
Investment on On-The-Job Training – A Real Options Approach

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Biographical Note

Pedro Alves Magalhães de Almeida was born in Porto, in 1995. He completed the BSc in Economics in 2017 at the School of Economics and Management of the University of Porto. He also studied for 6 months in Prague, at the Charles University, under the Erasmus program.

After completing the bachelor's degree, Pedro enrolled in the Master in Finance program of the School of Economics and Management of the University of Porto. The present dissertation is the last stage of the Master's degree.

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Abstract

We developed this model in order to understand when and how much investment is made in the training of employees and to ascertain the impact of certain factors, like costs, increase in productivity, generality of the investment and uncertainty, with the help of previous literature on on-the-job training and the Real Options framework, given the characteristics of this type of investment, which allowed us to have a different perspective on this important issue for companies. In order to give a better understanding of the model we present a numerical example so as to understand the impact of different variables by themselves and also their joint effect, by pairing them two by two.

Sumário

Desenvolvemos este modelo para entender quando e quanto investimento é feito na formação de funcionários e verificar o impacto de certos fatores, como custos, aumento de produtividade, generalidade do investimento e a incerteza, com a ajuda da literatura anterior sobre formação profissional, com a ajuda das Opções Reais, dadas as características deste tipo de investimento, que permitiu ter uma perspectiva diferente sobre esse importante assunto para as empresas. Para melhor compreensão do modelo, apresentamos um exemplo numérico para entender o impacto individual de diferentes variáveis e também o seu efeito conjunto, juntando as variáveis duas a duas.

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1 Introduction

Although there has been a lot of literature studying on-the-job training (OJT from now on) detailing its costs and benefits for both employers and employees, in my personal opinion a study of this phenomenon through the optic of real options has been lacking, given the characteristics of this type of investment.

We can see, generally, in this type of investment, the 3 characteristics that make the real options approach useful: uncertainty, flexibility and irreversibility.

Regarding uncertainty, the skills earned in this training may or may not become useful if some new technology becomes available that requires the specific skills taught by this training. On the other hand, those skills may be useful given the present technology, but if this technology becomes obsolete, the same may happen with the skills acquired through OJT.

As far as flexibility is concerned, like most investments, the employer has the option to either delay the investment, or even change the plan of training, because of technological changes for instance.

As for the irreversibility, when an employer provides training to a worker, it must either resort to outside trainers and pay them, or the training must be done by internal personnel. In this case, the person who is giving the training is, less productive or even stops producing in order to give the trainee proper guidance.

Considering the importance of human capital in general, and of OJT in particular, on economic growth, being a crucial factor on the growth of countries, firms and individuals, it is important for companies to correctly assess it, not only from a static perspective, but taking into account all the uncertainty surrounding OJT so that it represents an investment that creates value.

The aim of this dissertation is to come up with a real options model which may help employers make the decision to invest or not on OJT and aid them in calculating the best timing to do so, given the type of investment: general or specific. This model will have to take into consideration the economic environment, through the appropriate variables.

2 Literature Review

As we are studying a specific subject, OJT, in the larger field of Human Capital, which also includes schooling and education in general, it is important to know that the roots of this field go back at least to Smith (1904), who started to treat skills as capital, and developed later by Schultz (1962), Mincer (1962) and Becker (1962).

In a 2010 study by Albert et al. (2010) regarding the determinants of training, it was stated that workers with “high educational attainment, working in high-skilled non-manual jobs, and working in large firms and in certain service industries” were more likely to participate in training activities.

2.1 General and specific training

More related to our specific topic, Becker (1962) makes the distinction between general and specific training: while specific skills are only useful for a certain employer, general skills benefit multiple employers.

He also predicts in his model that, assuming a perfectly competitive market, general training, given its characteristics, results in both the costs and benefits being borne by the employees, with their wages reflecting this, being equal to subtracting the training costs to their productivity, while also having the benefit of a higher salary after the training, in the value of their productivity. Meanwhile, the company bears no costs and does not receive any benefit.

On the other hand, in specific training, costs and benefits will normally be shared by the workers and the employer, with the wage of the worker after the training being somewhere between the value of the marginal product for the current employer and the value of the marginal product in any other company. This type of training imposes losses upon separation: in case an employee quits, the employer loses part of its return on the investment; in case of a dismissal, a loss is imposed to the worker. When the benefits of the party initiating the separation are smaller than the losses imposed to the other part, we are talking about an inefficient turnover. To avoid this, division of the returns of the investment can be used in order to minimize inefficient turnover.

This, however, is not the only mechanism to minimize this issue. Mortensen (1978) presents another one: an agreement made before the separation that determines a compensation from the part initiating the separation to the other part, which has a loss without this kind of agreement.

Some empirical studies afterwards cast a doubt in Becker’s conclusion on general training. In their article, Frazis et al. (2007) developed a theoretical model which led to the conclusion that costs and benefits may be shared by employers and employees, if the training provided by employers raises mobility costs, if there are constraints in lowering wages (like minimum wage), if there is uncertainty about the value of training at competing employers or if any other market failures exist in the economy.

Nowadays, the consensus is that when the training is firm-specific, employers and employees share both costs and benefits, and that employees only pay and get all the returns from training if the labour markets are perfectly competitive, meaning there are not labour market failures, which is a very specific case. In all other cases there is room for both companies and employees to share both expenses and benefits, like it happens with specific training. The share of each part will depend on the impact that market imperfections have on the labour market.

2.2 Degree of formality

According to Frazis et al. (2007) the databases on OJT became more widely available in the two decades prior to their paper, which is a “survey of the recent literature on on-the-job training” and will be very helpful for this dissertation. Also, according to them, the databases on formal training are more widely available than the ones that include informal training, since for the latter both the employees and employers have more difficulty in recalling that type of training.

