decrease in each of the different models while we assist to an increasing uncertainty level. On the other hand, if the bidder chooses to include debtholders in the deal, helping partially with the acquisition, it is optimal to him to pay a higher premium when comparing with one that does not leverage the acquisition, *ceteris paribus*. In the unlevered scenario, we can see that for lower uncertainty levels, the difference between premiums tends to be higher. Remember what happened with the optimal coupon payment with high volatility, the global leverage was higher, allowing the new shareholders to transfer the risk associated to the acquisition to the debtholders and invest less of their own money. We can see the exact same thing here, the bidder is willing to invest in riskier opportunities since they do not bear all the risk, offering to the target higher premiums. From another perspective, we know that high volatility can provide greater profits than expected, 'higher the risk, higher the reward', but also higher potential losses. As long as the uncertainty towards the acquisition keeps increasing, the premium that the bidder is willing to offer will start to decrease, regardless of the possibility of using debt or not. The bidder tries to reduce the potential losses that the higher volatility could eventually bring, once it also increases the probability of that happening. However, leverage will always benefit the bidder in a way that it will be able to optimally increase the offered premium. Offering a higher premium will certainly ensure them to beat any competitor that may try to invest in the same target. In the same line of reasoning, new

shareholders can acquire companies with higher risk associated without necessarily increasing their exposure, paying the same premium than an all-equity investor would be willing to offer on a similar project but with lower uncertainty, and with that invest in opportunities that might reward them significantly better.

Looking to the right side, the understanding is pretty simple, showing us that the bidder is willing to give a higher premium when observing higher potential synergies from the acquisition, converting them in higher estimations for both models. We can see a little increase in the premium paid if there is debt associated to the deal, with both models showing the same moving trend. By using leverage to finance part of the acquisition, the bidder will have an advantage over potential competitors that might be interested in the same target but going all-equity. The truth is that debt provides to the bidder a more flexible investment decision, increasing its negotiation power and invest in better opportunities than its competitors that doesn't recourse themselves with debt[17]. It is clear to see that debt can benefit the investor in way more aspects than what we usually think it can, like the tax shields, higher management discipline or capability to undertake profitable growth opportunities, appearing as a powerful tool that cannot be disconnected to any investment. This negotiation power increase, associated to its higher investing capacity and lower required investment levels, really helps private equity firms to acquire struggling companies trough leveraged buyouts and enhances the chances of increasing the takeover activity.

[17] Dittmar et al. (2012) makes the distinction between financial buyer and corporate buyer. It shows that when corporate buyers compete with financial buyers for the same target, they usually pay lower premium. It corresponds to what the model predicts, with the financial buyer, expected to leverage the acquisition at its maximum, paying the highest premium. All other and lower debt-to-equity ratios used for the transaction will necessarily lead to a lower premium.

5. The Role of Mispricing

Mispricing can certainly influence the total acquisition payment, making it an important element when trying to avoid overpaying for the target. In this section we will see how the mispricing that hits the target firm can have an impact on the bidder's decisions[18]. It is assumed that the parameter capturing the target's mispricing in acquisitions (λ) will be always higher than zero. Note that for the sensitivity analysis for mispricing that will follow, whenever the parameter is equal to 1, it will show the fact that the mispricing is inexistent. Therefore, we included the new variable to the model, and it allowed us to quantify the impact of mispricing, comparing the results obtained when the target firm is not priced at its true value and when it is. Despite of the target being public or private, it is assumed that for each case, the mispricing involves the fact that both of them fail to expose their true value to outside investors due to a sequence of events in favor or disfavor of each company. For public firms, this mispricing can be easier to obtain since we can compare its fundamental value with its market price, showing how the market perceives the company, something harder for private companies where there is not a mechanism that can provide a way to obtain all the investors' feelings about the company. Nevertheless, even private firms will transmit a certain idea of what they worth to outside investors, that will be interpreted differently, regardless of their fundamental value. The bidder will use this information to price the target and will decide its investment based on its fundamental value. We assume the acquisition as friendly. With mispricing, optimal values for the offered premium, the investment timing and the annual coupon were determined as it follows:

$$\phi^* = \frac{\mathrm{K}_T + \mathrm{K}_B\,(\beta_1 - 1) + \dfrac{\mathrm{K}_T\,(\beta_1 - 1)\,\theta\left(1 - \dfrac{t\left(-\dfrac{t\theta}{b\beta_2 - t\beta_2 + t(\beta_2 - 1)\theta}\right)^{-1/\beta_2}}{t-1}\right)}{\lambda}}{\mathrm{K}_B(\beta_1 - 1) + \mathrm{K}_T\,\beta_1} \tag{11}$$

$$X^* = \frac{\beta_1}{\beta_1 - 1}\,\frac{\delta\,\mathrm{K}_T}{(1 - t)\,\lambda\,(\phi^* - 1)} \tag{12}$$