Despite this, Loewenstein and Spletzer (1999) refer to the EOPP (Employer Opportunity Pilot Project) survey, which tried to capture data on informal training at the beginning of the employees’ tenure. Still, most databases rely on formal training, given the additional difficulty to build reliable databases on informal training. The NLSY (National Longitudinal Survey of Youth) of 1993 is an example of this. The development of these databases is crucial to allow researchers to estimate the effects of training on the evolution of wages and productivity, variables essential to the development of our model.

According to Frazis et al. (1998) on a study analysing the results from the 1995 Survey of Employer-Provided Training in the US, for companies with 50 or more employees, there is a difference in the prevalence of the two different categories, with formality of training increasing with earnings, size of the company and tenure (the higher difficulty of dismissing/quitting a job increases the costs of separation, which may surpass the benefits from separation, thus reducing turnover and consequently increase the amount of training). Informal training has a high prevalence for all workers (90%).

The youngest and specially the oldest employees were less likely to have received formal training in the previous year. The total number of training hours followed a similar trend. The difference between these two groups and the remaining workers is especially significant in the average hours of formal training hours per worker. Furthermore, amongst the youngest workers, only 11% of their training hours were spent on formal training, significantly different from the rest of the workers.

“The relatively low hours of formal training among the very young provides some support for the notion that employers or employees (or both) may be delaying their investment in costly training until the employer-employee match is found to be a good one.” Frazis et al. (1998, p.10)

Additionally, black people tended to receive about the same formal training hours, on average, but a lot less informal training hours than white people.

Regarding education, almost 90% of the workers with a bachelor’s degree or higher received formal education, and this group having as well the highest average of formal training hours, but not when it comes to total training hours. Making a geographical comparison, there are significant differences in the degree of formality between different countries. According to Bassanini et al. (2005), in Europe formal training is required to attend some higher education in France, as well as in Austria, Ireland, Netherlands and Denmark, for example.

Comparing just OJT training as a whole, the percentage of employees taking some training per 12 months is higher in the Nordic Countries and lower in Eastern Europe, and this difference is substantial. For example, annual training participation varies between 10.8 % in Romania and 60.1 % in Sweden, while average annual hours per employee vary between 6.7 in Romania and 36.1 in Denmark. Training hours per participants are much

more homogeneous across countries, varying between a low 43 hours in the United States to a high 80 hours in Ireland according to data from OECD. According to the data, lower the average participation rate in training, higher the dispersion between the countries' different regions.

2.3 Division of costs and returns

Frazis and Loewenstein (2006) slightly modify a model previously presented by Hashimoto (1981) that minimizes the chance of an inefficient turnover by choosing an appropriate post-training wage, something that was previously pointed out by Becker (1962), but not formalized.

In the two-period model, it is not assumed that the other companies perceive all the gained marginal product by the worker through the process of training, meaning that the perceived value of the human capital of the worker can be inferior to the other companies than for the current employer. This can be the case if the training is not entirely general, or simply because the other employers do not fully realize the full potential of the given training. The model also considers that there can be a firm-specific shock that can make the effective value of the worker higher or lower than expected. However, this shock is assumed to have a mean equal to zero.

If the expected value of the worker for the company is lower than his wage, the company proceeds to dismiss the worker. Additionally, if the worker's wage plus the value that they put on working in that company (given by benefits besides the wage) is inferior to their alternative wage elsewhere, minus the moving costs of moving to that other job, the worker will quit his current job.

Given this values and variables, the authors proceeded to estimate the probability of a layoff and of a quit, and given those values, integrate them in an equation that determines the expected utility of the employee and of the employer (profit), given the appropriate discount rate.

The employer will then choose the wages of the first and second period in order to maximize its utility, given the restriction that employee's utility in the present employer is higher or equal than the utility elsewhere.

The optimal wage contract will then be the one that maximizes the sum of the gains of both parties, subtracting the utility of the worker elsewhere.

According to this model, there is only a way that the predictions of Becker are confirmed: when the general training is fully recognized by other employers and the cost from switching jobs is independent from the worker's human capital stock, the worker will have all the benefit (higher second-period wage) and bear all the costs (lower first-period wage).

2.4 Reasons why costs and benefits may be shared in “general” training

As stated before, Becker's predictions may not hold when there are certain market imperfections, making general training “specific” in economic terms. According to Acemoglu and Pischke (1999) if employers have some monopsony power, it will allow them to pay workers less than their productivity, meaning that employers can support some of the costs and get part of the benefit as well. They also demonstrate that the wage function increases less with the level of training than with productivity, what they call compressed

wage structure. The difference between the two curves is the profit that a company has while employing a certain worker, resulting in a company preferring to employ more skilled workers. This is one of the reasons that lead companies to invest in OJT.

In their article, they explain the reasons for this.

The first is the presence of transaction costs in the labour market, making it more difficult for workers to quit jobs and find new ones, but there are also transaction costs for companies to find new employees. So, both sides will have a surplus that will be shared by bargaining.

The second is the existence of asymmetry of information between the current employer and the others, since other companies may not notice the totality of the skills that a worker has acquired elsewhere. In that case, they are unwilling to give full compensation to the worker. Another source for this asymmetry of information was earlier indicated by Acemoglu and Pischke (1998): when ability and training are complementary, the more skilled workers benefit the most from training. This introduces asymmetric information, since a worker needs to be observed on the job so that the employer can fully recognize his ability.

Another important reason pointed out by the authors is that, most of the times, specific training help enhancing the effects of general training, both being complementary. This also leads to wage compression. These were the main reasons causing wage distortion, and consequently why companies and employees may share the benefits of general training.

2.5 Effects of training on wages and productivity

Regarding the effects on wages and productivity, Frazis et al. (2006) found high returns on both, and also that all researchers find that the returns on productivity are higher than the ones on wages.