[18] Analyzing the mispricing of the private equity firm, the bidder company, and how it influences the acquisition would be an interesting extension for future models.

$$c^* = \frac{K_T\, r\, \beta_1\, (\beta_2 - 1)\theta \left(-\frac{t\theta}{b\beta_2 - t\beta_2 + t(\beta_2 - 1)\theta}\right)^{-1/\beta_2}}{(1-t)\,(\beta_1 - 1)\,\beta_2\,\lambda\,(\phi^* - 1)} \tag{13}$$

Consider the same target firm from the base case scenario assuming now that the company is underpriced by 10%. If that was the case, the optimal investment timing would decrease from $ 91.12M to $ 74.39M. This underpricing made a significant change on the investment timing, allowing the bidder to proceed with the acquisition, once the generated cash flow would remain at $ 80M. Another important change is the increase of the premium that the private equity firm is willing to offer to the target, reaching 20%. The deal would also cost lower to the bidder, $ 1.575B from the base case scenario against $ 1.371B with a 10% underpricing. The optimal coupon payment would be no exception, decreasing to near $ 64.46M annually and corresponding to a decrease on the optimal leverage level to values around 73.6%.

So, those new triggers allowed us to quantify the effect of mispricing on the investment timing, optimal premium and optimal annual coupon payment. Additionally, here are the results about the total acquisition payment and the capital structure used for the acquisition:

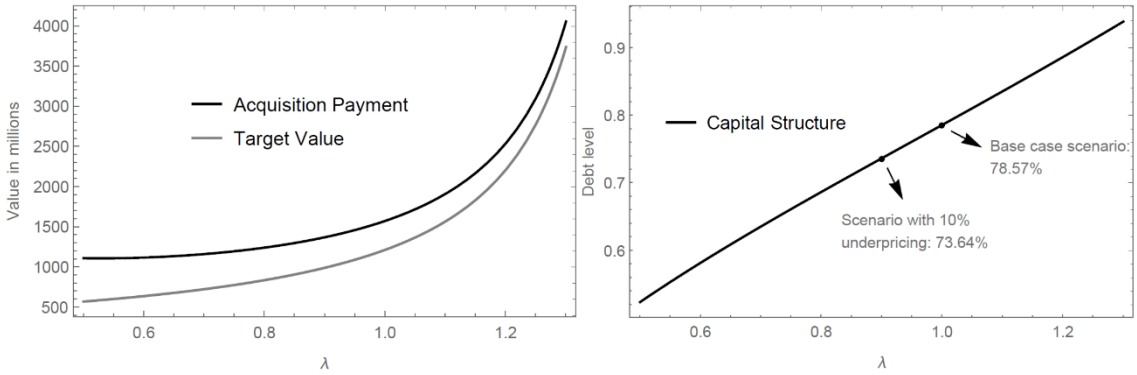

Figure 5: Optimal acquisition payment and optimal leverage level depending on the accuracy of the target's pricing.

As we can see on the left side of Figure 5, λ shows how the target company is perceived by the outside investors and how it is optimal to pay for the acquisition. As the company tend to be overpriced, (λ > 1), we see a great increase on the total acquisition payment. Once it is well seen by the outside world, the target's shareholders will be reluctant on selling their company, increasing the investment trigger for the bidder[19]. This will clearly delay the timing for investing, making the deal almost with no chances of happening if the mispricing keeps

[19] Remember that the investment trigger is defined target firm.

with that trend. On the other hand, this is where the target accepts the smallest premium, knowing it is the chance of one life to sell their company at that price if they hypothetically received an offer. We can see that the target value starts catching the optimal acquisition payment, showing the decrease of the premium offered as the overpricing keeps increasing. Similarly, if the target company is underpriced, the acquisition will tend to occur sooner since the total acquisition payment becomes lower mainly due to reduction of the optimal investment timing. However, this is where the target's shareholders are not willing to accept the usual premium paid for companies in such deals. They will require a higher payment due to the underpricing, possibly seen by target's shareholders as unfair by not revealing the real value of its company. On the bidder's perspective, there is a great chance they look for underpriced targets, knowing that it is the best way to reduce its total acquisition payment, regardless of the premium paid, knowing the fundamental value would remain unchanged. When looking on the capital structure used according to the mispricing variations, we can see that as the target tends to be overpriced, the debt level will tend to increase. The effect of mispricing on the optimal annual coupon, that tends to increase when the target goes from underpriced to overpriced, is way larger than the effect on the total acquisition payment, making the capital structure increase the amount of debt used.