Multiple regressions have been run using longitudinal data from the 1979 through 2000 waves of the 1979 cohort of the National Longitudinal Survey of Youth. Several of them were made by Frazis and Loewenstein (2006) in the attempt to take into consideration the ones they considered to be the major sources of bias: heterogeneity in productivity, in job matches, in wage growth between-person and between-job and in the effects of training; job promotions coincidental with training periods; incorrect functional form and measurement error. They arrived to an annualized rate of return of training in wages of 80%, considering the median 57 hours of training of the data used to estimate the regression. Considering costs of training other than wages of employees receiving the training, they arrived at a rate of 48%. They note that this rate does not take into consideration promotions, but concluded that a plausible rate of return considering promotions should be between 40 and 50%.

A more recent study was made by Lee (2009) using data from the Jobs Corps program, where eligible participants were divided in two groups: program group and control group, who were excluded from participating in the program for a 3-year period. Data collection was only made to a part of the program group and the totality of the control group. After a 4-year period the weekly earnings of the program group were approximately 227\$ and the ones in the control group were approximately 200\$, difference that was significant at a 5% level.

Görg et al. (2007) compared wages in foreign-owned companies in Ghana with other companies, and concluded that workers that received training in foreign-owned companies had

a higher wage growth than those on domestic-owned companies.

Albert et al. (2010) in an analysis of the effects of training on wages in 6 European countries (Spain, Portugal, Germany, UK and Italy) noted that effects on the southern countries, particularly in Portugal, were higher than in the other countries.

As for effects on productivity, they are more difficult to find, since there are very few studies that estimate productivity, since it is hard to measure. However the EOPP survey tries to measure it by asking employers to rate the productivity of the last employee hired by the firm from 0 to 100. Questions may arise if this is a good measure of true productivity, though. Bishop (1991) defends the use of this measure by noting that the coefficient of variation of output measured by the survey is similar to the ones found in studies that use physical measures of output. A study by Almeida and Carneiro (2006) found an average rate of return of 26% for firms providing training, against -7% of firms who did not.

The studies that compared the effects of training on wages and productivity have shown that training increased productivity more than wages. While this can be due to specific training to a certain degree, if the difference between the two effects was only due to specific training, the percentage of training that is specific would have to be much higher than the one found in studies like EOPP, where most respondents said that most of the skills learned through training were useful in other firms.

Frazis and Loewenstein found, using data from NLSY, that the rate of return of training was about 50% and concluded that it is higher than the effect on wages (both being very high), even though both estimates may not be very reliable, because of all the bias surrounding this kind of data.

Dearden et al. (2006), using data from British industries between 1983 and 1996, concluded that a 1% increase in training led to a 0.6% increase in productivity but of only 0.3% on wages, illustrating the previous point.

Regarding these effects, Barron et al. (1997) compared the differences between the reports of total hours of training of employers and employees and found a correlation less than 0.5, symptomatic of a substantial measurement error. According to the authors, this error may cause underestimation of the effect of OJT on wages and productivity. It was also found that companies report on average 25% more training than the workers do.

2.6 Other effects of OJT

Georgellis and Lange (2007), in their study of the German labor market, which aimed to perceive the effects of training on job satisfaction, revealed that the participation in training is heavily impacted by factors like age, gender, previous qualifications, marital status, nationality of the individual and the size of the firm in which the employee operates. Regarding the impact of training itself, the authors concluded that employer-sponsored training had a significant positive impact on job satisfaction of men, but not of women.

2.7 Methodology

McDonald and Siegel (1986), developed a real options model that considered the irreversibility of an investment decision, in which it considers that the present value of the future cash-flows (V_t) and the cost of the investment (F_t) both follow a geometric Brownian Motion as these variables are generally considered stochastic and the model uses the

option pricing method in order to estimate the optimal timing of an investment taking into consideration the irreversibility of the project.

Regarding option pricing models, the Black and Scholes (1973) model is nowadays one of the main models used either to evaluate financial put and call options and real options.

In order to take an investment decision we have to take into account: the uncertainty of this kind of investment, since we do not know with certainty the increase in the productivity of the workers that a certain investment will produce; the flexibility that exists, given the fact that this kind of decisions do not have a now-or-never characteristic, meaning that the company can wait for the ideal moment to make the investment decision, which means that if it is not ideal to make the investment immediately, the company may wait for more favourable conditions that may make the investment worthwhile; and the given irreversibility (at least partially) that arises from the fact that when the decision is made, if the surrounding conditions deteriorate, making the investment non-profitable, at least part of the investment cannot be recovered.

These are the three characteristics that make the Real Options methodology more appropriate than the static framework to evaluate this investment decision according to Dixit et al. (1994) in a book that further explored the initial approach of McDonald and Siegel (1986).

The approach of Dixit et al. (1994) allows companies to add an additional value, the value of the option to invest, when compared with the traditional static approach, and will be very important to further development of the methodology of this dissertation, which aims to incorporate the literature on Real Options with the literature on OJT.

3 Model

In the scenario presented in this dissertation, a company is faced with an option to invest in OJT in order to increase the productivity. Both the productivity increase generated by the training and the cost of this investment are subject to some sort of uncertainty. The first will depend on the individual characteristics of each employee and the latter, at least the component tied to the increase of the wage of the employees, will also depend on level of competition on the market itself, the portion of the training that is specific or general and either if the company resorts to outside trainers or internal ones.

The company will have to decide the timing of the investment, the amount of training provided in hours, if the training is more specific or more general, according to its needs, and the choice between internal or external trainers, although this model will only explore the timing and the amount of training provided, considering the other factors as exogenous to the model.

It is also important to take into consideration that the studies about this matter indicate that there are diminishing returns on this kind of investment, meaning that when an employee first receives training, it will have a certain impact on its productivity, while the subsequent training will gradually have a smaller impact.

The company will do the investment when it maximizes its results.