When looking to the investment trigger, we can see that mispricing can have a huge impact on the same investment decision. For the same target, the way the bidder should invest will be very different depending on how the market is pricing the company. Figure 6 shows the optimal investment timing when the target company is not priced as it should:

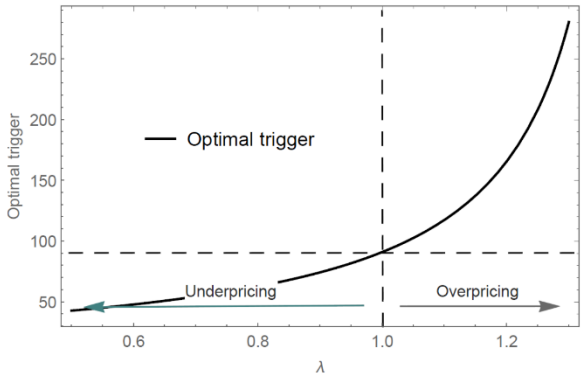

Figure 6: Optimal investment trigger reflecting different pricing possibilities for the target company.

The first thing we need to understand is that the optimal investment trigger tends to increase when observing the target going from underpriced to overpriced. It is quite obvious that the bidder will be reluctant to invest in a target overpriced. Remember that the

overpayment is one of the most critical elements that turn acquisitions into complete failures. However, as long as the target is underpriced, the investment trigger will be relatively constant, slowly increasing until reaching the point where the target's price equals its value. Once that limit is crossed, the optimal investment trigger starts to increase drastically, turning it nearly impossible to be reached and therefore we will see no acquisition at all. That second half of the Figure 6 shows clearly the fear of overpaying that every bidder keeps in mind and tries to avoid at all cost.

A similar analysis fits to the optimal premium that the bidder is willing to offer. As the target goes from underpriced to overpriced, we can see the reduction of the offered premium, also explained on the total acquisition payment. Besides the fact that the target is willing to accept even smaller premium while its overpricing increases, it also can be somehow a mechanism that allows the bidder to reduce eventual miscalculations about the potential gain of the deal. As the target price is approaching its fundamental value or even already exceeded it, the bidder starts to reduce its margin of error about the success of the transaction by reducing the premium that it is willing to offer. This is represented on the Figure 7 below:

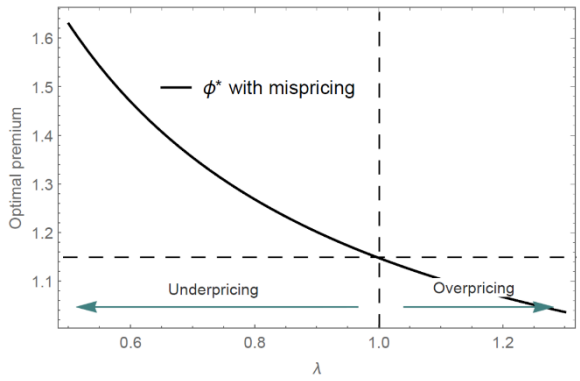

Figure 7: Optimal premium offered variations to different pricing possibilities for the target company.

6. Conclusion

The model provides the optimal investment triggers supposed to help private equity firms on their journey before the leveraged buyouts. It is given the optimal investment timing, showing to the bidder whether it is optimal to acquire the target company or to delay the acquisition. If it is optimal for the buyer to acquire the company, the model also provides the optimal amount of debt they should issue for the transaction and the optimal premium that should be paid to the target. The model is also capturing the effect of mispricing on the target firm, avoiding on its outputs eventual miscalculations that could lead to overpayment and consequently the failure of the deal.