The characteristics of the option embedded in this problem are the following: the company is the one that makes the decision to invest, so it is the owner of the option. The option is similar to an American call option whose underlying asset is the present value of the increase in the worker's productivity (gross project value) and whose exercise costs are the cost of the training (investment cost). The other inputs of the model are time to maturity, the volatility of the gross project value, risk-free interest rate and the opportunity cost of deferring.

3.1 Model Setup

The aim of the model hereby presented is to find out how companies make decisions on training of its employees, and how different variables influence the firm's decision to provide training to its employees.

In our setup we consider a firm which is considering to give OJT to one of its employees (the analysis can also be done for a group of employees), which in the present time has a salary of w_1 monetary units, while its current productivity is given by H . If the firm decides to invest in training of this employee, it will invest in k hours of training and this employee will have a gain of productivity of h per hour trained. Given the available literature in the field of OJT, specifically the model presented in Frazis and Loewenstein (2006), the salary (w_2) that the employee will receive after completing the training will depend on how specific or general the training provided is or how much of the increase in productivity is perceived by other employers. Considering γ the part of the increased productivity per hour of training which is perceived by other companies, w_2 will be equal to $w_1 + \gamma hk$, because the company will have to match the alternative wage of the worker elsewhere. Note that if $\gamma=0$, the training is entirely specific or its not observed by other employers and if $\gamma=1$, then the training is general.

The Gross Project Value (X) is considered to be a stochastic variable, which we assume

to follow a Geometric Brownian Motion:

$$dX = \alpha X dt + \delta X dz \quad (1)$$

where α is the risk-neutral drift rate, and is equal to $r - \delta$, dz is the increment of a Wiener process, and σ is the standard deviation. The firm is risk neutral and discounts against rate r , the risk-neutral risk-free rate and δ represents the opportunity cost for delaying investment, and is larger than 0.

3.2 Optimal investment and timing

Considering our basic setting, in which a company is deciding how many hours and when to invest in k hours of training, the payoff of this investment decision for the company will be:

$$V(X, k) = X(H + hk) - w_2 - ck^2 - XH + w_1 \quad (2)$$

Since,

$$w_2 = w_1 + \gamma hk \quad (3)$$

Then, by simplifying, we get:

$$V(X, k) = -k(ck + h(-X + \gamma)) \quad (4)$$

We consider the company first decides the number of hours to invest, and then when to invest. We can determine the number of hours that maximizes the value of the company if it decides to train its employees, through a simple optimization exercise, by maximizing $V(X; k)$ in order to k .

$$\max_k [V(X, k)]$$

The optimal number of hours for a given h and any X , is given by:

$$k^*(X) = \frac{h(X - \gamma)}{2c} \quad (5)$$

This equation informs us, *ceteris paribus*, that the higher the gain of productivity (h), the more the company is willing to train its employees (k^*), the higher the sensitivity of the cost to training to the number of hours trained (c), the lower the amount of hours trained (k^*) and the more general the training is (γ), the less investment will be made in training (k^*).

After the definition of the optimal number of hours of training for any X , the company must decide on the timing of investment in which it is optimal to invest in training.

As is standard in the literature, the value of the option to invest, $F(X, d)$, must satisfy the following ordinary differential equation:

$$\sigma^2 X^2 \frac{\partial^2 F(X, k)}{\partial X^2} + (r - \delta) X \frac{\partial F(X, k^*)}{\partial X} - r F(X, k^*) = 0 \quad (6)$$

The solution to the ODE takes the general form:

$$F(X, k^*) = A_1 X^{\beta_1} + A_2 X^{\beta_2} \quad (7)$$

where:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \quad (8)$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \quad (9)$$

As the payoff is that of a call option, i.e. increasing in X the solution, as is standard in the literature, takes the form:

$$F(X, k^*) = A_1 X^{\beta_1} \quad (10)$$

From now on, we will designate it as A and β (which must be greater than 1).

To determine the optimal timing X^* , we employ the value matching and smooth pasting conditions:

$$\lim_{X \rightarrow X^*} F(X^*, k^*) = V(X^*, k^*) \quad (11)$$

$$\lim_{X \rightarrow X^*} \frac{\partial F(X, k)}{\partial X} = \lim_{X \rightarrow X^*} \frac{\partial V(X, k)}{\partial X} \quad (12)$$

The value matching condition establishes that in the optimal moment of investment the value of the option to defer the investment is zero, where X^* is the optimal investment trigger, i.e. the value of the option to invest is equal to the present value of the risk-adjusted cash-flows the firm will receive. The final condition, equation 12 ensures that $F(X, d)$ is continuously differentiable along X , and that $F(X, d)$ and $V(X, d)$ meet tangentially when X is equal to X^* . Solving this system, we can show the option value of the decision to train employees is given by:

$$F(X, k) = \begin{cases} A^* \left(\frac{X}{X^*} \right)^\beta & \text{for } X < X^* \\ V(X, k) & \text{for } X \geq X^* \end{cases} \quad (13)$$

Where A^* is the pay-off at the optimal moment of investment and is given by:

$$A^* = k \left(-ck + \frac{2h\gamma}{c(-2 + \beta)} \right) \quad (14)$$

The optimal timing to invest is given by:

$$X^* = \frac{\beta\gamma}{-2 + \beta} \quad (15)$$

And the optimal number of hours of training, at the optimal timing will be:

$$k^*(X^*) = \frac{h\gamma}{c(-2 + \beta)} \quad (16)$$

Note that $\beta > 2$ is a condition that is needed for this model.

3.3 Numerical Example

In order to better analyse the model we need to attribute values to the different variables.

<i>Variable</i>	<i>Value</i>
X	1
H	100
h	10
w_1	100
c	5
α	0
r	0.1
σ	0.15
γ	0.5

Table 1: Base parameters

The chosen base parameters are indicated in the table 1 above, and in this case the trigger (X^*) is 1.15 and the optimal number of hours (k^*) is 0.65.

First of all we will take a look to the consequences of changes in the variables on the investment trigger.

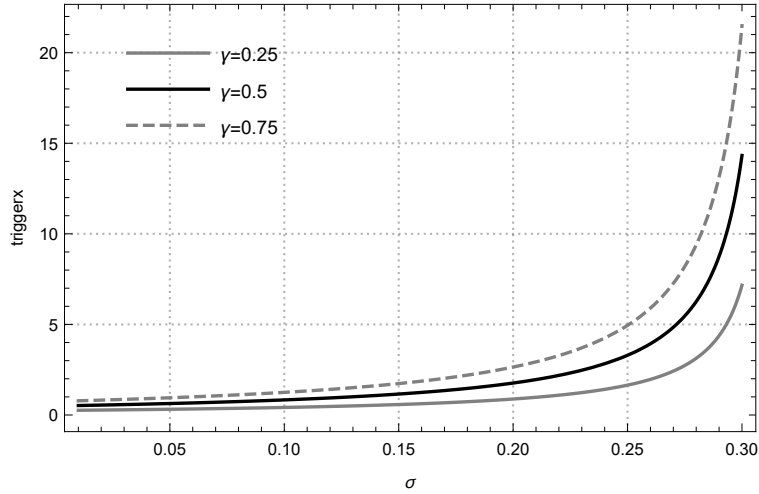


Figure 1: Investment trigger, uncertainty and generality of the training program

Figure 1 shows us that uncertainty will increase the value of waiting and thus leading to later investment. It also shows us that if the training is more specific (γ closer to zero) the trigger value will be reduced, meaning a sooner investment on training. For sufficiently high values of σ , the company will not invest in the training of workers because there is too much uncertainty. The company will then prefer to wait for better conditions in order to make the investment. For values of σ close to 0.3, there is an exponential increase in the trigger value of the investment.

The same conclusions can be drawn from Figure 2, where we can analyse the effect of both variables combined. The higher the generality level, given a certain uncertainty level of the training program, the higher is the trigger value for investing.

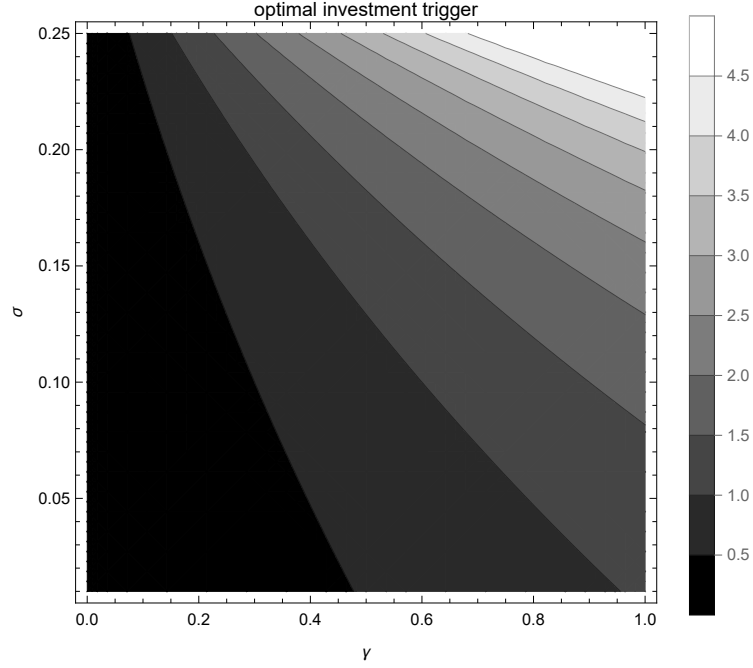


Figure 2: Investment trigger, uncertainty and generality of the training program

Now we will examine the effects of several variables on the dimension of training, or on the number of hours invested.

According to Figure 3 the higher the uncertainty and generality of the training, the higher will be the optimal number of hours of training.

The following Figure 4 is simple to interpret since it shows that if the productivity is higher and the cost is lower, the firms will invest in more hours of training.