We also saw the effect of uncertainty about the potential gains from the target firm after the acquisition, showing us that it is greater than the impact of synergies created. In fact, private equity firms, also known as financial buyers, do not seek for great synergies on the target, they rather take the maximum advantage of the debt benefits. We also saw that debt does not only help private equity firms in their acquisitions with the commonly known benefits as tax shields, discipline increase or higher capability in undertaking growth opportunities, it also helps and allows the buyer to increase the offered premium to the target, beating the competition when there are multiple bidders. The fact that financial buyers do not look for synergies the same way other buyers do, allows them to find a larger sample of good candidates suitable to be acquired through a leveraged buyout. The use of debt will increase its negotiation power and enhance the takeover activity.

Some other testable implications provided by my model would be interesting to observe, as the timing observed on acquisitions with the purpose to take the biggest advantages of synergies versus leveraged buyouts that tries to focus on debt benefits. It is obvious that both strategies will take some profits from the other side, leverage buyouts also seek for synergies and corporate acquisitions also include debt, but the focus of each one will really define different results for the success of each deal. Another useful test could be done to how the mispricing that hits the target firm affects the premium that the bidder is willing to pay or the optimal capital structure that it will have to choose to maximize the deal's worth.

7. Appendix

7.1 The optimal abandon trigger

The valuation of the option to abandon the company after the acquisition was a must for the model. The optimal coupon payment is a central element of this model and it was not possible to reach to a reliable answer without this option to abandon. It is also very valuable for the bidder to have this exit strategy available. So, we will define that same option and the associated abandon trigger. The acquisition is accomplished once the information acquired over the years ensure a sufficiently high accuracy for its estimations related to the future cash flows, and most importantly, to its future synergies. Then, the option to abandon will be exercised by the new shareholders only when the cash flows generated do not meet the initial expectations and fall below a certain level, the abandonment trigger[20]. The acquirers know exactly how to react to this eventuality well before the investment is made. The option value for the abandonment after the acquisition, $F(X_A)$, following the standard arguments[21], must satisfy the following usual ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 {X_A}^2 \frac{\partial^2 \mathrm{F}(X_A)}{\partial {X_A}^2} + (r-\delta)X_A \frac{\partial \mathrm{F}(X_A)}{\partial X_A} - rF(X_A) = 0$$

The general solution of the previous equation come as:

$$F(X_A) = A_1 {X_A}^{\beta_1} + A_2 {X_A}^{\beta_2}$$

where A_1 and A_2 are constants to be determined and

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

[20] Note that the abandonment trigger for the new shareholders will not be the same than some existing ones. After the acquisition, the optimal abandonment trigger will be influenced by the synergies effect.

[21] Dixit and Pindyck (1994) for more details.

All other variables are already explained when on the section where the model is developed. Then, to determine the solution for the optimal abandonment trigger post acquisition, we must set the following boundary conditions:

$$F(\infty) = 0$$

$$F(\mathrm{X}_A{}^*) = \left(\frac{c}{r} - \frac{\mathrm{X}_A{}^*}{\delta}\right)(1-t)$$

$$F'(\mathrm{X}_A{}^*) = -1$$

The first boundary condition stands for the fact that as the cash flows generated by the target keep increasing, the value associated to the option to abandon the company freshly acquired also tends to zero, since there will be no point of leaving a performing company where the probability of this going wrong is none or minimal. The second boundary, also known as the 'value matching' condition, represents the net value that the new shareholders will receive by leaving the company to the debtholders at the optimal abandonment timing. By exercising the option, the new shareholders will receive the value of the remaining debt, not directly although, but they won't need to pay that to the debtholders as initially agreed before the acquisition. On the other hand, they will lose the value of the company determined by the present value of current cash flows at the abandonment trigger. In that case, at the trigger, the value of the option must equal the net present value that the new shareholders will receive by leaving the company to debtholders. The last boundary, known as 'smooth-pasting condition' or 'high contact condition', ensures that the two value function tangentially meet at $X_A = X_A{}^*$.

Then, we need to set A_1 equal to zero so that we can respect the first boundary condition once $\beta_1 > 0$. Otherwise, as $X_A$tends to +∞, $A_1\ X_A{}^{\beta_1}$ will tend to +∞ as well and violate that condition, remember the general solution. After some standard calculus with the remaining boundary conditions we find the solution for the abandonment trigger value as:

$$\mathrm{X}_A{}^* = \frac{\beta_2}{\beta_2 - 1}\frac{\delta c}{r\theta}$$

8. References

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