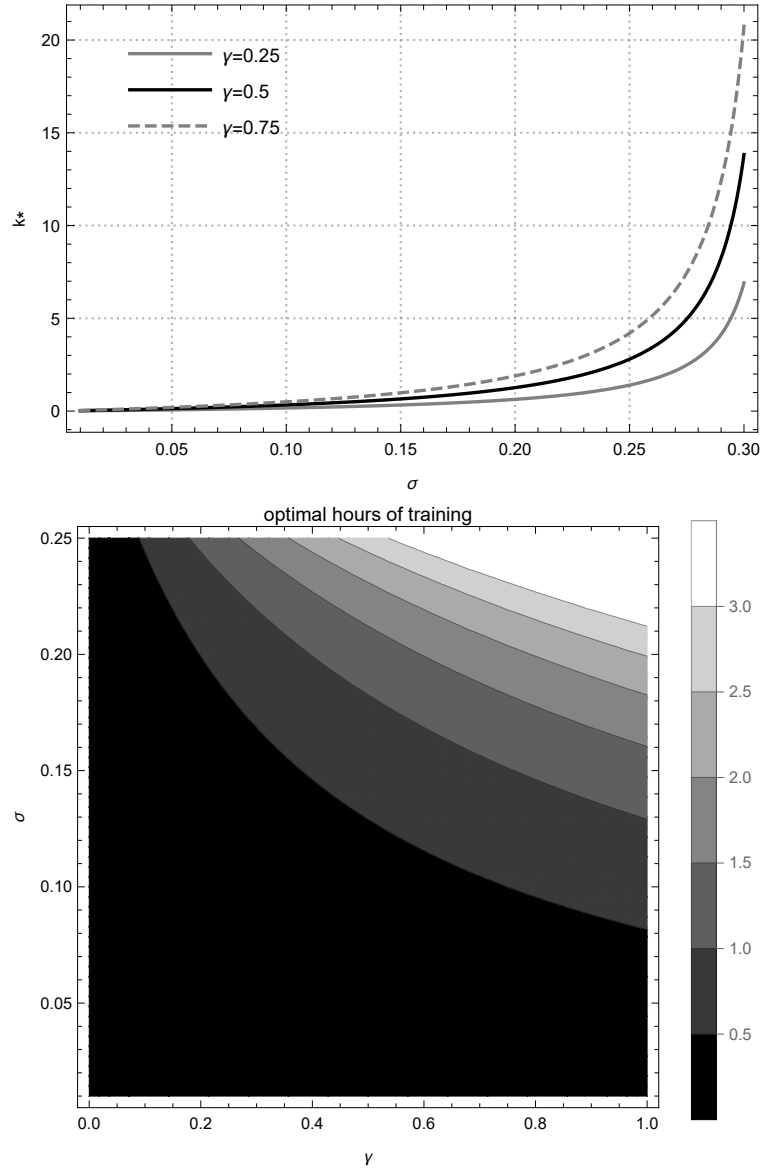


Figure 3: Hours of training, uncertainty and generality of the training program

Figure 5 shows us the conjunct effects of cost of training and uncertainty, showing that an increase in the cost of training and a corresponding increase in the uncertainty can lead to a similar optimal number of hours.

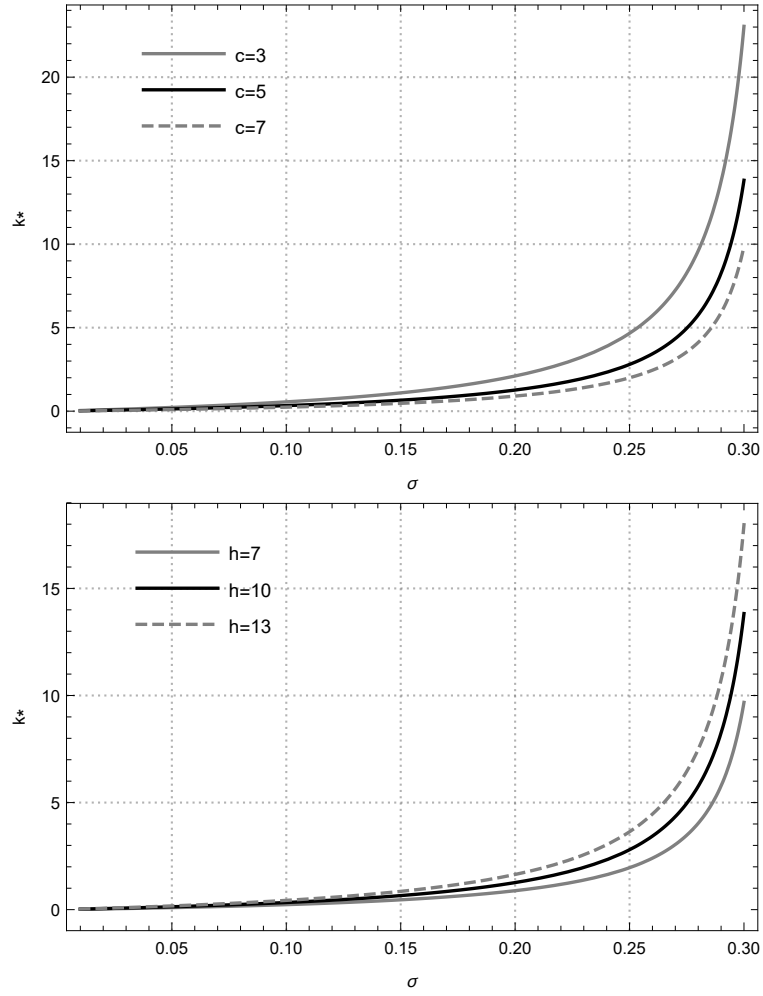


Figure 4: Hours, uncertainty, cost and raise of productivity of the training program

Figure 6 shows us the conjunct effects of the raise of productivity of the training program and uncertainty, and we conclude from it that for higher levels of uncertainty, the number of hours of training will be more impacted by the increase of productivity that the said training program enables.

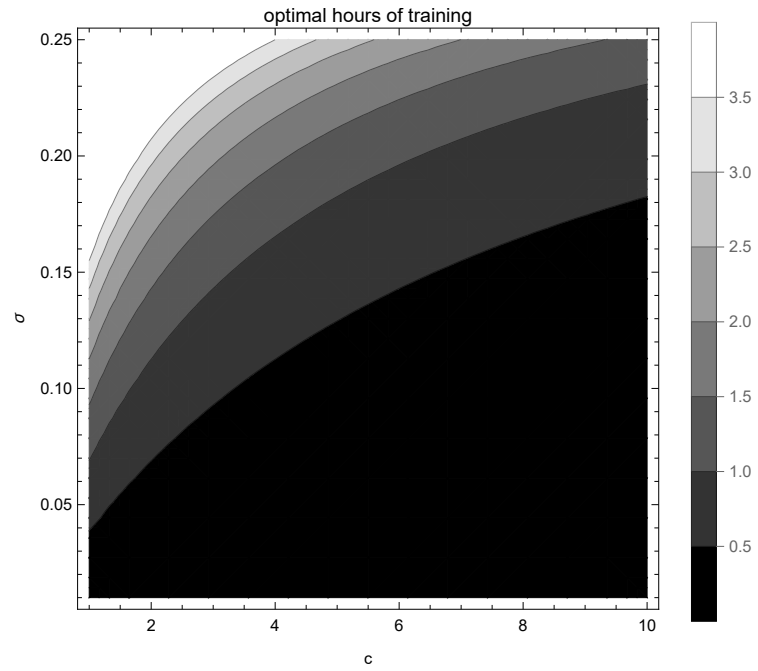


Figure 5: Hours, uncertainty and cost of the training program

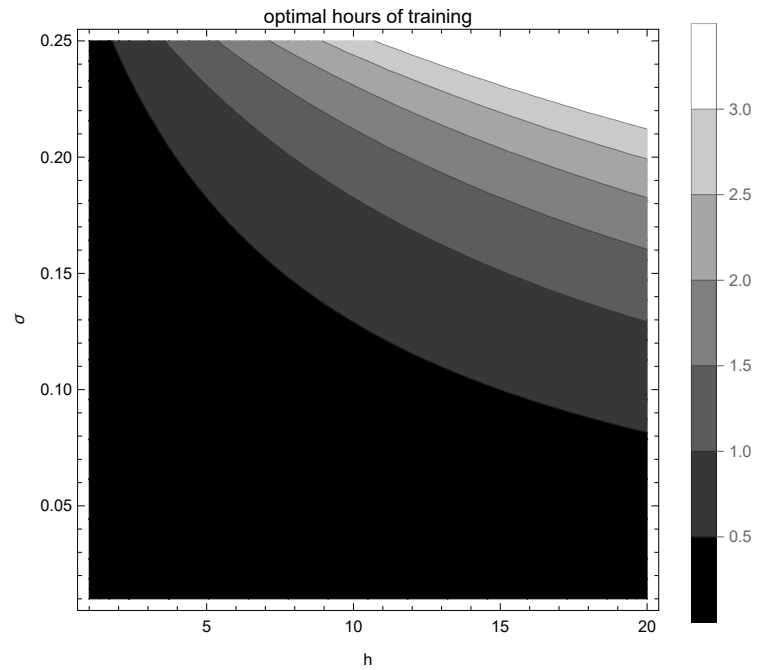


Figure 6: Hours, uncertainty and raise of productivity

Figure 7 shows us the conjunct effects of the raise of productivity of the training program and generality of the training program, concluding that the generality has a very

similar impact to the raise of productivity, and the number of hours of training will be more impacted by both variables, higher the other one is.

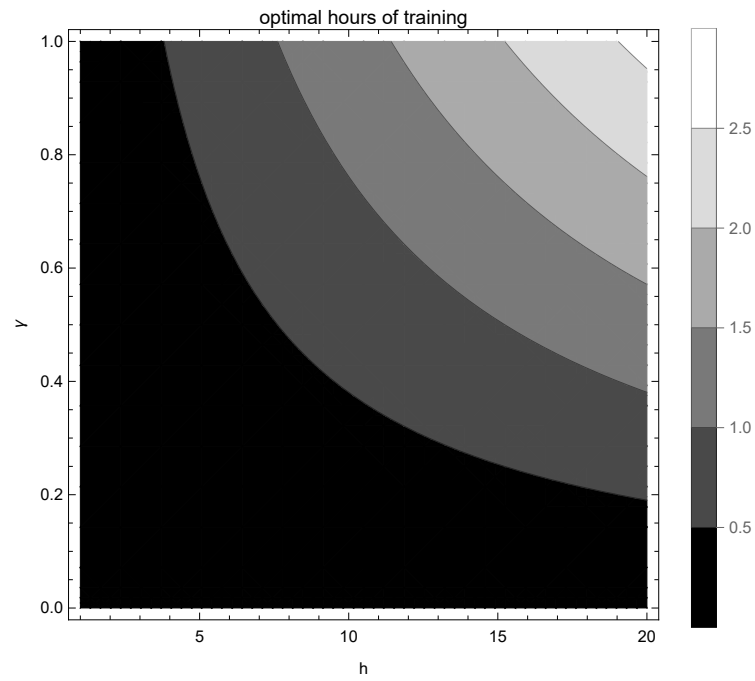


Figure 7: Hours, generality of the training program and raise of productivity

Figure 8 shows us the conjunct effects of generality of the training program and cost, concluding that the variable cost has a huge impact in the optimal number of hours in our example, because for low c , a change in the generality will increase significantly the number of hours, while for higher c , the impact of generality will decrease significantly.

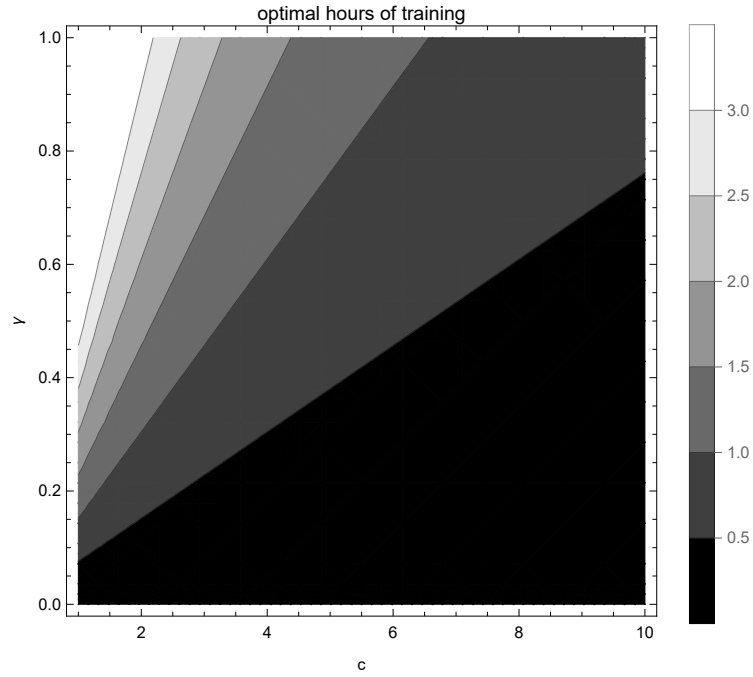


Figure 8: Hours, generality of the training program and cost

At last, figure 9 shows us the conjunct effects of the raise of productivity of the training program and cost, the conclusions being very similar to the ones of figure 8, since both the generality and increase of productivity have a similar impact on the optimal number of hours. So, for low levels of cost, the impact of the raise of productivity is higher when compared the impact of the same variable with high costs.

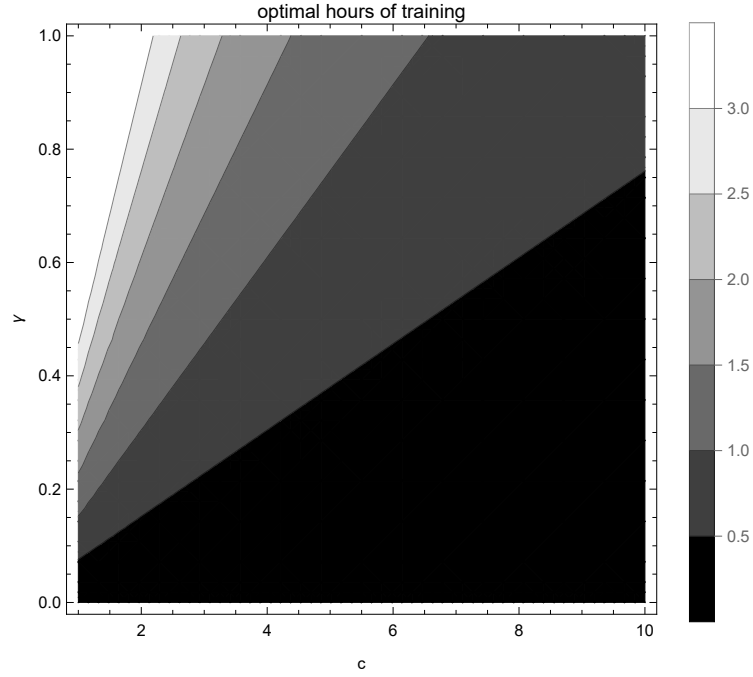


Figure 9: Hours, raise of productivity and cost

4 Conclusions

The analysis of previous literature on the field of OJT allowed us to build this model, which permitted us to understand the effects of different variables on both the timing and the amount of hours of training a certain company offers, and how those influence companies' decisions of providing training for their employees.

Both the generality of training and the uncertainty surrounding the company have a positive impact on the investment trigger, meaning the increase in these variables will lead to a later investment in training.

Regarding the effects on the amount of hours of training provided by the firms, uncertainty, generality of the training and raise of productivity have a positive impact on the optimal number of hours, while the cost has a negative impact on this factor.

This means that generality of training and the uncertainty will delay the investment, but when made, this investment will be bigger.

References

- Acemoglu, D. and Pischke, J.-S. (1998). Why do firms train? theory and evidence. *The Quarterly Journal of Economics*, 113(1):79–119.
- Acemoglu, D. and Pischke, J.-S. (1999). The structure of wages and investment in general training. *Journal of political economy*, 107(3):539–572.
- Albert, C., GARCÍA-SERRANO, C., and Hernanz, V. (2010). On-the-job training in europe: Determinants and wage returns. *International Labour Review*, 149(3):315–341.
- Almeida, R. and Carneiro, P. (2006). Costs, benefits and the internal rate of return to firm provided training. *The World Bank Policy Research Working Paper Series*.
- Barron, J. M., Berger, M. C., and Black, D. A. (1997). How well do we measure training? *Journal of Labor Economics*, 15(3):507–528.
- Bassanini, A., Booth, A. L., Brunello, G., De Paola, M., and Leuven, E. (2005). Workplace training in europe.
- Becker, G. S. (1962). Investment in human capital: A theoretical analysis. *Journal of political economy*, 70(5, Part 2):9–49.
- Bishop, J. H. (1991). On-the-job training of new hires. In *Market Failure in Training?*, pages 61–98. Springer.
- Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, 81(3):637–654.
- Dearden, L., Reed, H., and Van Reenen, J. (2006). The impact of training on productivity and wages: Evidence from british panel data. *Oxford bulletin of economics and statistics*, 68(4):397–421.
- Dixit, A. K., Dixit, R. K., and Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton university press.
- Frazis, H., Gittleman, M., Horrigan, M., and Joyce, M. (1998). Results from the 1995 survey of employer-provided training. *Monthly Lab. Rev.*, 121:3.
- Frazis, H., Loewenstein, M. A., et al. (2006). Wage compression and the division of returns to productivity growth: Evidence from eopp. Technical report.
- Frazis, H., Loewenstein, M. A., et al. (2007). On-the-job-training. *Foundations and Trends® in Microeconomics*, 2(5):363–440.
- Georgellis, Y. and Lange, T. (2007). Participation in continuous, on-the-job training and the impact on job satisfaction: longitudinal evidence from the german labour market. *The International Journal of Human Resource Management*, 18(6):969–985.
- Görg, H., Strobl, E., and Walsh, F. (2007). Why do foreign-owned firms pay more? the role of on-the-job training. *Review of World Economics*, 143(3):464–482.
- Hashimoto, M. (1981). Firm-specific human capital as a shared investment. *The American Economic Review*, 71(3):475–482.

- Lee, D. S. (2009). Training, wages, and sample selection: Estimating sharp bounds on treatment effects. *The Review of Economic Studies*, 76(3):1071–1102.
- Loewenstein, M. A. and Spletzer, J. R. (1999). General and specific training: Evidence and implications. *Journal of Human Resources*, pages 710–733.
- McDonald, R. and Siegel, D. (1986). The value of waiting to invest. *The quarterly journal of economics*, 101(4):707–727.
- Mincer, J. (1962). On-the-job training: Costs, returns, and some implications. *Journal of political Economy*, 70(5, Part 2):50–79.
- Mortensen, D. T. (1978). Specific capital and labor turnover. *The Bell Journal of Economics*, pages 572–586.
- Schultz, T. W. (1962). Reflections on investment in man. *Journal of political economy*, 70(5, Part 2):1–8.
- Smith, A. (1904). An inquiry into the nature and causes of the wealth of nations, cannan edition. *London: Meuthen*.